

DEVELOPING A PROBLEM SOLVING APPROACH
TO PRIMARY MATHEMATICS TEACHING:
A CASE STUDY

THESIS

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ABSTRACT

According to recent research a focus in teaching mathematics to children is the development of problem solving abilities. Problem solving means the process of applying mathematical knowledge and skills to unfamiliar situations.

A case study was done using a problem solving approach to the learning and teaching of mathematics with a sample of teachers registered at the Umlazi College For Further Education. These teachers were familiar with the traditional approach of teaching mathematics through drill and practice methods.

The new syllabus that is to be implemented emphasises a problem solving approach to the teaching of mathematics. This study set out to implement a problem solving approach with primary school mathematics teachers so that they would be somewhat prepared for the innovations of the new syllabus. Workshops were conducted using an action research approach with discourse and practice leading to reconstruction with improvements.

Early theorists like Piaget and Bruner offered ways of understanding children's learning, to help the teacher develop his teaching. Dienes introduced an element of play and Dewey spoke of the importance of experience. Dienes and Dewey show the first positive signs of recognising the importance of social interaction in the learning situation.

Social interaction lays emphasis on language and discussion in the mathematics classroom. A social constructivist model of teaching and learning was used for the research.

This research includes a study of the established ideas on developing a problem solving approach to mathematics teaching. These ideas were incorporated into the workshops that the group of teachers attended. During the workshops teachers were gradually exposed to the essence of problem solving techniques through much group discussion and doing practical exercises, which they could then implement in their classes. The teachers reported back at each subsequent workshop.

A non-participant observer evaluated the development at the workshops. The workshops' success was evident from the change in the teachers' attitudes and behaviour as well as their feedback of what transpired in the classroom. They reported on the change in their roles as information suppliers to facilitators where the thinking process was focused on, rather than the importance of a correct answer. In the workshops the teachers themselves moved from passive listeners to active participants.

It would appear from this preliminary investigation that through using a problem solving approach in workshops, in-service teachers can benefit constructively from this approach and will attempt to use it in their own teaching.

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PREFACE

I wish to thank the following people:

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CHAPTER 1

INTRODUCTION

1.1 Background and Motivation

It has been stated that developing a problem solving ability is an important aim in teaching mathematics (MASA 1988), (Cockcroft 1982, N.C.T.M. 1989). Mathematics is considered to be useful when it can be applied to a particular situation and this is the basis of problem solving.

By problem solving is meant the process of applying mathematical knowledge and skills to new and unfamiliar situations. The purpose of encouraging pupils to solve problems is that they will acquire and be able to use the process of mathematical thinking, so that they will put these processes to work whenever they are needed. Problem solving strategies involve posing questions, classifying data, pattern searching, inferring, analysing and validating results. A problem solving approach to learning mathematics is suggested in the position paper on a new mathematics curriculum for South African schools (MASA,1988). The focus is on active mathematical thinkers. It is assumed that pupils can achieve this by open-ended investigation, application and problem solving. Discussions between the teacher and pupils and between the pupils themselves is suggested as a means of furthering the problem solving process.

The Umlazi College For Further Education is a teacher-training college whose purpose is to upgrade the qualifications of teachers in the field. Its task is to equip teachers with skills needed to improve their teaching.

In the researcher's experience, primary teachers registered at the Umlazi College appear to teach mathematics by using rules through drill and practice methods. There is a need, therefore, to find ways of improving their teaching. According to James (1990) there is no-one who cannot do mathematics, "it's the teaching that makes the difference".

An assumption of current research (Hilton 1989, Olivier 1989, Laridon 1990) is that if the teaching of mathematics is based on a problem solving approach then the pupil is more likely to be led into the successful application of mathematics. In order to teach via a problem solving approach, teachers have to revise their fundamental beliefs of how children learn mathematics at the primary school stage. An issue of teachers' views concerning the nature of mathematics is implicitly addressed in this study. There has to be a change in attitude towards mathematics by the teacher as well as by the pupils - teachers have to believe that pupils can solve problems on their own and pupils also have to believe that they are capable of solving problems (Olivier 1990).

Appropriate problem solving approaches for mathematics learning need to be drawn up in consultation with teachers,

since they are the ones who are aware of the context in which the method is to be put into practice.

1.2 Statement of the Problem

The proposed new primary school syllabus for standards 2 to 4 is to be implemented by some departments in 1993. It is based on the Working Document distributed to educational institutions in South Africa by The Department of Education and Culture under the administration of the House of Assembly. This document emphasises a problem solving approach. If teachers are going to be guided by the Working Document, they will have to be familiar with the approaches to problem solving.

Thus, there exists a need to develop an approach to teaching problem solving that meets the needs of the teacher in the primary classroom. This case study sets out to implement a problem solving approach with a selection of local primary school teachers from Black schools.

1.3 The Research

In order to meet the needs of these particular teachers, the action research method (Cohen & Manion 1980) was used. In keeping with the cyclical process (Grundy 1987:146) of action research, where discourse and practice lead to reconstruction with improvements, the interaction with the teachers was

facilitated through the informal setting of workshops. The development and the evaluation of the project were conducted in the following way:

- a) by developing an appropriate problem solving teaching approach through the cyclical process of action, discussion and reflection
- b) an internal evaluation by getting teachers to report on the trial runs at their school through questionnaires and interviews and
- c) an external evaluation carried out by a non-participant observer.

After the group became aware of the theory of using a problem solving approach, they solved problems in the workshop sessions and then applied this approach in their own classrooms. In follow-up workshops it was necessary to make use of their feedback to adjust the methods to the prevailing conditions. From this it can be seen that the cyclical nature of the methodology cannot be clearly separated from the findings.

CHAPTER 2

REVIEW OF RELATED LITERATURE

2.1 How Children Learn Mathematics

In order to be able to teach effectively one needs to have a clear understanding of how children learn, especially with respect to the subject being taught. This applies to the teaching of mathematics. Baroody (1987:5) says that if mathematics is taught without considering the cognitive factors then "...many children will learn and use mathematics mechanically ...and some will develop learning difficulties." Theories of learning also help in developing curriculum material. Baroody (1987:15) says that "educational practice that overlooks how children learn mathematics may deter meaningful learning..."

2.1.1 Early Theorists

If Piaget's and Bruner's theories of how children learn mathematics are considered, and the gap between the two is filled, then a well-rounded idea of how children learn mathematics, based on the theory of the time, will be developed.

Piaget believed that learning is linked to biological

development (Liebeck 1984). Learning depends on and comes with the child's mental growth. Mathematical concepts cannot be taught. Learning mathematics occurs spontaneously. Piaget divided the learning that occurred in a child into four stages, according to age. These are the sensori-motor (0-2), pre-operational (2-7), concrete operational (7-11) and formal operational (11 onwards) (Clifford 1981:104-105).

Since the senior primary child is between 7 and 11 years of age, this stage will be focused on. During the concrete operational stage, Piaget says that the child is able to perform tasks using concrete objects. Conservation of number, length, mass, area, weight and volume are developed. The ability to put order into things is seen. Souviney (1989:20) characterised this stage by the ability to carry out mental pictures and to group objects according to their attributes.

According to Piaget, children can only work with assumptions in problem solving during the formal operational stage. It is the intention of the researcher to show that the ability to solve problems can be developed at an earlier level, given the correct support and guidance by the teacher.

Tests carried out by McGarrigle (Hughes 1987:19) show that Piaget's tests may not be reliable for placing children into the above definite categories since his tasks involved the use of language. The children may not have fully understood what the adult was saying. They may also have answered as they

thought the adult expected them to answer, e.g. the child was told to watch carefully while one of two equal sticks was moved to a new position and then the child was asked "Are they still the same length or is one longer than the other?" (Shuard 1986:62). The child may have believed that he had to respond as if a change had been made. Although Piaget took into account the child's experience, he underestimated the role of social interaction and language in learning. The role of language will be discussed later where its importance will be emphasised.

Bruner disagreed with Piaget's belief (Resnick & Ford 1984:113) that there was a need to wait until the child was ready to be taught the various aspects of mathematics. There is no need to wait for a child to reach a certain age before a particular concept is taught. The beginnings of a complex concept may be presented to a child at an early stage. This brings in the idea of teaching a concept in a spiral fashion, so that it is developed over a period of time. Using Bruner's spiral curve (Resnick & Ford 1984) the elementary aspects of a concept are first introduced, then built upon at various stages. Thus Bruner's model is cyclic in nature.

While Piaget advocated that children will only be able to perform tasks involving abstractions when they reached the stage of formal operations, Bruner believed that given appropriate instructions, these children could learn to perform and understand these activities at an earlier age. The

focus of Bruner's work was on concept development. According to Bruner's theory, the concepts which children acquire to represent the world, develop in three modes :

- a) enactive - where the child interacts directly with the physical world. Past events are represented through motor responses.
- b) iconic - the child works with mental pictures from the real world
- c) symbolic - at this stage the child manipulates symbols. This is mainly possible through the use of language (Liebeck 1984:241).

Bruner argued that if the intellect developed in the order of enactive-iconic-symbolic then new concepts should be taught in this order. Both Piaget and Bruner advocated that learning moves from concrete to abstract.

Dienes accepted both Piaget's and Bruner's work and developed appropriate mathematical principles for teaching. His theory (Liebeck 1984:243) sees learning as a process of increasingly intricate play. According to Dienes, there are two types of play:

- a) primary - involving manipulation and investigation of materials for its own sake and
- b) secondary - trying to build with materials, discovering patterns and forming rules or generalisations from the patterns found.

He believed that children are constructivists by nature. By this is meant that children should be allowed to piece together a picture of reality from their own experiences of the real world (Resnick & Ford 1984). Teaching materials enable them to have the concrete experience and perform tasks as Piaget mentioned earlier.

According to Dienes the learning cycle of concept development begins with free play using mathematical materials. Children's experiences are then structured to sharpen the concept. During this structured play, concepts are gradually abstracted. Children must be helped to find ways to talk about their findings. Abstracting their findings from the concrete materials can be done by drawing pictures or graphs. The next stage would be to attach mathematical symbols to the concepts. Children should be allowed to go back to the concrete stage at any time in order for the symbolism to remain connected to the concrete experiences.

Dewey felt that learning comes from experience (Reys, Snyman & Lindquist 1984:36). The importance of experience is highlighted in the constructivists' view of learning. Experience could be from the real world or from models used in class. Early learning moves from specific examples to generalisations. Although models may be used, teachers have to guide and direct children to form abstractions. In her chapter on "The Construction of Understanding", Shuard (1986:73) describes the learning process as a transition from one form

of thinking to another. A child will construct his understanding on the basis of his own experience. If, as a result of his lack of experience, he makes inappropriate generalisations, the teacher should assist the child to correct the misconceptions.

Reys, Snyman & Lindquist (1984) speak of "learning bridges" as a means of providing a path that can be travelled many times - in either direction - "to reach greater understanding". The bridges form a link between the concrete and the abstract. Mathematics learning requires a building onto what the child already knows. His previous knowledge has to be meaningful in order for the developmental process of mathematics learning to occur. This idea endorses those of Dienes and Dewey where learning comes from experience and the teacher helps to provide these experiences. Pupils need to be actively involved, be it physical or mental, to sustain motivation which, in turn, helps the learning process.

2.1.2 Types of Understanding

Skemp (1973) describes two types of understanding that a child can achieve:

- a) relational understanding - when the child knows exactly why he is doing what he is doing. Relationships are then formed.
- b) instrumental understanding - when a child merely applies the rules and does not know why he is doing,

what he does.

There is evidence to show that slow learners often use instrumental understanding when dealing with place-value (Ginsburg 1977, Denvir 1982, Smith 1990). Rules are learnt without reason. Children finally mix up the rules when there are too many to remember. When a child is confronted with a problem and has a set of rules to use, he is not likely to consider how it works. His reward is the tick that he will get for the right answer. Ginsburg (1977) points out as a result of his interviews with children who experienced learning difficulties, that their informal techniques were sound, but when they used "formal, school-derived techniques" they did badly. When a child tries to remember rules, these rules tend to cloud each other and the result is that they cannot be remembered or that they become distorted.

If, on the other hand, relational understanding is to be achieved, it would require more effort on the part of the teacher and take a longer time since different relationships have to be recognised. Once relationships are made, learning is more permanent (Denvir, Stolz & Brown 1982:43). An important characteristic of mathematics learning is making the connection between a set of understandings and the symbol system. The teacher will be able to note a persistent error in the child's work. Denvir, Stolz & Brown (1982:48) explain that children also learn by becoming aware of inconsistencies in their thinking which they try to resolve.

Skemp's description of understanding is similar to Baroody's (1987) two theoretical approaches to learning, which he calls the absorptive theory and the cognitive theory. The absorptive theory can be likened to instrumental understanding. Facts are memorised so that they can be recalled at a later stage and learning is viewed as being passive. Associations are formed and knowledge increases by memorising new associations. This type of learning places a great burden on the child to continuously remember rules. A clear understanding of what is being done may never be achieved.

The cognitive theory is akin to relational understanding. Information is connected by relationships. For learning to take place insight and meaning are required. The child internalises new information by relating it to his previous experiences, and new information is linked to what is already known through the process of assimilation. Previously isolated information is connected by what is called integration. This approach is used for meaningful learning. Knowledge gained can be transferred to new situations, and is therefore useful in problem solving situations. When children are working on a problem which interests them, they spend a considerable amount of time on it and they become enthusiastic when they discover thinking strategies. They may consider this to be a "detective game" (Baroody 1987), which is really a problem solving strategy.

The interaction between what the child already knows and his

experience with new ideas involves the processes of assimilation and accommodation (Skemp 1973). To understand the relationship, the concept of a 'schema' must be explained. This is a mental structure or an idea in a child's mind. To acquire more knowledge the learner makes use of what he already knows. When a new idea is encountered, the learner looks to his previous experience and assimilates the new idea in terms of what he already knows. If the new idea is different to his past experience then an adjustment occurs where the schema is reconstructed or re-organised. This is known as accommodation.

2.1.3 Constructivism as a Theory of Learning Mathematics

The Cockcroft Committee in the United Kingdom emphasised the view that "people actively construct understanding for themselves" (Shuard 1986:63). Shuard makes the point that mathematics will be remembered better if it makes sense to the learner.

The Committee argued that when the analytic approach to computation is used it is possible that learning takes place without reasoning. Ginsburg (1977) said that mathematics is then considered to be an isolated game without a link to reality. This prescriptive method induces a passive, dependent attitude while the constructivist's approach induces an active self-reliant, creative attitude towards learning (Human 1989).

In using Piaget's (1970) and Skemp's more recent views on learning mathematics, Olivier (1989) gives his perspective on constructivism. This is stated as a person's ability to learn, which depends on the quality of ideas he is able to bring to that experience. Knowledge gained depends on the child's pre-existing knowledge which will interact with his experience in the situation. The child actively participates in the construction of his knowledge. This rationale underpins the design of the Working Document and is on which the problem solving approach was based in this study.

The basic tenets of constructivism (Clements & Battista 1990) are that:

- 1) knowledge is actively created or invented by the child,
- 2) children create new mathematical knowledge by reflecting on their physical and mental actions,
- 3) there are individual interpretations of the world which are influenced by experience and social interactions and
- 4) learning is a social process.

The constructivist's idea of learning is that learning leads to changes in schema. There is an "awareness of the interaction between a child's current schema and learning experience..." (Olivier 1989). The teacher has to see things from the child's point of view. In order for a child to acquire knowledge in this way, he has to make the knowledge

his own. This can be achieved by discovering, reflecting and negotiating. The underlying theories of constructivism provide a basis on which a problem solving approach can be developed, so that the learner develops concepts for himself as a result of his experiences.

Yackel (1990) quotes von Glasersfeld's interest in "developing ways of thinking in the student". Yackel re-iterates that the constructivist acknowledges that pupils will interpret situations in different ways. Pupils can communicate these various ideas and interpretations by interacting with each other.

Laridon (1990) states that the principles in the new syllabus (Appendix C), on which this study is based, encapsulate constructivism. The perceptions of some teachers on the old curriculum (MASA 1988) are tabled as follows:

- standard algorithms were not well understood
- 'word sums' that children encountered were not realistic
- pupils don't really understand fractions and merely learn the rules of manipulation
- the curriculum is repetitive with no challenge for the more able pupil

When one looks at the above comments one can see the need that existed for the change of focus in the present curriculum. Yager (1991) speaks of the need for interaction of the encountered information and how the pupil processes this

information. He says that "all learning is dependent on language". He develops this idea further when he promotes group learning where pupils discuss their approaches to problems.

With the constructivist's approach the emphasis is on the child constructing meaning for himself. Bruner's theory as well as Dienes' teaching materials require the child to be actively involved in order to internalise concepts so that they have meaning for the individual. Dewey's idea of learning coming from experience is also in keeping with the constructivists' theory.

2.2 The Importance of Language

Donaldson (Hughes 1987:21) says that the child has to attend to the words of the teacher and at the same time think about the language used which is independent of the context. He calls this "disembedded thinking". Mathematics in school is disembedded from the immediate context. This makes learning of the subject difficult. Words used in mathematics frequently have a different meaning in everyday English. Shuard and Rothery (1984:27) use the word "difference" to illustrate this:

If a child is asked "What is the difference between 7 and 10?" he could say that one is even and one is odd. Such a child will not realise that the expected answer was supposed to be 3. When such words are encountered, and if the meaning is not

clarified, misunderstandings will arise.

When a child has encountered a difficulty or expresses a misconception, the best way to remedy this is to determine his method of solving the problem. This can be achieved by talking to the child. Talking allows the teacher to detect systematic errors that a child may make. Analysis of these errors can form the basis on which remedial work may be designed. A child's learning is linked to his understanding of language. Wood emphasised that it is important to find out where the child "is at" (Wood 1988:205). Talking allows the teacher to discover how the child is thinking. This also fits in with the constructivists' idea of children constructing relationships for themselves where they can fit pieces of information together to form a whole that has meaning. A concept, according to Skemp (1973), should not be explained by means of a definition. The child must be allowed to encounter examples of the concept through a range of tasks and experiences. By allowing a child to look for "sameness" from his many examples and verbalising this "sameness" he will form a definition for himself. This allows the processes of accommodation and assimilation to occur.

In the ten practical principles of helping children learn mathematics, Reys, Snyman & Lindquist (1984) state that one of the principles is: "Verbalisation is an integral part of learning mathematics". Children must be able to use the language of mathematics orally before they are able to use the

symbols which form the greater part of the subject. A child needs to tell the teacher what he knows or does not know. Talking about mathematics enhances one's understanding and develops the student's interest. This argument is further strengthened by the Bullock Report of 1975 (The Mathematics Association 1987:2) which points out that children are able to talk and listen much better than they can read and write. For this reason talking and listening in a mathematics class will help develop skills and understanding. Language-based mathematics teaching can be linked to specific uses of language stipulated in the Report.

Even though this Report laid down a policy for "language across the curriculum" (Shuard 1986:4) this was not taken into account for mathematics teaching at the time. Hence the Cockcroft Report of 1982 made reference to language and communication :

"There is a need for more talking time...ideas and findings have passed on through language developed through discussion, for it is this discussion after the activity that finally sees the point home" (Paragraph 306).

Vygotsky's main concern was the relationship between language and thought. He believed (Souviney 1989:22) that mental operations are the result of active social interaction. Learning is best achieved when the learner is engaged in co-operative activities. The teacher should therefore provide

support and guidance to his classes. He will organise the pupils into groups to interact with each other and this active interaction results in learning occurring.

In her model for learning and teaching mathematics, Hoyle (1988) adopts Vygotsky's proposal that a child's cognitive development is characterised by an actual development level and a level of potential development. The potential development is enhanced by the use of language and support provided by the teacher.

Yager (1991) says that a human being's experience is influenced by social interaction with other human beings. In terms of teaching, the use of language on its own is not a means of transferring information. Language must have meaning. It is used to help make sense of our world. Children try out new words that they may have heard. By encouraging children to experiment with the language and discussing the meanings and use of words encountered in mathematics, the children's understanding of mathematical terms can be improved. Children learn by co-operating, sharing and listening. Instead of a teacher repeating an answer in order to re-inforce an idea he can allow the pupils to give different answers to a question. This will allow children who did not know the answer to benefit, not only from hearing their peers giving the answer, but by having the concept repeated.

Murray (1990) adds that language helps the thinking process

and that as concepts get more abstract the individual is more dependent on language. The purpose of language, given by Ashworth (1985) when considering "language across the curriculum", applies to the learning of mathematics as well, as it:

- facilitates learning by enabling any form of knowledge to be classified and provides for this classification to be transmitted;
- enables experiences to be interpreted and inter-related;
- allows thinking, reasoning, predicting and exploring of alternatives to occur;
- promotes communication, understanding and self expression.

The evidence above suggests that discussion amongst students also helps learning. Communicating one's ideas with others can help to clarify them. During a discussion ideas are interrelated and interchanged to make concepts clearer. In this way the gaps that may exist can be bridged. Skemp (1973:122) says that discussion "stimulates new ideas". For these discussions to take place, there should be a friendly and uninhibited relationship amongst the pupils and between the teacher and pupils.

2.3 The Teaching of Mathematics

As a philosophy, the socio-constructivist's view (Ernest 1991:82) is that objective knowledge of mathematics is social. It is the result of shared rules and understanding as well as

interaction with members of a group.

Skemp's view that instrumental understanding is not effective for learning, is echoed by Denvir, Stolz & Brown (1982:52) who say that it has been found that "the teaching of skills in isolation is not effective". In his investigations into the way children handle mathematical laws, Smith (1990) found that children had difficulty in using the four rules of number. He says that those who used the rules did not necessarily understand them. In order to change this situation, he refers to the National Curriculum Council's non statutory guidance of encouraging pupils to develop their own methods of doing calculations. This is in line with the constructivist's approach of the child building meaning for himself.

When teaching early number skills, instead of using rules, pupils need to be allowed to handle physical objects and to talk about what they are doing. Denvir (1982:55) suggests the use of a wide range of appropriate apparatus and materials. Such apparatus can be made by the teacher so that even a teacher in a rural school can provide his pupils with a situation that will foster talking. Only after they understand exactly what they are doing on the concrete level should they move into informal recording. Here they should develop their own ways of recording. The child must experience success in order to build his self-confidence and reduce anxiety. The teacher has to engage in careful planning to develop the pupils' interest in the work to be done. Active learning

allows for "the acquisition of concepts" (Dossey 1988:291). It follows therefore that the teacher and learner must be actively involved in the learning process. Teachers should question pupils in order to encourage participation.

Kamii & Lewis (1990) offer the following principles of teaching based on constructivism:

- * encourage pupils to invent their own ways of adding and subtracting numbers rather than telling them how;
- * encourage pupils to exchange points of view rather than reinforce correct answers and correct wrong ones;
- * encourage pupils to think rather than to compute with paper and pencil.

A mathematics lesson should not involve the transfer of information alone, there should be something to be discovered by the child. If pupils help to formulate the problems they may take more interest in solving them. Mathematics must be seen as relevant to the child's attention and pupils must be allowed to experience and experiment.

The idea of "learning through experience" (Watson 1976:51) has been emphasised since 1955 in Britain, but in South African schools it is still not being used to its full extent. This is reflected in the "problems" that appear in school textbooks. These problems very often do not relate to the child's life experience. It appears that the problem in British schools mentioned by Watson (1976:65) and, in the

researcher's opinion, quite prevalent in local Black schools, is the fact that teachers in the primary schools are non-specialists who themselves experienced anxiety in mathematics while they were at school. Thus although they have a negative attitude to the subject they are still responsible for guiding the mathematical development of their pupils.

A pre-requisite for improving mathematics learning is a change in the children's attitude to learning the subject (Biggs 1985:52). By using the constructivist's approach, instruction will allow for the development of the pupils' ideas in mathematics (Clements & Battista 1990). Abstraction of ideas occurs through interaction with the mathematics task and with other students. The teacher provides guidance and support during this interaction. Children need to know why they are learning mathematics so that they can make use of it instead of merely practising calculations. Pupils must be encouraged in their work. If a wrong answer is given the child must be guided by the teacher towards the correct way of thinking. The use of materials may be encouraged, but, allowance should be made for pupils to decide whether they want to use them or not. Less emphasis on written work and more on investigative work can be used to foster the pupils' interest in the subject.

The National Council of Teachers of Mathematics endorses the "Position Statement" giving the twelve components of essential mathematics (Iris 1989). These are problem solving,

communicating mathematical ideas, mathematical reasoning, applying mathematics to everyday situations, alertness to the reasonableness of results estimation, appropriate computational skills, algebraic thinking, measurement, geometry, statistics and probability.

Some of the above components are also found in the Cockcroft Report (1982) which states the six elements of successful mathematics teaching as being:

1. exposition
2. discussion
3. practical work
4. practice of skills and routines
5. problem solving
6. investigation

The Cockcroft Committee also states that the major aims of teaching mathematics is the understanding of mathematics which can be achieved through practical work, problem solving and investigation. From the sample of teachers involved in the research, exposition and the practising of skills and routines have been the most commonly used methods of teaching mathematics. Problem solving and investigative work, in the researcher's opinion, have not been given due attention in most South African schools. It is for this reason that the researcher has focused on this area of learning and teaching.

2.4 Practical Work

Evidence suggests that if practical work is done in isolation and not linked or loosely linked to formalisation then children resort to memorising the formulae in order to reach a solution (Hart 1981, Dickson, Brown & Gibson 1984). The teacher ought to make an effort to link the two and ensure that the pupils see the connection between practical work and the formal methods.

Practical work needs to be well-structured. There should be clear stages of progression and these should be followed by discussion to ascertain whether learning has occurred (Shuard 1986). Practical work can be used in mathematics teaching with pupils of all ages.

In overcrowded classes, practical work can be cumbersome since the teacher is unable to easily walk around the classroom. An example of this is when the classroom is so full of pupils that the desks are arranged so that the teacher is unable to walk around the classroom. The teacher is unable to devote his attention fairly around the class and not much discussion occurs. In the researcher's experience in Black schools, where the average number of pupils in a class is 60, practical work is non-existent. In such cases practical work could be incorporated into group work so that the teacher will be able to communicate more easily with the groups. The group work issue will be discussed later.

2.5 Summary

From the various theories of learning stated above, it can be seen that the idea of a developmental process exists. Children's learning starts with making use of the senses. The concrete stage allows the child to touch, feel and smell the physical teaching aids used by the teacher to explain or demonstrate a concept. The child manipulates these objects and makes them part of his life-world or experience. An example of this is the use of the Dienes' Blocks, when teaching place value. The child physically exchanges ten little cubes for one long piece to allow him to understand what happens when we have ten or more units in the units column. The same idea is used when moving to the hundreds column or to the thousands column.

During the concrete stage, the child is encouraged to verbalise his activities as well as his ideas. This talking helps in his understanding of what he is doing. Once the child exhibits signs that he understands, he can record his knowledge. This may involve either an appropriate drawing or diagram, or the introduction and use of symbolic notation. Adequate and rich experiences in the concrete domain should enable the child to transfer his knowledge from the concrete to the abstract. Throughout the stages between the concrete and the abstract, the teacher is only able to help his pupils if he knows how they are thinking. This is achieved through the use of language.

It would seem that discussion between pupil and teacher, and between pupil and pupil is a necessary constituent for adequate and meaningful learning to take place. The teacher uses language to transmit his ideas to the class, while the pupils use language in their process of learning. The constructivist theory of learning, where the person through interaction builds meaning for himself is important. Another aspect as a direct outcome of discussion is the social interaction that helps in the learning situation. Analysing mathematics together can be like a detective game. This favours a problem solving approach supported by child-centred and language-based methodologies.

CHAPTER 3

PROBLEM SOLVING PROCESSES

Of the twelve components stated in the report by the National Council of Teachers, six can be undertaken by a teacher using a problem solving approach to the teaching of mathematics. This approach includes problem solving, communicating mathematical ideas, mathematical reasoning, applying mathematics to everyday situations, an alertness to the reasonableness of results, and estimation. The teaching of mathematics is following this trend in the U.S.A.

Two of the six elements from the Cockcroft Report from the Britain are problem solving and investigation. There is no clear separation between problem solving and investigation. Thus these two, together with discussion and practical work allow four of the six elements of successful teaching from the Cockcroft Report to be covered by the problem solving approach.

From both the American and British investigations into finding ways of improving the teaching of mathematics, there seems to be a move towards using a problem solving approach to teaching. In South Africa, too, the new syllabus also shows evidence of moving towards the use of the problem solving approach. Since problem solving is concerned with process

rather than content, it need not be confined to the mathematics lessons or syllabuses but may be taught across the curriculum.

Practice with independent problem solving should begin early. In the previous mathematics syllabus there was an over-emphasis on computational skills. This is no longer necessary since we have machines in the form of calculators and computers to do these. Rather there is a need for mathematics teaching to address the real problems that exist.

3.1 Mathematical Problem Solving

With the rapid advances in technology which is having a significant effect on the nature of careers and work, it is almost impossible for pupils to learn everything they need for the future. Students must be equipped with the ability to learn things that are not yet known. They need to be able to analyse a problem and find a way of solving it.

In order for an activity to be termed problem solving, the individual or group solving the problem, must want to find a solution to the problem which is not readily accessible by merely recalling algorithms. According to Lumb (1987), in problem solving the role of the teacher is no longer that of "information-provider", but becomes that of "question-asker" and "resource provider".

3.2 Routine and Non-Routine Problems

Routine problems are word or story sums that appear at the end of sections in a textbook. Non-routine problems cannot be solved by straight-forward application of algorithms.

The story sums may be one-step story problems which are also referred to as simple, translational problems (Charles & Lester 1982:7) e.g. One fish costs 52c how much should Susan pay for 5 such fishes. Multistep Story problems (Souviney 1989:65) are as follows:

Alan bought two stamps from Nick, one for \$10 and another for \$14. Later he sold the first stamp for \$12 and the second for \$16. How much profit did Alan make on the sale?

Charles & Lester (1982:7) call these complex translational problems.

Souviney(1989:65,66)gives possible ways of introducing non-routine problems as:

- * Headline problems where the student supplies the story to go with the headline.
- * No-value problems where no values are specified and students may supply their own values if they wish.
- * Fill-in value problems where students fill in the values before they solve them.

Souviney's non-routine problems also cover Charles & Lester's process problems, applied problems and puzzle problems.

Routine problems appear in the syllabus under the various sections such as estimation, decimal fractions and common fractions. Non-routine problems can be found in the sections on solving problems. Routine problems have always been in the syllabus. It is the non-routine problems that are being focused on in the Working Document (Appendix D).

3.3 Developing Problem Solving Strategies

Stoker (1989) believes that although investigation does not make teaching easier for the teacher, it provides better learning opportunities for the pupils. Shuard (1986:87) gives Kilpatrick's five techniques used by teachers to teach problem solving. These are:

- * Osmosis - pupils have many problems to solve and while solving them they acquire techniques,
- * Memorisation - pupils memorise algorithms which they apply to the problems without any guidance of how to tackle new problems,
- * Imitation - pupils learn problem solving techniques by imitating those used by an author of a book,
- * Co-operation - small groups discuss ideas of how to tackle the problem and
- * Reflection - pupils reflect on their own techniques.

In the researcher's opinion memorisation and imitation cannot make problem solving techniques transferrable. Thus they may not show signs of the child having learnt anything. Osmosis,

co-operation and reflection are better techniques to develop problem solving abilities. These techniques were incorporated in the planning of the workshops.

In line with Bruner's learning theory we see children progressing from the enactive (concrete) to the iconic (pictures) to the symbolic stages. It may be helpful for the child to go from the verbal problem to drawing a diagram (iconic) or table which will assist him in solving the problem (symbolic). This developmental process will be used in the workshops.

Based on research that has been done over the years, Reys, Snyman & Lindquist (1984:23) have several broad generalisations to make on the teaching of problem solving :

- * Problem solving strategies can be specifically taught
- * No one strategy is optimal for solving all problems
- * Teaching a variety of strategies gives children a repertoire from which to choose when solving a problem
- * Students need to be faced with problems in which the way to solve them is not apparent
- * Children's problem solving achievement is related to their developmental level

Some of the above aspects will be highlighted in the workshops.

A widely used model for teaching problem solving is that of Polya (1957), which has four phases. These are understanding

the problem, devising a plan, carrying out the plan and looking back. Three approaches to teaching problem solving based on Polya's model exist (NCTM Yearbook 1989). These are:

3.3.1 Teaching about Problem Solving

The teacher teaches Polya's problem solving model. The pupils are aware of their progress in terms of these stages as they solve problems. It involves a vast amount of discussion about how problems are solved.

3.3.2 Teaching for Problem Solving

The teacher focuses on ways of applying the mathematics learnt in the solution of problems. Pupils are given the opportunity to transfer what they have learnt by applying it to the solution of problems.

3.3.3 Teaching via Problem Solving

The teacher teaches a topic of the syllabus by starting with a problem situation. Techniques are developed by the pupils during the lesson. Using this approach non-routine problems are transformed into routine ones.

If the approach of teaching about problem solving is used, then problem solving will be a topic taught in isolation. With teaching for problem solving, the problem solving approach will only be used after a session is taught. This is similar to exercises given at the end of a lesson. Such "problems" only involve the application of the section recently taught.

Pupils have to merely follow the rules given by the teacher without necessarily understanding what they are doing.

The Yearbook of the NCTM (1989) prefers teaching via problem solving since it is in keeping with the recommendations of the NCTM's Standards Commission of 1987 that:

- (a) mathematics concepts and skills can be learnt in the context of solving problems;
- (b) the development of higher-level thinking processes be fostered through problem solving experiences;
- (c) mathematics instruction takes place in an inquiry-oriented, problem solving atmosphere.

Non-routine problems cannot be solved immediately. Pupils need to first develop confidence in themselves before they can experience success in solving problems. Souviney (1989:68) suggests the following ways in which the teacher can encourage the pupils to solve problems:

- # provide an environment that encourages students to take risks
- # value all answers as being potentially useful
- # praise pupils for their efforts
- # evaluate the quality of the pupils' efforts
- # allow group work during problem solving sessions
- # encourage pupils to work on tasks at home
- # systematically integrate the problem solving sessions into the mathematics curriculum
- # introduce a systematic problem solving plan

ask appropriate questions during each session

To achieve some of the above the teacher has to be convinced through the workshops, of the importance of pupils' talking in the mathematics class. Pupils should be allowed to give their opinions on a solution and encouraged to discuss their solution with other pupils. Active participation may allow pupils to go beyond the required solution. This is the beginning of investigation or the extension of a problem.

3.3.4 Burton's Method

Burton (1984) divides the above steps into four phases which he calls entry, attack, review and extension. During entry the pupil will try to understand what the problem is about. Once this has been achieved, he will attack it. He will not find the solution easily and thus will get "stuck" a few times. At the point of getting "stuck" the pupil will return to the entry phase to re-assess what he has interpreted. In review the pupil examines the solution that he has arrived at. If the solution is found to be lacking in any way, the pupil will return to the entry or attack phase. The resolution is then written up to be understood by others. Extension leads the pupils to further investigation. Extension and review are interrelated and can be equated to Polya's "Looking Back".

Burton (1984) has a list of organising questions that coincide with his phases:

Entry

- # What does the problem tell me?
- # What does the problem ask me?
- # What can I introduce to help get started?

Attack

- # Can I make connections?
- # Is there a result which will help?
- # Is there a pattern?
- # Can I discover how or why?
- # Can I break down the problem?
- # Can I change my view of the problem?

Review-Extension

- # Is the resolution acceptable?
- # What can I learn from the resolution?
- # Can I extend the resolution?

The above questions will be used to assist the teachers to progress through the problems in the workshops.

3.4 The Problem Solving Approach

The researcher's approach at the workshops was based on Polya's 4 phases of problem solving. Other established ideas as stipulated below were incorporated into the workshop sessions.

3.4.1 Understanding the Problem

Before a problem can be solved, a clear understanding of what the problem is all about is necessary. This can be developed by allowing the pupil to re-tell the problem in his own words. A clear focus on the language is needed for this. The problem should be linked to real life and to the experience of the child in order for it to be meaningful to him.

Concrete objects can be useful to make a concept clear. A list of the given facts and related information can be made. Pupils can state the goal in their own words. Pupils tend to work better in groups where discussion of the problem is possible. They can compare the present problem with previously solved ones. They need to note the stated conditions and restrictions as well as the implicit conditions. At this stage the teacher can use questions to stimulate the pupils' understanding.

Schroeder & Lester (NCTM 1989) stress the value of understanding as an aid to problem solving. The problem-solver has to "internalise the information in a problem" by developing a representation of the information. It is through understanding that a decision can be taken on the strategy to be used. Understanding allows the progress being made towards the solution to be evaluated. A clear understanding will enable the solver to decide on the reasonableness of the answer. Understanding promotes the transfer of knowledge so the learner can apply his newly gained knowledge to other

situations, including an extension of the particular problem.

3.4.2 Selecting a Strategy (Devising a Plan)

The teacher's role at this stage is to offer hints. Students should be allowed to find a strategy on their own but must not be left to struggle on their own for too long, since this decreases their motivation.

A summary of Souviney's strategies follows:

- * Guess and Test - make a guess and test your answer to see if it satisfies the conditions of the problem.
- * Substitute Simple Values - so that the structure of the problem is focused upon. Once a solution is reached with simple numbers the more complex ones can be substituted and the procedure can then be repeated.
- * Divide the Problem into Subtasks - where each part is more manageable than the whole problem.
- * Conduct an Experiment - experiments require a physical representation of a problem situation. Data has to be well-organised and used to look for patterns that may exist.
- * Designing a Model -this can cover the essential features of the problem.
- * Draw a Sketch or Graph - helps to visualise a solution.
- * Make a Systematic List or Table - which could be used to Look For a Pattern.
- * Construct a General Rule - that describes a pattern.
- * Work backwards - by starting with the known end state.

Charles and Lester (1982) cover the same areas in terms of strategy but they divide their strategies into two main types, namely, general strategies and helping strategies. Details of these are described in the chapter on methodology (Chapter 5).

3.4.3 Carrying out the Plan

In order to carry out a solution the child must have the self-confidence to be persistent. If a table, sketch, list or graph is used it is better for this to be accurate. When a particular strategy is chosen the pupils should be encouraged to stick to it until they have good reason to believe that it will not work. Emphasis should be laid on thinking things through and discussing the steps of the solution with the group.

3.4.4 Looking Back

Looking back (Taback 1988) does not mean simply evaluating the answer, but focusing on how the solution was arrived at. This can be done by allowing one or two members of the group to explain to the class how the strategy was employed, discussing the form and validity of the answer and seeing if the solution can be used in an extension of the problem.

3.4.5 The Role of Questioning during Problem Solving

One of the most important tasks of the teacher is to help his students. This task is not quite easy; it demands time, practice, devotion and sound principles (George Polya, 1957).

The above quotation emphasises that the teacher has an active role to play in problem solving. The pupils' learning can be directed by observing and questioning pupils. During the workshops the importance of the teachers' role will be developed.

Understanding

The teacher may ask questions (Charles & Lester 1982:37) to help the pupil understand. Groups can formulate questions for their own use.

Some questions which Souviney (1989) considers to be appropriate are:

- # What do you know about the problem?
- # Is it like any other problem you have solved?
- # What are some reasonable answers? Could there be more than one answer?
- # Does your answer make sense? Can you convince me?
- # What can you do to find out?
- # What is wrong with that answer?

Devising a Plan

The teacher can direct the pupils by encouraging them to link strategies to those used before. Souviney (1989) gives Polya's suggestion that questions be phrased in "a manner that they could have occurred to the students themselves" and offers the following list:

- # What is the goal? What are you trying to find?
- # What information is given? What do you know?
- # What special conditions or restrictions apply?
- # Can you remember a problem with a similar unknown or goal?
 Could you use its solution to help in any way?
- # Can you restate the problem in your own words?
- # What are some possible and reasonable answers?
- # If you can't solve this problem, can you first solve a similar one?
- # Did you use all the given information?
- # Is there unstated information that could be useful?
- # How do the stated conditions restrict the solution?

3.4.6 The Use of Small Groups

Problem solving has been found to work better in small groups than with individuals (Charles & Lester 1982). Small groups reduce the anxiety that can occur with problem solving. Ideas can be shared in groups and this can lead to discussion which is vital for problem solving. Co-operation as well as conflict that may occur in a group helps the quality of problem

solving. In large classes it is easier to monitor and assist students in a group where each group would be given more time than if the teacher had to attend to each pupil individually.

In using groups, teacher direction is important. The teacher should help to identify a group captain so that the most dominant child is not always in charge. The teacher should question those pupils who seem not to be involved in the group's activity. The teacher has to prevent some pupils' ideas from being censored by others in the group and also has to ensure that everyone in the group understands the group's solution.

The participants at the workshops will work in groups to maximise self activity. This will also show them how to conduct group activities in their classrooms.

CHAPTER 4

METHODOLOGY

4.1 Selecting the Sample

460 students who registered for mathematics in the Senior Primary Teachers' Diploma were asked to fill in a questionnaire (Appendix A). In response to questions (1a) and (1b) only those teaching mathematics from standards two to five were selected. This group consisted of 128 teachers. The level of school mathematics attained by these teachers ranged between standard six to standard eight.

It was necessary to have a group of teachers who had easy access to the College, which is based in Umlazi, so that they could attend the Saturday workshops (referred to in Chapter 1) on a regular basis. Thus 40 teachers who lived in and around the Umlazi-Durban area were chosen and sent letters of invitation to be involved in the project. As a result of these invitations 26 teachers responded by attending the first workshop. At the first session with the teachers, times for subsequent workshops were chosen to suit all participants present so that they would become regular attendants at the workshops.

4.2 The Nature of the Research

The method of research used was action research (Cohen and Manion 1980:175), based on a convenience sample (Cohen and Manion 1980:76) of about 20 teachers from the primary schools in and around Umlazi. In keeping with Lewin's (1946) belief that rather than asking experts from the outside to establish how a particular programme could be used by teachers, the sample themselves were required to analyse, conceptualise, discuss and plan the best way of carrying out the programme.

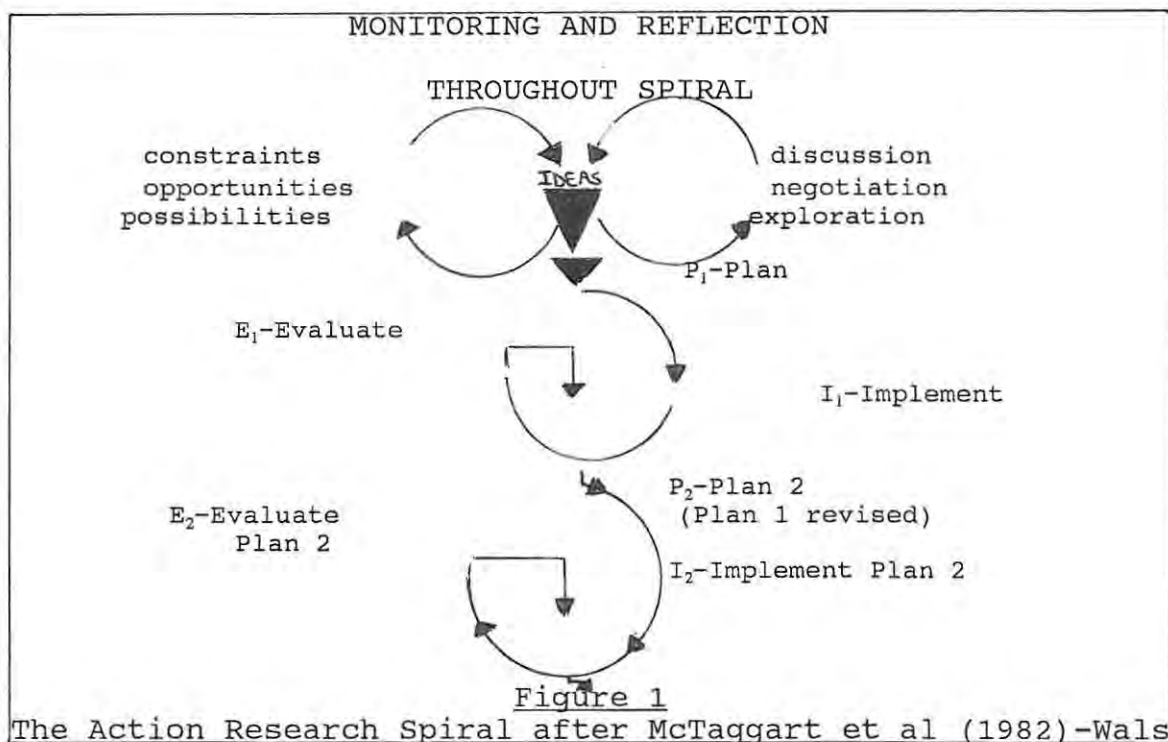
A justification for action research comes from McNiff (1988) who advocates this type of research as a means by which the teachers become actively involved as participants in their own educational process, "a process which shows one person's ideas develop and may be used by another to move his own ideas forward" (McNiff 1988:21). It is used as a means of bringing together both the theory and the practice of education. In action research the teacher researches his own classroom practice. The purpose of action research is to improve the educational process.

Action research involves change that takes place as a result of intervention. By being aware and critical of their practises, teachers are encouraged to improve their practise.

Action research differs from applied research in that it is participatory and thus a useful research method for aiding the

empowerment of the teacher. Applied research is more rigorous whilst action research "interprets the scientific method much more loosely, chiefly because its focus is a specific setting" (Cohen & Manion 1980:209).

The participants in this research critically reflected on their action with the view to improving what they had done. Collaboratively, the researcher and the participants held nine workshop sessions during 1991. According to Wals (unpublished) the process begins when participants decide to address problems that affect them. The problem at hand was how best to implement the problem solving approach in the primary school classroom. Having isolated the problem, they had to understand it, find ways of trying out various methods and identify constraints that might exist.



McTaggart (cited in Wals) states that in the first loop of the action research spiral, participants have to develop a plan, implement the plan and evaluate its effectiveness. This shows the systematic nature of action research which McNiff (1988) speaks about. The evaluation caused participants to move into the second loop of the plan. Figure 1 shows the spiral effect of action research, and how it was implemented in this study.

Various theories of learning in terms of how children learn mathematics were discussed and used as a means of developing problem solving strategies. Reference was made to the work of Piaget, Bruner, Reys, Dienes, Skemp and others. The importance of language was focused on, since it plays a vital part in developing problem solving strategies (Wood 1988:205, The Mathematical Association 1987:2, Hunter 1990, Stoker 1991).

During a series of trial runs, using a process approach, data on what existed in the classroom, was gathered from the sample of teachers through the use of interviews and questionnaires (See Appendix B). Using these methods for data gathering, together with observations made by a non-participant observer, a record of events that occurred at the workshops was obtained. In this way use was made of investigator triangulation, a recognised technique in action research (Cohen and Manion 1980:209).

CHAPTER 5

THE WORKSHOPS FOR IN-SERVICE TEACHERS

The sample of teachers attended workshops on using the problem solving approach on Saturday mornings.

5.1 Workshop 1

The need for and relevance of the project was established by examining the principles in the Working Document for Standards 2 to 4. The principles as stated in the syllabus were focused on. The problem solving approach is inherent in sections 1.3, 1.4, 1.6b and 1.7 of the Document (See Appendix C).

The need for the problem solving approach was also seen in the aims of the syllabus. The teachers' attention was drawn to the fact that aspects of problem solving appeared in every year of the syllabus from standards two to four.

The importance of mathematics in everyday life was then determined. The fact that any school day cannot be spent without using mathematics in one form or another was established. These include getting up at a certain time in order to be at school on time, paying one's bus fare and determining the correct change and knowing how long before the morning break. The importance of mathematics in the work place

was also discussed.

Piaget's four stages of development were mentioned by the researcher and the participants felt that their pupils fell into the concrete operational and formal operational stages. In discussing how concepts are introduced in the junior primary stage Bruner's three modes of representing the world were seen to fit in exactly with what was being done. There was the transition from working with physical quantities to pictures and then to symbols. The participants were in agreement with Dewey's theories that learning comes from experience. Thus it is the teacher's task to expose the pupils to experiences that will enhance their learning.

The teachers then had to determine what a "problem" is. Charles & Lester's (1982:5) definition was chosen to be the starting point:

"A problem is a task for which:

1. The person confronting it wants or needs to find a solution.
2. The person has no readily available procedure for finding the solution.
3. The person must make an attempt to find a solution."

In the discussion that followed, emphasis was placed on the person really wanting to find a solution. If a problem exists and the pupil is not interested in it or its solution, it is

not a problem to that pupil. The fact that the solution is not readily available shows that the pupil cannot use any of the previously learnt algorithms immediately. The immediate use of an algorithm makes it no longer a problem but an "exercise" like those found in most textbooks. To get all the answers in an exercise correct there is not necessarily a need to understand what is being done. The pupil who can imitate the steps done by the teacher can achieve this. If the pupil is to make an attempt to find the solution, such a solution must be within her/his grasp. The child must also be made to understand that if s/he cannot see the answer immediately, it does not mean that s/he will never be able to do so. This situation is supported by defining problem solving, as stated in chapter 1, "as the process of applying mathematical knowledge and skills to new and unfamiliar situations".

In the wider context of the word "problem", the following are the types of problems (Charles & Lester 1982) that a teacher is likely to encounter in a mathematics class.

5.1.1 Drill Exercises

456

x 34

Such exercises provide pupils with practice. They are used after a section is taught. They are not really problems if the objective is to develop mastery, but if pupils do not know how

to do them, then they can be classified as problems.

5.1.2 Simple Translational Problems

Tim has 6 marbles and Sipho has 8. How many more marbles does Sipho have than Tim?

Simple translational problems deal with events from the child's world. They link the real world with the mathematics classroom. They are used to reinforce students' understanding of mathematical concepts.

5.1.3 Complex Translational Problems

Aero chocolates come in packs of 4. A carton has 36 packs. Mr Gumede, the shopkeeper, bought 720 chocolates. How many cartons did he order?

Complex translational problems differ from simple translational problems in that they may involve more than one operation.

5.1.4 Process Problems

A table-tennis club had a tournament. 11 of its members took part. If every member played one game against each other member how many games were played?

The solution to a process problem requires the solver to think

about the problem and to work out his own plan to solve it. There is no set method of finding the solution.

5.1.5 Applied Problems

How many examination booklets will be needed by your school for the final examination?

These problems are linked directly to the real world and the solver must find his solution in the real world. With such problems pupils have to make use of a variety of mathematical processes such as comparing, analysing, classifying and estimating. Pupils are made aware of the value and usefulness of mathematics in everyday problem situations.

5.1.6 Puzzle Problems

Draw 4 straight line segments to pass through all 9 dots in the figure below. Each line must be connected to an end point of at least one other line segment (Charles & Lester 1982:7).



Puzzle problems do not necessarily involve mathematics. They allow for flexibility in their solution. The pupil has to extend his boundaries of mathematical thinking to obtain a solution.

The above description of the types of problems, together with the examples, were used as the basis for discussion. It was determined that only drill and practice and simple translational problems are being used in class. The teachers said that one reason for not using complex translational problems was the complexity of the language and the difficulties encountered in "making the pupils understand" the problem. A solution to this difficulty was found in class by taking the problem from the abstract to the concrete level. Multi-link cubes were used to represent the chocolates in the problem, plastic packets were used to show packs of four, whilst large shopping packets were used to represent the cartons. The mathematical operations were then written down as each step was worked through.

The teachers could not visualise the process problem. To determine the solution, the event was acted out while three members of the group made a recording of the number of games. The group then worked out how to find the total number of games from their recorded values. The participants were asked: "What would you do if it were not possible to act out the situation?" After first saying that a solution could not be reached, it was suggested by the group that a sketch could be

used.

Discussion then revolved around the applied problem and the method chosen involved making a table of the needs of every class. A note was made of the 4 different methods that were used in the 4 types of problems.

It was decided by the group that puzzle problems should be tackled only after they were familiar with process problems. They also felt that the day's activities would not be taken to class at that stage. They wanted more exposure to the problem solving approach to feel confident enough to try them out in class.

5.2 Workshop 2

As a result of the questionnaire on teaching methods (Appendix B) the researcher discovered the following regarding what currently exists in the classrooms of the members of the group. The sample also felt that although this pertained to their classes it could apply to other Black schools in their areas.

The number of pupils ranges from 50 to 80.

The medium of instruction is English and Zulu.

Everyone felt that mathematics should be taught in English.

This was because :

* they felt that English is used more widely out of school

- * mathematical terms are not clear or precise in Zulu
 - * they believed that English was better for competing with others when they left school
 - * English is used for content subjects like History and Geography, and if it is used across the curriculum the child would cope more easily
- # Group work is hardly used in the normal mathematics classroom. If it is used at all it is done as a separate section once a week.
- # Concrete objects are not used very often.
- # All participants teach mathematics using rules.
- # Pupils do not answer/participate easily. Reasons given for this were:
- * they don't understand what is being asked
 - * they don't understand what is being taught because of teaching by rules
 - * they are not fluent with mathematical terms.

The fact that many teachers teach using rules is endorsed by Kline (1973:5) and illustrated by the following example:

To add $\frac{5}{4}$ and $\frac{2}{3}$ we write $\frac{5}{4} + \frac{2}{3}$

Students are then told to find the least common denominator i.e. smallest number into which 4 and 3 divide exactly. This is 12. Now divide 4 into 12 to get 3 and then multiply the numerator (5) by 3. Similarly divide 3 into 12 to get 4 and multiply the numerator 2 of the second fraction by 4. We now

have:

$$\frac{15}{12} + \frac{8}{12}$$

"One now sees easily that the sum is $\frac{23}{12}$ ".

The traditional curriculum does not pay much attention to understanding but relies on drill. To apply the above steps to

$$\frac{7}{x + 6} + \frac{5}{2x - 1}$$

a student will have to memorise each step which will now be more complicated. Students imitate the teacher and the textbooks. Learning takes the form of memorisation. The various processes are therefore disconnected and in the students' view, topics are unrelated.

The traditional curriculum has become too archaic and thus it lacks elements that will motivate the learner. The beauty of mathematics is not seen when one is struggling to master the subject (Kline 1973). There has been a failure to give meaning to mathematics. South Africa has a society with diverse cultural backgrounds. Traditionally, textbooks have been written by a small group of people (first world) who have used examples from their backgrounds and have assumed that this could be projected into any environment (even third world). The problems in these textbooks do not appeal to the child. This is highlighted by, for example, a problem of how long it would take to fill a swimming pool at a given rate, a pool being a luxury which most children do not have. Textbooks are also inadequate in that reasons for steps are not given or

are too brief to make sense. If the teacher adheres rigidly to the textbook and does not expand the ideas for the learner, the child generally resorts to memorisation and imitation.

Mathematics should not be seen as a series of logical deductions. Kline(1973:48) quotes Henri Lebesgne as saying:

"No discovery has been made in mathematics, or anywhere else for that matter, by an effort of deductive logic; it results from the work of a creative imagination which builds what seems to be truth, guided sometimes by analogies, sometimes by an esthetic ideal, but which does not hold at all on solid logical bases. Once a discovery is made logic intervenes to act as a control; it is logic that ultimately decides whether the discovery is true or illusory; its role, therefore, though considerable, is only secondary."

This is in keeping with the problem solving approach where one allows the children to use their imaginations to find a solution to a problem. There does not have to be any definite order in how they go about finding their solutions. They should then be able to verbalise their methods and determine the feasibility. (Does it always work?)

The participants said that they found word problems a bugbear because the children did not want to read too many words. It was agreed that before the teacher can commence with the

problem solving approach, a positive attitude in the learner had to be developed. These two points, together with the need to focus on language which was established by the questionnaire, resulted in the researcher highlighting some of the readiness experiences (Charles & Lester 1982) that could be carried out by teachers in class. These were tried by the participants themselves in this session:

- Tell the children to close their eyes. Then read a story problem and tell them to imagine clearly what is described.
- Give each group a picture and have the members of that group make up a story around the picture which involves addition or subtraction. Let one person from the group relate this to the class.
- Give an addition and subtraction sentence (separately) and have the pupils make up a story from the sentence.
- Give a story problem and have the pupils write a similar problem by changing the setting slightly such as changing the objects in the problem or changing the quantities.
- Make a list of places where the children encounter numbers, for example the grocer's shop, and discuss what the numbers are used for.
- Let the children tell a story and draw a picture telling a story.

Using the above experiences the pupils should be able to visualise mentally the components of the problem. It was noted that all the above could not be achieved in one lesson but that they should be developed progressively. After the

children have made up their own mathematical problems they will not be intimidated by similar problems in their textbooks.

Although Charles & Lester (1982) place readiness experiences as being appropriate for grades 1 and 2 in the British context and this would be equivalent to our substandard A and substandard B, these were to be tried with the senior primary pupils. The justification for this comes from Hughes (1987:32) where he shows that there is a social-class difference in children's mathematical abilities.

The teachers were to try the readiness experiences in their classes and report back at the following session.

5.3 Workshop 3

In this session the teachers reported that they tried the readiness experiences and found that their pupils responded positively. The pupils were keen to make up stories involving mathematics from pictures that the teachers provided and many of the children were motivated to bring pictures of their own. They reported that more children participated in the lesson than was usually the case, by telling stories and then drawing pictures that told the story. This was the first step in getting children to talk mathematics. The readiness experiences thus began to establish a positive attitude in the mathematics classroom and the children were able to visualise

the components of problems.

In creating an atmosphere in class that is conducive to problem solving there are three types of experiences to consider (Charles and Lester 1982:16). The readiness experience has been dealt with. The other two are:

- activities for exploring problem solving strategies and
- solving various problems and discussing their solutions.

Strategies used for solving problems can be divided into general and helping strategies. General strategies are related to the overall attack of the problem, whereas helping strategies are specific ways of handling the particular problem. Charles and Lester (1982) tabulate the strategies as follows:

Table 1

<u>GENERAL STRATEGIES</u>	<u>HELPING STRATEGIES</u>
Look for a pattern; generalise	Re-read the problem
Use deduction (or induction)	Look for key words and phrases
Work backwards	Write down important information
Guess and check	Make a list or table
Solve a similar problem first	Use a picture, object or graph
Write an equation	Use simpler numbers

The group recognised that they had already used some of these. When using these strategies the teacher must bear in mind that he/she will have to sequence the problem solving activities by moving from easy to difficult problems. Participants were advised to evaluate the difficulty of the problem given in

terms of the students' performance by asking such questions as - was it too difficult / was it too easy? To start off with, it is easier to integrate the problem solving programme with sections of the syllabus that the children are already familiar with or have already completed. Although this may seem like teaching for problem solving, it should only be used at the early stages to develop confidence in the pupil and the teacher. The teacher can then move on to 'teaching via problem solving'.

Polya's four steps for problem solving were noted and related to the Working Document:

*Understanding the problem - this is illustrated by the example for 5.2.1(d) of the syllabus (Appendix D) where the concentration is on comprehension and not the solution of the problem.

*Devising a plan - refers to the strategies for solving problems which are specified in the syllabus, such as using concrete objects, acting it out, drawing a sketch, making a list, substituting smaller numbers for those given, trial and error, eliminating possibilities, making a table, working backwards and looking for a pattern. Most of these strategies are accompanied by an example to illustrate how they can be applied.

*Carrying out the plan - the method chosen to solve the problem - is referred to in the syllabus as recording the solution.

*Looking back - refers to evaluating the solution in

terms of the estimation, whether it makes sense or by substitution.

In the discussion based on the examples given in the syllabus the teachers were able to identify the strategies that could be used. Participants were provided with the following table (Charles & Lester 1982):

Table 2 - Problem Solving Stages and Sample Activities

Stage	Activity	Example
1. Problem comprehension and goal analysis	Given a story, make up a question that can be answered.	"Make up a question for this story. Then find the answer."
2. Plan development	Given a problem without numbers, tell how to solve it.	"Joe has ___ plants and plans to put them in ___ rows. How many should he plant in each row?"
3. Plan implementation	Given a table, make up a problem that can be solved using a table.	"Make up a story problem that can be answered using a table."
4. Solution evaluation	Given a problem and its answer, use estimation to determine the reasonableness of the answer.	"Estimate to decide which answer is reasonable. Ned's Ice Cream Shop sold an average of 37 milkshakes a day for 21 days last month. How many milkshakes did they sell for the month?" A.58 B.777 C.677

The teachers used the activities that were discussed in this workshop and were able to implement the first three stages of the table without any difficulty. They found solution evaluation to be more difficult to implement since they had not considered studying the reasonableness of their answer in the past. They saw the value of such an exercise in eliminating carelessness in calculation, and felt that they

saw the need to include estimation in all aspects of their work.

5.4 Workshop 4

During this session actual problems to be solved were analyzed. To start off with the problem called "Put it on the Table Please" was used. The group was provided with an addition table as shown below:

	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

They were also given a piece of tissue paper to draw a rectangle like the one in the table. In the given rectangle the group was asked to determine which the corner numbers were and which the inside numbers were. They then did the following:

Work out the sum of the corner numbers $5 + 8 + 7 + 10$

Work out the sum of the inside numbers $7 + 8$

Now move the rectangle to another place on the table and find

the two sums. Write them down. Look at the sums of the people on either side of you. What do you notice? Fill in yours and your neighbours' answers in a table like this:

top left-hand number	sum of corner numbers	sum of inside numbers
5	30	15

The table obtained looked similar to this:

6	34	17
10	50	25
7	38	19
11	54	27

In reply to the question "What do you notice?" the participants noted that the sum of the corner numbers were double the sum of the inside numbers. They were also able to predict the sum of the corner number given the sum of the inside numbers. When they were asked to predict the sum of the inside numbers from the number on the top left hand corner they could not do this immediately. The researcher re-wrote the table on the board since this was a whole class attempt at arriving at the solution. They were guided by questions and comments like: "Which sets of numbers are you working with? Write these down. Examine each set of numbers."

Thus the new table looked like this-

top left-hand number	sum of inside numbers
5	15
6	17

10	25
7	19
11	27

The first guess ventured was "top left-hand times three" from the first set of numbers. When this guess was tested for the next set it was found to be false. It was also noted that the guess was too large for all the other specific cases. This led the participants to look at how the inside numbers differed from "top left-hand times two". This column was placed in between the other two columns to get:

top left-hand number	x2	sum of inside number
5	10	15
6	12	17
10	20	25
7	14	19
11	22	27

By studying this new table the relationship between the middle and last column was found. Below is a transcript of what occurred.

Researcher: Is there a relationship between the columns?

Teacher 1: The last column is the middle column + 5.

Researcher: But what is the middle column?

Teacher 2: It is the top left-hand number x 2.

Researcher: So if the top left-hand number is 8, What will the sum of the inside numbers be?

Teacher 3 : $8 \times 2 + 5$.

Researcher: Let us call the top left-hand number 't'. If you know 't' then what will the inside sum will be?

There was much teacher-teacher discussion, after which, a group ventured to answer "It must be 2 times 't' plus 5". In response to the question "How do you know that this is correct?" , the group tested their prediction for other values of 't' and accepted that it was always true. The researcher did not insist on a sound mathematical explanation at this stage because she did not want the participants to lose the growing enthusiasm that they seemed to have.

Using the same addition table as above, any square of four numbers was chosen. To avoid any confusion an example was given.

3---4

4---5

Add the left hand numbers $3 + 4 = 7$

Add the right-hand numbers $4 + 5 = 9$

Make up a table like this:

Left-hand total

Right-hand total

7

9

Other squares of 4 numbers were used. "Fill in the table for them. Can you see a pattern? Why do you think it happens?"

The participants were able to see and verbalise the pattern

that they saw, but at first were unable to say why this was so. The researcher had to ask the following directed questions before the reasons for the pattern were clearly understood. "Look at the first row. How are the numbers different from each other? What about the second row? Link this to your table."

All the teachers used the addition table in their classes. Those who used the rectangular block over their table had their pupils make the table with 3 columns. A noticeable feature with this was that the teachers only compared the sum of the inside numbers and sum of the corner numbers. They did not draw their pupils' attention to the relationship between the top left-hand number and the sum of the inside numbers. Could it be that they were not confident enough to abstract the concepts to involve symbols?

Some teachers worked with the second aspect of adding left-hand and right-hand numbers and comparing them. They reported that their pupils had no problem doing this.

5.5 Workshop 5

On this day all the Senior Primary Teachers' Diploma students from Umlazi were present for lectures. The following problems were experienced: (a) Students not involved in the project wanted to be included; (b) There was no classroom available to conduct the workshop.

The group met in the open and it was decided to cancel the session. An informal discussion was held. The participants had started to use the problem solving tasks in class and found that their pupils had experienced difficulty with the language. It was decided to focus on ways of overcoming this problem in the next session.

5.6 Workshop 6

As a means of getting pupils to talk, group work was highlighted in this session. Most teachers ask questions with a specific answer in mind. If the first pupil does not give the expected answer, the teacher then goes to the next pupil. In the meanwhile pupils are trying to guess what the teacher's answer is.

In large classes the teacher cannot listen to all the possible answers. The following aspects of group work were considered. If a teacher is able to understand how a group is thinking about a problem, he will be able to ascertain more in a short space of time, than if he were to ask each pupil individually.

As already mentioned in 3.4.6 group work can be used as a means of overcoming some of the problems that exist. It can reduce the anxiety of problem solving where pupils can share ideas. This may lead to discussion among the pupils. The language that the pupils may use is of no consequence as long as they are able to express their opinion to others in the

group. Both co-operation and conflict within the group will help to develop problem solving techniques. Working in groups will allow the teacher to spread his time evenly around the class. The disadvantage of working in groups is that the more dominant pupils may control the group while the less dominant remain uninvolved. It is the teacher's task to move around the class during group work and to monitor the progress of the groups, to question those pupils not involved and to ensure that everyone in the group understands the group's solution.

The teacher's role at each of Polya's four stages was developed. In terms of understanding, either the teacher can ask questions, the group can formulate questions or the children can retell the story in their own words. When devising a plan the particular problem can be linked to related problems or strategies used before. Let the children suggest and share strategies. In carrying out the plan, encourage children to solve the problem on their own. If a plan does not work, encourage them to try an alternative. If they get stuck the teacher should check their understanding of the problem. Once the problem has been solved get the children to describe the strategy used in solving the problem.

The next two activities were done in groups to show how group work can be achieved.

The paper folding and cutting activity was recommended to be done with the pupils. Thin sheets of paper were to be used. A

first circle. They were shown that:

$$2 = 2^1$$

$$2 \times 2 = 2^2$$

$$2 \times 2 \times 2 = 2^3$$

These are called powers. Thus it follows that for no folds we have 2^0 and any base raised to the zero power is 1.

The solution to the following problem was attempted:

There are some rabbits and some hutches. If seven rabbits are put in each hutch, one rabbit is left over. If nine rabbits are put in each hutch, one hutch is left empty. Can you find how many rabbit hutches and how many rabbits there are?

By way of setting the scene to the problem the researcher spoke about rabbits in general. Their habits and habitat were discussed. The problem was given to the group to be read on their own. The problem was not clearly understood. The researcher then read it out to the class and provided concrete objects to be used. Rubber bands were used for hutches and counters represented rabbits. It was determined that there were two cases. Each case was dealt with separately and then combined because we were working with the same number of rabbits. Participants were divided into groups.

The researcher assisted by saying: "Imagine that you had one hutch, how many rabbits could you have to meet the first condition? Will this also meet the second condition?"

From this a table was drawn as follows:

number of hutches	1	2	3	4	5	6	7
rabbits for 1st condition	8	15	22	29	36	43	50
rabbits for 2nd condition	0	9	18	27	36	45	54

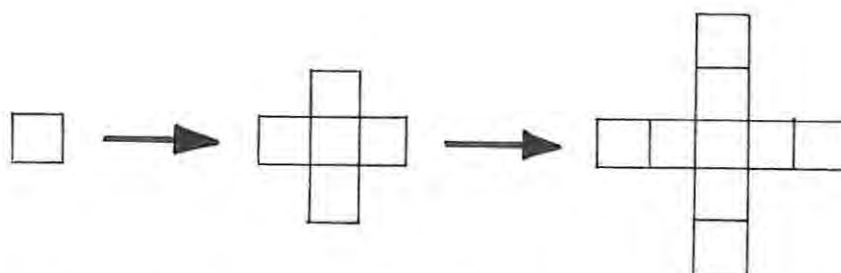
Much discussion revolved around meeting the necessary conditions. Some members of the groups had to continually remind others of the second condition. Although the groups finally arrived at the correct solution, as teachers, they did not feel confident enough to take such a problem to the classroom. The researcher decided that a problem involving similar principles as in the first condition of $7h + 1$ would be done at the next session to develop further understanding.

The participants tried the "Paper folding and cutting activity" with their own pupils in class. In the feedback provided by the teachers the researcher found that there was too much help from the teacher. Some teachers demonstrated the whole activity before allowing the pupils to try it. Comments like "They managed to guess the number of circles after I explained and showed them the pattern" shows that the teachers still want to be the person who can provide the child with the right answers. One teacher said that although she went up to six folds the class could not predict the answer for seven folds. The children here were used to being given a rule to use. It was only after one pupil said, "the pattern is multiplying" that they were able to give "answers for numbers we did not reach".

If pupils develop the ability of discovering patterns for themselves and then of finding out why the patterns are always true, they will be able to extend their knowledge beyond the limited scope of the syllabus. Using this "paper-folding" problem the children will be exposed to exponents earlier than was previously possible. This is in keeping with Bruner's theory (Resnick & Ford 1984:113) that the beginnings of a complex concept may be presented to the child at an early age.

5.7 Workshop 7

The following investigation was done by providing the class with illustrations and wooden blocks. Three groups were formed.



The groups were asked to make the 4th model and then do the following:

Express in words what you would do to make the next shape in the series.

How many blocks would you need to make the 10th model

the 47th model

the 100th model?

Thinking of any model in the series you like, how many blocks would you need to put out to make the model that

you are thinking of?

There was much discussion before a decision about the 10th model could be reached. One group looked at the pattern formed by the numbers:

```
1-----5
2-----9
3-----13
|         |
|         |
10-----41
```

and decided that the 100th model will have 41×10 blocks since $100 = 10 \times 10$. A member of the group was not happy because she felt that all the "answers should be odd". The fact that a member of the group could see a flaw in the argument and convince the rest that they were wrong, was a sign that critical thinking was in progress. The researcher then suggested that they describe the pattern of the blocks instead of looking at the numbers. "Describe to someone else how to make the second model."

From "one in the middle and 2 on each side," they continued with the 3rd, 4th and 5th models to reach the conclusion that you have one plus four times the number of the model that you want, that is, $1 + 4n$ or $4n + 1$. They were also able to explain why this was true.

After having done this the teachers were more comfortable in

trying to explain the rabbits and hutches problem. Thus to determine how many hutches there were they used numbers where the number of rabbits had to be seven times the hutches plus one.

hutches	1	2	3	4	5	6	7	8
rabbits	8	15	22	29	36	43	50	57

What had to be made clear was that the number of rabbits should be the same even though the arrangement had changed. They had to use the second condition as well. Since one hutch had to be left empty, if there was only one hutch then there would be no rabbits, but if there were 2 hutches there would be 9 rabbits.

hutches	1	2	3	4	5
rabbits	0	9	18	27	36

As soon as they obtained the number which also appeared in the table above it was tested to see if both conditions were satisfied. Although the teachers said that they understood the solution to the problem, they felt that it was too complex for their pupils and decided not to try it out in class.

5.8 Workshop 8

The investigation in the previous session using blocks was used by some teachers in class. They found that the children were unable to explain to their group in English, what had to be done, but they could explain in Zulu how to arrange the

blocks. The main idea, that the given number had to be multiplied by four and then that one had to be added to this product, was established. So in whatever language the explanation was done, the symbols finally used looked the same: $(4 \times n) + 1$. What was most important here, was that the children understood what was being done. Since understanding should be the mathematics teachers' main concern, the language used to reach this understanding is not of prime importance in this particular case.

To introduce the new problem for the session the researcher spoke about novelty items at school sports meetings. The group had no experience of these since their sports meetings did not include novelty items. The researcher explained the reasons for such items and it was accepted by the group that this was a good idea. The problem was called " A Greasy Pole".

At a sports meeting, a greasy pole climbing competition is held.

The greasy pole is 10 metres tall. One competitor climbs 5 metres in half a minute but then has to rest for half a minute and slips back 4 metres.

- a) Can you record the moves up and down in the form of a diagram?
- b) How many attempts does he have to make to reach the top?
- c) At this rate how long will it take him to reach the top of the pole?
- d) What is the total distance that he will have climbed

altogether by the time he reaches the top?

- e) How many metres will he have covered altogether by the time he reaches the top?

A member of the group felt that this problem was not suitable because he believed that the situation was not possible. He said that the competitor's hands would be too greasy after the first minute to continue. It pleased the researcher to see that the members of the group were now becoming critical of the problems and relating them to real-life. Other members of the group suggested to this participant that ways of overcoming this would have been reached before the event e.g. rubbing one's hand in sand at the beginning or rubbing the hands one at a time on the back of the pants. This problem was resolved in groups. There was confusion over (d) and (e). The groups felt that they were the same. A whole class discussion arose to explain the difference between "covered" and "climbed".

When this problem was done in class the participants also encountered problems with the pupils not understanding the language. This had to be explained by the teachers, after which the pupils could solve the problem.

The next two problems were used to show that there can be more than one correct answer. This will prevent the situation of the teacher always having one answer in mind and the children trying to guess that answer. It also makes the children

justify their answers by showing the others that they are true.

- a) There are five numbers. Two are equal. Three other numbers are also equal. The sum of the numbers is 72. What are the numbers?
- b) We have four numbers. All are different. One number has the same digits as the other but the digits are reversed. Two numbers have zeros in the place of the units' digit. The sum of the four numbers is greater than 80 and less than 100. What are the numbers?

The teachers reported that their pupils were fascinated by the fact that two or more answers could be correct yet different. The idea of being able to justify one's answer was established. The pupils were then prepared to venture an answer. The beginnings of the concept of conjecture in mathematics was developing, where conjecture is seen as "I am willing to guess an answer but I may change my answer when I have heard other pupils' reasoning for their answer". The underlying idea here is that pupils will learn to listen to each other.

5.9 Workshop 9

During this session the findings of the participants and the researcher were discussed to determine the best possible ways of implementing a problem solving approach in terms of integrating it with the present syllabus. These findings will be discussed in the next chapter.

CHAPTER 6

FINDINGS

Some of the findings of this research can be found at the points of their discovery in the chapter on Workshops. This was necessary because the nature of action research requires the implementation of the research to be adjusted to suit the observations noted.

In this chapter the researcher will attempt to consolidate the findings in terms of issues that have arisen out of the research.

6.1 The Interactive Learning Process

A definite change from the traditional learning situation was that the teacher no longer treated the child as an empty vessel to be filled (Shuard 1986). The pupils were actively involved in discovering meaning for themselves. This was evident from the first activity carried out in class. Instead of the teacher explaining how to do word problems, the pupils used pictures to make up their own word problems.

In order to determine the extent of the changes the researcher would have had to visit the schools during and after the workshop sessions. This was not possible. Reports from

teachers indicate that the group work allowed pupils to talk to each other about the problems that they were dealing with. The teachers no longer showed the pupils how to find the solutions. They allowed them to discover the patterns for themselves. The socio-constructivists approach (Yackel 1990) was brought into the foreground. The teacher supplied his pupils with teaching aids which they could use, but the decision to use them was left to the pupils. The thinking process was focused upon rather than the importance of a correct answer. Mathematics is effectively learned by experimenting, questioning, reflecting, discovering, inventing and discussing (Ahmed 1987). The non-participating observer found that the teachers were questioning each others reasoning when trying to solve a problem.

By experimenting with the blocks in Workshop 7, teachers were keen to discover the patterns for themselves. They also reported that their pupils could explain how the patterns occurred. The teacher has a major role to play in developing an atmosphere in class that is conducive to interactive learning. This interactivity occurred between the participant (learner) and the material, the participants themselves (learner and learner), and the participant (learner) and the researcher/teacher.

6.2 The Role of the Teacher and Teaching Styles

The teachers in this study showed that initially they had

difficulties with allowing their pupils to experiment. This was noted with the problems on paper-folding, the greasy pole and the blocks. They wanted to tell the pupils what to do. They wanted to help them in their thinking. Teachers will have to start accepting the fact that they do not know the answers to everything. Ahmed (1987) says that teachers "pre-empt pupils' decisions" because they want to lead them on the correct path and this is how the teachers worked at the beginning of the study. As they continued, they consciously started to hold back and gave their pupils a chance to grapple with the problem and come up with their own answers, according to feedback provided by the teachers.

By telling pupils how to do problems, they do not allow them to experiment with their own ideas. The teachers' role should be a supportive one, where the pupil is allowed to express his own ideas. Letting a child develop his own rules (Yackel 1990, Oliver 1990, Clements & Battista 1990) is a more fruitful learning situation than telling them what to do.

The teachers participating in the workshops preferred less challenging problems, e.g. when using the addition table from Workshop 4 they only compared the second and third columns since it could easily be seen that the one value was double the other value. The first and third columns required the learner to see the relationship, $2t + 5$. This was ignored when the problem was worked through with pupils.

These teachers have to be nurtured out of the situation where they only apply the rules that they know. Smith (1990) shows the flaws of allowing children to apply rules without understanding. It would appear that critical thinking must first be developed in the teacher so that it can be fostered in the child.

When the teachers were not totally confident about a problem, they did not take it to their classes. Watson (1971) comments on the fact that teachers who had not experienced success in mathematics had problems teaching the subject. When a challenging problem was proposed, they said: "the children won't manage' . Those teachers who were "brave" enough to try these problems with their pupils reported that they were surprised by how well their pupils coped. A problem given to the teachers without any previous discussion involved the prediction of the number of matchsticks required to continue the pattern:



BLOCKS

MATCHES

1

4

2

7

3

10

|

|

|

|

The teachers reported that their pupils gave their opinions.

They were also predicting answers and did not mind testing their conjectures.

In order for teachers to change there has to be a change in their convictions (Ahmed 1987). They should believe in changing their role from information providers to facilitators. The participants began to see their role as teachers differently by the end of the study. This change was also noted by the non-participant observer (Mkhwanazi) in his report, when he said: "The teachers said that they are no longer in a hurry to provide the pupils with the correct answers."

Arising from the activities and the discussions with the teachers in workshop sessions, it is felt that they need to consider the following as ingredients for a rich mathematical activity (Ahmed 1987,20):

- * It must be accessible to everyone at the start.
- * It needs to allow further challenges and be extendable.
- * It should involve children in speculating, hypothesis making and testing, providing or explaining, reflecting, interpreting.
- * It should not restrict pupils from searching in other directions.
- * It should promote discussion and communication.
- * It should encourage originality/invention.
- * It should encourage "what if" and "what if not" questions.
- * It should have an element of surprise.

* It should be enjoyable.

In the researcher's opinion, the above ingredients were attained to a limited degree. This was limited because the workshops were held over a short period. It is hoped that if this method is used continuously in the classroom, all the above ingredients could be eventually achieved.

6.3 The Use of Discussion

Children have to be trained to listen to each other and it should be the teacher's task to encourage this. Until recently it was believed that the teacher had all the answers so pupils listened only to the teacher. Teachers should show the pupils that by their listening to other pupils they may find that someone may have an idea that could help them to understand a situation better.

One of the main reasons for the teachers feeling that the children were developing a positive attitude to mathematics was that they found that pupils participated in the lessons. One participant commented that "pupils were also allowed to talk and share ideas among themselves. This was not done before in class". Another participant found that "every pupil has something to say during the lesson". By talking they were able to share ideas with each other. The idea of co-operative learning was that after working in a group, individual pupils felt confident enough to work on their own. As discussed under

the interactive learning process, the teachers found that the pupils did talk to each other and that real learning occurred. Before the pupils wrote their word problems, they verbalised the stories around the pictures. The teachers reported that pupils had something to offer in this session. It would seem that talking to the teacher and to each other allowed the pupils to gain understanding for themselves. It was found that after working in a group, individual pupils felt confident enough to work on their own.

The questionnaire (Appendix B), revealed that teachers have classes of 60 or more pupils. Group discussion gave the teachers a way of getting most of the children to talk in their groups. The importance of language, as stated by Vygotsky, Skemp and Murray (Chapter 2), makes it imperative that discussion takes place. The actual language used in discussion is not important as the purpose of discussion is to bring about understanding. Thus, when the teachers used Zulu to explain to each other in their groups, this was acceptable. The teachers said that their pupils also used Zulu in their group discussions because they were not confident enough with the English language to use it at all times.

6.4 Time

The teachers felt that the problem solving approach was time-consuming. Their pupils had to spend time pondering and exploring different ways of solving and determining the

validity of the solution. A suggestion from the group, which would take this into account, was that mathematics should be allocated double periods on the time-table instead of single 30-minute periods that presently exist.

In the workshops the teachers found that initially the problem solving approach was seen to take much time, but as the teachers gained practice less time was needed in developing problem solving strategies.

If the child, through the problem solving approach, is able to develop a rule when being taught a new topic, there is less chance of his forgetting it. On the other hand, a topic that is taught quickly by the teacher and subsequently forgotten by the pupil in a week may have to be re-taught, thus requiring more time. It can be argued that the problem solving approach does not really take more time.

Reys, Snyman & Lindquist (1984:24) say that additional time is gained "by organizing interactional activities so that some of the time allocated for practising computational skills is directed towards problem solving". Since the workshops were held on Saturday mornings, time was not a constraint whereas this was the case in the classroom. The teachers felt that the continuity required to solve a problem, was lost when they had to use only one period a day.

6.5 The Syllabus

In their initial training, it would appear that the teachers were not exposed to the psychology of mathematics teaching. Interviews with teachers revealed that they were not aware of the stages of development in a child. Their point of departure was "what had to be covered in the syllabus". There is a need to make the teacher aware of the level that the child is at and to start working from that point. There also has to be some form of diagnostic testing before new work is introduced. Carefully planned problem solving activities can be integrated into the syllabus. The two problems below are examples of these.

- (1) The number 12 has 6 factors: 1; 2; 3; 4; 6; 12. What other numbers have 6 factors? Which numbers have 2; 3; 4; 5 factors? Investigate.

- (2) Your local shopkeeper wants to supply your school with oranges. Each bag has 45 oranges. How many bags of oranges should you ask for, if each child gets one orange and there are 722 pupils in the school. How many bags would you need to give each child 2 oranges. (Note that pupils would not have done long division yet.)

As a new syllabus, the Working Document emphasises new aims in the teaching of mathematics. Learning is child-centred. The mathematical content is the same as that of the old syllabus,

but the emphasis is now on new teaching styles. The rationale of socio-constructivism is suggested to implement these teaching styles. A problem solving approach can be used throughout the syllabus.

6.6 Assessment

A direct consequence of implementing the syllabus would be some form of evaluation of this implementation.

To improve the pupils' problem solving ability, a regular evaluation of their progress is necessary. Charles, Lester & O'Daffer (1987) give the following points on evaluation and problem solving:

- * Evaluation is not synonymous with grading.
- * We should evaluate thinking processes as well as the correct answer.
- * We should always attempt to observe and question students while they solve problems.
- * We should match our assessment plan to our instructional emphases.
- * We should try to interview students on occasion.
- * Every student does not have to be evaluated in every problem solving experience.
- * Students should be informed of the teacher's evaluations plan.

As noted by the non-participant observer, the teachers at the

workshops progressed from passive listeners at the first two sessions to active participants. When a teacher did not understand an aspect of a problem, the other participants offered him/her their ideas on the problem.

While the form of assessment advocated by Charles, Lester & O'Daffer (1987) would suit a problem solving approach, the reality of the situation is that inspectors require a different form of assessment from the teacher. They want to see work done in the pupils' books. Exercises must be marked and the child's progress is determined by the number of sums he gets right. School inspectors have to be alerted to the move away from traditional teaching methods towards the problem solving approach.

A participant who has used this method in her everyday teaching noted that the pupils' marks in the quarterly test had improved. Such an improvement can be attributed to a number of possible causes, but, it is also in keeping with James' belief (1990) that by using the problem solving approach "...children's achievement should rise dramatically..."

An obvious development in the workshops was the change in the belief that every problem has one right answer. The teachers began to look for more than one way of solving a problem. There was a great deal of talking and critical questioning in the workshops. An example of this was that when their answers

differed, they wanted to test each other's answers. The teachers were prepared to explain how they arrived at their solutions. With this attitude in the classroom, it is hoped that the child will be guided to explore mathematics on his own.

6.7 Attitude

The teachers' initial responses (Appendix B) implied that their pupils generally did not enjoy mathematics. These pupils' dislike for word problems is endorsed by Ford (1990). Internalising word problems was achieved by encouraging pupils to create their own problems from pictures. Teachers reported the first signs of enthusiasm in their pupils when they created their own problems. Without being asked, the pupils brought their own pictures the next day to do similar problems. The next stage was to allow pupils to write their own problems. When this was done by Ford (1990), pupils began to solve each other's problems. The teachers reported that a similar situation had been created in their classes.

It has been stated earlier (Biggs 1985) that a prerequisite for learning mathematics was a change in the children's attitude. Such a change was noted by the non-participant observer (Mkhwanazi 1991) in his interviews with the teachers. They reported that the pupils were keen to use their own methods to solve problems. They also felt that the talking that occurred in class made the pupils relax. The classroom

was a happier place. Teachers listened to their pupils' comments and "explored the reasons for their responses".

The non-participant observer also noted a change in the attitude of the teachers. As the workshops progressed they became actively involved in the discussions that emerged. They critically analysed both theirs and the other participants' answers. The confidence that Watson (1976) found to be lacking was slowly being built up.

Enthusiasm for the subject will allow the teacher to introduce some mathematical concepts at an earlier stage in their classrooms. This was achieved with the "Paper-folding" problem involving exponents of 2. Pupils were able to predict what would happen with 6 folds, 7 folds... It was reported that the solution to the problem was arrived at by interaction with other pupils. With the present syllabus, such a problem is not attempted at the primary school level since these pupils do not encounter exponents.

6.8 The Use of English

The non-participant observer noted that the problem of using English was a major disadvantage. Most teachers felt that the use of English in the problems made it difficult for their pupils to understand them. One remark was that "The pupils of primary school level cannot express themselves in English. It would be much better if this medium of instruction started at

an earlier stage." It was generally felt by the participants (Appendix B) that English should be introduced earlier than at the standard 3 level.

Language is the basis of mathematics teaching (Murray 1990, Yager 1991, Reys 1984). In the first four years of the Black child's schooling the medium of instruction is the mother-tongue but in standard 3 the medium of instruction is changed to English. Since the teachers felt that mathematics should be taught in English, it would seem that if the child were exposed to the mathematical terms, he would encounter in English, at an earlier stage, he might cope better. One way of achieving this may be by adopting the approach of the Molteno Project.

The Molteno Project aims to support pupils to cope with the vocabulary required for standard 3. By the end of the first year the children can read and write in the mother-tongue language. The programme is developed in eight African languages. Work is based on the learner-centred model and children learn to work on their own. Rote learning and chorus answers are discouraged.

In the second year a special English course is used to develop their vocabulary. Other sections of the work are still done in the mother-tongue. This special bridging course is continued into Standard 1 and Standard 2. The children are able to read, write and talk in English. The vocabulary used in these

standards allows the child to become familiar with terms he will encounter in other subjects like Mathematics, Science and Geography.

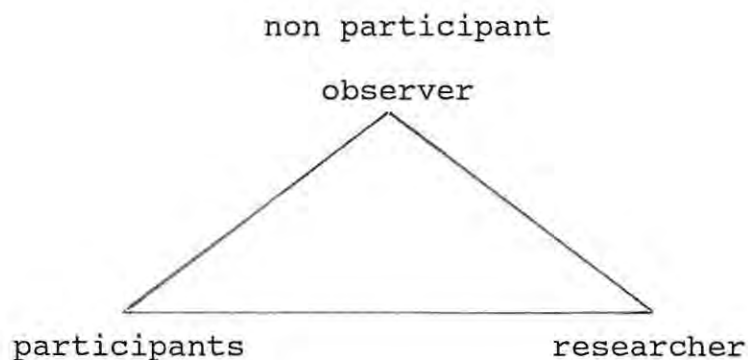
Group work is encouraged and discussion amongst pupils is prevalent. The child is actively involved at all times. Mathematics is introduced and related to real life situations by way of concepts such as fractions. Games are used to take away the stress from the pupils' learning. The trainers involved in the Molteno Project accept the importance of discussion in learning and have realized that the only way this can be achieved in the schools is if the children feel confident enough to use the language in which they are being taught. This will lead to an understanding of the concepts with which they are confronted. The underlying philosophies of the Molteno Project are similar to those of the problem solving approach.

6.9 Limitation of the Research

The sample selected consisted of competent mathematics teachers who were willing to change in order to improve their teaching. If this study were carried out with teachers who resist change there may not be as much success with the pupils.

Since the teacher-researcher believed in the use of the problem solving approach, she did everything possible to make

the project a success. Personal involvement in the success of the project is a shortcoming of action research. To provide another perspective, a non-participant observer was used in the evaluation. This resulted in triangulation, which is acceptable when doing action research (Cohen & Manion 1980).



A further limitation of action research is that it is not able to be generalised. Since this study involved the activities of a small group it is situational. It is hoped that the findings of this group may be applied to other groups and in the process a type of organic growth may occur in order to spread this new approach to the teaching of mathematics.

A problem solving approach cannot be made fully operative until the authorities such as principals, inspectors and planners accept that mathematics can be taught and learnt in this way.

CHAPTER 7

RECOMMENDATIONS

It would seem that a problem solving approach could be used to teach all aspects of the syllabus given in the Working Document. The most suitable way would be to use the constructivist's model in the learning and teaching of mathematics. In order to achieve this situation the following could be taken into account:

7.1 In-service Courses for Teachers

Such courses are necessary to allow the teacher to change his/her role to a facilitator in the learning process. The teacher should develop the ability to listen to his/her pupils. He also has to train his pupils to listen to each other. The importance of discussion needs to be emphasised. Yager (1991) speaks of in-service education involving conceptual change on the part of the teacher.

In-service courses should ensure that teachers are confident enough with the content of the subject matter, to enable them to allow pupils to explore new avenues in mathematics. During these courses, opportunities could be created to allow teachers to experience success in order to develop a positive attitude towards the subject. Teachers need to be actively

involved during the programme. Support should be given to the teachers as they introduce the new syllabus.

7.2 In-service Courses for Lecturers

Similar in-service courses as those held for teachers should be held for lecturers at colleges of education. This is necessary because, in the researcher's experience, the main criterion for choosing lecturers in KwaZulu colleges is that they have a degree. A lecturer who lectures to trainees for the primary school need not have taught in a primary school and in most cases has not taught at one. In-service courses could help such lecturers to use the problem solving approach, and when their students become teachers, it is hoped that they too would use this method with their pupils.

Some aspects given by Yager (1991) to illustrate the constructivist model, which can be emphasised during in-service programmes, are:

- # Seeking out and using student questions and ideas to guide lessons.
- # Accepting and encouraging student initiation of ideas.
- # Using student thinking, experiences and interests to drive lessons.
- # Encouraging students to test their own ideas.
- # Encouraging students to challenge each other's conceptualisations and ideas.
- # Using cooperative learning strategies.

- # Encouraging adequate time for reflecting and analysis and
- # Encouraging self-analysis and reformulation of ideas in the light of new experiences and evidence.

7.3 Bridging the gap between English and the Mother-tongue

The researcher recommends that the learner-centred techniques of the Molteno project be used to develop a path towards the confident use of English by the pupils in Standard 5. The underlying aim is that the child be totally fluent in his mother-tongue and that he develops the new language from a strong foundation. The fact that he has a good command of a language gives him the advantage to develop concepts for himself (Murray 1990).

7.4 Non-Routine Problems

In the workshops most of the problems used were of the non-routine type. The reason for this was that the researcher did not want the participants to be able to use algorithms that were known to them. If an algorithm cannot be used it is hoped that the learner will look for a new or alternate method to find a solution. This may result in non-traditional methods - of the kind suggested by Souviney in Chapter 3 - being used.

Mathematics can be found in geography, general science, health education and technical subjects in the primary school. Using non-routine problems is likely to allow the teacher to develop

problems across the curriculum. This emphasises the practical applications of mathematics.

As a result of the workshops, it is recommended that teachers encounter a problem solving approach to mathematics teaching by first using non-routine problems and then transferring the strategies of these problems to routine ones.

7.5 Assessment

It would appear that there needs to be a change in the form of assessment at school. It would seem unreasonable for the teacher to teach by using a problem solving approach and then to assess his pupils by using traditional tests. There should also be less emphasis on the quantity of written work. An alternative approach might be for the teacher to keep a record of continuous assessment throughout the year.

CONCLUSION

In spite of the limitations of this case study, the researcher was able to note a change in the attitude of the teachers and the development of a co-operative, purposeful atmosphere in the workshops. If this positive attitude can be developed and extended to the mathematics classroom, mathematics may not be the dreaded subject it is perceived to be at the moment.

It must be remembered that the use of a problem solving

approach cannot be achieved overnight. It will be a gradual process to convince teachers that their present, traditional methods are less relevant in today's society. To convince the majority of teachers, they will need to successfully experience the actual methods used as was done with the in-service teachers in the workshops. This study has shown how teachers can experience the benefits of using a problem solving approach to their teaching. It is clear that workshops for in-service teachers based on a constructivist model of learning is a possible method for exposing teachers to a problem solving approach in primary mathematics teaching. Yager (1991) says that:

"The constructivist model allows us to shed our immediate history of frustration with haphazard reforms and to move into the next century with enthusiasm, promise and excitement."

Suggestions for further research are:

1) The influence of language on problem solving.

A study can be made of the amount of discussion used in a class that is using a problem solving approach successfully as opposed to one that is not. Another aspect in this area is whether the use of English by second language speakers improves or retards their problem solving ability.

2) The effect of the use of problem solving strategies on the overall performance of the child.

The progress of a randomly chosen group of pupils can be followed to determine whether the child's ability to use problem solving strategies influences his performance in other subjects.

3) The influence of technology on the use of the problem solving approach.

Technology in the classroom has the ability to remove the drudgery of tedious calculations. This is likely to allow for the extension of problems, using complicated calculations that can be done by a computer. A study of this influence in the mathematics classroom could be made.

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APPENDIX A

STUDENT PROFILE

NAME: _____

STUDENT NUMBER: _____

ADDRESS: _____

TELEPHONE NUMBER: _____

1. For how long have you been teaching? _____
- 2a. Are you teaching mathematics at present? YES NO
- b. If yes, to which standard? _____
- c. What is the average number of pupils in your class? _____
3. What is the highest level of mathematics /arithmetic that you have passed at school? _____
4. Do you find mathematics difficult? _____
5. Do you enjoy doing mathematics? _____
6. Do your pupils enjoy doing mathematics? _____
7. Do you intend teaching mathematics in the future? _____

APPENDIX B

QUESTIONNAIRE ON TEACHING METHODS AND PUPIL PARTICIPATION

Please answer the following questions to the best of your ability. Be as honest as you can.

TEACHING METHODS

1. How many pupils are there in your class? _____
 2. At what standard do you teach mathematics? _____
 3. In which language do you teach mathematics? _____
 4. In which language do you think mathematics should be taught?
Say why. _____
-

5. Do you use group work in your teaching? _____
How often do you use group work? _____
6. Do you use concrete objects to teach concepts? How often?

7. Do you use mathematics by using rules? _____

PUPIL PARTICIPATION

Write a paragraph on pupil participation in your class. Have the following questions in mind.

Are your pupils actively involved while you teach?

When asked questions, do they answer easily?

Do you have to force them to answer?

Do pupils talk while trying to find an answer?

Are the pupils fluent with the terms used in maths?

APPENDIX C

PRINCIPLES FROM SYLLABUS FOR MATHEMATICS: STANDARD 2 - 4

- 1.1 Mathematics has an important place in primary school curriculum because it:
- (a) is an essential element of communication in the modern society;
 - (b) has a pervasive utilitarian value, making it essential for every citizen to be sufficiently numerate to cope with the everyday mathematics of number, measurement and space;
 - (c) has universal and cultural value in that it provides a broader insight into the patterns and relationships in the natural and man-made world.
 - (d) contributes to the development of logical thinking;
 - (e) has an inherent appeal which can provide pleasure and satisfaction through the solving the problems, puzzles and games; and
 - (f) provides a powerful tool for solving problems from diverse fields.
- 1.2 Since all pupils in the primary school take mathematics as a subject, the mathematics curriculum should:
- (a) have a sufficiently broad base to provide basic training for future study and careers;
 - (b) be experienced as relevant and worthwhile in itself; and
 - (c) cater for the range of pupil abilities.
- 1.3 The pupil learning mathematics is conceptualised as an active mathematical thinker who tries to construct meaning of what he is doing on the basis of personal experience and who is developing his way of thinking as his experience broadens, always building on the knowledge which he has already constructed.
- 1.4 In implementing the Mathematics Curriculum, due attention should be given, not only to the provision of mathematical knowledge, skills and concept in the planned curriculum, but also to the mathematical processes by means of which pupils are actively and productively involved in learning, e.g. comparing, classifying, describing, representing, pattern searching, inferring, analysing, proving and problem solving.
- 1.5 The mathematics curriculum for the primary school assumes universal access to calculators by all pupils at all levels. The calculator should be integrated into the curriculum as an important aid to be used in learning and doing mathematics.

- 1.6 With the availability of the calculator as a computational device the objectives for teaching pencil-and-paper algorithms in a calculator-integrated curriculum are:
- (a) to facilitate an understanding of number and the properties of operations, where the emphasis is unequivocally on understanding the procedures involved;
 - (b) to develop algorithmic thinking as an objective in its own right. Algorithmic thinking includes executing given algorithms evaluating the efficiency of different algorithms and designing algorithms.
- 1.7 Problem solving should be the central focus of teaching and learning mathematics. Not only is the ability to solve problems a major reason for studying mathematics, but problem solving provides a context for learning and doing mathematics.
- 1.8 Successful mathematics teaching should embrace a wide variety of styles and approaches which should include opportunities for:
- (a) direct teaching of individuals, groups and whole classes;
 - (b) activity-based learning;
 - (c) discussion between the teacher and pupil and between pupils themselves;
 - (d) application and problem solving;
 - (e) open-ended investigations; and
 - (f) consolidation and practice.

APPENDIX D

RELEVANT SECTIONS FROM THE WORKING DOCUMENT
STANDARD 2

5.2 Reading, under-
standing and solving
problems

In this section problem solving is seen as an end in itself. It should permeate throughout the whole syllabus by providing a context for doing mathematics and be used to:

- a) Motivate positive pupil attitudes about learning mathematics.
- b) Generate the need for the specific concepts and skills involved in the problem.
- c) Increase the understanding of mathematical content.
- d) Develop communication skills.
- e) Assist pupils to develop reading skills and make them aware of the decisions required in selecting the appropriate strategies.

5.2.1 Reading for
understanding

By systematically providing experiences to help pupils develop language processing skills in the area of mathematics children can improve their ability to solve problems.

* Pupils should develop language including symbols, to communicate mathematical ideas.

* Tasks which require pupils to read mathematics and respond to questions based on their reading should be an integral part of the mathematics programme.

(a) Reading with a specific purpose.

i) to get an overall
view of the setting;

* The first reading will not centre on the numerals but provide a setting for the situation.

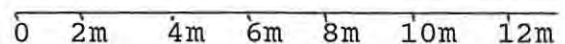
ii) to understand the
situation

Pupils read phrase by phrase to identify facts and relationships essential information that is

- provided in the problem and ignore the irrelevant data.
- iii) to identify the goal. * Pupils determine what is required to be found out and how they can find it.
- (b) Extracting information from different sources Pupils should have experience reading information from tables, charts, menu's, graphs, advertisements, reference books, maps etc.
- (c) Describing problems in pupils' own words.
- (d) Comprehension tasks related to the language of the situation where the solution is not paramount. e.g. Mother used 350g of flour from a 2kg packet. How many 500g containers can she fill with left over flour? Decide whether the following statements are true or false and whether they will help you.
 i) Mother used flour
 ii) There are 1000g in a kilogram
 iii) She will have to share the flour out.
- (c) Creating stories to match given models E.g. i) from a picture
 ii) From a number sentence

5.2.2 Strategies for solving problems

- Select suitable problems and encourage pupils to develop their own strategies. In discussion, however, the strategies listed in this section should be suggested.
- a) Using concrete objects
 b) Acting it out
 This helps pupils relate the problem to real life.
 e.g. 6 long-lost friends met at a party. Each shook hands with each other. How many handshakes were there?
- c) Drawing a sketch
 How this helps to visualise the situation, e.g. How many fencing poles will be needed to fence a 12 metre length which does not need a gate. A pole is needed every 2 metres.



d) Organising information into a list

E.g. How many different arrangements (orders) of the letters D, S and T can be made for a registration number in the Transvaal?

STD SDT TDS TSD DST DTS

e) Substituting smaller numbers for the given numbers in the problem

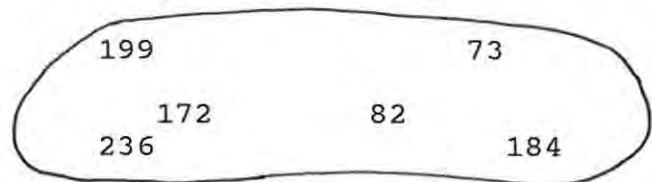
f) Trial and error methods

e.g. Which two numbers have a sum of 11 and a product of 24?
1,10 (No) 2,9 (No) 3,8 (Yes!)

g) Eliminating possibilities

Pupils decide which processes or solutions do not apply. e.g. Circle the number which is described by the following clues:

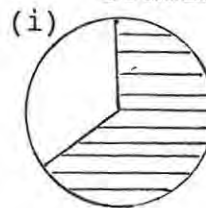
- i) it is divisible by 4
- ii) it is less than 200
- iii) the sum of its digits is 10



5.2.3 Recording the solution.

In recording the mathematical processes involved in solving the problem, a variety of methods should be allowed and encouraged.

E.g. Sam takes 1 hour to do his homework. If he completes his written work in 35 minutes, how long can he spend reading?



ii) No. of minutes
= 60 - 35
= 25

5.2.4 Evaluating the solution

iii) $\square + 35 = 60$ iv) $60 - 35 \rightarrow 25$
Check the solution

- i) against the estimation;
and/or
- ii) whether it makes sense;
and/or
- iii) by substitution.

STANDARD 3

- 6.1.5 The calculator as an investigational aid To be used at appropriate areas in the syllabus to investigate the nature of number and number processes.
- 6.1.6 Calculator games. Selection of suitable games to improve the pupils' flexibility with and understanding of number.
- 6.2 Reading with understanding and solving problems. Refer to the notes in the Std 2 syllabus, paragraph 5.2
- 6.2.1 Reading for understanding.
- (a) Consolidate the skill developed in std 2. Refer section 5.2.1
- (b) Identification of essential information from non-essential information.
- (c) Identification of insufficient information E.g. A sports shop has some cricket balls. They do not cost the same. What is the total value of the hockey balls and cricket balls in stock? What would you need to know to solve this problem?
- (d) Comprehension exercise on a given problem. E.g. The official motor cycle record over a measured kilometre from a flying start, set by Bill Johnson in 1962 is 361,4 km/h. From a standing start over a measured kilometre, David Hobbs set up a record of 197 km/h in 1947.
- 1) What does a flying start mean?
- 2) What does a speed of 197km/h mean?
- 3) It is possible to compare Johnson's record with Hobb's record? Explain.
- (e) Selecting a suitable number sentence from given options. A school hall contains 23 rows of 25 chairs each and 1 row of 19 chairs. How many seats are in the hall? Choose the correct number sentence for solving the problem.

- 1) $23 \times 25 \times 19$ 3) $(23 \times 25) + 19$
 2) $(23 + 25) \times 19$ 4) $23 + 25 + 1$

(f) Creating stories to match given models.

From number sentences
 $(230 + 195) - 201$

From a flow diagram
 $(?) \rightarrow +27 \rightarrow \times 3 \rightarrow 126$

6.2.2 Strategies for solving problems

Refer to the notes of section 5.2.2 in the standard 2 syllabus.

(a) Revise, in context the strategies in the std 2 syllabus.

(b) Further strategies

(i) Making a table

E.g. Ann has 4 coloured sweaters and 3 different coloured shorts. How many combinations can she make so that she wears different outfit each time?

black	red	pink	yellow	white
black	rb	bp	yb	wb
white	wr	pw	yw	ww
green	gr	pg	yg	wg

12 combinations

(ii) Working backwards

E.g. Jim got into the elevator (lift). He went down 5 floors up 6 floors and down 7 floors. Where did Jim get in?

By using a work back approach start with the 2nd floor and reverse the movements i.e.

$$2 + 7 - 6 + 5.$$

6.2.3 Recording the solution.

A variety of recording methods should be allowed and encouraged

E.g. The elevator problem above.

$$\begin{aligned} \text{i)} \quad & - 5 + 6 - 7 = 2 \\ & 6 - 5 + 6 - 7 = 0 \text{ x} \\ & 7 - 5 + 6 - 7 = 1 \text{ x} \\ & 8 - 5 + 6 - 7 = 2 \end{aligned}$$

$$\text{ii)} \quad 2 + 7 - 6 + 5 = 8$$

$$\begin{array}{l} \text{iii)} \\ 9 \\ ? \\ \\ 2 \\ ? = \underline{\text{8th floor}} \end{array}$$

- 6.2.4 Evaluating the solution Check the solution
- i) against the estimate; and/or
 - ii) whether it makes sense; and/or
 - iii) by substitution.
- (iii) Situations in which the remainder is discarded. E.g. Mrs Niven has 15m of sheeting material. How many sheets, 2m long, can she make?
- $15 \div 2 = 7,5$ sheets
She can make 7 sheets.

STANDARD 4

- 7.2 Reading, understanding solving problems. The developmental stages of problem solving have been outlined in the Std 2 and 3 syllabi. Pupils now refine and extend those skills and strategies to solve new, challenging situations where they apply their knowledge of mathematical concepts, methods and processes in both real world situations and purely mathematical situations.

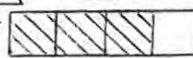
7.2.1 Reading for understanding.

- (a) Selected activities to reinforce the reading skills developed in standards 2 and 3. Refer to sections 5.2.1& 6.2.1

- (b) Translation of mathematical information. Translation means to transform information from one mode to another.

Verbal Symbolic
Verbal Iconic
Symbolic Iconic

E.g. The difference between two numbers is 12 --□- △ = 12

E.g. $\frac{3}{4}$ <-----> 

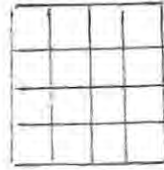
- (c) Reading and interpreting mathematical text from different sources. E.g. textbooks, newspapers, graphs tables etc.

7.2.2 Strategies for solving

problems

b) Systematic listing

E.g. How many squares can you find in this diagram?



Size of square	No. of squares
\square	16
2 by 2	9
3 by 3	4
4 by 4	1
	<u>30</u>

(c) Recognising pattern

E.g. $1 = 1$
 $1 + 3 = 4$
 $1 + 3 + 5 = 9$

Therefore, the sum of the first 10 odd numbers will be $10^2 = 100$.

(d) Solving a simpler problem

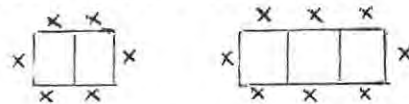
(i) Substitute smaller numbers to make the conditions of the problem more obvious

E.g. Change bulky decimal

(ii) Choosing simpler problems in order to perceive patterns

Tony's restaurant has 30 small square tables to be used for formal dinner. Each table can seat only one person on each side. If the tables are pushed together to make one long table, how many people can sit at a table?

Instead of drawing 30 small tables, consider a few.



Tables	1	2	3	...	30
People	4	6	8	...	?

by extending a pattern pupils can arrive at $30 + 30 + 2 = 62$ people.