A CRITICAL ANALYSIS OF SELECTED TEACHERS' PERCEPTIONS AND EXPERIENCES OF THE ROLE THAT VISUALISATION PROCESSES PLAY IN THEIR VAN HIELE LEVEL 1 TEACHING TO MIGRATE THEIR LEARNERS TO THE NEXT VAN HIELE LEVEL.

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ABSTRACT

Learning is a process that involves building on prior knowledge, enriching and exchanging existing understanding where learners' knowledge base is scaffolded in the construction of knowledge. Research on the teaching and learning of geometry in mathematics suggests that physical manipulation experiences, especially of shapes, is an important process in learning at all ages. The focus of the study was the migration of Grade 8 learners from one Van Hiele level to the next as a result of teachers incorporating visualisation processes and Van Hiele phases of instructions in their teaching. The study underpinned by the social constructivist's theory, therefore aimed at teachers developing visual materials and using Van Hiele's phases of instruction to teach two dimensional figures in Geometry.

The study was carried out in Namibia, Zambezi region in Bukalo circuit. It involved four schools, with 93 learners and three teacher participants. The research is an interpretive case study of a planned intervention programme, which took a four weeks to complete. Participating learners wrote a Van Hiele Geometric test prior and post the intervention programme to determine their geometric level of thought. Participating teachers all received training on visualisation in mathematics and the Van Hiele theory before the intervention. During the intervention, teacher planned and each taught three lessons on two-dimensional figures. Qualitative data was collected from classroom observation, stimulus recall interviews and focus group interviews. Quantitative data came from the pre and post-test of learners.

This study found that on average, Grade 8 learners who participated in the study were operating at levels lower than expected of pupils at their stage of schooling. This study also found that, visualisation processes and the Van Hiele phases are effective when used in geometry lessons to migrate learners from lower Van Hiele levels to higher. For teachers in the same circuit, partnership and planning of difficult topics on an agreed regular basis is recommended. When planning lessons teachers are encouraged to take advantage of the Van Hiele phases of instructions. This study thus recommends the incorporation of visualisation strategies of teaching geometry in particular at primary and lower secondary levels. Mathematics teachers are further encouraged to design visual materials such as Geoboards to use for every topic in geometry. Such visual materials should be carefully developed and evaluated to ensure that their use in the classroom is effectively linked to concepts under discussion in a given lesson.

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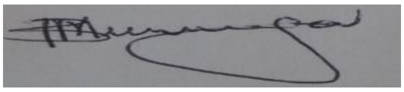
I dedicate this dissertation to my loving mother Rosalia Mubutu Kaela. Mother you have been my shepherd and my strength throughout this journey. Your guidance and trust in me has made me who I am today. This one is yours Ba Rosa. Mother, share this dedication with my grandmother Bana Masiye. Kuku you left while I was at Rhodes University campus completing this thesis. Rest in eternal peace Kuku.

DECLARATION OF ORIGINALITY

I, Ben Muyambango Munichinga, student number 15M8814, hereby declare that this thesis entitled "A critical analysis of selected teachers' perceptions and experiences of the role that visualisation processes play in their Van Hiele level 1 teaching to migrate their learners to the next Van Hiele level" is my own work, and a product of my research. It has not been submitted in any form to another institution. Where I have drawn on ideas of people from other publications or other sources, I have fully acknowledged these in accordance with Rhodes University, Education Department reference guide.



Mr. Ben Munichinga 30 November 2018

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CHAPTER ONE

CONTEXT OF THE RESEARCH

1.1 INTRODUCTION

This chapter presents the introduction to this study. I briefly highlight the background of this study, looking in particular at the rational for conducting the study. In presenting the background I will introduce the research questions that guide the study. Further in the chapter, I will give a glimpse of the study's theoretical framework and methodology before looking at the significance of the study. Finally, I will conclude the chapter with brief overview of the structure of the whole thesis.

1.2 BACKGROUND OF THE STUDY

Learning is the primary goal of any school and guiding learners to achieve during learning is thus the purpose of every teacher in the teaching profession. This has implications for every teacher in terms of how and what he/she teaches. The favoured approach to teaching and learning in the Namibian education system is based on a learner-centered paradigm as described in the ministerial policy document and the learner-centered conceptual framework (Namibia: MoE, 2003). Central to this is that learning in schools should therefore involve building on, extending and challenging learners' prior knowledge and experiences (Namibia: MoE, 2015).

According to Namibia: MoE, (2010), a learner-centred approach implies "a text-rich, visually and tactile-rich learning environment" (p. 27). The national syllabus for senior primary mathematics further outlines that mathematics uses its own specialised language that involves notations and symbols for describing numeric, geometric and graphic relations. This implies that the notion of visualisation is thus an important element in the teaching of mathematics in Namibia. The policy document further states that mathematical concepts should build on one another throughout all the phases, thereby creating a coherent structure (Namibia: MoE, 2014). In the context of the Namibian senior primary mathematics syllabus, the aim of mathematics is "to prepare all learners for present and future studies in mathematics and other related subjects" (p. 1).

Having taught mathematics for over eight years at high school level in Namibia, I have observed with frustration how many learners, particularly in the lower secondary phase struggle with recognizing simple plane figures, describing the properties of these plane figures and even to construct these shapes accurately, let alone make conjectures and possibly prove a basic geometrical result, such as the sum of angles in a triangle. My experience in teaching geometry is similar to that of the Dutch mathematics educator and researcher Van Hiele (1986), who pointed out that there are difficult moments that face every high school teacher in teaching geometry. According to Van Hiele (1986,), "learning mathematics means learning to think, and to be able to think precisely you should have attained the highest possible Van Hiele level in geometry."

The difficulties I experienced when teaching geometry to lower secondary learners led to purpose of conducting this study. A key element to this study is how visual representations of mathematical concepts in geometry at Van Hiele level 1 and 2 can enhance the development in learners' geometrical thinking. I therefore incorporated visualisation processes and the Van Hiele phases of instruction to help participating teachers plan and implement geometry lessons on two-dimensional figures. The Van Hiele theory is well known for its promotion of non-rote learning. Thus, constructivism as an epistemological theory aligned well with this study as the study aims at teachers developing visual materials and using Van Hiele's phases of instruction. This approach will thus require an active participation by teachers and learners throughout the implementation of this intervention programme.

I therefore find the use of visualisation processes, when incorporated successfully with the Van Hiele phases of instruction, to have potential in leading learners to learn geometry for conceptual understanding. In this study, I thus put a particular focus on how visualisation processes and Van Hiele phases were used by teachers in planning and presenting their lessons. My interest was also to understand firstly the geometrical level of participating learners before the intervention; and secondly how the participating teachers incorporate visualisation processes and Van Hiele's phases to teach two-dimensional figures in geometry; and thirdly to know the geometrical level of learners after the implementation of the intervention programme.

1.3 RESEARCH GOALS AND QUESTIONS

The aim of this study is to improve participating teachers' practices of teaching geometry through incorporating the Van Hiele phases of instruction and visualisation processes. The objectives of the study is through and intervention programme, seeks to improve grade 8 learners' performances at two-dimensional figures in geometry.

This study is inspired by some of the questions and gaps identified by Presmeg (2014) in the journal The ZDM Mathematics Education special edition of 2014. She inter alia asks the following questions:

- What aspects of the use of different types of imagery and visualization are effective in mathematical problem solving at various levels?
- What visualisation strategies do learners employ that enable them to construct meaningful conceptual content?

It is from the above identified gaps that I framed my own research questions as follows:

Research Questions

The three research questions that frame this study are:

- At what Van Hiele level of geometric thought are Grade 8 learners of the selected schools in the Bukalo circuit operating prior to and post the intervention program?
- How do teachers use visualisation processes and the Van Hiele phases to enhance the transition of learners' understanding from Van Hiele level 1 to level 2?
- What are selected teachers' perceptions and experiences of the role of visualisation in Van Hiele's model, as a result of participating in an intervention programme?

On teachers' perception, it is important to note that there is more than one meaning to the word 'perception'. As defined by dictionary, perception may mean the ability to see, hear, or become aware of something through the senses. While the word perception may also mean the way in which something is regarded, understood, or interpreted. This study thus focusses on the second definition and will explicitly focus on how teachers interpret and understand the role of visualisation in Van Hiele's model after participating in an intervention programme.

1.4 THEORETICAL FRAMEWORK

This study involves teachers and learners as core participants, thus social constructivism as an epistemological theory will inform the study. The Vygotsky's (1962) social constructivism theory centers on the notion that meanings are developed in coordination with others rather than separately within each individual. Hence Nodding, (1990, p. 7) points out that from a constructivist point of view, "all knowledge is constructed". For this study knowledge will be constructed through a collaborative interaction between learners and teachers, and through learner to learner interaction. In the constructivist classroom, the focus tends to shift from the teacher to the students. Driscoll (2005) argues that a classroom is no longer a place where the teacher ("expert") pours knowledge into passive students, who wait like empty vessels to be filled. In the constructivist model, the students are urged to be actively involved in their own process of learning. Teachers will provide learners with opportunities to construct knowledge by availing learning materials and guiding them to construct meaningful knowledge, while learners will work in pairs helping each other understand the learning content at hand. It should however be noted that despite having the same learning experience, each individual learners will base their learning on the understanding and meaning personal to them (Hill, 2002).

1.5 RESEARCH METHODOLOGY

This research took a form of a case study of a planned intervention programme using a mixed method approach of data collection. It is a case study as it is "a systematic and in-depth study of one particular case in its context' (Rule & John, 2011, p. 4, as referenced in Bertram & Christiansen, 2014, p. 42). The case is of a single mathematics intervention programme with three participating teachers in Grade 8 classes. The mixed method approach used a sequential design with the quantitative elements administered at the start and end of the intervention, however the qualitative approach was dominant. Quantitative data was obtained from a pre- and post-test, which Grade 8 learners wrote. I used the test result to determine the geometric level of learners according to the Van Hiele levels of geometric thought. Qualitative data was collected through reflective interviews, video- recordings of the teachers' lesson observations and focused group interviews.

The first step in deciding how you will analyse the data is to define a unit of analysis (Trochim, W, Marcus, S, Mâsse L, Moser, R, Weld, P 2008). The unit of analysis is the "who" or the "what" that you are analysing for your study. For this study, I analysed the video recordings of teachers in classrooms and learners' results of the pre and post-test. Therefore the one unit of analysis of this research intervention was the participating teachers' pedagogy practices and experiences during their engagement with the Van Hiele phases of instruction and the use of visualisation processes. While the second unit of analysis was the learners' performance in the pre and post-test result as compared to the control group.

This study was carried out at four different schools in the Bukalo circuit with three participants that were all Grade 8 mathematics teachers. The study was an intervention programme that consisted of: 1. Awareness workshop, approval, orientation workshop and pre-testing; 2. Planning and piloting; 3. Teaching of lessons, reflective sessions; and 4. Post-test, focus group interviews.I observed nine lessons (three lessons per teacher). After every observation, I interviewed the individual teacher where I focused on getting each teacher's experiences and of each lesson. At the end of the teaching programme, I met with the three teachers for a focus group interview, where the teachers reflected on the intervention programme and gave their perceptions of incorporating visualisation and Van Hiele's phases. Data collected was analysed according to my analytical frame work.

1.6 SIGNIFICANCE OF THE STUDY

My study focuses on two particular Van Hiele levels of geometric thinking and the transition between them, which are level 1 and 2. It is hoped that this study will inform policy makers and researchers with rich and meaningful information on the inclusion of the Van Hiele phases in their planning and deliberations. As teachers are important participants in this study, I hope that teachers in general will find this thesis interesting as they think about how best to teach the first level of Van Hiele (visual) and the transition to the next Van Hiele level 2 (analytic) as well as the use of visualisation strategies to help leaners attain mastery in geometry at these two levels. Furthermore, textbook authors will hopefully also find this study significant as it can inform them how to construct geometry tasks for appropriate Van Hiele level.

1.7 STRUCTURE OF THE THESIS

This thesis contains five chapters:

Chapter 2 (Literature review)

In this chapter I reviewed work from other scholars in relation to my study. In the chapter I gave an overview of three key concepts of this study, namely geometry, visualisation and Van Hiele Theory. I also gave a synthesis of the three concepts showing the link between them.

Chapter 3 (Methodology)

In this chapter I discussed the research methods used in the study. I started the chapter by describing the goals and research questions. This led to the research design strategies used in this study. I also presented the data collection tools and the analytical frame work which was used to analyse my data. Lastly in this chapter, I discussed ethical considerations pertaining to this study where the validity of the study was elaborated.

Chapter 4 (Data [presentation, analysis and discussions)

In this chapter I presented data that I collected and discussed my findings of the research study. I presented data collected from pre- and post-tests of participating learners, and the nine lessons taught by the three participating teachers. I also synthesised the test results of learners. From the presentation of the lessons, I discussed each teacher's lesson focusing on the visualisation processes used by teachers and the Van Hiele phase of instruction. I further gave a summary of how each of the visualisation process transpired as a result the intervention programme.

Chapter 5 (Conclusion and recommendations)

The last chapter of this thesis brings together the findings of the study in relation to my three research questions. The findings of this study are not generalised, hence in this chapter I presented the limitations pertaining this study. I also made some recommendations and suggestions for further studies. In the last part of this chapter I gave my personal reflections as a result of conducting this intervention programme.

CHAPTER TWO
LITERATURE REVIEW

2.1 INTRODUCTION

This chapter reviews literature relevant and related to geometry teaching and learning, the Van Hiele model and visualisation in mathematics geometry. The readings for this section therefore will be on three main concepts of the study, namely geometry, the Van Hiele model and visualisation processes in mathematics. The review starts with a general explanation of geometry. This will serve as a basis to discuss geometry from its origin, the types of geometry in schools and how it is perceived both abroad and in Namibia. In presenting this, I will review literature focusing on two-dimensional figures type which is taught at lower secondary level in Namibian schools.

Studying geometry has proven to be a challenge to some learners, in particular, beginners. Thus in this section of the literature review I will present one of the proposed models of teaching and learning geometry for success. Originating in the Netherlands in 1957, the Van Hiele model of geometric thinking provides guidance on how geometry lessons can be approached, and an easy way for students to better learn geometry with confidence. In this section therefore, I will provide a detailed description of the theory as presented by other scholars, looking at in particular its origin, the level of learning and the phases of instruction. The implication of the Van Hiele theory for both teaching and learning will also be discussed in this chapter.

In the last part of this chapter, I will discuss visualisation in mathematics. In discussing this, I will highlight on the history of the concept (visualisation) and how it gained momentum as a research topic in mathematics. I will then discuss visualisation in geometry, discussing about visualisation processes. I will end the chapter by discussing the three main concepts of my study (geometry, Van Hiele model and visualisation) showing how they complement each other.

2.2 GEOMETRY AS A DISCIPLINE IN MATHEMATICS

Geometry is one of the oldest branches of mathematics embedded in several ancient cultures such as Indian, Babylonian, Egyptian and Chinese, as well as Greeks (Jones, 2002). Furthermore, Jones (2002) points out that in these ancient times, geometry was mainly used to measure land, and in construction of religious and cultural artefacts. Overtime, Mathematicians and geometers found geometry a worthwhile branch of mathematics to study, which culminated in the compilation of Euclid's Elements as a systematisation of the geometric knowledge in 300 B.C. (Jones, 2002). Russell (2018) explains that geometry is a word derived from Greek. In Greek, "geo" means "earth" and " metria" means measure. From the earliest times therefore, geometry was associated with understanding (and visualising) the proportions and shapes of the world around us.

Sir Christopher Zeeman, quoted in the Royal Society and Joint Mathematical Council [Royal Society/JMC] (2001), as cited in King (2002, p. 12) explains that "geometry comprises those branches of mathematics that exploit visual intuition (the most dominant of our senses) to remember theorems, understand proof, inspire conjecture, perceive reality, and give global insight." The definition gives evidence that geometry is the visual study of shapes, sizes, patterns and positions, and that it occurs in most cultures and in almost all human activities.

Today geometry is taught as a branch in mathematics around the world. It is taught in every part of students' curriculum from kindergarten through matric and continues through colleges and post graduate studies. Geometry therefore, is an essential part of the mathematics curriculum that focuses on the development and application of spatial concepts through which children learn to present and make sense of the world (Thompson, as cited in Alex & Mammen, 2010, p. 203). Spatial concepts all relate to specific relationships between shapes and their contexts.

Nevertheless, the question of why we study geometry in schools and beyond can still be asked. There are many of such reasons why we study geometry. Piers Bursill-Hall (2002) explains that:

> 'Our reasons today are very deeply embedded in and reflect the science and scientific cultures of late 20th century Europe. Yet European culture has valued and studied geometry (and our benefactors have had their predecessors) over the last two and half thousand years, reaching back to such institution as Plato's *academy* in the Athens of the turn of 4th century BC (p. 289)'.

Sherard (1981, p. 20) notes that the knowledge of geometry remains a pre-requisite today for study in such fields as "physics, astronomy, art, mechanical drawing, chemistry (for atomic and molecular structure), biology (for cell structure), and geology (for crystalline structure)". These are few general views on why geometry is an important branch to study.

In the mathematics field, geometry lays a foundation for understanding topics such as arithmetical (numbers), algebraic, shapes and even statistical results. Sherard (1981) further points out that geometry have important applications to most topics in mathematics. As a result it has a unifying dimension in the entire mathematics curriculum. It is the basis for visualisation, arithmetical, algebraic and statistical concepts.

At the core of this study is a review of the teaching of geometry in the Namibian context through an intervention programme in three school in the Zambezi region. The aim is to report and provide evidence on teachers' practices, perceptions and experiences of teaching geometry incorporating the Van Hiele phases of instruction and visualisation processes. In simple language, school geometry is defined by Clement and Battista (2004, p. 420) as "the study of those spatial objects, relationships, and transformations that have been formalized (or mathematized) and the axiomatic mathematical systems that have been constructed to represent them". Further, The Universal class (2018), explains that:

> Beginning as infants, humans are attracted to patterns, designs and shapes. Parents reinforce this by often purchasing toys or mobiles with brightly colored shapes, pictures or designs. Babies are attracted to these items before they are able to reach, grasp or manipulate them in anyway. Later, toys are manipulated in such a way as to provide further hands on learning to develop these types of skills. These shapes and designs are the very foundational level of the mathematical field of geometry www.universalclass.com: retrieved (16-05-2018: 20:48)

At the most basic level, geometric principles occur all around us. At schools, students learn about angles, shapes, lines, line segments, curves, and other aspects of geometry that are in every single place you look, even on this page. As teaching is the basis for learning, it is therefore important that "ways are found not only to suggest changes in teachers' practices, but also to provide necessary support and assistance for such desired changes to manifest in the [mathematics] classroom" (Sanni, 2009, p. 39). Learning geometry can be viewed as a life-long skill that has a direct or indirect influence in all human lives. Jones (2002) writes that in schools, learning geometry helps students to develop the skills of visualisation, critical thinking, intuition, perspective, problem-solving, conjecturing, deductive reasoning, logical argument and proof. It is therefore important that teachers devises, ways and means that can see learners flourishing in this important discipline of mathematics (geometry).

Zhang, Ding, Stegall and Mo (2012) state that "students who struggle with learning mathematics often have difficulties with geometry problem-solving, which requires strong visual imagery skills (p.167). The authors view geometry and spatial sense as fundamental components of mathematics learning. Geometry as a branch of mathematics in itself, is full of interesting problems and surprising theorems and thus mathematics teachers should embrace it. Bleeker & Goosen (2009) assert that "geometry can be an effective tool in guiding learners towards abstract reasoning which we need in the wake of globalization with its challenges and opportunities" (p. 19). Geometry therefore needs to be presented in a way that stimulates curiosity and encourages students' learning and their attitudes towards mathematics. French (2004), as cited in Atebe and Schäfer (2008), asserts that students' general mathematical competencies have been closely linked to their geometric understanding. Table 2.1 below shows a sourced picture of two-dimensional shapes in Geometry.

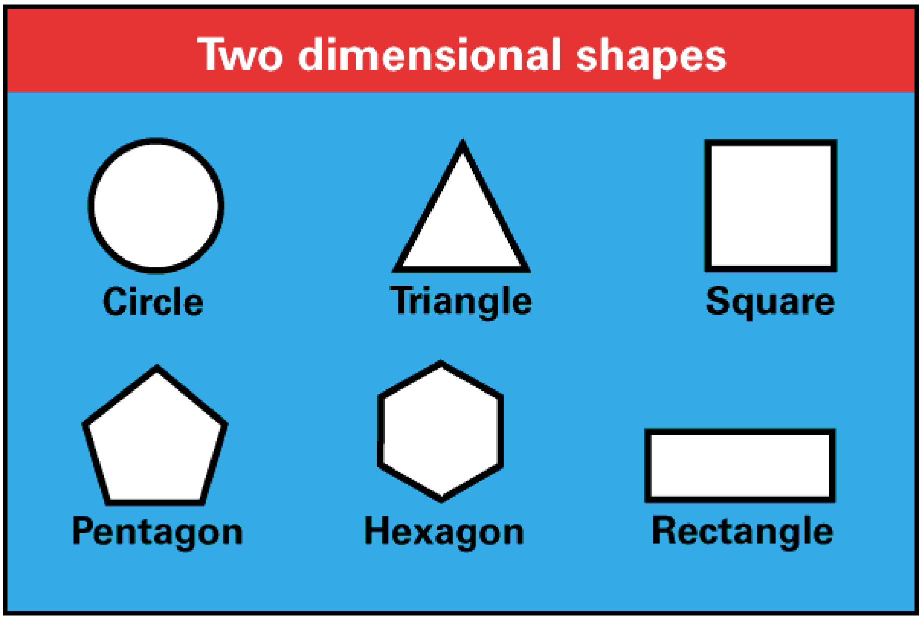


Figure 2. 1 Two dimensional shapes

The shapes shown in Figure 2.1 form the basis of early geometry studies. The Namibian Mathematics syllabus for Grades 5–7 defines geometry as "the mathematical understanding of space and shapes" (Namibia. Ministry of Education [MoE], 2010a, p. 2). Russel (2018) asserts that knowledge learned through a complete understanding of geometric principles can provide not only an increase in safety, but also an increase in the creation of tools and skill level enhancement, and thus basic geometry knowledge can be applied to many professions. Basic geometry in this context is about flat shapes such as triangles, quadrilaterals and circles; in general all such shapes which can be drawn on a piece of paper. Hence in the mathematics syllabus Grade 4 – 7, (NIED, 2015), two dimensional shapes are defined as "having measurable dimensions in two independent directions (length and width)' (p. 75). Figure 2.1 shows such kind of shapes.

Namibia, like other global nations, views geometry as a body of knowledge that is worth studying. There are several reasons why we study geometry in school and beyond. In the Namibian context there are several objectives for teaching geometry at each phase at school. In the Namibia's Mathematics syllabus Grade 4-7 phase, the general objectives listed suggests that by the end of learners' geometric study at senior primary phase (4–7) they will:

- Identify right angles and vertical, horizontal and slanting lines.
- Identify 2-D shapes and their lines of symmetry, and apply transformations to 2-D shapes.
- Identify and describe three-dimensional shapes.
- Give and follow directions on diagrams and in the environment.
- Name and classify angles and draw and measure angles smaller than 180˚.
- Classify triangles and quadrilaterals according to angles, sides and symmetry.
- Construct prisms from nets and sketch nets.
- Identify and name simple transformations.
- Use cardinal directions and alpha-numeric grids to describe position and movement.
- Name, construct and measure lines and angles.
- Describe, sort, name and compare different kinds of quadrilaterals and triangles.
- Draw circles and use circle terminology.
- Describe, sort and compare different kinds of pyramids and prisms.
- Identify and describe symmetry and transformations of geometric figures.
- Use the Cartesian coordinate system to describe and determine location.
- Identify and use benchmark angles and understand parallel and perpendicular lines.
- Differentiate between different kinds of quadrilaterals and between pyramids and prisms.
- Construct cubes and cuboids from nets.
- Use transformations to create composite two-dimensional shapes.

- Use alpha-numeric grids to determine position.

Adopted from the Grade 4-7 mathematics syllabus (Namibia MoE, 2015).

These objectives are fundamental to developing learners' geometrical conceptualisation at secondary level. However, in the syllabus the objectives listed above are not explicit about how geometry should be taught to learners. They only focus on what learners should be able to do. It is thus important to find interesting ways of presenting geometric topics on these objectives to learners. This study therefore, through and intervention programme, seeks to find useful visualisation approaches to teaching, which incorporates Van Hiele phases to teach two-dimensional figures in geometry at Grade 8 level.

2.2.1 GEOMETRY IN THE NAMIBIAN CURRICULUM

The Namibian mathematics curriculum is spread into four phases of schooling, namely the Junior Primary phase (0 –3), Senior Primary phase (4 –7), Junior Secondary phase (8 –9) and the Senior Secondary phase (10 –11). Geometry is part of the curriculum in all the phases. Learners from the senior primary phase are expected to enter the junior secondary phase with sufficient knowledge of basic geometry, the key aspect of this study. They should at least be able to visualise and recognize basic properties of geometrical shapes (two-dimensional) and measure lines and angles of geometric figures (Namibia. MoE, 2010a). However, a study carried out by NIED mathematics education officers in 2009 on the performance of learners in mathematics at upper primary level in Okahandja district of Namibia, revealed that more than half of the learners (53%) could not distinguish between different kinds of triangles and quadrilaterals (Namibia. MoE, 2009, p. 10). Distinguishing between shapes is part of basic geometry and an important level in the Van Hiele level of geometric thought. This alone signifies a problem on how geometry is taught in many Namibians schools.

Furthermore studies by Mateya (2008) and Dongwi (2013) in Namibian schools found that high school learners are performing below their expected levels. Many of them performed only at level one. Dongwi (2013) and Atebe and Schäfer (2011) further observed in their studies that, despite the clear expectations of the upper secondary school syllabus, learners should be operating at van Hiele level 3 or higher. Past researches indicates that most learners are only able to solve geometric problems that are at level 1 or level 2.

A summary in 2.1 below indicates two-dimensional topics in geometry covered at senior primary level. Adapted from the Grade 4–7 mathematics syllabus (Namibia MoE, 2015).

Table 2. 1: Summary of learning content of two-dimensional shapes in geometry

GENERAL OBJECTIVES Grade ….Learners will:	**GRADE 5, 6, 7 SPECIFIC OBJECTIVES** Grade …..Learners should be able to:
Topic 6: Geometry: Two-dimensional shapes	
Grade 5	
• know the basic properties of common regular quadrilaterals	• identify different kinds of quadrilaterals: square, rectangle, parallelogram and rhombus • describe differences and similarities between squares and rectangles; rectangles and parallelograms; squares and rhombuses in terms of - lengths of sides, - angles in shapes (limited to right-angles, angles smaller than or greater than a right angle) - lines of symmetry
Grade 6	
• know the properties of different kinds of quadrilaterals and triangles	• *Name basic shapes such as, squares, rectangles, rhombuses, parallelograms, trapeziums and kites, scalene, isosceles, equilateral, and classify them as either triangles or quadrilaterals.* • identify and classify triangles according to their sides: scalene, isosceles, equilateral, • draw lines of symmetry in triangles and quadrilaterals.
Grade 7	
• understand the properties of quadrilaterals and triangles • know the terminology for circles	• *describe properties, sort, name and compare different kinds of quadrilaterals (square, rectangle, rhombus, parallelogram, trapezium and kite), in terms of their length of sides, parallel or perpendicular sides, angles (right, acute, obtuse) and symmetry,* • describe, sort, name and compare different kinds of triangles in terms of their sides and angles, • *identify and name the following regular and irregular two-dimensional shapes: triangle, square, rectangle, rhombus, parallelogram, other quadrilaterals, pentagon, hexagon and circles in different*

	orientations and further classify them as concave or convex. • create circle patterns using a compass, • identify and use the following circle terminologies: radius, diameter, circumference, chord and semi-circle

The topics and lesson objectives outlined in Table 2.2 are the lessons that were taught during the intervention programme. The participating teachers specifically focused on these concepts and competencies. Even though circle terminologies form part of the Grade 7 Namibian syllabus, it could not be taught by the teachers, because the intervention period was not enough to cover all the topics under two-dimensional figures in geometry. It is important to note that only the topic in italic in figure 2.2 above were taught to the participating Grade 8 learners. The intervention involved Grade 8 learners because, by the time of this study, learners should have covered such topics in previous grades at senior primary phase. Details of this process is discussed in the methodology section. In the next section I highlight some challenges of geometry teaching.

2.2.1 PROBLEMS IN THE TEACHING OF GEOMETRY

Olkun, Sinoplu and Deryakula (2005) emphasise that improving students' geometric thinking is one of the major aims of mathematics education since geometric thinking is inherent to so many scientific, technical and occupational areas as well as in mathematics itself (p. 1). The perception of geometry course in school is that this is the place where students learn about proofs (Wu, 2010). This makes both the teaching and learning a problem to teachers and learners. Another problem faced is that the idea that developing Euclidean geometry (school geometry) from axioms is a good introduction to mathematics that has a very long tradition (Wu, 2010). But contrary to the pedagogical success of Euclidean geometry in schools, it is not dependent on axiomatic proof, but on the pedagogical strategies employed by teachers. It is the teacher who chooses the content, decides how to present it, and determines how much time to allocate to the learning content (Ding & Jones, 2006). Thus, geometry content, though heavily relying on the mythical entity called "proof", should be presented step-by-step in an interesting way which arouses young learners' or beginners' interests (Wu, 2010).

Burger and Shaughnessy (1986) pointed out some time ago that high school geometry is often taught in most high schools at a deductive level while most learners are only capable of reasoning informally about geometric concepts upon entrance into geometry. Nikoloudakis (2009) later also observes that similar research on the understanding of geometric concepts by learners has shown that learners in general find defining and recognizing geometric shapes and the use of deductive thinking in geometry problematic (p. 17). This in itself presents a problem as to why many high school learners fail to attain the highest possible level in geometry. Piaget (1975) as cited in Wirszup (1976), shares this sentiment: he maintains that geometry instruction begins too late and when it is eventually taught moves from a measurement (quantitative) position to the recognition of shape. Piaget further argued that this makes teachers ignore the qualitative phase of transforming spatial operations into logical ones (ibid).

A number of researchers have shared problems particularly attributed to the teaching and learning of geometry. One such researcher is Dongwi (2013) who stresses that "I often feel frustrated when teaching the lower secondary learners at the lack of geometry knowledge and experience many of them bring from the primary school". In her view, learners at primary schools do not spend enough time dealing with geometric ideas in a conceptual manner – their geometric understanding is often shallow and lacks conceptual understanding (p.12).

In my view as a junior secondary mathematics teacher, lessons on plain geometry are always viewed by learners as easy. I took the teaching of basic shapes such as polygons to be obvious. However, I have observed on many occasions that learners struggle with such topics involving plain geometry, let alone even to differentiate between triangles and quadrilaterals, regular or irregular polygons. Such problems can be attributed to how primary school teachers present lessons on plain geometry. Muhembo (2018) notes that teachers rely heavily on oral presentations which involve listing steps and formulas. Hence, Ding and Jones (2006) pointed out that effective instruction in geometry requires teachers to develop sound instructional strategies and knowledge of useful resources and activities. Effective mathematics teachers reflect on their connected mathematical knowledge bases, which include content knowledge, pedagogical content knowledge, conceptual knowledge and procedural knowledge (Luneta, 2013). It thus important for teachers to plan well what to teach, how to teach and seek out suitable visual materials to present lessons on plain geometry.

This research intervention aims at implementing a teaching programme that will incorporate Van Hiele phase of instructions and visualisation strategies of teaching. Visualisation processes will serve as an analytical framework of looking at the lessons. Below I will briefly explain the Van Hiele framework and then visualisation and visualisation in Mathematics.

2.3 THE VAN HIELE THEORY

Teaching school geometry has not always been easy to teachers of mathematics and thus this study makes use of a framework by Van Hiele (1950, 1959, 1986, and 1999) to inform and support teachers to better teach their learners from one level to another in geometry. The framework is a learning model that describes the geometric thinking that students go through as they move from a holistic visual perception of geometric shapes to a refined understanding of geometric proof (Van Hiele as cited in Genz, 2006, p. 4). A brief history on how the theory developed follows.

De Villers (1987) explains that the Van Hiele theory originated in the respective doctoral dissertations of Van Hiele-Geldof and her husband Van Hiele at the University of Utrecht, Netherlands in 1957. Van Hiele-Geldof unfortunately died shortly after the completion of her dissertation, and Van Hiele was the one who developed and disseminated the theory further in later publications. The model/theory was developed out of the frustrations both they (Dina Van Hiele-Geldof and her husband Pierre Van Hiele) and their students experienced with the teaching and learning of geometry (Genz, 2006). For example, Van Hiele (1986:39) explains that when teaching geometry, “it always seemed as though I were speaking a different language”.

2.3.1 KEY CHARACTERISTICS OF THE VAN HIELE THEORY

Pierre's dissertation mainly tried to explain why pupils experienced problems in geometry education (in this respect it was explanatory and descriptive); and Dina's dissertation was about a teaching experiment, and in that sense is more prescriptive regarding the ordering of geometry content and learning activities of pupils. The most obvious characteristic of the theory is the distinction of five discrete thought levels in respect to the development of pupils' understanding of geometry. Four important characteristics of the theory are summarized as follows by Usiskin (1982, p. 4):

- fixed order - The order in which pupils progress through the thought levels is invariant. In other words, a pupil cannot be at level *n* without having passed through level *n-1*.

- adjacency - At each level of thought, that which was intrinsic in the preceding level becomes extrinsic in the current level.
- distinction - Each level has its own linguistic symbols and own network of relationships connecting those symbols.
- separation - Two persons who reason at different levels cannot understand each other.

The Van Hiele model therefore describes the evolution of the kind of reasoning of a student in geometry. Usiskin (1982) indicates that many students fail to grasp key concepts in geometry, and leave the geometry class without learning basic geometric concepts. The model establishes a sequence of five levels of reasoning, labelled 1 to 5 in this paper. The five hierarchical levels of thinking suggested by Van Hiele, that learners go through when learning geometry are: visualisation, analysis, abstraction, deduction and rigor (Burger & Shaughness, 1986). A description by de Villiers (2010) on the general characteristics of each level is as follows:

2.3.2 VAN HIELE LEVELS

Van Hiele (1950, 1959, 1986 and 1999) provides an interesting framework that can inform and support teachers to move learners from one level to another when teaching geometry. The framework is a learning model that describes the geometric thinking that students go through as they move from a holistic visual perception of geometric shapes to a refined understanding of geometric proof (van Hiele as cited in Genz, 2006, p. 4). Although the Van Hiele model distinguishes between five different levels of thought and at high school the students are expected to reach level 4 by their final year, I shall, in this study, only focus on the levels 1 and 2 as they are pertinent for senior primary to junior secondary school geometry. de Villiers (2010) reflected on the general characteristics of each level as follows:

Level 1: Recognition/Visualisation level

The student at this level reasons about basic geometric concepts such as simple shapes, primarily by means of visual considerations of the concept as a whole, without explicit regard to properties of its components. For example, students recognise triangles, squares, parallelograms, and so forth by their shape, but they do not explicitly identify the properties of these figures (de Villiers, 1996). For example, de Villiers (1996) asserts that students recognise triangles, squares, parallelograms, and so forth by their shape, but they do not explicitly identify the properties of these figures. Pegg (1992) “While students may make mention of the length of sides or the size of angles, when directed to focus on these aspects they will not be used spontaneously without prompts” (p. 89). Pegg (1992) further indicates that for students at this level, a figure is a square, cube or rectangle because it looks like one. This is because students visually recognise figures by their “global appearance” (de Villiers, 1996, p. 2).

Recognition of shapes does not only happen, hence Pegg (1992, p. 90), identifies at least three categories within the first level. In the first category, for example, students can identify a rectangle; they can recognise it very easily because its shape looks like the shape of a window or the shape of a door. This means that the identification of shapes is based on a certain prototype. Further examples are: a cube is like a box or a dice; a rectangle is a long square; and parallel lines are like a door. The second category exists when students can identify certain features of a figure but not properties. These are such features as pointedness, sharpness, corners, and flatness. Students are unable to link these features to have an overview of the shape. The third category, which is the lowest, occurs when the student can focus only on a single feature.

Level 2: Analysis level.

Students analyse component parts of the figures, for example opposite angles of parallelograms are congruent, but interrelationships between figures and properties cannot be explained (Teppo, 1991).This means that students analyse figures in terms of their components and relationships among components, and perceive properties or rules of a class of properties of shapes empirically, but properties or rules are perceived as isolated and unrelated. According to Burger and Shaughnessy (1986), the student reasons about basic geometric concepts by means of an informal analysis of component parts and attributes.

At this level also, students begin to identify properties of shapes and learn to use appropriate vocabulary related to properties, but do not make connections between different shapes and their properties (Teppo, 1991). This implies that irrelevant features, such as size or orientation, become less important, as students are able to focus on all shapes within a class. For example, "an isosceles triangle can have two equal sides, two equal angles and an axis of symmetry but no property implies another" (Pegg, 1992, p.90). This means that the properties are seen as separate entities that cannot be combined together to describe a specific figure. Clements (2004, p. 62) gives an example, "if one tells us that the figure drawn on the blackboard has four right angles, it is a rectangle even if the figure is badly drawn." But at this level properties are not yet ordered, so that a square is not necessarily identified as being a rectangle, in other words, students at this level are unable to make short deductions.

Level 3: Ordering level/Abstract level

At this level, students logically relate previously discovered properties or rules by giving or following informal arguments such as "drawing, interpreting, reducing, and locating positions" (Feza & Webb, 2005, p. 38). Students at this level could begin to see "how one figure could be characterised by several different names" (Pusey, 2003, p.14). This is seen if the figures share the same properties, for example a square is seen as a rectangle, but a rectangle is not necessarily a square. Mayberry (1983, p. 59) states that "logical implications and class inclusions are understood". The role and significance of deduction, however, is not understood.

Level 4: Deduction level.

Grearson and Higgleton (1996), as cited in Siyepu (2005), describe deduction as a reasoning process by which one concludes something from known facts or circumstance, or from one's own observation. At this level, deduction becomes meaningful. For example, Hoffer (1981) explains that the student understands the significance of deduction and the role of postulates, axioms, theorems and proof. In level 4, learners go beyond just identifying characteristics of shapes; they observe the complete structure of geometry. They begin to develop longer sequences of statements and start to understand the significance of the deduction, role of axioms, theorems and proof. They are able to create a proof based on their own argument.

Level 5: Rigour Learners at this level start to reason formally about mathematical systems. They manipulate geometric statements such as axioms, definitions, and theorems. Non-Euclidean geometry can be studied at this level.

The main reason for the failure of the traditional geometry curriculum was attributed by the Van Hieles to the fact that the curriculum was presented at a higher level than those of the pupils; in other words they could not understand the teacher nor could the teacher understand why they could not understand (de Villers, 2004). The levels of thinking are presented in a hierarchical nature. They are logically structured to suggest that learners move from lower to higher levels of thinking in geometry. The current level is a prerequisite for the previous level. For example,

> The recognition of a figure at Level 1 feature is an essential prerequisite for Level 2. The consideration of properties at Level 2 will eventually lead to Level 3 understanding where students see relationships between them, i.e., how one or two properties lead to a third (Pegg, 1992, p. 21).

The fourth level leads to conceptual understanding of geometrical proof and the development of theorems and postulates.

It seems very hard to find a researcher on the Van Hiele model who has not needed to assess the students' Van Hiele level; this implies that the use of a test (written or oral) is key to do research on Van Hiele. The Usiskin's test (Usiskin, 1982) and the Burger and Shaughnessy's test (Burger & Shaughnessy, 1986) are the most frequently used, but both tests have some problems. For this study the Usiskin's test (Usiskin, 1982), which took the paper-pencil method is opted for, because it could be administered to many individuals and proved easy and quick to asses as level of reasoning of the students (Gutiérrez & Jaime, 1998). The test administered to participating learners was only used to determine the Van Hiele level of participating learners in the study.

It is also vital to note that two different numbering schemes are used by different researchers in the literature to identify Van Hiele levels of thinking (Senk, 1989, p. 310). The Van Hieles originally referred to Levels 0 through 4, a scheme consistent with the European system of numbering floors in a building: ground floor, first floor, second floor, and so on (Senk, 1989), which is still common in the construction of elevators in most parts of the world, Namibia included. However, when Wirszup (1976) and Hoffer (1979) brought the work of the Van Hieles to the attention of the American audience, they used a 1 through 5 numbering scheme (Senk, 1989). Despite the fact that the Namibian Education system is founded on the basis of the Cambridge Education system which is from Europe, in this study therefore, the numbering system 1 – 5 is used, as used in other research studies (Pegg, 1992; Mason, 1998; Siyepu, 2005; Genz, 2006; Atebe & Schäfer, 2008). The advantage of this numbering system is that it recognises and allows the researcher to use level 0 for students who do not function at what the Van Hieles referred to as the ground or basic level (level 1) (Wirszup, 1976). In this study therefore, level 0 is used to refer to those participating learners who failed to attain level 1 after test analyses.

Many researchers have pointed out that though the goal of most high school geometry courses is to have students reasoning at the deductive level (level 4) by the end of their schooling years (Groth, 2005), it is unfortunate to note that at most secondary schools, geometry is only taught up to level 3 (Cassim, 2006, p. 26). This is confirmed by Van Hiele (1986) who has once warned that lower secondary learners are only able to reach the third level of the Van Hiele levels, which is the ordering level. The claims above may add up to be the cause of poor performances in geometry in the Namibian national examination of both the (NSSC & JSC), in South Africa (de Villers, 1996), in the USA(Clements & Battista ,1992) and elsewhere in the world.

The levels in the Van Hiele model, however do not function independently. Therefore, to confirm the progression of students in the levels of thinking, Van Hiele-Geldof (1958, as cited in Fuys, Geddes and Tischer, 1988) stressed that learners cannot progress through the levels of thinking without proper instruction. Hence, it is important that the teachers' instruction is pegged at the appropriate Van Hiele level to enable learners to attain the highest possible level in their learning environments. Thus the Van Hiele model consists of phases of instruction which, “in the process of apprenticeship, lead to a higher level of thought” (Van Hiele as cited in Fuys et. al., 1984). Van Hiele (1986) recommended a set of instructional phases that teachers should follow in order to facilitate the students’ movement between the Van Hiele levels of geometric thinking.

The phases of instruction are: information, guided orientation, explicitation, free orientation and integration. Teachers are advised to guide their learners’ geometric conceptualization by employing these five phases of instruction in their practices (Van Hiele, as cited in Fuys, et. al., 1984; Mistretta, 2000; Clements & Battista, 1992; Groth, 2005; Ding & Jones, 2006; Serow, 2008; Abdullah & Zakaria, 2011). This would encourage learners to progress from lower to higher levels of thinking. This research intervention will make use of the Van Hiele phases in order to implement the teaching program which is aimed at improving learners’ conceptual understanding of basic geometrical shapes. A description of each phase of instruction is summarized below.

2.3.3 VAN HIELE PHASES

For the transition from one level to another to occur Van Hiele (1986) recommends a set of instructional phases that teachers should follow. Van Hiele-Geldof (1958, as cited in Fuys, et. al., 1984) stress that students cannot progress through the levels of thinking without proper instruction. Hence, it is important that the teachers’ instruction is pegged at the appropriate van Hiele level to enable students attain the highest possible level in their learning of geometry. The phases of instruction are: information, guided orientation, explicitation, free orientation and integration. A description of each phase of instruction is summarized below.

Phase 1: Information/Inquiry

During the information phase, the teacher provides inquiry-based activities in which learners carry out ‘experiments’ and make inductive reasoning and conjectures with regard to the objects learnt. The teacher at this phase introduces the correct required vocabulary and concepts necessary for completing a task and for the learners to become familiar with the working domain through discussion and exploration. Discussion takes place between the teacher and the learners to foreground the objects to be used. It is through this discussion that the teacher discovers how learners interpret the language, and then provides the necessary information to bring them to purposeful action and perception. Hence, Crowley (1990) reasons that the teachers engage with activities at Phase 1 so that they "learn what prior knowledge the students have about the topic while the students learn what direction further study will take" (p. 5). The teachers' task at this phase is to engage in discussion with learners, asking questions and trying to find out what learners already know about the topic (prior knowledge).

This study is about basic geometric shapes. In this phase, the teacher for example introduces the content to be learned, say a rhombus. The first thing the teacher will do, is to find out the learners' prior knowledge about the shape (rhombus). He/she does this by asking questions e.g 'What you know about this shape?' Or simply describe the shape? Hence Zhang et al. (2008) indicate that questions are applied to check learners' attention, evaluate rote learning, and even to stimulate their thinking and discussion. This type of question for example will definitely stimulate learners in discussion with their teacher, as the questions are open-ended and in addition they require responses after observations.

Phase 2: Guided Orientation

In the second phase, the teacher guides learners to uncover connections and to identify the focus of the subject matter. The teacher now specifically guides the learners to explore the objects of instruction in carefully structured tasks such as folding, measuring or constructing. In addition, learners will engage with concepts in order to begin to develop an understanding of them and the connections between them. The teacher's main task at this phase is to ensure that students explore specific concepts by providing them with activities that can guide the learners toward the relationships of the next level.

At this stage, the teacher is more involved in the learning processes. The teacher continuously directs learners in what to do. For example the teacher may provide learners with a rhombus (on paper) then guides them on what to do. The teacher may at this phase ask learners to fold the rhombus on its axes of symmetry and then ask them what they notice. Asking learners to carry out such activities can equally stimulate learners into more focused discussion with the teacher. However, the question may require focused responses of what learners observe after an action, hence new geometric terminologies may emerge e.g diagonal, line of symmetry etc. Thus, Crowley (1990) advices that at this phase much of the material will be short tasks designed to elicit specific responses.

Phase 3: Explicitation

In the third phase of the same lesson, learners learn to verbalise their understandings of the concepts and connections. This phase can be viewed as a peer learning stage, because learners maybe required to give feedback through verbalising their understanding. They become more conscious of the new ideas and express these in accepted mathematical language. While learners present, the teacher's task is to make sure that learners use the correct mathematical language with understanding. The teacher may do this by immediately monitoring when a word is used or noting down wrongly-used words and monitor afterwards. Hence, learners' new knowledge is formed through experience and integrating with past knowledge.

For the lesson on the rhombus the teacher may observe that correct terminologies and phrases such as 'The diagonals lie on the lines of symmetry'; 'There are two lines of symmetry'; 'Opposite angles are congruent'; 'The diagonals bisect the angles'; 'The diagonals bisect each other at right angles'; are correctly used. The learners' task at this phase is to demonstrate and explain how they obtained their findings.

Phase 4: Free Orientation

At this phase, relevant relationships are known to the learners. The free orientation phase allows learners to discover different ways of finding solutions to problems. The teacher assigns tasks for which there are several solution strategies and encourages the students to find their way to the solution.

For the lesson on the rhombus, at this stage the teacher may give learners incomplete rhombi and ask them to complete them. Clements (2004) advise teachers to carefully select appropriate materials and geometric problems for learners that require certain levels of thinking in order for them to successfully solve them.

Phase 5: Integration

The purpose of instruction is believed to be clear to the learners, hence there is less and less assistance from the teacher in the integration phase, and learners build an overview of the content studied. The teacher's task is to help learners reflect upon the observations they have made and that they begin to understand the overall structure of the concepts and where those structures fit in the scheme of formal mathematics. At this phase no new learning material is to be introduced to the learners.

The teacher could then ask learners to summarise the properties of a rhombus. Through summarising, learners begin to internalise the properties of a rhombus and by completion of this phase, learners will reach a new level of thinking. This new level of thinking replaces the previous level of thinking and learners are once again ready to repeat the five phases of learning at the next level of the Van Hiele model of thinking. The cycle keeps on repeating until learners attain the highest possible level of geometric thinking for the content under study.

As pointed out earlier in this review, learners must develop masterfully at each level before they are able to progress to the next level: “These levels are sequential, invariant, and hierarchical” (Clements 2004, p. 152). In this study therefore, participating teachers were required to observe the progression of learners from level 1 to level 2 through strategically planned teaching instruction. It was therefore required of teachers to make use of the Van Hiele phases of instruction to guide learners through their planned lessons. The five phases discussed above, when implemented successfully by the teacher, can support learners as they progress through the levels of geometric thought through sequential activities.

2.3.4 SUCCESSES OF THE VAN HIELE THEORY

Many studies all over the world have demonstrated that the Van Hiele theory can help improve the geometric understanding of learners (Vojkuvkova, 2012). Besides a significant amount of research studies into students’ understanding of geometric proofs, the Van Hiele theory stands out as one of the best recognised frameworks for the teaching and learning of geometry (Dindyal, 2007). Because of the increasing popularity of the theory, Mayberry (1983) asserts that the theory has often been considered as the foundation for curricula implemented in mathematics classrooms in many countries, such as Netherlands, Germany, Russia and United States of America. Since the mid-1980s there has been a growing interest in the area of teaching and learning geometry through the use of the Van Hiele model.

Back at home in Namibia, two studies involving the use of Van Hiele were located. Mateya (2008) recommends that the Namibian mathematics curriculum should also align itself with the Van Hiele theory. He adds that the teaching and learning of geometry should involve more hands-on activities that will actively engage the students. This he believes will enhance students’ conceptual understanding of geometric concepts. Similarly in her summary of her findings, Dongwi (2013) writes that “teachers saw the phases of instruction as a good pedagogical tool or template for planning and presenting lessons”. She further adds that teachers appreciated the application of the Van Hiele phases of instruction to teach circle geometry. They saw the teaching programme as a good lesson planner with reliable assessment of eighty-eight activities.

To learners, Dongwi (2013) asserts that the majority of learners followed instructions and seem to obtain the answers faster than expected. Though some researchers will argue that good learners' performances usually come as a result of good teaching, in her findings, Dongwi (2013) points out that most of the learners performed beyond basic expectations and they were actively engaged throughout the lesson presentations because of the integration made with Van Hiele theory when lessons were planned.

2.3.5 IMPLICATION OF THE VAN HIELE THEORY

Some studies have raised questions about some of the characteristics of the theory. Burger and Shaughnessy (1986, p. 45), for example, report that they failed to detect the discontinuity between levels, and found instead that the levels were dynamic and of a continuous nature rather than static and discrete. Fuys et al. (1988) also found students who progressed through the levels at different rates, or even oscillated between levels, for different geometric content. Gutierrez, Jaime and Fortuny (1991) found that students can sometimes develop more than one level at the same time. In other words, Van Hiele's broad statements "are not as black and white as they are often portrayed to be"' (Pegg & Davey 1998, p. 114).

Regarding levels, even Van Hiele (1986) himself doubted the existence or testability of levels higher than the fourth, and considered them as of no practical value. Yet other studies have suggested the addition of a pre-recognition level at the lower end (e.g., Clements & Battista, 1992). In this study the suggestion of a pre-recognition level was found to be worthwhile, as some participating learners could not attain level 1. This level is therefore referred to as level 0.

On the other hand, some researchers recognise the strength of the Van Hiele model of learning geometry. Malloy (2002, p. 143) for example, mentions one of the strengths of using the Van Hiele framework: "the progression of learners from one level to the next depends more on teaching than on the age of the learner". Hence, Van Hiele (as cited in Fuys et al., 1984), argues that when implementing the five phases of instruction, educators should be clear about the aim of the art of teaching geometry at each phase. That is, "precisely to face questions of knowing how these phases are passed through, and how help can effectively be given to students" (p.

250). Each instructional learning phase builds upon and adds to the thinking of the previous Van Hiele level (Genz, 2006). This implies that through physical manipulations of geometrical shapes (e.g. by using the geo-board) and pictures, students become competent at each level. At the first Van Hiele level, Dongwi (2013) for example, adds that it is recommended to use a variety of informal activities to provide appropriate conceptual knowledge for the formal activities at the next level. Thus age does not matter as opposed to visual exposure and experience through manipulations of shapes.

The implementation of the five Van Hiele phases has significant potential in the teaching of geometry. Freudental (1973), as cited in Patsiomitou and Emvalotis (2010) confirms that “good geometry instruction can mean… leading learners to understand why some organizations, some concepts, some definition is better than another” (p. 23). However, it is vital to first know if the current mathematics teachers are aware of this Van Hiele model, because it is the teachers who need to be aware of such implications for using the five phases of instruction. For example, it is the teacher who should place the learning materials at the learners’ disposal (Clements & Battista, 1992), and engage them in a discussion to talk and learn about their pre-knowledge. For this study, participating teachers received a workshop on the Van Hiele model and visualisation before they could begin the actual implementation of the intervention.

On assessment tasks for learners, Pegg (1992) recommends that when the teacher designs tasks and activities, such activities should be in line with the levels of thinking. To succeed in moving the students from the lower level to the higher levels, more sophisticated tasks and/or activities should be introduced (Siyepu, 2005). Teachers need to be aware of the types of questions at each level. Difficult questions do not however mean a higher level of thinking. Images (2007), further argues that teaching and assessing geometry content at one Van Hiele level when students are functioning at a different level (lower/higher) may hinder students’ learning. This happens because each level has its own language, hence the completion of each phase during the lesson is a pre-requisite.

The implications were vital to the intervention aspect of the study, as teachers were required to teach using appropriate language and set tasks in line with the Van Hiele theory. Therefore, to ensure a successful implementation of the teaching programme, teachers were made aware of the implications of the Van Hiele levels and phases as discussed above.

2.4 VISUALISATION

This section reviews literature that focuses on visualisation in mathematics. Visualisation is an important concept in mathematics and it is gaining popularity with scholars because it can be utilised in classrooms by teachers to enhance mathematical understanding in learners.

2.4.1 THE ORIGIN AND DEVELOPMENTS OF THE VISUALISATION CONCEPT IN THE FIELD OF MATHEMATICS

The concept of visualisation can be traced way back into the 18th century. Arnheim (1969) states that visualisation in mathematics began in the time of Leonhard Euler as a means of proving or establishing the existence of mathematics objects. Arnheim (1969) further explains that from that time more mathematical concepts began to be visualised, but later the act of visualising experienced a reduction in use in the 19th century, and only started gaining momentum when computers began to be more widely used in mathematics education.

Presmeg (1992) made a significant contribution to the topic of visualisation in mathematics. In her research-based article, Presmeg (1997) shows, by discussing how the concept of visualisation in mathematics gained momentum in Veszprem, Hungary when it was re-introduced by Alan Bishop. It is further explained in the article how the concept continued to gather more momentum in Paris in 1989 and also Mexico in 1990. Visualisation in mathematics then became a success in research in Assisi, Italy in 1991. From that moment onwards, many researchers, particularly in the fields of mathematics and science began doing research discussing teaching and learning in particular, which promotes the use of visualisation in mathematics and science classrooms.

In her article, Presmeg (1992) described visualisation in mathematics as a process whereby visualisation concepts are utilised for mathematics education. This preferred usage of the term 'visualisation' is presented in Piaget and Inhelder's (1971) research, and the position is taken that:

> When a person creates a spatial arrangement (including a mathematical inscription) there is a visual image in the person's mind, guiding this creation. Therefore visualisation is taken to include processes of constructing and transforming both visual mental imagery and all of the inscriptions of a spatial nature that may be implicated in doing mathematics (Presmeg, 1997).

As the visualisation concept gained momentum, a number of scholars started defining this useful concept in a more applicable way, as Auburn and Ausburn (1978) assert — that we live in an era of visual culture, which influences our attitudes, beliefs, values and life-style. It is thus important that in mathematics classrooms, the images used may influence learners' conceptualisation of mathematics concepts. For this study therefore, the term 'visualisation' will be discussed referring to its usage in the context of mathematics.

2.4.2 DEFINING VISUALISATION IN MATHEMATICS

According to Presmeg (1997), visualisation in mathematics is the process whereby visualisation concepts are utilised for mathematical understanding. Presmeg's (1997 definition of visualisation focuses on mathematics teaching and learning. Similarly Zimmerman and Cunningham (1991) define visualisation in mathematics as the intuition through pictures formed in the mind's eye. The two definitions are further aided by Kosslyn (1998) who defined visualisation as the creation of mental image of a given concept. I noted in these definitions the aspect of visualisation happening in the mind when doing mathematics. However, in the definitions, scholars do not mention how this process of visualisation happens.

Hence, Arcavi (2003) has expanded on the definition by including other aspects, such as the process and product and defines visualisation as:

> the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understanding (p. 217).

Arcavi (2003) views visualisation as a powerful tool that plays three major roles in the learning and teaching of mathematics: firstly, visualisation can support and illustrate essential symbolic results and possibly provide proof in its own right; secondly, visualisation can provide a way of resolving conflict between correct symbolic solutions and correct intuitions; and thirdly, visualisation can help learners to engage with concepts and meanings on a level that is not only symbolic and abstract.

Visualisation in mathematics is therefore/can be linked to the aspect of geometry in mathematics which allows one to understand and explore geometrical content via visualisation. While the process of visualisation in mathematics consists of two-dimensional drawings or buildings of three-dimensional models, I concur with Arcavi (2003) who points out that today visualisation mostly consists of using computers to make static two- or three-dimensional drawings, animations, or interactive programs. Therefore for this study the definition by Arcavi (2003) is preferred and will be referred to, however, not replacing hand-drawn objects for computer-generated.

2.4.3 VISUALISATION IN GEOMETRY MATHEMATICS

Imagery can be a powerful force for perception and understanding. Being able to 'see' something mentally is a common metaphor for understanding it (Bills 2002, p. 10). An image in geometry may be of some geometrical shape, or of a graph or diagram, or it may be some set of symbols or some procedure. Visualising geometry therefore means summoning up a mental image of geometrical shapes – seeing it in your mind (Bills, 2002). This can be observed when some people actually close their eyes and 'see' a picture, but for others it has much more to do with imagining than seeing. In mathematics geometry for example, one may try to picture a cube or a six-times table in numbers or a graph of sin x (Bills 2002). On the other hand, if you really want to grasp a concept or idea, struggling to visualise can be worthwhile. There are many aids to visualisation. Diagrams, symbols on paper, or physical apparatus often help.. Trying to 'say what you see' (or cannot see!) can be helpful, too. Visualisation and articulation go hand in hand (Bills 2002).

As Geometry is recognized as an important part of the kindergarten through Grade 12 mathematics curriculum, the National Council of Teachers of Mathematics (NCTM) (2000) argues that it is through geometry that children begin to develop an understanding of "geometric shapes and structures and how to analyse their characteristics and relationships" (p. 41). Part of this development includes the building of spatial visualisation skills. This involves building and manipulating mental representations of two- and three-dimensional objects and perceiving an object from different perspectives. Spatial visualisation is therefore viewed as an essential part of geometric thought as described by National Council of Teachers of Mathematics (NCTM, 2000, p. 41),).

The Principles and Standards for School Mathematics outline specifically what instructional programmes should enable children to do when leaving formal schooling (NCTM, 2000). Within geometry, one standard calls for students is to "use visualisation, spatial reasoning, and geometric modelling to solve problems" (p. 43). This standard focuses on the need to develop students' visualising skills through active exploration with physical objects and through technology. For middle school-age youth, programmes are encouraged to allow students to "examine, build, compose, and decompose complex two- and three-dimensional objects" (p. 237).

In his writing, Giaquinto (2007) postulates that "reliable justifications of belief can come from direct visual appraisal". He explains this as follows:

> Our initial geometrical conceptualisation of basic shapes depends on the way we perceive those shapes. In having geometrical concepts for the shapes, we have certain beliefs forming dispositions. These dispositions can be triggered by experiences of seeing or visual imagining, and when that happens we acquire geometrical beliefs. The beliefs acquired in this way constitute knowledge, in fact synthetic a priori knowledge, provided that the belief-forming dispositions are reliable (p. 12).

Therefore in geometry, employing visual support materials such as diagrams and sketches in the teaching and learning of geometry can assist learners to solve geometrical problems, present the solutions clearly, and check and interpret their results. It is thus important that educators use visual support materials strategically and correctly to tech geometry effectively. However, Acarvi (2003) cautions that "in different contexts, the 'same' visual objects may have different meaning even to the experts" (p. 232). Visualisation therefore, can be an aid to success in geometry when used correctly.

2.4.4 THE ROLE OF VISUALISATION IN MATHEMATICS LEARNING

Researchers have at many times agreed on the role visualisation play in learners' mind when solving mathematical problems. Ho (2010) and Zimmerman and Cunningham (1991) have noted that visualisation does not equate to just forming a mental image, but rather it is specifically about visualising a concept or problem rather than an idea. Similarly, Samson and Ndafenongo (2011) emphasises that the overall goal with visualisation is to help learners construct mental representations that correctly mirror mathematical relationships in instructional representation located outside the mind (p. 29). Visualisation is therefore seen as a process that happens in one's mind when faced with mathematical problem. However, in a mathematics classroom, visualisation does not only happen in learners' minds, but the process of visualising can be made easier by the teacher; thus visualisation in itself has features of being external, as in seeing graphics, charts etc.

Huang H, Liu Y and Chang H (2012) suggests that the primary aim of visualisation in the teaching of mathematics is to facilitate and support the pupils' solving process. He adds that visualisation helps in transforming a mathematical problem into the form of an image. This image enables the learners to better understand problems whose solution would otherwise be inaccessible without using visualisation. Visualisation is therefore intended to supplement other acts of solving mathematical problems. Thus Gilbert (2005) argues that from the educational point of view, visualisation provides information that other means of instruction do not, allowing students to develop deeper and richer conceptual understanding than they otherwise might have. In other cases, the visualisation activity is intended to serve as an aid in the solution of problems.

Particular to this study is visualisation in the mathematical teaching of geometry, namely two-dimensional shapes. In his writing on learning geometry with visualisation, Van De Walle (2010) refers to visualisation as:

> Geometry done with the mind's eye'. It involves being able to create mental images of shapes and then turn them around mentally, thinking about how they look from different perspectives predicting the results of various transformations. It includes mental coordination of two or three dimensions predicting the unfolding of a box (or net) or understanding a two dimensional drawing of a three-dimensional shape. (p. 429).

The focus here is on what learners can do when visualising mathematical concepts. It is important to note that it is not for learners to understand how the process of visualisation should happen, but on how to use the visual object to assist them in the learning or understanding of the mathematics topic being taught. A visualisation object could be a picture, a schematic diagram, a computer simulation, or a video. The student who uses the visualisation object can be said to be visualising. Such students, who use visual imagery in the absence of visualisation objects, are said to be introspectively visualising (Gilbert, 2005). Van Garderen (2005) makes a distinction between visual imagery and spatial imagery as follows:

Visual imagery is the representation of the visual appearance of an object, while spatial imagery is referred to as the relationship between objects and also their movement. Van Garderen (2006) further lists types of imagery which are pattern imagery, pictorial imagery, and schematic imagery. Of these types of imagery, van Garderen, (2006) asserts that schematic imagery is mostly seen as a powerful tool for success in mathematics, while pictorial imagery is less of a tool for success, particularly in mathematics.

2.4.5 THE ROLE OF VISUALISATION IN MATHEMATICS TEACHING

For teachers, visualisation approaches can be utilised to facilitate the work load, especially when incorporated with technology. Stolls (2011) points out that information technology can help teachers to focus less on memorising facts as well as performing routine calculations, but rather information and technology can give teachers more room for developing ideas, exploring consequences, justifying solutions and understanding connections between mathematics concepts when teaching. However, the use of the computer in visualising mathematical aspects requires a careful consideration. Information visualisation as argued by Chapman, Garnett and Jervis (2011), allows students to access mathematical knowledge integrating the symbolic re-constructive approach with a motor perceptive one. This approach is better preferred to the traditional approach where mathematical knowledge was a symbolic re-constructive approach and it was developed inside the interaction between the student and the teacher. Information visualisation makes it easy to represent visual images such as tables, graphs and drawings. Furthermore, the approach adds value to images, giving extra features such as colour.

My view is that, when teaching geometry, teachers should be aware of visualisation strategies that emphasises more on how to think than on what to think. For this study therefore, teachers will be encouraged to employ teaching strategies that promote the use of visualisation. Such strategies may include: display materials that convey concepts; display ideas visually in the classroom; encourage students to draw/sketch diagrams and images to explain or solve problems; ask learners to model concepts visually; model tasks that will encourage learners to discuss visual models then interpret them and debate what they mean; and encourage the use of concept maps as they can help learners make critical connections. Choices of what to do will depend on the teacher and topic of discussion.

In classroom settings, Debes (1969) identified thirty-five visual literacy skills that can be used to teach mathematics in a visual way. In its original context, the term “Visual Literacy” was coined in 1969 and defined by Debes (1969) as

> the ability to decode, interpret, create, question, challenge, and evaluate texts that communicate with visual images as well as words, and use images in a creative and appropriate way to express meaning.(p.25-27)

In the definition, action words such as ‘create’, ‘interpret’, ‘decode’, and ‘evaluate’ are used to strategically refer to what one can do when faced with visual images. Wileman (1980) defines visual literacy as the ability to read, interpret, and understand information presented in pictorial or graphic images. Yenawind (1997) defines visual literacy as the ability to find meaning in imagery (p. 845). He adds that it involves a set of skills ranging from simple identifications, determine the meanings of images and to display a set of skills in this process.

Another comprehensive definition of visual literacy is that presented by the Association of College and Research Libraries (ACRL) (2010):

> Visual literacy is a set of abilities that enables an individual to effectively find, interpret, evaluate, use and create images and visual media. Images and visual media may include photographs, illustrations, drawings, maps, diagrams, advertisements, and other visual messages and representations, both still and moving. Visual literacy skills equip a learner to understand and analyse the contextual, cultural, ethical, aesthetic, and technical components involved in the construction and use of images and visual media. A visually literate individual is both a critical consumer of visual media and a competent contributor to a body of shared knowledge and culture.

I adopted visual literacy skills and referred to them as visualisation processes. I made use of these visualisation processes during classroom observation. For the purpose of this study I focused only on five such skills and referred to them as visualisation processes. I chose five visualisation processes because in my view they could align well with the five Van Hiele phases of instructions. They are: *Designing visuals, manipulating visuals, interpreting visuals, analyzing visuals* and *visual language.* A brief discussion of each follows:

2.4.6 VISUALISATION PROCESSES

The visualisation processes will serve as the basis on which the lessons taught by participating teachers will be analysed. I will look at the five visualisation processes in each teachers' lesson. The coding criteria discussed in the data analysis chapter will be used together with the analytical tool, to assess the presence of these visualisation processes. Before the start of the intervention teaching programme, participating teachers will be made aware of these processes. They should make use of these processes strategically to develop conceptual understanding of concepts to be taught to their respective classes. Hanson, Silver and Strong (1988) point out that the use of visualisation processes is not taught in schools in any organized way. Thus there is a need to make participating teachers aware of what I will be looking for in their lessons.

Designing visuals

Initially, teachers have to evaluate learners' levels of understanding, and this may include learners' prior knowledge and abilities before designing materials to be used in class. Debes (1969) asserts that visual designs are of fundamental importance for the production of effective visual instructional materials. Fleming and Levie (1993) suggest that designers of visual images should take into account the basic principles of perception, so that they can produce teaching media that can communicate messages in a more efficient and effective way.

As a researcher, I will specifically be looking at the number and type of visual materials that teachers bring to their classes during lessons. The type of visual brought to class should be aligned with the subject content, and this will be observed early in the lesson because teachers are required to at least introduce the lesson with the use of their visual materials. Though participating teachers will jointly choose the visual material to be used, such materials should be grade, gender and user-friendly to learners.

In addition, as a teacher myself, I argue that there is nothing worse than a presenter struggling with their visual aids. Therefore, before the presentation of the lesson, I checked with each participating teacher that they were familiar enough with the visual tools to ensure that they were not disconcerted if something went wrong. Confident use of visual material will help marry them to their spoken presentation, helping them become part of an impressive lesson presentation. Finally, this visualisation process will be looked at by the audience, in this case the learners. It is important that teachers display the visual materials to be used in their lessons in such a way that every member of the class is able to see. If some members of the class are unable to see the displayed visual, it is important that the teacher changes the layout to enable every member of the class to see. This visualisation process could align well with Van Hiele's information or visual phase, because at this phase discussion takes place between the teacher and the learners to foreground the objects to be used. Furthermore Crowley (90) reasons that the teachers engage with activities at Phase 1 so that they "learn what prior knowledge the students have about the topic while, the students learn what direction further study will take" (p. 5).

Manipulating visuals

Manipulative cans come in a variety of forms and they are often defined as "physical objects that are used as teaching tools to engage students in the hands-on learning of mathematics" (Boggan, Harper & Whitmire, 2009). To accomplish this objective, the manipulative used must fit the developmental level of the child" (Smith, 2009, p. 20). Teachers therefore need to think critically about how to manipulate visual information to help learners to make a solid interpretation of its meaning (Rakes, 1999, p. 17). In addition, Rakes (1999) asserts that teachers should be able to manipulate different representations and concepts in order to assimilate them into learners' already existing knowledge to make sense of the geometrical idea. Just as people say 'seeing is believing' manipulating visuals as echoed by Lockridge (2014) is the effective use of teaching aids/visuals that allow for students to better comprehend material and therefore score better in tests.

For this study, under these processes, I will be looking at how teachers use different representations in their lessons. Teachers will be encouraged to use the visuals in a way that connects new concepts to what learners already know. In the Figure 2.3 below, an image of a tricycle and a quad-bike will used by a teacher to connect what learners already know to a new concept. In the first lesson of the intervention programme, teachers are to teach a lesson on naming basic shapes and classify them as either triangles or quadrilaterals. This visualisation process will be incorporated with phase 2 of Van Hiele. At the guided orientation phase allows teachers to guide their learners to uncover connections and to identify the focus of the subject matter. At this stage, learners are given the liberty to exchange ideas during classroom discussion.

For this lesson, under manipulating visuals as a visualisation processes, teachers will be required to connect existing concepts that learners know. It may be obvious that all learners by Grade 8 will know the two images as presented. The teachers' role is to allow learners to use the visual to discover the connection between the visual and what is to be taught. The teacher here may ask learners to connect all the wheels with straight lines in a tricycle and in a quad bike and observe the geometric shape that will come out.

Figure 2. 2: Visual images

As the two pictures shown, their meaning is different to the meaning they carry when examined in a mathematical context. It is after learners do the experiment (connecting the wheels with straight lines) that the teacher will engage the learners. The teacher at this point may ask a few questions to the learners: ’Did you connect all the wheels?’ ‘What did you discover?’ It is thus important that teachers take into account that if the visual material does not enhance what is to be taught, then the visual material is not worth bringing to class’ and thus logically trying to decompose its properties may further confuse learners. Debes (1969) asserts that teachers should use visuals in an orderly and logical way to avoid confusing learners. Manipulating visuals should simply be seen as extensions of the lesson presentation, hence the teacher needs to refer to the visual more than once during the lesson.

Visual language

Debes (1969) asserts that a visually literate person should be able to read and write visual language i.e. s/he should be able to decode and encode visual messages appropriately. However, unlike ideograms, Rakes (1999) stresss that teachers need to help students to become adept encoders and decoders of visual language so that they are equipped to deal with the increasing amount of computer-based information. Teachers can use techniques which promote the production of visuals to demonstrate learning. Furthermore, teachers can combine visual and verbal elements and present a visual language that can elicit learners’ feelings, emotions and attitudes. Under this visualisation processes, teachers should use strategies which will allow learners to produce meaning carried by the visual materials.

Visual language as a process is incorporated with the explicitation phase of Van Hile. At phase 3 of instructions, Dongwi (2013) argues that Learners learn to verbalize their understandings of the concepts and connections. They become more conscious of the new ideas and express these in accepted mathematical language. The concepts now need to be made explicit using accepted language. The teacher takes care that Learners advance the use of technical language with understanding through the exchange of ideas. Hence, learners’ new knowledge is formed through experience and integrating this with past knowledge.

Christian School Products (CSP) (2018) observes through research that the generation of students now moving into and through their educational system is by far the most visually stimulated generation that that system has ever had to teach. Research shows that sixty-five per cent of our students are visual learners (CSP, 2018). It is thus not arguable that the question of ’How smart are you?’ is now irrelevant. A more powerful new question is ‘How are you smart?’

As educators, there is need to recognize the nature of our students and prepare them for the world in which they will live and work. Christian School Products (CSP) (2018) therefore advises teacher educators to allow this understanding of the visual nature of students to influence teaching techniques and the educational technologies employed. Educators need to become Visual Teachers. But, who is a visual teacher? Christian School Products lists three characteristics of such a teacher as follows:

- ✓ *The Visual Teacher is an educator who embraces and models full spectrum visual literacy: The Visual Teacher understands the effects of visual stimulation on brain development and utilizes imagery where appropriate to enhance learning.*
- ✓ *The Visual Teacher utilizes graphic, image-rich technologies in his or her teaching: The Visual Teacher is proficient in the basics of contemporary image-making*
- ✓ *The Visual Teacher avoids passive learning experiences by bridging "seeing" and "doing" using appropriate projects, activities, and technologies: The Visual Teacher creates lesson plans and activities that reflect the Six Methods of Visual Learning, acknowledging that when we create and utilize images we will most likely be working in one (or some combination) of the following modes*: *Investigate, Chronicle, Express, Communicate, Inspire, Envision.* (Christian Schools products, 2018).

The characteristics mentioned above will be expected from participating teachers during the intervention programme. During the presentation of the lessons, four key indicators will be observed under this visualisation process as noted by Debes (1969). I will specifically look at strategies used by teachers that will allow learners to produce visuals when solving mathematical problems; how the teacher uses his/her visual material inter alia his/her verbal explanations; how the teacher responds to questions asked about the visuals by learners; and how consistent the teacher is with mathematical language and terminologies during lesson presentations. Finally, the visual material to be used should not only put new language out there, but also help to remind learners of the mathematical language used during the lesson.

Analyzing visuals

For better analysis of images or any visual materials, it is important for teachers to employ strategies to guide students through a close analysis of an image. Image-analysis should help students develop awareness of historical context, develop critical thinking skills, enhance their observation and interpretive skills, and develop conceptual learning techniques (Felten, 2008).

Carter (2013) mention, that [a]n approach that aims to classify and teach visuals according to their properties is more likely to assist students in making meaning and in transferring knowledge between visuals with similar properties.

To analyse, just as the word suggests, would mean to examine the nature of something — in this case the visual material, especially by separating it into different parts, in order to understand and explain it. For this visualisation process, the teacher needs to allow learners to fully decompose useful properties of visual material and then relate them to geometric concepts to be learnt in the lesson. Debes (1969) advises teachers to communicate to learners using visual material, use the visual material that decomposes properties of shapes, and to encourage learners to work with the visuals in terms of the concepts that are taught. These will be looked at during the teachers' presentations of the lessons. This visualisation processes will be incorporated with phase 4 of Van Hieles' phases of instructions. In the free orientation phase the teacher's role, according to Clements and Battista (1992, p. 431) is "to select appropriate materials and geometric problems" that require certain levels of thinking to solve geometric problems successfully. Teachers will required to assess learners with geometrical problems that requires them to use the visual materials in order to find solutions to different geometric problems.

Interpreting visuals

Avgerinou (2001) states that interpreting visuals includes applying critical thinking skills to visuals: identifying and evaluating the validity of information communicated in the visual message. It is the teachers who first need to properly explain the meaning carried by the visual materials, and then the learners. My favourite metaphor for this is that 'one cannot lead the blind, while blind'. Through teaching, teachers train learners on how to correctly interpret visual materials, hence Felten (2008) adds that "when teachers train learners well and allow them to practise visual skills they can develop the ability to interpret and employ diverse relationships of different visual forms effectively" (p. 60).

Interpreting visuals therefore will mean presenting a concept using visual aids, giving students something they can associate with the concept. The art is that students' can later recall the concept by bringing to mind the visual image used during the presentation, when faced with a similar problem. Debes (1969), on this visualisation process, encourages teachers to ask questions that prompt critical thinking and critical reasoning among learners about the visual material used. The teacher of this visual process may ask learners questions like: 'Did you express an idea about the visual material?' 'What conclusion can you make about the picture and the concept taught?' Such questions when used by teachers during the lesson presentation will help learners to properly explain and evaluate the visual material used, hence learning can take place.

For teachers, it is thus important to properly identify visual materials that can be presented clearly and smoothly, without complications. For this lesson on classifying triangles and quadrilaterals, in order for learners to gain a deep understanding of mathematical objects and visuals, they need to explain them by interpreting them in different ways that make sense to them. Learners can expound a shape by listing its properties and then relating the properties to actual objects to be learnt. Learners would list three wheels on a tricycle and relate that to the three sides of a triangle. This visualisation process will be incorporated with the integration phase of Van Hiele phases of instruction. At this phase Van Hiele advices teachers to allow learners to summarize the new understanding of the topic learnt and incorporate the appropriate language of the new level. It is however important that these summaries only include what the learners already know – no new material should be introduced at this stage. The purpose of instruction is now clear to the learners hence, there is less and less assistance from the teacher. By getting to this phase learners are believed to be ready for the next level and the circle of phases repeats in the next lesson.

For details on the incorporation of the visualisation processes and Van Hiela phase, see appendix g: lesson plans. The five visualisation processes as discussed above form part my analytical instrument which will in turn be my observable indicators outlined in detail in the methodology chapter.

2.5 THE RELATIONSHIP BETWEEN GEOMETRY, THE VAN HIELE MODEL AND VISUALISATION

In this section I will examine the relationship between the three concepts as discussed in this literature chapter. According to literature discussed on geometry, in particular its origin, geometry as a field in mathematics was first discovered dating way back to 2000 BC. Geometry was used for many purposes which were not even teaching, such as re-establishing values of properties after the annual flood of the Nile river. Literature provides evidence that geometry existed before the other two concepts of visualisation and the Van Hiele model. The two concepts (Van Hiele theory and visualisation) were developed in particular to make the learning and teaching of geometry easy for both learners and teachers. This is supported by Devilliers

(2010) who argues that traditional geometry was difficult to understand; hence the coming of the Van Hiele model, especially its origin, as stated by Van Hiele and his wife. The formation of the theory is based on personal experiences they had both had with their students in the teaching and learning of geometry. Students in particular faced difficulties in understanding geometric concepts.

Visualisation in mathematics was developed as a means of making geometry easy to understand for learners through visualising mathematical concepts, either on paper or in their minds. Thus, Presmeg (2014) described visualisation as a process of constructing and transforming both visual and mental imagery. This in itself involves imagining mathematics concepts through visualising them. The relationship between the three concepts can simply be put as: visualisation and the Van Hiele theory are methods or approaches in the teaching of geometry, and together manifest the conceptual understanding of geometry. Figure 2.4 shows the link between the 3 concepts.

Figure 2. 3: Relationship between visualisation, the Van Hiele theory and geometry.

2.6 THEORETICAL FRAMEWORK

This study is informed by Vygotsky's (1962) social constructivism. From a constructivist point of view, "all knowledge is constructed" (Nodding, 1990, p. 7). Simon (1995) adds that constructivism as an epistemological theory that asserts that "knowledge can be developed whether or not there is a teacher present or teaching is going on" (p. 116). Driscoll (2005), explains that constructivist teachers do not take the role of the "sage on the stage." Instead, the teacher acts as a "guide on the side" providing students with opportunities to test the adequacy of their current understandings. Further, students within a constructivist learning environment play a more active and responsible role in their own learning.

A key element in this study is how visual representations of mathematical concepts in geometry at Van Hiele level 1 can enhance the development in learners' geometrical thinking. Central to the Van Hiele model of teaching, is the promotion of non-rote learning. Therefore the constructivist teacher will encourage 'meaningful learning' in his/her classroom, which is active, constructive, long-lasting, and most importantly, this type of learning allows students to be fully engaged in the learning process. This study therefore argues for the successful implementation of visual teaching methods at the Van Hiele level 1(visualisation) to help learners migrate and grow in their geometric thinking. Teachers need to encourage learners to make use of visualisation processes to develop their conceptual understanding.

Serpil, Cihan, Sabri and Ahmet (2005) emphasise that by using a visualisation approach in teaching and learning, many mathematical concepts in general and geometric concepts in particular can become concrete and clear for students to understand. They further add that visualisation approaches can enable students to look at mathematics, which is "… usually seen as an accumulation of abstract structures and concepts from a different perspective" (p. 3). Thus, once concepts are visually clear to students, meaningful learning can take place and hence students can migrate up the Van Hiele's levels.

Brooks and Brooks (1993) state that rather than saying "no" when a student does not give the exact answer being sought, the constructivist teacher attempts to understand the student's current thinking about the topic. Therefore, through non-judgmental questioning, the teacher leads the student to construct new understanding and acquire new skills, and this is very significant for this study. Guzm'an (2002) adds that visualisation is extraordinarily useful in the context of the initial processes of mathematics as well as in that of the teaching and learning of mathematics. All these make very clear the importance of training our own visual ability and to introduce it to those whom we are trying to introduce mathematics to.

Social constructivism is thus central to my study because the intervention to be undertaken will require a classroom that is characterised by a collaborative learning approach. Yackel (2001) states that "Students construct their own meaning from the words or visual images they see or hear" (p.41). Thus, students need visual materials in order for them to construct meaningful knowledge. From a constructivist view, Pape and Tchoshanov (2001) highlight that the "necessary mapping between the concrete materials and the arithmetic algorithm (procedure) requires intensive social co-construction of meanings" (p. 123). Teachers and students need to co-construct their understanding of the steps of the mathematical operation while manipulating the materials (ibid.). Kilpatrick, Swafford and Findell (2001) add that interpretation and use of visualisation objects in teaching is integral to the process of understanding mathematics. Throughout the intervention programme, learning will be viewed as a process of peer interaction which is mediated and structured by the teacher. Students will be required to be actively engaged in all the intervention lessons and their progress will be assessed through the teachers' set tasks. The Van Hiele model aligns well with social constructivism as for example at phase 1 of instruction, the constructivist teacher will allow students to work with peers to allow them to learn from one another and even construct new knowledge together.

2.7 CONCLUSION

In this chapter I presented literature looking at the views of other scholars with regard to the three concepts of this study. In doing so, I presented literature discussing geometry as a discipline in mathematics. This allowed me to review literature regarding the teaching of two-dimensional figures at lower secondary level. In the second part of the review I discussed the Van Hiele theory, in which I reviewed that the theory could successfully allow teacher to present geometry in an interesting way to their learners. Visualisation formed part of the review, where I discussed the origin of the concept, and its role for learners and teachers when learning and teaching geometry. I then discussed the five visualisation processes that in turn will serve as my observable indicators of the lessons. Finally I concluded the chapter by synthesising the three concepts. The theoretical framework that underpins this study was also looked at.

CHAPTER THREE
RESEARCH METHODOLOGY

3.1 INTRODUCTION

In this chapter I describe and provide explanations that informed this research process. The objectives for conducting this research case study were threefold:; firstly, to determine the geometrical level of participating learners; secondly to implement a teaching programme that incorporates visualisation strategies and the Van Hiele phases of instruction; and thirdly to analyse participating teachers' experiences and perceptions after participating in the intervention programme. I see the three objectives as a journey, and worth reporting how they were all achieved and materialised.

To report on the mentioned journey above, I present this methodology chapter which is divided into four parts: The first part is an outline of the research goals, questions and orientation; the second part of this chapter discusses the research design strategies for the study; the third part presents the data collection tools and analysis framework; while the last part of this chapter discusses issues pertaining to ethics.

3.2 RESEARCH GOALS AND QUESTIONS

This study is inspired by some of the questions and gaps identified by Presmeg (2014) and *The ZDM Mathematics Education special edition* (2014, pp 151-157) journals which inter alia ask the following questions:

- What aspects of the use of different types of imagery and visualization are effective in mathematical problem solving at various levels?
- What visualisation strategies do learners employ that enable them to construct meaningful conceptual content?

From the themes of visualisation mentioned above I then framed my own research questions:

Research Questions

The three research questions that frame this study are:

- At what Van Hiele level of geometric thought are Grade 8 learners of the selected schools in the Bukalo circuit operating prior to and post the intervention program?

- How do teachers use visualisation processes and the Van Hiele phases to enhance the transition of learners' understanding from Van Hiele level 1 to level 2?
- What are selected teachers' perceptions and experiences of the role of visualisation in Van Hiele's model, as a result of participating in an intervention programme?

3.3 RESEARCH ORIENTATION

The research study is informed and conducted within the interpretive paradigm. Central to an interpretive paradigm is to understand the subjective world of human experience (Cohen, Manion & Morrison, 2011). The human experience in this case involved participating teachers' views and experiences on the use of visualisation processes and the Van Hiele phases of instruction to teach two-dimensional figure in geometry. The interpretive research perspective aims to capture and share the understanding that participants encounter of what they are teaching (Kilpatrick, 1998, p. 98). In this study I intended to investigate and understand the experiences of the participating teachers on the role and the use of visualisation in teaching for helping learners advance their understanding of geometry.

3.4 METHODS (CASE STUDY)

This research is a case study of a planned intervention programme. It is a case study as it is "a systematic and in-depth study of one particular case in its context" (Rule & John, 2011, p. 4) as referenced in Bertram & Christiansen, 2014, p. 42). The case was of a single mathematics intervention programme with three participating teachers, for Grade 8 learners. A case study approach can be easier to conduct for an individual researcher like myself "because it gives an opportunity for one aspect of a problem to be studied in some depth within a limited time scale" (Bell, 1993; Leedy & Ormrod, 2005). The van Hiele level of Grade 8 learners was a case of interest for me to study and understand whether, using visualisation approach of teaching incorporated with the Van Hiele phases of instruction, can help learners migrate from Level 1 to 2 and beyond in geometry. Basey (1999) argues that "a case study is the study of a singularity which is chosen because of its interest to the researcher". Therefore, in this case my particular interest was to adopt (with participating teachers) visualisation processes of teaching in order to help Grade 8 learners migrate from one Van Hiele level to the other.

Data collection processes of this research took a form of mixed method approach, and thus involved quantitative and qualitative methods. The two methods helped me to find answers to the research questions guiding this research. Quantitative data was collected from the pre and post-test, where numerical scores of participating learners was recorded and presented in the data analysis chapter. The aim of the tests was to determine learners' van Hiele level before and after the intervention programme. Qualitative that was collected from the lesson observation, stimulus recall interviews and a focus group interview with participating teachers. The aim of these interviews was to capture participating teachers' views, experiences and perceptions after participating in the intervention programme.

The one unit of analysis of this research intervention is the participating teachers' pedagogy practices and experiences during their engagement with the Van Hiele teaching model and the use of visualisation processes. The second unit of analysis is the change in learners' performance on the post-test as a result of the intervention programme as compared to a control group.

3.5 RESEARCH DESIGN

This research was conducted in four phases: *1. Consent seeking, Awareness workshop, approval, orientation workshop and pre-testing; 2. Planning and piloting; 3. Teaching of lessons, reflective sessions; and 4. Post-test, focus group interviews.*

3.5.1 Phase 1: Awareness (orientation) workshop and pre-testing

Consent letters were sent to four schools (including the control group) where the intervention was to be conducted. Similarly, permission letters were also sent to the regional director of education, the inspector of the Bukalo circuit. Upon approval from the director and the inspector, mathematics teachers in Bukalo circuit were invited for an awareness workshop. This was done through a correspondence I wrote to school principals, inviting Grade 6 –10 mathematics teachers for a workshop. Fifteen teachers attended the workshop. During the workshop session, the visualisation concept and the Van Hiele theory were introduced to mathematics teachers. Visualisation strategies involving solving mathematical problems were shared during the session. The Van Hiele theory was also introduced to teachers, in particular the implication of the theory. After the presentation, I launched the aim of this intervention programme and invited interested teachers to contact me for further information concerning my study.

Four teachers showed interest in the programme since they approached me. I then created a chat group with the teachers where more information was shared about the intervention programme. Through our chat group, we agreed on a day for me to conduct an orientation workshop and plan when to have learners write the pre-test. This was agreed upon and we met with the teachers. During our second meeting I introduced participating teachers to the visualisation processes that I was observing during lesson observations and we ended our day by setting a date for learners to write the pre-test. After setting the dates I gave each teacher consent letters to parents of grade eight learners. I had drafted two letters; one in English and the other in the local language (Silozi). All parents responded by allowing their children to be involved in the study. All acceptance letter which were returned to me were in English. (see appendix F: sample of consent letters to parents). Because of the distance between the schools, the tests could not be written on the same day. This allowed me chance to visit all the schools and administer the pre-test with the teachers in their respective schools. I marked the test and shared only the results with the teachers.

3.5.2 Phase 2: Planning and piloting

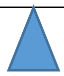
During this phase, I and the three participant teachers met again to plan how we were going to run the intervention programme. The fourth teacher was not invited this time because his class group served as the control group. Our discussion during the meeting involved the integration and harnessing of visualisation processes and the Van Hiele phases in the teaching of nine Grade 8 geometry lessons (three lessons per teacher) over a period of four weeks. The teachers and I developed a teaching programme in the form of carefully designed lesson plans. The lesson plans were designed following the Van Hiele phases of instruction. Visualisation teaching strategies were made explicit at each Van Hiele phase of instruction. The topics that were taught included naming basic geometric shapes, and classifying polygons and properties of quadrilaterals. I selected these topics because they are all covered under two-dimensional shapes in geometry at primary level. Though circles formed part of the topics under two-dimensional shapes, the topic was not covered due to time constraints. All the lessons were to be taught in the afternoons, unless in cases where the participating teacher had made arrangements at their schools. The Tables below in table 3.1 shows the planned schedule and provides a brief summary of the planned activities for each lesson from week 1 to week 3. The teachers were not limited to the visualisation activities in this summary; they were free to come up with other visual materials that align well with the following five visualisation processes: Designing visuals, manipulating visuals, visual language, analysing and interpreting visuals. (See appendix G: detailed lesson plans). These visualisation processes were incorporated as follows with the Van Hiele phases:Information phase/Designing visuals, Direct orientation/ Manipulating visuals, Explicitation/Visual language, Free orientation/Analysing visuals and Integration/Interpreting visuals: A detailed discussion of how each of these lessons was presented can be found in Chapter 4 of this thesis. The three teachers presented the same lessons to their Grade 8 learners.

The pilot of the test items was done with Grade Eight learners of my school. During the time administering the test I was looking at the questions which were perhaps challenging to learners in terms of the language used. I also paid attention to the duration of the test. No questions were asked by learners and they all completed the test within the give time. The test was thus found to be suitable for grade eights. The pilot test was also marked.

Table 3. 1 Planned lesson schedule for three weeks

<table>
<tr><th>Topic</th><th>Lesson Objectives</th><th>Visualisation strategies and Van Hiele phases activity in lessons.</th></tr>
<tr>
<td>Lesson 1: Naming basic geometric shapes</td>
<td>Describe, sort, name and compare different kinds of quadrilaterals and triangles</td>
<td>

Information phase/Designing visual: Teacher will provide learners with blank paper. The purpose is to conduct inquiry-based activities where the teacher will ask learners to draw any geometric shape they know. Feedback will be given by learners.
Direct orientation/ Manipulating: Teacher will provide a scenario to learners as follows: ‘Think of the world with only one shape! For example if everything we see could be in a form of a circle, what do you think this world would really look like? Allow learners to answer!!’(see lesson plan appendix)
Explicitation/Visual language: Teacher pastes picture of quad bike tricycle on the chalkboard. Then asks learners to join the wheels of the tricycle and that of a quad bike with straight lines and observe the resulting shape.
A tricycle will result in a trianglular shape while a quad bike will result in a quadrilateral shape. The teacher will use visual language that is consistent with mathematics, by calling a three-sided shape a triangle and not a tricycle, the same with a quadrilateral.
Free orientation/Analysing visuals: The teacher will have shapes such as a kite and a scalene triangle, and ask learners to paste them in the correct column, while using pictures of tricycle on the chalkboard to help them.

<table>
<tr><th>Shape</th><th>Name</th><th>Triangle</th><th>Quadrilateral</th></tr>
<tr><td></td><td>Triangle</td><td>✓</td><td></td></tr>
<tr><td></td><td>Quadrilateral</td><td></td><td>✓</td></tr>
</table>

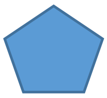
Integration/Interpreting visuals: The teacher will ask any member of the class to summarise the lesson. In the same way the teacher will do a summary by referring to the pictures and show how they can help to differentiate between triangles and quadrilateral.

</td>
</tr>
</table>

<table>
<tr><th>Topic</th><th>Lesson Objectives</th><th>Visualisation strategies and Van Hiele phases activity in lessons.</th></tr>
<tr><td>Lesson 2: classifying polygons</td><td>Identify and name regular and irregular polygons (only polygons with up to ten (10) sides will be taught) and further classify them as concave or convex.</td><td>

Information/ Designing visual: The teacher provides inquiry-based activities by pasting pictures of a tricycle and quad-bike on the chalkboard, then asks learners questions. What did we learn in the previous lesson? How many sides does a triangle have? What relationship did you notice between a quadrilateral and a quad bike?
Direct orientation/ Manipulating visual: the teacher will hand out geoboards to learners. The teacher will first establish rules for use of the geoboard. To begin, the teacher will ask learners to construct any shape from the previous lesson. this is done in order to help learners connect with the previous lesson and also to allow learners to know how to manipulate the visual material to be used in the lesson(geoboard)

Explicitation/Visual language: The teacher will then challenge learners and ask them to construct a five-sided shape. The next will be a six-sided shape, until they reach a ten-sided polygon. To define the word polygon, the teacher explains as follows: 'The word 'polygon' could be related to the word polygamy, meaning marrying more than one wife (Mabali),a common practice in our society'. 'Poly' means 'many'Thus the husband (pentagon) has 5 wives (sides). The teacher follows this order to and gives names to all polygons constructed until a decagon is reached.
Free orientation/Analysing visuals: Teacher asks learners to construct two four-sided shapes: one with equal sides and the other with one or two sides not equal. The teacher then introduces the concept 'regular' to mean polygons with equal sides and 'irregular' to mean polygons with sides not equal. The teacher does the same with concave and convex polygons and asks learners to construct five-sided concave and convex polygons. The teacher should first explain the two concepts before learners construct.
Integration/Interpreting visuals: In the last part of the lesson, the teacher asks learners to construct one polygon on their geoboards from what they have learnt. The teacher then asks learners to complete the table as shown below.

<table>
<tr><th>Polygon</th><th>Regular</th><th>Irregular</th><th>Concave</th><th>convex</th></tr>
<tr><td></td><td>✓</td><td></td><td></td><td>✓</td></tr>
</table>

</td></tr>
</table>

Topic	Lesson Objectives	Visualisation strategies and Van Hiele phases activity in lessons.
Lesson 3: properties of quadrilaterals	Describe, sort, name and compare different kinds of quadrilaterals (square, rectangle, rhombus, parallelogram, trapezium and kite), in terms of their length of sides, parallel or perpendicular sides, angles (right, acute, obtuse) and symmetry.	• **Information/Designing visual:** *the teacher provides inquiry-based activities. Teacher will provide geoboards, rubber bands and blank paper. The teacher then asks learners to construct any four-sided shape on the geoboard. The teacher ensures that learners construct four- sided shapes by checking on each group. Learners will give feedback on their shape by showing the whole class their shape and saying the figure's name.* • **Direct orientation/ Manipulating visual** *As each group has constructed a quadrilateral, the teacher ensures that all six quadrilaterals are constructed – at least one quadrilateral per group.* *On the chalkboard the teacher then pairs the four quadrilaterals as follows: square, rhombus, rectangle, parallelogram, trapezium, and kite.* *Tthe teacher then instructs learners to draw their quadrilaterals on blank piece of paper.* *Using a plain paper and a geoboard, learners should manipulate and find properties of their quadrilateral* • **Explicitation/Visual language:** *The teacher challenges learners by asking them to present their findings from the geoboard and the paper. Learners are to list as many properties as they can find. The teacher's task is to ensure that learners' representations are captured on the chalkboard, by listing all the properties mentioned by learners. The teacher also at this stage listens and controls learners' language, in particular their mathematical vocabulary.* • **Free orientation/Analysing visuals** *Now that all the properties of all the quadrilaterals are listed, the teacher then refers to how he/she paired the quadrilaterals.* *The teacher will at this point ask learners to find similarities and differences among the properties of quadrilateral as paired.* • **Integration/Interpreting visuals:** *In the last part of the lesson, the teacher concludes the lessons by looking at useful properties that carries useful concepts, like the diagonal.* *The teacher will ask learners to construct a diagonal on their figures using a different rubber band on their geoboard.*

3.5.3 Phase 3: Teaching intervention programme

The intervention programme consisted of three lessons spread over three weeks. All the lessons were video-recorded for the purpose of analysis. Since the three teachers taught the same lessons, the aim was not to compare teachers' pedagogical practices, but to see how visualisation processes and Van Hiele phases of instruction were used by the teachers in the lessons that they taught. I visited each teacher three times. Before the teachers started teaching, I held a preparatory session with the individual teacher to find out what the teacher had prepared before going to the classroom. Table 3.2 is a timetable that shows the dates and the time that I observed the individual teachers.

Table 3. 2 Intervention schedule

Teachers	**Dates**	**Time**
	Control Group	
Pre Test	11 April 2018, Thursday	14:00- 15:00
Post Test	20 April 2018, Friday	12:00-13:00
	Week 1	
Pre Test Group A	21 March 2018,Wednesday	14:00- 15:00
Lesson 1-Teacher M	26 March 2018,Monday	14:00- 15:00
Lesson 2-Teacher M	28 March 2018,Wednesday	14:00- 15:00
Lesson 3-Teacher M	30 March 2018,Friday	14:00- 15:00
Post Test	02 April 2018	14:00- 15:00
	Week 2	
Pre test	03 March 2018	14:00- 15:00
Lesson 1-Teacher S	06 April 2018,Fiday	14:00- 15:00
Lesson 2-Teacher S	09 April 2018,Monday	14:00- 15:00
Lesson 3-Teacher S	10 April 2018,Tuesday	14:00- 15:00
Post Test	11 April 2018, Friday	14:00- 15:00
	Week 3	
Pre test	12 April 2018, Monday	14:00- 15:00
Lesson 1-Teacher C	16 April 2018,Wednesday	14:00- 15:00
Lesson 2-Teacher C	17 April 2018, Tuesday	14:00- 15:00
Lesson 3-Teacher C	19 April 2018, Thursday	14:00- 15:00
Post Test	20 April 2018, Friday	14:00- 15:00
	Focus group Interviews	
Teacher-M, S,C	02 May 2018, Wednesday	12:00

All Teachers presented their lessons and a recall interview was done after the lesson. The interviews were transcribed word for word as they happened. Cohen et al. (2011) noted that “if the transcript is of the video tape, then this enables the researcher to comment on all of the non-verbal communication that was taking place in addition to the features noted from the audiotape" (p. 427).

3.5.4 Phase 4: Post-test, focus group interviews.

Phase 4 is well captured in Table 3.4, showing the intervention programme schedule. Below I explain how data from the recall interviews and the focus group interview was collected.

1. ***Stimulus response interviews***

These interviews entailed a conversation between each teacher and myself, where we viewed each lesson after presentation by the teacher. These interviews took the form of a one-on-one discussion with the each teacher to review each of their lessons. During these sessions, together with the teacher, we analysed the lesson while playing the video. I used my analytical framework in Table 3.4, looking at how each visualisation process happened. While interacting with the teachers, I engaged through questioning, how their experiences of incorporating the Van Hiele phases of instruction and visualisation strategies helped them to present their lessons. As indicated, stimulus recall interviews in this study were carried out after every lesson taught by the individual teacher. This was not only a timing factor but also created an opportunity for the teachers to not only reflect on their teaching, but also to provide them with an opportunity to prepare well for the next lesson.

2. ***Focus Group interviews***

These interviews were initially planned to be conducted at the end of every week, but due to the fact that some teachers had not presented their lessons, the time schedule was sequential per teacher. The questions for these interviews were directly related to my research questions. The interviews were recorded and transcribed word for word.

3.6 Research participants

This study was conducted with four participant teachers from four different schools in Bukalo circuit. Purposive sampling was used to select the four participants that were included in this research. Cohen et al. (2011) discuss that in purposive sampling a researcher hand-picks the participants to be included in the sample, on the basis of their typicality or possession of the particular characteristic being sought (p. 156). The four teachers were not necessarily hand-picked, but they willingly approached me to be part of the programme after the awareness workshop conducted with Bukalo circuit mathematics teachers. These teachers showed willingness to use visualisation in their teaching and to participate in the intervention programme.

All the four teachers had experience of teaching mathematics and they were still teaching mathematics to Grade 8 during the time of data collection for this study. While four teachers were part of this study (Teachers M, S, C and Y, see Table 3.5), Teacher Y, did not engage in the teaching of the lessons and thus the teacher's Grade 8 learners served as a control group. Teacher Y did not receive any further training on the use of visualisation strategies and the Van Hiele phases after the awareness workshop. Further, the teacher did not form part of the focus group interview. My initial plan is to visit Teacher Y at his school and train him how to incorporate visualisation and Van Hiele phase of instructions when teaching two-dimensional figures. Table 3.5 below shows the three participating teachers' qualifications and years of teaching experiences.

Table 3. 3 Qualifications and experience of participant teachers

Teacher	**Qualifications**	**Teaching experience**
Teacher M	B.ENG-Mining	2 year
Teacher S	BETD, Advanced diploma in mathematics	7 years
Teacher C	BETD- Maths and Science 8-10	12 years

3.7 RESEARCH TECHNIQUES

The research techniques that were used to collect qualitative and quantitative data in this study were observations, stimulus recall and focus group interviews. I also used the pre- and post-tests to collect quantitative data.

3.7.1 Observation

The purpose of observation through making use of video-recording was to capture the actual implementation of the teaching programme by individual participating teachers. Mare (2010) defines observation as "the systematic process of recording the behavioural patterns of participants, objects and occurrences without necessarily questioning or communicating with them (pp. 83-84). In the same way, I entered the class as a complete observer, without questioning or commenting during the teachers' lesson presentation sessions. The entire lesson my focus was on the teacher, recording the, words, actions and any other relevant occurrences during the lesson. I enjoyed this process as to me it meant the replay of work I have been doing with the participating teachers, from the awareness workshop to the planning of the lesson and then to the actual teaching. By simply getting to this stage of recording the teachers, as a researcher the process gave me hope of succeeding in this research journey, hence the excitement.

During the processes of observation, I video-recorded nine lessons of the three participant teachers. These videos were recorded by me. The video-recording focused entirely on the teaching process. I tried by all means to find an unobtrusive yet strategic position in every classroom that enabled me to capture the entire lesson. Recording the lessons was important because it allowed me an opportunity to go through the lessons afterwards by replaying the videos repeatedly. I analysed each video-recording using the instrument template discussed under the analytical framework in Chapter 3. I therefore report on all the nine videos recorded in Chapter 4.

3.7.2 Interviews

These took a form of conversation between me and participating teachers. Interviews can be done telephonically, face-to-face or via Skype, however these interviews took the form of face-to-face engagement. Questions were open-ended and were prepared before the interview session. During the interviews I tried being a good listener, spotting 'trigger- words' from the respondent. I did this so that I would be able to follow up on some important issues mentioned by the teachers. Furthermore, during the interviews I tried not to interrupt the respondent and only asked for additional information if the response was not satisfactory to me. I used two types of interviews for this study: stimulus recall interviews and focus group interviews. The procedure of these is discussed below.

Stimulus Recall Interviews

I did these interviews after each lesson presentation. Qualitative data was collected. I specifically targeted capturing participating teachers' experiences, and thus I started the interviews by asking the teachers to talk about their practices in general and then to share with me their experiences of the use of visualisation processes and the Van Hiele phases. I did three interviews with each teacher. During the interview sessions I replayed the recorded video of the lesson. I did this in order to remind teachers about specific occurrences during the lesson and then asked a question. I asked questions such as "What where you trying to accomplish here?"; "Why did you use this picture at this point?"; "Why did you use this word or statement there?"

The interviews in general were good as they provided me with first-hand information and I could ask for further clarity where possible. In addition, the recall interviews were easier for me to administer as I was guided by my research questions and thus only selected sections of the lesson where questions were asked. The recall open-ended interviews also proved to be one of the tools in research where the researcher can establish relationship with the respondent, as in my case the teacher would mostly give a smile when shown the video of the lesson. Furthermore, participating teachers kept in touch even after the implementation of the intervention program. My questions, though pre-determined, were mostly open-ended with follow up questions. (See appendix I for provisional questions)

Focus Group Interview

Similar to the individual recall interviews, a focus group interview was conducted with the three participating teachers as a group. The purpose was to capture participating teachers' experiences from the introductory workshop, and planning and implementation of the actual intervention programme. At the back of my mind, my aim was to ask questions that teased out teachers' experiences and perceptions of the use of visualisation strategies incorporated with the Van Hiele phases of instruction. I asked questions like "When you heard about visualisation, what came to your mind?"; "What did you like about the use of visual materials to teach geometry?"; "Did you find any problem with incorporating visualisation and the Van Hiele phases?" It was during this stage of the research study that the data collected provided insight into how the teaching programme worked for the teachers. My role as a researcher was to encourage and inspire the participants to share information as generously as possible. Questions were open-ended, although the process turned out to be more like a discussion. Our discussion was audio recorded with my cell phone and transcribed.

3.7.3 Pre-test and Post-test

To collect the qualitative data set, I administered pre- and post-test to four class groups of Grade 8. The four groups were from four different schools and are referred to in this study as group A, B, C and D. Group D served as the control group, thus they as a group received no teaching of the planned lessons of the intervention. As pointed out earlier, I arranged with the teachers of these class groups for the administration of the two tests (see appendix H: Test paper and memo). I administered both tests with the teachers of the participating class groups. During the tests, I read the instructions to learners. Learners were encouraged to use pencils for easy corrections should they make any mistake. The test contained six items, two of the items tested knowledge of Van Hiele Level 1 and the rest tested Level 2. The test took about sixty minutes to complete. Class group A had sixteen learners, groups B had twenty learners, and group C had twenty-six learners, while group D had thirty-one learners respectively. A total of ninety- three learners wrote the pre and post-test of the same items. The purpose of the pre-test was to determine participating learners' individual Van Hiele geometric levels. The pre-test was important at this point because it was the determining factor for this study to be carried out. The post test was to be used as an indicator of improved performance (if any), after implementing the teaching programme.

3.8 ANALYTICAL FRAMEWORK

The first step in the data analysis is to reduce data to enable the researcher to search for themes and identify patterns (Bertram & Christiansen, 2014, p. 119). Analysing data helps to bring about understanding of the data. For example, "making sense of data in terms of the participants' definitions of the situation, noting patterns, themes, categories and regularities" (Cohen et al., 2011, p. 537). The video recordings were analysed using the analytical instrument illustrated in Table 3.4 below. I analysed the videos and teased out the roles of the visualisation processes used by teachers on the basis of this framework. The stimulus recall interview was the medium with which the lessons were analysed, which then shed light on the answers to the research question.

The focus group interviews were analysed based on the themes from the stimulus recall interviews, as these interviews specifically focused on the participating teachers' perceptions on how the use of visualisation processes and the Van Hiele phases of instruction can improve learners' performances in geometry. The themes align with Debes' (1969) criteria used in the observable schedule.

The following visualisation processes drawn from the work of Debes (1969), as discussed in the context of this thesis, informed my observable schedule and analytical framework:

1. **Designing Visuals -** Ability to demonstrate the effective use of the visual elements of a mathematical concept (such as lines, shapes, angles and dimension).
2. **Manipulating visuals -** Making solid interpretations of visuals and using different visual representations.
3. **Visual language -** Ability to read and write visual language, decode and code visuals (Produce and interpret visuals).
4. **Interpreting visuals -**Ability to use visuals in a variety of ways and apply them critically.
5. **Analysing Visuals -** Ability to use visuals to illustrate and reinforce properties of different objects and shapes and arrange them to communicate effectively.

I analysed these five visualisation processes with a special focus on the observable indicators as outlined in the analytical instrument in Table 3.4. I used this instrument during observations and my interest was to see how the teachers used the different visualisation processes in their lessons and rate them as 'rich', 'medium' or 'no evidence' depending on my observations. The instrument is divided into five sections and they were all used in each teacher's lesson.

Table 3. 4. Analytical Instrument Tools used during observations

	Type of visualisation processes after Debes (1969)	**Rich evidence**	**Medium Evidence**	**No evidence**
Teacher used different representations during the lesson	*Manipulating visuals*			
Teacher demonstrated proper use of geometric concepts	*Manipulating visuals*			
Teacher used visuals in a way that allows learners to connect new concepts to what they already know	*Manipulating visuals*			
Teacher used visuals in an orderly and logical way to avoid confusing learners	*Manipulating visuals*			

Teachers' activity in the classroom (Observable indicators)	**Type of visualisation processes after Debes (1969)**	**Rich evidence**	**Medium evidence**	**No evidence**
Teacher brought to visual materials class	*Designing visuals*			
Visual materials brought to class are aligned with the subject content	*Designing visuals*			
Lesson was introduced with the use of visual materials	*Designing visuals*			
Teacher demonstrates the use of visual elements (lines, shapes, angles and dimensions)	*Designing visuals*			

Teachers' activity in the classroom (Observable indicators)	**Type of visualisation processes after Debes (1969)**	**Rich evidence**	**Medium evidence**	**No evidence**
Teacher used strategies that allow learners to produce visuals	*Visual language*			
Teacher used visuals in conjunction with his/her verbal explanations	*Visual language*			
Teacher responded to questions asked about the visuals by the learners	*Visual language*			
Teachers used visual language that was consistent with mathematical language and terminologies	*Visual language*			

Teachers' activity in the classroom (Observable indicators)	**Type of visualisation processes after Debes (1969)**	**Rich evidence**	**Medium evidence**	**No evidence**
Teacher properly identified and evaluated visual materials	*Interpreting visuals*			
Teacher asked questions that prompted critical thinking and critical reasoning among learners	*Interpreting visuals*			
Teacher concluded the lesson using visual materials	*Interpreting visuals*			

Teachers' activity in the classroom (Observable indicators)	Type of visualisation processes after Debes (1969)	Rich evidence	Medium evidence	No evidence
Teacher communicated to learners using visual materials	*Analyzing visuals*			
Teacher used visual materials that deconstructed properties of shapes.	*Analyzing visuals*			
Teacher encouraged learners to work with the visuals in terms of the concepts that were taught.	*Analyzing visuals*			

Tests analysis

To mark or analyse the pre and posttest, I used the Namibian grading system as described in the Mathematics syllabus for grades 8-9, (see Figure 3.7 next page). I opted for the Namibian grading system because the test was written by Namibian learners, who by Grade eight are used to this grading system. In addition this study will tell the story about performances of learners in selected schools in Namibia. For each item of the test, a forty per cent was required for a learners to be considered competent at each level. Learners who could not meet forty percent were assigned at a level below the tested level. Only two levels were tested, which resulted in two other additional levels. For learners who did not meet forty per cent in items testing level 1, were assigned level 0. Learners who could not get forty percent to be considered level 2, were assigned at Not level 2. According to the Namibian grading system a learners who achieves forty percent may not have achieved all the competencies, but such a learner's achievement is sufficient to exceed the minimum level of competency. The learners is in need of learning support in most areas.

Table 3. 5 Namibian grading system for grade 8-9

Grade	% Range	Grade descriptors
A	80%+	**Achieved objectives exceptionally well.** The learner is outstanding in all areas of competency.
B	70-79%	**Achieved objectives very well.** The learner's achievement lies substantially above average requirements and the learner is highly proficient in most areas of competency.
C	60-69%	**Achieved objectives well.** The learner has mastered the specific objectives and can apply them in unknown situations and contexts.
D	50-59%	**Achieved objectives satisfactorily.** The learner's achievement corresponds to average requirements. The learner may be in need of learning support in some areas.
E	40-49%	**Achieved the minimum number of objectives to be considered competent.** The learner may not have achieved all the specific objectives, but the learner's achievement is sufficient to exceed the minimum competency level. The learner is in need of learning support in most areas.
U	0-39%	**Ungraded.** The learner has not been able to reach a minimum level of competency in the objectives, even with extensive help from the teacher. The learner is seriously in need of learning support.

From: Mathematics syllabus grade 8-9(2015)

3.9 VALIDITY

Maxwell (2009) asserts that validation is an important way of ruling out the possibility of misinterpreting the meaning of what participants say and do, and the perspective they have about what is going on. Cohen, Manion and Morrison (2007) state that in qualitative research, validity should be addressed through the honesty, depth and scope of the data achieved. In this study I used triangulation (Gay, Mills & Airasian, 2012) to validate the data collected. To validate my data I used audio-visual recordings, stimulus recall interviews and focus group interviews with the same participants at different intervals.

In addition, Meriam (2009) states that the process of member checking involves a discussion of preliminary analysis and interpretations with the participant before making any conclusion. Member checking helped me to identify my own bias and misinterpretations of data. All the three participating teachers were involved collectively to share their views and perceptions about the interventions in the context of stimulus recall and focus group interviews. The pre-test and post-test were also moderated by each teacher to ensure the correct placement of learners in their geometric level of thought according to the Van Hiele model.

I adopted the test of Gutiérrez and Jaime (1998) which was developed in Spain, Valencia. Gutiérrez and Jaime (1998), attest that the validation of their test was the several pilot studies accomplished and the analysis made by several researchers with long expertise in the field of van Hiele theory. In addition upon adoption of the test, I sought input of two reputable persons in geometry for further member checking and moderation. One is a senior advisory teacher for mathematics at secondary and senior secondary schools in Namibia, who has worked with van Hiele theory at his postgraduate studies. The second is a mathematics education lecturer at a university in South Africa and has done his post graduate studies on Van Hiele model and is teaching a module in geometry to in-service teachers at this university. In addition this test was piloted by the researcher, and moderated.

3.10 ETHICS

Ethics in this study was aligned to Rhodes University ethical guidelines. (*See the attached Ethics clearance certificate)*

3.10.1 Respect and dignity

Participants in this research took part independently without being forced to do so. Their privacy, in particular their names will not be disclosed in this thesis. In addition, participants were informed of their right to withdraw from participating in the intervention should they wish to do so. In the same way I shared with participants my role, interest and reason for doing the intervention. This was done so that participants could be aware of their role and what was expected from them. In addition, I assured participants that data collected for the study would be kept confidential and anonymous. I also informed them that I would use pseudonyms (protection of participants, and site identities) in the final write-up of this thesis. By doing this, I believe to have helped participants feel respected throughout the research process

3.10.2 Transparency and honesty

This study was carried out in Namibia, in the might Zambezi region, within Bukalo circuit, involving four teacher at four different schools. The study also involved Grade 8 learners of the four schools. Therefore, prior the study, I wrote letters to the regional director of education (see Appendix B:), the inspector of the circuit (appendix C), principals of the schools(appendix D) seeking permission to conduct this study. All the letters I wrote were accepted, and permission was guaranteed. In addition, I also wrote a letter of consent to the parents of participating learners. With the help of the teacher, we managed to give all participating Grade 8 learners letters to their parents before the pre-test (appendix F). These letters were seeking permission for learners to write both pre- and post-tests and to be part of the class group during the presentation of lessons. Two letters were given to each learner, the one letter (Silozi) was a translation of the letter in English. All letters returned by learners were in English. Participating teachers were also given consent forms which they willingly completed before the start of the teaching programme. All teachers completed the forms and none of the teachers withdrew from participating. Member checking of the stimulus recall and focus group interviews was done in order to ensure that what was recorded was a true reflection of the teachers' responses.

3.10.3 Accountability and responsibility

During the actual data collection processes (interviews), I tried to be open to participants so that they felt free to talk about their experiences during the intervention. This was done by continuously thanking the participants for taking part in the study but keeping in mind the main aim of the study. During the analysis of my data I ensured a fair representation of gender, religion and hierarchy in power. This was achieved by ensuring that all voices of participants were represented as I had a female and two male teachers who taught the lessons.

3.10.4 Integrity and academic professionalism

I am a teacher and thus very interested in learners' performance in mathematics. Therefore my conduct during the research study reflected that of an academic and a professional teacher during my interaction with participants. My presence during the research process was made explicit to all participants and my position as a teacher and researcher did not in any way influence the findings. I ensured that my own biases, passion and assumptions about mathematics did not influence the findings by staying as objective as possible throughout the research process. As a control group was involved in the project, I undertook to expose all the learners in the control group and their teacher to the same intervention after my research project. This means that the teacher will receive further training on how to use and incorporate the visualisation process and the Van Hiele phases. I therefore declare that the final thesis is my own work and where I have used other peoples' work I have acknowledged and referenced them according to the Rhodes University reference guidelines. The findings from this study will be useful both to curriculum developers and mathematics teachers in Namibia.

3.11 CONCLUSION

In this chapter I presented my research paradigm, design and methodology according to which study is conducted. This research is a case study and it is oriented within the interpretive paradigm. The study makes use of a mixed method of data collection approach. The case study focused on three Grade 8 mathematics teachers' experiences of using visualisation approaches and Van Hiele phases of instructions to teach two-dimensional shapes in geometry. To collect qualitative data, I used video-recording, interviews and questionnaires. Quantitative data came from the pre- and post-tests. Finally, the chapter ended with a discussion on triangulation, limitations and ethical issues considered during the processes of collecting the data. In the next chapter I present my analyses and describe the findings of the collected data.

CHAPTER FOUR
DATA PRESENTATION ANALYSIS AND DISCUSSIONS

4.1 INTRODUCTION

In this chapter I present and discuss the findings of my research study. The main objective of the research was to analyse how the use of visualisation processes, when incorporated with the Van Hiele phases of instruction by teachers, can help learners move up the Van Hiele levels of geometric thought. This was done through a planned intervention programme. To begin the processes of the intervention, participating learners were tested through the use of a Van Hiele geometric test to determine their level. The same test was used to test learners after the intervention programme.

This chapter begins with the presentation of learners' results of the pre-test. This will be followed by the presentation and analysis of nine lessons (the same three lessons per teacher). During the presentation and analysis of the teachers' lessons, the focus was on visualisation processes, Van Hiele phases and other themes of interest that emerged from the lessons. I will then analyse the similarities and differences among the three participants according to my analytical framework of: designing visuals, manipulating visuals, analysing visuals, interpreting visuals and visual language. In the last part of this chapter, I share the results of the learners' post-test after their participation in the intervention programme. I compared the results of learners who received treatment (experimental group) to the control group class (the group which did not receive lessons during the intervention). The purpose of this comparison is to show if using visualisation approaches of teaching geometry can help to improve learners' performances in geometry.

During the presentation, the participants' actual work, words and responses from the lesson observations and interviews have been used to support the data presented and they are coded as follows: For lesson observations (**TCL1,** TC is for Teacher C and L1 stands for lesson 1, and I used Ls for all learners' responses**),** for stimulus recall interviews (**TCSRI1,** TC stands for teacher C and SRI1 stands for stimulus recall interview 1) and for focus group interviews (**TSFGI,** TS stand for teacher S and FGI1 stands for focus group interview**).**

4.2 PRESENTATION OF PRE-TEST RESULTS

The pretest was administered to a total of 93 learners. The test contained 6 items, and each item was marked and the results are presented below showing the performances of learners at each item of the test per class group.

Item 1 (a)

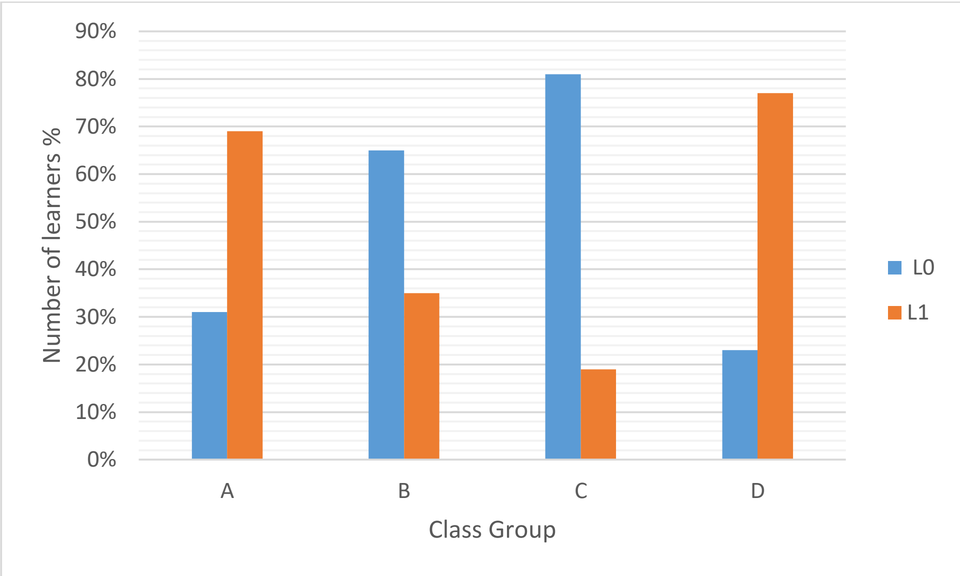


Figure 4. 1: Performance of learners at item 1(a)

From Figure 4.1 indicates that more than sixty percent of learners in class group A and D attained level 1 for this item. However, more than sixty percent in Groups B and C were at level 0.

Item 1(b-f)

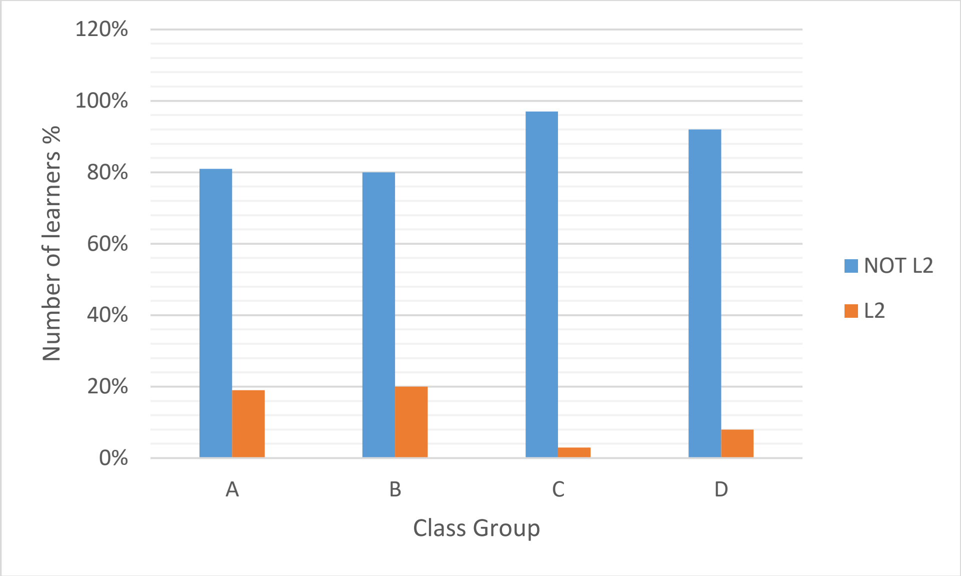


Figure 4. 2: Performance of learners at item 1(b-f)

From Figure 4.2, the majority of learners could not attain level 2. More than 80 percent of learners were not at level 2. The highest number of learners who could not attain level 2 was recorded in group C.

Item 2(a)

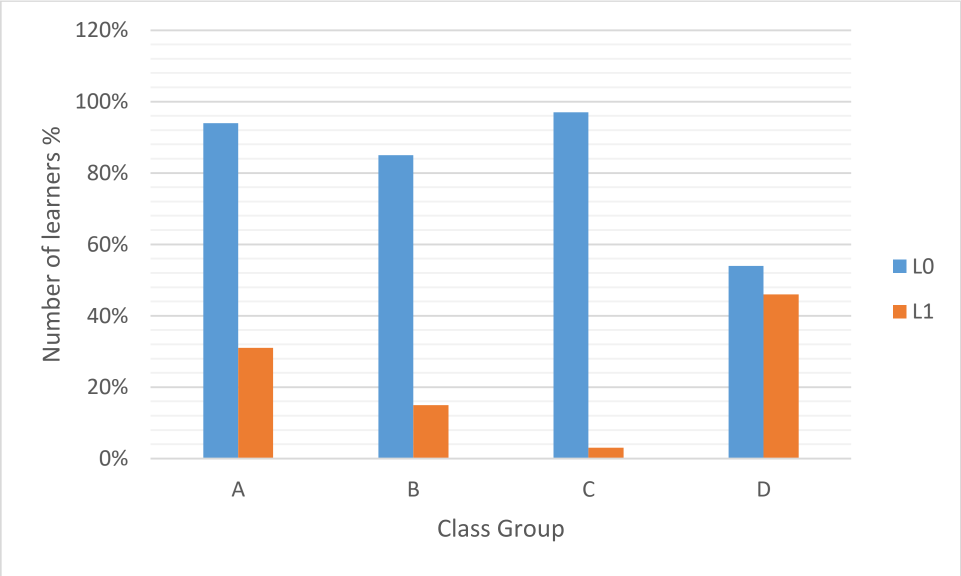


Figure 4. 3: Performance of learners at item 2(a)

For this item, more learners were operating at level 0, including learners in the control group. Figure 4.3 above indicates that more than eighty per cent of learners in groups A, B and C were at level 0, while more than fifty per cent in the control group were operating at level 0.

Item 2(b)

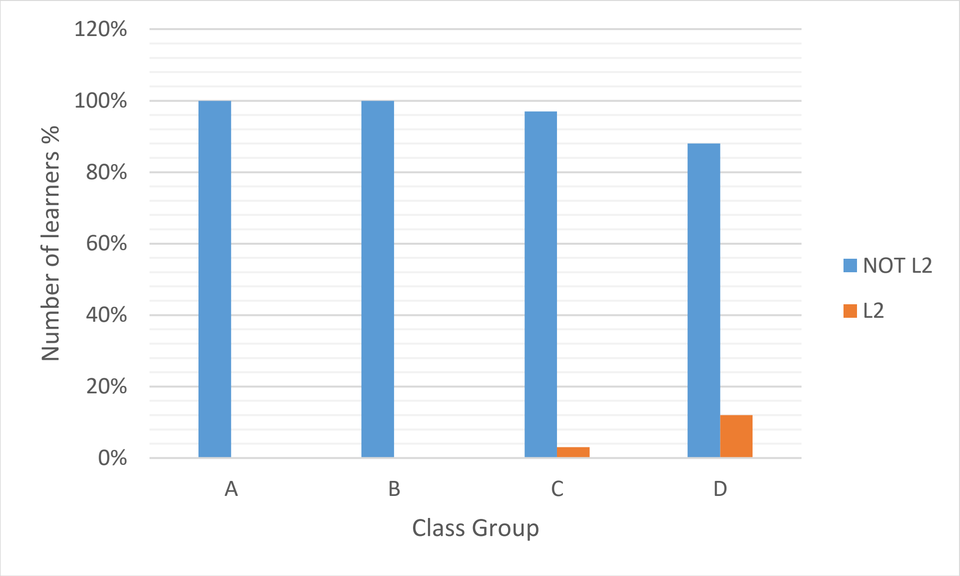


Figure 4. 4: Performances of learners at item 2(b)

All learners in class groups A and B were not at level 2 for this item, while in groups C and D more than eighty-five per cent of learners were not at level 2 as indicated by Figure 4.4 above.

Item 3

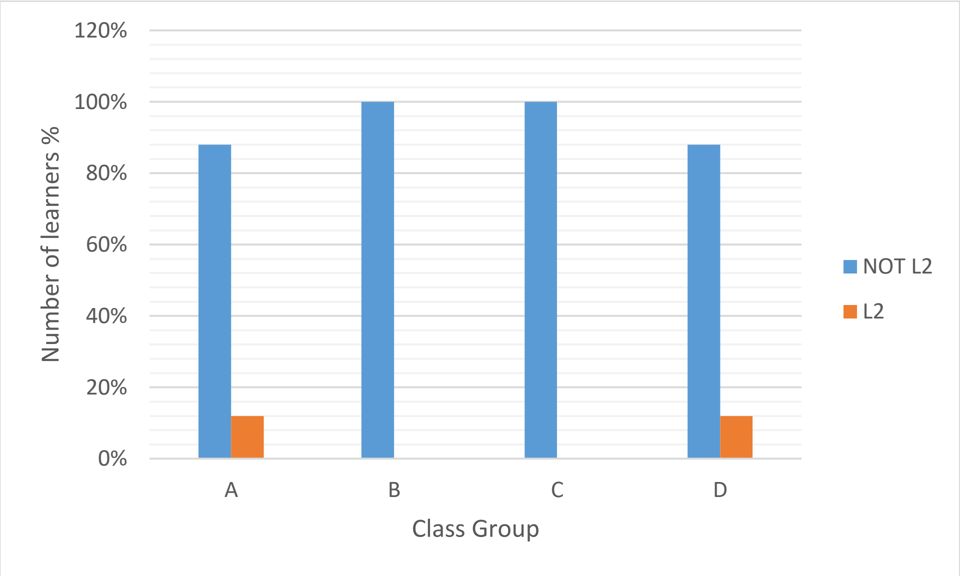


Figure 4. 5: Performances of learners at item 3

For this item, none of the participating learners were at level 2 in class groups B and C. In class groups A and D, more than eighty per cent of learners were not at level 2 in their geometric thinking as tested. Figure 4.5 above indicates learners' performances.

Item 4

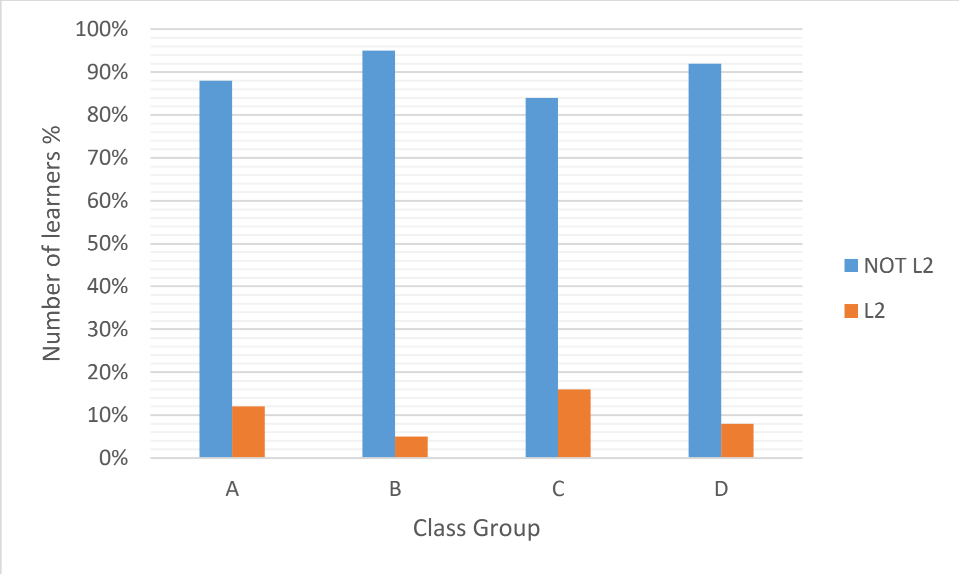


Figure 4. 6: Performances of learners for item 4

For item 4 as indicated on Figure 4.6 above, more than eight per cent of learners in each class group were not at level 2. Less than twenty per cent attained level 2 in each class group.

Item 5

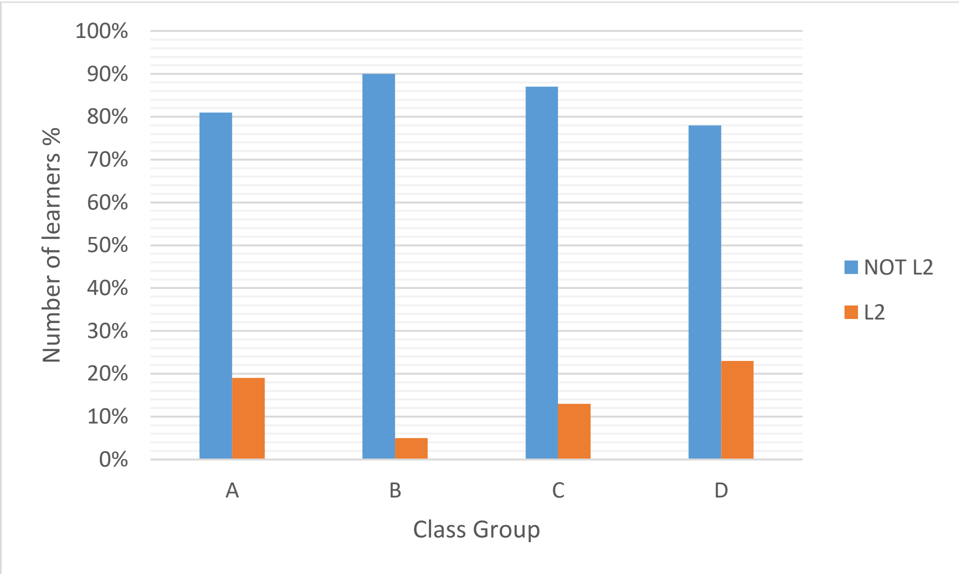


Figure 4. 7: performances of learners at item 5

Similar to item 4, Figure 4.7 above shows that more than eighty percent of participating learners were unable to attain level 2 in class groups A, B and C. In class D (the control group) just a little above twenty per cent of learners attained level 2.

Item 6

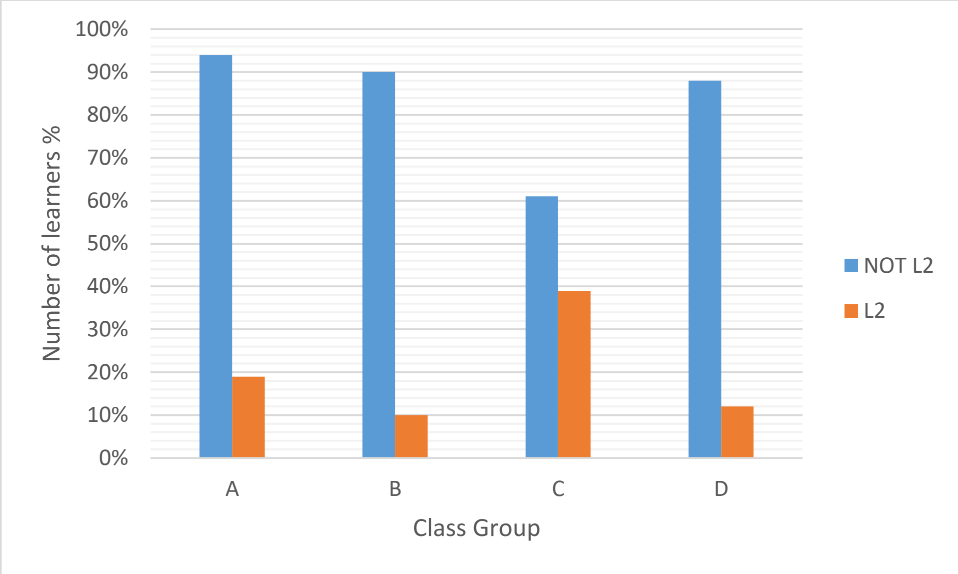


Figure 4. 8: Performances of learners at item 6

There were more learners who could not attain level 2, when compared to those who attained level 2 in all the class groups. Figure 4.8 shows that more than eighty per cent of learners in class groups A, B and D were operating below level 2, while in class C below forty per cent of learners were at level 2.

The pre-test was aimed at determining the geometrical level of thought of learners who participated in this study, according to the Van Hiele theory. To find out the Van Hiele geometric thought of learners, I used a Van Hiele test containing 6 items. Item 1(a) and 2(a) tested Van Hiele level 1 while items 1 (b-f), 2(b), 3, 4, 5 and 6 tested Van Hiele's level 2 as presented above.

For level 1 items 1(a) and 2(a), participating learners were found to be operating at pre-recognition level, referred to as L0 in the presentation. This means that they could not recognise plain figures. De Villiers (1996) points that at level 1, students recognise triangles, squares, parallelograms, and so forth by their shape, but they do not explicitly identify the properties of these figures. By Grade 8, participating learners should have passed the recognition level and are expected to be at the ordering level (level 3), as argued by Malloy (2002), who states that in an ideal world, students from pre-kindergarten through high school should learn to think and reason about geometry in a similar progression. Students in pre-kindergarten through Grade 2 would focus on the recognition/visualization level, students in Grades 2 to 5 on the analysis level, students in Grades 5 to 8 on the ordering level and Grade 8 and beyond would be on the deduction level. For the participating learners this was not the case as more learners were found to be way below their expected level.

For level 2 items, the majority of participating learners were found to have not reached level 2. Most of them could not identify properties of shapes and use appropriate vocabulary related to such properties. Teppo (1991), explains that students at level 2 can analyse component parts of the figures, for example opposite angles of parallelograms are congruent, but interrelationships between figures and properties cannot be explained. For level 2 items, the majority of learners failed to obtain a minimum of forty per cent in order to be competent at such level. In some level 2 items none of the participating learners could attain level 2.

From the presentation made above, it is evident that majority of learners were below each tested level including the control group class. For this study the participating learners appear to be between the precognition level and visual level. It therefore became important that help should be given to participating learners that could enable them to move from the precognition level to higher Van Hiele levels. I was equally inspired by Ding and Jones (2006) who state that effective instruction in geometry requires teachers to develop sound instructional strategies and knowledge of useful resources and activities. With this in mind, I found it suitable to implement an intervention program which incorporates visualisation strategies and the Van Hiele phases of instruction in order to help learners attain Van Hiele level 1 and 2 of geometric thought. The intervention included sound instructional strategies (Van Hiele phases of instruction) and it also involved useful resources and activities used to teach two-dimensional shapes (visual materials). I therefore went ahead and implemented the intervention program that particularly focused on level 1 and 2. In the next section, I present the lessons as presented by participating teachers which incorporated visualisation teaching strategies and the Van Hiele phases of instruction.

4.3 PRESENTATION OF LESSONS

In this section, I present nine lessons as taught by participating teachers. To teach the lessons, participating teachers used visualisation approaches to help learners attain level 1, 2 and beyond. In the lessons, the teachers also employed the Van Hiele theory — in particular the phases of instruction in order to help them plan and teach lessons. Each participating teacher taught three lessons. These lessons were the same for all the teachers.

4.3.1 A brief description of the lessons

The participating teachers each taught three lessons to Grade 8 learners on geometry. While the lessons were same, each teacher had his/her class group to teach. They all managed to teach the lessons, and below is a brief description of each lesson.

Lesson 1: The theme was geometry, and the topic was naming basic geometric shapes. The three teachers presented the lesson to their respective classes and the basic competency to be achieved at the end of this lesson was for learners to be able to name basic shapes and classify them as either triangles or quadrilaterals.

Lesson 2: was a continuation of lesson 1. The topic was classifying polygons. In this lesson the basic objective was for learners to be able to identify and name the following regular and irregular two-dimensional shapes: triangle, square, rectangle, rhombus, parallelogram, other quadrilaterals, pentagon, hexagon and circles in different orientations and further classify them as concave or convex.

Lesson 3: The topic for this lesson was properties of quadrilaterals. The basic objective of the lesson was for learners to be able to describe differences and similarities between squares and rectangles; rectangles and parallelograms; and squares and rhombuses in terms of: lengths of sides, angles in shapes (limited to right-angles, angles smaller than or greater than a right angle) and lines of symmetry.

4.3.2 Visualisation processes in the lessons presented

The overall aim of the study was to teach geometry lessons to Grade 8 learners using visualisation processes (of designing visuals, manipulating visuals, analysing visuals, interpreting visuals and visual language) in order to help learners migrate from one Van Hiele level to another. The Van Hiele theory, in particular the phases of instruction (of information, guided orientation, explication, free orientation and integration), were utilised by teachers during their lesson presentations. I therefore discuss the three lessons of each teacher following the structure below:

1. Visualisation processes: I present the extent of evidence observed against each visualisation process' criteria, as presented in the adopted analytical framework tool, and discuss each of the five visualisation processes.

2. Stimulus recall interviews: I present my one-on-one sessions with teachers which I conducted after each lesson observation, for the purpose of capturing teachers' perceptions and experiences of each lesson presented.
3. Focus group interviews: Here I report on participating teachers' personal experiences of using the Van Hiele phases and their experiences after taking part in the intervention programme.

Although each of the three teachers presented the same lesson, the above structure is presented in order to identify common and different visual teaching techniques used by the teachers in their classrooms. To identify these commonalities and differences, I present below the coding criteria which I used in order to identify the visualisation processes (discussed in Chapter 2) employed by teachers during their lesson presentations.

Coding descriptions

Rich evidence (RE)**:** Here the teacher demonstrates an abundance and big variety of the specific indicators. For example, under 'designing visuals', abundance would refer to the teacher bringing to the class three or more visual materials. It would also refer to the teacher engaging deeply with these visuals.

Medium evidence (ME)**:** Here the teacher only uses one or two visual materials and engages with them on a relatively superficial level of the specific indicators.

No evidence (NE)**:** Here the teacher does not use any visual materials and does not engage with any visual processes. I have abbreviated the visualisation processes as follows: designing visuals **(DV),** manipulating visuals **(MV),** visual language **(VL),** interpreting visuals **(IV)** and analysing visuals **(AV).**

For each visualisation process there are observable indicators that were observed and coded according to the coding descriptions above. For the analytical tools indicated on Table 3.6 in the methodology chapter the observable indicators are shown. There are four indicators for designing visuals, four indicators for manipulating visuals, four for visual language, three for interpreting visuals and three indicators for analysing visuals. It may be possible that a visualisation processes to have all three coding criteria of 'rich evidence', 'medium evidence' and 'no evidence', depending on how the observable indicator was used during the lesson. For such instances, the colours blue, orange and green may appear on one visualisation tool bar on the chart. Thus the bar showing such a visualisation process would be a ratio of the indicators.

The nine lessons as taught by teachers and analysis are presented below.

4.3.3 Lesson 1, Teacher M

4.3.3.1 Visualisation processes in the lesson

In this section I highlight how the visualisation processes occurred in Lesson 1 for Teacher M.

	DV	*MV*	*VL*	*IV*	*AV*
RE	*4*	*4*	*4*	*3*	*3*
ME	*0*	*0*	*0*	*0*	*0*
NE	*0*	*0*	*0*	*0*	*0*

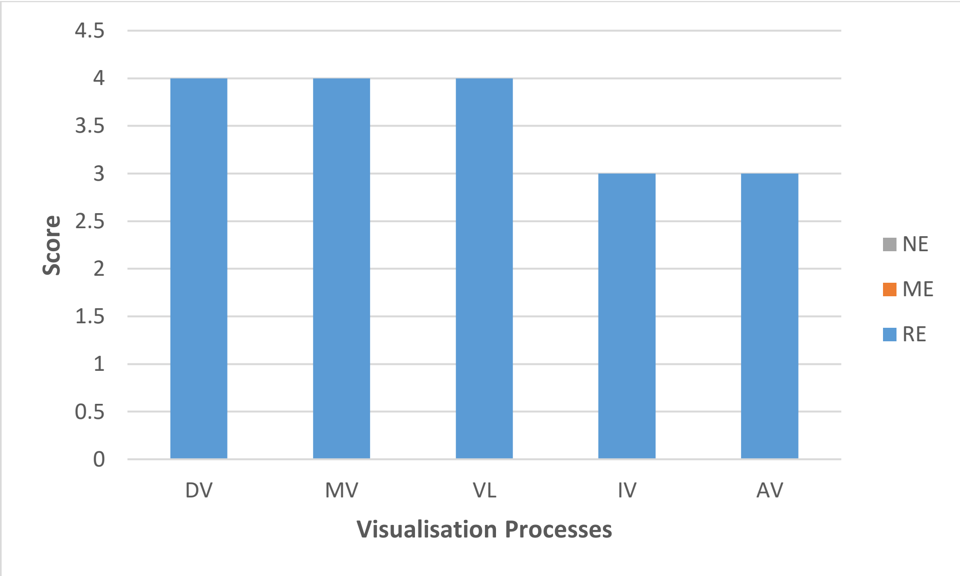


Figure 4. 9: Teacher M, Lesson 1: Visualisation processes, numerical scores

Figure 4.9 shows numerical score of the visualisation processes of Teacher M, in lesson 1. On the figure, rich evidence was recorded for all five of the visualisation processes. Below I describe how each of these visualisation processes occurred, resulting in the numerical scores shown in Figure 4.9 above.

Designing visuals: After writing Geometry' on the chalkboard, teacher M handed out blank sheets of paper to all learners in his class. On the paper, teacher M asked the learners to draw any geometric shape they knew. *Teacher M: "*

> *On that piece of paper I gave you, each should draw at least one shape that you know, and please try by all means not to copy from your friend. Use a marker pen and your shape should be visible from a distance.*

For teacher M this was a good introduction which was in line with Van Hiele's phase 1 of instruction as it states that it is by communicating with the learners that we learn about their pre-knowledge of the subject matter. Teacher M wanted to know the type of shapes learners already know.

All the visual material brought to class were aligned with the subject content. He brought blank paper which he gave learners to draw shapes they knew; he also brought pictures of a tricycle and a quad bike which were used to explain the mathematical concepts of 'triangle' and 'quadrilateral'.

Visual language: Teacher M continued with his lesson as he called to the front, five learners who presented their drawn shapes. As learners were presenting, teacher M encouraged them to show their shapes and say the names of these shapes. I observed through the learners' presentations, that their shapes included squares, triangles and circles, and others even drew shapes with more sides. Van Hiele asserts that after inquiry activities, the teacher should guide learners to uncover connections and to identify the focus of the subject matter. It was at this point that I observed Teacher M giving learners a scenario where he asked questions.

> *Teacher M: I want you to think now!!! Can you think of this world to have one shape? What I mean is if everything we see was in a form of a square like this one on the chalkboard. Everything around us, our clothes, our cars, I mean everything. What do you think our lives will be?*
>
> *Learner: sir the world will be very boring…*

As teacher M continued asking learners questions on how the world and our lives would be, many learners answered that the world indeed would be boring, and so would our lives as humans. Teacher M concluded

> *Surely the world would be boring, hence in geometry we study and appreciate spatial space around us.. In geometry therefore we study different shapes.*

The scenario given by teacher M helped learners to think more about why the study of shapes is important in our lives. To me this was an important aspect of using visual language that allows learners to produce visuals. For learners to answer the question of how the world would be, they needed to think and see all things in the form of a square for example, and make conclusions therefrom. By doing so, learners have to at some point make a connection between the teacher's verbal language and reality, in terms of visually seeing everything in the form of a square.

Manipulating Visuals: Teacher M then divided the learners into two groups according to the number of sides on the shape they had drawn earlier. Teacher M only wanted the class to be split into two groups of learners — those who had drawn three-sided shapes and those who had drawn four-sided shapes. As learners were moving and forming groups according to their shapes, I observed some confusion among some learners. Teacher M quickly noticed this, and gave them new blank paper to draw either three- or four-sided shapes. At this point Teacher M wanted to use a strategy that would allow learners to produce visuals, as he gave the learners who drew three-sided shapes the picture of the tricycle and those who drew four-sided shapes were given pictures of a quad bike.

> *Teacher M: with those picture I want you to use your markers and a ruler and connect all the wheels with a straight line. Make sure you connect wheels from the sides. Tell me what you have observed after connecting the wheels? What shape came up?*

Almost every learner in the class was seen observing in their groups, as one member of each group was connecting the wheels in the pictures. I noticed that the pictures used were big enough so that the learners could clearly see. Teacher M also pasted the two pictures on the chalkboard as he was explaining to learners what to do. Below is teacher M explaining what learners should do with the pictures of the tricycle and quad bike.

Figure 4. 10: Teacher M displaying visual material on the chalkboard

Interpreting visuals: Teacher M gave learners enough time to draw and observe what shape emerged after connecting the wheels. It is at this point that the teacher would immediately know if the visual material was properly identified and evaluated for the lesson. I observed the teacher focusing on two particular questions at this stage. He asked learners what shape emerged and how many sides the shape had. The two questions encouraged the learners to think about the pictures to the shapes (topic being taught).The questions were well answered by learners as the group with a quad bike responded that they had drawn a rectangle with four sides, while the group with a tricycle responded that they had drawn a triangle and it has three sides. For the Van Hiele phases, this could be referred to as the explication phase where the teacher takes care that learners advance the use of technical language with understanding through the exchange of ideas**.**

It is at this stage that the teacher introduced the new terminologies of triangle and quadrilateral. Teacher M said

> *now I want you to use and know the following two words as they are very much important and I shall ask you about them at the end of this lesson.*

He wrote quadrilateral and triangle on the chalk board. Teacher M explained the two concept by firstly defining the prefixes words in a quadrilateral and tricycle. He said

> *quad means four, as you can see the number of wheels in a quad bike is four, so you should remember quadrilateral as shapes with four sides like a quad bike has four wheels'.*

Analysing visuals: As teacher M continued to communicate with the learners with reference to the pictures of a quad bike and a tricycle, he used the time to deconstruct useful properties of quadrilaterals and triangles as he said

> *I want you to all know that all four-sided shapes are quadrilaterals, from the word quad bike with four wheels and all triangles have three sides like a tricycle that has three wheels. You must connect the number of sides to these two pictures.*

As teacher M was moving towards the end of his lesson, I observed the teacher taking out paper showing different shapes such as a kite, a square and a rectangle etc. The pages all showed three- and four-sided figures. The teacher showed learners a shape and asked if it was a triangle or a quadrilateral. As I continued to observe I noticed that the teacher was simply giving final tasks to challenge learners in the solving of problems as advised by Van Hiele in the last phase of instruction, free orientation. Learners were able to classify triangles shapes like right-angled triangle, scalene triangles. Squares, rectangle, rhombus were all classified as quadrilaterals.

4.3.3.2 Stimulus recall interview

In this section, I present the results from one-on-one in-depth interview which I did with Teacher M in order to gain a deeper understanding of what transpired during this lesson presentation. My interest was to find reasons behind the teacher's choice of the visual materials he used in this lesson. Below is an extract of our discussion:

> *R: Okay, why do you think the visual materials you used for this lesson were grade appropriate?*
>
> **TML1***: Yes, I am 100% sure that the pictures were suitable for learners, because I know that as were growing up all of us we like playing around with bicycles and so forth, in the streets, and I thought if I bring this on, they will be of interest to learners and maybe they would know what they are and they will be able to tell me what they are and quickly connect the pictures to the topic.*
>
> *R: how was it like to incorporate visualisation processes and the phases of instruction in your lesson? What worked what did not work well?*
>
> **TML1***: The way the processes and the Van Hiele phases were put together made it work easy. I did not think of them during the presentation, but yes I knew I was following the right order. I think everything worked just fine.*
>
> *R: What would you recommend from your lesson to other teachers?*
>
> **TML1***: I think teachers just need to always have some kind of visuals to help learners. Some learners don't really follow when we just teach definitions. It's funny for learners when they discover and connect what they learn in class to reality.*

The teacher felt that it was important to bring something of interest to the learners and what they already knew. He mentioned that for the visual materials that he had brought, even though some learners did not know their names (tricycle or quad bike), he was convinced that most of them had seen the visuals somewhere — even on television. He further explained that he was comfortable in using the visuals to teach the topic, as he (the teacher) had first-hand experience of playing with both a tricycle and a quad bike when he was growing up. Regarding the Van Hiele phases, the teacher indicated that it was the first time he was using the phases to teach, but he added that he was comfortable in using them in his lessons. He also recommended that other teachers should use the Van Hiele phases when planning.

4.3.4 Lesson 2, Teacher M

4.3.4.1 Visualisation processes in the lesson

	DV	*MV*	*VL*	*IV*	*AV*
RE	*4*	*4*	*4*	*2*	*3*
ME	*0*	*0*	*0*	*1*	*0*
NE	*0*	*0*	*0*	*0*	*0*

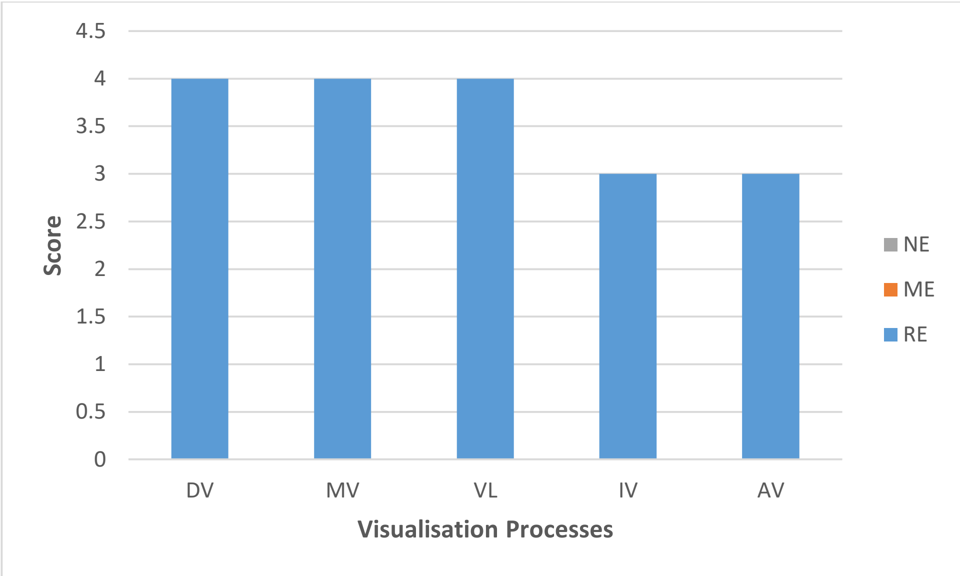


Figure 4. 11: Teacher M, Lesson 2 Visualisation processes, numerical scores

The Figure 4.11 shows a summary of the numerical scores of each of the five visualisation processes as per the coding criteria. On the figure, four of the visualisation process were coded ‘rich evidence’, according to the observable indicators of each visualisation process. One of the observable indicators of analysing visuals was coded ‘medium evidence’. Below, I present evidence of how each visualisation process occurred when incorporated with the Van Hiele phases resulting in the numerical scores presented.

Designing visuals: Teacher M brought to class all the visual materials required for this lesson. He brought pictures of a tricycle and a quad bike which he used in his introduction of the lesson, where he focused on the content learnt from the previous lesson. He also brought enough Geoboards for himself and his learners to use during the lesson. At this stage I observed that Teacher M was busy with the information phase according to the Van Hiele phases, as it is during this phase that the teacher places the learning materials at the learners’ disposal

(Clements & Battista, 1992) and engages them in a discussion, learning about their pre-knowledge. Hence, the learners were introduced to the subject matter to be studied at this stage. After the learners revealed what they had learnt from the previous lesson, teacher M distributed Geoboard to the learners.

Manipulating visuals: Teacher M instructed the learners to construct shapes on their Geoboards.

> *A Geoboard is used to explore properties of two dimensional shapes. Now on your Geoboards I want you construct any shape that you learnt from our previous lesson.*

My view is that the teacher used a Geoboard as visual material in a way that allowed learners to connect new concepts to what they already knew. Most learners were able to construct three- and four-sided shapes on their Geoboards. At this stage, I noticed that the teacher was guiding learners to uncover connections and to identify the focus of the subject matter. This was evident when two groups in his class constructed different shapes from the three- and four-sided shapes. One of the groups constructed a five-sided shape, while the other group constructed 3-D shape on the *geoboard* (see figure 4.12 below) Teacher M:

> *Waauuu, this is amazing, you have constructed a five sided shape, can I show it to the whole class?*

It is during this phase that the teacher guides the learners towards the content to be studied, while at the same time instilling the correct usage of geometric language. However, the learners' own language may also be accepted, but often under correction. At this stage I observed that the teacher used five-sided shape and not a pentagon, because the learners' own language was still acceptable.

Visual language: As learners were busy constructing different shapes, the teacher asked each member of the class to look at the Geoboard he had in his hand and observe the shape.

> *Look at my Geoboard, this five-sided shape is called a pentagon, and we have different kinds of pentagons, some with equal sides while others with one or all sides different in length. Can you construct on your Geoboard two pentagons, one with all sides equal and the other with one or all sides different.*

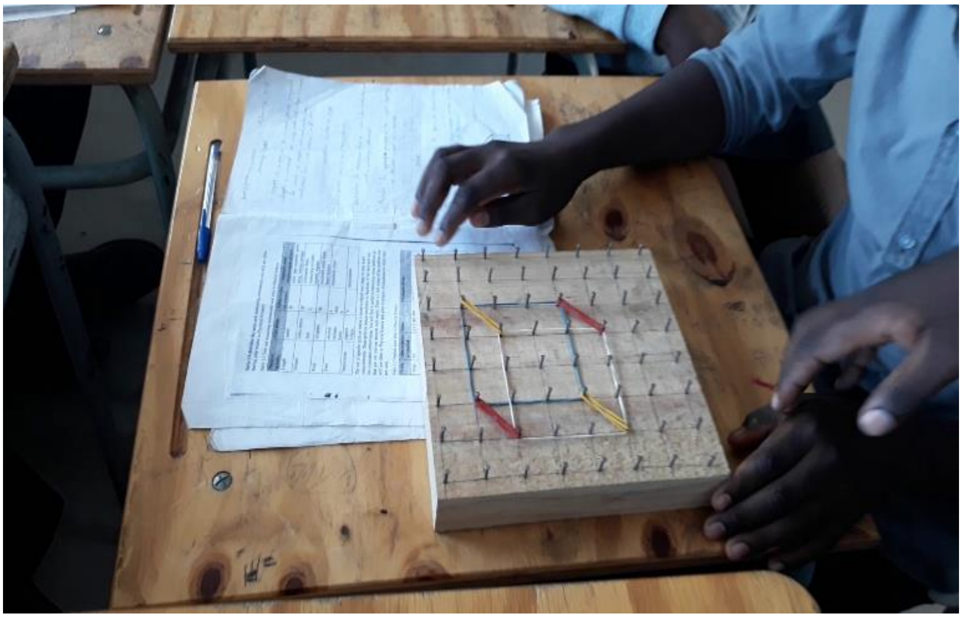


Figure 4. 12: A constructed 3-D shape by learners on the Geoboard

As I observed, I was enlightened to notice that is actually true that students can sometimes develop more than one level at the same time as argued by Gutierrez, Jaime and Fortuny (1991) when discussing the implications of the Van Hiele theory. I expected learners to reason and construct only up to the level of two-dimensional shapes. In other words, Van Hiele's broad statements "are not as black and white as they are often portrayed to be"' (Pegg & Davey, 1998, p.114) as evidenced by some learners in this class who already reasoned above other and constructed 3-D shape. It is important to note that teacher M introduced a geoboard to learners as only use to explore properties of 2-D shapes.

As I continued observing, I noticed that the teacher was trying to take care that the learners progressed to the use of technical language with understanding, through the exchange of ideas. As the learners were busy constructing two pentagons on their Geoboards, the teacher wrote 'regular' and 'irregular' on the chalkboard. He explained the concepts and encouraged the learners to use the words when referring to shapes. This, to my observation seemed that the teacher used a strategy that allowed learners to produce visuals. His strategy was to allow learners to first see how a regular or irregular shape looks on the Geoboard, before he explained the difference between the two concepts.

Analysing visuals: As the learners listened to the teacher explaining the difference between the concepts, Teacher M concluded his explanations by telling the learners to construct any shape with more than five sides.

> *now can you use your Geoboards and construct any shape with more than five sides, it can be seven, eight, nine or ten sides. Remember you should know if you shape is regular or irregular.*

This stage is known as the free orientation phase according to Van Hiele, where learners are challenged to solve problems. At this stage, learners needed to use their visual materials to construct different shapes. Debes (1969) would refer to this as the analysis visualisation process where the teacher encourages learners to work with the visuals in terms of the concepts that were taught.

As I observed further, learners were able to construct different polygons, some being regular while others were irregular. It is at this point that the teacher told the learners that a polygon is any shape joined by straight line and enclosed. As he explained the term polygon, the teacher used a cultural practice to help learners make visual conjunctions with his verbal explanation:

> *The word 'polygon' can be linked to the word polygamy, which means marrying more than one wife. This practice is common at our villages, like my grandfather has three wives'. So if we were in this class with him, he would be a triangle, because he has three wives and a triangle has three sides.*

To my observation, learners enjoyed the teacher's explanation as they joked about their relatives with more than one wife. Moreover, I observed the teacher to be constant with his mathematical terms as he labelled shapes with their geometrical names. For example a 6 sided polygon was labelled hexagon not as husband with six wives.

Interpreting visuals: Towards the end of the lesson, the teacher introduced another concept to learners, which to me was contrary to the Van Hiele theory where at the integration phase learners were to summarize their understanding of the topic. Similarly, the teacher also did not conclude his lesson using his visual materials as he was now teaching about concave and convex types of polygons. The teachers' feeling about this will be elaborated in during the interview session. It is in fact under interpreting visuals as processes where teachers are advised to conclude lessons using visual materials, hence medium coding on figure 4.11 above.

4.3.4.2 Stimulus recall interviews

During the interview session I was interested in finding out the teacher's experience of teaching using a Geoboard as a visual material, and how incorporating the phases of instruction had worked. My focus was to know an indicator from the teacher that convinced him that indeed learning took place. Below is our short discussion:

***R**: What would you specifically mention that shows learners have learnt one or two things in this lesson as a result of using the geoboard as a visual material?*

TML2*: mmmmm, I think that learners learnt a lot in this lesson as they could answer all my questions, like they could construct shapes on the Geoboard when I asked them to, they could mention the names of polygons depending on the numbers of sides. Some learners even extended and constructed 3-dimensional shapes on the Geoboard. So yes learners have learnt a lot and enjoyed the lesson.*

***R**: ok, now let's look at the focus of this programme. How did the incorporating the Van Hiele phase visualisation processes work?*

TML2*: mixing the phases and the tools is easy for me, but when I try to remember and follow each step or phase in the lesson it becomes a little difficult for me. It is working just fine to me.*

***R**: what exactly do you find difficult?*

TML2*: like for example to follow when to use the correct language, I think the language is on phase……….I cannot remember correctly, but I think the language should be at all phases.*

R*: lets just look the conclusion of this lesson…Yes here. You introduced new concepts*

TML2*:Oh yes I see that..actually I think the content for this lesson was too much, you see I could not complete the lesson on time.*

While the teachers' conclusion about incorporating visualisation strategies and the Van Hiele phases of instruction is that it is easy, I noted that indeed the teacher was not really concerned about which phase the lesson was. Instinctively though, he presented the lesson following the phases of instruction. When asked about the importance of teaching using visual materials such as a *Geoboard*, the teacher argued that when it comes to teaching shapes, the most important thing is the ability to distinguish shapes. He said that if, for example, one has seen a square and constructed it on a Geoboard, during examination time, when asked about the square it will automatically come to the learner's mind or memory and he/she will be able to differentiate it from other shapes and easily remember the properties. When asked why he could not conclude his lesson using his visual material, teacher M pointed out that the learning content for this lesson was a little more than it looked on the lesson plan.

4.3.5 Lesson 3, Teacher M

4.3.5.1 Visualisation processes in the lesson

	DV	*MV*	*VL*	*IV*	*AV*
RE	*4*	*4*	*4*	*3*	*3*
ME	*0*	*0*	*0*	*0*	*0*
NE	*0*	*0*	*0*	*0*	*0*

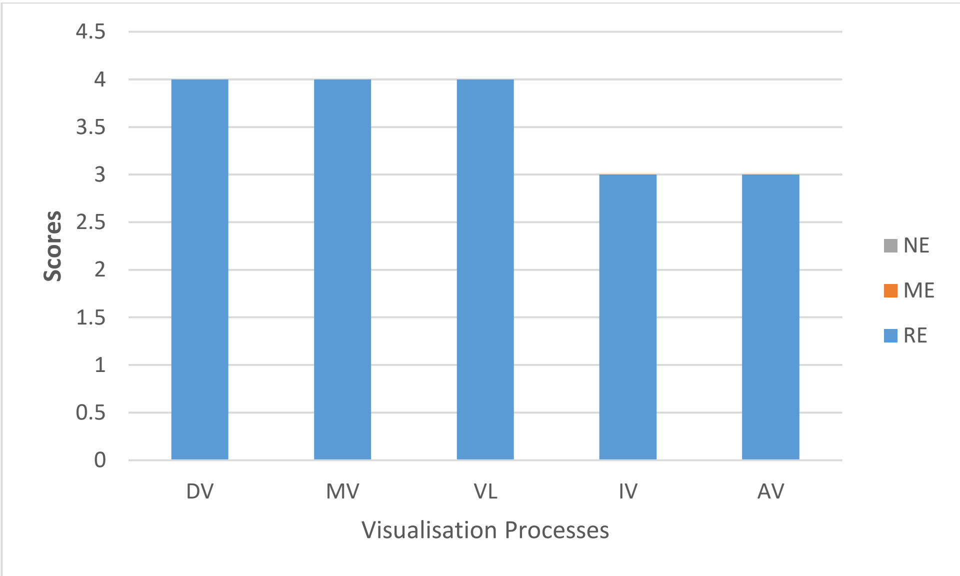


Figure 4. 13: Teacher M, Lesson 3: Visualisation processes, numerical scores

Figure 4.13 above indicates that all the five visualisation processes were coded 'rich evidence'. In this study, it means that teacher M used the processes well and all the indicators' objectives were met. Below I present the lesson

Designing visuals: For this lesson the teacher brought a *Geoboard* and rubber bands to class as his visual material to help him teach. As he introduced the lesson, he asked the learners what they had learnt in their previous lesson.

> *On your Geoboards construct for me a polygon which has five sides, it should be regular and convex. You can draw the shape first then construct it on your Geoboard'*

Most learners were aware of how a regular convex pentagon looked, but it took them time to make sure the sides were all equal when they were constructing on their *Geobaords*. The teacher then used a ruler and drew a regular pentagon on the chalkboard. Below is teacher M drawing his pentagon on the chalkboard.

Figure 4. 14: Teacher M drawing a regular pentagon

Manipulating visuals: Teacher M described a regular convex shape using his pentagon. He then asked the learners to remember the first lesson where they learnt about triangles and quadrilaterals.

> *do you still remember what we discussed in our first lesson? If so who can tell me what a quadrilateral is?*

A number of learners wanted to give their answers as to what they had learnt in the first lesson. They were able to define a quadrilateral. Teacher M then quickly instructed the learners to construct any quadrilateral on their *Geoboards*. By doing so, teacher M was fulfilling his role at phase 2 of the Van Hiele phases of instruction. This is because at this phase the teachers' main task is to ensure that students explore specific concepts by providing them with activities that can guide them towards the relationships of the next level. In this case, the next level was for the learners to explore properties of quadrilaterals. For visualisation processes, the teacher at this stage allowed the learners to use visuals materials in a way that allowed them to connect the new to what they already knew. This was so because by simply constructing quadrilaterals, learners were already exploring some properties of such quadrilaterals.

Visual language: At this point, the learners had finished constructing their quadrilaterals. I observed that the learners were all aware of quadrilaterals and successfully constructed them. Teacher M facilitated this processes as he went to each group and observed what they had constructed.

Teacher M paired the quadrilaterals as follows: square and rhombus, rectangle and parallelogram, trapezium and kite. Teacher M further assigned each group a quadrilateral. He asked them to first construct the quadrilateral on their *Geoboards*, and then find as many properties as they could find. Learners quickly constructed their quadrilaterals on their Geoboards. I observed that almost every member of the class was involved in constructing and the listing of properties. Teacher M allowed learners to present their findings. Below is a picture of learner presenting the properties of a square with a *Geoboard*. As the boy in the picture was listing the properties of a square, teacher M listened carefully and monitored the learners' technical language.

Figure 4. 15: Learner pointing and explaining properties of a square

Teacher M: show me the four lines which cut the square into halves!!

L: Here, 1, 2, 3 and this one four.

Teacher M: Ok, good. You are right, but those four lines have special names. These two are lines of symmetry and the two are diagonals (teacher M was pointing showing learners the lines on the Geoboard).

What Teacher M was doing at this stage (Explicitation phase) is very important in the developmental stages of learners' acceptable mathematics language. At this phase of learning it is advised that while learners present, the teachers' task is to make sure that learners use the correct mathematical language with understanding. The teacher may do this by immediately monitoring when a word was used or noting down wrongly used words and correcting them afterwards. Hence, learners' new knowledge is formed through experience and integrating it with past knowledge. Teacher M did as advised. For visualisation processes, Teacher M used a strategy that allowed the learners to produce visuals from the visual material (Geoboard). It was through the use of the Geoboards that learners discover important terminology regarding the topic.

Interpreting visuals: All the learners had completed presenting the properties they had found about their quadrilateral. Teacher M told the learners to remove all the rubber bands from their Geoboards. He then gave each group a small piece of paper that showed properties of different quadrilaterals. For example, Teacher M gave group A the properties of a rectangle as follows:

Construct a figure with the following properties

It has four sides

Opposite sides are equal

It has four angles

Two lines of symmetry

Two diagonals

Teacher M then asked group A to construct such a quadrilateral showing all the mentioned properties on their *Geoboard.*

This was a free orientation phase according to the Van Hiele phases of instruction as this phase allows learners to discover different ways of finding solutions to problems. The teacher assigns tasks for which there are several solution strategies, and encourages the students to find their own way to the solution (Clements & Battista, 1992). I also observed that Teacher M was encouraging learners to work with the visual material *(Geoboard)* in terms of concepts that were taught, and this was a ways of allowing the learners to analyse visuals according to Debes (1969).

Interpreting visuals: As the learners were reading the properties which were provided to them by the teacher, I observed that many of them were able to construct their quadrilaterals and show the properties. However, teacher M needed to be convinced that learning had indeed taken place. I observed teacher M inviting a boy to the front of the classroom and asked to summarise the learning content for the day. At this phase, no new learning material is to be introduced to the learners so the teacher concluded the lesson by asking the learners to summarise the learning content of the day using a Geoboard.

4.3.5.2 *Stimulus recall interviews*

With this lesson's follow up recall interview, I specifically focused on the teacher's experience on teaching quadrilaterals. I focused on this aspect because I wanted to know the teacher's perception of using visual materials to teach properties of quadrilaterals. Below is our short discussion.

> *R: In this lesson you taught about properties of quadrilaterals using visual teaching aid such as Geoboard, plain papers and chalkboard. What did you like most ad least about using the visual tools in this lesson?*
>
> **TML3***: Eeee, for me, what I liked most was the way learners participated in the lesson. Learners were very quick to find construct the shapes and to find their properties. I thought the properties of all quadrilaterals will take maybe 3 days, but I think the Geoboard made this look easy. I liked everything about the lesson.*
>
> *R: Can you describe perhaps the importance of using visualisation strategies to teach two-dimensional shapes?*
>
> **TML3***: I think, two dimensional shapes are the basics for geometry, so learners need to know them before they continue with their geometry study. So for me using visualisation can help learners to remember the shape easily and ..yes.*

Moving on with our discussion, Teacher M pointed out that a Geoboard is something he had never used before. However he agreed that it is a useful tool to teach properties of quadrilaterals. His learners were able to construct all the quadrilaterals. Teacher M further suggested that learners could benefit more if they were introduced to using a Geoboard as a visual tool at primary level. From Teacher M's input, I noticed that it is difficult to teach using visual materials on topics that the teacher has little knowledge about.

4.3.6 Lesson 1, Teacher S

4.3.6.1 Visualisation processes

	DV	*MV*	*VL*	*IV*	*AV*
RE	*4*	*4*	*4*	*3*	*3*
ME	*0*	*0*	*0*	*0*	*0*
NE	*0*	*0*	*0*	*0*	*0*

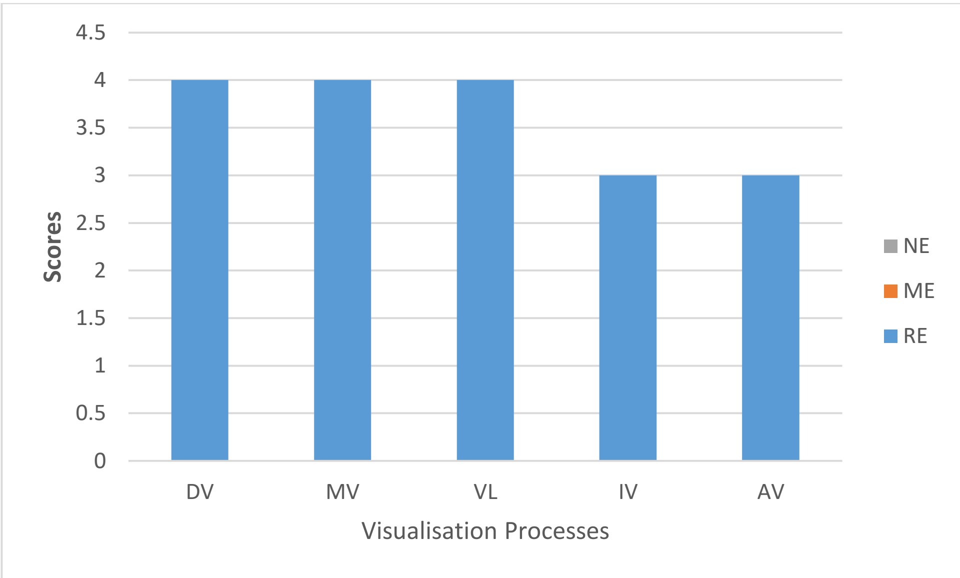


Figure 4. 16: Teacher S, Lesson 1: Visualisation processes, numerical scores

Designing visuals: In this lesson, teacher S used pictures of a tricycle, a quad-bike, blank paper and a chalkboard as her visual materials. To introduce the lesson, Teacher S gave out blank sheets of paper and asked the learners to draw basic shapes they know.

> *On those paper I gave you, I want you to draw any geometric shape you remember from Grade 7, 6 or 5. Draw a bigger shape that I can see from here'*

Teacher S provided the learners with markers to use so that their shapes could be seen from a distance. As she was walking around the class, she started verbally providing learners with words like "You can draw a square/a kite/a triangle.

I observed that the teacher conversed at length with her learners at this early introduction of the lesson. For Van Hiele this is known as the information phase and a very crucial phase of instruction as it is at this phase that teachers communicate with the learners in order to determine their pre-knowledge of the subject matter. Most learners in Teacher S's class were able to draw the basic shapes as expected. The learners presented their shapes and most of them were visible to all.

Manipulating visuals: After the learners' presentations, Teacher pasted pictures of a quad bike and a tricycle on the chalkboard. She asked learners for the names of the pictures shown. Most learners were unaware of the tricycle, but agreed that they had seen it. Teacher S then gave each pair a copy of a tricycle and a quad bike. She asked each pair to connect all wheels in a quad bike and in a tricycle. The learners used rulers and markers to connect.

> *Teacher S: After connecting those wheels, what did you observe? (Teacher S asked one pair to respond)*
>
> *L: The lines in this quad bike are four, and in this tricycle they are 3 lines.*
>
> *Teacher S: OK, very good another group?*

As Teacher S continued asking each pair to say what they had observed, she finally asked the whole class. "*How do we call shapes with four lines, which is in that quad bike?"* Most learners said it was a rectangle, and for the tricycle they said it was a triangle. It was at this point that Teachers S spelled out the main aim of her lesson. She said "*Today I want you to be able to classify between triangles and quadrilaterals."*

In doing all these activities with the learners, Teacher S was busy with the guided orientation phase. It is advised that at this phase much of the material should be short tasks designed to elicit specific responses. Joining the wheels was a short activity and required a specific response from learners. At the same time, Teacher S had critically thought about how learners would manipulate visual information from the pictures to help them to make a solid interpretation of their meaning. Through the pictures, learners were able to decompose useful properties of quadrilaterals and triangles especially that of number of sides.

Visual language: Teacher S was consistent with her mathematical language. She asked learners "*Which key property makes triangles different from quadrilaterals?"* Learners were able to respond, saying that it is the number of sides. Teacher S used the learners' responses to continue with her lesson. Teacher S used her visual materials in conjunction with her verbal explanations. For example, she would point at the number of wheels, counting one, two, three, and correspondingly count the number of sides in a triangle. Her language was clear as she told learners that

> *the visuals (poster) are not necessary the triangles or quadrilaterals, but are used as representations of the shapes and I 'only wanted you to use them to remember the shapes by the number of wheels'.*

For this lesson Teacher S made sure that the learners' technical language was developed with understanding through her verbal explanations**.**

Analysing visuals: Teacher S communicated to the learners using her visual materials, as she explained the properties of triangles and quadrilateral by using pictures of a tricycle and a quad-bike. The visual materials chosen for the lesson deconstructed properties of triangles and quadrilaterals.

Interpreting visuals: Teacher S identified and evaluated visual materials for the lesson. Her explanations were clear to learners as she use questions to help learners make connections. She asked learners

> *look at how a quadrilateral is, where have you seen the representation of a four-sided shape at your homes?*

This question was appropriate for learners as it helped them to visually connect something they had seen at home to what they learned at school. For this question, a girl in the class responded that *'Our garden is like a quadrilateral'*

Teacher S drew different representations of quadrilaterals and triangles and asked the learners to identify them. They were able to identify them as they first counted the number of sides then gave proper classifications. In her conclusion, the teacher used her visual materials as she drew columns on the chalkboard and asked the learners to classify different shapes as triangles or quadrilaterals. Below is the assessment activity teacher S used during her lesson. On the right-hand side of the chalkboard teacher S displays her visual pictures in figure 4.17 below, which in turn were to be used by learners in the table below to indicate whether a shape was a triangle or a quadrilateral.

Figure 4. 17: Teacher S displaying visuals and assessing learners on the chalkboard

4.3.6.2 Stimulus recall interview

In this session with Teacher S, I wanted to find out what she experienced after using such visual materials to teach the topic. Teacher S was quick to point out that the topic looks easy especially to teachers, but it is confusing to learners if they are just taught definitions of triangles and quadrilaterals without visual support to help them make a clear distinction between the two. Below is an extract of our conversation during the session.

> *R: Ok, Thank-you Ms S. so how did you find the use of visuals to teach the difference between triangles and quadrilaterals?*
>
> **TSL1***: Eeee for the pictures of tricycle and quad bike was good for the lesson. The pictures helped my learners to connect something they know. My learners also could just count the number of wheels and then, say ok, if it is 3 wheels, then it is a triangle and if it is 4 wheels then it is quadrilateral. My learners were able to do that with every shape I gave them.*
>
> *R: Did you find it difficult to incorporate visualisation strategies and Van Hiele phases during the presentation of the lessons?*

TSL1: *For Me, the phases of instructions I know them. It's how I teach my lessons but I do not really put much time when I plan the lessons. To combine the two was easy for me, it is like they are the same concepts.*

R: would you recommend someone to teach this topic using the visual materials you used today? If so why, or why not?

TSL1: *there are many materials a person can use you see, so for me I will prefer to use the two because they relate nicely to the topic and learners enjoy them.*

The teacher also indicated that it became easy for learners to find other shapes in the classroom that represented quadrilaterals especially. She pointed out that the learners were able to identify window frames, the chalkboard and the tops of their desks as forming quadrilateral shapes. Besides remembering the triangles and quadrilaterals by using images of quad-bike and tricycles, learners were extended to other visuals in the classroom, such as window frames, tops of their desks and the chalkboard to be quadrilaterals.

4.3.7 Lesson 2, Teacher S

4.3.7.1 Visualisation processes

	DV	*MV*	*VL*	*IV*	*AV*
RE	*4*	*4*	*4*	*3*	*3*
ME	*0*	*0*	*0*	*0*	*0*
NE	*0*	*0*	*0*	*0*	*0*

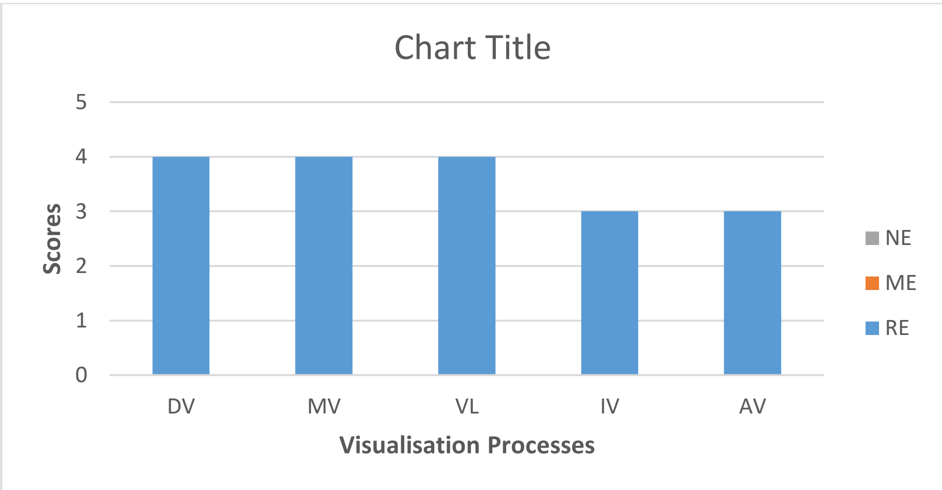


Figure 4. 18: Teacher S, Lesson 2: Visualisation processes, numerical scores

The numerical score on Figure 4.18 shows how Teacher S used visual processes in her lesson. The scores indicate that there was 'rich evidence' that teacher S designed, manipulated and analysed her visual material. The evidence of what was observed is presented below.

Designing visuals: Teacher S brought *Geoboards*, rubber bands and blank paper to class. She also used the chalkboard to present her lesson. In her introduction, teacher S pasted her tricycle and quad bike on the chalkboard. She did so to help learners remember what they had learnt in the previous lesson, as she asked *"Look at the pictures on the chalkboard. Who can tell me what we discussed in the previous lesson?"* A boy from the class responded *"We learnt about triangles and quadrilaterals. Triangles have three sides and quadrilaterals have four sides'*. This was a good summary of the main concepts that were taught by teacher S. In my observation this was a good start, as in the next activity the teacher gave *Geoboards* to learners and asked them to construct any shape from the previous lesson.

Manipulating visuals: The next activity for learners was easy as I observed that some learners started constructing different shapes before the teacher's instructions were complete. Teacher S's visual material *(the Geoboard*) allowed learners to connect new concepts to what they already know. This was evident because some learners found constructing quadrilaterals and triangles easy, and they started constructing other shapes. She used this to her advantage in her lesson presentation as she took a *Geoboard* where a learner had constructed a six-sided shape and showed it to the class.

Teacher S wrote the word 'polygons' as she said "*Look at what others have constructed Can you tell me if this is a quadrilateral or a triangle?"* Learners were not aware of the special name of the six-sided shape Teacher S had shown them, as I could hear and observe the learners arguing and others saying "*It's a quadrilateral and some said it is a square"*. To me this was the correct time that Teacher S used to guide learners to uncover connections and identify the focus of the lesson.

Visual language: As learners continued with their debate, Teacher S asked the learners to pay attention as she gave the correct name for the shape she had on the *Geoboard*. Teacher S said "*This shape is not a triangle or a quadrilateral, but it is a polygon."* In my observation, learners were not aware of what a polygon is. Teacher S demonstrated the proper use of visual language as she explained the word 'polygon' to learners. She stressed that a polygon should have straight lines which enclose to form a shape. She drew two lines on the chalkboard and asked learners if they could form a shape with two lines. As learners were thinking, a girl said "*It is impossible to make a shape with two lines."* The teacher then asked them if they could make a shape with three lines, and the whole class said yes, and they all drew triangles. Teacher S then explained the word ‘poly’ as referring ‘many’ and ‘polygon’ means a ‘shape with many sides’. However, “*You should know that different polygons have different names.”* Teacher S said.

Teacher S then gave names of polygons from three sides up to ten sides and encouraged learners to study and remember them. No questions were asked by learners.

Analysing visual: Moving on in the lesson, Teacher S encouraged her learners to work with the *Geoboard* to construct a rectangle and a square. She stressed that every group should have a square and a rectangle on their Geoboards. “*Forget about the names of those two shapes and look at their sides, what you notice?” teacher S asked the learners.* Learners were seen observing the two shapes and a boy said that “*the sides of the other shape are all equal, and the other one the two sides are only the same, but different to other two.”* At this point, Teacher S was again introducing two important concepts about shapes. She said ”G*ood observation -if the sides of a shape are all equal we call it a regular shape, and if one or all sides are not equal to the other then we call it an irregular shape.”* What was important here was that learners were able to analyse the shapes visually as they could see them.

Throughout the lesson the teacher used her visual materials and encouraged the learners to work with the visuals material as well in terms of the concepts that were taught. Teacher S later in the lesson, introduced two other concepts of concave and convex to classify polygons. She said for a polygon to be concave one of the corners should point inside the shape itself. Learners were able to follow her instructions hence in Figure 4.19 (next page) , a learner was able to draw a correct representation of a concave seven-sided polygon and construct it on the Geoboard.

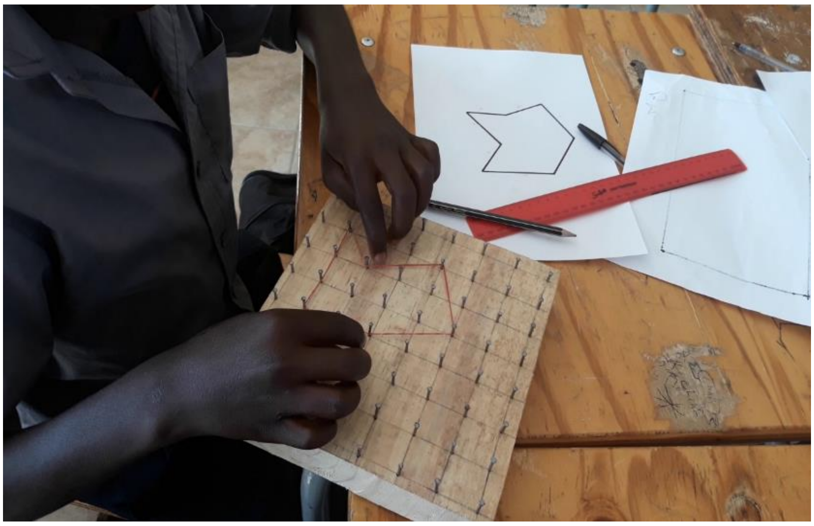


Figure 4. 19: Learner constructing his drawn shape on the Geoboard

Interpreting visuals: Teacher S evaluated her visual materials as she used them concurrently during her lesson presentation. For example, teacher S constructed a triangle on her Geoboard. She then asked the learners to identify it as a quadrilateral or a triangle. She then drew the same triangle on the chalkboard. She did this to persuade learners that they should know the names of shapes, be able to construct the shapes and finally be able to draw such shapes using ruler, pencil and paper.

4.3.7.2 Stimulus recall interviews

During the interview session with the teacher I wanted to find out the experience she had, using a Geoboard to teach the properties of quadrilaterals. Also, I wanted to hear her perceptions about incorporating the Van Hiele phases in her lessons. Below is a short extract of our session

> *R: Yes Ms. S. So teaching about polygon to grade 8 using a visual tool Geoboard. Can you share your experiences after using such a tool?*
>
> **TSL2***: Yes, I think so that learners learnt a lot in this lesson because of the way how I saw the learners engaged in doing all those constructions. Learners also could draw the shapes and construct them on their Geoboards, and also when I constructed a shape and ask for the name, my learners were able to say the correct names of polygon which is what I wanted to achieve. For me I liked teaching using the Geoboard, because most of the time learners were busy, even though they made noise but they were just arguing on how to construct correct shapes.*
>
> *R: What useful concept were you able to teach using the geoboard today?*
>
> **TSL2***: For me really today, I so that it is easy to teach using this Geoboard. Like, it is difficult to teach concepts of regular and irregular, and the other of con..What what… because on the Geoboard you just tell learners and they construct and they can see the difference. But when you try to explain, sometime the English also goes somewhere….The geoboard is very good, just that it is difficult to make.*

As I engaged more with the teacher on her experience of teaching quadrilaterals using a Geoboard, the teacher indicated how it made her lesson easy and interesting for learners. She argued that using a Geoboard reduced the amount of writing she normally does as everyone in the class was ‘hands on’ using the Geoboards.

4.3.8 Lesson 3, teacher S

4.3.8.1 Visualisation processes

	DV	*MV*	*VL*	*IV*	*AV*
RE	*4*	*4*	*4*	*3*	*3*
ME	*0*	*0*	*0*	*0*	*0*
NE	*0*	*0*	*0*	*0*	*0*

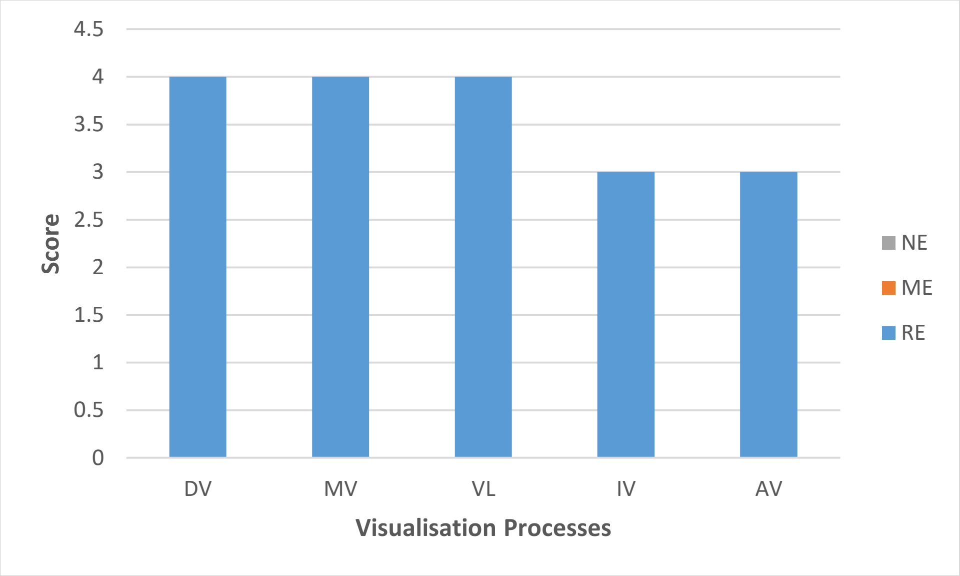


Figure 4. 20: Teacher S, Lesson 3: Visualisation processes, numerical scores

Designing visuals: Teacher S brought *Geoboards*, rubber bands and blank sheets of paper to class. She also used the chalkboard more to explain some concepts. To introduce her lesson, Teacher S gave the learners *Geoboards* and rubber bands and asked them to construct any shape from the previous lesson. I observed that most learners were constructing concave shapes with a number of sides. Teacher S moved around the class as she was observing what learners were doing. "*Ok I can see that you have nice shapes on your Geoboards, now can you remove all your rubber bands and construct for me any quadrilateral."* Teacher S's instructions were meant to get learners' attention and focus on the topic for the day which was already written on the chalkboard.

Visual language: Teacher S demonstrated an acceptable use of mathematical language as she explained with the use of shapes what she meant by 'properties'. She took a *Geoboard* from a learner who had constructed a square, showed it to the class and asked them "*What make this* *shape to be a square?"* For a moment the learners were silent as they looked at closely at the *Geoboard*. One boy answered "*It is a square because it has all the sides equal"* This was important for learners to first understand what they are to learn before they learn.

Teacher S then paired the quadrilaterals as square and rhombus, rectangle and parallelogram, trapezium and kite. She further divided the quadrilaterals among the six groups in her class. Each group was told to construct their assigned quadrilateral on the Geoboard and find as many properties on it as they could.

Analysing visuals: As the lesson went on, Teacher S asked the learners to present their findings. At this stage I observed that the teacher's questions were appropriately phrased and prompted critical thinking among learners. For example, as learners were presenting, they referred to the two diagonals of a kite as 'two inside lines'. Teacher S then drew a kite and with two such lines and asked learners "*How do we call the lines drawn inside this kite as shown here?"* Some learners said 'joining lines', and then teacher S introduced the new terminology of 'diagonal' and wrote it on the chalkboard. Below, Figure 4.21 shows Teacher S holding a visual representation of her kite. What is important here is to note that Teacher S uses visual material to explain the concepts to learners.

Figure 4. 21: Teacher S displaying and asking learners the names of the two lines in a kite

Everyday language was also observed when teacher S explained the opposite angles in a kite. She explained to a learner the word 'opposite by asking the leading question "*How do you call someone who is sitting directly facing you?"* The boy said '*opposite neighbour'.* The teacher capitalised on the word ‘opposite’ and pointed at the two opposite angles of a kite to explain what she meant by the term ‘opposite angles’.

Interpreting visuals: The teacher encouraged the learners to work specifically with their *Geoboards* in terms of the concepts that they were taught. Teacher S always encouraged the learners to show by pointing, the sides or angles which were equal if any, when they were presenting. At the end of the lesson, the teacher asked a member of the class to summarise the content learnt.

4.3.8.2 Stimulus recall interviews

During the interview session with Teacher S on her experience with the use of the visual materials, I particularly wanted to know if the teacher felt the need to bring extra visual materials to class.

> ***R****: Ok thank you! I also observed into the lesson, a part where you gave learners A4 papers and you encouraged them to look at A4 as a rectangle, do you think that was a perfect representation of a rectangle.*
>
> **TSL3***: Yes, because in actual terms an A4 papers is a rectangle and if we look at the properties of a rectangle and compare them to an A4 paper it is just the same, that’s why I used it in that capacity.*
>
> ***R****: OK let’s look at the Van Hiele phase did they help?*
>
> **TSL3***: I think the phase only help when planning the lesson, yes one must remember the circle of the phases, but for me they come natural because I know that now I need to change direction and focus on this or now I must test if learners understand.*
>
> ***R****: What would you recommend from your lesson for other mathematics teachers?*
>
> **TSL3***: For this lesson I would firstly recommend the Geoboard, it is really very nice to use. The other thing will be for them to plan well their lesson using the phases.*

In this lesson the teacher supplemented her *Geoboard* with a rectangular piece of paper where she encouraged learners to look for the properties of a rectangle. Teacher S argued that an A4 page is a rectangle and can be seen by learners visually. She agreed that visualisation as a concept is very useful in mathematics as it helps learners to see difficult concepts as very easy, because learners can touch and see what is being taught to them. Even though some visual materials are easy to find and use in class, Teacher S however notes that it takes time to always find visuals that can be linked to every topic in mathematics.

4.3.9 Lesson 1, Teacher C

4.3.9.1 Visualisation Processes

Figure 4.20 below shows the summary of the numerical scores of each of the five visualisation process as per the coding criteria. The way the teacher used each of these processes is discussed thereafter.

	DV	*MV*	*VL*	*IV*	*AV*
RE	*3*	*2*	*1*	*2*	*2*
ME	*1*	*1*	*2*	*1*	*1*
NE	*0*	*1*	*1*	*0*	*0*

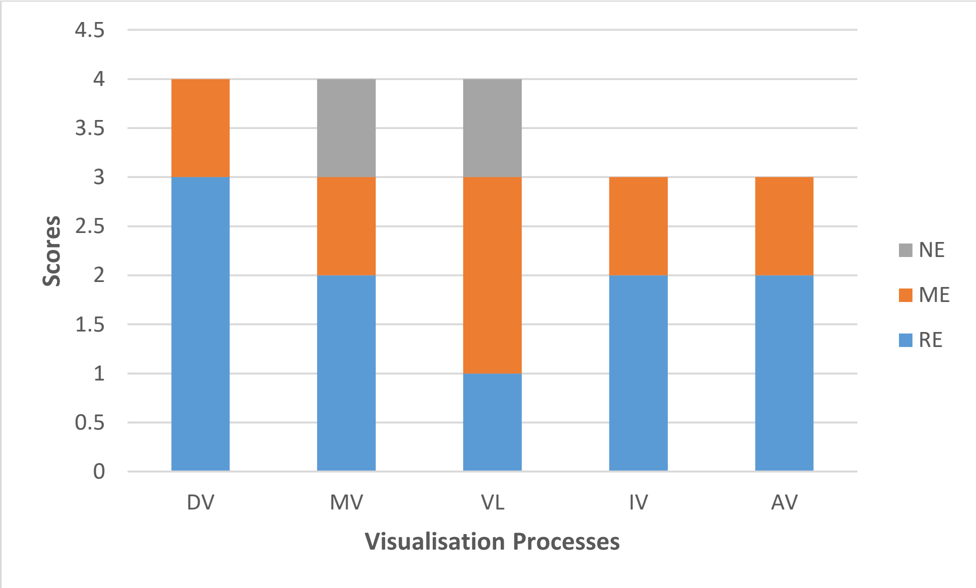


Figure 4. 22: Teacher C, Lesson 1: Visualisation processes, numerical scores

Figure 4.20 shows teacher C's lesson 1 visualisation scores. The figure indicates that no evidence was recorded under manipulating visuals and visual language. Medium evidence was recorded in all the visualisation processes. How this happened is discussed below.

Designing visuals: Teacher C brought to class visual materials such as blank sheets of paper and pictures of a tricycle and a quad bike. In his introduction, he asked the learners to use their blank paper to draw any geometrical shape they could remember.

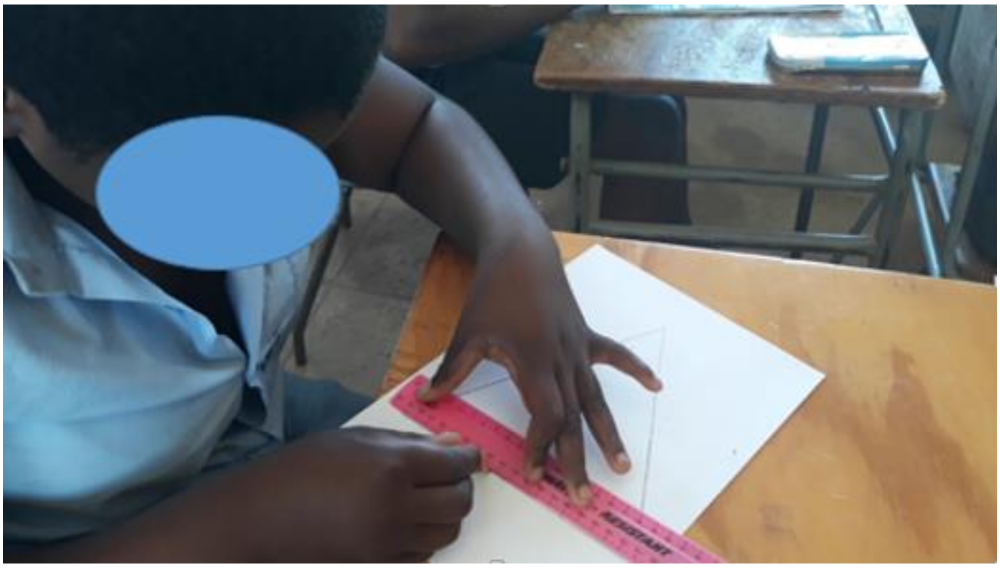


Figure 4. 23: A learner using materials as provided by the teacher to draw a geometrical shape

Manipulating visuals: When they had completed their drawings, teacher C asked the learners to show each other their pictures. He then asked four members from the class to come out in front and show other learners their shapes. While they were presenting these, teacher C encouraged the learners to say the names of the shapes they had drawn. I observed that most learners in the class drew squares, triangles, rectangle, circles and many other shapes. Many were aware of the basic geometric shape. Teacher C then said "*In today's lesson I want you to know and differentiate between triangles and quadrilaterals"* While this was the aim of the lesson, Teacher C did not lead learners to uncover the learning arena by themselves, nor did he use visual material to help learners make the connections.

Visual language: Teacher C then defined a triangle by writing the definition on the chalkboard. He wrote "*Triangles are shapes with three sides and quadrilaterals are all shapes with four sides."* Learners were to copy the definitions on the chalkboard. Teacher C then asked learners to list all shapes that have three sides. Only one boy was able to mention a right-angled triangle and an isosceles triangle. For shapes with four sides learners were able to mention at least four shapes. However, they did not mention a parallelogram and a trapezium.

Analysing visuals: Teacher C wrote the names of all the triangles and all quadrilaterals. He told learners that "*You have to know all these names."* I observed that learners were somehow not sure of what they were really taught, as I observed some learners even closing their eyes to memorise the names of the shapes written on the chalkboard.

Interpreting visuals: Although the teacher had brought some visual materials to class, not all of them were used during the presentation. It was only at the end of the lesson he showed learners the pictures of a tricycle and a quad bike. He then asked a boy in the class;

Teacher C: What are the names of these two things on these pictures?

Learner: This one is a motor bike this one is a walker.

Teacher C: OK, we have special names of this pictures, this one is a quad bike and this other one is a tricycle.

In conclusion, teacher said the two pictures should help learners to remember a quadrilateral ad a tricycle, because the name quadrilateral is from the name quad bike and it has four wheels. He said the same for triangles.

Teacher C's lesson was very short and did not use the visual materials as expected and planned. The Van Hiele phase were partially observed in the lesson plan but not fully put into practice. Below I tried to find out why this was so.

4.3.9.2 Stimulus recall interviews

R: do you think using visual materials to teach can help learners understand the concepts better?

TCL1*: I think it is important to always give chance to learners to relate what they are taught in class to something at their homes. This will always help them to remember even difficult concepts.*

R: In the lesson that you just presented, do feel you have utilised or used your visual material well?

TCL1*: No, I did not use them properly, I don't know why but I forgot them at some point in the lesson.*

In this lesson, Teacher C did not actually display his pictures of the tricycle and quad bike. However, in his conclusion of the lesson, he was holding the two pictures showing them to the class. He said that after teaching the concepts, it was good to finally relate what learners learnt to what they know. I further engaged with the teacher and asked if he was ready to continue with other lessons. He assured me that he was just a little nervous and that he was used to teaching without materials, but that he would make use of them. I also tried to ask him about the Van Hiele phases, as maybe he was not sure how to use them. The teacher said in fact he could remember the phases when he was teaching and he recommended them for others to use, especially when planning lessons.

4.3.10 Lesson 2, Teacher C

4.3.10.1 Visualisation Processes

Figure 4.22 below shows the numeric scores of the individual visualisation processes as per the coding criteria. The diagram indicates that 'rich evidence' was recorded under all visualisation processes as seen below.

	DV	*MV*	*VL*	*IV*	*AV*
RE	*4*	*4*	*4*	*3*	*3*
ME	*0*	*0*	*0*	*0*	*0*
NE	*0*	*0*	*0*	*0*	*0*

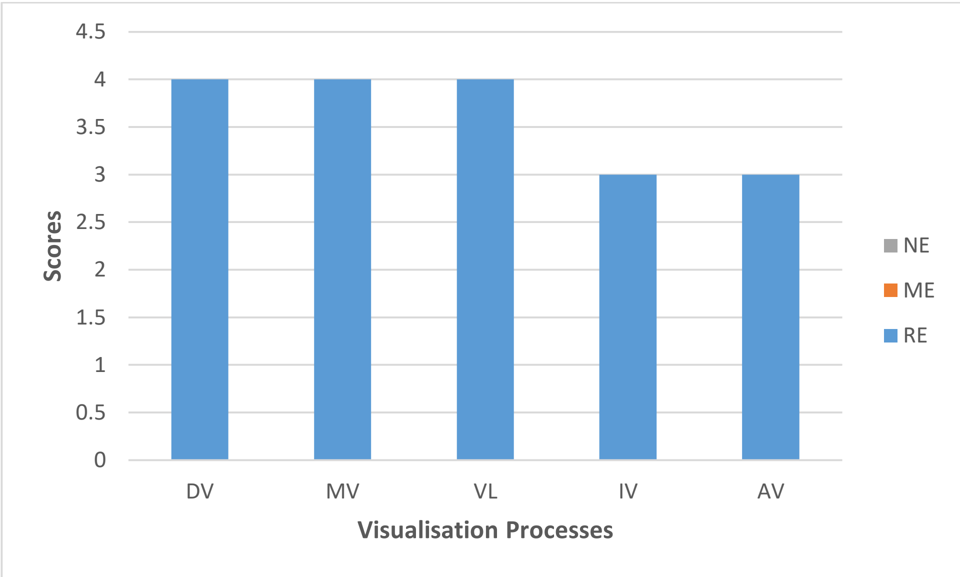


Figure 4. 24: Teacher C, lesson 2, Visualisation processes, numerical scores

Designing visuals: Teacher C brought *Geoboards*, rubber bands and blank sheet of paper to class. The teacher constructed shapes on the *Geoboard* using rubber bands to manipulate different shapes. In his introduction, he referred to the visual materials used in the previous lesson to help learners remember.

Manipulating visuals: Teacher C used different representations during his lesson. For example, the teacher constructed a kite on his *Geoboard*, he drew a kite on a page and also drew a kite on the chalkboard. All these representations of shapes are important especially when leading learners from concrete materials to abstract (on paper).

Visual language: Teacher C demonstrated proper use of visual language as he explained and responded to learners using simple language. Teacher C was observed responding to a question by a girl who asked how many sides a circle has. While not giving a straight-forward answer, Teacher C praised the girl for a good question and told all learners in the class to use their *Geoboards* and rubber bands and construct a circle, then count the number of sides. Learners were seen very busy arguing and stretching the rubber bands but they could not construct a circle on the Geoboard. Teacher C then said "*a circle has no sides.*"

Interpreting visuals: The teacher's questions during the lesson were properly and appropriately phrased and prompted critical thinking among the learners. For example, teacher C asked the learners to construct a quadrilateral and then divide it to form two triangles. He allowed learners to play with the *Geoboard* in order to get used to it before his main lesson presentation.

Analysing visuals: The teacher encouraged the learners to specifically work with their Geoboards in terms of the concepts that were taught. For example, the teacher encouraged the learners to show by pointing, the sides which were equal in a pentagon. Below is a picture of Teacher C confirming by counting the number of sides of one of the group's constructed pentagon.

Figure 4. 25: Teacher C seen confirming the number of sides of a pentagon

4.3.10.2 Stimulus recall interviews

During the stimulus recall interview that I had with Teacher C, he clearly stated that in most cases learners get confused in the naming of polygons. Getting to construct these polygons and attaching their names to the number of sides is very important as learners move on with their learning of polygons.

> ***R****: you concentrated much on the constructions and drawing of polygons in this lesson than their names. Why was that so?*
>
> **TCL2***: For me the picture is very important to learners. Learners need to get a clear picture of the polygons and then they can know their names, that's why the Geoboard was very good to use. It is just like us people, we need to know the face of a person then you will not forget the name.*
>
> ***R:*** *so how did find the use of visual material to teach this lesson as compared to the first lesson?*
>
> **TCL2***: I would say this lesson was good, because the Geoboard was used. Firstly learners enjoyed using the Geoboard so their attention was captured which is very important. Secondly I was able to teach those concepts which I thought they were difficult to teach like regular, concave, yaaa, all those topics were made easy by the use of the Geoboard.*
>
> *R: what would you recommend to other teachers from your lesson?*
>
> **TCL2***: from this lesson I think I would recommend the Geoboard to be used. It's very interesting and easy to use. Also I think the planning of the lessons are very important to use the phases.*

In his statements, Teacher C agreed and valued the powerful concepts that can be taught using *Geoboards*. He cites concepts such as symmetry, regularity, concave and convex that could be taught using a *Geoboard*. However, Teacher C was quick to point out that it could be a good idea if each learner could be provided with his/her own *Geoboard* to use. This would reduce the noise in the classroom. He further recommended the use of Geoboards to other teachers.

4.3.11 Lesson 3, Teacher C

4.3.11.1 Visualisation processes in the lesson

Figure 4.23 below shows the summary of the scores as per the coding criteria. The diagram indicates that there is evidence that visual representations were used under the following visualisation processes: Designing, manipulating, interpreting and analysing visuals.

	DV	MV	VL	IV	AV
RE	4	4	4	3	3
ME	0	0	0	0	0
NE	0	0	0	0	0

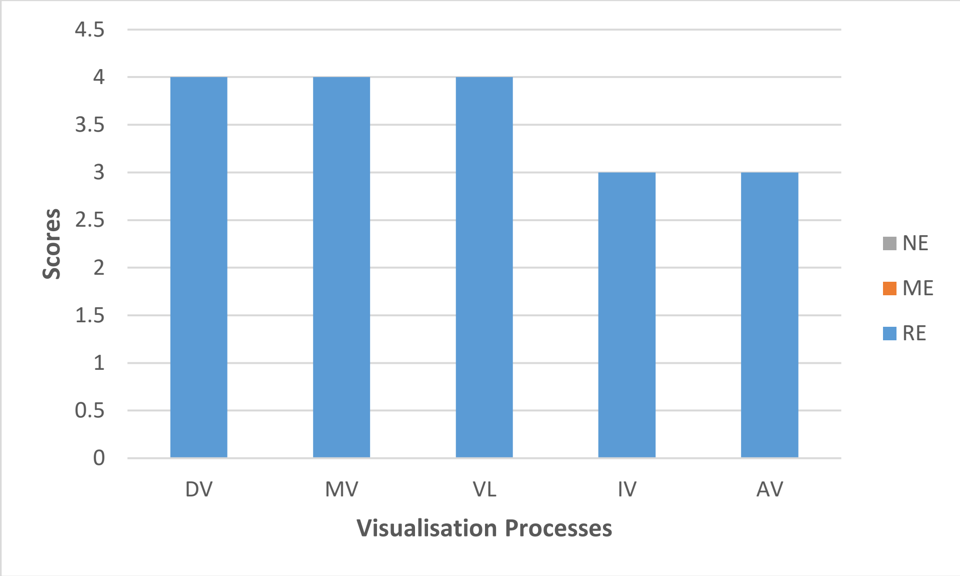


Figure 4. 26: Teacher C, Lesson 3: Visualisation processes, numerical scores

Designing visuals: The teacher brought *Geoboards*, rubber bands, and blank sheets of paper to class. He introduced the lesson by giving the learners Geoboards and rubber bands, and in groups he asked them to construct any quadrilateral from the previous lesson that they could remember. Learners were able to use the *Geoboard* and rubber bands as instructed and they constructed all the quadrilaterals.

Manipulating visuals: The teacher allowed the learners to work in groups. The visual material (*Geoboard)* learners to reconnect with what they had learnt in their previous lesson. Teacher C asked the learners to construct four quadrilaterals. Below is a picture of some constructed quadrilaterals by learners during the lesson.

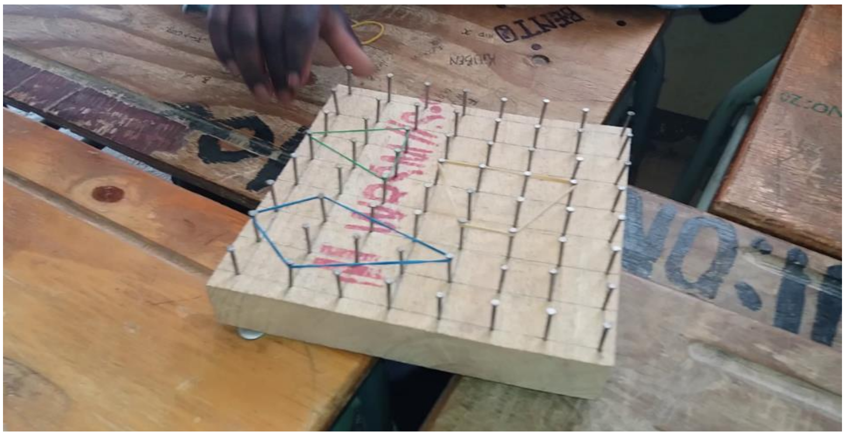


Figure 4. 27: Constructed quadrilaterals by learners

Visual language: To help learners recall the number of sides of quadrilaterals, Teacher C reminded the learners of the picture of a quad bike and asked them how many wheels it has. He reminded learners to always link the number of wheels of a quad bike to a quadrilateral. After his remarks, learners were able to construct the four-sided shapes.

Interpreting visuals: The visual materials identified for this lesson were suitable. The teacher used *Geoboards* to teach properties of quadrilaterals. The teacher asked the learners to give the properties of quadrilaterals they could find when constructing them on their *Geoboards.* Teacher C encouraged the learners to always be mindful of the properties as they were constructing the quadrilaterals. For example, teacher C would ask the presenter to point the four angles of a rectangle.

Analysing visuals: Teacher C drew some quadrilaterals on the chalkboard and explained further properties of shapes that had not been mentioned by the learners. The teacher was observed drawing a square, and further explained the properties such as lines of symmetry, angles formed by diagonals.

4.3.11.1 Stimulus recall interviews

For this session my aim was to allow the teacher to reflect on the lesson, particularly on how he found the use of a *Geoboard* to teach properties of quadrilaterals.

> *R: How did the use of the Geoboard as a visual material work in your lesson?*
>
> **TCL3***: For me, the Geoboard worked very well in the lesson. Teaching properties take long but now I could do it in one lesson.*
>
> ***R****: Why do you think it was easy this time sir?*
>
> **TCL3***:It was easy because learners could see what they were doing, they could easily find the properties on the Geoboard and so all of them wanted to present which made it easy for me.*
>
> ***R****: Were your learners able to find all the properties on their Geoboards?*
>
> **TCL3***: Yes I think all the properties of the shapes were uncovered on the Geoboard, yes, I think so.*
>
> ***R:*** *So, what would you recommend to other teachers from your lesson?*
>
> **TCL3***:Mmmmm, for this lesson teachers need just to use the Geoboard. It is the only tool which I think can help in teaching properties of any shape.*

Teacher C further stated that using visualisation processes in his classroom helped learners to see the reality of what he was teaching. Learners were able to see properly and they were able to work with the materials. Teacher C further recommended the use of *Geoboards* to other teachers, especially when teaching the properties of shapes. On the Van Hiele phases, teacher C argued that the phases helped him in the planning of the lesson to make sure that all the facets of Van Hiele's phases were included. During the presentation, the phases of instructions were easy to follow.

4.4 FOCUS GROUP INTERVIEW

In this section I present the participating teachers' views on the general implementation and their participation in the intervention programme. I also made use of this session to capture the participating teachers' experiences and perceptions of incorporating visualisation and the Van Hiele phase of instructions to teach their lessons. To capture, this I engaged with the three teachers and asked them three main questions which I will present below. Probing and follow-up questions were also asked during this process.

As a reminder for focus group interviews (**TSFGI,** TS stand for teacher S and FGI stands for focus group interview**). R** stands for researcher.

> **R:** ***What has interested you or not during the implementation of this intervention programme (anything from the first workshop, planning and teaching until today)?***
>
> **TMFGI***: As for me I was not present during the first workshop, but I was there when you taught us about the Van Hiele thing. I would say the whole programme was well organised and I have learnt a lot especially since I am new to the teaching world. What interested me the most was the visualisation concept because it also play a very important role science design? I am a designer and I like to kind of visualise things before I actually design them. To learners I think they enjoyed especially constructions on the Geoboard. yaaaa, yes thus all I can say!and also how to construct geoboard.*
>
> **TSFGI***: Myself I was present at the first workshop with your colleagues. At first I did not want to take part in this programme. I felt scared about the visualisation, but I learnt about the Van Hiele theory at UNAM so that one was fine. But then I thought when you video record us then the professors will be watching us. But anyway I liked the planning of the lesson even though it was long. The teaching was very fine and when I was teaching I was thinking of my learners that they should pass. I also learnt that this visualisation is not only to see things but also to think how the things are. I like this visualisation…*
>
> **TCFGI***: Eeeee, yes. Teaching was, but I think some lessons the content was too much. I learnt new things from the first workshop. I learnt how to construct and teach using a geoboard also.But I must say the sums the other man gave us to solve in the workshop*

almost scared me. I thought it was going to be tough but it is actually nice. I was introduced to new terminologies of Van Hiele and the other one, yaa visualisation. I learnt that visualisation is more than teaching aids we use in the class.

The teachers pointed out that they learnt new things from the intervention, in particular about visualisation. They agreed that visualisation is not just about seeing, but that it has to do with what a person is seeing, and how he or she interprets it. Another aspect that comes out strongly from the teachers is the knowledge learnt in particular that of making a *Geoboard*.

R: How did the use of the Van Hiele phases to plan your lessons help you during the presentation of lessons?

TMFGI*: For the Van Hiele phases was just like this practice of introduction, presentation and conclusion. I followed the phases of instruction from inquiry, directed, but then forget when I was actually deep in the lesson. But one will know that you have covered all of them because those phases allow you to introduce the lesson by asking questions, they also allow you to conclude the lesson. The phases also allows you to assess learners during the lesson. They really help during planning.*

TSFGI*: The Van Hiele theory I learnt it already. I use it in my lessons, but I normally don't take it serious. For now I have learnt to plan my lesson using the phases, those days I thought we just present the lessons using the phases. The phases also makes the lesson interesting because you move on and you cover all the aspects of the lesson. You see, the phases allows one to introduce the lesson, see what learners already know, assess them during the lesson and conclude the lesson. I like them.*

TCFGI*: yes, for me the Van Hiele phases of instructions helped me to make links between my lessons. I know that I use them every day but I think I did not know more about it, but it really helped me to make connections in my lessons. I cover everything during the time of teaching.*

To the participating teachers Van Hiele theory was a new theory they taught, in particular teacher M and C. However they note that Van Hiele phases has helped them a lot during the planning stages of the lesson. Though the phases remains visible during presentation, participating teacher found it easy to follow with the stages of every phase. It comes naturally if one is prepared teachers argued.

R: Do you think integrating Van Hiele phase and visualisation strategies has improved learners understanding of geometry?

TMFGI*: Yes, I think the 2 have a link and, because in the first phase I think there is visualisation because we use pictures to get the attention of learners. Visualisation itself is about seeing and making use of what you see to solve problems. I think using the 2 can help learners a lot and improve their performances in geometry. I also think that it is important to bring things that learners already know, but should be related to the* *topic. like the geoboard, ok the quad bikes those are good to use because their names are related to the shapes.*

TSFGI*: I definitely think that combining the 2 can really help to improve learners understanding. Geometry is something that needs learners to see and touch. For all the learners could see the shapes, they could construct them and now they have a permanent picture in their brains. The Van Hiele theory just make it easy for the teacher to plan and teach the lessons. Sometime I waste my time to think about teaching aids, but they are everywhere. Like the picture we used, they are very nice to teach the difference between quadrilaterals and triangles.*

TCFGI*: Visualisation and geometry is like a brother and sister. After the workshops and teaching I realised that the best way learners can learn geometry is through seeing what the teacher is talking about. Learners need to see the lines of symmetry not just to draw and give definitions. To use Van Hiele theory one has to read first and understand it very well, for me I understand but not so much I think.*

Participating teachers here agreed that visualisation processes and the Van Hiele phases have a place in teaching geometry. They pointed out that while seeing shapes is important, it is how you have planned for shapes to be seen by learners that matters. One needs to plan exactly when to let learners see the materials and when to manipulate them, that is important in geometry.

R: do you think the class you taught will improve in their performance when they write the post-test?

All the 3 teachers were confident that their learners would perfume very well.

4.5 GENERAL OBSERVATIONS ON VISUALISATION PROCESSES AND VAN HIELE PHASES AS USED BY TEACHERS

Designing visuals/Information phase: For the three lessons teachers were observed to have brought visual materials to class and hence rich evidence was mostly coded in the teacher lesson observation. All the materials brought to class by the teachers were mostly suggested during our lesson planning session, however the teachers were encouraged to source out more. All materials aligned well with the topics covered and with the subject content. The visual materials were also grade and user-friendly for the learners as no incidences of harm were observed throughout the intervention. I believe the visual materials chosen by the teachers are in line with what Fleming and Levie (1993) suggest — that designers of visual images should take into account the basic principles of perception, so that they can produce teaching media that can communicate messages in a more efficient and effective way. Visual materials should always be used to allow learners to make connections with prior knowledge and new content.

On the Van Hiele phases, information is known as the first phase of instruction. My observation was that teachers focused more on this phase during the planning and presentation of lessons than on other phases. During our planning sessions, the teachers always asked how they would introduce the lesson by using the visual materials. The teachers' concerns were in line with Crowley (1990), who reasoned that the teachers engage with activities at Phase 1 so that they "learn what prior knowledge the students have about the topic *while [italics added]* the students learn what direction further study will take" (p. 5).

Direct orientation/ Manipulating visual: In the second phase of instruction, teachers were more involved in the learning process. I observed that teachers were more involved as they guided learners towards the topic to be learnt in all their lessons. The teachers also used the visual materials to refer to geometrical concepts that learners were to learn during the lesson. For example in lesson 1 of this intervention, the teachers provided their learners with pictures of a tricycle and a quad bike and logically learners could connect the number of their wheels on the pictures to the number of sides of triangles and quadrilaterals. This was mostly observed with teachers M and S who showed by pointing out the number of wheels in a quad bike and the number of sides in a quadrilateral. The visual materials used by the teachers thus fitted the "developmental level of the child" as proposed by Smith (2009, p. 20). Similarly, the visual materials were used in a way that allowed learners to connect new concepts to what they already knew. The Geoboard use also connected learners' prior knowledge to the new concepts of 'regular', 'irregular', 'convex' and 'concave'.

Explicitation/Visual language: At this phase the teacher takes care that learners develop the use of technical language with understanding, through the exchange of ideas. Hence, learners' new knowledge is formed through experience, and by integrating this with past knowledge. My observation was that the teachers were well aware of their language and used terminologies that were grade and age appropriate. I observed this in teacher M's lesson when he simplified his language by integrating a cultural practice to explain what a polygon is. Teacher M further joked with learners that his grandfathers''husband's' name is 'pentagon' because he has 5-wives. Learners enjoyed the joke, but at the same time they learnt that a five-sided polygon is actually a pentagon. There was no use of confusing words during the presentations and teachers took time to monitor their learners' language. I observed teacher S telling a learner that a line that joins opposite sides inside a shape is called a diagonal and not a 'line half' as referred to by a learner in her class. Under this visualisation process participant teachers were expected to use language and terminologies appropriate to the level of Grade 8 learners. I noticed that teachers used strategies that allowed the learners to produce visuals. This I noticed was done through questions, as when Teacher S asked learners to think of anything at home that is in a form of a four-sided shape. The learners connected the language to a visual at their homes and they could say a garden is in a form of a quadrilateral.

Free orientation/Analysing visuals: As the name suggests, free orientation is a phase where learners are encouraged to use multiple ways to find answers to given problems. Through problem solving, learners' mathematical language develops, as the teacher provides capacity for logical thought. The teacher's role in this phase, according to Clements and Battista (1992, p. 431) is "to select appropriate materials and geometric problems" that require certain levels of thinking to solve geometric problems successfully. I observed teacher M providing learners with properties of shapes then use them to construct such shapes. This was a classical way of allowing learners to explore not only with the *Geoboard*, but with their reasoning.

On analysing visuals, participating teachers were mostly seen encouraging learners to work with their visual materials to complete tasks. This was mostly observed when learners were given time to present their findings. Teachers encouraged their learners to always show by pointing, for example, the equal lines in a square when they referred to it on the *Geoboard*. In addition, the teachers used visual materials that deconstructed properties of different shapes, the pictures in Lesson 1 deconstructed useful properties of quadrilaterals and triangles, while the *Geoboard* deconstructed a number of properties of different polygons.

Integration/Interpreting visuals: By getting to this phase (integration), learners are believed to have reached a new level of thinking. Thus they are able to summarise their previous learning content. It is important that learners sum up only what they have learnt and no new content is to be introduced at this phase. This was however not observed, in particular in the teachers' Lesson 2. But they argued that there was too much content to be covered in that lesson. When this happens it is difficult to summarise the learning content.

On the visualisation processes the teachers were observed to have always concluded their lessons with the use of visuals. I observed in particular, a Lesson 1 where the teacher pasted the pictures of the quad bike and tricycle and gave learners a task to do. While the learners were busy with the task, the pictures of the quad bike and tricycle were pasted on the chalkboard to help learners complete their task with the help of the visuals.

4.6 PRESENTATION OF POST-TEST RESULTS

The post-test results are presented in this section. The pre-test ad post-test results of each class group are combined to show the comparison in performance. All these are presented in form of bar charts. On the charts, each class group is represented by a letter. Thus I have groups A, B, C and D. It is important to note that group D served as the control group while A, B and C are experimental. For the levels 0, 1 and 2 are represented by L0, L1 and L2 respectively. NOT L2 represents the percentage of learners who did not attain level 2. Since the pre-test results were presented earlier, the interpretations will only focus on the percentage increase/decrease from the pre-test to post-test of each class group.

Item 1(a):

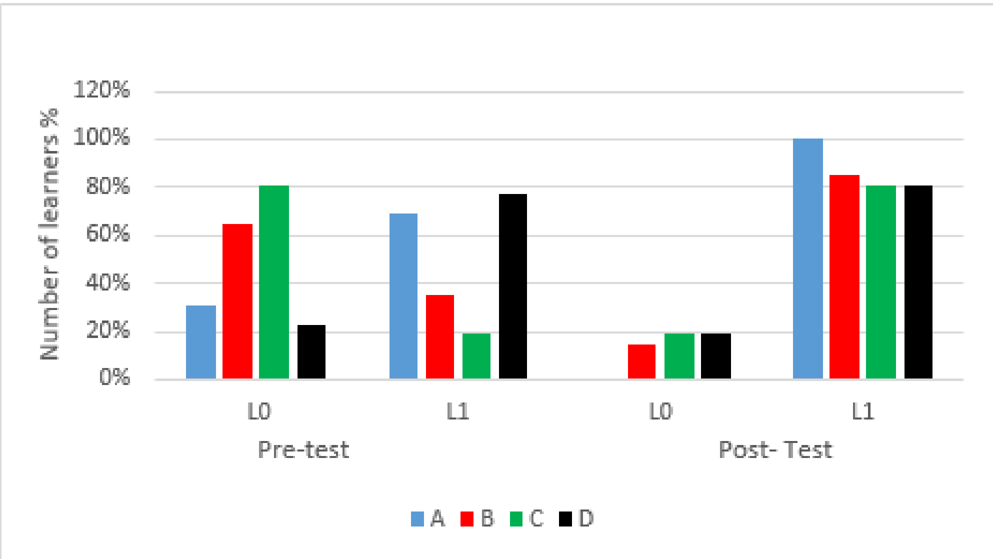


Figure 4. 28: Pre- and post-test results for item 1(a)

The Figure 4.28 shows that group A recorded a thirty-one per cent increase in learners' performance — from sixty-nine per cent in the pre-test to one-hundred per cent of learners attaining Van Hiele level 1 in the post-test. Group B recorded an increase of forty per cent — from thirty-five per cent in the pre-test to eighty-five per cent in the post-test. Group C recorded nineteen per cent of learners attaining level 1 in the pre-test, while in the post-test, there was an increase of sixty-one per cent to eighty-one per cent of learners attaining level 1. Group D, the control group, achieved seventy-seven per cent in the pre-test and recorded an increase of three per cent to eighty-one per cent in the post-test of learners attaining level 1.

For this study, incorporating the Van Hiele phases and visualisation approaches has had a positive impact of improving learners' performance at pre-recognition level to level 1.

Item 1(b-f):

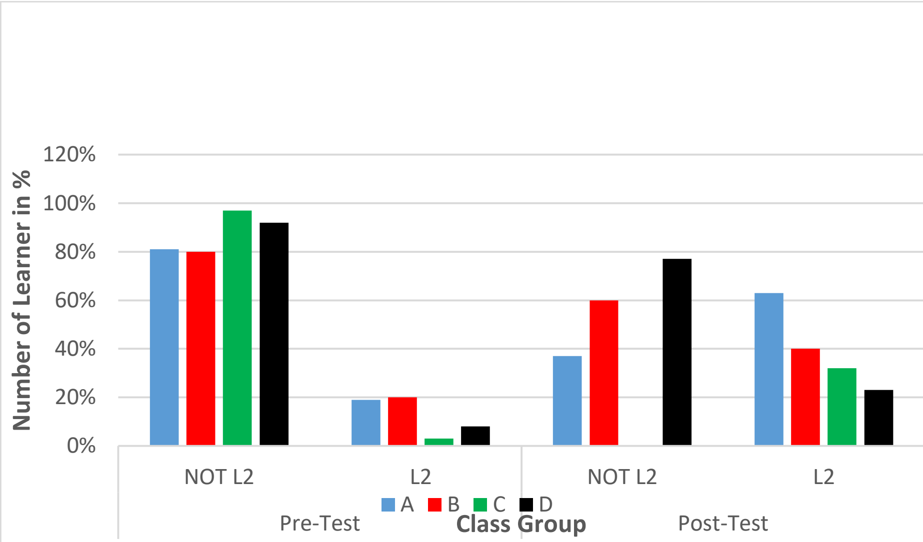


Figure 4. 29: Pre- and post-test results for item 1(b-f)

Figure 4.29 above shows that group A recorded a forty-four per cent increase in the learners' performance, from nineteen per cent in the pre-test to sixty-three per cent of learners attaining Van Hiele level 2 in the post-test. Group B recorded an increase of twenty per cent — from twenty per cent in the pre-test to forty per cent in the post-test, Group C recorded only three per cent of the learners attaining level 2 in the pre-test, in the post-test there was an increase of twenty-nine per cent to thirty-two per cent of learners attaining level 2.The control group, Group D ,recorded only eight per cent of learners attaining level 2 in the pre-test. There was an increase of fifteen per cent, to twenty-three per cent in their post-test of learners attaining level 2. The notable significant variations in the learners' performance shows that the use of visualisation strategies helped in improving learners' performances.

Item 2(a):

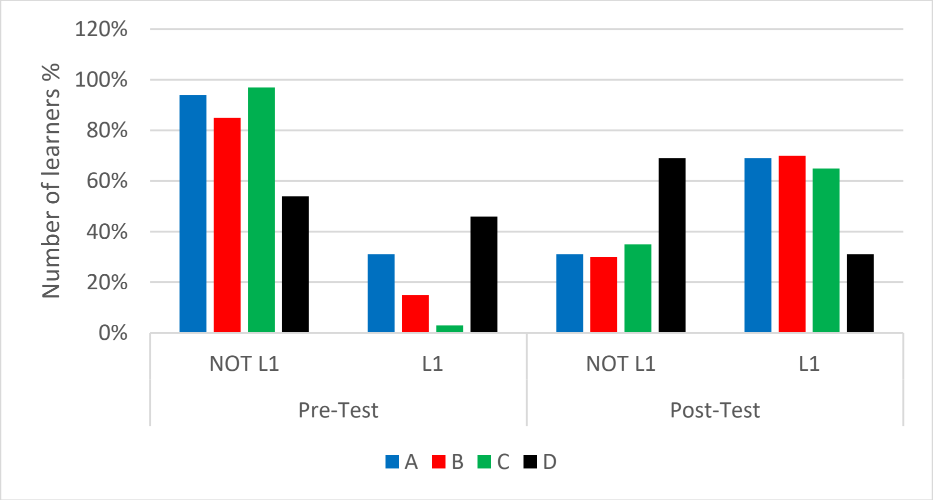


Figure 4. 30:Pre- and post-test results for item 2(a)

The Figure 4.30 shows that group A recorded a thirty-eight per cent increase in learners' performance, from thirty-one per cent in the pre-test to sixty-nine per cent of learners attaining Van Hiele level 1 in the post-test. Group B recorded an increase of tity-tive per cent, from fifteen per cent in the pre-test, to seventy per cent in the post-test. Group C recorded three per cent of learners attaining level 1 in the pre-test. In the post-test, there was an increase of sixty-two per cent, so sixty-five per cent of learners attained level 1. Control group D achieved forty-six per cent in the pre-test and a decrease of fifteen per cent to thirty-one per cent in the post-test of learners attaining level 1.

Comparing the class groups A, B and C to class group D, there is a clear variation in the learners' performances. The attainment to level 1 from level 0 of many learners in class groups A, B and C in the post-test when compared to the pre-test shows that visualisation strategies of teaching improve learners' performances.

Item 2(b):

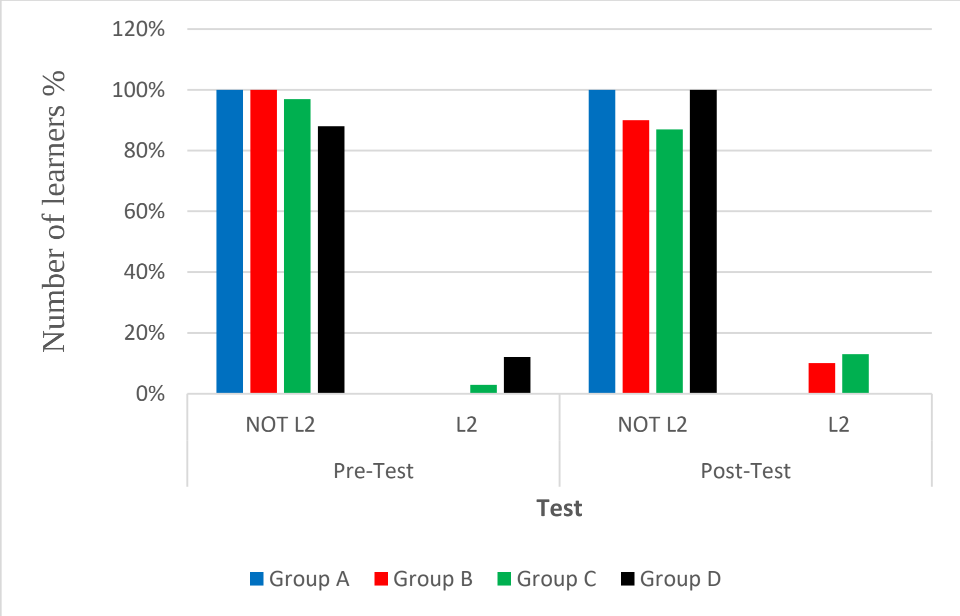


Figure 4. 31: Pre- and post-test results for item 2(b)

The figure 4.31 shows that group A recorded zero per cent in both the pre- per the post-tests of learners attaining level 2. Group B recorded an increase of ten per cent from zero in the pre-test to ten per cent in the post-test. Group C recorded only three per cent of learners attaining level 2 in the pre-test. In the post-test, there was an increase of ten per cent, to thirteen per cent of learners attaining level 2. Control D group recorded that only twelve per cent of learners attained level 2 in the pre-test, while a decline of twelve per cent was recorded in the post-test to record zero per cent.

For this item, none of learners in class group D attained level 2. Even in the other class groups who received teaching, a high number of learners could not attain level 2. This is an area of concern and it could be attributed either to the way the teachers presented their lessons, the visualisation materials used, or perhaps the language used for this item.

Item 3:

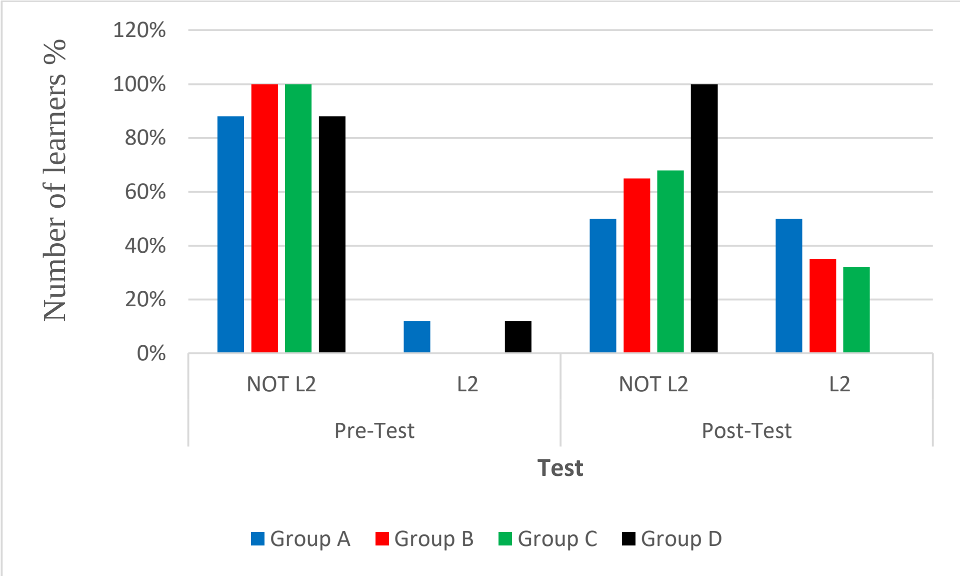


Figure 4. 32: Pre- and Post-test results for item 3

The Figure 4.32 shows that group A recorded a forty-eight per cent increase in learners' performance, from twelve per cent in the pre-test to fifty per cent of learners attaining Van Hiele level 2 in the post-test. Group B recorded an increase of thirty-five per cent, from zero in the pre-test. Group C also recorded no learners attaining level 2 in the pre-test. In the post-test for group C, there was an increase to thirty-two per cent of learners attaining level 1. Control group D achieved twelve per cent in the pre-test and a decrease of of twelve per cent in the post-test to achieve zero per cent of learners attaining level 2.

For this item, a shift in the learners' geometrical level was observed. This improvement in learners' performance, in particular in class groups A, B and C was recorded due to the use of visualisation teaching approaches and the Van Hiele phases employed by participating teachers.

Item 4:

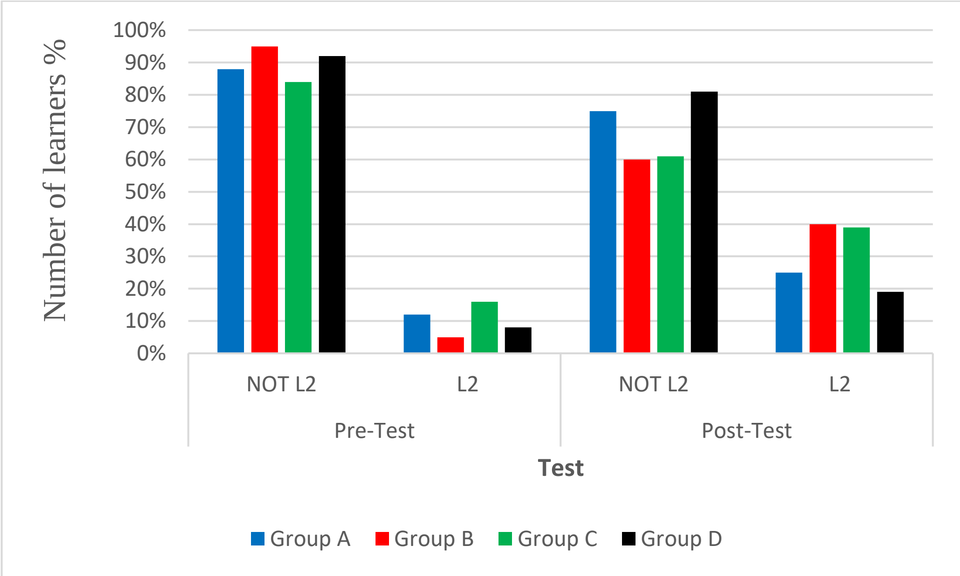


Figure 4. 33: Pre- and post-test results for item 4

Figure 4.33 shows that group A recorded a thirteen per cent increase in performance from twelve per cent in the pre-test to twenty-five per cent of learners attaining Van Hiele level 2 in the post-test. Group B recorded an increase of thirty-five per cent, from five per cent in the pre-test to forty per cent in the post-test. Group C also recorded sixteen per cent of learners attaining level 2 in the pre-test. In the post-test for group C, there was an increase of twenty-three per cent, to thirty-nine per cent of learners attaining level 2. Control group D achieved eight per cent in the pre-test and an increase of eleven per cent, to achieve nineteen per cent in the post-test of learners attaining level 2.

A notable increase in learners' performance was recorded in the post-test compared to the pre-test. Although an increase in group D was recorded (see Figure 4.28), the percentage of learners who attained level 2 is high in the other three classes, namely classes A, B and C.

Item 5:

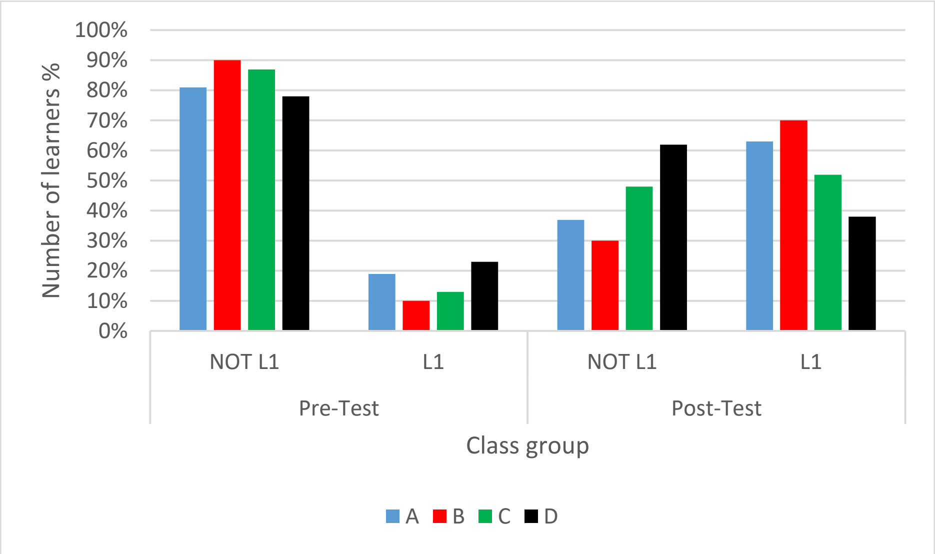


Figure 4. 34: Pre- and post-test results for item 5

Figure 4.34 shows that group A recorded a forty-four per cent increase in performance from nineteen per cent in the pre-test to sixty-three per cent of learners attaining Van Hiele level 2 in the post-test. Group B recorded an increase of sixty per cent from ten per cent in the pre-test to seventy per cent in the post-test. Group C also recorded thirteen of learners attaining level 2 in the pre-test. In the post-test for group C, there was an increase of thirty-nine per cent to fifty-two per cent of learners attaining level 2. Control group D achieved twenty-three per cent in the pre-test and an increase of fifteen per cent to achieve thirty-eight per cent in the post-test of learners attaining L2. The presented statistics above clearly show how the use of visualisation strategies and the Van Hiele phases of teaching can help to improve learners' performances in geometry.

Item 6:

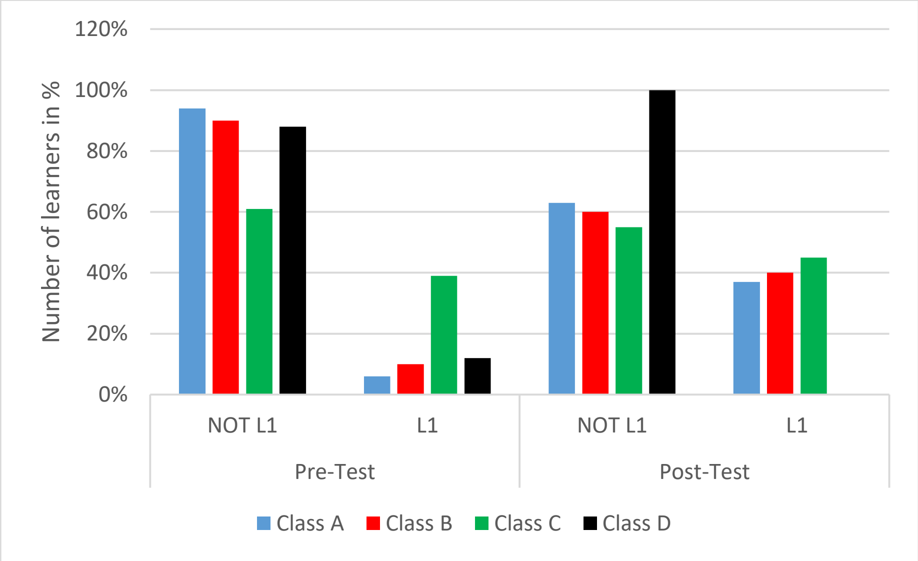


Figure 4. 35: Pre- and post-test results for item 6

The Figure 4.35 shows that group A recorded a thirty-on per cent increase in learners' performance, from six per cent in the pre-test to thirty-seven per cent of learners attaining Van Hiele level 2 in the post-test. Group B recorded an increase of thirty per cent, from ten per cent in the pre-test to forty per cent in the post-test. Group C recorded thirty-nine per cent of learners attaining level 2 in the pre-test. In the post-test, there was an increase of six per cent to forty-five per cent of learners attaining level 2. The control group recorded twelve per cent in the pre-test and a decrease of twelve per cent.

For this study therefore, there was a shift in learners' performances from Level 0 to level 1. Not so great a shift was recorded from NOT level 2 to level 2. The conclusion can be made that with time and constant use of visual material incorporated with Van Hiele phases of instruction, learners can indeed attain the highest level in geometry. Planning of lessons using the Van Hiele phases was very significant for this study and the participating teachers have further recommended its use when planning geometry lessons. Visualisation processes were also very significant. Teachers were at all times reminded of how and when to present such materials to learners. They agree that it is important that the materials used suits developmental stages of learners.

4.7 CONCLUSION

In this chapter I presented data pertaining the study. The chapter started with presentation of pre-test results which are presented on charts. The pre-test served as a gateway to the actual implementation of the intervention programme. The pre-test, of which there were six items, was analysed per item. After the pre-test I presented the actual teaching programme with the three participating teachers.

In presenting the lessons as taught by participating teachers, I focused on the visualisation processes and the Van Hiele phases of instructions. The two were of interest during the teaching because through them I could answer my research questions. Participating teachers' views on the incorporation of visualisation strategies and the Van Hiele phases of instruction were also presented in the section of focus group interviews where the three teachers gave their perceptions about visualisation and the Van Hiele phases.

I then presented a summary of the findings from the lessons in relation to the how the teachers used visualisation processes and the Van Hiele phases. I concluded the chapter by presenting the results of the learners' post-tests. This was also presented on graphs where I compared the results of learners in their pre-test to the post test.

CHAPTER FIVE
CONCLUSION AND RECOMMENDATIONS

5.1 INTRODUCTION

The main objective of this study was to use visualisation processes and the Van Hiele phases of instruction to teach two-dimensional figures in geometry. The focus therefore was the migration of learners from one Van Hiele level to the next as a result of teachers teaching with the use of visualisation processes. In this chapter, I present a summary of the findings, outline the significance of this study and possibly provide recommendations to other stake- holders in mathematics. In addition, in this chapter I will present the limitations of this study and suggestions for further research as well as my personal experiences after completing this study.

5.2 SUMMARY OF FINDINGS

There were a number of themes that came up during the process of data analysis. The summary of findings is presented below per research question:

Question 1

At what Van Hiele level of geometric thought are Grade 8 learners of the selected schools in the Bukalo circuit operating prior to and post the intervention program?

The pre-test results revealed that for items testing level 1, more than sixty per cent of learners were at level 0. This implies that the majority of learners in the experimental group were operating at level 0 (pre-recognition level), while more than sixty per cent of learners in the control group were operating at level 1. The majority of learners in this study performed below level 1 in the pre-test. This therefore necessitated the need for an intervention with teachers of these Grade 8 learners. For level 2 items, less than fifteen per cent of learners were operating at level 2 in both the experimental group and control group. This implies that about seventy-five per cent of participating learners were operating at Not level 2 in items testing level 2. Overall, this study found that on average, Grade 8 learners who participated in the study were operating at levels lower than expected of pupils at their stage of school education.

The post-test results indicated that for level 1 items, more than fifty per cent of learners in the experimental groups were now operating at level 1, while there was a decrease in the number of learners who attained level 1 in the control group from sixty-seven per cent to twenty-seven per cent, thus less than thirty per cent of learners were at level 1 in post-test. The decrease could be attributed to that these learners' teacher did not participating in the intervention. Thus, at the time of taking the post-test, the learners might have opted for other answer options since no correction or teaching was done and perhaps not sure of the correct answer.

For level 2 items there was an increase in the percentage of learners who attained level 2 in the experimental group. More than thirty per cent of learners were now operating at level 2 as compared to less than fifteen per cent in the pre-test who were at level 2. The control group class remained at less than fifteen per cent of learners operating at level 2. There was a difference of fifteen percent in the performance of learners when the experimental group was compared to the control group in the post-test. Besides a high number of learners who took part, and the limited time, visualisation processes and the Van Hiele phases has shown to be effective when used in geometry lessons to migrate learners from lower Van Hiele levels to higher.

Question 2

How does the use of visualisation processes and the Van Hiele phases enhance the transition for student understanding from Van Hiele level 1 to level 2?

Participating teachers said that lessons on two-dimensional figures should always be introduced with the use of visuals, as this allows learners to make links with their prior knowledge. The teachers agreed that the use of visual materials can help in guiding learners towards topics to be learnt during the lesson. On language, participating teachers asserted that help should be given to learners in order to advance their understanding of/and the use of technical language (mathematical language). This can be done through the use of visual language and integrating it with cultural practices where possible, to explain the main concepts in a given topic. Teachers in this study also noted that unfamiliar concepts on shapes should be simplified by selecting appropriate visual materials that can deconstruct useful properties of such shapes. Furthermore, teachers revealed that in order to enable the transition of learners from one Van Hiele level to another, learners should be encouraged to work with visual materials when solving problems throughout the lesson. Proper visual materials can one be effective when teachers plan for the lesson ahead using the Van Hiele phases of instruction.

Question 3

What are selected teachers' perceptions and experiences of the role of visualisation and Van Hiele's phases as a result of participating in an intervention programme?

Teachers said that they gained knowledge on how to design a Geoboard in particular as a visual tool to use when teaching two-dimensional shapes. The geoboard is a useful visual device that can be used effectively to introduce lessons and presentations. However the teachers pointed out that it takes time to design and they do not have enough time.

Pictures of a quad bike and tricycle were seen by teachers as very useful when used to teach classifications of quadrilaterals and triangles. The teachers asserted that the two pictures deconstructed useful properties of quadrilaterals and triangles. Further, the teachers also pointed out that the pictures were easy to use as learners have a social background of quad bikes and tricycles. This made it easy to link the pictures to the topic on classifying quadrilaterals and triangles.

When using visual language, participating teachers pointed out that during lesson presentations, they should use simplified language and integrate visuals to cultural practices where possible. To the participating teachers, visual language meant for example 'saying a word that allows one to form and interpret the word by forming an image in mind '. This was observed when the word polygon was linked to a cultural practice of marrying more than one wife — 'polygamy' — husband with many wives. So a 'pentagon' is that husband with 5 wives. This practice was observed to be effective to participating teachers ecause learners could easily remember the polygons' names.

Participating teachers established that incorporating visualisation processes in lessons can have a positive contribution in helping Grade 8 learners attain the next level in geometry. The teachers agreed that designing and making visual materials available during geometry lessons helps learners make connections to the real world, hence helping learners perform. Similarly, teachers also agreed that using visualisation processes, in particular visual language and manipulating visuals as processes, helps learners to develop and acquire geometrical concepts and terminologies. Acquiring mathematical terminologies can help learners understand questions which will result in answering many geometric problems correctly. Teachers also advocated that learners should be given opportunities to physically manipulate visual materials so that they can analyse and interpret the visual materials correctly in order to make meaning from such materials.

On the Van Hiele phases, participating teachers in this study agreed that effective planning using phases allows the incorporation of visualisation processes. When the phases and visualisation processes are used in geometry lessons, all stages of lesson presentation are considered, thus learners receive sufficient attention from the teacher. This in return has a positive influence on what learners learn in class.

Participating teachers also viewed the phases of instruction as a good strategy of planning and presenting lessons. They say that using the phases of instruction allows the teacher to do in-lesson assessments and language corrections. Teachers argue that assessment during lesson presentation is important because one is able to see if learners are following or not. This practice can be viewed as a practice of good teaching which will result in good learning. Teachers were able to navigate freely with the phases of instruction during the lesson presentations. They agreed that it is easy to move from one phase to another if one has used the phase during the planning stage of the lesson. This was evidenced on how teachers used language to explain concepts. Instead of waiting for the Explicitation phase of the lesson to allow learners verbalise their understanding, teachers similarly asked learners to verbalise what they knew about the lesson in the information phase.

5.3 SIGNIFICANCE OF THE STUDY

This study focused on two particular Van Hiele levels of geometric thinking and the transition between them. The use of visualisation processes incorporated with the Van Hiele phases of instructions were utilised by participating teachers in order to enable learner shift from one Van Hiele level to the other. It is thus hoped that this study, firstly informed participating teachers of the use of visualisation strategies to teach geometry; secondly, that the study contributed to participating teachers' increased ability to plan their geometry lessons using the Van Hiele phases of instruction. In the Namibian context this study will help inform policy makers and researchers with rich and meaningful information on the inclusion of the Van Hiele phases and visualisation processes in their planning and deliberations. Text book authors alike will find this study significant as they publish mathematics text books with pedagogical advice to teachers on teaching geometry, in particular two-dimensional figures.

5.4 RECOMMENDATIONS

While the findings of this study cannot be generalised to the entire circuit, region or Namibia at large, the results of this study can be used by anyone interested in the promotion of quality teaching strategies for mathematics. With this in mind I would like to make the following recommendations to mathematics stakeholders.

Mathematics teachers to form partnership with other teachers in the same circuit and plan mathematics lessons using the Van Hiele Phases. At these planning sessions, teachers may put forth difficult topics and plan the lessons jointly with other teachers and identify suitable visual teaching materials to use.

All mathematics teachers at primary and lower secondary level should jointly design Geoboards to use when presenting lessons on two-dimensional shapes. While participating teachers have raised the concern of Geoboards taking a long time to design and make, once it is made, a Geoboard has a long life span and can be extended to teach other topics such as transformation and area in mathematics.

Mathematics teachers should always look for, design and find visual materials to use for every topic in geometry. Such material should be well evaluated to ensure that it can be linked to the concept under discussion in the lesson. Further, a study should be conducted which will build on the findings of this study, involving a larger size sample.

5.5 LIMITATIONS

This case study research by nature involves a small sample of population. My case study involved four mathematics teachers of which only three received full training and ninety-three Grade 8 learners of which twenty-six did not receive teaching (control group), thus the results are not generalized. On lessons presented, this study only focused on a single geometry topic

— two dimensional shapes — thus the results of the tests cannot be generalised to the whole geometry discipline in mathematics. The teachers were only trained for a period of three days before they began teaching and being observed. The time for training the teachers was not sufficient enough for them to fully conceptualise visualisation processes and the Van Hiele phases of instructions. Although this study focused on teachers' pedagogical strategies, at times I had to capture both the teacher's and the learners' work during presentation. This was because the learners' work was a direct reflection of what the teacher was teaching. This study took place during school days, and teachers were tired from doing their departmental duties. I believe that the over-commitment of the teachers made them unable to fully commit themselves all to the intervention programme, especially in that they had to teach all their lessons in the afternoons. The teachers' responses were not as detailed as expected. They were seen to be in rush to go home.

The initial plan was to have a focus group interview after lesson 1 and lesson 2 and the final group interview at the end of lesson 3. This could not happen as teachers taught at different schools within a radius of eighty km from my school, therefore bringing the teachers together proved to be challenging due to finance and time constrains. Interviews after the lessons were also a challenging, as the lessons took place during the afternoons and the teachers were tired and hungry, thus they gave short frank answers. Further interrogations proved useless as they were in a rush to get home. Lack of research on Van Hiele and visualisation in the Namibian context made it difficult for me to draw from local experiences, in particular in Zambezi region.

5.6 SUGGESTIONS FOR FURTHER RESEARCH

There is very little research in the in the field of Van Hiele in Namibia. Similarly, the field of visualisation also was recently introduced to a few scholars in Namibia. This research involved a small sample of participants. I thus suggest a bigger sample to be used with the same research so that the findings can be generalised. Similar research can be conducted, but focusing on how learners use visualisation strategies when answering mathematics problems. Research which specifically focuses on visual manipulatives that can be used by teachers to teach different topics in mathematics, would help to solve problems pertaining to different disciplines of mathematics such as statistics, number sense, algebra and many others. Research should be conducted focusing on what learners are able to do at each Van Hiele phase of instruction in Namibia.

5.7 PERSONAL REFLECTIONS

This research study was not just pages written for the sake a qualification. In the process of writing there are things that I came across. For a period of two years, this research study has helped me grow in the following skills: writing, reading, listening and speaking. From the day of registration and undertaking this research project, I started writing, I started to read, to listen and to speak. I started to read for deeper understanding, I started writing to make my voice heard, I started listening for information, I started speaking to convey my feelings and experiences about this project research. When I look back to the first piece I wrote and presented for my proposal, I could not imagine this outcome, thanks to our website www.namaths.com, of which this presentation is part.

Doing this research has helped me understand why some things are the way they are, in schools, in classrooms and even in our personal lives. In the classes of the participating learners, I felt privileged to know their levels of geometrical thought through a geometry test and this has helped me understand why their performance is the way it is today. Research is a way of proving lies to support the truth. In addition, research is about knowledge, not only knowledge for the researcher, but for the participants alike. Participants in this research have learnt one or two things. When the participating teachers were first introduced to the visualisation theme and the Van Hiele theory, they were not very excited as the time they completed the intervention programme. After the intervention was completed, teachers, confessed that it was worthwhile being part of this journey.

5.8 CONCLUSION

Presented in this chapter are the findings after conducting this research. To teach geometry lessons successfully, visualisation strategies and the Van Hiele phases seems to be the way to go. I therefore presented some recommendations to other stakeholders in mathematics to pursue research in line with geometry teaching and learning.

In the last part of this chapter I presented a reflection on undertaking this study. Throughout the time I spent working on each chapter, I was reminded of my own work. In the end I feel honoured to have mastered every piece of writing in this thesis.

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APPENDICES

Appendix A: Ethical clearance

RHODES UNIVERSITY

Grahamstown • 6140 • South Africa

EDUCATION FACULTY • PO Box 94, Grahamstown, 6140
Tel: (046) 603 8385 / (046) 603 8393 • Fax: (046) 622 8028 • e-mail: d.wilmot@ru.ac.za

PROPOSAL AND ETHICAL CLEARANCE APPROVAL

Ethical clearance number 2017.12.08.10

The minute of the EHDC meeting of 05 December 2017 reflect the following:

2017.12.8 CLASS B RESTRICTED MATTERS
MASTER OF EDUCATION RESEARCH PROPOSALS

To consider the following research proposal for the degree of Master of Education in the Faculty of Education:

Mr Ben Munichinga (15m8814)

Topic: A critical analysis of selected teachers' perceptions and experiences of the role that visualisation processes play in their van Hiele level 1 teaching to migrate their learners to the next van Hiele level.

Supervisor: Professor M Schäfer
Co-Supervisor: Dr C Chikiwa

Decision: *Approved*

This letter confirms the approval of the above proposal at a meeting of the Faculty of Education Higher Degrees' Committee on the 5 December 2017.

The proposal demonstrates an awareness of ethical responsibilities and a commitment to ethical research processes. The approval of the proposal by the committee thus constitutes ethical clearance.

Sincerely

Ms Zisanda Sanda
Secretariat of the EHDC, Rhodes University
8th December 2017

Appendix B: Permission letter from the Director of education

REPUBLIC OF NAMIBIA

ZAMBEZI REGIONAL COUNCIL
Directorate of Education, Arts and Culture

Tel: 066/261902/917
Fax: 066/253187

Ngoma Road
Govt Building

Private Bag 5006
Katima Mulilo, Namibia

Enquiries: **Violet Nchindo**
Reference No:

ATT: Mr Ben Munichinga
Rhodes University

SUBJECT: PERMISSION TO CONDUCT RESEARCH STUDY IN SCHOOLS WITHIN BUKALO CIRCUIT: ZAMBEZI REGION

1.1 I acknowledge receipt of your letter dated 22 January 2018 as per above.

1.2 You are hereby informed that permission have been granted to you to visit schools and conduct your research, as per your letter.

1.3 By way of this correspondence, Inspector and Principals of **Bukalo Circuit** will be notified accordingly.

1.3 However, be advised that such permission granted to you must not at all interrupt the teaching and learning at the schools you intent visiting.

Thank you,

MINISTRY OF EDUCATION ARTS & CULTURE
Zambezi Regional Directorate
Private Bag 5006 · Katima Mulilo

AM SAMUPWA
REGIONAL DIRECTOR OF EAC

Appendix C: permission letter from circuit inspector

Republic of Namibia
Zambezi Regional Council
Directorate: Education, Arts and Culture
BUKALO CIRCUIT

Private Bag 5006, Katima Mulilo,

Enquiries: L. Limbo **Tel: 066: 252332/ 261900**

22 February 2018

Mr B. Munichinga
Rhodes University

Sir

Re: Permission to conduct research in schools of Bukalo Circuit

I do hereby refer to the above and would like to, in response to your request that has been subsequently approved by the Regional Director, likewise, hereby permit you to visit the respective schools, where your desired data is to be collected.

This brief shall also serve as your introduction to the respective Principals of such schools, permitting your entrance thereto, provided your activities shall not in any way derail the official school programs

I wish you success in your endeavoured program

I thank you

DIRECTORATE OF EDUCATION
Zambezi Regional Council
2018 -02- 2 2
Inspectorate
Bukalo Circuit
PRIVATE BAG 5006 - KATIMA MULILO

D.K. Limbo
IC

Appendix D: permission letters from school principals

[redacted]

Republic of Namibia
Zambezi Regional Council
Directorate of Education

[redacted]

<u>PERMISSION TO CONDUCT RESEARCH</u>

With reference to your letter dated **13 March 2018**, bearing the above subject, permission is hereby granted for you to conduct your research at the above institution.

For any related arrangements, please do not hesitate to contact us at the given address.

Yours

KN Tebuho
Principal

[redacted]

To whom it my concern

Re: <u>Permission to conduct a research study at our school</u>

Permission is hereby granted to you to conduct a research study as communicated in your letter dated 22/01/2018.

This memo shall therefore save as your introduction to the respective grade 8 you wish to conduct a pre and post-test.

Best wishes in your studies

Thank you

Mr A. Chunga (Principl)

[redacted]

2018 -02- 27

Ministry of Education

[redacted]

[redacted]

[redacted]

[redacted]

05 February2018

To: Mr Ben Munichinga

Rhodes University

Re: Permission to conduct research at school

I am writing to inform you that permission has been granted to conduct your research at school, provided this permission will not interrupt normal school programme.

I thank you

Yours faithful

[redacted]

2018 -02- 16

[redacted]

(R.Masule)

Principal

NSUNDANO JUNIOR SECONDARY SCHOOL
Private Bag 6016, Bukalo, Katima Mulilo Namibia
Telefax: (066) 254183,

Enquiries: Mr. Matomola L.M
Cell: 0811651272
Email: matomolalouis@gmail.com

02 April 2018

Mr Ben Munichinga

Rhodes University

Grahamstown

South Africa

Re: permission to conduct research study at school

With reference to your letter dated 22 January 2018, bearing the above subject, Permission is hereby granted to you to conduct your research study at the above institution entitled *'A critical analysis of selected teachers' perceptions and experiences of the role that visualisation processes play in their van Hiele level 1 teaching to migrate their learners to the next van Hiele level'.*

This permission should save as your introduction to the respective teacher and learners you wish to involve in your research, provided your activities shall not in any way interrupt the teaching and learning activities at school.

I wish you well in your studies

Thank you

....

L. Matomola

Principal

MINISTRY OF EDUCATION, ARTS AND CULTURE
Nsundano
Junior Secondary School
0 2 APR 2018
PO Box 6016 Bukalo
Telefax : 066-254183
REPUBLIC OF NAMIBIA

Appendix E: Participant teachers consent form

INFORMED CONSENT FORM

RHODES UNIVERSITY
Where leaders learn

Research Project Title:	A critical analysis of selected teachers' perceptions and experiences of the role that visualisation processes play in their van Hiele level 1 teaching to migrate their learners to the next van Hiele level.
Principal Investigator(s):	Mr.Ben Munichinga

Participation Information
• I understand the purpose of the research study and my involvement in it • I understand the risks and benefits of participating in this research study • I understand that I may withdraw from the research study at any stage without any penalty • I understand that participation in this research study is done on a voluntary basis • I understand that while information gained during the study may be published, I will remain anonymous and no reference will be made to me by name or student number • I understand that video recording (other data collection requirements particular to this research, e.g. test results, personal information, video recording) may be used • I understand and agree that the interviews will be recorded electronically • I understand that I will be given the opportunity to read and comment on the transcribed interview notes • I confirm that I am not participating in this study for financial gain

Information Explanation
The above information was explained to me by: Mr. Ben Munichinga
The above information was explained to me in English and I am in command of this language:

Voluntary Consent	
I, [redacted] hereby voluntarily consent to participate in the above-mentioned research.	
Signature:	Date: 29 / 03 / 2018

Investigator Declaration	
I, Ben Munichinga, declare that I have explained all the participant information to the participant and have truthfully answered all questions ask me by the participant.	
Signature:	Date: 29/ 03/ 2018

INFORMED CONSENT FORM

RHODES UNIVERSITY
Where leaders learn

Research Project Title:	A critical analysis of selected teachers' perceptions and experiences of the role that visualisation processes play in their van Hiele level 1 teaching to migrate their learners to the next van Hiele level.
Principal Investigator(s):	Mr.Ben Munichinga

Participation Information
• I understand the purpose of the research study and my involvement in it • I understand the risks and benefits of participating in this research study • I understand that I may withdraw from the research study at any stage without any penalty • I understand that participation in this research study is done on a voluntary basis • I understand that while information gained during the study may be published, I will remain anonymous and no reference will be made to me by name or student number • I understand that video recording (other data collection requirements particular to this research, e.g. test results, personal information, video recording) may be used • I understand and agree that the interviews will be recorded electronically • I understand that I will be given the opportunity to read and comment on the transcribed interview notes • I confirm that I am not participating in this study for financial gain

Information Explanation
The above information was explained to me by: Mr. Ben Munichinga
The above information was explained to me in English and I am in command of this language:

Voluntary Consent	
[redacted] ere arily consent to participate in the above-mentioned research.	
Signature:	Date: 29 / 03 / 2018

Investigator Declaration	
I, Ben Munichinga, declare that I have explained all the participant information to the participant and have truthfully answered all questions ask me by the participant.	
Signature:	Date: 29 / 05 / 2018

INFORMED CONSENT FORM

RHODES UNIVERSITY
Where leaders learn

Research Project Title:	A critical analysis of selected teachers' perceptions and experiences of the role that visualisation processes play in their van Hiele level 1 teaching to migrate their learners to the next van Hiele level.
Principal Investigator(s):	Mr.Ben Munichinga

Participation Information
• I understand the purpose of the research study and my involvement in it • I understand the risks and benefits of participating in this research study • I understand that I may withdraw from the research study at any stage without any penalty • I understand that participation in this research study is done on a voluntary basis • I understand that while information gained during the study may be published, I will remain anonymous and no reference will be made to me by name or student number • I understand that video recording (other data collection requirements particular to this research, e.g. test results, personal information, video recording) may be used • I understand and agree that the interviews will be recorded electronically • I understand that I will be given the opportunity to read and comment on the transcribed interview notes • I confirm that I am not participating in this study for financial gain

Information Explanation
The above information was explained to me by: Mr. Ben Munichinga
The above information was explained to me in English and I am in command of this language: YES

Voluntary Consent	
I, M. CHUNGA ....hereby voluntarily consent to participate in the above-mentioned research.	
Signature:	Date: 29 / 03 / 2018

Investigator Declaration	
I, Ben Munichinga, declare that I have explained all the participant information to the participant and have truthfully answered all questions ask me by the participant.	
Signature:	Date: 29 / 03 / 2018

Page **25** of **30**

Appendix F: Sample of consent from parents

PERMISSION LETTER TO PARENTS

RHODES UNIVERSITY
Where leaders learn

Ben Munichinga

08 February 2018

REQUEST FOR PERMISSION TO CONDUCT RESEARCH WITH LEARNERS

To whom it may concern

My name is Ben Munichinga, and I am a part-time M.Ed Mathematics student at Rhodes University (RU) in Grahamstown, South Africa and a mathematics teacher at Nsundano junior secondary School. I wish to conduct my research project in the mathematics class of your child [redacted]. Your child will be asked to write a mathematics test before my research projects starts and one after the research project is finished. I ask for your permission that your child participates in these tests. I also require your permission to video record your child's mathematics teacher teaching geometry. I will not video-record your child but in the event of filming your child by accident I undertake to blur his face so that he/she cannot be recognised.

Your child will benefit from this experience as he/she will engage in exciting ways of learning geometry and hopefully improve his/her performance.

Upon completion of the study, I undertake to provide you and the participating teachers with access to the research findings. If you require any further information, please do not hesitate to contact me on cell: 0812205365 or email: bmunichinga@gmail.com.

Thank you for your time and consideration in this matter.

Yours sincerely

Ben Munichinga (15M8814)

....

Please tick(√) appropriate box if you **accept** or **Do Not accept** my request.

Accept	√	**Do Not Accept**		**Name** [redacted]

PERMISSION LETTER TO PARENTS

RHODES UNIVERSITY
Where leaders learn

Ben Munichinga

08. February 2018

REQUEST FOR PERMISSION TO CONDUCT RESEARCH WITH LEARNERS

To whom it may concern

My name is Ben Munichinga, and I am a part-time M.Ed Mathematics student at Rhodes University (RU) in Grahamstown, South Africa and a mathematics teacher at Nsundano junior secondary School. I wish to conduct my research project in the mathematics class of your child [redacted]. Your child will be asked to write a mathematics test before my research projects starts and one after the research project is finished. I ask for your permission that your child participates in these tests. I also require your permission to video record your child's mathematics teacher teaching geometry. I will not video-record your child but in the event of filming your child by accident I undertake to blur his face so that he/she cannot be recognised.

Your child will benefit from this experience as he/she will engage in exciting ways of learning geometry and hopefully improve his/her performance.

Upon completion of the study, I undertake to provide you and the participating teachers with access to the research findings. If you require any further information, please do not hesitate to contact me on cell: 0812205365 or email: bmunichinga@gmail.com.

Thank you for your time and consideration in this matter.

Yours sincerely

Ben Munichinga (15M8814)

....

Please tick(√) appropriate box if you **accept** or **Do Not accept** my request.

Accept	✓	**Do Not Accept**		**Name**: [redacted]	**Signature**:..........

Appendix G: Lesson plans

Lesson 1

<table>
<tr><td>Teacher: ………………………..</td><td>Grade: 8</td><td>Date:

Day:</td></tr>
<tr><td colspan="2">Subject: Mathematics</td><td>Time:45-90min</td></tr>
<tr><td colspan="3">Theme: Geometry

Topic: Naming Basic Geometric Shapes</td></tr>
<tr><td colspan="3">Teaching materials & resources: Hand-outs of geometric Shape, Chalk board</td></tr>
<tr><td colspan="3">Learning objectives: Learners will be able to name basic shape and classify them as either triangles or quadrilaterals.</td></tr>
<tr><td colspan="3">Presentation of lesson: This lesson is sequenced and guided by the van Hiele phases of instruction: information, direct orientation, explicitation, free orientation, and integration. Visualisation processes are incorporated within each phase.

1. An appropriate short introduction:

Information phase:teacher provides inquiry-based activities.

Designing visual: Lesson is introduced with the use of visual material

To warm-up the class and gauge learners' understanding of basic foundation of shapes, teacher gives each learner a piece of paper and instruct them to draw any basic shape of their choice. Provide learners with verbal like, you could draw a square; a rectangle, kite etc. Give them about 3-5 minutes to do this. While they are drawing it could be a good idea to move around and encourage them to draw bigger shapes visible from a distance.

Direct orientation: teacher guides learners to uncover connections and to identify the focus of the subject matter.

Manipulating visual: Teacher uses visuals in a way that allows learners to connect new to what they already know

Select about 4-5 learners from the class to present their drawn shapes. Members of the class could first be asked to identify the shape before the owner. While they are presenting have these shapes drawn on the chalkboard with their names next to each shape. After the</td></tr>
</table>

presentation, this could be the best time to discuss why we study shapes? Here is something to share…….

> *'Think of the world with only one shape! For example if everything we see could be in a form of a square or circle, what do you think will this world really look like, if the classroom door was a circle, the chair would be circle, our windows could be circle and everything else as a circle. "You may pause and let learners tell what they think about the situation". Surely the world could be boring, hence in geometry we study and appreciate spatial space around us. In geometry therefore we study different shapes. Look around the classroom, what shapes do you see? Allow learners to answer!!!!!'*

2. **Presentation of subject matter & learning activities**

Explicitation:*The teacher takes care that learners advance the use of technical language with understanding through the exchange of ideas***.**

Visual language: *Teacher uses strategies that allow learners to to produce visuals*

The teacher identifies 2 learners from the class who have drawn 3 and 4 sided shape. Then divides the class into 2 groups. Group A= 3 sided shapes and group B= 4 sided shapes, if there are learners who have drawn shapes with more than 4 sides, allow them to form a group as well. Therefore group C= more than 4 sides. Engage in discussion with learners before giving them a definition of quadrilateral for example. Show them picture of a motor bike with four wheels and ask them its name. A 'quad-bike', yes it is a quad-bike, then ask learners to connect all the four wheels with straight lines. Then ask learners what shape is formed after joining the wheels. Therefore we can conclude that any four sided shape is called a quadrilateral maybe from the name 'quad'. Write the correct spelling on the chalkboard. Do the same for triangles by showing them a tricycle. Group C members would also want the name for their shapes (more than 4 sides' shapes). This could be the correct time to introduce a new vocabulary of '*polygons*'. A 5-sided shape is a pentagon, 6 is a hexagon. However without ignoring these learners tell them we 'will discuss this in more details in the next lesson', then focus on the topic for the day.

free orientation:*learners are challenged to solve problems*

Analysing visuals: *Teacher encouraged learners to work with the visuals in terms of concepts that were taught*

Hand-out new pages to group C members so that they draw either a 3 or 4 side shape of their choice and then let them join the two groups A & B. Time for a quick assessment. Have heading on the chalkboard, as; **shape**, **name**, **triangle** and **quadrilateral**. Teacher puts different kinds of shapes on the front desk and asks learners to paste and tick in the correct column as shown below. The teacher will have shapes such as kite, scalene triangle, equilateral triangle, square, rectangle, rhombus, parallelogram, isosceles triangle.

Shape	**Name**	**Triangle**	**Quadrilateral**
	Triangle	✓	
	Quadrilateral		✓

While learners are pasting, it is a good idea to get response from the class if shapes are correctly pasted.

3. Consolidation

Integration: *learners summarize their understanding of the topic*

Interpreting visuals: *The teacher concludes the lesson using visual materials.*

In their groups ask them to write a definition of a triangle and quadrilateral. It is important to write the final definition on the chalkboard. After the definition, pass through the names of shapes mentioned during the lesson, as appreciation, have as many learners mentioning names of shapes. Allow 2 or 3 learners to summarise the learning content for the day, while pictures of tricycle and quad bike are on the chalkboard.

Lesson 3

<table>
<tr><td>Teacher: …………………………………..</td><td>Grade: 8</td><td>Date:

Day:</td></tr>
<tr><td colspan="2">Subject: Mathematics</td><td>Time:45-90min</td></tr>
<tr><td colspan="3">Theme: Geometry

Topic: Properties of quadrilaterals</td></tr>
<tr><td colspan="3">Teaching materials & resources: Rubber bands, geo-board</td></tr>
<tr><td colspan="3">Learning objectives: Learners will be able to</td></tr>
<tr><td colspan="3">Presentation of lesson

1. An appropriate short introduction
Information: the teacher provides inquiry-based activities

Designing visual: Lesson is introduced with the use of visual material

Hand out geo-boards to learners with rubber bands. Recap on the previous lesson: polygons, regular polygons, and irregular polygons, classification of polygons as convex and concave. Have children construct shape they think from the previous lesson. Have a discussion with each group on the classification as above.

Direct orientation: teacher guides learners to uncover connections and to identify the focus of the subject matter.

Manipulating visual: Teacher uses visuals in a way that allows learners to connect new to what they already know

Ask learners to list quadrilaterals familiar with them; square, Rhombus, Rectangle, parallelogram, trapezium, kite. Teacher to assist if learners fail to list all quadrilaterals. Have each group construct a quadrilateral from the above listed. The teacher should construct a trapezium. The teacher then uses his/her geo-board and constructs a trapezium, at the same time explaining and demonstrating the properties of a trapezium to learners.

2. Presentation of subject matter & learning activities</td></tr>
</table>

Explicitation: *The teacher takes care that learners advance the use of technical language with understanding through the exchange of ideas.*

Visual language: *Teacher uses strategies that allow learners to to produce visuals*

It's now time for group A to present the properties of their shape (square)**.** Group B does the same for their Rhombus. Then it's time for quick look at the two shapes. Ask the class how the two shapes are the same, and how they are different. It is a good idea to list all the properties of a square and a rhombus and then encourage learners to look at similarities and differences in properties.

Then group C lists, all the properties of their shape (rectangle)**.**Group D also lists the properties of their shape (parallelogram)**.** Time for a quick look at the two shapes. How the two shapes similar and how are they different in terms of their properties? List all the properties of the two shapes, and allow learners, to figure out how the two shapes differ in terms of their properties.

free orientation:￿*learners are challenged to solve problems*

Analysing visuals: *Teacher encouraged learners to work with the visuals in terms of concepts that were taught*

Ask learners to look at the properties of each shape and then use them to draw the shapes. Ask learners to represent ad show all the properties as they draw. Learners may use geodot papers to draw the shapes. Have learners show each other their drawn shapes and encourage them to look ,at the properties of the shapes to see if they match with the properties

3. **Consolidation**

Integration: *learners summarize their understanding of the topic*

Interpreting visuals: *The teacher concludes the lesson using visual materials.*

Explain the vocabulary to learners for them to fully understand. Properties of shapes, what makes a square a square? It is its physical attributes, its characteristics that make it different from other shapes e.g the number of sides, the number of angles, its diagonals, the lines of symmetry etc.

- ✓ What is a diagonal?
- ✓ What is line of symmetry or rotational symmetry?

Lesson 2

Teacher: …………………………………	Grade: 8	Date: Day:
Subject: Mathematics		Time:45-90min
Theme: Geometry Topic: Polygons		
Teaching materials & resources: Hand-outs of geometric Shape, Chalk board and geo-board		

Learning objectives: Learners will be able to identify which polygons as regular, irregular, concave or convex (identify and name the following regular and irregular two-dimensional shapes: triangle, square, rectangle, rhombus, parallelogram, other quadrilaterals, pentagon, hexagon and circles in different orientations)

Presentation of lesson: This lesson is sequenced and guided by the van Hiele phases of instruction: information, direct orientation, explicitation, free orientation, and integration. Visualisation processes are incorporated within each phase.

1. **An appropriate short introduction:**

Information: *the teacher provides inquiry-based activities*

Designing visual: *Lesson is introduced with the use of visual material*

As a warm-up for the lesson, ask learners what they have learnt in the previous lesson. Questions like how many sides does a rectangle has. Is an isosceles triangle a quadrilateral or triangle? The teacher displays the pictures of tricycle and a quad bike to help learners recall.

Direct orientation: *the teacher guides learners to uncover connections and to identify the focus of the subject matter.*

Manipulating visual: *Teacher uses visuals in a way that allows learners to connect new to what they already know*

Time to have fun (constructions), have an overheard display of a geo-board. Establish rule for the class, for example geo-boards are only to be used for constructing shapes. You may demonstrate this by constructing a triangle. Dived learners in groups of 3 people per group. Hand out geo-boards and allow learners time to do free exploration with the geo-board. Display a triangle and a non-triangle and ask students to identify the shape you have created. Then ask learners to make 3 different triangles using 3 different colours of rubbers bands. Encourage learners to concentrate on ‘sides’, ‘edged’, ‘corner’, angle and so forth

2. **Presentation of subject matter & learning activities**

Explicitation: *The teacher takes care that learners advance the use of technical language with understanding through the exchange of ideas.*

Visual language: *Teacher uses strategies that allow learners to to produce visuals*

Remind learners of the previous lesson, where they had a 5 or 6-sided shape. Instruct each group to construct a 6 side shape on the geo-board. Allow each group to show their constructed shape to rest of the class. Then ask each group if the sides of their constructed shape are of equal length? It is a good idea to have the teacher’s hexagon having equal sides and show it to learners. Because all of us could construct a 6 side shape with different length of sides, therefore polygons can be classified into two categories. Polygons with all side length and angles equal are ‘regular polygons’ and those with different side length and angle size are ‘irregular polygons’

free orientation:*learners are challenged to solve problems*

Analysing visuals: *Teacher encouraged learners to work with the visuals in terms of concepts that were taught*

Time to have fun again. Start with triangles. Using different colour bands, construct a regular triangle and an irregular one. Let learners construct these two and then engage in a discussion with learners explaining what a polygon is. A polygon is any shape formed with straight lines. Triangle, quadrilaterals, pentagon, hexagon are all polygons. The name tells us how many sides the shape has. ***Something to share….***

> *‘The word ‘polygon’ could be from the word polygamy, meaning marrying more-than one wife. A common practice in our society’. Poly- means many. Thus the husband (pentagon) has 5 wives (sides).*

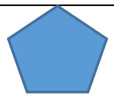
Now geo-boards aside, using a ruler and a pencil, construct 2 regular polygons and 1 irregular polygon having 7 sides. Now….. Represent your shapes on a geo-board. Allow learners to show each other their shapes either on paper or the geo-board.

Regular and irregular polygons can be classified as concave or convex!!!! What is a concave polygon? A concave polygon (straight sided) is concave where there are ‘dents’ or indentations in it (***Show this to learners***). You can remember it by con-“cave” (it has a cave in it!). The difference lies in the measure of angles. For a polygon to be convex, all of its interior angle must be less than 180 degrees. (Show this to learners).Allow each group to construct 1 concave and 1 convex polygon on their geo-boards. Time for a quick assessment. Have heading on the chalkboard as **Polygons, regular, irregular polygon, concave polygons and convex polygons, as shown below.** Paste different shapes on a chart. Learners should paste under polygon column their shapes and then tick the correct box. They may tick more-than once where applicable.

3. Consolidation

Integration: *learners summarize their understanding of the topic*

Interpreting visuals: *The teacher concludes the lesson using visual materials.*

Polygon	**Regular**	**Irregular**	**Concave**	**convex**
	✓			✓

Quick feedback on the class activity, reflecting on how learners engaged with activity. Summary of the learned content ask simple questions

-what is a polygon?

-draw a 5 sided convex polygon on the chalkboard.

Appendix H: Test paper and memo

RHODES UNIVERSITY
Where leaders learn

THE VAN HIELE LEVEL 1 & 2 GEOMETRY TEST

GRADE 8

Pre & Post-test

INSRUCTIONS:

1. Do not start until told to do so by your teacher.
2. Fill in the space below.

Full name:……*MEMORANDUM*……………………………………………

Name of school:…………………………………………………………………….....

Grade:…………………………… Date……………………………………

Age (in years) …………………………………. Gender:………………………...

3. This test will **NOT** be used for promotional purposes.
4. You will need **60 minutes** to complete the test
5. Use HB pencil and a ruler provided to you throughout.
6. Diagrams are not necessarily drawn to scale.
7. Please answer all questions.

>>

Turn Over

Item 1

a. Tick the box that corresponds with shapes given in Figure 1. Where applicable, you may tick more than one box for some of the shapes.

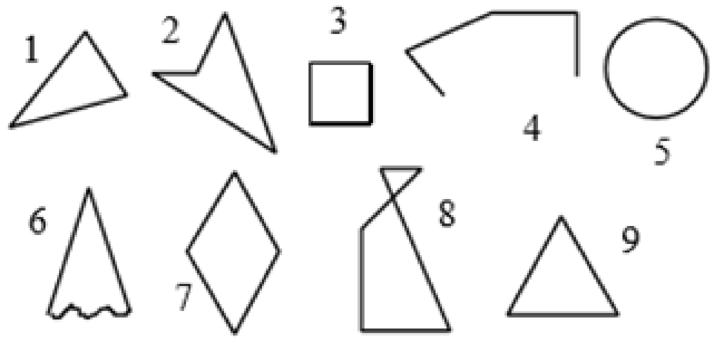


Figure 1

Shape	Polygon	Non-polygon	Triangle	Quadrilateral
1	✓		✓	
2	✓			✓
3	✓			✓
4		✓		
5		✓		
6		✓		
7	✓			✓
8	✓			
9	✓		✓	

14 marks

b. Write the numbers of the shapes that are not polygons and explain, for each of them, why it is not a polygon.

4: because the shape is not closed/ any relevant answer referring to the shape as not fully closed.

5: because it is a circle/any relevant answer.

6: because it has one line which is not straight/zigzag/bending or not all its side lines are straight.

6 marks

c. Write the numbers of shapes that are not triangles and explain why they are not.

2: because it has four sides/ it does not have 3 sides

3: because it has four sides/ it is a square/it does not have 3 sides

4: because it has four sides which are not enclosed/ it does not have 3 sides

5: because it does not have 3 sides/ it is a circle

6: because one of its sides is not straight

7: because it has four sides/ it does not have 3 sides

8: because it has more than 3 three sides/ 2 polygons which are combined

14 marks

d. Write the numbers of the shapes that are not quadrilaterals and explain why they are not.

1: because it has three sides/ it does not have four sides

4: the shape is not enclosed/

5: *because it does not have 4 sides which are straight*

6: *because it only has three sides/*

8: because it has more than four sides/ it is a combination of a triangle and a quadrilateral.

9: because it has three sides (which are closed).

12 Marks

e. Is shape 8 a polygon? Give a reason for your answer

Yes, it is a special type of polygon because it is a combination of 2 types of polygons (triangle & quadrilateral) which are enclosed, formed by straight lines or sides. it is a self-intersecting polygon. 2 Marks

f. Is shape 2 a triangle? If not so, explain.

Shape 2 is not a triangle because it has four sides. A triangle should have only 3 straight lines and enclosed.

2 Marks

Item 2:

a. Tick the box that corresponds with the polygon given in Figure 2. Where applicable, you may tick more than one box for some of the polygons.

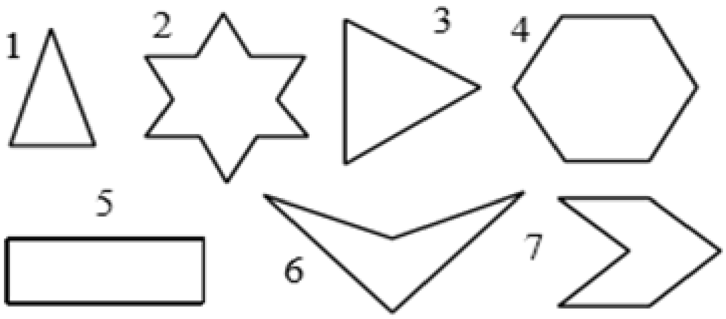


Figure 2

Polygons	Regular Polygon	Irregular Polygon	Concave Polygon	Convex Polygon
1		✓		✓
2		✓	✓	
3	✓			✓
4	✓			✓
5		✓		✓
6		✓	✓	

7		✓	✓		

14 Marks

b. For the following polygons 2, 4, 5 and 7 [in Figure 2], explain the choice of your tick.

Polygon 2

This is an irregular and concave polygon. It is irregular because the interior angles are not equal. It has some interior angles greater than 180^o while others are less than 180^o. Some of its sides look like they are 'pushed' in.

It is concave because it has sides caving inside forming interior angles greater than 180 degrees.

Polygon 4

This is a regular and convex polygon. The polygon has all its sides equal in length and angles equal in size. It is a convex because each of its interior angles is less than 180 degrees.

Polygon 5

This is an irregular and convex polygon. It is an irregular polygon because not all of its sides are equal in length. It is a convex because its interior angles are all less than 180 degrees.

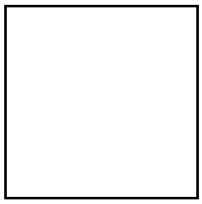
Polygon 7

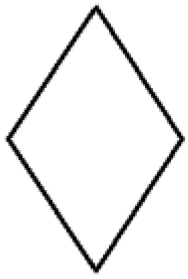
This is an irregular and concave polygon. It is irregular because not all of its sides are equal in length and angles in size. It is a concave polygon because one of its interior angles is bigger than 180 degrees and some of its sides look like they are 'pushed' in.

8 Marks

Item 3

Consider a Square and a Rhombus as seen





Square Rhombus

a. Write all the properties which are shared by ***squares*** and **rhombi**

- ✓ *Each has four equal sides*
- ✓ *a pair of diagonals*
- ✓ *Diagonals bisect each other and meet at 90^o*
- ✓ *Diagonals bisect the four angles*
- ✓ *Lines of symmetry order of 4*
- ✓ *Order of rotational symmetry*
- ✓ *Opposite equal angles*

Any four properties *4 Marks*

b. Write all the properties which are true for a square **but not** for a rhombus.

- ✓ *Four equal angle 90^o*
- ✓ *Two diagonals are equal in length*
- ✓ *four lines of symmetry*
- ✓ *Order of rotational symmetry of four*

Any three properties *3 Marks*

c. Write all the properties which are true for a rhombus **but not** for a square

- ✓ *opposite equal angles*
- ✓ *diagonals are not equal*
- ✓ *Two lines of symmetry*
- ✓ *Order of rotational symmetry of two*

Any two properties *2 Marks*

Item 4

Consider an **Equilateral triangle** and an **Acute triangle** (you may draw the two if you like)

a. Write all the properties which are common for equilateral and one of the acute triangles of your choice.

- ✓ *Have three sides*
- ✓ *Have three interior angles*
- ✓ *Sum of interior angles equals to* 180^o
- ✓ *All interior angles are acute*
- ✓ *Sum of any 2 interior angle is greater than* 90^o

Any 3 listed properties *3 Marks*

c. Write all the properties which are true for equilateral **but not** for the other acute triangles.

- ✓ *All three sides are equal*
- ✓ *All three interior angles measure* 60^o
- ✓ *Has 3 lines of symmetry*
- ✓ *3 rotational order of symmetry*

any two listed properties *2 Marks*

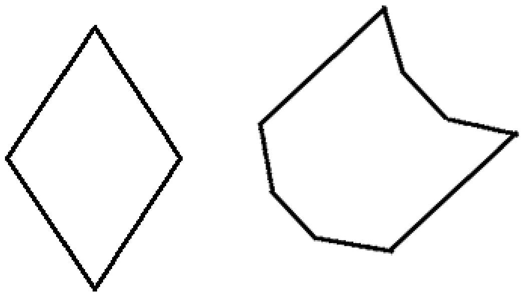
d. Write all the properties which are true for the other acute triangles and **not** for equilateral triangles.

- ✓ *Three sides not equal*
- ✓ *Three angles not equal*

2 Marks

Item 5

Refer to Figure (3a) and (3b) below.



-a- -b-

Figure 3

a) Figure 3a is a Rhombus. Make a list of all the properties that you find for this shape (you can draw to explain the properties).

- ✓ *Four sides equal*
- ✓ *opposite equal angles*
- ✓ *diagonals not equal*
- ✓ *Two lines of symmetry*
- ✓ *Order of rotational symmetry of two*

Any 3 listed properties *3 Marks*

b) Now look at shape 3b. Make a list of all the properties that you find for this shape (you can draw to explain the properties).

- ✓ *Has 8 sides*
- ✓ *Has 8 interior angles*
- ✓ *2 interior angles measure greater than 180^o*
- ✓ *Pair of opposite sides equal and parallel*

Any 3 relevant properties mention visible to describe the shape.

3 marks

Item 6:

The *diagonal* of a polygon is a segment that joins two non-adjacent vertices of the polygon. Now answer the following questions:

a. In a 5-sided polygon, the number of diagonals which can be drawn from each vertex is……**2**….and the total number of diagonals is…**5**……….

b. In a 6-sided polygon, the number of diagonals which can be drawn from each vertex is……**3**…..and the total number of diagonals is……**8**…….

4 Marks

End of Test

Appendix I: Stimulus-recall and focus group provisional questions

The following questions are designed to find out more about the lessons presented which incorporated visualisation processes and the van Hiele model. The purpose is to find out what worked well or not well during your lesson.

1. Did the teacher use a visual manipulative during the lesson? The name.
   ………………………………………………………………………………………
   ………………………………………………………………………………………
   ………………………………………………………………………………………
2. What geometrical concepts emerged from the use from this manipulative?
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3. Did the visual manipulative help learners in any way to learn or remember concepts?
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   ………………………………………………………………………………………
   ………………………………………………………………………………………
   ………………………………………………………………………………………
   ……………………………………………………………………………………...
4. Did learners get a chance to work with the visual material during class?
   ………………………………………………………………………………………
   ………………………………………………………………………………………
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5. Briefly reflect on the lesson generally (you may look at what worked well/ did not work, area of improvement, learners engagement/participation during the lesson)
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   ……………………………………………………………………………………..
6. Did the lesson help the learners move from van Hiele level 1 to van Hiele level 2. Please elaborate.
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   ....
   ....
   ....
   ....

Stimulus- recall interviews (provisional)

1. Together with the participating teachers we replayed and analysed the video recordings using the analytical tool discussed in the proposal. The format of the interview was a conversation where we teased out the role of visualisation processes during the lesson presentation as per the observable indicators elaborated upon in the proposal. We also discussed the teacher's responses to the questionnaire that they used to reflect on the lesson to elaborate on those reflections.