

**STUDENT TEACHERS' EXPERIENCES IN USING MULTIPLE  
REPRESENTATIONS IN THE  
TEACHING OF GRADE 6 PROPORTION WORD PROBLEMS: A NAMIBIAN CASE  
STUDY**

**A thesis submitted in partial fulfillment of the requirements for the degree of:**

**MASTER OF EDUCATION (MATHEMATICS EDUCATION)**

**of**

**RHODES UNIVERSITY**

**by**

**BOSMAN M. SIMASIKU**

**30 November 2012**

## DECLARATION OF ORIGINALITY

I, **Bosman M. Simasiku (Student number: 09S6100)** declare that this thesis “*Student teachers’ experiences in using multiple representations in the teaching of Grade 6 proportion word problems: a Namibian Case Study*” is my own work written in my own words. Where I have drawn on the words or ideas of others, these have been acknowledged using the reference practices according to the Rhodes University Education Department Guide to Referencing.

Bosman M. Simasiku

(Signature)

30 November 2012

(Date)

## **ABSTRACT**

This study investigated the experiences of four participating student teachers in using multiple representative approaches in the teaching of Grade 6 proportion word problems. The multiple representative approaches include the Between Comparison Method, the Within Comparison Method, the Diagrammatic Method, the Table Method, the Graph Method, the Cross-product Method, and the Oral Informal Method.

An intervention programme was organised, using workshops where student teachers were prepared to teach Grade 6 proportion word problems using multiple representative approaches. The teaching practice lessons of the four participating student teachers in two primary schools were video recorded, and the focus group interview was conducted at the University Campus.

With the exception of the Graph Method and the Cross-product Method, it was revealed that the multiple representative approaches were generally effective in the teaching of Grade 6 proportion word problems. The study further revealed that multiplicative relationships can be explored through using the different individual representative approaches. The study argues that the cross-product method is not the only way to teach Grade 6 proportion word problems. There are multiple representative approaches that should be used in conjunction with each other to enhance the teaching of proportion word problems.

Furthermore, this study revealed that a number of challenges were encountered when using multiple representative approaches. The challenges include difficulties with the English language, different and unique abilities of the learners, lack of plotting skills and the lack of proficiency in the learners' multiplication and division skills.

This study made recommendations on the integration of multiple representative approaches in the mathematics education curriculum and textbooks. It further recommended that in-service workshops for teachers and student teachers on the integration of multiple representative approaches in the teaching of Grade 6 proportion word problems should be initiated.

## **ACKNOWLEDGEMENTS**

I express my sincere thanks to the Almighty God for the strength and protection provided to me during the entire research period.

I also express my sincere thanks to Professor Marc Schafer who gave me academic and professional support during the entire research period. Without his support I would have given up this research because of all the challenges encountered.

I further express sincere thanks to the student teachers who volunteered to participate in this research. Without the student teachers this research would not have been possible.

I express sincere thanks to my family for their moral support, particularly my wife whose time I used to do this research.

I express sincere thanks to the Grade 6 learners and the staff of the two primary schools where this study took place. I also express sincere thanks to the Regional Director of the Ministry of Education in this region, and the Campus Director for providing me with permission to conduct this research.

Finally I express sincere thanks to the Ministry of Education Training sector Improvement Programme (ETSIP) for their financial support.

## **DEDICATION**

I dedicate this thesis to my beloved wife, Ms R. N. Sishingwa for her inspiration when I encountered challenges during the entire period of this research.

## TABLE OF CONTENTS

ABSTRACT.....	iii
ACKNOWLEDGEMENTS.....	iv
DEDICATION.....	v
TABLE OF CONTENTS.....	vi
LIST OF TABLES.....	ix
LIST OF FIGURES.....	x
ACRONYMS.....	xi
CHAPTER 1 INTRODUCTION TO THE STUDY.....	1
1.1 INTRODUCTION.....	1
1.2 CONTEXT OF THE STUDY.....	1
1.2.1 Difficulty of word problems.....	1
1.2.2 Dominance of the Cross-product algorithm.....	2
1.3 RESEARCH QUESTION.....	3
1.4 RESEARCH DESIGN.....	3
1.5 THE SIGNIFICANCE OF THE STUDY.....	4
1.6 LIMITATIONS OF THE STUDY.....	5
1.7 OVERVIEW OF THE THESIS.....	5
1.8 CONCLUSION.....	5
CHAPTER 2 LITERATURE REVIEW.....	6
2.1 INTRODUCTION.....	6
2.2 BACKGROUND.....	7
2.3 MULTIPLE REPRESENTATIONS.....	10
2.4.1 Proportionality.....	13
2.4.2 Algorithms.....	14
2.4.3 Multiplicative relationships and the additive approach.....	15
2.5 MULTIPLE REPRESENTATIONS IN UNDERSTANDING PROPORTIONALITY.....	17
2.6 FACTORS THAT AFFECT THE SOLVING OF WORD PROBLEMS.....	29
2.6.1 The story context of the problem.....	30
2.6.2 English Language as the medium of teaching and learning.....	31
2.7 CONCLUSION.....	33
CHAPTER 3 METHODOLOGY.....	35
3.1 INTRODUCTION.....	35
3.2 RESEARCH QUESTION.....	35
3.3 RESEARCH ORIENTATION.....	35
3.4 RESEARCH SITE AND PARTICIPANTS.....	36
3.5 RESEARCH METHODOLOGY.....	36
3.6 RESEARCH DESIGN.....	37
3.7 RESEARCH INSTRUMENTS.....	40
3.7.1 Observation making use of video-recordings.....	40
3.7.2 Focus group sessions.....	40
3.8 ANALYSIS OF DATA.....	41
3.9 ETHICS.....	42
3.10 VALIDITY.....	42

3.11 SUMMARY OF THE RESEARCH PROCESS.....	43
3.12 LIMITATIONS AND CHALLENGES.....	43
3.13 CONCLUSION.....	44
CHAPTER 4 FINDINGS AND DISCUSSIONS.....	45
4.1 INTRODUCTION.....	45
4.2 A BRIEF REMINDER OF THE MULTIPLE REPRESENTATIVE APPROACHES....	46
4.2.1 The Between-Comparison Method.....	46
4.2.2 The Within-Comparison Method.....	46
4.2.3 The Diagrammatic Method (Singapore).....	46
4.2.4 The Table Method.....	46
4.2.5 The Graph Method.....	47
4.2.6 The Cross-product Method/Cross Multiplication Method.....	47
4.2.7 The Informal Oral Method.....	47
4.3 DISCUSSION OF THEMES.....	47
4.3.1 Overall summary of using multiple representations.....	47
4.3.2 The effectiveness of the Between-Comparison Method.....	48
4.3.2.1 Exploration of the multiplicative relationship using the Between Comparison Method.....	49
4.3.2.2 The challenges faced when using the Between Comparison Method.....	52
The English language experienced as a challenge.....	52
The different and unique abilities of the learners as a challenge.....	53
Multiplication and Division as challenges.....	54
4.3.3 The effectiveness of the Within Comparison Method.....	54
4.3.3.1 Exploration of the multiplicative relationship using the Within Comparison Method.....	56
4.3.4 The effectiveness of the Diagrammatic Method (Singapore).....	57
4.3.4.1 Exploration of the effectiveness of the multiplicative relationship using the Diagrammatic Method (Singapore).....	59
4.3.5 The effectiveness of the Table Method.....	60
4.3.5.1 Questions used in the exploration of the emerging pattern in the Table Method.....	60
4.3.6 The effectiveness of the Graph Method.....	61
4.3.7 The effectiveness of the Cross-product Method/Cross Multiplication Method.....	62
4.3.7.1 Exploring proportionality using the Cross-product Method.....	63
4.3.8 The effectiveness of the Informal Oral Method.....	63
4.4 CONCLUSION.....	65
CHAPTER 5 CONCLUSION.....	66
5.1 INTRODUCTION.....	66
5.2 EXPERIENCES OF THE STUDENT TEACHERS USING MULTIPLE REPRESENTATIONS.....	66
5.2.1 The Between Comparison Method.....	66
5.2.2 The Within Comparison.....	66

5.2.3 The Diagrammatic Method.....	67
5.2.4 The Table Method.....	67
5.2.5 The Graph Method.....	67
5.2.6 The Cross-product Method.....	67
5.2.7 The Informal Oral Method.....	67
5.3 OTHER FACTORS.....	68
5.4 RECOMMENDATIONS.....	68
5.5 LIMITATIONS.....	69
5.5.1 Sample size.....	69
5.5.2 Time.....	69
5.5.3 Microteaching.....	69
5.6 SIGNIFICANCE AND FURTHER RESEARCH.....	69
5.7 REFLECTIONS.....	69
REFERENCES.....	71
APPENDICES.....	75
Appendix A: SACMEQ III table.....	75
Appendix B: Focus Group Schedule.....	76
Appendix C: Permission letter to the director.....	77
Appendix D: Permission letter to the principal.....	78
Appendix E: Consent form.....	79
Appendix F: Permission letter to the Deputy Dean.....	81
Appendix G: Worksheet proportion word problems.....	82

## LIST OF TABLES

Table 2.1: The Table Method (adapted from Van de Walle et al.).....	22
Table 2.2: Example 1 of a ratio table .....	23
Table 2.3: Example 2 of a ratio table .....	23
Table 2.3: Example 3 of a ratio table .....	23
Table 2.5: An example of a more structured ratio table (from Van de Walle et al).....	24
Table 2.6: A simple ratio table adapted from Cramer & Post.....	25
Table 3.1: The time frames for Phase 5.....	39
Table 3.2: Summary of the research process.....	43
Table 4.1: Summary of the participating student teachers' general impressions on using the multiple representative approaches.....	47
Table 4.2: Summary of the participating student teachers' responses regarding the effectiveness of the Between Comparison Method .....	49
Table 4.3: Participating student teachers facilitating the exploration of the multiplicative relationship using the Between Comparison Method.....	49
Table 4.4: Summary of the challenges faced by the participating student teachers when using the Between Comparison Method.....	52
Table 4.5: Summary of the responses of student teachers on the effectiveness of the Within Comparison Method.....	54
Table 4.6: The summary of experiences on the exploration of the multiplicative relationship using the Within Comparison Method.....	56
Table 4.7: Summary of the responses of the participating student teachers on the effectiveness of the Diagrammatic Method (Singapore).....	58
Table 4.8: Summary of challenges faced in the exploration of the multiplicative relationship using the Diagrammatic Method (Singapore).....	59
Table 4.9: Summary of the responses of the four participating student teachers regarding the effectiveness of the Table Method.....	60
Table 4.10: Questions used by the participating student teachers to help Grade 6 learners explore emerging patterns when using the Table Method.....	61
Table 4.11: The four participating student teachers' experiences of using the Graph Method.....	61

Table 4.12: Summary of the participating student teachers' responses regarding the effectiveness of the Cross-product Method.....62  
Table 4.13: Summary of the responses of student teachers regarding the effectiveness of the Informal Oral Method.....64

**LIST OF FIGURES**

Figure 2.1: A pictorial representation of proportion.....27

## **ACRONYMS**

BETD- Basic Education Teacher Diploma

ETSIP- Education Training Sector Improvement Programme

FGST1- Focus Group Student Teacher 1

SACMEQ- Southern and Eastern Africa Consortium for Monitoring Educational Quality

# CHAPTER 1

# INTRODUCTION TO THE STUDY

## 1.1 INTRODUCTION

This study sought to explore the experiences of the four participating student teachers in using multiple representative approaches in the teaching of Grade 6 proportion word problems. This chapter introduces this research by describing the context of the study and closes by presenting an outline (overview) of the whole thesis.

## 1.2 CONTEXT OF THE STUDY

### 1.2.1 Difficulty of word problems

At the College of Education where I teach upper primary mathematics education, student teachers have mentioned countless times that presenting problem solving to upper primary learners is difficult. This perception by student teachers motivated me to look deeper into this issue and conduct research with my student teachers.

The SACMEQ (Southern and Eastern Africa Consortium for Monitoring Educational Quality) III project report (Hungu et al., 2010) indicates clearly that Namibian Grade 6 learners struggle with solving word problems. An analysis carried out by Hungu et al. (2010) illustrates that the Grade 6 learners reaching the competent numeracy level in Namibia is less than 5% while those reaching the mathematically skilled level is less than 3%. The analysis further shows that the percentage of Grade 6 learners reaching the problem solving level is less than 1%. The Caprivi region, in which this research is conducted, registers 0% of Grade 6 learners reaching any skill level in problem solving (see Appendix A for further details). These results are of great concern and this study aimed at shedding some light onto this aspect of the Grade 6 mathematics curriculum and its teaching.

Hungu et al. (2010) suggest that overall mathematics competency spans a range of mathematical skills in numerous domains such as pre-numeracy, emergent numeracy, basic numeracy, beginning numeracy, advanced competent numeracy, as well as concrete and abstract problem solving skills. An essential skill required for all Grade 6 mathematics activities in solving word problems is the ability to translate and integrate “verbal, graphic or tabular information into an arithmetic form” (Hungu et al., 2010, p. 5) and vice versa. This skill is central to the solving of word problems as it involves the translation of verbal and written information into a mathematical language that enables mathematical manipulation

and operations. Although Makuwa (2010) reports Namibia as being one of the five SACMEQ countries that showed remarkable improvement in mathematics between 2000 and 2007, a closer examination of the results still reveals some worrying aspects. An analysis of the percentage of Grade 6 learners reaching the problem solving level for example, suggests that urgent intervention is still needed to improve mathematics understanding in Namibia.

Word problems in general are integral components of any mathematics curriculum, and proficiency in solving word problems effectively should therefore be central to any learning and teaching programme. This study specifically focused on gaining insights into the teaching of proportion word problems at Grade 6 level.

### **1.2.2 Dominance of the Cross-product algorithm**

It has come to my attention that most teachers in my region only use the cross-multiplication algorithm (also referred to as the cross-product algorithm) to teach proportion word problems in Grade 6. This mono-dimensional approach to teaching word problems is also predominantly used during micro-teaching and teaching practice by student teachers in the Caprivi region. There are valid reasons for only using this strategy in teaching Grade 6 proportion word problems. Some mathematics teachers told me that they only use this strategy because it saves time. They further asserted that using a multiple approach to teaching problem solving would be confusing to both the teacher and the learners. As many of these teachers served as mentors of our student teachers; it thus seemed obvious that student teachers on teaching practice also tended to adopt the use of only a single method to teach proportion word problems. There are however, many other methods to teach proportion word problems. Cramer & Post (1993) asserted that the use of multiple representations in teaching proportion word problems would support more meaningful and deep learning in solving these problems.

Kennedy, Johnson & Tipps (2008) warned mathematics teachers that “the cross-product algorithm should not be introduced until students have fully developed and refined their understanding that proportional relationships involve multiplicative relationships” (p. 334). They also acknowledged that the cross-product algorithm was only one of the strategies used to solve proportion word problems and should only be used when students have attained a deep understanding of multiplicative relationships. They do not discard the use of the cross-product algorithm entirely but suggest that it is only one of the multiple ways to solve proportion word problems. Cramer & Post (1993) stated that “when one has a superficial

understanding of a concept, it is easy to apply memorised rules in the wrong situation” (p. 344). The cross-product approach, in my view, lends itself to the memorisation of rules and thus can lead to a superficial understanding of proportionality. In sharing similar sentiments with Kennedy et al. (2008) on the use of the cross-product algorithm in solving Grade 6 proportion word problems, I wish to explore how a multiple representative approach could enhance the teaching of proportional word problems.

Cramer & Post (1993) stated that multiple representative approaches “give students the power to choose a strategy that best fit the data; the resources available and their personal preferences” (p. 346). I strongly agree that the personal preferences of students should be considered and not ignored when using multiple representative strategies to teach Grade 6 proportion word problems. Weber et al. (2010) valued the use of personal preferences and used a model that focused on “increasing the opportunities for different groups of students to participate in classroom mathematical activities” (p. 92). It is important for students to “see mathematics as a sense-making pursuit” (Brown, Collins & Duguid, 1989, p. 15) especially when allowed to use multiple representations.

### **1.3 RESEARCH QUESTION**

This study explored four participating student teachers’ experiences in using multiple representative approaches in the teaching of Grade 6 proportion word problems. The research question is:

What are participating student teachers’ experiences of using multiple representative approaches in the teaching of grade 6 proportion word problems?

### **1.4 RESEARCH DESIGN**

This research was a case study located in the interpretivist paradigm (Cresswell, 2003). It investigated the participating student teachers’ experiences of using multiple representative approaches in the teaching of Grade 6 proportion word problems. The purpose was to generate awareness amongst student teachers of the experiences of using multiple representative approaches.

This study was conducted at one of the Campuses of the University of Namibia and in two primary schools in the surrounding area. The four participating student teachers who

participated in this study were third year BETD student teachers. The four student teachers volunteered to participate in this study. Convenience sampling therefore informed the sampling process of both the student teachers and the primary schools (Newby, 2010).

### **1.5. THE SIGNIFICANCE OF THE STUDY**

This study is significant because it enriched the participating student teachers with knowledge and skills for integrating multiple representative approaches into their teaching of Grade 6 proportion word problems. It also generated an awareness of the challenges faced by participating student teachers when using multiple representative approaches in the teaching of Grade 6 proportion word problems. The rationale of the study was to address the growing concern of how to equip student teachers with the skills to adequately teach and help Grade 6 learners to be proficient in solving proportion word problems.

This study is also significant because it revealed that multiple representative approaches can be powerful and effective in the teaching of Grade 6 proportion word problems. Challenges were encountered, particularly when using the cross-product method and the graph method. It was also revealed that using multiple representative approaches facilitated a meaningful and conceptual teaching strategy. A further challenge will be to help the Grade 6 learners to be aware that not all word problems involve proportions. The study revealed that the use of multiple representations served as rich variations in the teaching of proportion word problems.

Moreover, this study revealed that it is very satisfying to use multiple representations in conjunction with each other. This implies that student teachers should avoid the use of only one representative approach. The challenge for teachers will be to use multiple representations consistently in their practice. It recommends that student teachers be trained in the effective integration of multiple representations in the teaching of proportion word problems. This implies that curriculum materials for mathematics education in teacher educational institutions should cover the integration of multiple representations in the teaching of proportion word problems. It also recommends that further research should be conducted into this method of teaching Grade 6 proportion word problems.

## **1.6. LIMITATIONS OF THE STUDY**

The findings of this study cannot be generalized because of the small number of the participants used and the short time taken to implement the multiple representative approaches. These limitations however provide an opportunity for other research to be conducted in Namibian Grade 6 classrooms where the number of participants and the time are substantially increased.

## **1.7. OVERVIEW OF THE THESIS**

The thesis is organized into five chapters:

**Chapter One** describes the context of the study, the research question, research design and further presents the outline of the entire thesis.

**Chapter Two** presents the literature review of this study. It discusses multiple representations, proportional reasoning in word problems, multiple representations in understanding proportion, and factors that affect proportion word problems and their solution strategies.

**Chapter Three** presents the methodology of this study. It describes the research question, the research orientation, the selection of the research site and participants, the research methodology, the research design, the research instruments used, data analysis procedures, research ethics and validity, and the limitations and challenges encountered.

**Chapter Four** presents the findings and discussions of this study. Much of the data is presented in tables and original quotes. I discuss the experiences of using multiple representative approaches by analyzing their similarities and differences.

**Chapter Five** presents a summary of the findings. It further discusses the significance and some limitations of the entire study. It also presents recommendations and concludes with some reflections.

## **1.8 CONCLUSION**

This chapter introduced this research by presenting the context of the study, the research question, the research design, the significance of the study, the limitations of the study and the overview of the thesis.

## **CHAPTER 2: LITERATURE REVIEW**

### **2.1 INTRODUCTION**

This study involves the participation of student teachers in developing an understanding of the use of multiple representations in the teaching of proportion word problems. It is hoped that this study will generate an appreciation of how multiple representations can enhance the teaching and learning of Grade 6 proportion word problems. Ozmantar et al. (2010) suggested that using multiple representations “can potentially create conditions for effective learning and lead to deeper levels of understanding the subject” (p. 20). They further suggested that “each individual representation provides students with a point of view through which they approach a problem and this in turn allows them to become more competent in handling mathematical problems” (Ozmantar et al., 2010, pp. 19-20). In my view it will also help student teachers to develop an understanding of the relevance of multiple representations in the teaching of Grade 6 proportion word problems. Above all, student teachers need to understand how multiple representations enrich the proficient teaching of Grade 6 proportion word problems.

Levav-Wayberg & Leikin (2006, p. 57) stated that “teachers seldom solve problems in different ways either for themselves or in their classes”. Their observations are based on evidence from studies in Israeli as well as in US classrooms. They stated that “in US classrooms, teachers rarely introduce their students to multiple-solution tasks” (Stigler & Hiebert, cited in Levav-Wayberg, 2006, p. 57) They claimed that their “study has shown that Israeli teachers as well rarely employ connecting tasks systematically in their classes” (Levav-Wayberg & Leikin, 2006, p. 57-58). In my own experience in Namibia I find that teachers rarely use different ways of teaching. For the purpose of this study the notion of multiple representations and using different methods/strategies are used synonymously. My study investigated the experiences of student teachers in using multiple representations in teaching Grade 6 proportion word problems. In particular, my study focused on using alternative methods to the commonly used cross-multiplication algorithm in teaching proportion word problems.

Kilpatrick, Swafford & Findell. (2001) made the observation that only using the cross-multiplication algorithm is problematic. They comment that “research suggests that

presenting the cross-multiplication algorithm before students understand proportion and can reason about them leads to the same kind of separation between strands of proficiency” (Kilpatrick et al., 2001, p. 244). They suggest that “more research is needed to identify the sequences of activities that are most helpful for moving from well-understood but less efficient procedures to those that are more efficient” (*ibid*). This study thus investigated selected student teachers’ experiences in using multiple representations in teaching proportion word problems.

This literature review discusses multiple representations, proportional reasoning in word problems, multiple representations in understanding proportion, and the factors that affect proportion word problems and the solution strategies. Multiple representations involve oral and written representations. Among the written representations are algorithms that are linked to algebraic representations such as the cross product, and some of its alternatives, “the within-comparison method and the between-comparison method” (Nielsen, 1998, p.39).

## **2.2. BACKGROUND**

The SACMEQ III project report shows that Namibia is one of the countries in Africa that experiences significant challenges in the teaching of Grade 6 problem solving, at both concrete and abstract levels (Hungu et al., 2010). This is a serious concern to all involved in mathematics education in Namibia because without problem solving skills, learners are not fully prepared for everyday life. A step forward in addressing this issue is that Namibia has incorporated problem solving as an integral part of the Grade 6 mathematics curriculum (Namibia Ministry of Education, 2006, p. 12-16). This is consistent with the suggestion made by Simons (1993, p. 5) that “problem solving should be an accepted integral part of any mathematics programme”. This implies that problem solving should be part of the curriculum from pre-school to tertiary level.

Furthermore, the SACMEQ III (Hungu et al, 2010) project report shows that the Caprivi Region is one of the 13 regions in Namibia with a score of zero percent in the Grade 6 learners reaching the level of problem solving. This percentage is not only worrying but demands that intervention programmes be initiated to remedy this critical situation. This study acknowledges that there is a problem with the teaching of problem solving, and thus wishes to investigate experiences of student teachers in using multiple representative approaches in the teaching of one major component of problem solving - proportion word

problems. It is important to do investigations of this type because valuable insights are provided into addressing this issue. Examining the experiences of student teachers in this aspect of proportional reasoning provides useful information about possible ways to teach with multiple representatives that may enhance the learning and teaching of proportion word problems.

It is interesting to note that the Grade 6 Mathematics textbook by Silver (2007) presents a number of problem solving questions that are intended to develop proportional reasoning in the learners. The following Grade 6 word problems adapted from Silver (2007, p. 59) are examples of proportion word problems:

12.3 *Mother says she needs 2 potatoes per person. If she is going to feed 8 people altogether, how many potatoes will she need?*

12.4 *If 1 brick is 8 cm high, how high will a pile of 5 bricks be?*

Proportion word problems form a major component of overall Grade 6 word problems. Problem solving skills are needed our entire lives. They are not only applied in mathematical situations, but in many situations where problems are encountered.

This study investigated the experiences of the student teachers in using multiple representative approaches in the teaching of Grade 6 proportion word problems. Typically, during microteaching and teaching practice, student teachers only use the cross-product approach to teach Grade 6 proportion word problems. Further, I have noticed that this is also the only approach used by experienced teachers to teach proportion word problems in local primary schools. This is a concern because the cross-product approach is used when the Grade 6 learners have little conceptual understanding of proportionality. The student teachers claim that they use this approach because it is a favourite and most simple approach which they learnt in their secondary education and at the college. This is worrying because there are many other meaningful approaches which could be adopted. This concern calls for an immediate intervention to introduce the use of multiple representative approaches in the teaching of Grade 6 proportion word problems. This study was aimed at generating awareness of using multiple representative approaches in the teaching of proportion word problems.

When they were engaged with multiple representative approaches student teachers were easily motivated and inspired. Through critical reflections of their personal experiences, they said that they became agents of change in the teaching of proportion word problems and

wanted to contribute to solving the challenge of problem solving in Namibian primary schools.

However, in my experience, some teachers are unfortunately reluctant to recognize and acknowledge the notion that learners are proficient problem solvers from a very young age. Pound & Lee stated that “problem solving is natural to young children because the world is new to them” (NCTM cited by Pound & Lee, 2011, p. 25). They further cited NCTM by stating that young children “exhibit curiosity, intelligence and flexibility as they face new situations” (*ibid*). They also stated that “the challenge at this level is to build on children’s innate problem-solving inclinations and to preserve and encourage a disposition that values problem-solving” (*ibid*).

These are inspiring remarks and should motivate teachers to recognize the importance of problem-solving for young children, including Grade 6 learners. I am convinced that a thorough knowledge of using multiple representative approaches in the teaching of proportion word problems would go a long way in making problem solving integral to any mathematics programme. I expect the use of multiple representative approaches to generate awareness in the student teachers, and the Grade 6 learners to value proportion word problems.

Some people argue that the SACMEQ III project report suggests that Grade 6 learners do not value problem solving. I would rather focus on the role that the teachers play in developing appropriate problem solving skills. The results might imply that it is actually the teachers who pay little attention to the section on word problems. The teachers could have little experience in teaching word problems and are thus frustrated by the negative reactions of the SACMEQ report. I argue that using only one approach in the teaching of problem solving is not enough to teach problem solving. Therefore this investigation of the experiences of student teachers in using multiple representations in teaching Grade 6 proportion word problems is an attempt to look at possible alternatives to using just one approach. The use of only one approach in the teaching of proportion word problems also leads Grade 6 learners to suffer from “anxiety, pressure and non-perseverance” (Simons, 1993, p. 5).

This study acknowledges that teaching proportion word problems to Grade 6 learners is not an easy task. However, this study provides some alternative ideas about using a multiple representative approach in teaching proportion word problems. It is important to take the advice by Simons who said that it is important that teachers teach in such a way that learners

“should not become over-anxious and worried at the prospect of not being able to cope with a problem situation” (1993, p. 5). The next section provides a detailed account of the importance of using multiple representations in the mathematics classroom.

### **2.3. MULTIPLE REPRESENTATIONS**

In their work on using multiple representations in teaching derivatives Ozmantar et al. (2010, p. 19) cite the National Council for Teachers of Mathematics (1989) which state that multiple representations “serve as lenses through which students interpret the problems and the solutions”. They noted that multiple representations are useful in constructing deeper understanding of derivatives; but state a pedagogical concern which is “not so much with teaching each representation but rather with teaching to translate between two or more representations which are introduced simultaneously” (2010, p. 21). They added by stating that “unless the links between and among the multiple representations are stressed, student experience difficulties in connecting the multiple representations by themselves” (2010, p. 20). My study is similar in design to the work of Ozmantar et al. (2010). It is however different in mathematical content and sample size. My study focused on proportionality in word problems as opposed to derivatives, and I worked with four participants as opposed to Ozmantar’s forty participants.

Amit and Fried (2005, p. 57) acknowledged the importance of multiple representations when they stated that “the general use for multiple representations in mathematics education hardly needs defending anymore”. They stated that they focused on “how ideas about multiple representations are realised in real classrooms” (*ibid*).

Amit and Fried (2005, p. 57) stated that the questions of their research were “quite complex for they concern not only students’ ability to work with multiple representations but also their interpretations of the meaning and value of what they are doing when they use multiple representations” (p. 57). However, they also argued that “in a learning environment where multiple representations have been fully taken into account by a well-informed teacher, learners may, nevertheless, fail to grasp the idea of multiple representations and why they are important” (*ibid*). It is thus important for teachers to use multiple representations not as a means to an end, but to facilitate an appreciation of how the multiple representations relate and connect to each other. They identified examples of multiple representations such as “algebraic, graphic, tabular, and verbal representations” (p. 58).

This research by Amit and Fried (2005) focused on the experiences of some Grade 8 students in using multiple representations in working with linear equations, rather than on the experiences of their teachers. It appears that although there is literature (Ozmantar et al., 2010; Amit & Fried, 2005; Kilpatrick et al., 2001; Levav-Wayberg & Leikin, 2006) available on the use of multiple representations in teaching, very few of them touch deeply on the experiences of student teachers and directly inform teacher preparation programs in mathematics education.

As did Amit and Fried (2005), who made use of videotapes in their research to reflect on multiple representations; I also used videotaped mathematics lessons as a source for reflections in semi-structured focus groups (Cohen, Manion & Morrison, 2007).

Kilpatrick et al. (2001) also recognised the importance of multiple representations when they stated that “communication about numbers, therefore, requires some form of external, such as a graph or a system of notation”. They added by saying that “the usefulness of numerical ideas is enhanced when students encounter and use multiple representations for the same concept” (p. 2). Kilpatrick et al. (2001) intimate that teachers should be encouraged to use multiple representations in their teaching and not solely rely on traditional algorithms. They describe algorithms as “step by step procedures for performing computation” (p. 2). They further added by saying that “an algorithm can be more or less useful to students depending on how it works and how well it is understood” (p. 2).

Kilpatrick et al. (2001) presented a “significant indicator of conceptual understanding” as “being able to represent mathematical situations in different ways and knowing how different representations can be useful for different purposes” (p. 119). They suggested that when adding fractions: “draw a picture or use concrete materials”, “a story”, “a number line”, and “renaming the fractions”, and they stated that “by operating on these different representations, students are likely to use different solution methods”(ibid). They further suggested that “this variation allows students to discuss the similarities and differences of the representations, the advantages of each, and how they must be connected if they are to yield the same answer” (p. 119).

Kilpatrick et al. (2001) strongly supported the use of multiple representations when explaining the five strands of mathematical proficiency. They stated that procedural fluency “supports the analysis of similarities and differences between methods of calculating”. These methods include: “in addition to written procedures, mental methods for finding certain sums,

differences, products, or quotients, as well as methods that use calculators, computers, or manipulative materials such as blocks, counters, or beads”. (p. 121). They further recommended that generating a mathematical representation “may be facilitated by making a drawing, writing an equation, or creating some other tangible representation” (p. 125). They observed that “with the help of representation-building experiences, children can demonstrate sophisticated reasoning abilities” (p. 129). They further stated that “students need experience in explaining and justifying the algorithm themselves with many different problems” (p. 130).

Kilpatrick et al. (2001) recognized that “the experience that people know and understand best is their own.” (p. 22). This sentiment by Kilpatrick et al. (2001) gave strength to my research because it reinforces the notion that personal experiences are helpful in gaining a deeper understanding of certain concepts.

Levav-Wayberg & Leikin (2006, p. 57) emphasized the importance of bridging the “existing gap between practice and theory of employing connecting tasks in school mathematics”. This in my view can be achieved by employing multiple representations in one’s teaching. I have noticed particularly in my teaching of upper primary Mathematics Education that student teachers have the tendency to only rely on one method to teach proportional reasoning. The student teachers inevitably use the cross-multiplication algorithm when presenting activities intended to develop proportional reasoning in Grade 6 children. I find very little evidence of any other kind of representation to teach proportionality in their teaching.

Levav-Wayberg and Leikin (2006) focused their study on “the development of teachers’ problem-solving performance as a result of systematic learning and their teaching practice associated with connecting tasks” (p. 57). My study involved the use of an intervention programme in which four selected student teachers were made aware of different multiple representations and then given the opportunity to teach them. A focus group session was then organized to obtain the experiences of those student teachers in using multiple representations in their teaching.

## **2.4 PROPORTIONAL REASONING IN WORD PROBLEMS**

### **2.4.1 Proportionality**

Cramer & Post stated that “many aspects of our world operate according to proportional rules” (1993, p. 342). They gave examples of proportionality that surface in the science and mathematics classroom. Kilpatrick et al. (2001) understood the importance of integrating context in formulating proportion problems. They remarked that “written symbolically, without labels, the statement becomes  $2/3=4/6$ ” which they “notice how important contextual framework is lost with this abstract notation” (Kilpatrick et al., 2001, p. 243). This statement implies that proportion problems should be framed within a context which is familiar to the students. Unfamiliar contexts may present difficulties similar to those experienced when abstract notation is used. Therefore, the use of examples of proportionality from the real world is important in assisting students to develop proportional reasoning. Cramer & Post advised mathematics teachers that “students need to see many problem situations that can be modeled and then solved through proportional reasoning” (NCTM (1989) cited in Cramer & Post, 1993, p. 342). In this research, when designing multiple representations of proportionality; I thus advised the student teachers to make use of contextual situations.

Kennedy et al. described proportionality “as involving the understanding of how quantities vary in relation to each other” (2008, p. 333). They gave a description of proportionality that is largely based on conceptual understanding and have the potential to “elicit more effective solution strategies” (Koedinger & Nathan, 2004, p. 146). I agree that the description of proportionality by Kennedy et al. (2008) is neither restrictive nor rote-learning based, because it is largely based on conceptual understanding which leads to effective strategies for solving proportion word problems.

Kilpatrick et al. (2001) stated that “proportional reasoning is complex and clearly needs to be developed over several years” (p. 244). They described proportion as “statements that two ratios are equal” and refer proportional reasoning as “understanding the underlying relationships in a proportional situation and working with these relationships” (p. 241). Further, they state that “proportional reasoning is based, first on the understanding of ratio” (*ibid*). They see a ratio as “a mathematical relationship that involves multiplication” (*ibid*). They clearly present three types of proportion problems which include “missing value problems; numerical comparison problems and qualitative comparison problems” (p. 243). They describe missing value problems as those that “present three values and ask the students to find the fourth or missing value”. “If 3 balloons cost \$2, then how much do 24 balloons cost?” (Kilpatrick et al., 2001, p. 243).

Kilpatrick et al. (2001) understand numerical comparison problems as those that “ask students to determine which of two given ratios represents more or less” (p.243). “Which is a better value: 3 balloons for \$2 or 24 balloons for \$12?” (Kilpatrick et al., p.243) is an example of numerical comparison problems. The qualitative comparison problem is: “What happens to the price of the balloon if you get more balloons for the same amount of money?” (ibid). They found that for the missing value and the numerically comparison problems “traditional textbooks tend to emphasize formal strategies from the beginning i.e. setting up a correct equation ( $3:2=24:x$ ), using a variable for the missing value, and using a cross multiplication algorithm ( $3x+48$  or  $x=16$ )” (Kilpatrick et al.,2001, p. 244). They then point out that “moving directly to the cross-multiplication algorithm, without attending to the conceptual aspects of proportional reasoning, can create difficulties for students” (2001, p. 244).

Kennedy et al. stated that students experience difficulties with “situations that require them to reason about proportional relations” (2008, p. 334). I too have observed this kind of difficulty over the years of working with my student teachers on teaching proportionality. However, this situation does not hinder mathematics education researchers from discovering alternative ways to deal amicably with this problem. Many researchers have spent time investigating whether it is the nature of the problem solving tasks or other factors that account for this difficulty.

#### **2.4.2 Algorithms**

Kilpatrick et al (2001, p. 196) described algorithms as “procedures that can be executed in the same way to solve a variety of problems arising from different situations and involving different numbers”. They suggested that algorithms become useful “if time is spent examining why algorithms work and comparing their advantages and disadvantages” (p. 196). They explained the advantage of inventing algorithms as “a kind of problem solving, and they must use reasoning to justify their invented procedure” (p. 197). They further state that “research suggests that students are capable of listening to their peers and to the teacher and of making sense of an algorithm if it is explained and if the students have diagrams or concrete materials that support their understanding of the quantities involved.”

Another algorithm that Kilpatrick et al. (2001) talk about is the use of arrays for multiplication and state that “arrays are powerful representations of multiplication” (p. 207).

They also say that “the expanded model permits students to function at their own level of comprehension and is likely to help them understand what they are doing” (p. 209).

A further algorithm considered by Kilpatrick et al. (2001, p. 211) is the “accessible Division Algorithm: take away copies of 46 until no more remain”. They explain that “the student builds up copies of the divisor until the dividend is reached and then reads off the quotient” (p. 212). This algorithm is supported by the abbreviated model: build up copies of 46 (ibid). Kilpatrick et al. comment that “the progression might begin with problem modelling and the use of easily understood concrete representations and algorithms...” (p. 213). Kilpatrick et al. (2001) recommend the use of classroom discussion as “a means of facilitating reflection by all children on the conceptual and notational features of arithmetic algorithms” (p. 213).

The cross-product method is also an algorithm. It is an algorithm used by many teachers to solve Grade 6 proportion word problems. Kilpatrick et al. (2001) assert that this algorithm should only be used when students understand the underlying concepts underpinning this algorithm. In this research I intend to use the cross-product algorithm, in conjunction with other algorithms to teach Grade 6 proportion word problems with selected student teachers.

### **2.4.3 Multiplicative relationships and the additive approach**

Kennedy et al. (2008) mentioned of the additive approach and multiplicative relationships as alternative strategies to the cross-product algorithm. Cramer & Post referred to the multiplicative relationship as “one critical mathematical characteristic of proportional situations” (1993, p. 342). They further stated that the “multiplicative relationship can be explored through tables, algebraic expressions, and coordinate graphs” (ibid). Kilpatrick et al. (2001) appear to strongly agree with Cramer & Post (1993) on strategies to explore the multiplicative relationship of a proportion.

Kennedy et al. (2008) observed that students solve proportion problems mostly by using the additive approach. They suggested that “teachers should counteract this by building students’ understanding of multiplicative relationships” (p. 334). Kennedy et al. (2008) saw the use of the additive approach as a method that contributes to the inhibition of good proportional reasoning in students. They recognized the importance of the additive approach, but warned mathematics teachers to use this approach more carefully. Kilpatrick et al. (2001) recognized

the additive approach as a way of facilitating students' reasoning by engaging them in building "composite units or units of units" (p. 243). They warned that "there is a danger, of course, in using this essentially additive building up process to generate equivalent ratios because students may not understand that the relationship is multiplicative" (p. 243). They said that "building from composite units does provide many students with a useful tool for working with proportional situations" (ibid). Therefore, in this research, I carefully considered the advice on using the additive approach and multiplicative relationships.

Kilpatrick et al. (2001) presented the first aspect of proportional reasoning as where "students' reasoning is facilitated as they learn to make comparisons based on multiplication rather than just addition" (p. 242). Kilpatrick et al. acknowledge the role that the additive approach plays but emphasize the use of multiplicative relationships with the aim of developing proportional reasoning. They also imply that it is necessary to know when and where the additive approach can be used before the multiplicative approach can be fully used (Kilpatrick et al., 2001). They do not completely discard the additive approach because it may be used to demonstrate the extent to which students can use it. However, it is also essential to demonstrate to the students that the additive approach may not be used to fully develop proportional reasoning skills. It may be used to help uncover the misconceptions between the additive approach and the multiplicative approach in relation to the development of proportional reasoning.

Kilpatrick et al. (2001) illustrated the additive approach and the multiplicative relationship by giving a problem of two marigolds, comparing their growth when they were 8 inches and 12 inches tall two weeks ago, and grew 11 inches and 15 inches respectively. The students who use the "additive or absolute comparison find the difference of 3 inches and conclude that each plant grew the same, 3 inches" (p. 242). Using "the multiplicative or relative comparison looks at the change relative to the original height" (Kilpatrick, 2001, p. 242).

Kilpatrick et al. (2001) stated that "students' reason is facilitated as they distinguish between those features of a proportion situation that can change and those that must stay the same" (p. 242). They pointed out that "in a proportion the numbers in the ratios can change but the multiplicative relationship must stay the same" (p. 242).

It is interesting to note that Kennedy et al. (2008) suggested the different methods that mathematics teachers may use when developing aspects of proportional reasoning in children doing mathematics. They mentioned the additive approach and the multiplicative approach as alternatives to the use of the cross-product algorithm in teaching students to reason proportionally. They acknowledged the use of the cross-product algorithm in secondary school children and suggest the use of the multiplicative approach for primary school children. They also mentioned the areas in which proportions are used inside and outside the mathematics classroom that may be helpful tools in guiding children to develop proportional reasoning.

It is also interesting to note that Cramer & Post (1993) examined the mathematical characteristics of proportional situations and make mathematical connections that enable students to solve problems. They also mentioned the importance of establishing multiplicative relationships of proportional situations when teaching proportion word problems. They shared similar sentiments with Kennedy et al. (2008) by stating that the cross-product algorithm is difficult for primary school children. They provided alternatives to the cross-product algorithm by discussing the table method and the graphing method in solving proportion word problems. They stated that a sound knowledge of the mathematical characteristics of proportional situations encourage students to use multiple ways to solve proportion word problems.

## **2.5. MULTIPLE REPRESENTATIONS IN UNDERSTANDING PROPORTIONALITY**

Amit and Fried (2005) described multiple representations as including “algebraic, graphs, tabular, and verbal representations” (p. 58). In this regard Kilpatrick et al. (2001) commented that generating mathematical representation “may be facilitated by making a drawing, writing an equation, or creating some other tangible representation” (p. 125). Ozmantar et al. (2010) cite the National Council for Teachers of Mathematics (1989) who describe multiple representations as different “lenses through which students interpret the problems and solutions” (p. 19). Although many researchers have different views on the description of multiple representations, there are similarities in their descriptions that are useful for my research.

In this research multiple representations refer to the different ways used to help learners to interpret and solve proportion word problems (Ozmantar, et al., 2010). In addition to the

cross-product algorithm, tables, diagrams, and graphs were used to teach proportion word problems and assist in presenting solutions to these problems. Moreover, “the between-comparison and within-comparison method” (Nielsen, 1998, p. 38) will also be used.

Kilpatrick et al. (2001) stated that “the aspects of proportional reasoning that must be developed can be supported through exploring proportional (and non-proportional) situations in a variety of problem contexts using concrete materials or situations in which students collect data, build tables, and determine the relationships between the number pairs (ratios) in the tables” (p. 244). Kilpatrick et al. (2001) added concrete materials as teaching and learning aids that support the development of proportional reasoning to the list given by Cramer & Post (1993). Kilpatrick et al. (ibid) made use of equations because the process of building up equations involved the formulation of algebraic expressions used by Cramer and Post (1993). All these authors appear to use multiplicative relationships as one of the relevant methods in finding solutions to proportion word problems. This research attempted to investigate student teachers’ experiences in using a variety of representations to teach grade 6 proportion word problems.

The example below illustrates the typical cross-product methodology in solving a proportion word problem:

*Mwaka works for N\$10 every week in Someli Shop. How much does Mwaka get as salary for January 2011 if she was employed on the 1<sup>st</sup> of January 2011 and paid on the 28<sup>th</sup> of January 2011?*

<i>Amount(N\$)</i>	<i>Week(s)</i>
<i>10</i>	<i>1</i>
<i>X</i>	<i>4</i>

*Cross-product algorithm: X N\$ multiplied by 1w = N\$10 multiplied by 4w*

$$X = N\$40.00$$

This algorithm can be followed with very little conceptual understanding of proportionality. It is clear therefore, that provided the pupil follows the cross-product algorithm step-by-step, the pupil will most likely arrive at the correct result. In my experience, and that of Kennedy et al. (2008), it is important to note however that “even pre-service teachers can have difficulty reasoning proportionally” (p. 334). It is of vital importance therefore that

educational institutions who train mathematics teachers ensure that student teachers are proficient in both the conceptual understanding of proportionality and its teaching.

There are many other methods and strategies that promote conceptual understanding of proportionality which can successfully be used to teach Grade 6 proportion word problems.. I argued that the teaching of a multiple representative approach is critical in the development of understanding proportionality in upper primary learners. Cramer & Post (1993) acknowledged that “students need to see many problem situations that can be modeled and then solved through proportional thinking” (p. 342). This statement emphasizes the need to expose student teachers to many proportion word problems that can be solved through multiple representations. My study focused specifically on the following different representative strategies:

- Cross-Product Algorithm,
- the Between-Comparison Method,
- the Within-Comparison Method,
- the Table Method,
- the Graph,
- the use of diagrams/models, and
- the Informal Oral Strategy.

### **The Between-Comparison Method**

Nielsen (1998) stated that the missing term of a proportion is found by making “a comparison between ratios of the proportion” (p. 38). This is illustrated as follows:  $2/3=8/?$

Nielsen stated that the learner identifies a multiplicative relationship between 2 and 8. In the proportion  $2/3=8/?$  the multiplicative relationship is 4. Therefore 3 is multiplied by 4 to get the missing term in the proportion. The missing term of the proportion is 12.

Van de Walle, Karp & Bay-Williams (2010, p. 364) referred to this method as the “between ratio” and described it as “a ratio of two corresponding measures in different situations”. It is interesting to note that they all use the words “ratio” and “corresponding” in their description of this method. The word “ratio” identifies this type of procedure and it is clearly distinguished from other procedures by using “corresponding measures in different situations”.

Van de Walle et al. (2010, p. 364) used an example of comparing similar rectangles. They said that “the ratio of the length of one rectangle to the length of another is a between ratio; that is between the two rectangles” (*ibid*). This is a comparison of the side of one rectangle to the side of another similar rectangle. For example the length of one rectangle is 10 cm and the length of another rectangle is 20 cm. It is important to note that “for all similar rectangles, corresponding between ratios will be equal” (*ibid*).

In this research the Between-Comparison Method is used as one of the alternative approaches to the Cross-Product method. The participating student teachers used the Between-Comparison Method as one of the multiple representative approaches in the teaching of Grade 6 proportion word problems. This method was used in conjunction with other representative methods in the attempt to support the exploration of proportion and enhance the teaching of proportion word problems.

### **The Within-Comparison Method**

Nielsen (1998, p. 38) stated that the missing term of a proportion is found by making a “comparison within the ratios” of the proportion. For example:  $2/3=8/?$

The learner identifies the multiplicative relationship between 2 and 3. In the proportion  $2/3=8/?$  the multiplicative relationship is 1.5 which multiplies 8 to get the missing term in the proportion. The missing term of the proportion is 12.

Van de Walle et al. (2010, p. 363) referred to this method as the “Within ratio” and describe it as “a ratio of two corresponding measures in the same setting”. It is interesting to note that they all use the words “ratio” and “corresponding” in their description of this method. The word “within ratios” identifies this type of procedure and it is clearly distinguished from other procedures by using “corresponding measures in the same setting”.

Van de Walle et al. (2010, p. 364) used an example of comparison of similar rectangles. They said that “the ratio of the length to width for any one rectangle is a within ratio; that is it is within the context of that rectangle” (*ibid*). This is a comparison of the length and width of one rectangle. For example, the length of the rectangle is 10 cm and the width of the same rectangle is 5 cm. It is also important to note that “for two similar rectangles, all of the within ratios will be equal” (*ibid*).

Nielson (1998, p. 43) stated that “in finding a missing term of a proportion, we have a choice between applying a Between-Comparison and a Within-Comparison”. He further stated that this choice depends on the “known terms” (*ibid*). Moreover, he stated that there is a choice in how a proportion is set up. He explained this choice by giving a proportion word problem: “3 chairs cost N\$225. What is the price of 8 chairs?” (Nielsen, 1998, p. 40). He stated that the proportion can be set up by considering “the ratio between the quantities equals the ratio between the prices” (*ibid*). He presents the fraction form of this proportion this way:

$$\begin{array}{ccc} \text{Number of chairs/number of chairs} & = & \text{price/price} \\ 3/8 & = & 225/? \end{array}$$

Van de Walle et al. (2010, p. 365) referred to this set up of proportion as that of using the “between ratios”.

Nielson (1998, p. 40) also stated that the proportion can be set up by considering “the ratio between quantity and price to be constantly the same” (*ibid*). This set up of proportions is referred by Van de Walle et al. (2010) as that of using the “within ratios” (p. 365). Nielsen (1998, p. 40-41) presented the fraction form of this proportion in this way:

$$\text{Number of chairs/price} = \text{Number of chairs/ price}$$

Van de Walle et al. (2010, p. 365) stated that given a proportion word problem, “the greatest difficulty students have is setting up a correct proportion or equation of two ratios, one of which includes the missing value”. They gave the questions which guide the process of setting up proportions: “Which fractions do I make? Where does the x go?” (*ibid*). In this research the participating student teachers were cautioned about this difficulty and were assisted to invent strategies to reduce this difficulty.

In this research the Within-Comparison Method is used as one of the alternative approaches to the cross-product method. The participating student teachers used the Within-Comparison Method as one of the multiple representative approaches in the teaching of Grade 6 proportion word problems. This method was used in conjunction with other representative methods. This research used multiple representations carefully.

## **The Table Method**

A learner finds the missing term of the proportion by constructing a ratio table (Van Etten & Adendorff, 2010). The ratio table consists of the word column, the first number column and the last number column that make up the proportion. Van Etten and Adendorff (2010) stated that “in a ratio table a new column is formed each time an existing value that fits the pattern is multiplied by another value” (p. 8). Table 2.1 below adapted from Van de Walle et al. (2010, p. 356) illustrates the table method.

Table 2.1: The Table Method (adapted from Van de Walle et al.)

Acres	5	10	15	20	25	
Pine trees	75	150	225			

Van Etten & Adendorff (2010) and Van de Walle et al. (2010) referred to this table as a “ratio table”. Furthermore, Van de Walle et al. (2010, p. 356) explained a ratio table as a way to organise information to “show how two variable quantities are related” (*ibid*). They stated that the “pattern that connects acres to pine trees ( $\times 15$ ) is the generative pattern and the multiplicative relationship between the values” (*ibid*). This research began with simple tables of values, and later as the Grade 6 learners become more proficient, comfortable, the complexity of the tables was be increased.

Van de Walle et al. (2010) discussed structured and non-structured ratio tables. They stated that “ratio tables may not always be in organized lists where a pattern can be found” (2010, p. 356). They further emphasised that “ratio tables can be used when only one ratio is known and you are trying to find a specific equivalent ratio” (2010, pp. 356-357). In this research participating student teachers used structured tables where a pattern can be explored by the Grade 6 learners.

They provided three solutions using ratio tables for the following proportion word problem (adapted from Van de Walle et al., 2010, p. 358):

*160 pounds on Earth is 416 pounds on Jupiter. If something weighs 120 pounds on earth, how many pounds would it weigh on Jupiter?*

The three solutions using ratio tables are provided below (ibid). Table 2.2, Table 2.3 and Table 2.4 give examples of solutions to the ratio problem.

Table 2.2.: Example 1 of a ratio table

Earth weight	160	80	40	120
Jupiter weight	416	208	104	312

Table 2.3.: Example 2 of a ratio table

Earth weight	160	80	40	120
Jupiter weight	416	208	104	312

Table 2.4.: Example 3 of a ratio table

Earth weight	160	20	100	120
Jupiter weight	416	52	260	312

These ratio tables represent the three solutions to the weight question on Earth and Jupiter. They are referred to as *not structured*. In this research the participating student teachers did not use complicated tables like the ones above; but they used ratio tables that were simple in which the multiplicative relationship could be easily explored using division or multiplication.

Van de Walle et al. also presented a *structured* ratio table which has the “notes column that shows what was done in each step” (2010, p. 358). Table 2.5 below is an example of a “more structured ratio table” (ibid):

Table 2.5.: An example of a more structured ratio table (from Van de Walle et al., 2010, p. 358)

	<b>Pounds</b>	<b>Cost</b>	<b>Notes</b>
A	1	4.25	Given
B	10	42.50	$A \times 10$
C	2	8.50	$A \times 2$
D	0.1	0.425	$A \div 10$
E	12.1	51.425	$A+C+D$
F	0.01	0.0425	$D \div 10$
G	0.03	0.1275	$F \times 3$
H	12.13	51.5525	$E+G$

This table was used to provide the solution to the proportion word problem below (ibid):

*Cheese is \$4.25 per pound. How much will 12.13 pounds cost?*

This type of table is too complicated for Grade 6 learners because it has several complex steps that require critical thinking beyond the abilities of most Grade 6’s. However, it may be offered to highly gifted Grade 6 learners. Cramer & Post (1993) provided a simple table that the participating student teachers used to teach Grade 6 proportion word problems. Table 2.5 below, adapted from Cramer & Post (1993, p. 345), is an example of a simple ratio table:

Table 2.6.: A simple ratio table adapted from Cramer & Post (1993, p. 345)

Y	3	6	9	12	15		
X	2	4	6	8	10	16	

They stated that “if the number of pairs are related proportionally, then the rate pairs should form a set of equivalent fractions” (*ibid*). This is an interesting observation made by Cramer & Post (1993). In this research the student teachers emphasised the use of equivalent fractions in designing a ratio table and its proportion word problems. The participating student teachers also used the table to explore proportionality.

Cramer & Post (1993) emphasized the importance of the table strategy by commenting that “constructing a table helps to identify the numeral relationship between the two quantities” (p. 345). This is consistent with the findings by Van de Walle et al. (2010) on using the table to solve proportion word problems. The work of McEldoon & Cochrane-Braswel (n.d.) also emphasized the importance of the table method. In terms of the table, they described functional thinking in terms of values amongst other multiple representations. They defined functional thinking as a “type of mathematical thinking which focuses on the relationship between two or more varying quantities” which is noted by a rule of correspondence” (McEldoon & Cochrane-Braswel, undated, unpagued). They explained the importance of functional thinking as “understanding the functional relationship between XY values” (*ibid*). They further explained how functional thinking about problems in a table form, requires reading values across columns or within one column. This explanation of interpreting a table of values may be used to distinguish between the *within-comparison* method and the *between-comparison* method by Nielsen (1998) and Van de Walle et al.’s (2010) *within ratios* and *between ratios*. The student teachers were also encouraged to use the table method to identify proportional situations and non-proportional situations.

In this study the participating student teachers emphasized the interpretation of the table of values by reading the values across columns and within one column. This entails that the table method will be used in conjunction with other multiple representations. However, the student teachers were advised not to use too many methods in addition to the table method. The participating student teachers explored whether the table method used in conjunction

with a reasonable number of multiple representations could enhance the teaching of Grade 6 proportion word problems.

### **The Graph Method**

This method entails the learner finding the missing term by drawing a linear graph from the table of values representing the proportional situation. The missing term is found by using the graph to locate the term.

Amit and Fried (2005) observed a teacher who taught a Grade 8 lesson on drawing a linear graph of a system of linear equations. The teacher informed the learners that the aim of the lesson was to “learn to translate  $x+y=6$  into another language” using sketches (Amit & Fried, 2005, p. 60). The teacher “neither wanted to write in the language of algebra nor to say in words, but to sketch it” (*ibid*). In this context, language refers to the type of multiple representations used by the teacher. When the teacher refers to the language of sketches the focus is on the graphical representation.

Amit and Fried (2005) found that learners consider drawing graphs as a “redundant exercise” that consumes much time when trying to draw “the numbers and checking and putting down the points” (p. 62). It would be interesting to see if the participants in this study would have similar experiences in the graphical representation. Van de Walle et al. (2010, p. 359) stated that “graphing ratios provides a powerful connection to algebra”. This notion would however not apply in this particular study. Grade 6 learners do not yet study algebra because of its abstract nature.

In this study the participating student teachers used tabular representations and graphical representations in conjunction to each other. This is consistent with Van de Walle et al.’s (2010, p. 359) notion that “any ratio table can be graphed”. The experiences of the participating student teachers in using the table method and the graph method would be noted with anticipation.

Van de Walle et al. (2010, p. 359) suggested a graphing activity that the learners could do. They stated that learners could be asked to draw a graph from “the ratios of sides in similar rectangles” (*ibid*). This graph could be used to “determine other equivalent ratios” of similar rectangles (*ibid*). They suggested that learners could graph similar rectangles  $3 \times 9$  and  $4 \times 12$  and use that data to find other equivalent ratios (*ibid*). The use of equivalent fractions was an important component in this study.

Cramer & Post (1993, p. 343) stated that “in all proportional situations, the numerical relationship between quantities can be expressed by a rule in the form  $y=mx$ , where  $m$  is one of the constant factors relating the two quantities”. The linear proportional graph has its points on the straight line that passes through the origin (Cramer & Post, 1993, p. 344). This informed the student teachers that the graph of  $y=mx$  represents a proportional situation. This implies that the graph representing a proportional situation can be used to explore the multiplicative relationship between the given values.

Cramer & Post (1993) further stated that a graph of the form  $y=x+a$  is not a proportional situation. This, therefore, tells us that a graph can be used to represent proportional and non-proportional situations. Therefore it is one of the many ways used to solve proportion word problems. The student teachers were aware that there is an obvious link between the table of proportion and the graph of a proportion.

### The use of diagrams/ models

The use of diagrams such as the bar model of the Singapore Curriculum is a “pictorial representation of known and unknown quantities and their relationships in a problem” (Pacific Institute for Mathematical Sciences, 2009, unpagged). In the context of proportional situations, the use of diagrams serves as powerful visual representations of the known and unknown quantities of a proportional situation. A proportional situation can be represented by pictorial models that are constructed by learners to obtain the answer to the problem. Figure 2.1 below gives an example of a pictorial representation of proportion.

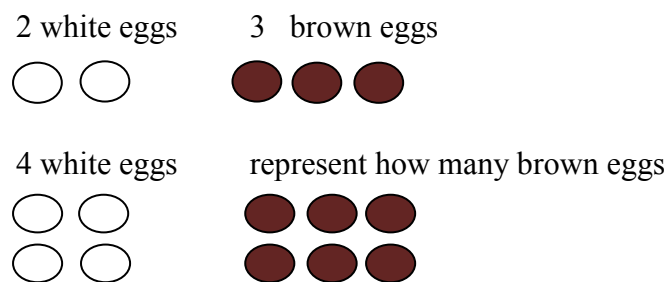


Figure 2.1: A pictorial representation of proportion

Van de Walle et al. (2010, p. 27) referred to this method as “a model for a mathematical concept”. They described a model as “any object, picture, or drawing that represents the concept or onto which the relationship for that concept can be imposed” (*ibid*). In this

research a model is any diagram drawn by the participating student teachers to represent proportionality extracted in proportion word problems. The student teachers facilitated the drawing of diagrams in the Grade 6 class to explore proportionality and use diagrams as a multiple representation. The Grade 6 learners were encouraged to draw diagrams which were familiar to them and which accurately reflected the proportionality under consideration.

Van de Walle et al. (2010, p. 27) warn teachers that they “will undoubtedly encounter situations in which they use a model that clearly illustrates an idea but a learner just doesn’t get it, whereas a different model is very helpful”. The participating student teachers were advised to be careful in selecting a model that will be used in exploring the solution to a proportion word problem. It is essential that the student teachers use familiar and simple models that are accessible by their Grade 6 learners. The student teachers were thus reminded not to ignore the preferences of the learners.

Van de Walle et al. (2010, p. 27) argued that it is “incorrect to say that a model illustrates a concept”. A model facilitates the understanding of a concept. The student teachers were also thus reminded that a model merely facilitates the exploration of proportionality. The learners still had to understand the underlying principles of the concept.

Van de Walle et al. (2010, p. 29) suggested that when teaching using models, mathematics teachers should avoid autocratic instructions such as “do as I do”. They stated that learners might pretend to be closely following the demonstration using models because they just want to impress the mathematics teacher. It is therefore advisable to give the learners an opportunity to demonstrate proportionality for themselves using their own models. In this research, participating student teachers were advised to give the learners an opportunity to demonstrate how their own models can be useful in exploring proportionality.

Van de Walle et al. (2010, p. 29) further suggested that the models should not be used by the learners as “answer-getting” models but as “tools used to explore the concept”. This is very sound advice as “answer-getting” deprives learners of deep mathematical thinking. A model should give the learners an opportunity to explore proportionality and be able to justify their reasoning with the aid of the model

### **The Informal Oral Strategy**

The student teachers were also encouraged to use informal oral strategies such as *trial-and-error* to solve proportion word problems before they used formal written strategies. Koedinger & Nathan (2004) supported the *trial-and-error* method as a legitimate informal strategy for solving word problems. They gave examples of informal strategies as the *unwind strategy*, and the *guess and test strategy*.

Cramer & Post stated that “multiple solution strategies give students the power to choose a strategy that best fit the data; the resources available and their personal preferences” (1993, p. 346). I strongly agree that student teachers have their personal preferences even when dealing with strategies to solve proportion word problems. Student teachers were therefore encouraged to use their personal preferences and make their own choices when using multiple representations in their teaching as they believed that this strategy has increasing benefits for the learners in understanding proportional situations.

## **2.6. FACTORS THAT AFFECT THE SOLVING OF WORD PROBLEMS**

There are many factors that can affect the solving of word problems. Two of them were considered:

- the story context of the problem
- English Language as a medium of instruction

### **2.6.1 The Story Context of the problem**

In this study the participating student teachers used simple contextual proportion word problems. They used familiar and simple contexts that are relevant to the Grade 6 learners. In this context the word “simple” does not mean proportion word problems that are not challenging, but those that represent situations that are known to the learners. Contextual proportion word problems are expected to help participating student teachers to avoid mathematical problems that “do not engage children because they are not real” or “they are too simple” (Pound & Lee, 2011, p. 28). Pound & Lee (2011, p. 28) cited Resnick (1988) who advised teachers using contextual word problems “to find ways to create in the

classroom situations of sufficient complexity and engagement”. They further stated that it “permits students (learners) to develop questions, not only solve problems posed by others” (ibid).

In this study the participating student teachers designed questions which were appropriate to the Grade 6 learners. They assisted Grade 6 learners to formulate own proportional word problems. This process was expected to instill some courage in the Grade 6 learners, so that they do not skip questions involving proportion word problems designed by the teacher. The added benefit of involving Grade 6 learners in formulating proportion word problems clearly helps them to identify proportional situations from non-proportional situations. If the Grade 6 learners succeeded in formulating their own proportion word problems, this will in turn boost the student teachers’ confidence in the teaching of proportion word problems.

Ollerton (2010) advised mathematics educators to avoid using real-life contexts that are unfamiliar to the learners. He stated that mathematics teachers should avoid using adult-life contexts in problem solving which the learners do not even understand. He stated that “teachers’ contexts are rarely those which children might be concerned with, or indeed even interested in” (Ollerton, 2010, p. 95). Ollerton’s (2010) argument is interesting and very relevant in the designing of proportion word problems. In this study student teachers should be familiar with the contexts which Grade 6 learners understand. Ollerton (2010) suggested that mathematics teachers should work together to design and discuss interesting and familiar real-life contexts with the Grade 6 learners.

Pound & Lee (2011) acknowledged the importance of word problems formulated from real-life contexts, but also suggested the use of mathematics story problems. These were problems that were not necessarily contextualized in real life situations, but were anchored in a fictional story context that the learners could relate to. They argued that using story context proportion word problems helped mathematics teachers to teach word problems in an interesting and exciting way. Using a story context, word problems have the added benefit of preparing Grade 6 learners for life skills as well as teaching mathematical skills. In this study student teachers did not use proportion story problems but used contextualized proportion word problems. The proportion story problems are placed in the phase that precedes contextualized proportion word problems. It is important to know the story context of the problem. McEldoon & Cochrane (undated) found that story contexts “help third grade students solving arithmetic problems over comparable symbolic contexts”. I therefore

advised the student teachers to make use of proportion word problems relevant to the context of the learners and they needed to ensure that the context was both relevant and familiar to the learners. In this study the big challenge that faces the participating student teachers is to match the chosen proportion word problem to the levels of abilities of the learners in the class. Ollerton however referred to matching activity as “nonsensical” (2010, p. 91). He suggested that “what we need is a range of starting points that every child can access together with a sequence of developmental tasks and questions that cater for different speeds and depths of cognition” (*ibid*).

### **2.6.2 English Language as the medium of teaching and learning**

English is the medium of instruction in Namibian schools. The Namibia Ministry of Basic Education Sport and Culture (2003, p. 2) stated that “the use of English as language of wider communication will only further enhance greater participation in social welfare activities”. In this research English was used as the language in which proportion word problems were taught and learnt. The student teachers used English to engage the Grade 6 learners in the multiple representations used in solving proportion word problems. The student teachers were advised that Grade 6 is the second year of using English as a medium of communication. Consistent with the Namibian language policy, the student teachers were therefore expected to teach proportion word problems in English and the learners were expected to give responses in English. It is however acknowledged that the learners at times find it difficult to understand English and thus the content of the proportion word problems. In those situations, the student teachers were expected to invariably code switch into the vernacular- a practice that they are very familiar with.

The use of appropriate language in the context of this study also refers to mathematical communication in a broader sense. Pound & Lee (2011, p. 40) stated that “the lack of talk about mathematics is linked to a lack of practice or rehearsal of ideas and therefore an absence of relevant vocabulary.” In this research study student teachers were made aware that a lack of talk on proportion word problems and multiple representations could be a barrier to learning. It needs to be emphasized that “much of the mathematics with which we present is written” and “the symbols and signs which are used are in themselves a third language.” (*ibid*).

In this research the student teachers and the Grade 6 learners were expected to be thoroughly familiar with all the appropriate vocabulary used in proportion word problems such as

*compare, the between-comparison, the within-comparison, the table, etc.* Koedinger & Nathan (2004, p. 133) stated that learners faced difficulties in “translating English or non-mathematical words”. They attributed the difficulty of the story problems to the “translate and solve strategy” (ibid).

Proportion word problems mostly use the language of multiplication and division. Anghileri (1991, p. 96) stressed that learners should “appreciate the nature in which multiplication and division are inherent, that they understand the language of such situations...” She suggested that “sharing activities provide experience of the language and concepts of multiplication and division” (ibid). She further stated that “exploiting terms such as lots of, each, equally, and fair are crucial” enriches learners with the language of multiplication and division i.e. the language of proportion word problems. It is important to note that “the idea of equal groups or portions are implicit and indicated by the word *each*” (Anghileri, 1991, p. 99). She commented that the learners “do not always understand the implication of this word and may ignore it in their solution of problems” (ibid). In this research the participating student teachers were of the words that learners find most confusing in proportion word problems and request them familiarize themselves with the meanings of these words. Moreover, the student teachers were advised to use small and manageable groups to overcome some of the language issues.

Anghileri (1991, p. 99) gave an example of a proportion word problem which does not have “key words that will not indicate that the operation is division”. The example is: “My dog eats 4 biscuits every day. How long will a packet of 64 biscuits last?” Therefore, it is important to identify those proportion word problems that have key words that represent the arithmetic operations and those that do not have. Grade 6 learners need to be able to identify key words, understand them and associate them with the relevant arithmetic operation. They should also be aware of proportion word problems with no useful key words that indicate the arithmetic operations to be used.

Anghileri (1991, p. 99) gave advice on using the word “altogether” and how it leads to misconception in proportion word problems. She gave an example of a word problem: “If Tom has 3 packets of marbles and each packet contains 15 marbles, how many marbles are there altogether?” She commented that learners who know that the word “altogether” is associated with addition may misinterpret this word problem and obtain a wrong answer.

The student teachers should explore with the Grade 6 learners the words that could lead to misconceptions in proportion word problems.

It is important to acknowledge that the difficulty with solving proportion word problems can be attributed to the low proficiency level of the learners in the language of proportion word problems. Those learners who are less proficient in the language of proportionality will have difficulty with the proportion word problems.

Anghileri (1991, pp. 100-101) acknowledged that even “high school children have difficulty identifying multiplication and division as appropriate operations to solve word problems.” This implies that exposing the learners to the language of proportion in foundation grades like Grade 6 is very important.

It appears that difficulties with the English language and the language of proportionality account for many of the misconceptions observed in using different solution strategies to solve proportion word problems. In this research I was thus mindful of the pitfalls associated with language on all levels.

## **2.7 CONCLUSION**

Multiple representations in Mathematics Education are useful strategies that help student teachers to teach Grade 6 proportion word problems.

In this literature review I discussed the problems of adopting a mono-dimensional approach to teaching proportion word problems, and considered some alternative methods such as:

- the Informal Oral Strategy
- the Between-Comparison Method
- the Within-Comparison Method
- the Table Method
- the Graph Method
- the use of diagrams/models

In the next chapter I discuss the methodology used for my study, and describe how I collected data to investigate the selected student teachers' experiences in using multiple representations in the teaching of Grade 6 proportion word problems.

## **CHAPTER 3      METHODOLOGY**

### **3.1 INTRODUCTION**

This study sought to explore the experiences of four selected student teachers of using multiple representations in the teaching of Grade 6 proportion word problems, through a qualitative research design. This chapter describes the research question of this case study, the research orientation, the selection of the research site and participants, research methodology, research design, the research instruments used, my data analysis procedures, research ethics and validity. Further, some limitations and challenges faced are also identified.

### **3.2 RESEARCH QUESTION**

My research question is: “What are selected student teachers’ experiences in the use of multiple representations in the teaching of grade 6 proportion word problems?”

### **3.3 RESEARCH ORIENTATION**

This research is located in the interpretivist paradigm (Creswell, 2003) because it sought to understand the personal experiences of four student teachers in using multiple representations to teach Grade 6 proportion word problems. This paradigm is appropriate for this case study because I made use of the subjective interpretations and perceptions my participants’ experiences when teaching word problems using a multiple representative strategy. Moreover, this paradigm allowed me to explore the experiences of the four selected student teachers in depth. Creswell asserted that “one cannot escape the personal interpretation brought to qualitative data analysis” (2003, p. 182). Wellington defined personal experiences as “perspectives and shared meanings” (2000, p. 16). I explored the “perspectives and shared meanings” of the four selected student teachers as collected in the focus group sessions and supported by data collected during the classroom visits.

The interpretivist paradigm acknowledges that an insider researcher “makes a difference to the observed and that reality is a human construct” (Wellington, 2000, p. 16). As an insider researcher this suggests that my presence in both the classroom and the focus group venue could have had an impact on the emotions and actions of the four selected student teachers and their Grade 6 learners. The four selected student teachers were inspired to reflect on, and challenge their practice on the use of multiple representative approaches of teaching Grade 6

proportion word problems. My presence and research project thus triggered a response which ultimately resulted in a changed practice. In addition, the four selected student teachers responded to my questions in a focus group session. As I knew the student teachers from my engagement with them at the University in their first year, they felt comfortable with my presence in the focus group sessions and the classroom visits.

The interpretivist paradigm opposes the ideas of the positivist paradigm which maintains that knowledge is “objective, value-free, generalizable and replicable” (Wellington, 2000, p. 15). I was not a stranger in both the focus group sessions and lesson presentations in Grade 6 classrooms. As an insider it was thus not possible to remain totally objective – the very nature of the study and the context was such that it could not remain value-free. The sample size was too small to claim any generalizability. Having said this, I do believe that the student teachers did not simply employ multiple representations just to please me, as would perhaps have been the case with a stranger.

### **3.4 RESEARCH SITE AND PARTICIPANTS**

The research was conducted at one of the Campuses of the University of Namibia and in two selected upper primary schools in the surrounding area. The participants in the research were four, third year BETD student teachers. The participants were selected on a voluntary basis from a pool of students specializing in teaching upper primary mathematics. Three of these student teachers were male and one was female.

Each of the four selected student teachers was allocated to a Grade 6 class in two different primary schools. The two primary schools were selected on the basis of their close proximity to the Campus and their frequent use by the University as teaching practice schools. Thus, both the sampling of the student teachers and the primary schools was informed by “convenience sampling” (Newby, 2010, p. 251). Convenience sampling is appropriate for this research because of using “data sources that just happen to be around” (ibid).

### **3.5 RESEARCH METHODOLOGY**

This study was conducted by using a qualitative case study (Simons, 2009) approach. The “case” of this study is the experiences of the four selected third year student teachers in using multiple representations to teach Grade 6 proportion word problems. The student teachers’ experiences constitute the unit of analysis in this study. The experiences of the student

teachers are complex and shaped by many factors that prevail at the Campus and in the selected primary schools.

This qualitative case study is underpinned by the interpretivist paradigm. This implies that the major focus is on the participating student teachers' subjective experiences of using multiple representations in the teaching of proportion word problems. These experiences are explored through focus group on the student teachers' teaching practice sessions in the two primary schools.

The focus group interview schedule was used to capture the experiences of the participating student teachers. Some video clips of the student teachers' lessons on using multiple representations in the teaching of Grade 6 proportion word problems were viewed to facilitate the discussions in the focus group.

### **3.6 RESEARCH DESIGN**

This study unfolded in 6 phases:

#### **PHASE 1- Pre-Workshop Activities**

Before the first workshop, on the 24<sup>th</sup> of October 2011, I visited the Mathematics Education lecturer of the University and the student teachers specializing in Mathematics Education. I told them about my research proposal and asked them whether they would be interested in participating in my study. I sought four volunteers once my research proposal was formally approved by Rhodes University and the Ministry of Education granted me permission.

Although it was not easy to secure four volunteers due to their involvement in other research projects and their initial sense of uncertainty, I managed to find four participants on the 23<sup>rd</sup> May 2012. I provided them with a letter of consent which they completed (Appendix E). I also informed them of the first workshop date and the entire research programme.

#### **PHASE 2- Workshop 1**

On the 13<sup>th</sup> June 2012 the student teachers were invited to the first workshop that took place in the office of the Assistant Faculty Officer at the University from 15:00 to 17:00. All four selected student teachers attended. I introduced them to multiple representative approaches that could be used in the teaching of Grade 6 proportion word problems. The multiple representative approaches that I included in this workshop were "the between-comparison

method, the within-comparison method, the diagrammatic method/Singapore, the cross-product method, the table method, the graphic method, and the oral informal method” – I discussed these in detail in Chapter Two, pages 19 -28.

I used the section entitled “multiple representation and proportionality” in my Literature Review (Chapter Two) of this thesis as workshop material. I gave this information material to the participants and used it as a basis for discussion and engagement. The participants appeared comfortable with the workshop on the multiple representative approaches because they were familiar with its content which they had studied in their first year of study at the University in 2010. The workshop also focused on how to craft, use and adapt multiple representative activities in the classroom. We ended the workshop with a task on developing some simple Grade 6 proportion word problems. With my assistance, the four selected student teachers designed three worksheets on simple Grade 6 proportion word problems (see attached Worksheets 1 to 3 in Appendix G). They were also asked to prepare lessons in which they used these worksheets, and pilot them in their teaching practice.

### **PHASE 3 – Micro-teaching/pilot phase**

In this phase the intention was for the four student teachers to present one microteaching lesson in front of their peers in their mathematics education classroom at the University. The rationale behind this was for the student teachers to pilot their lessons and then iron out any issues or problems based on the feedback they received from their peers. Unfortunately, due to time constraints we could not execute this plan. The students were heavily engaged in preparation for their May/June examinations and felt that they could not participate in this pilot phase. Instead we implemented the lessons in the two selected schools during the student teachers’ teaching practice after the May 2012 recess of the schools.

### **PHASE 4- Workshop 2**

In this phase I invited the four student teachers to a second workshop to reflect on the teaching process of Phase 2. This workshop took place in the office of the assistant faculty officer at the university on the 27<sup>th</sup> June 2012 from 15:00 to 17:00. We discussed ways to make improvements to the lessons using multiple representations. We touched on some of the challenges which they had experienced, such as contextualizing the proportion word problems to Grade 6. With my assistance the student teachers recognized that proportion

word problems can be contextualized into Grade 6 word problems on the theme of money and finance.

The feasibility of integrating all the multiple representative approaches into one lesson was also discussed. Two student teachers who tried integrating all the multiple representative approaches in one lesson reported that it was confusing to the Grade 6 learners and too long for a 40 minute lesson. The student teachers agreed to teach two lessons where they did not integrate all the multiple representative approaches. I advised them that they exercise flexibility in the amount of work to be covered, with reference to the multiple representative approaches - taking into consideration the time available, the learning outcomes and the stage of development of their learners. They suggested that a double period lesson of 40 minutes each would be an advantage if secured with permission from the support teacher. They further suggested that afternoon classes could also be organized. In short, the contributions of the student teachers were a key element in this phase.

### **PHASE 5 – Teaching in schools**

In this phase each of the four selected student teachers presented two lessons to Grade 6 learners using multiple representations. This coincided with their teaching practice. Each presentation was observed by me, a fellow student teacher, and in some cases, the support teacher too. The lesson presentations were video recorded. Table 3.1 shows the time frames for Phase 5 – Teaching in schools.

Table 3.1.: The time frames for Phase 5

<b>Student Teacher</b>	<b>Pseudonym of the school</b>	<b>Date</b>	<b>Time</b>
Student Teacher A	Mazoho Primary School	12 July 2012	07:50-09:10
		17 July 2012	07:50-09:10
Student Teacher B	Lihutu Primary School	16 July 2012	15:00-16:00
		18 July 2012	11:40-13:00
Student Teacher C	Lihutu Primary School	16 July 2012	16:15-16:50
		18 July 2012	08:30- 09:50
Student Teacher D	Mazoho Primary School	19 July 2012	10:10-11:30

			15:00-16:40
--	--	--	-------------

## **PHASE 6 – Focus group sessions**

In this phase the four selected student teachers were invited to a focus-group session at the University, where they reflected on their experiences. With permission from one of the student teachers the video of this student teacher’s lessons was played and through probing questions the experiences of the four selected student teachers were sought. The focus group was used specifically to reflect deeply on the use of multiple representations in the teaching of Grade 6 word problems.

### **3.7 RESEARCH INSTRUMENTS**

#### **3.7.1 Observation making use of video recordings**

The two lessons of each of the four selected student teachers were video recorded in the schools. The video recording of one student teacher was played back to the participants in Phase 6 and used as a tool to reflect on the experiences of teaching using a multi-representational approach to teaching proportion word problems. It was also used to provide evidence of the experiences shared by the four selected student teachers during the focus group interview. Koshy states that “one of the advantages of video recording is that it allows the researcher to observe an activity afterwards by matching the video without the disruptions of the classroom or time constraints” (2010, p. 96). I used a mathematics lecturer and an Integrated Natural Science lecturer to video record the sessions of the focus group interview because student teachers did not feel threatened by their presence. Moreover, the findings of this research will be helpful to this mathematics lecturer. Koshy further states that “video and film clips can often generate a good deal of discussion between observers and audiences with whom you will be sharing your findings” (2010, p. 96).

#### **3.7.2 Focus Group Sessions**

I designed a focus group schedule that I used as a tool to interview the four selected student teachers in order to seek information about their experiences of using multiple representations. The focus group interview, conducted in two sessions, was done after the completion of the classroom observations so that the four selected student teachers could reflect deeply on their teaching. Coles & McGrath (2010) recommended the use of focus

groups not only to gain access to more people but to discuss issues extensively. The first session of the focus group took place in the audio room of the University library and the second session took place in the office of the assistant faculty officer. The first session was captured on video by the Mathematics Education lecturer and the second session was captured by the Integrated Natural Science Lecturer. I transcribed both sessions of the focus group interview.

### **3.8 ANALYSIS OF DATA**

The focus group data on the use of multiple representations by four student teachers were coded and processed into categories and emergent themes. The four selected students were labeled as Student Teacher A, B, C, and D. Similar experiences on the use of multiple representations were grouped together. Unusual experiences on the use of multiple representations were also noted and put in the same tables as the similar experiences.

The themes that emerged from the research questions are experiences:

- on the effectiveness of using the Between–Comparison Method
- on the effectiveness of using the Within-Comparison Method
- on the effectiveness of using the Diagrammatic Method/ Singapore
- on the effectiveness of using the Cross-Product Algorithm
- on the effectiveness of using the Table Method
- on the effectiveness of using the Graphical Method
- on the effectiveness of using the Oral Informal Method

The transcribed data from focus group interviews that reflected similar experiences on each individual representation was colour-coded green; while unusual experiences were colour-coded red using the text highlight colour on the computer.

The colour-coded data was read many times and main ideas from each participating student teacher were extracted and tabulated. Similar ideas were easily identified in the tables because they appeared to be repeated in the tables and the colour coding was then removed. In some sections of the analysis, after reading the transcribed data many times, a summary of similar experiences were tabulated and then discussed using the deductive approach (Gay, Mills & Airasian, 2012). The tables were used as one way of organizing the experiences of the participating student teachers of using multiple representations in the teaching of

proportion word problems. The tables also served as a way to easily identify major and minor relationships in the experiences of the participating student teachers.

The most difficult part of the analysis was when the original transcribed data and the video had to be referred to back and forth many times. This method was used in order to understand the context of the experiences of the participating student teachers. I agree with Bernard & Ryan (2010, p. 109) that analysis is difficult because it “starts before you collect data – you have to have some ideas about what you’re going to study – and it continues throughout the research effort” as this was also my experience.

### **3.9 ETHICS**

I asked for permission to do this research from the Deputy Dean of the University Campus (see Appendix D), the Regional Director of the Ministry of Education in Caprivi (see Appendix E), the four selected students (see Appendix F), and the principals of the two selected primary schools (see Appendix G). Walford stated that permission to conduct research is “always provisional, as permission and trust can be withdrawn at any time by head teacher, teachers, parents or students” (2001, p. 34). It was with Walford’s (ibid) sentiments in mind that I sought permission to conduct my own research on in an ethical way.

The four student teachers were selected on a voluntary basis. The experiences of the four selected student teachers are treated as confidential information. The schools and the participants in this research remain anonymous. The workshops on multiple representations were conducted at a time and venue that was convenient to all the four student teachers. The teaching practice was done in the schools that are often used by the University institution.

### **3.10 VALIDITY**

The student teachers were given the transcripts of the focus group interviews to comment on whether they represented their genuine experiences of using multiple representations in their teaching. Throughout the research process I ensured that my own personal experiences were not foregrounded and did not act as a form of bias that interferes with the experiences of the participants. I also ensured that I remained transparent and honest throughout the research process. The experiences of student teachers were treated with respect and presented without any personal reservations.

I also ensured validity by using more than one research instrument - observations and focus group interviews. According to Cohen et al. (2007), the use of several research instruments in research provides a rich and in-depth collection of data that is trustworthy.

### 3.11 SUMMARY OF THE RESEARCH PROCESS

Table 3.2 below provides a summary of the research process

Table 3.2.: Summary of the research process

Phase	Activity	Technique/tool	Purpose	Data
I	Workshop 1		Crafting, using and adapting multiple representations; prepare lesson plans	
II	Micro-teaching/pilot phase	Video recording	Present micro-teaching lessons as a pilot study	
III	Workshop 2		Revise the use of multiple representations	
IV	Teaching in schools	Video recording	Present lessons during teaching practice	Visual audio data
IV	Focus group interviews	Video recording	Explore and capture teachers' experiences	Visual audio data and interview transcripts

### 3.12 LIMITATIONS AND CHALLENGES

As the research process unfolded, I experienced several limitations and challenges. Firstly, it took time to get permission to do this research and when permission was given the first semester teaching practice had already ended.

The student teachers did not immediately volunteer to participate in my research. By the time they volunteered, the University had already suspended lectures to give students time to fully prepare for the May/June special examinations. This situation forced us to cancel the microteaching that was scheduled in May 2012. It was a pity that we had to dispense with the pilot phase of the project.

The integration and contextualization of proportion word problems into Grade 6 word problems was a difficult process since proportion is not part of the grade 6 syllabus. However, we placed it in the word problems section of the curriculum under the theme money and finance. It was also difficult to make use of all the multiple representations in just two lessons.

In hindsight, if I were to do this research again I would do more than two workshops on the use of multiple representations in the teaching of Grade 6 proportion word problems. I would also allow the student teachers to teach grade 6 proportion word problems in four lessons. I would also study the Grade 6 scheme of work to determine well in advance when the proportion word problems could be integrated more fully into word problems involving money and finance.

During the video recording of lessons, I also encountered technical challenges with the video camera and the person who filmed the lessons. I had to use back-up cameras and cell phones to film the lessons when the first camera ran out of battery life. I also had to hire different people to the ones originally intended to film the lessons.

### **3.13 CONCLUSION**

In this chapter I outlined the research process by providing details of the research design and the instruments used. I also discussed issues of validity and ethics. I concluded the chapter with highlighting some of the limitations and challenges faced.

## **CHAPTER 4: FINDINGS AND DISCUSSIONS**

### **4.1 INTRODUCTION**

This chapter discusses the results and the findings of this study. This study set out to explore the experiences of student teachers in using multiple representative approaches in the teaching of Grade 6 proportion word problems. The multiple representative approaches include the Between Comparison Method, the Within Comparison Method, the Diagrammatic Method, the Table Method, the Graph Method, the Cross-product Method, and the Informal Oral Method. This chapter starts by briefly reminding the reader of each of these multiple representative approaches. It then presents an overall summary of the results before discussing in detail the experiences of the student teachers for each multiple representation.

The experiences of the student teachers for each multiple representation are discussed in themes as follows:

- the effectiveness of the Between Comparison Method,
- the effectiveness of the Within Comparison Method,
- the effectiveness of the Diagrammatic Method,
- the effectiveness of the Table Method,
- the effectiveness of the Graph Method,
- the effectiveness of the Cross-Product Method, and
- the effectiveness of the Informal Oral Method.

These were the themes that framed the focus group interviews and were supported by video recording of selected lessons. Within the above themes I also discuss the sub-themes which included:

- the exploration of the multiplicative relationship, and
- the challenges that were faced in using each individual representative approach.

## **4.2 A BRIEF REMINDER OF THE MULTIPLE REPRESENTATIVE APPROACHES**

### **4.2.1 The Between-Comparison Method**

In this method the missing term of a proportion is found by making “a comparison between ratios of the proportion” (Nielsen, 1998, p. 38). Van de Walle et al. (2010, p. 364) explained that this method makes use of a “ratio of two corresponding measures in different situations”. At the level of Grade 6, it is a method where quantities in different situations are compared. See page 19 for an example.

### **4.2.2 The Within-Comparison Method**

In this method the missing term of a proportion is found by making a “comparison within the ratios” of the proportion (Nielsen, 1998, p. 38). Van de Walle et al. (2010, p. 363) explained that this method makes use of a “ratio of two measures in the same setting”. At the level of Grade 6, it is a method where two quantities in the same setting are compared. See page 20 for an example.

### **4.2.3 The Diagrammatic Method**

In this method the missing term of a proportion is found by using diagrams that explore the concept of proportion. According to the Pacific Institute for Mathematical Sciences (2009, unpagged) this method is a “pictorial representation of known and unknown quantities and their relationships in a problem”. Van de Walle et al. (2010, p. 27) referred to the Diagrammatic Method as a method that uses “pictures or drawing that represents the concept or onto which the relationship for that concept can be imposed”. See page 21 for an example.

### **4.2.4 The Table Method**

In this method the missing term of a proportion is found by creating a table and using it to explore the concept of proportion. Van Etten & Adendorff (2010) and Van de Walle et al. (2010) referred to this table as a “ratio table”. Furthermore Van de Walle et al. (2010, p. 356) explained a ratio table as a way to organiseorganize information to show how two variable quantities are related. See pages 21 to 25 for examples.

#### 4.2.5 The Graph Method

In this method the missing term of a proportion is found by creating a graph to locate the missing term. Van de Walle et al. (2010, p. 359) stated that “graphs provide another way of thinking about proportions, and they connect to algebraic interpretations”. They further state that all graphs that represent proportional situations are “straight lines that pass through the origin” in which the slope of the line “is the ratio of the y-coordinate at any point with the x-coordinate of the same point” (*ibid*). See pages 25 to 27.

#### 4.2.6 The Cross-product Method/Cross-multiplication Method

In this method the missing term of the proportion is found by using the cross-multiplication algorithm. Kilpatrick et al. (2001, p. 244) stated that this method involves “setting up a correct equation using cross-multiplication algorithm, using a variable to represent the missing value and solve the equation to get the missing value. Van de Walle et al. stated that “if the cross-product approach is understood and presented as one strategy, and not necessarily the only approach or the best approach, students will be more likely to continue to reason and choose the strategy that makes sense given the context and the numbers involved in the problem” (2010, p. 365). See page 18 for an example.

#### 4.2.7 The Informal Oral Method

In this method the missing term of the proportion is found by trial-and-error which eventually leads to the development of mental calculations. Koedinger & Nathan (2004) referred to this method as the guess and test strategy. See page 28.

### 4.3 DISCUSSION OF THEMES

#### 4.3.1 Overall summary of using multiple representations

The presentation in Table 4.1 below gives a tabulated summary of key expressions of the four participating student teachers about how they felt using multiple representations.

Table 4.1: Summary of the participating student teachers’ general impressions on using the multiple representative approaches

Student teacher	The general impressions on using the multiple representations
A	The methods were quite effective.
B	The methods were effective though the learners were facing some problems on some parts.
C	The methods were effective because all in the end the learners were able to understand the methods
D	It was really indeed something which was very much enjoyable

Table 4.1 shows that the four participating student teachers' general impressions in using multiple representative approaches in the teaching of Grade 6 proportion word problems were positive despite the challenges they encountered. They shared their experiences very willingly in the focus group sessions. Student teacher A stated that the multiple representative approaches were "*quite effective*" and claimed that the Grade 6 learners "*seemed to have interest with the methods*". He used a metaphor to express the benefits of using multiple representative approaches by stating that "*there are many ways to kill a cat*".

Student teacher B stated that the multiple representative approaches "*were effective though the learners were facing some problems on some parts*". Student teacher B pointed out that the Grade 6 learners had problems because "*they did not know how to divide or multiply*". She further stated that the Grade 6 learners "*were always having problems with that, so that's what made some other methods a little bit difficult when they were working, but some methods were good for them and they were getting them fast*".

Student teacher C suggested that the Grade 6 learners "*were experiencing some problems here and there but the methods were effective because all in the end the learners were able to understand the methods*". He further stated an aspect he particularly enjoyed was that the multiple representative approaches "*arouse the learners' interest because the learners got interested in doing more problems for each one of the methods*".

Student teacher D observed that using multiple representative approaches "*was really indeed something which was very much enjoyable because it was like we were making an experiment to see whether it can work*". He said that "*it was also good because we used different methods*"...and that "*we know that one can learn better when he sees a picture and one can learn better when he reads the story*". He also commented that "*so it was very good because they had an opportunity to choose which methods they can learn better*".

#### **4.3.2 The effectiveness of the Between Comparison Method**

Table 4.2 below gives a summary of the participating student teachers' responses regarding the effectiveness of the Between-Comparison Method.

Table 4.2: Summary of the participating student teachers' responses regarding the effectiveness of the Between Comparison Method

Student teacher	The effectiveness of the Between Comparison Method
A	It was acknowledged as the easiest method
B	It was effective but not 100% effective
C	It was easier after taking some time to understand the method
D	The group using this method got the right answers

Table 4.2 above shows that the Between Comparison Method was generally seen as effective in the teaching of Grade 6 proportion word problems despite the challenges faced. The four participating student teachers responded positively on the use of this method.

Student teacher A said that the Between Comparison Method was one of the easiest methods he has used to teach Grade 6 proportion word problems. He found that as the second easiest method, it was quite effective. Although the use of this method was perceived to be effective, student teacher B observed that some Grade 6 learners in the class struggled to use the method. As a result she said that the Between Comparison Method was not entirely effective. Student teacher C said that he experienced the Between Comparison Method to be initially difficult, but as soon as he used the Boy to Girl Comparison (context) the method gradually became easier to use in teaching Grade 6 proportion word problems. Student teacher D said the group that used this method got the right answers.

#### 4.3.2.1 Exploration of the multiplicative relationship using the Between Comparison Method

Table 4.3 shows how the Between Comparison Method was used by the student teachers to explore multiplicative relationships.

Table 4.3: Participating student teachers facilitating the exploration of the multiplicative relationship using the Between Comparison Method

Student teacher	Facilitating the exploration of the multiplicative relationship
A	How many boys in 4 minutes? Learners knew already that if in 2 minutes there are 10, in 4 minutes they will be 20. So, by that Between Method they had to say what number did they use?
B	Then they compare the numerator and numerator like that. To get what they were doing on the other side they have to multiply or divide to see what number they should multiply to get the missing term.

C	So I told them like they should at least try to make sort of like equation whereby they will have the first equation for the statement itself.
D	They would tell me that I added just like $2+2+2+2$ , four of them. Then I would ask: what if they were 50?

Table 4.3 above shows the four participating student teachers' responses regarding the exploration of the multiplicative relationship using the Between Comparison Method. There is convincing evidence that Student teacher A used the Between Comparison Method to help Grade 6 learners explore the multiplicative relationship in a proportion. He narrated one of the unusual but exciting experiences he had with a group of Grade 6 learners who used the Between Comparison Method to work out a solution to a proportion word problem. The group of Grade 6 learners who used the Between Comparison Method already knew that if in 2 minutes there are 10 boys seen by the principal, therefore in 4 minutes there will be 20. He further remarked that using the Between Comparison Method the learners had to identify the number they used to obtain the answer. These remarks by student teacher A imply that the Grade 6 learners had to explore the multiplicative relationship between the given quantities and use it to obtain the answer. In this case, the multiplicative relationship which student teacher A is referring to is 5. The unusual and exciting experience of using the Between Comparison Method is that the learners used the reverse process to get the solution.

Student teacher A's interpretation of the reverse process that the Grade 6 learners used to show the solution demonstrates that the student teacher appreciated the efforts made by the learners. This builds confidence in the Grade 6 learners and keeps it sustained. It is good to get the correct answer but credit should also be given to the process of getting the answer. Student teacher A appreciated both the answer and the process of getting to the answer.

Student teacher B made use of the Between Comparison Method to explore the multiplicative relationship of proportional quantities. In her experiences of using the Between Comparison Method, she stated that the Grade 6 learners had to get the multiplicative relationship of the comparison of the numerators by multiplying or dividing. The multiplicative relationship was then used to find the missing term in the proportion. Student teacher B's statements referring to the multiplicative relationship suggest that Grade 6 learners were taught how to arrange the proportion with four values where one value was to be determined.

In the focus group interview student teacher C did not fully describe his exploration of the multiplicative relationship of the proportional quantities, but mentioned the first stages of this

exploration. The experiences of student teacher C suggest that the first stages of the exploration of the multiplicative relationship involved at least the first two steps of problem solving, namely understanding the question and devising the plan. In the first step of problem solving, student teacher C made sure that Grade 6 learners re-read the statement of the proportion word problem before they moved on to the question with the aim of identifying proportional quantities. In the second step of solving the problem he assisted the Grade 6 learners to devise a plan which they used to solve the proportion word problems. Student teacher C narrated the activities that were done in this second step by using expressions such as making a first equation and a second equation. He said that he told the learners that they should make “sort of like an equation, in which the first equation represents the statement itself whilst the second equation represents the question”. I refer to these activities of problem solving as activities of building up the proportion and then employing the Between Comparison Method to determine the multiplicative relationship.

Student teacher D’s experiences of using the Between Comparison Method suggest that he used probing questions and problem solving strategies to explore the multiplicative relationship of the proportional quantities. He started with simple questions that increased in complexity with the aim of engaging the Grade 6 learners in the exploration of the multiplicative relationship. He mentioned that he started with proportion word problems containing easily manageable quantities such as 2 and 4. Then he increased the complexity of the proportion word problems by using big numbers in place of the small numbers. He would ask the Grade 6 learners to explain how to get the answer. He acknowledged that the Grade 6 learners would tell him that they repeatedly added the same number, for example,  $2+2+2+2$ . This repeated addition strategy used by the Grade 6 learners is an exploration strategy to find the multiplicative relationship between 2 and 8. In this case, the multiplicative relationship is 4. The short form of  $2+2+2+2$  is 2 multiplied by 4. The next question asked about the extent to which the repeated addition of the same number could be used, by telling the learners to find the answer when the number of items increased to 50. He used follow-up questions to scaffold the learners into thinking of another strategy which could work when the quantities in the proportion word problems increased.

### 4.3.2.2 The challenges faced when using the Between Comparison Method

Table 4.4 below shows a summary of the challenges faced by the participating student teachers when using the Between Comparison Method.

Table 4.4: Summary of the challenges faced by the participating student teachers when using the Between Comparison Method

Student teacher	Challenges faced when using the Between Comparison Method
A	Misplacing quantities when building up a proportion English language as a medium of instruction
B	Some learners who can't think fast to see what happened between those numbers when they are comparing. Multiplication and division English Language as a medium of instruction
C	Determining division/ multiplication Misplacing quantities when building up a proportion Not reading the leading statement of the proportion word problem English Language as a medium of instruction
D	Misplacing quantities when building up the proportion Multiplication and division The multiplication of decimals

The challenges include the use of the English language different and unique abilities of the learners, setting up a proportion, proportion word problems, multiplication and division. Some of the above-mentioned challenges appear often in the experiences of the four selected student teachers, while some challenges are rare.

- **The English language experienced as a challenge**

All the four participating student teachers mentioned that the English language is a challenge to the Grade 6 learners especially when building up a proportion from a proportion word problem. Student teacher A argued that Grade 6 learners often failed to get the correct answer to the proportion word problem because they did not understand the question in English. In the focus group interview student teacher A referred to the English language as “*grammar*” and “*language*”. He further stated that being able to understand the proportion word problem requires learners to think critically to understand the language.

Student teacher B also acknowledged that using the English language is a very big challenge to the Grade 6 learners when teaching them proportion word problems. She asserted that English is the biggest challenge and she even referred to it as a “*problem*”. Student teacher C stated that English played a big role in learners’ understanding of proportion word problems. He gave an account in which he code switched to the learners’ home language to

help some learners to understand the proportion word problems. Student teacher C further added that Grade 6 learners often did not read the leading statement of the proportion word problems. He explained that some learners skipped reading the statement and only read the question part of the proportion word problems. His experience was that the learners who did not read the leading statement easily became confused. It could be argued that this reading problem is a result of a low proficiency in the English Language.

Student teacher D was silent on the issue of the English language as a barrier in the teaching of Grade 6 proportion word problems. However, some of the challenges that student teacher D faced seem to be related to language issues. For instance, student teacher D discovered that the learners had placed the quantities incorrectly in the setting up of a proportion to be used in the Between Comparison Method.

Student Teacher D was not the only one to identify the wrong placement of quantities as a challenge when setting up a proportion; for student teachers A and C had the same experience. Student Teacher A stated that the wrong placement of the quantities in the proportion was revealed by learners who tended to confuse the quantities, for example, “*they write apples under boys and boys under apples*”. Student teacher C stated that this challenge was evident when the Grade 6 learners placed the same quantity of minutes under the boys and vice versa. Misplacing quantities of the proportion account for some of the difficulties that are encountered when using the Between Comparison Method in teaching Grade 6 proportion word problems.

- **The different and unique abilities of the learners as a challenge**

Table 4.4 above shows that the different and unique abilities of the learners were experienced as a challenge in using the Between Comparison in the teaching of the Grade 6 proportion word problems. Student teacher B mentioned that problems were experienced with those learners who cannot think fast to see what happened between two numbers when comparing them. Student teacher C referred to the learners who cannot think fast as “*slow learners*”. This implies that these learners need special and additional attention when teaching Grade 6 proportion word problems.

- **Multiplication and Division as challenges**

Table 4.4 above further shows that multiplication and division were challenges which the four participating student teachers experienced in the teaching of Grade 6 proportion word problems. This implies that the learners needed a lesson on multiple strategies for multiplication and division which are relevant to their level of development. The Grade 6 learners should be introduced to concrete interpretations of multiplication and division before they are introduced to the abstract interpretations of these basic arithmetic operations (Kilpatrick et al., 2001). The Grade 6 learners struggled to figure out whether to multiply or divide when they used the Between Comparison Method to get the answer of the proportion word problem. Student teacher D noted that this challenge was intensified when multiplying numbers that involved decimals, for example money. The situation experienced by student teacher D indicates that multiplication of decimals should be first taught before proportion word problems. This shows that the hierarchical nature of mathematics is problematic when teaching proportion word problems.

### 4.3.3 The effectiveness of the Within Comparison Method

Table 4.5 below illustrates the responses of the student teachers regarding the effectiveness of the Within Comparison Method.

Table 4.5: Summary of the responses of student teachers on the effectiveness of the Within Comparison Method

<b>Student Teacher</b>	<b>Responses on the effectiveness of the Within Comparison Method</b>
A	He stated that the method was initially not effective but as familiar and contextualized situations were used it became effective.
B	She stated that this method was quite useful and effective because they used familiar concepts of fractions
C	He explained that learners did not experience many problems with this method because they compared it to the Between Comparison Method and the familiar fraction concept
D	He stated that this method worked well when familiar and contextualized situations were used

Table 4.5 above shows that the Within Comparison Method was generally effective despite the challenges that the four participating student teachers faced when using this method. Student teachers A and D made use of familiar and contextualized situations in the proportion word problems to make the method interesting and effective. This implies that it was difficult for the Within Comparison Method to be effective without using familiar and contextualized situations to explain the main features of the method. This finding reinforces the importance of contextualized proportion word problems. Student teacher A used the

context of gender in the proportion word problem, which was familiar to the Grade 6 learners. He clarified the features of the Within Comparison Method by stressing that when “*talking about boys it is just boys*” and when “*talking about girls it is just girls*”. Student teacher D made sure that the Grade 6 learners identified the two quantities compared in the proportion word problem and emphasized that when he said “*within Grade 6 D he meant just in that class*”. He also used the following expressions to clarify the features of the Within Comparison Method: “*if you are talking about dollars it is just within the line for Dollars*” and “*if you are talking about within the line of hours it is just within the line of hours*”. Student teachers B and C stated that this method was quite useful and effective because they used familiar fraction concepts. Besides that, Student teacher C stated that this method was effective because the Grade 6 learners compared it to the Between Comparison Method which they already knew. He explained that the Grade 6 learners compared “*numerator and denominator*” when using the Between Comparison Method, while using the Within Comparison Method the focus is on “*the numerator and denominator*”. This implies that related methods that are well-understood by the learners can be used to help understand the unfamiliar methods. This finding confirms the teaching strategy of moving from the known to the unknown.

These explanations appear to point to procedural fluency issues of the Within Comparison Method rather than conceptual understanding issues. This is understandable as the Within Comparison Method is characterized as a method.

The Within Comparison Method is referred by Van de Walle et al. (2010, p. 363) as “the ratio of two measures in the same setting”. In this case, the emphasis in the explanation of student teacher A is in the setting of “boys” and “girls”; while that of Student Teacher D is in the setting of “hours” and “dollars”. The emphasis in the explanation of student teachers B and C is in the fraction setting. The important mathematical idea that the participating student teachers attempted to explore is that “the two within ratios will be equivalent” (Van de Walle et al., p. 2010, p.363). This suggests that the multiplicative relationship was the same in each setting of the within ratio and can be explored so that it can be used to determine the missing value of the proportion.

### 4.3.3.1 Exploration of the multiplicative relationship using the Within Comparison Method

Table 4.6 below illustrates the student teachers' experiences regarding the multiplicative relationship using the Within Comparison Method.

4.6: The summary of experiences on the exploration of the multiplicative relationship using the Within Comparison Method

Student Teacher	Experiences on the exploration of the multiplicative relationship using the Within Comparison Method
A	He reported that multiplication was explained as repeated addition of the same number when some learners faced a challenge of multiplication.
B	She reported that the learners had a division problem, in spite of that they were guided by the question asking them to find the multiplicative relationship using the fraction.
C	He reported that the learners had a challenge of division when they attempted to find the multiplicative relationship.
D	He reported that the learners had a problem with the multiplication of big whole numbers and that of decimals.

Table 4.6 shows that all the four participating student teachers experienced some difficulties with learners when using multiplication and division problems when they used the Within Comparison Method. Student teacher A said that when the learners faced a problem with multiplication he described multiplication as repeated addition of the same number which was evident from the example he gave which is “ $4+4+4+4\dots$ eight times”. Student teacher A described multiplication as a “shorter way of addition”. This description of multiplication can be challenged because it is not every addition that meets this criterion; but it is only the repeated addition of the same number. All the examples which student teacher A gave, depicted multiplication as repeated addition of the same number.

Student teacher B said that she told the Grade 6 learners to find the multiplicative relationship using the fraction by examining the numerator and denominator. She claimed that the successful exploration of the multiplicative relationship by examining the fraction showed the effectiveness of the Within Comparison Method. She also maintained “so if they are dividing on this side with a certain number, that they also do to the other side”. She observed that some learners took “a long time to get the number they used to multiply or divide to get the

*denominator or numerator*". Student teacher C shared similar experiences as student teacher B when he mentioned that the Grade 6 learners did not know "*whether to divide or multiply*" in the exploration of the multiplicative relationship. Student teacher D stated that "*small numbers were not a problem because they [learners] could use the multiplication tables in their exercise books*".

The guiding questions that student teacher B used show that she prompted the Grade 6 learners to identify the multiplicative relationship using the Within Comparison Method and then use it to find the missing value of the proportion. This was an effective approach because it helped learners develop proportional reasoning. It was also noted that the multiplicative relationship in the given proportional situation remained constant - as stated by Cramer & Post (1993).

It was noteworthy that the four participating student teachers did not explore the additive relationship in the proportional situations given. This tendency was attributed to the limited availability of time and the level of development of the learners.

The observation that some learners took a long time to find the multiplicative relationship was a good indicator that individual learners are unique and have different abilities. Student teacher B referred to this observation as one of the challenges faced in teaching Grade 6 proportion word problems using the Within Comparison Method. It was good to recognize this observation as a challenge and prepare proportion word problems that are relevant and suited to their abilities. As observed with other methods, it is important that in proportion word problems, small numbers should initially be used. Above all, the proportion word problems should be interesting and challenging and should generally cater for the different abilities of the learners in the Grade 6 class.

#### **4.3.4 The effectiveness of the Diagrammatic Method (Singapore)**

Table 4.7 below shows a summary of the responses of the participating student teachers regarding the effectiveness of the Diagrammatic Method (Singapore)

Table 4.7: Summary of the responses of the participating student teachers on the effectiveness of the Diagrammatic Method (Singapore)

Student Teacher	Responses on the effectiveness of the Diagrammatic Method
A	He acknowledged the method as effective and enjoyable
B	She acknowledged the method as very effective
C	He acknowledged the method as easy and enjoyable
D	He acknowledged the method to be very easy and interesting

Table 4.7 above shows that the Diagrammatic Method was generally effective despite the challenges that the four participating student teachers faced in using this method. Student teacher A acknowledged that the Diagrammatic Method was effective and said that it is “*number one of all the methods*”. He reported that one of Grade 6 groups that used this method complimented themselves after getting the correct answer by “*saying well done Singapore*”. Student teacher A was amazed that the enthusiasm of this group caused them to forget their group name! Student teacher B said that this method was very effective because the Grade 6 learners liked and enjoyed drawing pictures. She said with excitement that the learners “*were enjoying drawing the apples, they were enjoying it*”. Student teachers C and D said that this method was “*easy*”, “*enjoyable*” and “*interesting*”.

The positive responses of the four participating student teachers on the effectiveness of the Diagrammatic Method reinforce the importance of diagrams in the teaching of proportion word problems. The diagrams provided the Grade 6 learners with a visual point of view through which they could explore proportions. Moreover, all the Grade 6 learners appreciated the use of diagrams. The level of appreciation of using diagrams varied from learner to learner as much as from one student teacher to the other. For example the student teacher A said this method was *effective* while student teacher B said it was *very effective*.

#### 4.3.4.1 Exploration of the multiplicative relationship using the Diagrammatic Method

Table 4.8 below shows the challenged experienced by the student teachers when using the Diagrammatic Method (Singapore).

Table 4.8: Summary of challenges faced in the exploration of the multiplicative relationship using the Diagrammatic Method (Singapore)

Student Teacher	The challenges experienced when using the Diagrammatic Method
A	He stated that a challenge was experienced when big numbers were used
B	She stated that a challenge was experienced when big numbers and a different context were used
C	He stated that a challenge was experienced when big numbers were used
D	He stated that a challenge was experienced with a key that shows one person represents ten people.

Table 4.8 shows that all the four participating student teachers experienced the same challenge when big numbers were used in the proportion word problems. Student teacher D explained that the Grade 6 learners were initially confused by expressions such as “*one person represents ten people*”. Student teacher A said that the Grade 6 learners “*got stuck*” when bigger numbers were used. Student teacher B observed that some learners drew apples in an attempt to provide the solution to a proportion word problem that did not even contain apples. Student teachers B and C said that the learners inquired whether they had to draw 120 boxes in an attempt to provide a solution to a proportion word problem with the number 120.

It can be frustrating for student teachers to come up against challenges like these. It is important to note however that “all that you actually see with your eyes is the physical object (diagram); only your mind can impose the mathematical relationship on the object (diagram)”<sup>22</sup> (Van de Walle et al., 2010, p. 27). Student teachers further used the diagrams to help Grade 6 learners explore the multiplicative relationship of proportional quantities. Student teachers need to realize however that learners do not always interpret diagrams in the same way as the student teachers. Van de Walle et al. (2010, p. 28) advise teachers to carefully examine whether models (diagrams) are “tools used to explore a concept” or are “answer-getting” models (diagrams). It is also important that the learners are allowed to make use of diagrams that they easily understand and identify with.

### 4.3.5 The effectiveness of the Table Method

Table 4.9 gives a summary of the responses of the student teachers regarding the effectiveness of the Table Method.

Table 4.9: Summary of the responses of the four participating student teachers regarding the effectiveness of the Table Method

Student Teacher	Responses on the effectiveness of the Table Method
A	He said that the method was very easy when the learners discovered the pattern between values.
B	She said that the Grade 6 learners found the method to be easier than other methods.
C	He said that it was a bit challenging to work with this method.
D	He said that this method was a bit easier or much easier

Table 4.9 shows that the Table Method was generally effective despite the challenges that the four participating student teachers faced in using this method. The four participating student teachers generally appreciated this method as a relevant strategy to use in the teaching of Grade 6 proportion word problems.

Student teacher A said that the Table Method was very easy to use once the Grade 6 learners discovered the pattern emerging from the given values of the proportion. He said that if the emerging pattern was addition by 2, the Grade 6 learners “just add by 2, by 2, by 2 just like that”. Student teacher B was amazed at the pace at which the different groups in her class completed the table of values of the proportion task after they determined the pattern. She remarked with surprise at the work done by the different groups: “*I found that they have filled up to 20, but I just told them to leave up at five because I wanted to use the other boxes to fill them up later*”. These experiences of the student teachers are consistent with those of Van de Walle et al. (2010, p. 357) when they say that it is an effective “strategy for solving a proportion”. The Within Comparison Method and the Between Comparison Method can work together with the Table Method. It can further be argued that these methods complement each other when used in the teaching of Grade 6 proportion word problems.

#### 4.3.5.1 Questions used in the exploration of the emerging pattern in the Table Method

Table 4.10 below shows the questions used by the student teachers to help their Grade 6 learners to explore emerging patterns in the Table Method.

Table 4.10: Questions used by the participating student teachers to help Grade 6 learners explore emerging patterns when using the Table Method

Student teacher	Questions used to help learners explore the emerging pattern between given values in the proportion
A	He used questions that led to the pattern being discovered
B	She used questions such as: “What is happening with these numbers? What do you think is the next, third number, the fourth number?”
C	He used questions such as: “By how many/much was the number increasing?”
D	He used questions such as: “If you look down here there is 60 and on top there is 12, so what happened to 60 for it to become 12?”

Table 4.10 shows that all the four participating student teachers used oral questions to prompt Grade 6 learners to generate a pattern emerging from the table of values of a proportion. The student teachers said that the Table Method was very effective. In using the Table Method, the four participating student teachers created a table and filled in the first values of the proportion, then invited learners in their respective groups to complete the table and examine the entered values, and then determine the emerging pattern. Once the pattern was identified the Grade 6 learners were advised to use the pattern to find the missing values of the proportion. Student teacher C said if the Grade 6 learners identified that “*it was increasing by 4.... then they kept adding 4*”. Student Teacher D said that “*if they multiplied 12 by 5 to get 60....so you all do the same with all the numbers*”.

#### 4.3.6 The effectiveness of the Graph Method

Table 4.11 illustrates the participant student teachers’ experiences of using the Graph Method.

Table 4.11: The four participating student teachers’ experiences of using the Graph Method

Student Teacher	Experiences of using the Graph Method
A	He said it was a bit difficult because the Grade 6 learners were doing the graph work for the first time
B	She said the learners had a problem with plotting the table of values
C	He said the learners did not know where to place the numbers on the graph paper
D	He said the learners did not have any knowledge of working with the graphs

Table 4.11 shows that it was generally difficult for the four participating student teachers to use the Graph Method in the teaching of proportion word problems. The difficulty of this method was attributed to the plotting of values that the learners had to do. It was problematic to draw and use the graph to find the solutions to the proportion word problems. The student

teachers generally found that the learners were not able to draw graphs. They said that the learners required some introductory activities involving the plotting of points and using simple number liners. This is consistent with the findings by Amit and Fried (2005) who found that learners consider drawing graphs as a redundant exercise that consumes time when trying to draw, checking and putting down the points. However, the student teachers said that when the graph of a proportional situation was already drawn the learners were able to solve the problems correctly. Student teacher C remarked with excitement that the Grade 6 learners “*could get the answer within some few second*”. Student teacher B also said that the Grade 6 learners knew how to use a graph to get the answer.

In order to overcome the difficulty of not yet being able to draw graphs, Grade 6 learners should perhaps be given the graphs to interpret and work with. The drawing of the graphs should however not be completely abandoned. It was suggested that a half-drawn graph can be brought to the Grade 6 class for them to insert just two points in order to complete it.

#### 4.3.7 The effectiveness of the Cross-product Method/Cross-multiplication Method

Table 4.12 is a summary of the student teachers’ responses regarding the effectiveness of the Cross-product Method.

Table 4.12: Summary of the participating student teachers’ responses regarding the effectiveness of the Cross-product Method

Student teacher	Summary of the effectiveness of the Cross-product Method/cross-multiplication
A	He said this method was not easy for the Grade 6 learners and he identified multiplication and the negative attitude of the learners as challenges
B	She said this method was effective because the Grade 6 learners were able to work with it and she identified multiplication and division as challenges
C	He said this method was much easier for the Grade 6 learners because they understood the procedure and he identified placing quantities wrongly in a proportion as a challenge
D	He said this method was not a problem to the Grade 6 learners and he identified multiplication and division as challenges

Table 4.12 shows contradicting views on the effectiveness of the Cross-product Method. Student teacher A’s views showed that this method was not effective hence it was very difficult for the Grade 6 learners to use. He said that some of the challenges included difficulties with multiplication and a negative attitude of the learners towards multiplication. He added by saying “*when they just hear cross-multiplication, that is multiplication, they don’t know how to multiply, it is just that fear*”. Fear of multiplication destroys the passion to

do multiplication, particularly cross multiplication. Apart from multiplication, division and the incorrect placement of quantities in a proportion were identified as further challenges in using this method. These sentiments about multiplication and division were important because they generated awareness of the problem and invited student teachers to seek better strategies for teaching multiplication and division. This challenged student teachers to address the issue of fear and come up with effective approaches such as using concrete examples of multiplication and division as suggested by Kilpatrick et al. (2001) that are relevant to the Grade 6 learners.

It was evident from Student teachers B, C and D's views that this method was effective because the Grade 6 learners understood the concepts underlying this hotly debated procedure. It was also evident that the concept such as cross-multiply was properly explained and related to the proportion. Student teacher B explained this method by saying "*just listen to what this method is called. It is cross,....if I cross this and that [points the directions]. This and that will multiply each other. These ones will multiply each other*". Student teacher C explained this method by saying "*I told them after you have drawn a cross write the pin-pointing arrows whereby the line which is running from left, I'm, top left to right bottom,....so if they are pin-pointed you write them down and you put the cross between the two*". It is interesting to note how the word "cross" was used to illustrate this procedure and, in this way, dispel misconceptions around it.

#### **4.3.7.1 Exploring proportionality using the Cross-product Method**

Student teacher B observed that some of the Grade 6 learners forgot to write the equal sign when writing down the cross-multiplied numbers. Some of the other errors they made were to put addition instead of a multiplication. Student teacher C explored proportionality when he explained that "*if you are multiplying  $1 \times 3$  on the left of the equal sign means that what is on the left is equal to what is on the right*". He said that it was effective because the Grade 6 learners "*knew that if they have 30 on the left [of the equal sign] they have to do something on the right to make sure to get 30*".

#### **4.3.8 The effectiveness of the Informal Oral Method**

Table 4.13 below illustrates the student teachers' responses regarding the effectiveness of the Informal Oral Method.

Table 4.13: Summary of the responses of student teachers regarding the effectiveness of the Informal Oral Method

Student teacher	Responses of student teachers on the effectiveness of the Informal Oral Method
A	He said this method was very easy because it didn't take much time for the Grade 6 learners to understand the method
B	She said this method worked well because the Grade 6 learners were able to answer questions correctly.
C	He remarked with surprise that it was like having already that prior knowledge to calculate mentally.
D	He said that the Grade 6 learners answered the questions correctly because he used small numbers.

Table 4.13 shows that the Informal Oral Method was generally seen as effective in the teaching of Grade 6 proportion word problems. Student teachers A and C were surprised at the mental ability that the Grade 6 learners demonstrated in answering oral questions. Student teachers B and D said the Grade 6 learners were able to answer oral questions correctly because the questions were simple. It is important that these questions should be of increasing difficulty even though they are meant to be done mentally. Simple questions are intended build the confidence of the Grade 6's so that more complex questions can be introduced. These responses show how the four participating student teachers appreciated the use of the Informal Oral method.

The Informal Oral method is essential in the teaching of Grade 6 proportion word problems as is illustrated by the student teachers' comments. Student teacher B said that the Grade 6 learners had invented their own way of doing mental calculations. She said with surprise that the Grade 6 learners *"just like visualize, I don't know how they were doing or mentally or just calculating mentally how we all do. Although each person has a different way of calculating that..."* Student teacher B said this method helps the Grade 6 learners *"to know their multiplication tables"*. Student teacher A added by saying this method *"develops the ability of the learners to think fast and respond in time"*. Student teacher D said that this method was *"useful because it made learners to think ....and visualize about the topic which you are going to teach about that day"* and it was used in the introduction of the lesson. It is evident from this remark that the Grade 6 learners developed proportional reasoning in the process of inventing their own strategies for mental calculation. Most of the strategies for mental calculation in proportional reasoning include multiplication and division.

All the four participating student teachers faced challenges in using the Informal Oral Method. These include “*poor listening skills*” and “*language*”. These challenges are related to language – both the English language and the language of Mathematics. Mathematics is a language that is used as a medium of communication. It is therefore important to allow learners to listen and speak this language.

#### **4.4 CONCLUSION**

This chapter presented, interpreted and discussed the experiences of the four participating student teachers in using multiple representative approaches. The multiple representative approaches that were used in the teaching of Grade 6 proportion word problems include the Between Comparison Method, the Within Comparison Method, the Diagrammatic Method, the Table Method, the Graph Method, the Cross-product Method, and the Informal Oral Method.

This study revealed that all the representative approaches used in the teaching of Grade 6 proportion word problems were generally seen as effective by the participants despite the challenges they faced. This study also revealed that there were conflicting views on the effectiveness of the Graph Method and the Cross-product Method, even though they were seen to be generally effective. Moreover, it was revealed that when these methods were used with Grade 6 learners it helped them to explore the multiplicative relationships of proportional quantities. It was also found that the multiple representative approaches complement each other.

This chapter also highlighted several challenges that affected the effectiveness of using multiple representative approaches. The challenges included the lack of multiplication and division skills, difficulties with using the English language, different and unique abilities of the learners, the negative attitude of the learners, poor listening skills, and lack of background knowledge on graphs.

The next chapter is the concluding chapter and presents a summary of the findings, followed by identifying some limitations of the study, the significance of this study, reflections and some recommendations.

## **CHAPTER 5      CONCLUSION**

### **5.1      INTRODUCTION**

This chapter presents a summary of the findings that emerged from the data collection process, in particular from the focus group interviews. The findings center on the research question of selected student teachers' experiences in the use of multiple representations in the teaching of Grade 6 proportion word problems. The multiple representations under focus were: the Between Comparison Method, the Within Comparison Method, the Diagrammatic Method, the Table Method, the Graph Method, the Cross-product Method, and the Informal Oral Method. This chapter further discusses some of the limitations encountered and makes some recommendations.

### **5.2      EXPERIENCES OF THE STUDENT TEACHERS USING MULTIPLE REPRESENTATIONS**

#### **5.2.1      The Between Comparison Method**

The four participating student teachers saw the Between Comparison Method to be generally effective in the teaching of Grade 6 proportion word problems despite the challenges encountered. It is an effective multiple representative approach which does not only serve as a good platform to understand proportions but also serves as a meaningful reasoning approach to explore the multiplicative relationships of the proportions. The participating student teachers appreciated this method as it facilitated deep understanding of proportional reasoning.

Some of the challenges encountered by the student teachers included difficulties with the English language, different and unique abilities of the learners, and lack of proficiency in the learners' multiplication and division skills.

#### **5.2.2      The Within Comparison Method**

The four participating student teachers saw the Within Comparison Method to be generally effective despite the challenges encountered. It did not only facilitate a meaningful representative approach but also encouraged the exploration of the multiplicative relationship of proportion.

### **5.2.3 The Diagrammatic Method**

The four participating student teachers saw the Diagrammatic Method to be generally effective despite the challenges encountered. The visual dimension of this approach was found to be particularly useful in the exploration of proportions. The visual emphasis aligns well with the Grade 6 learners' stage of development. This approach works well with any of the other multiple representative approaches in this study.

### **5.2.4 The Table Method**

The four participating student teachers also saw the Table Method as generally effective. It works particularly well when used in conjunction with the Between Comparison Method, the Within Comparison Method and the Graph Method.

### **5.2.5 The Graph Method**

The four participating student teachers saw the Graph Method to be generally difficult to use in the teaching of Grade 6 proportion word problems. The visual nature of this approach was highlighted as particularly useful in the exploration of proportions. As plotting of the graph is not in the Grade 6 curriculum, the learners found it difficult to draw the required graphs. Despite this, they found the interpretation of graphs very meaningful. The proportional relationship can be illustrated in table form, which the learners coped with.

### **5.2.6 The Cross-product Method**

The four participating student teachers were not unanimous in their experiences of the effectiveness of the Cross-product Method. Some found that this method worked well, while others did not. It was noted that this method requires a deep conceptual understanding of proportion. This method works well when used in conjunction with the Within Comparison Method and the Between Comparison Method

### **5.2.7 The Oral Informal Method**

The four participating student teachers saw the Oral Informal Method to be generally effective despite the challenges encountered. The notion of trial-and-error can be a very powerful approach in exploring proportions. It encourages learners to invent their own ways

of doing mental calculations. It also works well in conjunction with the other representative approaches in this study.

### **5.3 OTHER FACTORS**

This research found that the meaningful use of these multiple representative approaches also depended on other factors such as:

- Avoiding the use of only one representative approach i.e. using them in conjunction with each other.
- Using appropriate problem solving strategies.
- Using simple but appropriately contextualized proportion word problems that are familiar to the Grade 6 learners' real life situations.
- Using the relevant prior knowledge that Grade 6 learners bring to the classroom.
- Fluency in multiplication and division skills.
- Using cooperative learning.
- Encouraging and affirming the learners.
- Ability to understand the language
- Using relevant metaphors
- Ability to address the challenges faced when using the method.

### **5.4 RECOMMENDATIONS**

In order to introduce a multiple representative approach in the teaching of Grade 6 proportion word problems, I recommend the following:

- Student teachers should be trained in the use of multiple representatives;
- The curriculum should encourage a multiple representation approach to teaching proportions;
- In-service workshops should be run to train teachers in these approaches;
- Microteaching and teaching practice on the effective integration of multiple representative approaches in the teaching of proportion word problems should be encouraged;
- Textbooks should reflect multiple representations when dealing with proportions; and that

- Further research is conducted into this way of teaching.

## **5.5 LIMITATIONS**

### **5.5.1 Sample Size**

The size of my sample was very small compared to a similar study done by Ozmantar et al. (2010). If the scope of the study was larger, and there was more time I would have encouraged more student teachers to participate in this study.

### **5.5.2 Time**

The time to collect data was limited because student teachers were busy covering other topics in their studies. I would have liked to collect data on other topics in which proportion word problems could be integrated in Grade 6. Also the teaching practice time was limited, which resulted in limited opportunities to engage with the various methods.

### **5.5.3 Microteaching**

The initially planned microteaching was not done because student teachers were already in schools for teaching practice when they volunteered to participate in this study. A process of microteaching would have facilitated reflection and more careful planning for the actual teaching. I also would have liked to interview the individual student teachers after each microteaching session to better prepare them for their teaching session.

## **5.6 SIGNIFICANCE AND FURTHER RESEARCH**

Despite the limitations outlined above, this study highlighted that a multiple representative approach to teaching proportion word problems is worthwhile and should be sustained. The findings showed that, in this study, a multiple representative approach facilitates a meaningful and conceptual teaching strategy. It was acknowledged that due to the small sample size, the findings cannot be generalized. Further research should therefore be done in other contexts and in other grades.

## **5.7 REFLECTIONS**

The investigation to explore the experiences of the four participating student teachers was interesting and informative. It was interesting to see that the multiple representative approaches were generally effective in the teaching of Grade 6 proportion word problems despite the challenges encountered. It was also interesting to generate awareness that

multiple representative approaches can play an important role in the teaching of Grade 6 proportion word problems. The multiple representative approaches do not only serve as different points of view to understand proportions, but also enrich us to become competent in handling proportion word problems (Ozmantar et al., 2010). I highly appreciate the multi-dimensional approach that underpins the multiple representative approaches used to explore proportions.

It was an informative inquiry because I acquired and deeply engaged with the findings from this research. The positive experiences of the participating student teachers were very satisfying for me. The student teachers who used only one approach in the teaching of Grade 6 proportion word problems were frustrated by the negative reactions that the learners presented in their lesson. But after using a multiple representative approach, they felt more positive about teaching proportion word problems. I feel that student teachers should encourage their Grade 6 learners to invent their own multiple representative approaches to solve proportion word problems.

In this study, I not only engaged with the student teachers on a formal research level, but I also engaged with them less formally. It was interesting for me to obtain insights into their personal experiences and lives. It was very meaningful for me to share their challenges and assist in overcoming them.

This research was indeed an educative journey because it not only helped me to engage with the usefulness of the multiple representative approaches in teaching Grade 6 proportion word problems, but also with the conventions of the research process. This educative inquiry enriched me with the skills to critically interrogate the multiple representative approaches that are often taken for granted.

## REFERENCES

- Amit, M., & Fried, M.N. (2005). Multiple representations in 8<sup>th</sup> grade algebra lessons: Are learners really getting it? In H. L. Chick & J. L. Vincent (Eds.), *Proceedings of the 29<sup>th</sup> Conference of the International Group for the Psychology of Mathematics Education*, 2. (pp. 57-64). Melbourne: PME.
- Anghileri, J. (1991). The language of multiplication and division. In K. Durkin & B. Shire (Eds.), *Language in mathematical situations: Research and practice* (pp. 95-104). Milton: Open University Press.
- Bernard, H. R., & Ryan, G. W. (2010). *Analyzing qualitative data: Systematic approaches*. Los Angeles: SAGE Publications.
- Brown, J.S., Collins, A., & Duguid, P. (1989). Situated cognition and the culture of learning. *Educational Researcher*; 18 (1), 32-42.
- Cohen, L., Manion, L., & Morrison, K. (2007). *Research methods in education* (6<sup>th</sup> ed.). London: Routledge.
- Coles, A., & McGrath, J. (2010). *Your education research project handbook*. Harlow: Pearson Education Limited.
- Cramer, K., & Post, T. (1993). Making connections: A case for proportionality. *Arithmetic Teacher*, 40(6), 342-346. Retrieved May 18, 2011, from <http://www.jstor.org/stable/41195594>.
- Creswell, J. W. (2003). *Research dDesign: Qualitative and quantitative approaches*. London: Sage Publications.

- Gay, L.R., Mills, G.E., & Airasian, P. (2012). *Educational Research: Competencies for analysis and applications* (10<sup>th</sup> ed.). Boston: Pearson Education Limited.
- Hungi, N., Makuwa, D., Ross, K., Saito, M., Van Cappelle, F., Paviot, L., & Vellien, J. (2010). *SACMEQIII project results: Pupil achievement levels in reading and mathematics*. SACMEQ.
- Kennedy, L. M., Johnson, A., & Tipps, S. (2008). Guiding children's learning of mathematics: *Developing aspects of proportional reasoning: ratio, proportion, and percent*. Thomson Learning Inc. Retrieved May 03, 2011, from <http://books.google.com.na/books?id=mrFYTHOjPjYC&pg=PA334&dg=%>.
- Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics*. Washington: National Academy of Sciences. Retrieved May 03, 2011, from <http://www.nap.edu/catalog/9822.html>.
- Koedinger, K.R., & Nathan, M.J. (2004). The real story behind story problems: Effects of representation on quantitative reasoning. *The Journal of the Learning Sciences*, 13(2), 129-164. Retrieved May 12, 2011, from <http://www.jstor.org/stable/1466903>.
- Koshy, V. (2010). *Action research for improving educational practice* (2<sup>nd</sup> ed.). Los Angeles: SAGE Publications.
- Levav-Wayberg, A., & Leikin, R. (2006). Solving problems in different ways: Teachers' knowledge situated in practice. In Novotna, J., Moraova, H., Kratka, M., & Stehlikova, N (Eds), *Proceedings 30<sup>th</sup> Conference of the International Group for the Psychology of Mathematics Education*, 4. (pp. 57-64). Prague: PME.

- Makuwa, D. (2010). *SACMEQ policy issues series: What are the levels and trends in reading and trends in reading and mathematics achievement?* Paris.
- McEldoon, K.L., & Cochrane-Braswell, C. (undated). *Effects of problem context on strategy use within functional thinking*. Nashville.
- Namibia. Ministry of Basic Education Sport and Culture. (2003). *The language policy for schools in Namibia: Upgrading African languages project (Afrila)*. Windhoek: MBESC.
- Namibia. Ministry of Basic Education Sport and Culture. (2006). *Upper primary phase: mathematics syllabus 5-7*. Okahandja: NIED.
- Newby, P. (2010). *Research mMethods for eEducation*. Harlow: Pearson Education Limited.
- Nielsen, O. (1998). *Mathematics education module 6: ratio and proportion, scale drawing, rate, bearings*. Ongwediva, Namibia: Ministry of Education and Culture.
- Ollerton, M. (2010). Using problem-solving approaches to learn mathematics. In I. Thompson (Ed.), *Issues in teaching numeracy in primary schools* (2<sup>nd</sup> ed.) (pp. 84-96). McGraw Hill: Open University Press.
- Ozmantar, M. F., Akkoc, H., Bingolbali, E., Demir, S., & Ergene, B. (2010). *Pre-service mathematics teacher' use of multiple representations in technology-rich environments*. *Eurasia Journal of. Math Science & Technology. Education*, 6(1), pp.19-36.
- Pacific Institute for Mathematical Sciences: Bar model method for problem solving in math*. (2009). (initial workshop draft). Retrieved May 12, 2011, from <http://www.pims.math.ca/educational-event/091006-bmmpsm>.

- Pound, L., & Lee, T. (2011). *Teaching mathematics creatively: learning to teach in the primary school series*. New York: Routledge.
- Silver, E. (2007). *Maths for life grade 6*. Windhoek: Macmillan Namibia.
- Simons, H. (2009). *Case study research in practice*. Los Angeles: SAGE Publications.
- Simons, M. (1993). *The effective teaching of mathematics*. London: Longman Group UK.
- Van de Walle, J.A., Karp, K.S., & Bay-Williams, J.M. (2010). *Elementary and middle school mathematics: Teaching developmentally* (7<sup>th</sup> ed.). Boston: Allyn & Bacon.
- Van Etten, B., & Adendorff, S. A. (2010). The ratio table as learning tool in grade 7. *Learning and Teaching Mathematics, 10*, pp. 8-13.
- Walford, G. (2001). *Doing qualitative educational research: A personal guide to the research process*. London: Continuum International Publishing Group.
- Weber, K., Radu, I., Powell, A., Mueller, M., & Maher, C. (2010). *Expanding participation in problem solving in diverse middle school in mathematics classroom*. *Mathematics Research Journal, 22*(1), pp. 91-118. Retrieved May 25, 2011, from Ebscohost database.
- Wellington, J. (2000). *Educational rResearch: Contemporary issues and practical approaches*. London: Continuum International Publishing Group.

## APPENDICES

### APPENDIX A

#### *Pupil Mathematics (SACMEQ III Project, 2007) by regions in Namibia*

<i>Region</i>	<i>Mean</i>	<i>Percentage of pupils reaching mathematics competency level</i>							
		<i>Level 1</i>	<i>Level 2</i>	<i>Level 3</i>	<i>Level 4</i>	<i>Level 5</i>	<i>Level 6</i>	<i>Level 7</i>	<i>Level 8</i>
<i>Caprivi</i>	457.9	5.6	48.9	33.5	9.3	2.2	0.7	0.0	0.0
<i>Erongo</i>	523.3	2.2	22.6	30.6	21.5	13.9	7.9	1.1	0.3
<i>Hardap</i>	483.1	6.7	36.5	32.3	13.6	6.5	3.3	0.3	0.8
<i>Karas</i>	510.3	3.3	23.3	35.8	21.5	8.0	5.7	1.6	0.8
<i>Kavango</i>	455.6	6.8	47.7	33.8	9.3	1.5	0.7	0.2	0.0
<i>Khomas</i>	522.7	2.1	19.6	35.5	25.1	8.1	8.0	1.7	0.0
<i>Kunene</i>	478.2	4.5	40.7	35.6	11.0	3.2	4.1	0.9	0.0
<i>Ohangwena</i>	447.8	7.4	52.8	31.0	7.9	0.6	0.3	0.0	0.0
<i>Omaheke</i>	468.3	4.0	40.7	42.9	10.4	1.6	0.5	0.0	0.0
<i>Omusati</i>	450.2	6.6	51.4	34.4	6.2	1.1	0.2	0.0	0.0
<i>Oshikoto</i>	457.2	6.4	50.3	33.1	6.6	1.5	1.0	1.0	0.2
<i>Otjozondjupa</i>	488.6	3.7	31.1	39.1	19.2	4.9	1.5	0.5	0.0
<i>Oshana</i>	474.8	3.9	41.1	33.7	14.9	4.2	2.2	0.0	0.0
<i>Namibia</i>	471.0	5.4	42.3	34.0	12.2	3.4	2.2	0.5	0.1

*Adapted from SACMEQ III Project, 2007*

## **APPENDIX B**

### **The Focus Group Schedule**

#### **Background**

This focus group interview is on the experiences of the 4 selected student teachers in using multiple representations in the teaching of grade 6 proportion word problems. In this interview they will be referred to as Student Teacher A, B, C and D.

#### Questions

1. What are your general impressions on using the multiple representation in the teaching of grade 6 proportion word problems?

2. What are your experiences in using each of the following method as a multiple representation in the teaching of grade 6 proportion word problems:

- (a) The Between Comparison Method
- (b) The Within Comparison Method
- (c) The Cross-product Method
- (d) The Diagrammatic / Singapore Method
- (e) The Table and Graphical Method
- (f) The Oral Informal Method

3. What are the challenges you faced in using the multiple representation in the teaching of grade 6 proportion word problems?

4. What are the evidences from the video that show your experiences?

5. What are your recommendations on these multiple representations

## APPENDIX C

Box 39

Katima Mulilo

22 December 2011

To: The Director  
Ministry of Education  
Caprivi Region  
Katima Mulilo

### **Request for permission to conduct a research in Mathematics Education in the Katima Circuit**

I am a Rhodes University Student doing MEd in Mathematics Education. I am kindly seeking for permission to conduct a research in Mathematics Education in the primary schools in the Katima Circuit where student teachers will be placed in the first semester of 2012. I intend to video record the grade 6 classrooms where student teachers specializing in Mathematics Education will do their teaching practice.

My research is in the field of multiple representations in Mathematics Education. It is entitled “Student teachers’ experiences in using multiple representations in the teaching of grade 6 proportion word problems: A Namibian case study”. I intend to observe research ethics as outlined in the attached research proposal.

Thanking you in advance.

Yours Faithfully

Bosman Simasiku  
Assistant Faculty Officer (0814100595)

**APPENDIX D**

Box 39  
Katima Mulilo  
Republic of Namibia  
11 June 2012

To: The Principal

.....  
.....  
.....

Dear Sir/Madam

I am kindly requesting for permission to present two lessons at your school with my student teachers on the grade 6 Word Problems that are intended to develop proportional reasoning in the learners. The two lessons will be put on the video that will be used in discussing the experiences of the student teachers in using multiple representations to teach grade 6 proportion word problems.

The data will be used in the research entitled “experiences of student teachers in using multiple representations in teaching proportion word problems”. The confidentiality of the data will be observed.

Your support is highly appreciated

Yours Faithfully

Mr Bosman Simasiku

0814100595

## APPENDIX E

Box 39  
Katima Mulilo  
Republic of Namibia  
11 May 2012

Dear Student Teacher

I am conducting a research on the “experiences of student teachers in using multiple representations in teaching grade 6 proportion word problems”. I am looking for student teachers who are going to volunteer to be partners in this important research in Mathematics Education. Student teachers will be required to do the following:

Phases	Activities	Dates
Phase 1	A workshop on multiple representations	23 April 2012
Phase 2	Microteaching using multiple representations (Proportion Word Problems Integrated in the lessons)	
Phase 3	A second workshop on multiple representations	27 June 2012
Phase 3	Teach two lessons in the local Primary School	12 July 2012 19 July 2012
Phase 4	Focus group Interview on the multiple representations	20 July 2012

The dates will be discussed in the first workshop on multiple representations

If you are interested kindly complete the attached form and return it to Mr B Simasiku.

Your response is highly appreciated

Yours Sincerely

Mr Bosman Simasiku

.....

.....

Dear Mr B Simasiku

I .....kindly volunteer and accept to be part of this research on the “experiences of student teachers in using multiple representations to teach grade 6 proportion word problems.

Yours sincerely

**APPENDIX F**

Box 39

Katima Mulilo

22 December 2011

To: The Deputy Dean  
University of Namibia

.....

**Request for permission to conduct a research in Mathematics Education in the Katima Circuit**

I am a Rhodes University Student doing MEd in Mathematics Education. I am kindly seeking for permission to conduct a research in Mathematics Education in the primary schools in the Katima Circuit where student teachers will be placed in the first semester of 2012. I intend to video record the grade 6 classrooms where student teachers specializing in Mathematics Education will do their teaching practice.

My research is in the field of multiple representations in Mathematics Education. It is entitled “Student teachers’ experiences in using multiple representations in the teaching of grade 6 proportion word problems: A Namibian case study”. I intend to observe research ethics as outlined in the attached research proposal.

Thanking you in advance.

Yours Faithfully

Bosman Simasiku  
Assistant Faculty Officer (0814100595)  
UNAM Katima Mulilo Campus

## APPENDIX G

Worksheet: Word problems involving money and Finance

Grade 6 proportion word problems

1. One apple costs N\$4.00 at Katima Pick and Pay.

- (a) How much is the cost of 2 apples?
- (b) How much is the cost of 5 apples?
- (c) How many apples cost N\$ 12.00?
- (d) How many apples cost N\$ 40.00?

2. Majory works 12 hours at Greenwell Garage to get an amount of N\$ 60.00.

- (a) How many hours will she work to get N\$ 30.00?
- (b) How many hours will she work to get N\$ 15.00?
- (c) How much will she get if she works 4 hours?
- (d) How much will she get if she works 24 hours?

3. The cost of petrol at Katima Shell Petrol Station is N\$ 10.00 per litre.

- (a) How much is the cost of 3 litres of petrol?
- (b) How much is the cost of 5 litres of petrol?
- (c) How many litres of petrol cost N\$ 120.00?