EXPLORING OUTDOOR MATHEMATICS LEARNING FOR CONCEPTUAL UNDERSTANDING THROUGH SMARTPHONES

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VUYANI SAMUEL POP

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Dr Clemence Chikiwa and Professor Marc Schäfer

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ABSTRACT

This study investigated how selected grade 11 mathematics learners used smartphones with the MathCityMap application to learn trigonometry outdoors for conceptual understanding. The aim of this research project was to explore outdoor mathematics learning for conceptual understanding using smartphones. This case study of grade 11 mathematics learners in Lejweleputswa District in the Free State Province, was informed by the Realistic Mathematics Education theory. The study is grounded within an interpretive paradigm and used the explanatory sequential mixed-method design. Forty-two grade 11 mathematics learners participated in the survey and from these 12 were purposively selected to participate in walking the mathematics trails and interviews. The findings revealed that, while the grade 11 mathematics learners acknowledged the significance and value of using smartphones for learning mathematics, they were prohibited from carrying or using smartphones on the school premises, as part of the school code of conduct. The preferred use of smartphones for learning mathematics was understandable, as the survey was conducted at a time when the COVID-19 pandemic and associated restrictions were still in place. The survey unearthed that among applications for learning mathematics, the MathCityMap application was not known by the learners who participated in the survey. Mathematics trails observations indicated that outdoor tasks were a source of mathematical concepts or formal mathematical knowledge, and enabled learners to reinvent mathematical ideas and concepts with adult guidance. Learners were able to make use of appropriate mathematical models and connections. The mathematics trails ignited robust discussions among learners, and prompted learners to draw from prior knowledge, and recognise and identify suitable mathematical models and shapes from the real-world objects. Learners were able to use multiple representations, make necessary mathematical links, and use their prior knowledge to enhance their trigonometry conceptual understanding. This study concluded that using smartphones with the MathCityMap application could enhance conceptual understanding of trigonometry. The implications for teachers are that learners should be exposed to outdoor mathematics learning using smartphones with the MathCityMap application to improve their conceptual understanding. It is hoped that the results of this study can be used by various stakeholders, who include, inter alia, mathematics subject advisors and teacher training institutions, to enhance learners' conceptual understanding of mathematics.

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DEDICATION

I dedicate this thesis to my spiritual father and my late uncle Tshepang Michael Phosholi and my uncle Tshediso Michael Masakala.

Thank you so much for instilling in me a culture of learning, hard work and giving me a strong and firm educational background. I will live to take your wish to fruition.

DECLARATION OF ORIGINALITY

I, Vuyani Samuel Pop, student number (20p2051), hereby declare that this study entitled 'Exploring outdoor mathematics learning for conceptual understanding through smartphones in grade 11 trigonometry in Lejweleputswa district, Free State province' is my own work, and a product of my research. It has not been submitted, in any form, to another institution. Where I have drawn on the ideas of people from other publications or other sources, I have fully acknowledged these in accordance with Rhodes University, Education Department reference guide.

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ACRONYMS/ABBREVIATIONS USED IN THIS STUDY

AMESA	Association for Mathematics Education in South Africa
CAPS	Curriculum and Assessment Policy Statement
DBE	Department of Basic Education
DHET	Department of Higher Education and Training
FET	Further Education and Training
GESF	Global Education and Skill Forum
GPS	Global Positioning System
GSMA	Global System for Mobile Communication Association
ICT	Information and Communication Technology
MCM	MathCityMap
NCS	National Curriculum Statement
NCTM	National Council of Teachers of Mathematics
NRC	National Research Council
NSC	National Senior Certificate
RME	Realistic Mathematics Education
RUESC	Rhodes University Ethics Standard Committee
SA	South Africa
SGB	School Governing Body
TIMSS	Trends in International Mathematics and Science Study
UNESCO	United Nations Education, Scientific and Cultural Organisation
UNICEF	United Nations International Children’s Emergency Fund
UK	United Kingdom

2D	Two Dimensional
3D	Three Dimensional

CHAPTER ONE
INTRODUCTION

1.1 CONTEXT OF THE RESEARCH

The study aimed to explore outdoor mathematics learning for conceptual understanding using smartphones in selected grade 11 mathematics learners by means of the MathCityMap application (MCM app) (www.mathcitymap.eu). The chapter begins with the research context and background, and its relation to smartphones using the MCM app in mathematics education followed by the research goals and questions. The chapter further provides a summary of the methodology, followed by the theoretical framework and the significance of the study. The limitations of this study are also identified before the chapter concludes with a short outline of the structure of the thesis.

1.2 BACKGROUND OF THE STUDY

The study focused on grade 11 mathematics learners from a secondary school in Lejweleputswa district, Free State Province. Prior to this study's intervention programme, it also surveyed 42 grade 11 mathematics learners from the same school on how they were learning trigonometry concepts outdoors. The Department of Basic Education (DBE) (2011), National Curriculum Statement (NCS) grades R–12 advocates a learner-centred approach to learning. It subscribes to an active and critical approach to learning rather than rote and uncritical learning of given truths. The mathematics curriculum is based on the constructivist approach, which is crucial in this study.

The DBE (2011), the NCS, grades R-12 further state that mathematics is a human activity, and real-life problems should be incorporated into all sections whenever appropriate, and that they should be realistic. This study is premised on Realistic Mathematics Education (RME) (de Lange, 1996). In the mathematics curriculum, a number of reasons necessitate the inclusion of real-world problems. For example, through such problems, learners can organise themselves and their activities responsibly. The Department of Basic Education (2011) aims to produce learners who can use science and technology effectively and critically demonstrate an understanding of the world as a set of related systems by recognising that problem-solving contexts do not exist in isolation. Despite its benefits and relevance, trigonometry continues to be a subject many learners struggle with, believe to be difficult and see as abstract compared to other topics in school mathematics (Weber, 2005; Gur, 2009; Sasman, 2011; Hussaini & Usman, 2017; Jelatu et al., 2019). This is mainly due to how trigonometry is taught, usually detached from reality (Martín-Fernández et al., 2018), leading to learners constantly asking why they need to learn it.

In South Africa (SA), the trigonometry content area has a weighting of 40 per cent in the Grade 12 Mathematics Paper 2 (DBE, 2011). Thus, failure by learners to understand trigonometry concepts has a ripple effect on their success in school mathematics (Sasman, 2011; Kamber & Takaci, 2017; Chigonga, 2017; Khuzwayo, 2019).

Moloi (2012) argues that quality mathematics learning in SA will continue to be a challenge unless teachers try to understand how their learners think and learn and recognise the learning experiences of their learners. A study by Rankweteke (2020) in Limpopo province found that many learners were passively engaged and listened to or watched the teacher without being actively involved in the learning trigonometry. The teacher-centred approach remains the dominant teaching strategy used in SA (Maile & Makofane, 2019), and learning is seen as a set of predetermined content to be learned without any critical thinking, engagement with, or exploration of the information (Jojo, 2019). Learners are simply expected to give the correct answer without any meaning or explanation for the answer being provided or discussed (Harber & Serf, 2006; Shanmugavelu, et al.2020; Thomas & Bartlett, 2011; Umugiraneza et al., 2017; Taylor, 2021). This practice is still prevalent in many schools, e.g., in schools where the researcher taught and currently teaches. The researcher's involvement in the Association for Mathematics Education in SA (AMESA) activities for many years and in numerous grade 12 intervention projects in the Free State province attest to the persistence of this practice. Therefore, there is a need to invest in an alternative learning strategy for mathematics. An essential work of a teacher is to plan for diversity or different learner abilities. The mathematics trail using a smartphone is characterised by a problem-based learning approach to mathematics and attends to this range of abilities (Jarvis et al., 2012).

Siyepu (2015) and Khuzwayo (2019) posit that errors displayed by South African learners in trigonometry are conceptual and procedural and observe that some learners do not understand that trigonometric ratios can only be applied in a right-angled triangle. Newton (2010) asserts that since conceptual rather than procedural learning leads to a greater understanding, trigonometry is too vital in mathematics to be taught solely in a procedurally or theoretically without clear reference to everyday life. It is, thus, crucial to provide learners with meaningful learning experiences derived from their environments to enhance their conceptual understanding (Weber, 2005; Martín-Fernández, et al., 2018, Pambudi, 2022). The change of scene from studying indoors enhances mathematical concepts (Brooks, 2022). The MCM app, among other things, helps learners locate real-world tasks in their environment using trigonometry concepts learned in grade 11. Ludwig and Cahyono (2016) suggest that the effective use of dynamic software and applications, such as the MCM app, can potentially improve learners' conceptual understanding of mathematics.

This study hoped that the participating learners' conceptual understanding would develop the relevance of trigonometry to life. The National Council of Teachers of Mathematics (NCTM, 2015) argues that 'it is essential that teachers and learners have regular access to technologies that support and advance mathematical sense making, reasoning, problem-solving and communication' (p. 1). The researcher proposes that cautious efforts must be made to recognise smartphones as tools to be used effectively and correctly for learning. Thus, this study focused on enhancing conceptual understanding of trigonometry through the MCM app on smartphones.

1.3 RATIONALE OF THE STUDY

This study investigated whether outdoor learning of trigonometry concepts using the MCM application had enhanced learners' mathematical conceptual understanding. The study intended first to help improve the researcher's practice as a high school mathematics teacher and contribute to innovative methods of teaching mathematics by their experiencing mathematics in their environments or places of interest. In Chirinda et al.'s (2021) study of the strategies used by 23 grade 12 South African mathematics teachers at various secondary schools, reported the need to be innovative (p.7), particularly to incorporate smartphones or applications for mathematics learning. Their study did not mention the MCM application and mathematics trail, but WhatsApp, YouTube, among others. Mwapwele et al. (2019) in their study about technology readiness and implications revealed that most teachers surveyed in South Africa, were optimistic about the use of Information and Communications Technology

(ICT) for teaching despite the existing financial, technical, and digital skills challenges at their schools.

1.4 RESEARCH QUESTIONS AND OBJECTIVES

The following research questions guided this study:

- How do learners use smartphones for mathematics learning purposes before participating in an intervention programme?
- How does using smartphones to learn mathematics through outdoor mathematics tasks enhance conceptual understanding of trigonometry concepts in a selected grade 11 class?
- What are the selected grade 11 mathematics learners' experiences and perceptions of using smartphones for learning trigonometric concepts after participating in the MCM project?

1.5 RESEARCH METHODOLOGY

The study was conducted within an interpretive paradigm. Central to the interpretive paradigm is understanding the subjective world of human experience (Cohen et al., 2011), how learning was enabled, and the associated experiences of the selected grade 11 mathematics learners when learning trigonometric concepts using the MCM application. Cohen et al. (2011) pointed out that data from the interpretive approaches to research must be authentic and reflect participants' experiences. This was ideal for this study, which intended to gain an in-depth understanding of how learning is enhanced from the learners' perspective. Furthermore, an understanding of the participants' experiences and perceptions of the use of smartphones for supporting conceptual understanding of trigonometry concepts was sought (area, angles, heights, and distance).

This research was a case study using the explanatory sequential mixed-method approach. According to Bertram and Christiansen (2014), 'a case study, is a systematic and in-depth study of one particular case in its context' (p. 42). In this study, it was a selected grade 11 mathematics class at a secondary school in the Lejweleputswa district, Free State Province. The unit of analysis was the simultaneous use of mathematics trails and smartphones with the

MCM app in learning trigonometry and participants' experiences on how using smartphones enhanced their learning and conceptual understanding.

The study used a mixed-method approach using both quantitative and qualitative approaches. The data was collected through a survey, mathematics trail observations and focus group interviews. Quantitative (closed-ended) research emphasised objectivity in measuring and describing phenomena using numbers, whereas qualitative (open-ended) designs emphasised gathering data on naturally occurring phenomena in the form of words. Using the qualitative approach, the researcher sought to understand how participants made sense of the situation (Merriam, 2015; Creswell & Clark, 2007). Qualitative methods sought to achieve a depth of understanding, while quantitative methods examined the extent of understanding (Patton, 2002). Quantitative research collects and analyses numerical data to find patterns and averages, make predictions, test causal relationships, and generalise results to wider populations. This is the opposite of qualitative research, which involves collecting and analysing non-numerical data, e.g., text, video, and audio (Bhandari, 2020). Using these approaches in this study provided an extensive understanding of how using outdoor tasks during learning through the smartphone increased understanding of trigonometry concepts in a selected grade 11 mathematics class.

Forty-two learners responded to the survey administered at the school to determine how learners used smartphones for mathematics learning. The survey was followed by an intervention programme that involved a workshop with the 12 purposively selected grade 11 mathematics learners to expose them to the MCM app. Immediately after the workshop, the study piloted the mathematics trail around the school, so participants gained some experience before the real trailblazing since the mathematics trail concept using a smartphone with the MCM app was new to them. After the pilot, they were observed walking the mathematics trail on grade 11 trigonometry concepts outdoors, using a smartphone with the MCM app. In total, they walked two mathematics trails around the school. The mathematics trails took two weeks, i.e., one per week. The four groups comprised three members each, and the researcher observed one group per day. The trail observations were videotaped.

Each video recorded mathematics trail observation was immediately followed by a reflective interview (focus group interviews) to establish the selected grade 11 mathematics learners' experiences and perceptions of using smartphones for learning trigonometry theories after participating in the MCM project.

1.6 SIGNIFICANCE OF THE STUDY

In the South African education curriculum context, this study can inform curriculum developers and mathematics education researchers about combining outdoor or real-life mathematics tasks using a smartphone with the MCM app in the design of policy guidelines. This project contributed to the researcher's knowledge and professional development as a high school mathematics teacher. The study further contributes to innovative and exciting ways of learning mathematics. Additionally, the results would aid the Department of Higher Education and Training in integrating outdoor mathematics tasks using a smartphone with the MCM app in its teacher training programmes to use technological devices such as smartphones with the MCM app in mathematics teaching and learning. The findings of this research may assist schools in making deliberate efforts to start viewing smartphones as tools that can be effectively and correctly used for learning purposes.

1.7 LIMITATIONS

The challenges confronted in the study included getting the surveys from the learners, as the researcher was not a teacher at that school. The researcher is a full-time teacher at another school, making it difficult to get the surveys from some learners because meetings could only be arranged with the participants after normal school hours. However, all the surveys were collected with the assistance of a WhatsApp group for easy communication because, at first, it was difficult to know the location of other participants, which wasted the limited time after school as the study had to happen outside normal school hours. The researcher had to ensure that the learners finished before dusk and transport those who lived far away from school. This was a challenge due to after-school commitments such as sports training sessions and extra classes. However, meetings were organised with the teachers concerned, and it was resolved.

The Free State Department of Education caused a delay of about four weeks before beginning the study as no one knew who was supposed to assist with the permission letter. The mathematics subject advisor provided the head office contacts in Bloemfontein, and they informed the researcher that he was not to conduct the study with the learners from the school where he taught, leading him to start looking for another school. After two weeks, permission was obtained from a secondary school. Consent letters were sent to the learners and parents, and they waited for the response, hence the delay. Eventually, the Free State Department of Education's head office gave permission, although it had to be collected in person, leading to a drive of 190km because telephonic assistance was not available.

The exploratory nature of the study did not allow the findings to be generalised. This study was conducted over a short period, focusing on a specific area of mathematics – trigonometry. The Free State Department of Education does not allow research in the third and fourth term of the school calendar and near examination time, especially when learners are involved. Despite these limitations, useful insights on the use of a smartphone with the MCM app to enhance the conceptual understanding of learners in trigonometry have been established.

1.8 OPERATIONAL DEFINITIONS

This section defines the key concepts in the study.

Trigonometry: a branch of mathematics that studies the relationship of angles, lengths, and heights (Coolman, 2015; Martín-Fernández et al., 2018).

Smartphones: cell phones based on operating systems that allow internet, video, touch screen and several other applications are called smartphones (Ballagas et al., 2006).

The **MCM app** stands for the MathCityMap App that combines the well-known mathematics trail idea with the current technological possibilities of smartphones **(**Ludwig, 2012).

Trailblazing refers to the act of walking the mathematics trails and solving the mathematical tasks assigned at each station outside the classroom.

Mathematics trail: a set of mathematical tasks or questions that are bound to objects from the real world, arranged as a trail that users follow (Shoaf et al., 2004).

Conceptual understanding: is an integrated and functional grasp of mathematical ideas as described by Kilpatrick et al. (2001).

1.9 THESIS OUTLINE

This thesis comprises five chapters.

1.9.1 Chapter One (Context of the study)

This chapter introduces the thesis, the background to this study, and the definitions used in this study.

1.9.2 Chapter Two (Literature review)

This chapter discusses the conceptual framework and literature that informed and shaped the analysis and interpretations of the results. It begins by defining the MCM app project (MCM) followed by debating the benefits and reasons for using smartphones in learning school mathematics and presents the status quo of mathematics learning in SA. Challenges experienced by mathematics learners in the learning of trigonometry in most schools in SA are discussed. The chapter concludes by discussing conceptual understanding and the RME theory underpinning this study.

1.9.3 Chapter Three (Methodology)

This chapter explains how the research was designed and carried out. It discusses the research orientation, methods employed, research design, data collection tools and sampling techniques used. It further describes issues pertaining to data analysis, validity, and ethics.

1.9.4 Chapter Four (Data presentation, Analysis and Discussion)

The chapter presents and discusses the results of this study. It analyses the data from the three research instruments (surveys, mathematics trail observations, and interviews–focus group) and concludes with a summary of the research findings. The analytical tools used to analyse the data are adapted from the work of Freudenthal (1973) and Kilpatrick et al. (2001) also presented in this chapter.

1.9.5 Chapter Five (Conclusions and recommendations)

Chapter Five concludes the study by summarising the dominant themes, drawing conclusions from the findings, and making recommendations for future research.

The study intended to contribute to creative methods of teaching and learning mathematics, particularly to enhance learners' conceptual understanding as they experience mathematics outdoors, using smartphones with the MCM app.

CHAPTER TWO
LITERATURE REVIEW

2.1 INTRODUCTION

A literature review is an important component of research, as it describes how the proposed study relates to previous research findings. It reflects originality, relevance of research and highlights the gap in the literature that the study seeks to address. Therefore, this chapter discusses mathematics learning in South Africa, trigonometry, use of smartphones mathematics learning, MathCityMap (MCM) project, mathematics trails, conceptual understanding in mathematics learning.

2.2 MATHEMATICS LEARNING IN SOUTH AFRICA

The learning of mathematics in South African schools faces many challenges evident from research conducted by the DBE, universities, and other independent research agencies (TIMSS, 2015; DHET, 2018). Many different strategies and initiatives, such as teach and assess, extra classes where learners are grouped according to their levels, Wits Maths Connect-Primary, Continuous Professional Development, DINALEDI project and resources have been invested into the teaching mathematics. However, there seems to be little improvement (Mapaire, 2016; Taylor, 2021).

Many factors are associated with the current state of mathematics education in SA. These include the abstract and complex ideas embedded in a given topic (Gordon, 2004; Zachariades et al., 2007). Some are pedagogical (Mabena et al., 2021), and others stem from overemphasising computational procedures without conceptual understanding (Gordon, 2004). Another factor is how learners negatively perceive mathematics and its presentation (Daniyan, 2015; Maile & Makofane, 2019; Jojo, 2019). Research shows that mathematics is often perceived as an abstract subject confined to the classroom without any relevance to the real world and predominantly learned from textbooks (Mitchelmore, 2004; Mutodi, 2014). This often leads to apathy among learners and little conceptual understanding and creativity in the subject (Sofowora, 2014; TIMSS, 2015; Grinshpan et al., 2019). The question is how those countries with consistently good results improve mathematics learning.

Many countries participating in the TIMSS (2015) reported initiatives for integrating ICT across the curriculum. According to Mullis et al. (2016), most of these countries current mathematics curricula emphasise integrating technology into mathematics learning. Notable examples are Germany, Netherlands, Australia and Egypt. Technology can improve mathematics learning (Goos, 2010; TIMSS, 2015; Radovic et al., 2018). The National Council of Teachers of Mathematics (NCTM, 2015) argues that 'it is essential that teachers and learners have regular access to technologies that support and advance mathematical sense making, reasoning, problem-solving and communication'. (p. 1).

Mullamphy et al. (2010) write that education should respond to societal needs to be relevant, and 'as the type of learner entering school changes, we are obliged as teachers to adapt our teaching styles to suit the new demographics' (p. 446). This study employed RME to bring mathematics closer to learners as it favours mathematical practices and assists in developing the conceptual understanding (Papadakis et al., 2021; Moura-Silva et al., 2020). Since 2012, many countries use the MathCityMap (MCM) project (www.MCM app.eu). Ludwig and Cahyono (2016) posit that the effective use of dynamic software and applications (Apps), such as the MCM app, can enhance learners' conceptual understanding of mathematics. Thus, this study investigated whether using smartphones utilising the MCM app enhances conceptual understanding of trigonometry and whether an authentic outdoor mathematics learning environment using smartphones is beneficial.

Educational researchers agree that learning is more than memorisation and information recall (Fisher et al., 2016). Deep and long-lasting learning involves understanding, relating ideas and connecting prior and new knowledge (Alexander, 1996). Learning is not something done to learners but rather something learners themselves do (Davis et al., 2008). In constructivist terms, learners construct their knowledge using situations presented to them (Bermejo et al., 2021). The metaphor of construction is implicit in the first principle of constructivism, where 'knowledge is not passively received but actively built up by the cognizing subject' (von Glasersfeld, 1989, p. 162). Thus, learning directly results from how learners interpret and respond to their experiences and improve their worldview (Greeno, 2006). The teacher's main task is to provide a conducive environment for learning to take place. This study intended to promote an active learning space where learners discuss and debate ideas while walking mathematics trails provided through the MCM app.

Mathematics in SA is in a poor state and mostly taught by rote learning and memory (Jansen, 2011; Gontyeleni, 2019; TIMSS, 2019; UNESCO, 2023). Studies to this effect (Jojo, 2019; Mlachila & Moeletsi, 2019; Reddy et al., 2020; Taylor, 2021) ascribe this situation to inadequate training in previous political dispensation. Some teachers are less innovative than others, and their teaching styles make it difficult for learners to learn and understand mathematics (Stols et al., 2015; Hendriana, 2017; Mangwende, 2019; Prodigy, 2023). Moloi (2012) argues that quality mathematics learning in SA will continue to be problematic unless teachers understand how their learners think and learn and recognise the experiences of their learners. In a study in a secondary school in the Eastern Cape, Chikiwa and Schäfer (2018) found that teachers predominantly expose learners to lower-order questions in trigonometry when code-switching. The outdoor tasks on MCM are structured so that almost all cognitive levels are covered, especially the higher-order questions (analysis, evaluation and creation). Studies in SA also reveal that teacher-centred pedagogies continue unabated as the prevalent instructional style in the twenty-first century, promoting learner-passivity (Maile & Makofane, 2019; Sikhakhane et al., 2020). This is because in a teacher-centred classroom, learning is perceived as predetermined content to be learned by rote, with little or no critical thinking, engagement, or exploration of the information (Jojo, 2019). Learners are expected to give the correct answer without any meaning or explanation for the answer being provided or discussed (Taylor, 2021). Despite the identified limitations of teacher-centred pedagogies, the practice remains prevalent in many schools (Sikhakhane et al., 2020).

There is a need to invest in alternative learning strategies to enhance mathematics learning. A key duty of any teacher is to plan for diversity or different learner abilities. The mathematics trail using a smartphone is a problem-based approach, and a system of learning mathematics and attending to this range of abilities (Van der Walle et al., 2007). Gontyeleni (2019) participated in the Global Education and Skill Forum (GESF) in Dubai (2019), where there were more than 2,000 delegates from 144 countries and realised that SA needs to move away from traditional methods of teaching and learning, where learners are only the receivers of content and teachers are the deliverers. She further states that teachers need to create opportunities for learners to gain autonomy, take charge of their learning, and can analyse and engage with the content to enhance their conceptual understanding. The GESF forum emphasises the use of technology in learning.

This study employed innovative learning of mathematics through outdoor tasks, where learners were walking mathematics trails using smartphones with the MCM app (MCM app) application. The MCM app, among other things, assists learners in locating real-world problems in their environment and solving them using trigonometry concepts they learned in grade 11. This study asserts that mathematics trails can develop learners' conceptual understanding of trigonometry and acknowledges the relevance and utility of trigonometry in real life.

2.3 TRIGONOMETRY

Trigonometry is a branch of mathematics that focuses on the relationship between and among angles, lengths, and heights (Coolman, 2015; Martín-Fernández et al., 2018). It is a fundamental topic in mathematics with several applications in other branches of mathematics, as well as in statistics, economics, surveying, architecture, carpentry, navigation, satellite, medical imaging, oceanography and engineering (Hoachlander, 1997; Weber, 2005; Martín-Fernández et al., 2018). It is an essential component outside mathematics, particularly in science, technology and engineering. Careers in physics, astronomy, computer graphics, optics and many branches of engineering require an understanding of trigonometric functions and their application (Weber, 2005).

In SA, the trigonometry content area has a weighting of 40 per cent in Grade 12 Mathematics Paper 2, which forms a large section of the grade 12 mathematics syllabus (DBE, 2011), i.e., a third of the curriculum. In SA, trigonometry is taught to learners in the Further Education and Training (FET) phase, and covers topics such as trigonometric ratios, reduction formulae, trigonometric identities, area, sine and cosine rules, trigonometric equations, double and compound angles, and trigonometric graphs (DBE, 2011). According to the Curriculum and Assessment Policy Statement (CAPS), trigonometry in grade 11 is taught in terms two and three. In grade 11, the following topics must be taught; reduction formulae, trigonometry graphs, equations, 2D shapes, sine, cosine and area rules. In grade 12, it is taught in the first and second terms. The topics included in grade 12 are compound and double angles, and 2D and 3D shapes.

The content specification shows progression in concepts and skills from grades 10–12 for each content area. However, in certain matters, the concepts and skills are similar in two or three successive grades. The explanation of content gives guidelines on how progression should be addressed in these cases. Each content area is divided into topics, and their sequencing within terms describes how content areas can be spread and revisited throughout the year. The order of topics is not prescriptive. However, teachers are advised to ensure that part of trigonometry, especially in grade 12, is taught in the first term and more than six topics in the first two terms to balance the assessment between papers 1 and 2. CAPS further states that the weighting of mathematics content areas serves two primary purposes. Firstly, the weighting guides time needed to adequately address the content within each area. Secondly, the weighting guides the spread of content in the examination (especially end-of-the-year summative assessment). Thus, failure by learners to understand trigonometry concepts has a ripple effect on their success in school mathematics (Sasman, 2011; Chigonga, 2017; TIMSS, 2019). The DBE (DBE, 2014) National Senior Certificate examination diagnostic report on Mathematics Paper 2 highlighted that ‘performance in the trigonometry section was a cause for concern as candidates performed poorly in questions that tested basic knowledge’ (p. 121). The National Senior Certificate examination diagnostic report of 2020 cited factors such as learners misinterpreting sine, cosine and tangent of an angle when their values were negative, failing to identify relevant quadrants and inability to recognise the reference angle (DBE, 2020). All these could be traced back to learners’ lack of conceptual understanding of trigonometric concepts.

Despite its relevance, trigonometry continues to be a challenging subject for many learners believing it to be difficult and abstract compared to other areas of mathematics (Weber, 2005; Gur, 2009; Hussaini & Usman, 2017; Jelatu et al., 2019). In a study conducted at a Zambian and South African university to explore final-year student teachers’ understanding of trigonometry as taught at the secondary school level, Malambo et al. (2021) found that some students could not use the appropriate trigonometric ratio to calculate the length of a triangle when that triangle was not drawn for them. Some failed to present accurate diagrams representing the information in the question. Others applied the cosine rule incorrectly, did not understand tangent functions and could not solve equations. Brijlall and Niranjan (2015) noted, from discussions with mathematics teachers and in their teaching experience, that some learners had difficulty choosing the correct trigonometric ratio for solving three-dimensional (3D) problems in mathematics. They further noted that the South African education system was essentially needs to improve the teaching of mathematics to bring South African learners to international levels of competence and ensure they have access to learning mathematics.

They argued that providing an active learning environment that enables learners to participate activity enhances learning. Khuzwayo (2019) observed that some learners did not understand that trigonometric ratios can only be applied in a right-angled triangle, and lack of resources hampered the teaching and learning of trigonometry and led to inadequate training of mathematics teachers (Chikiwa & Schäfer, 2017). Stuart (1998) quotes Williams, paraphrasing the Chinese proverb: 'Tell me mathematics and I forget; show me mathematics and I remember; involve me and I will be less likely to have mathematics anxiety'. This study intended to allow learners to walk the mathematics trail, have autonomy in their learning and share ideas, without the teacher describing each step to solve the problem. Learners were in touch with the real-world or real-life situations. In their study on learning and teaching trigonometry, Ngcobo, Madonsela and Brijlall (2019) mentioned that science and technology rely on applying trigonometry in real-life situations. Their study raised some pedagogical implications for teaching and learning of trigonometry. Mosese (2017) posits that the secondary mathematics pass rate is unsatisfactory in trigonometry functions and that the possible contributing factors are the traditional method of teaching and learning. Her study revealed that learners were mostly familiar with the routine type of questions and struggled with concepts that required deeper conceptual understanding in trigonometry.

Newton (2010) asserts that, since conceptual rather than procedural learning leads to a greater understanding of the content, trigonometry in mathematics should not be only taught procedurally and theoretically without clear reference to everyday life. Chigonga (2014) and Alex (2019) say the poor state of trigonometry learning in South African schools can be ascribed, in part, to ineffective teaching approaches. In studying six schools in SA, Spangenberg (2021) states that trigonometry is a crucial section in secondary school mathematics curricula because of its links to algebraic, geometric and graphical reasoning and is a precursor to calculus. However, she states that many teachers find it challenging to teach trigonometry due to insufficient academic content knowledge. Mechanistic or traditional mathematics learning is characterised by the teacher dictating the class, while the learners only receive materials and work on the questions assigned, making them passive learners. Therefore, it needs to be changed or replaced with more meaningful approaches, such as Outdoor Learning in Mathematics (Pambudi, 2022). Providing learners with meaningful learning experiences derived from their environments is crucial to enhancing theoretical understanding. Meaningful learning has been defined differently in the literature and varies from learners being able to understand what is learned to connecting with learners' daily experiences. Different contexts are employed to activate prior knowledge, connect to learners' personal worlds, show the value of mathematics beyond the classroom, set goals for/with learners, and create cross-curricular contexts. Practices to foster and support meaningful learning include collaboration and discussions, working independently (without the teacher's intervention) and experiential learning (outdoor learning) (Polman et al., 2021). The observation that learning that occurs in significant learning environments leads to greater understanding and motivates learners is shared by different theories (Loyens & Gijbels, 2008; Wardekker et al., 2012).

The MCM app, among other things, helps learners locate and solve real-world problems in their environment using Global Positioning System (GPS) on their smartphones. In this study, two mathematics trails were designed focused on trigonometry concepts learned in grade 11. After that, all the information and tasks were uploaded to the MCM web. Participating learners then accessed those mathematics trail tasks on the MCM app, guided by the GPS on their smartphones.

2.4 USE OF SMARTPHONES IN MATHEMATICS LEARNING

Cell phones based on operating systems that allow internet, video, touch screen and several other applications are also called smartphones (Ballagas et al., 2006; Godwin Jones, 2011). Smartphones were released in 2000. Today, smartphones are used in various contexts, including learning and knowledge sharing (Yu, 2012). Many institutions worldwide have recognised the importance of smartphones in learning (Dewah & Mutula, 2013). However, a study by Dalvit and Gunzo (2012) noted that, in most cases, learners use smartphones for social purposes.

2.4.1 The role and integration of smartphone in the outdoor mathematics learning

The role of smartphones in outdoor mathematics learning is to help learners become mathematically active by presenting tasks prepared by the teacher or author of a mathematics trail (Pambudi, 2022). Using smartphones to learn mathematics outdoors means giving learners autonomy over the learning process, which is not characteristic of mathematics classes. They make decisions independently, e.g., how to approach, interpret or solve a task, how to interpret the solution and interact with the application. They use smartphones to search for information and definitions on the internet, reach the task location, read the tasks and hints, identify the sample solution and get feedback (Durak et al., 2020).

Yosiana et al. (2021) note that the advent of smartphones provides a strong learning environment for teachers to explore, experiment and share their knowledge as educational practitioners. They note that the industrial revolution 4.0 has changed the role of the teacher, who was the sole supplier of knowledge. This has changed by using smartphones with higher levels of accessibility. They concluded that integrating smartphones into learning is an effective way of delivering mathematical content, developing self-reliance in learners and enhancing their understanding of trigonometry. Park (2011) defines the use of smartphones in mathematics as learning while on the move using portable electronic devices (smartphones). The learning is facilitated by easy-transportable digital tools.

In contrast to learning inside the classroom, learners are not bound to a single fixed location. In addition to using a smartphone, learning is also associated with an object in the real world. Lonsdale et al. (2004) speak of context-aware ubiquitous learning (u-learning).

U-learning is considered a subcategory of mobile learning and accesses information anywhere and anytime. Learning is integrated into the flow of daily activities. Unlike traditional rote learning, which treats learners as passive subjects receiving information, U-learning places the learners at the centre of the information they manage. The benefits include increased social skills, continuous and active learning, critical thinking, collaboration, development of research skills and learner autonomy. However, the same benefits can turn against the learners if they cannot find reliable sources of information. Therefore, to help learners develop this ability is imperative (Durak et al., 2020). Smartphones can be taken to the objects and support the users with maps, hints, feedback and communication tools. Although smartphones are used in every aspect of our daily lives (especially among learners), they play a minor role in education (Chen & Kinshuk, 2005). Scharaldi (2020) cites the benefits of using smartphones in mathematics education, such as trigonometry. 'We can enhance the learning process and make concepts come alive through engaging and interactive media' (no page). Sub-Saharan Africa remains one of the world's fastest-developing regions in mobile subscription access, with a mobile rate of 75 per cent in 2018 (GSMA, 2018). Many have investigated whether smartphones enhance mathematics learning in resource-challenged schooling contexts in Africa. Notable examples are Egypt, Namibia, Rwanda, Ghana, Kenya and Zambia (UNESCO, 2012; TIMSS, 2015;

Isaacs, 2019). Smartphones allow learners to improve their understanding of mathematics by choosing topics to study by reading background theory and worked examples, or the completion of actual examples individually. They also allow for collaboration since users can form groups, compete and message each other (Isaacs et al., 2019). It was discovered that the distance learning students at the University of Ghana found it convenient to use smartphones in their academic activities to enhance conceptual understanding (Darko-Adjei, 2019). Fabian et al. (2016) posit that, in the past decade, they have observed substantial change in mobile computing as schools have started incorporating smartphones in teaching and learning mathematics. They found that the potential benefits of using smartphones for learning include, among others, anytime-anywhere learning, bridging academically designed learning contexts and allowing learning to be situated in a real-world context, in this case, outdoor mathematics tasks. Their study indicates that smartphone learning in mathematics (published mathematics and smartphone learning studies) is not confined to developed countries, but also attracts publications from developing countries, such as SA. According to this study, SA ranks fourth together with the other two countries from 21 that were observed.

2.4.2 Smartphone successes and findings in learning mathematics in South Africa

In SA, there have been some successful smartphone learning interventions. For example, the Meraka Institute (a research institute) has been at the forefront of the innovative use of smartphones to support the teaching and learning of mathematics. The institute has developed a mobile tutoring system that runs on smartphones. Volunteers from the Department of Engineering at the University of Pretoria offer real-time mathematical support to high school learners on their smartphones at reduced rates (Kizito, 2012). In a study conducted in township secondary schools in Gauteng province, Manga (2018) noted that incorporating smartphones in teaching and learning of mathematics allows improved learner participation and continuous involvement using a smartphone as a 'textbook' and facilitates the learner-centred approach. The integration of smartphones caters for different learning styles. Chaka's (2021) study indicates an uptake in using smartphones for learning by South African schools in mathematics: smartphones as learning tools outside school and as supporting tools. In a study in Mafikeng, Northwest province, Mosese (2017) suggests the integration of smartphones as an innovative or alternative method to teaching and learning trigonometry. She observes an increasing awareness that interaction between humans and smartphones can facilitate effective learning and support conceptual development.

The COVID-19 pandemic widely affected education across the globe. In SA, the pandemic aggravated pre-existing inequalities in the education system. The outbreak has caused damage, and like any critical sector, education has suffered. One in five learners could not attend school, one in four could not attend higher education classes, and over 102 countries ordered nationwide school closures, while 11 implemented localised school closures. This was mainly for children in informal urban and rural settings, with household poverty also playing a critical role (Pretoria, 2021; Press Centre, 2020).

Chirinda et al. (2021) revealed that the WhatsApp platform is an essential tool to support mathematics learning outside the classroom. Mosese' (2017) findings also provided insights into how mathematics teachers became learners during COVID-19 emergency remote teaching, how they sought solutions to unfamiliar problems and acquired knowledge from the larger mathematics education community worldwide. Roberts (2019) notes that the South African DBE, the United Nations Children's Fund (UNICEF), SA and the Reach Trust created an m-learning platform called the UVS in 2012. This platform was accessed by 8,000 different types of handsets, including smartphones. The UVS used social networking to provide educational and psychosocial support services to secondary school learners, teachers and parents across SA. Chaka (2021) notes that smartphone learning studies have been conducted in SA, particularly at secondary schools, to showcase an exponential growth in smartphone learning in SA and different areas of Africa. The study further states that smartphone learning is gaining some popularity in SA.

However, mobile learning as educational practice is not yet incorporated into formal school curricula or used as a standalone form of learning. It is used informally as a supplemental or support form of learning. Fabian (2018) states that the ubiquity of smartphones has added a new angle to contextualising mathematics learning in bridging classroom learning and the real world. Kyobe and Van Belle (2018) agree that developments in smartphone functionality have created exceptional opportunities, enabling smartphones to have a supportive role in enriching learners' learning experiences. However, they note that the use of smartphones in SA has not yielded satisfaction for all learners, although smartphones generally improve learners' learning experiences. However, there is a need to consider hindrances to learners getting the most favourable returns from these devices.

2.4.3 Challenges in the use of smartphones in mathematics learning

Although recent studies urge smartphones to be considered as important tools to enrich learning, cautious implementation or incorporation into mainstream curricula is vital. Sinclair et al. (2016) advocate the use of smartphones in mathematics education, as it has become mainstream in some countries, but they also say that new technological developments over the past decade have led to new challenges in using technology in learning mathematics. Supandi et al. (2018) assert that using smartphones in mathematics learning in schools is a positive development, but there are challenges, such as the lack of awareness of the benefits and disadvantages of mobile learning. Additionally, there are problems related to policies that restrict or, in some cases, ban smartphones within school premises. Ludwig and Jablonski (2019) agree that using smartphones in classrooms is, unfortunately, not very popular due to the restrictions legislated by school administrations.

Although smartphones are widely used in every aspect of our daily lives (especially among learners), they have a minimal role in education (Chen & Kinshuk, 2005). There are also challenges associated with costs, e.g., some learners cannot afford to buy a smartphone or data and teachers do not realise the positive educational value smartphones can add and are cautious about learners using smartphones (UNESCO, 2012). In SA, a few cases reported in the media suggest that learners were using their smartphones to send 'bullying' messages, cheat in tests using SMS messaging and access pornographic materials and sex chat rooms (Popovac & Leoschut, 2012; Mavhunga et al., 2016). These reports have negatively influenced teachers' perceptions of smartphone use and led many educators to support banning smartphones from schools (UNESCO, 2012; Mavhunga et al., 2016).

Blanket bans on smartphones might seem to make sense. However, certain subjects like mathematics will miss out on the many modern advantages they offer, e.g., learners have access to many applications to assist them in their learning. (Drabwell, 2018; Prizm Institute, 2023). Some difficulties noted by Kizito (2012) are the small smartphone size compared to a laptop, which provides a wider screen and keyboard. Another is affordability because learners might need to download certain applications. Kibona and Mgaya (2015) postulated that despite the advantages of smartphones in learning, they are not always positive as some applications affect learners adversely due to their addictive nature. They further speculate that these applications distract and steal learners' time, which affects their academics negatively; excessive use of smartphones leads to complications such as vascular permeability, neck pain, musculoskeletal disorders and mouse brain damage.

Sarfoath, cited in Darko-Adjei's (2019), revealed that unstable or unreliable internet connectivity inhibits learners using smartphones as a learning tool. Smartphones freeze sometimes at a crucial moment, and the memory or space of certain smartphones get full. In the same study, it was found that 'intruding calls may come in during learning'. These factors greatly distract learning by deviating attention from the core purpose.

In their study, Agbo-Egwu et al. (2018) noted that over-dependence on smartphones affects learners' memory efficiency in mathematics. The pattern of learners' inability to recall basic mathematical facts, theorems, axioms and formulae indicates the negative influence of smartphone over-dependence. The participants in this study acknowledged that over-reliance on smartphones for simple recall poses a threat to mathematical prowess. There are challenges, especially with people living in rural areas because of the disparities in SA. For example, due to high unemployment and poverty (Statistics SA, 2019–2020), most learners cannot afford to buy data, airtime or even buy a smartphone.

2.4.4 Recommendations

This study recommends the deliberate or cautious use of smartphones for mathematics learning. Teaching and learning mathematics should emphasise conceptual understanding, and learners should be guided through measured technological integration, e.g., smartphones in mathematics learning to avoid over-dependence on smartphones (Agbo-Egwu et al., 2018). This study was a guided intervention using the MCM app through a smartphone to enhance conceptual understanding of trigonometry. 'We recommend that the University of Technology of South Africa should consider launching wireless networks in student areas such as residences, lecture rooms and libraries, enabling smartphones and other technological devices to connect to internet at any given time making (mathematics) learning more accessible to run on smartphones' (Msomi & Bansilals, 2018, p. 136). While smartphones can be a distraction, they can be used effectively as learning tools. The benefits smartphones bring to learning mathematics are indisputable (Kroeger, 2020; Yosiana et al., 2021; Pambudi, 2022). In meeting challenges that come with integrating smartphones into mathematics learning, parents, learners, teachers, school governing bodies (SGBs) and other relevant stakeholders should be cautious about the use of smartphones within the guidelines they set collectively. Smartphones can access knowledge and, if used appropriately, enhance learners' conceptual understanding (Kroeger, 2020). The MCM app using a smartphone was designed strictly for mathematics trails, and each learner had a unique role in a group. Learners did not deviate from the task and access unwanted websites or send 'bullying' messages but focused on a given task. Each group had two smartphones in case one became dysfunctional, and all learners were included The workshop ground rules were created and explained to participants so they knew what was expected.

Teenagers in SA account for 40 per cent of social media users (Lama, 2020). The use of smartphones in South African public schools in mathematics education is relatively recent. COVID-19 created an opportunity for smartphone use, particularly for educational purposes, like mathematics education and have an important role in rural areas where schools do not have proper infrastructure. The COVID-19 regulations of social distancing and restricting indoor gatherings to smaller numbers presented opportunities for teachers to become more innovative. An example is using the MCM app with smaller groups of learners while learning outdoors. Using the MCM app, this study focused on an authentic outdoor mathematics learning environment through smartphones to grow conceptual understanding.

2.5 MathCityMap PROJECT (MCM)

2.5.1 Outdoor mathematics teaching and learning

The outdoor environment provides valuable mathematics-related lessons for learners, in the form of shapes and patterns of foliage, buildings and structures, such as the school grandstand, flag poles and billboards. Mathematics becomes inspiring through hands-on activities outdoors. The change of scene from studying indoors boosts mathematical concepts (Brooks, 2021). The author further states that it makes them a leading international school in Singapore, as learners and parents find value in non-traditional classrooms, open-air spaces, and outdoor tasks. According to Brooks (2021), outdoor learning of mathematics has several advantages. Learners can explore mathematical concepts in the real world instead of just inside the classroom. More equipment and supplies can be made available, allowing learners to learn mathematical concepts on a much larger scale while promoting natural curiosity, team-building skills, a greater sense of independence, connecting their learning using prior knowledge to create new knowledge and between mathematical concepts and real-world applications. In the same study, it is postulated that outdoor mathematics tasks allow learners to engage in projects they cannot do inside the classroom. Common tasks include using mud, sticks, or chalk to mark areas for measuring and using an outdoor environment to work out problems, such as the height of the school flagpole, the angle of inclination or slope of a ramp. Outdoor learning occurs in many different settings close to schools, neighbourhood parks and green spaces, local buildings, and community resources, as well as within the school grounds (Education Outside the Classroom, 2005). Outdoor Learning Mathematics (OLM) is meant to be implemented outside the classroom by utilising the environment as a resource that guides learners to collect data and solve problems, by identifying, reinventing, and applying relevant mathematical concepts and models (Pambudi, 2022).

Laird et al. (2021) suggest that outdoor learning is an effective educational approach to increase learners' interaction and learning, and comprises activities in the playground, the oval, or the garden. Also, learners performed significantly better in outdoor mathematics activities than in similar indoor classroom activities. Another study found that exclusively learning mathematics inside the classroom hampered learners from fully understanding mathematical concepts (Haji et al., 2017). Mathematical tasks outside the classroom add a new dimension to the learning experience, increasing chances for multi-sensory competencies (Laird et al., 2021). What makes indoor mathematics tasks different from outdoor tasks? Do we play the same games we play indoors outside? The answer to the second question is generally no, as we do not simply repeat the indoor play outdoors. Outdoor play is different by nature and there is no need to replicate tasks we could do as easily indoors. In her book, (Teach Early Years) (Maths Outdoors EYFS-Maths activities to try outside), Dancer suggests that as we plan our outdoor mathematics learning, we must consider our early experiences. She posits that as we develop mathematics outdoors, we need to reflect on the mathematics experiences we provide indoors and how to extend them outdoors, complementing and enhancing indoor provisions and celebrating the qualities of the outdoor environment. Teachers need to identify opportunities for mathematics learning to enhance conceptual understanding of trigonometry and enjoy the outdoor environment collecting and sorting natural objects, measuring huge areas using strides, doing non-stand measures and employing heuristics. Dancer further states that some teachers make their planning more complicated by identifying different learning objectives for indoors and outdoors. Teachers should plan indoor and outdoor tasks to support these goals so that learners associated and/or revisit their learning through different experiences. The outdoor tasks should complement the indoor tasks.

Wisbey's (2018) study resonates with the work of early years pioneers such as Rousseau and Froebel, as well as Montessori, on outdoor mathematics learning. It shows that there has been a recent resurgence of interest in the UK in the outdoor environment for supporting learners' mathematical conceptual understanding. Outdoor mathematics learning is the connection with the natural world to help learners develop curiosity and conceptual understanding with the opportunity of experiencing the outdoors with structure and purpose. The outdoor environment is as important as the indoor environment, working in combination to support learning and enhance learners' knowledge. The same study further argues that teachers only need to look for opportunities outside. Pratt (2011) posits that, for learners to become proficient in mathematics, they should develop mathematical dispositions. He suggests that this can be supported by doing mathematics outdoors, enabling learners to develop the awareness that mathematics is all around them, to connect their learning and experiences of natural materials in their original place. Learners will count, measure, explore shapes and develop mathematical ideas through their imagination. Dewey's 'children learn from doing' theory (Mooney, 2000) means rather than saying 'learners will enjoy this', teachers need to ask the following questions when they plan tasks or activities for learners: How does this expand on what learners already know? How will this task help learners to grow? What skills are being developed? How will this task help learners know more about their world? How does this task prepare learners to live more fully? (Wisbey, 2018).

The benefits of outdoor mathematics learning include a sense of freedom without the constraints of the indoor environment, encouraging mathematical disposition, enabling learners to make useful associations in their learning, encouraging exploration and risk-taking and contributing to learners' self-image and confidence (Wisbey, 2018). Learning Outside the Classroom Manifesto (2006) lists many benefits of outdoor learning: '…it enables learners to construct their own learning, build bridges between theory and reality, it can lead to deeper understanding of concepts which are frequently difficult to teach effectively using classroom methods alone, develop skills and independence, enhance learners understanding, nurture creativity, develop skills and improve attitude to learning', among others (pp 1–5). Being outdoors allows learners to meet real-life objects. For mathematics education, you can create authentic tasks, e.g., what is the height of a certain building, how many stones have been used to build that wall over there, how much water is in that pond, and so on. Thus, Hans Freudenthal interpretated mathematics as a human activity (Freudenthal, 1973; Gravemeijer, 1994). According to Freudenthal (1991), mathematics must connect to reality and be relevant to society for it to remain valuable to humanity.

2.5.2 Mathematics trail

Mathematics trails are outdoor trails with checkpoints that consist of tasks that require many process competencies at once, like problem-solving, reasoning and proof, communication, connections, and representations of mathematical concepts. In the early 1980s, Blane and Clark devised connecting those tasks to form a so-called mathematics trail. A map is required to find all these tasks, a description of the tasks (both form the trail guide) and walk around and solve authentic mathematical problems. A mathematics trail is a set of mathematical outdoor tasks within walking distance. To solve the tasks, you will normally need tools like a measuring tape, which should be listed in the trail guide, and the connection between enactive actions (measuring objects), iconic representations (e.g., creating a sketch of the object), and the symbolic representation (e.g., formula = volume of an object) is valuable for learning mathematics outdoors (Ludwig et al., 2020). The tasks and routes designed by the teacher should be unique and interesting (Ludwig & Jablonski, 2019). The teacher must also provide a photograph that helps to identify the waypoint, the object of the task and a description of the task. However, the photograph should be captured so that the participant(s) must go in person to the site to do the task.

Over the last 30 years, there have been few studies on mathematics trails. The first was probably by Kaur (1992) in Singapore who reported that mathematics trails motivated learners since they found them more meaningful and fun than typical classroom lessons. Later, Toh and Lim (2006) let learners create mathematics trails in Singapore. The learners had fun and gained a new perspective on mathematics. Callenberg and Andersson (2014) interviewed learners after running a mathematics trail in Sweden. The learners stated that a mathematics trail was fun and helped them to discover mathematics in the environment. Cahyono et al. (2023) state that a mathematics trail is a set of mathematical tasks or questions bound to objects from the real world, arranged as a trail that users follow. Usually, they are within walking distance. An ordinary trail has about five to 12 waypoints, each with a mathematical task to be solved. A mathematics trail guide has a map that displays interesting locations and descriptions of different tasks to discover mathematics in the environment. Each task gets a number or code. Learners start trailblazing in groups of about three or four (Ludwig & Jablonski, 2019).

Shoaf et al. (2004) saw potential in mathematics trails, where participants work cooperatively, share ideas and experience mathematics in a non-threatening environment without teacher intervention. Trailblazing is defined as discovering new outdoor trails or making paths to communicate information about the trail. Trailblazing is a new, promising, exciting way of doing something outside the classroom (Jablonski et al., 2018). When trailblazing, each group starts at a certain point with a specific task. When the group finishes the task, they move clockwise to the point or task with the next number or code. Every group uses their own prepared smartphone with the MCM app, which contains all the trail's location and task information (GPS coordinates, hints, sample solution) and a book for sketches and calculations. When using the MCM application, smartphones activate GPS, leading learners to the task's location**.** When the learners reach a point, they read the respective tasks from their smartphones. They then plan together and solve the task, using stepped-up hints provided on the application in case they get stuck or do not understand. Learners have a platform to discuss, share ideas and find different solutions. Participants learn from each other (Jablonski et al., 2018). When they agree on the solution to the task, they compare their solution(s) with the ones on the MCM app. In this manner, the MCM app allows learners to take charge of their learning.

2.5.3 MathCityMap project (MCM)

Mathematics plays a pivotal role in daily and occupational life. However, it is commonplace that public understanding of mathematics is unsatisfactory, and few learners are involved in mathematics during their school days (Behrends, 2009). At school, mathematics is sometimes perceived as a difficult and abstract subject. Internationally, by the eighth grade, only about a quarter enjoy learning mathematics (Mullis et al., 2012). A variety of projects in numerous countries took place to raise public awareness of mathematics. The MATIS I Team from Goethe University Frankfurt has emphasised to this by developing the MCM app project (MCM app), a mathematics trail project around the city. Ludwig and Cahyono (2016) posit that the effective use of dynamic software and applications, such as MCM app (MCM app), has the potential to enhance learners' conceptual understanding of mathematics.

The MCM project combines the well-known mathematics trail idea with the current technological possibilities of smartphones (Ludwig & Jablonski, 2019). The idea of the MCM was first presented in 2012 by Prof Matthias Ludwig and his team at the Goethe University, Frankfurt, Germany, and this has grown steadily. MCM aims to allow teachers, learners, and individuals of any age the opportunity to experience mathematics outdoors and their environment from a new mathematical perspective (Cahyono, 2018; Jablonski et al., 2018). MCM project has two components. The first component is for the teacher and is a web portal (www.MCM app.eu) that serves as an open-access database for real-world or authentic mathematics tasks in the environment. The other component is for the learners and is the MCM app. It shows on a map, where the mathematical tasks are located in the environment and provides tasks, hints, feedback and a sample solution.

This approach is used by schools and other educational institutions in more than 20 countries. It is intensively promoted by a European consortium and primarily by the MATIS 1 working group of the Goethe University, Frankfurt. At present (as of July 2019), they have more than 3,200 registered users, and more than 9,000 tasks in the system. The range of tasks is from arithmetic to geometry, and functions to statistics. (Ludwig, 2019). Matthias Ludwig, the manager of this project (MCM), visited Rhodes University in SA to present the idea, on 16 July 2017 (https://mathcitymap.eu/en/mathcitymap-goes-south-africa.2/). In 2013, this project was implemented in Indonesia tailored to the country's situation. In the first study, the mathematics trails were designed in several locations (Cahyono et al., 2015). Subsequently, in the second study, learners carried out the mathematics trails and the researchers explored its potential involvement in mathematics. The MCM project is a mathematics trail project implemented around the city facilitated by using a smartphone application. An investigation was conducted to see whether this project provides opportunities for engaging students in meaningful mathematical outdoor activities. As a pilot project, MCM was implemented in Semarang, Indonesia. Findings indicated that the project successfully designed and offered an engaging mathematics trail for learners. Learners were highly intrinsically motivated to be involved in mathematics. They gained experience, conceptual understanding and eventually realised that mathematics was not just abstract, located in the classroom and textbooks, but was, and is, relevant to everyday life and occupation (Cahyono et al., 2016).

Mathematics has been defined as a human activity in the CAPS document (DBE, 2011) and by Hans Freudenthal (Freudenthal, 1973). The MCM app supports learners with feedback, stepped hints and sample solution/s to help them work independently on the mathematics trail tasks and feel more competent. However, this support does not mean every learner can solve all tasks. The social form of group work meets the psychological need for relatedness and is often the first choice for out-of-school learning (Gurjanow et al., 2020). The MCM app presents an alternative way of learning mathematics with tasks designed using outdoor objects and arranged in a sequential trail that learners can follow. The tasks should be realistic, i.e., it must be from a real object and solvable in various ways (Cahyono, 2018). Vos (2015) notes that an realistic task in mathematics should have an out-of-school origin and be certified by experts as a task in their field. The real-life object on a mathematics trail gives the out-of-school origin. While following the trail, learners have a platform to discuss ideas and come up with different solutions – hence each participant involved can learn from each other (Jablonski et al., 2018). The tasks and routes are designed by the teacher and should be unique and interesting (Jablonski & Ludwig, 2019). The researcher (teacher) uploaded all the information and tasks on the MCM web portal, and the learners walked these trails guided by the GPS on their smartphones (MCM app). However, some studies (Bray & Tangney, 2017; Pocan, Altay & Yasaroglu, 2023; Ludwig & Jablonski, 2019) have noted that these same smartphones can enhance conceptual understanding, within formal settings (classroom) and informal settings (outdoors), e.g., the MCM app. Since the MCM app uses smartphones to facilitate mathematics learning at objects in the real world, it can be classified as a tool for U-learning. Using real-life objects for mathematics learning benefits learners' conceptual understanding (Gurjanow et al., 2020). The teacher uploads information and tasks on the MCM web portal and awaits an expert panel's feedback from Goethe University on the validity and efficacy of the tasks. In this study, the researcher designed two mathematics trails focusing on trigonometry concepts learned in grade 11. Thereafter, all the information/tasks on the MCM web were uploaded. Participating learners then walked these mathematics trails guided by the GPS on their smartphones (MCM app). Using the MCM app enhances conceptual understanding in mathematics learning, as presented in the next section.

2.6 CONCEPTUAL UNDERSTANDING IN MATHEMATICS LEARNING

Conceptual understanding is part of the five interwoven mathematical proficiency strands identified by Kilpatrick et al. (2001). Conceptual understanding is an integrated and functional grasp of mathematical ideas, knowing more than isolated facts and methods because conceptual knowledge enables them to learn new ideas by connecting them to what they already know (Kilpatrick et al., 2001). Commenting on observations made from the South African context, Malatjie and Machaba (2019) posit that '… a learner has conceptual understanding when he or she is able to explain, describe and apply the same concept in different ways and in different situations' (p. 2). Thus, learning approaches that help learners to see the integration of concepts (that mathematical concepts are not isolated but connected are necessary. Learners exposed to such approaches may better understand why a mathematical idea is important and the contexts in which it is useful. When learners have conceptual understanding, they can organise their knowledge coherently, enabling them to learn new ideas by connecting them to what they already know (Wilson & Heid, 2015). Teaching for conceptual understanding presents several advantages to learners during and after learning. As argued by some researchers (Gilmore & Bryant, 2008; Wiggins, 2014), learners with conceptual understanding can identify and apply mathematical ideas that are key to a particular problem and explain why they are important.

Conceptual understanding indicators:

Four key conceptual understanding indicators adapted from Kilpatrick et al. (2001) are discussed next.

- ***Building on prior knowledge*:**

This concept refers to the teacher's ability to link the mathematics they teach and what learners already know. The indicator requires that learners learn mathematics with understanding and build new knowledge from prior experiences. Learners can use knowledge learned to generate new knowledge and solve unfamiliar problems (Kilpatrick et al., 2001). For example, learners can calculate the height of the school flag pole. They drew from the previous knowledge of the similarity of right-angled triangles and had to remember the proportionality of the corresponding sides and the appropriate procedure to use to make the side representing the flag pole height subject of the formula. This procedure was introduced to them in algebraic equations when they solved for x. However, they were assisted by the hints from the MCM app using a smartphone when they got confused.

Knowledge is not merely acquired but constructed. Teachers cannot transfer concepts into learners' minds, but new concepts must be constructed upon what learners already know. That is why examples are important when introducing a new concept. New ideas will increasingly depend on previous understanding. For example, understanding algebraic equations depends on the correct understanding of equal signs (Wiggins, 2014). The National Research Council (2002) states that in trying to recall a concept, prior experiences with diagrams, attributes and examples associated with the concept are required. Alreshidi, (2023) note that meaningful learning maximises the use of prior knowledge. Learners with more extensive academic preparation tend to have greater academic success and enhanced conceptual understanding at university or any higher institution (Kurlaender & Howell, 2012). Conversely, prior misconceptions or inaccurate knowledge can also hinder future development or conceptual understanding (Ambrose et al., 2010). Rach and Ufer (2020) posit that research has established the role of prior mathematical knowledge in successfully dealing with challenges in learning. Learners' prior knowledge from the MCM project workshop on navigation or MCM app using smartphones, such as the ability to access tasks, hints, sample solutions and feedback helped to ease the process of solving the mathematics trails.

- ***Representing mathematical concepts in different ways*:**

This indicator refers to the teacher's and learners' ability to represent the same mathematical concepts in a particular problem in diverse ways and explore the relationships with the learners. This, in turn, enables learners to have a deeper understanding. The National Council of Teachers of Mathematics (2000) and the National Research Council (1996) also emphasise using multiple representations in teaching mathematics, as these support learners in examining real-world phenomena. Suh (2007) stresses using representations to foster conceptual understanding. Goldin (2014) postulates that mathematical representations are visible or tangible productions, such as diagrams, number lines, graphs, arrangements of concrete objects or manipulatives, models, mathematical expressions, formulae and equations, or depictions on a smartphone or calculator, inscriptions and external representations, which can also refer to a person's mental or cognitive theories. Eckert (2022) states that mathematical ideas can be communicated in different ways. For example, a learner can use verbal, written, symbolic, iconic and visual representations to illustrate on mathematical concepts. In this study, mathematics trails prompted learners to represent trigonometry concepts in various ways, e.g., the roof and wall task. They had to calculate the angle between the roof verge board and the wall. They were initially mystified, but the hints made them imagine two congruent right-angled triangles. The task persuaded learners to represent the triangles in their minds (internal representation), assumed they did, and thereafter, drew the same triangles on a page (pictorial representation), but, first, they looked at the MCM app hints (symbolic or verbal representation). It was a verbal representation of the two congruent triangles, an internal representation, and a pictorial representation of the same triangles. Eckert (2022) further argues that smartphones can teach concepts. Using a variety of representations can enhance learners' higher-order thinking and understanding. Mainali (2021) states that representation is important for teaching and learning mathematics. The same study states that representation is a sign or combination of signs, characters, diagrams, objects, pictures or graphs. There are normally four modes of representation in mathematics: verbal, graphic, algebraic and numeric. Some are more dominant in education than others. Multiple modes of representations enhance teaching and learning, and learners need these four modes of representations or skill to be proficient in learning mathematics.

- ***Connecting concepts and ideas in mathematics*:**

This refers to the teacher's ability to discuss the similarities and differences of the various representations and explore how these connect. This, in turn, facilitates learners' mathematical conceptual development (Brunner et al., 1997) because the degree of learners' conceptual understanding is related to the richness and the extent of the connections they make. Simon (2020) suggests that a mathematical concept is not just a process of observing a pattern between input and output, or the outcomes of an empirical process, but also knowledge of the necessity of a certain mathematical relationship. Capraro and Joffrion (2006) assert that 'connections make mathematical concepts meaningful, memorable, and powerful' (p. 162). It is the process by which new mathematics knowledge becomes part of an individual's internal cognitive structure connecting with previous mathematics knowledge and integrating it with the internal network (Yang et al., 2021). The participants had to calculate the angle of depression of the school's grand stairs. The MCM app hints were useful suggesting a calculation of the angle of elevation. Participants established the mathematical link between the two angles as corresponding angles because they formed a Z-shape and were, therefore, equal. This also meant an association of trigonometry and Euclidian geometry.

Hiebert and Carpenter (1992) observe that mathematical concepts, ideas, procedures, or facts are understood if they are part of an existing internal network. They further state that understanding mathematics makes connections between ideas, facts, or procedures is not new. Networks of mental representations of mathematical concepts develop when new models are incorporated or new associations are formed and the individual's understanding is enhanced (Abdulrahman et al., 2020). Yu and Yang (2005) divide understanding into five levels (zero, common sense, logical, conceptual and endless). Logical levels entail deep understanding, which requires the connection of old and new knowledge. Zhang and Guo (2007) say mathematical understanding is the process by which learners create connections between different concepts and modify or expand the cognitive structures of their knowledge in these domains. Chen (1995) observes that understanding is a cognitive function that involves a search for connections and relationships between things until essential laws are ascertained. The researcher concurs with the authors and argue that understanding concepts and how they correlate aids the teacher in planning mathematics trails for learners.

- ***Extending mathematical concepts to real-life contexts*****:**

This requires the teachers to relate what they are teaching to new areas or daily life contexts, enabling learners to see the practical applications. This may require the teacher to give examples related to the learners' daily lives, thus making them aware of the utilitarian value of mathematics (Gafoor & Sarabi, 2015). Real-world strategies or tasks make mathematics relevant. Lynch (2019) notes that some learners avoid mathematics because they perceive it as arbitrary rules with no relevance to the real world. It is stated in this study that the best way to make mathematics relevant is to show learners how it connects to their lives by using real activities, in this case, mathematics trails. Mathematics requires learning that learners can use daily with real-world connections (Lynch, 2019). Minero (2018) notes that mathematics was all rote memorisation and pencil-to-paper equations, disconnected from the reality. However, teachers are realising the significance of making practical, relevant connections in mathematics. This study employed real-world tasks in a mathematics trail form. Learners were required to calculate the slope of the school grandstand, the area bound by the drainage pipes, the angle between the ground and the middle supporting pole of the school billboard, among others. There were five tasks per trail. Integrating mathematics concepts with real objects and other subjects, such as geography and physical sciences, where concepts like slope or gradient are common in those subjects, indicates that mathematics can occur outside of classroom (Minero, 2018). If real-world connections become standard practice at school, it is easy to imagine a world without that vexing question ‘when will I ever need mathematics?’ Before learners even ask, teachers can provide authentic tasks to learners. When real-world tasks are part of lessons, learners can realise how mathematics fit into their lives. Consequently, mathematics becomes a visible occurrence instead of abstract rules and concepts confined to the classroom.

Connecting mathematics to the outside world helps learners to make sense of what they are learning and solve real-world problems, in this case, the MCM project. The researcher uploaded the real-world tasks based on grade 11 trigonometry concepts on to the MCM web, and awaited approval from the reviewers. Both mathematics trails were approved, and the reviewers uploaded them to the MCM web portal database to be accessed by learners and other participants on MCM app.

Exclusive learning procedures and proofs (trigonometry in this case) without a good theoretical understanding leave learners ill-equipped to use their knowledge in general and later in life. It is necessary to pay attention to Roscoe’s (2014) observation that providing learners with meaningful learning experiences is a goal that all mathematics teachers should strive towards and can be promoted through using outdoor mathematics tasks in form of mathematics trails.

The conceptual understanding indicators discussed in this section form an integral part of the second analytical framework discussed in the data analysis section of Chapter Three. The analytical tool was used to analyse the data in Chapter Four. Since evidence from literature shows that mathematics learning for conceptual understanding, in particular trigonometry, was and still is a challenge for many learners worldwide, there is a need to invest in alternative, innovative and exciting ways of learning trigonometry. Research reveals that the use of the MCM app can enhance mathematical conceptual understanding (Ludwig & Cahyono, 2016). Learners can have robust discussions, autonomy, make necessary connections, come up with multi-representation and employ prior knowledge to find a solution to a mathematics trail task. This study asserts that improved understanding and effective use of the MCM app align well with the RME presented in the following section. At the centre of RME are outdoor mathematics tasks close to or within the learner's environment.

2.7 THEORETICAL FRAMEWORK: REALISTIC MATHEMATICS EDUCATION (RME)

The RME is an influential teaching and learning theory in mathematics education (de Lange, 1996). It was explored in SA and led to the Realistic Mathematics Education in South Africa (REMESA) project, a partnership between Freudenthal Institute (FI) and the mathematics education division of the University of the Western Cape (UWC) to introduce RME as a viable approach for school mathematics. This occurred during a period when curriculum changes were introduced to fit the educational ideals of the 'new' SA (Julie & Gierdien, 2016, 2019). RME's principles are also inferred in the CAPS, which aims to ensure that learners actively construct and apply knowledge and skills meaningful to their own lives. In this regard, the curriculum promotes knowledge in local contexts (in this case, mathematics trails) while being sensitive to global imperatives. One of the CAPS principles is to encourage active and critical learning, rather than rote and uncritical learning. CAPS define mathematics as a human activity (DBE, 2011).

Realistic Mathematics Education has its roots in Hans Freudenthal's interpretation of mathematics as a human activity (Freudenthal, 1973; Gravemeijer, 1994). Freudenthal emphasised the actual activity of doing mathematics, which he proposed should predominantly consist of organising or mathematising subject matter from reality. According to Freudenthal (1977), mathematics must connect to reality, stay close to learners and be relevant to society for it to remain valuable. Trailblazing was done around learners' intriguing environments, located by using the MCM app through a smartphone. This study concurred with Freudenthal in exposing learners to mathematics trails (real-world tasks). Learners discussed ideas, interpreted information, and responded to or represented information using mathematical model/s (a mathematical tool replicating real-world situation) and symbols.

The RME considers a mathematics classroom a place where learners can reinvent mathematical ideas, insights, knowledge, procedures, and concepts through mathematisation, exploration of real-world problems and not as a place to transfer knowledge of mathematics from a teacher to learners (Wahyudi, 2017). Learners should, therefore, learn mathematics by mathematising subject matter from real contexts as a source of development, applying their own mathematical activity rather than from the traditional view of presenting mathematics as a ready-made system with general applicability (Gravemeijer, 1994; de Lange, 2006). In so doing, learners go through stages referred to in RME as horizontal and vertical mathematisation (King & Rasmussen, 2000).

From Freudenthal's (1977) work, Treffers (1987) formulated the idea of two types of mathematisation by distinguishing 'horizontal' and 'vertical' mathematisation. Horizontal mathematisation is when learners use their informal strategies to describe and solve a contextual problem, and vertical mathematisation occurs when the learners' informal strategies lead them to solve the problem using mathematical language or to find a suitable algorithm (Treffers, 1987; Barnes, 2005). Jupri and Drijvers (2016) define mathematisation as an activity of transforming a realistic problem into a mathematical problem (i.e., horizontal mathematization), as well as solving the problem expressed in a model and interpreting the solution (i.e., vertical mathematization). The traditional formal and authoritarian approach to teaching mathematics that has dominated South African classrooms for a number of years, has not afforded learners many opportunities to use horizontal mathematisation (Venter et al., 2004; Harber & Serf, 2006). The risk is that when learners have entered that process without first having gone through a process of horizontal mathematisation, a strong possibility exists that, if they forget the algorithms they were taught, they do not have a strategy in place to assist them in solving the problem (Barnes, 2005). The DBE (2011), the NCS, grades R–12, further state that mathematics real-life problems should be incorporated into all sections where appropriate and be realistic, as contrived mathematics tasks perpetuate the notion that mathematics is abstract without any relevance to real life.

The RME theory is one that is constantly 'under construction', being developed and refined in an ongoing cycle of designing, experimenting, analysing and reflecting. The REMSA project suggests that the curriculum milieu impacts differentially on appropriation of the learning resources, in this regard the proximity of materials to the curriculum plays an important role (Gravemeijer, 1994; Julie & Gierdien, 2019). RME instructional design principles noted by Gravemeijer (1994, 1999) identified six principles of RME. First, is the guided reinvention through progressive mathematisation which requires that well-chosen contextual problems that offer learners opportunities to develop informal, highly context-specific solution strategies

(Doorman, 2001), be presented to them. When learners use their everyday language (informal description) to structure contextual problems into informal or formal mathematical forms, that is mathematisation. The teacher tries to compile tasks that lead to a series of processes resulting in the reinvention of the intended mathematics, in this case, mathematics trails uploaded on the MCM app (Barnes, 2005). The idea is not that learners to reinvent everything independently, but that Freudenthal's concept of 'guided reinvention' should apply (Freudenthal, 1973). This should, in turn, allow learners to regard the knowledge they create as knowledge for which they have been responsible and belongs to them. With guidance from MCM app tasks and stepped-up hints, the learners can construct their own mathematical knowledge (Sumirattana et al., 2017). Peters (2016) postulates that learners should be able to experience a process similar to the one mathematicians underwent. During the learning process, learners should be able to build their own mathematical knowledge. Mathematics trails allow a variety of solutions, reflecting potential learning routes since learners can access sample solutions on their smartphones. Mathematics trails were activities for constructing knowledge instead of transmitting the existing knowledge. Learners participated in reinventing mathematics through mathematics trails guided by the smartphone. Peters (2016) and Wahyudi (2017) note that it is crucial to underline that this principle, on the learning side, aims to allow learners to regard the knowledge they have constructed as their personal mathematical knowledge. This enables learners to realise the intended mathematics knowledge or skills by themselves.

Second is **the didactical phenomenology** principle advocated by Freudenthal (1973), which stipulates that, in learning mathematics, one has to start from phenomena meaningful to the learner, entreating some kind of organising to be done, and stimulating learning processes. According to Treffers and Goffree (1985), this principle should fulfil four functions: concept formation (to allow learners natural and motivating access to mathematics); model formation (to supply a firm basis for learning the formal operations, procedures and rules in conjunction to other models as the support for thinking); applicability (to utilise reality as a source and domain of applications); Practice (to exercise the specific abilities of learners in applied situations) (Barnes, 2005). The outdoor mathematics tasks are organised by the teacher, verified and uploaded by the team of experts into the MCM web portal, and later published on the MCM app for learners to walk the mathematics trails, reflected the principle. The tasks enabled learners to form mathematical models and employ relevant concepts. The outdoor tasks allowed learners an opportunity through smartphones to locate the tasks in the real world and used hints where they got stuck. Peters (2016) and Wahyudi (2017) posit that, according to a didactical phenomenology, in situations where a given mathematical topic (in this case, trigonometry) is applied should be investigated to unveil the kind of applications that have to be anticipated in the task and consider their appropriateness for progressive mathematisation.

Third, is **the principle of emergent or self-developed models.** Barnes (2005), Wahyudi (2017) and Sumirattana et al. (2017) note that this third principle for mathematics trail design in RME plays an important role in bridging the gap between informal and formal knowledge and further postulate that, to realise this principle, learners need to be given opportunities to use and develop their own models when solving problems. The term 'model' is understood here in a dynamic holistic sense and learners enhance their models by using their former models and their knowledge. Learners progress from what is termed a 'model-of' a situated activity to a 'model-for' more sophisticated reasoning (Gravemeijer & Doorman, 1999 as cited in Kwon, 2002). Learners revealed their mathematical ideas through mathematical models, e.g., while working on the school grandstand stairs task, calculating the angle of depression. They realised that there was a relationship between the angle of elevation and angle of depression. They formed a Z-shape, that implied they were corresponding angles and used a rectangular plane to illustrate the angle of depression. This is quite distinct from the past (and in many instances still current) practice in SA, where learners were presented with a model or algorithm by the teacher and then given repeated opportunities and problems to practise using that model (Barnes, 2005; Peters, 2016; Sumirattana et al., 2017). Learners could analyse tasks or problem situations, as well as construct simple and meaningful models or methods to solve the tasks.

The fourth principle, closely related to the third principle, is the **learners' contributions**. Learners (through production and construction) actively create mathematical materials based on facilities in the learning environment provided by teachers and actively solve tasks in their own way (Wahyudi, 2017). The mathematics trails were designed so that learners had to improvise or solve the tasks in various ways, e.g., by using a stick for measuring.

Fifth is the **use of interaction**. Interacting and communicating during the mathematics trails encouraged learners to verify and develop mathematical ideas. Learners got positive feedback from the MCM app immediately after typing their answers on the smartphone. Barnes (2005) and Wahyudi (2017) posit that the interaction among and between learners and the teacher is an essential component of RME because deliberations and collaboration enrich reflection on the work (Cheung & Huang, 2010; Sembiring et al., 2008; Van den Heuvel-Panhuizen, 2013). Peters (2016) says that in interactive instruction, learners are engaged in explaining, justifying, questioning alternatives, and reflecting. In a learner-centred environment, it is infrequent that learners pursue the same track and be on the same development level simultaneously. However, this principle provides learners with a platform to share their experiences, strategies, and interventions (Van den Heuvel-Panhuizen, 2013). Keeping learners interacting can result in learners being reflective, leading to a higher level of understanding. (Wahyudi, 2017 & Peters, 2016). All these were evident during the mathematics trails, where learners learned from each other, asked questions, elaborated, and illustrated using drawings and relevant examples. For instance, on the drainage pipes task, they had to calculate the area bound by drainage pipes. During interviews, some learners confessed that they were never taught sine, cosine and area rules; but through the hints, internet and sharing ideas, they managed to successfully complete the task. As much as there was an interaction among learners and between learners and environment, the smartphone played the role of a teacher through the MCM app providing hints, feedback, sample solutions, among others.

Last is the **use of topic relatedness**. The learning of a mathematical material is related to various mathematical topics (in this case, trigonometry) in an integrated manner and other learning areas or subjects. Mathematical structures and concepts are interrelated; usually, a topic discussion should be explored to support meaningful learning. (Wahyudi, 2017). During the walking of mathematics trails, it was observed that learners made the relevant and appropriate mathematical connections, e.g., while working on the school flag pole task, they used similarity of triangles, particularly the proportionality of the sides, and subsequently solved for the missing side using algebra.

RME is one of the learning approaches that address problems caused by traditional and abstract mathematical learning (Bray & Tangney, 2015). Misunderstandings and misconceptions related to learning mathematical concepts can be fixed or remedied by employing RME. (Laurens et al., 2018). The use of RME in the mathematics trails provides tasks tied to real-world objects, which assist learners in solving their problems and improving their understanding (Laurens et al., 2018). Laurens et al. (2018) and Maulina et al. (2020) investigated the difference between concept understanding of junior high school learners using RME and those using a traditional approach and discovered that effective use of RME enhanced their understanding. Research has proven that RME is a promising approach to remedy and improve learners' understanding of mathematical concepts (Armanto, 2002; Fauzan, 2002). RME has the potential to transform mathematics learning into interesting, interactive and meaningful for learners by presenting, in this case, trigonometry concepts through the mathematics trails, as opposed to mathematics in a classroom (teacher-centred). In the mathematics trails, smartphones act as facilitators to assist learners in locating and solving the tasks. The RME using interactive tools/material (MCM app) is believed to positively impact learners' cognitive achievement, particularly their understanding of mathematics (Bonotto, 2008). RME activities are mostly interactive (mathematics trails) and designed to enhance learners' conceptual understanding (Fauzan, 2002).

Mathematics is not for mathematicians per se. However, it features in daily life. Mathematisation helps learners connect ideas to rediscover mathematical concepts. The principles of RME include meaningful contexts (mathematics trails), development of models (enabling learners to transform realistic/real-world problems into mathematical problems), the recreation of mathematics concepts, the interaction between learners and the MCM app. These RME principles improve understanding as learners identify relevant, integrated mathematical concepts (Yuwono, 2007; Laurens et al., 2018).

The RME theory is suitable for this study because of its principles, especially how it deals with realistic problems and situations for learning purposes. This study employed smartphones to promote conceptual understanding through outdoor mathematics learning (mathematics trail) connected to the real world. RME's fifth principle emphasises interaction. As learners walked the mathematics trails, they were communicating, explaining, asking, responding to questions (among themselves) and reflecting, to achieve a smooth transition from realistic problems to mathematical problems.

2.8 SUMMARY

The purpose of this chapter was to present a critical review of the literature relevant to this study starting with a discussion of the current state of mathematics learning in SA. This was followed by defining trigonometry, discussing difficulties in learning trigonometry concepts, and presenting the role of smartphones in mathematics learning and the challenges of using smartphones. MCM project, outdoor mathematics, mathematics trails and the MCM app were also presented. The chapter concluded with a discussion of conceptual understanding and the RME theory that underpins this study. The next chapter presents the methodology applied in the research study.

CHAPTER THREE
METHODOLOGY

3.1 INTRODUCTION

The chapter discussed the research methodology that underpins this study's research process. The study employed an explanatory sequential mixed-methods approach in gathering and analysing quantitative and qualitative data. A case study design was used as the main research method. The chapter discusses the instruments used to collect data, i.e., surveys, observations and interviews (focus group). This chapter considers the selection of participants and how the data was analysed. The chapter ends by deliberating validity issues and ethical aspects used to improve the quality of this research.

3.2 RESEARCH RATIONALE AND QUESTIONS

This study investigated whether selected grade 11 mathematics learners' mathematical conceptual understanding while learning trigonometry through a smartphone using the MCM app was enhanced due to an intervention programme. The following research questions guided the study:

- How do learners use smartphones for mathematics learning purposes prior to participating in an intervention programme?
- How does using smartphones to learn mathematics through outdoor mathematics tasks enhance conceptual understanding of trigonometry concepts in a selected grade 11 class?
- What are the selected grade 11 mathematics learners' experiences and perceptions of using smartphones for learning trigonometric concepts after participating in the MCM project?

3.3 RESEARCH ORIENTATION

A paradigm is a set of ideas and beliefs that provide a framework or model that research can follow. A paradigm defines existing knowledge, the nature of the problem(s) to be investigated, appropriate methods of investigation, and how data should be analysed and interpreted

(Nickerson, 2023). Paradigms inform us how meaning is constructed from the data collected, based on our individual experiences. It is the lens through which a researcher looks at the world (Kivunja, 2017). Bertram and Christiansen (2015) note that a research paradigm represents a worldview that defines what is acceptable to research and how it should be done. The researcher explored the subjective understandings and interpretations of the 12 selected grade 11 mathematics learners concerning using smartphones to learn mathematics through outdoor mathematics tasks of trigonometry concepts using the MCM app. This study was located within an interpretive paradigm.

3.3.1 Interpretive paradigm

Ryan (2018) discusses interpretivism and notes its belief that individuals or groups build reality based on interactions with the social environment (p 8). This paradigm is unique because it focuses on understanding the lived experience of those who live it. The researcher is concerned with subjective reality (Van der Walt, 2022). Interpretivism is a sociological method of research in which an action or event is analysed based on the beliefs, norms and values of the culture of that society. It is a method used to analyse data related to human actions in sociology. According to interpretivists, these subjective interpretations are more valuable than natural laws. Interpretivists believe that reality is constructed by individuals and cultural beliefs regarding their actions rather than the physical occurrence of the actions themselves (Helper & Cloud, 2022). Brewer et al. (2014) used interpretivism with quantitative methods, i.e., a mixed-method analysis (Helper & Cloud, 2022). The interpretivist uses a naturalistic approach to data collection, such as interviews and observations. Primary data using these methods can be considered highly valid because it tends to be trustworthy and honest but cannot be generalised since personal viewpoints heavily impact it. People experience the same ‘objective’ reality differently and have individual reasons for their actions (Alharahshel & Pius, 2020; Bhattacherjee, 2012). Interpretivists use qualitative and quantitative research methods with the view that there is no one ‘right path’ to knowledge, rejecting the idea of one methodology (Nickerson, 2023). Despite its strengths, this paradigm has some disadvantages. One limitation is that the interpretivists aim to gain a deeper knowledge of phenomena within its complex context rather than generalise these results to other situations (Pham, 2018).

Creswell (2003) notes that researchers in the interpretive model discover reality through participants’ perceptions of the situation being studied. Central to the interpretive paradigm is understanding subjective human experience (Cohen et al., 2011), e.g., how learning was enhanced, and the experiences of the grade 11 learners learning trigonometric concepts using smartphones with the MCM app. Christiansen (2014) advises that data from the interpretive research must be authentic and reflect participants' experiences.

This study provided an in-depth understanding of how learning was enhanced from the learners' perspective and the participants' experiences using smartphones to improve their conceptual understanding of trigonometry (area, angles, heights and distance). A qualitative approach to analysing this data (using trailblazing and interviews) was employed.

3.3.2 Mixed-methods research approach

An explanatory sequential mixed-method approach was used for this study. Explanatory sequential design starts with quantitative data collection and analysis and then follows up with qualitative data collection and analysis, which leads to interpretation. (Creswell & Creswell, 2018). Mixed-methods research uses both qualitative and quantitative research and can assist in acquiring a complete picture compared to a standalone quantitative or qualitative study. Mixed methods enable research to put findings into context and add richer details, employing various methods to gather data on the subject can make study results more credible (George, 2021). Quantitative (closed-ended) research emphasises objectivity in measuring and describing phenomena using numbers, whereas qualitative (open-ended) designs emphasise gathering data about naturally occurring phenomena. In a qualitative approach, the researcher is interested in how participants understand their situation (Merriam, 2015; Creswell & Clark, 2007). Qualitative methods aim to achieve greater understanding, while quantitative methods seek to achieve a range of understanding (Patton, 2002). Quantitative research collects and analyses numerical data. It can be used to find patterns and averages, make predictions, test causal relationships and generalise the results to wider populations compared to qualitative research, which involves collecting and analysing non-numerical data, e.g., text, video and audio (Bhandari, 2020). In addition, Creswell (2014) posits that a mixed approach provides a more knowledge of the research problem than either of these methods alone. In this study, these approaches provided an in-depth understanding of outdoor mathematics tasks through smartphones during learning enhanced conceptual understanding of trigonometry concepts.

The mixed-method approach allowed the quantitative analysis of surveys, mathematics trail observations and interviews(focus group) using qualitative data. The results analysis prompted the intervention as they revealed that learners were not familiar with the MCM app to study trigonometry, and the mathematics trail was unknown to them. They learned trigonometry using textbooks and relied on the teachers. This added to the validity of the research project. This data informed the writing of this study's intervention programme. Learners' experiences of using the MCM app for learning mathematics through smartphones were analysed using a qualitative approach after taking part in an intervention programme.

3.4 DATA COLLECTION METHODS

This study was divided into two parts. The first part was a survey of 42 grade 11 mathematics learners at the school, and the second part comprised a case study of 12 grade 11 mathematics learners selected out of the 42 who participated in an intervention programme.

3.4.1 Survey

The survey administered to 42 Grade 11 learners to ascertain how learners learned trigonometry and used smartphones for learning purposes before the intervention. The school had two grade 11 mathematics classes with a total of 52 learners. Hence only 42 out of 52 learners undertook the survey and ten learners were not present. The survey had closed and open-ended questions (Appendix C).

According to Cohen et al. (2011), surveys gather data at a particular time to describe existing conditions or identify standards which can be compared. The survey revealed a need for a workshop and pilot session since the MCM app and mathematics trails were new to these learners. The workshop and pilot were important for exposure and to reduce anxiety during the intervention and to save time because the intervention was after normal school hours.

3.4.2 Case study (design)

The study followed a case study approach. According to Bertram and Christiansen (2014), 'a case study, is a systematic and in-depth study of one particular case in its context' (p. 42). Case studies are a qualitative method in which the researcher explores a programme, event, activity, process, or one or more individuals in-depth. The case(s) are bound by time and activity, and researchers collect detailed information using a variety of data collection procedures over a sustained period (Priya, 2021). The case in this study was a selected grade 11 mathematics class at one of the secondary schools in the Lejweleputswa district. The unit of analysis is the simultaneous use of mathematics trails and smartphones with the MCM app in learning trigonometry concepts and participants' views and experiences on how smartphones enhanced their learning for conceptual understanding.

3.5 SAMPLING AND PARTICIPANTS

The study participants were purposively selected. According to Cohen et al. (2011), in purposive sampling, 'researchers hand-pick the cases to be included in the sample that are satisfactory of their specific needs' (p. 156). This method allows the identification and selection of information-rich cases for my study (Patton, 2002). Purposive sampling intentionally selects participants based on their knowledge, characteristics, experience or other appropriate criteria (NCSC, 2023). The sample for this study was selected at two levels. First, 42 grade 11 mathematics learners at the secondary school were asked to respond to a survey. Responding to the survey was voluntary. The learners were in grade 11 and taking mathematics because the study is about grade 11 trigonometry concepts as per CAPS. Only participants willing and available to participate were included (Bernard, 2002). NCSC (2023) posits that convenience sampling concerns recruiting individuals, particularly because they are available to access or contact at a practical level.

The researcher used convenience purposive sampling (Etikan et al., 2016) to select 12 grade 11 mathematics students to participate in the intervention programme.

3.5.1 Research participants

The learners were from a secondary school in Lejweleputswa, Free State province. The researcher went in person and talked to each potential participant about the study. The 12 learners were selected on the basis that they met the following criteria:

- willingness to participate in the study (voluntarily).
- in grade 11 mathematics class; the study concerned grade 11 trigonometry concepts.

- available after school to follow the trails and participate intervention-related programme.

The participants and the researcher collectively devised the criteria to group participants. This was done as a guide to ensure a balanced group in terms of the availability of resources (smartphones). Each group needed two smartphones and three members (a scribe, time and tools keeper, reader of questions, hints and writer of answers on the MCM app, smartphone). Three members per group were to optimise participation and avoid deviation from unwanted websites or distractions. These criteria assisted learners in choosing members, and the researcher just supervised the proceedings. The roles were rotational, from task to task, to circumvent boredom and to share and gain experience from various activities associated with trailblazing.

3.6 RESEARCH PROCESS

Data in this study were collected in two parts, part A and part B

Part A

Phase 1: Survey designing, piloting and distributing.

The survey was piloted (Appendix C) with one grade 11 mathematics class at the school. Permission was requested to conduct a pilot session, which was granted and arranged with a colleague who taught that grade 11 mathematics class to use a few minutes for learners to complete a ten-question survey. It took learners five minutes, at most, to complete the survey. The researcher supervised the process, and the learners responded to all questions. No learner needed clarity on any question. This implied that questions were clear to the learners and there was no ambiguity. The main purpose of the pilot was to check for ambiguity, the time taken to complete the survey, adequacy of space to write down answers, especially open-ended questions. The survey provided instructions on how to complete the survey, which had both closed and open-ended questions. No name or any form of identity was noted, either for the school or participants. After the pilot, the survey was given to 42 grade 11 mathematics learners. The mathematics head of department assisted in administering the survey to ascertain how learners learned trigonometry, and how they used smartphones for learning purposes before the intervention.

Phase 2: Analysing the survey data, selecting participants, and designing the trail.

In phase 2, all the respondents' surveys were analysed; first, quantitatively using descriptive statistics, e.g., bar graphs and frequency tables and then qualitatively examining emerging themes. The participants did not know about the MCM application, so they did not have it on their smartphones. They did not know mathematics trails in learning trigonometry as textbooks and teachers informed trigonometry concepts, and they indicated that smartphones were not allowed on the school premises according to the school policy. Learning for conceptual understanding was not evident due to traditional methods of teaching. Convenience purposive sampling was used to select 12 grade 11 learners to take part in my intervention programme.

After selecting 12 participants, two mathematics trails comprising five tasks each were designed by the researcher and supervisor. The mathematics trails related to objects on the school premises and to grade 11 trigonometry concepts (heights and distance, angles, trigonometry ratios, sine, cosine and area rule). Objects in the real world were photographed (the location must be on when taking the pictures). The researcher wrote down the tasks, then hints and sample solutions for each task. The next process was to upload the tasks to the MCM website:

- the MCM app was downloaded, and
- all the tasks, hints, sample solutions and pictures of the objects were uploaded to the MCM app.

On the MCM app, the option to make the tasks public was selected. However, the experts from Germany at Goethe University had to approve these on the MCM app before they could be published – the process took a week. This was due to questions seeking clarity, recommendations, comments, adjustments, corrections, and additions. Eventually, all the tasks were approved, and these are now accessible on the MCM app.

3.6.1 Teacher's intervention planning and uploading of the tasks on the MCM app.

This section is motivated by the outcomes of the survey. First, more than one smartphone per group was required in case the one in use became dysfunctional, or the battery ran flat. Second, no one mentioned the MCM app and outdoor mathematics tasks, making the workshop and pilot necessary. Third, from the survey, data acquisition was a challenge for all learners, so each group needed a hotspot to download the MCM app and do the internet search. Fourth, we had to create rules as a collective during the workshop to ensure maximum participation. The workshop took us one day with the 12 selected grade 11 mathematics learners. The workshop had two phases: one in the classroom for downloading the MCM app and exploring the application, setting of the rules and another for piloting. Learners walked and solved tasks outside the classroom (piloting) but within the school premises. The workshop and the pilot were videotaped. Any possible distractions from social media through notifications and chatting with friends instead of learning were addressed in the workshop, i.e., no one was allowed to chat with friends or visit other websites irrelevant to the task at hand. There were only three members per group, each with a specific role on an alternating basis. Hence, no one could deviate from the group tasks.

The planning of outdoor activities and uploading the grade 11 trigonometry tasks was done with the help of the supervisor, who came from the Northern Cape to the Free State and spent the whole day assisting the researcher and we created the first trail of this study. Objects around the school yard were identified, and the trigonometry tasks developed, hints provided and a sample solution; the object was photographed with the smartphone with its location, uploaded to the MCM website, waited for the expert panel's feedback, approval and uploaded the tasks on the MCM website. We tested these tasks, and the experience enhanced the theoretical understanding of solving mathematics outside the classroom, which is not the same as inside the classroom. Prior knowledge from the classroom was necessary for appropriate mathematical relationships and connections. Teamwork helped because the supervisor could assist, when necessary, which motivated the researcher developing the second trail two weeks later. The researcher engaged learners at his school as part of the pilot to check how long it took them on a task and clarify any ambiguity. Also, someone was needed to assist with video recording so to allow for observation by the researcher. The study had two successful and approved mathematics trails on the MCM app, each consisting of five tasks. The intervention programme was based mainly on the data collected from the survey. There were three members per group, and it took two weeks to complete the process–one week for each trail. There were four groups, each assigned a day to walk the trail.

The mathematics trails were assigned codes and titles; below are the tasks that I designed for this study:

First trail

Title**:** Trigonometry at Unitas (Trail code: 016067)

For the tasks in this mathematics trail and pictures, refer to Appendices 5–9. The first mathematics trail consisted of five tasks:

- School flagpole (calculate the height),
- School billboard (The size of the angle that the supporting pole in the middle makes with the ground),
- Roof and wall (the magnitude of the angle that the roof makes with the horizontal),
- Pipes at angle (calculate the acute angle between the water pipe and the drainage pipe), and
- Drainage pipes (calculate the acute angle formed where the two drainage pipes meet).

Second trail

Title**:** Trigonometry at Lephola (Trail code: 258912)

The second mathematics trail also comprised five trigonometry tasks from grade 11 syllabuses. (Refer to Appendices 10–14):

- School grandstand (calculate the slope of the grandstand in degrees),
- Grandstand stairs (Find the angle of depression of the grandstand stairs),
- School flag pole (Calculate an angle of elevation of the school flag pole, from the observer standing 680 cm away from the flag pole),
- Drainage pipes (Determine the size of the area bound by drainage pipes), and
- Roof verge boards (calculate the magnitude of the angle that the two roof verge boards make).

The tasks were public, i.e., they had been approved by the team of experts from MCM headquarters at Goethe University and could be accessed by anyone who has downloaded

MCM app on their smartphone. All that one needed was the trail code and title to identify the trail from the other mathematics trails on the MCM app.

Part B

Phase 1: Orientation, workshop and piloting the trail.

The 12 learners selected in Part A were oriented on the background or context, rationale, and ethical matters of this study and invited for a workshop conducted at school. In this workshop, we discussed the MCM project, downloaded it on their smartphones, and demonstrated how to use the MCM app, follow the mathematics trails, used stepped hints and we provide the rules. After the presentation, they did the pilot on two tasks prepared beforehand, outside the classroom but around the school.

I observed how they used the MCM app but mainly focused on how they solved the mathematics trails. They asked questions like if they might use any tools, they deemed suitable to help solve the task. That is where they were made aware of everyone's safety and health. Some learners dominated the discussions, and it was made clear that teamwork is important and that they needed to allow each other to discuss each aspect. This pilot was videotaped and analysed to determine the effectiveness of the recording instruments, the clarity of the questions and pictures, and the time learners took to complete a task. Since it was the first time for the participants to do mathematics outdoors using smartphones, they needed exposure before implementing the intervention to reduce anxiety, among other things. The participants were actively involved, and it was at this phase that the roles of each member were outlined, and how these would be alternated or rotated. The roles are described next.

- First, one smartphone and timekeeper for reading tasks and hints.
- Second, another is responsible for carrying tools (measuring tape, stick) and helping in taking measurements.
- Third, a scribe for the group (write down all the calculations, procedures, and suggested mathematical models on a page).

However, all of them were part of discussions and decisions made.

Participants were divided into groups consisting of three members each. Each group had more than one smartphone for the intervention. Each group was assigned a day in the first week (for the first trail) and a day in the second week (for the second trail) to walk the first and the second mathematics trail.

Phase 2: Trailblazing

Learners were divided into four groups of three members in phase 1 before walking the trails. This ensured that everyone was actively involved. The mathematics trail observations took about two weeks in the first term of 2022. This data collection on mathematics trails followed after the workshop and pilot session in phase 1. The first week was for the participants to walk the first trail, and the second for the second trail. Each group was assigned a day per week to walk the trail. Four consecutive days a week were spent gathering data. Both mathematics trails had five tasks each (see part A, phase 2). They had to walk and do five tasks a day, and each trail would take 60 minutes.

The learners were observed walking the mathematics trails based on grade 11 trigonometry concepts, using the MCM app. The trailblazing was video recorded. This process assisted in answering research question two.

Phase 3: **Interviews**

Each videotaped mathematics trail was followed by a focus group interview with each group immediately after they completed the trailblazing, for about 30 minutes. The focus of the interviews was to follow up on what transpired during the process, the study participants' views and experiences and whether the MCM app enhanced the learning of trigonometry. These interviews were video recorded. Analysis of the recorded and transcribed interview data assisted in answering research question three of the study.

3.7 DATA COLLECTION INSTRUMENTS

This research used the following data collection tools: surveys, observations and focus group interviews. One reason for employing these three instruments together was to make up for any inadequacy in each one, a process named triangulation (Bertram & Christiansen, 2014; Nickerson, 2023).

3.7.1 Survey

The survey had ten questions with instructions, so the participants knew how to complete it. It allowed anonymity and comprised closed and open questions (Appendix C). It was administered to 42 grade 11 participants in a secondary school in the Lejweleputswa district, and all survey forms were returned. This survey aimed to obtain data on how learners learned trigonometry and used smartphones for learning prior to the intervention survey and collected various data, including learning methods, the use of smartphones for mathematics learning and applications related to mathematics. For instance, on the use of smartphones for mathematics learning, learners' experiences related to what extent they used smartphones were observed, and how they were extending what they extended learning to the real world so that they could apply prior knowledge among others. This data was analysed and used to inform the drafting of the intervention programme. The participants mentioned in the survey the applications they used for downloading previous question papers, WhatsApp and Facebook, but not the MCM app. WhatsApp and Facebook were mainly used for communication or clarity on certain concepts in class or during presentations. They were also used for reminders about homework, tests and activities. However, this experience was useful while downloading the MCM app and using it.

3.7.2 Observations

Cohen et al. (2011) expound that 'observation offers the investigator the opportunity to gather 'live' data from naturally occurring social settings' (p. 396). 'Observation' refers to 'a systematic process of collecting data that relies on a researcher's ability to gather data through his/her senses without questioning or communicating with participants' (Anthanasou et al., 2018, p. 91). Likewise, Bertram and Christiansen (2014) state that observation enables the researcher to report on things they have witnessed and recorded themselves instead of what other people tell them. The observations took two weeks, bearing in mind the policy of the Free State Department of Education (Appendix 2) to mainly keep learners in classrooms to be prepare for formal assessment with minimum interruptions. However, the more observations, the better the credibility of the results to provide the researcher with a better understanding of how a programme or activity works. In this case, it enhanced the understanding of how a smartphone could improve conceptual understanding and witness this in detail. It also allows the researcher to learn things that interview or focus group respondents might not reveal.

Observations can be valuable for interventions that depend on technical skills, such as smartphones (Harvey 2018). This blends well with the research where 40 tasks were video recorded, using the researcher's smartphone (20 in the first trail, and 20 in the second) as learners were trailblazing using smartphones. Some tasks in the second trail were the same as some in the first trail but tweaked to a higher cognitive level and others were completely new. Refer to the tables below for the schedule of the trailblazing:

Table 3.1: First Trail (Week 1)

Group	Date	Tasks	Time
1	22/03/2022	5	15h00 to 16h30
2	23/03/2022	5	15h00 to 16h30
3	24/03/2022	5	15h00 to 16h30
4	25/03/2022	5	15h00 to 16h30

Table 3.2: Second Trail (week 2)

Group	Date	Tasks	Time	Interviews
1	28/03/2022	5	15h00 to 16h30	16h30 to 17h00
2	29/03/2022	5	15h00 to 16h30	16h30 to 17h00
3	30/03/2022	5	15h00 to 16h30	16h30 to 17h00
4	31/03/2022	5	15h00 to 16h30	16h30 to 17h00

Data was collected on how the use of smartphones to learn mathematics through outdoor mathematics tasks enhanced the theoretical understanding of trigonometry concepts of the selected grade 11 mathematics learners. See Appendices A and B for analytical tools used.

3.7.3 Focus group interviews

An interview is a conversation gathering information involving an interviewer who coordinates the process and asks questions and an interviewee who responds to the questions (Rosanne, 2020). Group interviews capitalise on communication between participants to generate data (Kitzinger, 1995). The interviews were appropriate for this study because they allowed the collection of in-depth information on learners' opinions, experiences and feelings. At the end of the second mathematics trail, the researcher met with each group of three learners immediately to conduct the audio/video recorded focus group interview. Four focus group interviews were recorded, and the learners discussed how using smartphone mathematics learning through outdoor mathematics tasks impacted their conceptual understanding of trigonometry. Refer to (Appendix D) Table 3 below, to access the interview questions. More questions were to follow up on matters or issues needing more detail. For instance, question number 4, learners indicated that all the questions enhanced their conceptual understanding, but two groups mentioned that they did not know sine, cosine and area rules. However, through the hints and internet use, they successfully answered the questions. The questions aimed to ascertain if conceptual understanding was enhanced and why this was their experience. The four focus group interviews were conducted immediately after the second trail as indicated in the schedule–Table 2 above.

3.8 DATA ANALYSIS

Analysis is a close or systematic study of data, or the separation of the whole data into its parts, for the purpose of study (Bertram & Christiansen, 2014). Data analysis is a 'process of making sense out of data' (Meriam, 1998, p. 178). This study used a mixed-methods design, gathering data in two separate phases. The researcher started by collecting and analysing quantitative and qualitative data from the surveys, followed by collecting and analysing qualitative data from mathematics trail observations and interviews. There were three stages in this analysis process.

Stage 1- Survey: At this stage, the survey data was analysed quantitatively and qualitatively to ascertain how learners used smartphones for mathematics learning purposes prior to an intervention programme. Data from the surveys was analysed and presented quantitatively using descriptive statistics, i.e., bar graphs and frequency tables (Bhandari, 2020). The analysis was done before the observations. This was because survey data was to inform the intervention programme. Adhering to the ethical agreements, each participant's survey response was allocated a particular code. The codes ranged from P_1 to P_{42} (P_1 stands for Participant 1). Quantitatively, learners were asked if they used their smartphones for learning mathematics, and majority of the participants used smartphones when learning mathematics, particularly at home, but not at school due to the ban on smartphones on the school premises. Such a fairly high use of smartphones for learning purposes augured well with the preparations for the intervention. The researcher reviewed each survey, keeping a tally of participants who used smartphones to learn mathematics, and those who did not. The latter category formed the subheadings of the bar graph. The same procedure was employed with applications used for learning mathematics. The survey had another bar graph with bars for WhatsApp, Google, Facebook and others and revealed that from all the surveyed participants, none listed the MCM app and any outdoor mathematics learning application. As this was the application to be used in this study, a workshop on the MCM app was necessary to familiarise the participants with the application.

The data were also analysed qualitatively using thematic analysis, as Braun and Clarke (2006) explained. Open-ended questions allowed the researcher to make use of themes. I used different coloured pens for grouping themes, and the same or similar texts were the same colour. For example, learners were asked for what purpose they were using applications or smartphone; some mentioned to find information, others to understanding concepts and definitions better. The other question was whether they used smartphones when learning trigonometry concepts. They were to explain how they learned trigonometry concepts. These questions identified themes and the data to be analysed. Most learners mentioned using textbooks, the teacher, or Google to learn trigonometry concepts. The data indicated a teacher-centred approach, and signified the need for an innovative leaner-centred approach, in this case, the MCM app.

Stage 2- Observation: During this stage, the researcher qualitatively analysed the data from the mathematics trails video recordings, focusing on how using smartphones to learn mathematics through outdoor mathematics tasks enhanced the participant's conceptual understanding of trigonometry. The analysis focused on the conceptual understanding indicators and RME principles. In observing evidence, the analytical tool used, see Appendices A and B. These indicators have characteristics that suit this study, e.g., RME requires the use of real-world problems, hence mathematics trails. Kilpatrick et al. (2001) note that indicators help ascertain if conceptual understanding has been realised.

Stage 3: **Interviews:** Focus group interviews were used to search for learners' experiences and perceptions on the use of MCM app to learn and solve problems related to trigonometry concepts and thematic analysis (Braun & Clarke, 2006; Caulfield, 2019) to analyse the transcripts of the interviews. Caulfield (2019) posits that thematic analysis provides a highly flexible approach that can be modified for the needs of many studies, providing a rich and detailed yet complex account of data.

The researcher repeatedly read the data, looking for patterns related to selected learners' experiences using the MCM app to learn and solve tasks related to trigonometry concepts. In approaching this data, specific questions were in mind (connecting mathematics to prior knowledge and the real world, connecting concepts and ideas in mathematics, and using multiple representations among others), which were coded. Different colours highlighted the texts in the transcripts related to these themes. Certain themes emerged during the analysis as follows:

- appreciation of the use of smartphones
- endorsement of MCM app
- active 'participation' in trailblazing
- challenges in using the MCM app

The analytical tools (Appendices A & B) mentioned in stage 2 were used to analyse the interview data.

3.9 VALIDITY

Noble and Smith (2015) assert that validity refers to the integrity and application of the methods undertaken and the precision of how the findings accurately reflect the data. Surucu and Maslakci (2020) contend that validity is concerned with whether the measuring instruments measure the behaviour or quality they intend to measure and how well the measuring instrument performs its function. Validity focuses on the accuracy of research measures, checking how well the results correspond to established theories and other measures of the same concept (Middleton, 2019).

This study used more than one data source for triangulation purposes. Triangulation is the gathering of data from different sources to ascertain if what is gathered from one source contradicts or affirms the data from another source or the use of multiple methods to develop a comprehensive understanding of phenomena (Patton, 1999; Bertram et al., 2014). The data was collected using surveys, mathematics trail observations and focus group interviews. For example, in interviews, the same question was worded differently to see if learners would provide different answers. The interviews were video recorded to accurately reflect the participant's words. It was convenient to transcribe as it could be paused and rewound to ensure text was correctly captured. In ensuring the validity of the survey, analytical tool, interview and observation schedule, the supervisor reviewed them to check for any validity threats. The mathematics trail tasks were validated by a team of experts (Goethe University) before being attempted by learners and uploaded on the MCM web portal for other users who might be interested. This study was presented at the AMESA national conference as a short-paper presentation in 2022. We conducted a workshop on the MCM app for AMESA conference attendees; they walked trails that we designed at the conference before the workshop. Feedback and comments were used to enrich this study. In 2023, at the AMESA national conference, we presented an academic poster to further enrich the study and share good practices.

To ensure credibility, data was recorded technically (Bertram & Christiansen, 2014), including a video recording device to videotape mathematics trails and verbatim interviews. This technology has contributed to the credibility of this study's data because it was more accurate than the researcher having to make notes during the interview sessions or mathematics trail observations. Finally, the survey was piloted, as well as the mathematics trails, before implementing the intervention programme, to rectify any data collection mistakes or any ambiguities. Sauro (2015) notes that reliability is a measure of the consistency of metrics and there are four common ways of measuring reliability: inter-rater reliability, test-retest reliability, parallel form reliability and internal consistency reliability. Reliability measures the consistency of a set of research measures (Middleton, 2019; Indeed Editorial Team, 2023).

The study survey revealed points that prompted or informed the workshop, pilot session, and intervention. Some points emanating from the survey were that learners used traditional learning methods to learn trigonometry (teacher-centred), textbooks and learning was confined to the classroom. However, there was a fairly high number of smartphone users and applications (WhatsApp, Facebook, Google) but not the MCM app and outdoor mathematics tasks. The study targeted using the MCM app and the idea of outdoor mathematics learning, particularly trigonometry, for conceptual understanding. These outcomes proved the validity of the study. The mathematics trail observations based on grade 11 trigonometry and the focus group interviews added an element of reliability and validity as four groups walked the same mathematics trails as indicated in the mathematics trail schedule in Tables 1 and 2 above. The first trail occurred in the first week with five tasks, and the second mathematics trail the following week, with its five tasks. The results were similar in all the mathematics trails, which showed a likely high level of reliability in line with the existing measures and theory on the

MCM app. Ludwig and Cahyono (2016) posit that the effective use of dynamic software and applications (Apps), such as the MCM app can enhance learners' conceptual understanding of mathematics. That led to the validity of the study. The focus group interview outcomes concurred with the mathematics trails observations and corresponded well with the established measures and theories on smartphones for enhancing mathematical conceptual understanding. Kyobe and Van Belle (2018) observe that the developments in smartphone functionality have created exceptional opportunities for learning, enabling smartphones to play a crucial role in enriching learners' learning experiences. For example, participants mentioned that, after the trailblazing, they learned new concepts and smartphones and mathematics trails assisted them in better understanding trigonometry concepts and saw their applicability.

3.10 ETHICS

The study used ethical principles stipulated by the Rhodes University Ethics Standard Committee (RUESC), such as respect and dignity, transparency and honesty, accountability and responsibility, as well as integrity and academic professionalism. These are fully articulated in the subsections below:

Respect and dignity

Firstly, the researcher clearly explained the objective of the study to the participants before getting their consent and what was expected of them during the study. Participants were informed that they were free to withdraw from the study at any time and that the data gathered would remain confidential unless their consent was obtained to do otherwise. Codes were used in this study to protect the identity of the participants and the research site. For instance, in the survey, codes ranged from P_1 to P_{42} for survey participants (P for participant and subscript number allocated to an individual), and for the observations and interviews, study participants were allocated G_1 or B_3: Letter G stands for girl and subscript number 1 for group 1 and B stands for Boy from group 3.

Transparency and honesty

In order to conduct this intervention, the researcher first applied for and obtained ethical clearance (Appendix 1) from RUESC, a committee responsible for ethical approval at the university. Secondly, permission was requested from the Free State Department of Education (Appendix 2) to allow the research to be conducted at a school. The researcher then applied for permission from the school principal where the research would be conducted (Appendix 3) and requested the parents' consent (parent, learner consent form) (Appendix 4).

Accountability and responsibility

The data collected was securely captured on a laptop's hard drive (for the duration of this study), which is password-protected to prevent unauthorised access. Integrity was maintained at all times, not compromising Rhodes University's ethical standards. Positionality issues did not have any effect on this research study. The researcher formed a healthy relationship with the participants from their first meeting to explain the objective of the study and created a conducive environment. Thereafter, meetings took place in the workshop and pilot session. That is where it was realised that the participants were friendly and felt free to ask questions and work with the researcher. They even suggested a WhatsApp group for easy and efficient communication because the researcher did not teach at their school. My presence during mathematics trail observations did not cause them any discomfort. All learners shared their experiences with ease during interview sessions.

Integrity and academic professionalism

This thesis is my own work, and where the researcher used the ideas of other people and acknowledged them according to the referencing guide of Rhodes University. The collected data for this research study are presented the way they are, without any manipulations.

3.11 SUMMARY

The research paradigm that underpinned my study and guided my design and process was discussed in this chapter. It details how the phases used in the research design answer the research questions. Surveys, observations and interviews, which were the main data collection tools, were also discussed. Additionally, the data analysis and the sampling process of the study were described. The chapter concluded with a presentation of the validity and ethical aspects of the study. The chapter that follows presents data gathered from the abovementioned data collection instruments.

CHAPTER FOUR
DATA PRESENTATION, ANALYSATION AND DISCUSSIONS

4.1 INTRODUCTION

The aim of this study was to explore outdoor mathematics learning for conceptual understanding through smartphones in grade 11 trigonometry. The data analysed in this chapter was obtained from surveys, mathematics trails observations and interviews (focus group). The chapter begins with a presentation and analysis of survey data on how grade 11 mathematics learners at a secondary school in Lejweleputswa district, Free State, used smartphones for mathematics learning prior to participating in an intervention programme. The chapter presents and analyses data from the two mathematics trails walked by selected grade 11 mathematics learners. An analytical tool developed from Kilpatrick et al.'s (2001) conceptual understanding indicators (Appendix A) and RME principles (Appendix B) were employed to analyse the data to ascertain if the conceptual understanding had been realised from the two mathematics trails observed. Thematic analysis was utilised to analyse data from the interviews on learners' experiences using the MCM app in learning and solving related problems. The chapter concludes with a summary of the research findings from the research instruments mentioned herein.

4.2 THE SURVEY

The purpose of the survey was to answer research question 1: *How do learners use smartphones for mathematics learning purposes prior to participating in an intervention program?*

Data from the survey responses of the 42 grade 11 mathematics learners were first analysed quantitatively using descriptive statistics and qualitatively using thematic analysis. The purpose was to ascertain how learners learned trigonometry and used smartphones for learning purposes prior to the intervention. The data collected from the survey was also used to inform the appropriate planning for the workshop and the intervention programme. Adhering to the ethical agreements, each participant's survey response was allocated a particular code, from P_1 to P_{42} (P_1 stands for Participant 1).

4.2.1 Use of smartphones for mathematics learning (n = 42)

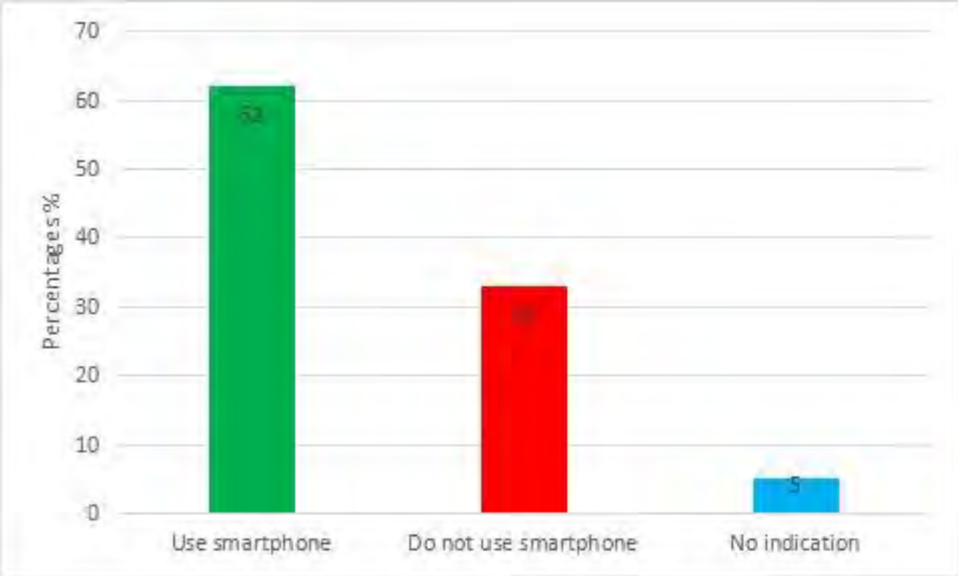

Figure 4.1: Learner use of smartphones for mathematics learning

Thirty-three per cent of participants said they were not using smartphones for learning purposes. They had smartphones but could not use them because smartphones were prohibited on school premises. The following quotations from the participants supported the statement: P_{39} said '*phones are not allowed at school'*, P_{40} said '*Because it is against the school rules'*. Some like P_{30} cited reasons like '*It distracts me'*. Most not using their smartphones cited prohibition by the school as the main reason. These results implied that learners owned smartphones and used them for learning mathematics even under these restrictive conditions. In this study, the availability of smartphones among learners was checked for the intervention, as at least two smartphones per three learners in the survey was required.

4.2.2 Applications used for learning mathematics

The survey also sought to uncover the types of applications used by learners while learning mathematics. As shown in Figure 4.2 below, Facebook had 43 per cent, followed by WhatsApp with 24 per cent usage. This is understandable as the survey was conducted when the COVID-19 pandemic and associated restrictions were still in place, although the restriction levels had already been eased. Learners were still using these two platforms for their online learning purposes.

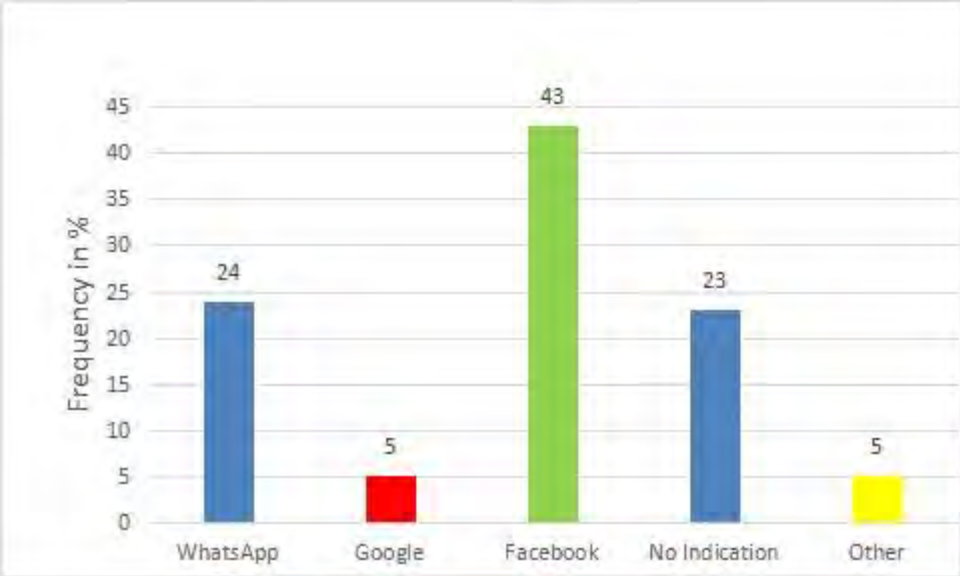

Figure 4.2: Application used by learners during learning

Among the participants, five per cent used other applications such as; P_3 said '*mobituta and Siyavula*', P_1 said '*mathematics mobile App'*, P_{12} said '*Hi mathematics and mobituta'*, P_{19} mentioned '*study smarter',* and P_{18} said '*foandmate*'. That they could clearly state different applications they were using meant that they were aware of and used different applications for mathematics learning. This was encouraging, as such experience would enable them to quickly adapt to the targeted application to be introduced during the intervention.

The study also revealed that none of the surveyed participants mentioned the MCM app and any outdoor mathematics learning application. Since this was the targeted application to be worked with in this study, there was a need for a workshop on the MCM app to expose the participants to the application.

4.2.3 Learner purpose for using the application of choice or smartphone

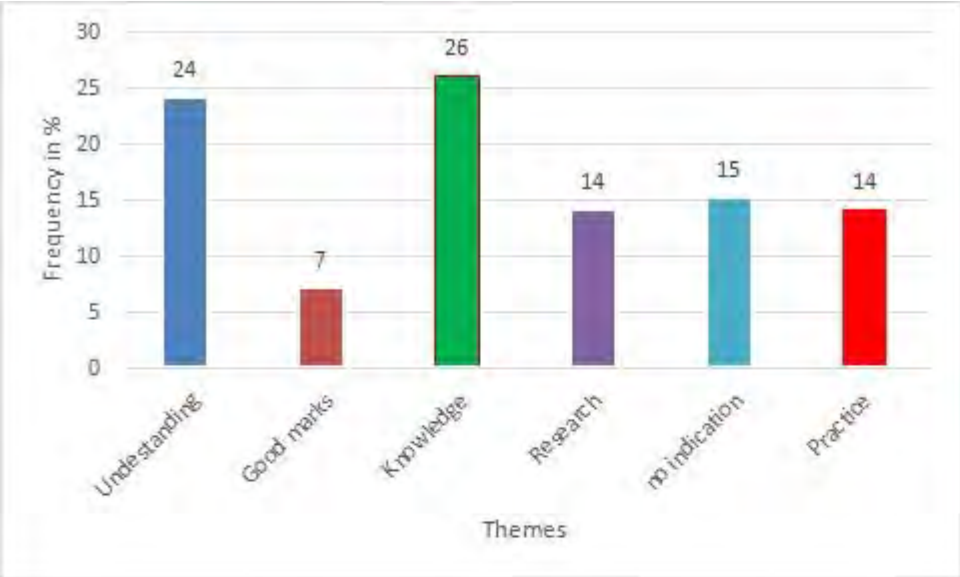

Figure 4.3: Learner purpose for using the application of choice or smartphone

Participants had different purposes for using the applications for learning mathematics. The survey findings in Figure 4.3 above showed that 24 per cent of participants used the application for understanding. P_{19} cited '*to understand the concepts'*. Twenty-six per cent used it for acquiring knowledge, 7 per cent for getting good marks (P_{33} mentioned '*to know more of the concepts and achieve good marks*'), and 14 per cent for research and practice. P_{14} said, '*searching for strategies I can use to solve complex mathematics problems*', P_8 listed '*to research school activities*'. The data showed that a fairly good number of the participants (24%) would be interested and inclined to engage in tasks that enhanced conceptual understanding, in this case, mathematics trails. P_{19} cited '*to understand the concepts*'. However, conceptual understanding requires, among other things, prior knowledge, therefore, a fairly good number of learners (26%) opted to use the application of choice for acquiring mathematics knowledge to solve the tasks during the mathematics trail. Among the learners, 14 per cent were interested in researching mathematics activities. P_8 listed *'to research school activities'*. Such desire would come in handy with the hints during the trailblazing.

4.2.4 After-school learning activities using smartphones

Learners indicated various learning activities engaged in after school, using their smartphones. Online classes led with 14 per cent, followed by group chats with 12 per cent, as in Figure 4.4 below. The online classes and group chats were mainly used during strict measures of COVID-19 pandemic as a means of teaching.

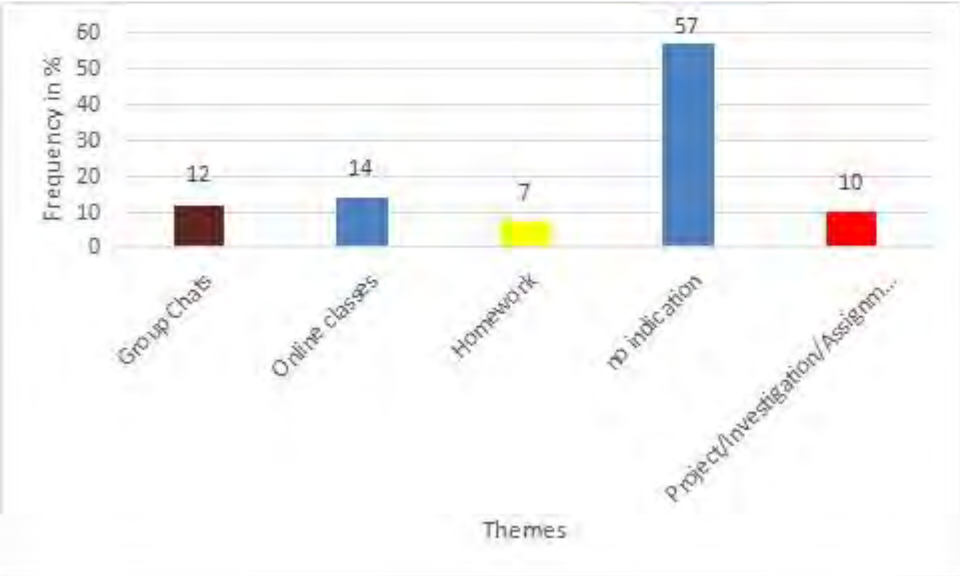

Figure 4.4: After school learner activities using smartphones

Figure 4.4 indicates the participants were not introduced to mathematics trails. Fourteen per cent of the participants listed online classes as the after-school activities they engaged in, and this was the highest percentage. This was due to COVID-19 implications on face-to-face learning. P_{40} said '*online classes'*. Twelve per cent mentioned group chats, and participants attested P_{15} mentioned '*group chats on WhatsApp when teachers want to finish activities they gave us in class, or asking questions where you didn't understand'*. Projects and homework counted for 10 per cent and 7 per cent, respectively. P_{39} said '*for doing homework*' while P_{17} mentioned '*investigations or assignments. I search for what had been given or maybe concepts I do not understand*'. However, a positive factor was that the participants were engaged in mathematics activities after school using smartphones, enabling them to take part in the workshop and, later on, to trailblaze after school during the intervention. Since the data showed that the participants were not exposed to mathematics trails, the workshop, pilot, and intervention were necessary to introduce them to the mathematics trail and walk or do one task outdoors as a pilot.

4.2.5 Learning trigonometry prior intervention

Figure 4.5 below revealed that participants learned trigonometry through known means besides mathematics trails. The data indicated that 52 per cent of participants used smartphones to learn trigonometry. P_8 said, '*Yes, to learn definitions of trigonometry concepts'*, P_{15} stated, '*to finish activities they gave us in class, or search something you didn't understand*'. Twenty-nine per cent used textbooks. P_7 mentioned, '*I learn trigonometry concepts by referring to the textbook examples'*. Seven per cent relied on the teacher – this is mostly because during COVID-19, teaching face to face was limited. P_{30} said, '*I use notes from my mathematics teacher'*. Twelve per cent made use of previous question papers. P_4 said, '*I look at my notes and examples used in or given in my classwork books'*, and P_{35} said, '*by use of textbook'*.

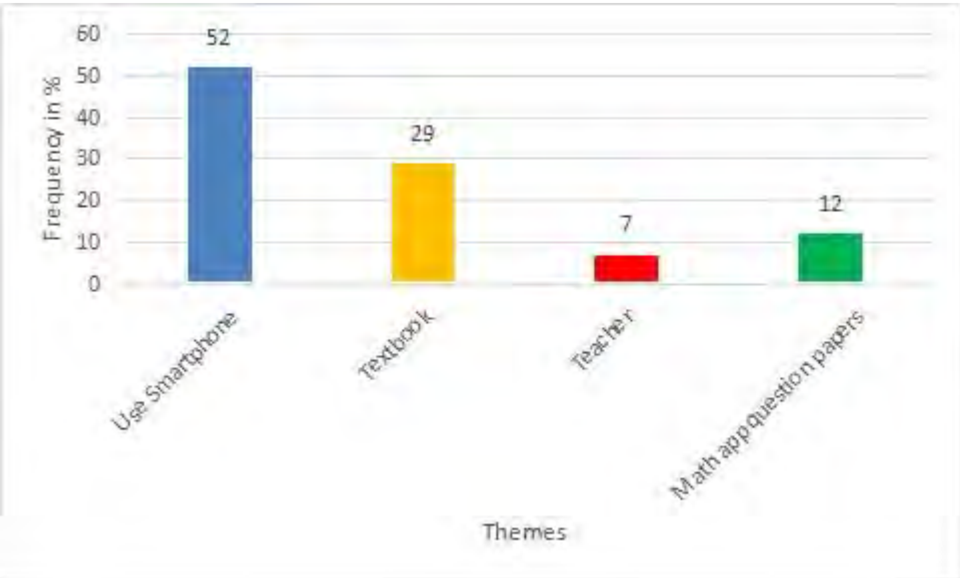

Figure 4.5: How participants learn trigonometry prior to the intervention

Figure 4.5 showed that participants needed a new approach to learning trigonometry rather than the traditional teacher-centred, textbook approach within the classroom environment to enable the participants to apply trigonometry concepts in real-life situations to enhance their understanding. Hence, participants must be exposed to the MCM app and outdoor tasks.

4.2.6 Learner challenges

The collected data demonstrated the factors preventing participants from using smartphones effectively for learning mathematics. Figure 4.6 below shows that 31 per cent mentioned the cost of data as a hindrance. P_{19} mentioned a '*lack of data for downloading trigonometry concepts*'. Ten per cent said school rules prohibited smartphone use. P_{11} said '*the smartphones are not allowed to be used on our school premises*'. In addition, load shedding was also mentioned as a challenge. P_{12} stated, '*most of the time is load shedding, where there will be no network to search for trigonometry questions*'. Also, 10 per cent cited social media as an issue. P_{17} remarked, '*distraction from social media, notifications'*, P_{33} mentioned personal reasons '*chatting with my peers instead of learning'*.

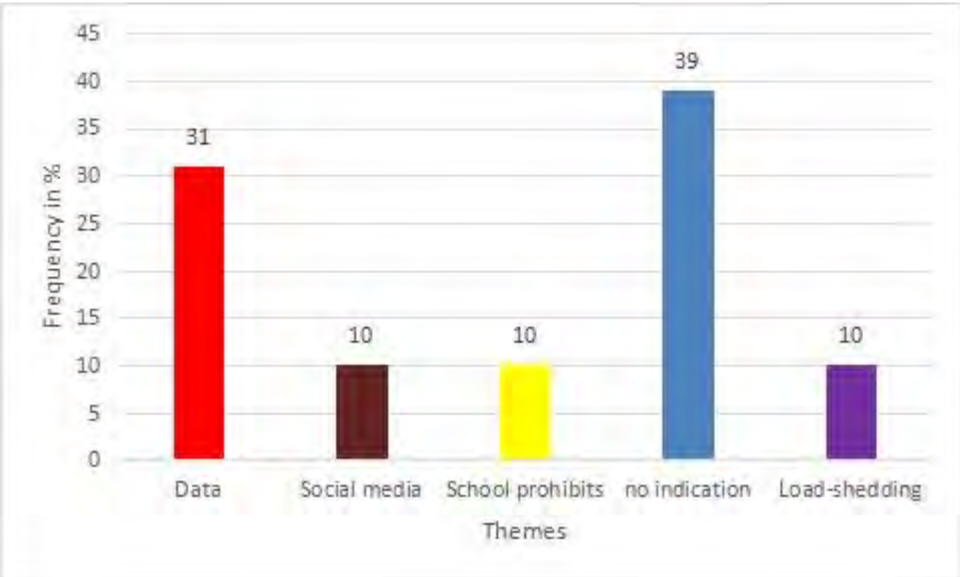

Figure 4.6: Learners' challenges

Mitigating measures

There was more than one smartphone per group (in case the one in use ran out of energy or stopped functioning). The researcher ensured enough data to hotspot those who might run out of data or did not have it. I also asked permission from the principal so the participants could use their smartphones for research purposes. I appealed to the DBE, School Governing Body, School Management Team and all relevant stakeholders at school to review the prohibition of smartphones at school because we live in the fourth industrial revolution. Research indicates that, if used effectively, smartphones can enhance conceptual understanding (NCTM, 2015; TIMSS, 2015; Cahyono & Ludwig, 2018; Shimakeleni, 2022). Load shedding and adverse weather conditions are, most of the time, beyond our control. Participants were asked to download the MCM app during the workshop prior to the intervention. Fortunately, the weather was favourable and the tasks were completed, although there were delays with one group because of cloudy weather, which lasted for less than three minutes. Some tasks could be done under a shelter or in enclosed buildings other than a classroom so that at least the group has something to work on, in case of adverse weather (proper planning or contingency plan).

4.2.7 Summary (survey)

The findings clearly showed there was a need to organise a workshop and conduct the research to introduce innovative ways of learning mathematics for conceptual understanding, where learners would be actively involved as they move from one task to another as opposed to classroom situation (teacher-centred approach). The study hopes that learners will realise the practical relevance of mathematics to the real world through the MCM app and outdoor tasks. Challenges such as lack of data, load shedding and prohibition of smartphones on the school premises were noted as reasons why some participants did not use smartphones for learning.

The workshop would help address these challenges by creating clear rules collectively. That would make compliance easy when participants were part of decision making. The participants downloaded the MCM app during the workshop. Fortunately, we did not experience load shedding disruptions; otherwise, we would have had to follow the load shedding schedules. The workshop selected three members per group with specific alternate roles to avoid idle members and unnecessary disruptions. The researcher met with the school principal to ask permission to conduct the research and allow participants to use smartphones for this purpose only and that the participants would be under supervision, given that the size of the group was manageable.

4.2.8 Interest in a new mathematics application

The collected data in Table 4.1 below revealed that 100 per cent of the participants were eager to be exposed to the new mathematics application to learn trigonometry. The question was: Would you be interested in being introduced to new applications for learning mathematics concepts? P_3 said, 'Yes'. P_{39} stated 'definitely', P_{10} remarked, 'Yes, as they would be helpful in improving our marks', and P_{41} said, 'Yes, I would like to use it'. These positive responses from the participants motivated the workshop and intervention alike. It showed the willingness and eagerness of the learners to participate in the intervention.

Table 4.1: Learners interest in MCM application

New mathematics application to learn trigonometry (MCM app)	Yes	No
Keen/interested	100%	0%

The findings of this survey revealed that rote learning, textbook and teacher-centred approaches were still predominant in these South African mathematics classrooms. P_{30} said, 'I use notes from my mathematics teacher', P_{35} stated, 'by use of textbook', and P_4 remarked, 'I look at my notes and examples used and given in my classwork books'. The teacher-centred approach was, and still is, the teaching strategy used judging from the observed mathematics classes (Maile & Makofane, 2019). The study also showed that learners perceived mathematics as a subject confined to a textbook or classroom. Today, smartphones are used in various assistive contexts, including learning and knowledge sharing (Yu, 2012). However, the school rules prohibit the use of smartphones on school premises. Thus, in this survey, the learners were asked what activities they engaged in after school using a smartphone. None of them listed outdoor tasks/activities. Instead, P_{20} said, 'online classes and they help me be ahead of my schoolwork', P_{27} said, 'the activities that actually gave me a challenge at school, I practice them and ask for help where needed'. Outdoor tasks can enhance theoretical understanding and keep learners actively involved and in charge of their own learning without a teacher standing in front of them. Cahyono & Ludwig (2018) posit that using dynamic software and applications such as the MCM app can strengthen learners' conceptual understanding of mathematics. Most participants mentioned that they learned trigonometry through the textbook, and it was presented to them by the teacher.

The study noted the challenges preventing learners from using the smartphone after or at school. P_{19} mentioned 'lack of data for downloading trigonometry concepts', P_{11} flagged 'the smartphones are not allowed to be used on our school premises', P_{12} said 'most of the time is load shedding where there will be no network to search for trigonometry questions', P_{17} stated 'distraction from social media, notifications', P_{33} mentioned 'chatting with my peers instead of learning'.

The National Council of Teachers of Mathematics (NCTM, 2015) argues that 'it is essential that teachers and learners have regular access to technologies that support and advance mathematical sense making, reasoning, problem-solving and communication' (p. 1). Therefore, cautious efforts need to be made to start viewing smartphones as tools that can be effectively and correctly used for learning. Training of teachers and learners is necessary in this regard. Learners need to be taught how to minimise distractions during learning times, while teachers may need training on best practices to optimise smartphones use for teaching. Yosiana et al. (2021) note that the advent of smartphones provides a strong learning environment for teachers to explore, experiment and share their knowledge as educational practitioners.

According to the data gathered in this section, learners indicated that they used textbooks, teacher notes and examples to learn trigonometry in the classroom before the intervention. This implied they used old methods and a teacher-centred approach, as opposed to outdoor mathematics learning using a smartphone. Hence, there was a need for the intervention to introduce innovative, vibrant, and active ways of learning.

4.3 OUTDOOR LEARNING OF TRIGONOMETRY

The observations of the walking of the mathematics trails were used to answer research question 2: *How does the use of smartphones to learn mathematics through outdoor mathematics tasks enhance conceptual understanding of trigonometry concepts, in a selected grade 11 class?*

Data from the two mathematics trails walked by the purposively selected grade 11 mathematics learners were analysed using an analytical tool developed from Kilpatrick et al.'s (2001) conceptual understanding indicators and RME principles. This was to ascertain whether this had been realised in the two mathematics trails observed.

This section reports on the analysis of the videos and interview excerpts simultaneously. It starts with videos analysis followed by focus group interview excerpts on every topic in this section. Each participant in a group had a distinct code: B_{3a} (B for Boy in Group 3 and *a* is for boy number 1, meaning there is another boy in Group 3 and that Boy is 3 subscript *b*) and G_2 (G for Girl in Group 2)

4.3.1 Learners' prior knowledge

Prior knowledge acts as a hinge between the learner's previous experiences and new information. Learners can use knowledge they understand to generate new knowledge and solve unfamiliar problems in the future (Kilpatrick et al., 2001).

The mathematics trail observations revealed that the outdoor tasks prompted learners to use known knowledge to solve unfamiliar tasks. Group 1, after reading the task from the MCM app, B_{1a} asked fellow group members which concept to employ to solve the task (see Excerpt 1 below). That meant that the mathematics trail task compelled learners to dig into their prior knowledge. G_1 asked, 'So, guys re tlo etsang?' (how are we going to solve this task?) They had to consult the first hint on the MCM app because they were stuck and hint number one suggested angle of elevation (see Excerpt 1 below). Then B_{1b} said, 'Angle of elevation. (B_{1b}: drew right-angled triangle on the page)'. Therefore, it was evident that hints from the MCM app assisted learners in recalling previous knowledge, and B_{1b} managed to draw the right-angled triangle. The prior knowledge was useful as learners successfully solved the unfamiliar task.

Excerpt 1: Group 1's discussion during Billboard Task (See Section 3.6.1)

> *G_1: Calculate the size of the angle that the supporting pole in the middle make with the ground, assume the ground is horizontal and leave your answer to one decimal place.*
> *B_{1a}: Re tlo sebedisang? (What concept are we going to use? she asked).*
> *G_1: Angle e mona yona mara keng? (What is the size of this angle here pointing?)*
> *B_{1a}: Ke 90 (is 90 degrees).*
> *G_1: Eya (yes, also agrees)*
> *B_{1b}: Yes, is 90 degrees because the base is horizontal.*
> *G_1: So, guys re tlo etsang? (so, guys what are we going to do? Meaning how are we going to solve this task?) hint number 1 ya rona ere re batle angle of elevation (hint number one says we need to calculate angle of elevation)*

B_{1b}: Angle of elevation. (They laughed) (B_{1b}: drew right-angled triangle on the page).

As shown in Excerpt 1 above, learners in group 1 had robust discussions during their attempt at the Billboard Task. During the discussion, the ability to recall the magnitude of a right angle was demonstrated, which was key to successfully solving the task. That is why B_{1b} said, 'Yes, is 90 degrees because the base is horizontal'. Hence, B_{1b} drew a right-angled triangle. It can be deduced that prior knowledge, such as trigonometry ratios in a right-angled triangle, assisted in solving the task effectively. These concepts were taught to learners in lower grades (9, 10 and including 11) (CAPS). This implied that the MCM app assisted the learners in remembering the appropriate prior knowledge for the given task, which we can infer as the ability to enhance understanding.

Excerpt 2: Group 3 discussion during Flag Pole task (See Section 3.6.1)

G_3: Calculate the height of the school flagpole in metres
B_{3a}: In metres? Now we have to convert cm into m.
B_{3b}: Yes, because right now it's in cm.
G_3: Kana re etsang? (What must we do again?)
B_{3b}: Do we divide or multiply?
B_{3a}: We divide
G_3: By100
B_{3b}: It is 8.074 m; let's check (meaning use MCM app) (after doing calculations with a calculator)
B_{3a}: It says not perfect but ok. (Feedback from MCM app)
B_{3b}: Kana hao sheba sample solution o etsang? (How do you check the sample solution? On the MCM app).

The data presented in Excerpt 2 indicated that learners recalled how to convert cm to m, from previous experiences in grades 8, 9 and 10 (CAPS) when dealing with conversions. B_{3a}: 'In metres? Now we have to convert cm into m'. As a result of Group 3 engagement, B_{3a} and G_3 remembered that they had to divide by 100 to do the conversion. The data showed that mathematics trail tasks enabled learners to use prior knowledge to solve the unknown by employing appropriate conversion processes, and subsequently got positive feedback from the MCM app. Learners used smartphones to access the sample solution from the MCM app, as they were curious about their results. They gained a better understanding after checking the sample solution from the MCM app (see B_{3b} Excerpt 2). From their discussions, using the MCM app sample solutions, previous experience and learners' responses, improved conceptual understanding was evident.

Excerpt 3: Group 3's responses mainly on the drainage pipes task.

> *$G_{3;}$ Ya (Yes), different triangles, like right-angled triangle, isosceles we were taught in grade 10.*
> *B_{3a}: Drainage pipes, when we were looking for an area of a triangle, we had to make use of the area rule.*

Excerpt 4: Group 2's responses on drainage pipes and (roof and the wall) tasks**.**

> *$B_{2b:}$ Task four we actually had to remember the properties of a rectangle.*
> *$G_{2:}$ …of a rectangle and a triangle*
> *B_{2b}: Because let me say on that building, we actually found a triangle and the rectangle at the same time.*
> *$G_{2:}$ …and task five too because we had to use different trigonometry ratios from the tasks we had done.*

During trailblazing (see Excerpt 3 and 4 above), learners highlighted how their previously constructed knowledge of different triangles (right-angled triangles, non-right-angled triangles) and the properties of rectangles from previous grades were useful, particularly where they had to use right-angled triangles to find suitable trigonometry ratios. The following quotation supports the latter statement: $G_{3;}$ 'Ya (Yes), different triangles, like right-angled triangle, isosceles we were taught in grade 10'. As further noted by $B_{2b,}$ task four prompted them to recollect the properties of the rectangle, and B_{3a} mentioned that they had to remember to use the area rule on drainage pipe tasks (non-right-angled triangle). The data from Excerpt 3 and 4 above demonstrated that outdoor tasks encouraged the use of prior knowledge and subsequently, it assisted learners to effectively solve the tasks. However, it should be emphasised that they had to consult with the MCM app for the hints, as mentioned in Excerpts 3 and 4 above. This meant that the smartphone assisted them in remembering prior knowledge to solve given tasks. B_{2b} 'Because let me say on that building, we actually found a triangle and the rectangle at the same time.' and G_2 '…and task five too because we had to use different trigonometry ratios from the tasks we had done', mentioned the latter tasks that prompted them to remember prior knowledge and this can be inferred to enhanced understanding of trigonometry concepts.

These findings indicated that learners used or referred to prior knowledge, which is a component of conceptual understanding because there was evidence of recollection of trigonometry concepts (see B_{2b}, G_3 and B_{1b}) and group 4 also remembered that B_{4b}: 'so what are we going to use? is it proportionality or similarity?' B_{4a}: 'ke similarity ebe o tloba proportionality' (similarity and then proportionality). Learners could distinguish which concepts were useful in a particular context for problem-solving (Kilpatrick et al., 2001). Wiggins (2014) posits that conceptual understanding helps learners understand how an idea or procedure is mathematically defensible, i.e., why the concept is justified when used (see G_3 and B_{3a}), and prior knowledge is useful.

In learning mathematics, learners' prior knowledge formed a base from which learning developed, as indicated by Group 4 below.

> *B_{4b} so we will convert it to metres using King Harry Died Miserable Death Called Measils one, zero zero (showing group mates on the page) so we are going to divide by 100.*

Learners who walked the trails demonstrated how prior knowledge helped them solve the outdoor problems. Group 4 demonstrated this

> *B_{4b}: ha re sebedise trig ratio yane ya tangent(let us use trigonometry ratio of tangent)*
> *B_{4a}: tan theta is = khana keng nthoe? (what is it again? I remember) opposite over adjacent.*

When Group 3 discussed the flagpole task, B_{3a} and B_{3b} indicated that prior knowledge of right-angle triangles and the similarity of triangles was required to solve this task (see Excerpt 2 above). Group 2 used appropriate trigonometry concepts to solve the unfamiliar task. The following quotations are from Group 2's discussion.

> *B_{2a}: wait wait lets identify our triangle first, what type of triangle do we have?G_2: ka nako yane rene re sebedisitse cosine rule neh? (last time we used cosine rule right?)*

B_{2a}: ya (yes), but it was a different task B_{2b}: I think the area rule will do because this is not a right-angled triangle.

The above shows that participants collaborated while walking the trails and used the MCM app to read the tasks and sample solutions. Therefore, the mathematics trails motivated the learners to move on to new material and solve tasks successfully. It can be deduced that a conceptual understanding of trigonometry was evident as prior knowledge is one of the interwoven competency strands.

4.3.2 Multiple representations of mathematical concepts

There was evidence from mathematics trail observations that different representations were employed. For example, from Group 2's discussions while walking trails, as shown in Excerpt 1 below. The learners' attempt to solve the real-world task indicated verbal representation, diagram and physical demonstration using a stick and the school grand stairs as external representations of the same trigonometry concept (angle of depression) (see Figures 4.7, 4.8 and 4.9). Learners used explanations, diagrams and non-verbal cues to solve this problem. This seemed to have assisted learners to solve the task effectively.

Excerpt 1: Group 2's discussions while walking trail (school grand stairs) (See Section 3.6.1)

B_{2a}: In simple terms angle of depression is the angle below x-axis, so when we talk about the angle of depression, we are referring to this (drawing a horizontal line on the page for fellow group members to see), that's the angle of the depression (pointing to the drawing on a page).

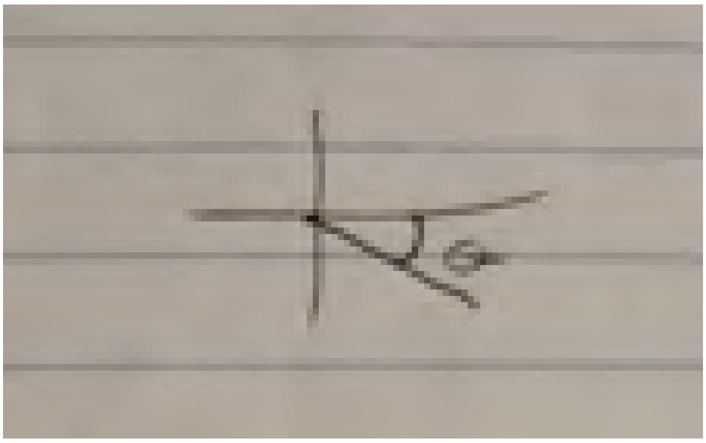

Figure 4.7: Angle of depression (Group 2)

B_{2b}: Ok let's go (heading the grand stairs). Angle of depression is this one (pointing at the angle between the stairs and the stick that he used as a learning aid).

Figure 4.8: Angle of depression (Group 2)

*B_{2a}: Look, let's say … (drawing on the page the sketch to represent the angle of depression and elevation in such a way that they form a Z-shape), angle of depression is below the x-axis akere (**right**?)?*
B_{2b}: Yes
*B_{2a}: Angle of elevation ke (**is**) this angle (pointing at the drawing on the page) that is above the x-axis, because angle of elevation ya (**it elevates**) eleveita eya hodimo (**it goes up**), ena e tsang? Depression e ya fatshe (**goes down**), you see?*
*B_{2b}: Ya (**yes**)*
B_{2b}: So that the line you are talking about is parallel to the horizontal surface.

B_{2b}: Employ the angle of elevation since angle of elevation and depression are alternate angles.

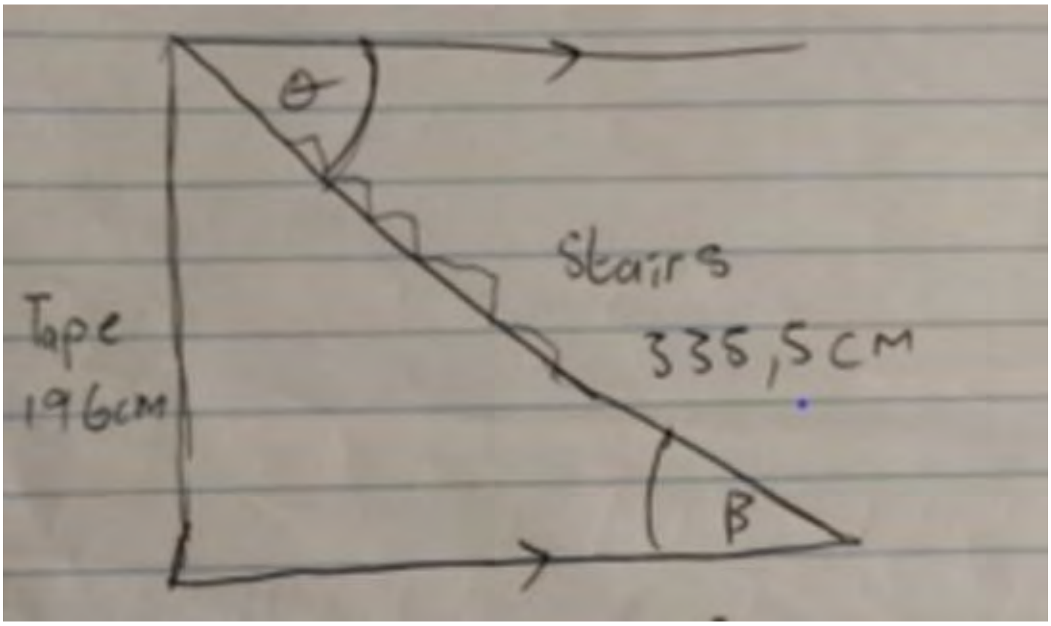

Figure 4.9: Angle of elevation and depression (Group 2)

B_{2a}: So, our beta is 35,7 degrees.
B1: checking the answer on the MCM app. They were happy to get the positive feedback they also checked the sample solution despite the fact that they got it correct.

The data presented in Excerpt 1 and in Figures 4.7, 4.8 and 4.9 above showed that learners used different representations, ranging from verbal representation, as B_{2a} said: ‘In simple terms angle of depression is the angle below x-axis, to diagrams’ (external representation) as shown in Figure 4.8. Learners have been able to successfully solve the tasks because they used different representations, one of the crucial elements of conceptual understanding. The data also indicated that mathematics trails necessitated learners using various representations. Learners used their smartphones to get feedback and checked the sample solution on the MCM app. B_{2a} said, ‘So, our beta is 35,7 degrees.’ B_{2b}: checking the answer in the MCM app. ‘They were happy to get the positive feedback and also checked the sample solution.’ Thus, smartphones came in handy in this learning situation of trigonometry concepts.

Excerpt 2: Group 3’s discussions while walking the trail (roof and the wall) (See Section 3.6.1).

B_{3b}: We calculate the bricks (reading the hints) like this (demonstrating) or bona ***(look)*** *if ever we calculate this (pointing at the bricks that forms the horizontal line of the wall), we going to get the base of that triangle over there (pointing at the top part of the wall, there was no triangle but he was talking his imagination).*

B_{3a}:(reading the hint) imagine two congruent right-angled triangles count and measure bricks that will form height of the two triangles. Take note of the cement in between.

B_{3a}: (took a page and started drawing right-angled triangles as per the hint)

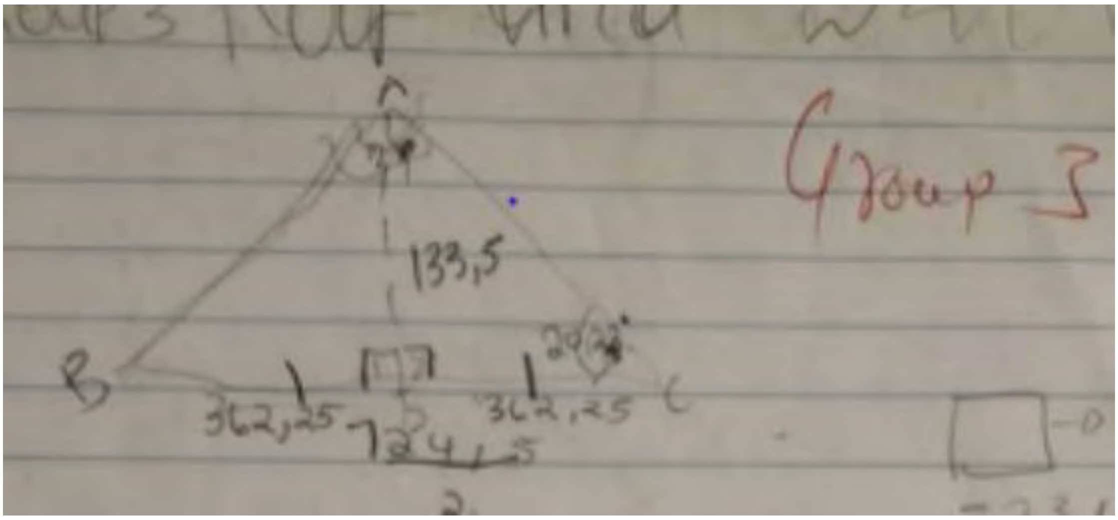

Figure 4.10: Two congruent right-angled triangles (Group 3)

The data presented in Excerpt 2 and Figure 4.10 above prompted the assumption that the MCM app hint prompted learners first to imagine congruent right-angled triangles (internal representation). B_{3a} read that (reading the hint) 'imagine two congruent right-angled triangles', then drew the triangles on the page as indicated in Figure 4.10. Figure 4.10 above shows the pictorial representation of the imagined trigonometry concepts of congruent triangles but in a right-angled representation. This data implied that learners could represent trigonometry concepts in different images using the smartphone, from internal to pictorial representation (see Excerpt 2 and Figure 4.10 above). They completed the task, hence G_3: 'Checked' (MCM app) B_{3b}: 'Well done' (answer from the MCM app). Conceptual understanding has many components, and multiple representations are one of the salient factors. As a result of the explanations and deliberations, Group 3 managed to represent the trigonometry ratio (tangent of an angle) verbally and in symbolic-numerical representation (see the following quotation from mathematics trail observations below and Figure 4.11).

*G_3: rena le opposite rena le adjacent (*We have opposite side and adjacent side)

B_{3a}: yes.

G_3 why re sebedisa tan theta? oho eya re batla angle, therefore, tangent theta is equal to opposite over adjacent (doing calculations). (Why are we to use tangent theta? Oh! I get it, we need to calculate the angle)

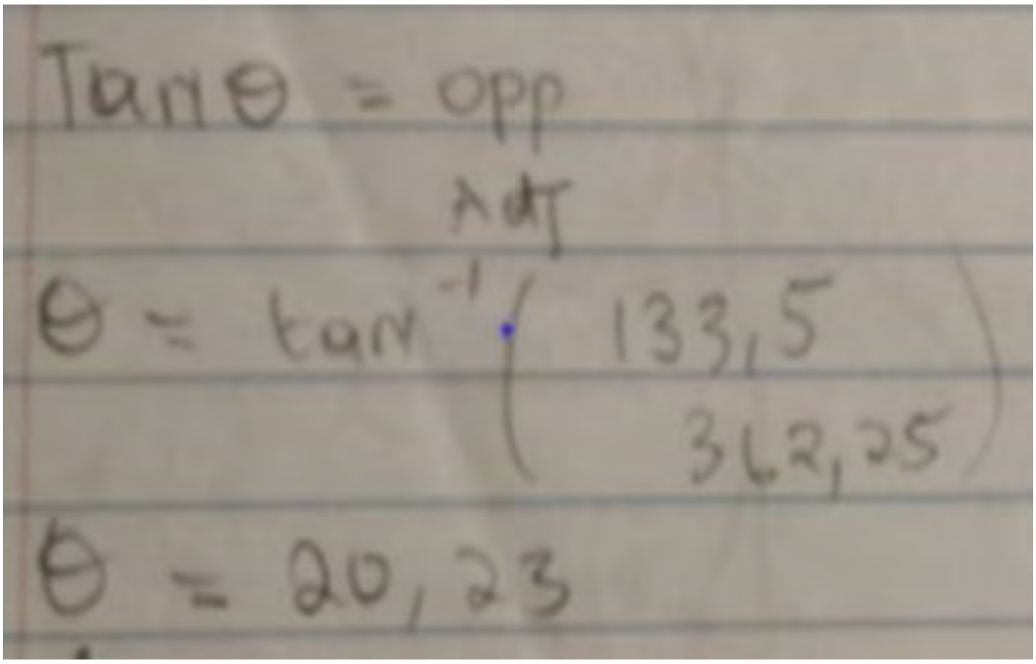

Figure 4.11: Trigonometry ratio (Group 3)

Mathematics trails using smartphones assisted learners in applying their ability to represent trigonometry concept in multiple ways as shown in Excerpts 1–2 and in Figures 4.7–4.11 above.

Excerpt 3: Group 4's responses mainly on drainage pipes task and (roof and the wall)

> *G_4: We made use of triangles (drainage pipe task), for example. In that way, we were able to utilise trigonometry in different ways, and also helped us to use trigonometric ratios, and we better understood how we can utilise them in different situations.*

Excerpt 4: Group 1's responses mainly on the school flagpole.

> *G_{1a}: ... is like most of the time we were drawing the shapes, according to what we read from the smartphone, saw, and measured. Yes, we also used equations.*
> *B_{3a}: we were able to utilise trigonometry in different ways and also get a better understanding on how we can utilise them in different situations.*

The data from the extracts above demonstrated that the participants used different mathematical representations, such as triangles (pictorial) and trigonometry ratios/equations (symbolic-numerical representations) to represent the same trigonometry concept in different ways (see G_4 and G_{1a} Excerpts 3 and 4). Although they could not be as explicit in their explanations as a teacher or more experienced individual, they confirmed the use of multiple representations of trigonometry concepts and were aware of the benefits. Hence, B_{3a} said, 'we were able to utilise trigonometry in different ways and also got a better understanding on how we can utilise them in different situations'. They recognised and acknowledged the benefit of using various representations, which they cited as a better understanding of mathematical concepts (see G_4 Excerpt 3). B_4 applauded the mathematics trails' ingenuity in arranging tasks that are different (different cognitive levels) and applied to multiple representations. B_{3a} said, (reading the hint) 'imagine two congruent right-angled triangles' (internal to/and verbal representation); the wall and roof task prompted learners to verbalise and then imagine the same trigonometry concept. After the internal representation, the same concept was depicted on a page B_{3a}: (took a page and started drawing right-angled triangles as per the hint) (see Figure 4.10 in Excerpt 2 above). The tasks on the MCM app came with stepped-up hints that assisted learners to represent the trigonometry concept in different ways and multiple representation is embedded in conceptual understanding. The mathematics trails allowed them to see and use multiple representations for deeper understanding (see Excerpt 3). Thus, the different representations were key in these occurrences. B_{3a} explained that 'we were able to utilise trigonometry in different ways and also get a better understanding on how we can utilise them in different situations'.

4.3.3 Connecting concepts and ideas in mathematics

Identifying and making connections between mathematical ideas is vital as it will make mathematics learning meaningful. It is a crucial point of conceptual understanding. However, one way to achieve this goal may be through mathematics trails. In this section, the data is presented and analysed under the sub-themes that follow.

4.3.4 Connection between mathematical ideas

Excerpt 1: Group 4's discussions while walking the trail. (School flag pole) (See Section 3.6.1)

> *G_4: Calculate the height of the school flagpole in metres (reading from the MCM app)*
> *B_{4b}: So, is it proportionality or similarity of triangles first?*
> *B_{4a}: Ke **(it's)** similarity ebe o tloba (and then) proportionality (writing down on the page)*
> *B_{4b}: So, we have calculated the height and the breadth of the stick so now we are going to for the breadth of the flag pole.*
> *G_4: We are cross-multiplying… then we divide both sides, the reason we divide both sides, we want to make AB subject of the formula.*
> *B_{4a}: AB = (pressing the calculator) …= 830, 91 cm*
> *B_{4b}: So, we will convert it to metres, using King Harry Died Miserable Death Called Measles (one, zero, zero, showing group mates on the page) so we are going to divide by 100.*
> *B_{4a}: check (now checking the MCM app)*
> *B_{4b}: We are right*
> *B_{4a}: Now let's check the sample solution*

According to the data in Excerpt 1, learners recognised and identified a link between similar right-angled triangles. B_{4b} asked fellow group members, 'So, is it proportionality or similarity of triangles first?' and B_{4a} replied, 'Ke (it's) similarity ebe o tloba (and then) proportionality'. This implied that the learners understood that two pairs of corresponding angles of the two triangles are equal, and the third pair is through the sum of interior angles of a triangle. They employed Euclidian geometry concepts, hence the similarity of triangles. Thereafter, they equated the ratios of the corresponding sides (Proportionality) and then linked algebraic equations (see Figures 4.12 and 4.13 below).

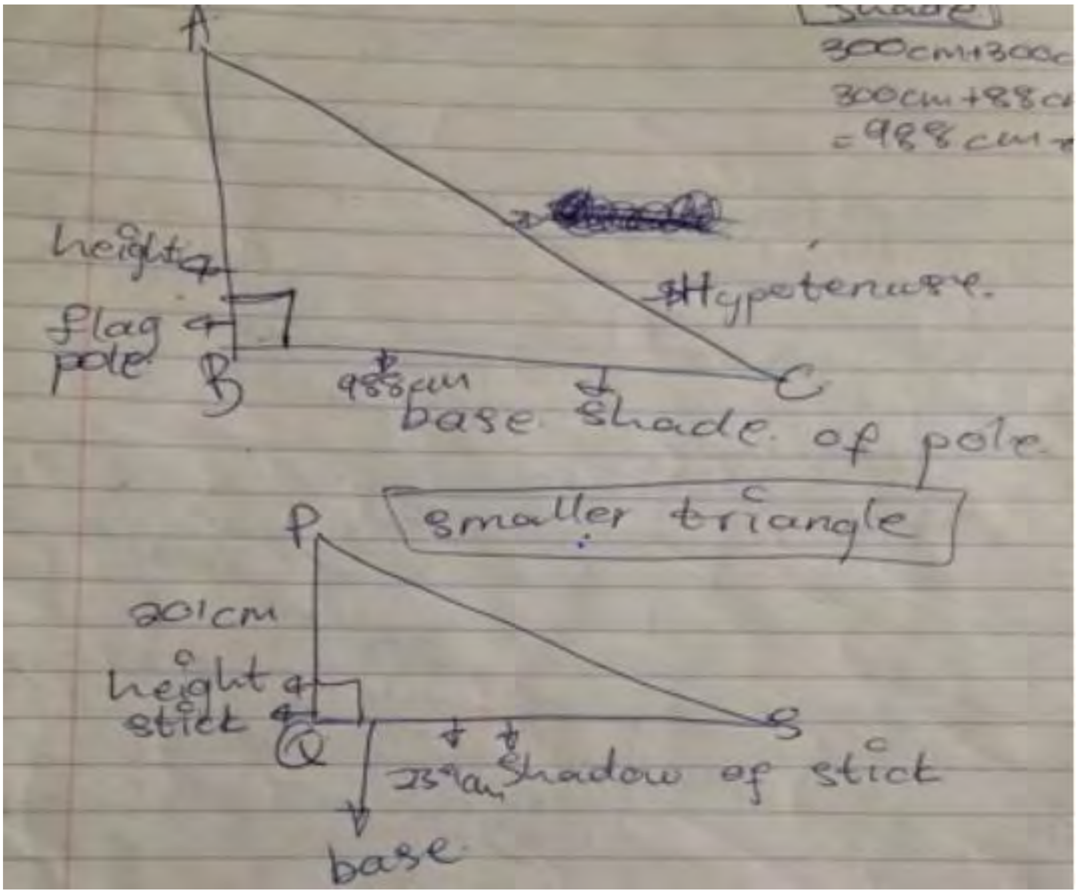

Figure 4.12: Similar right-angled triangle (Group 4)

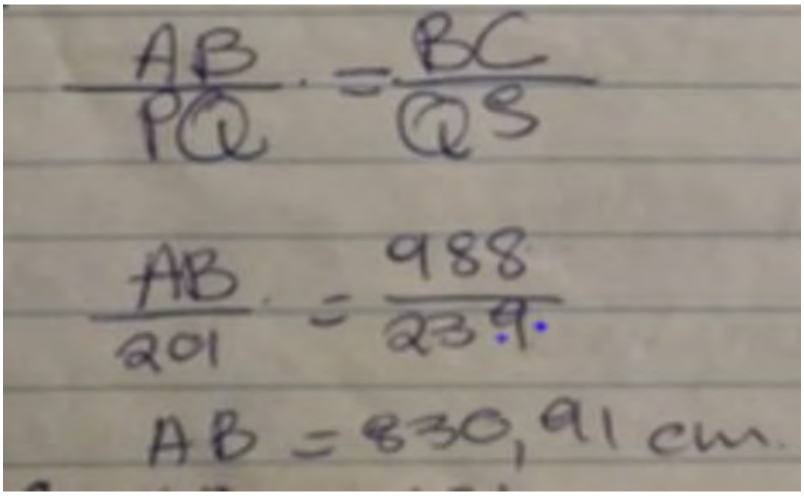

Figure 4.13: Proportionality and algebraic equation (Group 4)

The data in Excerpt 1 and Figures 4.12 and 4.13 demonstrated that mathematics trail enabled learners to make suitable connections within mathematics domains. G_4: 'We are cross-multiplying… then we divide both sides, we want to make AB subject of the formula.' Learners also made use of conversions, as noted by B_{4b}: 'So, we will convert it to metres, using King Harry Died Miserable Death Called Measles (one, zero, zero, showing group mates on the page) so we are going to divide by 100.' Considering mathematics as a subject, conversions is a topic on its own. They used the smartphone for feedback and to check the sample solution; they got feedback that they successfully solved the outdoor task (see B_{4b} in Excerpt 1). This implied that the outdoor task (mathematics trail) persuaded learners to recognise and identify appropriate connections and apply suitable procedures to solve the task effectively.

Excerpt 2: Group 1's discussions while walking the trail (school grandstand) (See Section 3.6.1)

> *G_{1a}: So, like question e re (says) calculate the slope of the grandstand in degrees give your answer in one decimal place, e re ke shebe (let me check the hint) hint number one: measure the horizontal distance of the wall.*
>
> *B_1: Are we supposed to draw a triangle or what?*
>
> *G_{1a}: Hint 3 says, use appropriate trigonometry ratio.*
>
> *B_1: Therefore, they are telling us that we use trigonometry ratios so therefore we can have a right-angled triangle.*
>
> *G_{1b}: Which trigonometry ratio are we going to use?*
>
> *B_1: Tangent*
>
> *G_{1a}: Yes, this is opposite over adjacent, then we have tan theta which is = Y over X. (writing)*
>
> *G_{1b}: 32.4 degrees. (They checked the answer on MCM app)*
>
> *B_1: Well done (the results from MCM app)*

The data in Excerpt 2 indicated that mathematics trails presented opportunities for learners to recognise and use links to solve the task on site. G_{1a}: 'Hint 3 says, use appropriate trigonometry ratio.' They were stuck, meaning they did not know what to do but the smartphone hints gave them a nudge in the right direction (see B_1 and G_{1a} in Excerpt 2). B_1 said, interpreting the hint 'Therefore, they are telling us that we need to use trigonometry ratios. So, therefore we can have a right-angled triangle'. This means they realised the mathematical relationships between the trigonometry ratios and right-angled triangles. G_1 asked which trigonometry ratio to employ and B_1 replied by saying tangent, resulting in the algebraic equation in Figure 4.8. The competence to make proper connections shows conceptual understanding. As noted by B_{2b}

from Group 2, 'Employ the angle of elevation since angle of elevation and depression are alternate angles.' It implies that the mathematics trail tasks prompted them to employ geometry concepts in this case.

Excerpt 3: Group 2's responses to the school billboard task.

> *B_{2b}: Yes, let me just take for instance there was this, I think it was task two where we had actually to use different trigonometry ratios in order for us to get the very same answer so that we can actually be accurate.*

Excerpt 4: Group 3's responses to the drainage pipes task.

> *G_3: Sir, the drainage pipes… the teacher did not teach us the area rule, cosine rule, and sine rule, but the trigonometry outside the classroom(mathematics trails) taught us the cosine rule, area rule, and sine rule. It will be very simple for us to apply in any situation.*

Excerpts 3 and 4 demonstrate that connections were made between mathematical ideas (see Excerpt 3). B_{2b} noted that they wanted to verify their views, so they used all three trigonometry ratios and got the same answer (sine x, cosine x and tangent x). Their ability to identify the relevant relationships prompted by the mathematics trail demonstrates their conceptual understanding and vertical mathematisation because they used different trigonometry ratios to confirm their answer, e.g., G_3 observed how they could understand and apply some rules they had not been taught in class. They could learn through the mathematics trail, with the task allowing them to make the appropriate connections and solve the tasks using the MCM app hints and the internet. However, learners had to analyse the real tasks and represent these informally, converting them to mathematical problems through the models, meaning they tapped into horizontal mathematisation. Hence, they successfully completed the task.

4.3.5 Link between mathematics and other subjects

Excerpt 1: Group 4's discussions while walking the trail (school flagpole) (See Section 3.6.1).

> *G_4: We are cross-multiplying… then we divide both sides by 163.5. The reason we divide both sides by 163.5 we want to make AB subject of the formula.*

B_{4b}: So, we will convert it to metres, using King Harry Died Miserable Death Called Measles (one, zero, zero, showing group mates on the page) so we are going to divide by 100.

Excerpt 2: Group 1's discussions while walking the trail (school grandstand) (See Section 3.6.1)

G_{1b}: The slope in degrees? Meaning we are calculating the angle. Are we not going to use theorem of Pythagoras?
B_{2b}: Employ the angle of elevation since angle of elevation and depression are alternate angles.

The participants' subject combinations included physical sciences and geography. The expertise or knowledge created or gained from the mathematics trails will be useful in those subjects. In Excerpt 1, G_4 applied the multiplicative inverse concept to make AB the subject of the formula. In physical sciences, e.g., this skill or knowledge is required and G_4 suggested a suitable mathematical procedure for the formula. Participants learned these skills in algebra. B_{4b} noted that the task required conversion expertise, cm to be converted to m. B_{4b}: 'So, we will convert it to metres.' This kind of connection also applies to geography in map work activities. The concept of slope is also familiar in physical science (see G_{1b} in Excerpt 2). Almost all subjects at school require some element of mathematics, like real-life situations (mathematics trails have supported this).

4.3.6 Extending mathematical concepts to real-life contexts

Relating mathematical concepts or ideas to everyday situations enables learners to recognise the practical value of mathematics learned in a classroom and improve their conceptual understanding. The following extracts, analysis and interpretation endorse this.

According to the data presented in Excerpt 1 (in Section 4.3.4), the participants were required to calculate the height of the school flagpole. The participants had to identify the relevant mathematical concepts to solve the real-world problem (mathematics trail) by employing appropriate connections. As a result, B_4 had to draw similar triangles to represent the school flagpole height as one of the sides of the similar right-angled triangles (Figure 4.12). However,

G_4 applied algebra to calculate the required side of the triangle, representing the school flagpole's actual height (Figure 4.13).

The mathematics trail (school grandstand) was solved mathematically by participants. They used the appropriate mathematical links by first establishing the right-angled triangle and the suitable trigonometry ratio to calculate the slope in degrees (see G_{1a}, B_1 and G_{1b} in Excerpt 2 in Section 4.3.4). The same happened with the school billboard task. Below are the extracts from group 2 mathematics trail (school billboard) observations.

> *B_{2b}: ...yes is 90 degrees because the base is horizontal and the pole is vertical.*
> *G_2: So, guys re tlo etsang? (so guys what are we going to do?) hint number 1 ya rona ere re batle angle of elevation (our first hint says we need to look for angle of elevation)*
> *B_{2b}: angle of elevation (He drew right-angled triangle on the page)*

The data on the school billboard above indicated that learners had discussions and were actively involved. Thus, proper mathematical connections were used for the task. All the mathematics trail tasks allowed learners to use informal means to solve the task, e.g., using a stick and converting the informal knowledge to formal mathematical knowledge (horizontal and vertical mathematisation). According to the data, learners realised that there was an angle of 90 degrees between the horizontal ground and the vertical pole. They established a right-angled triangle and used the proper trigonometry ratio to determine the angle of elevation. This shows that mathematics trails using a smartphone encouraged learners to relate mathematical concepts to real-life objects. The hints were useful – G_2 (our first hint says angle of elevation) and the mathematics trail task hinted that we should 'assume the ground is horizontal'. The task mentioned that the participants should assume the ground was horizontal (see school billboard task in Chapter 3). That information motivated learners to remember the correct concepts, in this case, a 90-degree and right-angled triangle. The learners recognised the relevance of mathematical concepts learned in the classroom and that these can be applied to real-life situations. Therefore, the MCM app made it possible for learners to relate mathematical concepts to real life.

4.3.7 Learners' excerpts from the mathematics trail observations (See Section 3.6.1)

B_{3a}: Ho chong re tlo qala ka base yane (that means we are going to start with the base) (meaning the top length of the wall) mejara (take measurements) (pointing at the horizontal part of the wall) kapo? eya, base ena e le kana le yane base e mona e le kana le yane (yes, this base is equal to that other base) (pointing at two opposite parallel sides of a rectangular wall saying those sides are equal and parallel),because this part of the wall forms a rectangle. (roof and the wall task)

G_2: Calculate the slope of the school grandstand in degrees.

B_2: slope, what is the slope?

G_2: (searching for the meaning from the smartphone internet and all listening to the voice from the smartphone.)

B_2: Oh ok, somehow ba batla re batle (they want us to look for) some sort of a gradient

G_2: Ne kere ke tjho jwalo (I share the same sentiments), I know the equation ya(of) gradient.

According to the data in Section4.3.7 above, learners recalled the properties of a rectangle after inspecting a certain part of the wall (see B_{3a}), i.e., they used their prior knowledge in this instance. Mathematics trails encouraged learners to connect mathematical concepts to real-world situations. The participants did not know what a slope was (school grandstand) B_2: 'slope, what is the slope?' The smartphone assisted them in finding the meaning (see G_2 and B_2 in Section 4.3.7) and after that, B_2 said, 'Oh ok, somehow ba batla re batle (they want us to look for) some sort of a gradient'. They were able to identify the connection or relationship between slope and gradient. As a result, they used suitable equations and procedures to solve the task (see G_2 in Section 4.3.7). B_{2a}: 'In simple terms, angle of depression is the angle below x-axis, so when we talk about the angle of depression, we are referring to this' (drawing a horizontal line on the page for fellow group members to see) (school grand stairs task).

4.3.8 Learners' extracts

G_4: We made use of triangles, (on drainage pipe task) for example. In that way, we were able to utilise trigonometry in different ways to get a better understanding and how we can utilise them in different situations.

G_1: On task number one, school flagpole, we used similarity of triangles to find the height of the flag pole whereby we had to create a small triangle using a stick as well as the big triangle using a shadow (see Chapter 3, group 1).

The data (see Table 4.b) indicated that learners could make appropriate mathematical connections to solve real-world tasks effectively (see G_1 in Section 4.3.8) indicating greater conceptual understanding. G_4 said that, ‘relating mathematical ideas to the real world enhanced their understanding’.

Connecting trigonometry concepts to everyday life situations through the MCM app was successful. Because the participants indicated an integrated and functional grasp of mathematical concepts, they solved the outdoor tasks. The analysed data revealed that learners used prior knowledge to complete the tasks.

The data presented in excerpts or extracts (quotations) implied learners’ competence in recognising and making the necessary connections in the mathematics trail. It also demonstrated that learners understood what they were learning in the classroom during the mathematics trails because they used well-integrated mathematics connections. It further meant that participants had informative discussions that informed appropriate connections. Therefore, by inference, the mathematics trail using a smartphone could dismiss the notion that mathematics is just an abstract subject confined to the classroom without any significance to real-life situations. The learners’ work demonstrated horizontal and vertical mathematisation because all the tasks were attached to objects outside the classroom but had to be solved using trigonometry concepts in the main.

4.4 TRIGONOMETRY TASKS TIED TO REAL-WORLD OBJECTS

Mathematics is an activity where learners actively construct knowledge. The RME further states that mathematics must connect to reality, close to learners rather than teacher-centred and relevant to society, to remain valuable. Mathematics trails using smartphones made it possible. The analysis of excerpts and interviews supported this.

4.4.1 Guided reinvention

Excerpt 1: Group 1’s discussion during Billboard Task (See Section 3.6.1)

> *B_{1a}: Re tlo sebedisang? (What are we going to use?).*
> *G_1: Angle e mona yona mara keng? (What is the size of this angle here(pointing)?)*
> *B_{1a}: Ke 90 (is 90 degrees).*

B_{1b}: Yes, is 90 degrees because the base is horizontal.

G_1: So, guys re tlo etsang? (So, guys what are we going to do? Meaning how are we going to solve this task?) Hint number 1 ya rona ere re batle angle of elevation (hint number one says we need to calculate angle of elevation)

B_{1b}: Angle of elevation. (B_{1b}: drew right-angled triangle on the page).

The learners identified the relevant mathematical concepts (see Excerpt 1 above). In this case, they ascertained the 90-degree angle between the vertical pole and the horizontal ground. B_{1b} said, 'Yes, is 90 degrees because the base is horizontal' and drew a right-angled triangle to employ a suitable trigonometry ratio to find the size of the angle of elevation (see B_{1b}, B_{1a} and G_1 Excerpt 1). They were guided by smartphones through hints, as noted by G_1, 'Hint number 1 ya rona ere re batle angle of elevation (hint number one says we need to calculate angle of elevation).' In Excerpt 1 above, the mathematics trail task indicated that participants should assume that the ground is horizontal (Chapter 3). This meant that mathematics trails were the source of mathematical concepts to be reinvented. They were initially confused, but the mathematics trail and smartphone assisted with a hint, a role that would normally be that of the teacher in a classroom. The hint unlocked suitable mathematical ideas or concepts to be utilised. G_1 said, 'trigonometry ratios or maybe we use sides first?', and then B_{1b} replied, 'looking for an angle of a triangle we have to go for sides first before we can have the angle, which is tangent to the power negative one'. Learners demonstrated their independence by deciding which trigonometry ratios and procedures without the teacher's help. The data suggests that the mathematics trail allowed learners to identify relevant mathematical concepts and ideas. It can be presumed that an integrated and functional grasp of mathematical concepts was enhanced.

Excerpt 2: Group 3 discussion during the Flagpole task (See Section 3.6.1)

G_3: Calculate the height of the school flagpole in metres

B_{3a}: In metres? Now we have to convert cm into m.

B_{3b}: Yes, because right now it's in cm.

G_3: Kana re etsang? (What must we do again?)

B_{3b}: Do we divide or multiply?

B_{3a}: We divide by 100.

B_{3a}: It says not perfect but ok. (Feedback from MCM app)

B_{3b}: Kana hao sheba sample solution o etsang? (How do you check the sample solution? On the MCM app).

The data in Excerpt 2 indicated that the mathematics trail allowed participants to recreate relevant trigonometry ratios and procedures to apply to calculate the height of the school flagpole. G_3 read the task from the MCM app, 'Calculate the height of the school flagpole in metres'. The task required the answer in metres but the measurements were in centimetres. B_{3a}: In metres? Now, we have to convert cm into m.' and B_{3b} answered, 'Yes, because right now it's in cm.' Learners discussed it, seeking clarity. G_3: 'Kana re etsang? (What must we do again?)' B_{3a}: 'We divide by 100.' This meant they found the answers to the questions, ascertained the relevant mathematical ideas, and solved the real-world task mathematically, as shown in the data (see B_{3b}, B_{3a} and G_3, Excerpt 2). B_{3a} checked the MCM app, 'It says not perfect but ok.' It implied that learners identified appropriate mathematical knowledge required to convert cm to m. Hence, they finished the task, and the smartphone guided them through sample solutions, to increase their conceptual understanding with alternative mathematical methods.

Excerpt 3: Group 3's responses mainly on the drainage pipes task (See Section 3.6.1).

G_3; Ya (Yes), different triangles, like right-angled triangle, isosceles we were taught in grade 10.
B_{3a}: Drainage pipes, when we were looking for an area of a triangle, we had to make use of the area rule.

Excerpt4: Group 2's responses on drainage pipes and (roof and the wall) tasks**.**

B_{2b}: Task four we actually had to remember the properties of a rectangle.
G_2: …and task five too because we had to use different trigonometry ratios from the tasks we had done and hints.

During the observations, G_3 indicated they had to identify the proper triangle to solve the task. B_{3a} realised that the area rule was suitable to determine the area bound by the drainage pipes. Therefore, the outdoor mathematics tasks enabled learners to use their trigonometry knowledge and imagine mathematical shapes and develop relevant properties or apply appropriate mathematical calculations guided by the hints. B_{2b} said, 'in Task four, we actually had to remember the properties of rectangles'. In task five, learners used appropriate trigonometry ratios assisted by MCM app hints (see Excerpt 4). Therefore, smartphones (hints) guided learners into discovering mathematical ideas to solve outdoor tasks.

4.4.2 Learners' emergent or self-developed models

Excerpt 1: Group 3's discussions while walking the trail. (Roof and the wall) (See Section 3.6.1)

> *B_{3b}: Droya this one (draw this one) (meaning a triangle)*
> *B_{3a}: (taking a page and start drawing a right-angled triangle after depicting the wall and the roof)*
> *G_3: So, height ya rona ho tlameha re constructe (so we need to construct our height)*
> *G_3: E tloba (H is going to be our height) height (for height) ya rona (feeding the drawing) akere (am I right)*
> *B_{3a}: So where do we start?(checking hints)*
> *B_{3b}: Ho chong re tlo qala ka base yane (that means we are going to start with that base)pointing at the part of the wall (meaning the top length of the imagined rectangle) mejara (take measurements) (pointing to the horizontal wall) kapo? eya, base ena e le kana le yane base e mona e le kana le yane (yes, this base is equal to that other base) (pointing at two opposite parallel sides of a rectangular wall saying those sides are equal)*
> *G_3:Tangent theta = opposite over adjacent. AB over BC and then AB is 126… (doing calculations)*
> *B_{3a}: 19.17*
> *G_3: Check (MCM app)*
> *B_{3b}: Well done (answer from the MCM app).*
> *G_3: Ha re shebe (let us look at the) sample solution (they checked the sample solution against their answer and trying to make sense of the answers from the app)*

In Excerpt 1 above, G_3 asked B_{3a} to draw the right-angled triangle, i.e., transforming the real-world task into a mathematical model and they subsequently solved the real-world task using another appropriate mathematical model, trigonometry ratio or equation (see B_{3b}), and G_3 noted, 'tangent theta = opposite over adjacent. AB over BC, and then AB is…' (doing calculations). Figures 4.14 and 4.15 below show the right-angled triangle derived from the diagram of the (roof and wall); then the equation of the employed trigonometry ratio.

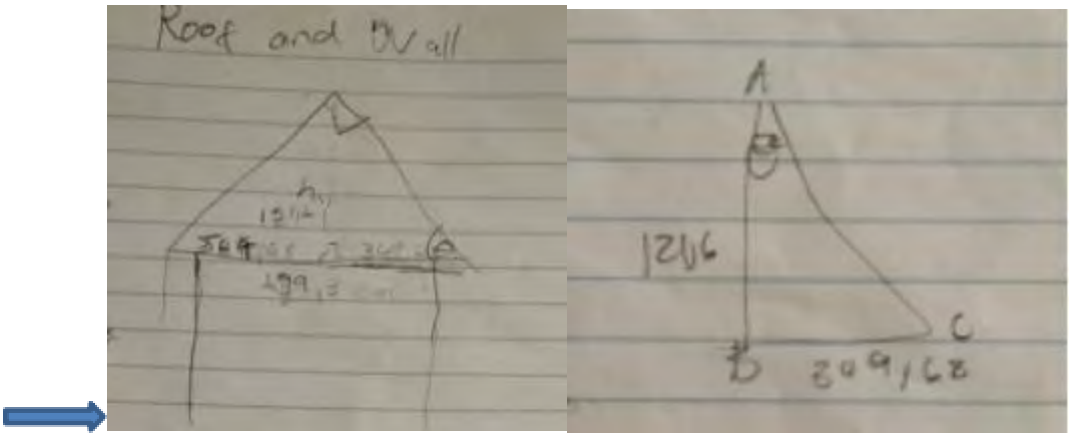

Figure 4.14: right-angled triangle (Group 3)

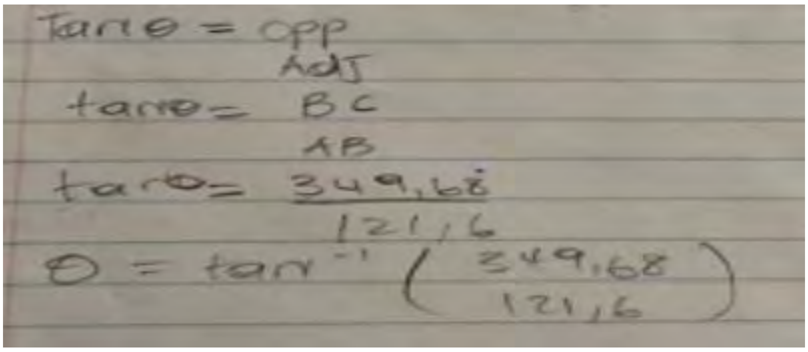

Figure 4.15 : Trigonometry equation (Group 3)

The MCM app on a smartphone had a crucial role (Figures 4.14 and 4.15) in guiding or assisting learners to use suitable mathematical models by providing hints. Hence, B_{3b} read from the MCM app:

> *Calculate the bricks (reading the hints) like this (demonstrating) or bona (look) if ever we calculate this (pointing at the bricks that form the horizontal line of the wall) we are going to get the base of that triangle (pointing at the top part of the wall).*

There was no triangle on the wall, but he imagined it, shared the idea with his group members and asked B_{3a} to draw the right-angled triangle on the page (see B_{3a} in Excerpt 1 and Figure 4.14 above). Participants developed relevant mathematical models on their own. They accessed the MCM app for feedback and samples solution to explore other appropriate models. G_3 said, 'Check MCM app'. The mathematics trail assisted participants in developing proper mathematical models that enabled them to solve the real-world task, implying an improved functional and integrated grasp of mathematical concepts.

Excerpt 2: Group 4's discussions while walking trail (Drainage pipes) (See Section 3.6.1)

> *B_{4a}:… wa bona lona hiso mistake e lo etseng hiso le itse dideng ke 90 degrees and hase 90 degrees (You see the mistake you committed here, is that you said there was an angle of 90 degrees here and yet, there was no 90 degree angle there) (taking the page correcting the triangle from right-angled to non-right-angled triangle, involving his group members, teaching them by demonstrating and showing them their mistakes.)*
>
> *B_{4a} :so re tlamehile re mejare ma side (so we have to measure the sides), wa tseba ho na lentho e ke e nahanneng re e bitsang (you know there is something I am thinking about something like a seven) something like seven(sharing his mnemonic device by using his fingers)seven, hiso re tlo user cosine rule(here we are going to use sine rule), so re tlo e bitsa(naming the triangle sides and angles) ke ABC di angle tsa rona re di ngola ka di capital letters neh so hiso re tlo ngola A. (our angles will be in capital letters and sides in small letters)*
>
> *B_{4b}: oh yane e re searchitseng (oh!the one we searched on the internet)*
>
> *B_{4a}: (yes). e kenye moo re bone (enter the answer on the MCM app)*
>
> *G_4: well done (feedback from MCM app)*

The data in Excerpt 2 above was from Group 4 participants who understood that B_{4a} corrected anothers' mistake of using the wrong mathematical model (right-angled triangle) in their attempt to transform the realistic problem into a mathematical problem (mathematisation). B_{4a} said:

> *You see the mistake you committed here, is that you said there was an angle of 90 degrees here and yet there was no 90 degree angle there (taking the page correcting the triangle from right-angled to non-right-angled triangle, involving his group members, teaching them by demonstrating and showing them their mistakes).*

The correct (non-right-angled) triangle is shown in Figure 4.16 below. Two members indicated they had never learned these trigonometry concepts (sine, cosine and area rule). Hence, they searched on the internet to confirm what B_{4a} shared with them indicated in Excerpt 2 above, B_{4b} mentioned, 'oh yane e re searchitseng' (oh! the one we searched on the internet). The MCM app helped them understand that the cosine rule was used in a non-right-angled triangle and confirmed the cosine rule formula (Figure 4.17). They then continued with the task confidently, using a non-right-angled triangle as a model to translate the outdoor task into a mathematical task. As a result, the task was solved, and the group received positive feedback from the MCM app. The hints played a significant part in assisting learners to identify the appropriate mathematical concepts (see Chapter 3).

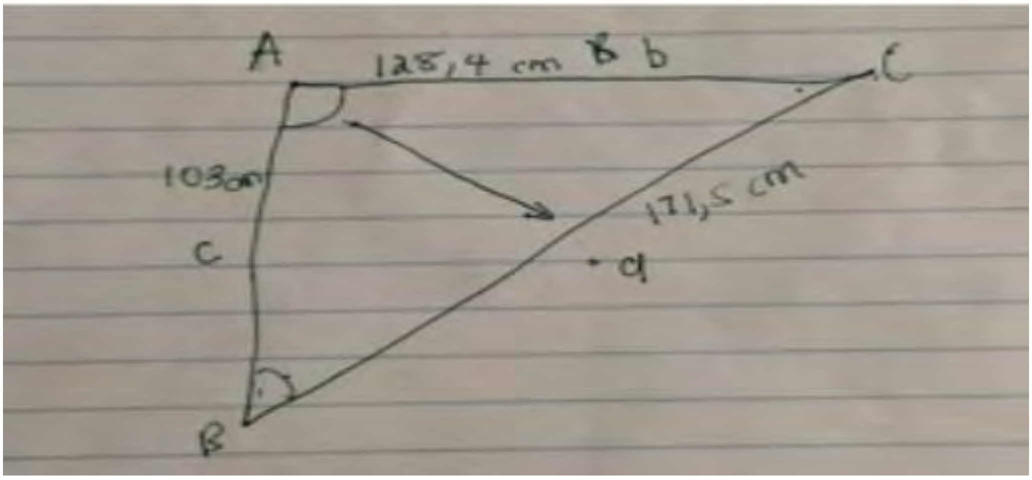

Figure 4.16: Non-right-angled triangle(Group4)

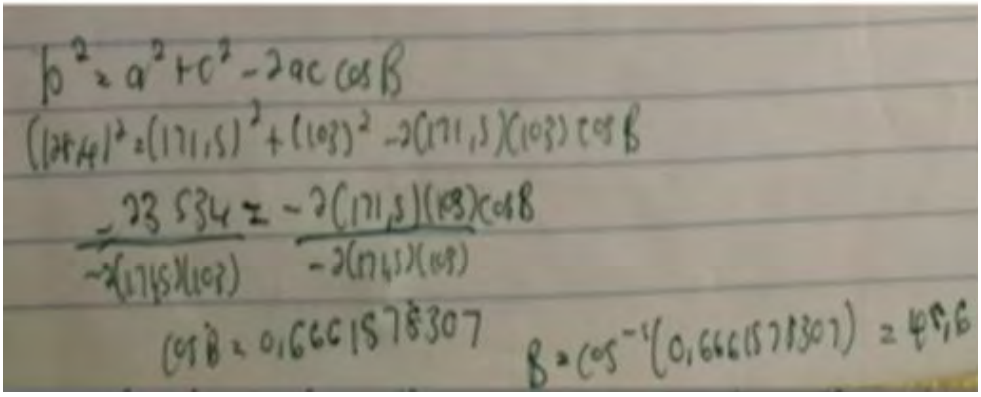

Figure 4.17: Cosine rule formula (Group 4)

The data indicates that mathematics trails allowed learners to develop and use appropriate mathematical models (see Figures 4.16 and 4.17). This process was essential to bridge the gap between informal and formal knowledge to enable the transformation from real-world task to a mathematical task (see Excerpt 2 above). Hence, the task was solved mathematically using the triangle and the trigonometry equation, as demonstrated in Figures 4.9 and 4.10. These models were developed by the learners without any help from a textbook or teacher, but they used the hints from the MCM app.

Group 2 drew appropriate mathematical models on the roof and wall task (Figure 4.18).

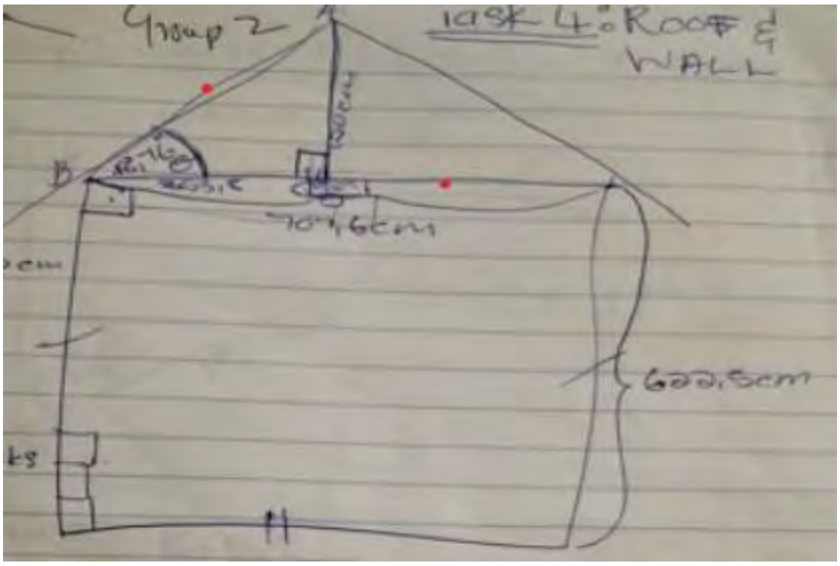

Figure 4.18: right-angle triangle and rectangle (Group 2)

The above also endorses emergent or self-developed models because the participants were confronted with the wall and roof without explicit or visible mathematical models (see Figure 4.19). The learners had to develop the relevant mathematical models (Figure 4.9) to transform the real-world task and solve it mathematically. They had informative discussions and asked questions when they needed clarity. Learners took charge of their learning process and relied on their ingenuity and the mathematics trail hints using a smartphone and when they confronted an issue, they made use of hints or Google search (Sections 4.3.1–4.3.3).

Figure 4.19: school wall and roof

Based on the data in Excerpt 1 and 2, learners identified and developed suitable mathematical models and transformed the realistic problem to a mathematical problem. As B_{4b} checked MCM app 'Well done!' (the results from the MCM app, Excerpt 2). This implied that mathematical ideas and concepts were not just learned to pass a test or exam but to solve real-world problems. Therefore, this task using a smartphone, created a platform for learners to express their mathematical knowledge, which enhanced conceptual understanding because they managed to develop appropriate mathematical models to solve the task.

4.4.3 Learners' quotations during mathematics trails observations (self-developed models)

> *B_{2a}: In simple terms angle of depression is the angle below x-axis, so when we talk about the angle of depression we are referring to this (drawing a horizontal line on the page for fellow group members to see) angle that's the angle of the depression (by drawing a horizontal line)* (see 4.3.2., Excerpt 2)
> *G_4: Then we divide both sides by 163.5, the reason we divide both sides by 163. we want to make AB subject of the formula.* (see 4.3.3., Excerpt 1)

According to the data in the quotation above, B_{2a} explained and described an angle of depression and drew a horizontal line (model) and said, 'it (angle of depression) is the angle from horizontal line down'. Therefore, he developed the horizontal axis of a Cartesian or rectangular plane as a model independently (Section 4.3.2, Excerpt 1, Figure 4.7). The mathematics trail triggered helpful discussions among learners and they took responsibility for their own learning. B_{2a} also said, 'Look, let's say' (drawing on the page the sketch to represent the angle of depression and elevation in such a way that they form a Z-shape). He illustrated that the two angles formed a Z-shape, meaning they are alternate angles (Section 4.3.2, Excerpt 1, Figure 4.9). B_{2b} applied the algebraic equation (trigonometry ratio) as another model to calculate the required answer to the question/task (Section 4.3.2, Excerpt 2, Figure 4.11). Therefore, the mathematics trails were vital in encouraging learners to develop appropriate mathematical models of their own accord to solve a real-world task mathematically.

The excerpts demonstrated that learners defended their thinking through mathematical concepts and self-developed models (see B_{2a}). For example, G_4 explained her reasons for applying the procedure. There was good communication among the group during mathematics trails as they developed the suitable models.

Figure 4.20: Learners constructing mathematical models

In Figure 4.20, the mathematics trails using smartphones prompted participants to personalise learning and bring an element of accountability to their learning. Hence, they created models themselves using pen and page. Thus, when the learner finishes secondary school, they can confront the real-world problems confidently and use mathematical models and concepts to make conclusions and defend their actions. Extending mathematical concepts to everyday life situations using a smartphone was not just for learners to actively engage in mathematics trails but also to learn the use of mathematical models and its advantages. This enabled the transition between real-world tasks and mathematical tasks because learners have a unique zone of proximal development. The learners learned by reasoning, justifying their answers and most of the time, using correct mathematical language.

4.4.4 Use of interaction

Knowledge is constructed through active involvement among or between people and objects. The following sub-headings describe the interactions that occurred during trailblazing by the learners:

Excerpt 1: Group 4's discussions while walking the trail (school flagpole) (See Section 3.6.1).

> *G_4:..calculate the height of the school flag pole in metres(reading the task from the MCM app)*
> *B_{4a}: Firstly, we will start by sketching our similar triangles (drawing on the page, big and small right-angled triangles)*
> *B_{4b}: So, we have calculated the height and the breadth of the stick so now we are going to for the breadth of the flag pole*
> *B_{4b}: So, is it proportionality or similarity first?*
> *B_{4a}: Ke (**it's**) similarity ebe o tloba (and then) proportionality (writing down on the page)*
> *G_4: We are cross-multiplying…*
> *B_{4b}: AB multiply by 163.5= 200.5 multiply by 724 cm*
> *G_4: then we divide both sides by 163.5, the reason we divide both sides by 163.5 we want to make AB subject of the formula.*
> *B_{4a}: AB = (pressing the calculator) …= 887, 84 cm*
> *B_{4b}: So we will convert it to metres, using King Harry Died Miserable Death Called Measles (one, zero, zero, showing group mates on the page) so we are going to divide by 100.*
> *B_{4a}: check (now checking the MCM app)*
> *B_{4b}: we are right*
> *B_{4a}: now let's check the sample solution, we are good.*

4.4.5 Interaction between learners and learning tools (smartphones and real-world objects)

The interaction of participants with smartphones was evident in that learners managed to locate the tasks through the navigation system, and when on site, they could read questions or tasks from the smartphone without any confusion; G_4 read the task as follows, 'Calculate the height of the school flagpole in metres (reading the task from the smartphone)'. Subsequently, participants cooperated on the task, as shown in Figure 4.21. They identified relevant mathematical models to solve the real-world problem (height of the school flagpole) mathematically (see B_{4a} and B_{4b}, Excerpt 1). In support of the latter statement, B_{4a} suggested,' 'firstly, we will start by sketching our similar triangles' (drawing on big and small right-angled triangles). B_{4a} further contributed mathematical ideas, 'Ke (it's) similarity ebe o tloba (first and then) proportionality', meaning that they needed to make sure or prove that the triangles are similar. First, before investigating the proportionality of sides – one side represented the height of the school flagpole (real-world object), most participants accessed hints from the smartphone for guidance. In their interaction, G_4 also made her submission of relevant mathematical procedure 'We cross-multiply… then we divide both sides by 163.5, the reason we divide both sides by 163.5 we want to make AB subject of the formula' and side AB is the height of the school flag (real-world object)(see Figure 4.22 below).

Figure 4.21: In action to calculate the height of the flag pole

The data in Excerpt 1 or the interactions presented improved conceptual understanding because participants made the appropriate mathematical connections. They created suitable models to transform real-world tasks to mathematical tasks, and how to do conversions, using their prior knowledge. The interaction was rewarding as it was a dynamic process compared to the passive classroom environment and participants realised the relevance of mathematics to real life. These were the benefits of using smartphones for learning as they helped learners with the real-world tasks. When they did not understand, they used the smartphone to search the for the meaning of the concept to enhance their mathematical understanding. G_3 said:

> *Sir, the drainage pipes task ... the teacher did not teach us the area rule, cosine rule and sine rule, but the trigonometry outside the classroom using a smartphone taught us the cosine rule, area rule and sine rule, it will be very simple for us to apply in any situation,' and she further stated, 'Sir, we used smartphone to search for explanation and meaning on internet of things we did not understand. Angle of elevation on task two school billboard task, ke hore (I mean) sir, smartphone helped us to find explanation on internet.*

Therefore, it was not smartphones per se that contributed to improved conceptual understanding of the participants, but the smartphone was pivotal to the whole process. When learners needed to locate the task, they used the smartphone for navigation, to search the internet and access the MCM app.

4.4.6 Interaction between learners and MCM app

There were times when learners did not know what to do during trailblazing and had to rely on the hints from the MCM app. The hints did not remove the learners' authority and responsibility to do the tasks on their own. The following extracts are instructive: G_1 said, 'hint number 3 says that use appropriate trigonometry ratio in a right-angled triangle, press shift in a calculator to find the angle in degrees' and B_{1a} made his inference based on hint number 3 and said, 'therefore they indirectly told us we use trigonometry ratios, so therefore we can have a right-angled triangle'. These extracts from the trail observation indicated the role played by the MCM app since the learners had to consult it. The hints unlocked the proper mathematical models to solve the tasks and transform real problems to mathematics problem. The stepped-up hints assisted learners in finding appropriate models (right-angled triangle) and encouraged them to recall suitable trigonometry ratios.

Another hint explored by Group 2, B_{2a} said, 'the first hint I think is correct it says to use the knowledge of similarity of triangles, sides in proportion to calculate the height of the flag pole first,' and G_2 commented after the reading of the hint saying, 'that's why I said we must calculate the height of the flag pole first before so that we know the height before re tlo etsa' (we do the subtraction of) subtract the height of the observer'. The MCM app hints assisted learners to remember and identify proper mathematical concepts and models and triggered robust discussions among learners improving their conceptual understanding.

The MCM app provided tasks, prompt feedback and sample solutions (see B_{4a} in Excerpt 1). B_{4a} asked 'check' (now checking the MCM app) and B_{4b} responded 'we are right', and in Group 3, G_3 checked the MCM app for feedback. Positive feedback was important because it confirmed they used the relevant mathematical concepts and appropriate models. They checked the sample solutions against their answer and tried to make sense of the answers from the app. All the groups checked the sample solution of each task irrespective of the feedback. The sample solution could provide an alternative answer and the learners gained more knowledge and new skills.

Figure 4.22 below shows part of the interaction.

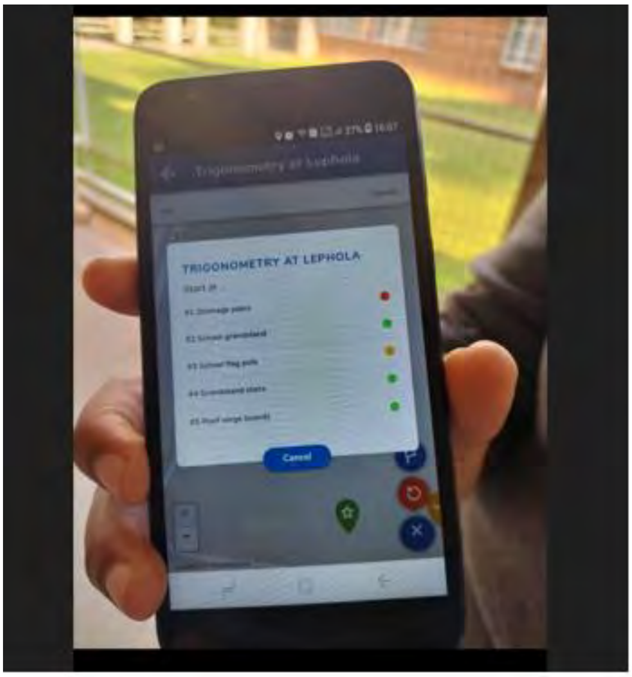

Figure 4.22: Interaction with the MCM app using a smartphone

4.4.7 Learners' interaction with each other

The interaction between learners triggered revealing discussions as shown in Figure 4.23 and in Group 4's interaction; B_{4a} remarked:

> *You see the mistake you committed here, is that you said there was an angle of 90 degrees here and yet there was no 90 degree angle there (taking the page correcting the triangle from right-angled to a non-right-angled triangle, involving his group members, teaching them by demonstrating and showing them their mistakes.).*

According to the group discussion, some learners misunderstood the task, leading to incorrect answers. However, B_{4a} realised the mistake and provided the solution so those who misunderstood learned the appropriate trigonometry rule for a non-right-angled triangle. Hence, G_4 said, 'but the trigonometry outside the classroom taught us the cosine rule, area rule and sine rule, it will be very simple for us to apply in any situation'. This data implied that learners' conceptual understanding was improved.

The deliberations went on, in other groups as well, and G_2 mentioned that:

> *the angle of depression is an angle between the horizontal line and the observation of the object from the horizontal line, did you get what I said?' and B_2 said 'no' However, B_2 intervened 'in simple terms angle of depression is the angle below x-axis, so when we talk about the angle of depression we are referring to this(drawing on the page for fellow group members to see).*

The data demonstrated cooperation and communication among the learners. Those who did not understand the concepts were assisted using concise examples and could correct their errors. G_{1b} asked 'Which trigonometry ratio are we going to use?' B_1 said, 'Tangent'. The data showed that learners could ask questions and actively took responsibility for their own learning.

Figure 4.23: Learners interacting during trailblazing

Excerpt 1: Groups' responses on tasks.

> *B_1: we were able to connect mathematical ideas because we could calculate things (required sides or angles).*
> *G_2: On task number two I mean task number one, a flagpole, we used similarity of triangles to find the height of the flagpole whereby we had to create a small triangle using a stick as well as the big triangle using a shadow.*

In the data presented in Excerpt 1 above, B_1 stated that, in their interactions in Group 1, they managed to develop relevant mathematical connections so they could calculate the required side or angle of a real object. This comment indicated a functional understanding of mathematical ideas since they had to make the appropriate mathematical links. G_2 said that they identified suitable mathematical models (similarity of right-angled triangles) to employ. This competence is crucial for transforming a realistic problem into a mathematical task transitioning between informal knowledge and formal mathematical concepts. The data implied that learner's conceptual understanding was improved.

Excerpt 2: Group 3's responses mainly on the drainage pipes task.

> *G_3: Sir is not the same as in classroom because ha o le (when) outside (I feel energetic, active and hands-on in my learning experience) wa e fila ntho ena after finding an angle o ka nna wa tswela pele waba tla (you continue to the next unfamiliar, interesting task) something, in a classroom o ngola feela (you write) just for the sake of writing ha nka qeta feela o sa tseba(without seeing the relevancy of the concepts in real life)…*
>
> *B_{3a}: …in other words, she is saying we become more energetic than in a classroom.*

The responses presented in Excerpt 2 demonstrated a learner-centred and active learning environment, as noted by G_3, 'Sir is not the same as in classroom because ha o le (when) outside'. The mathematics trail tasks created vibrant interactions compared to classroom situation where learners fail to see the relevance or mathematical concepts. Therefore, the learners actively constructed or reinvented mathematical ideas (see B_{3a} in Excerpts 2 and 1 above).

The data collected from Excerpts 1 and 2 demonstrated that learners developed appropriate mathematical models for transformations between the real-world and mathematics tasks using proper trigonometry ratios to solve the task (see Excerpt 1 above). The ability to make appropriate mathematical connections enhanced conceptual understanding. In Excerpt 2, G_3 implied that the teacher-centred and classroom environment deprived them of interactions characterised by discussions, and where learners to construct their own knowledge or reinvent mathematics ideas. This agrees with RME's main principles, i.e., a realistic mathematics approach has to allow learners to reinvent mathematical ideas and concepts with guidance (smartphone), and that mathematics trails were a source of mathematical concepts or formal mathematical knowledge. Participants recognised the necessary mathematical models and concepts needed to solve the outdoor tasks (see G_3 in Excerpt 2).

4.4.8 Summary

The process of connecting trigonometry concepts to everyday life situations through the MCM app could be inferred as a success while improving conceptual understanding from the analysed data. Because the participants indicated an integrated and functional grasp of mathematical concepts, they responded to the outdoor tasks and received positive responses from the MCM app. The data indicated that learners used their knowledge to complete the tasks and implied that learners had the competence to recognise and make the necessary links in the mathematics trail. It further demonstrated that learners understood better what they learned in the classroom during the mathematics trails. The participants had informative discussions and debates that informed appropriate decisions. By inference, the mathematics trail using a smartphone dismissed the notion that mathematics is an abstract subject, confined within the four walls of the classroom, without any relevance to real-life situations. The learners' work demonstrated horizontal and vertical mathematisation because all the tasks were attached to real objects outside the classroom but had to be solved using trigonometry concepts. Learners managed to develop appropriate mathematical models for transformation and proper trigonometry ratios to solve the task mathematically. The last-mentioned point is in line with one of the RME's main principles to say that realistic mathematics approach must allow learners to reinvent mathematical concepts with adult guidance (smartphone), and that mathematics trails were a source of mathematical ideas/concepts.

4.5 LEARNERS' PERCEPTIONS AND EXPERIENCES OF USING SMARTPHONES AFTER PARTICIPATION IN THE MCM APP PROJECT

The purpose of the focus group interviews was to answer the third research question: *What are the selected grade 11 mathematics learners' experiences and perceptions of using smartphones for learning trigonometric concepts, after participating in the MCM project?*

Learners' perceptions and experiences are presented in table form with the analysis following the tables.

4.5.1 Learners' appreciation of the use of smartphones

The learners in all groups recognised the importance of using smartphones in learning trigonometry and indicated that smartphones assisted them in understanding the tasks and how to locate the tasks in the real world.

Table 4.a shows some of the comments learners made supporting the importance of smartphones in learning.

Learner	Comments made by learners
B_{3b}	*Smartphones helped us to understand the content of the subject (mathematics) by providing hints on how to answer the questions*
G_1	*the smartphone navigated where to go to find all the tasks*
B_{2a}	*we used the smartphone to search for trigonometric functions/ratios*
G_4	*For instance, I did not understand application of trigonometry, in particular, how to use sine or cosine rules. However, a search on a smartphone regarding these topics helped to understand ntho tse re sa ditsebeng (things we did not know) like angle of elevation; nna e sale ke sa tsebe (I did not know it), sine rule, cosine rule but now since I used the smartphone, I now know it*

Learners explained the benefits they derived from using smartphones. For example, the ability to access hints for each task (see B_{3b}, in Table 4.a). Hints are important when learners reach an impasse while solving the tasks on site. Hence, the smartphone was convenient in that regard. As noted by G_1, smartphones helped to move easily from one task on site to another, which saved time. Internet searches on the smartphone were crucial to assist learners with definitions and examples of trigonometric concepts, especially for learners who were never taught the concepts in the classroom (see B_{2a} and G_4, Table 4.a). Therefore, the smartphones were key to their learning in that they acted as textbooks and teachers while learners were working on mathematics trails and had to learn the trigonometry concepts, such as cosine rule, sine rule and area rules independently using the internet. They could do the task and received positive feedback on the smartphone, which implied they made the appropriate mathematical connections and understood the concepts better (see B_{2a} and G_4, Table 4.a). The smartphones contributed to enhancing their understanding of trigonometry concepts.

4.5.2 Learners' endorsement and suggestions on the use of the MCM app

The learners expressed their willingness to use the MCM app in their learning of mathematics concepts, as one of their learning tools or aids. They listed the reasons that the MCM app assisted them in different ways and increased their understanding, as indicated by B_{2b} in Table 4.b below 'the MCM app is helpful to us because it enabled us to approach mathematical problem with different methods. The MCM app also has visual tasks which help learners who are visual thinkers and it can also help the abstract thinkers'. B_{3a} also said, 'We were able to utilise trigonometry in different ways and also get a better understanding on how we can utilise them in different situations.' The ability to represent mathematical concepts differently is an indicator of enhanced conceptual understanding.

Table 4.b indicates certain remarks by the learners on their experiences while using MCM app for learning trigonometry.

Participants	Comments made by learners.
B_{4b}	*The MCM app makes learning easy and enhanced our understanding making it easier to remember the content (concepts).*
B_{1a}	*Obviously, yes, we will use the MCM app again because it exposed us to new ways of learning angles, and properties of different types of triangles.*
B_{2b}	*The MCM app is helpful to us because it enables us to approach mathematical problem with different methods. The MCM app also has visual questions which help learner who are more visual thinkers, and it can also help the abstract thinkers.*
G_4	*…and considering that different people have different learning styles, the App offers learners different methods of learning, which allows them to think outside the box.* *like I said it not only help us realise that we don't only do maths in class, but maths is everywhere we can utilise the information we get in class to actually use it outside, thereby, connecting mathematics to the real world.*
B_{3a}	*We were able to utilise trigonometry in different ways and also get a better understanding on how we can utilise them in different situations.* *It can be used as a guide on some of the complex mathematics problem because it provides easy steps(hints) that one understand.*
B_{4a}	*I thought the numbers of the students who are doing mathematics was going down in SA; however, after using MCM app I feel somehow MCM app may change this scenario/perception in the near future.*
G_2	*... and actually having to realise that maths is not only in the classroom but everywhere. It is relatively easier to use MCM app to find/search for all the tasks.*
G_3	*Nka nepa (I will pass mathematics). There is joy in using the MCM app and perhaps it should be included as part of learning tools in the classroom or at school.*
B_{3b}	*Using the MCM app made me more interested in maths than I was before.*

The participants from all groups commended the MCM app's positive contribution to their learning of mathematics, such as enhanced conceptual understanding of trigonometry. B_{4a} and B_{4b} commented how it enhanced their mathematical conceptual understanding and were quoted as follows 'the MCM app make learning easy and enhanced our understanding making it easier to remember the content (concepts)'. Therefore, it suggested that the MCM app enabled learners to remember the relevant knowledge through the hints, e.g., 'MCM app can be used as a guide on some of the complex mathematics problems because it provides easy steps(hints) that one understands'. The participants recommended including or adopting the MCM app, as one of their learning aids in school mathematics. G_3 endorsed this: 'There is joy in using the MCM app and perhaps it should be included as part of learning tools at school.' The suggestion to adopt it as one of the learning aids in school mathematics was also prompted because they said that the MCM app could rekindle their interest in mathematics. B_{3b} said, 'Using the MCM app made me more interested in maths than I was before' (see B_3, $G_3$and B_4 in Table 4.b). Another reason to adopt MCM app was that some participants realised the benefits from using it, such as long-lasting memory and good marks in mathematics tests and examinations (see G_3, B_{4a} and B_{4b}, Table 4.b). B_{4a} remarked, 'I thought the numbers of the students who are doing mathematics was going down in SA; however, after using the MCM app, I feel somehow it may change this scenario/perception in the near future'. G_4 and B_2 mentioned that the mathematics trails from the MCM app had the advantage of accommodating learners with different styles or abilities. G_4 said, '… and considering that different people have different learning styles, the MCM app offers learners different methods of learning, which allows them to think outside the box'. Therefore, the MCM app presented tasks that accommodated different cognitive levels.

The MCM app tasks helped to connect trigonometry concepts to the real world. For example, in some tasks, learners were required to calculate the height of the school flagpole, and other tasks required them to calculate the slope of the school grandstand (see Chapter 3). As a result, learners realised that mathematics is not confined to the classroom without any relevance to the real world and other fields. G_2 concurred and stated that '… and actually having to realise that maths is not only in the classroom but everywhere. It is relatively easier to use MCM app to find/search for all the tasks.' Therefore, learners demonstrated concerted or integrated use of trigonometry concepts by using different methods and representations (see B_{3a}, B_{2b} and G_4 Excerpt 2). Thus, the MCM app was useful and made it possible for the learners to increase their understanding of trigonometry. B_{2a} noted the convenience of the MCM app for providing hints and sample solutions (see Table 4.a). Sample solutions were essential so that they could learn from their mistakes or find different approaches. Some learners indicated that the MCM app allowed them to think outside the box through the tasks tied to the real world (see G_2, B_{3a} and G_4 in Table 4.b).

4.5.3 Learners' participation in the trailblazing.

All the groups were actively involved while walking the mathematics trails, using smartphones in learning trigonometry concepts (Table 4.c) Learners indicated that mathematics trails helped to connect trigonometry concepts with the real world, and they learned from each other. B_{1a} and B_{1b} mentioned that 'We counted the blocks and after that we used the trigonometry ratios to get that angle of which that mix reality and trigonometry which makes it one point,' and 'It was task two where we had actually to use different trigonometry ratios in order for us to get the very same answer so that we can actually be accurate.' G_1 added 'We were able to calculate and identify the lengths of different mathematics shapes, in order to finally use trigonometry concepts, and we were able to use them with real-life situations.' This demonstrated that participants were actively involved and shared and reinvented mathematical ideas.

Table 4.c: Shows some of the comments leaners made in connection with their participation in trailblazing

Participants	Comments made by learners.
B_{1a}	*It was task two where we had actually to use different trigonometry ratios in order for us to get the very same answer so that we can actually be accurate.*
B_{1b}	*We counted the blocks and after that we used the trig ratios to get that angle of which that mix reality and trigonometry which makes it one point.*
G_1	*We were able to calculate and identify the lengths of different mathematics shapes, in order to finally use trigonometry concepts, and we were able to use them with real-life situations.*
B_{2b}	*We were somehow innovative; it really helped us to understand the concept better.* *task four we actually had to remember the properties of a rectangle.*
G_2	*It taught me hore (that, our) di trigonometry ratios tsa rona di (are important) important and made me understands better.*
G_3	*It actually helped to enhance participation, using the MCM app as in the classroom we are passive most of the time. We were able to think outside the box. We actually had to use hints that helped us to be more active and participating.*
B_{3b}	*We were taught that SOH CAH TOA, like in a paper and pen, we were able to use it to get the height of the flagpole.*
B_{4b}	*All of us participated one said we should this and the other said we should do that and at the end of the day we had all of our ideas combined.*

	The trigonometry outside the classroom taught us the cosine rule, area rule it will be very simple for us to apply it.
B_{4a}	*... and somehow solve mysterious problems.*
G_4	*I said it did not only help us realise that we don't only do maths in class, but maths is everywhere we can utilise the information we get in class to actually use it outside.*
B_{2a}	*Used different scenarios for us to have deeper understanding; did with the flag pole we managed to use the similarity of triangles.*

Mathematics trails using smartphone persuaded all learners to take part and cooperate in their respective groups, while solving the tasks because of the number in each group. B_4 supported this, and said 'All of us participated, one said we should do this and the other said we should do that and at the end of the day we had all of our ideas combined. The trigonometry outside the classroom taught us the cosine rule, area rule and it will be very simple for us to apply it.' There were only three members per group. Each had a role and, most importantly, issues to debate, questions and answering those questions with relevant mathematical knowledge (see B_4 and G_3 Table 4.c). Hence, G_3 said that 'It actually helped to enhance participation, using the MCM app. As in the classroom we are passive most of the time. However, with MCM app we were able to think outside the box, we actually had to use hints that helped us to be more active and participate'. The stepped-up hints assisted learners to remember suitable knowledge and started discussions among learners, increasing their understanding. As noted by B_2, in one of the tasks they had to remember the properties of a rectangle. B_3 recognised that he needed to link what he learned in the classroom with a real-world problem to find the solution. The data indicated that learners used prior knowledge and appropriate connections to solve the real-world tasks mathematically. Proper connections make mathematics meaningful and practical. The ability to remember the properties of a certain mathematical structure (triangle and rectangle) and to make correct mathematical connections implies that they improved their conceptual understanding (see B_{2b}, G_2 and B_{2a} Table 4.c).

Mathematics trails linked trigonometry concepts to the real world and assisted learners in making sense of mathematical ideas and the relevance of what they were learning in a mathematics classroom, and subsequently to solve real-world tasks (see G_4 and B_{4b} in Table 4.c). Hence, the mathematics trails were significant and this data showed that mathematics trails improved the learners' understanding.

4.5.4 Challenges encountered by the participants while trailblazing

The challenges are categorised in the following sub-headings

Table 4.2: Internal factors on the MCM app

Learner	Challenges encountered
B_{2a}	*The navigation. I don't know how to put it. However, I feel it was supposed to show us an arrow not that bling bling thing.*
G_3	*We actually not used to using navigators most of the time so it was somehow new to us using navigator and we find that bit of a challenging.*

The data in Table 4.2 above indicated that some participants had a challenge with the navigator. B_{2a} said it was not specific or clear in terms of exact direction. It was time-consuming, as some tasks were unclear because they related to the walls of the buildings that were concealed. So, if one was not familiar with the environment, it would even take longer to locate the task. Hence, he (B_{2a}) suggested that the navigator should have an arrow to clearly show the direction to take. G_3 shared similar sentiments (Table 4.2). However, frequent use of the MCM app may solve this challenge.

The listed challenges can be avoided by frequent use of the MCM app. In that way, learners become familiar with the navigation and its features. That means, teachers should, at least at the end of each topic, expose learners to MCM app tasks on the topics taught or to be taught in class. Alternatively, this can be done once per term, if it is time-consuming for other teachers.

Table 4.3: External factors on the tasks during mathematics trails

Learner	Challenges encountered
B_1	*The challenge that we were faced with, I may say it was on the tasks out of reach, whereby we had to just improvise, and tasks like verge board, by just seeing it; it was a little bit difficult and somehow new task to us. (meaning it was for the first time they were confronted with outdoor tasks questions or task out of reach for them, to not use measuring tape etc)*

According to Table 4.3 above, some tasks seemed challenging for the participants, particularly those out of reach for them to take measurements (see B_1 in Table 4.3). Such tasks prompted learners to think outside the box. They had to develop appropriate mathematical connections, and proper representations to solve the real-world problem mathematically. However, it took them a while to solve these kinds of tasks. Hints, as well as trial and error methods, assisted learners in solving the tasks. Some groups managed to successfully solve these types of tasks. See quotation below on the verge board task .

> *B_{3a}: 19.17(answer), please check with MCM app.*
> *B_{3b}: Well done. (Feedback from MCM app). (They checked the sample solution against their answer and trying to make sense of the answers from the app)*

Group 3 attempted this task and solved it successfully. The researcher noticed that these questions were a challenge for all the groups (during trail observations); it took all the participants a while to solve because learners were not familiar with them (see B_1 in Table 4.5). Learners need to be exposed to such tasks frequently as they created challenges, and the mathematics trail does this by providing hints and sample solutions. Therefore, such tasks need to be given to learners to solve more often using the MCM app.

Table 4.4: External factors on the trigonometry concepts during trailblazing

Learner	**Challenges encountered**
B_{1a}	*I have discovered I don't know trigonometry formula (meant sine rule, cosine rule and area rule).*
G_1	*We did not know what is trigonometry and how to apply it. (they meant to say sine, cosine and area rule). The teacher did not teach us.*

According to Table 4.4, participants did not know sine, cosine and area rules, as these trigonometry concepts came from the hints as they were trying to find a way to solve the task on drainage pipes (see B_{1a} and G_1 in Table 4.4). They said the teacher did not teach them (G_1 Table 4.4). This implied that learners were entrenched in the traditional approach of learning and teaching (teacher-centred approach). Hence, teachers need to introduce an alternative approach, such as the MCM app, where learners must find the trigonometry concepts or reinvent mathematical concepts through real-life tasks (mathematics trails). With the MCM app, learners are guided by the hints on a smartphone to reinvent relevant mathematical ideas. Table 4.3 above presented an quotation from one of the groups that were not be taught the said trigonometry concepts in the classroom. However, after using the MCM app and the internet, they solved the task effectively without the teacher's assistance. However, it took them some time to complete compared to other tasks in the mathematics trails (see G_4 Table 4.a).

Therefore, learners experienced productive challenges in that they used hints and the internet to solve the task. It further implied that learners could reinvent mathematical ideas guided by a smartphone, using the MCM app or internet.

It is suggested that teachers use the MCM app, so that learners can take responsibility for their learning and dissuade them from relying only on the teacher when there are alternatives to learning trigonometry concepts.

Despite the challenges, learners realised that taking part in this intervention programme was a good idea. There was a positive shift in their view of trigonometry concepts. They had to be in charge of the whole process, without a teacher instructing them. From the learners' data or excerpts presented in this chapter, the learners attested that their conceptual understanding was enhanced. The data presented in this chapter indicated that, due to the intervention, the participants understood better when using the MCM app for trigonometry learning. Gontyeleni (2019) participated in the GESF, which took place in Dubai (2019), where more than 2,000 delegates from 144 countries attended, realised that SA needs to move away from traditional methods of teaching and learning, whereby learners are only the receivers of content and teachers are deliverers. She further states that teachers need to create opportunities where learners gain autonomy, take charge of their learning and are able to engage with the content in ways that enhance conceptual understanding. The summit (GESF) emphasises the use of technology in learning. This study employed RME principles and smartphones using the MCM app, enabling learners' autonomy and letting them take responsibility for their mathematics learning.

CHAPTER FIVE
CONCLUSIONS AND RECOMMENDATIONS

5.1 INTRODUCTION

This study investigated how selected grade 11 mathematics learners used smartphones with the MCM app in grade 11 trigonometry for conceptual understanding. The aim was to research the effective use of the MCM app by using smartphones for conceptual understanding outside the classroom. The chapter concludes this study by presenting the following: a summary of the findings, the contribution of the study, recommendations to scholars of mathematics education and policymakers, limitations and suggestions for further research.

5.2 SUMMARY OF FINDINGS

Research Question One

How did learners use smartphones for mathematics learning purposes prior to participating in an intervention programme?

The findings from the survey data revealed that grade 11 mathematics learners recognised the significance and value of using smartphones for learning mathematics, but they were prohibited from using smartphones on the school premises as part of the school code of conduct. They used smartphones mainly at home. However, after engaging the principal on this matter, he gave consent to the 12 participants because they participated in this study. The prevalent use of smartphones for learning mathematics is understandable, primarily because the survey was conducted when the COVID-19 pandemic and associated restrictions were still in place, although the restriction levels had been eased. The survey also revealed that, among the applications mentioned, the MCM app was not listed as one of the applications learners used for learning trigonometry.

When responding to activities done after school or at school, mathematics trails were not mentioned. Learners exclusively received mathematics tuition in the classroom using textbooks, teacher's notes and examples to learn trigonometry prior to the intervention. They used traditional methods and teacher-centred approaches as opposed to outdoor mathematics learning using a smartphone. Hence, there was a need to introduce more innovative and active learning as opposed to the passive classroom situation. This necessitated a workshop, pilot and an intervention with some learners in the school, particularly focusing on the use of smartphones using the MCM app for learning trigonometry for conceptual understanding.

Research Question Two

How does the use of smartphones to learn mathematics through outdoor mathematics tasks enhance conceptual understanding of trigonometry concepts in a selected grade 11 class?

Mathematics trails observations indicated that outdoor tasks were a source of mathematical concepts or formal mathematical knowledge and allowed learners to reinvent mathematical ideas with adult guidance (hints from the MCM app had the role of adult guidance). Learners could transform realistic problems mathematically by using appropriate mathematical models. Outdoor mathematics tasks created useful discussions and prompted them to use prior knowledge, relevant mathematical connections, multiple representations, and recognise and identify appropriate mathematical models and shapes from real-world objects.

The process of connecting trigonometry concepts in everyday life through the MCM app using a smartphone can be seen as successful in that it improved conceptual understanding in accordance with presented and analysed data. The participants indicated an integrated and functional grasp of mathematical concepts, solved outdoor tasks and received positive responses from the MCM app. The data presented confirmed that learners used prior knowledge to complete the tasks and that they showed competence in recognising and making the required mathematical links in the mathematics trail. It further demonstrated that learners improved their understanding of that they learned in the classroom during the mathematics trails because they effectively solved unfamiliar tasks through well-integrated mathematics connections and had informative discussions. The mathematics trail, using a smartphone, helped eliminate the idea that mathematics is an abstract subject, confined to the classroom, without any relevance and significance to real life. Learners implied that the teacher-centred classroom environment did not allow healthy discussions, enthusiasm and self-construction of knowledge or reinvention of mathematical ideas.

An example is the school flagpole tasks, where they were required to calculate the height of the school flagpole. MCM app hints came to their rescue, hint number 1 said to use the similarity of triangles, prior knowledge acquired in grades 9 and 10. Thereafter, learners had to use proportionality, linking appropriate mathematical concepts to employ algebraic equations to solve the unknown side of the bigger right-angled triangle that represents the height of the school flagpole. This demonstrated an integrated and functional grasp of mathematical ideas. Learners understood, as a result of trailblazing, why the mathematical idea was important and context where it could be useful, helping them to work out a similar , more complicated problem in the second trail.

Mathematics trails and MCM app hints in all tasks prompted learners to read and imagine. For example, the roof and wall task asked the participants to imagine congruent right-angled triangles (internal representation) and depicted the triangles on the page (pictorial). By so doing, they indicated multiple representation aptitude. MCM app hints assisted learners to find a solution during trailblazing and use sample solutions to enhance their understanding. The mathematics trails created engaging interactions among learners. During mathematics trail observations on grandstand stairs task, 'calculate the angle of depression', the learners mentioned that there was a relationship between the angle of depression and the angle of elevation, Z-shape form (i.e., corresponding angles). During the trailblazing, the smartphone facilitated the learning process, making the process learner-centred and active.

Research Question Three

What are the selected grade 11 mathematics learners' experiences and perceptions of using smartphones for learning trigonometric concepts, after participating in the MCM project?

Interviews revealed that learners' perceptions had changed. They understood that the trigonometry concepts learned in the classroom were relevant and could apply to real life. They could transform realistic problems or tasks into mathematical tasks, reinvent ideas or concepts, use multiple representations, make necessary mathematical links, and use knowledge to increase their understanding.

Data presented and analysed demonstrated that proper mathematical connections were made between mathematical ideas. Participants said they had to verify their thinking or answer, by employing all three trigonometry ratios and getting the same answer (sine x, cosine x and tangent x). In the interviews, learners highlighted that the teacher had not or did not teach them sine, cosine and area rule. However, the mathematics trail taught them these concepts. It implied that the task allowed learners to make appropriate mathematical connections and they successfully solved the tasks with the assistance of the MCM app hints and the internet. During interviews, participants indicated that they had to first identify the triangle to solve the area bound by drainage pipes. They calculated that the area rule was appropriate to determine the area bound by the drainage pipes as the triangle was a non-right-angled triangle. This showed that outdoor mathematics tasks enabled learners to utilise suitable trigonometry knowledge or ideas to solve the task.

All participants commended the MCM app's positive contribution to their learning of mathematics, such as enhanced conceptual understanding of trigonometry. Participants realised that this intervention programme was a good idea. There was a positive shift in their conception of trigonometry concepts. They took charge from the beginning to the end of the mathematics trail, without a teacher giving instructions.

This study concludes that the use of smartphones with MCM app has the potential to enhance conceptual understanding of trigonometry.

5.3 CONTRIBUTION OF THE STUDY

In the South African education curriculum context, this study can encourage curriculum developers and mathematics education researchers to incorporate outdoor or real-life mathematics tasks using a smartphone with the MCM app into the design of policy guidelines. This project contributed to the researcher's knowledge and professional development concerning outdoor mathematics learning using a smartphone, as the researcher is employed as a high school mathematics teacher. The study further sought to contribute to innovative and exciting mathematics ways of learning, especially to enrich mathematical conceptual understanding of learners.

The results of this study could aid the Department of Higher Education and Training to integrate outdoor mathematics tasks using a smartphone with the MCM app in its teacher training programmes to adequately prepare teacher trainees to embrace the use of technological devices, such as the MCM app in mathematics teaching and learning. The findings of this research might assist schools in making deliberate efforts to start viewing smartphones as tools that can be effectively and correctly used for learning purposes, particularly to enhance learners' mathematical conceptual understanding.

5.4 RECOMMENDATIONS

The researcher is cognisant of the fact that the findings of this study cannot be generalised to the entire South African education system. It benefits those who wish to use smartphones for outdoor mathematics learning for conceptual understanding of trigonometry in a learner-centred approach. Based on the findings of this study, some recommendations are made to important stakeholders. Deliberate or cautious means are required to start viewing smartphones and mathematics trails as tools to be effectively and correctly used for learning purposes. Training of teachers and learners is necessary in this regard. Learners need to minimise distractions during lessons while teachers need to be trained on best practices to optimise smartphone use for teaching. We live in the fourth industrial revolution and the advent of smartphones provides a strong learning environment for teachers to explore, experiment and share their knowledge as educational practitioners.

Curriculum developers should consider including and making outdoor mathematics tasks using smartphones, particularly with the MCM app, compulsory in teaching and learning trigonometry. This is because the findings of this study show that the combination of well-known mathematics trails ideas with current technological possibilities of smartphones helps learners to enhance their understanding of trigonometry. Therefore, this authoritative information (making MCM app and smartphone learning compulsory) has to be emphasised in official documents. Additionally, universities training teachers should ensure that teacher trainees are well prepared to embrace the use of technological devices, such as smartphones with the MCM app in mathematics teaching and learning.

Another recommendation to teachers is that teaching and learning of mathematics should be given enough attention, with an emphasis on conceptual understanding. Learners should be guided through measured technological integration, such as smartphones in mathematics learning, to avoid over-dependence on smartphones. The suggestion would be to use the MCM app at least once per term and incorporate all the concepts dealt with during the term. Also, to emphasise group discussions during trailblazing and use of hints from the MCM app.

In meeting challenges that come with integrating smartphones into mathematics learning, parents, learners, teachers, SGB and other relevant stakeholders, should consider using smartphones appropriately within guidelines set by themselves as a collective. Smartphones are a gateway to knowledge and, if used appropriately, have the potential to enhance learners' conceptual understanding.

The findings from this study are that outdoor learning of mathematics brings several advantages. Learners can explore mathematical concepts in the real world instead of just inside the classroom, the great outdoors is more accommodating of equipment and supplies, allowing learners to learn mathematical concepts on a larger scale. Outdoor mathematics learning can motivate, promote natural curiosity among learners, develop team-building skills, give a sense of independence, allow learners to make useful links in their learning as they use prior knowledge to create new knowledge and make functional connections between mathematical concepts and real-world application through outdoor learning. In addition, exclusively learning mathematics inside the classroom hampers learners from fully understanding mathematical concepts. Therefore, teachers are encouraged to make use of outdoor mathematics with the MCM app as and when they make their planning (lesson plan).

The suggestion is that teachers expose learners to the MCM app, so that learners take responsibility for their learning, as this may dissuade them from relying solely on the teacher when there are alternatives to learning trigonometry concepts for deeper understanding.

5.5 LIMITATIONS

The challenges confronted in the study included getting the surveys from the learners, as the researcher was not a teacher at that school. The communication challenge between the researcher and participants prompted us to have a WhatsApp group. There was limited time after normal school hours, and participants had to be transported home after the mathematics trail.

Despite all the challenges mentioned, there were challenges in analysing and interpreting data at first, as this was the researcher's experience. However, that was addressed by consulting the people with the know-how and guidance from the supervisors.

The researcher had a challenge in accessing some scholars on internet (Google scholar), and as a result, could not access information required at the time and had to resort to the limited sources available. Load shedding impacted the study delaying work at times and the internet would sometimes be slow or unavailable. The researcher is a full-time employed teacher and undertook this study after hours.

Notwithstanding these limitations, useful insights on using smartphones with the MCM app to enhance mathematical conceptual understanding of learners in trigonometry were derived from this study.

5.6 SUGGESTIONS FOR FUTURE RESEARCH

The researcher recommends the following since this study was only done on a small scale. Future research should include more research sites and larger sample size to enable the findings to be generalisable in a wider context. Also, learners' and teachers experiences on using the MCM app on smartphones in learning trigonometry and understanding concepts, the SGB's views on this type of learning being carried out on the school premises, and whether utilising the MCM app in other areas of mathematics will yield positive results.

5.7 PERSONAL REFLECTIONS

This research journey has been characterised by experiments, trials and challenges and the first time the researcher has engaged in proper academic research. There were many challenges due to personal and financial constraints.

The process entailed much reading of journals, articles, books and attending conferences and presenting some parts of my study at the mathematics conferences. The reading and experiences enriched the knowledge of the topic and enhanced academic writing competencies.

The study allowed the exploration of other innovative and exciting ways of learning trigonometry for conceptual understanding, to share with my fellow mathematics teachers and, most importantly, expose learners to the MCM app. The researcher will encourage other mathematics teachers to use the MCM app, not only for trigonometry but for all mathematics. It is my wish to urge all stakeholders at school, particularly the SGBs, to allow smartphone use on the school premises to enhance conceptual understanding of mathematics.

5.8 CONCLUSION

This study indicates that outdoor mathematics learning using smartphones for conceptual understanding is beneficial to learning trigonometry as using the MCM app prompted learners to make mathematical connections, multiple representations, use prior knowledge, reinvent mathematical ideas, see the practical value of mathematics and not confine mathematics to the classroom. This chapter presented a summary of the key findings, contribution of the study, limitations, recommendations and suggestions for future research.

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APPENDICES

Appendix A: Analytical framework to capture how learners solve tasks, using smartphone with MCM

Conceptual understanding indicators	Code	Learners' response or computations in relation to mathematics trails tasks.
Extending mathematics in real life context	ER	There is evidence of identifying relevant mathematical concepts with given meaningful problems situated in real life, interpreting the mathematical solution in relation to realistic situation. Learners calculating the height of the school flagpole- here learners used similarity of right-angled triangles and algebraic equation (proportionality) to calculate the side which represents height of the school flagpole in real world. Learners' drawing from things they use everyday
Representing mathematics concepts in different way	RD	There is evidence that learners formulate the problem into various mathematical models/symbols to describe the relevant relations involved. Learners demonstrated the ability while working on the school grandstand stairs, calculating the angle of depression. They made use of rectangular pane to illustrate the angle of depression. They even demonstrated it by putting a horizontal 2 m stick on the rail of the grandstand stairs to create the horizontal to show the angle of depression from the horizontal stick down. They eventually used a Z-shape drawing to display the relationship between angle of depression and angle of elevation. Use of diagrams and sketches by learners.
Connecting concepts and ideas in mathematics	CI	The problem in the model gets solved and the learner reflects on the solution process (discuss solution with fellow learners in a group) The angle between the roof verge board task- learners had to imagine two identical right-angled triangles (MCM app hint).

		However, those triangles were located in a rectangular wall, from there, learners associated the triangles with trigonometry ratios and solved the trigonometry ratio algebraically because they formed equations (angles, rectangles are also solved in Euclidean geometry) Thus, connections within mathematics e.g., with other topics Connections with the everyday, connections with other learning areas particularly in physical science they make use of angles and mathematical shapes such as right-angled triangles.
Building on prior knowledge	BK	There is evidence of what learners already know and drawing from past experiences, for instance geometry shapes like triangles, rectangles, squares, solving equations or writing down known formulae. In all the mathematics trail tasks, learners indicated use of prior knowledge. For instance, they had to calculate height in metres, meaning convert the centimetres to metres. The learners had to call to mind how to do conversions. They used a mnemonic device called ‘King harry’ taught to them in the lower grades.

Appendix B: Analytical framework to analyse mathematics trail or Outdoor tasks for realistic mathematics learning

Realistic Mathematics Education Indicator	Code	Nature or characteristics of mathematics trail.
Guided Reinvention	GP	Learners exploring the phenomena of everyday life- the context should be real, relevant, and challenging for learners. Learners identify relevant mathematical concepts. All the mathematics trail tasks were tied to real world objects, for example drainage pipes task. Learners were able to recognise that the area bound by the drainage pipes resemble a non-right-angled triangle and that they needed to use cosine rule and thereafter, area rule to calculate the required area. They were guided by the hints from the MCM app to re-invent or generate these relevant mathematical concepts.
Emergent or Self-develop Models	SM	Learners use and develop their own models, from informal to formal. For all the outdoor mathematics tasks, learners transformed realistic tasks to mathematical tasks through appropriate mathematical models. One such example dealt with school Billboard task, where they applied a right-angled triangle as mathematical model, trigonometry ratio equations, and got to solve the problem mathematically and interpreted it back to realistic solution.
Use of interaction	UI	There is evidence of interaction among the learners themselves, interaction with the learning tools, and with the environment. Communication- ask and respond to questions, reflect to achieve progression from informal to formal, learners managed to locate the tasks through the assistance of navigation system, and when on site they were able to read questions or tasks from the smartphone without any ambiguity, confusion, as G_4 read the task as follows, *Calculate the height of the school flag pole in meters (reading the task from the*

		smartphone). There were times where learners did not know what to do, they reached impasse during trailblazing and there was no teacher to intervene but the hints from the MCM app. Interaction among learners, triggered illuminating discussion as B_{4a} remarked '***You see the mistake you committed here, is that you said there was an angle of 90 degrees here and yet there was no 90 degree angle there*** *(taking the page correcting the triangle from right angled to non-right angled triangle,*
Topic Relatedness	UR	There is evidence that learning of mathematical material is related to various mathematical topics in integrated manner, connections to other fields. The participants' subject combinations included physical sciences and geography. The expertise or knowledge created or gained from the mathematics trails can come in handy in those subjects too. G_4 in Excerpt 1 applied multiplicative inverse concept to make AB subject of the formula. In physical sciences, for example, this skill or knowledge is required. conversions expertise, cm to be converted to m. *B_{4b}: So, we will convert it to meters.* This kind of connection is also applied in geography during or on map work activities.

Appendix C: Survey

Survey for Grade 11 Mathematics Learners.

General information

Instruction:

- Use a cross(X) to select an appropriate response in each question where necessary.

What is your gender? Male……… or Female………

1.During your learning of mathematics do you make use of a smartphone?

Yes…… or No……
If yes, how do you use it? ....
If no, why? ....

2.Which application/s do you use for learning mathematics concepts?

WhatsApp ………..
Facebook ………...
Google Search ………
Or any other application, specify…………………………………………………...
3.If you use any of the above, please explain how?...
....

4.For what purpose are you using the application/smartphone?

……………………………………………………………………………………………….

5.Are there any school activities you engage in where you use a smartphone after school? Yes …. Or No ….

6.If Yes, which ones and how? And if No, elaborate…………………..................

……………………………………………………………………………………..………

……………………………………………………………………………….

7.Do you use smartphone when learning trigonometry concepts? Yes……. Or No……

8.If No, please explain how you learn trigonometry concepts?

…………………………………………………………………………………………………

……………………………………………………………………………….

9.If yes, please write the aim/purpose…………………………………………………

……………………………………………………………………………………………

10. What are the factors that prevent or prohibit you from using smartphone in learning trigonometry concepts?...

…………………………………………………………………………………………………

………………………………………………………………………………

11.Would you be interested in being introduced to new applications for learning mathematics concepts?

The END

Thank you for your time in answering this survey.

Appendix D: Focus group interview questions

SEMI- STRUCTURED INTERVIEW SCHEDULE

I anticipate the study participant to answer the following questions during the interviews. However, some questions to be asked will be guided by what will unfold during trail blazing.

1.Did the math trail tasks help you to learn trigonometry? Did you manage to identify any connections? If yes, give an example. And if no, elaborate.

2.How important is the use of smartphones in the learning of trigonometry concepts?

3.Do you think you managed to connect trigonometry concepts to real life context? Explain your answer.

4.Considering the tasks in your math trails, which task do you think helped you as a learner in understanding the concept better/deeper? Why?

5.Is there any task in the math trail that prompted you to make use or draw from prior knowledge? Please provide example/s and identify where this happened?

1. Do you think that using the smartphone with MCM application to learning trigonometry concepts helped you to represent mathematical concepts in different ways? Elaborate and give examples please.

7.How did the use of smartphones using MCM enhance your active participation in the learning of trigonometry concepts? Please explain and give examples.

8.Are you willing to utilize the smartphone with MCM again in learning trigonometry concepts? Please substantiate your response.

9.Did the math mathematics trail allow you to connect/extend mathematical concepts and ideas to the real world? Give an explanation.

10. What challenges did you come across as you were walking the trails?

In the future, would you like to learn mathematics in this way? Why?

Appendix 1: Ethical approval

RHODES UNIVERSITY
Where leaders learn

Rhodes University, Education Faculty
Research Ethics Committee
PO Box 94, Makhanda, 6140, South Africa
Tel: +27 (0) 46 603 8393
Fax: +27 (0) 46 603 8028
email: e.rosenberg@ru.ac.za

https://www.ru.ac.za/researchgateway/ethics/

28 March 2022

Vuyani Samuel Pop

Education Department

g20p2051@campus.ru.za

Dear Mr Vuyani Samuel Pop

Re: Exploring outdoor mathematics learning for conceptual understanding through smartphones in grade 11 trigonometry. A case study of one school in Free State province.

APPLICATION NUMBER: 2021-5146-6393

This letter confirms that your research ethics application has been reviewed and **APPROVED** by the Education Faculty Research Ethics Committee (EF-REC). Your permission letter(s) where applicable have been received and you are free to proceed with your study.

Approval is granted for 1 year. An annual progress report is required in order to renew approval for an additional period. You will receive an email notifying you when the progress report is due.

Should any substantive change(s) be made during the research process, that may have ethical implications, you should notify the Education Faculty REC Chair via email. This includes changes in investigators. The REC Chair will advise as to whether a new application is necessary.

Do keep this clearance letter secure and accessible throughout your study and after its completion. It will be needed when a thesis is examined and when publications are submitted to journals.

Please also submit a brief report to the REC Chair on the completion of the research. This can be done via email. The purpose of this report is to indicate whether the research was conducted successfully and whether any ethics-related matters arose that the committee should be aware of, in order to guide future studies.

Sincerely,

Prof Eureta Rosenberg

Chair: Education Faculty Research Ethics Committee

Appendix 2: Permission and notification for research

Enquiries: M.Z. Thango
Ref: Research Permission: V.S. Pop
Tel. 051 404 8808
Email: MZ.Thango@fseducation.gov.za

education
Departm nt o Education
FREE STATE PROVINCE

35141 Hani Park
Welkom
9463

Dear Mr. V.S. Pop

PERMISSION TO CONDUCT RESEARCH IN THE FREE STATE DEPARTMENT OF EDUCATION: LEJWELEPUTSWA DISTRICT

This letter serves to inform you that you have been granted permission to conduct research in the Free State Department of Education within the Lejweleputswa Education District. The details in relation to your research project with the Rhodes University are as follows:

Topic: Exploring outdoor mathematics learning for conceptual understanding through smartphones in grade 11 trigonometry in Lejweleputswa district, Free State province.

1. **List of schools involved:** Lephola Secondary School.
2. **Target Population:** Twelve learners doing Mathematics in grade 11 at the selected school.
3. **Period of research**: From the date of signature of this letter until 30 September 2022. Please note that the department does not allow any research to be conducted during the fourth term (quarter) of the academic year. Should you fa behind your schedule by three months to complete your research project in the approved period, you will need to apply for an extension. The researcher is expected to request permission from the school principals to conduct research at schools.
4. The approval is subject to the following conditions:
   - 4.1 The collection of data should not interfere with the normal tuition time or teaching process.
   - 4.2 A bound copy of the research document should be submitted to the Free State Department of Education, Room 101, 1st Floor, Thuto House, St. Andrew Street, Bloemfontein or can be emailed to the above-mentioned email address.
   - 4.3 You will be expected, on completion of your research study to make a presentation to the relevant stakeholders in the Department.
   - 4.4 The ethics documents must be adhered to in the discourse of your study in our department.
5. Please note that costs relating to all the conditions mentioned above are your own responsibility.

Yours Sincerely,

Mr. MZAMO W. JACOBS
DIRECTOR: QUALITY ASSURANCE, M&E AND STRATEGIC PLANNING

DATE: 08/03/2022

Enquiries: M.Z. Thango
Ref: Notification of research: V.S. Pop
Tel. 051 404 8808
Email: MZ.Thango@fseducation.gov.za

education
Department of Education
FREE STATE PROVINCE

District Director
Lejweleputswa District

Dear Ms. Zonke

NOTIFICATION OF RESEARCH: PERMISSION TO CONDUCT RESEARCH PROJECT IN LEJWELEPUTSWA DISTRICT

This letter serves to inform you that Mr. V.S. Pop has been granted permission to conduct research in the Lejweleputswa District under the auspices of Rhodes University. The details in relation to the research project are as follows:

Topic: Exploring outdoor mathematics learning for conceptual understanding through smartphones in grade 11 trigonometry in Lejweleputswa district, Free State province.

1. **List of schools involved:** Lephola Secondary School.
2. **Target Population:** Twelve learners doing Mathematics in grade 11 at the selected school.
3. **Period of research**: From the date of signature of this letter until 30 September 2022. Please note the department does not allow any research to be conducted during the fourth term (quarter) of the academic year nor during normal school hours. The researcher is expected to request permission from the school principals to conduct research at schools.
4. **Research benefits:** This study is intended to first help the researcher to improve his own practice as he is a high school mathematics teacher. It is also intended to contribute to innovative and exciting ways of teaching mathematics especially to enhance learners' understanding as they experience mathematics in their environments or places of interests. Furthermore, the researcher will share the outcomes and the expertise from the study with his colleagues at work, clusters, PLCs, workshops (organised by subject advisors) and at AMESA Free State gatherings.
5. Strategic Planning, Policy and Research Directorate will make the necessary arrangements for the researchers to present the findings and recommendations to the relevant officials in the Department.

Yours Sincerely

Mr. MZAMO W. JACOBS
DIRECTOR: QUALITY ASSURANCE, M&E AND STRATEGIC PLANNING

DATE: 08/03/2022

RESEARCH NOTIFICATION. V.S. POP. 08 MARCH 2022. LEJWELEPUTSWA DISTRICT

Strategic Planning, Research & Policy Directorate Private Bag X20565 Bloemfontein, 9300 Thuto House, Room 101, 1st Floor, St Andrew Street, Bloemfontein

Appendix 3: Permission for research Lephola Secondary school

09 FEBRUARY 2022

RE: REQUEST FOR PERMISSION TO CONDUCT AN EDUCATIONAL RESEARCH WITH GRADE 11 MATHEMATICS LEARNERS AT LEPHOLA SECONDARY, IN LEJWELEPUTSWA DISTRICT.

TO WHOM IT MAY CONCERN.

Dear Sir/Ma'am.

This communique' serves to affirm that the permission has been granted to **Mr. Vuyani Samuel Pop** who is a registered **Master's Degree** student with **Rhodes University (Student No: 20p2051)**, and a Mathematics teacher at **Unitas Secondary School**. This permission is granted on the basis of conducting a **Mathematics Education Research Study** with grade 11 Mathematics learners in Lejweleputswa District (**Lephola Secondary School**) for a period of about six weeks, for **ONE and half hours** after school (**FEBRUARY 2022**).

His study will involve **12 grade** 11 Mathematics learners, which will be selected according to their interest in technology, availability after school, and their willingness to participate in the study. The study topic is: **Exploring Outdoor Mathematics Learning for Conceptual Understanding through Mobile Smart-Phones in grade 11 Trigonometry**: A case study of one school in Lejweleputswa District, Free State Province.

NB: This study will be conducted on the school.

Yours in Education

Mr. Motseki A.L
Principal

Appendix 4: Letter for request for participation to learners

RHODES UNIVERSITY
Where leaders learn

Enquiries: Vuyani Samuel Pop
Cell number: 082 767 6653

Dear Learner

Re: Request for your participation in a research on the use of Math City Map (MCM) application for conceptual mathematics learning.

I am Vuyani Samuel Pop, a part-time master's Degree student at Rhodes University, SA and a mathematics teacher at Unitas Secondary School. You are hereby requested to take part in the above-mentioned study. If you agree, you are also requested to give your permission for participation in this study which intends to observe how you as a learner can use MCM application as a learning tool to enhance/improve mathematical conceptual understanding.

The study will require you, in groups of three, to walk the trigonometry math trails that I have set within the school premises. As you walk the trails, you will be stopping at designated points doing some mathematics tasks. I will video record you walk these trails and will interview you as a group at the end of walking each math trail. Two math trails will be observed in a period of about two weeks. However, each group will be required to stay behind after school for one and half hours once per week. Kindly be informed that your involvement in this study is voluntary. It is therefore your right to decide whether as a learner should be part of the group of three where I am going to observe or not. Your identity will be treated with high degree of confidentiality and anonymity, and data collected will not be used for any other purposes other than this study.

If you have any questions about this research, please feel free to contact me at 082 76 76653, vuyani.pop@gmail.com, or my supervisors Professor Marc Schäfer at m.schafer@ru.ac.za and/or Dr Clemence Chikiwa at clemence.chikiwa@spu.ac.za.

Lastly, if you agree to participate in this research, please complete the consent form below.

I …………………………………………………………………… (Full names of a learner), hereby confirming that I understand the contents of this document and the nature of the research. I am giving permission to be part of the class/group where the study will be conducted.

Signature(learner)………………………………

Appendix 5: Figure (a) School flag pole

Figure (a): Task 1: **School flag pole**: Calculate the height of the school flag pole in metres.

The smartphone screen on the left hand side shows a task; just underneath the tasks is a space for the participants to enter their answer using smartphone keyboard. The three bulbs are for the hints; participants press the first for hint number one, the second for hint 2 and third one for hint 3. Just after the hints there is check button to check sample solution. The first orange buttons is for the main menu and second orange button directs you to the next task. Participants get feedback immediately and thereafter the sample solution

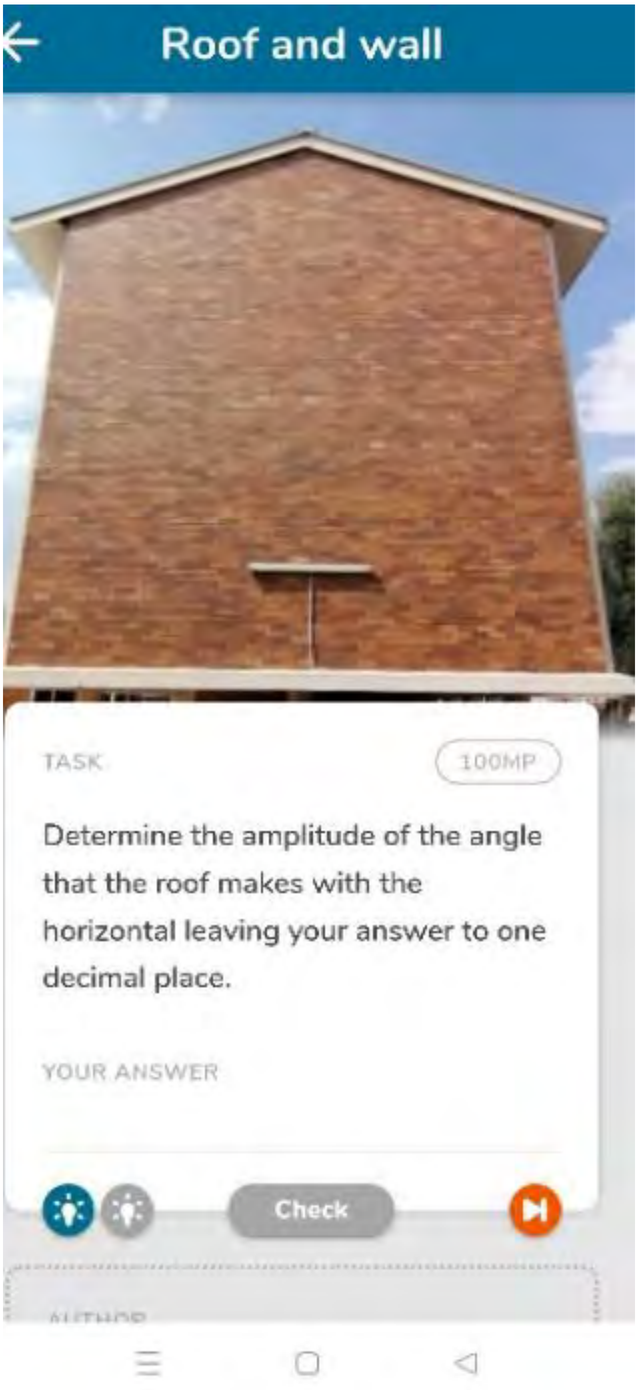
Appendix 6: Figure (b) School billboard

Figure (b) Task 2: **School billboard**: What is the size of the angle that the supporting pole in the middle makes with the ground? (Assume the ground is horizontal). Give your answer in one decimal place.

The smartphone screen on the left hand side shows a task; just underneath the tasks is a space for the participants to enter their answer using smartphone keyboard. The three bulbs are for the hints; participants press the first for hint number one, the second for hint 2 and third one for hint 3. Just after the hints there is check button to check sample solution. The first orange buttons is for the main menu and second orange button directs you to the next task. Participants get feedback immediately and thereafter the sample solution.

Appendix 7: Figure (c) Roof and wall

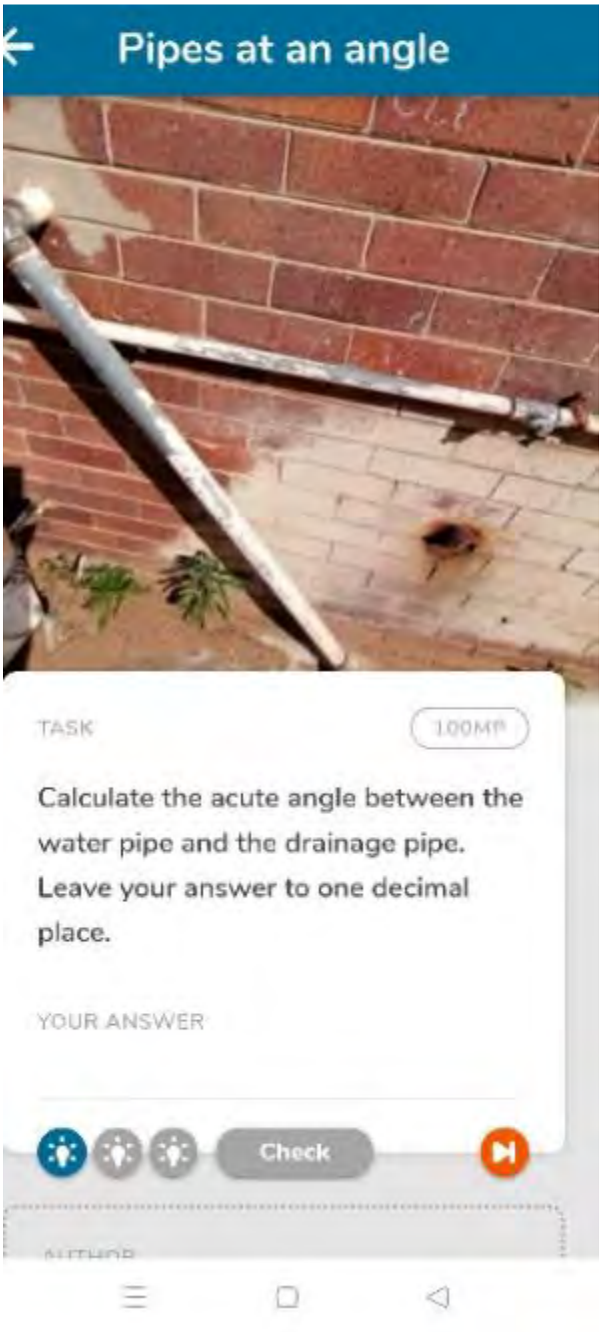
Figure (c): Task 3: **Roof and wall**: Determine the magnitude of the angle that the roof makes with the horizontal. (leave your answer to one decimal place)



The smartphone screen on the left hand side shows a task; just underneath the task is a space for the participants to enter their answer using smartphone keyboard. The three bulbs are for the hints; participants press the first for hint number one, the second for hint 2 and third one for hint 3. Just after the hints there is check button to check sample solution. The first orange buttons is for the main menu and second orange button directs you to the next task. Participants get feedback immediately and thereafter the sample solution

Appendix 8: Figure (d) Pipes at an angle

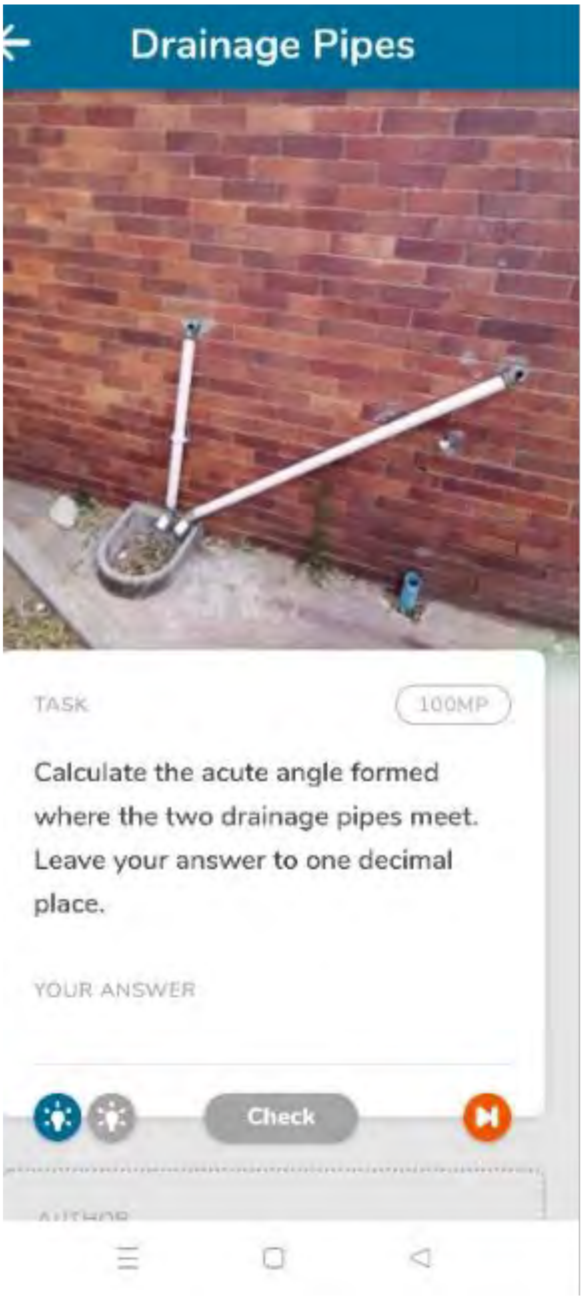
Figure (d): Task 4: **Pipes at an angle**: Calculate the acute angle between the water pipe and the drainage pipe. (leave your answer to one decimal place)



The smartphone screen on the left hand side shows a task; just underneath the tasks is a space for the participants to enter their answer using smartphone keyboard. The three bulbs are for the hints; participants press the first for hint number one, the second for hint 2 and third one for hint 3. Just after the hints there is check button to check sample solution. The first orange buttons is for the main menu and second orange button directs you to the next task. Participants get feedback immediately and thereafter the sample solution.

Appendix 9: Figure (e) Drainage pipes

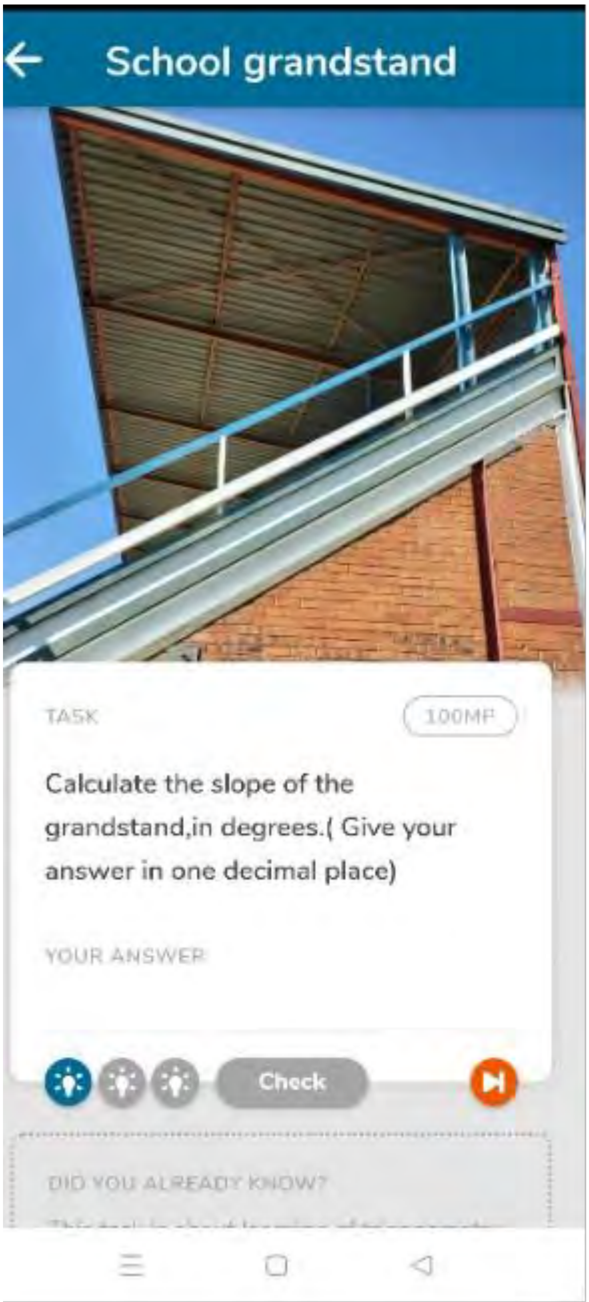
Figure (e): Task 5: **Drainage pipes**: Calculate the acute angle formed where the two drainage pipes meet. (Leave your answer to one decimal place).



The smartphone screen on the left hand side shows a task; just underneath the tasks is a space for the participants to enter their answer using smartphone keyboard. The three bulbs are for the hints; participants press the first for hint number one, the second for hint 2 and third one for hint 3. Just after the hints there is check button to check sample solution. The first orange buttons is for the main menu and second orange button directs you to the next task. Participants get feedback immediately and thereafter the sample solution.

Appendix 10: Figure (a) School grandstand

Figure (a): Task 1: **School grandstand**: Calculate the slope of the grandstand in degrees. (leave your answer in one decimal place)



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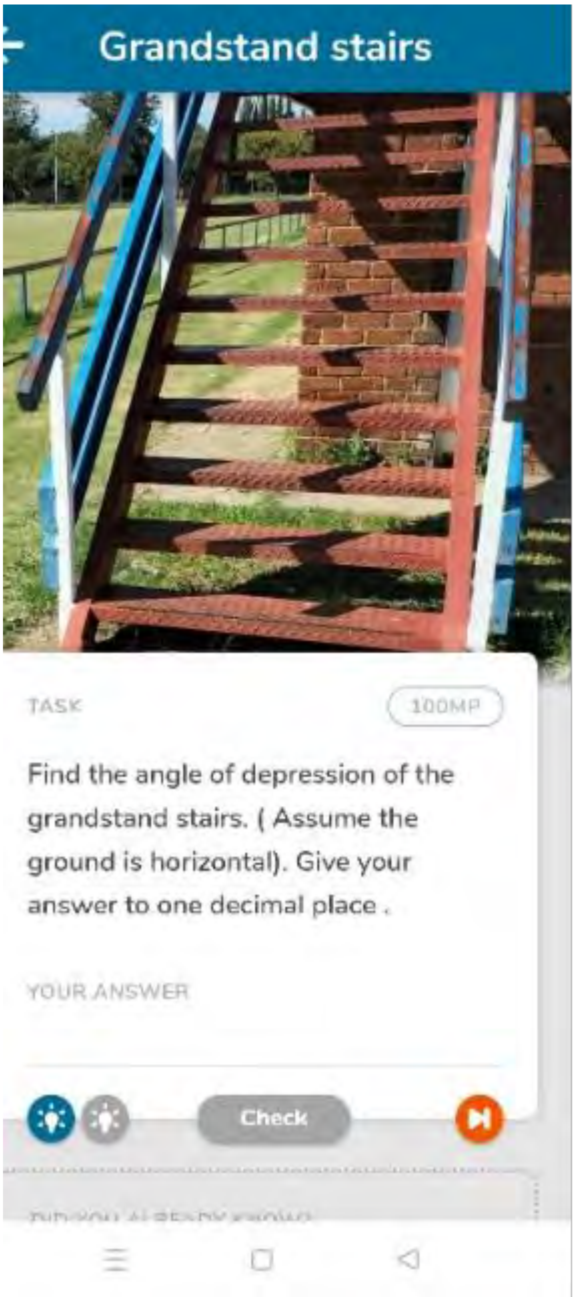
Appendix 11: Figure (b) Grandstand stairs

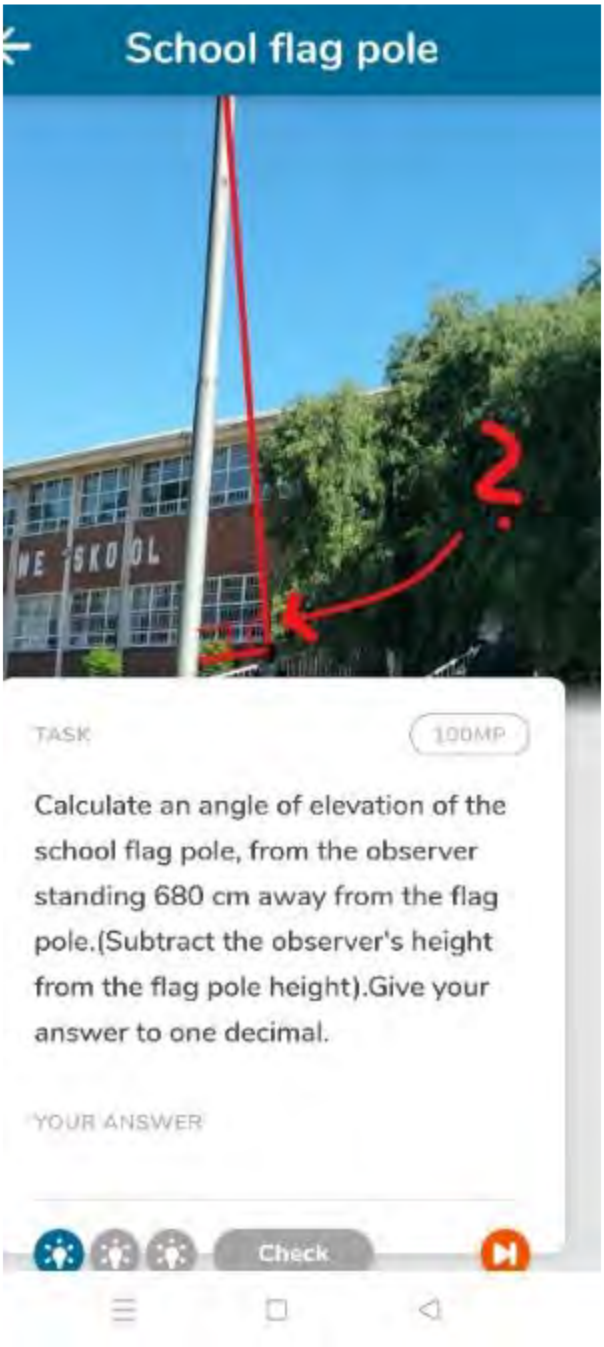
Figure (b): Task 2: **Grandstand stairs**: Find the angle of depression of the grandstand stairs. (Assume the ground is horizontal). Give your answer to one decimal place.



The smartphone screen on the left hand side shows a task; just underneath the tasks is a space for the participants to enter their answer using smartphone keyboard. The three bulbs are for the hints; participants press the first for hint number one, the second for hint 2 and third one for hint 3. Just after the hints there is check button to check sample solution. The first orange buttons is for the main menu and second orange button directs you to the next task. Participants get feedback immediately and thereafter the sample solution

Appendix 12: Figure (c) School flag pole

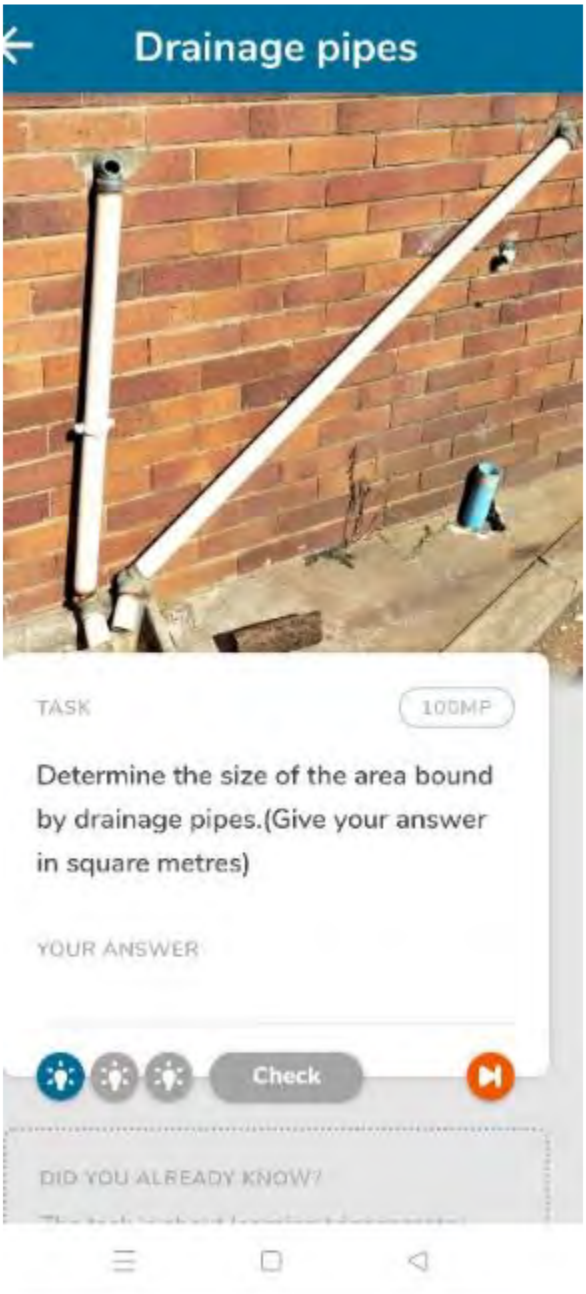
Figure (c): Task 3: **School flag pole**: Calculate an angle of elevation of the school flag pole, from the observer standing 680 cm away from the flag pole. (subtract the observer's height from the flag pole height). Give your answer to one decimal place.



The smartphone screen on the left hand side shows a task; just underneath the tasks is a space for the participants to enter their answer using smartphone keyboard. The three bulbs are for the hints; participants press the first for hint number one, the second for hint 2 and third one for hint 3. Just after the hints there is check button to check sample solution. The first orange buttons is for the main menu and second orange button directs you to the next task. Participants get feedback immediately and thereafter the sample solution.

Appendix 13: Figure (d) Drainage pipes

Figure (d): Task 4: **Drainage pipes**: Determine the size of the area bound by drainage pipes. (Give your answer in square metres)



The smartphone screen on the left hand side shows a task; just underneath the tasks is a space for the participants to enter their answer using smartphone keyboard. The three bulbs are for the hints; participants press the first for hint number one, the second for hint 2 and third one for hint 3. Just after the hints there is check button to check sample solution. The first orange buttons is for the main menu and second orange button directs you to the next task. Participants get feedback immediately and thereafter the sample solution.

Appendix 14: Figure (e) Roof verge boards

Figure (e): Task 5: **Roof verge boards**: Calculate the magnitude of the angle that the two roof verge boards make. (Assume the roof verge boards are not produced/protruding the wall)

The smartphone screen on the left hand side shows a task; just underneath the tasks is a space for the participants to enter their answer using smartphone keyboard. The three bulbs are for the hints; participants press the first for hint number one, the second for hint 2 and third one for hint 3. Just after the hints there is check button to check sample solution. The first orange buttons is for the main menu and second orange button directs you to the next task. Participants get feedback immediately and thereafter the sample solution

Appendix 15: Certificate of Language Editing

35 Narnia Lane

Knysna 6570

Tel: 0835906762

imarion100@gmail.com

2 February 2024

DECLARATION OF PROFESSIONAL EDIT

Document Title: EXPLORING OUTDOOR MATHEMATICS LEARNING FOR CONCEPTUAL UNDERSTANDING THROUGH SMARTPHONES

I declare that I have edited and proofread this document. My involvement was restricted to language usage and spelling, completeness and consistency, referencing style and formatting of headings and layout, captions and Table of Contents. I did no structural re-writing of the content. The writer was provided with the corrections/amendments which required action. The corrected document was subsequently proofread and a number of additional corrections were advised.

The undersigned takes no responsibility for corrections/amendments not carried out in the final copy submitted for examination purposes.

Sincerely,

Marion Pfeiffer

Freelance Copy-editor and Proofreader

Professional Member, CIEP UK

Full member, PEG and SAFREA

Professional EDITORS Group