TEACHERS' USE OF AUTHENTIC TASKS THROUGH MATHEMATICS TRAILS IN A MOBILE LEARNING ENVIRONMENT TO FACILITATE CONCEPTUAL TEACHING.

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ABSTRACT

The teaching and learning of mathematics in Namibia is confronted by various forms of challenges that require continuous search for effective pedagogical strategies and approaches to enhance mathematical understanding. Some of the ways include using real and authentic outdoor activities and technological tools, such as smartphones, for teaching and learning purposes. The need to use authentic and realistic tasks in outdoor settings in the teaching of mathematics has strong support from the literature. Moreover, many recent reforms in education challenge teachers across all subjects to use modern and up-to-date technologies to complement and support existing approaches to teaching. Smartphones, in particular, offer new opportunities in the evolution of technology-enhanced learning by allowing teaching and learning to occur in authentic and realistic contexts that extend to real-life environments.

This qualitative case study proposes a practical framework that can facilitate mathematical understanding in teaching through the implementation of authentic and realistic outdoor tasks by using the Math City Map (MCM) project on a smartphone. The study aims to analyse and understand how mathematics teachers can create and implement authentic and realistic tasks in an outdoor mathematics trail to facilitate the conceptual teaching of area, volume, ratio and proportion topics, within the context of the Realistic Mathematics Education theory (RME). The study is framed within the RME theory and the iPAC (personalisation, authenticity, collaboration) mobile pedagogical framework. The research process is underpinned by an interpretivist paradigm. Data was collected from eight selected teachers through observations and interviews and analysed using frameworks derived from the RME and iPAC mobile pedagogical theories and the emergence of common themes.

The findings suggest that the integration of smartphones and mathematics trails have pedagogical benefits in mathematics teaching and can facilitate the use of outdoor tasks that are connected to learners' realities. The study argues that while MCM mathematics trail tasks can be difficult to create, it was worthwhile for teachers using them to conceptually teach the selected topics. It is therefore hoped that the findings of this study contribute towards the use of outdoor mathematics trails and smartphones in the teaching of mathematics.

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"I can do all things through Christ which strengtheneth me" Philippians 4:13

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DEDICATION

This thesis is dedicated to the most important people in my life:

my wife

Marcelina Katumbu Matengu,

who has been my constant source of love and support over the years,

my special children

Gideon Matengu, Fernanda Lumba and Marcelina Manga

You are my pride and joy, and I hope you will always remember that.

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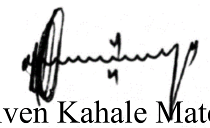
who gave me a good upbringing and made many sacrifices for me.

I will always cherish and be indebted to them.

DECLARATION OF ORIGINALITY

I, **Given Kahale Matengu**, student number **15M8765**, hereby declare that this thesis entitled “Teachers’ use of authentic tasks through mathematics trails in a mobile learning environment to facilitate conceptual teaching” is my own work, and a product of my research. It has not been submitted in any form to another institution. Where I have drawn on ideas of people from other publications or other sources, I have fully acknowledged these in accordance with Rhodes University, Education Department reference guide.



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ABBREVIATIONS AND ACRONYMS

AG	Agency sub construct
AP	Activity Principle
CC	Co-Creation subconstruct
CO	Conversations subcontract
CU	Customisation subconstruct
FGI	Focus Group Interview
GP	Guidance Principle
GPS	Global Positioning System
GRN	Government Republic of Namibia
ICT	Information Communications Technology
INP	Intertwined Principle
IP	Interactive principle
iPAC	Personalisation, Authenticity and Collaboration
LCE	Learner Centered Education
LP	Level Principle
MCM	Math City Map
MEAC	Ministry of Education, Arts and Culture
NCTM	National Council of Teachers of Mathematics
PSU	Problematic Smartphone Use
RAILING	Realistic, Activity, Interaction, Level, Intertwined, and Guidance principles
RI	Recall Interviews
RME	Realistic Mathematics Education
RP	Realistic Principle

SAARMSTE	Southern African Association for Research in Mathematics, Science and Technology Education
ST	Setting/Task sub construct
TO	Tools subconstruct
TR	Trail (eg TR1 means Trail 1)
UNESCO	United Nations Educational, Scientific and Cultural Organization
VITALmaths	Visual Technology for the Autonomous Learning of Mathematics
VR	Video Recording

CHAPTER 1

INTRODUCTION

The purpose of this chapter is to orient the reader to the background, context and goals of this research study, which focuses on how authentic/realistic tasks placed in mathematics trails can be used for conceptual teaching of area, volume, ratio and proportion topics. In this chapter I briefly address the theoretical underpinnings, methodology and significance of the research. The chapter ends with a brief outline of all the chapters as an overview of the study.

1.1 BACKGROUND AND CONTEXT OF THE STUDY

The use of mobile technologies in the teaching of many subjects is gaining traction globally and has attracted widespread research in this area over the years (Hsu et al., 2013). However, the question of how exactly mobile technologies can support the teaching of mathematics remains a concern. For example, Ludwig and Jablonski (2019), note that "with new technology in schools, the question has always been, how do teachers work with the new tools, how do they use them?" (p. 904). A contributing factor to this concern is what seems to be a paucity of research that explores teacher support and training in the use of mobile learning, (Ekanayake & Wishart, 2015), henceforth referred to as m-learning. Royle et al., (2014) echoed the same sentiments that "the place of mobile technology in education is still unclear and not enough instances of teacher development with mobiles are available to allow for analyses that inspire confidence" (p. 30). Hence, Baran (2014) and Kearney et al. (2019) recommend that more research is needed in this area to understand how conceptual teaching can be facilitated in m-learning environments.

In Namibia, even though the Information Communications Technology (ICT) National policy has been in existence for the past 24 years (Namibia: Ministry of Information, Communication and Technology [MICT], 2009), there has been no clear conceptual framework to guide the adoption of m-learning in secondary schools (Osakwe et al., 2017). From my experience over the past 17 years as a high school mathematics teacher, I have also taken note that the broad education curriculum in Namibia gives no direction on the use of m-learning technologies in the pedagogy of secondary school subjects. Further, I have observed with concern how mobile devices such as smartphones for example, have become a source of contention as to whether they should be allowed in school contexts or not (Osakwe et al., 2019; Narib, 2019). As a result, m-learning is seldom considered or used in teaching and learning contexts in Namibian schools.

Following the coronavirus (COVID-19) pandemic, remote learning has emerged as a new and novel way of learning, gradually replacing the traditional face-to-face methods (Daniel, 2020). Distance learning has become an inevitable reality, and thus Namibia and many other countries now have to implement m-learning in education. I argue that although m-learning has been ignored for a long time, it offers rich opportunities for innovative ways to deliver successful teaching and learning contexts in today's times and in the era of the post-COVID-19 pandemic. M-learning can provide collaborative learning mediums in which learners link up and interact with their classmates and teachers (Ozdamli, 2012; Karimi, 2016).

The MathCityMap (MCM) project is one of the more recent and unique digital apps that can be used as a medium of instruction in an m-learning environment when teaching mathematics. The award-winning project was developed in 2012 by the MATIS I Team from the Goethe University, Frankfurt, Germany (Cahyono et al., 2015). The project introduces new ideas of combining mathematical outdoor tasks, through trails, with mobile technology (Ludwig & Jesberg, 2015). Cahyono et al. (2015) add that "the use of digital technologies in m-learning environments has the potential to support teachers in facilitating outdoor mathematics teaching and learning processes" (p. 7). One of the benefits of using the MCM project as an m-learning tool is to place learning outdoors, which can provide opportunities for learners to experience mathematics in real world contexts.

On the other hand, it is widely accepted in the literature (eg, Uzunboylu & Ozdamli, 2011; Neilson & Campbell, 2017) that using realistic problem tasks to teach mathematics can enhance learners' conceptual understanding. Weiss et al. (2009), describe the mathematics that is rooted in real-world contexts as "authentic" mathematics. Vos (2018) considers mathematics tasks to be authentic if they have out-of-school origins and a "certification of provenance" (p. 8). Vos (2018) explains that in the context of school mathematics, the certification of provenance is realised when learners are aware of the importance of the task or activity to certain people or professions. Teaching in outdoor settings can assist learners to link difficult concepts they encounter inside their classroom to their everyday knowledge, which in turn can motivate them to appreciate the subject (Sugimoto et al., 2017). Baya'a and Daher (2009) argue that engaging learners in real world issues through authentic/realistic situations can enhance their understanding and application of mathematical ideas. Quitadamo and Brown (2001) support this view by stating that authentic situations and scenarios stimulate learners' learning. It excites and motivates them.

Notwithstanding these benefits, research (eg, Vos, 2011, 2015) shows that school mathematics seldom offers these types of tasks to learners. Teaching outdoors still appears to be an uncommon practice among mathematics teachers, and this has often led to learners' limited experience of how the mathematics learned in the classroom can resonate with the real world they live in (Tangney et al., 2010; Gijsbers et al., 2020). It is even more concerning to note that many secondary school learners do not see how mathematics relates to their lives (Gijsbers et al., 2020). This can have negative consequences because a perceived lack of relevance can lead to a loss of interest and even a complete withdrawal from the subject (Fitzmaurice et al., 2021). When mathematics teaching fails to create connections between what is learned in school and the real world, learners cannot see the importance of why they have to learn mathematics in the first place. Consequently, in such teaching contexts, learners can often simply lose interest in the subject and eventually resort to rote learning of mathematical formulas and procedures (Lauchande, 2001), without conceptually attaching meaning and understanding to what they are taught.

Kilpatrick et al. (2001) found conceptual understanding to be a key element of mathematical proficiency. Thus, the facilitation of conceptual teaching should be the fundamental goal of all instruction in mathematics if learners are to understand difficult and abstract concepts. Furthermore, it is proposed that mathematics instruction, in the context of developing conceptual understanding, forefronts visualisation. This promotes reasoning and enhances understanding of abstract concepts (Arcavi, 2003; Duval, 2014) and, this study argues, forms the basis of authentic/realistic tasks. Mudaly and Naidoo (2015), define visualisation as "the ability to form and negotiate a mental image necessary for problem-solving in mathematics" (p. 44). Thus, visualisation can stimulate and enable the human mind to "see the unseen", both in the apparent and hidden objects, trends and patterns of the world around us (Rudziewicz et al., 2017).

My approach in this research study is to introduce, to a group of eight selected secondary school mathematics teachers, mathematics trail ideas and explore how they can use the realistic tasks placed in the trails to teach the selected topics for conceptual understanding. These realistic tasks were all designed within the context of the MCM math trail project: see: https://mathcitymap.eu/en/tag/project/ (Ludwig & Jablonski, 2019).

1.2 RESEARCH GOALS AND QUESTIONS

The goal of this research study was to analyse and understand how eight selected mathematics teachers can implement authentic tasks in a mathematics trail to facilitate the teaching of area, volume, ratio and proportion for conceptual understanding.

Main research question

In pursuance of the above goal, the study was guided by the following main research question:

In the context of a mobile learning environment, how can teachers implement authentic tasks in a mathematics trail for conceptual understanding of selected mathematics topics?

Sub-research questions

The main research question was then broken into the following sub-research questions:

1. In the context of running mathematics trails and solving the MCM project tasks, in what different ways do selected teachers make use of outdoor authentic tasks for conceptual teaching of area, volume, ratio and proportion topics?
2. What are the selected teachers' experiences and perceptions on the design and implementation of mathematics trails using smartphones within the MCM platform?

It was thus the purpose of this research study to propose a practical framework that could facilitate conceptual teaching through the implementation of authentic and realistic outdoor tasks by using the MCM project. The study aims to analyse how the use of authentic mathematics tasks in an m-learning environment can facilitate the conceptual teaching of area, volume, ratio and proportion. The study's theoretical framework is underpinned by the Realistic Mathematics Education (RME) theory and the iPAC (Personalisation, Authenticity, Collaboration) mobile pedagogical framework.

1.3 THEORETICAL FRAMEWORK

This research work is underpinned by the Realistic Mathematics Education (RME) theory and the iPAC mobile pedagogical framework. RME is a domain-specific instruction theory for mathematics that was first introduced and developed by the Freudenthal Institute in the Netherlands in the early 1970s (Freudenthal, 1973). The RME movement argued that for conceptual understanding to take place, mathematics should be taught in such a way as to become useful for solving everyday life problems. So, at the heart of this theory is the notion that mathematics is a human activity and that in order to be of human value, the subject must be connected to reality, be close to learners and relevant to their experiences of society

(Gravemeijer, 1994). One of the determining characteristics of the RME theory is an approach that foregrounds realistic contexts. These contexts can come from the real world, the fantasy world or the formal world of mathematics, as long as they are meaningful and experientially real for learners. RME aims to help learners develop their own mathematical tools and procedures through a process of mathematisation, which involves moving from the informal to formal and from the concrete to abstract or vice-versa. The theory also emphasises the importance of social interaction and reflection in learning mathematics (van den Heuvel-Panhuizen & Drijvers, 2014).

The RME theory resonates well with the ideas of mathematics trails and authentic tasks used in this study; for both aim to help learners learn mathematics by engaging in realistic problem situations that they can imagine and relate to in their own lives. In RME, there are six principles that should guide the design and implementation of mathematics learning activities, namely the *reality principle* (RP), the *activity principle* (AP), the *interactivity principle* (IP), the *level principle* (LP), the *intertwinement principle* (INP) and the *guidance principle* (GP) (Van den Heuvel-Panhuizen, 2001). These principles were used, in the study, to develop a hybrid analytical tool for analysing the data on the different ways the selected teachers made use of the outdoor authentic/realistic tasks for conceptual understanding of the selected topics.

The iPAC mobile pedagogical framework is a theoretical model that identifies the distinctive pedagogical features of m-learning (Kearney et al., 2012; Kearney & Maher, 2013; 2019). The acronym iPAC stands for Personalisation, Authenticity and Collaboration, which are the three signature pedagogies of m-learning. Central to the framework is the concept of 'time-space' which influences the way learners experience the three distinctive pedagogical features in m-learning environments (Kearney & Maher, 2019). This means that time and space are not fixed or static, but rather dynamic and flexible. In other words, learners can choose when and where they want to learn and how they want to learn. This allows them to personalise their learning experience according to their preferences, needs and goals. Therefore, in this study, when designing m-learning environments, teachers were guided by the three pedagogical features of the iPAC framework. I use the framework to analyse and interrogate the participants' m-learning experiences and perceptions of their engagement with learners in formal and informal settings and schedules (or time-space configurations) (Nortvig, 2014; Kearney & Maher, 2019).

1.4 METHODOLOGY

This research study is oriented within the interpretive paradigm. Cohen et al. (2007) assert that the interpretive paradigm can help us "to understand the subjective world of human experience" (p. 21). Hence in this research study I use the interpretive paradigm to understand the selected teachers' lived actions and perceptions of using authentic/realistic tasks placed in mathematics trails to teach the topics of area, volume, ratio and proportion. This approach allowed me to enter the selected participants' worlds to understand their actions and experiences in this regard. The qualitative method of data collection and analysis was used. A cohort of eight mathematics teachers from three different secondary schools (three from school A and C, and two from School B) were purposively and conveniently selected to create realistic mathematics trail tasks that they used to teach the concepts of area, volume, ratio and proportion outdoors, within the RME theoretical framework.

The research design of the study consisted of three cycles that were designed and contextualised within the MCM project ideas. A total of 38 realistic tasks on the topics of spatial measurements, ratio and proportion spread across eight different mathematics trail routes were created for the purpose of this study (see Appendix Ten). Each of the eight teachers who took part in this study led a group of six to eight Grade 9 learners to walk, locate and solve tasks in the trails created for the MCM app project at their respective schools. All three cycles were sequenced in the following five phases:

Phase 1: Creation of the tasks for the MCM project

Phase 2: Submission of tasks to the MCM web portal for review

Phase 3: Implementation of the mathematics trails (video recordings)

Phase 4: Reflective Interview sessions

Phase 5: Reflection – Focus Group Interviews (FGIs)

The collection of data took place from Phases 3 – 5. The data generation methods used were observations, semi-structured reflective interviews (RI) and FGIs. The observations and interviews were video recorded and audio taped, then transcribed for analysis. In Phase 3 of each cycle, the trail activities of one teacher from each of the three schools were video recorded. The videos were aimed at capturing the different ways in which the teachers used the MCM realistic tasks to teach the selected topics within the context of the RME teaching principles. In Phase 4, semi-structured interviews in the form of reflective interviews (RIs) were conducted and audio recorded to solicit the teacher's experiences of the design of the authentic/realistic tasks for the MCM app project and how these tasks were used by the learners (using smartphones) to locate and solve the outdoor tasks. In the fifth phase, the three teachers (only two in Cycle 2) whose trail activities from each school were video recorded, came together and reflected on the whole process of creating the tasks and on how their learners solved the same tasks. This was done in the form of FGIs to generate more data on the teachers' experiences of using realistic tasks to teach outdoors. In total, there were 21 data sources presented and analysed for the purpose of this study and these were comprised of eight video recordings (VRs), eight RIs and three FGIs. For ethical reasons, pseudonyms were used and the names of participating teachers and their respective schools were withheld to ensure anonymity.

1.5 SIGNIFICANCE OF THE STUDY

This section demonstrates how this research study contributes to the existing knowledge and practice in the field of mathematics education. According to Marshall and Rossman (1999), the significance of any qualitative study should convince the reader about its links with "important theoretical perspectives, policy issues, concerns of practice, or persistent social issues that affect people's everyday lives" (p. 34). The significance of this study's contribution lies in its contribution to aspects of theoretical perspectives, policy issues and concerns with practice.

On the theoretical perspectives, Fabian et al. (2016) express concern that the lack of studies that link pedagogical theories to technology-based learning has caused a "gap in terms of discussion on how mobile technologies support the learning process in mathematics" (p. 97). Therefore, this study aims to address the gap in global research that explores teacher support and training in the use of m-learning technologies in secondary school mathematics (Baran, 2014; Ekanayake & Wishart, 2015; Kearney & Maher, 2019).

On a practical note, this study is significant in the sense that its findings will contribute to reducing the gap that exists in bridging formal classroom mathematics and the mathematics that learners encounter in the real world (Sawaya & Putnam, 2015). It is worthwhile noting, as some studies in this line of research have maintained (eg, Hakadiva-Vatileni, 2016; Vos, 2011, 2015), that the use of authentic/realistic outdoor tasks that are connected to real life situations is seldom practiced in secondary school mathematics, both in Namibia and beyond. Thus, this study has the potential to make a novel contribution to the area of how secondary school mathematics teachers can make use of authentic/realistic outdoor tasks that are connected to real life situations in the teaching of certain mathematics topics, using the RME teaching principles. Furthermore, the study is also significant because of its intended purpose to introduce the MCM app to selected secondary school teachers in Namibia, a move that can strengthen and encourage the use of mobile technologies, particularly that of smartphones, as well as outdoor mathematics trail activities, in the teaching of secondary school mathematics.

The significance of this study on policy issues is focused on its contribution to the development and adoption of an m-learning framework in Namibian secondary schools, which seems to be absent at present. Osakwe et al. (2017) reveal that there is a lack, or invisible use, of mobile technologies such as smartphones in the teaching of Namibian secondary school mathematics. Therefore, it is hoped that the findings of this study will inform policies and practices, primarily in the Namibian context, on the best ways mobile devices such as smartphones can be used in teaching and learning contexts of mathematics education.

1.6 THE STRUCTURE OF THE THESIS

This thesis consists of seven chapters and the remaining six are succinctly outlined below:

Chapter 2 – Literature review

This chapter offers a review of relevant literature and introduces the conceptual framework that underpins my research study. The study's conceptual framework rests on four main pillars: m-learning technologies, mathematics trails, authentic/realistic tasks and visualisation in mathematics education. Other additional conceptual constructs that inform this study are also explored and these are spatial measurements, ratio and proportion.

Chapter 3 – Theoretical framework

This chapter introduces the theoretical underpinning of this study. The study is positioned within the RME theory and iPAC mobile pedagogical framework. In the chapter I discuss key theoretical notions such as the RME teaching principles, mathematisation, personalisation, authenticity and collaboration. In the discussion I systematically highlight how these theoretical underpinnings align with the conceptual and methodological approaches of the study.

Chapter 4 – Methodology

In this chapter, I discuss and justify in detail the research paradigm and choice of methodological approaches of this study. In the chapter I explain how the research was conducted; starting with how the data was collected, analysed and presented. In addition, the validity and reliability of the study as well the ethical protocols observed are discussed. I end the chapter by highlighting the challenges encountered during the research process.

Chapters 5 and 6 – Data analysis and presentations

In Chapters 5 and 6, I present an analysis of the data findings to provide answers for the main research question of this study. Chapter 5 addresses the findings to the first sub-question which was about how the teachers used the MCM tasks to teach the selected topics of area, volume, ratio and proportion, within the RME theory. Chapter 6 is based on the findings of the second sub-research question that focused on the experiences and perceptions of the participating teachers' design and implementation of outdoor trails using smartphones, with their learners within the MCM platform. Data was vertically and horizontally analysed and categorised into predetermined themes of the six RME teaching principles and the six sub-constructs of the iPAC mobile pedagogical framework. Other emerging themes were identified to seek further insights to the answering of the main research question. These were visualisation aspects in the study and the teachers' views on the challenges of the MCM app project and teaching outdoors.

Chapter 7 – Conclusion and recommendations

In this last chapter of my study, I consolidate the findings of the study with reference to the original research question, within the contexts of the conceptual and methodological frameworks. I further interrogate both the implications, limitations and significance of the study. I draw final conclusions and make recommendations for improvement of practice as well as avenues for further research.

CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

As a high school mathematics teacher for 17 years, I have struggled to understand and apply the ideas of m-learning technologies in my teaching practice. Although education authorities urge teachers to use the latest technology in their teaching, many of us do not know how to implement it effectively. This is also true for authentic tasks in mathematics education. One of the rationales of the Namibian Mathematics Junior Secondary School Syllabus is that “the study of mathematics at the junior secondary level contributes to the learner’s ability to think logically, work systematically and accurately and *solve real-world problems*” (Ministry of Education, Arts and Culture [MEAC], 2015, p. 1). Still, these recommendations are rarely implemented in many mathematics classrooms.

Therefore, this chapter introduces and discusses the key concepts of this study and reviews literature relevant to them. The concepts include m-learning technologies in education, the Math City Map (MCM) app project, authentic tasks, mathematics trails, visualisation and teaching for conceptual understanding. The chapter ends with the discussion of concepts related to spatial measurements ratio and proportion. Although I support the use of m-learning (especially smartphones) in the context of creating and solving authentic tasks based on the selected topics using the MCM project in my literature review, I am also aware of the challenges that come with this move, hence a critical review of this literature from my part.

2.2 MOBILE LEARNING TECHNOLOGY IN MATHEMATICS EDUCATION

There has recently been a considerable body of research that examines the current reforms of use of technology and its effects in teaching and learning contexts (Schuck & Maher, 2018; Chowdhury, 2020; Hanafizadeh et al., 2019). At the forefront of this reform is the development of new mobile technologies, which has become a young and popular research field in education (Milošević et al., 2015; Kearney & Maher, 2019). Some studies (eg Pate, 2016) suggest that this trend reflects a ‘new professionalism’ that enhances learning experiences in academia. Therefore, “it is important that teachers become accustomed to, and familiar with the use of technology as a resource in mathematics to improve the quality of teaching and learning”

(Venketsamy & Wilson, 2020, p. 169), as this will prepare and empower them to meet the changing needs of learners.

Yet, notwithstanding increased advances in technology and the amount of research invested in unearthing the potentialities of mobile technologies in education, research in mathematics education on this topic is still limited (Beatty & Geiger, 2010; Borba et al., 2016). There is sufficient evidence to support that m-learning in schools has not yet yielded substantial improvement in the instruction of many subjects because of its poor acceptance from teachers (Şad & Goktas, 2014; Kafyulilo, 2014; Thomas et al., 2014; Ozdamli & Uzunboylu, 2015). Teachers are still oblivious of the fact that in today's world, mobile devices, particularly smartphones, have become part of our daily living and that almost everyone (including learners) now has access to them (Barbasa & Vale, 2020). Ostensibly, this has contributed to a lack of appropriate pedagogical models and strategies for m-learning in mathematics education.

Therefore, the successful adoption of m-learning in schools depends much on the teachers' willingness and acceptance of incorporating this type of technology in their teaching practice and this can only be possible if teachers are aware of the concrete and ongoing benefits of m-learning (MacCallum, 2010). As such, MacCallum and Jeffrey (2013) believe that "the adoption of mobile technology will largely depend on whether students and educators believe that mobile technology fits their particular needs" (p. 303). Likewise, schoolteachers as agents of change are crucial role players in the teaching and learning process and their acceptance, understanding and utilisation of m-learning technologies in their everyday teaching can guide the learners' evolution from personal to mathematical meanings (Lieban & Lavicza, 2019a; 2019b).

2.3 DEFINING MOBILE LEARNING

Mobile learning (m-learning), just like online learning, e-learning, or homeschooling, is a branch of distance education (Bozkurt, 2019), and the United Nations Educational, Scientific and Cultural Organisation [UNESCO] (2002, p. 22) defines distance education as

> any educational process in which all or most of the teaching is conducted by someone removed in space and/or time from the learner, with the effect that all or most of the communication between teachers and learners is through an artificial medium, either electronic or print

Defining m-learning is contested as the term can be described in numerous ways (Coyle et al., 2007), meaning different things to different people (Cowan & Butler, 2013). However, despite the ambiguity of the term, the definition of m-learning seems to hang on two key elements: the type of learning that uses mobile devices to access learning (Sharples, 2007; Osakwe et al., 2017; Schuck & Maher, 2018) and learning that revolves "around the mobility of the learner'' (Pachler et al., 2010, p. 6). A review of the combined works of Sharples et al. and their colleagues reveals a strong line of research into conceptualising m-learning. Their early research focused on the device as a key factor for enabling m-learning (Sharples, 2000; Sharples et al., 2002) and its potential for supporting lifelong learning. However, they soon realised that the mobility of the learner was more important than the device. This led them to consider m-learning from the learner's perspective, and to define it as:

> any sort of learning that happens when the learner is not at a fixed, predetermined location, or learning that happens when the learner takes advantage of the learning opportunities offered by mobile technologies. (O'Malley et al., 2003, p. 6)

This definition, however, does not exactly match with the idea of the project used in this study, because the learning environment in the MCM-project is predetermined (Ludwig & Jesberg, 2015).

Therefore, in the context of my study, I prefer to adopt the definition proposed by Park (2011) and Crompton (2013) which describes m-learning as a process of learning that happens across multiple contexts, through social and content interactions, using personal electronic devices. This definition is an evolution of the one from the works of Sharples and colleagues. Three key aspects need further discussion in this definition, and these are (1) learning that happens *across multiple contexts*, (2) through *social* and *content interactions*, and (3) using *personal electronic devices*.

The learning that happens *across multiple contexts* in this case can refer to both fixed and predetermined locations. In the current age of personal and technical mobility, learning as it was traditionally known, is now taking an approach towards mobility and more visibility in the context of physical space and technology (Sharples et al., 2009). Contrary to traditional ideas that have always confined learning to fixed locations, bounded times and teacher-based, m-learning is broadening the horizon of learning to contexts of learning taking place anywhere, anytime and by anyone using mobile devices (Khan et al., 2015; Sharples et al., 2019). According to Hanif et al. (2018), m-learning "provides an opportunity to change the existing classical learning strategies and offer students much more flexible ways of managing their learning experience" (p. 644). This means the use of m-learning allows learning to happen in any location and at any convenient time, which in turn reduces the number of constraints of static spaces such as school, classroom and home, where learning has traditionally been confined to (Milošević et al., 2015).

The definition also supports the premise that learning is a *social* activity where learners interact, share knowledge and negotiate meaning with others in a democratic learning environment (Vygotsky, 1978). To this effect, m-learning can create platforms where learners share and engage in meaningful learning that helps them to reflect on their understanding, refine their knowledge and have a shared understanding of the content (Baharom, 2013). These platforms can further supplement and even replace traditional face-to-face teaching and bring about learning methods that lead to "an expansion of the spaces and times of learning, with students learning outside the places of formal education and the hours of formal timetables" (Pegrum et al., 2013, p. 67).

Furthermore, my preferred definition of m-learning (Park 2011; Crompton, 2013) emphasises the use of *personal electronic devices* as facilitators of the learning process. Mobile devices are electronic devices that are mobile and portable; meaning they are easy to carry and can be used anywhere and at any time (Keengwe et al., 2014; Osakwe et al., 2017). Studies identify electronic devices such as smartphones, cellphones, laptop and notebook computers, tablet PCs and Personal Digital Assistants (PDAs) as suitable for the use of m-learning purposes because of their portability and mobility (Marwan et al., 2013; Traxler, 2007; Wali et al., 2008; Abidin et al., 2015; Osakwe et al., 2017). The portability of these devices provide flexibility in learning at any given time and place (Sarrab et al., 2012). Moreover, their use in educational contexts has been found to facilitate effective teaching as well as help learners access learning materials in a more collaborative and interactive way (Eng et al., 2016; UNESCO, 2012a). Also, the devices develop learners' creativity and innovation of ideas on specific subject matter (Pachler et al., 2010).

Research findings (eg, Sandberg et al., 2011; Karimi, 2016) identify the benefits of m-learning as learning that can take place everywhere and any time, thus allowing learners to participate actively in collaborative learning environments (Nassuora, 2012; Ozdamli, 2012). Rather than being passive recipients of knowledge, m-learning has also been found to support rich, blended and dynamic learner-centered educational learning environments in which learners become active and interactive (Thomas et al., 2013). As a result, meaningful learning is achieved because learners learn at their best cognitive ability (Bonnici et al., 2016). Additionally, systematic reviews of the literature indicate a tendency to use mobile devices to support contextualised outdoor learning in mathematics and reveal more opportunities for learning outdoors (eg, Crompton, 2013; Crompton & Burke, 2015: 2018; Crompton et al., 2019).

One other advantage of m-learning is that it can be used as an alternative to replace textbooks and worksheets in the form of mobile e-books (Saputra et al., 2018). An e-book is an "electronic representation of a book" (Garrish, 2011, p. 1). Textbooks and worksheets have been widely and predominantly used in the teaching and learning of many subjects, including mathematics. However, with the advancement of technology, some learners tend to find textbooks and worksheets less practical and more boring (Saputra et al, 2018). Moreover, a previous study by McLean and Kulo (2013) found that the textbooks' relatively large size makes them less preferred over the much smaller, faster, cheaper and interconnected digital devices such as smartphones, for example. Therefore, the adaptation of textbooks to follow technological progress is an easier way of learning. According to Suputra et al. (2018), m-learning in the form of e-books follows the idea of a pocketbook; whereas in this case instead of a small-printed book, a mobile device can be used to help learners store information or to access such information from the internet. Nowadays mobile devices are fully integrated in our daily lives and in the lives of learners from a very young age and consequently, it is an excellent opportunity to use the devices for the learning purposes.,

2.3.1 Mobile technology-smartphones

A wealth of literature studies has documented that the technology embedded in m-learning offers a new dimension in the evolution of technology-enhanced learning by allowing learning to occur in realistic contexts and extend to real environments (Silander & Rytkönen, 2005; Kurti et al., 2008; among others). Compared to other mobile devises, the rapid growth of smartphones in terms of availability, popularity, and user friendliness among people (particularly the youth) throughout the world (Franklin, 2011; Hossain & Quinn, 2013; Bouhnik et al., 2014), undoubtedly makes them to be one of the best choices for m-learning implementation. According to Rashid and Elder (2009), smartphones "only require basic literacy" (p. 1) and this makes them easy to operate. Their diverse functions have made them an indispensable part of life worldwide (Amez & Baert, 2020).

Research on the use of smartphones in mathematics suggests that the devices can extend mathematical thinking and enhance problem-solving procedures (Tangney et al., 2010; Subramanya & Farahani, 2012) as well offer an exploration of mathematics in the real-world contexts (Kearney & Maher, 2013; Sincuba & John, 2017). The devices have also been found to create new and significant opportunities of linking the learning to out-of-school and other meaningful contexts (Sawaya & Putnam, 2015; Cahyono, 2017). According to Greefrath and Siller (2017), "digital tools [including smartphones] can be of great assistance for teachers and learners, particularly in connection with real-world problems" (p. 530). Sung et al. (2016), add that in mathematics learners can learn better and access important information in different places and in a more appealing way by using mobile devices, which are becoming a powerful tool for both indoor and outdoor education. Further, smartphones enable the presentation of mathematical concepts through creating settings where learners engage with the subject in meaningful ways (Jung & Conderman, 2013; Lew & Jeong, 2014). According to Parsons (2014a), because the use of smartphones in the learning of mathematics enhances authentic real-world and cultural settings, this can in turn create opportunities for learners to personalise and have autonomy over their learning.

More findings from studies undertaken within the VITALmaths project (Visual Technology for the Autonomous Learning of Mathematics) consistently show that the use of smartphones increases learners' participation and self-motivation towards mathematical tasks as learners can download mathematics-related video clips on their own and with ease (Ndafenongo, 2011; Hyde, 2011; Kangwa & Schäfer, 2019). In addition, Genossar et al. (2008) also suggest that mobile environments can enhance experiential learning by not only making dynamic mathematical applications more accessible, but also by enabling learners to perform tasks that are more relevant and connected to their experiences.

2.3.1.1 Challenges associated with the use of smartphones in school contexts

The use of smartphones for school teaching and learning purposes is not without its challenges. Teachers appear to be unmotivated with the use of smartphones as pedagogical tools. This is because alongside opportunities, the use of mobile technologies such as smartphones in educational contexts can also present challenges. It is generally perceived that smartphones disrupt teaching and learning processes, and this has led to their limited use in school setups (Thomas et al., 2013). The challenges associated with smartphone use in secondary schools include knowledge, moral and legal related issues. According to Ozdamli and Uzunboylu (2015) teachers are still not in favour of, or are reluctant to adopt this technology in their teaching, because of inadequate training in the use of mobile phones. Kafyulilo (2014) and Thomas et al. (2014) claim that teachers fear that learners may use the devices in inappropriate ways. Common occurrences of disruptions such as ring tones, texting, playing games and the use of social media platforms during class time are reported to be among the inappropriate use of smartphones among learners. I refer to the literature for more detail: Trilling and Fadel (2009) and Yen et al. (2009).

Furthermore, ethical concerns such as cheating, sexting, harassment and cyberbullying are among the commonly cited reasons why the use of smartphones is seen as inappropriate in school setups (Keengwe et al., 2012; Dyson et al., 2013; Thomas et al., 2013). These concerns infringe on people's rights and result in legal actions. While these risks may sometimes happen outside the school environment, the results can be detrimental to the social and academic development of learners (Keengwe et al., 2014). To ignore these ethical concerns would be wrong; however, it would also be unwise to deter teachers and educators from exploring and adopting m-learning ideas as the pedagogical benefits seem to outweigh the involved risks (Dyson et al., 2013; Hinduja & Patchin, 2011). Thus, a calculated and careful implementation of m-learning can potentially limit or avoid the possible risks.

Furthermore, current research (Amez & Baert, 2020; Vezzoli et al., 2021) suggests that the overuse of smartphones can create mental health, social, interpersonal and academic issues. Particularly, spending too much time on social media can lead to the overreliance on the devices. Lepp et al. (2014), report that the excessive use of smartphones can affect learners' academic performance. This is because access to the internet leads to easy access to social media platforms such as Facebook and WhatsApp applications and these platforms can disrupt learning, especially if used during school hours.

More recently, Zhou and colleagues have used the phrase "Problematic Smartphone Use" (PSU) to describe the misuse and overuse of smartphones that may lead to health problems such as addiction, depression, obsessions, compulsions, dependence and lack of sleep (Zhou et al., 2022). The researchers add that the disadvantages resulting from PSU oftentimes affects the academic performance of learners and more worrisome, is the fact that only a "few studies explore the relationship between PSU and subject-specific performance such as mathematical achievement" (Zhou et al., 2022. p. 2). Because of this, PSU is one of the most cited reasons why many schools do not allow the use of smartphones in Namibia. The argument has always been that learners' misuse and overuse of smartphones affects their academic performance.

Cook. et al. (2011) state that the transformative power of smartphones in changing traditional teaching practices, in addition to their role in the transition to more mobile and dynamic learning environments is not recognised. From the teachers' perspective, the use of smartphones is time consuming, especially when learning how to use them effectively in their respective subjects (Battista, 2008). Perhaps the predominant reason why there is limited to no use of smartphones in many schools is because teachers do not see the value of using the devices in teaching and learning contexts. To reiterate, the argument I am forwarding here is that teachers need to recognise the potential of smartphones for learning purposes, not simply for social or recreational use. I draw this from a claim advanced by some study sources (eg, Royle & Hadfield 2012; Royle et al., 2014) that it is challenging to recognise that digital devices that are widely used by people for social or leisure purposes can also be used for educational purposes.

Another challenge advanced by Varanasi et al. (2020) is that many state governments and schools' leadership are worried that smartphones will distract teachers from their duties, work outside of work hours and increase stress and anxiety. I argue that this could be the reason why the MEAC in Namibia appears not to be present and therefore silent, on the issue of using smartphones in schools in this country. This is despite the ongoing current global debate on the matter. I discuss this in detail in the following section below. Therefore, awareness workshops and training on empowering teachers to learn and understand how to use the devices to foreground the underpinnings of teaching pedagogies and assist in delivery of the curriculum content is necessary (Koehler et al., 2007; Royle et al., 2014).

2.3.1.2 The use of mobile devices in Namibian mathematics classrooms

As far as the review of literature on m-learning in Namibia is concerned, only a few existing studies have focused on the promotion of using smartphones in secondary school contexts (eg, Ngololo, 2012; Osakwe et al., 2017; Osakwe et al., 2019). This is an indication that there is a scarcity of research that presents the educational benefits of smartphone use in secondary school subjects, including mathematics. Osakwe et al. (2017) and Osakwe et al. (2019) reveal that there is a negative stance on the use of smartphones in Namibian schools. In a similar argument that has emerged, Muyoyeta (2018), observes that the use of smartphones in Namibian schools is hindered by the view that it promotes unethical and unruly behaviour among learners, contributing to their failure rate.

To date, the issue of mobile phones in schools has been a long-standing debate that has sparked a divide of opinions within the education sector in the country (see media reports Ekongo, 2010; Haidula, 2015; Narib, 2019). In response, many schools have reacted to this debate by formulating internal policies that ban the possession and use of mobile phones on school grounds (Osakwe et al., 2017). It is concerning to note that a growing number of literatures shows that the education of many other developing African countries such as Rwanda, Nigeria, Zambia, Ghana, Cameroon and Sierra Leone, for example, also do not allow the use of mobile phones in schools (Kangwa, 2016; Muhideen et al., 2019). The devices are generally perceived as a distraction to teaching and learning and as having no particular significance or notable contribution to teaching and learning.

It is noted that many schools have banned the use of mobile phones in Namibia. However, I argue that the move to restrict the use of mobile devices in schools is a misinformed one, and thus can be challenged. There is sound evidence from literature pointing out that m-learning technologies can enhance teaching and learning when correctly used. For example, Muhideen et al. (2019) postulate that m-learning is currently an extremely promising technology as a learning tool, thus preventing the use of mobile devices such as smartphones in schools for fear of possible misapplication is a misplaced move in this digital age. In their study on the use of mobile phones in schools Baya'a and Daher (2009) recommend that:

> in spite of the disruption that these devices could cause in the classrooms, we believe that banning them from schools is not the solution. We should keep studying the pedagogy behind the use of mobile phones in the actual educational environment and develop appropriate activities that utilize these devices efficiently and profitably in the learning process. (p. 13)

Other proponents of using smartphones in schools (eg, Chen & Tzeng, 2010; Philip & Garcia, 2015; Osakwe et al., 2017) also argue that despite the associated challenges, if well regulated, the use of smartphones can yield a significant impact on teaching and learning. As already argued, smartphones have transformative benefits of changing traditional teaching practices, as well as play a key role in the transition to more mobile and dynamic learning environments. Therefore, allowing the use of smartphones in schools may easily lead to the integration of m-learning, which in turn can convert schools and classrooms into playful learning environments where learners are motivated to engage and learn unconsciously (Kangas et al., 2017).

As alluded to earlier, perhaps the predominant reason why some secondary schools have banned the use of smartphones could be a result of a lack of policy frameworks that guide the implementation of m-learning in the country. As the custodian of education in the country, the MEAC seems not to be doing enough in directing the use of mobile phones in schools. I say so because despite the current ongoing debate about using smartphones in school setups, the government has currently not clearly positioned itself on the matter. I therefore argue that there is a need for a government policy and action plan to support the implementation of m-learning in Namibian schools.

M-learning in schools can only be successful if the government supports its implementation (UNESCO, 2012b). As stated in Muhideen et al. (2019) and Osakwe et al. (2017), a lack of government policy in developing countries hinders the development, application and exploration of m-learning in their education. Further, Khan et al. (2015) hypothesise that "the fruitful results of m-learning can be obtained through the mutual understanding, cooperation and partnership between stake holders, involvement of private companies in the education sector" (p. 916). Clearly, these literature sources strongly suggest that there is a need for the involvement of stakeholders, particularly the Government Republic of Namibia (GRN) through the MEAC and teachers, to promote the implementation and application of m-learning in Namibian secondary schools. Thus, it is crucial that relevant policies are put in place to guide the implementation and embedding of m-learning in teaching and learning contexts (Borrás & Edquist, 2013).

Today, the famous words of Gates (2009), spoken over a decade ago, still ring true: "[t]he world of education is the sector of the economy so far, the least changed by technology. Ten years from now that won't be the case". In addition, Milošević et al. (2015) also predicted that in future m-learning will become mandatory and necessary to cope with the modern way of learning and be able to keep pace with the times and technology. Fascinatingly enough, today these assertions have become evident. Consider for example, the global COVID-19 pandemic that made it difficult to continue with normal teaching and learning as it used to be. As a precaution to reduce the chances of humans infecting each other with the virus and slow down its spread, learning institutions including schools, strived to limit or do away with face-to-face teaching methods by maximising the use of online learning platforms (Bozkurt, 2019; Pozo et al., 2021). To ensure the continuity of education, learning institutions had to revisit their mode of teaching and learning approaches and opted for online platforms rather than the complete closure of schools and universities (World Bank, 2020). So, in agreement with the above assertions, this study attempts to offer opportunities for Namibian school teachers and beyond, to improve and enhance their teaching strategies using the MCM project within the m-learning context.

2.3.2 Mobile learning−mathematics trail

2.3.2.1 The context in m-learning

Context can be defined as "the formal or informal setting in which a situation occurs; it can include many aspects or dimensions, such as environment, social activity, goals or tasks of groups and individuals; time" (Brown et al., 2010, p. 4). Other studies (eg, Borasi, 1986) defined a context as "a situation in which a problem is embedded", and its role is to "provide a problem solver with the information that may enable the solution of the problem" (p. 129). The term context is found to be used in two ways; *context of tasks* and *contexts of learning environment* – for example in or outside the classroom (Wedege, 1999; Sullivan et al., 2003). The context of the task refers to "the choice of the situation in which the mathematics is embedded" and environment is used to mean "the learning environment in which the task is used" (Sullivan et al., 2003, p. 118). Sealey and Noyes (2010) concluded that mathematics should be presented in some *context*, be purposeful and enable learners to acquire awareness of the important role of mathematics in society.

Considering the above selected definitions of context, Brown (2010) and Sharples (2016: 2019) posits that when learning becomes mobile, location becomes an important context in terms of the physical whereabouts of the learners. Hence, the mobility of learners and the types of tasks they engage in should benefit them, be it in or outside the classroom. Apart from creating good learning contexts, most research on the topic contends that teaching mathematics in outdoor settings can also offer teachers new ways of introducing new mathematical concepts (Nilsson et al., 2009) and attract new teaching methods that can help learners engage in meaningful, functional, and creative ways of learning (Uzunboylu & Ozdamli, 2011).

Notwithstanding the potential benefits that lie in teaching mathematics outdoors, teachers still find it difficult to find meaningful approaches that can allow them to teach mathematics in outdoor contexts. Consequently, learners are rarely exposed to the real world around them, which in turn presents difficulties for them to decontextualise real life situations or problems (Tangney et al., 2010). Hence a need for awareness projects such as the MCM app project on how to teach outdoors. This study explores a mathematics trail within the MCM project as one of the approaches to teaching and learning mathematics outside classroom contexts (Ludwig & Jesberg, 2015).

2.3.2.2 What is a mathematics trail?

Mathematics trails are outdoor walks where one can discover and solve mathematical problems about real objects in their environment (Shoaf et al., 2004). Thus, a typical mathematics trail is a planned walking trail or path that contains four to eight outdoor tasks at specific stops or stations (Vale et al., 2019), where each stop contains at least one problem to solve. The tasks in mathematics trails are designed in such a way that they can only be solved by being at the spot where the task is, because to solve them one has to obtain data from the actual object(s) in the task, for example, actual measurements and dimensions. So, in this way, trails can be used for educational purposes by organising them at (or nearby) schools for learners to experience mathematics in the real world. Subsequently, learners' understanding of their living environment can deepen through recognising forms, shapes, mathematical connections and the use of numbers in everyday life situations.

2.3.2.3 Background and nature of mathematics trails

Mathematics trails were first introduced in Australia in the early 80s and were originally created for family vacation activities to tour the city of Melbourne, through a sequence of mathematical problems connected to interesting objects and places within the city (Blane & Clark, 1984). The trails were designed in such a way that the participants could encounter problems that helped them to discover the mathematics embedded within the environment (Baumann-Wehner et al., 2020). As families and visitors toured the city through mathematics trails, they got to see hidden and invisible mathematics, and even learned more about the historical places and architecture found within the city. In addition to the idea of unearthing the mathematics hidden within the environment, mathematics trails were also created for the purpose of popularising mathematics. Popularising mathematics is viewed as "sharing of mathematics with a wider public" and encouraging people to be more active mathematically (Howson & Kahane, 1990). The popularising of mathematics in society can lead to mathematics literacy; a literacy that is described as the "knowledge to know and apply basic mathematics in our everyday living" (Ojose, 2011, p. 90).

Studies confirm that learners have a negative attitude towards mathematics and that the subject is not the most popular topic among adults either (Kollosche, 2018). It is common knowledge that many adults have a phobia about mathematics and that they neither like mathematics, nor understand its purpose within the school curricula or the after-school life in general (Ojose 2011; Barbosa & Vale, 2016). Thus, through the mathematics trail initiative, people (including learners) can walk and analyse, through a 'mathematical eye', their community, connecting some of its details with exploration tasks and research in mathematics education. This in turn, can popularise mathematics as a discipline and inculcate a positive attitude towards the subject.

Originally, mathematics trails were not created for teaching purposes, but to "popularize mathematics in society" (Jablonski et al., 2018, p. 115). However, the idea was adopted over time in teaching setups whereby schools took advantage of the existing trails and integrated them into their mathematics learning programmes (Blane, 1989). The success of this idea followed the adaptation and application of the programme in different locations, whereby the trail idea later appeared in several cities (Cahyono & Ludwig, 2019). Therefore, mathematics trails exemplify a worldwide collection of projects that aim to popularise mathematics through out-of-school activities. The provision of doing mathematics in out of school contexts, extends the time spent thinking about mathematics and mathematics problems. Mathematics trails have been increasingly recognised as adjuncts to improving mathematics education in the schools, and tend to connect back into school (Blane, 1989).

2.3.2.4 Mathematics trails as teaching and learning tools.

The learning in mathematics trails is oriented within the context of outdoor activities, and according to Fabian et al. (2016), exploring outdoor tasks can become an adventurous, interesting and an easy way of teaching, learning and experiencing mathematics. Malone (2008) adds that learning outside the classroom encompasses the "opportunities initiated by teachers and/or students to engage with alternative learning settings to complement and/or supplement the formal indoor classroom curricula" (p. 7). According to Trafton et al. (2001), this kind of learning can help students to be more interested and involved in mathematics and to develop a positive and realistic view of mathematics as a useful discipline.

Barbosa and Vale (2020) assert that "math trails have great potential for making more visible the connections between mathematics and everyday life, specifically the environment that surrounds us" (p. 47). What this means for mathematics teachers is that the trail idea can be used to expose learners to their living environments and help them discover certain places with mathematical perspectives (Lavicza et al., 2020). Cahyono (2017) asserts that "taking mathematics outside the classroom allows learners to experience mathematisation processes in solving problems around them and not only in textbooks" (p. 51). In a similar sense, Gainsburg (2008) and Wernet (2015) argue that teachers and learners do not consider traditional textbooks as a reliable source for meaningful connections to 'real-world' mathematics, thus rending them as less important. This line of argument is corroborated by Ludwig et al. (2020), who also see mathematics trails as having the potential to provide authentic and relevant mathematical problems to learners, in contrast to mathematics tasks in schoolbooks.

Richardson (2004) emphasises that because they occur outside of the classroom, mathematics trails can create an atmosphere of *adventure* and *exploration* and, at the same time, give learners the opportunity to solve and pose problems. Learners can also be afforded the opportunity of making use of the knowledge learned in the classroom to answer questions encountered in the trails and this can allow them to realise the applicability of mathematics in real life (Bonotto, 2010; Caldeira et al., 2020). In other words, as they engage in trail activities, learners gradually become more aware and more attentive to the mathematics that surrounds them in everyday life and most importantly, how this mathematics relates to what they learn in the classroom (Barbosa & Vale, 2016).

Furthermore, Barbosa and Vale (2020) envisage that "math trails make mathematics come alive engaging the participants cognitively, emotionally and physically, which is why they can be associated with active learning" (p. 51). In other words, with trail activities, mathematics steps out of the textbook and into the learners' real lives. So, learners benefit from outdoor activities by leaving the classroom and walking around to do tasks cooperatively in teams (Zender & Ludwig, 2016). Leaving the classroom and exploring their community with mathematical eyes can be a very motivating experience for the learners (see Figure 2.1 below). Hannula (2006) defined motivation as a potential to direct behaviour that is built into the system that controls emotion. In this regard, Cooper and Dunne (2000) noted that "children will find mathematics more interesting and relevant to their concerns if techniques of calculations are taught in the context of consumption, work, or at least via textual representation of such contexts" (p. 2)

Figure 2.1: A diagrammatic representation of the learner’s engagement in a mathematics trail. Adopted from Zender & Ludwig (2016)

Empirical studies conducted in Germany (Zender, 2019) and Indonesia (Cahyono, 2018) found that mathematics trails have a positive impact on learners’ performance. Moreover, in the context of mathematics education in schools, mathematics trails offer great opportunities for the application of mathematics in real, authentic situations, as well as the modelling of activities that precede calculations (Gurjanow et al., 2019). Furthermore, aspects of cognitive, physical and social engagement stand out as some of the greatest benefits that learners get from mathematics trails (Hannaford, 2005). According to Vale and Barbosa (2018) and Barbosa and Vale (2020), the interaction between these three dimensions as facilitated by mathematics trails brings about active learning, which in turn, can promote positive attitudes towards the learning of mathematics.

Mathematics trails can be created anywhere; however, Cahyono (2018) advises that “when determining a math trail location, developers should consider several things, such as safety, local culture, and history” (p. 56), for this information adds value to the educational attributes of the trail. Furthermore, to avoid boredom among learners when walking the trails and solving the tasks, Shoaf et al. (2004) suggest making “the problems independent of one another so that trail walkers will be encouraged by a fresh problem at each stop" (p. 12). Also, care should be given to the distance between the tasks, the time it will take to solve each task and the kinds of tasks or problems, as they affect the length of the entire trail and the participants’ interest in the activity (Cahyono, 2018).

2.3.2.5 Challenges associated with the creation and running of mathematics trails.

Mathematics trails can be designed and customised in lots of ways, so it is up to the teacher to decide what fits best for his or her learners. However, designing tasks for the mathematics trail has been found to be challenging. Ludwig et al. (2020) point out that the preparation, organisation, and implementation of the tasks can be a cumbersome and time-consuming process for teachers. Of course, this should not come as a surprise, because Smith et al. (2008)

pointed out that "mathematical tasks that give students the opportunity to use reasoning skills while thinking, are the most difficult for teachers to implement well" (p. 132). Likewise, the mathematics involved in the MCM tasks, the degree of challenge or the diversity of the nature of the task is always difficult to determine, particularly for teachers inexperienced in this teaching method (Cahyono, 2018; Barbosa & Vale, 2020).

Nevertheless, by means of the digital tool of the MCM app and website, these challenges can be mitigated. For example, when creating a task on the website, one can tag relevant keywords that indicate the mathematics knowledge involved in the task – eg, measure, count, geometry and so on. The task can also be assigned a grade level (see Table 2.1) to help determine the degree of challenge of the task. For instance, a task assigned to a Grade 11 level means that it will likely be too difficult for a Grade 9 learner. Moreover, the nature of the tasks involves estimating and measuring variables, calculating and comparing sizes, areas and volumes and solving problems, thus carrying out essential elements of mathematising and mathematical modelling (Buchholtz, 2017). This can easily be done when creating a task in the MCM project in the option of how the answers can be given.

Furthermore, Baumann-Wehner et al. (2020) raise two concerns on the running of trails, saying that firstly, it is always difficult for the teacher to divide his or her time to help all learners at the same time, due to the spatial separation of the tasks within the trail and this results in not helping learners who get stuck with some questions. Secondly, sometimes the tasks are challenging to such an extent that slow and struggling learners become overwhelmed, making their outdoor experience a negative one (Baumann-Wehner et al., 2020; Edelmann & Wittmann, 2012). To mitigate the challenge of not having to attend to all learners during the trail, Ancochea and Cárdenas (2020) recommend that learners be organised in small and manageable groups of four to six learners to allow collaboration (between learners) as well as interaction with the teacher. On the other hand Canyhono (2017) says that the feature of hints in the MCM app can help learners who may not have an idea of what to do when solving the problem tasks.

2.4 THE MATH CITY MAP (MCM) PROJECT

The MCM-project is a relatively new digital m-learning initiative that combines mathematics trail ideas with technology (computers, internet & smartphones). The project has the mathematics trail concept at its core and offers an educational mobile app-supported mathematical guide that directs learners or any other interested persons to find and solve the tasks outdoors. An educational app is a web-based or downloadable software application that typically works on a mobile device and is designed to support learning (Bouck et al., 2016; Papadakis et al., 2017: 2021). The MCM app is an open-source tool that can be accessed and used by anyone for free, and its operations are easy to understand (Gurjanow et al., 2017). Thus, with the MCM app, learners or users get to locate and solve mathematical problems that are connected to hidden real-life objects using Global Positioning System (GPS) enabled electronic devices such as smartphones (Ludwig & Jesberg, 2015). The use of GPS technology in particular is crucial to the practical implementation of the MCM project as the idea is for the GPS function to pinpoint the geographical location of the hidden tasks in the trail. Clough (2010) appraised mobile technologies that use GPS to support collaborative informal learning focused on location. In this regard, Osakwe et al. (2017) voiced the need for collaborative learning using mobile technology in Namibian high schools, which according to him, is long overdue.

Further, mobile end devices with their location-independence are ideal for extra-curricular learning environments and create a mediating context between the physical and the social environment, for example when multimedia and interactive apps such as the MCM app are used (Buchholtz, 2023). Thus, smartphones within the context of the MCM project can also offer a marked continuity of learning experience across different learning contexts, and with the availability of GPS-technology, learning can be extended to places where learners are motivated to solve interesting mathematics problems (Kearney & Maher, 2013; Sincuba & John, 2017).

The MCM app project is one of the many mathematics applications that has been recently developed. Kay and Kwak (2018) reveal that there are roughly 750 000 free and fee-based apps that are available in the domain of education, and most of these apps are found on popular stores such as Google play and Apple's App Store. However, educational researchers bemoan the fact that some of these apps have no guarantee of educational value (Larking, 2014: 2015; Papadakis et al., 2017; 2020; Kay & Kwa, 2018). Therefore, this makes it difficult and even overwhelming to identify and select appropriate apps that can be conceptually used in this regard (McGarth, 2013; Alon et al., 2015; Bouck et al., 2016; Papadakis et al., 2017).

Moyer-Packenham et al. (2015) assert that "an important goal for mathematics education is the design and selection of mathematics 'apps'" (p. 42). Particularly, the question of alignment between mathematics apps and the curriculum is critical to the selection of the right ones to be used (Larking, 2014). So, echoing Calder's (2015) question of "what is the [major] motivation of app designers?" (p. 236), one can also ask about the motive behind the creation and design of the MCM app project in the first place. A further expansion of this question can also lead to questions such as how reliable and worthwhile is the app for mathematics learning? Does the app help learners to conceptually understand the subject (deep understanding related to the meaning of mathematics) and do its procedural sequential steps help learners to solve mathematics problems? (Larkin, 2014; 2015).

In response to the above questions, several studies have attempted to address the challenge associated with the selection of appropriate mathematics apps by developing metrics that classify the apps in four different categories, namely practice-based, constructive, productive and game-based metrics (Grandgenett et al., 2011; Handal et al., 2016; Alon et al., 2015; Ebner 2015). In relation to this, I notice that the MCM can be classified to fit all these four categories. For example, the practice-based metric in an app helps learners to learn content and apply specific mathematics skills (Handal et al., 2016; Ebner 2015). The MCM app is designed for the purpose of meaningfully helping learners learn specific mathematics concepts related to varying topics such as geometry, statistics and probability, algebra and arithmetic (Cahyono & Ludwig, 2019). Moreover, one of the criteria of the MCM app when uploading the tasks on the portal is to digitally attach a relevant topic that features a connection to school mathematics topics (Jablonski et al., 2018), (refer to Table 2.1).

The constructive app metric category focuses on exploration (Handal et al., 2016; Murray & Olcese, 2011), making sense of new information (Grandgenett et al., 2011), skill acquisition and data management (Domingo & Gargante, 2016) as well as the active manipulation of ideas and concepts (Keengwe 2013). In the same manner, the MCM app uses GPS technology to help guide learners on their mathematics trails, and in the process, learners explore the mathematics around them, take appropriate measurements and make sense of the available information through the mathematisation process, to solve the tasks (Ludwig & Jesberg, 2015; Zender & Ludwig, 2016; Cahyono & Ludwig, 2019). In other words, the MCM app project allows learners to actively engage with their environment and different mathematics concepts when walking the trails and solving the tasks.

Kokotsaki et al. (2016) declare that the use of productive apps is grounded in project-based learning theory. Project-based learning is a type of learning that engages learners in complex and real-world tasks where learning is autonomous, constructive, investigative, collaborative and communicative, reflecting real-world practices (Blumenfeld et al., 2000). By their very nature, MCM tasks are connected to real life objects and situations, thus reflecting the aspect of real and authentic mathematical life situations.

Lastly, the game-based category describes the application of a game element in apps, while at the same time involving the learning of specific mathematics concepts (Riconscente, 2013; Ebner 2015; Kay & Kwak, 2018). Empirical studies within the MCM project have demonstrated how game-based the MCM app is. Among them is Gurjanow et al. (2017), who say that the MCM app project is modified to look game-like in order to interest and intrinsically motivate learners to work on the mathematics trails. The game elements of the MCM app involve feedback on the user's action like points, levels, leader board badges and quests (Zichermann & Cunningham, 2011). According to Lembrér (2013), the social interaction aspect in playing games have the potential to enable learners to create mathematical knowledge.

Furthermore, Gurjanow et al. (2017) assert that with the MCM app, to prevent guessing when uploading answers on the mobile app interface, learners are awarded points when giving the correct answer on their first attempt. To this effect, Barbosa and Vale (2020) add that the gamification feature in the MCM app is a game changer that encourages learners to use greater care to finding the correct answers in order to gain more points. Moreover, the hiding of the tasks within the surrounding area of learners' environment and the process of finding these tasks through the guidance of GPS technology using smartphones, evokes a feel of the 'hide and seek game' when walking the trails.

The MCM project uses supporting and technical output tools of *mobile phone application* (mobile app) and a *website portal* to digitalise mathematics trails. The smartphone app is designed for the navigation and running of trails whilst the web portal is used for teachers and users to create the tasks and trails. The core feature of the app is to provide access to tasks and trails which were created on the portal, while on the other hand the web portal creates a community of users who interact and share mathematics trail work – see Figure 2.2 below (Baumann-Wehner et al., 2020). Therefore, establishing an online community of practice for mathematics trailers and potential users who share and use trails is another idea of the project (Gurjanow et al., 2020).

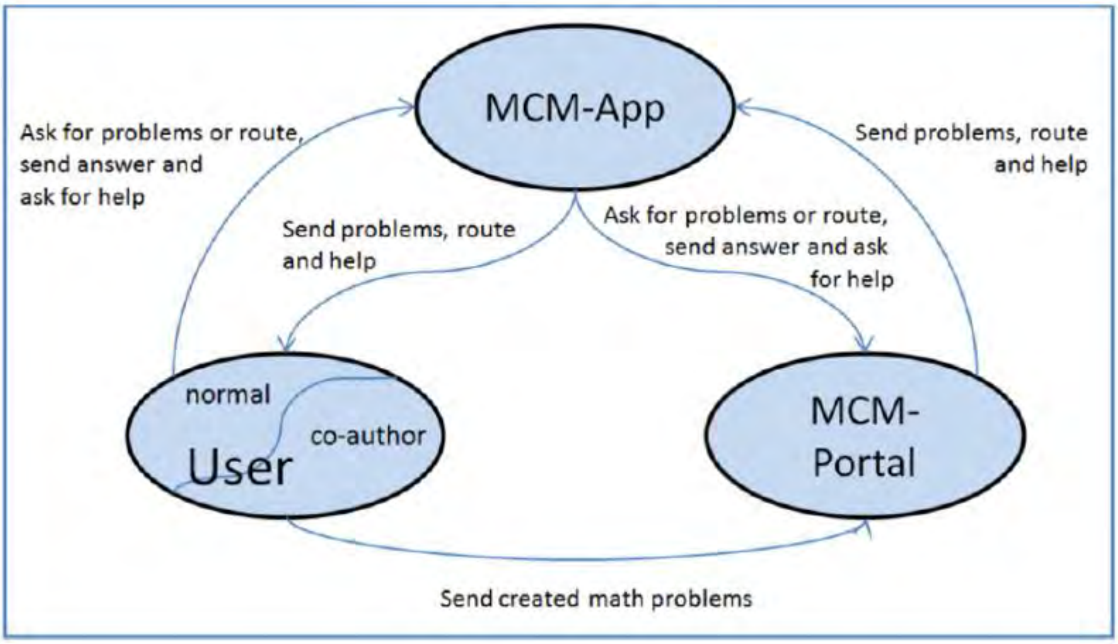
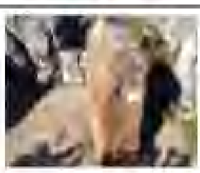


Figure 2.2: The technical structure of the MCM-Project
Source: Ludwig & Jesberg, 2015, p. 2779

Jablonski et al. (2018) comment that "in the MCM web portal, one can access published MCM tasks and trails, spotted in many different countries all over the world" (p. 117). Real world and authentic questions such as "*How many bricks make up this wall*?" or "*What is the height of this pole*?" can be encountered along the path of a mathematics trail within the MCM project. Consider, for example, the image of the Table Mountain monument from a South African trail in Cape Town that appears in Figure 2.3 below, illustrating the featured question (calculating the mass of the stone monument). Clearly, the designer of this task included the picture as a way to identify with precision the location of the task. Moreover, when solving this task, one needs to be at the site of the object and take necessary measurements, thus this presents the users with opportunities to explore the mathematics in their environments.



Task: Table Mountain's Monument (Cape Town, South Africa)

Definition of the task: Calculate the mass of the stone monument. Give the result in kg. 1 cm^3 of granite weighs 2,6 g.

Figure 2.3: Image of the MCM task on Table Mountain's Monument
Source: Jablonski et al. 2018

2.4.1 The background

The current review of the MCM web portal (www.mathcitymap.eu) indicates that as of 28 November 2023, the project had 25400 users who created 73399 tasks along 15285 trails in 93 countries. The tasks are an integration of different mathematics topics. The project is mainly active in Europe, Asia and South America, with few tasks in Africa and other parts of the world. South Africa has 572 tasks and 153 trails, while Namibia has 91 tasks and 16 trails. The project was introduced at Rhodes University, South Africa in 2017 and later in Namibia. However, the project has not been fully implemented in Africa yet. There is only one published study by Shimakeleni (2021) that investigated the MCM project in Namibia, and a few ongoing studies in South Africa and Namibia, including this one.

2.4.2 Technical implementation of the MCM project

2.4.2.1 Steps for preparation

Following the guidelines provided by the MCM project catalogue of criteria (refer to Table 2.1), teachers or users can create tasks that are linked to several locations within the learners' reach (school or town). The tasks are then connected to the appropriate GPS coordinates and uploaded to the MCM database through the portal website. Information such as grade level, tools required to solve the task (eg calculator, measuring tape), a visual picture or photograph of the object of the task and stepped hints to help learners solve the tasks, if need be, can be added. When creating a task, hints are an integral part of the criteria of good questions, for they guide learners to solve the problem on their own with the assistance of the app (Cahyono et al., 2015). Further, a PDF with multiple solutions to the tasks should also be entered in the portal (Cahyono, 2017). According to Baumann-Wehner et al. (2020), in the MCM portal a trail becomes a collection of existing digital tasks that can be viewed privately or publicly. The uploading of tasks to the portal is subject to review by the project's assessors, and depending on the outcome and recommendations, the tasks can finally be made available for public viewing on the MCM website.

The public viewing of tasks on the website means everyone, including learners, can view the tasks anywhere and anytime using smartphones, and can further be directed to the specific location of the tasks. When solving the tasks, the app provides learners with a sample solution that can be viewed immediately or after failing to get the correct answer on at least the sixth attempt. This way, learners can always complete a task with a correct solution, even if they were unable to find the solution themselves (Baumann-Wehner et al., 2020). Thus, when creating the task, it is a must for authors to provide the sample solution to the question.

Table 2.1: Catalogue of criteria to be followed when creating tasks in the MCM program.

Adopted from Jablonski, et al. (2018)

CRITERIA	
Uniqueness	The task should include a picture/image which will make it easier to identify the object of the task in question and what the task is about.
Attendance	The task should be *authentic* by connecting it to real life objects. Therefore, the question should not provide enough information to solve the task in the classroom, but at the location of the object.
Activity	Learners should be engaged and actively involved when solving the task. In other words, the task should make learners busy, eg by measuring, counting and calculating.
Multiple solutions	The task should be solved in multiple ways through the choice of mathematical modelling.
Reality	The task should be connected to real life objects. It should have meaningful relevance and not appear too artificial to learners.
Hints	Every task should provide at least one hint in terms of solving the task. Jesberg and Ludwig (2012) argue that stepped aids have a positive impact on learning performance, experience and communication.
School math and tags	The task should feature a connection to school math. Therefore, one can use tags with relevant key words and assign them to a grade.
Solution formats	The solution should be representable in one of the solution formats provided by MCM: interval, exact value and multiple choice. Especially for modelling tasks, the interval seems particularly relevant as it ensures that learners refrain from minor deviations in the solution, as through measuring differences or different mathematical models. In this format, one defines a green interval for correct solutions and an orange interval for incorrect, but acceptable ones. Solution values that do not fit into these intervals receive negative feedback and the player is asked to retry.
Tools	The task should be solved without special and extraordinary tools apart from calculator, measuring tape etc.
Sample solution	One should provide a sample solution including measured data (only visible in the portal and in the solution PDF) for teachers, in order to talk about the tasks in the following lessons and analyse typical errors.

2.4.2.2 Settings of the activity

Supporting tools such as the MCM mobile phone app can then be used by learners or users to follow the trail and trace the hidden outdoor tasks using GPS-enabled smartphones or a Google snapshot of the map, and solve the mathematical problems encountered along the path (Cahyono, 2017). The application works on Android and Apple smartphones and can be downloaded from the portal, Google Play Store or Apple stores. Alternatively, the author of the task can capture the map of the trail from Google Maps and upload it to the portal for the users to follow (mostly offline) when tracing the tasks. This study opted to use the GPS function over the printed map for the purpose of exposing learners to this latest technology that exists in smartphones. The app, through GPS coordinates, guides learners to the exact location of each task. The types of tools needed to solve the tasks are also shown. When arriving at the site, learners can then access the question, work out the problem, upload the answer and get feedback to the answer, right on the spot (Cahyono, 2017).

2.4.3 The potential benefits of using the MCM project in Namibian secondary schools.

The idea behind the MCM project is inspired by the mathematics trail as an innovative approach to teach outdoor mathematics supported by technology. As earlier discussed in Section 2.3.2.5 of this review chapter, in the past, it was rather challenging to create mathematics trails. Normally, tasks were manually put together into a trail guide which also contained a map overview and a title page of the trail, but this was however, found to be a time-consuming process (Gurjanow et al., 2020). So, with the emergence of the MCM app, tasks can now be easily connected to each other remotely in order to create digital mathematics trails (Baumann-Wehner et al., 2020).

Furthermore, the MCM approach encourages and motivates learners to be actively involved in outdoor mathematics activities, which in turn creates a constructivist learning environment. Barbasa and Vale (2020), report findings from a study that involved 48 participants who used the MCM app to do a mathematics trail in the city centre of Viana do Castelo, Portugal. The results of the study show that 60% of the participants did not know any digital resource they could use to explore mathematics outdoors. The results of the study indicated that most of the participants did not know of any digital resources for exploring mathematics outdoors. This may reflect the situation of Namibian teachers as well. However, the MCM project offers an innovative resource strategy for teachers who want to use smartphones to engage their learners in outdoor mathematics activities.

The MCM project has been reported to have a positive impact in supporting teachers and learners in the process of teaching and learning mathematics outside the classroom (Cahyono & Ludwig, 2019; Ludwig & Jablonski, 2019). According to Cahyono (2017), in the MCM project, learners interact with each other and with their environment during the process of solving the tasks, and by so doing learners are given the opportunity to construct their own mathematical knowledge in the context of real-life settings.

The interactions and collaborations that happen when solving tasks within the MCM project have a positive impact on the learning of mathematics, particularly if done within the context of a constructivist view of learning. According to Barbosa and Vale (2016), meaningful teaching consists of adopting an exploratory teaching approach that promotes the conditions for learners to construct their own knowledge, hence the MCM app was designed for that very purpose. Constructivism emphasises learner-centered, learner-directed, teacher-scaffolded and authentic tasks (Dagar & Yadav, 2016). Thus, one characteristic of constructivist teaching is teaching that features a learner-centred approach (Weimer, 2002; 2013). According to Albin et al. (2020), despite calls from existing instructional leadership documents that support learner-centred teaching in Namibia (including the Classroom Observation Instrument, the Learner Centered Education (LCE) policy and the National Policy Guide for the Secondary Phase), teachers still struggle to implement LCE in their teaching practice.

As Nyambe (2008) and Albin et al. (2020) observe, the implementation of LCE in Namibia has been difficult. This challenge is also experienced in other developing sub-Saharan countries such as South Africa, Botswana, Malawi, Uganda and many more (Chisholm & Leyendecker 2008; Schweisfurth, 2011; 2013). In fact, the implementation of LCE has been found to be a global challenge (du Plessis, 2020) and because many teachers do not understand it, they fail to apply its ideologies in their teaching practice (Kapenda, 2008; Schweisfurth, 2013; Albin et al., 2020). So, the adoption of the MCM project ideas in some Namibian secondary schools may potentially help mathematics teachers understand the LCE theory and even enable them to translate the concept into practice. There are studies that show that learners' behaviour when using smartphones fosters LCE, which in turn can enhance their cognitive ability to undertake inquiry-based and discovery learning (eg, Pegrum et al., 2013; Kabanda & Brown, 2017). Thus, when implemented within the Namibian context the MCM project can trigger teachers' trajectories of transforming their practice from teacher-controlled approaches to helping learners achieve autonomy in learning (Albin et al., 2020).

2.4.4 The review process of the MCM project tasks

The MCM project has a rigorous review process that ensures tasks meet certain standards and that quality is guaranteed before publication (Gurjanow et al., 2020). This review process is identified as an important feature of the project in terms of the quality of tasks and the professional development of teachers as task designers. The review process is composed of four steps (see Figure 2.4 below), where the first step is the creation of the task in the portal. By default, only registered users or authors are allowed to create and upload tasks. The second step is the initiative of the direction lights that give feedback on the technical quality of the task. The lights include three primary colours which are red, yellow and green. If the task shows a red or yellow light for one or more of the criteria, then it means that a specific criterion needs improvement for the task to be submitted. The third step involves the submission of the task to the reviewer team and only when all the task criteria are marked green, can a task be considered ready for submission (Jablonski et al., 2018).

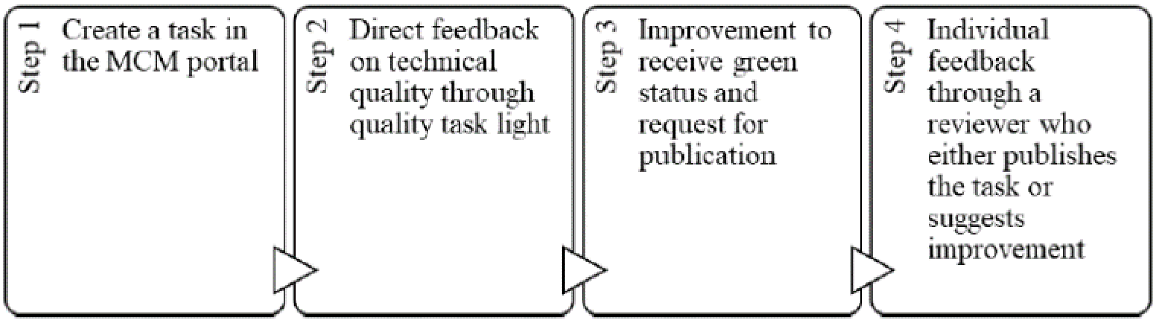


Figure 2.4: The MCM review process in four steps
Source: Jablonski et al. 2018

In the final step, a qualified reviewer evaluates the task and verifies its suitability for inclusion in the MCM database and public display on the website (Jablonski et al., 2018). Depending on how the reviewer assesses the task in line with the outlined MCM task criteria, the task will either be accepted or rejected with recommendations for improvement (Baumann-Wehner et al., 2020). The author of the task is then notified on the outcome of the review process via email. After working on the recommendations to improve the task the author can again request publication by resending the improved task for reviewing. So, because of the strict criteria in place, half of all created tasks in the MCM project have not yet been published, and this is a guarantee that only the appropriate and uncompromised tasks that correspond to the mathematics trail idea and the MCM concept are published and shared with the wider community (Jablonski et al., 2018).

2.4.5 The cognitive demand of the MCM tasks

Zender and Ludwig (2016) point out that what makes the tasks in mathematics trails stand out compared to that of schoolbook tasks is their level of challenge. A good task is viewed as a task whose cognitive demand aligns and challenges the learners' level of thinking. The cognitive demand of a task is described as the "amount of effort a student needs to expend to think about a problem, and mathematical tasks are categorized in two levels, low and high cognitive demand" (Candela, 2016, p. 315). The cognitive demand of a task is further categorised into four different sublevels, namely memorisation tasks, procedures without connections tasks, procedures with connections tasks and doing mathematics tasks (Stein & Smith, 1998; Smith & Stein, 1998). The memorising and carrying out of procedures without connections to concepts or meaning are under the category of low cognitive demand tasks while procedures with connections to concepts and doing mathematics are characterised as high demand tasks (Stein & Lane, 1996; Kessler et al., 2015).

The MCM app technology enhances the quality of tasks created under the guidance of its criteria and offers new opportunities that attempt to enable teachers to design challenging and relevant mathematics trail contextualised tasks for their learners (Lavicza et al., 2020). Regarding the cognitive demand of the MCM tasks, Barbosa and Vale (2016; 2020), emphasise that the tasks in the MCM app are highly demanding tasks because a mathematics trail is an exploratory type of learning that actively engages learners when solving the tasks. The physical movement, cognitive challenge and social interaction in a mathematics trail demands the cognitive and intellectual involvement of learners. Furthermore, the design of these tasks requires solid grounding in mathematical content, knowledge and experience (Cahyono et al., 2015).

Of course, it cannot be said that MCM project tasks are deliberately made difficult for learners to solve in order to test their intellectual capacity, for this would defeat the purpose of the cognitive aspect of tasks. As per Codreanu et al.'s (2020) observation, MCM tasks are designed according to the learners' level of thinking, hence one of the criteria is for the task to be linked to a specific grade level (refer to Table 2.1). This is meant to safeguard the balance between the cognitive demand of the tasks and the level of the learners' ability to solve the tasks. The lower the grade level the less cognitively demanding the task is, and vice versa. So, with the MCM tasks, it is important to consider the balance between authenticity and the level of cognitive demand, while at the same time striving to allow learners to link the practical task to the required conceptual knowledge.

Pioneers of task design (Doyle, 1983; Hiebert & Wearne, 1993) stated that high cognitively demanding tasks should always draw learners' attention to specific concepts as well as provide certain information surrounding the concept. The physical and social engagements that happen in the MCM trail tasks promote active learning, which in turn helps learners to cognitively cope with the tasks on the trail (Shoaf et al., 2004; Hannaford, 2005). Also, the fact that MCM tasks are real, life-based problems means they cannot be solved by memorisation but instead by the application of mathematisation processes whereby learners cognitively think between the real world and mathematical world when solving the tasks (Ludwig & Jesberg, 2015; Zender & Ludwig, 2016; Cahyono & Ludwig, 2019).

One other important principle of highly demanding tasks is that they require learners to access relevant knowledge and experiences and make appropriate use of them when working through the tasks (Bauer & Prenzel, 2012). Similarly, when solving MCM trail tasks learners need to make use of prior knowledge and appropriate connections for them to understand and make sense of the problem. When creating the tasks one of the criteria is that the tasks engage learners actively and they should be solved in multiple ways (see Table 2.1). In other words, the task should keep learners busy by means of taking measurements, counting, calculating and so on.

2.4.6 Criticisms of MCM and dissenting views

The implementation of the MCM app project comes with its own challenges. Firstly, it appears that the app has not yet achieved its intended purpose of improving the traditional method of creating trails. According to Gurjanow et al. (2017), although trails can now be created using the latest technology, the process itself remains a challenge for task designers; for more time and effort are still needed to create the tasks for the trails. Secondly, when used within the Namibian context, the MCM app project can become an added workload for teachers. Local studies (eg Amutenya, 2016; Shavuka, 2020) indicate that within the new revised curriculum, Namibian secondary school teachers are overloaded with work and this has presented them with a challenge of not completing the syllabus on time. Gurjanow et al. (2017) acknowledge that the MCM app project "takes a lot of time and endurance to create the amount of material [trail tasks] and to encourage others to do the same" (p. 470). Beside school hours, teachers often do not have enough time for other activities, and this can only mean that teachers' involvement in the MCM app project can be limited.

The third challenge associated with the implementation of the MCM project resides in the fact that the app offers little freedom for own ideas (Gurjanow et al., 2017). This is because of its automated feedback feature that gives specific answers for the solution of the tasks. Unfortunately, this gives no room for learners to interrogate and question answers that they do not agree with. Possibly, this can limit the intellectual autonomy of learners when it comes to providing arguments that judge the app's solutions. Kilpatrick et al. (2001) argue that learners need to be provided with mathematical situations that challenge them to argue, justify and explain their ideas "in order to make their reasoning clear, hone their reasoning skills and improve their conceptual understanding" (p. 130). Therefore, the MCM app seems to lack in this area and this can compromise the improvement of learners' mathematical argumentative skills.

The fourth challenge is that when solving the MCM tasks, the use of materials such as measuring tools, calculators or levels are needed (Jablonski et al., 2018). I contend that materials like measuring tapes can be difficult to find in schools and acquiring these tools only for the purpose of the MCM app project can sometimes be a demotivating factor for teachers to use the app in their teaching practice. The fifth challenge is that Namibian teachers are often not motivated to use technological devices such as smartphones, let alone allowing learners to make use of these devices (Osakwe et al., 2017). Moreover, according to a report on a recent four-day national conference on education, digital learning and transformation, lack of electricity and connectivity for devices remains a challenge in most rural Namibian schools (Nakale, 2022). Therefore, the implementation of the MCM app in rural schools with no internet and electricity can become a total headache to overcome.

Despite these challenges however, the literature shows that there is a need to overcome or mitigate them because of the great benefits of the MCM app project. Promoting the healthy use of smartphones and contextualising the learning of mathematics in authentic situations are vitally important for learning mathematics.

2.5 AUTHENTICITY OF TASKS IN MATHEMATICS EDUCATION

2.5.1 Defining authenticity

The term 'authentic mathematics' is often used to mean mathematics that is rooted in real-world contexts. However, there seems to be little consensus in the literature on the definition of authenticity in education, particularly in mathematics. Some studies define authenticity in education as tasks that are like real life, but not exactly the same (Palm, 2006), meaning the task, situation or activity is not the original reality, but intends to reflect reality. Other studies look at authenticity as tasks, situations or activities that originate from an out-of-school reality; indicating the authenticity cannot be added because it is already present even before it is used for educational purposes (Vos, 2018). This suggests that authenticity can refer to both real (original) and realistic (representational) problems or situations. In other words, authentic tasks are realistic, but not all realistic tasks are authentic.

Although Vos (2011; 2015; 2018) reiterates that authenticity should refer only to originals, in this study, I argue and agree with Newmann et al. (1995) and Palm (2006) that in mathematics education, authenticity should reflect how the problem, based on a real-world situation, matches the reality, whether it is actual or potential. Whether viewed from the context of realistic representations and/or real (original) situations, the goal is the same, and that is to engage learners in higher order thinking and active learning (Segers et al., 2003; Boshuizen et al., 2004). In his explanation of the term authenticity Palm (2002) writes that:

> "Authentic task" refers to one in which the situations described in the task compares favourably with a real-life situation outside the world of school mathematics. In addition, the task situation is truthfully described and the conditions under solving the task takes place in the real situation are simulated with some reasonable comparison in the school situation. (p. 7)

According to Gulikers and her co-authors (Gulikers et al., 2005), an authentic learning environment is

> a context that reflects the way knowledge and skills will be used in real life. This includes a physical or virtual environment that resembles the real world with real-world complexity and limitations and provides options and possibilities that are also present in real life. (p. 509)

In support of the above assertions Herrington et al. (2014) maintain that "authentic learning is a pedagogical approach that situates learning tasks in the context of future use" (p. 401).

Clearly, what these authors imply is that authentic-oriented teaching prepares learners for life beyond the classroom by exposing them to real-life contexts or problems.

Therefore, authenticity in mathematics education measures how *realistic* a problem situation is (Bonotto, 2007). Wernet (2017) defines *realistic* contexts as "scenarios potentially drawn from everyday life or career settings, in contrast to imaginative contexts, which are intentionally fanciful" (p. 74). In his study, Hakadiva-Vatileni (2016) used the term 'everyday' to refer to out-of-school non-professional practices, but not to specialised occupations. In addition to this line of reasoning, Wiliam (1997), points out that "a realistic context is characterised by the extent to which it is shared by students, and it fits with the mathematical structures being taught" (p. 8). So, in the same manner, this study views authenticity as a measure of how realistic MCM mathematics trail tasks and situations can be, particularly how the nature and context of the tasks involve real-life situations (Paredes et al., 2020).

The realistic aspect of authentic tasks gives learners an experience of solving problems that a person in a real-world situation would solve and uses the same tools the person in the real world would use. Moreover, learners are also exposed to the same conditions a person in the real world would encounter (Burton, 2011). Subsequently, learners are made to see the usefulness of specific ideas and skills being taught and how they can benefit from the use of these skills in real life settings (Sullivan et al., 2003). Some studies (eg Lavicza et al., 2020) observe that the absence of connections and application of skills and knowledge learned in schools often lead to mathematics being considered as distant from reality.

To further understand the concept of authenticity, there is also a need to look at the term in response to the question of "authentic to what"? (Arter & Spandel, 1992; Messick, 1994). This question implies that authenticity is a relative concept that can only be defined in relation to its resemblance to something (Honebein et al., 1993: Gulikers et al., 2008). For example, a copy resembles the original document it was copied from and can only be certified to be real or not, in relation or comparison to the original document. The (Cambridge English Dictionary, n.d.), affirms this point of view when it defines authenticity as something that is real or true, and reflects the original. On the other hand, other dictionaries (eg, Merriam Webster online dictionary, n.d.), contradicts this point of view (to some extent) by stating that an object is authentic if it is real, actual, and does not imitate the true origin, and it is from this point of view that Vos (2015: 2018) argues that a copy will remain a copy and thus cannot be referred to as an original.

In reconciling the contradiction that exists between origin and copy aspects in the definition of authenticity, Palm (2006) acknowledges that it is not always possible to exactly resemble or simulate all aspects in real life in an authentic mathematics task. Learners are not professionals and thus cannot be expected to do exactly what professionals do in real work environments. As argued by Vos (2018), this means that some aspects of the original task will have to be reduced for educational purposes. Although not entirely original, the tasks will remain authentic and offer learners experiences that will give them a "'feel' of what real professionals do with real mathematics in the real world" (Vos, 2018, p. 11). In this way, the degree of authenticity relies more on the structure and nature of the task than on the setting of the task.

Therefore, the discussion surrounding the concept of authenticity should focus on the degree or extent to which a given task resembles reality in terms of the context of the event, the questions asked, tools used and the purpose to be achieved (Gulikers et al., 2004a; 2004b; Palm, 2006). In this line of thinking, (Gulikers et al., 2004a; 2004b; 2006;) provide five dimensions that comprehensively define the term 'authenticity' in mathematics education. The dimensions form part of a framework that argues that the degree of authenticity of an assessment task depends on the degree of resemblance of the task to the professional practice situation it relates to. With these dimensions, Gulikers and colleagues connect the reality of tasks to the place of work, that is, the professional work environment.

The first dimension is the assessment task(s), which measures how the given task resembles that of the real profession or situation. The authors assert that in a mathematics assessment the problem task should confront learners with "activities that are also carried out in professional practice [work of place]" (Gulikers et al., 2004a, p. 71). Arguably, when school tasks resemble the reality of what is encountered in real life situations, learners are accorded the ownership of the tasks and the enthusiasm to develop solutions to these tasks (Savery & Duffy, 1995). Learners are motivated to do the task because they can see how the knowledge gained from engaging in the task might help them in future.

The second dimension is the physical context in which the task takes place. The question here is to what degree the context in which learners must perform the task is like that of the real work environment. Usually, the real place presents the realities of the task compared to the staged environment (Gulikers et al., 2006). Moreover, the tools needed to solve the task in a school or learning environment and the time taken to solve the problem may somehow differ to that of the real setups, which in turn affects the resemblance of the task in terms of its context.

For example, learners can have more or less time to complete a task, compared to the real-life work environment.

The third dimension is the social context of the assessment. This is concerned with the extent to which the social engagement and cooperation involved in the task reflects that of the real workplace (Gulikers et al., 2004a; 2004b). This principle looks at how learners work together to accomplish a given task and considers questions such as whether the learners demonstrate the spirit of teamwork that is needed in a real situation if faced with a similar problem. Their cooperation in solving school tasks prove that they can collaborate with their colleagues or workmates in a real work environment. Working together in real work environments is often the rule rather than the exception. In his research study, Silvey (2019) argues that it is a mistake to think that learners would see the importance of mathematics without engaging in some level of social interaction (both peer-to-peer and learner-to-teacher). In an authentic assessment situation, the social processes of the assessment should resemble the social processes in the reality of the similar situation (Gulkiers et al., 2008). Thus, collaboration should be integral to the task, both within the school task and the real world, rather than being achievable by an individual learner.

The fourth dimension emphasises the result or form that defines the output of the assessment: here the question is how the modality of the solution of the school task is similar to that of the real work environment. Gulikers et al. (2004a; 2004b) argue that the results should resemble the same quality as expected in a workplace. Also, learners should be able to present and explain their work or findings on the task to other people, either in an oral or written form to demonstrate and prove that the solution is a true reflection of their intellectual efforts (Wiggins, 1989). In this way learners also demonstrate their deeper level of reasoning using causal conjunctions, such as ‘because’, ‘as’ and ‘in order to’ (Bragg & Herbert, 2017). Brown et al. (1996, p. 1066) found that it is necessary that learners present and explain their work to others because:

> Understanding is more likely to occur when a student is required to explain, elaborate, or defend his or her position to others; the burden of explanation is often the push needed to make him or her evaluate, integrate, and elaborate knowledge in new ways

The fifth criteria is that of the assessment criteria. This concerns the explicit nature of the criteria learners need to follow when solving the tasks. Are the task criteria clear enough to learners or are the learners finding it difficult to understand what is expected of them? Do the criteria of the task resemble the reality of the real workplace where the task is likely to be found or are they more artificial than real? (Gulikers et al., 2006).

On a similar note, and perhaps most significant to that of Gulikers et al. (2004a; 2004b; 2006), Palm (2006) also provides a framework that can guide the creation of authentic and realistic school mathematics tasks – see Table 2.2 below. The framework contains several aspects that I consider useful in this study for designing the outdoor authentic and realistic tasks.

Notice how both Palm's (2006) and Gulikers et al.'s (2004a; 2004b; 2006) frameworks support the central viewpoint of authenticity in mathematics tasks as a resemblance to real life situations and problems requiring learners to work with authentic events or situations using authentic data strategies for authentic purposes (Vos, 2018). This means authentic tasks in mathematics should present problems with a relation to everyday life (Palm, 2006) and most importantly, learners must see how the task(s) resembles their future line of work (Gulikers, 2004a; 2004b; 2006) such that they think about or imagine themselves in the context when finding solutions to the tasks (Van den Heuvel-Panhuizen, 2005). In other words, in authentic tasks learners are presented with problem situations that reflect what is encountered in real life setups. At the same time the tasks present ways and procedures for how learners should mathematically approach these real-life problems.

Table 2.2: Description of Palm's (2006) aspects of authentic mathematics tasks

Task character	Description
Event	The event or problem described in the task has or is likely to occur.
Question	The question posed in the task is likely to be posed during the real-life event.
Information/data	Values provided in the task are realistic, the amount of information provided reflects whether certain information would actually be available in the situation and the specificity of that information.
Presentation	Mode (eg written, verbal, use of mathematical representations) and language of task presentation is accessible to students and does not stand in the way of interpreting the problem.
Solution strategies	The availability and plausibility of strategies for solving a contextual task reflects strategies available and are plausible in the situation being simulated.
Solution requirements	There is a close alignment between the mathematics and level of accuracy required for the school task and the realistic situation. Also, the types of assumptions allowed and reasonable in the realistic situation should be considered acceptable for students solving the task.
Circumstances	The social context of solving the problem in the classroom should reflect how such a problem would be addressed outside the classroom. It is divided into sub-aspects: availability of tools (eg, calculator or map), guidance, time constraints, opportunities to collaborate and discuss, and consequences of success or failure.
Purpose	The purpose of solving the task and the purpose of solving the associated real-world problem are clear to students. Further, these purposes should be reasonably aligned so that students can make reasonable assumptions about requirements and constraints.

On the other hand, Vos (2018) asks the thought-provoking question of "how can a theory-heavy subject like mathematics be connected to real life"? (p. 2). Of course, one way this can be possible is by using tasks that are connected to reality. As learners solve the tasks it is important that teachers encourage them to make connections between concepts and applications by embedding school mathematics knowledge into out-of-school life (Vos et al., 2007). In this way the mathematics that seems abstract in the classroom becomes meaningful to learners as they link and emulate the concepts to their lived experiences of real-life situations.

2.5.2 The question of relevance

When defining authenticity, *relevance* is another term that appears in the literature. Building on the work of other researchers (eg Nyabanyaba, 1999; Ernest, 2005; Jablonka, 2007), Hernandez-Martinez and Vos (2018) coined relevance as "a connection of four issues: *Relevance of what*? *Relevance to whom*? *Relevance according to whom*? and *Relevance to what end*?" (p. 248). These questions elucidate a need to understand why we learn certain things in mathematics. It is important to note that many secondary school learners appear to not be aware of the importance and relevance of learning mathematics (Vos, 2018; Hernandez-Martinez & Vos, 2018; Gijsbers et al., 2020). This is a cause for concern because "a perceived lack of relevance can result in disinterest and even disengagement with mathematics entirely" (Fitzmaurice et al., 2021, p. 1). More literature sources also echo the same sentiment that when learners find mathematics irrelevant to them, they begin to dislike the subject (Mujtaba et al., 2015; Reiss & Mujtaba, 2017). Hence, my approach in this study is to use authentic and realistic outdoor tasks that are relevant to the realities of learners, to help them understand the value of learning the topics such as area and volume.

On the other hand, Smart and Rahman (2008) assert that learners can enjoy mathematics more when they find it to be relevant to their future careers. In one study (Attard, 2012) the researcher concluded that when learners enjoy and persistently do mathematics because of its relevance to them, positive engagement towards the subject is what follows. Moreover, learners are motivated to learn something when they fully understand its relevance to their life or future studies and careers (Mujtaba et al., 2014). A significant number of studies (Verschaffel, 2004; Tripney et al., 2010; Keogh et al., 2018; Vos, 2020) provide evidence that the irrelevance of mathematics to many learners is the result of the mismatch between the mathematics learned in schools and the required mathematics in the real world or workplace. Thus, to address this issue, it is recommended that authentic tasks have real-world relevance, matching as closely as possible the real-world tasks of professionals in practice rather than decontextualised or classroom-based tasks (Oliver & Omari, 1999; Herrington & Herrington, 2006).

In response to their own questions on relevance, Hernandez-Martinez and Vos (2018) explain that the first question of *"relevance of what?"* is an issue that points at a complex object. The question addresses concerns such as: do the school tasks used to assess learners relate to what they need to do in the workplace? (Gulikers et al., 2004a). When solving the tasks can learners consider relevant aspects of the everyday life context in order to attain the correct solution?

(Paredes et al., 2020). In the current study, this question points to the relevance of mathematics in solving the MCM app tasks.

The second question of relevance, *"relevance to whom?"* supposes that relevance is connected to a person – in the case of my study, the learners. Thus, the question to ask here is: how relevant are the MCM app tasks are to the learners? (Gulikers et al., 2004a). The third issue of relevance is *"relevance according to whom?"* which suggests that relevance is a subjective judgement that depends on the person judging. So, the question of who decides the relevance of what learners do in mathematics is important to consider. In the context of solving the MCM tasks, the teacher as the designer of the tasks, can determine and decide the relevance of the tasks to learners. However, Hernandez-Martinez and Vos (2018), caution that sometimes what the teacher considers relevant to learners might not be relevant to them; for relevance means different things to different people (Sealey & Noyes, 2010). Stillman and Brown (2014) and Stillman et al. (2013a; 2013b) advocate that instead of demonstrating the relevance of mathematics, teachers should allow learners to experience the usefulness of mathematics through modelling activities by solving real-life problems. It is only when learners individually see the usefulness of the subject that they can appreciate its relevance to them and their lives (Hernandez-Martinez & Vos, 2018; Brown & Stillman, 2017).

The last question, *"relevance to what end?"* concerns the clarity on the goal-directedness of relevance. Hernandez-Martinez and Vos (2018) stress that relevance must have a purpose, and this purpose should relate to the motives and motivation of what is being done. Using real-world contexts in mathematics can show learners and teachers how the subject is relevant to their lives and can also increase their motivation, interest and achievement. (Honey et al., 2014; Hoogland et al., 2018).

Therefore, the purpose of the MCM tasks in the teaching of mathematics is manifold. According to Ludwig and Jablonski (2019), the MCM tasks expose learners to the mathematics found within their environment and it also allows them to solve authentic and realistic tasks connected to real life. Further, learners can get to learn and experience how smartphones substantiate and authenticate learning environments. Furthermore, the tasks promote both autonomous and collaborative learning skills when solving mathematics-related tasks during school and in the real workplace.

2.5.3 The *authenticity* of MCM project tasks

The MCM project idea is entitled to its claim as authentic mathematics education (Ludwig et al., 2020). Jablonski et al. (2018) emphasise that the tasks created for a mathematics trail of the MCM project should be *unique*, *authentic* and have *multiple solutions*. The *authenticity* of tasks in the MCM project are evidenced by the way they are contextualised in real life setups, and at the same time, the questions are what real people within the context of the real world would ask (Vos, 2018). In other words, the questions are connected to real problems within real contexts. For example, let us consider the *authenticity* and *relevance* of the following mathematics task:

> *If 1 litre of paint covers 6 square meters of wall, how many litres of paint will you need to paint the exterior frontside of this building?*

According to the MCM criteria, this task would have been directly connected to an actual building within the learners' environment and for this reason, authenticity cannot be added because it is already there (Vos, 2018). Further, the situation problem of the task is realistic because anyone concerned about getting the building painted would ask the same question. So, the task is *authentic* because it is real and there is nothing artificial about it (Wernet, 2017).

On the other hand, the *relevance* of this task can be twofold; firstly, for one to paint the wall, it would be necessary to have an idea of how much paint to use, hence an informed decision of how much paint to buy. Buying too much paint is a waste of money and buying too little paint can become a total headache, so whoever needs to paint the wall would want to ensure that the estimate is accurate. Secondly, the *relevance* of the task is in its connection to the school curriculum; the task is in line with what learners are supposed to be taught in a classroom situation. One of the criteria when designing MCM tasks is for the tasks to feature a connection to school mathematics and grade (Jablonski et al., 2018). In this regard, from the viewpoint of the Namibian mathematics curriculum, this task would feature topics such as mensuration, measure of units and basic arithmetic calculations at the grade level of secondary school learners (Grades 8 and 9) (MEAC, 2015). Also, the task designer would always ensure that the cognitive demand of the task aligns with the learners' grade level, ensuring that the task is not too difficult or easy for learners.

2.6 THE CONSTRUCT OF CONCEPTUAL UNDERSTANDING.

Rittle-Johnson and Alibali (1999) defined conceptual understanding as "explicit or implicit understanding of the principles that govern a domain and of the interrelations between pieces of knowledge in a domain" (p. 175). Thus, in the domain of mathematics education, conceptual understanding is seen as a comprehensive understanding of mathematical concepts, operations and relations (Kilpatrick et al., 2001; Chinnappan, 2014). According Sierpinska (1994), people demonstrate understanding by achieving a sense of order and harmony, where there is a sense of a 'unifying thought', of simplification, of seeing some underlying structure and that in some sense, a feeling that the essence of an idea has been captured. In other words, understanding something means it becomes part of us and we own it (Swan, 2014).

Studies (eg Sierpinska,1994; Swan, 2014) identify and list five operations that are at play when understanding things during the learning process. The first is the identification operation, where learners pay attention to the concept and then try to name and describe the concept. The second is the discrimination operation where learners strive to see the similarities and differences of a particular concept to that of others. The third operation is to generalize the concept by looking at the general properties of the cases of the concept. The fourth operation involves synthesizing the concept by perceiving the unifying principle of the concept. The fifth principle is the representation of the concept, where learners demonstrate their understanding of the concept by representing it in different ways, and this done can be through verbal, visual and/or symbolic means (Swan, 2014).

Furthermore, from the five interdependent strands of mathematical proficiency of Kilpatrick et al. (2001), conceptual understanding is said to be the catalyst of the other four strands: *procedural fluency*, *strategic competence*, *adaptive reasoning,* and *productive disposition*. This however does not mean that conceptual understanding is more important than the other strands, but it is viewed as key in becoming more proficient with the other strands (Mavani et al., 2018). For example, *procedural fluency* is a skill in carrying out procedures flexibly, accurately, efficiently and appropriately (Byrnes & Wasik, 1991; Kilpatrick, 2006), and developed through an iterative process of conceptual knowledge (Rittle-Johnson et al., 2001). It is as equal a partner of mathematical knowledge as conceptual understanding. However, the learning of mathematical calculations and algorithms without understanding is not helpful as learners can easily forget or incorrectly apply computational methods when solving problems (Kilpatrick, 2006; Rittle-Johnson & Koedinger, 2009; Moru & Qhobela, 2013). According to Froneman et al. (2015), procedural knowledge that is supported by conceptual understanding results in a meaningful application of procedures that can be remembered better and used more effectively.

2.6.1 Mathematical connections and the development of conceptual understanding

Learners' ability to make *connections* in mathematics is crucial for conceptual understanding (Anthony & Walshaw, 2009) and this suggests that when learners connect mathematical ideas, their understanding deepens and lasts longer. Eli et al. (2013), described a mathematical connection as

> a link (or bridge) in which prior or new knowledge is used to establish or strengthen an understanding of relationship(s) between or among mathematical ideas, concepts, strands, or representations. (p. 122)

Mathematical connections can be identified through the application of mathematics to contexts outside the classroom (the links between mathematics, other disciplines or the real world) as well as the interconnections between ideas in the subject itself (Blum et al., 2007; Blum, 2011). Therefore, the ability of teachers to accurately choose and apply approaches of teaching that promote mathematical connections is crucial for learners' development of conceptual understanding in mathematics (Lestari & Surya, 2017). Hence, it is important that teachers capitalise on the pedagogical opportunities that exist in mobile outdoor learning environments. Sugimoto et al. (2017), voice that pedagogies that connect learners to real-world contexts, especially those focusing on learners' own knowledge and experiences, can promote conceptual understanding and is a motivation for learning.

Furthermore, the Principles and Standards for School Mathematics (National Council of Teachers of Mathematics [NCTM], 1989) highlight the same importance of establishing connections: "when students connect mathematical ideas, their understanding is deeper and more lasting, and they come to view mathematics as a coherent whole" (p. 4). This can only be possible when teachers carefully identify and incorporate real-world connections to the tasks that have the potential to deepen the learners' understanding of specific concepts in the subject. While solving the tasks, teachers can then assess the learners' understanding of the incorporated concepts and find ways to support them (if need be) by promoting the use of connections. On the other hand, teachers are advised to not make the connections on behalf of learners but to initiate and let the learners see the connections between the mathematics they have been taught and its connection(s) to the real world for themselves (Gainsburg, 2008; Silvey, 2019).

With the MCM tasks, Barlovits and Ludwig (2020) say that "depending on the task formulation, working on MathCityMap task leads to a deeper understanding of mathematical concepts" (p. 57). Because MCM tasks are based outdoors, abstract concepts can be clearly understood because the environment is familiar to the solver (Moss, 2009, Reit, 2020; Crompton, 2020). Studies (Jansen & Bartell, 2013; Walkington et al., 2013) show that when tasks reflect familiar contexts and situations it becomes easier for learners to draw their knowledge from such contexts and situations to support successful problem solving.

Neilson and Campbell (2017) observe that learners' mathematical reasoning and understanding of concepts is improved by their careful observations and measurable predictions in scenarios that are within or related to real-world setups. There is also strong evidence that teaching outdoors can meaningfully help learners bridge the difficult science, technology, engineering and mathematics (STEM) concepts that are abstractly presented in textbooks (Kortland, 2007; Sugimoto et al., 2017). Similarly, learners' engagement during the MCM trails is characterised by numerous activities such as the probing of questions, identifying relevant quantities and information, choosing what to neglect and what to focus on, naming of items, making assumptions regarding their relations, predicting, approximating, simplifying, estimating and sketching diagrams, graphs and tables to collect data, measure, make statistical inferences, compute, interpret, verify, revise and so on (for review see Fessakis et al., 2018; Cahyono & Ludwig, 2019; Gurjanow et al., 2019; Cahyono et al., 2020).

2.6.2 The use of hints to support the understanding of mathematical concepts.

It is worth noting that the use of hints in the MCM project play a scaffolding role of reminding and directing learners towards the key concepts to be used during the problem-solving process. According to Jo (1993), "hints are often referred to as cues, clues, prompts or facilitators" (p. 1), and their purpose is to provide guidance that can lead learners to the correct solutions. Literature studies (eg Teong, 2003; Harskamp & Suhre, 2007) point out that the use of hints in mathematics is recommended for learners who struggle to remember, on their own, specific concepts needed to solve mathematical problems.

When teachers encourage struggling learners to solve the MCM problem tasks following the provided step-by-step hints from the task database (Baumann-Wehner et al., 2020), the connections between the apparent and hidden concepts that are embedded in the real-world objects and problem situations become clear to the learners. Hints serve as stepped aids that help learners to see the connections between the object concerned and the mathematical concepts related to it. Moreover, hints provide information and cognitive support that stimulate accurate problem-solving skills (Jo, 1993). Harskamp and Suhre (2007) add that using hints as aids increases learners' problem-solving skills, which in turn improves their learning performance, experience and communication.

The hints in the MCM app are more beneficial to learners who get stuck in the process of solving the problem tasks or have no idea on how to solve the tasks (Canyhono, 2017). In this way the learners as users can autonomously work on their own without guidance from the teacher. Not surprisingly, it has been noted that sometimes learners tend to rely more on the aid of hints than putting more intellectual effort into solving the tasks. Thus, the MCM app administrators introduced the awarding of points when one obtains the correct answer on the first attempt of answering the question (Gurjanow et al., 2017). In this way, problem solvers of the MCM project (including learners) are encouraged to solve the task problems without relying on the hints.

2.7 VISUALISATION IN MATHEMATICS EDUCATION

The ability to think and reason visually is recognised as an important cognitive tool in understanding mathematics concepts and processes (Rivera, 2011). Literature presents many definitions of visualisation, but I find the one proposed by Arcavi (2003) to be more in line with this study. According to Arcavi, visualisation is the

> ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds, on paper, or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understandings. (p. 217)

From this definition, two aspects emerge: visualisation as a product (the object, the 'what' of visualisation, the visual image) and visualisation as a process (the activity, the skill, the 'how' of visualisation) (Bishop, 1989). As alluded to by Nardi (2014), this means the process of visualisation can be external or internal. The external feature includes the use of visual artifacts such as displays of static images (eg physical manipulatives, drawings, charts and diagrams) and dynamic representations such as animations (eg virtual manipulatives), whilst the internal process of visualisation involves the formation and manipulation of mental images (Cohen & Hegarty, 2007).

In the context of this study, visualisation can further be thought of as a "mental outcome of a visual display that depicts an object or event" (Gilbert et al., 2008, p. 30). With this definition in mind, I want to build from the idea of visualisation as a 'mental outcome' of what is seen, by agreeing with Duval (2014) who says that "visualization is the recognition, more or less spontaneous and quick, of what is mathematically relevant in any visual representation given or produced" (p. 160). For me, what this means is that visualisation can enable one to see and recognise mathematical representations in the world around us.

Further, the process of visualisation involves mental images, and Gutiérrez (1996) asserts that a "mental image is a mental representation of a mathematical concept or property containing information based upon pictorial, graphical or diagrammatic elements" (p. 6). Studies (eg Mudaly & Naidoo, 2015) identify the ability to form and negotiate mental images as key to solving problems in mathematics. In other words, mathematical reasoning that is based on mental images can help learners to easily connect real situations to appropriate mathematical situations (Liu et al., 2019). Hence, in this study, the focus is on teaching that connects real-world objects with their mathematical representations through visualisation, for the purpose of helping learners have a deeper understanding of the mathematics around them.

Presmeg (1986; 1989; 1992) has gone a step further into the analysis of the visual strategies used by learners by developing a list of kinds of visual imagery used by learners: 'concrete, pictorial imagery', 'pattern imagery', 'memory images of formulae', 'kinesthetic imagery', and 'dynamic imagery'. Presmeg recommends that these visual strategies should be harnessed in learners in order to help them reach their full learning potential; for they are what makes visualisation an important skill for supporting the learning of mathematics, particularly the problem-solving aspect.

2.7.1 The convergence of visualisation and m-learning technologies

Spikol and Eliasson (2010), claim that with mobile devices such as smartphones, a powerful medium for visualisation is the learners' environment, which facilitates a connection between abstract mathematics concepts and the real world. The mobility of smartphones can facilitate an active network of learning where learners move in and out of different learning spaces, investigating mathematics properties within their environment (Fabian et al., 2018). A possible result of such a learning environment is the potential to facilitate the visualisation and understanding of mathematical concepts as learners match the abstract concepts with their concrete representations.

For example, the area and volume concepts can easily be understood when the abstract concepts are matched and aligned to the perceived and concrete objects that relate to the ideas of area and volume in real life. Unfortunately, as noted by Waisel et al. (2008), often learners drastically underuse this visualisation advantage. Therefore, Brown (2015) drew attention to the importance and necessity of teachers to understand and enact the cognitive visualisation that exists in the integration of real-world tasks and technology. According to her,

> many affordances useful in facilitating the solution of real-world tasks are available in Technology-Rich Teaching and Learning Environments (TRTLE's). Of particular use are those allowing visual image generation by technology (p. 431)

Experts agree that visually stimulating computer environments, for example, allows learners to become immersed in their own knowledge construction (eg Malabar & Pountney, 2002; Sullivan, 2012). Moreover, learners can create mental models for the problem situation and refine them through technology-generated external visualisations that lead to the integration of the mental model and the image on the screen. According to Borba (2012), this can lead to a seamless interplay between the mental model and the image on the screen, where the user can switch back and forth between the two in a dynamic process. Thus, the same can be argued with digitally stimulated learning environments. The visualisation that is generated from digital technology can impact on learners' perceptions and motivations by drawing on their prior knowledge, providing them with multiple presentation modalities, moving them from shallow to deeper learning and allowing them the opportunities to apply and build their own mental models (Paas & Sweller, 2014; Birt & Cowling, 2017).

Of particular interest is the photography feature on smartphones that can serve as a powerful tool for learning mathematics. The use of photography in mathematics education has been found to motivate learners and increase their interest and understanding of content through the connections between mathematics and everyday situations (Meier et al., 2018; Rizzo et al., 2019). Vale and Barbosa (2020) refer to photographs created for mathematics content as *mathematical photography* and *problem pictures*, whereby a depiction of a phenomenon or situation is accompanied by one or more questions, or a problem based on the context of the photograph. Image-based questions in mathematics can stimulate learners' curiosity in answering the questions, as well as increase their engagement during the problem-solving process (Bragg & Nicol, 2011). It is further argued that taking a photo/image creates an affective connection between everyday situations and mathematical concepts, which engages learners with the tasks (Barbosa & Vale, 2018; Vale & Barbosa, 2019; 2020).

2.7.2 The convergence of visualisation and real-world tasks

Using the earlier example of the painting task (refer to Section 2.5.3 above), let me illustrate how visualisation can be inherent in the solution of the MCM authentic and realistic tasks. Let us assume that the wall to be painted supposedly includes one door and two windows. First and instantly, the mere sight of the wall is likely to elicit the formation of concrete imagery (pictures-in-the-mind) of the geometrical shape that is related to it. Drawing from Stillman et al. (2004) and Duval's (2014) points of view, the cognitive processes that can happen here may be triggered by what learners see and hear (including words), in order to coordinate all possible representations of the wall to the concept of a rectangle. This visual feature can then help learners initiate a visual mapping of the solution to the problem situation. Among other things, this may include the abstract function of constructing memory images of the formulae of a rectangle (Presmeg, 1986), as well as possible multiple mental procedures of solutions to the problem.

The visual planning of the solutions can then be guided by the following questions: what are the tools needed to measure the dimensions of the wall and what units should be used? Should the surface areas covered by the door and windows be included in the paint to be used? If not, how can the amount of paint to be bought be determined? Specifically, in the context of problem solving, Deliyianni et al. (2009) discuss visualisation as "the understanding of the problem with the construction and/or the use of a diagram or a picture to help obtain a solution" (p. 97). To this effect, Fabian et al. (2018) suggest that to engage learners in productive visual thinking, teachers need to ask learners, at regular intervals, how they see mathematical ideas in certain situations. One way to enhance such a visualisation process is for teachers to encourage the use of rough drawings or sketches for the purpose of helping learners communicate and refine their mental thinking (Waisel et al., 2008).

Phillips et al. (2010) claim that "the common-sense view is that visualisations provide realistic depictions of the world" (p. 1). When learners make connections between mathematical topics and the real world, they explicitly employ visualisation for effective learning (Presmeg, 2014). Learners' ability to generate and use visual representations can enable them to solve real life problems. In other words, and particularly in the context of real-world tasks, visualisation can support problem solving (Carden & Cline, 2015). Fischbein (1987), for example, commented that "a visual image not only organises the data at hand in meaningful structures but is also an important factor guiding the analytical development of a solution" (p. 104). Furthermore, authors like Cobb et al. (1992) used the term *dualism* to explain the concept of visualisation in relation to connections between mathematics in learners' minds and their environment. In particular, the authors discuss that visualisation serves as a link between what learners think (images in the mind) against what they physically see (real objects). Zazkis et al. (1996) also adds to this discussion by stating that

> Visualization is an act in which an individual establishes a strong connection between an internal construct and something to which access is gained through the senses. An act of visualization [that is, the process of visualization] may consist of any mental construction of objects or processes that an individual associates with objects or events perceived by her or him as external. (p. 441)

Nemirovsky and Noble (1997) credit this point of view on visualisation to be more precise because of how it includes the act of visualisation to either the learner's 'mind,' or 'some external medium. In other words, visualisation serves as the means for travelling between what is physically seen to how it is interpreted in the mind. In their review, Cobb et al. (1992), argued that the "overall goal of instruction is to help students construct mental representations that correctly or accurately mirror mathematical relationships located outside the mind in instructional representations" (p. 4). Thus, in the same line of thinking, the process of visualisation helps learners interpret and understand their world through imaginary visualisation that helps them attach meaning to what they see. Thornton (2001) describes this process using the phrase 'mathematical power', which he explains as:

> the capacity to make connections, both between mathematical objects and concepts and between mathematics and the physical world. Visual thinking, whether in the form of concrete images, pattern images or dynamic images, has a key role to play in the development of students' mathematical power. (p. 256)

Hence, in this study, the focus is on teaching that connects real-world objects with their mathematical representations through visualisation, for the purpose of helping learners have a deeper understanding of abstract mathematical concepts.

2.7.3 Spatial reasoning and real-world objects

Spatial reasoning is another concept that needs discussion to understand how visualisation helps in connecting real world objects and the learners' imagery. Spatial reasoning is the ability or skill to visualise objects in three-dimensional space and manipulate and rotate them into different arrangements, as well as to mentally imagine what is inside these solid objects, according to the existing literature (Lohman, 2000, 2005; Hegarty & Waller, 2005; Titus & Horsman, 2009). Newcombe (2010) contributes to this definition by viewing spatial reasoning as a skill or ability to think about "the location of objects, their shapes, their relations to each other, and the paths they take as they move" (p. 30). What this means is that we reason spatially when our interpretation of how we see things helps us understand the world around us, including how we interpret what we see in relation to the mathematics we know.

Ojose (2011) advocates that understanding our world includes the awareness of navigating through space and through constructions and shapes. It also requires that we understand the relationship between shapes and images (or visual representations) such as between a real city and photographs and maps of the same city. The author further emphasises that the understanding of how three-dimensional objects can be represented in two-dimensions, how shadows are formed and interpreted, and what perspective is and how it functions, is also a necessity.

Literature on the emergence and use of the MCM app project within the teaching context substantiates evidence that the walking of mathematics trails and the use of digital GPS maps can strongly contribute to the development of learners' spatial orientation of their environment (Barbosa & Vale, 2020). When learners walk through the trails and solve the hidden tasks, they start to see, through a 'mathematical eye', objects that they could have valued less before, and begin to appreciate the relevance of these objects within the environment. According to Fowler et al. (2021), our first interactions with, and learnings about, our environment are primarily spatial and the importance and complexity of these spatial understandings increase throughout our lives. Thus, learners' understanding of maps are usually affected by their understanding of the surrounding world. Uttal (2000) claimed that learners often face difficulties in comprehending the features of a three-dimensional world when they are represented on a two-dimensional surface. Therefore, applying maps as analytical tools to explore and identify patterns can enhance learners' spatial reasoning skills.

A growing number of literature studies report overwhelming evidence of the benefits of spatial thinking in school contexts. For example, in his research on spatial visualisation, Hegarty (2010) discovered that spatial reasoning enables learners to undertake processes such as visual representations, and mental manipulation of both seen and unseen objects, use mental representations to solve problems, position objects in the environment, navigate and communicate visual stimuli. Möhring et al. (2021) and other study sources (eg Mix & Cheng, 2012; Wai et al., 2009; Hodgkiss et al., 2018; Gilligan et al., 2019; Hawes & Ansari, 2020) also claim that spatial thinking is linked to learners' increased mathematical achievements, skills and proficiency.

It is further envisaged that spatial visualisation plays an important role in the development of mathematical reasoning. Hegarty and Waller (2005) assert that spatial visualisation abilities are important for both constructing and comprehending abstract spatial representations in mathematical problem solving. Revina et al. (2011) in their study, claimed that spatial visualisation tasks help learners to develop their conceptual understanding of volume measurement, which in turn leads to space perception. The NCTM (2000) also emphasises that spatial reasoning harnesses the understanding of the concepts of perimeter, area and volume.

Research has shown that spatial vocabulary can enhance learners' spatial reasoning and problem-solving skills in various contexts (Mix & Cheng, 2012; Farran & Atkinson, 2016). Spatial vocabulary is the use of words that describe the location, size, shape, and features of objects and their relations (Feist & Gentner, 2007). For instance, words like 'next to' and 'in front of' can help learners conceptualise the relative positions of objects, while words like 'on' and 'in' can help them refine their spatial categories. Spatial vocabulary can also draw learners' attention to the relevant spatial attributes of shapes and objects, such as the angles, edges, and corners, when they are copying constructions or learning mathematical concepts (Bower et al., 2020). Therefore, using spatial vocabulary in teaching contexts can support learners' spatial understanding and development.

On the other hand, some research studies point out that even though we live in a three-dimensional world, mathematics textbooks still present learners with two-dimensional visualisation (for a review, see Turgut & Nagy-Kondor, 2013; González, 2015; Ng, 2017; Lavicza et al., 2020). This makes it difficult for learners to understand the relationship that exists between real objects around them and the mathematics they learn, for they are always forced to understand the shapes in two-dimensional representations. Idris (1998) argues that many mathematical topics such as geometry require visualising abilities, but because many learners are accustomed to visualising in two-dimensional perspectives it often becomes challenging for them to visualise and interpret real-world objects that are three-dimensionally oriented. Hence, the need to harness spatial visualisation to help learners visualise, manipulate and understand the objects that makes up the world we live in.

2.7.4 The convergence of visualisation and conceptual understanding

Most of the literature on visualisation is aligned with the visualising of concepts as inherent in human beings (Hershkowitz, 1989; Sierpinska,1994; Gray 1999; Marmo et al., 2010; Swan, 2014). For example, Hershkowitz (1989) argued that as human beings "we cannot form an image of a concept and its examples without visualizing its elements" (p. 61). In fact, Kosslyn (1996) argued that people work with images in the brain as a whole and as parts, meaning that our understanding of things depends on how we internally view them. Early psychologists such as Piaget held to the belief that humans do not understand the world directly but possess only internal representations of it. In other words, perception influences the way we interpret and understand things. Piaget and Inhelder (1967) describe perception as

> the knowledge of objects resulting from direct contact with them. As against this, representation or imagination involves the evocation of objects in their absence or, when it runs parallel to perception, in their presence. It completes perceptual knowledge by reference to objects not actually perceived. (p. 17)

Here we should understand that perception plays a key role in our ability to process information; for we find it much easier to reason spatially and in imagery than we do to process data in a flat list (Larkin & Simon, 1987), and significant parts of our memory are reliant on perceptual manipulation (Baddeley & Hitch, 1974). Therefore, exposing learners to preliminary activities involving perception and action within the physical world, and reflection on both perception and action, can develop early mathematical concepts in learners (Gray et al., 1999). Further, the understanding and solving of geometrical problems requires the interaction of imagery and conceptual understanding (Mhlolo & Schäfer, 2013). Thus, visualisation can enhance and stimulate our perception to "see the unseen", both in the apparent and in hidden objects, trends and patterns in the world around us (Rudziewicz et al., 2017).

There is also evidence that the use of concrete-pictorial imagery motivates and enables learners who struggle to understand mathematical concepts, by clarifying to them the structure of the problem and assessing the reasonableness of their results (Campbell et al., 1995). Thus, visualisation can play a significant role in the development of thinking or understanding mathematical concepts, and also make abstract material more concrete (Sevimli & Delice 2011; Tasova & Delice, 2012).

2.8 UNDERSTANDING THE CONCEPTS OF MEASUREMENT

Next, I review literature on spatial measurements (area and volume). Over the decades, studies have maintained that the ability to visualise spatially relates to the reading and understanding of two-dimensional (area measure) and three-dimensional (volume measure) representations and objects (Piaget et al., 1960; Ben-Haim et al., 1985; Barrett et al., 2011; Barrett et al., 2017). Further, it is also important to note that the learning of area and volume measurements depends on the understanding of the measure of length. According to Matsuo et al. (2020), learners who cannot compare length always struggle to understand and compare the measures of area and volume. These three spatial measurements involve the coordination of two-dimensions of learners' experience – continuous- space and discrete number.

Traditionally, measurement has been thought of as "the assignment of a numerical value to an attribute of an object, such as the length of a pencil" (NCTM, 2000, p. 44). Bishop (1988) claimed that "measuring… is concerned with comparing, ordering, and with quantifying qualities" (p. 34). Barret et al. (2011) also claim that the act of measuring involves the comparative reasoning of relating objects to some unit object and reporting its size in terms of those units. The historical and ethnographical account of measures indicates that early measurement units originated from concrete sources of tangible phenomena and that they have been often tied to particular contexts (Cooperrider & Gentner, 2019).

In the case of spatial measurements, the length unit was historically and culturally based on the artifact spans of the body such as finger-widths, hand-breaths, arm-lengths and the foot or step count. The area unit was measured by the number of days an ox could plough in a day, for example (Behnke, 1980). On the other hand, reasoning such as the amount of liquid a pot or vessel could hold influenced the measurements of commodities like the volume of milk, butter and honey (Marealle, 1963). According to Cooperrider and Gentner (2019), over time, measurement units underwent a shift from highly concrete to highly abstract as we now know them. And notably, today, instead of applying measurement in a relative and discrete way, the concept is determined as an absolute value using standard units.

Accordingly, empirical studies with a focus on measurement have advanced the early teaching and learning of length measurement and this knowledge continues throughout primary and secondary education (Baiduri, 2019; Siti Nur Annisa & Lim, 2021). Also, these studies agree that the initial instruction of measures should transit from concrete and familiar discrete methods of measurement to using conventional measuring tools. In their recent findings, Samara and colleagues confirm that the early teaching of measurement should be sequenced in such a way that children first learn how to compare lengths and measure with nonstandard units (eg hand spans and foot counting) before transitioning to the use of standard units of measuring (Sarama et al., 2022).

Other literature sources in this area also attest that in their early years of schooling, children need to be familiarised with discrete ways of measuring (Piaget et al., 1960; Solomon et al., 2015; Levine et al., 2009). The sequence of teaching measurement by starting with nonstandard units and then moving to standard units is an appropriate approach because children's early understanding of measurement is mainly perceptual, and this makes it easier for them to focus on the attribute being measured. Moreover, once a measuring concept is well developed, it becomes easier to use standard units, argue Gaoseb and Kasanda (2007).

2.8.1 The length measurement

The measure of length is a linear measurement that is viewed as "a comparative property of objects that embodies the amount of one-dimensional space between endpoints of the objects, which can be compared or quantified" (Szilágyi et al., 2013, p. 538). Thus, the learning of length measurement in school mathematics involves a one-dimensional property that quantifies the length or distance between two endpoints of an object or space (Gómezescobar et al., 2020; Welch et al., 2022). Comparing and ordering measurements of different objects to determine and distinguish between their heights or lengths is the most basic and fundamental competency that is usually taught at an early age (Szilágyi et al., 2013). Montague-Smith et al. (2012) and Lembrér (2013) stress that measurement cannot be done without making a comparison, as measurement is about making comparisons.

Work in Spain in this area supports the potential value of teaching the measure of length through discrete real situations of comparing the lengths of objects before transiting to the use of conventional tools. For example, Spanish authors like Gómezescobar et al. (2017) reason that "children can develop cognitive skills and abilities when they are offered real situation in which they can work hands on and develop measurement intuition" (p. 1378). Further, work from the USA also supports the importance of using non-standard units (spans, bars, etc.) while making measurements of length. Teaching in a way that builds a link between counting the units and using standard units allows teachers and learners to focus on reasoning rather than simply counting (McDonough & Sullivan, 2011).

Using her research investigation of the teaching of standard units of length, capacity and mass in her home country, Mwale (2022) argues that teachers in Malawi need to make instructional tasks on length measurements more challenging to encourage independent thinking and more reasoning from learners, rather than following direct instructions from teachers. A South African-based study (Feza-Piyose, 2012) claimed that learners lacked the foundational knowledge of spatial measurements, and this was shown by how some participants could not determine how many centimeters the length of a pencil was.

In Namibia, studies such as that of Nambira et al. (2009) found that learners struggled in the topics that are attributed to measurement (length, mass and capacity) and that the mathematical abbreviations for the units of these measures were a problem for learners to identify. Gaoseb and Kasanda (2007) concluded that because of the lack of a good foundation in the knowledge of mathematics in Namibia, secondary school learners still do not have a sense of estimating and differentiating between spatial measurements (eg centimetre, square centimetres or hectare and cubic centimetres). A recent study with a focus on learning measurement outdoors, Shimakeleni and Chikiwa (2022), found that code switching when solving measurement problems can help learners to understand the conceptual underpinnings of the topic of measurement. The authors recommend that in a country where the dominant language of learning and teaching (LOLT) is English, it should be necessary for teachers to consider the use of the local language to support the instruction of measurement.

International literature contends that learners often face difficulties when dealing with length measurement and one such difficulty is the conceptualisation of the unit and origin of measurement (Solomon et al., 2015; Tan-Sisman & Aksu 2016; Congdon et al., 2018). Learners tend to incorrectly align the ruler and the object at one rather than zero, and this means that they perceive the hatch marks, and not the space interval between two marks on the ruler, as units of measurement. (Solomon et al., 2015; Welch et al., 2022). At times learners determine the measurement of an object by reading the end of the ruler without considering the starting value aligned with the beginning of the object (Gómezescobar et al., 2020), which can be an indication that their understanding of measurements is influenced by the notion that any point on the ruler can be the origin of the measurement. Furthermore, learners struggle to relate measures when using different units (converting one unit of measure to another), including naming the unit when reporting a measure (Barrett et al., 2017; Coskun & Bostanci, 2022; Sarama et al., 2022).

Errors such as the overlapping of units and mixing up the units of length with other units of measurement are also reported as common occurrences when learning the measurement of length (Tan-Sisman & Aksu, 2016; Welch et al., 2022). On the other hand, Barret et al. (2017) argue that sometimes learners may know how to correctly place and align an object at zero and read the measure of a ruler, but still, this does not mean that they understand how or why a ruler works in relation to the concept of measurement. Learners can still fail to understand that when using a conventional standard tool like a ruler to measure, any point can act as the start or end point. Therefore, McDonough and Sullivan (2011) argue that without an awareness of the concept of unit, incorrect measuring can always occur.

2.8.2 The area and volume measurements

Area measurement can be described as the quantification of the amount of space within a two-dimensional, region or closed surface of a figure (Sarama & Clements, 2009; Weisstein, 2016; Sibanda & Machaba, 2022). Smith III & Males (2016) define area as “the quantity of two-dimensional (2D) space enclosed in shapes with closed boundaries, whether they lie on a plane or nonplanar surface” (p. 20). Generally, the measure of area marks an important and advanced transition in the teaching and learning of measurement. As said earlier, the understanding of area develops later than that of length (Montague-Smith et al., 2012) and in contrast to length, measuring area (and even volume) involves a shift from the use of conventional tools like rulers to numerical computations and formulas (Zacharos, 2006). Just as the measurement of length, to understand the measurement of area learners need to go through a learning experience of comparing the area of various sizes of surfaces by using identical and non-identical surfaces as tools (Putrawangsa et al., 2014).

The topic of area measurement connects to other mathematical topics such as multiplication, fractions, algebraic multiplication and composition of geometric figures, among others

(Outhred & Mitchelmore, 2000; Sarama & Clements, 2009; Clements et al., 2018). Also, the topic can be found in other subjects (particularly STEM subjects) (Razzouk & Shute, 2012). Further, the topic of area measurement connects to practical applications of daily life (Clements & Sarama, 2007a), such as the kitchen, laboratory, when driving or travelling by air, conducting population studies and in carpentry, for example (Chiphambo & Mtsi, 2021). Therefore, if the concept of area measurement is not well developed, learners may likely perform poorly in mathematics and even other science subjects. Consequently, this may disengage them from understanding and appreciating areas of life that require the knowledge of area.

Research indicates that learners struggle with the conceptual understanding of area because they are made to master and apply the formulas rather than to understand the concepts underlining the applications (Clements & Sarama, 2007a; 2007b; Clements et al., 2018). Cullen et al. (2018) suggest that the knowledge of area measurement in learners is supported by their ability to structure space. Sarama and Clements (2009, p. 296) define spatial structuring as "the mental operation of constructing an organisation or form for an object or set of objects in space, a form of abstraction, the process of selecting, coordinating, unifying, and registering in memory a set of mental objects and actions". According to these researchers, to understand area measurement, one needs to integrate the experiences of learning about unit concepts and spatial structuring of two-dimensional space.

Reviewing literature on area measurement inevitably leads to the concept of surface area. It is also envisaged that the manipulation of area in two-dimensional shapes reinforces the learners' understanding of three-dimensional measurements (Sarama & Clements, 2009; Chiphambo & Mtsi, 2021). Simply put, the knowledge of surface area relies on the learners' understanding of area measurement. Mbedzi and Samson (2013) define surface area as the number of identical square faces on the outer surface of a shape. In three-dimensional shapes or objects, surface area can be calculated by unfolding or opening the net of the object, which is a process of changing the object from three-dimensional to two-dimensional or flat figure.

In the analysis of area measurement tasks in their study, Tan-Sisman and Aksu (2016) found that learners face the following errors and misconceptions:

a) believing that area is not constant, under partitioning;
b) counting the lines around a shape for area;
c) point-counting for area;
d) confusing area with perimeter;

e) using the perimeter formula for area;
f) believing that area equals to length plus width;
g) using units of length/volume measurement;
h) using the volume formula for surface area;
i) believing that surface area equals to length plus width plus height;
j) confusing surface area with volume; and
k) believing that a shape has more than one surface area. (p. 1313)

On the other hand, Sasanguie et al. (2013) discuss the concept of volume as the quantity of three-dimensional space occupied by a liquid, solid or gas. Sibanda and Machaba (2022) describe volume as "the space occupied by a substance or enclosed by a surface" (p. 121). More previous studies (eg Dickson et al., 1984; Van de Walle et al., 2010) describe the calculations of volume as being internal and/or external. The external aspect of volume involves measuring the space occupied by a three-dimensional object while the internal volume has got to do with the amount of liquid or other pourable material the object can hold. Understanding the concept of volume measurement is also important to learners as it can provide them with a variety of contexts to expand their understanding of arithmetic, geometric reasoning and spatial planning (Battista, 2003; Lesh & Lehrer, 2003).

Just like area measurement, learners also encounter difficulties in their volume calculations and these include treating three-dimensional shapes as two-dimensional ones, counting surfaces or seeing units of cubes, counting the cubes in three-dimensional arrays incorrectly and confusing the concept of volume with that of surface area (Battista 2003, Owens & Outhred, 2006). Consistent to Tan-Sisman and Aksu's (2016) findings on area and Battista (2003) and Owens and Outhred's (2006) findings on volume, other recent studies (eg Chiphambo & Mtsi, 2021; Sibanda & Machaba, 2022) confirm that even secondary school learners confuse the general understanding of surface area with that of volume, including interchanging their formulas. The findings further indicate that learners also confuse the units of surface area and volume measurement. Sibanda and Machaba (2022) point out that sometimes the confusion between surface area and volume is caused by the phrases "space covered" and "space occupied" in the definitions of area and volume measurements. If the conceptual understanding of these phrases is not well mastered, learners may tend to link these two concepts and treat them the same, unless sufficient mastery of facts and concepts is prioritised.

Furthermore, scholars attribute the confusion between the concepts of area and volume measurement to the use of procedural algorithms and memorisation of formulas without understanding the underlying meaning of the formulas (Strutchens et al., 2003; Huang & Wirtz, 2011; Smith III et al., 2016; Crites et al., 2018). Specifically, the learning of area and volume measurements is found to be based on rote learning where teachers' focus is much more on how to use the formulas of area and volume than on providing learners with more experiences, with the exploration of formulas (Battista, 2003; Kamii & Kysh, 2006; Clements et al., 2018).

2.8.3 The learning of spatial measurements (length, area and volume) in the Namibian curriculum

In Namibia, the curriculum directs that length measurement be introduced to children as early as six years old, and that the teaching should focus on informal units of measurements through comparing objects and using appropriate vocabulary such as long/short, longer/shorter, longest/shortest, as well as the use of non-standard units such as hand spans, palms, footprints and paces (MEAC, 2015). This directive is in line with the international literature that records that the instruction of length measurement should start from children as young as five years old and that the focus of teaching should be on the iteration of non-standard units (Piaget et al., 1960; Szilágyi et al., 2013; Solomon et al., 2015; Sarama et al., 2022). The goal here is to help learners identify, recognise and understand appropriate units of measurement and instruments of length (as well as other types of basic measurement knowledge eg mass, capacity, area, time and money).

In the senior primary phase (Grades 4 – 7) learners are expected to demonstrate an understanding of using the units of length and solve relevant problems in theoretical situations and in applications to everyday life. Furthermore, learners must learn how to estimate, measure and compare familiar objects in the metric units of length: millimetre, centimetre and metre. It is also in this phase of their schooling where learners should be introduced to the concepts of perimeter, area and volume of regular and irregular figures (MEAC, 2015).

In the junior secondary phase (Grades 8 – 9), learners are expected to recognise and convert between various standardised units of measurement and solve problems related to measurement in theoretical situations or in applications to everyday life. Teachers should also introduce learners to the calculations of perimeters and areas of regular and irregular plane figures as well as volumes of solids such as cubes, cuboids and cylinders. In addition to this, the curriculum

(MEAC, 2015) also stipulates that learners should learn how to convert between units of length measurement and volume.

I acknowledge that Grade 9 learners (the focus grade of this study) may already have been introduced to the concept of length measurement, however, it is important to note that most Namibian secondary school learners lack the foundational knowledge of mathematics (including measurement) (Ilukena & Schafer, 2013; Hamukwaya & Haser, 2021). Moreover, the exploration of spatial measurement tasks in this study within the MCM tasks is uniquely designed to connect the abstract learned in the classroom to the outside world. In this way, the teacher participants of this study are expected to challenge their learners to not only apply the formulas but understand the principles behind the procedures they undertake. In other words, the tasks can expose learners to skills needed to grasp the conceptual understanding of geometrical measurement (Vasilyeva et al., 2009; Hannighofer et al., 2011; Crites et al., 2018).

Furthermore, the MCM tasks can allow the combined teaching of the length, area and volume concepts. Currently, the Namibian curriculum presupposes that spatial measurement of length, area and volume are to be taught in an isolated sequence instead of integrating these dimensions by focusing centrally on unit operations. Contrary to this order, international literature advocates that to develop a coherent understanding of spatial measurement the concepts of length, area and volume must be integrated with one another (Van den Heuvel-Panhuizen & Buys, 2008; Steffe & Olive, 2010; Barret et al., 2017). Matsuo et al. (2020) further provide evidence that learners who cannot compare length are also likely to struggle to compare area, suggesting that the comparison of length, area and volume is closely related. Therefore, the MCM app can potentially integrate these three conceptual strands of geometrical measurements through two of its many features, namely, the tags and sub questions.

2.8.4 Understanding the concepts of ratio and proportion

Ratio and proportion are seen to be useful concepts that are applicable to real life situations and the concepts link to numerical and concrete mathematics of arithmetic and algebra, both in elementary and secondary education (Post et al., 1988; Fuson & Abrahamson, 2005; Lamon, 2007). Ratio is defined as a multiplicative relationship between two values (Ekawati et al., 2015). Livy and Vale (2011) also describe ratio as a comparison between two quantities. On the other hand, Tourniaire and Pulos (1985) documented that proportion is a statement of equality of two ratios. In addition, Kilpatrick et al. (2001) also view proportion as “statements that two ratios are equal" and that "proportional reasoning is based, first on the understanding of ratio" (p. 241).

Boyer et al. (2008) articulate that reasoning proportionally requires "understanding the multiplicative relationships between rational quantities" (p. 1478), and according to Fujimura (2001), this type of reasoning is a mathematical thinking that is particularly difficult for both children and adults. In a similar sense, Lamon (2007) argued that

> … of all topics in the school curriculum, fractions, ratios and proportions arguably hold the distinction of being the most protracted in terms of development, the most difficult to teach, the most mathematically complex, the most cognitively challenging, the most essential to success in higher mathematics and science, and the most compelling research sites. (p. 629)

Several authors identified the tendency to solve ratio and proportion problems using additive or subtractive rather than multiplicative methods as one of the major mistakes when solving ratio and proportion tasks (Kilpatrick et al., 2001; Lamon 2005; Kennedy et al., 2008; Boyer & Levine, 2012). Some of these authors also (eg Kilpatrick al., 2001; Kennedy et al., 2008) agree that the failure to promote multiplicative situations can lead to the confusion between additive and multiplicative approaches to solving ratio and proportional problems. Furthermore, studies also report that isolating the teaching of the concepts of ratio and proportion from other related topics such as fractions, rate, scale, measurement, geometry and algebra, for example, is another cause for the difficultness of understanding the ratio and proportion concepts (Lamon 2007; Boyer & Levine, 2012; Burgos & Godino, 2020).

It has been concluded that just as with learners, teachers also struggle with the aspect of reasoning proportionally; for they do not seem to understand the conceptual underpinnings of the topics of ratio and proportion (Thompson & Thompson, 1996; Lobato et al., 2011). Among other reasons, Hilton et al. (2016) state that this can be caused by the teachers' poor planning and lack of preparedness as well as their inability to differentiate between additive and multiplicative situations. Subsequently, this leads to their failure to strategically help learners recognise or solve such situations. Simply put, ratio and proportion are difficult concepts for both learners and teachers.

The traditional strategy of cross-multiplication (sometimes referred to as the rule of three strategies) seems to be the most common approach to teaching about ration and proportion. According to some studies (eg Charles-Ogan & George, 2015) the use of this method alone can lead to superficial understanding and difficulties in the future development of proportional reasoning. Mahlabela and Bansilal (2015) argue that even when the cross-multiplication method is successfully used, many learners still carry out the operations without understanding how and why the algorithm worked. And to these authors, this is an indication that the cross-multiplication strategy is often used as a shortcut to teach ratio and proportion problems. Thus, they (Mahlabela & Bansilal, 2015) recommend that to conceptually help learners understand ratio and proportion, the topics should be taught alongside linear functions, for functions define the relationship between the quantities.

Furthermore, Kilpatrick et al. (2001), also advocate that "moving directly to the cross-multiplication algorithm, without attending to the conceptual aspects of proportional reasoning, can create difficulties for students" (2001, p. 244). Of course, apart from using the cross-multiplication algorithm strategy to solve ratio and proportional problems, literature identifies other strategies like the between-comparison method, the within-comparison method, the table method, the graph method, the use of diagrams/models and the informal oral method (Nielsen, 1998; Koedinger & Nathan, 2004; Amit & Fried, 2005; Van de Walle et al., 2010). In this line of argument, a study that was carried out a decade ago before the overhaul of the Namibian curriculum in 2015 to the current existing curriculum, found that Namibian teachers preferred to use the graph, table and cross-multiplication methods, for in their view, the two methods complemented each other (Simasiku, 2012). In addition, the study also revealed that the cross-multiplication strategy is not an easy fit for the teachers because it needs a deep conceptual understanding of proportion and ratio.

In my teaching career during the past 17 years as a high school mathematics teacher, I can attest that the cross-multiplication method and table method are the most used strategies in the secondary phase. This could be partly due to the persistent appearance of examples on ratio and proportion that use these two methods in the recommended and prescribed textbooks for secondary school mathematics (eg $y = mx + c$ *to success*). On the other hand, the curriculum does not pinpoint a specific strategy to use in this regard. Instead, the curriculum only proposes that learners be introduced to proportional reasoning from the junior through to the senior secondary phase (Grades 8 – 12), and that the instruction should focus on ratio and rates and apply the concepts to solving real-life problems (MEAC, 2015). In fact, the topic of proportion is only introduced in Grade 9 where learners are taught the difference between direct and indirect/inverse proportion.

I observe that there is a scarcity of tasks that feature ratio and proportion topics in the MCM task data base, hence a need to add more of these tasks. Studies have asserted that ratio and proportion are mathematical concepts that have many applications in real life (Ben-Chaim et al., 2012; Phuong & Loc, 2020; Mahlabela, 2012; Charles-Ogan & George, 2015). It is even asserted that proportional reasoning is a "pervasive activity that transcends topical barriers in adult life" (Ahl et al., 1992, p. 81) and yet the reality indicates that many adults are not proportional thinkers (Lamon, 2007). Therefore, apart from school contexts, the MCM app could also be useful in advancing the literacy of proportional reasoning among adults through its readily available tasks.

2.9 CONCLUSION

This chapter presented the conceptual constructs that frame this study. It discussed the concepts of m-learning, the MCM project, authentic tasks, mathematics trails, visualisation and spatial measurements in relation to their benefits and challenges in this study's context. The review showed that teachers can potentially learn from using mobile technologies such as smartphones in the MCM app project and visualisation processes to teach spatial measurements, ratio and proportion in authentic and realistic tasks, if they can overcome the difficulties involved. The next chapter explains the theoretical framing of this study.

CHAPTER 3

THEORETICAL FRAMEWORK

3.1 INTRODUCTION

The purpose of this chapter is to examine the theories that are relevant to and ground my study. I discuss these theories in relation to how they inform the design and analysis of the study. The pertinent theories are the *Realistic Mathematics Education* (RME) theory and the *Mobile Pedagogical Framework* (iPAC – Personalisation, Authenticity and Collaboration). I use the RME theory to drive the design and implementation of authentic and realistic tasks in this study's m-learning environments and the analysis of the data for conceptual teaching. The iPAC mobile framework directs the implementation and analysis of smartphones as mobile technologies in the study.

3.2 REALISTIC MATHEMATICS EDUCATION

The origins of the RME theory dates back to the early 1970s when a Jewish-German-born Dutch mathematician and his friends saw a need to reform the teaching of mathematics. The researchers viewed mathematics as a human activity (Freudenthal, 1973; Gravemeijer, 1994), with the notion that in order to be of human value, mathematics must be connected to reality, be close to learners and relevant to their experiences of society (Freudenthal, 1973; Van den Heuvel-Panhuizen, 2001; Ginsburg et al., 2004). The theory was the result of a curriculum framework that was developed for schools in the Netherlands, and also in response to the New Math textbook series entitled *Mathematics in Context* (Fauzan, 2002). The RME oriented curriculum propagates an approach to mathematics education in which learners are presented with learning activities that are 'experientially' real to them (Gravemeijer, 1994). This has been found to be effective when teaching is done in realistic and authentic contexts that encourage the development and application of concepts which are meaningful to learners (Van den Heuvel-Panhuizen, 2003). Thus, one of the determining characteristics of the RME theory is an approach that foregrounds realistic contexts.

The theoretical underpinnings of RME offer learners an opportunity to think mathematically in their everyday living. According to Gravemeijer (1994), in RME, everyday problems are starting points for mathematical problem solving, which in turn renders the subject as an activity. Barnes (2005) points out that in RME theory, problems can arise from either real-world situations, or 'imagined' realities with a purpose of converging the mathematics learned in the classroom with the real world. My study explores a combination of both real-world contexts and some aspects of imagined realities, in a sense that the tasks used are connected to real-world objects and at the same time some of the questions asked also reflect imagined realities. According to Lestari and Surya (2017), apart from the real relationship with the world, realistic contexts can also refer to the real problem situation in learners' minds. Van den Heuvel-Panhuizen (2003) says that:

> the term 'realistic' refers more to the intention that students should be offered problem situations which they can imagine … than that it refers to the 'realness' or authenticity of problems. However, the latter does not mean that the connection to real life is not important. It only implies that the contexts are not necessarily restricted to real-world situations. The fantasy world of fairy tales and even the formal world of mathematics can be very suitable contexts for problems, as long as they are 'real' in the students' minds. (pp. 9–10)

In a nutshell, Van den Heuvel-Panhuizen (2003) argues that RME involves both problems based on real-world situations and problems that learners perceive as real. According to this view, RME is a learning approach that uses contextual problems that relate to the real world, whether physical or imagined. The focus of RME theory is on relational and conceptual understanding, rather than on rote learning. Makonye (2014) supports this view by stating that RME helps learners to see the close connection between mathematical conceptual knowledge and mathematical procedural knowledge. This implies that realistic and authentic contexts enable learners to actively engage in problem solving and construct their own meaning and understanding of mathematical concepts.

3.2.1 Progressive mathematisation

The theoretical underpinning of RME is developed alongside the idea of progressive mathematisation as an important aspect of learning mathematics. Biccard and Wessels (2011) refer to mathematisation as "translating from the real world to the mathematical world" (p. 378) and vice versa. In other words, mathematisation is when something is rendered mathematical even though it is not necessarily supposed to (Jablonka & Gellert, 2007). The process of mathematisation allows learners to use school mathematics to solve real life problems. This translates the process into an activity of **doing** mathematics (Drijvers, 2020). Hence, in RME, mathematisation is seen as a human activity and can best be learned by doing it (Freudenthal, 1973), whereby learners engage in activities for the purpose of generality, certainty, exactness and being concise (Rasmussen & King, 2000). In addition, Arcavi (2002) views the process of mathematisation as an idea that bridges the gap between everyday mathematics and academic mathematics.

Treffers (1987) recognised two interlocking components of mathematisation, namely *horizontal* and *vertical* mathematisation. These are sometimes described as "two-way mathematisation" (Van den Heuvel-Panhuizen, 2010, p. 3). The components are what distinguish the RME approach from other learning and teaching approaches (Treffers, 1987; Van den Heuvel-Panhuizen, 2010). Several authors maintain that the process of mathematising horizontally involves the process of developing mathematical meaning from out of school experiences, whereas vertical mathematisation is the formalisation of activities within the domain of mathematics as a subject. For example, Van den Heuvel-Panhuizen (2003) explained that "to mathematize horizontally means to go from the world of life to the world of symbols; and to mathematize vertically means to move within the world of symbols" (p. 12). Selter and Walter (2020) also say that horizontal mathematisation is the processes of bridging the real-world contexts to formal symbolic mathematics and vertical mathematisation is concerned with the process of doing activities *within* the formal symbolic realm.

In addition to the discussion of the difference between horizontal and vertical mathematisation, Figure 3.1 below shows the cyclic character of the mathematisation process developed by De Lange (2006). Based on this cycle, the first step of mathematisation (1) entails the learners working on understanding the problem or task at hand by identifying concepts that are relevant to mathematics within the problem. Then from the identified mathematical concepts, the second step (2) follows the sieving away of irrelevant elements that exist in reality by formulating the problem into a mathematical model. In the third step (3), learners solve the problem while at the same time reflecting on the process used for the solution of the problem. The last and fourth step (4) involves the interpretation of the mathematical solution against the original and realistic situation (De Lange, 2006; Suaebah et al., 2020).

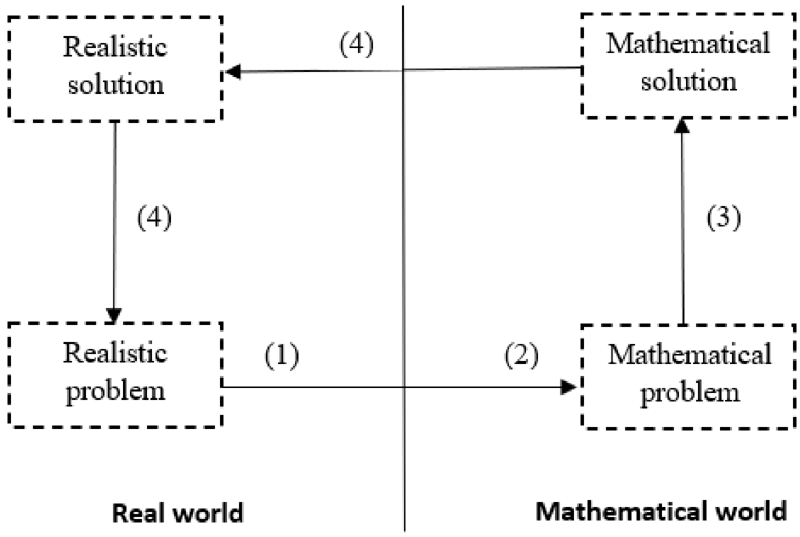


Figure 3.1 The mathematisation cycle
(Adopted from De Lange, 2006, p.17)

According to Suaebah et al. (2020) the focus of the first two steps is on horizontal mathematisation where the realistic problem is changed into a mathematical symbol, whereas the third step is the journey through the process of vertically mathematising the problem by using mathematical symbols to solve the problem. The fourth step of the cyclic character involves the changing of the solution from mathematical symbol to realistic solutions. Clearly, the whole process of mathematisation means that learners use their informal strategies to describe and solve a contextual problem, which in turn can enhance their understanding of using the appropriate mathematical language and application of suitable algorithm (Menon, 2013; Jablonka & Gellert, 2007; Loc & Hao, 2016; Arifin et al., 2021).

Literature studies within the RME theory (eg see Hershkowitz et al., 1996; Barnes, 2005) advocate that to promote mathematical conceptual understanding, learners should first undergo the process of horizontal mathematisation before entering the vertical mathematisation process. It is noted that the careful use of informal strategies to describe and solve contextual problems before using mathematical language leads to a better understanding of concepts and procedural knowledge. According to Treffers (1993), this method of mathematisation “let[s] the rich context of reality serve as a source for learning mathematics” (p. 89). Arcavi (2020) supports the use of horizontal mathematisation as a springboard for vertical mathematisation because it “builds on students’ knowledge from outside mathematics as a main resource and it relies on their common sense and their capacity to harness ad hoc intuitive and non-formal strategies as the main steppingstones upon which to build further” (p. 88). Furthermore, Makonye (2014) claims that the interconnection of horizontal and vertical mathematisation helps learners to see the close relationship between mathematical conceptual knowledge and mathematical procedural knowledge. This means, in RME theory, ‘realistic’ and authentic situations should be departure points for learning algorithmics and procedures in mathematics education.

It should not go unnoticed, however, that there are particular features of RME that have been criticised for disregarding the mechanistic aspects of learning. For example, Wittmann (2020) argues that the RME approach prioritises horizontal over vertical mathematisation, and according to De Bock et al. (2020), this results in too much freedom given to learners in their quest to find their own methods of problem solving. To this effect, De Bock et al. (2020) and colleagues favour an approach that encompasses a balance in this two-way mathematisation processes. On the other hand, den Heuvel-Panhuizen (2020) counteracts this criticism by arguing that

> excessive freedom that is supposedly given to students to construct their own solution methods is a wrong interpretation of the RME aim to break with the mechanistic approach of solving particular types of problems always in the same manner, but instead stimulate students to choose a solution strategy that suits the problem the students have to solve. (p. 13)

Another criticism advanced by Selter and Walter (2020), is concerned with the limited interpretation of the concept of context. The authors argue that studies within RME limit the definition of context to learners’ realities with less consideration to pure numerical contexts. According to them, numbers can also be used as realistic contexts. However, on the contrary, Arcavi (2020), appears to appreciate the realistic context approaches because these contexts change the learning of mathematics from highly procedural and rule-oriented to using the real-world contexts as starting points for mathematisation.

Connected to this two-way mathematisation, are the six core teaching principles that direct the operationalisation of RME in teaching and learning contexts: namely the *Reality Principle* (RP), the *Activity Principle* (AP), the *Interactivity principle* (IP), the *Level principle* (LP), the *Intertwinement principle* (INP), and the *Guidance principle* (GP) (Van den Heuvel-Panhuizen & Drijvers, 2014). These principles reflect on how children should learn mathematics and on how mathematics should be taught. I use these principles to inform my analytical framework which I discuss in Chapter 4 of this study.

3.2.2 THE RME PRINCIPLES

3.2.2.1 The Reality Principle

The Reality Principle in the RME approach proposes that the learning of mathematics should start from contextual real-life problems and that learners must be able to draw from such contexts an understanding of 'what to do and why' during the development of the schematisation process (Van den Heuvel-Panhuize & Drijvers, 2014). The underlining principle here is that when teaching a particular topic, instead of starting from the abstraction of concepts, learners must first be exposed to real-life or realistic activities that relate to and contain concepts of the topic. When the activities are immersed in rich contexts that require mathematical organisation, learners are placed in the position of building mathematical conceptual structures from such contexts (Van den Heuvel-Panhuizen & Drijvers, 2014).

I have personally observed the common practice in some mathematics classrooms (including my own) of first defining abstract concepts before aligning such concepts to specific activities. Oftentimes, these activities do not conceptually enrich learners because of how they are not oriented in rich contexts that can be mathematised. Clearly, I recognise that this practice is not in line with the RME's Reality Principle because such teaching does not start from problem-oriented situations to enable learners to apply school mathematics when solving real-life problems (Van den Heuvel-Panhuizen & Drijvers, 2014; Kaur et al., 2020). It is in this context that Peters (2016) argues that in Namibia, learners can only benefit from the RME theory if their teachers are capable and knowledgeable in initiating the use of this teaching approach.

3.2.2.2 The Activity Principle

The Activity Principle stresses that learners should be active participants in their own learning (Van den Heuvel-Panhuizen & Drijvers, 2014), meaning that rather than being passive listeners and receivers of knowledge, learners should interact with the content through active participation in generating ideas (Salman, 2009). One way to do this is for teachers to create learning platforms that can actively engage learners in constructive ways. The MCM project as a mobile technology-oriented tool, fits well in this case study as empirical findings (eg Shieh, 2012; Crompton 2013; 2020) reveal that incorporating technology into learning activities of mathematics is often more hands-on and promotes active learning approaches that can improve learning focus and learners' understanding.

Therefore, in alignment with the activity principle in this study, when implementing and teaching authentic and realistic tasks of spatial measurements, ratio and proportion, it is important that teachers trigger learners' active involvement during the walking of the trails and solving of the tasks. This may contribute substantially in enabling learners to solve the tasks given to them with confidence. Barbosa and Vale (2018; 2020) reiterate that the supported use of the MCM app can motivate learners to actively participate and cooperatively work together, which in turn can promote positive attitudes and autonomous learning towards the subject. Moreover, one of the criteria in solving the MCM trail tasks is that the teacher ensures that the task keeps learners busy by involving them in activities such as measuring, counting and calculating (Jablonski et al., 2018).

3.2.2.3 The Interactive Principle

With the Interactive Principle, RME theorises that the learning of mathematics is a social activity. The interaction between learners and teachers, and between learners with one another is viewed as a factor that develops and improves the learners' communication skills of argumentation, critique, and justification in mathematics education (Van den Heuvel-Panhuizen & Drijvers, 2014). This principle coheres with Vygotsky's (1978) sociocultural learning theory which states that learning is a social activity where leaners create knowledge through interaction with peers and the teacher. Similarly, in the MCM mobile environment, the very process of interacting and communicating with others when solving real-life related problems encourages learners to verify and develop mathematics ideas. Therefore, in this case study, teachers are encouraged to interact with their learners to monitor and guide them when experiencing difficulties during the process of progressive mathematising. Furthermore, teachers are to encourage learners to interact with their fellow learners for the purpose of helping each other to solve contextual problems and construct the needed knowledge thereof (Trisnawati et al., 2018).

3.2.2.4 The Level Principle

The Level Principle acknowledges that the process of learning mathematics involves various levels of understanding that learners need to pass through. To this effect, learners go through different levels of understanding to find related concepts and strategies that help them solve realistic situation-related problems. These levels include the "ability to invent informal context-related solutions, to the creation of various levels of shortcuts and schematisations, to the acquisition of insight into how concepts and strategies are related" (Van den Heuvel-

Panhuizen, 2010, p. 5). Furthermore, the informal learning of mathematics to the formal and abstraction of concepts transits through three levels identified as situational levels (using informal and everyday knowledge to solve problems), referential levels (using models or representations to make sense of the problem situations) and general levels (abstraction and generalisation of concepts, models or representations). These levels reflect the process of progressive mathematisation, from informal to formal reasoning (Van den Heuvel-Panhuizen & Drijvers, 2014)

The formation of models is an important and integral aspect of the Level Principle for it helps to bridge the gap between informal, context-related mathematics and the more formal mathematics. Studies articulate that in order to fulfil this bridging function, models have to shift from a model of a particular situation to a model for all kinds of other, but equivalent, situations (Van den Heuvel-Panhuizen, 2003). Hence, for teaching calculation algorithmics in mathematics, the Level Principle is reflected in the didactical method of progressive schematisation (Treffers, 1982a; 1982b).

Therefore, in this study the Level Principle equips my teacher participants with the knowledge of understanding learners' "use of models or bridging by vertical instruments" (Zulkardi, 2002, p. 3). Using the Level Principle, teachers can monitor the learners' progressive mathematisation process of how they organise contextual problems, try to find the mathematical aspects of the problems and discover regularities and relations, and how all this leads to the formulation of models and formulas for the development of mathematical concepts.

3.2.2.5 The Intertwinement Principle

The Intertwined Principle suggests that mathematics strands are not isolated from each other, but rather connect within the domain and across other disciplines. This means that mathematical domains and concepts are interrelated and interdependent, and that they also interact with other fields of knowledge (Van den Heuvel-Panhuizen & Drijvers, 2014). Thus, during instruction, it is the task of the teacher to help learners see these connections without having to follow any prescribed order. For example, domains of number, geometry, measurement and data handling are not considered as isolated curriculum chapters but as heavily integrated. Sirait and Azis (2017) insist that providing knowledge about the relationship between mathematical concepts with other concepts or in everyday life, is especially needed by learners to solve problems encountered in everyday life. For mathematics to be meaningful to learners, the theory of RME stresses that through the Intertwined Principle, teachers should create and harness teaching contexts that enable learners to see conceptual relationships between mathematics and other disciplines and real life (Van den Heuvel-Panhuizen & Drijvers, 2014).

As discussed earlier, promoting connections between mathematics, other disciplines or the real world is important for the development of learners' conceptual understanding and construction of knowledge (Anthony & Walshaw, 2009; Presmeg, 2014; Silvey, 2019). The intertwinement between concepts or between topics in mathematics makes the teaching and learning of the subject holistic rather than compartmentalised (Revina & Leung, 2018; Yilmaz, 2020). Therefore, in this study, when solving MCM tasks teachers are urged to provide learners with opportunities to see how the concepts intertwine with each other within the subject and other fields.

3.2.2.6 The Guidance Principle

The Guidance Principle recognises the important role played by teachers in providing appropriate guidance and instructions that enable learners to reinvent mathematics. According to Van den Heuvel-Panhuizen and Drijvers (2014), the RME theory approach provides "an opportunity for students to rediscover mathematical ideas and concepts with adult guidance through exploring various situations and real-world" (p. 376). The concept of scaffolding in this context can help us understand the perspective of the Guidance Principle within the RME theory. A classic definition of scaffolding from Wood et al. (1976) states that scaffolding is ''the process that enables a child or novice to solve a problem, carry out a task, or achieve a goal which would be beyond his unassisted efforts'' (p. 90). The authors further characterised scaffolding as an

> interactive system of exchange in which the tutor operates with an implicit theory of the learner's acts in order to recruit his attention, reduces degrees of freedom in the task to manageable limits, maintains 'direction' in the problem solving, marks critical features, controls frustration and demonstrates solutions when the learner can recognise them. (p. 99)

From this perspective, it was expected that the teacher participants of this study would skilfully guide conversations during the solving of the MCM trail tasks by drawing learners' attention to different strategies of solving the problems. Particularly, the mathematisation process is one area that has been found to be difficult for learners (Buchholtz, 2017), thus Van den Heuvel-Panhuizen (2020) suggests that the process be done under the guidance of the teacher. Just as with the scaffolding principle, Zolkower et al. (2015) recommend that teachers should always balance between loosening or tightening their guidance, considering the degree of challenge of the task at hand.

In addition, Van den Heuvel-Panhuizen and Drijvers, (2014) warn that the RME Guidance Principle should not in any way conflict with the Activity Principle. At first reaction, one could conclude that the two principles are at loggerheads with each other. By their ideologies, it seems that the Guidance Principle embraces a teacher-centred approach, whilst outwardly, the excessive time and freedom given to learners in the Activity Principle favours a learner-centred approach. Thus, this can make it difficult for teachers to switch between the two in order not to favour one over the other. Actually, this was also one of my concerns during the implementation of the MCM trail tasks in this study where the teacher participants were faced with the challenge of striking a balance between the Guidance and Activity principles. Moreover, the fact that Namibian teachers are accustomed to teacher-centred approaches (Albin et al., 2020; Kapenda, 2008), it is normal to anticipate that the teachers could find it difficult to shift from a teacher-centred to a learner-centred approach in their quest of implementing the RME Guidance Principle (Van den Heuvel-Panhuizen, 2010).

3.2.3 RME theory and the MCM app tasks

The design and implementation of authentic tasks in this study aligns well with the underpinnings of RME theory. For example, the tenets of Reality and Activity principles can guide the creation of experimentally real tasks for learners to engage in immediate and personally meaningful mathematical activities (Fredriksen, 2021). This in turn grants learners the opportunity to use mathematics as a tool to organise problems in realistic contexts, a process called horizontal mathematisation (Van Den Heuvel-Panhuizen, 2003). Furthermore, conceptual understanding of spatial measurement, ratio and proportion can then be attained through the schematisation of different models formulated by learners through the guidance of the teacher. In RME, this process is often referred to as learners' guided reinvention of mathematics (Drijvers, 2020; Van den Heuvel-Panhuizen & Drijvers, 2014).

In addition, the framing, particularly the six core teaching principles discussed, guided the preparation, planning, presentation and reflection of learning materials in the study. The tasks, whether from realistic or original contexts, were linked to mathematics with reality through outdoor activities with the aid of the MCM app project (Cahyono & Ludwig, 2019). During the walking of the trails, the six principles were carefully observed and implemented by the participating teachers, and this brought relevance to the tasks in the study. The principles also guided the engagement of learners in the tasks and ensured that learners were helped to mathematise and reinvent mathematical ideas and concepts through the guidance of teachers when exploring various situations and real-world problems within the context of the MCM app project (Ulandari et al., 2019).

Doorman (2001) advocates that in RME, learners should be presented with well-chosen contextual problems that offer them opportunities to apply informal strategies for the purpose of expanding their conceptual understanding. Instead of being passive receivers of knowledge, the RME theory urges teachers to give room to learners in taking the lead of finding their own strategies to solve problems, thus enabling them to construct their own mathematical knowledge. This, however, does not mean that learners are to be left on their own in the learning process, but rather are guided by teachers to construct knowledge through progressive mathematisation. Using this process, learners can develop mathematical knowledge from informal solution methods that can then facilitate the creation of formal and general rules (Gravemeijer, 1994).

Again, the RME approach advocates that teachers provide opportunities for learners to explore strategies and methods that suit them when solving the tasks. Although literature in this line of teaching approach recommends that teaching starts from informal setups that are familiar to learners' experiences before dealing with abstract mathematics and symbols (eg Menon, 2013; Loc & Hao, 2016; Arifin et al., 2021), learners may still be left to choose their own strategies that suit their understanding. So, in this case, some learners may choose to vertically mathematise, meaning using formal strategies such as algorithms and symbols to solve the problems, whereas others may use informal strategies that are clearer to them than algorithms and symbols (horizontal mathematisation). In this way, learners can be allowed to navigate their own strategies as far as when asked to explain, they can do so without difficulty (Barnes, 2005).

Furthermore, in this study, visualisation supports the learners' cognitive movement between the real world and the world of symbolism (Brown, 2015). In this way, the convergence of visualisation and mathematisation can enable learners to see the not-so apparent mathematical concepts and patterns that exist in the real world when solving the authentic and realistic tasks within the MCM project (Rudziewicz et al., 2017).

The six RME principles discussed above form an integral part of the analytical framework of this study – see the analysis section under the methodology chapter.

3.3 THE iPAC MOBILE PEDAGOGICAL FRAMEWORK

The iPAC framework, developed by Kearney and colleagues (Kearney et al., 2012), draws its underpinnings from a socio-cultural perspective. The framework focuses on the three signature pedagogies of m-learning: Personalisation, Authenticity and Collaboration. These are the meanings of the acronym iPAC. Compared to other frameworks on m-learning, the iPAC framework puts pedagogy at the center of m-learning, rather than technology, to examine how it can support learning (Kearney & Maher, 2013). Kearney et al. (2019) observe that "the pedagogical characteristics of m-learning have been given less consideration in the literature than the technical aspects" (p. 752). In other words, a pedagogical perspective on m-learning is often overlooked in the literature, and there is no clear or rigorous method to capture it. Therefore, the iPAC framework was designed to evaluate or measure how learners use mobile technologies in learning activities. See Figure 3.2 below, showing the three constructs of m-learning - Personalisation, Authenticity and Collaboration.

Traditional learning is often limited by the physical settings and the scheduled times of the learning activities, such as classrooms and school periods. M-learning, on the other hand, can overcome these limitations of time and space (Khan et al., 2015; Sharples et al., 2019). Therefore, according to Phelan (2017), the iPAC framework proposes m-learning that incorporates a time-space continuum to capture this distinctive feature. Central to the framework is the concept of 'time-space' which influences the way learners experience the three distinctive pedagogical features in an m-learning environment. Kearney et al. (2015) explain that the concept of time-space is "the organisation of the temporal (scheduled/flexible; synchronous/asynchronous/polysynchronous) and spatial (formal/informal, physical/virtual) aspects of the m-learning environment" (p. 49). The constructs are explained using sub-themes that pinpoint to critical features of m-learning from a pedagogical point of view: personalisation (learner agency and control), authenticity (situated learning experiences), and collaboration (connections to people and resources).

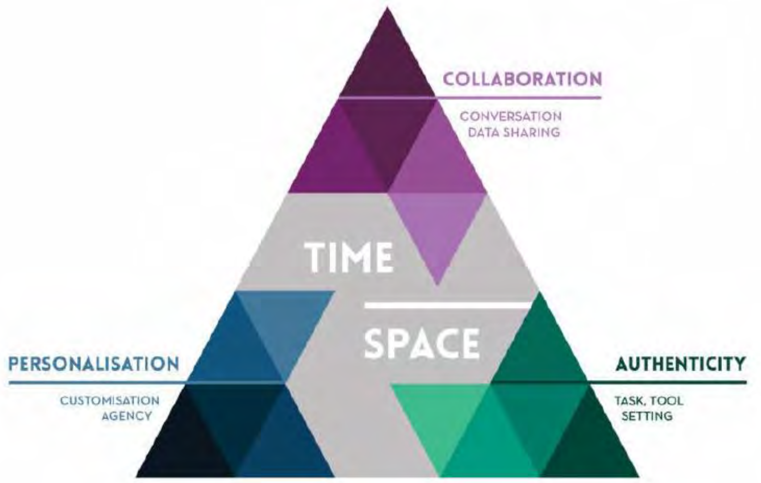


Figure 3.2: An illustration of the three constructs of iPAC
(Kearney & Maher, 2019)

3.3.1 The personalisation construct

Mobile devices are highly personal in nature, and this makes them potentially support personalised learning in many ways. Thomas et al. (2019) explain that personalised learning is a form of instruction that tailors teaching according to the individual needs of learners. Therefore, in the iPAC framework, the personalisation construct focuses on the agency and customisation of learning materials that supports m-learning (Kearney et al., 2012). Kearney et al. (2015), further explain that

> High levels of personalisation would mean the learner enjoys a high degree of agency in appropriately designed m-learning experiences, with the ability to customise and tailor both tools and activities, driven by a strong sense of ownership. (p. 49)

When learners customise their own learning by controlling the way in which they use the available technologies at their disposal (such as smartphones for example), their level and sense of ownership increases (Schuck et al., 2016). Eventually, this can lead them to choose their own resources and local information for learning. On the other hand, the sub construct of agency can be realised through activities that engage learners in meaningful learning tasks (France et al., 2021). What this means is that using smartphones as pedagogy tools within the MCM m-learning environment can significantly activate and strengthen learners' agency of their own learning and subsequently increase their will power on choices of where, what and how they learn. For example, learners can freely choose the places (physical or virtual), pace and time of their learning contexts (Koenraad, 2019).

3.3.2 The authenticity construct

The *authenticity* feature concerns the settings, tasks and tools of m-learning in both formal and informal settings. Kearney and Maher (2013; 2019) advance that apps designed for teaching purposes should engage learners in critically analysing online content texts or images within real-life situations. The authenticity feature is thus purposed to support authentic learning using settings, tasks and tools that are closer to learners' realities (see Figure 3.2 above). Koenraad (2019) supports the idea that m-learning should provide learning and teaching environments that are flexible and responsive to the learners' needs, preferences and contexts, where the tasks are designed to help learners have smooth and seamless learning experiences rather than complicated and challenging ones. Moreover, mobile devices such as smartphones and their relative apps have been found to extend and enhance learning to authentic and real-world settings (Parsons, 2014b). In the same manner Kearney and Maher (2013) claim that the authenticity feature of the iPAC framework should allow the meaningful use of mobile devices in order for learners to explore mathematics in real-world contexts. Alternatively stated, the authentic construct focuses on how realistic and relevant the m-learning activities are and how learners use apps and tools that mimic real-world practices.

3.3.3 The collaboration construct

The third principle, *collaboration*, consists of conversations and data sharing, which depicts the way learners "engage in negotiating meaning [and] making rich networking connections to other people and sharing information and resources across time and space" (Kearney et al., 2012). The sub-construct of conversation suggests that mobile technology allows learners to engage in a dialectical way with each other and with their teachers, whereas the data sharing recognises the inclusive experience of data production of mobile devices. According to Koenraad (2019),

> m-learning allows students to enjoy a high degree of collaboration by making rich connections to other people and resources mediated by a mobile device. Social interactions, conversations, and dialogue are fundamental to Vygotskian learning. Sharing conversational spaces mediated by mobile devices can be conducive to timely, personally tailored feedback from teachers, as well as rich peer interactions, leading to learners' negotiated meaning-making (conversation). In shared, socially interactive environments, learners can consume, produce, and exchange information and (self-generated) resources with peers, teachers, and other experts (data sharing). (p. 231)

From the above assertions, it can be induced that in m-learning platforms, collaboration is experienced through working together and sharing ideas through discussions. Kearney and

Maher (2019) note that conversations initiated from collaborations can be extended from in-person to more “flexible, online conversational spaces of using apps” (p. 143). Moreover, it is also important to mention that the networks created by m-learning do not only end at learners collaborating with fellow learners and teachers within their school environment but can also be extended to connecting with mathematics experts from larger communities through the third space contexts (Schuck et al., 2016; Burden & Kearney, 2017; Kearney et al., 2020).

3.3.4 The iPAC framework and the MCM app

In this study, when designing m-learning environments, teachers are guided by the three pedagogical features of the iPAC framework. For example, the use of smartphones to access the needed information when locating hidden tasks in the MCM trails can grant learners a sense of ownership. On their own, learners are left to navigate the environment in search of the given tasks and find ways to modify the tasks to suit their ways of solving them. Consequently, the experiences that arise from this type of an m-learning context can then help learners to personalise their own learning.

The authenticity feature is an integral concept of this study. Generally, smartphones are used as supporting tools that link learners to specific outdoor tasks that are connected to mathematics trails. Thus, their portability and wireless nature extends the learning environment beyond the classroom to authentic and appropriate situations and contexts (Naismith et al., 2004). This framework also provides a useful lens to explore the use and implementation of technology and outdoor trails using the MCM platform in the study. Further, I use the framework to analyse and interrogate the participants’ m-learning experiences and perceptions of their engagement with learners in formal and informal settings and schedules (or time-space configurations) (Nortvig, 2014; Kearney & Maher, 2019).

In this study, the participating teachers also use the collaborative feature of the MCM app to facilitate the engagement of learners in meaning making, through rich networking connections with their peers, teachers and experts who are outside their physical contact (Schuck et al., 2016; Burden & Kearney, 2017; Kearney et al., 2020). The technical use of the MCM app can initiate conversations and sharing of information and resources related to the outdoor trails among learners and teachers. Furthermore, the immediate feedback provided by the MCM app fosters a collaborative learning environment where learners feel that their contributions towards the MCM app project are equally appreciated. The capturing and sharing of images from their immediate environment using smartphones can also promote learners' sense of agency in their learning trajectories.

3.4 CONCLUSION

In this chapter I established the theories that frame this study. These are the Realistic Mathematics Education theory (RME) and the Mobile Pedagogical framework (iPAC). In my discussion, I unpacked the common perspectives and characteristics of the RME theory which include the process of mathematisation and the six teaching operationalisation principles of RME, namely *Reality Principle*, *Activity Principle*, *Intertwinement principle*, *Level principle*, *Interactivity principle* and *Guidance principle*. Furthermore, the iPAC, a research-inspired, digital pedagogical framework that emphasises a socio-cultural perspective of learning (Kearney et al., 2020), was thoroughly discussed in this chapter. The three iPAC constructs, namely *personalisation*, *authenticity* and *collaboration* were also discussed, and I showed how these (together with the six RME principles) inform this study. In the next chapter, I outline and discuss the research methods of the study.

CHAPTER 4

RESEARCH METHODOLOGY

4.1 INTRODUCTION

This chapter explains the research design and methodology of this case study, which aimed to analyse and understand how eight selected mathematics teachers implemented authentic tasks in a mathematics trail to facilitate the teaching of area, volume, ratio and proportion for conceptual understanding. The chapter begins with outlining the research orientation and paradigm that guided the study. Then, it describes the sampling and data collection processes and the tools used for collecting and analysing data. The chapter also discusses the validity and reliability of the study, and the ethical issues that were considered during the data collection. The chapter concludes by highlighting some research challenges encountered when carrying out this research study.

4.2 RESEARCH ORIENTATION

This case study used an interpretive research paradigm and a qualitative research framework to explore the experiences and actions of selected mathematics teachers who used the MCM realistic tasks to teach concepts of area, volume, ratio and proportion. An interpretive paradigm is a world view that seeks to understand the subjective world of human experience and how meaning is constructed and negotiated in social practices (Patton, 2002; Cohen et al., 2007; Cohen et al., 2011). As an interpretivist researcher, I aimed to understand and describe the subjective meaning making of the teacher experiences and their actions, rather than to predict what their experiences would be (Bertram & Christiansen, 2015). To do this, I had to immerse myself in the data related to the participating teachers' actions and experiences of designing and implementing the MCM tasks.

The choice for this research approach provided me with an appropriate platform to examine how the participating teachers interacted with the MCM authentic and realistic tasks in the context of the RME teaching framework and how they interpreted and enacted these tasks in their quest to conceptually teach the selected topics. The approach enabled me to construct meaning from the participants' experiences of how conceptual teaching can be facilitated through authentic and realistic tasks that are implemented in m-learning platforms. Using the interpretive paradigm positioned me in the subjective world of the participants' actions and perspectives regarding their interactions with mobile technologies in outdoor trail activities (Cohen et al., 2011; Cohen et al., 2018). In this way I was able to understand the meanings and interpretations of the eight selected teachers' social realities of creating the MCM tasks and implementing them within the RME teaching principles. The RME theory acknowledges that learning is a social and contextual activity that involves interactions with others and with the environment (Van den Heuvel-Panhuizen & Drijvers, 2014). Thus, through observations and interactions with the selected teachers, I could comprehend how they interpreted or made conjectures when creating and implementing the MCM tasks.

4.3 RESEARCH METHODOLOGY

4.3.1 Qualitative case study

This research adopted a case study method to investigate and analyse a single case that captured the complexity of the object of study (Stake, 1995). A case study method is a detailed analysis of an individual circumstance or event that is chosen because something new is in operation (Newby, 2010). The case of this study consisted of a cohort of eight mathematics teachers who designed and implemented mathematics trails in and around their schools. Yin (2009) ascertains that the case study method allows researchers to retain the holistic and meaningful characteristics of real-life events, in my case the use of mathematics trails and smartphones. In this context, as the teachers designed the trail tasks and implemented them within the RME theoretical framework and the MCM m-learning environment, this study aimed to make sense of how the teachers used the tasks to teach for conceptual understanding.

The study followed a qualitative research approach because of my desire to understand in-depth the selected participants' meaning making of the case of using authentic/realistic tasks for conceptual teaching (Merriam, 2002). The research approach was mainly qualitative although a small element of quantitative analysis also took place. Qualitative research is based on the subjective assessment of attitudes, opinions and behaviour of participants (Kothari, 2004). Qualitative research also has the advantage of encompassing in-depth inquiry and subjectivity, thus providing thick descriptions of phenomena (Cohen et al., 2007). Therefore, following this approach assisted me as a researcher to understand the social phenomenon of using realistic tasks to teach outdoors for conceptual understanding of spatial measurements, ratio and proportion topics from the participants' point of view. This decision was informed by one of the strengths of using a case study approach, which Cohen et al. (2007) described as allowing researchers to observe effects in real contexts.

4.3.2 The unit of analysis

The units of analysis of this study were twofold.

1) The observed interactions of the participating teachers (ie teaching) during the mathematics trails related to topics of area, volume, ratio and proportion using the RME principles through mathematics trail activities.
2) Their perceived experiences of the design, use and implementation of authentic and realistic tasks in outdoor trails using smartphones, within the MCM platform.

Grünbaum (2007) explains that the unit of analysis is the knowledge the researcher wants to obtain from the case, which usually comes from the participants or their actions. Yin (2009) states that the unit of analysis is the data and the analysis of data in relation to the research questions.

4.3.3 Research site and sampling

Pillay (2006) argues that "the quality of a piece of research not only stands or falls by the appropriateness of methodology and instrumentation but also by the suitability of the sampling strategy that has been adopted" (p. 50). Therefore, to select my research participants, I used purposive sampling in combination with convenient sampling. Mertens (2005), suggests that to gather rich information, researchers should purposively select samples that allow them to study a case in-depth when working within the interpretive paradigm. In addition, Cohen et al. (2011) recommend that convenience sampling can also be opted for, to easily access participants who are conveniently near and available for participation in the study. With this advice in mind, I purposively and conveniently selected and worked with a cohort of eight mathematics teachers from three different secondary schools in Kunene Region, in north-east Namibia. My choice for combining these two sampling methods was that purposive sampling would allow me to focus on specific aspects of the phenomenon under investigation and to obtain rich and detailed data from the participants. Convenience sampling, on the other hand, allowed me easy access to the participants without spending too much travelling time and financial resources to work with them. To this effect, the criteria of selecting the participants was mainly influenced by these two sampling methods.

The criterion of choosing the participants conveniently was that at least three teachers were to teach at the same school. The reason for this was to allow these teachers to form a small group or cluster to work together as a team to design and implement the MCM tasks and trails.

Therefore, the schools were conveniently selected based on the number of teachers, availability and accessibility (Cohen et al., 2011). Moreover, I have a good relationship with the management and the teachers of the three schools I had in mind, which facilitated a smooth recruiting and buy-in process for the participants and their learners. Also, due to the COVID – 19 restrictions that were imposed on public gatherings including schools at the time, I had to recruit participants who were willing to participate in the study and were accessible through personal networks in order to overcome logistical barriers that arose from the pandemic.

I considered purposive sampling to ensure that the conveniently sampled teacher participants were qualified to teach Grade 9 mathematics content level. In addition, these teachers were also to have the basic knowledge of operating GPS enabled smartphones. This technological device was central to my study as the MCM project in m-learning environments cannot fully be realised without it. Although I worked with Grade 9 learners in the implementation of the trails, the learners' participation was voluntary. They were only used by the teachers to implement the mathematics trails. I did not use the learners per se, to gather data. The data used for the study emanated from the teachers. Even though I used Grade 9 learners, the participating teachers did not necessarily have to be teachers of Grade 9 as such, as long as they were qualified to teach Grade 9 mathematics.

Adopting two methods of sampling contributed to the validity of the study. By only choosing convenient sampling, for example, I may have missed out on important criteria that were necessary for quality of the data. Similarly, by only choosing purposeful sampling, I may have missed criteria that would also have compromised the quality of the data.

It should also be noted that purposive sampling and convenience sampling are both non-probability sampling, which means that the sample is not randomly selected from the population and may not be representative of the population characteristics or distribution. I address further issues of validity and reliability of the data under the Validity section (refer to Section 4.7 below). The using of both the purposive and convenience sampling in this study helped me as a researcher to balance the quality and quantity of the collected data.

It is worth mentioning that for my empirical data collection process I chose to work in the context of Grade 9 classes because, firstly, according to the new and current broad Namibian curriculum, Grade 9 is now an exit grade – from the junior phase (Grades 8-9) to the senior secondary phase (Grades 10-12). Secondly, the region where this study was conducted mainly consists of junior combined schools (Grades 1-9). So, looking at this situation, the use of Grade

9 classes gave me a high chance of selecting three appropriate schools that were needed for this study without much difficulty. The careful selection of my schools, therefore, was important to obtain a deep understanding of using authentic/realistic tasks to teach for conceptual understating, rather than to make empirical generalisations from a large population sample (Patton, 2002).

4.4 RESEARCH DESIGN

4.4.1 Introductory workshop

The research design of this study began with a once-off workshop session that introduced the nine participants (later reduced to eight) to the following concepts:

- the objectives and ideas of the MCM project;
- the notion and examples of authentic/realistic tasks;
- the process of developing and designing such tasks; and
- the concept of a mathematics trail and the principles of the RME theory.

The participants also learned how to use the MCM app and how to create and upload the tasks on the system's database. They also learned the rules and guidelines for operating the app. Moreover, this workshop helped me to establish a good and mutually respectful rapport with the teacher participants before I started collecting data.

4.4.2 The MCM cycles

The workshop session was then followed by three MCM cycles which each unfolded in five similar phases:

1) the creation of tasks for the MCM project;

2) the submission of the tasks to the MCM web portal for review;

3) the implementation of the mathematics trails;

4) the reflective interviews sessions; and

5) the focus group interviews session (see Figure 4.1 below).

The first MCM cycle served as a pilot for checking the data collection process and tools and helped to refine the analytical instruments of the study. To test the suitability of the chosen instruments, a pilot phase took place from November 2021 to February 2022.

The collection of data took place in the Phases 3 – 5. Figure 4.1 below shows the implementation order of the first two cycles and the phases at which the data collection process happened.

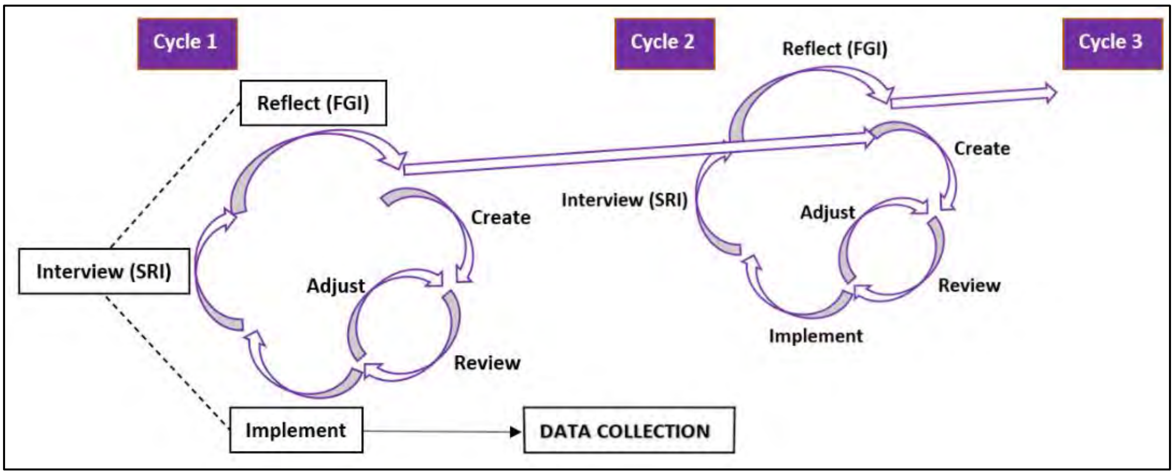


Figure 4.1: First and second MCM cycles as was implemented in the study.

4.4.2.1 PHASE 1: The creation of the tasks for the MCM project

In Phase 1, the three participating teachers (two for school B) and I created four to six authentic and realistic tasks at the teachers' respective schools. The tasks were based on real-life objects and situations in the schools and their surroundings. We jointly created the tasks following the MCM criteria of task design (refer to Section 2.4.2.1). We also ensured that the tasks were aligned to the mathematics junior level (Grades 8-9) of the Namibian curriculum. Moreover, the tasks were limited to topics of area, volume, and ratio and proportion (See Appendix Ten). To create the tasks for each school's trail we went outside and collected ideas. We took pictures of objects and determined the measurements of these objects. With this information at our disposal, we then started with the creation of the tasks for the MCM portal. We followed the same order for all the eight mathematics trails of this study. The tasks of the pilot mathematics trail for each school (Cycle 1) were from the school's surroundings. As the MCM cycles progressed, we expanded the locations of the tasks and explored other interesting places that were safe and easily accessible to the learners. Eight mathematics trails with a total number of 38 tasks (see Appendix Ten) on the topics of area, volume, ratio and proportion were created for the purpose of this study – see Table 4.1 below.

Table 4.1: Number of tasks in the trail of each school.

	Cycle 1		Cycle 2		Cycle 3	
	Trail	No. of Tasks	Trail	No. of Tasks	Trail	No. of Tasks
School A	TR1	5	TR4	5	TR6	4
School B	TR2	6	TR5	4	TR7	4
School C	TR3	6			TR8	4

4.4.2.2 PHASE 2: The submission of tasks to the MCM web portal for review

The MCM project requires that the tasks be reviewed by experts before they are published for the public on the app's website. Therefore, we submitted all the tasks of this study for review and revised them, based on the experts' feedback. After designing all the tasks for a specific trail, we created the digital version of the mathematics trail. However, we did not have to wait for the reviewers to use our tasks. We could still use the tasks in a trail that were not yet public and share the trail with the learners or users by giving them the corresponding code to access it. Thus, some of the tasks and trails that we used in this study, especially in the third cycle, were unreviewed. This was due to the delay from the task reviewers, which sometimes took longer than expected – a finding that was also highlighted by the teachers in their RIs (refer to Chapter 6 Section 6.4.3).

4.4.2.3 PHASE 3: The implementation of the mathematics trails

In the implementation phase, each school's three participant teachers divided their Grade 9 learners into three groups and led them along the same mathematics trail that they created as a group. To avoid overcrowding and for data collection purposes (see Section 4.5), we assigned each group to one of the three teachers (two for school B). This also helped us comply with the COVID-19 public gathering restrictions. For ethical reasons, we gave all Grade 9 classes of each school the chance to participate in at least one mathematics trail across the three cycles. This was possible because the selected secondary schools had only one or two Grade 9 classes. During the walk of the trail, each group solved at least two tasks that were spaced apart from the other trail walkers. If the number of tasks in the trail were not divisible by three, with for example four tasks, I would ensure that the group that was being recorded solved at least two tasks in that trail for data collection purposes.

Moreover, to ensure wider participation, we conducted the trail activities in the afternoons after the regular classes. This suited the schools' existing programmes, which required learners to attend afternoon studies. During the workshop, I informed the teachers that they would manage how learners walked and solved the trails. So, at the start of each trail, the teacher in charge allocated roles such as note taking, measurement taking and calculator and smartphone operators to the learners. Each teacher guided, supervised, taught and interacted with the learners during the trail and its activities. Richardson (2004) noted that this approach can work well because the trails are designed to stimulate mathematical communication among learners and teachers. In this phase, I began the process of collecting data through observations and video-recordings– see the data collection section for more details.

4.4.2.4 PHASE 4: The recall interview sessions (RIs)

Phase 4 of each cycle involved scheduled RIs as part of my data collection process. I conducted a RI with the teacher who taught and ran the trail, who was the focus of the VR in each cycle. For example, Table 4.2 under data collection shows that one teacher from each school was interviewed at the end of the implementation phase of each cycle.

4.4.2.4 PHASE 5: The reflective focus group interview session (FGI)

At the end of each MCM cycle, I conducted an FGI with the three teachers who participated in the trail walks, who were video recorded in Phase 3 for each school. The FGI aimed to elicit the teachers' experiences of the trail activities.

4.5 DATA COLLECTION

Three research instruments were used for data collection. These were an observation schedule, an interview schedule and FGI. The data collection of this study occurred from Phases 3-5 (see Figure 4.1 above). In Phase 3, VRs of the teachers' application of the RME principles while walking on the trails and solving of the tasks at each school were taken. In Phases 4 and 5 reflective interviews and FGIs were conducted and audio recorded to solicit the teachers' experiences of using mathematics trail tasks to teach outdoors.

4.5.1 Observations

For the purpose of collecting data on the implementation of the MCM tasks and how the selected teachers used the RME principles to conceptually teach the topics of area, volume, ratio and proportion, the trail walk of one teacher at each school per cycle was video recorded – refer to Table 4.2. Marshall (2006) defines observations as "the systematic noting and recording of events, behaviours, and artefacts in the social setting chosen for study" (p. 98).

During the observations, I acted as a non-participant observer. This means that I did not take part in the activity that I was observing. I wanted to have an unbiased and objective view of the participants and the settings, so I detached myself from the trail activities. I was involved in the design of the MCM tasks, but not in their implementation in the trails. Bertram and Christiansen (2015) warn that "it is practically impossible to observe everything that is happening in any situation, especially one where there are different interactions going on among a number of people" (p. 94). Therefore, I used a video camera to record all the interactions of the participating teachers with their learners in the trails.

The focus of my observations was on how the teachers made use of the RME principles in facilitating the trail activities. For example, during the observation and VR process, I concentrated on the teachers' interactions with their learners and how, within the framework of the teaching principles of the RME theory, the teachers used the trail activities to teach the selected topics of area, volume, ratio and proportion. The trail in this study was the medium or means of teaching, thus the focus was on how teachers engaged their learners in the trail activities. Although learners were not the focus of this study, it was important that their interactions with the teachers were also captured in order to have a holistic view of how they were engaged by their teachers to learn the concepts of the selected topics. Consent to include the learners in the VRs were thought after from their parents and legal guardians – see Appendix Five.

An observation schedule was developed, depicting the six RME teaching principles, namely the Realistic Principle (RP), Activity Principle (AP), Interactive Principle (IP), Level Principle (LP), Intertwined Principle (INP) and Guidance Principle (GP). Essentially, the aim of these outdoor observations was to establish how the principles were used by the participant teachers to teach the selected topics using the MCM realistic tasks in the trails. Eventually, the trail activities of all the eight teachers were video-recorded and transcribed for analysis.

4.5.2 Reflective interviews (RI)

The use of RIs helped me to gather data that could not be directly observed from the VRs (Patton, 2002). I conducted a one-on-one *reflective interview (RI)* with the teacher whose trail was video recorded in the implementation phase (see Table 4.2 below). Hancock and Algozzine (2006) note that interviews are a very common form of data collection in case study research as they enable the researcher to obtain rich, personalised information from the individuals or groups. Bertram and Christiansen (2015) also describe an interview as "a conversation between the researcher and the respondent" (p. 80). To keep the teachers in unending conversations, motivating them to discuss their thoughts, feelings and experiences about the MCM m-learning environment (Cohen et al., 2011), the interviews of this study were structured in the form of semi-structured interviews (see Appendix One). The main purpose of the RI was to elicit the teachers' views and perceptions of how they viewed the pedagogical aspects of using smartphones within the MCM m-learning environment. To this effect, an interview schedule was also developed to establish how the teachers viewed the use of smartphones in an m-learning environment of the MCM project – see Appendix One.

The interview schedule contained several seed questions that were aimed at evoking the teachers' thoughts to reflect on how they viewed the MCM m-learning environment from the perspectives of the iPAC mobile framework. The seed questions were often supplemented with further probing questions which were derived from participants' responses and found to be useful tools for digging deeper into the experiences described by the participants (Kear, 2012). During further probing I tried, as far as possible, to re-use the participants' own words in the questions in an attempt "to maintain coherence with the participants' descriptions of their experiences, and to clarify and check if I was understanding what they wanted to convey" (Etherington & Bridges, 2011, p. 13). This strategy formed part of validating my data obtained from the interviews with the teacher participants – refer to Section 4.7.1 on triangulation.

4.5.3 Focus Group Interviews (FGI)

The FGIs, in the reflection phase of each cycle, were also audio recorded. Carey (1994) defines an FGI as "using a semi-structured group session, moderated by a group leader, held in an informal setting, with the purpose of collecting information on a designated topic" (p. 226). As was reiterated by Morgan (1996), the interactions during the FGIs enabled me to generate and source data that pertained to the teachers' experiences of the MCM project and the use of authentic/realistic tasks in m-learning settings. The FGI was done at the end of each design cycle, and it involved the three teachers who were video recorded and interviewed in all the trails of each cycle – see Table 4.3 below for finer details.

Table 4.2: The allocation of teachers and a summary of the data collection sources.

	MCM CYCLE 1			MCM CYCLE 2			MCM CYCLE 3		
School	**A**	**B**	**C**	**A**	**B**	**C**	**A**	**B**	**C**
Teachers	Kamwi Sinvula Luke	Anna Calvin Betty	Moses Joshua	Kamwi Sinvula Luke	Anna Calvin Betty	-	Kamwi Sinvula Luke	Anna Calvin Betty	Moses Joshua
Video recording (8 recordings)	Kamwi	Anna	Moses	Sinvula	Calvin	-	Luke	Betty	Joshua
1-1 Reflective Interviews (8 interviews)	Kamwi	Anna	Moses	Sinvula	Calvin	-	Luke	Betty	Joshua
Focus group Interviews (3 interviews)	Kamwi, Anna, Moses			Sinvula, Calvin		-	Luke, Betty, Joshua		

4.6 DATA ANALYSIS

Data analysis is a process of making sense of raw data and communicating the essence of what they reveal (Patton, 2002). According to Cohen et al. (2007) this process involves organising and explaining the data in terms of the participants' perspectives, noting patterns, themes, categories and regularities. Newby (2010) adds that data analysis is an exercise to "get our data to release the information we need to answer our research question" (p. 395). Therefore, the analysis of this case study involved three types of data: VRs of the trail activities of the eight teachers who participated in the study, reflective semi-structured interviews with each teacher conducted individually and focus-group interviews with groups of three teachers per cycle, also semi-structured. The data was analysed using themes and categories derived from the literature, as well as emergent themes from the data itself, as explained in more detail below.

Maxwell (2005) stresses that "we should never collect data without substantial analysis going on simultaneously" (p. 95). Thus, the data analysis of this study happened parallel to the data collection process. For example, what this meant was that as I began to gather data of the VRs from the first design cycle of the study (pilot stage), I also started to analyse the data as it came in. This helped me to reflect on my methods and strategies and to keep refining the analytical tools for the subsequent cycles. Merriam (2009) observes "[t]he joint collection and analysis of data is essential in qualitative research" (p. 131). The data analysis of this case study followed the four stages of the generic protocol of qualitative analysis proposed by Newby (2010): (a) preparing the data, (b) identifying basic units of data, (c) organising data and (d) interpreting data.

4.6.1 Preparing the data: data transcriptions

The analysis of data for this case study began with the description of the trails contained in each MCM cycle, based on the data collected from various sources. For instance, data from all sources were carefully examined and transcribed word for word. I transcribed all the data myself, which enabled me to interact with the data at an early stage. The data transcriptions helped me to immerse myself in the study's data, which in turn enhanced my comprehension of the data findings. Due to the nature and design of my study, I had to pay close attention to data collection, spending time watching the video-recorded trail walks repeatedly so that I could devise a post-observation interview plan for each teacher. Kawulich (2017) advises that "once audio recordings or VRs are converted to text, it is the responsibility of the researcher to read through transcriptions along with the media source to ensure that the transcriptions are correct, prior to beginning analysis" (p. 773). So, after transcribing the data, I verified that the transcriptions were accurate by reading through and checking them with the research participants.

4.6.2 Identifying basic units of data: The analytical frameworks and coding

The next step in my data analysis process was to create analytical tools that I used to analyse the collected data, namely the RAILING and iPAC analytical frameworks. Coral and Bokelmann (2017) explain that an analytical framework helps to "organize research and provide a general list of areas or variables that will be used in any type of analysis" (p. 1). I used the analytical frameworks to analyse the data from the video observations and the one-on-one RIs, which were the two main research instruments.

4.6.2.1 The RAILING analytical instrument.

I developed the first coding structure, which I named RAILING, based on the six teaching principles of the RME theory as categories of observable indicators. The instrument was derived from my theoretical framework discussed in Chapter 3 and consisted of the six principles of the RME theory. RAILING is an acronym of **R**-Reality principle, **A**-Activity principle, **I**-Interactive principle, **L**-Level principle, **IN**-Intertwined principle and **G**-Guidance principle (see Table 4.3 below). The instrument provided observable indicators that aimed to link the principles to authentic/realistic tasks, conceptual teaching and visualisation processes. For example, to analyse how the Activity Principle (AP) was used to facilitate conceptual teaching during the mathematics trail activities, I considered four indicators coded AP1, AP2, AP3 and AP4. The indicator AP1 measured how often teachers ensured that learners were actively engaged in the solving of the tasks and discussions of specific mathematical concepts embedded in the tasks. The indicator AP2 helped me understand how the teachers created a conducive learning atmosphere within the trail to help learners be productive and comfortable with the teacher and each other. The AP3 indicator focused on if and how teachers created opportunities for learners to interact with each other, dividing responsibilities among them, and allow them to share ideas, strategies and solutions. The AP4 helped me to determine how the teachers assisted the learners to independently work on their own without help from the teacher. See Table 4.3 below for more detailed and finer descriptions of the other observable indicators.

To create data from VRs, Goldman et al. (2007) recommend that the researcher should strategically select video segments from an available corpus and use them for a specific analytic purpose. Therefore, to support the existence of RME principles in the trails, I chose to transcribe episodes that contained events of the use of the principles. I then used the extracts or excerpts of these transcripts together with screenshots of the teachers and their learners' interactions as supporting evidence of the existence or use of the principles in the trails. The transcripts of all the eight VRs were colour coded to look for similarities and differences as well as odd occurrences related to the six principles of RME. The findings that emerged from the analysis of VRs provided answers to the first sub research question of this study on the different ways the selected teachers made use of authentic outdoor tasks for conceptual understanding of area, volume, ratio and proportion topics.

Furthermore, the RAILING analytical framework used a Likert-type scale with descriptions such as *never*, *sometimes*, *very often* and *always*. In the Likert-type scale analysis, I employed basic statistical tools to illustrate the outcome of this analysis. Table 4.3 below shows the meaning of each description in the context of this study and Table 4.3a shows the coding descriptors for Table 4.3. Yin (2009) observes that the theoretical orientation in a case study helps to focus attention on certain data. Thus, the theoretical framework of RME guided my case study analysis of the first sub research question.

Table 4.3: The RAILING analytical instrument in Likert-type scale

RME principle/ Category	Description	Specific Observable Indicator: links with conceptual understanding. The teacher:	Descriptive Frequency			
			Never	Some-times	Very often	Always
RP	The use of meaningful real-life contextual problems that are related to specific concepts of mathematics' topics and themes	**RP1:** highlights the specific mathematical concepts that are embedded in the task				
		RP2: links the tasks that are in the trail to real life situations and explains to learners how such problems are likely to happen in real-life events.				
		RP3: highlights real-world aspects of the task to trigger learners' imaginations.				
AP	Learners actively participate in the learning process while walking the trails and give solutions to the problem tasks	**AP1:** actively engages learners in solutions to the tasks and the discussions on the concepts that are embedded in the tasks.				
		AP2: creates a conducive atmosphere during the trail so that learners engage in an easy and productive way.				
		AP3: creates opportunities for learners to interact with each by dividing responsibilities among them and allow them to discuss ideas, strategies and solutions.				
		. **AP4:** encourages learners to independently work on their own without help from the teacher.				
IP	Learning within the trail and the solving of the problem tasks is	**IP1**: allows learners to freely produce their solutions to the problem tasks without intervention.				

RME principle/ Category	Description	Specific Observable Indicator: links with conceptual understanding. The teacher:	Descriptive Frequency			
			Never	Some-times	Very often	Always
	conducted as a social activity where learners interact among themselves as well as the teacher.	**IP2**: promotes interactivity among learners during the trail and gives solutions to the problem tasks.				
		IP3: poses questions that lead to discussions that elicit the visualisation of specific concepts embedded within the authentic tasks.				
		IP4: engages learners in discussions that improves their communication skills of argumentation, critique and justification.				
		IP5: encourage the use of tools such as smartphone, calculator & tape measure to increase learner's participation				
LP	The use of models, concrete objects, manipulatives, schematisation, and tools to develop the learners' ability to mathematise both in a horizontal and vertical way.	**LP1:** is cognizant of the learners' current levels of understanding.				
		LP2: explains concepts at the right cognitive demand level of learners.				
		LP3 establishes the learners' prior knowledge.				
		LP4: demonstrates to learners how the tasks can be solved by using more than one strategy.				
		LP5: uses models to scaffold learners in understanding specific concepts.				
		LP6: uses models to encourage the visualization of specific concepts.				

RME principle/ Category	Description	Specific Observable Indicator: links with conceptual understanding. The teacher:	Descriptive Frequency			
			Never	**Some-times**	**Very often**	**Always**
		LP7: identifies learners' strengths and weaknesses in order to guide them towards the desired learning outcomes.				
		LP8: gradually develops learners' mathematical reasoning from informal and intuitive to formal and abstract.				
INP	The use of authentic tasks/problems to integrate topics with other strands or subject matter.	points out to learners the connections that exist between INP1: concepts within a specific topic/domain.				
		INP2: relates concepts to other strands.				
		INP3: relates concepts to other subject matter.				
		INP4: connects concepts to the real world.				
GP	The teacher guides the learners to use their informal models or strategies instead of directly use the formal ones	**GP1**: skillfully guides and promotes the exchange of ideas between learners and with the teacher.				
		GP2: poses open-ended questions to harness constructive discussions during the trail.				
		GP3: uses clear and understandable language to guide learners.				
		GP4: guides learners to understand specific and difficult concepts.				

RME principle/ Category	Description	**Specific Observable Indicator: links with conceptual understanding.** The teacher:	**Descriptive Frequency**			
			Never	**Some-times**	**Very often**	**Always**
		GP5: uses artifacts as guiding tools to help learners understand certain concepts and solve problems.				
		GP6: facilitates the transmission of mathematical knowledge among learners.				

Table 4.3a: The coding descriptors for Table 4.3

Coding	Categories	Descriptions (evidence of the use of the RME principle)
0	Never	There is no evidence of the use of the RME principle
1	Low	There are 1-2 incidences of the use of the RME principle
2	Medium	There are 3-4 incidences of the use of the RME principle
3	High	There are more than 4 incidences of the use of the RME principle

4.6.2.2 The iPAC analytical instrument

The one-on-one interviews were analysed using codes and indicators based on the adapted version of Kearney et al.'s (2012) and Kearney and Maher's (2019) mobile pedagogical framework. This framework consists of six subconstructs that fall under three principles: Personalisation, Authenticity, and Collaboration. The subconstructs are agency and customisation for Personalisation; setting/tasks and tools for Authenticity; and conversations and co-creation for Collaboration. The framework was applied to the interview transcripts to explore how the teachers perceived the pedagogical benefits of using smartphones in the outdoor environment of the MCM project. The analysis process involved two steps: Firstly, I collected the quotes and statements from the interviews that reflected the teachers' views on their learners' use of smartphones and the MCM app. Then, I coded each statement according to the indicator(s) that matched the subconstructs of the framework (see Table 4.4 below). This allowed me to identify all the statements from the teacher participants that indicated the presence of the principles in the trails. Secondly, I created a table based on the frequency of the coded statements to rate and scale the subconstructs of each principle. I counted the number of times each code appeared in the statements and compared it to the total number of codes for each subconstruct. For example, the conversation subconstruct (CO) of the collaboration principle had three codes (CO1, CO2, CO3). If only one code (eg CO1) was found in the statements, I rated the sub-construct as low. If two codes (eg CO1 and CO3) were found, I rated it as medium. If all three codes were found, I rated it as high. I repeated this process for all six subconstructs, adjusting the rating scale according to the number of codes for each subconstruct.

Table 4.4: The iPAC rubric for evaluating the pedagogical advantages of m-learning tools of the smartphone and MCM app

Scale/ Category and description	**Subcategory**	**Code**	**Possible evidence in the interviews** In the eyes of the teachers did:	**Yes**	**No**
PERSONALISATION	Agency (AG)	AG1	the learners have a choice in the routes they could take?		
		AG2	the learners have a choice of learning content?		
		AG3	the learners have control over the tools e, smartphone, tape measure?		
		AG4	the learners have a choice of their pace of learning?		
		AG5	the learners have a choice of methods to use?		
	Customisation (CU)	CU1	the app guide the learners to locate the tasks?		
		CU2	the learners tailor the app(s) settings to their preferences eg customised location on/off, camera/microphone access, time limit settings?		
		CU3	the learners receive individualised and/or group information through the app about their environment eg, information about the trail/task?		
AUTHENTICITY	Setting/Task (ST)	ST1	the learners learn in a realistic, virtual space?		
		ST2	the learners engage in learning content that was relevant to them?		
		ST3	the learners work like experts eg, collect data using GPS like a geographer; measure objects like a builder.		

Scale/ Category and description	Subcategory	Code	Possible evidence in the interviews In the eyes of the teachers did:	Yes	No
	Tool (TO)	TO1	the learners engage in activities related to everyday life eg, carrying out measurements?		
		TO2	the learners learn serendipitously in an unplanned way eg, searching for information on the internet?		
		TO3	the learners use tools that replicate those of real-world practitioners?		
		TO4	the learners use tools and their operations that were familiar to them eg, smartphone?		
COLLABORATION	Conversations (CO)	CO1	the learners talk about the work displayed on the device screen with others around them?		
		CO2	the learners discuss the work online with others eg, the app's reviewers?		
		CO3	the app create shared, socially interactive environments for learning?		
	Co-Creation (CC)	CC1	the learners work together to create a digital product eg, co-created a video or photo?		
		CC2	the learners communicate and exchange information with peers, teachers and experts?		
		CC3	the learners contribute to existing digital content eg, tagging a photo, commenting on a blog post?		
		CC4	the learners share digital content eg, a video, photo, document, or insert answers in the app?		

4.6.2.3 Emerging themes

In addition to analysing the interview data with the iPAC pedagogical mobile framework, data material from both the RI and FGI were transcribed and coded, leaving room for emerging themes. Cohen et al. (2007) cautions that when analysing qualitative data, sometimes "the codes themselves derive from the data responsively rather than being created preordinately" (p. 478). Thus, apart from the predetermined category themes that were created from the literature review of this study, there were other recurring themes that emerged during the vertical and horizontal analysis, and these were clustered into two thematic categories, namely visualisation aspects in the study and the teachers' views on the challenges of the MCM app project and teaching outdoors. The coding of these themes emerged as a result of the participants' experiences of the design and implementation of outdoor trails using the MCM platform.

4.6.3 Organising data into structured narratives of vertical and horizontal analysis and emerging themes

The next step in my data analysis was to organise the data and present it. This was done in three phases and is thus presented in three sections in the following chapters.

4.6.3.1 The narrative (vertical) analysis

The aim of narrative analysis in qualitative research is to explore the individual stories of the participants and how they make sense of their own experiences (Elbaz-Luwisch, 1997). Narrative analysis does not seek to generalise, but to understand the unique perspective of each research participant (Kelchtermans, 1993, Foster, 2006). Newby (2010) suggests that narrative analysis can help the researcher to gain insights into the participants' worlds. Therefore, in this study, I used narrative inquiry to present the data vertically, allowing the participants to share their own experiences and the meanings they derive from them. Graebner et al. (2012) contend that using the participants' own words can enable the researcher to capture their subjective experiences and interpretations more accurately. Thus, in this study, I used the teachers' narrative experiences of walking on the mathematics trails and solving the MCM tasks as a way of conveying their actions and events. To preserve the coherence and integrity of the participants' actions and responses, I presented the data in a chronological and holistic manner, using a story telling style of reporting to capture the wholeness (Cohen et al., 2007) of my participants' actions and words.

4.6.3.2 Horizontal analysis

Horizontal analysis, sometimes referred to as 'cross-case displays' focuses on sieving out common patterns and also, remarkable differences or recurring themes across the narratives of the different participants (Miles & Huberman, 1994; Hunter, 2010). In this study I used the horizontal analysis to summarise the main issues and themes that arose out of and across the participant teachers. This allowed me to identify and develop apparent patterns or differences in how the teachers used authentic and realistic tasks to teach mathematics within the context of the MCM m-learning environment. As recommended by Lieblich et al. (1998), when analysing the data horizontally, I hoarded the data into pre-determined topic categories of the six RME principles for VRs and the six subconstructs of the iPAC mobile pedagogical framework for the one-on-one RIs. These topic categories were collected from the narratives of the eight different participants. This approach was useful because this group of teachers shared a common interest of using smartphones to teach for conceptual understanding the topics of area, volume, ratio and proportion, within the MCM app project and the RME theory.

4.6.4 Interpreting the data

I followed the above structure to analyse the data and answer the research questions. This approach enabled me to explore how the teachers positioned themselves when they applied the MCM realistic tasks to conceptually teach the topics of area, volume, ratio and proportion, based on the RME theory. From an interpretive perspective Borko et al. (2007), state that "interpretive researchers attempt to capture local variation through fine-grained descriptions of settings and actions, and through interpretation of how actors make sense of their sociocultural contexts and activities" (p. 4). Therefore, to interpret the teachers' actions and narratives of using authentic/realistic tasks and smartphones for teaching the selected topics, I connected the different actions and statements from all the eight teachers to form a holistic interpretation. However, I should hasten to acknowledge that, by making such narrative connections on behalf of the teachers, I am not implying or claiming that I know my participants' narratives better than they do. I am cognisant of Hunter's (2010) warning that "representing and interpreting another's voice is not a simple task and needs to be done with respect and humility" (p. 50). So, my role as a researcher was to "interpret the [participants] stories in order to analyse the underlying narrative that the storytellers may not be able to give voice to themselves" (p. 227). According to Newby (2010), in social sciences research, we can understand peoples' worlds from their perspective by examining their lives, actions and statements. Therefore, in this study, as I analysed and interpreted how the teachers used the RME teaching principles to teach the topics of spatial measurements, ratio and proportion, I paid close attention to their actions and statements to understand how these related to the answering of the research question. This allowed me to explore and make explicit connections between the participants' experiences and actions, and to include any supporting narrative data which the participants may have missed. Figure 4.2 below shows the summary of the methodological approach used in this study.

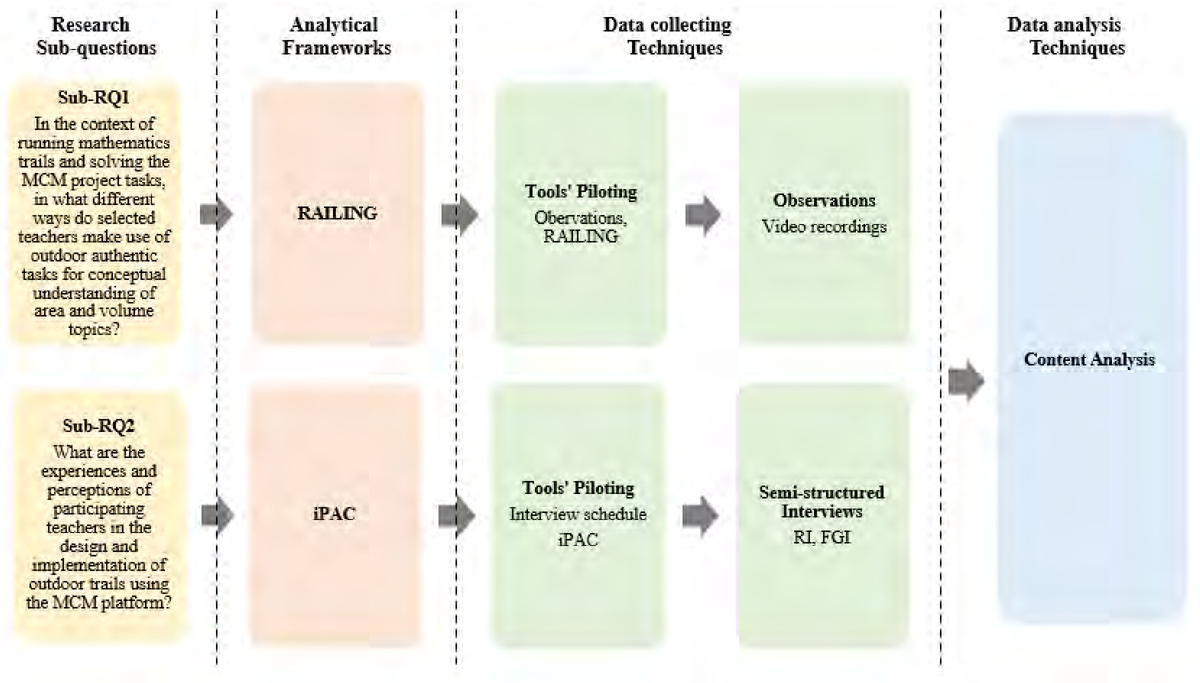


Figure 4.2: Sub-research questions, analytical frameworks, data gathering (including the tools involved) and data analytical techniques.

4.7 VALIDITY AND RELIABILITY

In qualitative research, validity and reliability are concerned with how reasonable and appropriate a study appears to be. Validity is conceptualised in terms of the trustworthiness, rigour and quality of the study (Golafshani, 2003), whilst reliability is concerned with the consistency, dependability, repeatability and stability of the instrument in question (Bannigan & Watson, 2009). According to Polkinghorne (2005),

> the trustworthiness of the data depends on the integrity and honesty of the research. Their production process needs to be transparent to the reviewers and to those who would use the findings in their practices. (p. 144).

Therefore, in this study, I presented and interpreted the data in a truthful and precise manner, consistent with the research objectives (Patton, 2002). To ensure the validity and reliability of the findings, I employed various strategies such as triangulation, member checking, thick descriptions, piloting and prolonged engagement in the field.

4.7.1 Triangulation

Golafshani (2003), identifies triangulation as a key strategy to test or improve the validity and reliability of a study and the evaluation of the findings thereof. Creswell and Miller (2000) define triangulation as "a validity procedure where researchers search for convergence among multiple and different sources of information to form themes or categories in a study" (p. 126). I thus used triangulation to corroborate the evidence of findings from the different data sources in observation videos, RI and FGI-audio recordings. In line with Plano-Clark and Creswell (2015), I also compared the actions and statements of all eight participants to cluster the similarities and differences that existed from the different data sources. This was done by posing several questions to validate each participant's responses and actions and that of their colleagues. This provided me with an accurate representation of the data that pertained to the lived realities of the participants' actions and experiences of using authentic and realistic tasks placed within the MCM mathematics trails, to teach the selected topics.

4.7.2 Member checking

After transcribing all the interviews for this study, I did member checking by giving each participant their own transcripts so that they could confirm the validity of the data findings (Maxwell, 2009; Merriam, 2009). Tracy (2010) claims that member checking is not only about letting the participants verify their responses, but is also about

> seeking input during the processes of analysing data and producing the research report... sharing and dialoguing with participants about the study's findings, and providing opportunities for questions, critique, feedback, affirmation, and even collaboration. (p. 844).

Therefore, I requested the participants' feedback to validate the data findings of this study and to make sure that the items that I identified and classified and that formed the coherent structure of the findings were valid. In this way, according to LeCompte (2000), I was able to determine whether the overall findings matched the participants' perspectives and actions and whether the results were clear and useful.

Another form of member checking that ensured the credibility of this study was reflected in what Thomas (2003) termed as peer debriefing or "stakeholder checks" (p. 7). This required providing opportunities for peers – those who are skilled in qualitative research and show specific interest in the research, to comment on the categories or interpretations of the findings of this study. So far, the emerging findings of this study have been published in a paper article that was submitted as a manuscript for the annual Southern African Association for Research in Mathematics, Science and Technology Education (SAARMSTE) 2024 conference to be hosted by the International University of Management in Namibia. The manuscript was peer reviewed by two qualified reviewers, and the feedback I got helped me to reconsider some of my interpretations and to deepen my insights into how the teachers used authentic/realistic tasks created for the MCM app to conceptually teach the topics of area, volume, ratio and proportion.

In addition, my supervisor for this study played an essential role in validating the research process, by evaluating my arguments regarding the data analysis and presentation, field observation schedule/tool and interview questions. Throughout the process, he offered different critical perspectives to issues, but also improved both the research process and my engagement with the project.

4.7.3 Prolonged engagement in the field

The fact that I spent a reasonable amount of time observing the eight teachers' design and implementation of the MCM realistic tasks in their natural settings, added to the credibility of the observation process of this study (Maxwell, 2008). I spent almost two years with all the participants designing and implementing realistic tasks for the MCM app, thus, I believe that this prolonged engagement allowed me to build trust with the participants, which in turn made them act naturally and freely share their thoughts on the study project. Creswell and Miller (2000) claim that in qualitative research the purpose of prolonged engagement in the field would be to build trust with the participants and "to establish rapport, so that participants are comfortable disclosing information" p. 128). In my study, although I only had one RI with each teacher participant, my prolonged interaction with each one of them before and after the interview sessions greatly contributed to their comfort in sharing with me their thoughts and perceptions. Moreover, I noticed that the data collected from the second and third cycles of this study were more detailed and richer than those from the previous cycle(s) and this can be attributed to the diminishing boundaries that could possibly have existed in the previous circle(s).

4.7.4 Thick description

A thick description involves providing as much detail about the data collection process as possible. Thompson (2001) argues that a thick description provides the necessary context for understanding meanings in a qualitative research study. It allows interested readers to share the researcher's experience of the whole data collection process. Hence, in this study, I strived to present every relevant detail of the teachers' actions and perceptions of how they used authentic/realistic tasks in the MCM mathematics trails to teach the selected topics for conceptual understanding. Firestone (1993) explained that: "factors unimportant to the researcher may be critical to the reader. They are more likely to show up in designs stressing the careful depiction of the setting studied" (p. 22). To this effect, to demonstrate that my study was unbiased, credible, and that I did not manipulate or misinterpret participants' words and actions, I used the participants' original transcribed quotes from the video and audio interviews to support the themes that emerged from the data analysis. Ponterotto (2006) articulates:

> It is the qualitative researcher's task to thickly describe social action, so that thick interpretations of the actions can be made, presented in written form, and made available to a wide audience of readers. Without 'thick description', 'thick interpretation' is not possible. Without 'thick interpretation', written reports of research will lack credibility and resonance with the research community... It is the thick interpretive work of researchers that brings readers to an understanding of the social actions being reported upon. (p. 542)

Furthermore, Morse et al. (2002) state that "a good qualitative researcher moves back and forth between design and implementation to ensure congruence among question formulation, literature, recruitment, data collection strategies, and analysis." (p. 17). Thus, part of my validity process was to ensure that there was coherence and a consistent alignment between my research questions, the data collection techniques and analytical instruments.

4.7.5 Pilot the first cycle of the study

For reliability purposes, I devoted the first design cycle of this study to pilot the creation and implementation of the MCM project tasks, the data collection instruments and the analytical instruments (Pietersen & Maree, 2011). As a result of my pilot, both of my two analytical frameworks, the RAILING and iPAC mobile pedagogical frameworks were refined and adapted for the improvement of the implementation of the other two cycles that followed.

4.8 ETHICAL CONSIDERATIONS

Cavan (1977) defined ethics as "a matter of principled sensitivity to the rights of others, and that 'while truth is good, respect for human dignity is better" (p. 810). Polkinghorne (2005) advises that the welfare of the participants should be a primary concern in qualitative research, and that: "In addition to maintaining the confidentiality of participants, researchers need to proceed with sensitivity and concern for their needs and desires" (p. 144). My study adhered to research ethics by ensuring the following ethical principles outlined below:

I sought permission for access to the three secondary schools where this research was conducted. I wrote an official request letter to the Regional Director (RD), to permit me to conduct research at the three different identified secondary schools in the region (see Appendix Two). After permission was granted from the first gatekeeper, who was the RD (see Appendix Eight), I then approached the three school principals in person to request permission (see Appendix Three), discuss the purpose of my study and present the permission letter from the RD that allowed me to conduct research in their respective schools. Thus, each of the three school principals served as gatekeepers in the process of accessing the research sites of this study. After permission was granted from the above-mentioned office bearers (see Appendix Nine), I sought consent from the participating teachers that I recruited for my research project (see Appendix Four).

I contacted each of the eight (previously nine) individual teachers to make an appointment for an informal conversation. During my meeting with each teacher, I clearly explained the purpose of the study to them, what was expected of them, and how they were going to benefit from participating in the study, as well as the risks that may have arisen from their participation in the study. Marlett and Emes (2010) who, in reflecting on the ethics of their own work recommended that for participants to be aware of the potential danger and advantages of their participation in a research study, the researcher needs to be clear about the purpose of their research and how they are going to use the findings of the study. In this way, the authors argued that the researcher should always strive "to be transparent and honest about what is expected and what safeguards are in place for everyone" (p. 135). Before the commencement of the study, I made it clear to the participants that participation in the research was completely voluntary, and that they could withdraw from the study at any time if they were uncomfortable with anything related to the study. This was in line with Cohen et al. (2007) who insist that the principle of informed consent implies the participant's right to freedom and withdrawal from the study. Lyons and La Boskey (2002) wrote that "[o]f particular concern are safety and privacy. If people are to share their meanings of experience, they need to be assured... that their work will be confidential" (p. 23). Therefore, to protect, guarantee anonymity and safeguard the confidentiality of interviewees in the study I used the pseudonym of Kamwi, Anna, Moses, Sinvula, Calvin, Luke, Betty and Joshua respectively throughout the study. Also, the names of the schools, learners, or principals were not disclosed in this study.

When all the arrangements regarding the teacher participation were done, I then requested the participant teachers to administer, on my behalf, the parent/guardian consent letters, and learner assent forms (see Appendices Four and Five). The parent/guardian consent letter was translated into the local language, Otjiherero (see Appendix Six). The task of translating and duplicating the letters for the learners of each school was done by me. In the letters, I informed the parents about the purpose of the study and that the focus of the observations was limited to teachers only. However, it was highly likely that the learners were going to feature in the videos, of which I assured the parents that their children's identity was not going to be revealed in the presentation of findings of this study or any other future use of the data, for example in conference presentations. I also assured the school principals, the teachers and the parents that walking on the mathematics trails was not going to disturb any official school activities as it was planned to be executed in the afternoons. It should be noted that some parents did not return the consent letters and I did not include their children in the intervention.

I demonstrated accountability in this research study by safeguarding the collected data during and after the research project. In my interactions with the participants, I was conscious of facilitating an atmosphere and ethos that was democratic, non-threatening and trusting. Any data gathered during this research will be solely and strictly confined and used for the sole purpose of this research project. During and after completion of the study the raw research data will be appropriately kept for five years on my personal computer encrypted with a password and a cloud facility as a backup storage. Apart from my supervisor, no data that contains confidential information and private details of participants will be given outside the scope of this study. However, if data will be needed for verification purposes from someone who was not part of the study, full consent and authorization will be sought by me. If verification is required, I will continue to use pseudonyms to guarantee the anonymity of the participants and their schools, for example school A, B, C, and Kamwi, Anna, Moses and so forth for teachers.

Such confidentiality initiatives and data storage measures are all in the interest of ensuring and protecting the privacy and anonymity of the participants.

As a teacher and an equal partner with the participants, I was aware that my position as a PhD student may have influenced their reactions and behaviours during the study. Most of the participants had diplomas and degrees in education, so they may have felt inferior or intimidated by my presence. This may have affected their responses. To address this, I informed the participants that I was also learning how to use the MCM app to teach authentic tasks. I thus positioned myself as a learner researcher and explored with the participants how to teach the topics of area, volume and ratio and proportion using the authentic tasks we designed for the MCM project. Throughout my engagements with the teachers in this study, I avoided expressing my own thoughts and opinions on what we were doing, to prevent bias. As Clandinin (2006) stated, "we must do more than fill out the required forms for institutional research ethics boards" (p. 52). To this effect, I upheld the high levels of conduct and represented the Education Department of Rhodes University beyond the signing of the consent forms.

4.9 CHALLENGES ENCOUNTERED

The first challenge I experienced in this research journey was that the schools where the study was located did not allow learners to possess smartphones in the classroom or the school environment. It was believed that learners used these devices in inappropriate ways that caused disturbances to the teaching and learning processes or harm to other learners. Generally, the devices were seen as a contributing factor to low performance in schools. Therefore, as I anticipated, it was not easy for me to gain access to one of the schools, as the school principal thought my study was overlooking the disadvantages of smartphone use among learners in schools. I still vividly remember the principal of this school jokingly remarking, "*Matengu, it looks like you are campaigning that learners should use smartphones during lessons*". The stigmatisation against the use of smartphones among learners in Namibian schools is a common occurrence. So, to mitigate this challenge, I diplomatically sensitised the regional director, school principals and participant teachers about the benefits of using smartphones in teaching contexts. In the consent letters to the gatekeepers, I was upfront and explicit about using smartphones, the heart of my study. From the literature point of view, I argued against biased internal policies that did not allow the use of smartphones in teaching/learning contexts (even though such policies were not sanctioned by the MEAC). Thus, I was critical about the negative attitude and practices created by the said policies towards smartphones, through the introduction of the MCM app.

The second challenge was the creation of the MCM tasks, which were not easy for us to design. Also, the reviewing process of the tasks took longer than expected. The task reviewers usually took two to three weeks to review the submitted tasks. As a result, this affected the participants' enthusiasm about creating more tasks for the trails. Some tasks in some trails were still pending the approval for public viewing at the time of my thesis's submission. The third obstacle was that conducting a research study during the novel COVID-19 pandemic was a challenging experience for me. The lockdowns and closures of schools for most part of 2021 affected the schedule of my observations, which compelled me to adjust my timeline of the planned data collection from January 2021 to November 2021. However, whenever I managed to convene meetings for the design of the tasks and implementation of the trails, I followed all the protocols and guidelines necessary as stipulated by the schools, the MEAC, the GRN and WHO, in order to avoid putting the research participants at risk.

4.10 CONCLUSION

This chapter presented the methodological approaches I followed to conduct this research study. I detailed the research paradigm, the research method, the research design and the data collection techniques I used. I moreover explained how I analysed the collected data. I also discussed measures that were put in place to ensure the validity and reliability of the research findings, as well as the ethical measures to ensure that the research participants' rights were protected. In the next chapter, I report on the findings that emerged from the VRs of the teachers' use of authentic/realistic tasks in the mathematics trails with their learners.

CHAPTER 5

ANALYSIS OF THE TEACHERS' USE OF AUTHENTIC TASKS IN THE MATHEMATICS TRAILS

5. 1 INTRODUCTION

In this chapter, I present findings on the investigation of how teachers used authentic tasks in an m-learning environment to teach mathematics for conceptual understanding, by interrogating evidence of the six RME principles. The aim of this study was to analyse and understand how eight selected mathematics teachers can implement authentic tasks in a mathematics trail to facilitate the teaching of area, volume, ratio and proportion for conceptual understanding. In pursuance of this goal, the study was guided by the following main research question: *In the context of a mobile learning environment, how can teachers implement authentic tasks in a mathematics trail for conceptual understanding of selected mathematics concepts?* This research question was then broken into sub-questions:

1. In the context of running mathematics trails and solving the MCM project tasks, in what different ways do selected teachers make use of outdoor authentic tasks for conceptual teaching of area, volume, ratio and proportion topics?
2. What are the selected teachers' experiences and perceptions on the design and implementation of mathematics trails using smartphones within the MCM platform?

So, in response to the above research questions, I structure and discuss the findings of this study in **two main chapters** (Chapter 5 and Chapter 6). These chapters are presented according to the research sub-questions of the study. In Chapter 5 I report findings on the question of different ways teachers used outdoor authentic tasks to facilitate conceptual teaching, and in Chapter 6 I present findings on teachers' experiences of the design and implementation of mathematics trails using smartphones within the MCM project. Data for the first research sub-question (Chapter 5) is from VRs (observations) of the trails and solving of tasks, whereas data for the second research question (Chapter 6) is from the interviews – RI and FGI.

5.2 REALISTIC MATHEMATICS EDUCATION PRINCIPLES (RME)

The present chapter starts with a presentation of both the vertical and horizontal analyses. The vertical analysis is focused on how each teacher participant implemented the six operational principles of the RME theory for conceptual teaching within the mathematics trail, while the horizontal analysis is an analysis across all the eight participating teachers. Both analyses were based on recordings of eight video sets that depicted the implementation of the six principles of the RME theory during the trail walk and solving of the MCM outdoor authentic and realistic problem tasks.

The RME theory advocates that in order to be of human value, mathematics must be connected to reality, close to learners and relevant to their experiences of society (Freudenthal, 1973; Ginsburg et al., 2004). It is within this context that this study foregrounded the design and solving of authentic tasks connected to learners' realities using the MCM app project. In using the RME theory in this study, I developed an analytical framework (tabulated in Section 4.6) consisting of the following categories: RAILING (**R**-Reality principle, **A**-Activity principle, **I**-Interactive principle, **L**-Level principle, **IN**-Intertwined principle, **G**-Guidance principle). To operationalise the framework, I introduced colour schemes for each of the six core teaching principles of RME categories of indicators. For example, four codes under the broad category of Interactive principle (IP), namely IP1, IP2, IP3 and IP4 were identified in turquoise. I therefore analysed and coded each video using the appropriate colour codes. Thereafter, I used the colour coding to deconstruct and analyse the teachers' actions and words that helped learners to understand concepts connected to authentic, outdoor tasks of the MCM project. My interest was on how the teachers used the principles to conceptually teach the selected topics.

In order to support the use of RME principles in the trails, I transcribed some portions and episodes of the VRs that I found showed evidence of the principles by highlighting them with colour codes. Thus, in support of my claims of the existence of the principles, I posted the extracts or excerpts of these transcripts as evidence. Also, I inserted screenshots from the videos as evidence of the existence or use of the six core teaching principles of RME in the trails.

Before unpacking the implementation of the six core teaching principles of RME in this study I briefly want to remind the reader what each principle entails. The focus of analysis of each principle during the walking of the trails and solving of the MCM tasks was:

Realistic Principle (RP): this principle concerns the use of meaningful real-life contextual problems that are related to specific concepts of the relevant mathematics topics and themes. In analysing the principle, I looked at the extent to which (with the help of the teacher) learners were able to apply and refer the concepts involved in the tasks to real-life examples.

Activity Principle (AP): this principle advocates that learners should actively participate in their learning process. I focused here on how teachers encouraged and created opportunities for learners to actively engage in the trail walks and solve the MCM tasks.

Interactive Principle (IP): in order to reach a higher level of understanding, the learning of mathematics must be seen as a social activity, thus the interaction between teachers and learners as well as among learners themselves, should be conducted to develop and improve the communication skills of argumentation, critique and justification. In the context of this study, I analysed the Interactive principle by looking at how learners interacted among themselves and with their teacher(s) during the trails and solving of the tasks.

Level Principle (LP): involves using models, schematisation, concrete objects, manipulatives or tools to scaffold learners in the learning of mathematics. I analysed this principle by looking at how teachers helped learners pass various levels of understanding of the different mathematical concepts involved in solving the MCM tasks.

Intertwined Principle (INP): this principle suggests the incorporation of different curriculum strands and mathematical connections across school subjects simultaneously, without having to follow any prescribed order. Here I looked at the extent to which the concepts involved in topics of measurements (area, volume, ratio and proportion) integrated with other mathematical topics and disciplines. Also, I looked at how the teachers made this integration evident to learners during the trail and solving of the tasks.

Guidance Principle (GP): this principle states that teachers should provide appropriate guidance and instructions to enable learners to reinvent mathematics. The analysis of this principle involved the measure of how teachers guided learners on the application of the MCM app, location and solving the tasks. In particular, I concentrated on how teachers guided the learners from the informal to formal use of strategies to solve outdoor tasks of area, volume, ratio and proportion.

Table 5.1: The coding criteria

Coding	Categories	Descriptions (evidence of the use of the RME principle)
0	Never	There is no evidence of the use of the RME principle
1	Sometimes	There are 1-2 incidences of the use of the RME principle
2	Very often	There are 3-4 incidences of the use of the RME principle
3	Always	There are more than 4 incidences of the use of the RME principle

For ease of reading and interpretation of this data presentation and analysis, I first present a visual presentation of the extent to which the principles were used in the trail (see Table 5.1 above). Thereafter I discuss how each of the six principles were used by the teacher concerned.

The following is the outline of this chapter:

Section 5.2.1 – MCM Cycle 1: Brief overview of the mathematics trails

Section 5.2.2 – Data presentation and vertical analysis of individual participants

Section 5.2.3 MCM Cycle 2: Brief overview of the mathematics trails

Section 5.2.4 – Data presentation and vertical analysis of individual participants

Section 5.2.5 – MCM Cycle 3: Brief overview of the mathematics trails

Section 5.2.6 – Data presentation and vertical analysis of individual participants

Section 5.3 – Horizontal analysis across participants

Section 5.4 – Conclusion

5.2.1 MCM Cycle 1: Brief overview of the mathematics trails

The design of the first MCM cycle of this study included three mathematics trails (one from each school of the participating teachers). The creation of the tasks in each trail commenced immediately after the workshop training on the MCM app and the RME principles. Notice that the design and implementation of Cycle 1 served as a pilot phase with a twofold purpose: firstly, to acquaint the participating teachers with the principles and ideas of the MCM app project. This cycle opened the stage for reflection on the creation and implementation of trails, which inevitably led to some adjustments. Also, for the first time, the practical application of the MCM app project was introduced to the teacher participants. It was also the first time that they could experience what it means to teach mathematics outdoors. Secondly, the teachers' engagement with the operations of the app and their walking of the trail(s) in this cycle were used to test the effectiveness of the developed analytical instrument, namely the RAILING. The developed coding system and analytical instrument were gradually refined during the analysis of the pilot study data, which in turn sharpened my data gathering and analysis protocol.

The mathematics trails of Cycle 1 had the following characteristics:

- School A (Exploring School A Trail) – consisting of five tasks.
- School B (Exploring School B Trail) – consisting of six tasks.
- School C (Exploring School C Trail) – consisting of six tasks.

All the tasks in the trails were created within the vicinity of the respective school environment and in each trail only two tasks were video recorded for data collection. The offline map in Figure 5.1 below shows the locations of each task, so the first task that was solved is labelled as 1 on the map, and the second task as 2, respectively. All 17 tasks in this cycle were based on the topics of measuring area and volume. One teacher from each school participated in creating and implementing the trails in Cycle 1.

Figure 5.1: The offline trail maps for each school in Cycle 1

5.2.2 DATA PRESENTATION AND VERTICAL ANALYSIS OF INDIVIDUAL PARTICIPANTS

In this section, guided by the six principles of the RME theory, I analyse the teachers' interactions with their learners during the implementation of the trails and solving of the tasks. The following is the outline of the analysis:

1. Profile and coding of the teacher.
2. Brief description of the tasks encountered and solved during the trail walk.
3. Analysis with RME principles: visual presentation of bar chart
4. Discussion of the operationalisation of the RME teaching principles

5.2.2.1 Trail 1 of School A (Teacher 1: Kamwi)

Profile and Coding

I used the pseudonym **Kamwi** to identify the teacher who was video recorded in the first trail of School A. The trail is named "*Exploring School A Trail*" and is coded as **TR1A** in the study and the MCM app code is **679425**. Kamwi has been in the teaching profession for the past seven years now and holds a Batchelor of Education (Mathematics and Science) from the University of Namibia. Kamwi is currently teaching Mathematics Grade 8 to 9 and Physics Grade 10 to 11. The video data for Kamwi is coded as **KV$_1$** and it is 38 minutes long.

There were five learners under the care of Kamwi during the walk of the trail – 3 boys and 2 girls. Kamwi and his learners solved two tasks of the trail. These tasks were based on the topics of area, volume and surface area – see Table 5.2 below. The learners shared and used one smartphone in their walk on the trail. Kamwi's group spent an estimated time of one hour to walk and solved the two selected tasks in the trail (see points 1 and 2 in Figure 5.1 above).

The implementation of the RME principles: Kamwi

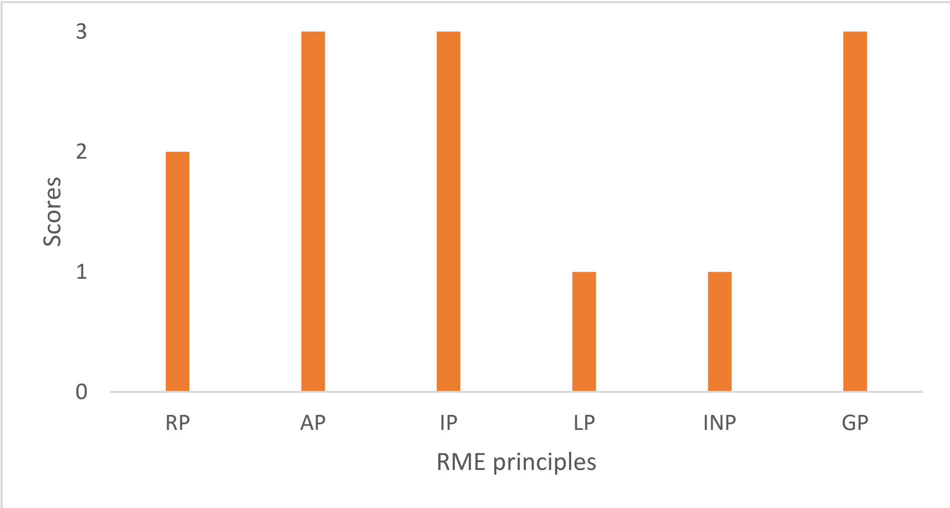
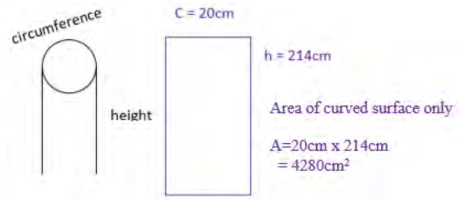


Figure 5.2: Numerical scores of the implementation of the RME principles by Kamwi.

The graph in Figure 5.2 above is a visual representation of the numerical scoring of the implementation of the six operational principles of RME theory for Kamwi. Each principle is measured by 0 – 3 points according to the coding criteria in Table 5.1 above. The analysis of data from Figure 5.2 shows that the AP, IP and the GP were the most used RME principles in Kamwi's implementation of the mathematics trail. The RP was moderately utilised whilst the LP and INP were least observed in the trail.

Table 5.2: The two tasks encountered and solved by Kamwi's group of learners in Trail 1 of School A

TASK TITLE	TASK DEFINITION	OBJECT	SAMPLE SOLUTION	HINTS
The lamp Pole Task Code 3740862	If you want to repaint this lamp pole to red, calculate the surface area that needs painting to the nearest cm².			1. The lamp pole is a solid cylinder. What does the net of a cylinder look like? 2. Take note that the part that needs painting is the curved surface, so this excludes the two circles. 3. The net of the curved surface part translates to a rectangle shape, hence, the area of the curved face is length × width, which is circumference × height
The Electricity Box Task Code 0836109	Find the surface area of the metal sheet that would be needed to replace the door of the electricity box. Write your answer to the nearest cm².		Measurements: Length 39.1cm; Width 27.5cm Area = length × width. = 39. 1 × 27.5 = 1075.25 = 1075 cm²	1. Will the needed metal sheet be the same size and shape as the current old one? 2. What shape is the current metal sheet?

The Reality Principle

The RP was well attended to in this trail for it was clearly observed that Kamwi made attempts to connect the reality aspect of the tasks by providing examples linked to real life (RP2). For example, during the solving of the first task, Kamwi was heard saying the following to emphasise the importance of determining the surface area that needs to be painted in real life:

Guys, okay listen, be careful there, we are going to paint this part (holding the pole in his hand) … this and that side … (moving his hand up and down the pole), isn't? we are just going to paint this part … it means this is the area we want. Let us say you want to invite someone to come and repaint this pole, at least you need to give the area that person needs to cover in order to come with enough paint, (RP1) isn't? so you need to give the correct surface area to that person. So, what can we do now? **KV1**

Figure 5.3: Kamwi highlights the RP in the task of painting the pole.

In this excerpt, Kamwi is using realistic examples to stress that in reality one needs to know the area that needs painting in order to acquire the correct amount of paint for the job (RP2). Another incidence where Kamwi highlighted the authenticity of what learners were doing in the trail was observed in the solving of the second task. This task needed learners to "*find the surface area of the metal sheet that would be needed to replace the door of the electricity box*". During the process of helping learners solve this task, Kamwi reiterated that the learners should imagine themselves being the ones to replace the door of the box in order to figure out what to do (RP3). Her urging learners to use their imaginations is evidence of the use of the RP in the trail.

The Activity principle

Data from **KV1** shows strong evidence that learners in this trail were actively involved in the trail activities and solving of the tasks. Right from the start of the trail, Kamwi encouraged his learners to get involved by allowing them to be in charge of the trail activities (AP1). It was clearly observed that most of the time learners were on their own, figuring out how to find the tasks on the MCM Google map. This was evident by how they located the positions of the tasks by themselves without interference from the teacher. Learners were at all times glued to their smartphones sharing ideas about where to find the locations on the map – see Figure 5.4 below. This in turn added to their increased participation as it was evident that they had to work together in order to correctly interpret the MCM Google map (AP3).

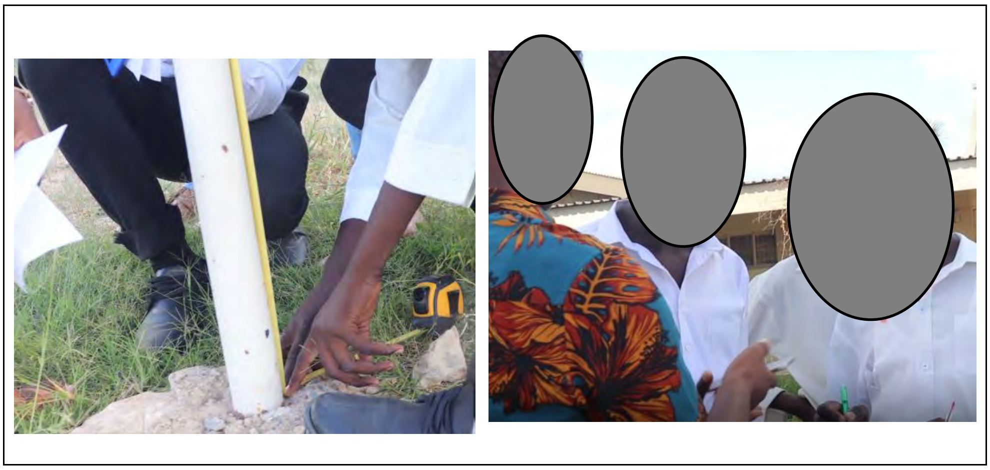


Figure 5.4: Learners actively solving their first task of the trail and also figuring out the location of the next task using the smartphone.

At the start of the trail, I observed that learners were struggling to find and locate the position of Task 1. It was evident that they had a good idea of what was expected of them in order to find the task, by how quickly they figured out the different places in the school environment on the map. Furthermore, they debated on the location of the tasks and how to solve the tasks. What is important is that these debates were productive, meaning that they clearly agreed where the task was located, how to get to the task spot, how to solve the task and the division of responsibilities among them – for instance, who was to read the question and check the answer from the smartphone, carry out the measurements, record the measurements and carry out the calculations on the calculator (AP1). These engagements were hands-on and kept learners busy most of the time during the trail.

Interactive Principle (IP)

The predominance of the IP in this trail was evident by the high degree of interactions between learners and learners, and between learners and the teacher. From the beginning of their trail walk, the learners were left to freely discuss and decide where to go and what to do without the intervention of the teacher (IP1/IP2). Kamwi encouraged his learners to find answers themselves and to also critique and justify the solutions to the tasks before uploading them on the MCM app (IP4). Furthermore, it was evident that Kamwi's frequent posing of probing questions contributed greatly to his interaction with the learners as well as their ability to see the embedded mathematical concepts in the tasks (IP3).

The Level Principle (LP)

There was only one incident where the LP was used by Kamwi in this trail. This was when he modelled the net of a cylinder by folding an A4 page to form a cylindrical shape (LP5:LP6) – see Figure 5.5 below. In this particular incidence the teacher was demonstrating to learners that they only needed to consider the outside curved part of the cylinder when calculating the required area to be painted, and to also understand that the top and bottom parts of the cylinder (circles) were not part of the area to be painted. This incidence of using an A4 page to model the net of a cylinder was brought about by the learners' concern regarding how they were going to calculate the surface area of the pole without first finding its radius. The learners' reasoning here was that if they were to calculate the surface area of the pole, they first needed to find the radius by measuring it at the top/bottom of the pole.

Figure: 5.5: Kamwi modelling the net of the cylinder to explain the concept of surface area

In their attempts at finding the radius, some learners suggested that they find the radius through measuring it at the top or bottom of the pole, which meant that the learners understood the concept of diameter as being a special chord that passes through the centre of a circle. Other learners attempted to find the radius by placing the tape measure around the pole, which was an indication that they did not understand that this was leading them to find the circumference of the pole and not the diameter. This showed that the learners lacked understanding of the properties of a circle, in particular parts of a circle.

According to my observations, the learners had difficulty in finding the diameter/radius of the pole because there was no way that they could measure it with an uneven top and bottom – see Figure 5.6 below. Also, the learners had misconceptions regarding the parts of a circle, by confusing the concepts of diameter and circumference of the pole. So, to help his learners clearly understand what they needed to do in this particular case, the teacher applied the LP of the RME theory to model the cylindrical shape of the pole, which that was at their level of understanding (LP3:LP7). The teacher used a paper modelling approach to explain to learners that they only needed a rectangle part of the net of a cylinder to work out the area needed for painting (LP2). Also, in his explanation the teacher emphasised the fact that when calculating this part of the net, learners did not need to find the diameter or radius.

Figure 5.6: The top and bottom part of the pole learners could not measure

Intertwined Principle (INP)

In this trail, the IP was evident in the way that tasks integrated topics from other strands and subject matter. For example, apart from the topics of measure and area, the solving of both of the tasks included conceptual knowledge of rounding off – See Figure 5.7 below. For each question, learners were supposed to round off their final answers to the nearest square centimeters, and it was observed that they found this easy to do.

TASK 1: Find the surface area of the metal sheet that would be needed to replace the door of the electricity box. Write your answer **to the nearest cm².**

TASK 2: If you want to repaint this lamp pole to red colour, calculate the surface area that needs painting **to the nearest cm²**

KV_1

Figure 5.7: Questions from Task 1 and Task 2 showing the integration of the rounding off topic into the topics of measure and area

However, according to my observations, although the connection of the topics of length and area measures to the rounding off was apparent in the tasks, it cannot be said that the teacher referred to these connections in his engagement with the learners. The same can be said regarding the connections between these concepts and other strands and subjects. Kamwi did not highlight the connections that existed between the measurement and area concepts to the other mathematical concepts and other subjects to show the relationship that exists between them. On the other hand, there was sufficient evidence indicating that the teacher made reference to the concepts embedded in the tasks with real life examples (INP4). See the extract inserted on RP.

Guidance Principle (GP)

I observed that the teacher provided guidance to learners where and when it was needed during the trail and the solving of both tasks. For example, during the solving of the first task, learners were mistaking the unit of a centimetre with that of metres on the measuring tape, and this led them to the misconception of converting the measurements that were already in centimetres to the same unit of centimetres. In Figure 5.8 below, Kamwi is seen explaining to the learners the units that are on the measuring tape and why the units should not have been changed to the same unit already on the measuring tape (GP5).

Another misconception where the teacher guided learners was on calculating the curved surface area of an open cylinder. As was reported on for the LP, it was also evident that learners failed to understand that they could have found the radius of the pole by measuring the circumference and use the formula $C = 2\pi r$ to find the radius. Further, it was observed that they did not need the radius to calculate the curved surface area. All what they needed was to measure the distance around the circumference and the height of the pole and the use these dimensions to calculate the required surface area, using the formula that the area of a rectangle is equal to $lenghth \times breath$ (See the sample solution for the task in Table 5.2).

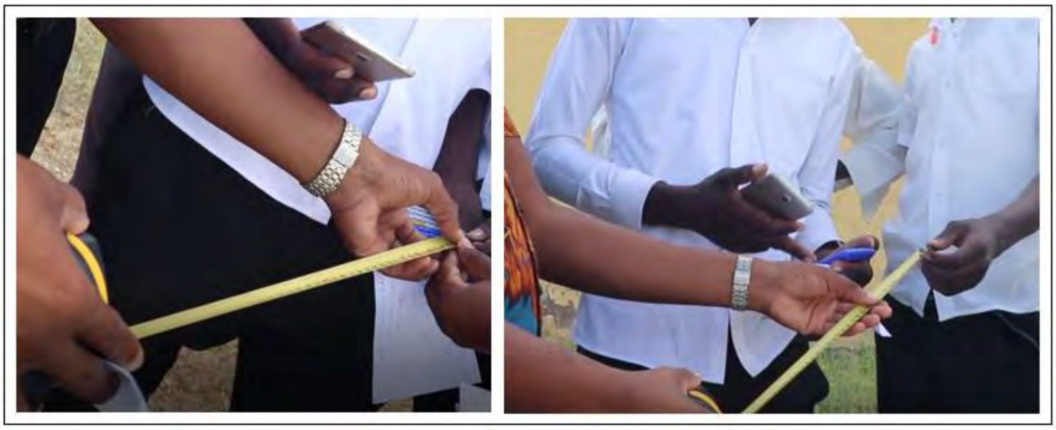


Figure 5.8: The teacher helping learners understand the units on the tape measure

Figure 5.9 below is an excerpt of a conversation from **KV$_1$** between the teacher and his learners in an attempt to help them understand what they were supposed to do in order to find the circumference of the pole.

TEACHER	Here there's no way you are going to measure the radius, so what should we do?
LEARNER A	But how sir? area of a circle is πr squared, so we need to find the radius for us to calculate the area.
LEARNER B	But it's true sir how can we do it without radius? So, I think we should just measure this way (*putting the measuring tape straight across the pole*).
TEACHER	Yes, here there is another way you can find the radius without measuring it on top or down. Okay, who can show me the circumference here? (GP2)
LEARNER A	the circumference is on top, so we need to remove that black thing on top for us to measure it, isn't? thing (*pointing at the top of the pole*)
TEACHER	We don't need to do that, look here (*holding the pole and moving his hand around it*). Is this not the circumference? I mean what is circumference? Listen here guys, who can tell me what is circumference? (GP2)
LEARNER C	Circumference is the distance around the circle.
TEACHER	Yes, so is this not the distance around this pole, which is in a circle shape? So, it is form here that you can also use the formula of $2\pi r$ to find radius povi (*isn't not so*)? If you have circumference then you can calculate radius by making r to be the subject of the formula. But also, here you need to understand that you don't need to find radius in order to calculate the circumference, because you can just directly measure it. (GP1/GP2). **KV$_1$**

Figure 5.9: An extract showing Kamwi guiding learners on how to find the circumference.

It was after this conversation that the learners seemed to understand how to solve the problem. It should however be noted that despite this attempt and many others from the teacher (including the MCM app hints), learners still failed to find the correct answer for the first task.

5.2.2.2 Trail 1 of School B (Teacher 2: Anna)

Profile and Coding

The pseudonym Anna is used to identify the teacher participant who was observed and video recorded during trail 1 at School B (coded **TR1B**). The MCM app trail code for TR1B is **0312123** and is named "*Exploring School B Trail*". Anna is a qualified mathematics teacher with a Diploma in Education (BETD) from the former Caprivi College of Education, the current Katima Mulilo UNAM campus. Anna has been in the teaching profession for the past 16 years now and is currently teaching mathematics and physical science to Grades 8 and 9 learners. The video data source from Anna is coded as $\mathbf{AV_2}$.

Eight learners (four boys and four girls) were under the guidance of Anna during the implementation of the first mathematics trail at her school. Anna and her group of learners solved two tasks of the first trail of School B – (see points 1 and 2 in Figure 5.1). It took approximately 45 minutes for Anna and her learners to walk and solve the two tasks on the trail.

The implementation of the RME principles: Anna

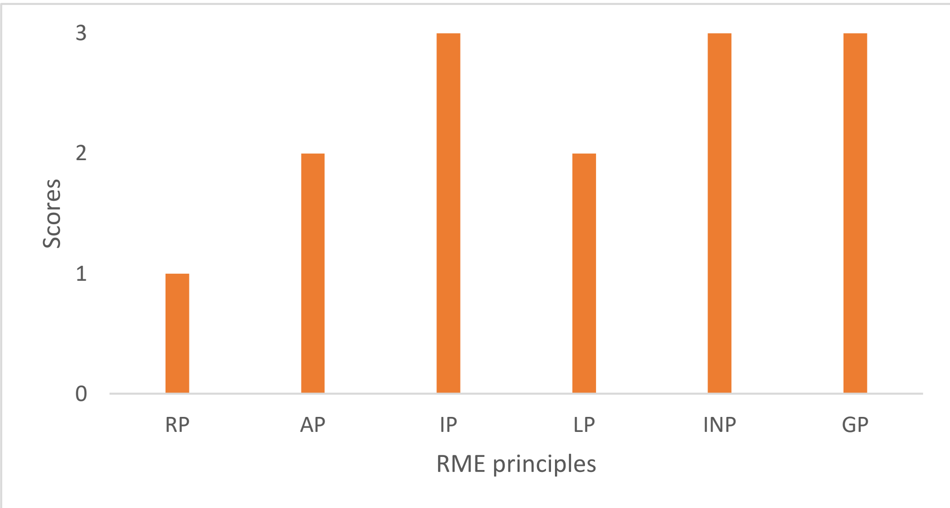


Figure 5.10: Numerical scores of the implementation of the RME principles by Anna.

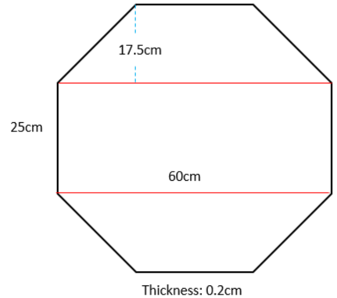
The graph in Figure 5.10 above shows that Anna's use of the six RME principles was dominated by the IP, INP and GP. These three principles scored a maximum of three scores each while the AP and LP show a score of two each. The RP was the least used in the trail.

The Reality Principle (RP)

According to my observations, there was only one incident where the teacher referenced the aspect of rounding off in real life. Data from **AV_2** shows that when solving the second task of the trail, the learners carried out measurements of the length and width of the patio in order to find the total area to be tiled. When calculating the total number of tiles to cover the floor of the patio, learners reached an answer of 187,22. The problem then was regarding how to round off this calculation in order to determine the total number of tiles. Whatever their answer was before rounding off, learners were supposed to round up their calculation to the next full tile in order to allow sufficient tiles for cutting.

However, in this case, instead of rounding up their answer to 188 tiles, learners rounded off to 187 tiles by using the basic principle of rounding off numbers to the nearest whole number (see Figure 5.11 below). It was at this point that Anna reminded the learners that they have to round up their answer to allow an extra tile which will be used to cover the 0,22 section. From this scenario, it can be inferred that the learners failed to apply the RP of rounding off quantities. In this case, the 0,236 fraction of a tile was left out simply because it was below the 0,5 benchmark of rounding off. This means that a part of the patio would be untiled. So, in order to cover the whole patio with tiles one extra complete tile would be needed.

Table 5.3: The two Tasks encountered and solved by Anna’s group of learners in Trail 1 of School B

TASK TITLE	TASK DEFINITION	OBJECT	SAMPLE SOLUTION	HINTS
The traffic road sign task	Calculate the amount of metal that makes up this traffic road sign board. Write your answer to the nearest cubic centimetres.		 A =2(Trapezium)+Rectangle $= 2\left(\frac{1}{2}(a+b)h\right) + (l \times b)$ $= 2\left(\frac{1}{2} \times (25+60) \times 17.5\right) + (60 \times 25)$ $= 1487.5 + 1500$ $= 2987.5$ $V = 2987.5 \times 02$ $= 1787.5$ $= 1788cm^3$	1. The stop sign is in the shape of a regular octagon. Therefore, divide the octagon into shapes you know. 2. The octagon can be divided into three shapes: two identical isosceles trapeziums and one rectangle. The area of a trapezium is ½ × (sum of parallel sides) × ⊥height and the area of a rectangle is l × w. 3. The "amount metal" means the volume of the road sign board. Therefore, after you get the total area of the three shapes multiply the answer with the thickness (the thickness represents height) of the sign board to get the volume.
The patio Task	If a tile measures 40cm × 40cm, how many tiles will be needed to cover the surface area of the patio?		**Measurements** length 747cm; width 401cm. Area of patio = l × w = 747 × 401 = 299547cm² Area of 1 tile = 40 × 40 = 1600cm² Number of tiles = 299547÷1600 = 187.2168 = 188 tiles	1. In what shape is the patio? 2. Divide the area surface of the patio with the area of one tile to find the number of tiles needed. 1. 3. Be careful how you round off your final answer to get the total number of tiles.

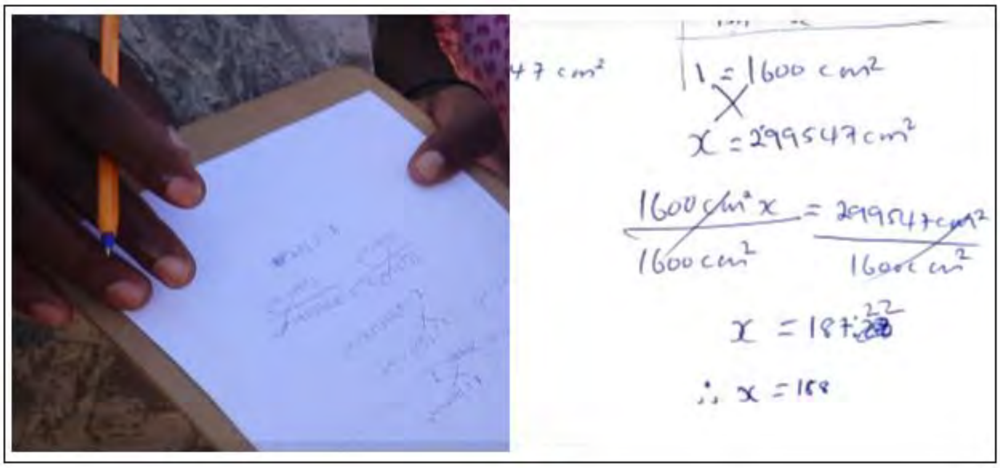


Figure 5.11: Learners' answer of rounding of the number of tiles to the nearest whole number.

Notice again that before Anna corrected the learners regarding the way they rounded off their calculations, some of them felt that it was better to leave the answer as it was, namely 187,22 and this led the teacher to caution them that the tiles would have to be bought as whole numbers, not as cut off pieces (RP2). She emphasised the following;

> Look here guys, you cannot leave your answer as 187. 22 … you need to round it off to 187 or 188. So where are you going to round it off, up or down? Remember this point two two (0.22) means that you will need a piece of a tile in order to complete the whole area of the patio, and in the shop, you can only buy a complete tile not a half or a piece of it. You cannot go in the shop and say I only want a small piece so break this tile for me, no! (RP2)
>
> **AV_2**

Figure 5.12: An extract of Anna applying the RP on rounding off.

In Figure 5.12, to clearly explain her point, Anna stressed that in reality the 0,256 of a tile that they were about to leave out, represented a part of a tile that would be needed to completely cover the whole patio floor. Otherwise, their rounding off to 187 instead of 188 tiles would mean that a small portion of the patio would not be covered by the 187 tiles (RP2).

The Activity Principle (AP)

The AP was one of the three RME principles that was effectively employed by Anna. She and her learners were seen actively engaged walking the trail and solving of the two tasks. The **AV_1** data source shows that the learners were excited about the whole idea of finding the tasks on the trail and solving them. It was clearly observed that the learners were engaged in productive discussions, where they often agreed on the locations of the trail tasks (AP1). The solving of the second task was observed to engage learners in the discussion of concepts related to surface area and proportion (AP1). Furthermore, all the learners, including the teacher, were seen taking part in measuring the dimensions of the patio and discussing the solution to the problem (AP2) – see Figure 5. 12.

Figure 5. 13: Anna supervising her learners while they measured the length of the patio

The Interactive Principle (IP)

In Figure 5.14 below, from **AV$_2$**, I present evidence of the interaction among learners and between the teacher and learners during the trail.

TEACHER	This is the map of School B (*mentions the name of the school*), is it not so (*learners agree*). Now do you see where this task is? (*again, learners show agreement*). Where do you think it is?
LEARNER A	At the governor (*meaning the governor's office*).
TEACHER	So, do you understand now understand this map? (*learners show agreement*). So, do you see that we are here? (*pointing at the position of the map on the phone*). Where are we?
LEARNER B	In front of the staff room.
TEACHER	Correct, so if this is in front of the office, where do you think this is? (*pointing at the position of the task, and for a moment, learners appear confused*).
LEARNER C	Oh…, I think this is at the gate, that side (*pointing at the boys' hostel entrance gate*).
LEARNER A	Yes, it is at the boys' side.
TEACHER:	Good, now which way should we go, this way or that side? (*pointing in two opposite directions… learners point at the same direction and echo this way this way sir*)
TEACHER	Okay let us go there. So, as you start walking you will see that this thing (*Live GPS blue light*) will start moving. And you will use this to show whether we are going at the right place or not, and when we get there you should click here to see the image of the task so that you know exactly what you are looking for. (IP2/IP3)
	AV$_2$

Figure 5.14: An excerpt of a conversation showing the interaction between Anna and her learners

This interaction was initiated by the teacher at the start of the trail when she let learners decide where to go in order to find the first task (IP2). From the start, the learners had difficulty interpreting the MCM app, and this ignited a debate of where to start and where to go. It was after the teachers' interaction with the learners that they managed to figure out where to go and what to do. In this particular discussion, the IP is evidence of how Anna engaged learners to help them understand and interpret the MCM app. Most important is the outcome of the above discussion that led to the correct interpretation of the MCM app, hence the learners knowing what to do next. Generally, it was observed that Anna ensured appropriate communication between her and the learners. This ensured their direct involvement with the MCM app and led to a positive social interaction between the three parties – teacher, learners and the MCM app (IP4).

The Level Principle (LP)

The use of the LP in this study was evidenced by how Anna directed her learners to divide the regular octagon shape into known shapes whose areas were easier for them to calculate (LP5) – see Figure 5.15 below. When solving the task, it was observed that the learners mathematised the problem by first discussing the appropriate method to use for finding the solution. At first there was a difference of opinion as to the shape of the road sign board and the correct approach to use when solving the problem. It was only after Anna intervened that learners were able to correctly identify the type of geometrical shape they were dealing with – see the excerpt on the GP above (LP4). Thereafter, the learners drew a rough sketch of the shape (octagon) which they then divided into three known shapes (a rectangle and two isosceles trapeziums) (LP5:LP6). This was then followed by identifying the formulas of the involved shapes. In all these processes there was minimal intervention from Anna in helping learners to identify the shape and how to divide it. What clearly emerged in all this is that the learners were able to move within the abstract world of symbols through the fluent recall and use of formulas, and I would refer to this as mathematising vertically (LP8).

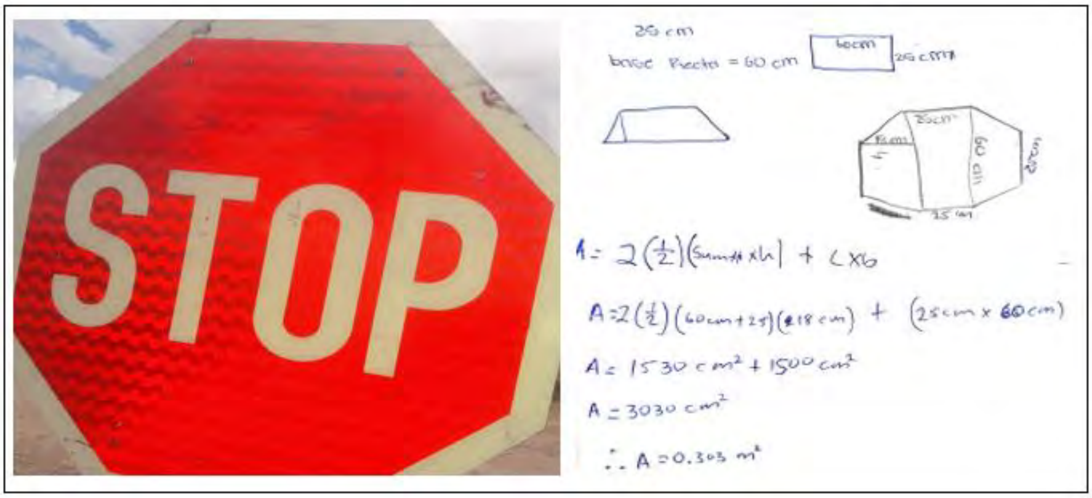


Figure 5.15: Learners' solution to the traffic road sign task

However, I nevertheless noticed that there were certain concepts that were confusing to the learners, which led them to incorrectly use the formulas. For example, it emerged that they struggled to identify the parallel sides on the trapezium shape when calculating the area of the cross-sectional base. So, the teacher had to take them through the properties of a trapezium using the original sign board object. With the aid of their rough sketch of a trapezium, the teacher asked the learners to show her the corresponding parallel sides and the height of a trapezium on the sign board (LP5:LP6:LP2). Apart from the confusion and misapplication of parallel lines and height of the trapezium, the learners demonstrated an understanding of the problem and how to solve it. It must be noted however, that when learners uploaded their solution(s) on to the MCM app, the feedback kept on indicating that the solution(s) were wrong. After several attempts by both the teacher and learners, it was discovered that the uploaded question from the system was incorrect, for the question instructed learners to give their final answer in square centimetres instead of cubic centimetres.

Another interesting observation here was how one learner suggested that instead of using the trapezium formula, they should divide the trapezium into triangles and a rectangle and then use the formulas of these shapes to find the area of the whole trapezium. Although this was not done, I see this suggestion to have been an attempt to vertically mathematise the problem where the learners' thoughts ventured into the world of mathematical symbols and algorithms. Also, this shows that the learners had more than one strategy to calculate the area of a trapezium.

The Intertwined Principle (INP)

The INP was evidenced by how Anna guided her learners to understand the concepts involved when calculating the area of a trapezium. First, after noticing that they were struggling to figure out the properties of a trapezium, Anna used a practical example to link the concepts of parallel, perpendicular and sloping sides to that of the original sign board they were dealing with (INP4). For example, the excerpt below (Figure 5. 16) shows the teacher linking the concepts of parallel from the trapezium to that of the actual building.

> Okay, what is the difference between parallel and sloping sides (*learner gives different responses*), look at the top of that roof (*pointing at a house opposite the school fence*) … do you see that those zincs (*iron sheets*) are sloping down … what about this trapezium? Which sides look like those zincs here (*pointing at the learners' sketch*) … (INP4)
>
> **AV$_2$**

Figure 5.16: Shows an excerpt of how Anna applied the INP.

Furthermore, Anna asked the learners to remember that just as in a triangle or a parallelogram, the height of a trapezium should also be perpendicular to the parallel sides (INP1). Several other concepts like opposite, quadrilateral, vertex and angles were mentioned in the process of solving the sign board task. Through a series of probing questions, the teacher strived to help learners to understand these concepts and how they related to different quadrilateral shapes. It was clearly seen that Anna was using the original sign board to physically show and explain the concepts to learners (INP4). Another important observation to report on here is how Anna captured the learners' attention by getting them to look around at their surroundings and identify other objects or physical patterns with properties such as parallel, opposite, perpendicular and equal (INP4).In response the learners pointed at the walls of buildings and roofs.

The Guidance Principle (GP)

TEACHER	What shape is this?
LEARNER A	It's a pentagon…, ah no mam it's an octagon.
TEACHER	Okay, so for you to easily find the area of this octagon, what should you do? (*all learners seem to not know what to do*). Look here, you should divide this shape into smaller shapes that you know. So what shapes can we here, for example if we cut this part of the octagon what shape are we forming (*points at the trapezium part of the sign board*) (GP2)
LEARNER	It's a rhombus. (*other learners laugh while others disagree*).
TEACHER	No (*mentions the learner's name),* how can you say this is a rhombus. A rhombus is a shifted square. Is this a shifted square? What kind of a shape is? (*learners still fail to recognise the shape*). Okay, draw the shapes we will form after dividing this shape *(referring to the whole octagon shape*). How many shapes will we have? (GP2:GP4)
LEARNER C	Three… (*meaning three shapes*).
TEACHER	So what shapes are they going to be? (GP1)
LEARNER A	This is going to be a rectangle (*pointing at the middle shape*)
TEACHER	And the other two shapes (*learners remain quiet*) What type of a shape is this where you have only two parallel sides? Come on people don't tell me that you don't know this. Please, don't disappoint me here (GP1:GP2)
LEARNER B	It's a trapezium mam.
LEARNER D	Yes mam, this is a trapezium (*and all other learners agrees*)
TEACHER	Good! So how do you calculate the areas of these shapes?
LEARNER	A rectangle is length times breath.
LEARNER E	A rectangle is half times base times height.
LEARNER B	No, that is for a tringle.
LEARNER A	But we can also have a rectangle and a triangle on this trapezium…(GP1/GP2) **AV2**

Figure 5.17: A conversation where Anna guided learners on how to divide the octagon into a rectangle and two trapeziums

During the trail and solving of the tasks Anna offered guidance to the learners where it was necessary. As mentioned earlier on the RP, the teacher used a practical example that enabled learners to understand that an answer of 187,22 was supposed to be rounded up to 188 tiles, not 187 (GP4). Also, Anna skilfully guided learners to understand and differentiate between parallel, perpendicular and sloping sides (GP4). Through a series of guiding and probing questions the teacher evoked discussions among the learners that helped them to make their own conclusions (GP2). Throughout the trail and the solving of the tasks it was clearly observed that Anna avoided providing learners with direct answers to their questions (GP6). For example, the excerpt in Figure 5.17 above shows a conversation where Anna used the LP to guide learners in dividing the octagon into a rectangle and the two trapeziums.

It was clear that after the above conversation, the learners understood what they were supposed to do. This was seen by how easily they divided the octagon sketch into the shapes of a rectangle and two trapeziums. It is also important to mention here that as the learners failed to point out properties such as parallel, perpendicular height and sloping sides, Anna kept on posing questions that guided them to identify the properties of an octagon as they appeared on the original sign board object (GP2:GP4).

5.2.2.3 Trail 1 of School C (Teacher 3: Moses)

Profile and Coding

The first trail of School C in this cycle is coded as **TR1C**. The trail is named *"Exploring School C Trall*" and the MCM app trail code is **2516091**. The first participant teacher to be recorded in Trail 1 of School C is given the pseudonym Moses. Moses is a secondary school teacher of mathematics and physical science to Grade 9, 10 and 11 learners at School C. He holds a Batchelor of Education in Mathematics and Science Education from the University of Namibia and has been in the teaching profession for the past five years now. The VR from this trail is coded as **MV**$_3$ and is 42 minutes long.

Six learners (two girls and four boys) were part of Moses's group during the first trail at school C and the group solved two tasks on the trail – (see points 1 and 2 in Figure 5.1 above and Table 5. 4 below). Moses and his learners took approximately one hour to walk and solve the two tasks of the trail.

The implementation of the RME principles: Moses

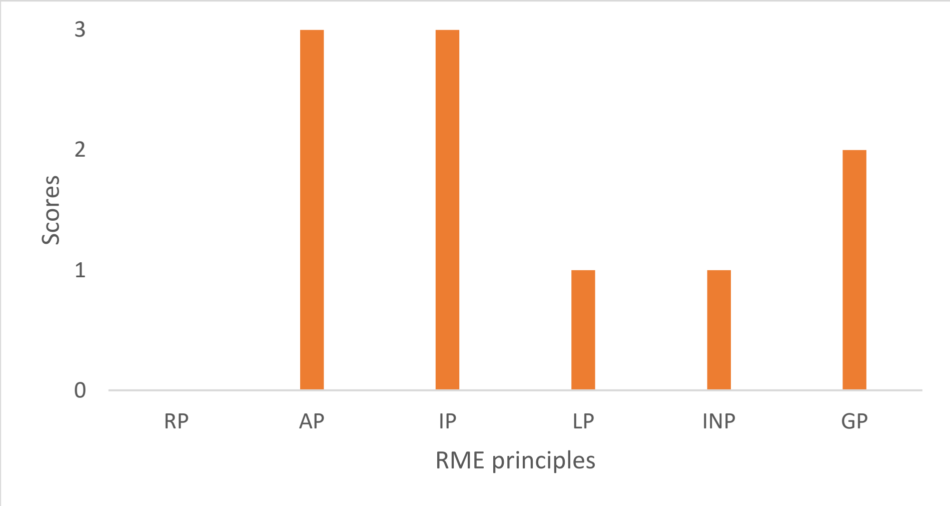
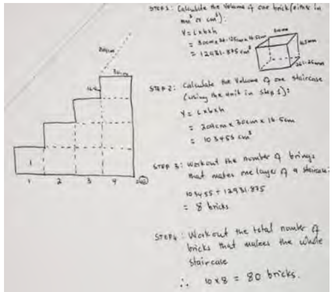
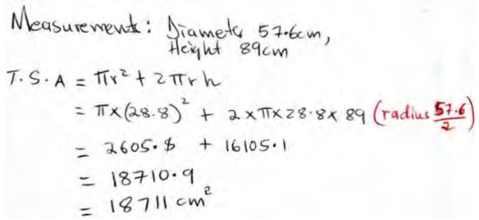


Figure 5.18: Numerical scores of the implementation of the RME principles by Moses.

The graphical summary in Figure 5.18 above shows that the AP and IP were the most used RME teaching principles in this trail, while the RP, LP and INP were observed and rated to be the least used. On the other hand, the GP was observed to be moderately used on the trail. Here is a detailed discussion of how each principle was employed to conceptually teach area and volume topics on Moses' trail.

Table 5.4. The two Tasks encountered and solved by Moses's group of learners in Trail 1 of School C

TASK TITLE	TASK DEFINITION	OBJECT	SAMPLE SOLUTION	HINTS
The staircase Task Code 1531547	Assuming that the size of one brick is 300mm x 261. 25mm x 165mm, work out the total number of bricks that make up these staircases.			1. A brick and a tier of a staircase are both in the shape of a cuboid. 2. The formula of calculating the volume of a cuboid is V = l x b x h. 3. Remember to use the same unit of measurement when calculating both the volume of a brick and a staircase (mm^3 or cm^3).
The dustbin Task 1 Code 3668229	If you were to replace this dustbin with a new one, work out the amount of metal sheeting needed to replace the bin. Give your answer to the nearest square cm.			1. The dustbin is in the shape of a cylinder. 2. Consider the net of a cylinder and identify which faces should be involved when calculating the surface area. 3. T.S.A = $\pi r^2 + 2\pi rh$

The Reality Principle (RP)

The RP in this trail was among the three least-used principles - see Figure 5.18 above. I observed that Moses was at odds to connect and explain the reality aspects of the tasks to be solved by the learners. Apart from being connected to real life objects of a dustbin and staircase, there were no attempts from the teacher to explain or provide examples of how and where the tasks can be used in real life. It therefore cannot be claimed that these tasks were used to educate learners about solving problems they face in real life. However, the problem situations of both the dustbin and staircase tasks still remained meaningful in that the tasks gave learners an opportunity to use real-life tools such as a tape measure tape to collect data from the real objects the dustbin and stairs. This in turn enabled the learners to attach meaning to the mathematical constructs they developed while solving the problems. For example, the solving of the tasks promoted real-life skills such as mental calculations, estimation and measuring techniques and the competent use of calculators (RP2).

The Activity Principle (AP)

Moses treated learners as active participants during the learning process of solving the MCM tasks. For example, he kept on encouraging them to find the solution to the stair task by themselves without expecting answers from him (AP1). The excerpt in Figure 5.19 below shows how Moses encouraged the learners to actively work together in order to find the solution:

Okay guys, I want you to work out the answer to this question on your own, I will not tell you what to do… work together and find the answer on your own AP1 **MV_3**

Figure 5.19: shows an excerpt of how the teacher encouraged learners to find a solution to the staircase task

The **MV_3** observation source also shows that during the trail, the MCM app succeeded in creating a conducive learning environment for engagement (AP2). The learners discussed, for example, what dimensions to measure, where and how to place the measuring tape and what unit of measurement to use. Apart from the names for the quadrilaterals, they frequently used mathematical terms such as parallel, angle, straight line, perpendicular and of equal length. It is therefore likely that because of the frequent use of these terms and how they linked them to the actual object tasks, their understanding of the concepts as related to calculating area and volume was deepened and became experientially real (AP1).

Interactive Principle (IP)

The IP emerged from the data on several occasions in this trail. The focus of these interactions was on the interpretation and understanding of the MCM app to find the location of the tasks as well as on the solving of these tasks (IP2). One example is when Moses, in helping learners solve the stairway task, started to question the more general ideas of learners when determining the type of shape the learners were dealing with to determine the correct formula to use (IP3) – see Figure 5.20 below. The learners were referring to the brick as a rectangular shape instead of a cuboid as they were supposed to, hence the teacher's intervention to correct them:

TEACHER:	What did you say this shape is? (*pointing at the first layer of the staircase*).
LEARNER A:	Efro…, this is a rectangle (*speaking in the mother tongue language*).
LEARNER B:	No this is a cube? (*disagrees with learner A*).
TEACHER:	Okay, wait a minute, in what shape is the first layer of this staircase?
LEARNER C:	Mam, this is like a block which is… yes, I think this is a cuboid.
TEACHER:	Good, so how do we find the number of bricks in this first layer **MV3**

Figure 5.20: Moses's interaction with his learners when correcting their misconception of the shape of a brick

Another observed key aspect of the IP in this trail was how the MCM app offered ample opportunities for interaction. As the six learners only had one smartphone between all of them, this situation made them work together and interact with each other. This turned out to be positive, for often all the six learners were seen bent over the screen together (see Figure 5.21 below), discussing and planning their actions (IP5). However, sometimes the learners carrying the smartphone did all the work of directing the team while the other learners simply walked along.

Figure 5.21: Learners interacting while using a smartphone to find the location of the next task to solve

The Level Principle (LP)

The **MV3** recordings show incidences where the teacher allowed the learners to apply their own strategies in solving the given tasks. For example, on the stairway task, instead of solving the task as expected on the sample solution (refer to Table 5.4), the learners opted to determine the number of bricks that make up one of the steps by repeatedly moving a single 30cm ruler along the length of the step (LP4) – See Figure 5.22 below.

Figure 5.22: Learners using the strategy of repeatedly moving a ruler to determine the number of Bricks.

When Moses asked why they were using a ruler to measure (by repeatedly placing it on the surface of the stair) instead of counting the number of bricks, the discussion from the **MV3** data source shown in Figure 5.23 below ensued.

The learners here were formulating their own solutions to the problem by mathematising the reality of the problem into an approach and situation that was simple for them to explain (LP2). I would consider this to be horizontal mathematisation, in the sense that the learners applied their informal strategies to solve the problem even before considering moving within the world of mathematics. Evidently, as with the teacher, we did not expect them to determine the number of bricks through an informal means of using a ruler to determine the number of bricks as they did. So, the learners' counting experience led to a situation model that stimulated their counting and use of indirect addition as an informal solving strategy (LP8).

LEARNER A	Efro, the question says that the measurements of one brick is $\mathbf{300mm} \times \mathbf{261.25mm} \times \mathbf{165mm}$… so this means the length of the brick is 300mm.
TEACHER	… and so, what does that mean?
LEARNER A	300mm is the same as 30cm, which is the size (*length*) as this ruler (*holding the ruler and showing it to the teacher*).
TEACHER	Now, why do you have to use this ruler to find the number of bricks from here to there (*pointing from one end of the staircase layer in length to the other*), I mean, why not just use the measuring tape to measure the whole of this length and then divide it by the length 300mm, which is the length of one brick according to the question?
LEARNER B	Here it's easy for us to find out how many bricks from here to there by just placing the ruler step by step like this… (*at this point the learner places the ruler by the end of the stairway and marks the floor with a pencil at the end of the ruler, and again places the ruler at the marked place to make another measurement*).
	MV3

Figure 5.23: Learners explaining their informal strategy of determining the number of bricks in a stairway

Another incident where the LP was observed was in the dustbin task where the learners calculated the surface area of a cylinder. It was observed that they used the net of a cylinder to explore the formula for calculating the total surface area (TSA), $2\pi r(r + h)$, of a cylinder. Figure 5.24 below presents a conversation where Moses, together with his learners, used the LP to explain and understand the $2\pi r(r + h)$ formula of calculating the TSA of a cylinder. In their conversation, notice how they mathematised the formula from formal mathematisation to informal. First, they discussed the formula from a mathematical point of view (symbols) and then moved to comparing the formula to the model of the net that was drawn on the ground (LP5) (see Figure 5.24). Also, notice that both the learners and the teacher compared and connected the formula to the actual bin in front of them (LP4).

LEARNER A	The formula is two pie r, open bracket r plus h inside the bracket. But this one is open on top ($\boldsymbol{2\pi r(r+h)}$)…, but now we are not going to include the top circle because its open here.
TEACHER	Okay, so what formula are you going to use if its open like this? (*referring to the dustbin*)
LEARNER A	Its 2pie r h plus pie r squared ($2\pi rh + 2\pi r^2$)
TEACHER	No, what is the meaning of this $2\pi rh + 2\pi r^2$ Okay, can you explain more. Where is $2\pi rh + \pi r^2$coming from?
LEARNER B	Sir, the thing is like this (*sketching the net of a cylinder on the ground*) but we only out one circle here because its open. (LP4)
TEACHER	Okay, good the net is fine. Okay look, when you remove this bracket (*meaning to expand* $2\pi r(r+h)$) you will get $2\pi rh + 2\pi r^2$. And then you can rewrite it this way, $2\pi rh + \pi r^2 + \pi r^2$. Look, this $2\pi r^2$ means there are two circles, which is this πr^2 and another one was supposed to be the other side, but remember, as learner B said, we are not going to put it here because its open, are together (*learners shows agreement*). (LP3, LP4, LP5) **MV3**

Figure 5.24: The emergence of the LP during the mathematisation (formal to informal) of the formula for calculating the total surface area of a cylinder.

Notice also how, in Figure 5.25 below, the learners' solution to the dustbin task was to model a net of a cylinder as with the previous task.

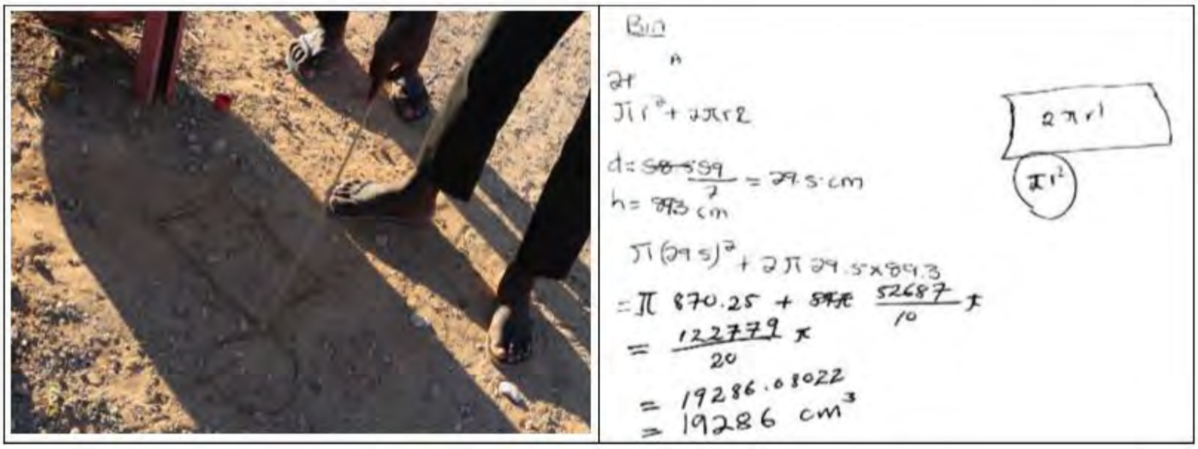


Figure 5.25: One of the learners sketching the net of the dustbin.

The Intertwined principle (INP)

I noticed the integration of one domain of mathematics with another in this trail when the learners combined skills from the topics of counting and measurement. Although not extensively intertwined, it was still clear that the teacher allowed them to integrate their knowledge of numbers, counting and measurement in their solution to the given problem task (INP1:INP2). The activities involved in solving these tasks gave the learners an opportunity to use various mathematical tools and knowledge. For example, I saw them applying a combination of skills including mental arithmetic, estimation, counting, calculator usage and measuring, to find the solutions to the tasks.

The Guidance principle (GP)

The teacher's interaction with learners was one of monitoring and guiding whenever they experienced difficulties, so as to create a fun learning environment. On one or two occasions Moses took a proactive role of guiding his learners through different scenarios that had the potential to shift their understanding from a lower to a higher level. For example, when they confused the three-dimensional shape of a brick to be a two-dimensional rectangle, I noticed the teacher asking the learners to differentiate between two-dimensional shapes and three-dimensional ones. This, in turn, helped them to understand that a brick cannot be a rectangle because a rectangle is a two-dimensional shape (GP3:GP4).

5.2.3 MCM Cycle 2: Brief overview of the mathematics trails

There were only two schools that participated in the implementation of the second MCM cycle of this study which meant that the second cycle had only two trails. Due to circumstances beyond my control, the participant from the third school was unable to take part in both the creation and implementation of the tasks. The two trails of this cycle were created outside but not far from the boundaries of the two schools. Figure 5.26 below shows the offline location map of each trail of Cycle 2.

The mathematics trails of Cycle 2 had the following characteristics:

- School A (Exploring the other side of Opuwo Trail) – consisting of five tasks.
- School B (Math math and math) – consisting of four tasks.

Figure 5.26: The offline trail maps for each school in Cycle 2

5.2.4 DATA PRESENTATION AND VERTICAL ANALYSIS OF INDIVIDUAL PARTICIPANTS

5.2.4.1 Trail 2 of School A (Teacher 4: Sinvula)

Profile and Coding

The participant teacher who was video recorded in **TR2A** is identified by the pseudonym Sinvula. The highest qualification Sinvula holds is a post graduate certificate (ACE) in Mathematics Education from North-West University. With seven years of teaching experience, Sinvula is responsible for teaching mathematics Grades 8 to 9 at School A. Data from Sinvula's VR is coded as **SV4** and is 53 minutes long. It is worth mentioning here that **TR2A** was selected the trail of the month for November 2022 – (https://mathcitymap.eu/en/trail-of-the-month-exploring-the-other-side-of-opuwo/). The name of the trail on the MCM app is "*Exploring the other side of Opuwo*" and its digital code is **5612172**.

In this trail, Sinvula led a group of seven learners (four boys & three girls) who solved two MCM tasks connected to the topics of area, ratio and proportion – see points 1 and 2 in Figure 5.26 above and Table 5.5 below. Just as in the other trails from the previous cycle of this study, each group of learners also shared and used one smartphone while walking this trail. Sinvula and his learners spent approximately an hour and 20 minutes on this trail.

The implementation of the RME principles: Sinvula

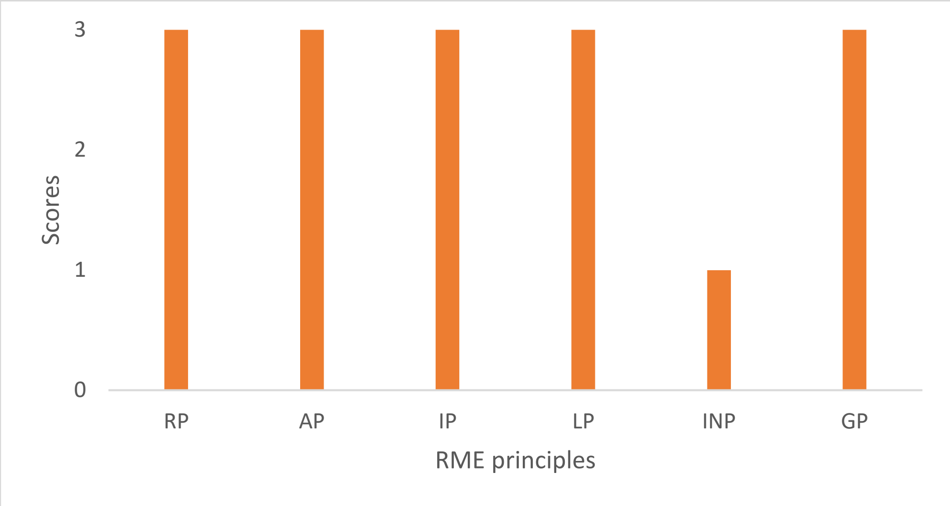
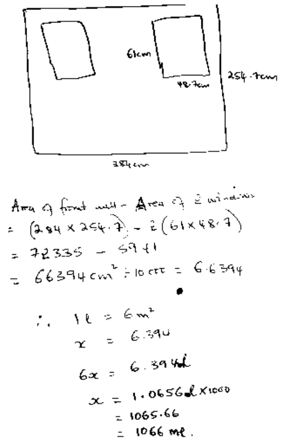


Figure 5.27: Numerical scores of the implementation of the RME principles by Sinvula.

The above graph in Figure 5. 27 summarises the extent to which teacher Sinvula implemented the six core teaching principles of the RME theory in TR2A. According to the graph, apart from the INP, the other five RME principles, namely the RP, AP, IP, LP and GP were used more than three times in the trail. What follows is a discussion of how each principle was used to teach the topics of area, ratio and proportion.

Table 5.5: The two tasks encountered and solved by Sinvula's group of learners in Trail 2 of School A.

TASK TITLE	TASK DEFINITION	OBJECT	SAMPLE SOLUTION	HINTS
The painting wall Task Code 8954735	If 1 litre of paint can cover 6 square metres of the wall, how many millilitres of paint will you need to paint the exterior front side of this building? Give your answer to the nearest millimetres.			1. First calculate the area of the wall (rectangle) and then subtract the area of the two identical window spaces. 2. Use direct proportion to determine the amount of paint needed for the wall. 3. Do not forget to change your final answer from litres to millilitres and round off to the nearest 100.
Age of the camelthorn tree Code 5654724	Determine the age of this camelthorn tree. In this part of the country, a camelthorn tree with a circumference of approximately 100cm is about 30 years old. Assume that the circumference grows proportionally. Give your answer to the nearest year.		Circumference of the tree = 221. 5 cm. 100cm = 30 years 221.5 cm = x (cross multiply) 100x = 6645 x = 66. 45 = 66 years old.	1. Measure the current circumference of the tree. 2. The age of the tree is proportional to its circumference.

The Reality Principle (RP)

The use of the RP in this trail was seen in the following incidences: first, the teacher reminded learners that when painting an actual building they need to know how much paint is needed to cover the wall, hence the need to work out the actual area of the wall in question (RP2). An extract from **SV4** in Figure 5.28 below presents evidence of how Sinvula emphasised the reality aspect of the painting task to learners.

Enue Zemburukee (*Remember guys*), when you want to buy paint for this wall you cannot just go to the shop before finding out how much paint you need to cover the wall. So, first of all, it's very important to calculate the amount of paint you need so that you buy the correct quantity of paint. So, say you were asked to paint this wall, how will you calculate the amount of paint you should buy if 1 litre can only cover 6 square meters of this wall (touching the wall)? What are you going to do to find the paint? … if you want to paint your house you should know how much paint you should use. You cannot just go to the shop and buy a lot of paint, what if it is more than what is needed? it will be a worst. **SV4**

Figure 5.28: Sinvula emphasising the RP of the painting task.

The second incident centred on the exclusion of the open window spaces when calculating the area to be painted. Clearly, learners understood that when painting the wall, the window spaces were not to be included in the painting, hence the reason why they decided to subtract the areas of the two windows from the area of the entire wall (RP1, RP2) – see Table 5.5 above. The third incident was observed when there was a question of whether to paint the frames of the windows or not. When measuring one of the window spaces, a learner asked why they were including the frames of the windows as part of the area not to be painted (RP2). This question led to the following excerpt of the discussion in Figure 5. 29 below:

LEARNER A	… Why are we not supposed to include this part of the window frame when painting the wall? (*pointing at the head, jamb and sill of the window*). I think this part also use to be painted, isn't so (*speaking in the vernacular language*)?
TEACHER	What do you think people? Do you hear what (*mentions the name of the learner*) is saying? What do you think?
LEARNER A	Yes, we also need to paint this part…
LEARNER C	No, when people use to put the windows on the house this part use to remain in the same colour it came with?
LEARNER B	What are you saying you? (*speaking in the vernacular language and questioning Learner B*)
LEARNER C	Yes, it's true… say you paint this house in white colour, this window frame together with the glasses will not painted. Just go to any house even at school you will see that this part is not painted.
LEARNER A	I think Learner C (*mentions the name of the learner*) is right.
TEACHER	Okay, do we agree to what Learner C is saying? (*Learners indicate agreement*).
LEARNE D	Yes sir, here we should just paint the wall and not any other parts of the window frame (*speaking in the vernacular language*).
	SV_4

Figure 5.29: Sinvula engaging learners on the realistic aspect of painting the window frames.

The discussion as to whether to include the window frames or not presented evidence of how the learners connected the reality aspect of the painting task to the actual profession of painting buildings. The learners reflected on the actual practice of painting buildings and recalled that an authentic builder or painter would not paint the frames in the similar situation (RP2). What is evident in the above excerpt (Figure 5.29) is that the teachers' interaction in helping learners understand this reality aspect of the task was minimal. It was the learners who started the discussion of whether to paint or leave out the window frames when painting the wall and also concluded, on their own, that in reality the two window frames were not to be painted.

The fourth incident was observed when Sinvula directed the learners to place the tape measure on the foundation line mark and not at the bottom of the building when they were measuring the height of the wall. In his explanation, the teacher used the RP depicted in the excerpt of Figure 5.30 below in order to help learners understand the part of the wall to measure and calculate the area.

You know that buildings have two different colours? There is this bottom colour below this line which separates the foundation and the wall going up. For example, look at that house … which colour, how many colours are there? (*learners answer that they are two colours, grey and maroon*). Yes, so the grey colour is on the foundation and maroon on the main wall of the house. So, when you paint this wall you will not have the same paint from top to bottom, the foundation is always having its own paint. So, you should know what to do here when measuring the height of the wall (RP2).

SV_4

Figure 5.30: Sinvula discussing the RP of two different colours in the painting task

It should be noted here that the task itself was not specific to what part of the wall was supposed to be painted. Nevertheless, the teacher's point of view was valid when considered from the reality perspective of painting buildings in real life.

The fifth incident involving the application of the RP from the **SV_4** data source was when learners were impressed by how they could calculate the age of the tree in mathematics, a reality that seemed new to them – (RP1). Their excitement induced them to clearly comment as in Figure 5.31 below:

LEARNER A	Sooo, I never knew that we can actually calculate the age of a tree in mathematics.
LEARNER B	Sir, does it mean that in reality this tree is 60 something years old?
TEACHER	Well, not really, we are just estimating how old it could be. Remember, the question says "assume". So, to assume means to estimate or guess kind of. But what do you think the age of this tree is?
LEARNER B	100?
LEARNER C	100 is too much…,
LEARNER D	Maybe 50, 60 or 70 there…
TEACHER	You can see that this answer is not far away from the answer we found here.
	SV_4

Figure 5.31: An excerpt that shows how learners were amazed by the prospect of calculating the age of the tree.

The Activity Principle (AP)

The AP tenet of the RME theory was shown to be used effectively throughout the entire trail. Sinvula allowed learners to work independently on their own to find the solution with minimal intervention from him (AP4). Many concrete examples can be drawn from the **SV_4** data source to show that the learners were actively engaged in collecting data by carrying out measurements of physical objects. One such example is shown in Figure 5.32 below where they are seen actively engaged in the measurements of the objects of the tasks in order to find solutions to them (AP2).

Figure 5.32: The emergence of the AP through the measurements of the dimensions of objects involved in the two tasks.

The Interactive Principle (IP)

The IP adds to the list of the most used principles of RME theory in the trail of **TR2A**. Throughout the trail and task solving there were positive interactions observed between learners and learners, and learners and the teacher IP2. The learners asked questions when they did not understand and the teacher was always ready to respond and help them. I also observed that the interaction between Sinvula and his learners was characterised by a sense of humour IP2. For example, one of the learners passed a joke about how she was going to tell her grandfather the age of the tree he usually sat under during the day – see Figure 5.33 below:

LEARNER A	Sir, this tree is big and it looks like the one grandpa used to sit under during the day at home (*speaking in the vernacular language*), now that I know how to calculate the age of the tee, I can't wait to go home this coming home weekend to go and tell him how old the tree.
LEANER B	Me I am going to tell people all the ages of trees in our village.
TEACHER	Well, you cannot go around and guessing the age of every tree you see… (*laughing*).
	SV4

Figure 5.33: shows an excerpt of the IP was characterised by a learner's joke

The other learners, including the teacher, laughed at learner A and B's comments above. To me, this was one of the socially light-hearted moments that provide evidence that the learners enjoyed the interactions on the trail because they were at ease and felt comfortable with each other's company, including the teacher (IP2).

The Level Principle (LP)

The use of the LP emerged from this trail when the learners sketched the wall with the two window spaces in it (see Figure 5.34 below). Clearly, this approach helped them to easily see that the wall to be painted was in the shape of a rectangle and led to them recalling the formula for the area of the rectangle (LP6).

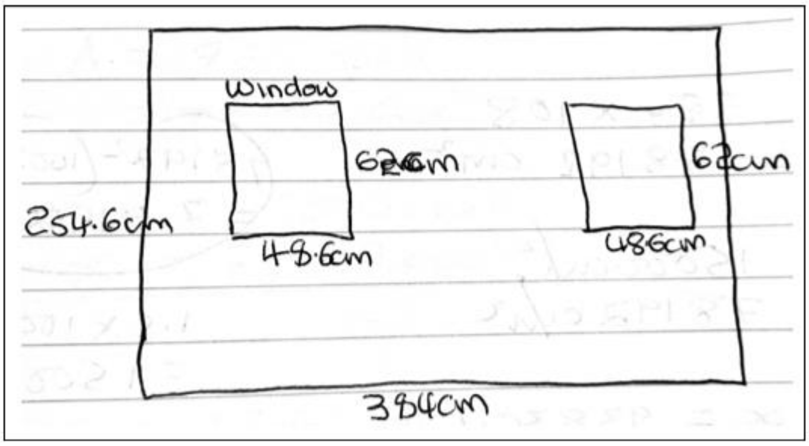


Figure 5.34: A learner's model of the wall to be painted and its windows

Below is an excerpt of the conversation between Sinvula and the learners when modelling the wall of the building to be painted.

TEACHER	Remember when you learned about the area of the shaded and unshaded parts of shapes… So which one is the shaded part on this picture? (*pointing at the learner's sketch*). (LP3)
LEARNER A	I think the part we are going to paint is the shaded part … this one without these two windows *(also pointing at the sketch on the paper*).
TEACHER	Good, so the question can be, calculate the area of the shaded part, isn't? (*learners show agreement*). So how can you find the shaded part here?
LEARNER B	Yes, we should calculate the area of the big rectangle and subtract the area of these two rectangles.
TEACHER	…and then what? I mean how is this the same as to this question of painting this wall?
LEARNER A	Sir, these small windows they represent the unshaded parts which should be removed from the whole area of this wall. So, all the area we are going to paint is the unshaded part in this question we are doing. **SV_4**

Figure 5.35: How the LP was used through the learners' sketch of the wall.

It should be noted that when Learner A was explaining the shaded and unshaded area concepts to the teacher, she alternatively gestured at the actual wall of the building and their sketch on the paper (LP6). She kept on referring to properties that made their sketch similar to that of the actual wall (see Figure 5.35 above). In this way, the LP was used to connect the properties that existed between the wall/windows and the geometrical shape of a rectangle. I observed that this resulted in an approach that helped learners to easily calculate the area that needed to be painted. The process of identifying properties from the actual wall that were relevant to the geometrical shape of a rectangle led to the formulation of a mathematical model of drawing a sketch to solve the problem (LP5). This can be inferred to as the application of mathematisation where learners used the available information to think between the actual wall of the building (real world) and the application of the shaded and unshaded areas and the rectangle formula to solve the task at hand (mathematical world) (LP8).

The Intertwined principle (INP)

The INP only appeared once in this trail. The **SV_4** data source shows that when learners were solving the painting task, Sinvula strived to help them see the links between the properties of a rectangle and the actual wall. This was done through linking the concepts of shaded and unshaded areas to the realistic situation of painting the wall where the windows were regarded as the shaded area that was supposed to be subtracted from the entire wall rectangle – see Figure 5.35 above. Therefore, the INP was evident through the linking of geometrical concepts of area to the actual, real building (INP4).

Moreover, both the two tasks in this trail involved more than one concept for solving them. For example, the concepts in Task 1 were measurement, area, proportion and rounding off, and in Task 2 there were concepts of measurement, parts of a circle (circumference), proportion and rounding off.

The Guidance principle (GP)

Sinvula skilfully guided learners on this trail. It was observed that most of the time the teacher provided guidance to learners in such a way that learners were always challenged to find the answers on their own, for example, see Figure 5.29 on the RP. More incidences can be cited where Sinvula applied the GP in this trail and one such an incident can be found in the following excerpt in Figure 5.36 below:

TEACHER	Wait a minute, what is the unit of your answer again?
LEARNER A	Its cm sir
TEACHER	Why do you say its cm? (GP2)
LEARNER A	Because we were measuring the circumference of this tree in cm.
TEACHER	But what does the question say? I mean what are you looking for? (*There is a long pause*)
LEARNER B	Ooh no… the answer should be in years, so it should be 66.26 years
	But the question says you should write your answer to the nearest years. What does that mean?
LEARNER C	The answer is 66 and 2 months…
TEACHER	Hmm, come on guys, nearest years not in years and months.
LEARNER	Okay sir, the answer is 66 years.
TEACHER	Yes, the tree should be 66 years old… you know your answer, how can you say cm, you are looking for years. So, your final answer must be in years.
	SV4

Figure 5.36: The emergence of the GP in TR2A.

In the above excerpt it is evident that learners did not understand or were simply confused about the type of unit to use on their answer. So, instead of telling them that they cannot use the unit of cm to represent the age of the tree, Sinvula posed a series of guiding questions to help the learners see their mistake (GP2). As can be noted in the extract, it was only after the teacher's guiding questions that learners rectified their mistake.

TEACHER	If the question was like write your answer in years and months, what was going to be your answer?
LEARNER A	The answer will be 66 years 6 months
LEARNER B	No, (*mentions Learner A's name*), I think we should say 0.6 times 12 because there are12 months in one year. (*explaining to the teacher and other learners while working it out*). So, the answer is 66 years and …
TEACHER	Good, it disturbed me to hear those 6 months, just because it is 66.6… because that's not how you change it to years and months. So, you should always be careful when it comes to converting the units of time (GP1)

Figure 5.37: Sinvula guiding learners on how to convert years to months.

Again, in the above excerpt, it can be noted that Sinvula was bothered by Learner A's response that the age of the tree was 66 years and 6 months. Although this was not part of the intended concept for learners to learn, the teacher quickly noticed that the learners had a misconception about converting years in fraction form to years and months. So, the teacher quickly engaged learners on this aspect and provided the necessary guidance as can been seen in Figure 5.37 above (GP2).

5.2.4.2 Trail 2 of School B (Teacher 5: Calvin)

Profile and Coding

Trail 2 of School B was coded **TR2B** and the MCM app code is **0216073**. The name of the trail is "*Math math and math*"!. The participant teacher was identified by the pseudonym Calvin. Calvin is a novice teacher in the teaching profession with less than two years of teaching experience in secondary school mathematics. He holds an honours degree in Mathematics and Biology Education from the University of Namibia. The data from Calvin's video is coded as **CV5** and the time frame of the video was 27.3 minutes. Calvin had eight learners consisting of five boys and three girls. The two tasks solved in this trail (see points 1 and 2 in Figure 5.26) were based on the topics of area and surface area – see Table 5.6 below. The learners in this trail also shared one smartphone and the trail lasted for 39 minutes.

The implementation of the RME principles: Calvin.

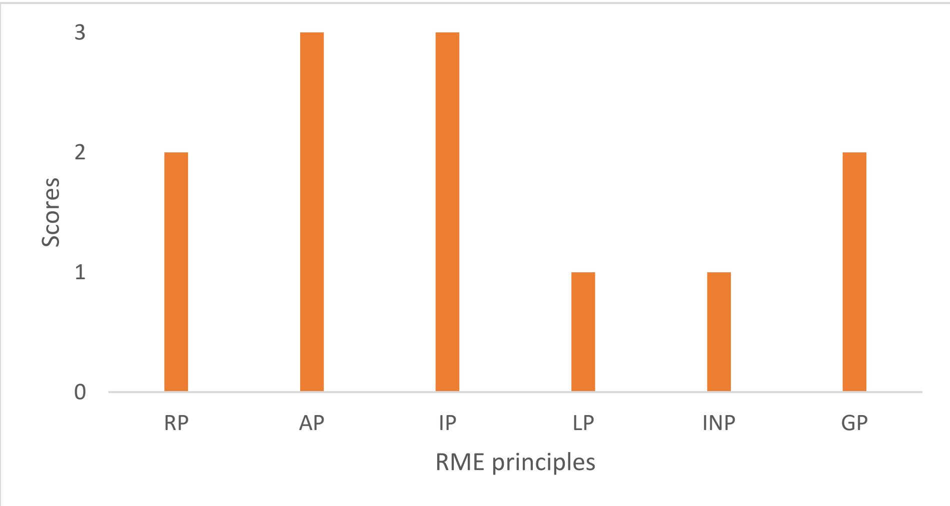


Figure 5.38: Numerical scores of the implementation of the RME principles by Calvin.

Figure 5.38 above shows that in the trail of TR2B the AP and IP were the most used principles. The RP and GP were moderately used, while the LP and INP were the least used. Following is the detailed report of how each of the principles were used by Calvin and his teachers.

Table 5.6 The two tasks encountered and solved by Calvin's group of learners in Trail 2 of School B

TASK TITLE/ CODE	TASK DEFINITION	OBJECT	SAMPLE SOLUTION	HINTS
The dustbin task Code: 3435989	Calculate the volume of the dustbin to the nearest litre.		Measurements: Diameter of drum cylinder = 57.2 cm Height of drum cylinder = 90 cm Volume of cylinder = $\pi \times r \times r \times h$ = $\pi \times 28.6 \times 28.6 \times 90$ (Radius =½ Diameter). = 231273 cm^3 Therefore 231273 ÷ 1000 = 231. 273 (1000cm^3 = 1 litres). = 231 litres	1. What shape is the dustbin? 2. Do not forget to change your answer to litres: 1000 cm^3 = 1 litre
The sewer task 1 Code: 4641949	Assume that you are asked to re-plaster the top surface of this sewer drain. Calculate the area to be plastered to the nearest square centimetre.		Measurements rectangle: length 162. 5cm and width118 cm circle: Diameter 77cm Solution Area of rectangle - Area of circle A = (length × width) - (πr^2) = (162.5 × 118) - ($\pi \times 38.5^2$) (radius = 77÷ 2) = 19175 - 4656.626 = 14 518.374 cm^2 = 14 518 cm^2	1. The area of the top concrete surface minus the area of the circular metal shape: the opening. 2. Area of rectangle = length × width Area of circle =πr^2

The Reality Principle (RP)

The RP in this trail unfolded as follows: the teacher made a comparison between the dustbin the learners were working on, with other big dustbins located at the entrance of the school hostel and the CBD of the town of Opuwo (RP2). Figure 5.39 below shows an excerpt of a discussion that highlights aspects of RP that emerged from this comparison.

TEACHER	Okay, why do we need to know the volume of this dustbin? By the way, what is volume?
LEARNER A	Volume is the amount of space of an object
TEACHER	Good. So, why do we need to know the volume of this bin?
LEARNER A	For us to know how much rubbish we should put in it.
TEACHER	And…? I mean why should one go to the trouble of calculating the volume of the bin the way we are doing now? (*No learner seems to know the answers*) … okay, say you have some rubbish to dispose of, you have to know whether they will fit in this bin or not, isn't so? You cannot just bring any amount of dirty and throw it in this bin without looking at whether it will fit or not, isn't so? (*learners show agreement*)
LEARNER B	Sir, like the one at school is bigger than this one, so you can put a lot of rubbish in it and it will not be full like this one, right?
TEACHER	Yes, you are right, and that's the reason the town council brought it to the school because its big and as a school we can put in rubbish as much as we can for the whole month before they come and pick it up. **CV$_5$**

Figure 5.39: An Excerpt showing the emergence of the RP in TR2B.

In Figure 5.39 above, the question of why learners needed to calculate and know the volume of the dustbin led to an incidence of comparing the amount of space between the dustbin the learners were working on and dustbins in other places. As can be seen in the extract, Learner A's response to the teachers' question shows that the learners understood that the concept of volume is related to the amount of space occupied by the dustbin (RP1). The example of the amount of waste or litter that was used, to compare the capacities occupied by these bins is close to the learner's reality. All around them, these learners are confronted with their surroundings full of dirt and waste that pose a health hazard to the community. So, knowledge about why these dustbins are placed where they are and how much waste is needed to fill them up, is vital (RP5).

In another excerpt (see Figure 5.40 below), Calvin concludes TR2B by posing questions that led to the learners reflecting on the reality aspects of the two tasks they had just worked on. The teacher encouraged his learners to always relate the mathematics they have learned from school textbooks to the real objects and situations they encounter in real life (RP2).

TEACHER	So, what did you learn in this task?
LEARNER C	What I learned is don't be afraid to take decisions and risks.
TEACHER	Okay, who else?
LEARNER A	Me I learned that we can calculate the volume of dustbins and other things so that we know how much things we can put inside them.
TEACHER	Okay, you need to relate what you learned from the class … you see what is in your textbooks, what you learn in the class, area, volume, you have use that in real life, for example, the carrying capacity of a dam, or the tank of water … of anything, this is how you can apply that knowledge. So, when you solve these problems of the MCM app, you need to know what type of math I need to apply here, how can I solve this… for example, sometimes you will be given an area to put interlocks, and then you will be given the dimensions of brick and the question would be how many bricks will be needed?
	CV_5

Figure 5.40: An excerpt showing how Calvin used real examples to conclude the trail TR2B.

The Activity Principle (AP)

The AP was observed as among the most used RME principles in this trail. Most of the times learners worked independently, with little intervention from the teacher (AP4). The **CV_5** data source shows recurring patterns of learners engaged in discussions of interpreting and understanding the MCM app, carrying out measurements of the objects involved in the task and discussing solutions to the tasks (AP2). For example, in Figure 5.41 below, the learners are actively working together in carrying out measurements in the two tasks.

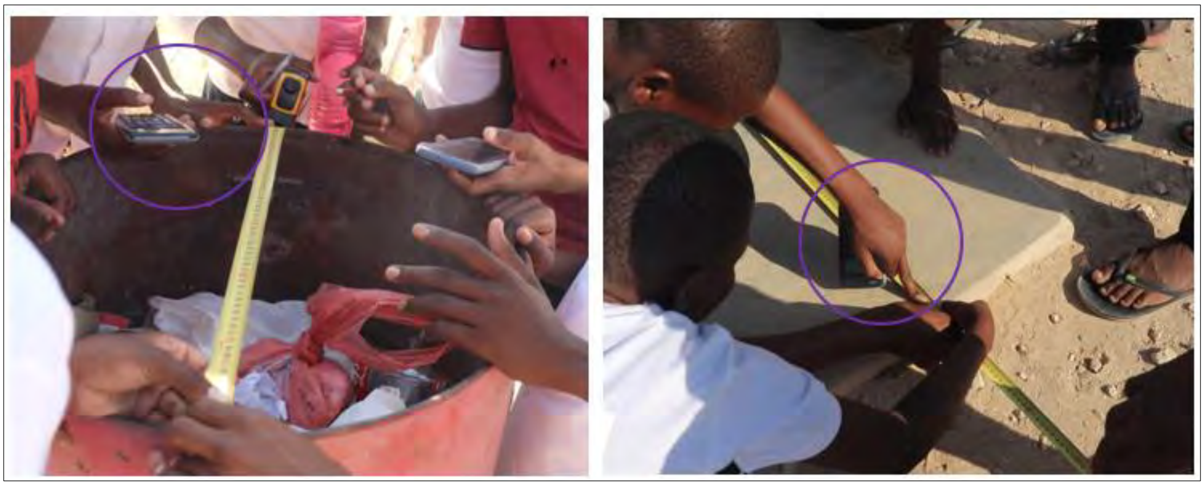


Figure 5.41: Learners actively carrying out measurements while solving the tasks.

Throughout the trail, the learners were continuously working together, although there were two to three learners who seemed detached from others. The teacher took note of these learners and found ways to encourage them to be part of the discussions. Refer to the LP in Figure 5.5, to see on how Calvin directly posed questions to Learners D and F in order to solicit their responses and be part of the discussions (AP2, AP4).

On the other hand, in Figure 5.41 above, one learner can be seen taking readings on the tape measure to determine the length of the diameter, while holding a calculator in his hand at the same time. To me this is an indication that this learner was multitasking, while the work of reading the measurement of the length of the dimeter could have been done by someone else. There were other learners who were not doing anything and could have taken the measurements or operated the calculator. Notice again that the same learner can be seen reading the measurements on the sewer object task. In fact, throughout the TR2B and solving of the two tasks, I observed that this particular learner was taking part in the measuring activity while simultaneously operating the calculator. It is likely that the teacher did not notice what was happening, or else he would have intervened as expected. Also, I observed that the teacher, right from the start of the trail, did not instruct and encourage learners to share the responsibilities of operating the calculator, carrying out the measurements, writing down the measurements and carrying the smartphone.

The Interactive Principle (IP)

Alongside AP, the IP was also highly and effectively used in this study. The active learning that emerged on several occasions in this trail was inevitably the result of a social engagement that facilitated interactions where the learners collaboratively worked together with each other and with the teacher. It was clearly observed that the learners actively exchanged ideas, approaches and solutions to the tasks (see Figures 5.38, 5.39 above), which in turn promoted the social engagements of presenting, discussing, justifying and connecting mathematical ideas and solution strategies that possibly allowed for sharing and developing mathematical meanings IP4.

Moreover, I also noticed that the teacher was concerned about the learners' safety during the trail. For example, as can be seen in Figure 5.42 below when the learners arrived at the site of the dustbin task, the teacher was heard saying the following to them:

> Please do not touch anything in the dustbin, remember there is dirty inside here … and don't stand here, this is in the road … come this side where there are no cars passing.
>
> **CV_5**

Figure 5.42: An excerpt showing the teacher taking care of the learners' safety on the trail.

The teacher also encouraged the learners to be confident, by taking risks and expressing their ideas without fear (IP3) – see Figure 5.43 below.

> Guys, what's wrong? Are you afraid to fail… if you fail you will use another different method…You have the answer with you here … what is confusing you? I mean how will you find out if you are correct or wrong if you don't try out your answer? Why are you doubting yourselves? Remember we are learning here, so if the answer is wrong you will try again until you get it right. Just enter this answer, and if it is incorrect you can ask for the hints… what are you afraid of? Are you afraid of getting the wrong answer po?
>
> **CV_5**

Figure 5.43: Shows an excerpt of Calvin encouraging learners to be confident in their work.

In the above excerpt, it can be noted that when the learners doubted the correctness of their answer to the dustbin task, Calvin insisted that they simply go ahead and test it on the MCM app. It was clear that they lacked confidence in their answer, and this resulted in their hesitation to enter it on the app. The teacher's attempt can be interpreted as encouraging his learners to be confident in their work and to always take risks in executing their ideas (IP2). On the other hand, the advocacy of learners to test the answer without thinking about whether it was correct or not, can also mean that the teacher was encouraging the learners to guess the solution to the task, something that would not be recommended.

Another observation of note here, is how the teacher ensured the participation of passive learners during the trail and by directing questions at them. For example, in excerpt in Figure 5.47 under the LP, Calvin directed questions to Learners D and E respectively because they seemed to be distant and not actively interacting with others (IP2). It can therefore be concluded that the teacher was consciously aware of each of his learners' levels of participation and encouraged those he deemed not to be actively involved in the ongoing discussions.

The Level Principle (LP)

The LP was one of the most used principles in this trail. There were notable incidences where the principle was observed being implemented and one was when the teacher used gestures to visualise and model the cylindrical shape of the dustbin (LP6) (see Figure 5.44 below). It was evident that Calvin wanted his learners to understand the geometrical shape of the dustbin, which was a cylinder. Also, in order to calculate the volume, the learners first needed to recall the formula before determining the dimensions involved (LP3).

Figure 5.44: The teacher using hand gestures to visually model the shape of a cylinder.

In Figure 5.44 above, the teacher can be seen gesturing with his hands to show and model the curved face of the cylinder in an attempt to explain that volume is the space occupied and bounded by this curved face (LP2, LP5). Further, I also observed that it was from this line of approach that the following discussion in Figure 5.45 started.

LEARNER A	… so, we are going calculate the volume of the cylinder?
LEARNER B	Yes, and it is area times height.
LEARNER A	Which area? There is no area here (*speaking in Otjiherero language*)
LEARNER A	...but the formula of calculating the volume of the cylinder is area times height.
LEARNER C	Yes, volume is equal to area times height (*confirms another learner*).
TEACHER	Okay, I am interested in Learner A's question. Where is the area you are talking about here?
LEARNER C	But sir, we were taught that volume is equal to area times height.
TEACHER	Yes, I am not saying that you are wrong, but I want you to show me the area here … okay, first, which one is the height of this shape. Yes …. (*mentions Learner Ds' name*).
LEARNER D	It's this one sir. (*pointing and moving her hand along the height of the bin*)
TEACHER	And the area?
LEARNER E	… area it is this distance around the bin…
TEACHER	No, how do we call this distance around … how do we call the distance around the circle?
LEARNER B	It's circumference sir.
TEACHER	Exactly, so where is the area then? (*Suddenly learners looked confused and no one could show the teacher the location of the supposed area*) … Okay you guys should understand that when we talk about area on the formula of calculating volume, we don't mean any other area that comes to your mind, we mean the area of the base of the shape ... and by the area of the base I mean the area of the face where this entire bin stands on or rests upon. So, on what shape will this dustbin stand on when we put it down on the surface or ground so that the top part is the same as the bottom part? Yes … (*mentions Learner F's name*)
LEARNER F	It is a square sir.
LEARNER A	No, it's a circle, it will drop and rest down the way it is (*speaking in Otjiherero language*).
TEACHER	You see, that's why I said that you should know the type of shape that forms the base of the solid you want to calculate the volume (*mentions Learner F's name*) because the formula you will use should always depend on the its base ... this case you need to use the area of a circle, which is?
LEARNER C	Pi r squared.
	CV_5

Figure 5.45: Calvin using probing questions to solicit the learner's prior knowledge.

The above excerpt in Figure 5.45 shows that Calvin used the learners' prior knowledge to explain to them the concept of 'area of base' (LP1:LP3). It was evident that the learners did not fully understand the concept of the area they were supposed to multiply with the height in order to find the volume. They seemed not to link the area concept to the corresponding shape in question, which was supposed to be the base of the whole solid shape or prism. In other words, it appeared that learners did not know which formula to use which was supposed to be determined by the type of base shape of the prism.

The other interesting incidence here is how the parts of the centre of a circle were used to understand the relationship between diameter and radius. As can been seen in Figure 5.46 below, Learner B was holding a pen to visually show or locate the centre of the circle (circled red) on top of the dustbin (LP6). While holding the pen, Learner B explained to the others that where he placed the pen was the centre position of the circle. I observed further that as Learner B held the pen, another learner (Learner A) started explaining to the others the relationship between radius and diameter, by visually showing, through gestures, how the two radii meet at the centre (located by Learner B with a pen) to form a diameter (LP5, LP6). It should however not go unnoticed here that in his explanation, it emerged that Learner B had a misconception of the meaning of r^2, and fortunately, this did not escape the teacher's attention – see the GP of this TR2B for more detail.

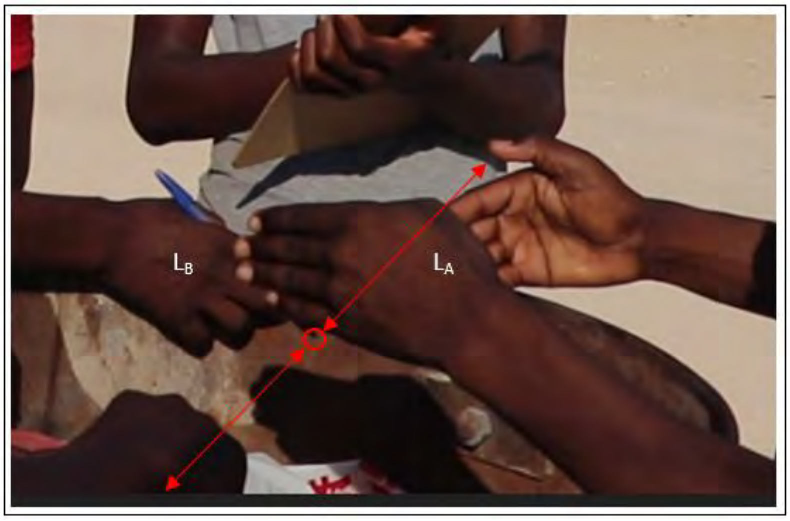


Figure 5.46: Learner A and Learner B visualising the positions of the centre and radii of a dustbin.

The Intertwined principle (INP)

The existence of the INP in TR2B was enriched by the teachers' probing questions that enabled learners to see the connections between concepts and topics and how these related to the real world. Two notable aspects of IP from the **CV$_5$** data source can be referred to this regard. The first aspect emerged in the concept of rounding off. In both the tasks, the learners were required to round off their answers to a given degree of accuracy. In Figure 5.47 below, one can see how Calvin helped the learners understand the intertwinement of the concepts of rounding off with calculator skills (INP1).

A closer analysis of the excerpt reveals that Calvin linked calculator skills and rounding off concepts in order to explain how the answer given in terms of π and a decimal fraction were the same (INP1). So, the teacher explained to learners that the only way to change the answer in terms of $\pi$was to change it using a calculator. Thus, knowing how to do that on their calculator was a vital skill. For example, in the solution to the dustbin task, the answer on the calculator display appeared as $\frac{29241\pi}{400}$ and yet the question asked for the answer to be rounded off to the nearest litre. So, what this meant was that the $\frac{29241\pi}{400}$ was supposed to be changed to a decimal fraction and then rounded off to the nearest whole number. Figure 5.47 below shows an extract of the discussion on changing the answer in terms of π to that of decimal places.

TECAHER	Remember, the question says you should write the answer how?
LEARNER A	In decimal.
TEACHER	Yes, so how do you do that?
LERANER A	We should change this answer to a decimal fraction.
TEACHER	And how are you going to do it?
LEARNER B	We can change it here sir (*pressing on the* ***sd*** *key of the calculator*).
TEACHER	So, what is the answer now?
LEARNER B	229…
LEARNER A	No, I think its 230 because the question says to the nearest whole number.
TEACHER	Okay, now tell me, what is the difference between this answer and this one (*referring to the two answers o f* $\frac{29241\pi}{400}$ *and 230*)
LEARNER C	230 is rounded off.
TEACHER	Okay, do you see this answer here (*referring to* $\frac{29241\pi}{400}$) is the exact answer to this question but 230 is not exact… this is like an estimated answer. Remember that we learn rounding off from the topic of estimation. I mean what is estimation? You guys are also doing physical science, so what is to estimate.
LEARNER D	To estimate is to give a sensible guess about the measurement of something.
	CV5

Figure 5.47: An excerpt of the discussion on changing the answer so that it is in terms of π to that of decimal places.

The second aspect of INP was observed when the teacher tactically moved between the concepts of base area, faces of solids two- and three-dimensional shapes. As can be noted in Figures 5.47 above and Figure 5.48 below, Calvin linked the topics of area and volume, rounding off and estimation, decimals and common fractions and how to convert between the two, using certain skills on the calculator (INP2). It should also be noted how the teacher shifted the discussion to link the properties of two-dimensional to that of three- dimensional shapes where concepts such as faces, sides, length, width and height are discussed, although not in any depth (INP2).

Teacher	Do you remember a cube? (*Learners indicates agreement*), what is its base?
Learner A	It's a square sir.
Teacher	Good, and a triangular prism?
Teacher A	It's a rectangle.
Teacher	No, its not a rectangle… by the way how many faces does a triangular prism has?
Learner E	Sir, what do you mean by faces?
Teacher	Okay, who can tell us, what do we mean by faces? (*learners look confused again*)
Learner	They are the sides of the shape sir.
Teacher	No, sides are not the same as faces. Okay for you to know the difference between sides and faces you should know the difference between two- and three-dimensional figures. For example, is this dustbin a two- or three-dimensional shape?
Learner A	It's a three-dimensional shape sir.
Teacher	Why?
Learner A	… because it have height.
Teacher	Okay, that can be part of the reason, but when we say three dimensional, we simplify mean that the shape has three dimensions, in other words they can be measured in three dimensions. For example, these shapes have length, width/breath, and height, so as you can see the shapes have three numbers you will need to use in order for you to find the volume.
	CV_5

Figure 5.48: An excerpt showing how Calvin linked different concepts and topics that were connected to the solving tasks.

In addition, it was also noted how Calvin, while solving both tasks, emphasised that rounding off to the nearest litre or square metres meant rounding off to the nearest whole number (INP1). So, it was clear that the teacher intertwined all these concepts in the trail because they were fundamental to the solving of the tasks in order for the learners to obtain the correct answers from their calculations.

The Guidance principle (GP)

Recurring patterns that show that Calvin guided the learners towards approaches that enabled them to find the correct solutions to the two tasks existed throughout the data source of this trail. The GP aspect was clearly found in the excerpts above. Of all these incidences, one incidence of interest where the GP was used, was when Learner A held a misconception regarding the meaning of r^2. Learner A was seen and heard explaining to the others that the reason they were using the formula $\pi r^2 h$ to calculate the volume of the dustbin was because a circle had two radii – see Figure 5.49 below. This statement could only mean that Learner A misinterpreted r^2 as $2r$, an indication that the learner did not understand the meaning of r^2. So, in helping this learner (and arguably other learners as well), the teacher explained the difference between r^2 and $2r$ (GP3). Below is an excerpt that shows how Calvin guided learners to understand the difference between r^2 and $2r$.

LEARNER A	The reason why we say radius squared, because its two radius… one radius starts from here and the other one ends here So, together they meet here at the centre to form one diameter. CV_5.
TEACHER	Wait a minute… what do you mean when you say you are going to use pi r squared because there are two radius? (*learners look at each other and the teacher*). Well, don't look at me you heard what Learner A said most.
LEARNER A	Sir because there are 2 radius in one diameter.
TEACHER	So, that means r squared is one diameter?
LEARNER A	Yes sir.
LEARNER B	I think r squared means *r* times *r* (*somehow the learner looks unsure*)
TEACHER	Yes, it means r times r, and not r 2r. so what is the meaning of 2r? (GP1:GP2)
LEARNER C	2 times r.
	Good. So, in other words r plus r. Remember how many radius are in one diameter?
LEARNER B	2
TEACHER	Yes, so what does that means… in one diameter there are two radius.
LEARNER A	But it's what I said sir.
LEARNER B	No, you said r squared *(speaking in otjiherero)*.
TEACHER	Yes, so when you said in r squared there are two radius then it means you don't understand the meaning of r squared. Remember, we use two pi radius when calculating circumference not area or pi diameter because in one diameter there are 2 radius. On area we use r^2. In Grade 7 or 8 you should have learned where r squared comes from. It doesn't mean 2 radius. So r squared means r times r and 2 pie r means r plus r which is equals to one diameter (GP3 **CV_5**

Figure 5.49: An excerpt where Calvin guided the learners to understand the difference between r^2 and $2r$.

Again, in this conversation you will notice that the teacher uses probing questions to guide learners to understand the difference between r^2 and $2r$ (GP2). Also, the link between radius and diameter was addressed by the teacher to help learners understand that r^2 cannot mean one diameter because a diameter is when two radii are added together, not multiplied (GP3).

Figure 5.50: Shows the teacher guiding learning on the best approaches of solving the tasks.

More incidences such as the ones shown in Figure 5.50 above from **CV5** are presented here to show the demonstration of the GP in the trail. Calvin can be seen guiding learners on how to find the area to be plastered by reminding and guiding them to use the concepts of shaded and unshaded areas (GP4). Also, in the same Figure 5.50 the teacher guides the learners on the best approach to solve the dustbin task. It should be noted that learners in this trail managed to get the correct answers on their very first attempt of entering it on the MCM app, hence, there is no report on the use of hints as a guiding tool in this trail.

5.2.5 MCM Cycle 3: Brief overview of the mathematics trails

All three participant teachers from schools A, B and C participated in the creation of tasks and implementation of the trails for this cycle. The tasks in the three trails were also created outside the school premises. Figure 5.51 below is a screenshot of the Google map that shows the location of the three trails and their respective tasks in Cycle 3 of this study.

The mathematics trails of Cycle 1 had the following characteristics:

- School A (The math in Opuwo trail) – consisting of four tasks.
- School B (Math is fun-Opuwo Trail) – consisting of four tasks.
- School C (Come we do math trail…) – consisting of four tasks.

Figure 5.51: The offline trail maps for each school in Cycle 3

5.2.6 Data presentation and vertical analysis of individual participants

5.2.6.1 Trail 3 of School A (Teacher 6: Luke)

Profile and Coding

The participant teacher in the third trail of School A (**TR3A**) in this study is identified as Luke. Luke is a diploma holder (BETD) in Mathematics and English Education from the former Ongwendiva College of Education – now the Hikepunye Pohamba University of Namibia campus. Luke has been teaching mathematics to junior secondary phase learners (Grades 8 to 9) for the past 19 years now. Luke's video data is coded as **LV$_6$**. TR3A is named as "*The math in Opuwo*" trail and is coded as **1516102** on the MCM app system. The video length of TR3A is 32 minutes and 23 seconds. Luke had six learners under his care (three girls and three boys) and these learners solved two tasks on the topics of area, ratio and proportion – see points 1 and 2 in Figure 5.51 above and Table 5.7 below. Luke and his learners used one smartphone as a mobile tool to access the MCM tasks. The approximate time taken to walk TR3A and solve the two tasks was 50 minutes.

The implementation of the RME principles: Luke.

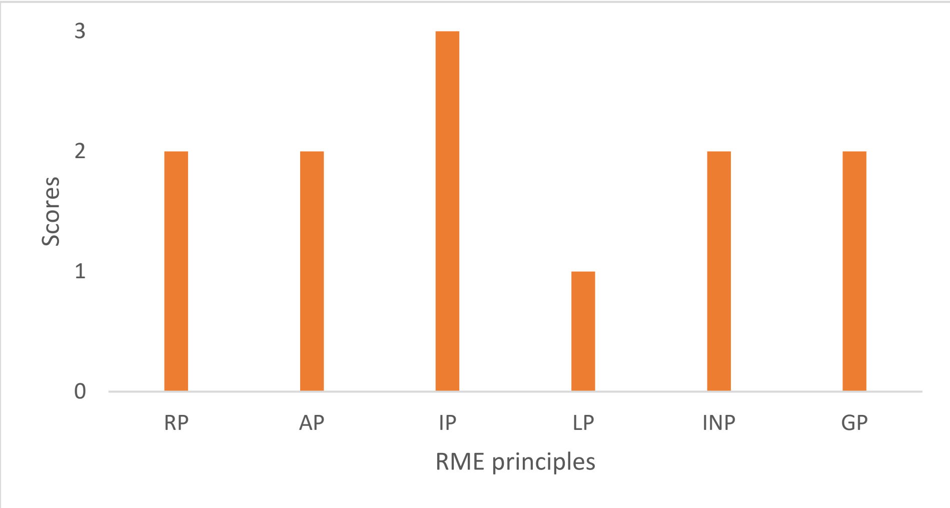
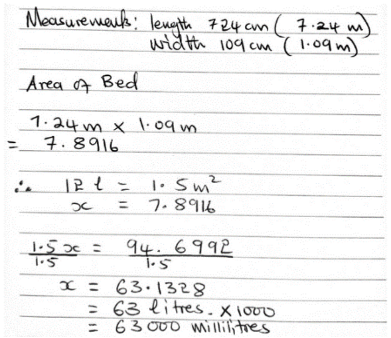
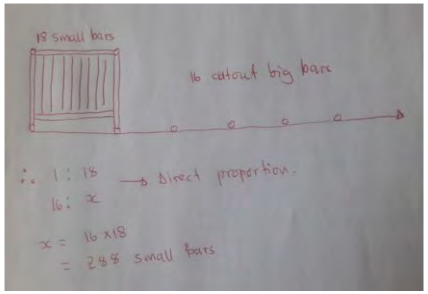


Figure 5.52: Numerical scores of the implementation of the RME principles by Luke.

Figure 5.52 above shows that on the TR3A trail the AP and IP were the most used and evident principles. The RP and GP were moderately used, while the LP and INP were the least used. Following is the detailed report of how each of the principles were used by Calvin and his teachers.

Table 5.7: The two tasks encountered and solved by Luke's group of learners in Trail 3 of School A

TASK TITLE	TASK DEFINITION	OBJECT	SAMPLE SOLUTION	HINTS
The lawn task Code 6866513	If 12 litres of water can cover an area of 1.5 square meters, work out how much water will be needed to water this raised lawn bed, to the nearest millilitre.			1. The bed is in the shape of a rectangle: $A = l \times w$ 2. Use the direct proportion approach to find your answer. 3. Do not forget to convert your final answer to millilitres.
Metal railing task Code 5936108	Work out the number of small metal bars that were cut from the rest of the metal railing.			1. Count the number of small metal bars that are between the two big bars and compare them to the rest of the bars that were cut off. 2. The number of small metal bars between two big metal bars is proportional to the number of small cut off bars.

The Reality Principle (RP)

As can be seen in Figure 5.52 above, the RP was one of the RME principles that was well attended to in this trail. I clearly observed that Luke managed to link the two tasks to the learners' reality. For example, during the solving of the lawn task, the teacher reminded the learners that in reality if one was asked to water the lawn bed, it would be important to know how much water to use (RP2). Figure 5.53 below is an excerpt showing how Luke emphasised the importance of the lawn task in real life.

Look at this grass, it looks dry nee? And needs water nee? You know water use to finish here in Opuwo, so it's important that you don't use a lot of water to water this grass, isn't not so? Or otherwise, you will waste water … or say this was a lawn at your house, I mean your parents' house. If you overwater it your parents will get angry because you are finishing water… remember here in town you pay for water, therefore it's important that you use water wisely, isn't?   **LV_6**

Figure 5.53: An excerpt showing the relevance of the lawn task to the learners

Some of the things in this example might not really be close to the learners' lives, for example, in this part of the country most of the learners reside in rural areas where one does not pay for water. Moreover, the learners might not even relate to the issue of watering a lawn because it simply does not exist in their world. Still, it should be noted that Luke was making learners aware of issues surrounding the watering task (RP2).

The other incidence of interest on RP was when the learners were measuring the dimensions of the lawn bed. They easily identified that the bed was in the shape of a rectangle, however, they did not know where to place the measuring tape when measuring the dimensions. In Figure 5.54 below, at first the learners wanted to measure the length of the bed by starting behind the brick kerb of the lawn bed (Figure 5.54a). When Luke saw the learners' mistake, he stopped and questioned them as to why they put the tape measure where it was. As can be seen in Figure 5.54b and 5.54c, the teacher explained the correct way of taking the measurements (RP1).

Figure 5.54: Luke using the RP to explain to learners the correct way of taking measurements

In addition, when explaining where to start their measurements, in Figure 5.55 below Luke emphasised the following regarding the measurement of the length of the bed when determining the area to be watered, which I argue also confirms the existence of the RP in this trail (RP2).

These bricks are not part of the area of the garden…, remember we said you need to preserve water, isn't? so when you are watering you don't need to water this part of the garden where there is no grass. So where should you put the measuring tape? Here or here? (*referring to picture b and c on* ***Figure 5.53***) **LV6**

Figure 5.55: Shows an excerpt of Luke emphasising the RP of the boundary of the lawn grass

One more incidence that also proves the existence of the RP in TR3A emerged when the learners' debated the relationship between millilitres and litres.

LEARNER A	See one litre is equal to one millilitre, (*other learners look at him with disapproval, and he again repeats the same statement*) one litre is equal to one millilitre, I remember very well.
LEARNER B	One what?
LEARNER A	One litre
LEARNER C	No, its 10 litres… 1000…
LEARNER C	Its 1000. Look, a 750ml container bottle is smaller than a 1 litre bottle… you see in a one litre bottle of cooldrink there will be 2 bottles of 500 millilitres (*speaking in Otjiherero*)
LEARNER A	Yes, you are right, so this answer we should divide it with 1000? **LV6**

Figure 5.56: An excerpt showing the emergence of RP, on the relationship between litres and millilitres

In the above excerpt in Figure 5.56 above, it is shown how Learner C used real, practical examples of container sizes to compare the units of litres and millilitres (RP1, RP2). It should also be noted that it was after this comparison that I observed that Learner A was convinced that one litre is not equivalent to one millilitre. This can only mean that this example helped Learner A and others understand the relationship between litres and millilitres.

The Activity Principle (AP)

Similar to the trends that have been reported in this study on the AP of other teacher participants, the AP in TR3A unfolded when learners engaged in discussions on how to find solutions to the tasks. For example, Figure 5.57 below shows the learners being actively engaged in activities of collecting the data necessary to solve the tasks through measuring, counting and calculating (AP1: AP2).

Figure 5.57: Shows learners involved in taking measurements and counting activities

The Interactive Principle (IP)

There was a positive interaction that existed between learners as well as between them and the teacher. During the trail the teacher encouraged the learners to work together by sharing responsibilities (IP2). For example, in Figure 5.58 below, the teacher instructs the learner holding the smartphone and trying to do the calculations at the same time, to give the calculator to someone else. Luke encouraged the learners to work together in order to finish the work on time (IP2:IP5).

Give the calculator to someone to calculate, you cannot be holding the phone and at the same time doing the calculation … guys, help him to calculate, he cannot do everything… come on let us work together and finish this task. **LV6**

Figure 5.58: An excerpt of the teacher encouraging learners to work together

The other aspect of IP that is presented in Figure 5.59 below shows an incident of learners' interest in proceeding to the next and third task of the TR3A. As can be noted in the excerpt, despite the teacher hesitating to continue to the next task, the learners insisted until the teacher gave in to their persistence. It should be noted from the excerpt that the learners did this at the expense of missing their dinner as it was time for them to go and eat. So, what this means is that the trail tasks were so interesting that the learners opted to continue even though they might miss their dinner at the hostel's dining hall (IP2).

TEACHER	Very good, you got it correct.
LEARNER A	Now the second one… (*meaning another task to work on*)
TEACHER	No, we will stop here. Time is gone and you need to go and eat. So, what did you learn?
LEARNER C	No sir, let us just do the second one
TEACHER	I can see that you are now interested, and what about food, you just heard the bell ringing
LEARNER B	Its fine sir, the food will wait sir, people are not going to eat now, while people are still waiting to be served we can just do one more task… we will find them (*speaking in Otjiherero)*
TEACHER	Okay, let us check the next task we can do, the one that is not far from here. **LV6**

Figure 5.59: An excerpt showing learners interest in TR3A as an aspect of the IP

The Level Principle (LP)

The LP was not very evident in this trail. The only incidence where the principle emerged can be found in the following excerpt in Figure 5.60

Learner A	Its 108 cm
Learner C	Its cm or m, I think it should be 108m
Learner A	No, it's 108cm, look from here to here is a ruler, another ruler, and another one, so when it reaches here its one meter … so this should be 108cm or 1.08m **LV6**

Figure 5.60: An excerpt showing how the LP was used as Learner A explains the relationship between litres and millilitres

In the above excerpt, it can be noted that Learner A corrected Learner C on the measurement unit. Probably, Learner C was confusing the units on the tape measure and Learner A opted to use the idea of a 30cm ruler, something that was familiar to them (LP1), to try and explain why the answer was supposed to be in millilitres and not metres as argued by Learner C. So, without using the actual 30 cm ruler, Learner A pointed out the exact positions equivalent to the 30 cm ruler on the tape measure – see Figure 5.61 below. What this means is that Learner A used the LP to make visual the 30 cm length of a ruler on the tape measure to explain to Learner C to why the answer was 108 cm not 108 m (LP6). Learner A's use of a nonstandard and familiar method of measurement to compare the units of centimetres and metres helped other learners to visually mathematise the relationship between these two units.

Figure 5.61: How Learner A used his finger to mark the positions on the tape measure

The Intertwined principle (INP)

The intertwinement between concepts, topics and the real world was seen when different topics were used together in order to find the answers. For example, for the learners to find the solution to the task of the lawn, they had to combine the topics of measurement, area, proportion and the conversion of units (INP1). Firstly, they had to obtain the measurements of the dimensions involved – length and width (the measurement topic). Secondly, the learners used these dimensions to calculate the area of the physical lawn (the area topic). Thirdly, the learners used the given ratio in the question to proportionally find the amount of water that would be needed to cover an area of the lawn bed (the proportion topic). Fourthly and lastly, the learners changed their answer that was in litres to millilitres as was required by the question in the task (the converting units topic). Figure 5.62 below shows how the topics were connected to find the final solution to the lawn task (INP1).

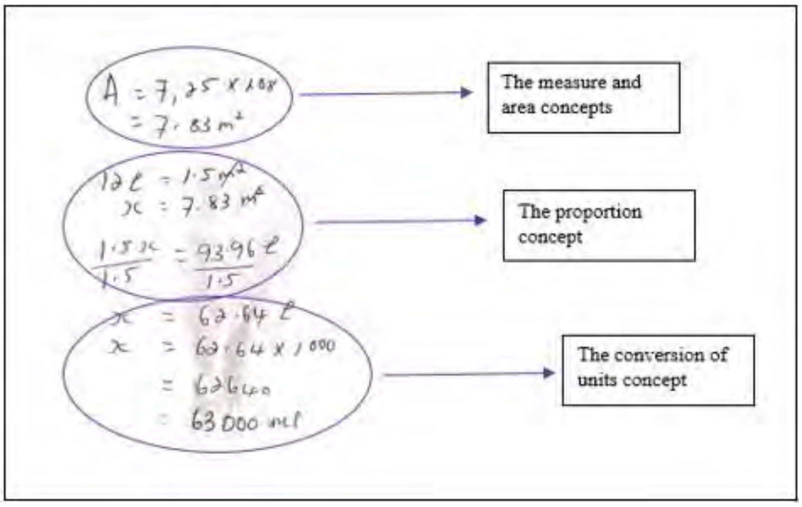


Figure 5.62: The intertwinement of concepts towards the solution of the Lawn Task

The Guidance principle (GP)

The implementation of the GP was evident in TR3A in how Luke guided the learners to understand concepts that were embedded in the tasks they were working on. Every time learners got stuck in their working or needed further understanding on certain concepts, the teacher would step in, but of course with caution and not to simply give the correct answers. For example, Figure 5.63 below shows how Luke guided his learners step-by-step to understand the concepts of parallel lines when they were measuring the width of the bed (GP3). This excerpt follows after the learners decided to measure the width in the middle of the bed instead of at the side kerb.

TEACHER	So, why measure your width here and not at the end there?
LEARNER B	Because we think here and there they are the same?
TEACHER	So, what makes them to be the same? (*there is a moment of silence*).
LEARNER	Because here and the other side they are parallel to each other
TEACHER	Good, they are parallel, so what do you know about parallel lines? (*there is a moment of silence before Learner A responds*)
LEARNER A	They do not meet because the distance between them is the same
TEACHER	Good, they never meet he is right, why? Because the distance between them is the same, that's why even if you measure here you will still get the same distance as when you measure the other side there. So, that's why we also say that on a rectangle the opposite sides are equal. This side is the same as the other one… and when you calculate area or perimeter, sometimes you will be given only one length and one breath, why? It' s all you need because you will know that the other length or breath will be just the same the measurements you are given (GP1, GP2:GP4) **LV6**

Figure 5.63: How Luke guided learners to understand the concept of parallel lines

Throughout TR3A, the teacher used clear and precise language with an appropriate tone to guide learners, ensuring that they understood the concepts he wanted them to learn. Luke also sometimes repeated himself and even code switched between English and Otjiherero (this can be clearly noted in some of the presented excerpts). It was evident that Luke played the role of a facilitator who provided prompts to learners whenever they hesitated and encouraged them to come up with their own answers (GP2). This in turn created learning opportunities.

5.2.6.1 Trail 3 of School B (Teacher 7: Betty)

Profile and Coding

Betty is the pseudonym given to the teacher participant who was video recorded in the third trail of School B (coded **TR3B**). Betty is a qualified mathematics teacher with a BEd Honours in Natural Sciences from the University of Dar es Salaam (Tanzania). Betty has 12 years of teaching experience, and she is Head of Department for Natural Sciences at School B. She is currently responsible for teaching mathematics to Grades 10 to 12 learners.

TR3B is coded **5716072** on the MCM app and named "*Math is Fun-Opuwo*". Five boys and two girls were assigned to Betty during the implementation of this trail at School B. Betty and her group of learners solved two tasks – see Table 5.8 below. Take note that the pole task contains a support task. Support tasks can help break down more complex tasks into manageable units. The video data source is 38 minutes long and is coded as **BV$_7$**. It took approximately 45 minutes for Anna and her learners to walk and solve the two tasks in the trail (see points 1 and 2 in Figure 5.51 in Section 5.2.5 above).

The implementation of the RME: Betty

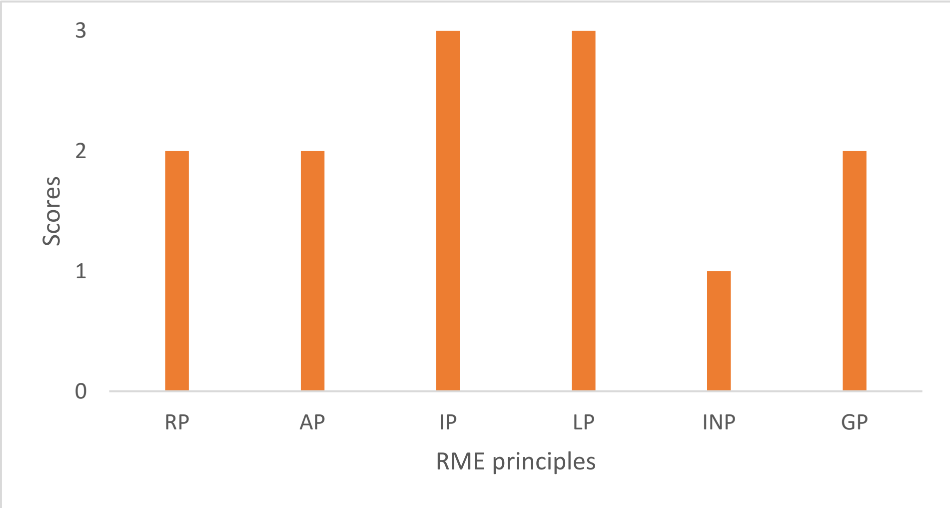


Figure 5.64: Numerical scores of the implementation of the RME principles by Betty.

The graphical summary in Figure 5.64 above shows that the IP, LP and GP were the most evident RME teaching principles in this trail, while I noticed that the INP was used the least. On the other hand, the RP and AP were moderately used in the trail. Below is a detailed discussion of how each principle was employed to conceptually teach area and volume topics in Betty's trail.

Table 5.8: The two tasks solved by Betty's group of learners in Trail 3 of School B.

TASK TITLE	TASK DEFINITION	OBJECT	SAMPLE SOLUTION	HINTS
The pole Task Task 5866064 Code 8966065	If you were to dig a hole to put in this street light pole (by submerging the concrete part below the surface of the sand), what would be the depth of the hole? **Support Task 1 of 1:** **The volume Task** Work out the volume of this concrete block. Give your answer to the nearest cm^3.		Height of concrete block: 64.2cm Measurements: length 48.5 cm, width 64.2cm, height 43.7 $V = l \times w \times h = 48.4cm \times 64.2cm \times 43.7cm = 140219.64 = 140220\ cm^3$	1. The height of the rectangular block determines the depth of the hole. 2. Height of block when upright = depth of hole. 1. The concrete block is a cuboid. 2. Volume of cuboid = $l \times w \times h$
The Volume Task 1 Code 0466062	Work out the total volume of the two concrete blocks to the nearest cm^3.		Measurements Big block: length 214.5cm, width 61cm, height 11cm Small block: length 155.5cm, width 53.5cm, height 11cm $V = V(\text{Big block}) + V(\text{Small block}) = (l \times w \times h) + (l \times w \times h) = (214.5cm \times 61cm \times 11cm) + (155.5cm \times 53.5cm \times 11cm) = 143929 + 91511.75 = 235440.75 = 235441cm^3$	1. Both the two blocks are cuboids. Volume = length × width × height. 2. Total volume means adding the volumes of the two blocks together.

The Reality Principle (RP)

The application of the RP was observed during the solving of the pole task. The first incident emerged when the learners were measuring the length of the concrete block to determine the depth of the hole to be dug. Figure 5.65 below shows that the learners measured the height of the concrete block up to the end (level B) instead of level A. It appeared that before this pole was uprooted from the ground, the only part that was submerged in the soil was the part from A to B.

Figure 5.65: The learners applied the RP in their measurements.

Instead of assuming that the part of the concrete that was submerged in the ground before the block was uprooted was the section A to B, the learners measured the length of the whole block. So, presumably to test learners, the teacher asked them why their measurements were ending at B and not A. In Figure 5.66 below is an excerpt of a conversation around the measurement of the depth of the concrete block. In this excerpt, in response to the teacher, Learner A highlights the reality aspect of why their measurements did not end at point A (RP5). It was evident that the learners were conscious of the reality aspect of obtaining the correct measurement that would allow them to correctly determine the depth of the hole.

TEACHER	Why do you measure up to here (*referring to level B*) and not here?(point A)
LEARNER A	Efro, this block ends here (B), not here (A)
TEACHER	So, what? you can see a line here which means you can just end here. I mean you can cut it from here (A) and remove this part (*referring to the concrete part between A and B*). This will even make the hole smaller, which means less work for you.
LEARNER A	No, mem (*laughing*) that will be too much work to do. We are going to cut it with what? I think its better to dig a big whole hole so that it can all go in. Cutting this thing will be a lot of work than just digging a hole. **BV$_7$**

Figure 5.66: Learner A's response to the teacher regarding the reality of determining the depth of the hole.

The second incident of the RP worth mentioning here is how the learners allowed a 1 cm variance in their measurements of the three dimensions needed to calculate the volume of the hole (RP1) see Figure 5.67 below. Notice that for the length 66.6 cm, they added 1cm to get 67.6 cm, and this was the same with the other two dimensions (height and width). Notice again that this was something overlooked by the teacher (and the researcher) on the formulated memorandum of the pole task for the MCM app – compare the answers on Table 5.8 above and Figure 5.68 below.

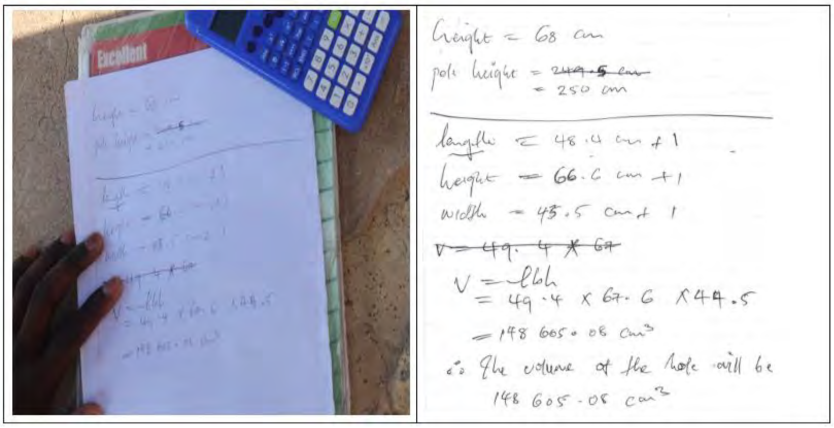


Figure 5.67: How the learners added 1 cm variation to their measurements of the concrete block

When Betty asked the learners why they were adding 1 cm to the dimensions, Learner A responded as follows (RP5):

… for it to fit in the hole we will add 1 number to each measurement **BV$_7$**

Figure 5.68: The learners' reasoning for adding 1 cm to the dimensions of the hole

Again, the above incident shows that the learners were extremely conscious about the authenticity of the hole task. The fact that they considered that in reality, when digging the hole, they needed to make it a bit bigger than the block itself in order to allow it fit in without difficultly, is evidence enough of the existence of the RP in the trail (RP2).

The Activity Principle (AP)

The AP in this trail was seen in the way learners engaged in measuring and calculating activities (AP1). They worked together to locate the positions of the two tasks and to find the solutions to the tasks. Figure 5.69 below shows the learners taking part in the measurement activities and solving the two tasks of the trail.

Figure 5.69: The learners' active engagement in measurement during the trail

This shows that the learners understood the application of the MCM app and easily located the tasks on their own. Furthermore, it was observed that the learners did not struggle with the solving of the two tasks. This said, there was an incident where they failed to connect the concept of height to the statement 'depth of the hole' (AP1). – see Figure 5.71 under LP.

The Interaction Principle (IP)

There was a high level of interaction between learners and between learners and the teacher in this trail. Betty appreciated the way learners interacted amongst each other, by saying the following:

I like the way you are debating; it simply means you are learning together. Now I like his question, how deep means what? Does that really mean that you should calculate the volume of the block? (IP1) **BV7**

Figure 5.70: Betty's acknowledgement of the learners' positive interactions on the trail.

Throughout the trail, the learners actively engaged with each other and took part in all the discussions (IP1). There was good coordination and a positive attitude towards the sharing of ideas and responsibilities among the learners. Everyone was doing something towards finding solutions to the tasks (IP2). The use of a smartphone, calculator and tape measure, I noticed, brought the learners together and even triggered discussions that contributed to the lively participation of learners in this trail (IP5:IP3), see Figure 5.71 below.

Figure 5.71: The learners interacting and gathered around a tape measure and calculator.

The Level principle (LP)

When the learners were debating about the meaning of the word ‘depth’, Betty used the LP to help learners understand the meaning of the word and why they were not supposed to calculate volume as depth. The following excerpt in Figure 5.72 shows the implementation of the LP in this regard.

TEACHER	I remember we did these types of questions in class, whereby sometimes you are given the volume of water and then you are asked to find how deep the water in the tank is… so, how deep means what? (*almost all the learners respond to say that it means height*) Yes, so you see, don't have to go far… How deep, that simply means how deep will it go down in the ground. From the ground level how deep will it go down?
LEARNER B	Friends, I think we don't need a formula here, we should just measure the height of this brick and then we will know how deep it will go down the hole (*Speaking in Otjiherero).*
TEACHER	Listen to what your friend is saying, you just measure straight… you don't need to calculate anything. Its like if I am asking you, how high will this pole be… what are you going to do? (LP2)
LEARNER A	Measure the thing (*meaning the pole*)
TEACHER	You just measure how high it is and that is it. Listen the pole will go up, the … the what? (*the concrete, responds Learner A*), yes, the concrete will go down… so how deep (*pointing the concrete block*) and how high (*pointing at the pole*).
LEARNER B	So, we just write the height as it is (*other learners show agreement with excitement*)
LEARNER C	This ka statement was confusing us… how deep, me I thought it was volume, but now I understand it… yes just write height, we don't have to measure it again.
TEACHER	Good, what if the question was how high the pole will be? Okay first find out how high the pole will be (LP2). **BV_7**

Figure: 5.72: An excerpt showing the implementation of LP using the learners' prior knowledge.

In the above excerpt, the learners were helped to reflect on their prior knowledge on volume and finding a missing height in order to figure out how to do the task at hand (LP2). It was clear that learners confused the meaning of depth and volume. It seems that to the learners, the question of how deep the hole was going to be, meant finding the amount of space the block was going to occupy in the ground. Hence, the teacher asked them to remember previous lessons on similar questions.

The teacher asked the learners to remember what they were taught for similar questions. (LP3). Further, Betty used a comparison between the height of the pole and the depth of the hole in order to hold the concrete block. This comparison was meant to help the learners understand the meaning of the phrase 'depth of the hole', in contrast to their notion linking depth to the volume concept. As a matter of fact, after this comparison, it was observed that the learners clearly understood what depth means in this context, as can been seen in Figure 5.72 above.

For the volume task, the object of the task was a combination of two cuboids joined together. To solve this task, the learners applied the LP by first calculating the area of each shape separately and then adding the answers. It was evident that they understood the concept of total, hence, they added the volumes of the two shapes together. By splitting the object into known shapes (cuboids), it made it easier for them to find the solution to the tasks (LP4).

The Intertwined principle (INP)

The implementation of the INP did not happen much in this trail. Of course, there were concepts embedded within the two tasks, but it appeared that the teacher did not put much emphasis on how these concepts were related to each other within the discipline of mathematics and other subjects. The only time it appeared that the LP was employed was when Betty explained how the height of the concrete block was related to the depth of the hole. What I see here was a connection of the concept of height to the reality of a deep hole (INP4).

The Guidance principle (GP)

The GP was seen in how the teacher guided her learners to make measurements and interpret data from the smartphone, tape measure and the calculator in this trail. Also, Betty guided learners in understanding their preferred approaches to finding the solutions to the tasks (GP1). For example, after they decided to separately calculate the volume of each cuboid concrete block in the second task, the teacher asked them to explain why they were doing so (GP2) see (Figure 5.73 below). Apart from saying that total referred to finding the volume of each and adding the answers together, learners seemed to not have any other mathematical reason for separating the two shapes. This led to the following conversation:

TEACHER	We don't know the name of this shape, or do we?
LEARNER B	It's a cuboid mem
TEACHER	No, I mean the two combined together as it now.
LEARNER B	No mem,
TEACHER	So even if the question was not going to say work out the total volume. we were still going to separate the two blocks because as it is now we do not what formula to use here because we do not have a mathematical name of this shape … or have you learned something like this … say from Grade 8 and 9? I tell you, even in Grade 12 you will not come across the name and formula of this type of a solid shape. So, in the way we can find its volume is to divide it into shapes that we know, which are
LEARNER A	cuboids
TEACHER	Good. **BV$_7$**

Figure 5.73: An excerpt showing Betty guiding learners in the volume task.

The teacher stayed as close as possible to the learners in this trail to offer individual and group guidance to them (GP1). However, it should also be noted that Betty's closeness to the learners was at odds with the AP because it appeared to me that she did not always allow enough time for the learners to notice their own mistakes and rectify them before her stepping in. For example, on the pole task, I observed that Betty intervened in the middle of the learner's discussions about whether the phrase 'the depth of the hole' meant volume or not. If only the teacher could have waited and allowed the learners to figure out on their own that they were wrong: an approach that would have been more consistent with the principles of RME.

Nevertheless, it was still evident that the teacher had good intentions to guide the learners through the solving of the tasks. Moreover, Betty guided learners on how to operate the MCM app, especially by finding the hidden tasks by following the live GPS pointer. On the other hand, it should also be noted that the stepped hints were not used in this trail as the learners managed to get the correct answers to the solutions to the tasks on their very first attempt.

5.2.6.2 Trail 3 of School C (Teacher 8: Joshua)

Profile and Coding

Trail 3 of School C was coded **TR3C** (MCM app code: **4516109**) and is named "*Come we do math…*" trail. The participant teacher was identified with the pseudonym Joshua. Joshua has been teaching mathematics for the past six years now and holds a Diploma in Education from the Institute of Open Learning (IOL). The data from Joshua's video is coded as **JV**$_8$ and the time frame of the video was 42 minutes. Six learners, consisting of two girls and four boys, participated in this trail under the care of Joshua. The two tasks they solved were on the topics of measurement, volume, ratio and proportion – see points 1 and 2 in Figure 5.51 and Table 5.9 below. Joshua and his learners walked and solved the two tasks in approximately 1 hour, 8min.

The implementation of the RME principles: Joshua.

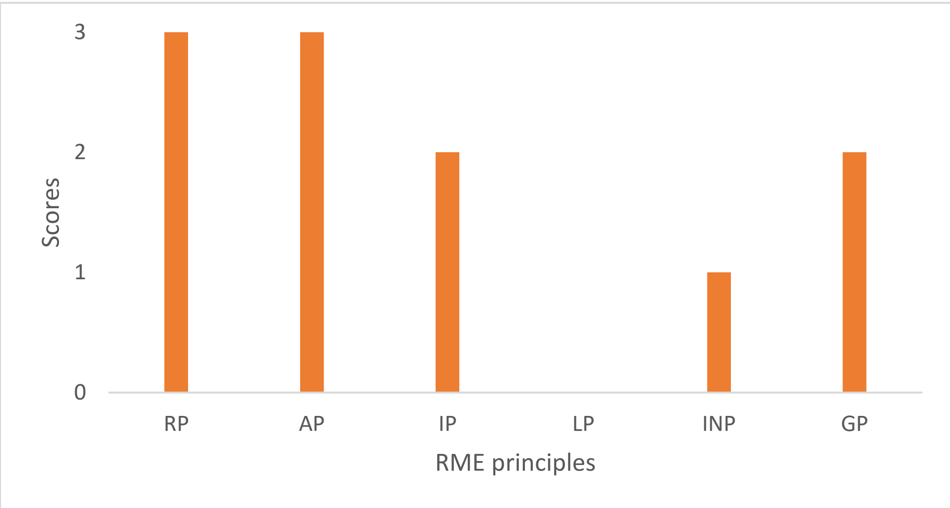


Figure 5.74: Numerical scores of the implementation of the RME principles by Joshua.

Figure 5.74 above shows that in the trail of TR3C the RP, AP, IP and GP were the most used principles. The graph also shows that there was no observed incident of LP and that INP only occurred once.

Table 5.9: The two tasks solved by Joshua's group of learners in Trail 3 of School C.

TASK TITLE	TASK DEFINITION	OBJECT	SAMPLE SOLUTION	HINTS
The road curbs task Code 4568235	Work out how many of these complete kerbs will be needed to cover a distance of 2 km.		Measurements: the length of one kerb is 88cm. Change 2 km to cm: 2×100 000 = 200 000cm 200 000 ÷ 88 = 22.7cm = 2272.72 complete curbs. = 2273 complete curbs	1. Measure the distance (length) of one kerb. 2. Divide the 2km (changed to cm) with the length of one kerb.
The pipe culvert task Code 1868230	Work out the amount of concrete that makes up this culvert pipe. Give your answer to the nearest cubic centimetre.		Measurements: Diameter (outer) 87.5cm Diameter (inner) 77.1cm Therefore: radius (outer) 43.75cm radius (inner) 38.55cm V = Outer - inner V = $\pi r^2 h - \pi r^2 h$ = $\pi \times 43.75^2 \times 245.6 - \pi \times 38.55^2 \times 245.6$ = 1476843.1 - 1146639.8 = 330203.3 = 330203cm²	1. The culvert is a cylindrical shape. 2. The volume of a cylinder is $\pi \times r^2 \times h$. 3. The amount of concrete is equal to volume of whole shape minus volume of open space.

The Reality Principle (RP)

According to my observations, there were incidents where the teacher alluded to the reality of the tasks the learners were working on. For example, data from **JV_8** shows that when the learners were solving the road kerb task, the teacher was heard emphasising that in reality, if they were working in a road construction company, they would have to calculate the number of kerbs using the same approach (RP2). see Figure 5.75 below.

> If you were working at Roads Authority or a roads construction company, the way we are calculating now to find how many of these small bricks, I mean kerbs will be needed for a distance of 2 km or even 10 km, isn't? you can see that we are using ratio and direct proportion. So, what does this means? The longer the distance the more of these kerbs we will need and the shorter the distance the less we will need.
>
> **JV_8**

Figure 5.75: An excerpt showing the teacher relating the learner's approach to solving the road kerb task to that of the real profession of constructing roads.

Furthermore, on the pipe culvert task, I heard the learners discussing what the possible use of such a big pipe in real life could be (RP2). Some learners suggested that it could be used as a dustbin by positioning it upright, while other learners suggested that the pipes were used to transport water when connected together. For the latter suggestion, a comment from one learner worth noticing is presented in Figure 5.76 below:

> I think these pipes are supposed to be connected together and the water from NAMWATER or even sewage water passes through the opening. Sometimes you will find these pipes by the bridge and then water from the river will pass through when it rains.
>
> **JV_8**

Figure 5.76: An excerpt showing the AP

These discussions are evidence of the existence of RP in this trail. Both the learners and the teacher contemplated the connection between the tasks and real professions. The teacher linked the reality aspect of the kerbs on the road to mirror the real profession of a road constructor by emphasising that in the same way the learners were solving the problem, a real road constructor would determine the number of kerbs. (RP2).

Another aspect revealed about the RP in the task of counting the road kerbs was that, after writing 2260 as their final answer with no decimals (see Figure 5.77 below), Joshua asked the learners to explain why they chose 2260 and not 2259 (when rounding the decimal 2259.887). As expected, the learners' answer was that it was because the digit after the decimal point was more than 5. This indicated that it was just by chance that the learners got the correct answer of 2260, otherwise with their logic of rounding off decimals, they would have obtained the wrong answer.

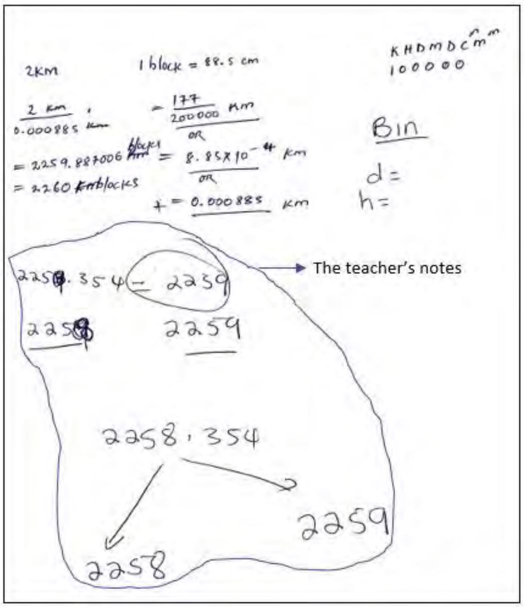


Figure 5.77: Learners' work and the teacher's notes(circled) on the solution to the road kerb task.

Joshua then asked the learners to tell him what the number of kerb blocks would be if they rounded off 2259.354 instead of 2259.887 as shown on the calculator screen. The learners said that it would be 2259 kerb blocks. The teacher then explained the following:

> You should round up your answer to make sure that you get enough kerbs that will cover the 2 km distance. Otherwise, if you round it to 2259 that it means that there will be ka small distance will remain because of the 0.354 kerb you removed. So, I would rather you have 2260 kerbs, which is more than enough to cover the whole distance of 2 km than have 2259, which will be less by one kerb where you will need to cut a piece to complete the kerb (RP1).
>
> **JV$_8$**

Figure: 5.78: The teacher explaining the RP of rounding off physical quantities.

I find the excerpt in Figure 5.78 to be rich in the RP of RME. The teacher is explicitly explaining to learners how to round off numbers in the context of real-world situations (RP2). Here we find the concepts of rounding down and rounding up. As can be noticed, the teacher emphasised that it was not possible to have 2259 and 0.345 kerb blocks, so the result needed to be rounded off to the next whole number of kerb blocks. He further emphasised that if they rounded off the number of kerb blocks based on the theory of rounding decimals, they would not meet the need of covering the required distance, because parts of the road would remain uncovered by the kerb blocks. Thus, he proposed having 2260 instead of 2259 kerb blocks. He argued that it was better to acquire more road kerb blocks than less, because there might not be enough.

The Activity principle (AP)

There is strong evidence that the learners in this trail were actively involved when walking on the trail and solving of the tasks. From the start of the trail, they were active participants in all the activities involved. The learners were seen taking measurements and engaging in discussions about finding solutions to the tasks (AP1) – see Figure 5.79 below. I clearly observed that most of the time the learners figured out themselves how to find the tasks on the MCM Google map (AP2). This was evident by how learners on their own located the positions of the tasks without the interference from the teacher.

Figure 5.79: The learners' active engagement in measuring objects in TR3C.

The Interactive principle (INP)

Especially for the pipe culvert task, the learners took a while to understand what they were supposed to do. They did not interact and share ideas as with the other task. At first some learners suggested that they consider opening the net of the pipe, an approach that could have led then to calculating the surface area. Others suggested that they simply find the difference between the outer and inner radius and use that in the formula $V = \pi r^2 h$, which also could have led them to the wrong answer. The latter approach would work if the learners were to use $V = \pi(R^2 - r^2)h$, where R is the outer radius and r the inner radius. It was after deliberations that they came to agree on using the method and approach of first finding the volume of the whole pipe and then deducting the volume of the hollow space (IP1). This reasoning led them to using the formula of $V = \pi r^2 h - \pi r^2 h$. See Figure 5.80 below for their detailed answer.

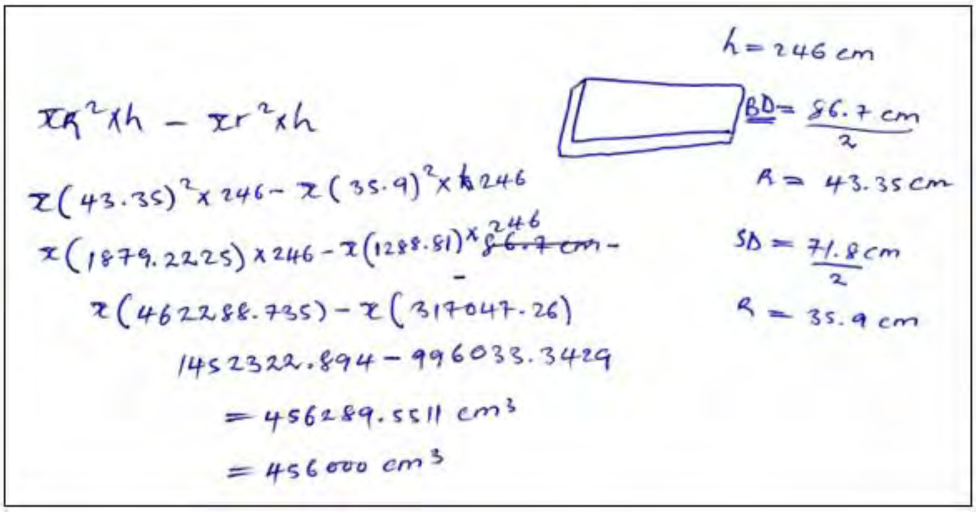


Figure 5.80: The learners' solution to the pipe culvert task

What I observed is that the solution to the pipe culvert task was reached as a result of positive interactions between the learners IP2. As already reported, other methods were suggested, then together, the learners interactively weighed them up and found them wanting, until the method shown in Figure 5.80 was agreed on as the best option. It is also important to report here that at one stage they used a smartphone to research the formula for the volume of pipes in different contexts. At some point in their research, the formula $V = \pi(R^2 - r^2)h$ popped up. It was evident that the learners did not understand this formula and that it was unfamiliar to them in this factorised format. Also, because the teacher seemed to not bother explaining the formula to the learners, suggests that he was also not familiar with this particular formula.

The Level Principle (LP)

Incidences that show the emergence of the LP were rarely observed in this trail. It cannot therefore be reported that the principle was evident in the trail. Although there were situations where the principle could have been explored, the teacher did not take the opportunity to help the learners understand certain concepts. For example, when they came across the formula $V = \pi(R^2 - r^2)h$, the teacher could have stepped in to help them understand how this factorised formula could be used ($V = \pi(R^2 - r^2)h$ = unfactorized formula). Eventually the learners opted for the format $V = \pi r^2 h$ because it was what they were familiar with. Perhaps it would have helped if the teacher had explained to them the concepts behind the formula $V = \pi(R^2 - r^2)h$ and how they could have used it to calculate the volume of the concrete of the pipe culvert.

The Intertwined Principle (INP)

There was only one incident where the INP was observed on the TR3C trail and that occurred when the learners connected the mathematical concept of volume to the word 'amount' (INP4). During the solving of the pipe culvert task, learners engaged in a discussion of the meaning of the word 'amount'. Although some of them had a misconception of interpreting the word 'amount' as total surface area, it was observed that two of them convinced others that the amount is concerned with volume – see Figure 5.81 below.

Learner B	I think we should cut here and open the net (*other learners show agreement*)
Learner A	But what does amount mean? Let us read the question again and understand what it says.
Learner C	Work out the amount of concrete that makes up this culvert pipe … amount of concrete … (re-reading to understand the question). Yes, I think this means calculating the surface area.
Learner A	No, I don't think its surface area, I think it's volume.
Learner D	Why do you say its volume? but the only way we can find the amount of concrete here is to open the net of this shape. This is a cylinder most.
Learner E	I think Learner A is right, amount does not mean surface area.
Learner B	But how can you calculate this amount of concrete if you don't open the net? Remember what we learned on the surface area of a cylinder. Here its open so there are no circles. So we just need to find the area from here to here… (touching the inner and outer surface of the pipe.
Learner A	But what does amount mean?
Learner E	Look, if the question says what is the amount of water in this bottle or tank, what does that mean?
Learner A	Eee, amount of water in the tank? Remember volume is like capacity which means the amount of liquid in a container, isn't not so? (other leaners look at each other and also at the teacher)
Teacher	Well, don't look at me … don't you think what they are saying is right? I like the example Learner A is using… amount of liquid in the tank or container? What does that mean? (INP4)
Learner	Yes, I think it means volume, but how can we calculate volume on this shape? **JV8**

Figure 5.81: A discussion on how the concept of volume was linked to the word 'amount'.

From the above excerpt, it can be noted how the concept of volume was linked to authentic settings like finding the amount of concrete of a hollow pipe and the amount of water in a tank or container (INP4). It should be noted how Learner A reminded others of the meaning of volume by linking the term to capacity, a concept found in the subject of physical science.

The Guidance Principle (GP)

The existence of the GP in this trail can be attributed to how the teacher was always available to learners whenever they needed his help during the trail and the solving of the tasks. It was observed that Joshua avoided providing learners with direct answers to their questions but only stepped in when there was a need to do so (GP1). For example, the excerpt in Figure 5.81 above on INP shows that Joshua allowed the learners to debate the meaning of amount in relation to the concepts of volume and total surface area until he saw the need to contribute to the discussion. Even so, I noted that Joshua did not directly tell learners what to do, but instead he posed a question that helped learners to reflect on what they were doing and then decide how to solve the task (GP2).

Also, in Figure 5.78 under RP it can be noted that the teacher directed leaners on how to round off numbers that represented physical quantities. Joshua guided the learners to understand that rounding off is not only about looking at the next digit to the right to see if it is more or less than five, to round down the number if is it less than five and round it up if it is more. The teacher emphasised that sometimes the context of the question will require a rounding up and not down, especially when dealing with physical quantities like the number of kerb blocks (GP3).

5.3 HORIZONTAL ANALYSIS ACROSS PARTICIPANTS

This section consolidates and summarises the analysis of the data presented above by analysing the data **across** the eight participants and discussing the findings with the reviewed literature for possible answers to the first research sub-question: *In the context of running mathematics trails and solving the MCM project tasks, in what different ways do selected teachers make use of outdoor authentic tasks for conceptual teaching of area, volume ratio and proportion topics?* The aim of this section is to obtain deeper insights into how the teachers facilitated conceptual teaching of area, volume, ratio and proportion topics using the MCM trail tasks. I do this by comparing different perspectives of the teachers' interactions with their learners in the mathematics trails. Based on the work of Van den Heuvel-Panhuizen (2001) on the six principles of RME, I identified several observable indicators that I categorised under the six broad RME indicators, (refer to RAILING analytical instrument Table 4.3). The horizontal analysis of the data, category-wise, showed that the participants demonstrated the ability to use the mathematics trail tasks to teach for conceptual understating within the context of RME-based instruction. Nonetheless, there was little evidence of actions that suggested the application of the LP and INT principles in the trails. The following Figure 5.82 summarizes how the eight teachers used the six RME principles in the mathematics trails.

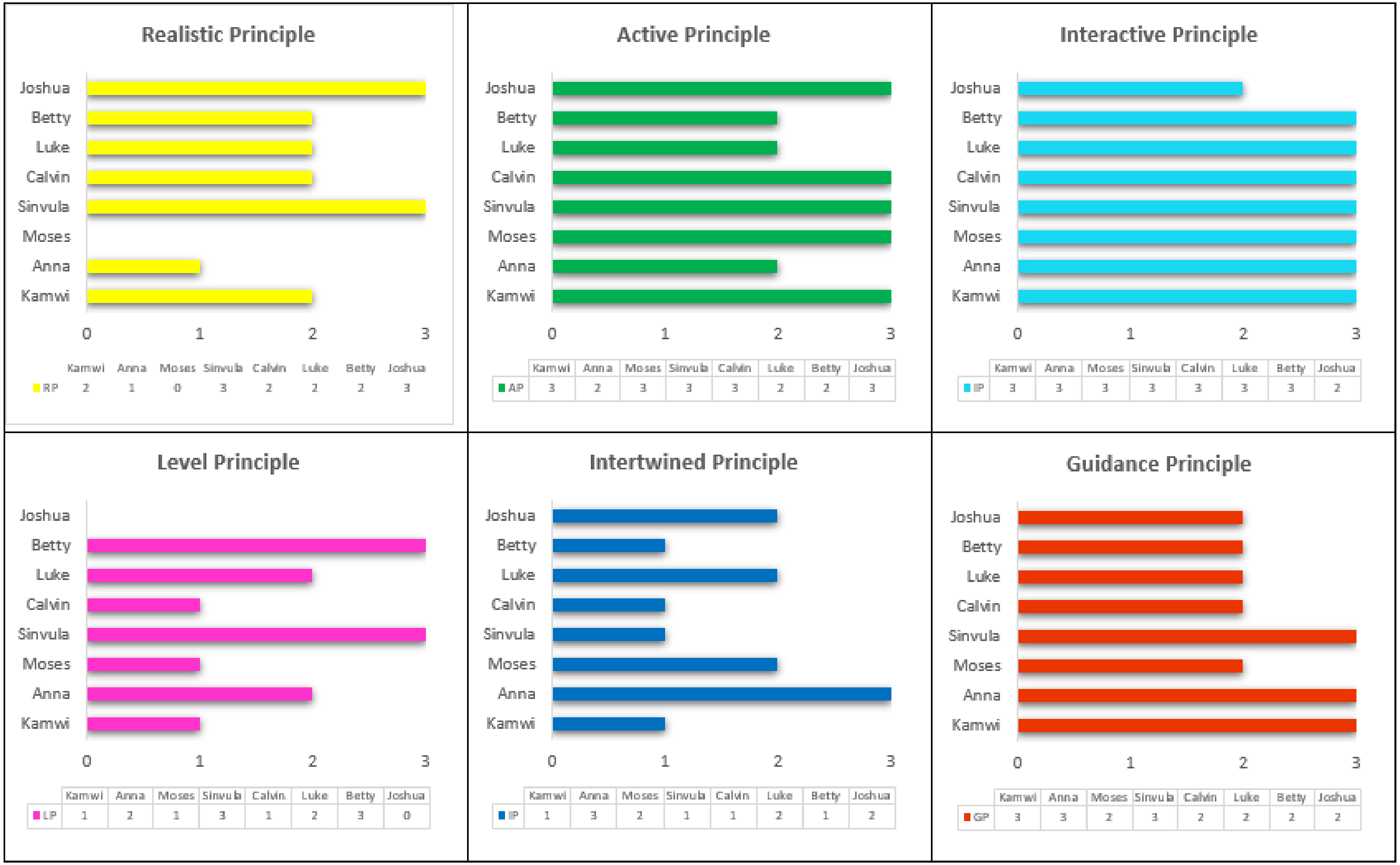


Figure 5.82: The teachers’ use of the RME principles in the mathematics trails

5.3.1 The Reality Principle – RP

The RP principle advocates that realistic problem situations should be the starting point for teaching mathematics, and that learners should be able to apply mathematical skills to solve real-world problems (Van den Heuvel-Panhuizen, 2001).

5.3.1.1 Synopsis

Kamwi (KV_1):

- highlighted the concept of area in both the two tasks (RP1);
- provided examples on how the tasks can be linked to real life (RP2); and
- urged learners to imagine themselves in the application of the tasks (RP3).

Anna ($AV_{2)}$:

- highlighted the concepts of area and volume in the tasks (RP1);
- linked the rounding off of tiles to physical quantities (RP2);
- highlighted that the first task can be solved in multiple ways (RP4); and
- probed learners to justify or explain their reasoning of rounding off (RP5).

Moses (MV_3):

- No evidence reflected in the trail.

Sinvula (SV_4):

- linked the painting task to the actual painting of a building (RP2);
- highlighted the concepts of shaded and unshaded areas in the first task (RP1); and
- linked the painting task to the age of the tree (RP1).

Calvin (CV_5):

- compared and linked the dustbin task to the realities of learners' community (RP2);
- linked the concept of volume to the term 'amount of space' (RP1);
- reflected on how the mathematics learned from the textbook link to situations (RP2).

Luke (LV_6):

- reminded learners about the reality of watering and water usage (RP2);
- used a practical example to compare units eg container (RP1, RP2); and
- highted the concepts of ratio and proportion in the tasks (RP1).

Betty (BV_7):

- probed learners on their method of measurement in the second task (RP5);
- asked learners to explain the 1cm variation in their measurements (RP5); and
- linked the term 'depth of the hole' to the concept of height (RP1).

Joshua (JV_8):

- linked the road kerb task to a real profession (RP2);
- highlighted the concept of ratio in the road kerb task (RP1);
- discussed the uses of the culvert pipe in real life (RP2);
- connected the concept of volume to the term 'amount of concrete' (RP1) and
- linked the concept of rounding off to physical quantities (RP2).

5.3.1.2 Discussion

In line with the RP principle, first, it is important to point out that following the MCM criteria, all 38 MCM tasks contained in the trails of this study were intentionally attached to real objects. Therefore, it can be said that the tasks were situated in realistic problem situations and that the RP was already ensured and activated when the teachers used the tasks for teaching purposes.

Second, apart from Moses, all the other participant teachers used the tasks to help learners understand the concepts of area, volume, ratio and proportion and the application of these concepts in real life. For example, Kamwi and Sinvula used the realistic situations of the tasks to discuss and estimate the area of the surfaces of physical objects as well as (in some instances) the amount of paint that would be needed to cover these surfaces. Anna and Joshua used the tasks to engage learners in discussions that were centred on how the rounding off of physical quantities such as the number of tiles and road kerb blocks can be done. The two teachers emphasised that in reality, they cannot have a fraction of a tile or road kerb, so they needed to round up to the next whole number. Calvin and Betty used the dustbin task and concrete blocks to discuss the concept of volume. On the other hand, Luke used the lawn bed task to discuss the concepts of area and proportion, and he also used the metal rail bars task to discuss the concept of ratio and proportion. Since the tasks were situated within the learners' environments and familiar to them, the teachers could easily connect them to the learners' realities by using the RP in their teaching while walking on the mathematics trails. In this way, the teaching of the topics of area and volume, ratio and proportion started from real-world contexts where teachers helped learners to use the contexts and relate them to the mathematics learned from the classrooms or textbooks.

Third, this study suggests a possible answer to Vos's (2018) question of how to connect mathematics, a theory-heavy subject, to reality. A feasible answer is to use mathematics trail tasks as teaching contexts that are framed within the RME theory. This way, the abstract mathematics in the classroom becomes meaningful to learners as they link and emulate the concepts to their real-life experiences and situations, as shown by this study. Following the recommendations of some research studies (eg Stillman et al., 2013a; 2013b), the teachers in this study let the learners experience the usefulness of mathematics by solving real life problems. This was in line with Hernandez-Martinez and Vos (2018) who argued that learners can only appreciate the relevance of mathematics if they can see how it relates to their own realities.

5.3.2 The Activity Principle – AP

With the AP, Van den Heuvel-Panhuizen and Drijvers (2014) state that learners should take an active role in their own learning of mathematics as this will enable them to reconstruct mathematics by engaging in, and reflecting on activities that develop concepts and mathematical understanding.

5.3.2.1 Synopsis

Kamwi (KV_1):

- interactively engaged learners in discussion concepts embedded in the tasks (AP1);
- encouraged learners to oversee the trail activities (AP2);
- encouraged learners to work on their own AP4); and
- allocated responsibilities to learners and encouraged them to work together (AP3)

Anna (AV_2):

- helped learners to effectively carry out measurements (AP2; AP4);
- engaged learners in interactive discussions of concepts embedded in the tasks (AP1); and
- allocated responsibilities to learners and encouraged them to work together (AP3)

Moses (MV_3):

- encouraged learners to work on own (AP4); and

- encouraged learners to work together in carrying out the trail activities (AP2).

Sinvula (SV_4):

- encouraged learners to work on their own (AP4);
- encouraged learners to oversee the trail activities (AP2); and
- allocated responsibilities to learners and encouraged them to work together (AP3).

Calvin (CV_5):

- encouraged learners to work on their own (AP4);
- encouraged learners to oversee the trail activities on their own (AP2); and
- allocated duties to learners and encouraged them to work together (AP3).

Luke (LV_6):

- interactively engaged learners in discussions of concepts embedded in the tasks (AP1); and
- encouraged learners to oversee the trail activities (AP2).

Betty (BV_7):

- interactively engaged learners in discussions of concepts embedded in the tasks (AP1); and
- encouraged learners to oversee the trail activities (AP2)

Joshua (JV_8):

- interactively engaged learners in discussions of concepts embedded in the tasks (AP1); and
- encouraged learners to oversee the trail activities (AP2).

5.3.2.2 Discussions

The AP of RME was the most-used principle in this study. All the participant teachers ensured the active participation of learners in the trails by allowing them to oversee the trail activities of locating the tasks and solving them. In the mathematics trails, the teachers played a minimal guiding role and allowed the learners to engage in physical movements and hands-on activities. These activities included measuring the dimensions of different objects, drawing sketches of the objects involved and using a smartphone with the MCM app to locate tasks. Furthermore, the teachers encouraged their groups of learners to work together and engage in interactive discussions, which I argue contributed to the liveliness of learners in the trails.

So, as recommended by Van den Heuvel-Panhuizen and Drijvers (2014), the teachers' engagements in the mathematics trails of this study demonstrate that the teachers did not treat learners as passive recipients of knowledge. Instead, the teachers allowed their learners to create and use mathematical concepts, tools and procedures to make sense of the problem situations through active measurements, carrying out of algorithmic calculations of area and volume, and discussions. In this way, the teachers scaffolded the learners in the development of deeper understanding of spatial measurements, ratio and proportion and fostered their mathematical creativity, curiosity and confidence. The teachers also gave their learners opportunities and freedom to choose their own methods and strategies of solving the tasks.

Furthermore, most of the teacher participants in this study ensured that their learners worked together by assigning them roles such as coordinator, note taker, measurer, phone and calculator operators. van den Heuvel-Panhuizen (2001; 2003), observes that when teachers treat learners as active participants in their learning process, learners' interactions with the subject matter content increases and subsequently this can develop deeper and more flexible understanding of mathematical concepts.

5.3.3 The Interactive Principle – IP

The IP of RME highlights the importance of social interactions in the process of learning mathematics. The principle, views mathematics as both an individual and a social endeavour (Van den Heuvel-Panhuizen, 2001).

5.3.3.1 Synopsis

Kamwi (KV_1):

- encouraged learners to find answers on their own (IP1);
- encouraged learners to critique and justify their solutions to the tasks (IP4);
- posed questions that led to the discussions and visualisation of concepts in the tasks (IP3); and
- encouraged the use of artifacts (smartphone, tape measure and calculator) to increase interactivity in the trail (IP5).

Anna (AV_2):

- encouraged learners to find answers on their own (IP1);
- promoted interactivity among learners during the trail activities (IP2); and
- allowed the use of smartphone, tape measure and calculator to increase interactivity in the trail (IP5)

Moses (MV_3):

- encouraged learners to find answers on their own (IP1);
- promoted interactivity among learners during the trail activities (IP2);
- posed questions that led to the discussions and visualisation of concepts in the tasks (IP3); and
- allowed the use of the smartphone and tape measure to increase interactivity in the trail (IP5).

Sinvula (MV_4):

- encouraged learners to find answers on their own (IP1);
- posed questions that led to the discussions and visualisations of concepts in the tasks (IP3);
- allowed the use of smartphones and tape measure to increase interactivity in the trail (IP5); and
- urged learners to share their ideas without fear (IP2).

Calvin (MV_5):

- encouraged learners to find answers on their own (IP1);
- promoted interactivity among learners during the trail (IP2);
- posed questions that led to the discussions and visualisations of concepts in the tasks (IP3); and
- allowed the use of smartphone and tape measure to increase interactivity on the trail (IP5).

Luke (LV_6):

- promoted interactivity by urging learners to work together (IP2);
- encouraged the use of artifacts (IP5); and
- aroused the learners' interest to engage in the trail tasks (IP2).

Betty (BV_7):

- encouraged learners to find answers on their own (IP1);
- promoted interactivity by urging learners to work together (IP2);
- encouraged the use of artifacts (IP5); and
- posed questions that led to the discussions and visualisations of concepts in the tasks. (IP3)

Joshua (JV_8):

- encouraged learners to find answers on their own (IP1); and
- promoted interactivity among learners during the trail and solving the tasks (IP2).

5.3.3.2 Discussion

In this study, the IP was the most dominant principle in all eight trails. Perhaps the active learning that emerged on several occasions from the trails was inevitably the result of social engagements that facilitated interactions among learners. The teachers encouraged their learners to work together and find answers to the tasks on their own without depending on their teachers. Further, the teachers used the realistic problem tasks to involve learners in solving questions related to topics of spatial measurements, ratio and proportion, and in the process the learners exchanged ideas, approaches and solutions to the tasks. I argue that the frequent use of probing questions contributed to and harnessed the productive discussions that were observed in the trails. The questions solicited interactive conversations about what dimensions to measure and the units to use on the tape measure, and how and where to position the tape measure.

Furthermore, the learners not only interacted with each other but also with their teachers and tools such as the smartphone, tape measure, calculator and the MCM app. Concepts such as quadrilaterals, parallel lines, angles, straight lines, perpendiculars, equal length, area, volume and many more related concepts were explored and interrogated when learners solved the mathematics trail tasks. Interactions were generated through mobile devices, physical objects and the communication between learners and their teachers. According to Vygotsky (1978), these types of active interactions can play an important role in the cognition development of learners. Vale and Barbosa (2019) advanced the idea that learning arises from experiences and interactions among the cognitive, social and physical aspects. The physical and social interactions that happened in the mathematics trails of this study were harnessed by the regular use of mathematics concepts and how these concepts were connected to the mathematics learned from the classroom, Therefore, I conclude that because of these interactions, it is probable that the learners' understanding of the concepts of spatial measurements, ratio and proportion were enhanced and became experientially real to them.

Moreover, during the trails I observed that some learners had difficulty in locating the first task or finding its solution. Nevertheless, as they engaged in interactive discussions and shared their ideas, they gradually began to understand what was expected of them and became more confident in solving the problems. This supports the claim by Van den Heuvel-Panhuizen and Djivers (2014) that "interaction evokes reflection, which enables students to reach a higher level of understanding" (p. 523). So, this was more evidence that the interactions that occurred in the trails of this study were crucial for helping learners comprehend the mathematical concepts related to the realistic situations they encountered. Through participating in a dynamic learning environment of mathematics trails, learners reflected on their methods and improved their conceptual understanding.

5.3.4 The Level Principle – LP

With the LP, RME theorises that in the process of learning mathematics, learners go through different levels of understanding to find related concepts and strategies that could help them solve realistic situation-related problems (Van den Heuvel-Panhuizen & Drijvers, 2014).

5.3.4.1 Synopsis

Kamwi (KV_1):

- explained concepts at the learners' level of understanding (LP2);
- established the learners' prior knowledge (LP3);
- used models to scaffold the learners (LP5);
- used models to encourage the visualisation of concepts (LP6); and
- identified learners' strengths and weaknesses (LP7).

Anna (AV_2):

- established the learners' prior knowledge (LP3);
- used a sketch to scaffold the learners understanding of concepts (LP5);
- used a drawing to encourage the visualisation of concepts (LP6); and
- developed learners' mathematical reasoning from informal to formal abstraction (LP8)

Moses (MV_3):

- used models to encourage the visualisation of concepts (LP6);

- developed learners' mathematical reasoning from informal to formal abstraction (LP8); and
- used a sketch to scaffold the learners understanding of concepts (LP5).

Sinvula (SV_4):

- established the learners' prior knowledge (LP3);
- used a sketch to scaffold the learners understanding of concepts (LP5);
- developed the learners' mathematical reasoning from informal and to formal (LP8); and
- used models to scaffold the understanding of concepts (LP5).

Calvin (CV_5):

- established the learners' prior knowledge (LP3);
- used gestures to model and make concepts visual (LP5),;
- used gestures to scaffold concepts (LP2, LP5); and
- encouraged the visualisation of concepts (LP6).

Luke (LV_6):

- explained concepts at the learners' level of understanding (LP2); and
- used models to encourage the visualisation of specific concepts (LP6).

Betty (BV_7):

- explained concepts at the learners' level of understanding (LP2);
- established the learners' prior knowledge (LP3); and
- demonstrated the multiple ways of solving the tasks (LP4).

Joshua (JV_8):

- No evidence of the principle on the trail.

5.3.4.2 Discussion

According to Van den Heuvel-Panhuizen and Drijvers (2014), the implementation of LP requires that teachers organise the learning content in such a way that learners use their everyday knowledge to solve problems that will enable them to transit to acquiring insights into how concepts and strategies are related. In this study, the LP was the least used principle in the trails. It was clearly seen that where the principle occurred, the learners relied on their own informal knowledge to solve problems, not because the teachers encouraged them, but because they devised their own strategies.

The findings revealed that Kamwi was the only teacher who explicitly employed the LP in his interactions with his learners. Kamwi used an A4 page to model the net of a cylinder and explained to his learners how this related to calculating the surface area of the pole to be painted (refer to Figure 5.5. In other words, the teacher used his everyday knowledge of painting things (situational level) to demonstrate that learners only needed to consider the outside curved part of the cylinder when calculating the required area to be painted. Also, I noticed that the attempt was aimed to help learners understand that the top and bottom parts of the cylinder (circles) were not part of the area to be painted.

Furthermore, Kamwi used the same paper model to represent the transition from a solid cylinder to its open net, particularly the curved surface. Eventually he concluded by explaining to the learners the abstract reasoning behind the formula of calculating the surface area of a cylinder (refer to Figure 5.6). I therefore argue that in this scenario, the teacher attempted to shift the learners' understanding of surface area from solving problems in real life situations to seeing the connections between ideas and methods of working with algebraic formulas of surface area of cylinders.

Another notable LP incident that was not thoroughly explored by the teacher (probably because it was the learners' initiative) emerged from Moses' learners when they solved the stairway task using the iterative method of measurement by using a 30 cm ruler to count the number of bricks (of 30 cm length) that made up the top stair (refer to Figure 5.22). Note that the learners used a ruler because of the question that indicated that the length of the brick was 300 mm, of which the learners correctly converted to 30 cm. So, instead of using a tape measure, the learners used a ruler as a nonstandard unit to determine the number of bricks. Although this incident was not the teachers' idea, Moses however helped the learners to determine the number of bricks in the top step. Thereafter, the teacher used the learners' informal knowledge to build up their understanding of calculating the volume of the whole step and dividing the answer with the volume of one brick. The other teachers used the LP sparingly, as it was observed to be minimal in the trails.

5.3.5 The Intertwinement Principle – INP

The INP suggests that there are connections between different mathematics topics and themes and that the topics should not be treated as isolated strands. The principle is grounded on the idea that mathematics is a coherent and connected discipline, and that learning mathematics involves building a network of mathematical concepts and skills (Van den Heuvel-Panhuizen & Drijvers, 2014).

5.3.5.1 Synopsis

Kamwi (KV_1):

- linked the concepts of spatial measures to learners' real-world experiences (INP4).

Anna (AV_2):

- integrated the properties of a trapezium and a parallelogram (INP1); and
- linked the properties of a trapezium to an actual building (INP4).

Moses (MV_3):

- encouraged the integration of numbers, counting and measurement concepts. (INP1:INP2).

Sinvula (SV_4):

- linked the concept of area to the reality of panting a house (INP4).

Calvin (CV_5):

- intertwined the concept of rounding off and that of calculator skills (INP1); and
- integrated the topics of spatial measures, rounding, estimation, decimals, common fractions and two- and three- dimensional objects (INP2).

Luke (LV_6):

- encouraged learners to integrate the topics of measurement, area, proportion and the conversion of units (INP2).

Betty (BV_7):

- connected the concept of height to the reality of the depth of a hole INP4.

Joshua (LV_8):

- linked the concept of volume to physical objects (INP4).

5.3.5.2 Discussion

Apart from integrating concepts of area, volume, ratio and proportion to real-world objects and situations, this study shows that the integration of these concepts with other subjects (INP3) was not common. While the design of the tasks included the intertwinement of two or more topics/concepts, teachers did not make any reference to these connections in their interactions with the learners. The same applied to the connections between these concepts and other subjects. Not much emphasis was put on the relationships that existed between area, volume, ratio and proportion concepts and other mathematical concepts and other subjects. On the other hand, some teachers used real life examples to relate to the concepts in the tasks. For example, the AV_2 data source revealed that Anna linked the shape of a trapezium to that of an actual building which was nearby, to help the learners understand the concepts involved in finding the area of a trapezium (refer to Figure 5.17). After realising that learners had difficulty understanding some properties of a trapezium, Anna used a practical example to relate the concepts of parallel, perpendicular, and sloping sides to the original sign board problem they were working on. Also, the teacher connected the concept of parallel from the trapezium to the actual building as can be seen in the excerpt in Figure 5.16.

Generally, there was sufficient evidence that the teachers integrated concepts from the topics of spatial measurements, ratio and proportion to that of other topics in mathematics. For instance, now and then learners had to round off their final answers to the nearest degree of accuracy in each question, which they did without difficulty. According to Van den Heuvel-Panhuizen and Drijvers (2014), integrating topics as the learners did in this study, can enable them to see the relevance and usefulness of different mathematical topics and tools for solving problems in various situations. I therefore argue that integrating the concepts of area, volume, ratio and proportion can foster learners' conceptual and procedural fluency, exposing them to multiple representations and connections among concepts.

5.3.6 The Guidance Principle – GP

The GP in RME theory acknowledges the importance of teachers giving appropriate guidance and instructions that helps learners to rediscover mathematics. Teachers are implored to take an active role of guiding learning towards situations that can act as a catalyst to achieve shifts in learners' understanding when solving realistic contextualised problems (Van den Heuvel-Panhuizen & Drijvers, 2014).

5.3.6.1 Synopsis

Kamwi (KV_1):

- used a tape measure to help learners understand the difference between the units of centimetres and metres (GP3);
- promoted the exchange of ideas on the trail (GP1);
- used clear language for learners to understand (GP3); and
- guided learners in understanding the relationship between radius and circumference (GP3)

Anna (AV_2):

- guided the learners to understand the concepts of rounding off, parallel and perpendicular, and how these were related to the area concept (GP3);
- posed open-ended questions that helped learners share ideas (GP1; GP2); and
- used the learners' sketch to help them understand the concepts of a trapezium and a rectangle (GP4).

Moses (MV_3):

- used the actual brick to explain the difference between two- and three-dimensional shapes (GP4); and
- guided learners through the understanding of the concepts of net, surface area and volume (GP3).

Sinvula (SV_4):

- guided learners on how to round off in years and months (GP3); and
- provided prompts to guide learners solve the tasks (GP2).

Calvin (CV_5):

- posed a series of questions to trigger the discussion surrounding radius and diameter (GP1; GP2);
- guided learners to understand the difference between radius and diameter in the context of their algebraic representations r^2 and $2r$ (GP3); and
- used the actual objects to explain the concepts around shaded and unshaded areas, volume, radius and diameter (GP4).

Luke (LV_6):

- provided prompts to guide learners to solve the tasks (GP2);
- posed questions that evoked group discussion of the concepts of parallel and opposite (GP2; GP3); and
- used a physical object (lawn bed) to explain the meaning of parallel sides (GP4).

Betty (BV_7):

- promoted interactive discussions among learners which led them to finding their own solutions to the tasks (GP1); and
- posed questions that provoked discussions on additive answers for volume (GP2).

Joshua (JV_8):

- avoided giving straightforward answers by asking questions that required learners to think and work towards answers (GP1; GP2); and
- guided learners to understand the concepts of rounding up and rounding down ((GP3).

5.3.6.2 Discussion

Research studies (Van den Heuvel-Panhuizen, 2001; 2003; 2005) suggest that teachers should give appropriate guidance and instructions that could help learners to rediscover mathematics. Particularly in the context of mathematics trails where learners can be exposed to real-life problem situations that can be mathematised, Van den Heuvel-Panhuizen (2020) suggests that the process be done under the guidance of teachers. All the participants in this study demonstrated an ability to guide learners on what to do in the trail activities by using appropriate materials that aligned to learners' cognitive levels of understanding. For example, when the learners confused the shape of a brick for a rectangle, Moses asked them to distinguish between two-dimensional and three-dimensional shapes. This helped the learners to understand that a brick cannot be a rectangle because a rectangle is a two-dimensional shape.

Anna, on the other hand, used the GP to guide the splitting of the octagon sketch into the shapes of a rectangle and two trapeziums. At first, the learners were struggling to point out properties such as parallel, perpendicular, height and sloping sides, so Anna used probing questions to scaffold them into identifying the properties of an octagon as they appeared on the original sign board. The other teachers also gave guidance to learners where and when it was needed. Probing questions were used to stimulate discussions among learners that helped them to reach their own conclusions. These discussions greatly contributed to a better and enhanced understanding of several mathematics concepts that were encountered during the solving of the tasks.

On the other hand, The MCM app GPS function enabled the learners to find the math trail tasks in their respective environments without depending on the guidance of the teachers. This means that the teachers only guided the learners in solving the tasks through the mathematisation processes. Perhaps the teachers' guidance in the solving of the trails led to little or no use of the hints feature of the app. It was interesting to note that only two groups of learners used the hints as a scaffolding tool in solving one of their two tasks. Even so, it should be noted that on the part of Anna's group of learners, they only used hints because of the question that wrongly asked (see the incident under the Level Principle in Section 5.2.2.2 with Anna's learners).

In addition, the teacher participants of this study assisted their learners when they had difficulties, but it was clearly observed that they did not provide direct answers to the learners. The teachers prompted learners on what to do, clarified concepts and advanced suggestions in the form of prompts. This approach was consistent with Cahyono's (2017) research study, who suggested that teachers should gradually reduce and stop their guidance and interaction in mathematics trails, so that learners can appreciate learning for its own sake. The aim of guiding learners is to boost their confidence to build more complex knowledge independently. Therefore, I argue that the interactions between the teachers and the learners in this study enabled the learners to solve problems and enhanced their skills for solving other similar problems in the future by themselves. This scaffolding method is widely supported in the literature (eg Vygotsky, 1978), as it can enable learners to attain the Zone of Proximal Development, which is the optimal level of learning they are capable of.

Moreover, I noticed that teachers were careful about how they used the GP lest they interfere with the learners' active participation and collaborations on the trails. Previous findings (eg Van den Heuvel-Panhuizen, 2020) have cautioned that when using the six RME principles, teachers should be careful of the possible conflict between the tenets of GP and AP. The teachers in this study were aware of this and made sure that their guidance was appropriate and that they did not treat learners as empty vessels but allowed them opportunities of being actively involved in their own learning.

5.4 CONCLUSION

The main purpose of this chapter was to provide an in-depth analysis of the participating teachers' use of mathematics trails in the context of RME theory to teach the topics of area, volume, ratio and proportion. From the results of this research study, I can conclude that math trail tasks in RME-based instruction helped the teachers to teach for conceptual understanding. In the RME-based mathematics trails, learners encountered realistic problem situations that were meaningful and relevant to them, and I believe that these tasks supported the connection from real-life objects and situations to mathematics learned in the classroom. Also, I argue that learners were motivated to engage with their local environment and understand the mathematics that existed around them because the trails were located within the school grounds. Therefore, I argue that using mathematics trail tasks can positively change learners' negative perceptions about the subject's relevance, and this can also address Fitzmaurice et al.'s (2021) concern about learners not understanding the usefulness of learning mathematics.

It was clearly observed that the teachers used the mathematics trails to encourage the learners to be active participants in their learning process. Moreover, the teachers provided learners with actual tools (tape measures and calculators) to measure and calculate the area and volume of objects in the tasks, just as real practitioners would have used them in similar situations. Also, I noticed the teachers used the trails to engage learners in interactive discussions with one another and with the teacher.

When creating the math trail tasks, the teachers included cross-curricular concepts and topics that related to other subjects and provided opportunities for learners to practice transferable skills. Furthermore, the mathematics trail tasks provided learners with problems that required them to use geometric concepts, properties and formulas necessary for the calculation of area and volume. This is evidence that the trails presented learners with problems that required the use of data handling skills such as collecting, organising, modelling and interpreting the data related to area and volume. However, apart from the real-world connections and other concepts within mathematics as a subject, it appeared to me that in their actual teaching, the teachers struggled to connect the area, volume, ratio and proportion concepts to other subjects.

Many of the trail tasks that were designed for this study were based on area and volume concepts. The teachers used these tasks to teach the embedded concepts by helping the learners to progress from informal to formal levels of mathematical reasoning and communication. The findings of this study also reveal that the ratio and proportion-based tasks were rarely used in the trails. Out of the 38 mathematics trail tasks that were created for this study, only eight tasks had the concepts of ratio and proportion embedded in them (see Appendix Ten). This resulted in very limited discussion and engagement on these topics/concepts during the implementation of the trail tasks of this study. This corroborates and aligns with the findings from other previous research studies that reported that teachers and learners usually struggle with the underlining principles of solving ratio and proportion-related problems (Lobato et al., 2011; Hilton et al., 2016; Burgos & Godino, 2020), and thus possibly avoid them.

The teachers acted as facilitators in the mathematics trails by providing learners with appropriate scaffolding and feedback that supported their learning process. For example, the teachers provided learners with clear instructions and expectations for the mathematics trails, such as the goals, rules and roles. Also, the teachers used prompts and questions to elicit the learners' prior knowledge of area and volume concepts, which in turn encouraged them to continue working on the problems through recalling their prior knowledge, making assumptions and checking their results. In the following chapter, I present an in-depth narrative analysis on the teachers' experiences of teaching in the mathematics trails using smartphones within the context of the MCM mobile environment.

CHAPTER 6

TEACHERS' EXPERIENCES OF PARTICIPATING IN THE DESIGN AND IMPLEMENTATION OF OUTDOOR TRAILS USING THE MCM PLATFORM.

6.1 INTRODUCTION

In the previous chapter, I reported and analysed findings on the different ways in which teachers used authentic outdoor tasks to teach for conceptual understanding of the topics of area, volume, ratio and proportion. The aim of this chapter is to analyse and answer the second sub research question of how the participant teachers experienced the design and implementation (using smartphones) of the MCM-based trail tasks. The chapter presents data from RIs and FGIs. The chapter is structured into three main sections: Firstly, I vertically present the narrative analysis of the teachers' experiences of implementing the MCM app project and the use of smartphones in the study. Secondly, I horizontally summarise the analysis across all eight participants by looking at patterns of interactions, similarities and differences. Thirdly, I present and discuss the emerging themes that drawn from the VRs, RI and FGI data sources of all eight participants, to seek further insights to the answering of the main research question.

I primarily used the iPAC (Personalisation, Authenticity and Collaboration) m-learning framework to analyse the reflective RI (tabulated in Chapter 4). Again, I want to remind the reader what the three signature pedagogies of the iPAC framework are:

Personalisation: learners can choose what, how, when and where they learn (Agency – AG). Learners can also customise their learning experiences to suit their preferences and needs (Customisation – CU). For example, they can use different apps, tools and settings to learn about a topic that interests them.

Authenticity: learners can engage in real-world tasks and problems that are relevant and meaningful to them (Settings/tasks – ST). Also, learners can use mobile devices as tools that real-world practitioners use (Tools – TO). For example, they can use a Google Map app to navigate an environment.

Collaboration: learners can communicate and share information with other people using mobile devices (Conversation – CO), and that they can also work together on projects and activities that require teamwork and creativity (Co-creation – CC). For example, they can use a smartphone to discuss a topic or a video app to create a presentation with their group.

To apply the iPAC analytical framework and seek evidence of the three principles above, I firstly collected the relevant quotes and statements from the interviews that showed the teachers' views on the learners' use of smartphones and the MCM app. Then, I coded each statement according to the observable indicator(s) that matched the three principles of the framework. In this way, I could identify all the statements that indicated the presence of the principles in the trails.

Secondly, to rate and scale the sub constructs of each principle, I created a table based on the frequency of the coded statements. I counted how many times each code appeared in the statements and compared it to the total number of codes for each subconstruct. For example, the conversation subconstruct of the collaboration principle had three codes (CO1, CO2, CO3). If only one code (eg CO1) was found in the statements, I rated the sub construct as Low. If two codes (eg CO1 and CO3) were found, I rated it as Medium. If all three codes were found, I rated it as High. I repeated this process for all six subconstructs, adjusting the rating scale according to the number of codes for each subconstruct.

The following is the structure of the chapter:

Section 6.2 – Data presentation and vertical analysis of individual participants

Section 6.3 – Horizontal analysis: discussion of findings

Section 6.4 – Emerging themes: further insights into the main research question

Section 6.5 – Conclusion

6.2 DATA PRESENTATION AND VERTICAL ANALYSIS OF INDIVIDUAL PARTICIPANTS

6.2.1 Kamwi

Kamwi's reflections:

- Well, the fact that we had only one phone, which was a deliberate move of making the learners to work as a group (CO1;CO3).
- I think the cooperation based on the task that was given was proper, they worked together by aiding and correcting each other (CO3).
- The learners compared the visual image they saw on the phone with the real object (CO1).
- You see from the first step the learners used the phone to find the tasks (CU1), they solved the tasks using the phone and even checked their answer on the phone (CU3; CC4).
- They searched for the formulas (TO2) … I mean they literally used a phone to solve mathematic problems that were connected to reality with objects that they can see every day (TO1), I mean it was wonderful experience for me, and I also believe, for my learners.
- Because of using one phone learners had to come together and agree on which direction to take (CU1).
- This phone became their guide, and they followed the directions as they appeared on the map from the phone (CU1) (ST3).
- With the app, the learners solved real problems, like the painting question... something that happens in real life (ST1).
- I think the task of replacing the door was important because that electricity box door needs replacement, it is danger the way it is like that (ST2).
- Learners took actual measurements of the pole the same way anyone would have done to find the area of the pole to be painted (ST3) This phone became their guide, and they followed the directions as they appeared on the map from the phone (CU1) (ST3).
- Yes, it was very easy for them because they were using the phone in a proper way (TO4).
- They were searching for things that were academically needed (TO2).

Figure 6.1: Evidence of the iPAC principles from Kamwi's interview.

Table 6.1: Ratings of m-learning scenarios for Kamwi

Scale	*Sub-scales*		*Use of MCM app*
Personalisation	Agency	(AG)	Low
	Customisation	(CU)	Medium
Authenticity	Setting/Task	(ST)	High
	Tool	(TO)	High
Collaboration	Conversation	(CO)	Medium
	Co-creation	(CC)	Low

6.2.1.1 Personalisation

Kamwi's RI revealed that there was low agency and medium customisation on the learners' levels of personalisation (see Table 6.1 above). The following transcript shows why Kamwi held the view that his learners did not demonstrate sufficient levels of personalisation within the MCM mobile environment.

> You know our learners are not independent, they like teachers to explain and re-explain things to them. So, they cannot just come there and start doing things on their own without the blessing of the teacher. So, I think if there was no teacher there, they were not going to do anything at all. In fact, what I am trying to say is that although somehow somewhere learners were helping each other by correcting each other on what is right and wrong. But somewhere somehow, they were too much on the teacher. They always expected me to say something in everything they were doing. I kept on reminding them that if they cannot get the correct answer, they can still submit so that they get the hint from the system than waiting for me to explain the questions to them. They were just there struggling as if they cannot use the app itself to get help answering the questions. They also wanted me to explain to them the hints that are already there already. I think they are very much dependent on the teacher than on themselves and the app.

Figure 6.2: An extract from Kamwi's RI explaining the low to medium levels of personalisation among his learners in the trail TR1A

In Figure 6.2 above, the transcript shows that Kamwi was aware of the potential of personalisation created by the MCM app m-learning tool, but the learners were not. According to Kamwi, the learners were not exercising their choices, agency or self-regulation, and they were not customising their learning experiences. The learners relied on him to explain and re-explain things to them, rather than using the app itself to get help through the hints feature. In other words, the customisation of the learning content was lacking because the learners preferred the teacher-centred approach to learning than exploring the learning content on their own. The extract also suggests that the teacher tried to encourage the learners to be more independent and autonomous in solving the tasks on their own, but they resisted. It appears that the learners lacked self-regulation in their learning as they often needed the teacher's approval or permission to start or continue their activities.

6.2.1.2 Authenticity

The analysis of Kamwi's RIs revealed that he viewed the authenticity construct as high on both the subconstructs settings and tools. According to him the MCM app presented realistic contexts for his learners by providing them with realistic tasks that were experiential to them. For example, he highlighted how both the painting and electricity door tasks were real in the sense that the learners always saw these activities in their community. Further, Kamwi emphasised how the learners engaged in the real-life activity of taking measurements of both the electricity box and lamp pole objects "*the same way anyone would have done to find the area of the pole to be painted*" (ST3). This implied that in the MCM mobile environment, the learners used tools in the same way they would have been used in real professions ie measuring the lengths of objects.

Further, Kamwi alluded to how his learners used the GPS function of the app to locate tasks: the process which inevitably led them to reading and interpreting the Google map that was displayed on the interface of the phone's screen (CO1). Furthermore, Kamwi's statement in Figure 6.1 above, "*they were searching for things that were academically needed*" (TO2), strongly suggests that the learners used the smartphone as a learning tool to access information on the internet.

6.2.1.3 Collaboration

Kamwi indicated that the levels of collaboration in the trails were low to moderate. The collaborations were enhanced by the learners' use of the phone to communicate with the MCM app. The learners followed the live GPS to the tasks (CO1). Further, Kamwi hinted that his learners visualised and used the image of the object in the task from the phone to help them identify the real task in their immediate environment (CO1). Furthermore, the teacher expressed that using one phone helped the learners to work together by correcting each other's mistakes through communicating and exchanging information in meaningful interactive discussions (CO3). They shared their strategies, reasoning or artifacts with each other or with the teacher. The only key indicator shared by Kamwi on co-creation in the trail was that the learners used the smartphone as an interface communication device to share their answers with the app's database system (CC4). This suggests that the learners co-constructed knowledge and created a shared understanding of the tasks through the aid of the smartphone, in the MCM mobile environment.

6.2.2 Anna

Anna’s reflections:

- It was very interesting for them, and I think they enjoyed working with the phone and the app (AG3).
- I did not tell them what to do, they decided to start with that task on their own (AG1).
- I think finding those hidden tasks was an interesting part for them (AG1; ST3).
- They managed their time very well; they did not spend much time on solving those tasks (AG4).
- The app was very realistic, or I can say [that] it was very reliable because it led us to the location of the tasks the tasks (ST1).
- Besides, it was also showing us real life things like for example Task 2 … learners were supposed to find the area of that patio (TO1).
- I think they were connected to reality because they gave learners a picture of real-life situations like in the case of a stop sign it's appearing in real life, so I think the questions were realistic to learners (TO1).
- With the phone, the learners worked as a group and shared their ideas (CO1)
- It was really a good arrangement to just use one phone because the learners worked in groups to share and develop new knowledge (CC2).
- When one learner said something, others could build new ideas on his idea. So, if it was individual work, it was not going to work (CO3).
- As they uploaded their answer in the ‘check answer’ of the app I could see they anxiously waiting for the response (CC4).

Figure 6.3: Evidence of the iPAC principles from Anna’s interview.

Table 6.2: Ratings of m-learning scenarios for Anna

Scale	*Sub-scales*		*Use of MCM app*
Personalisation	Agency	(AG)	Medium
	Customisation	(CU)	Low
Authenticity	Setting/Task	(ST)	Low
	Tool	(TO)	Low
Collaboration	Conversation	(CO)	Medium
	Co-Creation	(CC)	Medium

6.2.2.1 Personalisation

Anna reported that in her view, her learners had autonomy over their learning. She believed that the learners could do things on their own without constant supervision or guidance from the teacher (see Figure 6.3 above). The learners showed enthusiasm for participating in the trail and completing the tasks. She also noted that her learners had full ownership of the phone and mastered its functions (AG3). The learners chose which task to start with and managed their time well in the trails (AG4). Moreover, Anna mentioned that the learners knew how to use the tape measure, except for some confusion with the units. She also recalled how the learners wanted to keep working even when it was time to stop, indicating that they enjoyed the trail activities of the MCM environment.

6.2.2.2 Authenticity

The authenticity principle rated with low levels in both the settings and tools aspects within the trail. Figure 6.3 above shows how Anna appreciated the authentic and realistic contexts that the MCM app offered to learners. She said that the trail tasks and contexts matched the learners' interests and experiences. For instance, she remarked "*the app was very realistic, or I can say [that] it was very reliable because it led us to the location of the tasks*" (ST1). This statement indicated that Anna's learners participated in learning activities that connected to their everyday lives. Anna also mentioned that besides using the smartphone as a technological tool in the trail, the learners used a calculator to perform calculations similar to those of real professions.

6.2.2.3 Collaboration

Anna's RI showed that using a smartphone with the MCM app fostered a collaborative learning environment (CO3) where learners shared ideas and information with their friends and teacher

(CC2). However, Anna also expressed her concern that her learners were unhappy with the app's feedback of marking their answer to the sign board task as wrong. She said, *"…you could see that the learners were disappointed that the app said their answer was wrong… I think they expected the app to change its answer and admit that it was wrong, and they were correct.*" To me, this suggests that although the app can facilitate conversations between learners (CC4), it lacks live exchange of information. It seems that only the task creators can modify the tasks and not anyone else, especially not the learners.

6.2.3 Moses

Moses' reflections:

- …did you see that those learners used a ruler instead of the measuring tape we gave them? I think the chose their own way of measuring that staircase (AG5; ST3).
- All the tasks in our trail were linked to the learners' real-world knowledge (TO1).
- A building constructor will also do the same measurements in order to know the total number of bricks needed to make those steps (stairway) (TO1; TO3).
- The learners used the phone very well and it did not give them any problems (TO4).
- They did not misuse the phone by playing around with it unnecessarily or visiting the Facebook page… (AG3).
- The learner put the phone on silent mode because of some WhatsApp messages that were coming in (CU2).
- There was teamwork, and the learners were always on the phone and discussing (CO3; CO1).
- I think using one phone is the best, it helped the learners to work together… you know some learners work best in groups than on their own (CO3; CC2).
- The learners were discussing through the trail activities, and they were all actively involved (CC2).

Figure 6.4: Evidence of the iPAC principles from Moses's interview

Table 6.3: Ratings of m-learning scenarios of Moses

Scale	*Sub-scales*		*Use of MCM app*
Personalisation	Agency	(AG)	Low
	Customisation	(CU)	Low
Authenticity	Setting/Task	(ST)	Low
	Tool	(TO)	High
Collaboration	Conversation	(CO)	Medium
	Co-Creation	(CC)	Low

6.2.3.1 Personalisation

Moses believed that his learners showed their ability to choose how they learned in the trail by choosing a ruler instead of a tape measure to find out how many bricks were in the stair way. He said, He said, "*did you see that those learners used a ruler instead of the measuring tape we gave them? I think they chose their own way of measuring those staircase*" (AG5). In the context of the personalisation principle, what this means is that the learners exercised agency over their own learning. A ruler was not given to learners for use during the measurements of objects, and yet Moses's learners decided to use it in the place of the measuring tape. Also, using a ruler as an iterative way of measuring appeared to be an easy way of solving the problem for learners. This can be an example of customisation because it shows that the learners chose an approach that matched their prior knowledge, experience and skills, rather than challenging themselves with measuring the entire top step stair and dividing by the length of one brick. The teacher also commended the learners' ability to customise the smartphone's settings: "*The learner put the phone on silent mode because of some WhatsApp messages that were coming in*" (CU2). Moreover, in Moses's view that his learners did not visit any social media pages during the trails. This was evidence that they used the phone appropriately and for educational purposes (AG3).

6.2.3.2 Authenticity

Moses appreciated how the MCM app gave learners authentic contexts that matched their experiences and how the tasks they solved were relevant to their realities (TO1). For instance, in Figure 6.4 above, the teacher linked the stairway task to the-real-life scenario of a builder who had to estimate the number of bricks required before starting the construction. This demonstrates Moses's belief that the MCM app engaged the learners in realistic contexts where they were exposed to real life building scenarios (TO3; ST3). In other words, he saw a direct connection between what learners did and what professionals did in their actual work. He also noted that operating the smartphone was not a problem for the learners, which meant that they were familiar with its settings (TO4).

6.2.3.3 Collaboration

Moses claimed that sharing one phone among the learners on the trail fostered teamwork (CO3). He cited some examples from his trail activities to show how the learners often gathered around the phone and discussed the content related to the location of the tasks and the meaning of the questions (also refer to Figure 5.21). (CO1). He contrasted this with the scenario of each learner having their own phone and argued that using one phone was more conducive to social interactions. He said "*I think using one phone is the best… really, having many phones was going to make these learners work on their own, you know some learners work better in groups than on their own*" (CO3; CC2). Moreover, Moses noted that the MCM m-learning environment offered collaborative activities such as walking the trails, discussing the questions, and measuring the objects in the environment. He observed that these collaborations enhanced the learners' personal learning processes and motivation, as the learners were excited after getting the correct answers.

6.2.4 Sinvula

- Sinvula's reflections:They decided on their own the questions to solve and how to solve them (AG1;AG5).
- Painting is everywhere, it's something that we do as people every day, its applicable in real life (ST1;ST2; TO1).
- Learners measured the walls of the building the same way real builders use to do it (ST3), even that measuring tape was very easy for them to use (TO4), they were just like real builders (TO3).
- Even in building people also use calculators to calculate things (TO3), especially the volumes and areas of things.
- The learners were very surprised that we can calculate the age of a tree in mathematics, that was very exciting and real task for them (ST1;ST2).
- I was very surprised that the leaners could understand and follow that GPS on the phone (CO1), following properly until arriving at the task (CU1). I think it was very easy … they didn't even struggle to locate the place (TO4).
- The teamwork was very high … (CO3).
- Learners were sharing ideas on how to measure the circumference of the tree (CC2).
- Other learners were confirming the answer that appeared on the calculator (CO1).

Figure 6.5: Evidence of the iPAC principles from Sinvula's interview.

Table 6.4: Ratings of m-learning scenarios for Sinvula

Scale	*Sub-scales*		*Use of MCM app*
Personalisation	Agency	(AG)	Low
	Customisation	(CU)	Low
Authenticity	Setting/Task	(ST)	High
	Tool	(TO)	Medium
Collaboration	Conversation	(CO)	Medium
	Co-Creation	(CC)	Low

6.2.4.1 Personalisation

Table 6.4 above shows that Sinvula's RIs revealed that in his view, learners lacked agency and customisation, which means they had little control and choice over their own learning process and outcomes in the MCM m-learning environment. There was however one incident highlighted by the teacher where he said the learners chose the two tasks to solve in the trail and how to solve them (AG1). Apart from that Sinvula shared how the learners seemed not to be actively involved, and that he often had to push them to increase their working pace as they appeared to be slow. Sinvula reported that his learners were however fascinated by how the smartphone enabled them to tour their environment in the quest of finding the hidden tasks (CU1).

6.2.4.2 Authenticity

To the question of how the MCM app project presented authentic and realistic contexts to learners, Sinvula responded that the app created settings that were realistic to learners. He said that the learners could relate both the painting and the age of the tree tasks to their own realities and the community they lived in (ST1; ST2). Also, the measuring tape used replicated the actual measuring tape used in actual building and construction projects. The learners used the tool the same way a real builder or constructor would have (TO3).

6.2.4.3 Collaborations

Sinvula shared that there were medium to low collaborations created by the MCM app in his trail. Figure 6.5 above shows three statements that represent the observable indicators of learners conversing with each other over the information (Google Map and GPS) displayed on the smartphone screen (CO1) and how the app created a shared and socially interactive environment for learners where they discussed and shared ideas (CO1; CC2) pertaining to the suitable strategies for solving the tasks. For example, the learners discussed how and where to measure the circumference of the tree, whether on the bottom of the stump or in the middle part. The discussions also included the use of verbal and non-verbal cues to coordinate their actions and demonstrate to each other and the teacher the concepts of shaded and unshaded areas (the wall and windows).

6.2.5 Calvin

Calvin's reflections:

- The learners were even measuring other objects on their own trying and exercising some questions they can ask and how solve them (AG2).
- They searched for information of how to calculate the volume of a cylinder (AG2;. TO2)
- The learner who was responsible of operating the phone, made sure that the phone was always on silent mode and that it didn't dim off… (CU2) the guy was even now and then changing the brightness of the phone depending on the sun (CU2).
- There was no need to have a calculator because the phone had everything learners needed, the questions were in the phone, the calculator, even the information, a phone is like a textbook of its own (TO3; AG3).
- This app can even take my place as a teacher, because most of the time the learners worked on their own without me (AG1;. AG3)
- The learners talked and listened to each other (CO3).
- They were sharing information on how to solve the problems (CC2).
- In our town there are a lot of sewages and dustbins all around, so these questions were very relevant to learners (TO1; ST2).
- You know what makes these tasks interactive is how the learners were actively engaged in activities like measuring and calculating the areas of the shapes (TO3; CO3).
- The learners debated on the meaning of the questions from the app (CO1).

Figure 6.6: Evidence of the iPAC principles from Calvin's interview.

Table 6.5: Ratings of m-learning scenarios for Calvin

Scale	*Sub-scales*		*Use of MCM app*
Personalisation	Agency	(AG)	Medium
	Customisation	(CU)	Low
Authenticity	Setting/Task	(ST)	Low
	Tool	(TO)	High
Collaboration	Conversation	(CO)	Medium
	Co-Creation	(CC)	Low

6.2.5.1 Personalisation

Calvin's views in Figure 6.6 above suggest that the learners had low to medium levels of agency and customisation, respectively. The teacher claimed that his learners were in control of the mobile device they were using in the trail. The learners used the smartphone easily to locate the tasks in the trails as well as find solutions to the same tasks (CU1). The figure also presents evidence (eg TO3; AG3) that shows that the phone was more than just a mobile device in the hands of the learners. According to Calvin, the smartphone served many educational functions such as a calculator, an information hub and even the potential to substitute the teacher in the trail activities (AG2). In this regard it is interesting to note that Calvin's learners tailored the smartphone to suit their own personal preferences such as changing the settings, appearance, apps and content (CU2).

6.2.5.2 Authenticity

Calvin observed that the MCM app provided authentic contexts and tools that replicated real-life practices. Calvin commented that the dustbin and sewage tasks given to learners through the app's system were very familiar and relevant to learners (ST2). He reflected on how these objects are all over the immediate environment of the learners and that the learners understood their importance to the people of the community (TO1) very well. Thus, to him, knowing how to calculate the volumes and capacities of these objects is important in mathematics. Moreover, Calvin acknowledged that a tool like a measuring tape was efficiently used by the learners to carry out measurements without difficulty, a feature that means the learners used the tool in a professional manner (TO3).

6.2.5.3 Collaboration

In Figure 6.6 above, Calvin highlighted how his learners collaboratively worked as a group by giving each other chances to talk and listen to each other (CO3). This shows that the learners engaged in active conversations that were created by the MCM app m-learning environment. Also, the learners relied on each other and shared their perspectives on how to solve and complete the tasks (CC2). Furthermore, Calvin also hinted that the learners supported their arguments and opinions on how they understood the questions as they appeared from the MCM app on the smartphone screen (CO1) after reading the tasks, which means that they exchanged ideas and reached common understandings. This view agrees with what was observed in Calvin's VR (CV_5) that the learners demonstrated maturity in their discussion as they did not argue or talk over each other. This means that they respected and positively responded to each other's contributions and suggestions.

6.2.6 Luke

Luke's reflections:

- The learners like yesterday learners were taking the actual measurements and also recording them down as notes (ST;: TO1).
- They used the measuring tape very well; I mean they know how to take measurements (TO3).
- Those two tasks were real to learners (TO1).
- On the lawn task they were even talking about real containers… (ST1).
- There was division of labour, and the learners were really working together (CO3).
- They understood how the app worked and they were also very familiar with that phone, a Samsung is not like an iPhone, it's very easy to use (AG3;TO4).
- They had a choice of choosing the questions they wanted to do in the trail … (AG1).
- They discussed very well because they were listening to each other all the time (CC2; CO3).
- The task of watering the garden was very relevant to learners because it taught them not to waste water (ST2).
- The GPS was very interesting, it accurately directed learners to those tasks without getting lost (CU1).

Figure 6.7: Evidence of the iPAC principles from Luke's interview.

Table 6.6: Ratings of m-learning scenarios for Luke

Scale	*Sub-scales*		*Use of MCM app*
Personalisation	Agency	(AG)	Low
	Customisation	(CU)	Low
Authenticity	Setting/Task	(ST)	High
	Tool	(TO)	High
Collaboration	Conversation	(CO)	Low
	Co-Creation	(CC)	Low

6.2.6.1 Personalisation

Luke's RIs suggests that the learners had low levels of agency and customisation in their own learning. Apart from choosing the tasks to do in the trail and the task to start with (AG1) it appears that the learners did not have much control over the other aspects of agency in the trail.

The teacher also added that the customisation subconstruct seemed to have only happened when the learners used the GPS feature of the MCM app to locate the two tasks (CU1).

6.2.6.2 Authenticity

Luke observed that the learners' participation in realistic contexts, such as solving the lawn bed task, fostered the authenticity principle according to the iPAC framework. The task required the learners to calculate the amount of water needed to irrigate the lawn (ST3), which helped them relate it to the real-world advocacy of water conservation (ST2). Luke also noted that the learners collected data through measurements and note taking, mirroring the practice of field work experts (ST3; TO3). The learners used tools such as a tape measure correctly to obtain accurate and reliable data. Furthermore, the teacher noted the ability of his learners to handle the smartphone and understand the MCM app … "*they understood how the app worked and they were also very familiar with that phone, a Samsung is not like an iPhone, it's very easy to use*" (AG3), remarked Luke.

6.2.6.3 Collaboration

Luke observed that the trails had low levels of collaboration in both the conversations and co-creation subconstructs. However, he also noted that the smartphone use enhanced the teamwork in the trail (CO1) as learners communicated (CC2) about the task locations by using the app's Google map. When asked how one smartphone facilitated data sharing among learners, Luke said that they engaged in product conversations where they listened and spoke to each other, taking turns (CO3). Luke believed that this helped learners share and build on each other's ideas and opinions.

6.2.7 Betty

Betty's reflections:

- Learners chose their own methods without my interference (AG5).
- They took the correct measurements (TO1;ST3)), solved the problem, uploaded the answer (CC4) and got a positive response from the app (CU3) and their answer was correct without any help from the teacher (AG2).
- The learners decided on their own which task to start with (AG1).
- They spent as much time as they wanted (AG4).
- They agreed on their own on who should do the measurements, operate the calculator, and carry the phone (AG3).
- It was very easy for them to find the tasks with the help of the GPS of the phone (ST3).
- The way they were using that phone, they were very comfortable, it's like they knew what they were doing (CU2;TO4).
- The picture of the object in the phone really made it easy for learners to identify the object to be measured (CO1)
- The learners were really working together (CO3).
- That one smartphone we were having really bring them together (CO3) because they were like looking at the task together (CO1) and they have to work together to find the solution (CC2).
- The learners always agreed on the same answer before uploading it (CC4).
- When teach learners that volume is the amount of space occupied by an object, it becomes easy for them to understand because they can see what you are talking about (TO1).
- The platform brought us together [teachers and learners] where we can learn mathematics in our environment (CO3).
- The learners can share their answers on the app (CC4; CC2), and other people as far as Germany can be able to see their answers.

Figure 6.8: Evidence of the iPAC principles from Betty's interview.

Table 6.7: Ratings of m-learning scenarios for Betty.

Scale	*Sub-scales*		*Use of MCM app*
Personalisation	Agency	(AG)	High
	Customisation	(CU)	Medium
Authenticity	Setting/Task	(ST)	Low
	Tool	(TO)	Medium
Collaboration	Conversation	(CO)	Medium
	Co-Creation	(CC)	Medium

6.2.7.1 Personalisation

Betty rated the personalisation construct of the iPAC framework as medium for customisation and high for agency. Her responses in the RI with me suggests that her learners enjoyed a high level of agency because she allowed them to have some control and choice over their own learning, by choosing their preferred methods and approaches to solving the tasks (AG5). Betty alluded to her observation that learners managed to get answers on their own with minimal to no help from her (AG2). Furthermore, Betty revealed that before the start of the trail, her learners (on their own) decided to choose the two tasks they were going to solve as well as the order to solve the tasks (AG1). In addition to this, the teacher gave the learners an opportunity to choose and share their roles in the trail activities, although it appeared that everyone wanted to be in charge of the smartphone (AG3). The high level of agency was also identified by Betty's comment that her learners spent as much time as they wanted on the tasks without her reminding them about time management. In effect, this means that the learners had control of their own learning pace on the trail of the MCM m-learning environment.

According to Betty, the medium customisation level of the personalisation construct in the trail appeared to be as a result of the automated feedback they received from the app's data base after the entering of their answer (CU3). Furthermore, Betty noted that her learners used the smartphone with ease. They also tailored the phone and app's settings to preferences that were easier for them to operate (CU2).

6.2.7.2 Authenticity

The settings where the tasks were contextualised were found to be meaningful to learners. Betty mentioned that when she explained the concept of volume to learners, it was easy for them to understand her because they could physically see the object that represented the concept (TO1). This suggests that the MCM app presented realistic contexts to the learners and that her learners could see and appreciate the relevance and meaningfulness of the task as it appealed to their interests, backgrounds and experiences. Further, when asked about how she felt about the learners' comfort with using the phone in the trails, Betty was quick to say that her learners had no problem with the operations of the phone. It appeared to her that the learners were familiar with the applications of the phone and found it easy to operate on their own. Betty also stressed that the learners carried out precise and correct measurements (TO1), which can mean that they used the measuring tape (ST3) in the same way a real professional, like a builder for example, would have done.

6.2.7.3 Collaboration

Betty observed that the learners' interactions with each other on the trail were moderate for two reasons. Firstly, she noticed that some learners were not as engaged as they should have been, so she encouraged them to join what others were doing. Secondly, as shown in some presented transcripts in Figure 6.8 above, the teacher was also impressed by how using only one phone fostered teamwork on the trail (CO3). Betty attributed the sharing of one phone to the increased collaboration and communication among the learners as they solved the tasks. The medium level of the co-creation subconstruct resulted from sharing and communicating between her learners, the teacher, the app's virtual interface and other people (CC2). Betty appreciated that other people could virtually access and view the same tasks that her learners worked on (ST1).

6.2.8 Joshua

Joshua's reflections:

- The learners were able to insert their answers in the app and get the feedback right on the spot (CC4).
- It was easy, and learners were happy… they were just on their own and free unlike the classroom where they're will just be seated and answering questions from the teacher (AG1;AG3).
- The learners used the phone to search for information (CU3;TO2).
- The questions were real to learners (ST2).
- I think their measurements were fine that's why they got the correct answers (TO1; TO3).
- I did not tell the where to start, but they just randomly chose the question to start with (AG1).
- It was not difficult for them to find the tasks because they just followed the GPS arrow (CO1: AG3; ST3)
- learners can do everything on the phone, they can write on it (CC1), so they don't need a pen or paper, they can take pictures (CC1), they don't need a camera, they can also calculate using the calculator in the phone (AG3)
- they did not face any technical problems with the phone and the internet was just fine (AG3: TO4)

Figure 6.9: Evidence of the iPAC principles from Joshua's interview.

Table 6.8: Ratings of m-learning scenarios for Joshua

Scale	*Sub-scales*		*Use of MCM app*
Personalisation	Agency	(AG)	Medium
	Customisation	(CU)	Low
Authenticity	Setting/Task	(ST)	Medium
	Tool	(TO)	High
Collaboration	Conversation	(CO)	Low
	Co-Creation	(CC)	Medium

6.2.8.1 Personalisation

Joshua's RIs suggest that the learners had low to medium levels of agency and customisation in their own learning. As shown in Figure 6.9 above, it is stated that the learners found the app easy to use and that they enjoyed more freedom outdoors than in the classroom. However, it is unclear whether this freedom was related to their educational experiences. Joshua also highlighted that using the MCM app enabled the learners to interact with their environment by choosing where to go (AG1) and how to solve the problem tasks in their own ways. The learners' happiness and excitement about the app and the phone indicate that their personalisation levels were enhanced, although not much, as shown in Table 6.8 above.

6.2.8.2 Authenticity

The teacher believed that the MCM app provided realistic contexts and tasks for learners. He said that the GPS function guided the learners to real objects in their environment (ST3;CU1) where they performed activities that simulated real-life problems and help them to image how they would solve them (TO1;TO3). The activities involved taking actual measurements and doing calculations on volume in relation to real life situations (ST1). Joshua also reported that the learners had no difficulty using the smartphone and the measuring tape, and that they enjoyed working with the device in the MCM context.

6.2.8.3 Collaboration

Similar to the other teacher participants, Joshua also appreciated the collaborations that were initiated by the MCM app. He said that the app enabled his learners to interact and share their answers with others. He added that "*the learners could enter their answers in the app and receive immediate feedback*". This was an indication that the learners also interacted with the app's system to share and receive digital content in respect of their quest to solve the tasks (CC4). Moreover, Joshua also mentioned the multifunctionality of the smartphone that learners could have utilized, such as a writing pad, a camera and a calculator. However, he clarified that his learners did not actually use these functions, but he was just pointing out the potential benefits of them.

6.3 HORIZONTAL ANALYSIS ACROSS PARTICIPANTS

The teacher participants' views and experiences of creating and using the MCM-based trail tasks were grouped according to each subconstruct of the iPAC (Personalisation, Authenticity, Collaboration) mobile framework's main principles. The aim was to explore how the MCM m-learning environment supported the three pedagogical principles of the iPAC mobile framework in the study.

6.3.1 The personalisation construct

In terms of personalisation, the study revealed that the use of smartphones to explore the MCM app offered low to moderate levels of agency and customisation to learners.

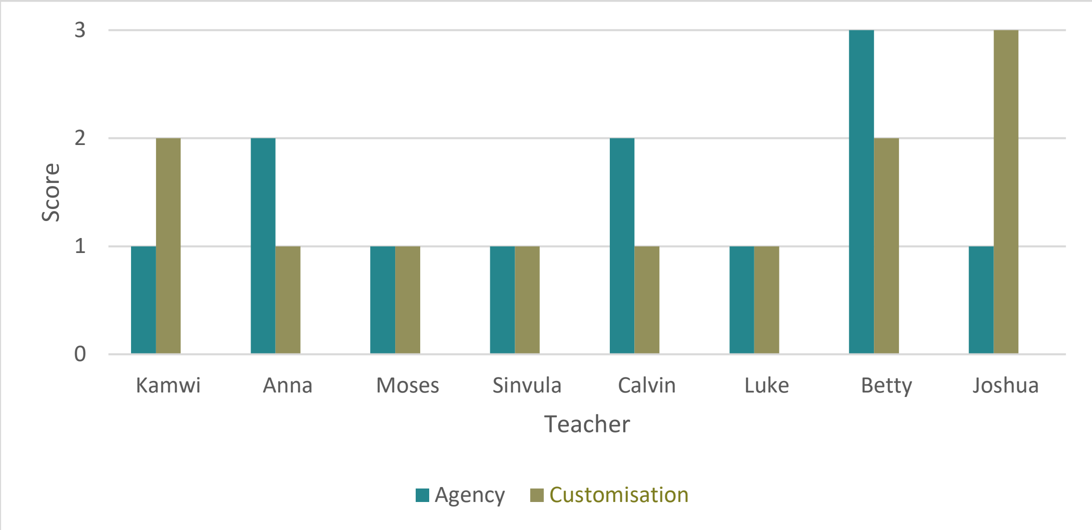


Figure 6.10: Numerical scores showing the emergence of the personalisation construct in the trails.

Figure 6.10 above shows that out of the eight teacher participants, five of them viewed the agency of learners as low in the trails. Two thought that the learners' agency was moderate and only one perceived her learners to have enjoyed high levels of agency in the study. The teachers' general views were that the learners had limited to moderate choices and control over the tasks to solve and the order of solving them (AG1). For example, Anna, Sinvula and Luke had the following to say regarding the learners' choices in the trails:

Anna: I did not tell them what to do, they decided to start with that task on their own.
Sinvula: They decided on their own the questions to solve and how to solve them.
Luke: They had a choice of choosing the questions they wanted to do in the trail.

Figure 6.11: Some teachers' statements regarding the learners' choices in their trails

The three teachers' views in Figure 6.11 suggests that from the tasks that were contained in the trails of this study, learners had a choice of selecting the tasks they preferred to solve and left out the ones they did not want to do. This could also mean that the learners' choices were influenced by the level of difficulty of the tasks, where the learners preferred to work on the tasks that they thought were easier for them to solve. Perhaps this could be the primary reason why most of the tasks solved by the learners in the trails were tasks based on the topics of area and volume rather than ratio and proportion. On the other hand, in terms of agency, this can also mean that by selecting the area and volume tasks over the ratio and proportion ones, the learners demonstrated autonomy over their learning content.

It is noteworthy to point out that the observable indicator of the learners' choice in the content of learning on the trails was not explicitly mentioned by any teacher (CU1). This could mean that the smartphone device did not provide learners with opportunities of being in control over the learning content in the mathematics trails. Perhaps, this was because the learning content of the MCM trail tasks can only be created and modified by the task creators who are the teachers and researchers, thus, the learners had to strictly follow the sequence of activities as was designed by their teachers. As was reported earlier in the previous section, Anna shared her experience on how an answer wrongly entered in the app's data base could not be altered and how this left her learners disappointed as the app's feedback indicated that they were wrong, when in actual fact they were correct.

Further, the findings also revealed that the learners had control over the timing and pace of the trail activities. For example, Anna shared that "*they* [learners] *managed their time very well; they did not spend much time on solving those tasks*". On the one hand, this means that the learners had a sense of urgency and challenge, which ultimately enhanced their engagement and interest to work efficiently within a limited period time. On the other hand, a statement from Luke: *"... they [learners]spent as much time as they wanted"* could mean that some groups of learners were not conscious of how much time they spent in solving the tasks. This shows that learners were flexible in choosing how to manage their time in the trails. The learners had options to extend or shorten the time limit according to their needs.

The RIs revealed that in terms of the customisation sub construct, the smartphone tool offered little to no opportunities for learners to adjust the settings of their device. Only Joshua, out of the eight teachers, said that his learners personalised their learning by changing the volume and the screen brightness (AG3). The other teachers, particularly Kamwi and Betty, did not think that their learners customised their learning experience according to their preferences and needs. The only common customisation feature that most of the participants mentioned was how the app let the learners upload their answers on the MCM's database and receive immediate feedback on their answers.

The learners personalised their learning by using their smartphones to find formulas on the internet that matched the shapes they were calculating. For example, Calvin, Joshua and

Kamwi noted that this was a way of customising their learning and making it more personal, as they learned something new and unplanned while doing the trail tasks. According to Suputra et al. (2018), smartphones are ideal tools for accessing information from the internet. Mobile devices not only enable communication, but also make it easy to find information. As Sung et al. (2016) point out, mobile devices are a powerful tool for learning both indoors and outdoors. They enhanced learning and enabled them to access important information in different locations and in more appealing ways. This also supports Norris and Soloway's (2011; 2015) view that in m-learning settings, teachers are not the source of information, but learners can access information by themselves. Kamwi's remark that he was not needed in the second task because the learners solved it on their own without his assistance demonstrated this notion in this study.

6.3.2 The authenticity construct

Authentic learning was evident in the participants' mobile practices in various ways – see Figure 6.12 below.

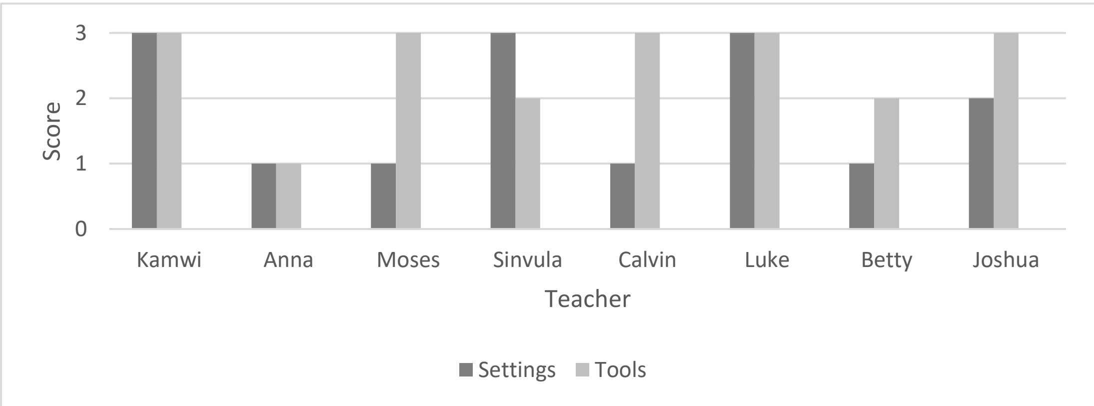


Figure 6.12: Numerical scores of the emergence of the authenticity construct in the trails.

The MCM app provided the learners with in-situ realistic contexts where they worked in similar ways to real practitioners like builders and constructors to collect measurements of physical objects and calculate their areas and volumes. For example, Figure 6.13 below provides evidence of the teachers' appreciation of the authenticity of the MCM app to learners' realities:

Anna: The app was very realistic, or I can say [that] it was very reliable because it led us to the location of the tasks the tasks.

Joshua: The questions were real to learners.

Calvin: In our town there are a lot of sewages and dustbins all around, so these questions were very relevant to learners.

Moses: All the tasks in our trail were linked to the learners' real-world knowledge.

Kamwi: With the app, the learners solved real problems, like the painting question... something that happens in real life.

Figure 6.13: Some teachers' statements regarding the learners' choices in their trails.

In line with these views, Kearney et al. (2019) also noted that using smartphones in the learning of mathematics enhances authentic real-world and cultural settings, which in turn can create opportunities for learners to personalise and have autonomy over their learning. In this study, smartphones supported the learners in locating the authentic tasks that were contextualised in realistic settings within their environment. Furthermore, the learners used their smartphones to check whether the solutions to the tasks were correct or not. In this way, they were able to be involved in rich and contextualised tasks that focused on the concepts of area, volume, ratio and proportion. According to MacDonald et al. (2020), mathematics is a STEM subject that should encourage learners to explore, think about and solve problems that are relevant and meaningful to their lives, instead of giving them tasks that are abstract or disconnected from reality. So, the teachers in this study believed that their learners used smartphones in a genuine way to relate to realistic problems and solve them.

In addition to the above benefits, the teachers also commended the MCM m-learning environment for allowing learners to interact with actual tools (tape measures and calculators) that were used to measure and compute the area and volume of objects in the tasks, just as real practitioners could have used them in similar situations. For instance, Sinvula, Kamwi, Moses and Luke were impressed by how the learners measured the wall and windows of the building they were working on with the tape measure, just as real builders would do. Based on this finding, I can therefore claim that learners who used measuring tapes to measure and calculate the spatial dimensions of various objects and shapes in this study acquired practical skills that are useful for their future careers or daily lives. Furthermore, these real-world tools could have possibly enhanced the learners' critical thinking and problem-solving skills by requiring them to apply their knowledge to real scenarios.

Moreover, the teachers also observed that their learners were comfortable using smartphones. Luke suggested that this might have been because the Samsung smartphone model that the learners used on the trail (along with other learners and their teachers) was more user-friendly than other mobile devices such as iPhones, for example. Rashid and Elder (2009) state that smartphones only need basic literacy, which makes them easy to operate. Similarly, the teachers in this study considered the smartphones used in the trails to have been simple for learners to use. This finding is consistent with studies (eg Eneje, 2021; Simonova et al., 2015) that claimed that the learners' familiarity with the mobile device they use for learning influences how much they can adjust most features and enjoy their practice and learning. Some of the learners in this study were able to easily customise the settings of the smartphones they were using according to their preferences because they knew how to use the devices.

6.3.3 The collaboration construct

The MCM app project enabled high-quality conversations in the m-learning activities.

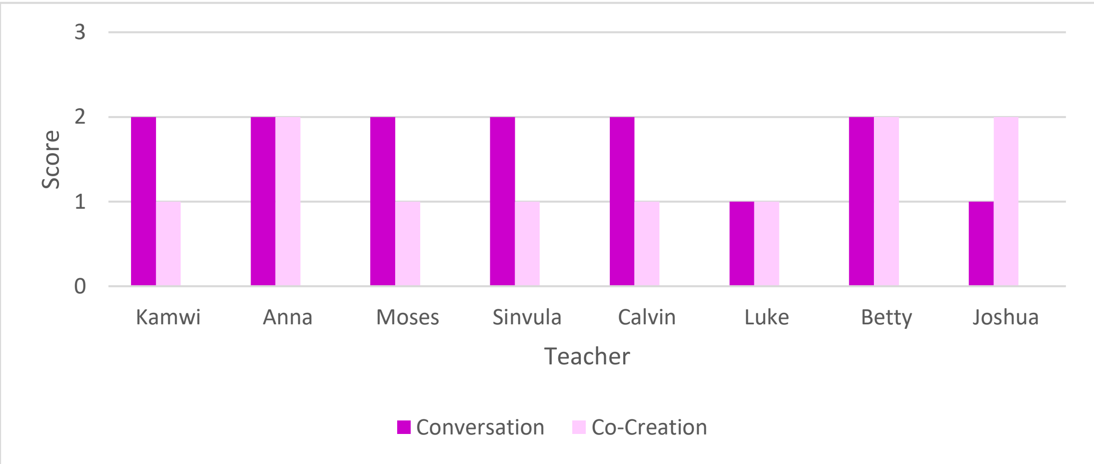


Figure 6.14: Numerical scores of the emergence of the collaboration construct in the trails.

As Figure 6.14 above shows, most of the teachers rated the conversation subconstruct highly, except for Luke and Joshua who gave low ratings. The teachers reported that the MCM trails, which used smartphones as mobile tools, allowed their learners to talk with their peers and teachers. The conversations were mainly about the MCM app information that appeared on the smartphone screens. For example, Kamwi said that "*the learners compared the visual image they saw on the phone with the real object*". Betty also said that "*the picture of the object in the phone really made it easy for learners to identify the object to be measured*". Sinvula agreed that the learners discussed the MCM Google map information that was displayed on the smartphone screens. The teachers felt that the app created a social and interactive learning environment where learners shared and exchanged ideas on how to solve realistic problems that were presented in the trails. Therefore, the smartphone screens provided a visual guide that helped the learners' conversations in finding and solving the tasks.

Moreover, the teachers reported that the discussions were conducted in a respectful and collaborative manner, where the learners valued each other's ideas and opinions. For example, Luke said that the learners participated in productive conversations, taking turns to listen and speak, which helped them to exchange and build on their perspectives. This aligns with Baharom's (2013) findings that m-learning environments create opportunities for learners to share and engage in meaningful learning that enables them to reflect on their understanding, refine their knowledge and reach common ground on the content.

This study also showed that the MCM m-learning environment supports the principle of Gulikers et al. (2004a; 2004b; 2006) that learners should work together as a team in realistic contexts, just as they would in real-life workplaces. The teachers shared that the learners' cooperation in solving the MCM tasks demonstrated that they had the ability to collaborate with their colleagues or workmates in a real-work environment. One example of how the learners collaborated was given by Betty and Luke, who reported that their learners assigned different roles to themselves. This was consistent with Barbosa and Vale's (2020) observation that the MCM mathematics trails naturally encouraged group work and sharing of responsibilities, such as using the smartphone, measuring, recording data, calculating or working together towards a common goal. Therefore, the authentic tasks and situations of this study mirrored the social processes of the assessment that matched the social processes in the reality of a similar situation.

On the other hand, it is worth noting that none of the teachers mentioned the observable key indicator of CC1, which involved learners working together to create a digital product using a smartphone. I suggest that this implies that smartphones in the context of the MCM m-learning offered little or no opportunities for online peer discussions or sharing and exchanging digital content. This could be due to the design of the app that only allows users to upload the content and interact with the reviewers of the tasks. On the other hand, Joshua highlighted the multifunctionalities of smartphones, such as writing pad, camera and calculator, and how learners could have used these (particularly the camera) to explore the pedagogical benefits of co-creating the learning of area and volume concepts. However, the learners in this study did not seize this potential advantage of using smartphones. Joshua's views are in line with the argument made by Kearney et al. (2019) that learners can work together using mobile devices (in the case of this study, smartphones) to create digital content and exchange information, data and artifacts.

6.4 EMERGING THEMES – FURTHER INSIGHTS

In this section, I present and discuss the themes that emerged from the data sources (VRs, RIs, and FGIs) of all eight teacher participants of this study to seek further insights to the answering of the main research question of: *In the context of a mobile learning environment, how can teachers implement authentic tasks in a mathematics trail for conceptual understanding of selected mathematics concepts.* I identified the following themes that were not covered in the vignettes discussed above but are important for my participants' sense-making process. The themes were: 1) Visualisation aspects in the study; and 2) The teachers' views on the challenges of the MCM app project and teaching outdoors.

6.4.1 The visualisation aspects in the study

6.4.1.1 Visualising with sketches

The first major theme that emerged from all the data sources of this study was that of the visualisation process. The teachers encouraged their learners to use sketches as visualisation tools to understand the concepts of area and volume. For example, Anna used the octagon sketch to make the properties of a trapezium to her learners and show how the properties lead to the formula for its area: $\frac{1}{2} \times (a + b) \times h$ (refer to Figure 5.15). The learners drew the shape of the octagonal sign board to represent the dimensions and measurements of the geometrical shapes they were working on. From the sketch, the learners were able to divide the octagon into three shapes whose areas they already knew. In this way, they could easily visualise and remember the formulas of the involved shapes. Moses' learners also modelled the net of the cylindrical dustbin by sketching the net on the soil and the paper (refer to Figure 5.25). This helped the learners to clearly see and understand the abstract reasoning for the formula that involved only two shapes: a circle and a rectangle ($\pi r^2 + 2\pi rh$), instead of the complete formula of $2\pi r(r + h)$ – consisting of all three faces of the net of a cylinder (two circles and one rectangle).

In addition, the SV_4 showed that Sinvula encouraged his learners to sketch the object task they were working with (refer to Figure 5.34). The sketch scaffolded the discussions about the concepts of shaded and unshaded areas. I clearly observed that the learners could easily visualise the area to be painted on the actual wall in relation to their sketch on the paper. The learners could also relate the geometrical shapes they learned in the classroom to the actual building.

What was commonly observed from the solving of the trail tasks is that in order to find the areas and volumes of the object tasks, the learners sketched the shapes and labelled their dimensions. Then they used the formulas for the areas of the shapes by determining and substituting the respective dimensions in the formulas ie lengths, widths, heights, parallel sides and so on. In some instances, the learners divided complicated shapes into simpler shapes, such as triangles, rectangles and trapeziums, and found their areas separately. In this way, they could easily see how the formulas for the areas of the shapes were derived from the formulas for the areas of other shapes.

The above reported aspects of visualisation processes that emerged from the use of sketches in this study are consistent with Waisel et al. (2008), who suggested that using sketches as visual representations enhances learners' communication skills and refines their mental thinking. The mere sight of a wall, for example is likely to elicit the formation of concrete imagery (pictures-in-the-mind) of the geometrical shape that is related to it.

According to Duval (2014), the learners' cognitive processes can be activated by what they see and hear to match all possible representations of the physical objects to the concepts of related geometrical shapes. As suggested by Fabian et al. (2018), the teachers in this study engaged their learners in productive thinking by asking them questions about how they viewed mathematical ideas in the sketches they had drawn. In this way, the learners could communicate their thinking more clearly and support their reasoning more convincingly by using the visual artifacts of sketches as evidence of the mathematical concepts of area and volume.

6.4.1.2 Visualising with smartphones

Another aspect of visualisation that emerged in this study is visualisation initiated by using smartphones. Firstly, throughout the trails, the learners used the smartphones' live GPS to walk the trails and connect to the physical objects in their environment. Secondly, the images from the phones helped the learners to compare and explore the physical images within their environment. During the RIs, some teachers confirmed this observation with the following remarks – see Figure 6.15 below:

Kamwi: The learners compared the visual image they saw on the phone with the real object.

Kamwi: This phone became their guide, and they followed the directions as they appeared on the map from the phone.

Sinvula: I was very surprised that the leaners could understand and follow that GPS on the phone.

Figure 6.15: Statements that show how smartphones aided visualisation on the trails.

The statements in Figure 6.15 above indicate that the learners visualised and used the images of the object tasks that were displayed on their smartphones to locate and identify the physical objects of the tasks in their immediate environment. The video data sources also confirmed this finding of the learners using the maps and GPS function to find the hidden tasks on the trails. For example, the excerpt from AV_2 (refer to Figure 5.14) shows that Anna questioned her learners to check their knowledge of interpreting the MCM Google maps and how to use the GPS to find the tasks in their environment. In all the trail walks of this study, it was clearly observed that all the other teachers also encouraged their learners to use the Google maps and live GPS features to navigate their environment. Therefore, similar to Vale and Barbosa's (2020) ideas on the use of maps and GPS in mathematics, it is likely that in this study, during the process of locating the tasks with the help of the GPS and needing to recognise their position in space, the learners' spatial orientation was positively developed.

6.4.1.3 Visualisation and photographs

The photograph feature of the MCM app also enhanced the learners' visualisation of the object tasks they were working on. For example, during his RI session, Kamwi hinted that his learners visualised and used the image of the object task from the phone to help them identify the real object in their immediate environment: "*The learners compared the visual image they saw on the phone with the real object*". During the trail, Anna was also heard telling her learners how they could use the image of the object to locate the actual object on the trail: " … *and when we get there you should click here to see the image of the task so that you know exactly what you are looking for*" (AV_2). In line with Szucs et al. (2013), these activities fostered the learners' visual-spatial memories and visualisation abilities by showing visual images on their smartphone screens in order to interact with the physical world.

Many other instances emerged from the trail activities of this study where the learners used what Vale and Barbosa (2020) term 'mathematical 'photography' or 'problem 'pictures' to illustrate a situation with one or more questions, or a problem related to the photograph's context. According to Bragg and Nicol (2011), image-based questions in mathematics can arouse learners' interest in solving the questions, and also make them more involved in the problem-solving process. I therefore argue that in this study, the use of problem pictures connected the learners to their environment and enhanced their spatial navigation skills.

The visualisation in the trails can also be compared to Arcavi's (2003) who talked about mathematical visualisation in a more symbolic and profound way, as seeing the invisible, not only what is visible but also what is hidden and becomes a tool for learners to learn mathematics. So, in this study, as the learners walked the trails, it was probable that they started to see the objects in their environment with a mathematical view that helped them appreciate the relevance of these objects in their environment. Betty confirmed this observation in her RIs when she said: "*When you teach learners that volume is the amount of space occupied by an object, it becomes easy for them to understand because they can see what you are talking about*". Perhaps for the first time, the learners could see for themselves, the mathematics that was embedded in the objects that existed in their environment. This aligns with Vale and Barbosa's (2020) view that using photos or images in mathematics can help teachers and learners use the real world as a source and context for developing mathematical thinking and problem-solving skills, as well as understanding what learners focus on visually. This was the case in this study, where the MCM images helped the learners learn mathematics better by making them pay more attention to the hidden math in everyday objects and situations.

6.4.2 The teachers' views on the challenges of teaching outdoors

The teachers' highlighted some challenges that were associated with the design, walk and solving of the MCM trail tasks in this study. Firstly, the challenge of time limitation was cited by Moses as follows: "*We only solved two questions out of six in the trail, and yet it took us more than an hour, imagine how much time it was going to be if we were to do them all.*" Anna also added that learners spent too much time on one task and that the teachers could not rush them because they wanted to see whether they would manage to get the correct answers or not. The video observations confirm that the time spent on the trails ranged from 40 minutes to an hour and 20 minutes for solving only two trail tasks. For example, the teacher participants Kamwi, Moses and Sinvula spent an hour and more on the trails with their learners.

Secondly, designing engaging and meaningful tasks that challenged the mathematical thinking of learners appeared to be a challenge. For example, during the design of the tasks in the first and second trails, the teachers struggled to strike a balance between making the tasks too easy or too hard for learners. This was also confirmed by Kamwi during the FGI, when responding to the question of why he thought his learners failed to solve the first task. He remarked: "*I think the first question was just too difficult for them*"; meaning that the pole task was a challenge for learners. On the other, Kamwi felt that the second question was too easy for the learners because of how they solved it on their own without seeking his help. Anna and Moses were also of the opinion that the questions solved by his learners were not so easy for them – see Figure 6.15 below.

Moses: "The task of calculating the surface area the dustbin was not easy for these learners"

Anna: "I think the patio task was easier than the stop sign, learners really struggled to solve that question

Figure 6.16: Statements suggesting that the tasks were of low cognitive demand for the learners

The teachers' perceptions on the degree of challenge of the tasks shows that some tasks were too easy for learners while others were difficult. This finding aligns with that of Barbosa and Vale (2016) who observed that the design of the tasks of the MCM app are not an easy process at different levels, particularly from the point of view of the mathematical knowledge involved, whether in the degree of challenge or in the diversity of the nature of the tasks. Also, the diversity of the nature of the tasks was difficult to determine. For example, it is noticeable that there were fewer tasks based on ratio and proportion compared to the ones on spatial measurements in all eight trails created for this study. This was because the participant teachers, together with the researcher, found it difficult to create tasks on the topics of ratio and proportion. Also, it was noted how task reviewers were inconsistent with their recommendations on questions that involved answers with intervals. Joshua complained, as evidenced in Figure 6.16:

It was a challenge coming up with the answers for the questions, especially the intervals story. The people were telling us two different things, Okutja *(like)* we should use 3 to 7 %, and another one again will say 3 to 5%. And calculating these limits is not an easy job.

FGI

Figure 6.17: Joshua's sentiments on the inconsistency of the task reviewers on the variation of the interval tasks.

What Joshua was referring to here was that on some occasions, a task reviewer would recommend a variation of between ±3% and ±7%. On the other hand, another reviewer would recommend a ±3% and ±5% deviation. For example, Figure 6.17 below shows varying responses from two different reviews on the variation percentage of the interval answers.

Reviewers message (Philipp Larmann: larmann@math.uni-frankfurt.de):
Hello [redacted]

Thank you for creating this task. However, before we can publish it, you would need to make a few adjustments.
We still have the following points to note:

- Answer format and solution (Interval too generous):
Please revise your solution interval. Since the required data is relatively easy to collect from the object, you can reduce both the orange and the green interval. Consider which measurement errors could occur during data collection or which are still acceptable. Adjust the interval limits accordingly. General rule for medium sized objects: Deviation of +-3% per measured value for green interval and +-7% per measured value for orange interval.

Please revise the task with regard to the points of criticism mentioned and resubmit the request for publication.

Thank you and best regards
Philipp Larmann

Reviewers message (Sina Wetzel: wetzel@math.uni-frankfurt.de):
Dear [redacted]

thanks for creating this nice task. However, we cannot publish it yet as your interval is quite small, usually it should account for more variation like 3% for green interval and 5% for yellow intervall (of the input values, not of the result).

Best,
Sina Wetzel

Figure 6.18: Screenshots of two varying responses from two different reviewers

Thirdly, the creation of the tasks demanded more time and effort from the teachers. Data from both the RIs and FGI shows that the teachers found it difficult to design the tasks because of their need for planning, preparation and coordination. For example, during the FGI Anna stated that the tasks took time to create, especially that they had to use the actual measurements of the objects to create answers to the tasks. Other studies (eg Jones & Pepin, 2016; Gurjanow et al., 2017) found that creating mathematics trails can be challenging for teachers, notwithstanding the fact that trails can now be created using the latest MCM technology, however the process itself remains a challenge for the task designers. More time and effort are still needed to create the tasks for the trails.

Fourthly, some of the teacher participants were of the view that incorporating MCM trail tasks in the school curriculum would be enormous task to overcome. For example, Kamwi shared his concern that if mathematics trail ideas were to be integrated into the curriculum, implementing them was going to be a challenge practically because of the time they take. Gurjanow et al. (2017) acknowledged that the MCM app project, takes much time and perseverance to create the amount of material needed and to encourage others to do the same. Therefore, when used within the Namibian context, the MCM app project can become an added workload for teachers.

Fifthly and lastly, the high summer temperatures in Namibia made the preparation and walking of the trail somewhat challenging. For example, Anna admitted: "*It was hot, and I was sweating… I think it would be good to do math trails during the time of Winter because then it is cool*". Moses added that "*You know, before we go out, we should use the weather app to see if it will be hot, or rainy, or even windy… you know sometimes Opuwo is too dusty*". These recommendations resonate well with that of Cahyono (2018), who advised that during high temperatures, it is important to identify a suitable time for outdoor activities such as the MCM mathematics trails. Cahyono further emphasised that MCM activities should be conducted in the morning or afternoon when the temperature is not too high in comparison to midday.

6.5 CONCLUSION

This chapter provided an in-depth analysis of how the participating teachers made sense of their interactions with the MCM app project tasks aimed at teaching for conceptual understanding of the topics of area, volume, proportion and ratio. The analysis of the RIs was based on the iPAC m-learning framework, which offered a new perspective on integrating technology as more than just tools for learning.

The personalisation construct of the iPAC framework consisted of two elements: customisation and agency. The analysis showed that the teachers perceived both elements to be at low to medium levels, meaning that the learners had limited to moderate choices over the tasks they solved in the trails and the time they spent on them. Moreover, the app guided the learners to locate and find the hidden tasks in their environment using its live GPS function.

The authenticity construct was more evident in the subconstruct of tools than in the subconstructs of settings and tasks, although all of them were significant. The MCM trail tasks enabled the learners to learn in relevant and realistic contexts and to use real objects, such as tape measures tapes and calculators, to simulate the practices of real professionals. The learners also used smartphones as mobile tools to search for information related to the area and volume formulas of shapes on the internet.

The collaboration construct was more prominent in the subconstruct of conversations than in the subconstruct of co-creation. The learners engaged in discussions based on the information displayed on the interface of the MCM app that appeared on their smartphone screens. However, apart from typing their answers on the smartphone and uploading them on the app's database, the smartphones and the MCM app did not provide opportunities for the learners to co-create digital content, such as videos and photos. The analysis revealed that this was because the MCM app was designed to only allow users (task designers) to remotely interact with the app's database and task reviewers.

The analysis also revealed how the learners used sketches, images, GPS and Google maps from smartphones to enact visualisation processes. The visualisation processes that existed in the trails and solving the tasks of this study afforded learners the opportunity to visualise and see mathematical patterns related to area and volume concepts in the physical objects existing within their environment. The analysis also identified some challenges that the teachers faced in designing and implementing mathematics trail tasks in this study. These challenges included time constraints, difficulty in planning, creating, and executing the trails, and security and safety concerns such as unfavourable weather conditions. In particular, the formulation of the tasks was not an easy process for the teacher participants, which could be attributed to their lack of experience in designing well-balanced tasks that matched the learners' cognitive demand. In the next chapter, I conclude the study by summarising the findings and recommending some ways of effectively using authentic tasks to teach for conceptual understanding of the topics of spatial measurements and ratio/proportion within RME based instruction and the MCM m-learning contexts.

CHAPTER 7

CONCLUSION AND IMPLICATIONS

7.1 INTRODUCTION

In this chapter, I conclude this thesis on my research project by consolidating the findings of the study with special reference to the main research question and sub-questions. I briefly summarise the key findings from the analysis of data, and within the context of the theoretical and methodological frameworks elaborated on in Chapters 3 and 4. In addition, I present an overview of the significance of the findings, point out the limitations of the study and recommend future research opportunities that emerged from this study. I conclude the chapter by sharing personal reflections on the research project that other researchers may relate to and learn from.

7.2 REVIEW OF THE RESEARCH AIM AND QUESTIONS

The aim of this study was to analyse and understand how eight selected mathematics teachers can implement authentic tasks in a mathematics trail to facilitate the teaching of area, volume, ratio and proportion for conceptual understanding.

In pursuance of this goal, the study was guided by the following main research question:

Main research question

In the context of a mobile learning environment, how can teachers implement authentic tasks in a mathematics trail for conceptual understanding of selected mathematics concepts?

This research question was then broken into two sub-questions:

Sub-research questions

1. In the context of running mathematics trails and solving the MCM project tasks, in what different ways do selected teachers make use of outdoor authentic tasks for conceptual teaching of area, volume, ratio and proportion topics?
2. What are the selected teachers' experiences and perceptions on the design and implementation of mathematics trails using smartphones within the MCM platform?

7.3 KEY RESEARCH FINDINGS

The research findings of this case study are presented as responses to the specific research goals and questions.

7.3.1 RME principles evidenced by the teachers (and the learners) when they walked the trails and solved the MCM-based tasks.

This case study used six RME teaching principles to analyse how eight mathematics teachers used authentic/realistic tasks to teach spatial measurements, ratio and proportion. An analytical framework derived from the works of Van den Heuvel-Panhuizen (2001; 2003; 2010) and Van den Heuvel-Panhuizen and Drijvers (2014) guided and informed the data analysis of the various video datasets. The details of the coding structure, RAILING, was discussed in Section 3.2.2. RAILING enabled me to reorganise my data and extract information to address the first sub-research question which reads:

In the context of running mathematics trails and solving the MCM project tasks, in what different ways do selected teachers make use of outdoor authentic tasks for conceptual teaching of area, volume, ratio and proportion topics?

The following is a summary of how the teachers used each of these principles to teach the topics of area, volume, ratio and proportion for conceptual understanding.

7.3.1.1 The reality principle

This research study showed that the MCM-based mathematics trails successfully enabled the teachers to design tasks that connected the mathematics learned in the classroom to real-life objects and situations. The teachers then used these tasks to teach and explain the concepts of area, volume, ratio and proportion in a meaningful way. They did this by helping the learners to compare the realistic problem situations in the trails to the mathematics in the classroom. In this way, the teaching in the trails supported the learning of mathematics from everyday problems before shifting to the abstract learning of concepts. It was evident that as learners walked the trails and solved the tasks, the teachers frequently asked questions to make the learners aware of the mathematics embedded in their surroundings. The teachers also used examples from everyday life to show how the trail tasks were relevant to real life situations. They used several illustrative examples such as calculating the amount of paint needed to paint certain objects, the number of tiles needed to cover certain areas of floors, the amount of space needed to fill up water tanks and dustbins, as well as the capacities of sewage tanks. Throughout the trail activities, the teachers encouraged the learners to interact with their local environment and understand the mathematics in it. Moreover, it was evident that the learners walked the trails and easily found the tasks because the tasks were placed in familiar places such as the school grounds and neighbourhoods.

7.3.1.2 The activity principle

According to the RME theory, learners should play an active role in their own learning, as this helps them to re-invent the subject content by engaging in and reflecting on activities that develop concepts and mathematical understanding. This study revealed that the learners worked on and solved the trail tasks collaboratively, with minimal guidance from their teachers. From the beginning of the trails, the learners participated actively in the discussions of how to locate and solve the tasks, and teachers fostered teamwork among the learners by encouraging everyone to contribute to the trail activities. To ensure effective teamwork, some teachers assigned the learners roles such as coordinator, note taker, measurement taker, phone operator and calculator operator.

The teachers granted their learners the opportunity and freedom to use their own methods and strategies to solve the tasks related to area, volume, ratio and proportion. The teachers also answered the learners' questions promptly and guided them when they needed help. This study showed that the teachers used the AP to enhance the collaboration, communication and social skills that were involved in the mathematics trails. This important finding implies that the learners were encouraged to discuss and exchange ideas, strategies, and solutions, as well as learn from each other about the concepts of area, volume, ratio and proportion. The learners also engaged actively in hands-on activities of measuring the dimensions of different objects. In this way, the teachers supported the learners in developing a deeper understanding of spatial measurements, ratio and proportion, and stimulated their mathematical creativity, curiosity and confidence.

7.3.1.3 The interactive principle

In this study, the IP was the most dominant principle evident in all eight trails. The findings show that the trails facilitated social interactions among the learners that led to active learning. It was evident that the learners exchanged ideas, approaches and solutions to the tasks, which promoted presenting, discussing, justifying and connecting mathematical ideas and strategies that possibly allowed for sharing and developing the understanding of area, volume, ratio and proportion. The teachers also used the IP to foster interactivity among the learners by frequently posing questions that elicited the concepts embedded in the trail tasks. Moreover, the teachers' use of the IP resulted in interactive conversations about what dimensions to measure and the units to use on the tape measure, and how and where to position the tape measure.

Furthermore, the learners interacted not only with each other but also with their teachers and tools such as the smartphone, tape measure, calculator and the MCM app. Concepts such as quadrilaterals, parallel lines, angles, straight lines, perpendiculars, equal length, area, volume and many more related concepts were explored and interrogated when solving the mathematics trail tasks. The findings suggested that because of the regular use of these concepts in the trails and how teachers connected them to the mathematics learned from the classroom, it was, according to the teachers, probable that the learners' comprehension of spatial measurements was enhanced and became experientially real to them. Additionally, the GPS function of the MCM app was a useful and instrumental tool that made the trail activities more interactive and engaging for the learners. It was clear that the sharing of one phone brought the learners together over the phone's screen to share ideas on where to find the locations of the tasks. This in turn increased the participation of the learners as it was evident that they had to work together to correctly interpret the MCM map.

7.3.1.4 The level principle

Another key finding of this study is that the teacher participants used everyday knowledge to help the learners build their knowledge of the formulas of area and volume of the geometrical shapes that were involved in the trail tasks. The teachers did this by using paper folding, sketches on paper and the ground, to model the abstract definitions of the properties related to the spatial measurement concepts. This is the first level of progressive mathematisation which states that the learning of mathematics should start from informal and intuitive before moving to the formal and abstract. In this study, the learners experienced area and volume concepts in realistic and meaningful situations where they used their everyday knowledge of painting walls, installing tiles, filling cylindrical containers with water and counting objects to understand the definitions of area and volume informally and intuitively.

The transition to the referential level (see Section 3.2.2.4) was observed when teachers helped the learners understand the use of symbols and notation to represent and manipulate area and volume. For example, the teachers stressed how the learners should write and express area and volume in square and cubic units. Moreover, the learners performed arithmetic operations with area and volume (and sometimes ratio and proportion) and the teachers introduced some of these operations using paper models, sketches and drawings. Along the way, the learners learned the rules and properties that governed the operations, for example finding the area of a trapezium by multiplying half by the sum of the parallel sides and the height and finding the volume of solid figures by multiplying the area of the base by the height. As a result, some of the learners were able to reason abstractly and generalise the concepts of area and volume by demonstrating the ability to express the concepts as ratios and proportions (general level). Therefore, in this study, using the LP, the participant teachers shifted the learners' understanding of area and volume from solving problems in real- life situations to seeing the connections between ideas and methods of working with the algebraic formulas of volumes and surface areas of different physical objects.

7.3.1.5 Intertwined Principle

This study found that the teachers rarely used the INP, with only a few of them explicitly connecting the properties of area and volume calculations with the physical object tasks the learners were working on. For example, Anna gave a practical example to show how parallel, perpendicular, and sloping sides were related to the original problem of the sign board. She also tried to link these concepts to the sign board and the trapezium properties to real-life patterns such as sloping roofs. Other concepts like the shaded and unshaded areas were used in other trails to illustrate how buildings are painted in real life. Nevertheless, the INP was somewhat evident in the trails because of how the design of the tasks integrated topics from other strands and subjects. For instance, besides volume and area measures, the mathematics trail tasks also involved rounding off and converting units. In most of the MCM tasks, the learners had to round off their answers to the nearest given accuracy level, which they did easily. However, the teachers did not emphasise these connections in their interactions with the learners. The same was true for the connections between these concepts and other mathematical concepts and subjects. The teachers did not stress the relationships between area and volume measures and other areas of mathematics. Overall, there was some evidence that most of the teachers used real-life examples to relate to the concepts in the tasks.

7.3.1.6 The Guidance Principle

The key findings on the GP in this study included the way in which teachers monitored and guided the learners on what to do by using appropriate material that aligned with the learners' cognitive level of understanding. The GP positioned the teachers as facilitators in the mathematics trails by allowing them to provide the learners with appropriate scaffolding and feedback that supported their learning process. For example, the teachers provided the learners with clear instructions and expectations for the mathematics trails, such as the goals, rules and roles. When the learners made mistakes in applying the principles of area or volume calculations, the teachers helped them to use the logarithms correctly. Also, the teachers used prompts and questions to elicit the learners' prior knowledge of area and volume concepts, which in turn encouraged them to continue working on the problems by recalling prior knowledge, making assumptions and checking their results. Using probing questions to scaffold the learners' understanding of the properties of the involved shapes was an effective strategy to help the learners see the properties as they appeared in the physical objects. The probing questions also stimulated discussions among the learners that led them to their own conclusions, which improved their conceptual and procedural knowledge of the area and volume concepts.

The teachers supported the learners throughout the trail activities, but they did not provide them with direct answers to their questions. Most of the time, the teachers were careful not to interfere with the learners' active participation and collaboration in the trails. Previous findings (eg Van den Heuvel-Panhuizen & Drijvers, 2014; Van den Heuvel-Panhuizen, 2020) have cautioned that when using the six RME principles, teachers should be careful of the apparent contradiction between the GP and AP of the RME theory. Therefore, the findings indicate that the teachers in this study were aware of this and made sure that their guidance was appropriate, and that they did not treat the learners as empty vessels. This supported the principle of the GP in RME which is based on the idea of 'guided re-invention' of mathematics, where teachers help the learners to discover and develop mathematical concepts, tools, and procedures by themselves, rather than telling them directly.

In sum then, this study demonstrated how the six RME teaching principles were applied in a coherent and integrated manner. The principles complemented each other by enabling the teachers to teach the selected topics more meaningful, relevant, and engaging way for the learners during the exploration of the trails and the solution of the tasks.

7.3.2 The iPAC constructs as evidenced by the teachers (and learners) when they walked the trails within the MCM m-learning environment

In answering the second sub-research question, I used the iPAC (Personalisation, Authenticity and Collaboration) m-learning framework to analyse the reflective RI (See Table 4.4).

The second sub-research question reads,

What are the experiences and perceptions of participating teachers in the design and implementation of outdoor trails using smartphones in the context of the MCM platform?

The summary of the findings about each of the above constructs, as articulated by the teachers is discussed below.

7.3.2.1 The personalisation construct

A key finding on the personalisation construct of the iPAC framework in this study, according to the responses of the teachers, revealed that the MCM m-learning environment offered the learners some choices over the learning content in the trails, but only within the topics of area, volume, ratio and proportion that were designed by the teachers and researcher. Not unexpectantly, the teachers said that the learners preferred the tasks on area and volume more than those on ratio and proportion. Another finding that emerged out of the analysis of the RI was that, according to the teachers, the learners could not create or modify the learning content themselves and they had to follow the order of the activities as planned by their teachers. Furthermore, they said that the learners also had control over the timing and pace of the trail activities and they could adjust the time limit according to their needs. However, some learners spent more time than expected on the tasks. The GPS function helped the learners to find the hidden tasks easily and made them more excited and motivated to do mathematics outdoors.

On the customisation sub construct, the findings indicated that according to the teachers, the learners had little or no opportunities to change the settings of the smartphones as mobile tools. Except for Joshua, none of the teachers thought that the learners adjusted the devices to their preferences. However, a notable aspect of the customisation subconstruct was how the learners used the internet app to search for information related to the formulas of area and volume of the shapes they were working on. This finding demonstrates how the learners adapted smartphones to their learning needs and learned in a serendipitous and unplanned manner.

7.3.2.2 The authenticity construct

The subconstruct of tools showed more evidence of authenticity than the subconstructs of settings and tasks, but all of them were important. The learners used smartphones to find authentic tasks that were related to realistic settings in their environment. The tasks helped the learners to learn in meaningful and real contexts and to use actual objects, such as tape measures and calculators, to simulate the practices of real professionals. The teachers said that this finding suggests that the learner participants of this study learned a life skill of taking measurements, and also a practical skill that may be useful for future careers that involve measuring. The teachers added that using these tools may have improved the learners' critical thinking and problem-solving skills for real life applications. Furthermore, the learners also used smartphones as tools to search the internet for information about the formulas of area and volume of shapes using smartphones. Also, with the same smartphones the learners checked their answers to the tasks to see if they were correct or not. This indicates that the learners engaged in rich and contextualised tasks that focused on the concepts of area, volume, ratio and proportion. One more key finding that was shared by the teachers was that of the authenticity principle of this study, which was that the learners were familiar and comfortable with the type and features of the smartphones they were using. According to the teacher participants this made the device more authentic and realistic for them.

7.3.2.3 The collaboration construct

The subconstruct of conversations showed more evidence of collaboration than the subconstruct of co-creation. The teachers observed that the app fostered a social and interactive learning environment where the learners shared and exchanged ideas on how to solve realistic problems posed in the trails. Specifically, the learners engaged in discussions based on the information displayed on the interface of the MCM app that appeared on their smartphone screens. The smartphone screens served as a visual guide that stimulated the learners' conversations when finding and solving the tasks. Furthermore, the teachers reported that the discussions on the trails were respectful and collaborative, where the learners valued each other's ideas and opinions.

Evidence from the co-creation subconstruct was insufficient according to the teachers, in the sense that the learners were not able to co-create digital content, such as videos and photos using smartphones and the MCM app. The analysis of the RI indicated that this was because the MCM app was designed to only let users (task designers) interact remotely with the app's database and task reviewers. This suggested that smartphones in the MCM m-learning context provided minimal or no opportunities for online peer discussions or sharing and exchanging digital content. As stated earlier, this could be related to the design of the app that only permits users to upload the content and interact with the reviewers of the tasks.

7.3.3 Summary of findings from the themes that emerged

7.3.3.1 The visualisation aspects in the study

Visualisation in mathematics is widely known as an ability that allows us to see the unseen. It helps us to go beyond what is visible to our eyes by not only seeing what is in front of us (physical), but it also helps us to see and understand what we mentally imagine in our minds, on paper or technological tools. This way, we can share, think about and enhance the knowledge of the information and ideas that come from such a visualisation process (Arcavi, 2003). The data analysis in this case study revealed three types of visualisations: *sketching*, *smartphone use* and *photography*. The teachers used sketching as a way of helping the learners visualise the concepts of area and volume. The learners drew sketches of unfamiliar shapes and divided them into familiar ones, which helped them remember the formulas for finding the surface areas and volumes of the shapes for example. For instance, Anna's learners sketched an octagon sign board and split it into three shapes (two trapeziums and a rectangle) (refer to Figure 5.15). The teacher then used one of the trapezium sketches to show the learners how to calculate the area of a trapezium and how to relate the sketch to real-world buildings.

This study used smartphones to enable visualisation in mathematics learning, by allowing learners to access the device's live GPS and connect to physical objects in their environment. The learners could also use the photography feature of the MCM app project to visualise and identify the real object tasks in their immediate environments, by using the images of photographs (object tasks) that were captured and tagged to the MCM tasks. The teacher participants encouraged their learners to use the Google maps and live GPS features to navigate their surroundings and compare and explore the images on their smartphones with the real objects around them while walking the trails. The analysis of this visualisation category in this study showed that the images used in the MCM app were similar to what Vale and Barbosa (2020) called mathematical photography or problem pictures, which can enhance mathematics learning by locating the tasks with the help of the GPS and recognising one's position in space, while fostering the learners' spatial orientation. Consequently, the three visualisation processes that occurred in the trails and task solving of this study enabled learners to see mathematical patterns related to area and volume concepts in the physical objects within their surroundings.

7.3.3.2 The teachers' views on the challenges of teaching outdoors

Lastly, the analysis of the data in this case study revealed that the planning and actual implementation of the MCM-based mathematics trails faced several challenges. The teacher participants reported difficulties in time management, designing, creating, executing the trails and coping with unfavourable weather conditions. Although only two tasks were solved in each trail of the eight teachers, the learners still spent too much time on these tasks. Moreover, the reviewing process of these tasks by the MCM project's task reviewers took longer than expected. The teachers also found the tasks difficult to design because they required a lot of planning, preparation and coordination. This raised some concerns about how these mathematics trails could be effectively integrated into the school curriculum, especially considering the heavy workload of Namibian mathematics teachers. Furthermore, the teachers struggled to balance the difficulty level of the tasks with the learners' ability to solve them. It was difficult for the teachers to align the cognitive level of the tasks with the learners' level of understanding.

Another notable problem with the MCM app project was the conflicting feedback from the task reviewers on the modifications of the tasks created by the teacher participants of this study. For instance, two reviewers gave different suggestions on the variation of answers that involved the intervals (refer to Figure 6.17). The teachers also mentioned that the high summer temperatures in Namibia (especially in the northern part of the country where this study was conducted) made the preparation and walking of the trails rather challenging. The trail participants faced unfavourable weather conditions such as hot and dusty winds.

7.4 LIMITATIONS OF THE STUDY

In this case study, I explored how teachers used authentic tasks in an m-learning environment to teach the topics of area, volume, ratio and proportion for conceptual understanding. I applied the six RME principles and the iPAC (Personalisation, Authenticity, Collaboration) as frameworks to analyse the evidence of the teachers' practices and perceptions. I worked with a small sample of eight teachers from one circuit of one region, which may not reflect the characteristics and experiences of the general population of secondary mathematics teachers in the country. Therefore, the findings of this case study cannot be generalised to other cases beyond the scope of this study. The situations observed and reported by the participant teachers may differ from other teachers I did not have access to, especially if such differences are influenced by the teachers' actions in designing and implementing worthwhile mathematics trail tasks that are to develop and enhance the learners conceptual understanding of the topics of spatial measurements, ratio and proportion. However, instead of generalisability, I aimed for transferability in this case study. Transferability means that the findings and conclusions of a case study can be applied to other situations, based on the researcher's understanding of the phenomena within its context (Rule & John, 2011). To achieve transferability, I provided thick descriptions of the teachers' interactions with their learners and the context of the case study, following the suggestion of Cohen et al. (2007). Thus, by using in-depth analysis and rich descriptions, I hoped to increase the transferability of the results of my case study.

Another limitation of this study was that it only collected data from the teacher's actions and views on the use of authentic tasks based on the RME theory and the iPAC m-learning framework. The learners who took part in the study might have had different experiences and opinions from their teachers. Moreover, this study only involved three schools in the Opuwo circuit of Kunene region that had at least three Grade 9 mathematics teachers at the same school. Therefore, the teachers were selected because they met the sampling criteria of this study, not because they were representative of the population. Furthermore, this study faced challenges due to the COVID-19 pandemic, which disrupted the normal operations of schools. The data collection period was extended from one year (2021) to almost two years (November 2021 – June 2023) because of frequent school closures caused by virus infections among learners and teachers. This made it hard to meet with the research participants, who were often busy catching up with the backlog of work. It was also difficult to organise the trail activities and RIs due to the high work pressure on both the participants and the researcher. I must admit however, that I used the opportunity created by the closure of schools to rigorously engage with and review the literature of this study. Lastly, one of the teacher participants from School B was transferred at the beginning of the data collection, leaving me with only two teacher participants at that school.

7.5 SIGNIFICANCE AND CONTRIBUTIONS OF THE STUDY

Bertram and Christiansen (2015) maintain that research should be of benefit either directly to the participants, or more broadly to other researchers, or to society at large. A review of pertinent literature relating specifically to the use and integration of authentic tasks and m-learning in Namibian mathematics secondary education revealed (a) a lack or invisible use of mobile technologies such as smartphones in the teaching of Namibian secondary school mathematics; (b) the absence of a clear framework that guides the use and implementation of m-learning technologies within the broad education curriculum (Osakwe et al., 2017); and (c) the use of authentic outdoor tasks that are connected to real life situations is seldom practiced in secondary school mathematics, both in Namibia and beyond (Hakadiva-Vatileni, 2016; Vos, 2011; 2015). Therefore, this study is significant because it contributes to filling the identified gaps mentioned above. In particular, the study contributes towards a critical discourse about the development and adoption of an m-learning framework in secondary schools, that presently seems to be absent. The study is also significant because of its intended purpose to introduce the MCM app to selected secondary school teachers in Namibia, a move that can strengthen and encourage the use of mobile technologies (particularly that of smartphones) and outdoor mathematics trail activities, in the teaching of secondary school mathematics. Further, the study findings contribute to the international literature of understanding how the potential of the nexus that exists between m-learning technologies and outdoor trail authentic tasks can facilitate conceptual teaching. Furthermore, as the study foregrounds visualisation as one of the conceptual constructs, the findings contribute to this area of research that connects interactive technology with visualisation as an epistemological tool.

7.6 IMPLICATIONS

This research study has implications for both theory and practice, as well as further research.

7.6.1 Implications for theory

This study has theoretical implications for the use of authentic tasks, m-learning, and RME in mathematics education. It reveals the benefits and challenges of using MCM mathematics trails as realistic tasks that connect mathematical concepts to real-world situations. It also shows how teachers can use smartphones and mathematics trails creatively and innovatively to enhance their teaching practice and learners' engagement. Moreover, the study demonstrates how m-learning can foster learners' mathematical understanding and interest by creating a realistic and contextualised learning environment. Therefore, this study provides empirical evidence on how m-learning technologies and authentic tasks can be integrated and applied in the context of RME-based instruction, using MCM trail tasks. The study can also generate new research questions and directions for further investigation on the development and improvement of the MCM mathematics trail programme and on the evaluation of its impact on various aspects and areas of mathematics and m-learning technology theories other than the RME theory and the iPAC m-learning technology.

7.6.2 Implications for practice

This research study can contribute towards improving the teaching of the topics of area, volume, ratio and proportion for conceptual understanding using a mobile app-supported math trail programme that incorporates realistic and engaging tasks based on the principles of realistic mathematics education (RME). It demonstrates how MCM can be used by schools and the public to explore and appreciate mathematics in their environment and to enhance their motivation, engagement, and achievement in mathematics. The study found that the teachers who participated in the MCM programme used authentic tasks that aligned with the RME principles and that challenged and stimulated the learners' mathematical thinking and problem-solving skills. Based on these findings, I recommend that various stakeholders, such as the government, education authorities, tourism departments, museum managers, travel agencies, mobile phone service providers and even mobile device manufacturers and sellers, support and promote initiatives such as the MCM programme in Namibia. I also suggest that secondary school mathematics teachers use the MCM app as a source of continuous assessment (CA) activities, such as practical investigations and projects, and that policy makers and curriculum designers integrate the MCM app project and the use of authentic tasks, smartphones and RME in the mathematics curriculum and standards. The findings of this study suggest that the MCM app can reduce the stigma associated with using smartphones in Namibian schools, and can increase the awareness and use of RME, m-learning technologies and authentic tasks in mathematics education.

7.6.3 Implications for further studies

This case study focused on teachers' practices and perspectives of teaching for conceptual understanding of the topics of area, volume, ratio and proportion, using MCM realistic tasks within the RME based instructions. Some possible directions for future research are:

- Conducting a similar study with a focus on learners' perspectives of how they view the effects of the MCM m-learning environment in learning the topics of spatial measurements, ratio and proportion.
- Repeating the current study with respect to critically evaluating the creation of ratio and proportionality topics and tasks, as the design and implementation of trail tasks involving these topics were found to be challenging.
- Repeating the current study with primary mathematics schoolteachers and/or different topics other than area and volume.
- Developing and improving the MCM app project in different places and conditions and evaluating the programme's impact on various aspects and areas of studies, such as learners' motivation, performance and beliefs.
- Conducting longitudinal research using much larger samples in Namibia to interrogate the effectiveness of using outdoor MCM trail tasks in the context of RME-based instruction.

7.7 PERSONAL REFLECTIONS AND CONCLUDING REMARKS

The data collection of my study involved working with eight secondary school mathematics teachers from three different schools in the Opuwo circuit of Kunene region, Namibia, who agreed to participate in my study. I conducted observations that were accompanied by VRs, RIs and FGIs to collect data on the design and implementation of the MCM tasks to teach the topics of area, volume, ratio and proportion for conceptual understanding, using the RME theory. However, my data collection was not without difficulties. The creation of the tasks and organisation of the mathematics trails was not an easy task. The COVID-19 pandemic had a significant impact on my research, as it disrupted the normal functioning of the schools and imposed various health and safety restrictions. I had to adapt to the changing situation and modify my research plan accordingly. For example, I had to postpone some workshops on the creation of the tasks and implementation of the mathematics trails, as well as some interviews.

Another challenge I faced was the use of technology in the mathematics trails. Initially, I had planned that the teachers use their own smartphones to download the MCM app so that they fully understood its operations. However, only one teacher was able to download and install the app on her phone, and the other teachers were hesitant and claimed that they did not have enough space in their phones, a reason I deemed as an excuse. I wanted the teachers to experience and explore first-hand how smartphones, through the application of the MCM project, can support their (and their learners) mathematical inquiry and communication in the outdoor settings. I also had to deal with some ethical issues such as obtaining consent, ensuring privacy and preventing misuse of the devices.

Despite these challenges, I mostly had positive experiences in my research journey. One of them was the task design experience, which involved collaborating with the teachers to create authentic and meaningful tasks for the mathematics trails. I learned a lot from the teachers' expertise and insights, and I also shared some of my own ideas and resources with them. We had fruitful discussions and feedback sessions, and we managed to develop tasks that aligned with the curriculum and the learners' interests. The walking of the trails in the natural environment of the town of Opuwo was another remarkable experience in this research journey. I enjoyed walking with the teachers and the learners and observing how they engaged with the tasks and the surroundings. I also appreciated the opportunity to experience different places and contexts, such as the three school environments, the different locations of the Opuwo and the locations inhabited by the local Ovahimba people. I found that walking outdoors offered a rich and diverse source of mathematical phenomena and problems and was also a fun and motivating way of learning mathematics.

The teacher participants' experiences were also a valuable part of my research journey. I developed a good rapport and trust with the teachers, and I appreciated their willingness and enthusiasm to participate in my study, despite the many challenges that arose from the COVID-19 pandemic and its aftermath. I learned a lot from their perspectives and experiences, and through this study, I also supported them in their professional development. I found that the teachers were open to new ideas and approaches of using the RME theory and the iPAC m-learning framework, and they were keen to improve their practice and enhance their learners' learning outcomes.

The smartphone use experience was another interesting aspect of my research journey. The teacher participants were fascinated by how smartphones could be used in the context of teaching and learning mathematics. Both the learners and teachers enjoyed using smartphones to explore their surroundings and look for the hidden object tasks. Smartphones offered many advantages, such as providing access to information, tools and media, enabling collaboration and sharing, and increasing engagement and motivation.

The last but not the least experience I want to mention is from the SAARMSTE. At the early stage of my research study in 2020, I attended an online SAARMSTE research school where a team of two experts from the universities of Witwatersrand and Nottingham helped me to refine my research ideas. I received valuable feedback and advice on my research. I found the programme to be very helpful and supportive, and I also contributed to the programme by presenting my research ideas and participating in the activities.

Furthermore, I also networked with an expert from Sol Plaatje University to present and share my research ideas to his students, particularly those of the RME theory and the MCM app project. These experiences indicate that apart from my main supervisor, I also collaborated with other experts in the field of my research study and learned a lot from them.

In conclusion, my research journey of this PhD study was a challenging but rewarding experience. I faced some difficulties and obstacles, but I also had many opportunities and achievements. I learned a lot from my research and I also contributed to the field of mathematics education. I hope that my study will inspire other researchers and practitioners to explore the potential of authentic and realistic MCM tasks and mathematics trails for enhancing mathematical learning and engagement in outdoor settings.

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APPENDIX ONE: Semi-structured interview schedule

These seed questions were administered to the eight research participants of this study.

In your opinion, during the walking of the trails and solving of questions,

1. Did the learners have a choice of learning content? please explain.
2. Did the learners have choice of routes to take in the math trails or not? please explain.
3. Did the learners have control of the tools or not, eg, smartphone, tape measure? please explain.
4. Did the learners have a choice over their own learning pace or not? please explain.
5. Did the learners have a choice of methods to use to find solutions to the tasks or not? please explain.
6. How did the smartphone guide the learners where to find the tasks? please explain.
7. Did the learners easily locate the hidden tasks or not? please explain.
8. How did you see the tailing of the device's settings to the learners' preferences? eg, customised location on/off, camera/microphone access, time limit settings.
9. Did the learners receive individualised and/or group information through the app about their environment or not? eg, information about the trail/task, please explain.
10. How do you evaluate the learning outcomes of the MCM app in both realistic and virtual environments?
11. Was the MCM m-learning tasks relevant to learners? please explain.
12. To what extent did the learners follow the experts' strategies when solving the tasks? Please explain.
13. Did the learners engage in any unplanned learning activities while walking the trails and solving the tasks? If so, please describe them.
14. How did you use the outdoor tasks to teach the topics of area, volume, ratio and proportion?
15. How did the learners use tools such as tape measures and calculators to replicate those of real-world practitioners?
16. Were these tools familiar or strange to learners? please explain.
17. How did smartphones aid learners' discussions of the concepts of area, volume, ratio and proportion?
18. Did smartphones create shared and socially interactive environments for learning? please explain.
19. Did the learners create ant digital product(s) using smartphones? if yes, please explain how this happened.
20. How did the learners interact with each other and their teacher(s) during the learning process?
21. Did the learners contribute to the existing MCM digital content? please explain?
22. Did the learners share any digital content during the walk and solving of the trail tasks?
23. In your view and experience, in what ways, during the walking of the trail and solving of the tasks, did you help learners understand concepts related to the topic of **(name of topic)**?
24. Can you say that learners clearly understood what was taught to them during the running of the trails and solving of the tasks? Please elaborate and provide examples.
25. What was your experience of creating the tasks for the MCM app?

26. How easy or difficult do you think were the tasks for learners to solve? Please explain.
27. On a scale of *easy*, *moderate*, and *difficult*, what can you say about the way learners had used the GPS enabled smartphones to access the tasks? Please elaborate and provide examples.
28. How did you provide the learners an opportunity to use smartphones in collaborative ways? Please provide examples.
29. Do you think learners found the running of the trails and solving of the MCM tasks to be of interest to them? Please elaborate and provide examples.
30. Do you think that the solving of the MCM project tasks equipped your learners with the skills necessary to solve real life problems? Please elaborate and provide examples.
31. In your view and experience, did the use of the MCM authentic tasks add to the learners' conceptual understanding of **(name of the topic)**? Please elaborate and provide examples.
32. In your view and experience, when tracing the hidden tasks, did the use of smartphones increase learner participation? Please elaborate and provide examples.
33. In the context of the MCM project, to what extent (low, medium, high) were the learners in charge of the trail activities?
34. Do you think the use of smartphones in the walking of the trails helped learners develop a sense of ownership towards their own learning? Please elaborate and provide examples.
35. In your experience what were some of the constraining factors when using smartphones in your teaching? Please elaborate.
36. To what extent (low, medium, high) do you think the authenticity of the tasks in the MCM project were realistic and offered problems encountered by real world practitioners?
37. In your view and according to your experience, what challenges do you think are associated with the use of the MCM project in teaching mathematics outdoors? Please elaborate and provide examples.
38. How did you find the general use of smartphones in this study? Please explain.
39. Are you going to use smartphones again in your teaching? Please explain.
40. Would you recommend or introduce the use of the MCM app to learners or other mathematics teachers at your school or the region at large? Please explain.

APPENDIX TWO: Letter of consent to the director of education

Rhodes University

Drosty Road,

Makhanda,

South Africa

6139

Regional Director

Kunene Regional Council

Directorate: Education, Arts and Culture

Private Bag 2007

Khorixas

11 January 2021

Dear Mrs Angeline A. Jantze

REQUEST FOR PERMISSION TO CONDUCT RESEARCH

I am a registered PhD student in the Department of Education at the Rhodes University. My supervisor is Prof. Marc Schäfer.

The proposed topic of my research is: *Teachers' use of authentic tasks through mathematics trails in a mobile learning environment to facilitate conceptual teaching*. The objectives of the study are

(a) To analyse and understand how selected mathematics teachers, in the context of a mobile learning environment, can implement authentic tasks in a mathematics trail to facilitate conceptual teaching.

(b) To promote the use of mobile technologies, particularly that of smartphones, as well as outdoor trail activities in the teaching and learning of mathematics through the introduction of the Math City Map project.

I am hereby seeking permission to conduct research on nine mathematics teachers from three different secondary schools in Kunene region. The field work of the study is projected to last for at least one year-2021. The following are the minor risks that can arise from conducting this study, and included is how I plan to minimize and mitigate through them:

- The running of the trail activities will involve Grade 9 classes. So, in order not to disturb normal classes, I want to assure you that the running of the trails will be executed in the afternoons, and at times that do not collide with the school activities.
- the selected learners will be taught in an outside environment that may not be safe compared to the usual inside classroom environment. The safety of learners will be of high priority in this project. Therefore,

as a safety measure, the tasks will be created in a free risk environment. Further, the mathematics teacher and I will always supervise where and what learners are doing during the running of the trails.

- The use of smartphones in the teaching and learning of many subjects in our region and the country at large is not yet an accepted norm, neither a common practice. However, you will notice in my research proposal that literature argues that given their potential to support collaborative and contextualised learning, when regulated and used in the right way, smartphones may address some of the concerns in mathematics teaching such as didactic approaches and de-contextualised material removed from real-world settings. Hence, the need for conducting this study; to find ways on how, as mathematics teachers, can best make use of mobile devices in the pedagogy of the subject. From this perspective, I thus want to request that the participants (teachers & learners), may be allowed to explore and use smartphones within the school environment during this study.

To assist you in reaching a decision, I have attached to this letter:

(a) A copy of an ethical clearance certificate issued by the University
(b) A copy of the research instruments which I intend using in my research
(c) A copy of my research proposal

Should you require any further information, please do not hesitate to contact me or my supervisor. Our contact details are as follows:

Given Matengu: +264812026224 ***Prof Marc Schafer (PhD): +27834095580***

givenmatengu@gmail.com M.Schafer@ru.ac.za

Upon completion of the study, I undertake to provide you with a feedback.

Your permission to conduct this study will be greatly appreciated.

Yours sincerely,

Given K. Matengu

PhD student

Mathematics teacher (Mureti SS, Opuwo Circuit).

APPENDIX THREE: Letter of consent to principals of participating schools

This letter was used to seek permission from the three school principals of the selected participants.

Rhodes University

Drosty Road,

Makhanda,

South Africa

6139

The school principal

Name of school and address

Date

Dear (**name of principal**)

REQUEST FOR PERMISSION TO CONDUCT RESEARCH

I am a registered PhD student in the Department of Education at the Rhodes University. My supervisors is Prof. Marc Schäfer.

The proposed topic of my research is: *Teachers' use of authentic tasks through mathematics trails in a mobile learning environment to facilitate conceptual teaching*. The objectives of the study are

(c) To analyse and understand how selected mathematics teachers, in the context of a mobile learning environment, can implement authentic tasks in a mathematics trail to facilitate conceptual teaching.

(d) To promote the use of mobile technologies, particularly that of smartphones, as well as outdoor trail activities in the teaching and learning of mathematics through the introduction of the Math City Map (MCM) project.

I am hereby seeking permission to conduct research on three mathematics teachers from three different secondary schools in Kunene region. The field work of the study is projected to last for at least one year-2021. The following are the minor risks that can arise from conducting this study, and included is how I plan to minimize and mitigate through them:

- Although learners are not the focus of this study, it will be important to also capture the interactions that the teacher has with the learners on the trail. Hence the reason why learners will feature in the video recordings. I will seek consent from both learners and their parents/guardians in this regard. The identity of the school and the participants will be anonymous, and no part of the collected data will be used for any other purpose other than this study.

- The running of the trail activities will involve Grade 9 classes. So, in order not to disturb normal classes, I want to assure you that the running of the trails will be executed in the afternoons, and at times that do not collide with the school activities.
- the selected learners will be taught in an outside environment that may not be safe compared to the usual inside classroom environment. The safety of learners will be of high priority in this project. Thus, as a safety measure, the tasks will be created in a free risk environment. Further, the mathematics teacher and I will always supervise where and what learners are doing during the running of the trails.
- My study will involve the use of smartphones as mobile devices. I therefore request that the participants (teachers & learners), be allowed to explore and use smartphones at the school environment during this study.

To assist you in reaching a decision, I have attached to this letter:

(d) A copy of an ethical clearance certificate issued by the University
(e) A copy of the research instruments which I intend using in my research
(f) A copy of my research proposal
(g) A copy of the permission letter from the education director of Kunene region.

Should you require any further information, please do not hesitate to contact me or my supervisor. Our contact details are as follows:

Given Matengu: +264812026224 ***Prof Marc Schafer (PhD): +27834095580***

givenmatengu@gmail.com M.Schafer@ru.ac.za

Upon completion of the study, I undertake to provide you with a feedback.

Your permission to conduct this study will be greatly appreciated.

Yours sincerely,

Given K. Matengu

PhD student

Mathematics teacher (Mureti SS, Opuwo Circuit).

APPENDIX FOUR: Informed consent letter to participants

PARTICIPANT INFORMED CONSENT

INFORMED CONSENT DECLARATION

(Participant)

Project Title: **Teachers' use of authentic tasks through mathematics trails in a mobile learning environment to facilitate conceptual teaching.**

Given K. Matengu from the Department of Education, Rhodes University has requested my permission to participate in the above-mentioned research project.

The nature and the purpose of the research project and of this informed consent declaration have been explained to me in a language that I understand.

I am aware that:

1. The purpose of the research project is to

    (a) analyse and understand how selected mathematics teachers, in the context of a mobile learning environment, can implement authentic tasks in a mathematics trail to facilitate conceptual teaching.
    (b) promote the use of mobile technologies, particularly that of smartphones, as well as outdoor trail activities in the teaching and learning of mathematics through the introduction of the Math City Map project.

2. The Rhodes University has given ethical clearance to this research project and I have seen/ may request to see the clearance certificate.

3. By participating in this research project, I will be contributing towards the development of a clear practical framework that will guide the adoption of mobile learning technologies in my teaching practice, the region, the country at large and beyond. Furthermore, the introduction of the *Math City Map* project has the potential to empower me with the creative and innovative ways of using smartphones and mathematics trails to teach mathematics.

4. I will participate in the project by designing mathematics tasks from topics of Area, Volume and Ration & Proportion that are connected to real world objects and situations, as well as use such tasks to teach Grade 9 learners through the implementation of mathematics trails. I will also use mobile technologies such as smartphones in the creation and teaching of the tasks.
5. My participation is entirely voluntary and should I at any stage wish to withdraw from participating further, I may do so without any negative consequences.

6. I will not be compensated for participating in the research, but my out-of-pocket expenses will be reimbursed.

7. There may be risks associated with my participation in the project. I am aware that

a. the following risks are associated with my participation:
    - the resistance and stigmatisation that may arise from colleagues and the school management because of allowing learners to use smartphones in the learning and teaching contexts.
    - The unexpected risks that may arise from teaching learners in an outdoor environment other than the comfort of the classroom.
b. the following steps have been taken to prevent the risks:
    - permission have been sought after to allow the use of smartphones in the study from the regional director and the school principal.
    - I have the consent of the parents/guardians to allow their children (learners) to explore the mathematics trails and solve the tasks in an outdoor environment.
c. there is a 10% chance of the risk materialising

8. The researcher intends publishing the research results in the form of journals and conference. However, confidentiality and anonymity of records will be maintained and that my name and identity will not be revealed to anyone who has not been involved in the conduct of the research.

9. I will receive feedback in the form of a written report regarding the results obtained during the study.

10. Any further questions that I might have concerning the research, or my participation will be answered by Given K. Matengu, cell: +264812026224, email: givenmatengu@gmail.com.

11. By signing this informed consent declaration, I am not waiving any legal claims, rights or remedies.

12. A copy of this informed consent declaration will be given to me, and the original will be kept on record.

13. Request to take pictures, video and voice recording for this study

I, ………………………………………………………………………….. have read the above information / confirm that the above information has been explained to me in a language that I understand, and I am aware of this document's contents. I have asked all questions that I wished to ask, and these have been answered to my satisfaction. I fully understand what is expected of me during the research.

I have not been pressurised in any way and I voluntarily agree to participate in the above-mentioned project.

…………………………………. ………………………… ………………………….

Participants signature **Witness** **Date**

APPENDIX FIVE: Informed consent letter to parents and guardians of learners – The English version

PARENT AND GUARDIAN'S INFORMED CONSENT

INFORMED CONSENT DECLARATION

(Parent or Guardian)

Project Title: **Teachers' use of authentic tasks through mathematics trails in a mobile learning environment to facilitate conceptual teaching.**

Given K. Matengu from the Department of Education, Rhodes University has requested my permission to allow my child/ ward to participate in the above-mentioned research project.

The nature and the purpose of the research project, and of this informed consent declaration have been explained to me in a language that I understand.

I am aware that:

14. The purpose of the research project is to

    (c) analyse and understand how selected mathematics teachers, in the context of a mobile learning environment, can implement authentic tasks in a mathematics trail to facilitate conceptual teaching.
    (d) promote the use of mobile technologies, particularly that of smartphones, as well as outdoor trail activities in the teaching and learning of mathematics through the introduction of the Math City Map project.

15. The Rhodes University has given ethical clearance to this research project, and I have seen/ may request to see the clearance certificate. [Certificate number: **2021-2787-5962**].

16. By participating in this research project my child/ward will be contributing towards the development of a clear practical framework that will guide the adoption of mobile learning technologies in the region, the country at large and beyond. The introduction of the Math City Map (MCM) project will create an outside learning environment where my child will be challenged and motivated to solve mathematical problems linked to real life situations and objects. The MCM project will also expose my child to creative, innovative and contextualised ways of using mobile technologies such as smartphones in his/her learning of mathematics. Furthermore, my child will have an opportunity to be taught topics of Area, Volume and Ratio & Proportion in a way that may help him/her to clearly understand the concepts.

17. My child/ward will participate in the project by solving authentic tasks using tools such as smartphone, calculator and measuring tape, while walking along the path or trail within the school environment and the surrounding area.

18. My child's participation is entirely voluntary and if my child/ward is older than seven (7) years, s/he must also agree to participate.

19. Should I or my child/ward at any stage wish to withdraw my child from participating further, we may do so without any negative consequences.

20. My child may be asked to withdraw from the research before it has finished if the researcher or any other appropriate person feels it is in my child's best interests, or if my child does not follow instructions.

21. Neither my child nor I will be compensated for participating in the research

22. There may be risks associated with my child's participation in the project. I am aware that

    a. the following risks are associated with participation: my child will be taught in an outside environment that may not be safe compared to the usual inside classroom environment.
    b. the following steps have been taken to prevent the risks: the safety of my child will be of high priority in this project. Thus, to ensure the safety of my child, the tasks will be created in a free risk environment. Further, the researcher and my child's mathematics teacher will always supervise where and what my child and other learners are doing during the running of the trails.
    c. there is a 10% chance of the risk materialising.

23. The researcher intends publishing the research results in the form of journals and conferences. However, confidentiality and anonymity of records will be maintained and that my or my child's/ward's name and identity will not be revealed to anyone who has not been involved in the conduct of the research.

24. I will not receive feedback regarding the results obtained during the study.

25. Any further questions that I might have concerning the research, or my child's participation will be answered by **Given K. Matengu**, cell: +264812026224, email: givenmatengu@gmail.com.

26. By signing this informed consent declaration, I am not waiving any legal claims, rights or remedies that I or my child/ward may have.

27. A copy of this informed consent declaration will be given to me, and the original will be kept on record.

I, ………………………………………………………………………….. have read the above information / confirm that the above information has been explained to me in a language that I understand and I am aware of this document's contents. I have asked all questions that I wished to ask and these have been answered to my satisfaction. I fully understand what is expected of my child during the research.

I have not been pressurised in any way to let my child take part. By signing below, I voluntarily agree that my child/ward ……………………………………………………………… (**insert name of child**), who is …………….. years old, may participate in the above-mentioned research project.

…………………………………. ………………………… ………………………….

Parent/Guardian's signature **Witness** **Date**

APPENDIX SIX: Informed consent letter to parents and guardians of learners – The local language (Otjiherero) version.

OMAITAVERERO WOVANENE OVAKWATE NOVATIZE WOVANATJE.

OMAITAVERERO WORUYANO

Omunene omukwate poo omutize womuatje

Epu: Ozomitiri okuhonga Ovivarero kovanatje vosikore amaveungurisa oviungurisiwa vyouye wakandino

Given K. Matengu okuza korupa rwomahongero, kosikore yokombanda (Rhodes University) watjiti omaningiriro kwami, mbiyandjere omwatje wandje makare norupa rwomakondononeno wotjirihongwa hi.

Omuhingo nomurari wongondononeno ndji vyahandjaurwa kwami meraka ndi mezuu.

Ami me tjiwa kutja:

1. Epu rokukaendisa ongondononeno ndji oyo ku:
   (a) Paha ondjiviro nokutjiwa kutja ozomitiri nda toororwa okuhonga ovivarero, mave ungurisavi oviungurisiwa vyouye wakandino momahongero wawo.
   (b) Okutunduuza omaungurisiro woviungurisiwa vyakandino tjimuna ouyendjezewa, navyarwe mbi ungurisirwa pendje netuwo romerihongero mozongondjero zokuhonga ovivarero.
2. Osikore indji yokombanda o Rhodes University ya itavera komaningira wokukaendisa ongondononeno ndji, nu ami wina orutuu ndwe ndji yandjera okukaendisa ongondononeno ndji mberu muna.
3. Mokukara norupa mongondononeno ndji, omwatje wandje mayandja oruvara komekuriro woviungurisiwa vyouye wakandino morupa rwovivarero morukondwa, mehi na wina komahi wopendje. Omatjiukisiro woviungurisiwa mbi ma ye yandjere ovahongwa okurihongera pendje pu ma pe tjiti kutja omwatje wandje ma tokwa nakara nongondjero yokuzengurura omauzeu wovivarero mbya tjama noviungurisiwa vyowatjiri mbi ungurisiwa i ovandu. Oviungurisiwa mbi mavi yandjere omwatje wandje meripure oukoto, marire onongo, makare nondjiviro yokuunngurisa oviungurisiwa vyouye wakandino tjimuna ouyendjezewa okurihongerako ovivarero. Okukaenda komurungu, omwatje wandje makara noruveze rwokuhongwa omapu omengi nga tjama kovivarero.
4. Omwatje wandje ma kara norupa mo projeka ndji okukondja okuzengurura omauzeu wovivarero a ma ungurisa ovina tjimuna ouyendjezewa, okarekene navyarwe ama kaondja ongondoroka nosikore.
5. Omakarero norupa womwatje wandje omeriyandjerero we omuni, neye wina tjeri kombanda yozombura hambombari, eye maso okuriyandjera omuni kokutja makare norupa.
6. Ami poo omwatje wandje tji mba zeri okunanununa omwatje mongondononeno ndji, ete ma tu tjiti nao nokuhina okuzunda omahongero inga.
7. Omwatje wandje mayenene okuningirwa kutja meri nanunune mongondononeneo ndji ngunda a I hi yeya komaandero, tji ma pemunika kutja okouwa womwatje ngo o ku isamewa mongondononeno ndji, poo eye tje hina kukongorera omazeva.
8. Ami poo omwatje ka tunakusutwa mokukara norupa mongondononeno ndji.
9. Mapeyenene okukara omaumba nga tjama nomakarero worupa mongondononeno indji ko muatje wandje. Ami me tjiwa kutja:
   a. Omaumba otja inga maye yenene okukarapo momakarero we norupa: Omuatje wandje mahongerwa pendje netuwo romahongero, opo pumapeyenene okuhina ku kara nondjeverero ndji ta pi okukapita momatuwo womahongero mu vehongerwa aruhe.
   b. Ovina mbi mavi kongorere kehi mba mavi kongorerwa okukondja okutunduuza omaumba: Ondjeverero yomwatje wandje marire otjina otjitenga ku ma ku tarewa mongondoneno ndji. Ma pe heewa ko kutja, omwatje wandje ma kara kehi yondjeverero ombwa orondu ongondononeno ndji mai tjitirua potuveze pepehakara omaumba pu pe ta pi. Okukaenda komurungu, omukondonone nomitiri ndji honga omwatje wandje ovivarero mave tjevere omwatje wandje navarwe pu ma veungurire ongondononeno yawo.

c. Pena ozoperesende omurongo (10) zoumba okuyenena okutjitwa.

10. Omukondonone una ondero yokupitisa omaziriro wongondononeno ndji mozo kurande nokotungovi
11. Ami hina kupewa eziriro okuza kongondononeno ndji ngunda a i hiya manuka.
12. Kondjiviro yokomurungu ndji me hepa ouhunga nongondononeno ndji, poo no muatje wandje, omaziriro ma yezu ku **Given K. Matengu** konomora ndji: +264812026224, Orungovi: givenmatengu@gmail.com.
13. Moku twako omunwe kembo komaitaverero woruyano ndwi, ami hina ku ka tjita omaningiriro wakangamwa otjina, omausemba poo otjina atjihe ngamwa omwatje wandje tji mayenene okukara natjo.
14. Ami mepewa otjiherengururwa tjomaitaverero woruyano ndwi. Nu orutuu ndwi orukarerere maru pwikwa.

Ami,

……

…………………………………. mbarese ombuze ndja tjangwa pombanda mbo, ame itavere kutja ombuze ndja tjangwa pombanda mbo mba handjaurirwa meraka ami ndi mezuu nawina avihe mbya tjangwa morutuu ndwi. Ami mba pura omapuriro ayehe ngu mbari ame zeri okupura nu ayehe yazirwa pumbari ameundjire ko. Ami metjiwa mbi mavi undjirwako komwatje wandje mongondononeno ndji.

Ami hiya ninikiziwa momuhingo umwe poo omukwao kutja omwatje wandje ma kare norupa. Ami okutwako omunwe kembo kehi mba, mberiyandjere omuni kutja omwatje wandje ………………………………………………………………………………………………………. (hitisa ena romwatje), wozombura …………………, makare norupa mongondononeno ndja ytjangwa pombanda mbo morutuu ndwi.

…………………………………   …………………………………………..

Omunene omukwate/omutize womwatje   Ohatoi

Omayuva ……………………………………..

APPENDIX SEVEN: Informed consent letter to school learners.

CHILD PARTICIPANT'S ASSENT FORM

INFORMED CONSENT DECLARATION

(Child participant)

Project Title: Making mathematics relevant: the use of smartphones in outdoor trails.

Researcher's name: Given K. Matengu

Name of participant: …………………………………………………………………………

1. Has the researcher explained what s/he will be doing and wants you to do?

   YES NO

2. Has the researcher explained why s/he wants you to take part?

   YES NO

3. Do you understand what the research wants to do

   YES NO

4. Do you know if anything good or bad can happen to you during the research?

   YES NO

5. Do you know that your name and what you say will be kept a secret from other people?

   YES NO

6. Did you ask the researcher any questions about the research?

YES NO

7. Has the researcher answered all your questions?

YES NO

8. Do you understand that you can refuse to participate if you do not want to take part and that nothing will happen to you if you refuse?

YES NO

9. Do you understand that you may pull out of the study at any time if you no longer want to continue?

YES NO

10. Do you know who to talk to if you are worried or have any other questions to ask?

YES NO

11. Has anyone forced or put pressure on you to take part in this research?

YES NO

12. Are you willing to take part in the research?

YES NO

_________________________ ____________________

Signature of Child **Date**

APPENDIX EIGHT: Permission to conduct research in the Kunene region – from the regional director.

REPUBLIC OF NAMIBIA
KUNENE REGIONAL COUNCIL
DIRECTORATE: EDUCATION, ARTS AND CULTURE
DIRECTOR'S OFFICE

Tel: 09264 67 - 335000
Fax: 09 264 67 -332226

Private Bag 2007
KHORIXAS

Ref: 13/2/9/1

03 March 2021

Mr. Given Matengu

Dear Sir

REQUEST FOR PERMISSION TO CONDUCT RESEARCH

Your letter dated 01 March 2021, bears reference.

Permission is granted to carry out your research for your PhD on "**Teachers use of authentic tasks through mathematics trails in a mobile learning environment to facilitate conceptual teaching in Kunene Region.**

You have to consult and present the approval to the respective principals. This activity should not interrupt the normal curriculum activities.

We humbly request you to share your research findings with the Directorate.

Thank you for your understanding in this regard.

Yours faithfully,

03/03/2021

MRS. ANGELINE. A. JANTZE
DIRECTOR OF EDUCATION, ARTS AND CULTURE
KUNENE REGION

"*Ensuring that every child has access to quality Inclusive Education; promoting and preserving Arts and Culture*"

APPENDIX NINE: Permission to conduct research in the Kunene region – from the three school principals.

REPUBLIC OF NAMIBIA

MINISTRY OF EDUCATION, ARTS AND CULTURE – KUNENE REGION
[redacted] – OPUWO CIRCUIT

Enq: [redacted]
Cell:
Email:

[redacted]

Dear **Mr G.K. Matengu**

Permission to conduct research.

I acknowledge receipt of your request for permission letter dated 06 April 2021 to conduct research at [redacted]

- Permission is hereby granted you to conduct your PhD research at the school.
- As indicated in the request for permission letter, you will have to seek permission from the concerned teachers and learners for their participation in your study.

I appreciate the intention of you sharing the findings of your study with the school.

Yours in Education

Principal
[redacted]

MINISTRY OF EDUCATION
KUNENE REGION
OFFICE OF THE PRINCIPAL
2021 -04- 08
TE [redacted]

REPUBLIC OF NAMIBIA

KUNENE REGIONAL COUNCIL

DIRECTORATE OF EDUCATION

[redacted]

Inspectorate: Opuwo

Ensuring that every child has access to quality education

[redacted]

Tuesday, 23rd March 2021

To: Mr Given Matengu
Mathematics Teacher
Opuwo

Dear Sir

RE: REQUEST FOR PERMISION TO CONDUCT RESEARCH

Your letter dated 12th March 2021, has reference.

Permission is hereby granted to carry research in the school. My office have given you permission to engage with the three teachers, but the final say lies with them.

As indicated in your request letter, please I will be glad if you are willing to share your findings with my office.

Sincerely yours

[redacted]

Kunene Regional Council
Directorate Education, Arts and Culture
The Principal
[redacted]
2 ? MAR 2021
P.O. Box 6
Opuwo
Tel: 065-273003 Fax: 065-273033

REPUBLIC OF NAMIBIA
KUNENE REGIONAL COUNCIL
DIRECTORATE: EDUCATION, ARTS & CULTURE
DIRECTORATE: EDUCATION

25 March 2021

Dear Mr G.K. Matengu

SUBJECT: Re-request to conduct research at

Your letter dated 23 March 2021 bears reference.

I hereby grant you permission to carry out your PhD research project at
However, please take note that the participation of teachers and learners depends on their consent and willingness.

I look forward to receiving the copy of your final thesis as promised in the request letter.

I wish you the best in your academic journey

Yours sincerely

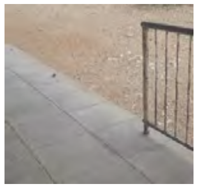
DIRECTORATE OF EDUCATION, ARTS AND CULTURE
KUNENE REGIONAL COUNCIL
25 MAR 2021

The School principal

APPENDIX TEN: Authentic/realistic tasks created for the MCM mathematics trails of this study.

	Area, Ratio	*Area*	*Area, Volume*	*Area*	*Area*	
TR1A 679425	**The lamp Pole Task** (3740862) If you want to repaint this lamp pole to red colour, calculate the surface area that needs painting to the nearest cm².	**The Electricity** Box Task (0836109) Find the surface area of the metal sheet that would be needed to replace the door of the electricity box. Write your answer to the nearest cm².	**The roof task (5654825)** Calculate the volume of the roof of this object to the nearest cubic centimetres.	**The Iron sheet Task** (3954830) Calculate the surface area of the iron sheet to the nearest 10 cm².	**The Trapezoidal Task** (1856193) Assume you are to repaint this trapezoid shaped block of concrete, workout the surface area to be painted to the nearest 100 square centimetres.	
						
	Area	*Area, Volume*	*Area, Volume*	*Area*	*Area, Volume*	*Area, Ratio, Proportion*
TR1B 0312123	**The window frame Task** (4542037) Calculate the area reserved for the space of the window frame in square meters to the nearest three decimal places.	**The hollow cylinder Task** (0312123) Calculate the amount of concrete needed to replace this hollow concrete cylinder. Give your answer to the nearest 100 cubic centimetres.	**The traffic road sign task** (4741172) Calculate the amount of metal that makes up this traffic road sign board. Write your answer to the nearest square centimeters.	**The Electricity box Task 1** (3954829) Calculate the surface area of the doors on this electricity box to the nearest square centimetre.	**The metal door Task** (5754831) Work out the amount of metal needed to put a door on this door frame. Write your answer to the nearest square centimeters.	**The Veranda Task** (0636112) If a tile measures 40cm × 40cm, how many tiles will be needed to cover the surface area of the veranda?
						

TR1C 2516091	*Area*	*Area, Volume, Ratio*	*Area, Volume*	*Area*	*Area*	*Area*
	The concrete Task (0770719) Assuming that you are to repaint the top part of this concrete block to a different colour. Work out the area of the part to be painted to the nearest square cm.	**The staircase Task** (1531547) Assuming that the size of one brick is 300mm x 261. 25mm x 165mm, workout the total number of bricks that makes up these staircases.	**Water drainage Task** (0370709) Work out the volume of this water drainage culvert pipe to the nearest $100cm^3$.	**The Trapezoidal Task** (1370706) Calculate the painted surface area. Give your answer to the nearest square cm	**Area of sewage** (531779) Calculate the area of the top concrete. Answer to the nearest 10 square cm.	**The dustbin Task 1** (3668229) If you were to replace this dustbin with a new one. Work out the amount of metal sheet needed to replace the bin. Give your answer to the nearest cm^2.
						
TR2A 5612172	*Area*	*Area, Volume*	*Ratio, Proportion*	*Area, Volume*	*Area, Ratio*	
	The road Sign Task (0441948) Calculate the surface area in red colour reflection to the nearest 10 square centimetres.	**The Sewage Task 3** (2551597) Assuming that the depth of this sewer is 3 meters. Calculate the capacity of the sewer to the nearest litres.	**Age of the camelthorn tree** (5654724) Determine the age of this camelthorn tree. In this part of the country, a camelthorn tree with a circumference of approximately 100cm is about 30 years old. Assume that the circumference grows proportionally. Give your answer to the nearest year.	**Volume of the water tank** (7830036) The school caretaker wants to know the capacity of this water tank when filled with water. Give the answer in liters.	**The painting wall Task** (8954735) If 1 liter of paint can cover 6 square meters of the wall, how many milliliters of paint will you need to paint the exterior frontside of this building? Give your answer to the nearest millimeters.	
						

	Area, Volume	*Area*	*Area*	*Area, Volume*		
TR2B 2516091	**The dustbin Task.** (3435989) Calculate the volume of the dustbin to the nearest litre.	**The door Task.** (6754824) Work out the area covered by the door to the nearest hundred square centimetres.	**The sewer Task 1** (4641949) Assume that you are asked to replaster the top surface of this sewer, calculate the area to be plastered to the nearest square centimeter.	**The Block Brick Task** (6954827) Calculate the amount of concrete that makes up this brick block. Give your result to the nearest ten cubic centimetres.		
						
	Area, Volume	*Area*	*Area, ratio, proportion*	*Ratio*		
TR3A 1516102	**The volume Task 3** 3770695 Work out the volume of this sewage tank if its depth is 2m. Write your answer in cubic meters to the nearest 1 decimal place.	**The concrete cylinder pipe Task** (4556189) Calculate the amount of paint that makes up this cylindrical pipe to the nearest cubic centimetres	**The lawn grass Task** (6866513) If 12 liters of water can cover an area of 1.5 square meters, work out how much water will be needed to water this raised lawn grass bed to the nearest milliliter.	**Metal railing Task** (5936108) Work out the number of small metal bars that were cut from the rest of the metal railing.		
						

	Area, Volume	*Area, Proportion*	*Area, Volume*	*Measurement*		
TR3B 5716072	**The Volume Task** 1 (0466062) Work out the total volume of the two concrete blocks to the nearest cm³.	**The painting Task** (5654833) If 250ml of paint covers an area of 1.5 squared meters. How many liters of paint will you need to paint this section of the wall? Write your answer to the nearest thousandth	**The drainage culvert task 1.** (3642295) When it heavily rains in Opuwo the velocity of water through one of these pipe drainage culvert is 6 m/s. What is the flow rate of the water in one pipe to the nearest liter per second (L/s)?	**The pole Task** (5866064) If you were to dig a hole to put in this street light pole (by submerging the concrete part below the surface of the sand). What would be the depth of the hole?		
						
	Area, Volume	*Area, Volume*	*Area*	*Ratio*		
TR3A 4516109	**Pipe Culvert Task** (1868230) Work out the amount of concrete that makes up this culvert pipe. Give your answer to the nearest cubic centimeter.	**The Tank stand Task.** (2870738) Calculate the amount of concrete that makes up this tank concrete stand. Give your answer in cm³ to the nearest 2 decimal places.	**The road sign Task 1** (2770727) Calculate the amount of metal used to make this road sign board. Write your answer to the nearest square cm.	**The road curbs Task** (4568235) Workout how many of these complete curbs will be needed to cover a distance of 2km.