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A PRELIMINARY INVESTIGATION INTO THE DEVELOPMENT OF
COMPUTER ABILITY, MATHEMATICS CONCEPTS AND PROBLEM
SOLVING TECHNIQUES USING TURTLE GEOMETRY

by

JOHN STOKER

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"We start with the simplest vocabulary of images, with "left" and "right" and "one, two, three," and before we know how it happened the words and numbers have conspired to make a match with nature: we catch in them the pattern of mind and matter as one." (Bronowski)

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CONTENTS

ACKNOWLEDGEMENTS

GLOSSARY

	<u>Page</u>
Chapter 1. AN OVERVIEW OF THE INVESTIGATION	1
Chapter 2. HOW CHILDREN LEARN MATHEMATICS	8
Chapter 3. MATHEMATICS AND COMPUTER PROGRAMMING	29
Chapter 4. TURTLE GEOMETRY	50
Chapter 5. METHODOLOGY	67
Chapter 6. THE INVESTIGATION	79
Chapter 7. CONCLUSIONS	103
Appendix	119
References	122

GLOSSARY

Calls: each time the end of a procedure is reached it tells the computer to execute the commands again, or move to another procedure.

Command: an instruction that performs in a particular way.

Learning metaphor:

the Turtle is seen as a computational metaphor. It is seen as a tool for analysing and understanding one's own thinking. It is argued that the hurdle in learning is to make explicit notions of relevance and irrelevance the learner is already employing.

LISP: a programming language used in the field of artificial intelligence and the basis for many of the ideas in LOGO.

Match: setting a problem that is just beyond the learner's present level of ability. (Hunt)

M.I.T. Massachusetts Institute of Technology.

Playing turtle:

a physical simulation of the turtle movements which help the learner to think through a procedure.

Procedure: the conceptual building blocks of constructing a LOGO program.

Re-planning or debugging:

changing a procedure so that it works as intended.

State: refers to the position and direction of the turtle.

Turtle talk:

the commands and procedures that turn and move the turtle.

A video-cassette of the edited version of the teaching sessions will be found in the main library of Rhodes University.

CHAPTER ONE

1. AN OVERVIEW OF THE INVESTIGATION

- 1.1 Introduction
- 1.2 Active Learning
- 1.3 LOGO Learning Environment
- 1.4 The Teaching Sessions

AN OVERVIEW OF THE INVESTIGATION

1.1 Introduction

The impact of computer technology has not yet been felt in South African primary schools. Yet it is believed that the effect of microelectronics on society will cause a revolution as significant as the industrial revolution. Whereas the industrial revolution amplified and extended the power of human muscles, the new microcomputer technology will mostly be concerned with amplifying and extending the power of the human mind. In order that the future generation might come to terms with the rapidly changing conditions, it is suggested that children of school age benefit by gaining experience with the new technology.

It was against this background of challenge that I began to explore various uses of the new technology. The Cockcroft Committee (1982) investigation considered many aspects of mathematics teaching in primary and secondary schools in England. They gave a clear indication of the place of the microcomputer in mathematics teaching when they made this significant observation,

"There can be no doubt that the increasing availability of microcomputers in schools offers considerable opportunity to teachers of mathematics to enhance their existing practice and also to work in ways which have not hitherto been possible." (para. 402)

The aspect of 'doing' mathematics 'in ways which have not hitherto been possible' caught my attention. I saw the microcomputer as a powerful tool which would extend the children's range of mathematical experiences through the use of simulations. Seymour Papert (1980) propounds an exciting vision of education for the future which consists of collaboration between computers and children. His philosophy does not allow for computers programming children through drill and practice methods. He envisages the child programming the computer and mastering the powerful technology by using a highly active problem solving method. Besides learning problem solving skills, the aspect most emphasized by Papert (1971, 1972, 1980) is that through building and experimenting with computer programs, a child ought to gain new mathematical insights in the topic under investigation.

1.2 Active Learning

It is necessary to make a distinction between learning styles. Choosing between active learning on the one hand and drill and practice methods on the other hand implies different theories about the way children learn and, consequently, the way they can best be taught. This was a key issue which strongly influenced the direction this investigation was to take. My belief was that the computer ought to focus on teaching through guided, active exploration rather than through the use of drill and practice routines. My task, therefore, was to devise situations using

microcomputer technology which would encourage active exploration and investigation in mathematics. The Cockcroft report indicates that problem solving methods ought to be undertaken on the computer in the primary years when it says,

"We believe that special attention should be paid to the development of programs for mathematical activities which will encourage problem solving and logical thinking in a mathematical context." (Cockcroft, 1982 para. 409)

If the method to be used is 'problem solving and logical thinking', the key notion is problem decomposition. In other words problem solving takes place within a context and is broken down into parts. The parts are solved and then combined to solve the whole. The question is how can this be implemented in computer program form suitable for primary school children?

1.3 LOGO Learning Environment

LOGO provides an appropriate environment so that a child can explore his problem solving capability. LOGO is a computer language designed for children. LOGO uses 'turtle graphics' which is a geometrical modelling language based on Papert's 'turtle geometry' (Papert, 1971). Using turtle graphics the child is able to command a mechanical robot to draw pictures or patterns.



At first these may be simple shapes such as squares, triangles and hexagons. Later these basic elements can be combined to form complex constructions. Drawings are made by a pen attached to the robot which leaves a trail as it moves. The robot or turtle is controlled by a microcomputer and all commands are typed at the computer keyboard. The immediacy of a physical floor turtle means that children are able to relate to its slow movements more easily. It allows children to build up problem solving procedures.

An alternative to the 'floor turtle' is the 'screen turtle' where the drawing is done on the monitor of the computer. The screen turtle can be moved in just the same way as the floor turtle and similarly will leave a trail on the screen.

My concern, initially, was to examine how children transferred from using the 'concrete' floor turtle to the

more abstract screen turtle. My other concern was how they would manage in moving from the horizontal plane of the floor to the vertical plane of the screen turtle.

1.4 The Teaching Sessions

The teaching situation was planned for eight sessions, each taking one hour. The sample for the investigation consisted of six primary level pupils, two boys and four girls all aged about twelve years. The events in each session were video-recorded for reference and a non-participant observer monitored all the sessions. He held discussions with the children at the conclusion of each session, which were also video-recorded.

Using LOGO the pupils were able to explore shapes and combinations of shapes. They were required to plan their procedures on paper prior to making their drawing. An essential aspect of the philosophy which underlies the use of LOGO with children is the idea that pupils should be in control of their own learning. Commands for such concepts as 'repeat' and 'variable' were introduced only in response to a need discovered by the pupils.

It was during the teaching sessions that I realised that my initial hypotheses were perhaps not the most important issues in this investigation. Comments made by the pupils in conversation, in addition to those made by the observer

caused me to reflect on the significant issues which actually emerged.

The objective of this study became an investigation into how twelve year old pupils learned mathematics concepts through their experience of LOGO computer programming.

CHAPTER TWO

2. HOW CHILDREN LEARN MATHEMATICS

- 2.1 Introduction
- 2.2 Understanding Mathematics
 - 2.2.1 Activity and Experience, the Key to Understanding
 - 2.2.2 Language and the Formation of Key Mathematical Concepts
 - 2.2.3 Critiques of Piaget's Theory
 - 2.2.4 Accelerating Progression Through Piaget's Stages
 - 2.2.5 Conclusion
- 2.3 Bruner's Theory of Learning
- 2.4 Gagne's Theory of Learning
- 2.5 Papert's Computer Model

CHAPTER TWO

HOW CHILDREN LEARN MATHEMATICS

2.1 Introduction

At a time when primary school mathematics teaching was under serious review in England, the Association of Mathematics Teachers (1956, p. viii) made this significant statement,

"There exists in each human mind the power to recognise order, to distinguish a whole and its part, and to combine wholes to make new and distinct wholes. These are fundamental patterns of mathematical thinking."

Because of its very nature, however, it is one thing to teach mathematics but quite another for children to learn mathematical skills and concepts.

Twenty years later, in their comprehensive survey of primary mathematics Williams and Shuard (1976, p. 12) make the same point when they write,

"mathematics is concerned with structures and operations, i.e. with mental images and the ways in which they can be manipulated in the mind. In other words mathematics depends on thinking."

Both Williams and Shuard and the Association of Mathematics Teachers are concerned with the issue of the child's patterns of thinking in mathematics. In looking at the views of the learning theorists, I will concentrate on the aspect of how

the primary school child develops these patterns of thinking or 'mental images'.

2.2 Understanding Mathematics

The Nuffield Primary Mathematics publication, "I do and I understand" (1967) says this,

"At all times and at all levels children should have a real understanding both of the problem involved and the possible ways it might be approached."

What is meant by understanding? This word has been used in such a variety of educational contexts and about so wide a range of subjects that any attempt to give it an easy definition would not help. Let us say when a teacher begins with mathematical material which is familiar (to the child) and slowly works towards the more complex, he is making understanding easier for the pupils. Understanding is different from rote learning. Understanding emphasises principles and concepts. If the principles are understood, then information can be derived from other sources without memorization taking place. This means the child can apply the information which he has learned to a variety of different situations. In other words his learning has meaning because he is able to transfer it to new situations.

To distinguish between learning with understanding and learning without understanding Skemp (1976, p. 25) uses the term relational understanding. By this he means "knowing

both what to do and why". The acquisition of skills purely by rote he calls instrumental understanding, or "rules without reasons".

Skemp uses the following analogy to draw out the differences between the two types of learning:

"A person with a set of fixed plans can find his way from a certain set of starting points to a certain set of goals. The characteristic of a plan is that it tells him what to do at each choice point: turn right out of the door, go straight on past the church, and so on. But if at any stage he makes a mistake, he will be lost: and he will stay lost if he is not able to retrace his steps and get back on the right path.

In contrast, a person with a mental map of the town has something from which he can produce, when needed, an almost infinite number of plans by which he can guide his steps from any starting point to any finishing point, provided only that both can be imagined on his mental map. And if he does take a wrong turn, he will still know where he is, and thereby be able to correct his mistake without getting lost; even perhaps to learn from it.

The analogy between the foregoing and the learning of mathematics is close. The kind of learning which leads to instrumental mathematics consists of the learning of an increasing number of fixed plans, by which pupils can find their way from particular starting points (the data) to required finishing points (the answers to the questions). The plan tells them what to do at each choice point, as in the concrete example. And as in the concrete example, what has to be done next is determined purely by the local situation. (When you see the post office, turn left. When you have cleared brackets, collect like terms.) There is no awareness of the overall relationship between successive stages, and the final goal. And in both cases, the learner is dependent on outside

guidance for learning each new 'way to get there'.

In contrast, learning relational mathematics consists of building up a conceptual structure (schema) from which its possessor can (in principle) produce an unlimited number of plans for getting from any starting point within his schema to any finishing point. (I say 'in principle' because of course some of these paths will be much harder to construct than others.)"

Skemp (op. cit. p. 23) follows this up with a discussion on the advantages of relational and instrumental learning:

"Instrumental

1. Instrumental mathematics is easier, initially, to acquire. 'If what is wanted is a page of right answers, instrumental mathematics can provide this more quickly and more easily.'

2. The rewards are more immediate and apparent. Pupils enjoy a feeling of success when they get a page of right answers."

This can be dangerous for both pupils and teachers. It is inevitable for the teacher to 'turn where the light glows brightest'. It is perhaps true of girls more than boys and particularly in the primary school where it is more possible (See Noble, 1974).

"3. Pupils can get the right answer to a routine question presented in a familiar way more quickly since they do not have to think it out.

Relational

1. It is more adaptable to new tasks - pupils do not have to learn a new method for each type of problem.

2. It is easier to remember, although it is harder to acquire because different relationships have to be recognized. However once relationships are made the learning is more permanent.

3. Seeing the relationships is more satisfying than having to learn meaningless procedures, and therefore more motivating.

4. Each idea grasped is a growth point for further relationships to be developed."

According to Skemp, unless a child has relational understanding (i.e. he can see relationships), he cannot make deductions using principles or tackle problems for himself. The instrumental learner is required to operate from memory. This is particularly true for slower children who have poor memories.

My stance, based on the evidence above is that understanding, I suggest, should precede rote learning, or formal exercises.

2.2.1 Activity and Experience, the Key to Understanding

The environment is vital to the child's mathematical development. He engages in a series of interactions that constitute experience. During this interaction of activities and explorations concepts are formed. Yet concept formation is not an automatic process. It is formed by selecting and piecing together experiences through perceptions. Children use their perceptual powers to act on things, to discriminate and conceptualize by image and thought. Piaget (1972) believed that activities were vital because mathematical

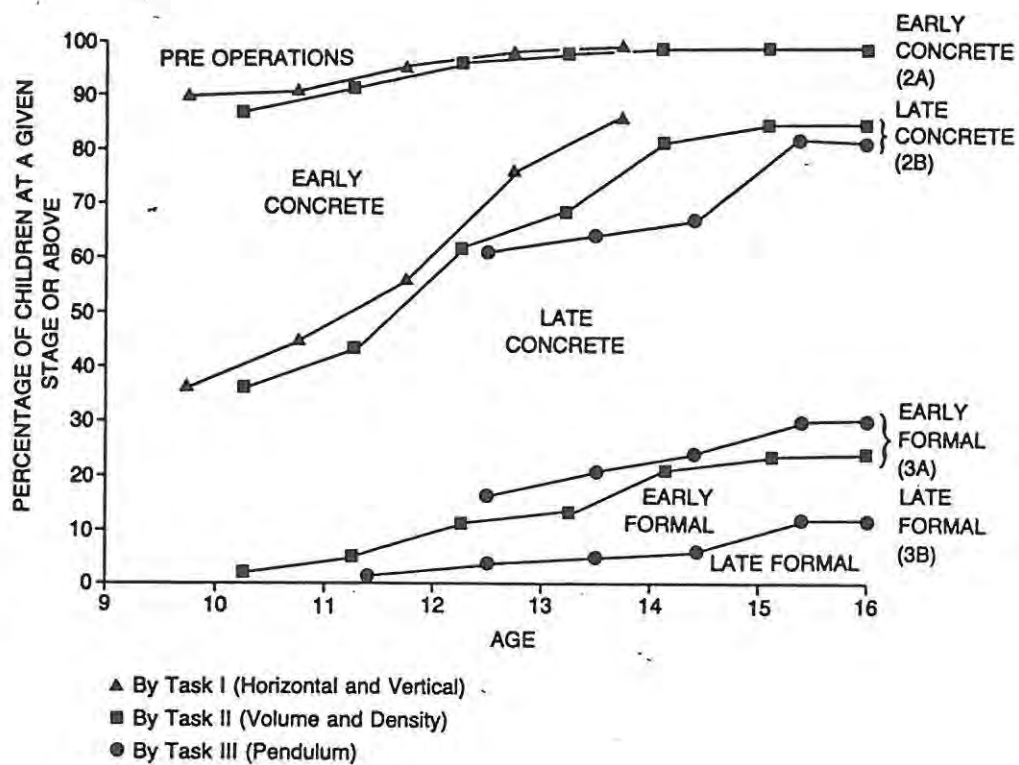
thought resulted from experience. He saw pupils as learning from two types of experience. By physical experience he meant activity with objects to discover the properties of the objects themselves. For example making different shapes of the same area on the geoboard. By logico-mathematical experience he meant the child's mental activity relating to the physical task eg. that different-shaped regions can enclose equal amounts of space (conservation of area). These actions are internalised symbolic operations. In other words, logic and mathematics develop beyond a level of just physical activities. Logico-mathematics overtakes physical experience which allows for higher order concepts to be formed.

Piaget stated that a child goes through three main stages of development of this type of logico-mathematical thinking, namely pre-operational, concrete operational and formal operational. A child who is pre-operational has no real understanding; reasoning is inconsistent, for example a number of counters remains the same regardless of their arrangement. The next is called concrete operational. At this stage the child has complete understanding but at a concrete level; reasoning is consistent provided there is reference to concrete objects. For example, a child who is concrete operational may not be able to make sense of a graph which relates pressure to temperature since the graphical representation is far removed from the physical changes it represents. At the formal operational level there is complete understanding at an abstract level i.e. the child is able to reason with reference to logical definitions.

Recent investigations with British children by Shayer and Adey (1981) indicate that the stages of development occur much later in a majority of pupils than has been suggested by Piaget. He said that most children were pre-operational from about 2 to 7 years, concrete operational from 7 to 12 years and formal operational from 12 years. By contrast Shayer and Adey (op. cit.) found that between 12 and 15 years only 10 to 20% of the group had reached the stage of formal operations. This means that 'slow learners' are likely to be in the concrete operational level during their entire school career on specific tasks.

Shayer (1981) selected three Piagetian tasks (horizontal and vertical, volume and density, pendulum) for distinguishing between concrete and formal operational thinking. He administered these tasks to a large sample of secondary school children in Britain. It was on the results of this investigation that he said the proportion of British children who reach the full formal operational stage before leaving school is not more than 20%.

The graph below shows the proportions of children in a sample of British schools who are at the four Piagetian stages; early and late concrete and early and late formal. They were tested by using the three Piaget-type tasks of horizontal and vertical, volume and density and the pendulum. It shows a fair degree of correspondence between the proportions of pupils defined as late concrete or early formal on specific tasks. This provides evidence to the existence of these stages when pupils tackle specific tasks.



(Bell et al, 1983).

Dienes (1960) showed that an acquaintance with a number of experiences illustrating a concept can facilitate learning. This evidence reveals that a single, concrete exemplar might cause a perceptual block where the child cannot generalise the concept to other instances. Dienes is convinced that it requires more than one exemplar to grasp a concept. Therefore variety of experiences should be contrived for each mathematical concept.

The main theme in this section has been to show the importance of activity and experience as the necessary foundation for the formation of mathematical concepts and

patterns of thinking. Pupils should not be seen as passive recipients of whatever information is meted out to them. They should be actively involved in the learning process in such a way that what they learn becomes a part of them. But not everyone agrees with all the findings of Piaget's theory.

2.2.2 Language and the Formation of Mathematical Concepts

Piaget (1954) gives a general view of the relationship between language and thinking. He states that,

"... language and thought are linked in a genetic circle where each necessarily leans on the other in independent formation and continuous reciprocal action."

On the other hand Vygotsky (1962) sees ego-centric speech as a transitional stage from vocal to 'inner-speech', that is the ability to think in verbal terms. The sub-vocal movement of the lips is sometimes observed in pupils when difficult material is to be mastered. This use of language is clearly helpful to the learner. Cockcroft writes, (1982, para. 306)

"Language plays an essential part in the formulation and expression of mathematical ideas"

It would seem that language plays an essential role in the development of higher order concepts. Both Piaget and Vygotsky provide evidence to show that concept formation and language are inextricably linked.

Stephens (1977) emphasises the need for a varied pattern of communication. The learning of mathematics involves discussion, or the 'negotiating' of mathematical meaning, for and by each pupil. By contrast, individual programmes, says Stephens, tend to isolate children from one another and reduces the amount of meaningful discussion as a result. Stephens emphasises the need to develop a teaching method which allows mathematical dialogue to take place, and for the teacher to engage in 'negotiation' with the pupils. An article by Hanley (1978) discusses two main themes. One is that the oral verbalization of mathematical ideas should be encouraged. The other is that some guidance about the choice of words for concepts should be given and that these concept words should be applied carefully and not used ambiguously. He maintains that the best learning situations are those where language can be used freely as the interactive medium and that the teacher has a vital role to play in fostering this interaction.

It would appear from the above evidence that the children's use of language (oral dialogue), has a significant part to play in the development of mathematics concepts.

It was the findings of Stephens and Hanley that directly influenced my approach in the teaching session of this study. Cognisant of their recommendations I planned that the pupils should work together in pairs, so as to promote oral dialogue. In this way I envisaged plentiful discussion between pairs of children which would help clarify their own concepts.

By arranging the pairs of children around the 'turtle-table' it was my intention to allow 'negotiation' (Stephens) to develop in a natural way. Concept words such as 'procedure', 'program' and 'external angle' would be introduced in the appropriate context to ensure understanding. By providing a 'climate' and a 'setting' for discussion it was hoped that the pupils would talk to their peers, or to the teacher about their difficulties and possible solutions.

2.2.3 Critiques of Piaget's Theory

Although the field of cognitive psychology has advanced beyond Piaget's innovative theories by revising and extending them, his fixed stages of development remain the same. However, Travers (1982) cites some of the limitations of his original theory.

One of these includes the relationship between language and thought, and it is claimed that Piaget does not make this relationship clear, particularly where there is believed to be thought without language. It is difficult to identify in the theory what takes the place of language.

Perhaps a major criticism is that Piaget failed to give proper recognition to the fact that the ability to perform logically is tied to situations which are familiar.

This means that experience plays a much bigger part than was recognised by Piaget. Travers says that "experience is a

very important component of logical effectiveness." (1982, p. 213).

Piaget realised that people may reach a certain stage and then cease to develop because they are not sufficiently challenged by relevant problems. He expected everyone, provided the environment was right, to become high-level logical thinkers. If this were true then it would leave little opportunity for individual differences.

Ennis (1975) doubts whether concrete thinkers can be clearly differentiated from formal thinkers, if Piaget's vague criteria are used.

Another criticism is that Piaget's theory is over elaborate. For the busy teacher this may well be true.

2.2.4 Accelerating Progression Through Piaget's Stages

Many of the studies that attempted to improve children's performances on classical Piagetian tasks were unsuccessful. It would appear that training has little effect on children's level of thinking. It is interesting to note that Piaget (1972) himself, expresses doubts about the efficacy of trying to accelerate cognitive growth.

There is growing evidence, however, that suggests that it is possible to teach children to solve problems belonging to a higher level of thinking. The evidence shows that by

changing certain features of Piagetian-type tasks, children are able to achieve a level at an earlier age than stated by Piaget (see Gelman 1972, Bryant and Trabasso 1971, Donaldson 1978). This means that it might be possible to teach children to control variables before the child has reached the appropriate level of development. The question is how can it be done?

According to Hunt (1969) the important factor in education is always to pose problems that are slightly beyond the learner's capability. This idea of setting a problem just beyond the learners present level of development is what Hunt calls 'match'. Lovell (1971) attempted to apply this principle of 'match' directly to the problem of mathematics learning in the primary school. Hunt (1969) maintains that the value of 'matching' learning to pupil capabilities lies in improving the quality of the child's thinking.

2.2.5 Conclusion

The work of Piaget, Dienes and Skemp has had an impact in formulating a philosophy for primary school mathematics. The leadership provided by Piaget in the field of developmental psychology has transformed the view of how children think and hence how they can best learn. Despite criticism of Piaget's theory it still provides an appropriate rationale for how children learn. Hunt's view of 'match' extends Piagetian principles to the classroom. Before accepting the Piagetian

perspective, however, other learning theories need to be considered.

2.3 Bruner's Theory of Learning

Farrel and Farmer (1980) make the point that Piaget provides us with the evidence we need as to how learning takes place. But they say that we need to look at other types of learning that results from intentional attempts to change behaviour. They take the behaviourist's view that learning is a more or less permanent change in human abilities which is not due simply to growth.

It is at this point, therefore, that a consideration of the work of Bruner (1966) becomes important. He indicates how teachers can directly support and promote learning. He set out a theory of instruction which would increase a child's active learning of most appropriate skills, processes and knowledge within his culture. Here the pupil employs techniques in order to better understand the world. He identifies three modes by which the learner can represent the world. The first is through action the (enactive mode), the second is through visual or other sensory organisation (the iconic mode) and the third is through words or symbols (the symbolic mode). Bruner suggests that there is a link between the learner's cognitive development and the role the teacher can play in promoting this development.

Bruner (1966, p. 21) writes,

"... the heart of the educational process consists of providing aids and dialogues for translating experience into more powerful systems of notations and ordering ..."

He emphasises the need for the child to be active in the learning process. The key concepts are again those of activity, experience, individualism and in general, child-centredness.

Bruner also maintained that there were ways of presenting difficult concepts so that children could understand them, provided they were presented in a manner appropriate to the level of the child. This concept of a 'spiral curriculum' has applicability so far as concepts of 'function' and 'variable' in mathematics are concerned.

Bruner writes, (1966, p. 44)

"... Any idea or problem or body of knowledge can be presented in a form simple enough so that any particular learner can understand it in a recognizable form."

It must be remembered however, that what might be appropriate for a pupil at one level of development might not be appropriate for a pupil at a more advanced level, particularly in teaching abstract mathematical concepts.

No discussion on how children learn would be complete without mention being made of the work of Gagne (1977). He describes eight different kinds of learning beginning with simple types and ending with complex ones. They are signal learning, stimulus-response learning, chaining learning, discrimination learning, concept learning, principle or rule learning and problem solving.

Stimulus-response learning derived from Gagne's model has two advantages. First, it emphasises the precise skilled nature of the responses involved and secondly it implies that the learned connection is instrumental in satisfying some motive. Connecting a set of responses (S-R's) with planned sequential steps is called chaining, hence chaining-learning. In this type of learning Gagne maintains that individual motor chains that are learned become the components of more complex motor skills. These are later organised into performances which practice helps to perfect.

Contrary to the explanation made earlier of how a child develops concepts, Gagne shows that a child can learn through verbal association. He is told, for example, whilst being shown a cube, "this shape is called a cube". If the conditions are right, the next time a child sees a cube he will be able to name it, says Gagne. But the conditions are important. The child for instance may not discriminate the object as a stimulus, and may not say the correct name of cube (eg. block, square). The act of naming a cube is a

chain made up of a series of links. Depending upon the stimulus that the object creates, the child recalls the associated name by memory.

Siann and Ugarebu (1980, p. 95) give a clear account of how Gagne's rule learning is applied to mathematics. They conclude with the comment that "effective learning of rules lie in making pupils' learning as interesting and as meaningful as possible."

To conclude this section Donaldson (1978, p. 122) draws our attention, rightly, to the important role of the teacher in raising a child's consciousness of his own learning development. She maintains it is the teacher's role to help children "to learn to be conscious of the powers of their own minds" and to help them "decide to what ends they will use them". This is the dimension that Bruner's theory of instruction has added to Piaget's theory of intellectual development.

The Cockcroft Commission suggests that for the learning of mathematics to be effective, the material needs to make sense to the child. The report also goes on to say,

"The primary years are a time when children are not only acquiring the skills of language and number but are also experiencing a variety of methods of learning; they are learning to think, to feel and to do, to explore and to discover." (Cockcroft, 1982, para. 367)

This approach to learning advocated by Cockcroft gives further support to the findings made by Piaget, Dienes, Skemp

and Bruner. In my view these psychologists have underpinned an appropriate philosophy for primary education. Cockcroft has further provided us with clear guidance on how children in primary school today should set about learning their mathematics.

Having discussed the views of relevant learning theorists it became necessary to see how these theories could be applied to learning through the computer.

2.5 Papert's Computer Model

Papert (1980) developed LOGO which he considered a powerful tool for learning and thinking in mathematics. Let us consider the characteristics of LOGO that make it educationally valuable for children. LOGO is a simple yet powerful language. Underlying its simplicity and ease of use is a philosophy of how children learn. Rooted in the theory of Piaget, LOGO is designed to place the child in a setting where the pupil controls both the learning environment and the technology. The learner becomes the authority, instructing the computer to follow commands. Building LOGO procedures encourages pupils to plan their work, develop a logical sequence and then test it. Abstract thinking is encouraged when the children consider new possibilities and learn what to anticipate from a given procedure. The child is provided with a means of recognising the strengths and weaknesses of his or her own thinking. In teaching the computer how to 'think' the child has the opportunity to

develop and sharpen his own reasoning process, in a logical and systematic manner. Papert (1980) would say that the child is "engaging in the PROCESS of thinking". The pupil has in fact created a learning environment in which he can experiment, investigate, estimate and finally see concrete examples of what he has generated. In other words LOGO has provided the child with a learning metaphor (See Glossary).

Although Papert has based his LOGO work on Piaget's theory of the child interacting with the environment, he maintains that it is Piaget's epistemological aspects that offer greater opportunities for education. Papert (1980, p. 156) writes,

"I think these epistemological aspects of Piaget's thought have been underemployed because up until now they offered no possibilities for action in the world of traditional education. But in a computer-rich educational environment this will not be the case."

Papert's interpretation of epistemology is based on the idea of the child programming the computer. In so doing the child is engaged in intellectual model-building or 'thinking about thinking'.

Papert (1980, p. 166) makes the point most clearly when he says,

"Given my background as a mathematician and Piagetian psychologist, I naturally became most interested in the kinds of computational models that might lead me to better thinking about powerful developmental processes: the acquisition of spatial thinking and the ability to deal with size and quantity. ...The kind of developmental questions I was interested in needed a dynamic model

for how intellectual structures themselves could come into being and change. I believe that these are the kind of models that are most relevant to education."

The 'Turtle' (See Chapter Three) is a computer controlled robot that makes up part of the 'LOGO environment'. Papert (1980, p. 11) sees the turtle as "being good to program and good to think with". He regards LOGO as a comprehensive computer model. It teaches programming, employs graphics and encourages new approaches to problem solving with the pupil initiating learning and interacting with his own program.

I believe that Piaget has formulated a general solution to the problem of how mathematical concepts develop. Bruner has given guidance to teachers on how to aid concept development. But it is the union of Piaget's theory with Papert's computer-based model that provided me with an appropriate rationale for use in the teaching sessions.

CHAPTER THREE

3. MATHEMATICS AND COMPUTER PROGRAMMING

- 3.1 Introduction
- 3.2 LOGO
- 3.3 Programming and Mathematical Rigour
 - 3.3.1 Mathematical Exploration and Programming
 - 3.3.2 Key Mathematical Topics
 - 3.3.2.1 Function and Variable
 - 3.3.2.2 Spatial Concepts
 - 3.3.3 Programming as a Context for Language and Problem Solving
 - 3.3.4 Conclusion
- 3.4 The Use of Metaphors
- 3.5 Summary

CHAPTER THREE

MATHEMATICS AND COMPUTER PROGRAMMING

3.1 Introduction

The issue being addressed in this chapter is whether there is a relationship between a child learning to program a computer and learning to think mathematically about a problem. In both programming and in mathematics it is necessary for the learner to describe a situation or process in the most precise and logical terms possible. It is then necessary for him to test that description for correctness and make relevant changes.

If the above is true, then it can be argued that the development of a child's ability to think mathematically might be encouraged by learning to write programs that describe a mathematical procedure. This argument is strongly supported by Papert (1973) when he says that certain kinds of programming experience certainly do enhance the pupils understanding of mathematics. This is particularly true in an interactive computing system which provides feedback about the validity of an attempted description. It is this activity of planning and testing a hypothesis which is considered the key to building mathematical concepts. Papert (1972) stresses the important difference between the work of a primary school child and that of a mathematician. He maintains that the difference is not in subject matter but in the fact that the mathematician is actively engaged in

working at a situation that is personally meaningful to him.

He says,

"Being a mathematician is no more definable as 'knowing' a set of mathematical facts than being a poet is definable as 'knowing' a set of linguistic facts. Some modern mathematical education reformers will give this statement a too easy assessment with the comment: 'Yes, they must understand, not merely know'. But this misses the capital point that being a mathematician, again like being a poet, or a composer, or an engineer, means doing, rather than knowing or understanding."

Papert's ideas expressed in his book 'Mindstorms' represent an exciting vision of education for the future in which he sees collaboration between computers and children. His vision does not allow for computers programming children, rather the reverse. He envisages the child programming the computer, mastering the powerful technology and working with important concepts related to science and mathematics. To enable children to build their own thinking structures Papert designed a computer language called LOGO. An essential aspect of the rationale which underlies the use of LOGO with children, is the premise that pupils should be in control of their own learning.

Feurzeig and Papert (1969) maintain that children should be taught programming skills in order to develop mathematics concepts. To substantiate this view they postulate the following four claims:

- i) that programming provides some justification for, and illustration of, formal mathematical rigour,

- ii) that programming encourages children to study mathematics through exploratory activity,
- iii) that programming gives insight into certain mathematical concepts, and
- iv) that programming provides a context for problem solving and a language with which a pupil may describe his own problem solving.

These claims will be examined more fully later in the chapter.

3.2 LOGO

Work in Artificial Intelligence is strongly based on the idea that constructing and modifying programs is an appropriate methodology for studying the interpretation of visual information. The computer language most associated with the programming approach is LOGO. It is an interactive procedural language derived from a high-level language used in the field of Artificial Intelligence called LISP (McCarthy, 1962).

According to Wills (1981) LOGO has a number of features which make it suitable for the children to use in the study of mathematics.

- " i) It is non-threatening and appealing with the focus on a pet-like 'turtle' and with emphasis on graphics rather than overt mathematics.
- ii) Focus on the robot provides a concrete representation of the pupils' programming.

- iii) It has a simple starting level of a few English commands which make it an easily accessible language.
- iv) Each command produces a concrete, visible response.
- v) Because of the emphasis on graphics (Turtle Geometry) there is visible re-planning or 'debugging'.
- vi) The language can be extended. It is easy to define new procedures and so tailor the language to the demands that one makes of it.
- vii) The language has a highly consistent syntax which uses an 'economy of concepts'.
- viii) The robot itself provides an interesting introduction to process control and procedure building."

Although the first three claims made by Feurzeig and Papert (1969) are not specifically related to LOGO this discussion will focus mainly on LOGO since it is the language that has been most used in the departments of Artificial Intelligence at the Massachusetts Institute of Technology and at the University of Edinburgh.

The issue to be faced now is whether there is sufficient evidence to confirm or deny the particular claims made by Feurzeig and Papert for children learning mathematics concepts through programming.

Ross and Howe (1981) maintain that mathematical rigour reduces ambiguity in mathematical communication. This applies to both the child's understanding as well as the ability to define a problem. The value of this rigour is that it forces the learner to produce an explicit description of the process to be carried out. Working with paper and pencil, in mathematics, the child may make many mistakes of which he may not be aware. This is because he must both specify the instructions and execute them himself. An advantage of LOGO programming is that even poorly thought out instructions will produce an observable response. The computer will say either there is a clerical mistake or it will carry out an unexpected series of actions. Ross and Howe (1981) state,

"In LOGO semantic mistakes are rare, because most of the actions are visible and immediate - very few instructions control the context of actions."

In seeking evidence to support the claim for rigour made by Feurzeig and Papert (1969) we turn to the research conducted by Howe and O'Shea (1976). They describe how some of the twelve year old children in their study exhibited marked gains in self-confidence and became more "mathematically argumentative". From this it was implied that the children were searching for greater refinement, hence the need to be more rigorous.

Evidence also comes from a further study by Howe, O'Shea and Plane (1979) in which they spent a year teaching LOGO to a group of eleven year old boys and then spent a year teaching them mathematics using LOGO. The results of this group was compared against a control group who had no LOGO programming exposure. From a questionnaire submitted to all the pupils it emerged that the teachers rated the boys in the experimental group as being significantly more able to argue rationally about mathematical issues and to be able to explain their own mathematical difficulties.

It must be mentioned, however, that as the experimental group worked outside the school, it is not certain that the result was due entirely to the LOGO programming.

Results from the Brookline project at MIT (1979) also tends to confirm the claim. The case studies made of the participating children in the project suggest that the pupils settled into a "steady, step-by-step, regular checking" approach.

There was no control group built into the design so that comparisons could be made. It is difficult, therefore, to substantiate the findings of this project when the evidence is largely anecdotal.

In Edinburgh, du Boulay (1978), working with primary school students in training produced evidence in favour of the claim that programming enhanced mathematical rigour. In a series of experiments he studied the effects of LOGO programming on

the mathematics ability of teachers in training. He reported that some of them came to value making explanations clear. The students noted the confusions which might arise in a child's mind if an explanation is not made explicit. Although du Boulay's sample consisted of only a few adults with a self-confessed 'block' about mathematics his evidence is more convincing because his subjects had the chance to make their observations prior to his investigation.

The limitation of du Boulay's findings is that there were only a few candidates in the sample and these were not randomly selected. Any generalisations made need to be tentative and seen within a specific context.

It could be argued that programming with its need for rigour and precision of expression might inhibit intuitive forms of mathematical thinking described by Kruteskii in the Cockcroft report (1982). Papert would argue that the first analysis of the problem may well be intuitive, producing an initial plan of action which is then gradually refined using more formal methods into the precise program.

The evidence, such as it is, arising out of research conducted at Edinburgh and MIT tends to favour the claim made by Feurzeig and Papert that programming (LOGO) does lead to formal mathematical rigour.

3.3.1 Mathematical Exploration and Programming

There is strong evidence that programming can be a useful tool for conducting investigations in mathematics. With reference to investigational methods Cockcroft makes this point in favour of them:

"The idea of investigation is fundamental both to the study of mathematics itself and also to an understanding of the ways in which mathematics can be used to extend knowledge and to solve problems in many fields."

(Cockcroft 1982, para. 250)

In particular, programming can allow a pupil to experiment with dynamic processes and patterns. Almost all the researchers using LOGO programming give examples of children doing geometric exploration. In fact the final report of the Brookline project (Papert, 1979) is filled with the geometric explorations of the children in the case studies. Another example is given by Howe, O'Shea and Plane (1979). They cite the boy who was trying to construct a LOGO procedure that would draw a house using the procedure of triangle and square. At first he produced a faulty procedure which caused the triangle to appear inside the square.

Nonetheless he went back to experimenting with the faulty procedure until he produced an interesting design using the faulty procedure.

According to Papert (1972) the computer system can be designed to embody a mathematical system. In the case of the turtle it is a geometry which the pupil can explore. This

principle of embodying a mathematical system in a piece of apparatus is not confined to computers but is true of all structural apparatus.

The Dienes (1973) Multibase Arithmetic Blocks (MAB) for exploring place value and numeration is an example of such an embodiment system. The teaching of abstract ideas through a particular model or embodiment of those ideas is, according to Dienes, an effective teaching approach. Both Dienes and Papert argue that the conceptual understanding arises naturally out of the activities of children using a mathematical embodiment.

A pertinent comment on the use of embodiments comes from Bell et al (1983) when they say,

"... the conclusion of this research is that constructive activities in an embodiment provide a useful starting point for the development of analytical thought leading to an abstraction, ..." (p. 194).

The type of help required by pupils programming is similar to that required by pupils using structured materials such as MAB. Dienes (1973) gives six stages of abstraction in the process of understanding a mathematical system. The child may start by playing with the structural apparatus, in this case the turtle. Then via a succession of activities (for example finding a route through a maze), they come to understand the principles and constraints built into the apparatus. The turtle will only do what it is commanded to do.

By representing this structure symbolically the child is in a position to examine it, then plan and re-plan a procedure. Later he will be able to examine the properties of the symbolic system itself (the program). Finally the pupil will be able to formulate theorems in the system under investigation, for example, the Total Turtle Trip Theorem to be described later.

Another major problem is one familiar to all teachers, namely how much guidance should the teacher give? An instance might be a child working on a program that includes the use of variables. If the teacher gives explicit instructions on how to build such a procedure then the enjoyment which comes from making a discovery is removed and the consequent level of understanding may well be reduced. Yet on the other hand guidance is necessary if the child is to feel he is making progress if he cannot do so on his own. Such a dilemma requires the teacher to decide which compromise to make.

Exploratory activity in mathematics appears to fall into two categories:

i) The one approach is guided discovery with clear objectives which the pupils are expected to attain. Guided discovery is closely bound up with problem solving in which the pupils investigate a prescribed task to discover which aspects in a problem are relevant to its solution. This is the approach favoured by Howe and the Department of Artificial Intelligence in Edinburgh (Ross and Howe, 1981).

ii) The other approach is one of free investigation. Here pupils set out to discover relationships

without being led to systematically searching for them. Open exploration is the method advocated by Papert in Mindstorms (1980). The Brookline project, for example, gave anecdotal evidence of it, describing situations in which children changed their goals when they encountered something interesting. Says Papert (1980, p. 214) "The best learning takes place when the learner takes charge."

The claim that programming activity helps to allow mathematical exploration seems to enjoy much support.

3.3.2 Key Mathematical Topics

3.3.2.1 Function and Variable

One of the key mathematical concepts mentioned by Feurzeig and Papert (1969) in connection with LOGO is that of 'function' and 'variable'. They argue in programming a number of issues arise concretely including:

"the many roles of 'x' in algebra:
sometimes it appears to be a number,
sometimes a subtly different kind
of object called a variable, and
other occasions it is to be treated
as a function" (p. 7, their underlinings).

They suggest that programming helps to clarify these concepts. Feurzeig and Papert (1969) taught LOGO to children of average ability aged between seven and nine years old. The children learned to write and 'debug' simple procedures. They claimed,

"Children of this age do acquire meaningful understanding of concepts like variable, function and formal procedure (though not in those words) through their experience with programming."

In Papert's (1980) 'Turtle Geometry' a number of other key concepts are given vivid illustration. In particular rotation, translation, state, state change operator, angle as rotation are all used by the pupils in the course of constructing a variety of plane figures. A study undertaken by Milner (1973) also showed promising results. He worked with a small group of eleven year old children of varying ability, teaching them to construct recursive programs to generate number series. The tests included evaluating expressions containing variables whose values were given and finding the value of variables under specific conditions. According to Milner the experimental group made significant gains in their test scores, whereas the control group did not. Milner maintains that the procedures written by the children and their explanations of the actions of the procedures provide evidence of the children's knowledge of variables. As there were only eighteen children in the sample one cannot generalise from these results.

3.3.2.2 Spatial Concepts

An investigation was conducted by McGinley (1980) in which eleven to thirteen year old children used LOGO over a ten week period. They studied such topics as directed numbers and fractions. The LOGO group was tested against groups

taught the same topics using conventional methods over the same period. The post-tests included measures of spatial and relational thinking. Although the LOGO group showed significant benefits, the distribution of scores within the LOGO group was bimodal. This result suggests that some children gained much from using LOGO whilst others gained very little. In speculating on the bimodal outcome I suggest that it may have been due to the distribution of high and low abilities of the children in the LOGO group. Another reason for the result might be because the high scoring pupils were well advanced in their thinking according to Piaget, whereas the low scorers were not (See Chapter Two).

The results from these investigations do support the claim that programming does assist children to gain insights into certain mathematical topics.

3.3.3 Programming As a Context for Language and Problem Solving

Feurzeig and Papert (1969) argue that programming gives the pupil many opportunities to solve problems, often of his own devising. The pupil can be given insight into his own problem-solving processes by using the record of his dialogue with the computer as an indication of his own thinking. Ross and Howe (1981) say that of all the claims made by Feurzeig and Papert (1969) this is the most contentious. They cite three assumptions being made,

" i) that people use the same context-free study skills to solve problems in different domains (Polya 1957),

ii) that certain of these skills are made explicit more readily through programming activity than through mathematical activity,

iii) that programming provides memorable paradigms for certain other skills that cannot otherwise be made explicit easily."

By contrast mathematics problem-solving often yields sparse documentary evidence of the process of solution unless special steps are taken. In programming, however, the idea of 'debugging' and the value of investigating mistakes comes out clearly. In this way programming can provide both a context in which problems may be posed and solved as well as a language for describing the problem-solving process. Not only does Papert (1972) wish to give children insight into their own thinking but he also wishes them to change the emphasis in mathematics education away from teaching particular pieces of mathematics towards teaching the activity of doing mathematics. In this way the nature of mathematics is brought out. It means the authority of the subject lies not in the authority of the teacher, but in the subject itself. Skemp (1975) elaborates on this aspect of mathematics and the teacher. Clearly there must still be explanations and suggestions for work from the teacher but much of the responsibility for what happens in a session now rests with the learner.

In a study conducted by Statz (1973) nine to eleven year old children were taught programming for a year. Her thesis was that children taught this way would be better on a set of prescribed problem-solving tasks than a control group who learned no programming. The results were not convincing. The experimental group did marginally better than the control group. Statz argued that the fault lay not with the programming skills required by the pupils but with the nature of the tests.

It appears that the difficulty of studying programming as an aid to problem-solving is that the pupil must first have some understanding of the subject matter. For example Papert and Goldstein (1972) looked at the errors in this sum:

35	Child reasons:
<u>+35</u>	5 + 5 = 10, Writes 10 and
610	3 + 3 = 6, Writes 6.

They said,

"... given access to a vocabulary for programs, plans and bugs, we believe that student and teacher could be articulate about describing the algorithm used by the student in reaching the '610' answer, in identifying the bugs, and in debugging the addition program (in the student's head) to yield the correct result."

The child appeared not to understand the concept of exchange (carrying) and was therefore unable to apply the relevant mathematical concepts. The lack of success in this case may not have been one of planning but one of mathematics.

The dilemma for the teacher is which comes first the mathematics concept or the programming ability?

3.3.4 Conclusions

If we refer to the research evidence produced on mathematics and programming we see few encouraging signs. In defence of their claims Papert argues that mathematics tests are usually bound to a specific context. If this is the case they do not measure those concepts that programming activities are aiming to develop. In support of this view Papert produces considerable evidence of a non-statistical nature which encourages the belief of 'mathematics through programming'.

What can be said with confidence however, is "LOGO is almost certainly the best educational language." (Ross and Howe, 1981). A lot more work needs to be done in order to produce convincing evidence even though it is a truism that results in educational research that are statistically convincing can be prone to weaknesses.

Reading the non-statistical case studies of Papert and his colleagues is nonetheless encouraging. It is because of Papert's claims that I went ahead with my investigation, recognising that there was a need for practice and not just theory.

According to O'Shea (1982) LOGO can contribute to a learner's mathematical development through metaphor (See Glossary). He argues that the programming activity serves as metaphors for mathematical concepts and mathematical reasoning. This, he says, is particularly relevant when it comes to such concepts as 'functions'. The program serves as a metaphor for the type of mathematical reasoning that involves generalisation.

Howe and O'Shea (1976) set up an investigation with average ability eleven year old boys in Edinburgh. Their goal was to determine how the children's mathematical skills were affected by the acquisition of a set of learning metaphors. Following Papert's example (Papert, 1973) they constructed a learning environment which comprised LOGO programming language and the turtle. A child communicated with the computer in order to change and run LOGO procedures. By using these devices the child provided himself with sets of metaphors which functioned both as tools for learning and for explicating the learning process.

Howe and O'Shea found that the children used four different situations in which metaphors were usefully invoked:

- "1) Metaphors in teaching. In the teaching situation Howe and O'Shea employed metaphors to help the child acquire LOGO programming concepts.
- 2) Metaphors in programming. When a child applied a programming concept in the construction of a procedure, he used this concept as a metaphor for a

wider class of processes than that explicitly embodied in the procedure.

3) Metaphors in communication. In understanding or explaining some problem the child invoked a metaphor without actually constructing a procedure.

4) Metaphors in interpretation. In their research Howe and O'Shea maintain that metaphors helped them to explicate and interpret the pupil's problem solving and learning processes."

(Research Paper No. 92).

As a result of this investigation Howe and O'Shea argue that learning and applying LOGO concepts made it possible for the pupils to acquire powerful metaphors. The pupils could then use these to achieve a deeper understanding of processes such as their own problem-solving activity, their own mathematics work at school and their own learning.

But, says O'Shea (1982), LOGO users have to date not performed 'stunningly better' at school mathematics than non-LOGO learners. One reason for this, he claims, is that much of the metaphorical content of LOGO is inappropriate to school mathematics. He says that most school work is not based on creative problem-solving but rather on the recall of particular algorithms. So the powerful problem-solving strategies that LOGO supports such as top-down decomposing of problems in sub-problems will not necessarily be helpful. Even more problematic is the fact that many algorithms taught in school depend on physical layout on paper. As we have seen earlier a pupil who cannot 'carry' will not obtain insight from a LOGO program.

LOGO was designed to help the learner express mathematical processes as computer programs. The acquisition of metaphors successfully follows the work of Piaget and the philosophy of expressing knowledge as procedures. The acquisition of these metaphors confirms the theory that the learner is a model builder. I concur with Heikinen (1983) when he makes this pertinent comment:

"Thinking about computers in education ultimately means thinking about education not about computers."

3.5 Summary

LOGO is an interactive programming language designed to be easy to learn. Many LOGO implementations simulate the action of the turtle by portraying it on the screen, allowing young children to quickly learn how to write programs that produce line drawings whose angle and other mathematical properties can be studied through observation, experiment and discussion.

Feurzeig and Papert (1969) made four claims for their approach to using LOGO. They argue that first, mathematics can be made both enjoyable and useful by basing it on the pupil's own programming projects. Second, that writing a computer program that models a mathematical process is an excellent way of explaining that process to oneself. Third, undertaking programming projects gives valuable practice in problem solving. Finally, that the problem solving techniques appropriate to LOGO, such as breaking down

problems and re-planning them, are concrete examples of a useful skill which the pupils can learn to apply in other areas of mathematics.

In general the research evidence does suggest that LOGO programs do assist in understanding the mathematical ideas that they embody. This was the key issue which was addressed.

A further issue was the fact that LOGO can contribute to a learner's mathematical development via metaphor.

4. TURTLE GEOMETRY

- 4.1 Turtle Graphics
 - 4.1.1 Turtle Talk
 - 4.1.2 Procedures
 - 4.1.2.1 Iteration
 - 4.1.2.2 Introducing the Concept of a Function with One Variable
 - 4.1.2.3 Sub-Procedures
 - 4.1.3 Debugging a Procedure
 - 4.1.4 Recursion
 - 4.1.4.1 Introducing the Concept of a Function with Two Variables
 - 4.1.4.2 Variables
 - 4.1.5 Total Turtle Trip Theorem
 - 4.1.6 Play Turtle
 - 4.1.7 Microworlds
 - 4.1.7.1 The Dependent Function
 - 4.1.8 Conclusion

CHAPTER FOUR

TURTLE GEOMETRY

4.1 Turtle Graphics

Turtle graphics was first developed by Papert and others at MIT as part of the programming language LOGO. At the heart of turtle graphics is a new way of doing mathematics called turtle geometry based on the movements of the turtle. Papert describes turtle geometry in these terms,

"To put this in perspective recall that you probably encountered at least two styles of doing geometry: Euclid's style (primarily logical in structure) and Descarte's style (primarily algebraic). Turtle geometry is a new style matched to the computer."
(Byte, 1980 p. 234)

This new-style geometry emphasises transformations in local space rather than using a fixed global reference point. By local is meant "it deals with geometry a little piece at a time", says Abelson and di Sessa (1981). When the Turtle moves, it does not move in respect of a globally-defined origin. The only requirement is how to move the turtle in relation to itself and the allocated space. For example, the turtle in drawing a circle deals only with the small part of the plane that surrounds the current position that it is in.

The Cartesian co-ordinate representation for a circle is $x^2 + y^2 = r^2$. By contrast with the turtle, $x^2 + y^2 = r^2$ relies on a global co-ordinate system to define its properties. On this issue Abelson and di Sessa (1980, p. 14) write,

"And defining a circle to be the set of points equidistant from some fixed point is just as global as using $x^2 + y^2 = r^2$. The turtle representation does not need to make reference to that 'faraway' special point, the centre."

Turtle geometry is more local, whereas co-ordinate geometry is more global.

The physical context for the floor turtle consists of a large flat surface (in this case the turtle table). The floor turtle is a computer controlled robot that is capable of moving forward and back relative to a previous position and direction. It is also capable of rotating about its central axis. The screen turtle is a representation of the floor turtle on the computer screen. It is a triangular point that moves about the screen using commands similar to those for the floor turtle.

The place and direction are the turtle's geometric state. At any particular time the turtle is at a particular place and facing in a particular direction. It is significant that these two components of the turtle's state are independent. It has a pen which can be up or down which enables the turtle to draw the geometric state that it is programmed to do either on the screen or on the floor at any particular time.

4.1.1 Turtle Talk

By 'turtle talk' is meant the commands used for doing turtle geometry. Even without previous computer experience it is

possible to move the turtle so that it will make a particular shape. For example,

```
FORWARD 100
RIGHT 90
FORWARD 100
RIGHT 90
FORWARD 100
RIGHT 90
FORWARD 100
RIGHT 90
```

The FORWARD command makes the turtle move in a straight line for 100 turtle units and the RIGHT command tells the turtle how many degrees and in which direction to turn.

In describing the effects of these commands it can be said that FORWARD changes the turtle's position and RIGHT changes the turtle's heading. These simple commands make turtle graphics easy for a child to 'get into'. The use of everyday language to tell the computer to do things that the child already knows, such as turning and moving, is an appropriate starting point for the child.

4.1.2 Procedures

Turtle geometry would be less exciting if it did not allow us to teach the turtle new commands. For example,

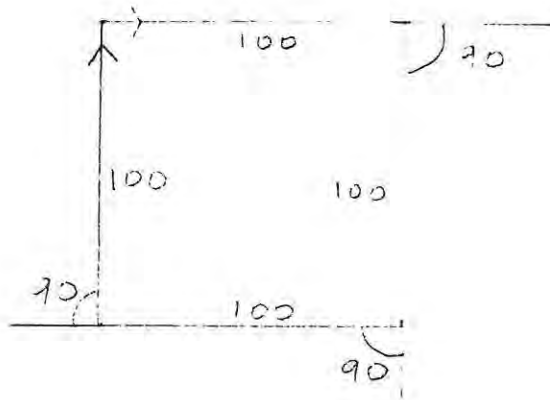
```
TO SQUARE
FORWARD 100
RIGHT 90
FORWARD 100
RIGHT 90
FORWARD 100
RIGHT 90
FORWARD 100
END
```

This is an example of a procedure. The first line of the procedure (the title line) determines the name of the procedure. The rest of the procedure specifies a list of instructions the turtle is to carry out in response to the square command. In other words these are directions telling the computer how to describe a square. The word TO, informs the computer that the next word SQUARE, is being defined and that the number of lines constitute its definition.

When it is indicated by the word END, that the definition is complete the machine will respond with SQUARE DEFINED.

Now if PENDOWN SQUARE is typed into the computer the turtle will carry out the procedure leaving a trail either as a floor turtle or as a screen turtle.

Unlike the direct command above the turtle moves in one continuous movement because a procedure has been made and stored in the memory of the computer. The drawn route would look like this,



Getting pupils to construct their own procedures is a vital step as these are the conceptual building blocks of LOGO programs.

4.1.2.1 Iteration

Iteration is a REPEAT command. When drawing symmetrical or regular shapes it becomes tedious typing the instructions repetitively. A more concise method of telling the turtle to draw a square would be by using iteration or REPEAT.

```

TO SQUARE
  REPEAT 4
    FORWARD 100
    RIGHT 90
  END

```

Besides saving typing work the child using REPEAT is starting to organise the movements of the turtle. He is beginning to understand how procedures can be built.

4.1.2.2 Introducing the Concept of a Function with One Variable

In the example of constructing a square the length of the side in turtle units (d) is being held constant. The learner is thus, in effect, working with a function with one variable which is the amount of rotation (θ°) and in this instance has been chosen as 90° degrees. If θ is chosen as say 60° degrees then a hexagon will result, if (d) is still held constant.

4.1.2.3 Sub-Procedures

LOGO allows the child to link procedures, that is, to get the turtle to follow one set of instructions and then to move directly to another. For example,

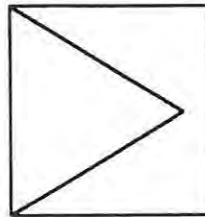
```
TO HOUSE
  SQUARE
  TRIANGLE
END
```

Here the child has linked the two sub-procedures of SQUARE and TRIANGLE to make a new procedure called HOUSE. Papert (1980, p. 60) says,

"One does not need a computer to draw a triangle or a square. Pencil and paper will do. But once these programs have been constructed they become building blocks that enable a child to create hierarchies of knowledge."

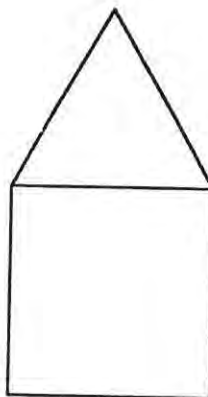
4.1.3 Debugging a Procedure

Very often a child's first attempt to draw a house results in a triangle inside the square (a) instead of on top of it.



(a)

Replanning or 'debugging' a procedure, as it is called, becomes a normal part of building a program. The debugging process helps a child understand the program. In this case the bug can be fixed by inserting a RIGHT 30 between SQUARE and TRIANGLE as in (b).



```
TO HOUSE  
SQUARE  
RIGHT 30  
TRIANGLE  
END
```

(b)

The learner has made progress without being criticised for an error in drawing. The pupil is encouraged to study the bug and rectify the error.

Often several options are available in designing a procedure. LOGO is not simply a matter of getting it right or not right. The LOGO language using graphics encourages the learner to correct the errors in a program so that the intended shape can be completed.

4.1.4 Recursion

In making HOUSE a procedure was made which 'called' both the other procedures, namely SQUARE and TRIANGLE. 'Calls' means that when the end of the procedure is reached it moves to another procedure. It is also possible to make a procedure which 'calls' itself. This is known as recursion. Take for example a program such as this:

```
TO POLY : SIDE : ANGLE
  FORWARD : SIDE
  RIGHT : ANGLE
  POLY : SIDE : ANGLE
END
```

This POLY program says go FORWARD some fixed amount, turn RIGHT some fixed amount and repeat this sequence over and over. The final line keeps the process going over and over by including "do POLY again" as part of the procedure. Harvey (1982, p. 166) writes about recursion and mathematics;

"It's hard to explain in a simple way why recursion is important. The idea behind recursion though has profound mathematical importance. By allowing a complicated problem to be described in terms of simpler versions of itself, recursion allows very large problems to be stated in a very compact form."

The POLY procedure has two inputs SIDE and ANGLE. This procedure generates a fascinating series of patterns as variable inputs are typed in. One advantage of this procedure is that the last line of POLY can be changed to make another interesting procedure. The new procedure is now,

```
TO POLYSPI : SIDE : ANGLE
FORWARD : SIDE
RIGHT : ANGLE
POLYSPI : SIDE +3 : ANGLE
END
```

```
e.g. POLYSPI 1 120
      FD 1 RT 120
      FD 4 RT 120
      FD 7 RT 120
      FD 10 RT 120
      etc.
```

The effect of POLYSPI is that each step is 3 units larger than the previous one. Systematic increments in the variable are now included in the programs which produces the pattern shown in Figure 1 on p. 63.

4.1.4.1 Introducing the Concept of a Function with Two Variables

Here (θ) varies and (d) varies.

In the example shown above (d) was chosen as 1 and (θ) as 120° . Variation of both these variables will result in the different shapes shown below.

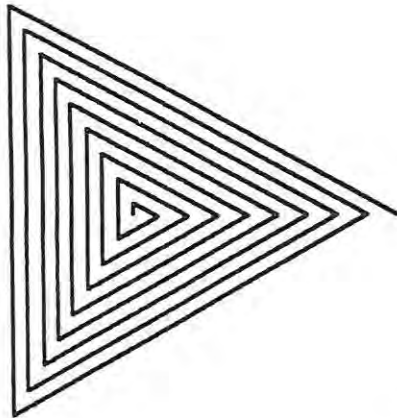
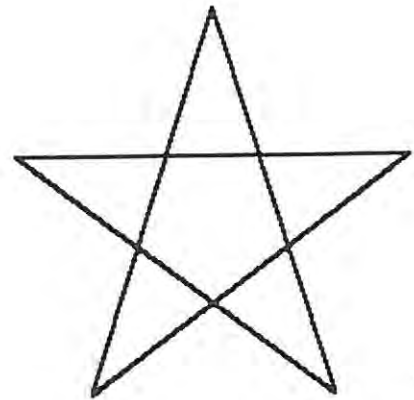
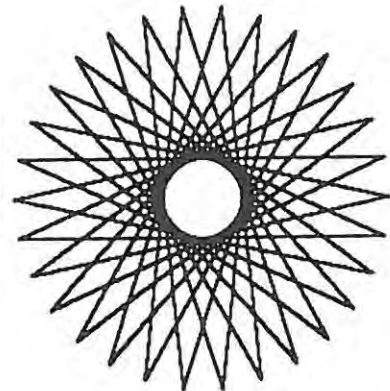


FIGURE 1



SIDE 85 ANGLE 144



SIDE 100 ANGLE 156

4.1.4.2 Variables

In the POLY and POLYSPI programs SIDE and ANGLE are of variable magnitude. The words on the title line preceded by " : " are names of the inputs, rather like the variables x and y in algebra. By keeping the SIDE variable constant and changing the ANGLE variable in the POLYSPI program the patterns on p. 60 are produced. The possibilities for creating patterns using two variables are endless.

Papert (1980) states that the variable is a key mathematical concept whose understanding is facilitated by the turtle. In 'turtle talk' he says variables are presented as a means of communication. 'Turtle talk' enables the learner to "create a procedure with an input". For example,

```
TO STEP : DISTANCE
FORWARD : DISTANCE
RIGHT 90
END
```

The command STEP 100 instructs the turtle to go forward 100 and turn right 90 degrees. If the input is changed to STEP 200 it will go forward 200 and then turn 90 degrees. This symbolic naming through a variable is an important mathematical concept for a child to know. The concept of variable coupled with the concept of recursion offers pupils two sources of power for constructing their own LOGO environment.

4.1.5 Total Turtle Trip Theorem

When the turtle travels along a route and ends in the same direction in which it started, the total amount of turn is 360 degrees. This is the proposition of the Total Turtle Trip Theorem. Total turning is the central concept here. All total journeys made by the turtle turn through 360 degrees, as do the three 120 degrees of the equilateral triangle and six 60 degrees of the regular hexagon. In the POLY program for example an angle input of 360/n draws a regular n-sided polygon.

By comparison with the Euclidean counterpart "The sum of the internal angles of a triangle is 180 degrees" Papert (1980, p. 76) makes the following observation,

"... in the context of LOGO computers the Total Turtle Trip Theorem is more powerful : The child can actually use it. It is more general : It applies to squares and curves as well as to triangles. It is more intelligible : the proof is easy to grasp."

4.1.6 Play Turtle

By more "intelligible", Papert means that a child is in a position to 'play turtle'. By that he means that the child can 'walk' the plan through i.e. simulate the movements of the turtle. The child who wants to make an approximation of a circle is asked to move his body as the turtle would move. Such movements might lead him to say, "When you walk in a circle you take a step forward and you turn a little." From

this physical movement comes a verbal description which is only a step away from a turtle procedure:

```
TO CIRCLE
REPEAT
FORWARD 1
RIGHT 1
END
```

We have seen from the Total Turtle Trip Theorem that the circle closes when the turtle has turned through 360 degrees. The result is an approximation of a plane curve with constant curvature. Playing turtle refers to the thinking related to a procedure before programming it. As Papert (1980, p. 76) describes it, "... it is a model for the general habit of relating mathematics to personal knowledge." Playing turtle according to Papert (1980) helps to relate and unify knowledge gained through the physical act of walking out the turtle route. This internalization is the basis of clear concept development in the individual.

4.1.7 Microworlds

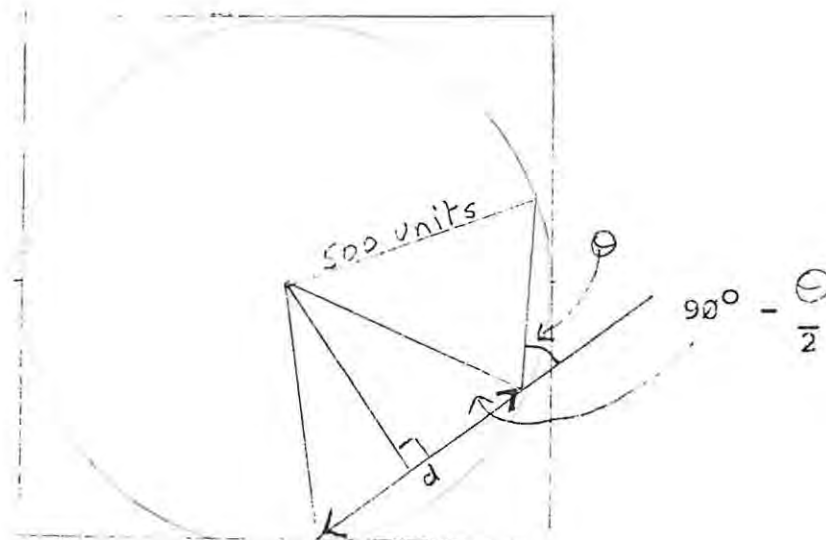
The term microworlds as used in a LOGO context is a well-defined but limited learning environment. The concept of microworlds stems from research in the field of Artificial Intelligence. It is difficult to probe directly the thinking processes of a child, but by limiting our attention to a small area it is possible to find aspects that can be modelled.

In the context of turtle graphics it is possible to define a microworld, for example, all the designs that can be drawn with the procedure POLYSPI. The POLYSPI microworld shows how a variable can be changed incrementally whilst holding the other variables constant. This provides a clear model how particular patterns may be generated through the dimensions of variation. Piaget would place this activity at the formal level of operational thought (See Chapter Two). The idea is important because it relates to other problem-solving situations.

4.1.7.1 The Dependent Function

If, for example, one function is dependent on the other we can solve a problem such as this:

Given a square of paper of side 1000 turtle units what is the relationship between the side (d) of an n-gon and the degrees of rotation of the turtle θ so that the polygon does not exceed the square of paper?



We have $f : n \rightarrow \theta$ with $\theta = f(n) = \frac{360^\circ}{n}$

and $g : \theta \rightarrow d$ with $d = g(\theta) = 1000 \cos(90^\circ \frac{\theta}{2})$

So for $h : n \rightarrow d$,

$$d = k(n) = g[f(n)] = 100 \cos(90^\circ \frac{180^\circ}{n})$$

$h = g \circ f$: the function h , is a composite of g and f .

d is a variable dependent on n .

4.1.8 Conclusion

Turtle Geometry was developed for children. The computer provides an environment in which the pupils learn by doing and thinking about what they are creating. The pupil investigates the capabilities of the computer and LOGO by planning procedures and re-planning them to remove the bugs. The computer serves as a tool for creating mathematical shapes through the use of the turtle. Playing turtle helps the child's understanding of that procedure.

The appeal of turtle graphics depends largely on the emergence of designs made by building procedures. These procedures can be developed and extended through the pupils' understanding of the powerful mathematical concept of variable. The other powerful (Papert) mathematical concept is that of the Total Turtle Trip Theorem which states that the complete turn of the turtle amounts to an integer multiple of 360 degrees.

The LOGO microworld provides a limited environment in which there are important concepts to be understood. Turtle geometry provides a rich environment for the child to understand certain mathematics concepts. It enables him to learn how to solve particular problems and gives him a method of becoming aware of his own thinking.

5. METHODOLOGY

- 5.1 Two Traditions
 - 5.1.1 Illuminative Evaluation: Reasons for Selection
- 5.2 Data Collection Methods
 - 5.2.1 Classroom Observation
 - 5.2.2 Video Recording
 - 5.2.3 Triangulation

CHAPTER FIVE

METHODOLOGY

5.1 Two Traditions

One of the problems before me in this investigation was the question of how to evaluate the teaching sessions. My attitude towards the evaluation of the research was in fact influenced by fundamentally different views about the nature of the educational process itself. Two traditions dominate the field of evaluation.

The first emphasises measurement and prediction. In this "experimental" style of research the emphasis is on rigorous testing of hypotheses. By changing selected features of the experimental situation whilst controlling others, it is possible to observe changes in behaviour, which are then interpreted in terms of the predetermined hypotheses. The interpretation of the results depends on the reliability and validity of the tests made as well as the control of as many extraneous variables as possible. It has been called the "experimental", "scientific", "objectives" and sometimes the "agricultural-botany" model. It was Parlett (1974, p. 14) who coined the phrase "agricultural-botany model" which he described in these terms,

"... appropriate for testing fertilizers on carefully-tended fields of crops at agricultural research stations, but inapplicable and incongruous for monitoring how innovations become absorbed and adapted in a diversity of school settings."

I would go further and say that the evaluation of such an activity as human learning must necessarily call for the interplay of a wider variety of human qualities than is normally considered in a scientific investigation.

Arising out of disenchantment with the experimental tradition comes the "hermeneutic" model. This approach has found favour with classroom observers and those interested in the sociology of knowledge. It is a style of evaluation that can best be described as "interpretive", because it relies on explanations of people's actions. The researcher's task is to understand man as viewed by the anthropologist. Bynner and Stribley (1979) see it like this,

"It is best symbolized by the school of research known as ethnography which emphasises naturalistic observation of phenomena in the field and seeks insights into social behaviour gained from data which are as unadulterated as possible by the procedures the researcher employs and the preconceptions he brings with him." (General Introduction)

The second model emphasises meaningful description and interpretation. It is concerned with emergent effects as the evaluation proceeds. It has been called the "illuminative", "participant observer" or "social-anthropological" model.

Clearly the issues that concern the researcher using these ethnographic methods are the accuracy of the information he collects, the problem of the role he adopts in collecting it, and the generalizability of his findings outside the particular research setting.

In sharp contrast to the ethnographic style, the experimental style provides a model for the logic of scientific enquiry, which no social researcher can completely disregard.

In short, the fundamental question asked by the experimental evaluator is, "does the innovation perform as intended?" On the other hand the illuminative evaluator asks, "what happens when the innovation is introduced?" These, in fact, were the very questions I had to ask myself prior to making a decision about the style of evaluation to be used.

5.1.1 Illuminative Evaluation: Reasons for Selection

In my investigation I recognised the complexity and subtlety of the teaching/learning processes taking place. I needed to employ an evaluation strategy that would take into account modifications and departures from the original instructional concept. I also wanted to take into account the social-psychological environment in which the pupils and the teacher co-operated together.

As far as possible I wanted to determine the variety of influences operating in the teaching sessions. This would include the pressures, the successes, the fears and the failures and the degree of co-operation whilst the learning activity was in progress.

Illuminative evaluators make the criticism here that the experimental-style evaluators have considered only the

innovation and have not taken the whole learning milieu sufficiently into account. They point out that new innovation produces unintended outcomes.

It is suggested by Parlett and Hamilton (1977, p. 12) that the style of evaluation has a bearing on the outcome of the innovation. They write,

"Students do not confront 'knowledge' in naked form; it comes to them clothed in texts, lectures, tape-loops etc. These form part of a wider set of arrangements for instructing, assessing and counselling which embody core assumptions about how knowledge and pedagogy should be organised.

... Both teaching and learning are profoundly influenced by the type of assessment procedures in use."

Illuminative evaluation rests upon a general research strategy not upon a standard methodological approach. I too, was looking for a research strategy that was sufficiently flexible to cope with such variables as the size of the group, the use of computer technology, the role of the evaluator and other factors. I could see that in this investigation the evaluator would not need to attempt to manipulate or control situational variables. He would accept the scene as he found it, with all its complexities included. I could also see that the evaluator would have to unravel these complexities, to separate the significant from the insignificant and to look for relationships. He would concentrate on 'process' within the learning milieu.

According to Parlett and Hamilton (1977), illuminative evaluation should properly pass through three stages, evaluators observe, inquire and seek to explain.

In reaching an interpretation I would deem it necessary to take into account evidence from other sources such as other observers. Without doubt the feature that attracted me to this style of evaluation was its flexibility and its adaptability to circumstances.

It was on the following factors that I made my decision to base the investigation on illuminative evaluation:

i) It was the nature of the investigation that defined the evaluative method to be used - not vice versa. The choice of research techniques that I used were determined not by doctrine but by the need to gather data in a particular setting. In this case the data had to be gathered from six pupils and a teacher, using computer technology.

ii) Different techniques were combined to throw light on the computer innovation. No one research method with its own inevitable built-in limitations was used exclusively or in isolation. The approach I used may be referred to as "triangulation" (Cohen and Manion, 1980). The different methods used for data collection were by a non-participant observer, video recordings and participant observation.

iii) The investigation progressively focussed on specific aspects of the milieu. Beginning with an extensive base of information about the innovation, the evaluators reduced the breadth of the enquiry to give concentrated attention to certain issues which emerged. After each teaching session new information was used to clarify and re-define the problem issues as the investigation progressed. The course of the study could not have been charted in advance. Through progressive re-focussing it was hoped that new and important factors would be given their due weight.

5.2 Data Collection Methods

5.2.1 Classroom Observation

There are two principal types of observation, participant observation and non-participant observation. In the former the observer engages in those activities he sets out to observe. In this investigation it was I, the teacher, who acted as the participant observer.

A non-participant observer, on the other hand, sets himself apart from the group he is actually investigating. In this case the non-participant observer was Dr. A. Penny who sat at the back of the classroom recording the exchanges and events that took place between pupils and between teacher and pupils. But in a setting where one is observing children and one is interested in what they are doing it is difficult for the observer who wishes to remain covert not to act as a

participant. If Dr. Penny had not participated it would have been difficult to explain his presence as he was very obvious to the actual participants.

This was an unstructured observation study. By that I mean there were no prepared observation schedules, as I wanted Dr. Penny to monitor events as they occurred, taking into account the pupils' verbal and non-verbal cues.

It was felt that this form of research was eminently suitable to participant observation if one considers the views of Bailey in Cohen and Manion (1980). He says there are advantages in using participant observation. In the observation study the investigator is able to discern ongoing activities as they occur and therefore can take note of the most important features. As the research takes place over a period of time the investigator can develop a more intimate and informal relationship with those he is observing, in this case the children.

On the other hand participant observation studies are not without their critics. It has been said that the account that emerges from participant observation is "subjective, biased, impressionistic, idiosyncratic and lacking in the precise quantifiable measures that can be found in experimental research" (see Cohen and Manion, 1980, p. 104). The observer might lose his perspective and become blind to the events he is supposed to be investigating.

It was for these reasons that I decided it was necessary to have a non-participant observer present at all the teaching sessions.

As well as having a non-participant observer present, sophisticated equipment in the form of video cameras was used in order to make the data capable of review and available to other viewers.

5.2.2 Video Recording

Video-recording the teaching sessions would support the idea put forward earlier on illuminative evaluation, that the nature of the phenomena to be studied should determine the method and by implication the techniques for observing it. As visual phenomena were to be investigated, a visual data collecting method was used. I felt the videotapes would capture both audio and visual information.

Adams and Biddle in Cohen and Manion (1980, p. 118) maintain that there are distinct advantages for using videotape recordings in classrooms.

"First, it provides an extremely comprehensive record of classroom behaviour that can be preserved for subsequent analysis. Second, what they term the fidelity of the system is good, that is to say the cameras can deal with conventional classroom settings, and microphones are able to pick up the greater proportion of the public utterances that take place. Third, the stop-rewind facility of the tape recorder permits sequences of behaviour to be viewed and reviewed at will during data-coding."

In this instance the reviewing of the videotapes was found to be particularly useful because of all the valuable information that emerged.

I elected to use video-recordings as part of the data collecting strategy for the following reasons. I felt that the tapes would provide a unique view of the teaching sessions which could be used for extensive analysis. As the teacher I would be aware throughout the replay of the total time-scale of the lesson. It would therefore be possible to make comparisons across the lesson of different aspects of it. Many behaviours of the pupils that I missed in a lesson would be clear on the videotapes. Perhaps most important was that the recording did not misinterpret or misrepresent their classroom.

It would be possible to look at the same lesson time after time so that it would allow reliability checks to be made by and between observers. These interpretations could be compared to assess the reliability of the interpretations, particularly where certain events have been claimed to be significant. These could be examined to establish the criteria for their significance.

This dissertation includes an edited version of all the videotapes that were made. Counter numbers have been provided so that the reader can easily locate the 'significant' events on the videotape (See Appendix).

5.2.3 Triangulation

Classroom observation techniques by a non-participant observer and a complete video-recording of the classroom events were the methods used to increase the validity.

The idea of using three methods of data collection in the study is known as triangulation from techniques used in navigation.

By analogy, triangular techniques in the social sciences strive to explain more fully the richness and complexity of human behaviour by studying it from more than one viewpoint.

The making of inferences always calls into question validity and requires the researcher to check that the data supports that inference. Becker in Bynner and Stribley (1978) gives some possible forms of checking the frequency and distribution of phenomena. I undertook to establish the validity of data by triangulation. Using the two methods of observer participation and video-recording it was possible to identify significant behaviour by means of two data sources, in addition to my own. Cohen and Manion (1980) argue that the use of a triangular technique helps to overcome the problem of 'method boundness' as it has been termed. When the phenomena in the investigation is recorded using alternative methods then the data collected is likely to be valid. (See Cohen and Manion 1980, for a full discussion on triangulation methods).

In using a participant observer, a non-participant observer and video-recordings I attempted to establish reliable and valid methods for selecting significant events in the teaching sessions.

CHAPTER SIX

6. THE INVESTIGATION

- 6.1 The Setting: The Floor Turtle
- 6.2 Session One
 - 6.2.1 Session Two
 - 6.2.2 Session Three
 - 6.2.3 Session Four
 - 6.2.4 Session Five
 - 6.2.5 Summary
- 6.3 The Screen Turtle
 - 6.3.1 Session Six
 - 6.3.2 Session Seven
 - 6.3.3 Session Eight
 - 6.3.4 Final Session: Pupil Demonstration and Observer Discussion

CHAPTER SIX

THE INVESTIGATION

This chapter describes the events that took place in the teaching sessions. There were seven one hour sessions with a final 'teach-back' at the end of the programme. Throughout this chapter reference will be made to the counter numbers of the edited version of the video-recording, so that the reader can identify what is being described in the text.

The six children who participated in the investigation were two boys and four girls, all from Standard Five at Kingswood College. The class teacher selected the pupils using the criterion of how well the children would co-operate together in their work. Their names were Nicola and Karen, Rhonda and Sheilagh and David R. and David P.

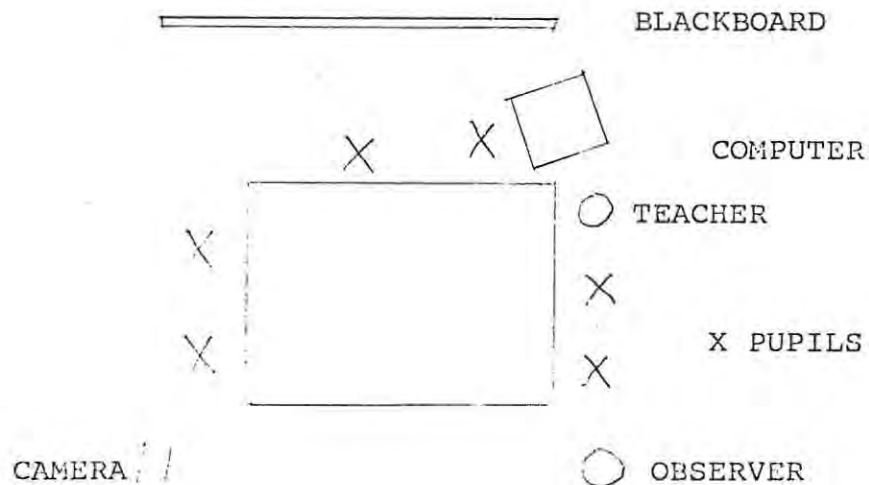
6.1 The Setting: The Floor Turtle

A deliberate attempt was made to create a setting that would allow the pupils to focus on the movements of the floor turtle, as well as follow the programs that had been planned on the blackboard. Getting the children to sit in pairs round the turtle table was designed to encourage communication with each other across the table and also with the teacher who sat with the children.

On one side of the turtle table was placed the micro-computer to which the turtle was connected (See Appendix 1, counter number 002).

The video camera was controlled from outside the room through a transparent window. Suspended above the turtle table were three microphones for recording the dialogue.

The observer was placed at a desk within the room in a position where he could hear what was said round the table and also see what the children were doing. He also had easy communication with the teacher. Although he was sitting at the rear of the room there was opportunity for him to become engaged in the whole teaching-learning situation.



The significance of the setting played a crucial part in the development of interaction between pupil and pupil and between teacher and pupils.

I collected the children from Kingswood school and squeezed the six of them into the car. They sat quietly for most of the ten minute journey unsure of what to expect.

For ease of identification we pinned on name tags and sat ourselves round the turtle table. The children selected their own partners. The only positions to remain fixed were those of the observer and the teacher.

The pupils were introduced to the turtle and the computer, and were encouraged to suggest how the turtle could be moved. In this initial phase I found it easier to teach the children the commands FORWARD and BACK, LEFT and RIGHT turn. They encountered no difficulties typing the commands into the computer using One Key LOGO. The advantage of using OK LOGO was that it reduced the tedium of typing experienced by children using a keyboard for the first time.

All that was required of the pupil was to type the initial letter of the command and the full command would appear on the computer screen. Working in pairs they concentrated on moving the turtle over a set distance to become familiar with the turtle unit as a measurement of distance. This unit of measurement was unknown to the children so that they required experience before making accurate estimations of distance. At this stage they were confined to right angle turns only.

Not all their estimations worked out, so this required further debate in order to debug the original plan. The amended version of the procedure would be recorded on the blackboard. (Appendix 1, counter number 4.5 to 042 shows Rhonda and Sheilagh planning a route through a maze, then debugging a procedure).

The group found this joint decision-making activity non-threatening and great fun.

I realised as did the pupils that soon they were going to need to rotate the turtle through an angle other than a right angle. In the video recording of this episode a child can be heard to say, "we have just been doing this sort of thing - we have been doing bearings in maths". Although it had recently been done in school it still caused difficulties for some of the children when they physically turned through angles of various magnitude related to the points of the compass. Rotation through four right angles (360°) laid the foundation for the Total Turtle Trip Theorem which they would require in later sessions. It also provided them with a concrete metaphor of the turning of the turtle which would be necessary in planning future programs. (See Appendix 1, counter number 042 to 066).

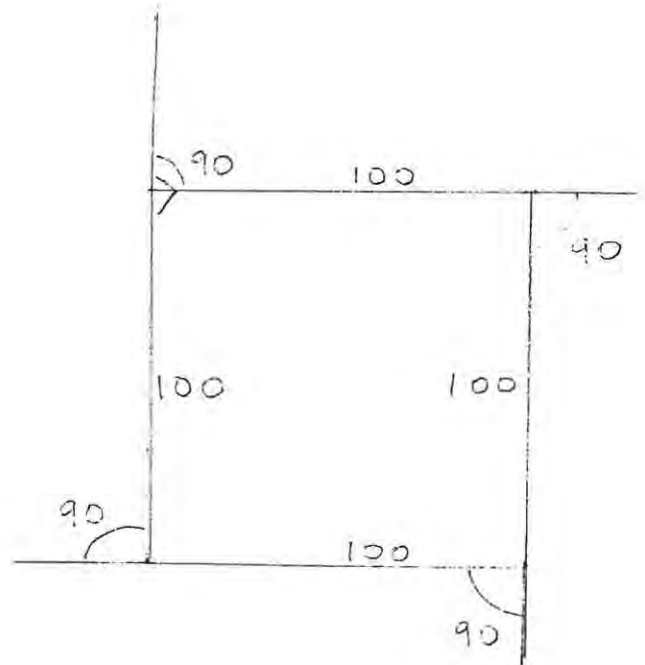
The first session ended with a discussion between Dr. Penny and the children. The return journey to school was very animated, with the children excitedly discussing the events of the afternoon. One girl asked if the turtle could make one long continuous journey instead of a set of discrete

journeys as it had done that afternoon. They were anticipating the next stage of procedure-building.

6.2.1 Session Two

The initial task set was to plan a procedure for a square. To assist the pupils in planning their program I suggested that they draw the diagram on the blackboard. The suggestion was ignored. The problem was theirs and they chose to solve it in their own way. Having worked through a solution Karen and Nicola wrote up their procedure for the square. Guidance was given for the instruction 'making a procedure'. The program read like this:

```
M  SQ  (Make square)
F  100  (Forward 100)
R  90  (Right 90)
F  100  (Forward 100)
R  90  (Right 90)
F  100  (Forward 100)
R  90  (Right 90)
F  100  (Forward 100)
R  90  (Right 90)
F  100  (Forward 100)
R  90  (Right 90)
E      End
```



This program showed the group that the turtle could be instructed to combine the commands into one continuous movement. As the program was long it took Nicola some time to type it in. I asked them to reconsider writing the procedure so that it could be shortened. The children's

thinking was revealed in the discussion with the teacher on how to plan a shortened procedure. David R. and Nicola wrote the shortened procedure for the square using the concept of repetition or 'times' as it is called in OK LOGO. They called their procedure 'SP',

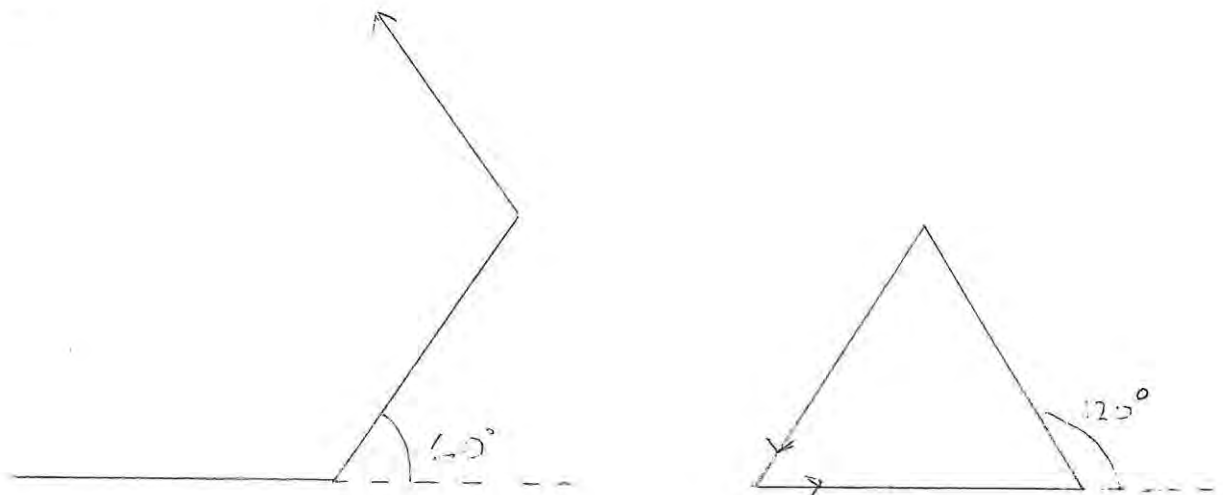
```
M  SP  (Make SP square)
F  100 (Forward 100)
R   90 (Right 90)
E      (End)
T4PSP (Times four procedure SP)
```

(See Appendix 1, counter number 066 to 110 for the full sequence of building a square procedure, first without repetition and then planning a procedure using the idea of repetition).

As the children appeared to have grasped the idea of using repetition in program writing, I posed the task of planning a procedure for an equilateral triangle thinking that they would be able to apply this concept in writing a program. I did not anticipate the difficulty they would experience. It was not a programming problem they encountered but a mathematical one.

Although the children were able to define an equilateral triangle with accuracy they experienced difficulty deciding on the size of the angles of rotation. One pair planned a procedure with three different size angles. They suspected this was incorrect yet were not sure how to debug the procedure because they had not conceptualized the problem.

Of their own accord they 'played turtle' with no prompting from the teacher. In fact they used their own strategies in searching for a solution to the problem which had become theirs. My role as teacher diminished as they addressed the problem which had now become very important to them. They programmed the turtle to move through the internal angles of the triangle and not through the 'turtle angle' which was the exterior angle. The error in their thinking can probably be accounted for by the misapplication of the rule that 'the three angles of a triangle add up to 180 degrees'. I suspect it was also due to having just drawn a square whose interior and exterior angles are all 90 degrees. The successful completion of the square did not confront the pupils with this interior/exterior angle distinction. (Appendix 1, counter number 110 to 120). The effects of physically turning through 360° in the previous session had not been transferred to this problem. It required me to intercede and deliberately focus their attention on the exterior angle before they realised that the turtle had to travel through 3 x 120 degrees = 360 degrees, which is the concept underlying the Total Turtle Trip Theorem. The diagram below illustrates the point:



The activity of finding the exterior angle of an equilateral triangle assumed considerable importance to the children. There was much argument, debate and negotiation between them before they committed themselves to testing their procedure. It would seem that this was a vital barrier that the children had to work through for themselves. Once they mastered $360/n$ for the exterior angle of a regular polygon, they were not confused by this mathematics concept in the sessions that followed.

Before running their program for the equilateral triangle Sheilagh said it was a pity the turtle could not leave a trail. That seemed the appropriate moment for the pen to be fitted to the turtle. The first turtle drawing was made and it was of an equilateral triangle. The drawing made a fitting close to the session. (See Appendix 1, counter number 129 to 147).

On the journey back the children could not contain their excitement at the afternoon's events and declared their enthusiasm for the floor turtle.

6.2.2 Session Three

In this session the pupils were set the task of drawing a flag. Once again the children set about creating a solution to the problem in their own way. It was significant that they incorporated 'playing turtle' (See Glossary) as a problem-solving technique before committing themselves to the

program, an indication of concrete-operational stage thinking. Rhonda and Sheilagh produced a procedure and a drawing for their flag. (See Appendix 1, counter number 147.5 to 157). Whilst the girls were busy with their program the boys produced a program that would repeat the original flag procedure. Before trying it David R. showed the class what route the floor turtle would take prior to the event. It was satisfying for the group as a whole to see the turtle produce the drawing that David had predicted. They were well into the idea of using recursion as a programming technique.

To reinforce the idea of the turtle travelling 360 degrees when creating an n-gon, I used a circular protractor. The children examined the amount of turn in prescribed angles in preparation for drawing various shaped polygons. (See Appendix 1, counter number 190 to 205).

To incorporate the mathematical idea of the Total Turtle Trip Theorem and the programming idea of recursion, the group was set the task of drawing a regular hexagon. Rhonda and Sheilagh produced this program:

```
M    Y
F 100
L   60
E
TGPY
```

Whereas the two Davids produced one program that 'called' another:

```
M   PEX   (Perfect hexagon : David R.)
```

```
F   100
```

```
R   60
```

```
E
```

```
M   RUN
```

```
T6P PEX
```

```
E
```

```
P   RUN
```

David R.'s clear explanation of the program reflected a thorough understanding of both the mathematics concept of the exterior angles of a regular hexagon, and the programming concept of one procedure 'calling' another. (Appendix 1, counter number 205 to 213).

6.2.3 Session Four

Each pair was set a different task. Using the circular protractor to help them, they had to create procedures for a trapezium, a rhombus and a simple cross. The pairs took turns to enter their procedure into the computer and test and debug the program if it needed it. Little guidance was required from the teacher and often I withdrew totally and allowed the children to continue on their own. It was interesting to note how proficient they had become in

planning the procedure, making a trial run, then running through the program with the pen down.

The next task was to combine sub-procedures into a new program. For instance one pair had to make a house using the stored procedures of triangle and square. Another pair had to make a rocket using the sub-procedures of triangle, square and trapezium.

One of the pairs preferred to create one entire procedure and not use the sub-procedures already stored in the computer. Through discussion they recognized it was easier to 'call' two sub-procedures to make the new procedure. Nicola and Karen designed the following program:

```
M HOUSE          "the house that Nicola built"  
P TRI  
P SQ  
E
```

(See Appendix 1, counter number 213 to 230).

Nicola and Karen also created a recursive program for their George Cross (G.C.)

```
M CROSS  
P ARM  
L 90  
E
```

M GC
T4P CROSS
E

These drawings indicated that the children had developed confidence in combining procedures. They were able to design programs that were recursive.

6.2.4 Session Five

Before the children arrived I programmed into the memory of the computer a set of sub-procedures. These included: triangle, square, rhombus, trapezium, arc etc. From these sub-procedures I asked the pupils to create new procedures. I anticipated that given a set of microworlds the children would be in a better position to plan their own procedures. This, in fact, was not the case. Although they tackled their tasks with enthusiasm they found great difficulty combining the sub-procedures into the new program. The children had not created the presented sub-procedures themselves and were therefore not fully familiar with them. In my ignorance I had created the set of microworlds which in this situation worked against itself. The boys struggled to plan and debug a program that was not theirs in origin. It was disappointing, considering the progress that had been made.

To introduce the idea of an externally created microworld at this stage was an ill-judged decision on the part of the teacher. In retrospect, I realise that I should have allowed

the children time to create their own microworlds in keeping with the philosophy of Papert (1980).

6.2.5 Summary

After five sessions with the floor turtle I reflected on the concepts that the children had developed. The pupils:

- i) learned LOGO commands for moving and rotating the turtle.
- ii) engaged in the process of planning a procedure - the building block of programming.
- iii) used iteration in building procedures.
- iv) had experience in using the idea of the Total Turtle Trip Theorem.
- v) created sub-procedures which could be incorporated into other procedures.
- vi) designed programs that used recursion.

As the pupils had reached this level of achievement I decided to transfer from the floor turtle to the screen turtle. I felt they were ready for it.

6.3.1 Session Six

The setting for the screen turtle consisted of three microcomputers arranged in a tight semi-circle, with two children working together on a computer. The task was first to familiarise themselves with slightly different commands. Then they had to build procedures for the regular hexagon, square and equilateral triangle. They also had to acquaint themselves with the turtle unit on the computer screen.

Both the observer and myself watched for difficulties that might have arisen from using a floor turtle with its slow, physical, three-dimensional movements, then adapting to the screen turtle's quick two-dimensional movements. This factor appeared to cause the children no difficulty, neither did moving from the horizontal plane of the floor turtle to the vertical plane of the screen turtle.

Having experienced building a procedure with the floor turtle, they transferred this skill with ease to the screen turtle and were impatient to get on and build other procedures. The girls set about making a house using the sub-procedures of square and triangle and the boys built a rocket using square, triangle and trapezium.

Video counter reading 230 to 239 shows the error in the house procedure of Karen and Nicola. They debugged the program and another error emerged, but they kept at it despite the

program not running as they intended it to. Significant aspects of using the screen turtle were the speed at which the children entered a procedure and the speed of the resultant response. I felt that the immediate feedback helped maintain interest and motivation. Although the program did not run as they hoped it would, the girls nonetheless persevered at resolving the problem. Video counter readings 239 to 245 show the boys designing a rocket procedure. In the first instance there is an error, in the second the error is corrected but the rocket is on its side, and the third shows the final correct rocket procedure.

The conversation with Dr. Penny (245 to 260) indicated some of the pupil difficulties. For Karen and Nicola it was debugging the house procedure. For Rhonda and Sheilagh it was establishing turtle distances and for the two Davids it was getting the rocket to face the right direction.

Despite these difficulties they were enthusiastic about the screen turtle. They felt they each had a turn more quickly, that they could understand it, it was faster and seemed more exciting. They were unanimous in saying the procedure-building ideas were the same as the floor turtle.

In the car they again voiced their enthusiasm for the turtle, this time the screen turtle. Judging from their comments they had clearly been ready for the transfer.

The pupils were more confident in building procedures. They were set the task of making an approximation to a circle. Each pair produced this procedure:

```

TO CIRCLE
REPEAT 360 [ RT 1 FD 1 ]
END

```

This produced a very slowly-drawn circle. I then asked if they could speed-up the drawing of the circle (See 261 to 271). Left with the problem they planned and tested different procedures which included:

```

TO CIRCLE
REPEAT 36 [ RT 10 FD 10 ]
END

```

```

TO CIRCLE 1
REPEAT 30 [ RT 12 FD 5 ]
END

```

In planning these procedures the children used the factors of 360 degrees to satisfy the Total Turtle Trip Theorem.

It was suggested to the pupils that they try to spin the circle they had made, in other words make a recursive procedure that would 'call itself'. The dynamic aspect of

creating a recursive pattern caused interest and excitement (271 to 281).

Later in the session the children were introduced to a one-variable function. It was suggested to them that they design a procedure that would accommodate squares of various sizes, building on from what they already knew. The ease with which the children created these programs was significant. It was a painless experience. For example,

```
TO SQ
REPEAT 4 [FD 60 RT 90]
END
```

then,

```
TO RSQ : SIZE
REPEAT 4 [FD : SIZE RT 90]
END
```

Rhonda and Sheilagh made squares and circles of variable size (283.5 to 320). The boys posed the problem of whether they could then spin a square of variable size. This meant using the concepts of recursion as well as function with one variable.

The program was developed out of the previous two procedures and looked like this:

```

TO SPINSQUARE : SIZE
RSO : SIZE
RT 20
SPINSQUARE : SIZE
END

```

6.3.3 Session Eight

This session developed the theme of variables. Once again, starting from the procedures they had already internalised, they planned a procedure for an n-gon. Once the n-gon was created they used recursion to spin the shape. This logical development flowed naturally from their own exploratory needs and was found to be exciting and certainly within their capability. The sequence of procedures that emerged looked like this:

```

i) TO TRI
   REPEAT 3 [ FD 50 RT 360/3 ]
   END

ii) TO SQ
   REPEAT 4 [ FD 50 RT 360/4 ]
   END

iii) TO HEX
   REPEAT 6 [ FD 50 RT 360/6 ]
   END

```

```

iv)   TO POLY   : SIDE
      REPEAT   : SIDE [ FD 50 RT 360/ : SIDE ]
      END

```

```

v)    TO SPIN  : SIDE
      POLY    : SIDE   RT 10
      SPIN    : SIDE
      END

```

The pupils understanding of one-variable function grew out of their activities and was not imposed or withheld by the teacher.

The final stage was when the side was varied and the angle of rotation as well. This introduced the concept of a two-variable function which could be considered well beyond the ability of pupils in secondary schools. It is not included in the Senior Certificate Mathematics syllabus. The two-variable concept emerged naturally in the course of working through the logic implicit in creating this program.

```

TO SHAPE : SIDE : ANGLE
FD       : SIDE
RT       : ANGLE
SHAPE   : SIDE : ANGLE
END

```

Of the group it was the boys who brought the most insight into explaining this program. When they came to run it they had difficulty in holding one of the variables constant.

Because of this they were unable to detect a pattern which might have been revealed if they had used a more systematic approach.

By using turtle geometry the abstract concept of two variables was seemed not to be beyond the comprehension of these twelve year old pupils. This accords with Hunt's (1969) theory of 'match' related to Piaget's stage development. Given more time to investigate and run their procedures I suspect they might have internalised the notion of running two mathematical variables in a program. I felt that they had the understanding but were restricted through lack of practice.

6.3.4 Pupil Demonstration and Observer Discussion

In this, their final session the children demonstrated to four external, adult observers the concepts that they had developed over the course. Commencing with the floor turtle they reviewed the mathematics concepts and programming techniques that they had acquired. These included programs from the square to programs for two-variable functions.

Arising out of this demonstration was a discussion with the four observers led by Dr. Penny. The following issues emerged:

- i) The turtle provided a dynamic means of communicating mathematics concepts.

- ii) The visual and tactile impact was important for establishing understanding.
- iii) It was an advantage for the pupil to be able to correct errors quickly and accurately and easily.
- iv) Turtle geometry provided a suitable vehicle for introducing primary children to computers.
- v) Turtle geometry provided a powerful introduction in the important mathematics concept of function.
- vi) Doubt was cast as to whether primary children should in fact be learning about variables. Debate ranged on whether time would be better spent in developing more simple mathematics concepts such as proportion. The findings from Hart's (1981) Concepts in Secondary Mathematics and Science (CSMS) project were quoted to substantiate this view.
- vii) An observer (the headmaster) was surprised to see how his 'average' pupils tackled problems with confidence and understanding.
- viii) The children expressed themselves with precision, clarity and confidence, which reflected thorough understanding and possession of the necessary skills.

- ix) The observers were amazed and surprised at the class teacher's (Kingswood) rating of the children in terms of school mathematics lessons. It did not compare with their own perceptions of the children.
- x) Doubts were expressed about the speed of operation of the screen turtle and how this might limit understanding for some children. The slow beginning with the floor turtle was considered a useful aid to learning.

(The entire discussion was recorded on audio-cassette and is included to assist the reader.)

7. CONCLUSIONS

- 7.1 Aspects of Significance: The Teacher's Perspective
 - 7.1.1 The Method of Teaching
- 7.2 The Observer and the Observed: The Observers Perspective
- 7.3 Concluding Summary
- 7.4 Suggestions for Further Research

CONCLUSIONS

7.1 Aspects of Significance:

The Teacher's Perspective

In using turtle geometry I attempted to create an environment in which the pupil's task was not to learn a set of formal rules but to engage in the process of program building. Howe (1979) likens this process to building a Meccano model.

To the beginner a Meccano set is a confusing collection of metal pieces that have to be correctly assembled in order to successfully construct a model. What does a child need to learn so that he can create a model? He must first become familiar with the components; angle girders, strips, fishplates, spring clips, trunions and axles. He must learn how to fit the parts together in a meaningful way. Building a complex model such as a gantry crane requires him to plan an assembly sequence, building the sub-parts of cab, hoisting-carriage and platform separately, before bringing them together into a co-ordinated whole. Should there be any mismatches between the plan and its execution, modifications need to be carried out to overcome the problem.

The analogy of the Meccano builder illustrates two key issues that need to be addressed in deciding whether mathematics can be taught successfully in conjunction with programming. The first is an assessment of how well the children learned

turtle geometry and the second is how well the children learned to program.

In this investigation I found that the computer developed both mathematics concepts and computer programming. The children needed to understand the mathematics concepts so these could be incorporated into a program. At the same time they needed to understand the techniques of compiling a program to illustrate a mathematical idea. The development of mathematics concepts through programming were complementary activities. These two aspects were of great significance to me. Like using a Meccano kit of metal parts for creating a model, the turtle became the electronic model building kit. Perhaps this will become the kit of the future?

7.1.1 The Method of Teaching

The way in which the computer is used in a classroom reflects a teacher's philosophy about teaching. Some see it as a tool for computation, or a tool for processing information. Others see it as a tool for creating and testing models. I see it as a tool for describing processes, procedures and algorithms. Papert (1980) would go so far as to say it is a tool to be used in an open-ended fashion for introducing a child to new ideas, new concepts, new techniques and a new outlook.

Papert bases his philosophy on what may be described as "natural learning" after Piaget (1972), where the child controls his own learning through interaction with the environment.

On the other hand the model of teaching presented by Howe (1979) at Edinburgh is much more structured in so far as he based his teaching on worksheets designed for self study. His approach to teaching was much more after the findings of Bruner (1966).

My style of teaching incorporated aspects from both Papert and Howe. Like the latter I deliberately set the children goals to achieve, although these goals often arose from their own declared needs. At the same time I used the children's natural tendency to learn, in other words I used the child's ability to exert control over his own activities. Arising out of this situation were what Stenhouse (1975) calls 'principles of procedure'. It implies that the goals centre around the process of learning, rather than around the product. Observations of the children indicated to me that where the children exercised control, their activities developed rationally in response to both mathematical and programming events. They developed their own algorithms for mathematics and for programming.

I was conscious of the external pressures operating in the teaching sessions (video camera and observer), yet I attempted to maintain enthusiasm through my perception of the whole teaching situation. The video tapes will give the

'reader' an opportunity to verify this. As this work proceeded I was pleased to see how the children became less dependent on me as 'the authority', which allowed them to evolve and act upon their own ideas. The subject matter was not seen as the possession of the teacher and we (Dr. Penny and I), were not seen by the pupils as having sole access to it. I saw my role as being sensitive to the pupils' reactions and interpretations, to value these, and to allow the pupils to develop the activity according to their own needs. This way I felt they gained control of the process of their own learning while I became part of the pupils' internal dialogue. Giving children the opportunity for conjecture, discussion and dilemma was for me the key to the pupils control over their learning. The observations by Dr. Penny and myself, suggested that the children were afforded this opportunity. With them saying,

"I wonder what will happen ...?"

How can we change this ...?"

If we do this, then"

they were defining their own problems.

This approach rejected the view that computer knowledge was absolute and objective, knowledge that was possessed by the teacher and therefore to be replicated in the pupils. Given real problems to tackle, the learning that took place and the knowledge gained was not a copy of the teacher's knowledge, but a reconstruction of it. The pupils' perspective of that knowledge was therefore their own.

The Observer's Perspective

Sitting at the back of a classroom an observer is often overwhelmed by information emerging from the teaching/learning situation. Sometimes the meaning of what is seen is not immediately apparent and therefore may be passed over. The observer cannot always be certain what is important or relevant at the time. When the situation observed is complex, ambiguous or emotionally charged, then it becomes more difficult for the observer. It is this dilemma that the observer (Dr. Penny) finds himself in and opens his description of the teaching sessions with this quotation from R.G. Collingwood (1951),

"I begin by observing that you cannot find out what a man means by simply studying his spoken or written statements, even though he has spoken or written with perfect command of language and perfectly truthful intention. In order to find out his meaning you must also know what the question was (a question in his own mind, and presumed by him to be in yours) to which the thing he has said or written was meant as an answer."

This is how the observer describes how he viewed the question:

"You begin with the premise that the teaching of mathematics needs improvement. It is unlikely that anyone would disagree. The trouble begins when we try to take action."

In attempting to take action we uncover ambiguities and diverse positions. Dr. Penny maintains that the need lies in changing mathematics teaching:

"An increasing demand that more people be able to respond to the mathematical aspects of their work and environment means that we (teachers) have to find ways of relating the present changing face of mathematics teaching to these demands. This is not to say that mathematics teaching must adapt to meet these demands. To suggest so would lead to a too ready acceptance of quickly offered solutions."

Perhaps the key question is this,

"Should we not be more critical of offered solutions and theory and more careful in our enquiry?"

Here Dr. Penny is on the horns of a dilemma. He attempts to answer his question on the one hand and yet on the other hand not fall into the trap of declaring that ready answers exist.

"Whilst you were trying to induct a group of children into a set of techniques, you were in my view, mainly concerned with the interpretation of what was happening and how one consequently acts."

He goes on to say,

"I am trying to avoid falling into the trap of thinking that there are answers ... the children initially thought there were such 'things' to be 'teased' out of you and me. We did not try to bluff them. Yet there is a problem here, that if one tries to be honest the result can be disheartening to those expecting help."

From his position as observer Dr. Penny commented on the total learning milieu. In doing so he focussed on significant events arising out of the situation. He describes his role in these terms,

"I did not see my task as that of achieving objectivity. I saw myself working within the diversity of situations to focus on emergent themes and areas of significance. I perceived my role as an additional person, distanced from the action, to whom the pupils and the teacher might speak in order to clarify what I thought was going on."

Looking at the whole learning environment the observer found the setting of significance when he wrote,

"The setting was yours; your beliefs, values about teaching and learning are embodied in it. My presence, however perceived by you and the children, did disturb the intricate network of interaction in that situation, as did the T.V. camera."

It is significant that Dr. Penny uses the word 'inspiration' and not 'motivation' in describing the teaching style, when he says,

"The setting, the given problems, the exciting change of situations form and medium, all provided **inspiration** (I am not a motivation fan). Each child was building on his/her own mathematics. It was related to his/her world.

I am looking at you the teacher. There is a brief exposition, a sharing of the problem, skilfully including a recap of previous work and an attempt to get the pupils to define their problem. Because it was difficult how can we know it was successful? There was the setting which encouraged discussion. You generated discussion, sharing etc., but sometimes the pairs were not listening to other

pairs' presentation because they were too involved in their own terms.

The children worked at their own speed yet were encouraged to follow your plan. But various avenues of work were open to them. The decision and consequences of their actions were immediately known to them."

The possession or control of knowledge was an issue that concerned both Dr. Penny and me (see previous section). I viewed the control of knowledge as an aspect of my own teaching method. This is how he saw it,

"I feel you were engaged in an enterprise aimed at:

- i). articulating the authority of experience - yours, Papert's etc. and
- ii) enabling the pupils to learn new ways of interpreting this situation.

You were also, on reflection, drawing attention to the inadequacy of any theory of learning and teaching which concerns itself only with the end product. All your actions embodied a theory which was largely idiosyncratic."

As regards his perceived role of observer, Dr. Penny acted as a 'mirror' to me. Although he started out as the non-participant observer, he became to the class a secondary source and confidante. His role as observer, critic and evaluator diminished as the programme ran its course. In the final analysis he viewed his part in the investigation in these terms:

"I saw myself there to offer legitimacy in an exercise which was fraught with difficulties."

7.3 Concluding Summary

The observer and I agreed that within the investigation the children utilized a range of concepts and skills.

7.3.1

In 'being mathematicians' (Papert) the children utilized such mathematical concepts as:

- estimation of turtle units and angles
- magnitude of angles
- interior/exterior angle relationship
- the sum of the exterior angles of a regular polygon is 360°
- similarity, same shape but different size
- symmetry, mirror image through reverse direction of angles

- curves made up of infinite line segments as in the algorithm for the circle
- functions with one and two variables
- properties related to construction of regular shapes
- developing mathematical algorithms

7.3.2

In working with the computer the children felt comfortable using the following programming techniques:

- typing LOGO commands
- transforming a prepared plan into a working program
- sequential procedure building
- editing and debugging
- developing programming algorithms

7.3.3

The socio-psychological aspects that emerged from the investigation were:

- the children became responsible for their own learning
- the children were in control of their own knowledge (the mathematics became personal)
- the turtle served as a metaphor for learning
- the children engaged in conjecture and generalization

7.3.4

Computer modelling as a teaching style involved the children in:

- articulating in precise terms how problems should be tackled
- investigating for themselves the mathematics concepts required for building a program.
- clear conceptualization of a problem if a program was to run as they intended it to

- expressing mathematical processes as computer programs which generated visual experiences.

I leave the final word to Winston when it comes to computers and problem solving:

"Answering questions about knowledge is the first step in learning to be an expert at any sort of problem solving. What kind of knowledge is involved? How should it be represented? How much is there? What knowledge specifically? Concepts uncovered to cope with these questions when creating computer problem solvers appear to be good for amplifying human intelligence as well."
(Winston 1979, p. 254)

7.4 Suggestions for Further Research

7.4.1

There is very little documented research evidence related to children first using the floor turtle, then moving on to use the screen turtle. Generally it is a case of using either the floor turtle or the screen turtle.

Having seen the children transfer with ease from the floor turtle to the screen turtle, I suggest it was because they thoroughly understood the techniques of procedure building. These techniques I suspect were established by means of the floor turtle. This is a hypothesis that needs exploring. If it was found that procedure building was better understood by primary children using a floor turtle, it would serve as a

useful guide to teachers who are considering introducing a computer into a school. If, on the other hand it was found that children find no difficulty using the screen turtle for procedure building, that too would be helpful information.

7.4.2

Another avenue for further research would be to conduct the same type of investigation described in this study, only over a longer period of time. It would be interesting to see the progress children made, as well as assess their attitude towards mathematics over an extended period.

7.4.3

From the class teacher's point of view an appraisal of the methods used to integrate computer technology into the normal school programme would indeed be useful. So much research to date has been of the laboratory-type, which excludes the difficulties of curriculum innovation. Very often the innovation is placed in the hands of overworked teachers. Further research in this area would provide the teacher with guidelines on how to introduce computer innovation into the classroom. This would be of practical value to the teacher.

7.4.4

It has been acknowledged that computer technology is likely to influence the lives of all people in South Africa in the future. We see signs of this happening now, not only in white schools but in black schools as well. It would be a worthwhile study to see if the computer could in some way bridge the cultural and knowledge gap between the developed and developing racial groups. Using turtle geometry might be just one method of doing it.

7.4.5

And what of research in teaching? This study provided an ethnographic model for classroom research. Using the procedures of ethnomethodological research incorporates the teacher's interpretations of what is occurring, with direct observation of events. Carefully implemented research of this type could generate descriptions of the distinctive nature of specific classrooms. This model of research has great potential for generating knowledge that is useful and relevant to teachers. The approach focuses on the total situation and might serve as a useful model for conducting research on how teachers view teaching. This model of research might provide teacher educationists with a method of improving initial or in-service training.

7.4.6

Of more direct mathematical relevance is the use of the turtle (both floor and screen) in the development of the function concept. Controlled educational experiments are fraught with difficulties. Nevertheless it would be interesting to make random selections from two groups comparable in mathematical ability and of the same sex at the Standard Seven level, and have them both taught a sequence of lessons on functions by the same teacher. One group could have enrichment via the turtle, emphasizing the "function" aspects. Although "one-variable" functions would be taught an exploratory sequence involving "two-variable" functions could also be tried, but not forced.

APPENDIX 1

TRANSCRIPT OF VIDEO-RECORDING (EDITED VERSION)

BETAMAX FORMAT

COUNTER READING

Start

End

000

TITLE: TURTLE TALK

002

Group round turtle table

003

The floor turtle

BUILDING A PROGRAM

004.5

042

Planning a program (Rhonda and
Sheilagh)

Debugging a program (Rhonda and
Sheilagh)

042

066

Turning though angles

066

110

Procedure for building a square (Karen
and Nicola)

Shortening the procedure (David R. and
Nicola)

110

120

Rhonda: procedure for equilateral
triangle

120

147

Loading pen into turtle

Trial run without pen

Turtle drawing equilateral triangle

<u>Start</u>	<u>End</u>	
		<u>USING REPETITION</u>
147.5	151	Rhonda and Sheilagh playing turtle
151	157	Turtle drawing flag (Rhonda and Sheilagh)
157	190	David R. explains repetition of flag on blackboard
190	202	Using circular protractors to measure the amount of rotation (Sheilagh)
202	205	Group discussion
		<u>COMBINING PROCEDURES</u>
205	213	David's (two) drawing of a hexagon David P.'s explanation of procedure
213	230	Discussion on building House (Nicola and Karen)
		<u>SCREEN TURTLE</u>
230	239	Building a house. Error (Karen and Nicola) Building a house. Second error (Karen and Nicola)
239	241	David's together. Rocket-building. Error
241		David's together. Rocket on side
	245	David's together. Rocket corrected
245	260	Discussion with Dr. Penny
261	271	Speeding up circle. Teacher's explanation

<u>Start</u>	<u>End</u>	
		<u>RECURSIVE PATTERNS</u>
271	281	Dauids together. Spinning the circle. Nicola. Spinning the circle
281	283	Rhonda and Sheilagh spinning the square

USING VARIABLES

283.5	300	Teacher's explanation of variable
300	304.5	Rhonda and Sheilagh. Variable size squares
304.5	316	Rhonda and Sheilagh. Variable size circles
316	319.5	Dauids together. Variable sides pattern
319	320	Group working

CHILDREN IN TURTLE GROUP

Nicola and Karen
Rhonda and Sheilagh
David R. and David P.

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