AN INVESTIGATION INTO THE KNOWLEDGE A GRADE ONE TEACHER USES TO DEVELOP THE NUMBER SENSE OF LEARNERS WITH MATHEMATICS LEARNING DIFFICULTIES

A thesis submitted in fulfilment of the requirements for the degree of

MASTER OF EDUCATION

(Mathematics Education)

of

RHODES UNIVERSITY

By

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June 2019

DECLARATION

I, Kirsty Fleming, hereby declare that the work in this thesis is my own and where ideas from other writers have been used, they are acknowledged in full using referencing according to the Rhodes University Education Guide to References. I further declare that the work in this thesis has not been submitted to any university for degree purposes.

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SIGNATURE DATE

ABSTRACT

Learners in South Africa continue to underperform in the international and national mathematics benchmarking tests. In the primary school, poor performance in mathematics is viewed as an indicator of limited number sense. Since the end of Apartheid, there has been a proliferation of classroom-based research that attempts to explain why learners are underperforming and find solutions to the problem. Research that seeks to explain learner underperformance attributes poor learner performance to social-economic issues, teachers' poor content and pedagogical knowledge, the complexity of the Language of Learning and Teaching, and insufficient support for learners with Mathematics Learning Difficulties (MLD). With regards to the latter, research suggests that Foundation Phase teachers are not equipped to assist learners with MLD develop their number sense. This qualitative case study aims to investigate the knowledge that an expert Foundation Phase teacher draws on, in the process of teaching, to assist learners with MLD develop their number sense. Data generated from observations and interviews with a Grade One teacher was analysed using Rowland, Turner and Thwaites' (2013) Knowledge Quartet. The study found that the participant Grade One teacher employed all four categories of the Knowledge Quartet when developing her learners' number sense. In particular, she placed strong emphasis on vocabulary development as a means of circumnavigating MLD when developing number sense in a Grade One mathematics lesson. She demonstrated knowledge of: the importance of vocabulary in learning mathematics; how to develop the learners' understanding of mathematics vocabulary (and concepts); and how to adapt her approach to support the number sense development of learners with MLD. This research has value for teacher education programmes, both pre- and in-service, as it highlights the knowledge that a Grade One teacher draws on as she develops the number sense of all her learners, including those with MLD.

DEDICATION

In memory of my grandpa, Mr James Brownlee McGhie Fleming. You were, and continue to be, an inspiration to me.

ACKNOWLEDGMENTS

This study would not have been a success without the unwavering support and encouragement from those who stood by me through the whole process. I would like to specifically acknowledge:

- My supervisor, Dr Lise Westaway. Without your unwavering support, critique and commentary, I would not be where I am today. Thank you for working alongside me to get this study to where it is. Without our constant meetings and emails, I do not think I would have achieved as much as I did both professionally and personally. Thank you for all that you did.
- To my other half, Matthew Liesenberg. Thank you for always being there for me through the good times and the bad. I know I was not always an easy person, especially at the end, but you were always there to take my mind off it all when I got too stressed and gave me the space to learn and grow.
- My mum and dad, Carrol and Seamus Fleming, thank you for always believing in me and what I can do. Having you support my chosen path has been one of the greatest gifts you could have ever given me. I hope that through this all I continue to make you proud of the person I aim to become.
- To my sister, Shannon Fleming, and cousin, Nicole Jackman, thank you for always being available when I needed you. Your love, support, and encouragement is what pushed me through.
- To my in-laws, Angela and Johann, you have been a rock through this whole process and a much-needed support system. Thank you for everything you have done for me throughout the years.
- To all my friends and family, for being there for me in any small way, I cannot thank you enough. This journey would not have been possible without the strong support system behind me.

- A special mention to Cathy and Chloe, who have walked this road before me or alongside me, your support is something I cannot describe. Without our weekly chat sessions, I would have been lost. I hope to one day repay you for all that you have given me in the last two years.
- Lastly, but most importantly, Elizabeth. I sincerely thank and appreciate you for everything that you have given me. Without you this study would not have been possible. Thank you for taking me in to your classroom and taking me under your wing, and for sharing your knowledge and wisdom with me. Thank you for being available when I needed you and for always being willing to share information with me no matter the circumstance.

Contents

List of Abbreviations

ANA	–	Annual National Assessments
ANS	–	Approximate Number Systems
BEd	–	Bachelor of Education
CCK	–	Common Content Knowledge
COCO	–	Count One Count All
EC	–	Eastern Cape
FoNS	–	Foundational Number Sense
HK	–	Horizonal Knowledge
KCC	–	Knowledge of Content and Curriculum
KCS	–	Knowledge of Content and Students
KCT	–	Knowledge of content and teaching
KQ	–	Knowledge Quartet
LiEP	–	Language in Education Policy
LoLT	–	Language of Learning and Teaching
MKfT	–	Mathematics Knowledge for Teaching
MKiT	–	Mathematics Knowledge in Teaching
MLD	–	Mathematics Learning Difficulties
OTS	–	Object Tracking Systems
PCK	–	Pedagogical Content Knowledge
PGCE	–	Post Graduate Certificate in Education
PRILS	–	Progress in International Reading Literacy Study
SACMEQ III	–	Southern and Eastern Africa Consortium for Monitoring Education Quality III
SACMEQ IV	–	Southern and Eastern Africa Consortium for Monitoring Education Quality IV
SA.DBE	–	South African Department of Basic Education
SA.DOE	–	South African Department of Education
SCK	–	Specialized Content Knowledge
SMK	–	Subject Matter Knowledge
TIMSS	–	Trends in International Mathematics and Science Study

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CHAPTER 1: CONTEXT

While classroom-based research was not common in South Africa prior to 1994, there has been a proliferation of research in primary schools subsequently (Hoadley, 2012). This context is focused on providing dominant explanations for learner underperformance in both literacy and numeracy. In this research, the performance of learners in national and international benchmarking tests is used as a starting point. The research aims to explain learner underperformance as a result of Foundation Phase teachers' content and pedagogical knowledge, using literature that shows that many teachers do not have the necessary pedagogical knowledge to develop Foundation Phase learners' number sense, especially learners with Mathematics Learning Difficulties (MLD).

1.1 LEARNER PERFORMANCE IN MATHEMATICS

In South Africa, research has identified several factors that influence learner performance. Both international and national benchmarking tests attest to the fact that South African learners are underperforming in mathematics. This can be seen in the results obtained from the Progress in International Reading Literacy Study (PIRLS), Trends in International Mathematics and Science Study (TIMSS), and the Southern and Eastern Africa Consortium for Monitoring Education Quality III (SACMEQ III) tests, which all indicate that South African learners' level of achievement is one of the lowest worldwide (Carrim, 2013).

The SACMEQ III reports that the average percentage of Grade 6 learners who are considered functionally innumerate[1] throughout South Africa is 40.17% and 50.34% in the Eastern Cape (Spaull, 2011). Put differently, these learners on an 8-point scale[2] are below Level 3 and thus, do not have basic numeracy (Spaull, 2011, Moloi, 2005, Department of Education (SA.DBE, 2010) or number sense which is critical for developing an understanding of mathematics (Graven, Venkat, Westaway & Tshesane, 2013). Soudien (2013) comments further on learner

[1] Functionally innumerate refers to when a child "cannot translate graphical information into fractions or interpret common everyday units of measurement" (Spaull, 2011, p. 34)

[2] The different competency levels look at the stages children need to cover before being considered fully competent in mathematics. Level 1 is pre-numeracy, Level 2 is emergent numeracy, Level 3 is basic numeracy, Level 4 is beginning numeracy, Level 5 is competent numeracy, Level 6 is mathematically skilled, Level 7 is concrete problem-solving, Level 8 is abstract problem-solving (Spaull, 2011; Moloi, 2005).

performance by stating that the results from the 2001 national Grade 3 systematic assessment indicated that the average score for numeracy, countrywide, was 30%. As Stott (2017) notes, this low level of basic numeracy has a negative impact on learners as they progress through school. This is due to learners barely attaining the required minimum score for their grade, which means they do not have a sufficient understanding of mathematics.

On a national and provincial scale, learner performance in the Annual National Assessments (ANA) indicates that in the period from 2012 to 2014 there was a slight increase in learner performance in mathematics in Grades 1, 2, and 3 (SA.DBE, 2014). As illustrated in Table 1, disaggregating the results according to provinces shows that the average percentage of learner performance in the Eastern Cape is generally lower than the National percentages (SA.DBE, 2014).

	2012		**2013**		**2014**	
	National	EC[3]	National	EC	National	EC
Grade One	68	65.2	60	56.2	68	64.5
Grade 2	57	55.2	59	59.9	62	63.7
Grade 3	41	40.5	53	50.6	56	52.2
Grade 4	37	35.3	37	32.6	37	34.8
Grade 5	30	28.1	33	29.1	37	32.2
Grade 6	27	24.9	39	33.0	43	36.8

Table 1.1 - Grade One – 6 ANA results for 2012, 2013 and 2014 (SA.DBE, 2014)

Moloi (2005) states that 44% of learners in South Africa who have been in formal schooling for six years are performing at the same level as expected of a learner who has been in formal schooling for three years. This links with Spaull and Kotze (2015), who highlighted the fact that the gap between learners in Grade 3 in Quintile 5 schools are three years ahead in their learning than their peers in Quintiles 1-3 school. In South Africa, the quintile system is used to denote the social-economic status of the community that the school is in. Learners at Quintile 5 schools attend schools in affluent communities, whereas learners in Quintile 1-3

[3] Eastern Cape

schools are from economically disadvantaged communities. Spaull and Kotze (2015) maintain that as learners' progress through the schooling system, the performance gap between learners in Quintile 5 schools and learners in Quintiles 1 – 3 school increases.

As seen in Table 1, there is a decrease in learner performance as learners' progress to the higher grades. In 2014, in the Eastern Cape, the average score moved from 64.5% in Grade One to 36.8% in Grade 6 (SA.DBE, 2014). This substantial decrease as learners move up the schooling system is reflected in 2012, 2013, and 2014. The results of the ANA thus indicate that learners are underperforming throughout the Foundation and Intermediate Phases.

Attempts to address the poor performance of learners becomes difficult as they move up the schooling system. Graven, et al. (2013) argue that the pressure teachers face when it comes to keeping up with the curriculum in the Intermediate Phase means that addressing issues that are a result of a poor instruction in the Foundation Phase becomes a secondary issue. In addition, Barber and Mourshed (2007) claim that placing learners who are not performing with teachers who lack the necessary content and pedagogical knowledge required to teach mathematics only exacerbates the problem of poor learner performance.

1.2 RESEARCH THAT EXPLAINS LEARNER UNDERPERFORMANCE

Research that explains learner underperformance can be broadly categorised into five inter-related categories: learner underperformance and socio-economic factors, learner performance and teachers' knowledge, learner performance and teacher education, language and learner performance, and knowledge of MLD and learner performance.

1.2.1 Learner performance and socio-economic factors

Whilst learner enrolment in schools has increased, the ANA results as highlighted in Table 1, suggest that Foundation Phase learners are not receiving foundational support in their mathematics development. This is exacerbated by the high levels of inequality in South Africa, where a growing proportion of the population lives in poverty (Graven, 2013). Gumede (2013) writes that poverty and underdevelopment influence learner performance as it can result in "poor adjustment to school, increased repetition of grades and dropping out of school" (p. 69). In addition, international research has indicated that language plays a large role in learner performance in the South Africa. As a result of the Language in Education Policy[4], many learners from low socio-economic status environments are learning in an additional language; that is, a language that differs from their home language. This impacts learner performance as they are learning, in the case of mathematics, a complex subject in a language that they are not completely fluent in (Graven & Venkat, 2017).

As Spaull (2011) argues, South Africa has a bimodal education system where learners in affluent areas get a better-quality education than those living in low socio-economic status communities. Whilst the 1995 White Paper on Education and Training states that learners in the well-resourced, elite schools are succeeding throughout their formal schooling there is still the issue that millions of school children are learning in schools in poor areas that are under-resourced (Hacking, 2013). Adler (2017) agrees with this by stating that the availability or resources, which are currently unequally distributed in South Africa, does have an impact on achievement.

Hacking (2013), Chisholm and Chilisa (2012), Graven (2013) and Adler (2005) all attribute this inequality to the legacy of Apartheid. Despite numerous efforts to redress the inequalities of the past, South Africa has one of the highest levels of economic inequality and educational inequality in the world. This has negatively affected learner performance, specifically in mathematics (Graven & Venkat, 2017).

1.2.2 Learner performance and teacher knowledge

One of the main issues that affects learner performance is that of teacher competence. Carrim (2013) argues that teacher competence, and lack of preparation, often indicates a lack of content and pedagogical content knowledge. Adler (2005) and Kazima, Pillay and Adler (2008) concur that teachers' mathematics and pedagogical knowledge impacts on their teaching, and thus, learner performance.

Research has indicated that there are high numbers of primary school teachers who have insufficient content knowledge. Venkat and Spaull (2014) report on the analysis of Grade 6 teachers' mathematical competence. Four hundred and ninety-eight Grade 6 teachers

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[4] This is elaborated on further in *Language in South Africa and learner performance*

volunteered to write the SACMEQ III test that was given to their learners. The results indicated that 79% of the teachers' knowledge of mathematics content was below Grade 6 and Grade 7 level. The results for the Grade 6 teachers located in the Eastern Cape showed that 17% of the teachers had the content knowledge required to teach Grade 6 mathematics. In addition to this low percentage, many of the teachers in the Eastern Cape declined to take part in the test which could be a result of their not being satisfied with their level of content knowledge (Venkat and Spaull, 2014).

When comparing the efficiency and effectiveness of South African teachers with teachers in other African countries, it is clear that the lack of content knowledge, along with other affecting factors, does have a direct effect on learner performance (Carnoy & Arends, 2012). When contrasting learner performance with that of their respective teachers, the teachers who had higher scores on the SACMEQ III mathematics test were found to be more effective, resulting in their learners obtaining higher scores in the test. Whilst the results of teachers' insufficient content knowledge, as shown in the SACMEQ III data focuses on Grade 6 teachers, they indicate that there is a possible correlation between teachers' content knowledge, their teaching capabilities, and learner performance. This is expected as Taylor (2008) notes "teachers cannot teach what they do not know" (p. 24).

In a study conducted in the United States of America, McCray and Chen (2011) indicate that most early childhood teachers lack the necessary content knowledge to teach mathematics and many are fearful of teaching mathematics. They maintain that this is aggravated by teachers' lack of confidence in their mathematics teaching abilities. In South Africa, the concern with teachers' content and pedagogical practices is also evident in the early years. Feza (2016) found that some Grade R teachers lack the basic skills to develop learners' mathematical knowledge. Seemingly, she argues that these teachers have insufficient content knowledge, a limited understanding of what number sense[5] is and little knowledge of how to develop learners' number sense (Feza, 2016). Jordan, Dyson and Glutting (2011) found that learners who enter Grade One with weak mathematics skills will be more disadvantaged than

[5] Number sense is elaborated on in Chapter Two. For the purpose of this chapter, number sense refers to the abstract outline of number information that is needed to understand number, number relationships, strategies for calculating, and solving number-related problems (Way, 2011).

learners who start the year with strong mathematics skills. They stress that it is important for Grade R teachers to develop learners' mathematics skills prior to formal schooling.

Drawing on various case studies in South Africa that focus on explaining learner underperformance, Hoadley (2012) concurs that learner performance is a result of teachers' poor pedagogical knowledge and practices. The purpose of the Count One Count All (COCA) study, conducted in the Western Cape, was to identify how Foundation Phase teachers develop their learners' number sense. The study found that many teachers are not equipped with adequate pedagogical knowledge to develop learners' number sense (Hoadley, 2012). While the meaning of number sense is highly contested, many researchers agree that it is vital for developing an understanding of mathematics (Anghileri, 2000). The COCA study found that teachers were not equipped to develop their learners' competence to make quantitative judgements and work flexibly with number and numerical computations. These are regarded as key characteristics of number sense that both learners and teachers require (Greeno, 1991).

Poor pedagogical practices limit the possibility of learners developing number sense. Hoadley (2012) suggests that part of the problem is that teachers tend to encourage the use of concrete methods for calculating rather than developing more abstract methods (Hoadley, 2012). Similarly, Graven, et al. (2013) found that teachers tend to progress learners from concrete methods, such as unit counting, to the formal, abstract algorithms, without developing alternative strategies, thus mitigating the development of learners' number sense. Venkat and Spaull (2014) concur arguing that teachers tend to use highly procedural methods and as such, encourage rule following rather than the development of learners' number sense.

Nel and Grosser (2016) suggest that insufficient content knowledge, poor pedagogical practices, and a lack of learner support, all impact on learners' performance. They claim that "inflexible teaching and assessment approaches that do not cater for diverse learner needs and styles" (p. 83) can cause a breakdown in learning and teaching in the classroom. Teachers require specific pedagogical knowledge and assistance to deal with learners who need additional support (Bird, Moon and Storey, 2013). In an attempt to explain why teachers have poor content and pedagogical knowledge, Feza (2015*)* suggests that the problem lies with the quality and appropriateness of pre-service and in-service teacher education programmes in South Africa.

1.2.3 Learner performance and teacher education

Adler (2005) and Kazima, Pillay, & Adler (2008) maintain that teacher education institutions play a significant role in supporting the development of the content and pedagogical knowledge that teachers require to teach mathematics. Poorly trained teachers are most likely to provide their learners with a less than satisfactory education (Dembélé & Miaro-II, 2003). The implication of poor teacher education programmes is that teachers battle to facilitate learners' conceptual understanding of mathematics (Dembélé & Miaro-II, 2003).

In a study done in Malawi, Kazima, & Mussa (2011) identified that qualified teachers, who have had good teaching practice experiences in their pre-service teacher education, enter schools aware of the challenges they will face; whereas teachers who do not have the necessary training are not equipped with the necessary skills to face the challenges that exist within the classroom. However, research also indicates that despite learning, for example, progressive methods during their teacher education programmes, teachers tend to revert to methods influenced by their beliefs of what mathematics is and how it should be taught (Raymond, 1997; Handal & Herrington, 2003).

Gallie (2013) suggests that the quality of teacher education programmes needs to be considered when identifying the factors of poor learner performance. The content that teacher training programs provides teachers with, in relation to pre-service and in-service training, needs to be re-evaluated to avoid the constant 'workshopping' of teachers to try solve learner performance issues (Hacking, 2013). Graven and Venkat (2017) point out that there are several factors that teachers need to contend with, such as "lack of teacher content knowledge" and "incoherent presentation of concepts" (p. 13). Teachers require support when developing concepts for numeracy learning and remediation that does not result in teachers being 'workshopped'.

Feza (2015) argues that teachers in South Africa who are employed at more affluent schools tend to have a better understanding of number sense development than teachers in low socio-economic status schools. She argues that this is a result of the quality of the teacher education programmes teachers were enrolled in. This may have been the case historically when teacher education institutions were segregated by race. A pre-service teacher education research study, coordinated by the Joint Education Trust, was conducted in 2015 (Bowie, 2014). It was a desktop review of the content and conceptions of teaching in the Bachelor of Education (BEd) and Post Graduate Certificate in Education (PGCE) curricula from five South African universities. The results of this research confirmed that "there is no agreement among mathematics teacher educators on exactly which types of courses are likely to best meet teachers' needs" (Bowie, 2014, p. 105). In other words, all five teacher education institutions had different views of the content and pedagogical knowledge that should be taught and at what level. This research has led to a number of research projects, such as the Primary Teacher Education: Number Sense project which aims to develop a set of common standards and a shared understanding of a quality mathematics teacher.

1.2.4 Language in South Africa and learner performance

During the apartheid era, learners were taught in their mother tongue from Grade 1 to Grade 4 (Setati, 2002). Thereafter, English and Afrikaans, which were the official languages in South Africa at that time, were phased in as the Language of Learning and Teaching (LoLT) (Setati, 2002). Post-1994[6], twelve languages, including sign language, were recognised as official languages in South Africa. The increase in official languages led to the development of the Language in Education Policy (LiEP) (1997), which dealt specifically with the question of language in schools. The LiEP stated that learners (and parents) have the right to 'choose' the preferred LoLT. Parents of children who spoke an indigenous language had a 'choice': either their children were taught in a second language (e.g. English or Afrikaans) from Grade 1; or they were taught in their home language till the end of Grade 3. From Grade 4 onwards they would then be taught in English or Afrikaans (Setati, 2002). However, a disclaimer was added to the LiEP: Learners and parents may choose provided the "school uses the language of learning and teaching chosen by the learner, and where there is a place available in the relevant grade, the school must admit the learner" (SA.DoE, 1997, p.2). While policy suggests that learners and parents have the right to select their preferred LoLT, this is not the case for all. With the LoLT in the majority of Quintile 1-3 schools being an indigenous language up to Grade 3, teachers in the higher grades are often required to make use of code-switching when

[6] Post-1994 refers to the official end of apartheid

teaching (SA.DOE, 1997; Setati, 2002; Setati & Adler, 2000). As can be seen in Table 2 below, a high percentage of learners who do not speak English regularly at home. It is worth noting that almost one-fifth of learners in the Eastern Cape never speak English at home.

	Never	Sometimes	Most of the time	All the time
Eastern Cape	18.2	61.0	10.4	10.4
South Africa	11.8	64.0	10.7	13.4

Table 1.2 - Percentage of learners who speak English at home in the Eastern Cape and South Africa (adapted from SACMEQ IV Report (SA.DBE, 2017))

In South Africa, English is the language of economics and politics. Given the hegemony of English, many parents want their children to be taught in English. Parents thus work to send their children to Quintile 4 and 5 schools, where English is the LoLT. Whilst these schools are largely multicultural, English is the LoLT. Setati & Adler (2000) and Setati (2002) maintain that this has an impact on the learning of the learners. Yet the recent PIRLS study shows that learners in Quintile 5 schools, irrespective of home language, out-perform learners in Quintile 1-3 schools (Howie, Combrinck, Roux, Tshele, Mokoena, & McLeod Palane, 2017).

According to the 2016 Progress in International Reading Literacy Study (PIRLS), the literacy rate of learners in South Africa indicates that 78% of Grade 4 learners cannot read for meaning. This means learners cannot infer from a text, extract meaning, and explicitly state what has occurred in a text. These results, compared to the 50 other participating countries, indicate that South African learners scored the lowest marks in the study (Mullis, Martin, Foy, & Hooper, 2017; Howie et al., 2017). Howie et al. (2017) identified several factors underpinning the poor performance of the South African learners in the PIRLS study: 75% of the learners who participated in the test were from low socio-economic background, and 94% of the learners were at schools that lacked resources. Furthermore, 62% of the learners did not have access to a school library (Howie et al., 2017). These factors all have a negative impact on the opportunities learners have to learn and develop their language competence. This is supported by the SACMEQ IV report (SA.DBE, 2017). The SACMEQ IV report indicates that 57% of learners in South Africa do not have access to a library and 3.7% of the learners, who do have access to a library, are not able to borrow books from their school library. These studies reveal that learners in South Africa do not have sufficient opportunity to develop their LoLT language inside or outside the classroom which. This puts them at a disadvantage.

1.2.5 Mathematics Learning Difficulties and learner performance

MLD refers to the difficulties that learners may experience in different areas of mathematics. It is not a disability as it does not affect the learning process on a long-term basis (Gersten, Jordan & Flojo, 2005). In other words, MLD tends to be temporary (Gersten et al., 2005). If a learner is identified as having a MLD when learning to count there is a higher chance of him or her experiencing complications when grasping counting concepts. The order-irrelevance principle, as defined by Gelman & Gallistel (1978), states that counting order is not relevant (e.g. whether the learner counts from left to right or right to left) but what matters more is whether a learner can count without repeating an item in the selection. Gersten et al. (2005) maintain that learners who have not grasped the order-irrelevance principle (i.e. they are assigning multiple number names to a single object) lack flexibility in their counting strategies and that this has implications for learning mathematics. Jordan, Glutting, & Remineni (2010) reiterate this when they identify that weaknesses in counting and number comparisons can lead to learners experiencing mathematics difficulties.

It is important to note that there is no single way to identify and support learners with MLD as all MLD are different. If a teacher is not aware, or equipped, with the necessary knowledge to identify and support learners with a MLD, it is likely that these learners will underperform as they move up the schooling system. Oktaç, Roa Fuentes & Rodríguez Andrade (2011) write that a teacher acknowledging and adjusting their teaching approach to accommodate the different learning processes used by learners with MLD does not mean that the teacher is neglecting one student over another. Rather, the teacher is finding a way to meet all the needs of every learner in their classroom, whether they be gifted learners or experiencing MLD. Research has indicated that learners in the Foundation Phase are more responsive to interventions from an early stage (Graven & Venkat, 2017).

Stott, Mofu and Ndongeni (2017) indicate that the use of written tests is not sufficient in identifying learners' strengths and weaknesses, rather there is a need for teachers to utilise a set of diagnostic tools that can be used to identify specific problems. Gervasoni (2004) suggests that teachers should utilise a method, for example one based on Vygotsky's zone of proximal development, to identify what learners know and develop learning opportunities based on learners' prior knowledge.

1.3 PROBLEM STATEMENT

As indicated above, there is an on-going concern about learners' performance in mathematics in South Africa. Research has shown that the pedagogical knowledge and content knowledge of teachers has an influential role in learners' performance. Whilst research has identified the poor pedagogical knowledge of teachers in South Africa as a possible explanation for learner performance, there is a need to investigate the knowledge that competent teachers draw on to assist learners who experience difficulties when learning mathematics. For this reason, my research attempts to identify the content and pedagogical knowledge that Foundation Phase teachers require to effectively work with learners who are likely to perform poorly when developing their number sense, specifically those learners with a MLD.

1.4 RESEARCH GOALS AND QUESTION

My research focuses on the content and pedagogical knowledge that is needed to support learners who have MLD, particularly in relation to number sense development.

1.4.1 Research Goals

The research aims to identify the content and pedagogical knowledge that is required for developing learners' number sense during mathematics lessons and the knowledge that a teacher needs to adapt their teaching to the benefit of learners with a MLD. This research contributes to two areas of study in South Africa that are under-researched; that is, the mathematics knowledge for teachers require to teach mathematics and MLD.

1.4.2 Research Question

The following question lays the basis for the study:

What content and pedagogical knowledge does a Grade One teacher use to assist learners with Mathematics Learning Difficulties to develop number sense?

As shown above, five key explanations for learner underperformance have been highlighted. Firstly, there is a discrepancy in the quality of education learners receive in affluent socio-economic communities and those in low socio-economic communities. This is primarily a result of the legacy of apartheid. Secondly, many teachers have insufficient content knowledge and poor pedagogical skills. Thirdly, the lack of content and pedagogical knowledge has been attributed to the teacher education system. Fourthly, the LoLT for many of the learners is not their first language. Finally, many teachers are not adequately prepared to assist learners with MLD and specifically in relation to number sense development.

1.5 OUTLINE OF THE THESIS

My research thesis consists of six chapters which are structured as follows:

Chapter One provides the context of the study. It highlights the crisis in the South African education system by examining learner performance in national and international research. The evidence for poor performance in mathematics can be explained by several factors. One of the factors that is central to my research is the apparent poor pedagogical knowledge of many Foundation Phase teachers. The research suggests that many Foundation Phase teachers do not have the knowledge required to develop learners' number sense and support learners with MLD.

Chapter Two explains the importance of number sense and how the impact of a poor number sense development can lead to a prevalence of MLD in the Foundation Phase. Literature on ways to develop number sense with learners who have MLD is discussed throughout the chapter. In so doing, the influence of language in supporting learners with MLD is highlighted.

Chapter Three presents the theoretical framework. Shulman's (1986, 1987) Pedagogical Content Knowledge (PCK), Ball, Thames, and Phelps' (2008) Mathematics Knowledge for Teaching (MKfT) and Rowland, Turner, and Thwaites' (2013) Knowledge Quartet are reviewed. Based on the critiques of Shulman (1986, 1987) and Ball et al. (2008), the Knowledge Quartet of Rowland et al. (2013) was chosen as the analytic and explanatory framework for this study.

Chapter Four provides a description of the methodology that was followed throughout the study. The research is a qualitative case study which is informed by an interpretivist orientation. In order to identify the content and pedagogical knowledge that a Foundation Phase teacher uses to assist learners with MLD, a well-established, 'expert' teacher was observed over a period of five weeks. Lessons, observations and interviews were used to generate the data. Data analysis involved both emic and etic coding.

In **Chapter Five** the data is presented and analysed using Rowland et al.'s (2013) KQ. The codes identified as part of the KQ are used to highlight key findings in this research.

Chapter Six provides a conclusion for the study. In this chapter, the findings, limitations and opportunities for further research are discussed.

CHAPTER 2: CONCEPTUAL FRAMEWORK

2.1 INTRODUCTION

The international and national benchmarking tests show that many South African learners do not have the number sense required for success in school and everyday life. As noted in Chapter One, there are many explanations for this. Of significance to my research are the explanations relating to teachers' poor content knowledge and insufficient pedagogical skills, specifically in relation to language development and working with learners with MLD. In this chapter, I present various conceptualisations of number sense and consider how language competence, specifically vocabulary, impact on number sense development. In addition, I examine the role of MLD on number sense development and draw on research to make suggestions of how teachers can support learners with MLD develop number sense

2.2 NUMBER SENSE

Mathematics plays an important role in society. It is a subject that prepares learners to negotiate the numerical demands in society. With this in mind, mathematics is an important tool for the advancement of society. Mathematics also supports individuals in becoming active citizens, that is individuals who can think numerically and spatially in order to interpret and analyse everyday situations and solve problems (Sa'ad, Adamu & Sadiq, 2014). The success of individuals in interacting and understanding our numerical world is, in many respects, dependent on the quality of mathematics education received at school. Number sense is regarded as key to learners' mathematics performance in primary school.

Dehaene (1997) and Spelke (2000) maintain that infants are biologically endowed with number sense. This means that number sense is innate in that it 'exists' prior to birth. There are two particular number sense systems which infants are born with. These are the approximate number system (ANS) and the object tracking system (OTS). Dehaene (1997) and Spelke (2000) regard these systems as core knowledge. From birth, infants draw on the ANS to identify the differences in magnitudes between groups of objects (Mazzocco, Feigenson and Halberda, 2011; Dehane, 1997). In this sense, infants are able to subitise, estimate and determine 'more' and 'less' without any mathematical language. Drawing from the ANS, the OTS indicates that infants are able to identify and keep track of objects that are quantifiable from zero to three (Piazza, 2010; Feigenson, Dehaene, and Spelke, 2004). This core knowledge system provides the background for the development of quantification and addition and subtraction with small numbers.

Lakoff and Nunez (2001) affirm that, from a very early age, babies can perform rudimentary numerical distinctions. As Dehaene (1997) mentions "we are born with multiple intuitions concerning numbers, sets, continuous quantitates, logic and the geometry of space" (p. 245 - 246). This means that humans are born with the innate ability to perform simple arithmetic and the ability to subitise small numbers (Dehaene, 1997; Lakoff and Nunez, 2001). Dehaene (1997) maintains that school mathematics provides learners with "arithmetic techniques" as well as helping learners identify "links between the mechanics of calculation and its meaning" (p 139).

Haylock (2010) indicates that learning mathematics can be made to be fun and enjoyable. This enables learners to grasp mathematical concepts and skills in a relaxed environment. It assists learners to make sense of problems and develop appropriate solutions to solve them. As such, it promotes creative and imaginative thinking. Key to developing an understanding of mathematics in the Foundation Phase is the development of number sense. 65%, 60% and 58% of mathematics teaching time in Grade One to Three classes are allocated to Number, Number Operations and Relationships respectively (SA.DBE, 2011).

Number sense is a highly contested and elusive concept. As Witzel, Riccomini and Herlong (2013) affirm, number sense has several descriptions that have developed over time and as number sense related research has increased. Howden (1989) who appears to have developed the concept of number sense out of the Cockcroft Report, refers to number sense as being a 'friendliness with numbers'. This description is extended by Greeno (1991) who writes that number sense refers to "several important but elusive capabilities, including flexible mental computation, numerical estimation, and quantitative judgment "(p. 170). The fluidity and flexibility of understanding and working with numbers is also identified as an important component of number sense by McIntosh, Reys, and Reys (1992) and Gersten and Chard (1999). Drawing on this, Gurganus (2004) states that number sense is the awareness of numbers and understanding that there are diverse meanings for numbers. The NCTM (1989) builds on this and refers to number sense as "an intuition about number that is drawn from all varied meanings of number", for example, understanding the numerosity of numbers (NCTM, 1989, p. 39).

Gersten and Chard (1999) maintain that number sense is the ability to understand the meaning of numbers and perform mental mathematics. Jordan (2007) provides more clarity on the 'meaning of numbers' as related to number sense by indicating that it is "the ability to grasp and compare quantities (6 versus 8); internalize counting principles (the final number in a count indicates the quantity of a set, numbers are always counted in the same order); and estimate quantities on a number line" (p. 64). Berch (2005) expands on the understanding of the 'meaning of numbers' in the development of number sense. Drawing on the work of McIntosh et al. (1992), Dehaene (1997), and Yang and Reys, (2001) he acknowledges the complex nature of number sense. Berch (2005) writes that number sense includes an

> understanding [of] the meaning of numbers to developing strategies for solving complex math problems; from making simple magnitude comparisons to inventing procedures for conducting numerical operations; and from recognizing gross numerical errors to using quantitative methods for communicating, processing, and interpreting information (p. 333 – 334).

In South Africa, the Department of Basic Education (SA.DBE, 2011) provides teachers with an explanation of what number sense is. They state that number sense is developing an understanding of numbers, their differences, their sizes, the relationships that numbers have, the different representations of number, and the different operations that are involved with numbers use (SA.DBE, 2011). This links with what Graven et al. (2013) identify number sense as being. They state that number sense goes beyond the ability to count and identify numbers; rather it is being able to understand number relationships, perform calculations mentally, and use numbers outside the classroom (Graven et al., 2013). Courtney-Clarke and Wessels (2014) support the view expressed by Graven et al. (2013) that the ability to make sense of numerical situations in everyday life supports the development of number sense.

The development of learners' number sense from birth has an influence on the learners' foundational number sense (FoNS). FoNS is integral in promoting mathematical competence, which is typically evident when the learner first starts school (Sayers and Andrews, 2015). This links with Aunio (2006) who mentions that a lot of "preliminary mathematical knowledge develops in early childhood, providing a basis for later formal mathematics learning" (p. 1). This foundational number sense and its subsequent development is an important part of mathematics education in schools (Anghileri, 2000). When number sense is promoted in the classroom, learners will develop a more successful understanding of mathematics (Anghileri, 2000; Yang & Reys, 2001).

Taking the above into consideration, number sense can be regarded as a conceptual framework of number information that is needed to be able to understand number, number operations and relationships, and to solve complex mathematics-related problems in and out of school (Witzel et al., 2013; Berch, 2005; Gersten and Chard, 1999; Greeno, 1991; NCTM, 1989; Graven et al., 2013).

Drawing on the conceptualisations that have been identified above, number sense has particular characteristics. These include:

1. An awareness of the relationship between number and quantity;
2. An understanding of number symbols, vocabulary, and meaning;
3. The ability to engage in systematic counting, including notions of cardinality and ordinality;
4. An awareness of magnitude and comparisons between different magnitudes;
5. An understanding of different representations of number;
6. The use of flexible and efficient strategies for calculating;
7. An awareness of number patterns including recognising missing numbers;
8. The ability to communicate mathematically with understanding.

(Sayers and Andrews, 2015; Black, 2014; SA.DBE, 2011; Aunio, 2006; Garganus, 2004; Gersten and Chard, 1999; McIntosh, Reys, Reys, Bana, & Farrell, 1997; McIntosh et al., 1992; Greeno, 1991)

Based on this it can be identified that number sense does not refer to one individual aspect of mathematics (i.e. the ability to count or the ability to perform number operations), rather it encapsulates a variety of mathematical characteristics that a person needs to realise in order to develop a complete understanding of mathematics.

2.3 NUMBER SENSE AND MATHEMATICAL UNDERSTANDING

Number sense development is critical in understanding mathematics at school and in everyday life (Haylock, 2010; Graven et al., 2013). It enables learners to make sense of problems, find appropriate solutions, and promotes creative and critical thinking. Research shows that developing number sense plays a vital role in developing learners' mathematics abilities in the Foundation and Intermediate Phases (Graven et al., 2013).

Research has indicated that number sense is linked with the retrieval of facts from as early as Grade One (Witzel et al., 2013). Hopkins and de Villiers (2016) identify that it is important for children to develop efficient retrieval-based strategies in their early years of education. Retrieval-based strategies are the techniques to quickly access, and without error, mental mathematics facts (i.e. to know that 6+8=14 from memory) (Gersten and Chard, 1999). This is particularly important in solving calculations where children can break down numbers in order to calculate (Hopkins and de Villiers, 2016). Retrieval capabilities are linked with number sense development, where learners with good number sense can understand number representations and are competent with number operations (Black, 2014, Way, 2011, SA DBE, 2011).

Learners who have mathematics learning difficulties (MLD) often struggle with some basic mathematics processes such as developing retrieval-based strategies that are taught in the classroom (Hopkins and de Villiers, 2016). Therefore, learners who experience MLD need to be encouraged to develop alternative retrieval-based strategies in order to "circumvent what is considered to be the typical route of retrieval development" (p. 312). If retrieval-based strategies rely on solving problems using a prescribed strategy (such as a drill and practice approach to fact-retrieval) then the unlikeliness of learners with MLD developing their own flexible problem-solving strategies increases (Gersten and Chard, 1999). Encouraging learners to adapt their learning to a way that suits them allows them to discover strategies that work for them. This, in turn, encourages learners to be more flexible in their general problem-solving abilities[7] as they are not taught one specific strategy. Rather, they are encouraged to

[7] Problem-solving abilities refer to the learner's ability to solve word problems and calculations and work with different number representations efficiently and effectively (Black, 2014; Way, 2011; SA.DBE, 2011; Hopkins & de Villiers, 2016)

develop a variety of strategies for calculating (i.e. strengthening learners' subitising skills) (Hopkins and de Villiers, 2016).

Knowing how to count is considered an important skill for learners to grasp. It is seen as integral to comprehending the ordered number system (Weitz & Venkat, 2017; Witzel et al., 2013). Counting, as identified by Gelman and Gallistel (1978), relies on five principles that influence its development. These are the:

- the one-to-one principle which involves learners being able to assign a number name to each object that has been counted;
- stable order principle identifies that number words are used in a set order (i.e. 1, 2, 3 and note 3, 1, 2);
- the cardinal principle expands on the first two and states that the last number counted is the number of items in the collection that has been counted;
- the abstraction principle states that the cardinal principle can apply to both tangible and non-tangible objects all the while being able to distinguish with what has been counted and what still needs to be counted; and
- the order-irrelevance principle determines that the order in which the items are counted (left to right or right to left) is not relevant as long as all the items are counted once.

The above principles are divided into the 'how to count' and 'what to count' principles. The first three principles listed above are the 'how to count' principles and the latter two refer to the 'what to count' principles (Gelman and Gallistel, 1978).

International research shows that there is a relationship between a learner's ability to count in pre-school and their performance when they enter Grade One (Manfra et al., in Weitz & Venkat, 2017). The ability to count is complex as it requires two processes that the learner needs to grasp. The first process involves the ability of the learner to distinguish between what has been counted and what still needs to be counted (Weitz and Venkat, 2017). The second process requires the learner to count items one at a time. In order to count successfully, learners need to be able to connect the item that has been counted with the corresponding number word (Weitz & Venkat, 2017). Due to counting being more intensive and requiring a higher level of cognitive understanding from learners (than some might initially think) it is an important factor in determining a learner's numerical competence and number sense (Weitz & Venkat, 2017).

Therefore, it can be surmised, from the research above, that learners who are unable to count successfully or develop inadequate retrieval-based strategies will struggle to develop number sense and are likely to perform poorly in mathematics. This could also allude to the possibility of the development of an MLD if not addressed timeously.

2.4 MATHEMATICS LEARNING DIFFICULTIES (MLD)

Gersten and Chard (1999) propose that there is a relationship between number sense and learning deficits. Whilst a small percentage of the population experiences a mathematics learning disability, such as dyscalculia, these differ from MLD. Mathematics learning disabilities are generally more biologically based and require additional assistance and intervention as they tend to persist over a long period of time (Mazzocco, 2007). MLD differ in that they are generally an effect of teaching. For example, a learner could struggle with a specific concept (e.g. doubling) and only be affected for a short period of time (Mazzocco, 2007).

Jordan et al. (2010), Weitz and Venkat (2017) and Courtney-Clarke and Wessels (2014) identified that number sense can be a good predictor for later mathematics performance where learners who lack the foundational understanding of mathematics experience difficulties later in their schooling years. It is more difficult to 'remediate' MLD learners who do not have the necessary foundations in mathematics as they proceed up the schooling system. The evidence from the ANA results, provided in Chapter One, attests to the fact that learner performance worsens as a learner advance up the schooling system. Jordan et al. (2010) found that learners who enter Grade One with foundational knowledge of number sense are more likely to progress through their primary school years without experiencing MLD (Jordan et al., 2010). If number sense is not developed accordingly it can influence learners' retrieval abilities, working memory, and understanding of cardinality concepts (Jordan et al., 2010, Hopkins, 2016). Learners with an MLD can ultimately grasp basic mathematical concepts in the Foundation Phase, provided they have a competent teacher.

2.5 IDENTIFICATION OF MLD

Despite learners not being aware that they are using mathematics in many of their daily activities, Mazzocco (2007) writes that “mathematical thinking permeates the daily activities of a young child” (p. 40). If an MLD is identified early in a learners’ schooling, then an intervention can be put in place to assist the learner in overcoming the difficulty. Early interventions assist in avoiding problems later in his/her schooling career (Mazzocco, 2007).

Different teaching strategies can be used to develop learners’ number sense. (Gurganus, 2004). Courtney-Clarke & Wessels (2014) suggest that teachers should teach in a manner that focuses on the learners and the solution strategies they develop in order to promote a conceptual understanding of mathematics. This method of teaching is difficult for many teachers as it challenges the idea of rote teaching and the repetitive practice of number operations. Gurganus (2004) identifies that if number sense is not present, and rote learning of rules and taught procedures is more prominent in the classroom, learners with MLD will have less chance of being successful in mathematics.

Graven and Venkat (2017) explain that low performing learners, who require additional support, are often not challenged in the classroom. In this way, learners with MLD are not given the opportunity to engage in more challenging mathematical ideas. Teachers tend to offer a ‘dumbed down’ mathematics education to learners with MLD, and they appear to confuse a temporary MLD in a specific area of mathematics, with an inability to do mathematics.

Gervasoni (2004) suggests that learners with MLD can have a variety of learning needs that have to be addressed, and teachers need to have the necessary knowledge to assist these learners. Hopkins (2016) suggests that identification of learners’ learning styles and adjusting the teaching methodology accordingly will support learners who have an MLD in grasping the particular concepts they are struggling with. Graven and Venkat (2017) indicate that a remediation intervention needs to be highly structured and take the individual needs of the learners into account. These interventions are seen to be extremely beneficial as they can bridge the ‘ability’ gap between the learners within the classroom (Graven & Venkat, 2017).

In order to be able to identify and assist learners who experience MLD in their classroom, teachers need to be able to identify learner errors as they occur. Research has shown that the teachers' ability to interpret their learners' work is essential in the process of remediation (Sapire & Shalem, 2016). If a teacher can determine the frequency of error making, and identify if there is a pattern, it will support them in determining the approach s/he should take when assisting the learner (Stott, 2017; Sapire & Shalem, 2016; Ball et al., 2008).

Stott (2017) noted that assessing a learner's proficiency in mathematics enables the teacher to identify to what extent a learner has mastered the mathematics concepts and assists in determining the course of action that needs to be taken to develop their understanding and proficiency in mathematics. However, due to the pedagogical load teachers experience, errors that individual learners make are often ignored. Rather, teachers tend to re-teach the work using the same pedagogical strategies (Sapire and Shalem, 2016).

Research on MLD has highlighted several potential sources that could cause MLD in the classroom. These include:

- The *language of mathematics*: Mathematical language is often ambiguous. If a learner does not understand the required terminology, they are unlikely to achieve mastery of mathematical concepts.
- *Spatial skills*: The ability to approximate and estimate is hampered if a learner is not able to recognise the differences in size, distance or amount.
- *Memory skills*: The ability to perform simple arithmetic is constrained if learners do not have a variety of retrieval strategies.
- *Working memory*: Working memory is linked to memory skills more generally. A learner has to rely on their memory when completing a task (i.e. when doing simple addition, the learner needs to remember the two numbers and where to start counting). If they are not able to do this, it can impact their mathematical thinking.

  (Hopkins & de Villiers, 2016; Mazzocco, 2007; Geary, 1993; Gersten et al., 2005; Dehaene, Spelke, Pinel, Stanescu & Tsivkin, 1999; Dowker, 2005)

In order to assist learners in overcoming an MLD, teachers require the ability to identify the potential sources of the MLD and know how to deal with the potential MLD.

2.6 IMPACT OF LANGUAGE AND VOCABULARY ON MATHEMATICS

Language plays an integral role in learners' mathematical understanding and the development of number sense. Fritz, Balzer, Herholdt, and Ragpot (2014) maintain that language is central to the development of number sense, as it is the medium required to articulate mathematical understanding. For example, language is important in learning to count properly. Learners need to be able to match the quantity of a set of objects to the number word and the number numeral before deemed 'able to count'. The importance of vocabulary and meaning[8] (e.g. number words meaning) has been identified as a possible characteristic of number sense (Huang, Spelke, & Snedeker, 2013).

Haylock (2010) argues that poor mathematics results are often associated with poor reading and language skills. The language in which mathematics is presented to learners has the possibility of being confusing as it is considered ambiguous. Jordan and Levine (2009) acknowledge that of the learners who experience an MLD more than half of them also experience reading and language related difficulties. Ernest (2011) acknowledges that the "unique formal language and symbolism of mathematics provides an extra set of problems in learning mathematics" (p. 79). Ernest (2011) expands on this by stating that in mathematics there are words that have different meanings to those used in everyday contexts and there are some words that have more than one meaning. For example, in an everyday context, the word 'product' refers to an item in a shop, but in mathematics, the word 'product' signifies the result of a multiplication calculation. This difficulty with the different meanings of words in mathematics and in everyday life is compounded for learners whose home language differs from the Language of Learning and Teaching (LoLT). Until they gain mastery in the LoLT then the probability of them experiencing difficulties is more likely. Cohen and Manion (1997) state succinctly, "a child can neither learn specific skills nor develop his [or her] potential ability until he [or she] can learner to speak, understand, read and write the language that is used in school" (p. 192). This is supported by Mushin, Gardner and Munro (2013), who write that

> students' success … in communicating what they understand about a mathematical concept depends not only on their capacity to use and understand the language

[8] Refer back to the possible characteristics of number sense on page 17.

> associated with that concept but also on the degree to which they understand the language of instruction itself (p.416).

In order for learners to be able to count and order numbers correctly, they need to be able to understand the "spoken and written language of the numeration system" (Wigley, 1997, p. 113). Morin and Franks (2009) identify that learners who have language impairments often experience difficulties in mathematics. A limited mathematics vocabulary effects not only counting, but also

> number naming and sequencing (i.e., rote counting) and number processing; phonological memory (i.e. storage and representation of speech sounds) and grammar (i.e., use of sentence structure and word endings to convey meaning) relate to numerical performance; [and] number sequencing and retrieval (Morin and Franks, 2009, p. 111).

In addition, learners with language and vocabulary related difficulties can experience problems with comparison concepts such as 'same', 'different', 'more', 'less' and 'fewer' (Clements and Samara, 2009; Mushin et al., 2013).

Wigley (1997) proposes that to avoid mistakes and errors in the classroom, there is a need for teachers to be explicit in their teaching of mathematics-related language. Teachers should be focusing on the development of learners' mathematics language and not just looking at isolated mathematics terms (Clements and Samara, 2009). Wigley (1997) suggests that this shift in focus could reduce the possibility of learners encountering mathematical errors that are a result of confusing the everyday and mathematical meanings of the terms used.

There are 12 official languages in South Africa (including sign language). The Language in Education policy suggests that parents have the right to choose the language that they would like their children to be educated in (SA.DOE, 1997; Setati & Adler, 2000). This is an instance of policy not being practical as it is not always feasible nor possible for parents to make such a choice. Many parents would like their children to be taught in English as this is the language of business and politics. Many teachers use English as the LoLT in the classroom which can affect many of the learners, especially given the multilingual nature of many South African classrooms (Setati, 2002; Setati & Adler, 2000). As a result of this, many learners are learning in a language that is not their home language. If these learners are not proficient in the LoLT, they may experience some difficulties when learning mathematics (Mushin, Gardner and

Munro, 2013) There is much research that has found that in South African primary school (Grades 1- 6) underachievement in mathematics is more prevalent in learners for whom English is not their first language (Taylor, Vinjevold, & Muller, 2003; Atweh, Bose, Graven, Subramanian, & Venkat, 2014; Taylor & Coetzee, 2013; Essien, 2018). However, there is also extensive evidence from the numerous benchmarking tests reported on in Chapter One, that learners at Quintile 5 schools perform far better, irrespective of home language, than learners in Quintile 1-3 schools (Spaull & Kotze, 2015).

2.7 ASSISTING LEARNERS WITH MLD

There are various methods that teachers can use to assist learners with MLD. Dowker (2005) writes that learners should undergo assessments to determine their strengths and weaknesses in relation to the different mathematical concepts. The use of standardized tests (e.g. British Abilities Scale, and the Weschler Intelligence Scale for Children) can help in measuring learners' abilities and aptitude and whether they have mastered particular mathematical components (Dowker, 2005). Weaver (in Dowker, 2005) advocated for the use of differentiation in the instruction and remediation of mathematics. This is mainly due to the fact that:

> arithmetic competence is not a unitary thing but a composite of several types of quantitative ability e.g. computational ability, problem-solving ability, etc., that these abilities overlap to varying degrees, but most are sufficiently independent to warrant separate evaluations, and that children exhibit considerable variation in their profiles or patterns of ability in the various patterns of arithmetic instruction
>
> (Weaver, quoted in Dowker, 2005, p 326)

Taking this into consideration, when working with learners who have MLD the teacher should then make use of 'good teaching', that is, differentiated learning strategies to meet the instructional needs of the learners (Dowker, 2005).

Ernest (2011) and Cockroft (1982) suggest that in order to circumvent language-related problems the teacher should ensure that there is an emphasis on the use of mathematics discourse in the classroom. This means that the teacher should introduce concepts and topics in the classroom in a way that takes into consideration, and develops, the learners' language skills and mathematics understanding. Cockroft (1982) proposes that the teacher should make sure that the materials used are easy to understand by the learner and the structure and vocabulary do not hinder the learning process. Allowing the learner to ask questions for clarification develops his or her proficiency and ability to articulate his or her mathematical ideas. This, in turn, has a positive influence on the development of his or her understanding of mathematics (Ernest, 2011).

Gurganus (2004) and Black (2014) suggest that teachers when working with learners, use teaching strategies that ensure that the learners can:

- give meaning to numbers, this can be done by identifying objects that associate with a number such as a bicycle having two wheels or a tricycle having three wheels;
- make use of different materials that can be used for number representation, such as cards, dice, bottle tops, rulers, and number charts; use numbers from everyday activities, such as sports scores; and
- look at numbers in different representations, such as decimals, as a currency, or in metres versus centimetres.

In addition to the above, an emphasis on the development of number sense requires that learners be given the opportunity to explore and solve problems through a variety of strategies and then to justify their choice of strategies verbally and in writing. This encourages learners to find a method that works for them and share their method with others who might have done it differently (Black, 2014). The ability to justify one's reasoning in solving numerical problems verbally them indicates the learner is developing number sense.

Understanding the learners in a class, their backgrounds, and mathematical competence can lead to the development and appropriate implementation of instructional programs that will assist learners in overcoming their mathematical difficulties. Jordan and Levine (2009) acknowledge the importance of curriculum programs that develop number sense in small groups in preschool and kindergarten. This gives the teacher the opportunity to offer more 'individualised' instruction and implement appropriate interventions before it becomes too late. Another important factor when working with learners who experience an MLD is for the teacher to understand how mathematical understanding develops (Dowker, 2005). The teacher is then able to identify the relationship that early numerical abilities have on mathematics later in the learners' schooling career.

Whilst number sense is not a new area of interest, the role that teachers have in developing learners' number sense is one of great importance. Teachers need to adequately develop learners' ability to make sense of mathematics (McIntosh et al., 1997). By focusing on the growth of learners' number sense, teachers can adequately determine and assist learners with MLD in the classroom. Being aware of possible concepts that learners struggle with results in the early identification and remediation of any difficulties that learners may experience. McIntosh et al. (1997) suggest that teachers should encourage learners to engage and reflect on why they have selected a problem-solving approach as a means of identification and remediation. This will allow teachers to gauge their learners' understanding whilst they are participating in activities that challenge their thinking and develop their number sense (McIntosh et al., 1997). In addition to this, acknowledging that each learner is different, and that the development of number sense is dynamic, will influence the approach that a teacher uses in the classroom.

2.8 CONCLUSION

Number sense is regarded as key to learners' mathematical development and use of mathematics in and out of school. While there are numerous conceptions of number sense, most researchers agree that number sense involves an understanding of the meaning of numbers, the relationships between numbers, ability to calculate using flexible and efficient strategies, and the ability to apply such knowledge and understanding to various computational settings (including everyday life) (McIntosh et al., 1992). Children with MLD often have difficulties developing their number sense. In Grade One many MLD develop when learners are not able to count properly or when they have poor retrieval-based strategies. Another reason for the development of MLD and poor number sense is linked to the use of language in the classroom. This occurs on three possible levels: (1) the LoLT differs from the home language of the child; (2) the language of mathematics, particularly the terminology, is inconsistent with everyday meanings of the terms; and (3) learners are not given sufficient opportunity to explain and justify their solution strategies verbally and in writing. It is in the process of explaining one's thinking that learners develop number sense. One of the key components of number sense is thus the ability of learners to express themselves mathematically.

In Chapter Five, I present the analysis of my data showing how a Grade One teacher develops the number sense of children with MLD. In particular, the data focuses on the awareness of the relationship between number and quantity, and the awareness of magnitude and comparisons between different magnitudes, and the ability to communicate mathematically with understanding.

Having provided the conceptual framing of this thesis, I present the theoretical framework in Chapter Three. This framework provided the analytic and explanatory tools for identifying the knowledge that a teacher draws on to assist in the number sense development of learners with MLD.

CHAPTER 3: THEORETICAL FRAMEWORK

3.1 INTRODUCTION

The focus of my research was to identify the content and pedagogical knowledge that teachers use to support the number sense development of learners with MLD. In so doing, I chose to adopt Ball, Thames and Phelps's (2008) Mathematics Knowledge for Teaching (MKfT) framework. It was my understanding that this framework would provide me with the analytic and explanatory tools to answer my research question. However, during the process of data collection and data analysis, I soon realised that the MKfT framework had a number of limitations. I thus decided to change my theoretical framework to one developed by Rowland et al. (2013). The framework, which they refer to as the Knowledge Quartet (KQ), had a number of advantages, primarily, it enabled me to identify the mathematics knowledge, both content and pedagogical knowledge that a teacher draws on in teaching mathematics and in assisting learners with MLD. In addition, the KQ focuses on the mathematics and pedagogical knowledge of the teacher in the process of teaching. Rowland et al. (2013) thus refer to it as the Mathematics Knowledge *in* Teaching framework (MKiT) or the KQ.

In developing an understanding of the KQ, I trace the work on teachers' mathematics subject knowledge and pedagogical knowledge historically and into the present. The chapter begins with an examination of the seminal work of Lee Shulman (1986, 1987). Thereafter, I explain the MKfT framework of Ball et al (2008). I consider the limitations of Ball et al's (2008) framework before elucidating Rowland et al.'s (2013) KQ or MKiT framework. The KQ provided me with the necessary methodological tools to analyse my data. This forms the basis of Chapter Five where I present the analysis of my data.

3.2 SHULMAN'S PEDAGOGICAL CONTENT KNOWLEDGE

Shulman (1986) was concerned that historically teacher education programmes tended to focus either on pre-service teachers' content knowledge or pedagogical knowledge. Based on this, he identified two issues: firstly, he questioned the knowledge that teachers have that assisted them in transitioning from expert student to novice teacher; and secondly, he wondered how they transform their knowledge of the subject into a form that could be understood by the learners they teach.

Shulman (1987) identified seven categories of teacher knowledge which he regarded as unique to teaching. These categories include the knowledge teachers have of the: (1) *content* that is being taught; (2) *curriculum* when selecting programs and resources that they require; (3) *learners and their characteristics* (including learning competencies); (4) *context* in which learning is occurring, taking into consideration the communities and cultures that influence learning and teaching; (5) *educational ends, purposes and values* that the teacher has to meet; (6) general *pedagogics* needed to understand the role and principles of classroom management and the organization of learning in the classroom; (7) and lastly, *pedagogical content* knowledge which is specific to the practice of teaching. It is this seventh category that Shulman regarded as most profound and which he referred to as a "special form of professional understanding" (Shulman, 1987, p. 8) that is unique to teaching. An amalgam of the first three categories is regarded by Shulman (1987) as PCK, while the latter four categories refer to general teacher knowledge.

For Shulman (1987) PCK is particularly important as it "identifies the distinctive bodies of knowledge for teaching" (p. 8). PCK involves the relationship between content knowledge and pedagogical knowledge (RAND Mathematics Study Panel, 2003). PCK is better understood as the knowledge that a teacher has, that is, both the knowledge of content and how to teach, and how the teacher creates a link between these two types of knowledge (Shulman, 1987). As illustrated in Figure 3.1, Shulman (1986) PCK includes subject matter content knowledge, pedagogical knowledge and curricular knowledge.

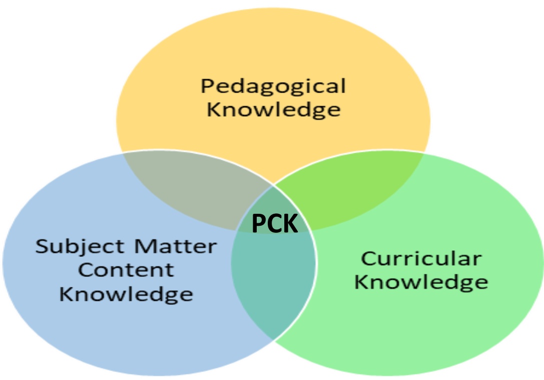

Figure 3.1 - Illustration of PCK (adapted from Shulman, 1986, 1987)

Subject matter content knowledge refers to the knowledge that a teacher has of a specific subject. It includes both the substantive and syntactic structures of that knowledge. The syntactic structures of subject matter knowledge refer to the specific content of that subject and the rules that generate the subject and govern its legitimacy (Shulman, 1986). Substantive structures verify the different representations of concepts and principles and how they are organized as the content of the subject (Shulman, 1987). Teachers should understand both the syntactic and substantive structures of the subject (Shulman, 1986).

Curricular knowledge is the knowledge that a teacher draws on to design learning programmes. It incorporates all aspects of teaching, that is, how a subject is taught and learned, and the materials utilised in the process. There are two aspects related to curricular knowledge. Firstly, lateral curriculum knowledge, which is based on the teacher being able to relate what is being taught in one subject to other subjects. Secondly, vertical curriculum knowledge which is constructed based on the teacher's awareness of what is taught in both the previous and later years of schooling. In this way, the teacher integrates what is taught with what the learners have learned, are learning, and will learn (Shulman, 1986).

Pedagogical knowledge is the knowledge that goes beyond the content of a subject. It acknowledges what makes learning a subject area difficult or not. It enables the teacher to identify the preconceptions that a learner has before learning a new concept and considers how a teacher is able to adapt and build on those preconceptions in order to engage with the learner. In addition, it recognises how the teacher encourages learning and understanding in the classroom (Shulman, 1986).

As depicted in Figure 3.1, subject matter content knowledge, pedagogical knowledge, and curricular knowledge all combine to form a teacher's PCK. While Shulman's work was heralded for introducing the concept of PCK, it is not without critique.

3.2.1 Critique of Shulman's PCK

Meredith (1995) argues that Shulman's PCK needs to be extended to incorporate various forms of teaching. S/he maintains that Shulman's PCK

> seems to imply one type of pedagogy, rooted in particular representations of prior knowledge. Most of the research posits a teacher-directed, didactic model of teaching. PCK does not seem to encompass alternative views of teaching, which, for

> instance, conceive of learners as autonomous agents, constructing their own understanding of the subject matter (p.176).

Put differently, Meredith (1995) implies that Shulman's PCK does not allow for alternative teaching approaches where learners are able to construct their own sense of understanding. However, based on my reading of the work of Shulman, it is not clear that he is promoting teacher-directed pedagogies. A teacher who enables learners to construct an understanding of mathematics has to be as knowledgeable of content, pedagogy and curriculum as one who is teacher-directed.

Meredith (1995) critiques Shulman's PCK suggesting that it implies that mathematical knowledge is "absolute, incontestable, unidimensional and static" (p.184). Rather, she suggests that PCK should also conceive of subject knowledge as "multidimensional, dynamic and generated through problem-solving" (p.184). She thus proposes a very different kind of knowledge for teaching (Meredith, 1995, p. 184). Whether Shulman did propose a view of knowledge that is absolute and unidimensional is not clear. What is evident is that teachers, no matter what their teaching approach and/or beliefs about teaching and learning, require PCK. It seems as necessary for a constructivist teacher, who regards mathematics knowledge as subjective, and continually open for review, to have PCK. For example, teachers who align themselves with constructivist pedagogies should be able to identify learner errors and misconceptions and know which models and representations will assist learners in correcting their misconception.

Ball at al. (2008) maintains that the distinction between what Shulman terms content knowledge and pedagogical knowledge remains unclear. This is further hindered by there being no clear interaction between the different categories that make up teachers' knowledge (Petrou and Goulding, 2011). Furthermore, Shulman's (1986) work focused on high school teachers and on all the school subjects. Identifying this as a limitation, Ball, Lubienski and Mewborn (2001) focused their research on primary school teachers of mathematics (This led to the development of the Mathematics Knowledge for Teaching (MKfT) framework.

3.3 BALL'S MATHEMATICS KNOWLEDGE FOR TEACHING (MKFT)

Shulman's PCK lays the groundwork for thinking about the knowledge teachers require to teach competently. Hill, Ball, & Schilling (2004) take Shulman's PCK further. They argued that "specific measures of pedagogical content knowledge and mathematical content knowledge were not yet in place in mathematics education" (Mohr, 2006, p. 219). Ball et al. (2008) define MKfT as the mathematical knowledge needed to perform the recurrent tasks of teaching mathematics to students" (p. 399) with examples of this being

> explaining terms and concepts to students, interpreting students' statements and solutions, judging and correcting textbook treatments of particular topics, using representations accurately in the classroom, and providing students with examples of mathematics concepts, algorithms, or proofs
>
> (Hill, Rowan, & Ball, 2005, p. 373)

Hill, Rowan, and Ball (2005) and Hill et al. (2004, 2008) argue that the knowledge that a teacher has strongly influences learner performance in mathematics. They state that previous research on teachers' knowledge has focused on their qualifications and skills. However, as shown in Chapter One, "teacher effects on student achievement are driven by teachers' ability to understand and use subject-matter knowledge to carry out the task of teaching" (p. 372).

Hill et al. (2008) acknowledge there is a relation between what a teacher knows, how a teacher comes to know, and how a teacher can teach what they know in a meaningful way. Ball et al. (2008) explain that while there is a need for teachers to know the subject they are teaching, having knowledge of the subject is not sufficient on its own. They thus sought to identify the knowledge that teachers require to teach mathematics in primary schools. In so doing, they developed the MKfT framework. As suggested by Ball and Bass (cited in Masingila, Olanoff and Kimani, 2018), the MKFT framework is important as it:

> a) supports teachers in unpacking mathematical concepts, skills, and procedures, (b) allows teachers to connect mathematical ideas within and across mathematical domains, (c) prompts teachers to communicate mathematically in ways that children can understand and use, and (d) promotes teachers using practices applicable to the discipline of mathematics (p. 2).

Using a framework such as Ball et al.'s (2008) MKFT provides a multitude of opportunities for researchers as they can identify the knowledge teachers draw on in the classroom, how they use this knowledge and the effect it has on their learners (Bobis, Higgins, Cavanagh, & Roche, 2012). This knowledge can be emphasised in pre-service and in-service teacher education to support and improve the mathematics teaching of primary school teachers.

Ball et al. (2008) argue that it is imperative to map out exactly what MKfT entails and how it can be used effectively in teaching. In so doing, Ball et al. (2008) and Hill and Ball (2009) elaborate on Shulman's framework to identify domains of knowledge within two categories: are Subject Matter Knowledge (SMK) and PCK. These categories are depicted in Figure 3.2.

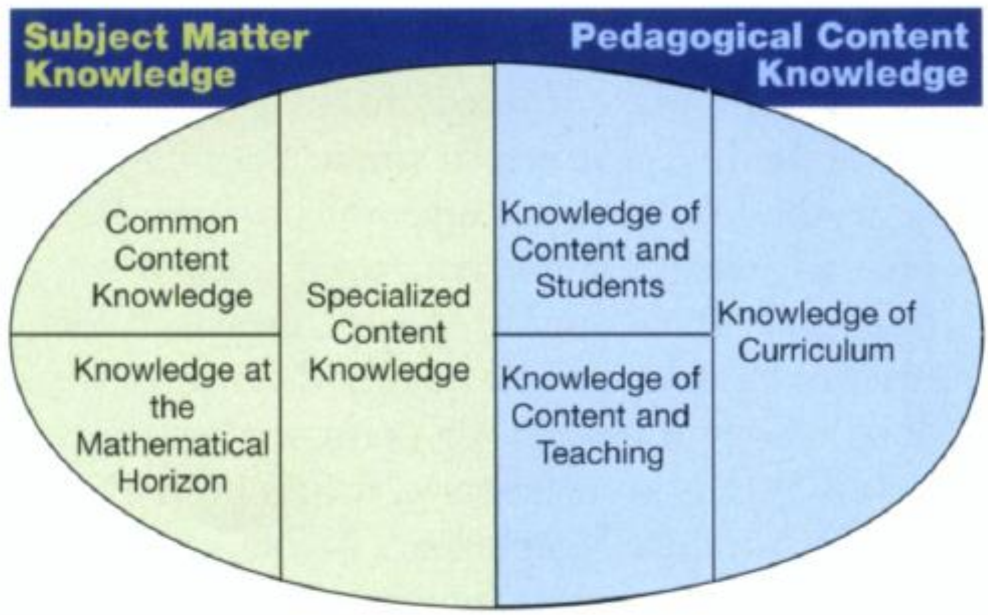

Figure 3.2 – The Mathematics Knowledge for Teaching framework (Hill & Ball, 2009, p.70)

3.3.1 Subject matter knowledge

As highlighted in Figure 3.2, the SMK category consists of the three domains: Common Content Knowledge (CCK), Specialised Content Knowledge (SCK) and Horizon Knowledge (HK) that a teacher has of the subject that is being taught. These domains refers to the teacher's general understanding of mathematics, the classroom specific understanding of mathematics, and the awareness of the mathematics requirements throughout the curriculum.

3.3.1.1. Common Content Knowledge (CCK)

Edwards, Hyde, O'Connor and Oldham (2015) state that CCK is the mathematics knowledge that most adults should have. CCK is the knowledge of mathematics that is used in everyday life by people who are not necessarily mathematics educators and it is the set of skills that can be used in settings that are not related to teaching (Ball et al.,2008). Whilst CCK is referred to as 'common' it does not mean that everyone has an understanding of this knowledge. Rather it is the knowledge of mathematics that is common amongst those that use and understand mathematics (Ball et al., 2008). This means that adults who work in sectors that require some mathematics knowledge have better mathematics CCK than adults who do not use mathematics explicitly in their work.

Teachers require a general understanding of the work that is being assigned to learners, hence the importance of CCK in teaching. CCK refers to the knowledge that a teacher must have to identify if an answer is correct or not, define concepts, explain tasks and carry these out with ease (Ball & Hill, 2009). Teachers should be able to identify inaccuracies in textbooks and worksheets as well as utilise the correct terms and notation when explaining and demonstrating the content to the learners (Ball et al., 2008). Without CCK a teacher is unable to identify the mathematical importance and focus of a task.

3.3.1.2. Specialized Content Knowledge (SCK)

Campton and Stevenson (2014) identify SCK as the mathematical knowledge that goes beyond CCK. It concerns the mathematics knowledge that is specifically needed in the classroom. It is the knowledge that is used solely for teaching specific content and is not required, or commonly used, outside of the teaching setting (Ball et al., 2008). Ball et al. (2008) state "the demands of the work of teaching mathematics create the need for such a body of mathematical knowledge specialized to teaching" (p. 401). With SCK teachers should also be able to "explain why a procedure works, [present] mathematical ideas and [find] examples and representations of mathematics" (Edwards et al., 2015, p. 38).

3.3.1.3. Horizon Knowledge (HK).

HK includes a teacher's knowledge of curriculum content throughout the schooling system. This enables a teacher to understand learners' prior knowledge and to build foundations for concepts learners will learn in the future (Ball et al., 2008).

This knowledge is necessary for sequencing the content to be taught in a manner that facilitates optimal learning (Nolan, Dempsey, Lovatta & O'Shea, 2015). HK takes into consideration the "peripheral vision" (Hill & Ball, 2009. p. 70) of mathematics that is required in teaching. It is necessary to navigate the curriculum and making decisions on what and how topics will be taught, while simultaneously considering what will be taught in later years. In the Foundation Phase, HK enables the teacher to recognise that what is taught in Grade One mathematics has a relation to what is taught in Grade 3 (Ball et al., 2008). For example, when it comes to using the number line, the Grade One teacher will start with a structured number line from 0 to 10. By the time the learner has reached Grade 3 they would be using an unstructured number line as a representation for addition and subtraction of two three-digit numbers.

To ensure that the teacher is well equipped to teach mathematics in the classroom, they require not only SMK, but also PCK.

3.3.2 Pedagogical content knowledge

The PCK category consists of the knowledge that is more specific to the teaching and learning process. These domains are: Knowledge of Content and Students (KCS), Knowledge of Content and Teaching (KCT), and Knowledge of Content and Curriculum (KCC). The focus is how the subject is taught, how learners learn the subjects and what the teacher needs to do to meet the requirements of the curriculum. As Ball et al. (2008, p. 400) state "teaching requires knowledge beyond that being taught to students". These domains are highlighted in Figure 3.2.

3.3.2.1 Knowledge of Content and Students (KCS)

KCS refers to how the teachers matches the mathematics content knowledge to the learner. It requires the teacher to know both the mathematics content and the learner. With reference to the learner, KCS suggests that teachers need to be knowledgeable of their learners' interests, levels of competence and capabilities as this has an influence on how the content is taught and how it is received (Ball et al., 2008; Hill & Ball, 2009) Furthermore, they need to know what motivates learners and how to push their thinking in a manner that facilitates learning (Nolan et al., 2015).

Knowledge of learners assists the teacher in anticipating any concepts that learners will find "confusing, interesting and motivating" (Nolan et al., 2015, p. 55). It is necessary knowledge for interpreting learners' work and emerging ideas (however incomplete they may be) (Nolan et al., 2015; Ball et al., 2008). With reference to the work that a teacher assigns to learners, Ball at al. (2008) state that a teacher needs to "anticipate what students are likely to do with it and whether they will find it easy or hard" (p. 401).

3.3.2.2 Knowledge of Content and Teaching (KCT)

KCT is similar to KCS but it considers the knowledge teachers have of content and how to teach it. Teachers' KCT entails "an interaction between specific mathematical understanding and an understanding of pedagogical issues that affect student learning" (Ball et al., 2008, p. 401). When planning a lesson, a teacher needs to consider what they will teach, how it will be taught, how the they will deal with incorrect representations, and how the they will respond to learners' contributions (Nolan et al., 2015; Ball et al., 2008). In the case of learner contributions, the use of KCT will allow the teacher to determine which comments, answers and questions are important to pursue and which ones will require attention at a later stage. In addition, teachers need to be able to determine whether specific representations and materials will be advantageous or disadvantageous to the teaching and learning process (Ball et al., 2008).

3.3.2.3 Knowledge of Content and Curriculum (KCC)

KCC incorporates knowledge of what content needs to be taught in the classroom (Ball et al., 2008). This knowledge enables the teacher to determine which materials could be used to ensure the learning of mathematics content and meet the curriculum requirements for the specific grade (Hill and Ball, 2009; Ball et al., 2001). KCC includes: identifying what learning and teaching materials are required; determining the integrity and usefulness of textbooks; workbooks and other resources; and being able to adapt these materials to the benefit of teaching and learning mathematics (Ball et al., 2008).

While the MKfT framework has, and is, being used extensively in mathematics education research (e.g. Chikiwa, Westaway, and Graven, 2017), it is not without its limitations.

3.3.3 Critique of Ball and colleagues' MKfT

Ball et. al. (2008) acknowledge that MKfT is not without its challenges. They write that there differentiating between the different knowledge domains is not a simple process. They maintain that there is often confusion in distinguishing between CCK and SCK, and KCT and KCS. Their concern is echoed by Thanheiser, Browning, Moss, Watanabe and Garza-Kling (2010) and Hurrell (2013) who acknowledge that the distinction between the domains are unclear.

Thanheiser et al. (2010) maintain that there is overlap between the knowledge categories as domains in one category influence domains in the other. For example, KCS, which is part of the PCK category, influences the SCK, which is part of the SMK category. As Thanheiser et al. (2010) note "a connection to children's mathematical thinking may cross the boundaries between specialized content knowledge and pedagogical content knowledge" (p, 3). This lack of clarity makes it very difficult to analyse teachers' MKfT.

Hill et al. (2008) acknowledge that there are gaps in the theoretical knowledge that should be considered when analysing teachers' MKfT. These gaps have given rise to a number of questions that should be considered by researchers:

> How does knowledge get expressed in instruction? Do teachers with stronger knowledge offer a qualitatively different form of instruction by focusing students' attention on mathematical meaning, as many hypothesize? Or, do these teachers simply avoid the mathematical errors made by less knowledgeable teachers? In what tasks of teaching does teacher knowledge make itself most apparent?
>
> (Hill et al., 2008, p. 431)

Whilst Ball et al.'s (2008) MKfT framework elaborates on Shulman's (1986, 1987) PCK, there are some areas of Shulman's work that have been omitted from the MKfT framework. Of Shulman's (1987) seven broader categories, Thanheiser et al. (2010) identified two categories that Ball et al. (2008) did not include in the MKfT framework. These are *knowledge of educational contexts* and *knowledge of educational ends*. Firstly, teachers need to have an a clearly defined end goal that is connected to the curriculum requirements. Having clearly defined outcomes guides the teacher with regards to curriculum coverage and in developing opportunities for the learners to achieve the outcomes. Secondly, teachers, in planning for teaching, should consider the context in which the teaching and learning process occurs and the all influential factors, both inside and outside the classroom, that effect the pedagogical endeavour. This criticism however, may be unfair as the MKfT framework developed out of Shulman's PCK and not the seven broader categories.

Rowland, Turner and Thwaite (2013) critique the MKfT framework for its seemingly limited scope for analysing teacher knowledge in the process of teaching. They developed a framework known as the Knowledge Quartet, which analyses teachers' mathematics knowledge *in* teaching (MKiT). While this framework was initially developed by Rowland, Turner and Thwaite (2013) to assist in analysing and explaining pre-service teachers' MKiT, in recent years, it has also been used to examine in-service teachers' MKiT.

While my initial intention was to use the MKfT framework of Ball et al. (2008), I realised, when I started analysing my data that it was very difficult to isolate my participant-teacher's knowledge in accordance with the MKfT framework. While I realise that the work of teachers, and their knowledge of mathematical content and pedagogy, is expressed in an integrated manner in the classroom, my intention was to identify which domains of MKfT were important when developing the number sense of learners with MLD.

Having struggled with the MKfT framework, I decided to change to the KQ framework as it enabled me to identify the knowledge my participant-teacher use in supporting learners with MLD *in* action. In that way, it allowed me to analyse the content and pedagogical knowledge of my participant-teacher as it unfolded in the naturalistic environment of a Grade One classroom.

3.4 ROWLAND, TURNER AND THWAITE'S KNOWLEDGE QUARTET

The Knowledge Quartet (KQ) builds on the work of both Shulman (1986, 1987) and Ball et al. (2008). It examines the specific knowledge that a teacher draws on in the classroom during the teaching and learning process. In other words, the KQ focuses on teachers' mathematical knowledge in the process of teaching. In many respects this emphasis is different from that of MKfT where the emphasis is on the knowledge that a teacher requires to teach.

Like Shulman (1986, 1987) and Ball et al. (2008), the KQ acknowledges that teachers require a combination of mathematics subject matter knowledge and pedagogical knowledge to teach (Rowland, Turner, Thwaites, 2013). Rowland et al. (2013) elaborates on the MKfT framework by emphasising that beliefs about the nature of mathematics, and teaching and learning also play an important role in the classroom. "All aspects of teachers' knowledge and beliefs come together as a resource from which to draw both in planning and in the act of teaching" (Petrou and Goulding, 2011, p. 19). The inclusion of teachers' beliefs as part of teachers' knowledge repertoire separates the KQ from Shulman's PCK and Ball and colleagues' MKfT. It acknowledges that teachers' beliefs are important in teaching mathematics. Teachers draw on their pre-existing beliefs in the classroom, and their beliefs are also influenced by their "new 'theoretical' knowledge [and the] 'practical' advice received from various quarters" (Rowland, Huckstep and Thwaites, 2003, p.9).

The KQ consists of four knowledge categories, namely, foundation, transformation, connection, and contingency. Rowland et al. (2013) developed these categories from their analysis of pre-service teachers' mathematics teaching. As mentioned earlier in this chapter, the KQ has also been used extensively in recent years to analyse the mathematics teaching of in-service teachers (e.g. Rowland, 2014).

3.4.1 Foundation

Foundation is the first category in the quartet. This category, unlike the other three, considers the knowledge that the teacher already possesses (Rowland, 2014). It is "rooted in the foundation of the teacher's theoretical background and beliefs" (Turner and Rowland, 2011, p. 200). It focuses on the knowledge that the teacher has acquired, at school, during their pre-service education, and during their in-service teacher education. This is knowledge that influences the role of the teacher in teaching mathematics in the classroom (Rowland, 2014; Turner and Rowland, 2011). Significant components in this category include the knowledge and understanding teachers have of content and pedagogy, as well as their beliefs about pedagogy and the nature of the subject they teach (Petrou & Goulding, 2011). Furthermore, it includes the knowledge the teacher has acquired to: identify quality and appropriate textbooks/workbooks; knowing various methods to complete calculation procedures; and identify learner errors (Turner and Rowland, 2011).

The next three categories (transformation, connection, and contingency) draw on the foundational knowledge of the teacher.

3.4.2 Transformation

Transformation knowledge is informed by foundation knowledge. It "concerns knowledge-in-action demonstrated both in planning to teach and in the act of teaching itself (Petrou & Goulding, 2011, p.18). In other words, transformation knowledge differs from foundation knowledge as it considers how a teacher is able to adapt the knowledge they have acquired in a way that promotes learning and conceptual understanding (Rowland et al., 2014).

Transformation knowledge is the category that focuses on the knowledge of a teacher that is specifically directed towards the learner (Rowland et al., 2013; Rowland, 2014). It is informed by numerous resources that teachers have at their disposal (e.g. handbooks, textbooks, articles and the internet). This knowledge, together with teachers' foundation knowledge, shapes the pedagogical approach of the teacher (Rowland, 2014). Transformation knowledge refers to the examples teachers give, the procedures they promote in the mathematics lesson, and the representations and models utilised to engender learning in the mathematics classroom. As such, it consists of the amalgam of the content to be taught, the needs of the learners, and how these affect the planning and implementation of a lesson or a series of lessons (Turner and Rowland, 2011).

3.4.3 Connection

The third category of the KQ is *connection.* Connection binds together the decisions and choices teachers make during the lesson. It "concerns the coherence of the planning or teaching displayed across an episode, lesson or series of lessons" (Rowland, 2014). More specifically, this category relates to the teacher's ability to make decisions that influence the progression of lessons that have been planned. It acknowledges that mathematics has no "discrete topics" (Weston, Kleve, Rowland, 2012, p 180) as concepts build on each other. Therefore, connection determines the teacher's consideration of the integrated nature of mathematics. It is the knowledge teachers require to anticipate concepts that the learners may find easy or difficult and provides insights for scaffolding mathematics concepts as to promote sense-making (Weston et al., 2012).

Connection does not refer solely to the sequencing of lessons from one concept to the next but also includes the developmental nature of the activities and exercises assigned to the learner (Rowland et al., 2014). The teacher needs to be cognisant of each activity selected and how it assists in the development of learners' understanding of mathematical concepts.

3.4.4 Contingency

*Contingenc*y is the final category of the KQ. It is "informed by the other three dimensions in the KQ … and is about situations in mathematics classrooms, which are not planned for" (Kleve & Solem, 2015, p. 3052). This category focuses on the teachers' ability to think on their feet when it comes to unplanned or unexpected moments in the classroom (Weston et al, 2012; Kleve & Solem, 2015).

Weston et al. (2012) and Kleve and Solem (2015) maintain that the knowledge that is categorised in the contingency category refers to the teacher's ability to adapt and deviate from the original plan; taking into consideration the ideas, questions, and answers that have been supplied by the learners. This relates to the teacher being able to recognise when the learners are contributing to the lesson in an unplanned manner (incidental teaching opportunities), and to use those moments as teaching and learning opportunities. As Rowland and Zazkis (2013) acknowledge, "teaching involves dealing with unpredictable, *contingent* events in the classroom. With this perspective on teaching, mathematical knowledge beyond the immediate curricular prescription is beneficial and demonstrably essential" (p. 138).

Therefore, a teacher needs to be able to make decisions, based on incidental events, and learner questions and answers in the classroom. How teachers respond is influenced by the knowledge that they have available to themselves, that is their foundation knowledge (Rowland, 2014)

In summary, each of the above-mentioned category of the KQ have components that are coded according to the knowledge the teacher requires in the classroom during the process of teaching. These component codes are elaborated on in Table 3.1 below.

Category	Associated Component
Foundation	The teacher's: • theoretical underpinnings of mathematics pedagogy • beliefs about the nature of mathematics, and teaching and learning • content knowledge • knowledge of mathematical procedures and strategies • knowledge of mathematics terminology • knowledge of appropriate textbooks and other teaching and learning support materials • ability to identify learner errors
Transformation	The teacher's: • use of explanations and demonstrations in the mathematics lesson • choice and use of instructional materials • choice of representation • choice of examples
Connection	The teacher's ability to: • make connections between procedures • make connections between concepts • anticipate complexity • make decisions about sequencing • recognise conceptual appropriateness
Contingency	The teacher's ability to: • respond to learners' ideas • make use of opportunities when teaching • deviate from the agenda • provide relevant insights

Table 3.1 – The categories of the Knowledge Quartet and associated components (Rowland, 2014; Rowland et al., 2013; Petrou and Goulding, 2011)

3.5 CONCLUSION

In this chapter, I trace, historically, the subject and pedagogical knowledge that teachers require to teach mathematics. The work of Shulman (1986, 1987) is central to this chapter as he has been cited as the person who first raised the question about the knowledge teachers require to teach well. His PCK framework was generic in that it focused on all subjects. Ball et al. (2008) elaborated on the work of Shulman and developed the MKfT framework. This framework was developed specifically with primary school mathematics teachers in mind. I initially began analysing my data using this framework, but soon realised that the domains are all integrated. I then turned to the KQ of Rowland et al. (2013). The value of this framework is that each category is analytically distinct and that it focuses not only on the knowledge teachers need for teaching, but also, the knowledge teachers draw on *in* the process of teaching. This is referred to as the Mathematics Knowledge in Teaching (MKiT) framework or KQ.

CHAPTER 4 – RESEARCH METHODOLOGY

4.1 INTRODUCTION

My research seeks to identify the content and pedagogical knowledge that is required for developing learners' number sense in Grade One, specifically when teaching learners with MLD. It thus asks the question: *What content and pedagogical knowledge does a Grade One teacher use to assist learners with Mathematics Learning Difficulties to develop number sense?*

This chapter discusses the research design and methodology that was used throughout the research process. My research was a qualitative case study. Data was generated through observations of a Grade One teacher teaching mathematics. The observations were recorded using videos, field notes, and interviews (casual conversations and a formal, semi-structured interview). I analysed the data using both an emic and etic approach. Firstly, I identified codes and categories emergent from the data and secondly, I used the knowledge categories from Knowledge Quartet (Rowland et al., 2008) to code the data. Special attention was given to validity and ethics at the end of the chapter as these were central in ensuring the quality of the data in my research study.

The above-mentioned processes are further elaborated on throughout this chapter.

4.2 BACKGROUND TO THE RESEARCH

I decided to look at the knowledge teachers draw on in their classrooms when teaching mathematics. My original aim was to focus on the MKfT of a Grade R and Grade One teacher. I was particularly interested in how mathematics learning, and any necessary interventions, develop from Grade R to Grade 1 in a context where teachers are required to meet the outcomes of the national curriculum. However, due to the fact that the mathematics learning in Grade R is not structured, I decided to focus my study on Grade 1.

These interests led to the development of the research aims for my study. Firstly, how does a teacher use their content and pedagogical knowledge to assist learners with mathematics learning difficulties (MLD) develop their number sense, and secondly, what mathematics and pedagogical knowledge does a Grade One teacher draw to meet the needs of all her learners.

The results from my research has the potential to inform pre-service and in-service teacher education programmes in South Africa.

As indicated in Chapter 1, the reason for choosing a well-established, expert teacher was to develop a sense of her mathematics and pedagogical knowledge *in* the process of teaching. My intention was not to highlight any possible deficits in her knowledge, but rather ascertain what knowledge she uses while developing the number sense of learners with MLD.

4.3 RESEARCH ORIENATION

Qualitative research is broad and includes a variety of methods that can be used by the researcher. Taylor, Bogdan, and DeVault (2016) identify that "qualitative methodology refers in the broadest sense to research that produces descriptive data" (p. 7). As stated in Merriam (2009), the term qualitative is used as an umbrella term for naturalistic and interpretive methodologies. Within qualitative methodologies there are three different approaches that a researcher can ascribe to, these are: inductivist; constructivist; and interpretivist (Bryman, 2012). An inductivist approach is one where theory is used to inform the understanding of phenomena; a constructivist approach acknowledges that phenomena are a result of social interactions which are in a constant state of change; and an interpretivist approach privileges understanding phenomena through observation and interpretation of participants experiences without attempting to instigate any form of change (Bryman, 2012; Merriam, 2009).

Qualitative research supports "an array of interpretive techniques which seek to describe, decode, translate, and otherwise come to terms with the meaning, not the frequency, of certain more less naturally occurring phenomena" (Van Maanen, 1979, in Merriam, 2012, p. 13). Qualitative researchers study a phenomenon in its natural setting with the aim of interpreting it (Denzin & Lincoln, in Merriam, 2009).

My research is underpinned by an interpretivist orientation. As explained by Merriam (2009), the main goal of interpretive research is to allow the researcher to understand a phenomenon and the meaning that the phenomenon has for the participants involved in the phenomenon. Research underpinned by an interpretive orientation incorporates both a phenomenological and ethnographic approach. The phenomenological approach identifies how an experience can transform the consciousness of people, while ethnographic research enables the researcher to immerse him/herself into an environment to observe and understand (Merriam, 2009, Bryman, 2012). My research incorporates an ethnographic approach as I spent five weeks in a Grade One class trying to understand how the teacher supports the number sense development of learners with MLD.

Thomas (2010) identifies that people's experiences influence their perspective of the world. He asserts that a researcher who adopts an interpretivist approach acknowledges that there are no specific paths to knowledge, as knowledge is fallible and relative (Thomas, 2010). The interpretivist orientation provides a subjective understanding rather than an 'objective' explanation of a phenomenon (Ritchie, Lewis, Nicholls & Ormston, 2013). My research is an interpretive study because I aim to identify what content and pedagogical knowledge my participant-teacher draws on in assisting learners with MLD develop their number sense. I want to understand what influences the teacher when making decisions in the classroom.

In this interpretive research, I used a case study method to identify the mathematics content and pedagogical knowledge that a Grade One teacher draws on in the process of teaching to assist learners with MLD develop their number sense.

4.4 CASE STUDY METHODOLOGY

My research is a case study. The purpose is to provide understanding about an issue or topic that ultimately leads to broader knowledge and understanding of the phenomenon studied (Borman, Clarke, Cotner & Lee, 2006). It is a method used to "contribute to our knowledge of individual, group, organizational, social, political, and related phenomena" (Yin, 2009, p. 21). A case study assists in identifying 'how something happens'. In my research, this refers to how the teacher assisted learners with MLD develop their number sense in her classroom. The use of case study enabled the development of an in-depth understanding of the phenomenon being researched (Yin, 2006). As Yin (2006) explains, case studies allow the researcher to identify important topics that might not be identified in the initial data collection process.

Stake (1995) states that a case study must focus on a 'thing' (e.g. a person or program) that is a specific, complex, functioning and well-bounded and not something that can be generalised. In addition, Bryman (2012) maintains that data is commonly collected in a single location with a focus on the phenomena in that location. As such, case study research focuses on “a single entity, a unit around which there are boundaries” (Merriman, p. 40) and where the data is collected and interpreted based on the context of the case (Creswell, 2013). Based on this explanation of a bounded case study, the case that I investigated was the content and pedagogical knowledge a Grade One teacher uses to assist learners with MLD develop their number sense and how they utilise this knowledge. The location chosen was a local primary school where the participant is a well-established teacher who is knowledgeable about MLD.

As Creswell (2013) suggests, case studies allow the researcher to use multiple forms of data collection. This can be done through using single sources of information, such as interviews, or through the use of multiple sources of information, such as observations and document analysis (Creswell, 2013; Yin, 2009). In this research, I used interviews, observations and field notes. In using a variety of methods to collect and analyse the data, I was able to ensure that I developed an in-depth and comprehensive understanding of the case.

4.5 THE SAMPLE

Sampling involves identifying individuals where the area of interest that is being studied will be most likely to occur (Mertens, 2005). More succinctly put, sampling, in my study, was done by identifying a teacher who is regarded as competent and knowledgeable in her field. This teacher would then have the knowledge and expertise to provide me with the relevant information for my study. In this research, my sample was a Foundation Phase teacher in a school in the Eastern Cape. The teacher I observed is considered to be highly competent in working with learners who have MLD. According to the local teaching community and her direct colleagues, this teacher is considered to be knowledgeable in identifying learners with MLD and assisting them in developing number sense. Essentially, she understands both the subject matter and pedagogical knowledge required to develop learners’ number sense and how to support them in this developmental process (Findell, 2009). A further reason for her being regarded as a competent teacher relates to the multiple roles and responsibilities she has. Elizabeth (the participant-teacher) occupies various leadership positions in the school in which she works. She is the Grade Head for Grade One and the Foundation Phase Head of Department. Her leadership is not limited to the immediate school context, she a cluster leader in the district, part-time lecturer at the university and professional teacher development facilitator.

4.6 THE RESEARCH SITE

The research was conducted in a former Model C school in the Eastern Cape in South Africa. The school is a well-resourced school which accommodates learners from Grade R to Grade 7. It is a multi-racial fee-paying school. The learners have access to a library, computer centre, hall, swimming pool, and a variety of sports fields (netball courts, tennis courts, and hockey fields). Most of the learners that attend the school are isiXhosa first language speakers despite the LoLT being English. Elizabeth's Grade One classroom is well-resourced and well-organised. The resources in her classroom are mainly sourced or made by herself. There are also resources that have been provided by the Education Department. Elizabeth has been teaching at the school for sixteen years and has been teaching in Grade One for eleven years. There were 30 girls in her classroom at the time of the study.

4.7 DATA COLLECTION PROCESS AND TECHNIQUES

The data collection process occurred in three phases: introduction; observation of lessons; and formal interviews. These phases are outlined in Figure 4.1 below.

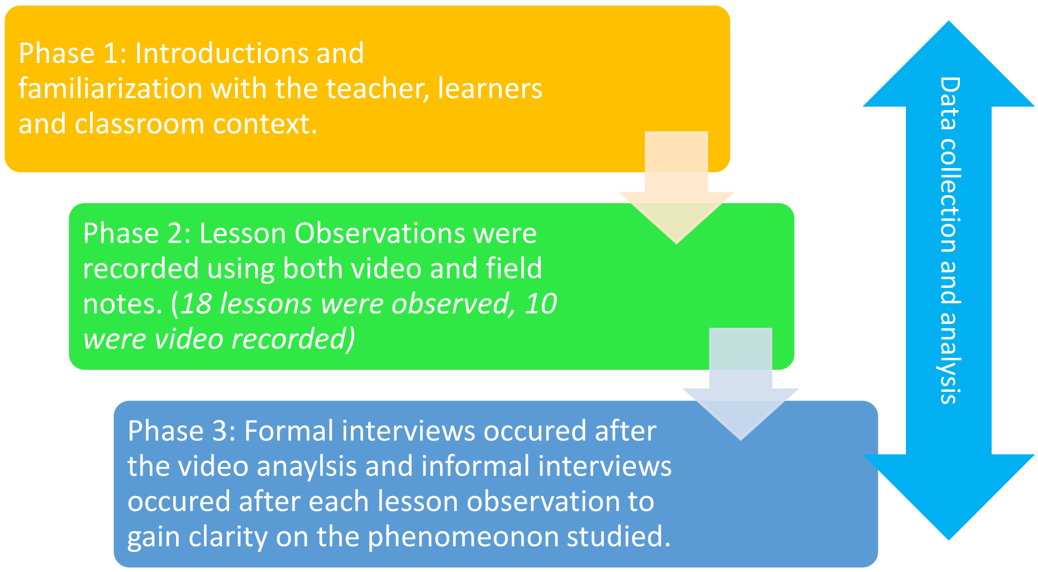

Figure 4.1 – Illustration of the data collection process

Phase one consisted of two processes. The first was to meet the teacher and inform her of the research, and the second was to familiarise myself with the classroom environment and the learners.

In phase one, I had my initial meeting with Elizabeth to get to know her better and fully understand her role in the classroom and in the school. This initial meeting was informal. The focus was to engage with her in a manner that was not guided by pre-determined questions and ideas. We had a conversation about my research topic which allowed me to gain insight into areas of interest that I had not considered. The purpose of phase one was to understand Elizabeth's views on teaching and learning mathematics in Grade One, the issues that she faces, and what she deemed to be important in teaching mathematics.

During the initial meeting, I discussed the process that I would use throughout the research period. I indicated that I would be recording some of the lessons and that my focus would be on her rather than the learners.

In addition to my meeting Elizabeth, I sought to familiarize myself with the classroom context and enable the learners to become accustomed to having me in their classroom. The purpose of doing this was to limit the chances of the learners seeing me as a novelty in the classroom. This phase lasted three days.

The focus of phase two was to observe Elizabeth teach. I spent three days a week in the classroom observing her teaching mathematics. I did not go in on a Monday and a Friday as Elizabeth told me that she could not guarantee that I would see a full mathematics lesson due to other commitments (Monday usually involved preparing the learners for the week and Friday was often the catch-up day or used for fun activities). Elizabeth's teaching followed a set formula: whole class teaching on the mat followed by small group activities on the mat. While some of the learners were on the mat with Elizabeth, the others were working at their desks. The learners did not work as a group in these small group activity sessions, rather they join Elizabeth on the mat so that she could give them more focused attention in accordance with their perceived needs. The mathematics lessons are elaborated on in Chapter Five.

4.8 DATA GENERATION TOOLS

I made use of three different data generation tools: observations; field notes; and interviews. To support the data gathered through the observation process, I made use of two approaches to interviewing: casual conversations and semi-structured interviews. These will be expanded on further in this section.

4.8.1 Observations

Observation, according to Cohen, Manion, and Morrison (2000), is a meaningful method of data collection as it allows the researcher to collect data of both verbal and non-verbal behaviours. This allows for the identification of behaviours over an extended period. The researcher is then able to create a more natural environment in an attempt to reduce researcher bias.

Research indicates that learners who enter the Foundation Phase with poor number sense are more susceptible to experiencing MLD (Jordan, Dyson and Glutting, 2011; Feza, 2016). Elizabeth indicated that in her experience learners with MLD can be identified early in the school year. As a result of this, the classroom observations took place at the beginning of the school year.

I observed Elizabeth in her classroom to ascertain how she draws on her mathematics and pedagogical knowledge to assist learners with MLD. These observations were a means of gathering data to establish how the teacher supports learners with MLD to develop their number sense. To ensure consistency in capturing data, I video-recorded ten lessons, that is, two lessons a week over a period of five weeks. (Merriam, 2009). Whilst intrusive to the classroom routine, video recordings allowed me to record what was happening in the classroom. They enabled me to review the lesson afterwards and identify details that could have been missed in real time.

I videoed the lessons in consultation with Elizabeth. We discussed when I would record the lessons and for how long. The camera was focused on Elizabeth the entire time as I was only interested in her actions and responses and not those of the learners in her class. I took detailed observation notes of 15 lessons. This included the 10 recorded lessons. The videos were used as a visual reference point for post-lesson analysis as well as a source to generate interview questions.

The focus of my observations were the daily mathematics lessons in Elizabeth's classroom. My interest was on how Elizabeth taught mathematics, either to the whole class or during the group mat activities. As a result, I only attended the formal mathematics lessons. I was not interested in the incidental mathematics opportunities that occurred throughout the school day. Elizabeth suggested I join the class at particular times to fit in with her timetable and to ensure that her school day was not affected by my research.

According to Creswell (2013), observation is "one of the key tools for collecting data in qualitative research" (p. 135). Observation requires that the researcher immerse themselves into the research environment and observe the behaviours and interactions of the participants (Bryman, 2012). Observation can be done in different ways. Creswell (2013) identifies these as a) complete participant, b) participant-as-observer, c) non-participant observer, and d) complete observer. In qualitative research, a researcher can change the type of observation that they use, from the aforementioned list. The ability to do this is acknowledged as an indicator of a good researcher (Creswell, 2013). For the purpose of this research, the initial observation was done as a non-participant observer where I observed the lessons as an outsider. As such, I did not have any direct involvement in the lessons. Once my presence in the classroom had been established and 'normalised', I took on the role of participant-as-observer where I participated in some of the classroom activities, especially the lessons that took place on the mat.

Taylor et al. (2016) maintain that it is vital to develop a rapport with the participants, establish a relationship with the participants, assist where needed and be respectful of what the participants share with you regardless of differing opinions (Taylor et al., 2016). Once my presence in the classroom was established, I was assisted by answering learners' questions and correcting learners where needed. I also participated in mat work activities. On some occasions, when I had completed my observations of Elizabeth, I facilitated a group on my own. She would give me instructions on what the activity was about and how to facilitate it. This gave me the opportunity to fully understand what Elizabeth does on a day-to-day basis with the learners who experience MLD. This involvement led to conversations with Elizabeth where she would discern if I was able to identify various issues without her pointing them out beforehand.

4.8.2 Field notes

I made use of field notes to document my observations that were not part of the video recordings and supplement the information that was captured on the video recordings (i.e. making note of areas of interest to refer to in the video recordings). I made detailed notes while watching the lessons that had been recorded. This allowed me to identify the subtler responses and actions of the teacher (i.e. a gesture or action), which could have been overlooked in 'real time' (Merriam, 2009; Creswell, 2013). These were accompanied by my personal reflections.

Field notes are the written accounts of the observations that the researcher does (Merriam, 2009). These notes were written during the mathematics lessons and then typed up using a word processor after each lesson. I did this to ensure that I had a soft copy of all my notes. I made use of field notes to identify possible areas of interest during the lesson and the informal interactions that occurred between the teacher and learners during the lesson. The field notes were accompanied by personal reflections. These reflections provided commentary on what I had observed and areas of particular interest that I wanted to get clarification on from Elizabeth.

The videos, field notes and personal reflections were transcribed. These transcriptions were then used for analysis purposes to produce data. However, the transcriptions served a second purpose where they were used to generate possible questions that I could ask Elizabeth during our formal interview.

4.8.3 Interviews

Phase three consisted of the interviews and conversations that occurred between Elizabeth and me. The conversations were often casual and occurred after each lesson. Elizabeth and I would discuss the lesson and what I had observed. The formal interview at the end of the observation period was done to ascertain Elizabeth's beliefs on teaching and the approaches that she uses in her classroom. This information formed part of the data generating process.

There were two types of interviews used: casual conversations and semi-structured interviews. Casual conversations and in-passing clarification, as described by Rubin and Rubin (2012), generally take place during the observation process where the researcher and participant have "spur-of-the-moment chats" where the "conversation sometimes moves onto a topic relevant to research" (p. 30). Elizabeth and I had many casual conversations throughout the data-gathering period. She would explain her actions, particularly when she sensed that the learners were confused. She did this to ensure that I understood the rationale for her actions. I would document these conversations in my field notes. The conversations were held at times that were suitable for Elizabeth and took place in her classroom. The classroom proved convenient as Elizabeth often referred to the learners' workbooks when elaborating on key aspects of the lesson and to identify examples of issues that she has encountered.

The interview at the end of my observation period was pre-arranged. The purpose was to elicit information from Elizabeth that was relevant to my research (Bryman, 2012). I decided to leave the interview until phase three so that I could observe Elizabeth teaching first. I wanted to become familiar with how she teaches mathematics to the Grade One learners in her class. This enabled me to identify areas of interest and ask questions that would provide clarity on aspects of her practice (Maxwell, 2013; Rubin & Rubin, 2012).

The in-depth interviews provided an "understanding [of the participant's] perspectives… experiences, or situations as expressed in their own words" (Taylor et al., 2016, p. 102). I used a semi-structured interview format. Semi-structured interviews are where the researcher has a set of questions to guide the interview but is able to ask clarification questions and probe where necessary (Bryman, 2012). These interviews allowed for a more relaxed environment where Elizabeth was able to share her knowledge and express her views and opinions on each question. During the interview, I was able to develop and identify points of interest for clarity without being influenced by the rigidity of a structured interview (Denscombe, 2007). My intention was to ascertain "what, when, how, why, or with what consequence" (Rubin & Rubin, 2012, p. 32). I prepared questions beforehand based on the field notes and transcribed video-recordings. Using the questions as a guiding tool, I was able to have a more flexible approach to the interview. The semi-structured interview format also enabled Elizabeth to influence the direction of the interview based on what she deemed relevant as well as opening the conversation to other questions and possibilities that would not have initially been considered (Rubin & Rubin, 2012). This allowed her to give a detailed explanation of why she did something in a particular way. It also provided me with the opportunity to identify links between what Elizabeth said and what Elizabeth did in the classroom, that is, what she did while teaching mathematics.

The formal interview was audio-recorded to ensure that I did not miss any information that was pertinent to the research. Whilst the recording of the interview led to a more in-depth transcription of data (Bryman, 2012), it also allowed me to make note of all the important points that had been identified as well as identify points of interest that may have been missed through the more traditional method of note-taking. The recording of the interview also ensured the accuracy of the data that was recorded.

The casual conversations and interview were used to ascertain the knowledge Elizabeth used in developing the number sense of learners with MLD. It afforded me the opportunity ask for clarification as to how she adapted her teaching to assist learners with MLD.

There were some limitations that I had to contend with during the data collection process. Due to the restrictions of the school timetable, casual conversations with Elizabeth could not lapse into teaching time as this would affect Elizabeth's schedule. Along with this, the formal interview had to be organised for a time that was agreeable for both Elizabeth and me. During the casual conversations and interview, there were interruptions that would disrupt the flow of the conversation. This often resulted in information being overlooked.

4.9 DATA ANALYSIS PROCESSES

Qualitative research often leads to extensive amounts of data that can be overwhelming. In order to reduce the effects of an excessive amount of data at the end of the data collection period, I worked in an iterative manner with the data. By that I mean, that I analysed the collected data during the process of collection (Bryman, 2012; Merriam, 2009). This resulted in all the data gathered during the research process being condensed and the relevant data isolated.

The data from the formal interview and observations were transcribed and coded. Due to there being 10 lessons recorded, I selected 4 videos to transcribe and analyse. I chose these videos because they were reflective of Elizabeth's general mathematics teaching practice. I transcribed each video using the VLC media player program that allowed me to reduce the speed of the video data. I was able to capture the interactions during the lessons without omitting any information. I transcribed the interview with Elizabeth using a similar process to the videos where the speed of the audio file was reduced. The Audacity program assisted me in accurately transcribing the interview data. All transcriptions were recorded in a Microsoft Excel spreadsheet according to the turns of each speaker (Table 4.1 and Table 4.2).

I analysed the data using both an emic and etic approach (Maxwell, 2013). Emic refers to the process where the coding for data analysis is obtained from the data itself. Themes are identified as they emerge from the data. In other words, the coding is not influenced by frameworks or theories. I realise that we always bring our own knowledge and experiences into the process of data analysis, but I did not use the Knowledge Quartet framework to analyse the data initially. I wanted the data to 'speak for itself' (Geertz, 1973). Table 4.1 provides an example of the emic coding.

Turn	Speaker	Utterance	Key words (Emic Coding)
1	Teacher	A and B, pick up your containers, the rest of you put them down please, now swop. Which one do you estimate is the heaviest A?	Using resources Questioning Developing an understanding of vocabulary
	A	Points to her original container of orange lentils	
2	Teacher	That one. What do you think B?	Questioning
3	B	This one (Points to her original container)	
4	Teacher	Okay, so B, pop it in. *(B puts the container in the balance scale)* A, pop it in. *(A puts the container in balance scale)* Okay, so which is the heaviest?	Questioning Terminology/concept
5	B	Points to orange lentils	

Table 4.1 - Example of emic coding of data (taken from V4)

Thereafter, I used an etic approach to code the data by drawing on the Knowledge Quartet framework. The components of each of the Knowledge Quartet categories were aligned with the relevant data. While the Knowledge Quartet framework assisted in the analytical process, I am aware that not all of Rowland et al.'s (2013) components are relevant to this study. I am also cognizant that there are possible components that Rowland et al. (2013) might not have identified that may relate specifically to my research. Hence, my decision to adopt an emic approach to analysing my data first.

Turn	Speaker	Utterance	Key words (Emic Coding)	Knowledge Quartet (Etic Coding)
1	Teacher	A and B pick up your containers, the rest of you put them down please, now swop. Which one do you estimate is the heaviest A?	Using resources Questioning Developing an understanding of vocabulary	Knowledge of the appropriate terminology and mathematical language *(foundation)* The use of instructional materials *(transformation)* Knowledge of the purpose of mathematics *(foundation)*
	A	Points to her original container of orange lentils		
2	Teacher	That one. What do you think B?	Questioning	
3	B	This one (Points to her original container)		
4	Teacher	Ok so, B pop it in. *(B puts the container in the balance scale)* A, pop it in. *(A puts the container in balance scale)* Ok, so which is the heaviest?	Questioning Terminology/concept	Make decisions about sequencing *(connection)* Knowledge of the appropriate terminology and mathematical language *(foundation)*
	B	Points to orange lentils		

Table 4.2 - Example of etic coding of data (taken from V4)

The data from my research is presented and analysed in Chapter Five.

4.10 VALIDITY

Validity is concerned with the meaningfulness and trustworthiness of the research. As such, it examines whether the research identifies what is intended (Drost, 2011). Factors that influence the validity of the research are transferability, credibility, confirmability and data dependability (Bryman, 2012; Yin, 2009).

To confirm and increase the validity of my research, I used multiple data collection techniques. As such, I ensured that I triangulated the data. Yin (2009) states that the use of multiple sources of data in case studies is considered to be “a major strength of case study data collection” (p. 135). It is through the process of collecting data from numerous sources that the data is verified (Guba, 1981). In my research, I used observations, field notes, and interviews to triangulate the data. Triangulation was used to analyse whether the information gathered in my observations correlated with what was obtained from the interviews and field notes. The use of transcribed audio and video recordings allowed me to have a constant source of data to refer to as the need arose.

With case study research, the researcher needs to be able to explain how a phenomenon occurred and what information can support this (Yin, 2009). Theories that were related to the field of study were used to ensure internal validity. This was done to ascertain whether the data reflected what had been already been identified by these theories (Bryman, 2012). It was important that all observations and recordings that were made and transcribed were transcribed accurately as my intention was to develop an in-depth understanding of how Elizabeth develops learners with MLD number sense.

I asked Elizabeth to member check the data for me. This is a process where the researcher gives the participant the opportunity to read the interview and lesson transcripts in order to ensure their credibility and confirmability (Birt, Scott, Cavers, Campbell, & Walters, 2016; Merriam, 2009). This was done to determine that the researcher’s findings are believable and have not been influenced by any personal values or opinions (Bryman, 2012). Elizabeth and I would sit and discuss the lessons and the data that I had captured together to ensure that I had correctly interpreted what she said and did during the lessons and formal interview.

Transferability refers to the ability to ‘transfer’ the findings of the research to other contexts and dependability considers if the data can apply to other time frames (Bryman, 2012;

Merriam, 2009). However, due to this research being a bounded case study, the transferability and dependability of the findings into other contexts and time frames are very unlikely. In other words, the study was context specific and the results are not generalisable from one teacher to the next. The content and pedagogical knowledge that teachers require to develop the number sense of learners with MLD is possibly generalizable. However, the knowledge Elizabeth drew on is context specific because Elizabeth worked in an affluent state school.

4.11 ETHICS

Ethics refers to "norms for conduct that distinguish between acceptable and unacceptable behaviour" (Resnik, 2015, p.1). This means how the research process is approached, in a respectful manner, influences the research process in terms of: avoiding issues such as plagiarising and misrepresenting the data that is collected; respecting those that are participating in the research process; and holding the researcher accountable should there be any breach in the ethical conduct of the research or researcher (Resnik, 2015). For research to be valid, the ethics of the researcher and the research should be carefully considered. The researcher needs to ensure that all ethical principles are considered as ethical issues can occur at any time during the research process (Bryman, 2012). Due to this possibility, the researcher needs to take various precautions to ensure that any risk that can occur during the research process is minimized (Bryman, 2012; Taylor et al., 2016; Merriam, 2009).

Due to the nature of research in education being a 'delicate process' (as often it revolves around learners), I had to ensure that any potential ethical concerns that may have occurred were considered. This was done by adhering to the Research Ethics Guidelines of the Faculty of Education at Rhodes University. Prior to beginning my research, my research proposal was granted ethical clearance (Appendix 1). Being clear about the intentions of my research at the beginning of the research process, I ensured that the participant was protected and that any issues or questions the participant had were addressed prior to the commencement of the research (Bryman, 2012). Thus, for the purpose of this research, permission was sought from Rhodes University Faculty of Education's Higher Degrees Committee, the school, Elizabeth and the parents of the learners in Elizabeth's class.

In qualitative research, ethical issues can arise in the methods used to collect the data. Cresswell (2013) states that ethical issues “are likely to emerge with regard to the collection of data and in the dissemination of findings” (p. 230). As Taylor et al. (2016) state, qualitative researchers are involved in the day-to-day lives of the participants they are researching and, in so doing, they tend to observe the person at their best and their worst. Therefore, it was important for me to establish a good rapport with Elizabeth (and the learners in her class) and “promote values that are essential to collaborative work” (Resnik, 2015). Before the start of the research process in Elizabeth’s class, I provided her with assurance of confidentiality and anonymity (Taylor et al., 2016). In my study, I began establishing a rapport through the initial meeting with Elizabeth and the principal of the school. In this meeting, I sought permission to conduct research both in the school and in the chosen classroom.

Informed consent provides the participant, and other parties who may be linked to the research, with the information required to make an informed decision on whether they would like to participate in the research or not (Bryman, 2012). Obtaining informed consent from all parties involved, safeguarded Elizabeth (and the learners) from any deception throughout the research process (Bryman, 2012). All stakeholders were made aware of my research intentions, who I intended to observe, the length of observation, and how I proposed to gather the data.

In my initial meeting alone with Elizabeth, we discussed the purpose of the research and I reiterated that she has the right to withdraw from the study at any point. Confidentiality and anonymity were ensured during this meeting and in the information and consent letters given to the principal (Appendix 2), Elizabeth (Appendix 3), and parents (Appendix 4 and Appendix 5). Elizabeth kindly distributed and collected the parent consent letters for me. These letters explained the purpose of my research, who my intended participant was, and how I would go about collecting my data. The relevant stakeholders were required to sign the letters to indicate their consent.

Confidentiality requires protecting the privacy and anonymity of the stakeholders involved in the research process (Creswell, 2013). Confidentiality in my research was assured through the consent form and the initial meeting. I informed Elizabeth that any information gathered during my observations and interviews with her would be protected from all possible eventualities, such as loss, theft or unauthorized access and use of the data.

Throughout my research, I ensured that Elizabeth and the school that she teaches in remained anonymous. I have safeguarded this anonymity with the use of pseudonyms. However, I am aware that, due to her position within the local community, there is the chance that she could be recognised by those who read this thesis. However, through consultation with my supervisor and Elizabeth, we did not deem this an issue that would cause her harm. Elizabeth is regarded as an expert teacher in the community. My research sought to understand what she does and why and not to criticise her or her practice.

During the interviews, I made sure that Elizabeth's dignity was maintained and that she did not feel embarrassed at any point (Merriam, 2009). Our interviews and conversations were guided either by events that had transpired in the classroom during my observation or when Elizabeth provided me with details from events that occurred on days that I was not in the classroom.

With observation, the researcher should avoid using any form of deception, such as making use of covert observation methods that the participant has not consented to (Bryman, 2012). I was always clear as to when I would be observing, and the participant was aware when I was taking field notes or recording the lesson. I always left my field notes open on the table when walking around the classroom so that Elizabeth could read them if she so wished. I did not want to make her feel uncomfortable at any point during the data collection process.

All the hardcopy versions of field notes and interview responses during the data collection process were kept in a secure place that no other person has access to. The hardcopy notes were backed up on to the computer where they were stored in a password protected folder. This ensured that no unauthorized access was possible. These safeguarding methods were put in place to avoid any tampering with or modification of the data that had been gathered.

4.12 REFERENCING CONVENTIONS IN MY THESIS

The following referencing conventions are used to refer to my data throughout.

Interview referencing format	
I1	Interview 1
Kirsty	Researcher
Elizabeth	Participant
A/B/C	Refers to learners that Elizabeth is interacting with. *While my focus is not on the learners, it is necessary to differentiate between the different learners Elizabeth interacts with.*
Lesson referencing format	
FN1, T1	Field note 1, turn 1
V1, TT12-13	Video 1, turns 12-13
V4, T4	Video 4, turn 4

Table 4.3 – Interview and lesson referencing conventions

The following conventions have been used to indicate when Elizabeth is commenting or providing a demonstration in Chapter Five.

Bolded writing	This indicates Elizabeth's responses or comments from video observations and interview.
Italicised writing	This indicates actions or gestures used by Elizabeth or the learners during a lesson
…	Indicates text that was difficult to hear, has been omitted or is missing.
[ ]	Used to indicate where the researcher has added text for clarity

Table 4.4 – Referencing conventions for Elizabeth's discourse and for actions

4.13 CONCLUSION

My research focused on an individual teacher, Elizabeth. It sought to identify the content and pedagogical knowledge that Elizabeth drew on as she assisted learners with MLD develop their number sense. The research adopted a case study method as my interest was a single teacher and a single phenomenon. The case study method enabled me to explore Elizabeth's practices in depth. Observations, interviews and field notes were used to generate the data.

The observations and interviews were transcribed immediately after each lesson or interview so that I could work with the data in an iterative manner. I collected and analysed the data simultaneously, particularly in relation to the emic coding process. The etic coding occurred after the data collection process. In Chapter Five, I present and analyse the coded data. The first section of Chapter Five presents the analysis from the emic coding process. The second section of Chapter Five focuses on the etic coding. This was done by drawing on the initial emic codes.

CHAPTER 5: DATA PRESENTATION AND ANALYSIS

5.1 INTRODUCTION

This chapter attempts to answer the research question: What content and pedagogical knowledge does a Grade One teacher use to assist learners with MLD when developing their number sense? This question emerged from my reading of literature that explains poor learner performance in mathematics. Chapter One explains that the poor learner performance of learners in mathematics can be attributed to several factors, such as the socioeconomic environment of the school, teachers' insufficient content knowledge and poor pedagogical practices, the LoLT and lack of support for learners with MLD. This suggests that teachers require knowledge to assist learners with MLD. It is in the process of identifying what knowledge teachers draw on to assist learners with MLD to develop their number sense, that teacher education programmes can ensure teachers are able to assist learners with MLD.

In Chapter Two, I argue that number sense is central to the primary school curriculum and thus key to learners' performance. While the concept, number sense, is contested, I show that researchers agree (more or less) that number sense in the foundation phase includes eight broad components. In this chapter, I focus on three of the components of number sense. These are an awareness of the relationship between number and quantity, an awareness of magnitude and comparisons between different magnitudes, and to talk about mathematical concepts with understanding. I have chosen to focus on these as they are most evident in the data that emerged while I was observing Elizabeth.

Knowledge of how to develop learners' number sense is necessary for improving learner performance, particularly with learners with MLD. Chapter Two provides an overview of several challenges that learners with MLD experience. The potential challenges that Elizabeth focuses on in her lessons include mathematics language, working memory and memory skills. Human, van der Walt and Posthuma (2015) state that knowledge of MLD and the specific challenges learners with MLD have, needs to be considered before developing and implementing any form of intervention.

In this chapter, I present and analyse the data that was collected to investigate the specific knowledge that a Grade One teacher draws on when assisting learners experiencing MLD. I begin the chapter with a brief overview of the context of the classroom and the observed lessons (number 5.2). Thereafter, I present my analysis of the observation and interview data, and my field notes (number 5.3). This analysis is based on inductive reasoning and the use of emic coding. I then present an analysis identifying the number sense components evident in Elizabeth's classroom (number 5.4). Lastly, I analyse the data using my chosen theoretical framework (number 5.5), the Knowledge Quartet (see Chapter Three). The four categories of the KQ, namely foundation, transformation, connection, and contingency knowledge have assisted me in identifying the knowledge that Elizabeth draws on while developing the number sense of learners with MLD.

5.2 THE STRUCTURE OF THE MATHEMATICS LESSONS IN ELIZABETH'S CLASSROOM

Elizabeth's mathematics lessons begin with the whole class sitting on the mat in front of the classroom. The learners are either seated in rows facing Elizabeth or seated in a large circle on the mat with Elizabeth sitting in the circle with them. The lesson begins with Elizabeth introducing the topic for the day. This is followed by Elizabeth explaining to the learners the activities they will need to complete at their desks. Elizabeth writes the activities on the board in the order she wants the learners to complete them. This is done to ensure productivity and limit interruptions throughout the lesson. There are on average three activities throughout the lesson. Elizabeth explains **"you set tasks for them to do that are enjoyable, um I call them 'high-value task', the things they really want to participate in, that are easy enough to complete without your input, then you can effectively teach on the carpet" (I1, T16)**. Whilst the learners are working at their desks, Elizabeth calls a group of learners to the mat. This small group activity provides an opportunity for Elizabeth to interact with each learner during the mathematics lesson.

In the first six weeks of the year, the learners are grouped randomly allowing Elizabeth to monitor each learner before grouping them according to 'ability' levels[9]. When selecting learners to come to the mat Elizabeth says that she **"starts randomly until [she] can see 'who is who', and who grasps concepts easily, and who has got the vocabulary for what you are trying to teach. [But] the groups change all the time depending on who's doing what and**

[9] This refers to the learner's perceived mathematics ability which is subject to change throughout the year as the learner gets stronger or starts to struggle.

who is grasping concepts" (I, T18). In Elizabeth's class, the class is divided into three groups with no more than ten learners in a group.

The mathematics lessons in Elizabeth's classroom last between 60 and 90 minutes. The time for the mathematics lessons alternate. On Monday and Wednesday, mathematics is the first lesson of the day, and on Tuesday and Thursday, it takes place after break time. This is done to maintain a structured learning environment and to ensure that mathematics and languages get the same amount of attention and emphasis. The lessons begin with the learners singing a song, counting in 2s, 5s, or 10s and, in some cases, watching a video that introduces the mathematics topic for the day. This whole class activity lasts between twenty and thirty minutes. Thereafter, the learners go to the mat in groups. These last between ten and twenty minutes per group, depending on the concept that is being taught.

While one of the groups is on the mat with Elizabeth, the rest of the class are completing activities at their desks. When one group is finished on the mat they move to their desks and the next group is called to the mat. During this changeover period, learners have the opportunity to approach Elizabeth with questions about their work. Queries about the activities did not only happen during the changeover period. There were many occasions when learners would walk up to Elizabeth while she was working on the mat with a group and ask her a question. This would disrupt the mat activity. However, being the beginning of the Grade One year, Elizabeth was still working at getting her learners into the various classroom routines.

5.2.1 Whole class interaction on the mat

I have chosen to present three examples of whole class lessons in this chapter. Vignette 5.1, 5.2, and 5.3 are typical examples of the interactions between Elizabeth and the learners during the whole class teaching stage of the lesson. These observations occurred at the beginning of each lesson when Elizabeth introduced the entire class to the mathematics topic.

Vignettes 5.1 and 5.2 are from the same day and focus on the same concept. In vignette 5.1, Elizabeth makes use of a learning opportunity to introduce the learners to 'opposites'. Elizabeth and another Grade One teacher came to school dressed the same but in opposite colour combinations. For example. Elizabeth wore a black shirt and white pants and the other teacher wore a white shirt with black pants.

Vignette 5.1 Introducing the concept of 'opposites'

Elizabeth takes advantage of the fact that she and her colleague are wearing clothes of the same colours but in a different combination. She decides to use this as an opportunity to teach the learners about 'opposites'. Elizabeth is wearing a black shirt and white pants and the other Grade One teacher is wearing a white shirt and black pants. Elizabeth invites the other teacher to join her in her classroom. *Elizabeth starts the lesson by asking the learners if they notice anything when looking at the two teachers. The learners begin making several identifications, such as Elizabeth is the shorter of the two, Elizabeth is wearing different coloured sandals, Elizabeth's hair colour is different. Whilst these observations are correct, they are not what Elizabeth expects. To get their attention she starts to gesture and points to the clothes that both teachers are wearing.* *One of the learners then notices that the teachers are wearing the same colours but in opposite combinations. However, she does not use the term 'opposite'. Through questioning (i.e. "what's the word?" "and there is a word") one of the learners eventually pronounces that the term is 'opposite'.*

Vignette 5.2 focuses on the prepared lesson on 'opposites'. This lesson begins after the teaching and learning opportunity described in vignette 5.1.

Vignette 5.2 A continuation of the lesson on 'opposites'

The observation begins with Elizabeth reading a book to the learners. The central theme of the book is 'opposites'. Each double page had two words written on it that are 'opposites'. (e.g. 'day' and 'night', 'young' and 'old', 'short' and 'long'). Throughout the story, Elizabeth encourages the learners to perform the actions that corresponded with the words in the book. For example, with 'up' and 'down', Elizabeth asks the learners to first stand up and then sit down. She does this several times. When it came to the difference between 'young' and 'old', Elizabeth asks the learners to think about the differences between themselves, being 'young', and herself, being 'old'. Through a process of enquiry, Elizabeth encourages the learners to identify different terms that relate to the ones in the book (e.g. 'grown up' and 'children' as opposed to 'young' and 'old'). Along with asking the learners to complete various actions for different words, Elizabeth uses hand gestures to illustrate the differences between words (e.g.

between 'tall' and 'short'). Once she has read the entire book to the learners, Elizabeth reads the book again, but this time she reads it in unison with the learners.

After the class have read the book together, Elizabeth asks the learners if they can think of any other examples of 'opposites' (i.e. 'short' and 'long', 'up' and 'down'). To consolidate the concept further, Elizabeth draws on the learners' prior knowledge by using examples of 'opposites' from the measurement lesson that she taught the previous day. Elizabeth asks the learners to think about possible 'opposites' that they remembered from their measurement lesson (e.g. 'heavy' and 'light'). Elizabeth reminds the learners about the routine[10] they do in the morning and encourages the learners to identify other 'opposites' (e.g. 'left' and 'right'). At the end of the whole class session, the learners watch a video that has a song about 'opposites'. They sing along to the song.

Vignette 5.3 was a whole class lesson on the mat where the learners were seated in a circle around a pile of unifix blocks. In this lesson, Elizabeth makes use of manipulatives to introduce the learners to the concept of 'less'.

Vignette 5.3 Introduction of the concept of 'less'

Elizabeth starts the lesson by asking a learner to take 5 blocks from the pile of unifix blocks which are placed on the carpet in the middle of the class. The learner takes five blocks from the pile. Elizabeth asks another learner to take one block less from the pile.

Before allowing the second learner to pick up the blocks, Elizabeth asks the class what they think the word 'less' means. Elizabeth uses this opportunity to clear up any confusion the learners have with defining the terms 'less' and 'more' by asking what 'more' means. One learner states that it means the amount "is getting bigger". Elizabeth asks the class for an explanation of the term 'less'. After asking several learners (with response ranging from "'less' means 'behind', "it's getting shorter" etc.), one learner gave the correct answer ("'less' refers to a smaller number"). Elizabeth allows the second learner to take her blocks. She takes four blocks (i.e. one less than five). Elizabeth continues to ask specific learners to take one block less than the learner before them.

[10] The routine is Elizabeth's own and involves the learners pointing to the left and right and then to the front and back. Through conversations with Elizabeth, she said the routine is to get the learners to cross their mid lines and the order of the actions varies.

Once the fifth learner takes one block from the pile, Elizabeth asks a learner to take less than that. She does this to see if the learners understand what 'less' means. The learner, whose turn it is, responds that she will pick up nothing because "there is nothing smaller than one".

Once the group completes the activity with the unifix blocks, Elizabeth produces a poster with the word "less" on it (see Figure 5.1). The font on the poster gets smaller for each letter. She asks the learners why she has written the word in this manner. She is trying to encourage them to see the link between the font size and the notion of something becoming 'less' or becoming 'smaller'.

All three vignettes (5.1, 5.2, and 5.3) focus on the concept, 'opposites'. Elizabeth begins this series of lessons trying to get the learners to first understand the notion of 'opposites'. She does this by drawing on the learners in and out of school experiences. Once she thinks the learners have a sense of 'opposites' generally, she draws their attention to the concept of 'less'. The concepts of 'more' and 'less' are central to the development of number sense as they focus on developing an understanding of the magnitude of numbers and comparisons between different numbers.

For Sayers, Andrews and Björklund Boistrup (2016) one of the key components of number sense includes an "awareness of magnitude and of comparisons between different magnitudes" (p.4) Central to developing an understanding of comparisons between magnitudes is the use language like 'bigger than', 'smaller than', 'less' and 'more' (Sayers et al. 2016).

5.2.2 Small group interaction on the mat

The group mat activities are an opportunity for Elizabeth to observe and interact with the learners to ascertain the extent of their number sense (and mathematics more generally) and the vocabulary necessary for number sense development. For all group mat activities, the learners are seated in a circle with Elizabeth. The resources are placed either in the middle of the circle or next to Elizabeth.

Vignette 5.4 summarises a small group interaction on the mat on 'mass'. In this vignette, Elizabeth makes use of a balance scale and small plastic containers of the same size which are filled with varying objects. These include dried macaroni, Lego bricks, balloons, lentils, and beans. The containers are filled to different levels (some being filled all the way to the top of the container and some only being filled halfway) with items that are 'heavy' (e.g. beans) or 'light' (e.g. balloons). She uses the containers to introduce the learners to the concepts of 'heavier' and 'lighter'. This lesson elaborates on her lessons on 'opposites'.

Vignette 5.4 Small group interaction on the mat introducing the concept of mass.

> *The learners sit in a circle around a collection of containers and a balance scale. The learners select a container and hold it in their one hand. She then asks the learners to swop the container with the person next to them. Elizabeth asks the learners to estimate which container, out of the two they have held, is the 'heaviest'. She does this with all the learners. Each learner points to their original container (i.e. the one they chose), regardless of whether it is the 'heaviest' of the two.*
>
> *Elizabeth asks the learners to put their containers into the buckets of the balance scale. They take turns to do this. Elizabeth asks them to identify which container is 'heaviest' and which is 'lightest'. Elizabeth gives each learner a chance to put their containers into the balance scale and tell her which of the containers is 'heaviest' and 'lightest'.*
>
> *The containers are returned to the centre of the circle. She does the same activity again, this time with each of the learners taking different containers. Elizabeth takes a different approach to ensure that the learners don't guess the weight of the containers. She tells the learners to choose a container, one that they had not selected beforehand. She instructs the learners to swop their container with a different learner. This time the learners take turns to hold their container and that of their partner simultaneously with one container in each hand. Elizabeth asks the learners to predict which of the two containers they think is 'heaviest'. The learners take turns to put the containers in the balance scale buckets and see whether their predictions are correct.*

In all of the above vignettes, Elizabeth emphasises the use of language and the development of vocabulary in mathematics. She does this for two reasons: (1) she maintains that vocabulary development is a necessary component of number sense development; and (2) many MLD originate from a misunderstanding of the mathematics terminology. Given that the home language of the vast majority of her learners differs from that of the LoLT, she takes great care in ensuring that all the learners have a clear understanding of the mathematical concepts and related vocabulary.

5.3 INTERPRETATION AND ANALYSIS OF OBSERVATION DATA FROM WHOLE CLASS AND GROUP MAT ACTIVITIES

Elizabeth regards the first six weeks of the Grade One year as extremely important. It is the period where she is able to ‘get to know’ the learners and their perceived ‘abilities’, whether they **“are left-handed, are they right-handed? Are they ambidextrous? Are they confused about which hand to use? Can they effectively cross the midline? Can they use a pair of scissors? Those kinds of things” (I1, T12).** As well as the aforementioned information, this introduction period is vital in determining how well the learners cope with developing their number sense and whether there are any clear, identifiable difficulties.

12	Elizabeth	In those six weeks, first of all, you want to get structure and routine and get to know the child and give them a chance to settle down. And you want them, you want to know for sure they are left-handed, are they right-handed? Are they ambidextrous? Are they confused about which hand to use? Can they effectively cross the midline? Can they use a pair of scissors? Those kinds of things. Can they follow instructions? Just depending on what they can do, those things you start to watch and observe and question.
13	**Kirsty**	Moving on. Do they have to do, sort of almost have a standardised test that they do? Or is it as you said, just general observation?
14	Elizabeth	Just every day observing, intervening, scaffolding, trying to see until you actually know your class and your children.
15	**Kirsty**	Um, so I think a big one is, once you've seen learners who have certain issues or difficulties, how do you work with those, those learners without losing control of the rest of the class? Is there a specific sort of approach you take, or do you try to incorporate it into your general teaching?
16	Elizabeth	It has to be part of your general teaching. Um, so for example, it is not mathematics, our spelling, our phonics. Everybody learns with simultaneous oral spelling. That's the method. Because that's the method that will best support children with difficulties with reading and

		spelling and the children who don't have difficulties will benefit because they'll be able to use those skills to decode even larger words than they should be. So that's the one point. The second point is that you try and make, by grouping them, you try and make it as inclusive and around their 'abilities' and who they are. And if you set tasks for them to do that are enjoyable, um I call them high value task the things they really want to participate in, that is easy enough to complete without your input, then you can effectively teach on the carpet and, um, work on children with difficulties without it being an, having an impact on your discipline. The trick is that the activities mustn't be so easy that they complete instantly, and they must want to spend time on them because they value the activity.
17	**Kirsty**	Okay, that then leads on to the next question. When you call them to the mat in their groups, is there a specific, sort of, 'method to the madness'? Or do you just, or is there specific groups, or do you just call them randomly?
18	Elizabeth	You start randomly until you can see 'who is who' and who grasps concepts easily and who has got the vocabulary for what you are trying to teach. Um, but the groups change all the time um, depending on who's doing what and who is grasping concepts. As you go along they develop and they grow and maybe someone isn't developing as fast as someone else so you change around and then the next person will catch the other and then if you've got children like we've got now, very, very much ahead of their peers, then they need additional activities to, to keep them interested and stimulated.

(I1, TT12-18)

Gervasoni (2004) argues that knowledge of the learners' needs and 'abilities' is necessary for number sense development. As Elizabeth states, the first six weeks are essential in finding out who the learners are and what their 'abilities' are. She says, **"you want to get structure, and a routine and get to know the child and give them a chance to settle down" (I1, T12).** By grouping the learners according to their 'abilities', and moving them to a different group when need be, Elizabeth recognises that learners develop their understanding of particular mathematics concepts at different times. Elizabeth acknowledges **“the groups change all the time um, depending on who's doing what and who is grasping concepts. As you go along, they develop and they grow and maybe someone isn't developing as fast as someone else so you change around and then the next person will catch the other” (I1, T18).** Knowledge of the mathematical development of learners allows Elizabeth to make informed decisions on how to progress from one concept to the next. This aligns with Dowker (2005) who states the importance of meeting the needs of learners through differentiating both content and the learning process.

19	**Kirsty**	What are the main sort of difficulties that you encounter in, sort of, the process of those first six weeks and then leaving out as well in mathematics, specifically, and what are the possible explanations behind that.
20	Elizabeth	Um, well if we're going to put it in context of this class and this year, establishing vocabulary and what it needs and when you explain what you have to do. I mean, it is August (referring to a previous year) and we have somebody who still doesn’t know what is required when I say plus.
21	**Kirsty**	Oh wow!
22	Elizabeth	Um, they've yet to grasp the word more and always has to be, [interruption] it has to be paraphrased every time. You have to say, “if I want so and so and two more” if we're working on the number line, then “what does more mean” “it's what?” and then eventually I manage to drag out ‘bigger’. So which way are the numbers getting ‘bigger’? Um, I've never quite had it like this. Um, where children still don't grasp the words ‘more’, ‘less’, ‘bigger’, ‘smaller’, ‘add’, ‘subtract’, ‘plus’, ‘minus’.

(I1, TT19-22)

Moving from one concept to the next is dependent on the learners in each ‘ability’ group having a clear understanding of the concept that is taught. If a learner struggles with a concept such as counting and number order, they will struggle with completing tasks that require them to identify the magnitude of numbers (i.e. 4 is less than 5). As Elizabeth mentions **“if you say to them um, think 7, 6, what's the next number? So, if they can't do**

1,2,3,4,5,6,7,8,9,10, 10, 9, 8 you can't apply that to any of the other number range" (I1, T28). Elizabeth says that in order to understand how larger numbers are generated, the learners need to know the numerals from nought to nine and be able to count in ones up to ten. If they can do this, the learners will be able to see the pattern necessary to generate numbers (e.g. 7, 17, 27, 37). Therefore, developing and solidifying an understanding of counting to ten and recognition of the number names and numerals is necessary before progressing to work with larger numbers. There can be times when, despite numerous attempts, a learner does not grasp a concept. Elizabeth states that when this happens the concept **"has to be paraphrased every time" (I1, T22).** By this, she means that she has to rephrase the question or statement to ensure the learners understand. For example, if learners don't understand the term 'more', she may need to say 'it gets bigger'**.**

Developing the learners' understanding of mathematics can be done in a variety of ways. In Elizabeth's classroom, the two main approaches that were observed were: (1) the use of language, specifically the development of mathematics-related vocabulary; and (2) the use of resources to develop an understanding of the language and related concepts.

Adler (2000) posits that learning and teaching support materials (LTSM) are important in developing mathematical concepts in the classroom. She explains that the use of LTSM is not limited to material objects. To aid the development of an understanding of mathematical concepts, Elizabeth makes use of a variety of resources. The resources provide the learners with an opportunity to consolidate what they have learnt. Elizabeth states that **"if you have enough practice and it's well consolidated before you move on then the next concept is quite easy" (I1, T56).**

Adler (2000) suggests that there are three kinds of resources: human resources, material resources and socio-cultural resources. Human resources refer to the teachers pedagogical knowledge; material resources encapsulate the variety of objects in the classroom such as whiteboards, computers, textbooks, manipulatives, posters, and everyday objects; and socio-cultural resources include language and time (Adler, 2000). Elizabeth selects resources based on how they support the development of conceptual understanding. These resources are used to facilitate the learning process and provided the learners with the opportunity to explore and develop their number sense. In Table 5.1, I list the resources that Elizabeth used while I was in her class.

Human Resources	Material Resources	Socio Cultural Resources
Teacher – including teacher's pedagogical knowledge	Technologies – whiteboard, computer	Language – First language, second language, Language of learning and teaching (LoLT), verbalisation, communication
	School mathematics materials – textbooks, other texts, unifix blocks, balance scales, shapes.	Time – the structure of timetable, length of lesson periods etc.
	Mathematical objects – number lines, posters,	
	Everyday objects – clothes	

Table 5.1 - Classification of the resources Elizabeth uses (adapted from Adler, 2000)

5.3.1 Types of resources

Gurganus (2004) and Black (2014) suggest that one of the strategies to assist learners with MLD is to make use of different resources. Elizabeth develops her learners understanding of mathematics through the constant use of resources. Resources are used to develop the learners' vocabulary which assists with their general understanding of concepts.

5.3.1.1 Books

One of the resources that Elizabeth uses in her classroom is a series of books that are geared towards developing learners' number sense. These *NumberSense*[11] books (Figure 5.2) allow the learners to consolidate what is taught in class. As mentioned in the extract below, Elizabeth notes that one of the many benefits of the books is that the **"children really enjoy it. They love the programme and love working in books and there is no resistance. Everybody likes the books" (I1, T48).**

[11] The title appears as one word

46	Elizabeth	Well, the resources are the same. Um, the number sense programme is a bit different. Um, and I have learnt a number of things that I did not know previously. The website, if you don't know, tells you exactly what, what is required and what they're trying to develop on that page. The nice thing is that, is there is so much repetition that eventually most people get there.
47	**Kirsty**	Something sticks.
48	Elizabeth	And the other thing is the children really enjoy it. They love the programme and love working in books, and there is no resistance. Everybody likes the books. Um, one of the interesting um, concepts they have is that the equals sign means ‘the same’, ‘the same as’. Um, because traditionally to teach, teachers would say equals was, and this 1+2 equals the answer is 3. Um, they are saying 1+2 is the same as 3.
49	**Kirsty**	Mmm, that's very interesting
50	Elizabeth	And they turn sums backwards from the very beginning 3 is the same as something + 1 and 3 is the same as 2 + something. Um, and that has caused major difficulties especially for my class because they're the babies, the outliers principle I explained to you, and their sense of directionality isn't well established and so every time they want to read it from left to right and not everything, in fact, a lot of the book is not done from left to right. Um, so they have this um, counting … sometimes it's counting in 2s, sometimes it's counting in 5s, sometimes it's addition and subtraction but it's like a snake
51	**Kirsty**	Oh, I remember it goes around.
52	Elizabeth	So, what happens is they complete the first line correctly and then they complete the second line correctly but in the wrong order and then they complete the third line correctly. So, I am forever sending back trying to fix the middle. So, I have started to try to highlight it and say to them watch the snake, watch what the snake is doing so that you follow it

(I1, TT46-52)

The benefits of the *NumberSense* books in Elizabeth's classroom are twofold. Firstly, the books use a different approach to mathematics than the traditional approach in the classroom. This is evident when Elizabeth says, **"one of the interesting, um, concepts they have is that the equals sign means 'the same', 'the same as'. Um, because traditionally to teach, teachers would say equals was, and this 1+2 equals the answer is 3. Um, they are saying 1+2 is the same as 3" (I1, T48).** Elizabeth highlights that this change in the representation of a sum **"caused major difficulties especially for my class because they're the babies, … and their sense of directionality isn't well established and so every time they want to read it from left to right" (I1, T50)**. In using these books, Elizabeth is then able to identify areas that require assistance and subsequently provide the learners with the necessary support.

Secondly, the books provide the stronger learners with extra practice. This is evident when Elizabeth states **"the number sense programme has definitely pushed our children into number ranges I wouldn't have considered before and an order I would not have considered. And it does seem to have merit and value because out good children, the children who are more mathematical, really have developed their knowledge and skills exponentially" (I1, T34).** Due to the layout of the books and the constant repetition of activities and concepts, learners are prompted to develop their automaticity of basic concepts. This resonates with the notion that with enough practice the learners will be able to move on to the next concept.

5.3.1.2 Manipulatives

In addition to the *NumberSense* books, Elizabeth uses several different manipulatives. These include:

- shapes for counting, shape identification and colour identification;
- white beans for counting and sharing;
- unifix blocks for counting, and working with concepts such as 'less' and 'more';
- a book, which was used to introduce and consolidate 'opposites';
- a balance scale and containers filled with different objects for 'heavy' and 'light'.

Having access to resources gives the learners the opportunity to visualise what they are learning.

In vignette 5.3 Elizabeth makes use of concrete objects to explore the concept of ‘less’. The unifix blocks assist Elizabeth in getting the learners to represent and visualise ‘one less’. This can be seen in the interaction between Elizabeth and a learner depicted in the excerpt below. Elizabeth is questioning what ‘less’ means before the learner picks up any blocks. As Elizabeth remarks, when introducing new information to the learners you need to **“try and explain it, unpack it enough that they develop understanding” (I1, T58).**

13	Elizabeth	What do you think this word ‘less’ means A?
14	Learner A[12]	Like you don't take ‘five’, you take ‘four’
15	Elizabeth	You don't take ‘five’, you take ‘four’. Pat yourself on the back. Well done

(V3, TT13-15)

Learner A’s response, **“like you don’t take ‘five’, you take ‘four’” (V3, T14)**, indicates that she understands the meaning of ‘less’. Learner A then represents ‘less’ visually using the use of the unifix blocks.

Another example of the use of concrete objects can be seen in vignette 5.4 where the learners are able to interact with the concept of ‘mass’ by weighing containers filled with different items on a balance scale. The interview excerpt below highlights that the learners were not selecting containers based on an educated guess or estimation. Instead, the learners all suggested that the container they had chosen was heavier than that of their peer. Elizabeth did acknowledge that the learners’ responses were not a true reflection of their understanding.

12	Elizabeth	Which is the heaviest?
13	Learner A	That one *(A points to the container of beans in B hand and B points to the container in A's hand)*
14	Elizabeth	*Speaking to B* Why are you choosing this one? (P*ointing to macaroni container in A's hand)* Why do you think this one is ‘heavier’? (*Pointing to macaroni container)* … Is it because it is the one you chose?

[12] Learner A is not a particular learner. I use Learner A and B to indicate that these are different learners responding to a question. Learner A in one excerpt is thus not necessarily the same learner in a different excerpt.

15	Learner B	Yes
16	Elizabeth	Okay A, pass her both please. *(A passes B her container)* Hold them both, feel them both. Now, which is the 'heaviest'? Go B. You can't choose a bottle to be 'heavy' just because it's the one you took … So, B which is the 'heavy' one? *(B points to beans in the down bucket)* The beans … Which was the 'lightest' one? *(A points to the Macaroni in the up bucket)* The macaroni

(*V4, TT12-16)*

In this excerpt, Elizabeth identifies that the learners' approach to the activity is not as she had hoped. She asks the one learner **"why are you choosing this one? Why do you think this one is 'heavier'? Is it because it is the one you chose?" (V4, T14).** This is remedied when Elizabeth tells the learners to **"hold them both, feel them both. Now which is the 'heaviest'?"** and following up with **"you can't choose a bottle to be 'heavy' just because it's the one you took" (V4, T16).** According to Ernest (2011), mathematics proficiency is enhanced when learners are given the opportunity to respond to questions (Ernest, 2011) and communication enriches learners number sense (Black, 2014). During a casual conversation with Elizabeth after the lesson, she indicates that the initial confusion with the learners selecting their original containers is not one she anticipated before the lesson. However, using an alternative approach to the lesson, Elizabeth is able to engage with the learners with the concept of 'mass' and ascertain of whether they have grasped the concept.

5.3.1.3 Vocabulary

Fritz et al. (2014) state that language is central to the development of number sense as it requires learners to verbalise their thinking and understanding. Therefore, how the teacher communicates with learners plays a vital role in them understanding the terminology and concepts when they are introduced. One overarching category evident in the data is the use of 'hooks'. 'Hooks' assist in promoting vocabulary development and conceptual understanding.

5.3.1.3.1 Hooks

As Ernest (2011) and Cockroft (1982) proposed, development of vocabulary in the classroom can be assisted by emphasising discussions that centre on appropriate mathematical language. Elizabeth uses a number of strategies, which she refers to as 'hooks', to assist the learners in the mathematics lesson. These included: songs, rhymes, routines, books, links to everyday life, posters, and gestures and actions. When discussing the importance of language development in mathematics, Elizabeth states that "**mnemonic hooks" (I, T58)** are one of the methods she uses. Hooks, such as a **"song or it could be sharing something out" (I, T58)** assist the learners to remember what they were learning**.** As Black (2014) maintains, learners need to have to opportunity to explore and verbalise their understanding when developing their number sense. 'Hooks' are utilised to allow the learners to **"access the vocabulary" (I, T58)** which, in turn, encourages them to **"remember what to do until they get to the stage of automaticity when they don't need the hook and they leave it behind" (I, T58).** This means that through the constant repetition of the 'hook', and encouragement for the learners to use the 'hook', they are able to complete the activity by themselves.

In the excerpt below, Elizabeth explains the use of 'hooks' as a means for developing learners' mathematics vocabulary. She discusses the possible methods that can be used as well as highlighting a positive outcome of a 'hook' being used in mathematics.

58	Elizabeth	… Um, and the third thing is to try and find mnemonic 'hooks' that will allow them to, to access the vocabulary because the hook is good. They will remember what to do until they get to the stage of automaticity when they don't need the 'hook' and they leave it behind. So right now um, we're 'halving' or 'sharing' and we doing both and we're now presented those who are not sure of the difference between 'doubling' and 'halving', because essentially they are 'opposites', will say to me is this me and you, because when I taught it to them we used some sweeties and we shared it out one for me, one for you
59	**Kirsty**	Okay
60	Elizabeth	So, by using sweeties which they weren't going to share unevenly I was able to teach 'halving'.
61	**Kirsty**	Okay

62	Elizabeth	… and the ones that don't need the 'hook' start and they 'halve' and if they by mistake they 'double' I say to them "you're meant to be 'halving' here but you're 'doubling'. What should you be doing?" and sometimes the 'hook' is a song. Um, so, um, I'm definitely not singing it for you.
63	**Kirsty**	*[Laughs]*
64	Elizabeth	But the 'doubling' I taught them with a song and it's a very catchy thing and they remember it. So, the hook can be a song, or it could be sharing something out.

(I1, TT58-64)

'Hooks' in Elizabeth's classroom can be reinforced in several ways. By creating a connection to everyday life, using resources, and using gestures, Elizabeth is able to create a meaningful connection for the learners to latch on to when a concept is being introduced to them.

In vignette 5.1, Elizabeth makes use of everyday life as a teaching moment to create a 'hook'. By asking the learners to compare the outfits of the two teachers, Elizabeth wants her learners to engage with the concept of 'opposites'. Elizabeth thus encourages the learners to apply and develop their understanding of 'opposites' in circumstances that are not necessarily considered to be related to mathematics. In other words, 'opposites' are not in and of themselves, a mathematics concept but it forms the basis for concepts such as 'heavy' or 'light', 'less' and 'more').

The link between the classroom and everyday life can be seen when Elizabeth uses a storybook to introduce and reinforce the term 'opposite' by assisting the learners to recognise and relate to everyday examples (i.e. 'day' and 'night', 'up' and 'down', 'awake' and 'asleep'). Whilst the book is a material resource, the content of the book has an explicit link to everyday life and activities/actions that the learners do. It highlights activities and comparisons that the learners would encounter outside of the maths lesson, and outside of the classroom (e.g. sit down and stand up). Cockroft (1982) maintains that teachers should use materials that the learners will understand. This is echoed by Gurganus (2004) and Black (2014) who acknowledge that when working with learners with MLD, giving meaning to numbers and concepts by associating them with everyday items or activities can benefit the teaching and learning process. Thus, when a teacher is able to link everyday experiences with the curriculum content then the probability of ensuring understanding increases.

Elizabeth makes use of a variety of material resources in her lessons (e.g. poster, unifix blocks, a book). In vignette 5.2, Elizabeth read a story on 'opposites' to the learners. The story was used to assist the learners in understanding what the term 'opposites' means and what opposites are. If the learners to do not understand these meanings (e.g. 'heavy' and 'light' in measurement, 'less' or 'more' on the number line), then there is the possibility that they will struggle with more complex concepts later in their schooling. This aligns with Clements and Sarama (2009) who state that developing learners' language competence is central to the development of number sense. According to Elizabeth, this is especially evident with **"children whose home language isn't English. Every year you need to establish the vocabulary to teach mathematics because if you don't understand the mathematical vocabulary how are you going to access the learning?" (I1, T36).** Mathematics language has a unique structure and if the learners do not understand the language then the probability of confusion heightens (Ernest, 2011; Jordan and Levine, 2009). Therefore, teachers need to be explicit in the use of number-related language. Learners develop this language by using the mathematical register and engaging in a discussion (Clements and Samara, 2009; Ernest, 2011; Wigley, 1997).

Elizabeth uses a poster that she created with the word 'less' on it to stimulate the learners' understanding of the term and concept. Each letter of the word is written in a smaller font than the previous one, indicating that the word 'less' means something is getting smaller (Figure 5.1).

Figure 5.1 - 'Less' poster in Elizabeth's classroom

In the excerpt below, Elizabeth draws the learners' attention to the font on the poster.

49	Elizabeth	Okay, so look at my word. Let's sound it out (*Elizabeth holds up 'less' poster with the letters getting smaller)*
50	Whole Class	L. E. SS. *(sounded out phonetically),* 'less'
51	Elizabeth	Why do you think I wrote it like this on the computer for you? *(Long pause)* Tell me, why did I write it like this A?
52	Learner A	Because it is getting smaller
53	Elizabeth	Because it is getting smaller

(V3, TT49-53)

Through questioning, Elizabeth encourages the learners to identify why she has created the poster in that specific manner. Learner A responds saying **"because it is getting smaller" (V3, T53).** Creating a visual representation of the word assists the learners to visualise the concept 'less'.

However, despite using these resources before, Elizabeth acknowledges that there is the possibility of the resources not being successful. Elizabeth states that "**the 'less' and 'more' [posters] haven't worked as well as they would have done in the past. And I haven't even gone for the word 'after' yet. 'Before' and 'after'. Because 'less' and 'more' have caused me so much trouble" (I1, T66).** Clements and Sarama (2009) acknowledge that language related difficulties can cause problems when learners encounter comparisons like 'less' and 'more'.

Elizabeth is concerned that if they do not have the required vocabulary, they will not understand what the term means and thus not grasp the concept. As a result of this, Elizabeth aims to prevent possible MLD in her classroom by not adding **“more words into [the learners’ vocabulary] when they haven’t mastered just those few” (I1, T68).**

While Elizabeth makes use of a variety of resources to assist in developing the learners’ understanding, she is cognizant of the fact that these resources do not necessarily work for all the learners. In her efforts to develop each learner’s understanding of the necessary vocabulary and the concepts, she also makes use of gestures.

Gestures can be defined as being “specific movements, usually of the hands and arms” (Alibali, Nathan, Breckinridge Church, Wolfgram, Kim, and Knuth, 2013, p. 426) and actions as the process of doing something.

Elizabeth uses gestures and actions as a ‘hook’ during her mathematics lessons. In vignette 5.2, Elizabeth encourages the learners to use various actions to support their vocabulary development and conceptual development. Elizabeth asks the learners to ‘stand up’ and ‘sit down’ repeatedly to support their understanding of the meaning of opposites (as noted in the excerpt below).

10	Elizabeth	Up. What’s the opposite of up?
11	Learner A	Down
12	Elizabeth	Think…. down. Stand up, stand up. Come on. Are you standing up? (*Uses upward hand gestures to get learners to stand. Learners stand)* … Do the opposite *(Learners sit)* … Up, stand up (*Uses upward hand gestures to get learners to stand. Some learners stand*). Up come on. Come lazy butts up! Up! Up! Up! *(Learners stand) …* Sit down, down, down, down. *(Learners sit down)*

(VR2, TT10-12)

Elizabeth makes use of gesture to encourage the learners to perform various actions. The use of gesture reinforces the ‘opposites’ referred to in the book she read to the class (vignette 5.2). She instructs the learners to **“stand up, stand up” (V2, T12)** and then tells them to **“do the opposite” (V2, T13).** While telling the learners what to do, she uses her hand to indicate that the learners should 'stand up' or 'sit down'. As she gestures to the learners, they perform the action. This process assists in reinforcing both the words and their meanings.

In summary, the emic analysis of the observation data highlights the importance of developing learners' mathematical understanding by emphasising vocabulary development. There is a great deal of emphasis put on the use of resources to assist learners development of the necessary vocabulary and concepts. Vocabulary is considered a significant resource in the classroom (Adler, 2000). Adler (2000) refers to three types of resources, material, social-cultural and human resources. She draws on her own experience and knowledge (human resources), books posters and manipulatives (material resources) and language (socio-cultural resources) to develop learners' knowledge of vocabulary and understanding of concepts.

The importance of developing learners' vocabulary in mathematics is evident throughout the data. The approach Elizabeth takes is one where learners are constantly exposed to the necessary vocabulary in various contexts. The use of 'hooks' through actions and gestures, connections to everyday life, books, posters and manipulatives reinforces the connections learners can use when referring back to a concept that has been learned.

5.4 DEVELOPMENT OF NUMBER SENSE IN ELIZABETH'S CLASSROOM

Through analysing the data, there is evidence that Elizabeth strives to develop the number sense of the learners with MLD. As discussed in Chapter Two, number sense is a highly contested concept that is crucial in understanding number relationships and solving mathematics-related problems (Witzel et al., 2013; Gersten and Chard, 1999). Whilst I highlighted eight components of number sense in Chapter Two, in this research only three components emerged from the data. Elizabeth focused on developing the learners' number sense by increasing their understanding of vocabulary and meaning, their awareness of magnitude and comparisons, and their ability to communicate mathematically with understanding.

In her classroom, Elizabeth puts emphasis on developing the learners' ability to *communicate mathematically with understanding.* Ernest (2011) and Cockroft (1982) proposed that by emphasising discussions in the classroom, mathematical language and vocabulary are developed. These discussions encourage learners to verbalise their thinking and develop their understanding of the concept. This is evident in vignette 5.4 where Elizabeth asks the learners to give a reason for why they thought a specific container was ‘heavier’. She asked **“why are choosing this one? Why do you think this one is ‘heavier’?” (V4, T14).** Further evidence of Elizabeth engaging with the learners and encouraging them to share their thinking is seen in vignette 5.3. Elizabeth shows the learners the ‘less’ posters and asks, **“why do you think I wrote it like this on the computer for you?” (V3, T51)** to which the learner responds, **“because it is getting smaller” (V3, T53).** Encouraging learners to communicate mathematically is developed in conjunction with *developing their vocabulary and meaning.*

As indicated in Chapter One, learners in South Africa have poor literacy skills. This affects the development of their vocabulary specifically, and language development more generally (Howie et al., 2017). Jordan and Levine (2009) acknowledge that a large percentage of learners who experience MLD, experience reading and language related difficulties. My research established that language, specifically vocabulary, play an integral role in mathematical understanding. Elizabeth indicated that **“establishing vocabulary” (I1, T20)** is one of the main difficulties that she encounters in her classroom. She further elaborates on this by saying that **“you need to establish vocabulary to teach mathematics” (I1, T36)** as it is an integral part of **“access[ing] the learning” (I1, T36).** By focusing on vocabulary and meaning in her classroom, Elizabeth is developing the learners’ number sense. During my observation, Elizabeth first introduced the learners to the concept of ‘opposites’ which was a precursor to ‘less’ and ‘more’. It was her intention to make the link between the concepts. However, despite focusing on developing vocabulary and meaning, learners can still experience difficulties which then results in the teacher not being able to move on to the next concept. As Elizabeth mentions **“to add…more words into [the learners’ vocabulary] when they haven’t mastered just those few” (V3, T68)** would cause even more difficulties for the learners.

There are instances where Elizabeth has to assist learners to develop their memory skills to retrieve vocabulary and number information. Elizabeth provides her learners with ‘hooks’ that they can refer to, to remember a concept or a term. As she explains, the ‘hooks’ are a way for the learners to **“access the vocabulary” (I, T58)** in a way that is meaningful to them. When a learner is unsure, she simply reminds them to, **“remember what to do” (I, T58)** until they have reached a state of automaticity where they no longer require the assistance of the teacher or the 'hook'. However, there are occasions where Elizabeth says a concept **"has to be paraphrased" (I1, T22)** to encourage the learners to retrieve the correct vocabulary (i.e. 'it gets bigger' is used for 'more'). The reinforcement of the 'hooks' results in the development of retrieval strategies that will assist them with retrieving mathematics facts that are required to complete simple arithmetic later on (Hopkins and de Villiers, 2016; Witzel et al., 2013). The ability for learners to efficiently retrieve vocabulary is necessary for developing their number sense (Black, 2014).

Drawing on their work with the approximate number system (ANS), Mazzocco, Feigenson and Halberda (2011) and Dehaene (1997) indicate that infants have the ability to approximate. However, the introduction of the terms 'more' and 'less' into the learners' vocabulary can cause difficulties. These difficulties are particularly evident when Elizabeth is developing the learners' *awareness of magnitude*. When Elizabeth introduces the concept of 'less', she begins by asking the learners to tell her what 'less' means (vignette 5.3). One learner responds saying it means **"more" (V3, T6).** This suggests that for some learner there could be confusion with the definition of 'more' and 'less'. This results in Elizabeth having to ask what 'more' means with a second learner responding that is means **"bigger" (V3, T8).**

Research has proposed that there is a relationship between poor number sense and the prevalence of learning difficulties (Gersten and Chard, 1999). Elizabeth's approach to assisting learners with MLD develop their number sense has a strong focus on developing learners' vocabulary and providing them with the tools to remember and access this vocabulary in an efficient manner. As Ernest (2011) highlights, developing learners' vocabulary allows them to communicate effectively and proficiently which can have a positive effect on their mathematical understanding.

5.5 AN ANALYSIS BASED ON THE KNOWLEDGE QUARTET

Whilst the above analysis is based on an emic analysis, that is, with the identification of codes emergent from the data, I will now move to the etic analysis. An etic analysis uses codes that are informed by the selected theory to highlight areas of interest. For this research, the etic analysis was informed by the four components of Rowland et al.'s (2013) KQ. The intention of the etic analysis is to identify the knowledge that Elizabeth draws on in developing her Grade One learners' number sense and in order to support those learners with MLD.

As explained in Chapter Three, the four knowledge components of the KQ are: foundation, transformation, connection and contingency.

5.5.1 Foundation

The foundation knowledge component of the KQ refers to teachers' pedagogical and content knowledge; the knowledge that the teacher has acquired through the various levels of schooling, their professional qualifications (both pre-service and in-service) and experience (Chapter Three) (Rowland et al., 2014). Foundation knowledge is based on education-related theories, teachers' content and pedagogical knowledge, and their ideas and beliefs about learning and teaching mathematics. This acquired knowledge forms the basis of the work of teachers. In this research that refers to the development of number sense of learners with MLD. Table 5.1 below outlines components of foundation knowledge.

Foundation knowledge consists of:
The teachers': • Knowledge and understanding of mathematics • Knowledge of theories relating to teaching and learning of mathematics • Knowledge of the conditions under which learners best learn mathematics • Beliefs about the nature of mathematics • Beliefs about how mathematics should be taught and learned • Knowledge of how to select and adapt resources (e.g. textbooks/workbooks) • Knowledge of various calculation procedures • Knowledge of the purpose of mathematics • Knowledge of the content in the curriculum • Knowledge of learner errors and misconceptions • Knowledge of the appropriate terminology and mathematical language

Table 5.2 - Components of foundation knowledge

Elizabeth has *an awareness of the purpose* of mathematics that informs her mathematics lessons. She is familiar with the expectations of the curriculum for Grade One (CAPS) and how to support the learners in meeting the curriculum requirements. This is evident when she talks about learners practising and consolidating what they have learned before moving onto the next concept. She guides them to do mathematics without the assistance of 'hooks' or manipulatives. The overarching purpose, during the time that I was in her class, was developing the learners' vocabulary. As Elizabeth states **"you have to be conscious of vocabulary which you know from the past experience that children struggle with" (I1, T58).** This she regards as key to developing an understanding of mathematical concepts, which she supports by saying **"if you don't understand the mathematical vocabulary how are you going to access the learning?" (I1, T36).** Elizabeth equates her role in the classroom to that of a translator as she aims to help the learners make sense of the vocabulary required. When relating mathematics to learning another language, Elizabeth's view is **"how would you understand what is going on without a translator? It is the same to me with mathematics" (I1, T78).**

The emphasis that Elizabeth places on language development, and in particular, the development of mathematical vocabulary suggests that she values the importance of social knowledge. Piaget (1965) argues that there are three kinds of knowledge that learners need to develop in order to understand mathematics. These are physical knowledge, social knowledge and logico-mathematical knowledge.

The purpose of each lesson relates to a different mathematical concept that Elizabeth teaches in each lesson. For example, in her lesson on 'opposites', she focuses on 'less' and 'more' and 'heavy' and 'light'. In the lessons described in vignette 5.2 and vignette 5.3, Elizabeth makes use of a variety of resources to assist the learners in making sense 'opposites'. She uses a book and everyday experiences to assist the learners in grasping the meaning of 'opposites'. She makes use of manipulatives, such as unifix blocks to teach 'more' and 'less' and containers filled with different objects to teach 'heavy' and 'light'. This suggests that Elizabeth seeks to develop the learners' physical knowledge, whilst simultaneously providing them with the terminology they need.

The use of manipulatives also suggests that one of Elizabeth's *beliefs about mathematics* is that children learn mathematics by engaging with concrete materials. The use of these materials assists the learners to develop their own strategies to complete activities.

Placing the learners in small groups indicates that Elizabeth acknowledges that learners have different 'abilities' and *believes that children learn best when they are paced in small groups* with children who are, more or less, at the same level. Whilst the learners are grouped according to their 'abilities', these are fluid. Learners move between groups depending on their competence with each particular mathematics topic. Elizabeth reiterates this when she says that **"as you go along they develop and they grow and maybe someone isn't developing as fast as someone else so you change around and then the next person will catch the other" (I1, T18).**

While *textbooks***,** even the national workbooks, are not used in all Foundation Phase classrooms, Elizabeth makes use of a set of books, known as '*NumberSense*' (Figure 5.2). These are books developed in South Africa by Aarnout Brombacher, a former mathematics teacher. Elizabeth maintains **"the *NumberSense* programme has definitely pushed out children into number ranges that [she] wouldn't have considered before and an order [she] would not have considered" (I1, T34).** Not only are these books a source of consolidation for the learners that develop their **"knowledge and skills exponentially" (I1, T34),** they provide Elizabeth with new information and alternative approaches to mathematics that she would not have considered before. Elizabeth mentions that through the use of the *NumberSense* books she has **"learned a number of things that [she] did not know previously" (I1, T46)** and with the access to the website she is able to get clarification. In many respects, Elizabeth is suggesting that she continues to learn, and in so doing, her foundation knowledge is enhanced.

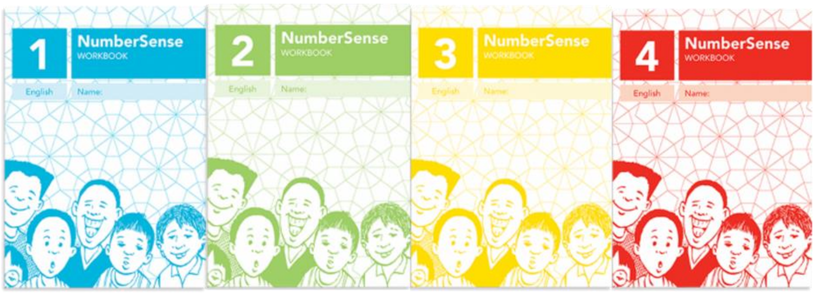

Figure 5.2 NumberSense workbooks for Grade One (Brombacher & Associates, 2019)

Errors in the classroom are a natural part of the learning process. Over the observation period, there were many times where errors were made by the learners. Stott (2017) and Sapire and Shalem (2016) acknowledge that, when working with learners with MLD, the teacher needs to recognise the frequency of error making and identify if there is a pattern. This can then assist them in determining the approach to take in supporting the learners. As Elizabeth mentioned, over time she has **"buil[t] up a bank of [those words] that could be the tricky ones, and [she is] always on the lookout for another one that could be giving them some trouble" (I1, T58).**

In Table 5.2, I provide a summary of the foundation knowledge that Elizabeth draws on when developing number sense in her classroom.

Elizabeth's foundation knowledge	
Knowledge of the purpose of mathematics	• Understanding of what the curriculum requires learners to know • Knowing to focus on the vocabulary necessary to learn mathematics • Knowing the purpose of each lesson and how each lesson builds on from the previous lesson • Knowledge of the curriculum content
Knowledge of the conditions under which learners best learn mathematics	• Mathematics is learned through practice • Children learn mathematics when they are placed into 'ability' groups • Children learn mathematics when they are given concrete materials • Children learn mathematics when they understand the terminology/vocabulary • Children learn mathematics when the mathematics content is linked to their prior knowledge and everyday experiences • Children learn mathematics when they are assigned tasks that they enjoy and are of **"high value**" (not too easy or difficult) • Children learn best when they are enjoying what they do
Knowledge of learner errors and misconceptions	• Knowing what possible errors learners make
Knowledge of the appropriate terminology and mathematical language	• Development of mathematics terminology through the introduction and use of terms such as 'less', 'opposites', 'heavier', 'lighter'

Knowledge of how to select and adapt resources (e.g. textbooks/workbooks)	• Selecting workbooks because of the support they provide learners. For example, using the *NumberSense* books as they support the development of number sense in the classroom.
Beliefs about the nature of mathematics	• Mathematics is a language • Mathematics is something we 'do'

Table 5.3 – Elizabeth's foundation knowledge

Foundation knowledge provides the 'theoretical' foundations for the other three knowledge components.

5.5.2 Transformation

Transformation knowledge draws on foundation knowledge. It focuses on how the teacher is able to adapt the knowledge they have developed through their education (schooling, pre- and in-service education). More specifically it looks at how the teacher uses foundation knowledge to select appropriate representations, examples and materials to use when teaching mathematics. Table 5.3 outlines the key aspects of transformation knowledge.

Transformation knowledge
The teachers' ability to: • demonstrate concepts • choose appropriate representations • choose relevant examples • use a variety of instructional materials effectively

Table 5.4 - Components of transformation knowledge

There is a strong emphasis on transformation knowledge in Elizabeth's classroom. This is evident when Elizabeth introduces the focus of a lesson to her learners. As described in vignette 5.2, Elizabeth introduces the concept of 'opposites' by first reading a book to the learners. This this she does to familiarise the leaners with the broad concept before moving on to mathematical terms such as 'less' and 'more'.

When introducing a concept to the learners, the *choice of examples* Elizabeth uses can have an influence on the learners' understanding. These examples need to be mathematically correct and contextually realistic to meet the aims and purpose of the lesson (Rowland et al., 2013). Elizabeth selects resources that are relevant to the concepts being taught and relevant to the learners' experiences. When working with the concept of 'opposites', Elizabeth introduces the learners to the vocabulary necessary to develop their understanding of the concept. She goes further by drawing on learners' prior knowledge to encourage them to give her examples of 'opposites' by saying **"can you think of another 'opposite'?" (V2, T86)** and **"we did one with the balance scale the other day" (V2, T90).** From using what the teachers were wearing and a reading book introduces the concept of 'opposites', Elizabeth progressed to using concrete materials (e.g. unifix cubes) to develop learners' understanding of 'less' and 'more'. The progression in the development of an understanding of a concept demonstrates her understanding of *how learners learn (foundation knowledge)*.

Elizabeth meets the aims of her lessons through her choices of *instructional materials* when introducing and developing on a concept. During the period of observation, Elizabeth made use of more than one form of instructional materials to develop learners' understanding of 'opposites': 1) she introduces "opposites" by focusing on the clothes that her and her colleague were wearing; 2) she reads a book on 'opposites'; and 3) she introduces 'less' using unifix blocks. When teaching the concept of 'less', Elizabeth's choice of resources (i.e. clothing, reading books, unifix blocks, balance scales) highlights the importance of providing various opportunities for learners to make sense of mathematics.

The use of the 'less' posters as a visual representation of the concept allowed the learners to see what the word means, that is, that the items become fewer. Similarly, the use of the unifix blocks provided a space for the learners to demonstrate their knowledge of 'less', allowing Elizabeth to determine which learners understood the concept and which learners were struggling.

The transformation knowledge that Elizabeth draws on in her classroom is summarised in Table 5.4.

Elizabeth's transformation knowledge	
Demonstration by the teacher	• Using gestures to encourage learners to participate and complete actions.
The use of instructional materials	• Using instructional materials such as reading a book on 'opposites' and providing visual stimulation with the 'less' poster 'less'. • Using Unifix blocks when teaching 'less' and 'more' and the containers and balance scales when teaching 'mass
The teacher's choice of examples	• Knowledge of examples that will develop and support the learners understanding such as the reading book depicting 'opposites' • Selecting examples that may not have originally been considered to represent the concept being taught such as what the teachers are wearing to show 'opposites'

Table 5.5 – Elizabeth's transformation knowledge

5.5.3 Connection

The connection knowledge category of the KQ refers to the teacher's ability to plan and enact a lesson in a coherent manner. Like transformation knowledge, this category requires the teacher to draw on their foundation knowledge. This involves the teacher's ability to determine the conceptual appropriateness of the lesson and whether there are connections between previous concepts and future concepts. This category ensures learners understand the concept that is being taught. The components of connection knowledge are outlined in Table 5.5 below.

Connection knowledge
The teacher: • Recognises conceptual appropriateness • Makes decisions about sequencing • Makes connections between calculation procedures • Makes connections between concepts • Anticipates complexity

Table 5.6 - Components of connection knowledge

Elizabeth is aware that there are times when *decisions about sequencing* lessons and when to progress from one concept to the next need to be carefully considered. Elizabeth sequences her lessons by first introducing the concept to the entire class before working with small groups on the mat. It is in the process of working with small groups that she is able to give learners more individualised attention. This is evident when she reads a book of 'opposites' to introduce vocabulary and then progresses to mathematics-specific vocabulary of 'less' and 'more' and 'heavy' and 'light'. She highlights that until a concept is solidified, and the learners are able to complete activities with confidence, progressing to the next concept can cause unnecessary confusion. When discussing the concepts of 'less' and 'more', Elizabeth acknowledges that **"if we haven't solidified this for the bottom group so to add more [confusion], more words into it when they haven't mastered just those few" (I1, T68).**

Elizabeth's comment that **"you kind of build up a bank of [those words] that could be the tricky ones, and then you're always on the lookout for another one that could be giving them some trouble" (I1, T58)** is evidence of her anticipating complexities.

The connection knowledge that is evident in Elizabeth's classroom is indicated in Table 5.6.

Elizabeth's connection knowledge	
Anticipate complexity;	Elizabeth: • Builds up a repertoire of vocabulary that she anticipates learners might struggle with
Make decisions about sequencing	• Makes decisions when to progress from one concept to the next by taking into consideration learner understanding.

Table 5.7 – Elizabeth's connection knowledge

5.5.4 Contingency

The final category of the KQ refers to situations in the classroom that are unplanned. It refers to the times when teachers' are expected to 'think on their feet'. Contingency knowledge draws on the previous three knowledge categories. Table 5.7 indicates the components of contingency knowledge.

Contingency knowledge
The teacher: • Responds to children's ideas • Makes use of incidental opportunities when teaching • Deviates from the agenda • Provides teacher-relevant insight

Table 5.8 - Components of contingency knowledge

During the observation period, Elizabeth drew on her contingency knowledge in several ways. During a casual conversation, Elizabeth mentioned that her style of teaching encourages learners to **"discover and figure it out through questioning".** This is evident when discussing the 'less' poster where Elizabeth asks, **"tell me, why did I write it like this A?" (V3, T51)** and the learner responds with **"because it is getting smaller" (V3, T52)**. However, despite the learners demonstrating an understanding of the poster, Elizabeth acknowledges that "**the 'less' and the 'more' [posters] haven't worked as well as they would have done in the past" (V3, T66).** This awareness and insight into the effectiveness of resources influence the approach that she takes in the classroom.

Throughout her teaching, Elizabeth is able to *deviate from the agenda*, allowing her to make use of *incidental opportunities when teaching.* She made use of the fact that she and her fellow Grade One teacher were wearing the same coloured clothes, but in opposite combinations (vignette 5.1). In so doing, she *deviates from her lesson plan* Likewise, during the lesson on 'mass', she had to develop an alternate way of 'testing' which containers were 'heaviest' when the learners kept saying that the containers they chose were 'heaviest' (vignette 5.4). This knowledge can aid her in developing *insight* into knowing when something works or not.

The contingency knowledge that Elizabeth draws on in her classroom is described in Table 5.8 below.

Elizabeth's contingency knowledge	
Make use of opportunities when teaching	Elizabeth: • Makes use of unplanned teaching opportunities, such as identifying the clothing the teachers are wearing were in 'opposite' combinations.
Deviate from the agenda	• Takes alternative approaches to teaching when learners are not understanding (e.g. determining which container was 'heaviest')
Provide teacher relevant insight	• Acknowledges that resources or activities that have been selected do not always work such as the 'less' poster and the approach to weighing the containers during the 'mass' activity.

Table 5.91 – Elizabeth's contingency knowledge

In summary, having worked with the KQ to analyse the observation data, it is evident, as Rowland et al. (2013) suggest, that there are links between the different categories. Foundation knowledge informs transformation, connection, and contingency knowledge. Nel and Grosser (2016) and Hoadley (2012) highlight that a teacher's pedagogical and content knowledge is important when supporting learners and improving their performance. This is further emphasised with the knowledge required for the process of teaching and that ability to make connections and use that knowledge as a resource (Rowland et al. 2013).

In using the KQ it became evident that Elizabeth continually draws on her foundation knowledge as it shapes how and what she teaches, and how she reflects on her teaching. This in turn influences her beliefs on how learners learn mathematics and how best to support and foster this learning in the classroom. Her development of the learners' mathematics vocabulary is influenced by her belief that mathematics is a language, and this impacts her approach to developing the number sense of learners with MLD. Elizabeth's transformation knowledge is evident through her purposeful selection of examples and instructional materials. By ensuring that her demonstrations are contextually appropriate, Elizabeth clearly guides the learners' number sense development. Based on her experience in teaching mathematics, Elizabeth draws on her connection knowledge when anticipating complexities in her classroom. Her awareness of vocabulary that creates confusion in the classroom influences her teaching approach. In addition, Elizabeth's knowledge of her learners' 'abilities' influences her task-related sequencing decisions.

The contingency knowledge in Elizabeth's classroom is influenced by her transformation and connection knowledge. She is able to adapt her teaching and use alternative strategies when confronted with unplanned situations. Given the extent of her foundation knowledge, Elizabeth is able to deviate from her lesson to assist learners in making connections and sense of the mathematical concepts and related vocabulary.

5.6 CONCLUSION

In this chapter, I analysed the data using both emic and etic coding. The emic coding identified the importance Elizabeth places on language in developing learners' number sense. Evident from the data is Elizabeth's view that in emphasising mathematics language, a teacher is able to avoid many MLD. To support the learners' language, and particularly their vocabulary, she uses a number of 'hooks', such as, actions and gestures, connections to everyday life, books, posters and manipulatives. The study found that the participant Grade One teacher employed all four categories of the Knowledge Quartet when developing her learners' number sense. In particular, she placed strong emphasis on vocabulary development as a means of circumnavigating MLD when developing number sense in a Grade One mathematics lesson. She demonstrated awareness of the importance of vocabulary in developing number sense. She knew how to develop the learners understanding of the vocabulary (and concepts) and how to adapt her approach when necessary.

CHAPTER 6: CONCLUSION

6.1 INTRODUCTION

The research process and presentation of this thesis was guided by the question:

What content and pedagogical knowledge does a Grade One teacher use to assist learners with Mathematics Learning Difficulties to develop number sense?

This question allowed me to uncover the necessary knowledge that a Foundation Phase teacher (Elizabeth) draws on to develop the number sense of learners with MLD.

Chapter One provided the context for the research. National and international benchmarking studies have identified that learners are underperforming in mathematics. This poor performance is attributed to a number of factors, such as the socio-economic status of the school, teachers' insufficient content knowledge, poor pedagogical practices, the LoLT, and the lack of support for learners with MLD. The dominant explanation for learner performance relates to teachers' insufficient pedagogical and content knowledge in South Africa. Similarly, a challenge many teachers face is in supporting learners with MLD in their classrooms. This led to the interest in teacher knowledge, and particularly the knowledge teachers require to develop the number sense of learners with MLD. Research suggests that learners with poor number sense will experience difficulties when learning mathematics in school.

In Chapter Two, number sense is conceptualised as having a number of components. Drawing on a vast body of literature, I suggest that number sense includes: an awareness of number and quantity; understanding of number symbols vocabulary and meaning; engaging in systematic counting (including notions of cardinality and ordinality); awareness of magnitude and comparison between different magnitudes; multiple forms of representation of number; development of efficient calculating strategies; awareness of number patterns; and communicating mathematically and with meaning (Sayer & Andrews, 2015; Gersten & Chard, 1999; McIntosh et al., 1997; McIntosh et al., 1992).

The goal of my research was to observe and identify the knowledge an expert Grade One teacher uses to assist learners with MLD develop their number sense. As highlighted in Chapter Two, MLD refers to a short-term difficulty that a learner may experience in different areas of mathematics (Gersten et al., 2005). In order to ascertain what knowledge is required in the process of teaching to support learners with MLD develop number sense, five weeks was spent observing and interviewing a Grade 1 teacher about her practice.

The value of my research is to inform pre-service and in-service teacher education programmes as to the content and pedagogical knowledge that a Foundation Phase teacher requires to develop the number sense of learners with MLD.

6.2 MY THEORETICAL FRAMEWORK

At the beginning of the research process, Ball et al.'s (2008) MKfT was selected as the theoretical framework. However, as mentioned in Chapter Three and based on the initial analysis, Ball et al.'s (2008) MKfT framework lacked the necessary clarity to distinguish between the different domains (Thanheiser et al., 2010). Due to my interest in the knowledge a teacher draws on in the classroom in the process of teaching, I selected the Mathematics Knowledge *in* Teaching framework, otherwise known as the Knowledge Quartet. The KQ was ideal for my research as it enabled me to identify the knowledge a teacher draws on 'in action'.

The KQ consists of four categories: foundation, transformation, connection, and contingency. These were the etic categories I used in analysing the data. The data was coded using the components from each of the four categories. It is important to note that due to the period of observation being short, not all of the components of the KQ categories were identifiable in the data. However, the data highlights the broad spectrum of knowledge that Elizabeth draws on in her classroom to assist learners with MLD develop their number sense.

6.3 LIMITATIONS OF THE KNOWLEDGE QUARTET

I found it very difficult to ascertain the mathematics content knowledge that the teacher was drawing on in her lesson. While it was clear that the teacher has a deep understanding of the content that she is teaching, the knowledge that underpins her knowledge of the content in the curriculum is not clear. The KQ did not assist me in identifying this knowledge. For Coskun and Bostan (2018), as the focus of the KQ is on how teaching happens in the classroom, it can be difficult to clearly distinguish between a teacher's subject matter and pedagogical knowledge.

Due to the KQ framework being used to identify the knowledge a teacher draws on in the classroom, it was difficult to identify some of the components within the categories. For example, it was particularly challenging to detect the teacher's beliefs as these are not necessarily visible in the classroom. Furthermore, identifying beliefs "requires making inferences about individuals underlying states [which is] fraught with difficulty because individuals are often unable … to accurately represents their beliefs" (Pajares, 1992, p. 314). This resulted in further reading into what the framework was inferring when discussing the beliefs of the teacher. It was important to understand what constituted as teacher beliefs as I did not want to confuse my beliefs as the researcher with those of Elizabeth.

6.4 KEY FINDINGS FROM THE RESEARCH

In my research, three components of number sense development emerged from the data. Elizabeth works towards developing the Grade One learners' *awareness of magnitude and comparisons* through concepts such as 'less' and 'more' and 'heavy' and 'light'. However, the differences in meaning between terms such as 'less' and 'more' caused confusions, despite the innate ability to approximate from birth (Mazzocco et al., 2011). As a result of this, Elizabeth places a strong emphasis on learners' ability to *communicate mathematically with meaning and understanding.* This development occurs in conjunction with the development of learners' *vocabulary and meaning*. This was evident when Elizabeth would introduce terms and concepts ('less' and 'more') and encourage the learners to define the term and demonstrate their understanding through reasoning.

I found that Elizabeth places a strong emphasis on the development of vocabulary and language. By acknowledging that the development of number sense shares similarities with learning another language, Elizabeth creates strategies and teaching methods that can be used to support the learners' vocabulary development. Like Morin & Franks (2009), Elizabeth asserts that poor language skills and vocabulary can influence a learner's mathematical understanding. I found that there is an emphasis on the use of resources in Elizabeth's classroom. She uses resources unifix blocks, balance scale and containers, posters, and the *NumberSense* workbooks as tools to assist learners develop their vocabulary in the classroom.

Elizabeth works on developing all her learners' mathematics vocabulary to try and counteract the possible presence of MLD in her classroom. It can be suggested that Elizabeth's approach to developing learners' number sense is to develop a strong foundational understanding to assist in reducing the prevalence of MLD as the learner progresses through the schooling system. Elizabeth achieves this by drawing on her pedagogical content knowledge and the wealth of knowledge that she has accumulated from experience. Each of the KQ categories plays an important role in Elizabeth's teaching and influences how she assists learners with MLD develop their number sense.

The foundation knowledge category highlighted the pedagogical content knowledge that influences Elizabeth. As is evident in Chapter Five, Elizabeth's knowledge of the purpose of mathematics influences the approaches she utilises in the classroom. Her beliefs on how learners learn mathematics, which I inferred from the data, informs how she structures the development of learners' understanding, and the emphasis she puts on vocabulary development. Elizabeth's focus on developing learners' mathematics vocabulary is influenced by her belief that mathematics is a language. As a result, her approach to developing the number sense of learners with MLD is based on how best she can support the learners in her classroom. This support of her learners manifests through her ability to transform and implement her pedagogical content knowledge into meaningful teaching and learning opportunities.

Elizabeth transforms her foundation knowledge through the purposeful selection and use of instructional materials to support her learners in developing their number sense. The examples, representations, gestures and action cues that she uses, assist learners in developing their understanding of the concept that is being taught. Elizabeth is able to make connections between concepts (i.e. 'opposites', 'less' and 'more', and 'heavy' and 'light'). Further connections are made based on how she progresses from whole class activities to group mat activities during a lesson. Her Contingency Knowledge is demonstrated through her ability to make decisions 'in the moment'. Her capacity to identify teaching opportunities in everyday experiences and acknowledge when tried and tested methods of teaching are not working is an example of how she draws on her contingency knowledge.

Rowland et al. (2013) reason that the transformation, connection, and contingency knowledge are all influenced by the knowledge that teachers develop in school and during their pre- and in-service teacher education (foundation knowledge). My research shows that contingency knowledge emerges from transformation and connection knowledge. In Figure 6.1, I illustrate the relationship between the KQ categories that emerged from Elizabeth's classroom.

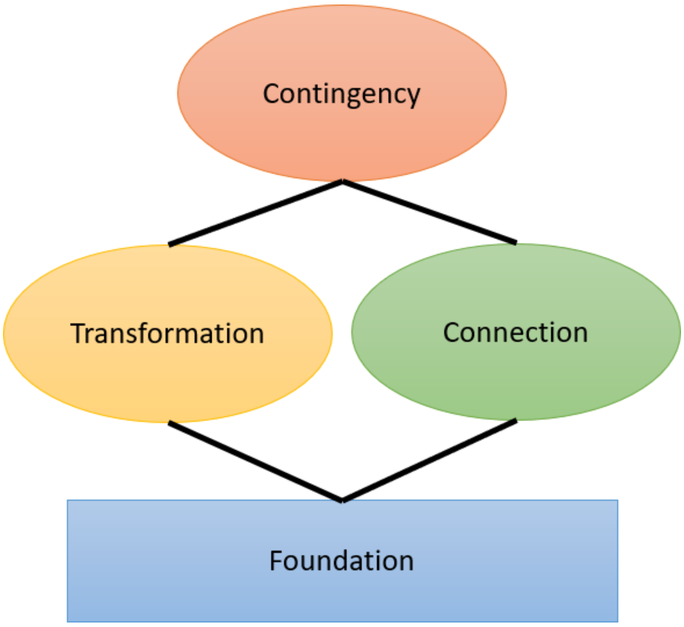

Figure 6.1 The relationship between the KQ categories

In Elizabeth's classroom the contingency category, whilst influenced by the foundation category, is more reliant on the connection and transformation categories. This is due to the contingency category referring to how teachers change and adapt their lessons in the process of teaching. It is often necessary for teachers to 'think on their feet' and make changes to the examples, instructional materials, and sequencing of the lesson when the initial lesson plan is not supporting the learning process.

Through this research, I have found that the level of competence of an expert teacher can be attributed to his or her experiences in the classroom. Through my interactions with Elizabeth during the period of study, she was constantly providing me with extra information and texts to read that have an influence on her teaching. It is evident that she is constantly updating her knowledge so that she can better support the learners in her classroom.

6.5 INSIGHTS EMERGING FROM THE RESEARCH

My research showed the importance of observing expert teachers to gain vital information regarding the knowledge Foundation Phase educators draw on to assist learners with MLD to develop number sense. This knowledge stems from the teacher's pedagogical and content knowledge. Teacher education programmes for pre-service and in-service teachers need to consider the importance of this knowledge and incorporate it into the mathematics method courses. Furthermore, developing learners' mathematical vocabulary should be considered a fundamental part of mathematics courses in teacher education programmes. Without a sufficient grasp of the vocabulary required to learn mathematics, teachers may not be equipped with the necessary tools and skills to develop learners' mathematics vocabulary and remediate MLD.

6.6 RECOMMENDATIONS FOR FUTURE RESEARCH

Based on my research, I suggest that observing 'expert' teachers and identifying the mathematics and pedagogical knowledge they use in the process of teaching will be beneficial for pre-service and in-service teacher education programmes. I also suggest that researching the knowledge that teachers use in the other grades in the Foundation Phase to develop the number sense of learners with MLD would be useful. This is a result of the possible differences related to the topics taught.

My research is based on a case study at well-resourced school in the Eastern Cape. It may be useful to explore this topic in other contexts to gain deeper insight into the knowledge that successful Foundation Phase teachers draw on when developing the number sense of learners with MLD.

6.7 A FINAL WORD

My research process has made an impact and assisted me in many facets of my life. Through the research process: I have become aware of my capabilities as a researcher; my knowledge on MLD in the Foundation Phase classroom has grown exponentially; I have gained more knowledge on teaching in the Foundation Phase; and my respect and appreciation for the work of Foundation Phase teachers has grown. Through this experience and opportunity, new interests and avenues for further research have emerged. The insight that I have gained through this process will hopefully, guide me in my chosen path, that is, working with Foundation Phase learners, pre-service teachers and conducting further research on learners' with MLD. I look forward to being able to share and develop this knowledge further in the near future.

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Appendix 1 – ETHICAL CLEARANCE

RHODES UNIVERSITY

Grahamstown • 6140 • South Africa

EDUCATION FACULTY • PO Box 94, Grahamstown, 6140
Tel: (046) 603 8385 / (046) 603 8393 • Fax: (046) 622 8028 • e-mail: d.wilmot@ru.ac.za

PROPOSAL AND ETHICAL CLEARANCE APPROVAL

Ethical clearance number 2017.6.04.01

The minute of the EHDC meeting of 7 September 2017 reflect the following:

2017.6.04 CLASS B RESTRICTED MATTERS
MASTER OF EDUCATION RESEARCH PROPOSALS

To consider the following research proposal for the degree of Master of Education in the Faculty of Education:

Kirsty Ann Fleming (12F3913)

Topic: The Mathematics Knowledge for Teaching (MKfT) that enables teachers to identify and assist learners with Mathematics Learning Difficulties (MLD) in relation to developing their number sense.

Supervisors: Dr L Westaway

Decision: *Approved*

This letter confirms the approval of the above proposal will be noted at the meeting of the Faculty of Education Higher Degrees' Committee on 5 December 2017.

The proposal demonstrates an awareness of ethical responsibilities and a commitment to ethical research processes. The approval of the proposal by the committee thus constitutes ethical clearance.

Sincerely

Prof Marc Schäfer
Chair of the EHDC, Rhodes University
6 Novembe 2017

Appendix 2 – PRINCIPAL CONSENT FORM

RHODES UNIVERSITY
Grahamstown • 6140 • South Africa

Dear Principal

Re: Permission to conduct research in Elizabeth's classroom

My name is Kirsty Fleming, a Masters in Education student in the South African Numeracy Chair at Rhodes University. I am currently working on a research project that seeks to identify the Mathematics Knowledge for Teaching that a teacher requires to identify and assist learners who have different needs in number sense development. Given the nationwide concern about learner performance in mathematics in South Africa, this research has the potential to make a valuable contribution to the education of teachers in South Africa. My research is being supervised by Dr Lise Westaway (Senior Lecturer in the Education Department at Rhodes University).

The research I wish to undertake seeks to identify the domains within the Mathematics Knowledge for Teaching that Foundation Phase teachers draw on in order to promote the development of children's number sense. Elizabeth has been identified for this research as she is a teacher who occupies leadership positions in the school (Head of Department and Grade Head) and outside the school.

Due to the nature of the research, I would like to observe Elizabeth in the context of the classroom. While the children in Grade One at Kirsty's Selected Primary School[13] will be in the classroom and interacting with Elizabeth, they are not the focus of my research. I wish to observe Elizabeth teach mathematics on a daily basis for a maximum of six weeks. These lessons will be video recorded. While the video camera will be focused on Elizabeth, there is a possibility that images of learners working on the mat with her will be captured on the video. The focus of the research is what the teacher says and does during her mathematics lessons.

[13] Pseudonym for the School

As a research scholar in the Education Department, I am bound to the ethical principles of the Education Faculty's Higher Degrees Committee. This has implications for how I carry out my research. In compliance with the Education Faculty ethical standards, the video data will be treated with the utmost confidentiality. The only people who will be able to view the videos will be the Principal, Elizabeth, me and, if necessary, my supervisor (Dr Lise Westaway). Viewing of the video material will only take place in my presence and in relation to my research. No video data will be available for any other purposes other than my research. Having done research before I am very aware of the importance of confidentiality and anonymity in the research process. In the thesis, pseudonyms will be used so that there is no mention of the name of the school or the participating teacher.

I hereby request permission from you, as principal of Kirsty's Selected Primary School, to conduct research in Elizabeth's classroom. I would like to commence my research on 12 February 2018.

Thank you for your co-operation.

K Fleming

MEd Scholar

Rhodes University

Grahamstown

6139

I, the undersigned, ___________________________, consent to the above research being conducted by Kirsty Fleming from Rhodes University in Elizabeth's Grade One classroom.

Signature: _________________________

Date: _________________________

Appendix 3 – TEACHER CONSENT FORM

RHODES UNIVERSITY
Grahamstown • 6140 • South Africa

Dear Elizabeth

Re: Permission to conduct research in your classroom

My name is Kirsty Fleming, a Masters in Education student in the South African Numeracy Chair at Rhodes University. I am currently working on a research project that seeks to identify the Mathematics Knowledge for Teaching that a teacher requires to identify and assist learners who have different needs in number sense development. Given the nationwide concern about learner performance in mathematics in South Africa, this research has the potential to make a valuable contribution to the education of teachers in South Africa. My research is being supervised by Dr Lise Westaway (Senior Lecturer in the Education Department at Rhodes University).

The research I wish to undertake seeks to identify the knowledge that Foundation Phase teachers draw on in order to promote the development of children's number sense. You have been identified for this research as you are a teacher who occupies leadership positions in the school (Head of Department and Grade Head) and outside the school.

Due to the nature of the research, I would like to observe you in the context of the classroom. While the children in Grade One at Kirsty's Selected Primary School will be in the classroom and interacting with you, they are not the focus of my research. I wish to observe you teach mathematics on a daily basis for a maximum of six weeks. These lessons will be video recorded. While the video camera will be focused on you, there is a possibility that images of learners working on the mat with you will be captured on the video. The focus of the research is what you, as the teacher, say and do during your mathematics lessons.

As a research scholar in the Education Department, I am bound to the ethical principles of the Education Faculty's Higher Degrees Committee. This has implications for how I carry out my research. In compliance with the Education Faculty ethical standards, the video data will be treated with the utmost confidentiality. The only people who will be able to view the videos will be the Principal, yourself, me and, if necessary, my supervisor (Dr Lise Westaway). Viewing of the video material will only take place in my presence and in relation to my research. No video data will be available for any other purposes other than my research. Having done research before I am very aware of the importance of confidentiality and anonymity in the research process. In the thesis, pseudonyms will be used so that there is no mention of the name of the school or the participating teacher.

I hereby request permission from you, as the Grade One teacher, to conduct research in your class.

Thank you for your co-operation.

K. Fleming

MEd Scholar

Rhodes University

Grahamstown

6139

I, the undersigned, ___________________________, consent to the above research being conducted by Kirsty Fleming from Rhodes University in my Grade One class.

Signature: _________________________

Date: _________________________

Appendix 4 – INFORMATION SHEET FOR PARENTS

RHODES UNIVERSITY
Grahamstown • 6140 • South Africa

INFORMATION SHEET FOR PARENTS OF LEARNERS IN ELIZABETH'S CLASS.

My name is Kirsty Fleming, a Masters in Education student in the South African Numeracy Chair at Rhodes University. I am currently working on a research project that seeks to identify the knowledge that a teacher requires to identify and assist learners who have different needs in number sense development. Given the nationwide concern about learner performance in mathematics in South Africa, this research has the potential to make a valuable contribution to the education of teachers in South Africa. My research is being supervised by Dr Lise Westaway (Senior Lecturer in the Education Department at Rhodes University).

The research I wish to undertake seeks to identify the domains within the Mathematics Knowledge for Teaching that Foundation Phase teachers draw on in order to promote the development of children's number sense. Elizabeth has been identified for this research as she is a teacher who occupies leadership positions in the school (Head of Department and Grade Head) and outside the school.

Due to the nature of the research, I would like to observe Elizabeth in the context of the classroom. While the children in Grade One at Kirsty's Selected Primary School will be in the classroom and interacting with Elizabeth, they are not the focus of my research. I wish to observe Elizabeth teach mathematics on a daily basis for a maximum of six weeks. These lessons will be video recorded. While the video camera will be focused on Elizabeth, there is a possibility that images of learners working on the mat with her will be captured on the video. The focus of the research is what the teacher says and does during her mathematics lessons.

As a research scholar in the Education Department, I am bound to the ethical principles of the Education Faculty's Higher Degrees Committee. This has implications for how I carry out my research. In compliance with the Education Faculty ethical standards, the video data will be treated with the utmost confidentiality. The only people who will be able to view the videos will be the Principal, Elizabeth, me and, if necessary, my supervisor (Dr Lise Westaway). Viewing of the video material will only take place in my presence and in relation to my research. No video data will be available for any other purposes other than my research. Having done research before I am very aware of the importance of confidentiality and anonymity in the research process. In the thesis, pseudonyms will be used so that there is no mention of the name of the school or the participating teacher.

The Principal, and Elizabeth have agreed, in principle, to this research. Before I commence with this research, I require permission from the parents of the children in Elizabeth's class.

I would appreciate it if you could return a signed copy of the attached consent form by Friday 9th February 2018.

Thank you for your co-operation.

K. Fleming

MEd Scholar

Rhodes University

Grahamstown

6139

Appendix 5 – CONSENT FORM FOR PARENTS

RHODES UNIVERSITY
Grahamstown • 6140 • South Africa

CONSENT FORM FOR PARENTS OF THE GRADE ONE LEARNERS IN ELIZABETH'S CLASS

I, the undersigned, ____________________________, have read the attached information sheet on the research to be conducted by Kirsty Fleming from Rhodes University.

I understand that:

- this research will be conducted in my child's teacher's class
- the teacher will be video recorded
- my child may appear in the video recorded data, but that this data will only be used for the research
- the video data will only be viewed by the researcher, Elizabeth, and Dr Lise Westaway
- my child's anonymity will be respected

The research will begin on Monday 12 February 2018.

I will be grateful if you could return this consent form by Friday 9 February 2018.

Signature of parent / guardian / caregiver

…………………………………………

For further information, please contact:

Researcher: K. Fleming 072 832 9796

Supervisor: Dr L. Westaway 073 337 1484