

**INVESTIGATING A WAY OF TEACHING TRANSFORMATION
GEOMETRY IN GRADE 9 APPLYING VAN HIELE'S THEORY AND
KILPATIRCK'S MODEL: A CASE STUDY**

A thesis submitted in partial fulfilment of the
requirements of the degree of

MASTER OF EDUCATION
(Mathematics Education)

Of

RHODES UNIVERSITY

by

NOKUZOLA HLALELENI-GEJA

NOVEMBER 2014

ABSTRACT

Transformation geometry has been neglected in our schools because teachers are often not proficient enough to teach it, as it was not part of the syllabus during their training. The study investigates effective ways of teaching transformation geometry in grade 9, applying van Hiele's theory (1986) of geometry teaching and learning and Kilpatrick's model of mathematical proficiency. The teaching programme activities require consistent use of physical manipulatives by the teacher for effective teaching, learning and understanding of geometric concepts. The type of study is a case study. Data collection tools are: - baseline evaluation, teacher & learner interviews (pre & post programme intervention) and observation (pre & post) during the implementation of the teaching programme. Results were analysed both qualitatively and quantitatively. My research findings show some improvement of learner performance after the application of the programme. Baseline evaluation shows that some learners attained below and above 30%. Interviews showed that some learners had problems before the implementation of the programme and some problems were eliminated by the use of the programme activities and learning progression was evident.

Learner performance showed that learners had acquired some knowledge and critical thinking and reasoning skills, reflection skills, communication through LOLT improved, commitment to activities of the programme and teaching practice had improved. Learner performance showed that a learner can be in two different levels at the same time. Consistent use of manipulatives resulted in effective teaching and learning of geometry in grade 9.

The results of this research support other researchers' views of teaching geometry using van Hiele's theory (1986) and Kilpatrick *et al.* (2001). Shaw (2002) argues that teaching geometry with manipulatives enhances conceptual understanding by the learner. In my opinion, it also promotes immediate intervention by the teacher as soon as the learner picks an incorrect object. The project enhanced and improved levels of communication between the learner, teacher and others in the classroom.

ACKNOWLEDGEMENTS

I would like to express my sincere gratitude to the following people who had put an effort to my work and made this thesis a success:-

- To my supervisor Dr Bruce Brown for his patience, support and guidance to make this project a success.
- To the Rhodes Department of Mathematics for organising sessions organised by Prof Marc Schafer, I thank him so much.
- To the EDO of the circuit, who had motivated and supported me in pursuing this research in selected schools in the neighbourhood.
- To principals of schools where the study was conducted for allowing me to conduct the pilot programme in their schools. To the principal of my school and the SGB for their understanding and patience when I requested permission to go out and perform research activities to other schools and disrupting the school programme sometimes.
- To the teachers involved in the research who sacrificed their time helping me to pursue the research project under their tight schedules.
- To all learners of schools who worked long hours on allocated activities of the teaching programme.
- To Mr Rob Kraft for helping me with academic writing without complaining.
- Ms Judy Cornwell for editing and proofreading my thesis.
- To the Eastern Cape Department of Education for awarding a bursary this year.

DEDICATION

I dedicate this work to my parents, my father Zamble George Hlaleleni and my mother Nomvula Nonagethe Hlaleleni who motivated and supported me at all times to pursue my studies. To my late husband Xoliso Stanford Geja, my only daughter Iviwe Apendule and my two sons Athenkosi Bhutise & Indibabale Silindokuhle and family, for the continued support they have put on me during the period of study.

DECLARATION OF ORIGINALITY

I, Nokuzola Hlaleleni – Geja, declare that this thesis is my own work which is written in my own words. In aspects whereby I used other people's ideas to clarify concepts, quotation marks and referencing have been used according to the Rhodes Departmental guidelines.

SIGNATURE:

DATE:

STUDENT NO. : g00g3747

LIST OF TABLES

Table	Page
1. Plan of the Teaching Programme	25
2. Participants involved during research study	36
3. Make up activity results for Activity 1(a)	50
4. Summary of research learners' results for Activity 2.....	50
5. Summary of learners' abilities in shapes & transformation	51
6. Summary of Activity 3 results	53
7. Learners results for Activity 4	56
8. Results of Activity 5	58
9. Summary of Activity 6	63
10. Summary of Activity 7	66
11. Summary of Activity 8	70

LIST OF FIGURES

Figure	Page
1. Results of learners for baseline evaluation	45
2. Learners' results for Activity 1(a) & 1(b)	49
3. Learners' results for Activity 2(a) & 2(b)	53
4. Learners' results for Activity 3 & 4	56
5. Results for Activity 5 & 6	59
6. Results of learners for Activity 7	65
7. Learners' results for Activity 8	71
8. Learner D's performance	75
9. Results of Learner A's performance for all activities	86

ACCRONYMS & ABBREVIATIONS

1. SMT – School Management Team
2. NCTM – National Council of Teachers of Mathematics
3. NAEP – National Assessment of Education Progress
4. TIMSS – Third International Mathematics and Science Study
5. LOLT – Language of Learning and Teaching
6. IQ - Interview Questions
7. TP – Teaching Programme
8. LO – Lesson Observation
9. LTP – Letters To Parents
10. LTEDO – Letter To the Education Developing Officer
11. LA – Learner Activities
12. LOPEDO – Letter Of Permission from Education Developing Officer
13. L1 – Learner 1, 2, 3, 4, 5
14. LA – Learner A, B, C, D, E, F, G, H
15. TB – Teacher B
16. TC – Teacher C
17. EDO – Education Developing Officer
18. PLO – Pre- Lesson Observation
19. SMT – School Management Team
20. NCTM – National Council of Teachers of Mathematics
21. NAEP – National Assessment of Education Progress
22. TIMSS – Third International Mathematics and Science Study
23. LOLT – Language of Learning and Teaching
24. IQ - Interview Questions

25. TP – Teaching Programme
26. LO – Lesson Observation
27. LTP – Letters To Parents
28. LTEDO – Letter To the Education Developing Officer
29. LA – Learner Activities
30. LOPEDO – Letter Of Permission from Education Developing Officer
31. L1 – Learner 1, 2, 3, 4, 5
32. LA – Learner A, B, C, D, E, F, G, H
33. TB – Teacher B
34. TC – Teacher C
35. EDO – Education Developing Officer
36. PLO – Post Lesson Observation

LIST OF GRAPHS

Graph	Page
37. Results of learners for baseline evaluation	46
38. Learners' results for Activity 1(a) & 1(b)	51
39. Learners' results for Activity 2(a) & 2(b)	54
40. Learners' results for Activity 3 & 4	58
41. Results for Activity 5 & 6	61
42. Results of learners for Activity 7	67
43. Learners' results for Activity 8	73
44. Learner D's performance	77
45. Results of Learner A's performance for all activities	90

TABLE OF CONTENTS

Abstract	(i)
Acknowledgement	(ii)
Dedication	(iii)
Declaration of originality	(iv)
List of tables	
List of figures	
Acronyms and abbreviations	
List of graphs	

CHAPTER 1: INTRODUCTION OF THE STUDY

1.1 Introduction to the chapter	1
1.2 Background & rationale to the research study	1
1.3 The research goals	2
1.4 Research questions	3
1.5 Thesis overview	3
1.5.1. Chapter 1	3
1.5.2. Chapter 2	3
1.5.3. Chapter 3	3
1.5.4. Chapter 4	3
1.5.5. Chapter 5	4
1.5.6. Chapter 6	4
1.5.7. Chapter 7	4

CHAPTER 2: LITERATURE REVIEW

2.1. Introduction	5
2.2. Historical background in S.A. education system levels	5
2.3. Van Hiele's levels	6
2.4. Characteristics of van Hiele's levels	9
2.5. The strands of proficiency	10
2.6. What is transformation geometry?	14
2.7. Using manipulatives in the classroom	15

2.8. Learning transformation geometry	16
2.9. Conclusion	18

CHAPTER 3: THE TEACHING PROGRAMME

3.1. Introduction	20
3.2. Design and structure of the Teaching Programme	21
3.3. Sample of activities of the Teaching Programme	23
3.4. Plan of the teaching programme	25
3.5. Learner progression through the Teaching Programme	30
3.6. Conclusion	30

CHAPTER 4: METHODOLOGY

4.1. Introduction	31
4.2. Research goals	31
4.3. Orientation, design and methods	31
4.4. Van Hiele's levels	32
4.5. Data collection tools	34
4.6. Research sites and participants	36
4.7. Sampling techniques	36
4.8. Ethical issues	36
4.8.1. Anonymity	37
4.8.2. Right to withdraw	37
4.9. Profile of participants	37
4.10. Validity and reliability	38
4.11. Conclusion	38

CHAPTER 5: DATA ANALYSIS

5.1. Introduction	39
5.2. Table of participants	39
5.3. Learner analysis	39
5.3.1. Baseline evaluation results	39

5.4. Learner interviews	40
5.4.1. Pre-programme learner interviews- School B	40
5.4.2. Post-programme learner interviews- School B	41
5.4.3. Post learner interviews- School C	41
5.5. Teacher analysis	42
5.5.1. Pre-lesson observation (Teacher B)	42
5.5.2. Post- lesson observation (Teacher B)	43
5.5.3. Pre-programme interview (Teacher B)	45
5.5.4. Post-programme interview (Teacher B)	46
5.5.5. Teacher interviews	48
5.6. Impact on learners of the use of levels in teaching	48
5.7. Importance of manipulatives in teaching	49
5.8. Analysis of activities	51
5.8.1. Activity 1(a) & 1(b)	51
5.8.2. Activity 2	52
5.8.3. Activity 3	56
5.8.4. Activity 4	60
5.8.5. Activity 5	61
5.8.6. Activity 6	63
5.8.7. Activity 7	67
5.8.8. Activity 8	69
5.9. Findings	73
5.10. Discussion of Learner D's work	75
5.11. Impact of the programme on learners	77
5.12. Weakness of the programme	80
5.13. Support given by the Teaching Programme	81
5.14. Using van Hiele's levels & phases to determine learners' geometric thinking	83
5.15. Using Kilpatrick's strands of learning in the classroom	83
5.16. Conclusion	84

CHAPTER 6: RESULTS & DISCUSSION

6.1. Introduction	85
6.2. Main findings	85
6.3. Analysis & findings about the impact van Hiele's theory had on learning	92
6.4. Analysis of the impact of Kilpatrick's model (2001) on learning	94
6.5. Using Kilpatrick's strands to determine geometric thinking	96
6.6. Relationship between two theories	96
6.7. Findings about using manipulatives during research in the classroom	97
6.8. The value of the two theories in the classroom during interviews	97
6.9. Conclusion	98

CHAPTER 7: CONCLUSION

7.1. Introduction	99
7.2. Summary of findings	99
7.3. Limitations	101
7.4. Avenues for further study	101
7.5. Personal reflection	101
7.5.1. Knowledge of learners	102
7.5.2. Teacher development & empowerment	103
7.5.3. Mathematical language	104
7.5.4. Financial constraints	104
7.6. Research conclusions	105
7.6.1. Learner development	105
7.6.2. Baseline evaluation	105
7.6.3. Problems encountered during implementation	105
7.6.4. Constraining factors	106
7.6.5. Challenges experienced during data collection	106
7.5.11. Recommendations	107
7.5.12. Conclusion	108

Appendix A: Letters

- (i) Request permission from schools
- (ii) Request permission from the Education Developing Officer
- (iii) Response from the EDO
- (iv) Response from the principal
- (v) Request permission from parents

Appendix B: Interviews

- (i) Pre-programme interview- Teacher B (TB)
- (ii) Post-programme interview- TB
- (iii) Pre-programme learner interviews- School C
- (iv) Pre-programme learners interviews-School B
- (v) Post- programme learner interviews -School C
- (vi) Post-programme learner interviews- School B

Appendix C: Teaching programme activities**Appendix D: Make up assignment****Appendix E: Learners' work/Pictures****Appendix F: Pre & Post lesson observation transcriptions****Appendix G: Van Hiele assessment table**

CHAPTER 1: INTRODUCTION AND OVERVIEW

1.1. Introduction

This chapter introduces the research study. It includes the introduction, the research goals and research questions. It then provides a brief overview of the remainder of the thesis.

1.2. Background and rationale to the study

I started teaching in the former Transkei in the early nineties, when mathematics learners were struggling to get teachers who specialized in mathematics. It was evident in the marking centers of that time that learners had a barrier to learning mathematics, especially geometry. In my experience in these rural schools, learner performance in geometry is generally poor and as a result, learners hate geometry and mathematics, thinking that mathematics is a difficult subject.

Teaching mathematics in an abstract manner seems to lead to memorization of facts for marks and to pleasing the teacher without understanding the meaning of what the learner is learning. I also think that the constant use and dependency on textbook activities encouraged the teacher not to plan lessons to be taught in the classroom. I wanted to know what causes learners and some teachers to hate teaching and learning geometry in the classroom; hence I decided to research this topic about transformation geometry.

This research is motivated by the high failure rate in mathematics, especially in grade 9 geometry in the past and present. The reason might be because of teaching and learning methods that do not effectively engage learners in the learning process and may lead to barriers to learning mathematics. The use of the teaching programme which involves the proper use of manipulatives in geometric activities in the classroom may provide some solution to the problems cited above. Attending regular workshops on the topic could also make a difference to the teacher before the teacher gets an opportunity to upgrade qualifications in mathematics.

As a member of the School Management Team, I noticed that many teachers are dependent on the prescribed textbook examples and teach without researching the topic and developing practical experience to enhance lesson understanding.

Transformation geometry has been introduced as a new topic in the senior phase. It was not in the syllabus when most teachers were trained in the training colleges, and thus seems neglected most of the time at this level.

It also appears difficult for learners to tackle it at FET level where it is also part of the curriculum. I wanted to research effective ways of teaching transformation geometry in a manner that is interesting and motivating to both teachers and learners.

I wanted to know what causes the challenges and negativity that people have in teaching and learning mathematics in the classroom and how it may be possible to eliminate these challenges. I wanted to help learners and teachers with activities that could reduce negative attitudes towards mathematics, which results in stereotypes that mathematics is difficult, is only for ‘clever people’ and that geometry is problematic and boring. I also think that the use of approaches building research into mathematics and geometry teaching could result in more effective teaching and learning of transformation geometry in the classroom.

Some of the reasons why teachers dislike transformational geometry and geometry in general, could be those I cited above. This dislike results in a poor foundation in geometry at primary level; namely, learning mathematics by rote and possibly copying work from other learners for the purpose of getting a pass mark and moving on to the next class. The results of this approach to teaching will thus follow our learners wherever they go.

In my view, when learners understand the concepts under discussion they will be able to more confidently answer questions posed. The activities of the programme will enable the learners (from those of lesser ability to those of greatest ability) to remain engaged with the content, thus providing for differentiated instruction.

This project researches the implementation of a teaching programme that was designed according to the levels and phases of van Hiele’s (1986) theory and the strands of proficiency proposed by Kilpatrick, Swafford, and Findell (2001). The activities are planned in such a way that manipulatives are consistently used in the classroom, in order to teach the concepts to learners. This consistent use of manipulatives may have a positive effect on learners for conceptual understanding. My hope is that this research will contribute to improved teaching practice and enhanced learner understanding.

1.3. Research goal

The goal of this research is to investigate a way of teaching transformation geometry in grade 9, with the use of manipulatives that apply van Hiele’s (1986) theory and the model of Kilpatrick *et al.* (2001).

1.4. Research questions

The research goal will be pursued by answering the following specific research questions:

- How did the implementation of the teaching programme influence the teacher's classroom practice and the teacher's understanding of learners' learning styles?
- How did the implementation of the teaching programme influence the learning of transformation geometry in the classroom?
- How did the support programme for teachers influence the teaching and learning of transformation geometry in the classroom?

1.5. Thesis overview

1.5.1. Chapter 1

This chapter presents the background and rationale of the study which includes (i) The context of the research, (ii) The research problem, (iii) The research questions and goals, and (iv) The thesis overview.

1.5.2. Chapter 2

This chapter reviews relevant literature to the research study. This includes a discussion of the frameworks developed by van Hiele (1986) and Kilpatrick *et al.* (2001), as well as a discussion of the use of manipulatives for learner understanding of concepts.

1.5.3. Chapter 3

This chapter deals with the development of the teaching programme with learner activities on geometry and transformation geometry, as a tool of the research study. The activities of the teaching programme include the use of manipulatives in the classroom. Aims and marking rubrics for each activity are included.

1.5.4. Chapter 4

This chapter details the research methodology used in this study. The research is a qualitative case study. The chapter describes the methodology used, in terms of orientation and design of research tools. It describes how participants were selected and who they are, where research has taken place, methods of data collection and analysis and how sampling was done.

1.5.5. Chapter 5

This chapter presents the research data and provides an initial thematic analysis. It includes a question by question analysis of programme activities and an analysis of learners' work.

1.5.6. Chapter 6

This chapter describes and discusses the results of the research study. It summarizes all the findings of the research. It also states clearly what the significance of study was, what limitations the study had, and the recommendations of the study.

1.5.7. Chapter 7

This chapter sums up the research, discusses what worked well with the teaching programme and why as well as what did not work well and why. It is the concluding chapter of the research project and discusses avenues for further study and includes a personal reflection.

CHAPTER 2 – LITERATURE REVIEW

2.1. Introduction

This chapter reviews literature relevant to my research study. It includes the historical background of mathematics education in South Africa. It also includes a brief explanation of van Hiele's (1986) levels and phases of learning; then discusses the strands of Kilpatrick *et al.* (2001), the nature of mathematics and transformation geometry and the challenges facing the teaching of mathematics. A discussion is also given of the use of manipulatives for conceptual understanding in the geometry classroom and the space for manipulatives in the curriculum. Finally, the relevance of the van Hiele (1986) and Kilpatrick *et al.* (2001) models to this research, and the benefits of using the two theories are presented.

2.2 Historical background of mathematics in the South African education system

Khuzwayo (2000) argues that mathematical education for Blacks in South Africa has never been in a healthy state. The quality of mathematics teaching in Black schools has always been questioned. There was a shortage of adequately trained secondary level teachers. Gaulin (1985) and Kirby (1990) argue that geometry has been neglected in education and students lack academic experience with spatial displays and appropriate skills and strategies for dealing with them.

I fully agree with the statements above. They remind me of my time as a student, being taught by less proficient teachers in teaching mathematics who were allocated to teach it because it had to be taught. Teaching and learning was theoretical at my school and no resources were ever used. This teaching and learning of mathematics was influenced by the structure of the mathematics classroom, the availability of educational resources and the political environment during the apartheid era, aligning well with the discussion of Khuzwayo (2000).

The Third International Mathematics and Science Study (TIMSS) and the National Assessment of Education Progress (NAEP) have collected data that show that South African learners' performance in geometry at all levels is quite alarming (Lapan, 1999). This triggered my research interest, to investigate a way of improving it through the use of a teaching programme that included the use of manipulatives. Lapan (1999) stated that we can improve students' knowledge and abilities to visualise and reason about the spatial world in which they live. He further argues that spatial instruction can be effective once basic skills have been mastered by the

learners. According to Kilpatrick *et al.* (2001, p. 120), manipulatives can help learners understand mathematical concepts well, help them avoid many critical errors and connect arithmetic and geometry as a link to more advanced mathematics. Soedjadi (2000) supports my hypothesis and the experience I had of seeing teachers teaching geometry without the use of manipulatives by stating that ‘abstract concepts and formulas are often introduced by teachers without paying much attention to aspects such as understanding, reasoning and logic’.

In my experience, some teachers depend only on textbook activities in the classroom, without relating these to the practical context used and seen daily at home and in the community. Kirby and Boutler (1999) argue that using a textbook alone in the classroom is disadvantageous to learners, because learners only memorize and follow given procedures without understanding, learning only for marks and pleasing the teacher. In doing this, learners learn procedures by rote without relating them to any appropriate form of conceptual knowledge (Brown & Burton, 1978 and Brown & van Lehn, 1982). They further state that learning by rote seems to lead learners to keep information for only a short time in their memory, because learners do not understand the concepts nor make any connections and relationships that could help them to recall the work when needed.

In contrast to this, it is argued by Carpenter (1975, p. 113) that, “Conceptual knowledge involves a rich network of relationships between pieces of information, which permits flexibility in accessing and using the information”. Manipulatives promote conceptual understanding of the learner (Kilpatrick *et al.*, 2001). Clements and Battista (1992) argue that there are learners who can’t even recognize shapes; that is, they are still at a pre-recognition level and can’t even reason at the level below the basic level of van Hiele’s taxonomy (the level of recognition).

2.3. Van Hiele’s levels

According to van Hiele’s theory (1986), learners learn geometry by progressing through the following levels:

- a) Level 1: Recognition/Visualization. At this level learners identify, name, and compare shapes according to their appearance but cannot identify properties of shapes. A learner can recognize and describe what they see in the mirror when seeing their image in it. They know that it is themselves that they see in the mirror without further explanation about what else they notice in terms of

the direction in which they and the image are facing

- b) Level 2: Analysis. At this level learners are able to give properties of shapes through observation and looking at relationships that might be there in shapes that they are comparing. In this case a learner is able to say that the original object looking at the mirror is looking at different directions / positions with the mirror image i.e. one is looking forward while the other is looking backward. No connections are made at this level.
- c) Level 3: Informal deduction. At this level learners form generalizations about previously learned rules, develop informal arguments to show generalizations to be true and make connections among figures. Learners are able to recognize relationships between and among properties of shapes. They use informal deduction language, they can do problem solving e.g. all objects and images are alike, what the object does also is done by the image e.g. blinking, smiling etc. If a mirror is broken, there will be no image shown or reflection will take place
- d) Level 4: Deduction. At this level learners are able to prove theorems, make formal deductions and relationships on work given.
- e) Level 5: Rigor. At this level the learner is able to establish theorems in different systems of postulates and compare or analyze deductive systems, (van Hiele, 1986). Non-Euclidean geometries can be studied and different systems can be compared (Clements, 2003).

Although van Hiele identified five levels, level 5 (Rigor) is not relevant to my learners as it refers to levels of geometry beyond the Further Education and Training curriculum. It is stated by Bennie and Newstead (1999) of the Malati project that levels 1 and 2 have an impact in the learning of transformation geometry and I fully agree with these views.

The levels named above work hand in hand with the phases established by van Hiele (van Hiele, 2001, Mason, 1998). These describe how learners may be helped to master geometry in a step by step fashion, with the aim of understanding the concepts under discussion before carrying out the procedures – a manner of teaching supported by Kilpatrick *et al.* (2001). Van Hiele has some input about transformation geometry

as shown in the table given below.

Slaugnessy (1995) has placed the study of isometric transformation in the van Hiele's framework as follows:-

Van Hiele levels	Characteristic thinking	Thinking objective	How?
1. Recognition	Pupil has a global, non-mathematical view of transformations	To recognize each isometry as a movement, and to perform isometries using appropriate manipulatives. To recognize the invariance of shape and space under an isometry.	By exposing them to use more manipulatives and do a lot of sorting.
2. Analysis	Pupils consider the properties of an isometry	To explicitly use the properties that characterize an isometry. To discover new properties of isometry by experimentation. To discover composition of isometries. To use mathematical notation & vocabulary for isometry.	By exposing them to do more experimentation of properties.
3. Ordering	Students can discover and use properties and relations between	To justify and use properties of isometry. To use and understand formal definitions of	The learners should be given extra work to go and investigate

	isometry. Can follow mathematical reasoning & make informal arguments.	isometries. To understand and use the intersection of perpendicular bisectors to determine the centre of rotation. To understand simple proofs.	themselves properties and relationships between isometry.
--	--	---	---

Bennie and Malati (1999) of the Malati project argue that van Hiele’s levels have an impact in transformational geometry especially levels 1 & 2 as follows and I fully agree with them.

2.4. Four characteristics of van Hiele’s (1986) levels

According to van Hiele (1999, p. 310), “School geometry that is presented in a similar axiomatic fashion assumes that students think in a formal deductive level whereas it is not the case, and they lack prerequisite understandings about geometry. This lack creates a gap between their level of thinking and that required for geometry that they are expected to learn”. Van Hiele and his wife noticed that there were problems pertaining to the learning of geometry in mathematics by learners. They found out that the learner has to work through levels of learning, in order to make sure that each geometric concept under discussion is mastered by the learners. The levels discussed above could influence the learning of geometry in schools in the following ways:

- a) These levels are sequential and a student has to pass through all prior levels to arrive at a specific level.
- b) Levels are not age dependent in the way Piaget (1988) described development. These levels have no age restrictions as even adults and teachers follow them when learning geometry and may be at level 1 or 2 depending on what geometry they have acquired (van Hiele, 1986). “Development is more dependent on instruction than on age or biological maturation, and types of instructional experiences can foster or impede development”. Van Hiele (1999, p.311).
- c) Geometric experiences have the greatest influence on advancement through

the levels.

- d) Instruction and language at a higher level than the level of the student may inhibit learning.

Van Hiele (1986), in discussing the learning of geometry, recommends that teachers should follow the following phases when planning lessons:

- a) *Information*: The teacher checks the prior knowledge of learners, linking and preparing them for the new lesson to be discussed.
- b) *Guided orientation*: The teacher gives instructions to learners on hands-on activities, and gives them time to think and explore, applying the skills they can remember.
- c) *Explication*: The teacher at this level introduces relevant mathematical terminology and gives learners a chance to describe what they have learned in their own words, linking this together with conceptual understanding.
- d) *Free orientation*: At this stage learners are asked to make relationships between concepts; they are asked to investigate and solve problems given by the teacher, on their own.
- e) *Integration*: Learners are asked to link what they have learned with what is new, and to make their own summaries and discover relationships between objects.

2.5. Strands of proficiency

The five strands of mathematical proficiency suggested by Kilpatrick *et al.* (2001) are the following:

- a) *Conceptual understanding* which refers to an integrated and functional grasp of mathematical ideas. “Students with conceptual understanding know more than isolated facts and methods. They understand why a mathematical idea is important and the kind of contexts in which it is useful. Knowledge is organized into a coherent whole which enables them to learn new ideas by connecting those ideas to what they already know. It also supports retention as facts and methods learned with understanding are connected, easier to remember and use and they can be reconstructed when forgotten. Learners

monitor what they remember and try to figure out if it makes sense”.

(Kilpatrick *et al.*, 2001, p.118). “When learners have grasped the concept, they are able to make connections between two or more problems”.

(Kilpatrick *et al.*, 2001, p.119). This helps the learners to avoid making many critical errors when solving problems, and results in students having less to learn because they can see similarities between superficially unrelated situations.

- b) *Procedural fluency*. Kilpatrick *et al.* (2001, p.115) argue that, “Procedural fluency is a skill in carrying out procedures flexibly, accurately, efficiently and appropriately, and it requires understanding of the learner”. “Procedural fluency refers to knowledge of procedures, knowledge of when and how to use them appropriately and skill to use them flexibly, accurately and efficiently.” Kilpatrick *et al.* (2001, p. 121). It is needed to support conceptual understanding and mental methods of calculation. I fully agree with Silver (1982) when arguing that procedural knowledge must rest on a conceptual knowledge base that forms a support system for procedural knowledge.
- c) *Strategic competence*. Kilpatrick *et al.* (2001, p.124) argue that, “Strategic competence is the ability of the student to formulate mathematical problems, represent them and solve them”. Skemp (1978) argues that learners who possess relational understanding will be better able to solve novel problems. Relational understanding leads to better transfer and wider application of mathematical knowledge.
- d) *Adaptive reasoning*. Kilpatrick *et al.* (2001, p. 129) argue that, “Adaptive reasoning is the capacity to think logically about relationships among concepts and situations”. This includes being able to show proofs and justify statements in a logical manner. In my experience, this strand is lacking in our classroom as learners do not want to give reasons for the statements they have made. This has been affected by the the quality of mathematics teaching in Black schools as argued by Kuzwayo (2000) that it has always been questioned and there was a shortage of adequately trained secondary level teachers.
- e) *Productive disposition*. Kilpatrick *et al.* (2001, p.131) argue that, “Productive

disposition refers to the tendency to see sense in mathematics to perceive it as both useful and worthwhile, to believe that steady efforts in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics”.

Strands a, b and c above may have a positive impact in transformation geometry; hence they are used in this research. Kilpatrick *et al.* (2001, 121) argue about the need to use the strands for effective teaching and learning of geometry in the classroom.

According to Kilpatrick *et al.* (2001, p. 369), “*The teacher needs to have a clear vision of the goals of instruction and what proficiency means for the specific mathematical content one teaches. Teachers need to know what they teach as well as the horizons of mathematics – where it can lead and where the students are headed with it. The teacher must be able to interpret students written work, analyze their reasoning, and respond to different methods they might use in solving problems. That is, teachers need to be proficient in the subject they teach*”.

Proficiency in teaching, according to Kilpatrick *et al.* (2001, p. 369) is related to effectiveness, consistently helping learners learn worthwhile knowledge (mathematical content). They argue that, “Teachers’ professional development should be high quality, sustained and systematically designed and deployed to help all students develop mathematical proficiency”. Kilpatrick *et al.* (2001, p.370-372) argue that teachers need to have a knowledge base of teaching mathematics which involves the following:

- a) Knowledge of mathematics which includes knowledge of mathematical facts, concepts, procedures and relationships among them. This also includes knowledge of mathematics as a discipline, the nature of discourse in mathematics and the norms and standards of evidence that guide evidence and proof. The teacher will take into consideration the goals of instruction and provide the basis for discriminating and prioritizing those goals. The teacher will be able to understand concepts correctly and perform procedures accurately, as well as understand the conceptual foundation of that knowledge. Kilpatrick *et al.* (2001, p.371).
- b) Knowledge of students’ abilities will help the teacher with the knowledge

of how learners learn mathematics, the general knowledge of how various mathematical ideas develop in children over time as well as specific knowledge of how to determine where in a developmental trajectory a child might be. This helps the teacher to be familiar with common difficulties students have with certain mathematical concepts and procedures, and it encompasses knowledge about learning and about sorts of experiences, designs and approaches that influence students' thinking and learning. Kilpatrick *et al.* (2001, p.371).

- c) Knowledge of instructional practice will provide the teacher with knowledge of the curriculum, knowledge of tasks and tools for teaching important mathematical ideas, knowledge of how to design and manage classroom discourse, and knowledge of classroom norms and values that support the development of mathematical proficiency. Kilpatrick *et al.* (2001, p.372).

Effectiveness as argued by Kilpatrick *et al.* (2001) depends in enactment on the mutual and interdependent interaction of the 3 elements: content, teacher and the students. Teachers need to prepare their lesson beforehand, have their own programmes to use in the classroom everyday and use manipulatives for understanding and quality teaching and learning. Kilpatrick *et al.* (2001, p.10) support the statement above by arguing that, "Effective programmes of teacher preparation and professional development help learners understand the mathematics they teach, how their students learn that mathematics and how to facilitate that learning".

It is not enough for students to understand how to perform mathematical tasks, they must appreciate why each of these ideas and relations work the way they do. Skemp (1971) argues that well constructed knowledge is linked together so that when one part of a network of ideas is recalled for use at some future time, the other parts are also recalled; hence learners should have a firm foundation of conceptual knowledge. The importance of teacher proficiency as cited by Kilpatrick *et al.* (2001) is supported by other researchers (Usiskin, 1987; Swafford *et al.*, 1997; Clements, 2000) who state that the level of understanding that students achieve for any concept is limited by the level of understanding of their teacher. Thus, a teacher who never specialized in teaching mathematics may not be proficient enough to teach it and this would affect

learners' performance in the classroom. Usiskin (1987, p. 20) argues that, "We cannot expect elementary school teachers to teach a broadened curriculum in mathematics if at college they have only taken arithmetic".

"Proficiency for all demands that fundamental changes be made concurrently in the curriculum, instructional materials, classroom practice, teacher preparation and professional development. These changes require continuing, coordinated action of policy makers, teacher educators and parents." (Kilpatrick *et al.*, 2001, p.10).

Proficiency in teaching, according to Kilpatrick *et al.* (2001) entails versatility i.e. being able to work effectively with a wide variety of students in different environments and across a range of mathematical content.

Reynolds and Muys (1999) argue that effective teaching is signified by ways of providing opportunities to learn, such as quality of classroom management, and time-on-task. Achievement is improved when teachers create classrooms that include effective question-answer and individual practices, and substantial feedback to students (Brophy, 1986; Brophy & Good, 1986; Borich, 1989). Using appropriate assessment techniques in the classroom as stated in chapter 3 might give the teacher the effective feedback desired on what the learners know, do not know and inform the teacher on what to do next to achieve the required set goals.

2.6. What is transformation geometry?

Transformation means change; when a figure moves from its original place, the size, shape and the area of the figure do not change and the two figures are congruent or equal in all respects. Transformation geometry is a branch of geometry. It is the study of rigid motion i.e. translation, rotation, reflection and congruence or rigid invariance (Williford, 1972).

Douglas *et al.* (1994) argue that transformation geometry involves the physical or mental manipulation of shapes to new positions or orientations. It involves three types of rigid motions – slides (translation), flips (reflections) and turns (rotations) (Howe, 1998). Movements are slides (translations) such as when a figure is moved in a page. A translation moves every point of a figure or space by the same amount in a given direction. Flips (reflections) are when a figure is turned over in 3-D & turns (rotations) are when a figure is rotated without being flipped. Movements can be qualitative (slides) and quantitative, when sliding the figure 2 units horizontally and 3 units vertically.

Kirby and Boutler (1999) argue that transformation geometry is a subset of geometry in which students learn to identify and illustrate movements of 2-D & 3-D shapes. Learners would normally be expected to know the 2D and 3D shapes and their properties. Transformational geometry is considered important in supporting children's development of geometric and spatial thinking (Hollebrands, 2003; Jones & Mooney, 2003). Teachers need to be proficient in teaching it for better understanding of the concepts by learners.

Boutler and Kirby (1994) argue that transformation geometry involves the physical or mental manipulation of shapes to new positions or orientations. Williford (1972) further argues that skills gained in transformational geometry are the basis of much spatial thinking in subjects such as art, sculpture and architecture which require manipulating designs and models, and creating blueprints.

New technologies in education are placing more emphasis on visual and spatial skills; those required to inspect, encode, transform and construct information in visual displays. Students are asked to navigate through virtual spaces; these developments acquaint learners with the underlying nature of visio-spatial ability, how it is related to academic performance and how it can be improved (Kirby & Boutler, 1999). This links well with transformation geometry as learners would be asked to transform shapes using their visio-spatial ability and accuracy in movements, making sure that they are applying the transformations required at an appropriate place with correct coordinates.

2.7. Using manipulatives in the classroom

Manipulatives, as argued by Schweyer (2000), are any objects or materials from the real world that children move around to: show a mathematics concept; develop mathematical ideas and critical thinking; and promote active participation in the classroom. The use of manipulatives has no age limit and is suitable for all ages and ability groups of learners in the classroom as argued by Schweyer (2000).

Manipulatives enhance the abilities of students at all levels to reason and communicate (Shaw, 2002). The use of manipulatives gives learners the opportunity to describe what they see and to identify connections and differences. Shaw (2002) argues that manipulatives help students to develop conceptual understanding of mathematical ideas by representing the ideas in multiple ways. Also, if the student fully understands the concept or idea, then fear of the subject is lessened.

Manipulatives and models are valuable resource tools for engaging students in the language and communication of mathematical ideas and concepts (Shaw, 2002).

“Students need opportunities to test their ideas on the basis of shared knowledge in the mathematical community of the classroom to see whether they can be understood and if they are sufficiently convincing” (NCTM, p.61).

Kilpatrick *et al.* (2001, p. 354) argue that, “Manipulatives also help students correct their own errors, provide valuable support for student learning when teachers interact over time with students to help them build links between the object, the symbol and the mathematical idea both represent”. Working with manipulatives deepens understanding of concepts and relationships, makes skills practice meaningful and leads to retention and application of information in new problem-solving situations (Shaw, 2002). The use of manipulatives “Develops students’ relational thinking and generalized ideas” (Moyer-Packerman, Salkind, & Bolyard, 2008, p. 204) and as a result students demonstrate gains in mathematics achievements and understanding (Moyer-Packerman, Salkind, & Bolyard, 2008). When used well, they can enhance student understanding, enable teachers and students to have a conversation that is grounded in a common referential medium and they can provide material on which students can act productively provided they reflect on their actions in relation to the mathematics being taught” (Kilpatrick *et al.*, 2001, p.353).

2.8. Learning transformational geometry

Mastery of 2-D patterns is a necessary foundational skill for those involved in genetic research, molecular biology, studies in crystal structure and decorative arts (Howe, 1998). It is argued by Bennie and Newstead (1999) of the Malati project, that van Hiele’s levels have an impact in transformational geometry as mentioned in section 2.3.above. Bennie and Newstead (1999) further argue that, “Much of this work on transformations has however been restricted to the perceptual level i.e. pupils have been given opportunities to physically manipulate figures using cut-outs, paper folding, geo-boards, tracings etc. Learners are seldom challenged to transfer their perceptual understanding to a representational level, i.e. to perform the operations mentally. It is the ability to perform transformations mentally that is valuable in mathematics”.

Learners have to use their senses when dealing with transformation, in particular sight as they have to see the objects given. I fully agree with Gutierrez (1996) when arguing about visualization that it is the kind of reasoning ability based on the use of visual or spatial elements, either mental or physical, performed to solve problems or to prove properties. Transformation geometry involves visualization and observation by learners. Visualization involves the integration of four components which are: mental images, external representations, processes of visualization, and abilities of visualization. Students must be able to visualize a shape or problem depending on the problem to be solved or image to be created, and choose among several visual abilities with different foundations.

Finally, Bennie and Newstead (1999) argue that teaching and learning of transformation geometry can be valuable in the following ways:

- a) As a means to develop spatial skills.
- b) As one method for studying plane geometry.
- c) As a means to integrate mathematical topics which have traditionally been studied separately e.g. the study of plane figures using co-ordinate geometry.
- d) As a topic of study on its own right.
- e) The transformations themselves can be regarded as the objects of study.
- f) Transformations can be used to explore other concepts, but the properties of transformations can be explored in their own right.

Vygotsky's theory developed in the 1920's and 1930's described learning as the process by which a student achieves a mature understanding of concepts and functions. Vygotsky (1962, p. 82) argues that, "A concept is more than the sum of certain associative bonds formed by memory, more than a mere mental habit; it is a complex and genuine act of thought that cannot be taught by drilling but can be accomplished only when the child's development itself has reached the requisite level. This requires teachers to scaffold instructions to students".

Learning of transformation geometric concepts is truly a process and learners need to understand all the concepts. I fully agree with Vygotsky's above argument because as learners are involved with hands-on activities, it gives them an opportunity to communicate verbally when answering probing questions asked by the teacher and in

writing, showing and improving their creativity and understanding as they also learn the meanings of terms in their mother tongue and the medium of instruction.

The National Council of Teachers of Mathematics (NCTM) Principles and Standards for School Mathematics (2000, p. 16) states, "Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well". The principles above are also relevant and apply to the South African education system too.

To be able to determine what students know and then to determine what they need to learn requires content knowledge at or above what is expected of students, hence Kilpatrick *et al.* (2001) emphasize the importance of teacher proficiency in the subject one teaches so that the teacher is enabled in their efforts to help learners learn mathematics in an efficient way.

NCTM principles and standards for school mathematics (2000, p.41) provide a description for the geometry standard that states: "Instructional programs from pre-kindergarten through grade 12 should enable all students to:

- a) Analyse characteristics and properties of 2D & 3D geometric shapes and develop mathematical arguments about geometric relationships.
- b) Specify locations and describe spatial relationships using co-ordinate geometry, and other representational systems.
- c) Apply transformations and use symmetry to analyse mathematical situations.
- d) Use visualization, spatial reasoning, and geometric modelling to solve problems. The principles above link with suggestions that the current South African curriculum makes in that learners should be able to recognise 2D and 3D shapes at foundation phase level to prepare them for the upper levels of learning.

2.9. Conclusion

In conclusion, this chapter presents the review of literature in geometry and transformation geometry and motivates the use of manipulatives in teaching for conceptual understanding, to make the lesson clear to learners through manipulating the objects at hand.

Kilpatrick *et al.* (2001, p. 369) argue that, "Teachers must have a clear vision of the goals of instruction and what proficiency means for the specific mathematical content they are teaching. They need to know the mathematics they teach as well as the

horizons of that mathematics, where it can lead and where their students are headed with it”. “Students need to be motivated to engage productively in mathematics lessons. The students should also interact with teachers and with their mathematical tasks” (Kilpatrick *et al.*, 2001, p.339).

Manipulatives, as argued by Kilpatrick *et al.* (2001, p.353) can enhance students’ understanding when used well and enable teachers and students to have a conversation that is grounded in a common referential medium.

CHAPTER 3: THE TEACHING PROGRAMME

3.1. Introduction

The research required a planned teaching programme with activities to be implemented in the classroom. The teaching programme consists of geometric activities in general and specific activities on transformation geometry. The focus is on activities based on translation, reflection and rotation with the use of manipulatives to build up learners' understanding from what they know to what they do not know and is new to them. The teaching programme has principles, structure and sample activities of learners which are level based.

The aim of the research is to find ways of teaching transformation geometry in the classroom that incorporate the following:-

- a) Building on prior-knowledge: Learners prior knowledge will influence what they learn, starting from what they know to what they do not know, which will give the teacher a chance to see gaps and know where to start filling those gaps that learners already have.
- b) Baseline evaluation questions would inform the teacher as to what learners know and what they do not know; introductory questions preceding the lesson could also be of good help to the teacher and good feedback from the learners.
- c) Use of manipulatives in the classroom will help the teacher to explain concepts better than when teaching without using them: "Manipulatives require careful use over sufficient time to allow students to build meaning and make connections" (Kilpatrick *et al.*, 2001. p. 353). Giving learners a chance to use manipulatives themselves, and engaging them in hands on activities will give them a sense of responsibility and independence in order to see what happens when they play with the manipulatives - finding answers on their own instead of depending fully on the teacher.
- d) The principled use of van Heile's (1986) levels of understanding and phases of teaching, as well as Kilpatrick *et al.*'s (2001) strands of proficiency to orient and guide the teaching and learning in the programme.

Assessment of learners at the end of the learning unit would also help in knowing whether learners have understood the concepts discussed or not. The teacher would also think of diagnostic ways of helping learners improve areas of misconceptions.

The activities of the teaching programme require sharing information acquired both verbally and in writing. This will widen the horizons of communication by learners. Learners will give feedback to classmates in some activities. They will be given a chance to present what they know and how they did an activity in class using manipulatives; the classmates will ask questions and give constructive comments on how it should have been done if they find out that there are errors in the activity, helping the presenter to take note of what was expected of him/her in future activities related to that one. This will give learners an opportunity to learn from their peers and be able to accept the correction given by others. Proficiency takes time to develop in the classroom as Kilpatrick *et al.* (2001) argue, depending on strategies used by the teacher and the type of learner the teacher is teaching, and whether they own their learning or not, it develops continually during learning time and not only at the end.

3.2. Design and structure of the teaching programme

The teaching programme consists of a variety of activities accommodating the strands, levels and phases of the theorists mentioned above (for full details see Appendix C). The teaching programme is designed and structured according to certain guiding principles. Before designing the teaching programme, a baseline evaluation was carried out to check the knowledge of the topic that learners had already acquired. The activities of the teaching programme were implemented after the baseline evaluation was written. This enabled effective planning of the teaching programme activities.

A baseline evaluation is a standardized test, a battery of tests, or observational procedures designed to establish the attainment levels of learners at a significant point (<http://www.encyclo.co.uk/visitor-contrib>). All the activities are level based according to van Hiele's levels (1986) and place each learner at the level at which they function. Each activity has a marking rubric together with the role of the teacher per activity and what is expected of the learner in each activity. Almost all the activities require the use of manipulatives for conceptual understanding and effective

teaching and learning. Learners had to move from one level to the next in activities of the programme. In order for learners to meet the requirements of the level concerned they need to demonstrate mastery of activities based on that level.

The programme starts with simple activities at low van Heile levels, in the expectation that learners could understand them easily before introducing the complex ones.

Activities in the programme involved the transformation of manipulatives that can be described in terms of slides (translation), turns (rotation) or flips (reflections) that leave the figure unchanged in terms of its shape and size. Learners will be involved in hands-on activities in order to understand and be familiar with the concepts, as Williford (1972) argues that knowledge can be gained at a basic level through personal involvement and interaction.

3.2.1. Structure of the teaching programme

The following is a diagram illustrating the principles of the teaching programme.

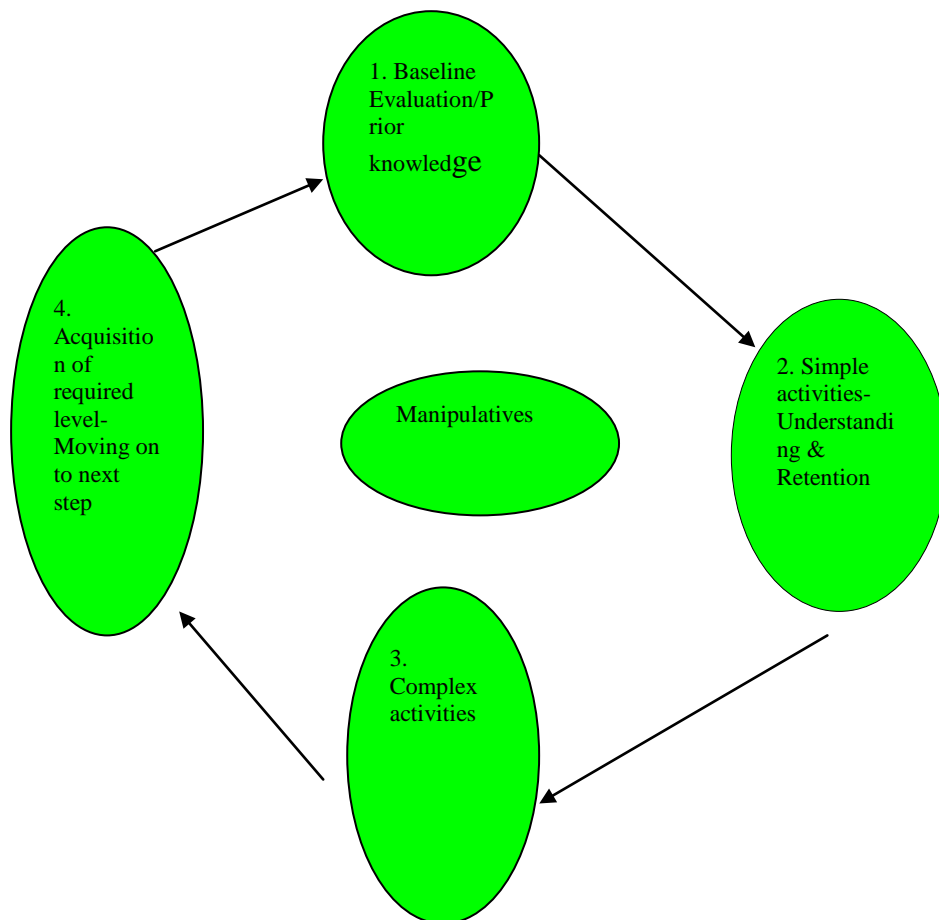


Figure 1: Principles of the teaching programme

3.3. Sample of activities of the teaching programme

The activities were planned and prepared according to the levels and phases of van Hiele's theory (1986) and the strands of Kilpatrick *et al.* (2001). The activities had thinking levels which a learner was expected to achieve and move on to another level based on performance and understating of what is asked by the question. Activities encouraged learners to solve problems and to think critically about the problem instead of just giving answers, as the goal was conceptual understanding and placing to a certain level.

The following is an example of one of the activities in the teaching programme.

Questions were structured as in this example, but the style differs from question to question. The questions were followed with a marking rubric in some activities (see Appendix C for full details of the programme activities).

Activity 1(a) - level 1 activity

Learners are engaged in an activity of sorting shapes. Answers given to the questions below led the researcher to assign levels to the learner depicted by the way the learner answers and reasoned about the questions.

Sort out the shapes given for activity 1 and give reasons why you sorted them the way you did. Record your findings. Probing questions such as the following are asked:

- Pick a square from the shapes given.
- How do you know that you have picked a square? Give reasons for your answer.
- Is a square a rhombus? Why and why not?
- What is a rhombus? Why do you think that the shape is a rhombus?
- What is the name of shape no.2?
- Why do you call the shape above a triangle?
- Pick a rectangle.

1(b)-Level 2 questions

- How do you know that it is a rectangle and not a square?
- Is a square a rectangle? Why do you say so?
- Is a triangle a quadrilateral? How do you know that?

- Have you ever seen quadrilaterals in real life other than in a book? Where and where else?
- What kind of shapes do you normally see around you? If you do not see shapes, how can you notice the presence of shapes wherever you go.
- Can you differentiate between shape no.7 and shape no.9?
- Give the difference in your own words.

Marking the activities according to the table [Refer to Appendix J] gives information about the level each learner is in. The mark and level will inform the teacher where to start in order to help learners master the requirements of the levels they appear to be in. More activities per level will be given to help learners catch up with the levels they missed and iron out the misconceptions they might be having in each level. Use of manipulatives will guarantee the improvement of learners in these activities and engage learners in hands on in the activities asking them questions throughout.

Activity 3

The aim of this activity is to check learners' prior knowledge about the transformation geometry they did in grade 8. It also tests whether learners understand the terms and are able to express themselves in their own words, to check where they are in terms of knowledge and where to start to help them understand the transformations from a very basic level to a complex one according to grade 9 syllabus.

If the learners still struggle with the activity, they need to be given a chance to make many drawings, explaining the terms above and matching the name with the drawing so that they gain more understanding of the meaning of each.

Activity 4

The aim of this activity was to check learners' prior knowledge and understanding about shapes, reasoning and symmetry. Learners were asked to draw shapes, butterflies, trees and letters of the alphabet and divide them into two or more equal parts and to differentiate between objects or letters that show symmetry, and those that do not. Activity 4, emphasizes the transfer of the learners' perceptual understanding to a representational level involving performing transformations mentally, (Refer to appendix C for further details on activities of the programme).

The activities of the teaching programme require consistent use of manipulatives which require learners to use objects brought to the class for better understanding of the lesson or concept under discussion. It is envisaged that the consistent use of manipulatives in classroom activities will help to revive the dormant skills of teachers such as thorough planning and lesson preparation before going to class. This could revive teachers, in my opinion, to be diligent in using manipulatives in order to bring home new concepts to learners and give them better chances of making sense of what is being taught in the classroom, and link it with what is taking place in their everyday environment.

3.4. Plan of the teaching programme

The plan of the teaching programme below guides and informs the reader about what the programme entails (Refer to Appendix C).

Table 1: The teaching programme

Day	Activity	Goal	Strand/Level
1	Engage learners in an activity of sorting out shapes according to appearance, number of sides & name. Ask probing questions like ‘how do you know that you have picked up a square? Why don’t you call it a rhombus?’ These questions are testing learners’ prior knowledge of shapes and what knowledge they have about shapes.	Review of previous grade’s work. Developing an experience with hands-on activities in geometry. To prepare learners to be able to give properties of shapes in the second level and third level. Questions asked do not need deeper insight, they just prepare the learner for the understanding of concepts to be taught, with the aim of gaining skills to be able to differentiate one shape from another, and the ability to work with others in the classroom in a good atmosphere.	Conceptual understanding. Visualization or Recognition

2	Learners are given a task to classify shapes given, giving properties of shapes copying and filling in the given table. This is done in groups or pairs	Assessment of the developmental level the students are in; students have fun and enjoy what they do; promoting interest that might develop towards a liking of the subject. Knowledge of properties of shapes will also be known by the teacher. Integrating properties of shapes that are common with the shape that has been picked by the teacher. This encourages them to work in a co-operative atmosphere with an aim of learning from each other and helping each other when one experiences a problem, together with respecting the ideas of others.	Analysis Procedural fluency
3	Description of Properties with sketches.	Give learners a chance to communicate breaking barriers of language by using the LOLT. Skills in oral communication and in writing; application of language use.	Strategic competence. Informal deductions. Conceptual understanding
4	They are given the opportunity to draw lines of symmetry of different objects such as a butterfly, tree, letters of the alphabet; polygons are given	To test their knowledge and skills and how they apply them in the classroom, can they think critically about the activities given or not? Are they able to draw lines of symmetry of different objects or not? If not, how can they be helped to overcome these barriers and apply the knowledge they have learned correctly in the classroom.	Conceptual understanding. Procedural fluency
5	Creation of an assessment instrument such as a checklist for each learner with 4 level	Check their level of understanding of the work discussed earlier so as to devise	Conceptual understanding

6	<p>descriptors to check whether they have mastered the work done or not</p> <p>Introduction of transformation geometry concepts such as reflection, rotation, translation, enlargement, reduction, glide reflection etc. Simple questions are posed such as, ‘What do you understand by the word transformation?’ ‘What is actually taking place when a figure is transformed?’ ‘Are there any differences you notice when you transform an object/figure in different forms? If yes, what are they?’ ‘Name anything that you think can transform in your experience.’ This can test how they apply knowledge based on experiences of what they know, understand and can do; can they think logically or not? What can be done to help them achieve the stated goal?</p>	<p>a means for catch-up activities to be in line with other members of the group.</p> <p>Give them a chance to manipulate objects to get a deeper understanding of what each term means and how each transformation takes place.</p>	<p>Conceptual understanding.</p> <p>Visualization.</p> <p>Procedural fluency</p> <p>Strategic competence.</p> <p>Adaptive reasoning</p>
7	<p>Learners are given a chance to visualize what each activity entails, explore the work as they are engaged in hands-on activities.</p>	<p>Test how they apply knowledge based on their experiences of what they know and understand; can they think logically or not? What can be done to help them achieve the stated goal?</p>	<p>Visualization</p> <p>Conceptual understanding</p>
8	<p>Learners are engaged in transformation geometry activities and are asked to perform rotation activities of different kinds.</p>	<p>Knowledge of the word/concept rotation, observation skills to develop conceptual understanding</p>	<p>Conceptual understanding</p>
9	<p>Continuation of activities on rotation, stressing the terminology used such,</p>	<p>Let them be able to link concepts, and understand the terminology being used e.g.</p>	<p>Adaptive reasoning</p> <p>Conceptual</p>

	clockwise, counter-clockwise and that anti-clockwise means the same thing as counter-clockwise using different shapes.	clockwise is linked with the clock and how it moves so that they gain a clear picture of the concept 'clockwise' and anti-clockwise/counter-clockwise as the opposite of clockwise and demonstrate it practically in the clock that has been brought in the classroom. This can develop their logical thinking, understanding of concepts taught, and retention of what has been taught.	understanding. Fluency. Strategic competency
10	Practical activity on reflection. Learners are asked to take mirrors and look for their reflections and comment on what they see in the mirror or water, and also go outside to view themselves on parked cars and say what type of transformation they see there and record everything down in their class work books.	To develop conceptual understanding, procedural fluency, strategic competence, adaptive reasoning as they are conversing and thinking about what is taking place in each activity and record down as evidence of having participated in the activity. Understanding, formation of relationships between types of transformations, observation skills develop, knowledge also is expected to be gained on each type of transformation.	Conceptual understanding, Fluency Competence Adaptive reasoning. Recognition Analysis Informal deductions
11	Continuation of reflection activities represented on a Cartesian plane.	Knowledge of the concept and understanding.	Conceptual understanding
12	Translation activities starting with a simple activity of sliding a book practically before using shapes and pictures	Understanding – knowledge, skills attained, critical & creative thinking	Conceptual understanding Adaptive reasoning
13	Translation activities continue and are represented on a Cartesian plane, use of ordered pairs and naming should be done by learners.	Knowledge of labeling quadrants correctly; ability to see change in an object and its ordered pairs. Skills gained, logical thinking	Adaptive reasoning. Formal deduction

14	More activities on translation on a Cartesian plane, learners are asked to label the ordered pairs on letters given, what happens to the ordered pairs when a figure is translated from one place to another? Do the ordered pairs remain the same or are there any changes that can be noticed?	Knowledge of being able to visually translate the object from one place to another, ability to write ordered pairs correctly, ability to answer questions related to the given figure under translation.	Conceptual understanding Fluency Competence Adaptive reasoning
15	Activities on reflection are discussed deeply and learners are given the opportunity to reflect a variety of figures and notice the changes that take place when a figure is reflected, also the way the figure appears should be noticed.	Learners are expected to gain knowledge of reflection as a form of transformation; Skills relating to being able to transform the figure or word in a reflective manner, notice changes that might occur.	Conceptual understanding
16	More activities on reflection and glide reflection are discussed and learners are given many activities to work on reflection checking whether they have mastered it or not and check areas of misconceptions around group work and individual activities.	Understanding of learners is observed through written work, Skills learners gain may help them read and reflect words and figures they come into contact with in the future.	Conceptual understanding
17	Revision of all the activities done in the classroom starting from symmetry, translation, reflection, glide reflection & rotation.	To check knowledge and skills learners have acquired on transformation geometry, what problems they might still be having and design some strategies around dealing with them so that all learners remain in the same boat in terms of understanding, although they might not be at the same level.	Conceptual understanding Analysis Informal deduction
18	Assessment in the form of an activity to make sure that what has been taught and revised has been understood by learners. After the activity they will be	Investigation skills of learners will be assessed at this stage, their understanding, retention, integration with related themes and topics.	Fluency Competence Adaptive reasoning.

engaged in a post interview about the whole topic of transformation geometry and how it can help them in a real life context.		
---	--	--

3.5. Learner progression through the Teaching Programme

The teaching programme is designed in such a way that learners should be able to develop the strands of Kilpatrick *et al.* (2001) and van Hiele's (1986) levels through the phases of learning. Learners who didn't perform well during baseline evaluation will be exposed to additional activities to help them gain the conceptual understanding that seemed to have been lacking in the baseline evaluation. The teaching programme will involve consistent use of manipulatives to enhance student understanding as argued by Kilpatrick *et al.* (2001, p.353).

3.6. Conclusion

The activities of the programme are designed in such a way that the teacher will be able to see learner progression as there is a baseline evaluation preceding the programme. Activities are level based and start with simple geometric activities up to complex ones, involving consistent use of manipulatives for further understanding. Activities on their own will encourage the learner to assess him/herself and see what s/he knows about the concept as shown by the level the learner has achieved.

CHAPTER 4: METHODOLOGY

4.1. Introduction

“Methodology is the achievement of coherence by applying methods that complement each other to produce data and findings that match the purpose of the research and answer the research questions” (Henning *et al.*, 2004, p.36).

This chapter describes the methodology used in the research study. It includes a discussion of the research orientation and the design of the research tools. The research is a case study in the interpretive paradigm. The research is a qualitative investigation conducted within this paradigm as it is argued that it affords the researcher an opportunity to understand and interpret the world in terms of its actors (Cohen, Geddes & Tischler, 2000, p. 180).

The chapter explains how participants and research sites were selected, how data was collected, how the process of analysis was followed, what problems were experienced and how were they curbed during the research. Finally it discusses the research ethics I (the researcher) adhered to.

The chapter also discusses data analysis, which is the way of looking for "underlying themes and other patterns" characterizing the case more broadly than a single piece of information can reveal (Leedy & Ormrod, 2005, p.136). This involves “working with data, organizing it, breaking it down, synthesizing it, searching for patterns that might be there, discovering what is important, what is to be learned and deciding what to tell others.” (Southwood & Spanneberg, 2000, p. 60).

4.2. Research goal

To research the possibility of enhancing teachers’ practices of teaching transformation geometry by means of a supported teaching programme that includes the use of manipulatives, based on van Hiele’s levels of understanding of geometry, phases of teaching geometry and Kilpatrick’s model of mathematical proficiency.

4.3. Orientation, design and methods

This research is a qualitative investigation conducted with an interpretive orientation. “This affords the researcher an opportunity to understand and interpret the world in terms of its actors.” (Cohen, Geddes, & Tischler 2000, p.180). The interpretive paradigm emphasizes the understanding and interpretation of the subjective (classroom) experiences of the participants involved i.e. (learners and teachers) in the

study, as argued by Connole (1998). In order to use the levels of van Heile (see Section 2.3) to understand the learner’s activity, the following table was used for analysis:

Table 12: van Hiele’s level assessment table

LEVELS	PROBING QUESTIONS	POSITIVE RESPONSE	NEGATIVE RESPONSE
1. Recognition/Visualization. Learners sort shapes according to appearance without mentioning their properties	Pick a square, how do you know that it is a square? Pick up a rhombus. How do you know that you have picked a rhombus?	When a learner is able to pick the correct shape and answer questions that it looks like the shape wanted.	When a learner is confused of which shape to pick, picks the wrong shape and can’t give a correct answer or keep quiet during the questioning time
2. Analysis. Learners are able to give properties of shapes without making any connections or relationships. They reason in terms of properties and do not understand the relationships between properties at this stage.	Is a rectangle a square? How do you know? Is a triangle a quadrilateral? Why?	If the learner is able to give the properties of shapes asked. If they are able to support their statements.	No knowledge of properties. No reasons given or no answer given on properties and no response.
3. Informal deduction / Ordering. At this level learners are able to order logically the properties of shapes. They are able to make connections, relationships and see	Is a parallelogram a rectangle? Why? What is the relationship between the two? Is there any	If learners are able to give correct responses to the questions asked and are able to give good reasons for their	If learners are unable to see any connections between the shapes compared or they can’t see any difference

differences between properties of shapes.	<p>difference you see between the two shapes?</p> <p>What is the difference between the two?</p> <p>Is a square a rhombus? Why and why not?</p> <p>Is a square a rectangle? Why do you say so?</p>	<p>answers.</p> <p>Yes, and give the difference.</p> <p>Yes. All sides are equal.</p> <p>Yes. All angles are equal and are right angles.</p>	<p>between properties of shapes.</p> <p>No. It looks different. Yes without reason to please the teacher.</p> <p>No response at all.</p>
---	--	--	--

Three different forms of data were collected in this research: (a) observation (pre & post), (b) interviewing, and finally, (c) assessment, where learners were tested on activities that they had done. Information was gathered through classroom observations before the programme was implemented, after workshopping the programme with teachers and after implementation with learners. Activities such as class work, tests, homework, assignments and projects were used to gather data on the teaching and learning strategies used. Interviews also took place before and after the implementation of the programme, reflecting on teacher practice in the classroom. The programme was implemented in the classroom by the researcher and one additional teacher. The following stages were used:

Pre-Observation stage: I first observed a lesson presentation by the teacher in the classroom before the implementation of the teaching programme. This pre-observation helped me to understand how the teacher taught and how learners participated and understood the lesson presented to them by the teacher.

Pre-Interviewing stage: I interviewed both learners and the teacher at pre-interview level to provide data on what learners knew about transformation geometry before the teaching programme was implemented. I also wanted to find out the teacher's

understanding and how the teacher taught, including what was taught, what problems were experienced in their classroom, whether manipulatives were used, etc.

Workshopping: After the two preliminary stages, I conducted a mini workshop with the two teachers I intended to work with (only one of whom carried out the teaching), discussing the levels and strands suggested by the two theorists van Hiele (1986) and Kilpatrick *et al.* (2001).

Post-observation: This was a lesson observation of how the teacher taught after the programme had been introduced and workshopped.

Post-interview: I had a post interview session with the teacher and with the learners. Interviews took place after the lesson observation with regard to the effectiveness of the programme activities in the classroom. The interviews investigated problems experienced during implementation of the programme, how the learners performed in the set activities, and how the levels helped the teacher and the learner in terms of monitoring performance in the classroom. Participants were also encouraged to discuss their ideas and find out what worked well or not in the programme, as well as identifying possible improvements. Learners preferred their mother tongue throughout the interview sessions, hence the translations given. Answers in brackets translate what the learner says in their mother tongue into English.

Assessing stage: Learners were assessed almost daily to check their responses to the activities in the programme and the challenges they experienced during the program implementation. The aim of this stage was to identify the level at which each learner functioned in these level based activities.

4.5. Data collection tools

This research was a qualitative investigation where data was gathered in the form of interviews and observations that took place before and after the implementation of the teaching programme; to give a complete picture of what happened before and after.

The tools below show how data was collected in each stage:

Pre-interviews. These gave the researcher feedback on the knowledge learners have about the topic before the implementation of the teaching programme, what they know and what they do not know which provided the researcher with information on where to start with the activities of the teaching programme. (Refer to appendix B, Pre-interviews). Bray (2007, p. 68) argues that, “A pre-observation interview probes the teacher’s lesson image for the lesson to be observed. This includes elaboration of

details regarding the goals and activities of the lesson as well as the elaboration of anticipated students' response including potential difficulties and teacher plans to address them. In addition, this interview asks the teacher to consider ways in which lesson planning was influenced by his/her particular teaching context".

Post-interviews. These gave information on how much knowledge learners had acquired about the topic, after it had been taught (Refer to Appendix B, Post interviews). Post interviews in this research will focus on whether the teaching programme had any influence in changing the way the teacher approached their teaching. Post observation interviews, according to Bray (2007, p. 69) are designed to elicit a teacher's general reaction following the observed lesson as well as elaboration of her instructional thinking at particular points in a lesson.

Pre-lesson observations. These gave information on how the teacher was teaching before the introduction of the theory.

Post-lesson observations. These gave the researcher information about whether the teacher changed her teaching style after the workshop, how learners performed when programme activities were implemented in the classroom, what differences the activities have shown to learners, and what challenges the programme implementation caused in the classroom. These observations were designed to elicit a teacher's general reaction following the observed lesson as well as elaboration of her instructional thinking at particular points in a lesson (Bray, 2007, p. 69).

Activity sheets. Learners were given papers daily to write on the given activities, providing feedback to the teacher on what had been taught in the classroom on a particular day. These were used by learners when writing/ implementing activities in the classroom. These were only collected from learners of school C who participated in all the activities of the programme, (Refer to Appendix E).

Rubrics. Tools that helped the researcher in marking the activities fairly for all learners, (Refer to Appendix C-Teaching Programme).

Informal journal of lessons. This is a notebook where I wrote notes on what to do and when and which helped with planning and reporting.

Workshop notes. A notebook for keeping records of meetings and a mini-workshop that was held at the beginning of the year.

Informal meetings held. These included the opinions I shared about the project to be introduced in the classroom, opportunities we might be missing as teachers,

difficulties encountered before, during and after the implementation of the programme and gains the programme had brought in teaching and learning.

4.6. Research sites and participants

I, the researcher was the participant teacher in school C; and I taught an after hour programme with the learners in grade 9 class. The research was carried out in a district in the Eastern Cape where I chose two neighbouring schools. School B had 20 learners and is a Junior Secondary school from grade R to 9, and school C is also a Junior Secondary school from grade R to 9 with 60 learners in one congested class. Research focused solely on grade 9 classes in both schools. I planned to select 30 learners in school C and work with them but conditions were not favorable and so I had to work with the whole class of 60.

4.7. Sampling Techniques

Sampling for interviews was done randomly from learners of school B & C before the programme started, for participation before and after the implementation of the teaching programme activities. Only 5 learners were selected randomly for pre & post interviews. Participation in classroom activities was done by all learners in the classroom. Sampled learners' work was used for analysis and interpretation. Teacher B was also involved in interviews and classroom observation before and after the implementation of the teaching programme.

4.8. Ethical issues

I assured the participants that they would remain anonymous throughout the research and confidentiality was kept throughout. I wrote letters (Refer to appendix A (i) - (v)) requesting permission to conduct research at school B & C to the Department of Education. Letters were addressed to the Education Developing Officer, to the principals of schools of school B & C, and to the parents of learners who were willing to participate in the research asking for permission to pursue the research. Learners from both schools were very enthusiastic to participate in activities conducted throughout the research process.

I informed them that participation was voluntary and that they were free to withdraw anytime during the course of study should they feel comfortable to do so.

4.8.1. Anonymity

Before the implementation of study, I wrote letters of permission to the EDO, principals of schools, parents and discussed the issue of anonymity. It was clearly stated in letters that confidentiality will prevail throughout the research process and that all participants will remain anonymous, hence research sites were referred to as School B & C with teachers B & C.

4.8. 2. Right to withdraw

I told all the participants that the research is not binding them to anything as they participate voluntarily and that they have the right to withdraw at any time (Dane, 1990). Learners participated as the whole class in activities given and concepts had to be taught to the whole group as time was limited and the concepts are examinable. School A withdrew itself from the programme and school B didn't finish all the activities.

4.9. Profile of participants

Teacher B

She has taught mathematics at School B Junior Secondary School for 20 years. She is a qualified mathematics teacher with the following qualifications:

- a) Primary Teachers Diploma at Bensonvale College and
- b) ACE in mathematics obtained at Rhodes University. She is very passionate about teaching mathematics to learners.
- c) She is the Head of Department at school B and we have been working together since the commencement of the pilot programme.

Pre-programme classroom observation and interviews showed that Teacher B never taught transformation geometry in grade 9 before and she was going to teach it for the first time through the implementation of the teaching programme (Refer to Appendix B (i) - Pre-interviews line 009).

Teacher C (The researcher)

She has taught mathematics at School C Junior Secondary School since 2006. She had been teaching mathematics in the senior phase ever since she started teaching in 1990 at a Junior Secondary School. She has the following qualifications:-

- a) Primary Teachers Diploma obtained at Mount Arthur College.

- b) B.A. degree obtained from UNISA.
- c) BED in Management from Potchefstroom University.
- d) Bed Hons in Math Education & ICT from Rhodes.
- e) Computer literacy certificate.
- f) Completing Masters in Math Education at Rhodes.
- g) She is the Head of Department at school C.

4.10. Validity & reliability

Validity is an important key to effective research. It is measured by clear goals and objectives (Moskel & Leydens, 2000). The goals of this study are achievable. Validity in qualitative research has principles such as the following, to qualify it to be reliable: the researcher is part of the researched world, the researcher is the key instrument of the research- hence the analysis of the work the researcher taught, data collected are presented in terms of the respondents rather than the researcher and seeing and reporting the situation through the eyes of participants (Lincoln & Guba, 1985; Bogdan & Biklen, 1992).

The interviews, lesson observations before and after the programme was implemented as well as learners' work provided a rich source of data that contributed to the validity and reliability of the results. Observations of participants participating in classroom activities assisted the researcher in collecting data from lived situations (Cohen, et al., 2000). Data analysis refers to a rigorous and methodical exercise based on practical evidence gathered through interviews and observations (Danemark *et al.*, 2002). The analysis of these multiple data sources allowed triangulation of the results, which will enhance their validity (Cohen & Morrison, 2000).

4.11. Conclusion

This chapter deals with how data was collected, what instruments and tools were used during collection, sampling of participants, ethical issues, profiling of participants and validity and reliability of the research.

CHAPTER 5: DATA ANALYSIS

5.1. Introduction

This chapter presents a detailed analysis of the data collected in learners' performance per activity and in the learner and teacher interviews of the study. The teacher is identified as TB and the learners were identified as LA-LH on assessment tasks and as L1 to L5 on interviews, for both schools participated. The teacher analysis will focus on teacher B and will be based on classroom observations (pre & post) and pre & post interviews with the teacher. Learner interviews for both schools B and C will then be analyzed. Finally, the analysis of learner activity data for the School C learners will be presented. School B learner activity data was not analyzed because teacher B did not complete all the teaching activities.

5.2. Table of participants

The table given below shows the number of learner participants, ages and gender that were involved during the research process from both schools.

Table 2: Participants involved during the research study

Grade 9	Enrolment	Age	Males	Females	Total
School B	20	14 - 18	9	11	20
School C	60	14 – 20	40	18	58

5.3. Learner analysis

5.3.1. Baseline evaluation results

Only the learners in school C wrote the baseline evaluation. The test took place before the activities of the teaching programme were implemented, to check the prior knowledge of learners about the concepts of transformation geometry and geometry in general. Most learners achieved less than 30% and their responses and marks obtained showed that they seemed to have gaps in both geometry and transformation geometry conceptualization.

The graph (figure 1) below shows that almost all selected learners scored at about and below 30% in the baseline evaluation (BE), except for learner B (LB) who has a good basic understanding of geometry and learner E (LE) has an acceptable basic understanding. Other learners seem to have a poor understanding as they were not able to achieve more than 30% in this assessment of their basic knowledge.

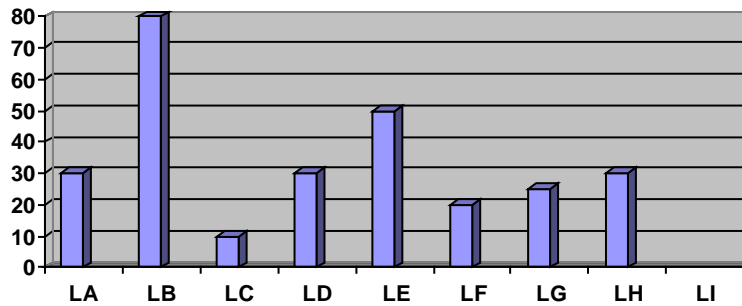


Figure 1: Results of learners for baseline evaluation

5.4. Learner interviews

5.4.1. Pre-interviews (Learners) - Schools B & C

Learners of both schools B & C were asked the same questions during pre- and post-interviews. All the learners of school B seemed to know little about geometry, contradicting what Teacher B said during her pre-interview session (Appendix B (i) PLO, Line 002-TB). She stated that she had taught transformation geometry in grade 8 in 2010. Learners seemed to have forgotten what had been taught as they couldn't answer correctly any questions relating to the nature of geometry, they were unable to identify a single geometric shape in the classroom or outside and they claimed to have never seen geometric shapes in their environment. They seemed to have forgotten the meaning of the word "shape".

Extract 1: Geometric shapes- Appendix B (i) PLO, Lines 135, 143 & 153.

Interviewer: Have you ever learned geometry before? (*Wawukhe wafunda na ngegeometry ngaphambili?*)

L1: (*No*), (Refer to Appendix B (i), Line 135).

Interviewer: Njena nje eklasini zange ukhe ubone nto tu? (Do you mean you had never seen any geometric shapes in the classroom?).

L1: *No* (Refer to Appendix B (i), Line 143).

L2: *It's about signs into endiyi-understandayo (I understand it to be about signs,)* (Refer to Appendix B, Line 153).

Interviewer: Yes, what else can you say?

L1: *That if signs are not the same subtract and take the sign of the bigger number.*

Interviewer: What do you think about geometry?

L1: *Ndicinguba igeometry ingakuthatha ikubeke kwindawo eright (that geometry can*

put you at a right place).

Interviewer: For an example?

L1: *Ine careers ezininzi (it has many careers).*

Interviewer: Yes? Give examples.

L1: *Ogqirha ne pilot (doctors and pilots).*

It seemed as if the learners in school B were not familiar with the term “geometry”. One learner responded by linking geometry to calculating with signed numbers and another indicated that geometry may open up good careers, but could not say what geometry involved. In contrast to this, learners of school C answered questions with understanding of what geometry entails and were able to give examples using objects in the classroom and also outside the classroom and in the neighbourhood.

Extract 1: Geometric shapes-Appendix B (i) - PLO. Line 109 & 110

L5: (Stammering) *‘I..i..i.geometry is about shapes (Stammering) I...geometry i...it let us learn more about shapes. Shapes and sizes of them and how to draw them. Yes, windows’* (pointing at the window).

L1: *‘Geometry is about shapes. When you are...eh...learn about geometry is about shapes and sides, how sides are measured, that these things sides you will understand that this side and one side how sides are measured’.*

5.4.2. Post-interview (learners from school B)

Learners answered questions better than during the pre-interview sessions although they were unable to support their statements and generally provided only yes/no responses. As an example, learner responses show that almost all the learners from school B had mastered the translation that was taught on that day.

Extract 2: Appendix B (iv), Lines 305-307)

Interviewer: What did you like most about the activities?

Line 305-L1: *Translation*

Interviewer: Why did you like it the most?

Line 306-L1: *Because you can take a shape to another different position*

Interviewer: Yes, what else?

Line 307- L1: *(No answer)*

5.4.3. Post-interview (learners from school C)

During the post-programme interview session, learners from school C claimed to have loved the level based activities they were engaged in and further said that they helped them to improve performance during June examinations to higher percentage levels.

Extract 3: Improving learner performance, Appendix B (v), Lines 300 – 304).

Interviewer: Did the levels help you?

L4: *Yes, because I'm the one of the learners who passed maths during June exams. I obtained 70%, I got level 6 in maths',*

Interviewer: What did you like about being placed in certain levels per activity?

L4: *'Because in June exams I improved in mathematics.'*

One learner from school C mentioned the importance of storing information one gets and laying a foundation for the next grade,

Extract 9: Storing information. Appendix B (v) - Line 250

L1: *'I've gained some work that I am going to do in grade 10 so I must keep that, it is some of the activities in mind that when I'm arriving in gr.10 and they ask about the activities I become very clever'* (Refer to Appendix B, Line 248).

5.5. Teacher analysis

The teacher analysis will focus on teacher B (TB) observations (pre & post) and pre & post interviews.

5.5.1 Pre-lesson observation.

The purpose of observing a lesson before the teaching programme implementation was to gain insight into what and how the teacher was teaching, how the lessons were planned and how the teacher facilitated learning.

The teacher was teaching transformation geometry for the first time in grade 9, teaching the types of transformations. The teacher was tense, teaching without the aid of manipulatives and did all the talking, while learners were silent. After the lesson observation TB confessed that she was teaching the concept in grade 9 for the first time.

The teacher appeared to be unclear about the content to be taught, as evidenced by her

misapplication of some of the terminology of transformation geometry in her teaching. For example, she consistently called a translation (slide) a flip, until I asked her: “Which one is a slide and which one is a flip?” The teacher appeared to dislike teaching transformation geometry as stated in **Extract 13: TB, Appendix B (i), Line 011**. *“It’s one of the topics that doesn’t make me to be happy, transformations, I don’t want to say I don’t like it because I just want to say it’s challenging because I have to do it liking or not”*. She said that she had never taught it in grade 9 before the teaching programme. The teacher did almost all the talking, while learners listened attentively and silently without making any contribution. The teacher seemed to be nervous with the fact that she was being observed and recorded. As a result, she showed some hesitancy in her teaching, for example she did not appear to be sure which letter of the alphabet she could give learners to reflect on, vacillating between a B and a C before finally opting for F.

5.5.2 Post lesson observation (PLO).

Teacher B and the learners at this stage appeared to show interest in the lesson which was presented in an interesting manner. They answered questions asked and were active throughout the lesson. The teacher seemed to be well prepared and showed proficiency and self confidence. She appeared sure of the concept under discussion and did not make many errors. The extract shown below is the evidence of what took place in the classroom.

Extract 4: Lesson presentation- Appendix H. Lines 1-4

TB, Line 001: I am trying to look at another example that can help us. Reflection again, let me use an example of water. Look at the water, what part of the body you see first on the water? Whose picture do you see?

Learners (chorus): *Your face, self*

TB, Line 002: *(Drew an F on a Cartesian plane on the board that is inaccurate and axis were not labelled. There was no T-square at school so she had to use her free hand). Reflect that picture horizontally. Which direction is horizontal? (A learner came and translated letter F and not reflected it as the teacher asked. Another one came and reflected letter F incorrectly).*

TB, Line 003: It must look the same as the image. Who can help us? (*Another child drew it correctly*). Let's give her a big hand. (*Learners clap their hands*). The distance must be the same but face different directions. Can you see the change?

TB, Line 004: Let me give you a C, B. In your own words translate this letter (*Erased letter C and said*), 'Reflect the same letter F downwards from the 1st quadrant to the 4th quadrant. Translate this letter; use a ruler to make a drawing of a Cartesian'.

Below is a summary of what took place in the classroom at school B during post lesson observation. The teacher introduced the lesson as follows:

Extract 5: Lesson introduction-Appendix C, PLO, lines 7-8

Teacher B: There are 4 types of transformation or examples of transformation. What are they?

Learners: (Chorus) *Translation, Rotation, Reflection and Enlargement.*

Teacher B: Today we'll only tackle translation and reflection. When we talk of translation it means that there will be a movement that is going to take place in the form of Let me open this door sideways which means I am taking this door from this position to another. There is going to be a movement or changing position, which means I am sli...

Learners: (Chorus) *Sliding..*

Some learners during the lesson introduction seemed not to understand the meaning of the word 'shape' and examples associated with the word, but the majority appeared to know and respond positively to the question asked, helping those who did not know as in L3 and L4. Some learners still have a problem of language usage that causes them to misunderstand the question although they may know the answer. This is evident in the following extract.

Extract 6: Geometric shapes- Appendix C, PLO, Lines 1- 3

TB: Can you give me examples of geometric shapes you know?

Learner 1: *Breadth.*

TB: Is breadth a shape or figure? Who can help us? (Refer to PLO, line 02).

Learner 2: *Breadth.*

Learner 3: *Parallelogram.*

Learner 4: *Triangle.*

Learner 5: *Square.* (Refer to PLO, line 03).

Learners seemed to have understood the three types of transformation under discussion and used the manipulatives that were there, when appropriate. The teacher tried to explain the meaning of words to familiarize the learners with concepts under discussion and used manipulatives and gestures, as in translation using a sliding door (Appendix C, PLO, line 08) and rotation using a clock.

Extract 7: Types of transformations- Appendix C, Lines 13-15

TB: Let us look at the table, I have these objects so you can come and take the object and say if it is a slide, a flip etc. Come to the table.

Learner 1: (Picks a sliding phone, slides it to the class silently). (Refer to PLO, line 13).

TB: (*Asks the whole class*) what movement do you see?

Learners: (Chorus) *Slide.* (Refer to PLO, line 14)

Learner 2: (*Picks a wall clock*) Shows to the class without saying a word.

Teacher B: What transformation is it?

Learners: (*Chorus*) *Rotation.* (Refer to PLO, line 15).

5.5.3. Pre-interview Teacher B (TB)

Teacher B mentioned in her interview that learners had done transformation geometry in grade 8 in 2010. She said that she had taught it thoroughly to the same learners in grade 8 and never taught it in grade 9, hence she took it casually and didn't have many practical examples. This can be seen in her response: '*First time in grade 9 yes. You know I took it very simple this year than last year, last year I brought apparatus such as paint and paper*' (Appendix B(i), Pre-interview, Line 010-TB). The teacher confirmed that she was teaching transformation geometry for the first time in grade 9 because it is challenging and made her feel unhappy: '*It's one of the topics that doesn't make me to be happy, transformations, I don't want to say I don't like it because I just want to say it's challenging because I have to do it, liking or not.*' (Appendix B (i) Pre-interviews, Line 011). Transformation geometry has been in the curriculum for a long time even in the previous curricula but the teacher never taught it in grade 9. She appeared less confident with this mathematical concept and almost

ignored the teaching of this topic (possibly because of her lack of proficiency), leaving learners with gaps on the concepts.

5.5.4. Post-interview (Teacher B)

The teaching programme appeared to benefit teacher B and (indirectly) her learners because it helped and encouraged her to teach transformation geometry in her grade 9 classroom. She confirmed that she felt more confident about herself than before the implementation of the teaching programme. When asked about how she felt after the lesson she responded as follows, *“Yho, eh. You know when you deal with shapes you need to be more confident because these are basics for grade 10.”* (Appendix B (i) Pre-Interview, Line 003-TB). She expressed the opinion that learning took place during the implementation of the programme activities and that learners had gained some knowledge. She responded as follows in Line 004, *‘They manage to reflect shapes given at least although the time has been short giving me hope that it would be easy next time when doing other types of transformation’*.

The teacher answered questions in a relaxed and less nervous manner than in the pre-interview. She said that the programme had helped her to prepare her lessons and do homework when she was not familiar with the concept (Appendix B (ii) Post-Teacher interview Line 014-TB). She confirmed during post interviews that the questioning style fitted the level of the class (Appendix B (ii) Post-interview Line 036-TB) *‘You know the style is suitable for their level. Questioning help them as they are specific. It is easy for them to answer or give possible answers or what is required mostly or lately’*.

The teacher confirmed in (Appendix B (ii) Post interview, line 035 TB) below that utilizing one-on-one interactions helped to unpack what the learners know and do not know, as some learners are not eager to answer questions in front of everyone. *‘Yes there are possible improvements just because that if you sit down you’ll find that they have knowledge. Some they express themselves better when you talk to them. They are lazy in writing and in research and when I try to find out why they give me maybe a brilliant explanation and try to find out a follow up, you’ll find out that there is knowledge’*.

It appeared as if the programme has encouraged TB to monitor the learners when they

were working so that they remain engaged to their work in the classroom. Monitoring learners in the classroom encouraged and committed the learners to work hard doing what the teacher asked them to do. The teacher responded as follows in (Appendix B (ii) Post interviews, Line 038 -TB), *‘Although the learners as I said before they are negligent, but if you monitor them in their work and see that look that each and every individual that is committed and engaged at his work is the duty of you as the teacher to see to it that and check it is where the meter stick or measuring instrument and help those who didn’t understand well and show appreciation’*.

TB also used available resources to develop conceptual understanding. The teacher responded as follows to the question asked about the importance of resources in (Appendix B (ii) Line 025- TB): *“You know, it brings/helps the learners to understand more fully what the lesson brings to them”*.

Finally, the teacher stated that she had learned that teaching is a commitment and one needs to dedicate oneself to achieve better outcomes– responding as follows in Line 038, *‘I’ve learned that e-h in this programme and the lesson I’ve learned that in life everything needs commitment and engagement. You must commit and also engage yourself in doing that as you continue doing the activities you’ll gain the interest, you’ll enjoy working with us’*.

5.5.5. The post lesson observation

Using actions and gestures in mathematics can also promote better conceptual understanding in learners as it happened to TB.

Extract 8: Rotation, Appendix H, PLO-Lines 10-11- TB)

TB: In rotation when you rotate you..... *(The teacher used her finger as if she was drawing a circle showing a turn by actions and gestures)*. Learners understood the action and replied as the class.

Learners (in chorus): *It is a turn* (Appendix H, PLO, Line 011- TB).

Extract 9: Reduction & Enlargement, Appendix H- Line 12- TB)

TB: I have something small and I want it to be visible, so the meaning of enlargement *(using actions spreading arms sideways)*.

Learners: *(chorus) Big*

TB, Line 20: We can take photos by a photographer (*She looks on the wall and saw an old picture hanging there and said to class*), Look at this, sapha, sapha (bring it to me), Mrs Geja here is another drawing. This is ...enlargement of Madiba's photo. We know Madiba; can we see that his ID photo is smaller than the picture? Can you see that our life is full of mathematics? You will see transformation in the field of architects and engineers (Appendix H, PLO- Line 20-TB).

Learners were also given the opportunity to present group activities to the whole class using the chalkboard. Learners seemed to have understood concepts discussed and were able to carry out procedures related to translations (Appendix H, PLO Line 002 - TB). They worked hard on their own and I noticed that they seemed to have developed a positive attitude towards their work and developed independence and eagerness to work individually.

5.5.6. Teacher interviews

The pre-programme interview discussed above showed that the teacher used to neglect transformation geometry teaching in grade 9 and started teaching it because of the influence of the teaching programme. Teacher B stated that she used to ignore teaching it; hence there were errors during presentation. (Refer to Appendix B, Pre & Post interviews)

In my opinion, the post-programme interviews indicated that the teacher seemed to have worked through the activities of the teaching programme in terms of planning: the last lesson was well planned and presented well. Teacher B cited the importance of using resources and the problem of scarcity of resources at her school. TB confirmed in the post-interview session that when manipulatives were used in class, they brought home the concept to learners and gave a clear understanding of the concept under discussion. She said in (Appendix B (ii) PPI, Line 045- TB), '*You know, it brings/helps the learners to understand more fully what the lesson brings to them*'.

5.6. Impact on learners of the use of levels in teaching

Classroom experience of implementing level based activities brought change and encouraged diligence, hard work, a willingness to improve levels and patience in

learners in class as every learner wanted to know the level to which one belonged. The change was noticeable in the lesson observation, as discussed above. Learners also mentioned in the interview that they appreciated this feedback: *'I liked it because it showed that you can improve if you do not understand the activity'* (Extract 12: Appendix B (iii), Line 268). Those who failed an activity were eager to be given an additional activity to work on to boost them to the next level. Learners also developed patience of doing the work on their own because of the influence of the programme following the teacher's example: *"That if I didn't understand at the first time I do it again and understand, I improve"*. (Extract 11: Learner understanding, Appendix B(iv), Line 269).

These activities also appeared to encourage learners to work hard to be in line with others in the group; they became more observant and committed to what they did in the classroom, (Post-interview, Appendix B (iii) Line 346) *'At first I didn't know how to check co-ordinates, I saw from other examples how to do it'*.

Learners developed a positive attitude of seeing the importance of mathematics as they worked on activities of the programme daily and set their future career goals. *'I've gained to be cleverer because there is no chance of being a pilot if I don't pass there to know this in grade 10'* (Appendix B, Line 344, Post-interview): . They were able to focus more effectively and correct mistakes made as they worked according to allocated times so that they couldn't miss the work or teaching that could follow up after the activity: *'Because I will know where my mistake is and focus on it'* (Appendix B, Line 353).

5.7. Importance of manipulatives used

Manipulatives seemed to have made the lesson understandable to learners as they manipulated resources themselves. Learners seemed to have understood the concepts under discussion as they were able to answer questions asked and identify the resources which can flip, rotate and slide on their own. It was easy for the teacher to see who was following or not following the lesson and the teacher was able to correct errors quickly when the learner picked up a wrong object for the transformation under discussion.

Consistent use of manipulatives and other objects led to effective teaching and meaningful learning. These enabled learners to learn with understanding and make connections which led to internalized conceptual action. (Refer to Appendix E,

Picture no.1, 17, 12, 5 & 14). Picture no.1 shows that the learner seemed to have lacked understanding; Picture no. 17 shows some improvement because of the use of manipulatives; Picture no. 12 shows understanding when drawings are used; Picture no.5 & 14 show that learners seem to have understanding and some are able to cut and paste objects (Picture no.5) and express their reasoning abilities in words, showing connections they see between two or more objects given.

There were instances where learners were able to use floor tiles to show translation and rotation on a Cartesian plane; and use different types of cell phones that slide, flip and rotate as well as clocks, mirrors and drawings to help them to understand the concepts better. Such hands-on learner involvement in the use of manipulatives enabled the learners to think and communicate actively throughout the lesson, (Refer to Appendix E, Picture no.12 & 14).

Extract 15: Manipulatives- Appendix H- PLO, Line 13-15

TB: Asked the learners to pick up manipulatives on the table, show them to the class, make a movement related to each object and the class gave the type of transformation described by the movement.

L1: (*Picks up a sliding phone, slides it to class.*)

TB: What movement or transformation took place?

Learners: (Chorus) *Slide.*

Learning by touching seemed to be more meaningful than using theory alone, as they did the activities practically on their own. Also, it was easy for them to correct each other immediately when one picked up the item that didn't match the transformation asked.

Extract 10: Using manipulatives - Appendix C, PLO, Line 018

L2: (*Picked up a phone that doesn't flip, slide or turn, showed it to the class.*)

TB: What transformation is that?

Class: *No transformation* (in chorus and no hands were up), (Refer to Appendix H, PLO, Line 018).

The lack of teaching resources hindered effective teaching and learning in the classroom as the teacher had to use free-hand drawing of a Cartesian plane (which

was thus inaccurate, with uneven spaces). This is because the school doesn't have T-squares to help the teacher show learners practically on the chalkboard how accurate spaces should be drawn and measured (Refer to Pre-Lesson Observation, Appendix H, Line 002).

5.8. Analysis of activities

5.8.1. Activity 1 (a) & 1 (b)

The activities given to learners were analyzed question by question, checking learners' responses per question, and whether the learners understood or didn't understand the concepts discussed. The marks obtained by each learner per activity are given, to show whether learners were progressing or not. The following is an analysis of some selected activities completed by the learners. The analysis is organised in tables and double bar graphs. Activity 1 (a) is a sorting activity which involves visualization. Activity 1 (b) is a level 2 question activity which requires the learner to think and give properties of shapes without making any connections. There is an assessment table with assessment requirements to meet the activity levels. (Refer to assessment Table 5).

Activity 1(a) & 1 (b) have been grouped together and results show that learners still have problems with knowing properties of shapes, sorting is easy and requires less thinking than properties. In Activity 1(a), 57 learners of school C wrote the activity, 46 passed (80, 7%) and 11 (19, 3%) did not and so were given a make up activity. Table 3 (make up activity) and Figure 2 given below show how learners performed in the make up activity.

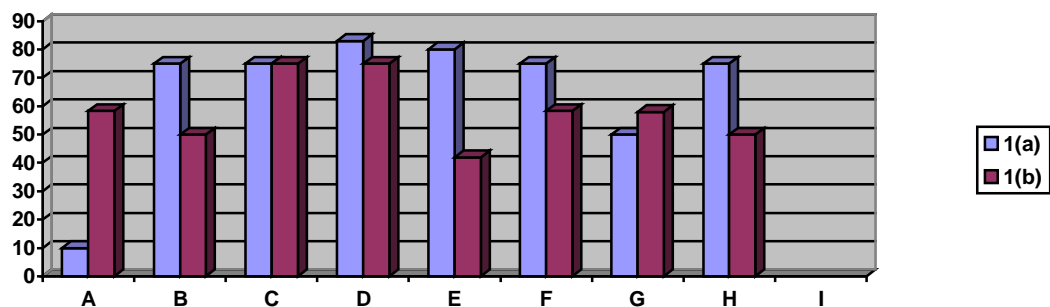


Figure 2: Learners' results for activity 1(a) & 1(b)

Learners performed well at 45% and above in these activities except for LA who scored below 20% in activity 1(a). Their average results in 1(a) were better than in

activity 1(b), some learners who performed well in activity 1(a) didn't do well in activity 1(b) although they passed it. This was expected, because activity 1(a) was level 1 and activity 1(b) was level 2, requiring properties of shapes.

Learner A did not perform well in activity 1(a) but worked hard in activity 1(b). Learner A attained a higher level in activity 1(b) which shows gradual improvement. Learner G also improved in activity 1(b) which requires properties of shapes. There was also a gradual improvement by learner G in activity 1(b).

Learner A and other 10 learners were given a make up activity which is Appendix D. (Refer to Appendix D). Shown below is a table for a make activity written by the 11 learners including LA.

Table 3: Make up activity results for activity 1(a)

No. wrote	Age	Males	Females	No. passed	% passed	No. Attained Level 1
11	17-20	7	4	11	100	11

All learners who wrote the make-up activity passed. It seemed as if these learners were able to use this activity to improve their understanding to the point that they passed and so were able to catch up with the class.

5.8.2. Activity 2

This activity was about naming and giving properties of shapes without making any connections, which is Level 2 analysis (van Hiele, 1986). Both activities look at conceptual understanding and procedural fluency as argued by Kilpatrick *et al.* (2001). The aim is to look at which concepts are mastered and which are not and what help can be given to learners to master the concept. The table below informed the teacher on what to assess for this activity level.

Marking activity 2 according to the table (Appendix J) gave information about the level each learner was in. The mark and level informed the teacher where to start in order to help learners master the requirements of the levels they were expected to be in. More activities per level were given to help learners catch up with the levels they missed and iron out the misconceptions they might be having. Use of manipulatives supported the improvement of learners in these activities and engaged learners in

hands on activities. They were also asked questions throughout the activity.

Table 4: Assessment table for activity 2

Level	Specific aim	Children may need...
1	Transition from level 1 to 2 starting to identify properties of shapes from level 1 to level 2	Sorting knowledge, be able to recognize shapes
2	Generating properties. No connections at this level	Knowledge of properties of shapes
3	General properties of shapes can be given. They can make connections and differences	Know properties of shapes. Be able to make connections. To be observant when dealing with shapes

Fifty five learners out of 60 registered learners wrote the activity, 5 withdrew themselves from the programme and 43 performed well and achieved the minimal requirements to be at level 2, obtaining 78, 2% to qualify for Level 2. But 12 learners (21, 8 %) were unable to give properties of shapes and were still on Level 1 (recognition). Table 4 below gives the summary of what took place in class for activity 2. Attained means achieved the minimal requirements set for each activity. If the learner gives correct answers expected, s/he achieves/ attains those requirements.

Table 5: Summary of learner results for Activity 2

School	No. wrote	Males	Female	Number Achieved Level 2	Number Below Level 2	Age	No. registered
C	55	33	22	43	12	14-20	60

The following is an example of Level 1 (recognition), and Level 2 (analysis) questions asked of learners in the classroom. Learners' responses led the learner to be identified as working at Level 1, 2 or 3, according to the criteria for van Hiele's (1986) levels of learning.

The consideration of the teacher was - **Can learners identify shapes?** The answer is “Yes” and “No”. Some could and others could not, depending on content knowledge and observation skills acquired.

- a. Some couldn't distinguish between a rhombus and a parallelogram, a rhombus and a pentagon, between a pentagon and an isosceles trapezium, a hexagon and an octagon, and between a square and a rectangle.
- b. Some had a problem with both naming and identification of shapes. School B learners couldn't identify shapes in class during interviews.
- c. Counting the number of sides was not a problem at all to all learners.
- d. ‘Attained’ means those who had achieved the required van Hiele Level 1 (recognition), and Level 2 (analysis), and were able to progress to the next level. ‘Not attained’ refers to those who did not meet the requirements of van Hiele level (1986) and therefore remained at the current level until s/he writes and passes a make-up activity preparing the learner for the next level. Use of a drawing was also suggested so that the learners could express themselves in a drawing form rather than in words.

The table below shows whether learners were able to identify shapes or not, know the properties of shapes or not and whether they were able to distinguish between objects that rotate, translate and reflect or not, from both schools at different levels of learning. The table shows that all learners are able to sort and translate shapes. Learners from school B and LA of school C were unable to link shapes with drawings. The reason might be because learners had not mastered the properties of shapes

Table 6: Summary of learner abilities in shapes and transformation

School C	Recognition	Properties	Sorting	Link with drawing	Translation	Reflection	Rotation
LA	Able	Unable	Able	Unable	Able	Able	Unable
LB	Able	Able	Able	Able	Able	Able	Able
LC	Able	Able	Able	Able	Able	Able	Able
LD	Able	Able	Able	Able	Able	Able	Able
LE	Able	Able	Able	Able	Able	Able	Able
School B							
L1	Able	unable	Able	Unable	Able	Unable	Unable
L2	Able	Unable	Able	Unable	Able	Unable	Unable

L3	Able	Unable	Able	Unable	Able	Unable	Unable
L4	Able	Unable	Able	Unable	Able	Unable	Unable

This table was used to record learners’ knowledge and understanding of shapes, how learners think and reason about shapes, and whether their reasoning and thinking is critical and reflecting the level they are in and the information they have gathered. It also relates to van Hiele’s levels of thinking and provides information about whether learners have mastered the visualization/recognition level or not.

Learners’ prior- knowledge of shapes will determine the way they respond and the response will inform the researcher about the level in which each learner is in according to van Hiele’s theory (1986). Learners’ prior knowledge was thus the initial benchmark for teaching to van Hiele’s levels to get insight into learners’ thinking approaches .This informed the researcher about where learners were and provided the foundational level for teaching to prepare the learner to master the first two levels of van Hiele, namely:-

- a) Visualization- where learners sort shapes according to appearance without mentioning their properties.
- b) Analysis / description - where learners are able to give properties of shapes without making any connections or relationships. The student at this level is able to reason about geometric shape in terms of its properties. The student now sees geometric shapes as a collection of properties. They can recognize and name properties of geometric figures, but they do not yet understand the relationships between these properties and between different figures. (Hoffer, 1981; van Hiele, 1986,; Mason, 1998).

One objective of the teaching was to enable learners to progress to the informal deduction / theoretical level – where they are able to make connections, and identify differences and relationships between shapes, showing that they have a conceptual understanding of shapes according to Kilpatrick et al (2001). Pegg (1995) argues that the student at this level can logically order the litany of properties of figures previously identified, and begins to perceive the relationships between these properties and between different figures.

The learners performed well in this activity. About 78.2% of the class on that day had passed the activity. Those who couldn’t make it were eager to be given another

activity to catch up with the class. Generally, the majority of learners (7/8) performed above 40% in Activity 2(b) and attained 87.5%, better than (5/8) in Activity 2(a) where they obtained 62.5%. The difference is that in Activity 2(a) some learners (3/8) performed above 80% i.e. at 37.5%, while in Activity 2(b) only LD attained above 80%. And those who did better in Activity 2(a) seemed to perform well even in 2(b). This may suggest that those who were proficient on Level 2 were able to engage well with the Level 3 questions, but those who weren't proficient, were not able to engage well.

Learners who appeared proficient were those who were able to give correct properties of shapes, see differences and connections between two or more similar shapes belonging to the same group e.g. a square and a rectangle, a rhombus and a parallelogram etc. They performed well and their performance showed that they have knowledge and reasoning abilities.

Activity 2(a) is an extension of Level 2 activities and Activity 2(b) consists of Level 3 questions according to van Hiele's (1986) levels for those who are able to analyse to move to informal deductive reasoning. This seems to be showing a better performance at Level 3 than at Level 2. Learners A, C & H performed below 40% in activity 2(a). The lowest achiever in Activity 2(b) obtained 33.3% and others are above 50% and this shows that they progressed well.

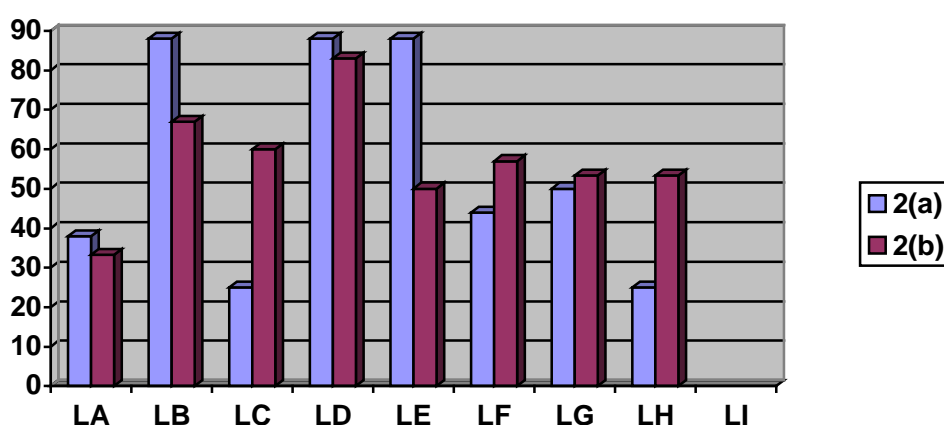


Figure 3: Learners' results for Activity 2(a) and 2(b)

5.8.3. Activity 3

The aim of this activity was to check learners' prior knowledge about transformation geometry from grade 8, to assess learners' understanding of terminology and their

ability to express themselves in their own words, to check where they are in terms of knowledge and where to start to help them understand the transformations from a very basic level to a complex one according to grade 9 syllabus. If they struggled with this activity, the teacher would need to engage them in other activities that would help them understand with the aid of manipulatives and code switching where necessary. The table given below is an assessment guideline the teacher used to assess learners' understanding.

Table 7: Using theory to inform the lesson

Level	+ Response	- Response	Mixed feelings	Teacher's role
1. Can recognize transformation according to appearance.	Recognize transformation without any doubts	Can't recognize transformation. Nothing making sense. No response.	Still has a doubt about which. Not sure what to do. Confused and looking at what others do.	More practical activities, more probing questions, more individual attention.
2. Recognize in terms of properties. Have meaning of each transformation	Understands the meaning of each transformation and how it works.	Can't understand the meaning of each in terms of properties. No response	Confused, not sure or copying from others	More activities, more probing encouraging them to talk. Give more clear explanation. Give motivational talks to boost

				their self-esteem.
3. Can make connections, see differences between transformations. Make connections with real life visual representations.	Knows the transformations well. Can make connections and differences between transformations. Associate the transformation with real life	Unable to make connections and differences between different transformations. No response or attempt to try to give an answer	Not sure Confused Frustrated Copying others' work.	More level 3 activities & probing until they see the connections and differences. More motivational talks to bring back self confidence and their self – esteem.

This activity required learners to define transformation geometry terms that they should have learned in Grade 8 as the foundation of what they would learn in Grade 9. Table 6 below shows the performance of those that wrote and submitted the activity on that day. The results show that the majority of learners who wrote understood the concepts and drew sketches where possible, (Refer to Appendix E, Picture no. 15).

Table 8: Summary of Activity 3 results

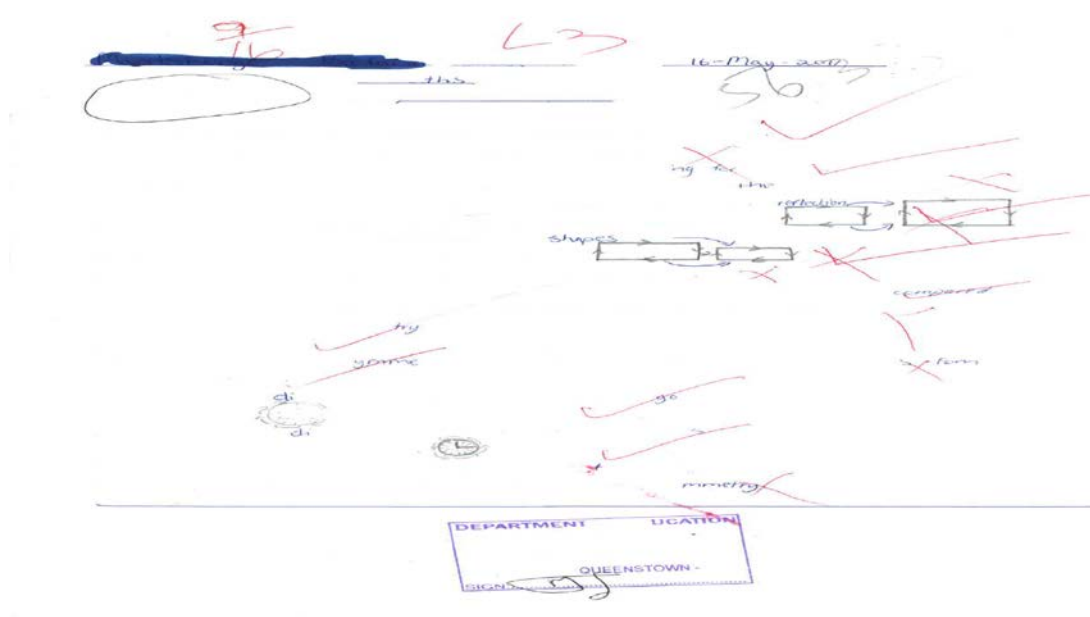
No. in class	No. wrote	Males	Females	Attained	Not attained	% Passed	% Failed
60	22	6	10	16	6	72,7	27.3

The activity required the learners to provide sketches with brief descriptions of what each concept means. The table above shows the performance of learners who wrote and submitted the activity. Only a few learners (22/60) did this activity in school C and submitted it, 23 of them didn't submit the activity, 3/22 who wrote submitted it although they didn't finish it (Refer to Appendix E, Picture no. 12); and, 2 learners

were unable to express themselves in writing and diagrammatically, i. e. they were unable to give brief description of concepts required.

Learners who wrote and submitted the activity performed well as the percentage passed was 72, 7%. Those who didn't finish the activity failed, some finished it but didn't give what was required by the question and failed (27.3%). The feedback from the learners' work below shows that learners seemed to have gaps in measurement.

Picture no. 15



The language of teaching and learning (LOLT) seemed also to be problematic, in particular understanding the concepts taught and being able to express oneself in one's own words. Also, showing an understanding of concepts in class as in the activity below, as well as during interviews in the LOLT, hence the learners preferred to use their mother tongue, especially in school B during pre- and post-interviews, (Refer to Appendix E, Picture no. 1 & 4). The following is a sample from Activity 3 of a learner's work written in his/her own words. This shows that this learner has a problem in expressing himself in the LOLT.

Extract 12: LA's work, Appendix E

Learner A wrote this and couldn't pass the activity:-

Transformation is something that you transform it. Translation is something that is unlike to me other that you drawn. Reflection is something that has shown likely.

Glide reflection is something that is unlike the reflection not is glide. Similarity is the thing that is similar. Horizontal symmetry is something that is a line is horizontal but symmetry.

Vertical line of symmetry is a line that is vertically symmetry. Clockwise direction is something that you have in your head. Anti-clockwise direction is something that is anti or clock that you has on your mind.

There were no sketches provided showing what the learner wanted to say about each concept and some concepts were left blank. The results above show that the learner A seemed to not have grasped the concepts and needed more time and a make-up activity that could help him understand transformation geometric concepts better and be able to express himself using mathematical language.

All learners scored above 40% which shows that they seemed to partially understand the concepts required by Activity 3 and the questions asked in the activities of the teaching programme. One exception was learner G who appeared to be still struggling with terminology.

5.8.4. Activity 4

This is an extension of Activity 3 which required transformation geometry terminology. This activity was two-fold; learners had to show symmetry on letters of the alphabet, trees and also in shapes. 49 learners out of 60 wrote the activity, with 42 passing it; therefore 85.7% of learners who wrote the activity attained the required level, while 14.3% did not. This was not the true reflection of the whole class as there were 11 absentees on the day of writing the above activity. School B did not do the above activity.

Table 9: Learner results for Activity 4

No wrote	Males	Females	Attained	Not attained	No. in class	School
49	34	25	42	7	60	C

Figure 4 below shows that learners performed well in both Activities 3 & 4, especially Activity 4 which was about identifying symmetry in shapes, letters of the alphabet and objects (Refer to Appendix E, Picture no. 18 & 22 for those who didn't finish and failed the activity).

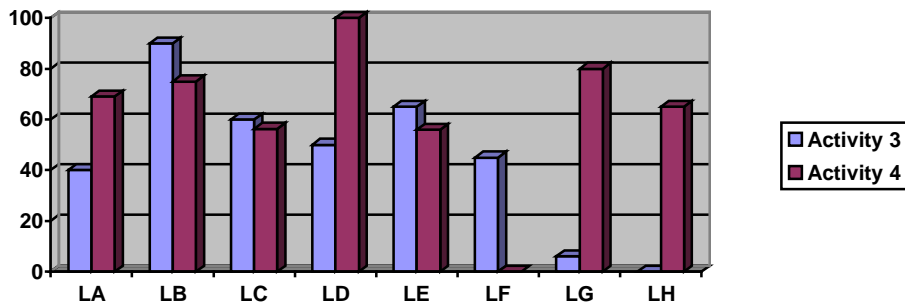
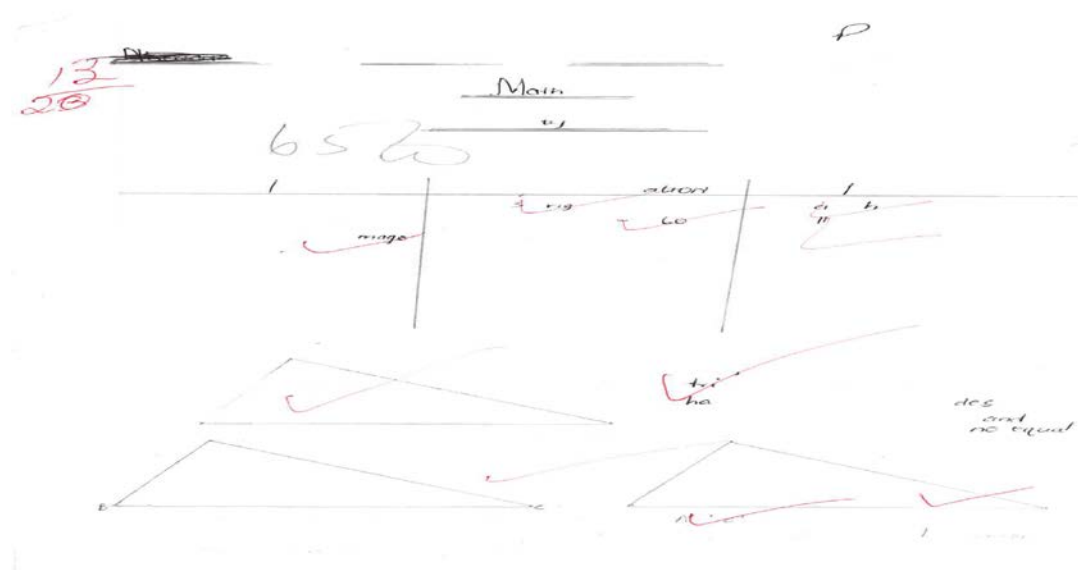


Figure 4: Learners' results for Activity 3 & 4

5.8.5. Activity 5

Learners were asked to identify manipulatives on the table and give the type of transformation relating to each object. The second part of the activity was to draw and cut out a scalene triangle, translate it, reflect it and rotate it. Knowledge of types of triangles was assessed, and a drawing of a correct original object was expected from them as well as a drawing of the transformed shape. Some learners did well in this activity (Refer to Appendix E, Picture no.5 below); and others didn't do well (Refer to Picture no. 10 below). Below is an example of a learner's work showing what the learner did in the activity.

Picture no.5



Many learners were not able to draw a scalene triangle, they drew a right-angled

triangle, some drew an isosceles triangle and that had affected their marks as they had to show transformation that could take place; hence they obtained lower marks (Refer to Appendix E, Picture no.10 below).

The teacher at school B used this activity for the observation lesson and presented it as planned. During the lesson, it was observed that the learners were able to identify objects that flip, turn, and slide although they didn't want to express themselves verbally about the objects they had picked up (Refer to Appendix H, PLO). Drawing a scalene triangle was also a problem to the majority of the learners in this class.

Picture no. 10

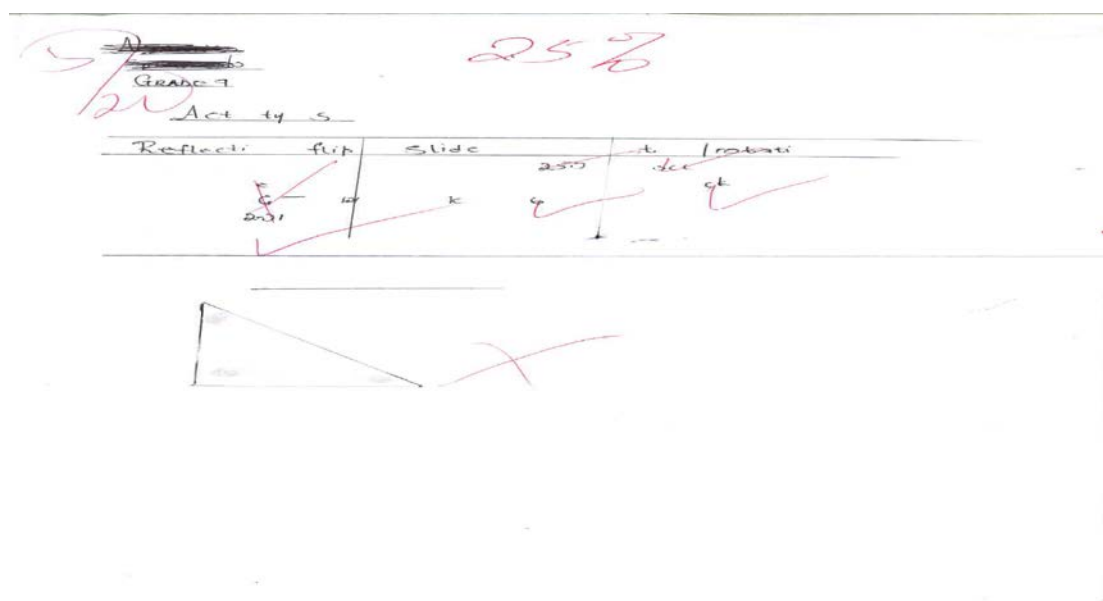


Table 10: Results of Activity 5

School	No. in class	No. wrote	Males	Females	Attained level	Not Attained
C	60	56	35	21	7	49

This was an individual activity and learners seemed not to have enough conceptual knowledge to be able to do the activity. They struggled with drawing a scalene triangle as discussed above because they did not know it, couldn't recognize it as a shape and its properties. Because of this, they couldn't translate, rotate or reflect the triangle.

It seems as if learners did not know what to do and as a result they did not do the

second part of Activity 5, hence they failed it, (Refer to Picture no, 10 above). This learner didn't finish the second part of the activity. Some couldn't even write what they had identified from the resources that were manipulated i.e. whether it was a flip, slide, or turn and wrote something different from what was required by the question. This led me to go back and do revision of what we did in the first quarter on triangles as they were unable to draw the scalene triangle required by the question. Learners' work images shown above show that the learners seemed to lack an understanding of triangles.

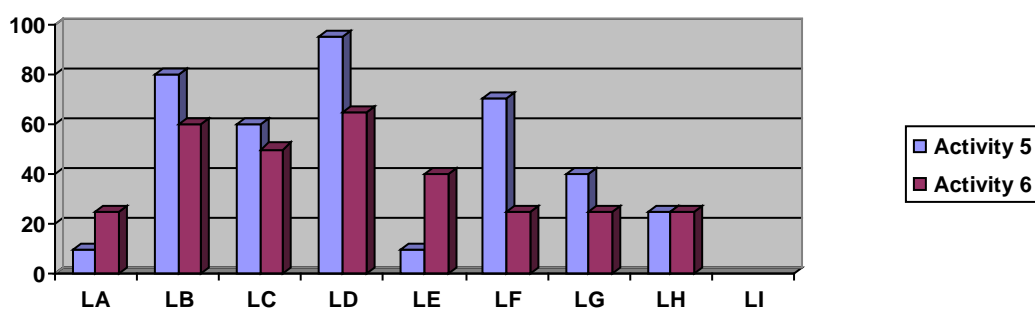


Figure 5: Results for Activity 5 & 6

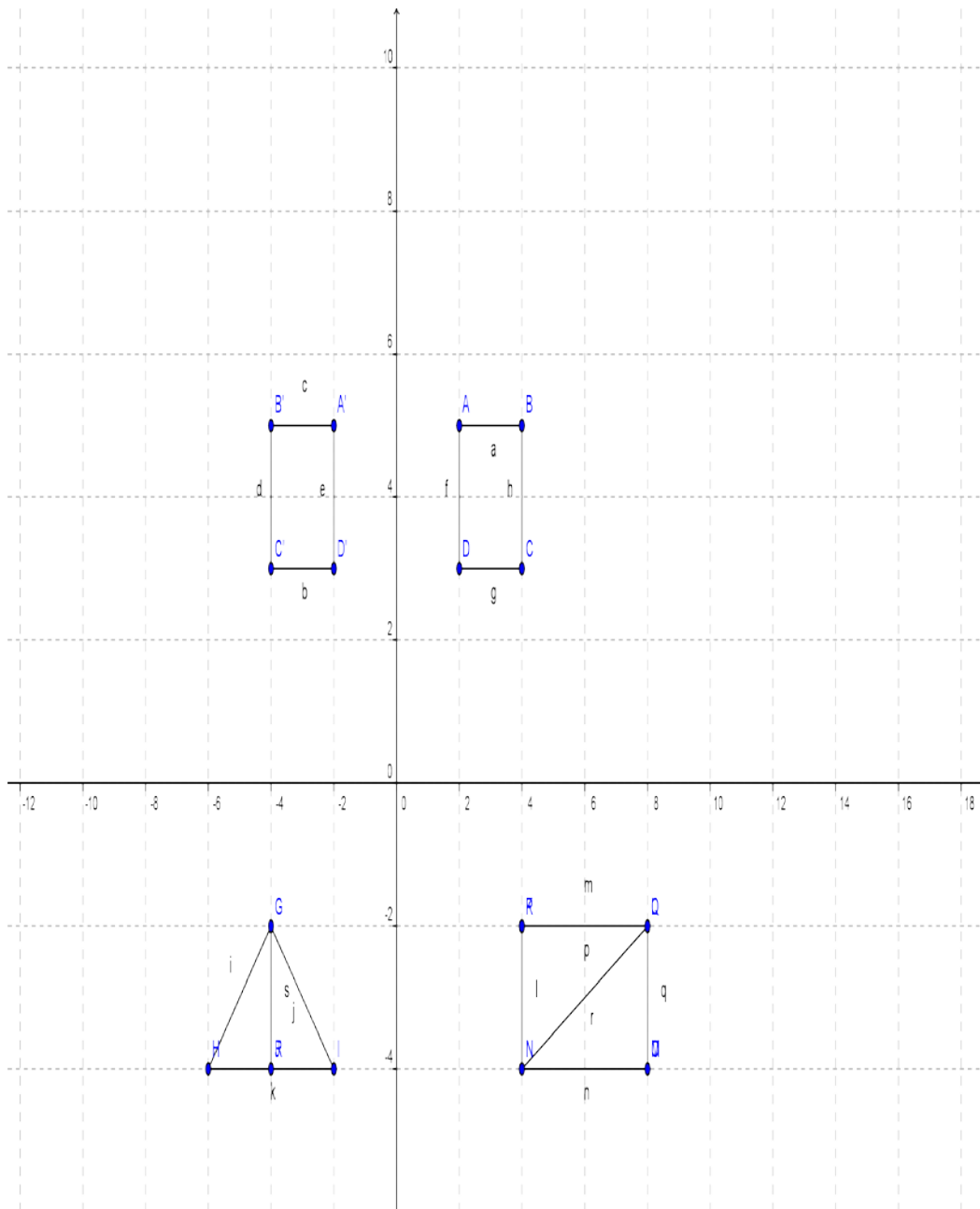
5.8.6. Activity 6

The aim of this activity was to check learners' understanding of types of transformation using known objects in their context. It also aimed at testing the knowledge acquisition of co-ordinates/ ordered pairs to learners. Given below is an example of activity 6 question given to learners.

Activity 6 questions

The objects shown below 'move' in a space. Write down whether the statement / movement is a translation, a reflection or a rotation.

- An airplane flying in the sky. What transformation is shown by this statement?
- What transformation is represented by the shapes in the Cartesian plane given below? Write down the ordered pairs of each image point.

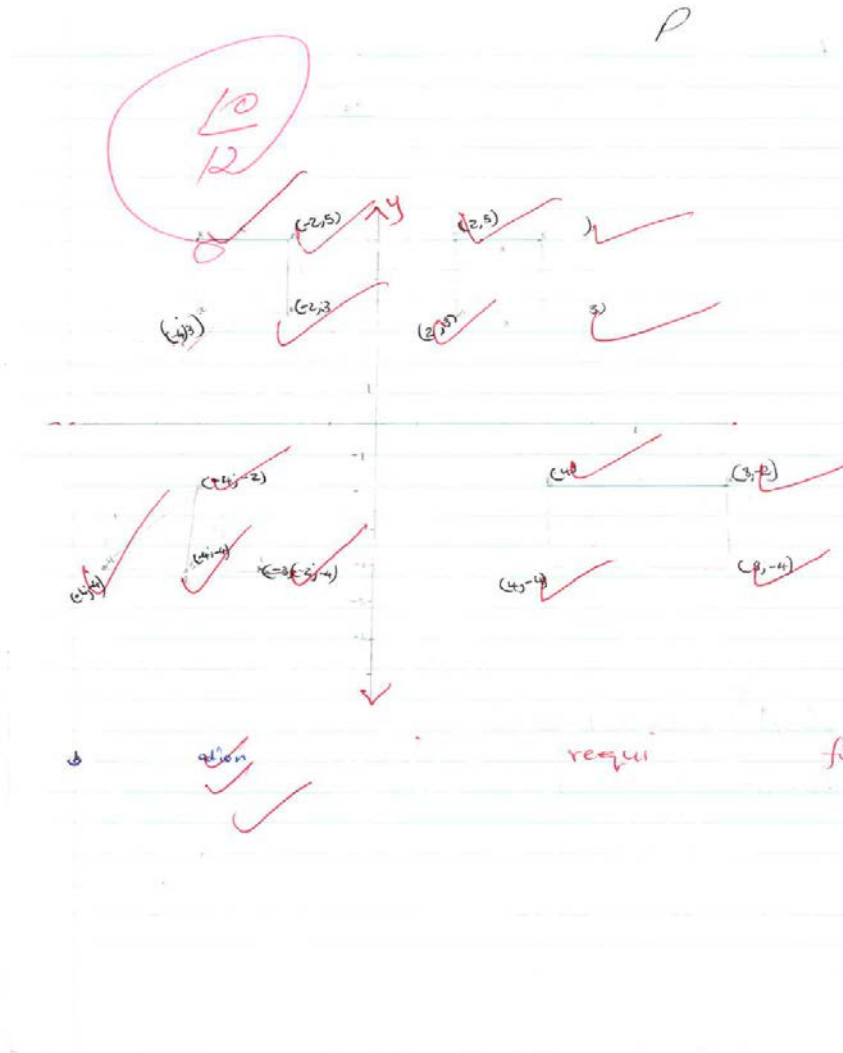


- c) The wall clock ticking from 6h00 to 6h30. What transformation is that?
- d) Crossing the river on a boat. What transformation is that?
- e) The car traveling on the road. What transformation is formed?

Learners seemed to have struggled with Activity 6, where 36/53 learners scored below 40% and did not pass the activity, but 17/53, (32%) passed the activity. Learners were asked to give the transformation represented by each statement (a, c, d & e) above. Most of them were able to answer 6(a) correctly, some answered correctly almost all the questions as in Picture 26 below, some didn't do the rest of the

questions and answered 6(b) requiring ordered pairs. (Refer to Picture 25 below). Some learners who passed the activity seemed to have understood the concepts of transformation and wrote correct ordered pairs in correct places as in Picture no.26 below.

Picture no. 26

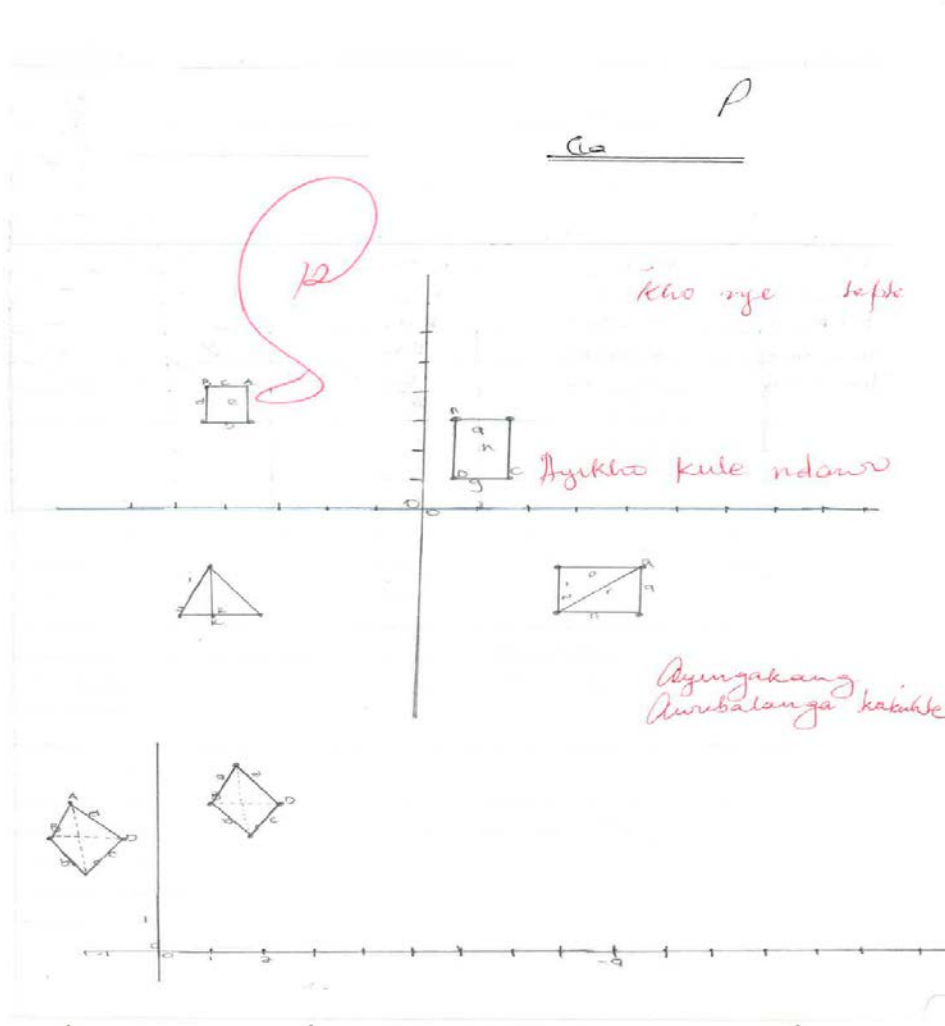


Picture 25 shows that the learner drew different figures in different places, incorrect spaces from those used in the question. This seemed to have resulted in writing incorrect co-ordinates in terms of (x; y). The learner seemed unable to identify which co-ordinate is x or y, and which one to write first. The axes were not labeled and might have been confusing when the graph was rotated into a landscape position.

Pictures no.7 & 13 in Appendix E show that the learners also seemed to have a problem with co-ordinates of each point, instead of reading the points in a plane, they

combined two points and give an ordered pair; e.g. in quadrilateral ABCD there should be 4 pairs of ordered pairs: A (x; y); B (x; y); C (x; y); D (x; y).

Picture no. 25



Learners with this problem of co-ordinates ended up writing incorrect 4 pairs of ordered pairs taking the x co-ordinate of point A pairing it with the x co-ordinate of point B, y co-ordinate of point A with that of point B even in point C and point D. They would also forget that the numbers they have chosen are both co-ordinates of x in the first pair, sometimes they took one x co-ordinate from point A & B and one y co-ordinate and formed an ordered pair. Sometimes the learners would omit to label the points. Because labeling guides the learner to move from one point to another, this resulted in having two pairs of ordered pairs instead of four and that was the reason why they were incorrect. Many learners (53/60) wrote Activity 6 even those who generally did not hand in their work. Only 17 learners passed the activity (32%) while

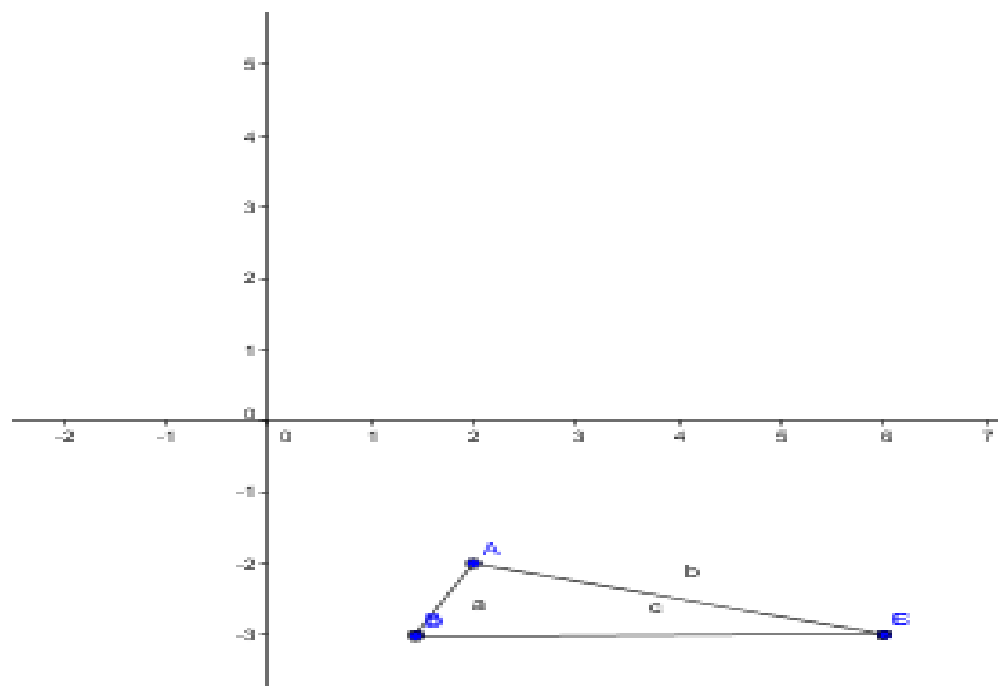
36 learners (68%) failed the activity. So the many failures may just be that the others who generally did not complete activities did complete this one and failed it and because their work was marked and their contribution was noted, while in the earlier activities it wasn't. Most learners who failed seemed to have misunderstood the questions asked as in Picture 25 above where the learner only focused on giving co-ordinates of points in a graph and ignored other questions.

Table 11: Summary of Activity 6

School	No. in class	No. wrote	Males	Females	Attained level	Not Attained
C	60	53	31	22	17	36

5.8.7. Activity 7

The following is a discussion of activity 7. Learners were given a Cartesian plane with an object to transform, following the requirements of the question. They were asked to translate the shape in the plane given below, 4 spaces to the right and record the co-ordinates that are formed after translation.



Learners who passed seemed to have understood the activity as the graph and table below show that most learners received 40% and above except for LA who attained 30%. Learners C and F tied with 50% which is the highest score obtained by learners. Learners who failed the activity seemed to have not carried out the instruction given to move the object 4 spaces to the right, instead some translated it one space to the

right as in Picture no. 24. Other learners translated it to the first quadrant, as in Picture no. 23 & 28. No ordered pairs were given by learners who failed the activity, the Cartesian plane was incomplete and axes were not marked.

There is some indication that in all the activities done by learners in the classroom, some learning took place, even if it was only a little and there is a high failure rate in both schools in these last activities. The misconceptions identified show that further work needs to be done to help learners understand concepts better in the classroom.

Picture no.28

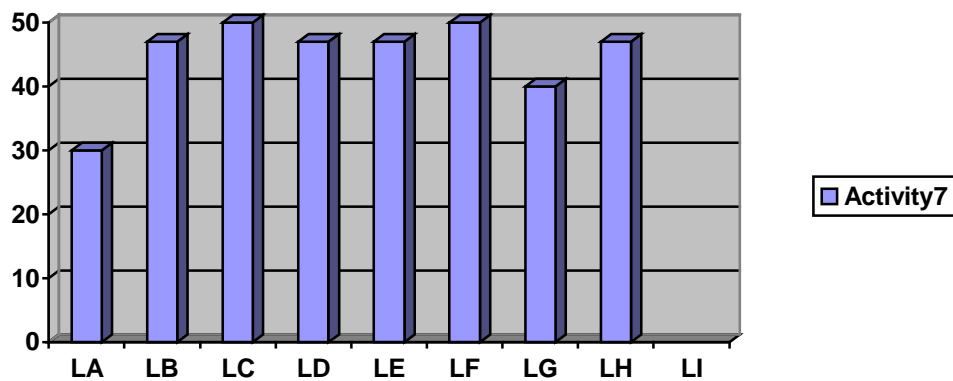
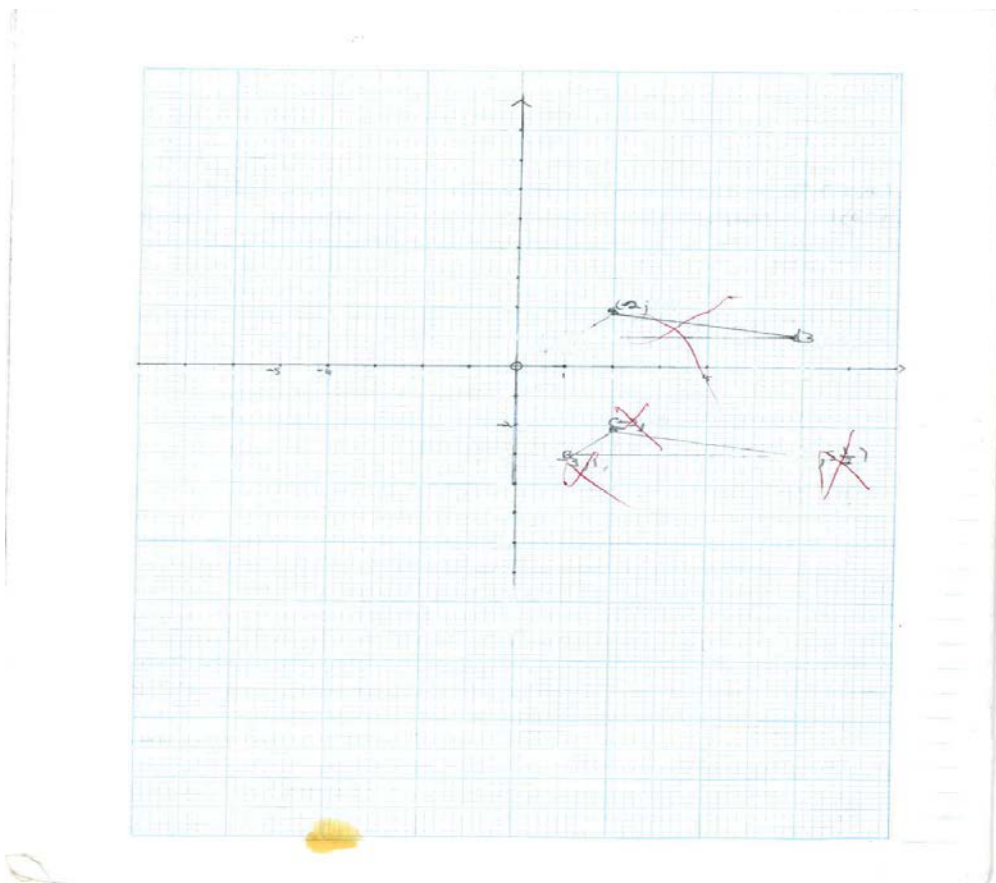


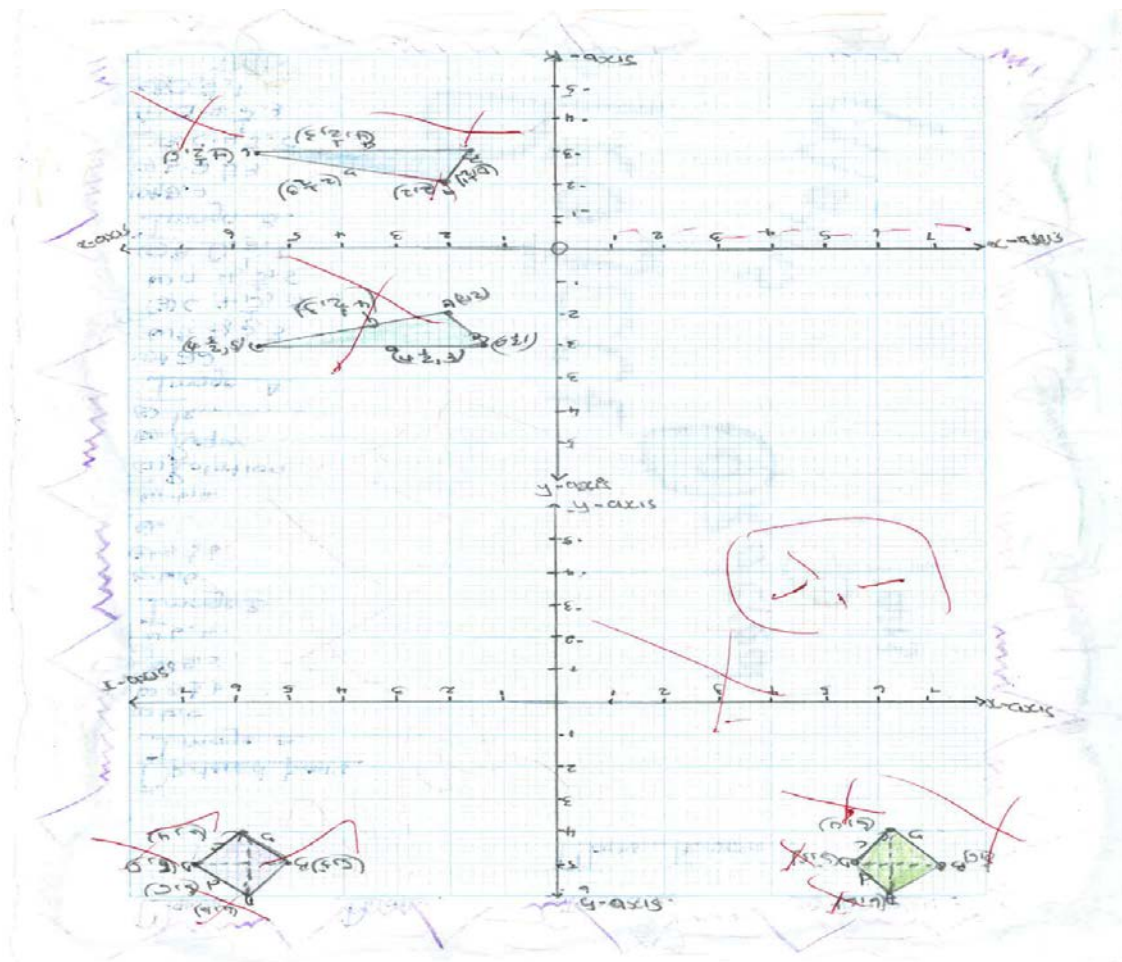
Figure 6: Results of learners for Activity 7

Table 12: Summary of Activity 7

School	No. in class	No. wrote	Attained level	Not Attained	Total
C	60	54	26 (48%)	28 (52%)	54
B	20	20	4 (20%)	16 (80%)	20

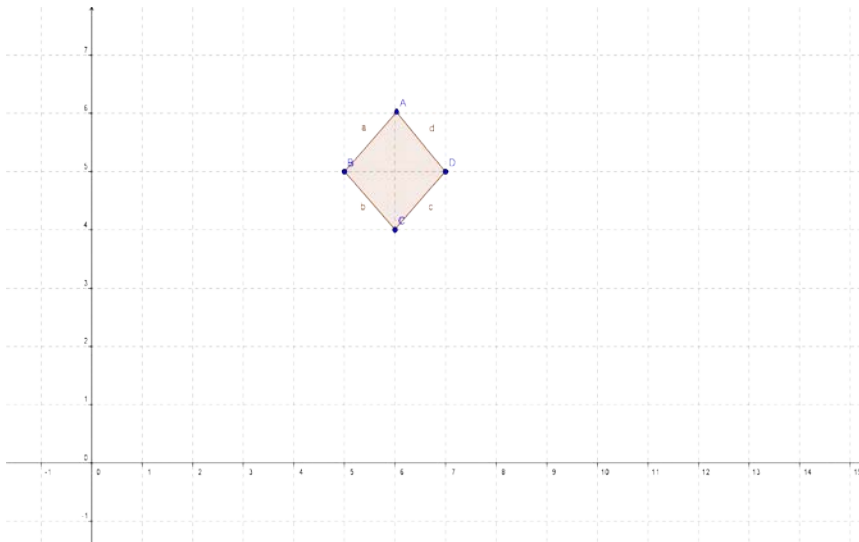
A problem evident in both schools was that learners were unable to draw the Cartesian plane correctly with its negative and positive numbers in the relevant quadrants, (Refer to Appendix E, Picture no. 23, 24 & 25).

Picture no.29



5.8.8. Activity 8

This activity is aimed at helping learners understand co-ordinates of points in the Cartesian plane, the terminology used in transformation geometry and finding relationship between rotation and reflection.



1. Give the ordered pairs of figure ABCD above.
2. Rotate figure ABCD from the point which is the centre of rotation 180 degrees counter-clockwise. Record the findings.
3. Reflect the original object ABCD downwards and record the findings.
4. What is the relationship between question 2 & 3 above? Write answers in your own words.

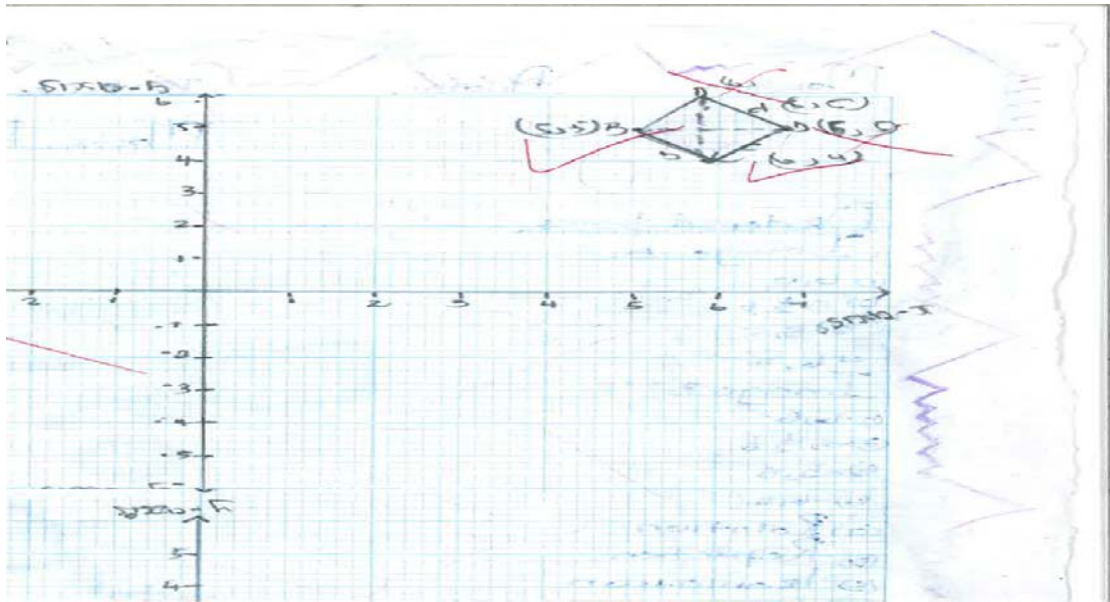
The learners' work, and the table and graph below show that some learning took place and some learners seemed to have understood the work that was done in the classroom. Learners seemed not to have carried out the instructions required by the question well, as they generally did only one part of the activity and ignored the rest. (Refer to Appendix E, Picture no.30 below).

Learners' work showed that there seemed to be some gaps in connection with drawing graphs accurately, making use of a ruler, labeling the graph correctly and writing negatives in the 3rd quadrant. In addition, learners' observation of the transformed object seems to have resulted in learners writing incorrect co-ordinates. Unfinished work seemed also to be a contributory factor towards low achievement, (Refer to Appendix E, Picture no. 27 & 11).

Picture no.11 (Refer to Appendix E) shows that the learner is able to draw a graph accurately, label it and count the spaces correctly. The learner seemed to have understood transformation and writing correct ordered pairs and was able to reflect the shape although s/he didn't fully carry out the instruction. The question required the original object to be reflected downwards and then rotated 180 degrees anti-clockwise. The learner also skipped question 4 that required a relationship between

question 2 & 3.

Picture no. 30



Learners such as the ones whose work is shown in picture 31 below seemed to be familiar with the terminology.

Picture no. 31

(a) Clockwise Direction is the movement that moves as watch wall, or is the direction that moves as watch wall.

(b) Anti-clockwise Direction is the movement that doesn't move as watch wall.

(c) Mirror Symmetry look as:

Mirror Image

Mirror Symmetry

Table 13: Summary of Activity 8

No. wrote	No. passed	% Passed	No. Failed	% Failed	No. Enrolled
49	22	45	27	55	60

It was evident when learners were doing activity 7 & 8 that they seemed to lack measurement and drawing skills. Some were unable to draw a correct Cartesian plane, label it and write correct co-ordinates in correct places, as in Picture no. 23, (Appendix E). Another issue that was evident was that of uncertainty around the x-axis and the y-axis. Thirdly, learners did not appear to grasp the importance of the order in an ordered pair, i.e. starting with the x co-ordinate then following with the y co-ordinate. If the ordered pair is (3; 1) they would incorrectly write (1; 3), (Refer to Picture nos. 9, 13 & 23 Appendix E).

Special attention from the teacher was required for learners work with quantifying translation and rotation. Learners seemed to battle with positioning, when required to translate a specific number of spaces to the right, left, up and down as shown below, (Refer to Picture no. 24, Appendix E). Some learners also made the mistake of calling a translation a reflection and rotation when required to transform the shape or object.

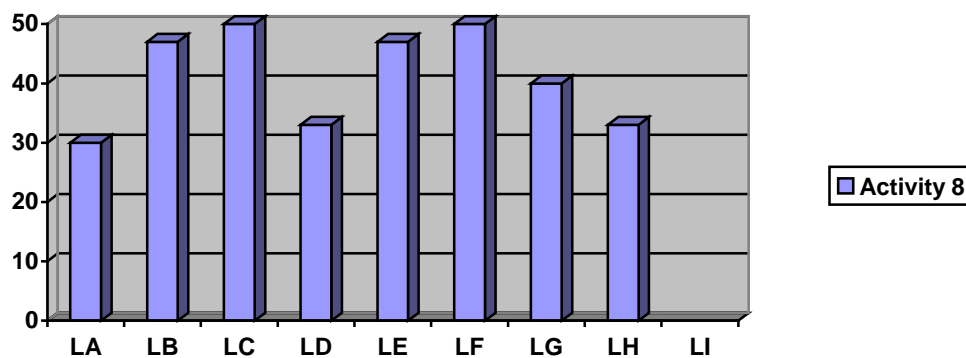


Figure 7: Learners' results for Activity 8

Although the activities showed that learning had taken place, most learners did not fully achieve Level 3, which is informal deductive reasoning (van Hiele, 1986). Not a single learner could go beyond Level 3 to Level 4 which is abstract deductive systems. Because this is not taught at school, this transition was not necessary and not be expected. Some remained at Levels 1 and many at level 2, i.e. (visualization and

analysis), (Refer to Picture no's 1, 2 & 3).

5.9. Findings

It appears as if many of the learners in school C compared shapes according to **equality of sides**. They looked at the lengths of the sides of each shape when finding similarities or connections and concluded that the sides of shape A are **equal** or **not equal** to the sides of shape B, Refer to Appendix E (Picture no. 14, 16 & 17).

Learners did the same thing when finding equal angles especially in the case of a rectangle and square, where they just said that all angles are 90 degrees. **Equality of angles** was also common even when giving properties of isosceles and equilateral triangles, learners just gave the properties without thinking or checking what the question asked.

Learner A found properties of shapes by counting **the number of sides** a shape has, and that is common in distinguishing quadrilaterals, three-sided shapes and other polygons. The picture no. 32 below of Appendix E explains further what learner A did in the activity.

Picture no.32

deg - May - 2011

33, 370

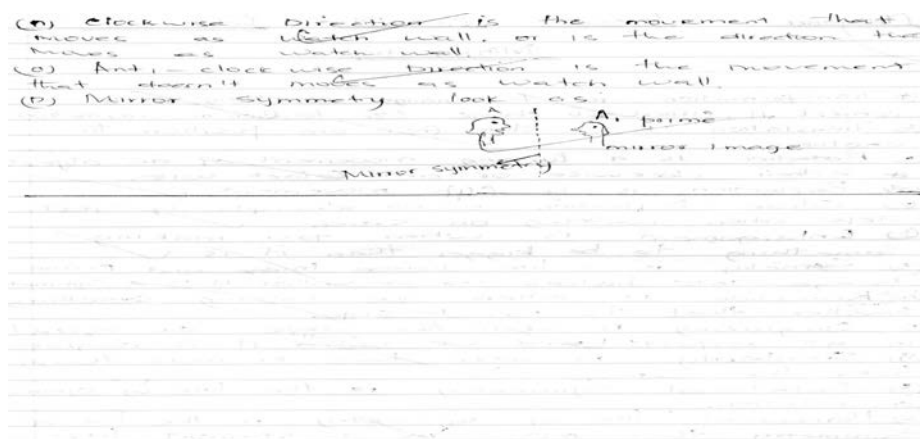
32

	PROPERTIES	
1. qbn	it has 3 sides	3
2. triangle	it has 3 sides	3 Sides
3. Right angle triangle	that are all equal	3 Sides
4. LAGON	3 Equal	3 Sides
5. Squadr	have all sides	4 Sides
6. all in on quadrilateral	it have 4 equal sides	4 Sides
7. um	it have 4 Sides	4 Sides
8. 950	it have 4 equal sides	4 Sides
9. 4	it have 6 sides	6 Sides
10. 30	are not equal	3 Sides
11. 12	it have 12 sides	12 Sides
12. 13	it have	13 Sides
13. 14	have	14 Sides
14. 15	equal sides	15 Sides
15. Pentagon	have	5 Sides
16. 17	equal sides	17 Sides

Some learners saw **connections** between a rhombus and a square in terms of **equality of sides** and that both have 4 angles, without mentioning that pairs of opposite sides are equal and parallel therefore they are both parallelograms. (Refer to Appendix E, Picture 14). Some learners did not see parallelism in a square, Refer to Appendix E, (Picture no. 4, 20 & 21).

Some seemed to be unable to write properties in words and preferred to furnish a drawing, rather than trying to explain what they wanted to say, (Refer to Picture no. 15 & 16, Appendix E) and Picture 31 below.

Picture no. 31



Some learners were able to see that the difference between a rhombus and a square is with the angles, as the square has all angles 90 degrees while the rhombus doesn't.

One learner measured the sizes of the obtuse angles a rhombus has, to show his understanding that one pair of angles is acute and the other one obtuse. When comparing a rhombus and a parallelogram, they only saw unequal sides, and few of them noticed parallelism.

The majority of the learners did not see **adjacency in shapes** as in a kite; and equality of **opposite sides or angles** as is the case of a parallelogram and rhombus. Learners were unable to distinguish between a rhombus and a kite. They all were able to recognize a 3D shape which is a rectangular prism and count the number of faces and sides because they saw rectangles in it.

Almost all learners were able to identify types of **transformation** and what each meant, but the problem lay with giving examples of transformations of specific shapes. Some did not know types of triangles hence they had a problem with drawing a scalene triangle, cutting it and transforming it according to the types of transformation. The only class of specific triangles that some learners knew is the

class of **right-angled triangles**. Some learners saw this as a special triangle because it has no equal sides and angles like a scalene triangle, hence some used it to do transformations. All learners were able to identify **symmetry** from objects, letters of the alphabet and shapes, although finding the number of lines of symmetry that a shape has was still a challenge to them.

5.10. Analysis of the responses of particular learners: (LA & LD)

Here follows an analysis of the responses of a particular learner, (LA) showing his performance in selected activities of the teaching programme. At the beginning of the teaching programme Learner A appeared as if s/he had not understood the properties of shapes and was only able to identify shapes by counting the number of sides a shape has. The performance improved gradually in activity 3 where s/he was able to express conceptual understanding, (Refer to Picture 32(i), Appendix E). The improvement might be the result of understanding or connection in activities 1-3. The performance declined towards the end of the teaching programme in activities (4-8) which were different to the first three activities. In Appendix E, Picture 32(i) showed that the learner gained some knowledge and was able to express him/herself defining concepts in his/her own words using the LOLT.

Learner D (LD)

The performance of this learner seemed to show some growth. He was among those who didn't do well in the baseline evaluation and he showed improvement on activities following the baseline evaluation. In these activities he became able to do the following:

He was able to see the following properties in shapes: equality of sides, counting the number of sides a shape has parallelism, equality and inequality of angles as in the case of a rhombus. He saw connections in some shapes but was unable to describe the differences he saw between two or more shapes, preferring to use drawings, (Refer to Picture no.15 &17, Appendix E).

He was able to show that the opposite angles in a rhombus are equal, by using a drawing, and not words. He was also able to briefly describe transformation geometric concepts in his own words using the LOLT, (Refer to Picture no.6, Appendix E). His description was accompanied by sketches where necessary, to elucidate the concept or term further (Refer to Appendix E, Picture no. 15).

He knew the properties of transformation and was able to recognize lines of symmetry in shapes, objects and some letters of the alphabet, (Refer to Picture no. 16, Appendix E).

This learner knew the types of triangles and their properties. He was able to distinguish between an object and its image by writing a prime in the label of the image, (Refer to Appendix E, Picture no. 26). He was able to draw a correct Cartesian plane with axes and to write correct ordered pairs with x and y co-ordinates in the correct order, and not vice-versa. He had no problem with mathematical language and the terminology used and was able to describe concepts in his own words using the LOLT. The graph below shows how **LD** progressed per activity during the implementation of the activities of the teaching programme.

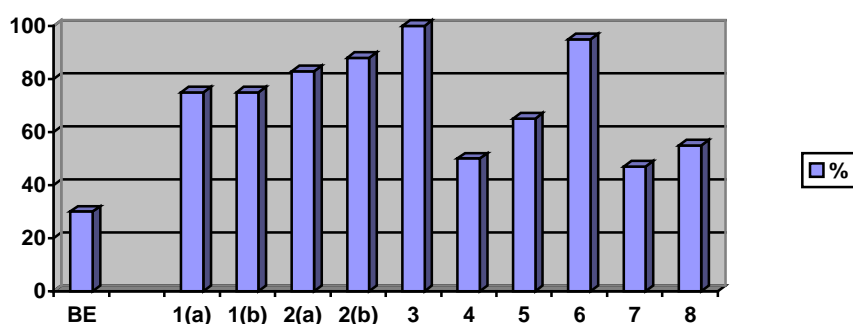


Figure 8: Learner D performance

The graph above shows two sequences of learning or mastering the tasks learnt in the classroom. The learner showed gradual improvement in activities 1(a) to 3 as the activities had connections which led to understanding the concepts taught. Activity 3 showed that the learner had mastered the concepts taught. The performance declined in activity 4 as it was different to the former activities done earlier. Learner performance in activity 4 showed that the learner was dealing with new concepts and didn't master them immediately although he managed to perform at 45%.

The performance of the learner improved over the sequence 4-6 as the learner developed to master the similar aspects of the activities. In activity 7 the performance of the learner declined as new concepts were introduced again. Activity 8 had connections with activity 7 and some improvement in performance was evident. This shows that this learner masters the concepts well when there are connections in activities and improved performance more than when new concepts unfamiliar to him

were introduced. The graph shows that the learner learns gradually at his own pace and has no problem with the LOLT.

5.11. Impact of the programme on learners

5.11.1. Strengths

Learners reported the following responses to the programme:

Learners' reflections on levels and feedback: Learners compared their marks daily on activities done in the classroom and they did reflect orally on what went well and what did not go well and why. Using the levels in this way, helped them to monitor their own progress before being told by the teacher or given feedback; hence they were eager to be given make up activities to work on in order to proceed to the next level when requirements are met. Refer to Appendix B (iii), L1 line 258: *'Yes, I think that those levels were upping my marks'*. Oral pre- and post-interviews also provided feedback and evidence to the teacher on what learners know in mathematics.

Discipline: Their attitude towards mathematics seems to have changed to be more positive, they seemed to show curiosity towards learning mathematics and they felt bad when underperforming. Hence they asked for make-up assignments to close the gap and catch up with others in the classroom to attain the required level. Refer to Appendix B (iii), L3 Line 76: *'I think it is an interesting thing in mathematics'*. L3 Line 77: *'Yes, because it is interesting and challenging'*.

Motivation: Seeing positive results in their work through close monitoring of learning and immediate feedback, promoted greater eagerness to learn mathematics and to own their learning. The outcome of the activities showed that Kilpatrick's strands are really interwoven, in that one strand cannot function well without the other, and are helpful if used well in the classroom activities. Learners who understood concepts would be able to carry out procedures using correct strategies while reasoning strategically in the classroom. Kilpatrick *et al.* (2001) argue that learners see sense in mathematics, perceive it as both useful and worthwhile and see themselves as effective learners and doers of mathematics.

Reasoning: The activities seemed to have challenged learners to think critically and logically (Refer to Picture 14, Appendix E), that is why some tried to explain concepts using drawings when unable to use words. (Refer to Appendix E, Picture no. 15 &

16). Learners were free to reach their own conclusions around concepts by associating shapes with objects (Refer to Picture 12, Appendix E).

Language development: Most learners from school C were able to describe transformation geometry in their own words using the LOLT, and furnish some drawings where necessary, (Refer to Appendix E, Picture no. 6, 12 & 15) whilst some from school B couldn't do this. The LOLT was used well by the majority of learners in school C even during the interview. The programme catered for the levels of van Hiele (1986) which involves the use of language throughout in all levels and phases. Some learners are still struggling with speaking the language, (Refer to Appendix B, Learner interviews).

5.11.2. Impact of the two theories in the teacher's teaching practice

Teaching using the programme with activities that take account of levels, strands and phases has taught me patience and better understanding of how learners learn and that each learns at his/her own pace. Van Hiele's levels and phases helped the teacher to see the level in which each learner belongs and that seemed to have encouraged the teacher to provide individual attention to each member of the group. Levels have shown the teacher the gaps the learners had so that s/he could intervene effectively to close them.

The use of the two theories during research has encouraged teachers to prepare their lessons thoroughly and the activities accommodated the level that each learner was in. Lesson presentation using manipulatives seemed to have helped the learners understand the concepts of the lesson better, through being actively involved in their manipulation during the lesson.

This use of van Hiele's levels and phases and Kilpatrick's strands seemed to have motivated the learners to develop a love of mathematics and decreased the level of absenteeism as most of them attended school every day in order to catch up with the daily activities performed in the classroom and to know at which level they were. Activities motivated school C learners to express themselves in the LOLT (Language of Learning and Teaching), and although they struggled, they tried their best even during the interviews to express themselves in the medium of instruction.

Van Hiele's levels (1986) and phases and Kilpatrick *et al.*'s strands (2001) have helped the teacher to know the thinking level of each learner, whether the learner has

grasped the concept or not, and whether s/he is able to carry out procedures or not before moving further. This encouraged the teacher to give more individual attention to learners in the class although it is time consuming. There were learners who were below Level 1 and had to be given more activities to boost them to Level 1, especially the newcomers.

The research, levels and strands seemed to have motivated the learners to develop a love of mathematics and encouraged good attendance. When they fell below the level they became frustrated and pressurised the teacher to give them other make up activities to work on. Although they struggled with the language, they tried their best to understand the concepts, especially if the teacher, did some code switching to explain the concepts further in their language, (Refer to Appendix B, Learner interviews of school B).

Their observation and writing skills have developed gradually and seemed to improve, especially those who do not like to answer questions out loud in the classroom. They are still striving to make connections and differences between concepts and shapes although none of them could go beyond Level 3.

5.11.3. Impact of the Teaching Programme on teachers

The engagement with the activities of the programme seemed to have helped the teacher to achieve the following:

- a) It appeared to have helped the teacher in selecting tasks, in choosing when to give the task and how to present it in an effective manner for better learner understanding of concepts in the classroom. Teacher B did not have a lesson plan with activities to be done in the classroom, she was completely dependent on the textbook and she was not sure which letter to choose and give the learners to reflect. In line 003 (Pre-lesson observation), Teacher B said, *"Let me give you a C or a B to translate this letter in your groups"*. (She erased letter C and B and asked learners to reflect the same letter F she used as an example when she started the lesson). In line 004 (Pre-lesson observation), TB gave the following instruction when teaching reflection, *"In your groups translate this letter (erased letter C), reflect the same letter F downward from the first quadrant to the 4th quadrant. Translate this letter using a ruler to make a drawing of a Cartesian plane. Use 1cm space, reflect it and see what the image will look like.*

b) According to Kilpatrick *et al.* (2001), of which I fully agree with concerning the programme, the teacher needs to have a clear vision of the goals of instruction and what proficiency means for a specific mathematical content that one teaches. Teachers need to know what they teach as well as the horizons of mathematics, where it can lead and where the students are headed with it. This research has taught me that the teacher must be able to interpret the students' written work, analyse their reasoning, and respond to different methods they might use in solving problems. The motive behind this statement is achieving better mathematics understanding which leads to better results.

The consistent use of manipulatives seems to have encouraged teachers to plan their lessons beforehand thoroughly, and organise manipulatives before the lesson is presented. As teacher B said earlier, preparing lessons before going to class makes the lesson easy and understandable. The teaching programme appears as if it has promoted individual attention as the teacher has to know each learner's problems and strengths every day. Awarding levels to learners individually, on activities done daily, resulted in knowing each learners problems.

5.12. Weaknesses of the programme

Development of co-ordinate notation

This was not catered for by the programme as the programme focus was solely on effective strategies for teaching and learning transformation geometry. But the learners' weakness in using coordinate notation became evident throughout the programme. Occasionally in some activities, these learners wrote ordered pairs incorrectly confusing the x co-ordinate with the y co-ordinate which shows they have not grasped the concept yet and are unable to carry out procedures and other strands, (Refer to Appendix E, Picture no.9).

Language of Teaching and Learning

Learners did not display proficient use of English (which was the LOLT) during the programme and this may have had an effect on the research. The teacher had to use their mother tongue (isiXhosa) to explain concepts for them to understand better and that alone was time consuming.

Language issues during research: Learning in a second language which is unfamiliar to the learner was a problem throughout the research process, although the language is the medium of instruction. Learners had problems with understanding questions posed to them. The teacher had to code switch to help learners understand what was being asked. During interviews at school B, learners couldn't express themselves in English; they replied in their mother tongue and that shows that some learners still have communication problems using the LOLT, hence the transcription is in both languages in pre- and post-interviews at school B.

The problem with the use of the LOLT was evident even during interviews where learners couldn't express themselves and preferred to use their mother tongue when answering questions (Refer to School B learner interviews). Some kept quiet until the teacher rephrased the questions in isiXhosa. Some used phrases like "was didn't know" when answering questions, which shows that there is a problem of language in our rural schools. Learners also have a problem with spelling when using LOLT or they read words incorrectly.

5.13. Support given by the Teaching Programme to improve teaching

- a. At the end of the programme the teachers seemed to have gained preparation skills, in that lessons were planned according to the levels and phases of van Hiele (1986) and Kilpatrick's strands (2001) so that the teacher knows more accurately the level of each learner fits, what the learner knows and doesn't know and what the teacher could do to help the learner to improve in the classroom.
- b. Teachers became more aware of the complexity of the learning process. As a result, the programme has motivated teachers to be more patient and accommodate the problems that learners needed to work through to develop their understanding. It also provided some insight into how to deal with the challenges of learning, so that learners could move from one level to another, (Refer to Appendix B, TB Line 006).
- c. Preparation skills gained by teachers appeared to lead them to be able to reflect orally and in writing as reflective practitioners. During pre- and post-interviews, teachers were able to express themselves verbally on questions posed about the programme as they were part and parcel of it.

- d. Presentation: Lessons were learner-centered. Learners were given opportunity to present some feedback from activities to the whole class, what they had done in their groups and that showed great improvement to what happened during pre-lesson observation where the whole lesson was done by the teacher without involving the learners.
- e. The use of manipulatives in the classroom seemed to improve learner understanding as they learned the concepts under discussion practically, involving learners throughout the lesson. The observation skill of learners improved while using manipulatives in the programme in so much that one learner from school B confirmed during post interviews that she also had a problem with knowing how to check co-ordinates at first but she observed the teacher how she did it and she also mastered it: (Appendix B (vi) L3, Post interviews, line 360) *“At first I didn’t know how to check co-ordinates. Interviewer: Who told you? L3: I saw from other examples how to do it.”*
- f. The programme appeared to have helped the teacher to see the importance of monitoring the learning progress in the classroom. Also the importance of dealing with learners individually to see how each learner progresses without the help of others in a group.
- g. Teaching transformation geometry using these activities seemed to be a positive experience to both teachers and learners. As a result, it seems to have motivated teachers to be more committed and engaged in their mathematics teaching in general, and the teaching of transformational geometry in particular.
- h. Teachers seemed to have gained more knowledge and understanding about the learning and teaching of transformation geometry through engagement in the activities of the programme. Teacher B (Refer to Appendix B, line 045) made examples of how transformation geometry integrates well with other learning areas such as Arts & Culture, EMS and Social Sciences and that convinced me during the post-lesson observation that she has understood the topic well.

5.14. Using van Hiele's levels & phases to determine learners' geometric thinking

The planning for this was discussed in detail in chapter 3. As I was introducing the teaching programme to learners in the classroom with activities classified into the levels of abilities that learners have, I found out that there were learners who were between the pre-recognition and recognition phase meaning that they are between level 0 and 1 according to van Hiele (1986). They were able to identify and recognize only basic shapes such as a square, rectangle, triangle and circle.

My understanding of the van Hiele levels (as a researcher involved in teaching the activities of the teaching programme), led me to teach the learners in a way fitting to their level and to work with them to recognise other shapes and to be able to give the properties of some shapes, before proceeding to more analytical and deductive work. As a result, learners were able to move from this basic level of understanding, to more advanced levels, and thus experience success and positive progress in geometry learning. In line with van Hiele's theory, this work involved encouraging the learners to think and reason as well as to express themselves orally and in writing in the classroom.

In addition, as stated by learners during post-interview sessions, identifying the levels attained seemed to give learners a chance to improve if they had performed badly in the previous activity through the use of appropriate make up activities (See L2 of school C). L3 (line 353 of school B) said during a post-interview session that levels help learners know where their mistakes are and give the learner a chance to focus on them.

5.15. Using Kilpatrick *et al.*'s strands of learning in the classroom

The use of Kilpatrick *et al.* (2001) strands in the classroom could yield positive results as the learner understands the concept before carrying out the procedures in many instances. Mastery of concepts and carrying out procedures give learners ability to formulate and represent concepts diagrammatically as they did in picture (15 & 16, Appendix E); laying a foundation for adaptive reasoning abilities for logical thinking, explanation and reflection. This had been clear even during learner interviews that effective communication depends on understanding the question and concept under discussion. (Refer to Appendix B (iii) to B (vi), Learner interviews). Some learners

communicated well while others could not. Kilpatrick's strands could bring positive results if they could be used in the classroom, giving learners a chance to work co-operatively with the teacher to fit in to the best level until they reach the highest level. By so doing mathematical knowledge can be developed in all learners when correct methods and lesson preparation is up to standard to all schools.

5.16. Conclusion

The data collected, discussed and analyzed in this chapter were in the form of observations before and after the programme commenced, together with interviews. Pre and post lesson interviews showed that there are learners who can't express themselves in the language of teaching and learning in grade 9 (Refer to Pre & Post learner Interviews of school B). Intervention will be needed to help these learners to use the language for everyday use in the classroom. A number of additional challenges were identified that accompanied the use of the teaching programme with activities in the classroom. These have been discussed in this chapter.

The data collected showed that successful learning of geometrical concepts appropriate to the learners' current level of understanding of geometry is possible and that this is experienced as positive by the learners. But because of poor prior geometric knowledge, learners need to develop a great deal more knowledge of geometry and transformation geometry. The analysis showed that the use of van Hiele's theory and Kilpatrick *et al.*'s strands could bring positive results if they could be used consistently and co-operatively in schools.

CHAPTER 6: RESULTS & DISCUSSION

6.1. Introduction

This chapter focuses on the results of the investigation throughout the period of study before and after the implementation of the teaching programme in the classroom. In this chapter, detailed comments are made in relation to the research questions:

- a. How did the implementation of the teaching programme influence the teacher's classroom practice and understanding of learners' learning styles?
- b. How did the implementation of the teaching programme influence the learning of transformation geometry in the classroom?
- c. How did the support programme for teachers influence the teaching and learning of transformation geometry in the classroom?

These three questions complement each other; one deals with showing evidence of how the teaching programme influenced the teacher's practice of teaching transformation geometry in the classroom and the second deals with the influence of the teaching programme on the learning of transformation geometry. The last one deals with teacher development in terms of content knowledge, techniques and skills of teaching transformation geometry and understanding different learning styles of learners. The chapter provides a discussion of the main research findings of the study.

6.2. Main findings

6.2.1. General effects of implementing the learning programme

6.2.1.1. Consistent, well planned, well prepared teaching that is meaningful to learners has a powerful effect on enabling committed learning by learners. The preparation of lessons beforehand by the teacher shows commitment to his/her work and this appeared to have a positive effect on learners. Also the teacher became confident about the topic to teach. Such well prepared lessons by committed teachers appeared to result in improvements in the quality and meaning of teaching and learning in the classroom. Prepared lessons show preparedness to teach by the teacher who used to go to class unprepared and unsure before the implementation of the programme (Extract 13, Appendix B(i) Line 011), then the teacher was prepared and teaching for conceptual understanding, procedural fluency, strategic competency, adaptive reasoning as argued by Kilpatrick *et al.* (2001).

Teaching with the use of manipulatives in all concepts resulted in meaningful lessons that were understandable to learners, making sense of what was taught (Kilpatrick *et al.*, 2001, p. 118). Learners had to visualise, order and analyse the concepts taught as stated in van Hiele's (1986) levels.

Learners appeared to learn by imitating their committed teacher who became a good and a dedicated example to them. The example set and tasks given to them lead them to work hard and observe deadlines given by the teacher. The statements above relate to effective teaching and learning which Kilpatrick *et al.* (2001, p.9) argues that, "Effectiveness depends on enactment, on mutual and interdependent interaction of the three elements: content, teacher and students". Learners of school C showed commitment in their learning as it is indicated by graphs summarising their learning in chapter 5 above. Learner interviews in both schools B & C show that learners were committed in the programme together with their teacher in answering questions asked on transformation geometry.

Before the implementation of the teaching programme, the teacher seemed to be unsure of what and how to teach transformation geometry. The concept made the teacher feel uncomfortable and less confident in class before the implementation of the programme, as she said during a pre-observation interview (Refer to Appendix B (i) Line 010 & 011). Kilpatrick *et al.* (2001) argue that proficiency in the subject one teaches is important and that teaching for mathematical proficiency leads to effective teaching and good results. "Effective teaching that fosters the development of mathematical proficiency over time can take a variety of forms each with its own possibility" (Kilpatrick *et al.*, 2001, p. 8).

In fact, the teacher used to ignore the topic and teach other concepts because she was not sure how to approach it. She said during interviews, as discussed in chapter 5 that she was teaching transformation geometry for the first time in grade 9 (Refer to Appendix B (i), line 010); as a result she didn't include or bring the necessary manipulatives in the classroom and taught some examples orally during pre-observation lessons.

Meetings held unpacked the concept of transformation geometry, how to teach it and what manipulatives to bring per activity to bring home the concept. These meetings helped in rebuilding the confidence lost by the teacher as s/he became certain of what to teach, when and how to teach it. After the meetings were held, having the

programme at hand, and with the support of the researcher, the teacher was able to teach the concepts of transformation geometry with confidence.

6.2.1.2. Importance of lesson planning and preparation

Lesson planning gave teachers time to think carefully about the topic to teach, how to teach it and when to teach it, taking into consideration all possible problems the lessons might be having depending on the approach used in the classroom. The goals of the lesson were taken into account and the lesson was designed to reach them. One of the ways of making sure that goals are achieved is by assessing the learner to see whether the lesson was a success or not and what to do next.

The confidence of the teacher increased as a result of meetings held discussing the content and the teacher was sure about what to teach and how to teach the concept. Effectiveness as argued by Kilpatrick *et al.* (2001) was evident that it really depends on mutual and interdependent interaction of the teacher, content and students. The teacher's lessons were not prepared when we first met - possibly due to the teacher's uncertainty about the content and how to teach it. She used to just take the examples and activities from the textbook without first trying them for herself.

The results of not preparing beforehand were evident in her uncertainty in the classroom, for example, not being sure which letter of the alphabet (B, C or F) to choose for the activity requesting the transformation of the letter by translating, reflecting or rotating it (Refer to Appendix B(i)- Pre-Lesson Observation, Line 004).

6.2.1.3. Usefulness of van Hiele's levels for the teacher's practice

Van Hiele's (1986) levels and phases helped the teacher to prepare and present the lessons in ways that helped learners to understand the concepts under discussion. Activities given to learners do not determine the level, but the children's responses to the activity determine the level of the learner. Assigning levels to responses helped the teacher to know which level the learner is at, and so what activities and teaching responses would be effective for the learner.

The responses of learners in the activities of the programme did help the teacher to know the learner as an individual, what the learner knows, how the learner thinks, what s/he doesn't know and needs to know. These gave the teacher a chance to research and develop other activities that could help to iron out the problems that

learners might be having with the concept. This preparation and the teaching based on it could then be done according to the different needs of the learners.

Van Hiele's levels (1986) and phases have a positive impact on learning of transformation geometry and may lead to effective teaching and learning when planning lessons to be taught in the classroom. They could inform the teacher about how the learner is progressing per concept taught and what help the learner needs on the topic discussed. It was evident that a learner can be in two or more levels depending on their understanding of the concepts under discussion (Refer to LC of school C, who is in different levels of learning in different activities. In activities 1 & 2 s/he is in level 2 while in activity 3 s/he is in level 1).

6.2.1.4. Effects of the programme in the classroom

The activities of the teaching programme encouraged the teacher to track how the learner is coping or struggling with the concept under discussion. Most of the activities were individually based and required feedback from the learner. This gave the learner a chance to express him/herself in writing without any help from other learners. This also informed the teacher about the performance of each learner. Individual attention given to learners helped both the teacher and learners to develop diligence, patience, a good teacher-learner relationship and enjoyment of the learning. One on one sessions with learners helping them with problem areas, enhanced their desire to work harder than they used to for better results and that brought the teacher and learners very closely together. The teacher became more willing to sacrifice time and engage deeply in their teaching of mathematics and helping learners with different learning problems.

Engaging learners in interviews before and after the implementation of the programme enabled the researcher to know what the learners knew and did not know about transformation geometry, what to include and what to exclude from the activities of the teaching programme. This also motivated learners to work hard and develop competing attitudes that led them to pass at the end of the year, (Refer to Appendix B (v)-Post-interviews L4, line 300, 301 & 304).

The teaching programme seemed to have helped the teacher to develop in the following ways:-

- a) The teacher gained knowledge and understanding of the learning styles of learners and that they learn at different paces.

- b) The teacher developed more patience (Refer to Appendix B (i)- pre-lesson observation, line 006) in helping learners understand the concepts under discussion and stopped rushing to be in line with the pace setter or the school's year plan and so leaving learners behind with no conceptual understanding.

6.2.1.5. Concrete experience – manipulatives and learning

The consistent use of manipulatives in the classroom contributed to effective teaching and learning of geometry and transformation geometry as learners appeared to learn concepts with understanding when manipulatives were used' (Refer to Post Lesson Observation, line 13 - 20). The programme helped the teacher to base the activities of the programme in a context that the learners understood, i.e. shapes they see every day in the community and at school, objects such as mirrors, and models of clocks, cars, cell phones that slide, flip and rotate.

Consistent use of manipulatives in each activity encouraged the teacher to plan ahead, research the topic to be discussed before teaching it and collect all the necessary resources required by the lesson to make it interesting and understandable, (Refer to Appendix C, Activity 5 of the programme).

Extract 14: Teacher B, Appendix B (ii), Line 013. *“Let us look at the table. I have objects that are displayed here. You can come and take the object and say if it is a slide, a flip, a turn etc”. Brother Bongani has a good example of enlargement. He uses a camera which takes pictures and makes them small. There are some adjustment knobs changing the size of the object.*

Learners' participation in this activity was good and learners picked up and named objects shown to them.

Heddens as quoted by Schweyer (2000) on the use of manipulatives argues that the results are a better understanding and retention of mathematics, a decrease in mathematics anxiety and a heightened confidence level among students who have really made a lesson their own. Activity 5 seemed to be a success and interesting to learners as they manipulated and discussed the objects that were familiar to them and in their environment.

Learners understand better when concrete objects that they know are brought into the classroom and related to their context, as argued by Schweyer (2000). Shaw (2002) argues that manipulatives help students develop conceptual understanding of

mathematical ideas by representing the ideas in multiple ways. "Manipulatives can enhance student understanding. They can enable teachers and students to have a conversation that is grounded in a common referential medium and they can provide material on which students can act productively, provided they reflect on their actions in relation to the mathematics being taught." (Kilpatrick *et al.* 2001, p.353). Refer also to Extract 14: TB, Appendix B (ii), Line 13 above.

6.2.1.6. Teacher's knowledge of the subject

The teacher's lack of content knowledge as well as knowledge of teaching the content led to poor commitment to teaching transformation geometry and poorly prepared and implemented teaching. This was evident with Teacher B before the implementation of the teaching programme and she had confirmed that she was teaching the topic for the first time in grade 9.

Kilpatrick *et al.* (2001) argues about the importance of proficiency in the subject one teaches although it takes time to develop; also the importance of teachers' professional development.

Thorough lesson preparation seemed to have improved self confidence, acceptance of the gap the teacher might be having and developed a positive attitude in the teacher and that appeared to have improved proficiency in the subject.

Extract 15: TB, Appendix B (ii), Line 019.TB *"Transformation is there in our daily lives as we have seen that we are making this paper fold into two halves, this is reflection. If we open and close our books we are doing reflection. Look at our faces in the mirror, we are doing what...?"*.

Learners: (*Chorus*) Reflection

6.2.1.7. Explicit learning progression

a) Learner understanding of geometry

Learner performance in class and post-interviews informed the researcher that learners initially seemed to have little understanding of geometry according to van Hiele's levels of learning. For example, when learners were asked to name any shapes they know, some said "breadth" (Refer to Post Lesson Observation, Line 001 & 002: school B). Some learners who knew the answer helped those who did not know by giving correct answers, and that showed that some learning did take place. (Refer to Appendix H, PLO, L3-L6, Line 3. Some learners were very shy and quiet during pre-

programme interviews but there was an improvement during post-programme interviews and L3 made an extra mile of communicating with the LOLT throughout the interview and that was impressive. (Refer to Appendix B (vi), L3, Lines 348-364). Learner interviews and activities done encouraged them to communicate orally and in writing in the language of teaching and learning.

(b) Different levels per concept

Responses given by learners to questions asked show acquisition of knowledge on concepts discussed and that determines the level the learner might be in. I found (in full agreement with Mayberry, 1983) that a learner can be at different levels of performance per activity per topic, depending on conceptual understanding of the topic under discussion at the time. Figure 9 below contains data showing LA's performance in activities of the programme. LA has been in different levels per activity. Data collected shows that in 1(a) he was at Level 2, activity 1(b) performance dropped to Level 1 as he was unable to give properties of shapes. In activity 3 he was up again at Level 2 preparing himself for Level 3 but the data shows that his performance declined again in activity 5, 6, & 7 and he had no task for activity 8.

Topics vary on their levels of difficulty and that leads to different levels of attainment. Levels are ways of thinking that vary from concept to concept and from one activity to another. Data collected and graphs given in chapter 5 above show that the learner performance is different per activity depending on conceptual understanding of the learner and that has an impact on the level a learner has attained. Even the way of thinking and reasoning learners displayed during interviews is not the same (Refer to Appendix B (iii) – B (vi), some responses were answered more effectively than others depending on the type of questions posed.

Teachers need to be able to understand which level the learner is at, for different subfields. Global attainment of the level may then occur once the learner has reached that level in enough local fields so that these local levels then generalize easily to other unfamiliar local fields (and so we have a more or less global functioning at this level).

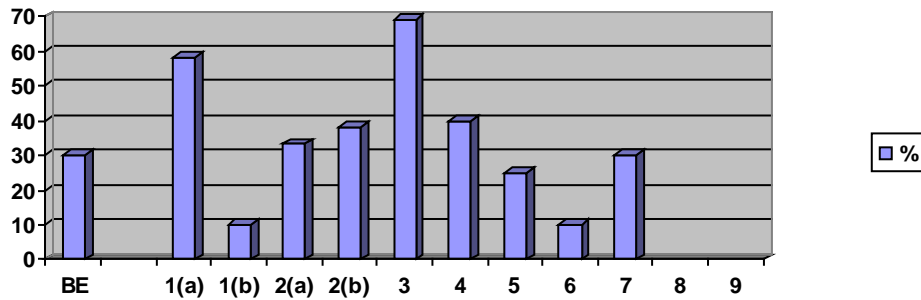


Figure 9: Results of LA's performance for all activities

(c) Attitudes of learners

As a teacher and researcher on activities I found out that the teaching programme had its pros and cons. The positive effect the programme had on learners was that it motivated them to learn transformation geometry, it kept them busy all the time and motivated them to do their homework and projects. The negative side of it was that activities were time consuming to learners. The best part was that working hard helped them to pass at the end of the year through diligence, complete engagement and commitment to activities given in the classroom, (Refer to L3 in Line 76 & 77: Post interviews). The learners confirmed that mathematics became more interesting and challenging.

Learners were able to describe transformation geometry terminology in their own words and furnished drawings where necessary trying to show what they had to say about the question, (Refer to Appendix E, Picture no. 6, 12, & 15). This relates to Kilpatrick *et al.*'s adaptive reasoning as, "Adaptive reasoning is the capacity to think logically about relationships among concepts and situation. Learners are able to display reasoning ability if they have sufficient knowledge base, if the task is understandable and motivating and if the context is familiar and comfortable" (Kilpatrick *et al.*, 2001, p.173), (Refer to Appendix E, Picture no.14).

6.3. Analysis and findings about the impact of van Hiele's theory on learning

Van Hiele's levels seem to have a positive impact of improving learner performance in the classroom during the implementation of programme activities.

1. The levels provided useful feedback to the learners about what and how they were learning. That is, they made it possible for the learners to evaluate where

they stood and to be able to decide for themselves if they needed to do more to master the work. Knowing the level the learner is at influences the learner's eagerness to do more activities that tally the required level of the activity. Dropping from level 3 to level 2 thinking shows that the learner had not mastered the concepts based on activities that required a level 3 way of thinking and the learner needs more activities that could help him/her to attain the requirements of level 3 thinking.

2. The presence of 'make up' activities that the learners could do (as a routine feature of the teaching programme), gave them a way of responding to improve their learning, when they saw that they hadn't fully mastered the learning; this could be done in a non-threatening way.
3. Combined, (1) and (2) above made it possible for the learners to take control of their own learning and they did this quite effectively. They make sure that they work harder to maintain the thinking level and be on par with their classmates at all times.
4. It appears as if van Hiele's theory assesses the level of performance of the learner per activity and that the levels of the learners vary from activity to activity depending on learner understanding and the familiarity of the specific concepts used in the activity. As learner performance is activity based, and levels can fluctuate, especially if learners do not have a firm foundation of the concept, it may at times appear as if learners have never learned about the geometric topic or concept required by the activity.
5. Learners seemed to drop from Level 3 to Level 1, depending on the requirements of the activity in the question. This goes against the characteristic of the theory which cites that van Hiele's levels (1986) of geometric reasoning are general and sequential.
6. The learner sometimes seems to be working in two different levels at the same time. For this reason, it appears to be difficult for the teacher to be sure which level the learner is at. Since learners are in control of their own learning and are always comparing themselves with others in terms of performance in the classroom, whenever feedback stated that s/he had worked

below the required thinking level and standard; that challenged the learners to request for an additional activity that could take them to the required level. Development is a process hence the learners moved from Level 1 to Level 2, but when I introduced Level 2 activities some failed to maintain the level and remained at Level 1, (Refer to LA in figure 9 above). Some of those who have passed Level 2 and progressed to Level 3, failed Level 3 activities and remained in Level 2.

7. It appears from the research that the van Hiele (1986) phases of learning encourage the teacher to work very closely with individual learners, facilitating learning in the classroom. Effective teaching and learning truly depends on enactment as Kilpatrick et al 2001 argues that it depends on mutual interdependent interaction of the three elements - (the teacher, the content and the learner). They also bring out the importance of the use of language throughout the learning process, thus giving the learner a chance to develop communication skills which are important keys to learning in the classroom.
8. From this research, it appears that knowledge of the levels of van Hiele (1986) may provide a form of extrinsic motivation to learners, providing them with effective feedback about their learning and encouraging them to become eager and willing to learn and perform better each time the activity is given to them. The levels and phases of van Hiele need a long time for the learner to develop effectively and I found out that proper development will never be effective in two months of implementation of the activities but needed more time. Data collected, discussed and analysed in chapter 5 & 6 show that learning took place and a lot needs to be done to develop the thinking level abilities of learners for quality and effective teaching and learning in the classroom.

6.4. Analysis of the impact of the Kilpatrick *et al.* (2001) model of learning

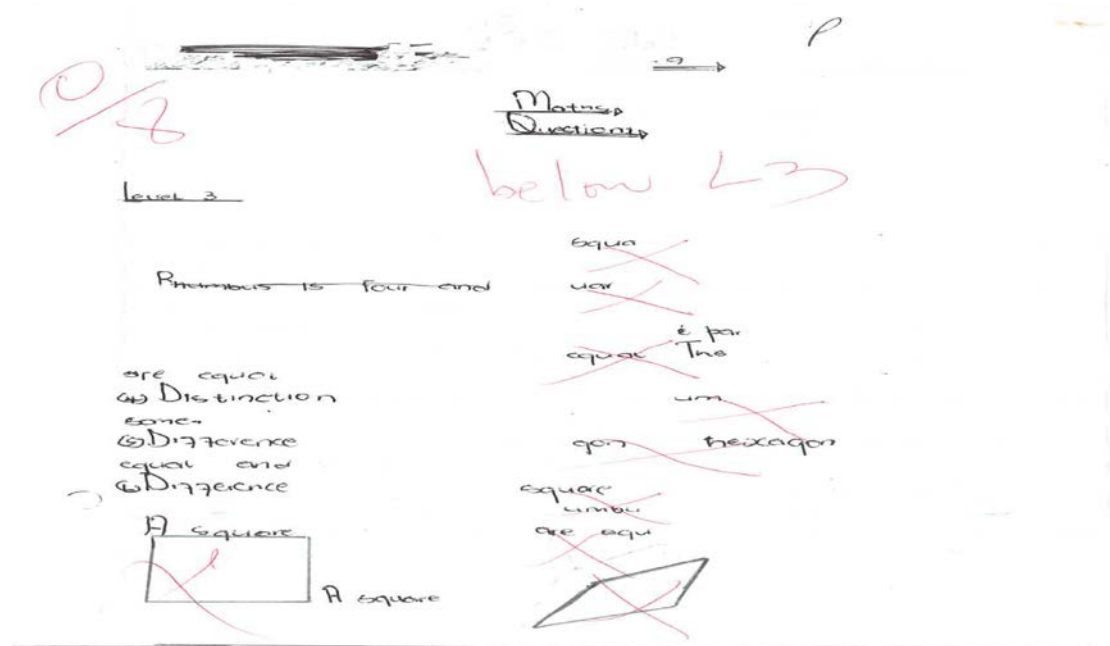
Not all strands were included in the programme focus for this research, because of time constraints. But conceptual understanding, procedural fluency, strategic competence and reasoning did play a role in making sure that the learners mastered the concepts and were ready to carry out procedures as expected. Although some were

struggling, helping them through gave them an understanding of what was expected of them.

As teaching and learning of the teaching programme was based on the application of Kilpatrick’s model in the classroom, it appears as if some learners learned better when they first had an understanding of concepts and expressed their understanding in drawing form as in Picture no.12 & 15, and other strands fell into place. Others were able to start learning by working on procedures and were able to describe the concept as necessary, (Refer to Picture no. 5, Appendix E).

To me this shows that learners learn differently although the major important yardstick to learning is understanding what is being taught. Some learners seemed to be learning by rote as it sometimes became difficult to link what they had learned to related instances, but they tried to reproduce what they had learned even if it was irrelevant, (Refer to Picture no. 21 & Picture no.1, Appendix E). Kilpatrick et al (2001) stresses the importance of understanding the concept first before carrying out the procedure not rote learning.

Picture no.21



Some learners (School B) seemed unable to give reasons for their statements and that was evident during pre- and post-interview sessions with learners and in the activities of the programme that required the learner to give reasons supporting their statements.

It appears as if they were not ready to share their views openly with other people. This showed that the 4th strand (adaptive reasoning) as argued by Kilpatrick (2001) that learners should at this stage be able to think logically, reflect what they had learnt, explain and justify their statements is not well developed in the classroom. The use of van Hiele's theory (1986) and Kilpatrick *et al.*'s model (2001) encouraged the use of LOLT in the classroom as learners of School C did and knowing the level of thinking the learner is in per concept, showing conceptual understanding.

6.5. Using Kilpatrick's strands to determine geometric thinking

The theory seemed to help the teacher when assessing learners to see whether concepts introduced were understood or not, whether learners were able or unable to follow procedures, whether they were strategically competent or incompetent, or whether they were able or unable to indicate their reasoning abilities to show and justify their understanding of the concept. Feedback from learners seemed to show that some learners understood the concepts taught, were able to carry out procedures, were able to apply and use relevant strategies as they were thinking and reasoning logically about the concept under discussion, (Refer to Appendix E, Picture no. 14 & 6).

6.6. Relationship between the two theories

This was discussed in detail in chapter 4. In my view, both theories are related and are useful in the classroom. They both contribute to helping the teacher to teach in an effective and efficient manner, providing insight into how the learners are progressing and where their shortcomings are so that the teacher is able to see what to do in future to help learners overcome the problems they have.

The research data results showed that both theories contribute to helping the learner to learn geometry with understanding so that they could be able to recall and use what they had learned in the past and apply it in future situations. (Refer to Chapter 5.13 & 5.14 above).

In this research, it appears as if lessons planned and prepared with proper consideration of the thinking levels to which each learner belongs and applying the phases and strands of learning seemed to produce achievable outcomes. These outcomes were conceptual lesson understanding and ability to carry out procedures as Kilpatrick *et al.* (2001) argue. That had occurred with the learners of school C, where

at the end of the lesson when learners displayed levels of conceptual understanding it became easy for them to carry out procedures and other strands fell into place more easily. (Refer to Appendix E, Learners' work).

6.7. Findings about using manipulatives during research in the classroom

These appeared to make the lesson understandable to learners as they manipulated resources themselves, as was argued on pp.18 & 88 above. Learners seemed to have understood and performed better when the teacher used manipulatives during teaching; they were able to answer questions asked during the lesson and were able to associate the manipulative with the types of transformation under discussion, (Refer to PLO, lines 13-20).

It seemed also to make it easier for the teacher to see who was following or not following the lesson and the teacher was able to correct the error immediately when the learner picked up a wrong object for the transformation under discussion. To the learners, learning by touching became more meaningful than using theory alone and they seemed to enjoy the lesson more as it was not boring and involved them as active participants during teaching and learning. (Refer to PLO, line 18, where a learner picked up an object that didn't slide, flip or turn and showed it to the class, when the teacher asked the type of transformation there was no answer). The teacher showed them another object that shows reflection which is a laptop and asked the transformation that takes place. Learners in chorus said it is a reflection example.

6.8. The value of the two theories in the classroom during interviews

The use of the two theories in this study helped me see what the learners know, do not know and need, as discussed above. The use of observation and interviews also provided more information on what is needed by learners and teachers in each school. Both theories show the importance of communication between the learner and the teacher, the learner and another learner and the learner and the mathematics content orally, diagrammatically and in writing.

Schools need to stress the importance of using the language of teaching and learning (LOLT) at all times in relevant classes to enable learners the ability to express themselves with confidence in the classroom.

School B learners were unable to communicate in the LOLT in grade 9 before the implementation of the programme; they were also unable to identify shapes in the

classroom and around (Refer to Appendix B). School C learners were able to identify shapes, know properties of shapes but unable to make connections and make informal and formal deductions based on them. Some school C learners were able to furnish drawings to describe a concept if they can't do it in words to show understanding as discussed in chapter 5 above. Although their responses on activities couldn't take them beyond level 3, some couldn't move to level 2 and remained in level 1; learning did take place despite the challenges experienced in the classroom.

6.9. Conclusion

It appears as if some of the learners were able to show their competencies in recognition, analysis and informal deduction and were able to reach Level 3 of van Hiele's theory (1986). Learners were beginning to adjust to the manner of teaching in the programme when it ended and were still at the beginning of applying strands suggested by Kilpatrick *et al.* (2001) in transformation geometry.

The research showed that teaching and learning of mathematics is a challenge to both teachers and learners and it requires diligence, motivation and dedication by both parties in order to produce positive results.

It appears that the programme was effective in enabling learners to develop some levels of improving proficiency in expressing their understanding in the LOLT as some of them were able to answer questions fluently during pre- and post-interviews. This led to an ability to answer questions asked in programme activities with understanding and it appears as if that is the reason why they were able to attain the performance levels they could attain in geometry learning.

At the end of the programme, the teacher was also still in the process of learning about what to do in order to be proficient in the subject one teaches and how to do it. Mathematics teaching and learning can be improved through the use of manipulatives. Learners need to be motivated by both teachers and parents so that they develop a positive attitude towards learning mathematics.

CHAPTER 7 – CONCLUSION

7.1. Introduction

This chapter sums up the research project, focusing on the main findings. It also provides a brief summary of limitations and makes recommendations for further research. The research findings emerge from the research questions about the teaching programme.

7.2. Summary of findings

7.2.1. Commitment of learners. Learners became committed to work hard on the level-based activities of the programme. These activities showed progression from general exposure to simple geometric shapes and their properties, to complex transformation geometric activities. Learners appeared to have been motivated by the level based activities in that they developed competing attitudes and a willingness to work hard to achieve the next level, as shown by their eagerness to request make up activities should they not make the level required by the activity, in order to remain on par with others in the classroom.

This eagerness to work also improved class attendance as the activities were done daily and also improved pass rates at the end of the year even though some didn't hand in their tasks for marking.

The programme was well planned and well prepared and it appeared as if this consistency resulted in learners becoming committed to learning during its implementation. Each activity had a purpose of engaging learners to it, an assessment table attached for marking, relevant manipulatives per activity and some scaffolding by the teacher if needs be. This helped the learner to know what was expected of him/her by each activity.

7.2.2. Effective learning with understanding. This happened in this project because of the use of Kilpatrick's strands and van Hiele's levels and phases of learning geometry. The indicators that the learner has understood the concept, is able to carry out procedures, and is able to solve a problem, led to the attainment of a certain level that the activity is based on. Learners were able to make connections between concepts and were able to show conceptual understanding by means of diagrams as the theory states, (Refer to Appendix E, Picture no. 14, 6, 5 & 15).

The change that took the learner from one level to another showed that learning did take place during the implementation of the programme and that using these two theories together brings meaningful teaching and learning in the classroom and the learners' work is evident of that. The structure of the activities of the programme was different when compared to other transformation geometric activities that learners used to work on in the past; these were thoroughly planned and prepared and based closely on the context of learners, and the levels and phases of van Hiele and Kilpatrick *et al.*' strands.

Learner performance at first showed that learners had little understanding of geometry when level-based activities were introduced to them but had the potential to develop this when strands were also applied in the classroom. Learner performance later showed that there was a change in the teaching and learning strategy of geometry in the classroom through the implementation of theory in the classroom. The use of levels led to individualized teaching and learning as the teacher was able to monitor the progress of each learner individually, (Refer to Appendix E, Picture no. 1-27). That also helped the teacher to know the learning style of each learner in each activity, what the learner knows and doesn't know and what intervention strategies the teacher could take to help the learner understand the concept.

7.2.3. The importance of specific goals. Specific goals such as improving teacher's practice and content knowledge of the subject were achieved and this change was evident as the teacher was able to plan, prepare and present the concept of transformation geometry for the first time in a meaningful manner to learners, (See **Teacher B** in pre-programme interviews in Appendix B). Theories used in this research also helped the teacher to take into account that learners have levels, phases and strands that need to be developed by the teacher to make sure that effective teaching and learning has taken place with the aid of using manipulatives.

7.2.4. Different levels. A learner can be in different levels depending on the particular concept under discussion. These multiple levels show that the learner can be in Level 1 in one concept and in Level 3 in another, depending on the learners' prior knowledge and experience about the concept, and not that s/he is poor in mathematics, (Refer to Figure 9, LA's performance results p. 90).

Knowing the thinking levels (Van Hiele, 1986) at which the learners were at, helped the teacher to track and monitor the progress of each learner, focusing on areas the learner needed assistance with and giving the teacher an opportunity to think of various intervention strategies of helping the learner understand various concepts. This showed me how important the use of levels and strands are in the teaching and learning of geometry in the classroom.

7.3. Limitations

The research was limited to two schools in which it took place. It was also limited to the short period of time of implementation of the activities in the classroom. The implementation of the activities of the teaching programme to a limited number of learners does not guarantee its effectiveness although it had positive and negative results in the classroom.

School B learners only did one activity and couldn't do the rest of them. The analysis of activities and findings were only based on school C learners' work as school B couldn't do the activities because of the pace set by that time that the teacher had to adhere to, not the teaching programme that was not from the department of education.

7.4. Avenues for further study

The use of manipulatives in the activities of the teaching programme in the classroom had a positive impact in promoting effective teaching and learning in the classroom as discussed in chapters above. The application of van Hiele's theory and Kilpatrick *et al.*'s model in this case study provides an opportunity for further research in the following areas:

- a) Using van Hiele's theory on children working on multiple levels in the classroom in different ways.
- b) Investigating the causes of fluctuations in learners' levels of performance in geometry classrooms applying van Hiele's theory.
- c) Investigating the importance of language use in geometry classrooms applying van Hiele's theory and Kilpatrick's model.

7.5. Personal reflection

This research to me as a novice researcher was a challenge and interesting at the same time because there are many new things that I learned from both teachers and learners

through conducting it, things I used to overlook in the classroom. The application of the programme appeared to bring both positive and negative effects of teaching and learning in the classroom.

7.5.1. Knowledge of learners

The project has given me an opportunity to better understand how learners learn mathematics in the classroom. The opportunity I had of allocating levels of thinking per activity helped me to have a better understanding of them and mathematics which means that it has developed my practice in the classroom. The strands and phases have helped me as a teacher and researcher to know how learners learn mathematics, how they think mathematically through answering oral and written activities, what questions to ask that can promote mathematical thinking, how to engage them in activities that will arouse their interests and curiosity in mathematics and what approaches can influence their thinking, learning and understanding of mathematics in the classroom?

The activities conducted in the classroom have taught me as the novice researcher the following lessons about the learners that I used to overlook:

- a) How bad learners feel when they are underperforming according to a set standard.
- b) How eager they become to be given more activities that can help them reach the required level in order to catch up with others.
- c) How willing they become to work individually and get assistance from the teacher by displaying their honesty on what they know and what they do not know, overcoming their habit of copying from friends. It was amazing to see them very much committed to their work as if they were preparing and writing examinations.
- d) Since they knew that the activities were measuring one's level in the classroom, that has improved learner attendance as no learner wanted to miss the activities and knowing where s/he is on a particular activity.
- e) I have also noticed that using manipulatives promoted active learner involvement and made the activity easier to understand and that eliminated a slow writing pace. Not all of them liked the activities or performed well

(e.g. LA) as they were challenging and demanded critical thinking, but since it was done for the first time I saw that the programme could work well if it could be given a chance and a long time in the classroom.

The learners developed diligence and enthusiasm to learn mathematics as each of them was eager to know the levels s/he belonged to every day and that developed a positive attitude towards learning as well as a willingness to learn. That was evident when learners obtained a mark that was below the required level for the activity, and they came for a make-up activity that could boost them to obtain the level so that they are on a par with others in terms of conceptual understanding.

What I like most about van Hiele's theory is that it gives the teacher clear understanding of what the learners know about the concept and where they are in terms of understanding the concept under discussion. It evaluates the learner per concept and the learner can perform differently in different learning concepts; which means that if the learner performs badly in geometry that does not mean that the learner is poor in mathematics just because s/he is not proficient in geometry. What I have also found out was that van Hiele's theory motivates learners to be observant and to look for similarities and differences at all angles, applying the prior knowledge the learner already has.

7.5.2. Teacher development and empowerment

As I have alluded in the statement above, I had an opportunity to know the learners better and the project has developed my teaching practice in terms of their thinking abilities and levels of performance they had shown in the activities they participated in. As I was dealing with an overcrowded class of 60, the project seemed to have developed my monitoring skills as I had to know each learner's performance in each activity especially when they were doing the activities as individuals, to make sure that they do not copy from each other and that they give a true reflection of what they know and do not know.

My patience levels also developed every day as all learners needed to be given a chance to prove themselves i.e. their competence and incompetence in each activity. The project has empowered me with planning skills and the importance of using a baseline assessment before the introduction of a new concept, checking how much knowledge they have about the concept to be discussed; also the application of theory

using manipulatives for better learner understanding. This also seemed to improve teacher-learner relationships we had in and outside the classroom as the learners developed a love of the subject and saw the importance of it in their lives as a failing subject.

As I have been working with teachers, I developed communication strategies, skills and I had to be patient and selfless as a source of information to the participants and I used to provide help in whatever way I could (Refer to Appendix H, Post Lesson Observation). The programme has helped me with planning in that I had to plan the school visits in advance for lesson observation and the interviews I had to conduct at each visit. This had to be done in such a way as to eliminate any inconvenience I might have caused to the school by making prior arrangements with the mathematics teacher and the principal of the schools.

Post-lesson observation in school B has shown me that the teacher has gained some proficiency in the topic of transformation geometry than she performed in the past before the programme activities were implemented; therefore it appears as if to me that the teaching programme has developed the teacher professionally in the classroom, which the video clip confirms, (Refer to Appendix H, PLO).

7.5.3. Mathematical language

The activities of the programme and the interviews have taught me that not all learners understand LOLT especially when one is being observed, and then code switching may need to be used, because the additional language vanishes when one is expected to give an immediate answer, especially during interviews. This opens the gate to use the mother tongue because what is important is to understand what the learner knows irrespective of which language the learner is comfortable with using. I had found out that learners seemed to be less familiar with mathematical language but they tried their best to express themselves showing their own understanding.

7.5.4. Financial constraints

At first I never imagined that the project could have a negative impact on my budget, but as time went by I began to feel it when I had to spend money on travelling, buying material for manipulatives to be used in the classroom and hiring a photographer to take pictures for me while doing lesson observations and interviews.

7.6. Research conclusions

7.6.1. Learner development

The activities of the programme encouraged learners to give brief description of concepts and terms in their own words and that gave those learners who do not like answering questions orally an opportunity to write down their views and that developed their writing skills. In some instances they preferred to furnish explanations as a drawing and as a result their drawing skills also developed.

There were times when they had to present to the whole class what they had done in groups and that developed their presentation and communication skills while learning from one another. This has taught me that learners are keen to learn from each other and praising them for the effort they have shown motivated them and boosted their self esteem and willingness to learn more from both the teacher and their peers in a relaxed atmosphere. They are enabled to do this without feeling threatened when they do not know the correct answer as learning to my view is not about giving correct answers only but about learning from others and from one's own experience.

7.6.2. Baseline evaluation

I have found out that it is the tool the teacher uses to check learners' prior-knowledge before introducing the activities in the programme. I have found out that although it is not a 100% perfect tool, it does help to know what the learners know and do not know as mentioned earlier in previous chapters. I have found out that a variety of questions should be used to accommodate all learner abilities. The activities of the programme were dependent on the results of the baseline evaluation.

7.6.3. Problems encountered during implementation

The programme was demanding and there was lots of work expected to be done by both the teacher and the learners in the classroom. This included a lot of marking to be done daily and make up activities that had to be available for different types of learners at all times. Lots of activities to be done by learners led some of them to cheat as they decided to copy from others which wasted the teacher's time as they had to re-do the activities so that the teacher could allocate the level they are in.

Withdrawal of participants from the programme slowed the progress of the programme and some participants dodged some of the activities and did only those they felt comfortable with. This has affected the analysis of data as only one school

managed to do all the activities of the programme.

The lack of resources at the schools was a problem and it became the duty of the teacher to see to it that manipulatives were there for the smooth running of the programme and for the benefit of the learners before the lesson commenced. The programme was expensive for the schools as it demanded lots of photocopies per learner every day.

The teaching programme used in the classroom applying theories has its own shortcomings which were discussed above in this chapter concerning notation and language of teaching and learning some learners are not familiar with. This was evident during interview sessions (pre & post) where learners could not answer the questions correctly or kept quiet for a long time, wasting time allocated for the questions and the video clip.

7.6.4. Constraining factors

- a. Pace setters used at schools, as sent from the department of education indicating what is to be done and when, are constraining factors, as supervisors question the slow progress coverage of the pace setter by the teacher, because of teaching transformation geometry for a long time instead of moving to other topics. As a result the teacher may be labelled as a lazy person.
- b. Knowledge of voluntary participation and withdrawing from the programme anytime encouraged learners not to submit certain tasks for marking and that created problems of not being able to know the level each learner belongs to. This delayed the pace and progression of the activities and level allocation per activity. It was problematic to include them in the next activity of the programme after missing a day's activity, at the same time they couldn't be left out as the activities took place in the same classroom they were in.
- c. Extra time taken by the implementation of the activities of the programme discussing only transformation geometry was also a problem for the other concepts.

7.6.5. Challenges experienced during data collection

Participants: Dealing with people who knew that they were not bound by the programme and had a right to withdraw at any time was difficult at times. One

teacher withdrew herself from the programme because of the workload she had, as the programme seemed to be too demanding to her in the classroom. Learners also had a tendency to absent themselves because they knew that the marks for the activities won't be recorded as their CASS mark, not knowing that they missed the most important activities.

Disappointment caused by participants and school sometimes demanded that the programme activities be stopped to use the ones in the textbooks to catch-up with the pace setters given by the department. This delayed the post-observation and post-interview sessions of the programme.

Resources: The lack of resources was a problem to participants as the activities required the use of manipulatives and as a result teachers had to struggle to collect these to make lessons successful. It was worse with activities on a Cartesian plane as these needed to be copied but schools do not have photocopying machines and learners had to spend time drawing the objects.

7.6.6. Recommendations

Trial use of the activities of the programme had given me the following suggestions that may have an impact on mathematical teaching in schools:

- a) Promoting the use of manipulatives in mathematics lessons would promote better understanding of topics under discussion and enhance challenging questions and answers by learners in the classroom. Manipulatives promote a relaxed atmosphere to all learners, arouse their interest and promote eagerness to communicate ideas in the classroom. This was observed in the classroom and is supported by theory. In my classroom, manipulatives promoted learner interest to communicate openly and to correct one another when mistakes were made before the intervention of the teacher.
- b) The inclusion of theory (van Hiele's levels and Kilpatrick *et al.*'s strands) could improve teachers' lesson planning and preparation of mathematical topics in all phases as a guide on what to teach, when to teach and how to teach.
- c) If baseline evaluation of all mathematics concepts could be carried out before introducing them, results would inform the teacher about the concept and what

to prepare beforehand. I think that the mathematics pass rate could improve in all grades no matter what the levels of intelligence are, as learners would be able to display their level of competency per activity.

- d) If learner interviews could be given space and done regularly with learners, this would not only develop their oral communication skills, but could help the teacher as a form of oral reflection feedback on what learners know about the concepts discussed and also encourage them to own their learning as some seemed not to.
- e) Effective ways of teaching transformation geometry begin with laying a firm foundation of geometry to learners using manipulatives, probing questions that would lead to critical and creative thinking, and exposing them to drawings from the beginning of the year. The communicated ideas would lead to problem solving and understanding of the concepts under discussion when put together.

In conclusion, the use of theory, use of manipulatives in the classroom, use of transformation geometry terminology, use of LOLT at all times in the classroom and a little code switching where necessary, planning and lesson preparation and involving learners all the time are effective ways among others of improving teaching of transformation geometry in the classroom.

7.6.7. Conclusion

This chapter had summarized the whole research project. Since the aim of the teaching programme was to improve the quality of teaching and learning of transformation geometry in the classroom, results showed that some learning did take place in the classroom. Learners were able to display their competencies in recognition, analysis and informal deduction up to van Hiele's Level 3.

Mathematical teaching and learning can benefit from consistent use of manipulatives for effective, meaningful teaching and learning of geometry in the classroom.

REFERENCES

- Anderson, et al. (1989).** Classroom Instruction. In M.C. Reynolds (Ed.), *Knowledge base for the beginning teacher*, (pp.101-115.). Washington DC: American Association of colleges for Teacher Education.
- Andi Stix, D., (1992).** *Pictorial Journal Writing in Mathematics*. (Ed.).
- Aspinwal, L. & Shaw, K. (2002).** When visualization is a barrier to Mathematics Teacher, 95, 714-717
- Bennie, K. & Newstead, K. (1999).** Obstacles to implementing a new curriculum. In M.J. Smit & A.S. Jordaan [Eds.], *Proceedings of the National Subjects Didactics Symposium* (pp. 150-157). Stellenbosch: University of Stellenbosch. Retrieved 2 February 2009 from:
<http://academic.sun.ac.za/mathed/MALATI/ConferencePapers.htm>.
- Boutler, D.R. & Kirby, J.R. (1999).** Spatial ability and transformation geometry. *European Journal of Psychology of Education*. Vol.XIV. No. 2. pp. 283-294.
- Borich, G. (1996).** *Effective teaching methods*. (3rd Ed.). New York: Macmillan.
- Bray, W.S. (2007).** *A study of Teacher Transitions to a Reform-based Mathematics curriculum*. North Carolina USA: Chapel Hill. Pro Quest Publishers.
- Brown, J.S., & Burton, R.B. (1978).** Diagnostic models for procedural bugs in basic mathematical skills. *Cognitive Science*, 2, 155-192
- Brown, J.S. & Van Lehn, K. (1982).** Towards a generative theory of “bugs”, In T.P. Carpenter, J.M. Moser, & T.A. Romberg(Eds.), *Addition and subtraction: A cognitive perspective* (pp. 117-135). Hillside, N.J: Lawrence Erlbaum Associates.
- Cantrell, D.C. (1993).** Alternate paradigms in environmental education research: The interpretive perspective. In R. Mrazek (Ed.). *Alternate paradigm in environmental education research*. Troy, Ohio: NNEE.
- Carpenter, T.P. (1975).** Measurement Concepts of First & 2nd grade Student. *Journal for Research in Mathematics education.*, pp. 3-13.
- Clements, D.H. (2003).** Teaching & Learning Geometry. In J. Kilpatrick, W.G. Martin & D. Schifter (Eds.), *Research Companion to Principles and Standards for School Mathematics* (pp.151 178). Reston, VA: NCTM.
- Clements, D. H. & Battista, M.T. (1992).** *Constructivist learning and teaching*. *Arithmetic Teacher*, vol.38.no.1.

- Clement, J. (2000).** Analysis of clinical interviews: Foundations and model viability. In AE Kelly & RA Lesh (Eds.). *Handbook of research design in mathematics and science education*. London: Lawrence Erlbaum Associates Publishers.
- Cohen, L., & Morrison, K. (2000).** *Research methods in education*. (5th Ed.). London: Routledge.
- Connole, H. (1998).** *Handbook of early childhood literature*.
- Connole, H. (1998).** 'The research enterprise'. In H. Connole, B. Smith & R. Wiseman (Eds.), *Study guide: Issues and methods in research* (pp.17-42). Geelong: Deakin University.
- Danemark, B., Ekstrom, M., Jakabsen, L., & Karrison, J. (2002).** *Explaining society. Critical realism in the social sciences*. London: Routledge.
- Dane, I. C. (1990).** *Research methods*, Pacific Grove, Ca: Brooks Cole. 348(xviii).
- Darling – Hammond, L. & Ball, D.L. (1997).** *What Policymakers need to know and be able to do?* Prepared for the National Educational Goals Panel.
<http://www.nepg.gov> INEPG/Reports/ high standards.htm.
[http://www.google.com.Data](http://www.google.com>Data)
- Douglas, R., Boutler, D. R. & Kirby, J.R. (1994).** *The journal of Educational Research*. Vol.87, No.5 (May –June, 1994) pp.298-303: Heldref Publications.
- Doyle, W. (1986).** Classroom organization & Management. In M.C. Wittrock (Ed.). *Handbook of Research on teaching*. 3rd ed., pp. 392-431). New York: Macmillan.
- Ernest, P. (1995).** Values, Gender and Images of Mathematics: A philosophical Perspective, *International Journal for Mathematical Education in Science & Technology*, 2(3) 449-462.
- Fuys, D. (1985).** Van Hiele Levels of Thinking in Geometry. *Education and Urban Society*. 17(4) 447-462.
- Gutierrez, A. (1996).** Visualization in 3-D geometry: In search of framework; in L. Puig and A. Gutierrez (Eds.). *Proceedings of the 20th conference of the international group for psychology of mathematics education*. (Vol.1, pp.3-19). Valencia: Universidad de Valencia.
- Henningson, M. & Stein, M.K. (1997).** Mathematical tasks and students cognition: Classroom-Based Factors that support inhibit high-level Mathematical thinking and Reasoning. *Journal for Research in Mathematics Education*, Vol.28. pp.524-549.

- Hollebrands (2003).** High school students understanding of geometric transformation in the context of technological environment. *Journal of Mathematical Behaviour*, 22(1), 55-72.
- Kapur, J.N. (1970).** Change in mathematical education since the late 1950's – Ideas and realization. *Developments in School Mathematics in India during the last two decades*, 9 (2), 245-253.
- Kirby, J.R (1985). & Boulter, D.R. (1990).** *Spatial ability and Transformation Geometry*. European Journal of Psychology of Education. Vol.14 (2), 283-294,
- Kirby, J.R., & Boulter, D.R. (1999).** Spatial ability & transformation geometry. *European Journal of Psychology of Education*, 14(2), pp.283-294.
DOI:10.1007/BF03172970.
- Kilpatrick et al, (2001).** *Adding it up. Helping children learn Mathematics*. Washington: National Academy Press.
- Khuzwayo, H.B. (2000).** *Selected views and critical perspective: An account of mathematics education in South Africa from 1948-1994*. Unpublished PhD thesis. Aalborg University: Denmark.
- Lappan, R.D., Glenda & Bouch, Mary, K. (1998).** Developing algorithms for adding and subtracting fractions. In: *The teaching & Learning of Algorithms in School Maths*. Morw, L.J. & Kenny, M.J. (Eds.). Reston, VA: NCTM.
- Lappan, R.D. (1999).** *Along came the sky*. Reston, VA: NCTM.
- Leedy, P.D., & Ormrod, J.E. (2005).** *Practical research: Planning & design* (8th Ed.). New Jersey: Pearson Prentice-Hall.
- Leydens, J.A., & Moskel, B.M. (2000).** Scoring rubric development: Validity & reliability. *Practical Assessment, Research & Evaluation*, 7(10). [Available online: <http://pareonline.net/getvn.asp?v=7&n=10>]
- Mason, M. (1998).** The van Hiele levels of geometric understanding. In McDougal Little (Ed.), *Professional handles for teachers: Geometry*, (pp.4- 8). Boston: McDougal Little / Houton-Mifflin.
- Mason, M. (2003).** *The van Hiele level of geometric understanding*. Virginia: University of Virginia.
- Maxwell, J, A. (2005).** *Qualitative research design: An interactive approach* (2nd Ed.). Thousands Oaks: Sage.
- Mayberry, J. (1983).** The van Hiele levels of geometric thought in undergraduate preservice teachers. *Journal for Research in Mathematical Education*, 14(1), 58-69.

- McNiff, J.** (1996). *You and your action research project*. London: Routledge.
- Moyer-Packenham, P.S., Salkind, G., & Bolyard, J.J.** (2008). Virtual manipulatives used by R-8 teachers for mathematics instruction: Considering mathematical cognitive and pedagogical fidelity. *Contemporary Issues in Technology & Teacher Education*. [Online serial], 8(3). Available from <<http://www.citejournal.org/vol8/iss3/mathematics/article.cfm>>Retrieved 09.09.08.
- National Council of Teachers of Mathematics.** (2000). *Principles and Standards for school mathematics*. Reston: VA, NCTM.
- Nixon, R.C. (Ed.).** (1987). *Euclid revised*. Oxford: Clarendon Press.
- Piaget, J.** (1964). 'Development & learning' in Ripple, R.E. & Rockcastle, V.N. (Eds.). *Piaget Rediscovered: Report of the conference on Cognitive Studies & Curriculum Development*, Ithaca: Cornell University Press, pp.7-20.
- Reynolds, D., & Muijis, D.** (1999). The effective teaching of mathematics: A review of research. *School leadership & Management*, 19, 273-288. doi: 10.1080/13632439969032.
- Schafer, M. & Atebe, H.U.** (2008). *As soon as the four sides are equal, then the angles must be 90*.
- Senk, S.L.** (1989). Van Hiele levels and achievement in writing geometry proof. *Journal for Research Mathematics Education*, 20(3); 309-321.
- Silver, E.A.** (1982). *Conceptual Understanding and Procedural fluency*. Washington:
- Shaw, M.J.** (2002). *Essential characteristics of Effective Mathematics Instruction*: Houton Mifflin.
- Shulman, L.S.** (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, Vol.15 (2), pp.4-14.
- Shulman, L.S.** (1987). Knowledge and teaching: Foundation of the new reform. *Harvard Educational Review*, Vol.57, pp.1-22.
- Skemp, R.R.** (1978). *Handbook of International Research in Mathematics Education*. 2nd Edition.
- Skemp, R.R.** (1978). Relational understanding and instrumental understanding. *Arithmetic teacher*, 26(3), 9-15.
- Skovesmose, O.** (2006). *Critical Mathematics Education for the future*. Denmark: Aarlborg University.
- Soedjadi, R.** (2000). *The feature of mathematics at our primary school*. Indonesia, Surabaya State University.

- Southwood, S., & Spanneberg, R. (2000).** *Rethinking the teaching & learning of mathematics*. Pretoria: Mc Printer.
- Swafford, J.O., Jones, G.A. & Thornton, C.A. (1997).** Increased knowledge in geometry and instructional practice. *Journal for research in Mathematics Education*., 28(4), 467- 483.
- Teppo, A. (1991).** Van Hiele levels of geometric thoughts revisited. *Mathematics teacher*, 84(3), 210-222.
- Usiskin, Z. (1982).** Van Hiele levels and achievements in secondary school geometry. Chicago: Chicago University Press.
- Van Hiele, P.M. (1986).** *Structure and Insight*. F.L: Academic Press.
- Van Hiele, P.M. (1986).** *Structure & insight: A theory of mathematics education*. Netherlands: Academic Press.
- Vygotsky, L.S. (1978).** *Mind in society: The development of higher psychological processes*. (Edited by M. Cole, J. Scribner, V. John-Steiner, & E. Souberman). Cambridge, M.A: Harvard University Press.
- Wertsch, J.V. (1997).** “*Vygotsky and the formation of the mind*”. Cambridge.
- Wikipedia. (2006).** *Mathematical manipulatives*. Retrieved March 23, 2007 from http://en.wikipedia.org/wiki/math_manipulatives.
- Williford, H.J. (1972).** A study of Transformational Geometry Instruction in Primary grades. *Journal for Research in Mathematics Education*, Vol.3, No.4. (Nov. 1972), pp.260-271.
- Yiu-Kwang, M. & Hing-Keung, L.** *Teaching transformational geometry via dynamic geometry software*. Department of Mathematics, Hong Kong Institute of Education.

APPENDIX A (i): Letter requesting permission to schools

Sir/Madam

RE: REQUEST TO CONDUCT RESEARCH TO GRADE 9 LEARNERS

I am a Rhodes University Masters student who is researching about transformation geometry in grade 9. I would be very pleased if you could allow me to pursue this research at your school in 2011. I promise that the information and participants will remain confidential between the above institution and Rhodes University. Participants are free to withdraw anytime should they feel dissatisfied.

I hope that this request will receive your favourable consideration.

Yours faithfully

Nokuzola Hlaleleni-Geja

APPENDIX A (ii): Letter requesting permission to the EDO

Sir / Madam

RE: REQUEST TO CONDUCT RESEARCH

I am a Rhodes University Masters student who is researching about transformation geometry in grade 9. I would be very pleased if you could allow me to pursue this research at your circuit in 2010 and 2011. I promise that the information and participants will remain confidential between the above institution and Rhodes University. Participants are free to withdraw anytime should they feel dissatisfied.

I hope that this request will receive your favourable consideration.

Yours faithfully

Nokuzola Hlaleleni-Geja

APPENDIX A (iii): Response from the EDO

THE SUPERVISOR
DEPARTMENT OF EDUCATION
RHODES UNIVERSITY
GRAHAMSTOWN

Sir/Madam

RE: CONFIRMATION OF THESIS TEACHING PROGRAMME

This serves to confirm that Ms Nokuzola Hlaleleni-Geja requested to implement a teaching programme with activities for her Masters Degree thesis in 2010. The activities were implemented in May 2011 in selected schools in the district.

We hope that the implementation will bring change to both teachers and learners that could influence the entire district.

Yours faithfully

Circuit Manager

APPENDIX A (iv): Response from the principal

THE SUPERVISOR
DEPARTMENT OF EDUCATION
RHODES UNIVERSITY
GRAHAMSTOWN

Sir/Madam

RE: CONFIRMATION OF THESIS TEACHING PROGRAMME

This serves to confirm that Ms Nokuzola Hlaleleni-Geja requested to implement a teaching programme with activities for her Masters Degree thesis in 2010. The activities were implemented in May 2011 at our school and other schools in the district.

We hope that the implementation will bring change to both teachers and learners that could influence the entire district.

Yours faithfully

Principal

APPENDIX A (v): Letter requesting permission from the parent

Mzali obekekileyo

Ndicela imvume yakho ngomntwana wakho ongu.....ukuba undikhululele yena ukususela nge 14h30 ukuya ngo15h30 yonke imihla malunga nomsebenzi othe chatha ekufuneke ewenzile kunye nam ngala maxesha ndiwakhankanye ngentla.

Ezi zifundo ke zoqhuba yonke imihla ngaphandle kwangoLwesihlanu okanye naluphi na usuku endinokuthi ndixakeke ngalo. Uvumelekile ukumrhexisa umntwana wakho ukuba akuyithandi le nto yeli xesha nangaliphi na ixesha uthanda.

Ngethemba lokuba sakusebenzisana kakuhle kulo mba.

Ozithobileyo

N. Geja (Mathematics teacher)

APPENDIX B (i): Pre- Programme interview – Teacher B

Interviewer: What were you teaching today?

Line 001- TB: *Transformation geometry*

Interviewer: What were specific goals of the lesson?

Line 002- TB: *To introduce transformation geometry presenting them on a Cartesian plane using the scale. It was continuous from grade 8, for today we have to know we have to know those types of transformation, how to place the shape, not to spoil from anything from the problems given. For example they have to listen to the question posed e.g. horizontally, vertically, upward, downward. This has to do with direction, they must have knowledge of direction, know how to count spaces from the shape transformed (laughing).*

Interviewer: How do you feel about today's lesson?

Line 003-TB: *Yho..eh.you know when you deal with shapes you need to be more confident because these are basics for grade 10.*

Interviewer: What do you think they have gained?

Line 004-TB: *They manage to reflect shapes given at least although the time has been short giving me hope that it would be easy next time when doing other types of transformation.*

Interviewer: What mathematical skills have they gained?

Line 005-TB: *It's measurement, eh..even counting.*

Interviewer: How do you feel when students do not give you the correct answer or don't seem to understand you during and after lesson presentation?

Line 006-TB: *Mh..I don't want to say I feel disappointed, I become happy when children not do what I expected because they give the reflection maybe of the instruction I gave them which means there is more effort I must put to make that lesson to be more effective just..mh.. It's one of the values that we need as mathematics educators that we must have tolerant and be patient. We must not be impatient with them when they don't get you well, we must be patient with them and try to do you work, try to find some means to make that lesson to be effective.*

Interviewer: What provision do you normally have to help learners understand the lesson? Those who haven't understood the lesson you must have provision for them, so what provision do you normally have?

Line 007-TB: *Normally I give them some maybe group work, homework, individual*

work, then from those activities I want to see, eh..their understanding; what was their understanding when they work as a group, in pairs or individuals. Then I get to know what type of help that is needed as you know that there are three types of learners we are dealing with e.g. fast learners, normal learners and slow learners.

Interviewer: Why is helping learners learn mathematics so important to you? Why?

Line 008-TB: *Because life is full of mathematics, if maybe we don't use maths in our life would be boring. Let me say some are going to be decorators of wedding halls of any gathering or function that maybe wanted to be attractive so they have to know the spacing of ornaments or flowers they must not get together, they must give space in between so that the whole hall will have to be having that looking.*

Interviewer: What challenges you think you have experienced when preparing this lesson?

Line 009-TB: *When you look at transformation geometry on its own, you will find that it contains all the LO's (Learning Outcomes) just because there is numbers, shapes, patterns, measurement and also data. The information is there so it requires a lot of research, get some information somewhere somehow.*

Interviewer: Were you teaching this lesson for the first time in grade, Mh.. you taught it in grade 8 so what was different today when teaching from what you did last time?

Line 010-TB: *First time in grade 9 yet. You know I took it very simple this year than last year, last year I brought apparatus such as paint and paper.*

Interviewer: Did you enjoy teaching this lesson? You do not like it.

Line 011-TB: *It's one of the topics that doesn't make me to be happy, transformations, I don't want to say I don't like it because I just want to say it's challenging because I have to do it liking or not.*

Interviewer: What challenges do you think learners have about this lesson?

Line 012-TB: *Mh..it's that if ever they don't know how to do spacing co-ordinates, 1st quadrant, 3rd quadrant they are going to experience a lot of problems just because it includes those co-ordinates when the shape moves/transform from this point or direction. It must have those, the co-ordinates won't be the same, they will change so if ever the learners are not clear about how to use their co-ordinates they are going to be misled somewhere somehow.*

Interviewer: How are you going to help them overcome the challenge identified above?

Line 013-TB: *It's to see to it that they know the nature of quadratic sorry Cartesian plane, they know the shape and listen and understand the mathematical language.*

Interviewer: What influence do you think you have that can help them develop a positive attitude towards mathematics?

Line 014-TB: *E.eh, it's not to...to...to help the learners develop a positive attitude, is that you must not panic in front of learners even if there is something you don't know. Tell them that I'm going to make further research that will make you understand much better. You know the challenges of maths teachers have panicked in front of learners. If you panic in front of learners that positive attitude will be lost.*

Interviewer: What opportunities did you have to develop yourself professionally towards learning all skills towards mathematics? Tell me about the programmes you have undergone upgrading yourself in this field.

Line 015-TB: *Although I don't have some..maybe let me say I tried to apply to some institutions but at this time I didn't have any replies that ensures me what to do, but I'm looking forward to do Bed Hons in maths just to upgrade.*

Interviewer: But you upgraded yourself after college?

Line 016-TB: *Yes, I did ACE specializing in maths.*

Interviewer: How often do you get support from the DoE concerning today's topic?

Line 017-TB: *They have tried their best to conduct workshops more especially this LO we are busy doing.*

Interviewer: How often do they visit the school to monitor the implementation of the topic i.e. transformation geometry?

Line 018-TB: *Very scarce (laughing)*

Interviewer: Do you think that these visits and meetings we had and intend having can help you iron out the problems you experience on this topic?

Line 019-TB: *A positive expectation that you are going to help me as you do these visits and maybe if you do them regularly we'll know each other and as you know that each one has his own SWOT analysis, you will help me with my weaknesses and encourage me with my strengths.*

Interviewer: How has the context helped you when preparing this topic? i.e. the situation here at school

Line 020-TB: *The context?*

Interviewer: Mh... the environment.

Line 021-TB: *Mh, you know as I said before maths is full of...eh. our life is full of*

maths. It has helped me a lot that even if we are taking this into consideration but maths eh do help us in our lives that there are changes so we must take a change in a positive way that when you transform a thing you are not spoiling it but you are making it attractive not boring.

Interviewer: How much support do you receive from the principal and SMT concerning this visit and teaching of this topic?

Line 022-TB: *Maximum support*

Interviewer: What about the other teachers?

Line 023-TB: *Also maximum support just because they appreciate your visits.*

Interviewer: Why did you use resources in this lesson? (Both laughing)

Line 024-TB: *Resources help you to know what you are doing; they give you direction, then the rest you'll get from the class that what is needed most or more is needed.*

Interviewer: What you don't like about resources?

Line 025-TB: *None, I don't have any just because the lesson depends on resources although they are giving the same knowledge but the approach is not the same.*

Interviewer: What do you mean by saying the approach is not the same?

Line 026-TB: *Maybe the other book is approaching it another way round although the knowledge is the same but the approach differ.*

Interviewer: Has the DoE provided support in providing you with resources concerning this lesson?

Line 027-TB: *Yes, there are maths sets from the DoE.*

Interviewer: How useful will this lesson be in students' lives in future?

Line 028-TB: *It was useful to the learners as I said that it brings/help them to take change with an ease that change doesn't spoil the nature of an object it just changes direction and position, so whatever change comes to their lives they must know that change doesn't take their lives it just takes position and direction. Their life is the same.*

APPENDIX B (ii): Post programme interview – Teacher B

Interviewer: How did the first geometric activity help you to lay a foundation of future geometric topics including transformation geometry? Did those activities do lay a foundation of future geometric topics?

Line 001-TB: *Yes they did. Firstly I can say we start by learning from the known they know geometric figures, they named them by their correct names by looking at the shape count the number of sides, measuring them and give the names of of that particular shapes according to appearance. (She talked about the activities of the programme as if they are of her own initiative- see video)*

Interviewer: What went well?

Line 002-TB: *The 1st activity that of naming shapes went well, but the 2nd one when learners is their own turn to define or explain, you'll find that they don't want to express themselves their knowledge. They give you a short or insufficient knowledge although they know. They don't want to come out.*

Interviewer: Why did they perform well in the sorting activity?

Line 003-TB: *Nothing challenging in the activity*

Interviewer: What didn't go well? Why?

Line 004-TB: *Negligence, the 1st problem is negligence & carelessness, laziness.*

Interviewer: Do you think that the activities of the programme could be useful in future in the classroom?

Line 005-TB: *Very much.*

Interviewer: Why do you think so?

Line 006-TB: *You know in maths the information that you have now determined e.h your progress in the near future.*

Interviewer: What possible improvements do you have in mind about the programme that we have been using?

Line -007-TB: *Yes there is possible improvements just because that if you sit down you'll find that they have knowledge. Some they express themselves better when you talk to them that they are lazy in writing and in research and when I try to find out why they give me maybe a brilliant explanation and try to find out a follow up, you'll find out that there is knowledge.*

Interviewer: Let's go back to the activities that they did in the classroom. Do you

think that there is a need for some improvements in terms of the questioning style?

Line 008-TB: *You know the style is suitable for their level. Questioning help them as they are specific it is easy for them to answer or give possible answers or what is required mostly or lately.*

Interviewer: What have learners gained from all activities they participated in so far?

Line 009-TB: *They have gained knowledge about this transformation and it will be easy to apply transformation in their lives now their eyes are opened that when they look at thing they will see maths. In any movement that is taking place in their lives they will see/know where to locate this movement in the knowledge that they have regarding the lesson that we have.*

Interviewer: What have you gained as a teacher about the programme and activities that are there?

Line 010-TB: *I've learned that e-h in this programme and the lesson; I've learned that in life everything needs commitment and engagement. You must commit and also engage yourself in doing that as you continue doing the activities you'll gain the interest, you'll enjoy working with us.*

Interviewer: What problems have you encountered during the implementation of the activities?

Line 011-TB: *You know, not that much but there are problems that you'll find out that some of the learners don't have resources. They only rely on when you draw on the board whereas that is not accurate when it is their turn somebody doesn't have a ruler or pencil and have to wait for somebody else's ruler or pencil.*

Interviewer: Did the programme bring change in your classroom?

Line 012-TB: Yes

Interviewer: In what way?

Line 013-TB: *Although the learners as I said before they are and negligent, but if you monitor them in their work and see that look that each and every individual that is committed and engaged at his work is the duty of you as the teacher to see to it that and check it is where the meter stick or measuring instrument and help those who didn't understand well and show appreciation.*

Interviewer: Did the levels per activity help the learners to perform better in the classroom?

Line 014-TB: *Yes, you'll find that some learners come from different schools, they are new in this grade and when you repeat or do it again you make them understand.*

You give those that are in grade 8 and didn't understand it very well another chance of understanding and give them a chance to be confident.

Interviewer: Did you implement the activities in the programme in your classroom?

Line 015-TB: *Yes*

Interviewer: What did you do with learners from other schools that didn't do transformation geometry in grade 8?

Line 016-TB: *Give them more activities so that they will be in the same level, then you'll find that those already know enjoys helping others.*

Interviewer: What do you think was the importance of resources when dealing with all the activities of the programme?

Line 017-TB: *You know, it brings/helps the learners to understand more fully what the lesson brings to them. I can quote some integration e.g. Arts and Culture, EMS and H.S.S.*

APPENDIX B (iii): Pre – programme learner interviews- School C

Appendix B (iii): Learner 1 of school C

Interviewer: What do you understand about geometry?

Line 029-L1: *Geometry is about shapes. When you are...eh...learn about geometry is about shapes and sides, how sides are measured, that these things sides you will understand that this side and one side how sides are measured.*

Interviewer: Oh, what is geometry all about?

Line 030-L1: *As I said it's about shapes, that's all I know.*

Interviewer: Do you like geometry?

Line 031-L1: *Yes, very much because geometry figures to understand an understanding of shapes.*

Interviewer: What do think about geometry? What comes in your mind when someone talks about geometry?

Line 032-L1: *I think that it's a ...like when you build a house make sure that all sides are equal so that the foundation and wall of a house do not fall down mos if measured all sides are equal in your house.*

Interviewer: Have you ever learned geometry before? How was it?

Line 033-L1: *Yes, it was hard in the first time. I didn't understand very much but in the second time I liked it.*

Interviewer: What were you doing in geometry classes? Just say whatever you have been doing.

Line 034-L1: *I remember in grade 7 we were doing shapes like a hexagon, a square all those shapes I remember that it is good to learn geometry, very good.*

Interviewer: Why do you think it is necessary to learn geometry?

Line 035-L1: *Because we must know each and every thing that before you do a thing you must measure so you will know the size of it.*

Interviewer: Have you ever seen geometry in the school environment or in the neighbourhood?

Line 036-L1: *I've seen geometry in school, in class not in the environment, I have never seen them.*

Interviewer: Give a few examples that you see in the classroom.

Line 037-L1: *The door, two sides are equal and other two sides are equal.*

Interviewer: Have you ever done transformation geometry before?

Line 038-L1: *No, I have never done it.*

Interviewer: What do you think it is all about?

Line 039-L1: *I think it is about transforming a shape to another shape, I think so.*

Interviewer: Do you like transformation geometry/ would you like it?

Line 040-L1: *I would like it if I see it.*

Interviewer: You have never seen it, but you like it?

Line 041-L1: *I think that I would like it if you can teach me.*

APPENDIX B (iii): Learner 2 of school C

Line 042-Interviewer: What do you understand about geometry?

Line 043-L2: *In geometry we are talking about shapes.*

Interviewer: Yes, What do you think about geometry?

Line 044-L2: *I think it is a thing of mathematics.*

Interviewer: Do you like it?

Line 045-L2: *Yes, I like geometry.*

Interviewer: Why?

Line 046-L2: *Because it makes me feel happy.*

Interviewer: How?

Line 047-L2: *Because I can draw shapes, whatever shapes.*

Interviewer: Have you ever learner geometry before?

Line 048-L2: *Yes.*

Interviewer: When?

Line 049-L2: *In grade 8.*

Interviewer: How was geometry in class? Did you enjoy or managed to enjoy it?

Line 050-L2: *I enjoyed it.*

Interviewer: What were you doing in geometry classes in grade 8?

Line 051-L2: *Transformations.*

Interviewer: What else did you do?

Line 052-L2: *Supplementary angles and alternate angles.*

Interviewer: Do you think it is important to learn transformation geometry? Why?

Line 053-L2: *Yes, it helps me to know how to measure shapes.*

Interviewer: Have you ever seen any geometric shapes in the classroom, at home or in the neighbourhood?

Line 054-L2: *Yes, e.g., cubes*

Interviewer: Cubes only?

Line 055-L2: *Rectangles*

Interviewer: Rectangles only? What else have you seen?

Line 056-L2: *Triangles*

Interviewer: Have you done transformation geometry before? What is it about?

Line 057-L2: *Yes, it is about changing the shape to something new.*

Interviewer: Do you like it / not? Why do you like or not like it?

Line 058-L2: *I like it, it helps to learn to measure shapes.*

APPENDIX B (iii): Learner 3 of school C

Interviewer: What do you understand about geometry?

Line 59-L3: *I think the word geometry what I understand is that in maths when you talk of geometry you are talking about shapes.*

Interviewer: What do you think about geometry?

Line 060-L3: *I think it is an interesting thing in mathematics.*

Interviewer: Do you like it? Why?

Line 061-L3: *Yes, because it is interesting and challenging.*

Interviewer: Have you ever learner geometry before? Where? and when?

Line 062-L3: *Yes, at Emaqwatini J.S.S in grade 8 we were taught about geometry.*

Interviewer: Yes, you were taught about geometry and then it was closed.

Line 063-L3: *We were taught about shapes and the names of shapes and how they are formed.*

Interviewer: How was it in the classroom?

Line 064-L3: *It was good to learn geometry in class.*

Interviewer: What were you actually doing?

Line 065-L3: *I was doing geometry; we were exploring a lot of shapes.*

Interviewer: Do you think it is important to learn geometry? Why?

Line 066-L3: *Yes it is, because it helps you learn a lot about shapes.*

Interviewer: Why?

Line 067-L3: *It helps to know them and how they are formed.*

Interviewer: What did you do about shapes?

Line 68-L3: *I learn about shapes and talk about them.*

Interviewer: Are shapes useful in your life? In what way?

Line 069-L3: *Yes, because they are used to make a lot of things.*

Interviewer: E.g.?

Line 070-L3: *Doors and window frames.*

Interviewer: Have you ever seen any geometric shapes at school, home or in the neighbourhood?

Line 071-L3: *Yes, I have seen it at school.*

Interviewer: Where?

Line 072-L3: *It's a door frame.*

Interviewer: At home?

Line 073-L3: *It's a window frame.*

Interviewer: Only?

Line 074-L3: *No there are many things.*

Interviewer: What do you understand about transformation geometry?

Line 075-L3: *It means to change something into something else, e.g. recycling. When we take something ordinary and make it something new, I think it is so.*

Interviewer: Have you ever done transformation geometry before?

Line 076-L3: *No, never.*

Interviewer: Do you think you like transformation geometry? Why?

Line 077-L3: *Yes, because it helps a lot to transform things into other interesting things.*

APPENDIX B (iii): Learner 4 of school C

Interviewer: What do you know about geometry?

Line 78-L4: *I understand that geometry is about shapes.*

Interviewer: What is it all about? Can you please dwell more?

Line 079-L4: *Geometry is about shapes, when you measure something like rectangle or triangle...*

Interviewer: What do you think about geometry?

Line 080-L4: About shapes.

Interviewer: Do you like it? Why do you like it?

Line 081-L4: *Yes I do, because it makes us learn how to do things if we make sides that they must be equal so that it can't fall down. How to weigh things.*

Interviewer: Have you ever learned geometry before? Where and when?

Line 082-L4: *Yes, at Sodidi J.S.S in 2010.*

Interviewer: How was it in the classroom?

Line 083-L4: *We are doing triangles and rectangles and cubes. We measured so that we can see if they are equal.*

Interviewer: Do you think it is important to learn geometry?

Line 084-L4: *Yes it is important because if you do things you do something that you must see that it is equal to other side so that it so that it cannot fall down.*

Interviewer: Have you ever seen geometry in the school environment or in the neighbourhood / community?

Line 085-L4: *I've seen at school.*

Interviewer: What did you see?

Line 086-L4: *Chalkboard.*

Interviewer: So at home there are no geometric shapes?

Line 087-L4: *Yes they are e.g. like as I said the door at home.*

Interviewer: Only the door at home?

Line 088-L4: *Windows.*

Interviewer: Ok eh... have you done transformation geometry before?

Line 089-L4: *No.*

Interviewer: Ok, what do you think transformation geometry is about? Just the word 'transformation'.

Line 090-L4: *Transformation geometry is about to change shape to another shape.*

Interviewer: Do you think you like transformation geometry?

Line 091-L4: *Yes, I like it.*

Interviewer: Why?

Line 092-L4: *Because as I said I like it because it makes things easy.*

Interviewer: Yes, how?

Line 093-L4: *When you do something you see it that this side is wrong so that you can measure it.*

APPENDIX B (iii): Learner 5 of school C

Interviewer: What do you understand about geometry?

Line 094-L5: *(Stammering) I...i..i.geometry is about shapes*

Interviewer: Right, what do you think about geometry? Just feel free to use the mother tongue if you are not ok with the medium.

Line 095-L5: *(Stammering) I...geometry i...it let us learn more about shapes.*

Interviewer: Yes, what else? Please speak louder I can't hear you.

Line 096-L5: *Shapes and sizes of them and how to draw them.*

Interviewer: Do you like geometry? Why?

Line 097-L5: *(Stammering) Yes, i... because if someone can ask you what is this shape you can tell him/her what is it because you have learned it.*

Interviewer: Have you ever learner transformation geometry before? Where? and when?

Line 098-L5: *Yes maam, here at school in grade 8 last year 2010.*

Interviewer: What were you doing in geometry classes?

Line 099-L5: *(Stammering) We drew, we drew shapes and counted sides.*

Interviewer: What else? Just say everything you did.

Line 100-L5: *We drew cuboids and prisms.*

Interviewer: What else?

Line 101-L5: *We count them*

Interviewer: Have you ever done any calculations in geometry?

Line 102-L5: *Yes*

Interviewer: Are you sure?

Line 103-L5: *Yes.*

Interviewer: Do you think it is important to learn geometry? Why?

Line 104-L5: *Yes, because sometimes when you are older when you are going to do something that needs to be measured you can measure it by using shapes.*

Interviewer: How can geometry help you with your life?

Line 105-L5: *It can help me when I'm older when I want an architect to build my house I can tell him what kind of a house I want and how can it be measured.*

Interviewer: Have you ever seen any geometric shapes at school, at home and in the community / neighbourhood? Can you show us in the classroom?

Line 106-L5: *Yes, windows (pointing at the window)*

Interviewer: What kind of shape is a window?

Line 107-L5: *A square*

Interviewer: Are they all squares?

Line 108-L5: *No rectangle.*

Interviewer: Have you done transformation geometry before? When?

Line 109-L5: *Yes in STD 5 we transformed shapes from one place to another without changing sides*

Interviewer: So, what do you think transformation geometry is about?

Line 110-L5: *Transformation geometry is about transforming shapes from one place to another.*

Interviewer: Give us one example of a transformation.

Line 111-L5: *Rotation*

Interviewer: How does it go?

Line 112-L5: *It turns around.*

Interviewer: Do you think you like transformation geometry? Why?

Line 114-L5: *Yes, because you can be able to change something/shape from one place to another.*

APPENDIX B (iv): Pre-programme interviews-School B

Appendix B (iv): Learner 1 of school B

Interviewer: What do you understand about geometry?

Line 115-L1: *It's about i signs, to understand uyazi ukuthi isigns zimi/zibekwa kanjani (how signs are placed).*

Interviewer: Yes, what else can you say?

Line 116-L1: *That if signs are not the same, subtract and take the sign of the bigger number.*

Interviewer: What do you think about geometry?

Line 117-L1: *Ndicinguba igeometry ingakuthatha ikubeke kwindawo eright (that geometry can put you at a right place).*

Interviewer: For an example?

Line 118-L1: *Inecareers ezininzi (it has many careers)*

Interviewer: Yes? Give examples

Line 119-L1: *Ogqirha ne pilot (doctors and pilots)*

Interviewer: Do you like geometry? Why?

Line 120-L1: *Yes, ndiyayi-understanda (I understand it)*

Interviewer: Have you ever learned geometry before? Wawukhe wafunda na nge-geometry ngaphambili?

Line 121-L1: *No*

Interviewer: Zanguyive tu? (You have never heard about it?)

Line 122-L1: *Yes*

Interviewer: Do you think it is very important for you to learn geometry?

Line 123-L1: *Yes*

Interviewer: Why?

Line 124-L1: *Yiyon'izandinika i-information eninzi ukuya eyuniversity nase high school. (It will give me enough information to go to university and high school).*

Interviewer: What are you going to do with the information?

Line 125-L1: *Ndizokwazi ukusebenza ngayo, mh... ndikwazi ukufunda kakuhle (so that I can learn very well)-(long silence).*

Interviewer: Do you think it is necessary/ important to know geometry? Why?

Line 126-L1: *Yes*

Interviewer: Have you ever seen any geometric shapes in class, at home and in the

community /neighbourhood?

Line 127-L1: *Ndandiyibone kwine newspaper ehig school ne question paper (I saw them in a high school newspaper and question paper).*

Interviewer: Here in the classroom, are there any geometric shapes?

Line 128-L1: *Yes encwadini (in the book)*

Interviewer: Njena nje eklasini zange ukhe ubone nto tu? (Do you mean you never seen any geometric shapes in the classroom?)

Line 129-L1: *No*

Interviewer: Can you give examples of geometric shapes e.g. you can even touch a shape you think it is geometric

Line 130-L1: *(She looked around and said no)*

Interviewer: Have you done geometry before?

Line 131-L1: *No*

Interviewer: What do you think what transformation geometry means?

Line 132-L1: *Inento ezine....ezine multiplication (It has something to do with multiplication)*

Interviewer: Ucinga inoba itsho nhe? (Do you think it is about that?).

Line 133-L1: *Yes*

Interviewer: Ayikho enye into? (Is there anything to say?).

Line 143-L1: *Yes*

Interviewer: Do you think you like transformation geometry?

Line 135-L1: *Yes*

Interviewer: Why?

Line 136-L1: *Inikisa umdla (It is interesting)*

Interviewer: So, uyazithanda izinto ezinika umdla? (Do you like interesting things?)

Line 137-L1: *Yes*

Interviewer: Why?

Line 138-L1: *Zindenza ndibe nomdla kakhulu kwimaths (They revive my interest in mathematics).*

Appendix B (iv): Learner 2 of school B

Learner 2 made the following responses during pre interview session

Interviewer: What do you understand about geometry?

Line 139-L2: *It's about signs into endiyi-understandayo (I understand it to be about signs)*

Interviewer: Do you like geometry?

Line 140-L2: *Kancinci (A little)*

Interviewer: Do you have problems about it?

Line 141-L2: *Yes*

Interviewer: What type of problems?

Line 142-L2: *Andiyi-understandi kakuhle (I don't understand it well).*

Interviewer: Have you told the teacher about your problems? Why?

Line 143-L2: *No. Andiyi-understandi and andifane ndiyitshone mna imaths, xa ndizibhalela ndiba right (I don't understand it, I never failed mathematics and when I write it I get correct answers).*

Interviewer: Have you learned geometry before?

Line 144-L2: *No*

Interviewer: Are you sure?

Line 145-L2: *Yes.*

Interviewer: Do you think it is very important to for you to learn geometry?

Line 146-L2: *Yes*

Interviewer: Why?

Line 147-L2: *If ndingafuman'umsebenzi obalayo ithi kanti andikwazi kubala umzekelo; Ugqirha iipilisi zakhe ziyabalwa ukuba ipatient itye ngaphi. (If I could get employed in a job which requires counting e.g. being a doctor I'll be able to count the number of tablets a patient needs).*

Interviewer: Have you ever seen any geometry at home, school and in the community?

Line 148-L2: *(Long silence nodding head) No*

Interviewer: Have you done transformation geometry before?

Line 149-L2: *No*

Interviewer: Ucinga ukuba inoba ingantoni? (What does it entail)?

Line 150-L2: *Inento edibanisa nemali. (It has something to do with money).*

Interviewer: Eli gama lithi, 'Transform' zange ulive tu ebomini bakho? (Have you

ever heard about the word, ‘Transform’) in your life?

Line 151-L2: *(Silence) transform, transform – Sekhe ndaliva (I heard about it)*

Interviewer: Phi? (Where?)

Line 152-L2: *Yonk’indawo sekhe ndaliva (I heard about it everywhere)*

Interviewer: Kwakusithiwa yintoni? (What did you hear about?)

Sukubek’isandl’emlonyeni. (Please do not cover your mouth with your hand)

Line 153-L2: *Kukutransformer (To transform)*

Interviewer: Do you like transformation geometry?

Line 154-L2: *Yes Miss*

Interviewer: Why?

Line 155-L2: *Xa uyicacisa Miss ungaya kude nemaths (When explained properly it can take you far)*

Interviewer: Mh, Theta konke okucingayo sukoyika (just say everything, don’t be scared)

Line 156-L2: *Ungafuman’ijobs ngemaths, zininzi izinto onokuzenza xa unnemaths (You can get jobs; you can do many things when you have mathematics).*

Appendix B (iv): Learner 3 of school B

Learner 3 made the following responses during interviews:

Interviewer: What do you understand about geometry?

Line 157-L3: *I understand signs*

Interviewer: What is geometry about?

Line 158-L3: *It’s about signs*

Interviewer: Sides or signs?

Line 159-L3: *Signs*

Interviewer: What do you think about geometry?

Line 160-L3: *Ndicing’inoba ingento zecareers (I think it has something to do with careers)*

Interviewer: Ukuthini kwazo? (What about them?)

Line 161-L3: *Ukuba mhlawumbi okanye le career ukuyo uza kuyenza okanye ofuna ukuba yiyo njengogqirha, unesi engineer (That maybe you want to be a doctor, nurse and engineer).*

Interviewer: Do you like geometry? Why?

Line 162-L3: *Yes, because it tell us more about maths.*

Interviewer: Have you ever learner geometry before?

Line 163-L3: *No.*

Interviewer: Are you sure?

Line 164-L3: *Hayi ndakhe ndafunda (No I learnerd about it)*

Interviewer: Wawufunda ntoni? Phi?

Line 165-L3: *Isigns*

Interviewer: Nini?

Line 166-L3: *Aph'enext door kwa grade 8 (Nextdoor in grade 8)*

Interviewer: How did you feel about it in class?

Line 167-L3: *Bendiziva ndinomdla kuba kwakumnandi ngokusasifunda ngayo. (I was interested in it when we learnerd it).*

Interviewer: Why do you think it was good?

Line 168-L3: *Kuba ibisixelela ngezinto ezininzi zemaths (It told us many things of maths)*

Interviewer: Ucinga kubalulekile ukufunda ngegeometry ebomini bakho?

Line 169-L3: *(Long silence) Yes*

Interviewer: Why?

Line 170-L3: *(Long silence)...Ngoba isfundisa ngezinto ezininzi zakwamaths (It teaches us a lot of mathematical stuff)*

Interviewer: Mh...mh, nika umzekelo (give examples)

Line 171-L3: *Ngesigns (It's about signs)*

Interviewer: Mh...mh what else?

Line 172-L3: *Silence.....*

Interviewer: Ok, have you ever seen geometry at school, at home, in the classroom and neighbourhood?

Line 173-L3: *No, kodwa yayikhona iposter enezinto zegeometry (but there was a poster with geometric shapes).*

Interviewer: Yayinantoni, injani?

Line 174-L3: *Zandiyiqwalasele (I did not observe it)*

Interviewer: Don't you have another example?

Line 175-L3: *No*

Interviewer: Can you show us any geometric shape in front of you, behind you, under your feet?

Line 176-L3: *No, chalkboard behind me.*

Interviewer: Fine, oh...have you done transformation geometry before?

Line 177-L3: *No*

Interviewer: Are you sure?

Line 178-L3: *Yes*

Interviewer: What do you think it is about?

Line 179-L3: *No answer*

Interviewer: Zanguve nasekhaya xa nidlala nithi something has transformed, zange khe nithethe ngayo? (Have you ever heard when you played at home saying something has transformed?).

Line 180-L3: *Sakhe sathetha (We talked about that)*

Interviewer: Nanisithini? (What were you saying?)

Line 181-L3: *Sasithetha ngopopayi abadla ngokutransformer (We talked about pictures which used to transform on TV)*

Interviewer: Njani? Xa kutheni? (How and why?)

Line 182-L3: *Ukuze zibe nepower (To gain power).*

Interviewer: Kukuthini ukutransformer and zitransformer njani? (What does transform mean and how does it take place).

Line 183-L3: *No answer*

Interviewer: Ok, do you think you like transformation geometry or learning about popeye when they transform, could they influence you in loving learning geometry?

Line 184-L3: *Yes*

Interviewer: Do you like popeye when they transform?

Line 185-L3: *Yes*

Interviewer: But you don't like maths things when they transform?

Line 186-L3: *Yes I am curious/ (Ndiyayirhalela)*

Interviewer: Why?

Line 187-L3: *Ndifun'ukubona ba zitransformer njani (I want to see how they transform).*

Appendix B (iv): Learner 4 of school B

Learner 4 responded like this during pre interviews:

Interviewer: What do you understand about geometry?

Line 188-L4: *Geometry, geometry, geometry had many careers.*

Interviewer: Many careers?

Line 185-L4: *Yes*

Interviewer: Like?

Line 186-L4: *If you want to get work you can getting to.*

Interviewer: What is actually about before career?

Line 187-L4: *About multiplications*

Interviewer: And what?

Line 188-L4: *(Silence), numbers, multiplying numbers*

Interviewer: Do you like it?

Line 198-L4: *Yes*

Interviewer: Why do you like it?

Line 190-L4: *To get a job, you can get a job*

Interviewer: Just about loving it?

Line 191-L4: *Yes*

Interviewer: Can I get a job by loving geometry?

Line 192-L4: *Yes*

Interviewer: How?

Line 193-L4: *(Long silence) Ungafuman'ulwazi (You get information)*

Interviewer: About what? Ulwenzeni olu lwazi? (What will you do with the information?)

Line 194-L4: *No answer*

Interviewer: Ok, mh...have you ever learned about geometry in previous classes?

Line 195-L4: *Yes*

Interviewer: Kwakusithiwa ni? (What was it all about?).

Line 196-L4: *Ine multiplication and division (It has multiplication and division)*

Interviewer: Is it necessary to learn geometry?

Line 197-L4: *Yes, maybe at work you get questions about geometry.*

Interviewer: What if you could not finish school but are having geometric knowledge? How would you do?

Line 198-L4: *Long silence- Let us pass*

Interviewer: Have you seen geometric shapes at home, school?

Line 199-L4: *Yes*

Interviewer: What were they? Could you see things with geometry in class, under

your feet, behind you and above your head?

Line 200-L4: *No answer*

Interviewer: In mathematics we learn about transformation geometry, have you heard about things that can transform?

Line 201-L4: *No zandive (I've never heard about them)*

Interviewer: Can you think about things that can transform?

Line 202-L4: *Ukutransformer?*

Interviewer: Yes

Line 203-L4: *No*

Interviewer: Would you like transformation geometry if someone could teach you about it?

Line 204-L4: *Yes*

Interviewer: Why?

Line 205-L4: *Maybe I could get information.*

Appendix B (iv): Learner 5 of school B

Learner 5 responded as follows;-

Interviewer: What is geometry all about?

Line 206-L5: *Signs*

Interviewer: Is it about sides/science?

Line 207-L5: *Signs*

Interviewer: What about the signs?

Line 208-L5: *To be a doctor*

Interviewer: What about them?

Line 209-L5: *(Long silence) – I saw it on TV (Scratching head without answering)*

Interviewer: What is that called?

Line 210-L5: *No answer*

Interviewer: Do you like geometry? Why?

Line 211-L5: *Yes, it has many things like nurses*

Interviewer: How? Do nurses have something to do with geometry?

Line 212-L5: *When they give medication to people they count pills and number of times of taking medication per day.*

Interviewer: Why do you associate it with geometry?

Line 213-L5: *Because of maths*

Interviewer: Do you like geometry? Why do you like it?

Line 214-L5: *Yes, many things need counting*

Interviewer: What things? Call things by name please.

Line 215-L5: *Engineers*

Interviewer: Are engineers geometry? So engineers are geometry?

Line 216-L5: *Yes*

Interviewer: Have you learned about geometry before?

Line 217-L5: *No*

Interviewer: You never heard it even in grades 4 to 9?

Line 218-L5: *Yes*

Interviewer: Where did you know things named above?

Line 219-L5: *From a grade 10 learner*

Interviewer: What did he say?

Line 220-L5: *When writing class work*

Interviewer: How do you think geometry is in class?

Line 221-L5: *Iright (It is right)*

Interviewer: In what way?

Line 222-L5: *Many people can be engineers*

Interviewer: Will it be good for you? How?

Line 223-L5: *Kakuhle (Nice)*

Interviewer: How?

Line 224-L5: *Because I want to be a lawyer*

Interviewer: Have you learned about transformation geometry before?

Line 225-L5: *No*

Interviewer: What do you think it is all about?

Line 226-L5: *Maths*

Interviewer: What about it?

Line 227-L5: *About signs*

Interviewer: Would you like it if someone can teach you?

Line 228-L5: *Yes, because it is about maths.*

APPENDIX B (v): Post-programme learner interviews from school C

Appendix B (v): Learner 1 of school C

Following is an interview of learners from each school and it will be followed by a summary of what other learners said and the analysis from both schools.

Interviewer: What did you like most about the activities of the activities of the programme?

Line 245-L1: *I like most about those activities because there was nothing I do know about transformation geometry so those activities I learn a lot from them that when you look at yourself in the mirror I see a line of symmetry.*

Interviewer: What type of transformation you get if you look at the mirror?

Line 246-L1: (He looked up for sometime) *I don't remember it Maam.*

Interviewer: What you didn't like about the activities?

Line 247-L1: *Some of them were tough but I tried my best to... so that I can get the mark.*

Interviewer: What do you think you have gained from the activities?

Line 248-L1: *I've gained some work that I am going to do in grade 10 so I must keep that it is some of the activities in mind that when I'm arriving in gr.10 and they ask about the activities I become very clever.*

Interviewer: Did you have problems during the implementation of the activities?

Line 249-L1: *Yes, some of them but not all of them.*

Interviewer: Did you have any problems during the implementation of activities?

Can you think of 1 or 2?

Line 250-L1: *Some of them were shapes that were the same but the names were not the same.*

Interviewer: Did you tell your teacher about your problems?

Line 251-L1: *No Maam, because I was scared to do so*

Interviewer: Scared of what?

Line 252-L1: *Because you teach me geometry and you will say, 'Hey January, you were not listening to me' I was scared of that.*

Interviewer: Oh you were scared of being shouted at? (laughing)

Line 254-L1: *Yes*

Interviewer: What did you like about being placed in certain levels per activity? Do

you still remember the levels 1, 2, 3 etc?

Line 255-L1: *I think level 2 was tough; I like some of the levels.*

Interviewer: What about level 3 questions?

Line 256-L1: *Some of the questions but not all of them.*

Interviewer: Do you think that those levels have helped you? In what way?

Line 257-L1: *Yes, I think that those levels were upping my marks.*

Appendix B (v): Learner 2 of school C

Interviewer: What did you like most about the activities in transformation geometry?

Line 258-L2: *I liked i..translation (stammering)*

Interviewer: Why?

Line 259-L2: *Because it helped me to place a thing from one place to another (stammering)*

Interviewer: Come again please

Line 260-L2: *It helped me to move a thing from one place to another.*

Interviewer: Is there anything you didn't like?

Line 261-L2: *No maam*

Interviewer: What do you think you have gained?

Line 262-L2: *When you turn something it must be on a point or circle (stammering)*

Interviewer: Mh..go on

Line 263-L2: *So that it cannot change to another shape.*

Interviewer: Did you have problems during the implementation of activities?

Line 264-L2: *I had them in translation and reflection*

Interviewer: Did you tell the teacher about your problems?

Line 265-L2: *No*

Interviewer: Why?

Line 266-L2: *I was scared to ask*

Interviewer: What did you like about being placed in certain levels per activity?

Line 267-L2: *I liked it because it showed that you can improve if you do not understand the activity (stammering)*

Interviewer: Mh..did those levels help you?

Line 268-L2: *Yes*

Interviewer: In what way?

Line 269-L2: *That if I didn't understand at the first time I do it again and understand.*

Interviewer: Go on

Line 270-L2: *I improve*

Interviewer: Mh...so you improved?

Line 271-L2: *Yes.*

Appendix B (v): Learner 3 'R' of school C

Interviewer: What did you like most about the activities?

Line 272-L3 'R': *I liked coz many angles I was didn't know and many triangles I was didn't know that's part I would like.*

Interviewer: And that was what you liked most?

Line 273-L3 'R': *And you told us many things we didn't know from intermediate phase you tell us more about.*

Interviewer: Which activities you didn't like?

Line 274-L3 'R': *Level 2 activities*

Interviewer: What was wrong with them?

Line 275-L3 'R': *The triangles were the same but the names were not the same, that's what I didn't like the most.*

Interviewer: What have you gained from the activities?

Line 276-L3 'R': *I gained many things coz sometimes we see the triangles they look like but their names are not alike.*

Interviewer: Go on - (qhuba)

Line 277-L3 'R': *That's why I gain most things and some triangles look alike but some do not look alike.*

Interviewer: Did you have problems?

Line 278-L3 'R': *No*

Interviewer: Are you sure?

Line 279-L3 'R': *Yes*

Interviewer: What did you like about being placed in certain levels...?

Line 280-L3 'R': *Level 1*

Interviewer: No, listen to the question first, do you remember the activities have levels e.g. L1, L2 L3, etc

Line 281-L3 'R': *I like because I would learn more things*

Interviewer: Go on (qhuba) you can speak Xhosa

Line 282-L3 'R': *That's all Miss*

Interviewer: You said more things, like what?

Line 283-L3 'R': *Like some of the angles I was didn't know and some triangles was didn't know that why I like Level 1.*

Interviewer: You want to be placed in Level 1?

Line 284-L3 'R': *No, I want to improve other levels*

Interviewer: Did those levels help you?

Line 285-L3 'R': *Yes*

Interviewer: How?

Line 286-L3 'R': *Some levels I was didn't know and some triangles I was didn't know, that's why I would like...*

Interviewer: Let us speak isiXhosa now. How did the levels help you when you were placed from one level to the other? (Zikuncede entweni eza levels ubekiwe kwilevel nganye umane ususwa ubekwe kwenye)?

Line 287-L3 'R': *Zindincede ekubeni ezinye izinto bendingazazi ndaye ndibuzwa, ndaye ndapRACTIZA so indincede kwezo nto. (They helped me on things I didn't know, I practiced and that is how the levels helped me).*

Interviewer: So, ibaluleke ngantoni into yokususwa kwenye ilevel uye kwenye? (Why is it important to be placed from one level to another?)

Line 288-L3 'R': *Ibaluleke ngokuba upRACTIZE into xa ungayazi (To practice when you don't understand a thing)*

Interviewer: So, xa ungapRACTIZANGA awubekwa kwilevels? (So, if you are not practicing you don't stand a chance of being placed in certain levels?)

Line 289-L3 'R': *Kaloku kufuneka upRACTISE ukwenzela uzokwazi...ndizakuthi njani na miss (In order to know you need to practice, what am I going to say...)*

Interviewer: Thetha ngesiXhosa (Use your mother tongue isiXhosa)

Line 290-L3 'R': *Kufuneka Miss unto..., ndizayichaza njani? (I need Miss to...how do I describe it?)*

Interviewer: Ndithe thetha ngesiXhosa (I said say it in Xhosa)

Line 291-L3 'R': *Funeka miss, uthe kufuneka xa uprovileyo nhe? (Did you say miss when I proved nhe?)*

Interviewer: Ndithe ilevels zikuncede entweni? (I said how did the levels help you?)

Line 292-L3 'R': *Zindincede ngokuya bendibhala iJune exams izinto ezininzi ndiye ndazazi pha kweza levels besizenza ekuqaleni. (They helped me when writing June*

exams, I knew a lot of things from those levels we did earlier).

Appendix B (v): Learner 4 of school C

Learner 4 answered questions as follows:

Interviewer: What did you like most about the activities?

Line 293-L4: *I like geometry, I like reflection because you see your image in the mirror and I like translation because it is like a sliding cell phone, rotation e.g. a watch.*

Interviewer: What you didn't like?

Line 294-L4: *Those activities that have similar shapes because I did not understand them.*

Interviewer: You didn't understand it?

Line 295-L4: *Yes*

Interviewer: Did you tell the teacher about it?

Line 296-L4: *No*

Interviewer: Why?

Line 297-L4: *No answer*

Interviewer: Is your teacher eating people?

Line 298-L4: *No*

Interviewer: What did you like about being placed in certain levels per activity?

Line 299-L4: *Because in June exams I improved in mathematics.*

Interviewer: Did the levels help you?

Line 300-L4: *Yes, because I'm the one of the learners who passed maths during June exams.*

Interviewer: Ok, keep on talking

Line 301-L4: *I obtained 70%*

Interviewer: 70%?

Line 302-L4: *Yes*

Interviewer: Keep on talking

Line 303-L4: *I got level 6 in maths*

Interviewer: Level 6?

Line 304-L4: *Yes*

Interviewer: So, you are above average (laughing together)

APPENDIX B (vi): Post-programme Learner interviews from school B

Following are the responses of a few learners from school B.

Appendix B (vi): Learner 1 of school B

Interviewer: What did you like most about the activities?

Line 305-L1: *Translation*

Interviewer: Why did you like it the most?

Line 306-L1: *Because you can take a shape to another different position*

Interviewer: Yes, what else?

Line 307-L1: *(No answer)*

Interviewer: What you didn't like about the activities?

Line 308-L1: *No*

Interviewer: Why did you like everything?

Line 309-L1: *Because ndifunde zonke*

Interviewer: What do you think you have gained?

Line 310-L1: *Measure shapes*

Interviewer: You didn't measure shapes, did you?

Line 311-L1: *No*

Interviewer: Uyilibele (You forgot it)

Line 312-L1: *Yes*

Interviewer: Did you have problems during implementation of activities?

Line 313-L1: *Yes*

Interviewer: Did you tell the teacher about your problems?

Line 314-L1: *No*

Interviewer: Why?

Line 315-L1: *Ndiyamonqena (I don't feel like telling her)*

Interviewer: So, what problems did you have?

Line 316-L1: *Andiziqondi kakuhle (I don't understand them quite well)*

Interviewer: What did you like by being placed in certain levels? (Uthande ntoni?)

Line 317-L1: *Nothing*

Interviewer: Did the levels help you?

Line 318-L1: *Yes*

Interviewer: How?

Line 319-L1: *No answer*

Appendix B (vi): Learner 2 of school B

Interviewer: What did you like most about the activities?

Line 320-L2: *Ndithande itransformation (I liked transformation)*

Interviewer: They are about transformation geometry, what did you like?

Line 321-L2: *Translation*

Interviewer: Why?

Line 322-L2: *Because in our life we always use translation*

Interviewer: In what way?

Line 323-L2: *Because xa ulapha uyatranslater uye kwenye indawo (Because when you are here you translate from one place to another)*

Interviewer: What you didn't like about transformation geometric activities?

Line 324-L2: *(Long silence)...Ayikho*

Interviewer: What did you gain?

Line 325-L2: *Ukuba ndazi ba itranslation isebenza xa kutheni. (To know when to use translation)*

Interviewer: Isebenza xa kutheni ke? (When does it work actually?)

Line 326-L2: *Xa uhamba ukuze uyokuba kwenye indawo uyatranslater. (When moving from one place to another it is translation)*

Interviewer: Did you have problems during implementation?

Line 327-L2: *No*

Interviewer: Did you understand everything?

Line 328-L2: *Yes*

Interviewer: Are you sure?

Line 329-L2: *Yes*

Interviewer: But you said earlier on that you didn't gain anything, how did you understand if you haven't gained anything?

Line 340-L2: *Hayi ndiye ndagainer. (No I did gain something)*

Interviewer: What did you like by being placed in certain levels? Those activities were graded according to levels.

Line 341-L2: *What levels Maam?*

Interviewer: Those levels stated in the questions, did you see or remember them?

Line 342-L2: *Yes*

Interviewer: What did you like about them?

Line 343-L2: *Translation*

Interviewer: What did you like about the levels?

Line 344-L2: *No answer*

Interviewer: Ucinga beziza kunceda entweni ezi levels? (In what ways would the levels had helped you?)

Line 344-L2: *Zindincede ngokuba ndiyayazi ukuba xa utranslater wenza ntoni (They helped me because I know what I do when doing translation).*

Interviewer: Zikuncedile phofu?

Line 346-L2: *Yes*

Interviewer: How?

Line 347-L2: *Ukuba ze ndazi ba xa kutranslatwa kutranslatwa njani. (To know how translation is done).*

Appendix B (vi): Learner 3 of school B

Interviewer: What did you like most about the activities?

Line 348-L3: *I likes translation*

Interviewer: Why?

Line 349-L3: *Because it makes us to be active.*

Interviewer: How and in what way?

Line 350-L3: *Check accuracy with the drawing and what sizes are important.*

Interviewer: Yes, do you want to say something more?

Line 351-L3: *Mh....No*

Interviewer: What you didn't like about the activities?

Line 352-L3: *Nothing, ndizithande zonke (I liked all of them)*

Interviewer: Whoau, you must be a good student, you like everything that Maam gives you. What causes you to like everything?

Line 353-L3: *Because it tells us about careers, it has to do with career fields.*

Interviewer: Which one has to do with careers?

Line 354-L3: *Even translation have, because I want to be a pilot in future.*

Interviewer: What have you gained?

Line 355-L3: *I've gained to be more clever because there is no chance of being a pilot if I don't pass there to know this in grade 10.*

Interviewer: Did you have problems during implementation?

Line 356-L3: *Yes*

Interviewer: What problems?

Line 357-L3: *At first I didn't know how to check co-ordinates*

Interviewer: Did you tell Maam?

Line 358-L3: *No*

Interviewer: Why?

Line 359-L3: *I was shy*

Interviewer: Who told you?

Line 360-L3: *I saw from other examples how to do it.*

Interviewer: What did you like about being placed in different levels? Do you remember the activities were having levels?

Line 361-L3: *No, I didn't see the levels.*

Interviewer: You do not know the level you are in?

Line 362-L3: *Yes*

Interviewer: Let's say you are at level 1, 2, or 3. Would the placing have helped you?

Line 363-L3: *Yes*

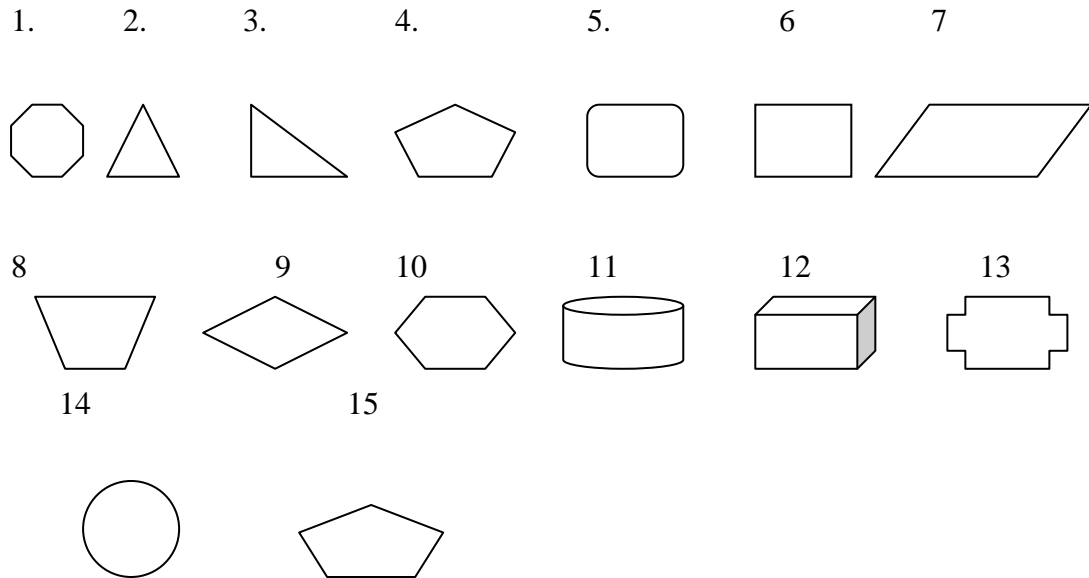
Interviewer: Why?

Line 364-L3: *Because I will know where my mistake is and focus on it.*

APPENDIX C: TEACHING PROGRAMME ACTIVITIES

Activity 1

Learners are engaged in an activity of sorting out shapes.



Sort out the shapes given for activity 1 and give reasons why you sorted them the way you did. Record your findings. Probing questions like the following are asked:-

- Pick up a square from the shapes given.
- How do you know that you have picked a square? Give reasons for your answer.
- Pick up a kite from the shapes given.
- Pick up a rhombus from the shapes given.
- What is a rhombus? Why do you think that the shape is a rhombus?
- What is the name of shape no.2?
- Why do you call the shape above a triangle?
- Pick up a rectangle.

LEVEL 2 & 3 QUESTIONS

- How do you know that it is a rectangle and not a square?
- Is a square a rhombus? Why and why not?

- Is a square a rectangle? Why do you say so?
- Is a triangle a quadrilateral? How do you know that?
- Have you ever seen quadrilaterals in real life other than in a book? Where and where else? How did you know that they are quadrilaterals? Make different drawings of shapes you know as quadrilaterals.
- What kind of shapes do you normally see from shapes given in activity 2 above? Give reasons why you classify them as that.
- Can you differentiate between shape no.7 and shape no.9?
- Give the difference in your own words.

VAN HIELE’S LEVELS – ASSESSMENT TABLE

LEVELS	PROBING QUESTIONS	POSITIVE RESPONSE	NEGATIVE RESPONSE
1. Recognition/Visualization. Learners sort shapes according to appearance without mentioning their properties	Pick a square, how do you know that it is a square? Pick up a rhombus. How do you know that you have picked a rhombus?	When a learner is able to pick the correct shape and answer questions that it looks like the shape wanted.	When a learner is confused of which shape to pick, picks the wrong shape and can't give a correct answer or keep quiet during the questioning time
2. Analysis. Learners are able to give properties of shapes without making any connections or relationships. They reason in terms of properties and do not understand the relationships between properties at this stage.	Is a rectangle a square? How do you know? Is a triangle a quadrilateral? Why?	If the learner is able to give the properties of shapes asked. If they are able to support their statements.	No knowledge of properties. No reasons given or no answer given on properties and no response.
3. Informal deduction / Ordering. At this level learners are able to order logically the properties of shapes. They are	Is a parallelogram a rectangle? Why? What is the relationship between	If learners are able to give correct responses to the questions asked and	If learners are unable to see any connections between the shapes compared

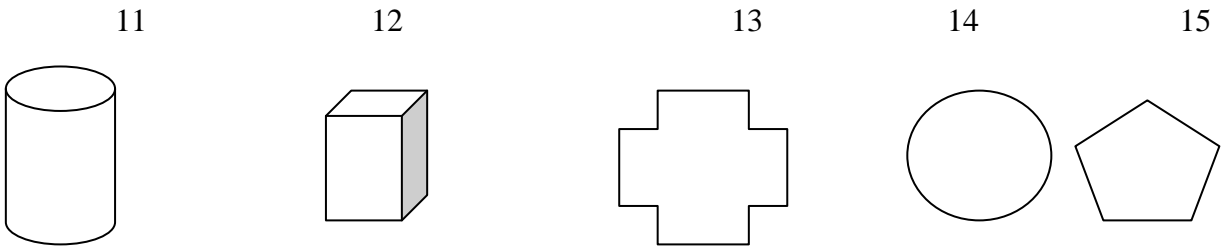
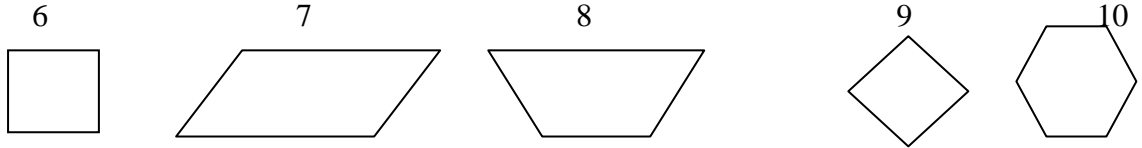
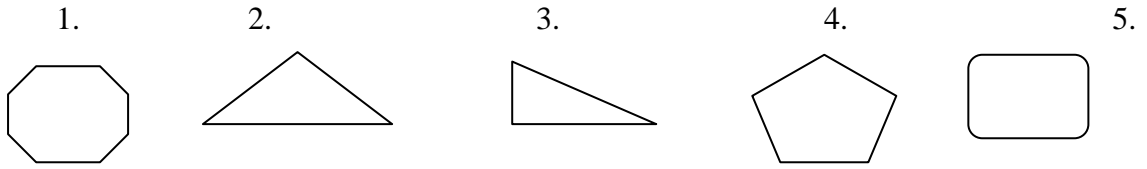
<p>able to make connections, relationships and see differences between properties of shapes.</p>	<p>the two? Is there any difference you see between the two shapes? What is the difference between the two?</p> <p>Is a square a rhombus? Why and why not?</p> <p>Is a square a rectangle? Why do you say so?</p>	<p>are able to give good reasons for their answers.</p> <p>Yes, and give the difference.</p> <p>Yes. All sides are equal.</p> <p>Yes. All angles are equal and are right angles.</p>	<p>or they can't see any difference between properties of shapes.</p> <p>No. It looks different. Yes without reason to please the teacher.</p> <p>No response at all.</p>
--	---	--	---

TEACHER'S ROLE

Marking the activities 1 & 2 according to the table (Appendix J) will give the information about the level each learner is in. The mark and level will tell the teacher where to start in order to help learners master the requirements of the levels they suppose to be in. More activities per level will be given to help learners catch up with the levels they missed and iron out the misconceptions they might be having in each level. Use of manipulatives will guarantee the improvement of learners in these activities and engage learners in hands on in the activities asking them questions throughout.

ACTIVITY 2

Give the names and properties of all shapes given in question 1 mentioning the number of sides that are there in each shape, using the table given below. Do not re-draw the shapes, just copy and complete the table provided below.



NAME OF SHAPE	PROPERTIES	NO.OF SIDES
1.		
2.		
3.		
4.		
5.		
6.		
7		
8.		
9.		

10.		
11.		
12.		
13.		
14.		
15.		

Marking rubric

Level	Specific aim	Children may need...
1	Transition from level 1 to 2 starting to identify properties of shapes from level 1 to level 2	Sorting knowledge, be able to recognize shapes
2	Generating properties. No connections at this level	Knowledge of properties of shapes
3	General properties of shapes can be given. They can make connections and differences	Know properties of shapes. Be able to make connections. To be observant when dealing with shapes

LEVEL 3 ACTIVITIES / QUESTION 2 EXTENSION

1. What are connections between a square and a rectangle?
2. What differences can you identify between a square and a rectangle?
3. Differentiate between a rhombus and a parallelogram. What are the connections?
4. Give a distinction between a trapezium and a kite.
5. What is a difference between an octagon and a hexagon?

6. Give a difference between a square and a rhombus.
7. What are the connections you see between a square and a rhombus?
8. You can make drawings to explain yourself clearly.

Purpose & teacher's role

The purpose of this activity is to expose learners to more shapes so that they master level 1 and 2 of van Hiele as they will recognize the shape by appearance and give its properties. This activity will prepare the learners for activity 3 whereby they will not only give properties of shapes but be able to make relationships and informal deductions. If learners do not master the activity above, more activities will follow and give learners a chance to learn more. Marking will be done according to the table given above.

ACTIVITY 3

Revision of grade 8's work

In your own words describe what do you understand about the following concepts? You can accompany your description with some sketches refreshing your memory from grade 8's work.

- a) Transformation
- b) Translation
- c) Rotation
- d) Reflection
- e) Glide reflection
- f) Enlargement
- g) Symmetry
- h) Reduction
- i) Congruency
- j) Similarity
- k) Rotational symmetry
- l) Horizontal line of symmetry
- m) Vertical line of symmetry
- n) Clockwise direction

- o) Anti-clockwise direction
- p) Mirror symmetry

ACTIVITY 4

- I. Imagine you seeing a butterfly flying and spreading its wings. Draw a picture of a butterfly and divide it into two equal halves. What do you notice? What is the name of the line dividing the butterfly into two equal halves? Give only the name of the line not in full sentence.
- II. Draw a picture of your friend and divide it into two equal halves. What findings do you get? Record them in your workbook.
- III. Draw a tree and see if it can divide into two equal halves and record your findings.
- IV. Write letters **M, W, N, H, E, X, Z, A, V, F, O, R, C, S, Q, T, D, Y, G** etc and see which ones have a **vertical line, horizontal line** of symmetry and which ones **have no line** and group them together.
- V. Draw the following polygons and show how many lines of symmetry each has:
 - 1. a square
 - 2. a rhombus
 - 3. a parallelogram
 - 4. a trapezium
 - 5. scalene triangle
 - 6. equilateral triangle
 - 7. a rectangle
 - 8. a kite
 - 9. an isosceles triangle
 - 10. an octagon
 - 11. a pentagon
 - 12. a heptagon
 - 13. hexagon
 - 14. decagon
 - 15. nonagon

Purpose of the activity

The aim of this activity is to check learners' prior knowledge and understanding about symmetry. At the end of all these activities surely the learners would have mastered the term as they will also draw sketches showing symmetry.

ASSESSMENT TABLE

LEVEL	+ RESPONSE	- RESPONSE	TEACHER'S JOB
1. Visualization Recognition of shapes	Can recognize symmetry without making sense of their meaning. They just see a line of symmetry that looks like any other line they have seen before. Line not important to them at all	No recognition at all. Not sure of what they see. Have no knowledge of shapes at all Have not noticed the existence of a line	Make drawings with a line at the centre and probe them- what they see? How many divisions do they see? What do they notice after the line has been drawn?
2. Analysis in terms of properties. No understanding of connection at this level. Understanding of what symmetry is.	Understand symmetry. Able to represent it in drawing form. Able to label the drawing when needs be. Know the meaning of the word without having problems	Doesn't know the meaning. Can show it correctly in a drawing. Confusing to him. Show no interest.	Give many activities for drawing symmetry. Ask them to make own drawings and draw line of symmetry. Encourage them to read, write and draw to make sense of the meaning of the word "symmetry"
3. Order/Informal deduction. At this level connections and differences are made between shapes. Different lines of	Is able to show symmetry in different shapes and letters. Knows connections and differences between symmetry of	Unable to show symmetry. Unable to make connections and differences	Encourage them to draw sketches and draw symmetries of different shapes, compare them and see what is the same and

symmetry to different polygons / objects are drawn	different shapes.		what is different about them.
--	-------------------	--	-------------------------------

ACTIVITY 5(a)

Make a collection of manipulatives brought in the classroom such as watch models, cell phones switched off, papers, mirrors, cocky pens, rulers, car models, windmill model, books, and papers folded in halves etc, and place these on a table.

Ask learners to pick up resources that slide, flip and turn from the collection on the table.

If there are problems, demonstrate a slide and call it translation, demonstrate a turn – rotation and demonstrate a flip- as reflection. Code switching can also be done at this stage because the aim of the activity is to let learners understand the terminology used in transformation geometry.

ACTIVITY 5(b)

Ask learners to draw and cut out a scalene triangle. Then translate, rotate and reflect the triangle and record the results they get. Describe in their own words, what happens to the object when they transform it in their own words.

GOAL: Understanding transformations such as rotation, translation and reflection at different levels

ASSESSMENT TABLE

Level	Specific aim	+ Response	- Response	Remedial
1. Recognition & Conceptual understanding	That learners can be able to differentiate a flip from a slide, and	Pick all resources that flip, slide, turn with ease	Unable to pick. Doesn't understand the concepts.	More manipulative activities should be given to help

	from a turn			them master the concepts
2. Analysis in terms of properties & fluency in carrying out procedures given	Knowledge of what each transformation is and how does it operate.	Understands properties in terms of terminology. Knows meaning of each transformation. Attempts to do the transformation with eagerness.	Not sure about properties and terms. Not sure about the meaning of each transformation	Give them more examples to expose them in different examples of transformations and how each works
3. Ordering-informal deductions, making connections & differences between objects and shapes. Strategic competence in performing given activities	Check learners' critical & creative thinking	Able to differentiate movement of turn to a slide and a difference between a reflection and a translation. Able to give reasons why a transformation is a flip, a turn or a slide.	Can't see the difference between different transformations. Can't make any distinction and reason why a flip is not a reflection. No response. Copying from others	More individual attention should be given to check how the learner thinks

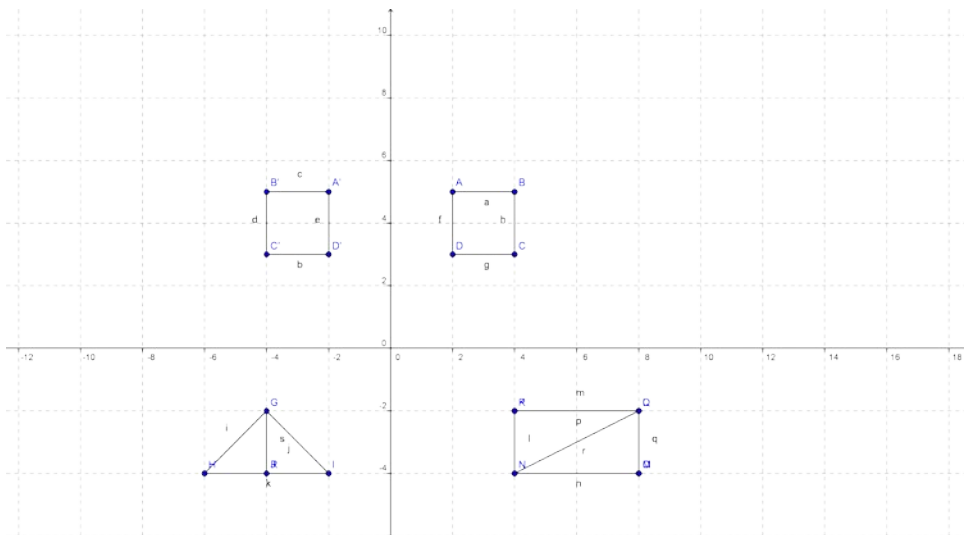
ACTIVITY 6

Procedural fluency, analysis & guided orientation

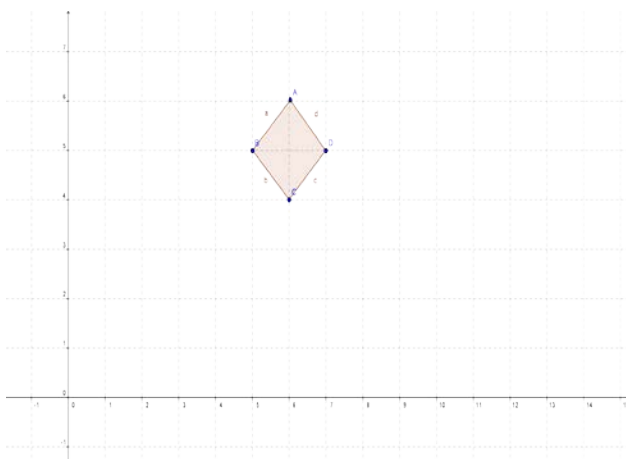
AIMS: At the end of these activities learners are expected to be able to carry out procedures and rules associated with the concept, be able to carry out instructions given, explore the work as they are engaged in hands on activities and give properties of shapes if needs be.

The objects shown below 'move' in a space. Write down whether the movement is a translation, a reflection or a rotation.

- An airplane flying in the sky. What transformation is shown by this statement?
- What transformation is represented by the shapes in the 1st and 2nd quadrants in the Cartesian plane given below? Write down the ordered pairs of the shape in the 1st quadrant and ordered pairs of the shape in the 2nd quadrant. Which shape is the image of the other from the two shapes given in the 1st and 2nd quadrant?
- Write all the ordered pairs of the two triangles given in the third quadrant.
- Give the ordered pairs of the shape given in the 4th quadrant. Give the name of the shape in the 4th quadrant.



- Imagine that you are looking at yourself in the mirror. Imagine that the picture you see in mind is the image of yourself. Reflect the image below and draw its mirror image. Write down the ordered pairs of both images.



- f) The wall clock ticking from 6h00 to 6h30. What transformation is that?
- g) Crossing the river on a boat. What transformation is that?
- h) The car traveling on the road. What transformation is formed?

Strands	Levels	Phases	Mastered	Not mastered	Remedy
Procedural fluency	Analysis	Guided orientation	When a learner has mastered the strands, levels and phases s/he is expected to master without difficulty and is willing to work co-operatively with the teacher and eager to ask questions and know more.	When a learner is unable to perform procedures, has not mastered the level and phase, is not willing to work with the teacher, is bored and doesn't even want to ask questions for further understanding. Is not motivated to learn the transformations.	Give them more activities to practice fluency, master analysis and see connections between shapes and properties given. Ask them questions that will encourage and force them to think critically and deeply about the problems given trying to master them through practice. Motivate them to learn more about the concepts that are taught in the classroom and want to investigate more about them.

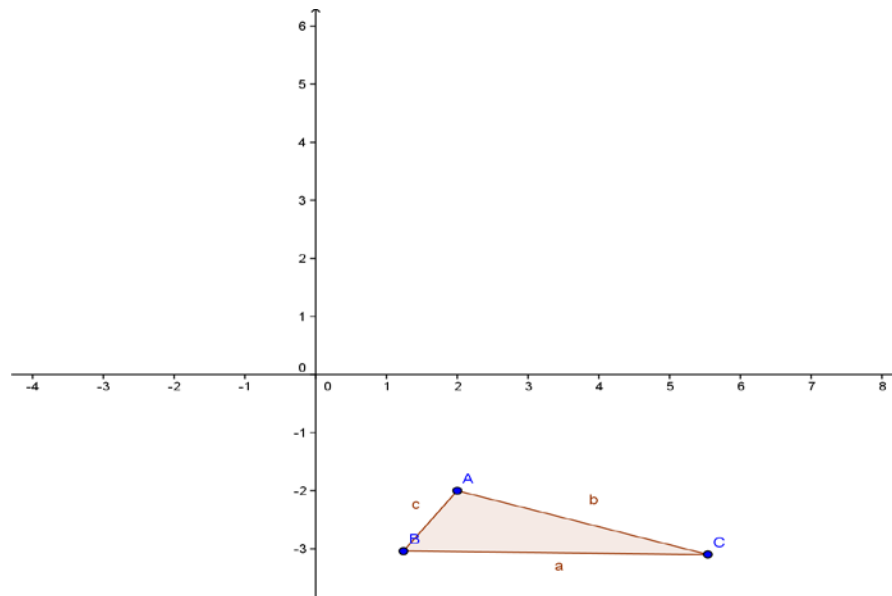
ACTIVITY 7

Strategic competence, informal deduction & explicitation

Aim: - To make sure that learners are able to carry out procedures correctly, are able

to make informal arguments to show generalizations to be true, making connections and be able to explain/describe what they learned in their own words.

Translate the shape in the plane below 4 spaces to the right and record the coordinates that are formed after the translation.



Strands	Levels	Phases	+ Response	- Response
Conceptual understanding	Visualization	Information	Learners have grasped the concepts of transformation geometry, can recognize what transformation they see in the mirror or paper, are able to describe & sort the transformation they see in their own words	Are struggling and are not sure of which transformation is given. Are also struggling to describe what they see and have done in their own words. Needs lots of assistance in sorting and identification
Procedural fluency	Analysis	Guided orientation	Are they able to carry out procedures? Do they have knowledge of when & how to use them appropriately &	Learners are not proficient enough to carry out procedures, how, when and where to use them

			skill in performing them flexibly, accurately & efficiently?	
Strategic competence	Informal deduction	Explicitation	Problem solving stage, informal arguments develop to show generalizations & make connections, can explain & describe what they learned in their own words	Are struggling in solving problems given. Are not sure of how to make connections between different transformations given. Need assistance.

Teacher's role: - To make sure that the above aims are fulfilled and if there are learners experiencing problems in any of the columns above, more activities should be given to learners to iron out the gaps that might be there. Learners need to be exposed to do lots of activities so that they understand the concepts and procedures correctly and accurately

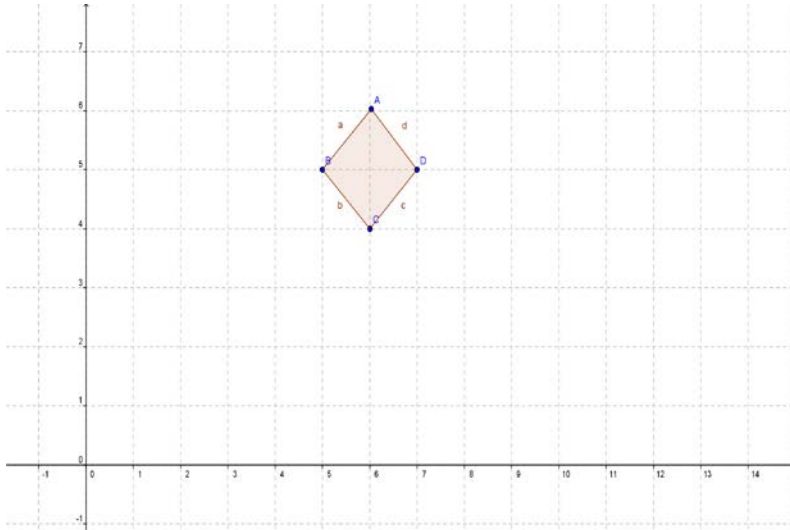
ACTIVITY 8

Adaptive reasoning, deduction & free orientation activities

Aim: - To check whether learners are able to think logically, can make formal deductions and relationships on work given and can solve and do investigations given to them by the teacher.

1. Give the ordered pairs of figure ABCD below.
2. Rotate figure ABCD from point which is the centre of rotation 180 degrees counter-clockwise. Record the findings.
3. Reflect the original object ABCD downwards and record the findings.
4. What is the relationship between question 2 & 3 above? Write it in your own words.

(a)

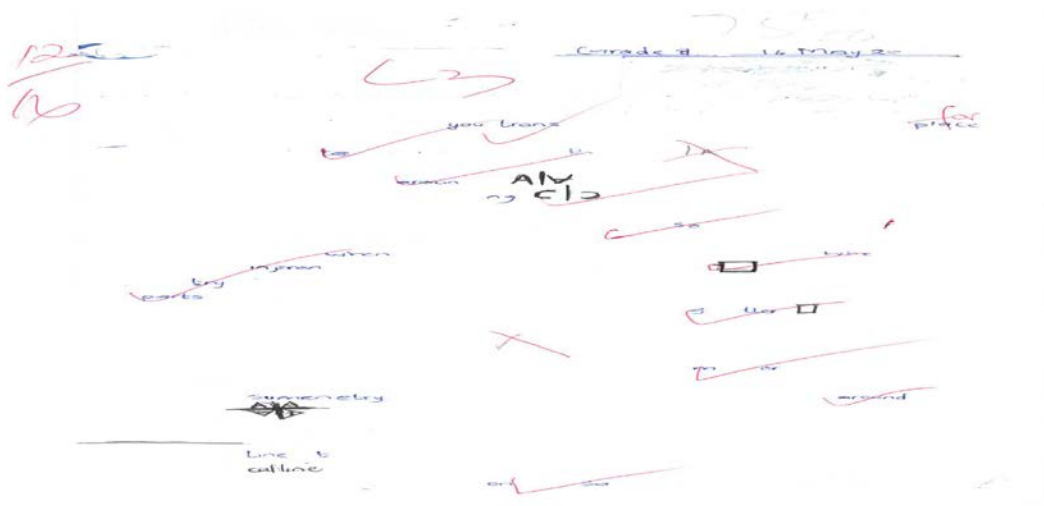


Marking rubric

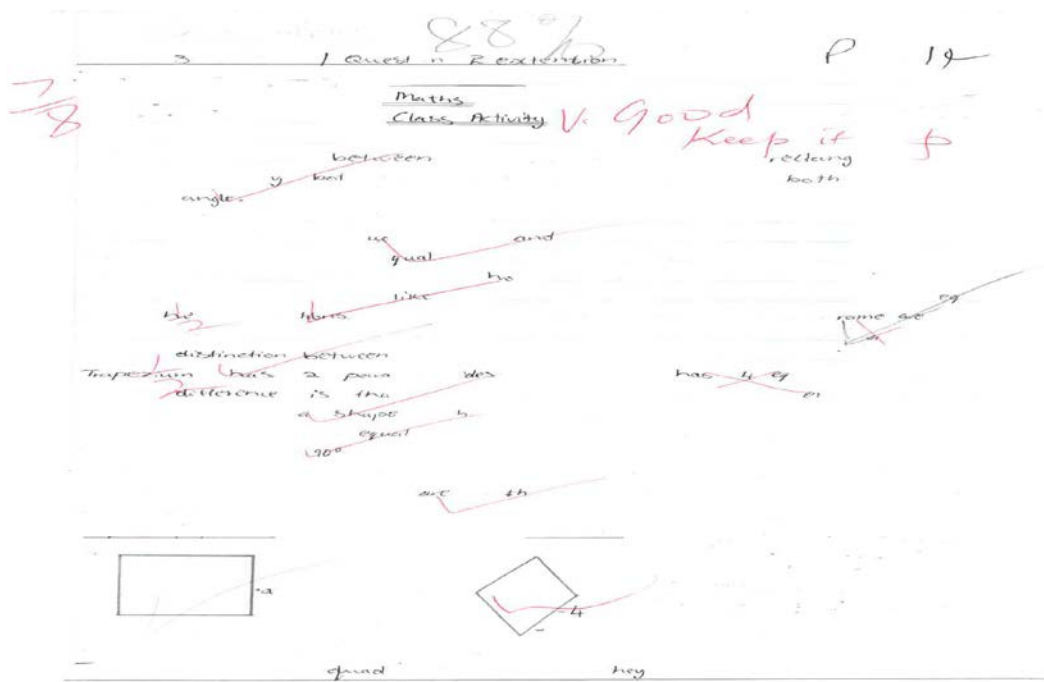
Van Hiele's level	Van Hiele's phases	Kilpatrick's strands	Teacher expectations
Deduction	Free orientation	Adaptive reasoning	That students are able to make formal deductions at this stage and be able to make links on work given to them by the teacher and see what is related and what is not.
Able to prove theorems, make formal deductions & relationships on work given.	Make relationships between concepts, can solve problems and do investigations given by the teacher on their own.	Are able to think logically about the relationships among concepts.	To be problem solvers who are able to think logically and critically about the problem given.

Teacher's role: - To help those who still struggle at this stage with more activities that can help those learners with barriers at this stage to master the work prepared for this level.

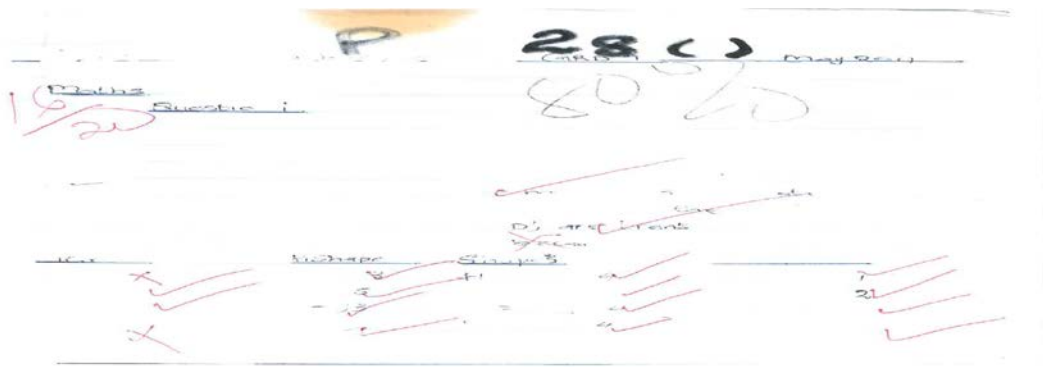
Picture 12



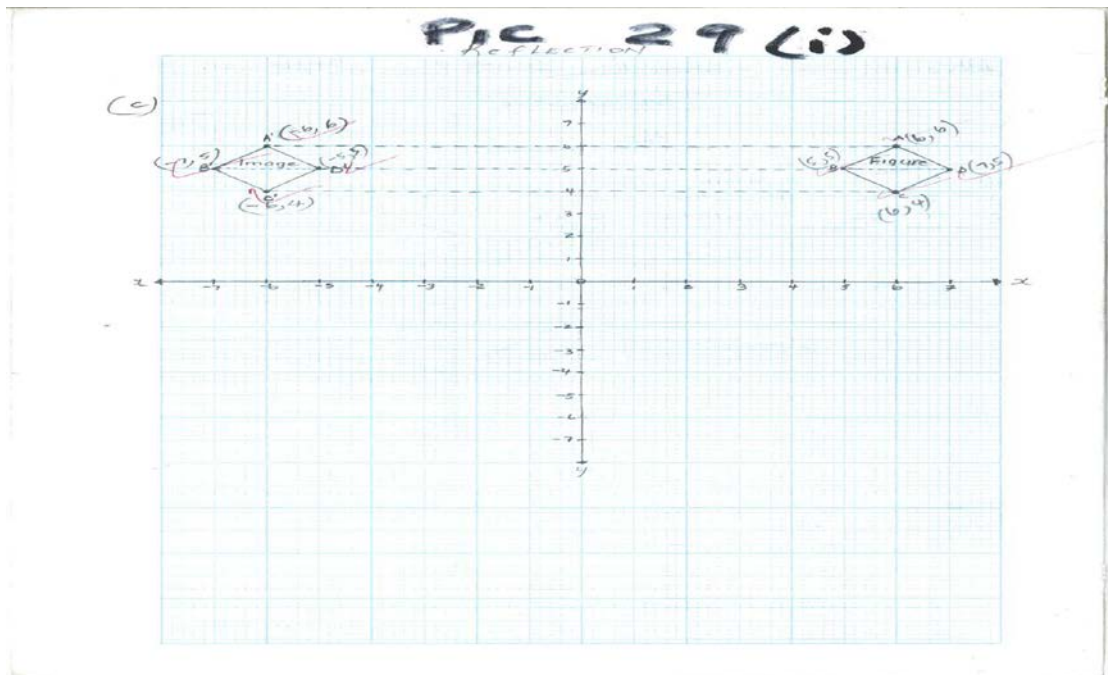
Picture 14



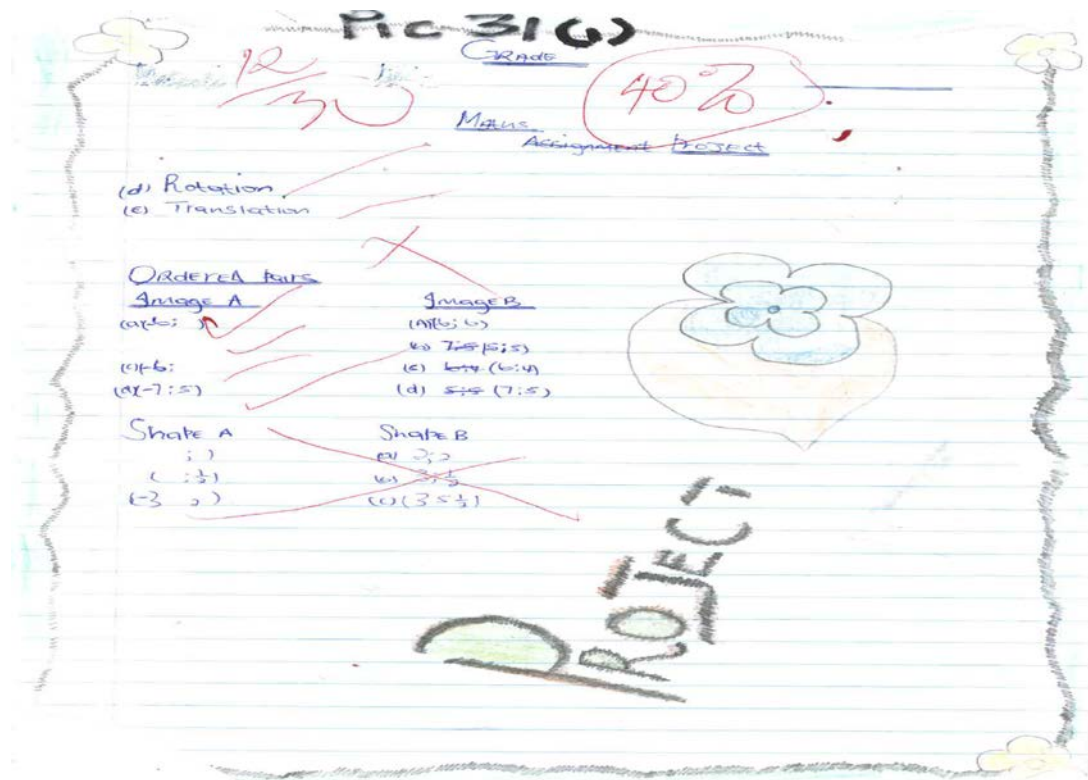
Picture 28



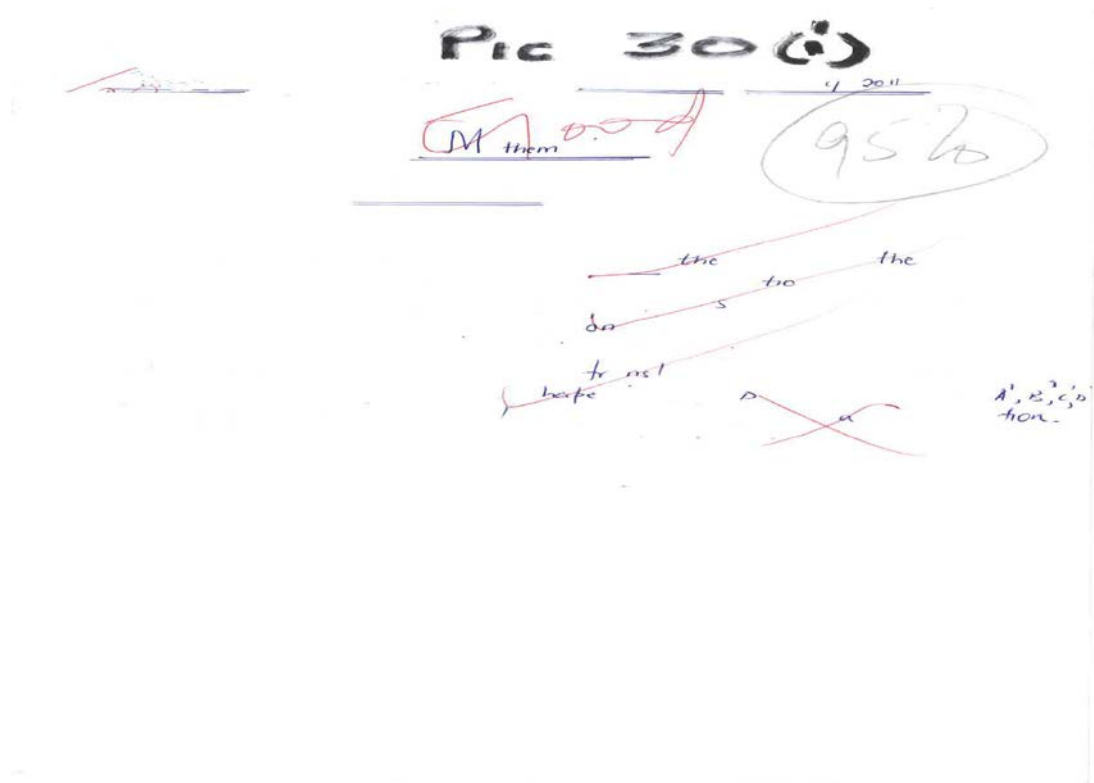
Picture 29(i)



Picture 31(i)



Picture 30(i)



Picture 32

deg - May - 2011
33,3%

c 32

	PROPERTIES	
1. gbr	it has 3 sides	3
2. triangle	it has 3 sides	3 sides
3. right angle triangle	that are not equal	3 sides
4. agon	3 equal sides	3 sides
5. agon	have 4 sides	4 sides
6. Squatre	it have 4 unequal sides	4 sides
Quadrilatera	it have 4 equal sides	4 sides
um	it have 4 sides	4 sides
agon	it have 4 equal sides	4 sides
agon	it have 6 sides	6 sides
7. agon		3 sides
8. agon		12 sides
9. agon	are not equal it have 13 sides	13 sides
10. agon	it have 5 sides	5 sides
11. agon	have equal sides	5 sides
12. agon		5 sides
13. agon		5 sides
14. agon		5 sides
15. agon		5 sides

30

Picture 32(i)

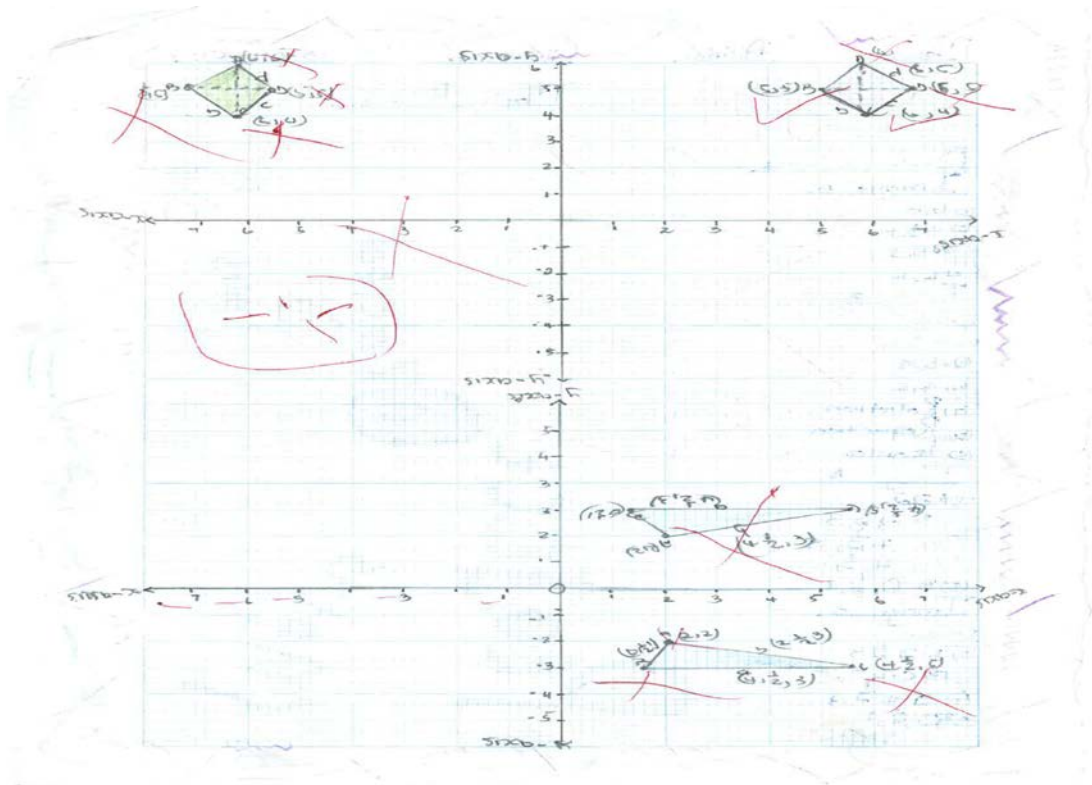
11/16 8j: 69% Grade 9 Pic 32(i)

Maths
Activity 2

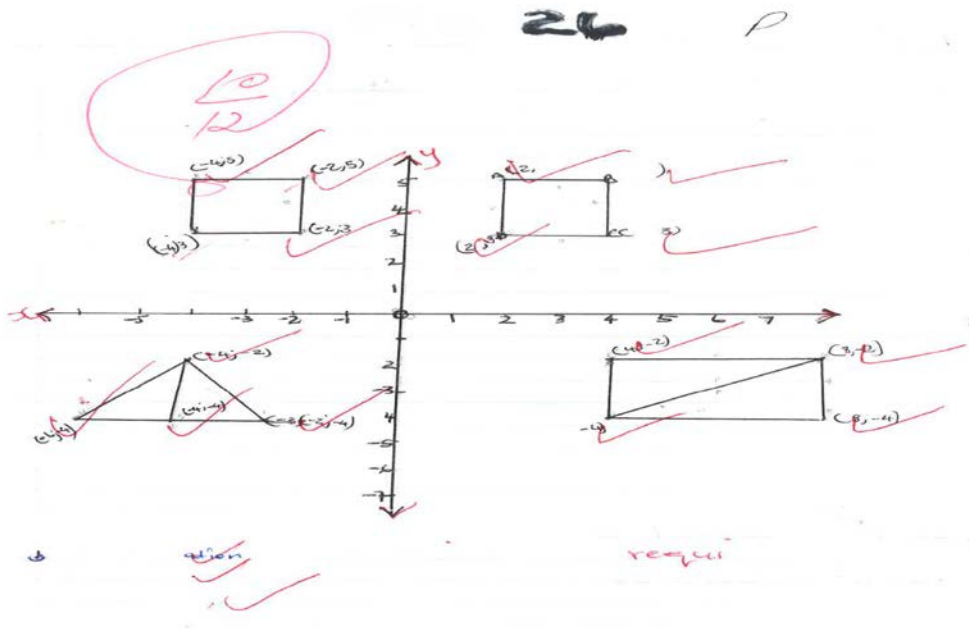
that that can be are
side side
ng at
something is reflected
something
small
at
one y
Le
point
to
the Le
polygon

Simple
up

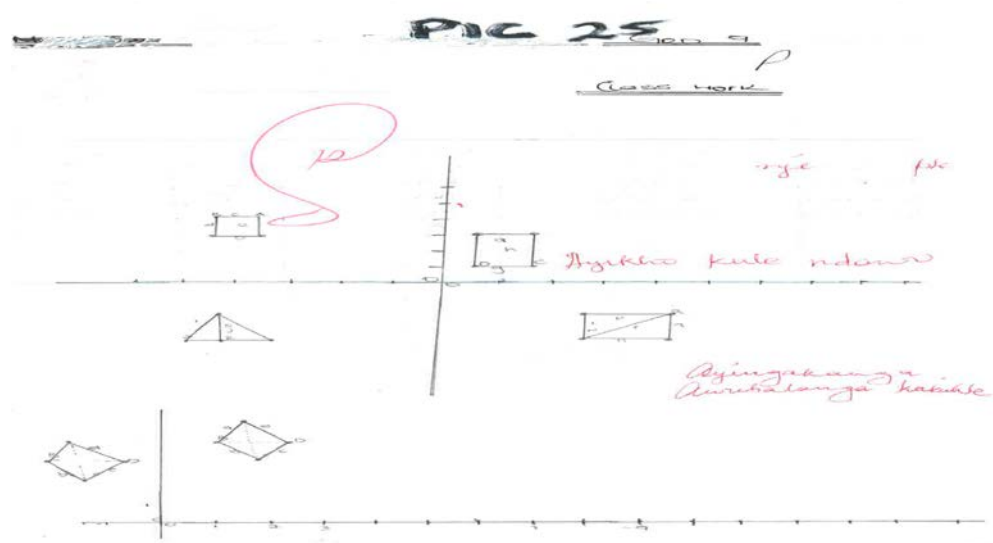
Picture 27



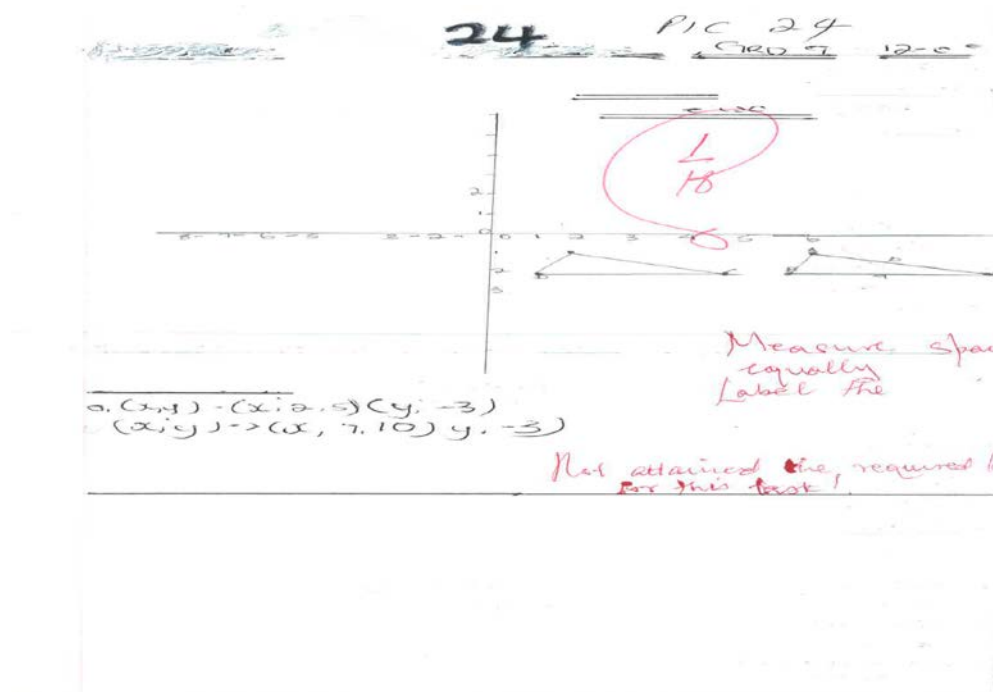
Picture 26



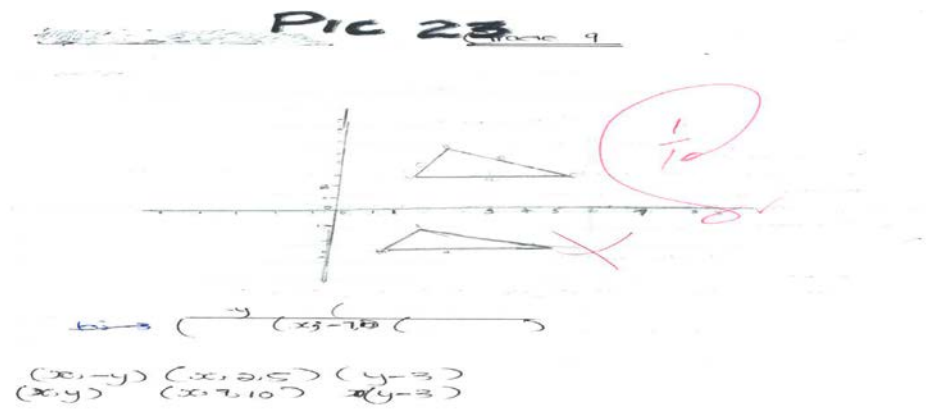
Picture 25



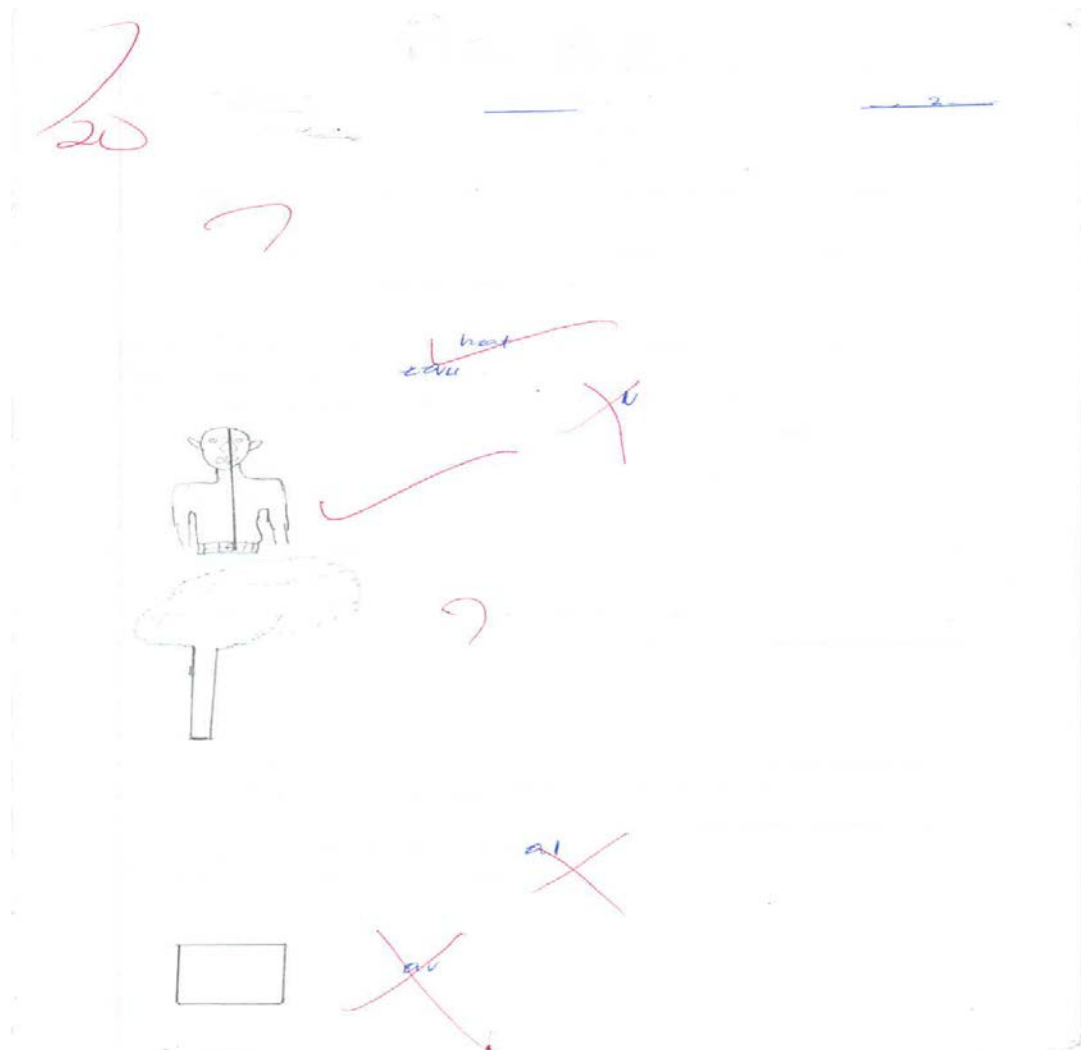
Picture 24



Picture 23



Picture 22



Picture 17

$\frac{7}{12}$ **Pic 17** 17
19-May-2011
42%

~~7 has 10~~
~~10~~ Becca two
a shape with
three si

$\frac{8}{10}$ is not
Q16, because

long
have
first 10.
quadrilateral
then
instead
Arc
shape
put

80%

Picture 16

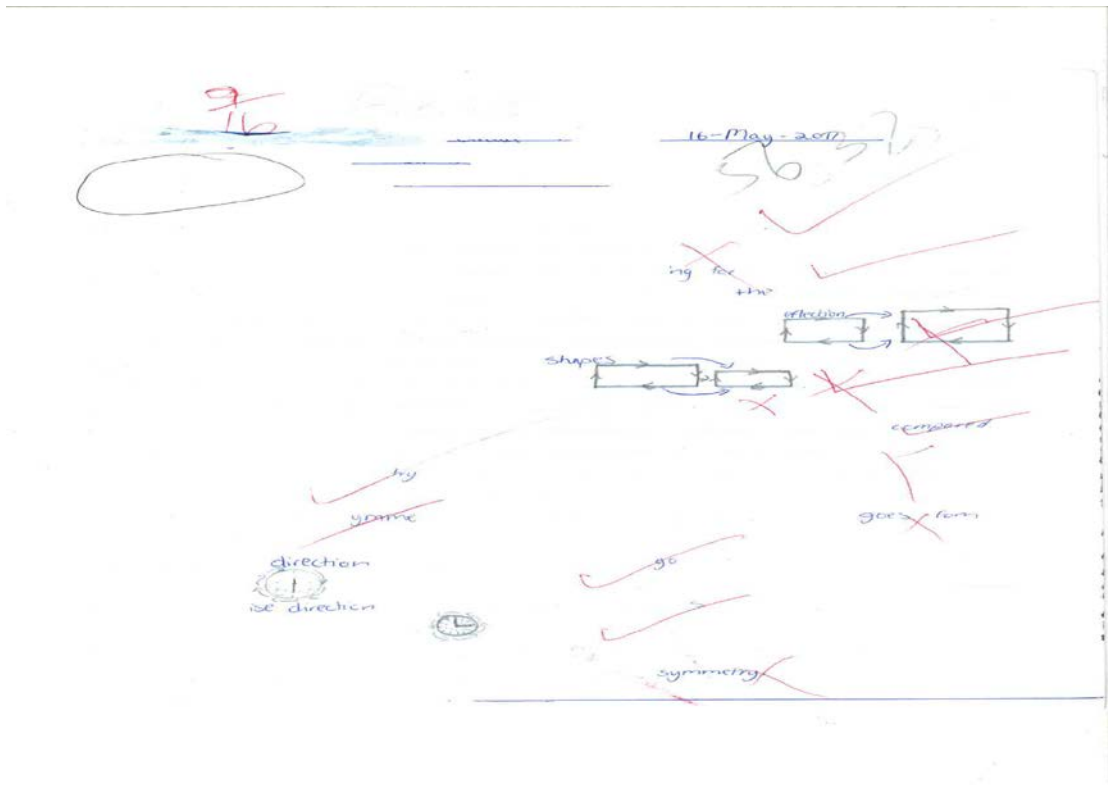
$7 \frac{1}{2}$
 $\frac{1}{20}$ **Pic 16**
Maths 19-May-2011
65%

butterfly can
two equal

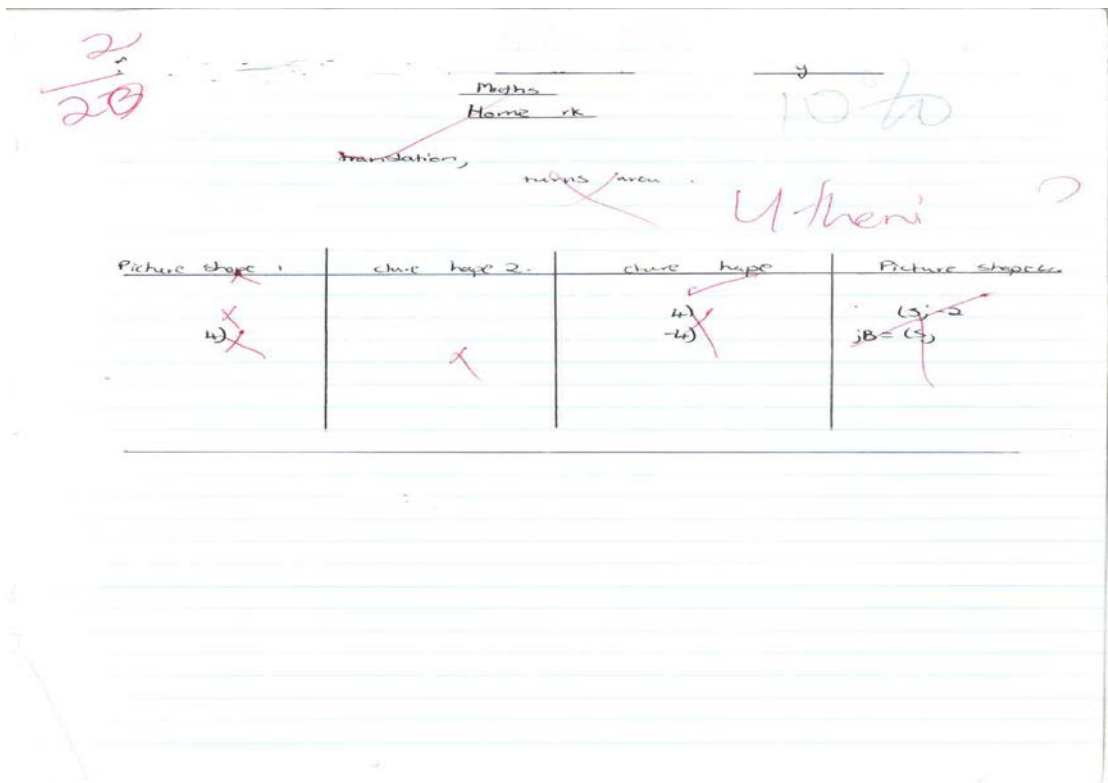
equal
my
no

apples

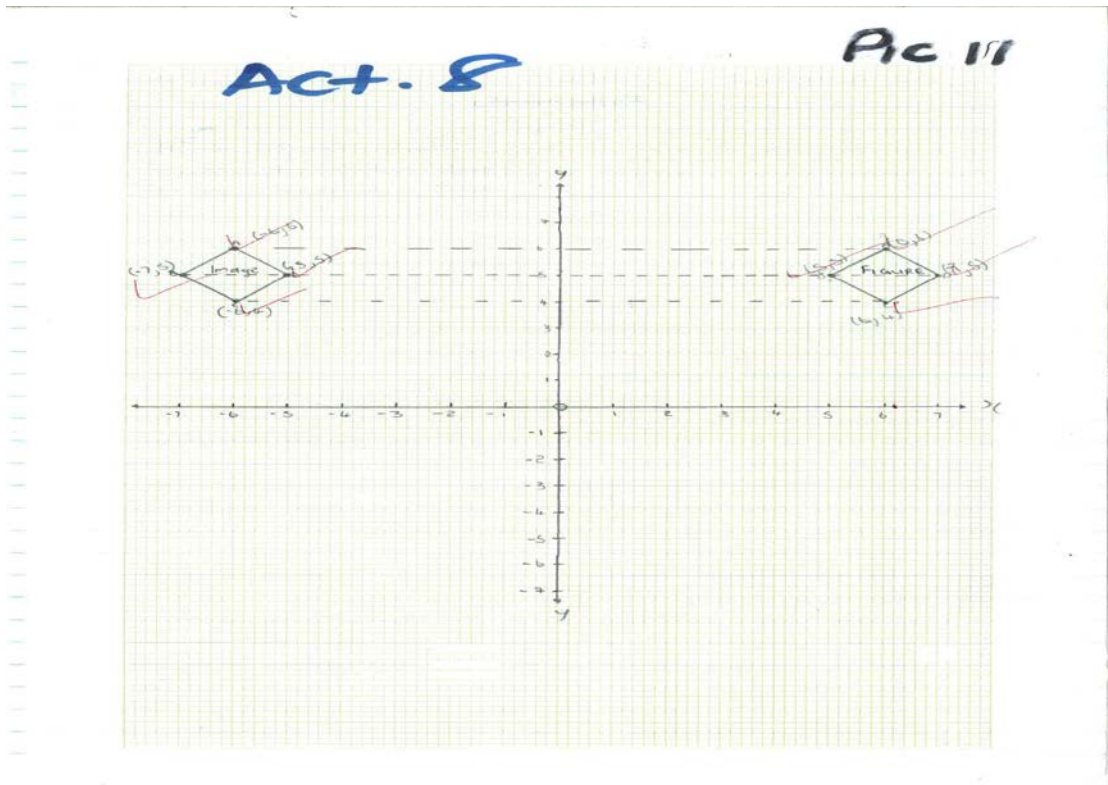
Picture 15



Picture 13



Picture 11



Picture 10

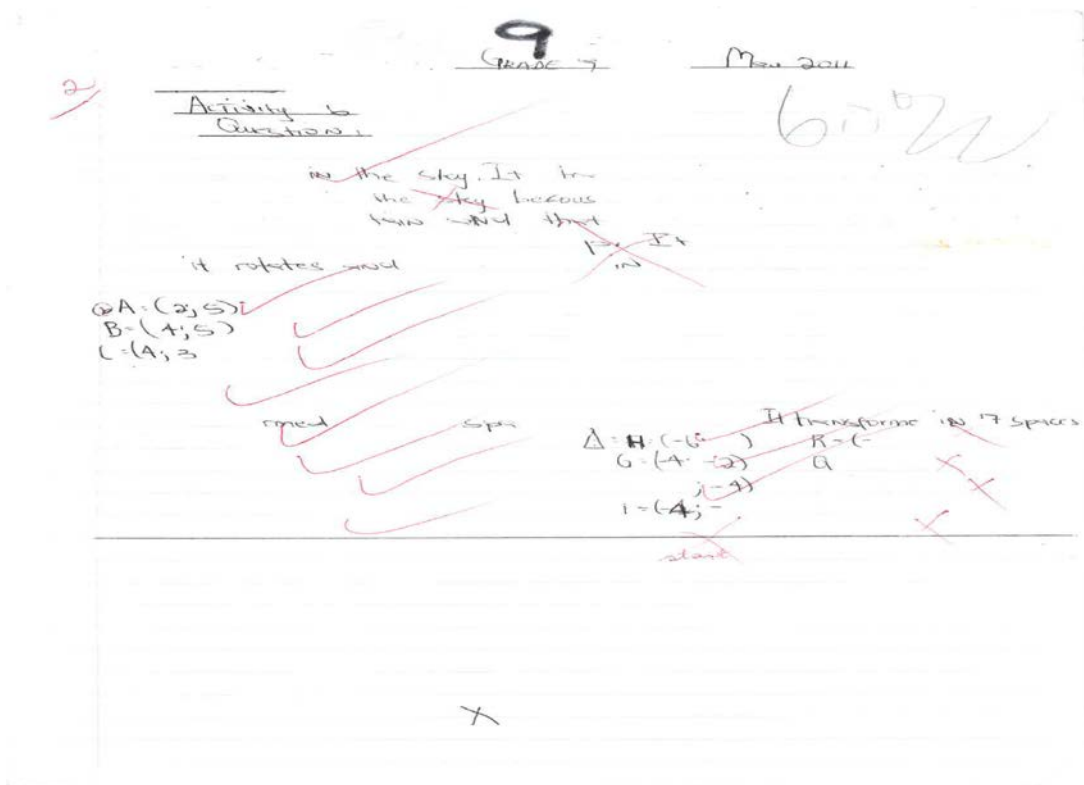
25%

Grade 7
Act 4.5

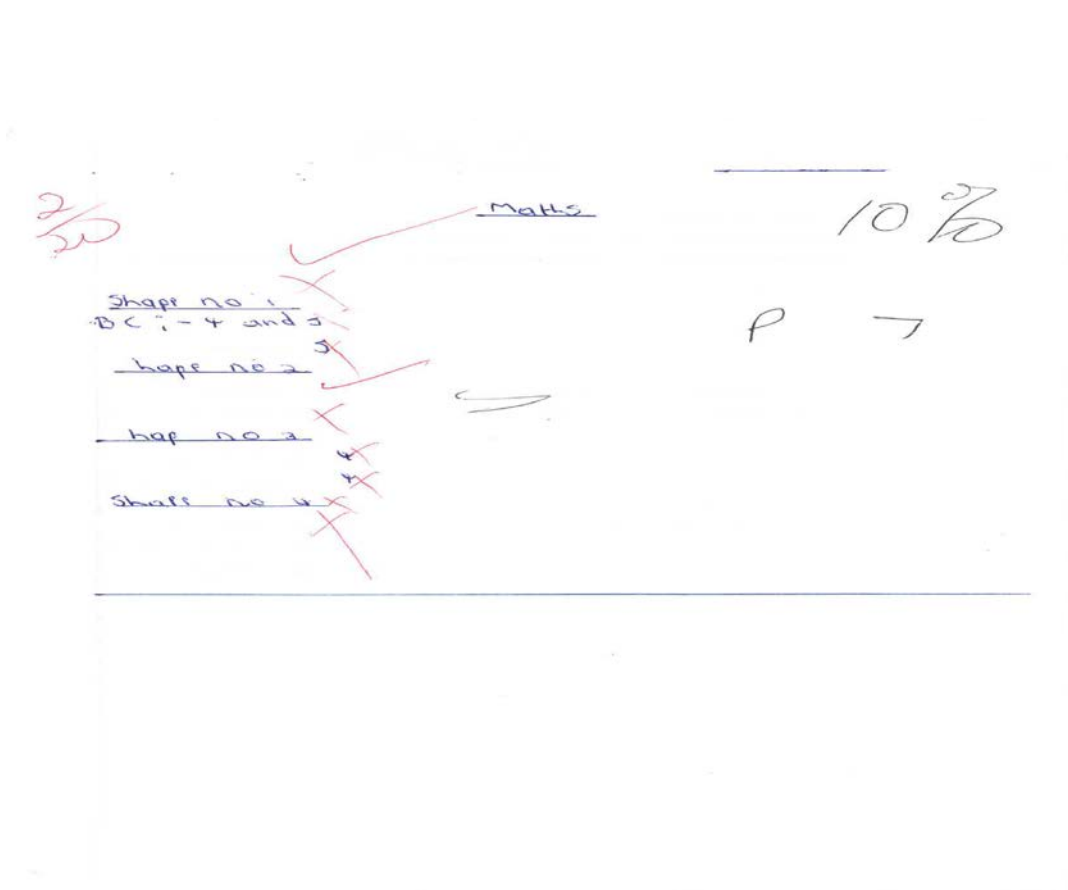
Reflec	flip	Slide	turn/rotate
F	T	k	25%
25%			dec

4

Picture 9



Picture 7



Picture 4

Pi 4
Grade _____
Date 12 May 2011

Math
Activity

4

0/8

below

Rectangle
two sides
Equal sides
and kite two

octagon is a shape that have 8 sides and
it have 6 sides.
6 the different is a square it have 4
two sides are Equal

Square one bus

below L3

Picture 3

Pic 3
Grade _____
Date 1 May

P

7/12

Shape number 6
1. number to square because

is a kite
is a Rhombus because it have
equal sides and one angle is not equal with
2. because it has 4 equal sides.
2. shape number 10 is a Rectangle

no Equal sides and Equal angles
cause a Rectangle has 2 sides are Equal and
cause has no Equal sides and no Equal angles
seen in a school bus

6. it is a Rectangle
7. shape number 7 it is like a Rhombus and shape
it is like kite but they are
the Rhombus and it shape number
they have 4 Equal

Picture 2

Grade _____
 Learning _____
 Date _____

B s Lin Evaluation

Q s i L

1) BABONILE E OB
 2) AMBULANCE E WA
 3) F F
 4) F F
 5) A A
 6) C C
 7) G G

~~on~~

~~7~~

Picture 1

~~0/0~~

SIRADET

~~VE~~

~~1. A square is a the shorts line n skirt~~

rectangle
 are equal
 2. The rec

~~the time is the number~~

~~oppor~~

~~ng~~

~~Deval~~

~~ax~~

~~ax~~

~~ax~~

below L3

APPENDIX F (i): PRE - LESSON OBSERVATION TRANSCRIPTION: TB

TB, Line 001: I am trying to look at another example that can help us. Reflection again, let me use an example of water. Look at the water, what part of the body you see first on the water? Whose picture do you see?

Learners (chorus): *Your face, self*

TB, Line 002: *(Drew an F on a Cartesian plane on the board that is inaccurate and axis were not labelled. There was no T-square at school so she had to use her free hand).* Reflect that picture horizontally. Which direction is horizontal? *(A learner came and translated letter F and not reflected it as the teacher asked. Another one came and reflected letter F incorrectly).*

TB, Line 003: It must look the same as the image. Who can help us? *(Another child drew it correctly).* Let's give her a big hand. *(Learners clap their hands)* .The distance must be the same but face different directions. Can you see the change?

TB, Line 004: Let me give you a C, B. In your own words translate this letter *(Erased letter C and said)*, 'Reflect the same letter F downwards from the 1st quadrant to the 4th quadrant. Translate this letter; use a ruler to make a drawing of a Cartesian plane. Use 1cm space, reflect it and see how the image will look like. Work in groups of 6 and less'.

TB, Line 005: Ok, the time is up. Khanindiboleke laa ncwadi. *(She paged the textbook looking for other examples for sometime.)*

TB, Line 006: Which group to go to the board and give us feedback? *(A group representative reflected letter F correctly).*

TB, Line 007: Let us look at the board and see if it is correct. Next time we will use geometric figures, letters and co-ordinates. Let us look at page, page, page, page *(No page number was found).*

APPENDIX F (ii): POST LESSON OBSERVATION TRANSCRIPTION- TB

TB, Line 001: Good morning learners, how are you?

Learners: Fine thanks

TB, Line 002: Sit down. We are going to start the types of geometric transformation.

Can you give me examples/types of transformation geometry?

L1: Breadth

TB: (*Repeating what the learners had said*) Breadth, is breadth a shape or figure?

Who can help us? Alwaba

L2: Breadth

L3: Parallelogram

TB, Line 003: In which family of quadrilaterals does a parallelogram belong?

L4: Rhombus

L5: Square

L6: Triangle

TB, Line 004: Who can give me the type of triangle we learned about?

Learner: Isosceles triangle, right angled triangle, acute angled triangle.

TB, Line 005: Now it's enough. We are going to..to...learn about transformation, (*writing on the board*). What 'trans means'? Any name that begins with 'trans' to check your vocabulary.

Learners: (Chorus) Translation, transport, transfer.

TB, Line 006: Yenza ntoni itransport? What is taking place by trans? By just saying 'trans' can I be in Queenstown? What is taking place with 'trans'?

TB, Line 007: (*Writing*) Movement/changing position. There are types of transformation or examples of transformation, what are they?

Learners, (*chorus*) translation, reflection, rotation, enlargement

TB, Line 008: Today we'll only tackle translation and reflection. When we talk of translation, it means that there is a movement that is going to take place in the form of(*not finished the sentence*). Let me open this door sideways which means I am taking this door from this position to another. There is going to be a movement or changing position which means I'm sli...

Learners (*chorus*) Sliding.

TB, Line 009: So eh..let me write this letter 'F'. When I want it to be in this position (*pointing*) I am sliding it and write F1.

TB, Line 010: Reflection, what is reflection? The meaning of reflection is aFlip. When you flip you... (*Using actions and gestures*).

Learners: (*chorus*) pull or push.

TB, line 011: In rotation, when you rotate you (*actions and gestures*) “Turn” and the learners in chorus say, ‘Turn’.

TB, Line 012: I have something small and I want it to be visible, so the meaning of enlargement (*using actions*).

Learners: (*chorus*) Big

TB, Line 013: Let us look at the table. I have these objects that are displayed here. This is a paper, you can come and take the object and say if it is a slide, a flip etc. Brother Bongani has a good example of enlargement. He uses a camera which takes pictures and makes them small. There are some adjustment knobs changing the size of the object. Come to the table.

L1: (*Picks a sliding phone, slides it looking at the learners in class*).

TB, Line 014: What movement?

Learners: (*Chorus*) Slide.

Another learner picks a clock and looking at the class while lifting it up.

TB, Line 015: What transformation is that?

Learners: (*Chorus*) Rotation

TB, Line 016: Come quick, quick, quick, take any object and face them.

Learner: (*Picks an exam pad, opens it up facing the class*)

TB, Line 017: What transformation is that?

Learners: (*Chorus*) Reflection

Learner: (*Picks up a phone that does not flip, rotate or slide and show it to the class*)

There was no hand up and the teacher thought of showing them another object herself that learners have been ignoring.

TB, Line 018: Oh, let me take this (*referring to the laptop on the table*). I like technology because it makes my life easy. What transformation is this?

Learners: (*Chorus*) Reflection

TB, Line 019: Yes, reflection. As I’m going to be punching letters, the information will be reflected to the screen. Now I want to ask you this question,” Why do you learn transformation? Tell me the purpose of transformation”. (*No answer from learners*). Transformation is there in our daily lives as we have seen that we are

making this paper fold into two halves, this is reflection. If we open and close our books we are doing reflection. Look at our faces in the mirror, we are doing what...?

Learners: (*Chorus*) Reflection

TB, Line 020: We can take photos by a photographer (*She looks on the wall and saw an old picture hanging there as said to class*) Look at this, sapha, sapha (bring it to me), Mrs Geja here is another drawing. This is ...enlargement of Madiba's photo. We know Madiba; can we see that his ID photo is smaller than the picture? Can you see that our life is full of mathematics? You will see transformation in the field of architects and engineers.

APPENDIX G: Lesson observation transcription - TB

Pre-lesson observation transcription

TB, Line 001: I am trying to look at another example that can help us. Reflection again, let me use an example of water. Look at the water, what part of the body you see first on the water? Whose picture do you see?

Learners (chorus): *Your face, self*

TB, Line 002: (*Drew an F on a Cartesian plane on the board that is inaccurate and axis were not labelled. There was no T-square at school so she had to use her free hand*). Reflect that picture horizontally. Which direction is horizontal? (*A learner came and translated letter F and not reflected it as the teacher asked. Another one came and reflected letter F incorrectly*).

TB, Line 003: It must look the same as the image. Who can help us? (*Another child drew it correctly*). Let's give her a big hand. (*Learners clap their hands*). The distance must be the same but face different directions. Can you see the change?

TB, Line 004: Let me give you a C, B. In your own words translate this letter (*Erased letter C and said*), 'Reflect the same letter F downwards from the 1st quadrant to the 4th quadrant. Translate this letter; use a ruler to make a drawing of a Cartesian plane. Use 1cm space, reflect it and see how the image will look like. Work in groups of 6 and less'.

TB, Line 005: Ok, the time is up. Khanindiboleke laa ncwadi. (*She paged the textbook looking for other examples for sometime.*)

TB, Line 006: Which group to go to the board and give us feedback? (*A group representative reflected letter F correctly*).

TB, Line 007: Let us look at the board and see if it is correct. Next time we will use geometric figures, letters and co-ordinates. Let us look at page, page, page, page (*No page number was found*).

Post-lesson observation transcription (PLO)

TB, Line 01: Good morning learners, how are you?

Learners: Fine thanks

TB, Line 02: Sit down. We are going to start the types of geometric transformation. Can you give me examples/types of transformation geometry?

L1: Breadth

TB: (*Repeating what the learners had said*) Breadth, is breadth a shape or figure?

Who can help us? Alwaba

L2: Breadth

L3: Parallelogram

TB, Line 03: In which family of quadrilaterals does a parallelogram belong?

L4: Rhombus

L5: Square

L6: Triangle

TB, Line 04: Who can give me the type of triangle we learned about?

Learner: Isosceles triangle, right angled triangle, acute angled triangle.

TB, Line 05: Now its enough. We are going to..to...learn about transformation, (*writing on the board*). What 'trans' means? Any name that begins with 'trans' to check you vocabulary.

Learners: (Chorus) Translation, transport, transfer.

TB, Line 06: Yenza ntoni itransport? What is taking place by 'trans'? By just saying 'trans' can I be in Queenstown? What is taking place with 'trans'?

TB, Line 07: (*Writing*) Movement/changing position. There are types of transformation or examples of transformation. What are they?

Learners, (*chorus*) translation, reflection, rotation, enlargement

TB, Line 08: Today we'll only tackle translation and reflection. When we talk of translation, it means that there is a movement that is going to take place in the form of(*not finished the sentence*). Let me open this door sideways which means I am taking this door from this position to another. There is going to be a movement or changing position which means I'm sli...

Learners (*chorus*) Sliding.

TB, Line 09: So eh..let me write this letter 'F'. When I want it to be in this position (*pointing*) I am sliding it and write F1.

TB, Line 10: Reflection, what is reflection? The meaning of reflection is aflip. When you flip you...(*using actions and gestures*).

Learners: (*chorus*) pull or push.

TB, line 11: In rotation, when you rotate you (*actions and gestures*) "Turn" and the learners in chorus say, 'Turn'.

TB, Line 12: I have something small and I want it to be visible, so the meaning of enlargement (*using actions*).

Learners: (*chorus*) Big

TB, Line 13: Let us look at the table. I have these objects that are displayed here. This is a paper, you can come and take the object and say if it is a slide, a flip etc. Brother Bongani has a good example of enlargement. He uses a camera which takes pictures and make them small. There are some adjustment knobs changing the size of the object. Come to the table.

L1: (*Picks a sliding phone, slides it looking at the learners in class*).

TB, Line 14: What movement?

Learners: (*Chorus*) Slide.

Another learner picks a clock and looking at the class while lifting it up.

TB, Line 15: What transformation is that?

Learners: (*Chorus*) Rotation

TB, Line 16: Come quick, quick, quick, take any object and face them.

Learner: (*Picks an exam pad, opens it up facing the class*)

TB, Line 17: What transformation is that?

Learners: (*Chorus*) Reflection

Learner: (*Picks up a phone that does not flip, rotate or slide and show it to the class*)

There was no hand up and the teacher thought of showing them another object herself that learners have been ignoring.

TB, Line 18: Oh, let me take this (*referring to the laptop on the table*). I like technology because it makes my life easy. What transformation is this?

Learners: (*Chorus*) Reflection

TB, Line 19: Yes, reflection. As I'm going to be punching letters, the information will be reflected to the screen. Now I want to ask you this question," Why do you

learn transformation? Tell me the purpose of transformation”. (*No answer from learners*). Transformation is there in our daily lives as we have seen that we are making this paper fold into two halves, this is reflection. If we open and close our books we are doing reflection. Look at our faces in the mirror, we are doing what...?

Learners: (*Chorus*) Reflection

TB, Line 20: We can take photos by a photographer (*She looks on the wall and saw an old picture hanging there as said to class*) Look at this, sapha, sapha (bring it to me), Mrs Geja here is another drawing. This is ...enlargement of Madiba’s photo. We know Madiba; can we see that his ID photo is smaller than the picture? Can you see that our life is full of mathematics? You will see transformation in the field of architects and engineers.