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*An exploration of problems experienced in the interpretation of word problems by
grade 12 learners*

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ABSTRACT

This research sought to investigate the problems experienced in the interpretation of word problems by senior secondary school learners, in particular to see how the language used in the articulation of word problems affects the interpretation. The study was conducted in a school in the Oshikoto region of Namibia, a school located in a semi-rural area of Namibia, and selected owing to the accessibility of the required participants. The research was located within an interpretive paradigm focusing on a study sample of 40 learners from a specified class in the selected school. Data were collected through written tests and a semi-structured interview based on written tests, and a comprehensive descriptive analysis of test results was prepared.

The findings of the study indicate that the language in which the word problem was articulated did not make a difference. The performance in both English and Oshindonga tests was almost the same. The findings also indicate that vocabulary, syntactic interpretation, semantic relationships, algebraic skills, and practical sense making in relation to real-life are all important for the successful interpretation and solving of word problems.

In view of these findings, the study has provided valuable insights into aspects of the teacher education curriculum that need to be revisited in order to improve the training of teachers in teaching word problems.

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ACRONYMS USED IN THIS STUDY

CWI:	Consistent with Interpretation
DNEA:	Directorate of National Examinations and Assessment
DVI:	Definite Validation of Interpretation
NIED:	National Institute for Educational Development
DI:	Different Interpretation
NoI:	No Information
MOE:	Ministry of Education

Pencil, ink marks and highlighting ruin books for other readers.

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CHAPTER ONE

INTRODUCTION AND BACKGROUND TO STUDY

1.1 Introduction

This chapter provides an overview of the study. It starts by explaining why research of this nature was undertaken. The chapter highlights the background detail of the study, research questions, research approach and context, and finally gives a brief summary of the structure of the whole thesis.

1.2 Background to the study

In the area of learning mathematical word problems, one understands and masters a variety of mathematical skills, knowledge, concepts and processes in order to investigate and interpret numerical and spatial relationships and patterns that exist in the world (Schoenfeld, 1992). Everybody uses mathematical word problems in their daily lives (Bernardo, 2002).

In my experience as a mathematics teacher, I have noticed that word problems play a vital role in questions which are used in assessing learners' progress, particularly in higher grades. Also, the main problem that I have encountered while working with learners in the mathematics classroom is solving word problems correctly. Therefore, I became interested in the question of why learners perform poorly in solving word problems. My learners all study mathematics through English as the medium of instruction; and it is their second language.

Problem in solving word problems is not a new issue in mathematics education, and it has attracted many researchers, who have approached it from different angles (Schoenfeld, 1992). Most researchers agree that "solving word problems in mathematics is influenced by linguistic factors" (Bernardo & Mariss, 2005: 1).

Some claim that factors such as “irrelevant numerical and linguistic information, mathematical terminology and the vocabulary level contained within the word problems can make the wording particularly difficult to understand” (Garderen, 2004: 226). However, Bernardo and Mariss (2005: 6) feel that “solving math word problems may not be a linguistically dependent phenomenon”. Instead it was “likely to be associated with the problem modelling strategies that students were taught by their previous teachers”.

According to Frey and Hohn (2002), finding a solution to word problems involves several phases: firstly, problem translation in which factual information in the problem statement is interpreted followed by problem integration in which knowledge of problem types is used to form an integrated structure of the relationships within the problem. The next phase is solution planning, in which learners select a solution procedure, and then use this to solve the problem and get the answer. After this process a learner will try to make sense of the answer by using solution execution and monitoring. The interpretation of problems in order to ultimately form an expression that leads to the solution depends on how well one understands the language used in those word problems (Warren, 2003).

Warren (2003: 3) states that “the main reason children have problems with solving word problems in mathematics is the learning of vocabulary”. That is, a child needs to understand the vocabulary used and should be able to attach the meaning to everyday life. Having such an understanding could enable them to solve the word problem correctly. Other studies (Hubbard, 2003) indicate that translating word problems into expressions that lead to solution strategies not only depends on cognitive development but on various environmental factors. For example, how clearly the instructions are introduced to learners and also how well language used can be understood by learners (Hubbard, 2003).

Bernardo and Mariss (2005) explain that students performed better if the word problem was presented in their first language than if it were in their second language. This was because learners had a better comprehension of the problems, and thus the translation of the word problems to an expression that led to solution was more likely to be successful.

These different points of view prompted me to investigate the role of the language in which given word problems are articulated in the interpretation of word problems.

Sense-making in word problems (Schoenfeld, 1992) is something that needs to be addressed, particularly in the Namibian context where many learners study mathematics through English, which is their second language. This research aims to provide some insight into the role that language plays in the interpretation of word problems in order to form solution strategies. In particular, it may provide information about how learners make sense of word problems that are stated in English and in their home language.

1.3 Research question

The aim of this research was to investigate the interpretation of word problems by learners, and in particular to see how the language used in the articulation of word problems affects their interpretation. To achieve this goal, I investigated the following questions:

- How successful are learners in solving word problems stated in English or in their mother tongue?
- How do learners interpret, or make sense of, word problems stated in English or in their mother tongue?
- How do learners' interpretations of the word problems relate to their solution attempts?

1.4 Structure of the thesis

This document is structured as follows:

Chapter One provides an introduction to the study. It begins with background of the study and explains what prompted me to undertake this research. It also describes the research questions.

Chapter Two gives an overview of the literature that shaped and informed this research, and provides the foundation to the study.

Chapter Three gives an overview of my research design and the methodology followed. I start by providing a brief overview of the research site, including location and regional context. I then describe the design and use of the different data collection tools such as tests and semi-structured interviews to investigate the problems experienced in interpreting word problems and developing solution strategies.

Chapter Four presents the analysis of the data collected using instruments described in Chapter 3.

Chapter Five discusses the findings in the light of the research questions and the literature reviewed in Chapter 2.

Chapter Six formulates a conclusion to the study. It provides the summary of the main findings, a number of implications and recommendations, and the areas that need further study in the same field so as to explore the other issues that emerge in this investigation. It also indicates the potential value of carrying out this work in Namibia.

CHAPTER TWO

LITERATURE REVIEW

Mathematics and word problems

2.1 Introduction

This chapter gives an overview of the literature that shaped and informed this research, and provides the foundation to the proposition for this study. My proposition for investigation in this study is that *the particular language used in stating word problems could be an important factor contributing to poor performance in solving such problems.*

I start by defining what a word problem is, and also indicate the importance of word problems in mathematics. As this study is located within the Namibian context, I then give a brief overview of word problems in Namibian education policy. This is followed by discussion of semantic relations. I explain what the semantic structure of a word problem is, and how it plays a role in learners' ability to interpret and identify the correct solution strategies. Then

- I analyze the language and interpretation of word problems by looking at the representation of the problems, and show how language plays a role in translating word problems into solution strategies.
- I discuss the conflict between the semantic setting and language issue by discussing how learners develop the understanding required to interpret a word problem. In this section I discuss the implications of the 'semantic' and 'language' approaches for this study.
- I discuss related research in the study of bilingual word problems. In this section I look at specific results about 'comparisons' of comparative work discussed earlier in this chapter.
- I end this chapter by discussing the role played by algebraic skills in the interpretation and solving of word problems.

2.2 Word problems

According to Durkin (1991), word problems play a vital role in assessing learners' progress, particularly in grades 8-12 of high schools. This indicates that many mathematical exercises are given to learners as word problems. For example, two or more related pieces of information are presented, and the learners' task is to supply a missing number or required number by performing the appropriate mathematical operations and skills. Here is a typical example "*A bottle has a mass of 280g when empty. When it is full of water, the total mass is 540g. What is the total mass, when the bottle is half full of water?*" Frobisher (1994) states that word problems are excellent classroom examples of the task-environment, goal-oriented views of problems. Unfortunately, many learners perform poorly in solving word problems.

2.2.1 What is a word problem in mathematics?

Sam and Valentine (2004) define a word problem as a description of a problem situation in which some quantities are made explicit in the problem, while others are implicit. On the other hand, Stapel (2000) defines a word problem as a question which needs the use of mathematics skills in order to achieve a solution, but in which the requirement of procedures has first to be extracted from within sentences. For example, "*a Mathematics class has 5 more girls than boys and there are 31 learners altogether. Find the number of boys in the mathematics class.*" Here, a learner needs to translate the problem text into algebraic expressions, as in "the number of girls in the classroom can be represented by $x + 5$, and the number of boys can be represented by a variable x ." Then a learner needs to form an equation, such as $x + x + 5 = 31$. It will be after forming an equation that a learner can manipulate it to find the number of boys in the classroom.

This research states that "the word problems are often not particularly novel, being frequently simply another way of providing practice of simple algorithms. Indeed, they often only require the application of the four rules, which are addition, subtraction, multiplication and division" (Stapel, 2000: 172).

For the purpose of this research, I define a word problem as a statement describing a situation and a question, given in words (in sentence form), which requires learners to do the following:

- Develop an understanding of the relationship between quantities given.
- Know the meaning of all or some key concepts used in the problem (this includes the key words or terms).
- Make sense of the problem text and see if it requires application of real life considerations when finding the solution.
- Interpret and develop solution strategies.
- Use those strategies to solve the problem.

2.2.2 Two types of word problem

For this research, I explore and discuss two types of word problems. The first type is abstract and the second is contextual. The next section discusses these two types.

2.2.2.1 Abstract word problems

This type of word problem is referred to as a “story problem” set in the words using a language such as English, but lacking real life context (Toom, 1999). This is more common in abstract mathematics because it is often found at the end of a set of algorithmic exercises in many mathematics textbooks. Learners are expected to apply mathematical algorithms in a pseudo real-life context (Polya, 1957).

For example, *Petrus thinks of a number. He subtracts three from the number, doubles the answer and then gets an answer of 12. Find the number Petrus was thinking of.* It must be noted that this example does not provide any real-life context. It is the absence of a context that makes it abstract by this definition. This form of word problem can also be referred to as a mathematical question which is in symbolic mode, but expressed in words.

2.2.2.2 A contextual word problem

“A problem is contextualized if the mathematical practices in which students engage are integrated in a larger array of meaningful practices” (Roth, 1996: 487). This means contextual word problems are any problems that arise naturally in real world contexts and need mathematics skills for their solution. These problems are thus set in a real world context. Such problems can vary from being very simple, one-step arithmetic word problems, to highly complex problems. For example, “*what will be the effect on the climate in Namibia if the world's exhaust emission remains at the current level?*” This question needs one to use a higher order of critical thinking. And it should be answered by a student who understands what ‘exhaust emission’ is in everyday life.

Both types of word problems (contextual and abstract) can vary from simple, one step calculations to highly complex interpretations and operations. Roth (1996) states that, problem solving of these types is a complex cognitive activity requiring students to see relationships in order to gain meaning and interpret them correctly in order to develop a solution strategy.

2.3 The importance of word problems in mathematics

This section discusses the importance of abstract and contextual word problems in mathematics. Abstract word problems are less relevant than concrete ones for real life, but are essential in mathematics for the development of critical thinking when approaching problems in practical life. Abstract problems can be used to train students to think creatively and develop problem-solving abilities (De Corte, Greer & Verschaffel, 2000).

Greeno and Riley (1987) suggest that through interpreting and solving contextual word problems learners see how they can use mathematics in everyday life, as they help learners with the following:

- They help them understand the part which mathematics plays in the world around them.
- They show pattern and relationship in mathematics.

- They enable learners to recognise when and how a situation may be represented mathematically.
- They develop in learners the ability to apply mathematics in other disciplines.
- They develop the ability to understand, interpret and make sense of everyday situations in mathematical terms.

2.4 Word problems in Namibian education policy

According to Namibian education policy, one of the major goals in mathematics is to “provide all learners with opportunities to shape the conditions which govern their access to knowledge, skills and understanding in classrooms and schools” (Namibia. Ministry of Education (MoE), 2006: 5). Learners are viewed as active mathematical thinkers who try to construct meaning and make sense for themselves of what they are doing on the basis of their personal experience (Namibia. MoE, 2006)

National subject policy (Namibia. MoE, 2006) states clearly that in learner-centred education, word problems, with various structures, are recommended to be used, and learners should find solutions by first making sense of the problem text, and then interpreting it in a way that leads them to a solution strategy.

National subject policy (Namibia. MoE, 2006) specifically underlines the importance of the use of word problems in the Namibian mathematics curriculum:

Mathematics is one of the subjects with high demands in today's competitive world. We always apply mathematics' word problems in everyday life, for instance, when we do shopping (cheap/expensive), when we travel (distance, speed and time), when we do planning, etc. Hence everybody needs a basic knowledge of mathematics. In the mathematical area of learning, learners understand and master a variety of mathematical skills, knowledge, concepts and processes, in order to investigate and interpret numerical and spatial relationships and patterns that exist in the world. Word problems in mathematics help learners to develop accuracy as well as logical and analytical thinking, and apply them to other areas of learning and real life.

(pg. 3)

The policy stresses that by solving word problems in the classroom, learners could be equipped with skills to use mathematics in everyday life situations such as shopping or calculating of speeds, distances or time. For example, “*Jeff takes 10 minutes to walk 1 kilometer. Find his average walking speed in kilometers per hour*”. Or “*When the mass of fertilizer in a bag is reduced by 30kg, I need to buy 10 more bags to get 75kg of fertilizer. What was the mass before the reduction?*” These are some examples of word problems that are used in Namibian classrooms. It is therefore very important for learners to understand and be able to interpret the problems in order to represent a word problem correctly. This enables them to apply mathematics skills in their everyday life.

Contextual word problems could help learners to recognise the relationship between what they learn at schools and what they bring to schools from home. According to Bernardo and Mariss (2005), word problems are an integral part of mathematics education in most parts of the world because these problems allow students to apply their mathematics knowledge and skills to real world situations. To solve a word problem correctly, a student needs to understand and interpret problems in way that leads him/her to obtain correct solutions. Fennema and Romberg (1999) claim that learners need to be able to construct relationships between quantities in the text.

2.5. The role of semantic structure in understanding a word problem

In this section, I discuss what semantic structure is and the skills required to understand the structure underlying word problems. I then discuss the role played by the semantic structure of word problems in learners’ ability to interpret and identify the correct solution strategies required to solve the word problems. I conclude by introducing the debate about language versus mathematical concepts.

2.5.1 What is semantic structure?

From a mathematical point of view, semantic structure can be defined as the relationship between the words and meaning of the statement given. According to De Corte, Greer and Verschaffel (2000), semantic structure refers to the meaning of the statements in the problem and their interrelationships. “By semantic relations we refer to the conceptual knowledge about increases, decreases, combinations and comparisons involving sets of objects and others” (Cummins, 1991: 262).

Fennema and Romberg (1999) claim that, taking the view of semantic structure and relations, learners are required to use questions like: “what are you talking about or what is asked here? What is being given here? What is talked about? What are the key concepts in this statement? And what are the relations between words and the things talked about?” (pg 20).

These are some of the questions that are suggested to help learners to get the meaning of a statement given and also help learners to develop an understanding of the statement and develop correct relationship between quantities. Similarly, Garderen (2004) says that learners are expected to match their problem solving strategies to the structure of the problem. The section below explores the role of semantic structure of word problems on learners’ ability to interpret and identify the correct solution strategy.

2.5.2. Roles of semantic structure of word problems on learners’ ability to interpret and identify correct solution strategies.

Research shows that learners are required to understand structures underlying word problems. Sam and Valentin (2004) indicate that a skilful solution process of a word problem starts with the construction of a representation of the basic semantic relationships between the main quantities in the problem.

Bernardo (2002) argues that learners understood a word problem when they are able to form an accurate mental representation of the different quantitative elements of the problem and the relationships among the elements that are relevant for solving the problem. For instance, “*Vaino has 3 toffees. Hanna also has some toffees. Vaino and Hanna have 9 toffees altogether. How many toffees does Hanna have?*” He claims that if students are able to infer the part-whole relation between the 9 toffees that Vaino and Hanna have altogether, and the toffees that each of them has on his/her own, then this helps them to solve the problem correctly. The part-whole relation is clearly, but not explicitly stated (De Corte & Verschaffel, 1991).

De Corte and Verschaffel (1991) argue that if students do not have relevant schemata of knowledge, then there is no way they can infer the relation between the distinct given quantities, and this may lead them to interpret each problem sentence separately and get wrong solutions. This research indicates that semantic relations in the problems between the two parts and the whole could be made more explicit. For example, “*Vaino and Hanna have 9 toffees altogether. Three of these toffees belong to Vaino and the rest belong to Hanna. How many toffees does Hanna have?*” This example does not directly support this research, but it shows that rewording the problem may reduce the representational errors that are made in the standard one.

Taking the views above, I argue that learners are only able to interpret a word problem and identify correct solution strategies required to solve it if they seek for a relationship between the quantities that are stated in the problem, and then create the relationship between what is asked and what is given.

The research by Polya (1957) indicates that there are many important aspects a student needs to consider in order to understand the problem before trying to solve it. First, he/she needs to start by understanding what the problem is asking. And he/she needs to start to figure out what information he/she already knows, and what he/she needs to know in order to develop a solution strategy.

Polya went further and suggested that for students to understand the structures underlying word problems and identify the solution strategy required to solve them, they should do the following:

“Ask: what is the unknown? What are the data? What is the condition? Ask: is it possible to satisfy the condition? Is the condition sufficient to determine the unknown? Or is it insufficient? Draw a figure, introduce suitable notation. Separate the various parts of the condition. Ask: can I write them down?”

(pg. 164)

It is after students asked themselves the questions above, then they could have a good understanding of the problems that would lead them to look for the connection between the data and the unknown. This means they could start by identifying relationships between the quantities in the written text (the semantic relationships), and develop an interpretation and the solution strategies required. This is supported by Frey and Hohn (2002) who say that at this stage a solver is doing problem integration in which knowledge of problem types is used to form an integrated structure of the problem's relationships. However, Garderen (2004) indicates that students lack those skills.

Various researchers have suggested different skills that learners need to have when approaching word problems. And those suggestions help learners to comprehend the semantic relations underlying a given word problem text and its mathematical structure. Here are some of them:

- Polya (1957) states that a learner should begin by considering whether he/she has seen the problem before, or has seen the same type in a different form.
- “Things take meaning from the ways they are related to other things. Learners construct meaning for a new idea or process by relating it to ideas or processes that they already understand” (Fennema & Romberg, 1999: 20). It is therefore very important to think of the related problem, or see if he/she can remember any theorem that could be useful to solve the problem given. If there is a related problem that a learner can remember, then a learner can start thinking if he/she has solved that type of problem before, and use its result to solve a new problem (Polya, 1957).

- It is also important to check if it is possible to reword the problem in order to make it more explicit and understand it better than before.

To sum up this section, it is clearly shown that the semantic structure and relations underlying a given word problem text play a vital role in the interpretation and representation of quantities in arriving at correct solution strategies. The next section discusses the issue of language in interpreting and understanding word problem texts.

2.5.3 Language and Concepts

Durkin (1991) indicates that mathematics contains many challenges, which may become a barrier to learners' developing mathematical proficiency. Even though he does not mention conceptual understanding of word problems, he points to language as a contributing factor to success in mathematics.

De Corte and Verschaffel (1991) claim that students fail to understand and represent the problem because they do not have the appropriate situational problem schemata. But, even if students have this knowledge, incomprehension or misinterpretation of a particular concept, word or expression in the problem statement may prevent them from constructing an appropriate representation of the problem situation. For example, in the problem "*Vaino has 3 toffees. Hanna also has some toffees. Vaino and Hanna have 9 toffees altogether. How many toffees does Hanna have?*" students may misinterpret the word "altogether" as "each". This means that the semantic relationship is not the only factor contributing to the interpretation of a word problem: language is also a major factor to be examined.

2.6 Language in word problems

In solving mathematics word problems, learners must possess not only formal mathematical knowledge, but also linguistic and situational knowledge. Kilpatrick and Nesher (1990) consider that what makes a word problem difficult and intriguing is not its formal properties, but the way it is represented linguistically, and the way formal mathematical relations map onto it.

Kilpatrick and Nesher (1990: 60) claim that “understanding what is to be solved requires understanding the problem statement given in an oral or written form”. The question one could ask is what role language plays in order to understand and interpret the word problem correctly. Of course, different researchers could focus on different issues. But for this research, the issue of vocabulary, order of words and other related items are taken as the focal issues.

2.6.1 Vocabulary

Kilpatrick and Nesher (1990) suggest that to grasp the meaning of what is stated in the given word problems, the words need to be understood very well. Frey and Hohn (2002) argue that interference with comprehension of a word problem may be caused by difficulty with the words and context. This is likely to be common with students who speak English as a second language.

The research by McNutt and Wheeler (2001) indicates that many students have difficulty with solving a word problem, which could be caused by unfamiliar vocabulary. This may be applicable in the Namibian context, where learners study mathematics through English, which is a second language to them. Warren (2003: 3) states that “the main reason children have problems with solving word problems in mathematics is the learning of the vocabulary”. That is, a child needs to understand the vocabulary used and should be able to attach the meaning to everyday life. Having such an understanding could enable them to solve the word problem correctly.

Hubbard (2003) states that if the instructions are not clearly presented to learners, and if learners do not adequately understand the language in which a given word problem is articulated, then the interpretation of the problem into solution strategy will be unsuccessful. However, the research shows that even though students might know the vocabulary, it is also possible that “the arrangements of the words prevent some or all students from understanding the problem much better” (2003: 119).

2.6.2 Syntactic structure of the word problem

The research by McNutt and Wheeler (2001) states that the sequence of words (syntactic structure) in the problem text may prevent learners from interpreting the text correctly. This means that the order of words in the text plays an important role in understanding the text. If the syntactic structure is too complicated, this may affect interpretation of the text, which may lead to incorrect representation of the problems. The research by Hubbard (2003) clearly shows that learners use the word sequence in the story to form the equation.

The other issue with syntactic translation is the use of “inconsistent language” in the problem text (Xin, 2007: 347). Xin’s research indicates that students have difficulty in solving the word problems that involve “inconsistent language”, in which the use of a “key word” (e.g. “times”) does not cue the operation.

For example, in most cases a problem would ask students to find a solution using, for example, the multiplication operation; but what is stated in the text is a different operation, like division: “*Linda made 12 clay pots in one week. She made 3 times as many clay pots as Ben made in the same week. How many clay pots did Ben make in that week?*” Of course in interpreting the text the word “times” may be used as a cue for multiplication, but students need to use division to solve the problem correctly (e.g. $12 \div 3 = 4$ clay pots that were made by Ben). These problems are common in Namibian classroom mathematics exercises.

Conversely, in the problem “*Ben made 4 clay pots this week. Linda made 3 times as many clay pots as Ben. How many clay pots did Linda make?*” the word “times” is consistent with multiplication as the correct process to solve the problem (e.g. $4 \times 3 = 12$ clay pots). It was noticed that students find this type of word problem relatively easy to solve. This is because “students have schemata in which only those preferred story constructions fit” (Xin, 2007: 348).

Researchers point out that “students experience more difficulties with the ‘inconsistent’ language problems than problems with extraneous information” (Xin, 2007: 348). When students meet a word problem with inconsistent language, they have to reorganise the information presented, and in the process many of them make mistakes which lead them to get the wrong solution (Xin, 2007).

2.7 Language versus semantic structures

This section aims to distinguish between different interpretations caused by the semantic structures underlying the word problems, and different interpretations due to purely language issues. It draws on ideas from the semantic structures and language sections.

It has been clearly shown in previous sections that semantic structure includes the ways in which an interpretation of the text points to particular mathematical relationships, while the influence of language has to do with words and phrases (Garderen 2004). This is supported by Cummins (1991), who suggests that certain words and phrases are ambiguous to a child, and that use of such terms in a word problem leads to incoherent representations (a purely language issue).

For example, “*when a number is added to two, the answer is same as when fourteen is subtracted from three times the same number. What is the number?*” A child could come up with an expression like $2 + 3x = 14 - 3x$, $x + 2 = 14 - 3x$, $x + 2 = 14 - 3$ owing to the different understanding of the structure and relations underlying the problem text, and also to a language related problem. For example, the first expression, $2 + 3x = 14 - 3x$, implies that the child does not construct the correct relationship between quantities in the statement (e.g. the relationship between 2 and $3x$, or 14 and $3x$), and at the same time it is implied that the child does not understand “subtract from” (a language-related problem), resulting in $14 - 3x$.

The research by Cummins (1991) suggests that the common failure of students to represent the problem in the correct way is a result of their misinterpretation. For example, "*Tomas has 5 more oranges than Sara*" is misinterpreted as "*Tomas has 5 oranges*". This is a semantic relation issue, a lack of conceptual understanding of the problem text. Here the students seemed not to understand the comparative nature of the concept "5 more oranges than". This leads students to encounter problems when solving such word problems.

Apart from constructing an appropriate representation of the problem situation in terms of sets and set-relations, one must be in position of the interplay between the particular text and the person's knowledge about problem situations and linguistic terms in order for the construction of a problem representation to be correct.

(De Corte & Verschaffel, 1991: 79).

According to Schoenfeld (1991) the representation of a word problem in term of the semantic relations between the constituting elements is also seriously affected by the solver's knowledge of the unusual type of text that a word problem is, the arrangement of words in the text, and the scholastic context in which it is encountered. This has been discussed in the previous sections.

2.8. The effects on solving mathematical word problems stated in the student's first and second language

This section seeks to investigate whether being weak in reading and understanding the meaning of concepts stated in English or the home language has an impact on the interpretation of word problems. It will also investigate whether weakness in making meaning from more complex written sentences or in identifying relationships between the quantities in the written text indicates that language may be the problem. It draws on research that has been carried out on bilingual learners and word problems. The research indicates that linguistic factors influence the representation of word problem into the structures that lead to a solution strategy of the problem. For example:

- Bernardo and Marissa (2005) studied bilingual students in Papua New Guinea and gave students a test in English which was not their first language. The outcome of this study indicated that about 39 per cent of the errors were related to reading mistakes and comprehension errors.
- In addition to that, “Filipino-English bilingual students who were solving arithmetic word problems, were better at solving word problems presented in their native language (Filipino) than in their second language (English)” (Bernardo & Marissa, 2005: 2). The better performance in the Filipino word problems was evidence that having a better comprehension of the word problems will result in their correct representation. This was the reason why the students were performing better in the Filipino word problems.
- In a related study, Bernardo (2002) found bilingual students were poor in solving arithmetical word problems in both the first and second languages compared with the same problems presented in a purely numerical format. Clearly, linguistic factors either in the first or second language plays a great role in understanding and interpreting the word problems.
- In support of the above claim, Cummin and Swain (1986) claimed that learners can do well in the second language provided that the vocabulary used in the second language (in this case English) exists in their first language, as this is the only way they will be able to make meaning out of them.

Learners may have difficulty in interpreting mathematics word problems in that some vocabulary used in mathematics is difficult to attach to the words that exist in the learners’ everyday life. This could be one of the reasons that some researchers suggest that “the mind is less efficient in second language whatever it is doing, there is second language ‘cognitive deficit’, as it is sometimes called” (Cook 1991: 53).

- Research by Bernardo and Mariss (2005) indicates that students produced more expected answers by solving the word problems stated in the students’ first language (Filipino) than by solving the same ones in the student’s second language (English). They observed that students made technical errors in solving the word problem whatever language it was stated in.

Bernardo and Mariss (2005) conclude that students performed better if the word problem was presented in their first language than if it were in their second language because learners had a better comprehension of the problems and thus the translation of the word problems to an expression that led to a solution was more likely to be successful.

2.9 Algebra

In this section, I explore the use of algebra in solving word problems in mathematics.

2.9.1 Equality formulation

There are often two steps required to solve mathematical word problems using algebraic expressions. The first is to translate the wording into a numeric equation that combines smaller “expressions”. This requires an understanding of the meaning of algebraic variables (Hubbard, 2003). Hubbard argues that the symbols or variables representing the number of quantities in a problem text are used inconsistently. Sometimes students make errors when expressing the quantities in terms of a variable.

Hubbard’s research on the modelling of word problems leading to algebraic equations shows that writing an expression is simple, and does not require the creation of relationships and an understanding of equality. This is unlike the writing of equations, in which a relationship is implied, and an understanding of equality is required. This indicates that students could make more errors when constructing equations than just writing expressions.

Many word problems require the use of algebraic skills to find the solutions. The research indicates that to prepare for success in solving word problems using algebraic skills, students need to develop a relational understanding of the equal sign.

The research by Alibali, Grandau, Hattikudur, Knuth, Krill, McNeil, and Stephen (2006) indicates that sometimes students may have an inadequate understanding of the equal sign.

Instead of interpreting an equal sign as a relational symbol of mathematical equivalence, most students interpret the equal sign as an operational symbol meaning “finding the total” or “put the answer”. Students not only provide operational interpretations when asked to define the equal sign, but also rate operational interpretations such as “the total” and “the answer” as “smarter” than relational interpretations such as “equal to” or “two amounts are the same”.

(Alibali et al. 2006: 368)

The research stresses that by the time they reach higher grades, students are expected to have acquired an adequate relational understanding of the equal sign; but they continue to interpret it as an operational symbol at these grades. Alibali et al. (2006) argue that one contributing factor may be that students are not exposed to a sufficient number of equations with operations on both sides of the equal sign at previous grades.

2.9.2 Manipulation of algebraic equations

The research by Lima and Tall (2006) argues that many students solve and manipulate algebraic equations using the rule “change side change sign”. For instance, in transforming $5x - 7 = 2x + 5$ into $5x - 2x = 7 + 5$ and on reaching an equation of the form $3x = 12$, students require to move the coefficient of x to the other side of the equal sign and divide by it, in this case giving $x = 12/3$.

Such a solution involves a movement of the symbols, together with an extra technical element such as changing the sign to give the correct result or solution. The research by Lima and Tall (2006) shows that such a technique could be easily be rote-learned as meaningless embodied actions, shifting symbols and doing something else at the same time. It was noticed that students make a lot of errors in manipulating and solving a linear equation. For example, students may change sides without changing signs, or change the sign of the coefficient of x as they shift it to the other side (Lima & Tall, 2006).

This research by Lima and Tall also shows that students often refer to the use of rules to solve equations, but not many students understand that this refers to performing the same (inverse) operation on both sides of the equal sign. The language that many learners use often seemed to have an embodied meaning relating to actions performed on the symbols in the equation, such as: *“Pick this number and put it at the other side of the equals sign, or, I take off the brackets, or, The power two passes to the other side as a square root”* (Lima & Tall, 2006: 6).

These actions have underlying embodied foundations that relate not to real world activities, but to moving symbols around, with a mysterious twist to make things right (Lima & Tall 2006: 9). It seems as if students are more comfortable with trying to shift symbols rather than perform the same operation on both sides of the equal sign. The research by Lima and Tall argues that many students’ conceptions of equations and ways of solution are fundamentally based on arithmetic met before, where the equal sign is conceived as “something to do” to “get the solution”, and on what they recall from previous experience in algebra.

Similarly, students who offer an operational interpretation of the equal sign are less likely to solve algebraic equations with operations on both sides of the equal sign. In contrast, it was argued that it is possible that “a greater number of students actually understand the equal sign in a relational way, but they may be unable to demonstrate that understanding in an equation-solving situation, because the equations they typically encounter in schools frequently elicit the operational interpretation” (Alibali et al. 2006: 381). Hubbard (2003) claims that students can only manipulate equations correctly if they have an adequate understanding of equality.

We can therefore conclude that not only vocabulary, syntactic structure, and semantic structure and relation underlying the word problems involved in interpreting and solving word problems successfully but there are other aspects, such as algebraic skills. Language has its part to play, but it seems there are other many skills that students are required to have in order to tackle word problems.

2.10. Conclusion

In this chapter, I have briefly discussed what a word problem is and explored the historical background of the type of word problems used in this research. I discussed the theoretical framework showing the importance of mathematical word problems in the Namibian context and its policy. This chapter also dealt with the role of language usage in word problems, and the effect of semantic structures underlying word problems on learners' ability to interpret and identify the correct solution strategies.

I outlined the analysis of research studies on solving word problems among bilingual students, and discussed how algebraic skills play a vital role in solving word problems. The research methodology is discussed in the next chapter.

CHAPTER THREE

RESEARCH METHODOLOGY

3.1 Introduction

This methodology chapter explains how the researcher investigated the research question. The chapter discusses in detail the research design and methods that the researcher used to collect and analyze the data. Design and methods were selected carefully to suit the aims of the research study. The main aim was to investigate the interpretation and solution of word problems by learners, in particular to see how the language used in the articulation of word problems affects their interpretation and solution.

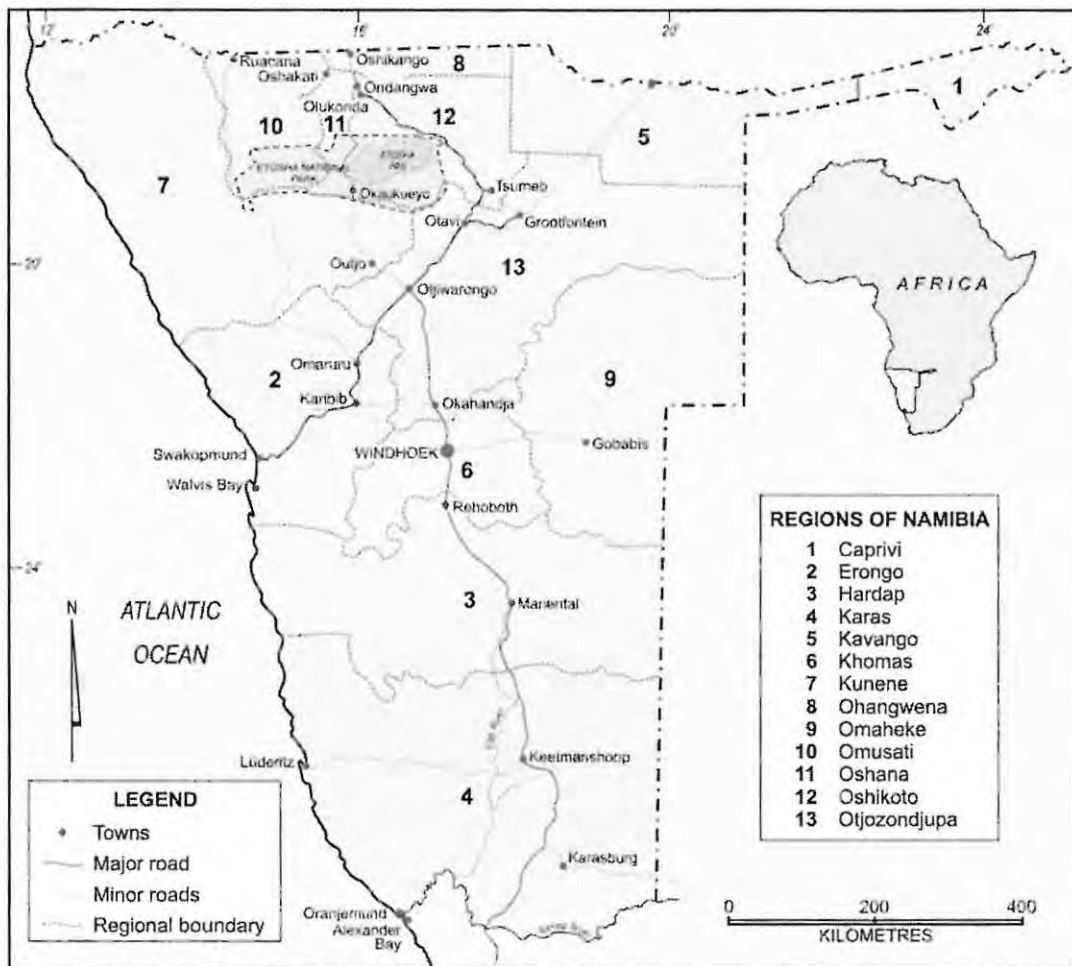
3.2 Research site

The study was conducted at a senior secondary school 30km from Ondangwa in the Oshikoto region of Namibia. The 54 per cent of learners in this school are from rural schools and areas. The learners' first languages can only be Oshindonga or Oshikwanyama. All learners study mathematics with English as the medium of instruction, which is their second language. The school has 870 learners. I have decided to conduct this research at the school where I was teaching, because there I had good access to as many learners as required for data collection. I used one specified Grade 12 class with 40 learners because this was the class that was regarded as having good mathematics learners as compared to other classes. For this reason, these learners were expected to work through a set of word problems in a reasonable time. Also these learners were more articulate, and so were better able to answer reflective questions.

3.2.1 Location and regional context

The Oshikoto region is in northern Namibia, and is one of the country's thirteen regions. The main languages spoken there are Oshindonga and Oshikwanyama. According to the education management information system for 2005, Oshikoto has 178 schools: 115 primary schools, 49 combined schools and 14 secondary schools (Namibia. MoE, 2005),

Figure 1: Regions of Namibia.



(Abrahams, 2006. Graphics Services Unit, Rhodes University, Grahamstown)

3.3 Research design

This research is a qualitative case study located within the interpretive paradigm. Cohen, Manion and Morrison (2000: 36) state that “the central endeavor in the context of the interpretive paradigm is to understand the subjective world of human experience”.

The main aim of this research was to investigate how learners make sense of word problems. The intention was to gain a deeper understanding of learners' interpretation of word problems, in particular to see how their interpretation may be affected by the language used in the articulation of these problems.

In qualitative research, one seeks to interpret, understand, explain and bring meaning to the collected data (Henning, Rensburg & Smit, 2004). Therefore, research design should show how all the major parts of the research study work together to try to address the central research question, from the planning of the enquiry to designing a strategy for data collection and analysis. "A good design points clearly to what the researcher intended to investigate, using the most appropriate way possible for that particular research" (Maxwell, 1996: 3). However, there is a concern that defining research design as a "pre-established plan for carrying out a study or as a sequence of steps to conduct a study may not be a compatible definition" (Maxwell, 1996: 4).

3.4 Research Methodology

The research methodology refers to all the tactics and strategies that I used to collect and analyze the data for this research study.

3.4.1 The Sample

Convenience sampling was used for this research. The research was conducted at the school where I was teaching, because there I had good access to as many learners as required for data collection. I used one specified grade 12 class with 40 learners because this was the class that was regarded as having higher performing mathematics learners than other classes. It was judged that this would be an advantage because these learners were expected to work through a set of word problems in a reasonable time. Also these learners were more articulate, and so were expected to be better able to answer reflective questions.

3.4.2 Data collection

I used two tools to collect data, namely tests and semi-structured interviews. In section 3.4.3 I describe how the tests were administered. In section 3.4.5 I describe how and why I opted for interviews as my second tool to collect data.

3.4.3 Tests

One class of grade 12 learners in the school where the research was conducted was asked to write two tests. The first test was set in English. The second test was the same as test one, but it was set in the mother tongue, Oshindonga, the first language at the school. The tests consisted of 10 questions to allow learners to finish answering them in 1 hour and 30 minutes (see appendix 2).

The grade 12 class of 40 learners was split into two equal groups, A (20 learners) and B (20 learners), for the purpose of writing the tests. In week one, group A first wrote the English test and at the same time group B wrote the Oshindonga test. A week later, the two groups switched, with group B writing the English test and group A writing the Oshindonga test. The plan to have two groups, each seeing the tests for the first time in a different language, was included in an attempt to reduce the effect of learners having seen the questions before in a particular language.

3.4.4 Choice and Alignment of questions

As a source of questions for the test design, I used different question papers from the school where I was teaching and from neighbouring schools. This enabled me to gather as many different questions involving word problems as possible. I then selected and justified appropriate and suitable questions for this research. I modified some questions to make them more appropriate for the study. I then asked one of my colleagues to check the appropriateness of the questions selected.

The selected questions were those that required learners to understand some key concepts so as to construct the relationship between the quantities stated in the text, and did not involve only a single contextualized operation. For example, a question such as “There are five sweets in a box. If Maria took 3 how many sweets are left in the box?” was not chosen for this study because it involves only a single operation. But a question like “When the mass of fertilizer in a bag is reduced by fifty grams, I need to buy ten more bags to get thirty-six grams of fertilizer. What was the mass before the reduction?” was suitable for this study because learners are expected to construct the relationship between quantities stated in the text; it did not only involve a single contextualized operation.

In this study, I used both contextual word problems and abstract word problems (see Chapter 2, page 7). I expected the use of both types of word problems to provide me with a broad understanding of how learners interact with different word problems.

3.4.5 Interviews

Interviews were also used as data-gathering tools because the test could not provide information about learners’ feelings and perceptions (Patton, 1990).

After the tests had been written I studied learners’ responses and made inferences about their possible interpretations of the questions. I formulated questions for the interviews based on this analysis.

The aim of interviewing learners was to find out how they went about answering the questions in the tests. I discussed the questions and their solutions with them in order to elicit information about their interpretation of the questions. I tape-recorded our discussions during the interviews, and later I prepared full transcriptions.

The information obtained from the interviews was used to:

- validate the inferences about interpretations that I made from the tests written by these learners, and correct these inferences where necessary.
- make inferences about interpretations of some written responses that I was previously not able to analyse.
- develop a deeper understanding and analysis of what was written in the tests.

The saturation model for interviewing was deliberately adopted from the start. It means interviewing as many learners as possible until one is no longer getting anything new, or is at least certain of having enough interesting data.

3.4.6 Pilot test of the instruments

The two instruments were piloted in order to determine how effective they were in gathering the data I needed to explore the research question—that is, how comprehensive they were and how accurate in collecting data. De Vos (1998) views the pilot study as “the dress rehearsal of the main investigation and . . . similar to the researcher’s planned investigation but on a small scale” (179).

Because the medium of instruction was English, initially the questions were in English. To obtain the mother-tongue test, I translated the questions from English into the mother tongue, Oshindonga. After translating the questions from English into the mother tongue, I then conducted my first phase of piloting, by giving my questions to my two colleagues to write as a test. I then gave a copy of the original questions to the teachers and asked them to comment on the accuracy of the translations and invited improvements where necessary.

In the second phase of piloting I piloted the tests on learners. I used 10 learners from other classes at the same grade. I asked five to write the first test, and five the second test. When I piloted the tests and interviews on selected learners, the experimental group was not aware of their participation in this study and the piloting process was conducted three months in advance. For this reason, the interaction between the pilot group and experimental group was not expected to unduly influence the findings obtained in this study.

Next, I conducted a pilot interview with three learners from those who wrote the pilot tests. I chose three learners because I hoped that this would be enough to provide me with the necessary feedback that I needed to make some adjustments, if necessary. My purpose in piloting the interview was to try to determine its duration, the suitability of the questions, and practical issues related to the use of a tape recorder.

As a result of my piloting the instruments, some questions were replaced completely as they did not appear to produce enough good information about interpretation. Some abstract questions were replaced with similar questions because there were many more abstract word problems than contextual word problems. These were replaced with contextual word problems. I made other adjustments to my questions like changing the order, and rephrasing.

As a result of the pilot interviews, I learned that I should probe more to get in-depth information, and also what to say and do and what not to say or do so in order not to influence the learners. Finally, through the pilot I learned how to transcribe an interview, and how to translate it.

3.5 Data analysis

According to Lankshear and Knobel (2004: 266) data analysis refers to “the process of organizing the collected pieces of information, identifying their key features or relationships systematically, and interpreting them meaningfully”.

The ways of identifying important features in data is always also informed by theory and is directly related to a research question for that particular study. Data analysis involves the selection, grouping and synthesizing of the collected information.

My analysis had two phases: the first was initial analysis of solution strategies. Here I analysed the written responses by individual learners, organizing and grouping all emergent strategies from written tests according to their similarities. I then verified or validated my interpretation of all strategies with the information I got from the interviews.

The second phase was thematic analysis. Here I used initial analysis to search for themes and select them, and ensure that my consolidated themes were relevant to the research problem. I then compiled these consolidated themes for every problem to ensure they were each well represented. Finally, I used the themes and example strategies relating to the themes to develop deeper insight into the research problem (Cohen, Manion & Morrison, 2000).

3.6 Triangulation

I used triangulation methods to enhance the validity of my research. Triangulation, according to Jacob (2000: 1), refers to “the application and combination of different research methodologies in the particular study of the same phenomenon”. Combining several methods or instruments assists the researcher to develop a deeper understanding of collected data and avoid biased interpretation of data coming from the use of a single method.

Triangulation helps one to detect errors made in analysis of collected data. Qualitative researchers seem to emphasize that the purpose of triangulation in a specific context is to obtain confirmation of findings through the convergence of different perspectives (Jacob, 2000).

According to Henning, Van Rensburg and Smit (2004: 127), “the qualitative researcher should make sure that the findings portray authenticity or internal validity of what is studied”. This is enhanced by means of triangulation. In other words, the researcher should ensure that the findings of the study do make sense to the readers and to the study itself.

For that reason, triangulation is very useful in research, to ensure that the data collected and analyzed is both valid and reliable. Although there are various kinds of triangulation, in this study I collected the data from written tests, and also by conducting interviews.

3.7 Ethical standards

I followed normal protocol in this form of research with respect to ethical standards. According to Callahan and Hobbs (1998: 1) the primary concern of a researcher should be the safety of the research participant. Privacy and confidentiality concerns should also be approached carefully.

I asked for permission to do my research from the school Principal in a formal letter. I also wrote to the Regional Director for permission to carry out my research in the school (see Appendix 4). I then informed the participants of the aims of the research and discussed with them how and when they would write the tests. We set up the dates together for both tests and interviews. I assured all participants and the school that all information would be treated as confidential. I made it categorically clear that everybody was free to withdraw from the research if they wished.

3.8 Limitations

The size of my sample was limited to 40 learners, and for that reason I cannot generalize the findings. Being a teacher of my participants, my learners were not comfortable enough to speak to me freely during the interviews. Recording the interviews made learners even more uncomfortable, even though I have tried to make the reasons for doing so clear to them.

3.9 Conclusion

In this chapter I described the research design I used and focused on the methods used to collect data in order to answer my research questions. Written tests and semi-structured interviews have been used as the main tools for collecting data in this study. I took into account the ethical standard, validity and limitations of the study, and explained the data analysis style that was administered.

CHAPTER FOUR

ANALYSIS OF DATA

4.1 Introduction

In this chapter, I report on my findings from the written tests which were in both English and Oshindonga. I use the data from the semi-structured interviews to validate the inferences about interpretations that I made from the tests written by these learners (see Chapter 3, page 28).

This chapter is presented as follows:

- In section 4.2 I present the initial analysis of the data.
- In section 4.3 I carry out a thematic analysis, building on the initial analysis to identify the most important themes relevant to the research question or goal. The themes identified are; vocabulary, syntactic structure, concept-related issues of the problem, failure to relate the problem to practice, equality formulation, manipulation of algebraic equations, inconsistent use of a variable, making sense of the statement without turning it into a useful solution strategy, and minor errors.
- In section 4.4, which is a full thematic analysis, I consolidate similar emergent themes from different problems under the same themes. I summarize all emergent themes using a table for each problem.

4.2 Initial analysis

4.2.1 Discussion of the approach

First, I analyse each learners' test responses in order to identify, organize and group the solution strategies used. Second, I look for pattern from individual responses as compared to other responses, and then group the solution strategies according to their similarities. Third, I draw a table and find the number of learners that used each strategy. Finally, I endeavour to explain why learners did it the way they did. This means I start making inferences from interpretations of learners' written responses.

I then use the information from the interviews to validate the inferences I made about learners' interpretations. I use four categories to show how my interpretations of solution strategies were validated. The categories were as follows:

- NoI – No Information: This problem was discussed but there was no information from interviews to validate the interpretation of solution strategy.
- CWI – Consistent with Interpretation: The information from interviews does not conflict with my interpretation.
- DVI – Definite Validation of interpretation: The information from the interview is definitely consistent with my interpretation of the solution strategy. What I said is what learners said.
- DI – Different Interpretation: The information from the interview was different and provided an alternative interpretation of the solution strategy. This alternative will be discussed.

In the following section, I will show the initial analysis of two problems in detail. The rest of the initial analysis is given in Appendix 1. For each problem, I start with a description of the task, and show this in both English and Oshindonga as it was presented to learners. I classify and justify the choice of each task, and then present the initial analysis in five parts:

- (i) Expected relationship or equation: Here, I discuss the expected relationship of quantities in each problem that would lead learners to the correct solution.
- (ii) Expected solution: Here I give the correct answer to each problem including all steps involved.
- (iii) Errors in strategies: Here I classify all the solution strategies used in each task, according to the errors that were made by learners. I classify errors into two categories:
 - Major errors – refers to serious interpretation errors which occurred due to complexity of the vocabulary, syntax, misunderstanding of semantic structures and relation underlying the problem text and solving incorrectly.

- Minor errors – refers to simple mistakes which occurred due to silly mistakes, which shows the lack of concentration.
- (iv) Unexpected solutions: Here I give the various answers from learners' works due to the errors in (iii).
- (v) Other: Here I show everything that had emerged in the learners' responses that I could not analyze.
- (vi) Finally, I draw a table of the proportion of all learners falling into each category identified in (iii), for both the English and the Oshindonga tests. These proportions are expressed as percentages with the actual number of learners given in brackets.

In my discussion in part (iii) I use the codes above to show where the interpretation of each strategy falls after the interviews. If there was no discussion of the strategy in the interviews, no validation code will be given. Finally I use the initial analysis of the errors in strategies in each task to identify emergent themes, and I draw a table to show the number of learners in each theme from both English and Oshindonga tests.

4.2.2. Detailed examples

4.2.2.1 Problem 1

Description of the problem in English and Oshindonga.

<p>When a number is added to two, the answer is the same as when fourteen is subtracted from three times the number. What is the number?</p>	<p>Ngele onomola oya gwethwa kumbali, eyamukulo olyafathana nuuna omuulongo nane gwakuthwa momulongo nandatu gwi indjipalekwa nonomola ndjoka. Onomola oyini ndjono?</p>
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This problem illustrated above is an abstract word problem and I expected many learners to interpret it into an algebraic expression before coming up with a solution. That is the reason I chose it to be used in this study.

i. Expected relationship or equation

As a first step, learners were expected to form the following relationship of quantities in the problem that would lead them to the correct solution. These relationships could be expressed using one of the following equations:

$$2 + x = 3x - 14 \quad \text{or} \quad x + 2 = 3x - 14.$$

In the tests in both languages, some learners constructed one of the equations above and they solved it correctly.

ii. Expected solution

All learners were expected to give the answer of 8. This could be attained from one of the equations in (i) as follows:

$$2 + x = 3x - 14$$

$$x - 3x = -14 - 2$$

$$-2x = -16$$

$$x = -16/-2$$

$$x = 8$$

iii. Errors in strategies

There were some *common errors* made by learners in interpreting and constructing a solution strategy for this problem.

(a) $x + 2 = 14 - 3x$ DVI

These learners followed the order of the words in the problem and wrote $14 - 3x$ instead of $3x - 14$. That is, because the sentence said “Fourteen is subtracted from three times the number” so fourteen came first in the statement, then “three times a number”; so the learners wrote first fourteen and then three times a number. This error was noticed in both English and Oshindonga tests. This interpretation was definitely validated by the learner interviews.

CN: Why does your equation look like this?

L1: Here, I started writing my equation, 2 plus x ... is the number which is added to two.. .

is equal to (kept quiet... here I confused, I wrote $14 - 3x$ instead of $3x - 14$ because the statement said 14 is subtracted from three times a number but it did not say three times a number was subtracted from 14.

CN: What made you confused, and write that expression?

L1: Sir.. (kept quiet) I think the problem was here.

CN: Where?

L1: It said 14 is subtracted from three times a number, maybe when I saw 14, I just wrote it and subtracted three times a number. But it is wrong.

CN: Hm! Do you want to say the confusion was brought by 14 which was written first before three times a number?

L1: Iyaa, Sir! I think so.

CN: OK.

(b) $2 + x = x(3 - 14)$ NoI

The error above was only noticed in the Oshindonga test, and it appeared to be a syntactic interpretation which caused the error. In the Oshindonga test, the sentence said “fourteen is subtracted from three times the number”. This appeared to have been interpreted as first $3 - 14$ and then the answer multiplied by a number. That is, learners appeared to read this as “fourteen is subtracted from three” before the word “times”. Hence, they wrote $x(3 - 14)$ instead of $3x - 14$. The interviews provided no validating information for this strategy.

(c) $2 + x = 3 + x - 14$ CWI

These learners made a single error by interpreting “times” as an addition rather than a multiplication. As a result they wrote $3 + x$ instead of $3x$. This caused them to get a wrong solution. The interviews were consistent with this interpretation, although they did not provide direct validation.

CN: How did you go about this question?

L4: I started with making equation from the sentence.

CN: How?

L4: the sentence said "when a number is added to two, the answer is the same as when fourteen is subtracted from three times the same number."

CN: Hm. Then?

L4: 2 plus x ... x is the number which is added to two ... is equal to three plus a number then I subtract 14.

CN: Three plus a number?

L4: No sir, three times a number.

CN: OK.

(d) $x + 2 = 3 - 14x$ NoI

I think these learners made a minor error and multiplied the variable x by 14 instead of multiplying 3 by variable x. Hence, they did not get the expected solution. There was no information relating to this interpretation in the interviews.

(e) $2 + x = 3x - 4$ DVI

These learners also made the silly mistake of writing 4 instead of 14, and as a result their final answer was wrong. This is not a language or reading problem. This error was noticed in both tests. This interpretation was definitely validated by the learner interviews.

L6: Here I made a mistake, sir. I wrote 4 instead of 14.

CN: Why did you make that mistake?

L6: I think because I was fast when I was writing. I always do those mistakes, but I do not know why I do them. I think maybe because of confusion, or sometimes I do not read very well.

CN: OK.

(f) $2x = 16$ then $16 \div 2 = 10$ DVI

These learners also made a silly mistake when they divided 16 by 2 to get 10 instead of 8. This can be regarded as a minor error made by many learners when solving mathematical problems. This interpretation was definitely validated by the learner interviews.

L6: Sir! I do not know why I got an answer of 10. Maybe I was confused, or I put a wrong number on the calculator.

CN: Do you make these types of error in mathematics test?

L6: Yes sir! But only sometimes.

CN: OK.

(g) $x + 3x = 14 + 2$ CWI

These learners made this error in the second phase of their solution attempts. Learners appeared only to think of bringing like terms together, not of what was needed to keep the meaning or equality of the statement above the same throughout the steps. They were therefore supposed to obtain $x - 3x = -14 - 2$ instead of the expression above. The interviews were consistent with this interpretation, although they did not provide direct validation.

L3: From there I bring like terms together on one side of equation and bring the x's bring to the left-hand side, and the numbers without x's on the right hand side of equal.

CN: Hm. And then?

L3: And then I added all like terms together.

CN: What was the value of x?

L3: 4

CN: OK.

iv. Unexpected solutions

Some learners failed to get correct solutions owing to various errors in the process. The following incorrect answers were given: 4, -4, 9, -1/6, 1/15, 3, 1/5, 10, -6.

v. Other

Nothing else emerged in this problem that I could not analyze.

vi. Table: 4.2.2.1a The proportion of learners in each category

Categories		English test	Oshindonga test
Number of learners doing the test in each language		100% (40)	100% (40)
Expected relationship or equation	(a) $2 + x = 3x - 14$	60% (24)	45% (18)
Major errors	(i) $x + 2 = 14 - 3x$	25% (9)	17% (7)
	(ii) $2 + x = x(3 - 14)$	-	20% (8)
	(iii) $2 + x = 3 + x - 14$	10% (3)	5% (2)
	(vii) $x + 3x = 14 + 2$	12.5% (5)	5% (2)
	$x + 3x = 14 + 2$	7.7% (3)	12.5% (5)
Minor errors	(iv) $x + 2 = 3 - 14x$	5% (2)	-
	(vi) $16/2 = 10$	2.5% (1)	-
	(v) $2 + x = 3x - 4$	5% (2)	7.5% (3)
Learners who obtained the expected solution		42.5% (17)	40% (16)
Learners who obtained unexpected solutions		57.5% (23)	60% (24)
Other		-	-

(the number in brackets shows the actual number of learners)

As can be seen in the table above, 60% of learners (24) constructed correct equations in the English test while in the Oshindonga test only 45% (18) constructed correct equations. This was because 60% of learners (24) made major errors in the Oshindonga test as compared to 52.5% (21) in the English test. However, 12.5% of learners (5) doing the English test made minor errors as compared to 7.5% (3) doing the Oshindonga test.

4.2.2.2. Problem 8

Description of problem 8 in both English and Oshindonga

Tomas bought four pieces of wood and each piece is 2.5m long. How many pieces of wood of length 1m can he get out of them?

Tomas okwa landa iipambu ine yiiti (iipilangi), noshipapu kehe oshi na uule woometa mbali netata (2,5m). Iipambu ingapi yuule wometa yimwe kehe ta vulu okumona mo?

The problem above is part of number and measurement. It is a contextual word problem. This problem requires learners to consider it in a real life practical situation in order to interpret it correctly and give the expected answer. I included this problem to see whether learners would be able to understand and interpret problems from a real life context.

i. Expected relationship or equation

All learners were expected to think that from a single piece of wood, 2.5m long, they would only get two pieces of wood 1 metre long. If they have 4 pieces of wood each of the same length then from each piece they could only get 2 pieces of wood from each, and a piece of 0.5m would remain in each case. In practical life one cannot attach a 0.5m piece of wood onto another piece of 0.5m to make one piece with a length of 1 metre.

ii. Expected solution

All learners were expected to give an answer of 8 pieces of wood. This could be attained by doing as follows:

$$2 + 2 + 2 + 2 \quad \text{or} \quad 2.5 \times 4 = 10 \text{ then } 10 \div 2 \\ = 8 \text{ pieces of wood.}$$

iii. Errors in strategies

Many learners made *similar errors* when they were solving this question.

(a) $2.5/1 = 2$ pieces DVI

These learners appeared to think correctly and showed that in 2.5m, the length of one piece of wood, one could only get 2 pieces of 1m. But they seemed not to think about the second important piece of information, that there were 4 pieces of wood. They appeared to think the answer they got was for 4 pieces of wood. On the other hand, they seemed not to focus on answering the question – merely giving as the answer the result of their first calculation. This interpretation was definitely validated by the learner interviews. I said:

CN: How did you go about this question?

L2: Sir, here, I first thought that 2.5m was for 4 pieces of wood.

CN: Why do you say that?

L2: Because when I read it now I think I am wrong. My answer here is wrong

CN: Why do you say your answer here is wrong?

L2: I thought I was given a total of 2.5m for 4 pieces of wood and then how many pieces of 1m wood can I get out of total. So each wood, each piece of wood will be 1m. So if you take 1m, 1m from pieces, then there will be a half of metre, which has no other piece, so you need to subtract that piece.

CN: Then what was your answer?

L2: 2 pieces of 1m and half (0.5m) remained, but this is wrong, sir

CN: OK.

$$(b) \begin{array}{l} 4 = 2.5 \\ x = 1 \end{array} \quad \text{CWI}$$

These learners appeared to understand the problem as follows: they thought the length of 4 pieces of wood was 2.5m, and then they asked how many pieces of wood 1m in length they could obtain from 2.5m. They then used a cross-multiply method to find the solution. These learners appeared not to understand this part of the question: "four pieces of wood and each piece is 2.5m long". Learners seemed not to understand the term "each", so they said the length of 4 pieces of wood is equal to 2.5m. This interpretation was definitely validated by the learner interviews.

CN: What is the total length of 4 pieces of wood?

L5: 2.5m.

CN: Then you multiply it by 4?

L5: Yes.

CN: Now did you get the total length or number of pieces?

L5: The pieces of 1m.

$$(c) \quad 2.5 \times 4 = 10/1 = 10 \text{ pieces} \quad \text{DVI}$$

Many learners appeared to interpret this problem from a theoretical perspective, and did not consider it in a real life practical situation. They did understand that 2.5m was the length of one piece of wood, and if they had 4 pieces the total length would be $2.5\text{m} \times 4 = 10\text{m}$. Then they thought to get pieces of wood of 1m they merely then have to cut the total of 10m into pieces 1m long, and getting an answer of 10 pieces. This could be correct in theory, but it was wrong in a real life practical situation. This interpretation was definitely validated by the learner interviews.

CN: How did you go about this question?

L4: It says "Tomas buys four pieces of wood and each piece is 2.5m long. How many pieces of 1m can he get out of them?" Then I make one piece is equal to 2.5 meters.

How many pieces of 1m can I get out of them? I was told one piece is 2.5m long.

I was asked how many one 1m pieces I can get from 4 pieces.

Then I have to make 1m is equal to 2,5m, then I take $4 = x$ in order to get the total pieces. I cross-multiply, $4 \times 2.5 = 10m$.

CN: $4 \times 2.5 = 10m$! Does it represent the total length of 4 pieces? Or does it represent the number of pieces that you can get?

L4: It is the length; then I divide 10 by 1 to get 10 pieces of 1m.

(d) $2.5 \times 4 = 6$ pieces NoI

These learners made the error of adding instead of multiplying. I think they may have added because they were not confident with multiplication, which shows a real mathematical weakness. There was no information relating to this interpretation in the interviews.

**(e) $1 = 2.5m$
 $x = 1m$ NoI**

These learners interpreted this problem as follows: 1 piece of wood has a length of 2.5m. How many pieces of 1m can they get from that? They have formulated the question but then not done anything to solve the problem. The interviews provided no validation for this strategy.

iv. Unexpected solution

There were several different answers from learners' work, as follows: 10, 6, 2, 1.6, 0.4, 6, 7 pieces of wood.

v. Others

There were some learners who did not answer this question.

vi. Table 4.2.2.2a The proportion of learners falling into each category

Categories		In English test	In Oshindonga test
Number of learners doing the test in each language		100% (40)	100% (40)
Expected relationship or equation	(a) $2 + 2 + 2 + 2$ or $2.5 \times 4 = 10$ then $10 - 2$ 8 pieces of wood	5% (2)	2.5% (1)
Major errors	(i) $2.5/1 = 2$ pieces	10% (4)	12.5% (5)
	(ii) $4 = 2.5$ $x = 1$	5% (2)	
	(iii) $2.5 \times 4 = 10/1 = 10$ pieces	65% (26)	85% (34)
	(iv) $2.5 \times 4 = 6$ pieces	5% (2)	
	(v) $1 = 2.5m$ $x = 1m$	10% (4)	
minor errors	-	-	-
Learners who obtained the expected solution		5% (2)	2.5% (1)
Learners who obtained the unexpected solution		95% (38)	97.5% (39)
Others		5% (2)	2.5% (1)

(the number in brackets shows the actual number of learners)

As can be seen in Table 4.2.2.2a, this question was extremely poorly interpreted in both tests. 95% of learners (38) in the English test interpreted and solved it wrongly (making major errors) as compared to 97.5% of learners (39) doing the Oshindonga test. Language usage did not make a difference at all because the performance in both tests *was* the same. In both tests many learners answered from a theoretical perspective. This means learners applied straightforward arithmetic procedures and produced an answer from the theoretical mathematics perspective rather than thinking in real-life terms to get an answer of 8 pieces of wood.

4.3 Emergent themes

In this section, I discuss the themes that emerged from the initial analysis of the data. Each theme is briefly discussed and then examples of strategies relating to this theme are given from the detailed analysis of problems 1 and 8. Additional examples from the other problems are also given, with reference to the initial analysis in Appendix 1. In the full thematic analysis, I consolidated similar emergent themes from different problems under the same theme. (For more detail see Appendix 1). Each consolidated theme identified in this way is discussed in this section.

4.3.1 Language related issues

4.3.1.1 Vocabulary

This theme includes all the issues emerging from the initial analysis related to learners' vocabulary. Learners may misinterpret the problem text owing to a misunderstanding of some of the key words or terms in the problem text. For example:

- 4.2.2.1 (iii.c), problem 1(c): Learners interpreted the term “times” as addition rather than multiplication.
- Appendix 1, problem 2 (a): Learners wrote $4x - 9$ instead of $4x + 9$. They interpreted “added to” as a subtraction.
- Appendix 1, problem 3 (b): They interpreted the term “decreased” as division and so wrote $3x/9$ instead of $3x - 9$.
- 4.2.2.2 (iii.b), problem 8(b): Learners appeared not to understand the term “each”. Hence, they interpreted that 4 pieces of wood had a length of 2.5m instead of understanding that “each piece of wood has a length of 2.5m”.

4.3.1.2 Syntactic structure

This section includes the issues emerging from the initial analysis related to the syntactic interpretation. Learners use the word sequence in the problem text to form algebraic expressions or equation. Here are the examples:

- 4.2.2.1 (iii.a), problem 1(a): Learners constructed the expression $14 - 3x$ instead of $3x - 14$, writing the order of the terms in the same order as the words.
- 4.2.2.1 (iii.b), problem 1(b): This error was only noticed in the Oshindonga test, where the sentence said “fourteen is subtracted from three times the number”. This was interpreted as $3 - 14$, and then the answer was multiplied. This was because learners understood that the words “fourteen”, “subtracted from” and “three” came before “times”. Hence they wrote $x(3 - 14)$ instead of $3x - 14$.
- Appendix 1, problem 6(f): Learners interpreted ‘7 lighter than a large piece’ as $7 - x$, rather than $x - 7$ which should express the mass of the medium piece in terms of x .

4.3.2 Semantic issues

4.3.2.1 Concept-related issues of the problem text

This section consists of all the issues emerging from the initial analysis, related to conceptual understanding of the problem texts. In these strategies, it was judged that learners sometimes misunderstood the meaning of the problem text, because of their misunderstanding of concepts important for the understanding of the text. This could have led them to a wrong interpretation of the problem. Here are the examples:

- Appendix 1, problem 4(e): The concept “younger than” was interpreted using the sign “+”. As result, they expressed the ages of Tuli as $x + 25$ instead of $x - 25$, and the age of the mother as $x + 2$ instead of $x - 2$. Learners appeared to understand that the father was 25 years younger than Tuli and 2 years younger than the mother.

There was a lack of understanding of the concept of comparison. This has led learners to create a wrong relationship between the ages of the three persons in terms of a variable x .

- Appendix 1, problem 5(b): Learners interpreted “Silo is five years younger than” as “Silo is 5 years old”. This shows that learners seemed not to understand the comparative nature of the concept “younger than”.

- Appendix 1, problem 5(d): The concept “five years younger than” was interpreted as “five times as old as Siphho”. This shows that learners seemed not to understand the type of relationship between the ages which was required. They seemed to confuse subtraction with multiplication. Thus they multiplied (e.g. $5x$) instead of subtracting (e.g. $x - 5$).
- Appendix 1, problem 5(e): Learners interpreted “Temba is twice as old as Siphho” as “Siphho was twice as old as Temba”. Learners appeared to reverse the ideas in the problem text.
- Appendix 1, problem 5(f): The concept “twice as old as” was interpreted as “two more than”. Learners appeared not to understand the difference between “twice as old as” and “two more than”. They appeared to confuse addition with multiplication. Thus they added (e.g. $x + 2$) instead of multiplying (e.g. $2x$).
- Appendix 1, problem 6(a): Learners interpreted the statement using ratio ideas. Learners seemed to think that “lighter than” and “heavier than” meant “a part of”. Thus they added the lighter parts to the heavier parts and got a total of 11, which represented the mass of the whole pizza. That was how these learners misunderstood the concepts in this problem and were led to the wrong solution. They took $7/11 \times 300\text{g} = 190.9\text{g}$ as the mass of the lighter piece of pizza, and $4/11 \times 300\text{g} = 109\text{g}$ as the mass of the heavier piece. They found the mass of the medium piece by adding the two masses and dividing by 3 to obtain 100g.

They needed to create the relationship between the masses of three pieces and express them first in terms of a variable. For example: small piece = $x - 4$, medium piece = x , and large piece = $x + 7$. Construct an equation combining all algebraic expressions: $x + x + 7 + x - 4 = 300$.

- Appendix 1, problem 7(c): Learners interpreted “twelve pencils costs sixty cents more than” as “12 pencils cost sixty cents”. These learners seemed not to understand the concept of comparison.
- Appendix 1, problem 10(a): Learners thought the quantities were directly proportional to each other and worked out the solution as $(2 \times 6)/3$ instead of understanding that the quantities in the problem were inversely proportional to each other and working out the solution as $(3 \times 6)/2$.

4.3.2.2 Failure to relate the problem to practice

This section consists of the issues emerging from the initial analysis of the tendency of learners not to consider or apply real-world knowledge when they were answering these questions. Here are the examples:

- 4.2.2.2 (iii.c), problem 8(c): Learners interpreted this problem from a theoretical perspective, but did not consider the problem in a real-life practical situation.
- Appendix 1, problem 9(b) and (c): These learners made sense of the problem, but did not think how practical the problem was with regard to the situation in real life. As a result, they resolved their result unrealistically, either rounding up (b) or not rounding up at all (c).

4.3.3 Algebra

4.3.3.1 Equality formulation

This case refers to a relational understanding of equality between two quantities in the problem – equality formulation. Learners were expected to construct algebraic equations connecting two algebraic expressions using an equal sign, where the equal sign would be interpreted as a relational symbol of mathematical equivalence rather than as an operational symbol. This section consists of all the issues emerging from the initial analysis related to equality formulation. Here are the examples:

- Appendix 1, problem 2(b): Learners did not understand how to connect two algebraic expressions to form an equation. They added two expressions of $4x + 9$ and $3x - 13$ and got $4x + 9 + 3x - 13$ instead of connecting them using an 'equal sign' and getting the equation $4x + 9 = 3x - 13$.
- Appendix 1, problem 3(d): Same problem as above.

4.3.3.2 Manipulation of algebraic equations

To solve a problem that had been formulated as an equality using a variable, learners would need to manipulate the equation. A correct solution would only be obtained if learners manipulated the equation through keeping equality in all steps. This section consists of all the issues emerging from the initial analysis related to this manipulation. Here are examples:

- 4.2.2.1 (iii.g), problem 1(g): Misunderstanding of algebra and failure to manipulate the equation to keep the equalities. Learners added $x + 3x = 14 + 2$ instead of applying the rule for manipulating the equation correctly: “change side change sign” and do it thus: $x - 3x = 14 - 2$. Learners did not change signs when they were shifting the numbers from one side of the equal sign to the other side.
- Appendix 1, problem 3(a) and (c): Misinterpretation of algebraic equalities. In (a) learners applied the rule for manipulating the equation correctly by changing the sign for 9 as they shifted it to the right-hand side of the equal sign, and got $9 + 1$.

But they did not change the sign for the coefficient of x as they shifted it to the left-hand side of the equal sign. They just added $3x + 2x$ instead of $3x - 2x$. In (c) they did not change the signs on both sides of the equal sign. They just applied ‘addition’ with the coefficients of x and wrote $3x + 2x$ instead of $3x - 2x$. On the right-hand side of the equal sign they just applied ‘subtraction’ and wrote $9 - 1$ instead of $9 + 1$.
- Appendix 1, problem 7(a) and (b): Learners in (a) made the error of writing $12x + 8x = 180$ instead of $12x - 8x = 180$. They did not change the sign for the coefficient of x as they shifted $8x$ to the left-hand side of the equal sign. In (b) they made the error of writing $p + e = 15$ instead of $p - e = -15$, and also by writing $12p + 8e = 60$ instead of $12p - 8e = 60$. Learners did not change the signs of the numbers when they were shifting them from one side of the equal sign to the other side.

4.3.3.3. Inconsistent use of a variable

This section consists of all issues emerging from the initial analysis related to inconsistent use of a variable. Learners use variables and get wrong solutions because the variable used seemed to represent more than one quantity used in the problem text. Here are the examples:

- Appendix 1, problem 4(a): In the relationship between Tuli's age and her mother's age, the variable x seemed to represent the age of Tuli but in the relationship between her father's age and her mother's age, the variable x seemed to represent the father's age.
- Appendix 1, problem 6(c) and (d): In (c) learners expressed the mass of the large piece as x , the medium piece as $x - 7$ and the small piece as $x + 4$. If the mass of the small piece was $x + 4$ this meant that x was representing the mass of the medium piece; but if the mass of the medium piece was $x - 7$, then x represented the mass of the large piece. That was how the inconsistent use of x came in. In (d) the learners expressed the mass of the large piece as $x + 7$, of the medium piece as x , and of the small piece as $x - 7 - 4$.

If the mass of the medium piece was represented by x , then the mass of the small piece could not be represented by $x - 7 - 4$ as this would mean that x represented the mass of the large piece. But if the mass of the large piece was $x + 7$, then x appeared to represent the mass of the medium piece.

4.3.4 Making sense of the statement without turning it into a useful solution strategy

This section consists of all the issues emerging from the initial analysis where learners made good sense of the problem in the initial step, but failed to provide correct solutions or the next steps that could lead them to correct solutions. Here are the examples:

- Appendix 1, problem 4(b): Mother was $x - 2$ years old and Tuli was $x - 25$ years old, but the learners did not use the age of the father (x years old) in the second step, which was $x - 25 + x - 2 = 78$ instead of $x - 25 + x - 2 + x = 78$. Learners started correctly by expressing the ages of the mother and Tuli in terms of x , but in the next step they did not use a solution strategy that could lead them to the correct answer.

- Appendix 1, problem 5(a): Learners expressed the ages all three persons in terms of x correctly. Temba was $2x$ years old, Silo was $x - 5$ years old, and Siphos was x years old. But in their next step, they wrote $2x - x - 5 = 31$ as if 31 years old was a different age between Temba's age and Silo's age. The learners were supposed to construct the equation $2x + x - 5 + x = 31$, combining all algebraic expressions correctly, as in the first step.
- Appendix 1, problem 6(g): Learners wrote $x + x - 7 + x - 11 = 300$, which was correct, but they did not know which algebraic expressions were representing the small piece, medium piece or large piece. They were supposed to show algebraic expressions of the mass of each piece, or show the mass of each piece after they had solved the equation.
- 4.2.2.2 (iii.a and d), problem 8(a), and (d): In (a) learners did understand that from a 2.5m piece of wood one could get only 2 pieces of wood, but they were unable to find the number of pieces they could get from 4 pieces. In (d) learners also understood that 1 piece of wood had a length of 2.5m, and started to find the number of lengths of 1m they could obtain from 2.5m, but they were unable to manipulate their strategy and find the number of pieces of 1m.
- Appendix 1, problem 9(a) and (d): Learners gave 329 and 333 boxes instead of the number of eggs. Learners should have realized that what they needed to calculate was not the number of boxes, but the number of eggs that could be in the boxes.

4.3.5 Minor errors

A minor error in this study means one made when a learner seems to understand the problem, but owing to a lack of concentration makes a silly mistake. This section consists of all the minor errors emergent from the initial analysis. Learners make silly mistakes throughout while they are tackling out mathematics problems. Here are the examples:

- 4.2.2.1 (iii.d), problem 1(d): learners multiplied variable x by 14 instead of 3.
- 4.2.2.1 (iii.e), problem 1(e): learners wrote 4 instead of 14.

- 4.2.2.1 (iii.f), problem 1(f): learners divided 16 by 2 and got 10 instead of 8.
- Appendix 1, problem 2(a): learners wrote $6x - 1$ instead of $6x - 13$.
- Appendix 1, problem 2(b): learners wrote $3x - 13$ instead of $6x - 13$.
- Appendix 1, problem 2(c): learners wrote x instead of $4x$.

4.4 Full thematic analysis

In this section, the themes identified and discussed in section 4.3 are used to organize and group the incorrect or incomplete solution strategies identified in the initial analysis of all the data, to create a full thematic analysis for each problem. Each of the consolidated themes identified in the full thematic analysis was discussed in section 4.3.

In the remainder of this section, the complete analysis is summarized in a table of themes relating to incorrect solutions for each problem. For each problem except 1 and 8, I start with a description of the task, shown in both English and Oshindonga as it was presented to learners and then classify and justify the choice of this problem, before providing the summary table. For full detailed analysis of these problems see Appendix 1.

4.4.1 Problem 1

A detailed discussion of the problem and solution strategies was given in section 4.2.2.1 and themes relating to solutions for this problem were given in section 4.3. A summary table is presented below.

Table 4.4.1a Themes emergent from Problem 1

THEMES	No. OF STRATEGIES	IN ENGLISH TEST	IN OSHINDONGA
Vocabulary	1	7.5% (3)	5% (2)
Syntactic structure	2	22.5% (9)	37.5% (15)
Manipulation of algebraic equation	1	25% (10)	17.5% (7)
Minor errors	3	12.5% (5)	7.5% (3)

(the number in brackets shows the actual number of learners)

4.4.2 Problem 8

Again, a detailed discussion of the problem and solution strategies was given in section 4.2.2.2 and themes relating to solutions for this problem were given in section 4.3. A summary table is presented below.

Table 4.4.2a Themes emergent from Problem 8

THEMES	No. OF STRATEGIES	IN ENGLISH TEST (%)	IN OSHINDONGA (%)
Vocabulary	2	10% (4)	-
Failure to relate the problem to practice	1	65% (26)	85% (34)
Making sense of the statement without turning it into a useful solution strategy	3	25% (10)	12.5% (5)

(the number in brackets shows the actual number of learners)

4.4.3 Problem 2

Description of Problem 2 in both English and Oshindonga

A certain number is multiplied by 4 and 9 added to it. The result is the same as when the same number is multiplied by 6 and 13 subtracted. Find the value of this number.

Onomola yontumba oyi indjipalekwa nane e taku gwedhwa omugoyi. Eyamukulo olya faathana uuna onomola ndjoka yi indjipalekwa nahamano e tamu kuthwa omulongo nandatu. Konga onomola ndjoka.

The problem illustrated above is an abstract word problem, and it was used in this study because it required students first to construct an algebraic expression. Learners are expected to follow the word sequence in order to construct the algebraic expression that will lead them to the correct answer.

Table 4.4.3a Themes emergent from Problem 2

THEMES	No. OF STRATEGIES	IN ENGLISH TEST	IN OSHINDONGA
Vocabulary	1	2.5% (1)	-
Minor errors	3	10% (4)	5% (2)
Equality formulation	1	2.5% (1)	15% (6)

(the number in brackets shows the actual number of learners)

4.4.4 Problem 3

Description of Problem 3 in both English and Oshindonga

If 3 times a number is decreased by 9 the result is the same as 2 times the same number when decreased by 1. Find the number.

Ngele onomola yi indjipalekwa lutatu oya shonopalekwa nomugoyi (mwa kuthwa omugoyi), eyamukulo olya faathana nonomola oyo tuu ndjono yi indjipalekwa lwaa li eta mu kuthwa yimwe. Konga onomola ndjoka.

The question above is an abstract problem and it was used in this study because it uses the term “decrease”. This is a common mathematical term, and as it is used in many questions it was appropriate in this study to see if learners could interpret it correctly. I included this question in the study to see if it is one of those terms that affect learners’ interpretations negatively.

Table 4.4.4a Themes emergent from Problem 3

THEMES	No. OF STRATEGIES	IN ENGLISH TEST	IN OSHINDONGA
Vocabulary	1	12.5% (5)	-
Equality formulation	1	-	12.5% (5)
Manipulation of algebraic equation	2	5% (2)	7.5% (3)

(the number in brackets shows the actual number of learners)

4.4.5 Problem 4

Description of Problem 4 in both English and Oshindonga

Tuli is 25 years younger than his father. His mother is 2 years younger than his father. Their total ages add up to 78 years. How old is Tuli?

Tuli omushona kuhe nomvula omilongo mbali nantano. Yina omushona kuhe noomvula mbali. Omvula dhawo adhihe kuumwe odhi li omilongo heyali nahetatu. Tuli oku na oomvula ngapi?

The problem above can be regarded as an abstract word problem or as a contextual word problem. It was used in this study because it needs learners to translate it into mathematical language. That is, learners are expected first to express the ages of Tuli, Father and Mother in terms of a certain variable before constructing an equation that will lead them to a solution.

Table 4.4.5a Themes emergent from Problem 4

THEMES	No. OF STRATEGIES	IN ENGLISH TEST	IN OSHINDONGA
Concept-related issues of the problem text	1	7.5% (3)	10% (4)
Inconsistent use of a variable	1	10% (4)	12.5% (5)
Making sense of the statement without turning it into a useful solution strategy	1	17.5% (7)	22.5% (9)

(the number in brackets shows the actual number of learners)

4.4.6 Problem 5

Description of problem 5 in both English and Oshindonga

Temba is twice as old as Siphon and Silo is five years younger than Siphon. The total of their ages is thirty-one years. How old is Siphon?

Temba oku vule Siphon lwaali, Silo omushona kuSiphon noomvula ntano. Oomvula dhawo kumwe adhihe odhi li omilongo ndatu nayimwe. Siphon oku na oomvula ngapi?

The problem illustrated above can be regarded as a contextual word problem. The reason for setting such a problem was to explore whether or not learners would use the terms “twice” and “younger” correctly, and to see if learners would construct correct algebraic expressions to represent the ages of the three people used in the problem text, and would connect the expressions correctly to come up with an equation that leads them to a correct solution.

Table 4.4.6a Themes emergent from Problem 5

THEMES	No. OF STRATEGIES	IN ENGLISH TEST	IN OSHINDONGA
Concept-related issues of the problem text	4	30% (12)	17.5% (7)
Making sense of the statement without turning it into a useful solution strategy	1	2.5% (1)	12.5% (5)

(the number in brackets shows the actual number of learners)

4.4.7 Problem 6

Description of Problem 6 in both English and Oshindonga

Marcus bought a pizza and cut it into three pieces. When he weighed the pieces, he found that one piece was seven grams lighter than the largest piece and four grams heavier than the smallest piece. The mass of the whole pizza was three hundred grams. What was the mass of each piece?

Markusa okwa landa opizza, e teyi tete miipambu itatu. Sho a vihi iipambu mbyono (itatu), okwa mono kutya oshipambu shimwe osha li shi vulike oohalama heyali koshipambu shi oshinene kwaayihe, nosha li oshidhigu shi vule oshipambu oshishona kwaayihe noohalama dhi li ne. Oshiviha shOpizza ayihe osha li oohalama omathele gatatu. Oshiviha shoshipambu kehe oshi thike peni?

The above word problem is a contextual problem. It is about measurement. I used it in this study to see whether learners would be able to understand, interpret and construct the relationship between the masses of the three pieces of pizza stated in the text.

Table 4.4.7a The themes emergent from Problem 6

THEMES	No. OF STRATEGIES	IN ENGLISH TEST	IN OSHINDONGA
Syntactic structure	1		10% (4)
Concept-related issues of the problem text	2	20% (8)	15% (6)
Inconsistent use of a variable	3	22.5% (9)	27.5% (11)
Making sense of the statement without turning it into useful solution strategy	1	-	7.5% (3)

(the number in brackets shows the actual number of learners)

4.4.8 Problem 7

Description of Problem 7 in both English and Oshindonga

An eraser costs 15 cents more than a pencil, twelve pencils costs sixty cents more than eight erasers. How much does one pencil cost?

Okadhimitho otaka kotha oosenda omulongo nantano shi vulike kopena yekala, oopena dhekala omulongo nambali otadhi kotha oosenda omilongo hamano shi vulithe uudhimitho uhetatu. Opena yimwe yekala oyi na ingapi?

The problem illustrated above is a contextual problem. It is more about money and a bit of a trick question that needs learners to think critically in order to construct the correct relationship between the quantities stated in the text. I included this problem to find out whether learners were able to understand, interpret and construct the correct relationship between the cost of erasers and pencils.

Table 4.4.8a Themes emergent from Problem 7

THEMES	No. OF STRATEGIES	IN ENGLISH TEST	IN OSHINDONGA
Concept-related issues of the problem text	1	52.5% (21)	57.5% (23)
Manipulation of algebraic equation	2	30% (12)	22.5% (9)

(the number in brackets shows the actual number of learners)

4.4.9 Problem 9

Description of Problem 9 in both English and Oshindonga

A box of 24 eggs cost N\$7,30. How many boxes can your teacher buy with a N\$100 note?

Okapakete komayi 24 Otaka kotha (gu) N\$7,30.
Uupakete ungapi omulongi gweni ta vulu okulanda nefo Iyo N\$100

The problem above is part of the curriculum topic of money. It is a contextual word problem. I used this problem in this study to see whether learners would be able to understand and interpret it in practical situations rather than from just a mathematical point of view.

Table 4.4.9a Themes emergent from Problem 9

THEMES	No. OF STRATEGIES	IN ENGLISH TEST	IN OSHINDONGA
Failure to relate the problem to practice	2	30% (12)	25% (10)
Making sense of the statement without turning it into a useful solution strategy	2	15% (6)	15% (6)

(the number in brackets shows the actual number of learners)

4.4.10 Problem 10

Description of Problem 10 in both English and Oshindonga

Three men take six days to complete a job. How long will two men take to complete a similar job?

Aalumentu yatatu oshe ya pula (otashi ya pula) omasiku gatatu okumana oshilonga. Otashi ka pula aalumentu yaali uulethimbo wu thuki peni, opo ya mane oshilonga sha faathana?

The two quantities in the problem above are inversely proportional, so that as the one increases, the other decreases. I included a question on inverse proportion to see whether learners would be able to make sense of the problem and recognize how the quantities in the problem related to one another.

Table 4.4.10a Themes emergent from Problem 10

THEMES	No. OF STRATEGIES	IN ENGLISH TEST	IN OSHINDONGA
Concept-related issues of the problem text	1	35% (14)	32.5% (13)

(the number in brackets shows the actual number of learner)

4.5 Conclusion

This chapter presented the initial analysis of data and then used it to identify themes. Each identified theme was briefly discussed, and then examples of strategies relating to this theme were given from the detailed analysis of problems 1 and 8. Additional examples from problems 2, 3, 4, 5, 6, 7, 9 and 10 were also given, with reference to the initial analysis in Appendix 1. The similar themes emerging from different problems under the same theme were consolidated. Each consolidated theme identified in this way was also discussed. Finally, a full thematic analysis was provided for each problem.

CHAPTER FIVE

DISCUSSION OF FINDINGS

5.1 Introduction

In this chapter I discuss the emergent main themes in greater depth in relation to the research questions and the literature reviewed in Chapter 2. The discussion in this chapter is structured by the following themes:

- Vocabulary
- Syntactic structure
- Concept-related issues of the problem text
- Failure to relate the problems to practice
- Equality formulation
- Manipulation of algebraic equation
- Making sense of the statement without turning it into a useful solution strategy

5.2 Vocabulary

According to Warren (2001) vocabulary plays a vital role in interpreting and solving word problems. Learners may misinterpret the problem text owing to their lack of understanding of some of the key terms or words. From the written tests, it was evident that some terms or words in the problem texts did affect learners' interpretation and led to wrong solutions.

For example, some learners interpreted the word "times" as "addition" rather than "multiplication". Thus in solving problem 1 (iii.c) they added $(3 + x)$ instead of multiplying. Learners seemed to understand the term "addition" better than "multiplication". This may have been the reason they added $(3 + x)$ instead of multiplying. This problem with these words was only experienced in the English test, not in the Oshindonga test. In the Oshindonga test, learners did understand the word "indjipalekwa" to mean "multiplication".

Another example from this study was when learners were asked “if 3 times a number is decreased by 9 the result is the same as 2 times the same number when decreased by 1”. Some learners interpreted the term “decrease” as “division”. Thus they represented it as $3x/9 = 2x/1$ instead of as $3x - 9 = 2x - 1$. This problem was again only experienced in the English test. This finding supports the research carried out by Nesher and Kilpatrick (1990: 18-19), which states that “the complexity of the vocabulary could lead to wrong representation of quantities stated in the problem text”.

5.3 Syntactic structure

Hubbard (2003) indicates that learners use the word sequence in the story to form the equation. In this study, there is evidence in the written tests in support of Hubbard’s finding. Thus learners were using the word order to write $14 - 3x$ instead of $3x - 14$. This was because the sentence said “Fourteen is subtracted from three times the number”, so “fourteen” came first in the statement, then “three times the number”. This error was noticed in both English and Oshindonga tests.

In the Oshindonga test, the evidence of using the order of the words to construct the equation is illustrated when the sentence said “fourteen is subtracted from three times the number”. This was interpreted as $3 - 14$, to be followed by multiplying the answer by a number. This was because learners understood that the words “fourteen”, “subtracted from” and “three” came before the word “times”. Hence, they wrote $x(3 - 14)$ instead of $3x - 14$.

The evidence seems to suggest that the influence of syntactic interpretation is worse in Oshindonga word problems than in English ones for these learners, where the problem has a similar form. The Oshindonga syntax for problems 1 and 6 encourages misinterpretation because the statement contains words that could have caused the confusion in comprehending the statement correctly. However, there are some problem types on which English syntax has a greater effect than the Oshindonga syntax.

5.4 Concept-related issues of the problem text

Before attempting to find a solution to the word problem, learners first need to understand the problem text and then develop an understanding of the relationship between quantities used in the problem text (Fennema & Romberg, 1999). In this study, some learners seemed to have a problem with understanding these relationships. For example, in problem 4 learners misinterpreted the term “younger than” by reversing the order of the relationship. Thus they represented the age of Silo as $x + 5$ instead of $x - 5$. They did not seem to understand that the term “younger than” implies “less than”. In problem 5, some learners interpreted the statement “Silo is five years younger than” as meaning that “Silo was 5 years old.” They did not seem to appreciate that “younger than” is a comparative concept. This problem was experienced in both English and Oshindonga tests which suggests that this is not simply an issue of vocabulary.

The concept of “twice as old as” also seemed to be a problem for many learners. Learners understood this to mean “two more than”. Learners did not see a difference between the two (see Chapter 2, page 18). Learners did not seem to understand the semantic relations underlying the given text.

This finding seems to be in contrast to the findings of some research carried out on the effects of stating the word problem in bilingual settings. According to Bernardo and Mariss (2005) students performed better if the problem was presented in their first language. In this study, stating the problem in the learners’ first and second languages did not make a big difference because the performance in both tests was quite close, and mostly the same difficulties were experienced in both tests.

I would be cautious in claiming that my results directly challenge those of Bernardo and Mariss because in this study, the overall performance in both tests (English and Oshindonga) was almost the same while in their studies students performed better in the test in which the problems were presented in their first language.

I think semantic structures and relations underlying the problem text at the secondary school level may have nothing to do with the language in which the problem is articulated, because the same problems experienced in the English test were experienced in the Oshindonga tests (the learners' first language). This was also supported by the findings from the interviews. One learner indicated that if you do not understand the problem, you just do not understand it; it does not matter in which language the problem is articulated.

5.5 Failure to relate the problems to practice

Mathematical ideas need to be understood in relation to practice, according to Mathematics National Policy (Namibia. MoE, 2006). Learners are required to use word problems to apply their mathematical concepts to solving problems in their daily life. In this study, the findings indicate that learners take mathematical knowledge in isolation. Sometimes they do not try to make sense of the problem in relation to practice. This was noticed in both tests. Articulating a word problem text in either English or Oshindonga seemed not to make any difference. The problems in both texts were exactly the same (see Appendix 1). This indicates that the symbolic representation was misleading.

The problem "Tomas bought four pieces of wood and each piece is 2.5m long. How many pieces of wood of length 1m can he get out of them?" was misleading because a typical abstract mathematical assumption did not apply if one considered certain realities in the problem context (Bernardo & Mariss, 2005). In this case, the assumption was that lengths could be added without complications. Many learners applied such a standard solution ($4 \times 2.5 / 1 = 10$). This problem was experienced in both the English and Oshindonga tests, supporting Bernardo's finding that the language used in mathematics word problems might not always be an important factor. The problem here lay with the tendency not to consider or apply real-world knowledge. The research clearly showed that learners did not consider a real-life application when they were solving the problem. Learners seemed to make sense of the problem, but they did not think how it related practically to the situation in real life (see Appendix 1, problems 8 – 10).

Furthermore, in problem 9 (see Appendix 1, [iii.c]) some learners gave 13.7 boxes, and some gave 14 instead of 13. This finding also clearly showed that learners did not consider real-life applications when they were solving this problem. They ignored realistic knowledge here and applied mathematics ideas from a theoretical perspective. The findings in the interviews have also shown that the way a learner understands a problem in English is the same way he/she understands it in Oshindonga.

5.6 Equality formulation.

Many of the abstract word problems used in this study required learners first to construct an equation as a first step in attempting to get solutions. Some learners did this correctly. However, there was one thing that emerged as a barrier for many learners to construct correct equations.

Learners had a problem with using the equal sign in terms of creating a relationship between two expressions. For example, individually constructing two expressions was not a big problem, for instance, $4x + 9$ and $3x - 13$; the problem was in connecting them to form an equation.

The research by Alibali et al. (2006) indicates that some students only interpret the equal sign as an operational symbol meaning “find the total” or “write the answer”, but they do not also interpret it as a relational symbol of mathematical equivalence. The findings from this study fully support that claim. For example, learners wrote $4x + 9$ and $3x - 13$, then said $4x + 9 + 3x - 13$, they connected the two expressions by using addition instead of using equality. This was not a language issue, but one of the meaningful interpretations of mathematical representation, linking algebra with reality in a meaningful fashion. Also, the problem had to do with lack of algebraic understanding needed to form equality. The problem lay not only with the formulation of algebraic equations, some of those who formulated correct equations then experienced problems with the valid manipulation of equations to get solutions. This is discussed in the next section.

5.7 Manipulation of algebraic equations

After learners constructed an equation from a word problem, they were expected to manipulate it to get a solution. In this study, it appeared to be a problem for many learners. Some learners constructed a correct equation but manipulating it to get a solution was a problem. This was not a language issue—learners could understand the problem very well and could construct a correct equation—but if they did not have the algebraic skill to manipulate equalities to keep the equalities up to the last stage, then they would still perform poorly in word problems. This can happen if the problem is given in English or Oshindonga. Manipulating equalities has nothing to do with how well learners comprehend the problem text. It requires the acquisition of algebraic skills including making sense of all steps involved in finding the solution.

5.8 Making sense of the statement without turning it into a useful solution strategy

According to Schoenfeld (1992) solving word problems requires learners first to make sense of the word problems. I agree with Schoenfeld (1992), but in this study it is shown that some learners made sense of the problem text and showed that they understood the problem, but that understanding did not contribute to a correct solution (see Appendix 1).

This means learners could make a sensible interpretation of a word problem, without being able to use this for a solution. This can happen if the problem is in either the learners' first or second languages. In this study, this problem was noticed in both tests (English and Oshindonga).

To illustrate the point above, in problem 5 (see Appendix 1, (iii.a)), some learners first represented the ages of three persons in terms of x and said Temba was $2x$ years old, Siphon was x years old, and Silo was $x - 5$ years old. This was making sense and seemed to be correct, showing that learners did understand the problem text and that the initial interpretation was correct. But the next step of $2x - x - 5 = 31$ was not useful for an expected solution. This was noticed in both the English and Oshindonga tests.

Thus failing this problem being given such an interpretation and representation, one cannot regard comprehension of the problem text as causing difficulty, or that the semantic relations underlying the problem text were complicated. The difficulty could be that of failing to make sense of the representation and seeing if that representation could lead to the correct answer or that learners sometime understood the problem but not what the question is that needs to be answered. Also, when learners got their final solutions they did not try to make sense of their solutions in relation to the first ages in terms of x . Exactly the same thing happened in problem 7 (iii.g), problem 9 (iii.a) and (iii.b), and problem 10 (iii.a), (iii.b) and (iii.c). For further detail see Appendix 1.

5.9 General Comments

The findings suggest that language articulating a word problem need not make a big difference in terms of performance at secondary level. This was because there were some factors that affected learners' solution strategies more in English test than in Oshindonga test. And there were some factors that affected learners' solution strategies more in Oshindonga test than in English test.

- Thus the average of 46.5 % of learners in the English test gave expected solutions, and in the Oshindonga test the average of 44.5% of learners gave expected solutions. There was no big difference in overall performance.
- The findings further suggest that the semantic relations and the mathematical structure underlying the problem text may have a negative effect in identifying correct solution strategies. The average of 27.5% of errors across the English test was related to semantic relations as compared to 25.0% of errors across the Oshindonga test. This shows that there was only a very slight improvement in the Oshindonga test in terms of comprehension and conceptual understanding.
- The average of 11.3% of errors across the English test was related to syntactic structures, while 23.8% of errors across the Oshindonga test were related to syntactic structures. This shows that learners were likely to make more errors in syntactic structure when interpreting the problems that were given in their mother tongue.

- In the problems that need realistic consideration, the average 47.5% of errors across the English test was due to failure to consider the problems realistically as compared to 55.0% of the same errors made in the Oshindonga test. This clearly shows that the tendency of not considering real world knowledge when solving word problems is high in the Oshindonga test.
- The findings show that sometime learners did make sense of problems, but used the wrong solution strategies because they lacked the capacity to use the right ones. The average of 12.0% of errors across the English test was related to making good sense of the problem, but learners failed to use the correct solution strategies. In the Oshindonga test, the average of 14.0% of errors across the test was related to the same problem.
- The findings show that learners need to understand all the key words in a problem text in order to interpret it correctly. However, this was not the main issue in this study. An average of 6.9% of errors across the English test was related to vocabulary, but only an average of 1.3% of errors across the Oshindonga test was so related. This indicates that many learners in the Oshindonga test do not misinterpret the word problem owing to a failure to understand the vocabulary used.
- The findings show learners required algebraic skills in order to interpret and solve word problems correctly. In both the English test (with an average of 13.6% of errors), and the Oshindonga test (with an average of 16.4% of errors), the errors were related to equation formulation and manipulation as well as the consistent use of a variable.
- The findings show that learners make minor errors while they are attempting to solve word problems. An average of 11.3% of minor errors was made in the English test, while in Oshindonga test the average was 6.3%. This suggests that learners may concentrate harder in a mother-tongue test.

The critical point about the findings in this research is that sometimes the first language may not be better used than a secondary language. In this study, semantic structures and relations underlying the problem text, equality formulation, manipulation of algebraic equation, failure to relate the problems to practice and making sense of the statement without turning it into useful solution strategy appear to be more important than language. The general comments above support this. I think the findings in this study have taken a direction against much of the literature. The language issue is obviously not simple, and research findings need to be carefully interpreted.

5.10 Conclusion

The themes identified have been discussed in some detail, linking them to both the research questions of the study and the literature on the topic. The discussion has shown that language usage in word problems is not the only factor that affects learners' attempts to find solution strategies. There are various ways of working that play a vital role in solving word problems. The discussion has also shown that there is a lot still to be done in order to help learners to do well when interpreting and solving word problems.

CHAPTER SIX

CONCLUSION

6.1 Introduction

In this chapter, I start by giving a summary of the main findings, and then give some implications and recommendations. I discuss briefly some limitations of the study, and I suggest areas for possible further research. Finally, I highlight a reflection on the entire research process and conclude the chapter.

6.2 Summary of Findings

- The overall performance in the English and Oshindonga tests was almost the same.
- Many errors learners made in the English test were the same as those made in Oshindonga test, but the percentage was different in both tests.
- Many errors were related to the semantic relations and structures underlying word problems. This was noticed in both English and Oshindonga tests, although they were slightly fewer in the Oshindonga test.
- Algebraic skills play a vital role in interpretation, representation and solving of word problems. The main algebraic skills required are formulation and manipulation of equalities and the consistent use of a variable. This was not a language issue.
- Many learners use their mathematical knowledge in isolation from its practical uses in life. Hence they ignore real world knowledge where it is required.
- Learners make minor errors throughout while interacting with mathematics problems, particularly in constructing equations. This was noticed in both tests.
- Learners often copy word sequences to develop their solution strategies (i.e. in constructing equations), and this habit seems to become worse in their first language.

In summary, whether word problems are stated in English or in the mother tongue may not make a significant difference at the secondary level and that other factors, such as semantic structures and relations underlying the problem text, inconsistent use of a variable, equality formulation, manipulation of algebraic equation and failure to relate the problems to practice may prove to be more significant.

6.3 Implications and Recommendations

This study suggests that there are several areas that educators need to take into account in order to address the difficulty learners experience in trying to solve word problems. It may be believed that language is the main problem that affects learner's interpretations of word problems, and the construction of correct solution strategies. But the findings from this research indicate that there are several reasons for learners' difficulties apart from language usage or articulation of the word problems. Therefore the following points need to be addressed:

- This research's findings revealed that comprehension is a critical aspect of mathematical word problem solving. The literature review shows or suggests that to grasp the meaning of what is stated in the given word problems, the words need to be understood very well. This point was supported by the findings in this study, chapter 4, page 47. Thus teachers must work to improve comprehension.
- The findings in this study, chapter 4, page 48 show that learners sometimes misunderstood the meaning of the problem text, because of their misunderstanding of concepts important for the understanding of the text. Therefore teachers are advised to explain clearly and explicitly the concepts used in the problem texts. For these problems they included: younger, twice, more than, younger than, and others. This could help learners become aware of the meaning of these concepts.

It could also be useful if learners could be helped to develop the ability to begin by explaining the problem in their own words. This will enhance their understanding and thus their ability to find correct solutions.

- The findings in this study show that not only language usage, but also a variety of other factors need to be taken into consideration when dealing with word problems and these factors are like: equality formulation and manipulation of algebraic equation. Algebraic skills require strong attention. Teachers are advised to make sure learners have mastered all algebraic skills, for example, the consistent use of variables, the creation of relationship between quantities, and the manipulation of equations based on an understanding of equality.
- The findings in this research proved that learners up to grade 12 still need to acquire the relational understanding of the equal sign. Teachers need to make sure learners at secondary level can not only provide an operational interpretation when asked to define the equal sign, but can also give relational interpretations such as “equal to”, or “two amounts are the same” (see Chapter 2, p. 20). This will help learners to construct an equation, for instance $3x + 4 = 2x - 7$, without making any error in the process.
- This study has shown that giving word problems in English or in the mother tongue at secondary level does not make a big difference in terms of understanding the problem. Of course there could be some terms that need to be understood in the mother tongue, but not many at secondary level. It seems that learners in grade 12 have sufficient vocabulary for the given problems, but the semantic structures and relations underlying problem texts do seem to play a major role in understanding them.

6.4 Significance of the Study

I think the results of this study have contributed to the mathematical understanding of word problems in the Namibian context in the following ways:

- Firstly, giving mathematics tests in both English and the mother tongue provided a good opportunity to evaluate language usage in word problems. It was necessary to investigate if the language articulating word problems makes a big difference in helping learners to interpret word problems correctly. The results will contribute to the debate on the issue of whether it is necessary or not to teach mathematics in the mother tongue.

- Secondly, exploring how grade 12 learners interpret and solve word problems was necessary in that it informs the educators at senior institutions of the mathematical level of students they receive. It also informs the curriculum developers of the need to look again at the curriculum and shape it to strengthen the standard of its products, and shows teachers how to help students to become competent word problem solvers.
- Thirdly, this study shows how abstract mathematical ideas may sometimes be misleading where real-life application is required to get a solution.
- Fourthly, the findings of this research show the importance of understanding and solving word problems by proceeding through the phases of problem translation, problem interpretation, solution planning, solution execution and solution monitoring.

6.5 Limitations of the Study

A potential limitation of my research was that it was an unfamiliar challenge for learners to answer mathematics questions given in their home language, because they have never seen them in that form before, either at junior or secondary level. The other limitations were:

- My participants were limited to 40 learners, and for that reason I cannot generalize the findings.
- Since I was one of their teachers, my learners may not have felt comfortable about speaking freely to me during the interviews.

6.6 Avenues for Future Research

The findings of this study suggest that more in-depth study in various areas need to be carried out in order to address the problem of getting solutions to word problems.

- Different word problems with different semantic structures and relations underlying word problems given in the same language are one of the issues needing in-depth exploration. Different learners from different areas and background need to be involved in such studies in order to give wide coverage to the findings.

- Research is needed in the teachers' colleges to see how teachers are trained to assist learners in understanding word problems and developing solution strategies. Teachers need to acquire the skills to be able to help learners in approaching word problems from different perspectives and contexts.

6.7 Reflection on the Research Process

The entire research process has been a fascinating and challenging experience to me. When I formulated my research question I decided to use tests and interviews as tools for collecting data. I gave tests in English and again the same tests in the mother tongue. I would not choose to do it like that in future if I happen to carry out the similar research.

Through carrying out this research:

- I have acquired relevant knowledge and skills for formulating relevant and appropriate questions, and conducting interviews with a sensitive regard for ethical considerations and confidentiality.
- Through using different tools for collecting data, I have learnt the importance of triangulation and evidence-seeking during the research process. This has enhanced the validity of the research results.
- Through attempting to write up my thesis I have acquired writing skills including data analysis, presentation and discussion in a logical and reader-friendly way.
- I studied part-time, which means the research process was challenging, and demanded a lot of sacrifice and time management to reach the due date. However there is a lot that could appreciate for, both personally and professionally.
- I studied through exposing myself to professional and academic literature, discussing with my supervisor as well as with colleagues, conducting interviews and the writing process. That was great achievement to me.
- The analysis process of my data was frustrating and difficult and I spent much time attempting to find the suitable way of doing the way but with assistance from my supervisor and colleagues I managed to get a framework of doing it.

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APPENDICES

APPENDIX 1

INITIAL ANALYSIS

4. 2. Problem 1

4.2.1 Description of the problem in English and Oshindonga

When a number is added to two, the answer is the same as when fourteen is subtracted from three times the number. What is the number?

Ngele onomola oya gwethwa kumbali, eyamukulo olyafathana nuuna omuulongo nane gwakuthwa momulongo nandatu gwi indjipalekwa nonomola ndjoka. Onomola oyini ndjono?

4.2. 2. Classification and justification of word problem

This problem illustrated above is an abstract word problem and I am expected many learners to interpret it into algebraic expression before coming up with a solution. That is the reason I have chosen it to be used in this study.

4.2.3. The categories used

(i) Expected relationship or equation

All learners were expected to form the following relationship of quantities in the problem that would lead them to the correct solution. These were mainly one of the following equations; $2 + x = 3x - 14$ or $x + 2 = 3x - 14$ as a first step. In both tests, some learners constructed one of the equations above and they solved it correctly. It was noticed in both English and Oshindonga tests. From the interviews with learners (section 1K and 1D), this interpretation falls in the category of DV.

(ii) Expected solution

All learners were expected to give the answer of 8.

$$2 + x = 3x - 14$$

$$X - 3x = -14 - 2$$

$$-2x = -16$$

$$X = -16 / -2$$

$$X = 8$$

(iii) Errors in strategies

There were some common errors made by learners in interpreting and constructing the solution strategy of solving this problem.

$$(a) x + 2 = 14 - 3x$$

These learners used words' order and wrote $14 - 3x$ instead of $3x - 14$. This was because the sentence said "Fourteen is subtracted from three times the number" so fourteen came first in the statement then three times a number, that's why the learners wrote first fourteen and then three times a number. This error was noticed in both English and Oshindonga tests. From interviews, the interpretation falls in the category of DV

$$(b) 2 + x = x(3 - 14)$$

The error above was only noticed in Oshindonga test and this was syntactic interpretation which caused the major error. For example, In Oshindonga test, the sentence said "fourteen is subtracted from three times the number". Was interpreted like $3 - 14$ and later multiplied the answer by a number. This was because learners understood that the word 'fourteen', 'subtracted from' and 'three' came first before the word 'times'. Hence, they wrote $x(3 - 14)$ instead of $3x - 14$. From interviews, the interpretation falls in the category of NoI.

$$(c) 2 + x = 3 + x - 14$$

These learners made a single error by interpreting 'Multiplication' as 'an addition' and as result, they wrote $3 + x$ instead of $3x$. This caused them to get wrong solution. From interviews, the interpretation falls in the category of COI.

$$(d) x + 2 = 3 - 14x$$

I think these learners made a minor error and multiplied variable x with 14 instead of multiplying 3 by variable x . Hence, they did not get the expected solution. From the interview, the interpretation falls in the category of DV

(e) $2 + x = 3x - 4$

These learners also did a silly mistake if it is not language or reading problem, they wrote 4 instead of 14, as result their final answer was wrong. This error was noticed in both tests. From the interview, the interpretation falls in the category of DV

(f) $2x = 16$ then $16 \div 2 = 10$

These learners also did a silly mistake when they divided 16 by 2, they got 10 instead of 8. This error can also be regarded as minor error and many learners do this type of mistake while interacting with mathematical problems. From the interview, the interpretation falls in the category of DV

(g) $x + 3x = 14 + 2$

These learners did this error in their second steps of attempting to answer this question. Learners only thought of bringing like terms together but they did not think the rules that needed to keep the meaning of the statement above the same through the steps. They were therefore supposed to do as follow; $x - 3x = -14 - 2$ instead of the expression above. From the interviews, the interpretation falls in the category of CWI

(iv) Unexpected solution

Some learners failed to get correct solution due to various errors in the process. As result the following answers were given; 4, -4, 9, -1/6, 1/15, 3, 1/5, 10, -6.

(v) Others

Nothing else emerged in this problem that I could not analyze.

(vi) The table of proportion of learners falling into each category

Categories		In English test (%)	In Oshindonga test (%)
Number of learners doing the test in each language		100% (40)	100% (40)
Expected relationship or equation	$2 + x = 3x - 14$ $X - 3x = -14 - 2$ $-2x = -16$ $X = -16 / -2$ $X = 8$	60 (24)	45 (18)
Major errors	$x + 2 = 14 - 3x$	22.5 (9)	17 (7)
	$2 + x = x(3 - 14)$	-	20 (8)
	$2 + x = 3 + x - 14$	7.5 (3)	5 (2)
	$2 + x = 3x + 14$	5 (2)	-
	$X + 3x = 14 + 2$	12.5 (5)	5 (2)
	$x + 3x = 14 + 2$	7.7 (3)	12.5 (5)
Minor errors	$x + 2 = 3 - 14x$	5 (2)	-
	$16/2 = 10$	2.5 (1)	-
	$2 + x = 3x - 4$	5 (2)	7.5 (3)
Learners who obtained the expected solution		42.5 (17)	40 (16)
Learners who obtained the unexpected solution		57.5 (23)	60 (24)
Others		-	-

(the number in brackets shows the actual number of learners)

The table above shows the percentages of learners who came up with an expected relationship or equation, solved and got an expected or unexpected solution in both English and Oshindonga tests. Also, it shows the percentages of learners who did conceptual and technical errors in both tests while they were attempting to construct or solve their equations.

As it can be seen in the table above, 60% of learners (24 learners) constructed correct equation in English test while in Oshindonga test only 45% of learners (18 learners) constructed correct equation. This was because 59.5% of learners (24 learners) did major errors in Oshindonga test as compared to 52.5% of learners (21 learners) in English test. However, 12.5% of learners (7 learners) doing the English test did minor errors as compared

to 7.5% of learners (3 learners) doing the Oshindonga test. There was no other thing emergent that could not be categorized.

4.3. Problem 2

Description of Problem 2 in both English and Oshindonga

A certain number is multiplied by 4 and 9 added to it. The result is the same as when the same number is multiplied by 6 and 13 subtracted. Find the value of this number.

Onomola yontumba oyi indjipalekwa nane e taku gwedhwa omugoyi. Eyamukulo olya faathana uuna onomola ndjoka yi indjipalekwa nahamano e tamu kuthwa omulongo nandatu. Konga onomola ndjoka.

4.3.2 Type and justification of word problem

The problem illustrated above is an abstract word problem and it was used in this study because it required students first to construct algebraic expression.

Learners are expected to follow the words' sequence in order to construct algebraic expression that will lead them to the correct answer.

4.3.3 The categories used

(i) Expected relationship or an equation

All learners were expected to form an equation of $4x + 9 = 6x - 13$. This was the main equation that could lead learners to get correct solution. Some learners formed this equation and solved it correctly. This was noticed in both tests.

(ii) Expected solution

All learners were expected to give a solution of 11.

(iii) Errors in strategies

Some learners did a mistake during interpretation and construction of equations in the first steps. Some learners did these errors in others steps as from the first one. The errors noticed were as follow:

(a) $4x - 9 = 6x - 1$

These learners made a silly mistake if it was not a language or reading problem, they wrote $6x - 1$ instead of $6x - 13$ and also they wrote $4x - 9$ instead of $4x + 9$. This mistake was also emerged in the first problem. From the interview, the interpretation falls in the category of DV

(b) $4x + 9 + 3x - 13$

These learners did two errors, the first error, they wrote $3x - 13$ instead of $6x - 13$. That could be a minor error or reading error if it was not language problem. The second error they failed to connect the two algebraic expressions of $4x + 9$ and $3x - 13$ together correctly. They put 'plus' instead of 'equal to'. From the interview, the interpretation falls in the category of DV and C respectively

(c) $x + 9 = 6x - 13$

These learners did a silly mistake in the first part of the equation by writing x instead of $4x$. As result the final solution was wrong due to the error in the first step. From the interview, the interpretation falls in the category of COI

(iv) Unexpected solutions

There were different answers given like 6 and 4 and all of them were wrong.

(v) Others

There were some learners left it blank.

(vi) The table of proportion of learners falling into each category

Categories	In English test (%)	In Oshindonga test (%)
Number of learners doing the test in each language	100% (40)	100% (40)
Expected relationship or equation $4x + 9 = 6x - 13$ $4x - 6x = -13 - 9$ $-2x = -22$ $x = -22/-2$ $x = 11$	77.5 (31)	80(32)

Major errors	$4x - 9 = 6x - 13$	2.5(1)	-
	$4x + 9 + 3x - 13$	2.5(1)	15(6)
Minor errors	$3x - 13$	2.5(1)	-
	$4x + 9 = 6x - 1$	7.5(3)	2.5(1)
	$x + 9 = 6x - 13$		2.5(1)
Expected solution		77.5(31)	75(30)
unexpected solution		17.5(6)	20(8)
Others		5(2)	5(2)

(the number in brackets shows the actual number of learners)

As it can be seen in the table above, this question was extremely well answered in both tests. 3 learners (7,5 % of learners wrote the test) made a major errors in English test of using ‘subtraction’ instead of ‘an addition’ and also of connecting two expressions with ‘an addition’ instead of using ‘equal sign’. While in Oshindonga test, 6 learners (15% of learners) did only a conceptual error of connecting two expressions with ‘an addition’ instead of using ‘equal sign’. However, 4 learners did technical errors in English test as compared to 2 learners in Oshindonga tests.

4.4. Problem: 3

4.4.1 Description of the problem 3 in both English and Oshindonga

If 3 times a number is decreased by 9 the result is the same as 2 times the same number when decreased by 1. Find the number.

Ngele onomola yi indjipalekwa lutatu oya shonopalekwa nomugoyi (mwa kuthwa omugoyi), eyamukulo olya faathana nonomola oyo tuu ndjono yi indjipalekwa lwaa li eta mu kuthwa yimwe. Konga onomola ndjoka.

4.4.2. Type and justification of problem

The question above is an abstract problem and it was used in this study, because it consist of a good term ‘decrease’ this is common mathematics term and it used in many questions, it was good in this study to see if learners interpret it correct. Or it is one of those terms affect learners’ interpretation negatively. This was the reason; I included such question in this study.

4.4.3. The categories used

(i) Expected relationship or equation

All learners were expected to form an algebraic expression of $3x - 9 = 2x - 1$ as a first step before getting the answer. Some learners started with an expected relationship or equation. This was noticed in both tests (English and Oshindonga tests).

4.4.3. 3. Expected solution

All learners were expected to give a solution of 8.

$$3x - 9 = 2x - 1$$

$$3x - 2x = 9 - 1$$

$$x = 8$$

(iii) Errors in strategies

There were some errors noticed in learners' works which were noticed in either the first step or other steps. All these errors led learners to get wrong solution.

$$(a) 5x = -10$$

It was noticed in Oshindonga test where a learner just added $3x + 2x$ together and then added -9 and -1 together. From the interview, the interpretation falls in the category of C.

$$(b) 3x/9 = 2x/1$$

Some learners interpreted the term 'decrease' as 'division' and constructed an equation of $3x/9 = 2x/1$. From the interviews, the interpretation falls in the category of C.

$$(c) 5x = 8$$

This error was done in the second step where learners took $3x$ and added $2x$ forgetting the relationship between $3x$ and $2x$. But they took 9 and 1 was subtracted from 9 , that was correct. From the interviews, the interpretation falls in the category of CWI

(d) $3x - 9 + 2x - 1$

Some learners constructed correct algebraic expressions of $3x - 9$ and $2x - 1$ but they failed to understand and constructed correct relationship between $3x - 9$ and $2x - 1$. They put the sign of a 'plus' instead of 'equal to'. From the interviews, the interpretation falls in the category of DV.

(iv) Unexpected solution

There were three different solutions which were wrong. These were -2 , 5 and 10 .

(v) Others

Some learners did not answer this question, they left it blank.

(vi) The table of proportion of learners falling into each category

Categories	In English test (%)	In Oshindonga test (%)
Number of learners doing the test in each language	100% (40)	100% (40)
Expected relationship or equation	$3x - 9 = 2x - 1$ $3x - 2x = 9 - 1$ $x = 8$	82.5(33) 90(36)
Major errors	$5x = -10$	2.5(1)
	$3x/9 = 2x/1$	12.5(5)
	$3x - 2 = 9 + 1$	
	$5x = 8$	2.5(1)
	$3x - 9 + 2x - 1$	
Minor errors	-	-
Expected solution	80(32)	75(30)

unexpected solution	18(7)	21(8)
Others	2.5(1)	5(2)

(the number in brackets shows the actual number of learners)

Table 4.4 indicates that this question was extremely well answered by many learners in both tests. There were few learners who interpreted the term ‘decrease’ as division in English test only. Some learners wrote correct algebraic expressions but failed to connect them correctly by using ‘equal sign’, this was only noticed in Oshindonga test.

4.5. Problem 4

4.5.1 Description of the problem 4 In both English and Oshindonga

Tuli is 25 years younger than his father. His mother is 2 years younger than his father. Their total ages add up to 78 years. How old is Tuli?

Tuli omushona kuhe nomvula omilongo mbali nantano. Yina omushona kuhe noomvula mbali. Omvula dhawo adhihe kuumwe odhi li omilongo heyali nahetatu. Tuli oku na oomvula ngapi?

4.5.2. Type and justification of the problem

This problem can be regarded as an abstract word problem and at the same time can be also regarded as contextual word problem. It was used in this study because it needs learners to translate it into mathematical language. It means learners are expected first to express the ages of Tuli, Father and Mother in term of a certain variable before constructing an equation that will lead them to a solution. This is the reason, I included it in this study.

4.5.3. The categories used

(i) Expected relationship or equation

All learners were expected to use a variable to express the ages of Tuli, Father, and mother in term of that variable. For example, Father is x years old, Tuli is $x - 25$ years old and then the mother is $x - 2$ years old. After these expressions is when learners could move to the next step of combining them, $x + x - 25 + x - 2 = 78$. Some learners followed correct procedures

and came up with correct representation. This was noticed in both English and Oshindonga tests.

(ii) Expected solution

All learners were expected to get 10 years the ages of Tuli

(iii) Errors in strategies

Many different errors were noticed in both English and Oshindonga tests.

(a) Tuli = x, Father = x - 25 and Mother = x - 2, then an equation of x - 25 + x - 25 = 78

The error occurred in the first step where they expressed the mothers' ages as $x - 2$ which was implying that the mother was 2 years younger than Tuli. From the interviews, the interpretation falls in the category of DV.

(b) Mother = x - 2 and Tuli = x - 25 then $x - 25 + x - 2 = 78$

The two expressions above could be correct provided that the ages of father is x years old. Learners said $x - 2 + x - 25 = 78$ which was wrong relationship because 78 years was the total ages for all three of them. From the interview, the interpretation falls in the category of DV

**(c) Cross multiple method using: $25 = 2$
 $78 = x$**

Some learners thought that 25 years was equal to 2 years therefore they thought to get the ages of Tuli, they needed to use cross multiple method as it is indicated above.

(d) Tuli = x, Father = x - 25, Mother = x - 23 and then $x + x - 25 + x - 23 = 78$

The expressions above implied that the father was 25 years younger than Tuli and also the mother was 23 years younger than Tuli. This was wrong relationship. From the interviews, the interpretation falls in the category of C.

(e) Tuli = x + 25, Mother = x + 2, Father = x and then $x + x + 25 + x + 2 = 78$

The expressions above shown wrong representation of the problem into solution strategy, this was due to the error in relationship between quantities, the expressions implied that Tuli was 25 years older than Father and also mother was 2 years older than Father. That was wrong relationship and representation. From the interviews, the interpretation falls in the category of C.

(iv) Unexpected solution

Various answers were seen. Some learners said, Tuli's ages was 2, 6, 27.5, 53, 17, 3, 51, 15, and 14 years old

(v) Others

There were few learners who did not answer this question.

(vi) The table of proportion of learners falling into each category

Categories		In English test (%)	In Oshindonga test (%)
Number of learners doing the test in each language		100% (40)	100% (40)
Expected relationship or equation	Father = x or Tuli = x Tuli = x - 25 father = x + 25 Mother = x - 2 mother = x + 23 $\therefore x + x - 25 + x - 2 = 78$ $3x - 27 = 78$ $3x = 78 + 27$ $3x = 105$ $X = 35$ years for father's Tuli is 35 - 25 =10 years old	60(24)	70(28)

Major errors	Tuli = $x - 25$ Mother = $x - 2$ $\therefore x - 25 + x - 2 = 78$	17.5(7)	22.5(9)
	Tuli = x Father = $x + 25$ Mother = $x - 2$ $\therefore x + x + 25 + x - 2 = 78$	10(4)	12.5(5)
	$25 = 2$ $78 = x$ Cross multiple step used	5(2)	
	Tuli = x Father = $x - 25$ Mother = $x - 23$ $\therefore x + x - 25 + x - 23 = 78$	2.1(1)	
	Tuli = $x + 25$ Mother = $x + 2$ Father = x $\therefore x + x + 25 + x + 2 = 78$	7.5(3)	10(4)
Minor errors	-	-	-
Expected solution		45(18)	53(21)
unexpected solution		42.5(17)	45(18)
Others		12.5(5)	2.5(1)

(The number in brackets shows the actual number of learners)

As it can be seen in the table 4.5 above, many learners in Oshindonga test came up with expected relationship or representation of ages of Tuli, father and mother in term of a variable x as compared to English test. However, not all those learners who came up with expected relationship or representation got a correct solution but some made conceptual errors in their seconds and third steps, resulting in wrong solution. 11 learners (27.5% of learners) did conceptual errors in English test while in Oshindonga test, 15 learners (37.7% of learners) did the same conceptual errors. No minor errors were noticed in both tests.

4.4. Problem 5

4.6.1 Description of problem 5 in both English and Oshindonga

Temba is twice as old as Siphon and Silo is five years younger than Siphon. The total of their ages is thirty one years. How old is Siphon?

Temba oku vule Siphon lwaali, Silo omushona kuSiphon noomvula ntano. Oomvula dhawo kumwe adhihe odhi li omilongo ndatu nayimwe. Siphon oku na oomvula ngapi?

4.6.2. Type and justification of word problem

The problem illustrated above can be regarded as an abstract and contextual word problem. The reason for setting such problem was to explore whether or not learners would use the term 'twice' and 'younger' correctly and also to see if learners would construct correct algebraic expression to represent the ages of three people used in the problem text. Also, to see if learners would connect the expressions correctly to come up with an equation that leads them to correct solution.

4.6.3. The categories used

(i) Expected relationship or equation

All learners were expected to represent the ages of Temba, Siphon and Silo in term of a certain variable first before constructing an equation. For example Temba = $2x$, Siphon = x and Silo = $x - 5$. They supposed to move on and construct an equation of $2x + x + x - 5 = 31$. Some learners did it correctly but there were some who did conceptual relationship errors.

(ii) Expected solution

All learners were expected to give 9years as the ages of Siphon

(iii) Errors in strategies

Various errors were noticed in both English test and Oshindonga test.

(a) Temba = 2x, Siphos = x, Silo = x – 5 and then 2x – x – 5 = 31

The first error, learners thought Siphos ages could be obtained when they subtract Silos ages from Tembas ages. Thats why they took Tembas ages and subtract Silos ages which would be equal to 31. They thought the different between Tembas ages and Silos ages was equal to Siphos ages. From interviews, the interpretation falls in the category of DV.

(b) Temba = 2x, Silo = 5 years and then 31 – 5 = 26/2 = 13 years old

The first error, there was no Siphos ages. Second error learners thought Silo was 5 years old. The third error, learners thought Temba and Siphos were age-mates. That was the reason for some learners to subtracted 5 years from total ages of 31 years and later took 26 divided by 2 to get 13 years for each (Siphos and Temba). From the interviews, the interpretation falls in the category of NoI.

**(c) Cross multiple method using: 5 = 2
31 = x**

This error was similar to the error made in problem 4. Learners interpreted that 5 was equal to 2 and 31 was equal to the ages of Siphos. That could be the reason to work it out as it shown above.

(d) Temba = 2x, Silo = 5x and then 2x – 5x = 31

The first error, the term ‘five younger than’ was interpreted as fifth as old as Siphos. The second error, learners thought that the different between Tembas ages and Silos ages is equal to 31 and that would be leaded to Siphos ages. From the interviews, the interpretation falls in the category of COI.

(e) Temba = x, Siphos = 2x, Silo = 2x – 5 and then x + 2x + 2x – 5 = 31

There was only a single mistake and this was a common mistake many learners do. Some learners interpreted “Teba is twice as old as Siphos” as it said ‘Siphos was twice as old as Teba’. Thats why they represented the ages of Siphos as 2x, Teba as x instead of Teba

to be $2x$ years old and Sipho to be x years old. From the interviews, the interpretation falls in the category of DV.

$$(f) \quad x + 2 + x + x - 5 = 31$$

This error is implied that the term 'twice as old as was interpreted as 'two more than', that's why they wrote $x + 2$ instead of $2x$. From the interviews, the interpretation falls in the category of C.

$$(g) \quad 2x - 5 = 31$$

The main error these learners did was that they did not express the ages of Temba, Sipho and Silo in term of a variable. That could be the reason of coming up with algebraic expression of $2x - 5 = 31$ which does not show any meaning in term of information stated in the text.

(iv) Unexpected solution

There were various solutions from learners' works and these were 12, 7.2, 14.4, 13, 14

(v) Others

Some learners left it blank, they did not answer it. And some just wrote their answers

(vi) The table of proportion of learners falling into each category

Categories		In English test (%)	In Oshindonga test (%)
Number of learners doing the test in each language		100% (40)	100% (40)
Expected relationship or equation	Temba = $2x$ Sipho = x Silo = $x - 5$ $2x + x + x - 5 = 31$	65(26)	65(26)

	$4x - 5 = 31$ $4x = 31 + 5$ $4x = 36$ $x = 9$		
Major errors	Temba = $2x$ Silo = 5 years $31 - 5$ $26/2 = 13$ years old	17.5(7)	5(2)
	$5 = 2$ $31 = x$ $5x = 62$	5(2)	
	Sipho = x Temba = $2x$ Silo = $x - 5$ $2x + x - 5 = 31$	2.5(1)	12.5(5)
	Temba = $2x$ Silo = $5x$ $2x - 5x = 31$	2.5(1)	
	Temba = x Sipho = $2x$ Silo = $2x - 5$ $X + 2x + 2x - 5 = 31$	10(4)	5(2)
	$X + 2 + x + x - 5 = 31$		7.5(3)
	$2x - 5 = 31$	5(2)	5(2)
Minor errors	-	-	-
Expected solution		53((21)	55(22)
unexpected solution		42.5(17)	35(14)
Others		5(2)	10(4)

(the number in brackets shows the actual number of learners)

As it can be seen in the table above, 65% (26) of learners in both test (English and Oshindonga) managed to construct correct relationship in term of a variable x between the ages of all three people used in the problem text. 7 learners in English test misinterpreted the concept of “5 younger than” as it was saying “5 years” while in Oshindonga test only 2 learners did the same errors. 3 learners in English test misinterpreted the concept of saying ‘ Temba was twice as old as Siphon’ as it was saying ‘Siphon was twice as old as Temba’. It seems that learners reversed the meaning. While in Oshindonga test only 2 learners did the same errors. No minor error was noticed in both tests.

4.7. Problem 6

4.7.1 Description of problem 6 in both English and Oshindonga

Marcus bought a pizza and cut it into three pieces. When he weighed the pieces, he found that one piece was seven grams lighter than the largest piece and four grams heavier than the smallest piece. The mass of the whole pizza was three hundred grams. What was the mass of each piece?

Markusa okwa landa opizza, e teyi tete miiipambu itatu. Sho a vihi iipambu mbyono (itatu), okwa mono kutya oshipambu shimwe osha li shi vulike oohalama heyali koshipambu shi oshinene kwaayihe, nosha li oshidhigu shi vule oshipambu oshishona kwaayihe noohalama dhi li ne. Oshiviha shOpizza ayihe osha li oohalama omathele gatatu. Oshiviha shoshipambu kehe oshi thike peni?

4.7.2. Type and justification of the word problem

The above word problem is an abstract and at the same time is contextual problem. It was about measurement. I used it in this study to see whether learners would be able to understand, interpret and construct the relationship between the masses of those three pieces of pizza stated in the text.

4.7.3. The categories used

(i) Expected relationship or equation

All learners were expected to represent the masses of all three pieces using a certain variable before constructing an equation that could lead them to answer. For example, medium piece = x , large piece = $x + 7$ and small piece = $x - 4$. The next step was to construct an equation combining all algebraic expressions; $x + x + 7 + x - 4 = 300$. Then it was after that they could solve that equation and get the required solutions. On the other hand, they could also construct an equation using the following expressions; large piece = x , medium piece = $x - 7$ and small piece = $x - 11$. There were some learners who constructed correct relationship between quantities stated in the problem text and came up with correct algebraic expressions. Some learners made conceptual relationship errors and failed to come up with correct algebraic expressions.

(ii) Expected solution

All learners were expected to give the following solutions; the mass of L-piece = 106g, the mass of M-piece = 99g and the mass of S-piece = 95g.

(iii) Errors in strategies

There were various errors made by learners when they were attempting to construct the relationship between the masses of all three pieces.

(a) Ratio ideas

Some learners interpreted it using the ratio ideas. They understood that by saying "7 lighter than" means a part of and "4 heavier than" means a part of. They added the lighter part to the heavier part and got a total of 11 parts which consist of lighter and heavier parts. They took $\frac{7}{11} \times 300\text{g} = 190.9\text{g}$ and $\frac{4}{11} \times 300\text{g} = 109.0\text{g}$. It noticed in English test. From the interviews, the interpretation falls in the category of DV.

(b) 300g/3

Many learners did like what is shown above. This was noticed in both English and Oshindonga tests. Learners thought that the pieces were three and the total mass of three pieces was 300g. They only took 300g and divided it by 3 and found that one piece is equal to 100g, regardless of what is stated in the problem text. From the interviews, the interpretation falls in the category of DV.

(c) Large-piece = x , Medium-piece = $x - 3(x - 7 - 4)$, Small-piece = $x - 7$ and then $x + x - 3 + x - 7 = 300$

These learners used a variable x to express the masses of each piece in term of x and showed how these pieces are related to one another. The first error these learners did was; $x - 7 - 4 = x - 3$ instead of $x - 11$. The second error, if the mass of large piece was xg then the mass of small and medium piece could not be $x - 7$ and $x - 3(x - 7 - 4)$ respectively but should be $x - 11$ for small piece and $x - 7$ for a medium piece. From the interviews, the interpretation falls in the category of C.

(d) Large-piece = $x + 4$, Medium-piece = x , Small-piece = $x - 7$ and then $x - 7 < x + 4 = 300$

If the mass of the medium piece was represented by x , then the large could not be represented by $x + 4$ but should be $x + 7$ and the mass of the small piece was supposed to be represented by $x - 4$. The relationship between masses in term of x was wrong. This error was noticed in Oshindonga test. From the interviews, the interpretation falls in the category of C.

(e) Large-piece = $x + 7$, Medium-piece = $x - 7$, Small-piece = $x - 7 - 4$ and then $x + 7 + x - 7 - 4x + 28 = 300$

These learners only did two errors. The first one, they represented the mass of large piece as $x + 7$ instead of x and then the representation of masses for the other two could be correct. The second error, they combined their masses to form an equation but it was wrongly done because in equation, there was $4x$, 28 which did not appear in any masses of these piece. From the interviews, the interpretation falls in the category of C.

(f) $7 - x + 4 + x = 300$

These learners interpreted '7 lighter than a large piece' as $7 - x$ and also '4 heavier than small piece' as $4 + x$. They did not express the masses of all three pieces in term of x . they only used '7 lighter than', '4 heavier than' and $300g$. They did not try to create the relationship between the masses of all three piece first. From the interviews, the interpretation falls in the category of C.

$$(g) x + x - 7 + x - 11 = 300$$

This equation was correct and could lead learners to get correct but they did not indicate the piece that was represented by each algebraic expression. They did not make the relationship between the masses explicitly. It was difficult for one to figure out which one was expressed by x , $x - 7$ and $x - 11$. From the interviews, the interpretation falls in the category of NoI.

$$(h) x + 7 + x + x - 4 = 300$$

There are two errors with the equation above, The first one, there is no any mass that could be represented by $x + 7$, x and $x - 4$, they could not create any relationship between those expressions. Second error, they did not show which masse was representing small piece, medium piece and large piece. From the interviews, the interpretation falls in the category of NoI.

$$(i) 3x + 11 = 300$$

These learners expressed the masses of these pieces as follow; x , $x + 7$, $x + 4$ and then $x + x + 7 + x + 4 = 300$. They did not construct correct relationship between the masses of all three pieces.

(iv) Unexpected solution

Different answers were seen and were as follow; Large-piece = 224g, Small-piece = 75g, 101g for each 100g for each, Large-piece = 101, Medium-piece = 101g and Small-piece = 94g, 297g for each, 99.9 or 100g was not specified, 98g was not specified, Large-piece = 103.3, Medium-piece = 100.3 and Small-piece = 96.3.

(v) Others

There were few learners who did not answer this question.

(vi) The table of proportion of learners falling into each category

Categories		In English test (%)	In Oshindonga test (%)
Number of learners doing the test in each language		100% (40)	100% (40)
Expected relationship or equation	$M - \text{piece} = x$ or $L - \text{piece} = x$ $L - \text{piece} = x + 7$ $M - \text{piece} = x - 7$ $S - \text{Piece} = x - 4$ $S - \text{piece} = x - 11$ $3x - 4 + 7 = 300$ or $3x - 7 - 11 = 300$ $3x + 3 = 300$ $3x - 18 = 300$ $3x = 300 - 3$ $3x = 300 + 18$ $3x = 297$ $3x = 318$ $x = 99$ $x = 106$	35(14)	45(18)
Major errors	$\frac{7}{11} \times 300$ $\frac{4}{11} \times 300$ 190.9 109.0 $\frac{299.9}{3}$	2.5(1)	
	$1p - 7 + 3p + 4 = 300$	2.5(1)	5(2)
	$\frac{300}{3}$	17.5(7)	15(6)
	Large piece = x Medium piece = $x - 3(x - 7 - 4)$ Small = $x - 7$ $x + x - 7 + x - 3 = 300$	2.5(1)	5(2)
	Large piece = $x + 4$ Medium = x Small = $x - 7$ $X + 7 < x + 4 = 300$	20(8)	17.5(7)
Larger = $x + 7$ Medium = $x - 7$ Small = $x - 7 - 4$ $X + 7 + x - 7 - 4x + 28 = 300$		5(2)	

	$7 - x + 4 + x = 300$	10(4)	
	$X + x - 7 + x - 11 = 300$		7.5(3)
	$X + 7 + x + x - 4 = 300$	2.5(1)	
	$3x + 11 = 300$		7.5(3)
Minor errors	-	-	-
Expected solution		32.5(13)	30(12)
unexpected solution		57.5(23)	62.5(25)
Others		10(4)	7.5(3)

(the number in brackets shows the actual number of learners)

As it can be seen in the table above, 16 learners did conceptual errors in both English and Oshindonga tests. No technical error was noticed in both tests.

4.8. Problem 7

4.8.1. Description of problem 7 in both English and Oshindonga

An eraser costs 15 cents more than a pencil, twelve pencils costs sixty cents more than eight erasers. How much does one pencil cost?

Okadhimitho otaka kotha oosenda omulongo nantano shi vulithe opena yekala, oopena dhekala omulongo nambali otadhi kotha oosenda omilongo hamano shi vulithe uudhimitho uhetatu. Opena yimwe yekala oyi na ingapi?

4.8.2. Type and justification of the word problem

The problem illustrated above is an abstract and at the same time one can regard it as contextual problem. It is more about money and a bit trick question that need learners to think critically in order to construct correct relationship between the quantities stated in the text. That was the reason I included this problem to find out whether learners were able to understand, interpret and construct the correct relationship between the cost of erasers and pencils.

4.8.3. The categories used

(i) Expected relationship or equation

There were only two ways which could lead learners to get correct answer. They were expected to use a variable to represent the cost of a pencil. For example, the variable x and then use the information in the problem text to create relationship between the cost of a pencil and an eraser like $12x = 60 + 8(x + 15)$ they could solve it. The second way, learners supposed to construct two a equations and solve them simultaneously. For example, $e - 15 = p$ and $8e + 60 = 12p$, so E stand for the cost of an eraser and P stand for the cost of a pencil. However, Different variable could also be used to represent the cost of a pencil and an eraser. There were only few learners who constructed these equations and solved them correctly. It was noticed in both tests. From the interviews, the interpretation falls in the category of DV.

(ii) Expected solution

All learners were expected to give an answer of 45 cents the cost of one pencil.

(iii) errors in strategies

$$(a) 12x + 8x = 180$$

Learners did the mistake in the second step of $12x = 60 + 8(x + 15)$, they removed brackets and got $12x = 8x + 180$. They tried to put like terms together and said $12x + 8x = 180$. This was wrong because the meaning of the problem was changed and as result they will not get the correct price of the pencil. They supposed to do it like $12x - 8x = 180$ in order to keep the meaning similar to what is in the first step. From the interviews, the interpretation falls in the category of DV.

$$(b) \begin{aligned} e + 15 &= e \\ 12p &= 60 + 8e \end{aligned}$$

Rearrangement

$$p + e = 15$$

$$12p + 8e = 60$$

These learners started well and the first step was correct but again changed the meaning of the problem text in the second step of rearranging the equations to solve them simultaneously. They supposed to say $P - E$ instead of $P + E$. In the other equation, they supposed to say $12p - 8e$ instead of $12p + 8e$. From the interviews, the interpretation falls in the category of C.

$$(c) \begin{aligned} 12 &= 60 \\ 1 &= x \end{aligned}$$

$$12x = 60$$

Cross multiple method was used

Many learners understood this problem in such way that 12 pencils cost 60 cents. Then they just said 1 pencil cost x and then use cross multiple method to get the cost of one pencil. They did not try to create the relationship between the cost of pencil and an eraser. From the interviews, the interpretation falls in the category of NoI.

$$(d) \begin{aligned} 12 &= 15 \\ 60 &= x \end{aligned}$$

These learners interpreted this problem that 12 pencil cost 15 cents and what they did not know is the number of pencil that cost 60 cents. These learners also did not create the relationship between the cost of pencil and eraser.

$$(e) 8 \times 15 - 12 \times 60$$

These learners understood and interpreted this problem as follow; the different between (8×15) and (12×60) is equal to the cost one pencil. From the interviews, the interpretation falls in the category of NoI.

(iv) Unexpected solution

Different answers were given by different learners. These answers were; 75cents, 5cents, N\$1.88, N\$1.20, 15cents, N\$3.75, N\$4.85, 9cents, 20 cents, 14 cents and 70cents. Some of these answers were noticed in tests, some in English test only and some in Oshindonga test only.

(v) Others

There some learners who did not answer this question and some left it half way.

(vi) The table of proportion of learners falling into each category

Categories		In English test (%)	In Oshindonga test (%)
Number of learners doing the test in each language		100% (40)	100% (40)
Expected relationship or equation	$12x = 60 + 8(x + 15)$ or an Eraser = x and $12x = 60 + 8x + 120$ a Pencil = y $12x - 8x = 60 + 120$ x = y + 15 $4x = 180$ $8x = 12y - 60$ $x = 45$ cents rearrange $x - y = 15$ $8x - 12y = - 60$ $8x - 8y = 120$ $- 4y = 180$ $y = 45$	15(6)	14(5)
Major errors	$X + 15 > x$ $12x + 8x = 180$ $P + 15 = E$ $12p = 60 + 8E$ Rearrangement $P + E = 15$ $12p + 8E = 60$	5(2) 25(10)	22.5(9)

	$12 = 60$ $1 = x$ $12x = 60$ Cross multiple method was used next	52.5(21)	57.5(23)
	$12 = 15$ $60 = x$ $X - 15 + 12x = 60 - 8$	5(2)	2.5(1)
	$8 \times 0.15 - 12 \times 0.6$		10(4)
Minor errors	-	-	-
Expected solution		10(4)	5(2)
unexpected solution		87.7(35)	92.5(37)
Others		2.5(1)	2.5(1)

(the number in brackets shows the actual number of learners)

As it can be seen the table 4.8 above, this question was extremely poorly interpreted correctly. 26 learners in English test did major errors while in Oshindonga test, 19 did major errors. No minor error was noticed in both tests. 35 learners got wrong solution in English test as compared to 37 learners in Oshindonga test, there was a difference of two learners.

4.9. Problem 8

4.9. 1. Description of the problem 8 in both English and Oshindonga

Tomas bought four pieces of wood and each piece is 2.5m long. How many pieces of wood of length 1m can he get out of them?

Tomas okwa landa iipambu ine yiiti (iipilangi), noshipapu kehe oshi na uule woometa mbali netata (2,5m). Iipambu ingapi yuule wometa yimwe kehe ta vulu okumona mo?

4.9.2. Type and justification of the problem

The problem above is part of number and measurement. It is a contextual word problem. This problem requires learners to consider this problem in a real life in practical situation in order

to interpret it correctly and give an expected answer. Therefore, I included this problem to see whether learners would be able to understand and interpret this problem from real life context.

4.9.3. The categories used

(i) Expected relationship or equation

All learners were expected to think that on the piece of wood, 2.5m long, they would only get two piece of wood of length 1m. if they have 4 pieces of wood then on each piece they could only get 2 pieces of wood from each and a piece of 0.5m would remain. In practical life one can not fixed 0.5m on the other 0.5m to make it one a piece of wood with a length of 1m.

(ii) Expected solution

All learners were expected to give an answer of 8 pieces of wood.

(iii) Errors in strategies

Many learners did similar error when they were solving this question.

$$(a) 2.5/1 = 2 \text{ pieces}$$

These learners thought that 2.5m was the length of pieces of wood and then they divided 2.5m by 1m and got 2 pieces. From the interviews, the interpretation falls in the category of DV

$$(b) \begin{array}{l} 4 = 2.5 \\ x = 1 \end{array}$$

These learners understood this problem as follow, they thought the length of 4 pieces of wood was 2.5m, and then they said how many pieces of wood of length 1m could they obtain from 2.5m? They then used a cross multiple method to find the solution. From the interviews, the interpretation falls in the category of DV.

$$(c) 2.5 \times 4 = 10/1 = 10 \text{ pieces}$$

Many learners interpreted this problem from theoretical perspective and did not consider the problem in a real life in practical situation. They did understand that 2.5m was a length of one

piece of wood and if they have 4 pieces of wood the total length would be $2.5\text{m} \times 4 = 10\text{m}$ and then they thought to get pieces of wood of length 1m, they have to cut off 10m in piece of length 1m and got an answer of 10 pieces of wood. This could be correct from theoretical perspective but it was wrong from real life in practical situation. From the interviews, the interpretation falls in the category of DV.

(d) $2.5 \times 4 = 6$ pieces

These learners did a technical error and I did not understand how they interpreted this problem. From the interviews, the interpretation falls in the category of NoI.

**(e) $1 = 2.5\text{m}$
 $x = 1\text{m}$**

These learners interpreted this problem as follow; they said 1 piece of wood has a length of 2.5m and then they thought if they have a piece of wood of length 1m, how many piece they can get out of that length. This was the reason, learners worked it out as it is shown above. From the interviews, the interpretation falls in the category of C.

(iv) Unexpected solution

There were many different answers from learners' work and were as follow; 10,6, 2, 1.6, 0.4, 6, 7 pieces of wood.

(v) Others

There were some learners who did not answer this question. And there were some of the work that did

(vi) The table of proportion of learners falling into each category

Categories		In English test (%)	In Oshindonga test (%)
Number of learners doing the test in each language		100% (40)	100% (40)
Expected relationship or equation	$2 + 2 + 2 + 2$ or $2.5 \times 4 = 10$ then $10 - 2$ 8 pieces of wood	5 (2 learners)	2.5 (1 learners)
Major errors	$2.5/1 = 2$ pieces	10 (4 learners)	12.5 (5 learners)
	$4 = 2.5$ $x = 1$	5 (2 learners)	
	$2.5 \times 4 = 10/1 = 10$ pieces	65 (26 learners)	85 (34 learners)

	2.5 x 4 = 6 pieces	5 (2 learners)	
	l = 2.5m x = 1m	10 (4 learners)	
Minor errors	-	-	-
Expected solution		5 (2 learners)	2.5 (1 learners)
unexpected solution		95 (38 learners)	97.5 (39 learners)
Others		5 (2 learners)	2.5 (1 learners)

(the number in brackets shows the actual number of learners)

As it can be seen in the table 4.8 above, this question was extremely poorly interpreted in both tests. 95% of learners (38 learners) in the English test learners interpreted and solved it wrongly as compared to 97.5% of learners (39 learners) doing the Oshindonga test. Language usage did not make a difference at all because the performance in both tests looked to be the same. In both tests many learners answered it from a theoretical perspective. This means learners applied straightforward arithmetic procedures and produced an answer of pieces of wood from the mathematics theoretical perspective rather than thinking in real life terms and applying realistic knowledge to solve this question to get an answer of 8 pieces of wood.

4.10. Problem 9

4.10.1 Description of the problem 9 in both English and Oshindonga

A box of 24 eggs cost N\$7,30. How many boxes can your teacher buy with a N\$100 note?

Okapakete komayi 24 Otaka kotha (gu) N\$7,30.
Uupakete ungapi omulongi gweni ta vulu okulanda nefo lyo N\$100

4. 10.2. Type and justification of the problem

The problem above is part of money. It is contextual word problem. I used this problem in this study to see whether learners would be able to understand and interpret it in practical situations rather than interpret and answer it from just mathematical point of view.

4.10.3. The categories used

(i) Expected relationship or equation

All learners were expected to take $100/7.30$ and get 13.3 but they should understand that there is no way they can buy a full box and a piece of a box but rather get a change.

(ii) Expected solution

All learners were expected to give an answer of 13 boxes and a change of N\$5.10.

(iii) Errors in strategies

Some learners did errors when they were answering this question, they did not think this question from its practicality situation in real life. The initial interpretation could be regarded as it was correct but when it comes to the final answer, they made a conceptual relationship error in sense that they did not think and how money are used in real life situation.

(a) 329 Boxes

These learners interpreted and work it out as follow; 24 eggs cost N\$7.30 and they said how many boxes they could buy with hundred dollars instead of saying how many eggs they could buy with hundred dollars. That was the reason they gave 329 boxes instead of saying 329 eggs and later they could find the number of boxes. From the interviews, the interpretation falls in the category of DV.

(b) 14 Boxes

These learners interpreted this question correctly and they have followed correct strategy of finding the answer but at the final step, they did not think how practical the problem was, they just applied mathematical rules and when they got 13.7 they rounded it to get 14 boxes. From the interviews, the interpretation falls in the category of DV.

(c) 13.7 Boxes

These learners interpreted this question correctly but they made error in the final step. It is impossible to buy 13.7 boxes but they supposed to understand that they could only buy 13 boxes and get a change that was not enough to buy a full box. From the interviews, the interpretation falls in the category of DV.

(d) 333 boxes

These learners find the cost of one egg and they got N\$0.30 for each egg and then they took $N\$100 \div 0.30$ and got 333boxes instead of eggs. That was how these learners made error. From the interviews, the interpretation falls in the category of DV.

(iv) Unexpected solution

There were different answers caused by conceptual relation error and were as follow; 13.7, 14, 329, 228.8 and 333 boxes

(v) Others

There were few learners who did not write anything on this question.

(vi) The table of proportion of learners falling into each category

Categories		In English test (%)	In Oshindonga test (%)
Number of learners doing the test in each language		100% (40)	100% (40)
Expected relationship or equation	$1 \text{ box} = N\$ 7.30$ $x = N\$ 100.00$ $7.3x = 100$ $X = 100/7.3$ $X = 13.7$ $\therefore \text{ can only buy 13 boxes}$	95(38)	100(40)
Major errors	329 Boxes	12.5(5)	15(6)
	14 Boxes	10(4)	17.5(7)
	13.7 Boxes	20(8)	7.5(3)
	333 Boxes	2.5(1)	
Minor errors	-	-	-
Expected solution		52.5(21)	59(23)
unexpected solution		45(18)	40(16)
Others		2.5(1)	2.5(1)

(the number in brackets shows the actual number of learners)

As it can be seen in the table above, this question was extremely well answered in both tests. 17 learners did conceptual errors in English test while in Oshindonga test, 16 learners did the same conceptual errors that were done in English test. No technical errors was noticed in both tests.

4.11. Problem 10

4. 11: 1 Description of the problem 10 in both English and Oshindonga

Three men take six days to complete a job. How long will two men take to complete a similar job?

Aalumentu yatatu oshe ya pula (otashi ya pula) omasiku gahamano okumana oshilonga. Otashi ka pula aalumentu yaali uulethembo wu thuki peni, opo ya mane oshilonga sha faathana?

4.11.2. Type and justification of the word problem

The two quantities in the problem above are inversely proportional, then as the one increases, so the other decreases. I therefore included a question on inverse proportion to see whether learners would be able to make sense with the problem and recognized how the quantities in the problem related to one another.

4.11.3. The categories used

(i) Expected relationship or equation

All learners were expected to understand that the quantities in the problem were inversely proportional to each other. Then they were expected to take 6×3 and get 18 days that would be taken by one man. Finally, they supposed to take $18 \div 2$ and get 9 days that will be taken by 2 men. Some learners used correct strategy and got correct solution. This was noticed in both English and Oshindonga tests.

(ii) Expected solution

All learners who followed or used the correct strategy should give any answer of 9 days

(iii) Errors in strategies

Some learners did not realize that the quantities in the problem were inversely proportional to each other, hence some learners thought the quantities were direct proportional to each other, that's why they ended up doing conceptual relationship errors between quantities.

$$\begin{aligned} \text{(e) } 3 \text{ men} &= 6 \text{ days} \\ 2 \text{ men} &= x \end{aligned}$$

Cross multiple methods was used

These learners understood that the quantities in the problem were direct proportional to each other. This was the reason for them to work it out as it is shown above. From the interviews, the interpretation falls in the category of DV.

$$\text{(f) } 6 \times 2 = 12 \text{ days}$$

These learners thought that if $3 \times 6 = 18$ days that would be taken by one man to complete a job, then $2 \times 6 = 12$ days that would be taken by two men to complete a job. From the interviews, the interpretation falls in the category of C.

(iv) Unexpected solution

There were various wrong answers. They were as follow; 12 days and 4 days.

(v) Others

Some learners in both English and Oshindonga tests did not write anything on this question.

(vi) The table of proportion of learners falling into each category

Categories	In English test (%)	In Oshindonga test (%)
Number of learners doing the test in each language	100% (40)	100% (40)
Expected relationship or equation	$(3 \times 6)/2$ 55(22)	52.5(21)

Major errors	3 men = 6 days 2 men = x Cross multiple method was used	35(14)	32.5(13)
	6 x 2 = 12 days	10(4)	12.5(5)
Minor errors	-	-	-
Expected solution		55(22)	52.5(21)
unexpected solution		45(18)	45(18)
Others		0	2.5(1)

(the number in brackets shows the actual number of learners)

The table above shows that 13 learners in English test interpreted the quantities used in this problem as they varies directly to each others while in Oshindonga test 12 learners interpreted it in the same was as in English test. No technical error was noticed in both tests.

APPENDIX 2

Ministry of Basic Education

Oshikoto Region

Name _____

Grade: _____

Mathematics test 1

2008

Time: 1 h and 30 minutes

Marks: _____

Additional materials to be provided by the candidate

- Ruler
- Calculator

Instructions to the candidate:

1. Write your name clearly on the answer paper
2. Answer all the question in the space provided
3. Show your work clearly

1. When a number is added to two, the answer is the same as when fourteen is subtracted from three times the same number. What is the number?
2. A certain number is multiplied by 4 and 9 added to it. The result is the same as when the same number is multiplied by 6 and 13 subtracted. Find the value of this number.

3. If 3 times a number is decreased by 9 the result is the same as 2 times the same number when decreased by 1. Find the number.
4. Tuli is 25 years younger than his father. His mother is 2 years younger than his father. Their total ages add up to 78 years. How old is Tuli?
5. Temba is twice as old as Siphoh and Silo is five years younger than Siphoh. The total of their ages is thirty one years. How old is Siphoh?
6. Marcus bought a pizza and cut it into three pieces. When he weighed the pieces, he found that one piece was seven grams lighter than the largest piece and four grams heavier than the smallest piece. The mass of the whole pizza was three hundred grams. What was the mass of each piece?
7. An eraser costs 15 cents more than a pencil, twelve pencils costs sixty cents more than eight erasers. How much does one pencil cost?
8. Tomas bought four pieces of wood and each piece is 2.5m long. How many pieces of 1-m wood can he get out of them?
9. A box Of 24 eggs cost N\$7,30. How many boxes can your teacher buy with a N\$100 note?
10. Three men take six days to complete a job. How long will two men take to complete a similar job?

Mathematics test 2

2008

Time: 1 h and 30 minutes

Marks: _____

Additional materials to be provided by the candidate

- Ruler
- Calculator

Instructions to the candidate:

4. Write your name clearly on the answer paper
5. Answer all the question in the space provided
6. Show your work clearly

1. Ngele onomola oya gwehwa kumbali, eyamukulo oiyafathana nuuna omuulongo nane gwakuthwa momulongo nandatu gwi injipalekwa nonomola ndjoka. Onomola oyini ndjono?
2. Okadhimitho otaka kotha ocenda omulongo nantano shivulithe kopena yekala, oopena thekala omulongo naambali otadhi kotha ozenta omilongo hamano shivulithe uudhimitho uuhetatu. Opena yimwe yekala oyina iingapi?
3. Temba okuvule Sipho lwaali, Silo omushona kusipo nomvula ntano. Omvula thawo kuumwe adhihe odhili omilongo ndatu nayimwe. Sipho okuna omvula ngapi?
4. Markusa okwa landa opizza, eteyi tete miipambu itatu. Sho a vihi iipamu mbyono (itatu), okwa mono kutya oshipambu shimwe osha li shi vulike oohalama heyali koshipambu shi oshinene kwaayihe, nosha li oshidhigu shi vule oshipambu shishona kwaayihe noohalama dhi li ne. Oshiviha shOpizza ayihe osha li oohalama omathele gatatu. Oshiviha shoshipambu kehe oshithike peni?
5. Ngele onomola yindjipalekwa lutatu oya shonopalekwa nomugoyi (mwakuthwa omugoyi), eyamukulo olya faathana nonomola oyo tu ndjono yi indjipalekwa lutatu ita mu kuthwa yimwe. Konga onomola ndjoka.
6. Okapakete komayi 24 otake kotha (gu) N\$7,30. Uupakete ungapi omulongi qweni ta vulu okulanda nefo lyo N\$100?
7. Tomas okwa landa ipapu ine yiiti (ipilangi), noshipapu kehe oshina uule woometa mbaali netata (2,5m). Iipambu ingapi yuule wometa yimwe kehe tavulu okumona mo?
8. Aalumentu yatatu oshe ya pula (otashi ya pula) omasiku gahamano okumana oshilonga. Otashi ka pula aalumentu yaali uulethimbo wu thike peni, opo ya mane oshilonga sha faathana?
9. Onomola yontumba oyi indjipalekwa nane eta kugwedhwa omugoyi. Eyamukulo olya fathana uuna onomola ndjoka yindjipalekwa nahamano etamu kuthwa omulongo nandatu. Konga onomala ndjoka.

10. Tuli omushona kuhe nomvula omilongo mbali nantano. Yina omushona kuhe noomvula mbali. Omvula dhawo adhihe kuumwe odhi li omilongo heyali nahetatu. Tuli oku na oomvula ngapi?

APPENDIX 3

Transcription of interviews

Learner 1 BN

1. When a number is added to two, the answer is the same as when fourteen is subtracted from three times the same number. What is the number?

CN: how did you go about this question?

L1: I started with making equation from the sentence.

CN: How?

L1: The sentence said “ When a number is added to two, the answer is the same as when fourteen is subtracted from three times the same number”

CN: Hm, then

L1: 2 plus x ... x is the number which is added to two .. is equal to three times the number then I subtract 14.

CN: Why did you subtract 14?

L1: because they say, the answer is same as when 14 is subtracted from three times a number.

CN: Hm, okey from there!

L1: from there I bring like terms together on one side of equation and the x, bring them on the left hand and the numbers on the right hand.

CN: Hm and then

L1: and then I subtract... solve x number first.

CN: what was the value of x?

L1: 8

2. A certain number is multiplied by 4 and 9 added to it. The result is the same as when the same number is multiplied by 6 and 13 subtracted. Find the value of this number.

CN: Explain how did you start answering this question?

L1: they said a number is multiplied by 4 then I take x as a number multiplied by 4 and then said 9 is added to it ($4x$), plus 9 then I make it equal to ... because they said the result "sama as when the same number multiplied by 6. I make $6x$ and put it in brackets.

CN: there is equal here, where did you find it?

L1: equal is the... I put it because the result is the same as

CN: do you want to say 'same as' means equal to.

L1: no. the results... I meant because they said the result is equal to as $6x - 13$. It means what is on left hand side of equal is same as the right hand side

CN: okey! Continue

L1: then I make x times 4 give me $4x + 9$ and on the other side I make $6x$ and give $6x - 13$. I bring the like terms together, then I find x .

CN: which is

L1: 11

3. If 3 times a number is decreased by 9 the result is the same as 2 times the same number when decreased by 1. Find the number.

CN: how did you go about this question?

L1: again I started 3 times a number, that means $3x$ and that said the number that $3x$ is decreased by 9 and decreased means subtraction. Then is equal ... because they say the result is same, then I put equal to 2 times means $2x - 1$ because the number is decreased by 1

CN: okey

L1: bring like terms together and solve for x

4. Tuli is 25 years younger than his father. His mother is 2 years younger than his father. Their total ages add up to 78 years. How old is Tuli?

CN: how did you go about this question?

L1: first I found how many ages do they have both of them by using x.

CN: mmh

L1: then they said "Tuli is 25 years younger than his father". Then I make $x - 25$ because the age of father is x

CN: mmh, but why did you say $x - 25$?

L1: because they said ... that is the ages of Tuli... they said Tuli is 25 younger than his father. Then the father's ages is x, then I make ... I take the ages of the father subtract 25 in order to get the ages of Tuli.

CN: mmh

L1: then they said...his mother is $x - 2$, kulya, I mean his mother is two years younger than his father, the ages of his father is x, then I make $x - 2$ that is the ages of his mother. Then from there I make my equation

CN: how?

L1: by adding the ages of Tuli, his mother and his father together and make it equal to 78 because here they said their ages end up to 78 years

CN: mmh from there

L1: then from there I bring like terms together and I find x, that is the ages of his father, then in order to get the ages of Tuli which is asked here, I make $35 - 25 = 10$ years, that is the ages of Tuli.

5. Temba is twice as old as Siphon and Silo is five years younger than Siphon. The total of their ages is thirty one years. How old is Siphon?

CN: How did you go about this question?

L1: again here, from the statement they said “Temba is twice as old as Sophi” that means the age of Temba is twice, twice means $2x$ and then the ages of Sophi is not given here I put x then the ages of Silo, they said is 5 younger than Siphoh, that means the ages of Siphoh is x and for Silo is $x - 5$. From there I make my equation by adding their ages together and make it equal to 31.

CN: mmh, from there

L: I bring the like terms together and I found x and I found that $x = 9$ and that is the ages of Siphoh

CN: Okey

6. Marcus bought a pizza and cut it into three pieces. When he weighed the pieces, he found that one piece was seven grams lighter than the largest piece and four grams heavier than the smallest piece. The mass of the whole pizza was three hundred grams. What was the mass of each piece?

CN: How did you start answering this question?

L1: again, here.. this is a trick question, first they said, the total mass of the pizza is 300g, is equal to 300g. then they said “the one piece was 7 grams lighter than the largest piece I will give some x because the largest piece is not given here and the medium piece I make $x - 7$ and small piece, here they said... and the small piece here here, they said it was 4g heavier than the smallest. It means the medium was 4g heavier than the smallest piece that means $7g + 4g = 11g$, and I make $x - 11$.

CN: why did you add $4g + 7g$?

L1: because 7g is the middle piece because here they said that the medium piece is 7g lighter than the largest piece. The smallest piece is 4g heavier than the smallest, that is the medium piece that 4g heavier than than the smallest one. If is heavier that means you have to add 4 to 7g so that you get the smallest.

CN: Hm

L1: from there I take... x is the largest piece and $x + 7$ is middle and $x + 11$ is the smallest and make them they are equal to 300g

CN: why?

L1: because they said the mass of the whole pizza is 300g

CN: Hm

L1: bring the like terms together

CN: Okey!

7. An eraser costs 15 cents more than a pencil, twelve pencils costs sixty cents more than eight erasers. How much does one pencil cost?

CN: How did you go about this question?

L1: here I make that 12 pencils cost 60 cents more than 8 erasers. That means the cost of 12 pencils I do not know, kulya.. I do not know how much and therefore I make 12 pencils times x make it equal to 60 cents plus.....

CN: how?

L1: there I make because they said eraser, that means one eraser is cost 15 cents more than a pencil. That means 60 cents and they said "12 pencils cost 60 cents more than 8 erasers" there I make $60 + 8 \text{ erasers times } x + 15 (60 + 8(x + 15))$

CN: where did you find $x + 15$? And what does it represent?

L1: $x + 15$ is represent the cost of the rubber, the eraser

CN: why did you put brackets?

L1: I put brackets because there is an equation

CN: from there

L1: I solve the equation and first I remove brackets

CN: Hm

L1: then bring the like terms together

8. Tomas bought four pieces of wood and each piece is 2.5m long. How many pieces of 1-m wood can he get out of them?

CN: how did you go about this question?

L1: first 2.5 there is 4 pieces of wood in order to find out how many piece does Tomas buy if they having one metre, I just make 4×2.5 and from here he is going to get 10 piece if they are having a length of 1m.

CN: If I listened to you very well you said 2.5 is a length of 4 pieces. Is it what you said?

L1: No the length of 2.5m Tomas is going to buy 4 pieces, if they are having the length of 2.5m and what is asked is how many will he buy? If they are Having 1m long. From there I make $2.5 \times 4 = 10$

CN: what did you get when you multiply 2.5 by 4?

L1: if the length is 1m, he is going to buy 10 pieces

CN: what is the total length of 4 pieces?

L1: 2.5m

CN: then you multiple it by 4

L1: yes!

CN: now, did you get the total length or number of pieces.

L1: the pieces of 1m

9. A box of 24 eggs cost N\$7.30, how many boxes can your teacher buy with a N\$100 note?

CN: how did you answer this question?

L1: here they are asking the number of boxes which a person going to buy if having a N\$100 and they are saying one box cost N\$7.30.

CN: mmh

L1: there, I make... I just take hundred divided by the cost of one box and I found that, she is going to buy 13 boxes in N\$100

CN: is that the reason you divide by N\$7.30.

L1: The reason is to get the number of boxes which you are going to buy with N\$100 not. And a number of one box is given. Therefore I just take 100 divided by number, the cost of one box.

CN: why did not use 24?

L1: 24 is not . it will not be possible to get the number of boxes which is going to buy... a person is going to buy, because 24 is just number of eggs which are in one box

10. Three men take six days to complete a job. How long will two men take to complete a similar job?

CN: how did you go about this question?

L1 there I found how many days which one man going to take by multiplying 6 days by 3 mans.

CN: why?

L1: to get a how many days, the number of days one person going to take to... to get the days which one man going to take, you have to take 6 times 3 then you find out if it was one man, he will take 18 days then take 18 days divided by 2

CN: okey! That was test 1. this is the second test and you are welcome to use both language

11. Okadhimitho otaka kotha ocenda omulongo nantano shivulike kopena yekala, oopena thekala omulongo naambali otadhi kotha ozenta omilongo hamano shivulithe uudhimitho uuhetatu. Oopena yimwe yekala oyina iingapi? (question 7 in English test.

CN: how did you go about this question?

L1: here they are saying a rubber cost 15 cents higher than a pencil. Higher than 12 pencils(read the whole sentence in oshiwambo), here I just make that $12x = 60$ cents then I divided by 12

CN: why?

L1:because they are saying 12 pencils... because is 60 cents higher than rubber

CN: mmh now here you took 12 times x, where did you find 12x

L1: x is the amount of... is the cost of the rubber, and I do not the cost of the rubber, then I make $12x = 60$

CN: where does it started in the statement?

L1: It stated when they said 12p cost 60c higher than 8 rubbers

CN: Hm!

L1: then, I make 60 divided by 12

CN: then you get the cost of pencil!

L1: yes!

12. Temba okuvule Siphon lwaali, Silo omushona kusipo novula ntano. Omvula thawo kuumwe adhihe odhili omilongo ndatu nayimwe. Siphon okuna omvula ngapi? (question 5 in English test)

CN: how did you answer this question?

L1(read) Temba “okuvule Siphon lwaali” that means Temba is $2x$ and they are saying “Silo omushona ku Siphon novula ntano” that means Silo is $x - 5$. and ages of Siphon is x then I add their ages together to form equation which is equal to 31

CN: okay

13. CN: I take you a little bit behind, lets look at questions 2 and this question 7 in the English test. Are they the same or different?

L1: they are the same

CN: why do you have different answers?

L1: is because of language

CN: what do you meant by saying 'language'? what do you want to say?

L1: one question is asked in oshiwambo and the other one is asked in English

CN: is the different?

L1: yes

CN: How?

L1: because if the question is asked in English is much better to understand than the one is asked in oshiwambo.

CN: why do you say that?

L1: is because.... For example when you are forming a equation which is asked in english is better than forming an equation to the question which is asked in oshiwambo.

CN: what did it make it better?

L1: is the words which is used

CN: do meant the words in oshiwambo are difficult to understand?

L1: you will understand them but you may not know what they meant?

CN: is it not the language that you speak fluently?

L1: it is

CN: but why is there difficult in understanding?

L1: because for examples when you are saying the number is divided in English, it may not be the same when it is asked in oshiwambo

CN: Okey! Now lets refers to this question. I did not say this correct or this one but looking at your works are different. Here you take 12 pencils, nothing here then 60 cents. Tell me what causes this different

L1: maybe is because the way I was understand the day before

CN: what do you meant by that?

L1: I meant the one which was asked in English I understand it better than the one in Oshiwambo

CN: why? I did not say this is correct, maybe this is correct one (refer to oshiwambo)

14. Ngele onomola yindjipalekwa lutatu oya shonopalekwa nomugoyi (mwakuthwa omugoyi), eyamukulo olya faathana nonomola oyo tu ndjono yi indjipalekwa lutatu ita mu kuthwa yimwe. Konga onomola ndjoka.

CN: Tell me how did you answer this question?

L1:(read the question), here I make an equation that if a number is multiplied by 3 and that is representing $3x$, then divided by $9 - 9$

CN: why did you divide?

L1: because here, they say 'shonopalekwa'

CN: okey

15. Tomas okwa landa ipapu ine yiiti (ipilangi), noshipapu kehe oshina uule woometa mbaali netata (2,5m). Iipambu ingapi yuule wometa yimwe kehe tavulu okumona mo?

CN: how did you go about this question?

L1: here, the length of 2.5m you get 4 pieces

CN: why?

L1: it stated in the question

CN: Hm

L1: here they are asking the length of 1m, I make 2.5m it will give 4 pieces and 1m give you x

CN: is it what it say?

L1: (read the question)

CN: lets look at these two questions, are they the same or different?

L1: they are the same

CN: why do you say they are the same?

L1: because the number of pieces which is in the other question is same as the number of pieces in this question.

Learners 2 D

- 1. If 3 times a number is decreased by 9 the result is the same as 2 times the same number when decreased by 1. Find the number.**

CN: how did you go about this question?

L2: I take three, three times x, I take x as a number and I multiply by x and I subtract 9

CN: why did you subtract 9?

L2: because the question say "three times a number is decreased by 9, so nine is subtracted from it.

CN: Oo then

L2: then I write is equal to $2x$ and subtract 1 from $2x$ because the question say "2 times the same when decreased by 1. one is subtracted from $2x$.

CN: why did you find equal?

L2: the same... the same means equal to

CN: okey

2. Tuli is 25 years younger than his father. His mother is 2 years younger than his father. Their total ages add up to 78 years. How old is Tuli?

CN: how did you go about this question?

L2: Okey, the question says Tuli is 25 younger than his father. So the ages of father is x . then I take $25x$ subtract 25 because Tuli is younger than, when it say younger than , is... it means subtraction. 25 is subtracted from x , x which is the ages of his father

CN: Hm!

L2: His mother is 2 younger then his father, 'iyaa' because father is x , x years old. Then I take x for father subtract 2 years

CN: which statement represented by $x - 2$

L2: Okey! I subtract the ages of mother $x - 25 - 2$ which is 2 for mother then the ages of mother is x , $25 - 2$ then you get 23. $x - 23$ is the age of her mother, his mother.

CN: the ages of Tuli is

L2: the ages of Tuli is $x - 25$

CN: why?

L2: because, the ages of Tuli is 25 younger than his father. The ages of father is x , so you have to subtract 25 from x

CN: mmh, okey then

L2: $x - 23$, this is the ages of mother because the ages of mother is the father's ages minus the ages of father (ooh) the father's ages plus the ages of Tuli. Iyaa $x - 23$ is... I got it from $x - 25$ then I subtract 2 years

CN: then from there

L2: I take x the ages of father plus the ages of Tuli which is $x - 2$, 25 I meant plus $x - 23$ the ages of mother, then that answer is equal to 78 years. If you add those x s together they will give $3x$ then subtract, $3x$ is the... when x , x , then $- 25 - 23 = - 48 = 78$. then $3x - 48 = 78$ then you add 48 to 78 then you got $3x = 126$, then I got x which is the ages of father is 42

CN: How did you get the ages of Tuli?

L2: because the ages of Tuli is the ages of father minus 25, so Tuli is $42 - 25 = 17$. Tuli is 17 years old.

CN: Okey

5. Temba is twice as old as Siphon and Silo is five years younger than Siphon. The total of their ages is thirty one years. How old is Siphon?

CN: How did you go about this question?

L2: Because Temba is twice as old as Siphon. Siphon is x years old, I gave him x , then, then I write $2x$ which is the ages of temba then I add x which is the ages of Siphon. Silo is 5 years younger than Siphon. Means Siphon's ages minus 5 years is equal to Silo's ages. If you add those years, $2x$ for Siphon, x for Silo and $x - 5$ for Silo then you got $4x - 5 = 31$ years old, because the question say their total ages is 31

CN: you said Temba's ages is $2x$! why do you say so?

L2: $2x$ is the ages of Temba

CN: why?

L2: because Temba is twice the ages of Siphon which is x . if you multiply x by 2 you get $2x$

CN: then from there

L2: if you add $2x$ plus x plus x again, you get $4x$ then minus 5 is equal to 31 ($4x - 5 = 31$)

CN: okey

6. Marcus bought a pizza and cut it into three pieces. When he weighed the pieces, he found that one piece was seven grams lighter than the largest piece and four grams heavier than the smallest piece. The mass of the whole pizza was three hundred grams. What was the mass of each piece?

CN: How did you start answering this question?

L2: this question was difficult, it was confusing me. I just take x

CN: x for what?

L2: x for the medium for the medium pizza, then I take large... the small pizza... I took the medium pizza then I subtract 7

CN: why did you subtract 7?

L2: because it says one piece is, was 7g lighter than largest piece, "ohaa" I means, lighter than largest piece so I just guess... I did n't know if I will get this.

CN: OO! Just proceed, you also have $x + 4$ what does it represent?

L2: x is the... x is the number of medium pizza then I add 4 because it says four gram heavier than small piece. I just take, the the the gram of the small one and add

CN: where does that stated in the question?

L2: (lough) I just quesss because it says 4 grams heavier than small, I didn't know

CN: so then you decided to add 4 to x, why? Don't subtract?

L2: the what... the statement say 4 gram heavier than, then small

CN: Did that say add?

L2: the thing is heavier, is not less, is not lighter

CN: then the next step

L2: I take x, $x + x - 7 + x + 4$ then I add all x together

$$3x - 3 = 300$$

$$X = 101$$

CN: okey

7. An eraser costs 15 cents more than a pencil, twelve pencils costs sixty cents more than eight erasers. How much does one pencil cost?

CN: you did not write anything here, why?

L2: I did not write because I was stuck, I don't know because the question say how much one pencil costs, so it was difficult for me to calculate one pencil how much it cost, so it was difficult serious.

CN: what was difficult there?

L2: this 12 pencil cost 60c more than 8 erasers, now I don't know how much it costs one eraser, so I don't know how to calculate the cost of one pencil if I don't know of one eraser

CN: okey

16. Tomas bought four pieces of wood and each piece is 2.5m long. How many pieces of 1-m wood can he get out of them?

CN: How did you go about this question?

L2: Sir here, I first thought that 2.5m was for 4 pieces of wood

CN: why do you say that?

L2: because when I read it now I think I am wrong... my answer here is wrong

CN: why do you say your answer here is wrong?

L2: I thought I was given a total of 2.5m for 4 pieces of wood and then how many pieces of 1m wood can I get out of total. So each wood, each piece of wood will be one metre. So if you take 1m, 1m from pieces, then there will be a half of metre, which has no other piece, so you need to subtract that piece

CN: then what was your answer?

L2: 2 pieces of 1m and half (0.5m) remained but this is wrong sir

CN: Ok

Learner 3 A G

1. Tuli is 25 years younger than his father. His mother is 2 years younger than his father. Their total ages add up to 78 years. How old is Tuli?

CN: How did you go about this question?

L3: I think you have to start with father. The father... Tuli is x years, then the father is 25, 25years more than Tuli, so you have to add 25 to x so that you find the ages of the father. Then the mother is 2 years younger than the father. So the mother you have to minus 2 years then you get $x + 25 - 2$. So Tuli is x and father is x

CN: Where did you find $x + 25$? Is it stated in the statement?

L3: no sir! Just that, sir you have to solve such equation, because you do not know the ages of Tuli. So you have to use x instead of particular number

CN: then from there, what was your next step?

L3: you have to add then all the ages together, you have to add x the ages of Tuli plus $x + 25$ the ages of father and then plus $x + 23$ from which $x + 25 - 2$ which is the ages of the mother is equal to 78 which is stated here. Then $3x$ then you add all x together get $3x$, then you add $25 + 23$ then you say $3x + 48 = 78$ then solve it and $x = 10$, so Tuli 10 years old

3. Marcus bought a pizza and cut it into three pieces. When he weighed the pieces, he found that one piece was seven grams lighter than the largest piece and four grams heavier than the smallest piece. The mass of the whole pizza was three hundred grams. What was the mass of each piece?

CN: How did you go about this question?

L3: here you have a pizza, imagine you have a pizza, then you cut into 3 pieces, is like you cut, not in the middle but next to the end. Then you leave the part in middle which is the big one. the part which is the big one is x . then the part which is the second largest is $x - 7 + 4$ then the part which is last... which is smallest is $x - 7$, so then their mass add up to 300g so you have to add those parts together, then you find x , to find the particular number.

CN: Now you take x , what does that x represent? Then plus also $x - 7$ plus $x - 7 + 4$

L3: x is represented the part of the big piece. $x - 7$ represent the piece of the second largest and $x - 7 + 4$ represent is the part of smallest

CN: why?

L3: because when you look here, $x - 7 + 4$ it may looks that this one os the big for rather than $x - 7$. but $x - 7$ is small because when 7.. negative 7 plus 4 you get negative 11 when you subtract $- 11$ from x then you get a small number than $x - 7$.

CN: where did you find this information? Which information from the statement did you use?

L3: It was here "he find that one piece was 7g lighter than the largest piece". so $x - 7$, the largest piece is x then that piece

CN: Can you also explain to me how use information to come up $x - 7 + 4$

L3: then they say the smallest piece is 4g. the largest is 4g heavier than the smallest. The largest piece is $x - 7 + 4$. this one you add 4 because now here the largest piece and the smallest is 4g. the largest piece is 4g heavier than the smallest, this statement one piece was 7g lighter than largest piece is for the second largest and that second largest is 4g heavier than the smallest.

CN: the second one!!!!

L3: the second one

CN: this one! Mmh!

L3: mmh!, then then you take x the number of the second largest/the amount $x - 7$

CN: that is the second largest!!!

L3: $x - 7 + 4$

CN: but here you add

L3: no! I think here it suppose to be $- 4$

CN: so you just put plus

L3: No it was a mistake

CN: it was a mistake!!

L3: yes!

CN: okey! Next step

L3: I add the total together

CN: mmh!!

L3: then I add $x + x + x$ you get $3x$, $-7 + - 11$ you get

CN: where did you get $- 11$?

L3: it was suppose to be $- 3$, positive

CN: mmh!!

L3: then you add all x together $3x = 300 - 4$ (18)

$X =$

CN: and what does that represent?

L3: this one represent the mass of the large piece, to find the smaller you have to subtract 7 and get 95 and then for the medium you have to subtract negative, you have to subtract negative. (quite) for the smallest you have to subtract -11 and for second largest subtract -7 and the for the big one is already here

4. An eraser costs 15 cents more than a pencil, twelve pencils costs sixty cents more than eight erasers. How much does one pencil cost?

CN: how did you go about this question?

L3: here I take the 0,60 which represent 60 cents is equal to 12 pencils and then one pencil is equal... one erasers cost 0.15 cents more than a pencil

CN: mmh!!

L3: then I take $12x$, and then a pencil is equal to $x - 0.15$ cents which is for the amount of an eraser.

CN: where does that one stated in the statement?

L3: it is not stated, just that the number of... you have to find this x , to not stated, so you will represent one pencil by x

CN: why did you subtract 0.15?

L3: is because an eraser cost 15 cents more than a pencil, so a pencil is less than, an eraser is more 0.15 cents than a pencil

CN: and then

L3: that means an eraser is one eraser is more than a pencil. Then you... you substitute an eraser with 0.15 cents by x and then a pencil, you substitute by 1, then you get x is bigger than $y - 0.15$, you get the value of the pencil.

CN: now!! What is the value of the pencil?

L3: the value of pencil is... is – 0.15!

CN: now while we are at that question lets look at that question. How did you go about this question? (you can speak Oshiwambo)

L3: Okathimbo kamwe okena ocends 60. dho oopena the kala thili omulongo nambaali, paife... 60 cents ota dhi topola nopena dhekala 12. to mono osipenidha yimwe, ithilinga intano. Ota kutiwa iye okadhimidho otaka kotha 15 cents shivulike ko pencil. Paife ko cents ntano oto gwedha ko 15 shaashi oka dhimidho ota kakotha... opencil... kotha oka dhimidho no 15. to mono kulya opena dhe kala otadhi kotha 20 cents.

CN: looking at these two question, are they different? or are they the same?

L3 they are different

CN: How different are they?

L3: mpano otaku tiwa okadhimidho ota ka kotha 15 cents shivulike kopena. Ano okadhimidho okevulike kopena no 15 cents. Here an eraser cost 15 cents more than a pencil, an eraser is the one which is more than a pencil

Learner 4 NC

- 1. Marcus bought a pizza and cut it into three pieces. When he weighed the pieces, he found that one piece was seven grams lighter than the largest piece and four grams heavier than the smallest piece. The mass of the whole pizza was three hundred grams. What was the mass of each piece?**

CN : how did go about this question?

L4: I was asked to calculate the gram of three pieces of pizza. I was told that I have to ... they are having three pieces one large, one middle, one smallest. I was told the medium one has.... Is... one piece was 7g lighter than the largest and 4g heavier than the smallest one. I take x as the the mass of large piece then x I minus one as a mass of the second piece. and the last one, I take, I add 7 to x x – 7 add 4, to get the mass of the small one.

CN: why do you add $7 + 4$?

L4: because I was told that 4 is... the medium, the medium one, the medium mass of medium pizza is..., is heavier than the small pieces.

CN: now $x - 7 + 4$, what does it represent?

L4: it represent the smallest

CN: why did you add?

L4: I think when you add 4 to 7, it will be.... When you minus 11 from x. just the same as to this one is heavier than the small one

CN: Okey then from there

L4: then I was I was told that when you add the them together then I get $3x - 18$ then $300 - 18$, then I get $x = 106$ is the grams of the biggest. At first I found that $x - 7$ is the gram of medium and $x - 11$ because I add $7 + 4$

2. An eraser costs 15 cents more than a pencil, twelve pencils costs sixty cents more than eight erasers. How much does one pencil cost?

CN: how did you go about this question?

L4: I was told that a pencil... an eraser is 15 cents more than a pencil. Then I decided to make x. I just divide a pencil with x while eraser I make $x + 15$

CN: why did you say $x + 15$?

L4: I thought here I make mistake I suppose to make a pencil $x + 15$ then eraser is equal to x

CN: before that!!can you explain what you did! And later explain the mistake that you did

L4: okey sir! I make a pencil a pencil equal to x, then eraser is $x + 15$, then I was told that 12 pencils cost 60c more than eraser, I make $12x = 8(x + 15) + 60$

CN: why did you multiply $x + 15$ by 8?

L4: I thought I was told, this one represent eraser, then I make $12x = 8(x + 15) + 60$, then I bring like terms together, the I get that $x = 45$ Or 90 cents

CN: Okey! Now you said earlier that you made mistake! Can you now tell me how it suppose to be?

L4: I thought I suppose to make a pencil is equal to $x + 15$

CN: then eraser

L4: eraser is x

CN: okey!

3. Tomas bought four pieces of wood and each piece is 2.5m long. How many pieces of 1-m wood can he get out of them?

CN: how did you go about this question?

L4: it say 'Tomas bought four pieces of wood and each piece is 2.5m long. How many pieces of 1-m wood can he get out of them?' then I make one piece is equal to 2.5 meters. How many pieces of 1-m wood can I get out of them? I was told one piece is equal to 2.5m long. I was asked how many one 1m I can get from 4 pieces. Then I have to make one metre is equal to 2,5m, then I take $4 = x$ in order to get the total pieces. I cross multiple, $4 \times 2.5 = 10m$.

CN: $4 \times 2.5 = 10m$! does it represent total length of 4 pieces? Or does it represent the number of pieces that you can get?

L4: is the length ,then I divide 10 by 1 to get 10 pieces of 1m

CN: okey!

4. A box of 24 eggs cost N\$7.30, how many boxes can your teacher buy with a N\$100 note?

CN: how did you go about this question?

L4: here I was told that a Box of 24 eggs cost N\$7.30. how many box can your teacher buy with hundred note

CN: Hm!!

L4: so here I have to found the.... First I have found the price of one box

CN: Hm!!

L4: I have to found the total, the price of each egg then I divided I make $7.3/24$ so that I can get the price of each then I get 0.30

CN: that is the price of one egg!!

L4: yes! Then I make 100 divided by 0.30 = 333 box. But I think this is not correct.

CN: is not correct

L4: yes sir!!

CN: why do you say it is not correct?

L4: because here, I did not calculate the number, I was asked how many boxes the teacher can buy with 100 note but here I calculate, how teacher, how many eggs he can buy with 100 note, each egg!

CN: so do you want to say 333 is wrong?

L4: yes!

CN: and what does that 333 represent?

L4: 333 box, I think 333 is representing the number of eggs in each, the total amount the eggs in boxes

CN: and do you say it is wrong?

L4: yes!!

CN: how do you suppose to do it?

L4: I think here, I think here I suppose to make... I was told that 24 eggs cost N\$7.30, then here I suppose to make N\$7.30 divide from 100, so that I get the number of box a teacher can get from 100 note.

4. **Okadhimitho otaka kotha ocenda omulongo nantano shivulike kopena yekala, oopena thekala omulongo naambali otadhi kotha ozenta omilongo hamano shivulithe uudhimitho uuhetatu. Opena yimwe yekala oyina iingapi? (question 7 in English test)**

CN: how did you go about this question?

L4: (read in Oshiwambo) “Okadhimitho otaka kotha ocenda omulongo nantano shivulike kopena yekala, oopena thekala omulongo naambali otadhi kotha ozenta omilongo hamano shivulithe uudhimitho uuhetatu. Opena yimwe yekala oyina iingapi?” I take 12 and multiply it by x , first I take x as an amount of pencil then I take $x + 15$ as an amount of eraser. Now I see that $12x = 60 + 8(x + 15)$ because I was told 12 pencil cost 60c more than 8 erasers and I took 60c and take a prize of erasers and multiply it by 8.

$12x = 60 + 8(x + 15)$ then I add like terms together and get that $x = 45$ cents which is an amount of one pencil

CN: Let’s look question 7 in test 1 and test 2? Are they the same or different?

L4: they are the same

CN: why do you have different answer here and there? (pointing answers in both papers)

L4: here maybe I make mistake when I come here, I suppose to minus but I just take $12x - 8x$.

CN: and here you say x , what does x represent?

L4: it represents an amount of pencil.

CN: $x + 15$

L4: is represent an amount of eraser

CN: I was told that you made mistake

L4: I was just confused

CN: which one did you write first

L4: this one

CN: okey!!

Learner 5: MH

- 1. Marcus bought a pizza and cut it into three pieces. When he weighed the pieces, he found that one piece was seven grams lighter than the largest piece and four grams heavier than the smallest piece. The mass of the whole pizza was three hundred grams. What was the mass of each piece?**

CN : how did go about this question?

L5: first because the question say that “Marcus bought a pizza and cut it into three pieces. When he weighed the pieces, he found that one piece was seven grams lighter than the largest piece and four grams heavier than the smallest piece. The mass of the whole pizza was three hundred grams. What was the mass of each piece?”

First I have to find small, medium and larger pieces. Okey then I say maybe the piece which is lighter... 7g lighter than the bigger piece and 4g heavier than the small piece is the medium then I make it to be x and then I subtract to find the.. to get the value if the small piece in form of x . I have to subtract 4 because it is 4g heavier than the small piece then to find the large piece I have to add because is 7.. 7.. 7 what, 7 lighter than the largest piece

CN: okey from there!

L5: from there I form an equation and then solve it

CN: how did you form it?

L5: I form it by adding all the grams in form of x together and equal to 300g because it says that mass of the whole pizza was 300g. then I find first x which is a medium piece, I got 99 and then I substituted x with 99 to find the value of other pieces

CN: Okey

- 2. An eraser costs 15 cents more than a pencil, twelve pencils costs sixty cents more than eight erasers. How much does one pencil cost?**

CN: why did not you write anything here?

L5: I did not understand it

CN: what do you not understand here?

L5: the whole question

CN: how?

L5: I do not know whether I am going to form an inequality or whether I am going to form an equation!

CN: Is that the reason you decided to escape it?

L5: yes! First I tried but I can't

5. CN: now lets look at this question "Okadhimitho otaka kotha ocenda omulongo nantano shivulike kopena yekala, oopena thekala omulongo naambali otadhi kotha ozenta omilongo hamano shivulithe uudhimitho uuhetatu. Opena yimwe yekala oyina iingapi? (question 7 in English test)" How did you go about this question?

L5: can I answer it in 'oshiwambo'?

CN: yes! You can

L5: (read the question and said) teningi iye andola opena yekala nayi kale 'y' yokadhimidho taka kala 'x' ndishi okwa tiwa okadhimidho ota ka kotha ocenda ntano shivulike kopena. Paife ndishi opena oyo 'y' opo... paife onda ningi owala andola $y + 15 = x$

CN: and then

L5: taku tiwa ishewe opena thekala omulongo nambaali otathi kotha 60c shivulike uudhimidho uuhetatu. Paife, mpano ote ningi opena ndishi okwatiwa omulongo nambaali andola.... Opo ndi mone oprice for 12 pincils, onda na okwi ndjipaleka omulongo nambaali nomwaalu gwo prisa yopena yimwe yekala ndjika kandi shi but I make it 'y' then 12y oshidhike pamwe nuudhimidho wu li uuhetatu. Tandii indjipaleke natango noprice yokadhimidho ndjoka kandishi. Ndjoka ndaningi 'x' tegwedhako natango omilongo hamano $12x = 8x + 60$. tandi shiningi iye shaampa simultaneously equation

CN: mmh!

L5: te yamukula

CN: how?

L5: tango onda ningi.... Sho nda yamukula, ando ndishi omiyaalu kadhidhike pamwe onda hogolola po x nenge y kulya which one should be equal then I go up to answer

CN: what was your answer?

L5: 45

CN: Now lets look at these two question, are they the same or different?

L5: are the same

CN: why did you not answer this one?

L5: maybe I understand this question in oshiwambo than in English

CN: okey

6. Tomas bought four pieces of wood and each piece is 2.5m long. How many pieces of 1-m wood can he get out of them?

CN: what is the total length of 4 pieces of wood?

L5: 2.5m

CN: then you multiple it by 4?

L5: yes

CN: now did you get the total length or number of pieces?

L5: the pieces of 1m

CN: Ok

Learner: 6 TB

The discussion with the first learner

- 1. When a number is added to two, the answer is the same as when fourteen is subtracted from three times the number. What is the number?**

CN: What is same as mean in this case?

L6: It is equal to

CN: How did it help you to construct an equation?

L6: I use letter x and say x plus two is equal to x times three minus fourteen and then x minus three x is equal to negative fourteen minus negative two x, is equal to negative sixteen and then divide negative two by negative two and also divide negative sixteen by negative two. The number is eight.

CN: Why did you use x?

L6: I use x for me to get the answer, to make up and construct an equation

2. An eraser costs 15 cents more than a pencil, twelve pencils costs sixty cents more than eight erasers. How much does one pencil cost?

CN: Can you explain to me what did you do first? Where did you start and why?

L6: First of all, I think, I think of the cost of one eraser and then I make 8 erasers, that is one eraser plus, is one, one eraser plus 15 cents and then I get a total number of 8 erasers. From there I take 12 erasers which is one eraser times 12 and then the answer I got I divided it with 12, I got number, the cost

CN: At the beginning you said you find the cost of one eraser. How did you find that? Is it given in the statement? Or how did you find it?

L6: In the statement they said that the pencil, 12 pencils are cost sixty cents more than 8 erasers. So I make it like this (keep quiet)

CN: How? You make it like this

L6: I make it like, 8, I find, they said, one eraser is 15 cents more than a pencil.

CN: Mh!

L6: I make it like one minus 15 cents and that number give me the total number of pencils

CN: Which number? You said one number, which number?

L6: I took, I took 0,45 – 15

CN: Where did you find 0,45? Is it stated in the statement given?

L6: No, I took it from my head

CN: How? How?

L6: (quiet for some minutes) I just think that when something is more than, it is something more than, you have to add, and so I have to subtract 15 cents from a number and that number when I add 15 it must give a total number of one eraser and pencil

CN: Okey

3. Tomas bought four pieces of wood and each piece is 2.5m long. How many pieces of 1-m wood can he get out of them?

CN: Can you explain to me what is asked here? And How did you tackle this question?

L6: They are asking for the, they are asking, how long one piece of wood it will be because here they said Tomas divide the wood into four pieces, and then each piece is 2,5m long. I make it like one piece is 2,5m if I have one 1m, how many pieces I will have

CN: Hm! How did you do that?

L6: I did it like two, 1m divided by 2,5m then I got 0.4

CN: Why did you take 1m divided by 2,5m? why did you do it like that?

L6: I construct an equation using x, I cross multiple

CN: Using x for what?

L6: Just in the sake of getting the rights answer

CN: What was your answer?

L6: My answer was 0,4

CN: 0.4 pieces, what is that meant?

L6: It (kept quiet)

CN: You said now, if you have 2,5m and then there are four pieces, then take 1m piece of wood. You said in four pieces of 2,5m you get 0.4 pieces, what does that 0.4 represent?

L6: 0.4 is a number of pieces of woods

CN: The pieces of wood that you get! Okey

4. Tuli is 25 years younger than his father. His mother is 2 years younger than his father. Their total ages add up to 78 years. How old is Tuli?

CN: What was your starting point? How did you start answering this question number 2?

L6: I number 2, I just form up a formula

CN: How did you use this information given to form up a formula?

L6: I used, because the ages is not mentioned there, I use x

CN: JA!

L6: I use x , then I form up a formula, they said, Tuli is 25 years younger than his father! Here I use $x + x - 25$,

L6: Can you explain to me that x now! What does that x represent? There is x , there is plus $x - 25$, there is also $x - 2$ and then give 78, Can you explain this?

L6: I use x just to make easy for, to form up an equation

CN: JA! But you have to use the information. Is it not so?

L6: Yes! Because there is no information here I use x so that I form up an equation

CN: What does x represent?

L6: is the, just to represent about total, is the number of his father, is the age of his father

CN: That is the ages of his father and then what about Tuli and her mother?

L6: Tuli is $x - 25$

CN: Why?

L6: Because they said that Tuli is younger, is 25 years younger than his father.

CN: Younger, what does it mean?

L6: Younger means smaller

CN: Hm! What about her mother?

L6: the mother is $x - 2$

CN: Why?

L6: because his father, because the mother is younger, is two years younger than the father

CN: Mh! And then after that, how did use 78?

L6: they said 78 is total amount of their ages, I use the total amount is equal to if you add these ages, it will give me 78, the total amount of their ages

Learner 7 BG

- 1. When a number is added to two, the answer is the same as when fourteen is subtracted from three times the number. What is the number?**

CN: What is same as mean in this case?

L7: Means equal

CN: How did it help you to construct an equation? Can you please tell me how did come up to the final solution?

L7: You take x and added two

CN: Why did you take x and add two?

L7: x , there, you take any letter to represent unknown number

CN: Hm!

L7: Then subtract x from 14 (kept quiet)

CN: Okey!

2. An eraser costs 15 cents more than a pencil, twelve pencils costs sixty cents more than eight erasers. How much does one pencil cost?

CN: Can you explain to me what did you do first? Where did you start and why?

L7: (Kept quiet) you take 12 multiple with x

CN: Why?

L7: Because you do not know the cost of the eraser and then you subtract 8 from $12x$

CN: Why did you subtract 8?

L7: 8 plus 15

CN: Where does it stated in the question? Which information in the question did you use?

L7: 8 for, represent 8 eraser then x plus 15 ($x + 15$) is the cost of one eraser

CN: Where does it stated here in the question?

L7: (kept quiet)

CN: Iyaa! 15 cents more than a pencil

L7: $x + 15$

CN: why? And what does it represent?

L7: is for one eraser

CN: So, one eraser is $x + 15$, why do you say that? Where does it stated in the question?

L7: More, more than

CN: Hm! Then just proceed!

L7: Then (kept quiet) 12 pencils is 8, Sixty here is the answer, is the answer you get and subtract 12

CN: Do you mean $60 - 12$?

L7: ($60 - 8x$) sixty minus 8 times x minus 15, then the answer you get 12

CN: Which information from the question did you use there?

L7: (kept quiet)

CN: Okey! May we can continue with the next question.

3. Tomas bought four pieces of wood and each piece is 2.5m long. How many pieces of 1-m wood can he get out of them?

CN: Can you explain to me what is asked here? And How did you tackle (answer) that question?

L7: First you multiple 2,5m with 4 because you bought 4 pieces of wood

CN: Why did you multiple 2,5m by 4?

L7: Because he bought four pieces of wood then multiple the length 2,5m

CN: Why did you multiple 2,5m by 4

L7: to get the total length of all pieces of wood

CN: What does 2.5m long represent?

L7: Represent one piece of wood

CN: Where does it state in the statement?

L7: In the statement said that each piece is 2,5m long

CN: So 'each' means what?

L7: "each" means one

CN: And then, how many pieces of wood

L7: You divide the answer you get when $(4 \times 2,5m)$, you divide it with one and then you get 10 pieces

CN: So you take $4 \times 2.5m$ because you 'each' means one and that is the reason you multiple it by 4

L7: yes!

CN: Then you divide your answer by one

L7: yes!

CN: Why did you divide it by one?

L7: To get, to get the number of pieces of 1m long

CN: Okey!

APPENDIX 4

P. O. Box 928

Ondangwa

Tel: 065 – 242 516 (W)

Cell: 0812755078

02 April 2007

The Director

Ministry of Education

Oshikoto Region

P/Bag 2012

Ondangwa

Dear Madam

**RE: REQUEST FOR A PERMISSION TO CONDUCT A RESEARCH AT UUKULE
SSS**

I, Christian Neshuku, a Mathematics Advisory Teacher, Ohangwena Region is hereby requesting for permission to conduct a research at Uukule SSS next term.

I am registered as a part-time student at Rhodes University, Grahamstown (student number 605n5286). I have been studying for a master's degree in mathematics education since January 2007. I would be most grateful if you would allow me to conduct my research at Uukule SSS and use the current grade 12A or 12B as the research site for the research report which I am required to write at the end of this year 2008. These are some of the classes I have been teaching before I joined Ohangwena Education Region as an advisory teacher for Mathematics last year on the 1st of October last year.

The aim of my research is **to explore the problem experienced in the interpretation of word problems in mathematics by grade 12 learners**. Should you agree to allow me to conduct my research at Uukule SSS and use one of the two classes as a research site, the class will be asked to write three special tests next term, some learners will be selected for an interview.

The school, learners and teachers will be assured of anonymity in the final research report and will be invited to proofread draft of the report to ensure that details are accurately recorded and reported.

Yours sincerely,

C. N. Neshuku (Mr.)

Consent Form

Christian Neshuku is hereby given permission to use either current grade 12A or 12B at Uukule S.S.S as the research site for the research report he is required to write for the completion of his master's degree. I understand that data for analysis will be collected from tests' results, and also from interviews with three learners and two teachers and that information from these may be used in the final report. I have been assured that my school, my learners and teachers will have anonymity in that report.

Ester Anna Nghipondoka

Director

Ministry of Education

Official stamp

