THE USE OF VAN HIELE'S THEORY TO EXPLORE PROBLEMS ENCOUNTERED IN CIRCLE GEOMETRY: A GRADE 11 CASE STUDY.

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ABSTRACT

The research presented in this thesis is a case study located in the interpretive paradigm of qualitative research. The focus is on the use of van Hiele's theory to explore problems encountered in circle geometry by grade 11 learners and making some policy recommendations concerning the curriculum structure and teaching of the geometry at all grades. The interpretation is based to the learners' background in geometry i.e. their prior knowledge and experience of learning geometry.

The study was carried out over a period of three years. The data collection process took a period of two months (April and May 2003) with a group of 21 grade 11 mathematics learners in a rural senior secondary school in the Eastern Cape. The researcher used document analysis, worksheets, participants' observation, van Hiele tests, a questionnaire and semi-structured interviews to collect data.

The study showed that the structure of the South African geometry syllabus consists of a some what disorganized mixture of concepts. It is not sequential and hierarchical and it sequences concepts in a seemingly unrelated manner. The study revealed that the South African high school geometry curriculum is presented at a higher van Hiele level than what the learners can attain. The findings of the study showed that many of the grade 11 learners were under-prepared for the study of more sophisticated geometry concepts and proofs. Three categories of reasons could be ascribed to this: Firstly, there was insufficient preparation of learners during the primary and senior phases. Secondly the study indicated that there is overload of geometry at the high school level in the South African mathematics curriculum. Thirdly, the over-reliance on the traditional approach to teaching geometry, poor presentation of mathematical technical concepts and language problems, were identified as possible additional reasons for the poor learner understanding of geometry in general and circle geometry in particular.

The study recommends that the structure of the South African geometry curriculum should be revisited and redesigned. Teachers should be empowered and developed to be more effective in teaching geometry through further studies in mathematics and in-service workshops. They should also be engaged in the process of implementing the van Hiele's theory in the teaching of geometry in their classrooms.

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As I reflect on my own achievement in reaching and completing this Masters, I think also of the spirit of the late Singela Gubhela Mkhize who has motivated me – as young school colleagues he professed that one day the two of us should go to university and become lecturers.

DEDICATION

This thesis is dedicated to both of my parents, my father "Donkiri" Tutu and my mother Nomight Rebecca Ntombentsha Siyepu. I cannot leave out my two late brothers Sidwell Mncedi and Abner Nkosimbini Siyepu.

To me it is certain that it was their wish to celebrate this achievement with me.

ACRONYMS

NCS	National Curriculum Statement
D.O.E.	Department of Education
OBE	Outcomes-Based Education
RS	Range Statement
PI	Performance Indicator
SO	Specific Outcome
LOLT	Language of learning and Teaching
SA	South Africa
CTA's	Common Tasks for Assessment
2D	Two dimensional
3D	Three dimensional
FET	Further Education and Training
RNCS	Revised National Curriculum Statement
π	Pie

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CHAPTER ONE

INTRODUCTION

1.1 INTRODUCTION

This chapter provides an introduction to my research study. It includes a rationale of the study and explains the fundamental goals of this research project. It also highlights the purpose, significance, and limitations of this study and ends with a brief overview of the thesis.

1.2 RATIONALE

Many high school students see geometry as the most problematic section in mathematics. This can lead to poor performance of learners in the geometry section of the curriculum. The essence of this study was thus to explore and tentatively suggest some reasons for the difficulties experienced by grade 11 learners in circle geometry.

1.3 PURPOSE OF THE STUDY

The aim of this study was to use the van Hiele theory as a tool to explore problems encountered by grade 11 learners in circle geometry. The study sought to thoroughly investigate problems experienced by grade 11 learners in circle geometry and make some recommendations that may lead to the improvement of the learning and teaching of geometry.

1.4 RESEARCH GOALS

In order to use van Hiele's theory as a tool to explore problems encountered by grade 11 learners in circle geometry, I pursued the following goals:

Goal 1: To determine the van Hiele levels of geometry thinking required by the grade 11 curriculum in South Africa.

Goal 2: To determine the van Hiele level of geometry thinking evident in 21 grade 11 learners.

Goal 3: To establish to what extent the curriculum requirements for grade 11 mathematics are compatible with the van Hiele levels demonstrated by the learners.

1.5 SIGNIFICANCE OF THIS STUDY

Globally teachers and learners regard geometry as the most difficult section in mathematics (Snyders, 1995). This study used the van Hiele theory to explore and investigate problems experienced by grade 11 learners in circle geometry in order to identify the causes of these problems. It used the findings to make recommendations where possible and suggest possible solutions.

The difficulties experienced with geometry affects many South African high school students who consequently perform poorly in the mathematics paper 2, which largely consists of circle geometry. It is evident that the South African geometry curriculum does not link elegantly with the van Hiele theory. This study tries to emphasise the necessity of the understanding the van Hiele theory by curriculum planners, policy makers and mathematics educators in order to effectively implement the mathematics curriculum.

1.6 LIMITATIONS OF THIS STUDY

The study focused only on circle geometry. The South African 1997 grade 11 geometry syllabus requires learners to be taught theorems and converses (see Appendix A), but due to time constraints I could not explore learners' problems with converses, hence the focus was only with theorems in geometry.

Generally geometry is regarded as problematic in all grades but I could not explore all the grades. Therefore I dealt with circle geometry only with grade 11 learners. Due to the nature of the study

I did not select a large sample. The sample initially consisted of thirty participants. It ultimately dropped to twenty-one.

This study is also limited to only one rural senior secondary school in the Eastern Cape.

1.7 THESIS OVERVIEW

1.7.1 Chapter two

Chapter two is a literature review of this study. It deals with the van Hiele theory and discusses some features and properties of the van Hiele levels and the van Hiele phases.

This chapter also deals with some of the causes of difficulties in high school geometry. It analyses the primary phase, teaching methods, and the traditional approach to teaching. It also explores the constructivist approach in the context of the van Hiele theory. It briefly addresses the role of language in the teaching and learning of geometry and gives a short critique of the van Hiele theory.

1.7.2 Chapter three

This chapter presents the research methodology used in this study. It describes the research orientation, qualitative research, case study, research site and participants. It also discusses the research ethics, data collection methods, validity and reliability of the study.It also deals with issues such as discipline, language and disappointments.

1.7.3 Chapter four

This chapter deals with the data analysis and findings of this study. Data analysis involved a document analysis of the syllabus, a discussion of the results of the analysis of the syllabus and the van Hiele theory, and the structure of the geometry curriculum in South Africa. It also involved an analysis of worksheets and van Hiele tests. It ends with an analysis of a questionnaire, understanding of mathematical technical concepts and analysis of interviews.

Chapter four also gives details of the findings based on each instrument or technique used to collect data.

1.7.4 Chapter five

It presents the conclusions of the study. It presents a summary of the findings, significance of this study, some recommendations, limitations, avenues for further research and reflections.

CHAPTER TWO

LITERATURE REVIEW

2.1 INTRODUCTION

The focus of this research study is the exploration of problems encountered in circle geometry by grade 11 learners. This chapter's literature review examines how other researchers and writers interpret learners' understanding of geometry. Globally, high school geometry has been noted as problematic for both teachers and learners (Snyders, 1995). The South African high school geometry syllabus requires proof of theorems and their application in solving riders (D.O.E., 1997a: 4). Researchers like Wirszup (1976), Hoffer (1981) and de Villiers (1987) indicate that, while the typical high school geometry course is taught at van Hiele level 4, most students taking it are situated at van Hiele level 1 (see 2.2.1). They further argue that this is the reason why many students find learning geometry proofs difficult. De Villiers (1997:40) indicates that "the van Hiele dissertation mainly tried to explain why pupils experienced problems in geometry education."

In this chapter I initially analyse and discuss the van Hiele theory in the teaching of geometry in the grade 11 learners. I then look at some causes of difficulties in high school geometry.

Some researchers put the blame of poor performance in geometry on the primary phase. They argue that students enter high school without enough skills and knowledge to cope with high school geometry. I discuss the insufficient preparation of learners in the primary phase (see 2.8.1). I then discuss the traditional teaching approach as one of the possible causes of the poor performance of geometry students (see 2.8.3). Many researchers, for example Sekiguchi (1996) recommend a constructivist approach as an effective method in the teaching of geometry (see 2.9). I therefore discuss the constructivist approach in relation to the van Hiele theory. Other researchers see language problems as one of the possible causes of the poor performance in geometry at higher levels. I critically discuss the role of language in the learning and teaching of mathematics, particularly geometry (see 2.10).

There is some confusion surrounding the numbering of the van Hiele levels. Some authors talk about level 0 as the first level and others talk about level 1 as the first level. I refer to this confusion in 2.2.1 and 2.11.

Some students and teachers see geometry as problematic, and geometry is regarded as the most difficult subject. Mahaffey and Perrodin (1973:15) confirm that "geometry tends to arouse fear in the most courageous elementary teachers". Further, the unpleasant experiences encountered cause many students to avoid any further mathematics (Mahaffey and Perrodin, 1973). My observations during twenty years of teaching mathematics and interacting with teachers concur with Mahaffey and Perrodin's view. Many students, teachers and other stakeholders tend to assume that geometry has been the work of outstanding human geniuses (Freudenthal, 1973). As a result, learners tend to refrain from learning geometry. This suggests that geometry is generally regarded as a difficult section in mathematics.

Many researchers in mathematics education have shown that learners experience problems in high school geometry. Fuys *et al.* (1988:4) allege that "the experience of secondary school mathematics teachers indicates that many students encounter difficulties in high school geometry, in particular in doing formal proofs." To my mind, the above also applies in South Africa. King (2002:178) asserts that "dissatisfaction with the secondary school geometry curriculum in South Africa and poor performance in geometry has been the topic of many discussions over the past decade or two among mathematicians and researchers." This implies that some amendments and innovations need to be made in the South African geometry curriculum.

In my experience as a mathematics teacher, I have noticed that many secondary school students and teachers do not have enough background in geometry to cope with the requirements of the curriculum. Their prior mathematics education has not prepared them to master high school geometry. Van Hiele (1986:40) supports the contention that "most teachers do not have the courage to present the subject matter of geometry in high schools." This difficult relationship with geometry does not motivate them to teach Higher Grade Mathematics. In my conversations with several mathematics high school teachers in rural areas, I have heard the claim that their students do not have the confidence to do Higher Grade Mathematics. Rossouw and Smith (1997) endorse the view that some teachers cannot cope with teaching students who require Van Hiele 3 (informal deduction) thinking (see 2.2.1). This supports the argument that teachers also do not have the confidence to teach Higher Grade Mathematics. Fuys *et al.* (1988) indicate that the van Hieles were greatly concerned about the difficulties their students encountered in high school geometry.

2.2 THE VAN HIELE THEORY

Many researchers suggest that the van Hiele theory is one of the best frameworks presently available for studying, teaching and learning in geometry (King, 2002). I will briefly discuss the van Hiele model using Pegg's (1995) framework:

- What are the van Hiele levels of geometry understanding?
- What are the features or properties of the levels?
- What are the five van Hiele phases?
- How can the levels be identified in the classroom?
- What are the teaching implications of the theory?

2.2.1 Van Hiele levels of geometry understanding

Poor performance of high school geometry learners has been a problem in many countries for many years (see 2.1). The van Hieles suggest that five levels of geometry understanding can be used in the teaching and learning of geometry. They claim that these levels should follow each other in a particular order and should occur sequentially. These levels are positioned according to their sequence and importance (Pegg,1995). I believe that teachers should use the van Hiele theory to prepare their learning units. The van Hiele levels of geometric thinking are:

Level 1 (Visualisation): Students recognize figures by appearance alone, often by comparing them to a known prototype, which is an original thing. A student identifies, names, compares and operates on geometric figures (for example: triangles, angles, parallel lines, and circles) according to their appearance. The properties of a figure cannot be understood at this stage. At this level, students make decisions based on perception, not reasoning. For example, at this level a figure is a square, cube or a rectangle because it looks like one.

Pegg (1995:90) suggests that "there are at least three categories within this level." The first category is the ability of the students to associate a geometrical shape with a known shape, for example, a cube is like a box or a dice, a rectangle is a long square, parallel lines are like a door (Pegg, 1995). The second category occurs when students can identify certain features of a figure (not properties) such as pointedness, sharpness, corners, and flatness; but are unable to link these features to have an overview of the shape. The third and lowest category is where the student can focus only on a single feature (Pegg, 1995).

Level 2 (Analysis): Analysis is the process of identifying and examining each element of an object, or features of it, in detail in order to understand it. Students see figures as collections of properties. They can recognize and name properties of geometric figures, but they do not see relationships between these properties (Mason, 2003). The properties are seen as discrete entities independent of one another. For example, an equilateral triangle can have three equal sides, three equal angles and three axes of symmetry but no property implies another. Another example is that in a circle where two radii are joined by a chord, an isosceles triangle is formed but that does not imply that the base angles are equal. In short, students at this level cannot make short deductions. The South African Revised National Curriculum Statement (RNCS) requires that students be at level 2 in the intermediate phase (10 –12 years) (D.O.E., 2002c).

Level 3 (Abstraction): An abstraction is an idea or principle considered or discussed in a purely theoretical way without reference to actual examples and instances. Students see at this level relationships between properties. At this level, student can create meaningful definitions and give informal arguments to justify their reasoning (Mason, 2003:4). Mayberry (1983:59) indicates that at this level "logical implications and class inclusions are understood." For example, in an equilateral triangle, the fact that all sides are equal is seen to imply that all angles are equal. In addition, relationships between figures are understood. For example, an equilateral triangle is an isosceles triangle because the set of all properties of an isosceles triangle is included in the set of properties of an equilateral triangle. When studying a circle, students can see that the angle in the centre doubles the inscribed angle if the same arc subtends them. Both the South African Curriculum syllabus (D.O.E.,1997a) and the RNCS (D.O.E., 2002c) require students in the senior phase to be at van Hiele level 3. Students at this level are able to do short deductions but the role and significance of formal deductions is not understood. Therefore students at this level cannot solve geometry problems that need to apply a chain of application of deductions.

Level 4 (Deduction): Deduction is the reasoning process by which you conclude something from known facts or circumstances or from your own observations (Grearson and Higgleton, 1996). At this level the significance of the role of deduction is understood. Students can construct proofs; understand the role of axioms, definitions and theorems. The students can develop proofs of theorems and the need for rote learning is minimized (Pegg, 1995). At this level, students should be able to construct proofs such as those typically found in all high school geometry class. In short, students should be able to give formal proofs of theorems and apply them to solve riders. A theorem is a statement that can be proved to be true by applying generally accepted ideas or theories (Grearson and Higgleton, 1996). The South African geometry curriculum (D.O.E., 1997a) requires high school students, grades 10-12 (16-18 years), to be at the van Hiele level 4. For example, at this level, a student should be able to prove that the angle between the tangent to a circle and a chord drawn from point of contact is equal to an angle in the alternate segment. This is one of the theorems to be mastered by grade 11 students for examination purposes (D.O.E., 1997a: 3). At this level the students are required to master and solve advanced, complex geometrical riders.

Level 5 (Rigour): Students at this level understand the formal aspects of deduction such as establishing and comparing mathematical systems. Students at this level can understand the use of indirect proofs and proofs by contra-positive, and can understand non-Euclidean systems (Mason, 2003). This work lies outside the scope of what is normally expected of secondary school students. I decided, therefore, not to focus on this level, as it is not within the scope of my research.

Mayberry (1983) and Usiskin (1982) in Pegg (1995:90) "found it valuable to refer to students who have not attained level 1 as thinking at level 0." Students at this level notice only a subset of the visual characteristics of shape, resulting in an inability to distinguish between figures (Clements and Battista in Mason, 2003). For instance, a child who regards the shapes of an egg and soccer ball as round objects is in the van Hiele level 0. At the same time, they may distinguish between triangles and quadrilaterals, but may not be able to distinguish between a rhombus and a parallelogram (Mason, 2003). In their original work, the van Hieles numbered the levels from 0 to 4. Researchers like Pegg (1995), and Burger and Shaughnessy (1986) started numbering the levels from 1 to 5 instead. This scheme allows for the pre-recognition level to be called level 0. However in this study I use the 1 to 5 numbering scheme.

According to Pegg (1995), the van Hiele levels represent a broad structure upon which a teaching learning program can be based. It is important to discuss a number of features attributed to the different levels.

2.2.2 Some features or properties of the van Hiele levels.

A student cannot attain a higher level without first passing through the lower level(s). That is, if the levels form a hierarchy, then a student performing tasks at level N but not at level N+1 would be expected to perform at all lower levels but not at any higher levels (Mayberry, 1983). However, students can pretend to be in higher levels by learning rules or definitions by rote or by applying routine algorithms that they do not understand (Pegg, 1995). In this thesis' exploration of the problems encountered in circle geometry I aim to ascertain the insight of students in their understanding of geometry. It will be explained that there is little insight development in rote learning (see 2.8.3).

Except perhaps in the case of exceptional students, to move a student from one level to the next requires direct instruction, and exploration and reflection by the student (Pegg, 1995). That is, students should be taught using an investigative approach. They should relate new information to what is already known, linking the known and the unknown (Etchberger and Shaw, 1992). They should collaborate by explaining, clarifying, elaborating, questioning and discussing possible solutions to the problem. Pegg (1995:93) indicates that "it takes time to move from one level to the next. Teachers must be prepared to allow time for this growth to occur." Teachers should understand that progress from one level to the next is more dependent on educational experiences than on age or maturation (Mason, 2003). Some experience can facilitate progress within a level or to a higher level.

Students may differ in their conceptual understanding. That is, students in the same level may not have an identical understanding of concepts. Pegg (1995:93) indicates that "once one concept has been raised to a higher level, it will take less time for other concepts to reach that level." In my view, this is possible if the concepts are interrelated.

Students need to confront a 'personal crisis of thinking' in moving from one level to the next (Pegg, 1995). That is, students should engage in more sophisticated thinking in moving from one level to the next (Pegg, 1995). Teachers should provide students with the problems linking the levels, and guide students to shift from one level to the next. Pegg (1995) emphasizes that students cannot be forced to think at a higher level. That is, a variety of teaching and learning strategies should be used to move students from one level to the next. Pegg (1995:93) further argues that "certain teaching strategies can inhibit such growth and place boundaries on students' potentials." This indicates that certain teaching methods and/or strategies can endanger the progress of the students. I propose, therefore, that teachers should select the most appropriate teaching methods and/or strategies to move students from one level to the next.

'Level Reduction' occurs when structures at a higher level are re-interpreted at a lower level. This usually occurs by making the structures at the higher level visible. The effect of this procedure, when it is teacher-directed, can be counter-productive, as it can remove the stimulus for students to attain a higher level. This can cause students to see no reason for learning the proofs of theorems.

Each level has its own language or linguistic symbols. People who reason at different levels speak different languages and, in general, cannot understand one another (Pegg, 1995). This language problem can occur even between students in the same classroom. Thus, very serious communication problems exist between students on one level and their fellow students, teachers, textbooks and exercises on another level. For example, a tangent in trigonometry is a ratio, and here means the opposite side divided by an adjacent side, whereas a tangent in Euclidean geometry is a line with one point of contact in a circle. This needs a thorough explanation. In a circle theorem, an angle in the alternate segment is equal to the angle between a chord and a tangent. This can create confusion for the student who also knows that equal alternate angles occur when two or more parallel lines are cut by a transversal. Teachers need to treat homonyms carefully and explain them thoroughly.

Each level has its own organization of relationships (Pegg, 1995). Teachers need to be aware that what may appear to be correct at one level may not be seen to be correct at a higher level, or vice versa. The most obvious example of this is that it is not until level 3 that a square is seen to belong to the set of rectangles.

The learning process is discontinuous. That is, having reached a given level, a student remains at that level for a time, as if maturing (Mason, 2003). Forcing a student to perform at a higher level or directing teaching at a higher level will not succeed until the maturation process has occurred. A student at a level 3 will not automatically understand mathematics lessons presented at level 4.

Rote learning or applying routine algorithms without understanding does not represent the achievement of a particular level (Pegg, 1995). That is, any information and/or knowledge learnt without insight cannot be regarded as the attainment of a certain level of understanding. In short, rote learning or applying routine algorithms without understanding cannot enhance learning. Rote learning of proofs of circle theorems cannot make learners correctly apply the theorems to solve riders.

In designing learning units, teachers should consider what the properties of the van Hiele levels imply in a mathematics classroom. According to Dina van Hiele-Geldof as cited in Pegg (1995), a student progresses through each level of thought as a result of instruction that is organized into five phases of learning. Pegg (1995) suggests that if it takes time for growth to occur, teachers could assist students to meet the challenge of the new level by the application of the van Hiele phases.

2.2.3 The van Hiele phases

The initial work in this area appeared in the doctoral thesis of Dina van Hiele-Geldof (Pegg, 1995). The main question posed in her study was: is it possible to follow a teaching approach that allows students to develop from one level to the next in a continuous process (Pegg, 1995). As a result of this work, five phases were identified that encourage students to move from one level to the next (Pegg, 1995).

The five van Hiele phases are summarized in Mason (2003) as follows: -
Phase 1: Information: Through discussions, the teacher identifies what students already know about a topic and the students become oriented to the new topic. This is similar to baseline assessment. The South African Department of Education (D.O.E., 2002d:2) describes baseline assessment as "assessment usually used at the beginning of a phase, grade or learning experience to establish what learners already know." It assists educators and teachers with the planning of learning programmes and learning activities (D.O.E., 2002d). That is, Phase 1 (information) is a way of ascertaining the prior knowledge of learners. Van Hiele elaborated further on the phases (Pegg, 1995). Van Hiele (1986) considered the study of a rhombus in which students move from level 1 to level 2 (Pegg, 1995). A rhombus was shown to the class. The class was asked to pick rhombuses from a collection of figures and identify rhombuses in composite figures. In my research I consider the study of circles in which students move from level 1 to level 4.

Phase 2: Guided Orientation: Students explore the objects of instruction in carefully structured tasks such as folding, measuring, or constructing. The teacher ensures that students explore specific concepts. That is, the teacher presents activities in the form of a learning unit to shift learners from one level to the next level (Pegg, 1995). Simple activities involving a rhombus are undertaken. These include folding and reflecting. Students are expected to notice things about the angles, sides and diagonals. In my study simple activities involving circles were undertaken. These included the construction and drawing of circles and their components. Students were expected to identify centres, circumferences, diameters, radii, sectors, tangents, chords, segments, arcs (minor arcs and major arcs) and secants.

Phase 3: Explication: Students describe what they have learned in the topic in their own words. The teacher introduces relevant mathematical terms. This serves to assess learners' understanding of the lesson and/or learning units previously taught so as to assess their progress and to improve their learning. This is done to monitor and support the process of learning and teaching. Students exchange ideas about what they have done and what they have found. They talk about the properties of the rhombus. Mathematical words are introduced to help promote accurate communication, and to clarify various aspects of the students' language. In my study, the use of construction is made to investigate properties of chords, tangents, arcs and angles in a circle. The developments of conjecture, in which students exchange ideas about what they have done and what they have found was promoted. They talked about the properties of the circle. Mathematical words are introduced to help promote accurate communication and to clarify various aspects of the students' language (Pegg, 1995). I, as a participant observer explained and clarified mathematics terms.

Phase 4: Free Orientation: Students apply the relationships they are learning to solve problems and investigate more open-ended tasks. This is to demonstrate learners' insight through the application of their understanding and/or knowledge to the solving of problems, especially non-routine problems. Open-ended tasks are given to the students. They are encouraged to seek their own solution path. For example, some vertices and sides of a rhombus are given and the rhombus has to be constructed. In this study open-ended tasks were given to the students. They were encouraged to seek their own solution path (Pegg, 1995). For example, two non-parallel congruent chords were given and four vertices of the chords at the circumference were drawn and one angle was given and students were expected to calculate the values of the angles in the diagram.

Phase 5: Integration: Students summarize and integrate what they have learned, developing a new network of objects and relations. The properties of the rhombus are summarized and memorized. In my study the properties of a circle were summarized and memorized. Students were expected to integrate all the axioms, postulates and theorems done and to write formal proofs and solve riders.

Van Hiele, in Pegg (1995), offers a more appropriate approach, which would be to replace (say) the rhombus as the centre of attention with more global concepts. In my study I used a variety of activities integrating various axioms, postulates and theorems done in lower classes up to grade 11 circle geometry theorems to explore problems experienced in learning circle geometry. Although the National Curriculum Statement (NCS) does not mention the van Hiele theory specifically, there is a relationship between the NCS and assessment standards for school mathematics and the van Hiele theory (see 2.7).

2.3 HOW CAN TEACHERS ASSESS STUDENTS' VAN HIELE LEVELS?

Many researchers like Mayberry (1983), Wilson (1985), Burger and Shaughnessy (1986), demonstrate methods and techniques that can be used to assess students' van Hiele levels. Mayberry (1983) designed a task using seven geometric concepts: squares, right angled triangles, isosceles triangles, circles, parallel lines, similarity and congruence. He designed the interviews by setting questions, which were validated by 14 mathematicians and mathematics educators who had a special interest and expertise in geometry (Mayberry, 1983). The interviews were on audiotape and a check sheet for rating responses was also used (Mayberry, 1983). I used this strategy in my research (see Appendix E). I adopted Mayberry's format, but formulated my own questions.

Burger and Shaughnessy (1986) administered the experimental tasks to each student in an audio taped clinical interview. The interviews were conducted in a separate room, during the time of the students' mathematics class. Only the students and interviewer were present (Burger and Shaughnessy, 1986). They lasted 40-90 minutes. The interviews consisted of eight tasks dealing with geometry shapes. The task involved drawing shapes, identifying and defining shapes, sorting shapes and engaging in both informal and formal reasoning about geometric shapes. Jaime and Gutierrez (1994) criticize the interviews as a time-consuming tool, which makes the test unsuitable for assessing many people. However, the great advantage of this test is that the information obtained from the interviews results in a deeper knowledge of the way students reason and, therefore, in a more reliable assessment of the van Hiele levels than that obtained by paper and pencil tests. Senk (1985) developed three non-overlapping forms of a test on proofs. Each form contained six items. The first required the students to fill in four missing statements or elements of reasoning in a proof, the second required translation from a verbal statement to an appropriate "figure", and to " know the required to prove" and the last four required the students to write full proofs. Usiskin in Wilson (1985) constructed a test to classify students into five van Hiele levels. The Cognitive Development and Achievement in Secondary School Geometry Project (CDASSS) van Hiele level test was a 25 item multiple choice with 5 foils per items per level. Jaime and Gutierrez (1994) criticize the Usiskin test, claiming that it is based on paper and pencil and there are some doubts about the possibility of measuring reasoning by means of multiple-choice items. They do, however, acknowledge that the multiple-choice test has its main advantage in the fact that it can be administered to many individuals and it is easy and quick to assess a level of reasoning in students (Jaime and Gutierrez, 1994).

Jaime and Gutierrez (1994) propose a framework for designing tests to assess the van Hiele level of reasoning. The framework is based on the consideration of the different key processes involved in each thinking level and the use of open-ended questions (Jaime and Gutierrez, 1994). Jaime and Gutierrez (1994) summarize the key process characterizing the van Hiele levels 1 – 4 as follows: -

Level 1: Identification of the family a geometric object belongs to.

Level 2: Definition of concepts understood from two different points of view.

Level 3: Reading of definitions that is: to use a given definition; and to state definitions that are formulated for a class of geometric objects; and classification of geometric objects into different families.

Level 4: Proof of properties or statement, or ways of convincing someone else of the truth of a statement.

2.3.1 Open ended items for assessing the van Hiele levels.

Paper and pencil-ended items, where the students can freely explain the reason for their answer, are more reliable than multiple-choice items for assessing the van Hiele level of reasoning (Jaime and Gutierrez, 1994). In my view, the best way to assess students' van Hiele levels is to integrate different techniques like tests, clinical interviews, and classroom-based activities, such as questionnaires, worksheets and open-ended questions. That is, the use of a number of different techniques can overcome some of the pitfalls and shortcomings of specific techniques. In the next section, I discuss the teaching implications of the van Hiele theory.

2.3.2 The teaching implications of the van Hiele theory

Teachers face great challenges in teaching mathematics well, particularly geometry. Van Hiele's theory provides reasons why teaching mathematics well is such a difficult task (Pegg, 1995). In order to teach geometry, teachers should have an understanding of the van Hiele theory. They should implement this theory and integrate it with the NCS and assessment standards (see 2.3.1).

2.4 TEACHING IMPLICATIONS OF THE VAN HIELE LEVELS.

Teachers should use baseline assessment to establish what learners already know. That is, teachers should identify the van Hiele level at which the students are. Van Hiele's theory emphasizes that learners who are at lower levels of thinking cannot be expected to understand instructions presented to them at a higher level of thinking (Teppo, 1991). Teachers should, therefore, be able to assess their level of presentation to suit the students' level. They should be able to locate the van Hiele level of each learner in a given topic. This will lead teachers to start instructions from where students understand their work (Pegg, 1995). Teachers should use individual attention to link the pre-existing knowledge of the learner with the newly presented knowledge. Pegg (1995) suggests that "a spiral curriculum should be used." That is, students should practise mathematical problems many times at increasing degrees of sophistication. This requires teachers who are qualified, competent, dedicated and caring (D.O.E., 2002d: 3). Teppo (1991:213) suggests that "students must pass through the learning periods to each level in succession to be able to develop an appropriate understanding of the mathematical concepts expressed at each level." Teaching should be done in a systematic order to scaffold the students sequentially. Teppo (1991) stresses that geometry strands should be effectively integrated during the learning period. Teachers should provide students with all the help they can (Pegg, 1995). They should keep in mind that teaching mathematics lies in the development of insight (Pegg, 1995). In the present South African situation, there is a dilemma because teachers have to teach for examinations on the one hand and for development of insight on the other. There is no relationship between examinations and the next grade. For instance, grade 9 teachers have to prepare students to cope with and /or pass the common tasks for assessment (CTA) as well as to prepare them to cope with the further education and training (FET) level. The same applies for grade 12 teachers who have to prepare students to cope with and /or pass matric examinations as well as to prepare them for the demands made by tertiary education level. Learning should enhance learning. This means high school education should prepare learners for the tertiary education level. At the present moment, there is no ready solution to this dilemma.

2.5 TEACHING IMPLICATIONS OF THE VAN HIELE PHASES

Dina Geldolf-van Hiele developed a teaching approach that allowed students to move from one level of understanding to the next in a continuous process (Freudenthal in Pegg, 1995) (see 2.2.3). The major strength of the van Hiele phases is their link with the level description. This offers teachers a chance to identify clear starting and ending points (Pegg, 1995). The phases can assist teachers to plan and develop learning units in a systematic teaching order.

The van Hiele theory places great importance on the role of language in moving through the levels (Pegg, 1995). The phases provide teachers with a clear approach. The student's own language should be used as the starting point in phase 1. Then as students proceed to advanced levels, more formal language and terminology should be introduced. According to Pegg (1995), there is a need for students, by the end of the topic, to be able to use correct mathematical terminology. That is, students should be guided to master the language appropriate to the subject matter. In summary, the phases advocate a gradual transition from the language of the students to the language appropriate to the subject.

The van Hiele phases separate simple straightforward tasks from those that are more difficult and open-ended. The role of the teacher is to ensure that classroom activities are designed to suit the van Hiele level of students. Then more sophisticated tasks and/or activities should be introduced so as to shift students from the lower levels to the challenging advanced levels 4 and 5. Hence, the role of the teacher is to assist students in finding relationships and links between different solution paths (Pegg, 1995). The van Hiele phases demonstrate means by which teachers could encourage students to have ownership of their mathematics (Pegg, 1995). That is, mathematics educators should guide their students in teaching to investigate and discover solutions on their own. They should not be taught to memorize already discovered procedures. Hershkowitz (1990:73) indicates that "memorisation is not considered to characterize any level." In short memorization is not regarded and/or recognized as meaningful learning. The van Hiele phases allow students the chance to develop ideas through explorations, discussions, sharing, collaboration and negotiations (Etchberger and Shaw, 1992, Pegg, 1995). That is, there is a relationship and a link between the van Hiele theory and outcomes based education as both theories adopt the hands-on enquiry approach.

The van Hiele phases offer teachers a chance to analyse their main teaching strategies. These phases could assist teachers to develop appropriate activities and/or tasks to link the already existing knowledge with the newly presented knowledge. The phases equip teachers to develop many suitable teaching methods, which enable students to develop insight into the mathematics they are taught. The van Hiele phases could help teachers to develop new and different ideas for teaching mathematics, particularly geometry.

2.6 SOME TEACHING IDEAS OF THE VAN HIELE THEORY

In my experience of learning and teaching mathematics, for students to grow in mathematics understanding it is important that they receive a wide range of learning experiences. That is, students should be provided with a variety of mathematical problems to be solved in order to develop meaningful learning. Pegg (1995) suggests some guidelines for implementing the van

Hiele theory when designing learning units. In summary his guidelines for each level are as follows:

Level 1

In geometry, classroom instruction should be logically based on the perception of geometric figures (Pegg, 1995). Geometric figures should be provided in both their standard and non-standard orientations. 'Standard' describes things that "are normal or typical or are most frequently used" (Grearson and Higgleton 1996: 952). For example, perpendicular lines are constructed solely of vertical and horizontal lines and the tangent in a circle is constructed at the bottom part of the circumference and the angle in a semi-circle is drawn in the upper segment of a circle. Pegg (1995) recommends the use of concrete material to highlight different orientations. He further emphasises that teachers must continually reinforce the recognition of a given figure in its different orientations if correct perceptions are to be developed (Pegg, 1995). Pegg (1995) suggests that useful activities include:

- Sorting and classifying shapes: the focus could be on what aspects have determined the classification of various figures: this could involve two-dimensional figures as well
- Identifying small shapes
- Constructing figures using various forms of graph paper: dot and grid paper would be suitable.

In my research I prepared activities for learners to identify a circle among other shapes (see Appendix C)

Level 2

In level 2 (analysis) properties of the figure play a leading role (Pegg, 1995). Pegg (1995) suggests several activities that are appropriate for this level:

- Sorting and classifying shapes in terms of properties. In this study activities from Serra (1997) were given to learners in order for them to discover chord properties, tangent properties, and arcs and angles.
- Identification of shapes given various groupings of properties.
- Algebra exercises involving properties.
- Construction of shapes using geometrical instruments.

In my research, the properties of a circle are established and these properties are not seen to be related (see Appendix B). In this study, several activities were given to students to construct circles with some properties to form conjectures as a way to develop understanding of theorems (see Appendix B). Pegg (1995) focuses his activities on the rhombus, but in this study the focus is on circle properties, circle theorems and the application of the circle theorems to solve riders.

Level 3

At level 3 (abstraction or ordering) the properties are seen to be related and, further, different figures are seen to belong to various classes of figures (Pegg, 1995). Activities for this level include:

- Discussing various descriptions of figures using different figures. For example, a square is a rectangle. In this study the different circle figures, that is circle diagrams for both standard and non-standard figures, are used.
- Creating "family trees" of figures; that is how one figure may be changed to become another.
- Providing missing lines in a short proof.
- Looking for and describing different ways to solve simple deductive tasks (see Appendix C van Hiele level 3 test).

My research explores several activities given to students to develop growth in the understanding of more sophisticated problems. Many circle geometry problems need to be established to encourage student to practise regularly. Much collaboration is needed at this level.

Level 4

At level 4, deduction is understood. Activities for this level include:

- Solving exercises involving deductive reasoning.
- Comparing different solutions to the same deductive exercise or theorem.
- Using and understanding the role of 'if' and 'only if'.

Students at this level should be able to prove theorems using any given diagram and be able to solve riders by applying meaningful understanding of theorems done. In my study, students were expected to apply circle theorems and other theorems, done in the lower classes, to solve riders. At this level, students were expected to integrate their knowledge to get different solution paths.

2.7 RELATIONSHIP BETWEEN NCS AND THE ASSESSMENT STANDARDS FOR SCHOOL MATHEMATICS WITH THE VAN HIELE THEORY.

Due to the legacy of apartheid in South Africa, a new curriculum has been established to address the educational imbalances of the past, and to ensure that equal educational opportunities are provided for all sections of our population (D.O. E., 2002b).
D.O.E. indicates that:

> Outcomes-based education (OBE) forms the foundation for curriculum in South Africa. By setting the outcomes to be achieved at the end of the process, OBE strives to enable all learners to reach their maximum learning potential. The outcomes encourage a learner-centred and activity-based approach to education.
>
> (D.O.E., 2002b:1)

Outcomes-based education, as underpinned by constructivism, encourages a learner-centred approach where learners are encouraged to seek information on their own.

In the NCS the outcomes require learners to be able to reflect on and explore a variety of strategies to learn more effectively (D.O.E., 2002b). The van Hiele theory shares the same principle that effective learning takes place when students actively experience the objects of study in appropriate contexts, and when they are engaged in discussion and reflection (Mason, 2003). Outcomes-based education as well as the van Hiele theory agree that using lectures and memorization as the main methods of instruction will not necessarily lead to effective learning. Pegg (1995) clarifies that rote learning or applying routine algorithms without understanding cannot be regarded as evidence of reaching a certain level of thinking or understanding. That is, rote learning is not an effective step in the learning process. Assessment standards of the Department of Education as well as the van Hiele theory describe the levels at which the learning outcomes should be achieved. In the next paragraph I discuss how the levels of assessment standards used by the NCS relate to those of the van Hiele theory.

2.7.1 How the levels of assessment standards used by the NCS relate to the van Hiele theory.

The NCS specifies the minimum standards of knowledge and skills to be achieved at each grade and sets achievable standards in all subjects (D.O.E., 2002b). Similarly the van Hiele theory has levels of thinking, which specify the minimum standards of knowledge and skills to be achieved at each level (see 2.2.1). I indicated earlier that the van Hiele levels of geometrical thinking are sequential and hierarchical (see 2.2.1). There is a common principle between assessment standards of NCS and the van Hiele theory that educators and teachers should consider the prior knowledge of learners and their development to ascertain learning potential. It is common to the NCS and the van Hiele theory that learners cannot skip a level and grade to master higher levels without first understanding lower levels.

In the NCS as well as in the van Hiele phases there is an emphasis on integration (see 2.3.2). The National Department of Education claims that "in adopting integration and applied competence, the NCS grades 10 to 12 (schools) seeks to promote an integrated learning of theory, practice and reflection" (D.O.E.,2002 b: 2). This is a strategy used in the assessment standard to promote a learner from one grade to the next. The van Hieles proposed the use of activities integrating the levels i.e. levels 1 to 4, when, say, designing the learning units for level 4. This study will be recommending that the Van Hiele theory and outcomes-based approach should be integrated in the teaching/learning situation to overcome difficulties encountered in learning geometry. That is, teachers should implement both the van Hiele theory and an outcomes-based approach in their teaching to achieve the maximum level of understanding to proceed to the next level or grade. Teachers should know the assessment standards as presented in the NCS as well as how to assess students' van Hiele levels.

2.8 WHAT ARE SOME OF THE CAUSES OF DIFFICULTIES IN HIGH SCHOOL GEOMETRY?

The van Hieles did their research in the Netherlands from 1957 to1984. Other researchers in America like Senk (1985) focused their research on the difficulties students encountered with writing proofs.

Senk highlights that:

> The van Hiele model, which posits the existence of discrete levels of geometry thoughts and ideas on how best to help students through levels, has been used to explain why many students have difficulty with the higher order cognitive functions, particularly proof, required for success in high school geometry.
>
> (Senk, 1985: 448)

The purpose of this research study is to explore and tentatively suggest some reasons for the difficulties students in grade 11 experience in circle geometry. Below a brief discussion as to some of the causes of difficulties encountered in geometry is provided.

2.8.1 Primary Phase

Many learning theories emphasize that prior knowledge and/or learning should be taken as foundations for the assimilation of new knowledge. Piaget argues that knowledge is constructed as the learner strives to organize his/her experiences in terms of pre-existing mental structures or schemes (Bodner, 1986). The van Hieles believe that secondary school geometry involves thinking at a relatively high "level" and students have not had sufficient experience in thinking at prerequisite lower "levels" (Fuys *et al.,* 1988). This suggests that the foundation laid for learners in primary school is not necessarily sufficient for them to cope with high school geometry.

Clements and Battista (1992) claim that, in the United States, learners enter high school not knowing enough geometry to undertake high school geometry courses. They also say that "this is caused by the curricula, which consist of a hodgepodge of unrelated concepts with no systematic progression to higher levels of thought, requisite concepts and substantive geometric problem solving" (Clements and Battista, 1992:42). In my view, this quote also applies in South Africa, as the South African mathematics curriculum is not systematic in its approach to building the skills required at higher levels.

According to de Villiers (1997), Russian research done in the late 1960s shows that poor performance in secondary school by students of geometry who had made good progress in other subjects was due to insufficient attention given to geometry in the primary phase of schooling. This suggests that the skills needed in geometry in senior secondary school are not being built in the primary phase. De Villiers (1997) asserts that this also applies in South Africa.

De Villiers claims that:

> The main reason for the failure of the traditional geometry curriculum was attributed by the van Hiele theory to be the fact that the curriculum was presented at a higher level than those of pupils, in other words they could not understand the teacher nor could the teacher understand why they could not understand.
>
> (DeVilliers, 1997:42)

Clements and Battista (1992) suggest that elementary and middle school students in the United States are woefully under-prepared for the study of more sophisticated geometry concepts and proofs, especially when compared to students from other nations. In my view, the above quotation applies in South Africa as well. Hoffer (1981:14) claims that "the van Hiele research indicates that for students to function adequately at one of the advanced levels, they must have mastered large chunks of the prior levels." He further argues that students enter high school geometry courses with insufficient background to enable them to work at level 4. The van Hieles indicated that writing proofs requires thinking at a high level, and that many students need to have more experiences in thinking at lower levels before learning formal geometry concepts (Mason, 2003:4). The primary school geometry curriculum should be designed to prepare learners for secondary school geometry. Freudenthal, cited in Van Niekerk (1995:7) claims that "learners experiences must lead to the development of sufficient geometry knowledge to enable them to cope with advanced geometry in later years." This suggests that geometry should be distributed across all phases, starting from the foundation phase. The learners' current level of cognitive development should be taken into consideration to scaffold learners' understanding, enabling them to reach their full potential. Nickson (2000:50) who asserts that "it is generally accepted that there is a relationship between cognitive variables of a spatial nature and learning related to geometric concepts" supports this. In short, the blame appears to be on the elementary and junior high schools that do not provide the learners with enough geometry background.

The next section focuses on the teaching method as another possible cause of poor understanding of geometry by the learners.

2.8.2 Teaching Methods

Understanding of any classroom lesson presented depends largely on the teaching method used. Teachers should be able to select the most appropriate teaching methods to enable learners to gain insight.

Presmeg (1991) argues that teachers appear to experience difficulty in making links between psychological learning and teaching theories such as the van Hiele theory and actual practice in their mathematics classrooms. Teachers should therefore be empowered to develop and gain insight in the application of cognitive and psychological theories. Sekiguchi (1996) suggests that current instruction processes of mathematical proofs have serious defects, and need to be changed. Some researchers like Senk (1985) indicate that problems with learning proofs in geometry are results of the traditional approach to teaching the subject.

2.8.3 Traditional approach

In my experience, too many teachers tend to neglect geometry in their teaching. The Department of Education (D.O.E., 2002a: 2) examiner's report indicated that "it could be seen from the candidates' performance that very little geometry is taught in some schools and in some others, the way it is taught is not conducive to good learning". Etchberger and Shaw (1992:411) indicate "mathematics at the elementary level has been taught as a series of steps to follow in order to get right answers." They further argue that children memorize certain algorithms in order to solve problems. Skemp in Etchberger and Shaw (1992:411) describes "the memorisation of rules without meaning as instrumental understanding." In this situation, children only know the procedure to be followed without knowing why it is followed. It has been declared that, while learners perform the operations and follow the procedures successfully; substantial evidence and classroom experience indicate that understanding of reasoning for using these algorithms is alarmingly lacking (Etchberger and Shaw, 1992). This suggests that teaching algorithms and procedures often leave learners without understanding.

Pegg (1995) stresses that the purpose of teaching mathematics should be to develop and to promote insight in students. To my mind, the traditional approach characterized by learners reciting the steps to be followed to solve mathematical problems without justification, cannot develop creative, critical thinkers and problem solvers. Pournara (2002) states that the procedural approach to mathematics has received much criticism all over the world. This suggests that a change in the traditional teaching approach is necessary. What should be done? The National Research Council and the National Council of Teachers of Mathematics in Etchberger and Shaw (1992:411) state that "to become a nation, of thinkers and problem solvers, teachers must move away from emphasizing how to as the goal in mathematics (and education in general) and into the why." They further suggest that teachers must reflect on what they are doing in the classroom and they (teachers) must equip children to solve problems, not merely showing them (children) the procedures and algorithms (Etchberger and Shaw, 1992).

The direct teaching of geometry definitions with no emphasis on the underlying process of defining has often been criticised by mathematicians and mathematics educators alike (de

Villiers, 1998). Learners should not passively receive information from the teacher; they should construct their information. Freudenthal (1973: 417) said that "betraying a secret that could be discovered by the child itself is bad pedagogics; it is even a crime." De Villiers (1998) highlighted that to increase students' understanding of geometric definitions, and of the concepts to which they relate, it is essential to engage them at some stage in the process of defining geometric concepts. The teaching of Euclidean geometry generally is dominated by the imposition of the theorems without any attempt to encourage learners to explore and discover conjectures and theorems.

De Villiers suggested that:

> The learners should either retrace (at least in part) the path followed by the original discoverers or inventors, or to retrace a path by which it could have been discovered or invented. In other words, learners should be exposed to or engaged with the typical mathematical processes by which new content in mathematics is discovered, invented and organised.
>
> (De Villiers, 1998:1)

Balacheff in Sekiguchi (1996:243) makes a similar case when he suggests that "the constructivist approach should be used to teach proofs in a geometry classroom."

In my view, at the conceptual level, the integration of the van Hiele theory and teaching approach and the OBE and constructivist approach of the South African classroom, should not be that problematic, because they are closely related in their underlying principles.

2.9 THE CONSTRUCTIVIST APPROACH IN THE VAN HIELE THEORY

In this section I discuss the interrelationship between the constructivist approach and the van Hiele theory.

Osborne and Wittrock in Driver and Oldham said that:

> The brain is not a passive consumer of information. Instead it actively constructs its own interpretation of information, and draws inferences from them. The brain ignores some information and selectively attends to other information...and is much more than a "blank slate" that passively learns and records incoming information.
>
> (Driver and Oldham, 1986:107)

Constructivism, and the van Hiele theory, emphasizes understanding as the purpose of education. Serman in Matthews (1992) suggests that the core epistemological theses of constructivism are:

1. The cognising subject does not passively receive from the environment, but actively constructs knowledge.
2. Coming to know is an adaptive process that organizes one's experiential world; it does not discover an independent, pre-existing world outside the mind of the knower.

Etchberger and Shaw (1992:411) declare that "true knowledge can only exist when it is constructed within the mind of a cognizing being." It has been indicated that a process of knowledge construction begins with sense perception (Etchberger and Shaw, 1992). That is, learners should use the senses of hearing, seeing or reading, touching, tasting, smelling, manipulating, and observing, as these all provide the learner with data. This can be applied to the van Hiele theory which claims that learning begins with recognition or visualisation (van Hiele level 1) (Pegg, 1995 and Mayberry. 1983). Etchberger and Shaw (1992:411) suggest that "cooperative learning is an ideal method that will assist students to construct knowledge and is a powerful tool to enrich learning." Van Hiele theory also encourages cooperative learning. According to Pegg (1995), students should be motivated to exchange ideas about what they have done and what they have found. This suggests that students should collaborate by explaining, justifying and negotiating meanings.

In a constructivist approach the teacher's role is to facilitate learning. The teacher must organise the classroom in such a way that it allows learners to enquire. This can be achieved by posing problems, creating a responsive environment and assisting the learners to achieve autonomous discoveries (Wood, 1988). Teachers should design a variety of activities to provide the student with an understanding of the breadth of the field under study (Pegg, 1995).

Dawe (1995) highlights that in the past, teaching was a matter of the efficient transmission of knowledge from teacher to student, dependent mainly on the mastery of the subject and students' capacity to learn. However, in a constructivist approach through carefully planned experiences, students are encouraged to be active participants in the construction of their own mathematical knowledge rather than passive receivers of the teacher's knowledge (Dawe, 1995).

Driver and Bell in Wood (1988) list five features of a constructivist perspective that could have an impact on teaching / learning situations in schools:

- Learners are not viewed as passive but are seen as purposeful and ultimately responsible for their own learning. Pegg (1995:89) indicates that "a student would exhibit insight when he or she could act adequately with intention in a new situation." He further argues that students must have ownership of their mathematics and be capable of providing answers to questions they have not been given previously (Pegg, 1995). Learners bring their prior conceptions to the learning situation.
- Learning is seen to involve an active process on the part of the learner. It involves the construction of meaning and often takes place through interpersonal negotiation. Pegg (1995) in considering the teaching implications of the properties of van Hiele levels, recommends that teachers should consider student-student interaction and activity-based instruction as an alternative approach to "teacher talk" (Pegg, 1995:95). Thompson cited in Chissick (2004:6) says that "mathematics is a subject that allows for the discovery of properties and relationship through personal inquiry." She also claims that learning through discovery has been one of the cornerstones of mathematics education theory for many years (Chissick, 2004).

This suggests that teachers should promote learning through inquiry as a tool to acquire insight.

- Knowledge is not 'out there' but is personally and socially constructed: its status is problematic. It must be evaluated by the individual in terms of the extent to which it fits with their experience and is coherent with other aspects of their knowledge. This is tantamount to what Pegg (1995) calls a "crisis of thinking" where learners cannot be forced to think at a higher level. This suggests that prior knowledge of learners should be considered as a link between pre-existing knowledge and new knowledge.

- Teachers also bring their prior conceptions to learning situations in terms of not only their subject knowledge but also their views of teaching and learning. These can influence their interaction in the classroom.
- Teaching is not the transmission of knowledge but instead involves the organization of situations in the classroom and the design of tasks in a way that promotes learning. Pegg (1995) in his analysis of the van Hiele theory suggests teaching ideas where teaching and learning with insight is promoted (see 2.5.3). As indicated earlier, the purpose of teaching mathematics should be to develop insight. To my mind the integration of the principles of constructivism within the OBE curriculum with the van Hiele theory can lead us (mathematics educators) to success. Fuys *et al.* (1988:84) claimed that "many failures in teaching geometry result from a language of a higher level than is understood by the student".

2.10 THE ROLE OF LANGUAGE IN THE TEACHING AND LEARNING OF GEOMETRY

The majority of South African high school mathematics classrooms use English as the language of learning and teaching (LOLT). In most schools in the Eastern Cape both teachers and learners do not use their own language for learning and teaching. Setati (1999) confirms this by saying that most schools, teachers and learners are not English first language speakers. The South African Department of Education (D.O.E., 1997b: 12) asserts that "meaning is central to communication." This suggests that the learners' ability to understand depends on the language used in the learning and teaching situation.
Robertson asserts that:

> Language theorists have for a long time postulated that one's ability to make sense of the world, and particularly one's ability to develop concepts and to communicate abstract ideas, is to a large extent determined by how well one is able to manipulate words.
> (Robertson, 2000:1)

Vygotsky (1978) regards language as a tool for thought and as central to learning. Robertson (2000:1) supports the view that "language is a very important determinant of thought structure". She further argues that the more complex a language is, the more complex the thought processes will be (Robertson, 2000). This proposes that language is an indispensable tool in learning and teaching.

In many countries such as South Africa, English has become an official language or lingua franca as well as the medium of instruction and learning in many schools and universities (Young, 1995). It has been indicated that it is difficult to think in an unfamiliar language (Macdonald and Burroughs, 1991). The use of English in many mathematics classroom compounds the problem of learning mathematics. In many schools English is a second language. It is therefore difficult for learners to understand both ordinary English language and mathematics language.

Pegg (1995:88) indicates that "the van Hiele theory places great importance on the role of language in moving through the levels." It is therefore important for mathematics teachers to be aware that they are also language teachers (Dawe, 1995). Teachers should be able to communicate effectively with the learners using both ordinary language and mathematical language. Teachers should know how to use English for mathematical purposes. This knowledge has been handed down over the centuries as mathematics has grown as an academic discipline (Dawe, 1995). Mathematics vocabulary involves many words, which were borrowed from other languages. Hoffer (1981:12) points out that "a geometry course probably stresses the use of language more than any other mathematics course." This indicates that there is a wide vocabulary for the students to learn. The transition from one level to the next requires understanding of basic technical terminology. This includes terminology like parallel, adjacent, perpendicular, bisect, intersect and equal as well as basic logical quantifiers such as 'at least one,' bisect 'each other' and 'concurrent'. Learning and understanding the meaning of these terms is even more problematic for second language learners. This emphasises that teachers should be able to explain the words to help learners gain understanding.

Pegg (1995:98) recommends "that the student's own language should be used as the starting point." This suggests that teachers should introduce mathematics concepts, terms and symbols gradually using code switching.

Dawe emphasized that:

> The need for teachers to be aware of the specific ways they use English for students to gradually express themselves in speech and writing with the vocabulary and symbolism of the mathematics register, cannot be over-emphasized.
>
> (Dawe, 1995:233)

This proposes that teachers should consider the language of learners and link it with the LOLT, which in this case is English. Pegg (1995:98) suggests that "teachers are encouraged to consider the student's language when developing ideas but there is also the need for students, by the end of the topic, to be able to use correct mathematical terminology." In my view the LOLT (English) should be used in formal teaching, but the teacher should also know the level of the learners in order to develop relevant activities to the appropriate language.

James (2000:21) also notes that "English is a global language of communication, literature, science and diplomacy." It is therefore necessary for teachers to use both natural English and mathematical English. The teacher must help students to make mental links between symbolism and the real world through speaking, reading, writing, drawing and acting-out their experiences (Dawe, 1995).

Macdonald and Burroughs indicate that:

> A thorough first language course get children off to a good start in education because the language provides a bridge between the child's home and the demands of the new environment of the school. In their own language, children at school can say what they think if they are allowed to speak the language they know. If children can use their own language, they can express their own ideas; they can be creative. But if children have to learn in a new language, they are put into a kind of prison.
>
> (Macdonald and Burroughs, 1991:3)

They have also argued that children's thinking develops most quickly and easily in their first language (Macdonald and Burroughs, 1991). They further argue that once children are well equipped mentally in their first language they can transfer their skills and knowledge to a second language with reasonable ease.

In conclusion the teacher should scaffold learners by using the relevant language to gain understanding of geometric terminology. The teacher should assist learners through an apprenticeship where they can gain a meaningful understanding of the international language of geometry specifically and mathematics generally.

2.11 CRITIQUE OF THE VAN HIELE THEORY

De Villiers and Njisane (1987) indicate that the use of a hierarchical classification might not be necessary for formal deductive thinking. They also suggest that the van Hiele theory needs refinement with regard to the levels at which deduction can occur, and propose that simpler intuitive deductive reasoning might be possible at levels lower than the abstraction level. They note that there is some confusion within the writings about the van Hiele theory as to where class inclusion is supposed to occur.

De Villiers (1987) suggests that the van Hiele theory uses a limited notion to proof, that is, learners who cannot see the meaning in terms of logical systematisation do not see the meaning of proof. He suggests that, if other meanings of proof were to be used, then this could possibly be done at lower van Hiele levels. Mason (2003) highlights that some mathematically talented students appear to skip levels, perhaps because they develop logical reasoning skills in ways other than through geometry. This contradicts van Hiele model's view that argued that a student cannot achieve one level of understanding without having mastered all the previous levels.

The van Hiele theory's placement of hierarchical classification at level 3 (e.g. square as a rectangle) is not an invariant. There is enough anecdotal evidence that in dynamic geometry, learners can learn to **see** a square as a special rectangle at level 1 simply by the dragging the rectangle until it becomes a square. Mason (2003) points out that in the van Hiele theory students cannot achieve one level of understanding without having mastered all the previous levels. In my experience of learning and teaching mathematics I have noticed that it is possible to prove and apply theorems without knowing short and precise definitions of certain concepts. For instance a student who knows long definitions with apparent unnecessary words (e.g. a rectangle is a quadrilateral with opposite sides parallel and equal, all angles 90^{o}) can understand or solve geometry problems that are situated at van Hiele level 4.

De Villiers reported that:

> Many students, who exhibit excellent competence in logical reasoning at level 3, if given the opportunity, still prefer to define quadrilaterals in partitions. (In other words, they would for example to define a parallelogram as a quadrilateral with both pairs of opposite sides equal, but not all angles equal).
>
> (De Villiers, 1998:6)

Burger and Shaughnessy (1986) have suggested, however, that the levels are not as discrete as suggested by the descriptions. Rather, it appears that learners can be in transition between levels and that they will oscillate between these during the transition period. There is also evidence that a learner's level of thinking might vary according to how recently a topic was studied (Mayberry, 1983; Fuys *et al.*, 1988).

The category of level 1 is the subject of some controversy (Senk,1989: 319). Van Hiele asserts that all students enter at ground level, that is, at level 1, with the ability to identify common geometric figures by sight. Researchers like Usiskin (1982) and Clements and Battista (1992), propose the existence of level 0, which they call pre-recognition. Students at this level notice only a subset of the visual characteristics of a shape, resulting in an inability to distinguish between figures (Mason, 2003: 5).

2.12 CONCLUSION

This chapter gave some background of what other researchers did in the field of geometry. Globally geometry is regarded as a most problematic section in mathematics. Many students find geometry difficult. This chapter discussed the van Hiele theory and its implications in the teaching and learning of geometry. It also presented some of the causes of difficulties in high school geometry. It presented a discussion on the integration of the van Hiele theory with the assessment standards of the NCS. It also analysed the relationship between the van Hiele model and a constructivist epistemology. Some critiques of the van Hiele theory were discussed.

This chapter also presented methods and techniques that can be used to assess student's van Hiele levels. It also provided some teaching ideas of the van Hiele theory. It recommended the use of the van Hiele levels and phases to develop learning units or lesson plans.

This chapter also recommended that teachers should integrate the assessment standards presented in NCS with the strategies of assessing students' van Hiele levels in order to implement the RNCS and NCS effectively.

The literature points out that some of the difficulties in high school geometry can be partly explained by insufficient preparation done in the primary phase. This study, inter alia, will confirm these earlier findings. This suggests that learners enter high school with inadequate knowledge to cope with high school work. The review above also examined the structure of the South African geometry curriculum and suggested that a traditional teaching approach could be another cause of the poor understanding of geometry. It also highlighted the issue of language problems in the learning of geometry. This chapter analysed the constructivist approach relative to the van Hiele model and suggested ways of integrating it into the teaching of geometry.

In terms of the critiques of the van Hiele theory, some research suggested that some mathematically talented students are able to skip levels. This chapter also dealt with the controversy about the van Hiele level 1 category. Some researchers talk of a pre-recognition level 0.

In the next chapter I will describe the research process of my study.

CHAPTER THREE

METHODOLOGY

3.1 INTRODUCTION

This chapter focuses on the methodology used in this research. The methodology is articulated in terms of orientation, design and process. The chapter discusses the interpretive paradigm, the qualitative research method, the case study and the selection of the research site and participants. The tools and techniques used to collect data are explained and clarified.

The research presented is based on work that was carried out over a period of two months (April & May 2003) with a group of 21 grade 11 mathematics learners in a rural senior secondary school in the Eastern Cape. The main purpose of this study was to explore problems encountered in circle geometry by grade 11 learners in relation to van Hiele's theory.

The research presented here, represents an attempt to understand, interpret and make recommendations relating to the problems experienced in circle geometry by grade 11 learners. The interpretation is linked to the learners' background in geometry i.e. their prior knowledge and experience of learning geometry.

3.2 ORIENTATION

This study is located within the interpretive research paradigm. Terre Blanche and Kelly (1999:123) indicate that "the interpretive approach is characterised by a particular ontology, epistemology and methodology". **Ontology** specifies the nature of reality that is to be studied and what can be known about it (Terre Blanche and Durrheim, 1999). The focus of this research is to explore problems encountered in circle geometry by the grade 11 learners. One of the objectives of this research is to compare grade 11 learners' understanding of circle geometry with the curriculum requirements as stipulated in the syllabus (D.O.E., 1997a). Through the exploration of problems encountered in understanding circle geometry I isolate the causes of these and offer some suggestions for improving learners' performance in learning of circle geometry. In an interpretive research paradigm, the given ontology assumes that people's subjective experiences are real and should be taken seriously (Terre Blanche and Kelly, 1999). Through using several tools and techniques to collect data, such as interviews and questionnaire, it became possible to explore the problems experienced by grade 11 learners in circle geometry.

Epistemology specifies the nature of the relationship between the researcher (knower) and what can be known (Terre Blanche and Durrheim, 1999). The majority of the research participants in the sample were my former students in the senior phase. I am well acquainted with their geometry background, with their knowledge and application of geometry in the senior phase syllabus. Using an epistemological approach, researchers can understand people's experiences by interacting with them and listening to what they tell us (Terre Blanche and Kelly, 1999). In the data collection process I interacted with learners in the sample by collaborating with them and negotiating the meaning derived from interpreting the data. Worksheets and interviews were used in the data collection process to interact with the learners as well as to obtain their ideas (see 3.5.5).

Methodology specifies how the researcher may practically go about studying whatever he or she believes can be known (Terre Blanche & Durrheim, 1999). Terre Blanche and Kelly (1999:123) recommend that "qualitative research techniques are best suited" to an interpretive approach. The data collection and analysis was thus qualitative in nature.

3.3 QUALITATIVE RESEARCH

This research deals with the exploration of problems experienced in the learning of circle geometry by grade 11 learners. Interpretive qualitative research relies on first hand accounts, tries to describe what it sees in rich detail, and presents its findings in engaging and in sometimes evocative language (Terre Blanche and Kelly, 1999). In this research my intention is to explore the problems encountered, to describe them in rich detail and to present the findings in an engaging manner.

Sherman and Webb in Southwood indicate:

> Qualitative implies a direct concern with experience, as it is 'lived' or 'undergone'...Qualitative research, then, has the aim of understanding experience.
> (Southwood, 2000:25)

Stakes (1999:39) indicates that "qualitative researchers treat the uniqueness of individual cases and contexts as important to understanding." He further argues that particularization is an important aim, because through it one comes to know the particularity of the case. Exploration of problems in this research focussed on a small number of participants. Jackson (1995:17) claims that "qualitative research is based on a small number of participants, or on an in-depth examination of one group." In this study close attention was paid to one small group of participants in order to provide enough time for discussions. This research is therefore in the form of a case study.

3.4 CASE STUDY

The research design can be described as a single site case study focussing on a group of 21 grade 11 learners from a rural senior secondary school in the Eastern Cape. Typical of interpretive/qualitative methodology, the case study is a "small information-sample selected purposefully to allow the researcher to focus in-depth on issues important to the study" (Cantrell in Southwood, 2000:37).

3.5 RESEARCH SITE AND PARTICIPANTS

On 3rd September 2002 I approached the school principal at the identified site of research and asked him to allow me to conduct research at his school. After we reached an agreement that I may conduct research at the school, we consulted the grade 11 learners in order to choose the sample from amongst them. We discussed the purpose of the research with these grade 11 learners (see1.3). I explained that this research is required for the partial completion of my master's degree. Initially the sample comprised of 30 participants but nine dropped out as there were clashes between data collection sessions and scheduled time for choir practice. The participants' ages ranged from 16 to 21 years. This big range is a result of learners' family backgrounds where some started schooling very late. For instance one learner started schooling at the age of eleven. Two learners in the sample started school at the age of nine. Then, six learners had repeated certain classes in the primary phase. The frequency Table 3.1 shows the differences in the ages in the sample.

Table 3.1 Frequency table showing age differences in the sample

Ages of participants	Number of participants
16	3
17	6
18	4
19	6
20	1
21	1
Total	21

The sample consisted of five boys and sixteen girls. Eight participants were from the 'academic class' (a class with mathematics and physical science as major subjects) and thirteen participants were from the 'commercial class' (a class with accounting, economics, and business economics as majors). They were all studying grade 11 geometry for the first time. The 21 participants were divided into 4 groups. Groups A, B, and C consisted of 5 members and group D consisted of 6 members. No specific criteria were applied to constitute the groups. I simply asked them to divide into 4 groups.

I based the choice of the site of research on the good relationship between the principal (who is also a mathematics teacher) and myself but the school is easily accessible and it is the nearest senior secondary school to my workplace. Convenience, access and geographic proximity should be considered when selecting the site of research (Yin, 1994). Learners' participation in this study was voluntarily.

3.6 RESEARCH ETHICS

I made it clear that for ethical reasons, the participants, the school and its staff are to be kept anonymous.
Christian asserted that:

> Proper respect for human freedom generally includes two necessary conditions. Subjects must agree voluntarily to participate. That is without physical or psychological coercion. In addition, their agreement to participate must be based on full and open information.
>
> (Christians, 2000:138-139)

This was to create a positive climate between the participants and the researcher in this case study.

3.6.1 Confidentiality and anonymity

Pseudonyms have been used for the sake of confidentiality. Confidential information is information, which is intended to be kept secret (Grearson and Higgleton, 1996). All personal data ought to be secured or concealed and made public only behind a shield of anonymity (Christian, 2000). Anonymity is the state of not being known, recognised or identified by name (Grearson and Higgleton, 1996). Lastly we agreed to use 23 afternoon sessions from 14h00 – 15h00. I started on 8th April 2003 and completed the sessions on 29th May 2003, using Tuesday, Wednesday and Thursday each week. I used a variety of methods to gather information.

3.7 STANCE

The term stance is used to refer to the personal 'positioning' of the research participants, of myself as the researcher, and the research relationship between the sample and myself (Southwood, 2000).

3.7.1 Myself as the researcher

In my role as researcher, I adopted what Ely in Southwood (2000:40) describes as an "ethnographic stance, a stance that demands an attitude that puts us into learning roles". As the starting time was 14h00 I had to leave my workplace at 13h30 to reach the research site in time. I had to rush, as I had to travel a distance of about 2km to reach the site. On my arrival I had to request that other grade 11 learners who were not in the sample vacate the room. Throughout the process my role was an interactive one. I facilitated the process as the participants worked with the tools and techniques used to collect the data.

For every session I had to prepare research tools and techniques to be used by the sample to collect data. At times I had to use the chalkboard to write instructions and had to supply learners in the sample with blank A4 paper to do research activities on. This research demanded that I consult many references in order to build relevant tools. For example I made use of Serra (1997) to prepare worksheets and Usiskin (1982) to prepare a questionnaire.

Initially all the participants showed great interest. As we proceeded some tended to dodge some activities, especially the tests. I reported the matter to the mathematics teacher who pleaded with them to write the tests even if they thought they would fail. I photocopied the grade 11 geometry syllabus to show that these activities may help them, as these activities are also part of the syllabus requirements. This was very beneficial as many participants in the sample then developed a serious attitude with more co-operation and concentration.

3.8 DATA COLLECTION METHODS

This part of the chapter focuses on the collection of data. It discusses the tools and techniques used to collect the data. It also justifies and explains the selection of these tools and techniques. I used document analysis, worksheets, participants' observation, a questionnaire, tests and semi-structured interviews to collect data.

3.8.1 Document analysis

I started the data collection by consulting both the grade 11 mathematics higher-grade syllabus and the mathematics draft NCS Grades (10-12) in order to ascertain the expectations of what is to be taught in grade 11-circle geometry. Hitchcock and Hughes (1995:215) emphasise that "qualitative researchers and fieldworkers are likely to pick up a wide range of documents from their research sites." I used syllabi documents with material on the van Hiele model by Pegg (1995), Burger and Shaughnessy (1986), Mayberry (1983) and Fuys *et al.* (1988). It is important for the researcher to be able to immediately recognise the significance of a particular document and what it has to say about a particular research (Hitchcock and Hughes, 1995). Van Hiele's readings were used to check their compatibility with the syllabus requirements. As indicated earlier, the aim of this research is to explore problems experienced by grade 11 learners in circle geometry (see1.4). Van Hiele readings were used in conjunction with the syllabus requirements to design relevant tools and techniques to collect data. Worksheets, a questionnaire, tests and interviews needed to be clearly and closely matched with both the requirements of the syllabus and with van Hiele theory. All these (worksheets, questionnaire, tests and interviews) should be based on the syllabus, that is, what the learners are expected to know. The van Hiele theory emphasises the hierarchical levels to be followed in the teaching of geometry. Research tests were designed to identify the van Hiele levels of the learners. It was therefore necessary for me to be conversant with the van Hiele readings or documents in order to design appropriate research tools. The grade 11 mathematics higher-grade syllabus (1997a) and the mathematics draft (at schools) National Curriculum Statement (NCS) (grade 10-12) (2002b) are attached as Appendix A.

3.8.2 Worksheets and Observations

Three worksheets were administered to the sample (see Appendix B).

The first worksheet was designed to discover **chord** properties of a circle. A constructivist approach was used so that learners could, through an investigative co-operative approach, discover chord properties of a circle and form conjectures. Formation of conjectures leads to the derivation of the theorems. This worksheet consisted of three investigations:

- Investigation 1 was to develop a conjecture that says: if two chords of a circle are congruent, then they determine two central angles that are equal.
- Investigation 2 was to develop a conjecture that says: a line drawn from the centre of a circle perpendicular to the chord bisects the chord.
- Investigation 3 was to develop a conjecture that says: the perpendicular bisectors of the two non-parallel chords that are not diameters intersect at the centre of a circle (see Appendix B).

The second worksheet was designed to discover properties of **arcs and angles** in a circle. It consisted of five investigations numbered as investigation 4, 5, 6, 7 & 8:

- Investigation 4 was to develop a conjecture that an angle in the centre of a circle doubles the inscribed angle if the same arc subtends them.
- Investigation 5 was to develop a conjecture that says inscribed angles subtended by the same arc are equal.
- Investigation 6 was to develop a conjecture that says inscribed angles subtended by the diameter are right angles.

- Investigation 7 was to develop a conjecture that says the opposite angles of a cyclic quadrilateral are supplementary.
- Investigation 8 was to develop a conjecture that says the exterior angle of a cyclic quadrilateral is equal to the interior opposite angle of a cyclic quadrilateral.

The third worksheet was designed to discover **tangent** properties. It consisted of three investigations numbered as investigation 9, 10 & 11.

- Investigation 9 was to develop a conjecture that says a radius is always perpendicular to a tangent.
- Investigation 10 was to develop a conjecture that says an angle between a tangent and a chord is equal to an angle in the alternate segment.
- Investigation 11 was to develop a conjecture that says tangents drawn from a common point outside a circle are equal.

The worksheets were also used during workshops conducted to observe learners' responses throughout the researcher-participants interaction. While participants were engaged in different tasks I observed them. Observation is regarded as the most popular form of collecting data in interpretive research as it takes place while things are actually happening and thus gives the researcher a close-up view of the action (Terre Blanche and Kelly, 1999). I interacted with the learners and facilitated while they were engaged with the investigations. Denzin and Lincoln (2000:18) assert that "qualitative researchers have assumed that qualified competent observers can, with objectivity, clarity and precision, report on their own observations of the world, including the experiences of others." In the process of observation I kept records of what actually happened, that is, I noted in my research journal problems, which were encountered as the sample group worked.

Denzin and Lincoln allege that:

> Qualitative research across disciplines seeks a method that would allow them to record accurately their own observations while also uncovering the meanings their subjects bring to their life experiences. This method would rely upon the subjective verbal and written expressions of meaning given by the individual studies as windows into the inner lives of these persons.
>
> (Denzin and Lincoln, 2000: 19)

I kept on asking questions for clarity and requesting interpretations of the information from the participants. This happened in an informal atmosphere as I engaged in conversation with the participants. Denzin and Lincoln (2000:19) confirm that "no single method can grasp all of the subtle variations in ongoing human experience." Worksheets and observation were used concurrently to collect data in this research. The purpose of these worksheets was to check the conceptual understanding of the participants and their ability to construct circles and its components accurately using ruler, compass, protractor and lead pencil. This also explores their ability to form conjectures that lead to the derivation of the theorems. The theorems used in this research study match those found in the syllabus (1997a) (see Appendix A).

Denzin and Lincoln indicate that:

> … Qualitative researchers deploy a wide range of interconnected interpretive methods, always seeking better ways to make more understandable the worlds of experience they have studied.
>
> (Denzin and Lincoln, 2000:19)

All circle geometry theorems except their converses were covered as prescribed in the syllabus 1997a by the worksheets. In the next section I discuss how the van Hiele tests were used as a technique to collect data in this research.

3.8.3 Van Hiele tests

I designed five tests based on the van Hiele levels (see Appendix C). These tests were constructed to determine the van Hiele levels of the participants. Each test focused on one van Hiele level to explore the problems of the participants in that particular van Hiele level. Through the use of tests, researchers have at their disposal a powerful data collection method for gathering data of a numerical rather than a verbal kind (Cohen, Manion and Morrison, 2000). These tests served as pointers for ascertaining how many learners in the sample were in a particular van Hiele level.

Kanjee alleges that:

> Researchers usually develop questions with the assistance of people knowledgeable in the subject area. With regard to the development of standardised tests, subject area specialists can be enlisted to assist with drafting questions.
>
> (Kanjee, 1999:293)

The ideas for developing the tests were obtained from the work of Senk (1985, 1989), Usiskin (1980,1982), Mason (2003), Burger and Shaughnessy (1986), de Villiers (1998) and Serra (1997). Without the adoption of the test format of the above-mentioned mathematics educators and mathematicians my research work would have been impossible. These tests are criterion-referenced as they provide the researcher with information about what a learner has learned; what he/she can do (Cohen, Manion and Marrison, 2000). In these tests, my interest was to find out on what van Hiele level the participants could be placed. The intention in a criterion-referenced test is to indicate whether students have achieved a set of given criteria (Cohen, Manion and Morrison, 2000).

Test 1 consisted of 10 questions. All questions carried equal marks and the total mark was 30. In the sample of 21 grade 11 learners, 18 learners wrote test 1. Three participants were absent. This test sought to find out whether the sample could differentiate a circle from other shapes (see question 1 of the van Hiele level 1 test in Appendix C). Another aim was to find out whether they could recognise different components of a circle as characterised in the van Hiele level 1 (see Appendix C). Van Hiele level 1 is characterised by visualisation where geometric shapes are recognized on the basis of their physical appearance as a whole (Crowley, 1995). Figures were drawn with the different components of a circle and learners were instructed to choose the correct one from among other components (see questions 2, to 10 in the van Hiele level 1 test in Appendix C). Basic components of a circle were to be identified in the van Hiele level 1 test. These components were a chord, diameter, radius, tangent, semi-circle, sector, arcs (major arcs and minor arcs), secant and concentric circles.

Test 2 consisted of 7 questions, each question carried 3 marks, and the total mark for this test was 27. In the sample of 21 grade 11 learners twenty learners wrote test 2. This test explored and tested both the van Hiele levels 2 and 3. The purpose of this test was to find out whether the grade 11 learners in the sample can recognise components of a circle as well as whether they are able to create meaningful definitions as required at the van Hiele level 3 (see 2.2)
Ohtani in de Villiers argues that:

> The traditional practice of simply telling definitions to students is a method of moral persuasion with several social functions, amongst which are: to justify the teacher's control over the students; to attain a degree of uniformity; to avoid having to deal with students ideas; and to circumvent problematic interactions with students.
>
> (De Villiers, 1998:2)

Vinner and many others as cited in de Villiers (1998:2) have presented arguments and empirical data that "just knowing the definition of a concept does not always guarantee understanding of the concept." It is for this reason that I constructed the van Hiele level 2 and van Hiele level 3 tests using Serra's (1997) ideas to allow learners to construct their own definitions (see test 2 in Appendix C). From the van Hiele theory, it is clear that understanding of formal definitions can only develop at level 3, since that is where students start noticing the inter-relationships between the properties of a figure (de Villiers, 1998). I constructed this test to allow learners to formulate definitions of concepts like diameter, radius, tangent, semi-circle, chord, minor arc, major arc and a secant.

Test 3 explored and tested only the van Hiele level 3. I designed test 3 to explore whether the learners can perceive relationships between properties and between figures as described in level 3 of the van Hiele model (see 2.2). At this level, learners can give information, argue to justify their reasoning and do short deductions (see 2.2.). Test 3 can also be found in Appendix C. The aim of this test was to explore the learners' ability to explore short deductions. This involves the application of axioms, postulates and theorems in the form of short and informal deductions.

Question 1 tested learners' ability to find the value of angle PRS if angle PQS $=110^{\circ}$ (see Appendix C test 3). This question demands application and understanding of the theorem that states that: an angle in the centre doubles the angle in the circumference if the same arc subtends them.

Question 2 tested the learners' ability to deduce that angle BAC $= 90^{0}$ if BC is a diameter. This requires understanding of the theorem that says an inscribed angle subtended by the diameter is a right angle (see question 2, test 3 in Appendix C).

Question 3 tested the learners' ability to apply two theorems:

(a) Base angles of an isosceles triangle are equal.

(b) Angles that are subtended by the same arc are equal.

Question 4 tested learners' ability to see and demonstrate that the opposite angles of a cyclic quadrilateral are supplementary.

Question 5 tested learners' understanding of the theorem that says a radius of a circle is always perpendicular to the tangent.

Question 6 explored whether learners can see that the exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.

Question 7 integrated two theorems

(a) Two radii of the same circle form an isosceles if a chord joins them.

(b) If the radii join two congruent chords then the central angles are equal.

Question 8 explored and tested learners' understanding of the theorem, which states that a line is drawn from the centre of a circle and bisecting the chord is perpendicular to the chord.

Question 9 tested learners' understanding of the theorem that says the tangents drawn from a common point outside a circle are equal.

Question 10 tested learners' understanding of the theorem, which states that a line drawn from the centre of a circle perpendicular to the chord bisects the chord.

Question 11 tested learners' understanding of the theorem that says an angle between a tangent and a chord is equal to the angle in the alternate segment.

Test 4 tested and explored learners' understanding of the proofs of the theorems. Using theorems and their applications to solve riders involves deduction. At this level (van Hiele level 4) learners are required to construct proofs, understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions (see 2.2.1). All 21 research participants wrote this test.

Question 1 required the learners to prove that a line drawn from the centre perpendicular to the chord bisects the chord. Because learners tend to memorise theorems together with the figures, I used my own figures (see test 4 in Appendix C).

Question 2 required learners to use the given diagram to prove that angle DBC = 2 angle DAC (see Test 4 in Appendix C). Learners were expected to prove that in a circle, the angle in the centre doubles the angle at the circumference if the same arc subtends them.

Question 3 required the learners (in the sample) to prove that the angle at the circumference of a circle subtended by a diameter is a right angle.

Question 4 required the learners to prove that the opposite angles of a cyclic quadrilateral are supplementary.

Question 5 required the learners to prove that the exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.

Question 6 required the learners to prove that a radius to a circle is perpendicular to the tangent at the point of contact.

Question 7 required the learners to prove that inscribed angles subtended by the same arc are equal.

Question 8 required the learners to prove that an angle between a tangent and a chord is equal to the angle in the alternate segment.

3.8.4 Questionnaire

A questionnaire is defined as "a group of written questions used to gather information from respondents" (Kanjee, 1999:293). The purpose of the questionnaire in this study was to explore participants' understanding of technical mathematical concepts. This was used to supplement the previously written tests that focussed on the van Hiele levels. Researchers usually develop questions with the assistance of people knowledgeable in the subject area (Kanjee, 1999). In this questionnaire I adopted the format used by Usiskin (1980) in the Cognitive Development and Achievement in Secondary School, Geometry Project (CDASSG). This is a 25 item multiple choice with 4 foils per item per level. My questionnaire consisted of 20 multiple-choice items with 4 foils per item per level (see Appendix D).

The questionnaire was highly structured. This means that learners were to select only the correct answer out of 4 possible given answers. The questions were closed. Closed questions prescribe the range from which respondents may choose (Cohen, Manion and Marrison, 2000). Kanjee (1999:295) says "closed questions do not allow the respondent to provide answers in their own words, but force the respondent to select one or more choices from a fixed list of answers provided."

The questionnaire explored the following technical mathematical concepts.

- Perpendicular
- Diameter
- Circle
- Right angle
- Supplementary angles
- An arc
- Tangent
- Isosceles triangle
- Cyclic quadrilateral
- Bisector angles
- Radius
- Tangent radius relationship in a circle
- Inscribed-centre angle relationship in a circle.
- Axioms of congruency
- Inscribed-diameter relationship in a circle
- The relationship of exterior and interior opposite angles in a circle
- Equal chords – central angles' relationship in a circle
- Chord-tangent relationship in a circle
- Equilateral triangle.

3.8.5 Interviews

An interview may be defined as "an interchange of views between two or more people on a topic of mutual interest to collect data for research purposes" (Cohen, Manion & Marrison, 2000:267). Terre Blanche and Kelly (1999) recommend that, an interview is the most appropriate tool to collect data in the interpretive approach. I designed the interviews to supplement the tests and the questionnaire used earlier to collect data. These interviews were semi structured, this means that questions were set but follow-up questions were also used to get more clarity.

Hitchcock and Hughes suggest that:

> The semi structured interview is the one which tends to be most favoured by educational researchers since it allows depth to be achieved by providing the opportunity on the part of the interviewer to probe and expand the respondent's responses.
>
> (Hitchcock and Hughes, 1995: 195)

It is favoured because the "interviewer asks certain major questions of all respondents, but each time they can alter the sequences in order to probe more deeply" (Hitchcock and Hughes, 1995: 157). The ideas for the interviews used in this study were obtained from the work of Mayberry (1983), Senk (1985, 1989) and Kuchemann (2003). The interviews were audiotape and recorded in a research journal.

It took four hours to conduct interviews with eight selected participants in the sample. The selection of the interviewees was based on their performance in the van Hiele tests, their response to the questionnaire, and their participation in the activities conducted during the process of data collection. Three of the interviewees were the best achievers, three were average achievers and two were the lowest and the second lowest achievers. Only the participants and the interviewer were present in the room.

The purpose of the interviews was to find out more about the understanding of the learners. My aim was to ascertain:

- The work covered in the primary phase of schooling because, as indicated earlier, learners in the primary phase do not appear to receive enough knowledge of geometry to cope with the demands of high school geometry (see 2.8.1).
- The time spent on geometry activities throughout their studies up to grade 11. It has been asserted that very little geometry is taught in some schools (see 2.8.3).
- The ability to translate a diagram or a figure into a verbal statement. Learning of mathematics depends on the language used in the classroom. The role of language is of vital importance. The ability to interpret mathematical symbols into verbal statements needs language understanding (see 2.10).
- The learners' conceptual understanding and their ability to do short deductions. Learners need to understand basic technical concepts in geometry in order to gain meaningful understanding. In chapter 2 I indicated that it is difficult for second language speakers to understand the technical terminology (see 2.10).

At the end of the data collection process I met with all the participants for a general overview of the data collection process. I asked the participants to give an evaluation of all the activities conducted in the data collection process. I did this in the form of an oral discussion. They indicated that they had initially seen the activities as irrelevant to their studies but soon realised the relevance of the activities as the process continued. Their explanations indicated that they felt they had benefited from the activities included in the data collection.

3.8.6 Triangulation

I used triangulation, which most writers on case study methodology consider to be vital to internal validity, especially those studies that seek explanatory outcomes (Hitchcock and Hughes, 1995). In this research only methodological triangulation was applied. This indicates the use, within a data collection format, of more than one method of obtaining information (Hitchcock and Hughes, 1995). The worksheets, observations, questionnaire, tests, and semi-structured interviews were used to explore the conceptual understanding of the sample. Terre Blanche and Kelly (1999) highlight that triangulation entails collecting material on the same aspect in as many different ways and from as many diverse sources as possible. The use of triangulation can, in addition, also help the researcher establish the validity of the findings through cross-referencing; the different perspectives obtained from different sources, or identifying different ways the phenomena are being perceived. The different techniques that were used in this study ascertained learners' understanding of geometry in terms of the van Hiele theory as well as their ability to apply their understanding to solve riders.

3.9 VALIDITY

As indicated above, I used the ideas of other researchers to validate my tools by adopting their format to formulate my own questions. Yin (1993:39) claims that "construct validity deals with the use of instruments and measures that accurately operationalise the constructs of interest in a study." He further argues that, because most instruments and measures are not necessarily as accurate as desired, a common strategy is to use multiple measures of the same construct as part of the same study. For this reason I used many techniques like document analysis, worksheet, observation, tests, the administering of a questionnaire and interviews.

The data collection techniques employed should fit, or be suitable for answering, the research question entertained (Eisenhart and Howe, 1992). One of the goals of this research was to determine the van Hiele levels of geometric thinking required by the grade 11geometry curriculum in South Africa.

For content validity I used Laridon *et al.* (1987) the grade 11-mathematics textbook and Serra (1997) to validate definitions, mathematical concepts and ways of proving theorems. This research focused only on circle theorems.
Cohen, Manion and Morrison claim that:

> For content validity the researcher must ensure that the elements of the main issue to be covered in the research are both fair representation of the wider issue under investigation (and its weighting) and that the elements chosen for the research sample are themselves addressed in depth and breadth.
>
> (Cohen, Manion and Morrison, 2000: 109)

In the worksheets and tests, the focus was to explore conceptual understanding, insight and ability to apply mathematical concepts to do calculations and prove theorems. Worksheets were validated as they were taken from Serra 1997. I adopted the test format used by Senk (1985), Burger and Shaughnessy (1986) and Mayberry (1983). In the questionnaire and interviews, the focus integrated prior knowledge of the sample and their understanding of the circle theorems. To validate the questionnaire I adopted the format used by Usiskin (1982). To validate interviews I adopted the format used by Jaime and Gutierrez (1994).

For external validity I compared my findings with results or findings of other researchers in research conducted within and outside the country. Cohen, Manion and Morrison (2000:109) state that "external validity refers to the degree to which the results can be generalised to the wider population, cases or situation."

3.10 RELIABILITY

Cohen, Manion and Morrisin (2000:117) suggest "reliability is essentially a synonym for consistency and reliability over time, over instruments and over groups of respondents." They further argue that reliability is concerned with precision and accuracy. For this reason I validated my tools by using other researchers' ideas. Bogdan and Biklen in Cohen, Manion and Morrison (2000:119) declare that "in qualitative research reliability can be regarded as a fit between what researchers record as data and what actually occurs in the natural setting that is being researched i.e. a degree of accuracy and comprehensiveness of coverage."

This study has used a range of instruments which have been used and tested by other researchers in this field. Further, the fit between the evidence of these instruments was checked against what was actually occurring in the natural setting through detailed on-site observations of subjects working on the geometry problems. In addition, interviews of selected subjects sought to further confirm the evidence of the other data sources.

3.11 ISSUES

In the process of data collection unexpected problems cropped up. I regarded them as issues (Southwood, 2000).

3.11.1 Discipline

For effective learning, learners should be disciplined to learn successfully and respond to the questions attentively and positively. One of the groups of five had issues with discipline. Their progress was very slow and they failed to concentrate on the task. They ate sweets and they had no mathematical instruments with which to work. They moved around the classroom to borrow mathematical instruments. They were not prepared to sacrifice time by staying for a few minutes after school. I reported the matter to the subject teacher who assisted me by talking to them clarifying the importance of this research in their studies. As a result this group settled down and subsequently focussed on the tasks at hand.

3.11.2 Language

It is important to point out that while English was the medium of communication it was not the first language of all the participants. All the participants were Xhosa speakers, so as result, I felt bound at times to code-switch for reason of clarity throughout the process of data collection. I also advised the participants to express themselves in the language they felt most comfortable in. The conversation between the participants and myself took place in a free atmosphere as I allowed them to speak the language with which they felt comfortable.

3.11.2 Disappointments

On the 28th April 2003 the participants and I took a decision to conduct the research study. I woke full of enthusiasm to work and explore with the group. Unfortunately I had a clash with one choirmaster who decided to involve some of the research participants in music practice. There was no alternative but to return home without doing any tasks. I then decided to proceed with the case study without the choristers. As a result sample group was reduced to 21 by nine participants.

3.11.3 Fears

Because of the loss of these nine participants, I developed a fear that the whole data collection process would be aborted. Then I developed a strategy to include the grade 11 mathematics syllabus in the research in order to show that what we were doing was also a part of the curriculum. This helped a great deal as I noticed that the participants developed a good spirit and attitude to the given tasks.

3.11.4 Finance problem

The materials used to collect data involved some unbudgeted expenses. I prepared material on a computer where I had to pay R5.00 per page. I then photocopied the material to make copies for all the participants plus two more copies for myself as the researcher and the mathematics teacher. This was costly yet unavoidable.

3.12 CONCLUSION

This case study focused on a group of 21 grade 11 geometry learners. In this study I used an interpretive research paradigm and hence I used mostly qualitative research approaches. I used multiple tools and techniques to collect data. These included document analysis, worksheets, observations, a questionnaire, tests and interviews. I validated my tools and techniques by using the work of other researchers.

This chapter also dealt with the research methodology, that is, techniques and tools to collect data. The chapter described the van Hiele tests that were used to determine the van Hiele levels that the participants were at. The chapter ended with a discussion on triangulation. It also highlighted issues of validity and reliability, and articulated some unexpected problems that were encountered.

CHAPTER FOUR

DATA ANALYSIS AND FINDINGS

4.1 INTRODUCTION

This chapter focuses on the process of analyzing data. Taylor and Bogdan (1998:141) state that "throughout analysis, researchers attempt to gain a deeper understanding of what they have studied and continually refine their interpretations." The data analysis presented here represents an attempt to understand, interpret and clarify the data collected for my project by means of the methods explained in chapter 3.

This chapter also analyses documents that were used as tools in the data collection process. These are the South African grade 11 mathematics higher-grade geometry syllabus (D.O.E., 1997a) and the NCS (2002b).
Cantrell in Southwood claims that:

> Analysis involves working with data, organizing it, breaking it down, synthesizing it, searching for patterns, discovering what is important and what is to be learned, and deciding what you will tell others.
>
> (Southwood, 2000: 60)

This research used worksheets to explore the learning process of a sample group. "A key principle of interpretive analysis is to stay close to the data, to interpret it from a position of emphatic understanding" (Terre Blanche and Kelly, 1999:139). Throughout the investigation process (see 3.8.2) I interacted with the sample group by talking to them, and familiarizing myself with their feelings and experiences. I also give detailed analyses of the van Hiele tests, questionnaire and interviews used in the research.

4.2 DOCUMENT ANALYSIS OF SYLLABUS

My starting point is the analysis of the grade 11 mathematics higher-grade geometry syllabus (D.O.E., 1997a). The core focus of this study is grade 11 circle geometry. The initial aim is to ascertain the syllabus requirements for circle geometry. The syllabus requires that all the theorems given in the syllabus must be proved but that only proofs of those denoted with an asterisk (and their converses where mentioned) will be required for examination purposes (see Appendix A). This research deals with the following theorems:

- If two chords in a circle are congruent, then they determine two equal central angles.
- The line segment joining the centre of a circle to the midpoint of a chord is perpendicular to the chord and, conversely, the perpendicular drawn from the centre of a circle to a chord bisects the chord.
- Corollary: The perpendicular bisector of a chord passes through the centre of a circle.
- The angle, which an arc of a circle subtends at the centre, is double the angle it subtends at any point on the circumference.
- The angle at the circumference of a circle subtended by the diameter is a right angle.
- Angles in the same segment of a circle are equal.
- Angles in equal segments of a circle are equal.
- The opposite angles of a cyclic quadrilateral are supplementary.
- The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.
- A tangent to a circle is perpendicular to the radius at the point of contact.
- If two tangents are drawn to a circle through a common point, then the distances between this point and the points of contact are equal.
- The angle between a tangent to a circle and chord drawn from the point of contact is equal to an angle in the alternate segment.

According to the NCS (D.O.E 2002b), these theorems have now been shifted to the grade 12 syllabus, but it had not yet been implemented at the time of the study. The South African geometry syllabus indicates that only proofs of theorems denoted by an asterisk (see Appendix A) will be required for examination purposes (D.O.E., 1997a). This may tempt mathematics teachers not to teach proofs of the theorems that are not required for examination purposes. The same reluctance may apply to the teaching of constructions, because the South African syllabus (D.O.E., 1997a:7) states "no constructions for examination purposes." As a result, teachers often neglect teaching constructions even though they are necessary in an inquiry-discovery-approach to develop insight in the learners (see 2 .9). The syllabus requirement of being able to prove theorems and apply them to solve riders suggests that the learners are expected to perform at the van Hiele level 4 (see 2.2.1).

Senk argues that:

> According to the van Hiele model, only if the student has reached level 4, the level at which thought concerned with such concepts as axioms, the converse of a theorem, and necessary and sufficient conditions will he or she be able to write formal proofs.
>
> (Senk, 1989:310)

In my view, learners in the lower classes often do not receive the relevant geometry knowledge to equip them to reach the van Hiele level 4 in grade 11. A perusal of the mathematics syllabi for circle geometry in the lower classes shows that little attention is given to circle geometry. Also the area of a circle is generally taught in a traditional manner. Regarding the components of a circle, only diameter, radius and circumference are taught in the intermediate phase. These are only taught to determine the perimeter and area of a circle. Very little is done to develop an understanding of circle theorems. As a result, learners tend to enter high school at the van Hiele level 1 with regard to circle geometry. To validate this observation, I used the format of the van Hiele method by Pegg (1995), Burger and Shaughnessy (1986), Mayberry (1983) and Senk (1989).

4.2.1 Discussion of the results of the analysis of the syllabus

The South African grade 11 geometry syllabus requires proof of theorems and their application in solving riders (see 2.1). Although the syllabus requires learners to be taught at van Hiele level 4, indications are that most of the learners in the sample group had only attained van Hiele level 3. Most of them enter grade 11 at van Hiele level 1 in circle geometry. Table 4.1 on page 77 indicates that most of the learners did not know the components of a circle (see 4.6.1). All the circle theorems require understanding of the components of a circle. The theorems deal with the relationship and properties of these components in order to prove the theorems. My view supports de Villiers' (1997) idea that the curriculum is presented at a higher level than that attained by learners (see 2.6.1). I also agree with Clements and Battista (1992) who assert that learners in the elementary and middle school are woefully under-prepared for the study of more sophisticated geometry concepts and proofs. Despite the syllabus requirement that learners should be taught at van Hiele level 4 I believe the evidence of this study showed that the grade 11 learners in the sample are only able to work at van Hiele levels 1, 2 and 3. The van Hiele model stresses that learners cannot skip a level (Pegg, 1995). Therefore, if learners are at van Hiele level 1 or even level 0 they are not ready to be taught at van Hiele level 4.

Teachers tend to rigidly use the syllabus as the guideline for preparing lesson plans. Perhaps they do not use baseline assessment to ascertain or understand the prior knowledge of the learners. One of the traditional teaching principles was that teachers move from the known to the unknown. Teachers tend to focus on what is in the syllabus. That is, they teach geometry content, as it is required by the syllabus. In my view teachers should use the van Hiele levels to link pre-existing knowledge with newly acquired knowledge. This means that if grade 11 learners are situated at van Hiele level 1, then they should gradually be moved from van Hiele level 1 to level 2, 3 and 4. Pegg (1995) and Mason (2003) indicate that learners cannot be forced to think at a higher level. This suggests that to teach proofs of theorems to the learners who are at van Hiele level 1 is not effective and productive, as they probably cannot understand them.

The South African syllabus (1997) says no constructions are required for examination purposes. Perhaps this tempts teachers to neglect the teaching of constructions, which are valuable tools that help learners acquire insight and a strategy to develop and form conjectures.

4.2.2 Structure of the geometry curriculum in South Africa

In my view the South African geometry curriculum lacks a systematic approach to make logical links between the lower grades and the higher grades in the teaching and learning of geometry in high school levels. Clements and Battista (1992) assert that, in the United States, the geometry curriculum consists of a disorganised or confused mixture of different, unrelated concepts. They further argue that "there is no systematic progression to higher levels of thought requisite for sophisticated concept and substantive geometry problem solving" (Clements and Battista, 1992: 42). In my view, this also applies to the South African geometry curriculum.

Grade 9 is currently used as an exit point where learners write common tasks for assessment (CTAs). Both CTA section A and CTA section B tend to neglect geometry or, if there is a question in geometry, it is given a little attention (see 2003 CTA and 2004 CTA as Appendices F and G). As I indicated earlier on, teachers tend to be examinations driven and emphasize in their teaching what is normally examinable. Because the CTAs contain little or no geometry, teachers will probably gradually shift away from teaching geometry and emphasize much of what is contained in the CTAs.

The South African policy document (D.O.E., 1997b), which is currently used in the foundation, intermediate and senior phases, gives too many confusing statements. For example, specific outcome 7 (SO 7) of the range statements (RS) 1.1 and 1.2 gives the instruction: represent objects in various forms of geometry and show links between algebra and geometry (D.O.E., 1997 b). I suggest this instruction is too vague for both foundation phase teachers and for learners. This perhaps confuses teachers who might not know exactly what to teach. Further, it is too complicated for the foundation phase, which has not encountered algebra or geometry; they only study mathematical literacy and numeracy. In specific outcome 2 (SO 2), RS 4.1 says teachers should explore tessellation. In my view learners cannot be taught tessellation without first being taught shapes, as knowing the use of shapes is a prerequisite for doing tessellations.

In the intermediate phase policy document, SO 3 RS 1.2 states: examine the history of measurement and the development of geometry. RS 3.2 says develop and use formulae in measurements in 2D and 3D. The formulae mentioned here were neither developed in the foundation nor later in the intermediate phases. My belief is that learners should be guided on how to derive a formula on their own without necessarily being informed about the application of the formulae to solve mathematical problems. In SO 8, RS 3.1, the performance indicator bullet 2 states: classify figures in terms of congruencies and similarity. In my view, the learning of congruencies and similarities requires understanding of angles, lines and shapes. This 1997b policy document was not designed in a manner to develop a systematic understanding of geometry. Both the foundation and intermediate phase policy documents mention nothing specific about a circle and its properties. This may tempt teachers not to teach a circle and its properties, as there is nothing that stipulates the teaching of a circle and its properties. As a result, learners will probably enter the senior phase at van Hiele level 1 in circle geometry.

In the senior phase policy document SO 3, SO 5, SO 7, and SO 8 deal with the learning of geometry. SO 3, RS 1.2 states: examine the history of measurement and the development of geometry (D.O.E., 1997b). Performance indicators, bullets 1 and 2, state: apply primitive methods used for measuring to solve problems involving perimeters, surface areas, volumes and angle measurements; and apply in a variety of ways the idea and use of angles as portrayed in dancing and other cultural activities. The fact that the policy document states that congruencies and similarities should be taught earlier than angle measurements endorses Clements and Battista's (1992) idea that the curriculum consists of a hodgepodge of unrelated concepts with no systematic progression to higher levels of thought. I would prefer to start with angle measurements in geometry far earlier than the introduction of congruencies and similarities.

It is only in SO 5 of the senior phase policy document where RS 2.3 states: identification, measurement of and use of π, including the circumference and area of a circle. The fact that other components of a circle and their properties are not mentioned perhaps may tempt teachers not to teach them. Therefore, this suggests that learners enter the Further Education and Training (FET) band at van Hiele level 1 in circle geometry. However, the syllabus requires learners to do formal proofs of theorems that are situated at van Hiele level 4.

In my view even the (RNCS) Grades R-9 is haphazard. D.O.E. (2002c: 86) states: classify angles into acute, right, obtuse, straight, reflex or revolution. This follows RNCS (D.O.E., 2002c: 80), which requires that learners be taught quadrilaterals, solids, polyhedra, spheres and cylinders. My view is that in geometry classification of angles should be taught earlier than the introduction of shapes, solids and polyhedra. The South African geometry curricula old and new need to be amended and revisited to help learners to cope with high school geometry. Almost all the geometry curriculum documents ignore circles for the General Education and Training (GET) phase.

4.3 ANALYSIS OF WORKSHEETS

I used informal analysis to analyze the three worksheets. Miles and Huberman in Southwood (2000:61) describe informal analysis as something "which occurred mostly during data collection and at one level may be described as cycles of thought about existing data and generating strategies for collecting further data." The process of observation occurred while the participants were constructing and calculating the problems in the worksheets. The aim was to find out where they encountered difficulties and to discuss and share their shortcomings.

Taylor and Bogdan suggest that:

> The best way to learn inductive analysis is by reading qualitative studies and articles to see how other researchers have made sense out of data. The researcher should study up not to find theoretical frameworks to improve on your data but to learn how others interpret and use data.
>
> (Taylor and Bogdan, 1998: 141)

I used the format of de Villliers (1993) to analyze worksheets and observations.

4.3.1 Analysis of Worksheet 1

The first worksheet consisted of three investigations, which were designed to discover **chord** properties (see 3.8.2 as well as Appendix B).

a) Investigation 1

The aim of investigation 1 was to develop a conjecture that says if two chords of a circle are congruent, then they determine two central angles that are equal (see Figures 4.1).

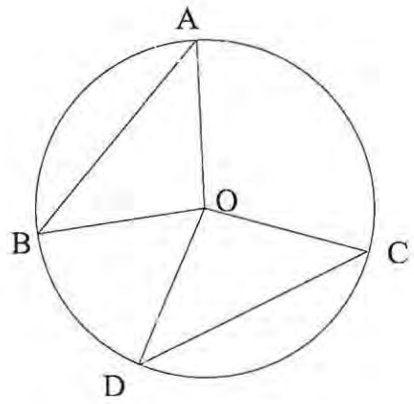


Fig 4.1 Central chord theorem

In Figure 4.1, if chords AB & CD are equal then the conjecture is that angle AOB = angle COD if O is the centre of the circle.

Construction and measurement

Only two learners in the sample had an understanding of the word construct, the others were unable to differentiate between 'draw' and 'construct'. I demonstrated the construction of a circle with the two congruent chords joined by the radius by showing an example to the group leaders who were elected randomly by the group members. One group indicated that they had problems with using a protractor to measure angles.

The RNCS grades R-9 (Schools) (D.O.E., 2002c) indicates that the use of a pair of compasses, ruler and protractor to accurately construct geometrical figures for the investigation of properties and design of nets should be taught in grade 7. The South African Mathematics syllabus (D.O.E., 1997a) requires that construction of regular polygons be taught in grade 7. This involves the construction of angles using protractors and the construction of sides using a pair of compasses.

The fact that these grade 11 learners were unable to construct and measure angles indicates that little emphasis was put on constructions in the senior phase, as required by the syllabus. As a result they entered grade 11 ill equipped for dealing with the grade 11-geometry syllabus. Five participants in group C displayed a lack of accuracy in measuring angles. For example, two angles that measured 84° were recorded as if they measured 85°. Then, as we discussed the problem they realised their mistake and rectified it because the angles around the common point did not add up to 360°.

Findings with respect to investigation 1

All the participants in the sample group could not understand the concept of 'conjecture' and were thus unable to generalize. It was difficult for the sample group to form a conjecture, even though the necessary conditions were sufficient. They stated clearly that they were not familiar with an approach to geometry that requires them to form conjectures. I explained that to make a conjecture is "the process of forming opinions or judgements without having all the facts" (Grearson and Higgleton, 1996:198). In mathematics, to make a conjecture is to generalize without giving a formal proof.

They managed to notice that angle AOB and angle COD share the same magnitude but they could not deduce from this that they are therefore equal. It was only after comparing their results with those of other groups that one learner came up with the idea that the central angles are equal.

There was also a lack of understanding that in an isosceles triangle the base angles are always equal. In fact only three learners in the different groups understood that if the radii of the same circle are equal, this implies that the base angles in a triangle thus formed are also equal. This indicated to me that the learners in the sample group were positioned at van Hiele level 2 (analysis) as they could see equal radii but were not able to deduce that this implies that the base angles are equal (see 2.2.1).

This investigation showed that some learners in the sample group did not have a solid understanding of an isosceles triangle and its properties. Some did not know how to use both a protractor and a compass to construct accurate circles and congruent circumscribed triangles. This suggested to me that some learners in the sample group were not exposed to an investigative, learning approach, which is facilitated by hands-on enquiry.

b) Investigation 2

The aim of investigation 2 was to develop a conjecture that says a line drawn from the centre of a circle perpendicular to the chord bisects the chord.

Construction and measurement

In this investigation only the five participants in group A understood the meaning of the concept 'perpendicular'. This suggests that there is a lack of conceptual understanding among the sample. I explained the concept perpendicular as two lines that intersect to make equal adjacent angles and /or intersect to form right angles around the point of intersection.

One learner in the sample group who understood the concept of perpendicular showed the group leaders how to construct a perpendicular line using a protractor. Thereafter all the participants in the groups managed to draw a perpendicular line from the centre of a circle to the chord.

Findings with respect to investigation 2

A perpendicular from the centre to the chord divides the chord into two equal segments, but the learners were not able to say that the perpendicular line bisects the chord because they did not understand the concept of bisection. At the same time they were not yet familiar with forming conjectures. This reflects a situation that the grade 11 learners in the sample group do not have the necessary vocabulary to acquire knowledge of geometry, as required by the syllabus (see 2.8.1).

c) Investigation 3

The aim of Investigation 3 was to develop a conjecture that says that the perpendicular bisectors of two non-parallel chords that are not diameters intersect at the centre of a circle (see Appendix B).

Construction and measurements

The five participants in group C were unable to construct the perpendicular bisector of the chords. I requested that one participant from another group assist them. In group, D with six participants, they were able to construct and find equal lines from the points of intersection to the circumference but they could not deduce that these lines were radii.

Findings with respect to investigation 3.

The learners in the sample group still struggled to make conjectures, even when the necessary conditions were sufficient and fulfilled. In this investigation, four groups noticed that lines formed from the point of intersection to the circumference are equal but they could not deduce from this that they are radii and the point of intersection is the centre of a circle.

4.3.2 Analysis of Worksheet 2

The second worksheet was designed to discover properties of **arcs and angles** in a circle. It consisted of five investigations, numbered as investigation 4, 5, 6, 7 and 8. (see 3.8.2 and Appendix B).

a) Investigation 4

The aim of Investigation 4 was to develop a conjecture that says an angle at the centre of a circle doubles the angle at the circumference if the same arc subtends them (see Appendix B).

Construction and measurement

As we (the researcher and the sample groups) proceeded with the investigations, learners became familiar with the constructions, although they displayed small problems of inaccuracy. I requested that three of the groups redo their constructions in order to become accurate and to make correct deductions.

Findings with respect to investigation 4

Three groups managed to form conjectures correctly. This means they managed to see that an angle at the centre of a circle doubles the angle at the circumference if the same arc subtends them. One group could not form a conjecture despite having a 120^0 angle at the centre and a 60^0 angle at the circumference. This indicates to me that they are positioned at the van Hiele level 2 (analysis) (see 2.2.1) as they can only see properties but are unable to see relationships.

(b) Investigation 5

The aim of investigation 5 was to develop a conjecture that says that inscribed angles subtended by the same chord are always equal (see Appendix B).

Construction and measurement

None of the participants understood the meaning of the concept inscribed angles. I explained that inscribed angles are angles that are formed by two chords that meet at a common vertex at the circumference of a circle. One group constructed chords inaccurately. As a result they realised that inaccurate construction results in a wrong conjecture.

Findings with respect to investigation 5

Only group C, with its five participants, did not manage to construct angles accurately and, as a result, they did not manage to form a correct conjecture. They did not start the chords from the correct points, that is, their chords were inaccurate. As a result they proved that inaccurate construction results in a wrong conjecture. The remaining three groups with sixteen participants constructed accurately and, as a result, formed correct conjectures. Learners in the sample groups became familiar with the formation of a conjecture and were able to see the relationship between properties. This reflects a gradual shift from the van Hiele level 2 (analysis) to the van Hiele level 3 (abstraction) (see 2.2.1).

b) Investigation 6

The aim of investigation 6 was to develop a conjecture that says an inscribed angle subtended by the diameter is a right angle (see Appendix B).

Construction and measurement

All participants in the four groups constructed a diameter and inscribed angles accurately and correctly. They showed a great improvement in their construction of accurate circle-diagrams. This was the easiest investigation for all of the groups. At this stage they (learners in the sample group) knew that in each and every investigation they had to form a conjecture.

Findings with respect to investigation 6

All participants in the groups easily noticed that all inscribed angles subtended by the diameter are right angles. Learners in the sample groups found this investigation easy, perhaps because of their familiarity with right angles and understanding of the diameter. I indicated earlier in the section on document analysis (see 4.2) that the syllabus requires learners to deal with the diameter, radius, and circumference in the intermediate phase. This investigation showed that learners were familiar with a right angle as well as a diameter. This suggests that perhaps learners can cope well with circle theorems in high school if all the components of a circle are taught in the lower grades.

c) Investigation 7

The aim of investigation 7 was to develop a conjecture that says opposite angles of a cyclic quadrilateral are supplementary (see Appendix B).

Construction and measurement

Initially the learners did not know what an inscribed quadrilateral was. I described it as a quadrilateral with its vertices touching the circumference of a circle. Thereafter the participants were easily able to draw a cyclic quadrilateral.

Findings with respect to investigation 7

The entire sample group managed to notice that the sum of the opposite angles of a cyclic quadrilateral are supplementary. As we proceeded with the investigation, learners in the sample group became familiar with the formation of a conjecture. As a result, they performed better in the last investigations of worksheet 2 than they did in the first ones. My view is that once learners are familiar with certain terms it becomes easier for them to make a conjecture. In the senior phase (grades 7 – 9) learners are taught that supplementary angles add up to 180°. This is applied in the lower grades to calculate angles in a triangle and to find one unknown angle using co-interior angles that are supplementary when two or more parallel lines are cut by a transversal. The learners in the sample here showed transfer of learning. This indicates that once learners become familiar with the technical mathematical terms, they soon grasp procedures used to calculate mathematical problem involving the known terms.

d) Investigation 8

The aim of investigation 8 was to develop a conjecture that says the exterior angle of a cyclic quadrilateral is equal to the interior opposite angle (see Appendix B).

Construction and measurement

This investigation exposed a language problem, as learners did not know that 'ex' means 'out'. I consequently explained that the exterior angle is an angle outside a quadrilateral which is formed when one side is extended. I also did a demonstration on the chalkboard. They then constructed a cyclic quadrilateral with the exterior angle.

Findings with respect to investigation 8

It became easy for all the groups to make a conjecture that the exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.

4.3.3 Analysis of Worksheet 3

A third worksheet was designed to aid the discovery of **tangent** properties. It consisted of three investigations, numbered as investigation 9, 10 & 11 (see 3.8.2 as well as Appendix B.)

a) Investigation 9

The aim of investigation 9 was to develop a conjecture that says a radius of a circle is always perpendicular to a tangent of the same circle.

Construction and measurement

One group struggled to understand that the point of contact is the same as the point of tangency. After an explanation all groups managed to construct a radius and a tangent in a circle.

Findings with respect to investigation 9

All the learners in the groups managed to make a conjecture that a tangent to a circle is perpendicular to the radius at the point of contact. The conjecture was probably easily made because learners had done some tasks in the lower grades involving radii. Secondly this investigation is related to investigation 2, which says a line drawn from the centre of a circle perpendicular to the chord bisects the chord. At the same time they had been using the terms radius and perpendicular many times. This suggests that once learners become familiar with mathematical concepts it becomes easy for them to link already existing knowledge with newly acquired knowledge.

b) Investigation 10

Investigation 10 was designed to develop a conjecture that says the angle between a tangent to a circle and a chord drawn from the point of contact is equal to an angle in the alternate segment.

Construction and measurement

All the groups constructed their diagrams correctly and accurately. This investigation showed a great improvement in the ability of the participants to make constructions.

Findings with respect to investigation 10

This investigation became problematic as learners tended to confuse an angle in the alternate segment with the alternate angle formed when two or more parallel lines are cut by a transversal. I demonstrated this by means of an example and explained how to identify an angle in the alternate segment. At least one participant in the sample group understood it and helped me to explain it to the others. They then formed a conjecture. In this case it was problematic to have the related terms 'an angle in the alternate segment' and 'an alternate angle' refer to the different terms or concepts. This indicates that teachers should differentiate between related terms that are homonyms.

c) Investigation 11

The aim of investigation 11 was to develop a conjecture that says if two tangents are drawn on a circle through a common point, then the tangents between this point and the points of contact are equal (see Appendix B).

Construction and measurement

All the learners in the groups constructed their diagrams correctly and accurately.

Findings with respect to investigation 11

All the groups managed to make a conjecture that says that if two tangents are drawn from a circle to a common point then the distances between the points of contact are equal.

4.3.4 Discussion of the findings with respect to the worksheets

The worksheets were designed to explore the understanding of basic concepts in the learning of circle properties. This involved the learners' ability to form conjectures. The ability to form conjectures was used as a strategy for shifting learners from the van Hiele level 2 (analysis) to the van Hiele level 3 (abstraction). For learners to be able to form a conjecture they should be able to deduce a conclusion from the inferences. The technical mathematical concepts explored in these worksheets were radius, congruent, chord, central angles, diameter, perpendicular, perpendicular bisector, obtuse angle, circumference, arc, inscribed angles, right angle, cyclic quadrilateral, exterior angle, tangent, and alternate segments. This exploration also involved checking learners' ability to use mathematical instruments to construct accurate diagrams. This also enabled one to assess background or prior knowledge of learners in mathematics.

4.4 INSUFFICIENT PREPARATION DURING THE PRIMARY AND SENIOR PHASES

The South African geometry syllabus requires teachers to teach naming and measuring of angles in the primary phase (D.O.E., 1997a). One learner in the sample group indicated that she did not know how to name angles when she used two alphabetical letters to name angles. This suggests that she passed all the lower classes without being able to name angles. In my view any person who is unable to name angles cannot do correct geometry calculations, as we need to notate angles in many calculations. This indicates that not much attention was paid to geometry in the primary and senior phases.

Some learners could not use a protractor to measure angles. This indicates that for these learners no hands-on approach was used to develop the skill of measuring angles. It also suggests that little time was spent in the lower classes to develop insight in the learning of geometry.

The literature review indicates that both learners and teachers see geometry as a difficult section of mathematics. The lack of understanding of these grade 11 learners in this sample suggests that some teachers seem to not pay much attention to the geometry syllabus requirements. Perhaps teachers in the primary phase do not have enough knowledge of geometry to teach successfully.

Geldenhuys (2000) asserts that some primary mathematics teachers do not have the relevant qualifications to teach mathematics. Due to a shortage of qualified mathematics teachers, under-qualified teachers are requested to teach mathematics or are sometimes forced by the authorities to teach it. As a result they spend little time on the teaching of mathematics and tend to avoid geometry as a difficult and/or boring section of mathematics (Pegg, 1995).

4.5 IMPACT OF THE TRADITIONAL APPROACH ON THE TEACHING AND LEARNING OF GEOMETRY

In my view and experience an inappropriate approach is generally used to teach geometry (see 2.8.3). Teachers mostly use a traditional approach, which is dominated by the teacher who tells. In my view, discovering geometry through an inductive approach is preferable to the telling method. Learners indicated that they are not familiar with using protractors, compass, rulers, rubber and lead pencils to discover conjectures in geometry. As a result, some of them could not place the protractor in the correct position to measure angles. In short, they are unable to measure angles. All the learners in the sample group told me that participating in this study was for them the first time that they experienced the formation of conjectures in the study of geometry. A constructivist approach that allows learners to construct their own definitions and discover relationships of the properties of a circle and other geometrical shape had not been used in their earlier learning.

The majority of learners in the sample group could not even use a compass to bisect lines and angles. This could perhaps be ascribed to the traditional approach of teaching where learners are told that to bisect means to divide lines and angles into two equal parts without being given a chance to use a compass to bisect lines and angles on their own. Some of them indicated that they just rote memorized definitions of the components of a circle, as they were unable to identify some components of a circle. They simply recite definitions as they are given in the textbook. My opinion is that teachers should adopt a more constructivist approach as espoused in the new South African curriculum dispensation. However, I support de Villiers' (1998) idea that learners should be exposed to or engage with a mathematical process in which new content in mathematics is discovered, invented and organised.

4.6 ANALYSIS OF THE VAN HIELE GEOMETRY TESTS

The van Hiele geometry tests comprised five tests, which were constructed to determine the van Hiele levels of the participants (see 3.8.3). I used formal analysis and coding to analyze the tests. Southwood (2000: 61) says that "formal analysis occurs mostly after data has been collected and involves sorting the data in some ways in order to make sense of it, to find out what meaning can be made of it." Bryman and Burgess (1994) indicate that codes serve to summarize, synthesize, and sort many observations made out of the data in order to provide a link between data and conceptualization.

4.6.1 Analyses of the van Hiele level 1 test

This section discusses the analysis of the van Hiele level 1 test (see Appendix C). Eighteen participants out of 21 wrote this test. Three participants were absent. This test comprised of 10 questions, each question carried 3 marks, and the total mark for the test was 30. All marks were converted to percentages. The scores are reflected in Table 4.1

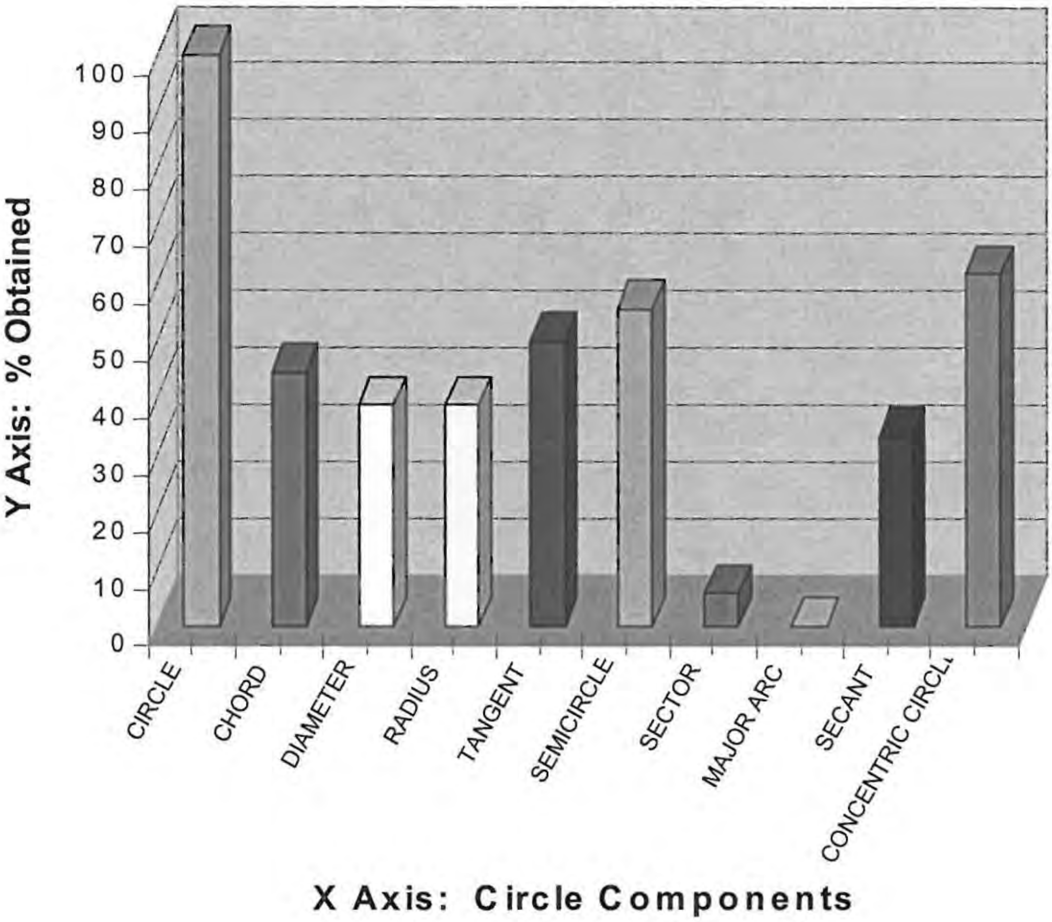
Table 4.1 Frequency table of van Hiele level 1 test

Names of learners	% obtained	Frequency	Cumulative Frequency
L 20	10	1	1
L9, L 13, L 17	20	3	4
L1, L 5, L 10, L 15	30	4	8
L 19	40	1	9
L 7	57	1	10
L 2, L 3, L 6, L 18, L 21	60	5	15
L 12	67	1	16
L 8, L 11	70	2	18
Total		18	

This test was designed to explore learners' understanding of geometry and to determine which of the learners can be placed at van Hiele level 1. The results indicate that 50% of the sample is not even at the van Hiele level 1 with regard to the circle geometry. These learners can differentiate between a circle and other shapes, but they cannot identify components of a circle.

This test tested and explored the extent to which learners were able to identify and recognize various geometrical concepts of the circle such as chord, diameter, radius, tangent, semi-circle, sector, arc, secant and concentric-circles as these are all fundamental components of circle geometry.

Figure 4.1 Histogram indicating learners' performance in circle components



As indicated in Figure 4.1 all the participants managed to identify a **circle** from amongst other shapes.

Only eight participants in the sample groups managed to identify a **chord** among the other parts or components of a circle. This indicates that these learners are not ready to be taught circle theorems, as three theorems in the syllabus require understanding of the chord (see 4.2). This suggests that the syllabus requires that learners be taught at a higher actual level than their level of understanding. In my view, learners should be taught the components of a circle before they are required to prove circle theorems (see 2.1). They also need to be taught circle properties and their relationships.

Only seven participants managed to identify a **diameter** from amongst the other components of a circle. This indicates that although the syllabus requires learners to be taught the various circle theorems, learners do not have the relevant background and enough knowledge to master circle theorems. The syllabus requires learners to be taught the proof of the theorem that states that the angle at the circumference of a circle subtended by the diameter is a right angle. This shows that much preparatory work is needed to assist learners in their understanding of formal proofs of theorems and their applications. The implication is that the South African geometry syllabus should be designed in such a way as to develop the necessary understanding for the learning of geometry.

Only seven participants managed to recognise and identify a **radius** from amongst the other components of a circle. Many theorems in the syllabus require learners to use a radius. To my mind learners cannot understand a theorem if they do not know the concepts that are used in the statement of theorem. This suggests that teachers should use the van Hiele theory to help learners gain insight in the learning of geometry theorems. I indicated in Chapter 2 that the transition from one van Hiele level to the next requires understanding of basic technical terminology (see 2.10). I propose that teachers should focus their teaching on developing a sound knowledge of the basic technical terminology before introducing the formal proofs as required by the syllabus (D. O.E., 1997a).

Nine participants managed to identify a **tangent** from amongst the other components of a circle. A tangent is one of the basic technical concepts that need to be explained before learners are taught circle theorems. This concept needs to be emphasised in teaching practice, as there are two concepts of tangents in mathematics, which are homonyms. For example a tangent in Euclidean geometry is not the same as a tangent in trigonometry (see 2.2.2). The South African geometry syllabus requires that learners know three theorems involving a tangent (see 4.2). This shows the importance of understanding the concept of the tangent before it is used in theorems proofs. This indicates that 50% of learners in the sample need to be exposed to concept development in order to gain an understanding of the theorems involving a tangent.

Ten participants were able to identify **semi-circles** in a circle. Here a language problem was encountered as many did not know that the prefix 'semi' means 'half.' All of the participants could identify a circle but only ten recognized a semi-circle. This also indicates that these circle concepts were not adequately taught in the lower classes. The South African geometry syllabus requires that learners be taught at a higher level, but learners do not have enough knowledge or skill to assimilate lessons presented at the van Hiele level 4 (D.O.E., 1997a). The indication is that teachers should develop the learners' understanding gradually from van Hiele level 1 to van Hiele 4. That is, they should use the van Hiele levels to assist learners to develop an understanding of geometry.

Eleven participants managed to identify **concentric-circles**. This is the only component, which was identified by more than 60% of the learners in the sample.

4.6.2 Analysis of the van Hiele level 2 test

The second van Hiele test tested both the learners' ability to identify components of a circle and their ability to construct their own definitions. This test allows learners to formulate definitions of concepts like diameter, radius, tangent, semi-circle, chord, minor arc, major arc and a secant (see 3.8.3).

The frequency Table 4.2 illustrates the performance of the learners in the van Hiele level 2 test. L indicates learners and D.N.W. did not write.

Table 4.2 Analyses of the van Hiele level 3 test

Learners	Percentage obtained	Frequency	Cumulative Frequency
L17	0	1	1
L5	11,1	1	2
L9	14,8	1	3
L1	22	1	4
L20	25,1	1	5
L21	37	1	6
L2 and L10	44,4	2	8
L19	29,6	1	9
L8	48	1	10
L12	51,9	1	11
L7	55,5	1	12
L6	62,9	1	13
L3	66,7	1	14
L18	92,5	1	15
L4	96	1	16
L11,L14,L15	96,2	3	19
L16	100	1	20
L13	D.N. W.	1	21

The performance of the learners in the van Hiele level 2 test shows that almost half of the learners in the sample group fall below a cumulative frequency of 50%. This indicates that almost all the learners could not construct their own definitions of the components of a circle, nor could they recall what they had learnt from textbooks. This indicates that they have not yet entered van Hiele level 3 (see 2.2.1). The van Hiele level 3 claims that learners should be able to formulate their own correct definitions (see 2.8.3). That they were unable to formulate definitions shows that they were not ready to learn proofs at van Hiele level 4 as required by the South African geometry syllabus (D.O.E.,1997a).

The performance of the learners in the van Hiele level 2 test suggests that learners in the same class may be at different van Hiele levels.

For example learner 17 obtained 0% and learner 16 obtained 100%. Learners 18, 4, 11, 14 and 15 obtained 92,5%; 96%; 96,2%; 96,2%; 96,2% respectively. This can create confusion in the classroom where a teacher may not even know where to start. The cumulative frequency of five learners shows that they fell below 30% while six learners are above 90% (see Table 4.2.). The poor performance is perhaps an indication that learners have not paid attention to the definitions of mathematical concepts, or that not enough time was devoted to baseline assessment in order to ascertain prior knowledge of the learner' grasp of mathematical concepts.

4.6.3 Analysis of the van Hiele level 3 test

The aim of this test was to test the learners' ability to deduce. This involved applications of axioms, postulates and theorems in the form of short and informal deduction (see 2.2.1.) The frequency Table 4.3 shows percentages obtained by the learners in the van Hiele level 3 test.

Table 4.3 Frequency table of the van Hiele level 3 test

Learners	% Obtained	Frequency	Cumulative
L1, L13	5	2	2
L5	27.7	1	3
L17	33.3	1	4
L9 and L19	16.7	2	6
L21	38	1	7
L15 and L8	38.9	2	9
L20	50	1	10
L7 and 10	55.6	2	12
L6	57.9	1	13
L12	64.8	1	14
L2	72.2	1	15
L18 and L16	83	2	16
L3	88.9	1	19
L4 and 14	94	2	21

The findings in the van Hiele level 3 test partly refutes the chronological aspect of the van Hiele theory. That is learners should master Van Hiele level 2 work in order to understand van Hiele level 3 work. For example learner 17 obtained 0% in the van Hiele level 2 test and she obtained 33,3% in van Hiele level 3 test. Learners 20; 2; and 10 obtained 25,1% 44,4% and 44,4% respectively in van Hiele level 2 while they obtained 50%; 72,2% and 55,6% respectively in van Hiele level 3 test. It is easy to notice that they obtained higher marks in van Hiele level 3 test than in van Hiele 2 test. This suggests that perhaps teachers focussed much more on calculations than definitions or learners do not pay much attention to definitions, as it is rare to find definitions in examinations. This may be a reason as to why some learners cannot define components of a circle whilst they see the relationship between the properties of the components of a circle.

Learners' performance in van Hiele levels 2 and 3 tests is not consistent. To obtain a high mark in the van Hiele level 2 test does not necessarily imply that a learner will cope with van Hiele level 3 work. For example learner 15 obtained 96,2% in van Hiele level 2 test and dropped to 38,9% in van Hiele level 3 test. This agrees with Burger and Shaughnessy (1986) who claim that a learner can be in transition between the van Hiele levels. The above frequency indicates that nine participants had no yet attained van Hiele level 3. That is, they could not see the relationship between properties. This shows that they are not yet ready to do geometry at van Hiele level 4. Short deductions are regarded as a prerequisite for the understanding of a chain of deductions, as required by the van Hiele 4. The van Hieles say that learners cannot understand van Hiele level 4 if they have not yet progressed through van Hiele level 3. This means a great deal of preparatory work needs to be done to raise them to the van Hiele level 4.

The frequency Table 4.3 shows that the performances of learners in van Hiele level 3 test range from 40% - 59%. This indicates that while they are in the van Hiele level 3, much effort is needed to move from van Hiele level 3 to van Hiele 4. The investigations in this research project have shown that the sample group cannot understand proofs of theorems and the applications needed for a chain of deduction.

Two participants managed to get an average in the range of 60% to 69%. In my view, these learners should be gradually introduced to sophisticated problems to move them from the van Hiele 3 to the van Hiele level 4(deduction). Much time is necessary to move learners from one level to the next (Pegg, 1995).

Six participants managed to pass within the range of 70% to 100%. In my view, these learners were ready to be taught at the van Hiele level 4. Table 4.4. shows the percentages of learners who were proficient in the theorems, which were explored in the van Hiele level 3 test.

Table .4.4 Proficiency in theorems.

Theorem	**Percentages of sample proficient in theorems**
1. An angle at the center doubles the angle at the circumference if the same arc subtends them.	47,62
2. Angle subtended by the diameter is a right angle.	76,2
3. Angles that are subtended by the same arc are equal	85,7
4. The opposite angles of a cyclic quadrilateral are supplementary.	33,3
5. The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.	61,9
6. A radius of a circle is always perpendicular to the tangent.	87,6
7.If two chords are congruent the central angles formed by the radii joining end-points of the chords are always equal.	23,8
8. A line drawn from the center of a circle bisecting the chord is perpendicular to the chord.	95,2
9. Tangents drawn from a common point outside a circle are equal.	95,2
10 A line drawn from the center of a circle perpendicular to the chord bisects the chord.	61,9
11.An angle between a tangent and a chord is equal to the angle in the alternate segment.	38,1

Table 4.4 shows that theorems differ in their level of complexity. Theorems 8 and 9 were the easiest for the learners as 95,2% of the learners got these theorems correct. This indicates that almost all the learners managed to understand these theorems. Theorem 7 was the most complicated one as only 23,8% of the sample group managed to do the short deduction based on this theorem. Theorem 11 was the second most complicated theorem, which also required short deduction. Only 38,1% of the sample group managed to see and name the angles in the alternate segments. Learners tend to confuse alternate angles that are formed when two or more parallel

lines are cut by a transversal with the angles in the alternate segment when two chords drawn from a point of tangency are joined by another (see question 11 in Appendix C).

Three learners could not differentiate between two consecutive angles in a cyclic quadrilateral and opposite angles. They claim that two consecutive angles of a cyclic quadrilateral are supplementary whereas the correct deduction is that opposite angles of a cyclic quadrilateral are supplementary. The fact that the consecutive angles of a parallelogram are supplementary causes this confusion. One learner mistakenly could not get a right answer as a result of wrong subtraction.

For theorem 1, three learners said that an angle at the center is equal to the angle at the circumference if the same arc subtends them instead of saying that an angle in the center doubles the angle at the circumference if the same arc subtends them. Another three learners treated these angles as if they are supplementary. To find the unknown angle at the circumference they subtracted the angle at the center from 180° and as a result, their answer was 70°. At least all the participants knew that there is a relationship between the angles although they were not certain of the exact relationship. This indicates that they were at least at van Hiele level 2 (see 2.2.1). The fact that they knew one thing implies another indicates apprehension know-how of van Hiele level 3.

With regard to the remaining theorems, numbered 2, 3, 5, 6 & 10 learners indicated an understanding and a readiness to proceed to van Hiele level 4 with the exception of a few individuals who had no facility for finding the relationship between properties. The percentages obtained in these theorems tests indicate that only few learners do not understand these theorems.

One surprising finding is that grade 11 learners use two alphabetical letters to name angles. My view is that these learners must have passed lower classes' examinations without passing mathematics. This also shows that their junior secondary and senior primary school neglected the teaching of geometry. The South African geometry syllabus indicates that the naming of angles should be taught in the intermediate phase (grades 4 to 6) (D.O.E., 1997a).

4.6.4 Analysis of the van Hiele level 4 test

The aim of this test was to explore the learners' ability to prove theorems in relation to the van Hiele level 4 (see 2.2.1). All 21 research participants wrote the test. The theorems explored are as detailed in Appendix C (see also 3.8.3). The frequency Table 4.4 shows the percentages obtained by the learners in the van Hiele level 4 test.

Table 4.5 Frequency table of the van Hiele level 4 test

Learners	% obtained	Frequency	Cumulative frequency
L1, L17	5	2	2
L21; L9; L12; L20	10	4	6
L6; L13	12,5	2	8
L5; L7; L19	13,8	3	11
L8	8,8	1	12
L15	28,8	1	13
L3; L4	31,3	2	15
L10	35	1	16
L14	38	1	17
L2	46,3	1	18
L18	63,8	1	19
L11	72,5	1	20
L16	97,5	1	21

This suggests that learners 18, 11, and 16 were at van Hiele level 4. Learner 16 was exceptional as he obtained 100%; 83%; and 97,5% in tests 2, 3 and 4 respectively. He did not write test 1 as he was not present when the test was written. Learner 11 obtained 70%; 96,2%; 64,8%; and 72,5% in the van Hiele level 1, 2, 3 and 4 tests respectively. Learner 18 obtained 60%; 92,5%; 83% and 63,8% respectively in the same series of tests. Their outstanding performance suggests that they are ready to be taught proofs of circle theorems.

In the van Hiele level 4 test a cumulative frequency of almost 86% of the learners in the sample group fell below 50%. This suggests that most of the learners in the sample who enter grade 11 are not yet ready to be taught circle geometry at van Hiele level 4. That is, they cannot cope with van Hiele level 4 circle geometry work. In the van Hiele level 4 test only four learners in the sample group showed consistency with the syllabus requirements.

Some learners showed consistency in these van Hiele tests. For example, learners 1; 13; and 17 performed badly in all the van Hiele tests. Learner 1 obtained 30%; 22%; 5%; 5%; in tests 1; 2; 3; and 4 respectively. Learner 13 obtained 20%; D.N.W.; 5%; and 12,5% in tests 1; 2; 3; and 4 respectively. Learner 17 obtained 20%; 0%; 33,3%; and 5% in the van Hiele level tests1; 2; 3; and 4 respectively. These learners fell below 40% in all the van Hiele tests. This indicates that the performance of these learners is not compatible with the syllabus requirements. This suggests that these learners perhaps had a poor geometry background and were ill prepared to cope with grade 11 circle geometry. Learners 3; 4; 6; 7; 12; and 14 performed well in van Hiele level 1; 2; and 3 tests but all fell below 40% in van Hiele level 4 test. This suggests that there is a big gap between van Hiele level 3 and van Hiele level 4. This probably supports Hoffer's (1981) claim that the van Hiele research indicates that for students to function adequately at one of the advanced levels, they must have mastered large chunks of the prior levels. Pegg (1995) confirms this, as he claims that many sophisticated activities and exercises should be given to the learners to shift them from one level to the next.

Table 4.4 shows that 80,9% of the sample group were not yet proficient at the van Hiele level 4. Only one participant obtained a range of 40% - 59% and another one obtained a range of 60% - 69%. The remaining two participants were outstanding as they obtained 72,5% and 97,5%.

Seventeen learners did not even understand how to start proofs of theorems as their performance shows low percentages obtained in the van Hiele level 4 test. According to the D.O.E.,1997a geometry syllabus theorems are supposed to be first presented in grade 10. The learners' performance in this study probably indicates that they had never done theorems before, with the exception of two participants. This suggests that geometry is neglected not only in the junior secondary and primary phase but also in high schools.

Seven learners could not translate a verbal statement into a diagram. For instance, question 3 required learners to draw their own figures or diagram to prove that an angle at the circumference of a circle subtended by the diameter is a right angle. They could not draw a correct diagram deduced from the given statement.

Five participants could not write the standard format of proving theorems as:

Given:
Required to calculate:
Construction if necessary:
Proof:
Statement:
Reason:

I suspect that the theorems were memorized by rote, as correct statements supported by incorrect reasons were given and there was no logic in many of the proofs. During some of the stages of the van Hiele level 3 test the learners made statements without supplying reasons.

There were times when the learners confused the given information with that which is required to prove. At some stages they wrote required to prove as a given statement, and at other stages wrote given as required to prove. This indicates that this section of geometry (proofs) had not been done thoroughly in the classroom.

The analysis of the van Hiele level 4 test indicates that much work needs to be done to move learners from van Hiele 3 to van Hiele level 4. Teachers need to dedicate more time on the translation of verbal statements to diagrams and vice versa. Mason (2003) indicates that progress from one level to the next is more dependent on educational experiences. Many activities and exercises should be practiced to gradually move learners from van Hiele level 3 to van Hiele level 4.

4.7 ANALYSIS OF A QUESTIONNAIRE

The purpose of the questionnaire in this study was to explore the learners' understanding of technical mathematical concepts (see 3.8.4). The frequency table 4.5 shows the percentages obtained by the learners in the questionnaire.

Table 4.6 Frequency table showing learners performance in a questionnaire

Learners	% obtained	Frequency	Cumulative Frequency
L19	25	1	1
L1; L17	30	2	3
L10;L12; L13	35	3	6
L18; L9: L11	50	3	9
L5	55	1	10
L21	60	1	11
L20	65	1	12
L15	70	1	13
L6	75	1	14
L2; L3	80	2	16
L4	85	1	17
L18; L14; L7	95	3	20
L16	100	1	21

Because the literature indicates that the transition from one level to the next requires understanding of basic mathematical technical terminology, this questionnaire was designed to explore understanding of technical mathematical concepts by the learners' in the sample group (see 2.10). This questionnaire also supplemented mathematical technical concepts that were not covered in the van Hiele tests 1, 2, 3 and 4.

Table 4.5 indicates that the cumulative frequency of 10 learners in the sample group fell below 60%. In my view learners need to understand technical mathematical concepts before they are taught formal proofs of theorems. The fact that almost half of the sample group showed poor understanding of technical mathematical concepts indicates that they were not yet ready to be taught circle theorem proofs. According to Pegg (1995) learners who have not yet acquired sufficient technical mathematical concepts cannot shift from van Hiele level 3 to van Hiele level 4.

Table 4.5 shows that five participants did not have sufficient understanding of technical mathematical concepts. As a result it can be difficult for them to understand theorems involving the concepts, as they need to understand these concepts in order to have an insight into the theorems. This also perhaps reflects that teachers did not give enough time to the development of these learners' understanding of technical mathematical concepts or that learners did not pay enough attention in learning of mathematical concepts.

Mathematics vocabulary needs to be developed to help learners gain insight in the learning of geometry (see 2. 10). In short, those participants who obtained less than 40% in the questionnaire were not ready to be taught theorems at van Hiele level 4.

Six participants obtained percentages in the range of 50% - 69%. This indicates that the vocabulary and symbolism of the mathematics register needs to be developed in these participants (see 2. 8). The percentages obtained indicate that while they were ready to be taught proofs of theorems, emphasis should be placed on the understanding of technical mathematical concepts. The poor performance of learners in technical mathematical concepts suggests that teachers do not regard understanding of concepts as a prerequisite for the understanding of proofs in theorems. As a result they do not devote enough time to the teaching of mathematical concepts. Concepts are only taught in an integrated manner during presentation or the teaching of theorems.

The results of the questionnaire indicate that the nine participants who obtained outstanding percentages had enough knowledge of mathematical technical concepts. Their results were: 70%; 75%; two at 80%, one at 85%, three at 95% and one at 100%.

In my view mathematical technical concepts should be treated as the cornerstone for understanding the proofs of theorems. For learners to have a meaningful understanding of theorem proofs, they need to master mathematical concepts. Table 4.7 indicates percentages of learners who mastered each concept.

Table 4.7 Learners performance in conceptual understanding

Concept	**Percentage of learners who indicated understanding of each concept.**
Perpendicular lines	28,6
Diameter	80,9
Right angle	66,7
Supplementary angles	95,2
An arc	61,9
Tangent	66,7
Isosceles triangle	75,4
Cyclic quadrilateral	66,7
Bisector angles	52,4
Radius	66,7
Tangent-radius relationship in a circle	42,9
Inscribed-centre angle relationship in a circle	42,9
Axioms of congruency	66,7
Inscribed-diameter relationship in a circle	61,9
The relationship of exterior & interior opposite angles in a circle	66,7
Equal chords-central angles relationship in a circle	85,7
Equilateral triangle	66,7
Properties of isosceles triangle	38,1

Table 4.7 indicates that these learners also had a poor understanding of some concepts, for example only 28,6% of the sample grasped the concept of perpendicular lines. Three of the theorems in the 1997 South African geometry syllabus require an understanding of perpendicular lines (see 4.2). They also performed badly in the concept test for a chord -tangent relationship in a circle. Table 4.6 indicates that only 28,6% of the sample group understood the chord-tangent relationship. That only 38,1% of the sample group know the properties of an isosceles triangle indicates that the properties of a triangle were not taught thoroughly in grades 7 – 9. The tests indicate that the majority understood the concept of a right angle as 95,2% of the sample group

recognized a right angle. In short, the questionnaire indicates that learners in the sample group need to develop an understanding of geometrical concepts and the components of the circle.

4.8 UNDERSTANDING OF MATHEMATICAL TECHNICAL CONCEPTS

Pegg (1995) indicates that each van Hiele level has its own vocabulary and language. Learners need to know certain concepts and certain symbols in order to shift from one level to another. As indicated in Tables 4.2 and 4.6, very few learners understood concepts such as perpendicular, arc, chord, and tangent.

Teachers should use the syllabus to tabulate the required mathematical terminology and compile a concepts glossary and give learners enough time to develop an understanding of mathematical technical concepts. It seems as if teachers do not give lessons that are designed solely to develop understanding of concepts and technical mathematical concepts.

Serra (1997) indicates that many activities could be used to develop understanding of geometrical concepts. This can only be achieved if teachers can shift from the traditional teacher-tell approach to a discovery approach, as proposed by constructivism (see 2.9).

Teachers should be informed of the van Hiele theory, as this will help them to develop and attain a more relevant approach for the teaching and learning of geometry. To make this possible, the van Hiele theory could be introduced to the teachers through in an in-service workshop, as not all teachers are able to further their studies in mathematics education. The mathematical technical concepts that need to be taught in order to shift learners from van Hiele 2 (analysis) to van Hiele level 3 (abstraction) are indicated in Chapter 3 (see 3.8.4 and Appendix D).

4.9 ANALYSIS OF THE INTERVIEWS

The purpose of the semi-structured interviews was to ascertain the geometry background of the learners. My aim was to discover:

- The work covered in the primary phase.
- The time spent in geometry activities throughout their studies up to grade 11.
- The ability to translate a diagram or a figure into a verbal statement.
- The learners' conceptual understanding and their ability to do short deductions (class inclusion).

This analysis was qualitative and exploratory in nature (Mayberry 1993). Eight learners in the sample group were selected to participate in the interviews. Only one interviewee claimed that she did shapes in grades 4 and 5. It is transcribed as follows:

Researcher: Phem in which grade did you start learning geometry?
Phem: In grade 7.
Researcher: Did you not do anything in geometry in the primary phase?
Phem: We did shapes like triangles, squares rectangle in grade 4 and 5 and then we did angles in grades 6.
Researcher: Did you do some calculations in geometry?
Phem: No

Almost 87,5% of the interviewees (7 out of 8 interviewees) claimed that they only started geometry in grade 7. In other words, seven interviewees indicated that they did not geometry in the primary phase. This supports de Villiers' (1997) claim that poor geometry performance in secondary school by learners who had made good progress in other subjects was due to insufficient attention given to geometry in the primary phase of schooling (see 2.6.1). One learner confirmed that she never did geometry in the senior phase, and had started it only in grade 10. For example Pretty said in the interview:

Researcher: In which grade did you start learning geometry?
Pretty: In grade 10
Researcher: Which sections did you learn?
Pretty: Triangles and quadrilaterals.
Researcher: In which grade did you start learning theorems?

Pretty: In grade 10.

Researcher: Did you do circle theorems in the lower classes?

Pretty: No.

Researcher: In which grade did you first learn circle theorems?

Pretty: In grade 11

Pretty claimed that she never learnt geometry in the lower classes. Situations like this need serious attention, not only by the teachers but also by administrators, principals, subject advisors and curriculum managers.

Six interviewees in the sample group were unable to translate a diagram into a verbal statement

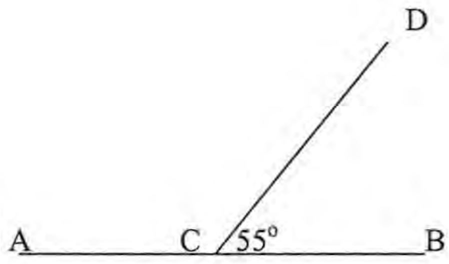


Figure 4.2

This indicates that learners in the sample group had not yet reached van Hiele level 4 and they were thus not ready to be taught circle theorems at van Hiele level 4.

In my view, the understanding of theorems is largely dependent upon the ability to translate a diagram into a verbal statement or interpret a verbal statement into a diagram. Six interviewees were unable to see and say that in Figure 4.2, DC meets AB at C and angle BCD = 55° and therefore what is the required to calculate is angle ACD. For learners to be able to understand problems providing theorems, they should be able to interpret the diagram in such a way that they see given information and are able to identify missing information, in this case the angle that needs to be calculated is angle ACD.

Only one interviewee out of eight interviewees could not understand or identify a diameter. She just said it is a straight line. Learners need to be exposed to the process of conceptualization in order to develop a meaningful understanding of technical mathematical concepts.

One interviewee was able to identify that an angle in a semi-circle is a right angle, but she could not furnish a correct reason. Perhaps she got the identification correct through guessing. One learner could not even realize that an angle in a semi-circle is a right angle; she instead said it was 180°. The remaining six interviewees identified that an angle in a semi-circle is a right angle. This indicates that these interviewees were only at van Hiele level 3 (abstraction) in this postulate. Two interviewees could not see the relationship between an angle at the centre of a circle and an angle at the circumference subtended by the same arc. They could not say that an angle at the centre of a circle doubles the angle at the circumference if the same arc subtends them. The other six interviewees could see the relationship between the angle at the centre of a circle and an angle at the circumference if the same arc subtends them.

Teachers tend to use one form of a diagram to teach and test this theorem (see Figures 4.3 a, b, and c)

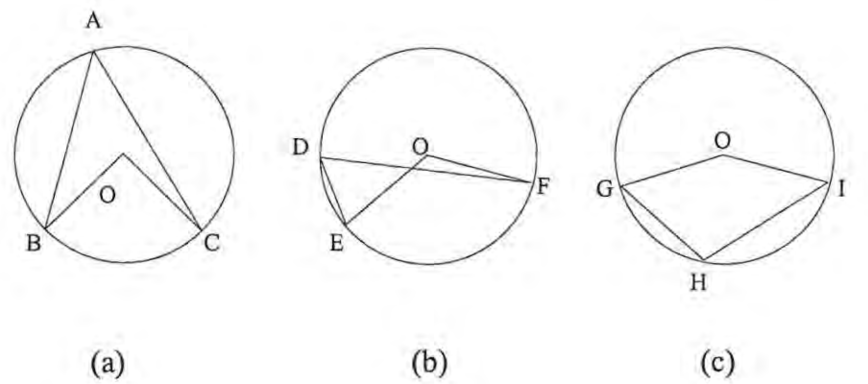


Figure 4.3

Teachers tend to use Figure 4.3a only to teach and test the understanding of the theorem that says an angle at the centre of a circle doubles the angle at the circumference of a circle if the same arc subtends them. Learners therefore tend not to see that angle FOE = 2 angle FDE in Figure 4.3b. It is hard for the learners to see that reflex angle GOI = 2 angle GHI. This suggests that teachers should use different orientations to teach geometry to familiarize learners with the different sophisticated activities and exercises. It was noticed in the interviews that some learners tended to feel comfortable when the diagram is familiar and depend greatly on this in order to make appropriate deductions.

This further confirms that the kind of foundations being laid at the primary and senior phases is inappropriate or inadequate. I believe that, in part, the fault lies with the teachers, as geometry is included in the primary syllabus. According to the National Department of Education syllabus learners are supposed to enter grade 7 at the van Hiele level 2, where all shapes and suitable vocabulary should have been covered in the primary phase (D.O.E., 1995).

The interviews showed that even in the senior phase geometry was neglected, as the majority of the interviewees were unable to translate a simple diagram into a verbal statement. For the past 20 years the translation of diagrams into a verbal statement in grade 8 has been in the syllabus, yet the grade 11 learners in this sample were unable to do it. This indicates that in practice little time was spent on teaching geometry in the senior phase.

All the participants interviewed were unable to define a theorem, even though they dealt with theorems several times. This indicates that geometry was not taught with insight. To my mind it is not wise to teach formal proofs of theorems without understanding the definition of a theorem (de Villiers 1987). Teachers should know that learners will not be able to master formal proofs of theorems if little geometry has been done in the lower classes. Hoffer (1981:14) indicates that "for students to function adequately at one of the levels, they must have mastered large chunks of the prior levels." This implies that much work needs to be done in both the primary phase and the senior phase to enable students to cope with high school geometry work. Hoffer (1981: 14) suggests that "students need geometry to have informal experiences prior to the introduction of formal proofs."

Many interviewees knew what a diameter was, or that an angle in a semi-circle is a right angle. The majority of the interviewees saw the relationship between an angle at the centre and an angle at the circumference subtended by the same arc.

During these interviews learners indicated that they have an understanding of circle theorems, but it was obvious they were still only at the van Hiele level 3. They were only able to do short deductions.

4.10 CONCLUSION

In this chapter I outlined my data analysis and gave details of the relationship of the syllabus requirements to the van Hiele levels. The current syllabi (D.O.E., 1997a, NCS 2002b) require grade 11 learners to be at van Hiele level 4.

As the purpose of this study is to explore problems encountered by grade 11s in circle geometry, I used worksheets to explore the learners' performance. The worksheets dealt with construction and measurement, accuracy and deduction. In the analysis of the worksheets the learners' responses and performance indicated a lack of conceptual understanding. They also showed insufficient preparation during the primary and senior phases. The fact that learners in the sample group were unable to construct and even position protractors to measure angles indicated insufficient preparation done in the primary and senior phases.

This chapter also postulated that poor performance of learners in geometry was possibly as a result of the traditional approach to the teaching and learning of geometry. The fact that learners were not given chances to explore, investigate and discover, and make conjecture restricted them in the learning of mathematics.

The test results show that learners in the sample group are at different van Hiele levels and, at the same time, have problems in conceptual understanding. I have indicated that theorems differ in their level of complexity. As a result, complicated theorems required much attention and teaching time. I also indicated that there is a big gap between van Hiele levels 3 and 4. A great deal of work needs to be done to prepare learners to move from van Hiele level 3 to van Hiele level 4.

In the questionnaire and interviews the responses confirmed that the sample had not received enough knowledge during the primary phase to cope with the high school geometry syllabus. The syllabus requires proofs of theorems and most of the learners in the sample are not yet ready to be taught at van Hiele 4.

CHAPTER FIVE

CONCLUSION

5.1 INTRODUCTION

This chapter provides the conclusions of the whole research project. It includes a summary of the research findings and highlights the significance of the study. It also presents some recommendations, and articulates limitations and avenues for further research. It ends with a personal reflection.

5.2 SUMMARY OF FINDINGS

Chapter four of this study gave details of the research findings. The findings discussed the structure of the geometry curriculum in South Africa, analysis of the syllabus, and insufficient preparation during the primary and senior phases. It also provided a brief overview of the impact of the traditional approach on the teaching and learning of geometry. It also covers the understanding of mathematical technical concepts and language problems.

5.2.1 Curriculum and syllabus

This study endorses other researchers' work like Clements and Battista (1992) who claim that the geometry syllabus consists largely of a disorganized or confused mixture of different, unrelated concepts (see 4.2.3). The fact that the syllabus has no apparent systematic progression to higher levels of thought makes geometry a complicated section of mathematics to learn and teach.

De Villiers (1997) indicates that geometry tends to be overloaded at the high school level in the South African geometry curriculum. This suggests that very little is done to develop an understanding of circle theorems in the lower classes. The South African geometry syllabus has other limitations (D.O.E., 1997a). There are cases where some aspects like constructions are not meant for examination purposes. As a result teachers tend to neglect this section in their teaching.

5.2.2 Syllabus and van Hiele theory

The current South African geometry grade 11 syllabus requirement is such that learners should be taught at van Hiele level 4 (see 2.1). This study indicates that most of the grade 11 learners in the sample group are only able to work at van Hiele levels 1, 2 and 3. Learners attain a lower level than the requirements of the curriculum. They do not reach the curriculum requirements. This study confirmed sentiments expressed by de Villiers (1997) that the curriculum is presented at a higher level than what the learners can attain. Many learners were not ready to be taught at the van Hiele level 4.

The van Hiele model stresses that learners cannot skip any van Hiele level in order to understand geometry (Pegg 1995). Therefore if, for example, learners are situated at the van Hiele level 1 they are not ready to be taught at the van Hiele level 4. The findings of this study indicate that 50% of the learners in the sample group are not even at the van Hiele level 1 with regard to circle geometry. They cannot identify components of a circle. For example, only one participant managed to identify a sector in a circle and none of the participants managed to identify a major arc in the circle. This suggests that the South African 1997 geometry syllabus requirements are not compatible with the van Hiele levels.

In the van Hiele level 3, (see 4.6.2 and 4.6.3) test learners' performance indicated that learners do better in short deductions and in class inclusion than in definitions. According to van Hiele theory, understanding of both definitions and short deductions suggests that learners are situated at van Hiele level 3 (abstraction), but in this study definitions were explored and tested in van Hiele level 2 (see 2.2.1). The findings of this study suggest that only six participants, representing 29% of the sample group were ready to be taught at van Hiele level 3. Only three learners, including an exceptionally outstanding one in the sample group, placed at van Hiele level 4.

5.2.3 Insufficient preparation of learners during the primary and senior phases

The study showed that many learners were largely under-prepared for the study of more sophisticated geometry concepts and proofs as illustrated by many learners in the sample group who were unable to position a protractor to measure angles. This suggests that learners enter high school not yet ready to be taught geometry at a high level. The same number of learners was not even able to name angles. They used two letters of the alphabet to notate angles. According to the South African geometry 1997 syllabus this is supposed to be taught in Grade 7. This suggests that perhaps little time was spent in geometry in the elementary and middle levels of schooling.

Students have shown that there is a problem of unqualified and under qualified mathematics teachers in the primary phase (Geldenhuys, 2000). This is one of the reasons why primary mathematics teachers at times pay little attention to the teaching of geometry. In my view it is difficult or impossible to teach something if one does not know it.

5.2.4 Impact of the traditional approach on the teaching and learning of geometry.

This study identified the traditional approach as a possible reason for the poor understanding of circle geometry. The fact that many learners at grade 11 could not use compasses and protractors to bisect and measure angles shows the lack of hands on enquiry in the classroom. This suggests that an inappropriate approach is generally used to teaching geometry. Teachers should have students discover geometry through an inductive approach using mathematical instruments to develop insight into the learning and teaching of geometry.

5.2.5 Understanding of mathematical technical concepts and language problems.

This study recognises that there is a tension between language issues and mathematical conceptualisation in geometry. It was at times difficult to determine whether a learner experienced difficulties in understanding a concept due to his/her lack of language skills, or due to his/her lack of mathematical conceptualisation skills. For example, this applied in a situation where learners could not differentiate between construct and draw.

Mathematics is a language on its own. There is a plethora of mathematical vocabulary and terminology that needs to be developed in the teaching and learning of geometry. Geometry teachers should prepare glossaries for each learning unit and lesson. This will assist learners to know relevant mathematical technical concepts. In this study the majority of the learners in the sample group indicated a lack of understanding of certain mathematical concepts like perpendicular, arc, chord and tangent. Terms like conjecture, inscribed angles, exterior angles and semi-circle tend to stretch or overlap in such a way that I could not distinguish with certainty whether learners had a conceptual or a language problem. That is, I could not say for certain learners whether they have a conceptualisation or a language problem. This suggests that teachers should allow learners to develop an understanding of mathematical technical concepts as well as mathematical language using a variety of activities to gain insight (Pegg, 1995).

5.3 SIGNIFICANCE

The purpose of this study was to explore problems experienced by grade 11 learners in circle geometry. Globally geometry is regarded as the most problematic section in mathematics for both teachers and learners (Snyders, 1995). My experience was that the high failure rate of mathematics at the matric level in particular in the Eastern Cape, is caused by the performance of learners in Paper II which is composed of much Euclidean geometry involving a lot of circle geometry. Circle geometry is the core of high school geometry as it dominates in the current geometry syllabus (D.O.E., 1997a).

The reasons for exploring problems experienced by grade 11 learners were to: identify these problems, some of their causes and to use the findings to make recommendations where possible, and to perhaps propose some solutions. I used the van Hiele theory as a tool to check compatibility of the syllabus with the van Hiele theory.

I hope that the findings of this study can offer some guidance to mathematics educators, mathematics subject advisors, and curriculum designers especially in the Eastern Cape.

5.4 RECOMMENDATIONS

The fact that participants enter high school not having enough knowledge to cope with circle geometry suggests that circle geometry should be introduced in all the schooling phases. My view is that learners should be situated at van Hiele level 1 in the foundation phase. That is, they should be able to recognize or visualize a circle at least at the end of the foundation phase. Then at the intermediate phase learners should be taught components of a circle. That is, they should be taught at van Hiele level 2. They should know properties even if they can not see the relationship between properties. Then at the senior phase (grades 7-9) learners should be taught at van Hiele level 3. That is, they should be taught ways or methods of noticing or seeing the relationship between the properties of a circle. They should be able to make short deductions. Then lastly, they should be taught at van Hiele level 4 in the FET phase.

In my view learners can cope with FET geometry, that is, with van Hiele level 4-circle geometry if they can enter high school situated already at van Hiele level 3. This implies that the structure of the geometry curriculum in South Africa needs to be revisited and redesigned in order to develop a systematic progression.

Revisiting and redesigning the structure of the geometry curriculum alone may not be sufficient. There are too many unqualified and under-qualified teachers who teach mathematics due to a shortage of qualified teachers. There is an urgent need to upgrade these teachers.

The Department of Education should empower teachers through furthering studies in mathematics and in-service workshops or courses. To my mind teacher centres should be revamped to assist teachers in understanding the new developments in the teaching of mathematics and geometry in particular. In my experience of 21 years working with inset-providers, (subject specialists employed to in-service teachers in their respective subjects) and interacting with mathematics teachers, I observed that there is a need for in-service courses to work out problem areas for the teachers. In my view there is a lack of appropriate mathematics inset-providers and subject advisors in the Eastern Cape.

The tension between language issues and conceptualization in geometry needs to be addressed. In my view this can only be solved if suitably qualified teachers teach mathematics. Teachers need to be empowered to develop appropriate teaching methods to address language problems. My view is that learners should be exposed in both word problems and problem solving. This should involve a situation where learners are given opportunities to translate verbal statements into diagrams and vice versa. To my mind integrating mathematical language and mathematical technical concepts is vital. The fact that learners in the sample group experienced problems with terms or concepts like adjacent, perpendicular, conjecture and inscribed angles indicates that learners need to develop the relevant and appropriate mathematical vocabulary. To develop understanding of mathematical vocabulary learners should be given opportunities to formulate their own definitions of mathematical concepts.

A constructivist approach should be used to teach meaningful mathematics. It is my view that constructions using protractors, rulers, compasses, led pencils and rubber should be used as a strategy of discovering geometry to form conjectures. Formation of conjectures is a prerequisite for introducing formal proof of theorems. I also recommend a hands-on inquiry approach as one of the appropriate teaching methods to develop insight in the teaching and learning of geometry.

5.5 LIMITATIONS

The fact that all the participants were not English first language speakers is a limitation of this study. It restricted learners not to express themselves freely and adequately. Although they were free to use any language of their choice, at times they were perhaps shy to ask questions for clarification. They indicated in a conversational discussion that at times they could not understand instructions correctly during the process of investigations.

The South African grade 11 geometry syllabus requires learners to be taught circle theorems and converses where mentioned (D.O.E.,1997a). Due to time constraints I could not include converses in my study.

The South African 1997grade 11 geometry syllabus does not only cover circle geometry. A limitation of this study is that it focused only on circle geometry.

Also due to time constraints, I did not include application of theorems to solve riders in this study. The last test (van Hiele level 4 test) only focused on formal proofs of theorems. There were not enough complicated problems in the data collection process to find out learners' ability to transfer and integrate knowledge. There is hence still a need to further examine more complex problems involving circle geometry.

5.6 AVENUES FOR FURTHER RESEARCH

This study focused in grade 11 circle geometry. I suggest that the full spectrum of the geometry syllabus should be researched. A variety of similar case studies will make a tremendous contribution to the understanding of problems experienced in geometry across all grades. Future Eastern Cape researchers should conduct similar studies in all grades.

The fact that language problems were identified as one of the causes in the teaching and learning of geometry I recommend that future researchers should conduct research to also explore language problems in the teaching and learning of geometry syllabus in all grades.

As I indicated in the literature review as well as in the findings, there is some evidence that the South African geometry curriculum lacks a systematic approach. I suggest that there should be research conducted to re-dress these shortcomings in the geometry curriculum.

5.7 REFLECTIONS

As a novice researcher I struggled to estimate correctly the time frames for finishing each chapter. For instance I initially thought the literature review would take one month when in practice it took two months before it was submitted to the supervisor for the first time. It took another month to revisit after the supervisor had marked it. Similarly, in chapter four the data analysis and findings took me almost three month to finish although I only estimated one month.

At times I got frustrated when I did not understand the comments of the supervisor. But with time and experience I came to understand the language and intent of this feedback. The entire project was challenging and at times frustrating, but it was a wonderful experience.

There were tensions and disappointments in this study especially during the data collection process. It happened that nine research participants were also choristers and the choirmaster decided to use the same time used in research for choir practice. There was no alternative but to exclude these nine participants. This was unfortunate as they had indicated a lot of interest in this research project.

This study assisted me to develop an understanding of time management, the value of computer literacy and the need to own a computer. In the end, far too much of my own monies were spent on using computer centres for typing and e-mailing. This study also taught me to be disciplined in my devotion to my work. However this also could create other problems. Thus for example, on one occasion I left a formal function early to get back to my studies and was criticized by my hosts for not staying on.

As geometry is my special field of interest, I gained much from this study. I developed a better understanding of different approaches in the teaching and learning of geometry. This study also uncovered for me with key barriers that need attention in geometry teaching, such as language issues and insufficient preparation done in the lower classes.

5.8 CONCLUSION

This chapter summarized the whole project. The findings were discussed and a number of the more important factors that cause problems in the learning of circle geometry at the high school level were highlighted. Recommendations were made to suggest possible ways of improving learner's performance in circle geometry. Limitations that restricted this research were also discussed.

I believe that with the benefit of an improved understanding of the learning problems and reasons uncovered by this study, and with implementation of its recommendations, one could make a substantial contribution towards solving the problems experienced by our high school learners with circle geometry.

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APPENDIX A

SOUTH AFRICAN 1997 GRADE 11 GEOMETRY SYLLABUS

(EUCLIDEAN GEOMETRY)

i. The following must be treated within the framework of a mathematical system. Hence only axioms in logic and definitions, axioms and theorems that occur in this list or in the lists for standards 7 and 8 may be used as reasons for statements in solving riders.

ii. Although all theorems must be proved only proofs of theorems denoted with an asterisk (and their converses where mentioned) in the following list will be required for examination purposes.

iii. Applications of any axiom or theorem in this list or in the lists for standard 7 and standards 8 may be set. (No constructions for examination purposes).

iv. Not more than <u>three tenths</u> of the marks for geometry will be given for bookwork in the examination.

v. A logical order of the following should be adhered to.

3.1 The theorem of Pythagoras (Without proof)

*3.2 the line segment join the centre of circle to the mid-point of a chord is perpendicular to the chord, and conversely, the perpendicular drawn from the centre of circle to a chord bisects the chord (Theorem).

3.2.1 Corollary: The Perpendicular bisector of a chord passes through the centre of a circle.

3.2.2 A unique circle can be drawn through any three points not in a line.

*3.3 The angle which an arc of a circle subtends at the centre is double the angle it subtends at any point on the circumference. (Theorem).

3.4 The angle at the circumference of a circle subtended by a diameter is a right angle and conversely if a chord of a circle subtends a right angle on the circumference, the chord is a diameter. (Theorem)

3.5 Angles in the same segment of a circle are equal and conversely, if a line segment joining two points subtends equal angles at two other points on the same side of the line segment, these four points are concyclic. (Theorem)

3.5.1 Angles in equal segments of a circle, or of equal circles are equal. (Theorem)

*3.6 The opposite angles of a cyclic quadrilateral are supplementary, and conversely, if a pair of opposite angles of a quadrilateral is supplementary, then the quadrilateral is cyclic. (Theorem)

3.7 The exterior angle of cyclic quadrilateral is equal to the interior opposite angle, and conversely, if an exterior angle of a quadrilateral is equal to the interior opposite angle, then the quadrilateral if cyclic. (Theorem)

3.8 A tangent to a circle is perpendicular to the radius at the point of contact, and conversely, a line drawn perpendicular to a radius at the point where it meets the circumference is a tangent to the circle. (Theorem)

3.9 If two tangents are drawn to a circle through a common point, then the distances between this point and the points of contact are equal. (Theorem)

*3.10 The angle between a tangent to a circle and a chord drawn from the point of contact is equal to an angle in the alternate segment, and conversely, if a line is drawn through the end point of a chord making with the chord an angle equal to an angle in the alternate segment, then the line is a tangent to the circle. (Theorem)

3.11 The following theorems:

3.11.1 The bisectors of the angles of a triangle are concurrent.

3.11.2 The perpendicular bisectors of the sides of a triangle are concurrent.

3.11.3 The medians of a triangle are concurrent.

3.11.4 The altitudes of a triangle are concurrent.

SOUTH AFRICAN 2002 GRADE 12 GEOMETRY SYLLABUS

(EUCLIDEAN GEOMETRY)

Assessment standards

We know this when the learner:

Grade 12

- Investigates the geometry of circles, accepting that a tangent is perpendicular to the radius drawn to the point of contact and makes conjectures and proves them within a local axiomatic system (including the following theorems:
  - The line drawn form the centre of a circle, perpendicular to a chord bisects the chord
  - The angle subtended by and arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle
  - The opposite angles of a cyclic quadrilateral are supplementary.
  - The tangent chord theorem.

APPENDIX B

DISCOVERING CHORD PROPERTIES

Investigation 1

Step 1 Construct a circle where the radius is strictly greater than 2cm. Label the centre O.

Step 2 Construct two congruent chords in your circle. (Use a compass to guarantee that they are congruent.) label the chords AB and CD

Step 3 Construct radii OA, OB, OC and OD

Step 4. With your protractor, measure angle BOA and angle COD.

Compare your results with the results of others near you. State your observations as your next conjecture.

(a) If two chords in a circle are congruent then they determine two central angles that are............equal.

(b) If two chords in a circle are congruent then their central angles are congruent.

Investigation 2

Step 1 Construct a large circle. Mark the centre.

Step 2 Construct tow nonparallel congruent chords that are not diameters.

Step 3 Construct the perpendiculars from the centre to

How does the perpendicular from the centre of a circle to a chord divide the chord. State your observation as a conjecture.

(c) The perpendicular from the centre of a circle to a chord or the bisector of the chord.

Investigation 3

Step 1 Construct a circle where a radius is strictly greater than 3cm and mark the centre.

Step 2 Construct two nonparallel non-congruent chords that are not diameters.

Step 3 Construct the perpendicular bisector of each chord and extend the bisectors until they intersect.

What is special about the point of intersection. Compare your results with the results of others near you. State your observation as a conjecture.

(d) The perpendicular bisector of a chord passes through the centre of a circle.

DISCOVERING ARCS AND ANGLES PROPERTIES

Investigation 4

Step 1 Construct a circle with a radius greater than 2cm

Step 2. Draw two radii OR and OC such that $90° < C\hat{O}R < 180°$ that is $C\hat{O}R$ can be any obtuse angle.

Step 3 Draw two chords CA and RA for A is a point in the major arc.

Step 5 Compare the size of the incscribe angle with the size of the central angle.

Compare your results with the results of other

State a conjecture

(e) Angle in the centre of a circle doubles the inscribed angle in the circumferences if they are subtended by the same arc.

Investigation 5

Step 1 Construct a large circle with a radius greater than 3 cm.

Step 2 select two points on the circle. Label them A and B.

Step 3 Select a point P on the major arc and construct inscribed angle APB.

Step 4 With your protractor, measure angle APB.

Step 5 Select another point Q on major arc APB and construct inscribed angle AQB.

Step 6 Measure angle AQB. How does the magnitude of angle AQB compare with the magnitude of APB.

Compare your results with the results of others.
State a conjecture.

(f) Inscribed angles subtended by the same arc are equal.

Investigation 6

Step 1 Construct a circle with radius greater than 2cm. Label the centre.
Step 2 Construct a diameter.
Step 3 Draw three inscribed angles subtended by the diameter in the same semicircle.
Step 4 Measure each angle with your protractor compare your results with the results of others and make a conjecture.

(g) Angles inscribed in a semi-circle are right angles.

Investigation 7.1

Step 1 Construct a circle with radius greater than 2cm.
Step 2 Construct an inscribed quadrilateral.
Step 3 Measure each of the four inscribed angles. Write the magnitude in each angle.
Step 4 Add consecutive angles.
Step 5 Add pairs of opposite angles.
Step 6 Compare your observations with the observations of those near you.
Step 7 State your findings as a conjecture.

(h) Opposite angles of a cyclic quadrilateral are supplementary.

Investigation 7.2

In investigation 7.1 extend one side of a cyclic quadrilateral and measure the exterior angle. Then what in special about it.

DISCOVERING TANGENT PROPERTIES.

Investigation 8

Step 1 Construct a large circle. Label the centre.

Step 2 Draw a line that touches the circle at only one point. Label the point T. Construct OR.

Step 3 Use your protractor to measure the angles at T.

Compare your results with the results of others near you. State your observations as a conjecture.

(i) Radius of a circle is always perpendicular to the tangent .

A tangent to a circle is perpendicular to the radius drawn to the point of tangency.

Investigation 9

Step 1 Draw a circle with radius greater than 2cm.

Step 2 Draw any tangent.

Step 3 Draw two non-congruent such that they share the point of tangency.

Step 4 Join tow different points of two chords to form a triangle.

Step 5 Measure two angles subtended by two chords in the alternate segments.

Compare your results with the results of others near you. State your observations as a conjecture.

Investigation 10

Step 1 Construct a circle. Label centre E

Step 2 Choose a print outside the circle and label it N

Step 3 Draw two lines through point A that appear to be tangent to the circle. Mark the points where the lines appear to touch the circle and label them A and G

Step 4 use your compare segment NA and NG

(Segments NA and NG are called tangent segments)

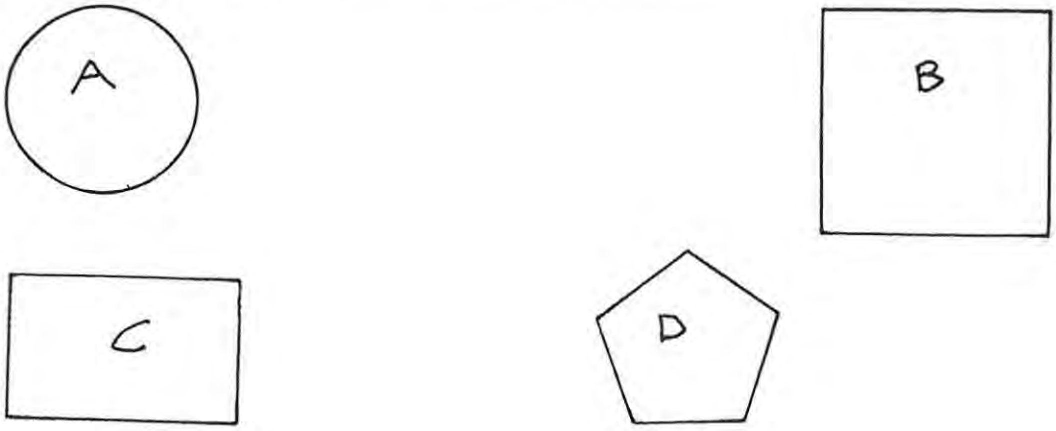
Compare your results with the results of others near you. State your observations as your next conjecture.

APPENDIX C

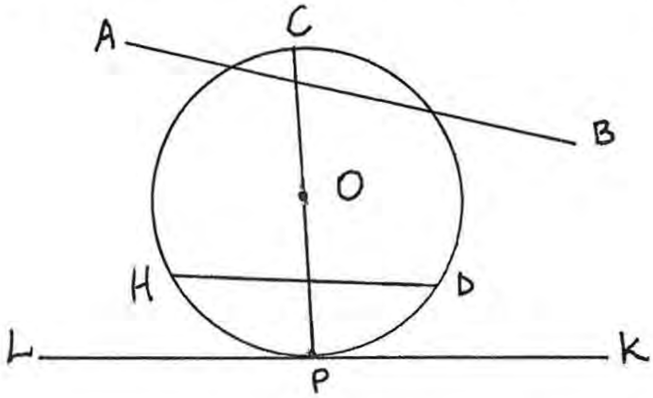
THE VAN HIELE TESTS

Van Hiele Level 1 Test

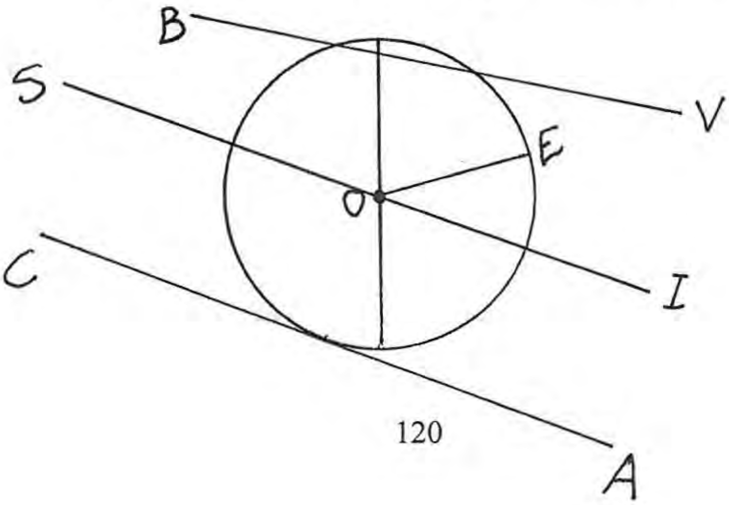
1. Which one of the following diagrams represents a circle?



2. Choose a line segment that represents a chord in the diagram below. Use O as the centre.



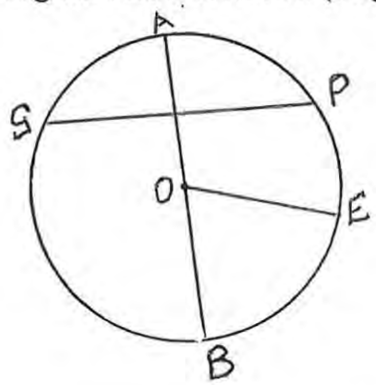
3. Choose a line that represents a diameter in the diagram below. Use O as the centre.



4. Use the diagram in question 3 to choose a line segment that represents a radius.

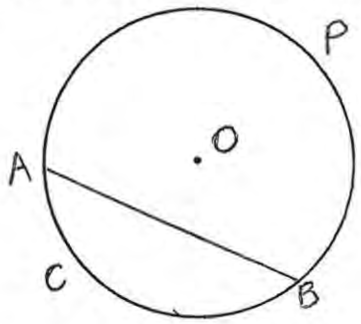
5. Use the diagram in question 3 to choose a line segment that represents a tangent.

6. Which one of the lines in the diagram below divides it (diagram) into semi-circles? Take O as a centre.

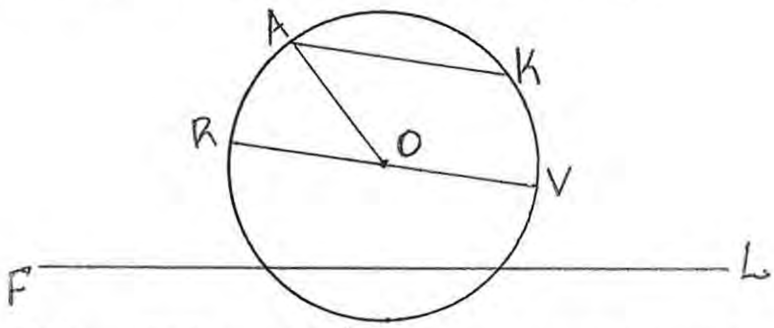


7. Use the diagram in question 6 and determine which lines form a sector in the diagram?

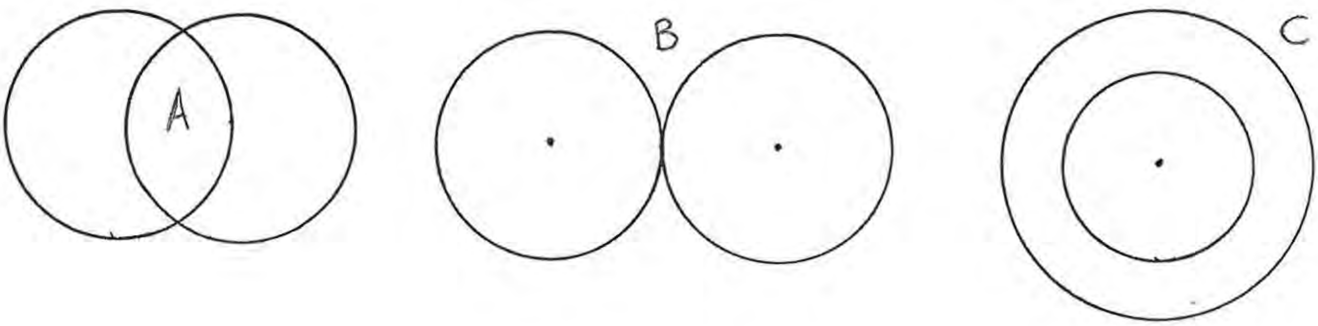
8. In the diagram below which arc is known as a major one? Use O as the centre.



9. In the diagram below which line is a secant? Use O as the centre.



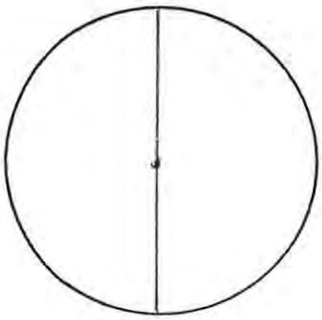
10. In the diagrams below which one of the following represents concentric circles?



Van Hiele Level 2 Test

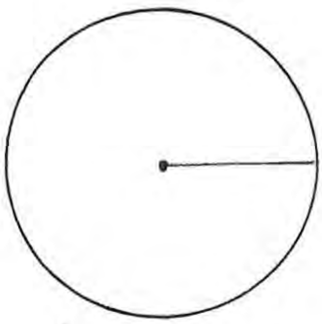
Define the line drawn in figure A.

A.



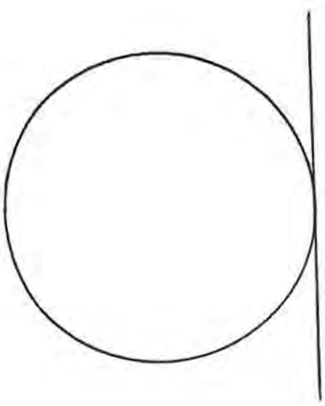
Define the line drawn in figure B.

B.



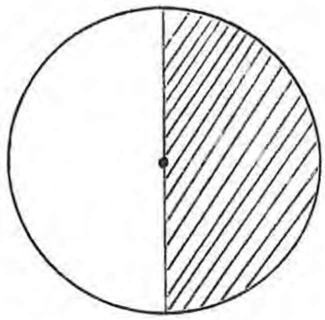
Define the line drawn in figure C.

C.



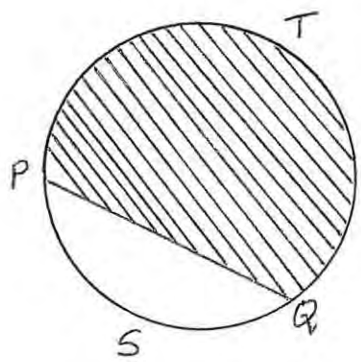
Define the shaded region of figure D.

D.



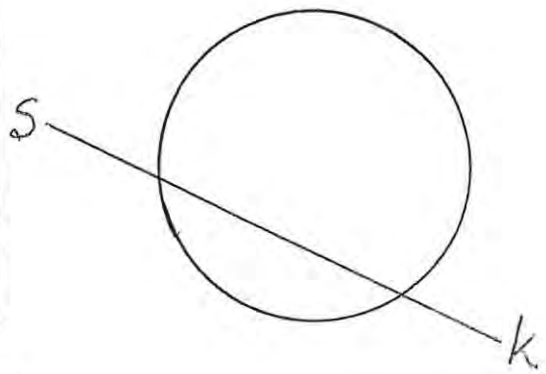
Define line PQ, $\widehat{PTQ}$ and $\widehat{PSQ}$.

E.



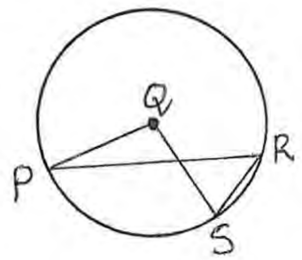
Define the line SK in figure F.

F.

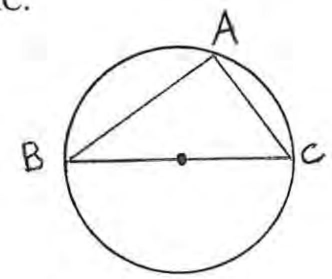


Van Hiele Level 3 Test

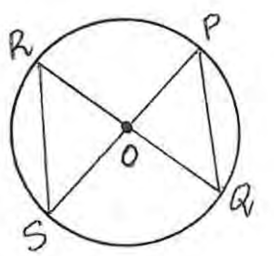
1. In a circle Q is the centre. Find $P\hat{R}S$ if $P\hat{Q}S = 110^{o}$.



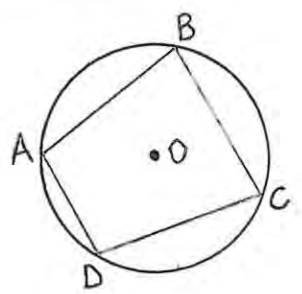
2. In circle below BC is a diameter; find $B\hat{A}C$.



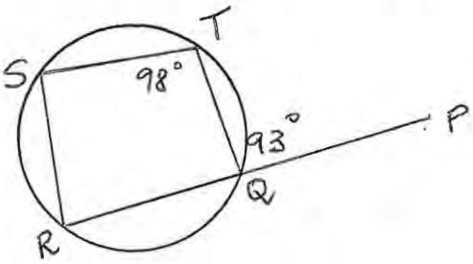
3. In circle below O is the centre. RQ intersect PS at O. $O\hat{P}Q = x$; find other angels that are equal to x.



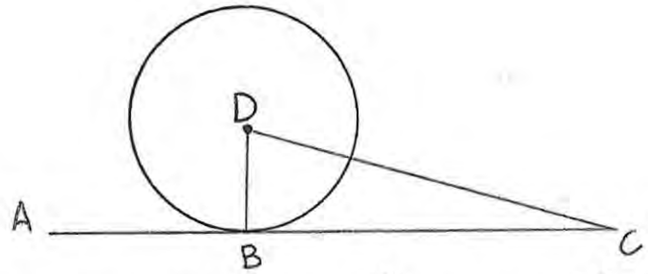
4. In circle below ABCS is a cyclic quadrilateral $A\hat{D}C = 82^{o}$; $B\hat{C}D = 96^{o}$. Find $D\hat{A}B$ and $A\hat{B}C$.



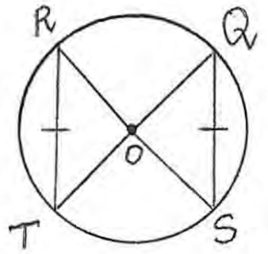
5. In the figure below $P\hat{Q}T = 93^\circ$ and $Q\hat{T}S = 98^\circ$. Find $T\hat{S}R$ and $S\hat{R}Q$.



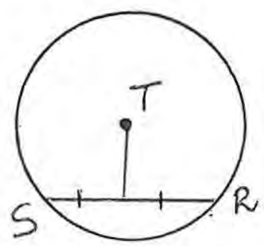
6. DB is a radius and ABC is a tangent. Find $D\hat{B}C$.



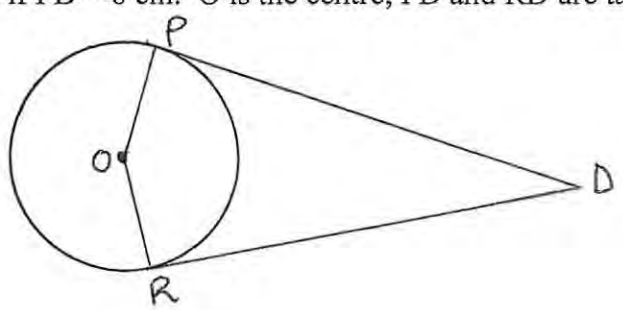
7. In circle below chords RT = QS; O is a centre. $O\hat{S}Q = 38^\circ$; find $S\hat{Q}O$ and $R\hat{O}T$.



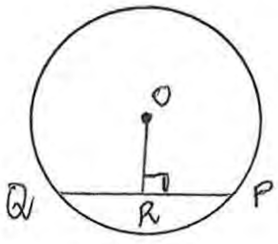
8. In the figure below find $T\hat{Q}R$ if T is the centre of a circle. SQ = QR.



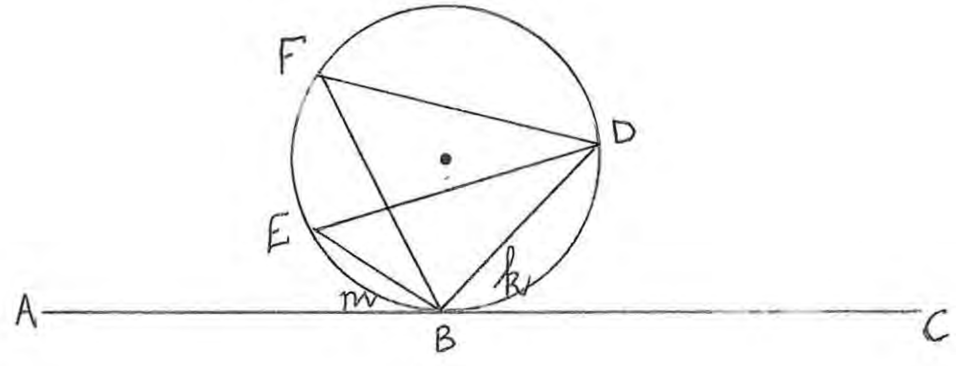
9. In the figure below find DR if PD = 6 cm. O is the centre; PD and RD are tangents.



10. Find QP if PR is 3cm, OP $\perp$ QR

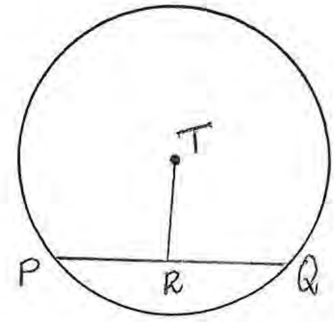


11. In the figure below ABC is a tangent; BE; FB and BD are chords from the point of tangency. DBC = k; $\hat{ABE}$ = m. Find other angles that are equal to k and m.



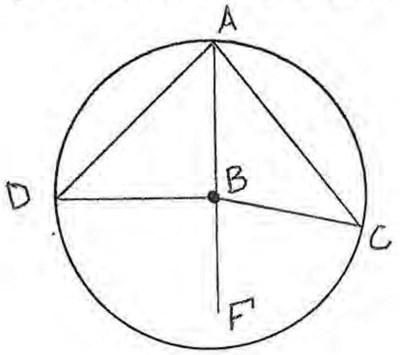
Van Hiele Level 4 Test

1.



In circle T; T is the centre; TR perpendicular PR. Prove that PR = RQ.

2.



In the figure above B is the centre of a circle BD & BC are radii DA & CA are chords that meet at A in the circumference. AB is extended to F.

Complete the table below to prove that 2 $\hat{DAC} = \hat{DBC}$

Given: In circle B; BD & BC are radii DA & CA are chords that meet at A in the circumference. AB is extended to F.

R.T.P.: $2\hat{DAC} = 2\hat{BDC}$

Construction: AF is drawn such that B $\in$AF

STATEMENT	REASON
In triangle BAC	
$\hat{BAC} + \hat{BCA} = \hat{CBF}$	______________________________

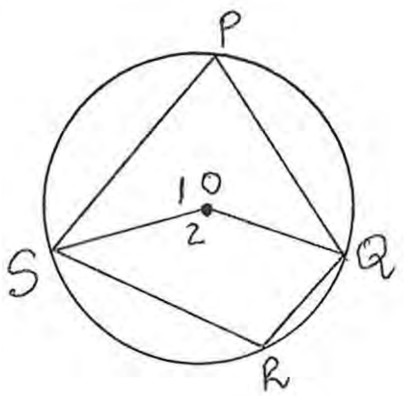
STATEMENT	REASON
$BA = BC$	________________________
$\widehat{BAC} =$ ________________________	$BA = BC$
In triangle ABD	
$BA =$ ________________________	Radii
$\widehat{BAD} = \widehat{BDA} = Y^{o}$	________________________
______ + ______ $= \widehat{DBF}$	Exterior angel of triangle = 2 interior opposite angles
$2\widehat{BAC} + 2\widehat{BAD} = \widehat{DBC}$	________________________
$2(\widehat{BAC} + \widehat{BAD}) = \widehat{DBC}$	
$2\widehat{DAC} = \widehat{DBC}$	

3. The angle at the circumference of a circle subtended by a diameter is a right angle.

Suppose you wished to prove the above statement

1. Draw and label a figure
2. Write, in terms of your figure what is given and what is required to be proved
3. Prove the theorem

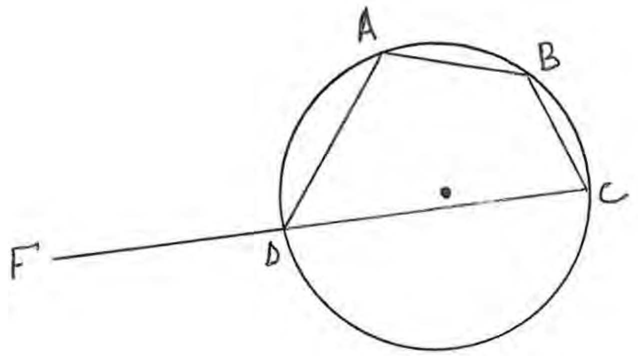
4.



In circle O, PQRS is a cycle quadrilateral, OS & OQ are radii

Use the given information in the figure above to prove that $\widehat{SRQ} + \widehat{SPQ} = 180^{o}$

5.

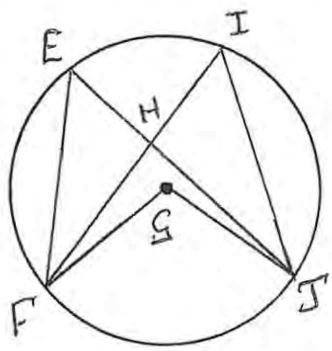


In the figure above ABCD is a cyclic quadrilateral CD is produced to E such that D $\in$ CE

Prove that $\widehat{ADE} = \widehat{ABC}$

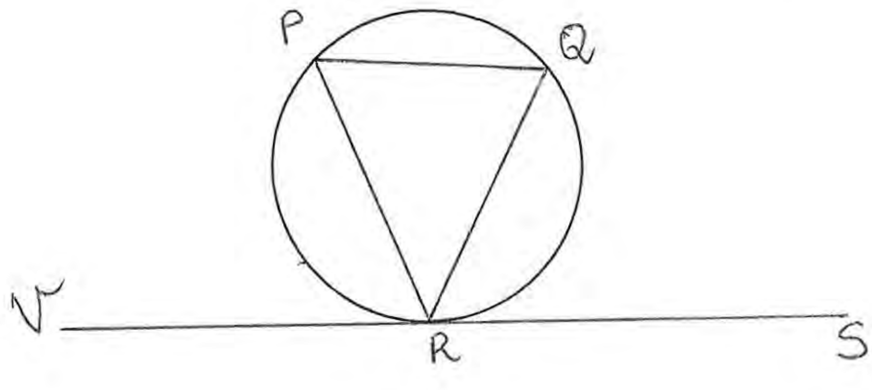
6. A tangent to a circle is perpendicular to the radius at the point of contact. Prove this theorem.

7.



Use the figure above with G the centre of a circle to prove that $\widehat{FEG} = \widehat{FIJ}$.

8.



Use the figure above to prove that angle QRS = angle RPQ. Follow all the necessary steps for formal proof of theorems.

APPENDIX D

Questionnaire

Instruction

1. Read each question carefully.
2. Choose the correct answer by indicating the correct letter.

1. Perpendicular lines
   (a) intersect to form four equal right angles
   (b) intersect of form only equal vertically opposite angles
   (c) intersect to form acute angles
   (d) do not intersect at all

2. A diameter
   (a) is a line drawn from the centre of a circle to the circumference
   (b) is a line joining any points of the circumference
   (c) is a line joining two points on the circumference passing through the centre of a circle.
   (d) is a line that cuts two points on the circumference

3. The plane figure produced by drawing all points exactly 6 cm from a fixed point is a
   (a) circle with a diameter of 6cm
   (b) circle with a radius of 6cm
   (c) circle with a chord of 6cm
   (d) square with a side of 6cm

4. Right angle measures
   (a) 180°
   (b) 360°
   (c) 90°
   (d) 270°

5. Supplementary angles add up to

(a) 60°

(b) 360°

(c) 180°

(d) 90°

6. Bisector angle is a/an

(a) line that divides one angle into two equal angles

(b) angle that is divided into two equal angles

(c) adjacent angle

(d) line that cuts two parallel lines

7. An arc is a

(a) part of the circumference

(b) portion between two radii

(c) relationship between circumference and radius

(d) half of a circle

8. Tangent is a

(a) line that cuts two points on the circumference

(b) line that has only one point of contract with the circle

(c) line that joins two opposite angles of a polygon

(d) distance around a circle

9. A chord joining two end points of two radii of the same circle form a/an

(a) equilateral triangle

(b) scalene triangle

(c) right-angled triangle

(d) isosceles triangle

10. In the following statements there is only one correct statement about a cycle quadrilateral. Choose the letter next to the appropriate correct answer.

(a) consecutive angles of a cyclic quadrilateral are supplementary.
(b) opposite angles cyclic quadrilateral area equal.
(c) opposite angles of a cyclic quadrilateral are supplementary.
(d) opposite angles of a cyclic quadrilateral are complimentary.

11. A line drawn from the centre of a circle to the circumference is a

(a) chord
(b) diameter
(c) radius
(d) sector

12. In the following statements, there is only one correct statement. Choose the letter next to the correct answer.

(a) a tangent to a circle is parallel to the radius of the same circle.
(b) a tangent to a circle is perpendicular to the radius of the same circle
(c) a tangent to a circle bisects the radius of a circle.
(d) a tangent of a circle is congruent to the radius of a circle.

13. An angle in the centre of a circle

(a) is equal to the angle of the circumference subtended by the same arc.
(b) is half of an angle at the circumference subtended by the same arc.
(c) doubles the angle at the circumference of a circle subtended the same arc.
(d) trebles the angle at the circumference of a circle subtended by the same arc.

14. Which one of the following in not the axiom or condition for congruency in triangles.

(a) side, side, side
(b) angle, angle, angle
(c) angle, angle, side
(d) side, angle, side

15. Angle in a semi-circle is a/and

(a) acute angle

(b) obtuse angle

(c) right angle

(d) straight angle

16. The exterior angle of a cyclic quadrilateral is equal to

(a) the interior opposite angle of a cyclic quadrilateral

(b) the sum of the two interior opposite angles

(c) the right angle in the alternate segment

(d) four interior angles of a cyclic quadrilateral

17. Central angles are equal if they are

(a) subtended by equal chords

(b) subtended by parallel non-congruent chords

(c) subtended by two equal tangent

(d) subtended by two equal secants.

18. Angle between a chord and a tangent is

(a) equal to any inscribed angle in a circle

(b) equal to the angle in the alternate segment

(c) equal to the adjacent angle made between a chord and a tangent

(d) equal 90°

19. Which is true? In an isosceles triangle

(a) all angles are equal

(b) base angles are equal

(c) each angle measure 60°

(d) each base angle measure 45°

20. What is not true? In an equilateral triangle

(a) all sides are equal

(b) each angle measures 60°

(c) medians are axis of symmetry

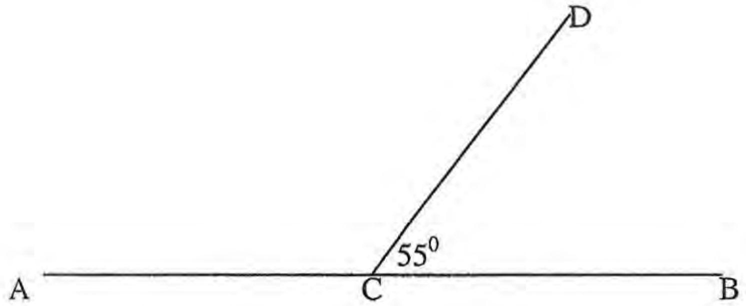
(d) if one side is extended the exterior angle of a triangle is always equal to 100°

APPENDIX E

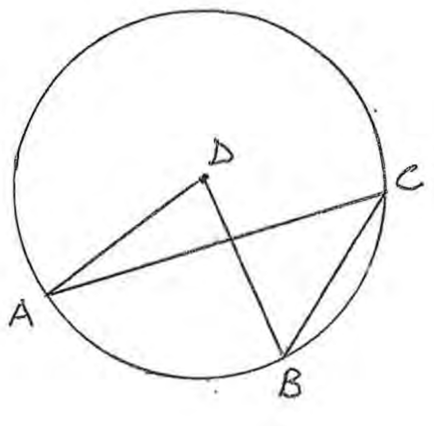
INTERVIEWS: Semi-structured interviews

Researcher:- In which grade did you start learning geometry?

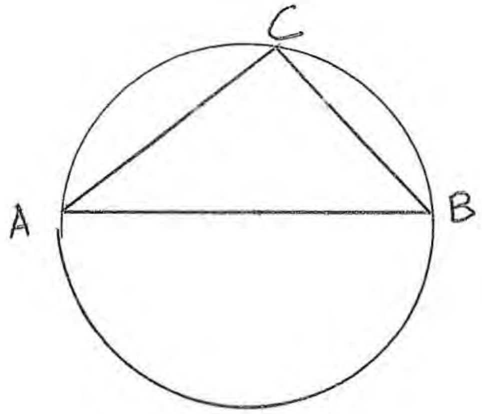
1. Which section did you do or learn?
2. In which grade start learning theorem?
3. Did you learn also circle theorem?
4. Can you translate the figure below by saying the given information and required to calculate.



5. In the CIRCLE below what is the relationship between angle ADB & angle ACB



6. What is the name of line AB in figure below if $A\hat{C}B = 90°$



APPENDIX F

COMMON TASKS FOR ASSESSMENT (CTA) – GRADE 9 (2003)

DEPARTMENT OF EDUCATION

DEPARTMENT OF EDUCATION

MATHEMATICAL LITERACY, MATHEMATICS AND MATHEMATICAL SCIENCES (MLMMS)

Common Tasks for Assessment (CTA)
Grade 9
2003

LEARNER'S BOOK

SECTION B

Time: 2 hrs

Marks: 80

No. Pages: 6

THE COMMON TASKS FOR ASSESSMENT (CTA)

PROGRAMME ORGANISER : Sustainable Living

FOCUS : How can the tourist industry use Robben Island as a tourist destination to contribute to sustainable living?

Note to the learner:

This is Section B of your CTA.

SECTION B:

What you should know:

- Section B consists of five questions.

What you need:

- Pen
- Ruler
- Calculator

Duration:

- Two hours

Instructions:

Follow instructions for each question carefully.

- Answer all questions.
- You are encouraged to show all your calculations.

QUESTION 1 (Recommended time: 30 min)

Robben Island does not have sufficient fresh water for its residents. A large number of tourists visit the island daily. The total daily water consumption during peak season is approximately 25 *kℓ* for both the residents of Robben Island and the tourists.

There are 150 residents of Robben Island. Each resident of the Island uses 80 litres of water every day. 1000 tourists visit the Island daily in peak season and 500 tourists daily in the off-season.

There are four reservoirs (large water tanks) on the Island, two big reservoirs and two smaller ones. Suppose that the big reservoirs have a capacity of 200 *kℓ* each and the smaller ones 100 *kℓ* each.

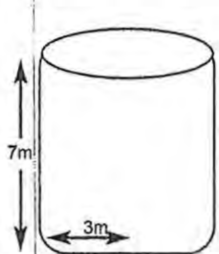
1.1 Calculate the daily water consumption of the residents of Robben Island. Express your answer in kilolitres. (3)

1.2 What additional daily increase in water consumption on Robben Island is caused by tourists in peak season? (2)

1.3 What percentage of the daily water consumption is used by tourists in peak season? (3)

1.4 Show through calculations that the water stored in the four reservoirs, if not refilled, will not be enough to supply water for December (peak season). (7)

1.5.1 Calculate the volume of water contained in a reservoir with the following dimensions: Radius of the base = 3 m, height of the reservoir = 7 m. $[\pi = \frac{22}{7}]$



1.5.2 Now express your answer in kilolitres. (4)

1.6 Is the reservoir in question 5 big enough to supply the extra water needed to make up the peak season shortfall? Give a reason for your answer. (2)

[21]

QUESTION 2 Recommended time: 15 min

A desalination plant, which removes the salt from seawater, provides fresh water to Robben Island. In the case of a breakdown in the plant, water is ferried across from the mainland using cable boats. The water tank on a cable boat can hold 50 $k\ell$ of water. Due to the high cost of operating the desalination process and the transfering of water from the mainland, the price per kilolitre of water on the island is R5.

2.1 Calculate the cost of water for the Smith family on Robben Island for the month of November. There are four members in the Smith family. Remember that the daily consumption of water on the island is 80 litres per resident. (4)

2.2 The generators to supply power used in the desalination plant run on diesel.

Calculate:

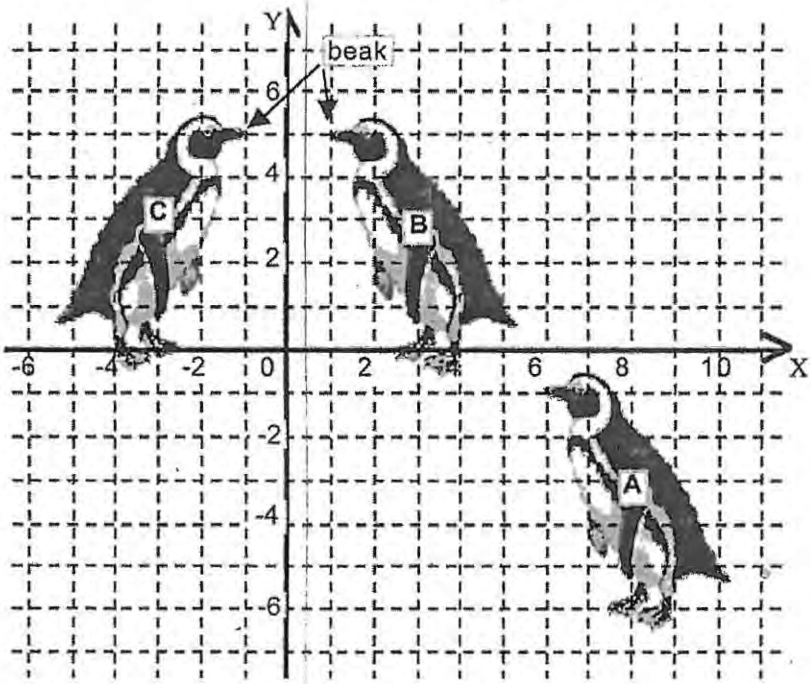
2.2.1 The daily running costs for November of one generator if it runs 8 hours per day and uses 12 litres of diesel per hour. 1 litre of diesel costs R4. (4)

2.2.2 The running costs of the plant for November if there are three generators. (3)

[11]

QUESTION 3 Recommended time: 25 min

Macy took some photographs of an African penguin moving around on Robben Island. She made a combined picture to show the different positions of the penguin. She drew an X- and Y- axis over the picture and prepared some questions to challenge you, and her brother Camille, skills and knowledge of movements and transformations.



3.1 Describe the transformation when the penguin moved from

3.1.1 position A to position B (3)

3.1.2 position B to position C (2)

3.2 If the co-ordinates of the point of the beak of the penguin in position B is $(x; y)$, write the co-ordinates of the point of the beak of the penguin in

3.2.1 position A in terms of x and y (2)

3.2.2 position C in terms of x and y (2)

3.3 If the penguin moves from position A to position D using the transformation given by $(x-7; y-4)$,

3.3.1 Describe the transformation of the penguin (3)

3.3.2 Write down the co-ordinates of the point of the beak of the penguin in position D. (3)

[15]

QUESTION 4 Recommended time: 35 min

Use the data in the table below to complete the following questions:

Maximum daily temperature (^{o}C) on Robben Island from 1 - 20 April 2003

21	21	16	18	17
16	21	17	18	21
22	23	16	21	22
16	21	17	20	23

4.1 Make a frequency table of the data. (6)

4.2 Determine the following characteristics of the data:

a) the range (1)

b) the mean (3)

c) the mode (1)

d) the median (1)

4.3 Would you choose the mode or the mean to predict the maximum temperature on a specific day in the 20 day period? Give a reason for your choice. (3)

4.4 Draw a graph of your choice to represent the data in the table. (5)

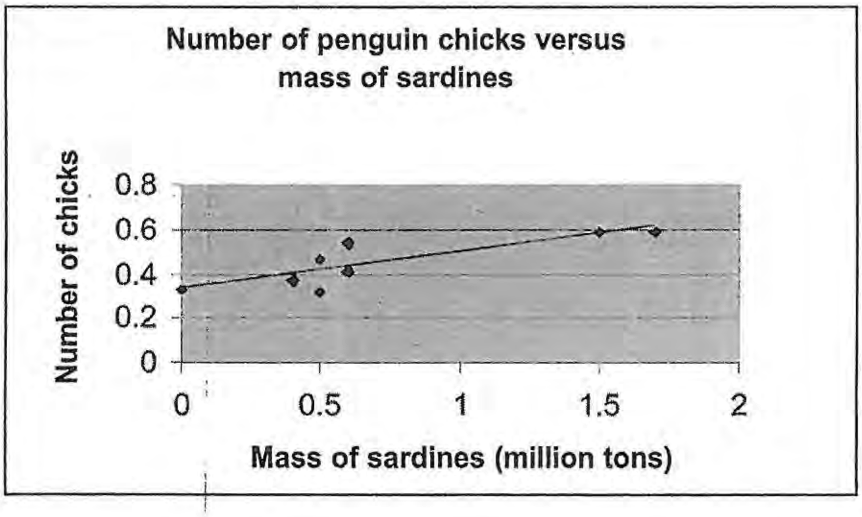
4.5 If the 23^{o}C in the table is changed to 30^{o}C,

4.5.1 How is the median affected? (1)

4.5.2 How is the mean affected? (1)

QUESTION 5 Recommended time: 15 min

Penguins, on average, lay two eggs per breeding season. In the initial stage of their growth these chicks are fed sardines. The scatter graph below shows the relationship between the mean number of penguin chicks fledged (successfully raised) per breeding pair and the mass of sardines. The straight line represents the best line of fit.



5.1 According to the best line of fit, if there are no sardines, the mean number of chicks raised per breeding pair will be 0,333...

5.1.1 Write 0,333... as a common fraction (1)

5.1.2 Explain what the number 0,333... in 5.1.1 above means. (1)

5.2 If the gradient of the line of best fit is $\frac{1}{5}$, show that the equation of this line is $3x - 15y + 5 = 0$. (4)

5.3 If the mass of sardines increases to 20 million tons, use the equation of the best line of fit to calculate the average number of chicks raised per breeding pair. (3)

5.4 Is your answer to question 3 above realistic? Motivate. (2)

5.5 What would happen to the penguin numbers if the fishermen depleted the stocks of fish in the sea? (1)

Total Marks: 80 **[12]**

APPENDIX G

COMMON TASKS FOR ASSESSMENT (CTA) – GRADE 9 (2004)

DEPARTMENT OF EDUCATION

TIRISANO WORKING TOGETHER

DEPARTMENT OF EDUCATION

MATHEMATICAL LITERACY, MATHEMATICS AND MATHEMATICAL SCIENCES (MLMMS)

Common Tasks for Assessment (CTA)

Grade 9

2004

LEARNER'S BOOK

SECTION B

Time: 2 hrs

Marks: 80

No. Pages: 8

THE COMMON TASKS FOR ASSESSMENT (CTA)

PROGRAMME ORGANISER : Culture and Society

FOCUS : The vehicle manufacturing industry

Note to the learner:

This is Section B of your CTA.

SECTION B:

What you should know:

- Section B consists of five questions.

What you need:

- Pen
- Ruler
- Calculator

Duration:

- Two hours

Instructions:

Follow the instructions for each question carefully.

- Answer all questions.
- You are encouraged to show all your calculations.

QUESTION 1

Recommended time: 20 min

The 2003 Annual Report of The National Association of Automobile Manufacturers of South Africa (NAAMSA) estimates that in 2002 vehicle exports contributed R17,2 billion to new car sales. The consultant has recommended that ZZ Motors export their cars to increase revenue.

Table 1: Sales of locally produced motor cars.

	1999	2000	2001	2002
Local sales	132 912	160 664	172 052	163 474
Exports	52 347	58 204	97 599	113 025
Total domestic production	185 259			276 499

* Adapted from NAAMSA Annual Report 2003

Study Table 1 above and answer the questions that follow:

1.1 Determine the total domestic production of motor cars for 2000 and 2001. (2)

1.2 What percentage of the total domestic production was exported in:

a) 1999 (Give the answer correct to one decimal place.)

b) 2002 (Give the answer correct to one decimal place.) (5)

1.3 Calculate:

(a) The increase in exports from 1999 to 2002. (2)

(b) The percentage increase in exports from 1999 to 2002.

(Give the answer correct to one decimal place.) (2)

1.4 Give two reasons for the percentage increase in car exports for the above period (1999 to 2002) in your own words. (2)

[13]

QUESTION 2

Recommended time: 20 min

National Auto wants to manufacture 5 700 Unity and 3 420 Sporty cars in a 6-month period. In the car manufacturing process, one of the final steps towards a finished car is called 'baking'. The car is placed in a large 'oven' for several hours. This is done so that the paint bonds with the metal surface of the car.

It takes 24 hours to bake a Unity and 36 hours to bake a Sporty.

2.1 Determine the total number of factory work-hours needed to bake the number of cars National Auto wishes to manufacture per month. (4)

2.2 ZZ Motors currently uses its baking ovens for 107 930 hours per 6-month period. It takes them 12 hours to bake each car they produce.

a) How many cars will the company have ready for delivery at the end of each 6-month period? (4)

b) Will ZZ Motors be able to meet the same target as National Auto of 5 700 Unity and 3 420 Sporty cars during the 6-month period? Give a reason for your answer through a calculation. (3)

2.3 How many additional cars can ZZ Motors have ready for delivery at the end of a 6-month period if it reduces the baking process to 9 hours per car? (4)

[15]

QUESTION 3

Recommended time: 45 min

ZZ Motors has provided parking only for their 12 directors, although there are open spaces in the parking garage. The labour union demands that management enlarge the existing parking garage to accommodate the vehicles of 309 workers.

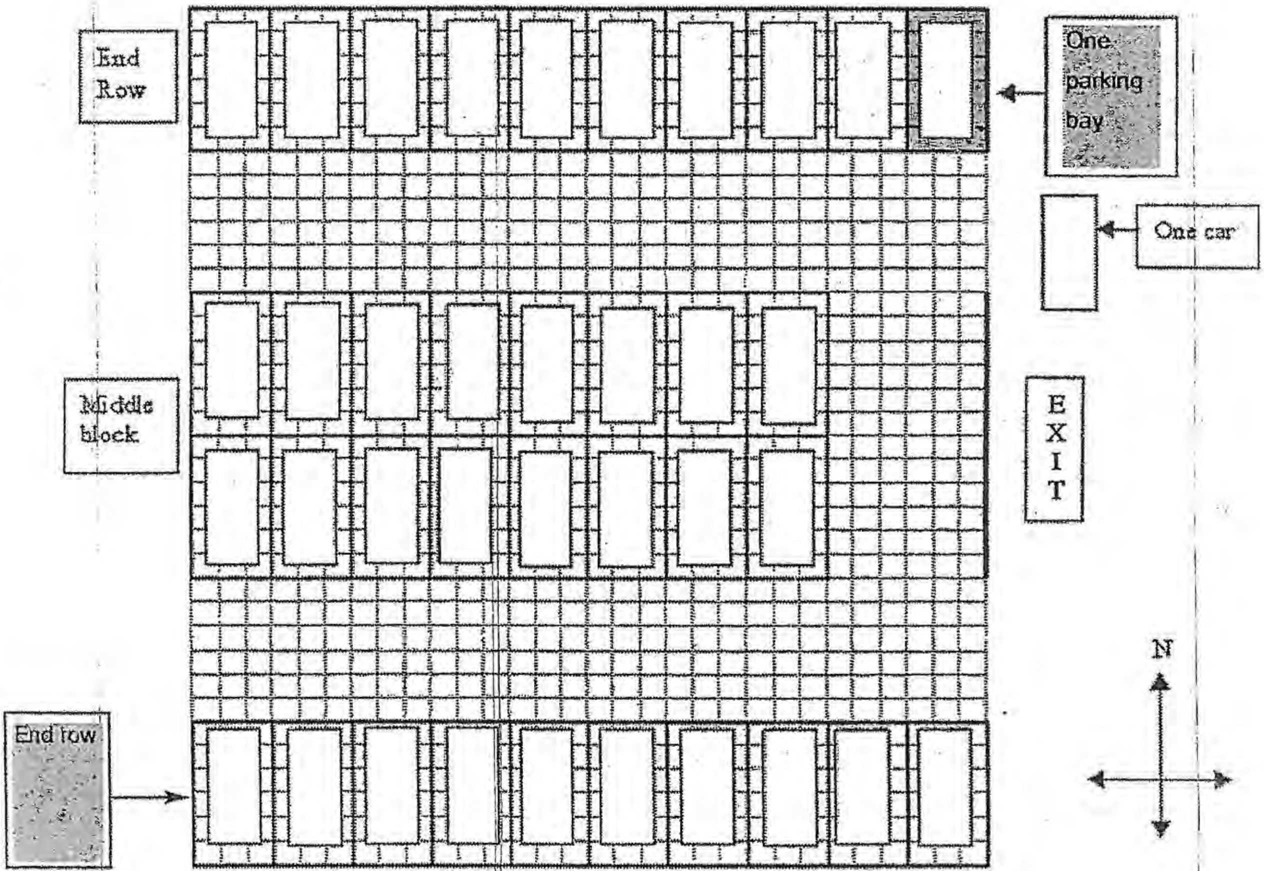
Specifications for the parking garage

- The driving lanes must be 6 meters wide to ensure enough space for driving between the parking bays.
- The area of one parking bay is 18 m^2
- The parking garage can only be extended to the south.

Floor plan of existing parking garage

The layout of the parking bays includes 2 end rows accommodating 10 cars each.

The middle block has two rows of parking 8 cars.



3.1 Use the given information to determine the following:

a) The scale of the floor plan (1)

b) The length and the breath of the parking garage (2)

c) The number of cars that can be parked in the parking garage (1)

3.2 Adding more middle blocks would increase parking accommodation in the garage. Redraw and complete the given table in your book to keep track of the number of cars that could be parked if more middle blocks were added.

x = Number of middle blocks	**1**	**2**	**3**	**50**	**x**
Parking bay in middle block	16		48		
Total no of parking bays in the garage		52		80	
y = Total floor space needed in m²	1080		2160		540x + 540

(8)

3.3 Show how the equation $y = 540x + 540$ is obtained for the calculation of the floor space, where y is the floor space and x represents the number of middle blocks that must be added. (4)

3.4 If 9 out of 10 workers own cars,
(Remember that there are 309 workers and 12 directors.)

a) How many parking bays are needed? (3)

b) How much floor space is needed in total? (2)

c) Calculate the number of middle blocks that must be added to provide parking for all staff. (3)

3.5 Calculate the length of the enlarged garage. (2)

3.6 Assume that the measurements of the enlarged garage are 30m by 324 m.

a) Would you consider extending the garage to 324 m to be the best solution? Give reason for your answer. (2)

b) What alternative solution would you suggest? Give reason for your answer. (2)

[30]

QUESTION 4

Recommended time: 35 min

- Study the graph provided below and answer the questions that follow:

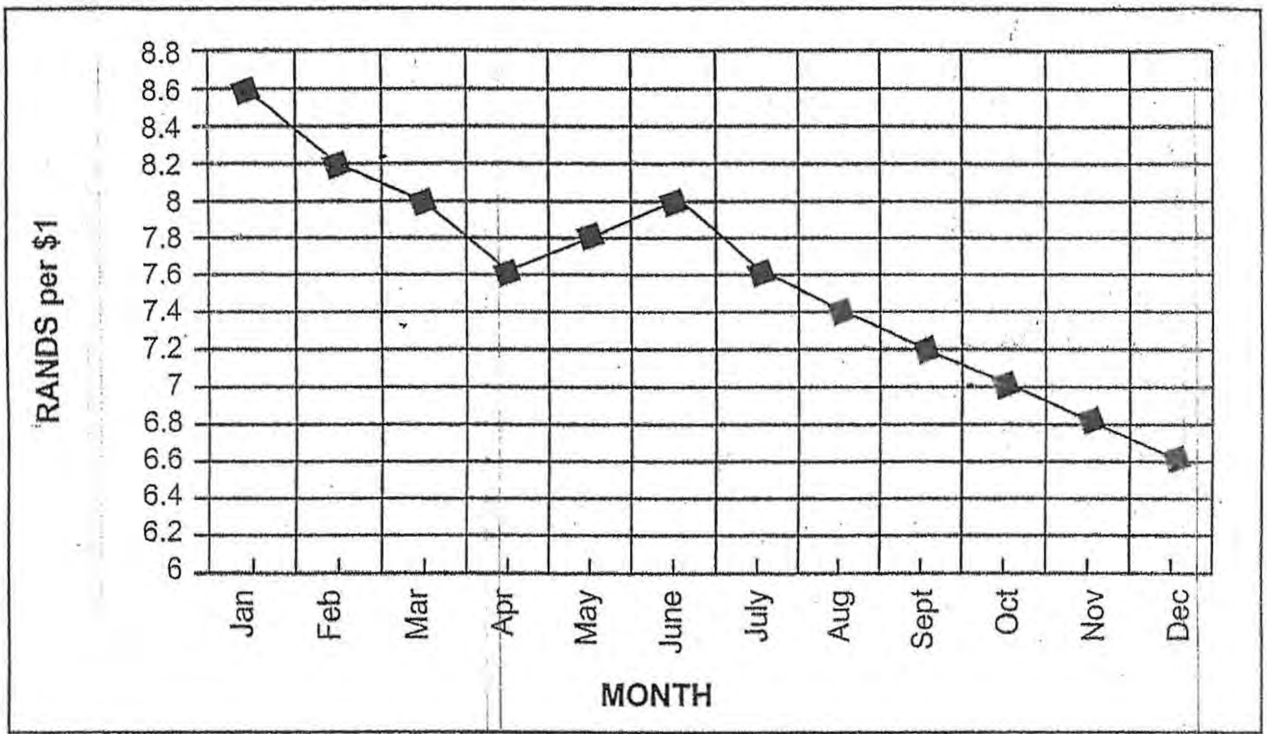


Figure 3: Rand – Dollar exchange rate for 2003

4.1 Determine how many Rands are required to purchase $50 in July. (3)

4.2 Determine how many Dollars are required to purchase R50 in June. (3)

4.3 Did the Rand increase or decrease in value against the Dollar in 2003? (1)

4.4 A car is purchased in South Africa at a cost of R456 000.

Its value in the United States is $60 000.

(a) Calculate the Rand – Dollar exchange rate.
(Ignore the transport and import fees) (2)

(b) During which month(s) could this car have been purchased? (1)

Read the information provided below and answer the questions that follow:

When the Rand gains in value against the United States Dollar, local sales of cars increase but export sales of cars made in South Africa decrease. When the Rand loses value against the United States Dollar, local sales of cars decrease but export sales of cars made in South Africa increase.

The following formula is used to calculate R, the total revenue (or income) produced from local and foreign sales, in millions of Rands.

$R = 20a\ (40 - 1{,}75a) - 17\ (10a + 70)$

a = exchange rate, that is, the number of Rands required to purchase $1

4.5 Simplify the above formula and show that the answer is

$R = -35a^2 + 630a - 1190$ (2)

4.6 Use the formula for R to complete the table below:

a	R3,00	R8,50	R12,00
R	(a)	1636,25	(b)

(3)

4.7 A car is exported to the United States at a cost price of R98 000.
Transport and other costs are added at 11% in Rand value.
In the United States a 23% import tax is added to this price.
The Rand to Dollar current exchange rate is R6,60 to $1.
How much will the car cost in the United States? (7)

[22]

TOTAL: 80