EXAMINING THE NATURE OF LEARNING WITHIN AN AFTERSCHOOL MATHEMATICS CLUB: A CASE STUDY OF FOUR LEARNERS

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by

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DECLARATION OF ORIGINALITY

I, Penehafo Olivia Kaulinge, student number G12K3134, hereby declare that this thesis is my own work and is the product of my own original research. It has not been submitted in any form for another qualification or any assessment to another University or Institution. Ideas, techniques, quotations, or any other material derived from the work of other people included in my thesis, published or otherwise, have been fully acknowledged in accordance with the Rhodes University Education Department reference guide.

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ABSTRACT

This study examined the nature of learning within an afterschool mathematics club established by the South African Numeracy Chair project. The study sought to establish what sort of progress in mathematical learning occurred in a grade 3 afterschool maths club, using assessment instruments associated with the Learning Framework in Number. The study also sought to understand the nature and effects of mentor mediation in the maths club, using Vygotsky's notion of the Zone of Proximal Development (ZPD) together with the notion and practice of scaffolding.

The study made use of a variety of data collection techniques, including one-to-one assessment interviews, task-based interviews and observations. In line with the case study approach adopted, four learners were selected for interviews. The assessment interview results revealed that, in terms of proficiency in early arithmetical learning, all four learners showed progress after spending four months in an afterschool maths club. Even though they were found to have advanced in their Strategies for Early Arithmetic Learning (SEAL), some of them were observed still using their fingers to support their counting. Such strategies were likely to mirror the teaching approaches used in their usual school mathematical lessons. The overall findings in terms of learners' proficiency and progress give rise to concerns about current number teaching practices in their school, which emphasize the standard written algorithm in the lower primary grades.

The study also made use of Vygotsky's notion of the ZPD to analyse the nature of mentor-peer mediation. Witnessing the learners' use of trial and error strategies during the task-based interview allowed both mentors to support learners through understanding their thinking, prompting them and encouraging them to reflect on their answers and develop more effective strategies. Learners progressed through the ZPD at different paces and in different ways, with 'aha' moments happening at different points for individual learners. Their progression in the ZPD seemed to depend on interaction among all participants, which varied according to what was contributed and what requested by each participant. The findings revealed that although there was evidence of learners achieving success at the tasks in task based interviews there were also there were also some learners who experienced difficulties. Additionally, in order to argue that learning was fully realised within the ZPD would have required follow up task based interviews to assess the extent to which learners were able to complete the tasks independently

without the scaffolding of mentors. This was not possible within the scope of this research but would be useful in future research.

DEDICATION

I dedicate this work to my late uncle Immanuel HelaoNghixulifwa. You never had any doubt I could do this; you encouraged me and provided support in countless ways. When I was too busy or tired to come for a visit, you never complained (at least not too much). My uncle didn't make it through this process, but he knew I could do it, long before I did. He was afraid his illness would stand in the way of my finishing but "Omwene iha kwatelwa ngee ta tenge okwe kudimbula shiveli". Uncle, I know you're looking down and smiling; I did it! Thank you for seeing the best in me.

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Opinions expressed and conclusions arrived at are those of the author and are not necessarily to be attributed to Rhodes University or the sponsors.

LIST OF ACRONYMS

BNWS	Backward Number Sequence
BTS	Base-Ten Arithmetical Strategies
DEST	Department of Education Science Training (Victoria, Australia)
DOE	Department of Education
ECD	Early Child Development
EO	Education Officer
NWS	Number Word Sequences
FNWS	Forward Number Word Sequence
LFIN	Learning Framework in Number
MLA	Monitoring Learning Achievement
MRP	Maths Recovery Programme
NI	Numeral Identification
NWA	Number Word After
NWB	Number Word Before
SANC	South African Numeracy Chair
SEAL	Stages of Early Arithmetical Learning
TIMSS	The Third International Mathematics and Science Study Repeat Survey
ZPD	Zone of Proximal Development

CHAPTER ONE - INTRODUCTION

1.0 INTRODUCTION

Studies have highlighted the fact that South Africa is facing what is termed a "maths crisis" (Fleisch, 2008). This is noted both locally and internationally. In tests on grade 8 pupils, South Africa came last in the 2003 Maths and Science indexes of the Trends in International Mathematics and Science study. South Africa also came last out of 40 countries in the 2006 Progress in International Reading Literacy Study. The poor grounding in numeracy and literacy in primary schools is regarded as a fundamental factor contributing to poor mathematical learning outcomes. Thus improving both the quality of the teaching and learning of mathematics and also the number of students with access to the foundation phase is among the top government initiatives aimed at addressing the numeracy challenge in the primary school.

This research study examines the nature of grade 3 learners' mathematical learning through their participation in an afterschool mathematics club. The afterschool mathematics club is one of the initiatives of the South African Numeracy Chair (SANC). The club consists of 10 learners of mixed ability (5 girls and 5 boys) at a township school in Grahamstown, South Africa.

1.1 BACKGROUND/CONTEXT

When it comes to the foundation skills of numeracy quality of learning outcomes, the mathematics performance of South African learners is amongst the lowest of middle income countries (that is, amongst those countries where such statistics are available). Despite improved matric results after decisive steps were taken to improve quality, there is a body of evidence to show that the quality of mathematics ability in the majority of primary schools remains poor in South Africa.

Moreover, learner performance tests have also revealed that most South African primary scholars have a poor grasp of elementary foundational mathematical concepts, as reflected in the 30% average scores for both the 2001 grade 3 numeracy national tests and the 1999 Monitoring Learning Achievement (MLA-project Department of Education, 1999) grade 4 numeracy project, in which South Africa had the lowest score amongst the 12 African participating countries (OECD, 2008). In the three international and highly regarded TIMSS tests the local grades 7, 8 and 9 maths scores were at the bottom of the maths league tables (Reddy, 2006; Howie, 2001).

Schollar (2008) notes that the majority of South African learners do not have an understanding and knowledge of the basic number bonds (algorithms) and place value in the base-10 number system, and cannot readily understand the meaning of multiplication and division. Results from the TIMSS 2002 grade 8 tests also highlighted that the South African curriculum emphasises the application of mathematics to real-life situations and multicultural approaches at the expense of understanding and mastering basic fundamental mathematics concepts and skills (Reddy, 2006). Such criticism is warranted, given the fact that the official curriculum (Curriculum 2005) in early 2000 was characterised by under-specification of basic knowledge and skills (Reddy, 2006).

South African learners' performance in maths and numeracy in regional, sub-regional, national and international tests is indeed problematic and reveals the dismal maths situation South Africa currently faces.

More than half of South Africa's high school pupils were found to be functionally illiterate and innumerate, and the situation at primary school level was even more dire, with only about 40% of grade 6 pupils meeting the standard for language achievement and 22% meeting the standard for maths. This incompetence was observed mostly in South African grade 8 l earners' performance in mathematics and science in TIMSS in 1995. The study revealed that South African learners' did not have the required basic mathematical knowledge and their scores in mathematics and science were significantly lower than those of other learners in all the other participating countries in the second study (Meier, 2011).

An example from recent classroom-based research indicates that numeracy skills are not being systematically developed in South African primary schools. The third International Mathematics and Science Study Repeat Survey (TIMMS-R) of worldwide trends in respect of scholastic performance in mathematics and science has also confirmed that South African mathematics learners' performance is significantly poorer than the vast majority of other participating countries in tests that measured basic mathematical skills (Howie, 2001, p. 18) . South Africa fared significantly worse than the other two African countries that participated in the survey, Morocco and Tunisia. The study revealed that South African learners did not have the required basic mathematical knowledge. Learners often acquire deficient, superficial, and rote learned knowledge of basic concepts (Maree & De Boer, 2003; Maree & Steyn, 2001; Maree, Aldous, Hattingh, Swanepoel, & Linde, 2006). The study further highlighted that learners did not only show a lack of understanding of mathematical and science questions but also an inability to communicate their answers verbally and in writing (Howie, 2001, p. 150).

Another study, the MLA project that was conducted in several African countries in 1999, measured the competencies of grade 4 learners in numeracy, literacy and life skills. South Africa's performance in all three areas indicated serious shortcomings compared to the other participating countries. Of the 12 participating countries, South Africa scored the lowest average in numeracy, the fifth lowest in literacy and the third lowest in life skills.

A national learner assessment programme was conducted by the Department of Education in 2004 (DoE, 2004) and focused on grades 3 and 6. Learners were assessed at grade 3 level in literacy, numeracy and life skills, and grade 6 learners were assessed in English and Mathematics. The performance of learners in these tests was extremely disappointing in all learning areas (Howie, 2001). The achievement rates of learners in the grade 6 evaluations were even worse than those in grade 3, with learners obtaining an average of 38% in language, 27% in mathematics and 41% in Natural Sciences.

In response to the poor performance of learners (particularly in numeracy) in primary education, the Department of Education launched numerous ambitious intervention initiatives and assessment programmes: the MLA Project (1999); the Quality Improvement, Development, Support and Upliftment Programme (2005); the Quality Learning and Teaching Campaign (2008); Southern and Annual National Assessment (ANA) (DoE, 2011). One of the main aims of the Department of Education was that by the year 2011, all learners would be able to demonstrate age-appropriate levels of literacy and numeracy in South African schools (Meier, 2011). In all these interventions, the key focus was on what learners should be learning in order to improve achievement levels (Meier, 2011).

In this study, the focus is on one particular initiative, taken by the Department of Science and Technology Education and private funders together with the National Research Foundation, which aims at developing several interventions to strengthen the foundation skills of numeracy among South African learners. In 2010, the government announced that the first priority would be to improve the quality of basic education, specifically in numeracy and literacy (Department of Basic Education [DBE], ANA-report, 2011). The establishment of Numeracy Chairs in universities across South Africa is an example of the various initiatives funded by these national research organisations to propel this agenda of quality basic education.

Since my research is conducted within one of the South African Numeracy Chair (SANC) afterschool maths clubs, I now turn to discuss background information about the SANC and the club.

1.2 SOUTH AFRICAN NUMERACY CHAIRS

The South African Numeracy Chair (SANC) at Rhodes University is one of several Chairs set up by the National Research Foundation in an attempt to improve the quality of teaching of in-service numeracy teachers at the foundation phase. The aim is to improve learner performance in primary schools through quality teaching and learning practices (Graven, 2011). One of the challenges that primary teachers face is that learners lack the necessary mathematical foundation to be learning effectively at the grade level in which they are placed (Graven & Stott, 2012 in process). Hence the Numeracy Chair at Rhodes University introduced an afterschool maths club programme, which involves regular learner intervention as one way of addressing the challenges (Graven, 2011).

This Chair project brings together a growing research community of both full-time and part-time doctoral and masters students. I am a full-time master's student conducting research within this project.

1.3 AFTERSCHOOL MATHEMATICS CLUBS

Maths clubs may be described as informal, afterschool, extra-curricular clubs that focus on developing a supportive learning community of inquiry, where learners' mathematical participation, engagement and sense-making are fore grounded (Graven & Stott, 2012). From this definition, one can deduce that maths clubs are viewed as a second site of learning where learners should be enabled to gain a surer grasp of mathematics and a chance to engage and participate actively in mathematical activities in a less structured environment.

While Graven (2011) has argued that maths clubs can offer opportunities for the creation of active engagement, negotiation and participation for learners, and where they can live out different stories, she cautions that this "is not to deny the need for mathematics classrooms to provide the opportunity for more participatory and positive learner identities." Graven stresses that the extra-curricular nature of such clubs might provide more freedom to focus on the deliberate construction of positive participatory mathematical identities, at the expense of covering the range of skills and knowledge required to 'get through' the curriculum (Graven & Stott, 2012, p. 96).

The empirical site for my research is one of the afterschool maths clubs within the SANC project. According to Graven (2011), the clubs are conceptualized as informal places where learning can take place during out-of-school-time. A pilot study of one club that took place from September to November 2011 (Graven, 2011) showed promising results in terms of learners' mathematical progression through participation in the club.

1.4 SIGNIFICANCE OF THE STUDY

The profound and persistent inequalities within our society, the poor quality of teaching and learning mathematics, and the poor mathematical performance of the majority of South Africans and Namibians, are the facts that constitute the rationale for this study. As part of the research community at the SANC at Rhodes University, I chose to carry out my study within an afterschool maths club. It was an ideal opportunity for getting involved in an activity that resonates with my work as an Education Officer (EO) for maths and science in Namibia. As an EO, I am responsible in part for new developments in the education system in Namibia.

As part of the SANC research community, my study aims to contribute to the growing area of research seeking solutions to the many numeracy education challenges faced in South Africa. The study aims to contribute to the literature about learning within afterschool mathematics programmes, particularly to help remedy the paucity of literature relating to young learner programmes. Graven (2011) points out that there is very little literature pertaining to learning within math clubs, especially regarding early learners. Both Mathematics Education Chairs at Witwatersrand University and Rhodes University run afterschool maths interventions as a key part of their research and development work. Additionally, while there is little research in this field in South Africa, there is none at all in Namibia. Thus the value of this study lies not only in my personal academic growth but, more importantly, in its capacity to provide insights into how to improve mathematics education in Namibia in the future.

Furthermore, in the area of numeracy learning and afterschool mathematics clubs, especially in primary grades in South African and Namibian schools, little is known about the actual number knowledge of children and what strategies they use to solve number problems. Although the government is insisting that learners spend more time doing mathematics, there is not much information on how one might achieve this. Thus I am hoping that the results of this study might reveal implications for an afterschool education policy both in Namibia and South Africa. For example, I have noted that most foundation phase learners stay at school until 14h00, and thereafter either go home or play in the playground. This time could possibly be used more effectively for extended mathematics instruction.

1.5 STATEMENT OF PURPOSE

Given the background of the problem of teaching and learning in the context of the South African 'maths crisis', I realised I needed to help to improve mathematical learning in primary grades. This study sets out to examine the nature of grade 3 learners' mathematical learning in an afterschool mathematics club. The study is therefore located in the broad area of searching for ways of improving learners' mathematical understanding.

1.6 RESEARCH QUESTIONS

The following specific research questions guided the study:

What is the nature and the extent of progress in mathematical learning (if indeed there is any) in a grade 3 afterschool maths club, and how do the SANC project assessment instruments enable me to assess this?

What is the nature of mentor to peer mediation in the afterschool maths club?

1.7 THE STRUCTURE OF THE THESIS

This research report consists of seven chapters. In this chapter I have discussed, among other things: the background of the study, the purpose of the study and its research questions, the rationale for and the significance of the study.

In Chapter 2 I offer a review of the literature that helped to shape and inform the study, and in Chapter 3 I discuss the theoretical perspective of the study. The theoretical framework is based on social constructivist and socio-cultural perspectives on learning. Other key features of this chapter are the notions of scaffolding and Vygotsky's Zone of Proximal Development.

Chapter 4 describes the study's design and the methods used in data collection and analysis.

The rationale for the choice of the methods and their appropriateness is explained to indicate how the study was conceptualized. An outline of key issues relating to ethical compliance is given, and issues of validity and reliability are discussed.

In Chapter 5 I present analyses of the quantitative data that was collected from the proficiency assessment interview. The learners' progression in terms of the various mathematical aspects demonstrated in the oral interview are highlighted and discussed quantitatively.

In Chapter 6 I present analyses of the qualitative data that was collected through observation and task-based interviews during the maths club sessions, and in Chapter 7, I draw conclusions about the nature of learners' mathematical learning in the case study of the afterschool maths club. In so doing I summarize the findings of the study. I also reflect on the value of my research and elaborate on the experience and knowledge I gained while working on this study. The weaknesses and limitations of the study are discussed.

CHAPTER TWO - LITERATURE REVIEW

2.0 INTRODUCTION

This study sought to analyse the nature of learning in an afterschool setting, in order to identify strategies that provide learning opportunities for students participating in a maths club. The main topics explored in this section concern the nature of afterschool programmes, specifically afterschool maths clubs; what is meant by mathematical proficiency, and the assessment of learners' mathematics or numeracy proficiency progress. The literature consulted will be useful in responding to both research questions.

A review of previous studies also provides a researcher with useful points of reference for identifying methods and techniques of gathering information. In light of the above, I reviewed the literature relating to the five main aspects of the study, namely:

2.1. Contextual overview: Mathematics education in South Africa
2.2. Need for extended learning time on numeracy
2.3. The nature of the afterschool mathematics club in this study
2.4. Numeracy and becoming mathematically proficient
2.5. Mathematics assessment

2.1 CONTEXTUAL OVERVIEW

2.1.1 Mathematics Education in South Africa

A major policy objective of the South African Department of Education is to provide all young people in South Africa with a strong foundation in numeracy and English literacy skills (DoE, 2011). The Department of Education has stated that "South Africa's economic survival depends largely on training sufficient numbers of secondary school learners and post secondary school learners in the key subjects of science and technology" (Delvare, 1995, quoted in Maree, et al., 2006, p. 7). The importance of sound numeracy skills for young children has also been widely acknowledged elsewhere (see, for example, Australian Association of Mathematics Teachers, 1997; Numeracy Task Force, 1998).

In South Africa and in other countries, the fact that careers depend on mathematics at the highest institutional level is recognized. Among the DoE's top priority targets was to increase the number of grade 12 learners passing Mathematics and Physical Science by improving the percentage of learners in grades three, six and nine in public schools who obtain the minimum acceptable mark in the National Assessments for Language and Mathematics (DoE, 2012). To support this, the DoE has introduced a strategic plan for 2011 to 2014. Within this the Department of Basic Education has established a four-pronged approach. Improving the quality of teaching and learning with a focus on literacy and numeracy and increasing access to, and performance in, mathematics and science is one focal point. This entails increasing access to quality materials and providing competent and professional teachers. Improving educational quality in schools and, specifically, improving learning outcomes, stands out as one of the greatest challenges. Without substantial improvements in learning outcomes, the future development of South Africa will be seriously compromised. As a result, there is increasing public pressure and strong emphasis by the government on improving learning outcomes. In particular, there is increased emphasis on better quality learning and teaching in early childhood development (ECD) and primary schooling. There is also a demand for improved performance and better learning outcomes in key subjects such as mathematics, science, technology and languages. The SANC at Rhodes University where my research is located is one strand of the growing research and development response to this demand.

A study done by Carnoy, et al. (2011) in South Africa found that the teaching of procedures is given most time in the lessons observed, with higher level thinking and reasoning skills accorded the least attention. Moreover, lessons are characterized by short explanations of concepts at the beginning of the class. This is followed by the teacher's explanation of the procedures (algorithms) required, with the focus on producing a correct answer. The study found that most lessons end with learners repeating the procedures individually or in groups. The study points out that although the efficient use of procedures is among the skills needed by a child in order to be proficient in mathematics, it is not sufficient for learning more advanced concepts. The study therefore recommends that in their lessons teachers should also teach why the procedures work and how to apply them in different areas within and outside mathematics. For this, teachers need to spend more time on each lesson and have adequate material to facilitate the exploration of concepts (Carnoy, et al., 2011).

2.1.2 What content knowledge will be learnt by the Foundation Phase Mathematics learner in the South African education system?

The *National Curriculum Statement Grades R - 12* (DoE, 2011a) sets out what are regarded as the knowledge, skills and values worth learning. It aims to ensure that learners acquire and apply knowledge and skills in ways that are meaningful to their own lives. In this regard, the curriculum promotes the idea of grounding knowledge in local contexts, whilst being sensitive to global imperatives.

Mathematics in the Foundation Phase covers five main content areas. Each content area contributes towards the acquisition of specific skills. The table below shows the general content areas of each topic as well as the specific content for grades 1 to 3. The skills which Foundation Phase learners are required to demonstrate are included in the second table, below.

In the general notes for grade 1-3 mathematics, the Curriculum and Assessment Policy Statement (CAPS) document (DoE, 2011) recommends the following: all concepts should be thoroughly taught before moving on to the next; the learners' abilities should be taken into account when deciding among possible alternative teaching methods; activities should be designed to consolidate knowledge on a concrete, semi-concrete and abstract level.

Table 2.1 Foundation Phase Mathematics Content Focus

Content area	Generals content focus	Foundation phase specific content focus
Numbers, Operations and Relationships	Development of number sense that includes: the meaning of different kinds of numbers; relationship between different kinds of numbers; the relative size of different numbers; representation of numbers in various ways; and the effect of operating with numbers	The range of numbers developed by the end of Grade 3 includes whole numbers to at least 1 000, and unitary and non-unitary common fractions. The number concept of the learner is developed through working with physical objects in order to count collections of objects, partition and combine Quantities, skip count in various ways, solve contextual (word) problems, and build up and break down numbers. Therefore, the teacher should: give the learner opportunities to work with number cards, number charts and number lines to recognize, read, write, count and order numbers; encourage the learner to say number names out aloud and to write numbers in words and symbols; give the learner opportunities to practice mental calculations; structure activities with an awareness that the learner can count by rote and recognize and say number names and symbols for larger numbers than the learner is able to use for calculation and solving problems; encourage the learner to appreciate and learn number names and symbols in the mother tongue (if not the language of instruction) and at least one other local language; and encourage the learner to judge the reasonableness of answers.

Table: 2.2 what is that expected to be covered in grades 2 and 3

	Grade 2	Grade 3
Whole numbers	Counts to at least 100 everyday objects reliably. Count forwards and backwards in 1s, 2s, 3s, 4s, 5s and 10s between 0 and 200. Recognizes number symbols and names between 0 and 200. Order, describe and compare 2-digits whole numbers. Represent odd and even numbers Recognize the place value to at least 2- digits. Add and subtract whole numbers with answers up to 100. Multiplication of at least 2-digit by 1- digit numbers Division of at least whole 1-digit by 1- digit numbers. Mental calculations with whole numbers including: Addition, subtraction and multiplication to at least 20 Problem-solving involving whole Numbers Uses the following techniques: a) building up and breaking down numbers; b) doubling and halving; c) using concrete apparatus e.g. counters; d) number lines Explains own solutions to problems. Checks solutions to problems given by peers.	Count forwards and backwards in 2s, 5s and10s between 0 and 500. Count forwards and backwards in 20s, 25s, 50s and 100s between 0 and 1 000. Recognize and identify number names and symbols between 0 and 1 000. Order, describe and compare 2-digit and 3-digit whole numbers. Represent odd and even numbers Recognize the place value of 3-digits. Round off to the nearest 10 Add and subtract whole numbers with at least 3 digits. Multiplication of at least 2- digit by 1-digit numbers Division of at least whole 2- Mental calculations with whole numbers including: Addition, subtraction, multiplication and division to at least 50 Problem-solving involving whole numbers Building up and breaking down numbers a) building up and breaking down numbers; b) doubling and halving; c) number lines d) rounding off in tens. Explains own solutions to problems. checks solutions to problems given by peers

I place these tables here to show what is intended to be learned by learners in the grade they are currently in and what they are supposed to have learned in the previous grade. This is useful for my study as it helps me when it comes to analysing the learners' mathematical proficiency progression.

2.2 NEED FOR EXTENDED LEARNING TIME ON NUMERACY

Studies have revealed that countries like South Africa have less instructional time during the day than countries like Japan, France, and Australia. For example, a study by Carnoy, et al. (2011) indicates that South African learners have an average of 52 lessons appearing in their notebooks compared to Botswana's average of 80 (Graven & Stott, 2012). Additionally Carnoy et al. argued that there should be an increase in South African learners' lesson time average, projecting a 6% gain in numeracy results (Graven & Stott, 2012). The DoE requested that educators take a closer look at how students spend their time while at school and at the amount of time they spend on schoolwork. The amount of time spent on mathematics has a decisive impact on learners' development of mathematical concepts and skills. The activities learners engage in should, however, not be "keep busy" activities, but should be clearly focused on mathematics as it is outlined in the curriculum (DoE, 2011).

Fashola (2003) argues that it is extremely worrisome that, generally speaking, there is no support system at home to reinforce skills acquired at school. Fashola (2003) found that students benefit most from extended school time, and recommends that school operating hours are extended and that the community and schools work together during this extended time. Maths clubs are designed to fill such a gap. They also provide the extended learning time that some students need in order to comprehend and practice skills introduced during the regular school day (Cosden, Morrison, Albanese, & Macias, 2001, cited in Fashola, 2003).

There are other studies in which scholars stress the importance of afterschool programmes as a way of extending time for learners, maths clubs being one of them. Continuous exposure to critical math concepts allows students to develop an understanding of these concepts over time. It also allows them to learn at their own individual pace and to continually review their work as they move through the programme. Such an approach has proven to be an effective way to maintain skills and develop an understanding of critical mathematical concepts (Baratta-Lorton, 1994).

2.3 THE NATURE OF THE AFTERSCHOOL MATHEMATICS CLUB IN THIS STUDY

In this section, I describe the afterschool maths club that forms the empirical field for this research study. I also attempt to delineate the advantages that afterschool maths clubs might provide for teaching and learning mathematics.

It has been suggested that for the effective learning of mathematics to occur, classes should provide opportunities for pupils to talk, be listened to, receive feedback, explain their knowledge, thinking and methods, and to suggest alternative ways of tackling problems. This is in line with the way in which the SANC maths clubs are conceptualized, as informal, extra-curricular clubs focused on developing a supportive learning community, where learners' active mathematical participation, engagement and sense making are the focus. Individual, pair and small group interactions with mentors are the dominant practices, with few whole class interactions (Graven & Stott, 2012).

According to Graven (2011), the clubs are conceptualised as informal places (see Table 2.3, below) where learning can take place in an out-of-school-time. This is one of the major features of the clubs (Graven & Stott, 2012). Maths clubs are thus viewed as a second site of learning, where learners are given the opportunity to make sense of their mathematics and granted the chance to participate actively in mathematical activities in a less structured environment. The table below, published in Graven and Stott (2012, p. 2), summarizes the differences between the club and the classroom environment.

Table 2.3 Formal vs. informal learning environments

Formal maths classroom / environment	**Informal club / maths environment**
Participation is expected as part of formal schooling (in-school-time)	Voluntary participation during out-of-school time
Less learner choice over the activities that they work on and engage with	More learner choice over the activities that they work on and engage with
Curriculum and assessment standards as a prescriptive framework strongly influencing choice of content and activities	Curriculum as contextual guide for what is nationally expected of learners but individual learner numeracy levels guide content and activities
Largely acquisition based and often driven by teaching for/to assessments	Participation based, where participants are active and engaged
Teacher led and much whole class teacher learner interaction	Many interactions are learner led with few whole class-mentor interactions and many one-to-one interactions between mentors and learners.
Assessment tends to be summative and results in ranked performance	Assessment is formative and integrated and is used to guide individual learning experiences for the participants
Prescriptive, teacher controlled classroom rules within general school rules	Negotiated socio-mathematical norms (Cobb, 1996 and Hunter, 2008) which may differ from in-school time rules

A second aim of the clubs is to promote mathematical proficiency. The conceptualization of the clubs, according to Graven and Stott (2011), has brought together the notions of participation and acquisition (Sfard, 1998). According to Graven and Stott (2012), the blended notion is by no means particular to the clubs and various studies provide mathematical examples where the two notions are being blended (Askew & Brown, 2004; Goos, Galbraith, & Renshaw, 1999; Jaworski & Potari, 2009). This focus on mathematical proficiency and evolving participation in the clubs is parallel to the SANC research focus (Graven & Stott, 2012).

2.4 NUMERACY AND BECOMING MATHEMATICALLY PROFICIENT

Before presenting the literature on the framework used in assessing primary learners' mathematical proficiency progression, I will first define what it me ans to be mathematically proficient. Kilpatrick, Swafford, & Findell (2001), observed that mathematical proficiency provides a way to think about mathematics learning in that it encompasses the key features of knowing and doing mathematics. Mathematical proficiency implies expertise in handling mathematical ideas. Learners who are mathematically proficient "understand concepts, are fluent in performing operations, exercise a selection of strategic knowledge, reason clearly and maintain a positive outlook towards mathematics" (Kilpatrick, et al., 2001, p. 116).

Graven and Stott (2011), argue that, given that the SANC projects work towards improving numeracy proficiency among learners, this same definition may serve as a guide for the design of club activities and assessment instruments. Graven and Schafer (2011), drawing on Askew, Brown, Rhodes, Johnson, & William (1997) and Kilpatrick, et al. (2001) define mathematical proficiency as:

> the ability to process, communicate and interpret numerical information in a variety of contexts overlaid with strands of numeracy proficiency: understanding numeracy concepts, computing fluently (practically, mentally and procedurally), applying concepts to solve problems (in creative and inventive ways), reasoning logically (in creative and inventive ways) and engaging with mathematics – seeing it as sensible, useful and do-able (enjoyment and passion) (p. 20).

According to the Kilpatrick, et al. (2001), mathematical proficiency is measured by the presence of the five strands that together form the mathematical proficiency variable viewed as necessary to learn mathematics successfully. The five strands of mathematical proficiency are:

1. *Conceptual understanding* – comprehension of mathematical concepts, operations, and relationships.
2. *Procedural fluency* – skills in carrying out procedures flexibly, accurately, efficiently, and appropriately.
3. *Strategic competence* – ability to formulate, represent and solve mathematical problems.
4. *Adaptive reasoning* – capacity for logical thought, reflection, explanation, and justification.
5. *Productive disposition* – habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

These strands should be seen not as individual goals but rather as interdependent and interwoven elements within a rich definition of proficiency. If any one of the five aspects is missing, the learning process is affected. Learners should be able to use the five strands of mathematical proficiency in an integrated manner, so that each strand reinforces the others. According to Kilpatrick, et al. (2001), promoting one strand while neglecting the others "has an implication as to how learners acquire mathematical proficiency, how teachers develop that proficiency in their students and how teachers are educated to achieve that goal" (p. 5). Since the strands are "interwoven and interdependent" (Kilpatrick, et al., 2001, p. 380) good teaching practices should not focus on just one or two but on all of them.

The importance of the notion of mathematical proficiency to this study is considerable as it provides a framework and language for describing learners' numeracy proficiency learning trajectories over the six-month research period, so as to address research question one. This is done by relating the strands of proficiency to the mathematical proficiency interview described in the methodology chapter.

2.5 MATHEMATICS ASSESSMENT

> *He who wishes to impart something to someone else will also want to find out what the other already knows, in order to build further upon this. And, if he has taught something, he will want to find out whether this has taken root. (Freudenthal, 1976a, quoted in Citation, 2010, p. 33); translation of original Dutch text)*

For the purposes of this study, I reviewed a wide range of research on early numeracy, including Cobb & Wheatley (1988); Steffe & Cobb (1988); Steffe (1992); Mulligan (1998), and others, but found the Wright, Martland and Stafford (2006) framework described in this section to be the most useful for my purposes. I used this framework as a data collection tool as well as the analytic tool for profiling the four learners' proficiency progression.

The assessment framework used in this study is called the Learning Framework in Number (LFIN). The LFIN helped me to obtain detailed information about the four learners' mathematical progression while they were participating in the afterschool maths club. The LFIN is a research-based framework for the assessment and teaching of number in the early years of schooling. The framework was developed and is used in the Maths Recovery Programme (MRP) by Professor Bob Wright and his colleagues (1992; 1996; 2006 & 2012). Since this is the primary instrument used to address my research question one, I expand upon it in the next section.

2.5.1 Mathematics Recovery (MR) Programme

Wright (2003) describes the MR programme as an approach to early number that comprises an interview-based assessment, documenting learners' current knowledge and administering instruction. As a foundation for assessment and teaching, the Mathematics Recovery Programme uses a Learning Framework in Number. MR is a programme that focuses on intervention and aligns strongly in terms of both theory and practice with the current cutting-edge approach to the mainstream classroom teaching of number in the early years. This implies that MR is applicable in every classroom context regardless of learners' ability.

The programme has its theoretical origins in the research of Les Steffe in the 1970s, '80s and '90s (Wright, 2003) and was developed by Bob Wright in Australia in the early 1990s (Phillips, Leonard, Horton, Wright, & Stafford, 2003). The programme is described as one of the few programmes developed to address young children's chronic failure in mathematics using an individualized teaching approach (Dowker, 2003). The Mathematics Recovery programme includes an intensive and extensive programme for teacher professional development (Wright, 2000). The four key features of this programme are early intervention, assessment, teaching and professional development (Wright, et al., 2006).

The MR programme has a detailed approach and uses specific tools to assess children's early number strategies and knowledge. It involves a one-on-one interview in which the learner is presented with groups of tasks, and where each group relates to particular aspects of early number teaching (Wright, 2003). The MR assessment is videotaped, thus there is no requirement for the interviewer to make written records of observations during the course of the interview. This prevents 'down time' for the student while s/he waits for the teacher to create some form of record (Wright, 2003). Video technology could be used to record the assessment and the teaching sessions for re-examination and later analysis, but in my study I chose to make use rather of my written record of observations made during the course of the interview. On the whole I found the extensive notes I made sufficient, but in retrospect there were times when reviewing the video would have been helpful.

According to Wright (2003), assessment is critical for ongoing learning and the development of teaching. He proposes two important broader purposes of assessment in early learning. First, assessment is able to provide a detailed description of the student's current knowledge of early number. He regards this as the bigger picture which is necessary in terms of the aspects of early number knowledge that can reflect on the assessment schedule. Secondly, assessment should lead to determination of the learner's level of understanding on the relevant tables in the framework of assessment and learning. The MR programme uses a model, developed from Steffe's research, of the stages of early arithmetical learning, and a model of the levels in base-ten arithmetical strategies (Cobb & Wheatley, 1988 quoted in Wright, et al., 2006).

One of the key elements of the MR programme is its LFIN framework for assessment of learning and teaching (Wright, et al., 2006). In this study, I was interested to know what the nature of the four learners' mathematical progression was (if there was any at all) through their participating in an afterschool mathematics club at one of the local schools. For this, I used the Wright, et al.'s (2006) assessment interview adopted by the SANC project, but adapted to include some assessment items from Askew, Brown, Rhodes, Johnson & William (1997) (see Appendix A). This individual interview assessment involves recording the learner's responses on a schedule. Overall the assessment interview and recording schedule includes 24 tasks, of which two thirds are from Wright, et al.'s (2006) MRP, while one third (namely: 2, 10, 11, 13, 14, 18, 19 , 23 and 24) are derived from Askew, et al. (1997).

Since the LFIN analytic framework is used as a data collection tool and for the analysis of all items assessed I expand upon it in the next section.

2.5.2 Learning Framework in Number (LFIN)

Learning frameworks provide a description of skills, understandings and knowledge in a sequence in which they typically occur, giving a virtual picture of what it means to progress through an area of learning (Bobis, Mulligan & Lowrie, 2009). In other words, they provide a pathway or map for monitoring individual development over time. The LFIN is an essential component of the MR programme. One of the important functions of the LFIN is to enable summative profiling of students' knowledge, typically on five aspects of early number teaching and learning (Wright, 2003; Wright, et al., 2006). The profiling shows in clear and simple terms how the student is progressing over time.

The framework is divided into four parts, A, B, C and D, and into 11 aspects or categories. Within those parts, the following six aspects are used in this study: Part A: Stages of Early Arithmetical Learning (SEAL) and Base-Ten strategies (BTS); Part B: Numeral Identification

(NI), Forward and Backward Number Word Sequences (FNWS/BNWS), Number Word After (NWA) and Number Word Before (NWB); Part C: Combining and Partitioning and subitizing; Part D. early multiplication and division (see Wright, et al., 2006).

The LFIN provides a description of the knowledge and skills characterizing major stages of development in each of these categories (Wright, et al., 2006). The stages are used as descriptions to profile students' knowledge in each key component. Such information then provides instructional guidance as to where each student needs to progress. An important step in a teacher's ability to utilise the framework in their instructional decision-making is their understanding of how all the components are interrelated (Bobis, et al., 2009). B obis, et al. (2009) add that a student's location in a framework can be utilized as a guide in determining the types of learning experiences that will be most useful in meeting the student's individual needs at that particular stage in their learning.

2.5.2.1 PART A: Stages in Early Arithmetical Learning

During the last three decades, the prevailing view of the teaching and learning of mathematics has shifted from being one of 'transferring knowledge' to one of 'constructing knowledge'. Along with this shift, the importance of counting in the development of children's strategies for solving number problems has been highlighted (Wright, et al., 2006). Table 2.4, be low, shows the model for the stages in Early Arithmetical Learning that was consequently developed.

Table 2.4 Model for the stages in early arithmetical learning (Source: Wright, et al., 2006)

Stages		Name Characteristic
0	Emergent counting	Cannot count visible items. The child might not know the number words or might not coordinate the number words with the items
1	Perceptual counting	Can count only visible items start from 1. Including seeing, hearing and feeling
2	Figurative counting	Can count concealed items but with at redundant (even after being shown that two groups of sweet each is 20 and you ask what is the answer the learner will still go and recount again.
3	Initial number sequence	Initial number sequence. The child count on rather than counting from one, to solve + or missing addends. May use the counting down to solve removed items. (count-back-from)
4	Intermediate number sequence	Count-down-to to solve missing subtrahend (eg. 17-3 as 16, 15 and 14 as an answer. The child is able to use a more efficient way to count down-from and count down-to strategies (count-back-to)
5	Facile number sequence	Uses of range of non-count-by one strategies. These strategies such as compensation, using a known result, adding to 10. Commutativity, subtraction as the inverse of addition, awareness of the 10 in a teen.

A necessary part of determining a child's strategies is to present the child with tasks or situations that are problematic for him or her. In such situations the child's goal, we assume, is to resolve the difficulty, i.e. to solve the problem as he or she constructs it. In doing so the child uses a current strategy or may construct a strategy which is novel for that child. Then the child's stage in early arithmetical learning (SEAL) is determined as shown in Table 2.1. SEAL is considered by Wright, et al. (2006) as the most important aspect of the LFIN.

2.5.2.3 PART A: Base-ten arithmetical strategies (BTS)

According to Wright, et al. (2006), children do not initially construct ten as a unit composed of ones (only later are they able to see this). Learners who attain stages 3, 4 and 5 of the SEAL are typically regarded as beginning to develop knowledge of the tens and ones structure of the numeration system. However, according to Wright, et al. (2006), learners are capable of solving addition and subtraction tasks involving two-digit numbers (that is from 10 onward) prior to their developing knowledge of the tens and ones structure. Table 2.5 below, shows the model used for the development of Base-Ten Arithmetical Strategies.

Table 2.5 Model used for development for development of Base-Ten Arithmetical Strategies (Source Wright, et al., 2006, p.9)

Level		Name Characteristic
1	Initial concepts of ten	Not able to see ten as a unit composed of ten ones. The child solves tens and ones tasks using a counting-on or counting-back strategy.
2	Intermediate concepts of ten	Able to see ten as a unit composed of ten ones. The child uses incrementing and decrementing by tens, rather than counting-on-by-one to solve uncovering board task. The child cannot solve addition and subtraction tasks involving tens and ones when presented as horizontal written number sentences
3	Facile concepts of ten	Child is able to solve addition and subtraction tasks involving tens and ones when resented as horizontal written number sentences by adding and/or subtracting units of tens and ones

2.5.2.4 PART B: Numeral identification (NI)

Numerals are the written and read symbols for numbers. Learning to identify, recognize and write numerals is an important part of early arithmetical development (Wright, et al., 2006).The NI is further described as written and read numerals e.g. "3", "4". Most children learn about numerals a little bit later than they learn about number words and number word sequences. They learn about numerals in almost the same way as they learn about letters of the alphabet. Learning to identify numerals is one of the important parts of early numeracy development (Wright, et al., 2006). The term ‘identify’ in LFIN refers to the request to “state the name of a displayed number”. One of the important aspects assesses in my study was learners’ ability to identify numerals or read number symbols, that is, to state the name of a displayed numeral correctly. This was determined using the outlined model for development of Numeral Identification (source Wright, et al., 2006, 2012), as in Table 2.6, below.

Table 2.6 Model for development model for development of Numeral Identification (Source: Wright et al, 2006, p.12)

Level		Name Characteristic
0	Emergent	Not able to identify some or all 1-digit numerals
1	One digit numerals	Able to identify 1-digit numerals
2	Two-digit numerals	Able to identify 2-digit numerals
3	Three-digit numerals	Able to identify 3-digit numerals
4	four-digit numerals	Able to identify 4-digit numerals and beyond

Wright and his colleagues argue that determining children’s levels is straightforward if based on whether a child makes errors or not. If a child makes no errors at all with two-digit numerals but makes an error with three-digit numerals, s/he will be judged to be at Level 2. Wright, et al. (2006), draw a distinction between counting and reciting a sequence of number words. While counting usually occurs in a problem-solving situation, merely saying a sequence of number words is not regarded as counting. Moreover children’s ability to say a sequence of number words is a prerequisite for developing the ability to count.

2.5. PART B: Forward Number Words Sequence (FNWS) and Number Word After (NWA)

FNWS refers to the learners’ ability to count forward a sequence of numbers as well as being able to identify numerals (Wright, et al., 2006). To assess learners’ FNWS, learners are presented with numbers and asked to start counting forward (e.g. 1 to 32) or to say the number that comes after “5”. To be able to count on, children need to be able to say forward number word sequences correctly without missing any number. Table 2.7, below, shows the model for construction of Forward Number Word Sequences.

Table 2.7 Model for Construction of Forward Number Word Sequences (Source Wright et al, 2006, p.9)

Level		Name Characteristic
0	Emergent	Not able to produce Forward Number Word Sequence
1	Forward sequences in the range 1 to 10	Able to produce the Forward Number Word Sequence from 1 to 10
2	Forward sequences in the range 10 to 30	Able to produce Forward Number Word Sequences in the range 10 to 30
3	Forward sequences in the range 30 to 50	Able to produce Forward Number Word Sequences in the range 30 to 50
4	Forward sequences in the range 50 to 100	Able to produce Forward Number Word Sequence in the range 50 to 100

5	Forward sequences in the range beyond 100	Able to produce Forward Number Word Sequence in the range beyond 100

Determining children's levels in forward and backward number word sequence is also straightforward, based on whether a child makes or doesn't make an error (Wright, et al., 2006; 2012). Wright, et al. (2006) add that it should be noted if the child seemed hesitant or was not fluent.

2.5.2.6 PART B: Backward Number Word Sequence (BNWS) and Number Word Before (NWB)

Similarly, to be able to count backwards, children need to be able to say correctly backward number word sequences without missing any number. Children's levels were determined according to the model for the construction of backward number word sequences as shown in the table below. Table 2.8, below, shows the model construction of Backward Number Word Sequences.

Table 2.8 Model for Construction of Backward Number Word Sequences (Source Wright et al ,2006, p.11)

Level		Name Characteristic
0	Emergent	Not able to produce Backward Number Word Sequence
1	Backward sequences in the range 1 to 10	Able to produce the Backward Number Word Sequence from 10 to 1
2	Backward sequences in the range 10 to 100	Able to produce Backward Number Word Sequences in the range 1 to 30
3	Backward sequences in the range 100 to 1000	Able to produce Backward Number Word Sequences in the range 1 to 50
4	Backward sequences in the range beyond 1000	Able to produce Backward Number Word Sequence in the range 1 to 100
5	Backward sequences in the range beyond 1000	Able to produce Backward Number Word Sequence in the range beyond 100

2.5.2.7 PART C: Combining and partitioning

Counting strategies are regarded as a vital component of learners' early number learning because they can form an important basis for the further learning of numbers (Wright et al., 2006). Children also develop knowledge of simple additions, e.g. involving two addends in the range one to five, which does not rely on counting. According to Wright (2000, p. 706), although teaching children to provide answers immediately to questions might be regarded as habituated, recent research provides strong indications that teaching children to habituate simple addition facts through combining and partitioning small numbers can significantly facilitate the development of advanced arithmetical strategies such as non-counting-by-one strategies (Wright, et al., 2006). The LFIN provides no pr otocols for profiling children's stages or level for this specific aspect according to the LFIN framework. Therefore for my study I made use of the scores obtained in the assessment interview to trace the four learners' proficiency progression over the four-month period.

2.5.2.8 PART C: Spatial Patterns and Subitizing

Experiences with spatial patterns can be important in early number (Wright, 2000). Children's strategies relating to this aspect of early number arise in settings involving spatial configurations of various kinds, such as random arrays. Von Glasersfeld (1992), cited in Wright, et al. (2006) describes the aspect of subitizing as the immediate, correct assignation of number words to small collections of perceptual items. He explains that when a young child says, for example, three, in response to a briefly displayed spatial array, the child may be doing no more than recognizing and naming a spatial arrangement.

For the aspects of combining and partitioning, there is no model for profiling children's stages or levels in the LFIN framework. Consequently for my study I made use of the scores obtained in the assessment interview to trace the four learners' proficiency progression over the four-month period.

2.5.2.9 PART D: Early Multiplication and Division

While early arithmetical strategies form the central structure of the framework, MR argues that other aspects such as multiplication and division should not be seen as compartmentalized or be developed in isolation. According to Wright (2000), in its initial form the LFIN did not address the aspects of multiplication and division; but in its revised form it includes sections written by Mulligan (1998) focusing on young children's strategies in multiplication and division. Children's levels were determined according to the model for the construction of multiplication and division strategies adapted from Mulligan (1998), as shown in Table 2.9, below.

Table 2.9 Model for construction of multiplication and division strategies (Source Wright, et al. ,2006, p.14)

Level		Name Characteristic
0	Initial grouping and perceptual counting	Able to model or share by dealing in equal groups but not able to see the group as composite units; count each items by ones
1	intermediate composite units	Able to model equal groups and counts using rhythmic, skip or double counting; counts by ones the number of equal groups and the number of items in each group at the same time only if the items are visible.
2	Abstract composite units	Able to model and counts without visible items i.e. the learner can calculate composites when they are screened, where they are no longer rely on counting by ones. The child may not see the overall pattern of composites such and "3, 4 times".

3	Repeated addition and subtraction	Co-ordinates composite unites in repeated addition and subtraction. Uses a composite unit a specific number of times as a unit e.g. 3 + 3 + 3 + 3; may not fully co-ordinate two composite units.
4	Multiplication and division as operations	Two composite units are co-ordinated abstractly e.g. "3 groups of 4 makes 12"; "3 by 4" as an array
5	Know multiplication and division facts strategies	Recalls or derives easily, known multiplication and division facts; uses multiplication and division as an inverse relationship.

This model consist of six levels of multiplication and division knowledge, as described in order of increasing sophistication of modelling, counting, grouping and sharing processes. At the same time, the development of multiplication and division as operations relies on the child's ability to co-ordinate composites, i.e. to use groups of equal groups as single entities.

2.6 Conclusion

In this chapter, I have discussed the relevant literature, focusing on literature in which the mathematical content knowledge required for grade 2 and 3 is described in terms of the South African education system. I have also discussed the need for extended time for numeracy and how additional time spent on mathematics is linked to increased student achievement in the regular classroom. I have also outlined how the literature pertains to the nature of the afterschool mathematics club.

I discussed what is meant by mathematical proficiency in terms of the five strands and how they are intertwined. In the last part of the chapter, I discussed aspects of numeracy assessment and detailed the Mathematics Recovery programme with its LFIN assessment framework. In Chapter 3, I describe the theoretical perspective that forms the basis of my study.

CHAPTER THREE - THEORETICAL FRAMEWORK

3.0 INTRODUCTION

The purpose of this study is to shed some light on the nature of mathematics learners' learning when they are participating in afterschool mathematics. The study describes the learning pathways of four of the ten learners who participated in a maths club. The intention of this chapter is to provide a theoretical framework for the study. To achieve this, I explore the following concepts: socio-constructivist and socio-cultural learning, social interaction, scaffolding and the Zone of Proximal Development (ZPD) as a semiotic field. This chapter also explains teaching and learning methods that influence learning in mathematics education, such as collaborative learning, active learning and social interaction.

3.1 THEORETICAL PERSPECTIVES THAT INFORM THIS STUDY

Two theories inform the notions of teaching and learning in this study: Vygotsky's (1978) socio-cultural theory, and the social constructivist perspective on learning. Social constructivist theory emphasizes the importance of social processes in individual knowledge building (Vygotsky, 1978), while socio-cultural perspectives emphasise learning from a cultural point of view and stress the interdependence of social and individual processes in the co-construction of knowledge. Socio-cultural approaches view semiotic tools or cultural amplifiers as personal and social resources, mediating between the social and the individual construction of meaning (Vygotsky, 1978).

My choice of these theoretical perspectives to illuminate this study has been informed by the empirical field of the study and my two main research questions. Graven and Stott (2011) have described afterschool maths clubs as being conceptualized around the blended notions of participation and acquisition. The clubs are conceived of as supportive inquiry communities where sense making, active mathematical engagement and participation, and mathematical confidence building are fore grounded (Graven & Stott, 2011).

The socio-cultural and social constructivism lenses allowed me to examine teaching and learning through the Vygotskian notion of the Zone of Proximal Development (ZPD). On the one hand, I used a socio-cultural lens to focus on the social aspect of the afterschool maths club and how it enables participating learners to access mathematical knowledge, while on the other hand I used a social constructivist perspective to explore learners' progression through various levels of proficiency.

Sfard (1998) observes that "nowadays educational research is caught between two metaphors, the *acquisition metaphor* and the *participation metaphor*" (p. 5). The acquisition metaphor characterizes learning in terms of knowledge gain and concept development, while the participation metaphor characterizes it in terms of participation in the activity of a community. Sfard argued that focusing exclusively on one metaphor and neglecting the other can lead to theoretical distortions. For example, exclusive reliance on the acquisition metaphor leads to the philosophical dilemma of trying to explain how individuals can want to acquire knowledge of something that is not yet known to them. On the other hand, exclusive reliance on t he participation metaphor makes explaining how knowledge is carried across contextual boundaries difficult. For Sfard (1998), the acquisition and participation metaphors offer complementary accounts of learning.

I perceive social constructivist theory in its individualistic form as swinging between the two metaphors, acquisition and participation. This is because within social constructivism there is the concept of *getting* (acquiring knowledge) in terms of how knowledge is described as constructed by learners (Sfard, 1998). Social constructivism and socio-cultural theory also contain the essence of the participation metaphor as they both emphasize the importance of culture and context in understanding what occurs in society: social constructivists argue that for meaningful learning to occur, individuals should be engaged in social activities.

Both social constructivism and the socio-cultural perspective are congruent with my assumptions about teaching and learning. They also reflect the context in which my data was gathered. In the next section I discuss the theoretical background of the socio-cultural and social constructivist perspectives on learning. Although these perspectives overlap and are by no means completely discrete, they will be discussed separately.

3.2 Some central features of the social constructivist and socio-cultural theory (neo-Vygotskian perspective)

In this section, I will discuss the theoretical perspectives which form the framework for this research, including the key concepts of Zone of Proximal Development and scaffolding.

In this study, I use the term "neo-Vygotskian" to refer to a theoretical approach to the study of learning and cognitive development which draws heavily, though not exclusively or literally, on the work of Vygotsky (Mercer & Fisher, 1993). Other scholars have called this approach "cultural psychology" (Mercer & Fisher, 1993), "socio-cognitive-developmental theory" (Staples, 2007) and "the socio-historical approach" (Newman, Griffin, & Cole, 1989, cited in Meira & Lerman, 2001). Mercer and Fisher (1993) posited that the essence of this approach is to treat learning and cognitive development as culturally based, not just culturally influenced, and as social rather than individualised processes. It highlights the communicative aspects of learning, whereby knowledge is shared and understandings are constructed in culturally-formed settings.

3.2.1 The socio-cultural theoretical background

Socio-cultural theories maintain that learning is an active process in which the context plays an important role. These theories developed from the work of Vygotsky (1978), who argued that learning is not just an individual matter but develops within a social environment (Goos, 2004). Socio-cultural theories place the social environment at the very centre of learning, for without it the "development of the mind is impossible" (Cole & Wertsch, 2001, p. 4). This is because learning is mediated. Vygotsky proposed that in the learning process, experts use tools to mediate learning. Cognitive development is not a direct result of activity, but is indirect: other people must interact with the learner and use mediatory tools to facilitate the learning process, and then cognitive development may occur. These tools are "psychological" (Vygotsky, 1978, p. 53) in nature, in that they are used to express thinking, and include language, signs, symbols, texts and mnemonic techniques.

The most significant socio-cultural tool is language, as it is a teaching tool and is vital in the process of developing higher psychological functions (Karpov, 2003). Mediator tools are first seen externally, as the expert teaches the learner how to use the tool, then internally as the learner begins to use the tool in performing other activities. In the internalization process, the tools modify and transform the learners' thought processes as they begin to use the tools to express their thinking.

As Edward (2009) claims, much of the theoretical basis for the pedagogic approach of using small group work in classrooms comes from the socio-cultural field. Collaborative group work, in which pupils work jointly on the same problem, is linked with ideas such as scaffolding and the ZPD. According to this framework, mathematics learning is seen as both an individual and a shared activity, during which learners should be encouraged to investigate, argue, justify and test conjectures (Cobb, 1995). Such activities need a classroom learning environment that encourages learners to question what teachers and peers say about a particular mathematical concept. Certain learning environments are stronger than others when it comes to supporting learning (Honing & McDonald, 2005). The fact that the learning environment is central to the socio-cultural perspective invites the researcher to take a close look at the informal afterschool maths club context.

3.2.2 The social constructivist perspective

Social constructivist theory is also strongly influenced by Vygotsky's (1978) work, which suggested that knowledge is first constructed in a social context and is then taken up by individuals (Eggen & Kauchak, 2004). According to social constructivists, the process of sharing each person's point of view, called collaborative elaboration (Van Meter & Stevens, 2000), results in learners building collective understanding that they could not build alone. This is in contrast with the individualistic approach (Lerman, 2001) and embodies a shift away from viewing mathematics learning as acquisition towards understanding mathematical learning as participation in the discursive and cultural practice of a community (Goos, 2004).

Central to Vygotsky's perspective on development and learning is that higher mental functioning in the individual derives from social life (Vygotsky, 1978, p. 128). In the first instance, language and other semiotic mechanisms (such as mathematical symbols, diagrams, gesture, stance) provide the means for ideas to be talked through and communicated on the social plane and, following the process of internalisation, language and other semiotic modes provide the tools for individual thinking.

Vygotsky (1978) argues that the relationship between language and thought is direct, and that cognitive development results from social communication. He describes the social construction of knowledge within what he called a Zone of Proximal Development (ZPD). In the next section, I will focus on Vygotsky's notion of the ZPD as this is the analytical tool that I used for analysis of paired task-based interviews with club learners.

3.3 ANALYTIC TOOL IN THIS STUDY: THE ZONE OF PROXIMAL DEVELOPMENT

The essential psychological asymmetry of the teaching and learning relationship is represented in two neo-Vygotskian concepts, the ZPD and scaffolding (Mercer & Fisher, 1997).

3.3.1 The Zone of Proximal Development and Appropriation

Despite the ZPD being a key Vygotskian concept, it has been interpreted and re-interpreted since his death in 1934 (Meira & Lerman, 2001). Different researchers use it in somewhat different ways, and it appears that there is no single best interpretation. It is unfortunate that Vygotsky did not live to elaborate and clarify his notion of the ZPD (Meira & Lerman, 2001).

Vygotsky (1978, p. 86) defines the ZPD as "the distance between a child's actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under guidance or in collaboration with a more capable person". The common conception of the ZPD presupposes interaction between a more competent person and a less competent person on a task, such that the less competent person becomes independently proficient at what was initially a jointly-accomplished task. Thus mediation of learners' activity and communication is a critical aspect of the ZPD.

The concept of the mediation of human actions (including "acts of thought") is central to Vygotsky's theorizing and is, perhaps, its defining characteristic (Moll, 2005). This mediation of actions through cultural artifacts, especially language in both its oral and written forms, plays a crucial role in the forming and developing of human intellectual capacities (Moll, 2005).The ZPD comes into being when one person acts as the mediator for another person who is not able to execute a particular action alone. The ZPD is more usually seen as a positive interaction whereby a person is able to do something with help, which alone s/he would find impossible. Scaffolding is the process by which someone supports another to work in the ZPD (Lock, 2005). Vygotsky advises that the best method of teaching uses the mediation method, which both guides and evolves through the social interaction that occurs during the learning activity. During this process the teacher does not impart knowledge. Rather she/he mediates learning through the social interaction between learner and teacher or mentor to learner.

Scholars such as Newman and Holzman (1993) and others have criticized the notion of the ZPD as a fixed space which the teacher (in my case, the mentor) must locate in order to teach successfully (Meira & Lerman, 2001). Thus researchers (Meira & Lerman, 2001; Lerman, 2001; Askew, et al., 2007) have proposed an extension of the ZPD beyond the determinate attributes of the learner or something of relatively fixed dimensions, so as to conceptualize the ZPD as a symbolic space in order to theorize learning.

Lerman (2001) describes collective participation in mathematical discourse and reasoning practices as pulling all participants forward into their zones of proximal development, which he terms a symbolic space, "an ever-emergent phenomenon triggered, where it happens, by the participants catching each other's activity" (p. 103). The ZPD can therefore be redefined as follows:

> The ZPD is an intersubjective space, created in activities, in which the participants teach each other and learn from each other, where the dialectic of thinking and speech is manifested, and where the individual's meanings encounter social meanings (sense) and purposes. This implies that the opportunity and possibility for learning does not exist prior to the event or activity. (Meira & Lerman, 2001, p. 200)

Meira and Lerman (2001) argue that Vygotsky himself used the ZPD as a symbolic space that captured the potential learning of children while collaborating in problem solving activities with an adult or peer. The ZPD is thus not a physical space but a symbolic space created through the interaction of learners that offers a useful tool for both researchers and teachers to use in analysing and designing environments for teaching and learning in school.

According to Lerman (2001), the definition of the ZPD as a symbolic space allows for an analysis of consciousness that incorporates affect and cognition. This is congruent with my study, which aimed at analysing learning based on the individual learner in a social interaction. Such a perspective provides a more flexible notion of the learning that is possible in each learning interaction, and this resonates with my own experience of working with learners and observing learners in the clubs. Other scholars such as Goos (2004) also work with this more fluid notion.

The expanded view of the ZPD argues that rather than being a fixed attribute of the learner, the ZPD is dynamically created and only emerges in the activity. In this view the ZPD is characterised as having transformative potential, a semiotic field for interaction and communication where learning leads to development (Newman & Holzman, 1997; Meira & Lerman, 2001).

This notion of the ZPD as a symbolic space coheres with the design of the club, which focuses on maximising each learner's participation through creating opportunities for peer interactions and mentor-learner interactions. In this study, I worked with the notion of the ZPD as a semiotic field for the emergence of diverse forms of communication and dialogue. I used the ZPD as an analytical tool for examining the nature of mediation of learning within the afterschool mathematics club. This was done by examining video-taped episodes from the task-based interviews of two pairs of learners participating in an afterschool mathematics club discussed in chapter 4.

3.3.2 Scaffolding within the Zone of Proximal Development

It is important to note that although there is a clear link between the concept of scaffolding and Vygotsky's work, the notion of scaffolding did not originate with Vygotsky. The scaffolding metaphor was first introduced by Wood, Bruner and Ross (1976) in their analysis of the role of tutoring in problem solving (Cole & Wertsch, 2001). The term *scaffolding* has traditionally been used to refer to the process by which a teacher or more knowledgeable peer assists a learner, altering the learning task so the learner can solve problems or accomplish tasks that would otherwise be out of reach (Collins, et al., 1989; Wood, et al., 1976, quoted in Cole & Wertsch, 2001).

Recently scholars such as Askew (2007) have drawn attention to the need to reconsider the viability of this notion of scaffolding within the contemporary mathematics classroom. Hunter (2009) proposes a re-conceptualization of scaffolding, moving away from viewing scaffolding as a tool that will give results (i.e. students' acquisition of mathematical knowledge) to viewing it as interactional strategies which scaffold learners' participation. This view seems to resonate with the practices I observed in mentor-learner interaction in the club and in the simulated task-based interview.

3.3.3 Learners' involvement in mathematics learning

Social constructivism is a learning theory that "emphasises that learning is an active social process in which individuals make meanings through interaction with each other and with the environment they live in".[1] To the social constructivist, knowledge is thus a product of humans and is socially and culturally constructed (McMahon, 2000).

Learning, according to constructivists, is associated with activity. Their perspective on learning highlights the role of learners' active involvement in learning: learning is not something that is done to a learner, but something the learner does (Fosnot, 1989, cited in Rowley, Jensen & Rowley, 2005). Learning therefore is connected to action; and in any given classroom, while the teacher might be the director, the student should be the actor (Rowley, et al., 2005).

Students should be encouraged to be involved in putting ideas into language, testing their understanding with peers and listening to and making sense of the ideas of other students. These interactions form the basis of the mathematics learning context, where learners are afforded an opportunity to get involved in aspects which might require cognitive actions such as negotiation, clarification, elaboration and consensus making (McRobbie & Tobin, 1997).

While a constructionist considers knowledge as an individually mediated product and the environment as that individual's prior experiences, a social constructivist, by contrast, views knowledge as a socially mediated product (Stahl, 2000). The environment here would be the other learners who bring to bear their prior experiences from different (cultural) contexts. The different cultural and experiential environments that learners represent will have a significant impact on the co-creation of knowledge mediated within that group (Palincsar, 1998). The social constructivist view of the environment therefore places more emphasis on the interaction with other group members than one's own prior experience (Ernest, 2006; Palincsar, 1998).

3.3.4 The Zone of Proximal Development as a framework for analysing learning

In this section, I discuss the notion of ZPD as an analytic tool because it is the concept that I used to analyse the nature of mediation of learners participating in an afterschool maths club.

Meira and Lerman (2001) assert that analysis of classroom communications necessarily involves interpretation according to a specific model. They use a model which involves identification of the two aspects they consider crucial to the ZPD as a symbolic space, namely, content-orientated

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[1] Unlike other perspectives, social constructivists view learning and knowledge construction as taking place in the social arena, in the space between people even though the end products are appropriated and internalized by those persons individuality (Ernest, 2006, p. 1).

language and communication-orientated language. Content-oriented language includes the teacher's instructive goals and the steering of learners towards conceptual learning in specific aspects of LFIN. Communication-orientated language refers to the process that is involved when the teachers and the children communicate with each other. Literature on the ZPD has pointed to two key aspects of the ZPD seen as a learning mechanism, teacher-student interactions and student-student interactions.

Vygotsky also analyses the notion of the ZPD in terms of equal status partnerships, noting that when children play together they are able to regulate their own and their partners' behaviour according to more general social scripts that involve assuming the perspective of others (Goos, 2004). Lerman (2001) proposed tracking the catching of others' ideas by looking for instances in the conversation or dialogue where the participants establish joint understanding, listen to one another, consider each other's utterances and find a common way to solve tasks. Goos (2004) adds that working in collaborative peer groups, students have an opportunity to own the ideas they are constructing and to experience themselves and their partners as active participants in creating personal mathematical insights.

Lerman (2001) argues that one can regard the ZPD as an explanatory framework for learning as a whole, both in intentional settings, such as schooling, and in informal settings, in other words, in all socio-cultural milieus. It recognises the fundamental asymmetry of the teacher-student relationship, and of peer interaction, an asymmetry often denied or underplayed by more individualistic approaches.

3.4 CONCLUSION

In this study I investigated the nature of mathematical learning of four learners participating in afterschool mathematics. My unit of analysis was an individual learner in a conversation or interaction with a peer and with a mentor. My wish was to explore how interaction in the club affords learners the opportunity to learn. The purpose of this chapter was to provide a conceptual and theoretical backdrop to the study. To that end, I have shown how the notion of the ZPD provides for analysis of the interaction between learners and mentor. More generally, in this chapter I have identified a theoretical and analytic framework for understanding mathematical learning. The Zone of Proximal Development (ZPD) forms the foundation for my analysis of mentor/learner interaction in the club sessions.

CHAPTER FOUR - RESEARCH DESIGN AND METHODOLOGY

4.0 INTRODUCTION

In the previous chapters, I discussed the theoretical and analytical framework that informed my research. In this chapter, I explain the route I took in planning and collecting information concerning the nature of the four learners' mathematical proficiency progression and the mediation of their learning in relation to their participation in an afterschool maths club in South Africa. This chapter includes a discussion of my research objectives; research orientation; research methods; research site and empirical field; data collection procedures (instruments and methods). The data analysis process includes trustworthiness: reliability and validity; and ethical considerations.

4.1 RESEARCH OBJECTIVES

The focus of this study was concerned with the nature of learners' mathematical proficiency progression and the mediation of learning while participating in an afterschool mathematics club. The study was conducted with a sample of four learners who attend a local school in Grahamstown, and who participated in an afterschool maths club. To this end, a number of specific research questions were posed (see below). Care was taken to ensure that research questions were clearly formulated, intellectually worthwhile, researchable and were used as a means to move from broad research to specific research (Mason, 2002, p. 19). As indicated in Chapter 1, this research addresses the following specific questions:

1. What is the nature of mathematical learning progression (if at all) and how do the assessment instruments used by the SANC clubs enable me to assess this?
2. What is the nature of mentor to peer mediation in the afterschool mathematics club?

4.2 RESEARCH ORIENTATION

The approach that I used was interpretive in nature drawing on qualitative methods with some elements of quantification in relation to learners' progression. As the name suggests, the interpretive paradigm is concerned with interpreting and understanding human actions. According to Merriam (2009, p. 213), the assumption underlying qualitative research is that reality is holistic, multidimensional and ever-changing; it is not a single, fixed, objective phenomenon waiting to be discovered and observed. The paradigm provides for multiple realities and the goal of my study was to understand a phenomenon that is not yet well understood. It assumes that the way to understand something is through studying it in its natural context (Cohen, Manion & Morrison, 2011). In my case the nature of mathematical learning and proficiency progression for the selected grade 3 learners was investigated within the context of learners participating in an afterschool mathematics club.

I chose to work in the qualitative paradigm based on my assumption that reality is constructed by individuals, interacting in their social worlds, and the meaning is embedded in the individuals' experiences and is mediated by my perspective as a researcher. Moreover my choice of qualitative methods also comes as a result of both my research goal and purpose. My research questions, theoretical framework and the way I view reality pointed towards an interpretive research design. This is because I needed an in depth understanding of the nature of learning in an afterschool maths club and how it enabled learners' mathematical learning and proficiency progression.

According to Merriam (2009, p. 38), an interpretive qualitative study "would be interested in (1) how people interpret their experiences, (2) how they construct their worlds and (3) what meaning they attribute to their experiences." Such an orientation allows researchers to gain access to individual points of view - their representations of reality rather than reality itself; thus it enables researchers to generate rich data or 'thick descriptions' (Geertz, cited in Stake, 2000, p. 439) of such meanings.

The qualitative interpretive paradigm with its characteristics described in this section such as a naturalistic setting, rich description and focus on understanding human behaviour, lends itself to an examination of the phenomenon of my research study.

4.3 RESEARCH METHODS: CASE STUDY AND SAMPLE

A case study approach consisting of four grade 3 learners aged 9 to 11 was chosen for my study. The study was conducted with learners participating in an afterschool maths club in a township school situated in Grahamstown, South Africa.

The case study design is flexible and adaptable (Schumacher & McMillan, 2001). It enables the researcher to closely examine data within a specific context (Merriam, 2009). Yin (2009) defines the case study research method as an empirical inquiry that investigates a co ntemporary phenomenon within its real-life context. He adds that case study research is known for dealing with research questions that involve *why, how* and *what.* Merriam (1998, p. 27) describes a qualitative case study "as an intensive, holistic description and analysis of a single instance, phenomenon or social unit".

My four case study learners were selected through purposive sampling (see 4.4.2). Schumacher and McMillan (2001, p. 401) describe purposive sampling as a way of "selecting small samples of information-rich cases to study in-depth without desiring to generalize to all such cases". A balance of two boys and two girls were selected from the club of ten learners. The two boys and two girls were suggested by the mentors as learners who regularly and actively participated in club sessions and who displayed a range of numeracy proficiency. The four learners were purposefully sampled according to their general performance (abilities) in class and their ability to articulate ideas. According to Cohen, et al. (2011), using a purposive and convenient strategy is suitable for small scale research because it is less complicated to set up and there is no attempt to generalize one's findings.

The four learners' abilities varied from high, medium to low ability. The sample size was considered adequate for performing the study which is a half thesis in nature. The use of a case study method as a framework for this study also relates to the nature and purpose of the research and my desire as a researcher to probe and gain more insight into a complex social phenomenon, in this case the nature of mathematical learning.

4.4 RESEARCH SITE AND EMPIRICAL FIELD

This study was conducted in the Grahamstown area, a town in the Eastern Cape, which is one of the poorer provinces in South Africa. The research is part of a broader study conducted by the South African Numeracy Chair research team at Rhodes University. The school is a township school situated on the edge of the township in Grahamstown. The majority of learners come from the township, but some learners are bussed in from surrounding farms. The dominant language in the area is isiXhosa. The medium of instruction at the school is English as historically it was a Catholic Church school. The school is a fee paying school, although at least 15% of learners are exempted or partially exempted from paying school fees. The maths club in which the four case study learners participated consists of 10 learners: 5 boys and 5 girls. All ten learners speak isiXhosa as their home language. The setting for the one to one assessment interviews and for both task-based interviews was the school library.

As explained above, I am researching within the broader research project of the SANC at Rhodes and, as such, I had the opportunity to research student learning within one of the afterschool clubs established in March 2012. These clubs meet weekly for just over an hour after school hours. In all the clubs, learners were selected for the clubs across performance levels by their class teachers.

After observing two of the SANC clubs for a month, based on various factors, I chose a particular club for reasons of convenience as it is in walking distance from the university.

4.5 DATA COLLECTION PROCEDURE (INSTRUMENTS AND METHODS)

As part of the data collection strategies, the learners were interviewed twice in terms of their numeracy proficiency (LFIN stages and levels) during the research, adapted from Wright, et al., (2006). The first was when the club first started while the second one was done after four months of club participation. This was done in order to collect the data for my research question one. Additionally, the four learners were regularly observed in club sessions and I used field notes, video and audio recordings throughout the observation period.

Finally, these learners were also interviewed in pairs using the task-based interviews in order to analyse how mediation of learning occurred. The use of multiple data sources was employed for the purpose of enhancing both validity and reliability and ensuring credibility through enabling triangulation (Cohen, et al., 2011). According to Merriam (2009), a key strength of the case study method involves using multiple sources and techniques in the data gathering process, i.e. observation, and individual task-based interviews with learners.

Below, Table 1 summarizes how and when the data were collected for this study

Types of data	**Period collected**
Assessment of proficiency progression First administration of the following instrument • Mathematical proficiency assessment interview (adapted from Wright, et al. and Askew, et al.)	April 2012

Types of data	Period collected
Observation of mediation • Club sessions • Field notes • Reflective journal • Task-based interviews	April to August 2012
Re-assessment of proficiency progression • Mathematical proficiency assessment interview	August 2012

Table 1 Summary of the data collected

4.5.1 Learners' mathematical proficiency progression

The assessment instruments discussed below focus on data collection for determining learners' evolving mathematical proficiency progress over time and are, thus, primarily connected to research question number one. The mathematical proficiency instrument is used by the SANC and was adapted with permission from the work of Askew and his team in the *Effective Teachers Of Numeracy* study conducted in England in the nineties (Askew, et al., 1997) and the work of Bob Wright and his colleagues in Australia (discussed in chapter 2). The assessment was carried out by way of an assembled instrument that combined questions (sometimes adapted) from both key works (Graven & Stott, 2012). The assessments comprise one instrument, namely, the mathematical proficiency assessment interview (See Appendix A).

4.5.1.1 Mathematical proficiency interview

The main purpose of the individually administered interviews is to enable one to track students' mathematical proficiency progression, typically on the different aspects of early number teaching and learning as identified in Wright, et al.'s (2006) LFIN framework. During the study, the assessment interview (see Appendix A) was administered twice, first in April and, then again in August. This interview was administered to individual learners and lasted about an hour for each learner. Detailed notes were made on learner methods and responses on a recording schedule. In the second round, the same oral, one-to one interview was administered to the four selected learners. The interview was divided into 3 parts, as outlined below, to identify learners' proficiency progression:

Part one (tasks 1-11): Numeral Identification (NI), Forward Number Words Sequence (FNWS) and Backward Number Word Sequence (BNWS), and Number Word After (NWA) and Number Word Before (NWB).

Part two (tasks 12-16): Strategies of Early Arithmetical Learning (SEAL), Base-Ten Arithmetical Strategies (BTS), combining and partitioning.

Part three (tasks 17-24): Subitizing, Multiplication and Division.

Learners' were then profiled in all the aspects as set out in tabular form according to the LFIN framework (see Chapter 2) and their progression was shown in stages or levels and sometimes in terms of percentages.

4.5.2 The nature of the mediation of learning

For this study, observation was crucial because the issue at hand is exploring and understanding how mediation in an afterschool maths club may enhance learners' learning. In this study, I made detailed field notes of the four learners participating in the club. Best and Kahn (2006) describe observation as a technique of obtaining data through direct contact with a person or group of persons. All sessions were videotaped in order to enable in-depth reviews of the interactions amongst participants. As a non-participant observer, I used field- notes to record specific events and observable data related to the four learners. Field notes are written records of observations, interactions, conversations, situational details and thoughts kept by a researcher during the period of study.

4.5.2.1 Club session observations with personal journal

To collect the data for my study, I chose the non-participant observation approach. This means I was not a participating member of the club with the four learners. Merriam (2009) describes a non-participant observer as an observer who is not directly involved in the situation to be observed. In other words, the observer is not directly involved in the world, but he or she is outside looking in and does not intentionally interact with, or affect, the observation.

However, I made sure that I gained access and was accepted by the individuals being observed through talking to them before or after the session, and checking that they were comfortable with my presence. Being an inside observer, though not part of the social life within the club, gave me an opportunity to gain an overall view of the interrelationships of factors (cultural context) of the club, such as children's social interaction with mathematical activities, children's interactions with others and how they relate to one another, the mentor and resources used for this club. That engagement afforded me the opportunity to generate thick descriptions, which would provide a clearer explanation and interpretation of events in the data analysis instead of depending on my own inferences.

Cohen, et al. (2011) explain that unstructured observations are far less clear on what they are looking for and, therefore, allow for observations of what is taking place before deciding on its significance for the research (p. 305). Observations are recorded as field notes (Cohen, et al., 2011, p. 311).

Following each session, I also wrote a reflective journal that allowed me to record the insights gained on the nature of learning and the mediation process as I observed it within the clubs. Journal writing has been defined as a useful tool for promoting critical reflection of experiences, as it is a means by which educators engage in learning and allows educators to discuss the link between theory and practice and, through this, explore new possibilities for being and acting (Cunliffe, 2004; Chirema, 2006).

4.5.2.2 Video and audio recordings

At first, I did not intend to use the video or audio recordings but rather relied on the field notes and reflective journal. However, as time passed, I went through the data and realized that I could not capture all the necessary data using only the field notes. I then turned to the video and audio recordings to supplement my field notes. I used video and audio recordings for recording of learner interactions between peers and with the mentor in the club. I focused on four learners using both video and audio recorders and positioned the camera to capture their discussions with other learners and the mentor. I recorded almost every club session within the 2nd term. In the second term, there were eight sessions and seven sessions in the third term (15 sessions in total). Only selected excerpts from club sessions have been transcribed to supplement the analysis.

Within each club session, carefully chosen tasks are intended to enable task-based mediated, focused discussion between individual learners and the mentor in order to invoke ZPDs (Stott & Graven, submitted paper). Recording these was thus an important aspect of my data collection and I used the ZPD as my lens for examining these interactions (discussed further below). However the business and noise levels of ten learners working in the club made detailed transcription of mentor-learner mediation difficult within the natural setting of the club. For this reason I chose to use simulated paired task-based interviews where learners were asked to work 'in the same way as they do in the clubs' but were recorded in a noise free environment. These task-based simulated sessions were designed and conducted by the mentors to maintain continuity of the club norms and the styles of activities and mediation interactions.

4.5.4 Task-based Videotaped Interviews

The task-based interviews were administered with four learners within the norms of the club, e.g. to allow a learner to express him- or herself freely and were conducted by the two club mentors. Each pair of learners was interviewed for almost an hour and each interview was videotaped.

In analysing the task-based interview, I focused on capturing the interactions between the mentor and the learners and how the mentor could hold learners attention. The task based interviews comprised a series of questions. Tasks were similar to assignments used in club sessions and included problem solving. The interviews were videotaped and later transcribed. The transcriptions consisted of descriptions of the child's words and actions during the classroom observation and the task-based interview. In the episode-by-episode analysis, each text or segmented transcription under each pattern was interpreted separately.

4.6 DATA ANALYSIS PROCESS

From the data gathering techniques that I used in my study, I have generated both quantitative and qualitative data. Hence, in analyzing the data, a broad mixed method approach to data analysis was used. This approach is in line with the types of data collected by the Chair in the other clubs. Much of the data collected using the instrument discussed here is analysed on an on-going basis and is used in a dialectical manner for both research and practice in the clubs and reflects that dialectical nature of research and development in the Chair (Graven & Stott, 2012).

4.6.1 Data analysis for research question one (1)

Quantifiable data was collected from the four learners through the individual assessment (interviews) and from this one was able to report on the extent of learners' progress across LFIN stages and levels. Both interviews for April and August were reviewed and transcribed. The transcriptions consisted of descriptions of learners' levels and stages obtained and, where necessary, words and actions during the interview. Based on the transcriptions, a description of each child's levels and strategies for each aspect was written and general insights concerning their strategies were noted. Furthermore, each child's stage and level was determined using the models in Tables 2.4 to 2.10 in Chapter 2. In describing strategies, notes were made about distinctive features of each child's strategies. I quantified learners' progress or lack of progress using the instruments discussed in Chapter 2. This quantification enabled the assessment and documentation of learners' number knowledge and the extent to which the progression occurred across the 9 aspects of early number knowledge in mathematics.

4.6.2 Data analysis for research question two (2)

For this research question, I collected qualitative data through observations of the learners' extracts from field notes and reflective journal writing. I gathered two sets of data to address my two major questions. Data collected from video recordings of the four learners from the task based interviews were transcribed and analysed using the ZPD as an analytic tool. Drawing on Lerman's (2001) paper, which relates to how to read and analyse the ZPDs from transcript data, I proceeded with the analysis of my data following my transcriptions of my session recordings. This implies that I searched for events of communication from the audio and the transcripts in which learners 'catch each other's thoughts' (Lerman, 2002, p. 64). Indicators for this include trying to establish joint understanding; listening to each other; considering each other's utterances and finding common ways for task solving. These are evidenced by, for example, re-voicing, requesting clarification and/or elaboration, providing new suggestions and so forth.

Transcription of the task-based interview indicated that a prevalence of scaffolding practices as described by DEST (2004) discussed further in Chapter 6, Section 6.2.1. The prevalence of these practices therefore led me to draw on the DEST's framework for describing and analysing the mediation process within the ZPD.

4.7 TRUSTWORTHINESS: RELIABILITY AND VALIDITY

A research study is trustworthy to the extent that there has been some rigor in carrying it out (Merriam, 2009, p. 209). Reliability and validity are important in any discussion of rigor in scientific research. According to Silverman (1993) and Maxwell (1992) (cited in Graven, 2002), validity is the key issue in the debate over the legitimacy of qualitative research. However, the standard for rigor in qualitative research differs from that of quantitative research (Merriam, 2009). Graven (2002) notes that issues related to validity and reliability might differ based on the nature of the research conducted, the philosophical and ontological assumptions of the researcher. In this research study, I employed multiple strategies to ensure and enhance validity and reliability of data.

Reliability is viewed as synonymous with dependability which indicates the extent to which results can be regarded as stable (Lincoln & Guba, cited in Merriam, 2009). According to Merriam (2009, p. 229), "reliability is problematic in the social sciences simply because human behaviour is never static". Graven (2002), posited that since categories are a discovery, it is unreasonable to expect others to discover the same categories as the researcher. Adler (1996), argues that reliability should be established as the extent to which these already discovered categories are recognizable to others.

In that respect, the researcher can use what Silverman (1993 cited in Adler, 1996), calls 'inter-reliability' which refers to getting others to look at the same data according to the established categories and to see whether they are recognizable in the data. The nature of my study made it easier for data collected to be accessible to both my supervisors (and club mentors). It was also possible for my supervisors to provide the means for checking the categories and themes that I used in the data. In terms of categories of scaffolding that I recognized, I worked with one of my supervisors to review my videotapes to see if she could recognize similar categories.

Validity refers to the degree to which a test measures what it is supposed to measure. According to Graven (2002), validity must take on different meanings and use different techniques in relation to qualitative research. McMillan & Schumacher (2001, p. 407), describe validity in qualitative designs as "the degree to which the interpretations and concept have mutual meanings between the participants and the researcher."

One way of strengthening validity is with respondent validation. Respondent validation refers to member checks (Lincoln & Guba, 1985, cited in Merriam, 2009). Merriam further described respondent validation as a systematically soliciting feedback about one's data and conclusions from the people you are studying. This helps to rule out the possibility of misinterpreting the meaning of what participants say and do. It also helps to avoid misinterpreting the perspectives they have on what is going on, and provides a way of identifying researchers own biases and misunderstandings of what was observed (Maxwell, 2005 cited in Merriam, 2009). In my case, learners (participants) for this study were too young to check validity but the two club mentors Debbie and Mellony were asked to check the data transcripts and interpretation from the transcript of the simulated task-based interview sessions and club sessions.

Riessman (1993, cited in Graven, 2002) argues that in order to make it possible for others to determine the trustworthiness of a study the author must describe how interpretations were produced; make visible what she/he did; be specific about how one accomplished successive transformations, and make primary data available to other researchers. This concurs with Merriam (2009, p. 229) who expresses that in providing rich description, a researcher plays a role in contextualising in such a way that readers will be able to determine the extent to which their situations match the research context, and, hence whether findings can be transferred.

This study has worked towards addressing trustworthiness by providing such descriptions and analysis close to the data. I used respondent validation (supervisor instead of learners) and reliability. I have also engaged with ethical issues in depth. By doing so I aimed at striving to achieve trustworthiness, rigor, descriptive and interpretive, and theoretical and explanatory validity as required in qualitative research.

4.8 ETHICAL CONSIDERATIONS

The most important thing to consider when conducting research is to inform the participants about how the research will be conducted or carried out and how they will be involved so that they can make informed decisions about their participation. I made it clear that the rights of participants in the research were respected and protected.

The study was conducted with due consideration of the ethical issues in research. Working with young learners in the maths clubs made it vital to pay attention to ethical issues. Before commencing the research study; the South African Numeracy Chair at Rhodes University, applied for ethical clearance from both the Eastern Cape Department of Education (for broader club research) and through my proposal I applied for ethical clearance through Rhodes University (for my specific study). Permission was also sought from the principal, teachers and parents of the participants.

Additionally, to adhere to ethical issues, I talked to the learners (participants) and explained to them about the focus of my study. I committed to ensure their anonymity and made it clear that their involvement in my research was voluntary. Thus, they were allowed to withdraw at any time from the research without prejudice and I assured them that I would, therefore, not reveal any data relating to their club participation. While permission to conduct the broader study had already been granted to the South African Numeracy Chair, I also made myself available to talk to the school principal, teachers and parents about my study. Pseudonyms were used in all my research writing and the participants were assured that the information provided would be stored in a secure place where only the researchers have access to it.

Consent letters were sent to the school principal and to parents whose learners participated in the research study requesting their permission. Letters were signed by parents. It was indicated in the consent letters sent to all parties that participation was voluntary.

4.9 CONCLUSION

This chapter outlined the research design, the methods of data collection, and the detailed research process used in this study. The methods used were informed by the research questions, context of the study and the theoretical framework. Also, since the procedure was to conduct research of a qualitative nature, the methods of data collection used were consistent with the dictates of qualitative research, such as detailed description of the data analysis employed in this study. Issues pertaining to the reliability and validity of this research as well as ethical considerations that guided me in the process were also discussed.

CHAPTER FIVE - RESULTS, ANALYSIS & DISCUSSION: MATHEMATICAL PROGRESSION

5.0 INTRODUCTION

In this chapter I present and discuss the information gathered from the assessment interviews were developed to track learners' progress in developing mathematical proficiency. The data collection methods and analysis are explained in Chapter 4. The information in this chapter was derived from a sample of four learners within the afterschool mathematics club through an analysis of their performance in the assessment interviews as described in Chapter 4.

For responses to interview items, the Wright, et al. (2006) framework was adapted and used. The framework consisted of eight categories, each with its own levels or stages. However, scores were used for some aspects such as combining, partitioning and subitizing in the absence of levels in the framework. Levels and stages were described earlier in Chapter 2.

This chapter is divided into four parts. I begin by describing the club in the study (drawing on my observations and field notes). Thereafter, I present the learners' background information, followed by a presentation of the findings for research question one which focused on the nature of mathematical learning progression (if at all) and how the assessment instruments used by the SANC clubs assisted to assess this. I then discuss the findings in terms of the nature of learners' mathematical proficiency progression as interpreted against the LFIN. The final part of the chapter highlights the overall findings in terms of the learners' mathematical proficiency progression. As previously mentioned in the methodology chapter, participants' pseudonyms names were used instead of real names. In the case of the mentors, their real names are used with their permission. The learners' names are Alleta, Kaino, Elia and Olien and the two mentors from the club are Debbie and Mellony.

5. 1 DESCRIPTION OF THE RESEARCH PARTICIPANTS

The club and the school in which my four case study learners participate were described in chapter 4. This section provides a brief profile of the four case studies of learners in the maths club.

5.1.1 Alleta is a ten year old girl. It was noticed from the beginning of the research that she was one of the learners who could engage well in the class tasks but often spoke only when addressed. She rarely spoke of her own accord at the start. The result from the proficiency interview in March and personal observation indicated that Alleta was one of the above average learners in the club. In the second to third month of the study, she opened up and spoke and responded freely. At some point she stood up to raise her thumb, in case the mentor did not notice her, to ensure her chance to answer. The most significant part of Alleta's proficiency progression was her way of moving away from inefficient counting strategies toward highly efficient counting strategies (see Table 5.4).

5.1.2 Kaino is a nine year old girl. I observed that she was a happy and easy going person although sometimes easily upset. She occasionally volunteered to translate into the learners' home language in cases where students did not understand an instruction. Kaino expressed herself freely and was always willing to answer questions. From my observation, it seemed that she often dominated any conversation with other learners. During the study she asked questions when she needed more information, for example, "is this how you want us to do it teacher?" (Calling the mentor). At one point, when she had to work with other learners who would sometimes solve a problem before her, her facial expressions showed frustration perhaps indicating an element of competitiveness in her nature.

5.1.3 Elia is a nine year old boy. During the initial part of the research period, Elia rarely expressed himself or raised his hand when the mentor asked a question. I also observed that, at first, he seemed to be a timid and reserved child who had little command of the English language. He did not do well in the first assessment based interview during the first round. However, after some time, Elia started taking an active role in various maths activities in the club. By mid-year, I observed that he participated more freely. For example, on some of the days he suggested which games could be played that day. The most significant thing about Elia was the potential and interest he showed in club activities in the last weeks of the research. This can be seen in the way he improved in his performance.

5.1.4 Olien is a nine year old boy. Olien was confident and articulate and showed good social skills. He was one of the learners who went beyond the mentor's instruction and came up with something extra. His attitude toward maths activities seemed very positive as indicated by his immediate focus on suggested activities.

In the following section I outline the data captured from the individual proficiency interview.

5.2 LEARNERS' MATHEMATICS PROFICIENCY PROGRESSION

Section 5.2.1 presents the results from the proficiency assessment interviews

5.2.1 Results from the proficiency assessment interviews

As mentioned earlier in both the literature review chapter and the methodology chapter I used the Wright, et al.'s (2006) assessment framework adopted by SANC project. The framework for assessing learners' mathematical proficiency progression was used between March and August 2012. One of the important aspects of the framework is that it enables summary profiling of learners' current knowledge, typically on the five aspects of early number knowledge (Wright, 2003).[2]

The assessment interviews consisted of three parts and were conducted in April and August to determine learners' stages and levels in the following aspects: Part one (tasks 1-11) consisted of numeral identification, forward and backward number word sequences and number word after and before. Part two (tasks 12-16) consisted of strategies of early arithmetical, Base-ten arithmetical learning strategies and combining and partitioning. Part three (tasks 17-24) consisted of subitizing and multiplication and division tasks.

For rich data and in order to determine the learner's stage or level, it was necessary to review the recorded schedules for the interview. This was done in order to determine the learners' stage and level. Notes were made about the different features of each learner's way of answering the questions. Video recordings would have enabled revisiting of data, however generally I found that the notes at hand were sufficient. In addition, each learner's level and stage was determined using the methods in Tables 2.3 to 2.5 in section 3 of the literature review chapter.

5.2.1.1 Numeral Identification (NI)

Numerals are the written and read symbols for numbers. Learning to identify, recognise and write numerals is an important part of early arithmetical development (Wright, et al., 2006). Learners' knowledge of numeral identification was assessed by displaying numeral cards

[2] I have mentioned the instrument has been adapted and modified by the SANC using their own input and the work of Askew, et al. (1997).

individually, and asking the learners' to name the numeral given number or whether the learner counted from 1 to respond to these tasks.

	Numeral identification	
	April	August
Kaino	4	5
Alleta	3	4
Olien	4	5
Elia	3	4

Table 5.1 learners' levels in numeral identification[3]

As shown in Table 5.1, none of the learners were below level 3 in this aspect at any point. In April, Alleta and Elia were at level 3, which means in April they were only able to identify numbers up to three digits long. However, in August they were both able to identify up to four-digit numerals within the range of 1000 and beyond. This implies that both Elia and Alleta, improved by moving from level 3 to level 4.

According to the LFIN, for a learner to be at level three she/he could only identify numerals in the range 1-100. During the April assessment interview, when Alleta and Elia were asked to say the number 1025, they experienced problems expressing it correctly. However, the other two learners, Olien and Kaino, were able to identify up to 4-digit numbers and thus were placed at level 4. In August Alleta and Elia both managed to advance from level 3 to level 4 by identifying numerals including 4-digit numerals while Kaino and Olien who were already at level 4, moved to level 5.

There are various possible explanations for these results. First, the initial interview was conducted in March, just two months after the school year started and a week or two after the club started. The second interview was conducted in August; 4 to 5 months after the school year had commenced and the learners had been in the club for at least 4 to 5 months. The afterschool maths club included teaching recognition and writing of the numerals from 1-100 and beyond and the use of flard cards. Hence, the improvement might be as a consequence of the work that they did in the club such as identification of numbers using flash cards. Within the club, learners

3 Levels in numeral identification: 1 One digit numeral -Able to identify 1-digit numerals, 2 Two-digits numeral -Able to identify 2-digit numerals, 3 Three-digits numeral -Able to identify 3-digit numerals' 4 Four-digits numeral -Able to identify 4-digit numerals

were also provided with extra books that they worked through solving different mathematical problems and one of the first books focused on numbers. However, the fact that the learners had attended four to five months of the club by the time of the second interview seems likely to be a contributing factor to the learners' mathematical proficiency progression.

5.2.1.2 Forward (FNWS) and Backward Number Word Sequence (BNWS), and Number Word after (NWA) and Before (NWB)

BNWS and FNWS refers to the learners' ability to count a forward or backward sequence of number words as well as being able to identify numerals (Wright, et al., 2006). The creation of the number word sequence requires more than generating a rote count. To assess learners' BNWS or FNWS, learners were presented with a starting number and asked to start counting forward or backward (e.g. 1 to 32 or count backward from 10). The learner could also be asked to identify which number comes before or after a given number. Table 5.2 be low shows the results of learners' levels in FNWS/NWA and BNWS/NWB

	Forward Number Words Sequence (FNWS)		Backward Number Word Sequence (BNWS)	
	April Level	August Level	April Level	August Level
Kaino	5	5	5	5
Alleta	4	4	5	5
Olien	5	5	5	5
Elia	4	5	4	5

Table 5.2 learners' scores and levels in FNWS/ after and BNWS/before[4]

In terms of FNWS and number words after**,** the table shows that none of the four learners was at a level lower than 4 in April and August. In April, Kaino and Olien were at the highest level on FNWS. They were both able to answer all the tasks on FNWS correctly within the range of 100 and beyond without dropping back. According to Wright, et al. (2006), for a learner to be judged to be at level 5, he/she may need to be able to say forward number word sequences correctly without missing any number in the range of one to 100. T his also applies to the backward number words sequence.

Elia and Alleta were only able to produce forward number word sequence in the range of 1 to 100 without falling back. Thus, in April their level in FNWS was 4. T hey both omitted some

4 For NWA and NWB, the child's level is determined using the model shown in chapter 2 table 3 and table 4 for BNWS and NWB, Table 2.1: Model for Construction of Forward Number Word Sequences

0 emergent,

numbers in the counting number sequence and they were slow at counting. They found it difficult to produce sequences when they were required to go from one-digit numbers to two/three-digit numbers. For example, when Elia was asked to count-on from 48 to 68, as it was recorded.

Overall, two learners, Kaino and Olien, were already at the highest level in April. That indicated they were facile with numbers in the range of 1 to 100 and beyond. The other two learners, Alleta and Elia, were at level 3. That indicated they were only facile in the range 1-30. In August, Alleta remained at stage 4 while Kaino and Olien remained at the highest level 5 joined by Elia who moved from level 4 to level 5.

Overall when the first interview was conducted in April, there were already three learners, Kaino, Alleta and Olien, who were at level 5 in BNWS. This implied that the three learners were already fluent in this aspect of BNWS up to 100 and beyond. Elia was the only one who was at level 4, because he was only fluent with numbers up to 100 but not beyond. On the other hand, by August, all four learners in the study were fluent with numbers in the range of 1 to 100 and beyond, including Elia.

The reason for improvement in both FNWS and BNWS for Elia may be attributable to various factors. Both the afterschool maths club and the CAPS (2011) for grade 3 mathematics included the standard study of basic numbers up to 9, counting groups of up to nine objects, as well as saying FNWSs up to 100 and beyond. Clearly, these teaching topics considered together go a long way toward children attaining the goal of being facile with the FNWS in the range 1-100 and beyond.

The fact that the learners did a mental session in every club session could also have been a contributing factor to the learners' improvement in these aspects. Wright (2009) argues that children's apparent lack of progress in some aspects of LFIN such as BNWSs during the first years of schooling could be explained in terms of classroom practices. Secondly, according to the CAPS document, learners in grade 3 are supposed to know how to count forwards and backwards in 2s, 5s and 10s between 0 and 500. They are also expected to learn how to count forwards and backwards in 20s, 25s, 50s and 100s between 0 and 1 000. In August, these learners had progressed quite far in grade 3, which means that they might have been able to learn how to count backwards or forward in these number ranges in class.

5.2.1.3 Strategies of Early Arithmetical Learning (SEAL)

For example, a learner is given a collection of counters, e.g. 8, 15, or 27, and asked to count how many in all. The mentor (interviewer) takes note of how the learner counts, whether the learner knows the number words him/herself and whether the learner correctly coordinates the number words with the items. The way the learner responds to the question, gives an indication of his or her current counting procedures.

Table 5.3 below shows learners' results in terms of stages of early arithmetical learning

	Strategies of Early Arithmetical Learning	
	April	August
	Stage	Stage
Kaino	3	4
Alleta	2	3
Olien	2	4
Elia	2	3

Table 5.3[5] learners' stages of the SEAL

The table shows that none of the four learners was found to be in the emergent stage (stage 1), during the first interview in April. Thus, most learners were able to answer the questions in April (although not quickly) and they depended on counting from one, rather than counting on, and did not make use of mathematical strategies. Hence, three of them were at stage 2, except Kaino, who was the only learner already at the advanced counting stage 3. The other three learners, Elia, Olien and Alleta, were at the figurative stage (stage 2) of early arithmetical learning.

As reflected in her interview, Kaino was rated at stage three in April because of her ability to use 'counting on' strategies in solving problems such as 16 + 9 rather than 'counting from' strategies. Although she used her fingers to symbolize number 9, it seemed she was able to count on, rather than counting from. Wright, et al. (2006) call this anticipation and capacity to keep track of the number of counts the hallmark of stage 3.

In comparison, the August results show that all four learners progressed up at least one stage. Olien progressed to stage 4 while Alleta and Elia moved to stage 3. Elia was able to use the counting on and backwards strategies. For example, when he was asked to count 12 – 5; he was

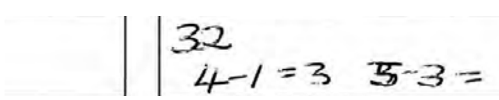
[5] Stages: 0 Emergent, 1 perceptual, 2 Figurative, 3, advanced-counting by ones and 4 Facile.

able to count backwards but he was still counting in 'ones'. According to Wright, et al. (2006), a learner at stage 3 must be able to use a counting-on rather than a counting-from strategy.

At one point Alleta was asked to add 10 to 294 (in task 10, adding/subtracting with tens). Although she struggled a bit, she managed to get the answers by using her fingers to count forward from 294 and she finally got to the correct answer. In the same task, she was similarly observed using a count-down strategy. This showed that she was at a different stage compared to the strategies she used in April. This stage of performance according to Wright, et al. (2006) was stage 4. A learner who is at stage 4 will be able to use a countdown-to strategy to solve missing subtrahend tasks. In another example, in task 10a) add 10 to 92, Kaino counted on from 92 and in the process, reverted to using her fingers while counting on to 102.

Moreover, as the table 5.4 reflects the early arithmetical learning stage model, during the April interview, the results show that three of the learners were at stage 2 (figurative counting) while only Kaino was at stage 3 (the initial number sequence). By August, of the 3 learners who were at stage 2, only Elia had advanced to stage 3. Alleta and Olien advanced two stages to stage 4. Kaino advanced to stage 4 from stage 3. Interestingly, 2 of the 4 learners advanced more than one stage by August, showing that 3 of the 4 learners (Kaino, Olien and Alleta) were at stage 4, the highest level in the SEAL model. In other words, all four learners were able to make progress in their strategies of early arithmetical learning stages between April and August.

One of the noted errors was in part two of the assessment interview, question 12, 43-15= on this question 2 out of 4 learners switched the order of the unit subtraction in order to subtract. Some learners were observed subtracting 10 from 40 and then 5 from 3 (as shown in the example below).



The subtraction of a larger unit digit from a smaller unit digit is noted by many researchers as one of the errors frequently made by children (e.g. Young & O'Shea, 1981; Brown &Van Lehn, 1982; Ashlock, 1982, as quoted in Wright & Rumiat, 2010). The teaching of the standard written algorithm was identified as a factor which contributes to this error. This has led to the suggestion that is would be better to defer the teaching of standard written algorithms until the later years of schooling, and to wait until children have a strong conceptual understanding of tens and ones. In the early years, it is better to support children to invent their own algorithms (Kamii, 1998, cited in Wright & Rumiat, 2010).

From table 5.3, we see that when the August interview was conducted, learners were in a better position to display their knowledge of counting strategies and had made progress. This might be as a result of time spent on this strategy in class. From personal observation, in almost every club session, the mentor made sure that she emphasized quick and efficient mental strategies for calculations with number. In an informal conversation with one of the mentors she said she did not necessarily follow the curriculum but rather focused on building on where the learners were in terms of their mathematical knowledge. Additionally, from that same conversation, it was noted that developing more efficient strategies was among the top priority topics learned in the club.

5.2.1.4 Base-ten Arithmetical Strategies (BTS)

Base-ten strategies refer to the learners' knowledge of tens and one structures of the numeration system (Wright, et al., 2002). To determine children's levels in base-ten arithmetical strategies, they were asked to solve different types of tasks such as add 10 to 92 or use ten-dot strips. The interviewer observed whether the child counted by tens or by ones.

Table 5.4 below shows the learners' level of development of base-ten arithmetical strategies.

	Arithmetical Strategies (BTS)	
	April	August
	Level	Level
Kaino	3	3
Alleta	3	3
Olien	2	3
Elia	2	2

Table 5.4[6] learners' levels of BTS

The results suggest that both Alleta and Kaino were already at facile level (level 3 or up) of base-ten in April and remained there in August. Here, they were able to see ten as a unit comprising

Levels of BTS: 1 Initial concept of ten, 2 Intermediate concepts of ten and 3 facile concept of ten

ten and were also able to solve addition and subtraction tasks involving tens, without using materials. Unlike other aspects of this interview, base-ten is one of the aspects where most learners rarely depended on visual material to get the answers. In both April and August Elia was found to be at the foundation concept of ten level, in which he was only able to see ten as a unit composed of ten ones. Moreover, being at level two of the base-ten arithmetical strategies shows that Elia found it hard to successfully solve at least 90% of the addition and subtraction sentence tasks when those tasks were presented as written problems. As table 5.4 shows, all four learners were able to see ten as a unit of ten ones. However, it was a challenge to assign levels to these learners as they relied on counting-on or counting back strategies to solve most of the addition and subtraction problems that involved tens and ones, more especially during the April interview.

Overall, there seems to have been no progression in terms of shift from one level to another (with the exception of Olien who progressed one level). However from an observational point of view there was a slight progression in terms of learners' strategies within the same levels. The shift in levels here refers to the learners' ability to improve the initial base-ten arithmetical strategies but the improvement does not make them to move to the next level. For example Kaino's base-ten strategies were at facile level (level 3) and again in the second round of the interview she was still at level 3. From the interview one could see that Kaino has improved in her strategies but based on the different criteria assigned she remained at level 3. Thus from looking at her work, one could see the shift within level 3. This shift could have occurred because of the strategies Kaino used during April and most likely depended on the materials she used compared to the ones she used in August. It was not so clear why these grade 3 learners or former grade 2 learners faced difficulties in solving two digit addition and subtraction problems. As per the curriculum document, these competencies were intended to be mastered at grade 2 level.

5.2.1.4 Combining and partitioning

Combining and partitioning refers to joining or separating collections. To assess learners' progression in these aspect learners were asked to give two numbers that add up t o 10 (combining) or asked to partition the number seven. Wright, et al.'s (2006) LFIN makes no provision for levels for assessing learners' progression in terms of combining and partitioning. Hence, to determine learners' progression in this regard, I needed to make use of percentage scores of correctness.

Table 5.5 below shows the average total scores of each of the four learners for partitioning and combining.

	Combining and Partitioning			
	April		August	
Kaino	100%		100%	
Alleta	75%		88%	
Olien	100%		100%	
Elia	100%		100%	

Table 5.5: Learners' scores for combining and partitioning

The results suggested that in April, three learners (Kaino, Olien and Elia) were able to answer all the questions on c ombination and partitioning. They were able to find the complement of a number to make 5 or 10 quickly and were able to give at least two partitions of a number, as this was the kind of question asked in the interview. One learner Alleta scored 75% and this shows that she was only able to find the complement of a number to make 5 without counting by ones. She was only able to give one or two partitions of a number. Alleta showed improvement in finding the complement of the number to make 5 or 10 but she was not as quick as the other three learners. Alleta was the only learner not to achieve 100% in August.

5.2.1.5 Spatial Patterns and Subitizing

The spatial patterns and subitizing refers to the assessment of whether the learner is able to instantly recognise and allocate a number word to a small group of perceptual items. To assess learners' progression in spatial and subitizing, the interview gave a range of tasks. For example learners were shown a card showing bags of apples for about 10 s econds and the learner was asked to say how many apples there were altogether.

Table 5.6 below shows learners' scores in spatial patterns and subitizing

	Subitizing	
	April	August
Kaino	100%	100%
Alleta	93%	93%
Olien	100%	100%
Elia	93%	100%

Table 5.6 learners' total average scores in subitizing

There was no model in LFIN for determining learners' levels in subitizing and spatial patterns. Hence, to determine learners' progression, I made use of percentage scores. As shown in table

5.6, the highest score was 100% and the lowest was 93%. Kaino and Olien scored 100% while Alleta and Elia had scores of 93% in April.

In comparing learners' August results with the April scores, the table shows that while there were only two learners with 100% scores in April, during August only Alleta did not score 100%. In summary, the table shows progress in terms of Elia's performance with a 7% increase in August, while the other three learners retained their April scores. Overall, subitizing was an aspect where learners performed well. Wright, et al. (2006) argue that activities that involve subitizing have an important role in young children's numerical development.

5.2.1.6 Early Multiplication and Division

Students' early knowledge of multiplication and division is based on the development of counting sequences, the skills of combining, partitioning and patterning and the students' ability to use equal groups.

Table 5.7 below shows learners' levels in early multiplication and division.

	Multiplication and Division	
	April	August
	Level	Level
Kaino	2	3
Alleta	2	2
Olien	2	2
Elia	1	2

Table 5.7 learners' levels in multiplication and division

It should be noted that multiplication and division *topics per se* were not tackled directly in the clubs although some repeated addition and sharing 'word' problems were regularly included. Thus perhaps learners drew on their prior-knowledge from previous grades, or from their work in the grade 3 c lassroom or their every day knowledge to answer some of the questions in this section e.g. show the 10x2 array of dots. Table 5.7 shows that in April at least three learners out of four were at stage 2, except Elia who was still at the initial grouping and sharing stage.

However, when the interviews were conducted for the second time in August, Kaino and Elia had advanced by one level while Alleta and Olien made no progress. Kaino moved from level 2 to level 3 and Elia moved from level 1 to 2.

Two possible explanations for this are that on both occasions when the interview was conducted, the club had not covered the topic of division and multiplication in detail, if at all. In a personal conversation with one of the club mentors, she indicated that it was not easy for them to cover those two topics, because, from their observations, learners still needed to consolidate the operations of addition and subtraction. There was also much ground to be covered as some learners seemed to be a grade level behind in some cases, so the club focused on some of the basic topics such as numeral identification before they moved to other topics including multiplication and division.

5.3 SUMMARY OF LEARNERS' MATHEMATICAL PROFICIENCY PROGRESSION BASED ON THE INTERVIEW

In terms of the interview-based assessment a summary of learners' progression is presented in Table 5.8 which shows learners' overall levels; scores and/stages in the different aspects of the LFIN for April and August.

	APRIL					AUGUST				
Learners	***K***	***A***	***O***	***E***		***K***	***A***	***O***	***E***	
Aspect (highest aspect level / stage)					*No learners at highest level*					*No learners at highest level*
NI (4)	4	3	4	3	2	4	4	4	4	4
FNWS (5)	5	4	5	4	2	5	4	5	5	3
BNWS (5)	5	5	5	4	3	5	5	5	5	4
SEAL (5)	3	2	2	2	0	4	4	4	3	0
BTS (3)	3	3	2	2	2	3	2	2	2	1
EARLY MULTI/ DIVISION (3)	2	2	2	1	0	3	2	2	2	1
Aspect (highest scores)					*No l's with highest scores*					*No l's with highest scores*
COMBINING & PARTIONING (100%)	100	75	100	100	3	100	88	100	100	3
SUBSITISING (100%)	100	93	100	93	2	100	93	100	100	3

Table 5.8 summary of learners' progression across all LFIN aspects

Table 5.8 indicates learner progression across all the LFIN aspects. We can see that there is general improvement across all LFIN aspects. For example in FNWS and BNWS, the number of learners at the highest level increased by 1 from April to August. Numeral Identification shows a pleasing growth from 2 learners at highest level in April to a full complement of 4 in August. The Strategies for Early Arithmetical Learning aspect shows the greatest progress where 2 learners progressed at least one level while two learners improved by two levels. However, none of them achieved the highest level of 5.

Although there is not a great deal of change shown in combining and partitioning, what can be seen is that Alleta increased her score by a substantial 13%. In subsitising, 3 learners achieved 100% and Elia increased his score by 7%.

What is interesting is that this progress across all LFIN aspects was made over the period from mid-April to mid-August, a period of just over 2 school months. Possible explanations for this progress could be the focus in the club sessions on a ctivities to develop efficient mental strategies, use of flard cards for place value work and for number recognition work.

5.4 CONCLUSION

This chapter presented and discussed the data collected in the interview-based assessment and was used to assess learners' proficiency progressions in terms of their basic number knowledge.

The data obtained from the various instruments was presented in tabular form with summaries. The chapter also presented, analysed and discussed the findings of the study, specifically in relation to research question two.

CHAPTER SIX - ANALYSIS AND DISCUSSION OF THE NATURE OF MEDIATION

6.0 INTRODUCTION

Chapter six deliberates the findings for the second research question. It presents the data collected from the club observations and the task-based interviews. As mentioned in Chapter 3, the social constructivist and socio-cultural perspective theories of learning were deemed appropriate for examining the nature of the mediation of learning within an afterschool maths club.

This chapter is divided into two sections. The first section concerns the discussion of the club as a learning context. This data was collected through observation and video recording of club sessions. In the second section, the notion of the ZPD to analyse the nature of the mediation of learners within the club was applied. The data used in the second section was collected through videotaping the two task-based interviews.

6.1 OBSERVATION: SETTING THE SCENE OF THE LEARNING ENVIRONMENT

This section presents a brief description of the club learning context as the empirical field for this research study. It was important to provide a picture of the learning context in which the learning took place. In total 12 club sessions were videotaped and selectively transcribed. A discussion of the various themes that emerged during the analysis of the videotaped sessions and my reflective journal follows.

6.1.1 Observation of the socio-mathematical norms of the maths club

> When the classroom environment in which students spend so much of their day is organized so that student-to-student interaction is encouraged, cooperation is valued, assignments and materials are interdisciplinary, and students' freedom to chase their own ideas is abundant, students are more likely to take risks and approach assignments with a willingness to accept challenges to their current understandings. Such teacher role models and environmental conditions honor students as emerging thinkers (Hill, 2002, p. 110).

As mentioned earlier the study was undertaken in an informal setting (afterschool maths club) at one of the township schools in Grahamstown. There were 10 learners in the club and four of them were selected as participants for my research. All the learners were English second language speakers.

According to Graven and Stott (2011), the curriculum is the contextual guide to what learners should learn but individual learners' numeracy levels guided the activities done in the club. In this respect the mathematical content covered in the club complemented what they did in the classroom. The club met weekly after school in 2012 except during school holidays. The sessions ran for about an hour and participation was voluntary.

On the basis of observation and the entries in my journal, it appeared that learners were encouraged to take responsibility for the direction and pace of their mathematical learning. For example different learners worked on different activities in pairs depending on their levels and pace and learners were encouraged to make decisions and suggest rules for the warm up dice and card games played at the start of sessions.

Most of the club activities I observed were done in small groups or pairs. Learners were encouraged to discuss and learn from each other. At the beginning of each session, the whole group came together to discuss the activity for the day. This enabled the mentor to explain or give instructions or to do 5 to 10 minutes of mental maths work. The group regrouped again at the end of the session for a few minutes to allow the learners to explain a discovery or for the mentor to consolidate learning. This activity was not always possible due to limited time or when their transport arrived early.

Interaction amongst learners and sharing of ideas was also common. Mentors were found to be encouraging and supportive. The mentors circulated among the pairs, mediating through questioning and actively intervening to challenge the direction of learners' thinking. At the beginning of the year boys and girls were not willing to work together but this improved, especially when they played games together.

The next section presents and discusses various principal norms observed during the analysis of the videotaped club sessions and my reflective journal.

6.1.2 Prominent norms observed

The data analysis process involved repeated engagement with the data using an iterative approach. During this process, various themes and common norms were visible. Norms promoted by both mentors were *a) fostering of "math talk; b) encouraging learner autonomy in the mathematics club c) establishing collaboration and co-operative working and d) modeling and nurturing positive attitudes and sense-making engagement.* While acknowledging that the emergent norms were not meant to be completely discrete as there was overlap, for the purpose of this study I discuss them separately. I focus on the topics/themes that proved to be significant aspects of the findings from both a social constructivist and socio cultural perspective.

6.1.2.1Fostering math talk

A strong thread running through the transcription sessions and field notes between March and August was that both mentors (Debbie and Mellony) maintained and created a maths talk inquiry environment and promoted discussion among the club learners. Learners were regularly encouraged to discuss problems. The mentors would generally introduce an activity and after clarifying instructions, the learners would be given a chance to work on their own or in pairs initially without the mentor's mediation. The mentors would encourage the learners to discuss their answers although this was not always immediately taken-up, particularly in the first few months of the club.

Moreover, learners were constantly encouraged to ask questions and explain their strategies to others whilst working together to solve mathematical problems. For example when a learner arrived at an answer the mentor would ask the learner to explain how she/he got an answer. (e.g. '*explain to the group how you got the answer'*). Additionally, mentors made an effort to elicit questions from learners and re-voiced learners' answers.

The following excerpt is an example of a mentor encouraging learners to explain:

M: Now when I say to you 8 plus what is 10, (talk to your friend)
Ls: 8 plus 2,
M: how do you know it, do you just know it or you also count 9, 10?
L: I know it
M: So if you know it for 8 and 2 then if I say to you 98 plus what is 100?
L: 2
M: How did you know it? How did you know it? Explain?

L: It was easy, 8 plus 2 is 10

M: So then 98 plus 2, how come is that 100?

Ls: Because 8 and 2 is 10 and 98 plus 2 is 100).

M: Okay. Alright. Okay.

The excerpt above shows an example of how the club mentors support learners to explain and justify their reasoning in the club as a way of fostering maths talk. The process of allowing learners to discuss each problem gave the mentor an opportunity to facilitate mathematical thinking. While learners were discussing their ideas the mentor could pick up when a learners' mathematical knowledge was developed or delayed. When learners struggled the mentors would usually work with that specific learner or pair of learners and engage with their thinking using questions and suggestions.

This concept is supported by Kilpatrick, et al. (2001), in their study that emphasizes the need for students to discuss their mathematical thinking as a way of increasing understanding. Additionally discussion can benefit learners by affording them opportunities to express their thinking in the learning context thereby making them feel less nervous to participate. Klibanoff, Levine, Huttenlocher, Vasilyeva, & Hedges (2006), discovered that teacher-facilitated "math talk" in the early years significantly increased children's growth in understanding of mathematical concepts (p. 59).

In analyzing the session transcripts I observed that the mentors constantly walked around helping learners in groups or on an individual basis. The transcripts revealed that the mentors tried to direct learners' answers often through re-voicing the learners' strategies to others.

M: Okay alright Sonia how did you do it so quick?

S: I said 100 plus 40 is 140 and 140 plus 5 is 145

M: See how she did it? She took the big number, what is the biggest number?

Ls: 100.

M: And then she added 40 and then she put 5 on the end. So that is a quick strategy

This example shows that the mentors used re-voicing as another strategy of increasing maths talk within the club. The process of *re-voicing* gave the mentors an opportunity to embed mathematical language into the discourse so that learners could further explain their thinking.

6.1.2.2 Encouraging learners autonomy in the mathematics club

From several transcripts I examined, another strong thread that emerged was that learners took an active role in the process of building their own knowledge and in understanding the activities. The types of questions or instructions used by both mentors showed that the learners were actively involved in their learning. For example, a mentor gave an instruction for an activity for bonds to 20. Learners formed pairs but they were also free to work on their own except that their end product should be compared and result in a shared understanding. Olien, one of the four case study learners decided to go beyond the given instructions by pushing the boundaries of the activity. In the example below, Olien went to the point where he found three numbers (rather than 2) to make 20. As reflected in the transcript below, this was not just in one row, column or diagonal (as given in the activity instructions) but in the form of an L-shape.

M: Aaaah you clever boy now you've found 3 hey? You got creative there I like that - that's creative thinking. Ok circle that one for me and I want you to show the others your creative thinking. The activity says circle 2 that make 20 but you found three (as she was talking to Olien). Okay, can I show you quickly what Olien did? (Mentor asking the whole class to look)

Class: yeees!.

M: Olien said ...I mean tell them loudly Olien!

O: I said 6 plus 4 plus 10 (pointing to 6, 4 and 10 in the L-shape in the grid)

M: Oh wow, then you did a fancy one hey? So, you said ok I'm not just going to look for one I'm also going to look for 3's. Is there anyone who can find another 3, be creative, and be creative. After you have found all the 2's see if you can find what Olien did.

In the example above Olien had the opportunity to influence the direction of the activity. By showing his work to the others the mentor was implicitly encouraging this autonomy and influencing the nature of participation in the club. Self-initiated activities can support learners to become masters of their own world (Baratta-Lorton, 1994). Studies have shown that learners who control their own learning were more independent, socially connected, self controlled, and self reliant. And when learners' chose the direction of their explorations and investigations, their discoveries were enhanced by fellow peers and teachers as they learnt collaboratively (Baratta-Lorton, 1994). In all the maths club sessions learners were also able to suggest the game they would like to play in the following club session.

The overall impression showed a degree of autonomy in the club learning context and at the same time the mentors also maintained communal understanding through sharing of ideas. This concurs with research which argues that the responsibility of learning should reside increasingly with the learner (Glasersfeld, 1989 cited in Ernest, 2006). Benson (2001) posited that maintaining autonomy is supported by the social constructivist tenet of active learning. According to Little (1994), learners' autonomy is "the product of interdependence rather than independence" (p. 435) which underscores the dynamics between collective and individual actions.

6.1.2.3 Establishing collaboration and co-operative engagement

Another prominent norm observed during the club teaching and learning session is both mentors and learners were mostly found working together and supporting each other to achieve a common goal. This was evident in most sessions as the mentor ensured that there was a shared understanding among learners. The mentor gave learners an opportunity to work in pairs or groups and to explore the mathematical concepts through discussion. For one example the mentor related:

Mentor: Okay. Yes, they can leave it there and watch here because I am going to get those who got it right to be helpers, okay, you guys stop, everybody's attention. I need you to be the helpers. ....you can compare the answers with a friend but you are not allowed to copy friend's answers.

The session's transcription shows that the children were given opportunities to work together in pairs or in small, cooperative groups and also to help others where possible. Although sometimes learners chose to work alone, they always shared their ideas at the end of the session or discussion. Collaboration is a key element in terms of the Zone of Proximal Development (Vygotsky, 1978) whereby students have the potential to achieve skills that they cannot attain on their own, but which become attainable when working together with others.

Learners mostly worked in pairs, with the support of mentors moving from one pair to another. It was evident that getting children to engage with each other in the activities was a bit of a problem more especially during the first months of the club. It was interesting to see that some of the learners started to enjoy sharing ideas in pairs and it became routine as time went on. For example learners like Elia were initially not open to speaking in a group or in pairs but towards the end of the study he was one of the learners who participated well in pair work.

6.1.2.4 Modeling and nurturing positive attitudes and sense-making engagement

Another key theme which occurs in almost every transcription of the club sessions was the mentors' ways of modelling positive attitudes. The afterschool maths club environment seemed to contribute not only to active and cooperative learning but also created room for learners to be comfortable and build on their increasing confidence. For example, shy students were observed to participate more readily, and were more enthusiastic than at the start of the club. This could be attributed to the fact that norms were established where learners were not allowed to make fun of others who made mistakes. The mentors always emphasized that it was acceptable to make mistakes in the club and that mistakes created good learning opportunities. Below is an example of the mentor emphasising that mistakes are a necessary part of learning in the club:

M: Now guys I want to make a point, did you notice how quickly you all learned that technique but I also want to point out that in the beginning when we made some mistakes over here with adding those 1000's and 100's, do you remember? And then we were helped and we learned, ah, okay not to do that. Did you notice from that learning how quick you became and you landed up being the first to finish the second time around, so what I want to point out is that making mistakes is sometimes very good, because it is in the making mistakes that we learn and you will never forget it now partly because of that mistake one remembers the right way okay so we must never be scared of mistakes in maths. Mistakes are your friends. When you make them as long as you learn from them they are wonderful things. Okay.

From my journal I noted that both mentors ensured that when posing a question they did not just give the opportunity to answer to those learners whose hands were up but included the other learners. They made sure they supported them by means of probing further and giving them hints to enable them to get the correct answer.

6.2 Learning within the Zone of Proximal Development: Analysis of task-based interviews

This section focuses on mentor-learner interaction in two paired task-based interviews with the four case study learners (2 boys and 2 girls). The ZPD and literature on scaffolding was used to describe and analyse the mediation of mathematics learning of the four learners engaged in task-based interviews.

For the purpose of analyzing the videos, it was essential to look at the mediation interactions between learners and also between mentors and learners in the context of their dialogue in a task-based interview. I looked for evidence from all the elements of the data-set to help draw inferences regarding the nature of mentor to peer mediation within the club and how it enhances the mathematical learning of the four learners.

My aim was to explore salient qualities of the ZPD that were mutually constructed in the interview context over time. I focused exclusively on the learning dialogues, with reference to learner actions and written productions during the task-based interview. This was done in the interest of clarifying incidences of the emergence of the ZPD. I selected only one episode from each pair of participants to enable in-depth interpretation of the emergence of the ZPD and the learning within.

My focus is on what I understand to be mediation of learning and the indicators that make it possible for me to recognize this. The ZPD was used as a tool for analysis for this study. Hence, the focus in this study is on how mediation interaction leads to the emergence of the ZPD and how that can support learning. To be able to describe and explain how learners learn in an afterschool maths club, I used video excerpts of the task-based interviews and selected two episodes where there was sustained conversation between learner and learner and/or also between mentor and learner. Additionally, the two task-based interview sessions were examined for the use of conversation, gestures or utterances as part of creating the ZPD that leads to mathematical learning. In every case, I viewed the videos a number of times, including in slow motion, in order to ensure the accuracy of the transcription and to isolate gestures and the combination of speech and gestures.

Drawing on how other researchers have analysed young learners' interactions, I used Meira and Lerman (2001) and also Lerman (2001) in looking for learners' attention being captured a) by the mentor; b) or by the fellow learners and c) learners' utterances and actions being directed and modified by the mentor or by their fellow learners. I also looked for instances where the utterances or actions of one student affected another, resulting in a modification of what they then said and did (Lerman & Vile, 1996, cited in Lerman, 2000, p. 11). In short, I searched for events of communication from the video and the transcripts in which students "catch each other's thoughts" (Lerman, 2001, Meira & Lerman, 2001). I used the clues to the socio-cultural setting which affected the creation, or otherwise, of the ZPD, to identify the imbalance in relationship in knowledge as well as the ways in which collaborative learning (working together) were framed by the mentor, the students, and other factors in the club.

I have chosen episodes that illuminate the emergence of the ZPD. The two episodes serve as illustrations of the types of activities and mediation occurring in the maths club. The chosen sessions were conducted by two mentors from the club namely, Mellony and Debbie. The paired interviews included a series of tasks given to learners. Learners were encouraged to solve the problems together; however they were also allowed to work on their own.

6.2.1 The process of selecting codes for analysis

In analyzing the video and transcripts of the task-based interviews, I used a schema of 12 scaffolding practices for effective numeracy mentors developed by the Department of Education

Science & Training in Victoria, Australia (DEST, 2004) (see table 6.1 below). This was because scaffolding emerged as a commonly repeated mediation practice by both mentors in both club sessions and in the task-based interviews even while mediation of learning did not always involve scaffolding.

Table 6.1: Scaffolding practices

12 categories of scaffolding practices (DEST, 2004)	
Mentor –learner	
Category:	***Descriptors***
Excavating:	drawing out, digging, uncovering what is known, making it transparent
Modelling:	demonstrating, directing, instructing, showing, telling, funnelling, naming, labelling, explaining (mentor models mathematical thinking)
Collaborating:	acting as an accomplice, co-learner/problem-solver, co-conspirator, negotiating
Guiding:	cuing, prompting, hinting, navigating, shepherding, encouraging, nudging,
Convince Me:	seeking explanation, justification, evidence; proving;
Noticing:	highlighting, drawing attention to, valuing, pointing to
Focusing:	coaching, tutoring, mentoring, flagging, redirecting, re-voicing, filtering
Probing:	clarifying, monitoring, checking
Orienting:	setting the scene, contextualising, reminding, alerting, recalling
Reflecting/Reviewing:	sharing, reflecting, recounting, summarising, capturing, reinforcing, reflecting, rehearsing (mentor asks students for : clarification; elaborations; justifying their response and strategies, mentor encourages reflection, self monitoring, and self checking)
Extending:	challenging, spring boarding, linking, connecting
Apprenticing:	inviting peer assistance, peer teaching, peer mentoring

In relation to the task-based interview in this study, I became aware of the absence of any explicit reference to evaluating in all the categories in the DEST framework above. I thus chose to add 'evaluating' as a descriptor of several categories such as *guiding*, *convince me, focusing, reflecting and noticing.*

Additionally, I also found that not all 12 codes were equally useful as a means of classifying and analyzing the selected excerpts for the study. The majority of interactions were between the mentor and learners with few interactions between learner and learner (especially in the case of the boys). Thus, in most cases, I have used only those scaffolding practices that focused on the mentor-learner interaction.

These codes helped in providing a language to describe and analyse what happened in the episode/dialogue. Additionally to mark where the ZPD emerged in the transcribed dialogue, I drew on the work of Lerman (2000, 2001, and 2012) and Meira & Lerman (2001).

Next, I present how the interviews were analysed by giving a summary of each selected episode with a description.

6.2.2 Analysis the nature of mentor-peer mediation

There were three tasks for each pair of learners (interviewees) to work through and each pair was interviewed by a different mentor. Debbie interviewed the pair of girls while Mellony interviewed the pair of boys.

Each task-based interview took an hour for each pair of learners. For analysis of the study, I divide selected tasks into episodes. Each episode consisted of an extended conversation about one part of the task. Episodes were then divided into segments and it was those segments that form the basis of the summary of each episode. An episode contained dialogues of learners focusing on a specific task or question related to a given tool. For the purpose of analysis, the segments were then divided into excerpts. I analysed two different episodes from the task-based interview, one from the girls and the other one from the boys.

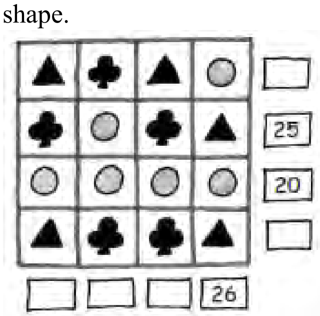
First, I present the episodes from the girls task-based interview. Here I focused on the activity of completing a logic puzzle while I focused on a mini number board puzzle activity for the boys.

Presentation of the episodes and excerpts included an indication of a turn (mentor and learners taking turn in a conversation), transcript of the dialogue, corresponding utterances and actions (in round brackets and italics) and coding. Transcript conventions included the following: turn numbers in subscript correspond to, and index, the onset of an action or gesture and the boundaries of overlapping talk are shown in round brackets and in italics, while the square brackets show the learners' writings or what the mentor or the learners pointed to. The sequences chosen from the data help to illustrate the features of teaching and learning that form the focus of this research study.

Excerpt 1: Girls working on the number logic puzzle

Below is the given task. Only one sheet was provided for both learners to work on.

In this table, each shape stands for a number. Find the number for each shape.



This episode involved Kaino and Alleta, working on the puzzle which required them to find what numbers each shape stood for. This task and episode was the third and last in the girls' interview. Debbie, the mentor and the interviewer, started the task by asking learners to identify the different shapes. Her introduction was not part of the planned activity, however, it served as a warm up introduction for the activity in which the mentor set the scene and invoked learners' prior knowledge about those shapes. Table 6.2 below provides a summary of the whole episode.

Table 6.2: Girls episode overview

turns	Segments	Description
1-25 00:10 to 01:47	1	Mentor and learners went through identifying different shapes in the task (e.g. circle, triangle and clubs). This was mostly done with the mentor asking a question relating to reminding the learners where they had seen the shape before and what it is called. Learners were mostly responding to the mentor's questions. The mentor wanted the learners to find out the value of each shape. Learners were required to make use of the information they got in the introduction such as the total given in the activity for some rows and columns. The mentor specifically directed learners to row 3 where all the shapes are the same and the total for the row is given. Learners were given a hint thus they both started looking at the grid and started whispering/counting. After a few seconds, Alleta shouted "5". Through questioning and checking of learners thinking the mentor encouraged learners to clarify their thinking. The mentor and the learners agreed that the value for a circle was 5.
26-49 01:51 to 05:09	3	After the learners found the value for the circle (as 5), the mentor asked the learners to find the value of the remaining shapes (clubs and triangles). The mentor suggested learners work together. The learners went on with task using the information they knew about the value of the circle. Here the learners were uncertain as to what to do thus when the mentor asked them to share what they were doing they indicated that they were not sure. In response to this the mentor started posing questions. In this segment the learners did not get the value of any shape however they were given the clue of looking at the column with 2 circles and 2 triangles.
50-78 05:10 to 07:15	4	In this segment, the learners were supposed to use the clue (26 total and 5 for each circle) they got in the third segment to work out the value of the two triangles in the last column. A few seconds after the instruction Kaino gave her answer (13) first without spending much time on calculating. Alleta however spent some time before giving an answer. The mentor followed up on what Alleta was doing by asking questions relating to justification, elaboration and clarification of what she was doing. In her responses, Alleta indicated that she was looking for a half of 16 after subtracting 10 (the value of the two circles) from 26 (the total value of the column). The mentor also tried to amplify what Alleta was doing or saying and asked Alleta to explain or justify her strategies. Alleta's final answer was 8 for each triangle.
79-128 07:16 to 13:38	5	Using the information they got (value for triangles and circles) learners were then supposed to find the value of the club. Learners started working individually using their fingers. While working on the problem, the mentor turns back to Kaino again and asked her "how did you get 13"? This was the 13 that Kaino got when they were trying to find the value for each triangle. Step by step then Kaino went through the strategies she used to get 13. While doing so the mentor helped Kaino to realize that her answer was not applicable for that specific question. Additionally the mentors made sure learners were at the same place (e.g. M: ok, can you see what she did, because with your way you didn't take those, you didn't think about what that was did you"?). Meanwhile Alleta was told to go ahead with finding the value for the club. The mentor used a series of questions to prompt and guide learner thinking and encouraged learners to explain their ideas and decisions. Learners were also being directed to reflect on their answers.

Overall, the episode showed interactions between the mentor and learner as they discussed (through questions and answers) ways to find solutions to the puzzle. As an example of a recurring pattern of mediatory interaction from this episode, we will consider excerpts 1 to 3.

In excerpt 1, the mentor directed learners about what to do and asked Alleta to explain her strategy. The mentor then repeated and expanded on Alleta's strategy (turn 22) and opened the discussion to further contributions from Kaino.

Keys: M-mentor, K-Kaino, A-Alleta, and Ls-both learners

Excerpt 1: girls, working on the logic puzzle (turn 16-25)

Turns	Transcript	Scaffolding practices: categories
16	M: Alright now if you look at this, what I want to know, I want to know for each of these shapes, what number they represent. I want you to work out. Each of these shapes has got a number but you don't have a lot of information, you have to work out what you think each of these shapes numbers could be. *(Mentor, put her hand up indicating to Alleta to wait)* So you are going to work it out together. *(Alleta, quickly put her thumbs up)*[7]	***Orienting**: setting the scene*
17	A: it is 5 plus	
18	M: oooh, okay. GO GO! *(excited voice)*	***Guiding**: encouraging , prompting and evaluating*
19	A: plus 5 are 10 and 5 is 15 plus 5 are 20 *(at the background Kaino, was counting in multiple of 5)*	
20	M: So this here is the total? *[pointing at total for circles]*	***Reflecting**: reflecting and* ***Noticing**: highlighting*
21	Ls: yes	
22	M: It is the sum so you are saying the circles are?	***Focusing**: re-voicing* ***Noticing**: highlighting*
23	A: 5.	
24	M: So put a 5 here next to the circle. Sorry I didn't ask you that Kaino, do you agree with that.	***Modelling**: directing* ***Reflecting**: reflecting and sharing*
25	K: yes	

In this excerpt, the aim was to work out the value of each of the three shapes in the puzzle given above. Debbie (the mentor) asked the learners to work out the answer individually or in pairs. Right after the mentor finished with giving the instruction, Alleta shouted "it is 5". While the mentor was busy giving the instruction, Alleta indicated that she already knew the answer (by putting her thumb up). From watching the video, the mentor's attention seemed to be caught by

[7] This indicates that she has an answer (this is a practice widely used in the club so that other learners could see, and the mentor could know that learner is ready to share)

Alleta's answer thus she turned towards her and started to prompt, guide and probe through questioning. At the same time the mentor also encouraged Alleta to go ahead with the explanation of her answer until it emerged in a clear form.

Thus I can say that there seemed to be little scaffolding needed at this point, more especially for Alleta who already knew the answer. Instead what the mentor did was only to confirm (evaluates positively) and allow Alleta to reflect on her initiation. For example in turn 16 and 17, just after the mentor had finished introducing the new task, Alleta started giving the answer. The mentor also made sure that in the end there was a joint understanding between the two learners. This can be seen in turn 24 when the mentor asked Kaino, "Sorry I didn't ask you that Kaino, do you agree with that?" and Kaino agreed.

From the transcript we can observe that the mentor's overall scaffolding practices were mostly to confirm through a joint understanding of the focus of the task through Alleta's pathway and checking through prompting. The mentor also allowed learners to reflect and orchestrated the sharing of ideas for example in turn 24. This seemed to lay a foundation for further activity in relation to finding the values of the other shapes.

There seems to be little ZPD created between learner and learner because there was no point where both learners regulated each other's behaviour although Kaino agreed with Alleta. According to Goos (2004), the ZPD can emerge in situations where learners are able to regulate their own and their partner's behaviour accordingly. On the other hand, the ZPD can also emerge where they are catching others' ideas for instance, in the conversation or dialogue where the participants establish joint understanding, listen to one another, consider each other's utterances and find a common way to solve tasks (Lerman, 2002). When the mentor suggested that the learners could work to together as indicated in the excerpt, Alleta already had the answer in the second turn but Kaino was not paying attention to what Alleta was doing.

After Alleta found the value for the circle, the mentor confirmed the information and asked them to work out a value for the other two shapes i.e. the clubs and the triangles.

Below is the second excerpt that shows the interaction between Alleta and the mentor.

Excerpt 2: girls' working on a logic puzzle turns 26-49

Turns	Transcript	Scaffolding practices: categories
26.	M: Now you have to work out what this club is and what this triangle is, with the other information that you've got.	***Focusing**: redirecting*
27.	Ls: Ok	
28.	M: Work together if you want. *(After, checking learners' work)*. [*Kaino, reoriented the paper to herself, while Alleta moved to reposition herself to see what is on the paper*]. That is a really good start getting that now you can see what is next.	***Orienting**: setting the scene* ***Guiding**: evaluating and encouraging*
29.	Ls: *(moved closer to each other but each one of them continues to count her own way)*	
30.	M: Talk louder if you can.	*Asking for the purpose of recording video (not to do with the learning process)*
31.	Ls: (5)	
32.	M: you can write here if you want to so if you want to put things next to those, it is up to you.	***Modelling**: directing*
33.	Ls: *talking quietly and adding using their fingers [Alleta rewrites the 5 on the paper]*	
34.	M: so tell us what you are thinking? How are you going to do the next bit?	***Probing**: monitoring* ***Guiding**: prompting*
35.	Ls: we don't know what is this number *(pointing at the paper)*	
36.	M: No you don't!	***Reflecting**: reinforcing*
37.	Ls: Yes,	
38	M: but what do you know what else, what other information do you have?	***Guiding**: prompting*
39	Ls: circles	
40	M: you have got circles!	***Guiding**: reinforcing*
41	Ls: yes	
42	M: so can I give you a little bit of an idea? I am not going to tell you how to do it. You know that this is equal to?	***Guiding**: prompting and hinting*
43	Ls: 20	
44	M: and each circle is? (with a smile on her face)	***Guiding**: prompts*
45	Ls: 5	
46	M: okay so here is another place where it has given us an answer, okay, so we have got one circle and another circle, how much does that equal?	***Guiding**: cuing, prompting* ***Modelling**: funnelling*
47	Ls: 10	
48	M: okay and what does the whole column equal?	***Guiding**: prompting and directing*
49.	K: 26	

Turn 26 of excerpt 2, the mentor focused the learners by asking them to find the value of the other two remaining shapes i.e. the triangle and the clubs. Again, the mentor suggested that the learners work together. From viewing the video I noticed that the learners moved closer to each other, however, they continued to calculate on their own, using their fingers.

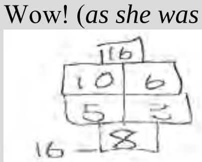
Learners were given time to think and reflect on the task where possible. Between turn 26 to 34, learners tried to figure out what to do for quite a long time (40sec) while the mentor tried to establish the new direction of the task through probing and guiding. When the learners were busy trying to figure out what to do, the mentor interjected and asked them how they were going to work out the value of the remaining shapes. In response the learners indicated that they were not sure what to do: "We don't know what this number is" (turn 35).

When the learners stated this, the mentor changed her ways of scaffolding to a more directed way. Thus between turn 35 to 45 the dominant scaffolding practices were guiding through prompting. Throughout, the mentor provided a series of simple prompting questions that learners were able to answer. Learners kept responding to the mentor's questions and thus one can argue that the mentor managed to keep the learners focused and working through questions to the point where the learners noticed how that approach could help them to proceed to the next step.

In turn 46, the mentor started with more directed scaffolding by modelling through redirecting and funnelling. The mentor asked questions to support the learners by enabling them to get the value for the triangle and the clubs. From line 46, the mentor explained, demonstrated and showed learners what to do. The mentor evoked the ZPD from the way she directed scaffolding. This matched what Lerman (2010) envisioned that the emergence of the ZPD depended on both the learning environment and the people in it. To recognise the emergence of the ZPD, Meira and Lerman (2001) and Lerman (2001) suggest that one needs to look for the utterances or actions where one person affects the other. For example in a dialogue, the mentor/learners primarily called for demonstration or catching each other's attention. Mercer and Fisher (1997) maintain that the ZPD for any particular child on any particular task can be established in the course of an activity. They added that key to establishing the ZPD space was the quality of the supportive interventions of a mentor.

In the following excerpt, the mentor asked Alleta what she was doing, but at the same time she was told to continue working (turn 59).

Excerpt 3: girls' task working on the logic puzzle turns 59-77

Turns	Transcript	Scaffolding practices: categories
59.	M: so what is your answer Alleta? [*Alleta starts with writing as shown here*]  Explain what you are doing Alleta? *(Alleta, continues to calculate)*	***Probing**: checking* ***Convince Me**: evaluating*
60.	A: I want a half	
61.	M: you want to half?	***Probing**: checking* ***Focusing**: re-voicing*
62.	A: yes	
63.	M: So what did you get? You said 26 minus 10 was 16 and now you are trying to work out what half of this is. Is that what you are trying to do?	***Probing**: checking and evaluating* ***Focusing**: re-voicing
64.	A: yes	
65.	M: So tell me, so tell me how you have done that,	***Probing**: asks for explanation*
66.	Ls: I can…….*(Kaino, talking at the background trying to correct Alleta)*	
67.	M: what do you think half is? Have you got your answer? Wow! (*as she was looking at Alleta's work):* 	***Probing**: checking* ***Noticing**: evaluating and valuing
68.	A:hhhm	
69.	M: So what are you saying Alleta?	***Convince Me**: seeking for explanations*
70.	A: I said 8 *(Kaino at the background: they are 8).*	
71.	M: Do you see what she is doing Kaino? *(mentor turns and faces Kaino)*	***Reflecting**: sharing*
72.	K: yes	
73.	M: I think what Alleta has done. [*pointing at Alleta's work]* (*As shown in turn 67).* I am just trying to say what she has done. 26 minus 10, so we have got 26 and we knew this was 10, so these 2 together were?	***Noticing**: highlighting* ***Focusing**: re-voicing*
74.	Ls: 16	
75.	M: and then you said if it is 16 together, then I have to find half of that?	***Noticing**: highlighting* ***Focusing**: re-voicing*
76.	Ls: yes	
77.	M: so you got 8. *(Kaino, staring at the window)* Do you agree with that Kaino?	***Reflecting**: drawing attention to, reflecting and sharing* ***Noticing**: pulling her back into the activity*
78.	K: yes	

In this excerpt the learners continued to work to find the value of the triangle. After a few minutes of directed scaffolding from the mentor Alleta tried to come up with a strategy to work out a value for the two triangles. As it can be seen from the excerpt, at this point the mentor noticed and re-voiced what Alleta was doing, and gave her a chance to explain what she was doing. This was evident in turn 59 to 70.

Therefore in this episode scaffolding was adjusted from a more directed support (in a previous excerpt) to a less supportive one to allow Alleta to finish her work and be able to explain what she was doing to the mentor and to Kaino. In turn 59 and 61 the mentor prompted and evaluated what Alleta was doing by asking her to explain what she was doing. At this time the mentor did not provide Alleta with enough time to finish with her work, as the mentor interjected while she started drawing the picture in turn 67. But through questioning, the mentor gave Alleta a chance to finish and go through her work until she got the answer (8) as seen in turn 70.

This excerpt infers that both the learners continue working within the ZPD, which was created in the previous excerpt. This was evident from turn 59 to 70, when Alleta was busy working on her strategy to get the value for each triangle. In excerpt 2, the mentor interacted in a directed way of scaffolding in which she modelled and allowed learners to make use of the information they had in the previous excerpt and to use it to get the value for each triangle. The information as reflected in column 4 was triangles =26. Thus in using that information Alleta came up with her strategy as shown in turn 67.

The next part of the chapter looks at the boys' working on the number board puzzle.

Boys working on mini number board puzzle

Task 1 – mini number board puzzle (boys task-based interview)

15	35	20	• Can you predict which row has the highest total? • Which three numbers total 38? • Are there other numbers that total 38? You can add together as many numbers as you like to total 38. • Find one number that is half of another.
6	14	8	
12	40	10	

Task 1 above was one of the 3 tasks done by both boys and girls during the task-based interviews. The transcription shows both mentor–learners' interaction between Mellony (mentor), Elia and Olien.

At the beginning of the interview, the mentor emphasized the need for working together to the learners and sharing as they usually do during the club sessions. She pointed out to the learners that there should be no difference between the way they behave during the club sessions, except that she (the mentor) wanted them recorded in a noise-free environment. The table below provides a summary of one of the episodes from task 1 of the boys' transcript. This episode was about learners trying to find an answer to question number 2 and 3 of task 1, as shown in the table above.

Table 6.3: summary of boys' selected episode

Turns	Segment	Description
1-46 (05:40 to 09:25)	1	The mentor started asking the learners to find three numbers that add up to 38 from the grid. Students looked through the grid and kept on brainstorming the answers. Learners also used their fingers to get the answers. The mentor asked learners to find more sets that add up to 38. In response to learners' suggestions the mentor asked questions and made comments such as "can you find other numbers that add to 38", "but 40 and 14 is already too big, is it giving you 38?". The mentor also asked learners to clarify their answers. Together with the learners, the mentor checked the learners' suggested answers. The segment goes on with the mentor continuing to ask questions relating to the learners answers and encouraging them to keep searching. Learners kept on suggesting different sets of numbers and respond to the mentor's follow up questions. Learners used their fingers to count throughout the segment.
47-64 (09:26 to 11:14)	2	In this segment the learners continue to search for three numbers that add up to 38. The mentor starts by highlighting to the learners that the strategy they were using was good for adding the tens nicely but they should find a way to get eight as the unit digit in 38. Students proposed different answers. The mentor repeated the learners' answers and asked learners to check if they get the 8 in the units. Learners kept searching, suggesting and using the suggested strategy of finding 8 in the unit to check their answers. The mentor asked questions in response to the learners' response such as "you are right; these two added up to 32, what do you still need to make it 38?" In this case when a learner gave the right answer the mentor asked him to convince the other learner.
65 – 95 (11:16 to 13:35)	4	This exchange continues with the learners suggesting various numbers while the mentor responded with guiding questions. Some of the questions were "what do you need to add to 30 to get 38?". The exchange went on with the learners searching for more numbers. When there were no more sets of three numbers that added to 38 in the grid, the mentor suggested looking for four numbers that add to 38.

In the following part of this section I present excerpt 1 to 3, which I see as illustrative of other interactions during the interview. The mentor introduces the task by asking the learners to look at the grid of numbers and find three numbers that add up to 38. The mentor emphasized the need for the learners to work together, however this rarely happened. Thus they ended up getting different answers. However, after each question the mentor encouraged learners to check whether they understood the logic of the each other's work.

In the first excerpt of the task episode, the mentor asked a number of questions and the learners responded to them. Below is excerpt 1 from the extended episode:

Keys: M-mentor, O-Olien, E-Elia, and Ls-both learners

Excerpt 1: boys' task working on the numbers grid of, turns 1-48

Turn	Transcript	Scaffolding practices: categories
1	M: Alright. Um, in this picture, there are, 3 numbers that add up to 38 *(Elia, facing the other side, then the mentor goes on)*, can you find three numbers that add up to 38? Can you see if you can add all of them and there might be more than one.	***Orienting**: setting up the scene*
2	E: 38? *(eyes open wide while he changes his body position to focus on the number grid)*	
3	M: Ja! Yes there might be more than one. *(Elia, places his thumb on the chest[8])*	***Orienting**: alerting* ***Noticing**: drawing attention to*
4	E: I know *(as he puts up his hand)*	
5	M: Have you found one already? *(as she put up her fingers to say wait to Elia)*	***Probing**: checking*
6	E: yes	
7	M: Write it down Elia. *(as she checks Elia's work and speaks with a tone of approval)[Elia wrote 20, 10 and 8]* M: *(facing Olien)*Olien write it down the one you found.	***Modelling**: directing and instructing*
8	Ls: *both nod their heads as they write*	
9	M: Very good. [*after looking at the learners' written work, as each learner writes the three numbers 20, 10 and 8*], M: Can you find another three numbers that add to 38 now? *(both learners continue to work out their answers in writing)*	***Guiding**: encouraging* ***Noticing**: evaluating and valuing* ***Orienting**: reminding*
10	E: I think it's this one *[Elia, pointing at 20 10 and 8](after counting with his fingers, and watches Olien as he is saying his answer)[pointing at number 40 and 14]*	
11	M: Adding to 38? (*asking*)	***Probing**: pressing for clarification* ***Convince me**: evaluating*
12	E: yes (*as he was nodding his head but looks a bit confused)*	
13	M: But [*pointing at 40]* plus 14 is already 54, we are already too big. Remember we are adding up to 38.	***Focusing** : evaluate and flagging* ***Orienting**: reminding*
14	O: I think it is this one *(as he is pointing at 15, 14 and 6 meanwhile Elia kept on nodding his head).*	
15	M: 38? [*pointing at Olien's work where he wrote 15, 14 and 6]*	***Focusing** : flagging*
16	Ls: aaah*(both learners continue to count but Olien starts counting orally)*	
17	M: Ahhh, let's add these three, *[pointing at 15, 14 and 6]*, what do you get? *(while she was pointing at Olien's work)*	***Modelling**: instructing*
18	O: I get (*pause*) uum, I get 34 (*counted numbers over again*)	
19	M: Okay, not quite, try again, the 10 and 10 was 20 and 5 and the 4 was?	***Reflecting**: evaluating* ***Modelling**: directing and funnelling*
20	L: 9 *(while counting using fingers)*	
21	M: plus 6	***Guiding**: prompting*
22	O: 34	
23	M: 9 plus 6, what is 9 plus 6?	***Guiding**: prompting*
24	O: 16 *(counting using fingers)*	
25	M: 9 plus 6 *(repeating, because the first answer was wrong)*	***Focusing**: evaluate and flagging*

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8 This indicates that he has an answer (this is a practice widely used in the club so that other learners could see, and the mentor could know that the learner is ready to share)

26	O: 15 *(with his hands on his mouth)*	
27	M: 15 okay so if you add all of these you can either go 10, 20, 6 and 4 is 10, 30, and 5 *(facing Olien).*	***Modelling**: funnelling*
28	O: (35),	
29	M: so it's not quite 38, do you agree?	***Probing**: evaluatingandchecking*
30	O: yes	
31	M: Keep looking. You've already found one row of 3, show me? *(before mentor finishes speaking, Elia. starts making shifting movements and getting excited to suggest an answer)*	***Guiding**: encouraging and nudging*
32.	M: Keep looking. M: Have you already found another row of 3, show me? (*before mentor finishes speaking, Elia. starts shaking asking to be given a chance)*	***Guiding**: encouraging and nudging* ***Convince me**: seeking evidence*
34.	E: This one, and this one, and this *[pointing to 15, 14, 8]*	
35.	M; Ahh, but what is 15 and 14?	***Focusing** : evaluate and flagging* ***Probing**: checking and calls for clarification (evaluates learners' understanding)*
36.	O: 29	
37.	M: Very good, but is 29 plus 8 is it 38? *(Pointing to Elia).*	***Guiding**: encouraging and prompting*
38.	E: no! *(with his eyes wide open looking at the grid)*	
39.	M: is 29 plus 8 is equal to 38? That's your conjecture, you are one short. [*Pointing to Elia]*	***Probing**: checks and calls for clarification (evaluating learners' understanding)*
40.	E: this one's *[pointing to the numbers, 14, 6 and 15*]	
41.	M: Ah, okay, these 2 gives you, what do these 2 give you (Elia's hands up)	***Probing**: checking*
42.	E:19 *(counting on his fingers)*	
44.	O: 20	
45.	M: 20 plus 15 is?	***Guiding**: prompting*
46.	O: 35	
47.	M: Is it giving you 38?	***Probing**: prompting*
48.	O: No (*nods his head while Elia counts using fingers)*	

In this excerpt the mentor began setting up the activity by posing a question for the learners to search for the sets of three numbers that add up to 38 from the number grid provided. In turn 6, Elia got the first set within a few seconds, which was 20, 10 a nd 8. After that, the mentor suggested learners keep trying in order to find the next set. The learners kept on guessing and counting in ones using their fingers, but without using a particular strategy. They seemed to be using a random trial and error method. In response to the learners' unsuccessful guessing the mentor supported them by catching their thoughts through probing by means of evaluating, prompting and encouraging which allowed learners to reflect on their answers. The mentor also suggested a more efficient strategy to be used for checking the suggested answers.

In addition, the mentor modelled, evaluated, highlighted and asked the learners to evaluate the sensibility of their suggestions. Based on the above excerpt, the mentor's overall pattern of using scaffolding was mostly guiding through evaluating learners' understanding and drawing the learners' attention to particular features of their own answers and asking learners questions to evaluate and reflect on their answers reminding them of the original question. The mentor continued to engage with the learners by encouraging and nudging them to keep on searching for more sets of numbers that add up to 38.

In the same excerpt the mentor checked learners' answers by asking them for clarification, for example in turn 16 and 17, after Olien suggested three numbers "15, 14 and 6", the mentor asked the learners to add all the three numbers together to see the result. Without telling him that his answer was wrong, Olien realized that there was something wrong with his answer and this could be seen from his facial expression, thus he indicated to the mentor that his answer was 34 not 38.

From this excerpt we see that the mentor prompted the students by asking them questions such as "what do you get?" or "is that giving us 38?" At the same time the mentor allowed learners enough time to reflect and think about the questions. Afterwards, the directness of the scaffolding increased as the mentor noted that the learners were not coming up with strategies that would help them to find the answers.

What was noticed from this excerpt was that the mentor repeatedly caught the attention of the learners through noticing their suggestions but flagging that they are incorrect. And so a ZPD was created between the mentor and the learner, pulling the learners into the ZPD. From the excerpt we can see that there was limited learner to learner interaction and so it was not a case of a learner pulled into the ZPD by a more capable peer. It was difficult to evaluate whether the ZPD could have emerged if the learners were left on their own without the mentor's interference. This was unlikely as, although both learners were involved in the same activity, they rarely looked at each other's suggested answers or work.

Additionally, we can see that (more especially from turn 31) both learners seemed to be randomly selecting numbers and then checking them, thus guessing and checking rather than developing any strategy. So at this point the mentor caught their thoughts, encouraged them to keep trying and required them to check the sensibility of their answers. However, it was noted that only at one point did the mentor suggest a more efficient strategy for checking answers (turn 19). Through this the mentor evaluated and responded to learners with prompts that indicated problems with their answers. Perhaps all of these were required steps (in establishing the need for strategy) before pulling them into the ZPD of developing more efficient strategies for identifying such possible numbers.

After several attempts to get the answer the mentor provided more directed scaffolding involving modelling through funnelling that enabled the learners to focus on the unit digits as shown in excerpt 2 below. Thus the mentor proposed a strategy which was to concentrate more on the units when adding up the three numbers:

Excerpt 2: boys' task working on the number grid turns 49-69

Turn	Transcription	Scaffolding practices: categories
49.	M: You are adding the 10s nicely together but to get 8, to get 38, the 8 is the unit part. Try to come up with a strategy.	***Reflecting**: summarizes and* ***Modelling**: funnels*
50.	O: I think this one *[as he points to 20, 15, 12 meanwhile Elia went on making wooh sound with his hands behind his head]*	
51.	M: Ah, okay, you are saying 20, then 10, and then 10, but if I add the units, what is 5 and 2	***Guiding**: prompting*
52.	O: 7	
53.	M: and a zero, so I am not getting the 8 in the units, can you see that, so what you are going to land up with these, …*[pointing at thirty, forty seven]*	***Modelling**: funnelling and directing* ***Reflecting**: evaluating*
54.	E: shoo…I get, this one, this one 20 [*pointing at 12 and 20]*	
55.	M: Okay, okay, that is 32, so what do you still need? *(facing to Elia)*	***Excavating**: drawing out* ***Noticing**: pointing to*
56.	Ls*:(no response but both with wide eyes looking or searching in the grid)*	
57.	M: you are right, these two add up to 32, what do you still need to make it 38?	***Noticing**: valuing* ***Excavating**: drawing out* ***Orienting**: reminding*
58.	E: I know it, I know it, wait, this one this one this one *[pointing to 12, 20 and 6] (after, 1min9sec 09:26 to 10:35)*	
59.	M: Woo hoo! (*excited tone*), write it down, write it down.	***Noticing**: valuing and evaluating* ***Guiding**: encouraging*
60.	E: starts to write *(while dancing/moving his body sideways)*	
61.	M: But now I want you to convince Olien, write it down first and then I want you to convince Olien	***Convince me**: seeking for justification* ***Reflecting**: sharing*
62.	E: *continues to write*	
63.	M: So Olien, he is saying that these three, do you agree with him? (*pointing to 20, 12 & 6)*	***Reflecting**: sharing*
64.	O: yes teacher *(playing with his fingers)*	
65.	M: do you?	***Reflecting**: sharing*
66.	O: yes *(as nods his head)*	
67.	M: Double check him (*talking to Olien*)	***Reflecting**: reinforcing*
68.	O: Because 20 plus 6 equals to 26 plus 10, 20 plus 10, equals to thirty six plus 12 equals to 18 *(in the process Olien got something wrong and he corrects himself meanwhile Elia looked at O and starts smiling)*	

69.	M: Very good! OK. Now see if you can find another one. I now notice that you are paying more attention to the units and you are finding them quicker *(clicks her fingers to illustrate)*	*Noticing: evaluating and valuing* *Guiding: encouraging* *Focus: re-voicing and summarising*

In excerpt 2, turn 49 the mentor acknowledged the learners' efforts of adding the tens together to get the three numbers that add up to 38. After the mentor realized that the learners were not coming up with a strategy that could help them to identify the numbers that add up to 38, she directed their attention to devise a strategy that could help them to get the eight as a unit digit in 38. Again the learners started random suggesting and guessing. It was not really clear if the learners used the suggested strategy because even the first suggestion that Olien made the units did not add up to 8. In response to that the mentor prompted the learners by asking them for example in turn 51 "Ah, okay, you are saying 20, then 10, and then 10, but if I add the units, what is 5 and 2?" Here the mentor allowed the learner to reflect on and think about their answer and focus on the units as a means of checking the correctness of the answers.

In turn 51, the scaffolding was increasingly directed as the mentor realized that the learners were still guessing and suggesting the answers without considering their units. From turn 53 to 58 scaffolding was adjusted to a more direct approach. The mentor's scaffolding practices were modelling by funnelling and noticing (pointing to) and excavating (drawing out).

Based on this it would seem that the ZPD was created for Elia but the extent of the creation of the ZPD for Olien seemed to be less clear. Based on the mentor's scaffolding practices such as guiding through prompting, hinting, encouraging and nudging that seemed to move learners' thinking from the strategy of concentrating on the tens towards paying more attention to the units. The way the mentor encouraged the learners to reflect on their answers which in turn gave them a chance to monitor their work allowed for the creation of the ZPD. This is in line with Goos' (2004) assumption of the centrality of scaffolding in the creation of the ZPD. At the same time the mentor asked Elia to convince Olien that his solution was correct, indicating that the mentor valued the social interaction with peers as the means of generating mathematical thinking. This resonates with Vile and Lerman's (1996) argument that creating a ZPD is more about mutual orientation of goals and desires than about the intended content of interaction (cited in Lerman, 2001, p. 9).

Excerpt 3 below is a continuation of the boys working on the task.

Excerpt 3: boys working on the number grid, turns 84 -90

Turns	Transcript	Scaffolding practices: categories
77.	M: To make 38 I would need another 30, am I right? That's what you did here *(points to 8)*, you said 8 and 30 *(pointing to 10 and 20), (Elia looking around)*	***Focusing**: re-voicing and evaluating* ***Reflecting**: reflecting*
78.	E: hh, (looking on the ground)	
79.	M: Can you find another 30 somewhere that you can add to that 8? (*Elia disengaged, sitting back in his chair*). There might not be one. *(Elia looks at the grid and points)* Aah, I think you've got that one there *(after looking at what Elia was pointing at)*	***Modelling**: directing* ***Noticing**: pointing to*
80.	E: 20 plus 15 plus 8 *(looking at mentor) (Elia puts fingers to lips and smiles and points to the 20 on the grid)*	
81.	M: Okay, but the 20 and the 15 is 35 already hey. *(pointing to the row of numbers)*, so we're a bit short there	***Focusing**: flagging* and evaluates
82.	E: (*Elia points to same row)* this one	
83.	M: To make 38 I would need another 30, am I right? That's what you did here *(points to 8)*, you said 8 and 30 *(pointing to 10 and 20 in the right column band of 20; 8 and 10), (Elia looking around)*	***Modelling**: directing and Focusing: re-voicing*
84.	M: that one gives you 28, maybe we can add 4 numbers together *(Elia counts on fingers against chest)*	***Focusing**: redirecting*
85.	E: I know it, I know it (*puts his hand up, smiles)*	
86.	O: this one is too big	
87.	M: You are right; it is too big *(to Olien)*. *(looks at Elia)* Yes	***Noticing**: valuing*
88.	E: You can say 6 plus 14 plus 8 plus 10 *(points to numbers as he was talking)*	
89..	M: Ahh, now you got 38 (*Elia very pleased with himself, big smile)* okay so you said 28 and another 10 is 38, so you found 4 numbers that you add together to get 38 (*Elia does a little dance in his chair)*, I like it, do you think there's any others? *(Elia writes the sum on his paper)*	***Noticing**: highlights and* ***Focusing**: flags and re-voices*
90.	M: what I think you boys have done very well. I like it, 4 numbers that make 38. Okay, should we move on to something else? You boys have done very well.	***Guiding**: encourages*

From the transcript it can be observed Elia contributes twice whilst Olien contributes only once. In turns 84, 87, 89 the mentor flags and re-voices and encourages the learners where necessary. For example "Ahh, now you got 38 okay so you said 28 and another 10 is 38, so you found 4 numbers that you add together to get 38, I like it, do you think there's any others?" There seemed to be no evidence that the boys used a strategy to find the answer. From the transcript it can be observed that the mentor's overall practice was more of noticing by highlighting and asking them to find how sensible their answers were by questioning and re-voicing.

6.3 DISCUSSION

From the excerpts discussed above, it was evident that both mentors' ways of scaffolding followed similar patterns. Both started with setting the scene before moving on to give the learners the opportunity to explore their own strategies and reflect on their own work before the mentor got involved and adjusted the scaffolding practices based on the way the learners worked on the task. Additionally, both mentors in these excerpts demonstrated similar scaffolding practices, more especially the way they handled the learners when learners seemed to be experiencing difficulty or where they seemed to hit a ceiling. For example, this was evident in excerpt 2 for the girls (turn 35-49), and in excerpt 2 for the boys (turn 49 to 59). In both cases, each mentor provided more directed scaffolding through modelling and guiding (primarily through directing, prompting, and funnelling) when learners indicated or it was apparent that they were not progressing. In the case of the girls this was when they expressed that they 'did not know' (line 35) while for the boys this occurred after several attempts to find three numbers indicated a persistent guess and check strategy which was not leading to success.

The on-going scaffolding and increased direction at these points seemed to keep all four of the learners focused on the mutual goals of the activity. As it can be seen with the girls, in excerpt 1, the learners found it hard to find a value for the triangle. But, immediately after the mentor adjusted the scaffolding style, Alleta managed to come up with a strategy that helped her to get the answer in excerpt 3. In another example with the boys Elia had also a problem finding the next set of three numbers which add up to 38 but soon after the mentor suggested the new strategy through directed modelling he got a correct set of three numbers and screamed 'I know it, I know it, this one, this one, this one' (turn 58).

In each of the excerpts however there were few learner to learner 'catching each other's' thoughts except when explicitly prompted to reflect and share by the mentor. Additionally in these excerpts learners appeared to be progressing through the ZPD at different paces and in different ways. 'Aha' moments happen at different points for different learners and they did not happen simultaneously even when the mentor addressed both learners. It appeared that mentor scaffolding of learners who had not yet suggested an answer often involved reflection in terms of sharing whether they agreed with the other learner's suggestion or the other learner's work. In this respect the nature of the scaffolding practices was different for the learner who suggested a strategy or answer first and the learner who was required to assess or agree with the learner who suggested and provided the answer first.

It must be noted that I have only analysed two episodes of two task-based interviews. Other task-based interviews with other club learners indicated slightly different scaffolding patterns where more learner to learner collaboration was evident (personal communication with mentors and interviewers). What seemed evident in these episodes however was that learner utterances guided the mentor scaffolding practices and the nature of the learning within the ZPD and that (in such a paired activity) progress was dependent on the interactions of all three participants and evolved differently depending on what was contributed and what was requested by each participant (learner, learner, mentor).

Thus while it appears that the ZPD was evoked for both learners (in each pair) as they continued to engage in common tasks with common goals, and reflected on each other’s answers and ideas (when the mentors call for reflecting and sharing), and although there was evidence of their achieving success at the tasks, the nature of their movement through the ZPD differed for each of the learners. Additionally, the nature of the scaffolding for the individual learners in each pair differed.

Analysing the task-based interview highlights the critical role of club mentors in constantly micro-adjusting their scaffolding practices, according to learners' ideas (in the pair and individually), participation and reaction to interactions. The analysis also serves to illuminate the complexity of the learning process. This has implications for the teacher development work in which I am involved in Namibia: important as it is to support teachers in mastering the range of scaffolding practices available to them when working with learners, the appropriate employment of these practices depends to a large extent on the individual nature of what each learner knows and contributes. This conclusion equally highlights the limitations of this research. While the in-depth analysis of these episodes and selected excerpts has enabled the researcher to demonstrate and explore the complexity of learning within the ZPD, it also serves as a reminder that the situation in a classroom where many learners are all demanding attention is very different, and that exploring scaffolding practices within a context so different could illuminate very different issues and challenges. This is an interesting avenue to explore in the future.

6.4 CONCLUSION

This chapter began with a thematic analysis of various norms promoted within the maths club environment. These norms were replicated in the task-based interviews in which learners were asked to behave as they did in clubs. In the interviews the identified club themes of ‘encouraging maths talk’, ‘learner autonomy’, ‘collaboration’ and nurturing positive attitudes’ were evident in various utterances, as discussed above. The task-based interviews enabled detailed analysis of the nature of learning on the part of the four case study learners in their pair-based interactions with the mentors. As stated at the start, pair-based activities and mentor interactions such as these with pairs of club learners were a common practice in the clubs. The club learning context and the excerpts illuminated how participation in an afterschool mathematics club enables mathematics learning for the four case study students.

CHAPTER SEVEN - CONCLUSION AND LIMITATIONS

7.0 INTRODUCTION

This chapter provides a summary of the keys ideas of the research findings and indicates some of the limitations and implications of the study. Chapters 5 and 6 presented and discussed the results of the study in terms of the first research question posed (see Chapter 1). This chapter presents cognate conclusions drawn from the major results, shares some insights produced by these results, and points to the limitations of the study.

The main purpose of this study has been to explore the nature of learning within an afterschool mathematics club for learners in the primary grades. My interest in the topic stemmed from my awareness as a researcher, a teacher and an education officer, of the poor performance of Southern African countries (including Namibia) in numeracy tests and in recent international comparisons of mathematics achievements. Research shows that intervention in the early childhood years can be effective in reducing disparities in mathematics achievement (Gervasoni, 2005; Wright, Martland, & Stafford, 2000; Young-Loveridge, Peters, & Carr, 1998).

In mathematics education, debates and research are generally informed by theories of learning. In this study, I employed the framework of Vygotksy's social constructivist and socio-cultural theory to analyse the nature of the mediation experienced by learners participating in an afterschool maths club through interactions with their mentors and peers (RQ.2). I also drew on Wright et al.'s (2006) framework for assessing the progression of four learners' mathematical proficiency over a period of four to five months. Issues of theory and methodology were discussed in Chapters 2 and 3.

A case study approach was implemented, drawing on a v ariety of data collection techniques, including assessment interviews, task-based interviews and observation. The data collected was examined and analysed in order to answer the two critical research questions:

1. What is the nature and the extent of progress in mathematical learning (if indeed there is any) in a grade 3 afterschool maths club, and how do the SANC project assessment instruments enable me to assess this?
2. What is the nature of mentor to peer mediation in the afterschool maths club?

The analytical frameworks (ZPD and the LFIN framework) conceptualised in Chapters 2 and 3 provided the necessary tools to analyse learners' responses.

7.1 CONCLUSION

In response to research question one, my findings as presented in Chapter 5 revealed various degrees of progress in mathematical learning. In terms of their progress in the different aspects of number knowledge, the findings indicated that the four learners progressed diversely in a range and combination of aspects after spending four months in the afterschool maths club. The findings indicated that among the four learners, progress in terms of early number knowledge was more clearly visible in learners who showed weak number knowledge at the beginning. For example Elia managed to progress in most areas except in BTS where he retained his level of achievement in April. On the other hand, the learners who achieved a high rating in the first interview or test did not slide back in terms of achievements but rather maintained their levels. Additionally, although there was marked improvement in the area of SEAL for all four learners, the findings reveal that not all learners were able to achieve the highest stage.

In response to the second part of my first research question, which concerns the extent to which the assessment instruments used by the SANC project enabled me to assess the four learners' progress in mathematical learning, my findings indicated that the two assessment frameworks did indeed enable me to assess the learners' progress in mathematical proficiency in terms of number knowledge, or lack of it, as observed during the analysis of both written and oral assessment interviews. The instrument allowed one to see different aspects and rates of progress in the LFIN framework.

According to the results of the qualitative analysis, it seems reasonable to assume that a club setting with regular interaction between mentors and learners supports and enables learning. The club observations revealed that the club learning environment supported learners' confidence to participate and this potentially played a role in promoting learning for these learners. For example, observation notes indicated that shy learners participated more readily in discussion and activities over time. Moreover, the club learning environment supported learning through the establishment of norms according to which learners were not allowed to make fun of each other's mistakes: it was in fact emphasised that it was acceptable to make mistakes in the club because they were a useful part of learning.

The learners' use of trial and error strategies during the task-based interviews allowed the mentors to support the learners through recognizing their thought processes and prompting and encouraging them to reflect on their answers and develop more effective strategies. The study suggests that more directed scaffolding of tasks, with accompanying enabling and extending prompts ready at hand, can facilitate learners' participation and movement within the ZPD. The ongoing scaffolding, including mentors' increased use of directives, kept all four learners focused on the mutual goals of the activity during the task-based interviews, and this was equally visible in club sessions.

The study revealed that learners progressed through the ZPD at different paces and in different ways. 'Aha' moments happened at different points for different learners – they did not happen simultaneously even when the mentor addressed both learners. The study suggests that the ZPD is affected by the intellectual quality and developmental appropriateness of the mediation. In other words, the extent of one's development within the ZPD is predicated in part upon how the 'more knowing other' organises, or *scaffolds*, the task at hand (Blanton & Stylianou, 2002).

It must be noted that I only analysed two episodes of two task-based interviews. Other task-based interviews with club learners have indicated different scaffolding patterns, with more learner to learner collaboration and recognising each others' ways of thinking (personal communication with mentors and interviewers). Progress is dependent on the interaction of all three participants, and evolves differently depending on w hat is contributed and what is requested by each participant (learner, learner, and mentor). While there was some evidence of learners achieving success at the tasks, a full realisation of the ZPD cannot be inferred from simply one session. Learning takes place over time, so in order to assess whether the ZPD was fully realised analysis of similar activities without the support of the mentors would have been required. Only then might one say that this mediation resulted in successful learning, i.e. movement through the ZPD that enables the learner to complete similar tasks independently. Indeed in subsequent club sessions or in take-home activities, learners were given activities to complete independently in order to assess whether learning had occurred, and the ZPDs that emerged in the session were fully realised. The detailed analysis of transcriptions in this study however enabled a rich description of the emergence of the ZPD in mentor-peer mediation and of the nature of the scaffolding.

Moreover analysing the task-based interview highlights the critical role both of club mentors in constantly micro-adjusting their scaffolding practices according to learners' ideas (in the pair activities and individually) and of learners' participation and reactions to interactions. This analysis illuminates the complexity of mediatory learning process.

7.2 LIMITATIONS OF THE STUDY

There are several limitations to this study, including the fact that the findings for this case study are not generalisable due to the small number of participants involved. The findings could thus be developed through further research that focuses on a broader sample of participants. Additionally it should be understood that the results and findings cannot be generalised to other learning contexts. However, since the selected afterschool maths club possesses some characteristics typical of both South African and Namibian informal and formal learning contexts, the results and discussion could be useful in providing insights into mediation practices and associated issues in similar learning contexts.

7.3 INSIGHTS FOR MY WORK IN NAMIBIA

The findings of this research indicate that afterschool intervention programmes such as maths clubs have the potential to improve learners' mathematical proficiency and learning. I am therefore determined to arrange the introduction of such clubs in Namibia.

The findings have several implications for the structure and design of intervention programmes. Further enquiry is needed into how children progress after several years in the lower grades, and it would be interesting too to investigate the effects of scaffolding over time and on learners' learning in a formal classroom context. Follow-up research involving video recordings of individual learners working independently on activities similar to those that I analysed from the task-based interviews would enable one to interrogate the nature of the learning over time. One could examine the extent to which the mediation enabled learners to succeed at similar tasks and activities independently in the future. I would also like to explore this further in future studies within the context of my home country, Namibia.

The study also suggests that Wright et al.'s (2006) LFIN framework is a useful way of assessing learners' progress in mathematics, and one that could inform our refinement of instructional design within the school curriculum and teachers' education in the Namibian context. My intention is to use this framework as a teacher development tool in my work as an Education Officer in Namibia, and to support teachers in conducting interviews with learners in order to make full use of the framework and its potential.

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APPENDIX A: ASSESSMENT INTERVIEW

SA NUMERACY CHAIR

RHODES UNIVERSITY Where leaders learn

MP
SOUTH AFRICAN NUMERACY CHAIR

INDIVIDUAL INTERVIEW for MATHEMATICAL PROFICIENCY

GRADE 3 - ENGLISH

LEARNER & INTERVIEW INFORMATION		© 40 minutes	Date	
Surname		First name		
Club		Gender	Male ☐ Female ☐	Age
Mentor		Interviewer		

Instructions in [bold brackets], what you say to the learner in *italics*
PLEASE WRITE IN BLACK OR BLUE PEN (NOT PENCIL)

PART ONE – Qs 1 to 11

Numeral identification, FNWS, BNWS, Counting by 10s & 100s, Place Value

Task 1: Numeral Identification — W/CU

[Use number cards to show each number to learner. Tick if correctly identified]
Tell me the name of these numbers

6		*11*		*20*		*99*		*101*		*208*		*300*		*1025*		*½*		*¼*	

Comments:

Task 2: Number Representations — A2/CU

[White card. Show 1 number line at a time] *Here is a number line. Tell me what number the arrow points to?*

	Wrongly positioned?	Correct
(a) 15		
(b) Approx. 90		

Comments:

Task 3: Forward counting number word sequences — W/CU

[Ask orally] *Start counting in ones from ____ and I'll tell you when to stop.*

	Skipped numbers	Last no counted correctly
(a) 1 to 32		
(b) 48 to 61		
(c) 93 to 112		

Comments:

SANC 2012 Adapted from Wright et al (2006) and Askew et al (2002) — Page 1

Task 4: Backward counting number word sequences — W/PF

[Ask orally] Example: *Count backwards from 3. . .Three, two, one.*

Now count backwards in ones from ___ and keep going until I say stop.

	Skipped numbers	Last no counted correctly
(a) 10		
(b) 23 to 16		
(c) 72 to 67		

Comments:

Task 5: Number word before — W/PF

[Use green number cards for each number] Example: *Which number comes just before 2? Now say the number that comes just before ___*

Note each answer

(a) 9		*(b) 11*	
(c) 20		*(d) 30*	
(e) 50		*(f) 100*	

Comments

Task 6: Number word after — W/PF

[Use green number cards for each number] Example: *Which number comes just after 1? Now say the number that comes just after ___*

Note each answer

(a) 4		*(b) 19*	
(c) 25		*(d) 32*	
(e) 70		*(f) 99*	

Comments

Task 7: Sequencing numerals — W/PF

[Show the green number cards face up in random order, asking the learner to identify each number as you put it out. Then say] *Can you place the cards in order? Start with the smallest number.*

	Note sequence learner laid cards out	Sequence correct
(a) Cards from 0 to 10		
(b) Cards from 46 to 55		

Comments:

Task 8: Perceptual counting — W/PF

[Ask learner to place out counters for a & b. Note how learner counts these and the number counted]

	Counts in 1s?	Counts in multiple? Say which?
(a) Place out 13 counters for me		
(b) Place out 18 counters for me		

Comments:

Task 9: Counting with incrementing tens — W/AR&CU

[Use pink strip cards. Show strip (a) then add others for steps b to e. Ask] *How many dots are there altogether?*

	Note Given Answer & How Answered	Correct?
(a) The 'four dot' strip		
(b) Add a 'ten dot' strip to the right		
(c) Add another 10 to make 24		
(d) Add another 20 to make 44		
(e) Add another 30 to make74		

Comments:

Task 10: Adding / subtracting with tens — A2&3/PF

[Ask orally]

	Note Given Answer	Correct
(a) Add 10 to 92		
(b) Add 10 to 294		
(c) Take 10 away from 50		
(d) Take 10 away from 700		

Comments:

Task 11: Adding with incrementing hundreds — A2&3/PF

[Ask orally]

	Note Given Answer	Correct
(a) Add 100 to 9		
(b) Add 100 to 932		
(c) Take 100 away from 400		
(d) Take 100 away from 634		

Comments:

END OF PART ONE

PART TWO – Qs 12 to 16

Early Arithmetic Strategies, Combining & Partitioning

Task 12: Horizontal sentences – Early Arithmetic Strategies W/AR

[Use blue sentence cards] *Tell me how you would work out the answer to:*

	Note Given Responses & How Answered	Correct
(a) 16 − 10 = □ *(b) So what is 16 + 9?*		
(c) 42 − 23 = □ *If correct ask, Do you have another way to work it out or check it?*		
(c) 43 − 15 − □ *Repeat the question above*		

Comments:

Task 13: Number Stories– Early Arithmetic Strategies A2&3/SC

[Ask orally] *I am going to read you some number stories. Answer the question at the end of the story.*

	Note Given Responses & How Answered	Correct
a) 12 people are on a bus and five get off. How many people are on the bus now?		
b) 22 people are on a bus. 13 are children. How many adults are there?		
c) 18 people were on a bus. 8 people get on and 3 get off. How many people are on the bus now?		

Comments:

Task 14: Number Stories– Early Arithmetic Strategies A2/SC

[Use pale yellow card with sums] *I am going to read you another number story. Here are some sums. Tell me which* ***sum*** *you would use to answer the question at the end of the story. I don't want the answer.*

	Given Answer	Correct
(a) There are 43 children in the class. 28 of the children are boys. How would you work out how many are girls?		

Comments:

Task 15: Non-count-by-ones – Early Arithmetic Strategies W/AR

[Use the orange calculation cards. Note how learner arrives at answers]

	Note Given Answers & How Answered	Correct?
(a) What is 9 + 3		
(b) Can you use that to help you work out 9 + 4		
(c) and 9 − 5		
(d) What is 7 − 5		
(e) Can you use that to help you work out 27 − 5		
(f) and 47 −		

Comments:

Task 16: Number combinations W/PF

[Ask orally]

	Note answers
I will say a number and you say the number that goes with it to make 5. *(a) 4* *(b) 0* *(c) 3*	
(d) Give me two numbers that add up to 10	
(e) Give me two other numbers adding up to 10	
(f) I have 7, how many more to make 10?	

Comments:

END OF PART TWO

PART THREE – Qs 17 to 24

Subitising, Multiplication and Division

Task 17: Visible items arranged in arrays – Subitising W/AR

[Use red dot cards. Show 1 at a time. Note how the learner counts & the given answer]

Tell me how many dots there are all together.

	Given answer	Counts in 1s/ multiples? Which multiple?		
(a) Show the 10 × 2 array of dots		1s	multiples	
(b) Show the 5 × 3 array of dots		1s	multiples	
(c) Turn (b) through 90 degrees		Recounts?	Instant answer?	

Comments:

Task 18: Visible items arranged in arrays – Subitising A2/AR

[Show the cake cards 1 at a time. Note how the learner counts & the given answer]

There are 5 cakes in each box. I am going to show you pictures of what the children bought for a short time. Look carefully and tell me how many cakes each one bought.

	Given answer	Counts in 1s / multiples? Which multiple?		
(a) Natasha		1s	multiples	
(b) Rajesh		1s	multiples	

Comments:

Task 19: Visible items arranged in arrays- Subitising A2/AR

[Show the apple cards 1 at a time. Note how the learner counts & the given answer]

There are 10 apples in a bag. I am going to show you pictures of what the children bought for a short time. Look carefully and tell me how many cakes each one bought.

	Given answer	Counts in 1s / multiples? Which multiple?		
(a) Dawn		1s	multiples	
(b) Gary		1s	multiples	

Comments:

Task 20: Equal grouping of visible items – Subitising and Multiplication W/CU

[Use orange circle cards. Place down four circles with three counters on each. Show the difference between circle and counter. Note how the learner counts & the given answer]

	Given answer	Counts in 1s / multiples? Which multiple?		
(a) *How many circles are there?*		1s	multiples	
(b) *How many counters in each circle?*		1s	multiples	
(c) *How many counters are there altogether?*		1s	multiples	

Comments:

Task 21: Equal grouping of visible items – Partition Division W/PF

[Place out a pile of 15 counters. Note how the learner counts & the given answer]

	Given answer	Works in 1s / multiples?		Which multiple?
(a) How many counters are there?		1s	multiples	
(b) Share them equally among 3 children.		1s	multiples	
(c) How many does each one get?		1s	multiples	

Comments:

Task 22: Equal grouping of visible items – Partition Division with RedistributionW/PF

[Place out a pile of 24 counters. Note how the learner counts & the given answer]

	Given answer	Works in 1s / multiples?		Which multiple?
(a) How many counters are there?		1s	multiples	
(b) Share them equally among 3 children.		1s	multiples	
(c) How many does each one get?		1s	multiples	
(d) Now share them equally among 4 children.				

Comments:

Task 23: A2/PF

[Ask orally]

	Given answer	Correct
(a) Three lots of three makes (or three times three, equals)		
(b) Four lots of five makes (or four times five equals)		
(c) What are ten sets of four?		
(d) What are ten sets of 70?		

Comments:

Task 24: A2/CU

[Show white marbles card] *Jane and Peter play a game of marbles. Here is a picture of the marbles they use.*

	Given answer	Correct
(a) Jane wins half of the marbles. How many marbles will she win?		
(b) Peter wins a quarter of the marbles. How many will he win?		

Comments:

END OF PART THREE