The nature of Visual Representations of multiplication and division exercises in nine Grades 1 to 3 South African textbooks

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Declaration of Originality

I, Tammy Irene Booysen (18b8559) declare that this thesis is my own work, written in my own words. Where I have drawn on the words or ideas of others, these have been acknowledged according to the Rhodes University Education Department referencing guidelines.

Tammy Irene Booysen   Date: 12 December 2022

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Dedication

I would like to dedicate this thesis to my late Grandmother, Mrs Florence Sarah Beam, who planted the seeds of the tree-bearing fruit today. Since birth, she has played an instrumental role in my education.

Abstract

Mathematics is a language that is rich in visual representations (Mudaly & Rampersad, 2010). Visual Representations assist us in developing our reasoning skills when solving a problem and our understanding of the relationships between concepts (Ozkan et al., 2018). This thesis focuses on the different visual representations (VR) in South African Foundation Phase mathematics textbooks and workbooks.

Textbooks and workbooks play an important role in developing an understanding of mathematical concepts for both teachers and learners (Harries & Spooner, 2000). While teachers generally rely heavily on textbooks, they were a key resource while schools were closed due to COVID-19 lockdown regulations.

The theory of Constructivism forms part of the theoretical framework for this study. Constructivism advocates that learners actively construct knowledge through experiences rather than passively receiving knowledge from the outside (Von Glaserfeld, 2001). Vygotsky believed that social interactions create experiences that facilitate the learning and meaning-making process (Vygotsky, 1978).

This case study is underpinned by an interpretivist paradigm as it sought to examine the nature of VRs in three Grades 1 - 3 textbooks/workbooks. My research approach is primarily qualitative with descriptive statistics to assist in developing a more comprehensive understanding of the research questions.

The study was guided by the analytic tool designed by Fotakopoulou and Spiliotopoulou (2008) which I adapted for Foundation Phase mathematics use. The framework provides insight on the type of VR, VRs relation to content, VRs relation to reality, the function of the VR and dimensionality of a VR.

While the workbooks had many more VRs than textbooks, the dominant type of VR in textbooks and workbooks are images. The VRs mostly have a strong relation to content with a realistic relation to reality as they were predominantly 2D representation of a 3D object that had an exemplifying function (type b).

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List of Abbreviations

2D	2 Dimensional
3D	3 Dimensional
CAPS	Curriculum Assessment Policy statement
DBE	Department of Basic Education
DeFT	Design, Functions and Task
FP	Foundation Phase
ICT	Information and Communications Technology
SACMEQ	Southern African Consortium for Monitoring Educational Quality
TIMSS	Trends in Mathematics and Science Study
VR	Visual Representation
VRF	Visual Representation Framework

Conference Proceedings and Presentations from This Thesis

1. Booysen, T. & Westaway, L. Exploring visual representations of multiplication and division in early years South African mathematics textbooks. Long paper presented at the Mathematics Education Research Group of Australasia (MERGA) in Launceston, 3-7 July 2022.

2. Booysen, T., Westaway, L. & Vale, P. The use of visual images in multiplication and division in four early years' mathematics textbooks. Short paper presented at the 30th conference of the Southern African Association for Research in Mathematics, Science and Technology Education (SAARMSTE), virtual conference in January 2022.

3. Booysen, T. The use of visual images in multiplication and division in three South African foundation phase mathematics texts. Poster presented at the 29th conference of the Southern African Association for Research in Mathematics, Science and Technology Education (SAARMSTE), virtual conference in January 2021.

4. Booysen, T. & Westaway, L. The use of visual images in multiplication and division in early years' mathematics textbooks. Oral communication presented at the 44th conference of the International Group for the Psychology of Mathematics Education in Thailand, 21-22 July 2021.

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Chapter 1: Introduction, Context and Rationale

1.1 Introduction and Background

This chapter introduces the thesis titled *The nature of Visual Representations of multiplication and division exercises in nine Grades 1 to 3 South African textbooks.* Chapter 1 describes the context and background of the South African education context. The chapter introduces the research goals, states the research questions and explains the significance of this study. Lastly, the chapter concludes with a brief overview of each chapter of the thesis.

1.2 Low Performance in South African Schools

Reddy (2013) states that South Africa has continuously been ranked in the lower section of the international rankings when compared to other countries. The unsatisfactory performance of South African learners on national and international benchmark tests in mathematics, such as the Trends in Mathematics and Science Study (TIMSS), the Southern African Consortium for Monitoring Educational Quality (SACMEQ) IV and Annual National Assessment, prompted me to investigate how multiplication and division are visually presented in mathematics workbooks and textbooks in the Foundation Phase (FP).

In 2019, 11 903 Grade 5 learners in South Africa completed an assessment for the TIMSS (Mullins et al., 2019). The TIMSS works with a point-categorised system. The categories on the scale are advanced benchmark (>625 points), high benchmark (<550 points), intermediate benchmark (<475 points) and low benchmark (<400 points) (Reddy et al., 2019). The TIMSS 2019 results point out that 37% of South African learners scored in the low benchmark category (Reddy et al., 2019). This means that the learners in this category demonstrated only basic mathematical knowledge of using numbers, measurement and geometry and data (Mullins et al., 2019). A further 16% of South African learners scored above 475 points, which is in the intermediate benchmark (Reddy et al., 2019). This demonstrates that 16% of Grade 5 learners are able to show and apply basic mathematical knowledge to solve problems. This is the ability to add, subtract, multiply and divide.

The SACMEQ IV is a study that involves 16 Ministries of Education from Southern and Eastern Africa. The SACMEQ IV assesses the schools' condition and the Grade 6 learners' and teachers' levels of performance in Mathematics and Literacy (SACMEQ, 2017). The SACMEQ IV results revealed that multiplication and division are areas that need attention within the South African FP curriculum as it is in this phase that the foundation for developing multiplication and division is set (Department of Basic Education [DBE], 2013, 2011). The SACMEQ IV achievement indicators range from level 1 to level 8. The levels are pre-numeracy (level 1), emergent numeracy (level 2), basic numeracy (level 3), beginning numeracy (level 4), competent numeracy (level 5), mathematically skilled (level 6), concrete problem solving (level 7) and abstract problem solving (level 8) as described with indicators in Table 1.1 (SACMEQ, 2017).

Table 1.1: A description of the SACMEQ mathematics basic competency levels

	Level	Descriptor	Competencies
BASIC MATH SKILLS	1	Pre- Numeracy	Applies single step addition and subtraction.
	2	Emergent Numeracy	Applies a two-step addition and subtraction involving carrying.
	3	Basic Numeracy	Translates verbal information into arithmetic operations.
	4	Beginning Numeracy	Translates verbal or graphic information into simple arithmetic problems.
	5	Competent Numeracy	Translates verbal, graphic, or tabular information into an arithmetic form in order to solve a given problem.
ADVANCED MATH SKILLS	6	Mathematically Skilled	Solves multiple-operation problems (using the correct order) involving fractions, ratios, and decimals.
	7	Concrete Problem Solving	Extracts and converts information from tables, charts and other symbolic Presentations in order to identify, and then solve multi-step problems
	8	Abstract Problem Solving	Identifies the nature of an unstated mathematical problem embedded within verbal or graphic information and then translate this into symbolic, algebraic or equation form in order to solve a problem.

Source: DBE SACMEQ IV South Africa report (2017, p. 33)

South Africa has been placed 6th out of the 16 countries in the SACMEQ IV mathematics score results (SACMEQ, 2017). Just less than 1% of South African learners achieved pre-numeracy levels. The majority of the Grade 6 learners in South Africa fall into levels 2 (14.1%), 3 (35.1%), 4 (20.3%) to 5 (14.8%). In other words, most of the learners in Grade 6 are able to perform basic maths skills that range from level 2 to 5 on the SACMEQ IV competency level (Table 1.1).

Spaull (2013) conducted a study on South African numeracy skills in schools and found that 58.6% of South African Grade 6 learners were not functionally numerate. According to Gal et al. (2020), UNESCO refers to the term 'functionally numerate' as a person's ability to use mathematics to be able to function in their context. For a Grade 6 learner, this may include multiplication of whole numbers up to 10x10, using multiplication and division as inverse operations and solving contextual word problems (DBE, 2011b). Although the above studies focus on the Intermediate Phase, it suggests that learners in the FP do not have the required knowledge of basic mathematics to perform well in the intermediate phase (Grade 5 and 6).

There are numerous reasons for poor learner performance. These include societal, home and school constraints. For example, the lack of water supply to the home and school environment links to the lack of availability of resources to the community. Many resources cannot reach the location of the communities because they are situated in isolated rural areas with poor road infrastructure (Mullins et al., 2019). Another reason for poor learner performance is high absenteeism and learners not being able to concentrate due to being hungry (TIMSS, 2019). In addition, teachers lack mathematical content knowledge and therefore are reliant on textbooks to facilitate the teaching process (Reddy et al., 2019). The lack of exposure to early numeracy resources of learners during their early years may result in a lack of numeracy readiness in the FP mathematics classroom. Poor early numeracy exposure is also one of the reasons given for underperformance in schools (Reddy et al., 2019).

1.3 Why Multiplication and Division?

The SACMEQ IV and TIMSS 2019 results have shown that multiplication and division are areas that have produced weak results which South African learners need to improve on (Bansilal, 2013). Bansilal (2013) suggests that the problem comes in the transition from addition and subtraction to multiplication and division in FP mathematics.

In the FP, the number, operations and relationships domain (which includes the four basic operations) is central to learning mathematics and the curriculum document suggests more time is spent on this domain than others (ie. 50%). Table 1.2 below presents the weighting of numbers, operations and relationships in Grades 1, 2 and 3. In Grade 1, the majority of the time (65%) is spent on the numbers, operations and relationships content area. This decreases slightly in Grade 2 and 3 to 60% and 58% respectively.

Table 1.2: Excerpt from the Curriculum Assessment Policy statement (CAPS) document of NOR in FP mathematics

WEIGHTING OF CONTENT AREAS			
Content Area	**Grade 1**	**Grade 2**	**Grade 3**
Numbers, Operations and Relationships*	65%	60%	58%
Patterns, Functions and Algebra	10%	10%	10%
Space and Shape (Geometry)	11%	13%	13%
Measurement	9%	12%	14%
Data Handling (Statistics)	5%	5%	5%
	100%	**100%**	**100%**

*In Grade R – 3, it is important that the area of Numbers, Operations and Relationships is the main focus of Mathematics. Learners need to exit the Foundation Phase with a secure number sense and operational fluency. The aim is for learners to be competent and confident with numbers and calculations. For this reason the notional time allocated to Numbers Operations and Relationships has been increased. Most of the work on patterns should focus on number patterns to consolidate learners' number ability further.

Source: DBE (2011a, p. 10)

The CAPS suggests that earlier grades focus on addition and subtraction explicitly. Multiplication and division are done implicitly (through repeated addition) in the earlier grades and more explicitly in the later grades. The reason for choosing multiplication and division as a topic in this study is because, in Grades 2 and 3, learners are introduced to multiplication and division calculations. Multiplication calculations include working out times tables and the inverse of multiplication. Learners can also draw on concepts such as sharing and grouping which are done in the earlier FP years. The learners would be expected to make use of the appropriate symbols (x, ÷, =) and multiply numbers 1 to 10 by 2,5,3 and 4 (DBE, 2011a).

The poor performance in mathematics in general and multiplication and division has urged the South African DBE to formulate intervention strategies for teacher educators in particular to improve learner performance in mathematics (Fleisch et al., 2011).

1.4 Textbooks and Workbooks in South Africa

There are two main primary texts used in South Africa, textbooks and workbooks. Ben-Peretz (1990) suggests that there is a reliance on textbooks as they are the leading pedagogic tool used by teachers in trying to understand the intended curriculum. While teachers generally rely heavily on textbooks, they were a key resource when schools are closed due to COVID-19 regulations. This research is particularly relevant considering how the 2020/1 school years unfolded as texts became increasingly central to learning as the vast majority of learners were required to use texts without assistance from teachers during the national lockdown periods.

1.5 Textbooks and Their Use in South Africa

A textbook can be defined as a book used in the study of a particular subject. The content in a textbook aims at covering instructional material that the curriculum is based on (Makgato & Ramaligela, 2012). According to Makgato and Ramaligela (2012), the publishers of commercial textbooks submit their textbooks to the DBE. The DBE evaluates the textbooks and compiles a list of those deemed suitable for schools to choose from. Suitability includes the content coverage of the subject and whether the concepts that are taught are clear and coherent, have relevant visual representations (VRs) and consist of appropriate assessment tasks (Makgato & Ramagligela, 2012).

1.6 Workbooks and Their Uses in South Africa

The DBE workbooks are compulsory in the learning of mathematics in South African (Grade R to 6) state schools are able to make their own choice about which textbook to purchase for their school from the national textbook catalogue and according to the budget per child. There is very little research on South African textbooks. There appears to be few articles by South African researchers on workbooks namely Hoadley and Galant (2016), Makgato and Ramaligela (2012), Flesich et al. (2011), and Mathews et al. (2014) on the South African DBE workbooks. These researchers look at how teachers use textbooks to develop their content knowledge and sequence and pace lessons of the workbooks.

The importance of VRs in textbooks is to complement the curriculum and assist learners in understanding the mathematics they are learning. Hence, the quality of the VRs is important as it assists in the visualisation process.

1.7 Visualisation

Visualisation plays an important part in how we make sense of the world (van Lieshout & Xenidou-Dervour, 2019). Visual representations (VRs) used in texts can assist learners in understanding mathematical concepts and it is a way of facilitating one’s thinking and reasoning (Arcavi, 2003). One of the purposes of VRs in a text is to help non-expert readers to understand what is being said (Fotakopoulou & Spiliotopoulou, 2008), which is important for FP learners.

1.8 Problem Statement

The aforementioned poor performance in mathematics, specifically multiplication and division in FP, the over-reliance of teachers on texts and the importance of VRs in texts though the limited research on the use of VRs in South African texts highlights the need for research on the quality of textbooks that support both teachers and learners. Thus, this study sought to explore the nature of VRs in South African FP Mathematics textbooks.

1.9 Research Aim, Goals and Significance of the Study

The study aimed to explore FP textbooks to analyse the visual representations in terms of the VRs used in relation to multiplication and division. It specifically explored the function of the VRs and the dimensions of the VRs in relation to supporting learner access to Mathematics ideas and their relevance to learners' reality.

This study was based on the VRs in textbooks. Through researching the nature of the VRs in the textbooks, I assessed and explored the quality of VRs used in FP textbooks that are relevant to the teaching of multiplication and division. It is my hope that the findings of this thesis may influence policy on i) textbook selection, and ii) the alignment between the curriculum and textbooks and their efficiency as a support tool for teaching multiplication and division. In addition, I hope to raise increased awareness amongst educators about the types and functions of VRs used in presenting multiplication and division in textbooks. I will do this through presenting at conferences and sharing the findings of my work on various platforms (eg. seminars, conferences and DBE indaba's).

It is also hoped that stakeholders should pay careful attention to the quality of a textbook with regard to the suitability, purpose and function of the VRs when assessing which textbook is appropriate to endorse at their school. The relevance of a textbook for the context of the South African child is salient as the learner should be able to relate to the content of the VR in the text.

1.10 Research Question and Sub-questions

Emerging form the above arguments I chose the following research questions:

1. What is the nature of the visual representations used to support South African FP learners' understanding of multiplication and division in South African FP textbooks?

1.1 What is the nature of the visual representations used to support FP learners' understanding of whole number multiplication?

1.2 What is the nature of the visual representations used to support FP learners' understanding of whole number division?

2. How does the nature of visual representations in texts compare to those promoted in the curriculum?

1.11 The Structure of the Thesis

This section provides a brief overview of the thesis.

Chapter 2 reviews the literature on visualisation and VRs in mathematics textbooks, as well as the role of textbooks in mathematics education. In addition, a broad view of multiplication and division is given. The chapter provides a summary of past research on multiplication and division and on research that promotes the use of VRs. Chapter 2 provides a literature review of the key concepts in this research, namely the importance of texts, defining visualisation, the use of visual representations in general (and in the case of multiplication and division specifically), views of effective teaching of multiplication and division and VRs promoted. From this literature review, I established the importance of VRs in mathematics textbooks.

Chapter 3 provides an overview of constructivism and the rationale for using it as the broad theoretical framework for this study. The Visual Representation Framework (VRF) by Fotakopoulou and Spiliotopoulou (2008) builds on a broadly constructivist view of learning and developing a schema for learning. I draw on examples from the selected textbooks, coupled with explanations from the pilot study to present the analytical framework and explain in detail the analytical tool explored in the pilot study.

Chapter 4 outlines the methodology and reasons why this specific method was suitable for document analysis. This study made use of an interpretivist paradigm. This chapter elaborates on the sample of the study and how the textbooks used in this study were chosen. The research consisted of six phases, namely: phase 1 – sample selection, phase 2 – selection of the Visual Representation Framework (VRF), phase 3 – conducting a pilot study on the Grade 4 DBE workbook, phase 4 – adapting the VRF analytical framework, phase 5 –conducting qualitative analysis of how the study makes use of qualitative analysis with descriptive statistics.

The study made use of an iterative data generation process which consisted of coding and re-coding the data collected. This study also used descriptive statistics in the data analysis process. This chapter explores the ethical considerations, validity and trustworthiness relevant to a study of this nature.

Chapter 5 presents the data analysis section for this study. This section explains the general layout of the various textbooks, followed by the primarily qualitative analysis of the three textbooks across Grades 1, 2 and 3. The data analysis section analyses the data of the VRs across publishers and in the CAPS document.

Chapter 6 presents the discussion, findings and recommendations of this study.

In the following literature review chapter, I discuss literature relevant to workbooks and textbooks, VR and multiplication and division.

Chapter 2: Literature Review

2.1 Introduction

A literature review is a culmination of research on the topic's findings (Creswell, 2012). This chapter provides a synopsis of the literature that has been researched in order to better understand visualisation in mathematics and texts relating to the topics of multiplication and division. In this chapter, the following is explored: the role of texts in learning mathematics, the term 'visualisation' in the context of mathematics and the concept of multiplication and division. As noted, South Africa contains two main key texts, namely workbooks and textbooks. In this study, from here onwards when referring to them collectively they will be called texts and when referring to only one I will refer to it as either workbooks or textbooks.

2.2 The Role of Texts in Learning Mathematics

Mathematics texts are used as support material (Fan et al. 2013). They contain many VRs which are used to communicate and clarify mathematical concepts. In the context where teachers make use of the texts to teach, or where the textbook is the teacher (i.e. during the lockdown as a result of the Covid-19 pandemic), the VRs used in the texts become crucial in supporting learning and sense-making.

Textbooks and workbooks play an important role in developing an understanding of mathematical concepts for both teachers and learners (Harries & Spooner, 2000). Texts assist learners by providing activities that allow learners to practise mathematics and provide teachers with a guiding framework in preparing classwork and homework activities that align with the curriculum (Nicol & Crespo, 2006). Teachers use texts when planning and implementing mathematics lessons (Thomson & Fleming, 2004). Nicol and Crespo (2006) state that textbooks form a framework that informs teachers about possible ways to teach the content in a particular sequence. Texts guide teachers in what needs to be taught, how it needs to be taught and in which order. Working in a South African context, Hoadley and Galant (2016) found that teachers use the texts to monitor what has been covered in the curriculum and to assist with learner assessment. As such, teachers rely on texts to support their pedagogical content knowledge (Makgato & Ramaligela, 2012).

Freeman and Porter (1989) suggest that how teachers use the textbook is dependent on the teacher's knowledge of the quality of the text. Teachers' selection of texts is influenced by their own preconceived ideas of what and how content should be taught. Teachers rely on texts and expect that the authors of these texts are competent experts in the field (Pehkonen, 2004). Nyariki and Krolak (2016) maintain that the distribution of learning materials such as textbooks has much value and may positively increase the level of quality teaching and learning in primary schools in Africa.

Texts do not only support teachers but are useful curriculum tools because they assist learners in supporting their understanding of concepts by offering VRs and symbols, together with experiences and language that assist learners in understanding mathematics (Hoadley & Galant, 2016; Liebeck, 1984). Mazumder et al. (2020) found that learner texts are of great value for learners as they affect how they construct knowledge.

In South Africa, several texts are used, but in FP, many teachers use workbooks only. In my review of the literature, there appears to be no research on FP mathematics textbooks in South Africa other than the national workbooks. Several South African researchers have analysed the DBE workbooks. Hoadley and Galant (2016) researched the use of the DBE workbooks as a curriculum tool which was used to support learning in South Africa. Their analysis suggests that the DBE workbooks are a teaching tool which provides structured learning activities for the learners to practise (Hoadley & Galant, 2016). The workbook consists of activities that resemble worksheets. The intention is that learners complete four worksheets per week (Fleisch et al.,2011).

Mathews et al. (2014) found that the workbooks lack an index page and do not address all the requirements of the CAPS. Their research on the use of the DBE workbooks in primary schools found that the majority of South African teachers did not use the DBE workbooks in the way that the DBE intended. Rather than using the workbooks as a resource to support learning and teaching, the workbooks are used to teach. Fleisch et al. (2011) concur that the DBE workbook should be incorporated into the teachers' lessons with activities which include, but are not limited to, the use of language, for example, code-switching, for learners to understand the concept. There are, however, researchers who critique teacher reliance on texts. This is explored below.

2.2.1 Challenges of reliance on texts

Researchers such as Fan and Zhu (2007) and Drews (2007) have found that there are gaps between the intended and actual curriculum covered in texts. Thus, texts should be used as a mediating resource rather than as a replacement for the teacher. Hence, Fleisch et al. (2011) assert that teachers need to mediate the textbook. If learners are exposed to resources, for example, textbooks, without mediation by the teacher, they may create problems and misunderstandings for learners (Mathews et al., 2014). The unfacilitated use of texts is not what is intended by the government's workbook initiative – *Department of Education Workbook Training Manual* (DBE, 2012).

Furthermore, Mathews et al. (2014) critiqued the DBE books and said that the textbook is not necessarily used in chronological order. Consequently, teachers select the pages that they think will be appropriate for the concept to be taught each day. In addition, Makgato and Ramaligela (2012) suggest that little research has focused on analysing the language levels of the book, especially when the language is English and not the home language of most learners.

The texts contain activities that make use of VRs that assist in understanding the concept presented. The following section explores visualisation and the importance of VRs.

2.3 Visual Representation/ Visualisation

Mudaly and Rampersad (2010) state that mathematics is a language rich in visuals. We use perception to make sense of the world around us (Arcavi, 2003). Mathematics texts contain many VRs which are used to support clarifying mathematical concepts. Visualisation is an essential component in understanding mathematical concepts and is necessary for conceptual and perceptual reasoning (Arcavi, 2003).

2.3.1 Visualisation is difficult to define

There are different terminologies used to define the process of visualisation. According to Presmeg (2019), defining the term visualisation has been tenuous as little research has been done on visualisation in mathematics education in the past. When searching for articles on visualisation the terminology used to describe VRs differ. Terms such as visual representation (VR) (Kaput, 1987), inscriptions (Roth, 2004) and representations (Kaput, 1987) are used synonymously. In my research, I use the term Visual Representation (VR).

2.3.2 What is visualisation

Research on visualisation appeared from my engagement with the literature to have started in the 1980s when researchers started questioning how people think and process information (Presmeg, 2019). Presmeg (1986) was one of the first mathematics researchers to conceptualise visualisation for mathematics teacher support material. She defined visualisation as a mental scheme that forms and illustrates visual or special information. Goldin and Kaput (1996) assert that visualisation is a part of what we imagine, and that visualisation is often not explicitly taught to learners in the same way as other methods of mathematics instruction, for example, problem solving.

Arcavi (2003) aligns himself with the view that visualisation is a cognitive process of understanding, interpreting (Zimmerman & Cunningham, 1991) and transmitting information in an aesthetically pleasing way (Arcavi, 2003). Arcavi (2003) draws on Zimmerman and Cunningham to explain that:

> Visualization is the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds, on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understandings. (p. 3)

McCormick et al. (1987) explain that visualisation is a method of seeing the unseen as visualisation makes the invisible things visible (Ozkan et al., 2018). The unseen may also involve mental representations. The VRs assists in our reasoning skills and our understanding of the relationships between concepts (Ozkan et al., 2018).

Visualisation is a cognitive process (Presmeg & Canas-Balderas, 2001). Piaget and Inhelder (1971) and Presmeg and Canas-Balderas (2001) concur that a VR is first constructed in a person's mind. Therefore, each individual creates a mental image (Presmeg, 1986). The image could be seen as an abstract image (memory image) or the visualisation of an object. An abstract image is a memory image of a concrete object or VR thereof, for example, a mathematics formula (Presmeg, 1986). In order to create a memory image the individual needs an opportunity to work with concrete representations. As Bruner (1960) notes, learners move from the enactive (concrete) to the iconic (metaphoric drawings) to the symbolic (symbols) when introduced to new mathematical concepts (McLeod, 2008).

Dreyfus and Eisenberg (1991) found it difficult to classify visualisation. They identified three categories, namely cultural, cognitive and sociological. Cultural visualisation (Dreyfus & Eisenberg, 1991) is based on the idea that we all come from different cultural backgrounds with different languages, beliefs and value systems. This stands in contrast to a cognitive perspective that suggests, like Piaget, that individuals have different cognitive abilities and that each of us has different types of knowledge specific to our context (Dreyfus & Eisenberg, 1991). Each cultural group has their own influence on visualisation as it is context-dependent (Dreyfus & Eisenberg, 1991). When one visualises a concept, the above-mentioned aspects influence our visualisation process. While visualisation supports learners' sense-making, VRs also assist in developing learners' visualisation skills.

The VR of an object does not need to be seen to be formed physically; it can be constructed as an imagined image identifiable to the individual (Presmeg, 1986). This refers to representations of abstract objects (Moyer, 2001). Sobbeke (2005) suggests that in mathematics classrooms, VRs support the learners to see mathematical concepts and ideas. For Crisp and Sweiry (2006), mental representations are salient in the teaching-learning process. Visualisations are not necessarily something one can physically touch and include abstract images, visual objects and gestures (Arcavi, 2003).

Examples of VRs include diagrams, graphs, words, symbols, patterns and pictures (Presmeg, 1986). When looking at VRs in texts it is important to look at the function of each VR. The function of a VR could be decorative, exemplifying, explanatory or complementary (Fotakopoulou & Spiliotopoulou, 2008). I elaborate on this in Chapter 3 as these categories were identified as particularly useful for my analysis.

2.3.3 The importance of visualisation in maths texts

Visualisation assists in helping learners, whether they are able to read or not, to understand and think about the content of the lesson and further communicate their ideas to the outside world (Fotakopoulou & Spiliopoulou, 2008). Dreyfus and Eisenberg (1991), Kim (2012) and Fotakopoulou and Spiliotopoulou (2008) argue that visualisation is a tool that can be used in solving mathematics problems.

The process of creating a VR of a mathematical problem assists in understanding the concept inherent in the problem (Spiliotopoulou-Papantoniou et al., 2009). The use of VRs is important to assist non-expert readers in understanding the problem (Fotakopoulou & Spiliotopoulou, 2008) and in explaining various mathematical concepts (Mazumder et al., 2020). Learners can benefit greatly from using VRs to make connections between and across mathematics concepts (Presmeg, 2016). Taking a visual approach to teaching and learning mathematics (Sobbeke, 2005) teachers should assist the learner in creating a mental representation and increase recall and improve problem solving skills (Mazumder et al., 2020).

In addition, it is suggested that learners also make their own VRs in their minds in order to visualise the problem. To be able to solve mathematical problems an image needs to be constructed on paper or in the mind (Presmeg, 1997). Polya (1973), cited in Rösken and Rolka (2006) suggests that a successful problem solving strategy is to "draw a figure". This strategy enables learners to visualise the problem. Through drawing the problem first, the problem is modified, and a picture is formed on paper and in the mind of the individual. Dreyfus and Eisenberg (1991) noticed that learners (and undergraduate students) are reluctant to make use of VRs (for example, drawing) when working out a problem. The result of their research indicated that few high school learners like to think using pictures. They maintain that there is a connotation of being a weak learner if a drawing is used to enable the learner to understand the problem.

Several studies have been conducted on the effects of the use of mathematical VRs (e.g., Ainsworth, 2006; Csíkos et al., 2011; Lowrie, 2001; Mazumder et al., 2020) and on VRs in other contexts (e.g. Fotakoupoulou & Spiliotopoulou, 2008). Csíkos et al. (2011) conducted a study on the effects of drawing to make meaning of word problems. In the study with 106 Grade 3 learners in the experimental group and 138 Grade 3 learners in the control group, learners were requested to work through 73 word problems, one learner at a time, making use of drawings to assist in obtaining a solution. The difference between the experimental and the control group is that the class teachers in the control group received no information about the aims of the study and teachers in the experimental group received the aims and material to equip themselves on the effects of drawing when working through word problems. Even though the teacher in the control group did not know the aims of the study, both groups made use of visualisation to solve a problem (Csíkos et al., 2011).

Mazumder et al. (2020) conducted a study on VRs in textbooks as a tool to create meaning. In this study, the authors analysed 15 Java online texts and examined the characteristics of explanatory diagrams focusing on variables, arrays and object diagrams. They suggested that people learn more from texts with VRs than from texts without VRs. Lowrie's (2001) study explored the relationship between different problem representations and mathematics measuring tasks. The study investigated how Grade 6 learners placed different problems across a visual to non-visual continuum when solving the problems. The findings indicated that the learners who used visualisation methods performed slightly better than those who did not.

Working outside of mathematics education, Fotakopoulou and Spiliotopoulou (2008) conducted a study on VRs in school Economics textbooks in Greek secondary schools. First, the textbooks were analysed to identify the characteristics of VRs in the textbooks. The findings were that more than half of the VRs in textbooks consisted of photographs (52% of VRs in the text had a decorative function). Thereafter, the authors interviewed 58 learners on their experience of the VRs by looking at two pre-selected visual representations together with a set of questions. The learners were given a set of questions to respond to. In one of the questions, the learners were asked to describe the content of the VRs and what each VR aimed to demonstrate. The authors concluded that learners found it difficult to read and ascribe a meaning to the VRs and had difficulty describing the VRs (Fotakopoulou & Spiliotopoulou, 2008).

Spiliotopoulou-Papantoniou et al. (2008) did a similar study on ICT textbooks in Greek schools. The study aimed to analyse the VRs in the ICT textbooks as well as gather the experiences of learners looking at two specific VRs from the assigned textbook. The results showed that seeing a VR influences a person's thinking and assists in the process of finding a solution (i.e. the product). Visualisation is a useful teaching and learning aid as it assists the person in reasoning mathematically when solving mathematics problems (Spiliotopoulou-Papantoniou et al., 2008). Spiliotopoulou-Papantoniou et al. (2008) state that visualisation also requires that learners interpret and reflect on the pictures and images to understand the mathematics topic being taught. The findings of the study indicate that the most popular VR type was a sketch-comic and snapshot with few VRs having an explanatory function. Similar to their study on the use of VRs in economic texts, they found that learners had difficulty recognising the main idea present in the VRs in Information and Communications Technology (ICT) texts (Spiliotopoulou- Papantoniou et al., 2008).

Furthermore, Ainsworth (2006) produced a paper on how multiple representations should be used to allow for optimal mathematical learning when learning about new concepts. The paper aimed at addressing different aspects of learning with multiple representations. Ainsworth (2006) explained that in the Design, Functions, Task (DeFT) framework, multiple representations are explored when problem solving. Ainsworth (2006) found that developing visualisation requires teacher facilitation of the learning process. In other words, teachers need to mediate the VRs as VRs are not sufficient in promoting understanding on their own. In her findings, Ainsworth (2006) found that exposure to multiple VRs enables learners to select the appropriate VRs for the task based on their individual preferences (Ainsworth, 2006).

The findings from the above-mentioned research suggest that VRs are a useful tool for developing an understanding of various topics. As Rösken and Rolka (2006) note, VRs reduce the complexity of the information being shared. Arcavi (2003) indicated that translating existing VRs can be difficult for learners and this difficulty may complicate the mathematical learning process, hence the importance of teachers mediating the VRs for the learners.

The following section explores various VRFs that are available to analyse VRs.

2.3.4 Visual representation framework

There are numerous frameworks used to analyse VRs. In this section of the literature review I include three VRFs. The first two are taken from the context of mathematics education while the latter was developed in the context of economics texts. I have chosen these three because DeFT is a well-known international framework that focuses on how representation impact learning. The framework by Makgato & Ramaligela (2012) is a South African one which assesses aspects of VR. However, I found it useful to look beyond math education to see what Fotakopoulou and Spiliotopoulou offered on VR. This framework encompasses aspects from both the DeFT and Makgato and Ramaligela frameworks (2012).

The DeFT framework looks at how learning can be influenced by a combination of representations. The first aspect of the DeFT framework is the design of the representation. The design of the VRs focuses on the number of representations, the information that each representation conveys, the form of the representation (picture, text, animation, sound or graph) and the sequence in which the representation should be presented in order to construct meaning. The second category of the DeFT framework is the different pedagogical functions in the representations (i.e. the number of inferences that can be made from a representation of the concept). The third category of the DeFT framework is to ascertain whether the cognitive task assesses the nature of the learners' learning when looking at the representations. Ainsworth (2006) suggests that understanding the function of multiple external representations is only possible when learners master the cognitive tasks associated with the concept.

When analysing learning materials, Makgato and Ramaligela (2012) assess the following three aspects as a criterion when selecting technology textbooks: (1) Does the textbook include content that is relevant to the curriculum? (2) Do the concepts show progression across the grade? (3) Does the learning material include real-life examples that are relevant to the sociological and cultural context of South African learners?

Fotakopoulou and Spiliotopoulou (2008) maintain in their framework, that there are different types of VRs, namely the image/text type, the relationship the representation has with the content on the page, the relation between the textbook and the learner's experiences (reality), and lastly, the function of the VRs within the text (Fotakopoulou & Spiliotopoulou, 2008). The VRF by Fotakopoulou and Spiliotopoulou (2008) is elaborated on below.

First, Fotakopoulou and Spiliotopoulou (2008) explain that a VR type can be either an image or a diagram. An image could be a sketch, a clip art image, a scratch (scribble) or a photograph. A diagram could be in the form of a table, graph, schematic representation and concept map (Fotakopoulou & Spiliotopoulou, 2008). The second category is the VR's relation to content (Fotakopoulou & Spiliotopoulou, 2008) – the VR may have no relationship, a weak relationship or a meaningful relationship to the content (Fotakopoulou & Spiliotopoulou, 2008). A meaningful relationship assists in supporting the content. The third category examines VRs in relation to reality where the VR could either be realistic or metaphoric (Fotakopoulou & Spiliotopoulou, 2008). The third category focuses on the function of the VRs, meaning that there are different uses for each representation. According to Fotakopoulou and Spiliotopoulou (2008), a VR's function could be decorative, exemplifying, explanatory or complementary. Decorative VRs contribute to the aesthetic beauty of the textbook/workbook. Exemplifying VRs provide examples of concepts while explanatory VRs offer additional information and enhance information explained in the text. Mayer et al. (1995) further explain that explanatory VRs increase the opportunity for recall and improve problem solving in learners. A VR with a complementary function provides information that cannot be found in the text (Fotakopoulou & Spiliotopoulou, 2008). I purposely chose the Fotakopoulou and Spiliotopoulou (2008) framework because it addresses a holistic view of the VRs in relation to the content and I was able to adapt the framework to suit the FP maths type of VRs. The holistic view of the framework includes the types of VRs, the content and context of the text as well as the intended meaning of each VR. For example, whether the VRs further explain a concept or whether they provide an example of the concept. To this end, the VRF of Fotakopoulou and Spiliotopoulou (2008) is discussed in more detail with examples in Chapter 3.

2.3.5 Critiques of research on Visualisation in education

Arcavi (2003) suggests that research into visualisation needs to include verbalisation. If a learner looks at a VR, they are required to first decode the VR. Once the VR has been decoded in the mind of the learner, the learner then explains the VR verbally. Through decoding and explaining the learner expresses their understanding of the concept. Much of the research that focuses only on VR is limited.

Thornton (2011) suggests that visualisation should be seen by mathematics education researchers as a continuum of VRs ranging from concrete to semi-concrete to abstract VRs rather than only as concrete or abstract. This refers to the different stages of development according to Piaget. Thornton (2011) finds it important that the learners develop visualisation at an abstract level after going through the other stages. In my research, I pay attention to both cautions.

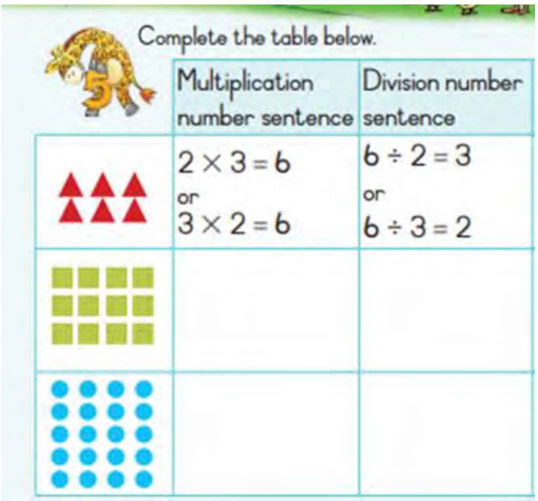
My study focused on multiplication and division and thus in the next section, I expand on the broad views within the literature on multiplication and division.

2.4 Multiplication and Division

Clark and Kamii (1996) assert that multiplication involves numbers that act upon each other in different ways. Multiplication is a scalar relation that involves a multiplier, multiplicand and the product. The multiplier is the scaling factor and represents how many times the operation should be iterated. The multiplicand is the number of objects in a set, and the product is the rescaled result. In the problem 3x5 =15 the number 5 is the multiplicand (number of objects in a set-multiplicative unit) and the number 3 is the multiplier (number of sets). The number 15 is the product. The understanding of multiplication is that a whole can be composed of different-sized equal groups (Milton et al., 2019). The inverse of this would be division, which involves decomposing a whole into equal groups (Milton et al., 2019).

2.4.1 Two broad views

There are two broad views as to the order in which multiplication and division should be taught. The first view is that mathematics is hierarchical and that the operations of addition, subtraction, multiplication and division should be taught sequentially (Harries & Barmby, 2007). In other words, multiplication should be taught prior to division. The second view, supported by Nunes and Bryant (1996), is that multiplication and division should be introduced to learners simultaneously so that the learners begin to understand the inverse relationship between the two operations (Vula & Berdynaj, 2011). The latter view maintains explicitly that mathematics is about studying patterns, making connections and identifying relationships. Figure 2.1 below provides an example of an activity that seeks to show the relationship between multiplication and division. In Figure 2.1 the VR of an array is indicated in the first column. The multiplication problems linked to the array are written in column two and the division sums are in column three. Columns two and three should provide the opportunity for learners to understand the inverse relationship between multiplication and division.


Complete the table below.

	Multiplication number sentence	Division number sentence
	2 × 3 = 6 or 3 × 2 = 6	6 ÷ 2 = 3 or 6 ÷ 3 = 2

Figure 2.1: An example of multiplication and division inverse relationship (DBE, 2011, p. 43; Grade 2)

Often, multiplication is viewed as an extension of repeated addition (Askew et al., 2019) and as such, multiplicative reasoning is viewed as an extension of additive reasoning (Askew, 2018). There are two perspectives on the role of additive relations (the relationship between addition and subtraction) in developing an understanding of multiplicative relations. The first perspective is evident in the CAPS curriculum (DBE, 2011a, b, c, d); these are the DBE CAPS documents for Grades R to 3, Grades 4 to 6, Grades 7 to 9 and Grades 10 to 12. Anghileri (2001) notes that counting in groups may improve learners' understanding of multiplication and division. Askew (2018) asserts that learners need to be developmentally ready for multiplicative relations as it involves a level of abstract thinking that is not evident in additive relations.

Array representations are one of the forms of VR that assist learners in developing multiplicative reasoning (Barmby et al., 2009). Figure 2.1. is an example of an array representation. In Figure 2.2, the 16 cabbages are planted in rows of four.

Figure 2.2: An example of an array representation (Mostert, 2011, p. 63; Grade 1, Text B)

2.4.2 What is multiplication?

Multiplication and division as a conceptual field (Vergnaud, 1983) is made up of functional and scalar relations. A function diagram shows the relationship between the quantities in the input and the output (Figure 2.3). Figure 2.3 is an example of a function diagram which focuses on multiplication by 2. To work out the answer the learner will need to start with the input which is 11 (which is the input or the number of objects in a group) and multiply it by the rule (X2), in order to get to the product (output).

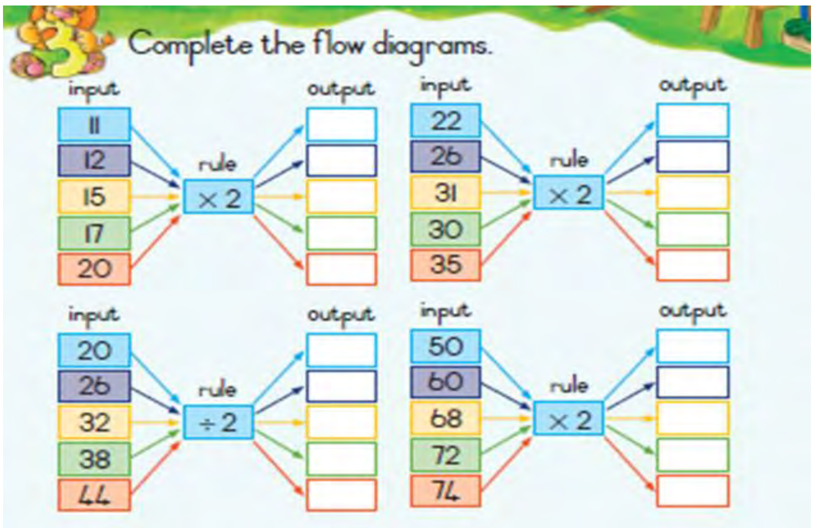


Figure 2.3: An example of a function relationship (DBE, 2011, p. 5; Grade3, Text A)

A scalar relation refers to the association *within* quantities (refer to Figure 2.4 below). A scalar relation is the relationship between the multiplier and the multiplicand. Shield and Dole (2013) state that it is important to develop a learner's understanding of creating links or relationships between concepts. This will allow the learners to use the knowledge immediately and equip learners with knowledge for future use (Shield & Dole, 2013). Shield and Dole (2013) suggest that teaching and making these links early in a learner's life is undervalued.

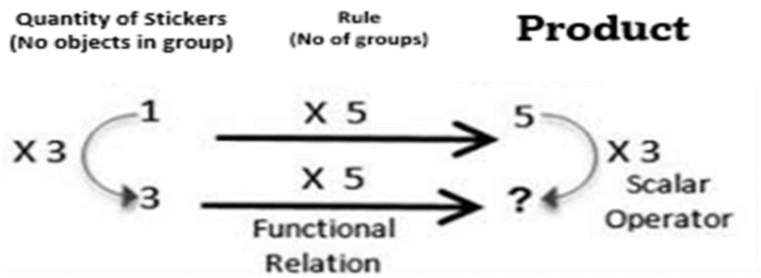


Figure 2.4: An example of Scalar relation and Functional relation

Scalar and functional relations are the prerequisites for teaching and learning ratios. Figure 2.4 above focuses on stickers and particularly the number of stickers in a set. The horizontal relation refers to the number of stickers in each set (1) and the number of sets (5). This represents a functional relationship where the input number (1) and the rule (X5) result in an output number (5). The appropriate operation would be 1x5=5. The second example of a function relation is that there are 3 stickers on each page and there are 5 pages (sets), how many stickers are there in total? In this case, the operation would be 3x5=? (15). The use of scalar and functional relations is necessary to think multiplicatively.

The vertical relation represents a scalar relation which occurs between quantities. The scalar relation would be solved by saying there is 1 sticker in the set and there are 3 sets (1x3= 3). The inverse of the above diagram would explain the scalar and functional relations diagram in relation to division.

The term 'multiplicative reasoning' broadly refers to the learning of multiplication. The next section explains multiplicative reasoning and explores how it is taught according to various researchers.

2.4.3 What is multiplicative reasoning?

Multiplicative reasoning (MR) is the ability of learners to see the relationship between the number of objects in a set and the number of sets in order to be able to identify the product. A composite unit is a number of objects in a set that is represented as one thing repeatedly added (Nunes et al., 2008). For example, with the calculation 3x5, there are 3 objects in 5 sets. The '3' is the composite unit. The inverse of this multiplication sum would be for the learner to work out a division operation, 15÷5=3 (partitive or equal groups) and 15÷3=5 (quotative).

2.4.4 Teaching multiplicative reasoning

According to Askew (2018), multiplicative reasoning can be developed through the teacher creating experiences of working with multiplicative situations. This can be done by showing learners how multiplication and division can be represented through different models, namely array representation, double number line or T-Table model. Making use of different representations (different ways) has been argued as critically important in formulating conceptual understanding (National Research Council, 2001).

Lamon (2005) acknowledges that multiplicative reasoning is difficult to teach and that it takes time for the learners' understanding to develop. It is through practice that new meanings come into being and that we move to think multiplicatively (Nunes et al., 2008). Developing a solid understanding of multiplicative reasoning is important for mathematics in later years (Askew et al. 2019). The more learners are required to think multiplicatively, the more the multiplicative reasoning schema is created. A multiplicative relations schema can be created by practising and seeing the relationship between ratios, proportions, fractions, cartesian products and percentages (Askew et al., 2019).

When teaching multiplication and division, it is important to be aware of misconceptions related to the topic. The VRs used in a text can assist (or not) learners in understanding mathematical concepts and misconceptions (Lamon, 2005). My research focuses on multiplication and division. Above, I have explored multiplication and next, I will explore the main ideas in division.

2.4.5 What are the main ideas of Division?

Division is dealt with as a separate strand to multiplication in the CAPS document (Askew et al., 2019; DBE, 2011a). It explores division through grouping and sharing with and without remainders (Kieren & Dreyfus, 1998). Heirdsfield et al. (1999) maintain that learners need to draw on their knowledge of multiplication to work out division problems. This suggests that multiplication should be taught before division and that division is the inverse of multiplication (Downton, 2013; Greer, 1992; Squire & Bryant, 2002; Vula & Berdynaj, 2011). The CAPS explains that before teaching division explicitly, teachers should teach repeated subtraction and sharing into equal parts, leading to division. Repeated subtraction and sharing are focused on in Grade 1 and then later on (in Grades 2 and 3) the concept of division emerges, often after multiplication has been taught.

2.4.6 Partitive and Quotative

There are two types of division, namely partitive and quotative division. Partitive division is similar to sharing objects into groups. In partitive division, the number of groups is known, and the size of the groups is not known (Fischbein et al., 1985). For example, if 10 sweets are shared equally between 2 children, how many sweets will each child receive (10÷2=?). The quotative model is when the size of the group is unknown and the number of groups is unknown (Fischbein et al., 1985). For example, there are 10 sweets. Each child receives 5. How many children received sweets? (10÷5=?).

When researching multiplication and division it is important to acknowledge the works that researchers have already completed on the topic. The following section acknowledges the contributions of researchers on multiplication and division.

2.4.7 Multiplication and division schemas

Young-Loveridge et al. (2013) completed a study on introducing multiplication and division to learners in their first year of schooling. By doing this they argued that the learners are developing schemas for multiplication and division based on repeated addition and subtraction.

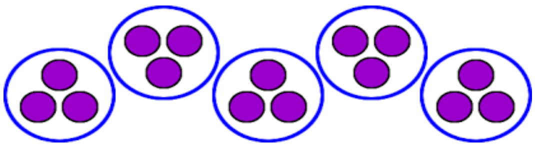
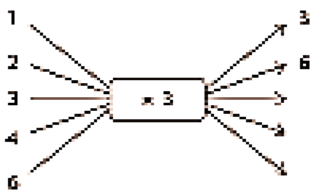
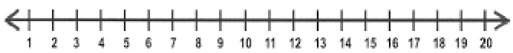
A schema is the way in which one's thoughts are organised. The schema of counting objects (which is the organisation of an activity) includes one-to-one correspondence (which is the ability of the learner to match one number word to each object) (Vergnaud, 2009). Vergnaud (2009) insists that schemas are important as they organise gestures and physical actions, interactions and conversations with people and reasoning. Multiplication and division schemas are developed by exposing learners to mathematical problems in which they can practice multiplication and division using different methods. Park and Nunes (2001) assert that learners need to develop a correspondence schema to identify the relationship between multiplication and division to solve multiplication and division problems.

Kosko (2018) researched which VRs third grade teachers (57) reported they use when solving multiplication and division problems in class. The researcher made use of surveys to collect the data. Kosko (2018) reported that the Grade 3 teachers identified equal groups and arrays as their preferred method to solve multiplication and division problems.

2.5 Chapter Summary of Literature Presented

The key VRs promoted in the research that informed the literature for teaching and learning multiplication and division reviewed above are summarised in Table 2.1 below.

Table 2.1: Literature that promotes the use of VR

VR	**Example of VRs**	**Examples of Researchers promoting the use of VRs**
Arrays	::::	Nunes & Bryant (1996) Barmby et al. (2009) Kosko (2018)
Repeated addition	3+3+3+3=12	Nunes et al. (2008) Askew (2018) Askew et al. (2019)
Groups and sharing		Fischbein et al. (1985) Kieren & Dreyfus (1998) Hierdsfied, Cooper, Mulligan & Irons (1999) Anghileri (2001) Milton et al. (2019)
Function diagrams		Vergnaud (1983)
Number lines		Richards (2014)

From Table 2.1 it is evident that a range of VRs is promoted in the research informed by literature for teaching and learning of multiplication and division and include arrays, repeated addition, grouping and sharing, function diagrams and number lines.

2.6 Concluding Remarks

The purpose of this chapter was to provide the reader with literature on visualisation in mathematics texts, specifically the concepts of multiplication and division. I attempted to define visualisation and its importance, and acknowledge the critiques of research on visualisation. The various visualisation frameworks in literature were also explored and reasons were given why the Fotakopoulou and Spiliotopoulou (2008) framework was suitable for this study. I explored the broad views of researchers on multiplication and division and summarised the chapter by tabling the different types of VRs that researchers have identified as relevant in the teaching and learning of multiplication and division (see Table 2.1). This includes multiplicative reasoning, quotative and partitive division. Visual representations (VRs) are clearly an important tool and as such are used in texts. The aforementioned literature forms the basis of the next chapter on exploring the theoretical framework that underpinned this research, namely constructivism. The next chapter also explains the analytical tool used to analyse this research.

Chapter 3: Theoretical Framework

3.1 Chapter Overview

The chapter will discuss constructivism as the chosen theoretical framework of the study. In particular, I will describe various concepts related to the broader constructivist theory, namely the mediation of signs and tools. The chapter will conclude by describing how constructivism was identified as a suitable theoretical framing for the present study. Constructivism emphasises the importance of active, meaningful discussions in the learning process in the classroom environment (Kukla, 2000) and that learners actively construct knowledge and develop schemas through engaging in learning activities. In their active interpretation and engagement with key ideas presented through VRs, learners develop schemas that assist them in doing multiplication and division.

3.2 Constructivism

A constructivist view of learning underpinned this study. Constructivism focuses on the theory of knowledge and knowing in teaching and learning (Jones & Brader-Araje, 2002). It suggests that the learner actively constructs knowledge through experimentation, action and problem solving (Sjøberg, 2007) rather than passively receiving from the outside (von Glaserfeld, 2001). Learning becomes a process, not simply a product (Jones & Brader-Araje, 2002). Ndlovu (2013) agrees and argues that mathematics learning is thus a complex process of constructing and reconstructing what one knows.

3.2.1 Knowledge is actively constructed

Gray and Macblain (2012) agree with Piaget and von Glaserfeld that constructivism is a learning theory in which learners are considered to actively construct their knowledge, views and ideas about the world from their everyday experiences. Smith (2013) asserts that constructivism rests on the belief that a child builds knowledge by acting on their experiences in the world and the meaning they have made from their experiences. The learner actively contributes to learning and creating knowledge (Selley, 1999). Learners develop mathematical knowledge by exploring, justifying, representing, discussing, experimenting, describing, investigating and predicting (Countryman, 1993). Learning has to do with developing the knowledge of a learner (Ndlovu, 2013).

3.2.2 Knowledge in mediation and knowledge construct

Many theorists have contributed to the conversation about the construction of knowledge. Vygotsky (1978) refers to how people are interconnected beings and construct meaning and acquire knowledge together (Baker et al., 2007; Sarama & Clements, 2009; Olusegun, 2015). Social interactions create experiences that facilitate the learning and meaning-making process (Vygotsky, 1978). Creswell (2009) explains that human beings construct meaning as they experience, live and engage with the world they live in (their reality). Learning is a two-way process, meaning it occurs between the teacher and the learner and between the learners themselves (Graven et al., 2015). Vygotsky's sociocultural learning theory shares many contextual assumptions but builds a notion of mediation and multiplicative reasoning by the More Knowledgeable Other (MKO) (Abtahi et al., 2017). In this study, VRs serve as a form of mediation where knowledge of the MKO is built into the design of the VRs (e.g. array representation).

Vygotsky (1978) asserts there is a social dimension to new knowledge. In the social interaction with an MKO, for example, the teacher assists in making meaning from the signs being used (Prawat & Floden, 1994). Vygotsky believed that social interactions create experiences that facilitate learning and meaning-making (Vygotsky, 1978). The teacher needs to promote talking, probing and investigating to make meaning of the information. In this research, the social environment is the classroom. When working with the textbooks, the teacher assists the learner in making meaning of the VRs in the texts. Constructivism is a learner-centred approach (Olusegun, 2015). Learners need to communicate, interact and instruct one another for them to develop and for learning to take place (Griqua & Schäfer, 2022). Therefore, learners need to create meaning as they are the originators and modifiers of knowledge (Boero & Guiterrezz, 2006).

The term 'mediation' is used in Vygotsky's theory (Veer & Valsiner, 1994). According to Daniels (2005), Vygotsky describes mediation as the process by which social and individual interactions shape one another. In the case of this research, the teacher mediated the learning process. Furthermore, Zayyad (2020) explains that a mediator helps a human modify their environment in order to get better action. A mediator is needed in the mediation process and their role is to guide and scaffold the learning process. Ideally, the teacher would present a lesson on the concept being taught and make use of the textbook to consolidate the knowledge. The teacher makes use of language to mediate the knowledge between the signs and tools found in the textbook (Jones & Brader-Araje, 2002). Portere and Briede (2021) explain that the mediation process is actually the learning process as the teacher becomes a facilitator of knowledge that assists the learners. Werstch (2007) suggests that mediation is not necessarily visible to the one completing the action with the tool. Mustafa et al. (2019) suggest that mediation can take the form of regulations, symbolic artefacts and gestures. The mediation which happens intentionally with awareness of the external object, people or signs is explicit (Werstch, 2007). Implicit mediation is when the mediation process takes place internally and is not the object with which reflection occurs (Wertsch, 2007).

3.2.3 Signs and tools

'Signs' and 'tools' are used by learners as symbols that assist in making meaning of the environment they are in and mediate the learning process (Daniels, 2005). Signs and tools mediate actions (Werstch, 2007). Vygotsky (1978) defines signs as internal psychological activity. A sign assists in the transitions between speech and actions (Veraksa, 2013) and assists with concept formation.

Vygotsky (1978) defines tools as human activity aimed at mastery. Tools have an indirect function on the object and assist in accomplishing the learning activity. Vygotsky (1978) explains that tools modify human activity and help structure people's thinking (Katić et al., 2009). The tool is externally orientated and demonstrates mastery (Vygotsky, 1978). Tools are primarily used by those who know how to use them.

Veraksa (2013) suggests that signs and tools each have different functions and meanings. Daniels (2005) explores the function of tools and explains that a tool has two functions. Firstly, it assists and introduces one to several new functions of a tool. A tool's second function is abolishing processes and replacing some functions with others (Daniels, 2005). In the context of this study, the VRs may assist in abolishing the process which the learner would have used to work out a problem. Because the VRs have a specific function, for example, a VR with an explanatory function, it now assists the learner in working out the problem.

In this study, the texts served as the tools and the signs are the VRs used for mediating learning. Daniels (2005) sees tools as cultural artefacts which control behaviour from the outside. Tools assist learners in accomplishing an activity, yet cognitive processing is still necessary to bring about learning (Vygotsky, 1978). Signs and tools are auxiliary means of solving a problem (Vygotsky, 1978). Learners construct meaning from what they hear and see

(Yackel, 2001). Visual representations (VRs), such as pictures, symbols and resources are essential in understanding multiplication and division concepts (Nghifimule, 2016) as they are the signs and tools used to mediate the learning process.

3.2.4 South African Curriculum and Assessment Policy Statement and constructivism

The National Senior Certificate has reported that the general aims of the South African curriculum take on an "active and critical" learning approach rather than "rote-learning". The CAPS document provides guidelines for teachers to use for classroom management. Here they refer to learners sharing/discussing ideas in groups in a cooperative manner (DBE, 2011a). The CAPS advises that the seating arrangements also be done in small groups (DBE, 2011a). This study took on a constructivist approach where learners were actively involved in their own learning. In the constructivist approach, a facilitator is used to assist in creating knowledge. The CAPS recommends creating knowledge with the help of a facilitator which could be the teacher or peer (DBE, 2011a).

3.2.5 A critique of constructivism

Liu and Matthews (2005) suggest that there is a dualism to constructivism, that is, whether it occurs individually or in groups. Liu and Matthews (2005) concluded that whether learning occurs in a social environment or not, they are closely interconnected. There is social collectivity in individual learning and development. Therefore, we need to acknowledge the role of social and collective learning and the role of individual learning (Resnick, 1996). Fox (2001) does, however, critique that constructivists imply "social constructionism" (p. 816).

Fox (2007) and Liu and Matthews (2005) argue that the constructivists' dismissal of passive learning, such as traditional didactics, implies that 'passive learners' cannot learn. Fox (2007) disagrees and maintains that a learner who is being passive could also be learning.

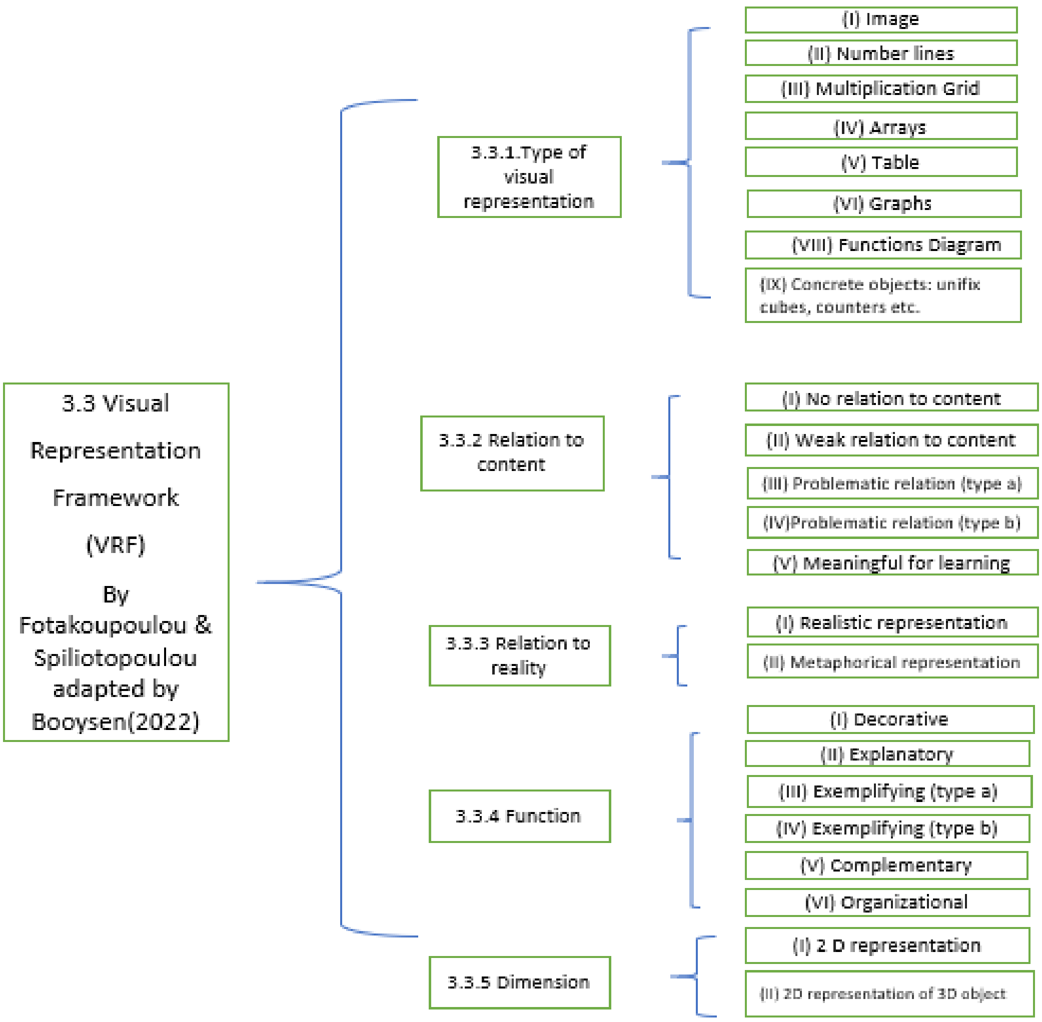
They maintain that there are learners who can engage in the learning process passively, for example, by sitting quietly and thinking about a maths problem. Because the learner is not active does not mean they are not processing information. Fox (2007) also criticises constructivism as it fails to address how the external world is bridged in the internal mind.

Ndlovu (2013) argues that a challenge of a constructivist approach to learning is that it takes time in the classroom context. The teacher is the facilitator of knowledge; therefore, they need to engage in the topic with the individual/group and probe and engage the learner for knowledge of the concept to develop over time. With constructivism, learners need guidance and support when constructing knowledge, and this requires carefully scaffolded processes that guide learners in constructing meaning.

3.3 Developing a Visual Representation Analytic Framework

As mentioned above, learners need to make use of signs and tools are needed to complete an activity. The tools in the case of this research were the VRs present in the textbooks and as suggested in the CAPS document. To analyse VRs, the VRF by Fotakopoulou and Spiliotopoulou (2008) was adapted for mathematical use in order to analyse the mathematics textbooks. Fotakopoulou and Spiliotopoulou (2008) designed an analytic tool to draw attention to the nature of VR in texts. In their analysis, they focus on the type of visual representation, the relation it has to content and reality and the function of each VR. This is reflected in Figure 3.1.

Figure 3.1: The Visual Representation Framework



Source: Fotakopoulou and Spiliotopoulou (2008 adapted by Booysen, 2022)

I expand on each of these five aspects of VRs below.

3.3.1 Types of visual representation

There are different types of representation in mathematics. Yilmaz and Argun (2018) explain that a visual image forms in the mind and creates a mental scheme in which visual information is received. When information is received it needs to be processed and interpreted for it to be understood. The first category under the types of VR is a visual image (see Figure 3.1). A visual image is interpreted and made sense of through the visualisation process. The process of visualisation enables the learner to form a sketch in their mind.

The second category under types of VR is a number line (see 3.3.1 (II)). A number line consists of a horizontal line with numbers which are evenly spaced. While the double numberline is more appropriate for multiplicative reasoning, the single numberline can be used to show multiples of a number (e.g., skip counting). This allows the learner to see the multiplicative nature on the numberline. Learners begin with a structured number line which is made up of units of 1. This is followed by a semi-structured number line where the numbers are in increments of, for example, 10 or 100. Once the learners have mastered the use of the semi-structured number line, they use an unstructured number line, that is, an empty number line.

A multiplication grid (see 3.3.1 (III)) is the third category under types of VR. Gierdien (2009) explains that multiplication grids are made up of columns and rows that are aligned. The alignment in the multiplication grid seeks to make learners aware that 2x6=12 and that 6x2=12, thus, helping the learner to understand the commutative property of multiplication and later, also making the discovery that division is the inverse of multiplication. When the student does the same sum in the 7 times table, they will find that the answer for 4x7 and 7x4 is the same (commutative). This shows that even if you change the order of the numbers being multiplied, the answer (product) does not change.

Barmby et al. (2009) asserts that the use of array representation (see 3.3.1 (IV)) assists in developing learners' multiplicative reasoning and understanding. Stott (2016) explains that an array is a set of objects which appear in a rectangular formation with rows and columns. Array representations support the learners in understanding multiplication, division and the inverse relationship between multiplication and division. There are various sums that learners can generate from this array including repeated addition (e.g. 5+5+5 or 3+3+3+3+3), multiplication (3X5 or 5X3) and division (15÷3 and 15÷5). Array representation assists in exploring the relationship between repeated addition and multiplication, the commutative relationship in multiplication and the inverse relationship between multiplication and division.

A function diagram (see 3.3.1 (VIII)) represents a relationship that numbers have with one another. A function diagram contains an input number, a rule and an output number.

A concrete object is a representation of a real object that can be touched, for example, beads (see 3.3.1 (IX)). There are examples of unifix cubes, beads and counters in the textbooks.

3.3.2 VRs Relation to content

This category explores the extent to which a VR is related to content and the concept of multiplication (e.g. repeated addition leading to multiplication) and division (e.g. repeated subtraction leading up to division). Relation to content is the second category of the VRF. The VR can be categorised as having no relation to content, weak or strong (see Figure 3.1) (Fotakopoulou & Spiliotopoulou, 2008) or a problematic (type a or b) relationship to content (see Figure 3.1) that influences learning.

When a VR has no relation to multiplication or division and therefore is suitable for the category of no relation to content. When the VR has a weak relation to the content it means that there is not a clear link between the exercise and the problem that needs to be solved. A VR with a strong relation to content assists and supports the development of the mathematical concept being taught.

The VRF analytical tool by Fotakopoulou and Spiliotopoulou (2008) did not include an option where an image relationship can be problematic. This is not a criterion in the analytical tool but during the analysis, I identified a few representations that I was not able to place under no, weak or strong relation to content as the representation was problematic. During the process of conducting the pilot study (DBE Workbook Grade 4 Book 1), I realised that there was an additional relationship in this category. A problematic diagram is one that is potentially meaningful with regard to content but can also cause confusion or develop learner misconceptions. Hence, I came up with the problematic (type a) which is when there is an error in the VR and problematic (type b) which is when the VRs cause unnecessary constraints for the learner (See section 4.7 in Chapter 4).

3.3.3 VR's relation to reality

The VR's relation to reality could either be realistic or metaphorical (Fotakopoulou & Spiliotopoulou, 2008). A realistic representation (see Figure 3.1) can be a photo or drawing of an actual object for example a flower. A metaphorical VR (see Figure 3.1) represents something or symbolises meaning (Fotakopoulou & Spiliotopoulou, 2008), such as tallies.

3.3.4 Function of each visual representation

The fourth category of the VRF is the function of each VR. The function of a VR could be decorative, exemplifying (type a and b), explanatory or complementary (see Figure 3.1) (Fotakopoulou & Spiliotopoulou, 2008).

The decorative function's direct purpose is to make the page aesthetically pleasing (Fotakopoulou & Spiliotopoulou, 2008). A VR that has an explanatory function assists in explaining the concept (Fotakopoulou & Spiliotopoulou, 2008). The main function of the image is to explain the concept with the hope that learners see the relationship between the numbers. The explanation may occur using, for instance, speech bubbles in textbooks. The original VRF by Fotakoupoulou & Spiliotopoulou (2008) contained the category exemplifying function. During the process of conducting the pilot study (DBE Workbook Grade 4 Book 1), I realised that there was an additional relationship in this category - those with a worked example and those that the learners needed to complete themselves. This lead me to create the two categories exemplifying function (type a) which consists of worked examples of the concepts that relate to the written language in the texts (Fotakopoulou & Spiliotopoulou, 2008) and exemplifying function (type b) is when a learner needs to complete the activity themselves (See section 4.7 in Chapter 4). A complementary VR complements a definition of the concept on a page (Fotakopoulou & Spiliotopoulou, 2008). A complementary VR provides extra information for the activity but does not necessarily support the learning. Learning will still be able to commence without the VR.

The analytical tool by Fotakopoulou and Spiliotopoulou (2008) did not include an option where the VRs were simply an organising device as noted in my pilot of the Grade 4 National Workbook. This prompted me to create categories relating to ordering in the FP textbooks. Herewith, my supervisors and I added the category "organising". This category was added to be able to classify the function of the VRs present in FP texts. Organising is thus the fifth category under the functions of the VRF. A VR with an organising function looks at the layout of a visual representation that will help organise information for the viewer to make sense of the thought process when looking at the visual representation.

3.3.5 Dimensionality of VRs

The fifth category focuses on the dimension of the visual. A VR can be either a 2-dimensional representation or a 2D representation of a 3D object. A 2D representation (see Figure 3.1) is a flat object. A numberline is an example of a 2D representation. The last category in the VRF is a 2D representation of a 3D object (see Table 3.1). This means that the VR is a 2D representation on a flat piece of paper; however, in real life, the concrete object would be 3D. For example, a photo or picture of a little boy would be a 2D representation of a 3D object.

3.4 Concluding Remarks

This chapter explained the theory of social constructivism and its relevance to this study. It also provided explanations of the different categories of the VR theoretical framework by Fotakoupoulou and Spiliotopoulou (2008) used in this study. This chapter provided the required information on the theoretical framework which is necessary for the next chapter that discusses the methodology and research design of the study.

Chapter 4: Methodology and Research Design

4.1 Introduction

The focus of this study was to explore the nature of VRs in three FP mathematics texts. The first phase of the research involved identifying my sample texts and grades to focus on. I chose textbooks and workbooks. The textbooks used focused on Grades 1 to 3. This chapter discusses the reasoning behind the methodologies used in this research study. The structure of this chapter includes the research goals, the research orientation and method, data collection, sampling procedure, data analysis and ethical considerations (trustworthiness, credibility, transferability, dependability, and conformability) undertaken in this study.

4.2 Research Goals

This study aimed to analyse the VRs used in nine FP texts. As noted in Chapter 3, my focus was the type of VRs, the VRs' relation to content and reality, and the function of the VRs in whole number multiplication and division sections of the textbooks. The following research questions framed the study:

1. What is the nature of the visual representations used for whole number multiplication and division in South African FP texts?

1.1 What is the nature of the visual representations used for whole number multiplication in South African FP texts?

1.2 What is the nature of the visual representations used for whole number division in South African FP texts?

2. How does the nature of visual representations in texts compare to those promoted in the research literature and the curriculum?

One aim of the study was to contribute to the discussion and evaluation of texts for selection and decide how they are used in schools. I intend to engage with various stakeholders (Department of Education officials, principals and teachers) in a range of forums (for example, conferences and DBE Indaba's) to share insights into what to look at when choosing a textbook for FP learners, specifically regarding the types and function of VRs in texts.

4.3 Research Orientation and Method

Paradigms are how we come to understand and know our world. Neuman (2006) suggests that a paradigm on which a research study is based is understood and made explicit in the research itself. An interpretivist paradigm underpinned my research. The interpretivist paradigm seeks to describe and understand how people make sense of their world, their lived experiences and their different realities to make meaning of a phenomenon (Bertram & Christiansen, 2017). Lukenchuk (2013) asserts that the interpretivist paradigm focuses on understanding, interpreting and meaning-making by drawing on qualitative data. In this sense, an interpretivist orientation suggests that knowledge is relative to the participants and this paradigm cohered well with my research interpretations and my theoretical framework. Thus, my analysis was based on an acknowledgement of the subjective interpretation of the CAPS and nine South African FP texts.

My research approach was primarily qualitative. However, I considered that I used qualitative and quantitative methods that would provide different but complementary insights. In combining qualitative analysis with quantification that enabled descriptive statistics, I developed a more comprehensive understanding of the research questions (Creswell, 2012).

4.3.1 Qualitative research

Qualitative research is a valuable investigation methodology for exploring and understanding a central idea (Cohen et al., 2007). Qualitative research is open-ended (Creswell & Plano, 2006) and allows for a thick description of the data. The qualitative research researcher seeks to answer *what, why* and *how* questions (Gay et al., 2012). This is reflected in the research questions informing my study as highlighted in Chapters 1 and 4.

Qualitative research requires an iterative process of data generation. This means that the researcher codes and re-codes the data while generating the data. The layers of analysis when investigating the different types of VRs allowed for in-depth, textbook-specific rich qualitative data that categorised the nature of the VRs in the textbooks. The open-ended nature of qualitative research is dependent on the researcher's reasoning and interpretation of the content.

A benefit of a qualitative approach was that it allowed me to understand and interpret the data generated from the study (Maxwell, 2012). While this, as noted above, is a strength, the use of qualitative data exclusively is also a weakness as the researcher may not know the reason for the choice of VRs used in the CAPS and textbooks. Krantz (1995) and Choy (2014) agree that qualitative data is not objectively verifiable as researchers may have had a keen interest in the topic and this could lead to bias. A further weakness of qualitative research is that the data collection and data analysis processes can be time-consuming.

Quantitative data typically includes surveys, correlation studies and experimental research (Apuke, 2017). Quantitative research refers to closed-ended information where the data collected is statistically analysed (Creswell & Plano, 2006). The quantitative methodology includes numerical values using mathematics and basic statistical methods (Apuke, 2017). This study did not use quantitative research per se, instead, it used quantification to support descriptive analysis. Thus, in this study, I used quantification to describe the prevalence of different VRs used within the texts. For example, I identified the frequency of the different types of VRs, the relation to content and reality and the function of each VR that related to whole number multiplication and division.

While Choy (2014) argues that data gathering in quantitative research is quick and precise, this was not my experience. The iterative process of data generation, coding and recording were time-consuming as I constantly had to revise my codes. I was thus only able to determine the frequency of the codes once I had finalised the codes. The data was collected rigorously using an adapted version of the VRF developed by Fotakopoulou and Spiliotopoulou (2008) (see Section 3.3, Chapter 3). This added to the study's reliability (Choy, 2014). A weakness of quantitative research is that it does not allow for the elaboration of people's in-depth experiences (Choy, 2014). The statistical nature of the research method can be challenging for readers to understand if they are not familiar with statistical procedures.

For the reasons mentioned above, I decided to primarily use qualitative research with quantification of frequencies of categories of VR use and for the generation of descriptive statistics for my study.

4.4 Phase 1: Textbook Choice (Sampling)

4.4.1 Sample selection

As noted, South Africa makes use of two primary texts, namely workbooks and textbooks. Sampling involves choosing a sample from the population, that is, the entire set of cases from which the sample is drawn (Taherdoost, 2016). The sample pertaining to my study consisted of Grades 1–3 DBE workbooks (1 per grade per semester), and two other popular publishers of Grade 1–3 textbook series (one per grade per year) and the CAPS document. I chose these texts because they are the most widely used texts in South African schools in the FP. The textbooks by the two publishers are the top-selling textbooks as indicated by the Publishers Association of South Africa (Schroeder, personal communication) and contain instructional texts and exercises. The DBE texts are supplied free of charge to all public schools in South Africa. They include exercises that the learners complete in the book itself and are intended to be used by the learners daily.

As noted above, this study made use of document analysis. Bowen (2009) states that document analysis is a systematic procedure for reviewing or evaluating documents referred to as texts in this study and included texts and images (Bowen, 2009). The process of document analysis involves examining and analysing texts so that knowledge is produced and understood (Bowen, 2009). The choice of textbooks and CAPS suggested the use of primary analysis. Primary analysis refers to analysing actual raw material (Cohen et al., 2018).

The advantage of document analysis in my research was that the content was readily available (Bowen, 2009). When doing this document analysis, the document's authenticity was considered (Cohen et al., 2018). The purpose for which the document is designed assisted in understanding the document's content and in turn influenced the credibility of the document that was analysed (Cohen et al., 2018). A disadvantage of document analysis is that it does not necessarily contain enough detail (i.e. the document may not include enough detail on a specific topic), the data sets may be incomplete, or it may be difficult to retrieve all the information needed to complete the document analysis. A disadvantage of the document analysis is that I did not interview the original authors of the texts to ascertain why they used the VRs they did. Another concern with document analysis is that the documents contain the authors' selective bias or misconceptions (Bowen, 2009).

In Phase 1 of the research, I spoke with a former president of the Publishers' Association of South Africa to identify the top-selling Grade 1–3 mathematics textbooks. He identified textbooks from two different publishers. I decided to include the DBE workbooks in my research as these books are delivered to every primary school in the country annually. Having identified the textbooks, I conducted a literature search to ascertain which textbooks had previously been analysed in research in South Africa. It appears that the research all focused on the DBE textbooks (e.g. Fan et al., 2013; Fleisch et al., 2011; Hoadley & Galant, 2016; Mathews et al., 2014 Mdluli, 2014). Based on my review, there was no research on the FP mathematics commercial texts in South Africa.

4.5 Phase 2: Selections of Framework

In Phase two of my research, I examined the VRF developed by Fotakopoulou and Spiliotopoulou (2008) in the context of FP mathematics. The authors of the VRF are high school science teachers and the purpose of their framework is to analyse science textbooks. I thus had to adapt the VRF for mathematics textbooks in the FP. I conducted a pilot study to ascertain what adaptations I needed to make to analyse FP mathematics texts.

4.6 Phase 3: Conducting a Pilot Study (Gr 4 DBE Text)

In Phase 3 of my research, I conducted a pilot study to ascertain the appropriateness of the VRF for my study. First, my supervisors and I coded and analysed a Grade 4 National Workbook (Terms 1 and 2) separately. We used the Fotakopoulou and Spiliotopoulou (2008) framework to identify the VRs in the whole number multiplication and division sections of the Grade 4 DBE textbook. In the Grade 4 text, the title of the text served as an indication of the focus of the content on each page.

4.7 Phase 4: Adapting the Analytic Framework

While doing the pilot analysis, it became apparent that aspects of the framework needed to be adapted. Some VRs did not fit into the categories that Fotakopoulou and Spiliotopoulou (2008) had identified in their VRF. I added a category to 'function of the VRs', which I named 'organising'. This category related to VRs whose sole purpose was to assist in organising the information for the learners (see Figure 4.1). In this example, the table and the colours assist the learners with the activity. The learners are expected to multiply the colour in the circle by the number with the same colour in the table. The first sum is 30X2. The table assists the learners in knowing where to write the answer (60).

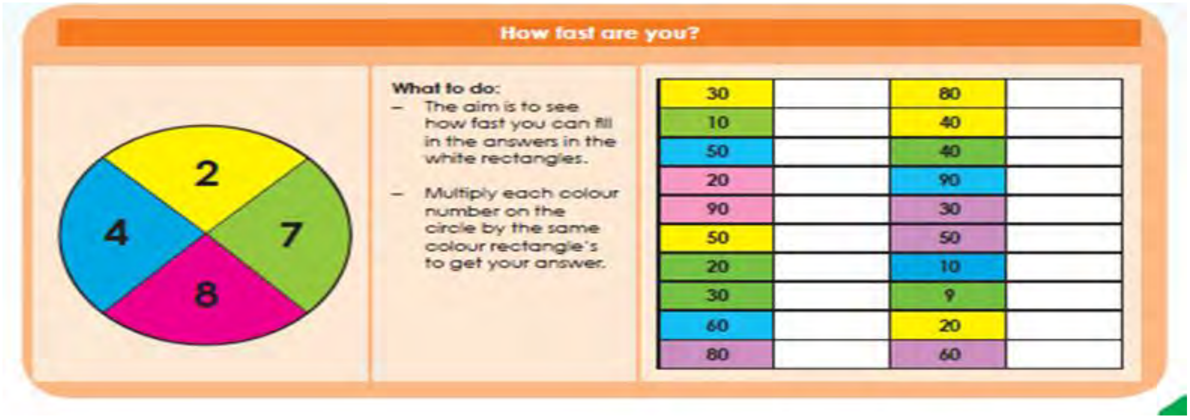


Figure 4.1: An example of a VR with an organising function (DBE, 2011, p. 71; Grade 4, Book 1)

An example of a VR with an organisational function was also evident in Figure 4.2. The purpose of the table is to organise the learners' thinking. The activity is organisational, in that it arranges the learners' writing so that they can identify the patterns.

1. Complete the table below.

Number	x 10	x 20	x 30	x 40	X 50
10					
20					
30					
40					
50					

Figure 4.2: An example of a VR with an organising function (DBE, 2011, p.118, Grade 4, Book 1)

From the pilot study, my supervisors and I also noticed that there were different types of VRs in the FP mathematics texts compared to high school science texts. The following subcategories for the different types of VRs emerged: image, number line, multiplication grid, arrays, tables, graphs, function diagram, unifix cubes and counters.

We also noticed that an adaptation was necessary for the category 'problematic relation to content' as there are VRs that were problematic because they had an error (problematic type a) and there were VRs that had a problematic relation to reality that caused an unnecessary constraint (problematic type b). We also noticed that the category 'exemplifying function' had two types of VRs with an exemplifying function. Exemplifying function (type a) is an example of a worked example and exemplifying function (type b) provides an example that the learners have to work out.

As mentioned in Chapter 3, the VRF examined the type of VR, the relation to content, the relation to reality, the function of the VR and whether it is a 2D or 2D representation of a 3D object that focused on pages of whole number multiplication and division explicitly. For the purpose of this research, I have not included doubling and halving in the analysis of the textbooks as there are no explicit links to whole number multiplication and division in the textbooks.

4.8 Phase 5: Qualitative Analysis (With Descriptive Statistics): Grade 1, 2, and 3 Texts

After the pilot study, I analysed, modified, refined and re-checked the nine texts and CAPS by capturing data on a spreadsheet (Figure 4.3).

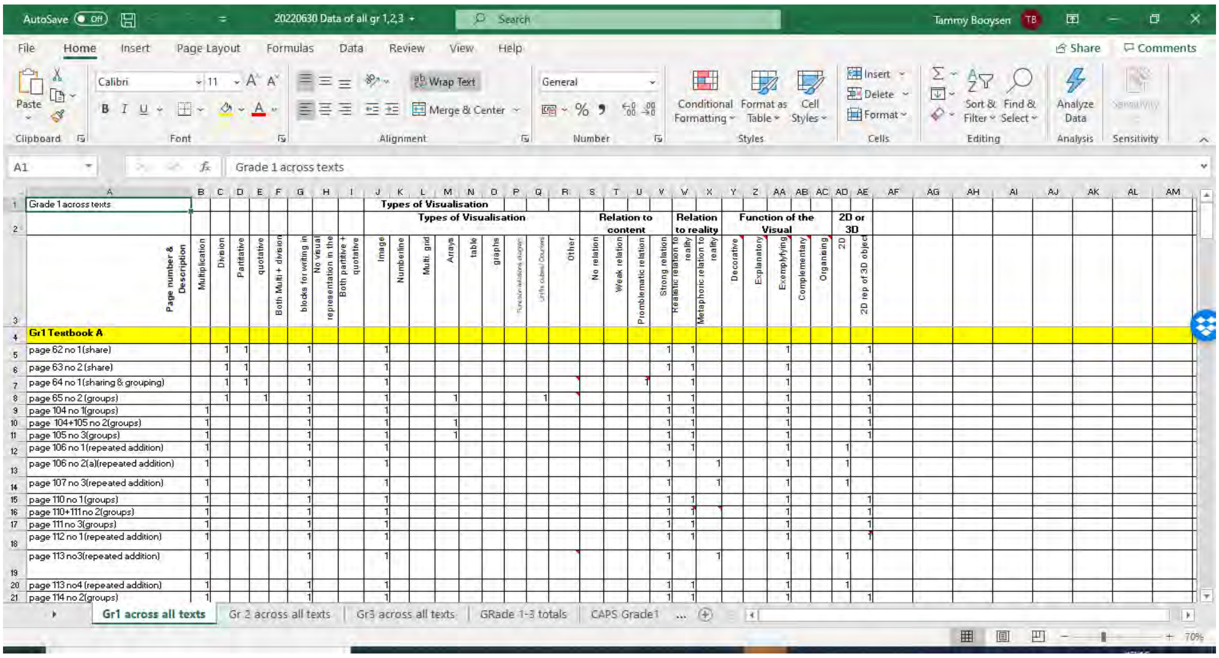


Figure 4.3: An example of the excel spreadsheet used to capture data

For the analysis process, I used deductive reasoning as I started with a set of categories provided by the framework developed by Fotakopoulou and Spiliotopoulou (Section 3.3 in Chapter 3) and my pilot study on a Grade 4 DBE textbook. The categories I used for the qualitative and quantitative analysis of the VRs were the type, the relation to content, the relation to reality and their function (Fotakopoulou & Spiliotopoulou, 2008).

In my research, I sorted the VRs into categories (Creswell & Plano, 2006) by examining the nature of the VRs in the FP texts, for example, the different types of VRs and the purpose of VRs in textbooks and CAPS using an adaptation of the Fotakopoulou and Spiliotopoulou (2008) framework. This analytic framework assisted in understanding and interpreting the VRs used in the whole number multiplication and division exercises.

I prepared, organised and coded the data according to the revised VRF independently and later checked for inter-rater reliability while comparing my own analysis with my supervisor's analysis. Hereafter, we talked through our coding differences and came to a consensus. The qualitative analysis consisted of analysing and classifying the VRs. Figure 4.4, taken from page 43 of the Grade 2 DBE textbook, provides an example of how I used the VRF. The VR in Figure 4.4 has a strong relation to content as its focus is whole number multiplication. It aligns with the expectations of the CAPS for mathematics (DBE, 2011) by linking repeated addition to multiplication and multiplication to division. The circles provide a metaphoric relation to the content and the flags provide a realistic representation. Furthermore, it has an exemplifying function (type a) as the author demonstrates how the problem can be solved using repeated addition, multiplication and division either by using the purple dots in the VR or the flags to get to the answer.

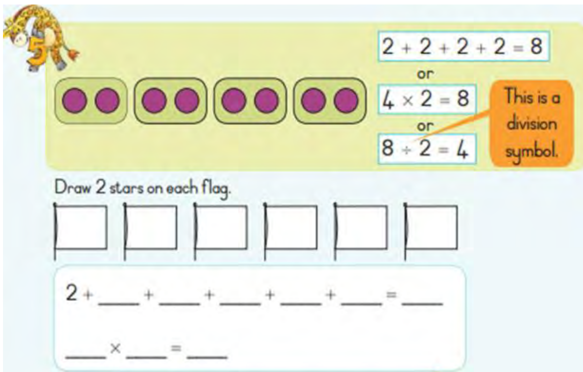


Figure 4.4: An example of a VR from Grade 2 DBE Book 2 (DBE, 2011, p. 43)

During the process of coding and re-coding the data based on the adapted version of the VRF framework, several debates emerged between my supervisors and me about which VRs should be included in the coding. Those led to further re-coding until a consensus was reached.

4.8.1 Types of visual representation

An image is a visual of a person or object such as a photo or shape. A diagram can be described as an outline, sketch or drawing that shows arrangements and relations between something, for example, number lines, multiplication grids, array representations, graphs, tables, function relations diagrams, unifix cubes, counters and beads.

4.8.2 Visual Representations relation to content

Figure 4.5 and 4.6 provide examples of VRs with realistic and metaphoric relations to content.

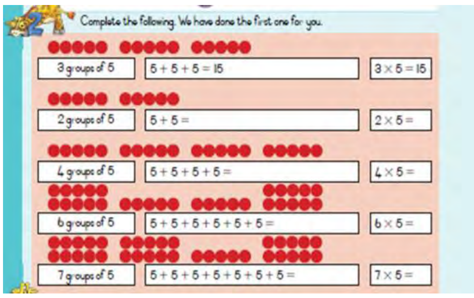


Figure 4.5: A metaphoric relation to content (Text A)

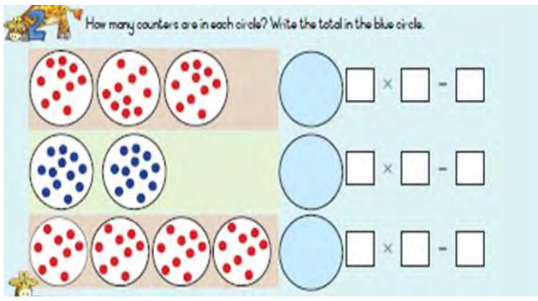


Figure 4.6: A realistic relation to content (Text A)

In Figure 4.5, the red dots are circles and thus have a metaphoric relation to content. In contrast, the text above the VR in Figure 4.6 describes the red and blue dots as counters. This example was coded as having a realistic relation to content as counters are objects that learners are exposed to in the classroom.

The array representation in Figure 4.7 has a problematic (type b) relation to content. It is not clear whether the multiplication sum should focus on the ‘buttons’ or ‘buttonholes’. The learners are likely to focus on ‘the multiplication sum’ when two multiplication sums are possible and not one single answer as the exercise suggests.

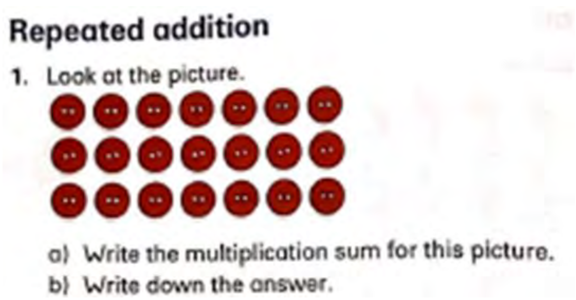


Figure 4.7: An array representation with a problematic (type b) relation to content (Text B)

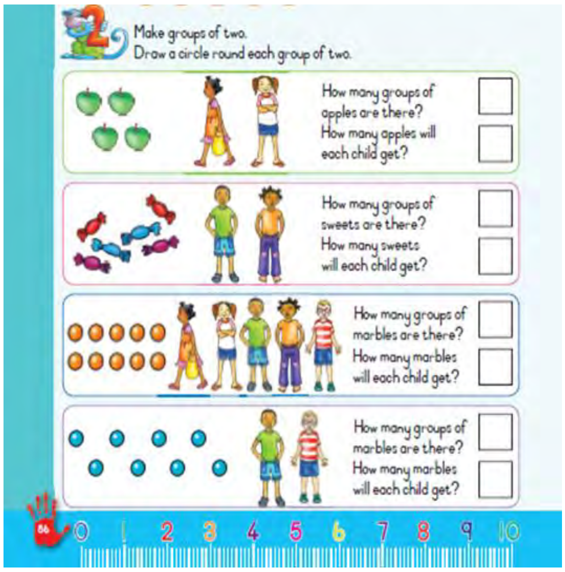


Figure 4.8: An example of a VR that has a problematic (type b) relation to content (Text A)

Figure 4.8 also demonstrates a problematic (type b) relation to content. The instruction suggests that learners make groups of two to ascertain how many groups there are and how many each child will get. The third and fourth rows present as confusing and ambiguous and it is unclear what is intended to serve an explanatory function.

4.8.3 The function of a Visual Representation

We also discussed what makes a VR explanatory. Figure 4.9 is an example of a VR with an explanatory function. The caption on the right explains how one may wish to count the sweets on each table. Figure 4.10 does not have an explanatory function as the VR presented only shows the headings, faces of the learners and the apples. Instead, this VR has an exemplifying function (type b).

Figure 4.9: A VR with an explanatory function (Text A)

Figure 4.10: A VR that does not have an explanatory function (Text A)

An example of a VR with an exemplifying function is included in Figure 4.11, whereas the example in Figure 4.12 does not have an exemplifying function. The first problem in Figure 4.11 no. 1a (monkeys) has an exemplifying function (type a) as it contains a worked example that explains how to solve the problem. Figure 4.11 no. 1b to d have exemplifying functions (type b) as they provide space for the learners to answer the questions.

Figure 4.11: An example of a VR with an exemplifying function

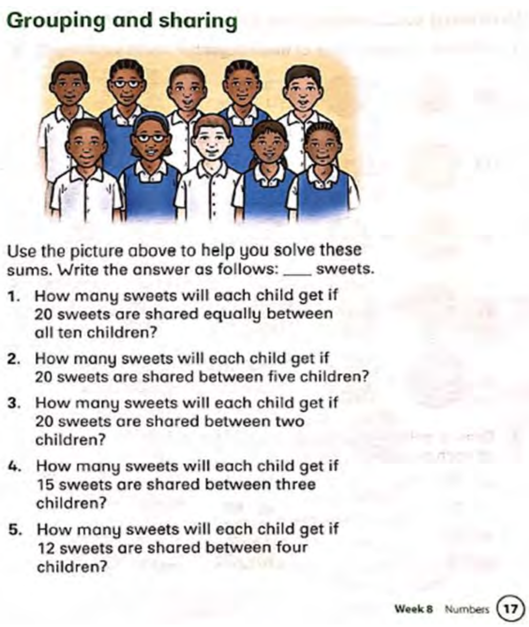


Figure 4.12: A VR that does not have an exemplifying function

Figure 4.12 above does not have an exemplifying function because the VR does not provide an example of the first problem. The VR of the learners in Figure 4.12 has a complementary function even though the wording tells the learner to use the picture (10 children).

Figure 4.13 is an example of a VR with a complementary function. Problem no. 1a may seem like it has an exemplifying function as the picture has the correct number of learners and there is a VR of the crayons that need to be divided amongst the learners. However, there are 11 crayons and not 10 as the word problem suggests.

Figure 4.13: A VR with a complementary function (Text B)

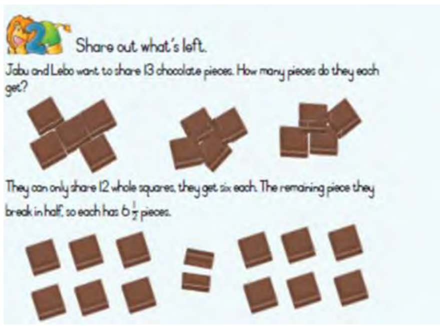


Figure 4.14: A VR that is not complementary (Text A)

Figure 4.14 may seem to have a complementary function as the problem can be solved without the VR that is present. However, the VR has an exemplification function as the pieces of chocolates exemplify the answer.

4.8.4 The dimensionality of Visual Representations

Unlike the VRF of Fotakopoulou and Spiliotopoulou (2008), there were no 3D visual representations for multiplication and division in the textbooks. There were 2D representations and 2D representations of 3D objects.

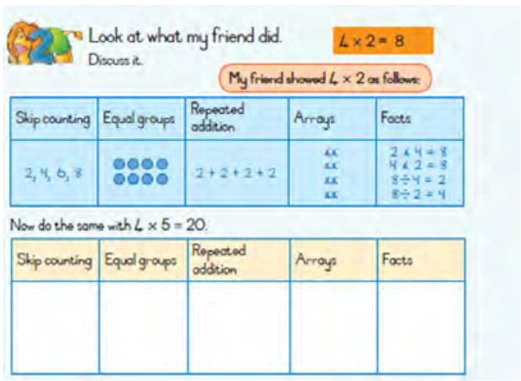


Figure 4.15: An example of a 2D VR

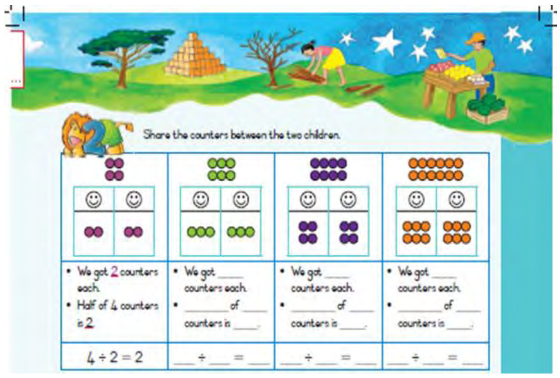


Figure 4.16: A VR that is a 2D representation of a 3D object

The circles in column 2 (equal groups) of Figure 4.15 is an example of a 2D representation. However, the circles in Figure 4.16 (no. 2) are explicitly referred to as counters in the instruction to the learners. Counters are thus coded as 2D representations of 3D objects as learners use them in the classroom.

4.9 Phase 6: Quantification Analysis: Grade 1, 2, and 3 Texts

Thereafter, I conducted the quantitative analysis. In descriptive statistical analysis, the data is presented in the form of tables and graphs. This involved tallying the frequency of the different codes across the textbooks (see Figure 4.3) and the CAPS document. As noted by Cohen et al. (2018), I found the analysis to be a recursive process that involved going back to re-code and re-check my initial analysis. Figure 4.3 provides an example of how I extracted the data from Text A using an Excel spreadsheet. All the data from the three textbooks were recorded on a different sheet according to the grade. In Figure 4.3, the page number of each whole number multiplication and division activity appears in Column A. The rows refer to the codes used from the adapted VRF.

I compared the VRs in the textbooks to the literature on whole number multiplication and division and the FP Mathematics CAPS (see Section2.4 in Chapter 2), specifically in relation to the topics of repeated addition leading to whole number multiplication and repeated subtraction leading to division in the Numbers, Operations and Relationships content area and the VRs used to mediate the concepts. In the CAPS curriculum, there is an explicit link between repeated addition leading to multiplication and repeated subtraction leading to division. This section of the analysis allowed the researcher to see the link between the CAPS curriculum and the exercises in the texts.

The data for this research was captured on an Excel spreadsheet. From the Excel spreadsheets, I developed graphs for each code (see Figure 4.17). I presented the data clearly with a VR of the different codes and textbook series. The purpose of the graphs was to identify the frequency of the VRs. I calculated the frequency by making use of the Excel spreadsheet (see Figure 4.17) and writing a '1' in the appropriate column which allowed me to calculate the total of each column and obtain the total number of VRs in each category. The frequency of the different characteristics of each VR was important as it allowed me to analyse and note which VRs were most or least represented across the nine textbooks and were promoted by the CAPS document.

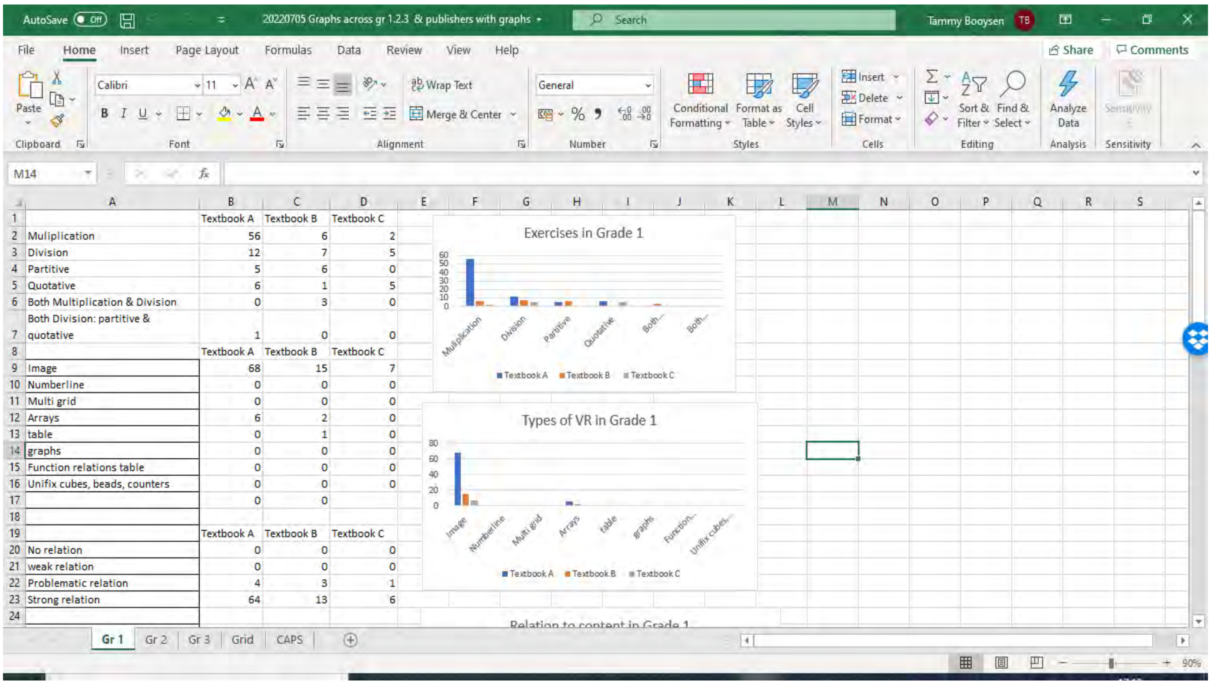


Figure 4.17: An example of an Excel spreadsheet and graphs

4.10 Ethical Considerations

This research proposal was accepted by the Education Higher Degrees Committee. The Ethics in Health Research report (Department of Health, 2015) indicated that a document analysis of documents in the public domain does not require ethical approval. This research relied on publicly available information and no human participants were involved. As a researcher, I paid particular attention to ensuring that I observed the appropriate ethical standards during the analysis and reporting (Bless et al., 2013) in order to ensure no harm to the authors of the texts. I only analysed what was in the text and made a conscientious attempt not to imply or assume anything about the authors of the texts. It is also important to note that I do not have any affiliation with the publishers or authors of the texts, and thus there was no conflict of interest.

4.11 Validity and Trustworthiness of the Study

Validity refers to the extent to which a concept 'measures' what it is said to measure, for example, the features it intends to explain, describe or theorise (Cohen et al., 2018). To ensure the validity of this study, I first piloted the VRF by Fotakopoulou and Spiliotopoulou (2008) with the Grade 4 DBE text for Terms 1 and 2 with my supervisors. This assisted in ascertaining if the VRF was suitable to analyse the FP textbooks to answer the research questions. As such, I drew on the inter-rater reliability to check that a sample of my analysis process was reliable and recognisable. This improved the trustworthiness of this study.

Trustworthiness is when the data has been analysed using the data analysis process and can convince the reader that the findings are of high quality. Trustworthiness incorporates the four concepts of credibility, dependability, transferability and confirmability (Bless et al., 2013). Shenton (2004, p. 64) explains credibility with the question, "how congruent are the findings with reality?" When doing qualitative research, there is an in-depth understanding of what is happening in the research that is being analysed (Bless et al., 2013). Lincoln and Guba (as cited in Shenton, 2004) emphasised that the researcher should provide sufficient contextual information about the methodology used in the study. This promoted thick description that assisted in the transferability and dependability of my study. Confirmability is when researchers show that the results come from the data collected and not the researcher's predispositions (Shenton, 2004).

I analysed the VRs across nine texts (three sets of Grade 1, 2 and 3 texts) from three different publishers; one set was produced by the state and the others were commercially produced. As a researcher, I was aware that this was my subjective view as I did not interview the authors of the texts. To ensure the validity of my analysis I used the same analytical framework across all texts.

Reliability measures the accuracy of the instrument used for the study (Heale & Twycross, 2015). I continuously developed and revisited the Fotakopoulou and Spiliotopoulou (2008) framework as I coded and re-coded the data. As a researcher, I attempted to ensure that the data collection and analysis procedure was as transparent as possible; this enhanced the credibility of the study.

4.12 Concluding Remarks

This chapter provided a rationale for the paradigm used for this research study. It included the sampling strategies and research methodology that was used when collecting and analysing data. This documentation analysis was grounded in an interpretivist paradigm. The next chapter presents the data analysis.

Chapter 5: Data Analysis and Findings

5.1 Introduction

This chapter presents the data analysis and findings of this study using the VRF explained in the previous chapter. This data analysis and findings chapter is divided into four sections. It explores 1) the general layout of each set of texts and the data analysis and findings of Grade 1 texts; 2) the data analysis and findings of Grade 2 texts; 3) the data analysis and findings of Grade 3 texts; and 4) the data analysis and findings of the CAPS curriculum. In addition, it discusses the limitations of the study and future recommendations.

5.2 General Layout of the Texts

Figures 5.1, 5.2 and 5.3 are examples of double pages from Texts A, B and C each from different publishers. The topic for all three double-page spreads is repeated addition leading to multiplication which is an expectation of the CAPS for Grades 1, 2 and 3 (DBE, 2011). The layout is the same in Grades 1–3 for each text. Figure 5.1 provides an example of a double-page for Text A.

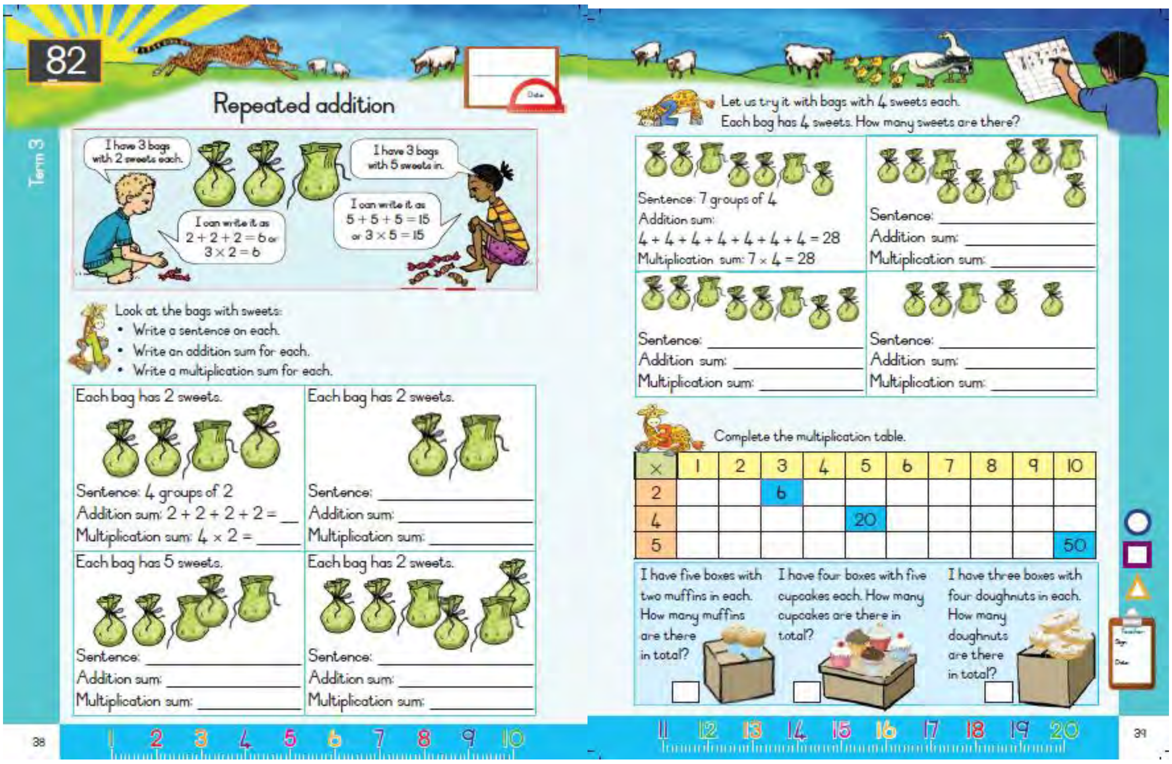


Figure 5.1: A double-page layout of Text A (Grade 2, Text A, Book 2, pp. 38–39)

The design of Text A is the same across all three grades. Text A is bright and colourful and the page borders are populated with pictures. At the top of each page is a picture that bears no relation to the activities. At the bottom of the page is a number line. The number line does not necessarily assist the learners with the activities on the page. There is an activity number (e.g., 82) on the top left-hand side of each double-page and the term (e.g., Term 3) in which the activities should be completed underneath the activity number. Each page has a heading (e.g., Repeated addition) which is most likely to be the topic for the lesson presented by the teacher. The numbering of each exercise is indicated by an animal with a number on it. The Grade 1 book has a monkey, Grade 2 book has a giraffe and the Grade 3 book has a lion. When learners are exposed to these texts with various VRs that do not focus on the activity, it is important that they have good figure-ground perception. In other words, they need to be able to look at the appropriate VRs related to the exercises they are completing and allow everything else on the page to move into the background. As the grades progress the spacing of the content becomes more compact, that is, the activities are closer to one another.

Figure 5.2 provides an example of a double-page spread taken from Text B. The content area for each page in Text B (e.g., numbers) and the topic (e.g., 'repeated addition' and 'grouping and sharing') appear at the top of the double-page spread page. At the bottom of the page, the week in which the learners should complete the exercises is noted. Figure 5.2 shows that repeated addition and grouping and sharing should be done in week 8.

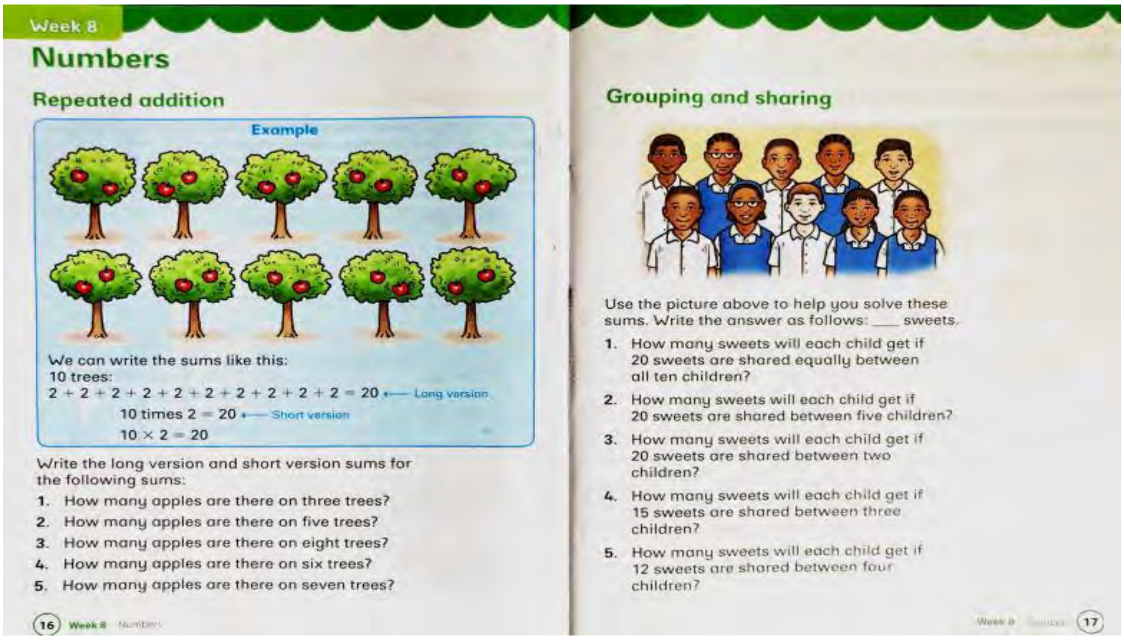
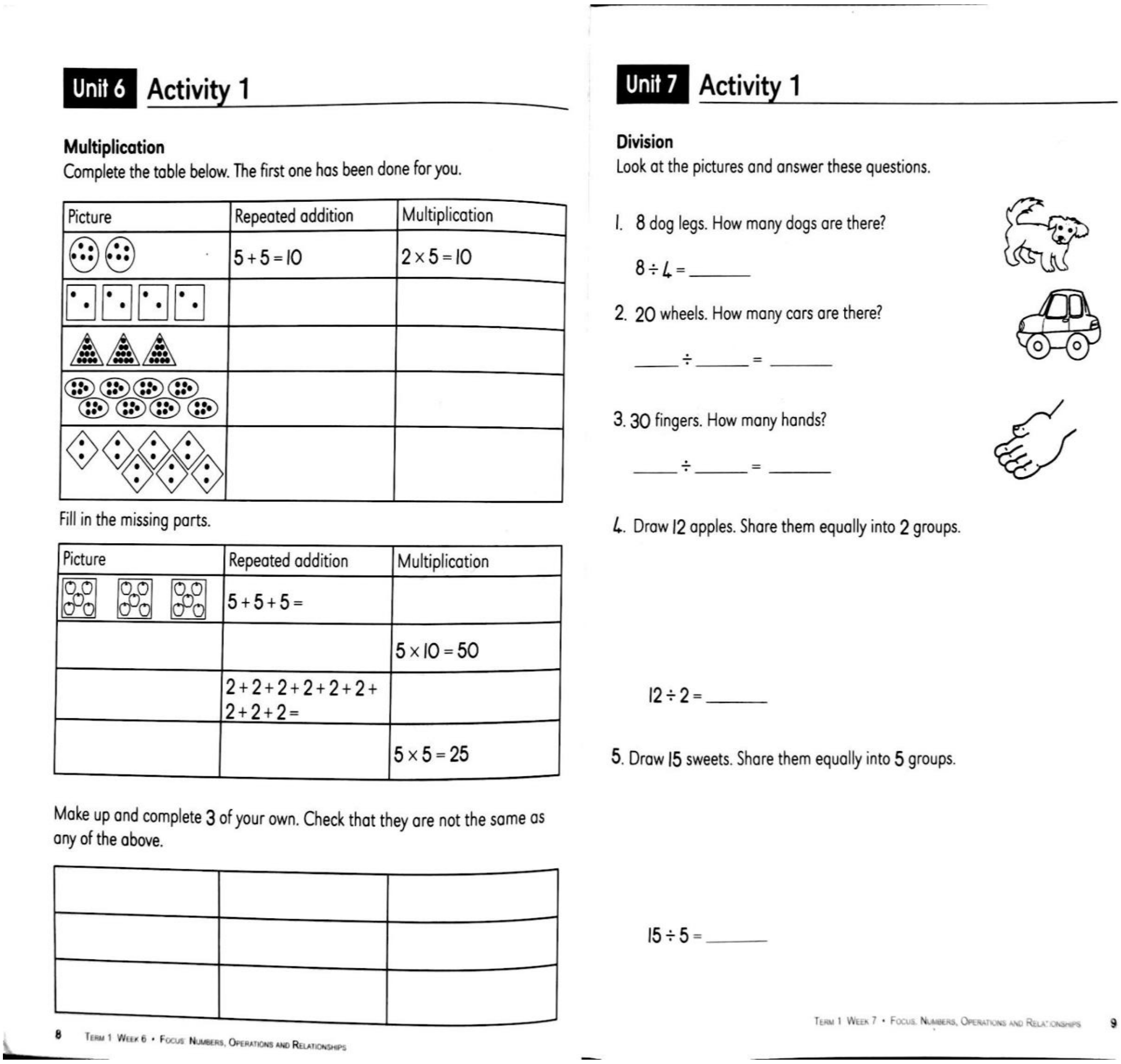


Figure 5.2: A double-page layout of Text B (Grade 2, Text B, pp. 16–17)

Typically, there is a colour VR at the top of each page that relates to the exercises below. This is usually followed by instructions on how to complete the exercise. There are about 5 questions (numbered 1 to 5) on each page. The expectation is that learners do not answer the questions in the text but provide the answers in a separate workbook.

Unlike Texts A and B, Text C is not in colour. In Text C the unit and activity number appear at the top of each page. The topic (e.g., repeated addition leading to multiplication) is written under the unit and activity number. The term and week during which the exercises should be completed and the specific mathematics content area to be addressed appear at the bottom of the page. The example in Figure 5.3 indicates that the content area is 'Numbers, Operations and Relationships', and the exercises should be completed in Week 8 of Term 1.



Unit 6 Activity 1

Multiplication

Complete the table below. The first one has been done for you.

Picture	Repeated addition	Multiplication
	5 + 5 = 10	2 × 5 = 10

Fill in the missing parts.

Picture	Repeated addition	Multiplication
	5 + 5 + 5 =	
		5 × 10 = 50
	2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 =	
		5 × 5 = 25

Make up and complete 3 of your own. Check that they are not the same as any of the above.

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Unit 7 Activity 1

Division

Look at the pictures and answer these questions.

1. 8 dog legs. How many dogs are there?
   8 ÷ 4 = ______
2. 20 wheels. How many cars are there?
   ____ ÷ _____ = ______
3. 30 fingers. How many hands?
   ____ ÷ _____ = ______
4. Draw 12 apples. Share them equally into 2 groups.
   12 ÷ 2 = ______
5. Draw 15 sweets. Share them equally into 5 groups.
   15 ÷ 5 = ______

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Figure 5.3: A double-page layout of Text C (Grade 3, Text C, pp. 8–9)

A single page is dedicated to each lesson. Typically, each exercise has VR on which the questions are based. In Text C it is not clear whether the learners should complete the exercises in the book or in their classwork book. If learners are expected to write it in the book, then it is problematic as the space for the learners to write in the text is small for FP.

5.3 Analysis of the Visual Representations in the Three Texts

For ease of reading I will explain each category in the VRF (as noted in Figure 3.1).

- A VR with a realistic relation to content can be a photo or drawing of an actual object for example a flower (Fotakopoulou & Spiliotopoulou, 2008).
- A VR with a metaphorical relation to content represents something or symbolises meaning (Fotakopoulou & Spiliotopoulou, 2008), such as tallies.
- A VR with a decorative function's direct purpose is to make the page aesthetically pleasing (Fotakopoulou & Spiliotopoulou, 2008).
- A VR that has an explanatory function assists in explaining the concept (Fotakopoulou & Spiliotopoulou, 2008).
- A VR with an exemplifying function (type a) which consists of worked examples of the concepts that relate to the written language in the texts (Fotakopoulou & Spiliotopoulou, 2008).
- A VR with an exemplifying function (type b) requires the learner to complete the activity themselves (Fotakopoulou & Spiliotopoulou, 2008).
- A complementary VR complements a definition of the concept on a page and provides extra information for the activity but does not necessarily support the learning (Fotakopoulou & Spiliotopoulou, 2008).
- A VR with an organising function looks at the layout of a visual representation that will help organise information for the viewer to make sense of the thought process when looking at the visual representation.

5.3.1 Analysis of Text A: Grade 1

The following section presents data that has emerged from Grade 1, Books 1 and 2 of Text A. Text A has 56 (82%) multiplication and 12 (18%) division VRs (see Figure 5.4). The division exercises include 5 partitive, 6 quotative and 1 combined partitive and quotative division example as shown graphically in Figure 5.4.

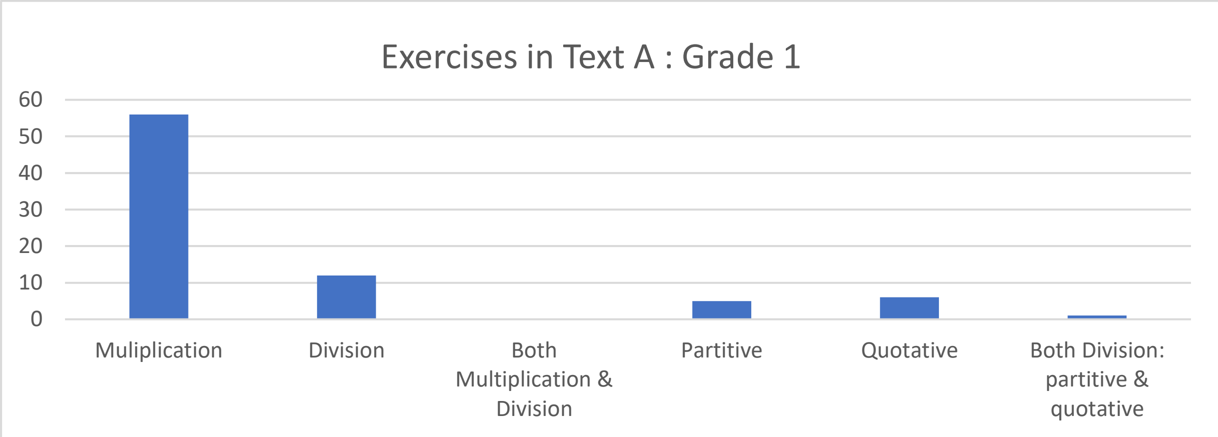


Figure 5.4: Multiplication and division exercises (Text A)

5.3.1.1 Type of visual representations

The most prominent type of VR in Text A are images (68) (92%), followed by array representations (6) (8%).

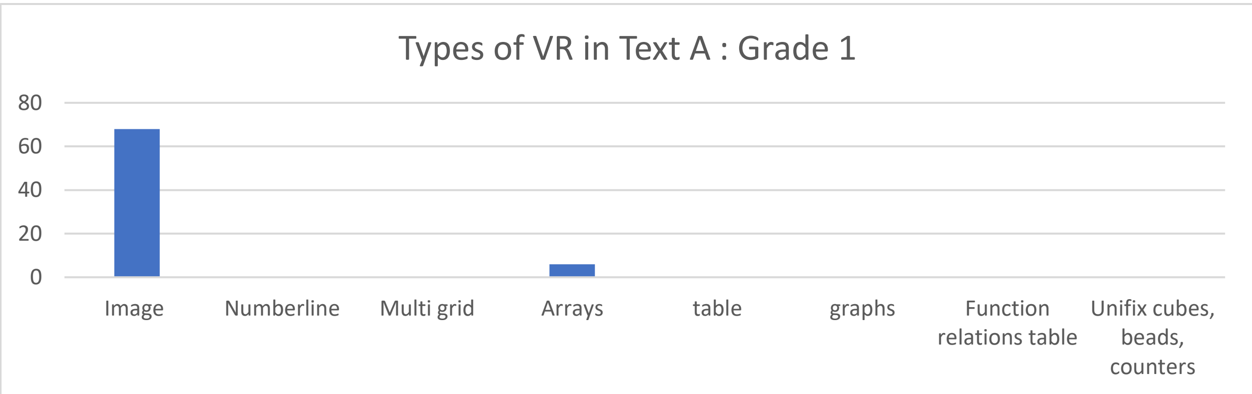


Figure 5.5: Types of visual representation (Text A)

An image is a picture of an object. For example, the exercises in Figure 5.6 depict an image of 2 monkeys, 3 bananas, 2 rabbits and 4 carrots. As noted above, the second most prominent VRs in Text A is an array. Figure 5.7 is an example of an array with 3 rows and 5 columns organised in a rectangular formation.

	
Figure 5.6: An example of an image (Grade 1, Text A, Book 1, p. 64)	***Figure 5.7: An example of an array representation (Grade 1, Text A, Book 2, p. 115)***

5.3.1.2 The visual representations' relation to the content

Figure 5.8 presents the extent to which the VRs relate to the content in Text A. There are 64 (94%) VRs that have a strong relation to the content and 2 (3%) VRs that have problematic (type a) and 2 (3%) VRs that have a problematic (type b) relation to content which are unnecessarily constraining. A VR with a problematic (type a) function is a VR that has an error (see Figure 5.10). A problematic (type b) function has unnecessary constraints (see Figure 5.11).

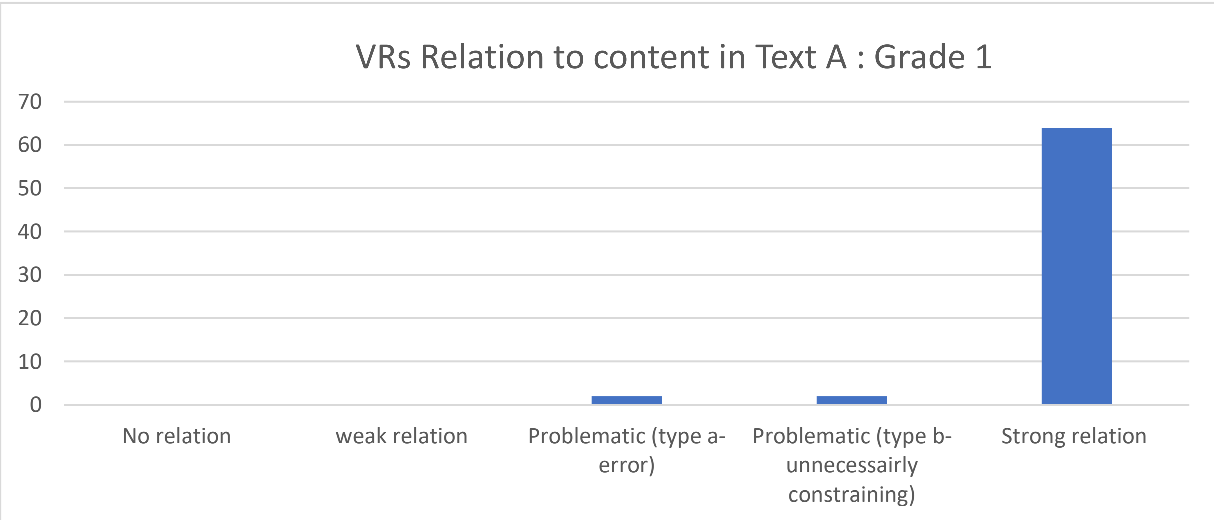


Figure 5.8: Relation to content (Text A)

Figure 5.9 is an example of a VR (i.e., the sweets) with a strong relation to content. In exercise no. 2A, the learner is required to form 1 group of 5. The structure of the VR assists the learner in forming 1 group of 5.

Figure 5.9: An example of a strong relation to content (Grade 1, Text A, Book 1, p. 118)

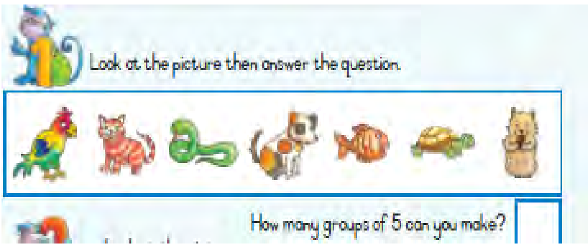


Figure 5.10: An example of a problematic (type a) relation to content (Grade 1, Text A, Book 2, p. 32)

Figure 5.11: An example of a VR with a problematic relation to content (type b) (Grade 1, Text A, Book 1, p. 118)

Figure 5.10 is an example of a VR with a problematic (type a) relation to content. The learner is supposed to look at the VRs and see how many groups of 5 they may create. In the VR, the animals are a mixture of those that are kept as pets and other animals. In addition, the last VR in the row of animals is an error as all the animals in the space are animals we keep as pets. However, it is difficult to see what type of animal this is. When doing the activity, the learners may only circle the animals we keep as pets.

Figure 5.11 is an example of a VR (i.e., the socks) with a problematic (type b) relation to content. While the instruction requires the learners to draw circles around the socks to make 2 groups of 5, the socks are organised into groups of 2. Such an image may confuse the learners as they could create 5 groups of 2 instead of 2 groups of 5.

5.3.1.3 The visual representations' relation to reality

The majority of VRs in Text A have a realistic relation to reality. There are 45 (66%) VRs in Text A with a realistic relation to reality and 22 (34%) VRs with a metaphoric relation to reality as seen in Figure 5.11.

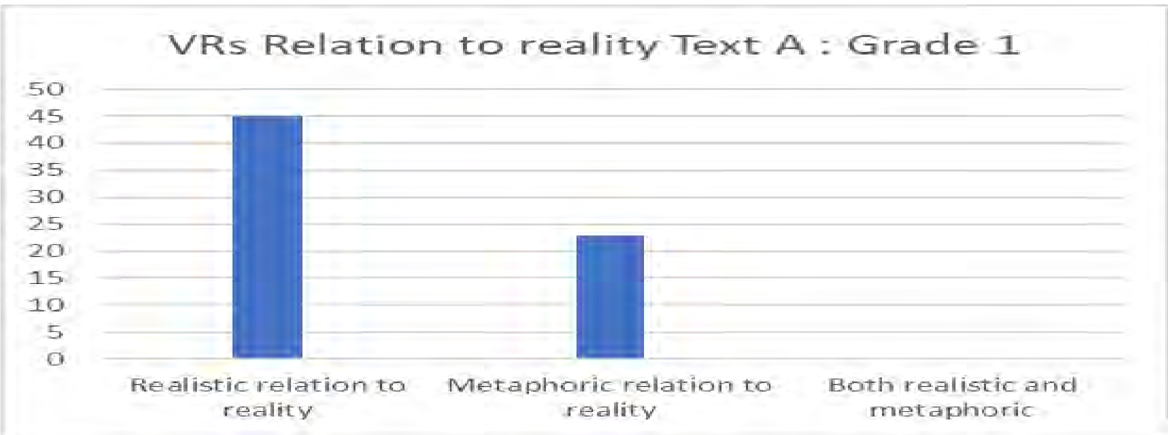


Figure 5.12: Relation to reality Grade 1 (Text A)

The VRs in Figure 5.13 have a realistic relation to reality. The image of the cupcake is a 2D representation of a 3D object. As such it is a VR that is categorised as 'real'. In contrast, Figure 5.14 is an example of a VR with a metaphoric relation to content. The circles are metaphoric as they are not representations of real objects. As noted in Chapter 4, there are instances where the circles could be regarded as having a realistic relation to content. This is in a situation where the instruction refers to the circles as counters.

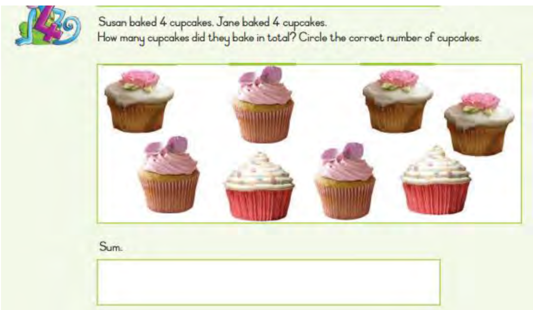


Figure 5.13: An example of a VR with a realistic relation to reality (Grade 1, Text A, Book 1, p. 117)

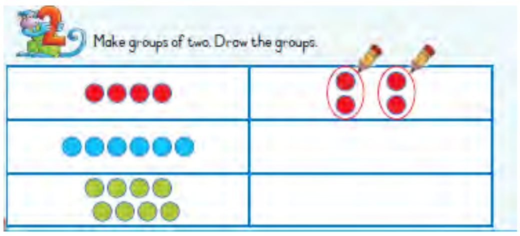


Figure 5.14: An example of a VR with a metaphoric relation to reality (Grade 1, Text A, Book 2, p.106)

5.3.1.4 The function of the visual representations

In Text A there are 67 VRs that have an exemplifying function (type b). That is, a VR of a worked example where the answer is provided. There are 27 VRs with an exemplifying function (type a). That is, a VR that provides a space for learners to do the example. Twenty-seven of the VRs provide worked examples (i.e., exemplifying function (type a)) (see Figure

5.15) and 67 of the VRs provided a space for the learners to do the example (i.e., exemplifying function (type b)) (see Figure 5.16).

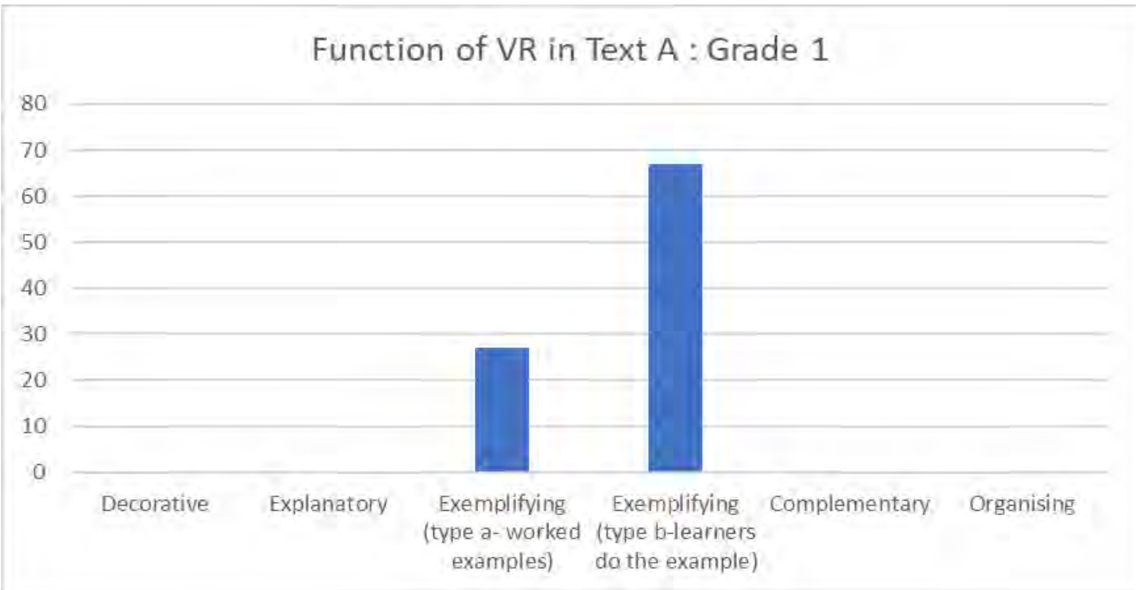
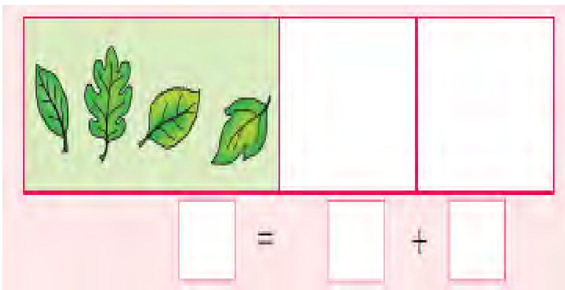


Figure 5.15: Function of VRs (Text A)

Figure 5.16 is an example of a VR that has an exemplifying function (type a) with a worked example. The sum is completed for the learners and the bears provide the explanation for the sum. Figure 5.17 is an exemplifying function (type b). In Figure 5.17, the exercise requires the learners to share the 4 leaves between the two boxes. The sum has not been completed.

Figure 5.16: An example of a VR with exemplifying function (type a) (Grade 1, Text A, book 1, p. 63)



5.17: An example of a VR with exemplifying function (type b) (Grade 1, Text A, book 1, p. 63)

5.3.1.5 The dimensionality of visual representations

Twenty-three (34%) VRs are 2D representations and 44 (66%) VRs are 2D representations of a 3D object (see Figure 5.17). The circles in Figure 5.14 are an example of a 2D representation, while the bears in Figure 5.15 are an example of a 2D representation of a 3D object.

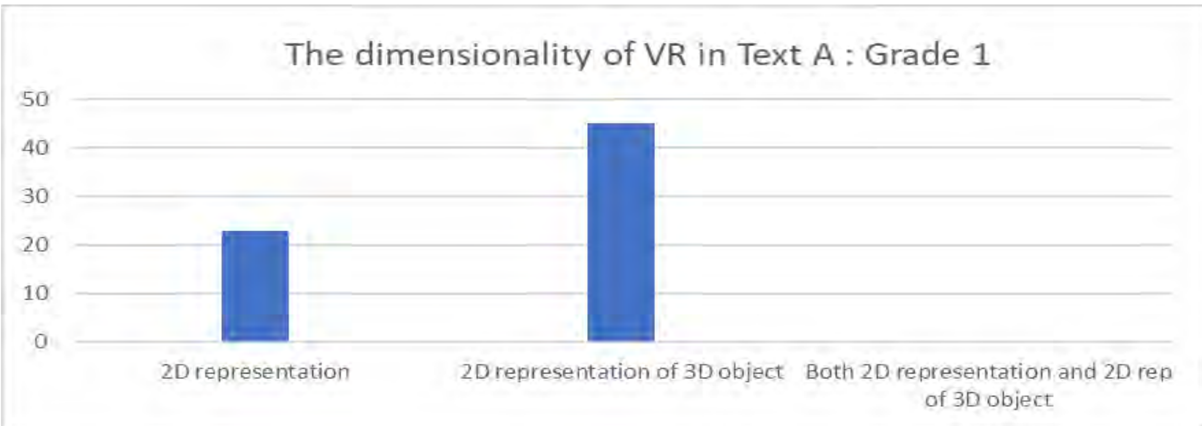


Figure 5.18: The dimensionality of VRs (Text A)

5.3.2 Analysis of Text B: Grade 1

There are 6 (37.5%) multiplication exercises and 7 (43.75%) division exercises in Text B (Figure 5.19). Of the division exercises in Figure 5.19, 6 focus on partitive division and 1 on quotative division. There are 3 (18.75%) VRs that draw the learners' attention to the relationship between multiplication and division. In other words, they relate to questions that focus on both multiplication and division.

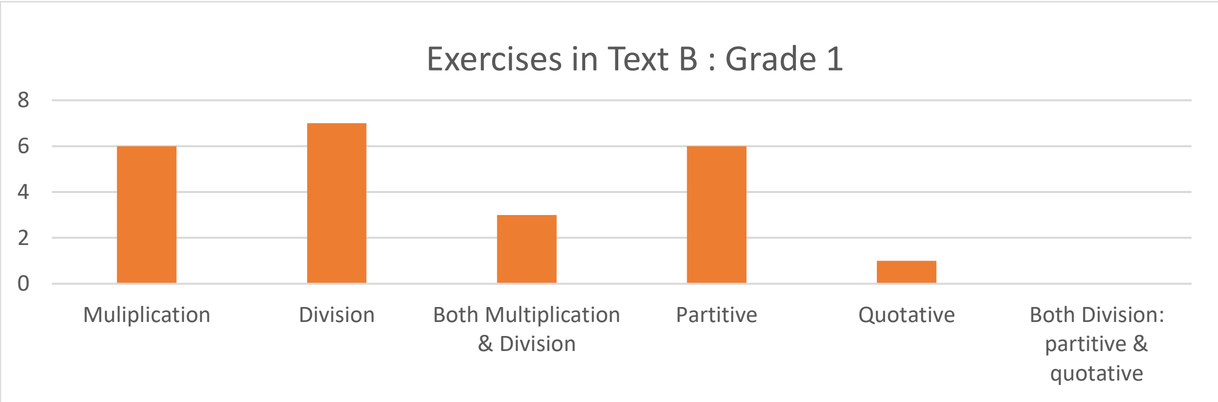


Figure 5.19: Multiplication and division exercises (Text B)

5.3.2.1 Type of visual representation

In this Grade 1 text, the most prominent type of VRs is images (15) (83%), followed by arrays (2) (11%) and tables (1) (6%) as indicated in Figure 5.20. The VRs that are categorised as an image, array representation and table appear in Figures 5.21, 5.22 and 5.23 respectively.

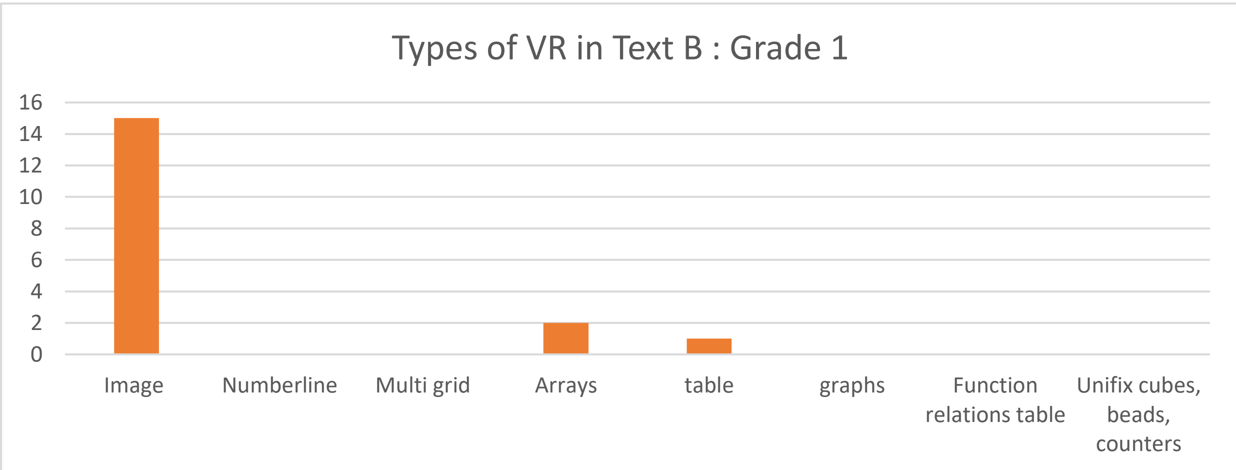
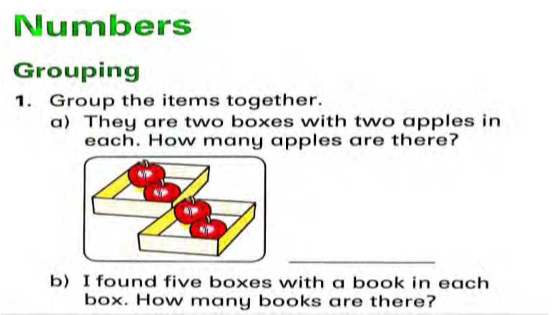


Figure 5.20: Type of VRs (Text B)

In Figure 5.21 the image of boxes with apples serves to assist the learners to “group the items together”. The exercise in Figure 5.22 requires the learners to “use your counters to solve these word problems”. The array diagram on the right-hand side is an example of a VR that can be used to solve word problems as it represents an array that consists of 20 trees planted in four rows with five trees in each row. The table in Figure 5.23 is the only example of a VR in the form of a table. The learners are expected to calculate the number of eggs in 1 nest, 2 nests, 3 nests and so forth.

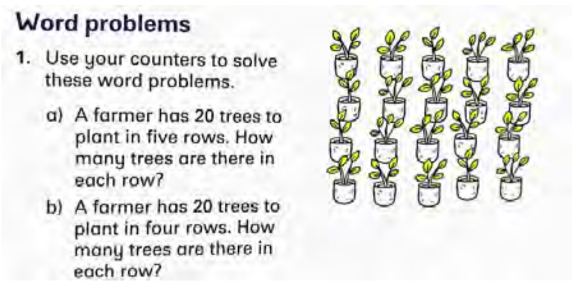


Numbers

Grouping

1. Group the items together.
   a) They are two boxes with two apples in each. How many apples are there?
   b) I found five boxes with a book in each box. How many books are there?

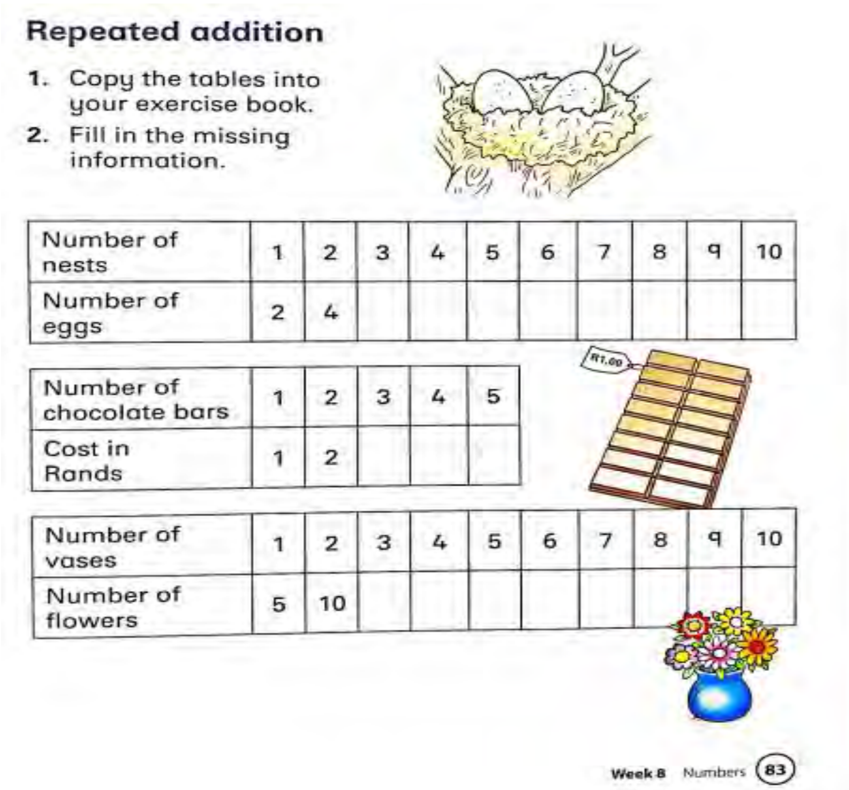
Figure 5.21: An example of an image (Grade 1, Text B, p. 18)



Word problems

1. Use your counters to solve these word problems.
   a) A farmer has 20 trees to plant in five rows. How many trees are there in each row?
   b) A farmer has 20 trees to plant in four rows. How many trees are there in each row?

Figure 5.22: An example of array representation (Grade 1, Text B, p. 85)



Repeated addition

1. Copy the tables into your exercise book.
2. Fill in the missing information.

Number of nests	1	2	3	4	5	6	7	8	9	10
Number of eggs	2	4								

Number of chocolate bars	1	2	3	4	5
Cost in Rands	1	2			

Number of vases	1	2	3	4	5	6	7	8	9	10
Number of flowers	5	10								

Figure 5.23: An example of a table (Grade 1, Text B, p. 83)

5.3.2.2 The visual representations' relation to content

There are 13 (81%) VRs in Text B that have a strong relation to content and 3 (19%) VRs that have a problematic relation to content (type a) (Figure 5.24). This means that the VRs have errors in them making the task potentially confusing for the learners (Figure 5.26).

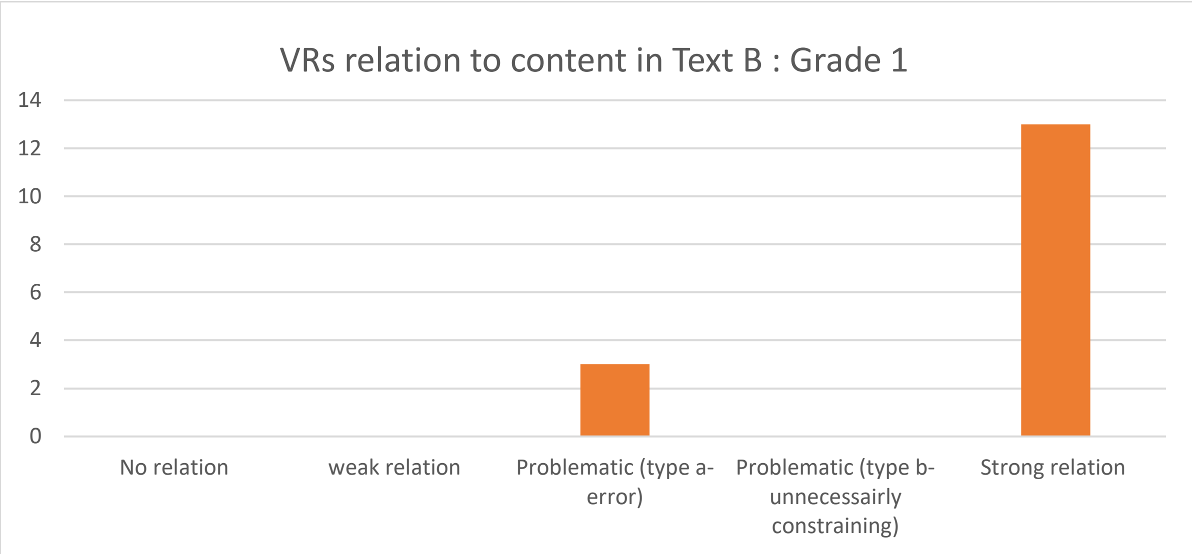


Figure 5.24: Relation to content (Text B)

Figure 5.24 is an example of a VR that has a strong relation to content. The mangoes relate directly to the word problem as the learners are required to estimate and count them before dividing them into groups of various sizes. By contrast, Figure 5.25 reflects a VR with a problematic relation to content (type a). In the first question, the learners are required to solve the following word problem: "There are five learners and 10 crayons. How many crayons does each learner get?" While the VR at the top of the page is an image of 5 learners, there are 11 crayons on the right-hand side of the page. In this case, the image of the crayons may be problematic (type a). While the learners may think they can use the crayons to solve the word problems, they do not provide the necessary support as the number of crayons differs from that in the word problems.

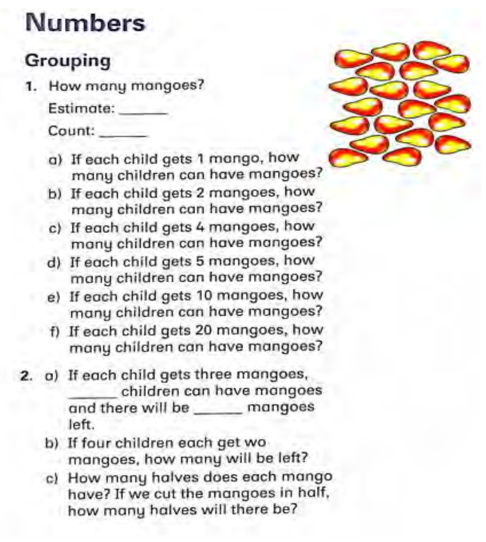


Figure 5.25: An example of a VR with a strong relation to content (Grade 1, Text B, p. 84)

Figure 5.26: An example of a problematic (type a) relation to content (Grade 1, Text B, p. 62)

5.3.2.3 The visual representations' relation to reality

In Text B, all but one of the VRs have a realistic relation to reality (15) (94%) (see Figure 5.27). The VRs in Figures 5.25 and 5.26 have a realistic relation to reality as they are examples of real-life objects (i.e., mangoes, learners and crayons). The table in Figure 5.23 is the only VR in Text B with a metaphoric relation to reality.

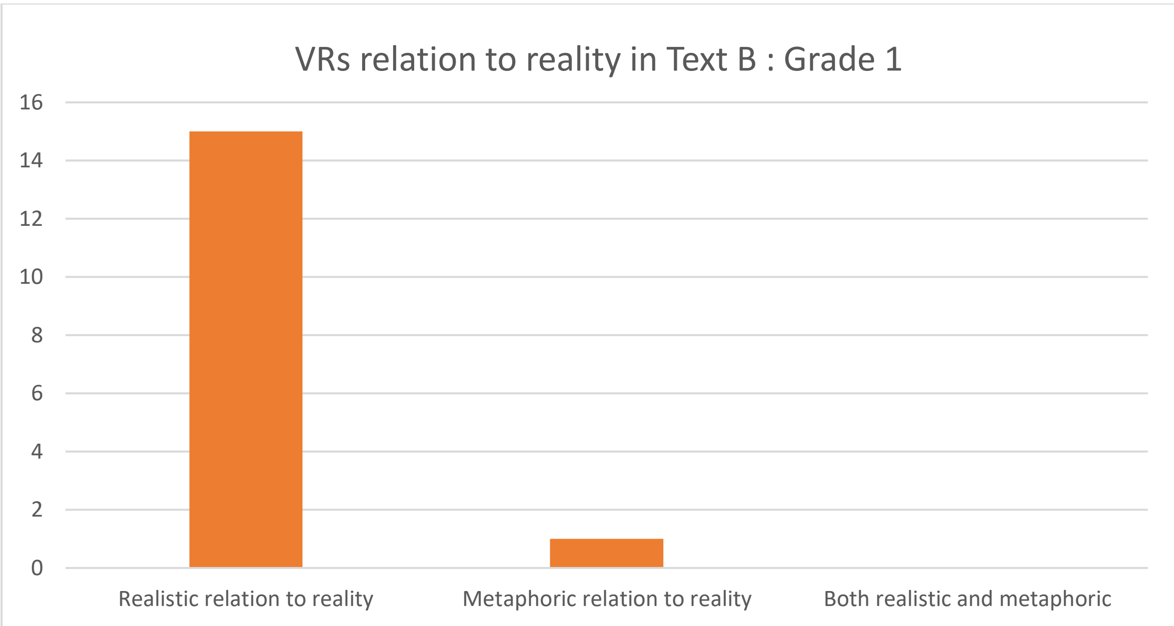


Figure 5.27: Relation to reality (Text B)

5.3.2.4 The function of the visual representations

The most prominent function of the VRs in this Grade 1 text is an exemplifying function (type b) (10) (62%), followed by a complementary function (6) (38%) (see Figure 5.28). An exemplifying function (type b) is when the learners write the sum in their books and there is no worked example for them.

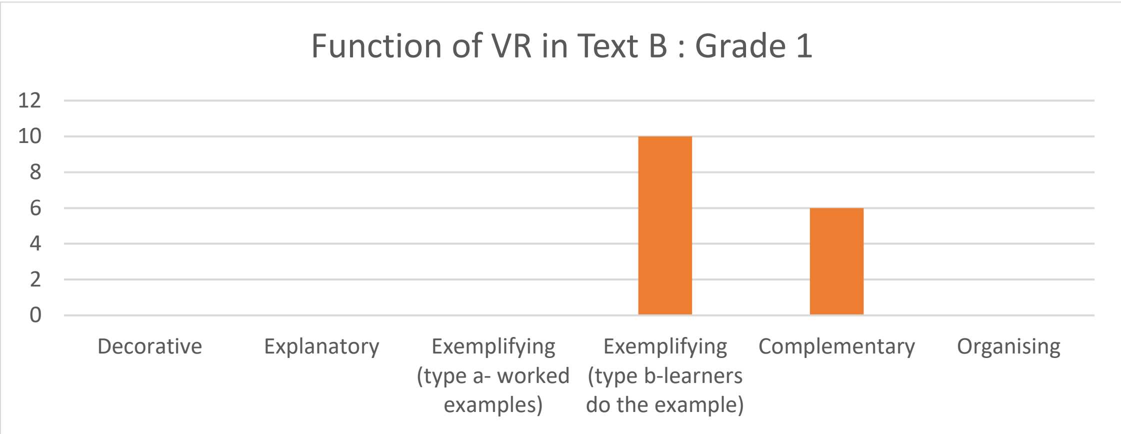


Figure 5.28: Functions of VRs (Text B)

An exemplifying function (type b) provides a space where the learner needs to do the sum. An exemplifying function (type b) presents an example that the learners need to complete (see Figure 5.29) as there is no worked example. In this example, the cakes on the table are shared between the two learners. The word problem in 1(a) requires that four cakes be shared between two learners and the word problem in 1(b) requires that five cakes be shared between the two learners. The VRs support the learners in solving the word problems.

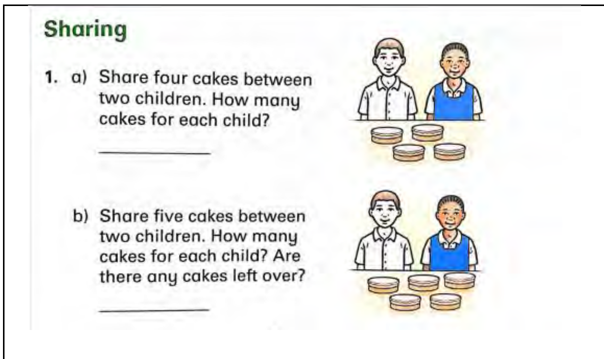


Figure 5.29: An example of an exemplifying function (type b) (Grade 1, Text B, p. 19)

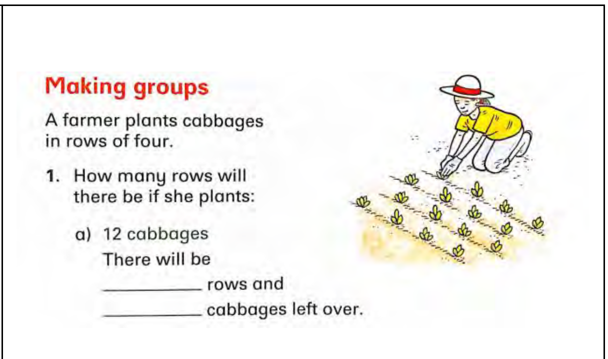


Figure 5.30: An example of a VR with a complementary function (Grade 1, Text B, p. 63)

Figure 5.30 is an example of a VR with a complementary function. The number of cabbages in Figure 5.30 does not correspond with the number of cabbages in the word problem. The VR provides context but does not assist in solving the word problem.

5.3.2.5 The dimensionality of visual representation

There is 1 (6%) VR in Text B that is a 2D representation and 15 (94%) VRs that are 2D representations of 3D objects (see Figure 5.31). The table in Figure 5.23 is a 2D representation. In Figure 5.29 the learners and the cakes provide an example of a VR that is a 2D representation of a 3D object.

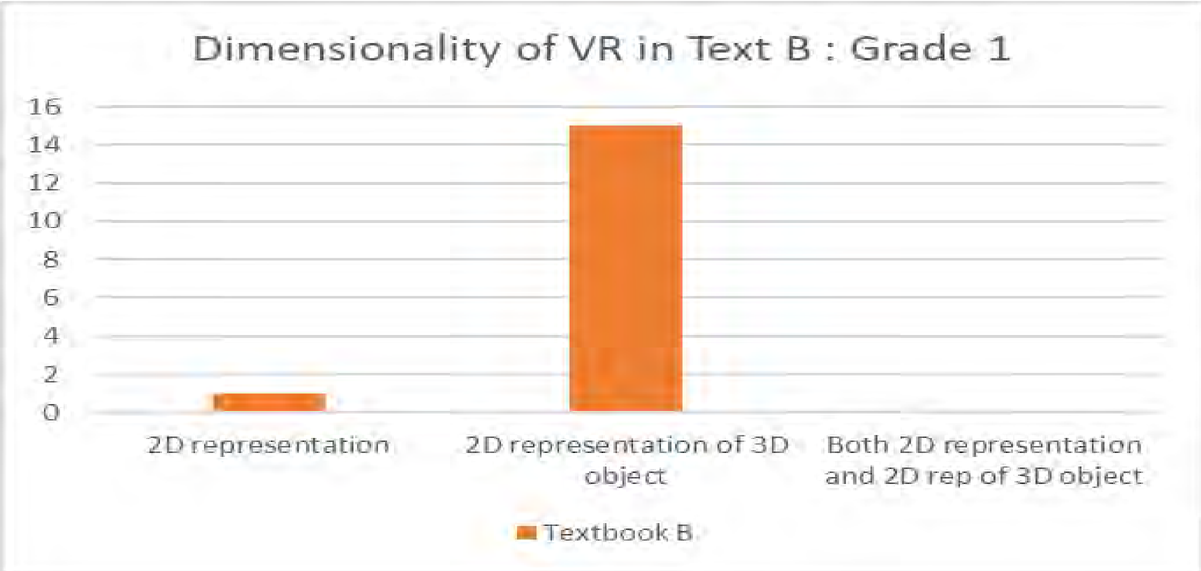


Figure 5.31: 2D representation and 2D representation of 3D object (Text B)

5.3.3 Analysis of Text C: Grade 1

The following section presents the data from Text C. There are 2 (29%) multiplication exercises and 5 (71%) division exercises in Text C (Figure 5.32). The type of division exercises (5) in Text C are all quotative as seen in Figure 5.29. There are no exercises that contain both multiplication and division.

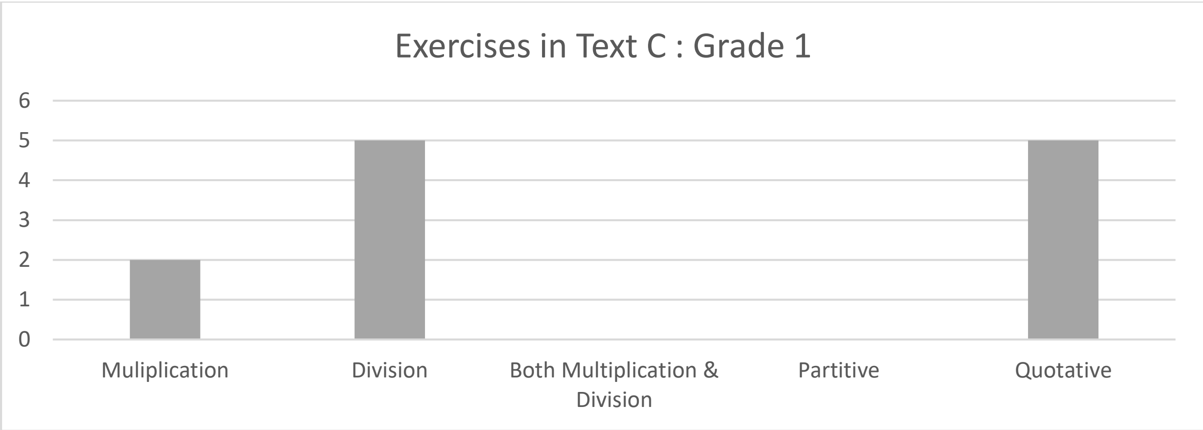


Figure 5.32: VRs of multiplication and division exercises (Text C)

5.3.3.1 Type of visual representation

All 7 (100%) of the VRs related to multiplication and division in Text C are images as noted in Figure 5.33 below.

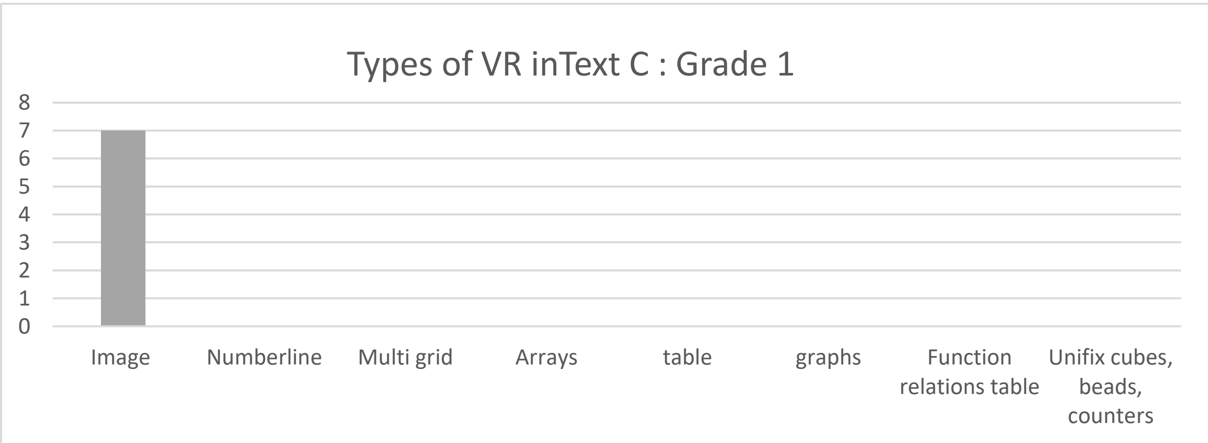


Figure 5.33: Types of VRs (Text C)

In Figures 5.34 and 5.35, the elephants, mice and cats are classified as images.

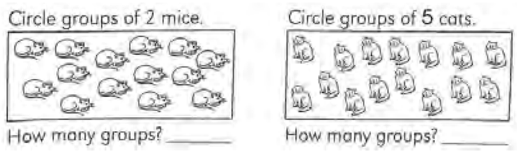


Figure 5.34: An example of an image (Grade 1, Text C, p. 121)

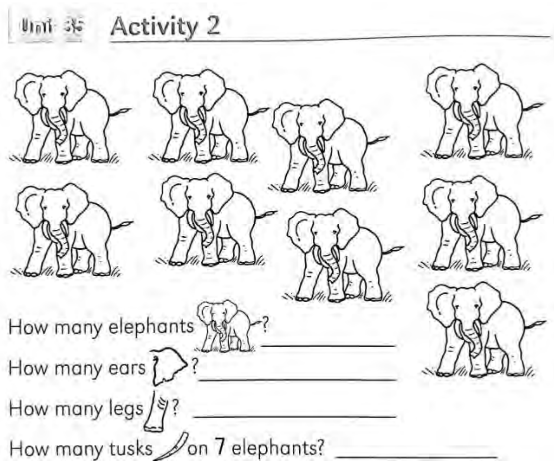


Figure 5.35: An example of a VR with a strong relation to content (Grade 1, Text C, p. 138)

5.3.3.2 The visual representations' relation to content

In Text C, all 7 (100%) VRs have a strong relation to content as indicated in Figure 5.36.

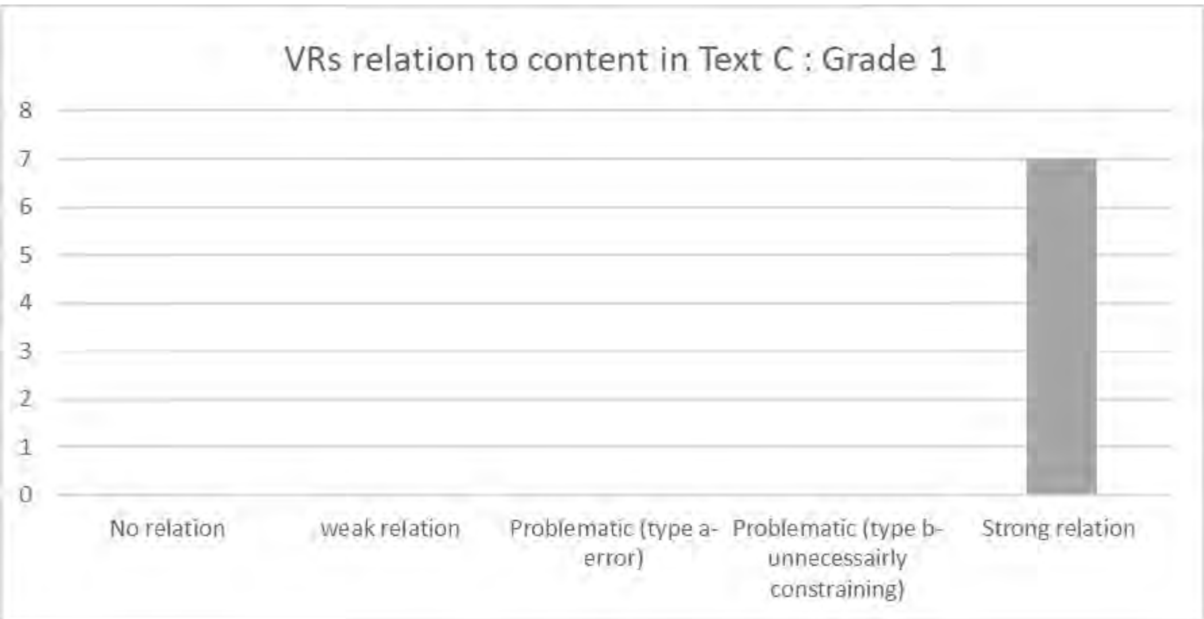


Figure 5.36: Relation to content (Text C)

Figure 5.35 is an example of a VR that has a strong relation to content because the learners can use the elephants in the picture to answer the questions. In this way, the VR supports the use of repeated addition or multiplication to answer the questions.

5.3.3.3 The visual representations' relation to reality

A VR can either have a realistic relation to reality or a metaphoric relation to reality (see Section 3.3.3 in Chapter 3). The bar graph in Figure 5.37 shows that all 7 (100%) of the VRs in Text C have a realistic relation to reality.

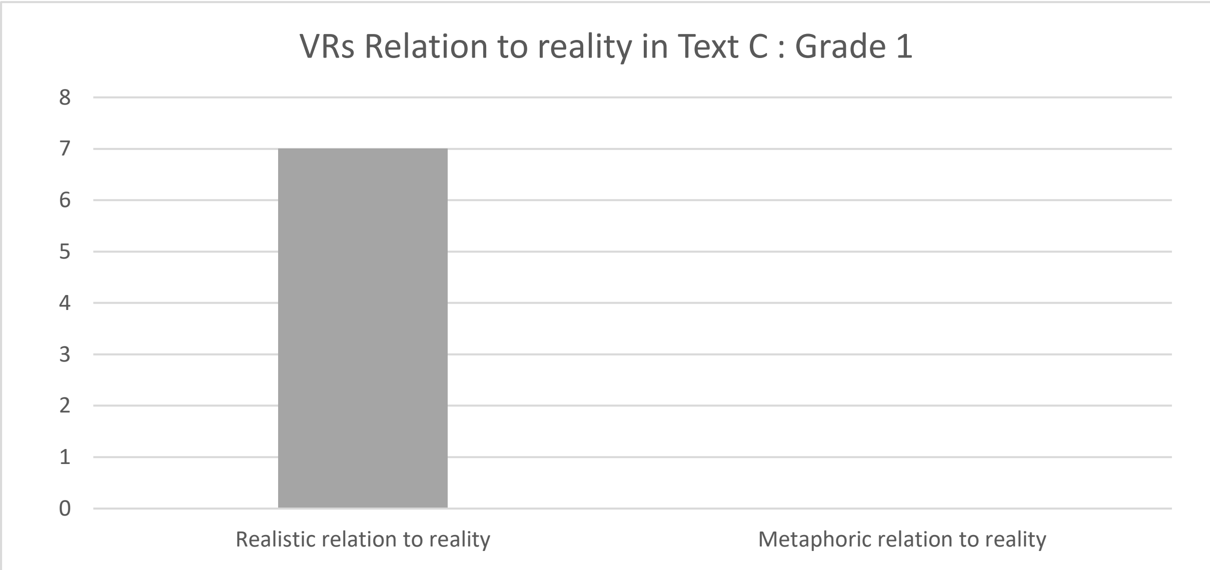


Figure 5.37: Relation to reality (Text C)

The VR in Figure 5.38 has a realistic relation to reality as the VR of the birds looks real.

Figure 5.38: An example of a VR with a realistic relation to reality (Grade 1, Text C, p. 121)

5.3.3.4 The function of the visual representations

In this Grade 1 text, all 7 (100%) VRs have an exemplifying function (type b) as seen in Figure 5.39. The learners are expected to complete the exercises themselves by using the VRs to guide them.

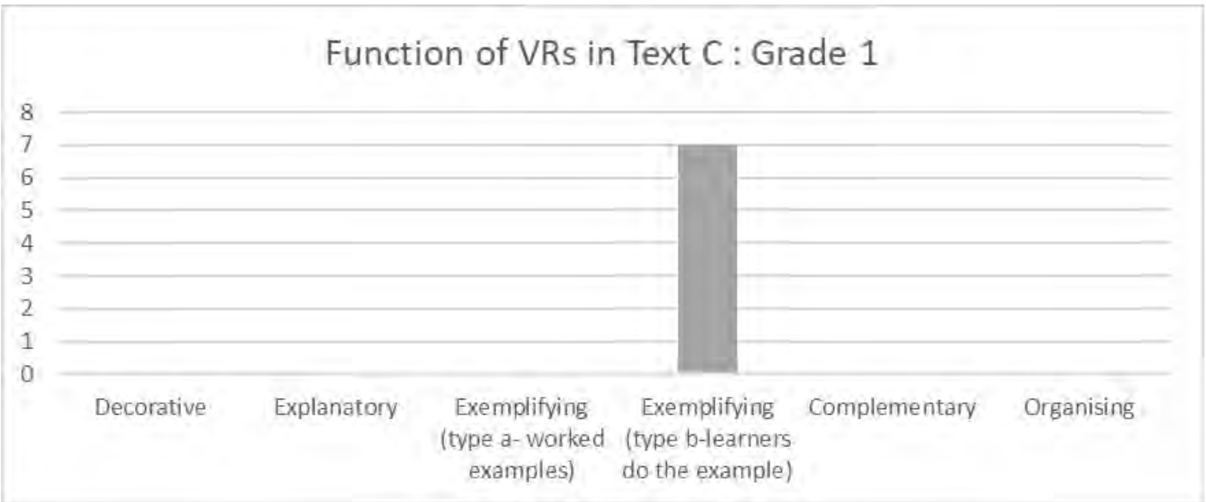


Figure 5.39: Function of VRs (Text C)

The VR in Figure 5.40 below illustrates an exemplifying function (type b) where the learners are required to make groups of two using the socks and trucks.

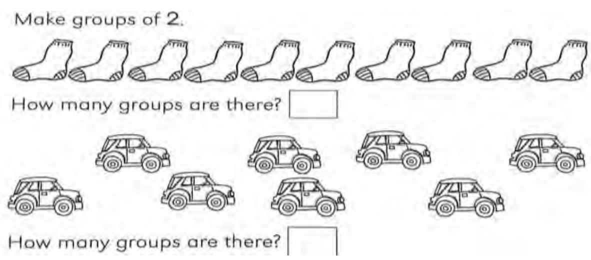


Figure 5.40: An example of an exemplifying function (type b) (Grade 1, Text C, p. 67)

5.3.3.5 The dimensionality of VRs

There are 7 (100%) VRs that are 2D representations of 3D objects such as the mice and cats as seen in Figure 5.33, the elephants in Figure 5.34, the birds in Figure 5.38 and the socks and cars in Figure 5.40.

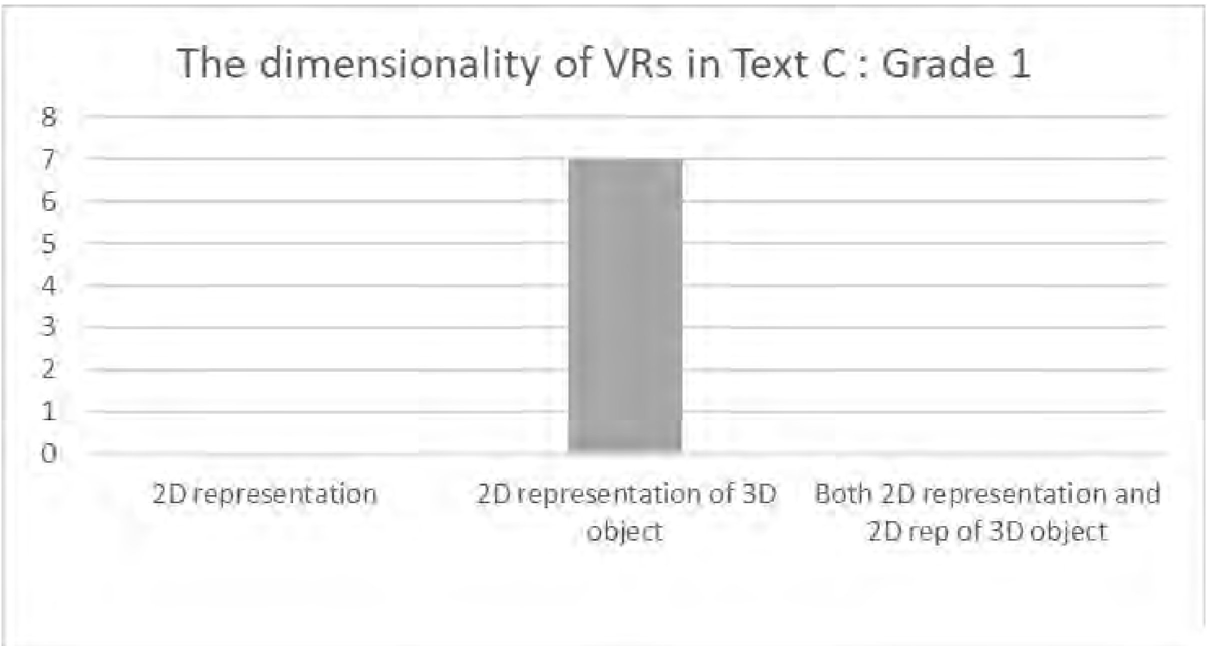


Figure 5.41: The dimensionality of VRs

5.4 Data Analysis Across the Three Grade 1 Texts

This section presents the analysis of the VRs across Text A, B and C in Grades 1 in relation to the multiplication and division exercises (see Figure 5.42). Across the Grade 1 texts, there are 64 (72%) VRs that only focus on multiplication exercises and 24 (27%) that only centre on division exercises. The division exercises consist of 11 partitive, 12 quotative and three exercises with both partitive and quotative division exercises. There are 3 (3%) VRs that consist of both division and multiplication exercises.

As evident in Figure 5.42, Text A has significantly more multiplication exercises (56) (82%) than Text B (6) (9%) and C (2) (3%). While there are few division exercises across the three texts, Text A has the most exercises (12) (50%) with a similar number of partitive and quotative examples. Text B contains 7 (29%) division exercises of which 6 are partitive and 1 quotative. Text C only has 5 (21%) quotative division exercises. Text A has an example of an exercise that includes both partitive and quotative division, and Text B includes 3 exercises where learners are encouraged to explore the relationship between multiplication and division.

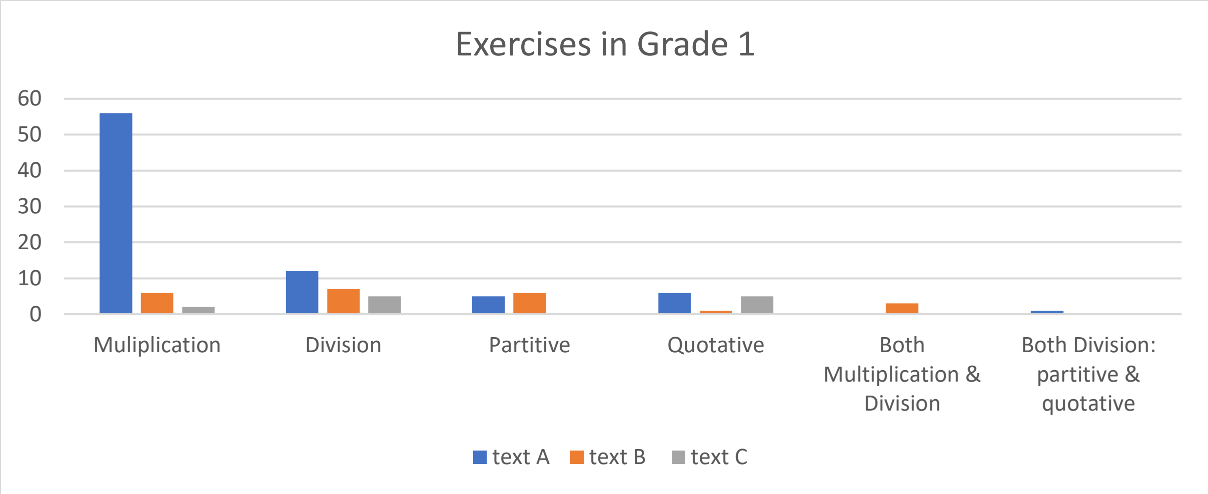


Figure 5.42: Multiplication and division exercises in Grade 1 texts

The most frequent VRs across the Grade 1 texts are images (90), followed by array representations (8), and a table (1) (Figure 5.42). Of the 90 images (100%) across all three texts, Text A has 68 (76%) images, Text B has 15 (17%) and Text C has 7 (8%) images. Text A has 53 more images than Text B and 61 more than Text C. Texts A and B both have arrays and Text B is the only text that has a VR in the form of a table. Worth noting is that Text C only has VRs that are images. The absence of an array is of interest given the emphasis on this in the literature as a powerful VR for supporting MR.

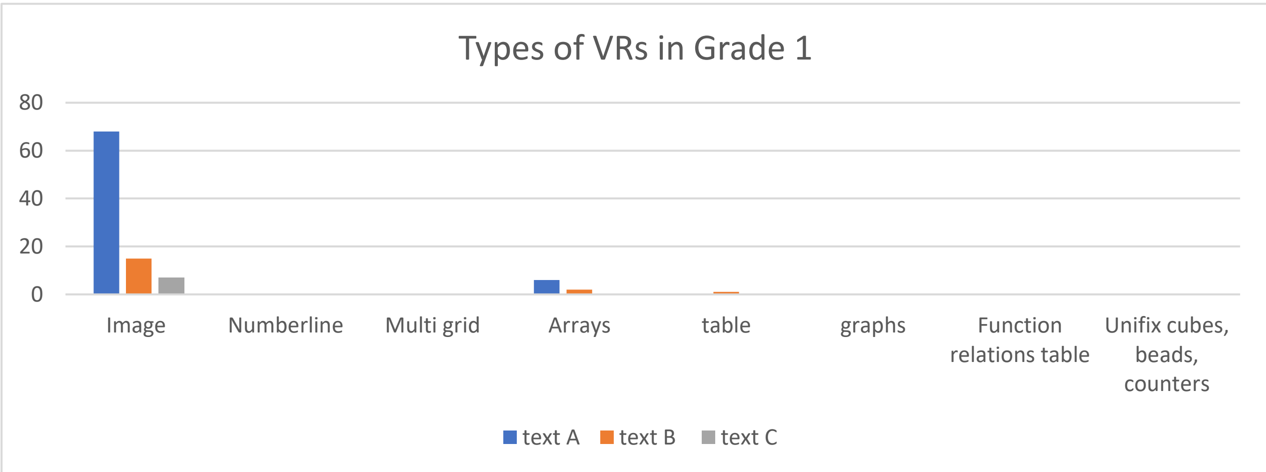


Figure 5.43: Types of VRs in Grade 1 texts

In the three Grade 1 texts, 84 (92%) VRs have a strong relation to content and there are 5 (6%) VRs with a problematic function (type a) and 2 (2%) VRs with a problematic function (type b) (see Figure 5.44). Text C has no VRs with a problematic relation to content. There are no examples where the VRs have a weak relation to content across all three texts.

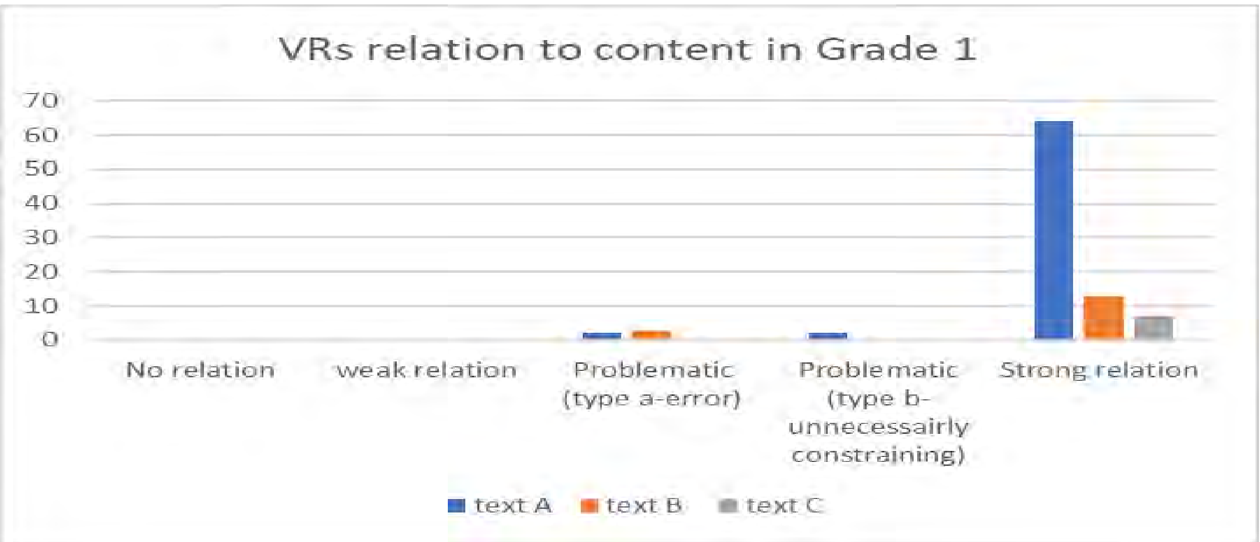


Figure 5.44: Relation to content in Grade 1 texts

More VRs in the Grade 1 texts have a realistic relation to content (67) than metaphoric relation to content (24). In other words, 74% of the VRs look like real-life objects. Forty-five of the 67 (66%) VRs with a realistic relation to reality are found in Text A compared with 15 (22.5%) in Text B and 7 (10.5%) in Text C (see Figure 5.45).

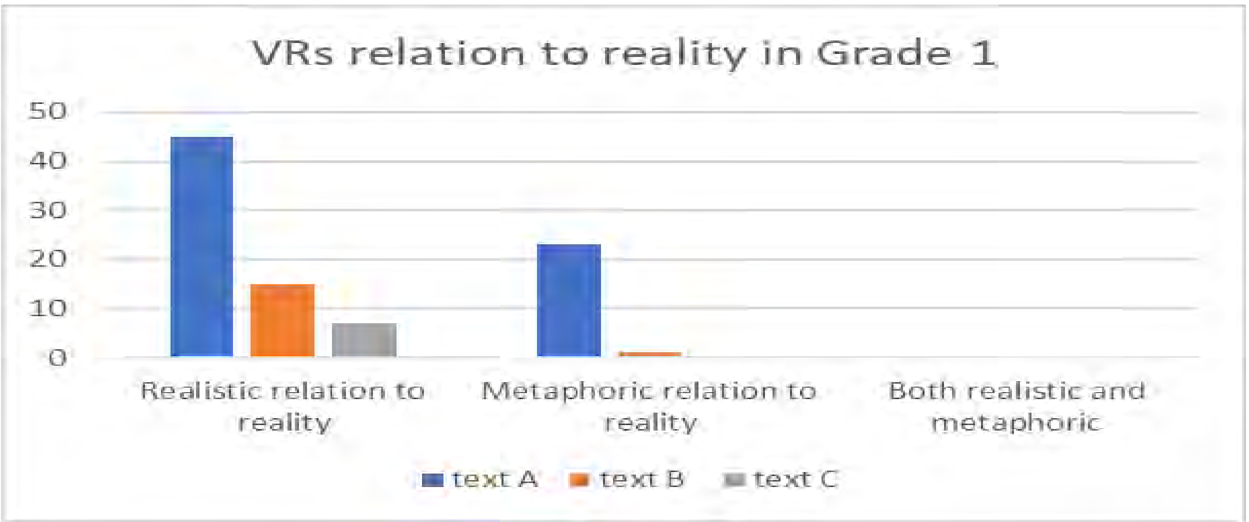


Figure 5.45: Relation to reality in Grade 1 texts

Of the 117 VRs in the functions category, the most common VRs in the Grade 1 texts are exemplifying (type b) (84) (72%), followed by exemplifying function (type a) (27) (23%) and complementary function (6) (5%). There are 6 VRs with a complementary function which is a VR that accompanies the exercise and provides additional information (see Section 3.3.4 in Chapter 3). These 6 VRs all appear in Text B. As such, 10 of the 16 (62.5%) VRs in Text B have an exemplifying function (type b) and 6 (37.5%) have a complementary function.

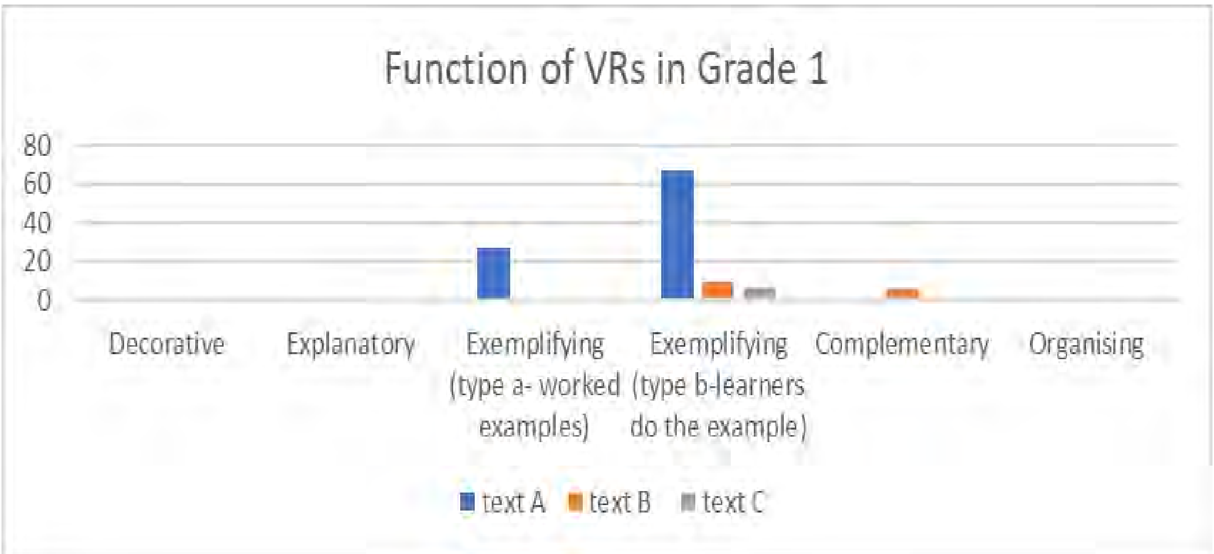


Figure 5.46: Function of VRs in Grade 1 texts

The majority of the VRs across the Grade 1 texts are 2D representations of 3D objects, that is, 64 of the 91 images (see Figure 5.47). There are 27 VRs that are 2D representations. This means that 70% of the total number of VRs are 2D representations of 3D objects and 30% are 2D representations. In terms of dimensionality, Text A has 68 (75%) VRs of which 37% are 2D representations and 63% are 2D representations of 3D objects. This stands in contrast to Texts B and C. Text B has 16 VRs, 15 (94%) of which are 2D representations of 3D objects and 1 (6%) 2D representation. The 7 VRs in Text C are 2D representations of 3D. This is not surprising given the emphasis on working with concrete objects in the FP (Thornton, 2011).

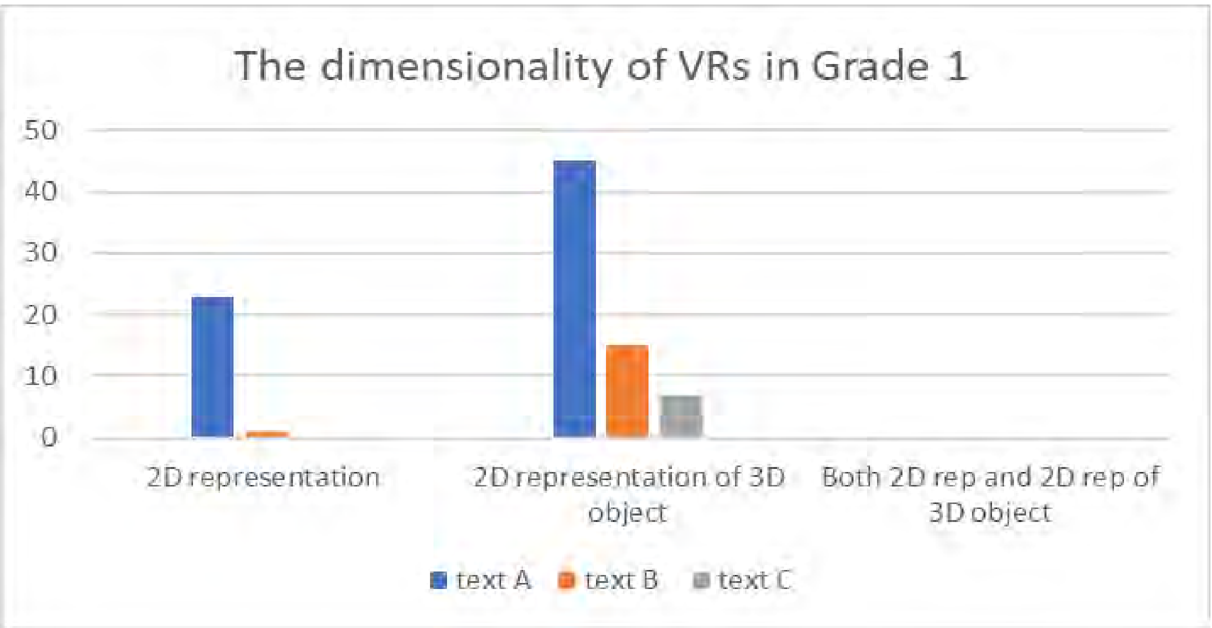


Figure 5.47: The dimensionality of VRs

5.4.1 Discussion of Grade 1 texts

In this section of Chapter 5, I aim to present discussion points based on the data presented above. Text A privileged multiplication as there is four times more multiplication than division exercises. By contrast, Text C has more exercises that relate to division than multiplication. The difference is not as significant in Text B. In the CAPS document Grade R learners are exposed to in-context problems using repeated addition and grouping and sharing (DBE, 2011, p. 20–21). However, with regard to context-free calculations, it is suggested that learners be introduced to repeated addition and grouping and sharing in Grade 1.

Exposing learners to a range of different representations is important to develop their conceptual understanding (Kilpatrick et al., 2001; Steinberg, 2005). From the analysis, it is evident that overall Text A contains more VRs than Texts B and C. The government-provided workbook (Text A) has the most VRs and reflects a greater variety of VRs. While the government-issued workbooks (Text A) are found in all schools, Text B and C are chosen by teachers. These two are regarded as the most popular texts in South Africa (according to sales) (see Section 4.4.1 in Chapter 4), yet they have very few VRs to support learners in developing an understanding of multiplication and division. This is concerning as learners use VRs to make sense of different mathematical concepts (Csíkos et al., 2011). However, even though learners are in Grade 1, it may be that the teachers who chose these texts did not expect them to provide scaffolding for learners' thinking and instead planned for that in the whole class teaching. However, this assumption became problematic in the context of COVID-19 where learners and their parents might have only had the text to work with. In schools that only use Texts B and/or C, learners are exposed to a significantly fewer number of VRs and VRs types. The onus is on the teacher to ensure that learners have the opportunity to work with models that develop their understanding of multiplication and division.

It is important for learners to be exposed to a variety of different forms of representation to be able to fully understand the concept. As explained in Chapter 2 (see Table 2.1) VRs that should be used to support teaching and learning of multiplication include images, number lines, array representations, function diagrams, unifix cubes, beads and counters. In using a wide range of VRs, learners' thinking is mediated (Fleisch et al., 2011). The three Grade 1 texts include VRs in the form of images, arrays and a table. Learners should also be exposed to number lines, multiplication grids and function diagrams.

In Grade 1 across Texts A, B and C the majority of the VRs have a realistic relation to content, followed by a metaphoric relation to content. In text C, there is no VR with a metaphoric relation to content.

Most VRs across Textbooks A, B and C have a strong relation to content. A possible reason for the majority of VRs having a strong relation to content could be that the VRs in the texts extend to real life which assists learners in not only understanding multiplication and division, but also, in some instances, seeing how mathematics relates to everyday life.

The most common function of VRs across Texts A, B and C is exemplifying (type b) as learners need to do the example presented in the texts themselves. This is a positive response to constructivism as through completing many examples themselves learners construct their multiplication and division schemas. Notably, there are no VRs with an explanatory function in Grade 1. Perhaps this is not surprising as not all Grade 1 learners are independent readers.

5.4.2 Analysis of Text A: Grade 2

The following section presents data that emerged from Grade 2 Books 1 and 2 of Text A. Text A consists of 57 (74%) multiplication exercises and 20 (26%) division exercises. Of the division exercises, all 20 are partitive division exercises as reflected in Figure 5.48.

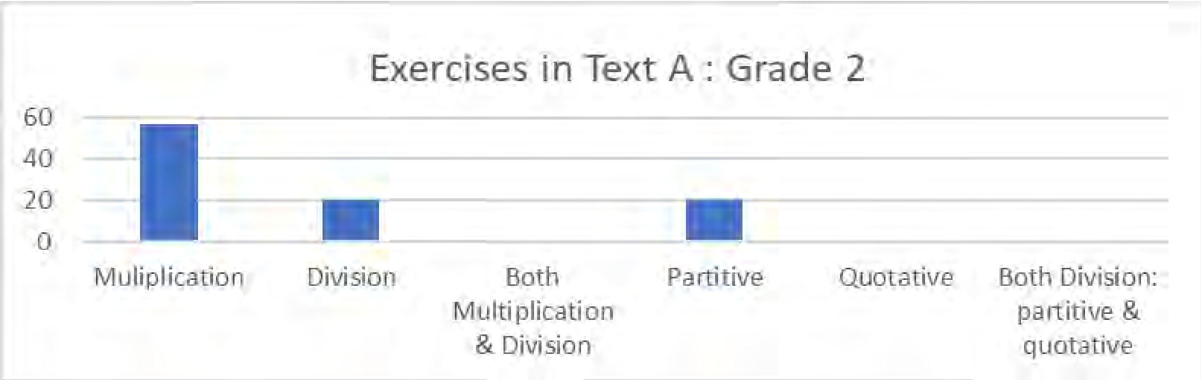


Figure 5.48: VRs of multiplication and division exercises (Text A)

5.4.2.1 Type of visual representation

There are 107 different types of VRs in Text A in Grade 2 as shown in Figure 5.49. The most frequent type of VRs in Text A is an image (71) (66%). The hands and feet in Figure 5.50 are examples of an image that can be found in the text.

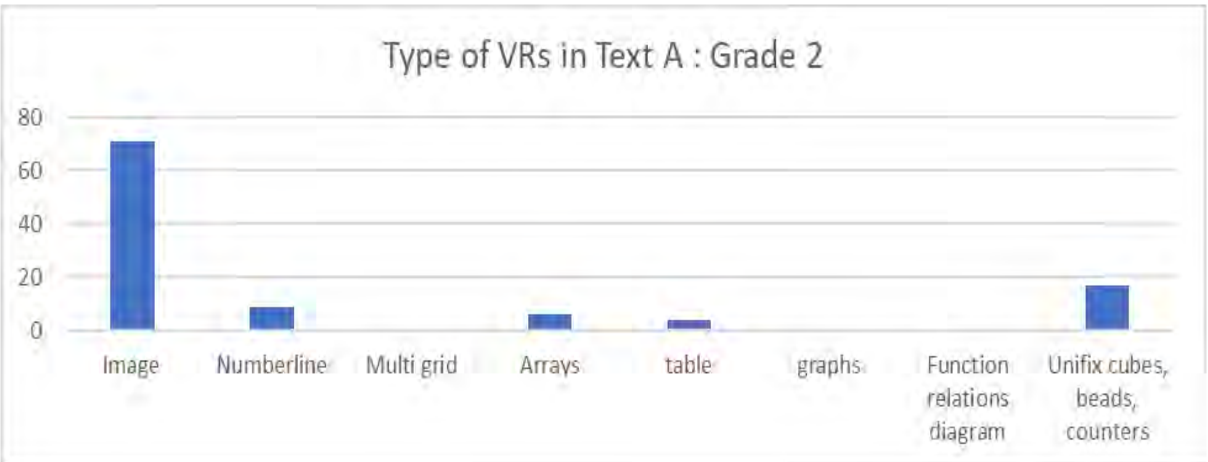


Figure 5.49: Types of VRs (Text A)

As noted in Figure 5.49, for Grade 1 texts, the most prominent type of VRs are 71 (66%) images. The second most prominent type of VRs in Text A are unifix cubes, beads and counters (17) (16%). Figure 5.51 is an excerpt from Text A used to illustrate what an example of unifix cubes in the text looks like. Of the 107 VRs in Text A, there are 9 (8%) VRs of number lines (Figure 5.49). Of the nine number lines, 3 include an image with the number line. The analysis also demonstrates that the types of VRs in Text A include array representations (6) (6%) and tables (4) (4%) as seen in Figure 5.49.

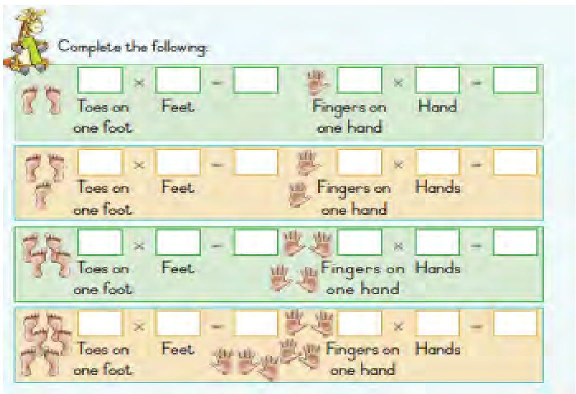


Figure 5.50: An example of an image (Grade 2, Text A, Book 2, p. 40)

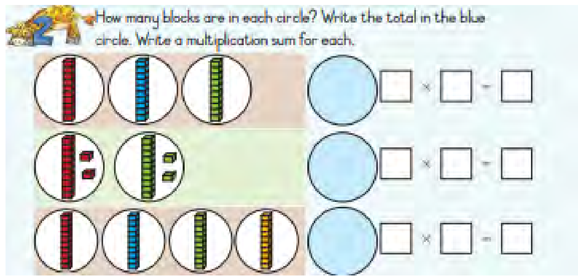


Figure 5.51: An example of unifix cubes (Grade 2, Text A, Book 1, p. 124)

Figure 5.52 is an example of a number line as it appears in the text. In Figure 5.52 the learners are expected to use the number line to demonstrate how many wheels 5 tricycles have. The learners would demonstrate this by looking at the VR of the tricycles. They would first draw 15 counters (ungrouped representation manipulatives), followed by representing it on the number line and writing the number of wheels on the tricycles using repeated addition and multiplication.

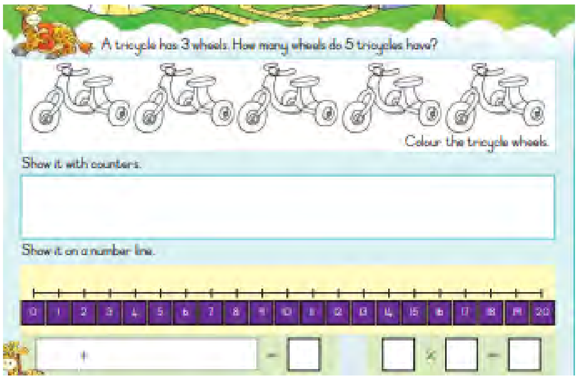


Figure 5.52: An example of a number line (Grade 2, Text A, Book 2, p. 115)

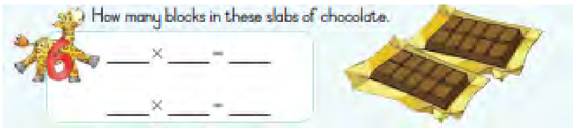
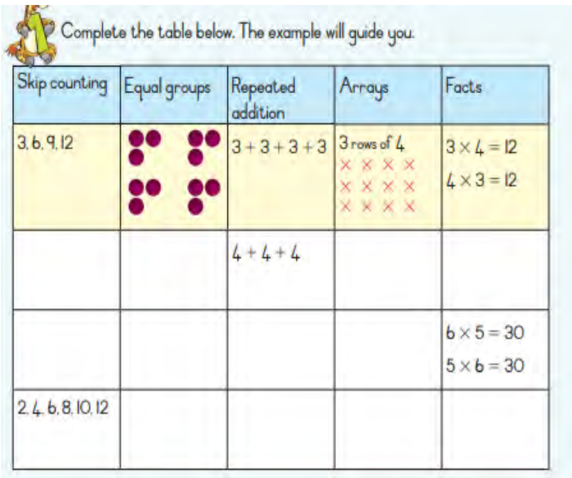


Figure 5.53: An example of an array representation (Grade 2, Text A, Book 2, p. 43)

The fourth most common VRs in Text A is an array (see Figure 5.53). Figure 5.53 is an example of an array representation where the learners are required to create a multiplication sum using 2 chocolate bars in which there are 2 rows and 5 columns each.

Figure 5.54 is an example of a table. These assist the learners in organising the information. In Figure 5.54 the instruction draws attention to the VR being in tabular format. Therefore the aim of this activity could be to draw the learners' attention to how a table works and the different representations in the table. These include skip counting, equal groups, repeated addition, arrays and multiplication.


Complete the table below. The example will guide you.

Skip counting	Equal groups	Repeated addition	Arrays	Facts
3, 6, 9, 12		3 + 3 + 3 + 3	3 rows of 4	3 × 4 = 12 4 × 3 = 12
		4 + 4 + 4		
				6 × 5 = 30 5 × 6 = 30
2, 4, 6, 8, 10, 12				

Figure 5.54: An example of an array representation and a table (Grade 2, Text A, Book 2, p. 106)

5.4.2.2 The visual representations' relation to content

The data presented below in Figure 5.55 is of the VRs' relation to content in Text A. Most of the VRs in this text have a strong relation to content – 73 (95%) of the 77 VRs have a strong relation to content (see Figure 4.48). Four (5%) VRs have a problematic relation to content (type b). This means that the VRs have unnecessary constraining factors such as in Figure 5.57 below.

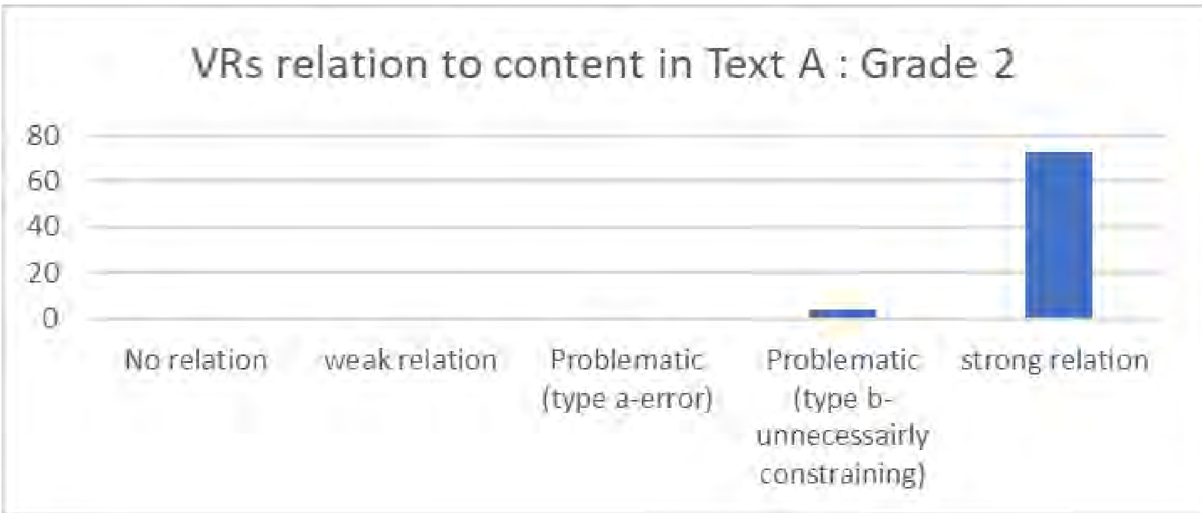


Figure 5.55: Relation to content (Text A)

Figure 5.56 is an example of a VR that focuses on multiplication in Text A that has a strong relation to content. The learners are required to multiply the feet and ears of the three different kinds of animals.

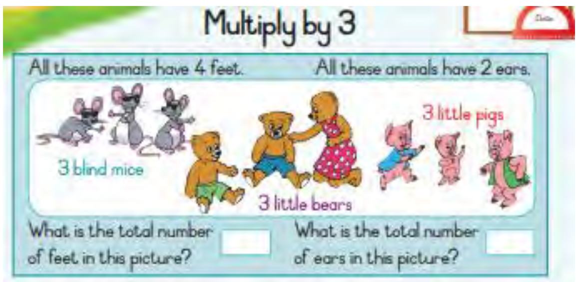
<table>
<tr>
<td>
</td>
<td>
</td>
</tr>
<tr>
<td>Figure 5.56: VR with a strong relation to content (Grade 2, Text A, Book 2, p. 104)</td>
<td>Figure 5.57: An example of a VR with a problematic relation to content (type b) (Grade 2, Text A, Book 2, p. 53)</td>
</tr>
</table>

Figure 5.57 is an example of a VR with a problematic relation to content (type b). In this word problem, the learners are expected to share the tea sets between the 2 children. However, the VR of the tea set in Text A is too small (in the printed text format) for the learners to distinguish between the teapots, cups and sugar bowls.

5.4.2.3 The visual representations' relation to reality

The data presented in Figure 5.58 shows whether the VRs have a realistic (58) (75%) or metaphoric (17) (22%) relation to reality. There are 2 (3%) VR that have both a realistic relation to content and a metaphoric relation to content.

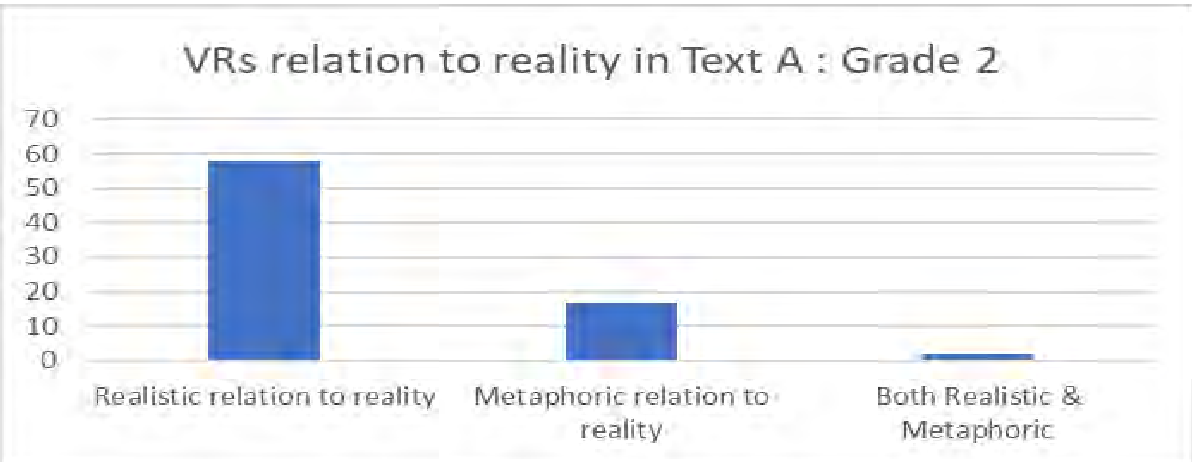


Figure 5.58: Relation to reality (Text A)

Figure 5.59 is an example from Text A that has a realistic relation to reality. The learners are required to multiply the apples and bananas by a given number. The orange circles in Figure 5.60 is an example of a VR with a metaphoric relation to reality.

Figure 5.59: A VR with a realistic relation to reality (Grade 2, Text A, Book 2, p. 104)	***Figure 5.60: A VR with a metaphoric relation to reality (Grade 2, TextA, Book 1, p. 110)***

Figure 5.61 is an example of a VR that illustrates a realistic relation to content (plates) and a metaphoric relation to content (circles). This is the only text in which such an example can be found.

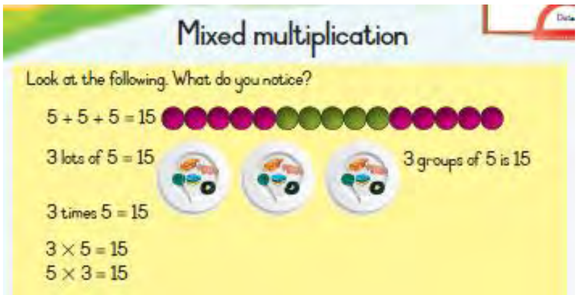


Figure 5.61: A VR with both a realistic and metaphoric relation to content (Grade 2, Text A, Book 2, p. 106)

5.4.2.4 The function of visual representations

Figure 5.54 displays the different functions of the VRs in Text A for Grade 2. The most common function of VRs in Text A is exemplifying function (type b) where learners need to complete the example themselves. Of the 90 VRs in Text A, 64 (71%) have an exemplifying function (type b). There are 17 (19%) VRs with an exemplifying function (type a). These are VRs that have an example already. This is followed by 6 (7%) that have a complementary function and 3 (3%) that have an explanatory function.

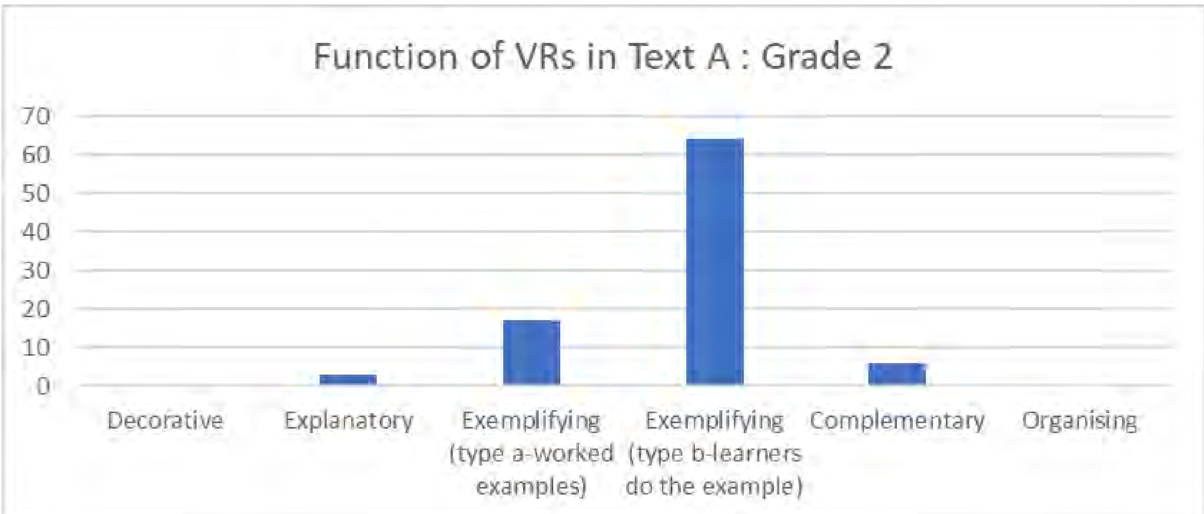


Figure 5.62: Function of VRs (Text A)

Figure 5.63 is an example of a VR that has an exemplifying function (type a). In Figure 5.63 the learners are given a worked example of how to complete the sums. The learners are expected to answer the question by using the purple circles. It appears that the intention of the exercise is to assist the learners in identifying the relationship between 'groups of', repeated addition and multiplication.

Six of the VRs have a complementary function. An example of a complementary function is provided in Figure 5.64. The VR of the pot assists the learners to recognise a three-legged pot if they are not familiar with it. However, the VR of the pot is not needed to answer the question.

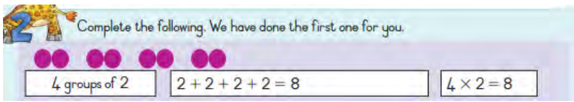


Figure 5.63: An example of an exemplifying (type a) function (Grade 2, Text A, Book 1, p. 60)

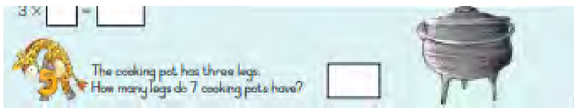


Figure 5.64: An example of a complementary function (Grade 2, Text A, Book 1, p. 107)

The third most prominent function of the VRs in Text A is an explanatory function. There are 3 VRs with an explanatory function. In Figure 5.65 the speech bubbles explain the thinking process that supports writing a sum as repeated addition and multiplication.

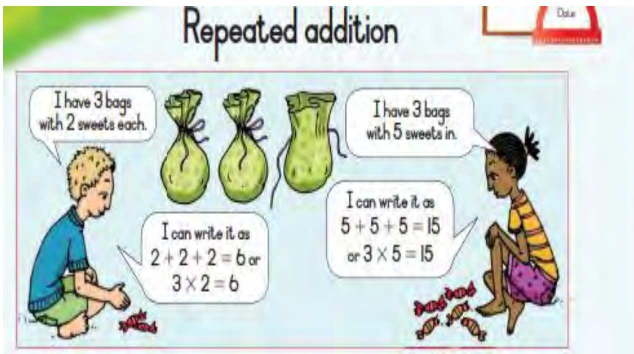


Figure 5.65: An example of an explanatory function (Grade 2, Text A, Book 2, p. 38)

5.4.2.5 Dimensionality of VRs

There are 17 (22%) VRs that are 2D representations and 58 (75%) that are 2D representations of 3D objects as noted in Figure 5.66. There are 2 VR (3%) that represent a 2D representation and a 2D representation of a 3D object. Figure 5.61 depicts both a 2D representation and a 2D representation of a 3D object in 2 (3%) VR. In Figure 5.61 the circles are 2D and the plates are 2D representations of 3D objects. This is the only VR with both 2D representation and 3D representation of a 3D object.

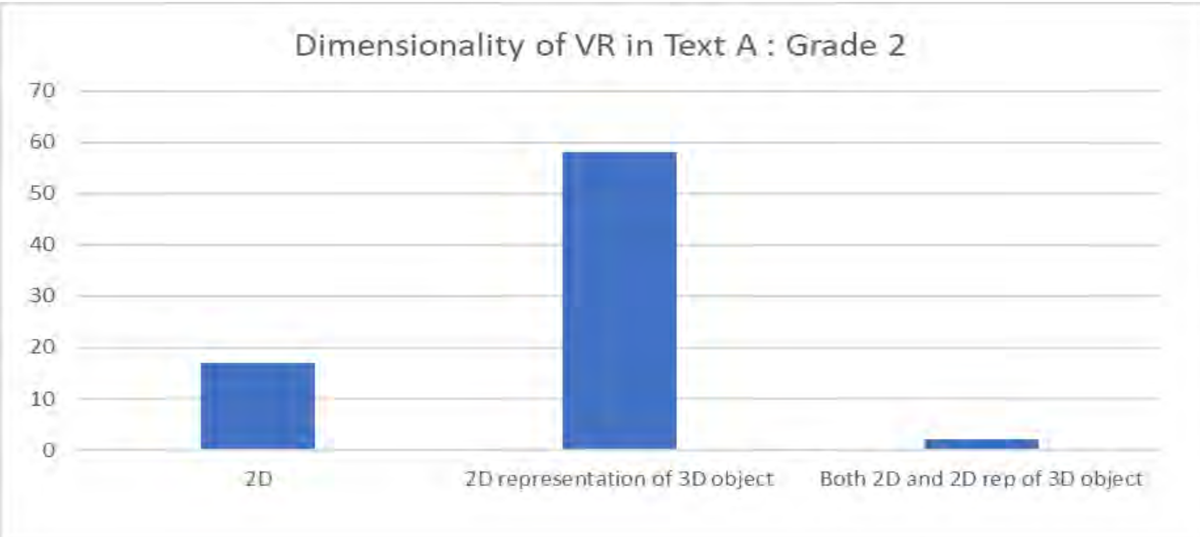


Figure 5.66: 2D representation and 2D representation of a 3D object (Text A)

The beads in Figure 5.67 are an example of a VR that is a 2D representation of a 3D object. The circles in Figure 5.68 are an example of a 2D representation.

Figure 5.67: An example of a 2D representation of 3D object (Grade 2, Text A, Book 1, p. 130)

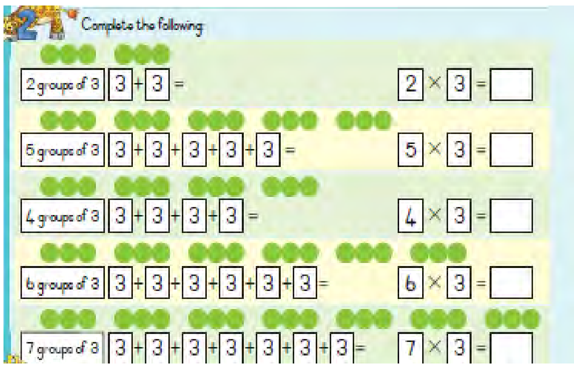


Figure 5.68: An example of a 2D representation (Grade 2, Text A, Book 1, p. 106)

5.4.3 Analysis of Text B: Grade 2

The following section presents the analysis of Text B in Grade 2. There are 11 (69%) VRs that support the development of multiplication and 5 (31%) that assist with division. There are only partitive division exercises (4) (Figure 5.67). There is 1 VR that is both partitive and quotative as depicted in Figure 5.69.

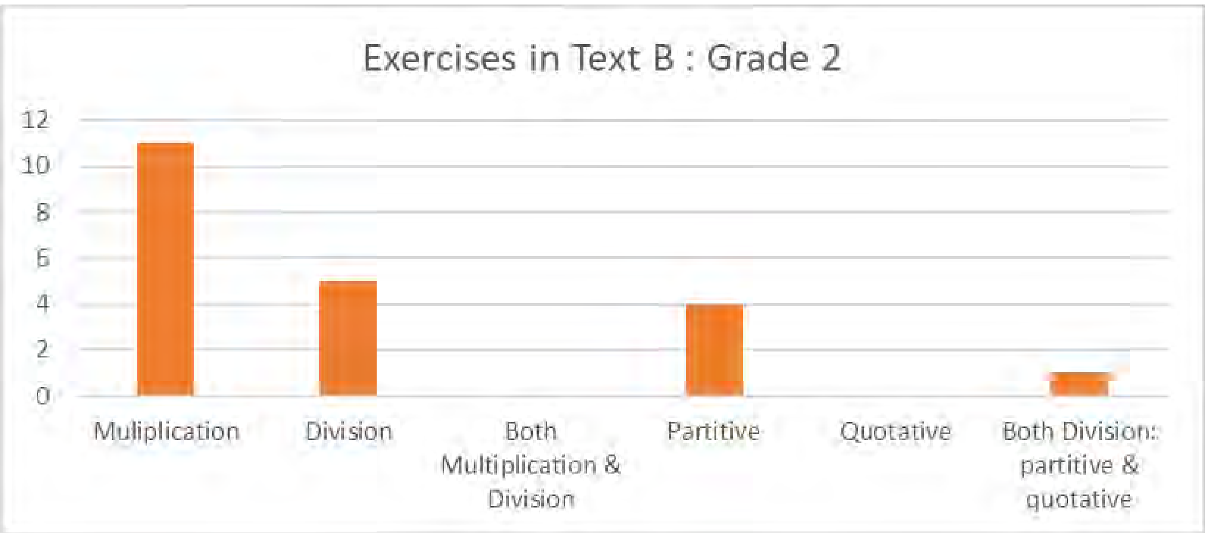


Figure 5.69: Multiplication and division exercises (Text B)

5.4.3.1 Type of visual representations

Of the 20 different types of VRs in this Grade 2 text, 16 (80%) of the VRs are images. Three (15%) of the VRs are arrays and 1 (5%) is of a number line (Figure 5.70). Images are thus the most prominent type of VRs. Figure 5.71 is an example of an image from Text B.

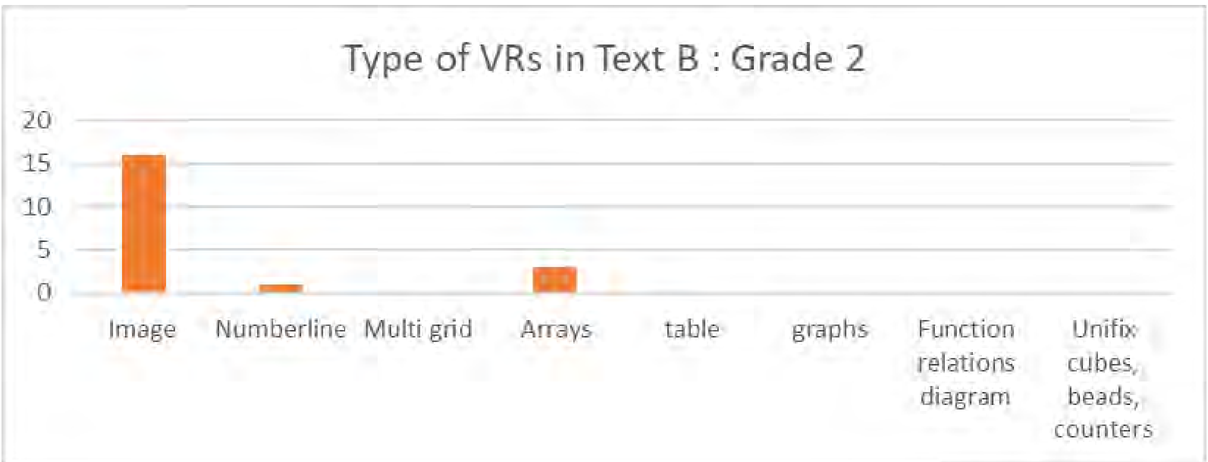


Figure 5.70: Type of VRs (Text B)

In this Grade 2 text, array representation is the second most common type of VRs. The array representation in the example in Figure 5.72 consists of 2 rows and 15 columns. The learners are required to count buttons and buttonholes in 2s.

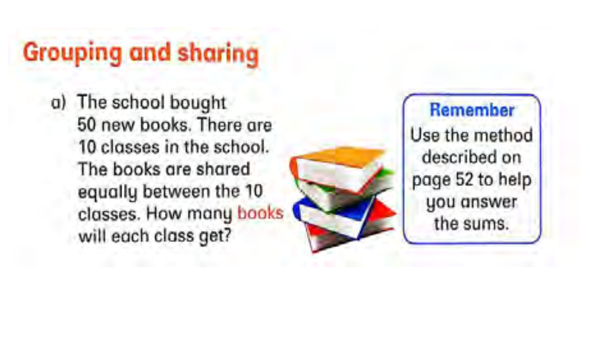


Figure 5.71: An example of an image (Grade 2, Text B, p. 61)

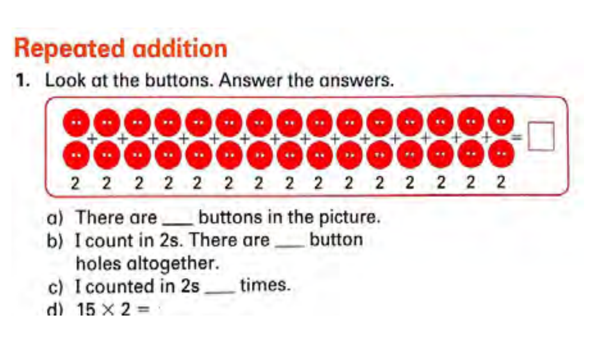


Figure 5.72: An example of a VR of an array representation (Grade 2, Text B, p. 60)

In Figure 5.73 the learners are expected to use the number line to complete the 5 calculations (a-e) that focus on repeated addition leading to multiplication.

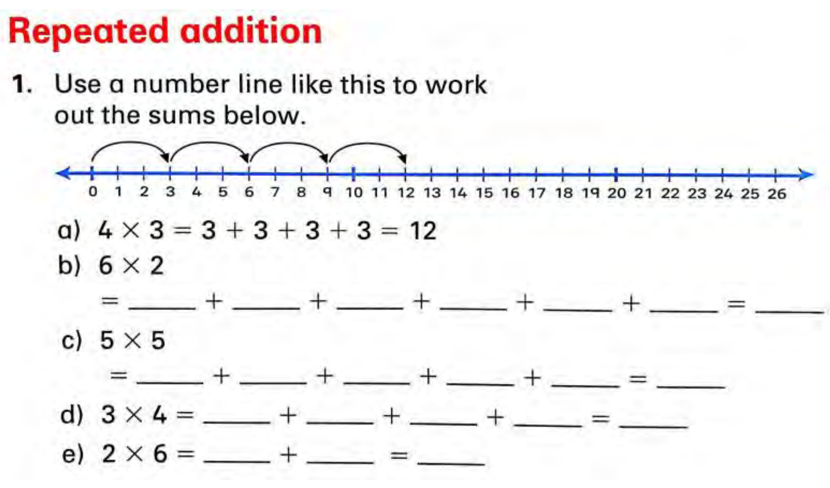


Figure 5.73: An example of a number line (Grade 2, Text B, p. 37)

Figure 5.73 is an example of a 2D representation. In this example, the number line serves as a 2D representation.

5.4.3.2 The visual representations' relation to content

Most of the VRs in Grade 2 of Text B have a strong relation to content. There are 14 (88%) VRs with a strong relation to content, 1 (6%) VR with a problematic (type b) relation to content and 1 (6%) with a weak relation to content. These are highlighted in the graph in Figure 5.74.

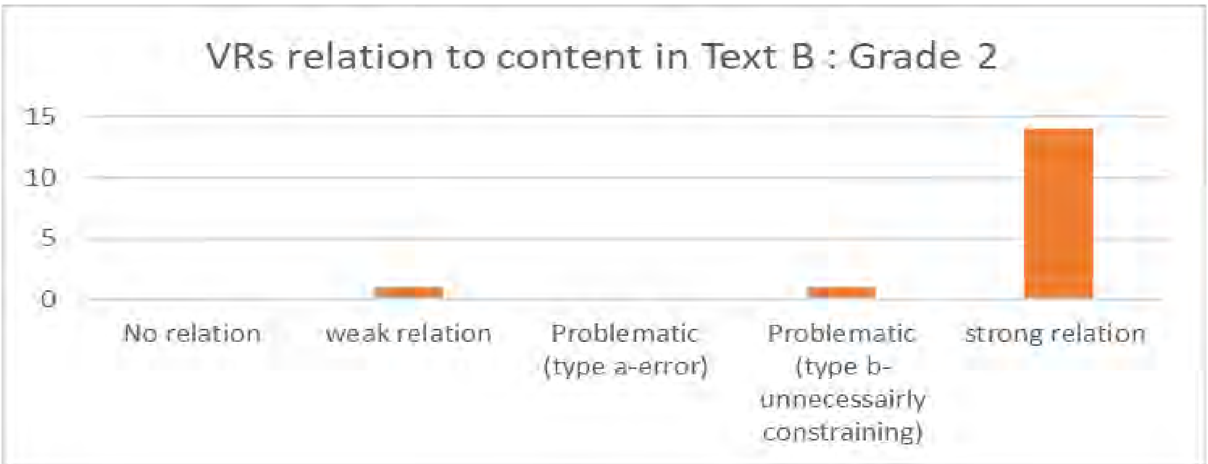


Figure 5.74: Relation to content (Text B)

Figure 5.75 is an example of a VR with a strong relation to content. The tricycles are important as they are necessary for the learners to answer the questions beneath them. Figure 5.76 is an example of a VR with a problematic (type b) relation to content. This VR presents unnecessary constraints. When writing a multiplication sum, it is not explicit what the learners should consider (for example, eyes, ears, hair, mouth and arms).

<table>
<tr>
<td>Look at the picture.

a) Write the multiplication sum for this picture.
b) Write down the answer.</td>
<td>3. Look at the picture.

a) Write the multiplication sum for this picture.
b) Write down the answer.</td>
</tr>
<tr>
<td>Figure 5.75: An example of a VR with a strong relation to content (Grade 2, Text B, p. 82)</td>
<td>Figure 5.76: An example of a VR with problematic relation to content (Grade 2, Text B, p. 83)</td>
</tr>
</table>

The VR in Figure 5.77 of a man and the tomato plant has a weak relation to content. The VR shows 3 rows of tomatoes, yet the related word problem refers to 6 rows of tomatoes. This VR provides context but does not assist the learners with the calculation (i.e., 70÷6).

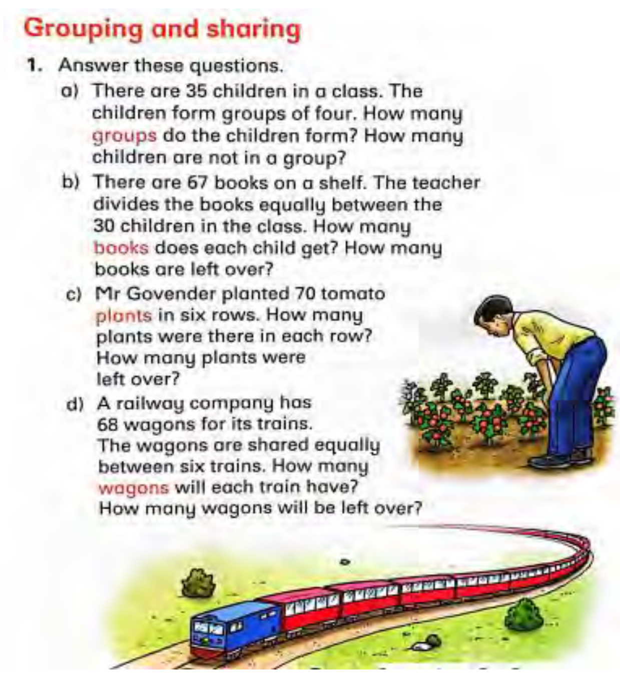


Figure 5.77: An example of a VR with a weak relation to content (Grade 2, Text B, p. 62)

5.4.3.3 The visual representations relation to reality

Text B has 14 (88%) VRs that have a realistic relation to reality and 2 (12%) have a metaphoric relation to reality as shown in Figure 5.78.

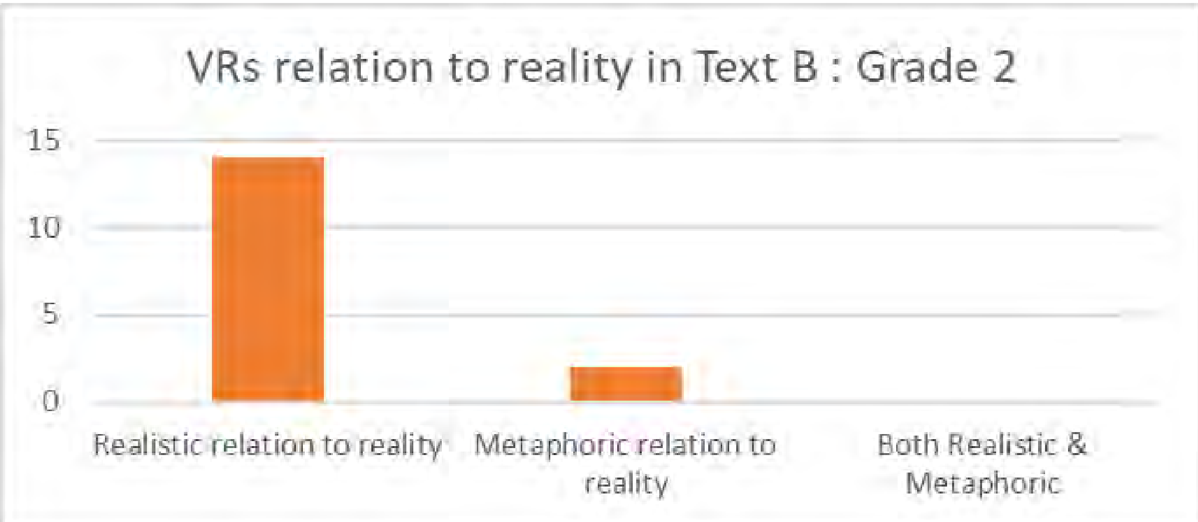


Figure 5.78: Relation to reality (Text B)

Figure 5.79 is a VR with a realistic relation to reality as the VR is of apples which learners are familiar with. Figure 5.80 has a metaphoric relation to reality as the red triangles are figurative.

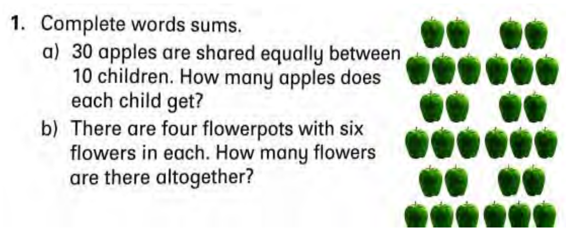
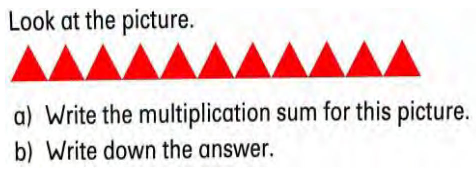
	

Figure 5.79: An example of a realistic relation to reality (Grade 2, Text B, p. 66)	***Figure 5.80: An example of a metaphoric relation to reality (Grade 2, Text B, p. 82)***

5.4.3.4 The function of visual representations

The most prominent function in this text is exemplifying (type b). Of the 18 VRs, there are 11 (61%) VRs that have an exemplifying function (type b), 5 (28%) have a complementary function and 2 (11%) with an exemplifying (type a) function in Text B for Grade 2 (Figure 5.81).

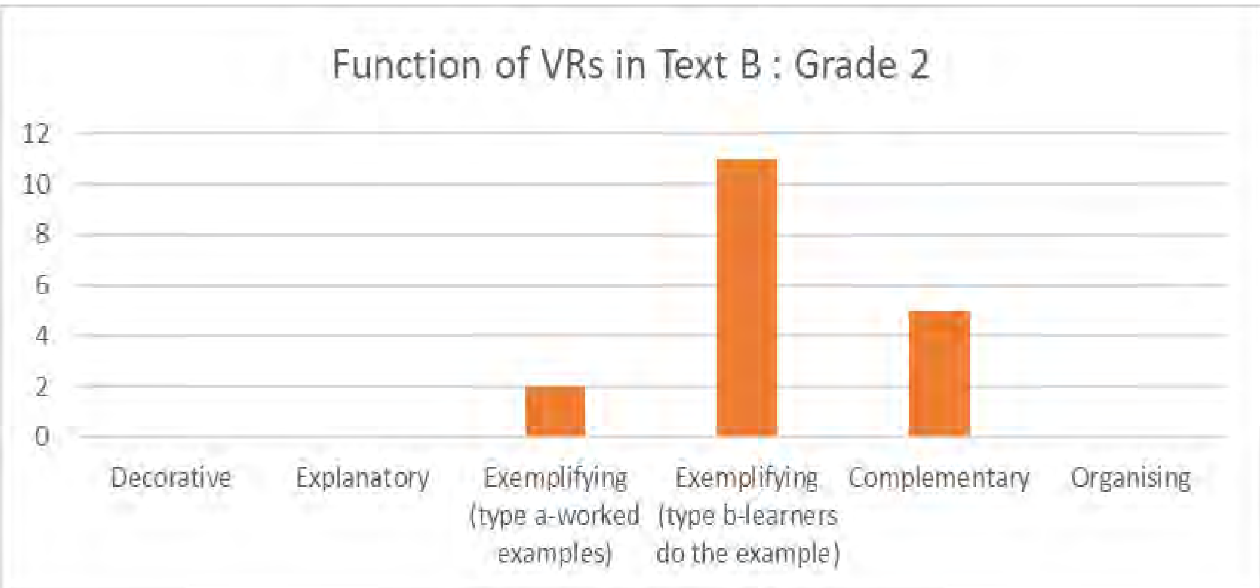


Figure 5.81: Function of VRs in Grade 2 (Text B)

Figure 5.73 is an example of a VR with an exemplifying function (type b) as the learners need to complete the exercises. In Figure 5.82 the VR of the octopus complements the word problem, in other words, it has a complementary function. This image of the octopus does not necessarily assist learners to answer the questions. Rather it provides context for the learner, such as 1 octopus has 8 legs.

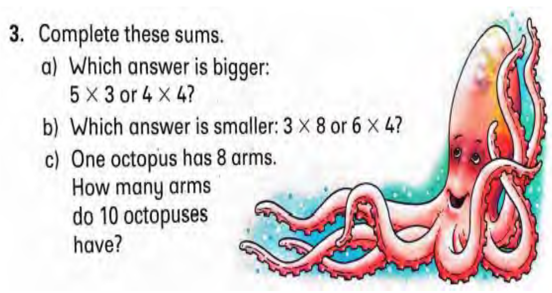
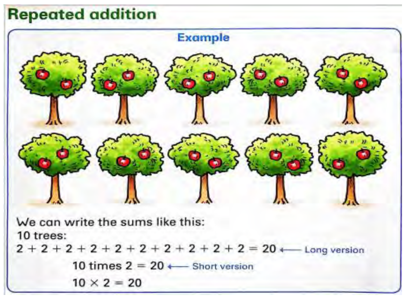
	

Figure 5.82: An example of a VR with a complementary function (Grade 2, Text B, p. 37)	***Figure 5.83: An example of a VR with an exemplification function (type a) (Grade 2, Text B, p. 16)***

An example of an exemplifying function (type a) VR is presented in Figure 5.83. This VR provides the learners with a worked example of the different ways in which they can solve the problem. The apples in the image are grouped in twos and this allows the learners to see how both repeated addition and multiplication can be used to solve the problem.

5.4.3.5 The dimensionality of VRs

There are 2 (12%) VRs that are 2D representations and 14 (88%) VRs that are 2D representations of 3D objects as illustrated in Figure 5.84.

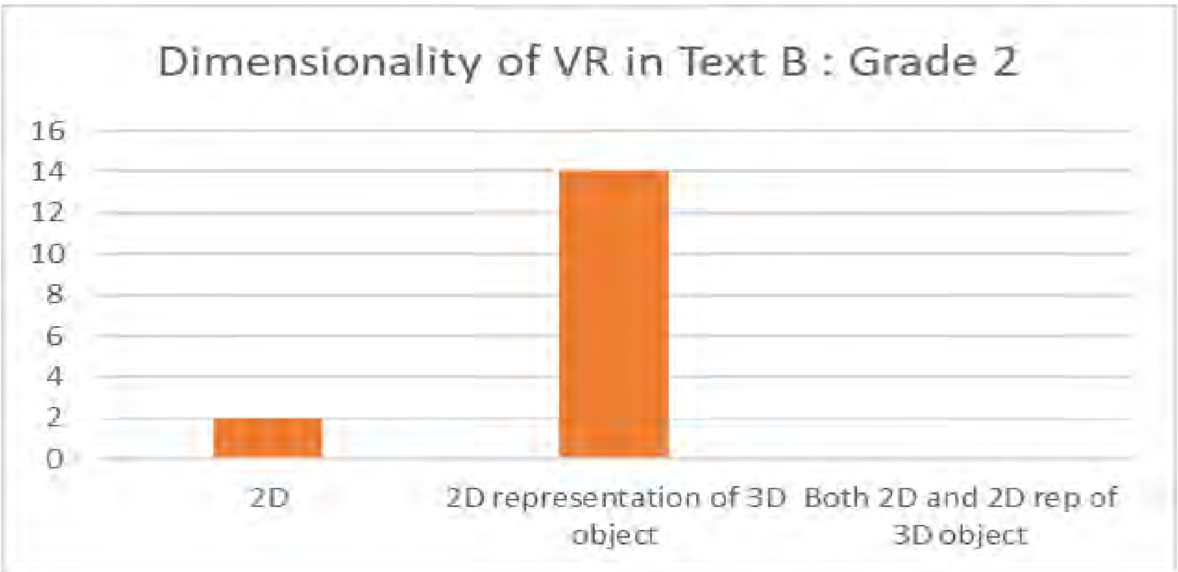


Figure 5.84: 2D representation and 2D representation of 3D object (Text B)

Figure 5.85 is an example of a 2D representation of a 3D object in Text B. The learners are familiar with tables and they have to use their knowledge that a table has 4 legs to answer the questions.

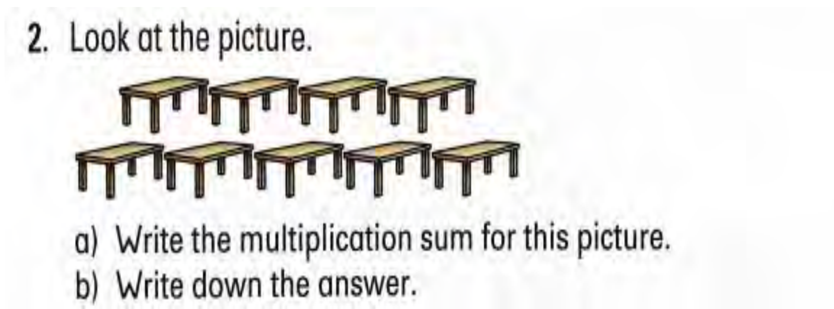


Figure 5.85: An example of a 2D representation of a 3D object (Grade 2, Text B, p. 83)

5.4.4 Analysis of Text C: Grade 2

The following section presents data that has emerged from Text C. Text C consists of 12 (80%) VRs that focus on multiplication and 3 (20%) VRs that focus on division. The division exercises include 2 partitive and 1 quotative example as highlighted in Figure 5.86.

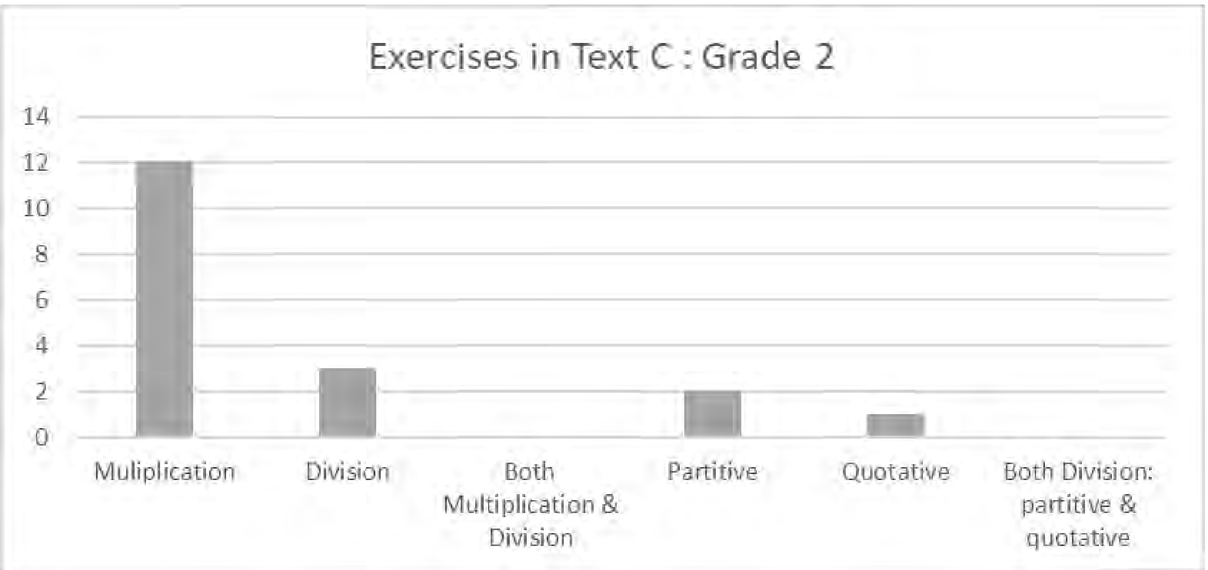


Figure 5.86: Multiplication and division exercises (Text C)

5.4.4.1 Type of visual representations

The most common type of VRs in Text C is images. Of the 17 VRs in Text C, there are 12 (70%) images in text C. This is followed by 3 (18%) number lines and 2 (12%) function diagrams as illustrated in Figure 5.87.

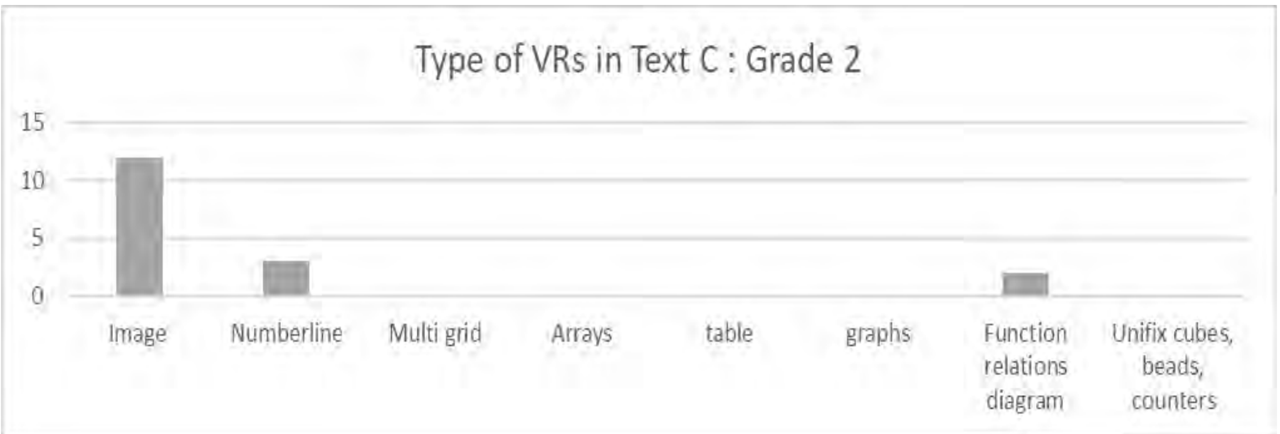


Figure 5.87: Types of VRs (Text C)

The image of the 4 dogs in a basket in Figure 5.88 is a typical example of an image in Text C. The learners have to identify the groups of dogs (number of baskets) and the number of dogs in each group (basket) in order to determine how many dogs there are.

The second most prominent VRs in Text C are number lines. In Figure 5.89 the learners are expected to make use of the number line to work out the answer to the multiplication sums below the number line.

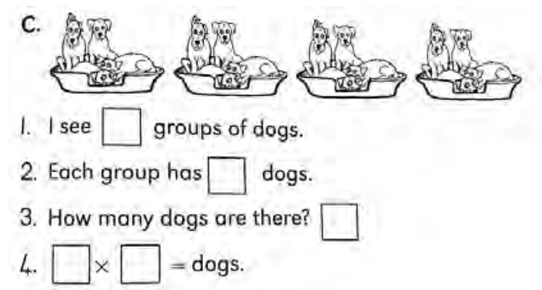


Figure 5.88: An example of an image (Text C, p. 23)

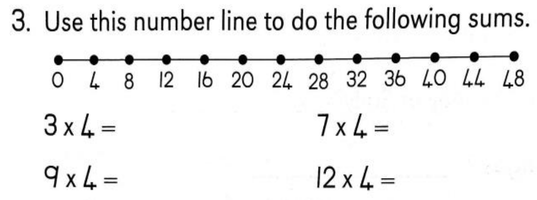


Figure 5.89: An example of a number line (Text C, p. 110)

The third most prominent VR in Text C is the function diagram. Figure 5.90 is an example of a function diagram. The input is the number in the second layer of the wheel, the rule is the number in the centre of the circle (e.g., X3) and the output is the answer that the learners insert in the outer circle.

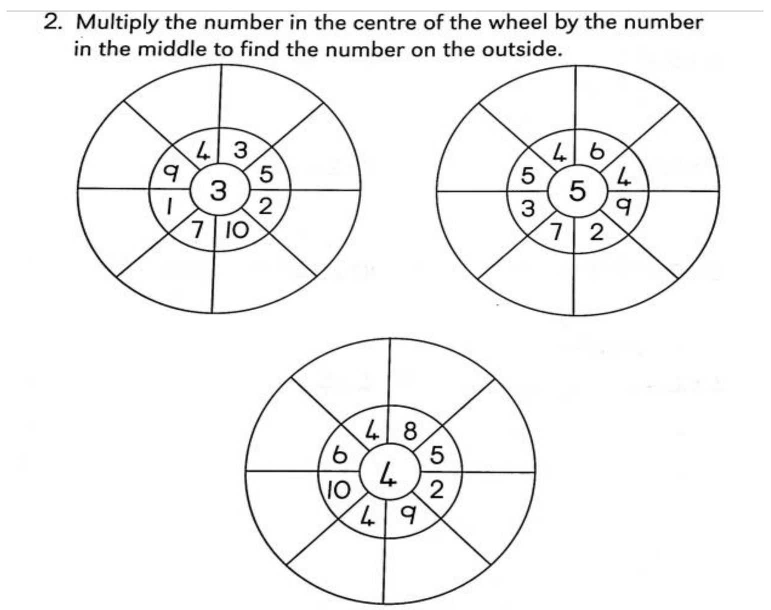


Figure 5.90: An example of a function diagram (Text C, p. 100)

5.4.4.2 The visual representations' relation to content

Text C consists of 14 (93%) VRs that have a strong relation to content and 1 (7%) VR that has a problematic relation to content (type a) as seen in Figure 5.91.

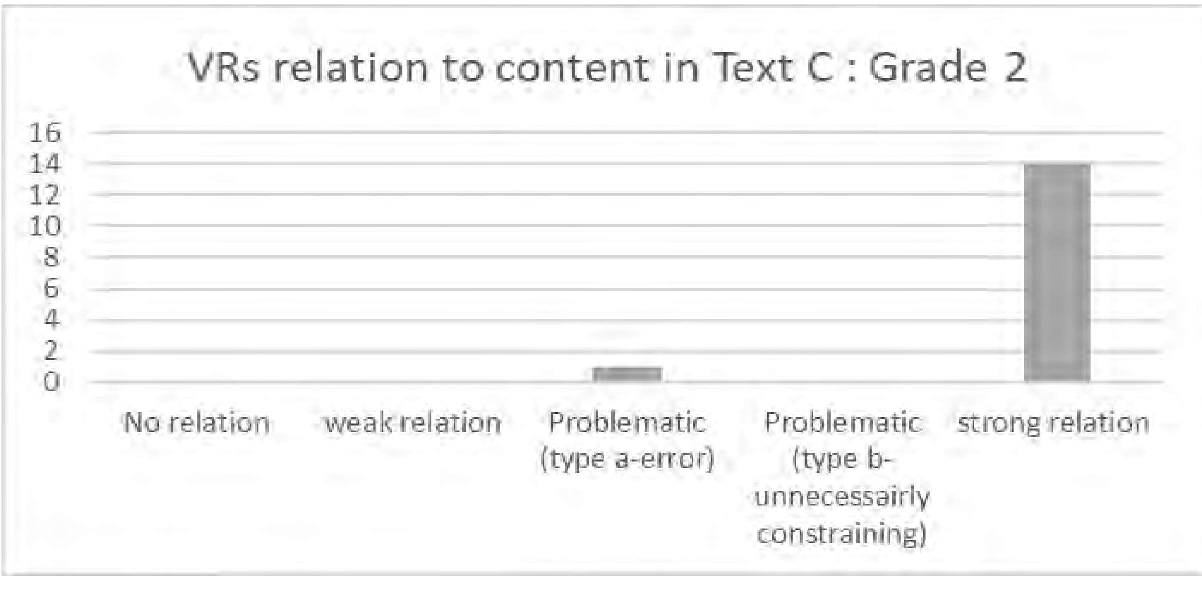


Figure 5.91: Relation to content (Text C)

Figure 5.92 has a strong relation to content as the VR of the kittens in each basket shows the learners that there are 3 groups of 5.

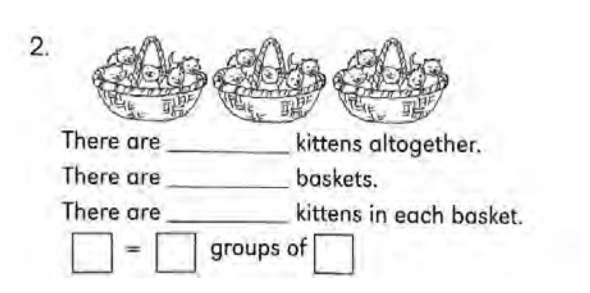


Figure 5.92: VR with a strong relation to content (Grade 2, Text C, p. 24)

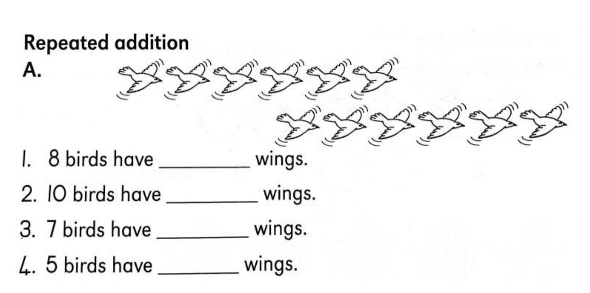


Figure 5.93: An example of a VR with a problematic relation to content (type a) (Grade 2, Text C, p. 20)

Figure 5.93 has a problematic relation to content (type a), there is an error in the exercise. There are 12 birds in the picture which bears no direct relation to the 4 questions the learners are required to answer.

5.4.4.3 The visual representations' relation to reality

There are 9 (60%) VRs with a realistic relation to reality and 6 (40%) with a metaphoric relation to reality as seen in Figure 5.94. Figure 5.88 includes a VR of dogs which is a realistic representation of reality, whereas Figure 5.81 is of a number line which has a figurative relation to reality.

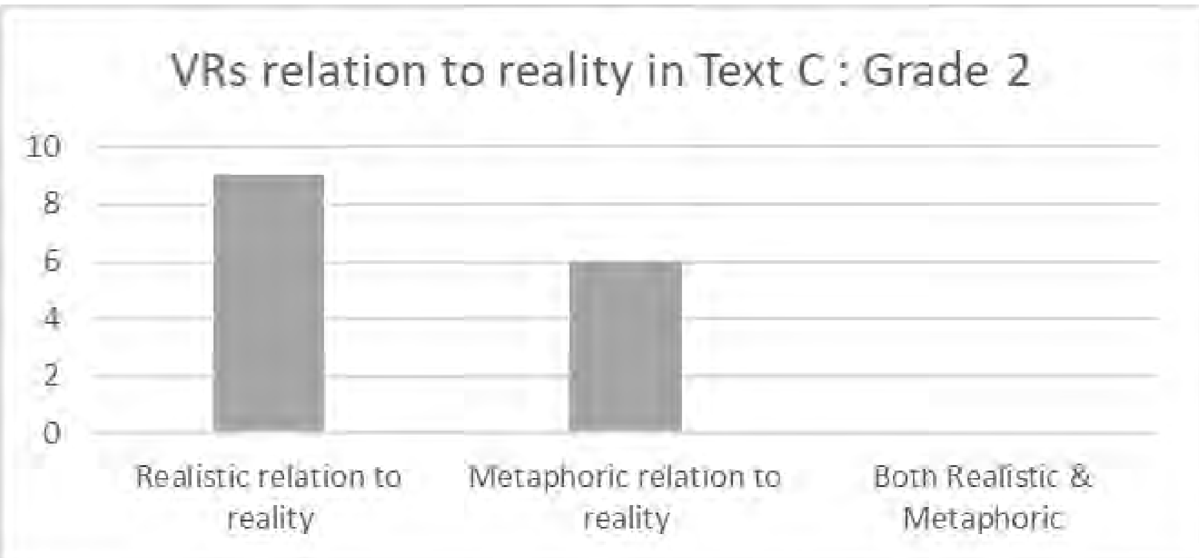


Figure 5.94: Relation to reality (Text C)

5.4.4.4 The function of visual representations

Most of the VRs in Text C for Grade 2 have an exemplifying function (type b) (14) (88%) followed by VRs with an exemplifying function (type a) (2) (12%) as indicated in Figure 5.95.

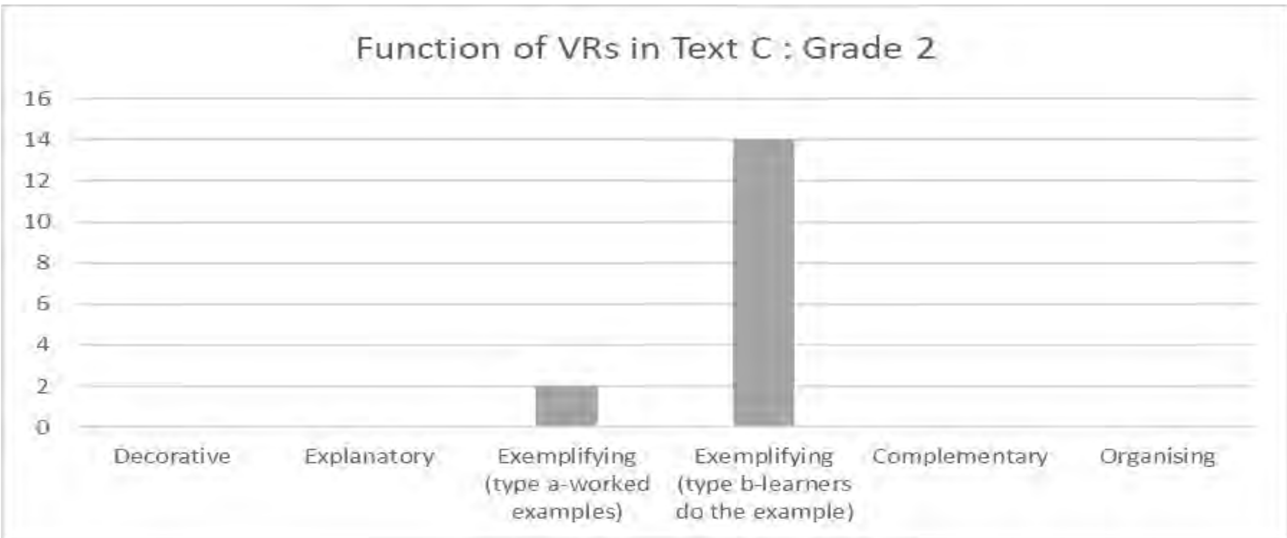


Figure 5.95: Function of VRs (Text C)

The first example of the exercise in Figure 5.96 demonstrates an exemplifying function (type a) of a worked example that shows how the problem should be solved using repeated addition. In exercises 2 and 3, the learners are expected to follow the example given in exercise 1 to perform calculations 2 and 3 themselves. This is a VR with an exemplifying function (type b).

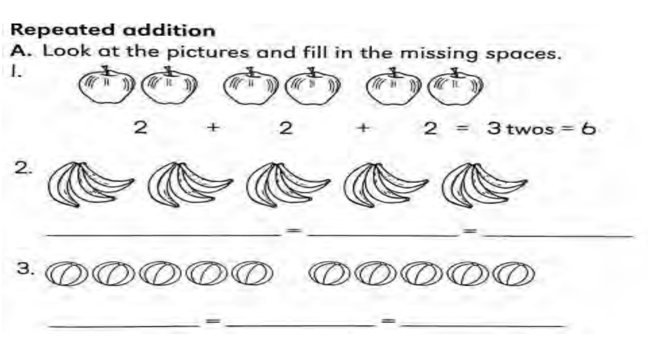


Figure 5.96: An example of an exemplifying a and b function (Grade 2, Text C, p. 19)

5.4.4.5 The dimensionality of a VRs

The number of 2D representations and 2D representations of 3D objects is similar in Text C. Of the 15 VRs, 6 (40%) are 2D representations and 9 (60%) are 2D representations of 3D objects as seen in Figure 5.97.

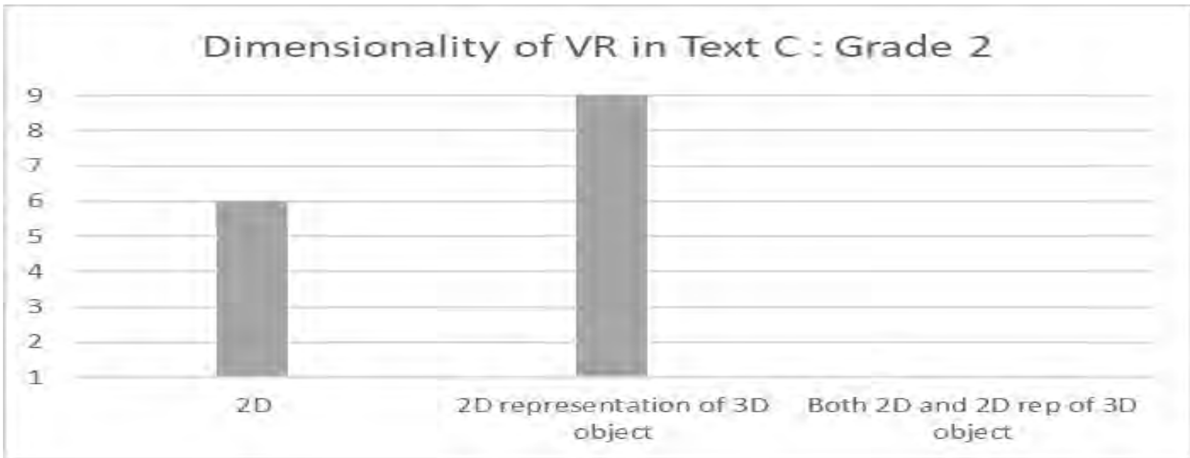


Figure 5.97: 2D representation and 2D representation of a 3D object.

The crabs in Figure 5.98 are a 2D representation of a 3D object, while the number line in Figure 5.99 is a 2D representation.

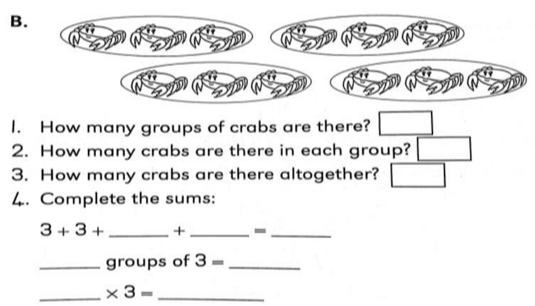


Figure 5.98: 2D representation of a 3D object (Grade 2, Text C, p. 22)

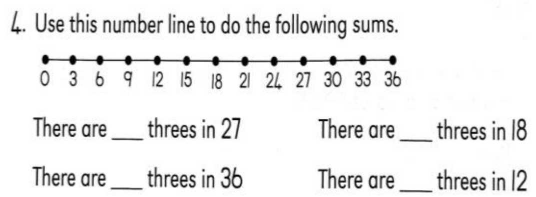


Figure 5.99: An example of a 2D representation (Grade 2, Text C, p. 110)

5.5 Data Analysis Across the Three Grade 2 Texts

This section presents the analysis of Texts A, B and C in Grade 2 in relation to the multiplication and division exercises (see Figure 5.100). Of the 108 VRs in the Grade 2 texts, there are 80 (74%) multiplication exercises and 28 (26%) division exercises. Of these exercises, there are 26 that are partitive, 1 quotative and 1 VR that is both partitive and quotative as illustrated in Figure 5.100.

As evident in Figure 5.100, Text A (57) (71%) has significantly more multiplication exercises than Texts B (11) (14%) and C (12) (15%). The division exercises in Text A (20) (71%) are all partitive. Of all the division exercises in Grade 1, Text B consists of 5 (18%) division exercises. Of the 5 division exercises in Text B, 4 were partitive and 1 quotative highlights the relationship between multiplication and division. Text C has 3 division (11%) exercises; 2 exercises which are partitive and 1 that is quotative.

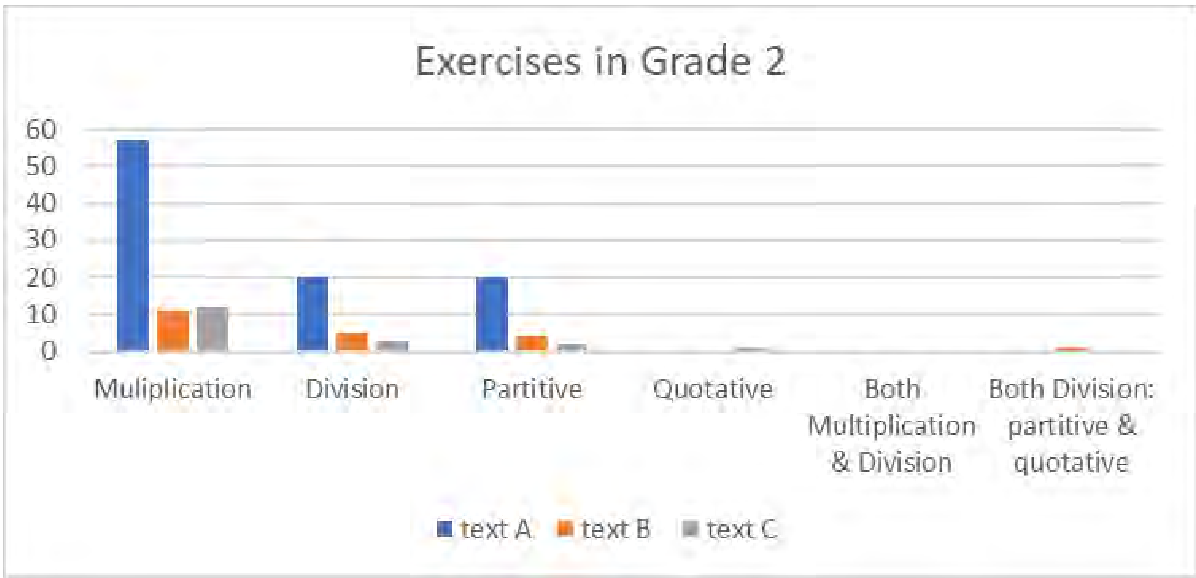


Figure 5.100: Multiplication and division exercises in Grade 2 texts

The most frequent type of VRs across the Grade 2 texts are images (99) (69%) as evident in Figure 5.101. Of the 99 images in Grade 2 texts, 71 (72%) images can be found in Text A, 16 (16%) in Text B and 12 (12%) in Text C. Of the 144 VRs in Grade 2 texts, 19 (12%) contain unifix cubes, beads and counters, followed by number lines (13) (9%), arrays (9) (6%), tables (4) (3%) and function diagrams (2) (1%).

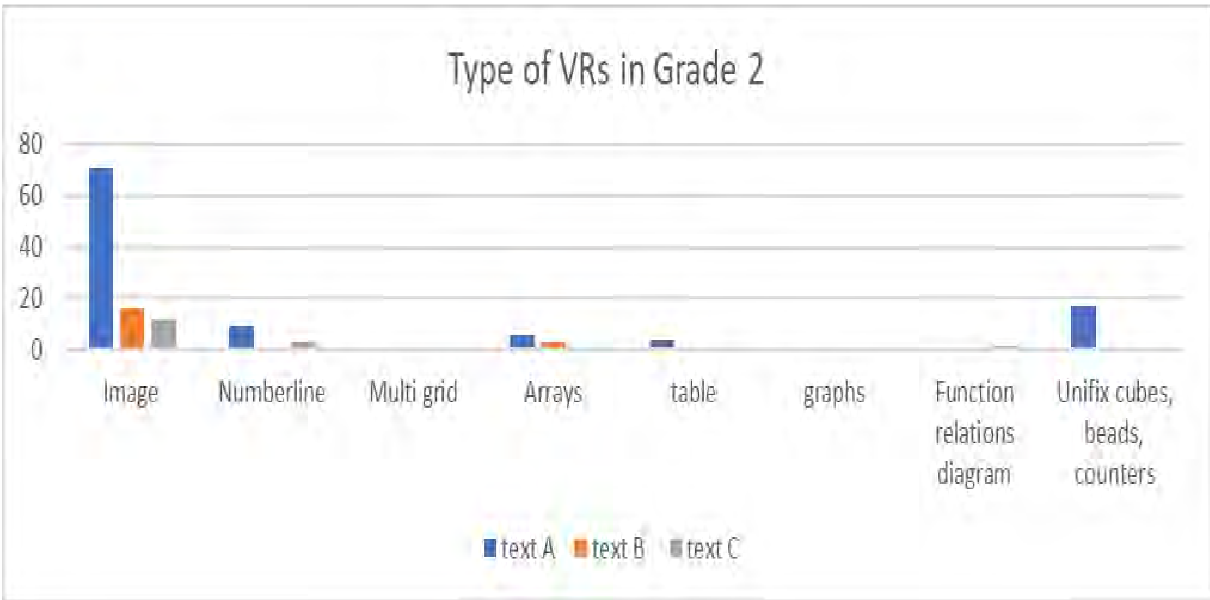


Figure 5.101: Types of VRs in Grade 2 texts

Of the 108 VRs in the three Grade 2 texts, 101 (93%) VRs have a strong relation to content as seen in Figure 5.102. There is 1 (1%) VR across the three texts that have a problematic relation to content (type a). This is only evident in Text C. There are 5 (5%) VRs across the three texts that have a problematic relation to content (type b). Four are evident in Text A and 1 VR in B. There is 1 (1%) VR that has a weak relation to content, found in Text B.

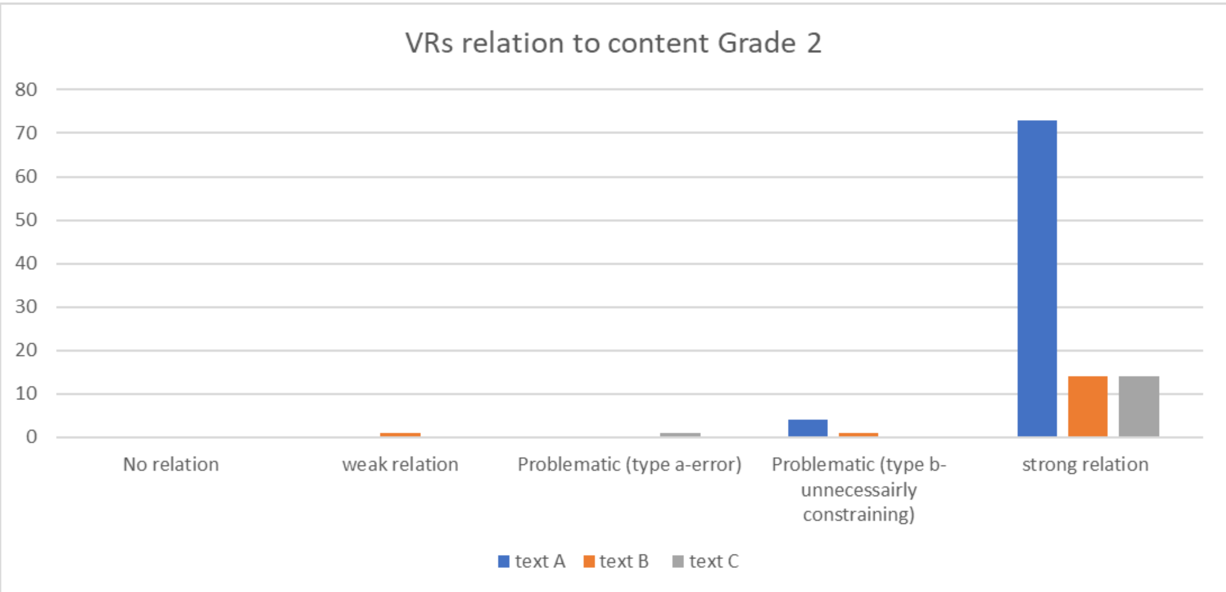


Figure 5.102: Relation to content in Grade 2 texts

Of the 108 VRs in Grade 2 texts, 81 (75%) have a realistic relation to reality, 25 (23%) have a metaphoric relation to reality and 2 (2%) have both a realistic and metaphoric relation to content as seen in Figure 5.103. Text A has the majority (58) (55%) of VRs that have a realistic relation to reality, compared to 14 (13%) of Text B and 9 (8%) of Text C (see Figure 5.155). Furthermore, across Grade 2, 17 (22%) of Text A, 2 (13%) of Text B and 6 (40%) of Text C have a metaphoric relation to content.

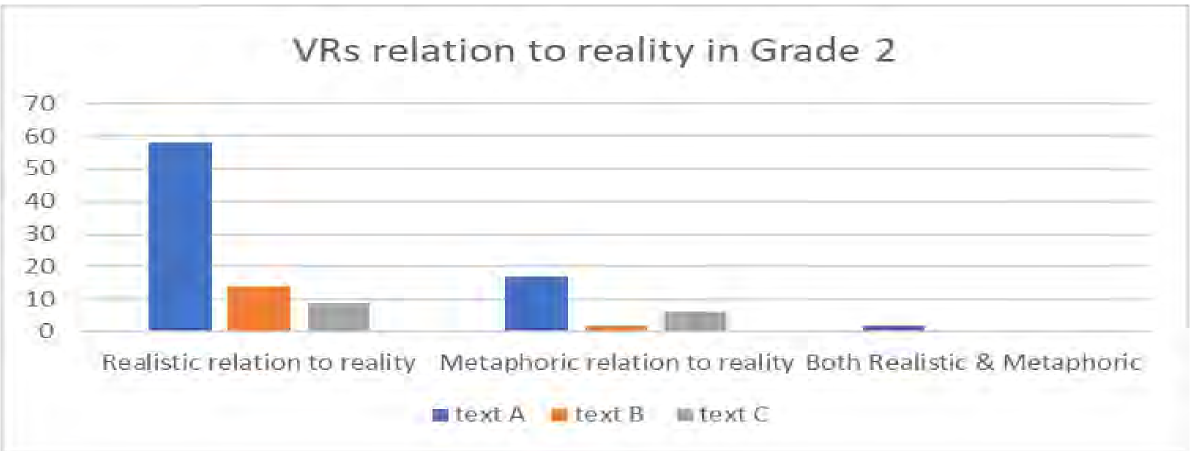


Figure 5.103: Relation to reality in Grade 2 texts

The most common function across the three Grade 2 texts is exemplifying function (type b). Of the 124 VRs in Grade 2, Text A has 64 (52%) VRs with an exemplifying function (type b). This stands in contrast to Texts B and C which have 11 (9%) and 14 (11%) VRs with an exemplifying function (type b). The second most prominent function in Grade 2 is an exemplifying function (type a) of which there are 21. There are 11 (9%) VRs with a complementary function across the three texts. Six of these are in Text A and 5 in Text B. There are no complementary VRs in Text C. Text A is the only text that contains VRs with an explanatory function (3) (2%).

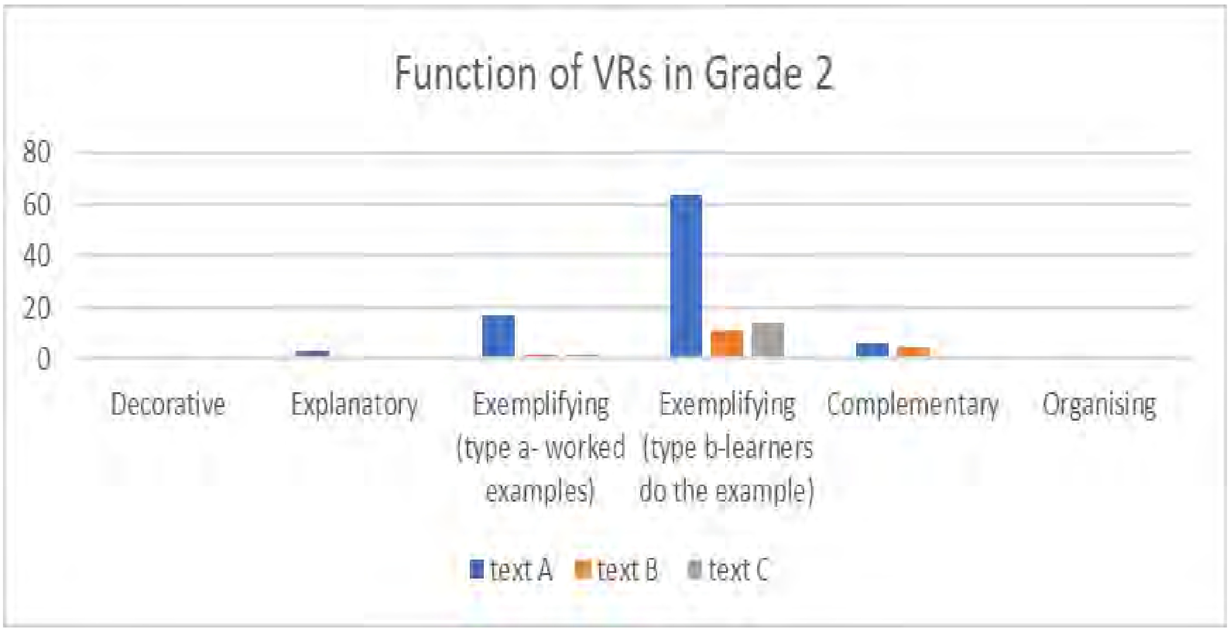


Figure 5.104: Function of VRs in Grade 2 texts

The majority of VRs across the three texts are 2D representations of a 3D object. Of the 108 VRs, there are 25 (23.1%) VRs that are 2D representations. Fifty-eight (53.7%) of the VRs that are 2D representations of 3D objects appear in Text A, while there are only 14 (13%) and 9 (8.3%) VRs in Texts B and C respectively. Text A is the only text that has a VR that is both a 2D representation and a 2D representation of a 3D object (2) (1.9%).

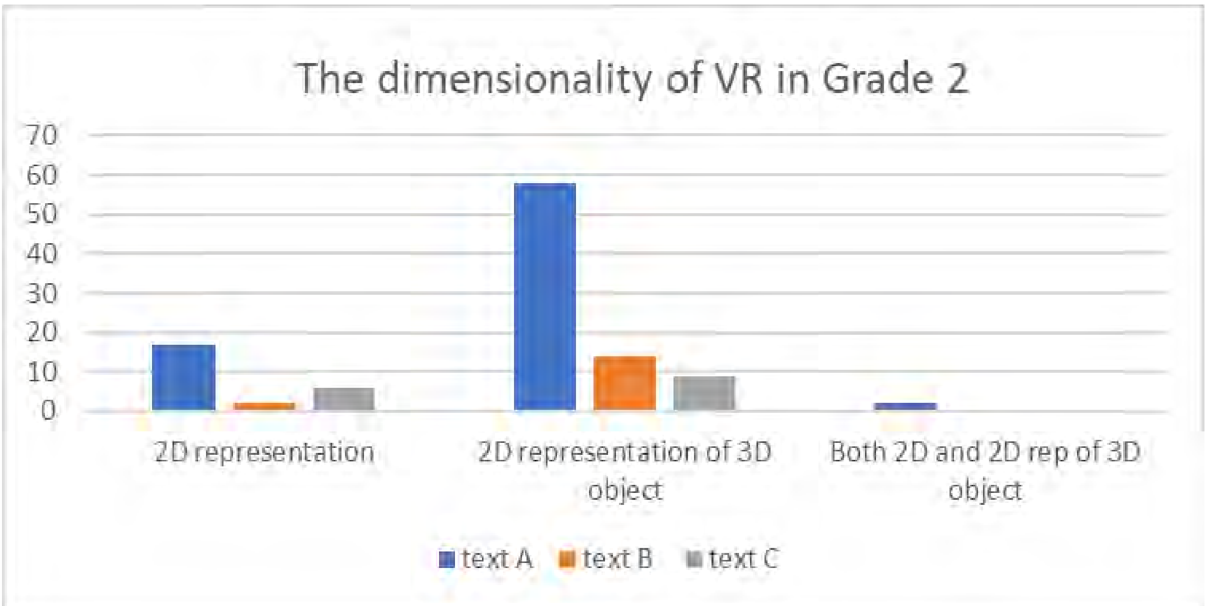


Figure 5.105: 2D representation and 2D representation of 3D object

5.5.1 Discussion of Grade 2 Texts

In Chapter 2, I argued that there are two perspectives on multiplication and division. The first perspective is that the operations should be taught sequentially (Harries & Barmby, 2007). The second perspective is that multiplication and division should be introduced to learners simultaneously so that learners can recognise the inverse of one another. The fact that no VR is used to deal with multiplication and division exercises simultaneously (see Figure 5.100) indicates that authors favour the former perspective of teaching these sequentially.

Vula and Berdynaj (2011) argue that mathematics is about studying patterns, making connections and identifying relationships and Askew et al. (2019) argue that multiplication is an extension of repeated addition. Nunes and Bryant (1996) maintain that multiplication and division be taught simultaneously as the inverse of one another. Harries and Barmby (2007) maintain the curriculum is based on a hierarchical approach as learners are first introduced to addition, then subtraction, then multiplication and after that division. In Grade 2, the learners are exposed to more multiplication exercises than division exercises across the three texts.

Similar to Grade 1, in Grade 2 there are many images in Text A, compared to Texts B and C. However, in contrast, there is a wider range of different types of VRs in the Grade 2 texts than those in Grade 1. A wide range of VRs allows for optimal mathematical learning when introduced to new concepts (Ainsworth, 2006). The different types of VRs include images of unifix cubes, beads and counters (Text A), number lines (Text A, B and C) and array representations (Text A and B). Tables (Text A) and function relation diagrams (Text C). The majority of VRs in Texts A, B and C have a realistic representation of reality and a strong relation to content. Fotakoupoulou and Spiliotopoulou (2008) suggests that conceptual understanding of a topic is aided if the VR has a realistic representation as it relates to a real-life object.

In the data analysis, it is evident that the function of the VRs found in most texts is exemplifying function (type b). An exemplifying function (type b) does not provide a worked example for the learners. It is important to note that Text A is the text with the most VRs with an exemplifying function (type a) compared to Text B and C which only contain two VRs with an exemplifying function (type b). This is important to note especially during the COVID-19 pandemic where parents were required to assist their children. There is one VR in Text A that has a VR which is both a 2D representation and a 2D representation of a 3D object.

5.5.5.1 Analysis of Text A: Grade 3

The following section presents data that emerged from Text A. Text A has 56 (70%) VRs that focus on multiplication, 13 (16%) on division and 11 (14%) VRs that highlight the relationship between multiplication and division as seen in Figure 5.106. Of the 13 division exercises, 8 are partitive and 5 are quotative division exercises.

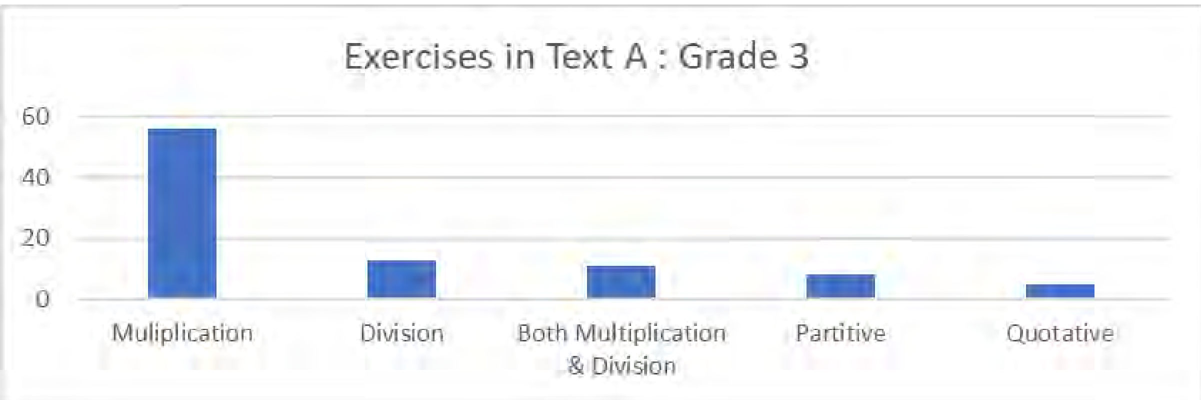


Figure 5.106: Multiplication and division exercises (Text A)

5.5.5.2 Types of visual representations

The type of VRs in Text A is presented below in Figure 5.93. Of the 118 types of VRs in Text A, the most prominent type of VRs are images (73) (61.9%). This is followed by 15 (12.7 %) VRs that are tables, 12 (10%) array representations, 9 (7.6%) number lines, 8 (6.7%) function diagrams and 1 (0.8%) VR that is a unifix cube (Figure 5.107).

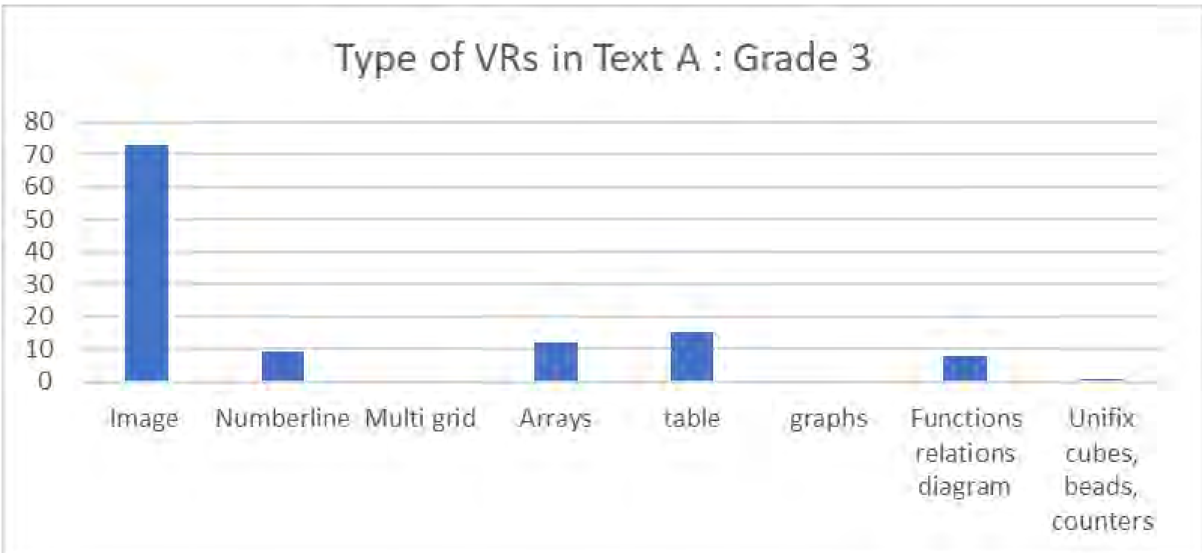


Figure 5.107: Types of VRs (Text A)

The most prominent VRs in Text A are images. In Figure 5.108 the learners in groups at the top of the page provide an example of an image. Figure 5.109 is an example of the second most frequent type of VRs, that is, tables. In this example, the learners are required to multiply the numbers in the top row by four. The top row of the table consists of the input number, the rule is X4 and the output number is recorded in the row below. For example, the first sum would be 1X4 and they would need to fill in the answer 4.

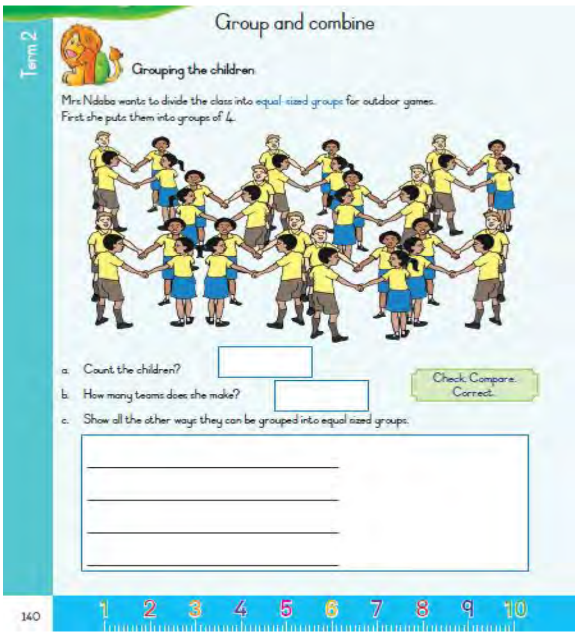


Figure 5.108: An example of an image (Grade 3, Text A, Book 1, p. 140)

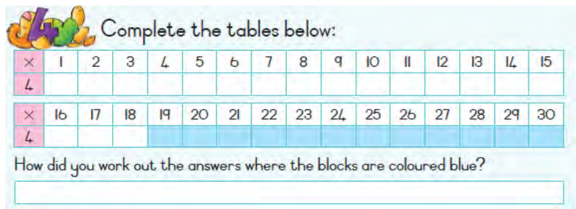


Figure 5.109: An example of a table (Grade 3, Text A, Book 2, p. 41)

The third most prominent VRs in this text is an array. Stott (2016) explains that an array is a set of shapes (or numbers) which appear in a rectangular formation with rows and columns. Figure 5.110 is an example of an array. The title of this page is 'Pave with Tiles'. The explanation along the top and bottom of the arrays explains how the array can be used for calculating.

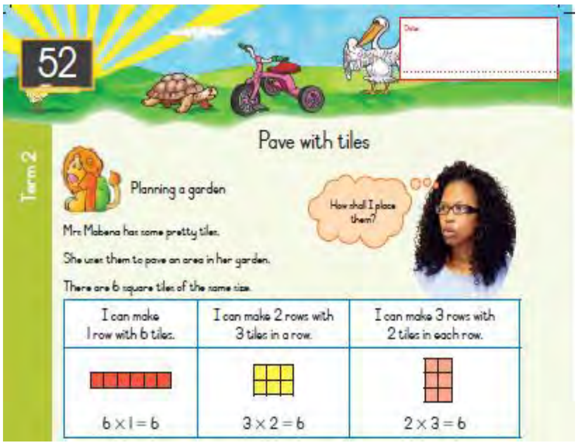


Figure 5.110: An example of an array representation (Grade 3, Text A, Book 1, p. 118)

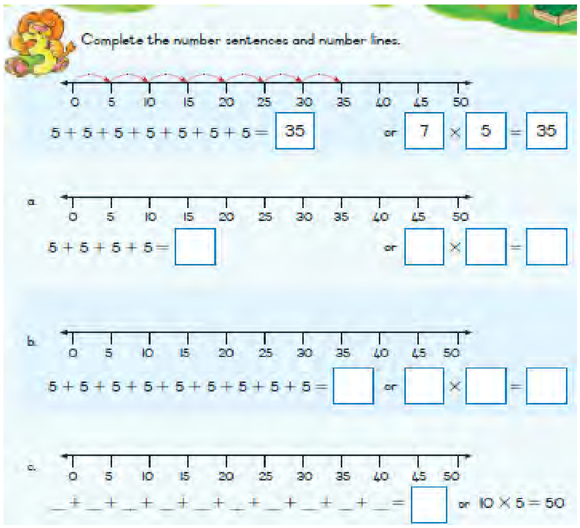


Figure 5.111: An example of a number line (Grade 3, Text A, Book 1, p. 55)

The fourth most prominent type of VRs in Text A is a number line. An example of a number line in Text A for Grade 3 can be found in Figure 5.111. The learners are given the sum in the form of repeated addition and are required to complete the sum on the number line. This exercise assists learners in identifying the link between, skip counting, repeated addition and multiplication.

The fifth most prominent type of VRs is a function relations diagram. Figure 5.112 provides an example of a function diagram. In the first example (top left) of Figure 5.112, the input (numbers on the left) will be multiplied by the rule (i.e., x5) and the output number (in this case, the answer) will be written down in the blank spaces on the right-hand side.

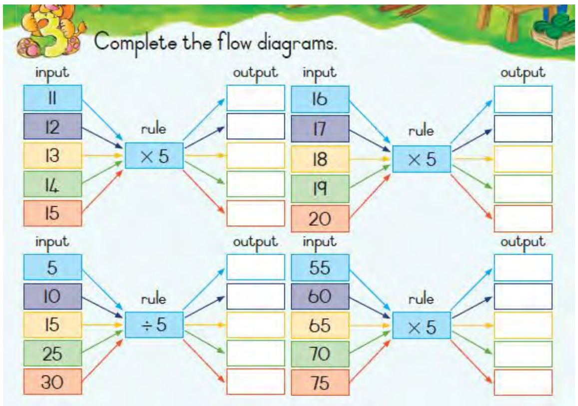


Figure 5.112: An example of a function diagram (Grade 3, Text A, Book 2, p. 29)

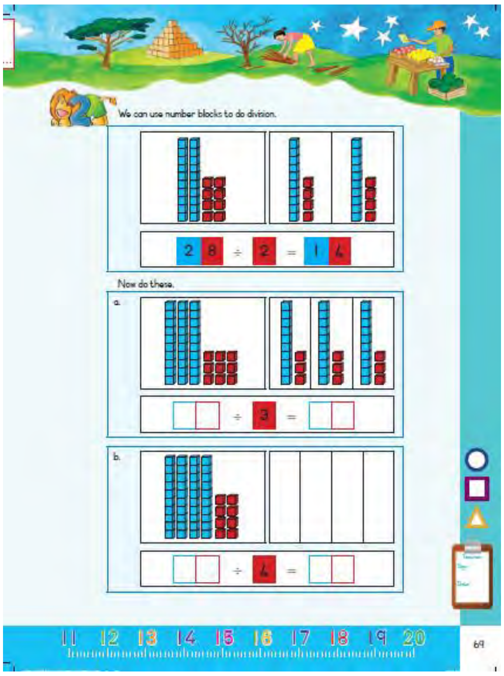


Figure 5.113: An example of unifix cubes (Grade 3, Text A, Book 1, p. 69)

The only example of unifix cubes in Text A appears in Figure 5.113.

5.5.5.3 The visual representations' relation to content

The data presented in Figure 5.114 is of the VRs' relation to content in Text A (80) (100%). Most of the VRs relate directly to the content in the text and have a strong relation to content (75) (94%). This is followed by 2 (2.5%) VRs with a problematic relation to content (type b), 2 (2.5%) VRs with a weak relation to content and 1 (1%) VR with no relation to content.

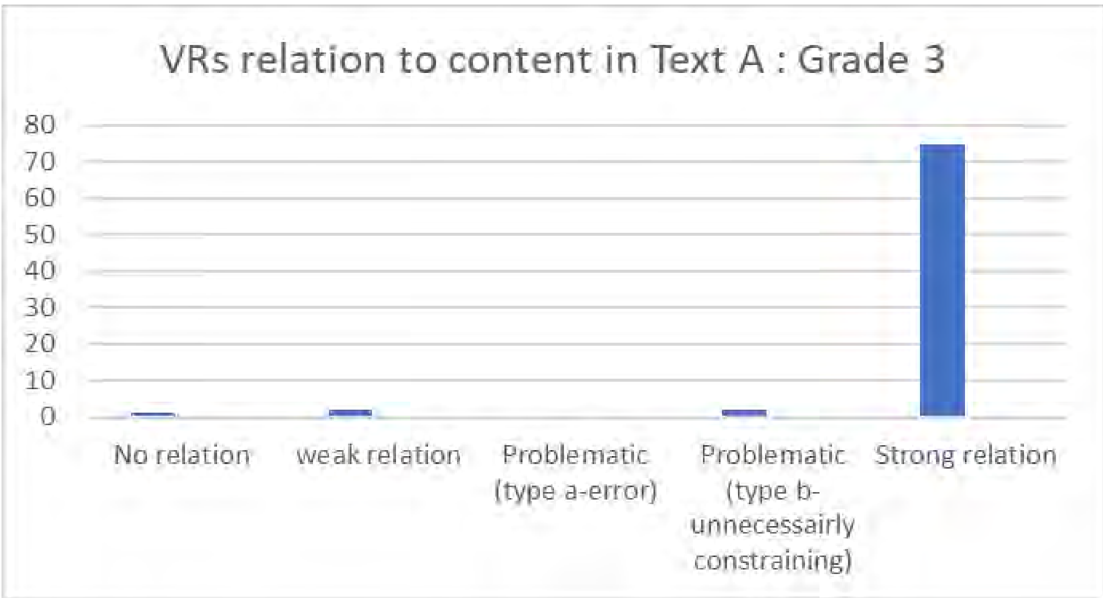


Figure 5.114: Relation to content (Text A)

Figure 5.115 is an example of a VR with a strong relation to content. The VR of the trees are providing explanations of the problem in the format of an array representation. This is a strong relation to the multiplication and division concepts as it shows the inverse relationship between the concepts in the exercise, which makes it a strong relation to content.

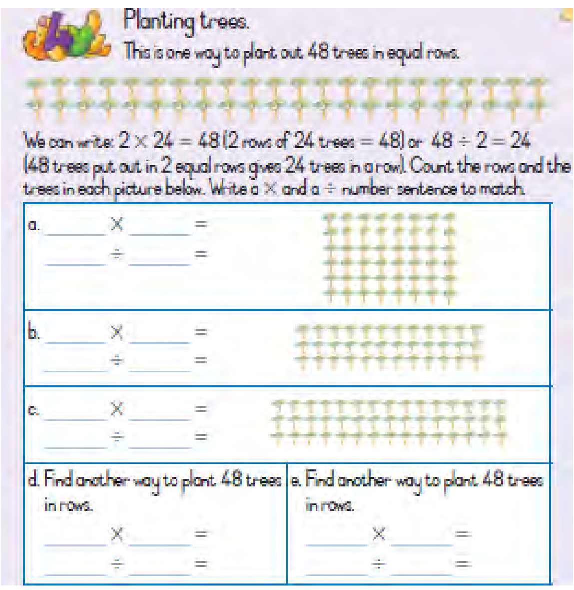


Figure 5.115: An example of a VR with a strong relation to content (Grade 3, Text A, Book 2, p. 109)

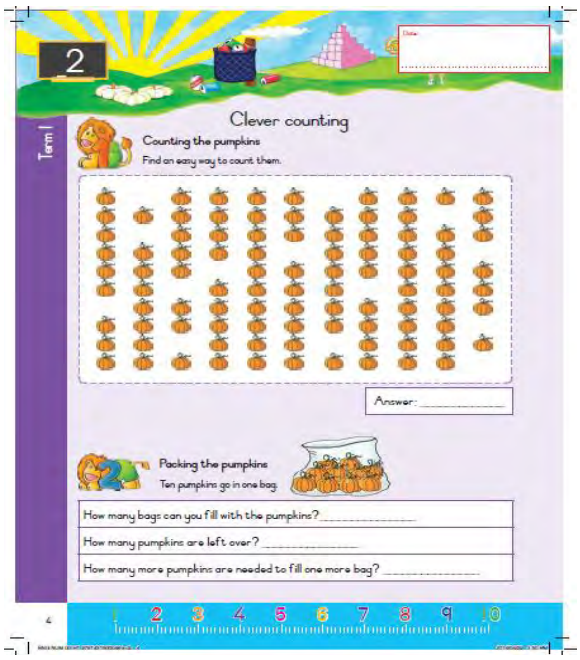


Figure 5.116: An example of a VR with a problematic relation to content (type b) (Grade 3, Text A, Book 1, p. 4)

The third most common relation to content in this text is problematic relation to content (type b). Figure 5.116 shows a problematic relation to content (type b) that has an unnecessary constraint (counting). The learner will need to know to refer to the first exercise which is number 1 (the lion with the number 1 on it) by counting the pumpkins in order to work out exercise 2 (the lion with the number 2 on it).

The third most common relation to content is a weak relation to content. Figure 5.117 is an example of a VR with a weak relation to content. There is only one 5-rand coin for the calculations in this exercise, the learners require more than one 5-rand coin.

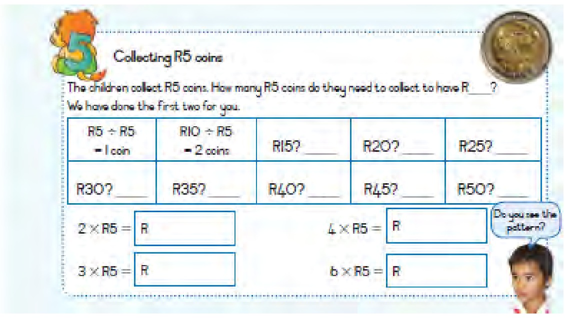


Figure 5.117: An example of a VR with a weak relation to content (Grade 3, Text A, Book 1, p. 121)

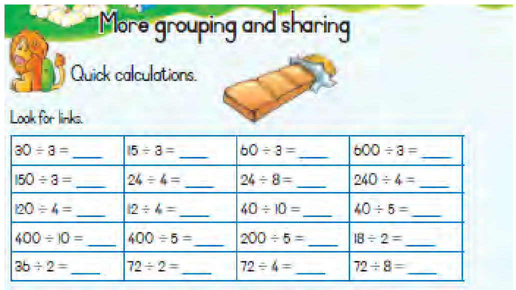


Figure 5.118: An example of a VR with no relation to content (Grade 2, Text A, Book 2, p. 128)

There is 1 VR in this text that has no relation to content. The chocolate in Figure 5.104 does not refer to any of the calculations.

5.5.5.4 The visual representations' relation to reality

The data in Figure 5.119 represents the relation to reality and whether the VRs have a realistic (50) (62%) (Figure 5.121) or metaphoric relation (30) (38%) to reality as seen in Figure 5.120.

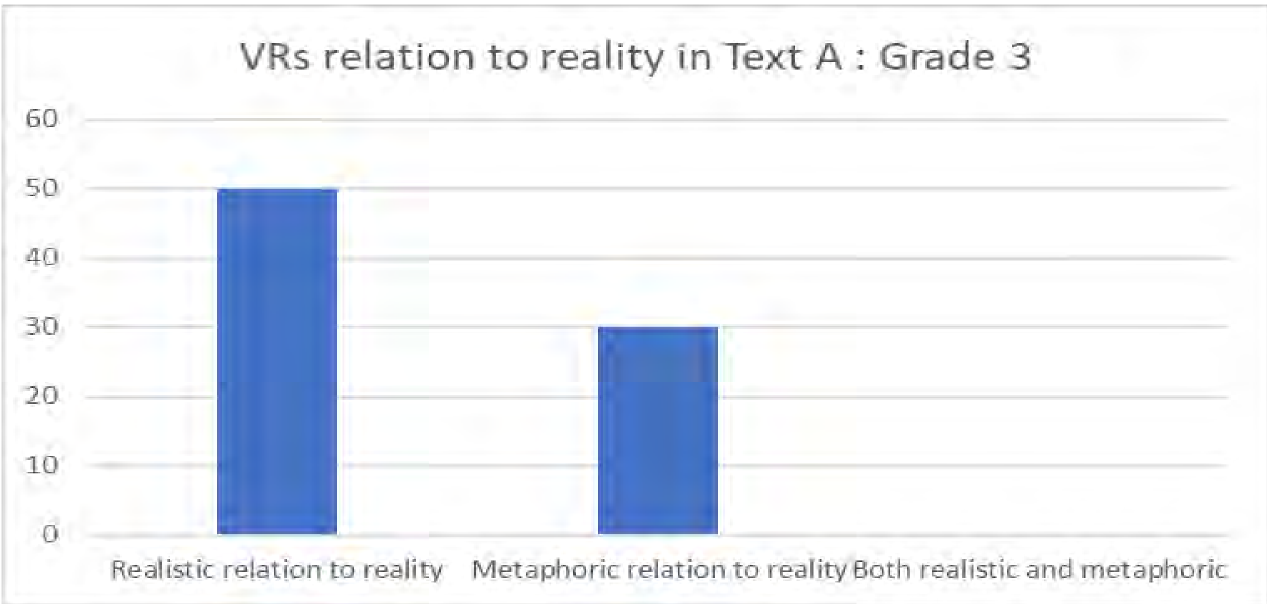


Figure 5.119: Relation to reality (Text A)

In Figure 5.106 the blue circles and crosses have a metaphoric relation to the content as they are shapes.

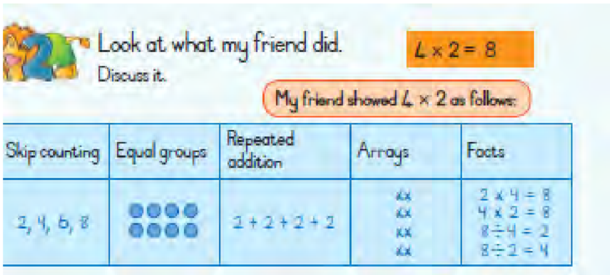


Figure 5.120: An example of a VR that has a metaphoric relation to content (Grade 3, Text A, Book, p. 38)

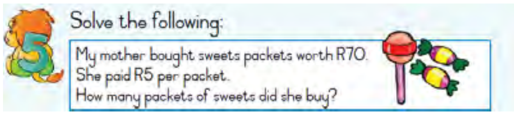


Figure 5.121: An example of a VR with a realistic relation to content (Grade 2, Text A, Book 2, p. 29)

Figure 5.121 is an example of a VR with a realistic representation of reality. The sweets in Figure 5.107 are representations of real objects that the learners can relate to.

5.5.5.5 The function of the visual representations

The most prominent function of the VRs in Text A is an exemplifying function (type b) (57) (58%). Of the 98 VRs, 21 (21%) have a complementary function, 17 (17%) have an exemplifying function (type a), 2 (2%) have an explanatory function and 1 (1%) has a decorative function.

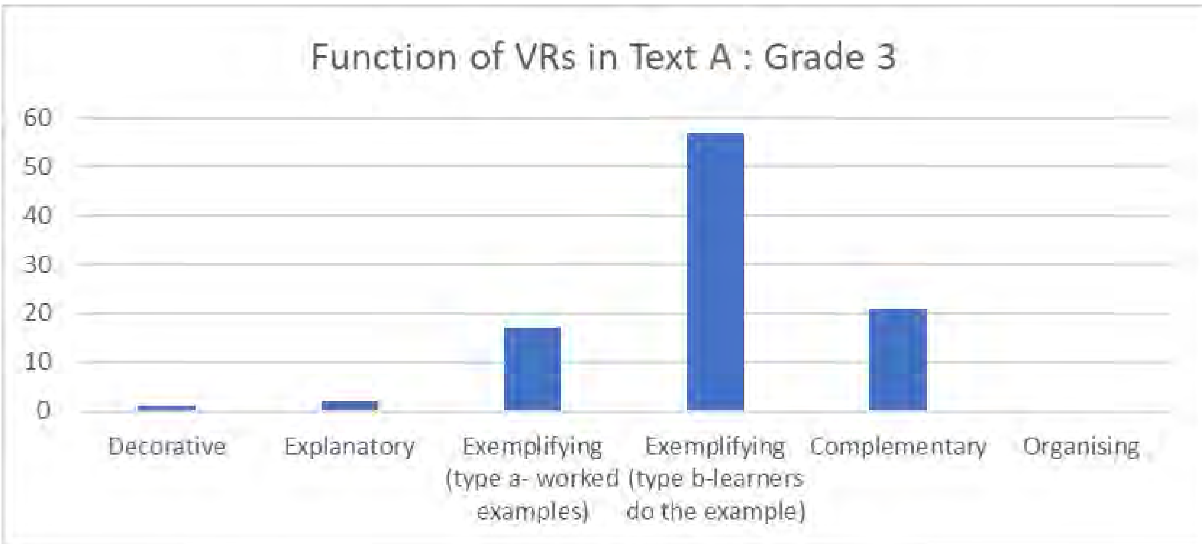


Figure 5.122: Function of VRs (Text A)

The VR in Figure 5.123 serves an exemplifying function (type b) as the learners are required to complete the example themselves. In contrast, the VR in 5.124 has an exemplifying function (type a) as a worked example is provided. The learners are able to draw on this example to complete the exercise "Write how many socks".

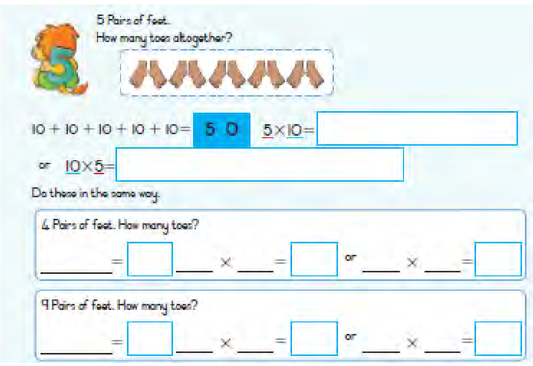


Figure 5.123: An example of a VR with an exemplifying function (type b) (Grade 3, Text A, Book 1, p. 53)

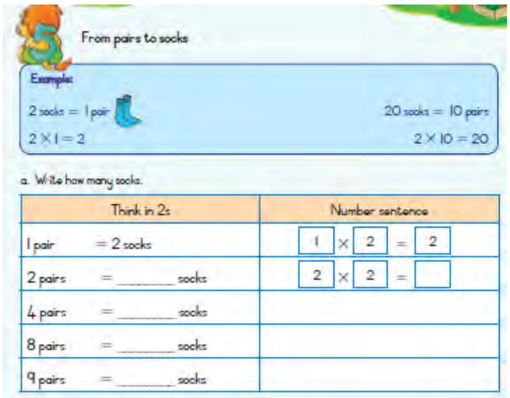


Figure 5.124: An example of a VR with an exemplifying function (type a) (Grade 3, Text A, Book 1, p. 59)

The second most prominent function of the VRs is a complementary function. The sweets in Figure 5.107 have a complementary function. However, if the VR of the sweets was not there the learners would still be able to solve the problem. The third common function is an exemplifying function (type a) as the text contains a worked example (see Figure 5.124).

The fourth most common function is explanatory. An example of an explanatory function can be seen in Figure 5.115 above. This is an explanatory function because there is an explanation of how an array representation works at the top of the page. The VR is combined with a written description of how to perform the calculations. Figure 5.118 is an example of a VR with a decorative function (1%). The chocolate at the top of Figure 5.118 acts as a decoration for the page as it does not link to any of the sums in the exercise.

5.5.5.6 The dimensionality of VRs

In Text A there are 50 VRs that are 2D representations of 3D objects (50) (62%) such as the chocolate bar in Figure 5.121. There are 30 VRs that are 2D representations (30) (38%). The circles in Figure 5.120 are an example of a 2D VR.

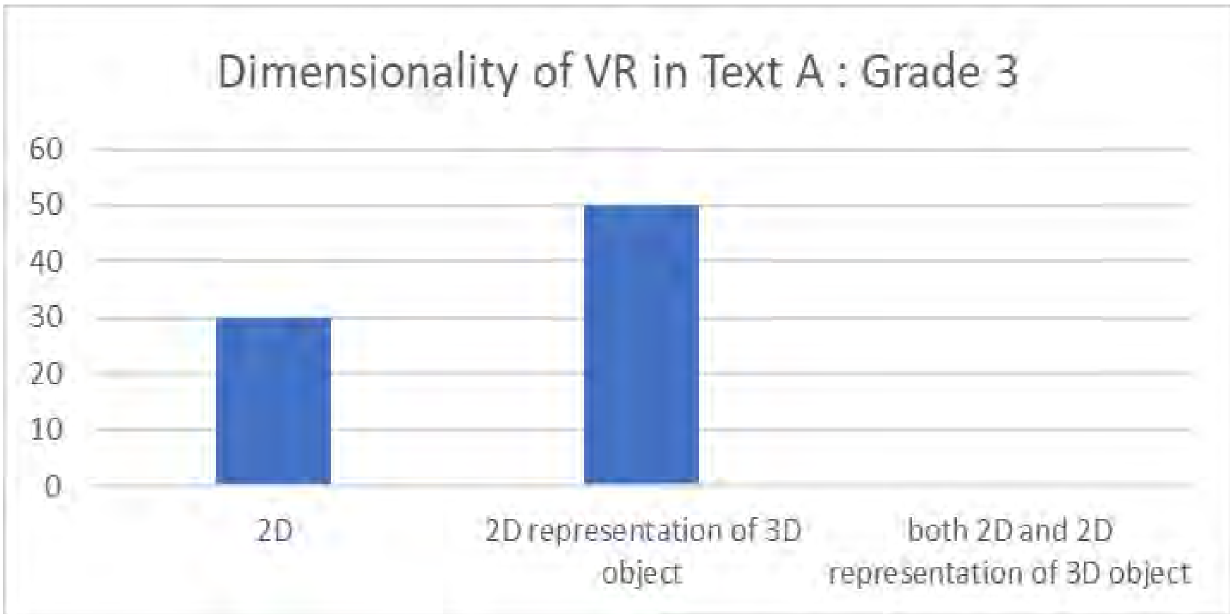


Figure 5.125: 2D representation and 2D representation of a 3D object

5.5.2 Analysis of Text B: Grade 3

The following section presents data that emerged from Text B. Text B consists of 12 (63%) multiplication and 3 (16%) division exercises. There are 1 partitive and 3 quotative division exercises in Text B. There are 4 (21%) VRs that support the development of an understanding of both multiplication and division (see Figure 5.126).

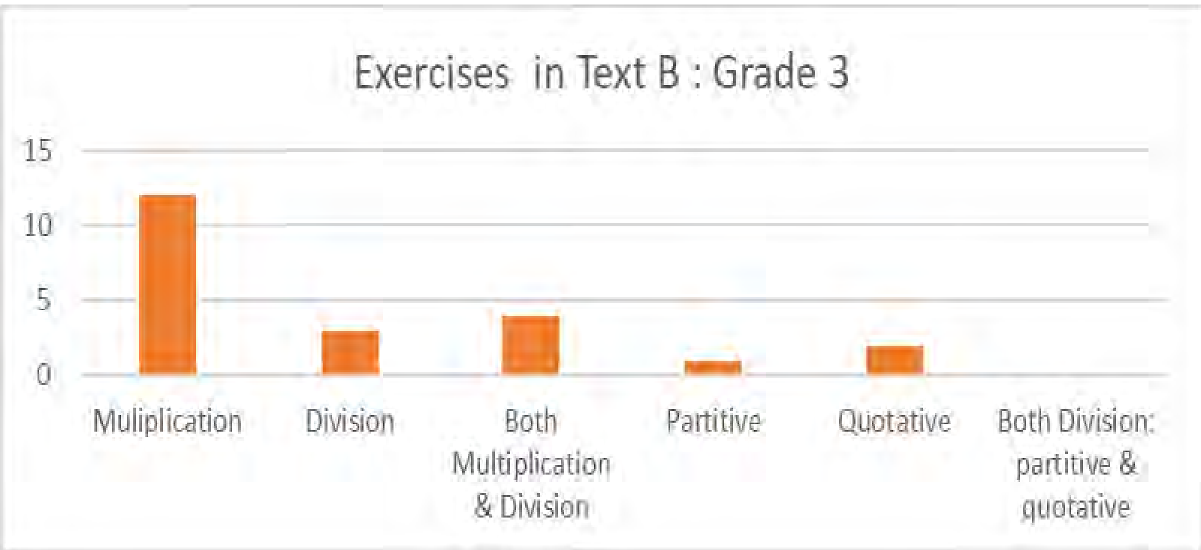


Figure 5.126: Multiplication and division exercises (Text B)

5.5.2.1 Type of visual representations

As illustrated in Figure 5.127, there are 19 images (70%), 3 (11%) number lines, 2 (7%) arrays, 2 (7%) tables and 1 (4%) unifix cube.

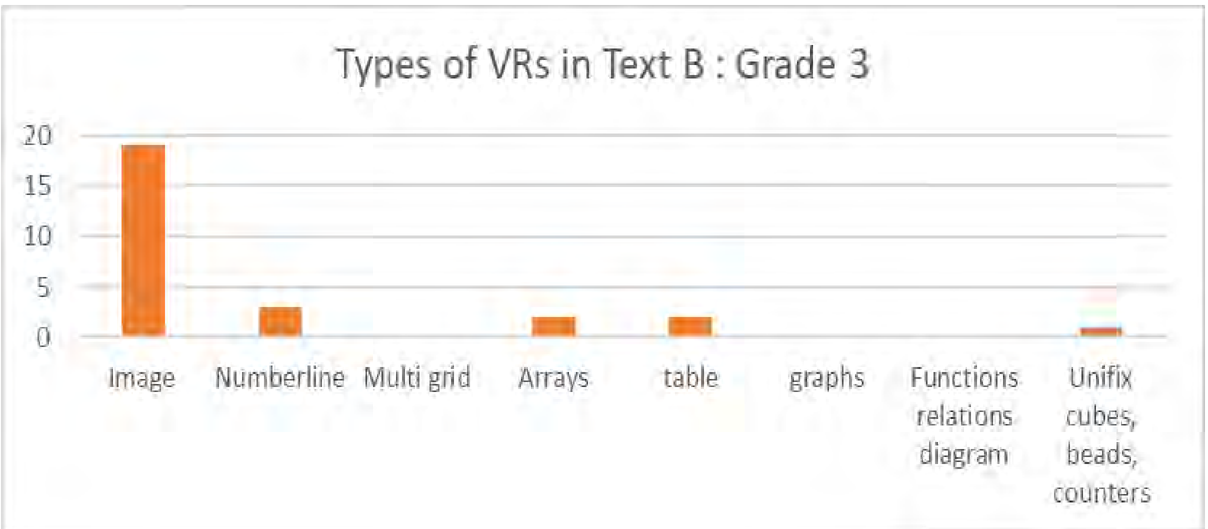


Figure 5.127: Type of VRs (Text B)

As highlighted above, the most prominent type of VR is an image. The party packs in Figure 5.128 and bananas in Figure 5.129 are examples of images found in Text B.

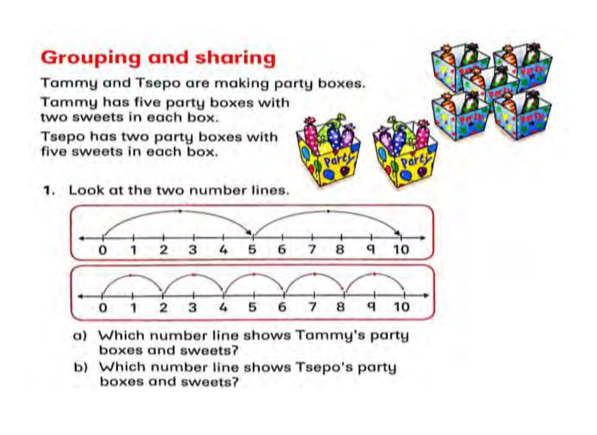


Figure 5.128: An example of an image (Grade 3, Text B, p. 39)

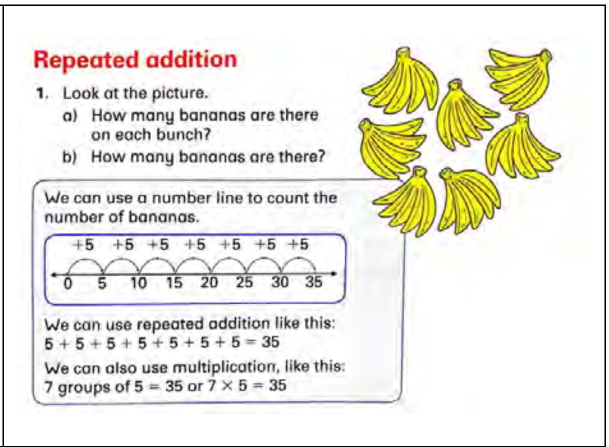


Figure 5.129: An example of a number line (Grade 3, Text B, p. 38)

The second most common VR is the number line. A number line can be explained as a horizontal line with numbers that are usually read from left to right (Gellert & Steinberg, 2014). An example of a number line can be found in Figures 5.128 and 5.129. In the example in Figure 5.115, the number line is used to represent the bananas. In this example, the intervals are made up of 5 units and the learners must add 5, 7 times, to get to the answer. This VR represents multiplication as skip counting and repeated addition.

The third most prominent type of VR in Text B is array representation. Figure 5.130 is an example of an array. The flowers are organised in an array representation of 2 rows and 4 columns.

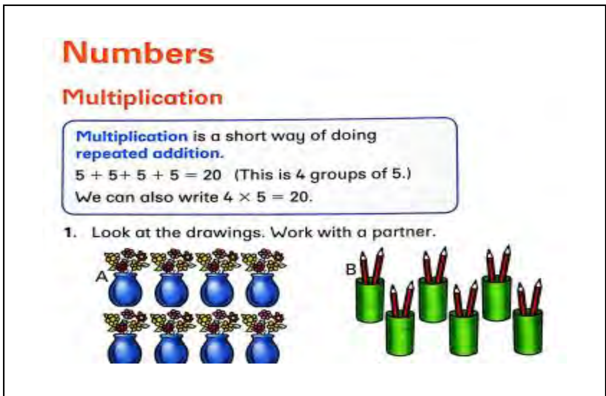


Figure 5.130: An example of an array representation (Grade 3, Text B, p. 60)

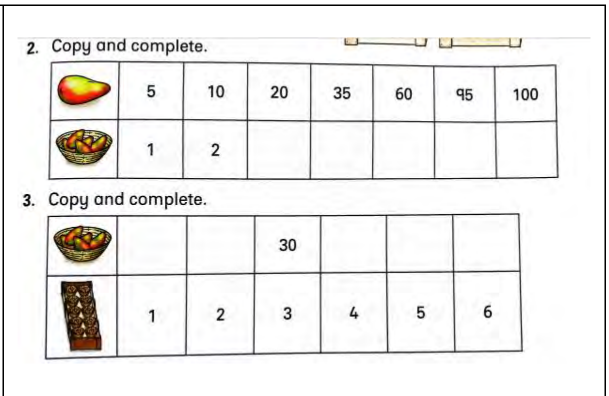


Figure 5.131: An example of a table (Grade 3, Text B, p. 83)

There are 2 VRs that are tables in Text B. These are on the same page as indicated in Figure 5.131. The least common type of VRs in Text B are unifix cubes. An example of an unifix cube can be seen in Figure 5.132.

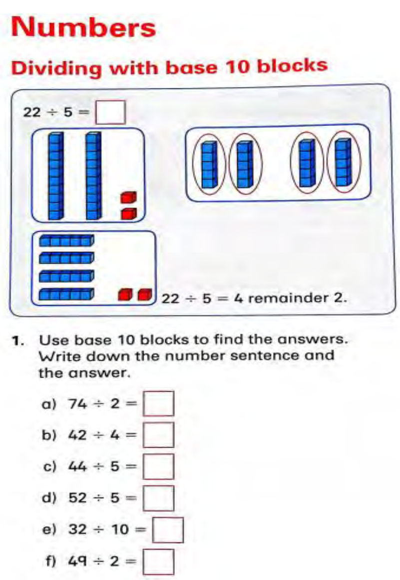


Figure 5.132: An example of unifix cubes (Grade 3, Text B, p. 40)

5.5.2.2 The visual representations' relation to content

Most of the VRs in Grade 3, Text B, have a strong relation to content (16) (84%). However, there are 2 (11%) VRs that are problematic relation to content (type a) and 1 (5%) VR with a problematic relation to content (type b) as seen in Figure 5.133.

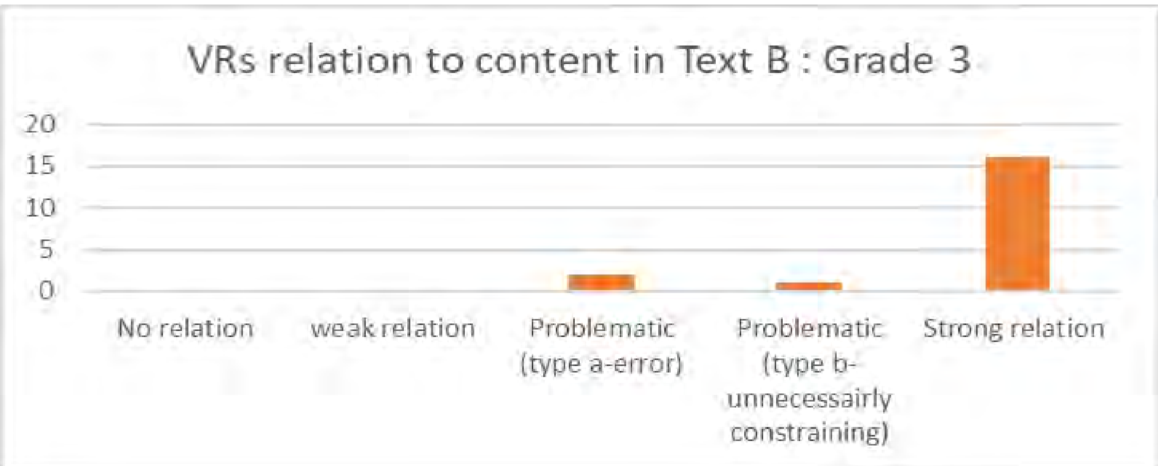


Figure 5.133: Relation to content in Grade 3 in text B

Figure 5.134 is an example of a VR that has a strong relation to content. A strong relation to content is one in which the VR has a direct relationship to the content in the text. In the example in Figure 5.120, the counters are used to support the explanation of repeated addition, repeated subtraction, multiplication and division and their inverse relationships.

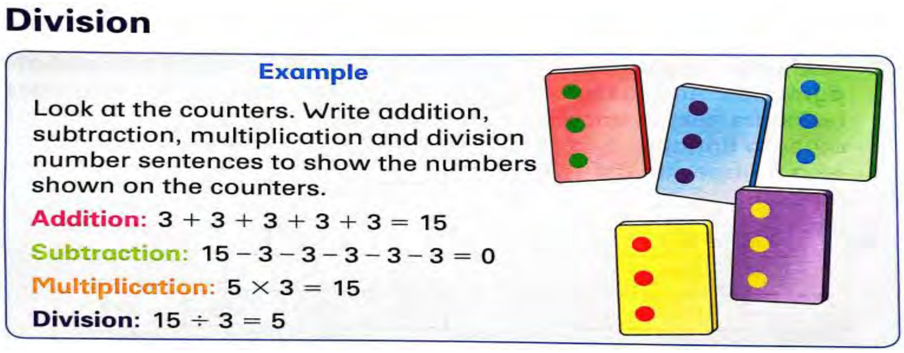


Figure 5.134: An example of a VR with a strong relation to content (Grade 3, Text B, p. 82)

Figure 5.135 is an example of a VR that has a problematic relation to content (type a). A VR that is problematic relation to content (type a) is one that has an error. The VR only has 2 boxes of mangoes as an example of what a box of mangoes looks like. The exercises a to c then use numbers (4, 20, 15 baskets) that differ from those represented in the picture (2 boxes). This has the potential to be problematic as a learner who is still calculating at a semi-concrete level will need to count the pictures to get to the answer.

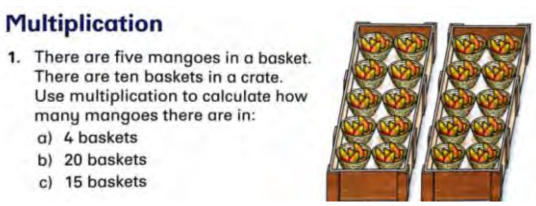


Figure 5.135: An example of a problematic relation to content (type a) (Grade 3, Text B, p. 83)

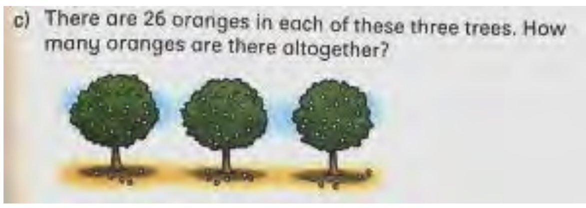


Figure 5.136: An example of a problematic relation to content (type b) (Grade 3, Text B, p. 83)

Figure 5.136 provides a problematic relation to content (type b) in that it has an unnecessary constraint. The problem states that the 26 oranges are in the tree. However, there are only 20 oranges in each tree and 6 lying underneath each of the trees. While the number of oranges associated with one tree is 26, they are not all in the tree. This may cause confusion as learners may only count the 20 oranges in the tree.

5.5.2.3 The visual representations' relation to reality

In Text B, 15 (79%) of the VRs can have a realistic relation to reality and 4 (21%) have a metaphoric relation to reality as shown in Figure 5.137.

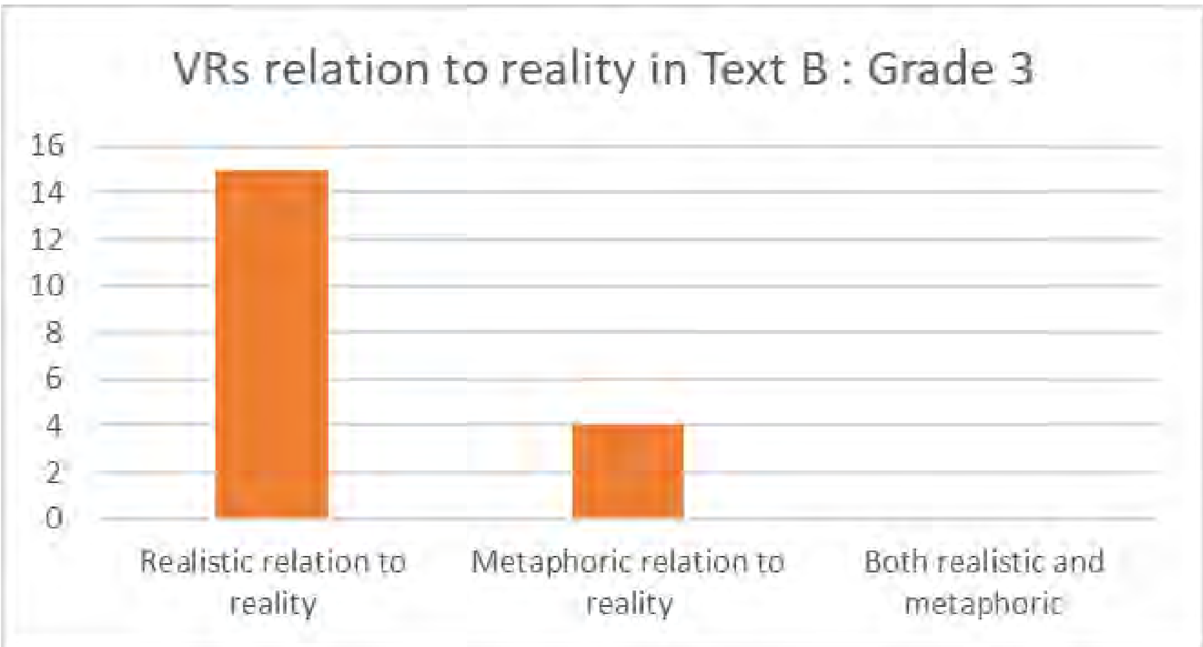


Figure 5.137: Relation to reality VRs (Text B)

A realistic relation to reality is when the VRs represent real-life objects. The images of flowers, pencils, eggs, grapes and fish in fish bowls all have a realistic representation of reality (Figure 5.138).

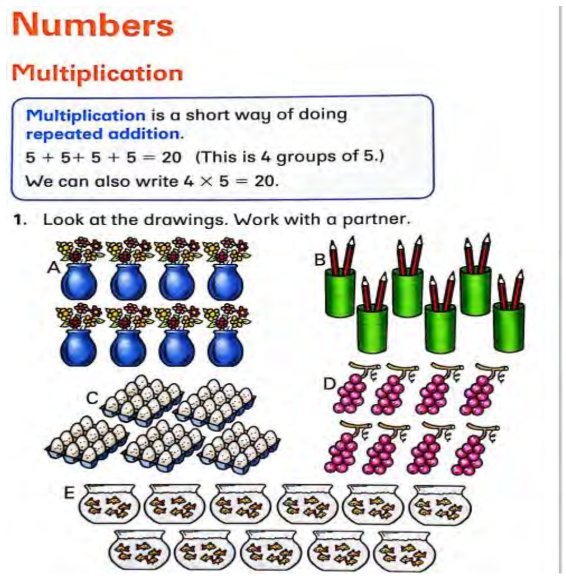
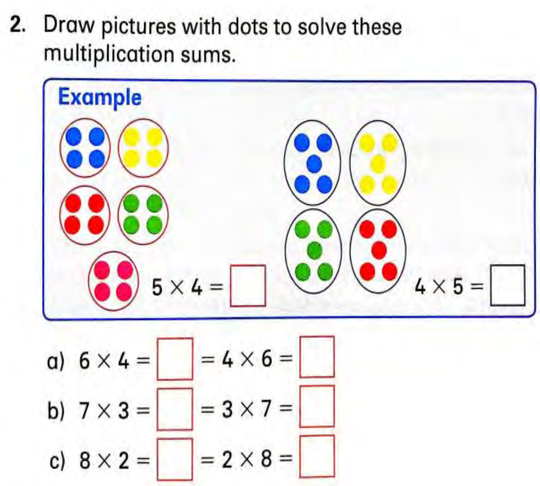
	

Figure 5.138: An example of a VR with a realistic relation to reality (Grade 3, Text B, p. 60)	***Figure 5.139: An example of a VR with a metaphoric relation to reality (Grade 3, Text B, p. 61)***

An example of a metaphoric relation to reality can be seen in Figure 5.139. This VR is metaphoric as the dots do not represent real objects.

5.5.2.4 The function of visual representations

Of the 23 VRs in Text B, there are 10 (43%) VRs with an exemplifying function (type b), 5 (22%) with an exemplifying function (type a), 5 (22%) with an explanatory function and 3 (13%) with a complementary function as seen in Figure 5.140.

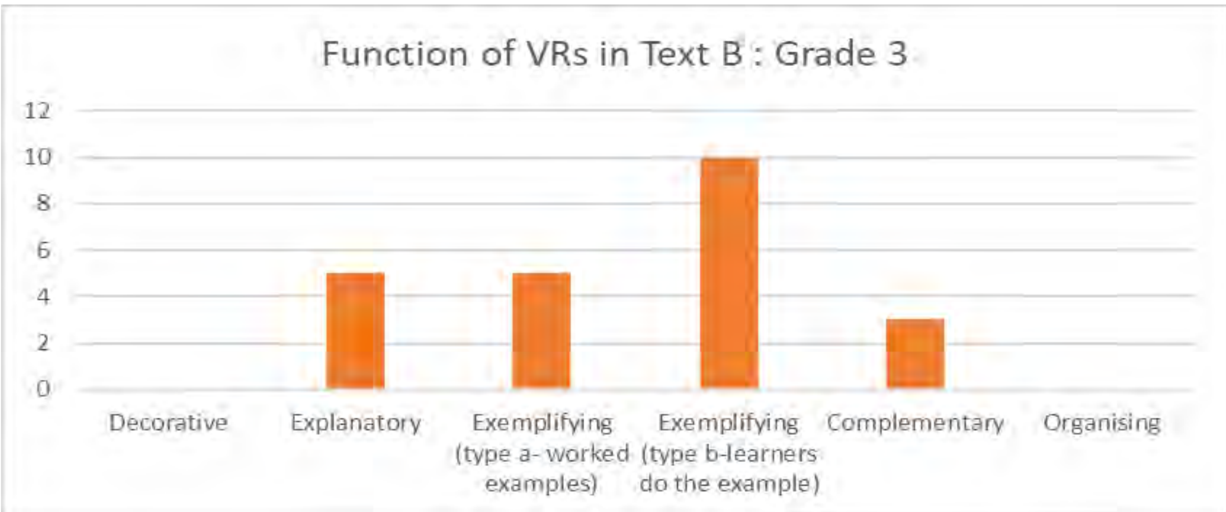
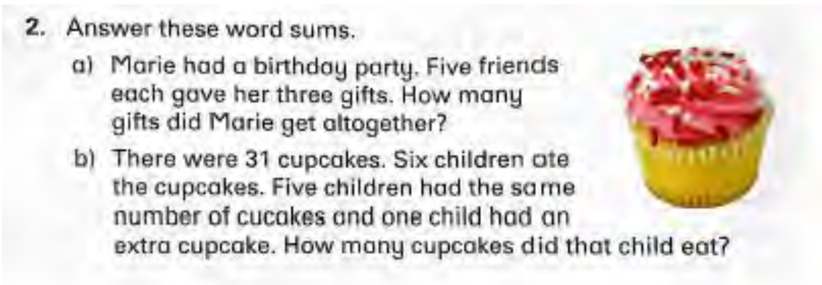


Figure 5.140: Function of VRs in Grade 3 in text B

The most common function of VRs in this text is exemplifying (type b) function. Figure 5.138 no. 1a to 1e is an example of an exemplifying function (type b) where learners need to complete the exercise independently. This is followed by exemplifying function (type a) (see Figure 5.139). An example of an exemplifying function (type a) can be found in Figure 5.139. The dots in the picture are arranged to illustrate the inverse of the multiplication sum (5x4 and 4x5).

An explanatory function is represented in Figure 5.134. In this example, the counters are supported by an explanation to demonstrate repeated addition, repeated subtraction, multiplication and division. Figure 5.141 is an example of a VR that has a complementary function. A VR with a complementary function provides information that is not necessary to solve the problem, in other words, it only provides context. The VR of the cupcake has a complementary function for exercise 2b. To work out the answer, the VR is not required as it does not supply any new information that is not in the word problem.


2. Answer these word sums.

a) Marie had a birthday party. Five friends each gave her three gifts. How many gifts did Marie get altogether?

b) There were 31 cupcakes. Six children ate the cupcakes. Five children had the same number of cucakes and one child had an extra cupcake. How many cupcakes did that child eat?

Figure 5.141: An example of a VR with a complementary function (Grade 3, Text B, p. 88)

5.5.2.5 The dimensionality of VRs

There are 4 (21%) VRs that are 2D representations and 15 (79%) VRs that are 2D representations of 3D objects.

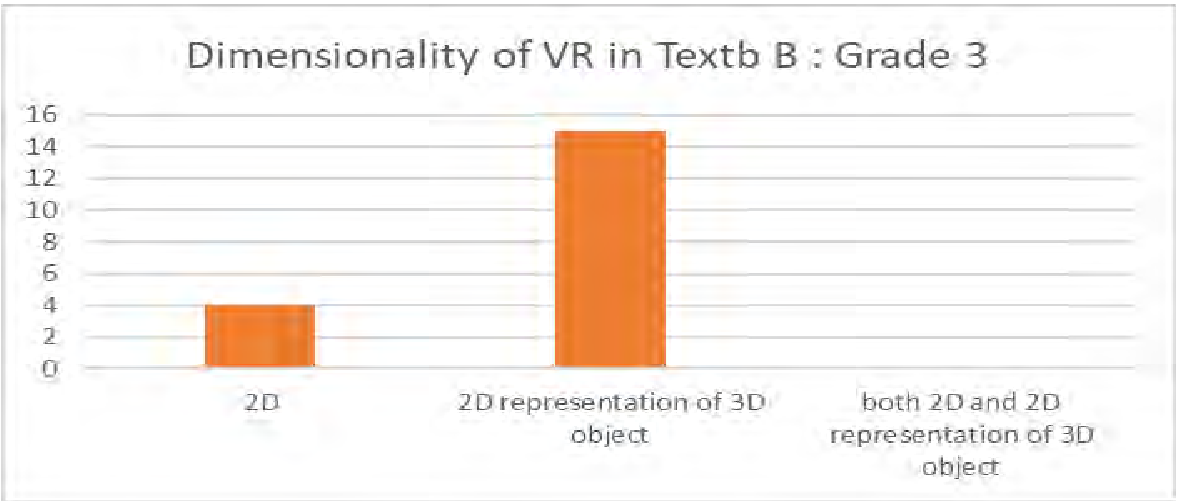


Figure 5.142: 2D representation and 2D representation of 3D objects

The majority of VRs in Text B are 2D representations of 3D objects. In Figure 5.136 the trees are 2D representations of 3D objects. By contrast, the dots in Figure 5.139 are examples of 2D representations.

5.5.3 Analysis of Text C: Grade 3

The following section presents data that has emerged from Text C. This text consists of VRs that focus on multiplication (8) (19%) and division (32) (76%). There are 2 (5%) VRs that promote both multiplication and division. Text C has 19 partitive and 13 quotative division exercises (Figure 5.143).

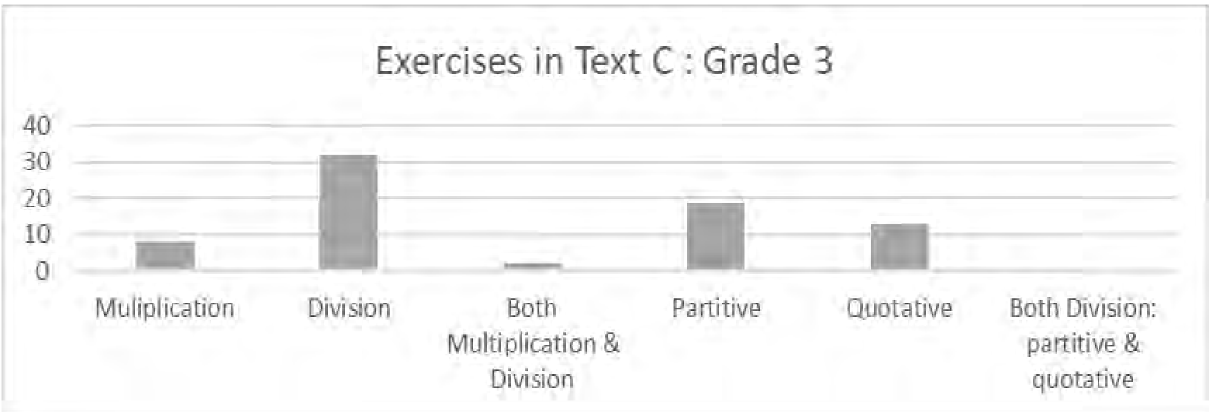


Figure 5.143: Multiplication and division exercises (Text C)

5.5.3.1 Type of visual representations

Of the 200 (100%) different types of VRs in this text, 33 (60%) are images and 9 (16%) are number lines. In addition, there are 6 (11%) arrays, 6 (11%) tables and 1 (2%) function diagram as illustrated in Figure 5.144.

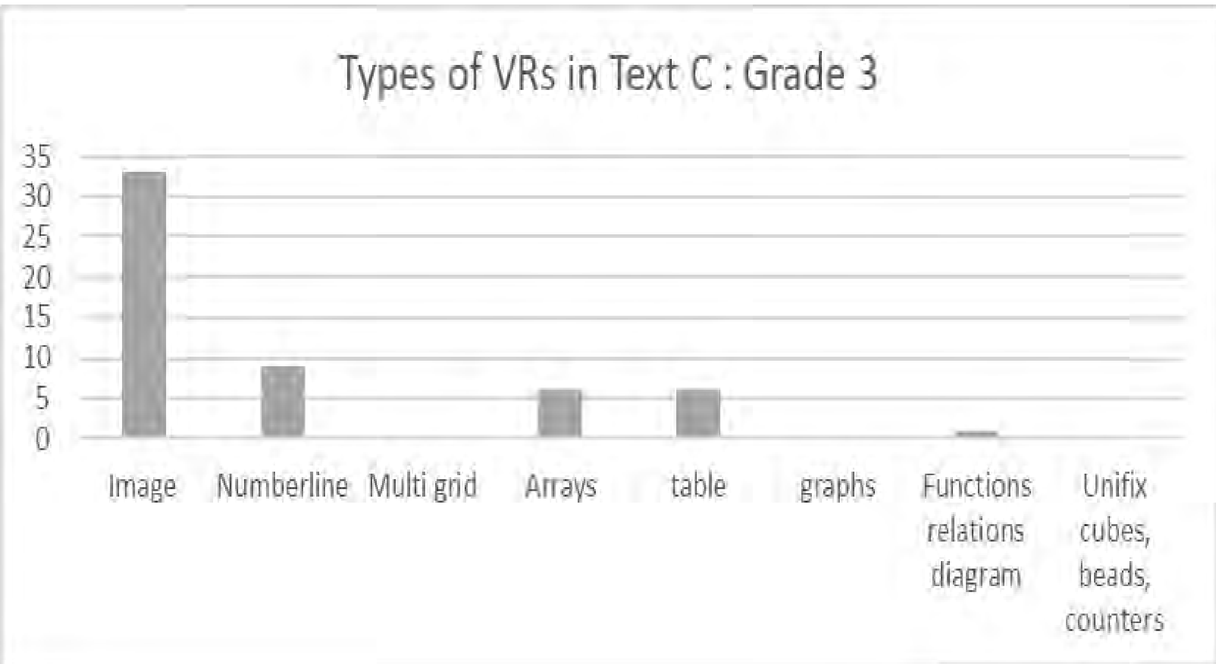


Figure 5.144: Types of VRs (Text C)

The most prominent type of VRs in Text C of Grade 3 is an image (60%). Figure 5.145 consists of images of kites, books, rockets, air balloons and birds. The second most prominent VR is the number line (16%). An example of a number line can be found in Figure 5.146. The number line makes use of repeated addition as a basis for multiplication. Using the number line, the learner is able to complete both the repeated addition and multiplication and form links between skip counting, repeated addition and multiplication.

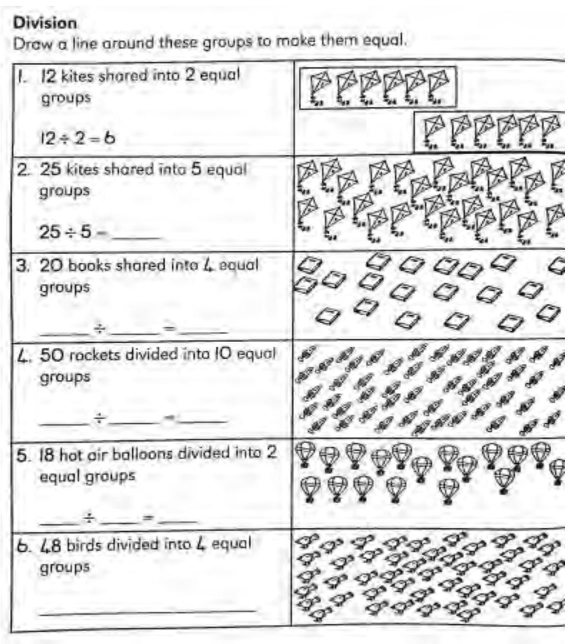


Figure 5.145: An example of an image (Grade 3, Text C, p. 36)

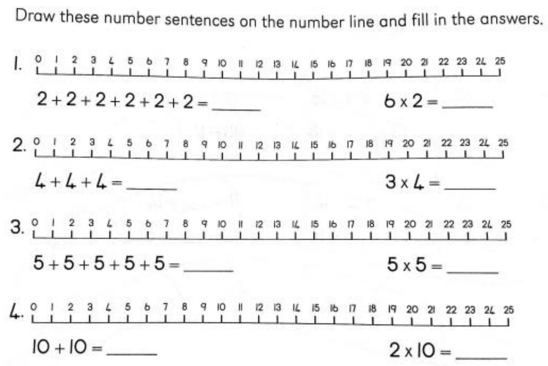


Figure 5.146: An example of a number line (Grade 3, Text C, p. 26)

The third most common type of VR is an array. An example of an array VR can be found in Figure 5.147. This array consists of 3 rows and 4 columns with 3 dots in each column. The multiplication and division sums on the left-hand side of the VR link to the array and demonstrate the relationship between multiplication and division. The fourth most prominent type of VRs is a table. Figure 5.148 is an example of a table in which the learners need to practice division.

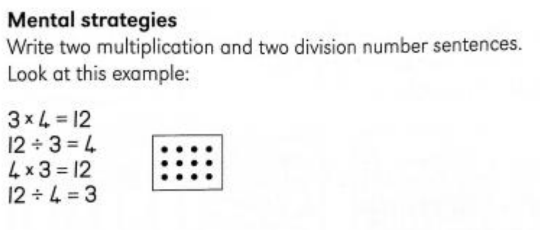
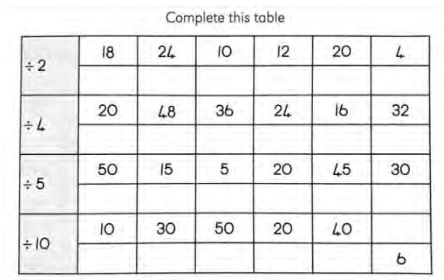


Figure 5.147: An example of an array representation (Grade 3, Text C, p. 68)


Complete this table

÷ 2	18	24	10	12	20	4
÷ 4	20	48	36	24	16	32
÷ 5	50	15	5	20	45	30
÷ 10	10	30	50	20	40	
						6

Figure 5.148: An example of a table division (Grade 3, Text C, p. 37)

In FP texts a function diagram may appear in the form of spider diagrams as presented in 5.149. CAPS refers to function diagrams as spider diagrams (DBE, 2011a). In Text C they are the least common type of VRs. In Figure 5.149 the first number 16 is the input which needs to be doubled (the rule) to get the answer (output).

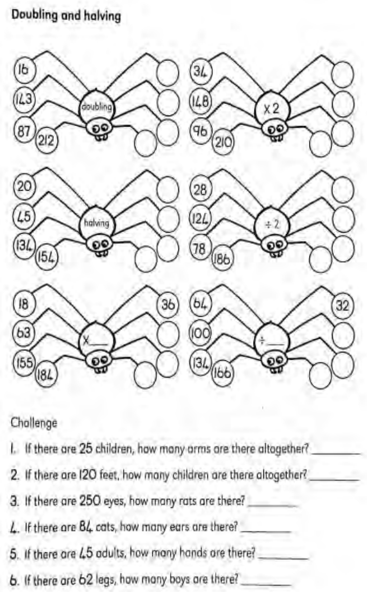


Figure 5.149: An example of a function relations diagram-spider diagram (Grade 3, Text C, p. 23)

5.5.3.2 The visual representations' relation to content

Text C has 39 (93%) VRs with a strong relation to content and 3 (7%) with a problematic relation to content (type b) as indicated in Figure 5.150.

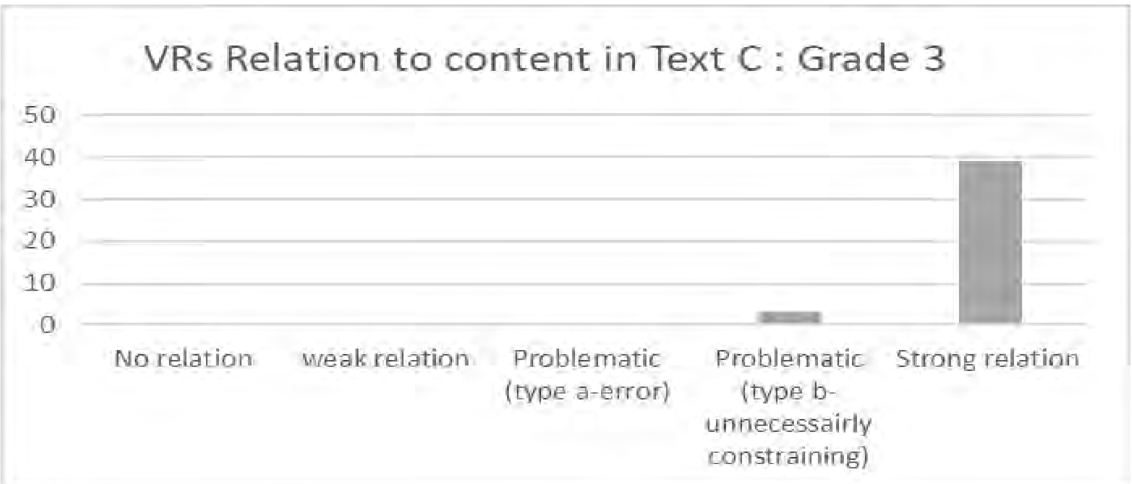


Figure 5.150: Relation to content (Text C)

Figure 5.151 is an example of a VR with a strong relation to content as the VRs (aeroplanes, stars, suns and moons) assist in solving the division problem.

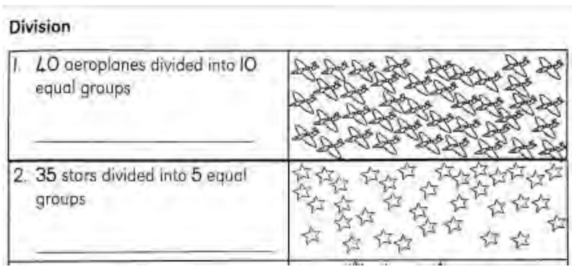


Figure 5.151: An example of a VR with a strong relation to content (Grade 3, Text C, p. 37)

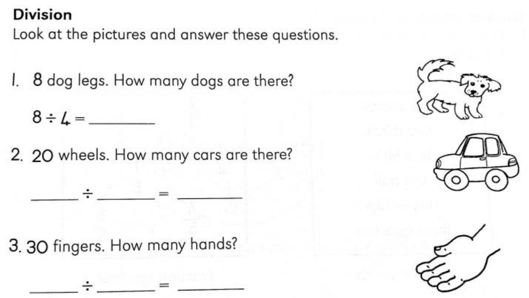


Figure 5.152: An example of a VR with a problematic relation to content (type b) (Grade 3, Text C, p. 9)

There are VRs that have a problematic relation to content (type b). A problematic relation to content (type b) VR is one which has an unnecessary constraint. An example of a VR that has a problematic relation to content (type b) can be found in Figure 3.152. In this example, the VR on the right-hand side does not correlate with the numbers in each word problem and may cause confusion for the learners when they try to solve the problem.

5.5.3.3 The visual representations' relation to reality

In Text C there are 28 (67%) VRs with a realistic relation to reality and 14 (33%) with a metaphoric relation to reality as noted in Figure 5.153.

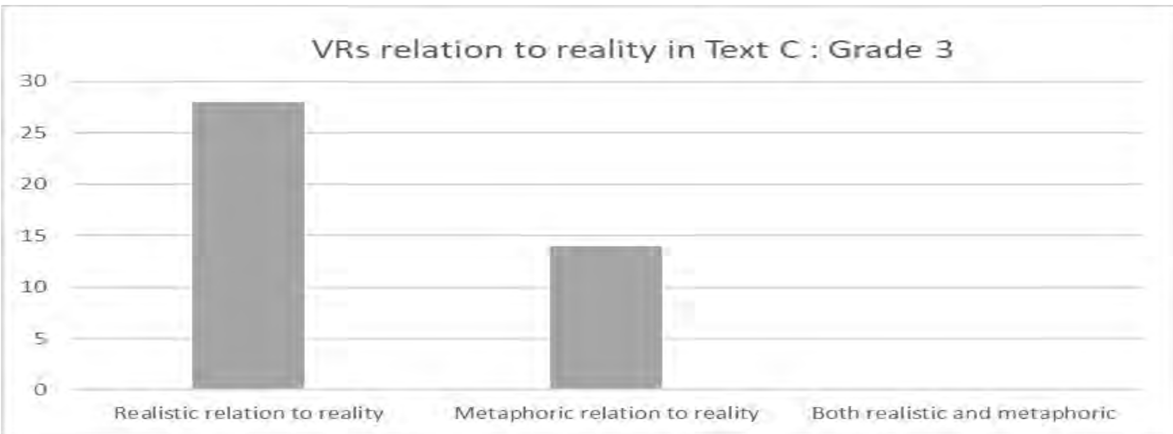


Figure 5.153: Relation to reality (Text C)

Figure 5.154 from Text C is an example of a VR with a realistic relation to content. The image of the fish is a realistic one.

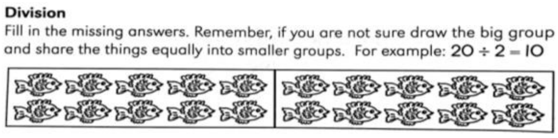


Figure 5.154: An example of a realistic VR (Grade 3, Text C, p. 38)

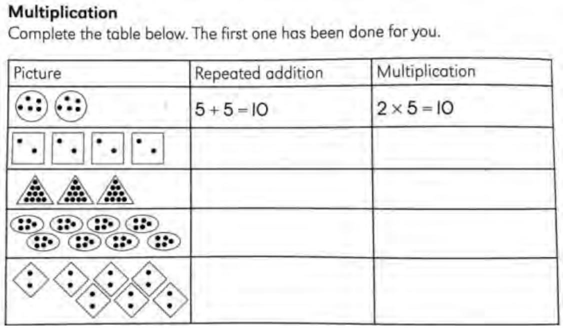


Figure 5.155: An example of a metaphoric VR (Grade 3, Text C, p. 8)

The dots in each shape in the example in Figure 5.155 have a metaphoric relation to reality.

5.5.3.4 The function of the visual representations

Each VR in the text has a particular function. There are 47 VRs in Text C. In this text there are 36 (76%) VRs with an exemplifying function (type b), 6 (13%) VRs with an exemplifying function (type a), 3 (6%) VRs with a complementary function and 2 (4%) VRs with an explanatory function (Figure 5.156).

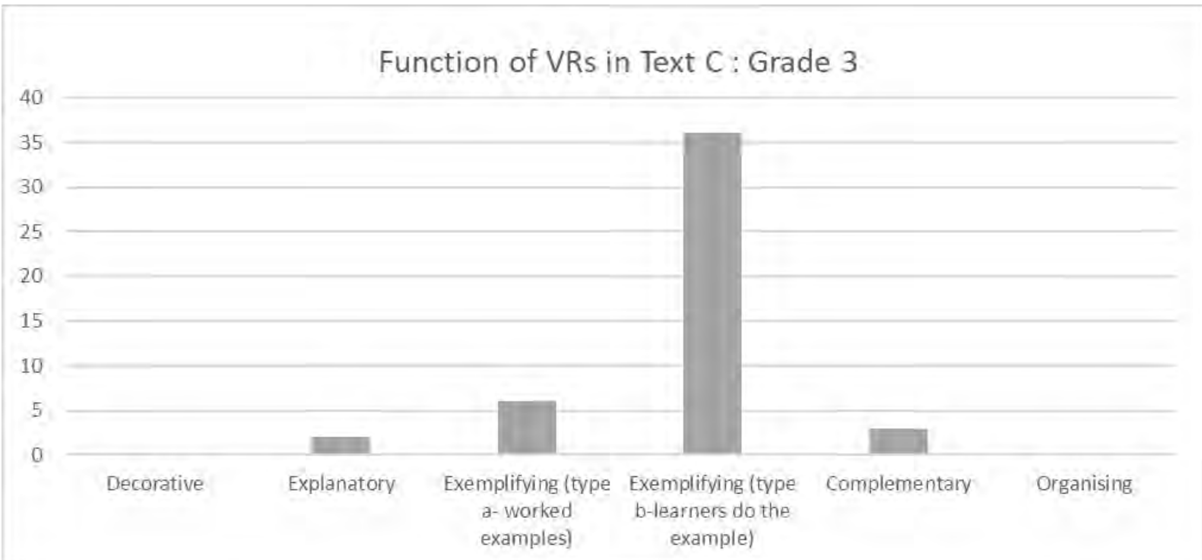


Figure 5.156: Function of VRs (Text C)

Figure 5.151 provides an example of a VR with an exemplifying function (type b). The learners are provided with an exercise that they need to complete. Figure 5.157 provides an example of a VR with an exemplifying function (type a). Figure 5.155 provides a worked example of how the learner will possibly need to solve the problem (5+5 and 2x5). The VR is a worked example of the concept being taught.

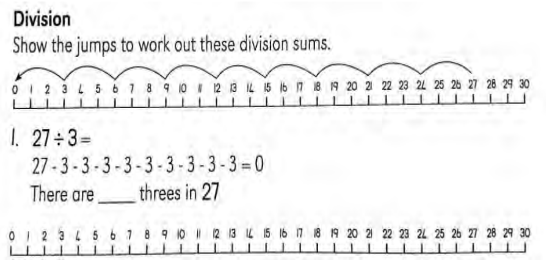


Figure 5.157: An example of a VR with an exemplifying function (type a) (Grade 3, Text C, p. 64)

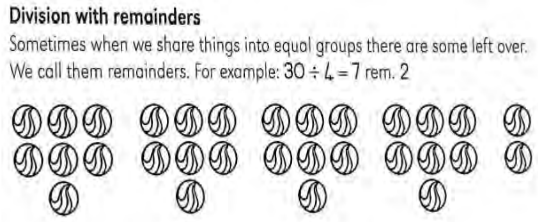


Figure 5.158: An example of VR with an explanatory function (Grade 3, Text C, p. 39).

Figure 5.152 is an example of a VR with a complementary function. The VR is an illustration of a 2D representation of a 3D object (dog, car, fingers) that accompanies the word problem on the left. However, the word problem is solved without the use of VRs.

The third most prominent VR has an explanatory function (Figure 5.158). A VR with an explanatory function is one where the text explains the problem and how the learner should solve it. The paragraph and VR of the marbles are meant to assist in explaining the problem to the learners.

5.5.3.5 The dimensionality of VRs

Text C consists of 14 (33%) 2D representations and 28 (67%) 2D representations of 3D objects (Figure 5.159). The majority of the VRs are 2D representations of a 3D object. The marbles presented in Figure 5.159 above is an example of a 2D representation of a 3D object. The dots in Figure 5.134 is an example of a 2D representation.

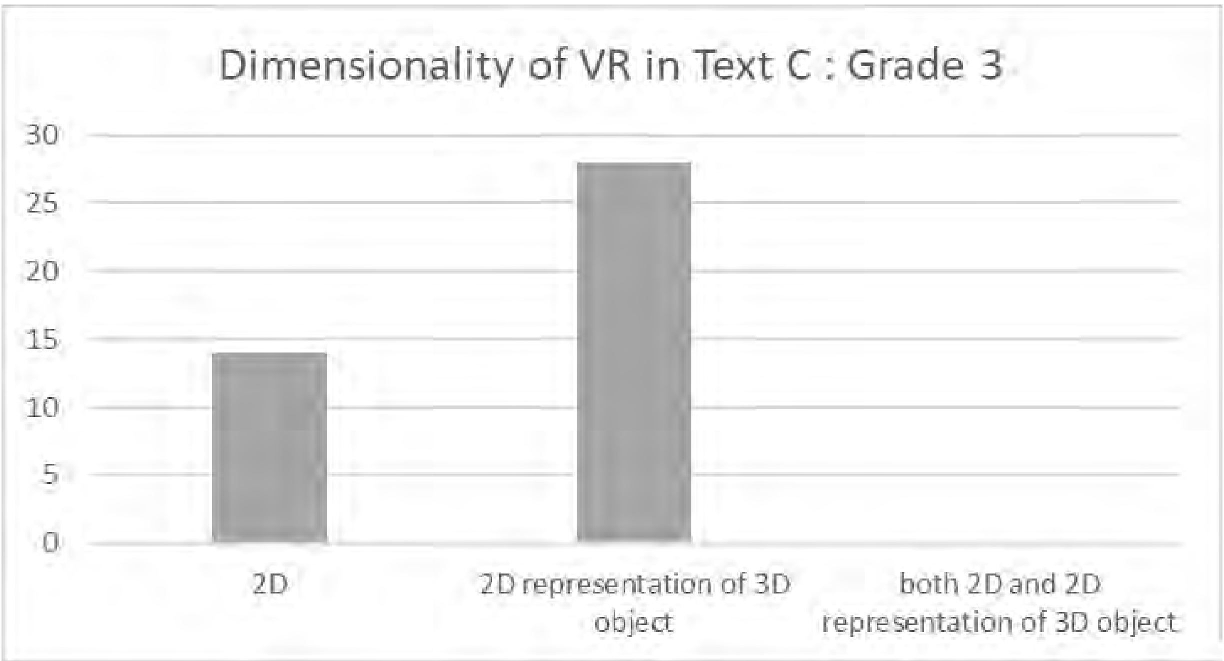


Figure 5.159: 2D representation and 2D representation of a 3D object (Text C)

5.5.3.6 Data analysis across the three Grade 3 texts

This section presents the analysis of Text A, B and C in Grade 3 on multiplication and division. There are 144 (100%) VRs in the exercises in the texts of which there are 76 (54%) VRs that are multiplication exercises and 48 (34%) that are division exercises across the three texts. There are 48 division exercises (28 partitive exercises and 20 quotative exercises). There are 17 (12%) exercises consisting of both multiplication and division. This is the first grade across all three texts where such mixed exercises appear.

As evident in Figure 5.160, Text A has significantly more multiplication exercises (56) (70%) than Text B (12) (63%) and C (8) (19%). There are fewer division exercises across the three texts, Text C has the most exercises (32) (76%). Followed by Text A (13) (16%) and B (3) (16%).

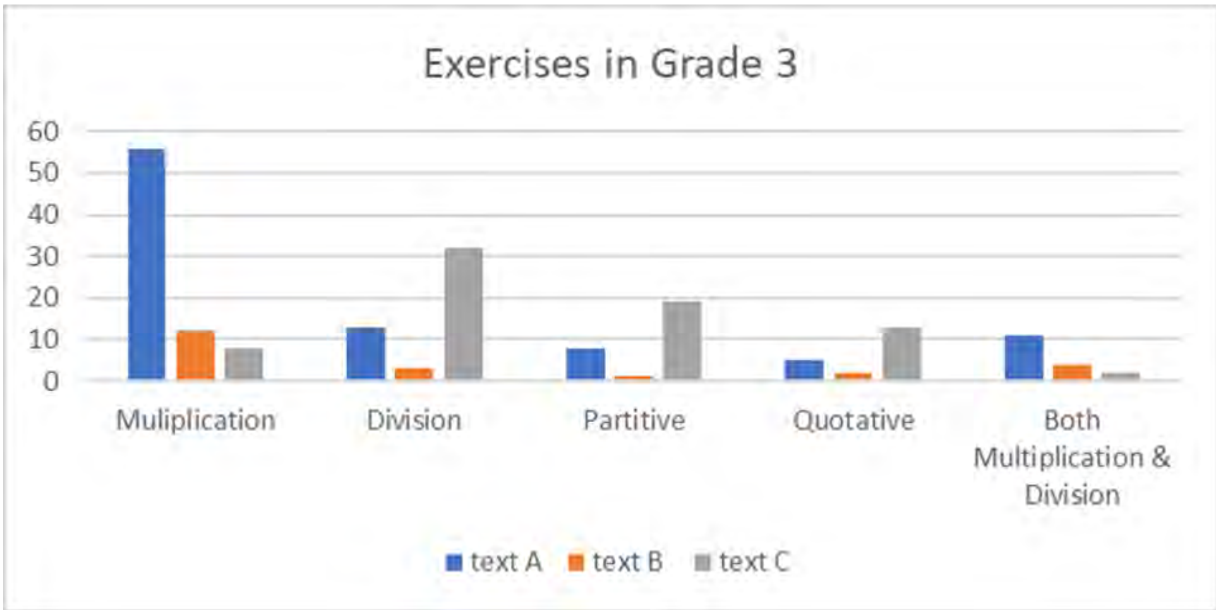


Figure 5.160: Multiplication and division exercises in Grade 3

Of the 200 different types of VRs across the Grade 3 texts the majority of VRs are images (125) (62.5%), followed by number lines (21) (10.5%) and tables (23) (11.5%). There are also array representations (20) (10%), function diagram (9) (4.5%), unifix cubes, counters and beads (2) (1%) as in Figure 5.148). There are a total of 125 (62.5%) images found across the Grade 3 texts. Text A has twice as many VRs as Text C and almost four times more VRs than Text B. Text A has 40 more images than Text C and Text B has 14 fewer words than text C.

All texts have number lines, array representations and tables. Texts A and C are the only texts in Grade 3 that contain function diagrams. Across the three texts, Text A contains 8 function diagrams (88%) and Text C contains 11%. It is worth noting that Texts A and B have single VRs that are unifix cubes, beads or counters.

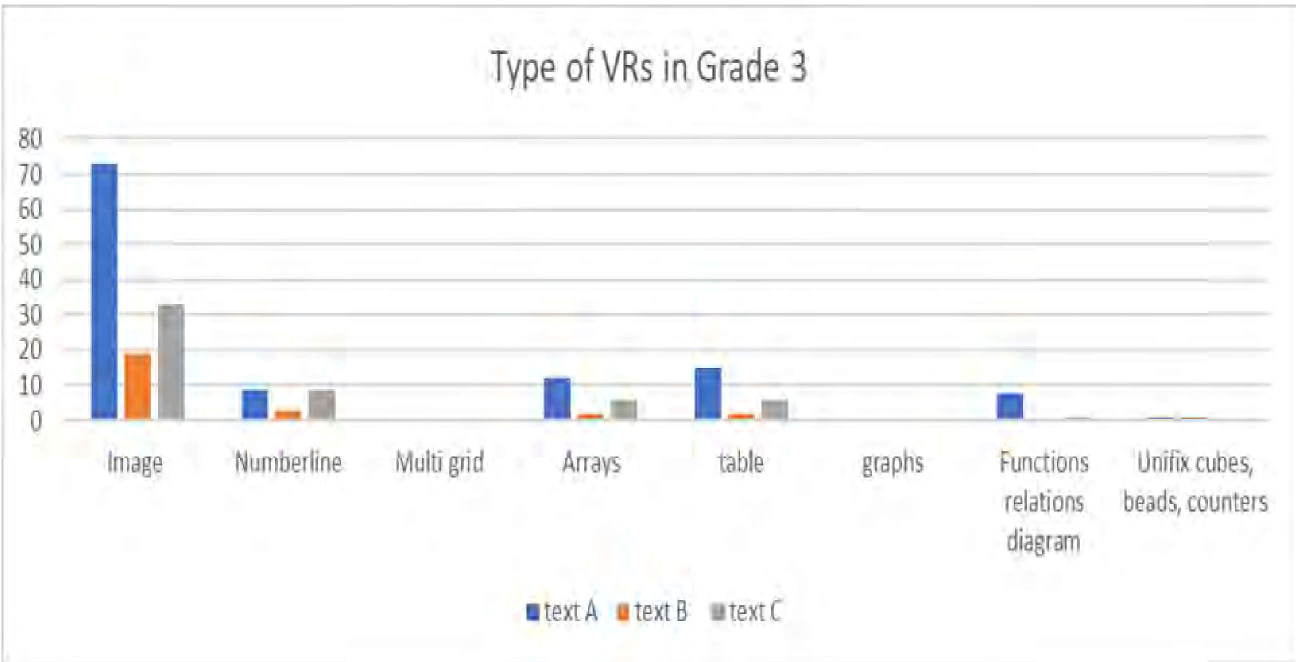


Figure 5.161: Types of VRs in Grade 3 texts

The most frequent relation to content is a strong relation to content (130) (92%). While there are more VRs in Text A than in Text B and C, of the 141 VRs there are 8 (6%) VRs across the three texts that have a problematic (a+b) relation to content. Of the 8 VRs with an exemplifying function, 5 have an exemplifying (type a) function and 3 have an exemplifying (type b) function. Furthermore, there are 2 (1%) VRs with a weak relation to content and 1 (1%) VR with no relation to content in Text A as illustrated in Figure 5.162.

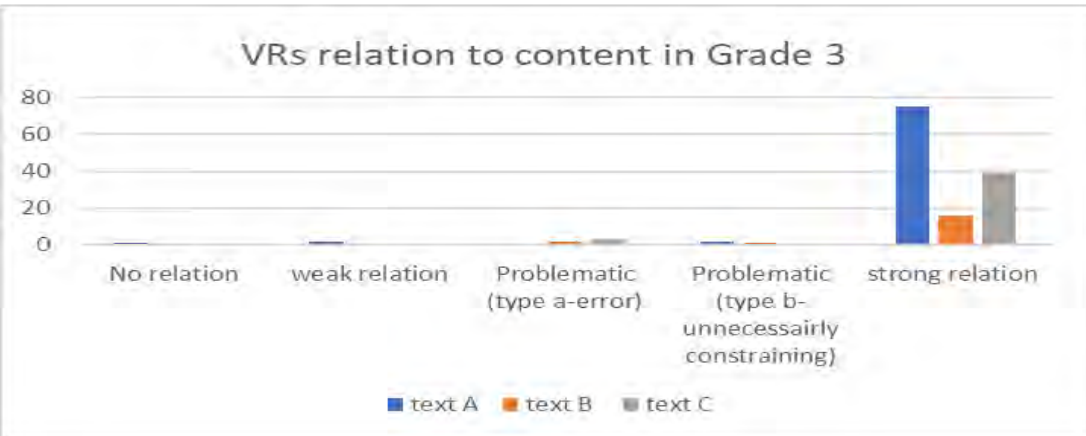


Figure 5.162: Relation to content in Grade 3 texts

Of the 141 VRs in the three texts, there are 73 (52%) VRs that have a realistic relation to reality and 68 (48%) VRs that have a metaphoric relation to reality (Figure 5.163). The majority of VRs have a realistic relation to reality, with Text A (30) (21%) having the most VRs. Of the 68 VRs in the category 'metaphoric relation to reality' across Grade 3 Text A consists of 74% (50), Text B consists of 6% (4) and Text C contains 20% (14). Text B has the least VRs with a metaphoric relation to content in Grade 3. There is no VR that includes both a realistic and metaphoric relation to reality.

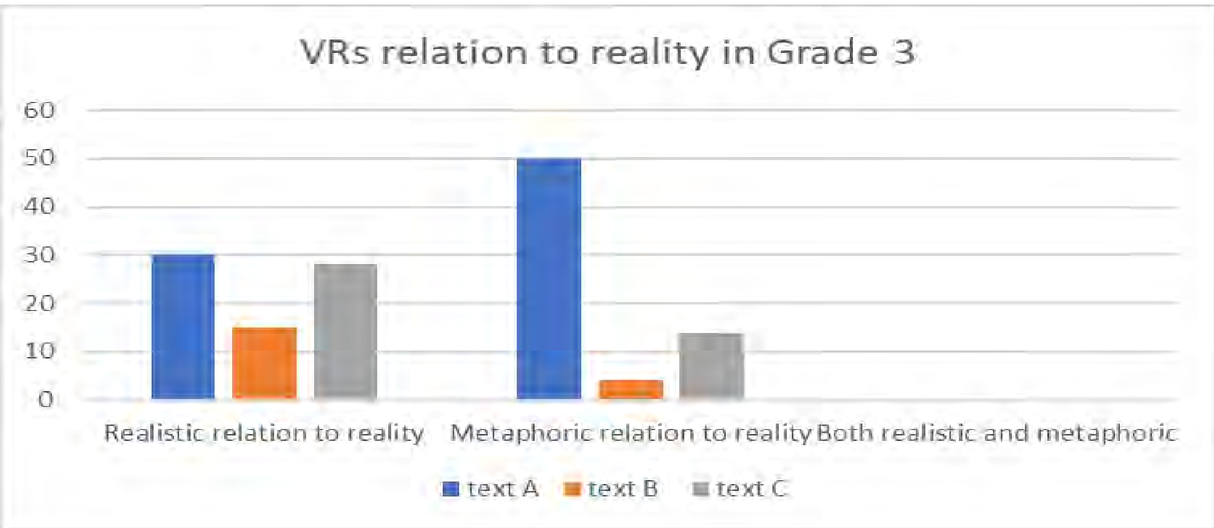


Figure 5.163: Relation to reality in Grade 3 texts

Of the 168 VRs, there are 103 (61%) VRs with an exemplifying function (type b) across the three texts, followed by 28 (17%) VRs that are exemplifying functions (type a). Across the Grade 3 texts, there are 27 (16%) VRs that are complementary. There is a total of 9 (5%) VRs that have an explanatory function and 1 (1%) has a decorative function only found in text A as depicted in Figure 5.163. Across the three texts, there is no VR with an organising function.

In Grade 3 Texts A, B and C contain VRs with an explanatory function (9) (100%). Across the texts, 56% (5) can be found in Text B, and 22% (2) in Text A and C respectively. Text A contains 1% (1) decorative function as seen in Figure 5.164.

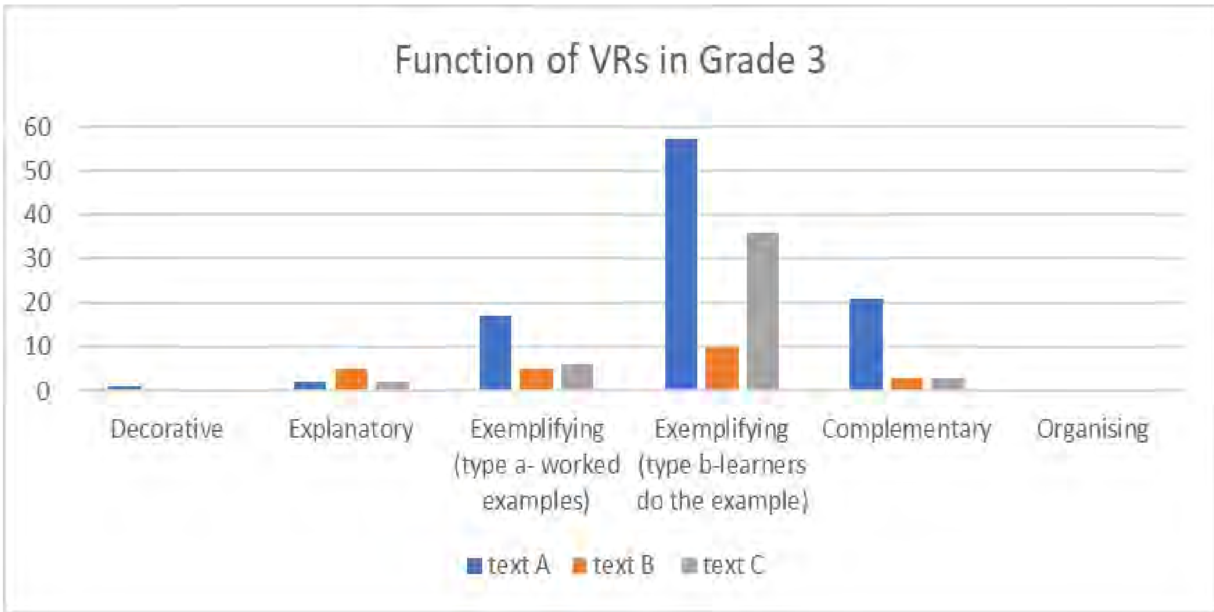


Figure 5.164: Function of VRs in Grade 3 texts

There are 141 (100%) VRs in the category dimensionality of VRs. The majority of VRs in Grade 3 are 2D representations of 3D objects (93) (86%). There are 48 (34%) VRs that are 2D representations across the three Grade 1 texts as presented in Figure 5.165.

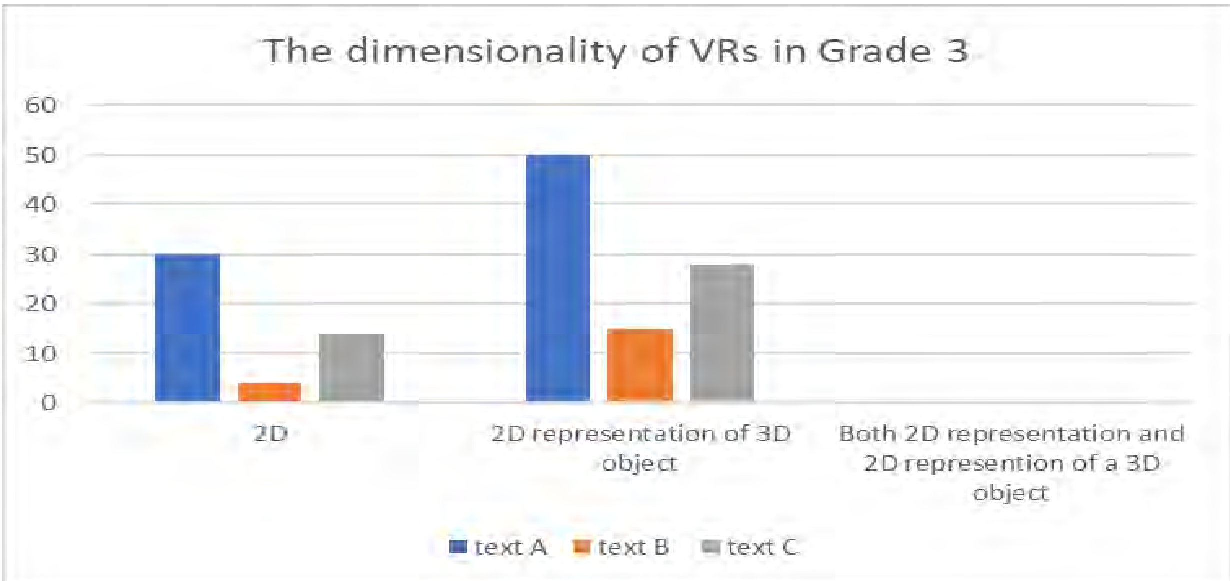


Figure 5.165: 2D and 2D representation of 3D object

5.5.3.7 Discussion of Grade 3 texts

In Grade 3, the CAPS document suggests that learners be exposed to multiplication and division. Similarly to Grades 1 and 2, Text A and B have more multiplication exercises than division exercises, except for Text C in Grades 1, 2 and 3 which contain more division exercises than multiplication exercises.

Taylor and Dyer (2014) have explained that teachers tend to show resistance to exposing learners to multiple different types of VRs. The reason is that the teachers are afraid the type of problem may be too complex for learners. Nevertheless, in Grade 3, learners are exposed to various types of VRs namely images, tables, number lines, arrays, function diagrams and unifix cubes as seen in Figure 5.154. Despite teachers' resistance to using multiple forms of VRs, Presmeg (2016) maintains that the multiple VRs assist learners to make the link between mathematical concepts. Likewise, Kilpatrick et al. (2001) suggest that representing mathematics in different forms is important for conceptual understanding.

It is worth noting there is a misalignment between the curriculum and Texts A, B and C when it comes to exposing learners to multiplication grids and graphs when doing multiplication and division. The curriculum requires learners to be exposed to graphs and multiplication grids from Grade 2 (refer to Table 5.2 below).

The majority of VRs across the three texts have a realistic relation to reality. This is consistent with the Grades 1 and 2 texts. Interestingly, Text A in Grade 3 has more VRs with a metaphorical relation to content. As Bruner (1960) notes, learners move from the enactive (concrete) to the iconic (metaphoric drawings) to the symbolic (symbols) when introduced to new mathematical concepts (McLeod, 2008). The images with a realistic relation to reality are all 2D representations of 3D objects, while the VRs with a metaphoric relation to content capture the iconic mode of representation. Thornton (2011) concurs with Bruner, arguing that the use of VRs should be seen as a continuum of VRs ranging from concrete to semi-concrete to abstract VRs rather than only as concrete or abstract.

The majority of VRs in Grade 3 have an exemplifying function (type b). A VR with an exemplifying function (type b) is where learners need to complete the exercise themselves. According to Nicol and Crespo (2006), texts contain exercises that assist in developing learners' mathematics schemas. As mentioned in Chapter 3, learners actively construct knowledge and develop schemas. Through the learners engaging with the learning exercises in texts learners develop a schema of multiplication and division. The more they are exposed to multiplication and division exercises the more the schema is developed.

Similarly to the study by Mazumder et al. (2020), there are few VRs that have an explanatory function in the DBE books. According to Mazumder et al. (2020), people learn more from texts that have an explanatory function with a VR than texts without VRs. An explanatory function elaborates on the thinking process necessary to solve the problem and increases the opportunity for learners to be able to recall and improve their problem solving strategies (Mayer et al., 1995). A suggestion would be to expand on the VRs that have a complementary function for Grade 1 as this is the first year of formal schooling and the learners could benefit from VRs that have complementary associations.

5.5.4 Data Analysis across Publishers

The following section illustrates the findings per text across Grades 1, 2 and 3 in each text.

5.5.4.1 Analysis of Text A: Grades 1, 2 and 3

Of the 225 VRs across Text A, there are 169 (75%) VRs that contain multiplication exercises. There are 45 (20%) VRs that contain division exercises and 11 (5%) VRs with both multiplication and division exercises. Of the division exercises, 33 are partitive and 11 are quotative. There is 1 exercise that has both partitive and quotative as seen in Figure 5.166.

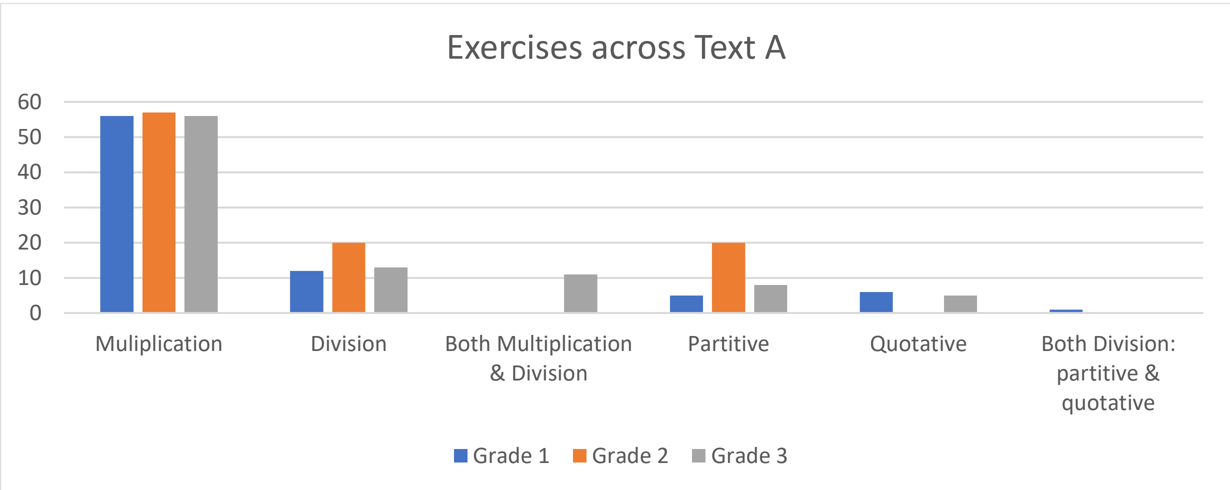


Figure 5.166: Exercises in Text A

- **Type of VRs in Text A**

Two hundred and twelve (71%) of the 299 VRs across the three grades in Text A contain 212 (71%) images. The number of images increases marginally from Grades 1 to 3 as shown in Figure 5.167. The learners are introduced to arrays in Grade 1. The focus of the array in Grade 1 is to support repeated addition, whereas, in Grade 3, the learners use the array for both repeated addition and multiplication facts. The use of number lines and tables first appears in Grade 2 and the function relations diagram in Grade 3.

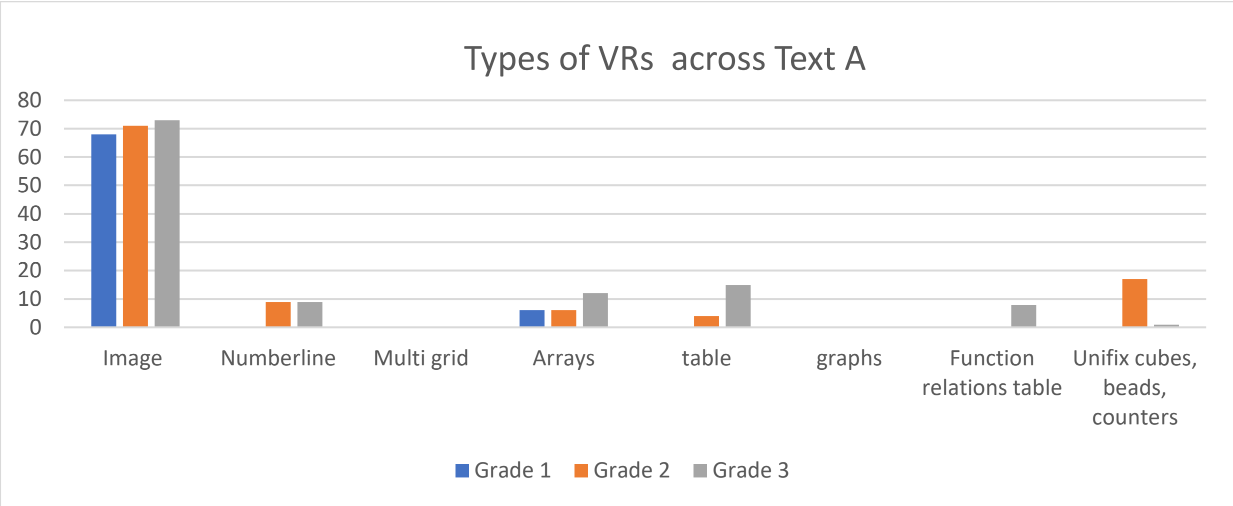


Figure 5.167: Types of VRs across Text A

- **The visual representations relation to content**

Out of the 224 VRs, 211 (94%) have a strong relation to content in Figure 5.168. There are 8 (4%) VRs with a problematic relation to content (type b) in all three grades, and 2 (1%) VRs with a problematic relation to content (type a) in Grade 1. Surprisingly, there are 2 (1%) VRs with a weak relation to content in Grade 3 and 1 (0.4 %) VR with no relation to content. The majority of the VRs, that is 211 out of 224, have a strong relation to content. In other words, the VRs should assist in developing learners' understanding of multiplication and division. The learners are able to form clear links between the VRs and the concept.

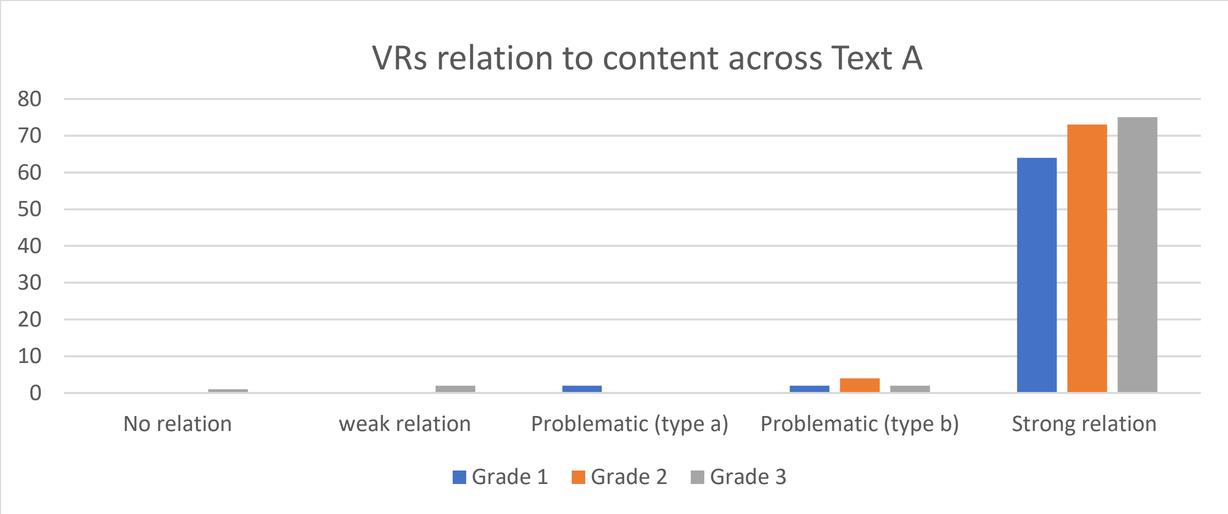


Figure 5.168: VRs relation to content across Text A

- **The visual representations' relation to reality**

The majority of VRs have a realistic relation to reality (153) (68%). Grades 2 and 3 contain more VRs with a realistic relation to reality than Grade 1. Makgato and Ramaligela (2012) maintain that VRs that learners can relate to aid the learning process. Interestingly, there are more VRs in Grade 2 that have a realistic relation to reality than in Grade 1. Of the 225 VRs, 70 (31%) have a metaphoric relation to reality. There are 2 (1%) VRs in Grade 2 that have both a realistic and metaphoric relation to content. In Grade 3, the majority of VRs have a metaphoric relation to reality as seen in Figure 5.169 below.

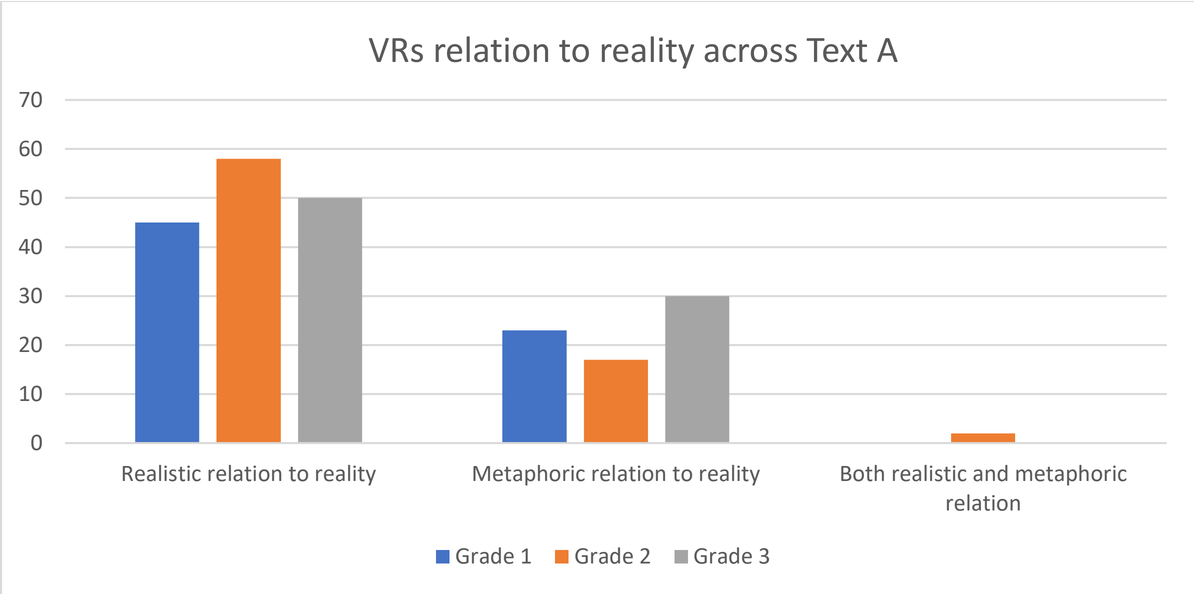


Figure 5.169: VRs' relation to reality across Text A

- **The dimensionality of VRs**

The most prominent VRs in Text A is a 2D representation of a 3D object (153) (68%). This is of significance as Grade R learners are introduced to 2D shapes and 3D objects. In the texts, I recognised that 2D representations of 3D objects are foregrounded, as only 70 (31%) of the 225 VRs in Text A are 2D representations (see Figure 5.170). There are 2 (1%) VRs in Grade 2 that have both a 2D representation and a 2D representation of a 3D object. This allows learners to see VRs in both the format of a 2D representation and a 2D representation of a 3D object.

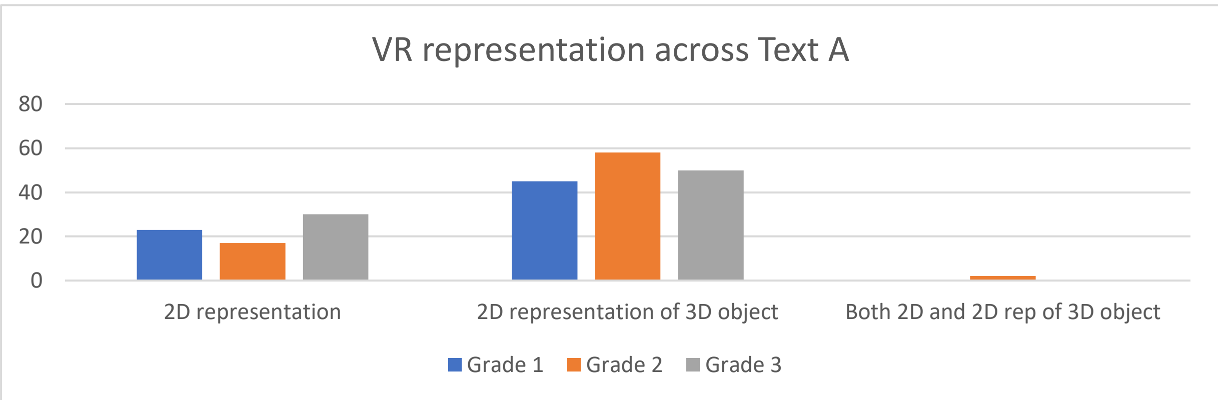


Figure 5.170: VRs representation across Text A

- **The function of the visual representations**

In Text A the most frequent function of a VRs across the Grade 1 to 3 texts is an exemplifying function (type b) (188) (66.7%). The number of VRs with an exemplifying function (type b) decreases marginally from Grades 1 to 3 as shown in Figure 5.171. In Grade 1, learners are provided with more exercises for learners to work out (exemplifying function type b) compared to Grades 2 and 3. According to Nicol and Crespo (2006), when learners practice the exercises from texts it allows learners to practise mathematics and provides learners with a worked example that the learners can refer back to when experiencing difficulty solving the problem themselves (exemplifying function type b). There are 61 (21.6%) VRs with an exemplifying function (type a). The reason for Grade 1 containing more VRs of worked examples than Grade 2 and 3 is that most learners in Grade 1 are not able to read independently so they make use of the worked examples to guide them in completing the rest of the exercise independently. Out of the 282 VRs, 27 (9.6%) VRs have a complementary function. There are more VRs with a complementary function in Grade 3 as the Grade 3 learners already have an idea of how to solve the problem. Therefore, the VRs only have a complementary function that provides additional information to complete the problem.

There are 5 (1.7%) VRs that have an explanatory function found in Grades 2 and 3. There are more VRs with an explanatory function in Grades 2 and 3 than in Grade 1 as these learners are able to read independently. The expectation is that learners in Grades 2 and 3 are able to read fluently and can make sense of the information provided in the VRs with an explanatory function to be able to use the information to solve the problem.

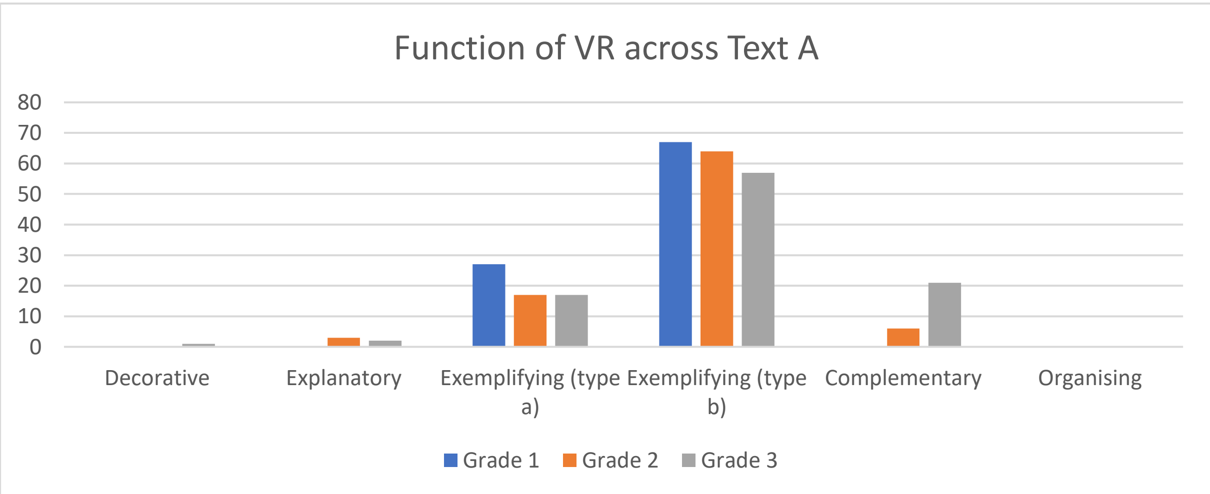


Figure 5.171: Function of VRs across Text A

- **Analysis across Text B: Grades 1, 2 and 3**

Of the 51 exercises in Text B, 29 (57%) VRs contain multiplication exercises and 15 (29%) VRs contain division exercises. In Text B, learners are exposed to more multiplication than division exercises in Grades 2 and 3 than in Grade 1. This is in accordance with Harries and Barmby (2007), who state that mathematics is hierarchical and that multiplication and division should be taught sequentially. There are 7 (14%) VRs that contain both multiplication and division exercises across Grades 1 to 3 in Text B. Of the division exercises, there are 11 partitive and 3 quotative. There is 1 VRs that is both partitive and quotative in Grade 2.

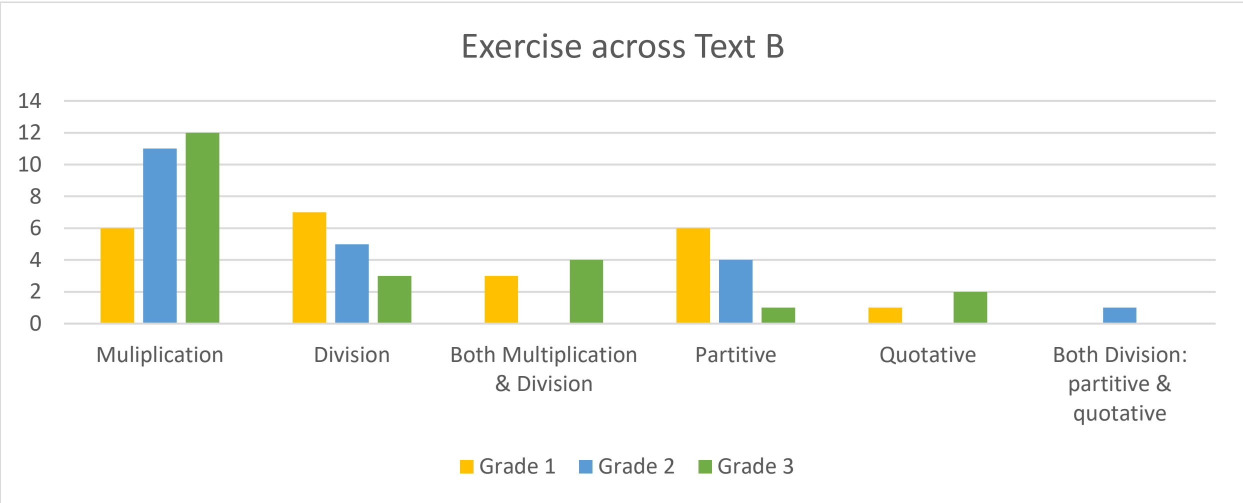


Figure 5.172: Exercises across Text B

- **Type of visual representations**

Of the 65 different types of VRs in Text B, the most common VRs are images (50) (77%). The number of images increased significantly from Grades 1 to 3. The next most prominent type of VRs in Text B is array representation (7) (11%) in Grades 1, 2 and 3, followed by number lines (4) (6%) in Grades 2 and 3. According to the CAPS document, number lines are used as a problem solving technique. Learners in Grade 1 use the number line to get to the answer and in Grade 2, number lines assist learners to record their thinking processes using number symbols. Text B also contains tables (3) (5%) and concrete objects such as unifix cubes (1) (1%) across Grades 1 to 3 (see Figure 5.173).

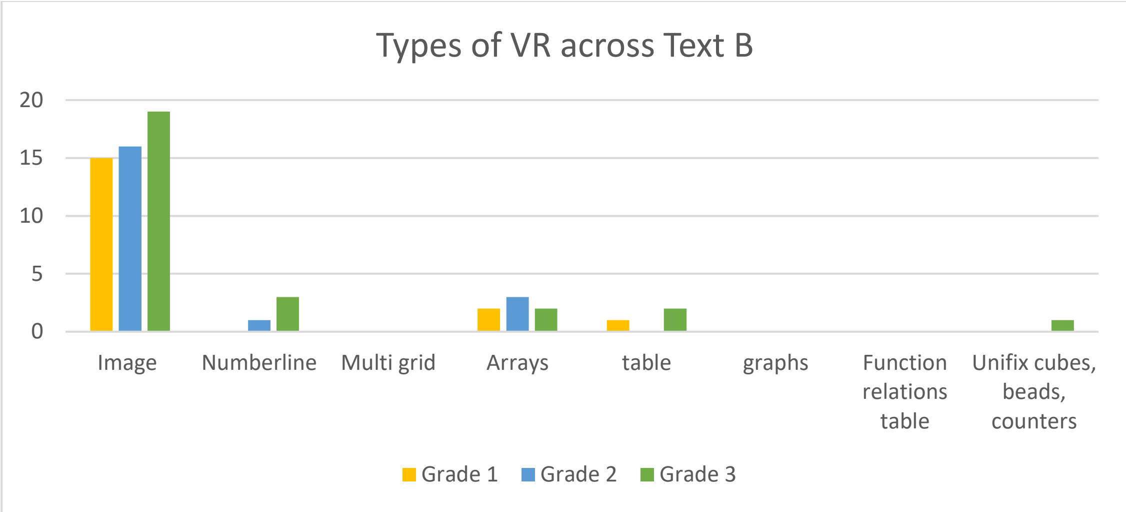


Figure 5.173: Types of VRs across Text B

- **The visual representations' relation to content**

Of all 51 (100%) VRs in Text B, the majority of the VRs have a strong relation to content (43) (84%). This shows that the majority of VRs relate to multiplication and division concepts and learners are able to make connections. In text B, there are 5 (10%) VRs with a problematic relation to content (type a). There are 2 (4%) VRs with a problematic relation to content (type b) in Grades 2 and 3 and 1 (2%) VR with a weak relation to content in Grade 2.

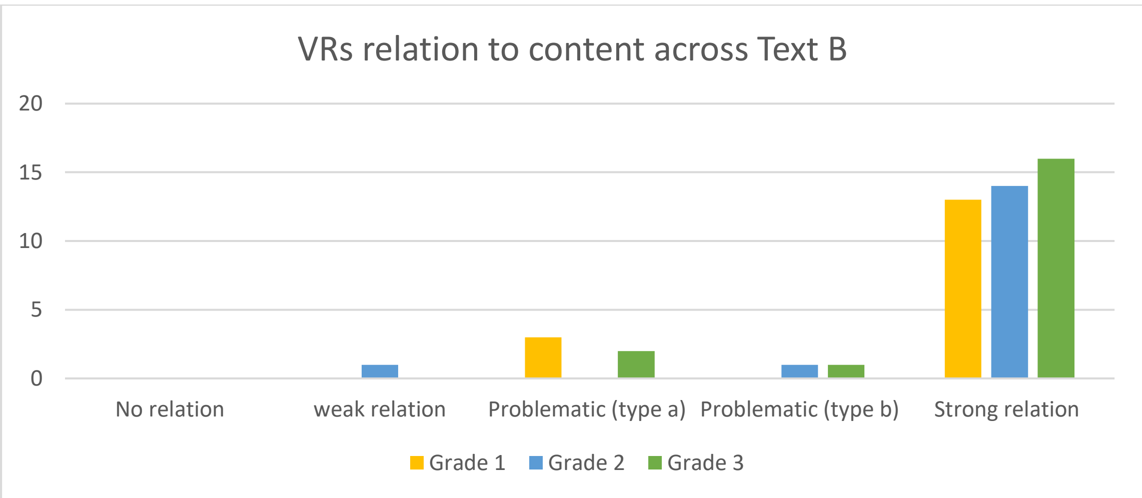


Figure 5.174: VRs relation to content across Text B

- **The visual representations' relation to reality**

Across Grades 1 to 3 in Text B, the majority of VRs have a realistic relation to reality (44) (86%). The VR looks like real-life objects. There are also VRs with a metaphoric relation to reality (7) (14%) (see Figure 5.175). The VR with a metaphoric relation to reality increases from Grades 1 to 3.

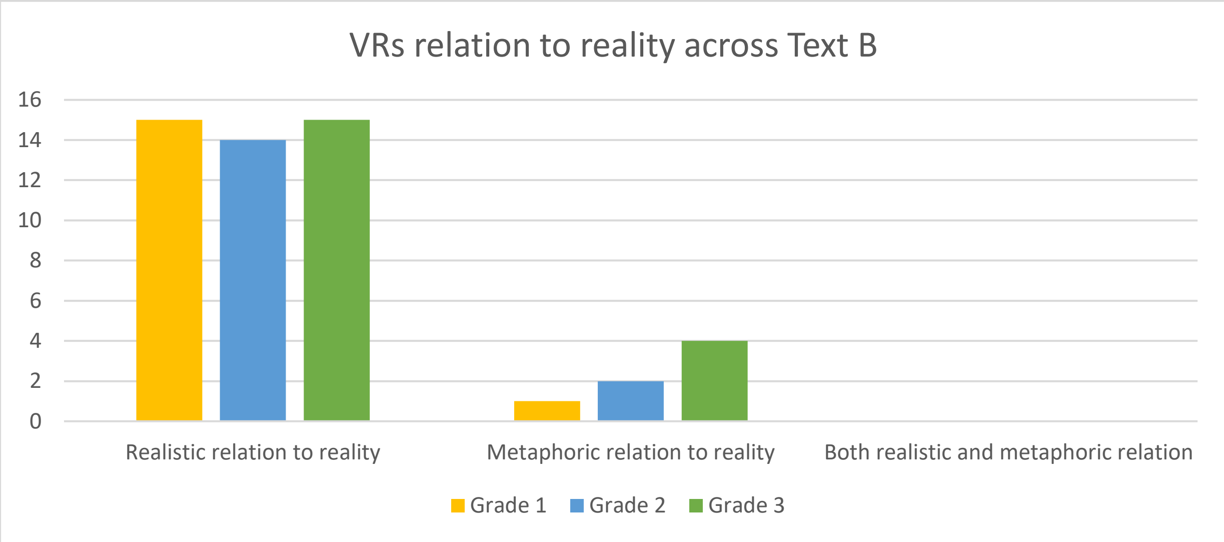


Figure 5.175: VRs' relation to reality across Text B

- **The dimensionality of VRs**

The most prominent VRs in Text B is a 2D representation of a 3D object (44) (86%). This is important because as learners get older they should be able to identify and draw 3D representations (DBE, 2011a). There are significantly fewer VRs that are 2D representations (4) (14%) in Text B.

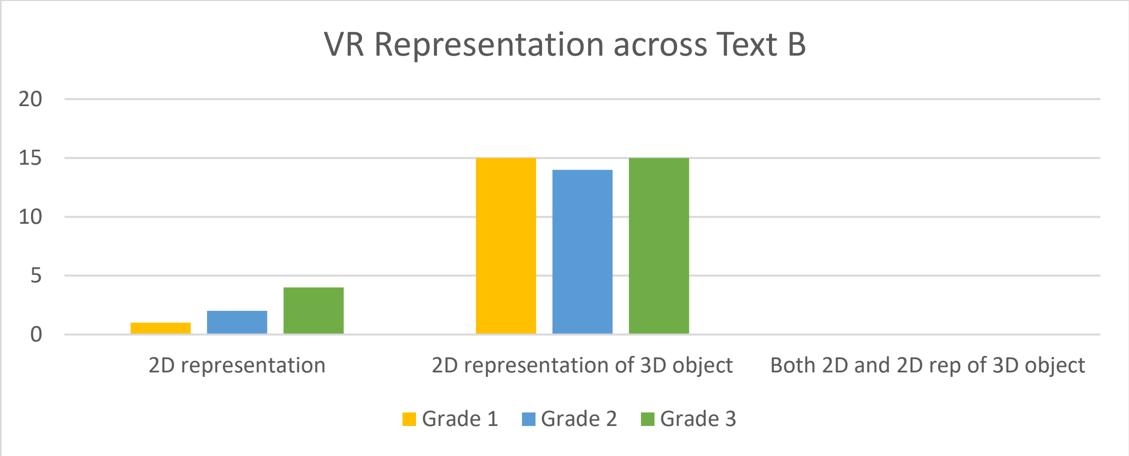


Figure 5.176: VRs representation across Text B

- **The function of the visual representations**

In Text B, the most common function of the VRs is an exemplifying function (type b) (21) (37%). The second most prominent function of the VRs is an exemplifying function (type a) (17) (30%). This is when a worked example is presented in the text. A VR with an exemplifying function (type a) assists non-expert readers (for example, Grade 1) in understanding the problem (Fotakoupoulou & Spiliotopoulou, 2008). Therefore, Grade 1 has the most VRs with an exemplifying function compared to Grades 2 and 3. Out of the 57 VRs, there are 14 (24%) VRs with a complementary function. The least common function of the VRs is an explanatory function (5) (9%).

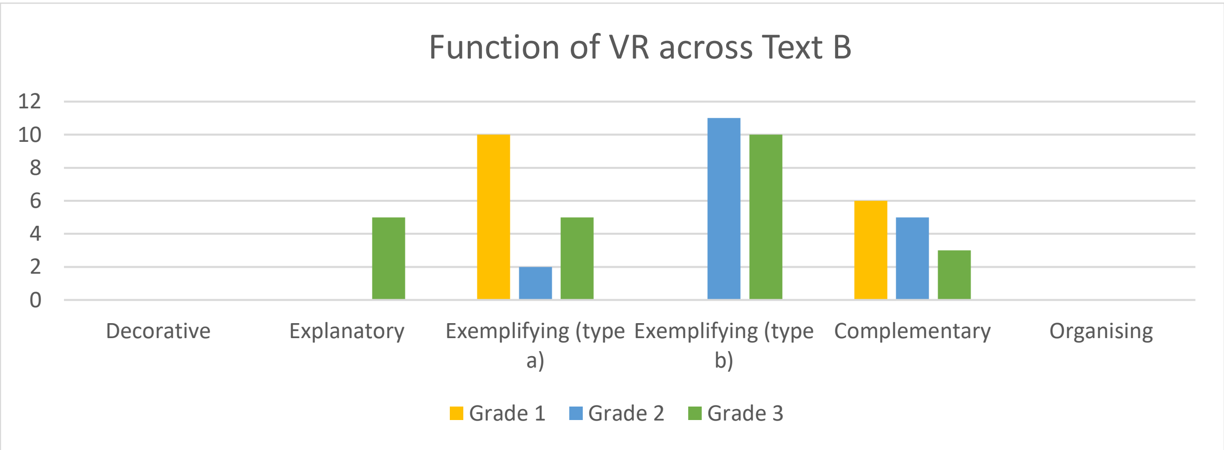


Figure 5.177: Function of VRs across Text B

- **Analysis of Text C: Grades 1, 2 & 3**

Of the 64 exercises in Text C, there are 22 (34%) VRs that contain multiplication exercises and 40 (63%) VRs that contain division exercises. There are 2 (3%) exercises that contain both multiplication and division exercises found in Grade 3 Text C. Of the division exercises, there are 21 partitive and 19 quotative.

Interestingly, Grade 3 Text C contains the most division exercises (32). The CAPS document explores division through grouping and sharing with and without remainders. In Grades 1 and 2, learners are expected to solve practical problems of sharing and grouping. In Grade 3, the learners are expected to make use of the knowledge of multiplication to work out division problems (Heirdsfield et al., 1999). The division sign is officially introduced in Grade 3 and the expectation is that the learner is able to construct a division number sentence to describe a grouping or sharing problem.

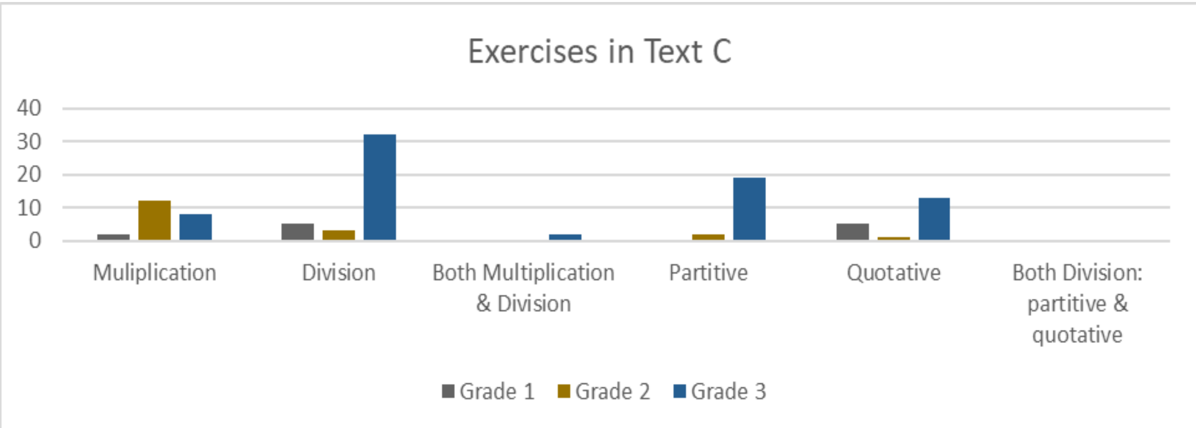


Figure 5.178: Exercises across Text C

- **Type of visual representations**

Taking a visual approach to teaching and learning mathematics (Sobbeke, 2005), teachers should assist the learner in creating a mental representation. Out of the 79 different types of VRs in Text C, images are the most common (52) (66%). Followed by number lines (12) (15%), array representation (6) (7.5%), tables (6) (7.5%) and function diagram (3) (4%).

The CAPS document states that learners need to be exposed to tables from Grade 2. In Text C, there are only exercises containing tables in Grade 3. This is little exposure to tables before the learners go to Grade 4. In the texts, the tables are often function diagrams that contain an input and output to get to the answer. The CAPS document refers to function diagrams as spider diagrams as noted in Grades 2 and 3 in Text C. The expectation is that learners are exposed to spider diagrams from Grade 2 in FP. It is worth noting that Text C is the only text that exposes learners to function diagrams in Grade 2.

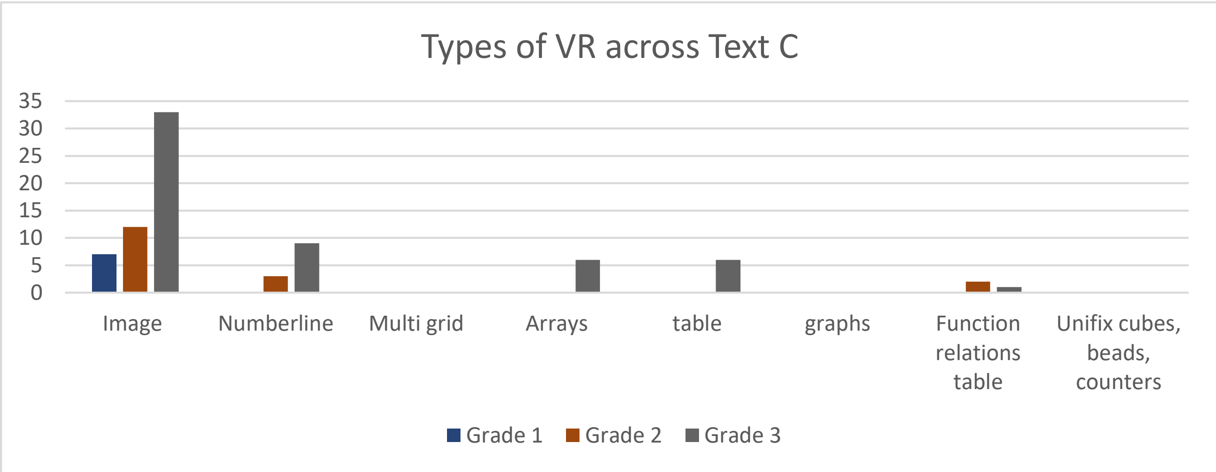


Figure 5.179: Types of VRs across Text C

- **The visual representations' relation to content**

The majority of VRs have a strong relation to content (52) (83%). There is a total of 63 (100%) VRs in the category relating to content. There are VRs that have a problematic relation to content (type b) (3) (5%) as well as a problematic relation to content (type a) (1) (2%).

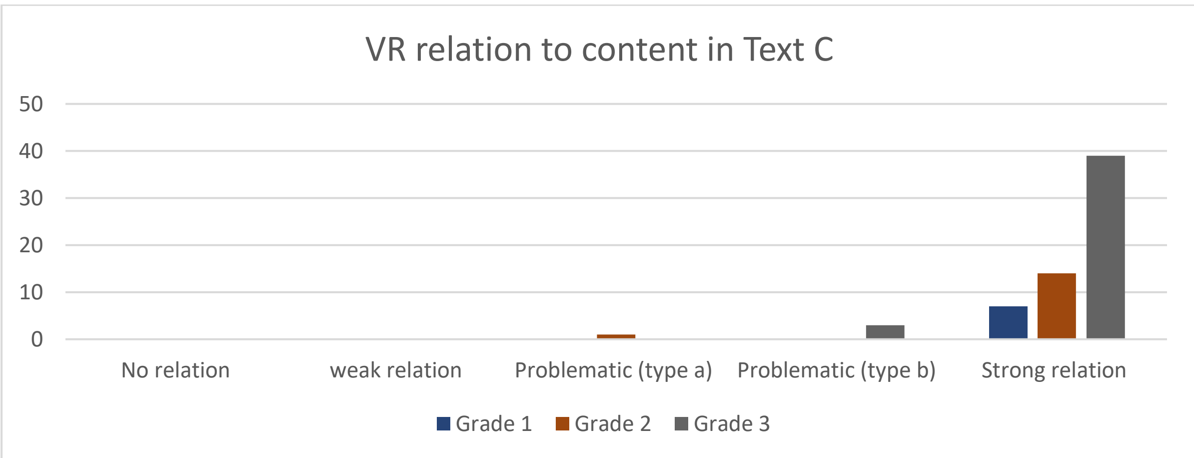


Figure 5.180: VRs relation to content across Text C

- **The visual representations' relation to reality**

Forty-four (69%) out of 64 of the VRs have a realistic relation to reality. There are VRs with a metaphoric relation to reality (20) (31%).

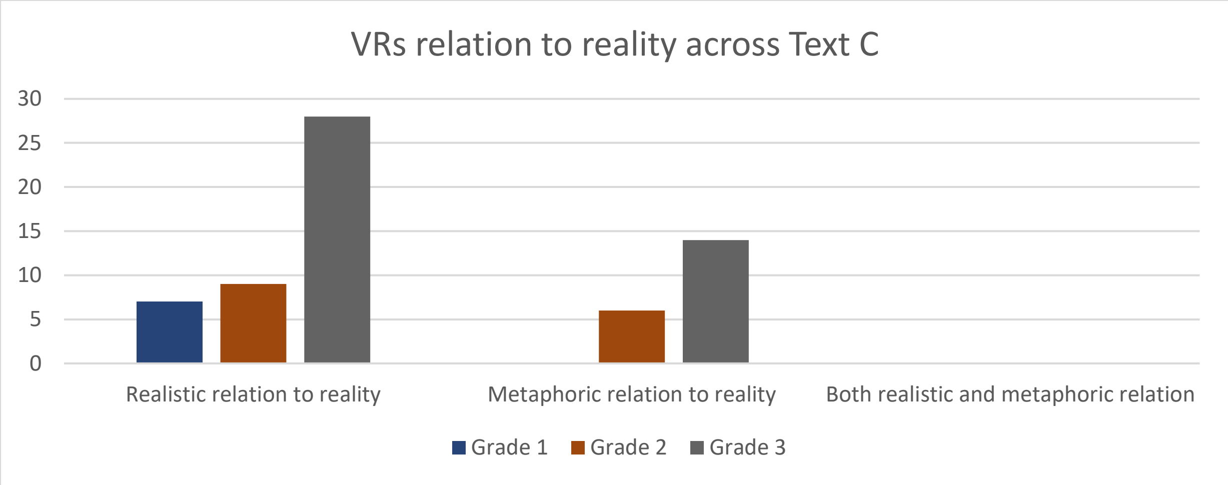


Figure 5.181: VRs' relation to reality across Text C

- **The dimensionality of VRs**

  The most prominent VRs in Text C is a 2D representation of a 3D object (44) (69%). Text C does contain 2D representations (20) (31%).

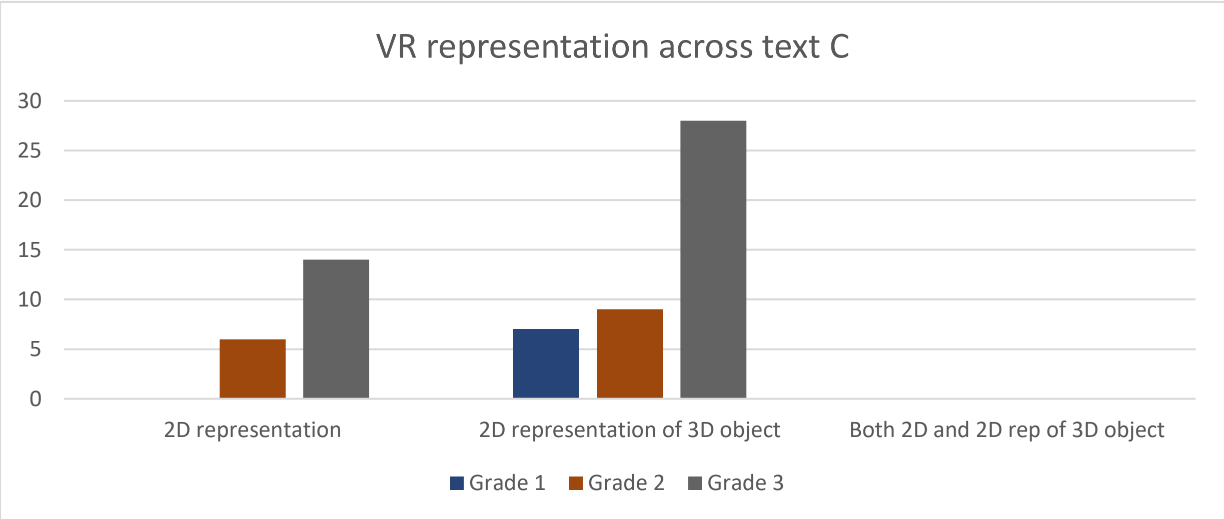


Figure 5.182: VRs representation across Text C

- **The function of the visual representations**

Out of the 70 VRs across Text C, the most common function of VRs is an exemplifying function (type b) (57) (81%), followed by an exemplifying function (type a) (8) (11%). There are 3 (4%) VRs with a complementary function and 2 (3%) VRs with an explanatory function. A VR with an explanatory function increases recall and improves problem solving skills in mathematics (Mazumder et al., 2020). The DBE intentionally has fewer VRs with an explanatory function in FP as the learners are not independent readers yet and the texts are often scaffolded with the help of a teacher (Fleisch et al., 2011).

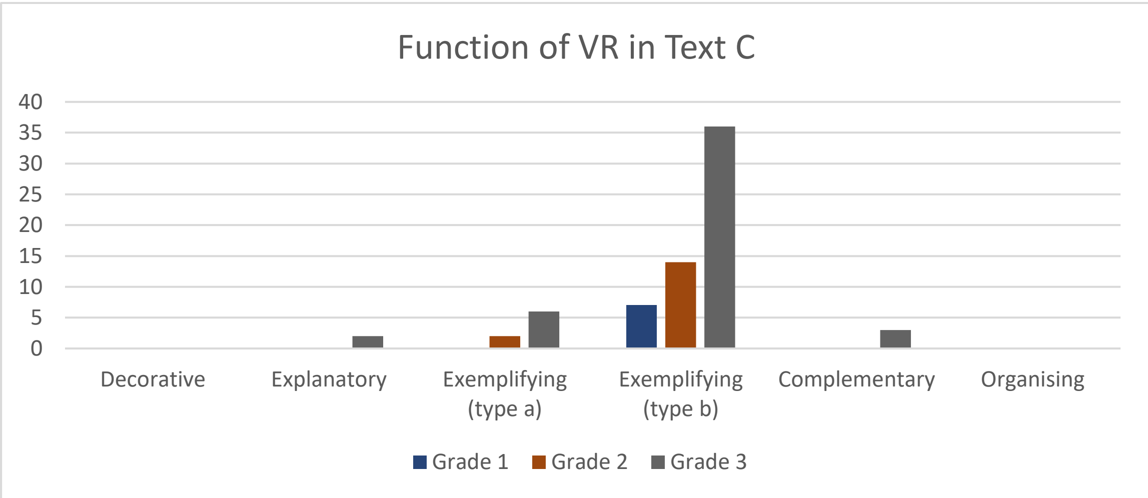


Figure 5.183: Function of VRs in Text C

5.6 Discussion Across Publishers

The following section presents the findings of the data analysis across Texts A, B and C. Texts A and B have many more multiplication exercises than division. The significance of this is that mathematics operations are taught in a sequence, multiplication is taught before division and, therefore, more emphasis is put on multiplication in Grades 1 and 2. McIntosh and Ramage (2011) found that when learners are fluent in multiplication, it reduces the cognitive load in learning new concepts such as division.

Text A (299) has a substantial number of different types of representations when compared with Texts B (65) and C (79). While all three texts have a range of different VRs, images are the most common VRs. The use of images in a text assists non-expert readers to make sense of concepts being taught and increases their awareness of visual literacy (Sieber & Hatcher, 2012; Ratnaningsih, 2019).

As noted in Chapter 2, learners should be exposed to a wide variety of VRs, which include number lines, arrays, tables and function diagrams (Askew, 2018). Texts A and C are the only texts with function diagrams which are all in the form of spider diagrams.

Arrays are regarded as important VRs for assisting learners to understand the commutative property for multiplication (3X6=6X3) and exploring the relationship between multiplication and division (Kosko, 2018). While array diagrams appear in all three grades in Texts A and B, they only feature in Grade 3 of Text C.

The proportion of VRs with a strong relation to content in Texts A and B is 94.2% and 93.75 % respectively, followed by Text B in which the proportion of VRs with a strong relation to content is 84%. The VRs with a strong relation to content are able to communicate the concept to the learners effectively (Fotakopoulou & Spiliotopoulou, 2008).

In Text B, 10% of the VRs have a problematic relation to content (type a) compared to 6% in Text C and 1% in Text A. The VRs with a problematic relation (type a) may be a result of final editing and printing. It is worth noting that the national workbook has proportionally the fewest problematic (type a) VRs that have errors.

The proportion of VRs that have a problematic (type b) in Texts A and B is 4%. There are no VRs with problematic (type b) in Text C.

The proportion of VRs that have a realistic relation to reality in Texts A, B and C is 68%, 86% and 69% respectively. Text B contains proportionally more VRs with a realistic relation to reality than in Texts A and C. A VR that has a realistic relation to content enables the learners to experience a representation that resembles their lived reality (Fotakopoulou & Spiliotopoulou, 2008). The proportion of VRs that have a metaphoric relation to reality is 31.1% in Text A compared to 31.25% in Text C and 14% in Text B. There are proportionally more VRs with a metaphoric relation to reality in Text C than in Texts A and B. In Text A there is 1 VR that has both a metaphoric and realistic relation to content – the proportionality of this VR is 0.8%.

Similarly to the above, the VRs presented in Texts A, B and C are predominantly VRs that are 2D representations of 3D objects. The proportion of the 2D representations in Text C is 31.25%, this is followed by the proportion of 2D representation in Text A (32.1%) and in Text B (14%). However, the proportion of VRs that are a 2D representation of a 3D object in Text B is 86.2%, this is followed by 69% in Text C and 68% in Text A.

Bruner (1966) in Mc Leod (2008) argues that learners should be exposed to three modes of representations namely the enactive, iconic and symbolic modes. The enactive mode is the exposure that learners get when working with concrete objects. Thereafter, learners move to the iconic where they are exposed to and make use of drawings to make sense of mathematics concepts. Learners move to the symbolic mode once they have had sufficient experience working with concrete objects and drawings. The section on dimensionality in this framework refers to learners first being exposed to the concrete object then a drawing of it in the text (2D representation of a 3D object) before making sense of the symbols.

The most common function of the VRs in Texts A, B and C is exemplifying (type b). This is an exemplifying function that provides an example that the learners need to complete. The proportion of VRs with an exemplifying function (type b) in Text C is 89% compared to 84% in Text A and 61% in Text B. The VRs with an exemplifying function (type B) provide structured learning activities for the learners to practise (Hoadley & Galant, 2016).

Missing from Texts A, B and C are examples of multiplication grids, graphs, double number lines and T-tables. According to the CAPS (DBE, 2011a) these are important models for developing learners' understanding of multiplication and division.

An argument could be made that Text A and B are better suited to novice teachers or for learners working independently or with caregivers at home, as they provide explanations and worked examples with opportunities for learners to practice. Furthermore, they provide teachers with a framework that may assist with the sequencing of the content (Nicol & Crespo, 2006). It is worth noting that the number of exercises in Text B have fewer VRs than Text A. Text A may assist with keeping track of learners' progress (Hoadley & Galant, 2016) as it contains considerably more VRs than the other texts. Texts A and B are more suitable for a novice teacher as it provides a worked example (and explanations) and provides multiple exercises that the learner can practice. According to Makgato and Ramaligela (2012), teachers rely on texts to support their pedagogical content knowledge.

Teachers can work from Texts A and B as a means of ensuring curriculum coverage. Text A may assist with keeping track of learners' progress (Hoadley & Galant, 2016) as it contains considerably more VRs than the other texts. Text C does not fulfil this function as it contains fewer exercises with VRs, therefore, the teacher will need to include their own variations to be able to cover the CAPS curriculum. However, Text C could well serve as support material to a teacher who already knows what the curriculum entails. Fan et al. (2013) similarly refer to texts as support materials that clarify concepts. There are fewer exercises in Text C, meaning that teachers will need to use this text to supplement their own exercises for the learners.

The next section provides a summary of the CAPS document and suggestions of VRs that can be used to solve multiplication and division exercises.

5.7 Relating the Texts to the CAPS Document Requirements

Table 5.1 presents a summary of whole number multiplication and division content for Grades 1 to 3 as prescribed in the CAPS document. When teaching whole number multiplication and division, the topics in the CAPS document provide a quarterly summary of the content that needs to be taught. The summary explores: (1) what needs to be taught across Grades 1, 2 and 3 and (2) the sequencing and progression across the grades. The CAPS document suggests that repeated addition leading to multiplication be taught using different strategies (e.g,. word problems, ratio, rate, array) across the three grades. The topic for division is grouping and sharing (with and without a remainder) using word problems. Table 5.1 also shows the progression of the word problems across the three grades. In the last topic of Table 5.1 below, I summarise the different calculation techniques the CAPS suggests teachers expose learners to in the different grades.

Table 5.1: Different calculation results suggested by CAPS

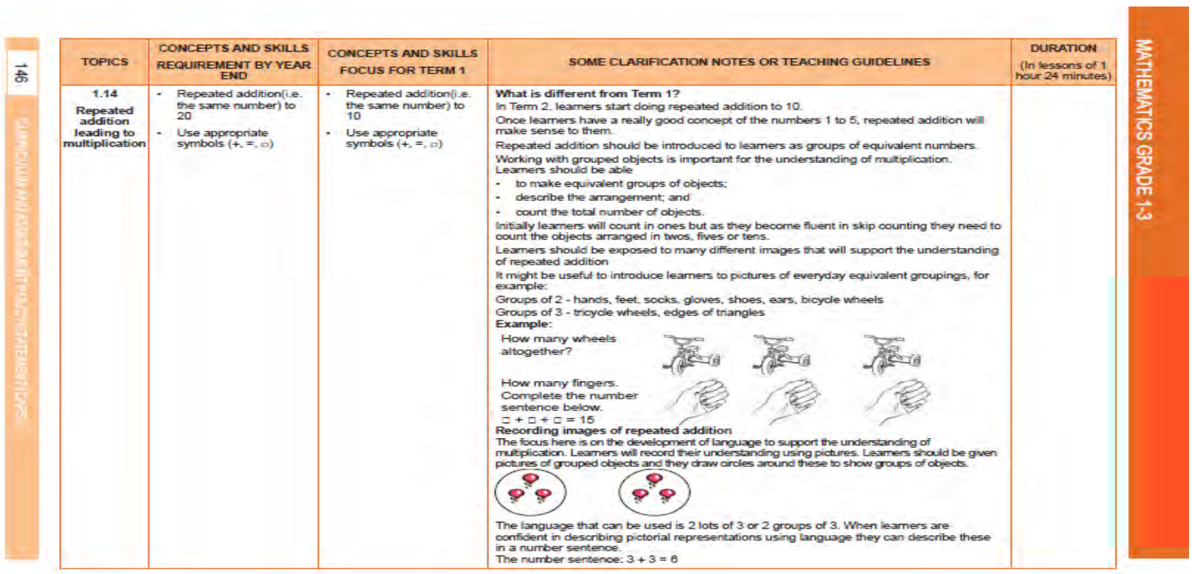
	Grade 1	Grade 2	Grade 3
	Context	**Context**	**Context**
Repeated addition leading to multiplication	Word problems Repeated addition using: groups (feet, hands, wheels, chairs etc)	Solve word problems and explain solutions such as: grouping-equivalent groups, multiplicative comparison & rectangular array	Solve word problems and explain solutions
			Ratio
	Rate	Rate	Rate
		Arrays	Arrays
		Multiplying numbers 1 to 10 (by 1, 2, 5, 3, 4 up to 50)	Multiplying numbers 1 to 10 (by 1, 2, 5, 3, 4 up to 50)
	Grids (Multiplication Grids)	Grids (Multiplication Grids)	Grids (Multiplication Grids)
		Tables	Tables
		Flow diagrams (function diagram)	Flow diagrams (function diagram), Rounding off, Flow diagram is used to record multiplication facts
	Context Free	**Context Free**	**Context Free**
	Repeated addition using appropriate symbols (+,-, =, □)	Use appropriate symbols (+,x,=, □) Multiplication number sentence and the Multiplication sign is introduced	Use appropriate symbols (÷, x,=, □)
	Groups of objects (eg. 2 groups of 3) with number sentences & pictures		

	Match number sentence to drawings		
Grouping and sharing (with remainder) leading to division	**In context**	**In context**	**In context**
	Word problems involving: equal grouping and sharing (up to 20)	Solve and explain practical problems: equal sharing and grouping using number senses	Solve and explain practical problems: equal sharing and grouping. Division sign is introduced
Grouping and sharing (without remainder)	Word problems involving equal grouping and sharing	Word problems involving equal grouping and sharing	
Division	Practical problems of sharing and grouping	Practical problems of sharing and grouping	Division sign is introduced Division number sentences can be used to describe grouping/sharing using numbers. Learners should become less dependent on drawings Use knowledge of multiplication to complete division (number relationships)
			Clue board
Calculation techniques	Concrete apparatus e.g., beads/counters/ objects/ number tracks	Concrete apparatus e.g., beads/counters/objects /number tracks	Concrete apparatus e.g., beads/counters/objects/ number tracks
	Draw pictures Using pictures Using numbers and arrows	Draw pictures multiple images	
	Build up and break down numbers	Build up and break down numbers: multiples of 10, place value, number pairs	Build up and break down numbers

	Number lines (get to the answer) Number lines (with concrete apparatus)	Number lines (help record their thinking. The start of the number line on the number learner is counting on from)	Number lines using number symbols and picture drawings
		Unifix cubes	
		Recording images that they can use to explain how a problem is solved	

Table 5.1: Summary of CAPS content on multiplication and division

Table 5.1 is a table of the content related to multiplication and division as prescribed in the CAPS document, that is, the content relating to the topic of repeated addition leading to multiplication and groups and sharing leading to division. I examined the clarification notes to identify the frequency of each topic, and the concepts and skills to be developed, and the VRs used to support the development of multiplication and division across the texts. Figure 5.184 provides an example of the clarification notes. The clarification notes, in the CAPS document, have been divided into the different grades with the same topics used as headings for each grade. The clarification notes contain written explanations and VRs that go with the written explanations to assist teachers in preparing for their lessons.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MATHEMATICS GRADE 1-3

TOPICS	CONCEPTS AND SKILLS REQUIREMENT BY YEAR END	CONCEPTS AND SKILLS FOCUS FOR TERM 1	SOME CLARIFICATION NOTES OR TEACHING GUIDELINES	DURATION (In lessons of 1 hour 24 minutes)
1.14 Repeated addition leading to multiplication	- Repeated addition(i.e. the same number) to 20 - Use appropriate symbols (+, =, □)	- Repeated addition(i.e. the same number) to 10 - Use appropriate symbols (+, =, □)	**What is different from Term 1?** In Term 2, learners start doing repeated addition to 10. Once learners have a really good concept of the numbers 1 to 5, repeated addition will make sense to them. Repeated addition should be introduced to learners as groups of equivalent numbers. Working with grouped objects is important for the understanding of multiplication. Learners should be able - to make equivalent groups of objects; - describe the arrangement; and - count the total number of objects. Initially learners will count in ones but as they become fluent in skip counting they need to count the objects arranged in twos, fives or tens. Learners should be exposed to many different images that will support the understanding of repeated addition It might be useful to introduce learners to pictures of everyday equivalent groupings, for example: Groups of 2 - hands, feet, socks, gloves, shoes, ears, bicycle wheels Groups of 3 - tricycle wheels, edges of triangles **Example:** How many wheels altogether? How many fingers. Complete the number sentence below. □ + □ + □ = 15 **Recording images of repeated addition** The focus here is on the development of language to support the understanding of multiplication. Learners will record their understanding using pictures. Learners should be given pictures of grouped objects and they draw circles around these to show groups of objects. The language that can be used is 2 lots of 3 or 2 groups of 3. When learners are confident in describing pictorial representations using language they can describe these in a number sentence. The number sentence: 3 + 3 = 6	

Figure 5.184: An example of clarification notes from the CAPS document (DBE, 2011, p. 146)

Table 5.2 provides a frequency of the CAPS calculation strategies present in Texts A, B and C. The grey shaded areas are topics not applicable for those grades. Given that Text A has the most exercises in Grades 1 to 3, it is not surprising that it has the most VRs that apply to the CAPS topics for multiplication and division. There are, however, instances where there are no exercises across the Texts B and C that make use of number lines in Grade 1. The CAPS document suggests that Grade 3 learners use clue boards as a calculation strategy when doing multiplication and division exercises, Grade 3 has 1 exercise that contains a clue board. In Grade 3, learners are expected to be exposed to long multiplication strategies. Consequently, none of the texts contain long multiplication for Grade 3.

*Table 5.2: Table of CAPS summary frequency across texts**

Frequency of CAPS criteria in Texts A, B and C		**Grade 1**			**Grade 2**			**Grade 3**		
		A	**B**	**C**	**A**	**B**	**C**	**A**	**B**	**C**
Repeated addition & subtraction leading to multiplication		32	6	3	30	6	9	44	10	5
Grouping and sharing leading to division		12	9	4	14	6	6	22	5	10
Problem solving strategies	Concrete apparatus e.g., Counters	5	0	0	9	0	1	1	1	0
	Drawing	2	0	0	7	0	0	0	0	0
	Picture of an object	41	10	6	27	11	6	34	9	9
	Number lines	0	0	0	8	1	2	9	3	2
	Build up and break down numbers				0	0	0	0	0	0
	Tables				1	0	0	36	1	4
	Multiplication grid				1	0	0	0	0	1
	Flow diagram (Function diagram)				0	0	1	9	0	0
	Array representation				1	1	0	10	2	2
	Round off to the nearest 10 (Gr3)							0	0	0
	Long multiplication							0	0	0
	Clue board							4	0	0

5.7.1 CAPS analysis of Grade 1

The CAPS for mathematics has a range of topics for learners from Grades 1 to 3. In this section of the data analysis, I will be drawing from the CAPS topics for multiplication and division and forming links to the content in Texts A, B and C for Grade 1.

The CAPS document describes repeated addition and repeated subtraction as a topic for Grade 1. In Texts A, B and C there are 32, 6 and 3 exercises respectively that focus on developing repeated addition leading to multiplication. There are no examples of repeated subtraction in any of the texts.

Grouping and sharing and equal grouping/sharing with and without remainders are topics included in the CAPS for Grade 1. Text A has 12 exercises that focus on grouping and sharing (Chapter 5, see Table 5.2). Text B has 9 exercises and Text C has 4 exercises.

The CAPS document presents calculation strategies that need to be used for Grade 1. These are concrete (e.g., counters, abacus, beads, bundles of 10), semi-concreted apparatus (e.g., number tracks, pictures of objects and number lines and number cards) and drawings. In Texts A, B and C the most common calculation strategy used for multiplication and division are pictures of objects, that is, 2D representations of 3D objects with a strong relation to reality (see Table 5.2).

5.7.2 CAPS analysis of Grade 2

The CAPS document provides the following topics for Grade 2, namely repeated addition leading to multiplication and grouping and sharing. The CAPS document describes repeated addition and repeated subtraction word problems as a topic for Grade 2. In addition, CAPS also states that multiplication should be taught in Grade 2 focusing on multiplying 1 to 10 by 1, 2,5, 3 and 4. The number range for this grade is from 1 to 50. In my analysis I have found that there are 30 exercises that involve repeated addition and subtraction in Text A, 7 exercises in Text B and 9 exercises in Text C.

The next topics in the CAPS and Texts A, B and C are grouping and sharing. This includes the use of arrays and grids. In Text A there are 14 exercises that include grouping and sharing. Text B has 6 exercises that include grouping and sharing and Text C there are 6 exercises that include both grouping and sharing in Grade 2.

According to CAPS, the calculation strategies for Grade 2 are concrete apparatus namely abacus, beads, unifix cubes as counters. Drawing and recording pictures to solve a problem is encouraged in Grade 2 as well as the use of number lines and building and breaking up numbers. In Grade 2, the most common calculation strategy in Texts A, B and C are pictures of objects. The second most common calculation strategy in Grade 2 Texts A, B and C are number lines. Text A is the only text that includes a table (1). Text C is the only text that contains a function diagram (see Table 5.2).

5.7.3 CAPS analysis of Grade 3

The CAPS document describes repeated addition leading to multiplication and long multiplication as a topic for Grade 3. The CAPS suggests that division be introduced in Term 1 using a function relations diagram. The terminology used in CAPS for this is spider diagram. The learners can also make use of a clue board, this is a board with division facts written on it.

The CAPS document describes repeated addition and subtraction as a topic in Grade 3. This can be covered when doing word problems with a number range from 1–99. In Text A there are 44 exercises that contain repeated addition leading to multiplication. In Text B there are 10 exercises that make use of repeated addition leading to multiplication, while Text C has 5 exercises that contain repeated addition leading to multiplication. The CAPS document suggests that multiplication can be introduced by using array representations, ratios, multiplication grids, tables, flow diagrams, functions diagrams and flow charts.

In Grade 3, Text A has 22 VRs of grouping and sharing leading to division. Text B contains 5 exercises and Text C contains 10 VRs of grouping and sharing that leads to division.

The CAPS document suggests that the following calculation strategies be used to solve multiplication and division problems in Grade 3. The most common problem solving strategy for Text A is tables (36) followed by pictures of objects (34). In Texts B and C, the most common problem solving strategy are pictures of objects followed by number lines in Text B and tables (4) in Text C.

5.7.4 Concluding remarks of Chapter

This chapter provides the data analysis of Grades 1, 2 and 3 texts for each publisher and across the grades. I analysed the texts across publishers and then I related the texts to the topics provided by the CAPS document. Chapter 5 forms the basis of Chapter 6 where I present my findings of this thesis, summarise Chapters 1 to 5 and discuss the implications from the study.

Chapter 6: Conclusion

6.1 Introduction

This chapter provides a summary of the key findings of this study with the the aim of answering the following research questions:

- What is the nature of the visual representations used for whole number multiplication and division in South African FP texts?
  - What is the nature of the visual representations used for whole number multiplication in South African FP texts?
  - What is the nature of the visual representations used for whole number division in South African FP texts?
- How does the nature of visual representations in texts compare to those promoted in research literature and the curriculum?

Chapter 6 provides a summary on the nature of VRs presented in FP texts in South Africa, the limitations of the study and the recommendations for future research in the field followed by concluding remarks and a reflection.

6.2 Summary of Chapters

In Chapter 1 (introduction) the main argument made is that teachers are reliant on texts as support material for teaching and learning of mathematics. The texts contain VRs that assist learners and teachers in understanding multiplication and division concepts.

The key argument of Chapter 2 (the literature review) is that VRs are important as they assist learners in creating connections between and across multiplication and division (Presmeg, 2006). The VRs assist in facilitating one's thinking (Arcavi, 2003) as visualisation is a cognitive process (Presmeg & Canas-Balderas, 2001). The use of VRs is important to assist non-expert readers in understanding mathematics (Fotakoupoulou & Spiliotopoulou, 2008), notwithstanding the important role that teachers play in mediating the learning of mathematics in the classroom.

In Chapter 3 (theoretical framework) I presented the theory of constructivism. Constructivism involves an active learning environment that develops schemas. Prawat and

Floden (1994) and Kukla (2000) argue that VRs assist in developing learners' multiplication and division schemas while engaging with various mathematics experiences.

In Chapter 4 (methodology) I explained that the research is a qualitative case study that includes the use of descriptive statistics. This case study was underpinned by an interpretivist paradigm as it sought to look at the nature of VRs in three Grades 1–3 texts. This study was executed over five phases.

- Phase 1: Textbook choice (sampling)
- Phase 2: Selection of framework
- Phase 3: Conducting a pilot study (Gr 4 DBE national workbook)
- Phase 4: Adapting the analytical framework
- Phase 5: Qualitative analysis: Grades 1, 2, 3 textbooks

In the methodology I argued how and why I adapted the VRF by Fotakoupoulou and Spiliotopoulou by creating three additional categories. The first related to the function of the VRs, the second to the VRs relation to content and the third to the function of the VRs. As I was analysing a Grade 4 text in my pilot study, I realised that what was missing from the Fotakoupoulou and Spiliotopoulou framework was a category of VRs with the function of assisting the learners to organise their work. I thus added an organising category. While analysing the Grades 1–3 texts I noticed that there were two types of errors in the VRs that I categorised as having a problematic relation to content. These I referred to as problematic (type a) and problematic (type b) relations to content. Problematic (type a) refers to VRs that present an error, whereas problematic (type b) refers to a VR that has unnecessary constraints. Likewise, when I categorised the function of each VR, I noted that there were two different types of VRs with an exemplifying function. I thus created two subcategories: exemplifying function (type a) and exemplifying function (type b). An exemplifying function (type a) is a worked example of a problem and an exemplifying function (type b) is an exercise that the learner should complete.

Due to the results of my pilot, and given that research is an iterative process of data collection and analysis, I made use of the VRF by Fotakoupoulou and Spiliotopoulou (2008) that I adapted to analyse the data in Chapter 5. I specifically looked at the multiplication and division exercises in all the texts, the VRs relation to content, the VRs relation to reality, the function of the VRs and the dimensionality of the VRs. In this chapter, I first presented the data and analysis of Grades 1, 2 and 3 of Text A and provided discussion of the text, followed by Texts B and C. Thereafter, I presented the data and analysis of Texts A, B and C across the 3 different texts (publishers). Hereafter, I provided a discussion on how the texts related to the CAPS document. The main argument developed and substantiated with data is that the government-provided textbook (Text A) had the most visual representations and reflected a greater variety of visual representations.

6.3 Key Findings That Respond to Research Question 1: The Nature of VRs

6.3.1 The nature of the multiplication and division exercises

For the purpose of this study, I analysed multiplication and division exercises in the texts. Multiplication was the dominant operation across all of the texts. Text C was the only text in Grades 1 and 3 that contained more division exercises than multiplication. As noted in Chapter 2, there are two distinct views as to when division should be introduced. The first view is that division should follow from multiplication (Harries & Barmby, 2007). McIntosh and Ramage (2011) concur with this and argue that when learners are fluent with the concept of multiplication it reduces the cognitive load in learning division. The second view is that multiplication and division should be introduced simultaneously as they are relational (Nunes & Bryant, 1996). With regard to the division exercises, partitive division was most dominant, with the Grade 2 texts only containing 1 quotative exercise.

6.3.2 Text A contains more Visual Representations

Mathematics is a language rich in visuals (Mudaly & Rampersad, 2010). Given that Text A had the most exercises in Grades 1 to 3, it is not surprising that Text A contained the most VRs across the 3 texts.

6.3.3 The nature of the visual representations

The most dominant type of VRs across the three texts were images. Mathematics is a subject that is rich in VRs (Arcavi, 2003). Images assist non-expert readers in making connections between concepts (Ratnaningsih, 2019). The second most dominant type of VRs were arrays, except for Text C where there were no examples of array representations in Grades 1 and 2. Barmby et al. (2009) suggest that array representations are important teaching tools that assists learners in developing their multiplicative reasoning. Arrays support an understanding of the relationship between multiplication and division, and the commutative principle for multiplication.

The vast majority of VRs were 2D representations of 3D objects, with a realistic relation to reality. As Bruner (1960) notes, learners develop their knowledge of concepts first through action, secondly through drawings and finally through symbols and words (Mc Leod, 2008). It is, therefore, not surprising that the proportion of VRs that were 2D representations of 3D objects was significant.

The findings indicate that most VRs had an exemplifying function (type b) across Grades 1 to 3 in all the texts. A VR with an exemplifying function (type b) is a VR that has an example that the learners need to complete. These are structured learning activities for the learners to practice multiplication and division activities (Hoadley & Galant, 2016).

6.4 Key Findings That Respond to Research Question 2: Coherence of VRs to CAPS and Literature

There appears to be general alignment between the literature, CAPS and the research findings of this study across the three texts in several respects. Learners should be exposed to a range of VRs to formulate their understanding of a concept. As noted in Chapter 2, the literature suggests that, in addition to the VRs found in the texts and CAPS, learners should also be exposed to the double number line and T-tables to develop their multiplicative reasoning skills (Askew, 2018). The CAPS suggests that learners in Grade 3 should be exposed to clue boards (DBE, 2011a). Notably missing from the texts were double number lines, T-tables and clue boards. There were, however, some VRs that the CAPS document expected learners to be exposed to in Grades 2 and 3 (Chapter 5, Table 5.2). Although CAPS suggests that learners be exposed to tables and arrays in Grades 2 and 3, learners are already exposed to these VRs in Grade 1.

There were substantially more multiplication exercises than division exercises in the texts. Furthermore, the results showed a missed opportunity in emphasising the connection between multiplication and division explicitly in the exercises presented in all the texts.

6.5 Contributions of this Study

In this study, the VRF by Fotakopoulou and Spiliotopoulou (2008) was adapted for FP mathematics use. In the analysis of the data, and as shown in Figure 6.1, the following contributions were made. The category problematic (type a) and problematic (type b) was added to the VRF. As the analysis presented data that contained errors (problematic type a) and information that caused unnecessary confusion (problematic type b).

The category exemplifying (type a) and exemplifying (type b) was also added as there were exercises with VRs that contained a worked example of what the learners needed to do (exemplifying type a) and exercises that learners had to complete themselves (exemplifying type b).

In category functions of VRs, I added the category organising function when doing the pilot study on Grade 4 texts. An organising function assists the learners in arranging the information in a manner so that the concept is understood.

Having completed my research, I realised that the results for the categories realistic/metaphoric relation to content and dimension of VR were the same. In hindsight, this is to be expected as VRs that are realistic would be 2D representations of 3D objects and VRs that are metaphoric would be 2D.

The above methodological contributions have been made to adapt the VRF of Fotakopoulou and Spiliotopoulou (2008) (see Figure 6.1) in order to suit mathematics texts. Other contributions of this study are the empirical findings that highlight both presences and frequencies and absences of key VRs promoted in the CAPS document and in the literature discussed in Sections 6.3 and 6.4 above.

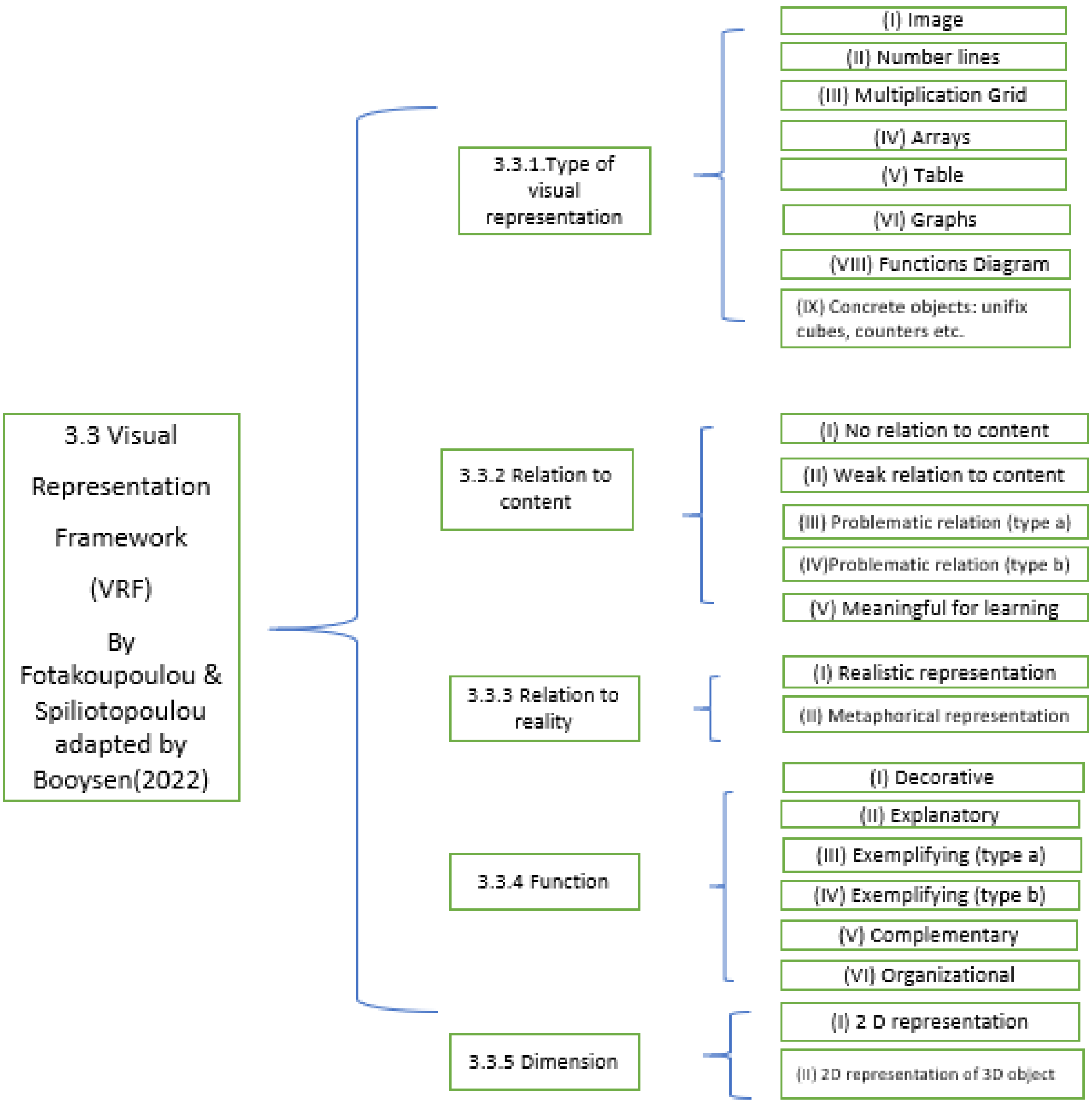

Figure 6.1: The Visual Representation Framework (Fotakoupoulou & Spiliotopoulou adapted by Booysen, 2022)

It is important to analyse the texts and pay careful attention to the functions of the VRs presented. In a future study I would recommend that researchers include the voices of the authors of the different texts, teachers and learners. It is important to speak to the authors of the texts to ascertain why they chose the particular VRs and what their intention of the different VRs was. Similarly, it is important to note the experiences of teachers and learners when interpreting the VRs in the text. I would suggest that researchers speak to teachers in the field who use the different texts and investigate how the VRs are mediated in the classroom. This will assist pre-service and inservice teachers to recognise what they need to look for when selecting a text.

6.6 Implications of this Study

The implications of this study is that all stakeholders, namely the DBE officials, teacher educators, pre-service and in service teachers will become aware of the VRs presented in texts when selecting texts for classroom use. They should consider the quality of the chosen text with relation to content and context of the learners and in relation to the curriculum. Similarly, when the authors of the texts design textbooks or workbooks they should make a conscious decision to design VRs that will have a specific function for the learner. This is so that the VRs have clear links to the content being taught and do not re-enforce common misconceptions of the concepts.

6.7 Limitations to this Study

A limitation of the study is that the sample of the texts was not representative of all the texts available in South Africa. The texts consisted of one government supplied workbook and two popular textbooks from well-known publishers. The reason for choosing these texts was that they are the most frequently purchased texts by teachers.

A second limitation to the study is that it did not explore how learners make use of VRs when solving mathematics problems and it did not explore the gestures used by teachers and learners when engaging with the texts in class. Exploring the gestures when learners and teachers engage with the texts can also be a recommendation for future studies.

6.8 Reflections

The journey of completing my Master's in Education has been a challenging yet liberating process. My academic literacy skills have improved immensely. During the last three years I have been exposed to a range of webinars, conferences and research communities in South Africa and globally that have influenced how I view mathematics and how it is taught in South Africa. Through these interactions I have gained a wealth of knowledge that has triggered me into thinking about different areas of education that will need to be monitored and researched in the future.

I am excited at the prospect of furthering my studies and enrolling as a PhD candidate. I envision delving into the concept of VRs in the FP by focusing on teachers' interpretations and experiences working with the VRs in these textbooks in their classrooms in schools. I will specifically look at if the teachers alert learners to the different uses and functions that VRs can have in texts.

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Appendices

Appendix 1: VR amounts of Text A, B and C

	Text A				Text B				Text C			
Exercises in texts	**Grade 1**	**Grade 2**	**Grade 3**	**A**	**Grade 1**	**Grade 2**	**Grade 3**	**B**	**Grade 1**	**Grade 2**	**Grade 3**	**C**
Multiplication	56	57	56	169	6	11	12	29	2	12	8	22
Division	12	20	13	45	7	5	3	15	5	3	32	40
Both multiplication & division	0	0	11	11	3	0	4	7	0	0	2	2
Types of VR	**Grade 1**	**Grade 2**	**Grade 3**		**Grade 1**	**Grade 2**	**Grade 3**		**Grade 1**	**Grade 2**	**Grade 3**	
Image	68	71	73	212	15	16	19	50	7	12	33	52
Numberline	0	9	9	18	0	1	3	4	0	3	9	12

Multi grid	0	0	0	0	0	0	0	0	0	0	0	0
Arrays	6	6	12	24	2	3	2	7	0	0	6	6
table	0	4	15	19	1	0	2	3	0	0	6	6
graphs	0	0	0	0	0	0	0	0	0	0	0	0
Function relations table	0	0	8	8	0	0	0	0	0	2	1	3
Unifix cubes, beads, counters	0	17	1	18	0	0	1	1	0	0	0	0
				299				65				79
VR Relation to content	**Grade 1**	**Grade 2**	**Grade 3**		**Grade 1**	**Grade 2**	**Grade 3**		**Grade 1**	**Grade 2**	**Grade 3**	
No relation	0	0	1	1	0	0	0	0	0	0	0	0
weak relation	0	0	2	2	0	1	0	1	0	0	0	0

Problematic (type a)	2	0	0	2	3	0	2	5	0	1	0	1
Problematic (type b)	2	4	2	8	0	1	1	2	0	0	3	3
Strong relation	64	73	74	211	13	14	16	43	7	14	38	59
				224				51				63
VRs relation to reality	**Grade 1**	**Grade 2**	**Grade 3**		**Grade 1**	**Grade 2**	**Grade 3**		**Grade 1**	**Grade 2**	**Grade 3**	
Realistic relation to reality	44	58	32	134	15	13	13	41	7	9	28	44
Metaphoric relation to reality	24	20	48	92	1	3	6	10	0	6	14	20
Both realistic and metaphoric relation	0	1	0	1	0	0	0	0	0	0	0	0
				227				51				64

Function of VR	**Grade 1**	**Grade 2**	**Grade 3**		**Grade 1**	**Grade 2**	**Grade 3**		**Grade 1**	**Grade 2**	**Grade 3**	
Decorative	0	0	1	1	0	0	0	0	0	0	0	0
Explanatory	0	3	2	5	0	0	5	5	0	0	2	2
Exemplifying (type a)	27	17	17	61	10	2	5	17	0	2	6	8
Exemplifying (type b)	67	64	57	188	0	11	10	21	7	14	36	57
Complementary	0	6	21	27	6	5	3	14	0	0	3	3
Organising	0	0	0	0	0	0	0	0	0	0	0	0
				282				57				70
VR Representation	**Grade 1**	**Grade 2**	**Grade 3**		**Grade 1**	**Grade 2**	**Grade 3**		**Grade 1**	**Grade 2**	**Grade 3**	
2D representation	25	24	33	82	1	1	4	6	1	7	14	22

2D representation of 3D object	43	54	50	147	15	15	13	43	6	8	28	42
Both 2D and 2D rep of 3D object	0	1	0	1	0	0	2	2	0	0	0	0
				230				51				64

Appendix 2: Research ethics committee approval letter

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12/11/2020

Tammy Booysen

Education Department

T.Booysen@ru.ac.za

Dear Mrs Tammy Booysen, Dr Lise Westaway, Dr Pamela Vale and Prof Mellony Graven,

Re: The use of visual images in multiplication and division in three South African Foundation Phase mathematics texts

APPLICATION NUMBER: 2020-2783-4771

This letter confirms that the above research ethics application has been reviewed and **APPROVED** by the Education Faculty Research Ethics Committee (EF-REC). Since the study does not involve human subjects, a permission letter is not required. The committee was also satisfied that you have considered potential conflict of interest in the review of the textbooks, and are satisfied based on your application that there is none. You are free to proceed with this study.

Approval is granted for 1 year. An annual progress report is required in order to renew approval for an additional period. You will receive an email notifying you at this, your Rhodes e-mail address, when the progress report is due.

Should any substantive change(s) be made during the research process, that may have ethical implications, please notify the Education Faculty REC Chair. This includes changes in investigators. The REC Chair will advise as to whether a new application is necessary.

Do keep this clearance letter secure and accessible throughout the study and after its completion. It will be needed when a thesis is examined and when publications are submitted to journals.

Please also submit a brief report to the REC Chair on the completion of the research. This can be done via email. The purpose of this report is to indicate whether the research was conducted successfully and whether any ethics-related matters arose that the committee should be aware of, in order to guide future studies.

Sincerely,

Prof Eureta Rosenberg

Chair: Education Faculty Research Ethics Committee