Exploring Foundation Phase pre-service teachers' mathematical pedagogical content knowledge for teaching additive mental mathematics strategies

A thesis submitted in fulfilment
of the requirements for the
degree of

MASTERS IN EDUCATION
at
RHODES UNIVERSITY

By

Marisa Luisa Kumm
G22K3960

ORCID ID: 0009-0008-5753-9433

03 March 2025

CONTENTS

ABSTRACT

Poor learner performance in mathematics remains a concern in South African primary and secondary education according to continuous national and international assessments. Many learners fall behind in the early grades without being provided the opportunity to establish a solid foundational number sense. This is an essential requirement to progress into the higher grades. Addressing the challenges of number sense in the Foundation Phase is mandatory. This study aimed to investigate third-year Foundation Phase pre-service teachers' content knowledge and pedagogical content knowledge of their additive reasoning strategies (a key aspect of number sense) at a private teacher education institution, guided by the question: What content knowledge and pedagogical content knowledge do third-year Foundation Phase pre-service teachers have of additive reasoning mental mathematics strategies? Several researchers have identified the need to develop pre-service teachers' mathematical content knowledge and pedagogical content knowledge. This study was guided by Shulman's (1986) constructs of teacher knowledge.

This qualitative, descriptive study, which formed part of the Mental Mathematics – Work Integrated Learning (MM-WIL) programme, collected data from an interpretivist perspective. The information for this study was gathered from participants who agreed to be part of the study. The study collected data through pre- and post-intervention questionnaires that had two parts. The first part assessed pre-service teachers' methods of solving four basic additive reasoning calculations (that lent themselves to using the strategies of bridging through ten, jump strategy, and rounding and adjusting). The second part of the questionnaire asked pre-service teachers to describe these strategies and how they would teach them, with the aim of understanding their additive reasoning skills and ability to use and teach these mental strategies.

Key findings are that many pre-service teachers do not have the content knowledge to solve basic calculations using efficient methods. These are skills that they are required to teach in the Foundation Phase classroom. The findings in this study highlight the urgent

need to address the weak content knowledge, pedagogical content knowledge and inefficient unit based counting methods of many pre-service teachers to improve learners' development and fundamental understanding of numbers that allows for flexible and efficient calculation.

STATEMENT OF ORIGINAL AUTHORSHIP

The work contained in this thesis has not been previously submitted at this or any other higher education institution. To the best of my knowledge and belief, this thesis contains no material previously published or written by another person except where due reference is made.

Signed: Date: 04 March 2025

ACKNOWLEDGEMENTS

I would like to express my appreciation to the many people who supported me and contributed to my academic journey.

Thank you to my family for your love, support and encouragement throughout my journey.

A very special thanks to my supervisors Prof. Mellony Graven and Dr Lise Westaway, for your invaluable guidance, expertise and patience. Not only have you helped me grow, but your mentorship has also shaped my research and enriched my academic experience.

I would also like to acknowledge the South African Numeracy Chair at Rhodes University, held by Prof. Mellony Graven, for assisting me, and the entire SANC team for the thoughtful, constructive feedback that has challenged me to refine my ideas. A special thanks to the administrator, Ms Zipho Maldaka for all the administrative assistance and travel arrangements throughout my study.

I am grateful to the academic institution and colleagues for granting me permission to do my research and accommodating my needs.

Lastly, to my critical friends, thank you for your collaboration and insightful discussions. Your support has been instrumental, and your contributions have enhanced the quality of this work.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ACRONYMS

ANS:	Approximate number sense
BEd:	Bachelor of Education
BTT:	Bridging through ten
CAPS:	Curriculum and Assessment Policy Statements
CCK:	Common content knowledge
CK:	Content knowledge
DBE:	Department of Basic Education
EGM:	Early grade mathematics
ENS:	Early number sense
FNS:	Foundational number sense
FP:	Foundation Phase
HCK:	Horizon content knowledge
INS:	Innate number sense
JS:	Jump strategy
KCS:	Knowledge of content and students
KCT:	Knowledge of content and teaching
MM-WIL:	Mental Mathematics – Work Integrated Learning
MNS:	Mature number sense
MSAP:	Mental Assessment Starters Programme
NS:	Number sense
NSES:	National School Effectiveness Study
PCK:	Pedagogical content knowledge
PSTs:	Pre-service teachers
PNS:	Preverbal number sense
R&A:	Rounding and Adjusting
SANC:	South African Numeracy Chair
SCK:	Specialised content knowledge
SMK:	Subject matter knowledge
TIMMS:	Trends in International Mathematics and Science Study
VNS:	Verbal number sense

CHAPTER ONE

1. INTRODUCTION, CONTEXT AND RATIONALE

1.1. INTRODUCTION

Many would agree with Ball et al. (2008, p. 404) that teachers knowing the subject they teach is essential, and that equipping teachers with the powerful forms of knowledge necessary for quality teaching remains beset with challenges. This is especially true of mathematics education, where underperformance is well-documented and researched. Graven and Stott (2012) suggest that many primary school learners in South Africa are not capable of working flexibly with numbers. Several students in the Foundation Phase (FP) (Grades R-3) and Intermediate Phase (IP) (Grades 4-7) use tally lines when solving calculations, implying a need to rely on concrete counting strategies (Ensor et al., 2009). Many researchers claim that South African underperformance in primary mathematics is partly attributed to the dependence on unit-based counting methods and performing the standard vertical algorithms without number sense (Graven et al., 2013; Schollar, 2008; Weitz & Venkat, 2013).

More recently, however, attention is turning to mathematics teacher educators and the preparation of mathematics teachers in primary schools. A range of recent research is pointing to weaknesses in primary pre- and in-service teachers' knowledge of mathematics (Bowie et al., 2019; Venkat & Spaull, 2015;). The core research objective for this research was to investigate the nature of FP pre-service teachers' (PSTs) mathematical content knowledge (CK) and pedagogical content knowledge (PCK) for teaching additive mental mathematics strategies and enhancing their PCK. This was based on the assumption that knowledge of effective and efficient calculation strategies is foundational to teacher competency and thus it is essential that this knowledge is established during pre-service teacher education.

This introductory chapter explains how the focus of this study emerges from and relates to some of the difficulties in primary mathematics teaching and learning, and in primary teachers' mathematical knowledge of the content they are required to teach. It also provides the rationale behind this study. It examines the urgent need to better equip pre-service primary teachers with CK and PCK as part of a comprehensive intervention strategy. It briefly describes the private institution where the study was conducted. The goal of this research is then outlined, followed by the related research questions. The chapter concludes with an outline of the thesis chapters that follow and concluding remarks.

1.2. THE SOUTH AFRICAN CONTEXT

International and South African research has highlighted the need to address the weak mathematics results in South Africa and improve mathematical performance (Spaull & Kotze, 2015; Wright, 2003). Targeted early intervention in mathematics has proved cost-effective and more beneficial than later intervention (Spaull & Kotze, 2015).

Major political, social and economic change has taken place in South Africa's education since the beginning of democracy in 1994. Despite the many policy initiatives and changes in the education system, the quality of education in South Africa is still so different between what is labelled as a functional or dysfunctional school (Fleisch, 2008; Modisaotsile, 2012; Van der Berg et al., 2011). Many well-resourced schools retain their heritage of cultural and financial character portrayed by the apartheid era (Du Toit, 2008). These functional schools achieve educational success similar to other developed countries (Van der Berg et al., 2011). However, (historically) disadvantaged, dysfunctional schools continue to serve mainly children of colour throughout South Africa (Maringe & Moletsane, 2015). The National School Effectiveness Study (NSES) confirmed that the Grade 5 students' results in traditionally black schools were far worse than Grade 3 learners in historically white schools (Taylor, 2021).

Inequality still abounds in the system of education in South Africa (Spaull and Jansen, 2019). Indeed, Morrow (2007) feared “that in some regions and sectors, the [education] system is close to total collapse” (p. 28). The seemingly poor quality of mathematics education, evidenced by ongoing underperformance, regularly takes centre stage (Reddy, 2016; Fleisch, 2008; Spaull, 2013; Spaull & Kotze, 2015). Results obtained in international assessments such as Trends in International Mathematics and Science Study (TIMMS) and The Southern and Eastern Africa Consortium for Monitoring Educational Quality (SACMEQ) have significantly increased the concern around the learners' performance. The TIMMS tests in 2002 included a sample of Grade 8 and Grade 9 learners which produced dismal results nationally. Ten percent of South African learners achieved a score higher than 400, which indicates that 90 per cent of learners in South Africa scored less than 400 scaled points. “South Africa and Ghana have the highest percentage of learners with a score lower than 400” (Reddy et al., 2016, p. 25). SACMEQ data show similar trends where significant disparities in performance across provinces and schools in high and low socio-economic areas were noted. Despite evidence of improvement over the last 20 years, performance differentials between (historically) disadvantaged and advantaged schools persist (Van der Berg & Gustafsson, 2019, p.25). Data from TIMSS reveals “a textured picture of South African mathematics achievement patterns over the last 20 years”, and “access to socioeconomic resources at the individual, home and school levels” continues to shape learner performance in mathematics (Reddy et al., 2019, p. 170). In TIMSS 2015, the Eastern Cape Province continued to rank lowest, showing that 74 percent of the learners in Grade 5 were not able to do basic mathematics (Reddy et al., 2016). It is worth remembering that improvement in mathematics remains differently distributed over the schooling system (Van der Berg & Gustafsson, 2019). However, the value of international assessments lies beyond just benchmarking learner performance. An important advantage of these international assessments is the diagnostic data they provide when compared to national results.

South Africa's results analysed by Schollar (2008) indicate that 60.3 percent of students in Grade 7 do not understand place value or the base ten system, and cannot solve problems using mental strategies. More recent studies agree that South African Grade 3 learners do not have the instant facts for bonds of ten and still calculate using concrete methods (Graven & Venkat, 2021). Graven et al. (2013) noted that learners in the intermediate phase still use tally lines to answer questions. The prevalence of weak number sense and continual use of unit counting (beyond the initial appropriate stages) is a possible factor which contributes to South Africa's poor performance in mathematics when compared globally (Graven et al., 2013; Weitz & Venkat, 2013).

Mention must also be made of the effect of COVID-19 on an already unstable education system in South Africa where achievement is continuously associated with race, financial, cultural and geographic background. Students from deprived backgrounds achieve at a lower level than students from privileged schools (Graven, 2014). This is evident when comparing the difference of 75 points in mathematics accomplishment between schools that pay fees and schools that do not pay fees in TIMSS 2019 (Reddy et al., 2020).

While the trends in the mathematics scores in TIMMS show some improvement prior to the commencement of the COVID-19 pandemic (Reddy et al., 2020), there is concern that COVID-19 had a negative effect on the progress of the system of mathematics education in South Africa (Soudien et al., 2022). While largely speculative, Soudien et al. (2022) examine existing studies to try and quantify learning losses due to the impact of COVID-19. They emphasise the crucial "relationship between increased social and educational provision and improved learner educational attainment" (2022, p. 308) after the upward trajectory of learner performance faltered after 2021 by an increase in unemployment and a deterioration in socio-economic conditions. Similarly, due to the lockdown brought about by COVID-19, they predicted that "the achievement gains made since 1994 would probably revert to the achievement levels recorded in TIMSS 2015 – a loss of five years of learning" (p. 317). They concluded that in addition to learning losses it is "likely learning loss has been experienced disproportionately by those who are vulnerable and less able to draw on the resources of the system" (p. 317). Consequently, "COVID-19 has thus laid bare both the inequalities in provisions needed to continue learning from home – such as funds, digital devices and data, adequate nutrition – and the disparity in how well teachers, learners and parents have been equipped to do so" (Soudien et al., 2022, p. 320).

1.3. TEACHER KNOWLEDGE

Researchers have identified that several teachers in South Africa lack CK and PCK, which many state as the primary cause for the shortfalls and underperformance in mathematics learning in the classroom (Bowie, 2019; Fleisch, 2008; Taylor & Taylor, 2013). Adler and Reed (2003) argue that while CK is important, it is not enough and emphasises the need to improve teachers' PCK to assist learners in making sense of mathematics. Research surrounding the abovementioned challenges has identified the pressing need to improve teacher content knowledge. As Morrow (2007) notes, "an underemphasis on the content of teaching is a prevalent and serious problem in schools and other institutions… [and that] in the case of any teaching, the teacher must know the content being taught" (p. 82). Teachers without the confidence or understanding of the concepts which they are required to teach, struggle to teach mathematics. Addressing these weaknesses in knowledge requires equipping teachers with CK and PCK (Shulman, 1987).

The problem with content knowledge is not limited to in-service teachers but is also evident in PST education. Taylor's (2021) reports on curriculum-based tests indicated that fourth-year Bachelor of Education students are unprepared to teach primary school mathematics. Similarly, Bowie et al. (2019) identified the lack of PST and in-service teachers' mathematical knowledge and skills as contributing factors to continued underperformance in national and international mathematics assessments. In this respect, Graven and Coles (2017) argue that pre- and in-service teachers should be supported in developing their mathematics knowledge and skills.

Universities are well positioned (and have a responsibility) to intervene and explore ways to adjust their teaching methods and bridge gaps in PST knowledge to ensure the best learning opportunities in FP classrooms.

In my experience as a mathematics lecturer at an independent teacher training college, I have seen first-hand the necessity to develop the CK of PSTs. Sizeable gaps exist in some PSTs' mathematics knowledge, involving elementary level fluency and additive reasoning. To demonstrate, I witnessed several second-year Bachelor of Education students (BEd), who counted on their fingers to complete the pattern on a baseline test that required them to count on in multiples of four (for example 12, followed by four-finger gestures, then writing 16).

Based on the preceding discussion on weak mathematics performance, further research into ways to bridge the gap and support PSTs in the South African context is necessary. At first, this prompted me as a primary mathematics educator to explore ways to create a space within my teaching to address some of the shortfalls in PST mathematical CK and PCK. Ultimately, it led to the current study in which I explore the use and value of a more structured and research-based approach to developing PSTs' number sense.

The setting for this socio-constructivist, interpretive research study was a private higher education independent college registered with the Independent Institute of Education (IIE) and the Department of Higher Education and Training (DHET). The higher education establishment in this study is situated in the Eastern Cape, South Africa. This research was conducted with a diverse range of third-year BEd. students from different socio-economic, cultural and language backgrounds.

I am a mathematics education instructor at the aforementioned higher education teacher education institute, studying for my master's in education at Rhodes University. Through experience, and supported by literature, I have observed that many BEd students do not possess the confidence, competence or conceptual understanding of mathematics, yet they are expected to develop these with the learners they teach. It is necessary to support PSTs in developing the skills and knowledge essential to do mathematics CK before expecting them to develop didactic skills in the subject PCK.

1.4. THE SOUTH AFRICAN NUMERACY CHAIR

This research study was guided with the support of the South African Numeracy Chair (SANC), at Rhodes University in Makhanda, supported by the National Research Foundation (NRF) and chaired by Professor Mellony Graven (SANC, 2021).

The SANC has numerous projects that research and merge development in mathematics education in sustainable and feasible ways (Robertson & Graven, 2018). The aims of the SANC include:

- improving the quality of teaching of in-service teachers at the primary school level
- improving learner performance in primary schools as a result of quality teaching and learning
- developing sustainable and practical solutions to the challenges of improving numeracy in schools
- providing leadership in numeracy education and increase dialogue around solutions for the mathematics education crisis (SANC, 2021).

As a master's research student, I participate in the greater Mental Mathematics – Work Integrated Learning (MM-WIL) project, which is led by Prof Venkat (Witwatersrand University) and Prof Mellony Graven (Rhodes University) from the South African Numeracy Chairs. My research takes the first two strategies of the Mental Starters Assessment Project (MSAP), currently being implemented by the Department of Basic Education (DBE) with Grade 3 teachers in schools (Graven & Venkat, 2021), into the Higher Education PST education classroom. The MSAP focuses on six calculation strategies to improve and develop students' additive reasoning and number sense. While this study only focuses on three of the strategies, the programme includes six strategies, namely bridging through ten, jump strategy, doubling and halving, rounding and adjusting, re-ordering and linking, addition and subtraction. One of the motives for the two Chairs introducing the MSAP with pre-service teachers (through the MM-WIL project) is because the 2021 national roll-out of the MSAP project for current in-service teachers revealed statistically noteworthy improvements across all provinces (Askew, Graven & Venkat, 2022).

1.5. RESEARCH GOAL AND QUESTIONS

The core research objective for this research study was to explore the nature of FP PSTs' mathematical CK and PCK for teaching additive mental mathematics strategies and enhancing their PCK.

The primary research question is:
What is the nature of FP PSTs' mathematical CK and PCK for teaching additive mental mathematics strategies?

Sub-questions include:

- What strategies do third-year PSTs use to solve basic additive reasoning calculations? Do they use these effectively? [data pre-CK questionnaire]
- Can PSTs describe the bridging through ten, jump, and rounding and adjusting strategies? If so, how do they explain them and how they might teach them? [data pre-PCK questionnaire]
- How might PST strategies shift following exposure to the bridging through ten and jump strategies in lectures? Does PST performance improve in relation to learning additional strategies? [data post-CK questionnaire]

1.6. CHAPTER OVERVIEW

My research thesis consists of five chapters organised as follows:

Chapter One presents the background and context for my study, primarily concerning the crises in mathematics education in South Africa as exhibited in national and international research. The primary reason relevant to this study is PSTs' lack of CK and PCK to teach mathematics in the FP classroom.

Chapter Two outlines the theoretical framing of the study and expands on Shulman's (1987) key construct of subject-specific knowledge required to teach in the classroom, which frames this study. This chapter also provides a review on the issues South Africa faces in mathematics education and in particular PSTs' mathematics education.

Chapter Three describes the methodology of my qualitative, interpretivist study that involved data collection phases that gathered CK and PCK of all third-year PSTs; however, only data collected from PSTs who signed consent forms was used in this study. The data collection method is a pre- and post-questionnaire with two parts, additive reasoning calculations and questions about strategies and how to teach them.

Chapter Four includes data presentation, findings and analysis. This data was collected in phases as mentioned above. The data was then analysed and presented in this chapter. I have endeavoured to deliver as "thick" a "description" as possible (Geertz, 1973) of the discoveries of this research. The objective was to deliver a transparent summary of the CK and PCK of PSTs. In addition, it reviews the possible need to teach mathematics in the Foundation Phase. Chapter Four is therefore significantly longer than the other chapters.

Chapter Five concludes my study. In this concluding chapter, I make available a summary of the procedure and findings of my study and conceptual framework. Furthermore, I discuss the findings considering various implications for pre-service primary mathematics teacher education and recommend avenues for further research.

1.7. CONCLUDING REMARKS

The shortcomings within mathematics education in South Africa are evident and, consequently, well-documented. However, any strategic intervention that aims to improve the current landscape needs to focus on equipping mathematics teachers with both CK and PCK. Whilst this is foundational, it has also been established that skilled teachers possess a type of CK that is “distinct from disciplinary content knowledge” (Ball et al., 2008, p.392). Without knowledgeable and competent teachers, one cannot hope to raise the mathematics standard amongst South African learners. In the following chapter, I unpack in more detail the types of knowledge required by teachers that is necessary, though not necessarily sufficient, for supporting student learning of flexible and efficient methods of calculation.

CHAPTER TWO

2. LITERATURE REVIEW AND THEORETICAL FRAMEWORK

2.1. INTRODUCTION

Chapter One outlines and discusses the crisis facing mathematics education in South Africa. Concern over the state of mathematics education and the underperformance of learners has led to a growing body of research exploring these concerns. Much of this has sought to identify possible explanations for learners' poor performance (Graven et al., 2013; Schollar, 2008; Weitz & Venkat, 2013). From this, it has become evident that the problem is complex and cannot be attributed to a single factor. Instead, many concerns have been raised (Fleisch, 2008). Of these, and with relevance to this study, is the increasing worry that teachers are often ill-equipped to teach mathematics. Adler (2005), in her work exploring mathematics teacher education programmes, identifies a lack of knowledge amongst mathematics teachers as a common concern. She observes, "The problem facing mathematics teachers currently in practice, as these courses suggest, is that they don't know enough mathematics" (p.9). Adler is not alone in her observation. Ongoing research supports the link between teachers' mathematical knowledge and learner performance (Schollar, 2008; Spaull, 2013; Spaull & Kotze, 2015), and yet challenges related to limited teacher subject knowledge abound.

I explore in this chapter both the inspiring work of Shulman (1986) and an elaboration of his work by Ball, Thames and Phelps (2008), where they focused their research on the importance of teachers' subject knowledge in education. Ball et al. (2008) strongly emphasised the need to thoroughly unpack and conceptualise what is meant by teacher subject knowledge. Shulman and Ball's frameworks highlight the composite nature of teacher subject matter knowledge. They provide valuable theoretical frameworks for analysing teacher knowledge, and whilst both are discussed here, Shulman's framework is used to analyse the data in this study.

The remainder of the chapter is then used to unpack the specific mathematical CK and PCK upon which my intervention and study are focused. This study was designed to address inefficient calculation strategies and an absence of a structural understanding of number, which is required for number sense development in Grade 3 learners. I explore here, its potential value as a tool for developing the CK and PCK of PSTs.

2.2. SHULMAN'S FRAMEWORK FOR TEACHER KNOWLEDGE

Shulman's (1986) concern over the shift he observed in educational research and policy, which emphasised the importance of pedagogical skill at the expense of subject-specific CK, sparked an effort to reform educational practice and marked a significant shift in focus within the field. The lack of focus on content expertise in research and educational practice left a gap, which he and his associates famously referred to as the "missing paradigm", and they argued that pedagogical skill without adequate CK was useless. Shulman argued that "high-quality instruction requires a sophisticated, professional knowledge that goes beyond simple rules such as how long to wait for students to respond." (Ball et al., 2008, p.391) and in his work, identified seven categories of teacher knowledge which he considered to be the minimum requirement for ensuring adequate comprehension of content amongst learners. These categories are:

- **content knowledge**; the amount of knowledge and organization of this knowledge in the mind of teachers.
- **general pedagogical knowledge**, with special reference to those broad principles and strategies of classroom management and organization that appear to transcend subject matter.

- **curriculum knowledge**, with a particular grasp of the materials and programs that serve as "tools of the trade" for teachers.
- **pedagogical content knowledge**, that special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding.
- **knowledge of learners and their characteristics**; prior knowledge and background of learner.
- **knowledge of educational contexts**, ranging from the workings of the group or classroom, the governance and financing of school districts, to the character of communities and cultures.
- **knowledge of educational ends**, purposes and values, and their philosophical and historical grounds.

(Shulman 1987, p.8)

Of these, PCK is particularly noteworthy. This entirely new category was a unique amalgam which "[bridged] content knowledge and the practice of teaching" (Ball, 2008, p.389). It referred specifically to knowledge that went "beyond knowledge of subject matter per se to the subject matter knowledge *for teaching*." (Shulman, 1986, p.9) and, in Shulman's (1987) words, was "the category most likely to distinguish the understanding of the content specialist from that of the pedagogue" (p.8). This knowledge, while underpinned by a thorough understanding of the subject in question, included knowledge of how to make that content accessible to learners. Consequently, it referred to a much deeper and more specialised understanding of content, allowing teachers to make important decisions about the "how" and "why" of teaching. While this requires a solid understanding of the subject in question, it also requires that a teacher possess additional knowledge and skills that are unique to teachers and allow them to skillfully and flexibly employ strategies that ensure learning takes place. In his words,

> "the key to distinguishing the knowledge base of teaching lies at the intersection of content and pedagogy, in the capacity of a teacher to

> transform the content knowledge he or she possesses into forms that are pedagogically powerful and yet adaptive to the variations in ability and background presented by the students."
>
> (Shulman 1987, p.15)

In defining this new category, Shulman (1986) included the following examples of PCK:

- knowledge of the most important topics within one's subject area.
- knowledge of the most useful forms of representation for those ideas.
- knowledge of the most powerful analogies, illustrations, examples, explanations and demonstrations for making the subject comprehensible.
- an understanding of what makes learning of specific topics easy or difficult.
- an understanding of the conceptions and preconceptions that students at different ages and backgrounds bring with them; knowledge of strategies most useful for addressing misconceptions.

(p.8)

At the time, Shulman's (1986, 1987) work around teacher knowledge allowed the education field to recalibrate itself. In fact, much of Shulman's effort was spent on using his research to advocate educational reform. In his writing, Shulman (1986) also expressed concern over teachers' limited subject knowledge that had its roots in their own earlier education. A deficit that begins in early education could, potentially, be a lot more difficult to overcome. In this vein, he asks "What pedagogical prices are paid when the teacher's subject matter competence is itself compromised by deficiencies of prior education or ability?" (p.8). This question is a pertinent one in the South African context, explained in Chapter One, where early education is often fraught with issues (Spaull, 2013; Spaull & Kotze, 2015; Schollar, 2008) that often have a domino effect within the system. Here, the often-limited mathematical knowledge of many PSTs is a result of deficiencies in their prior education. Presently, many who wish to become teachers experience significant shortfalls in subject expertise. In their work, Adler et al. (2005) observe that:

> "Teacher–learners bring increasingly diverse mathematical histories. In many countries prospective elementary teachers have learned limited mathematics in school. In countries where there are great shortages, even prospective secondary teachers are entering training with relatively poor mathematical experiences and performance at school. This reveals that we are dealing with different kinds and levels of under-preparedness, a phenomenon that extends into in-service teacher education." (p.3)

Shulman's (1986) conceptualisation of teacher knowledge sparked widespread interest, especially in PCK, which pointed to a crucial part of what teachers know and do, but had previously been unnamed and overlooked. This omission had a limiting effect on how teachers were prepared for the profession. However, despite its obvious importance, some have felt that the term called for further elaboration and have worked to expand and refine his conception of teacher knowledge. Ball et al. (2008) have done extensive work in this area, and while they recognise the value of Shulman's (1986) work, they suggest that defining the concept more clearly would allow for an even greater and more substantial impact – in "revamping the curriculum for teacher content preparation, in informing policies about certification and professional development, and in furthering our understanding of the relationships among teacher knowledge, teaching and student learning."

2.3. MATHEMATICAL KNOWLEDGE FOR TEACHING

While of the opinion that their work has relevance across subjects, Ball et al. (2008) have specifically situated their work in mathematics education, and have established a framework for understanding teacher knowledge that is based on extensive empirical testing and "that focused on the work teachers do in teaching mathematics" (p.390). They affirm the view that PCK extends beyond a good conceptual comprehension of mathematics and knowing that an educator's "work involves an uncanny kind of unpacking of mathematics that is not needed – or even desirable – in settings other than teaching" (p.400). They consider their framework to be a refinement of Shulman's framework, based on their classroom observations, and they propose additional categories of teacher knowledge (p.402) that incorporate new subdomains.

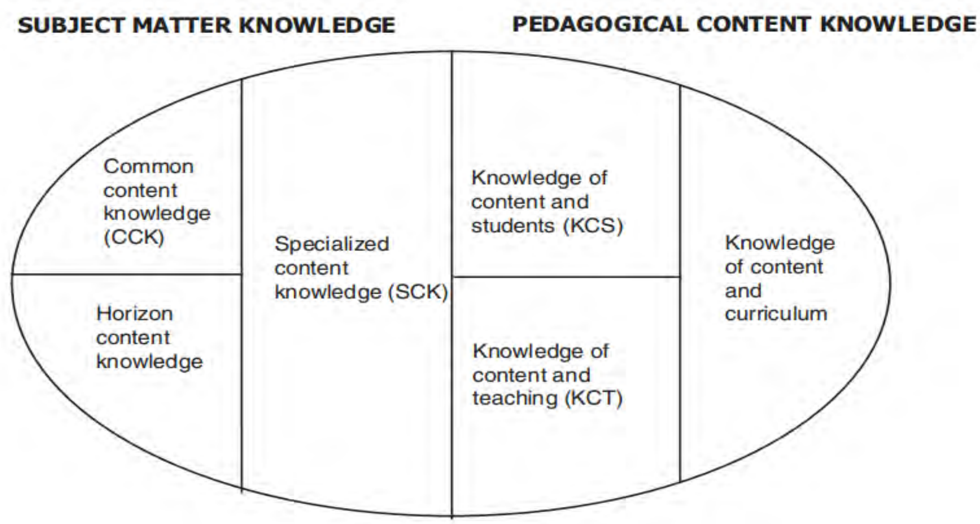
Ball et al. (2008) take two of Shulman's areas, namely content knowledge, and PCK and separate this knowledge into three domains. These delineations are more specific than Shulman's categories and include new additions too.

Within SMK there are three areas, namely common content knowledge (CCK), specialised content knowledge (SCK) and horizon content knowledge (HCK) (Ball et al., 2008). Unlike Shulman (1986), Ball et al. (2008) distinguish between CCK and SCK. While CCK talks about the fundamentals of a particular subject, SCK discusses the knowledge needed specifically to teach, for example, knowledge that allows one to analyse student errors with relative ease. They also include HCK in their framework. HCK refers to understanding how topics within a subject are related and the order in which they are introduced within a curriculum.

When unpacking PCK, they distinguish between "knowledge of content and students (KCS), knowledge of content and teaching (KCT), and knowledge of content and curriculum (KCC)." KCS expects teachers to merge their own knowledge of mathematical content and their knowledge of the learners to plan and anticipate the needs of the learners. KCT implies knowledge necessary to make important decisions about how to structure and design learning. Learning opportunities need to be carefully scaffolded, and teachers need to know which instructional models are best suited to a topic. This requires that knowledge of content and teaching interact in meaningful ways with knowledge of content and curriculum, which they feel could very well be a subcategory within KCT (p.402).

With the addition of new subcategories, Ball et al. (2008) are able to recognise more specifically which type of knowledge is being employed by a teacher. These categories can also be used to identify, more precisely, if there are areas of teacher knowledge that are more beneficial and more influential when it comes to learner success (p.405).

Figure 2.1 The Domains of Mathematical Knowledge for Teaching



(Ball et al., 2008, p.403)

2.3.1 Critique of Ball's Mathematical Knowledge for Teaching

Ball et al. (2008) expanded on Shulman's conceptualisation and developed methods of teachers' mathematical knowledge, stating that there is CK and SCK, and that SCK is the domain of teachers. Jaffer (2020) argues that the boundaries are blurred, and that specialised content knowledge is not only the domain of teachers, for example, a parent may be able to identify a child's error and be able to help a child. Furthermore, Petrou and Goulding (2011) state that Ball's conceptualisation does not recognise the value of teachers' viewpoints in the mathematics classroom. Teachers' beliefs will inform their own approach to teaching. For example, if teachers consider the memorisation of rules and steps to follow as a requirement when teaching mathematics, this viewpoint will guide their attitude and way of teaching.

Both Shulman and Ball's frameworks emphasise the unique and specialised nature of teaching, and in particular, teaching mathematics. However, it is the significant absence of CCK that forms the focus of this study, and, while Ball's additional categories have clear value, the choice was made to employ the constructs and terminology within Shulman's framework in the study. This is because it speaks directly to what is being researched, that is PSTs' mathematical CK and PCK to teach mental mathematics strategies. While my ultimate focus is on researching what PSTs know about mental strategies and how they use these mental strategies for calculating, I consider these mental strategies to be common content knowledge, and a key aspect of number sense. Thus, the next section begins with an exploration of what the idea of number sense means before considering the relationship between number sense, mental mathematics and mental strategies for calculating.

2.4. PRIMARY MATHEMATICS TEACHER EDUCATION

Early grade mathematics (EGM) has gained much research, intervention, and policy attention in recent years (Spaull &Taylor, 2022). The numeracy chairs at Rhodes and Witwatersrand universities have been linked alongside these policy and intervention directions. Furthermore, the huge gaps in teacher and PST knowledge have been linked to underperformance in primary education (Bowie et al., 2022). Venkat and Askew (2012) also discuss teachers' practices linked to EGM. Teachers use low level strategies such as concrete counting methods, which inhibit learners' development and continue to be seen as a constraint in South Africa's education.

With EGM in mind, research has focused attention on PST education of mathematical content knowledge (Fonseca et al., 2018). The low levels of development of content knowledge across BEd programmes are of concern (Bowie et al., 2019). Venkat et al. (2016) argue that performance levels of PSTs do not meet the criteria considered adequate levels of understanding for teaching primary mathematics. Subsequently the Mental Maths – Work Integrated Learning project aims to develop PSTs' additive reasoning skills and number sense.

2.5. NUMBER SENSE

Developing number sense (NS) plays a vital role in discussions within and on mathematics education, particularly early mathematics education, and while its importance is undisputed, a single definitive definition of the concept remains elusive. Consequently, Westaway and Vale (2021) describe it as “a nebulous and contested concept” (p.167) and draw our attention to a variety of definitions. Howden (1989) describes NS as a natural feel for numbers that improves as young learners manipulate and experience numbers in different ways and contexts. McIntosh, Reys and Reys (1992) argue that the learners learn NS in different ways as they think mathematically. Andrews and Sayers (2015) elaborate on the three types of NS, namely preverbal, verbal and foundational NS, as characterised by McIntosh et al. (1992), which focuses on the young learner.

Preverbal number sense (PNS), also referred to as innate or intrinsic number sense, is a cognitive skill that learners are endowed with at birth (Dehaene, 2001). Spelke (2000) refers to this innate number sense as core knowledge needed for future cognitive functioning. This innate number sense serves as the building block for developing new cognitive skills, which continues to later development in older children and adults.

Verbal number sense (VNS) refers to the knowledge of babies that improves as infants develop the language of mathematics such as counting words (Andrews & Sayers, 2015). Butterworth (2005) suggests that counting is a difficult skill involving learning the counting words in the correct sequence before counting concrete objects. Furthermore, Butterworth (2005) claims that learning to count is the first logical step or bridge that links the child’s innate ability of numbers as learners start to see the association concerning the number of objects and the number words. In addition, Butterworth (2005) states that children start school with prior knowledge and informal number ideas established through their experience counting and calculating numbers. As parents are the initial teachers, their background, attitude and knowledge of mathematics is likely to impact on a young child's early experience and introduction to number sense in a formal or informal setting (Sayers et al., 2019). Andrews and Sayers (2015) imply that this verbal NS that forms before starting school is the Foundational Number Sense (FNS) that is consolidated in the mathematics classroom through education in the first years of conventional education.

Andrews and Sayers (2015) identify eight interrelated components of FNS, namely recognition of numbers, counting systematically, discrimination of quantity, different forms of representation, estimation, basic arithmetic operations, and recognition as well as extension of number patterns. According to Andrews and Sayers (2015) there is a link between the components, and if these components do not connect, the child might not be able to count understanding the significance of numbers.

Within these definitions, and in search of greater clarity, is the proposal made by Whitacre et al. (2020), which suggests that the variety of explanations points to the polysemic nature of the concept. Based on extensive literature reviews on the topic, they identify three separate constructs that all go by the same name, that is, number sense. They identify these as innate number sense (INS), early number sense (ENS) and mature number sense (MNS) (p.104). As mentioned earlier, Spelke (2000) states that this INS is the building block needed to develop new reasoning skills that contribute to developing NS.

During early childhood as young learners interact in society, ENS develops as they learn verbal NS. These learnt skills comprise of number recognition, counting systematically, number relations, perception of quantity, comprehending different number representations, estimation and understanding basic arithmetic (Andrews & Sayers, 2015).

According to McIntosh et al. (1997), mature number sense (MNS) refers to students' ability to reason and understand number operations to be flexible in their thinking and choice of effective and efficient strategies when problem-solving. Whitacre (2015) explains MNS as the numerical skills learnt in formal education with the focus on two and three-digit whole numbers and rational numbers normally developed in the Intermediate Phase.

In contrast to INS, ENS and MNS can be learned in schools. While ENS focuses on number knowledge like counting and comparing objects using number words and numerals, and is closely aligned to school mathematics and the curriculum, MNS focuses on understanding and ability to flexibly manipulate numbers in different ways including counting strategies for mental calculations (Whitacre et al., 2020, p. 105). An extract from a table comparing these constructs highlights and contrasts key concepts and skills related to ENS and MNS:

Table 2.1: Early Number Sense versus Mature Number Sense

	Early Number Sense	Mature Number Sense
Key Concepts	Number recognition, counting, number patterns, number comparison, number operations, and estimation	Flexibility, mental computation, strategy selection, reasonableness of solutions

(Whitacre et al., 2020, p.101)

What is often found and evident within the definition afforded by the South African Mathematics Curriculum and Assessment Policy Statement (CAPS) (DBE, 2011) is a type of amalgam of ENS and MNS within definitions. Often, the lines between the two are somewhat blurred. In CAPS, for example, number sense is defined as the "knowledge of the relative size and meaning of different numbers, the relationships between numbers, and how to represent and calculate with numbers in different ways" (DBE, 2011). Here we see reference to skills from both ENS and MNS. Research suggests that ENS "provides an important foundation for the development of higher mathematical skills in the elementary years" (Whitacre et al., 2020, p. 109).

A further example of this is evident in the definition provided by McIntosh, et al. (1992) who state that “number sense refers to a person’s general understanding of number and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgements and to develop useful strategies for handling numbers and operations” (p. 3). They continue, noting that number sense is often said to include “knowledge and facility with numbers”, “knowledge and facility with operations” and “applying knowledge of and facility with numbers and operations to computational settings” (1992, p. 4).

Accompanying this growing appreciation for the importance of number sense, is the need to focus more on mental mathematics. While the ability to calculate mentally often requires number sense, it has the additional and important benefit of also developing number sense (Westaway & Vale 2021, p. 168).

2.6. DEVELOPING NUMBER SENSE THROUGH MENTAL MATHEMATICS

2.6.1. Mental Mathematics

It is a now widely accepted view that mental mathematics “underpins the development of an understanding of many mathematical concepts” (Harries & Spooner, 2013, p. 9), but as with the concept of number sense, it can often be difficult to find a clear and succinct definition of the term. Often, there are misconceptions surrounding the term due to the way in which it has been used in the past, where there was a particularly strong emphasis on memory and the recall of facts. Harries and Spooner (2013) offer a simple definition to begin with, stating that “it is about the way we think mathematically” (p. 10). Mental mathematics is also often contrasted with written mathematics, and it has been argued that “mental calculation…is more complex because it requires one to deal with entire numbers [and that]…In such a context, a deep understanding of number, operations and their relations is required, in addition to knowledge of basic facts and fact families" (Krauthausen, 1993; Heirdsfield & Cooper, 2004; Threlfall, 2002 as cited in Rathgeb-Schnierer & Green, 2019).

The argument that understanding relationships and connections in mathematics is vital has gained traction due to the wealth of research supporting this idea. One is regularly reminded that learners who can solve calculations easily and efficiently "are able to do so because they have constructed relationships among them and between addition and subtraction in general, and they use these relationships as shortcuts" (Fosnot & Dolk 2001, p. 98). In a similar vein, Harries and Spooner (2013) draw our attention to the study by Askew et al. (1997), which investigated the successful teaching of numbers. They remind us that research studies determined that the most effective teaching took place when teachers were able to see the relationships in mathematics and therefore were able to teach students to see the link and master the concepts.

Fosnot and Dolk (2001, p. 98) are convinced that "thinking about the relationships among [mathematics] facts is critical" and explain that a "child who thinks of 9 + 6 as 10 + 5 produces the answer of 15 quickly, but thinking, not memorization, is at the core." In this explanation, the child's appreciation of the relationship between numbers allows them to use an efficient mental strategy to solve the calculation with relative ease. Such strategies and the flexible ability to choose the most effective strategy to solve problems indicate that learners are developing MNS (McIntosh et al., 1997), which is considered in more detail in the following section.

2.6.2. The Role of Mental Strategies in Developing Mental Mathematics

Performing calculations mentally, then, is necessary if learners are to "develop a deeper understanding of number, number operations and the structure of a number" (Westaway & Vale 2021, p. 169). However, this deeper understanding also "encourages the use of strategies based on the particular sum" (Westaway & Vale, 2021, p. 169). Researchers have identified a wealth of strategies based on how children approach different types of calculations and problems, and in their review of the literature, Westaway and Vale (2021, p. 170) have drawn up a table which offers readers a useful synopsis of these different calculation strategies. Furthermore, they found that various names were used for the different strategies. For example, 26 + 32, the strategy of building up and breaking down 20 + 6 and 30 + 2; some researchers call this combining while other researchers refer to the strategy as decomposition, splitting or the ten-ten strategy. In the South African CAPS (2011, DBE, p.44) document the strategy is referred to as the 'building up and breaking down' method. For this reason, I have chosen to refer to this method as building up and breaking down, as stated in the CAPS document. Table 2.2 below includes the names used by 12 different researchers for this strategy.

Table 2.2: Researcher names for the building up and breaking down strategy for addition strategies typology (extracted from Westaway & Vale, 2021, p. 170) *

Carpenter et al.	Combining
Fushion et al. (1997)	Decomposition
Klein & Beishuizen (1998)	1010
Thompson (1999)	Splitting
Yackel (2001)	Breaking up both operands
Van den Heuvel-Panhuizen (2001)	Splitting
Selter (2001)	Hundreds, tens and units
Heirdsfield & Cooper (2004)	Separation
Heinze et al. (2009)	Split
Hopkins, Russo & Downton (2019)	Standard partitioning
Graven and Venkat (2019)	Split
Rathgeb-Schnierer & Green	Split

* Carpenter et al. And Rathgeb-Schnierer & Green dates are not given in the original.

It is now widely agreed that space ought to be created in the classroom where learners are encouraged to make connections and practise using these strategies. Fosnot and Dolk (2001), for example, encourage teachers to design lessons in a strategic fashion, picking problems “that are likely to develop certain strategies or big ideas” (p. 127).

2.6.3. The Mental Starters Assessment Project

The intervention on which this research is based, is built on the Mental Starters Assessment Programme (MSAP) that has been rolled out in Grade 3 by the DBE specifically to address prevalent inefficient calculation strategies and an absence of a structural understanding of number required for number sense. The MSAP is a carefully developed project designed to improve mental arithmetic amongst South African learners, but here, it is evaluated as a possible intervention for also developing the CK and PCK of PSTs whose competence has been compromised. Since this programme has shown positive results (Graven & Venkat, 2021; Askew et al., 2022), it was decided that it was worth investigating its value as a tool for developing the knowledge base of PSTs.

The MSAP focused on six curriculum aligned calculation strategies (bridging through ten, jump strategies, doubling and halving, rounding and adjusting, re-ordering and linking, addition and subtraction). The MSAP includes pre- and post-tests which aid teachers in diagnostically assessing where students are at the start of each unit and the extent to which students progress following the teaching of eight lesson starters per strategy (Graven & Venkat, 2021; DBE, 2021a; DBE, 2021b; Askew et al., 2022). These tests are accompanied by a teacher guide with scripted lessons and demonstrations which are accessed by a QR code. These are provided for each strategy and are designed to provide support for teachers in developing efficient additive reasoning calculation strategies and number sense. Key representations such as the part-whole diagram and the empty number line are used across multiple strategies to aid developing NS and a structural understanding of numbers.

While the programme is currently being implemented with learners across provinces by the DBE, a more recent focus is a parallel PST development, specifically developing competency in PSTs' use of these strategies with an aim towards strengthened foundational number sense and developing knowledge for teaching these strategies — along with researching various aspects of this process. The project was extended to PSTs as many researchers claim that South African underperformance in primary mathematics can be linked to teachers' weak CK and PCK. Furthermore, they suggest that lack of CK is in part attributed to either unit-based counting methods and/or standard vertical algorithms without number sense (Graven et al, 2013; Schollar, 2008; Weitz & Venkat, 2013).

2.7. CONCLUDING REMARKS

Chapter 2 has explored the value of CK to teach and examined two frameworks for unpacking teacher knowledge that are relevant to this research. The emphasis of the study is on the common content knowledge of PSTs; however, I use the terminology of Shulman's (1986) framework, which is germane to our narrow research focus on the development of mental strategies for calculating. These strategies are considered part of a developing number sense and are a significant component of common content knowledge needed for maths. In this chapter the concept of number sense, mental mathematics and the association between the two has been unpacked. The chapter concluded with an overview of the MSAP programme that is currently being implemented by the DBE across Grade 3 classrooms to support mental mathematics, and the MM-WIL programme that is taking this programme into PST education. My research contributed to the MM-WIL aims of understanding how the MSAP programme and related materials may be useful for supporting PSTs in the development of mental mathematics strategies and number sense more generally. In the following chapter, I will discuss the research design and methodology for this research.

CHAPTER THREE

3. RESEARCH DESIGN AND METHODOLOGY

3.1. INTRODUCTION

The previous chapter explored the work of Shulman, Ball and colleagues, then unpacked the mathematical constructs of CK and PCK that I use as a framework to analyse my data.

This study intended to be a study of the strategies that PSTs used and whether their knowledge of strategies could shift after extensive intervention. However, this originally planned intervention did not take place because of the COVID-19 pandemic, which I explain in more detail in Section 3.3. The challenge to conducting the intervention limits the extent to which this study could be about the intervention or possible shifts. While there is some useful data on how they shifted in the post-questionnaire, the focus of this study is on what strategies PSTs know and use.

This chapter begins with a brief overview that outlines and justifies this study's qualitative, interpretive research orientation and methodological approach to answer the following main research question: What CK and PCK do FP PSTs have of additive reasoning mental mathematics strategies?

The chapter then explains the research orientation and outlines the context of the research (along with the intended intervention), the sample (and gaining informed consent), the method of data collection and data analysis used. Limitations are highlighted and discussed. Validity is also discussed to provide transparency and an overview of this study.

O'Leary (2017, p. 85) defined methodology as "the framework associated with a particular set of paradigmatic assumptions" which researchers use to guide them through a study. The framework of a study outlines the tools used to "design research, collect data and associated theories of how and why they are to be used" (Stevenson-Milln, 2018, p. 37). The methodological decisions, influenced by the ontological and epistemological assumptions made in a study, must be appropriate to the research question and the research participants (O'Leary, 2017). The methodological approach that I chose to guide this study was a qualitative, interpretivist case study, as the research question aimed to understand and determine the level of CK and PCK of FP PSTs. Furthermore, it strives to understand the structure of knowledge that comes from my interaction with the participants, spending time at the research site and probing to obtain more detail (Creswell, 2018).

3.2. A QUALITATIVE, INTERPRETIVIST CASE STUDY APPROACH

As noted above my research adopts a qualitative, interpretivist case study approach. I provide a rationale for each of these choices below.

Van Maanen (1979) defines qualitative research as "an umbrella term covering an array of interpretive techniques which seek to describe, translate, and otherwise come to terms with the meaning, not the frequency, of certain more or less naturally occurring phenomena in the social world" (p. 520). Flick (2014) suggests that qualitative research analyses the "subjective meaning" (p.542) or the social construction of events or practices, by collecting, analysing and interpreting data. For the purposes of this research study, a qualitative, interpretivist orientation is appropriate.

The interpretivist orientation broadly underlines my research. Interpretivism is described by Bertram and Christiansen (2014), Creswell (2018) and Yin (2018) as typically linked to qualitative research, as a researcher aims to understand the knowledge obtained from interacting with the participants. The objective of interpretivist research is to comprehend the viewpoint of individuals in an ongoing study (Cohen, Manion & Morrisson, 2018). For this study, qualitative, interpretivist research is fitting as I aim to engage with PSTs in a joint intervention to explore and get to know their mathematical PCK for teaching additive mental mathematics strategies with the aim of improving it over time.

According to Mead (cited in Thomas, 2013, p. 108), the world around us is "socially constructed", and individuals might exhibit several views about the world established by their own practices of and within. This makes it difficult to understand or make sense of human experiences in specific ways. There is growing acceptance that there is no one, all-embracing certainty. My qualitative, interpretivist, research case study aimed to analyse and "understand … the lived experiences of humans" within the context of this study (Willis, 2007, p. 7). In other words, to know the CK and PCK of third-year FP PSTs.

The decision to use an interpretive paradigm draws on Cobb’s (2007) opinion that interpretivism can be valuable when doing a research study that explores and reflects on the "complexity and messiness" (p. 30) of a location to understand the occurrences.

Research also suggests some overlap between qualitative and interpretive research methods (Bevir & Kedar, 2008; Yanow & Schwartz-Shea, 2006). Interpretive research is distinctive in its approach to research design, standards of assessment, concept formation and data analyses. The qualitative research approach supports the interpretivist orientation, providing the opportunity to explore the context and complexity of this research (Mason, 2006) through observations, body language, class discussions and comments made by individuals in the classroom (Creswell, 2018). Interpretivism is an orientation that enables the researcher to develop a deeper understanding of human action through qualitative research, where there is no single viewpoint but many interpretations of events or actions (Cohen et al., 2018). The assumption of my study was informed through a broadly interpretivist interpretation concerning ontological and epistemological theories, keeping in mind the limitations of this research.

This qualitative, interpretive paradigm informed the assumptions regarding ontology and epistemology. Ontology discusses the study of the "nature of existence" (Willis, 2007, p. 9), while epistemology focuses on questions regarding knowledge or "what we know about reality… and how we know it" (Willis, 2007, p. 10). Interpretivism falls within the epistemological viewpoint that knowledge is intuitively and socially constructed. My qualitative, interpretivist research case study aims to understand PSTs' CK and PCK to narrate the observed data and answer the research question.

A case study examines aspects, events, people, processes, organisations, or social groups (Merriam, 1988) and is frequently used in qualitative research studies exploring 'bounded systems' (Merriam, 2009; Stake, 1995). In the words of Creswell (2007), “case study research is a qualitative approach in which the investigator explores a bounded system (a case) or multiple bounded system (cases) over time, through detailed, in-depth data collection involving multiple sources of information (e.g., observations, interviews, audiovisual material, and documents and reports) and reports a case description and case-based themes” (p.73). For this study, a case study method is appropriate because this research focuses specifically on gaining evidence of PSTs’ mathematical CK and PCK using a variety of tools.

Stake (1995) discusses three types of case studies: intrinsic; instrumental; and collective. An intrinsic case study is usually done when a researcher is interested in a particular case where the goal is to gain insight into specifics. An instrumental case study offers understanding of an issue or phenomenon, while a collective case study aims to understand multiple cases. My research is an intrinsic-instrumental case study. My aim was to understand a single phenomenon (mental mathematics knowledge for teaching) of a group of second year PSTs to gain insight into their mathematics CK and PCK in order to find ways to strengthen it. While I am aware that similar studies are currently being done at other teacher education institutions country-wide as part of the MM-WIL research programme, my study is a minor-scale research study where the findings are not generalisable

(Bertram & Christiansen, 2014; Creswell, 2018; Merriam, 2009) but contribute to this emerging body of knowledge about South African PSTs' mental mathematics strategies and how the MSAP materials can be used to support PSTs' knowledge.

3.3. THE CONTEXT OF THE STUDY AND THE INTENDED INTERVENTION

As indicated in Chapter One, I am a lecturer at a private institution in the Eastern Cape. This research was driven by my awareness that many of the PSTs I teach have limited knowledge of strategies and additive reasoning skills to compute simple mathematics calculations at the level that they are required to teach these skills. In the institution where I lecture, no pure mathematics courses are taught as part of the four-year Bachelor of Education degree. However, in years one to three, students participate in various courses that focus on the pedagogy of teaching mathematics in the Foundation Phase (Grade 1-3). In their fourth year they complete courses linked to other subjects.

These courses focus on the theories of teaching mathematical content rather than on the mathematical content per se. In my experience many PSTs do not have sufficient mathematical CK to teach the various content areas of the curriculum and in particular students have limited knowledge of various efficient strategies for calculation.

I planned to research the nature of their knowledge of efficient addition strategies and implement an intervention to broaden their repertoire of knowledge of three strategies, namely bridging through ten (BTT), jump strategy (JS), and rounding and adjusting (R&A), over a three-week period. These efficient strategies are built on the MSAP materials that PSTs will implement when teaching Grade 3 in schools. (The rationale for this focus is given in Chapters 1 and 2).

The initially planned three-week more extensive intervention was not possible. By the time at which I received ethical clearance, we were in lockdown due to the Covid-19 pandemic. Lecturers and students had limited time on campus. The policy (at the institution where the study took place) changed frequently and there was no clarity as to when students would or would not be permitted back onto campus on a more regular basis. I therefore made the pragmatic decision to conduct the pre- and post-questionnaire, provide intervention and collect data (five 50-minute sessions) in one week while this group of students were on campus.

While not ideal, as this was a stressful time for participants, I decided that it was the best option at the time, since future student online participation vs on-campus participation was an unknown. Furthermore, I felt that three weeks of probable online intervention and data gathering could be compromised by students using calculators or possibly searching for methods online. With this in mind, I made the choice to collect all my data in the one guaranteed week on campus. There was no time to analyse data and conduct interviews, so students were asked to comment on whether they felt they had or had not benefitted from the intervention.

My main research focus was on understanding strategies (see above). However, sub-question three stated:

- How might PST strategies shift following exposure to the bridging through ten and jump strategies in lectures?
- Does PST performance improve in relation to learning additional strategies?

I was thus interested in researching the extent to which providing an intervention that gave exposure to multiple additive reasoning strategies may expand PSTs' repertoires for solving basic additive reasoning calculations and their ability to teach these strategies. However, due to the rushed nature of the intervention and collection of data, there is a limitation to answering this sub-question. The PSTs had limited exposure to only two of the strategies (BTT and JS over two days) but it was not as substantial as initially intended (three strategies BTT, JS and R&A) over a three-week period.

3.4. SAMPLE AND SITE

I used purposive and convenient sampling in this research study (Merriam, 2009). My sampling was purposive in the sense that I purposefully chose to work with my 3rd year PSTs because this is their last year of mathematics pedagogy instruction in their four-year course. My sampling was convenient in the sense that I was lecturing these PSTs and wanted to establish their knowledge of CK and PCK. Coyne (1997, p. 624) portrays 'purposeful sampling' as the involvement and careful selection of "information-rich" participants from whom a researcher can learn much. Maxwell (2013, p.97) agrees that purposive sampling is an example of non-probability sampling where purposefully selected individuals can be informative rather than randomly selecting participants for statistical data collection.

As noted earlier, the sample for this study was a convenience sample purposefully chosen (Folgeman, 2002). The PSTs in my study are from different backgrounds with diverse home languages such as English, Afrikaans and isiXhosa. However, the language of learning and teaching (LOLT) in the mathematics education class is English.

All 80 third-year FP Bachelor of Education PSTs were invited to participate in the study on a voluntary basis and to complete the questionnaire. While all 80 PSTs were invited to participate, only 54 agreed and so my sample size was 54 female PSTs. There were no male PSTs in my class. However, due to absenteeism on different days of data gathering, 54 PSTs wrote the pre-questionnaire and 51 wrote the post- questionnaire. This sample size lends itself well to the use of descriptive statistics.

3.5. INVITATION TO PARTICIPANTS

As mentioned above, an invitation to participate in a MM-WIL Teacher Education for Primary Mathematics research programme was extended to the students (Appendix B). The letter highlighted the aim of the research study.

The invitation letter explained the collaboration with other higher education institutions in South Africa, addressing PSTs development, specifically teaching for number sense, while developing knowledge for teaching mental strategies. The letter also explained how teaching mental strategies could address the weak number sense and develop learners' mental calculation strategies, and how future teachers need to be competent in these strategies to improve learners' mathematics.

3.6. POSITIONALITY

I knew some students might feel vulnerable and pressured to participate as I am their lecturer. To reduce this effect, my position as lecturer and researcher was discussed, and students were assured they would not be obliged to take part in the study. I reiterated to all students their right to choose, that participating was entirely voluntary, and that they may remove themselves from the research study at any stage without prejudice, penalty or negative consequences. Figure 3.1 below highlights a section of the information shared with participants.

Figure 3.1 Participation information given to PSTs.

Participation Information

- I understand the purpose of the research study and my involvement in it. My lecturer has informed me that my participation in this study will not affect my final year mark.
- My participation is entirely voluntary and I understand that I may withdraw from the research study at any stage without any penalty or negative consequences.
- I will participate in the project by completing a questionnaire and the MSAP pre- assessment, prior to the teaching of the mental starters. On completion of the mental starters program, I will be required to complete a post-test on the strategies taught to establish the development of my knowledge of the mental strategies
- I understand that while information gained during the study may be published, my name will remain anonymous and no reference will be made to me by name.
- In terms of the Protection of Personal Information Act (No. 4 of 2013) it remains my right to request a detailed explanation from the research of exactly how confidentiality and anonymity of the data I provide will be achieved. It is also my right to know exactly how and for how long my personal information will be stored securely.

3.7. ETHICAL CONSIDERATIONS

The ethical considerations in this research included respecting participants' rights and ensuring their well-being while maintaining the integrity of this study. Crucial principles such as consent, confidentiality and voluntary participation were considered to ensure accurate data collection methods and minimise bias. Merriam (2009) recommends that "ensuring validity and reliability in research involves investigating ethically" (p. 209). Ethical issues could sometimes occur during a phase of the research procedure (Bloomberg & Volpe, 2008); therefore, precautions are necessary to minimise any associated risk (Cohen et al., 2011).

The key to ethics in this research study was to make certain that the methods were followed during the research to minimise harm and maximise benefits. Banegas and Villacanas de Castro (2015) suggest that ethical considerations embrace anonymity and confidentiality. I chose to allocate two letters to each PST to support anonymity. Permission was obtained from Rhodes University's Education Higher Degrees Committee and Research Ethics Committee to conduct my research

(Appendix A). Thereafter, I submitted my proposal to the ethics committee at the organisation where the research took place. Ethical considerations included requesting gatekeeper permission from the private PST education college where the study took place.

After obtaining permission from the private college where my study took place, invitation letters (Appendix B) were handed out to students, and I sought consent from the third-year PSTs to conduct the research (Appendix C). An informed discussion was held with the participants, considerations were explained, and it was emphasised that participation was voluntary without prejudice or penalty for non-participation. After that, written permission letters were obtained from PSTs who decided to participate in the study. PSTs were advised that pseudonyms would be used and that no identifying markers of the institution would be shared in future publications. This was to ensure that the dignity and confidentiality of the participants were always protected. The participants were also informed that the research findings would be available to them upon request.

3.8. VALIDITY

Validity, defined by Charmaz (2006), is the extent to which the study correctly denotes what the intention of research was. McMillan and Schumacher (2010) advise that validity refers to the "truthfulness of findings and conclusions... and the degree to which explanations are accurate" (p. 104). The validity of an interpretivist qualitative study is therefore understood as rigour and a trustworthy interpretation of data gathered. In other words, the trustworthiness of my study, to some extent depends on the validity of my interpretations but also can be judged by the extent to which the analysis resonates with the reader in terms of the data shared. According to Mead (cited in Thomas, 2013), “the world” around us is "socially constructed" (p. 108), and individuals have different perceptions based on experiences within their environment. As the researcher and author of this study, I sought to "understand… the lived experiences of humans" (p. 7), eliminate bias, remain vigilant, and increase the trustworthiness of the qualitative research by consistently conducting the data analysis and disclosing the methods of analysis with enough detail to enable the readers to determine the credibility of the process (Willis, 2007). My study has benefitted from my participation as a lecturer in a teacher education institution in the Mental Mathematics – Work Integrated Learning project (MM-WIL) of the two SA Numeracy Chairs at Wits and Rhodes University. My participation in annual meetings and annual conference presentations of this larger project has allowed me to continuously reflect and present my research to colleagues (in other higher education institutions) grappling with implementing the Mental Starters at their respective institutions.

Triangulation formed part of my study as I considered various data sources and several data collection methods to compare, "cross-checking" (Maxwell, 2003, p. 245) information. This was done to gain a clear and full understanding, minimising the risk of prejudice from a single data source (Ferrance, 2000; Willis, 2007). I was aware of the probability of "researcher bias" (Maxwell, 2003, p. 243) compromising the credibility of my research while coding and re-coding in collaboration with my supervisor. I constantly guarded against unconsciously projecting my pre-conceptions in discussions throughout my research study.

3.9. RESEARCH GENERATION TOOLS

As noted above, I used:-

- pre- and post-questionnaires (calculation items and open-ended questions).
- post-mental starters (written reflections).

The instruction given was: Reflect on your experience working with the MSAP BTT and JS and give reasons why you think mental mathematics strategies are or are not advantageous?

The pre- and post-questionnaires included calculation items and open-ended questions to collect data that allowed me to answer my primary overarching research question:

- What is the nature of Foundation Phase pre-service teachers' mathematical content knowledge and pedagogical content knowledge for teaching additive mental mathematics strategies?

  Table 3.1 highlights the relationship between the three research sub-questions and data collection methods.

Table 3.1: Relationship between questions and data collection methods

Research sub-questions	Data gathering methods
What strategies do third-year PSTs use to solve basic additive reasoning calculations? Do they use these effectively?	Pre-CK questionnaire – PART 1 (calculation items with methods explained).
Can PSTs describe the BTT, Jump, and Rounding and Adjusting strategies? If so, how do they explain them and how they might teach them?	Pre-CK questionnaire – PART 2 (open-ended questions).
How might PST strategies shift following exposure to the bridging through ten and jump strategies in lectures? Does PST performance improve in relation to learning additional strategies?	Post-PCK questionnaire (PART 1 and 2)

3.9.1. Pre- and post-questionnaires

A questionnaire in educational research can involve a mix of open-ended and closed-ended questions. This format allows response flexibility while ensuring standardisation. Creswell, (2018) and Yin (2018) both advocate for flexibility in methodology, ensuring that the research process remains responsive to participants' perspectives. In this study, I included open-ended questions to gather rich qualitative data. The questionnaire that I used for this study (see Table 3.2 and 3.3 below) began with four calculation questions with a space alongside for the PSTs to explain the strategy they used to do the calculation. Additional questions were asked in Part 2 of the questionnaire (Q5-10). The PSTs were asked to explain their understanding of bridging through ten, the jump strategy, and rounding and adjusting, and how they might teach these strategies. Willis (2007) encourages the collection and use of rich data to understand and accurately record the participants' responses. The questionnaires were not administered under test conditions. The questionnaires were handed out in a relaxed classroom setting with sufficient time for PSTs to think about and explain their method.

Table 3.2: Pre- and post-questionnaire PART 1 – Four calculation items (CK)

Solve the following calculations.	**Explain how you solved it.**
(1a) 36+8 =	(1b)
(2a) 47+29 =	(2b)
(3a) 63 – 24 =	(3b)
(4a) 98-99 =	(4b)

Table 3.3: Pre- and post-questionnaire PART 2 – CK and PCK

(5) Describe the bridging through ten strategy.
(6) How might you teach this strategy to a Grade 3 learner?
(7) What instant facts do learners need to know to be able to use bridging through ten?
(8) Describe the jump strategy and how you might teach it.
(9) What representation would you use to help learners understand bridging through ten and what would you write on the board?
(10) Describe Rounding and Adjusting and how you might teach it.

For the reflection the instruction given was:

- Reflect on your experience working with the MSAP BTT and JS.
- Give reasons why you think mental mathematics strategies are or are not advantageous?

3.10. RESEARCH DATA GATHERING PROCESS

As indicated previously, the unexpected impending shutdown limited the access to students on campus. The uncertainty of the way forward due to the COVID-19 protocols resulted in the gathering of all data within a week (five 50-minute sessions). The pre-intervention data collection included the gathering of data relating to PSTs' CK and PCK through administering the pre-questionnaire (Table 3.1 and 3.2) to the 54 PSTs who signed consent forms. The PSTs were told to work individually (at their own pace) and try to answer every question. I waited for every student to finish (25 minutes) before collecting all the questionnaires.

3.10.1. Day 1: Pre-questionnaire

As noted above, on Day 1 (session one of five) the first data collected was a non-time restricted and not under test conditions questionnaire that included (in part 1) four calculations that lent themselves to the strategies of BTT, JS and R&A. PSTs were asked to complete the following four calculations: 36+8, 47+29, 63 – 24, and 98+99, and in a box alongside state the method they used (as shown above in Table 3.1). As this was not under strict test conditions, occasional discussion or looking to see what others were doing occurred even while it was requested that PSTs should work alone. In part 2 (see Table 3.2) additional questions 5-10 asked students to describe various additive reasoning strategies and to explain how they might teach them. This was aimed to gather data on their CK and PCK of various strategies, respectively.

3.10.2. Day 2: Teaching Bridging Through Ten

Two 50-minute lecture sessions (session two and three) were allocated to exposing the students to the MSAP BTT strategy and teaching this efficient additive reasoning mental mathematics strategy. Teaching this strategy aimed to address the requirements of the FP PSTs for teaching mathematics. This study and lectures simultaneously focused on developing PSTs' PCK following Adler and Reed's (2003) argument that while CK is necessary, it is insufficient for teachers.

As mentioned, I exposed the students to the MSAP materials of BTT that they will implement if they teach Grade 3 in schools. I thus began, as in the guidelines for teachers on the use of the MSAP materials, with the pre-tests for the strategy. This gave the PSTs a sense of how these timed tests work. Initially I thought these MSAP tests may provide useful data. However, as learners do not show their methods and many were seen to be using hidden calculators, these did not provide useful or reliable data for answering the research questions. After the tests, the PSTs divided into groups of eight. Every person in each group was allocated one of the eight mental starters and given 10 minutes to familiarise themselves with the lesson's content. Once everyone had worked through their lesson starter, they had to teach it to the other seven group members. At the end of the 50-minute session, the class had a ten-minute break.

In the next session, the PSTs were regrouped based on their corresponding lesson starters. In other words, 'group specialists', all the students who taught lesson one in the first session grouped. This promotes collaborative learning (sharing among the group) as each group discusses how they would teach the allocated lesson to the rest of their peers. The originally intended two consolidation worksheets for the BTT strategy were not completed prior to the post-test due to the time constraints.

3.10.3. Day 3: Teaching the Jump Strategy

In session four, I once again followed the teachers' guidelines for use on the MSAP and commenced the session by exposing the PSTs to the MSAP JS pre-tests. Thereafter the PSTs were arranged into groups based on their performance in the previous two sessions. During the two 50-minute sessions (session four and five), the PSTs engaged in learning and micro-teaching their peers the eight jump strategy lesson starters from the MSAP programme in the same way that they did for the BTT strategy. At the end of the fifth session on Day 3, the pre-questionnaire was repeated as a post-questionnaire. Once again, this questionnaire was completed under non-time restricted, non-test conditions to determine any shifts in their CK and PCK.

As noted, the initially planned interviews were not possible due to the uncertainty and limited access to students, therefore an impromptu reflection was encouraged. No specific question was asked; the PSTs were just asked to write their thoughts and feelings about the MSAP and whether they felt it was beneficial or not. They just wrote their thoughts on the bottom of the questionnaire.

As noted previously, the initial plan was to collect data from the pre- and post-tests and questionnaires after substantial intervention over a three-week period. This data collection and intervention was rushed and took place over only three days, which was not ideal. While I thought that the tests might provide useful information, the rushed nature meant that the tests were not suitable data. There were major limitations to learning prior to the post-tests, therefore the main contribution for my thesis comes from the data collected on the pre-questionnaire. This data provided valuable insight into the CK and PCK of PSTs and the intervention needed to develop this knowledge.

As indicated in the sample above, only 51 of the 54 who wrote the pre-questionnaire answered the questions in the post-questionnaire. While I acknowledge the limitations, the data from the pre- and post-questionnaire and reflections were analysed to see possible shifts in CK and PCK.

3.11. DATA ANALYSIS

A thematic analysis was conducted (on the 54 PSTs' pre-responses and 51 PSTs' post-responses) in terms of the various strategies used to solve the four calculation items and the descriptions and explanation of how PSTs might teach these strategies. This data was obtained in the form of pre- and post-questionnaires. In terms of answering the third research sub-question on how their CK and PCK knowledge of various additive reasoning strategies may have shifted following (albeit limited) exposure to the strategies and the MSAP learning materials, similar thematic analysis was conducted, and this analysis was then compared with their responses in the pre-questionnaire and possible shifts identified and discussed.

Data analysis is a systematic method of examining and displaying data to make sense of it and determine valuable information that could be used to help and support decisions made (Bloomberg & Volpe, 2008). In research, collected data is selected, organised, recorded, interpreted, debated and presented (Brenner, 2006; Ryan, 2006; Willis, 2007). Merriam (1998) explains that data analysis is an iterative, back-and-forth process of examining sections of data to make meaning of the data collected.

To collect and analyse rich, relevant, meaningful data to interpret and understand the data, multiple methods and tools can be used (Maxwell, 2013; McCallister, 2018; Willis, 2007). In my study I analysed and studied the questionnaires with meticulous attention to detail and accuracy as I captured the data. The data was then displayed in the form of tables. This iterative process of capturing and checking allowed me to analyse and revisit the phases as I re-coded and re-analysed the data in different ways to acquire a holistic view of the data. Ferrance (2000), Merriam (1998) and Maxwell (2003) emphasise the need to be consistently systematic and organised, following an analytical framework as a guide when dealing with large amounts of qualitative data. This was most helpful when a thematic analysis of all PSTs' questionnaires was conducted to identify common themes.

Inductive coding was used to identify recurring patterns in the data (McMillan, 2010). Inductive coding allowed the data to 'speak for itself' (McMillan, 2010). This enabled me to thematically code PST responses into categories to analyse their knowledge of various strategies and their PCK in relation to these strategies. Descriptive summaries displayed as tables allowed for analysis of pre- and post-questionnaires.

3.12. LIMITATIONS

There were several constraints concerning my study. Firstly, my proposal and ethics approval were delayed, which meant that I had limited time with the students. Secondly, limitations and time constraints on campus, imposed by the university protocol due to the COVID-19 pandemic, influenced my decision to collect all my pre- and post-intervention data in one week. These constraints reduced intervention, and no time for individual interviews had an impact on my research study.

3.13. CONCLUDING REMARKS

In this chapter I have explained how a qualitative, interpretive paradigm has shaped the methodological decisions I made. I have explained the reasoning for this study and provided a rationale for the use of a two-part semi-structured questionnaire for data collection. I have explained the sample for my study and described my use of Shulman's constructs of CK and PCK to support the data analysis process. I have discussed the validity, ethical considerations, and several limitations of the study. The subsequent chapter presents an analysis of the data gathered and presents the findings of the study.

CHAPTER FOUR

4.1. INTRODUCTION

The previous chapters indicated that underperformance of mathematics in South Africa is often attributed to teachers' lack of both CK and PCK. In addition, the inefficient unit-based counting methods often dominate teaching and learning, preventing learners from developing a structural understanding of numbers that allows for flexible and efficient calculation. This chapter discusses the data in two main parts. I have structured these parts as follows: Part 1 of this chapter analyses the results and methods used to calculate the four questions on the pre-questionnaire and Part 2 examines the shift from the pre- to post-questionnaire on two strategies. As stated, this chapter summarises and analyses the findings of the data gathered to explore how PSTs perform on basic additive reasoning calculations, what strategies they use and if the PSTs use the strategies effectively. The initial intention was to investigate whether there was any improvement following intervention sessions and the way in which PSTs' CK and PCK developed (if at all) when engaged in mental mathematics.

As noted previously, only data obtained from PSTs who signed informed consent forms were used in this study. Fifty-four PSTs participated in the pre-questionnaire and 51 PSTs participated in both pre- and post-research data gathering methods. Thus, the tables in Part 1 focused on 54 PSTs (n=54) while Part 2 analysed 51 PSTs (n = 51).

In Part 1 of this data analysis chapter (section 4.2) I analyse the pre-responses to the first calculation items given in the first part of the questionnaire. For ease of reference, I refer to the four calculation items with the explanation of solution methods as the Content Knowledge (CK) questionnaire under the following headings:

4.2.1 PSTs' pre-session results on the four calculations.

4.2.2 Overview of methods/strategies used on the pre-questionnaire.

4.2.3 Error analysis on four calculations in the pre-questionnaire.

I will refer to the second part of six questions about methods (Table 3.3 gave these questions) as the Pedagogical Content Knowledge (PCK) questionnaire.

In the second half of Part 1 (section 4.3) I analyse the pre-responses to the further six questions that focus on exploring how one might describe and teach calculation strategies (bridging through ten, jump strategy, and rounding and adjusting). As noted, I refer to this second part of six questions as the PCK questionnaire, and analyse PSTs' pre-intervention descriptions of BTT, JS, R&A and how to teach it under the heading:

4.3. Analysis of pre-PCK questionnaire

Part 2 analyses the responses and possible shifts in knowledge from analysing pre- to post-questionnaire responses. In the first section (4.4) of Part 2 of this data analysis chapter I analyse the post-responses to the first four calculations in the first part of the questionnaire:

4.4.1 Shifts in frequencies correct/incorrect from pre- to post-questionnaire.

4.4.2 Shifts in methods from pre- to post-questionnaire.

4.4.3 Error analysis and comparison of the four calculations from pre- to post-questionnaire.

4.4.4 Analysis of the shift across the four calculations of PSTs who achieved 50 percent or less in the pre-questionnaire.

In the second section (4.5) of Part 2, I analyse the post-responses to the further six questions (which I refer to as the PCK questionnaire) that focus on exploring how one might describe and teach calculation strategies using the following headers:

4.5.1 Shifts in explanations from pre- to post-questionnaire on two strategies

4.5.2 PSTs' descriptions of their perceived growth in confidence to teach the strategies.

The chapter concludes with a discussion and comments.

Thus, in this chapter I present data to establish the number sense (NS), CK and PCK of PSTs before and after limited intervention, in an attempt to establish the needs of PSTs.

PART ONE: PRE-QUESTIONNAIRES

4.2. ANALYSIS OF PRE-CK QUESTIONNAIRE

In this section of the chapter, I analyse and share findings of PSTs' responses to the CK questionnaire. The first four research questions gathered information on PSTs' knowledge or use of specific additive reasoning strategies. The aim of these four questions was to establish to what extent the PSTs were able to answer additive reasoning calculations with their preferred method.

I have organised the analysis of PST data from the questionnaire as follows:

4.2.1 PSTs' pre-session results on the four calculations
4.2.2 Overview of methods/strategies used on the pre-questionnaire.
4.2.3 Error analysis on four calculations in the pre-questionnaire.

I begin with the section that looked at PSTs' results on the four addition and subtraction calculations, and their responses on the pre-CK questionnaire.

4.2.1. PSTs' pre-session results on the four calculations

My research questions focused on PSTs' knowledge or use of various additive reasoning strategies, but it is also useful to know the extent to which PSTs were able to successfully answer additive reasoning calculation items with whatever method they chose. Table 4.1 shows the performance results of the 54 PSTs on the four addition and subtraction problems A to D in the pre-questionnaires. All 54 PSTs attempted to complete the calculations and explained the method used. In other words, there were no blank spaces as every question was either answered correctly or incorrectly. (Graven, 2014)

Table 4.1: Frequencies correct/incorrect for pre-questionnaire (n=54)

	A 36 + 8	B 47 + 29	C 63 – 24	D 98 + 99
54 PSTs	PRE	PRE	PRE	PRE
Correct	51	49	43	48
Incorrect	3	5	11	6

All 54 PSTs answered all four questions, which, while administered individually, were not done under test conditions. As is evident in Table 4.1 most PSTs managed to obtain correct answers for all four questions. The percentage correct across the four questions were 94, 91, 80 and 89 percent respectively for A, B, C and D. 51/54 calculated A correct; 49/54 got B correct; 43/54 did C correct; and 48/54 answered D correct. While no PST got all four incorrect, there is some concern for those (at least 11 PSTs) who did not manage to answer all four correctly. Since the items are Grade 3 simple calculations requiring no problem solving or question interpretation, other than the occasional careless slip in calculation, one would wish that all PSTs could manage these questions accurately and efficiently. Question C, the subtraction question, was the most problematic, with one fifth (11/54 i.e., 20 percent) failing to get the calculation 63 – 24 correct.

4.2.2. Overview of methods/strategies used on the pre-questionnaire

Solving calculations both efficiently and accurately is important. Reviewing PSTs' methods used to solve calculation items A-D enables analysis of the efficiency (and accuracy) of the methods used. Table 4.2 shows the different methods that PSTs used in the pre-questionnaire calculation items as indicated by analysis of their written method in conjunction with their written statement about their method.

Some PSTs indicated a combination of methods, for example breaking down and the use of fingers. These statements were written in the right-hand column next to the four calculations A-D (see Figure 4.1 for example).

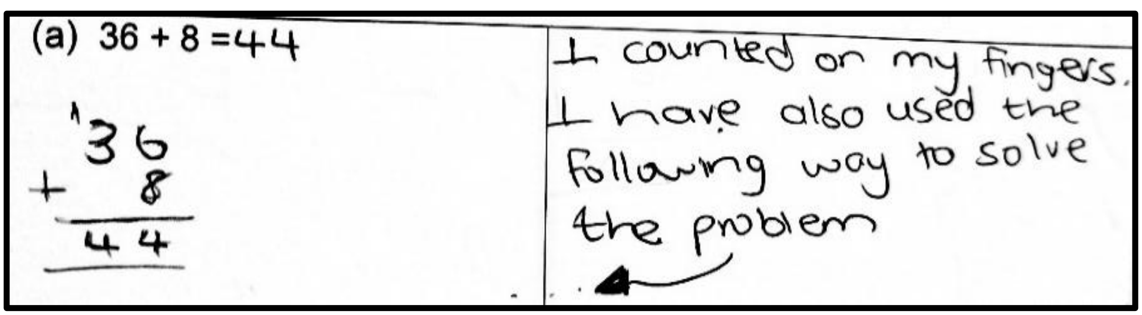
Table 4.2: Methods used in the pre-questionnaire (n=54)

PRE-QUESTIONNAIRE 54 PSTs	A 36 + 8	B 47 + 29	C 63 – 24	D 98 + 99
	PRE	PRE	PRE	PRE
Counted on with fingers	9	4	0	4
Counted on	4	0	0	0
Breaking down	13	24	23	18
Breaking down with fingers	1	1	2	0
Vertical algorithm	23	23	27	28
Vertical algorithm with fingers	1	0	1	0
BTT/number line	3	0	0	0
Jump Strategy	0	0	0	0
Rounding & adjusting	0	2	0	4
Answer only - no method indicated	0	0	1	0

Looking at Table 4.2 above, it is evident that the most dominant method across the four calculations is the vertical algorithm (used by over 46 percent of PSTs across the four items). This is followed by building up and breaking down (used by over 36 percent of PSTs across the four items). However, unit counting (counting with fingers and counting on) and a combination method including fingers was used by 13 percent of the PSTs for the addition calculations. Hardly any use the BTT and rounding and adjusting (R&A) method. Few PSTs in the pre-questionnaire used the efficient methods of BTT, JS or R&A (which B and D lend themselves well to). Unit counting (counting with fingers and counting on) and a combination method including fingers was used by almost a quarter of learners for item A, and four PSTs use this method for items B and D – though interestingly not for the subtraction calculation. While using this method is relatively quick for adding on eight, it is not an efficient method for adding 29 and 99.

In Question A (36 + 8) the most prevalent methods used, by 24 of the 54 PSTs (44 percent) was the vertical algorithm, followed by 13 PSTs (24 percent) building up and breaking down. That 24 PSTs (one with fingers) use the vertical algorithm and another 14 (one with fingers) use breaking down (i.e., 36 + 8 = 30 + 6 + 8 = 30 + 14) for a simple calculation involving adding a single digit to a two-digit number speaks to challenges in basic number sense. Understanding BTT and simple rapid recall facts such as 36 + 4 = 40; 8 = 4 + 4; 40 + 4 = 44 should enable such problems to be solved efficiently and mentally. Similarly, 13 PSTs (24 percent) counted on with or without fingers. Two additional PSTs (3 percent) used a mixed method, counting fingers, one with the vertical algorithm and the other used the building up and breaking down method. Only three PSTs (5 percent) used the more effective BTT method with or without the number line for the simplest item A (36 + 8). Figure 4.1 below is an example of a combination of methods used by PST TL, who used the vertical algorithm and fingers.

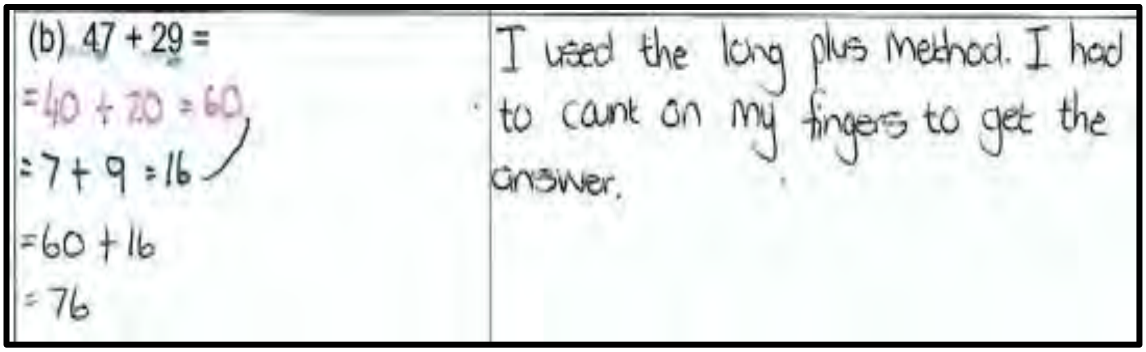
Figure 4.1 Correct calculation using the vertical algorithm with fingers.



PST TL

Two of the 54 PSTs (4 percent) used the rounding and adjusting method for Question B (47 + 29). Forty-seven participants (87 percent) used either the building up and breaking down method, one with fingers (as shown in Figure 4.2) or the vertical algorithm, and five PSTs (9 percent) used their fingers.

Figure 4.2 Combination of decomposing numbers and using fingers.



PST JB

Fifty-three PSTs (98 percent) used either the vertical algorithm or the building up and breaking down method with or without fingers for Question C, the subtraction question, and one PST (2 percent) gave no explanation for the method used. The vertical algorithm was the most used method by 27 (50 percent) of the PSTs in Question C, followed by 23 PSTs (42 percent) who used the building up and breaking up method, three (6 percent) used a mixed method, which included counting on fingers, and one PST (2 percent) gave no explanation of the method used.

The use of the breaking down method links to its emphasis in the curriculum where it “is one of the most important techniques in the Foundation Phase. Using this technique allows learners to split and recombine numbers to help make calculations easier” (SA. DBE, 2011, p. 133). The FP CAPS document recommends the introduction of this method from Grade 1 and suggests that learners “will largely be using this technique in the Intermediate Phase as well” (SA. DBE, 2011, p. 219).

For subtraction, the examples given in the CAPS document suggest decomposing in multiples of ten and counting backwards to make the calculations easier. For example, in Grade 3 the focus of the building up and breaking down method shifts to subtraction of two three-digit numbers with no example of regrouping. In other words, they only suggest “Breaking down a number into smaller parts to make a calculation easier. Most of the strategies that learners use involve breaking down numbers. They continue to do this with three-digit numbers" (SA. DBE, 2011, p. 389). See Westaway (2021) for a summary of the CAPS recommendations for this method from Grade 1-3.

In Question D (98 + 99), 28 participants (52 percent) used the vertical algorithm, 17 (31 percent) used building up or breaking down numbers as a method, one PST (2 percent) broke up both numbers into units and tens then broke each number up into a further nine groups of ten and counted on from ten, and four (7.5 percent) counted on with fingers. This left only four (7.5 percent) who used the efficient rounding and adjusting method. This resonates with (Venkat, 2014), where she critiques the over-reliance on traditional algorithms and advocates for more effective approaches.

Counting on with or without fingers was a method frequently used. While 51 of the 54 third-year PSTs (94 percent) calculated the first two-digit plus a single digit addition Grade 1 level question (36 + 8) correctly in the pre-questionnaire, nine (17 percent) wrote that they counted on their fingers and four (7 percent) counted on. Two of the four that counted on said that they counted on with fingers. This points to very basic levels of calculation strategies that learners are expected to move on from post-Grade 1. South Africa's Curriculum and Assessment Policy document (CAPS) for Grades 1 to 3 states "Use calculation strategies to add and subtract efficiently: Put the larger number first in order to count on or count back" (DBE, 2011a, p. 60) in preparation for the Intermediate Phase where fluent flexible calculation strategies are expected and the focus shifts to written methods of building up, breaking down and the vertical algorithm for addition and subtraction calculations (DBE, 2011b). Graven, (2002) emphasises the importance of teaching mathematics in ways that promote mental strategies. Fluency and conceptual understanding.

Questions B, C and D (the larger two-digit calculations) did not lend themselves to counting on (with or without fingers); nevertheless four, three and four PSTs respectively used this method for B, C and D. For B, C and D the vertical algorithm and building up and breaking down remained the preferred choice of method. Table 4.2 indicates that the strong performance of the majority of PSTs across the items is predominantly linked to the correct use of inefficient methods of calculation.

Applying the building up and breaking down method relates to its emphasis in the curriculum where it “is one of the most important techniques in the Foundation Phase. Using this technique allows learners to split and recombine numbers to help make calculations easier” (SA. DBE, 2011, p. 133). The Foundation Phase CAPS document recommends the introduction of the building up and breaking down method in Grade 1. In Grade 2 the focus is on place value, where learners are expected to break up numbers into tens, ones, multiples of tens and number pairs. The examples given in the CAPS document suggest decomposing in multiples of ten and counting backwards to make the calculations easier. In Grade 3 the focus of the building up and breaking down method shifts to subtraction of two three-digit numbers with no example of regrouping. The curriculum only suggests “Breaking down a number into smaller parts to make a calculation easier. Most of the strategies that learners use involve breaking down numbers. They continue to do this with three-digit numbers” (SA. DBE, 2011, p. 389).

While building up and breaking down is a method promoted in the curriculum, it is not particularly efficient for these calculations and is highly error prone for subtraction problems in which the units digit of the subtrahend (smaller number) is greater than the units digit of the minuend (as in the case of Question C 63 – 24). Possibly a less error prone, more efficient way to solve subtraction computations would be the JS with the BTT strategy (63 – 20 = 43; 43 – 4 = 43 – 3 – 1 = 40 – 1 = 39), which could be done on the number line in order to make sense of and better understand the calculation.

While the Foundation Phase curriculum promotes building up and breaking down as an efficient method, the Grade 3 CAPS document only suggests the breaking up method with either breaking up both numbers or only breaking up the subtrahend. This method can work well if each of the *respective* digits in the subtrahend are less than each of the respective digits in the minuend as in the example given in the CAPS (DBE, 2011, p. 390) below.

Table 4.3: An example in CAPS of breaking up numbers (DBE, 2011, p. 390)

Breaking up both numbers 389 − 137 = 389 – 137 = (300 +80 +9)−(100 +30+7) = (300−100)+(80−30)+(9−7) = 200 + 50 + 2 = 252	Breaking up one number 389 – 137 = 389 – (100 + 30 + 7) 389 − 100 → 289 − 30 → 259 − 7 = 252

Westaway (2021) notes that none of these examples (or others given) involve regrouping. In other words, there are no examples in the CAPS Grade 3 document that require learners to subtract with regrouping, for example 143 − 87 = ⊡.

The absence of caution in using the method when this is not the case is cause for concern and likely a contributing factor to the relatively poor results (one in five PSTs got the answer wrong) for the subtraction item (63 – 24). Indeed, this method is highly error prone for subtraction calculations such as item C, 63 – 24. Next, I review the error analysis.

4.2.3. Error analysis on pre-questionnaire

Table 4.4 below shows that 3; 5; 11 and 6 PSTs got items A-D incorrect respectively. Here I share some of the most common errors made on each of these items.

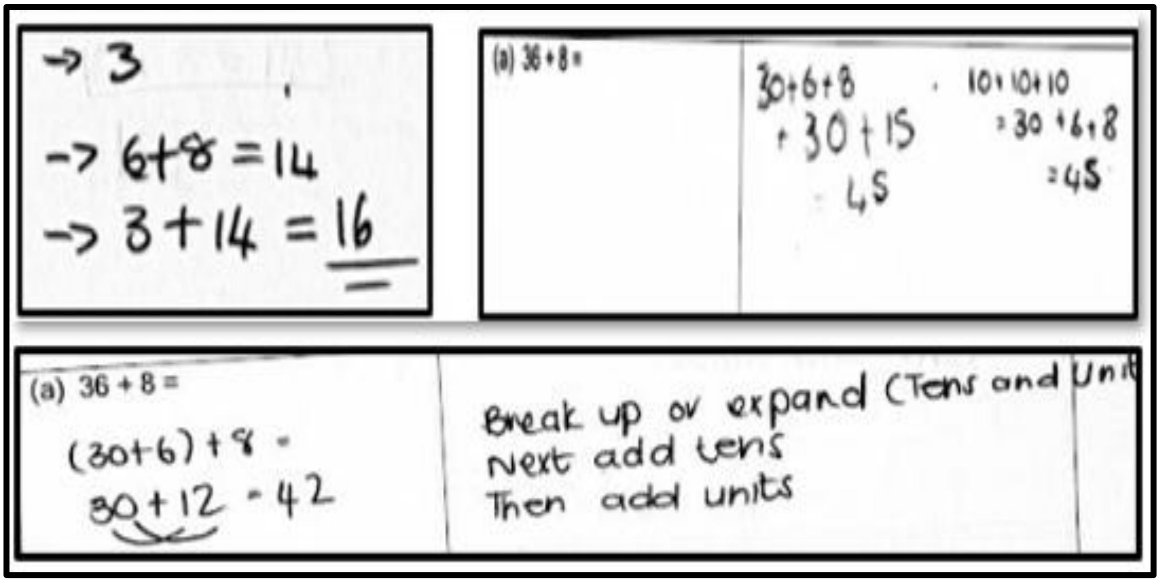
Table 4.4: Error analysis of pre-questionnaire (n=54)

ERROR ANALYSIS PRE-QUESTIONNAIRE	A 36 + 8	B 47+ 29	C 63 – 24	D 98 + 99
54 PSTs	PRE	PRE	PRE	PRE
Adding error	2	3	0	3
Place value error	1	1	0	0
Reversed numbers	0	0	7	0
Copy error	0	0	2	0
Minus 7 (3+4)	0	0	2	0
More than one error	0	1	0	3

Error Analysis Question A (36 + 8)

The building up and breaking down method was the strategy used for all three of the calculations with errors for Question A (36 + 8) in the pre-questionnaire. The errors appeared as adding errors, one with no concept of place value. Examples of these errors can be seen in Figure 4.3. In the first example (PST VJ), I can see that three is added in place of 30, while in the second example, PST KG broke up the number 36 (30 + 6) and then broke up the 30 further into three tens. The calculation was now 10+10+10 = 30+6+8 which while equivalent did not assist in the calculation and the PST then made a final calculation error answering incorrectly = 45. In the third error, PST LM explains her method used where she breaks up the number 36 into tens and units then makes an adding error while adding the units (i.e., 6+8 = 12) therefore she wrote: 30 + 12 = 42. Linking the 30 and 12 imply the learner is adding the tens, digits and the units' digits in the final step.

Figure 4.3 Adding errors using the building up and breaking down method.

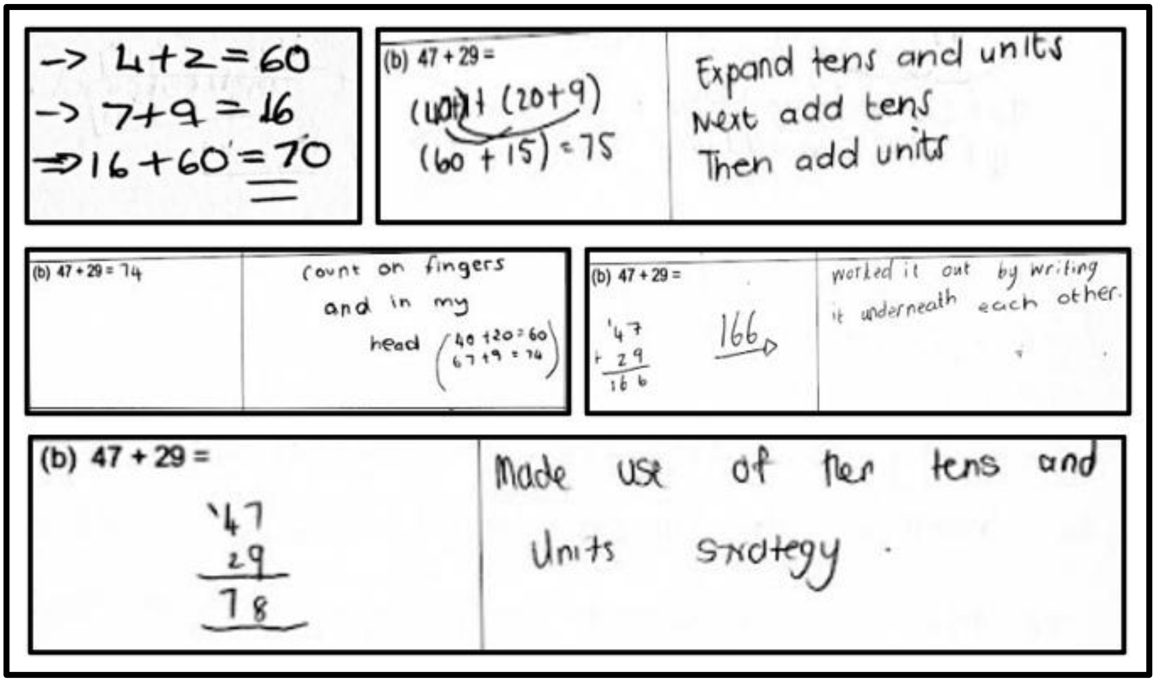


PSTs: VJ, KG and LM

Error Analysis Question B (47 + 29)

Five PSTs made errors (see Figure 4.4 below) in Question B, the more challenging two-digit addition question, which highlighted weak number sense. This depicts a lack of understanding of place value and very basic methods of manipulating numbers for calculations that could have been as simple as rounding and adjusting or bridging through ten. The first two examples (PST VJ & PST LM) were both adding errors while building up and breaking down the numbers. The third example shows PST AL using her fingers and making an adding error, namely 47 + 29 = 74. The vertical algorithm was used by PST KA to calculate the sum in the fourth example. The 29 was placed under 47 then the units nine and seven were added together to get 16 and the one was carried over. The tens four and two were then added together to get six tens and the one was carried down to get the answer 166. In example five, PST MP illustrates an adding error using the vertical algorithm where the units seven plus nine were added incorrectly to make 18 and one was carried over to the ten's column, the ten's column was added correctly, and the answer given was 78.

Figure 4.4 The five errors in Question B (47 + 29).

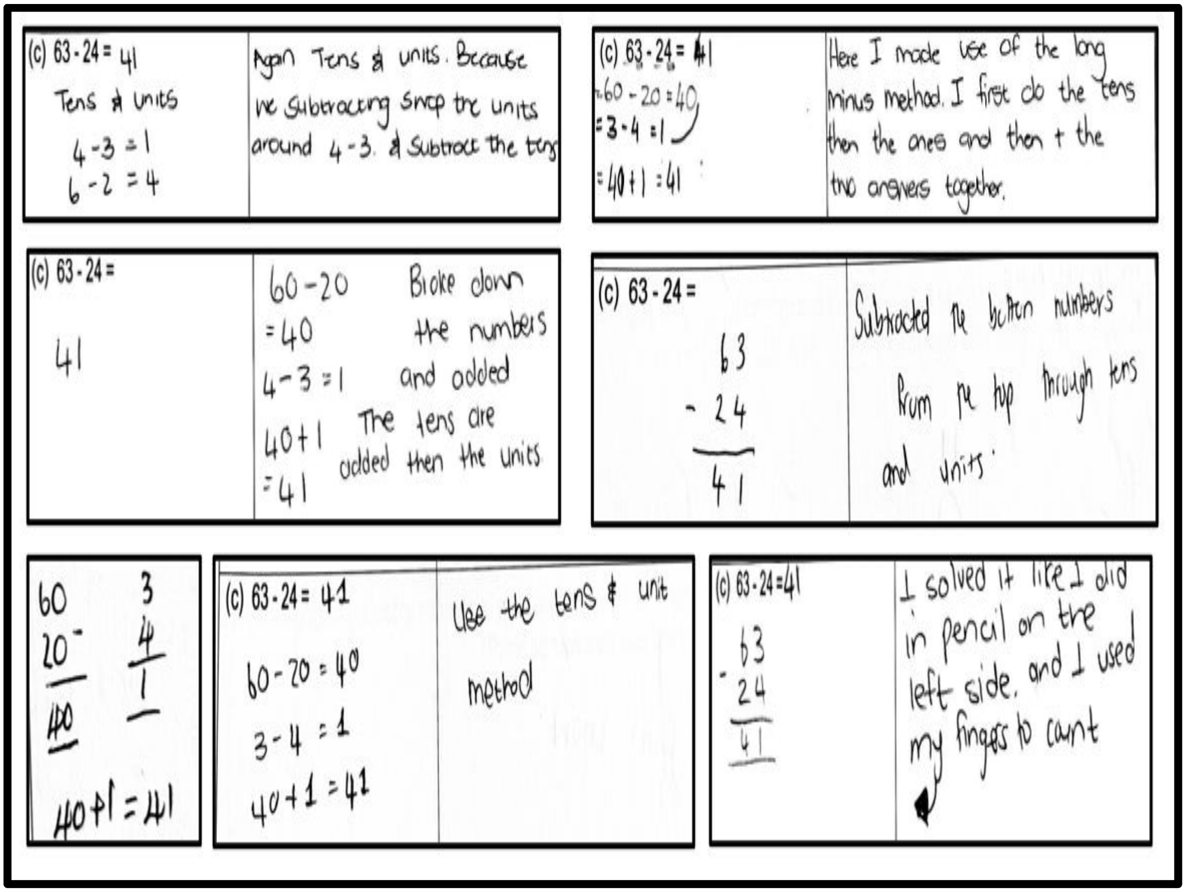


PSTs: VJ, LM, AL, KA, and MP

Error Analysis Question C (63 – 24)

Question C was the item where PSTs made the most errors. Analysis of the 11 incorrect answers (see Figure 4.5) revealed that the most prevalent error involved reversal of the units (in both the breaking down method and with the vertical algorithm). Seven PSTs (PST LG, PST JB, PST AF, PST MP, PST AB, PST SM & PST TL) reversed the units (shifting the problem from 63 – 24 to 64 – 23 to avoid the subtraction of a larger number from a smaller number, thus giving the answer as 41). Figure 4.5 below provides all seven instances of the reversal errors, with PST LG, in the first instance in the figure stating this reversal as her method ("Again Tens & units. Because we are subtracting, swop the units around four minus three and subtract the tens"). In the second example, PST JB believes the reversal error is an acceptable method and states use of the long minus method as the method used. The reversal of numbers is a common error that arises in subtraction where learners reverse the numbers to avoid subtracting the larger number from the smaller number. This can be an indication that the learners do not comprehend place value or have a good understanding of base ten and the number line. This lack of place value knowledge impacts later learning. This has been identified at different levels of study, including high school students in their early stage of formal algebra learning (MacGregor & Stacey, 1993).

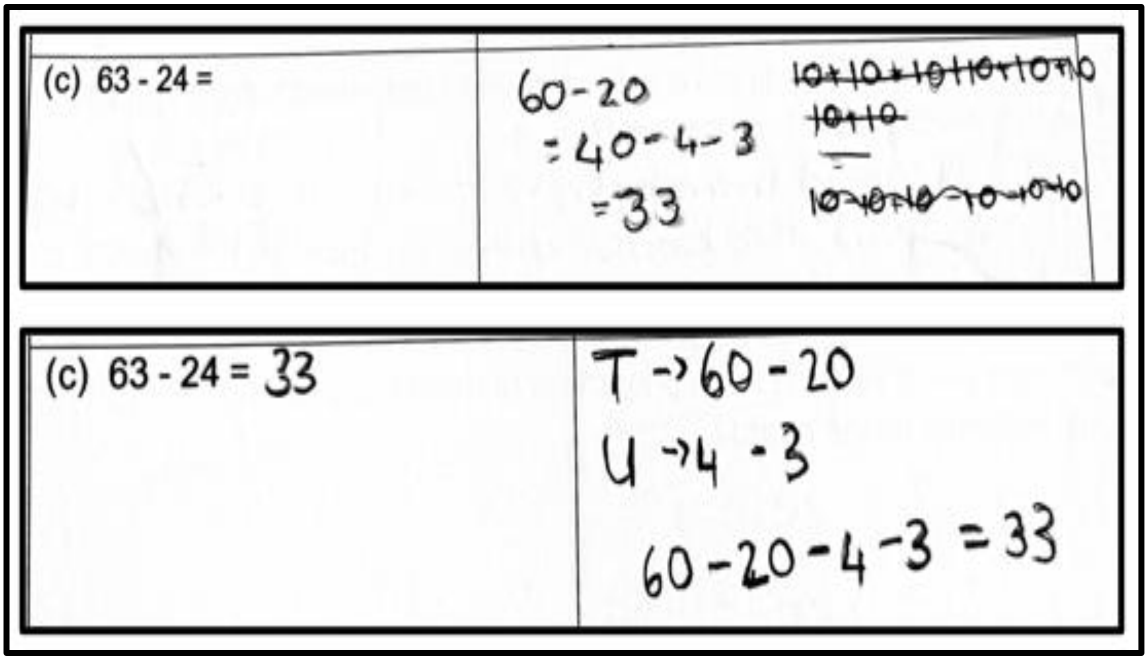
Figure 4.5 The seven reversal errors in Question C (63 – 24).



PSTs: LG, JB, AF, MP, AB, SM and TL

An additional two PSTs (KG & IS) made errors on item C. Both PSTs subtracted the tens, then subtracted the units of both the subtrahend and minuend (60 – 20 = 40 – 4 – 3 = 33); however, PST CE broke down the 60 and 40 into tens in order to subtract the 40 from the sixty. See Figure 4.6 below.

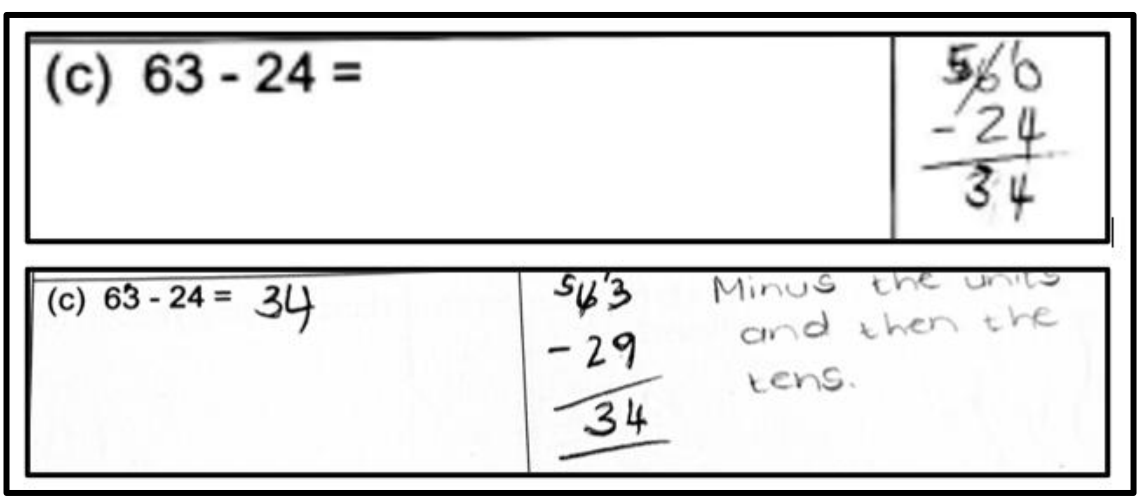
Figure 4.6 Two examples of errors using the breaking down method while subtracting the units of both numbers in Question C (63 – 24).



PST KG and PST IS

The last two of the 11 PSTs (Figure 4.7) who made an error on this item seemingly changed the problem. PST CE ignored the three in 63 and made a single digit subtraction error when working with the units (10 – 4 = 4) while PST CD made a copy error and instead of subtracting 24 subtracted 29.

Figure 4.7 Transferring the question to the algorithm in item C (63 – 24).

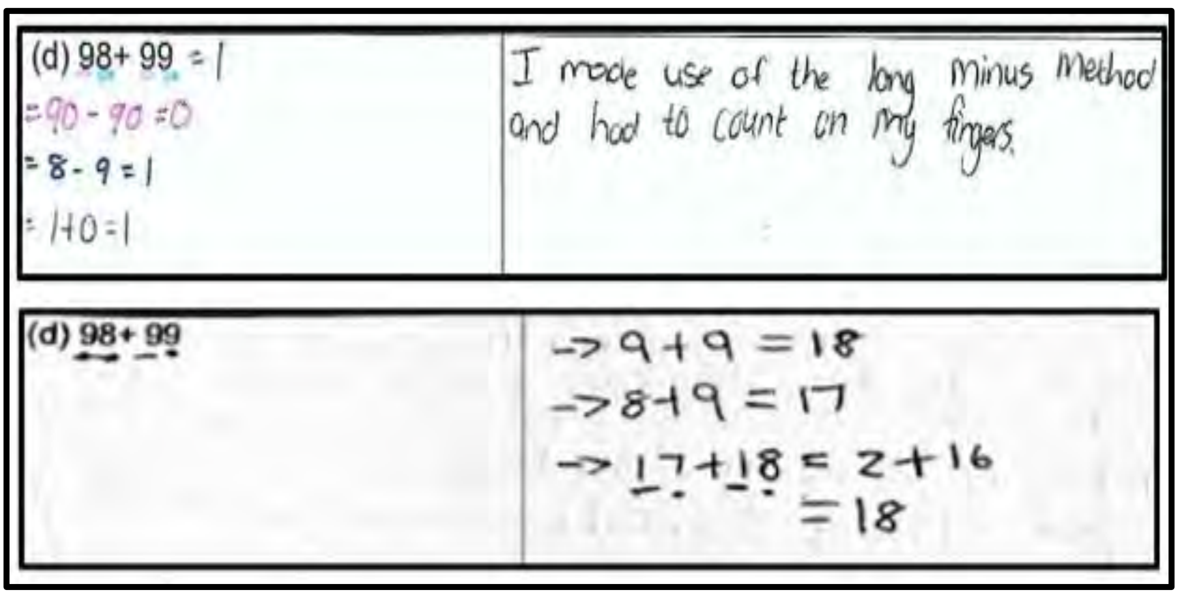


PST CE and PST CD

Error Analysis Question D (98 + 99)

As is evident in Table 4.2 above, the most prevalent method for item D (98 + 99, which lends itself well to rounding and adjusting) was the vertical algorithm method (28/54) followed by breaking down (18/54) – only four PSTs used the method of rounding and adjusting. The six incorrect answers for Question D (98 + 99) involved errors while using the former two methods. The four PSTs who used the rounding and adjusting method all got the item correct. The two entries in Figure 4.8 below show how two PSTs (JB & VJ) break down the numbers into units and tens; the first example, PST JB then makes two errors – she subtracts instead of adding (90 – 90 = 0) and answers eight minus nine as one, ignoring the fact that it is minus one. The second, PST VJ broke down the tens and units then failed to work with the tens as 90 and 90, instead simply calculated 9 + 9. The error was then repeated while calculating 18 + 17 answering 2 + 16 = 18. This indicates a lack of place value understanding.

Figure 4.8 Two examples of errors with the breaking down method, item D.

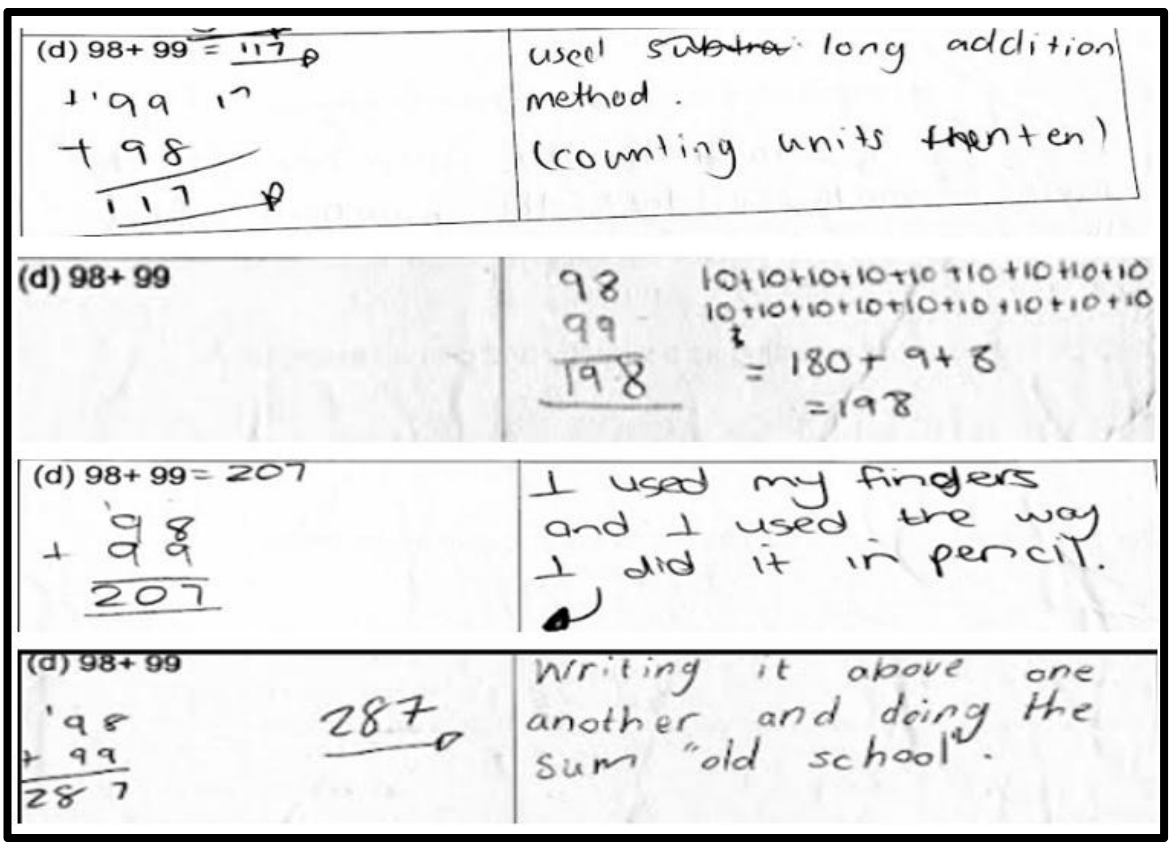


PST JB and PST VJ

Figure 4.9 below shows the four errors made while using the vertical algorithm for item D (98 + 99). Again, several errors are visible in the single digit calculations done as part of using the vertical algorithm. In the first example of Figure 4.9, PST AJ correctly adds the units to get 17, writes down 17 then ‘carries’ the ten over and writes one but does not know how to do the next stage of the algorithm. The next example shows how PST KG breaks down both numbers into tens then counts in tens to 180, thereafter nine and eight are added and the answer is 198.

The next two examples show similar errors in working with the addition of the tens and the carried over ten. It is unclear whether these are simply single-digit addition errors when totalling the tens. In the third example, PST TL used the vertical algorithm and counted on with fingers to get an answer of 207. Example 4 shows how PST KA added the units, wrote down seven and carried one over then added the tens together (9 + 9 = 18) and wrote down eight. She then added the one to the one that was carried over earlier to get the answer 287. Across the examples it seems PSTs know some aspect of the algorithm but do not know all steps and do not understand how the vertical algorithm works.

Figure 4.9 Four error examples using the vertical algorithm method, item D.



PSTs: AJ, KG, TL and KA

4.3. ANALYSIS OF PRE-PCK QUESTIONNAIRE

The questions (5 – 10 given in Table 3.3) that followed the four calculation items focused on the gathering of PSTs' PCK of three mental mathematics strategies (bridging through ten, jump strategy, and rounding and adjusting) before exposure to the MSAP. In these questions, PSTs were asked to describe BTT, JS and RA, and explain how they would teach it. This section focuses on PSTs' responses.

Analysis of 54 PST responses to these questions indicated that only two PSTs could correctly describe the strategy of BTT while no PST was able to correctly describe JS or R&A. Table 4.5 shows the frequency of four different types of PST responses to the request to describe the three strategies BTT, JS and R&A. The four types of responses were: they stated they did not know, they gave an incorrect explanation, they did not answer (left it blank), or they answered correctly (i.e., provided a basic description of the strategy).

Table 4.5: Frequency of PSTs able to describe the three strategies (n=54)

PART 2 PCK	Describe BTT	Describe JS	Describe R&A.
54 PSTs	PRE	PRE	PRE
Said “Don't know”	15	9	5
Answered incorrectly	24	25	31
Left it blank	13	20	18
Answered correctly	2	0	0

In the first question (describe the BTT strategy), 15 out of the 54 PSTs (28 percent) indicated that they did not know, an additional 24 (44 percent) attempted to answer but got it wrong. Thirteen (24 percent) left it blank and only two (4 percent) PSTs were able to describe the BTT strategy but did not know how to teach the strategy). No PST could explain the JS or R&A strategy correctly (see Table 4.5 above).

After completing the questionnaire many students commented that they had never heard about or been taught these strategies at school. Examples of PSTs' incorrect descriptions are given for each strategy below:

Incorrect descriptions of BTT

Here are four examples of the types of answers given by the 24 PSTs who attempted to describe BTT but did so incorrectly:

PST VJ "Counting the tens and units together…"

PST RB "Place value on tens & units, if no has 2 nrs its ten
If no has 1 it's a unit (counting in tens)"

PST LM "I am not 100's sure. Taking a guess…" The PST then explains the persistent breaking down, building up method.

PST LG "Not entirely sure what this means. I have my thoughts, but I can't recall hearing about this…"

This inability to describe the strategies or how to teach them clearly points to the need for further intervention.

Incorrect description of JS

Here are three examples of the types of answers given by the 24 PSTs who attempted to describe JS but did so incorrectly:

PST CE "the learners need to borrow by a number or add to a number"

PST BA "The number is increasing so for example the number to, you jump to more, you get to go. You can teach it by playing snakes and ladders or on a counting table …"

PST JV "The Jump strategy is where you add subtract numbers"

Incorrect description of R&A

Here are three examples of the type of answer given by PSTs who attempted to describe R&A but did so incorrectly:

PST AJ "Rounding is to help the number look more whole so that the number after The comma does not confuse the learners"

PST AF "To get the number closest to its correct value 54879 would become 55"
PST LG "…If the number is 53 we would either round up 55 or round down to 50"

As PSTs were unable to describe these strategies appropriately, they were not able to explain how they would teach a strategy they did not know. Thus, the absence of CK of the strategies of BTT, JS and R&A meant that very little data could be gathered on their PCK in relation to these strategies. Content Knowledge (CK) is a pre-requisite for knowing how to teach that knowledge.

For the two PSTs who were able to describe the BTT strategy, they described the strategy quite well, paying attention to the way in which getting to the multiple of ten makes it easier to add. PST LG wrote:

PST LG "Make numbers nearer to the closest 10s and then add so that it is easier to say 40 + something. E.g. 37 + 6 (take 3 from 6 = 3 to make 37+ 3 = 40 and then add remaining 3."

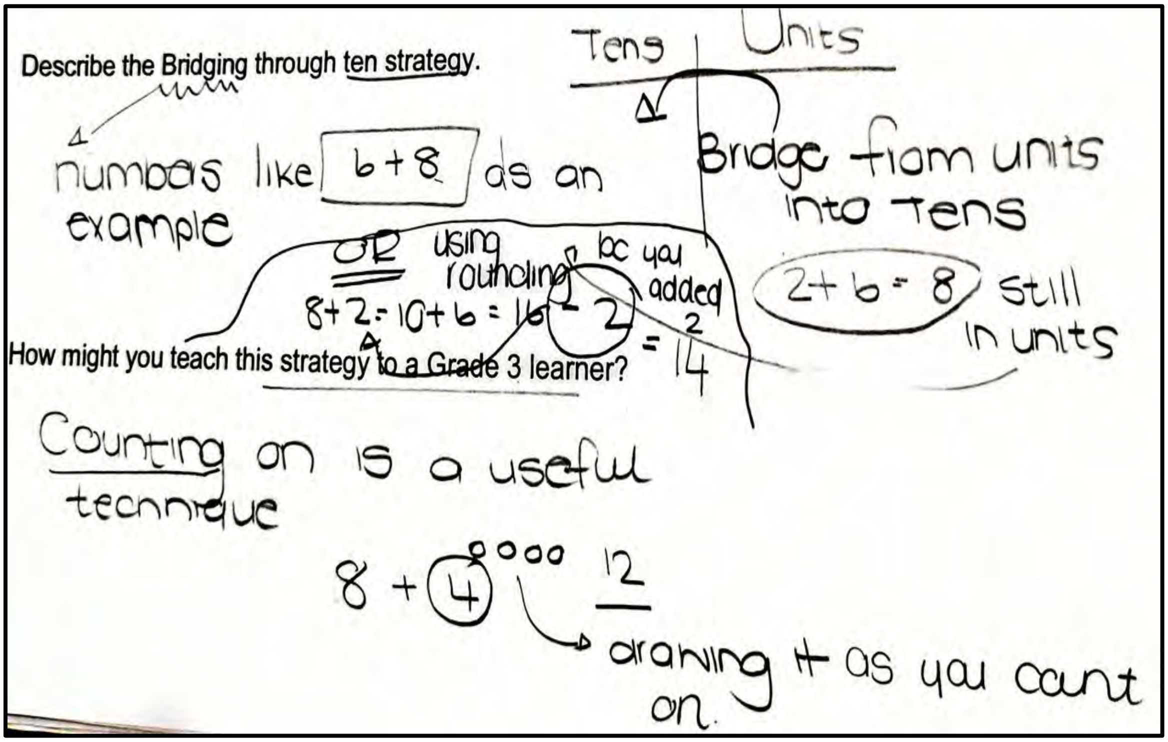
In answering "How might you teach this strategy to a Grade 3 learner?" PST LG provided a response that indicated some PCK in relation to tools and manipulatives that might support the development of the concept with learners:

PST LG "By using blocks or using their number charts. Using blocks allows you to break up whole numbers into pieces and then add."

The use of unifix blocks could be a useful way to show bridging through ten when adding 37 + 6 as in the PST's example. Of interest however is that the empty number line is not mentioned nor used in PST LG's description. The introduction of the use of the empty number line to PSTs for assisting learners using the BTT mentally could thus broaden this PST's repertoire of representations to support teaching this strategy.

PST CD provided a generally good description of BTT but made a calculation error in her example, then explains unit counting as a method to teach BTT. See Figure 4.10 below.

Figure 4.10 Description of BTT strategy.



PST CD

As noted above, PST CD was able to describe the BTT strategy with an appropriate example where she explains bridging to the nearest ten, i.e., she adds two to eight and gets ten; however, she makes a calculation error and adds six to ten to get 16 as the answer (e.g. 6 + 8 = 8 + 2 = 10 + 6). When asked how you would teach the strategy to Grade 3 learners, she states that she would use the counting on technique and draw it (the addend) as you count on. This would imply that PCK related to the way in which one might teach this strategy is limited. as stated by many researchers. Spaull (2013), Hoadley (2012), Fleisch (2008), Taylor and Taylor (2013) and Bowie (2019) have all indicated that educators have limited CK and PCK needed to teach mathematics.

PART TWO: SHIFT FROM PRE- TO POST-QUESTIONNAIRE

In Part 2 of this chapter, I analyse and share findings of PSTs' responses to the CK post-questionnaire and the PCK questionnaire.

4.4. SHIFTS IN CK FROM PRE- TO POST-QUESTIONNAIRE

In the CK post-questionnaire PSTs were again asked to solve the first four calculation items (36 + 8; 47 + 29; 63 – 24 and 98 + 99) and provide a statement of the method used.

I have organised the analysis of PST data from the post-intervention questionnaire as follows:

4.4.1 Shifts in frequencies correct/incorrect from pre- to post-questionnaire.
4.4.2 Shifts in methods from the pre-to post-questionnaire.
4.4.3 An error analysis and comparison of the four calculations from pre- to post-questionnaire.
4.4.4 Analysis of the shift across the four calculations of PSTs who achieved 50 percent or less in the pre-questionnaire.

In this section I looked at PSTs' results on the four addition and subtraction calculations and their responses on the post-CK questionnaire.

4.4.1 Shifts in frequencies correct/incorrect from pre- to post-questionnaire

This next section addresses the shift in frequencies correct or incorrect. Table 4.6 compares the frequency from pre- to post-questionnaire of PSTs who solved the four addition and subtraction problems A to D correctly and incorrectly. The intention was to do BTT, JS, and R&A but R&A was not a focus or addressed in this study due to the limited time. The intention was for the PSTs to have the option to use R&A but in fact they could have used jump strategy or BTT. Items B, C, and D all include the option of BTT and the option of the JS. I felt it was more appropriate to spend more time on the BTT strategy as it is included in all of the items. Table 4.6 below shows the pre- and post-results for the 51 students who completed both pre- and post-CK questionnaires.

Table 4.6: Frequencies correct/incorrect for pre- and post-questionnaire (n=51)

	A 36 + 8		B 47 + 29		C 63 – 24		D 98 + 99	
51 PSTs	PRE	POST	PRE	POST	PRE	POST	PRE	POST
Correct	48	51	46	48	42	43	48	49
Incorrect	3	0	5	3	9	8	3	2

The above table shows small shifts in frequencies of the 51 PSTs who got the answers correct from pre- to post-questionnaire, which is due to most PSTs getting correct answers in the pre-questionnaire. Below I discuss results for each question separately.

Question A (36 + 8): All 51 PSTs (including the three who got it incorrect in the pre-questionnaire) got it correct in the post-questionnaire. This is not the case for the other questions.

Question B (47 + 29): The table reveals five errors in the pre-questionnaire and only three errors in the post-questionnaire. Four PSTs who incorrectly calculated Question B in the pre-questionnaire successfully calculated it in the post-questionnaire, one made the same error in the pre- and post-questionnaire, and two PSTs answered correctly in the pre-questionnaire and incorrectly in the post-questionnaire. One PST used the same method for the pre- and post-questionnaire but made an addition error as underlined in their working that showed: 47+29; 40+20 = 60; 7+9 = 15; 60+15 = 75 in the post-questionnaire. The error of another student in her pre-test was that she answered the question correctly by using a partial building up and breaking down method; however, when she applied a combination of mental strategies she seemingly got confused. She attempted bridging through ten, writing: 47+29; 47+3 = 50 then added on six, omitting to add on 20, and answered 56.

Question C (63 -24): The subtraction problem was, again, most poorly answered in the post-questionnaire. The pre-questionnaire reveals nine errors while the post-questionnaire displays eight errors showing a shift by one error. However, four of the nine PSTs who answered incorrectly in the pre-questionnaire then answered correctly in the post-questionnaire, and three additional errors were made in the post-questionnaire (5+3=8 errors). An analysis of the eight errors in the post-questionnaire revealed that two PSTs who made errors in the pre-questionnaire, made the same reversal error (reverse the units to make the calculation easier). For example, 63 – 24 = 64 – 23 in the post-questionnaire. Three made different errors in both the pre- and post-questionnaire and three errors were made by PSTs who had answered correctly in the pre-questionnaire and made mistakes in the post-questionnaire. The continued poor performance by approximately one fifth of the PSTs on the post-questionnaire is cause for concern as being able to accurately calculate a two-digit subtraction will be core to what they must teach.

Question D (98 + 99): PST data for this item indicates three and two errors in the pre- and post-questionnaire respectively. Once more, the three PSTs who made errors in the pre-questionnaire improved and answered Question D correctly in the post-questionnaire and two errors were made by PSTs who answered correctly in the pre-questionnaire. Many calculation errors were made when the PSTs' focus shifted as they attempted a new strategy.

4.4.2 Shifts in methods from the pre- to post-questionnaire

Table 4.7 below compares the methods PSTs used for the four calculations in the pre- and post-questionnaire. I can see that the frequency of the dominant vertical algorithm method across the four calculations has reduced in the post-questionnaire. The building up and breaking down has, however, remained prevalent, with some increased use for three of the four calculations. This means the intended shift toward using efficient strategies of BTT and JS was not achieved by the intender.

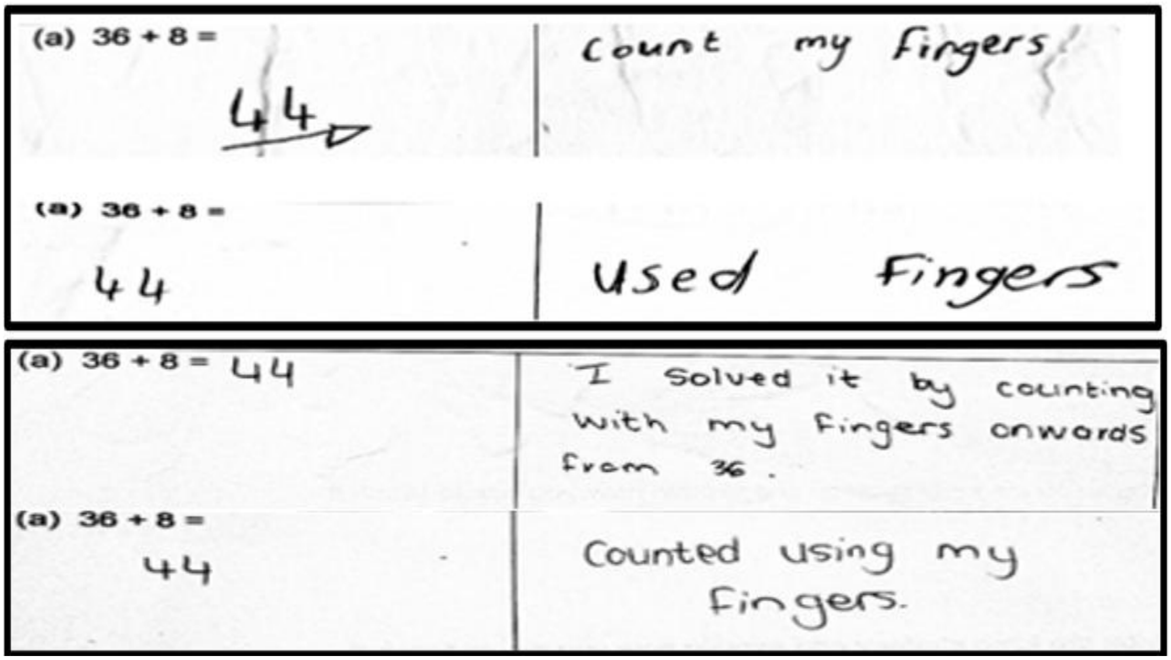
Table 4.7: A comparison of pre- and post-questionnaire methods (n=51)

	A 36 + 8		B 47 + 29		C 63 – 24		D 98 + 99	
51 PSTs	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Counted on with fingers	8	2	2	0	0	0	2	0
Counted on without fingers	4	2	0	0	0	0	0	0
Build up break down	12	15	25	22	23	25	18	22
Build up down with fingers	1	0	0	0	2	0	0	0
Vertical algorithm (VA)	22	7	22	12	25	13	27	11
VA with fingers	1	0	0	0	0	0	0	0
VA first then number line	0	3	0	0	0	0	0	0
BTT with number line	3	15	0	12	0	10	0	9
BTT without number line	0	5	0	0	0	0	0	0
Rounding & adjusting	0	0	2	5	0	0	4	9
Counted dots mentally	0	1	0	0	0	0	0	0
No explanation, just ans.	0	1	0	0	1	3	0	0

Unit counting (counting with fingers and counting on) reduced in frequency considerably, which was a pleasing shift (only two counted with fingers and two counted on). Across the four calculations only two of the 14 PSTs who used fingers in the pre-questionnaire, continued to use their fingers in the post-questionnaire. Figure 4.11 below shows the two examples of the correct answer for Question A in the post-questionnaire with the statement about continuous use of fingers to calculate.

This was matched by an increase from only three instances (in the pre-questionnaire) shifting to 51 PSTs using BTT across the four calculations in the post-questionnaire. All fifty-one PSTs (100 percent) answered Question A correctly in the post-questionnaire, 20 used BTT (five without the number line). In Questions B, C and D, 12,10 and nine PSTs respectively used the BTT strategy.

Figure 4.11 Continuous use of fingers in the post-questionnaire of two PSTs.



PST KA and PST TJ.

As noted above and seen in Table 4.7 above, the use of the BTT method increased from three to 20 in Question A, zero to 12 for Question B (47+ 29), zero to 10 for Question C (63 – 24), zero to nine for Question D, and an additional nine used rounding and adjusting for Question D (98 + 99). Of interest was that there was less use of the vertical algorithm method, 96 instances in the pre-questionnaire across items to only 43 instances in the post-questionnaire. Table 4.7 depicts a substantive shift from twenty-two to seven of the PSTs who used the vertical algorithm in Question A, 22 to 12 in Question B, 25 to 13 in Question C and 27 to 11 in Question C. However, the building up and breaking down method remained the preferred method of choice, with the majority of PSTs choosing this method post-intervention. Only Question B decreased by three from 25 to 22 PSTs using the building up and breaking down method to calculate 47 + 29.

4.4.3 Error Analysis & Comparison – pre- to post-questionnaire

Table 4.8 provides a summary of errors across items from the pre- to post-questionnaire (n=51). The table displays minimal improvement across the four items from pre- to post-questionnaire. Below I analyse and discuss each of the four items individually.

Table 4.8: Errors across four calculation items from pre- to post-questionnaire

Four calculations	A 36 + 8		B 47 + 29		C 63 – 24		D 98 + 99	
51 PSTs	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Adding error	2	0	3	2	0	0	2	2
Place value error	1	0	1	1	0	0	1	0
Reversed numbers	0	0	0	0	5	7	0	0
Copy error	0	0	0	0	2	0	0	0
Minus 7 (3+4)	0	0	0	0	2	0	0	0
More than one error	0	0	1	0	0	0	0	0
Added on number line	0	0	0	0	0	1	0	0
Total errors	3	0	5	3	9	8	3	2

Error Analysis Question A (36 + 8)

Three errors were made in the pre-questionnaire and no errors were made in the post-questionnaire for Question A (36 + 8).

Error Analysis Question B (47 + 29)

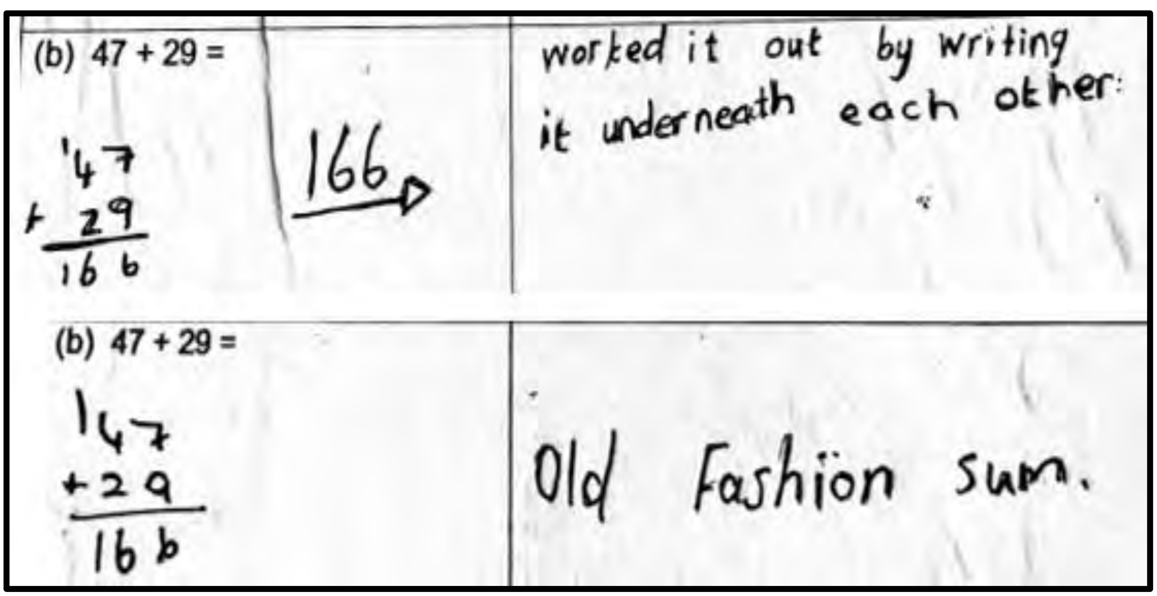
Forty-six PSTs (90 percent) calculated Question B, the more challenging, two-digit addition question correctly in the pre-questionnaire (five made errors) and 48 (94 percent) calculated the sum correctly in the post-questionnaire (three made errors). The shift was greater than the figures reveal, and four of the five PSTs improved. Only PST KA made the same error in the pre- and post-questionnaire; the other four PSTs who made errors in the pre-questionnaire answered the question correctly in the post-questionnaire and two additional PSTs, TM and LO, who answered correctly in the pre-questionnaire, made errors in the post-questionnaire (see Table 4.9 below).

Table 4.9: Error analyses from pre- to post-questionnaire for Question B (n=51)

Pre-test	KA	VJ	LM	AM	MP		
Post-test	KA					TM	LO

As mentioned earlier, PST KA made the identical error in the pre- and post-questionnaire. This is an indication that more intervention is required to improve PST KA's number sense. The addition of two, two-digit numbers highlighted weak number sense and a lack of understanding of place value as the same type of error was made in both the pre- and post-questionnaire, as can be seen below in Figure 4.12, where the first response is from the pre-questionnaire while the one under it is from the post-questionnaire.

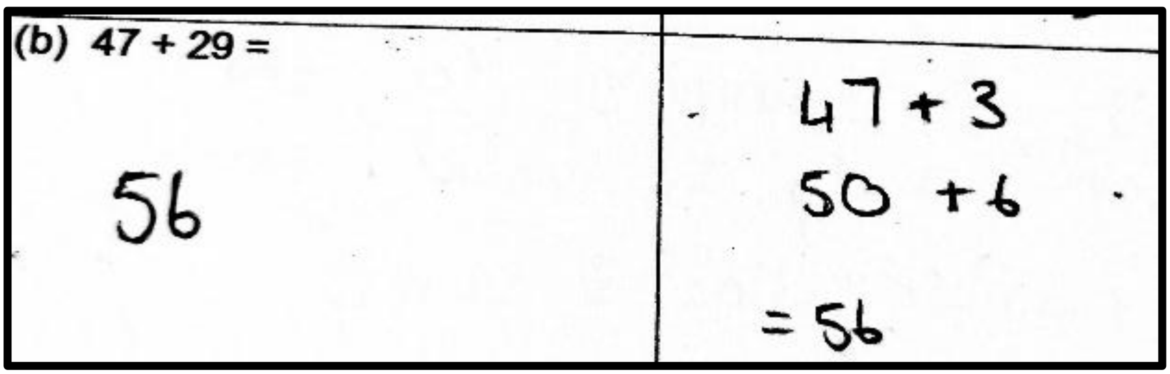
Figure 4.12 Pre- and Post-questionnaire answer (B) of one PST.



PST KA

In the next example (Figure 4.13 below), I can see that PST TM used the building up and breaking down method to answer Question B correctly in the pre-questionnaire. The calculation was (40 + 20 = 60, 7 + 9 = 16, 60 + 16 = 76). However, in the post-questionnaire an attempt was made to use a mental strategy. PST TM wrote 47 and added three to bridge to the nearest ten, then added the remaining six units to get 56 and forgot to add the 20 (see Figure 4.14). While the calculation was not complete, an attempt was made to use a more efficient strategy; however, this incomplete calculation indicates that more time is needed to practise calculations using the mental strategies.

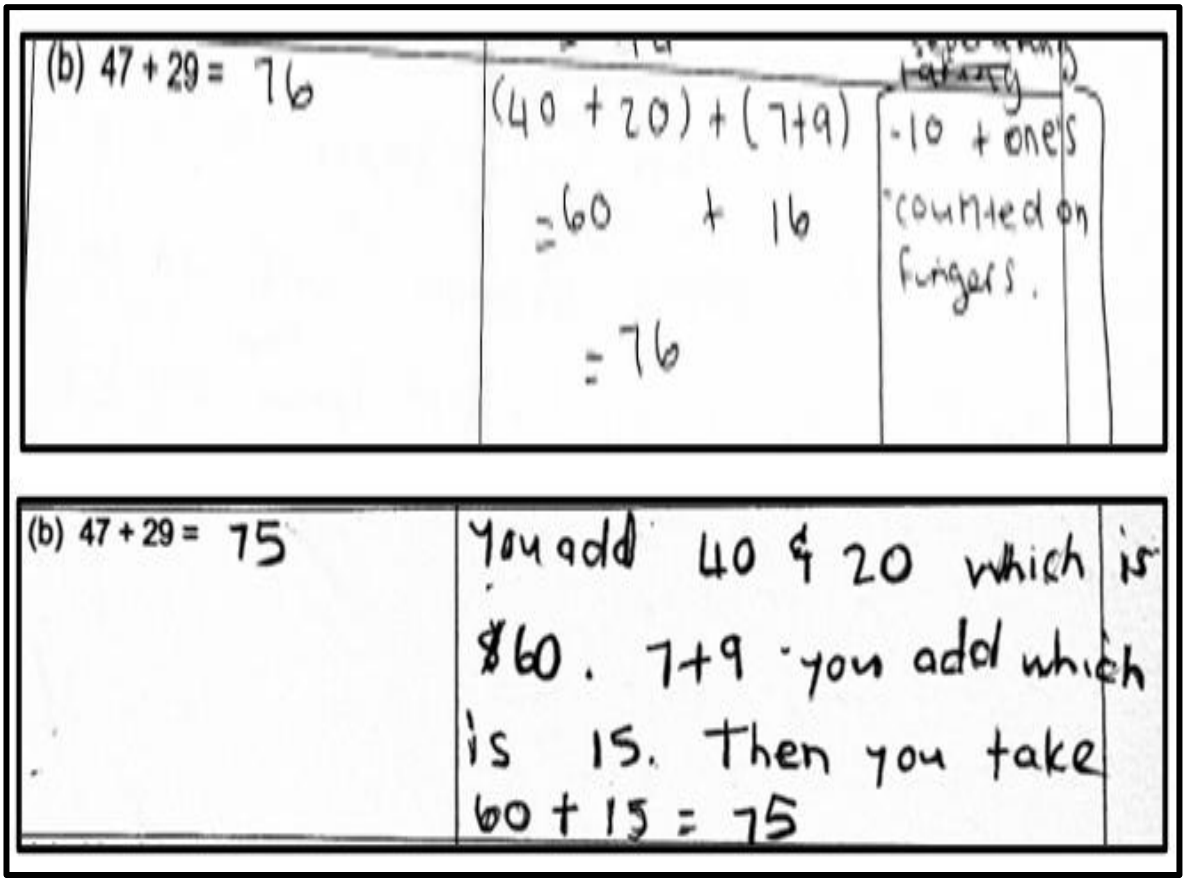
Figure 4.13 A failed attempt at BTT in the post-questionnaire.



PST TM

The other additional error in the post-questionnaire for Question B was made by PST LO who used a combination method, building up and breaking down with fingers in the pre-questionnaire and calculated the correct answer. In the post-questionnaire, my instruction was to calculate without using fingers. She used the same building up and breaking down method in the post-questionnaire, without fingers (40 + 20 = 60, 7+9 = 15, 60 +15 = 75) and added the units incorrectly. Figure 4.15 below shows PST LO's pre- and post-test calculations.

Figure 4.14 A comparison from pre- to post-questionnaire calculations.

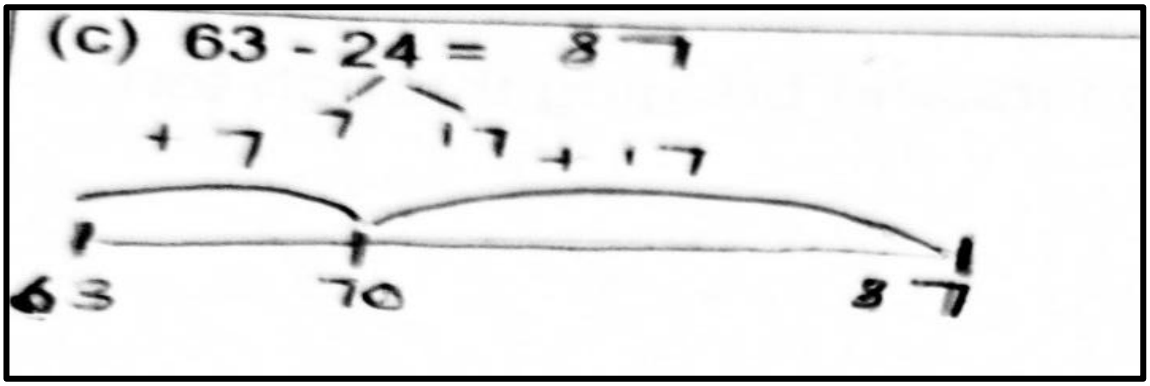


PST LO

Error Analysis Question C (63 -24)

Question C remained challenging, with nine errors in the pre-questionnaire and eight of the fifty-one PSTs (16 percent) calculating the subtraction calculation (63 – 24) incorrectly in the post-questionnaire. The eight incorrect answers revealed that seven PSTs again reversed the units (64 – 23 = 41) to do the calculation and one PST attempted the BTT method but changed the calculation to addition on the number line then took seven away from 24 and added it to 63, and added the remaining 17 to 70 to get 87. See Figure 4.15 below.

Figure 4.15 Changing subtraction to addition on the number line.



PST CE

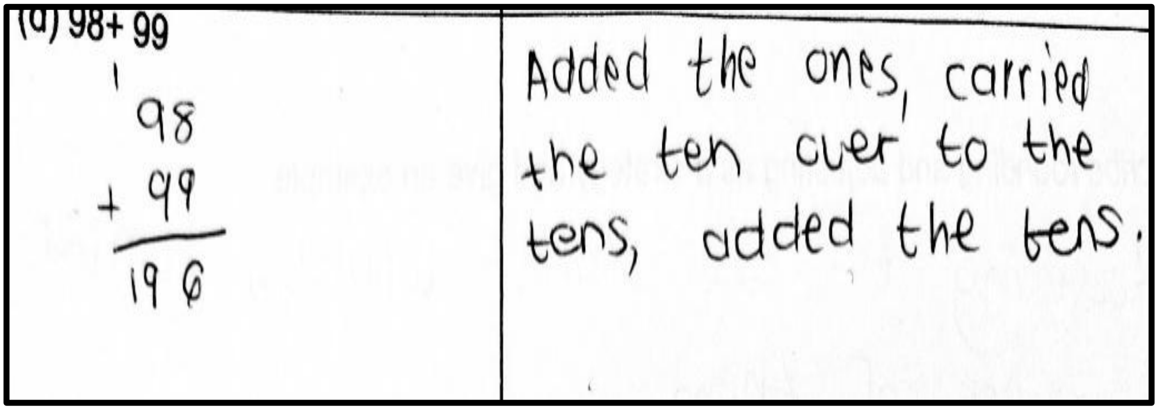
There were nine errors in the pre-questionnaire and eight errors in the post-questionnaire for Question C; however, two PSTs improved and one PST, who answered all her questions correctly in the pre-questionnaire, made an error in the third question on the post-questionnaire. Nevertheless, eight of the fifty-one PSTs (16 percent) persisted with errors where the reversal error was most common. This again indicates weak number sense.

Error Analysis Question D (98 + 99)

The three PSTs who made errors on Question D in the pre-questionnaire answered Question D correctly in the post-questionnaire and two PSTs who gave the correct answer in the pre-questionnaire, answered Question D incorrectly in the post-questionnaire

PST TV used the vertical algorithm method for the pre- and post-questionnaire but made a calculation error in the post-questionnaire (i.e., 98 + 99 = 196), see Figure 4.16 below. PST ZT did not explain her method, but also calculated incorrectly and gave the same answer (98 + 99 = 196).

Figure 4.16 Calculation error using the vertical algorithm in the post-questionnaire.



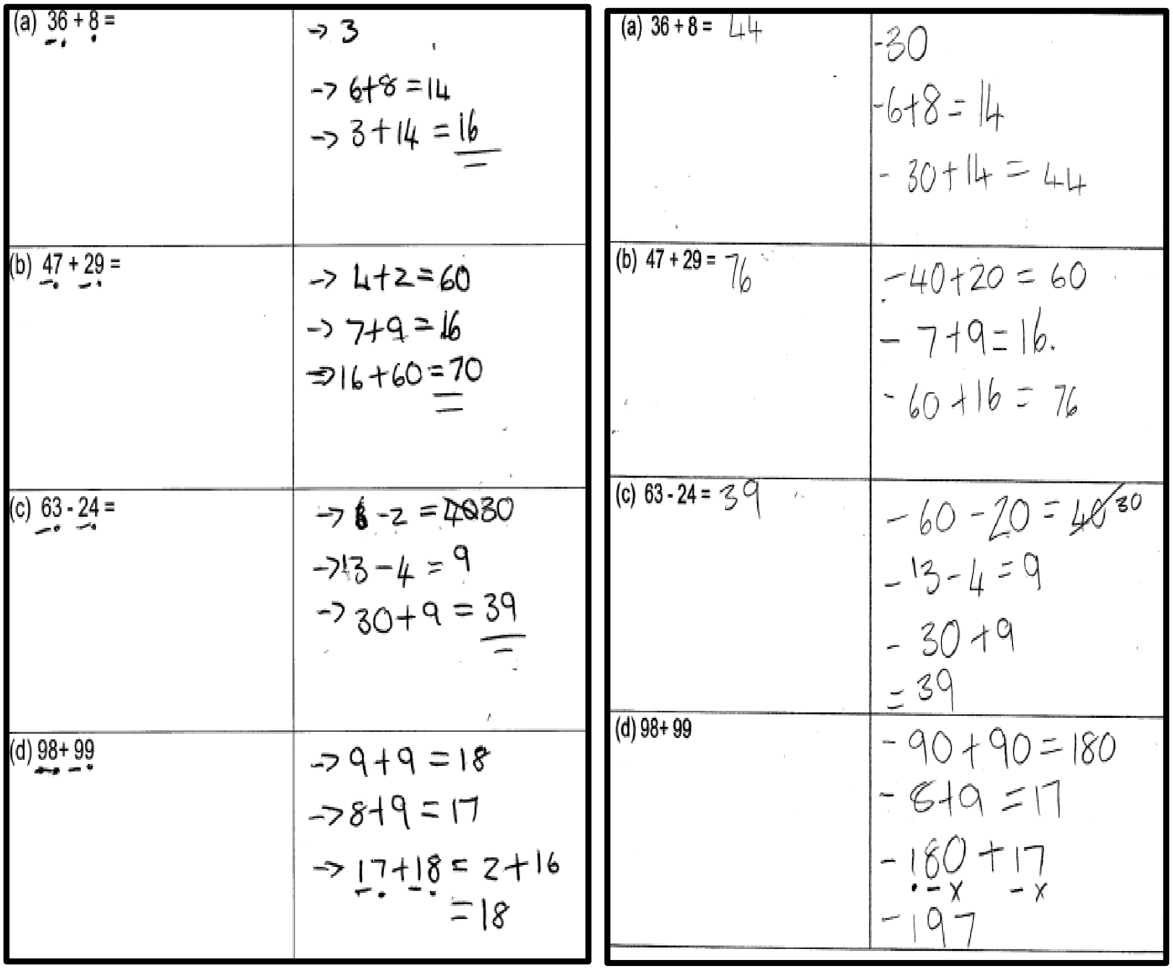
PST TV

4.4.4 Analysis of the shift across the four questions (A-D) of PSTs who achieved 50 percent or less in the pre-questionnaire

Two of the 51 PSTs achieved 50 percent or less where they answered two or more of the four questions incorrectly in the pre-questionnaire and, like most PSTs, showed an improvement. I chose to share their pre- and post-questionnaire results side by side so that shifts across all four items can easily be seen.

In Figure 4.17, I share PST VJ's calculations, who like most of the PSTs showed some improvement from pre- to post-questionnaire. In the pre-questionnaire, PST VJ achieved a score of 25 percent, answering three addition questions incorrectly but less prone to error on the more difficult subtraction question. I share this example because I can see an improvement in results from 25 percent in the pre-questionnaire to 100 percent in the post-questionnaire; however, like most other students, PST VJ continued to use a less effiecient method.

Figure 4.17 Shifts across all four items from three out of four incorrect to four correct calculations from pre- to post-questionnaire.



PST VJ

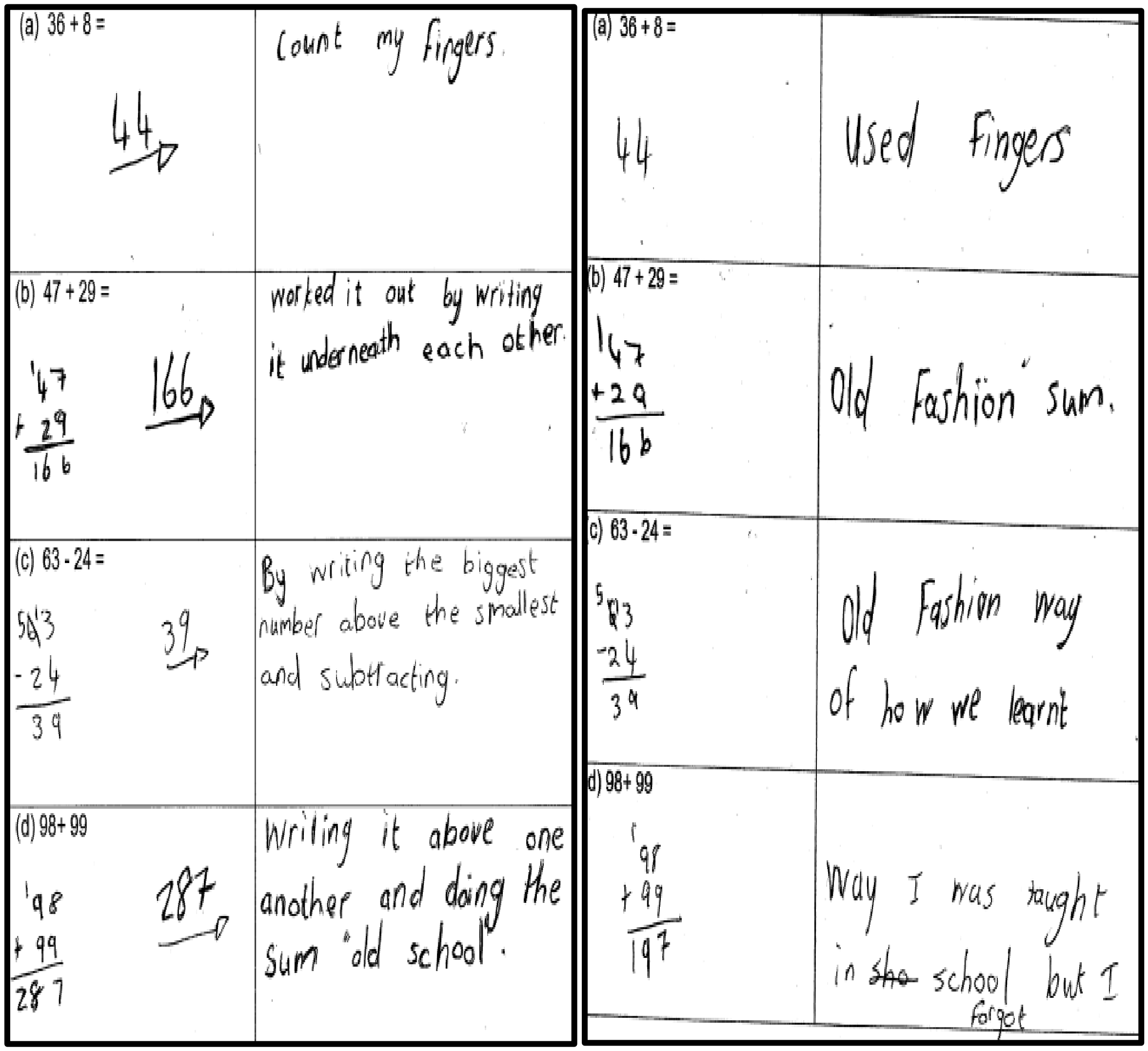
In Question A (36 + 8) and Question D (98 + 99) the breaking down method is used in both the pre- and post-questionnaire. However, in the pre-questionnaire a place value error is made in both calculations. In the calculation the units (6+8) are added together to get 14 then the three in the tens column is added as 3 units and another calculation error is made. The answer given for the calculation (14+3) is 16. In Question D (98 + 99) the nine in the tens column is again written as units (9+9) and added together to get 18, leaving off the zero. The units (8+9) are then added together to get 17. The two incorrect answers were then written down (17+18) and once again a place value error was made. The two ones in the tens column were added together and written down as ‘2’. The units ‘7’ and ‘8’ were then incorrectly added and the answer given was 16. The ‘2’ was then added to the 16 and the final answer for (98 + 99) was 18. In the post-questionnaire, while PST VJ persisted using a less efficient strategy for both questions A and D, there is some improvement as the place value is understood and represented correctly and the answer stated for the calculation is correct.

In Question B (47 + 9) the same method is used for both the pre- and post-questionnaire; however, in the pre-questionnaire the 40 and 20 are represented as 4 and 2 but added together and the answer is written down as 60. The units (7 + 9 = 16) are also added correctly. The error occurs in the third part of the calculation (16 + 60 = 70). In the post-questionnaire the method does not change but the calculation is without errors and the correct answer is stated. In the next question (63 – 24), both the pre- and post-questionnaires reveal the correct answer using the same stubborn breaking down method to calculate Question C, the subtraction calculation.

While there was a significant improvement from pre- to post-questionnaire from one out of four calculations correct (25 percent) to four out of four calculations correct (100 percent), the persistent use of the less efficient, highly prone to error, breaking down method was favoured.

While the results of most PSTs improved, some PSTs repeated their earlier methods and errors on some questions while improving on others. The next example (Figure 4.18 below) shows a marginal improvement of PST KA from pre- to post-questionnaire, with two out of four answered correctly in the pre-questionnaire (50 percent) and three out of four correct in the post-questionnaire (75 percent).

Figure 4.18 Shifts across all four items from pre- to post-questionnaire.



PST KA

I can see a slight improvement from pre- to post-; however, PST KA gets Question A right in both the pre- and post-questionnaires with continuous use of fingers as a method, so there is no improvement on the use of efficient strategies.

As mentioned earlier, the same place value error was made in Question B (47+ 29), and the same answer was obtained. The persistent use of the vertical algorithm remained the preferred method in both the pre- and post-questionnaires where the one was carried over to the tens column and brought down to represent 100 in the answer. Once more the "old fashion way of how we learnt it". This persistent use of methods taught at school can again be seen in Question C (63 – 24). PST KA gets the subtraction question right from pre- to post- with the continued use of the vertical algorithm.

At first glance the following question, Question D (98 + 99) shows an improvement from pre- to post-questionnaire, as PST KA answers incorrectly in the pre-questionnaire and obtains the correct answer in the post-questionnaire. While the scores show some improvement, the insistent use of the less efficient vertical algorithm was again the preferred method, which is highly prone to error. This type of error is evident in the pre-questionnaire where the units were added together, the seven was written down and one was carried over. The two nines in the ten's column were then added together to get 18 and the one that was carried over was added and 18 became 28 giving the final answer as 287. While the place value is a problem there is also a lack of number sense as 98 + 99 is almost 200 and 287 is almost 300. However, in the post-questionnaire, Question D was answered correctly with no improvement on the strategy used. These figures indicate that more intervention is required. In the next section, I analyse the second part of the questionnaire where PSTs describe their understanding of Bridging Through Ten, Jump Strategy, and Rounding and Adjusting, and how they might teach these strategies.

4.5 SHIFTS IN PCK FROM PRE- TO POST-QUESTIONNAIRE

In the post-questionnaire, the questions that followed focused on the gathering of PSTs' PCK of two of the mental mathematics strategies (bridging through ten and the jump strategy) after exposure to the MSAP. PSTs were again asked to describe BTT and JS, then explain how they would teach it.

The data was organised as follows:

4.5.1 Shifts in explanations from pre- to post-questionnaire on two strategies

4.5.2 PSTs' description of their perceived growth in confidence to teach the strategies.

As mentioned earlier, this section focuses on data from Part 2 of the questionnaire, which looked at shifts in PSTs' PCK of the mental maths strategies.

4.5.1 Shifts in explanations from pre- to post-questionnaire on two strategies

Please note that due to time constraints no intervention was done on the rounding and adjusting strategy.

The second part of the questionnaire included questions that asked PSTs to describe Bridging Through Ten, Jump Strategy, Rounding and Adjusting, and then separately asked how they might teach these strategies. Analysis of 51 PST responses to these questions indicated that only two PSTs could correctly describe the strategy of BTT while no PST was able to correctly describe JS or R&A. Table 4.11 shows the frequency of PSTs' responses to the request to describe two of the strategies, BT and JS, into four identified categories of response, namely: they stated they did not know, they gave an incorrect explanation, they did not answer (left it blank) or they answered correctly (i.e., provided a basic description of the strategy).

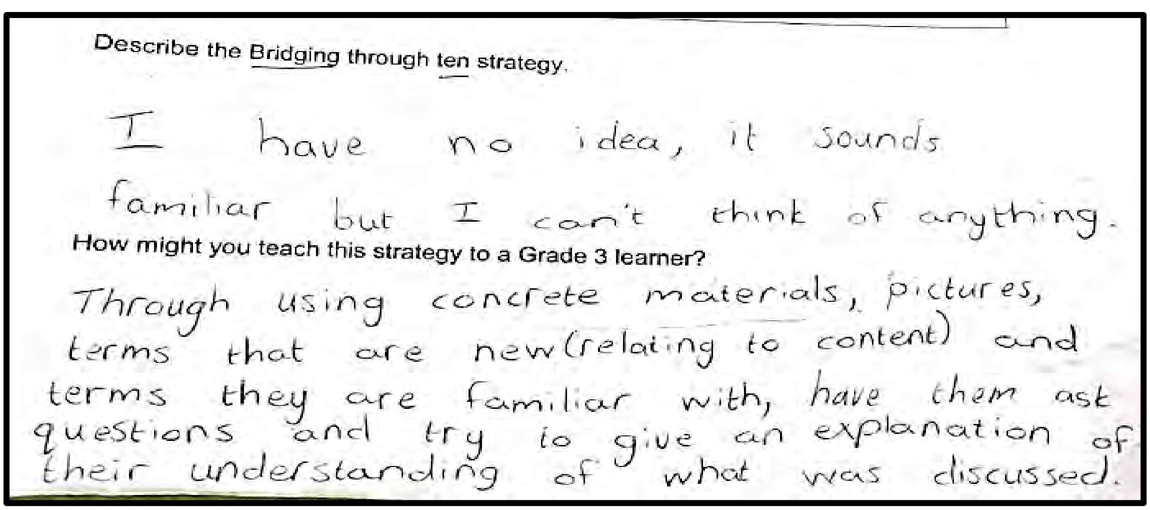
Table 4.10: Frequency shift from pre- to post-questionnaire of PSTs able to describe BTT and the JS (n=51)

PART 2 PCK	Describe BTT		Describe JS	
51 PSTs	PRE	POST	PRE	POST
Did not know	12	0	6	0
Answered incorrectly	24	6	25	12
Left it blank	13	5	20	10
Answered correctly	2	40	0	29

While the table shows some improvement, 40 and 29 of the 51 PSTs (78 percent) were able to describe BTT and JS post-intervention; however, only 34 of the 40 PSTs (who explained BTT correctly) were able to explain how to teach the strategy. I can see that more time needs to be allocated to teaching these strategies when 17 out 51 students (33 percent) were not able to explain how they would teach the strategies after working with the material over two 50-minute sessions per strategy. The next figure is evidence that not enough time was allocated to learning the strategies and how to teach them. I chose to display PST KA's pre- and post-answers alongside each other in order to draw a comparison on the question, describe BTT and how would you teach it.

In the pre-questionnaire PST KA's response to describe BTT was "I have no idea, it sounds familiar, but I can't think of anything." PST KA then explained that she would teach the strategy using concrete materials and pictures. Like many others there is improvement from pre- to post-questionnaire; however, more intervention is required. In the post-questionnaire PST KA gave an acceptable explanation for BTT then drew a beautiful, animated number line and attempted to explain how to teach BTT but did the calculation incorrectly. Her example was 57 + 12, she broke up the 12 into tens and units then incorrectly added 57 + 10 to equal 60 + 2. The answer she gave for 57 + 12 was 62. (See Figure 4.19 below)

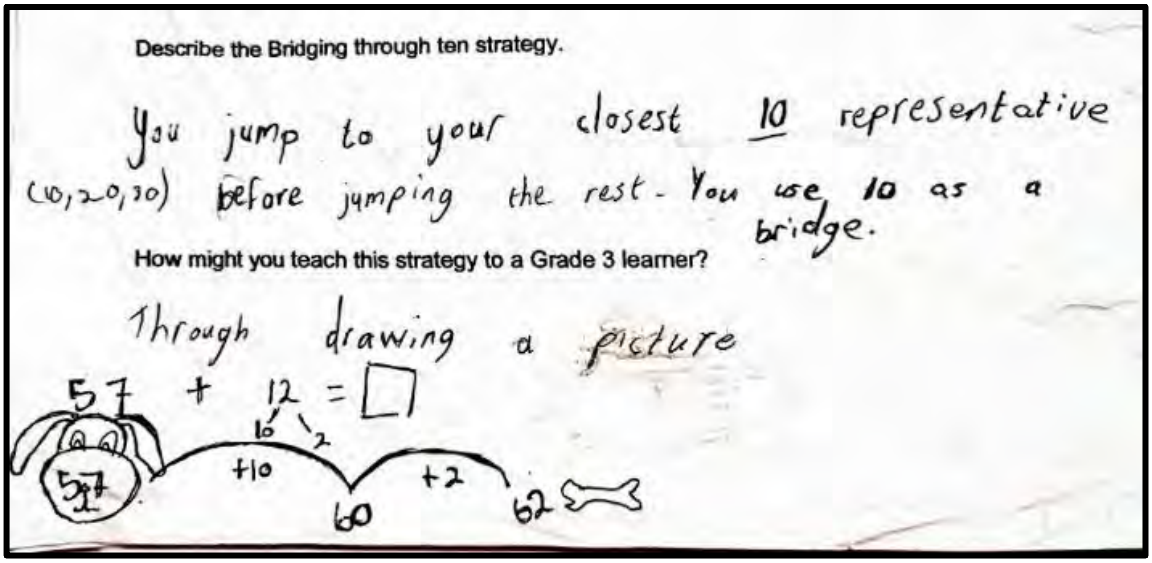
Figure 4.19 A comparison of pre- and post-description of BTT and how to teach it.


Describe the Bridging through ten strategy.

I have no idea, it sounds familiar but I can't think of anything.

How might you teach this strategy to a Grade 3 learner?

Through using concrete materials, pictures, terms that are new (relating to content) and terms they are familiar with, have them ast questions and try to give an explanation of their understanding of what was discussed.


Describe the Bridging through ten strategy.

You jump to your closest 10 representative (10, 20, 30) before jumping the rest. You use 10 as a bridge.

How might you teach this strategy to a Grade 3 learner?

Through drawing a picture

57 + 12 = □

10 2

57 +10 60 +2 62

PST KA

When the PSTs completed their adapted version of the post-questionnaire on strategic calculations, they were asked to reflect and write how they felt about the intervention programme. In the following section, I discuss PSTs' perceptions and perceived growth in confidence.

4.5.2 PSTs' descriptions of their perceived growth in confidence to teach the strategies

At the end of the last session PSTs were additionally asked to reflect and comment on their experience of working with the MSAP materials. It was the last session on campus and there was not much time allocated to reflection. This was due to the Covid-19 pandemic and the limited time on campus. Thirty-four of the 54 PSTs submitted written reflections. The instruction given was: *Reflect on your experience working with the MSAP BTT and JS and give reasons* why *you think mental mathematics strategies are or are not advantageous?* The responses were transcribed and analysed to identify themes. The findings were mostly positive indicating many gains from the interaction and MSAP resources; however, one neutral response indicated no need for intervention "I don't think this has made me see mental maths differently... I don't think the intervention was needed".

Seven themes were identified, namely fluency, confidence, strategic thinking, understanding, improved PCK, acknowledging usefulness, and needs more intervention. Many PSTs commented on more than one theme; some examples of these comments are listed and discussed below.

Sixteen of the 34 PSTs (close to 50 percent) commented that they have a better understanding of the strategies which made calculations easier.

> “The mental maths has helped improve my own maths…“ [PST-KJ].
>
> “… and the methods provided made it much easier to understand” [PST-SM].

Seven acknowledged the usefulness of the MSAP resources, which improved their own ability to calculate and teach the strategies. The first example that I use is the comment from PST VJ, who scored 25 percent, only answering one of the four question in the pre-questionnaire correctly. In the post-questionnaire she scored 100 percent.

> “The mental maths has had a positive effect on me. I think it will help a lot of learners.” [PST-JV]
>
> “Practicing the work in class has been beneficial and added confidence when making use of these concepts in maths.” [PST-LM]

Of interest is that nine of the 34 PSTs (26 percent) stated that more intervention is needed for them to effectively use the mental strategies to do calculations.

> “Still got confused between the different strategies…” [PST-AB].
>
> “… I think this new way will take some time getting used to.” [PST-KG]

Two PSTs reported increased speed and fluency using the mental strategies.

> “it makes doing the sums somewhat faster instead of using your fingers to count.” [PST-LG2]

> "I knew the older way of doing mental math and found my way to be slightly longer…" [PST-KG].

Four PSTs reflected on their increased confidence.

> "it gave me added confidence when making use of these concepts in Maths." [PST-LG2]
>
> "I feel more confident doing maths now." [PST-KJ]

Two PSTs commented on their own increased fluency and ability to think strategically.

> "It has changed the way that I think, plan and reason when I look at designing lesson plans it has made me rethink strategies." [PST-KA]
>
> "It made me think of how I can use the strategy and …" [PST-CP].

Eight PSTs described a growth in confidence to teach the mental strategies (PCK). This is encouraging, as Shulman (1987) recommends improving PCK.

> "I felt good about mental maths at the start but maybe not as well to teach but I see the benefit to how to be able to teach these strategies better." [PST-LG2]
>
> "I feel that this mental maths was actually fun and I gained from it as I learnt how to teach children different strategies." [PST-TM]

4.6 CONCLUDING REMARKS

While many PSTs had not fully mastered the strategies due to the limited intervention and time constraints, it was pleasing to see a shift in CK and PCK. An additional highlight of the study was the change in confidence and attitude of most PSTs from 'can't to can'. It was pleasing to note that even the stronger performing students (in terms of the four calculations) stated that they saw benefit in the mental strategies and commented on the benefits to understanding ‘how to teach’ the strategies.

The next chapter reflects on my research study and reviews how my findings respond to my research question located within the context of learner underperformance and teacher competency in mathematics.

CHAPTER FIVE

4. CONCLUDING DISCUSSION

5.1 INTRODUCTION

This final chapter reflects on my research study journey and discusses how my findings respond to my research question. I locate my research within the context of learner underperformance in both national and international assessment, and argue teacher competency in mathematics as a possible contributing factor to the underperformance of learners at school.

My findings concur with Spaull (2013), Hoadley (2012), Fleisch (2008), Taylor and Taylor (2013) and Bowie (2019), who highlight issues around teachers and pre-service teachers' limited CK and PCK needed to teach mathematics. I advocate that a collaborative intervention is required to develop the CK and PCK that PSTs need to teach mathematics. In addition, I reflect on my own journey as a researcher, discuss the extent to which the research goals were met, and examine implications which could result from this study, for future education in teacher training.

Based on my findings, which are discussed in the next section, my reflective conclusion discusses recommendations for future research opportunities which might extend from this study.

5.2 KEY FINDINGS IN RELATION TO THE RESEARCH QUESTIONS

My study sought to explore the nature of FP PSTs' mathematical CK and PCK for teaching additive mental mathematics strategies. To this end I addressed the following questions:

- What strategies do third-year PSTs use to solve basic additive reasoning calculations? Do they use these effectively? [data pre-CK questionnaire]
- Can PSTs describe the bridging through ten, jump, and rounding and adjusting strategies? If so, how do they explain them and how they might teach them? [data pre-PCK questionnaire]
- How might PST strategies shift following exposure to the bridging through ten and jump strategies in lectures? Does PST performance improve in relation to learning additional strategies? [data post-CK questionnaire]

In this section I summarise and discuss the findings for each of these research questions. Further, while not an explicit research question, I include PSTs' perceived growth in confidence that emerged from the post-intervention reflective paragraphs they wrote.

5.2.1 What strategies do third-year PSTs use to solve basic additive reasoning calculations? Do they use these effectively?

Data for these questions were gathered in the CK pre-questionnaire, and analysis focused on performance, strategies and errors.

PSTs' performance results on the four calculations

Most of the PSTs (80 percent) managed to get all four calculation items correct in the first questionnaire; however, very few used efficient methods of calculating. Of great concern is the 20 percent (11/54) of PSTs who did not manage to answer all four the Grade 3 easy calculation items (which do not require problem solving or question interpretation) correctly. The most challenging was the subtraction question. The error analysis (on methods used) that follows indicates possible reasons for the underperformance on the subtraction question. Additionally, the analysis suggests that hardly any PSTs used BTT, JS or R&A for the calculation items, and displayed preference for less effective methods, for instance unit counting, the vertical algorithm or breaking down (i.e., splitting up the tens and units to calculate).

PSTs' methods/strategies used on the four calculations

Calculating efficiently and accurately is of utmost importance. Studying PSTs' methods used to resolve the four calculation items A-D allowed for analysis of the efficiency (and accuracy) of the methods used. The data reveals that the performance of most PSTs across the calculation items is mostly linked to the use of inefficient methods of calculation (i.e. building up and breaking down and the vertical algorithm methods).

The absence of use of more efficient methods of calculating links with the weaker performance using the MSAP Grade 3 material. The low performance of many PSTs solving strategic calculating items points to weak number sense and insufficient knowledge of efficient additive reasoning calculation strategies.

Error analysis on the four calculations

The data on the questionnaire before intervention points to a small group of PSTs who do not manage the four calculations on the questionnaire with any method. That is, they make a range of errors even when they use sound yet less efficient methods of counting in ones, the vertical algorithm or the breaking down method for calculations that lend themselves well to BTT and the JS. Some of the most common errors made indicate the absence of fluency in basic addition facts (i.e. 6+8). Subtraction proved to be the most difficult calculation with 11 of the PSTs making errors. The most prevalent error was the reversal error where seven PSTs reversed the units, changing the calculation from 63 – 24 to 64 – 23 to avoid subtracting a larger number from a smaller number (the answer given was 41). This common error could be an indication that the learners do not understand the concepts of base ten, place value and the number line which impact later learning. This absence of place value knowledge has been detected at different levels of study, including senior school students in their primary stage of formal algebra learning (MacGregor & Stacey, 1993).

5.2.2 Can PSTs describe the bridging through ten, jump, and rounding and adjusting strategies? If so, how do they explain them and how they might teach them?

Data for these questions were gathered in the PCK pre-questionnaire. Fifty-four PSTs were asked to describe BTT, JS and R&A, and explain how they would teach it. Only two of the 54 PSTs could correctly describe the BTT strategy. No PST was able to correctly describe JS or R&A. This highlights the fact that these third-year PSTs (in their final year of mathematics education) had not been exposed to these strategies in their first or second year. After completing the questionnaire many students commented that they had never heard about or been taught these strategies at school. This is an indication that PCK is limited, as noted by many researchers. Bowie (2019), Fleisch (2008), Spaull (2013) and Taylor and Taylor (2013) have all indicated that teachers have limited CK and PCK needed to teach mathematics. It is thus important that pre-service teacher education addresses these strategies that are foundational to supporting number sense and developing a structural understanding of number.

5.2.3 How might PST strategies shift following exposure to the bridging through ten and jump strategies in lectures? Does PST performance improve in relation to learning additional strategies?

Data for these questions were gathered in the pre- and post-CK questionnaire, and analysis compared the shifts between these.

Shifts in CK from pre to post- questionnaire

The data which was analysed in Chapter 4 shows minor shifts in frequencies of the 51 PSTs who responded correctly from pre- to post-questionnaire. This is because most PSTs answered the pre-questionnaire correctly. However, Question C, the subtraction calculation remained problematic. The repeated weak performance by 20 percent of the PSTs on the post-questionnaire is grounds for concern as being able to correctly calculate a two-digit subtraction is essential to what must be taught.

Shifts in PSTs' methods/strategies used on the four calculations

The rate of recurrence of the dominant vertical algorithm method across the four calculations reduced in the post-questionnaire. However, the building up and breaking down method persisted, with some added use of it for three of the four calculations. As noted in Chapter 4, it was pleasing to see that the use of the BTT method increased across the four calculations.

Error analysis and comparison on the four calculations

The addition of two, two-digit numbers highlighted weak number sense and a lack of understanding of place value as the same errors were made in both the pre- and post-questionnaire. In some instances, there was an attempt made to use the more efficient BTT mental strategy. However, the incorrect (changing subtraction to addition) and incomplete calculations indicates a need for more intervention.

Analysis of shifts in PCK from pre- to post-questionnaire

A comparison of data from pre- to post-questionnaires shows some improvement, as 40/51 and 29/51 PSTs were able to describe BTT and JS respectively, post-intervention. However, only 34/51 PSTs were able to describe how to teach the BTT strategy. Seventeen out 51 students (33 percent) could not explain how to teach a mental strategy after working with the MSAP material over two 50-minute sessions allocated per strategy. This is evidence that insufficient time was allocated to learning the strategies and how to teach them.

5.2.4 PSTs' descriptions of perceived growth

As noted above, while not an explicit research question, PSTs were asked to write reflective paragraphs on their experiences of learning about the mental strategies. Thirty-four of the 51 PSTs submitted written reflections. The responses were mostly positive. Sixteen PSTs stated that they have a greater understanding of the two mental strategies, which made computations much easier. Two of the 16 PSTs commented on their own increased fluency and ability to think strategically, and eight PSTs mentioned a growth in confidence to teach the mental strategies (PCK).

However, of interest is that nine of the 34 PSTs (26 percent) indicated that more practice is needed for them to efficiently use the mental strategies to do computations.

5.3 LIMITATIONS

This study collected data from 54 PSTs at one higher education institution. This limited data can therefore not be generalised to PSTs in South Africa. However, my continuous participation in the MM-WIL, which involves teacher educators from nine teacher education institutions, indicates that my data is similar to issues emerging in other institutions. An added constraint of the focus of this study on the pre-questionnaire is that while I intended to gather data on PSTs' CK and PCK relative to mental mathematics strategies (prior to lectures focused on these strategies), the lack of CK of the mental strategies meant I was not able to gather meaningful data on their PCK. This indicates the importance of CK as a necessary but not sufficient condition for the development of PCK (a point also noted by Adler, 2005).

5.4 MY LEARNING THROUGH THE RESEARCH PROCESS

My journey as a researcher started with my commitment to learning, and the realisation that I was not preparing PSTs adequately to teach mathematics in the Foundation Phase classroom. Early challenges included my own fears and setbacks I faced grasping concepts and understanding research methodologies, which often felt overwhelming. However, my desire and determination to understand the needs of PSTs to teach mathematics, and the desire to improve my own teaching to meet these needs, helped me overcome my fears. I have changed the way I teach this subject. I now teach the maths concept as I would in a Foundation Phase classroom, and then I discuss the strategies that were used in that lesson. This change has had its own challenges as not all PSTs need to learn basic maths CK or how to calculate using the four operations (as some come with strong foundational mathematics knowledge). However, all PSTs in my study did need to learn about the different strategies for calculating efficiently as even those who could answer all calculations correctly tended to do so with the vertical algorithm even when this method was inefficient for the given calculation. Creating carefully planned groups that mix PSTs with stronger and weaker backgrounds has allowed the opportunity for stronger PSTs to teach the concepts to weaker peers, which provided a good learning experience for them, while those with weaker mathematics backgrounds had the opportunity to learn from their peers.

Ongoing engagement with my supervisor and critical friends was fundamental. Their guidance helped me embrace various challenges I faced, as a vital part of the research process in order to enhance my research skills. Each challenge has increased my determination and dedication to improving education and I remain motivated by the prospective influence of research on society.

5.5 RECOMMENDATIONS FOR FUTURE STUDIES

While studies into PSTs' mathematical CK are quite new in South Africa (see, for example, Bowie et al., 2019), the view that South African teacher education needs urgent attention is not. Carnoy and Chisholm (2008) raised concerns about the training of teachers in 2008, stating that "the relatively low level of mathematics knowledge that teachers have in all but the highest student [socioeconomic status] schools is somewhat troubling" (p. 33). Similarly, Taylor (2008) noted that "in the hands of teachers whose own conceptual frames are not strong, the results are likely to be disastrous where school knowledge is totally submerged in an unorganized confusion of contrived realism" (p.276). Taylor's view about teachers' weak mathematical content knowledge negatively impacting their ability to support learners' mathematical progression is widely held (Bowie et al., 2019; Jenßen et al., 2022).

There is increasing concern over primary in-service and PSTs' inadequate CK and PCK needed for teaching primary mathematics. Analysing the 2007 South African SACMEQ data represented nationally (from 401 mathematics teachers, teaching Grade 6 learners), Venkat and Spaull (2015, p. 121) found that "79% of grade 6 mathematics teachers showed content knowledge levels below the grade 6/7 band, and that the few teachers with higher-level content knowledge are highly inequitably distributed". This study highlights increasing challenges in the context of PST education, pointing to the need to adapt mathematics education courses accordingly. Similarly, Manuel (2022) found BED students "assessed at three universities scored a weak 54 percent for maths content-based tests meant for primary school pupils" (p. 1). Similar gaps in PST knowledge were documented in a study by Bowie et al. (2019) also across three universities. They, however, noted small improvements in knowledge as PSTs progress from first to final year, arguing that this indicates "a need for student teachers to revisit primary school mathematics in a way that provides a deep understanding of key mathematical concepts in order to be better prepared for future teaching careers" (p. 286).

The underperformance of many PSTs when solving strategic calculating items indicates limited knowledge of effective calculation strategies. This indicates a need to develop PSTs' repertoires of additive reasoning calculation strategies so that they can use these strategies effectively and efficiently. This will not only strengthen their own number sense but also strengthen their PCK and ability to teach these strategies, which are a requirement in the curriculum.

Considering the above, I recommend that further studies are needed on the development of efficient calculation strategies, as the MSAP and MM-WIL aim to do. PSTs should be exposed to these strategies at the start of their four-year BEd degree so that the use of the strategies can be built into their four years of study.

For the small group of PSTs in my study who could not answer any of the four calculations correctly (and likely exist in PST cohorts across South African institutions), I argue that developing efficient calculation strategies is not enough. This group of PSTs need specialised support in developing both the range of basic facts required for performing calculations and the use of effective calculation procedures and strategies, including the breaking down and building up method and the vertical algorithm.

5.6 CONCLUSION

This study focused on pre-service teachers' content knowledge (CK), pedagogical content knowledge (PCK), use of various additive reasoning strategies, and the extent to which PSTs were able to successfully answer additive reasoning calculation items with whatever method they chose. Findings indicate that many final-year PSTs are unprepared to teach mathematics in primary schools, and Taylor (2021) suggests that "unless initial teacher education (ITE) is reformed at the same time, continuous professional development (CPD) becomes a never-ending task of making marginal differences to the shortcomings of each successive cohort of qualified but incompetent teachers emerging from the universities" (p. 1). The consistent picture across these studies points to the critical need for greater attention to be paid to developing primary school teachers' mathematical CK during their in-service studies.

The intervention of this study, teaching efficient additive reasoning mental mathematics strategies, attempted to pay attention to the demands of FP PSTs for teaching mathematics. This said, the study and lectures at the same time focused on developing PSTs' PCK, following Adler and Reed's (2003) argument that while CK is essential it is not enough for teachers. They emphasise the need to simultaneously develop teachers' PCK to assist learners to make sense of mathematics. They further advise that teacher education institutions should avoid being 'complicit' in the continued production of the mathematics crisis.

Given that my findings cohere with findings in other studies indicating that many final-year PSTs are unprepared to teach mathematics in primary schools, I strongly recommend the need for intervention into PST courses and future studies on how to develop and equip PSTs to better teach mathematics and build a strong mathematics foundation for learners in the early years.

REFERENCES

Adler, J. (2005). Mathematics for teaching: What is it and why is it important we talk about it? *Pythagoras*, *62*, 2-11. https://doi.org/10.4102/pythagoras.v0i62.109

Adler, J., & Reed, Y. (2003). *Challenges of Teacher Development: An investigation of take-up in South Africa*. Van Schaik.

Andrews, P., & Sayers, J. (2015). Identifying opportunities for grade one children to acquire foundational number sense: Developing a framework for cross cultural classroom analyses. *Early Childhood Education Journal*, *43*, 257-267. https://doi.org/10.1007/s10643-014-0653-6

Askew, M., Brown, M., Rhodes, V., Johnson, D., & Wiliam, D. (1997). *Effective Teachers of Numeracy*. School of Education, King's College.

Askew, M., Graven, M., & Venkat, H. (2022). From what works to scaling up: Improving mental strategies in South African Grade 3 classes. In C. Fernández, S. Llinares, A. Gutiérrez, & N. Planas (Eds.), *Proceedings of the 45th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 27-34). PME: Alicante, Spain. https://www.researchgate.net/publication/362504391_From_what_works_to_scaling_up_Improving_mental_strategies_in_South_African_Grade_3_classes

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, *59*(5), 389-407. https://doi.org/10.1177/0022487108324554

Banegas, D. L., & Villacanas de Castro, L. S. (2015). A look at ethical issues in action research in education. *Argentinian Journal of Applied Linguistics, 3*(1), 58-67. http://wrap.warwick.ac.uk/69491

Bertram, C., & Christiansen, I. (2014). *Understanding research: an introduction to reading research*. Van Schaik.

Bevir, M., & Kedar, A. (2008). Concept formation in political science: An anti-naturalist critique of qualitative methodology. *Perspectives on Politics*, *6*(3), 503-17. https://doi.org/10.1017/S1537592708081255

Bloomberg, L. D., & Volpe, M. (2008). *Completing your qualitative dissertation: a road map from beginning to end*. Sage.

Bowie, L., Venkat, H., & Askew, M. (2019). Pre-service primary teacher's mathematical content knowledge: An exploratory study. *African Journal of Research in Mathematics, Science and Technology Education, 23*(3), 286-297. https://doi.org/10.1080/18117295.2019.1682777

Bowie, L., Venkat, H., Hannan, S., & Namome, C. (2022). TIMSS 2019 South African Item Diagnostic Report: Grade 9 Mathematics. Human Sciences Research Council https://www.timss-sa.org/wp-content/uploads/2022/05/TIMSS-2019_Grade-9_Maths-Item-Diagnostic-Report_FInal.pdf

Brenner, M. E. (2006). Interviewing in educational research. In J.L. Green, G. Camilli, & P.B. Elmore (Eds.), *Handbook of complementary methods in education research* (pp. 357-370). Lawrence Erlbaum. https://www.scirp.org/reference/referencespapers?referenceid=2758438

Butterworth, B. (2005). The development of arithmetical abilities. *Journal of Child Psychology and Psychiatry*, *46*, 3–18. https://doi.org/10.1111/j.1469-7610.2004.00374.x

Charmaz, K. (2006). *Constructing grounded theory: A practical guide through qualitative analysis*. Sage.

Cobb, P. (2007). Putting philosophy to work: Coping with multiple theoretical perspectives. In: F.K. Lester (Ed.) *Second handbook of research on mathematics teaching and learning* (pp. 3-38). Information Age.

Cockcroft, W. H. (1982). *Mathematics counts*. HM Stationery Office.

Cohen, L., Manion, L., & Morrison, K. (2018). *Research methods in education* (8th ed.). Routledge.

Cohen, L., Manion, L., & Morrison, K. (2011). *Research methods in education* (7th ed.). Routledge.

Coyne, I. T. (1997), Sampling in qualitative research. Purposeful and theoretical sampling; merging or clear boundaries? *Journal of Advanced Nursing, 26*(3), 623-630. https://doi.org/10.1046/j.1365-2648.1997.t01-25-00999.x

Creswell, J. W. (2007). *Qualitative inquiry & research design: Choosing among five approaches* (2nd ed.). Sage.

Creswell, J. W., & Creswell, J. D. (2018). Research design (5th ed.). SAGE Publications

Dehaene, S. (2001). Precis of the number sense. *Mind and Language*, *16*(1), 16-36. https://doi.org/10.1111/1468-0017.00154

Department of Basic Education [DBE]. (2011). Curriculum and Assessment Policy Statement Foundation Phase Grades R-3: English Mathematics. Government Printing Works. https://www.education.gov.za/Portals/0/CD/National%20Curriculum%20Statements%20and%20Vocational/CAPS%20English%20HL%20GRADES%20R-3%20FS.pdf?ver=2015-01-27-154201-167

Department of Basic Education [DBE]. (2019). Mathematics teaching and learning framework for South Africa: Teaching Mathematics for Understanding. Department of Basic Education. https://www.education.gov.za/MathematicsTeachingandLearningFramework.aspx

Department of Basic Education [DBE]. (2021a). Grade 3 Mental Starters and Assessments: Teacher Guide (Creative commons license) – all South African languages. https://www.education.gov.za/MSAP2022.aspx

Department of Basic Education [DBE]. (2021b). Grade 3 Mental Starters and Assessments: Print Masters (Creative commons license) – all South African languages. https://www.education.gov.za/MSAP2022.aspx

Du Toit, S. 2008. Parental choice in South African high schools - An urban Cape Town case study. PhD thesis. Bellville, South Africa: University of the Western Cape. https://etd.uwc.ac.za/bitstream/handle/11394/2397/Du%20Toit_PHD_2008.pdf?sequence=1&isAllowed=y

Ferrance, E. (2000). *Action research*. Brown University.

Fleisch, B. (2008). *Primary education in crisis: Why South African schoolchildren underachieve in reading and mathematics.* Juta.

Flick, Uwe. (2014). *An introduction to qualitative research* (5th ed.). Sage.

Fogelman, K. (2002). Surveys and Sampling. In M. Coleman, & A. Briggs (Eds.), *Research methods in educational leadership and management* (pp.93-107). Paul Chapman.

Fonseca, K., Maseko, J., & Roberts N. (2018). Students' mathematical knowledge in a Bachelor of Education (Foundation or Intermediate Phase) programme. In Govender, R. & Junqueira, K. (2018). Proceedings of the 24th Annual National Congress of the Association for Mathematics Education of SA (AMESA), 25–29. University of the Free State, Bloemfontein.

https://www.researchgate.net/publication/326271006_Students'_mathematical_knowledge_in_a_Bachelor_of_Education_foundation_or_intermediate_phase_programme

Fosnot, C. T., & Dolk, M. (2001). *Young mathematicians at work: Constructing number sense, addition and subtraction*. Heinemann.

Graven, M. (2002), Teacher learning and development in mathematical education: A South African case study. *Proceeding of the 26th Annual Conference of the International Group for the Psychology of Mathematics Education*, 3,245-252.

Graven, M. (2014), 'Poverty, inequality and mathematics performance: The case of South Africa's post-apartheid context', ZDM 46(7), https://doi.org/10.1007/s11858-013-0566-7

Graven, M. & Coles, A. (2017). Resisting the desire for the unambiguous: productive gaps in researcher, teacher and student interpretations of a number story task. *ZDM, 49*:881-893. https://doi.org/10.1007/s11858-017-0863-7

Graven, M., Venkat, H., Westway, L., & Tshesane, H (2013) Place value without number sense: Exploring the need for mental mathematical skills assessment with the Annual National Assessments. *South African Journal of Childhood Education*, *3*(2), 131-143. https://doi.org/10.4102/SAJCE.V3I2.45

Graven, M., & Stott, D. (2012). Conceptualising procedural fluency as a spectrum of proficiency. In S. Nieuwoudt, D. Laubscher & H. Dreyer (Eds.), *Proceedings of the 18th annual national congress of the Association for Mathematical Education of South Africa* (AMESA) (pp. 146–156). North-West University. https://www.researchgate.net/publication/236644737_CONCEPTUALISING_PROCEDURAL_FLUENCY_AS_A_SPECTRUM_OF_PROFICIENCY

Graven, M., & Venkat, H. (2021). Piloting national diagnostic assessment for strategic calculation. *Mathematics Education Research Journal, 33*(1), 23-42. http://dx.doi.org/10.1007/s13394-019-00291-0

Harries, T., & Spooner, M. (2013). *Mental mathematics for the numeracy hour*. Routledge. https://doi.org/10.4324/9781315068718

Hoadley, U. (2012). What do we know about teaching and learning in South African Primary schools? *Education as Change, 16*(2), 187-202. http://dx.doi.org/10.1080/16823206.2012.745725

Howden, H. (1989). Teaching number sense. *Arithmetic Teacher, 36*(6), 6-11. https://doi.org/10.5951/AT.36.6.0006

Jaffer, S. (2020). Assessment of number in foundation phase initial teacher education programmes in four South African universities. In Vale, P., Westaway, L., Nhase, Z., Schudel, I. *Book of Proceedings of the 28th Annual Conference for Research in Mathematics, Science and Technology Education*. SAARMSTE.

Jenßen, L., Möller, R., Eilerts, K. & Roesken-Winter, B. (2022). 'Pre-service primary teachers' shame experiences during their schooling time: Characteristics and effects on their subject-choices at university', Educational Studies in Mathematics 100, 432–455. https://doi.org/10.1007/s10649-021-10141-1

Manuel, M. (2022). Shock SA Education fail: Incoming teachers score 54% for primary maths, The South African. https://www.thesouthafrican.com/lifestyle/breaking-teachers-maths-students-2030-reading-panel-report-education-crisis/

Maringe, F., & Moletsane, R. (2015). Leading schools in circumstances of multiple deprivation in South Africa, *Educational Management Administration & Leadership, 43*(3), 347-362. https://doi.org/10.1177/1741143215575533

Mason, J. (2006). Mixing Methods in a Qualitatively Driven Way. *Qualitative Research, 6*(1), 9-25. https://doi.org/10.1177/1468794106058866

Maxwell, J. A. (2013). Qualitative research design: An interactive approach. Sage.

Maxwell, J. A. (2003). Designing a qualitative study. In T.E. Hendrick, L. Brickman, & D.J. Rog. *Applied research designs* (pp. 214-253). Sage.

McCallister, J. (2018). What is action research? https://study.com/academy/lesson/action-research-in-education-examples-methods-quiz.html

McIntosh, A., Reys, B., Reys, R., Bana, J., & Farrell, B. (1997). *Number sense in school mathematics: student performance in four countries.* Perth, Australia: Mathematics, Science & Technology Education Centre, Edith Cowan University.

McIntosh, A., Reys, B. J., & Reys, R. E. (1992). A proposed framework for examining basic Number Sense. *For the Learning Mathematics, 12*(3), 2-8. https://www.researchgate.net/publication/281091335_A_proposed_framework_for_examining_basic_number_sense

McMillan, J. H., & Schumacher, S. (2010). *Research in Education: Evidence-based Inquiry* (7th ed.). Pearson.

Merriam, S. (1998). *Qualitative research and case study applications in education*. Jossey-Bass

Merriam, S. B. (2009). *Qualitative research: a guide to design and implementation*. Jossey-Bass.

Modisaotsile, B. M. (2012). The failing standard of basic education in South Africa. *Africa Institute of South Africa (AISA) Policy Brief,* 72, 1-7. https://www.purpletod.co.za/docs/FAILING%20STANDARDS.pdf

Morrow, W. (2007). Learning to teach in South Africa. Human Sciences Research Council Press. http://hdl.handle.net/20.500.11910/6104

O'Leary, Z. (2017). *The essential guide to doing research* (3rd ed.). Sage.

Petrou, M., & Goulding, M. (2011). Conceptualising teachers' mathematical knowledge in teaching. In: Rowland, T., Ruthven, K. (Eds.) *Mathematical Knowledge in Teaching*. Springer. http://dx.doi.org/10.1007/978-90-481-9766-8_2

Rathgeb-Schnierer, E., & Green, M. (2019). Developing flexibility in mental calculation. *Educação & Realidade, 44*(2), 1–17. https://doi.org/10.1590/2175-623687078

Reddy, V., Kanjee, A., Diedricks, G., Winnaar, L. (2003). *TIMSS 2003: Mathematics and Science Achievement at South African Schools.* Human Sciences Research Council. https://www.hsrcpress.ac.za/books/mathematics-and-science-achievement-at-south-african-schools-in-timss-2003

Reddy, V., Isdale, K., Juan, A., Visser, M., Winnaar, L., & Arends, F. (2016). *TIMSS 2015: Highlights of mathematics achievement of Grade 5 South African learners.* Human Sciences Research Council. http://hdl.handle.net/20.500.11910/10673

Reddy, V., Winnaar, L., Harvey, J., Hannan, S., Isdale, K., Arends, F., & Juan, A. (2019). The South African TIMMS 2019 Grade 5 Results. *Building Achievement and Bridging Achievement Gaps*. http://hdl.handle.net/20.500.11910/19287

Reddy, V., Winnaar, L., Juan, A., Arends, F., Harvey, J., Hannan, S., Namome, C & Zulu, N. (2020). *TIMSS 2019:* Highlights of South African Grade *5 results in mathematics and science*. Human Sciences Research Council. https://hsrc.ac.za/uploads/pageContent/1044991/TIMSS%202019_Grade%205_HSRC_FinalReport.pdf

Ryan, A.B. (2006). Methodology: Analysing qualitative data and writing up your findings. Researching and writing your thesis: A guide for postgraduate students (pp. 92-108). Maynooth: Maynooth Adult and Community Education.

Schollar, E. (2008). Final report of the Primary Mathematics Project: Towards evidence-based educational development in South Africa.

https://www.zenexfoundation.org.za/wp-content/uploads/2022/03/Final-Evaluation-of-the-Primary-Mathematics-Research-Project-2004-2007.pdf

Shulman, L. (1986) Those who understand, knowledge growth in teaching. *Educational Researcher, 15(*2), 4-14. http://www.jstor.org/stable/1175860

Shulman, L. (1987) Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57*(1), 1-23. https://doi.org/10.17763/haer.57.1.j463w79r56455411

Soudien, C., Reddy, V., & Harvey, J. (2022). The impact of COVID-19 on a fragile education system: The case of South Africa. In F.M Reimers (Ed.) *Primary and secondary education during Covid-19* (pp.303-325). Springer. https://doi.org/10.1007/978-3-030-81500-4_12

South African Numeracy Chair Project (SANC). (2021). https://www.ru.ac.za/sanc/

Spaull, N. (2013). South Africa's education crisis: The quality of education in South Africa 1994-2011. Johannesburg: Centre for Development and Enterprise, 21(1), 1-65. https://nicspaull.com/wp-content/uploads/2011/04/spaull-2013-cde-report-south-africas-education-crisis.pdf

Spaull, N., & Jansen, J. (2019). *South African schooling: The enigma of inequality*. Springer. https://doi.org/10.1007/978-3-030-18811-5

Spaull, N., & Kotze, J. (2015). Starting behind and staying behind in South Africa: The case of insurmountable learning deficits in mathematics. *International Journal of Educational Development, 41*, 13-24. http://dx.doi.org/10.1016/j.ijedudev.2015.01.002

Spelke, E. S. (2000). Core knowledge. *American Psychologist, 55*(11), 1233-1243. https://psycnet.apa.org/doi/10.1037/0003-066X.55.11.1233

Stake, R. E. (1995). *The art of case study research.* Sage.

Taylor, N., & Taylor, S. (2013). Teacher knowledge and professional habitus. In N. Taylor, S. Van der Berg & T. Mabogoane (Eds.), *Creating effective schools: Report of South Africa's national school effectiveness study* (pp. 204–233). Pearson.

Taylor, N. (2021). The dream of Sisyphus: Mathematics education in South Africa. *South African Journal of Childhood Education, 11*(1), 1-12. https://doi.org/10.4102/sajce.v11i1.911

Thomas, G. (2013). *How to do your research project: A guide for students in education and applied social sciences*. Sage.

Van der Berg S., & Gustafsson, M., (2019). Educational outcomes in post-apartheid South Africa: Signs of progress despite great inequality. In N. Spaull & J Jansen *South African schooling: The enigma of inequality*. Springer. https://doi.org/10.1007/978-3-030-18811-5_2

Van der Berg, S., Taylor, S., Gustafsson, M., Spaull, N., & Armstrong P. (2011). *Improving education quality in South Africa.* Stellenbosch, South Africa: Department of Economics, University of Stellenbosch. https://resep.sun.ac.za/wp-content/uploads/2017/10/2011-Report-for-NPC.pdf. Accessed 07 December 2023

Venkat, H., & Askew, M. (2012). Mediating early number learning: Specialising across teacher talk and tools? Journal of Education, 56, 67–90. https://research.monash.edu/en/publications/mediating-early-number-learning-specialising-across-teacher-talk- Accessed 10 December 2023

Venkat, H. (2014) Revisiting the role of procedures in mathematics teaching: *Implications for teaching in South Africa. Education as Change*, 18(1), 19-34.

Venkat, H., & Spaull, N. (2015). What do we know about primary teachers' mathematical content knowledge in South Africa? An analysis of SACMEQ 2007 *International Journal of Educational Development. http://dx.doi.org/10.1016/j.ijedudev.2015.02.002*

Venkat, H., Askew, M., Abdulhamid, L., Morrison, S., & Ramatlapana, K. (2016). A mediational approach to expanding in-service primary teachers' mathematical discourse in instruction. Paper presented at TSG49: 13th International Congress on Mathematical Education. Hamburg, 24–31 July.

Weitz, M., & Venkat, H. (2013). Assessing early number learning: How useful is the Annual National Assessment in numeracy? *Perspectives in Education*, *31*(3), 49-65. https://www.semanticscholar.org/paper/Assessing-early-number-learning-%3A-how-useful-is-the-Weitz-Venkat/3a1492c7daec63688f11bc6824e1ce0453e8150e

Westaway, L., & Vale, P. (2021). Preservice teachers' noticing of children's addition calculation strategies. In M. Qhobela, M. M. E. Ntsohi, & L. G. Mohafa (Eds.), Book of Proceedings of the 29th Annual Conference of the Southern African Association for Research in Mathematics, Science and Technology Education (pp. 167-182). SAARMSTE.

Whitacre, I. (2015). Strategy ranges: Describing change in prospective elementary teachers' approaches to mental computation of sums and differences. *Journal of*

Mathematics Teacher Education, 18, 353–373. http://dx.doi.org/10.1007/s10857-014-9281-8

Whitacre, I., Henning, B., & Atabaş, Ş. (2020). Disentangling the research literature on number sense: Three constructs, one name. *Review of Educational Research, 90*(1), 95-134. http://dx.doi.org/10.3102/0034654319899706

Willis, J. W. (2007). *Foundations of qualitative research: Interpretive and critical approaches*. Sage.

Wright, R. (2003). A mathematics recovery: Program of intervention in early number learning. *Australian Journal of Learning Disabilities, 8*(4), 6-11. http://dx.doi.org/10.1080/19404150309546741

Yanow, D., & Schwartz-Shea, P. (Eds.). (2006). *Interpretation and Method: Empirical Research Methods and the Interpretive Turn*. M.E. Sharpe

APPENDICES

APPENDIX A: ETHICS APPROVAL

RHODES UNIVERSITY
Where leaders learn

Rhodes University, Education Faculty
Research Ethics Committee
PO Box 94, Makhanda, 6140, South Africa
Tel: +27 (0) 46 603 8393
Fax: +27 (0) 46 603 8028
email: e.rosenberg@ru.ac.za

https://www.ru.ac.za/researchgateway/ethics/

15 August 2023

marisa lnisa kumm

Education Department

g22k3960@campus.ru.ac.za

Dear Mrs Marisa Kumm

Re: Exploring Foundation Phase pre-service teachers' mathematical pedagogical knowledge for teaching additive mental mathematics strategies

APPLICATION NUMBER: 2022-5717-6915

This letter confirms that your research ethics application has been reviewed and **APPROVED** by the Education Faculty Research Ethics Committee (EF-REC). Your permission letter(s) where applicable have been received and you are free to proceed with your study.

Approval is granted for 1 year. An annual progress report is required in order to renew approval for an additional period. You will receive an email notifying you when the progress report is due.

Should any substantive change(s) be made during the research process, that may have ethical implications, you should notify the Education Faculty REC Chair via email. This includes changes in investigators. The REC Chair will advise as to whether a new application is necessary.

Do keep this clearance letter secure and accessible throughout your study and after its completion. It will be needed when a thesis is examined and when publications are submitted to journals.

Please also submit a brief report to the REC Chair on the completion of the research. This can be done via email. The purpose of this report is to indicate whether the research was conducted successfully and whether any ethics-related matters arose that the committee should be aware of, in order to guide future studies.

Sincerely,

Prof Eureta Rosenberg

Chair: Education Faculty Research Ethics Committee

APPENDIX B: INVITATION TO PARTICIPANTS

Dear Student Teacher

Invitation to participate in a research study - Mental Maths Work-integrated Learning (MM-WIL) in Initial Teacher Education for Primary Mathematics

You are invited to participate in a research study entitled Exploring Foundation Phase pre-service teachers' mathematical pedagogical knowledge for teaching additive mental mathematics strategies.

What I would be asking of you is that you participate in the Mental Starters Assessment Programme (MSAP) which consists of pre- and post-tests, eight lessons per strategy, micro teaching, a semi-structured interview and your reflections and opinions about the way the lessons unfolded.

Your participation in the research will be anonymous. I would not directly reveal your identity. The collection of this data will require between 30 to 40 minutes per lesson.

If you are willing to participate, I will explain in more detail what would be expected of you and provide you with the information you need to understand the research. These guidelines would include potential risks, benefits, and your rights as a participant. Once this study has been approved by Rhodes University's Ethics Committee (Faculty of Education), I will send you a copy of their letter of ethical approval.

Participation in this research is voluntary. A positive response to this letter of invitation does not oblige you to take part in this research, as you will have the right to withdraw at any time. To participate, I will ask you to sign a consent form to confirm that you understand and agree to the conditions, prior to any interview commencing. Please note that you would have the right to withdraw at any given time during the study without prejudice.

Thank you for your consideration in this matter.

Yours sincerely,
Mrs Marisa Kumm

APPENDIX C: INFORMED CONSENT FORM

Research Project Title: Exploring Foundation Phase pre-service teachers' mathematical pedagogical knowledge for teaching additive mental mathematics strategies.

Principal Investigator: Mrs Marisa Kumm

Participation Information

- I understand the purpose of the research study and my involvement in it. My lecturer has informed me that my participation in this study will not affect my final year mark.
- I understand that I may withdraw from the research study at any stage without any penalty.
- I understand that participation in this research study is done on a voluntary basis.
- I understand that while information gained during the study may be published, my name will remain anonymous and no reference will be made to me by name.
- I understand that I will be given the opportunity to read and comment on the transcribed interview notes.

Information Explanation

The above information was explained to me by Mrs Marisa Kumm in English and I am in command of this language.

Voluntary Consent

I, ____ hereby voluntarily consent to participate in the above-mentioned research.

Signature: __________________________________ **Date:** / /

Investigator Declaration

I, Mrs Marisa Kumm, declare that I have explained all the participant information to the participant and have truthfully answered all questions asked of me by the participant.

Signature: __________________________________ **Date:** / /

APPENDIX D: PRE- AND POST-QUESTIONNAIRE

An Adapted version of pre-test on strategic calculations.

Solve the following calculations. Explain how you solved it

Solve the following calculations.	Explain how you solved it
(1a) 36 + 8	(1b)
(2a) 47 + 29	(2b)
(3a) 63 – 24	(3b)
(4a) 98 – 99	(4b)
(5) Describe the bridging through ten strategy.	
(6) How might you teach this strategy to a Grade 3 Learner?	
(7) What instant facts do learners need to know to be able to use bridging through ten?	
(8) Describe the jump strategy and how you might teach it.	
(9) What representation would you use to help learners understand bridging through ten and what would you write on the board?	
(10) Describe rounding and adjusting and how you might teach it.	