EXPLORING HOW VISUAL MODELS CAN BE USED IN TEACHING MATHEMATICS FOR GROWING A PRODUCTIVE DISPOSITION IN GRADE 9 LEARNERS.

by

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ABSTRACT

This study explored how visual models can be used to teach mathematics to possibly grow selected Grade 9 learners' productive disposition. Underpinned by an interpretive paradigm, the study took the form of a mixed-method research case study. It was carried out at a combined school in the Ohangwena region, Namibia. The sample consisted of two sets of participants, namely two Grade 9 mathematics teachers and their learners. Data were collected through pre- and post-questionnaires, lesson observations, and stimulus-recall interviews. The study was framed by Vygotsky's social constructivism perspective. The study's results revealed that learners who identified themselves negatively with mathematics changed their views about mathematics after participating in a series of mathematics lessons where teachers used mainly visual models. Based on the findings, the study recommends that curriculum designers consider making visual models mandatory in the teaching of mathematics in Namibia.

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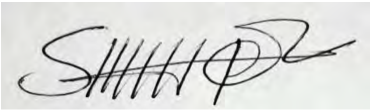
DEDICATION

I dedicate my work to all my children including Shalulile Andreas, and Paangwa. Guys, this is the standard I set for you. I achieved this despite hardship.

DECLARATION

I Shetunyenga Fillipus Shetunyenga, Student number 15s8833, declare that this thesis titled "*Exploring how visual models can be used in teaching mathematics for growing a productive disposition in Grade 9 learners*" is my own work and it has not been submitted before. Where I have used others' ideas, I have fully acknowledged and referenced them in accordance with Rhodes University Education Department referencing guide.



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List of Acronyms

BPFCU Building procedural fluency from conceptual understanding

CIER Contemporary Issues in Education Research

COVID-19 Coronavirus pandemic of 2019

DBE Department of Basic Education

EMGFL Establish mathematical goals to focus learning

EUELT Eliciting and using evidence of learners' thinking

FMMD Facilitating meaningful mathematical discourse

IRE Initiation Response Evaluation

ITPRPS Implement tasks that promote reasoning and problem solving

MBEC Ministry of Basic Education and Culture

ML Mathematical Literacy

NBCE National Curriculum for Basic Education

NCTM National Council for the Teaching of Mathematics

PPQ Posing purposeful questions

SPSL Supporting productive struggle in learning

UVM Use of visual models

CHAPTER ONE: INTRODUCTION

1.1 INTRODUCTION

In this chapter, I introduce the reader to the thesis. This chapter thus presents the context of the study. Thereafter, the research goal and questions are outlined, followed by the research methodology and rationale of the study. Finally, the thesis structure is outlined.

1.2 CONTEXT OF THE STUDY

In our present complex society, a person needs functional knowledge of mathematics to make informed decisions both as private citizens and as productive workers (Wilkins & Ma, 2003). Despite this, many learners still perceive mathematics as a challenge. For example, Fleisch (2008) reports a numeracy and mathematics crisis in South Africa. Research relates this crisis to consistent poor learner performance, evidenced in various international mathematical assessments such as the Trends in International Mathematics and Science Study (Howie, 2001) regional assessments (Carnoy et al., 2011) and national assessments such as the Annual National Assessment (Department of Basic Education [DBE], 2012). When students were asked to identify the factors contributing to this crisis, poverty, insufficient teaching time, lack of opportunity to learn, low homework frequency and poor teacher knowledge were some of the factors identified (Graven et al., 2013). Graven and Heyd-Metzuyanim (2014) have, however, tried to understand this crisis from the perspective of learners' identity and disposition. Their study's findings revealed that when learners are given appropriate access to mathematics sense-making environments and opportunities to negotiate and participate in that sense making, learners were able to re-author their mathematical learning disposition in ways that support their learning.

In their study, Graven et al. (2013) explored the nature of shifting learners' mathematical identity in various mathematical literacy classroom environments. They kept a track record of learners who showed the most negative experiences of mathematics in their first few years of primary school. When these learners talked about mathematics, they were observed using terms such as *failure, struggle, stress, nervousness, hate for maths, worry, extreme difficulty, lack of confidence and helplessness*. Of special concern was that many learners connected their negative mathematical image to their broader self-image. For example, one learner wrote "My struggle with maths also negatively impacted my self-confidence, and left me to feel like I was stupid and useless" (Graven & Buytenhuys, 2011, p. 498).

Being a secondary school teacher of mathematics, I have also made similar observations. I observed that Grade 9 learners' mathematics preference is very low. When they are given mathematical problems to solve, many do not show joy and confidence in solving them, nor do they show the courage to keep trying. They view mathematics as a difficult subject and find themselves incapable of learning mathematics and using mathematics to solve problems. In the interview that teacher colleagues and I conduct at the beginning of every term at school to discuss learners' termly achievement against their termly targets, most Grade 9 learners revealed that they did not achieve their target in mathematics because it is just "a difficult subject". To this, Wright et al. (2008) have cautioned that a lack of understanding of school mathematics and rare experiences of success in school mathematics may result in a negative attitude (disposition) toward mathematics.

In this regard, a matter of concern is that the Namibian Ministry of Education has implemented the revised curriculum (Namibia. Ministry of Basic Education and Culture [MBEC], 2016). One of the changes in the revised curriculum is the introduction of mathematics as a compulsory subject from Grades 1 to 11. In addition, from Grade 10, mathematics is offered at ordinary and high level, unlike in the previous curriculum where a core level was offered to cater for learners who could not cope with advanced mathematics (Namibia, NCBE, 2016). The Core and Ordinary level's content in the previous curriculum have been merged as Ordinal level content in the new curriculum, including some additional high level content, making it more advanced than before. Also, the use of a calculator in mathematics Grade 8 and 9 Paper 1 is now forbidden. In my view, this further contributes to learners' mathematics discomfort as many learners relied heavily on the calculator to solve mathematics problems, and were encouraged to do so in the previous curriculum.

Kilpatrick, Swafford, and Findell (2001) identified four essential strands for mathematical proficiency; conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition. The first four strands refer more on mathematical content knowledge, and the last strand, productive disposition, refers on something different. Kilpatrick (2001) defined productive disposition as the tendency to see sense in mathematics, to believe that steady efforts in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics. This strand is related not directly to mathematical knowledge but instead to the attitudes and beliefs of people toward mathematics as well as how they identify with being doers of mathematics. My concern about my learners low preference of mathematics and less courage to try mathematics problems seem to be caused by learners not being encouraged to develop productive disposition because learners who lack productive disposition may not see themselves as doer of mathematics nor see value in steady effort in mathematics. I thus see productive disposition as what my learners needed support to develop. Graven (2012) and Siegfried (2012) indicated the need for research studies on productive disposition. These authors oberved that productive disposition strand has been neglected in current research as many research projects have tended to focus on the first four only. During my research journey, I have only came across few studies on productive disposition. My study thus, argues that if no deliberate interventions are undertaken, the notion of growing a productive disposition in learners will remain neglected.

With regard to the importance of a productive disposition, Wright et al. (2008) wrote about how a learner's three-year gap in the early years of schooling expands to a seven-year gap in later years after about ten years of schooling. They however argued that young learners with mathematical difficulties react well to appropriate interventions if they are engaged and are positively disposed to these interventions. Venkat and Graven (2008) also echoed that learners develop entirely different stories of their mathematical disposition when given the opportunity for active participation, for meaningful learner engagement and sense making. This has inspired me to conduct an intervention study with selected Grade 9 teachers to explore how visual models in particular can be used to teach mathematics for a productive disposition.

1.3 STATEMENT OF THE PROBLEM

Many learners still perceive mathematics as a challenge. Most learners' mathematics preference is very low (Fleisch, 2008; & Graven et al 2013). When they are given mathematical problems to solve, many do not show joy and confidence in solving them, nor do they show the courage to keep trying. Despite this, in Namibia, the revised National curriculum have made mathematics a compulsory subject from grade 1-11. In Grade 8 & 9 Paper 1, the use of the calculator is now forbidden. In addition, from Grade 10 mathematics is offered at ordinary and high level unlike in the previous curriculum where core level was offered to cater for learners who could not cope with advanced mathematics (Namibia, NCBE, 2016). A study conducted has however tried to understand this crisis from the perspective of learners' identity and disposition (Graven and Heyd-Metzuyanim, 2014). The study revealed that productive mathematical disposition is integral to mathematical proficiency. However, this strand, productive disposition is neglected as many current studies focused on the four other strands. It's against this background that in this interventionist study, I explored how visual models can be used in teaching mathematics for growing a productive disposition in grade 9 learners.

1.4 RESEARCH GOALS AND QUESTIONS

1.4.1 RESEARCH GOAL

This study explored how visual models can be used to teach mathematics to grow selected learners' mathematical productive disposition.

1.4.2 RESEARCH QUESTIONS

In the context of an intervention programme that foregrounds the use of visual models in the teaching of mathematics, this study was guided by the following research questions:

1. What is the nature of selected Grade 9 learners' disposition towards mathematics prior to an intervention programme?
2. How do selected teachers teach for productive disposition as a result of participating in an intervention programme?
3. What are the selected learners' mathematical dispositions after their teachers' participation in an intervention programme?

1.5 RESEARCH METHODOLOGY

This study is a mixed-method research case study underpinned by the interpretive paradigm. Mixed-method research is a "research design where a researcher collects, and mixes both qualitative and quantitative data in a single study" (Creswell, 2014, p. 2). This study focused on two secondary school teachers and their learners. The teachers were Ms. Moshana and Mr. Kondja, at Ounonge Combined School (pseudonyms). They both teach Grade 9. As this study intended to research a shift in learners' productive dispositions as a result of their teachers participating in an intervention programme, a series of lessons which involved planning, implementing, observing and reflecting on their lessons for further refinement, were implemented. For each teacher, five lessons were observed and video recorded. A pre- and post-disposition questionnaire was administered to all learners who participated, to establish their mathematical dispositions before and after the intervention programme. A case study is a "thick and detailed description of a social phenomenon that exists within a real-world context" (Du Plooy-Cillier et al., 2014, p. 2). My case is visual models and how they are used in teaching mathematics. My unit of analysis is how this teaching affected selected Grade 9 learners' mathematical dispositions.

1.6 RATIONALE OF THE STUDY

The findings of this study may contribute to the understanding of innovative teaching approaches that promote mathematical proficiency in general and a positive disposition in particular. It will also create awareness among mathematics teachers, researchers, policymakers and curriculum designers about the role and impact of visual models in mathematics teaching and learning. In my own experience and my circle of peers, the explicit use of visuals other than only diagrams in textbooks, and visual teaching and its impact on teaching and learning of mathematics, is not very familiar. Through reading this study, mathematics teachers will get an opportunity to be introduced to the concept of visualisation and to experience the impact of visual teaching in mathematics and the impact this has on learners' productive dispositions.

1.7 STRUCTURE OF THE THESIS

1.7.1 Chapter One (Introduction)

This chapter presents an overview of this thesis which is structured into eight chapters. In this chapter, the context of the study is discussed. The research goals and questions are then set out, followed by the research methodology and rationale of the study. The chapter concludes with the thesis structure.

1.7.2 Chapter Two (Literature review)

This chapter unpacks the main concepts relevant to the study, namely visualisation, visual models in mathematics and productive disposition. The constructivism theory is then discussed.

1.7.3 Chapter Three (Methodology)

This chapter focuses on the research methodology used to answer the research questions. This chapter begins with the research orientation, followed by research goals and questions and the research design. Research participants, data collection tools, and data analysis are also discussed. The chapter ends with validity and ethical considerations.

1.7.4 Chapter Four (Presentation and analysis of pre-questionnaire)

This chapter presents and discusses quantitative data from a pre-questionnaire aimed at establishing learners' mathematical dispositions prior to the intervention programme. The presentation and analysis were done according to Siegfried's (2012) constructs.

1.7.5 Chapter Five (Presentation and analysis of lesson presentation)

This chapter presents and discusses the qualitative data generated from lesson observations. Five lessons were observed. Each lesson was analysed using Mann's (2014) teaching practices.

1.7.6 Chapter Six (Analysis of lessons per teaching practices

This chapter presents and horizontally analyses data from lesson observations per Mann's (2014) teaching practices.

1.7.7 Chapter Seven (Presentation and analysis of post-questionnaire)

This chapter presents and analyses quantitative data from a post-questionnaire which was aimed at establishing learners' mathematical dispositions after the intervention programme. The presentation and discussion were done according to Siegfried's (2012) constructs.

1.7.8 Chapter Eight (Conclusion and Recommendations)

This chapter concludes the study. It presents the summary of findings, significance and limitations of the study. It also makes some recommendations for further research. Lastly, it concludes with my personal reflection on my research journey.

CHAPTER TWO: LITERATURE REVIEW AND THEORETICAL FRAMEWORK

2.1 INTRODUCTION

The purpose of this study was to explore how visual models can be used to teach Grade 9 learners for mathematical productive disposition. In this chapter, I thus explore various literatures relevant to my study. Firstly, I discuss the concept of visualisation, beginning with its historical background, followed by its importance in teaching and learning, and then present and discuss some of the visual models that can be used in teaching and learning mathematics. Secondly, I unpack the concept of productive disposition with special emphasis on the indicators of productive disposition and the eight pedagogical practices that support mathematical learning for all learners. Lastly, I discuss constructivism, my study's theoretical framework.

2.2 HISTORY OF VISUALISATION

The concept of visualisation in education and psychology first appeared in the late nineteenth century (Phillips et al., 2010). Literature reports that the pioneer behind the development of this concept was Francis Galton, a British psychologist. He developed a 'breakfast table questionnaire' before 1880 (Phillips et al., 2010) which contributed to the initial developments and understandings of visualisation. His questionnaire consisted of nineteen questions aimed at establishing perceptions of people about their own experiences of visualisations and mental images that they experienced. This questionnaire was interestingly referred to as a 'breakfast table questionnaire' because the instruction to the respondents was: "before answering the question 1 to 5, think of some definite object – suppose it is your breakfast table as you sat down to it this morning – and consider the picture that rises before your mind's eyes" (Burbridge, 1994, p.443).

The questionnaire was administered to more than 300 participants including his friends, members of the Royal Society, the Royal Institution, the Royal Geographical Society, the Birmingham Philosophical Society, and schoolboys. His published analysis of participants' responses was however limited to 100 adult males and 172 schoolboys.

From the findings, Galton was surprised to learn that many of the scientists who completed the questionnaire indicated that mental imagery was not known to them (Phillips et al., 2010).

From this Galton assumed that these scientists had a mental deficiency of which they were unaware (Galton, 1880). Furthermore, Galton (1880) claimed to show that there is a hereditary link to the ability to construct mental imagery and without providing any explanation, he predicted that this ability has a relationship to race as well, citing the Bushmen. Even though Galton predicted that visualisation is an inherited ability, this ability is trainable and it is of great potential to education (Phillips et al., 2010).

Parallel to Galton's exploration of visualisation were a number of independent European psychologists (Phillips et al., 2010) who explored visual imagery and the relationship between image and thought. Their work was challenged by a group of German psychologists in Wurzburg led by Karl Marbe and Oswald Kulpe (Phillips et al., 2010). This group claimed that the European psychologists were reporting on imageless thoughts. This means that when the European psychologists were exploring the relationship between image and thoughts, the German psychologists were not convinced that all thoughts could be analysed as images, hence they developed the concept "imageless thought" to refer to such thoughts.

This academic war did not end there, as the Germans' results were further challenged by an American psychologist, Edward Titchenner, who claimed to have shown that all thought was accompanied by imagery (Jonson-Laird, 1998). This debate continued throughout the twentieth century. However, just before the end of the first half of the twentieth century, it was declared irrelevant by the upcoming behaviourist movement at that time. Thereafter in the second half of the twentieth century, the exploration of visualisation abated. As a result, reference to visualisation was rarely found in the research literature (Phillips et al., 2010). According to Phillips et al. (2010) out of 30 books on cognitive science written before the 1950s, visualisation could only be located in the indexes of fewer than three books.

In the second half of the twentieth century, with the rise of cognitive psychology, questions on experiences of visualisation were revived (Reisberg, 2006). Apart from interrogating experiences of visualisation, cognitive psychology research also focuses on the structure of the mental images, reviving the work of the European psychologists of the nineteenth century.

In the 1950s, research into the understanding of visualisation became popular. Phillips et al. (2010) report a publication on visualisation written in 1950, entitled *"The power of thinking"*, by Norman Vincent Peale. This opened the door for several studies and publications on visualisation, (see Figure 2.1). Some of the interesting titles of publication in the field of visualisation are:

- Why the use of mental visualization facilitates problem-solving.
- Seeing and visualization: it's not what you think.
- How can we distinguish between image and seeing?

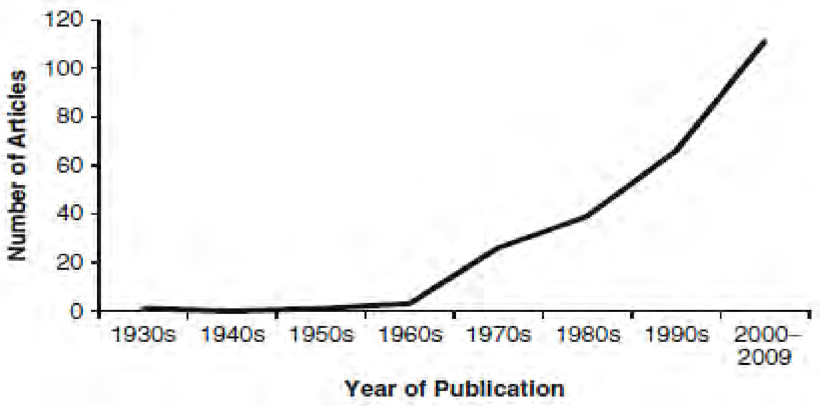


Figure 2.21: Articles on visualisation published between 1930 and 2009.

Adapted from (L. N. Phillips et al., 2010).

More recently, visualisation has been embraced by many scholars due to its significance and potential to enhance teaching and learning. Much has also been written about the value of visualisation (Bishop, 1988). Worth noting is that universities such as Rhodes University in South Africa have established Research Chairs which facilitate research specifically in the field of visualisation. I am a student in this Chair project and argue for a critical look at how the potential of visualisation on teaching and learning can be fully harnessed in the classroom and by curriculum designers to heed Duval's (1998) call for explicit integration of visualisation in the school curriculum. In the next section, I will unpack the concept of visualisation.

2.3 DEFINITION OF VISUALISATION

The fact that scholars are yet to reach a consensus on a single common definition of visualisation signifies the complexity of this concept. In the literature, various terms are used interchangeably. These include visualisation, visual representation, visual media, media literacy, visual communication skills, visual literacy, and illustrations (Griffin & Schwartz, 2005). Also, visual imagery, mental images, real objects, pictures, internal representations, external representations and schematic representations are some of the terms that are triangulated in the field of visualisation. In the discussion that follows I wish to narrow my focus particularly to visualisation in mathematics and mathematics education.

Scholars have defined visualisation differently. Zimmermann and Cunningham (1991) for example, described mathematics visualisation as the process of “forming images (mentally, or with pencil and paper, or with the aid of technology) and using such images effectively for mathematics discovery and understanding” (p. 3). Arcavi (2003) defined visualisation as follows:

> Visualisation is the ability, the process, and the product of creating, interpreting, use reflection upon pictures, images, diagrams, in our minds or on paper or with the technological tools, to depict and communicate information, thinking about and developing previously known ideas and advancing understanding. (p. 217)

These authors emphasise that mathematical visualisation is about an individual’s ability to create and process images and visual models for mathematical understanding. These authors also intimate that the effective use of visual models can enhance mathematics discovery and mathematical understanding (Ho, 2010). On the other hand, Kosslyn (1994) describes visualisation as a “ cognitive process involving visual imagery in which images are either generated, inspected, transformed or used for mathematical understanding” (p. 23).

To visualise a problem means to understand the problem in terms of mental images (Presmeg, 2006). She views visualisation as an aid to understanding problems – mathematical problems in particular. As highlighted by Kosslyn (1994), the mental process where images are generated, inspected, transformed and used, may or may not involve diagrams. In other words, as learners are working on a mathematical problem, they form mental pictures/images of what they are working on, either in diagrammatical or non-diagrammatical form, for the purpose of understanding.

Rieber (1995) clarifies that visualisation includes both internal and external representation. Internal representations refer to the mental imageries formed as imagined representations of a real/physical object or the mental picture of how a person views or understands a scenario, situation, concept or problem. In addition, mental representations may represent persons, scenes, situations, words, discourse, concepts and argumentations (Antonietti, 1999). Mental images may either be static or represent movement and transformation. External representations on the other hand include real objects (e.g. a cuboid), printed pictures, graphs, videos, films or animations.

According to Arcavi (2003), visualisation is recognised as a key component of mathematical reasoning as it allows deep engagement with concepts, problems or ideas. In addition, Zimmerman and Cunningham (1991) reiterated that mathematics visualisation is not merely mathematics appreciation through pictures or for illustration purposes and, as is highlighted by Arcavi (2003), is a means for in-depth meaning-making in an effort to understand and guide a problem-solving process and enhance creative discovery.

2.4 TYPES OF VISUALISATION

Visualisation appears in different forms. Rieber (1995) classified visualisation into two groups – internal and external visualisation. Internal visual representations refer to mental imageries that one forms in the mind as a representation of a story told, a description of a person or a mathematical problem (Kosslyn, 1994). External visualisations, on the other hand, include virtual and physical objects. Virtual objects are non-concrete in the physical sense but digital or dynamic artefacts that can be observed on a computer screen, for example, to resemble and simulate physical objects or abstract mathematical ideas (Matengu, 2018). An example of virtual objects are those generated by software such as *GeoGebra*, described by Mwiikeni (2016) as a dynamic mathematics software package, which can be used for teaching and learning mathematics from primary to university level.

Physical objects, which are the focus of this study, are objects that learners can see, touch and manipulate to facilitate the understanding of mathematical concepts. Boggan et al. (2010) describe physical objects as objects, which are used as teaching tools to engage learners in the hands-on learning of mathematics. This includes real physical objects such as dried beans, bottle caps, rectangular boxes, pictures, diagrams, cartoons and any other models that can be created as representations of abstract mathematical ideas (L. N. Phillips et al., 2010). This classification is summarised in Figure 2.1 below.

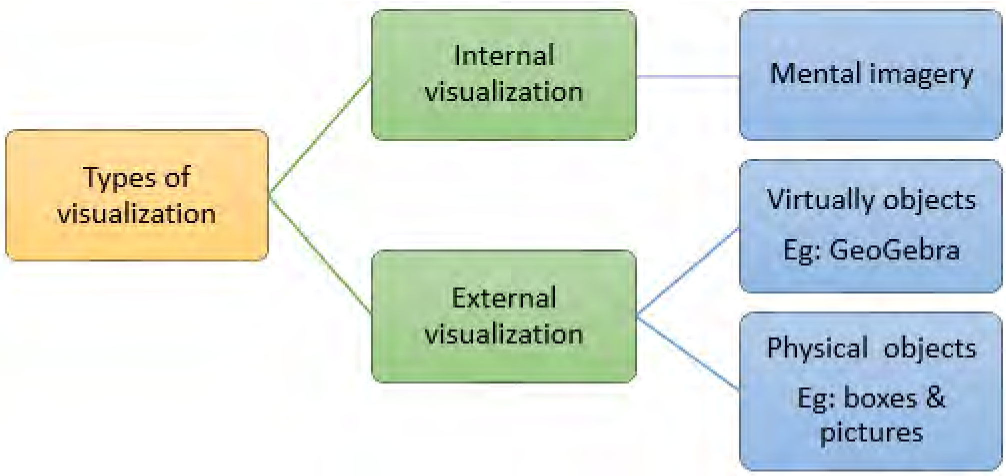


Figure 2.42: A visual summary of the types of visualisation.

Parts of our bodies can also be considered as visuals. Boaler et al. (2016) indicated that our fingers are visuals – they can be used to illustrate objects. A brain study by Berteletti and Booth (2015) revealed that there is a region in the brain called the somatosensory finger area. This region of the brain is a part of the complex distribution of networks in the brain that handle perception and representation of fingers. The somatosensory finger area makes it possible for one to see or imagine a representation of one's fingers or configurations of one's fingers in the brain, without seeing the actual physical fingers. The findings of their study further revealed that finger representations and figure-based strategies play a significant role in the learning and understanding of arithmetic mathematics in young children. Also, the use of fingers in representation strengthens the brain's capacity to handle knowledge. Hence, Berteletti and Booth (2015) encourage learners of any age to make use of finger representations when working on mathematical problems.

2.5 IMPORTANCE OF VISUALISATION

Even though scholars are yet to agree on the common definition of visualisation, most, if not all, have acknowledged the important role of visualisation, particularly in teaching and learning. Despite much research conducted on the role and importance of visualisation, Makina (2010) is however of the view that little has been done to extend these roles to mathematics education. A matter of concern is that the quality of learning that teachers provide depends largely on the pedagogical approach used in their classrooms.

The use of visuals in teaching and learning, particularly in the introduction of a mathematical concept, enables deep understanding to both the teacher who is introducing the concept and the learners who are using visual/s to think and to make sense of mathematics (Boaler et al., 2016). Arcavi (2003) acknowledged the importance of visuals too. He stated that visual representations help learners to advance their understanding of mathematical concepts and procedures, to make sense of problems and engage in mathematical discourse. Most mathematical concepts are abstract, which is a challenge to many learners. Due to the abstract nature of mathematics, people can have access to these ideas through meaningful and appropriate visual representations (Mann, 2014). Phillips et al. (2010) also stated that visuals can be used to take abstract ideas and make them concrete. Let's consider the following hypothetical scenario:

A cuboid has a perimeter of 30 cm. If its length is13 cm, work out its breadth

For many, this is an abstract scenario that should best be drawn to make sense. The cuboid could be sketched and the given dimensions labelled accordingly. In that way, the problem is concretised, as argued by Phillips et al, (2010). Having concretised the problem (by drawing the cuboid), the diagram of the cuboid will then guide the cognitive process which will lead to the development of the solution (Bishop, 1989).

According to Arcavi (2003), visual mathematics activities encourage deep engagement and new understanding. Visual brain activities can show learners that mathematics is an open and beautiful subject rather than a fixed, closed and impenetrable subject, as argued by Skovsmose (2008). Boaler et al. (2016) also argued that when mathematics is taught using a different approach from visualisation; learners develop a feeling that mathematics is inaccessible and uninteresting. They reiterate that learners who are taught using visual activities, particularly those learners who dislike mathematics and cannot do mathematics, change their minds positively toward mathematics and start to see mathematics as something worthwhile.

School curriculums should be designed in a way that responds to the societal needs of the learners. Currently, the world is rapidly advancing. However, in most curriculums – including the Namibian curriculum – the way to learn and understand mathematics visually is not very explicit. Boaler et al. (2016), raise the observation that many companies now have and use large amounts of data, known as big data, which requires people to make use of visualisations to process this data. Interpretations of the data will require new knowledge which is largely based on imagery rich in content and information, unlike in the past when knowledge was largely based on words and numbers. These authors thus suggest that visualisation may enhance the nature of knowledge needed in today's high-tech world.

2.6 CRITIQUE AND DIFFICULTIES OF VISUALISATION

Despite visualisation being so essential in teaching and learning, particularly in mathematics, it also poses some challenges. Rieber (1995) reiterated that even though visualisation is a powerful tool for solving mathematical problems, it is rarely promoted in learning instructions. On the other hand, in the curriculum where visualisation is emphasised, teachers are still observed giving more attention to verbal skills than visual skills and abstract reasoning over concrete reasoning, overlooking the idea of using visualisation. Thus, much still needs to be done regarding visualisation, as alluded to by Makina (2010). This is in agreement with Hanna and Sidoli (2007) who reaffirmed that a considerable amount about educational visualisation is still poorly understood.

Phillips et al. (2010) also acknowledged the importance and value of visualisation. They however raised a concern that the benefits of visualisation do not come automatically. It requires some experience to use it well. In other words, a person viewing an image, for example, should have a range of experiences and mental skills to interpret and use this image. Jonson-Laird (1998) stated that some cognitive action is required to move from what is on a picture/image to some internalised concept of what is represented, before interpretation can take place. Anderson-Pence et al. (2014) specified that visuals can only be useful to learners if they can create models themselves, interpret models and use them effectively for problem solving. It is thus important for teachers to realise that simply presenting visuals to learners does not guarantee learning. Teachers need to provide support and act as a catalyst to encourage and motivate learners' thinking. They need to mediate between what is visualised in the mind of the learners and the visual object or diagram itself.

Pictures, particularly in textbooks, can be very useful and are important resources. They provide important clues about the text and can reduce the cognitive demand/load. However, it has been found that some pictures in some textbooks do not really serve as a bridge between what learners already know and the desired outcome of the visualisation (Casselman, 2000). In other words, they become irrelevant. Also, when teachers are designing visual teaching aids such as pictures, for example, they need to be careful not to cluster them with unnecessary information that distracts learners from the real point of the picture. To this Casselman (2000) suggests that only the information that a diagram needs to make its point, should be placed on the diagram.

Many learners find it a challenge to form mental images through visualisation. Jones and Bills (1998) affirmed that one of the challenges when learning visually particularly relates to forming mental images and using those images in problem solving. It is also suggested by Love (1995) that in geometry, the relationship between "mental objects and physical objects is especially a difficult one" (p. 125).

The way individual people construct their reality is also a point of concern. Rieber (1995) pointed out that visual interpretations are filtered through one's prior knowledge, experiences and beliefs. This means that the use of visuals in classrooms does not guarantee that all learners will be directed towards the same understanding. This is because each learner's interpretation is based on his/her prior knowledge. Teachers can thus not assume that all learners will see the same message in a particular visual.

2.7 VISUALS AND LEARNING OF MATHEMATICS

The value and significance of using visuals in learning for all age groups has been emphasised widely in the literature. Rieber (1995), for example, confirmed that visuals are powerful cognitive tools for all people, and not just a select few. Phillips et al. (2010) described visuals as a good way of reminding learners what they have learned. These authors explained that an image remains in the long-term memory for longer than words. They also doubt whether there are other reminders superior to visualisation. Hence, in textbooks for young learners, more space is allocated to visuals than text. This ratio of pictures to text however decreases as learners progress to the higher grades. The reason for this relationship is that visuals help young learners in their formative years to learn fundamental concepts.

Phillips et al. (2010) asserted that the combination of visuals and text minimises learning difficulties and allude to the observation that reading text involves interpreting information and constructing mental pictures. For many learners, particularly second-language speakers, this is a challenge because they have to grapple with their second language as they are reading. The use of visuals can then provide clues to the readers, unburdening them from these challenges, and enabling them to focus only on understanding the concept. Additionally, the combination of visuals and text also serves as a motivation to the reader. From my own experience as an English second-language speaker, when reading journals which rely heavily on text – particularly those with high lexical density – I often feel tortured when reading through all the pages, paragraph by paragraph, in contrast to when I read a well-illustrated story.

Certain beliefs by people, however, often distract them from using visuals for learning. Some of those beliefs highlighted by Boaler et al. (2016) are:

- visuals are for lower grades;
- visuals are for young learners or for those who are struggling in mathematics;
- visuals can only be used in the introduction of more advanced or abstract mathematics;
- pictures and diagrams are for use by ordinary people and children, while words and mathematics symbols are for the serious professionals; and
- learners who prefer visuals are often labelled as having special educational needs.

These are however unsubstantiated claims or mere speculations that distract people from benefiting from the potential of visualisation.

Boaler et al. (2016) observed that mathematical knowledge is handled by the dorsal visual pathway, an area in our brains. A study on the human brain by Kucian et al. (2011) confirmed that the dorsal visual pathway comes into play when a student considers visuals or spatial representations of quantities. As they grow older and continue using visuals, the dorsal pathways become more specialised in processing visuals. Özkan et al. (2018) argued that visual thinking helps learners to understand events, processes, and objects around them, to further construct detailed and comprehensive concrete schemes in their brains. Visuals are essential for learners' cognitive development.

2.8 VISUALS AND TEACHING OF MATHEMATICS

The appropriate use of visuals in teaching has been advocated in the literature by various scholars and mathematics organisations. The National Council for the Teaching of Mathematics (NCTM) has long encouraged the use of multiple representations in students' teaching and learning of mathematics (NCTM, 2000). However, recent work by Boaler et al. (2016) reveals that mathematics is still taught as an almost entirely numeric and symbolic subject, with a multitude of missed opportunities to develop visual understanding. To this, they recommend that a good mathematics teacher is one who typically uses visuals, manipulatives and body language in teaching to enhance students' understanding of mathematical concepts. By body language they refer to the gestures that teachers make use of during teaching. For example, if the teacher is explaining mathematical concepts and in the process forgets the right term to use, she can use her hand to draw a shape in the air. An embodied cognitive study by Alibali & Nathan (2012) also concluded that as we gesture, we think mathematically. They thus recommend that teachers should make use of gestures to ground mathematical thinking alongside their verbal explanation.

Teachers are encouraged to consider visuals in their day-to-day lesson delivery (Boaler et al., 2016). There are several ways of doing this. Boaler et al. (2016) suggest that teachers can engage learners in visual thinking by giving them visual activities/questions and ask them to provide visual solutions to those questions. Secondly, a teacher may employ learners in visual thinking by asking them to present mathematical ideas in a multitude of ways, such as in picture form, using models, graphs and in doodles or cartoons. Another way is to give learners mathematical ideas and then ask them to draw what they see or how they visualise the idea. To illustrate how a teacher can give learners a mathematical idea and ask them to draw what they visualise, let us consider the following scenario:

Oshikwiila (traditional bread) is sliced into ten equal pieces and Ndina eats four pieces of this bread. Another same-sized loaf of *oshikwiila* is sliced into five equal pieces and Kondja eats two slices of this. Ndina said that she ate more *oshikwiila* bread than Kondja, but Kondja said they both ate the same amount. Use words and pictures to show that Kondja could be right.

In this way, learners may be compelled to draw similar pictures that represent the two *oshikwiila* loaves, divide them into the relevant fractions, and compare the amount eaten. The author mentioned above stressed that through drawing models learners can formulate their ideas and develop understanding.

Boaler et al. (2016) observed that when lessons simply rely on numbers only, learners respond that the work is either “easy” or “hard” or announce that they are “finished” after racing through a worksheet. However, when the same content is taught visually, the responses of the learners are quite different as they are far more engaged in the lesson (Makina, 2010). Boaler et al.’s (2016) findings confirm that when teachers provide visual experiences of mathematical ideas to their learners that were previously only encountered numerically and abstractly, their learners gain deeper insights into these ideas and they start to understand these ideas more meaningfully.

On teaching mathematics, Boaler et al. (2016) made three useful recommendations to teachers and parents, who in my view should be responsible for giving mathematics basic education at home. These recommendations are:

1. Encourage and celebrate students’ visual approach and do away with the idea that strong mathematicians (learners) are those who memorise, and calculate well and fast. In fact, fast calculation is neither good mathematics nor a need for high level mathematics (Schwartz, 2001). Rather, strong mathematics learners are those who think deeply, make connections and visualise (Boaler et al., 2016).
2. Teachers and parents should encourage finger use. In Boaler et al.’s (2016) view, successful mathematics users have a well-developed somatosensory finger area. This literally means that if we stop learners from using their fingers or learning finger strategies in mathematics, we are actually stopping an important part of their mathematical development.

3. Lastly, these authors recommend that mathematics teaching and learning should become more visual. They are concerned that teaching and learning mathematics is dominated by memorising mathematical facts and working through worksheets, with few or no visuals. I agree with these authors because in my experience of being a mathematics teacher for more than ten years, this is what we have been enforcing on learners. These authors thus urge teachers to use visual teaching because "there is not a single mathematical idea or concept that cannot be illustrated or thought about visually" (p. 5).

2.9 VISUAL MODELS IN MATHEMATICS

Within the field of mathematics, the concepts of visual models, representations, teaching aids and manipulatives are used interchangeably. For this study, I too did not make any distinction between the four. In this thesis I use the term 'visual models' to refer to all these ways of referring to visualisation.

When learners are presented with mathematical concepts or when they orally solve mathematical problems, they generate internal representations (Goldin & Shteingold, 2001) or use external representations (visual models) to help them understand and solve the problem. Internal representations are described by Goldin & Shteingold (2001) as "the internal system of representations that are created within a person's mind and are used to assign mathematical meaning" (p. 178). In this study, I have however focused on external representations, visual models in particular.

External representations (visual models) are defined by Goldin & Kaput (1996) as "physically embodied, an observable configuration such as words, graphs, pictures, equations, or computer micro worlds" (p. 400). From these authors' definition, two key concepts (physical embodied and computer micro world) further divide external representation into two main groups: concrete/physical and virtual models.

Virtual models are interactive web-based visual representations of a dynamic object that present opportunities for constructing mathematical knowledge (Durmus & Karakirik, 2006). There are various virtual models in mathematics, including *GeoGebra*, and geometry applets. These models enable learners' engagement as do physical models. In this study, I however focused on physical/concrete visual models.

Durmus & Karakirik (2006) defined concrete models as "physical objects such as base-ten blocks, algebraic tiles, unifix cubes, cuisenaire rods, fraction pieces, pattern blocks and geometric solids that can make mathematical abstract ideas and symbols more meaningful and understandable to students" (p. 120). Lesh (1979) describes physical visual models as concrete objects that are used as an intermediary between the real world and the mathematical world. Geometry rods, geoboards, isometric papers, symmetry mirrors, diagrams, graphical displays and symbolic expressions are all examples of physical visual models (Durmus & Karakirik, 2006). Also, our fingers –part of our bodies – are visual models. Fingers can be used to represent and solve arithmetic problems, particularly in lower grades. Worth mentioning is that arms are often used by teachers to gesture during teaching and this is also a type of visual model. This study does not, however, focus on a specific physical visual model. The choice of models in this study is based/derived from the intervention topics chosen by the teachers.

Literature says much about the importance of visual models in terms of teaching and learning and I have unpacked the importance of visual models in Section 2.5. It is worth noting that all learners need to be allowed to manipulate visual models. This claim was made by Swan & Marshall (2010) who plead that all children need concrete materials to aid maths activities for some time and eventually dispose them of as they become more adept in mental strategies. These authors take the view that the demonstration of visual models by the teacher is not sufficient for learners to realise their full potential. This study also argues that it is not in line with the constructivist view advocated by this study, which views learners as co-constructors of knowledge in interactive activities.

Presenting learners with equal opportunities to manipulate visual models has huge benefits. However, this does not guarantee conceptual understanding or a productive disposition for mathematics. Students who use visual models in a rote manner (Hiebert & Wearner, 1992), may fail to find the link between the visual model and the mathematical idea or develop an incorrect concept (Durmus & Karakirik, 2006). When this is the case, the purpose and use of visual models is nominal (Swan & Marshall, 2010) or useless as described by Kozulin (2003). To this, Durmus & Karakirik (2006) have suggested that a careful definition of the teachers' role and the potential of the task is required. Also, apart from carefully choosing the visual model, a simple design that enables easy manipulation should be adopted while addressing motivation at the same time.

Despite having an awareness of the potential of visual models, teachers are nevertheless observed using teaching approaches other than the visual approach (Phillips et al., 2010). Several variables are involved. One could argue that teachers do not influence the design of the curriculum. In other words, if the curriculum does not stress nor mandate the use of visual models, then teachers tend to believe that they are not compelled to do so. This is confirmed by Miranda and Adler (2010) who stated that in many African countries, including Namibia, there is no rigorous standardised measure to ensure that visual models are used for teaching and learning school mathematics. One could also argue that teachers do not have an epistemological/conceptual understanding of visual models, their role, their benefits for teaching and learning, and probably the understanding of how to use visuals in the teaching of mathematics. I concur with Boaler et al. (2016) who argue that there are no mathematical ideas/concepts which cannot be illustrated or thought about visually. I thus discuss hereunder some examples of visual models that can be used in teaching mathematical concepts.

2.9.1 Area models

Fractions are one of the challenging topics for learners in mathematics. This is evident in Katenda's, (2018) study conducted with teachers in the Khomas region, Namibia. Her findings revealed that teachers mostly introduce this topic to learners using symbolic notation rather than visual models. There are however many visual models that teachers can make use of to teach fractions. Linear and area models are some of the models identified by Sidney et al. (2019). For the moment I will only focus on area models which are described by Sidney et al. (2019), as a way of representing fractions as parts of whole shapes using either circular or rectangular shapes. Below I have demonstrated how to compare $\frac{2}{3}$ and $\frac{5}{6}$ using area models.

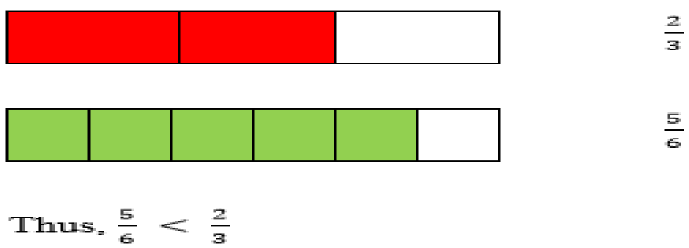


Figure 2.3: An area model

With area models, a whole shape is divided into equal parts. The denominator is represented by the total number of equal parts in a whole shape, while the numerator is represented by the total number of shaded parts as shown in the figure above. A recent study by Baek et al. (2017)

revealed that area models help to highlight students' part-whole conception of fractions and can successfully support children's visual representation of fractions and help them to understand the role of the common denominator in a fraction.

2.9.2 Bottle tops

Most of us open a bottle or any other container and throw the lid or tops away without noticing that it has many other uses. Cherif and Adams (2018) point out that for teachers, bottle tops have a multitude of educational and pedagogical uses, particularly in mathematics teaching and learning. These authors claim that bottle tops can be used to illustrate mathematical concepts such as addition, subtraction, division, multiplication, ratio, fractions, percentages, proportion, statistics and graphs. Figure 2.4 below demonstrates how bottle tops can be used in the addition of directed numbers.

Question: Work out $-5 + 7 = ?$

Figure 2.4: A visual illustration of $-5 + 7$

The use of bottle tops can be a motivation to many learners. Cherif and Adams (2018) confirm this and further suggest that one effective way of motivating learners to be actively engaged in their learning is to use relevant, familiar and practical objects in a way that they have never thought of before. Bottle tops are easily found in our environments. Thus, the use of a bottle top in teaching school mathematics can promote learners' engagement with mathematical concepts. Exposing learners, particularly young ones, to manipulatives such as bottle tops does not only help them to learn mathematics but also promotes a positive image of mathematics to learners, helping them discover that mathematics is real and fun (Boggan et al., 2010).

2.9.3 Algebraic tiles

In my experience algebra is one of the mathematical domains which is rarely taught visually. Below, Figure 2.5 shows how to teach algebraic concepts using algebraic tiles, according to Leitze and Kitt (2000).

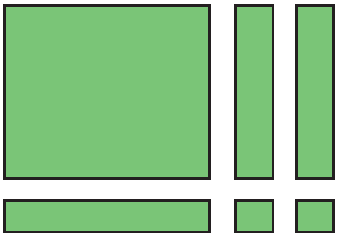


Figure 2. 5:Algebraic tiles

The algebra tiles model consists of large squares, rectangles and small squares as shown in Figure 2.6 below. The small square represents one unit tiles (number 1), the rectangles represent $\boldsymbol{x}$ and the large square represents $\boldsymbol{x^2}$.

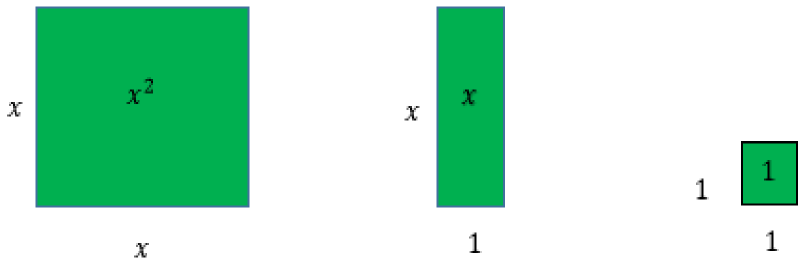


Figure 2.6: Different types of algebra tiles

The components of the algebra tile are designed in such a way that the length of the sides of x^2 are equal to the length of tile x. The width of the x tile is equal in length to the sides of the unit tile, but the length of the sides of the unit tiles is not an integral factor of the unit tiles. These algebra tiles can be constructed from any available material with one side red and green, for example, to show a positive value (green) and a negative value (red). Zero is illustrated by a positive x tile and negative x tile adjacent to each other, as illustrated below in Figure 2.7.

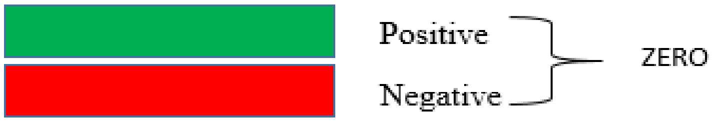
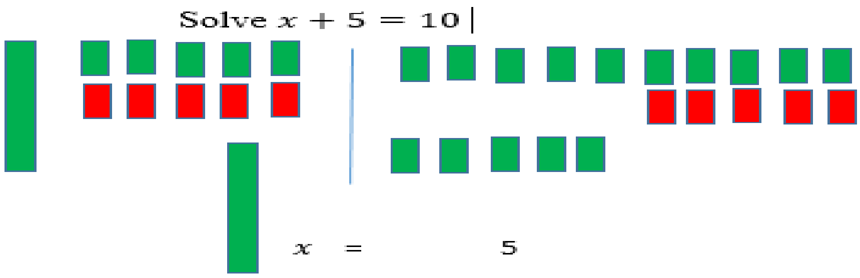


Figure 2.7: A visual illustration of zero (0)



To illustrate how algebra tiles can be used to concretise algebraic concepts, the following equation below in Figure 2.8 can be solved in this way:Figure 2.8: A visual illustration of an algebraic equation

There is value in using algebra tiles, as agreed by Leitze and Kitt (2000). They mention that a significant number of learners do not connect algebraic concepts with previously learned mathematical ideas – geometry for example – because they do not see the connection. Apart from concretising the algebraic concepts, algebra tiles help learners to combine algebraic and geometrical approaches to algebraic concepts. Additionally, Chappell and Strutchens (2001) also point out that the use of algebra tiles helps learners to connect algebra to other mathematical concepts such as number properties. Leitze and Kitt (2000) further commend the use of algebra tiles because the sequencing of instruction from the concrete level, through to the pictorial level and finally to the abstract or symbolic level, is easily achieved. Hence, through the use of algebra tiles, learners' cognitive demand/loads may undoubtedly be reduced as argued by Casselman (2000). It is worth noting that in the absence of algebra tiles, learners can draw sketches of algebra tiles to solve algebraic problems.

2.10 USING VISUALISATION MODELS TO GROW A PRODUCTIVE DISPOSITION

2.10.1 Productive disposition

In the present complex society, a person needs some level of mathematical proficiency to make informed decisions as both a private and a productive worker (Wilkins & Ma, 2003). Mathematical proficiency can be conceptualised in terms of Kilpatrick et al.'s (2001) five interrelated strands, namely conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition, the latter being the focus of this study. A mathematical productive learning disposition according to Kilpatrick et al. (2001, p. 131):

> ...refers to the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics. If students are to develop conceptual understanding, procedural fluency, strategic competence, and adaptive reasoning abilities, they must believe that mathematics is understandable, not arbitrary; that with diligent effort, it can be learned and used as they are capable of figuring it out.

From Kilpatricks et al.'s (2001) definition, a productive disposition may be simply defined as being positive and resilient toward mathematics, as argued by Carr and Claxton (2013). Possessing a productive mathematical disposition is integral to mathematical proficiency (Feldhaus, 2014). On the same note, Kilpatrick et al. (2001) indicate that students' dispositions toward mathematics is a major factor in determining their educational success. In contrast, having an unproductive mathematical disposition (the opposite of a productive disposition) can be a huge loss to learners, particularly in the Namibian context where mathematics is now compulsory from Grade 3 to s11 (MBEC, 2016), and learners are not motivated to take mathematics. According to Beilock et al. (2010), when demotivation and self-doubt occur, thinking and reasoning can be compromised. I agree with Beilock et al. (2010) and argue that a lack of self-confidence can be devastating for learners. This is further confirmed by Code et al. (2016) who state that confidence is known to affect students' willingness to engage with a task, the effort they spend in working on a task and the degree to which they persist when they encounter setbacks.

Feldhaus (2014) argues that the consequence of a lack of confidence, the avoidance of mathematics and performing poorly may be attributed to an unproductive mathematical disposition towards mathematics.

An unproductive mathematical disposition does not only affect performance; it also has a personal dimension. Graven and Buytenhuys (2011) report that some learners connect their negative mathematical image and disposition to their broader self-image. These authors quote a learner who wrote "my struggle with maths also negatively impacted my self-confidence and left me to feel like I was stupid and useless" (p. 498). On the same note, Feldhaus (2014) also highlighted that a negative disposition towards mathematics can increase mathematics anxiety. In this regard, Boaler and Selling (2017) have called for a paradigm shift in mathematical education that allows mathematics meaning-making, and make mathematics become more visible to learners. It is this visibility that makes mathematics real and accessible and could lead to a productive disposition towards mathematics. Hence this study is advocating for the teaching of mathematics using visual models to grow learners' productive dispositions.

2.10.2 Indicators of a productive disposition

Having unpacked the concept of productive disposition and its possible effects on learners both in and outside the classroom context, my story would be incomplete if I did not review the literature on how both productive and unproductive mathematical dispositions are manifested by learners.

It is argued that the fundamental purpose of education for the 21st century is not so much about the transmission of a particular body of knowledge, skills and understanding, but rather for the development of a positive learning disposition (Carr & Claxton, 2002). Heyd-Metzuyanim (2013) reports that most learners have what she terms "dependent ritual behaviours". By this Heyd-Metzuyanim means that most learners still rely heavily on teachers for solutions. The author further indicates that some learners have a ritualistic behaviour of blindly following rules, without thinking critically for themselves. In this regard, a recent study by Pérez (2018) revealed that students continue to experience school maths as rule-bound, linear and solitary, in which success is regarded in terms of arriving quickly at a single correct answer to please the teacher.

In South Africa, Mathematical Literacy (ML) was introduced as a new senior secondary school subject in 2006. In their study, Graven et al. (2013) tracked some learners in order to discover their experiences of mathematical learning, by asking them to write their own stories. Their findings revealed some very strong feelings about their experiences in mathematics. The learners' stories were characterised by terms such as failure, struggle, stress, nervousness, hate for maths, worry, extreme difficulty, lack of confidence and hopelessness. These stories, in my view, all illustrate that unproductive mathematical dispositions towards mathematics are very common among learners.

In contrast, learners with a productive disposition are confident in their knowledge and abilities (Kilpatrick et al., 2001). They see that mathematics is reasonable and intelligible, and believe that with appropriate effort and experience, they can learn. Boaler et al. (2016) indicate that when learners are given mathematical work, they express their emotions differently. Some say work is “easy” or “hard”, while some pronounce that they have finished after racing through the work sheet. However, learners with a productive disposition demonstrated readiness to learn, with their willingness and eagerness visible on all their faces (Carr & Claxton, 2002).

Other authors also use various terms to describe learners with a productive disposition. Bronfenbrenner (1979) described them as persisting in solving a task, giving their opinions during lesson presentations, contributing ideas and working collaboratively. Katz (1988) stated that learners with a productive disposition respond to situations in a certain way such as showing curiosity and friendliness. Goleman (1996) used seven terms to describe learners with a productive disposition. They are confidence, curiosity, intentionality, self-control, relatedness, communication, and cooperation. Claxton (1996) on the other hand, uses terms such as curiosity, mindfulness, selectivity, resilience, experimentation, reflectiveness, opportunism and conviviality.

In an attempt to develop a tool for assessing productive disposition, Siegfried (2012) identified eight indicators of productive disposition. These are beliefs, affects, mathematic identity, mathematic integrity, risk-taking, goal, motivation, and self-efficacy. For the purpose of my questionnaires however, I used the following seven Siegfried constructs: beliefs, affects, mathematic identity, risk-taking, goals, motivation, and self-efficacy. I replaced the goal construct by ‘engaging with tasks’ as this was very evident in my data (see Appendix A). Below I discuss the resulting/final eight constructs in detail.

2.10.2.1 Beliefs

Beliefs are the subjective knowledge that learners may possess towards mathematics (Furinghetti & Pehkonen, 2002). They are subjective in the sense that this knowledge is based on individual opinions rather than facts. These authors further postulate that individual beliefs are always changing because they are usually under continuous evaluation against the learner’s new experiences and with other learners’ beliefs. Learners’ mathematical beliefs can be determined by asking them questions such as: “what do you think math is about?”, “are there some learners who cannot learn maths or can everyone learn maths if they try”? (Muis, 2004). Muis (2004) further showed that indicators that learners have positive beliefs about mathematics (and thus positive productive dispositions) includes “everyone can do mathematics if they try”, “mathematics is a series of interconnected concepts”, “mathematical problems can be solved in a variety of ways or working on mathematical problems is more than simply to get the correct answer as quickly as possible”.

2.10.2.2 Affect

Affects refer to learners' attitudes and emotions toward mathematics (Jacobson & Kilpatrick, 2015). This includes whether learners like or dislike the subject and whether they find it interesting or boring.

Affects impact one's positive disposition differently. Positive affects help learners to continue working on and thinking about difficult mathematical problems, in addition to seeing their endeavours as useful and worthwhile. On the other hand, negative affects can lead learners to easily give up attempting to solve mathematical problems or to feel that there is no point in doing mathematics (Debellis & Goldin, 2006).

Learners' affects can be determined in various ways. Ruffell et al. (1998) highlighted that learners' affects can be determined. Through asking them for example, to describe when they enjoy learning mathematics, how they feel when engaged in mathematical activities or what they like least about mathematics. Teachers may also access learners' affects by interviewing them or analysing their conversations as they interact with other learners. Concepts such as interest or frustration reveal learners' mathematical dispositions (Gomez-Chacon, 2000).

2.10.2.3 Mathematical identity

Learners' mathematical identity refers to the narrative they create to explain themselves in relation to mathematics and their mathematical experiences (Lutovac & Kaasila, 2014). In other words, mathematical identity is how people see themselves in relation to mathematics. Identity in general consists of qualities people recognise in themselves or are recognised by others (Gee, 2001). These qualities emerge through interaction with other people.

Mathematical identity is one of the most important indicators of a productive disposition. Being able to see oneself as a doer of mathematics requires one to have a strong mathematical identity. In addition, Boaler (2002) has also asserted that having a mathematical identity does not only increase students' mathematical content knowledge but also makes them engaged in the practice of mathematics, thus developing a productive relation with mathematics.

Mathematical identity can be assessed by asking learners about their previous experiences with mathematics. To be specific, learners can be asked to explain whether their mathematical experiences were good or bad. Siegfried (2012) stated that observing how learners engage with mathematics and asking how learners envision mathematics fitting into their broader lives also gives a broader picture about their mathematical identity.

2.10.2.4 Risk-taking

Academic risk-taking involves learners engaging in adaptive behaviours (sharing ideas, asking questions, trying new things) that place them at risk of making mistakes or appearing less competent than others (Beghetto, 2009). Siegfried (2012) describes academic risk-takers as learners who can examine their solutions and exhibit a willingness to make mistakes. In general, when one is taking a risk means attempting a task in which you have not more than 50 per cent probability of succeeding. According to Clifford (1988), academic risk-takers exhibit three traits: they prefer difficult but doable tasks; they have a relatively high tolerance of failure; and they tend to use various flexible strategies to overcome obstacles.

Academic risk-taking can be assessed by observing learners' willingness to share mathematical ideas or interviewing them to establish their feelings about failure and their willingness to make errors.

2.10.2.5 Engagement with tasks

Tasks are the main components of teaching and learning (Mann, 2014). The literature identifies many different types of tasks (Yeo, 2017), among them being open-ended tasks and investigations. It is, however, the responsibility of the teacher to select and develop worthwhile and appropriate tasks and learning materials that will afford learners opportunities to develop mathematical understanding, competencies, interest and positive dispositions.

Learning depends on the type of tasks given, as the nature of tasks influences and structures learners' thinking and their degree of engagement. Skinner et al. (2008) reported that students who are engaged in learning have been found to invest greater effort and exhibit more persistence and determination in solving tasks, thereby contributing to higher-quality learning and better outcomes.

Mathematics is viewed as a challenging subject by many learners. However, engaging learners in well-designed tasks builds their self-confidence and by implication, their dispositions. Katz and Stupel (2015) discussed the potential of confident learners. They indicated that learners with confidence approach difficult tasks as a challenge to be mastered rather than as a threat to be avoided. Such learners set themselves challenging goals and maintain a strong commitment to them. In contrast, learners who lack confidence shy away from difficult tasks which they view as personal threats (Katz & Stupel, 2015). They have low aspirations, negative dispositions and weak commitment to their goals. They give up quickly in the face of difficulties and they are slow to recover their sense of confidence following failure. They can lose faith in their capabilities very quickly and can easily fall victim to stress and depression. To this, Katz and Stupel (2015) suggest that mathematics teachers should train young people to approach threatening situations when learning mathematics with confidence.

2.10.2.6 Motivation

Motivation is defined as the "inclination to do certain things and avoiding doing some others" (Hannula, 2006 p.165). In the context of mathematics, it is the force that drives a learners to willingly tackle mathematical problems despite setbacks. In psychology, a learner is either intrinsically motivated (motivation from within the learner), or extrinsically motivated (motivation from outside, for example, though rewards from a teacher or parents). Motivated learners, particularly intrinsically motivated ones spend more time on their work with little push from the teacher or parents. They also tolerate failure more than extrinsically motivated learners (Siegfried, 2012).

Mathematics is not a naturally motivating subject, especially for many of our mostly average and below-average learners in our schools. Traits of productive disposition are found mainly in intrinsically motivated learners. This includes perseverance, and the selection of challenging mathematical problems (Siegfried, 2012).

2.10.2.7 Self- efficacy

Self-efficacy is Bandura's (1986) concept which refers to individuals' beliefs and confidence. Bandura (1977) defines self-efficacy as a person's beliefs in his or her capability to successfully perform a particular task. On the same note, Heslin and Klehe (2006) explain that self-efficacy is a strong determinant of effort, persistence, strategizing and their subsequent training and job performance.

Self-efficacy plays a vital role in one's success. Heslin and Klehe (2006) singled out self-efficacy as one of the most powerful motivational predictors of how well a person will perform. These authors further clarify that a high degree of self-efficacy leads to people work hard and persist in the face of setbacks. Worth noting is that when learning complex tasks, high self-efficacy causes people to strive to improve their assumptions and strategies, rather than look for excuses such as not being interested in the task at hand. In the context of mathematics, this translates to those with high self-efficacies in mathematics rightfully seeing themselves as being able to learn mathematics and perform well in mathematical tasks.

On contrary, where self-efficacy is low, people often see negative outcomes as confirming the incompetency they perceive in themselves and they also tend to blame either the situation or another person when things do not go well (Heslin & Klehe, 2006).

2.10.3 Growing learners' productive disposition

In the previous section, I unpacked the concepts of productive and unproductive dispositions. In this section, I review the literature on growing learners' productive dispositions. Using the research of various experts in this field, I argue from two perspectives:

1) disposition and
2) visualisation (how the use of visual models can grow learners' productive dispositions).

Carr and Claxton (2002) argued that no single teaching method is sufficient on its own to change learners' dispositions. Many different approaches are required. This study will explore how visual models, as one approach, can be used in teaching mathematics to grow learners' productive dispositions.

Various authors have dwelt on how to grow learners' productive dispositions. Graven (2011) argued that growing learners' productive dispositions involves shifting learners' dispositions from passive learners to more engaging, confident and actively participating learners. Being passive means accepting whatever the teacher says without any input. I am hopeful by that using visual models in my intervention, the participating learners' productive dispositions will grow. I was inspired by Özkan et al. (2018) who argue that visual models help to create thoughts that we have not previously had, alleviating the notion of being passive. Using visual models makes information more meaningful, arousing learners' interest and making them actively participate in lessons.

Carr and Claxton (2002) postulated that growing learners' productive dispositions requires developing their learning capability. Learning capability in this context refers to the toolkit of learning: the skills, strategies and abilities which are required in learning. Carr and Claxton (2002) further clarified that developing learners' abilities breeds success and success may lead to building a productive disposition.

On the same note, Code et al. (2016) claim that growing learners' productive dispositions is more than about learning the required subject content knowledge. Learners need to gain domain-specific ways of thinking and doing mathematics. Makina (2010) spoke of critical thinking. She raised a concern that teachers usually succeed in teaching learners the content of a respective academic discipline, but teaching learners how to think effectively and critically about the subject (mathematics in this context) is often not achieved. In her view, critical thinking is essential in the learning of mathematics. She thus recommends visualisation as an important tool in enhancing critical thinking in mathematics. This form of teaching is in contradiction with the pedagogical approach to teaching where learners experience school mathematics as rule-bound (Pérez, 2018).

It is alluded to by many scholars that growing learners' productive dispositions requires a paradigm shift in mathematics education. For example, Pérez (2018) has requested teachers to do away with teaching approaches that focus more on learning procedures without any connection to meaning. Apart from having a deeper understanding of the mathematical knowledge they have to teach, teachers need to be skilled in ways that are effective in developing mathematics learning for their students (Mann, 2014). Mann (2014) further argues that there is a need to move from pockets of excellence in mathematics education to a system of excellence through eight pedagogical practices that enhance the ability of all students to learn at the highest possible level and, besides this, to become mathematical thinkers. These eight pedagogical practices have been defined to include the development of five interrelated strands that, together, constitute mathematical proficiency. Below I briefly discuss them, and how they can manifest in the classroom.

2.10.3.1 Establishing mathematical goals to focus learning

These practices involve setting clear mathematical goals to show that students are learning, positioning these goals with learning progress and using them to guide instructional decisions (Mann, 2014). His research stresses that at the lesson planning stage clear goals are established, where a teacher begins by clarifying and understanding the mathematics to be learned, and how it will unfold in during the lesson. In other words, the teacher should be able to anticipate how the lesson will transpire. This helps to shape or focus the lesson and influences the teacher's instructional decisions during the lesson.

Clear mathematical goals are demonstrated in the lesson when the teacher indicates and clarifies what learners are expected to learn and understand (Mann, 2014). Daily goals need to be displayed or written somewhere for learners to see. In the Namibian context, lesson goals and objectives are always indicated on the lesson plan as stated in the syllabus. Mann (2014) also cautions that mathematical goals that guide instruction should not just be a reiteration of standard statements, but must be linked specifically to the current classroom curriculum and student learning needs.

As to how good understanding of mathematical lesson goals and objectives manifest in lessons, Mann (2014) highlights that problems or examples used by the teacher should show a transition from learners' prior knowledge to more sophisticated knowledge and should show how mathematical ideals build on and relate to one another. In addition, Daro et al. (2011) stressed that teachers should use an ordered set of instructional experiences and tasks that are geared to stimulate the mental processes or actions that develop or progress in the desired direction.

There are more benefits in setting clear lesson goals. Apart from guiding the teacher to make instructional decisions during lessons, they also help students to understand the mathematical purpose of the lesson, and how the activities contribute to and support their mathematical learning. It is worth noting that establishing clear lesson goals focuses learners' attention on monitoring their progress towards the intended learning outcomes.

2.10.3.2 Implementing tasks that promote reasoning and problem solving

Effective teaching of mathematics uses tasks to motivate students to learn and help them to build new mathematical knowledge through problem solving (Mann, 2014). Mann (2014) recommends that teachers should provide learners with the opportunity to engage with tasks, and there should be sufficient work for learners to practise what they have learned. Selected tasks should range from easy to complex and should focus students' attention on a particular mathematical idea. The teacher should make use of different representations and tools to foster solving problems using various approaches and strategies.

In addition, the teacher should implement tasks that promote reasoning and problem solving. However, many tasks that promote reasoning and problem solving require high cognitive demand, hence learners may struggle in those activities. Regarding this, Mann (2014) stresses that teachers should provide suggestions or clues that will help learners to make progress in the task at hand, but he/she should avoid taking over the thinking of learners or provide a specific way to follow. It is worth mentioning that learners in a class come from different cultural backgrounds as alluded by Vygotsky (1978), therefore, the teacher should be cognizant enough to consider the context, culture, and language of learners in creating mathematical tasks. A task that draws on students' prior knowledge and experience allows them to strongly engage with the task, as they feel more connected. Lastly, mathematics is too abstract (Skovsmose, 2008), making it challenging for learners. Hence, the teacher should regularly encourage students to engage with activities and make use of multiple approaches and strategies to make sense of the problems and eventually arrive at the solutions.

2.10.3.3 Using correct mathematical representations

The use of various mathematical representations constitutes effective mathematical teaching (Mann, 2014). When students learn to represent, discuss, and make connections among mathematical ideas in multiple ways, they demonstrate a deeper mathematical understanding. Tripathi (2008) noted that using various representations is like examining the concept through various lenses, with each lens providing a different view and making the concept richer and deeper.

Teachers can promote the use of visual models by:

- introducing learners to forms of visual representations that can be useful to them,
- selecting tasks that allow students to make use of visual representations, and
- regularly encouraging students to write, make maths drawings or other visual representations to explain and justify their reasoning.

Mann (2014) also highlighted that students, especially young learners, also benefit from using physical objects or acting out processes during problem solving. It is however argued by Hiebert and Wearner (1992) that the use of visual models does not guarantee learning. Hence, teachers need to focus students' attention on the essential feature of the mathematical ideas depicted by the visual model.

2.10.3.4 Facilitating meaningful mathematical discourse

Mathematical discourse involves the purposeful exchange of ideas through classroom discussions, as well as through other forms of verbal, visual and written communication (Mann, 2014). The discourse in the mathematical lesson allows students to share ideas, clarify understanding, construct convincing arguments regarding why and how things work and develop a language for expressing mathematical ideas.

Teachers can facilitate meaningful mathematical discourse by engaging students in classroom discussions, building on their thinking. Even though discourse provides opportunities for students to learn what mathematics is and how it is done, developing that culture in the classroom is a challenging one. To this, Engle and Conant (2002) advised that teachers should determine how to build on and honour student thinking, while ensuring that the mathematical ideas at the heart of the lesson remain prominent in the classroom.

To coordinate a class discourse, the teacher can select students to present their mathematical work, sequence students' responses in a specific order for discussion, connect different students' responses, connect the responses to key mathematical ideas, use probing questions or provide clues to make connections among mathematical ideas (Mann, 2014). This helps to avoid an elaborate show-and-tell session, a situation described by Wood and Turner-Vorbeck (2001). Still on classroom discussion, Mann (2014) also highlighted that students should be accorded more opportunity to talk with, respond to and question one another in ways that support the mathematical learning of all students in class.

2.10.3.5 Posing purposeful questions

Effective mathematical teaching relies on questions that encourage learners to explain and reflect on their thinking. Posing purposeful questions allows a teacher to know what students know, prompts them to make mathematical connections, lets students explain and justify their reasoning and in return, develops the culture of classroom dialogue.

In posing purposeful questions, two critical issues should be considered: the type of question and the pattern of the questions. Mann (2014) highlighted four types of common questions in the classroom. They are:

- gathering information questions,
- probing thinking questions,
- making the mathematical visible questions, and
- encouraging reflection and justification questions.

Many scholars have researched classroom interaction. For instance, Mehan (1979) coined the term Initiation Response Evaluation (IRE). This pattern of asking questions involves the teacher asking a question to gather information, generally with the answer in mind; learners respond and the teacher evaluates the response. In the IRE questioning approach, learners respond without reasoning. Thus, IRE is widely criticised because learners are not given enough time to think about and reflect on their answers.

The other two forms of questioning are funnelling and focusing (Herbel-Eisenmann & Breyfogle, 2005). A funnel pattern of questioning involves a teacher asking a series of questions that direct the learners to the desired procedure or conclusion, while the focus pattern of questioning involves the teacher attending to what students are thinking, pressing them to communicate their thoughts and reflecting on the responses of their classmates.

The above highlights that the teacher can pose purposeful questions by using a variety of question types to assess the students' progress, gather evidence of students' thinking and ask students to reflect on and justify their thinking. The teacher may also use patterns of questions that focus student attention and thinking on the concept and extend students' current ideas to advance them (from point A to B). In addition, the teacher may also ask intentional questions that provide clues to students to examine and discuss the problem. Giving students sufficient time to formulate and give responses to questions is one of the fundamental aspects in posing purposeful questions.

2.10.3.6 Building procedural fluency from conceptual understanding

The development of conceptual and procedural fluency is vital in the effectiveness of teaching mathematics. Conceptual understanding means having an epistemological understanding of the underlying concept, while fluency means being able to choose flexibly among methods and strategies to solve a mathematical problem, while being able to explain your approach (Mann, 2014).

The two concepts conceptual understanding and procedural fluency are interrelated. Procedural fluency follows and builds on the foundation of conceptual understanding. When procedures are connected with the underlying concepts, students can remember the procedures and can apply them in new situations (Fuson et al., 2005).

To build procedural fluency and conceptual understanding, the teacher should ensure that learners are provided with opportunities to practise procedures and use their reasoning and strategies to solve the mathematical problems. The teacher should also ask students to explain their procedures and strategies, refine learner-generated strategies to more efficient procedures, and use visual models to support students' understanding of general methods (Mann, 2014).

Ramirez et al. (2013) have, however, cautioned that a rush in fluency by giving learners a problem to practise too soon undermines their confidence and interest in mathematics, thus contributing to a negative mathematical disposition. To this, the author suggested that students need to practise on a moderated number of carefully selected problems after they have established a strong conceptual understanding of a mathematical concept. The authors further suggest that practice should only occur after learners are able to explain and justify their reasoning strategies.

2.10.3.7 Supporting a productive struggle in learning mathematics

The focus on learners' struggles is a necessary component in the teaching of mathematics with understanding to struggling learners. Teaching that embraces and uses productive struggle leads to long-term benefits, with learners more able to apply their learning to new problem situations (Fuson et al., 2005).

Productive struggling is measured when a learner is making progress toward sense making, explaining, or proceeding with the task regardless of not getting the correct solution (Warshauer, 2011). This is in contrast to teachers who sometimes view their learners' frustration or lack of immediate success as an indicator that they have somehow failed their learners. Hence, they jump in to rescue them by simplifying the task or providing guidance step by step. To this, Mann (2014) views such rescue as undermining the effort of the learners, lowering their cognitive demand of the task and depriving them of fully engaging with the task.

To support productive learning in mathematics, Mann (2014) suggests that the teacher needs to anticipate what students might struggle with and prepare how best to support them productively. The teacher should give students sufficient time to struggle with the task and ask questions that scaffold their thinking, without stepping in to do the work for them. The teacher may also make students understand that confusion and errors are natural parts of learning by facilitating discussions on mistakes, misconceptions and struggles. Praising students for their efforts in making sense of the mathematical ideas and their persistence in reasoning through problems is also another way of supporting struggling learners productively.

2.10.3.8 Eliciting and using evidence of student thinking

Effective teaching of mathematics involves eliciting evidence of students' current mathematical understanding and then, as a teacher, using it to influence your instructional decision. In other words, eliciting evidence of students' thinking should be regarded as an essential component of formative assessment by every teacher. Many a time, teachers plan their lessons and teach according to that plan, without considering learners' thinking. In addition, in most cases they teach and wait for Friday or until the topic is completed, then give a task or test to assess whether students are making progress, which is too late. To this, Heritage (2008) suggest that each lesson needs to include intentional and systematic plans to elicit evidence that will show how students' learning is evolving toward the desired goal.

To elicit and use evidence of students' thinking, the teacher needs to know and identify what counts as evidence of students' progress in learning. Warshauer (2011) argues that progress in learning does not mean getting the correct answer but rather progress in sense making, reasoning, explaining and approaching problems, in addition to a growth in the confidence of learners to solve problems. During the lesson presentation, such indicators of progress in learning are when learners nod their heads and make sounds of agreement such as "eheee", "okay" and expressions of joy.

Regarding thinking, Mann (2014) points out that the teacher needs to elicit and gather evidence of students' understanding at a strategic point during instruction, interpret students' thinking to assess their mathematical understanding, reasoning and methods, and decide on the best approach to respond to students with probing and scaffolding questions. In addition, the teacher may use tasks that require students to discuss, present and justify their mathematical understanding. The teacher might also have students talk with their partners before the whole class discussion. When individual work is given, the teacher could ask learners to compare their answers, and then have the learners (particularly those with different solutions) discuss them with the whole class.

These eight pedagogical practices mentioned above, together with visual models as advocated by this study are central in growing learners' productive dispositions. The purpose of my study is thus to research how the combination of visualization knowledge with the above pedagogical practices may have a positive impact on the learners' mathematical dispositions.

2.11 THEORETICAL FRAMEWORK

2.11.1 Constructivism

Constructivism is a theory that is based on observation and scientific studies about how people might acquire knowledge and learn (Sarita, 2017). It has its historical roots in the work of a well-known French-Swiss developmental psychologist Jean Piaget, who advocated the idea of how individuals (learners) construct knowledge (cognitive constructivism) and Lev Vygotsky, a Russian psychologist who wrote about how children construct knowledge in a social sphere and how it is influenced by their culture and mediated by tools (Powell & Kalina, 2009).

As advocated by both Piaget and Vygotsky, constructivism is a teaching and learning approach based on the premise that learning is the result of mental construction (Sarita, 2017). In other words, students learn by comparing and contrasting new information against what they already know. Constructivism views learners as active agent and not passive receivers of knowledge in the process of knowledge construction (Liu & Chen, 2010). In other words, as learners confront their understanding in the light of the new information they encounter in their learning situation, they remain active throughout the process. They construct their knowledge rather than passively receiving it from the teacher. Phillips (1995) explained that this active process is characterised by applying current understanding, noting relevant elements in new learning experiences, judging the consistency of prior and emerging knowledge and modifying knowledge based on their judgements.

Constructivism theory further acknowledges that learners do not come to learning situations empty-headed or *tabula rasa* as referred to by Gupta (2017). They come with prior knowledge gained from their environment (unstructured knowledge) or their previous grades (structured knowledge) (Vygotsky, 1978). This existing knowledge serves as a foundation and influences the interpretation of new knowledge which learners construct from their learning experiences (Adams, 2006; Watson, 2001). When learners encounter new information, they enter a state of disequilibrium or cognitive conflict as they try to make sense of the new information. This happens because the new information does not fit into their schemas (thinking). Hence, they make sense of the new data by reconciling it with their existing knowledge by either changing their thinking to accommodate new knowledge or discarding the new information as irrelevant (Sarita, 2017).

Social constructivism as championed by Vygotsky (1978), extended the concept of constructivism by augmenting it with social interaction, culture and mediation tools as integral parts of learning. This study will only focus on the concept of social interaction and mediation.

Vygotsky (1978), believed that knowledge is constructed through social interaction with peers or with more knowledgeable others. More knowledgeable others in this context, refers to teachers and other learners who have greater knowledge and skills than others. According to Hurst et al. (2013), learning skills and concepts meaningfully occurs in an integrated context and not in an isolated and hierarchical manner. This contradicts the notion of perceiving learners as passive receivers of information from a teacher who is on top of the hierarchical structure in terms of learning.

Vygotsky (1978) observed that children pay greater attention to what their peers (friends and classmates) know or are doing than they do to adults. Thus, through social interactions, they freely raise their questions, challenge old knowledge and address misconceptions (Powell & Kalina, 2009). In a social sphere, Vygotsky (1978) argues that people's cognitive development occurs serially in two different phases: firstly, through social interactions on the social plane and secondly as an inner, internalised form. In other words, social interaction allows the transition of functions from the inter-personal plane to the intra-personal plane.

Vygotsky (1978), argued that the development of the child's higher mental processes depends on the mediation agents in the child's interaction with the environment. Mediation is the interaction between the teacher and learners that directly affects learners' understanding of mathematical concepts. Mediational tools are either abstract or concrete, used in cognitive change. Vygotsky (1978) proposed three categories of mediation: mediation through human beings, symbolic tools and mediation in the form of organised learning activities.

Various authors have dwelt much on humans as mediators of learning. Kozulin (2003) has acknowledged the importance of mediation. He stated that children who receive more mediation through interaction become better learners. He is however concerned about the poor understanding of mediation among teachers and parents. This is evident in Lehrer and Shumow's (1997) study which compared parents' and teachers' assistance to children solving mathematical problems. Their study's findings revealed that parents assist their children by directly telling them what to do, which is contrary to teachers' approach. Generally, teachers try to help learners to make sense of the problem and then encourage them to solve the problem independently. To bridge the gap between the teachers and parents' approach, these authors reported on the need for teacher-parent training on general types of mediation.

On mediation approaches, Tharp and Gillimore (1988) have highlighted that teachers can mediate learning by modelling, praise, critique, giving feedback and providing strategies for the organisation of students' work. In another study, Palinscar and Brown (1984) proposed a reciprocal teaching framework to develop literacy. In this framework, teachers and students took turns reading and discussing the text. Through this approach, a teacher aims to develop students' questioning, summarising and predicting skills and clarifying strategies. Bliss et al.,'s (1996) study on scaffolding in science teaching highlighted that approval, encouragement, structuration and organisation of students' work are all components of mediation.

Regarding symbolic mediation, Vygotsky (1978, p. 127), mentioned casting lots, tying knots and counting fingers as symbolic mediators. Casting lots occurs in a situation where doubt is caused by two equally opposing situations that cannot be solved naturally (Kozulin, 2003), but rather by the application of artificial tools, such as rolling a die or tossing a coin for example. Tying knots refers to the use of learning devices – visual models that aid information retention or retrieval in human memory. Finger counting shows how physical objects (fingers) can serve as external symbolic tools that organise cognitive function involved in the elementary arithmetic operation. Symbolic mediation may also include different signs, symbols, formulae and graphic organisers (Kozulin, 2003).

According to Vygotsky, cognitive development and learning essentially depend on the child's mastery of symbolic mediators. Kozulin (2003) raised a concern that some teachers fail to help students to find the essence or the instrumental part of the mediation tool. To this, he proposed that the use of mediational tools requires a different learning paradigm. Mediational tools should portray a deliberate action (Kozulin, 2003). In other words, its intention should be appropriated and internalised by learners. Rogoff (1995) coined the concept 'appropriation' which refers to changes which occur in individual learners as a result of partaking in a mediated activity.

The relationship between symbolic and human mediation is worth noting. According to Kozulin (2003), symbols may remain useless unless their meaning as cognitive tools are correctly mediated with the child. Furthermore, a symbolic tool may only fulfil its role if it is appropriated and internalised by the learners.

2.11.2 The constructivist view of teaching

This study intends to explore how visual models can be used to teach mathematics for a productive disposition. In other words, I am exploring how a teacher can teach Grade 9 learners using visual models in order to bring about a positive shift in Grade 9 learners' attitudes toward mathematics. To do this, I required theoretical and methodological lenses congruent with research (Wittrock, 1977). I thus used the constructivism theory as a lens to explore firstly how learners learn through social interactions and secondly how their learning is mediated by visual models, in order to bring about a positive shift in the Grade 9 learners' attitudes toward mathematics.

The constructivism theory has assured us that everyone can learn provided that they are given enriching activities that induce thinking. With the assistance of the teacher, learners engage with these activities and the teacher eventually withdraws assistance as learners gradually begin to work on their own. This signifies the need for visual models for learners to construct meaningful knowledge and to further aid information retention.

From a constructivist point of view, children learn more and have more enjoyment in learning when they are actively involved rather than passively listening to the teacher (Sarita, 2017). Enjoyment, active involvement and participation in mathematics lessons are viewed as important variables for predicting positive mathematical dispositions (Graven, 2012; Lee & Chen, 2010). However, learners tend to develop a negative disposition towards mathematics when they are not actively involved in the learning process or given appropriate access to a mathematics sense-making environment (Graven & Heyd-Metzuyanim (2014). To this, Venkat and Graven (2008) have postulated that learners develop entirely different stories of their mathematical dispositions when given the opportunity for active participation, meaningful learners' engagement and sense making.

2.12 CONCLUSION

This chapter was aimed at discussing that literature that informed my study. I began the chapter with the concept of visualisation and its role in teaching and learning mathematics. Indicators of productive disposition were also highlighted. Lastly, I discussed the theoretical framework of this study.

CHAPTER THREE: RESEARCH DESIGN AND METHODOLOGY

3.1 INTRODUCTION

This study aimed to investigate how visual models can be used to teach mathematics to grow selected learners' mathematical productive disposition. To achieve this goal, I used different research techniques to collect data and interpret these data, with insight from the literature and the theoretical framework discussed in the previous chapter. In this chapter, I briefly discuss the research paradigm which guided my research process. Thereafter, I discuss the research design, research participants, data collection tools and analysis of the research project. The validity and ethical issues that were considered in this study are also discussed.

3.2 RESEARCH ORIENTATION

Researchers approach problems from different worldviews. My study is underpinned by an interpretive paradigm. 'Paradigm' is Kuhn Thoma's (1962) denotes one or more mind-sets. Kuhn (1962) defined a paradigm as a way of looking at or researching phenomena, a world view, a view of what counts as accepted or correct scientific knowledge or way of working, or an accepted model or pattern. In simple terms, Mertens (2012) describes a paradigm as a set of beliefs, or lenses through which one views and understands the world.

Different paradigms contain different ontological, epistemological, and methodological views, hence each paradigm has different assumptions about reality and knowledge. The ontological position of interpretivism holds that reality is subjective and differs from person to person (Guba & Lincoln, 1994). These authors further stated that reality emerges when people consciously engage with objects – visual models in this context – which are already pregnant with meaning. Interpretivism views reality as individually constructed, hence there are as many realities as individuals. Despite many individuals' realities, the truth is however the consensus formed by the co-constructors (Pring, 2000).

The research approach of interpretivism therefore provided me with lenses to explore how visual models can be used in teaching mathematics for growing Grade 9 learners' productive dispositions. To be specific, this approach allowed me to make sense of the teachers' lessons. I needed to understand how both the participating teachers and learners individually understood visual models, how visual models helped individuals (teachers and learners) to understand mathematical concepts and how learners’ understanding of mathematics concepts as a result of visual models influences their dispositions toward mathematics.

The interpretive paradigm is inductive (Scotland, 2012). In another words, meaning is generated from data. Also, an interpretivist methodology is directed at understanding phenomena from an individual perspective through interaction between the researcher and participants (Guba & Lincoln, 1994). Thus, this approach further provided me with a methodological framework: the data collection tool used, how I designed the data collection tools and how I derived meaning from the data. It is worth noting that paradigms – interpretive in this context – exist to clarify and organise the thinking of researchers (Cohen et al., 2018).

3.3 MIXED-METHOD CASE STUDY

When we look at phenomena, do we put on our qualitative hats or quantitative hats? In viewing our world, we use all means and data at our disposal to understand a situation (Cohen et al., 2018) and we more naturally integrate rather than separate. This study thus adopted a mixed-method case study approach.

The case study has been defined differently by many scholars. But for this study, I adopted the definition of Tight (2010), who defined a case study as a detailed examination of a small sample and the in-depth investigation of a specific phenomenon from multiple perspectives, to catch its complexity and uniqueness. In the context of this study, my case is visual models and how they are used in teaching mathematics. My unit of analysis is how this teaching affected selected Grade 9 learners' mathematical disposition.

A mixed-method approach incorporates both qualitative and quantitative methods of data collection and analysis in a single study (Cohen et al., 2018). This approach allowed me to understand complex phenomena qualitatively and to explain the phenomena through numbers, charts and basic statistical analysis.

Several scholars have spoken widely about the potential of the mixed-method approach. Cohen et al., (2018) acknowledge that the world is mixed. It is not exclusively quantitative nor qualitative. These authors thus encourage researchers not to look at the world with a single lens but rather to integrate multiple views of making sense of the world. They further argue that bringing quantitative and qualitative data together in a single study gives a greater understanding of the problem in question, than either a quantitative or qualitative approach on its own provides. In my view, using only one kind of approach may not do justice to the issue in question.

On the same note, Greene (2008) acknowledges that there are many legitimate approaches to research. He further encourages the use of multiple approaches to research because a single approach on its own will only yield a partial understanding of the phenomenon being investigated. In support, Creswell and Plano Clark (2011) emphasise that multiple approaches to research increase the usefulness and credibility of the results found, allowing unexpected results to be found. The mixed-method approach provides a more complete picture of the phenomenon under study than would be yielded by a single approach, thereby overcoming the weaknesses and biases of a single approach (Denscombe, 2014). Denscombe further added that a mixed-method approach increases the accuracy of data and reliability through triangulation, reducing bias in the research and enabling comparison between strengths and weaknesses of the research strategies.

In this study, quantitative data were generated using pre- and post-questionnaires, while qualitative data were generated through lesson observations and interviews. Both the quantitative and qualitative data were interactively analysed. In other words, they were analysed in a mix, depending on how one type of data influenced the other.

3.4 RESEARCH GOALS AND QUESTIONS

3.4.1 Research goal

The main goal of this study was to explore how visual models can be used to teach mathematics to grow selected learners' mathematical productive dispositions.

3.4.2 Research questions

In the context of an intervention programme that foregrounds the use of visual models in the teaching of mathematics, this study is guided by the following research questions:

1. What is the nature of selected Grade 9 learners' dispositions towards mathematics prior to an intervention programme?
2. How do selected teachers teach for productive disposition as a result of participating in an intervention programme?

3. What are the selected learners' mathematical dispositions after their teachers participated in an intervention programme?

3.5 RESEARCH DESIGN

A research design is a plan drawn up for organising the research and making it practicable to answer the research question (Cohen et al., 2018). This study was conducted in four phases:

Phase 1: Disposition questionnaire

In this phase, a pre-disposition questionnaire was administered to the two selected teachers of Grade 9, learners who gave assent to participate in the study. The study was aimed at exploring how visual models can be used to teach mathematics to grow Grade 9 learners' mathematical dispositions. Thus, the pre-disposition questionnaire was administered to establish learners' mathematical dispositions before the intervention programme. Having established learners' dispositions before the intervention did not only provide me with concrete evidence of the nature of Grade 9 learners' mathematical dispositions but also enabled me to assess the shift in their dispositions.

Phase 2: Planning of the intervention programme

In this phase, I and the two selected Grade 9 teachers came together and planned how we would carry out the intervention programme. The planning involved identifying five topics from the Namibian Grade 9 syllabus that would be taught during the intervention programme. We made decisions based on two pieces of advice: firstly, by Norton (2009) who argues for the active participation of the participants in selecting the topics to be taught; and secondly by Boaler et al. (2016) who advise that there are no mathematical ideas/concepts which cannot be illustrated visually. Before topics were identified, I conducted a workshop to introduce the teachers to the concept of visual models, the types of visual models and how they can be used to teach mathematics. The topics chosen are indicated in Table 3.1 below.

Table 3.51: The five selected topics for the intervention programme.

Chosen topics	Suggested visual models
1. Directed numbers	Area models, number lines, bottle caps
2. Common and decimal fractions	Area models
3. Comparing, ordering and estimation	Area models, Set models
4. Algebra: removing brackets & solving equations	Algebraic tiles
5. Transformation	Algebraic tiles

Phase 3: Intervention (teaching programme), and disposition questionnaire

The intervention programme consisted of five cycles spread over 12 weeks (see Figure 3.1). Each cycle was characterised by a mathematical topic from the Grade 9 syllabus. The two selected teachers and I developed lesson plans for the agreed topics and implemented three lessons per cycle, of which I video recorded one per teacher. These lessons were carefully planned and designed with activities that make use of visual models. Since these lessons required more time, they were carried out when the teacher had double periods or during the last periods of the day. The video-recorded lessons formed the basis of my observation and the individual teachers chose which video-recorded lessons would be used for this thesis. In each cycle, the video-recorded lessons were analysed through a stimulus-recall interview with each of the two teachers, where we collaboratively analysed the lessons as per the analytical framework.

Further, at the end of each cycle, I administered a disposition questionnaire (see Appendix A: Pre- and post-disposition questionnaire.) to each of the participating learners. This questionnaire was aimed at assessing Grade 9 learners' mathematical dispositions immediately after each cycle. For ethical issues, learners were requested to use pseudonyms instead of their real names. However, to keep track record of each learner's mathematical disposition, I created a class list for each class. Before administering the disposition questionnaire, I read through the class list to remind them of their pseudonyms, in case if they had forgotten.

At the end of each cycle, the teacher and I met and planned for the next cycle. The cycles outlined above are somewhat reminiscent of the classical action research cycles described by McNiff and Whitehead (2011), as illustrated in Figure 3.1 below.

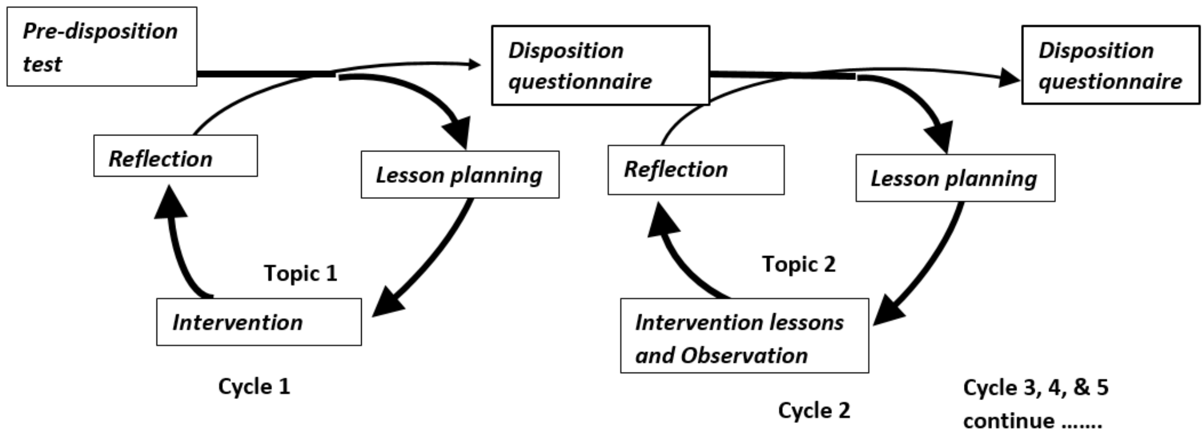


Figure 3.1: The action research lesson cycles

Phase 4: Post-disposition questionnaire and interview

At the end of the fifth cycle, I administered the post-disposition questionnaire to all the participating learners. This questionnaire was aimed at establishing Grade 9 learners' mathematical dispositions at the end of the intervention. I further conducted a one-on-one stimulated recall interview with each teacher at the end of each lesson. The purpose of this interview was to get the teachers' reflections on the lesson. This was further followed by a focus group interview (Cohen et al., 2011) with both the participating teachers to reflect on the entire intervention process. The purpose of the reflection was to establish whether it was worth participating in the study, what they learned, how the study impacted their teaching approaches and other general views about the study.

3.6 RESEARCH SITE AND PARTICIPANTS

This study took place at one school, Ounonge Combined School in the Ohangwena region, located in the northern part of Namibia. The real names for both the school and teachers in this study are not revealed to maintain anonymity and privacy. Pseudonyms are thus used for the teachers and the school. This school accommodates learners from the Pre-primary to Grade 11, with an entire population of 489 learners.

This study focused on two secondary school teachers: Ms. Moshana (T1) and Mr. Kondja (T2), both Grade 9 teachers at Ounonge Combined School (pseudonyms). Grade 9 is split into two groups, Grade 9A and Grade 9B. Ms. Moshana teaches Grade 7, Grade 9A and Grade 11, while Mr. Kondja teaches Grade 8, Grade 9B and Grade 10. Mr. Kondja is a highly experienced teacher with 17 years of mathematics teaching experience, while Ms. Moshana has ten years of mathematics teaching experience. The teachers' profiles are summarised in the table below.

Table 3.2: Participating teachers' profiles.

Teachers' name (pseudonyms)	Codes	School	Grade taught (mathematics)	Qualification	Working experience
1. Mrs. Moshana	T1	Ounonge CS	8 A, 8B, 9A, 11A & 11B	Grade 12 and BEd Hons	10 years
2. Mr. Kondja	T2	Ounonge CS	9B, 10A, & 10B	Grade 12, BETD, and BEd Hons	17 years

This set of two teachers was purposively selected. Purposive sampling is a strategy in which particular participants are selected deliberately to carry out important tasks that cannot be carried out in any other way (Taherdoost, 2016). The two teachers were chosen on the basis that they both teach Grade 9 mathematics classes. Also, the two teachers and I, on the basis of our experiences with these two classes, agreed that the learners in these classes generally were not very motivated and were negative about mathematics. Further, Cohen et al. (2018) caution that factors such as expense, time and accessibility frequently prevent researchers from gaining information from all the participants. Considering this advice, I selected these two teachers because they were accessible and very eager to participate. They also said they were energetic and motivated to be part of the study.

My second set of participants were the Grade 9 learners of the two selected teachers. These classes have a combined population of 70 learners. They all provided their assent to participate in the study.

3.7 DATA COLLECTION TOOLS

Selecting data collection tools is not a matter of preference, but a deliberate process in which the choice is based on the notion of fitness for purpose (Cohen et al., 2018). Thus, this study employed three data collection tools which were deemed fit for the purpose of collecting data for this research. These are disposition questionnaires, observations and interviews.

3.7.1 Disposition questionnaires

In this study, a disposition questionnaire was administered (see Appendix A: Pre- and post-disposition questionnaire.). A pre-disposition questionnaire was administered prior to this intervention study, while the post-disposition questionnaire was administered after the intervention programme. The purpose was to establish the nature of the selected Grade 9 learners' mathematical dispositions before and after the intervention programme. The individual questions for both the page-and-a-half long pre- and post-disposition questionnaires were the same.

Denscombe (2014) cautioned that it is important to avoid putting too much strain on the respondents of a questionnaire such as limiting the time to respond. I was also mindful that the language of the questionnaire was not in the learners' home language. I therefore read out each question and waited for the learners to tick their responses before proceeding to the next question. Although the individual questions of the two questionnaires remained the same, before I administered the post-questionnaire, I changed the order of the questions to avoid issues of predictability.

3.7.2 Observation

My second data collection tool was observations. Observations provide the researcher with first-hand information (Bertram & Christiansen, 2014). Furthermore, it also allows a researcher to collect live data in naturally occurring social settings (Cohen et al., 2011). I generated first-hand data by video recording ten lessons (one lesson per teacher per cycle). The focus of the observations and analysis of each lesson was on how the participating teachers made use of visual models in their teaching. The observation schedule I used (see Appendix B: Classroom observation and analytical tool) consisted of Mann's (2014) eight teaching practices (see Section 2.10.3).

3.7.3 Interviews

I used two kinds of interviews. Firstly, I conducted the one-on-one stimulus-recall interviews with each teacher after each video recorded lesson in each cycle. The purpose of this interview was to obtain individual reflections on the use of visual models and how they possibly impacted learners' productive dispositions. We also discussed what could possibly be done to refine the teaching approach in the next lessons.

Secondly, I conducted a focus group interview at the end of each cycle with both teachers together. A focus group interview is described by Denscombe (2014) as a small group of people who are brought together by a researcher to explore attitudes, feelings and ideas about a specific topic. The purpose of this interview was to obtain a collective reflection about the lessons in general, the use of visual models and how this intervention impacted on the learners' dispositions.

3.8 DATA ANALYSIS

As highlighted earlier, this study generated three sets of data. The first set was quantitative data from the pre- and post-questionnaire, the second set was qualitative data from lesson observations and the last set was qualitative data from the interviews (one-on-one stimulus-recall interviews). Quantitative data generated from the pre- and post-questionnaires were quantitatively analysed as per the disposition constructs discussed in Section 2.10.2 (see Appendix A: Pre- and post-disposition questionnaire.). To determine learners' dispositions prior to the intervention programme and a shift of disposition (if any), descriptive statistics such as tables and graphs were used to illustrate and mediate the meaning from this data.

The process of analysing qualitative data from the lesson observations began with transcribing the video recorded lessons. The transcriptions were then carefully analysed to identify the indicators of Mann's (2014) eight teaching practices criteria for developing and growing a productive disposition (see Appendix B: Classroom observation and analytical tool). The occurrence of these indicators was then presented on a frequency table and discussed according to the eight teaching practice criteria. Qualitative data from interviews were used to validate the qualitative data from the lesson observation.

This study generated a huge amount of data, resulting in lengthy presentations and analyses. It was my initial intention to report on the two teachers, Ms. Moshana and Mr. Kondja. Due to the limited scope and word count of the thesis however, the data analysis in Chapters Four, Five, Six and Seven will focus only on one teacher, Mr. Kondja. The answer to my research questions will thus only be derived from data collected from Mr. Kondja's lessons and his learners. The reasons for choosing Mr Kondja was that he had more teaching experience than Ms Moshana.

3.9 VALIDITY

Validity is the way in which people develop trust in the findings of a study (Du Plooy-Cillier et al., 2014). In other words, if a similar study is conducted, the findings should be more or less similar. To ensure validity, my data collection instruments were piloted beforehand. Piloting data collection instruments enabled me to test whether they are generating appropriate data to answer the research questions and guided me on how to amend and refine my instruments. Further, I made use of multiple data collection tools such as questionnaires, observations and interviews to enable me to triangulate the data. I also gave the interview transcripts back to the two teachers to ensure that these reflected exactly what they had said, or wanted to say.

3.10 ETHICAL CONSIDERATIONS

3.10.1 Respect and dignity

Respect for participants' rights was a priority in this study. Before the participants signed consent letters, I informed them that their participation in this study was voluntary and they could withdraw at any time. I was mindful and respectful of the teachers' chosen topics for the intervention programme and their specific lessons they wanted video recorded. The participants were informed that their lesson presentations would be video recorded, and they were asked to give consent for this. Participants were also assured of their privacy and anonymity and that information generated in this study would not be disclosed to third parties without their consent. Since this thesis will be accessed by the public, I assured my participants that their participation would not compromise their dignity in any way.

3.10.2 Transparency and honesty

This study upholds transparency and honesty by first obtaining permission to conduct the study from the directorate of education in the Ohangwena region and the school principal (see Appendix D: LETTER OF CONSENT TO THE REGIONAL DIRECTOR. Before participants gave their consent to participate in the study, the aim and the process of the study were carefully explained to them. At the same time, participants were also encouraged to give their honest opinions and experiences throughout the process of the study.

3.10.3 Accountability and responsibility

In this study, I exercised full accountability and responsibility for all data collected, by keeping them in safe documents both on my laptop and external hard drive to avoid any loss or misplacement of data. Questionnaires were stapled together and stored in a lockable cupboard. As a Head of the Department and supervisor of the participating teachers, I am aware that my authority might influence their participation in one way or another. To overcome this, a comprehensive plan of my study and purpose was explicitly given to the participants at the beginning of the study. The school has an existing culture of peer co-teaching where teachers of the same subject jointly present lessons together, to learn from one another. This practice minimised any fear which might have arisen due to my presence in their lesson presentation.

3.10.4 Integrity and academic professionalism

In this study, I demonstrated a commitment to integrity and academic professionalism by obtaining ethical clearance from Rhodes University's ethical committee to conduct my study. I explained my study to the participants clearly and honestly. Before they took part in the study, I obtained informed consent from all the participants while at the same time guaranteeing them the protection of their privacy.

Throughout the study, I have used peoples' work and ideas that I have acknowledged and referenced according to the Rhodes University referencing guide. Data were presented as collected without any distortion or fabrication to suit my assumptions. I thus declare that the final work of this thesis is my own, and it is not a reprint or published elsewhere.

3.11 CONCLUSION

In this chapter, I described the research orientation. I elaborated further on the methodology – namely a mixed-method case study – research design and selection of the participants. I also explained how data were collected and analysed. I then concluded the chapter by elaborating on possible ethical issues.

In the next chapter, I present, analyse and discuss the findings from the pre-disposition questionnaire.

CHAPTER FOUR: PRESENTATION AND ANALYSIS OF PRE-DISPOSITION QUESTIONNAIRE

4.1 INTRODUCTION

This chapter presents, analyses, and discusses the results of data collected from the pre-disposition questionnaire. These data were aimed at answering my Research Question 1: *What is the nature of selected Grade 9 learners' disposition towards mathematics prior to an intervention programme?* I begin the chapter by briefly explaining the nature of the questionnaire, how the results were analysed and I then end with the presentation and analysis of findings according to Siegfried's (2012) constructs for assessing productive disposition. As discussed in Chapter Three, the presentation and analysis of data (Chapter Four to Seven) focus only on Teacher 2, Mr. Kondja. This is due to the huge amount of data and lengthy analysis which was generated.

4.2 PRE-DISPOSITION QUESTIONNAIRE

Before the intervention programme, a pre-disposition questionnaire was administered to all the learners who participated. The purpose of the pre-disposition questionnaire was to establish learners' mathematical dispositions before the intervention programme. The pre-disposition questionnaire (see Appendix A: Pre- and post-disposition questionnaire.) is composed of Siegfried's (2012) six constructs and the emerged construct of 'engaging with tasks' for assessing productive dispositions. These are:

- beliefs about mathematics,
- affect toward mathematics,
- motivation,
- academic risk-taking,
- engagement with tasks,
- mathematic identity, and
- self-efficacy.

Each of these constructs has two to five indicators, and five levels of agreement which are strongly agree, agree, not sure, disagree, and strongly disagree. For the data to be meaningful and easy to understand, the percentage of respondents who agreed (strongly agree and agree) were merged, as was the data for disagree (disagree and strongly disagree). This then brought down the level of agreement to three: agree, not sure and disagree. This section thus presents and discusses the learners' responses per construct.

4.2.1 Beliefs about mathematics

The purpose of this indicator was to establish learners' beliefs about mathematics prior to the intervention programme. After the pre-test was administered, their responses were summarised as shown in the table below.

Table 4.1: Responses of learners' regarding their beliefs about mathematics

Indicators of beliefs about mathematics	Levels of agreement by percentage (%)		
	Agree	Not sure	Disagree
1. I believe that mathematics is a difficult subject	29	33	38
2. I believe that mathematics does not make sense	9	30	61
3. I believe that mathematics is a set of disjoint rules and procedures	20	25	55
4. I believe that mathematics is about real life	69	9	22

The data (see Table 4.1) reveals that prior to the intervention programme, 38% of the respondents indicated that mathematics is not a difficult subject. The rest – 29% of the respondents – indicated that mathematics is difficult, while 33% of the respondents were not sure. It is worth noting is that 61% of the respondents agreed that mathematics makes sense. Having 61% of the respondents indicating that mathematics makes sense and only 38% of the respondents indicating that mathematics is easy, might signify that some of the 61% of the respondents work hard to make sense of mathematics. In addition, the data further revealed that 69% of the respondents believed that mathematics is about real life, while 22% believed that mathematics is not about real life. This may suggest that the 62% (sum of agree and not sure) in Indicator 1 and the 45% (sum of agree and not sure) in Indicator 3 might refer to some of the learners who struggle with abstract mathematics as described by Skovsmose (2008). For example, they might not have been exposed to appropriate visual representations, which would have helped them advance their understanding of mathematical concepts and eventually help them make sense of mathematics. In my view, a learner with a superficial understanding of mathematical concepts may easily conclude that mathematics does not make sense and that it is a set of disjointed rules and procedures (Kilpatrick et al., 2001). The data further revealed that prior to the intervention programme, an average of 40% of the respondents had negative beliefs about mathematics. This is evident in Indicators 1, 2, and 3. Despite this, Furinghetti and Pehkonen (2002) argued that learners' beliefs are continuously changing because they are usually under continuous evaluation in relation to their new experiences.

4.2.2 Affect toward mathematics

The purpose of this construct was to determine learners' emotions, feelings, and attitudes toward mathematics before the intervention programme. The results of the pre-test are shown in the table below.

Table 4.2: Learners' responses regarding their affects towards mathematics.

Indicators of affect toward mathematics	Levels of agreement by percentage (%)		
	Agree	Not sure	Disagree
1. I enjoy mathematics lessons	58	19	23
2. I feel confused and frustrated in mathematics lessons	33	18	39
3. My experience with mathematics makes me hate mathematics	27	31	42
4. Mathematics is useful in our daily lives	64	14	12

The in Table 4.2) reveals that 58% of the respondents enjoy mathematics lessons while 23% of the respondents do not enjoy mathematics lessons. Enjoyment of mathematics lessons is one of the fundamental signs of a mathematical productive disposition. When learners begin to enjoy mathematics lessons, their curiosity and interest grows to an extent where they may develop good habits and attitudes. Among the 23% of the respondents who do not enjoy mathematics, 10% of the respondents strongly disagreed, suggesting that they do not enjoy mathematics lessons. Considering the 19% neutral respondents (see Table 4.2), these figures are worrying, mainly because in the Namibian curriculum mathematics has become a compulsory subject from Grades 1 to 11, and if one-fifth of the learners do not enjoy mathematics, this constitutes a big cohort of learners with a negative disposition. As learners move from Grade 9 to Grade 11, mathematics becomes more complex and harder. In terms of teaching, if nothing is done about learners' dispositions, the number who do not enjoy mathematics might increase.

The data further shows that only 39% are comfortable in mathematical lessons while 33% of the respondents said that they feel confused or frustrated in mathematics lessons. The percentage (27%) of respondents whose experiences made them hate mathematics tallies closely with those who feel confused and frustrated in mathematics lessons. In conclusion, these data show that before the intervention programme, many of the participating learners have negative feelings about mathematics. However, an encouraging observation is that a relatively big portion of these learners are not totally negative. They still have hope of doing well in mathematics and make use of mathematics in their daily lives as reflected in Indicator 4 (see Table 4.2).

4.2.3 Motivation

The purpose of this construct was to determine the learners' motivation to do mathematics. The pre-test results regarding learners' motivation prior to the intervention are shown in the table below.

Table 4.3: Learners' responses regarding motivation

Indicators of motivation	Levels of agreement by percentage (%)		
	Agree	Not sure	Disagree
1. I am eager to learn mathematics	60	15	25
2. I gain satisfaction when I accomplish the mathematical task	53	22	25
3. I enjoy doing mathematical problems	51	21	28
4. One day I would like to do a mathematics-related career	58	21	21

The data in Table 4.3 revealed that 60% of the respondents are eager to learn mathematics while 25% are not. The percentage of learners who are not eager to learn mathematics correlates with the percentage of learners whose experience with mathematics make them hate mathematics (see Table 4.2). This may suggest that 25% of the learners might have had a bad experience with mathematics. Not being eager to learn mathematics is one way that may lead to avoiding the subject. All these indicators suggest an unproductive mathematical disposition. According to Graven and Buytenhuys (2011) avoiding mathematics may not only contribute to poor academic performance, but may also affect learners' self-image. They cited a learner who wrote "my struggle with mathematics left me to feel like I was stupid and useless" (p. 498).

The data further shows that only 53% of the respondents gained satisfaction when they completed mathematical tasks. This figure is worrisome because more than half of all the participants are not motivated to persevere with mathematical problems until they succeed in solving them. It is worth noting however, that half (see Table 4.3) of the learners wish to one day do a mathematics-related career. This serves as a motivational basis to encourage learners to persist with learning mathematics, since it is fundamental knowledge in the 21st century.

4.2.4 Academic risk-taking

The purpose of this construct was to establish learners' willingness to take risks concerning their learning of mathematics.

Table 4.4: Learners' responses regarding academic risk-taking

Indicators of academic risk-taking	Levels of agreement by percentage (%)		
	Agree	Not sure	Disagree
1. I ask questions during mathematics lessons	41	22	37
2. I share strategies and solutions with my classmates	67	12	31
3. I try mathematical problems on my own	64	7	29
4. I try using more than one approach when solving mathematical problems	58	21	21

The data in Table 4.4 reveals that only 41% of the respondents ask questions during mathematics lessons while 37% do not ask questions. These figures may raise questions as to why they do not ask questions. To this, Boaler et al. (2016) recommend that mathematics teaching and learning should become visual, so that learners are actively engaged and ask questions as they seek clarification on concepts and procedures (Arcavi, 2003).

The data further showed that about 60% of the respondents try mathematical problems on their own, and they also share their solutions and strategies with their classmates. However, 30% of the respondents do not attempt mathematical problems on their own nor consult other learners. In their study, Graven and Heyd-Metzuyanim (2014) report on learners with a similar behaviour which they describe as 'dependent ritual behaviour'. These learners are reportedly reluctant to try mathematical problems on their own. When given tasks, they rely on others and the teacher for solutions. To this, Carr and Claxton (2002) suggested that apart from content knowledge, these learners need to be educated on how to develop a positive learning disposition.

Literature suggests that learners who are good at mathematics approach mathematics problems from several points of view and move flexibly from one approach to another until they find the solution (Mann, 2014). The data (see Table 4.4) shows that 58% of the respondents use more than one approach to solve mathematical problems, which is encouraging. Considering that Grade 9 content consists of basic mathematics (not complex), in my view the 42% of the respondents (who answered 'not sure' and 'disagree') who do not use more than one approach or are not sure whether they use more than one approach, is worrisome. This figure may rise as they move to upper grades where mathematics focuses more on application and proving (complex mathematics).

4.2.5 Engagement with tasks

The purpose of this construct was to understand learners' engagement with tasks. The results of the pre-questionnaire are shown in the table below.

Table 4.5: Learners' responses regarding goal setting

Indicators of engagement with tasks	**Levels of agreement in percentage (%)**		
	Agree	Not sure	Disagree
1. I learn from my mistakes	73	7	20
2. I enjoy engaging in challenging tasks	33	36	31
3. Even if the task is challenging, I don't give up. I keep trying	30	37	33

The data in Table 4.5 shows that 73% of the respondents learn from their mistakes. This shows that the majority of these learners view doing mistakes as part of the learning process. Mathematics can be challenging (Skovsmose, 2008) and mistakes are an inevitable part of the learning process. Embracing mistakes as learning opportunities cultivates resilience and perseverance (Kilpatrick et al., 2001). It is through resilience and perseverance that learners realise that steady effort in learning mathematics pays off, making them effective doers of mathematics (Kilpatrick et al., 2001). However, to Graven and Buytenhuys (2011), making errors and mistakes and achieving poor results affect learners' confidence. The consequences of a lack of confidence may thus lead to learners giving up on mathematics or avoiding mathematics. These could thus be some of the attributing factors as to why 20% of the respondents indicated that they do not learn from their mistakes.

On a positive note, the data shows that 33% of the respondents enjoy engaging with challenging tasks and 30% of the respondents showed determination to persist in trying challenging mathematical problems.

4.2.6 Mathematics Identity

The drive for this construct was to determine the meaning that learners attach to themselves in relation to mathematics. The results prior to the intervention programme are shown in the table below.

Table 4.6: Learners' responses regarding mathematics identity

Indicators of mathematics identity	Levels of agreement in percentage (%)		
	Agree	Not sure	Disagree
1. I am good at mathematics	16	41	43
2. My mathematics experience was not good	30	33	37
3. I learn mathematics better when I am with others	77	8	15
4. I can develop strategies for solving mathematical problems	18	35	47

The data in Table 4.6 reveals that 43% of the respondents view themselves as not good in mathematics. This figure is further followed by 41% of the respondents who are not sure of their mathematical identity, in other words, whether they are good or bad at mathematics. From my experience, when learners respond to the question of being good or bad in mathematics, they mostly base their judgement on their abilities to solve basic mathematical problems. However, mathematics can be challenging, with many setbacks. Thus, to Kilpatrick et al. (2001) a learner who is good at mathematics should demonstrate persistence and resilience by persevering through difficulties and setbacks. Therefore, having only 16% of the respondents who view themselves as good at mathematics might be a huge mountain to climb for the rest of the class, particularly as they will soon face the revised content in Grade 10 which I view as relatively hard.

The data further showed that 30% of the respondent's mathematical experiences were not good. A similar study was conducted by Lutovac and Kaasila (2014) on pre-service teachers' mathematical identity. Their study reveals that the teachers' negative mathematical identity was a result of mathematics being boring, taught in a teacher-centred way, they were given little motivation and the subject was abstract. Their findings further revealed that in mathematics, some learners were guided by beliefs such as "I am not meant for mathematics", and that some of their teachers view them negatively. They cited a teacher who hit a learner who did not study. In the context of my study, it is likely that if a focused interview was conducted on 30% of the respondents to establish why their mathematical experiences were not good, they could possibly point out similar experiences to the findings of Lutovac and Kaasila (2014).

The data also showed that 77% of the respondents learn mathematics best when with others. This finding concurs with Vygotsky (1978) who postulates that knowledge is constructed through social interaction. He pointed out that children learn more and enjoy learning more from their peers. The reason for this is that children pay greater attention to what their peers know and do, than they do to adults. To Graven (2012), enjoyment is one important indicator for predicting a productive mathematical disposition.

Lastly, the data shows that 18% of the respondents indicated that they can develop their own strategies for solving mathematical problems. This figure closely tallies with the percentage of respondents who view themselves as good in mathematics (16%). In simple terms, the data shows that prior to the intervention programme, less than 20% of the respondents had a positive mathematical identity, even though about 80% of the respondents acknowledged that they learn mathematics best when interacting with their peers.

4.2.7 Self-efficacy

This construct was aimed at establishing learners' perceptions of their own competence in mathematics prior to the intervention programme. The results of the pre-test are shown in Table 4.7 below.

Table 4.7: Learners' responses to self-efficacy

Indicators of self-efficacy	**Levels of agreement in percentage (%)**		
	Agree	**Not sure**	**Disagree**
1. I can learn mathematics	75	13	12
2. Everyone can learn mathematics	76	10	14
3. I am confident in my mathematical abilities	46	27	27
4. With time, I can accomplish difficult mathematical problems	58	23	19
5. If I put more effort, I can learn and understand mathematics.	81	9	10

The data in Table 4.7 shows that 75% of the respondents believe that they can learn mathematics. This shows that a large portion of these learners have hope and have not yet given up on mathematics. These results tally closely with the respondents (75%) who believe that everyone can learn mathematics. It is worth noting is that 81% of the respondents strongly believe that if they put more effort into their work, they can learn and understand mathematics. This is a positive note about these learners.

However, the percentage of learners who are confident in their mathematical abilities is relatively low, at 46%. Lack of understanding of school mathematics and rare experiences of success in school mathematics affect learners' confidence (Wright et al., 2008). This may suggest that they need a teaching approach which will help them make sense of mathematics, understand concepts, see how mathematical ideas build on one another and accumulate knowledge and skills that will boost their self-confidence and self-efficacy for doing mathematics.

4.3 CONCLUSION

In this section, I presented and discussed the data on the nature of learners' mathematical disposition prior to the intervention programme, as per the Siegfried's (2012) constructs for assessing productive disposition. This data were collected using the questionnaire, and the findings were aimed at answering my Research Question 1.

CHAPTER FIVE: PRESENTATION AND ANALYSIS OF LESSON PRESENTATIONS

5.1 INTRODUCTION

In this chapter, I present, analyse and discuss the results of the lesson observations. One teacher was observed teaching Grade 9 learners, of which five lessons were video recorded. The video recorded lessons were transcribed and coded, as discussed in Section 2.10.3. The data from the lesson observations were used to answer my Research Question 2: *How do selected teachers teach for productive disposition as a result of participating in the intervention programme?* The preliminary sections of the chapter include a brief overview of the five lessons. The findings per lesson are then discussed, focusing on how the participant teacher taught for a productive disposition under the following headings:

- Establishing Mathematical Goals to Focus Learning (EMGFL)
- Implementing Tasks that Promote Reasoning and Problem Solving (ITPRPS)
- Using Visual Models (UVM)
- Facilitating Meaningful Mathematical Discourse (FMMD)
- Posing Purposeful Questions (PPQ)
- Building Procedural Fluency from Conceptual Understanding (BPFCU)
- Supporting Productive Struggle in Learning (SPSL)
- Eliciting and Using Evidence of Learners' Thinking (EUELT)

The teacher's lesson reflection is presented at the end of each lesson.

5.2 LESSON OBSERVATIONS

Initially, I video recorded and transcribed the five lessons taught by the teachers. I then coded the ten transcripts according to my analytical framework discussed in Section 3.8. In this chapter, I now present and discuss my analysis of each lesson according to Mann's (2014) eight teaching practice criteria for developing and growing a productive disposition. The results are presented in chronological order as the lessons were presented.

5.2.1 Overview of Lesson 1 (Addition and subtraction of integers)

This is the first lesson I observed of Mr. Kondja teaching Grade 9B. The lesson was on the addition and subtraction of integers. In brief, when the teacher entered the class, he introduced the lesson, and wrote the topic on the chalkboard. He began engaging learners in the discussion of "what are integers?" The teacher had six prepared examples for demonstration purposes. These were: a) $4 + 3 = ?$; b) $4 + (-3) = ?$; c) $7 - 3 = ?$; d) $-7 - 3 = ?$; e) $8 - 6 = ?$; f) $8 - (-6) = ?$ He gave the first example and asked learners to solve it mentally. From the second example, Mr. Kondja demonstrated how to find the solution using three methods: the arithmetic, number line and bottle caps methods. Before the learners were introduced to the method of using bottle caps, the teacher grouped them and provided each group with bottle caps. The bottle caps were in two colours: red and black. After the teacher demonstrated the six examples, he gave classwork which was done in groups, while he moved around the class monitoring and assisting struggling learners. The lesson was 80 minutes long.

The analytical tool in Appendix B: Classroom observation and analytical tool was used to analyse the observed lesson and the results are presented in the tables below. As discussed in Section 3.7.2, the codes that I used for extracts from the lesson transcriptions are as follows: a teacher or learner's direct extract in Lesson 1: Paragraph 2, Line 5 is coded as L1: P2, L5.

5.2.1.1 Establishing Mathematical Goals to Focus Learning (EMGFL)

Table 5.1: Frequency of EMGFL in Lesson 1

Observable Indicators	Code of the Observable Indicator	Frequency
1. The teacher states the topic and lesson objective and clarifies what learners are expected to learn.	EMGFL, OI1	1
2. The teacher demonstrates a good understanding of mathematical learning goals and how the lesson will unfold.	EMGFL, OI2	11
3. The teacher refers during discussions to lesson goals and objectives during instructions.	EMGFL, OI3	4
4. The teacher uses ordered instructions and tasks that are aimed at achieving the lesson goals and objectives.	EMGFL, OI4	8
5. The teacher assesses and monitors learners' understanding toward the learning goal.	EMGFL OI5	8
Total		**32**

The data in Table 5.2 reveals that all five indicators of EMGFL were observed in this lesson, and they were observed 32 times in total. Indicator 2 (EMGFL, OI2) was observed most. This tells me that Mr. Kondja had a good understanding of the lesson objectives, and he knew how he would steer the lesson to achieve these objectives.

Indicators 4 and 5 (EMGFL, OI4 and 5) were the second most observed. Evidence of ordered instruction was observed when Mr. Kondja demonstrated -4 + 3 on the number line. He said "*Look at this one! From zero, where are we going? ''To the negatives",* said the learners. *"How many steps? " Four steps"*, said the learners. *"From there?" "To the positive*", said the learners. *"How many steps?" "Three steps",* said the learners [L1: P4, L1]. Regarding ordered tasks, the teacher prepared ten examples as shown in the lesson overview, of which six were for demonstration purposes and four were done as a class work. These examples were carefully structured from simple to complex, stimulating the learners' mental processes to progress in the desired direction, by establishing the lesson goals and objectives (Daro et al., 2011).

Indicator 1 (EMGFL, OI1) – stating of the topic – was least observed. I only observed Mr. Kondja introducing the lesson by saying "*Today we will talk about addition and subtraction of* *integers*", and then writing it on the chalkboard. In my view, introducing the lesson and writing the topic on the chalkboard is a good practice because it is an instrumental tool for the teacher to refocus teaching and for the learners to refocus their learning. However, the topic or the lesson goals are supposed to be briefly discussed so that learners will understand and develop a clear understanding of the mathematical purpose of the lesson (Mann, 2014).

5.2.1.2 Implementing Tasks that Promote Reasoning and Problem Solving (ITPRPS)

Table 5.2: Frequency of ITPRPS in Lesson 1

Observable Indicators	Code of the Observable Indicator	Frequency
1. Motivate learning of mathematics by using a variety of problems that build on and extend learners' understanding.	ITPRPS, OI1	8
2. Select tasks that can be solved in various ways, including the use of visuals.	ITPRPS, OI2	9
3. Give tasks on a regular basis that require a high level of cognitive demand.	ITPRPS, OI3	2
4. Support learners in solving tasks without taking over their thinking.	ITPRPS, OI4	2
5. Encourage learners to use various approaches to understand and solve problems.	ITPRPS, OI5	8
Total		**29**

According to my data in Table 5.3, all indicators of ITPRPS were observed in this lesson, of which the predominantly observed indicator was Indicator 2 (ITPRS, OI2). All the tasks used in this lesson were solved using three methods: the arithmetic method, the number line and by using bottle caps.

Indicator 1 and 5 (ITPRS, OI1 and 5) were the second most observed. On encouraging learners to use various approaches, two observations were made. Firstly, the teacher directly instructed learners to use various approaches. This evidence was observed when Mr. Kondja said *"Can I give you the class work now?" "Yes",* said the learners. *"We are only going to use bottle tops"* [L1: P11, L1]. By saying "*we are only going to use bottle tops*", is what I referred to as directly instructing learners. The second observation is what I described as indirect communication, whereby the teacher used the method for demonstrating during the lesson presentation. An example of this evidence was observed when the teacher said "*Now let's look at the meaning of 4+3 in terms of the number line"* [L1: P3, L1]. In this case, the teacher was not only using the number line, but he was also indirectly motivating learners to use it for adding integers.

The least observed indicators were ITPRS, OI3 and 4 – the use of tasks that require a high level of cognitive demand and support for the learners in solving tasks. A unique case was observed in this lesson. Two groups could not get the solution of $8-(-6)$ by using bottle caps. In this case the teacher was observed taking over their thinking by telling them what to do. This tells me that the support given to the learners depended on their prior knowledge. In my opinion, if the learner does not know what is going on, then it is incumbent on the teacher to guide the learner what to do.

5.2.1.3 Use of Visual Models (UVM)

Table 5.3: Frequency of UVM in Lesson 1

Observable Indicators	Code of the Observable Indicator	Frequency
1. Introduce forms of representation (visuals) that can be useful to learners.	UVM, OI1	7
2. Use representation (visuals) to explain, demonstrate, connect mathematical ideas and solve problems.	UVM, OI2	13
3. Focus learners' attention on the essential features of the mathematical ideas which arise.	UVM, OI3	5
Total		**25**

The data in Table 5.4 reveals that all indicators of UVM were observed in this lesson. The teacher's use of visuals was predominantly observed. Mr. Kondja privileged the use of visuals to help learners understand and solve problems on the addition and subtraction of integers. An extract below shows how he used visuals to explain $4+3$. Mr. Kondja said *"Ok, in 4+3, is 4 a positive or negative?" "Positive",* said a learner. *"Yes positive! When there is no sign like this (while pointing next to 4), its positive! And 3?" "It's positive",* said a learner. *"In terms of the number line, remember we start from zero! The sign tells us about the direction to go, either this or that (pointing left and right), while the number tells us the movement we must make. 4+3 means, from zero, that zero, we go to the positive, four steps (while pointing), then again to the positive three steps and where we end, that will be the answer* [L1: P4, L2].

The second most observed indicator was Indicator 1 (UVM, OI1). Mr. Kondja used six forms of representation in this lesson. They were writing [L1: P1, L2], a number line [L1: P2, L5], pointing [L1: P3, L7], visualising [L1: P9, L1], bottle caps [L1: P5, L1], sticks [L1: P2, L2] and finger counting [L1: P2, L2]. The use of multiple representations has long been encouraged as they ground the mathematical thinking of the teacher, alongside their verbal explanations (Alibali & Nathan, 2012).

The focusing of learners' attention on the essential features of the mathematical ideas depicted by the visuals (UVM, OI3) was the least observed indicator. This indicator was observed five times. The illustration below shows how Mr. Kondja focused learners' attention on the essential features. He said *"There is a topic where we deal with these red and black colours! What topic is that? Electricity neh? Have you done that?" ''Yes",* said learners. "*Red colour is positive and black is negative",* said the learners. *"Aaaayee"* (No), says the teacher. *"The black is positive and this red is negative!" "Noooo",* said the learners. "*Ok, no problem, this one in math we will call it (red) negative, while black is positive. If they are together like this, it's zero"* [L1: P6, L5].

5.2.1.4 Facilitating Meaningful Mathematical Discourse (FMMD)

Table 5.4: Frequency of FMMD in Lesson 1

Observable Indicators	Code of the Observable Indicator	Frequency
1. Engage learners in the lesson through class discussions and presentations, and through other forms of verbal, visual and written communication.	FMMD, OI1	20
2. Encourage learners to share ideas through class discussions.	FMMD, OI2	7
3. Connect learners' responses to key mathematical ideas.	FMMD, OI3	1
4. Use probing questions and clues to make connections among mathematical ideas.	FMMD, OI4	11
Total		**39**

These indicators looked at how the teacher facilitated discussions in the class. The data in Table 5.5 reveals that the learners' engagement in this lesson was high. They were engaged in the lesson discussion during teaching and through the use of visuals. The teacher illustrated each of the examples he gave by using three methods: arithmetic, number line and the bottle caps.

Learners were extremely active as they used the bottle caps. I observed that when the teacher used the arithmetic method, some leaners were less interested in the lesson, while some touched other things. However, when Mr Kondja used bottle caps, most learners put other things aside, touched the bottle caps and actively debated. Despite being asked to work as a group, most wanted to try the problems individually. These observations resonate well with Boaler et al. (2016) who also observed that when the lesson relies on numbers, learners respond that the work is easy or hard or announce that they are finished. However, when the same content is taught visually, they are far more engaged.

The second most observed indicator was the teacher's use of probing questions (FMMD, OI4). It was interesting to observe how he did this in solving $-4 + 3$ using the number line. He said *"Look at this one, from zero, where are we going?" "To the negatives",* said the learners. "*How many steps?" "Four steps"*, said the learners. *"From there?" "To the positive",* says learners. *"How many steps?" "Three steps"*, said the learners [L1: P4, L1].

The third most observed indicator was the encouragement of learners to share ideas in class (FMMD, OI2). The act of grouping learners and giving them bottle caps to solve problems on integers significantly enhanced the sharing of ideas. This observation raised a question in me as to why learners do not freely share ideas in class when they are asked to do so by the teacher, as they do when using visuals.

The least observed indicator of FMMD was Indicator 3 (FMMD, OI3) which was observed only once when the teacher said "*What do you have here?" "Positive 8 minus 6"*, said a learner. *"No, just say 8 minus 6"* [L1: P12, L1]. I view this as evidence of connecting learners' responses to key mathematical ideas because the teacher provided the learner with the correct mathematical concept.

5.2.1.5 Posing Purposeful Questions (PPQ)

Table 5.5: Frequency of PPQ in Lesson 1

Observable Indicators	Code of the Observable Indicator	Frequency
1. Use a variety of questions (to gather information, probe understanding, make the mathematics visible and ask students to reflect on and justify their reasoning).	PPQ, OI1	21
2. Partner questions to focus learners' attention and thinking on the concept.	PPQ, OI2	4
3. Give sufficient time for learners to formulate and offer responses to questions.	PPQ, OI3	4
Total		**29**

This set of indicators looked at how the teacher used questions during the teacher – learner interaction. The data in Table 5.6 shows that all indicators of PPQ were observed in this lesson. The most observed indicator was Indicator 1 (PPQ, OI1). I observed the teacher using questions to gather information, to probe learners understanding and to make mathematics visible. An example of how the teacher made mathematics visible was evident when he asked: *"How many negatives do you want to subtract?" "6"*, said the learners. *"Ok, you will bring in six- zero there! … Now you can take these six red caps"* [L1: P18, L1].

The evidence of the teacher using partnering questions to focus learners' attentions and thinking on a particular concept was observed four times. Partnering questions were observed when the teacher solved $-7 - 3$. After Mr. Kondja put out seven red caps (to represent -7, he asked *"Can we take three blacks here in seven red?" "No",* said the learners. *"If no black, what can we do? We just add zero! Remember zero cannot change anything! How many zero? Three zero, because we want to subtract three blacks. Ok! Like this. Now we will take the three blacks because we have them! What is the answer? Is the answer positive or negative?" "Negative"*, said the learners. *"1, 2, 3...10, negative 10! Is it clear?"* [L1: P9, L3]. In this case, the teacher asked continuous questions, and the learners provided the responses. This type of question is described by Mehan (1979) as a Initiate-Response-Evaluation (IRE) pattern. In the IRE, the teacher asks a question with the answer in mind, the learners respond and the teacher evaluates the response.

Evidence of the teacher allocating sufficient time for the learners to think and provide responses were observed in this lesson. Once the teacher had posed a question, he patiently waited for the learners to respond. In other word, there was always a dialogue between the teacher and the learners, particularly when he was assisting individual learners.

5.2.1.6 Building Procedural Fluency from Conceptual understanding (BPFCU)

Table 5.6: Frequency of BPFCU in Lesson 1

Observable Indicators	Code of the Observable Indicator	Frequency
1. The teacher provides learners with the underlying meaning of the concept.	BPFCU, OI1	10
2. Learners are provided the opportunity to practise procedures, including using their own strategies.	BPFCU, OI2	3
3. Learners are asked to explain their strategies and the teacher refines them to more effective procedures.	BPFCU, OI3	4
4. The teacher uses visual models to support learners' understanding of general methods.	BPFCU, OI4	13
Total		**30**

These indicators looked at how the teacher helped the learners to build fluency from conceptual understanding. The data in Table 5.7 shows that all indicators were observed in this lesson. The teacher's use of visual models to support learners' understanding was the most observed (BPFCU, OI4). As indicated earlier, three methods were used in this lesson. These multiple ways were used so that the learners could choose which method which best suit them in terms of sense making. The practice of using multiple ways to solve mathematical problems is recommended by Mann (2014), who said that learners should be able to approach problems from several points of view and be encouraged to switch among representations until they are able to understand the problem or find the solution to the problem.

Indicator 1 was the second most observed (BCFPU, OI1). The teacher explained key concepts throughout the lesson, including the meaning of $4 + 3$ in the context of the number line. He said *"In terms of the number line, remember we start from zero! The sign tells us about the direction to go, either this or that (*pointing left and right*), while the number tell us the movement we must make*. $4 + 3$ *means, from zero, that zero, we go to the positive, four steps* (while pointing), *then again to the positive three steps and where we end, that will be the answer"* [L1: P3, L1]. Instead of the teacher showing only the procedure of adding 4 + 3 on the number line, he explained the underlying meaning of it. This is in agreement with Fuson et al. (2005), who argued that learners will have a better retention of the procedure if it is built on underlying conceptual understanding.

Indicator 3 (BPFCU, OI3) was the third most observed. No learner came up with a new strategy. They all used the procedure given by the teacher. In other words, evidence of learners explaining their procedures was observed, however it was the procedure which was provided by the teacher that they adopted. Lastly, the learners were also given opportunities to practice. They were given three tasks which they completed before the end of the lesson.

5.2.1.7 Supporting Productive Struggle in Learning (SPSL)

Table 5.7: Frequency of SPSL in Lesson 1

Observable Indicators	Code of the Observable Indicator	Frequency
1. Anticipate what learners might struggle with and how best to support them.	SPSL, OI1	7
2. Help learners realise that confusion and errors are a natural part of learning, by facilitating discussions on mistakes, misconceptions and struggles.	SPSL, OI2	5
3. Give learners sufficient time to struggle with tasks.	SPSL, OI3	3
4. Praise learners for their efforts in making sense of mathematical ideas and perseverance in reasoning through problems.	SPSL, OI4	5
Total		**20**

These indicators were used to look at how the teacher supported struggling learners during the lesson. All four indicators of SPSL were observed in this lesson (see Table 5.8). Indicator 1 (SPSL, OI1) was observed most. Evidence of Indicator 1 I observed were how Mr. Kondja defined key concepts such as integers, asking the learners to complete labelling the number line and rewrite the expression. When rewriting the expression, first the teacher wrote $7 - 3$. Thereafter, he rewrote $(7) - (3)$ [L1: P10, L3]. Rewriting the expression by adding brackets showed that the teacher anticipated that in $7 - 3$, the learners might struggle to tell whether the

3 required black caps (positive) or red caps (negative). However, adding the brackets clearly showed that both 7 and 3 are positive, thus making it easier for struggling learners to understand the equation.

Indicators 2 and 5 (SPSL, OI2 and 5) were the second most observed. I observed the teacher facilitating discussion by asking the learners how they used bottle caps to add and subtract integers. For example, when the learners were solving $8 - (-6)$, he asked *"Remember 6 is red only*, *so you must only take who?" "Reds only*", said a learner [L1:P18, L2]. He indirectly gave learners the clue that they should subtract the 6 red caps from the eight black caps.

The teacher praised learners for their efforts in making sense of mathematical ideas, using words such as *"very good"*, and *"thank you".* Evidence of this was observed when he asked *"Apart from counting, how else could we get the answer*? *Any other way? Tell me?"* A learner replied, *"number line". "Very good, thank you,"* said the teacher [L1: P2, L4]. By saying *"very good, thank you",* the teacher praised the learner for understanding that $4 + 3$ can be also solved using the number line.

The least observed indicator was Indicator 4 (SPSL, OI4). Three tasks were completed as a classwork. The teacher ensured that all groups completed their work by giving them one problem per time. Giving one problem per time was however a boring exercise for the fast learners because they had to wait for the others.

5.2.1.8 Eliciting and Using Evidence of Learners' Thinking (EUELT).

Table 5.8: Frequency of EUELT in Lesson 1

Observable Indicators	Code of the Observable Indicator	Frequency
1. Let learners talk about the problem or their solution with their partner in front of the whole class.	EUELT, OI1	6
2. Respond to learners with questions and prompts, to probe, scaffold and extend their knowledge.	EUELT, OI2	12
3. Use learners' thinking to address mistakes and misconceptions around the concepts.	EUELT, OI3	5
4. Use tasks that require learners to explain, present and justify their mathematical ideas	EUELT, OI4	7
Total		**30**

The data in Table 5.9 shows that all indicators of EUELT were observed in this lesson. Evidence of Indicator 2 (EUELT, OI2) was observed most. One notable observation was that the teacher did not declare a learner's contribution or solution as wrong. However, when he found that the answer was incorrect, he used probing questions to guide the learners until they realised their mistakes. For example, when one group was solving $8 - 6$ using bottle caps, he asked *"How did you get 14?" "We want to get the answer*", said a confused learner. *"But you have 14 black caps?" "But, the expression says here eight black! And what is this one for?"* [L1: P16, L1]. Instead of telling them that they were wrong to have 14 black bottle caps, by using probing questions he guided them to relate the number of their bottle caps to the expression and to know what number to subtract.

The second most observed evidence was for Indicator 4 (EUELT, OI4). Six tasks were completed during the lesson. All these tasks required the learners to explain and justify their ideas. However, their explanations were more practical and clear when they used the number line and bottle caps. This concurs with Cherif & Adams (2018) who indicated that using practical, familiar objects in a way that learners have never thought of before, is one ways to motivate learners to actively engage with their learning.

The third most observed indicator was when the learners discussed mathematical problems and shared their solutions with their group members (EUELT, OI1). Grouping or pairing learners is viewed as one way of facilitating learner-learner interaction. However, I observed that the learner-learner interaction was more effective when it is based on visual models. In other words, visuals are like a catalyst which ignites and sparks the discussion in either pairs or groups.

The least observed indicator was Indicator 3 (EUELT, OI3). The teacher said *“Let’s go to physical science! There is a topic where we deal with this red and black! What topic is that?” Electricity neh? Have you done that?*” “*Yes”* said the learners. *“Red colour is positive and black is negative”,* the learners responded. *“Aaaayee,”* (No) replied the teacher. *“No”*, said the learners. *“Ok no problem, this one in math we will call it (red) negative, while black is positive. If they are together like this, it’s zero”* [L1: P6, L]. In this case the teacher knew that in science the colour red is used as positive, while black is negative. If he had started using these colours inversely in mathematics without explaining, it would have bred confusion and misconceptions. Thus, he deliberately brought it up to define how these colours would be used.

5.2.2 Reflection on Lesson 1

This section contains the transcript of the interview between Mr. Kondja and myself (SF) where Lesson 1 was reflected upon..

> *SF: How was the lesson?*
>
> *Mr Kondja: Eish! I do not know what to say! But to me the lesson was good. I am very happy with the learners’ participation.*
>
> *SF: How was their participation?*
>
> *Mr. Kondja: It was really good, and they cooperated well today. I wish it could be as such forever!*
>
> *SF: Have you experienced any challenges?*
>
> *Mr. Kondja: Not a major one! But yes, when you have someone recording you, it brings a difference in one’s teaching. In other words, I was a bit scared! Secondly, even though the learners participated well, they were too loud! I do not know if their loudness was a disturbance to other classes nearby or this is how we should teach mathematics. And lastly, time! Using more than one method requires time. Just imagine having two lessons but I was just rushing.*
>
> *SF: What can be done to improve the next lesson?*
>
> ***Mr. Kondja****: I think I need to plan fewer contents if I am to continue with this way of teaching, more especially when I have a single lesson.*

5.2.3 Overview of Lesson 2 (Removing brackets)

This was the second lesson I observed of Mr. Kondja teaching Grade 9. The lesson was on algebra – removing brackets. According to the Grade 8–9 syllabus, learners were already introduced to this topic in Grade 8. However, Mr. Kondja prefers to revise it again in Grade 9 to remind learners and to connect it easily to factorising. The teacher prepared five examples for demonstration. They were: a) $2(x+3)$; b) $3(x-3)$; c)$2(2x-3)$; d)$2(-x-3)$; e) $5(-x+1)$. The teacher used two methods: the arithmetic method and algebra tiles. Learners were grouped, and the teacher provided each group with algebraic tiles. The lesson was 40 minutes long.

The analytical tool in Appendix B: Classroom observation and analytical tool was used to analyse this lesson and the results are presented in the tables below.

5.2.3.1 Establishing Mathematical Goals to Focus Learning (EMGFL)

Table 5.9: Frequency of EMGFL in Lesson 2

Observable Indicators	Code of the Observable Indicator	Frequency
1. The teacher states the topic and lesson objective and clarifies what learners are expected to learn.	EMGFL, OI1	1
2. The teacher demonstrates a good understanding of mathematical learning goals and how the lesson will unfold.	EMGFL, OI2	9
3. The teacher refers discussions to lesson goals and objectives during instructions.	EMGFL, OI3	1
4. The teacher uses ordered instructions and tasks that are aimed at achieving the lesson goals and objectives.	EMGFL, OI4	3
5. The teacher assesses and monitors learners' understanding toward the learning goal.	EMGFL, OI5	5
Total		**19**

According to the data, all the five indicators of EMGFL were observed in this lesson too. According to the data in Table 5.9), Indicator 2 (EMGFL, OI2) was the most observed, as in Lesson 1. Having a good understanding of mathematical goals is an essential aspect in the effective teaching of mathematics. It sets the stage for everything (Hiebert & Wearner, 1992).

In other words, learners understand more deeply because instructions, tasks and the pedagogical approach is well guided by the lesson goals and objectives. The second most observed indicator (EMGFL, OI5) was the assessment of learners' understanding. Regarding assessment and monitoring, Mr. Kondja mainly used questions such as *"How do we do this?"* [L2: P1, L3], or *"How many x do you have there?"* [L2: P8, L3]. The teacher used these kinds of questions to assess learners' progress towards the lesson's intended goals.

The third most observed indicator was Indicator 4 (EMGFL, OI4) and the least observed were Indicators 1 and 3 (EMGFL, OI1 and 3). As in Lesson 1, the teacher introduced the topic and then wrote it on the chalkboard. During the lesson, he aligned the lesson' discussions to the lesson goals and objectives.

5.2.3.2 Implementing Tasks that Promote Reasoning and Problem Solving (ITPRPS)

Table 5.10: Frequency of ITPRPS in Lesson 2

Observable Indicators	Code of the Observable Indicator	Frequency
1. Motivate learning of mathematics by using a variety of problems that build on and extend learners' understanding.	ITPRPS, OI1	5
2. Select tasks that can be solved in various ways, including the use of visuals.	ITPRPS, OI2	6
3. Give tasks on a regular basis that require high levels of cognitive demand.	ITPRPS, OI3	1
4. Support learners in solving tasks without taking over their thinking.	ITPRPS, OI4	1
5. Encourage learners to use various approaches to understand and solve problems.	ITPRPS, OI5	4
Total		**17**

The data Table 5.10) shows that all indicators of ITPRPS were evident in this lesson. The most observed evidence was for Indicator 2 (ITPRPS, OI2). As mentioned in the lesson overview, this lesson was about removing brackets. The teacher demonstrated all the examples using two methods: multiplication and algebraic tiles.

Indicator 1 was the second most observed (ITPRPS, OI1). The teacher had five examples for demonstration purposes which ranged from easy to complex. He successfully presented them in a 40 minutes lesson, despite using two approaches for each.

Indicator 5 was the third most observed (ITPRPS, OI5).The teacher encouraged the learners to use various approaches in the class. Influencing a certain behaviour takes various ways, one of which can be repeatedly acting out a behaviour. The teacher using various methods repeatedly is, in my view, another way of encouraging learners to use them.

The least observed indicators were 3 and 4 (ITPRPS, OI 3 and 4). Few learners needed the teacher's support in this lesson. This could be because the learners possess a certain level of prior knowledge of this topic from the previous grade. From a different perspective, though, one could propose that perhaps the examples that the teacher prepared were not challenging enough.

5.2.3.3 Use of Visual Models

Table 5.11: Frequency of UVM in Lesson 2

Observable Indicators	Code of the Observable Indicator	Frequency
1. Introduce forms of representation (visuals) that can be useful to learners.	UVM, OI1	3
2. Use representation (visuals) to explain, demonstrate, connect mathematical ideas and solve problems.	UVM, OI2	5
3. Focus learners' attention on the essential features of the mathematical ideas which arise.	UVM, OI3	2
Total		**10**

As in Lesson 1, the data in Table 5.11) show that all the indicators for UVM were observed in this lesson. The use of visuals to explain ideas and solve problems was predominantly observed. In other words, the teacher made use of visuals to explain and solve mathematical problems in this lesson. The habit of using visuals to explain and solve problems has huge benefits to learners. Apart from helping them to make sense of mathematics, it also promotes a positive mathematical image among learners, helping them to discover that maths is real and fun (Boggan et al., 2010).

The second most observed was Indicator 1 (UVM, OI1). Three forms of representations were introduced: writing [L2: P1, L2], algebraic tiles [L2: P2, L1] and gesturing [L2: P2, L1]. The teacher introduced algebraic tiles by saying *"There is another way of solving this, using algebraic tiles*" [L2: P2, L1]. The least observed was the focusing of learners' attention on the essential features depicted by the visuals. To illustrate this, the teacher said *"This tile is brown in colour (positive), and if you turn it, it is red which means negative"* [L2: P3, L1].

5.2.3.4 Facilitating Meaningful mathematical Discourse (FMMD)

Table 5.12: Frequency of FMMD in Lesson 2

Observable Indicators	Code of the Observable Indicator	Frequency
1. Engage learners in the lesson through class discussions and presentations, and through other forms of verbal, visual and written communication.	FMMD, OI1	6
2. Encourage learners to share ideas through class discussions.	FMMD, OI2	3
3. Connect learners' responses to key mathematical ideas.	FMMD, OI3	2
4. Use probing questions and clues to make connections among mathematical ideas.	FMMD, OI4	2
Total		**13**

The data in Table 5.12) shows that all the indicators of FMMD were observed in this lesson. The predominantly observed indicator was the learners' engagement in the lesson (FMMD, OI1). In this lesson, the learners were actively engaged during the lesson presentation, in contrast to Lesson 1 where they were only attentive when they were using visual models. The teacher spent much of the lesson demonstrating how to remove brackets. Engaging the learners in classroom discussions gave them an opportunity to share ideas, clarify their understanding and develop a language for expressing mathematical ideas, and learn to see things from other perspectives (Mann, 2014).

The second most observed was Indicator 2 (FMMD, OI2). As in Lesson 1, the teacher encouraged the learners to share ideas by giving them work to do in groups. The least observed were indicators 3 and 4 (FMMD, OI3 and 4). The teacher connected the learners' responses to key mathematical ideas when the he asked *"How do we do this?"* The learners replied *"We* *multiply x by 2 and then 3 by 2." "The meaning of brackets means multiply"*, said Mr Konja [L2: P1, L4]. The teacher connected the concept of multiplication to brackets by saying *"brackets means multiply"*.

5.2.3.5 Posing Purposeful Questions (PPQ)

Table 5.13: Frequency of PPQ in Lesson 2

Observable Indicators	Code of the Observable Indicator	Frequency
1. Use a variety of questions (to gather information, probe understanding, make the mathematics visible and ask students to reflect on and justify their reasoning).	PPQ, OI1	7
2. Use patterns of questions to focus learners' attention and thinking on the concept.	PPQ, OI2	2
3. Give sufficient time for them to formulate and offer responses to questions.	PPQ, OI3	1
Total		**10**

The data in Table 5.13) shows that all three indicators of PPQ were observed in this lesson. As in Lesson 1, the most observed Indicator 1 (PPQ, OI1). The teacher used different questions, such as questions to gather information, probe to make the mathematics visible. Questions to gather information were the most frequently asked. Some the commonly observed questions were *"How do we do this in 2(x + 3)?* [L2: P1, L4], and "*what are …?"* [L2: P2, L2]. By asking *"how do we do this?"*, he wanted to establish whether learners had an understanding of how to remove brackets.

The second most observed Indicator 2 (PPQ, OI2). Two instances of pattern questions were observed in this lesson. In one case the teacher said *"Like in this example* $2(x + 3)$*, how will I do it using tiles? You take your tiles, how many* x *do we have? "One!"*, said the learners. *"Ok, you put it there. How many units do you have?" "Three units"* said the learners. The teacher started putting tiles below the expression $2(x + 3)$ as follows *"one, two, three tiles! Now, you are multiplying those tiles by who?" "By 2"*, the learners said. *"If you multiply those tiles by 2 how many tiles will you have?"* (Pointing at unit tiles). *"Six"*, said the learners [L2: P3, L5]. I interpret these questions as the IRE pattern.

Lastly, I noticed that the leaners were given sufficient time to formulate their responses and they gave their responses without being distracted by others.

5.2.3.6 Building Procedural Fluency from Conceptual understanding (BPFCU)

Table 5.14: Frequency of BPFCU in Lesson 2

Observable Indicators	Code of the Observable Indicator	Frequency
1. The teacher provides learners with the underlying meaning of the concept.	BPFCU, OI1	2
2. Learners are provided the opportunity to practise procedures, including using their own strategies.	BPFCU, OI2	2
3. Learners are asked to explain their strategies and the teacher refines them to more effective procedures.	BPFCU, OI3	2
4. The teacher uses visual models to support learners' understanding of general methods.	BPFCU, OI4	6
Total		**12**

The data in Table 5.14) reveals that all four indicators of BPFCU were observed in this lesson. As in Lesson 1, the most observed was Indicator 4 (BPFCU, OI4). To support the learners' understanding, Mr Kondja used two methods to demonstrate how to remove brackets, ie by multiplying and by using algebraic tiles.

Indicators 1, 2 and 3 (BCFPU, OI1, 2, and 3) were each observed twice. Regarding Indicator 1 (BCFPU, OI1), the teacher explained the tiles and the colours associated with the signs. For indicator 2 (BCFPU, OI2), he grouped the learners and gave them tiles to do the class activity. Lastly, for Indicator 3 (BCFPU, OI3), none of the learners came up with a new strategy for removing brackets. However, most explained well how they applied the procedure provided by the teacher when they used algebraic tiles.

5.2.3.7 Supporting Productive Struggle in Learning

Table 5.15: Frequency of SPSL in Lesson 2

Observable Indicators	**Code of the Observable Indicator**	**Frequency**
1. Anticipate what learners might struggle with and how best to support them.	SPSL, OI1	2
2. Help learners realise that confusion and errors are a natural part of learning, by facilitating discussions on mistakes, misconceptions and struggles.	SPSL, OI2	2
3. Give learners sufficient time to struggle with the task.	SPSL, OI3	2
4. Praise learners for their efforts in making sense of mathematical ideas and perseverance in reasoning through problems.	SPSL, OI4	0
Total		**6**

The data in Table 5.15) shows that three indicators of SPSL were observed in this lesson. They were all observed twice. For Indicator 1 (SPSL, OI1), the teacher said *"Look here, look here! I know the majority of you have a problem with signs – when you multiply, you forget about signs. But this one is showing you clearly!"* [L2: P9, L1]. This shows that the teacher knew that most learners have a challenge when dealing with negative signs. Thus, he asked the learners to pay attention as he addressed this challenge using algebraic tiles. The above extract is also evidence of the teacher facilitating discussions on mistakes, as in Indicator 2 (SPSL, OI2). Mann (2014) has indicated that supporting learners productively in their struggles is a sign of effective teaching. He further requests teachers to view struggle as an opportunity to examine the problem more deeply. Regarding Indicator 3, the teacher gave the learners one task at a time to ensure that all groups were finished before moving to the next one.

5.2.3.8 *Eliciting and Using Evidence of Learners' Thinking (EUELT)*

Table 5.16: Frequency of EUELT in Lesson 2

Observable Indicators	Code of the Observable Indicator	Frequency
1. Let learners talk about the problem or their solution with their partner in front of the whole class.	EUELT, OI1	2
2. Respond to learners with questions and prompts to probe, scaffold and extend their knowledge.	EUELT, OI2	1
3. Use learners' thinking to address mistakes and misconceptions around the concepts.	EUELT, OI3	2
4. Use tasks that require learners to explain, present and justify their mathematical ideas.	EUELT, OI4	2
Total		**7**

The data in Table 5.16) shows that all four indicators of EUELT were observed in this lesson. Indicators 1, 3, and 4 (EUELT, OI1, 3 and 4) were all observed twice. Indicator 1 (EUELT, OI1) was observed when the teacher gave the class work to do in groups. In their groups the learners mainly discussed the work using the algebraic tiles.

Evidence of Indicator 3 (EUELT, OI3) was observed when the teacher corrected learner by saying *"What is the answer?" "Negative 5 plus…."*, they said. *"Be careful, if you say -5 only without x, it's like you are referring to this small one, units! So, it is negative* $-5x + 5$ [L2: P10, L3]. The act of correcting the learner immediately is supported by Mann (2014). He said that it is important to identify and address potential learning gaps before errors or faulty reasoning becomes consolidated and more difficult to remediate.

Regarding Indicator 4 (EUELT, OI4), all tasks used in this lesson required the learners to explain and justify their ideas, particularly when they were using algebraic tiles. Evidence of Indicator 2 (EUELT, OI2) was shown when the teacher went to a group while they were solving $2(-x - 3)$ using algebraic tiles and asked them *"Is that negative $-x$? How many $-x$ do you have there?'' One or 2?"* [L2: P8, L5]. In this context, the teacher was correcting learners using probing questions.

5.2.4 Reflection on Lesson 2

This section contains the transcript of the interview between Mr. Kondja and the interviewer (FS), reflecting on Lesson 2.

SF: How was the lesson?

Mr. Kondja: I would say the lesson was good! Myself, I was fine today, I did not have any fear today. I am getting comfortable with the video recording.

FS: What about the learners?

Mr. Kondja: Learners started at a low pace, but they picked up later more especially when we started using algebraic tiles. So tiles added a vibe to today's lesson.

SF: Have you experienced any challenges?

Mr. Kondja: Yes, yes! Firstly, I had to admit that learners had a fear of algebra in general. That is why they were a bit quiet at the beginning. They were behaving as if they are facing a giant enemy, and that was a huge challenge to me! However, when I gave them tiles to work in groups, they were relieved, even though there were still some groups who were struggling.

SF: What can be done to improve the next lesson?

Mr. Kondja: I think with algebra I don't have to prepare more lesson content. It will just be a bit, then I will give them more time to practise. Also in this lesson, I did not prepare a worksheet, which sometimes consumes time when I have to write everything on the chalkboard.

5.2.5 Overview of Lesson 3 (Solving algebraic equations)

This was the third lesson I observed of Mr. Kondja teaching Grade 9. The lesson was on algebra – solving equations. After greeting the learners, Mr. Kondja began by recapping what was covered in the previous lesson (removing brackets). He solved $2(x + 3)$ using tiles to remind the learners what they had covered in the previous lesson. Then the teacher introduced the lesson and wrote it on the chalkboard. He had prepared seven examples, of which the first four were for demonstration purposes and the last three for class work in groups. These examples were: (a) $x + 2 = 5$; (b) $x - 2 = 5$; (c) $2x - 1 = 5$; (d) $4 - x = 5$; (i) $x - 1 = 5$; (ii) $5 - x = 5$; (iii) $3x - 2 = 11$. The teacher demonstrated these examples using two methods: arithmetic and using algebraic tiles. After he demonstrated the first four examples, he grouped the learners, gave them tiles and asked them to do the class work. Due to time constraints, only two groups managed to finish the first problem of the class work. The lesson was 40 minutes long.

The analytical tool in Appendix B: Classroom observation and analytical tool was used to analyse this lesson and the results are presented on the tables below.

5.2.5.1 Establishing Mathematical Goals to Focus Learning (EMGFL)

Table 5.17: Frequency of EMGFL in Lesson 3

Observable Indicators	Code of the Observable Indicator	Frequency
1. The teacher states the topic and lesson objective and clarifies what learners are expected to learn.	EMGFL, OI1	1
2. The teacher demonstrates a good understanding of mathematical learning goals and how the lesson will unfold.	EMGFL, OI2	9
3. The teacher refers discussion to lesson goals and objectives during instructions.	EMGFL, OI3	2
4. The teacher use ordered instructions and tasks that are aimed at achieving the lesson goals and objectives.	EMGFL, OI4	8
5. The teacher assesses and monitors learners' understanding toward the learning goal.	EMGFL, OI5	6
Total		**26**

As in Lessons 1 and 2, the data in Table 5.17 confirms that all the indicators of EMGFL were observed in this lesson. Indicator 2 (EMGFL, OI2) was observed most. The teacher's good understanding of the lesson goals and objectives was manifested in the examples he used. The examples show a transition from easier to more complex. He began with examples where a variable (x) was positive and ended with examples where the variable (x) was negative, making it easier for the learners to see how the positives and negatives related to each other.

Indicator 4 (EMGFL, OI4) was the second most observed. A variety of examples, as listed in the lesson overview, were used for learners to gain a deep understanding of equations. The least observed indicators were 3 and 1 (EMGFL, OI3 and 1). As in Lessons 1and 2, the teacher introduced the lesson and wrote the lesson objectives on the chalkboard.

5.2.5.2 Implementing Tasks that Promote Reasoning and Problem Solving (ITPRPS)

Table 5.18: Frequency of ITPRPS in Lesson 3

Observable Indicators	Code of the Observable Indicator	Frequency
1. Motivate learning of mathematics by using a variety of problems that build on and extend learners' understanding.	ITPRPS, OI1	5
2. Select tasks that can be solved in various ways including the use of visuals.	ITPRPS, OI2	6
3. Give tasks on a regular basis that require a high level of cognitive demand.	ITPRPS, OI3	2
4. Support learners in solving tasks without taking over their thinking.	ITPRPS, OI4	0
5. Encourage learners to use various approaches to understand and solve problems.	ITPRPS, OI5	4
Total		**12**

The data in Table 5.18 reveals that only four of the five indicators of ITPRPS were observed in this lesson. Indicator 2 (ITPRPS, OI2) was predominantly observed as was observed in Lessons 1 and 2. This agrees with Boaler et al. (2016) who argued that there are no mathematical ideas or concepts which cannot be illustrated or thought about visually and Mr Kondja was aware of this.

The second most observed Indicator 1 (ITPRPS, OI1). Using a variety of examples does not only expose learners to different ways of asking questions, but also helps them to thoroughly understand the concept. Indicator 3 (ITPRPS, OI3) was the least often observed. It is worth incorporating tasks that require high cognitive levels of thinking to encourage reasoning. However, too many of these questions affect learners' confidence, which eventually leads to mathematics anxiety (Feldhaus, 2014). Indicator 4 (ITPRPS, OI4) was not observed in this lesson. In this study, teacher support was measured mainly when the learners are doing tasks. Thus, the absence of evidence for ITPRPS, OI4 could be because of time constraints. Only two groups managed to finish the first problem in the class work, making it impossible for the teacher to render support.

5.2.5.3 Use of Visual Models (UVM)

Table 5.19: Frequency of UVM in Lesson 3

Observable Indicators	Code of the Observable Indicator	Frequency
1. Introduce forms of representation (visuals) that can be useful to learners.	UVM, OI1	4
2. Use representation (visuals) to explain, demonstrate, connect mathematical ideas and solve problems.	UVM, OI2	6
3. Focus learners' attention on the essential features of the mathematical ideas which arise.	UVM, OI3	2
Total		**12**

The data in Table 5.19) show that all the indicators of UVM were observed in this lesson. The most observed was Indicator 2 (UVM, OI2), also predominantly observed in Lessons1 and 2. The dominance of this indicator says more about the benefit of using visuals. One possible meaning of this is that the teacher might have discovered the usefulness of using visuals in teaching, hence his substantial use of them. The frequent use of visuals in explaining mathematical ideas has a significant positive impact. Mann (2014) states that learners who make use of visuals grow in their perceptions of mathematics as a unified and coherent discipline.

Indicator 1 (UVM, OI1) was the second most observed indicator. The teacher used four forms of representation, ie writing words and numbers on the chalkboard [L3: P1, L1], algebraic tiles to model and solve equations [L3: P1, L3] and hand gestures of pointing to show the transferring of a number to the other side [L5: P1, L5]. The act of using different forms of representation in teaching is viewed by Phillips et al. (2010) as a good reminder to learners of what they have learned. These authors claimed that an image remains in the long-term memory for longer than words.

The least observed was Indicator 3 (UVM, OI3). The most interesting evidence of focusing learners' attention was when the teacher demonstrated how to eliminate the negative sign in $4 - x = 5$ using tiles. He said *"Remember most learners have a problem when x is a negative like this one. Try if you can learn something here"* [L3: P9, L2]. From my experience in teaching equations, this resonated with me, as many of my learners do not eliminate the negative coefficient of x and thus do not solve the equation fully.

5.2.5.4 Facilitating Meaningful mathematical Discourse (FMMD)

Table 5.20: Frequency of FMMD in Lesson 3

Observable Indicators	Code of the Observable Indicator	Frequency
1. Engage learners in the lesson through class discussions and presentations, and through other forms of verbal, visual, and written communication.	FMMD, OI1	6
2. Encourage learners to share ideas through class discussions.	FMMD, OI2	2
3. Connect learners' responses to key mathematical ideas.	FMMD, OI3	0
4. Use probing questions and clues to make connections among mathematical ideas.	FMMD, OI4	5
Total		**13**

The data in Table 5.20) reveals that three indicators of FMMD were observed in this lesson, with Indicator 1 (FMMD, OI1) being observed the most. The learners were more engaged while the teacher was presenting the lesson. The second most observed was Indicator 4 (FMMD, OI4). The teacher used probing questions to involve learners in the lesson. An example of this was when the teacher said: *"Let us take an example (a) $x + 2 = 5$! What do you do? The way you were taught in your previous grade!"* [L3: P2, L2].

Indicator 2 (FMMD, OI2) was least observed because the time did not allow learners to complete their classwork, thus limiting their chances of sharing ideas in the class.

There was no evidence of Indicator 3 (FMMD, OI3). This could also be attributed to a lack of time. This reveals that when learners are not given sufficient time to engage with the task, teachers may miss a golden opportunity to hear learners' questions, reasoning, interpretations, misconceptions and to assess their progress in learning (Mann, 2014).

5.2.5.5 Posing Purposeful Questions (PPQ)

Table 5.21: Frequency of PPQ in Lesson 3

Observable Indicators	Code of the Observable Indicator	Frequency
1. Use a variety of questions (to gather information, probe understanding, make the mathematics visible and ask students to reflect on and justify their reasoning).	PPQ, OI1	8
2. Use patterns of questions aimed at focusing learners' attention and thinking on the concept.	PPQ, OI2	3
3. Give learners sufficient time to formulate and offer responses to questions.	PPQ, OI3	0
Total		**11**

The data in Table 5.21) reveals that two indicators of PPQ were observed in this lesson. As in lessons 1 and 2, the most observed was Indicator 1 (PPQ, OI1). The teacher used probing questions to integrate learners in the lesson.

The least observed was Indicator 2 (PPQ, OI2). The teacher used the IRE pattern of questioning (Mehan, 1979) during the lesson presentation. For example, in solving $x - 2 = 5$, he said *"Remember what types of two units do we have now?" "Negatives"*, said the learners. *"And how many are positives?" "5"*, responded the learners. *"Now we have modelled $x - 2 = 5$"*. Now *I have to do something here so that x can be alone! What do I do with this one?" "You add two positives"*, said the learners. *"Ok! Then also that side I add 2 positives* (while putting tiles on the board with glue). *Now the concept of zero comes in. Negative and positive gives me what?" "Zero"*, said the learners. *"...and then $-$ and + give me zero"* [L3: P6, L2]. This way of asking continuous questions kept learners focused on the procedure of solving $x - 2 = 5$. Evidence of Indicator 3 (PPQ, OI3) was not observed.

5.2.5.6 Building Procedural Fluency from Conceptual understanding (BPFCU)

Table 5.22: Frequency of BPFCU in Lesson 3

Observable Indicators	Code of the Observable Indicator	Frequency
1. The teacher provides learners with the underlying meaning of the concept.	BPFCU, OI1	2
2. Learners are provided the opportunity to practise procedure, including using their own strategies.	BPFCU, OI2	1
3. Learners are asked to explain their strategies and the teacher refines then to more effective procedures.	BPFCU, OI3	0
4. The teacher uses visual models to support learners' understanding of general methods.	BPFCU, OI4	5
Total		**8**

The data in Table 5.22) shows that three indicators of BPFCU were observed in this lesson, with Indicator 4 (BCFPU, OI4) observed the most. The teacher reinforced learners' understanding of equations by using algebraic tiles. The second most observed was Indicator 1 (BCFU, OI1). An example of how Mr Kondja explained the meaning of the equation was: *"Let us take an example: (a)* $x + 2 = 5$*! Remember this is a mathematical statement which says mmmm* (x) *plus 2, you get who?"* (While pointing at 5). *"5"*, the learners responded. *"Now we want to know who that mmmmm (x)!"* [L3: P2, L4]. Having been given the underlying meaning of the equation, learners may easily solve the equation using other means, such as prediction. The least observed was Indicator 2 (BCFU, OI2), which was only observed once. As indicated earlier, there was insufficient time for the learners to practise on more examples.

5.2.5.7 *Supporting Productive Struggle in Learning (SPSL)*

Table 5.23: Frequency of SPSL in Lesson 3

Observable Indicators	Code of the Observable Indicator	Frequency
1. Anticipate what learners might struggle with and how best to support them.	SPSL, OI1	2
2. Help learners realise that confusion and errors are a natural part of learning, by facilitating discussions on mistakes, misconceptions and struggles.	SPSL, OI2	1
3. Give learners sufficient time to struggle with tasks.	SPSL, OI3	1
4. Praise learners for their efforts in making sense of mathematical ideas and perseverance in reasoning through problems.	SPSL, OI4	0
Total		**4**

The data in Table 5.23) shows that three indicators of SPSL were observed in this lesson. The most observed was Indicator 1 (SPSL, OI1). Evidence of this indicator was observed when the teacher began the lesson by saying *"In the previous lesson, we did removing brackets. One of the examples,* $2(x+3)$ *."* The teacher used tiles to model $2(x+3)$ as he said: *"... twice of those diagrams, we will have another* x *, and another 3 units. Then our answer will be?"* "$2x+6$*"*, said the learners [L3: P1, L2]. Solving equations and removing brackets are two related topics. A learner needs the basics of removing brackets to successfully handle equations. Thus, beginning the lesson by recapping what was taught, is a good idea.

The least observed were Indicators 2 and 3 (SPSL, OI2 and 3). The teacher facilitated discussion on signs by saying *"Let me give you one with negatives,* 4- x=5. *We have 4 units, then you have a red* x *and 5 units. Remember most learners have a problem when x is negative! Try now if you can learn something here"* [L3: P7, L2]. The teacher seems to have given the above example intentionally to expose the common mistake of many learners. The teacher gave the learners one task at a time. Only when all the groups were finished with a task, he gave them the next task.

5.2.5.8 *Eliciting and Using Evidence of Learners' Thinking (EUELT).*

Table 5.24: Frequency of EUELT in Lesson 3

Observable Indicators	Code of the Observable Indicator	Frequency
1. Let learners talk about the problem or their solution with their partner in front of the whole class.	EUELT, OI1	2
2. Respond to learners with questions and prompts to probe, scaffold and extend their knowledge.	EUELT, OI2	6
3. Use learners' thinking to address mistakes and misconceptions around the concepts.	EUELT, OI3	2
4. Use tasks that require learners to explain, present and justify their mathematical ideas	EUELT, OI4	2
Total		**12**

The data in (Table 5.24) shows that all indicators of EUELT were observed in this lesson. Indicator 2 (EUELT, OI2) was observed most. The teacher hardly told or directly corrected the learners, but instead he used questions, creating a space for the learners to think and give their ideas. In this way, learners kept their focus on the lesson.

The least observed were Indicators 1, 3 and 4 (EUELT, OI1, 3 and 4). Evidence of learners discussing problems with each other was observed when the teacher grouped them. In addition, their discussions was more active as a result of using algebraic tiles. In other words, when learners were using visuals, they had more to talk about than when they did not have visuals.

In this lesson, the teacher heavily emphasized signs. Being a mathematics teacher, I have noted that many learners tend to ignore signs, the negative sign in particular. He gave emphasis to this by saying *"Can we now write what we have there?" "We have $-x = 1$",* said the learners. *"Are we done?" "Nooo"*, they replied. *"X is not a negative, x is x,"* said the teacher. *"Now how do I make that x is positive*? *What I do I just turn it*? *If I turn it, it will become what?" "positive"*, said the learners. *"Once I turn this one, I also turn the other side. And this one become what?" "Negative"*, said the learners. *"So then the answer is -1"* [L3: P8, L9].

Lastly, all the tasks used in this lesson required the learners to explain, present and justify their ideas. However, I noted that not all teaching methods make the mathematical problems explainable, presentable and justifiable as well as visuals do.

5.2.6 Reflection on lesson 3

This section contains the transcript of the interview reflection on Lesson 3, between Mr. Kondja and the interviewer (SF).

SF: How was the lesson today?

Mr. Kondja: The lesson was good. I am also learning.

SF: What have you learned?

Mr. Kondja: One of the interesting parts of today's lesson was at $-x = 1$. *When I used the algebraic tiles, it came out clearly that once we have negative x, we still have to do something to make it positive. In many cases, learners tend to ignore that negative sign, assuming that they are done. I am just hoping that they will never forget it anymore.*

SF: Have you experienced any challenges?

Mr. Kondja: Not really, apart from time. This type of teaching requires time. Like you have seen that when I was just about to give classwork, the bell ringed. Yes, I couldn't rush them to the classwork because they needed that conceptual understanding so they can apply it in other equations.

SF: What can be done to improve in the next lesson?

***Mr. Kondja**: The fact that this method is interesting to the learners, it's hard to withdraw it, despite requiring more time. Maybe we will consider arranging with the colleagues for an extra lesson. Otherwise, I will be using this method for introduction only, then once they master the concept, they will carry on with the mental method.*

5.2.7 Overview of Lesson 4 (Angles in parallel lines)

This was Mr. Kondja's fourth lesson I observed of him teaching the Grade 9s. The lesson was on angle properties, particularly the angle formed within parallel lines intersected by a transversal. After greeting the learners, Mr. Kondja began by asking questions on what learners had learned yesterday in mathematics. Three angle properties were taught previously. They were the complementary, supplementary and vertically opposite angles. After the learners identified these angle properties, he told them *"We will use these angle properties in combination with what we are going to learn today"*. He then introduced the lesson (angles in parallel lines), by writing the topic on the chalkboard.

The teacher then began a discussion, asking what parallel lines and transversal lines are. He then identified the three automatic angles formed when a transversal intersects parallel lines. From there he clarified what the learners should know by the end of this lesson. Thereafter, the teacher began to explain and demonstrate the three angles formed when a transversal intersects the parallel lines, one by one, giving the model to find each angle. After explaining all the angle properties, he gave the class an activity where he demonstrated how to find those angles, using the model he provided them with and showed them how to reason mathematically. Lastly, the teacher gave individual class work to the learners, while moving around the class monitoring them. While the learners were busy with the class work the bell rang, signifying the end of the lesson. The lesson was 40 minutes long.

The analytical tool in Appendix B: Classroom observation and analytical tool was used to analyse this lesson and the results are presented on the tables below.

5.2.7.1 Establishing Mathematical Goals to Focus Learning (EMGFL)

Table 5.25: Frequency of EMGFL in Lesson 4

Observable Indicators	Code of the Observable Indicator	Frequency
1. The teacher states the topic and lesson objective and clarifies what learners are expected to learn.	EMGFL, OI1	2
2. The teacher demonstrates a good understanding of mathematical learning goals and how the lesson will unfold.	EMGFL, OI2	9
3. The teacher refers discussion to lesson goals and objectives during instructions.	EMGFL, OI3	1
4. The teacher use ordered instructions and tasks that are aimed at achieving the lesson goals and objectives.	EMGFL, OI4	8
5. The teacher assesses and monitors learners' understanding toward the learning goal.	EMGFL, OI5	5
Total		**25**

The data in Table 5.25) shows that all the indicators of EMGFL were observed in this lesson. The predominantly observed indicator was (EMGFL, OI2). The teacher demonstrated a good understanding of the lesson goals by linking the previous lesson to the current lesson and he structured the lesson toward the intended goals. The second most observed evidence were for Indicator 4 (EMGFL, OI4). The teacher carefully used the instructions to drive the lesson toward the intended lesson goals and objectives. He did this by saying *"Let's start with alternate angle"* [L4: P5, L1], *"Now let's go to co-interior angle*" [L4: P9, L1], and *"Now let's go to the activity*" [L4: P10, L1].

The third most observed Indicator 5 (EMGFL, OI5) and the least were Indicators 2 and 1 (EMGFL, OI2 and 1). It is worth noting that in this lesson the teacher clearly stated learners' expectations, in contrast to the previous lessons where he simply stated the topic and wrote it on the chalkboard. To illustrate this, he said: *"In mathematics, when a transversal intersects parallel lines, three automatic angles come in! What are they? The first one is alternate angle, second one is corresponding angle, and the third one is co-interior angle. Those are the three automatic angles that come in.* ***Now, what are you supposed to know****? You need to know how they are! How they are means be able to see with your eyes in parallel! Then, how to find them"* [L4: P4, L1]. Clarifying what learners are expected to learn, not only serves as a guiding tool for learners, but also serves as an assessment tool for their learning progress (Mann, 2014).

5.2.7.2 Implementing Tasks that Promote Reasoning and Problem Solving (ITPRPS)

Table 5.26: Frequency of ITPRPS in Lesson 4

Observable Indicators	Code of the Observable Indicator	Frequency
1. Motivate learning of mathematics by using a variety of problems that build on and extend learners' understanding.	ITPRPS, OI1	7
2. Select tasks that can be solved in various ways including the use of visuals.	ITPRPS, OI2	7
3. Give tasks on a regular basis that require a high level of cognitive demand.	ITPRPS, OI3	0
4. Support learners in solving tasks without taking over their thinking.	ITPRPS, OI4	0
5. Encourage learners to use various approaches to understand and solve problems.	ITPRPS, OI5	7
Total		**21**

The data in Table 5.26) reveals that three of the five indicators of ITPRPS were observed in this lesson. It is worth noting that as in Lessons 1 to lesson 3, the teacher used various approaches to solve mathematical problems – which speaks well to the core aspects which gave birth to this study. This study was prompted by struggling learners who do not like mathematics due to what they termed as 'difficult' mathematics content, after the curriculum was reviewed. In support, Mann (2014) recommends that using various approaches, including the use of visuals, is the best approach for struggling learners and those with language problems Visuals in particular allow learners to participate meaningfully in the classroom, thus building their interest in mathematics, which may eventually positively shift their disposition in mathematics. The author further stressed that learners' success in solving problems depends on their ability to move flexibly among representations.

Evidence of Indicators 2 and 3 (ITPRPS, OI2 and 3) were not evident in this lesson. Despite the teacher having prepared the classwork, time constraint meant that it could not take place, thus making it impossible to assess evidence of Indicators 3 and 4 (ITPRPS, OI3 and 4).

5.2.7.3 Use of Visual Models (UVM)

Table 5.27: Frequency of UVM in Lesson 4

Observable Indicators	Code of the Observable Indicator	Frequency
1. Introduce forms of representation (visuals) that can be useful to learners	UVM, OI1	4
2. Use representation (visuals) to explain, demonstrate, connect mathematical ideas and solve problems.	UVM, OI2	7
3. Focus learners' attention on the essential features of the mathematical ideas which appears	UVM, OI3	2
Total		**13**

The data in Table 5. 27 shows that evidence of all three indicators of UVM were observed in this lesson. Indicator 2 (UVM, OI2) was observed the most. The dominance of this indicator proves the effectiveness of visuals in teaching mathematics. During the teacher's lesson reflection interview, he indicated that *"the lesson was good and learners actively participated"*. Considering the teacher's reflection and how often this indicator was observed might be an indication that the teacher realised the essence of using visuals. To this, Rieber (1995) asserted that visual are powerful cognitive tools for learning and understanding mathematics. Moreover, Mann (2014) stresses that struggling learners participate well in lessons where visuals are used because visuals assist them to follow the reasoning of their classmates and in the process, it helps them to develop their own explanations.

The second most observed was Indicator 1 (UVM, OI1). Four forms of representation were observed: writing [L4: P1, L4], gesturing [L4: P2, L5], drawing [L4: P3, L3] and models [L4: P5, L3]. Gesturing and modelling are shown on the figure below.

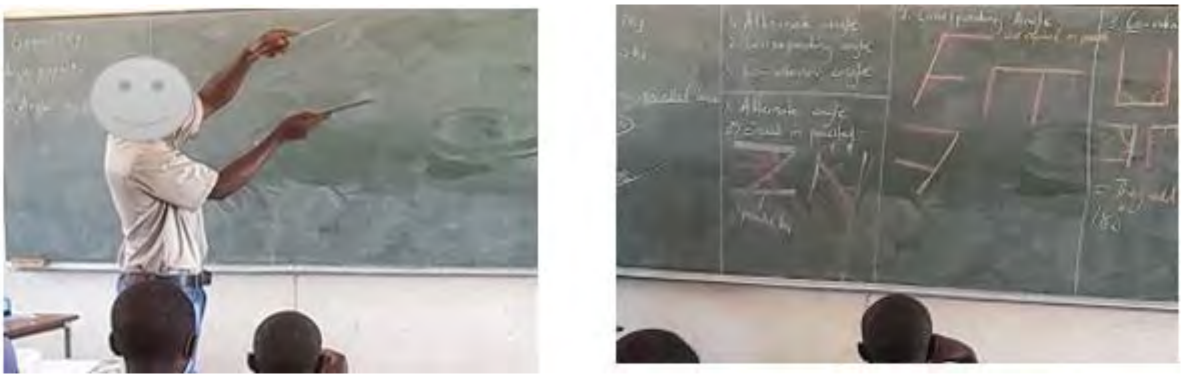


Figure 5.1: Mr. Kondja gesturing parallel lines and modelling angles.

The first part of figure shows Mr. Kondja modelling parallel lines, while the second figure shows the models for finding angles.

The least observed was Indicator 3 (UVM, OI3). He focused learners' attention on the Z-shape by saying *"You need to know what a Z-shape is. This part and this part are parallel lines"* (the teacher highlighted the lines using coloured chalk). *"So, what is the other part? This line? It's a transverse! What does it mean? It means if parallel lines are like this* (horizontal), *then you will have a Z-shape while if they are up like this, you will have an N-shape! Now which are alternate angles here? This one inside Z, these two angles are equal. If you have an N-shape, which one are alternate angles? This one and this are"* (showing the model). [L4: P5, L1]. As highlighted by Phillips et al. (2010) providing models does not automatically guarantee learning. Mr. Kondja thus explained and demonstrated to learners the composition of these models before showing the location of the angles. This action is in line with Jonson-Laird (1998), who pointed out that visuals need some cognitive action to point out the internalised concepts of the visual, so that learners can understand and use the model.

5.2.7.4 Facilitating Meaningful mathematical Discourse (FMMD)

Table 5.28: Frequency of FMMD in Lesson 4

Observable Indicators	Code of the Observable Indicator	Frequency
1. Engage learners in the lesson through class discussions and presentations, and through other forms of verbal, visual, and written communication.	FMMD, OI1	11
2. Encourage learners to share ideas through class discussions.	FMMD, OI2	2
3. Connect learners' responses to key mathematical ideas.	FMMD, OI3	1
4. Use probing questions and clues to make connections among mathematical ideas.	FMMD, OI4	5
Total		**20**

The data Table 5.28 shows that all the indicators of FMMD were observed during this lesson. The engagement of learners in this lesson was high. They were engaged in various ways, including learning how to reading mathematical concepts. The teacher said, *"Do you know how to read them?" "Yes",* said the learners.*" Iya, read the first one: alternate angle, said learners? Second one," "corresponding angles", said* the learners. *"... and the third one is co-interior angle",* said the learners [L5: P11, L6]. The second most observed was Indicator 4 (FMMD, OI4). An example of how the teacher probed learners was observed when he asked, *"What we have learned yesterday*?" "*Angle properties",* responded the learners. *"What are they?" "Complementary angle/right angle 90°, supplementary/straight angle, revolution (360°), and vertically opposite angles",* said the learners. *"These angle properties, we will use them in combination with what we will do today"* [L4: P1, L1]. The question *"What have we learned yesterday?"* I view as a probing question because the intention of the teacher was to link the angle properties learned yesterday to those to be taught in this lesson.

The third most observed indicator was for learners sharing ideas (FMMD, OI2). The teacher asked the learners to explain parallel lines. He said, *"What are parallel lines?" "Lines that are like this",* said Lonia*. "Any lines which are like this*? *Lonia, make it clear please!"* [L4: P2, L1].

The least observed was Indicator 3 (FMMD, OI3) – connecting learners' responses to key mathematical ideas. I observed this when the teacher said *"a is how many degrees? Remember we are going to give reasons! Ok, a is equal to how many degrees?" "$a = 120°$*, said the learners. *"Why? It is because ...",* a learner tried to explain. "*Noooo us mathematician don't say 'because', we just say 'vertically opposite angle' straight"* [L4: P11, L2]. The key mathematical idea in this context was the angle property, vertically opposite angle.

5.2.7.5 Posing Purposeful Questions (PPQ)

Table 5.29: Frequency of PPQ in Lesson 4

Observable Indicators	Code of the Observable Indicator	Frequency
1. Use a variety of questions (to gather information, probe understanding, make the mathematics visible and ask students to reflect on and justify their reasoning).	PPQ, OI1	16
2. Use patterns of questions aimed at focusing learners' attention and thinking on the concept.	PPQ, OI2	1
3. Give sufficient time for learners to formulate and offer responses to questions.	PPQ, OI3	0
Total		**17**

The data in Table 5.29 shows that two indicators of PPQ were observed in this lesson. The teacher used a variety of questions as in Lessons 1, 2, and 3. In addition, a funnelling question (Herbel-Eisenmann & Breyfogle, 2005) was observed in this lesson. This was observed when the teacher asked *"Now, let me say this angle is 72*, (writing in parallel lines), *which ones are the alternate angles here? Draw the model first! How? Like this* (drawing a Z-shape)*…."* [L4: P8, L1]. The funnelling question is very similar to the IRE pattern question. However, in a funnelling question, the teacher does not expect the answer from the learners.

The least observed was Indicator 2. The teacher used the IRE pattern of questioning to keep the learners focused. No evidence of Indicator 3 (PPQ, OI3) was observed because there was not enough time in the lesson.

5.2.7.6 Building Procedural Fluency from Conceptual understanding (BPFCU)

Table 5.30: Frequency of BPFCU in lesson 4

Observable Indicators	Code of the Observable Indicator	Frequency
1. The teacher provides learners with the underlying meaning of the concept.	BPFCU, OI1	5
2. Learners are provided the opportunity to practise procedures, including using their own strategies.	BPFCU, OI2	1
3. Learners are asked to explain their strategies and the teacher refines them to more effective procedures.	BPFCU, OI3	0
4. The teacher uses visual models to support learners' understanding of general methods.	BPFCU, OI4	5
Total		**11**

The data in Table 5.30 shows that three indicators of BPFCU were observed in this lesson. Indicators 1 and 4 (BPFCU, 1 and 4) were frequently observed. Regarding Indicator 1, the teacher explained key concepts throughout the lesson, ie parallel lines and the three models (Z, F, & U shapes) for finding angles in parallel lines. The teacher also used hand gestures to explain parallel lines.

The least observed indicator was for learners practising procedures BPFCU, OI2). Despite the teacher having prepared activities for classwork, time constraints did not permit leaners to practice the procedures. Lastly, there was no evidence of Indicator 3 (BPFCU, OI3) observed. This could be due to insufficient time to accommodate the classwork.

5.2.7.7 Supporting Productive Struggle in Learning (SPSL)

Table 5.31: Frequency of SPSL in Lesson 4

Observable Indicators	Code of the Observable Indicator	Frequency
1. Anticipate what learners might struggle with and how best to support them.	SPSL, OI1	9
2. Help learners realise that confusion and errors are a natural part of learning, by facilitating discussions on mistakes, misconceptions and struggles.	SPSL, OI2	1
3. Give learners sufficient time to struggle with tasks.	SPSL, OI3	1
4. Praise learners for their efforts in making sense of mathematical ideas and perseverance in reasoning through problems.	SPSL, OI4	2
Total		**13**

The data in Table 5.31 shows that all four indicators of SPSL were observed in this lesson. The most observed was Indicator 1 (SPSL, OI1). The evidence of this was observed when the teacher began the lesson by asking what they had learned previously. This was a way of reminding the learners, making it easier for them to use that knowledge in this lesson.

The second most observed was Indicator 4 (SPSL, OI4). The teacher praised the learners for locating the corresponding angles in parallel lines and for reasoning correctly. He said, *"a is equal to how many degrees?" "$a = 120°$"*, answered the learners. *"Why? It is because ...",* said a learner. *"Noooo us mathematician don't say 'because'. We just say?" "Vertically opposite angle",* replied a learner. *"Thank you that is how we speak mathematically",* said Mr Kondja [L4: P12, L4].

The least observed indicators were 2 and 3 (SPSL, OI2 and 3). The teacher facilitated the discussion on mistakes saying, *"Which angle is easy to find maybe? Coz when you are given a problem like this, you need to find those angles that do not need calculations! Which one maybe?" "a"*, said the learners [L4: P12, L1]. In this way the teacher was providing a possible technique of how best learners can approach and tackle the question, making them not to struggle so much,

5.2.7.8 Eliciting and Using Evidence of Learners' Thinking (EUELT)

Table 5.32: Frequency of EUELT in Lesson 4

Observable Indicators	Code of the Observable Indicator	Frequency
1. Let learners talk about the problem or their solution with their partner in front of the whole class	EUELT, OI1	0
2. Respond to learners with questions and prompts to probe, scaffold and extend their knowledge.	EUELT, OI2	4
3. Use learners' thinking to address mistakes and misconceptions around the concepts.	EUELT, OI3	1
4. Use tasks that require learners to explain, present, and justify their mathematical ideas.	EUELT, OI4	3
Total		8

The data in Table 5.32 shows that three indicators of EUELT were observed in this lesson. Evidence of Indicator 2 (EUELT, OI2) was observed most. As in Lessons 1, 2 and 3, the teacher used probing questions to teach. Mann (2014) acknowledges that asking questions is one way of eliciting learners' thinking, as it appears in their reasoning. The teacher then used their reasoning as a basis for making instructional decisions.

The second most observed was Indicator 4. Mr Kondja provided learners with angle properties and models for justifying their ideas/solutions. The least observed was Indicator 3 (EUELT, OI3) – using learners' thinking in teaching. The teacher asked, *"what are parallel lines?" "Lines that are like this",* said a learner. *"Any lines which are like this*? *Lonia, make it clear please!"* [L4: P2, L1]. The misconception around parallel lines which appeared in the learners' response is that they viewed parallel lines as any lines that are going in the same direction. Hence, the teacher addressed this misconception by saying "*Parallel lines are lines that are going in the same direction like these rulers*, *but the distance between them remains the same"* [L4: P2, L4].

5.2.8 Reflection on Lesson 4

This section contains the transcript of the interview reflecting on Lesson 4, between Mr. Kondja and the interviewer (SF).

SF: How was the lesson today?

Mr. Kondja: The lesson was good. Learners participated well. The classroom atmosphere was live and I liked it. Everyone was happy and smiling. I also liked how they were responding. I never saw how the lesson got finished.

SF: Have you experienced any challenges?

Mr. Kondja: No, no, no! Only maybe on some learners who were copying while I was teaching. However, after I told them to stop, everything was fine. I have learned something today, when the lesson is such good, the issue of discipline disappears. Since I stop those learners who were copying, I cannot remember reprimanding learners anymore.

SF: Any improvement for the next lesson?

Mr. Kondja: I was truly disappointed by the ink which is finished in the photocopy machine, hence I could not print out the worksheet for learners. So, I struggle to write the activity on the board. This has consumed most of my time, and also it creates a gap distracting learners and giving them a chance to think of other things not related to mathematics when I was writing on the board. They could be more focused if I could have a worksheet to give immediately after the explanation.

5.2.9 Overview of Lesson 5 (Reflection)

This was the fifth and last lesson I observed of Mr. Kondja teaching Grade 9. The lesson was on reflection, a subtopic under transformation. After greeting the learners, Mr. Kondja introduced the lesson by saying *"Today we will talk about transformation, and under transformation, we will focus on reflection"*. He wrote it on the chalkboard. After that the teacher began the lesson by defining transformation, giving topics falling under transformation for different grades. The teacher then wrote on the chalkboard, the two main objectives of the lesson, namely identifying reflected figures and reflecting figures.

Before the teacher demonstrated how to reflect figures, he grouped the learners, gave them mirrors and then asked one learner per group to draw a triangle in the summary book. The teacher used the mirror and the triangle to investigate the properties of reflection by placing it at different positions around the triangle. He focused on these aspects: a) position of image; b) nearest point; c) size; d) shape; and e) distance between the object and the line of reflection, and between the line of reflection and the image. After the investigation, the teacher summarised the properties of reflection and gave the class activities where the learners had to plot first before reflecting. As the learners were busy doing the class work, the teacher moved around the class assisting struggling learners and complimenting non-struggling learners. The lesson was 80 minutes long.

The analytical tool in Appendix B: Classroom observation and analytical tool was used to analyse this lesson and the results are presented on the tables below.

5.2.9.1 Establishing Mathematical Goals to Focus Learning (EMGFL)

Table 5.33: Frequency of EMGFL in Lesson 5

Observable Indicators	Code of the Observable Indicator	Frequency
1. The teacher states the topic and lesson objective and clarifies what learners are expected to learn.	EMGFL, OI1	2
2. The teacher demonstrates a good understanding of mathematical learning goals and how the lesson will unfold.	EMGFL, OI2	12
3. The teacher refers discussion to lesson goals and objectives during instructions.	EMGFL, OI3	5
4. The teacher use ordered instructions and tasks that are aimed at achieving the lesson goals and objectives.	EMGFL, OI4	9
5. The teacher assesses and monitors learners' understanding toward the learning goal.	EMGFL, OI5	12
Total		**40**

The data in Table 5.33 shows that all the indicators of EMGFL were observed in this lesson. The dominant indicators were Indicators 2 and 5 (EMGFL, OI2 and 5). The teacher defined all the key concepts, directed the lesson and monitored learners' progress toward the lesson objectives. The second most observed was Indicator 4 (EMFGL, OI4). The teacher structured the lesson in a coherent manner by first defining transformations, followed by an investigation of the properties of reflection and lastly the activity to assess whether the lesson objectives were achieved.

In this lesson it was worth noting the way the teacher integrated lesson goals and objectives in the lesson. He did this by saying *"Now we are moving to the last one. And this is the main objective of the lesson, to reflect the figure"* [L5: P24, L1]. This extract vividly shows how the teacher re-directed and re-focused learners' attention on the lesson goals and objectives as alluded to by (Mann, 2014).

The teacher showed strong evidence of Indicator 1 (EMFGL. OI1). Firstly, he stated the topic and wrote it on the chalkboard [L5: P1, L1], stated and wrote the lesson objectives on the chalkboard [L5: P3, L1] and then clarified what the learners were expected to know [L5: P3, L6]. These sets of skills demonstrated by Mr. Kondja are defined by Mann (2014) as effective in developing mathematical proficiency.

5.2.9.2 Implementing Tasks that Promote Reasoning and Problem Solving (ITPRPS)

Table 5.34: Frequency of ITPRPS in lesson 5

Observable Indicators	Code of the Observable Indicator	Frequency
1. Motivate learning of mathematics by using a variety of problems that build on and extend learners' understanding.	ITPRPS, OI1	5
2. Select tasks that can be solved in various ways. including the use of visuals.	ITPRPS, OI2	2
3. Give tasks on a regular basis that require a high level of cognitive demand.	ITPRPS, OI3	0
4. Support learners in solving tasks without taking over their thinking.	ITPRPS, OI4	10
5. Encourage learners to use various approaches to understand and solve problems.	ITPRPS, OI5	1
Total		**18**

The data in Table 5.34 reveals that four indicators of ITPRPS were observed in this lesson. Evidence of Indicator 4 was predominantly observed. Because the lesson was on reflection, the teacher provided the learners with coordinate points which they had to plot and join to come up with figures to reflect. Most learners struggled with plotting the points because they had covered this topic a couple of days ago, hence evidence of Indicator 4 was frequent. One of the extracts showing how the teacher supported learners was: *"Totadimwe, how did you plot this?" "I used this two and this…,"* said Totadimwe. *"Ok, look at your mistake, always the first coordinate is from the x-axis, and the second one is from the y-axis. Ok, on the x-axis, show me 2! Ok, hold it with this figure. Ok, then on the y-axis, where is 3?" "This one"*, said Totadimwe. *"Ok, hold it with this finger. Now where did this two lines meet?" "Here*", replied Totadimwe. *"Very good, that is where you have to put the point"* [L5: P21, L1]. This clearly shows how the teacher helped a learner without taking over his thinking.

The second most observed was Indicator 1 (ITPRPS, OI1), and the least observed was Indicator 5 (ITPRPS, OI5). Here, the teacher demonstrated how to reflect figures in two ways; by using a mirror and by counting units. Lastly, the data indicates that no evidence of tasks that require high level of cognitive demand were observed in this lesson. In my view, these learners were still at the infant stage of grasping the concept of reflection.

5.2.9.3 Use of Visual Models (UVM)

Table 5.35: Frequency of UVM in Lesson 5.

Observable Indicators	Code of the observable Indicator	Frequency
1. Introduce forms of representation (visuals) that can be useful to learners	UVM, OI1	6
2. Use representation (visuals) to explain, demonstrate, connect mathematical ideas and solve problems.	UVM, OI2	7
3. Focus learners' attention on the essential features of the mathematical ideas which appears	UVM, OI3	1
Total		**14**

The data in Table 5.35 reveals that all indicators of UVM were evident in this lesson. The data further showed that the use of visuals to explain ideas was the most observed (UVM, OI1). The extract below shows how the teacher used visuals to demonstrate the properties of reflection. He said *"Ok, put your mirror here!* (pointing) *"Ok, do you see something in that mirror?" "Yes",* said the learners. *"Now you have two pictures! Hashol? This one is the chalkboard is called the object! The original figure and how do we call that one you are seeing there?" "It's called the image",* the learners responded. *"I will start using those words now! Remember, object, image, line of reflection* (while pointing at them). *When you put your mirror here, where is the image? Show me on the chalkboard: All of you please! Which side?" "There",* said the learners while pointing to the left [L5: P5, L1].

The data also shows that the teacher used other forms of visual models useful to learners. These were writing on the chalkboard [L5: P1, L2], turning around to demonstrate rotation [L5: P2, L4], drawing figures on the chalkboard [L5: P2, L4], hand gesture such as moving his arms outwards to demonstrate enlargement [L5: P2, L4], hand gestures to demonstrate stretching [L5: P2, L6] and mirrors for demonstrating the properties of reflection [L5: P3, L2].

The least observed evidence was the focusing of learners' attention on the meaning appearing in the model. The teacher focused the learners' attention on the properties of reflection depicted by the mirror by saying *"When you put your mirror here, where is the image? Show me on the chalkboard: All of you please! Which side?" "There",* said the learners while pointing to the left. *"Here on the chalkboard, the nearest point on the line of reflection is C. Who is the nearest point there on the image?" "It['s] C",* said the learners. *"Here, the furthest point from the line of reflection is B, who is the furthest there on the image?" "It['s] B",* said the learners. *"... between the image and the object, which one is bigger?" "Equal",* the learners replied. *"On the shape, this one is a triangle, that one?" "Triangle also*", said the learners [L5: P6, L2]. In other words, the teacher used the mirror to show the learners that the position, size and shape of the reflected image remained the same.

5.2.9.4 Facilitating Meaningful Mathematical Discourse (FMMD)

Table 5.36: Frequency of FMMD in lesson 5

Observable Indicators	Code of the Observable Indicator	Frequency
1. Engage learners in the lesson through class discussions and presentations, and through other forms of verbal, visual, and written communication.	FMMD, OI1	20
2. Encourage learners to share ideas through class discussions.	FMMD, OI2	3
3. Connect learners' responses to key mathematical ideas.	FMMD, OI3	0
4. Use probing questions and clues to make connections among mathematical ideas.	FMMD, OI4	8
Total		**31**

The data in Table 5.36 shows that three indicators of FMMD were observed in this lesson. The data further reveals that the learners were highly engaged in this lesson, particularly in the investigation of the properties of reflection using the mirrors. As the teacher engaged learners in the lesson, he made use of probing questions and clues to help the learners make sense of reflection. The least observed indicator was the encouragement of learners to share ideas in the lesson which was observed three times (FMMD, OI2). This was observed mainly as a result of grouping learners and giving them mirrors to investigate the properties of reflection.

5.2.9.5 Posing Purposeful Questions (PPQ)

Table 5.37: Frequency of PPQ in Lesson 5

Observable Indicators	Code of the Observable Indicator	Frequency
1. Use a variety of questions (gather information, probe understanding, make the mathematics visible, and ask students to reflect on and justify their reasoning)	PPQ, OI1	15
2. Use patterns of questions aimed at focusing learners' attention and thinking on the concept	PPQ, OI2	6
3. Give learners sufficient time for learners to formulate and offer responses.	PPQ, OI3	2
Total		**23**

The data in Table 5.37 shows that all three indicators of PPQ were observed in this lesson, where the teacher used various questions such as information-gathering questions and probing questions. In addition, questions aimed at learners to reflect and justify their reasoning, were also observed. An example of these questions is *"If you put the mirror here, will you see the image there?"* [L5: P28, L1]. This question was aimed at a learner to reflect on whether he had drawn the image in the correct position.

The data further shows that the teacher used pattern questions to focus learners' attention and thinking on the particular concept. For example, when he was investigating the properties of reflection, he used the IRE pattern of questioning to keep the learners focused. Evidence of pattern questions were also observed in the four previous lessons.

The least observed was Indicator 3 (PPQ, OI3). It was observed only twice. On one occasion, the teacher said *"Are you all done? We want to move to the next step"* [L5: P18, L5]. This indicates that the teacher was making sure that they were done with the first step before moving to the next one.

5.2.9.6 Building Procedural Fluency from Conceptual understanding (BPFCU)

Table 5.38: Frequency of BPFCU in lesson 5

Observable Indicators	Code of the Observable Indicator	Frequency
1. The teacher provides learners with the underlying meaning of the concept.	BPFCU, OI1	8
2. Learners are provided the opportunity to practise procedures, including using their own strategies.	BPFCU, OI2	7
3. Learners are asked to explain their strategies and the teacher refines them to more effective procedures.	BPFCU, OI3	3
4. The teacher uses visual models to support learners' understanding of general methods.	BPFCU, OI4	11
Total		**29**

The data see Table 5.38 shows that all four indicators of BPFCU were observed in this lesson. The teacher used visuals to strengthen his explanation. For example, he used hand gestures (moving his arms outward) to support the learners' understanding of enlargement.

The second indicator most observed showed the teacher explaining key concepts (BPFCU, OI1). Some of the key concepts in this lesson were: transformation, reflection, graphs and equations. Regarding equations, he explained the meaning of $x - 2$ by saying *"What is the meaning of* $x = -2$ *again? You are going to cut which axis?" "x-axis*", replied the learners. *"At which number?" "At* -2*"* said the learners [L5: P21, L4]. Explaining the underlying meaning helped the learners to see sense in mathematics and to desist from the belief that mathematics does not make sense or it is just a set of rules and procedures (Kilpatrick et al., 2001).

Evidence of Indicator 4 showed the learners being given an opportunity to practise the procedure. They were actively involved in the investigation of the properties of reflection and thereafter they were given classwork to plot and reflect figures.

Lastly, there was no evidence of a learner coming up with a new strategy different from the approach given by the teacher. However, there was evidence of the teacher asking learners to explain how they plotted their points or reflected their figures.

5.2.9.7 Supporting Productive Struggle in Learning (SPSL)

Table 5.39: Frequency of SPSL in lesson 5

Observable Indicators	Code of the Observable Indicator	Frequency
1. Anticipate what learners might struggle with and how best to support them.	SPSL, OI1	8
2. Help learners realise that confusion and errors are a natural part of learning, by facilitating discussions on mistakes, misconceptions and struggles	SPSL, OI2	9
3. Give learners sufficient time to struggle with tasks.	SPSL, OI3	1
4. Praise learners for their efforts in making sense of mathematical ideas and perseverance in reasoning through problems.	SPSL, OI4	5
Total		**23**

The data in Table 5.39) shows that all indicators of SPSL were observed in this lesson. Many learners were struggling with plotting, mainly because the topic on plotting points was done a few days before. The teacher guided the learners who were struggling. . This further reminds us that if the lesson contains components covered a while beforehand, a short revision is required. Otherwise the teacher should expect the learners to struggle, as observed in this lesson. When they are subjected to this type of struggle it can affect their confidence, which may eventually lead to them developing a negative mathematical image and disposition (Graven & Buytenhuys, 2011).

Evidence of Indicator 1 (SPSL, OI1) was when the teacher anticipated the learners' struggles. Evidence was observed when the teacher explained transformation, giving the types of transformations. Giving a detailed explanation of transformation helped the learners to have a broader picture of transformation and at the same time, helped them realise that reflection is the easiest type of transformation. Regarding Indicator 4 (SPSL, OI4), in numerous cases the teacher praised the learners for understanding the properties of reflection through the investigation and during the activity. An example of this evidence was *"Put it here. Are you there?" "Yes",* said the learners. *"We are about to finish!! Where is the image? Show me show me!" "Down, down, down"*, said the learners. *"Ok, the nearest points here are C and B, eeee* *on the image?" "C and B!",* the learners exclaimed. *"Ok, is it still a triangle?" "Yes*", said the learners. *"Ok thank you, give yourself a round of applause!"* [L5: P8, L1].

The least observed indicator of SPSL in this lesson was the time allocated to learners to struggle with tasks (SPSL, OI3). This was only observed once. Despite the teacher having prepared two tasks, time allowed the learners to only complete one task. Moreover, as they were completing the first task, there was no evidence of the teacher rushing them or complaining that they were too slow.

5.2.9.8 Eliciting and Using Evidence of Learners' Thinking (EUELT)

Table 5.40: Frequency of EUELT in Lesson 5

Observable Indicators	Code of the Observable Indicator	Frequency
1. Let learners talk about the problem or their solution with their partner in front of the whole class.	EUELT, OI1	2
2. Respond to learners with questions and prompts to probe, scaffold and extend their knowledge.	EUELT, OI2	7
3. Use learners' thinking to address mistakes. and misconceptions around the concepts	EUELT, OI3	5
4. Use tasks that require learners to explain, present and justify their mathematical ideas.	EUELT, OI4	3
Total		**17**

The data in Table 5.40) shows that all four indicators of EUELT were observed in this lesson, with evidence of Indicator 2 (EUELT, OI2) predominantly observed. The teacher interacted with the learners during the lesson mainly by using probing questions to scaffold them. However, there was a case observed where a learner needed to be told what to do because he had little understanding of plotting. The second most observed indicator was the teacher using learners' thinking to address mistakes. This was observed when the teacher was helping a learner plot a point. He asked the learner *"Totadimwe, how did you plot this?" "I used this two and this…",* said Totadimwe. *"Ok! Look at your mistake, always the first coordinate is from the x-axis, and the second one is from the y-axis. Ok on the x-axis, show me 2, ok hold it with this figure. Ok then on the y-axis, where is 3?" "This one!"* said the learner. *"Ok, hold it with this finger! Now where did this two lines meet?" "Here",* replied the learner. *"Very good, that is where you have to put the point. Do the next one then"* [L5: P29, L1].

The third most observed indicator showed that all the tasks required the learners to justify their answers because they had to plot according to the coordinates and to reflect as per the instructions (EUELT, OI4). Lastly, evidence of learners sharing the problem with others was minimally observed because the class work was done individually.

5.2.10 Reflection on Lesson 5

This section contains the transcript of the interview reflecting on Lesson 5, between Mr. Kondja and the interviewer (SF).

> *SF: How was the lesson today?*
>
> *Mr. Kondja: The lesson was good. Learners were participating actively. So, no problem. Also, I did not have any disciplinary issues.*
>
> *SF: Have you experienced any challenges?*
>
> *Mr. Kondja: Yes! There was a group of learners at the back who were struggling with plotting. Since there was no better learner in the group to help others, I asked them to join other groups so they can learn from others. Some learners were too fast also, making it difficult for me, and rushing the struggling learners.*
>
> *SF: Any improvement for the next lesson?*
>
> *Mr. Kondja: I need to have a lot of activities to accommodate fast learners. Also, slow learners were not supposed to be in their group. Next time I will dictate the group composition or arrangement.*

5.3 CONCLUSION

In this chapter, I presented, analysed and discussed the data generated from the lesson observation. This was a vertical analysis to show fine details in order to report on what and how the teacher taught. The observation data were aimed at answering Research Question 2. I started this chapter with the profile of the participating teacher, followed by the presentation and discussion of data from each lesson. The overview of each lesson is also given before the presentation and discussion. The reflection for each lesson is also presented at the end of each lesson.

In the next chapter, I horizontally discuss the findings from my observations using the analytical tool in Appendix B: Classroom observation and analytical tool.

CHAPTER SIX: ANALYSIS OF LESSONS AND TEACHING PRACTICES

6.1 INTRODUCTION

In Chapter Five I presented, analysed and discussed the finding from the lesson observation, focusing on how the participant teacher taught for a productive disposition. In this chapter, I horizontally analyse and discuss the findings from the lesson observations in relation to growing a productive disposition. The finding from both chapter five and six are aimed at answering my research question two; *how do selected teacher teach for productive disposition as a result of participating in the intervention programme?* The discussion is done as per the eight teaching practices across all the lessons. The discussion was based on the summary percentage frequency of indicators of each teaching practices in each lesson.

6.2 ANALYSIS AND DISCUSSION OF DATA ACROSS THE LESSONS TAUGHT

6.2.1 Establishing Mathematical Goals to Focus Learning (EMGFL)

Table 6.1: Summary showing EMGFL in the five lessons

Observable Indicators	Lesson 1 Frequency (%)	Lesson 2 Frequency (%)	Lesson 3 Frequency (%)	Lesson 4 Frequency (%)	Lesson 5 Frequency (%)	Total
1. The teacher states the topic and lesson objective and clarifies what learners are expected to learn.	3.1	5.3	3.8	8	5	25.2
2. The teacher demonstrates a good understanding of mathematical learning goals and how the lesson will unfold.	34.4	47.4	34.6	36	30	182.4
3. The teacher refers discussion to lesson goals and objectives during instructions.	12.5	5.3	7.7	4	12.5	42
4. The teacher uses ordered instructions and tasks that are aimed at achieving the lesson goals and objectives.	25	15.7	30.8	32	22.5	126
5. The teacher assesses and monitors learners' understanding toward the learning goals.	25	26.3	23.1	20	30	124.4
Total	**100**	**100**	**100**	**100**	**100**	**500**

The data in Table 6.1 highlights the teaching qualities that Mr. Kondja displayed across the five lessons. On the whole, Mr. Kondja demonstrated a good understanding of the lesson goals and objectives, as can be seen from the high frequency levels of Indicator 2 (EMFGL, OI2), which was observed the most. This presumably influenced how he instructed and gave tasks to align to the lesson goals and objectives. It is thus not surprising that Indicator 4 (EMFGL, OI4) was observed the second most times (see Table 6.1 above). In addition, the teacher used both the instructions and tasks to assess the learners' understanding of the learning goals. In short, the data reveals that Mr. Kondja taught his lessons with a clear understanding of the lesson goals and objectives with well thought-through activities and problems to use in the lesson. The teacher sequenced his activities so that learners could easily see how mathematical ideas build and relate to one another. When learners start seeing how mathematical ideas build and relate to one another, they eventually begin to view mathematics as a coherent and connected discipline (Kilpatrick et al., 2001). Furthermore, the teacher developed good assessment tools for measuring whether the lesson goals and objectives were achieved (Mann, 2014). All these played a role in making the lesson interesting and to build confidence and contentment on the part of the teacher. This all contributes to learners building a productive disposition (Beilock et al., 2010; Kilpatrick et al., 2001).

6.2.2 Implementing Tasks that Promote Reasoning and Problem Solving (ITPRPS)

Table 6.2: Summary showing ITPRS in the five lessons

Observable Indicators	Lesson 1 Frequency (%)	Lesson 2 Frequency (%)	Lesson 3 Frequency (%)	Lesson 4 Frequency (%)	Lesson 5 Frequency (%)	Total
1. Motivate learning of mathematics by using a variety of problems that build on and extend learners' understanding.	27.6	29.4	29.4	33.4	27.8	147.6
2. Select tasks that can be solved in various ways, including the use of visuals.	31	35.3	35.3	33.3	11.1	146
3. Give tasks on a regular basis that require a high level of cognitive demand.	6.9	5.9	11.8	0	0	24.6
4. Support learners in solving tasks without taking over their thinking.	6.9	5.9	0	0	55.5	68.3
5. Encourage learners to use various approaches to understand and solve problems.	27.6	23.5	23.5	33.3	5.6	113.5
Total	**100**	**100**	**100**	**100**	**100**	**500**

Effective mathematics teaching uses tasks as one way of motivating learning. The data in Table 6.2 shows that Mr. Kondja frequently used a variety of tasks to motivate learning. In addition, he used various ways to solve those tasks, including visuals such as the number line, bottle caps and algebraic tiles, as observed in various lessons. Tasks are essential tools in focusing learners' attention on a particular mathematical idea. Well thought-through tasks (including those that require high cognitive demand) motivate learners to engage in tasks of different levels. It was through their engagement in tasks that the learners constructed meaning for the mathematical concepts they were learning (Vygotsky, 1978). To Vygotsky, learning takes place by doing tasks in context. In addition, it is through engaging with tasks that learners participate in mathematical activities and feel good about their mathematics. It is important that their interest in mathematical activities is aroused and sustained so that they see themselves as doers of mathematics (Kilpatrick et al., 2001) and see mathematics as a worthwhile activity.

The data further shows that the teacher frequently used different teaching methods to understand and solve problems. The use of different methods afforded learners an opportunity to engage with the tasks from different perspectives. Also, the act of using different methods in solving these tasks was inspirational to the learners and added to their curiosity. It also encouraged learners to adopt and try them, thus contributing to developing a positive disposition.

6.2.3 Use of Visual Models

Table 6.3: Summary of UVM in the five lessons

Observable Indicators	Lesson 1 Frequency (%)	Lesson 2 Frequency (%)	Lesson 3 Frequency (%)	Lesson 4 Frequency (%)	Lesson 5 Frequency (%)	Total
1. Introduce forms of representation (visuals) that can be useful to learners	28	30	33.3	30.8	42.9	165
2. Use representations (visuals) to explain, demonstrate, connect mathematical ideas and solve problems.	52	50	50	53.8	50	255.8
3. Focus learners' attention on the essential features of the mathematical ideas which appears	20	20	16.7	15.4	7.1	79.2
Total	**100**	**100**	**100**	**100**	**100**	**500**

From the data in Table 6.3, it emerged that the teacher frequently used visuals to explain mathematical ideas in various topics. The use of visuals in various topics aligns with Boaler et al. (2016) who claimed that all mathematics concepts can be illustrated or thought about visually. The frequent use of visuals reveals two things: firstly, that Mr. Kondja had discovered the effectiveness of visuals in explaining mathematical ideas, because in my view, one only clings to an approach if it is working best for them; secondly, that the frequent use of visuals to explain mathematical ideas led to learners' active participation and sense making of mathematical concepts, as highlighted in the Lesson 1 reflection. In terms of sense making and participation, Arcavi (2003) also confirmed that visuals help learners to advance their understanding of mathematical concepts, to make sense of the problem and to actively engage in mathematical discourse. In this study, seeing sense in mathematical problems, active participation and engagement in mathematical discourse are excellent indicators of learners growing productive dispositions.

The data further shows that the teacher made use of various representations that are helpful to learners during teaching. These included number lines, drawing and gesturing. They were used for various purposes, including the reinforcement and cementing of the teacher's explanations. According to Phillips et al. (2010), it is worth using representations because they assist for long-term memory processes – and are remembered for longer than words. This might have motivated learners to keep trying mathematical problems on their own, eliminating mathematical anxiety and changing their beliefs that mathematics is a difficult subject as viewed by many, and that it can be learned by all (Feldhaus, 2014).

6.2.4 Facilitating Meaningful Mathematical Discourse

Table 6.4: Summary showing FMMD in the five lessons

Observable Indicators	**Lesson 1 Frequency (%)**	**Lesson 2 Frequency (%)**	**Lesson 3 Frequency (%)**	**Lesson 4 Frequency (%)**	**Lesson 5 Frequency (%)**	**Total**
1. Engage learners in the lesson through class discussions and presentations, and through other forms of verbal, visual, and written communication.	51.3	46.1	46.2	55	64.5	263.1
2. Encourage learners to share ideas through class discussions.	17.9	23.1	15.4	10	9.7	76.1
3. Connect learners' responses to key mathematical ideas.	25.6	15.4	0	5	0	46
4. Use probing questions and clues to make connections among mathematical ideas.	28.2	15.4	38.4	25	25.8	132.8
Total	**100**	**100**	**100**	**100**	**100**	**500**

The data in Table 6.4 reveals that Mr. Kondja actively engaged learners in his lessons – particularly when using tasks. From the constructivist point of view, learning is socially constructed (Vygotsky, 1978). Children learn and enjoy learning more when they are actively involved, rather than passively listening to the teacher (Sarita, 2017). Through engagement, learners share ideas, construct convincing arguments on how mathematical ideas relate to one another and further develop their own language of mathematics. Enjoyment and active involvement in mathematical lessons are viewed as important variables in growing a mathematical productive disposition (Graven, 2012).

When learners were doing tasks, the teacher was moving around, giving support to learners. He often used probing questions and clues. This is in line with the constructivist principles which underpin this study. According to social constructivism, learning concepts meaningfully occur in an integrated context, either between peers or between the learner and the more-knowledgeable other, ie the teacher in this context (Vygotsky, 1978). Vygotsky further recognised human mediation as an integral part of learning. His view is that learners who receive support from the teacher who tries to help them make sense of the concept/problem without telling them what to do, become better learners. Having a guaranteed support – in this context during the learning process the teacher was moving around the class – safeguarded the learners' learning process because the teacher was purposefully there to give support to struggling learners. This might have encouraged many learners to try. Furthermore, the support the teacher gave to learners when they were stuck, in the form of probing questions, further helped the learners discover the science of mathematics, which can further lead to them developing an interest in mathematics.

6.2.5 Posing Purposeful Questions

Table 6.5: Summary of PPQ in the five lessons

Observable Indicators	**Lesson 1 Frequency (%)**	**Lesson 2 Frequency (%)**	**Lesson 3 Frequency (%)**	**Lesson 4 Frequency (%)**	**Lesson 5 Frequency (%)**	**Total**
1. Use a variety of questions (to gather information, probe understanding, make the mathematics visible and ask students to reflect on and justify their reasoning).	72.4	70	72.7	94.1	65.2	374.4
2. Use patterns of questions to focus learners' attention and thinking on the concept.	13.8	20	27.3	5.9	26.1	93.1
3. Give sufficient time for learners to formulate and offer responses to questions.	13.8	10	0	0	8.7	32.5
Total	**100**	**100**	**100**	**100**	**100**	**500**

The data in Table 6.5 shows that Mr. Kondja used a variety of questions during his teaching. As previously indicated, he mostly communicated to learners in question form. In other words, he did not tell the learners directly what they should do or how things should be done. I observed this practice as one way of inspiring learners to think, share ideas and to keep them focused on the lesson. Mann (2014) stressed that it is through this strategy that a teacher can ascertain what learners know, prompt them to make mathematical connections, let them explain and justify their ideas and in return, develop a culture of classroom dialogue and hopefully contribute to growing a productive disposition.

Apart from probing and gathering questions, the teacher also often used pattern questions. The most observed pattern questions were IRE patterns, where the teacher asked a question with the answer in mind. The learners then responded and the teacher evaluated the response (Mehan, 1979). Even though IRE pattern questions were mostly used to keep the learners focused, they made the lesson come to life. This promoted the learners' interest in the subject and willingness to engage with tasks (Code et al., 2016). Willingness is a strong indicator of growing a productive disposition. When learners have a strong will to participate in mathematical activities, it changes their entire self-image, confidence in particular. They begin to think and reason openly.

6.2.6 Build Procedural Fluency from Conceptual Understanding

Table 6.6: Summary showing frequency of BPFCU in the five lessons.

Observable Indicators	Lesson 1 Frequency (%)	Lesson 2 Frequency (%)	Lesson 3 Frequency (%)	Lesson 4 Frequency (%)	Lesson 5 Frequency (%)	Total
1. The teacher provides learners with the underlying meaning of the concept.	33.3	16.6	25	45.5	27.6	148
2. Learners are provided the opportunity to practice procedures, including using their own strategies.	10	16.7	12.5	9	24.2	72.4
3. Learners are asked to explain their strategies and the teacher refines then to more effective procedures.	13.3	16.7	0	0	10.3	40.3
4. The teacher uses visual models to support learners' understanding of general methods.	43.4	50	62.5	45.5	37.9	239.3
Total	**100**	**100**	**100**	**100**	**100**	**500**

The data Table 6.6 shows that Mr. Kondja often used visuals to support learners' understanding. This data is also in agreement with Table 6.3, Indicator 2 (UVM, OI2). The indicator here says more about the importance of visuals in the development of conceptual understanding and procedural fluency. Various authors have dwelt upon and written about the magic of visuals. For instance, Arcavi (2003) confirmed that visuals enhance deep understanding. Among the five topics taught by Mr. Kondja, three of those topics (integers, removing of brackets and equations) were rather abstract (Skovsmose, 2008). Phillips et al. (2010) acknowledge that abstract mathematical concepts are a challenge to learners. However, they suggested that these concepts can be meaningfully accessed through the use of appropriate visuals, as reflected in the data above. In other words, the frequent use of visual models confirms that visuals can indeed enhance mathematical understanding. This further means that Mr. Kondja's learners' understanding changed positively as a result of frequently using visuals.

Secondly, the data also shows that across the five lessons, the teacher taught for conceptual understanding of mathematical concepts. This is in agreement with Mann (2014) who recommended that learners need opportunities to practise procedures after they have established a strong conceptual foundation. In other words, learners with a strong conceptual foundation are able to explain their mathematical understanding, justify and explain their reasoning and also flexibly choose among different methods and strategies to solve problems. Ramirez et al., (2013) however caution that rushing learners to fluency undermines their confidence and interest, which can lead to mathematical anxiety and a negative disposition.

6.2.7 Supporting Productive Struggle in Learning

Table 6.7: Summary showing frequency of SPSL in the five lessons

Observable Indicators	Lesson 1 Frequency (%)	Lesson 2 Frequency (%)	Lesson 3 Frequency (%)	Lesson 4 Frequency (%)	Lesson 5 Frequency (%)	Total
1. Anticipate what learners might struggle with and how best to support them	35	33.3	50	69.2	34.8	222.3
2. Help learners realise that confusion and errors are a natural part of learning, by facilitating discussions on mistakes, misconceptions and struggles	25	33.4	25	7.7	39.2	130.3
3. Give learners sufficient time to struggle with tasks.	15	33.3	25	7.7	4.3	85.3
4. Praise learners for their efforts in making sense of mathematical ideas and perseverance in reasoning through problems.	25	0	0	15.4	21.7	62.1
Total	**100**	**100**	**100**	**100**	**100**	**500**

The data in Table 6.7 shows that across the five lessons, Mr. Kondja demonstrated strong evidence of being a proactive teacher. This means that he thought of and anticipated possible hindrances and constraints that may affect learners' learning. Minimising learning barriers in the learning process is a huge relief to learners in particular. When teachers are proactive, learners may develop willingness and confidence in learning, which may lead to them changing their perception about mathematics. The data further confirmed that in lessons where the teacher did not consider the learners' struggles and difficulties, the occurrences of mistakes and misconceptions was high. A good example was observed in Lesson 5 – Reflection. In this lesson, the teacher gave a task on reflection, but he gave coordinate points instead of a figure. Learners had to plot the point, join them to form a figure and then reflect the figure. Most learners struggled with plotting because the teacher did not think of doing a brief revision on plotting points to remind his learners. Such actions discourage learners to keep trying mathematical problems and it is viewed as one of the possible cause of mathematical anxiety (Feldhaus, 2014). To prevent this, Mann (2014) stressed that as teachers plan lessons, key components for them to consider are the leaners' struggles and misconceptions that might occur, as done by Mr. Kondja.

The data further reveals that Mr. Kondja actively assisted learners who were struggling and held discussions with them about the mistakes they made. Through these discussions learners learned to value struggle as an expected and natural part of learning (Mann, 2014). These simple successes after struggles may have inspired and encouraged learners to continue trying, making them see challenging work as an opportunity to learn and see themselves as effective learners and doers of mathematics.

6.2.8 Eliciting and Use Evidence of Learners' Thinking

Table 6.8: Summary showing frequency of EUELT in the five lessons

Observable Indicators	Lesson 1 Frequency (%)	Lesson 2 Frequency (%)	Lesson 3 Frequency (%)	Lesson 4 Frequency (%)	Lesson 5 Frequency (%)	Total
1. Let learners talk about the problem or their solution with their partner in front of the whole class.	20	28.6	16.7	0	11.8	77.1
2. Respond to learners with questions and prompts to probe, scaffold and extend their knowledge.	40	14.2	50	50	41.2	195.4
3. Use learners' thinking to address mistakes and misconceptions around the concepts.	16.7	28.6	16.7	12.5	29.4	103.9
4. Use tasks that require learners to explain, present and justify their mathematical ideas.	23.3	28.6	16.6	37.5	17.6	123.6
Total	**100**	**100**	**100**	**100**	**100**	**500**

The data Table 6.8 shows that Mr. Kondja often used questions to develop learners' thinking. This practice is encouraged by Mann (2014) who said that gathering evidence of learners' thinking should not be left to occur sporadically, but there should be a well thought-through plan to do so. Mr Kondja used various questions such as probing and gathering questions.

Through questioning, a teacher promotes learners' reasoning processes. It also assists in identifying learners' common mistakes, difficulties and misconceptions (Swan & Marshall, 2010). It is through mistakes and changes in misconceptions that learners note their shift learning.

The teacher used tasks to gain evidence of learners' thinking. As observed in various lessons, the teacher gave tasks both in groups and individually, through which the learners could reveal their thinking. This is also confirmed by Mann (2014) who said that a teacher can access learners' thinking by having all learners respond in writing and hand in their responses for further analysis.

The data further showed that the teacher addressed mistakes and misconceptions from the learners' point of view. Eliciting evidence of learners' current mathematical understanding and using it as a basis for doing mathematics is one of the effective approaches in teaching mathematics. In my view, it is always good experience when the teacher can say what is in a learner's mind. It helps learners acknowledge their thinking and reasoning as worthwhile, making them think more. On the part of the teacher, eliciting evidence of learners' thinking disrupts a teachers' traditional culture of sticking to their lesson plan when teaching. This flexibility allows the teacher to identify and address learning gaps and misconceptions immediately, before they become consolidated and more difficult to remediate (Heritage, 2008). Through addressing misconceptions, learners may vividly note their growth in knowledge, arousing their desire for learning mathematics, and hopefully contributing to their growth in mathematical productive dispositions.

6.3 CONCLUSION

In this chapter, I discussed the findings from the lesson observations according to the eight teaching practices across all the lessons taught. The discussion focused on how the teacher taught to grow learners' productive dispositions. The findings were aimed at answering Research Questions 2 and 3.

CHAPTER SEVEN: PRESENTATION AND ANALYSIS OF POST-DISPOSITION QUESTIONNAIRE

7.1 INTRODUCTION

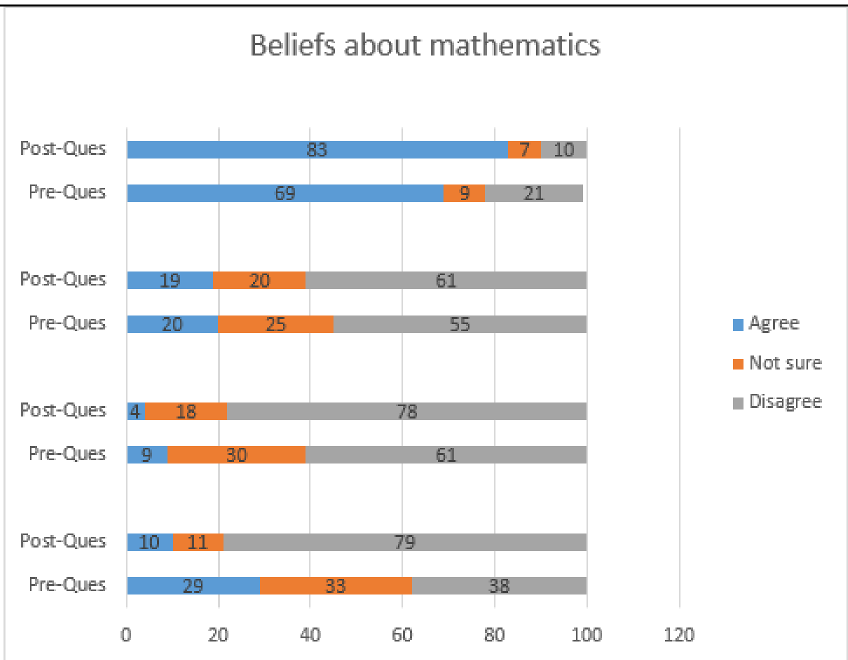
In the previous chapter, I presented, analysed and horizontally discussed the findings from the lesson observation, as per the eight teaching practices as discussed in Chapters Two and Three. In this chapter, I present, analyse and discuss the findings from the post-disposition questionnaire in comparison to the pre-disposition questionnaire. The findings from this chapter are aimed at answering my Research Question 3: *What are the selected learners' mathematical disposition after their teachers' participation in an intervention programme?* I thus discuss these results in relation to the findings from the lesson observations, Chapters Five and Six. The post-disposition questionnaire is also composed of Siegfried's (2012) six constructs and the emerged construct of engaging with tasks for assessing productive dispositions.

7.2 POST-DISPOSITION QUESTIONNAIRE AND COMPARISON TO PRE-DISPOSITION QUESTIONNAIRE

After the intervention programme, a post-disposition questionnaire similar to the pre-questionnaire was administered to all the learners who participated. The purpose of the post-disposition questionnaire was to establish learners' mathematical dispositions after the intervention programme. This section thus presents and discusses the learners' responses per construct. For each construct I have also included the results of the pre-questionnaire for comparison purposes and to remind us what the pre-questionnaire results told us.

7.2.1 Beliefs about mathematics

Table 7.1: Responses of learners' beliefs about mathematics in the pre- and post-questionnaires

4. I believe that mathematics is about real life	
3. I believe that mathematics is a set of disjoint rules and procedures	
2. I believe that mathematics does not make sense	
1. I believe that mathematics is a difficult subject	

The data in Table 7.1 shows that after the intervention programme, 79% of the respondents believe that mathematics is not a difficult subject, as opposed to 38% in the pre-questionnaire. This positive shift could be a result of a number of factors during the intervention programme. When learners responded that mathematics is not difficult, I deduced that they understood what the teacher taught them. During the intervention programme, the observation results in Table 6.1 (Summary showing EMGFL in the five lessons) showed that the teacher demonstrated a good understanding of mathematics. In addition, he used ordered instructions and tasks to help learners understand mathematical concepts. He used a variety of tasks with a variety of problems that built on and extended the learners' understanding of mathematics (see Table 6.2: Summary showing ITPRS in the five lessons). Through these teaching practices learners could easily gain mathematical understanding, which eventually changed their beliefs about mathematics.

The positive shift in learners' mathematical beliefs could also be a result of the teacher's use of visual models to develop learners' conceptual understanding of mathematics (see Table 6.6: Summary showing frequency of BPFCU in the five lessons). The teacher made extensive use of visuals in teaching all five lessons. Visual models are proven to be effective catalysts in enhancing deep mathematical understanding (Arcavi, 2003). I argue that it is through this newly gained conceptual understanding that learners positively changed their beliefs about mathematics. Notably, the data (see Table 6.7: Summary showing frequency of SPSL in the five lessons) also showed that the teacher showed strong evidence of being a proactive teacher, anticipating possible challenges and barriers that may affect learners' learning. This makes learning easier as learners are rarely exposed to errors and mistakes that may affect their confidence, hence developing a positive view of mathematics.

The data (see table 7.1 above) also shows that 78% of the respondents now believe that mathematics makes sense as opposed to 61% in the pre-questionnaire. People who are yet to discover sense in mathematics tend to rather conclude that mathematics is a set of disjointed rules and procedures. Hence, these findings reveal a parallel positive shift in respondents who now believe that mathematics makes sense and those that agree that it is not a set of disjoint rules and procedures. This positive shift is a result of using visual models in teaching. I thus agree with Arcavi (2003), and Kilpatrick et al. (2001) who argued that visuals help learners to make sense of mathematics. In Lessons 2 and 3, the teacher used algebraic tiles to teach the removal of brackets and solving of algebraic equations. In these lessons, the learners' participation and engagement with the task were high, due to the use of the algebraic tiles. In my view, active participation and meaningful engagement with a task may only occur if learners make sense of it. This is confirmed by Boaler et al. (2016) who argued that if teachers provide visual experiences of a mathematical idea for their learners that they previously only encountered numerically and abstractly, they gain a deeper understanding and start to see these ideas more meaningfully. It is through these meaningful ideas that learners begin to see sense in mathematics and begin to disbelieve that mathematics is a set of disjoint rules and procedures.

It is no surprise that a high percentage (83%) of the respondents now believe that mathematics is about real life (see Table 7.1). This positive shift is possibly attributed to the use of visual models from their communities, eg bottle caps. When the teacher introduced bottle caps as visual models to teach addition and subtraction of integers, most learners were surprised. They did not believed that bottle tops, that they mostly regard as useless in their communities, can be useful in mathematics, hence changing their views about mathematics as related to their everyday lives.

7.2.1 Affects toward mathematics

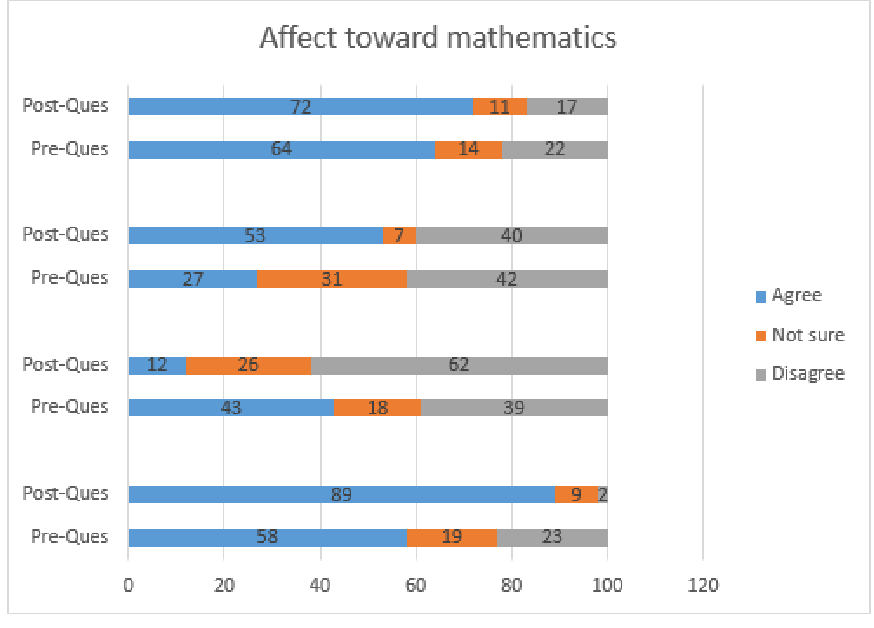
4. Mathematics is useful in our daily lives	
3. My experience about mathematics makes me hate mathematics	
2. I feel confused and frustrated in mathematics' lessons	
1. I enjoy mathematics lessons	

Table 7.2: Responses regarding the affects of mathematics.

The data (see Table 7.2 above) shows that after the intervention programme, 89% of the respondents now enjoy mathematics lessons as opposed to 58% in the pre- questionnaire. This positive shift can be attributed to a number of issues that happened during the intervention programme. Firstly, the use of visual models such as bottle tops and tiles attracted much of the learners' attention. Each learner was eager to explore using these visuals. In Lesson 1 for example, three approaches were used to teach addition and subtraction of integers: the arithmetic method, the number line method and the bottle cap method. Among the three approaches, the learners' curiosity and active engagement were observed more vividly during the third approach – bottle caps. This is evident in the Lesson 1 teacher's reflection when he said *"Eish! I do not know what to say ... I am very happy with the learners' participation"*. It was through this active engagement that learner began to enjoy mathematics. In addition, the use of multiple approaches to teach mathematical concepts may have contributed to this shift. Multiple approaches in teaching help learners to examine and understand mathematical concepts from different perspectives (Mann, 2014), thus making them enjoy mathematics lessons.

The data further showed that after the intervention programme, 62% of the respondents did not feel confused or frustrated in mathematical lessons as opposed to 39% in the pre- questionnaire. In mathematical lessons, confusion and frustration occur when learners are not given appropriate access to mathematics sense making (Graven & Heyd-Metzuyanim, 2014). This shift thus signifies that the use of visual models in teaching has helped learners to make sense of mathematics, hence a decline in the percentage of learners who feel confused and frustrated during mathematics lessons.

The percentage of respondents who hated mathematics due to their past experiences increased from 27% to 53%. From the graph, this shift occurred in the category of respondents who were not sure (see Table 7.2), since it dropped from 31% to 7%. This implies that as a result of the intervention programme, learners possibly began to reflect on their attitudes toward mathematics, concluding that they hated mathematics because of their past experience. Notably, the data (see Table 7.2) still shows that the majority of learners recognise mathematics as a useful subject in their lives. This finding concurs with Wilkins and Ma (2003) who asserted that despite mathematics being a challenging subject, we need mathematical knowledge to make informed decisions in our lives.

7.2.2 Motivation

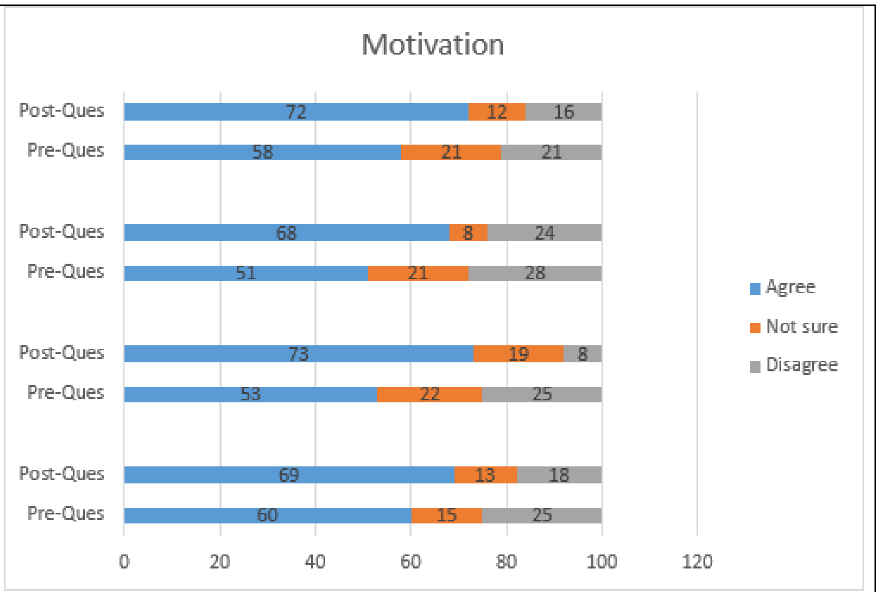
4. One day I would like to do mathematic related career	
3. I enjoy doing mathematical problems	
2. I gain satisfaction when I complete the mathematical task	
1. I am eager to learn mathematics	

Table 7.3: Learners' responses regarding motivation

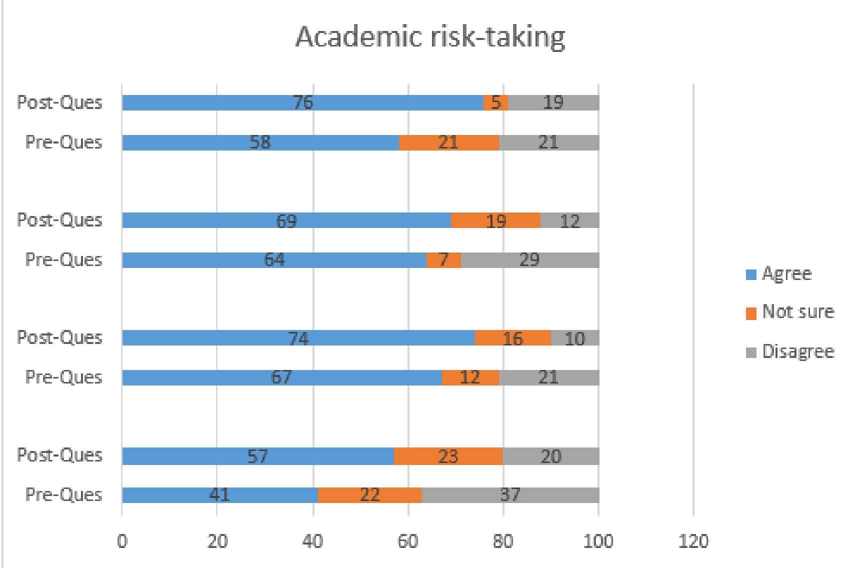
The data in Table 7.3 above shows that after the intervention programme, 69% of the respondents are now eager to learn mathematics as opposed to 60% in the pre- questionnaire. This positive shift shows that the intervention programme impacted their desire to learn mathematics. To be specific, learners' desire to learn mathematics could be raised by the conceptual understanding gained in the lessons. The teacher was frequently observed giving the conceptual meaning of key concepts (see Table 6.6 Summary showing frequency of BPFCU in the five lesson). Furthermore, in all the observed lessons, the teacher used visual models to make mathematics more visible and accessible to learners. All these may have contributed to learners' good understanding of mathematics, and hence grow their curiosity to learn more about mathematics.

The data further showed that 68% of the respondents enjoyed doing mathematical problems. From my observation, learners enjoy doing mathematical problems that they can sense of and where they can make progress. During the intervention programme, the teacher made use of visual models that significantly helped learners to make sense of mathematical concepts. However, much of their enjoyment of doing mathematical problems could be as a result of the nature of problems used by the teacher and the order in which they were given. According to Table 6.2 summarising ITPRS in the five lessons, the results show that the teacher used many examples. Secondly, these examples built on one another, growing learners' understanding from point A to B. On top of that, the teacher demonstrated how to solve these problems in various ways. In my view, these may all develop learners' love for doing mathematical problems.

It is not surprising that 72% of the respondents now indicated that they want to do mathematics-related careers as opposed to 58% in the pre- questionnaire. This shift could be justified by the learners' current status: 78% who believe that mathematics is not difficult, 78% who believe that mathematics makes sense (see Table 6.1 Summary showing EMGFL in the five lessons), 89% who now enjoy mathematics lessons and 72% who now recognise mathematics as a useful subject in their daily lives. The above Table 7.2 tells that they would not hesitate to do mathematics-related careers.

7.2.3 Academic risk-taking

Table 7.4: Learners' responses regarding academic risk-taking

4. I try using more than one approach when solving mathematical problems	
3. I try mathematical problems on my own	
2. I share strategies and solution with my classmate	
1. I ask question during mathematics lessons	

The data in Table 7.4 shows that after the intervention programme, 57% of the respondents ask questions during mathematics lessons as opposed to 41% in the pre- questionnaire. One of the attributing factors to this positive shift could be the nature of tasks used by the teacher. The results in Table 6.4 summarising the frequency of FMMD shows that across the five lessons, the teacher made strong use of tasks that require learners to explain and justify their mathematical ideas. Through this interaction, learners may not only develop a culture of clarifying and justifying their mathematical ideas but also develop a culture of asking their teacher or their peers to clarify issues that are not clear. In addition, the same table (Table 6.4 Summary showing FMMD in the five lessons) also reveals that the teacher made strong use of questions (eg probing) when responding to learners, instead of giving the answer. Vygotsky (1962) stated that learners learn by seeing a more-knowledgeable other, a teacher in this context. Consequently, the teacher's approach possibly influenced the learners – they adopted the teachers' culture of questioning issues.

The data further shows that after the intervention programme, 74% of the respondents shared their strategies and solutions with their classmates, as opposed to 67% in the pre- questionnaire. The magnitude of this positive shift (7%) correlates well with that of the teacher's efforts in encouraging learners to share strategies and solutions during teaching. For example, the results in Table 6.4 shows that the encouragement of learners to share ideas with the class was the second least observed. In addition, the results in Table 6.8 (Summary showing frequency of

EUELT in the five lessons) also show that the teacher letting learners discuss problems and their solutions with others was the least observed.

The data also show that after the intervention programme, 69% of the respondents try mathematical problems on their own as opposed to 64% in the pre-questionnaire. This positive shift could be a result of the assured support from the teacher during teaching. This is evident in Table 6.7 summarising the frequency of SPSL in the five lessons, where the teacher strongly demonstrated how to best support struggling learners. In my view, the fear of trying may only arise when one is not sure of where to get support when stuck. In addition, learners may hesitate to try mathematical problems on their own, due to the fear of making errors and mistakes. Moreover, the results in Table 6.7 further give strong evidence of the teacher helping learners to see errors, mistakes and confusions, as part of the learning process.

The impact of the teacher's use of multiple ways to teach mathematical concepts manifests in how the learners solved the mathematical problems. This is evident in Table 7.4 where 76% of the respondents have now adopted a culture of using more than one approach to solve mathematical problems as opposed to 58% in the pre-questionnaire.

7.2.5 Engagement with tasks

3. Even if the task is challenging, I don't give up. I keep trying
2. I enjoy engaging with challenging task
1. I learn from my mistake

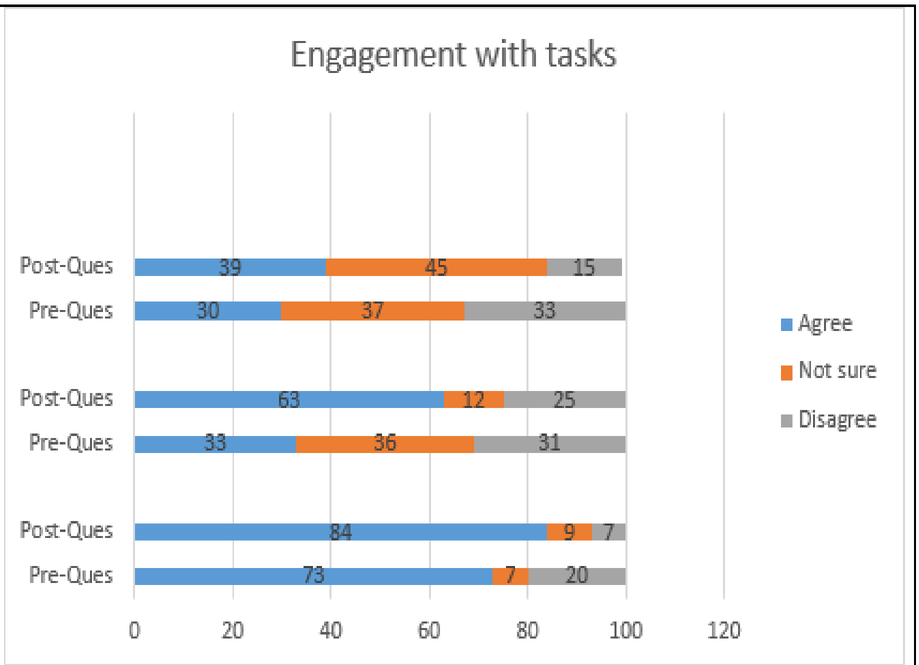


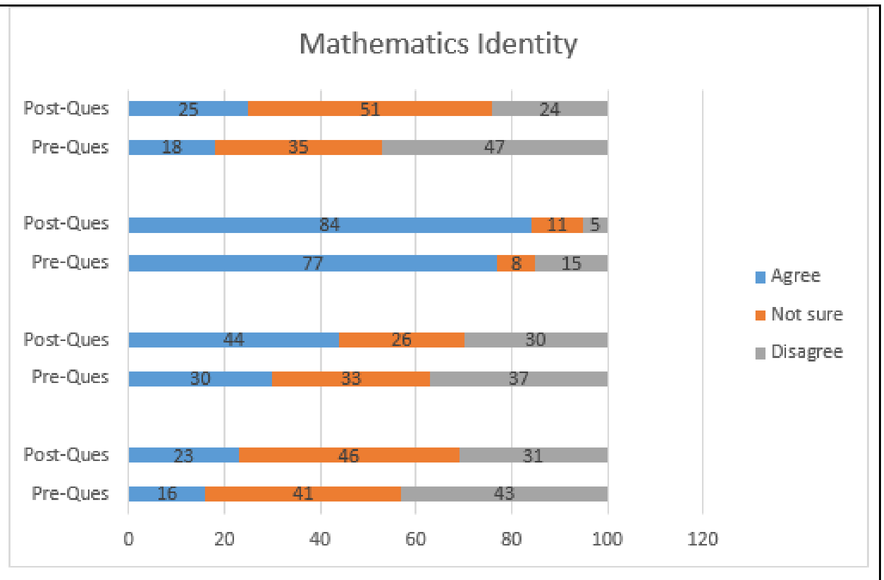
Table 7.5: Learners' responses regarding their engagement with tasks

The data in Table 7.5 shows that after the intervention programme, 84% of the respondents agree that they learn from their mistakes, as opposed to 73% in the pre-questionnaire. I view this as an indicator of learners acknowledging the value of making mistakes or experiencing failure as a means of gaining mathematical knowledge. When learners adopt this mind-set, they become more resilient (Kilpatrick et al., 2001) and willing to take risks, knowing that if they get stuck, they still gain valuable knowledge that will ultimately contribute to their overall learning.

The data further shows that after the intervention programme, 63% of the respondents now enjoy engaging with challenging tasks as opposed to 33% in the pre-questionnaire. This signifies a positive shift in their attitudes toward mathematics. As I alluded to earlier, if learners gain a good mathematical understanding, they become more confident in their abilities and begin to feel capable of tackling challenging tasks. During teaching, the teacher used many tasks, beginning from easy to complex and he also used visual models to help learners make sense of those tasks. Through this intellectual stimulation, learners may begin to enjoy challenging tasks. As a result of enjoying engaging with challenging tasks, 39% of the respondents now insist that even if the task is difficult, they will keep on trying.

7.2.6 Mathematics Identity

Table 7.6: Learners' responses regarding mathematics identity

4. I can develop strategies for solving mathematical problems	
3. I learn mathematics better when I am with others	
2. My mathematics experience was not good	
1. I am good at mathematics	

The data see Table 7.6 shows that after the intervention programme, 23% of the respondents pronounced themselves as good at mathematics. Being good at mathematics is an individual assessment that this study cannot substantiate with evidence, as there was no testing of their mathematical knowledge. However, generally, good learners in mathematics are those who have good mathematical knowledge and techniques to solve a variety of problems. Persistency and perseverance through difficulty, are also good characteristics of being good at mathematics (Kilpatrick et al., 2001). In the context of these results, this shift could be a result of learners' success in solving the various mathematical problems the teacher used (see Table 6.2 showing ITPRS in the five lessons). In addition, apart from giving sufficient tasks to practise, the teacher also ensured that learners have sufficient time to work on the tasks. Spending more time working on mathematical problems thus resulted in learners seeing themselves as effective in mathematics (Kilpatrick et al., 2001).

The data further shows that after the intervention programme, 44% of the respondents admitted that their mathematical experience was not good. Mathematical experiences involve several issues, including not enjoying mathematics or the inability to see sense in mathematics. In the intervention programme, the teacher however used visual models whose aim was to enable the learners to gain a deep understanding of mathematical concepts, which they possibly used to make comparisons with their experience.

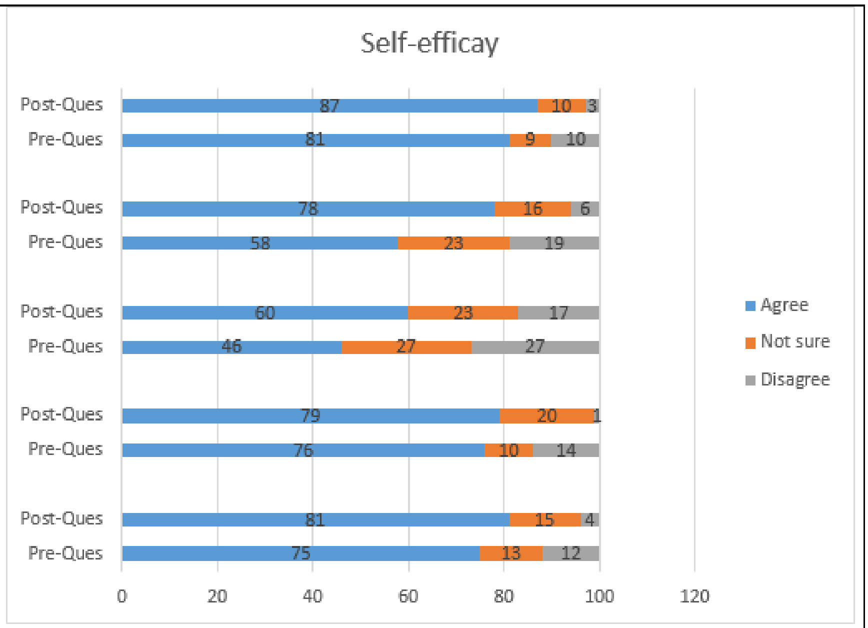
The data also shows that after the intervention programme, 84% of the respondents acknowledged that they learned better when with other learners, as opposed to 77% in the pre-questionnaire. This finding confirms Vygotsky (1962)'s stance that learners learn best when they socially interact with their peers. In the context of this study, the teacher prepared several strategies that could have contributed to this shift. Firstly, the teacher put the learners into groups, thus promoting teamwork and learning from one another. Also, he encouraged the learners to share ideas (see Table 6.4 showing FMMD in the five lessons) and discuss problems and share solutions with other learners in front of the whole class (see Table 6.8 showing EUELT in the five lessons). Learners who benefitted from this exercise may thus acknowledge learning in groups.

Notably, after the intervention programme, 25% of the respondents now view themselves as able to develop strategies for solving mathematical problems as opposed to 18% in the pre-questionnaire. This data shows that the use of visuals in teaching mathematics has indeed enhanced learners' understanding of mathematical concepts, because in my view, one may only get to the level of developing strategies for solving mathematical problems if they have a good understanding of the concept. In addition, Pérez (2018) described visual models as important tools for enhancing critical thinking. It is through critical thinking that learners come up with unique ways of solving mathematical problems.

7.2.7 Self-efficacy

Table 7.7: Learners' responses regarding self-efficacy

5. If I put more effort, I can learn and understand mathematics	
4. With time I can accomplish difficult mathematical problems	
3. I am confident in my mathematical abilities	
2. Everyone can learn mathematics	
1. I can learn mathematics	



The data in Table 7.7 shows that after the intervention programme, 81% of the respondents now see themselves as able to learn mathematics as opposed to 75% in the pre-questionnaire. This result thus shows that after the intervention programme, most learners gained confidence in their ability to succeed in mathematics. The shift in their ability to succeed might have grown as a result of continuous success in the given tasks (see Table 6.2 showing ITPRS in the five lessons). It could also be as a result of the joy that they had as they were using visuals to work on mathematical problems. Having 87% of the respondents see themselves as able to learn mathematics may have resulted in 79% of the respondents believing that everyone can learn mathematics.

The data further showed that 60% of the respondents have confidence in their mathematical abilities as opposed to 46% in the pre-questionnaire. This result means that if these learners continue to be exposed to teachings that integrate visual models, their confidence will be strengthened, making them succeed in mathematics because strong confidence drives learners' willingness to engage with tasks and spend more time working on tasks (Code et al., 2016).

The data further shows that after the intervention programme, 78% of the respondents now believe that with time they can accomplish difficult mathematical tasks, as opposed to 58% in the pre-questionnaire. This shows that the learners gained knowledge of the concepts taught. Also, it shows that they succeeded in doing the given tasks, hence it is hoped that with time, they will be able to accomplish difficult mathematical problems. It is worth noting that after the intervention programme, 87% of the respondents agree that by putting in more effort, they can learn and understand mathematics. Putting in more effort can be done in different ways, including spending more time working on mathematical problems and thinking critically (Heyd-Metzuyanim, 2013). To enhance critical thinking, the teacher used probing questions and clues to help learners make connections between mathematical ideas (see Table 6.4 showing FMMD in the five lessons). Success in seeing these connections may thus lead to learners concluding that one needs to put in more effort to understand mathematics.

7.3 CONCLUSION

In this section, I presented and discussed the data on the nature of learners' mathematical dispositions after the intervention programme, as per the Siegfried's (2012) constructs for assessing productive dispositions. This data were collected using the questionnaire and the findings were aimed at answering my Research Question 3.

CHAPTER 8: CONCLUSION AND RECOMMENDATIONS

8.1 INTRODUCTION

In this chapter, I present a summary of the findings of my study and provide some recommendations arising from these findings. The significance and the limitation of the study are also highlighted. Finally, I present the reflection on my research journey and the overall conclusion of the study.

8.2 OVERVIEW OF THE STUDY

The main goal of this study was to explore how visual models can be used to teach mathematics to grow selected learners' mathematical productive dispositions. To achieve this goal the following research questions guided the study:

1. What is the nature of selected Grade 9 learners' dispositions towards mathematics prior to an intervention programme?
2. How do selected teachers teach for productive disposition as a result of participating in an intervention programme?
3. What are the selected learners' mathematical dispositions after their teachers participated in an intervention programme?

Various data-gathering techniques were used to collect data to answer these research questions. For Research Question 1, a pre-disposition questionnaire was used to establish learners' mathematical dispositions prior to the intervention programme. For Research Question 2, I used lesson observation data to see how a teacher taught for productive disposition, as a result of their participation in the intervention programme. For the last research question, the same questionnaire was used as a post-disposition questionnaire to establish learners' mathematical dispositions after the intervention programme. In addition, teachers' lesson reflections were also deemed essential data and were used to answer the second and third research questions.

8.3 SUMMARY OF THE FINDINGS

The summary of the findings is presented below in relation to the research question. The findings of research questions 1 and 3 are presented together.

8.3.1 Research Question 2

How do selected teachers teach for productive disposition as a result of participating in an intervention programme?

8.3.1.1 Establishing Mathematical Goals to Focus Learning

In all the observed lessons, the teacher introduced the lessons and wrote the lesson topic on the chalkboard. For example, in Lesson 1, the teacher introduced the lesson by saying "*Today we will talk about addition and subtraction of integers"*. Displaying the topic on the chalkboard throughout the lesson helped learners to establish a clear focus for the lesson. It is a visual reminder to which both the teacher and the learners can stay on track (Mann, 2014).

However, in the fifth lesson, a change was observed. The teacher was observed writing both the lesson topic and the lesson objectives on the chalkboard and leaving them on the board for the whole duration of the lesson. This act ensured clear communication between the teacher and learners (Mann, 2014) as the topic provided a wider overview of what was to be covered, while the lesson objectives specified the specific learning outcomes of the lesson, affording learners an opportunity to reflect on what they learned and how it relates to the lesson objective.

In addition, the teacher was observed using the lesson objectives to redirect and refocus learners' attention during the lesson. This was evident when he said, *"Now we are moving to the last one. And this is the main objective of the lesson, to reflect figures"*.

8.3.1.2 Implementing Tasks that Promote Reasoning and Problem solving

In all the observed lessons, the teacher used various examples ranging from easy to complex. One notable observation was the design of the examples. For example, lesson 2's examples were: a) $2(x+3)$; b) $3(x-3)$; c)$2(2x-3)$; d)$2(-x-3)$; e) $5(-x+1)$. These examples were closely related with only a small difference between each one, which learners were able to establish and then solve the task/problem. To be specific, the teacher persistently used the same numbers (2 and 3) while changing the signs. This strategy made learners focus on one aspect of each example. Success in establishing these links or gaps was a joyful moment for most learners. Apart from gaining conceptual understanding, they also began to discover how mathematical ideas build on one another, making them realise that mathematics is a coherent and connected subject (Kilpatrick et al., 2001).

8.3.1.3 Use of Visual Models

In all the observed lessons, including the lessons on algebra, the teacher used visual models to explain and illustrate mathematical ideas/concepts. This finding is in line with Boaler et al. (2016) who states that there are no mathematical ideas/concepts which cannot be illustrated visually. As the teacher was using visual models to teach mathematics, most abstract mathematical concepts (Skovsmose, 2008) were brought to light. For example in the Lesson 1 example (b) $4 + -3$, the teacher solved it using bottle caps and the learners were amazed to see how the red cap (negative) and black cap (positive) eliminated each other (becoming zero) to give 1.

8.3.1.4 Facilitating Meaningful Mathematical Discourse

Learners were actively engaged in the lessons (see Table 6.4 showing FMMD in the five lessons) through class dialogue and doing tasks. Two notable practices which promoted learners' active participation and engagement in lessons were observed. Firstly, the learners were actively participating when they were solving tasks using visual models. This was further revealed by the teacher in his lesson reflection. For example in Lesson 2 he said *"Leaners started at a low pace, but they picked up later especially when we started using algebraic tiles. So tiles added a vibe to today's lesson".* The second notable practice was the teacher-learner dialogue. The teacher frequently communicated to learners using incomplete sentences, probing and giving clues, rather than giving them the exact information. For example in Lesson 1 the teacher asked: *"There is a topic where we deal with red and black colours. What is that topic?".* This approach helped learners to contribute to the lesson through thinking rather than passively receiving knowledge (Vygotsky, 1978).

8.3.1.5 Posing Purposeful Questions

As highlighted in the earlier findings, the teacher frequently communicated to learners in question form, inspiring learners to not only think, but also to keep focused on the lesson. Through this teacher-learner interaction, the teacher could evaluate what learners knew at the same time as they were engaging in thinking and providing explanations, in the process becoming active thinkers. This finding aligns to my theoretical framework, constructivism, which views learners as active agents in the process of knowledge construction (Vygotsky, 1978).

8.3.1.6 Building Procedural Fluency from Conceptual Understanding

The teacher encouraged the conceptual understanding of mathematical concepts using visual models as opposed to only using oral explanations. This was evident in the learners' active participation and engagement in the tasks. I argue that learners' active engagement in tasks is as a result of sense making.

8.3.1.7 Supporting Productive Struggle in Learning

The teacher demonstrated strong evidence of being proactive in most lessons. In other words, he thought of and anticipated possible hindrances that may affect learners' learning. For example in Lesson 1, he first wrote $7 - 3$. Thereafter, he wrote it as $(7) - (3)$. In $7 - 3$, he anticipated that learners would struggle to tell whether 3 was positive or negative. However, when he rewrote it in brackets, it clearly showed that both 7 and 3 were positive.

The consequences of not being were also revealed in this study. In Lesson 5 for example, learners were required to reflect a figure after plotting the given coordinates. The topic of plotting coordinates was however covered some weeks ago. As a result, most learners struggled to plot the points, which the teacher also acknowledged in Lesson 5's reflection by saying *"There was a group of learners at the back who were struggling with plotting".* To this Mann (2014) asserted that struggling learners may have gaps in their understanding due to previous difficulties or missed concepts, which may only be addressed through continuous revision.

8.3.1.8 Eliciting and Using Evidence of Learners' Thinking

The teacher elicited learners' thinking in various ways. Firstly, through teacher-learner dialogue, the teacher identified the learners' mistakes, difficulties and misconceptions. This was evident in Lesson 2 when the teacher corrected the learner by saying *"What is the answer*? *"Negative 5 plus….",* said a learner. Then the teacher corrected the learner by saying *"Be careful if you say -5 only without x. It's like you are referring to this small one, units!"*

Secondly, the teacher elicited learners' thinking through tasks. After the teacher gave a task, he moved around the class observing how learners were working on the task. If he found a mistake, he engaged the learner/s to establish how they did it instead of merely declaring the answer as wrong.

8.3.2 Research Questions 1 and 3

8.3.2.1 Beliefs about mathematics

The results of the pre-questionnaire showed that learners had predominantly negative beliefs about mathematics. In three of the four indicators of this construct, only 38% and less were positive (see Table 7.6 showing learners' responses regarding mathematics identity). For example, in the pre-questionnaire, only 38% of the respondents disagreed that mathematics is a difficult subject. However, in the post-questionnaire, the respondents who disagreed that mathematics is a difficult subject increased from 38% to 79%, signifying a positive shift of 41%.

On a positive note, 69% of the respondents in the pre-questionnaire indicated that mathematics is about real life. This showed that before the intervention, the majority of the learners knew the importance of mathematics in our daily lives. After the intervention programme, the results showed that this figure increased by 14%, from 69% to 83%. This finding concurs with Furinghetti and Pehkonen (2002) who postulated that individual beliefs are always changing as learners continuously evaluate them against their new experiences. In their findings, Graven and Heyd-Metzuyanim (2014) revealed that when learners are given appropriate access to mathematics sense-making environments, their beliefs about mathematics change.

8.3.2.2 Affects toward mathematics

Prior to the intervention programme, respondents showed mixed emotions toward mathematics. Results in Table 7.2 showing responses regarding the affects of mathematics revealed that 58% of the respondents enjoyed mathematical lessons, which in my view is a sign of productive disposition. At the same time, 43% of the respondents felt confused and frustrated in mathematical lessons. To Graven and Heyd-Metzuyanim (2014), learners' feelings and emotions are always changing, depending on the situation at hand. They cited that learners may experience anxiety and frustration when they are confronted with a complex mathematical problem. These figures however changed after the intervention programme. In the post-questionnaire, respondents who enjoyed mathematical lessons increased by 31%, from 58% to 89%. The respondents who do not feel confused or frustrated in mathematical lessons also increased by 23%, from 39% to 62%.

8.3.2.3 Motivation

The results of the pre-questionnaire showed that the majority of the respondents showed a productive disposition concerning motivation. In all four indicators of motivation, at least half of the respondents showed a positive motivation. For example, 60% of the respondents were eager to learn mathematics, while 58% wished to do mathematics-related careers. After the intervention programme, results in the post-questionnaire revealed a positive shift in all four indicators. The highest positive shift of 20% was recorded in the respondents who gain satisfaction when they accomplish mathematical tasks.

8.3.2.4 Academic risk-taking

Apart from motivation, more than half of the respondents showed a will to take academic risks by trying problems on their own and sharing their solutions and strategies with others. However, only less than half of the respondents (41%) indicated that they asked questions in the mathematics class. After the intervention programme, results from the post-questionnaire revealed a positive shift in all four indicators. The highest positive shift of 16% was recorded in the respondents who ask questions during mathematics lessons.

8.3.2.5 Engagement with tasks

Before the intervention programme, results from the pre-questionnaire revealed that learners' engagement with tasks was weak. For example, only 33% of the respondents enjoyed engaging with challenging tasks and 30% of the respondents said that they do not give up when trying challenging mathematical tasks. The majority (73%) of the respondents acknowledged that they learned from their mistakes. After the intervention programme, the post-questionnaire revealed a positive shift in all four indicators, with the highest increase of 30% recorded on the respondents who enjoy engaging with challenging tasks.

8.3.2.6 Mathematic Identity

Prior to the intervention study, the results in the pre-questionnaire revealed that the majority of the learners identified themselves negatively with mathematics. For example, only 16% of the respondents saw themselves as good at mathematics. However, the majority of the respondents acknowledged that they learned better when with others, concurring with Vygotsky (1978) who postulated that knowledge is socially constructed. After the intervention programme, a positive shift was recorded in all four indicators, with the highest shift of 14% recorded on the respondents who indicated that their mathematics experience was not good.

8.2.3.7 Self-efficacy

Apart from motivation and academic risk-taking, learners also showed a positive determination to learn mathematics (see Table 7.7 showing learners' responses regarding self-efficacy). In the pre-questionnaire, the majority of the learners were not confident in their mathematical abilities with only 46% being confident in their mathematical abilities. After the intervention programme, a positive shift was recorded in all five indicators, the highest of 20% was recorded on the respondents who acknowledged that with time, they can accomplish difficult mathematical problems.

8.4 OTHER DISCOVERY

This study has discovered that few studies were conducted on this strand, productive disposition. This finding is in agreement with Graven and Heyd-Metzuyanim (2014) who posist that productive disposition is neglected as many current studies focused on the four other strands. In namibia, a study like this has never been done before. In addition, the instrument (observation and analytical tool) I used was new.

8.5 RECOMMENDATIONS

At the regional level, there are senior education officers responsible for managing subjects which include providing training on successful teaching of the subjects. Based on the findings, I recommend that the senior education officers adopt Mann's (2014) eight teaching practices when training teachers on how to successfully present their lessons and how to foster a productive disposition in their learners. Teachers should also be encouraged to try these teaching practices in their day-to-day teaching because they serve as a model for presenting a successful lesson.

This study's findings acknowledge that visual models can enhance mathematical sense making and encourage a productive disposition towards mathematics. Despite that, the Namibian national curriculum is silent on the use of visual models in teaching. I thus recommend to the curriculum designers to consider making visual models mandatory in teaching. The syllabus should highlight possible visual models to be used per topic. To develop a love of using visual models in teaching mathematics, teachers should be trained in identifying visual models, designing and integrating them into their lessons. Phillips et al. (2010) cautioned that the benefit of using visual models does not come automatically, it requires teachers' experience to guide learners to make sense of them.

8.6 AREA OF FUTURE RESEARCH

This study was carried out at one school, focusing on two teachers, but reporting on only one. It would be interesting if this research was expanded to a bigger population. The study could for example, be conducted in more than one school and with more than two teachers.

Learners with negative attitudes are not peculiar to mathematics only, but in other subjects too. It would thus be interesting to see this study expanded to other subjects where learners seem to have lost interest.

In this study, learners' disposition was measured at the class group level. I recommend an in-depth longitudinal study where individual learners are followed in a series of lessons over a long period of time to determine their dispositional shift.

8.7 SIGNIFICANCE OF THE STUDY

This study explored how visual models can be used to teach mathematics to grow learners' productive dispositions. Participating teachers gained insightful knowledge about visual models in general, how to identify them and how to integrate them into the teaching of mathematics. In many cases, mathematics teachers are troubled by learners' poor participation in their lessons, lack of interest in the subjects and poor academic performance. Often, teachers view learners as not serious and irresponsible with their schoolwork, leaving little room for them to reflect on their teaching approaches. This study provides teachers with a different dimension – disposition in this context – from which they can understand learners and their performance in mathematics.

The Namibian education curriculum has been reviewed recently, making mathematics a compulsory subject from Grades 1 to 11. As subject advisers are busy conducting workshops empowering teachers on how to implement the curriculum, the findings of this study highlights to them, that apart from teachers having sound mathematics content knowledge, visual models in teaching mathematics enhance mathematics sense making. Thus, teacher training can be expanded to designing and sourcing visual models and how to use them to teach mathematics.

8.8 CHALLENGES AND LIMITATIONS OF THE STUDY

The study was conducted at the time when I assumed my official duties as the Head of Department at Ounonge Combined School. This has affected the study process and progress in many ways. For example, the recruitment process of participants took longer than anticipated because I was still at the stage of learning my new work environment and getting to know my teachers. The data collection process was also lengthy due to the interference of my office responsibilities. In addition, the world was brought to a standstill in 2020 by the pandemic (COVID-19) which affected our lives, including our schools. Schools were closed, putting on hold the data collection process until schools began with face-to-face teaching again.

Initially, I intended to conduct the study at two schools. Schools were however far apart so I ended up focusing on only one school. The data were collected during school hours. Considering that I am also a teacher, I could not fulfil my teaching obligations and travel to collect data during the day, unless I took leave.

The other challenge was the volume of data. I anticipated the study to be conducted over 12 weeks. I observed ten lessons in total, five lessons per teacher. Reporting on both teachers was thus a challenge, due to the limited scope and word count of the thesis. Thus, the data analysis only focused on one teacher. Since data were collected from such a small sample, the findings of this case study cannot be generalised to the other teachers, schools or regions in Namibia.

8.9 PERSONAL REFLECTION

I discovered my love and passion for mathematics while in primary school. However, I was unfortunate in that I did not have a teacher who could satisfy my desire for learning mathematics until I completed my first teaching diploma at the college. After I completed my BEd Hons in 2017, I developed an interest in doing my Master’s degree in mathematics education. I was however offered an opportunity to do a Master’s in Science Education which I completed in 2018.

In 2019, I then began my journey of pursuing a Master’s degree in mathematics education. It was a tough and rewarding journey. It was tough in the sense that I had to create time which I rarely had for my studies, as much of my time was occupied by school and office responsibilities. The language barrier was a challenge for me. It was difficult to put my ideas in English as accurately as I wished. It was however rewarding because it contributed to my professional growth, particularly in reading academic articles and expressing myself in English, writing and analyzing data. As a result of engaging with literature, I gained extensive knowledge which has positively improved my teaching. Even though this study has kept me away from my family for many days, it granted me an opportunity to meet fellow scholars from which I have learned much about research and other issues related to my professional career.

I am grateful for my supervisor's support and guidance throughout this academic journey. Most importantly, he was understanding. This study was supposed to be completed two years ago. Despite that, he did not give up on me. He understood my situation and supported me fully.

8.10 CONCLUSION

This chapter concludes the study. A summary of findings was presented and the significance and limitations of the study were also discussed. The recommendations are also outlined. Finally, a reflection on my experiences throughout my entire research journey concludes this chapter and thesis.

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APENDICES

Appendix A: Pre- and post-disposition questionnaire.

Keys: **SA** – Strongly Agree **NS**- Not Sure **D**- Disagree

Constructs	Questions (indicators)	Levels of agreement				
		SA	A	NS	D	SD
Beliefs about mathematics	1. I believe that mathematics is a difficult subject					
	2. I believe that mathematics does not make sense					
	3. I believe that mathematic is a set of disjoint rules and procedures					
	4. I believe that mathematics is about real life					
Affect toward mathematics	a. I enjoy mathematics lesson					
	b. I feel confused and frustrated in mathematics lessons					
	c. My experience about mathematics makes me hate mathematics					
	d. Mathematics is useful in our daily lives.					
Motivation	1. I am eager to learn mathematics					
	2. I gain satisfaction when I accomplish a mathematical task					
	3. I enjoy to do mathematical problems					
	4. One day I would like to do mathematics-related carrier.					

Academic risk taking	1. I ask questions during mathematics' lessons					
	2. I share strategies and solutions with my classmates					
	3. I try mathematical problems on my own					
	4. I try using more than one approach when solving mathematical problems.					
Engagement with tasks	1. I learn from my mistakes					
	2. I enjoy engaging with challenging tasks					
	3. Even if the task is challenging, I don't give up. I keep on trying.					
Mathematics identity	1. I am good in mathematics					
	2. My mathematics experience was not good					
	3. I learn mathematics better when I am with others					
	4. I can develop strategies for solving mathematical problems					
Self-efficacy	1. I can learn mathematics					
	2. Everyone can learn mathematics					
	3. I am confident in my mathematics abilities					
	4. With time, I can accomplish difficult mathematical problems					
	5. If I put in effort, I can learn and understand mathematics					

Appendix B: Classroom observation and analytical tool

Teaching practices	Observable Indicators The teacher:	Coding			
		No evidence	**Weak evidence**	**Evidence**	**Strong evidence**
Lessons goals and objectives	a. provides clear lesson goals and objective in the lesson plan or states it at the beginning of the lesson				
	b. uses lessons goals and objectives to guide teaching and learning				
Implement tasks that promote reasoning and problem solving	a. Provides interesting tasks				
	b. Links tasks to learners' prior knowledge.				
	c. Provide tasks that demands high level of thinking and reasoning				
	d. Motivate and reward learners during the lesson				
Use of visual models	a. Uses a variety of visual models (actions, drawings, pictures, words, physical				
	b. Use visual to make connections and explain abstract mathematical ideas/concepts				
Facilitating meaningful mathematical discourse.	a. Engage learners in the lesson and lesson' s discussions				

Teaching practices	Observable Indicators The teacher:	Coding			
		No evidence	Weak evidence	Evidence	Strong evidence
	b. Encourage learners to share ideas through classroom discussions				
	c. Use correct mathematical language to apprentice learners in mathematical discourse language				
Posing purposeful questions	a. Use questions that encourage learners to explain and reflect on their thinking				
	b. Use questions that make important mathematical connections				
Building procedural fluency from conceptual understanding	a. Focus on developing conceptual understanding by stressing the understanding of mathematical ideas				
	b. Focus on developing procedural fluency by understanding when and how to use a certain procedure.				
	c. Use a variety of approaches and strategies to problem solving				
Supporting productive struggle in learning	a. Move around the class to support learners learning				
	b. Support individual struggling learners				
Elicit and use evidence of student thinking	a. Use learners' mathematical understanding as basis in teaching and instruction				
	b. Respond to learners' mathematical understanding appropriately				

Appendix C: Ethical Clearance

RHODES UNIVERSITY
Where leaders learn

Human Ethics subcommittee
Rhodes University Ethical Standards Committee
PO Box 94, Grahamstown, 6140, South Africa
t: +27 (0) 46 603 8055
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e: ethics-committee@ru.ac.za

www.ru.ac.za/research/research/ethics
NHREC Registration no. REC-241114-045

30 April 2020

Shetu SHETUNYENGA

Email: g15s8833@campus.ru.ac.za

Review Reference: 2020-0933-3297

Dear Prof Schafer

Title: Investigating how visualization can be used to teach mathematics for A productive disposition

Principal Investigator: Prof Marc Schafer

Collaborators: Mr. Fillipus S Shetunyenga

This letter confirms that the above research proposal has been reviewed and **APPROVED** by the Rhodes University Ethical Standards Committee (RUESC) – Human Ethics (HE) sub-committee.

Approval has been granted for 1 year. An annual progress report will be required in order to renew approval for an additional period. You will receive an email notifying when the annual report is due.

Please ensure that the ethical standards committee is notified should any substantive change(s) be made, for whatever reason, during the research process. This includes changes in investigators. Please also ensure that a brief report is submitted to the ethics committee on the completion of the research. The purpose of this report is to indicate whether the research was conducted successfully, if any aspects could not be completed, or if any problems arose that the ethical standards committee should be aware of. If a thesis or dissertation arising from this research is submitted to the library's electronic theses and dissertations (ETD) repository, please notify the committee of the date of submission and/or any reference or cataloging number allocated.

Sincerely,

Appendix D: LETTER OF CONSENT TO THE REGIONAL DIRECTOR

Shetunyenga Fillipus S
P.O. Box 3290
Oshakati
Cell: 0812911661
19 March 2021

To: The Director of Education
Ohangwena Education Directorate
Private bag 88005
Eenhana

Dear Mr Isack Hamatwi

Subject: Request for permission to conduct educational research with Grade 9 mathematics' teachers at [redacted] Combined School.

I am Fillipus Shetunyenga, a part-time master student at Rhodes University (student number: 15s8833), and a mathematics and science teacher at Oluwaya Combined school. I am hereby humbly requesting your permission to conduct a mathematics education research study with Grade 9 mathematic' teachers at the above mentioned schools for a period of about twelve to fifteen weeks in the first and second term of 2021. This study is entitled: ***Exploring how visual models can be used in teaching mathematics for growing a productive disposition in Grade 9 learners***. It is under the supervision of Prof Marc Schafer, (Email: m.schafer@ru.ac.az).

Recently, the Ministry has implemented the revised curriculum whereby mathematics is now a compulsory subject from Grade 1-11. In addition, its content is more advanced than before. Being a mathematics teacher, I have noted that Grade 9 mathematics' preference is very low, and most learners seems to have a negative attitude towards the subject. This study thus aims to address this challenge by working with selected teachers' through an intervention action research programme. This study will engage selected teachers in action research cycles which involve identifying topics, collectively developing lesson plans using appropriate visual models, implementing these lessons and conducting reflections at the end of each cycle. This process is expected to benefit teachers as they are planning together and devising appropriate models and approaches to teaching and learning. The study will involve video recording and interviewing the two selected Grade 9 mathematics teachers. I will focus on how teachers are teaching using visual models during school normal hours. Each teacher is expected to present 15 lessons of which five lessons will be vide-recorded.

Lastly, I would like to assure your office that all research ethics principles will apply throughout the process of the study. The identity of the participants and their schools will be treated with a high degree of confidentiality and anonymity. The rights and responsibilities of the participants will be made clear to them, and pseudonyms will be used to protect their identity and that of the school. This study has been approved by the Rhodes University ethics committee. For further enquiry, contact the committee director, Mr. Siyanda Manqele, (Email: s.manqele@ru.ac.za.).

Thank you for your time, and your consideration in this regard will be highly appreciate

Yours sincerely

………………………………….. ……………………………………….

Fillipus S. Shetunyenga Prof Marc Schafer (supervisor)

(Rhodes student)

Please indicate your response by ticking [√] in the appropriate box below.

Permission given ☐ Permission not given ☐

Signature: ………………………………

Your cooperation will be highly appreciated

Appendix E: LETTER OF CONSENT TO THE SELECTED TEACHERS

Enquiries: Mr Fillipus SS

Cell number: 0812911661

Dear Sir/Madam

Re: Request to participate in my research study

I am Shetunyenga Fillipus S, a part-time Master student at Rhodes University, South Africa I am hereby requesting permission from you to be a participant of my research project that I will be conducting with two selected Grade 9 Mathematics teachers and their learners. The study involves observations (video recording) of five lessons per teacher and interview (focus group) at the end of the 5th cycle with all the participating teachers.

The study will concentrate on the use of visual models in teaching mathematics to grow learners' productive disposition. This study will help participating teachers to integrate visual models strategically in their lesson presentation. This study is an intervention research project. I hereby invite you to be part of this project. You and the other teacher will be expected to collectively identify topics for the intervention programme, develop lesson plans, implement the planned lessons and then reflect on the lesson presentation. You will be expected to present 15 lessons using visual models during normal school hours of which five preferred lessons (one per cycle) will be video recorded. This intervention programme will run for about 12 to 15 weeks in term one and two of 2020.

Your participation in this research study is completely voluntary and you can withdraw at any time you wish. I will ensure that your identity and views will be treated with high degree of confidentiality and anonymity. The data that will be collected will not be used for purposes other than this study. The findings may be used for conference presentations and articles. It is important that you are aware that this study has been approved by the Rhodes University director of ethical committee Mr Siyanda Manqele, email: s.manqele@ru.ac.za.

If you have any question about the research, please feel free to contact me at +264 812911661, she2ongwediva@gmail.com, my supervisor Prof Marc Schafer at m.schafer@ru.ac.za or the Rhodes University director of ethical committee Mr Siyanda Manqele, at s.manqele@ru.ac.za.

Yours Sincerely

………………….   ________________

Shetunyenga FS   Prof Marc Schafer (Supervisor)

Student (Rhodes University)

Please indicate your choice by completing the form below.

I ………………………………………… (Full name of the teacher), hereby confirming that I understand the content of this document and the nature of the research. I thus ……………………. (agree/disagree) to take part in this study.

Signature: ………………………………

Your cooperation will be highly appreciated

Appendix F: LETTER OF CONSENT TO THE PARENTS

Enquiries: Mr Fillipus SS

Cell number: 0812911661

Dear Parent

Request for participation in research on teaching mathematics using visual models.

I am Fillipus Shetunyenga Shetunyenga, a part-time Master student at Rhodes University, South Africa. You are hereby requested to give permission for your child to be part of the class where I am going to observe how his/her teacher uses visual models to teach mathematics to grow your child's positive attitude towards mathematics.

The study will consist of five cyclical lesson, of which one lesson per teacher will be video recorded. Your child will be expected to participate in pre and post-disposition questionnaires. The filming of lessons will only focus on the teachers, and not on learners. The camera will be placed at the back of the class and the faces of the learners will not be video recorded. Kindly be informed that your child's involvement in this study is voluntary, hence they are free to withdraw at any stage of the study. It is therefore the right of the parent to decide whether his/her child should take part in the disposition questionnaire or be part of the class where I am going to observe and video record. The identity of leaners will be treated with high degree of confidentiality and anonymity, and data collected will not be used for purposes other than this study. The thesis which will be produced from this study will be published and will be accessed through the Rhodes' library.

If you have any question about the research, please feel free to contact me at +264 812 911661, she2ongwediva@gmail.com, or my supervisor Prof Marc Schafer at m,schafer@ru.ac.za. This study has been approved by the Rhodes University director of ethical committee Mr Siyanda Manqele, email: s.manqele@ru.ac.za.

Yours Sincerely

FS Shetunyenga　　　　Prof Marc Schafer (Supervisor)

Student (Rhodes University)

If you agree your child to participate in this research, please complete the consent form below.

I ………………………………………… (Full name of parent/guardian), hereby confirming that I understand the content of this document and the nature of the research. I am giving permission to ……………………..………………… (Name of the child) to be part of the study.

Appendix G: LETTER OF ASSENT TO THE LEARNERS

Enquiries: Mr Fillipus SS

Cell number: 0812911661

Dear learner

Re: Request to participate in my research study

I am Fillipus Shetunyenga Shetunyenga, a part-time Master student at Rhodes University, South Africa. I am hereby requesting permission from you to be part of the class where I am going to observe how your teachers use visual models to teach mathematics to grow your positive attitude towards mathematics.

In order to do this I wish to film your teacher on five occasions. I will film from the back of the class and your faces will not appear in the film. Your identity will thus never be revealed. I have also asked your parents to agree that you can be part of this class.

All learners who will agree to take part in this study will be expected to voluntarily complete a disposition questionnaire. Information generated from this study will be used to produce my study thesis only, which will be published and accessed through Rhodes' library.

Kindly be informed that your participation in the study is voluntary. It is therefore your right to decide whether you should take part in the completion of the disposition test and questionnaire or not. I would however like to assure you that the data collected will be used only for the purposes of this study.

If you have any question about the research, please feel free to contact me at +264 812 911661, she2ongwediva@gmail.com, or my supervisor Prof Marc Schafer at m,schafer@ru.ac.za. This study has been approved by the Rhodes University director of ethical committee Mr Siyanda Manqele, email: s.manqele@ru.ac.za.

Yours Sincerely

……………………   ____________________

Shetunyenga FS   Prof Marc Schafer (Supervisor)

Student (Rhodes University)

I request you to indicate whether you are agreeable or not to be part of the lessons I wish to video record by ticking [√] in the appropriate box below.

Agree ☐ Not agree ☐ Signature: ………………………………

Your cooperation will be highly appreciated

Appendix H: EVIDENCES OF CODED LESSONS

Lesson 1 Transcript

Topic : Addition and Subtraction of Integers

The teacher entered the class. He cleaned the chalkboard and divide it into 4 parts. He then greeted the learners and then introduce the topic of the day by saying "today we will talk about addition and subtraction of integers. (*LGO: OIA)*" He wrote the topic on the board. After that he asked the class, "what are integers?"(*FMMD:OIA*) they are positive and negative number' says the learners. 'And zero," say another learner. The teacher rephrases the definition as "integers are positive, negative numbers including zero,'' and write it on the board (*FMMD:OIC)(BPFCU; OIA)*

The teacher gave the 1st example 4+3. He asked the class. " What is the answer'', '7', learners reply as a whole class. Then the teacher highlighted that, ''you can get 7 by counting on your fingers or sticks as we did in Grade 1- 3*(BPFCU: OIA, OIB,OIC*)'' Apart from counting, how else could we get the answer?*(FMMD: OIA)* says the teacher. Any other way/ tell me (*FMMD; OIC*? A learner replied, "number line." Very Good! Thank you! Says the teacher(*ITPRS: OID*). The teacher draws a number line using a meter ruler and put zero in the middle (*UVM; OIA*)(*FMMD; OIC*). ''Who can come and finish labeling?'' says the teacher(*FMMD: OIA).* All learners put their hands up. ''Ok, ok'' let's do this. Now just tell me what to write here?(*EUEST;OIA*)'' the teacher asked. "-1, -2, -3, -4, -5 say learners. ''And from here?' the teacher asked. "-1, -2, -3, -4, -5 say learners''(*FMMD: OIA*). Ok, ok, ok, thank you very much,'' teacher says(*ITPRS: OID*). Then the teacher highlights that 'from zero to the right are positive numbers, and from zero to the left are negative numbers'' clear?*(BPFCU: OIA, OIB)* Yes, says the learners.

Now let's look at the meaning of 4+3 in terms of the number line. But before, know that we always start counting from zero (*BPFCU; OIA,OIB*)'' Ok! In 4+3, is 4 a positive or negative ?, says the teacher.''Positive",learner. ''Yes positive!''(*PPQ:OIB)(EUEST; OIA)* when there is no sign like this (while pointing next to 4), its positive! And 3? ''it's positive'' says learner. ''Yes, it's positive,'' says the teacher(*BPFCU: OIA*). In terms of the number line, remember we start from zero! The sign tells us about the direction to go, either this or that (pointing left and right), while the number tell us the movement we must make'' says the teacher. 4+3 means, from zero, that zero, we go to the positive, 4 steps (while pointing), then again to the positive 3 steps and where we will end, that will be the answer(*BPFCU: OIA, OIB, OIC*). ''Can we do it now?'' ''Yes, yes sir (*FMMD; OIA*),'' learner. The teacher used colored chalks for the 3 steps, ending at 7. '' This is the answer,'' teacher.

He gave example 2, -4+3. Look at this one (*ITPRS: OIC*). From zero, where are we going? ''to the negatives,'' learners. ''How many steps? ''3 steps, learners. *(FMMD; OIA*). The teacher started drawing it. After finishing, the teacher emphasized that remember to draw your line clearly showing how you left and arrived at a point by putting arrows' (*BPFCU: OIA*)

Ok, from here, I am going to use method 3, bottle cap (while writing on the board). (BPFCU: OIC) The teacher took to the plastic with bottle caps and showed the class. ''Do you know this?'' Yes, learners (while smiling). Can I ask you to split into groups of 4? The teacher moved around the class, helping learners to form their groups. (*SPSL; OIA, OIB*)

''We only going to have 8 groups! 1,2,3,…….8. I will give this to each group, but for this group, I am going to use it for demonstration and give it later,'' teacher. Do you know where I picked them? teacher ''Nooo" learners. ''It's at cuca shops!! ''We have two colors, red and black! '' The red color is now for what? For what ? (While holding the cup). For what? Let's go to physical science! There is a topic where we deal with this red and black! What topic is that? Asked the teacher. Electricity neh? Have you done that? ''Yes," learners (*ITPRPS; OIB*) (*EUEST; OIA)*(*UVM; OIA*) ''Red color is positive and black is negative', learners. 'Aaaayee', teacher! This one is, black is positive and this red is negative! ''Noooooo! Learners'. Ok no problem, this one in Math we will call it (red) negative, while black is positive. If they are together like this, it's zero' (*UVM; OIB)(LGO; OIB)* The teacher called learners to hold caps, one red & one black while he is holding the combination, zero (*FMMD; OIA*) We are now going to apply this on integers. The teacher started from example one the example 2.

In example 1, 4+3, what do we have here? 4 what ? ''Blacks'', learner. ''plus, what? 3 what ? ''blacks'', learners. Together is? ''7'' learners, all black meaning positive (*UVM: OIA,OIB*)(*BPFCU; OIC*)

On the 2nd example: The problem starts here! What do we have here? -4, right? ''yes'', learners. (*FMMD; OIA*)(*EUEST; OIA*) Now which one is negative there? (While pointing on the 3 learners holding caps)(*FMMD; OIA)(UVM; OIB).* The red one, hasho? ''Yes '',learners. We take negative (4red caps), then add positive 3, put in one line. Ok, remember when you have a combination of red and black, what will happen? It is zero! Now here, how many combinations do we have? Are they two? ‘’No, 3’’ learners. (UVM; OIB) (BPFCU; OIA, OIC)

Ok, this is zero, this zero, this zero! What colour is this one? ‘’Negative,(*UVM; OIA, OIB*)” learners. How many caps? ‘’One’’, learners. That is the answer. ‘’uuh uuh uuh’’, learners laughing.(*EUEST; OIA*) Now how many methods do you have? ‘’3 methods, ’learners. You have 3! You can use your brain (COUNTING), You can use the number line or you can use bottle caps! Is it clear? (*BPFCU; OIA, OIB, OIC)*‘’ Yes’’ learners while smiling. (*EUEST; OIA)*

Can I go to the next example? I will do 2 more examples and then give you to practice. The next is

(4) + (-3). Think about the answer. One learner should ‘’zero’’. No! think, I said to think and think does not mean say! Think and tell your neighbor the answer!, (*FMMD; OIB*) teacher. After 2 minutes, ‘’Are you done?’’ ‘’Yeess,’’ learners. Ok tell your neighbor, just tell the answer!! I said tell your neighbor. Do not use the bottle cap plz!! Neighbors, did you hear? ‘’yees’’, learners. (*FMMD; OIB*)

Ok, first I will still do it with the number line. The teacher labelled the number line (*BPFCU; OIC*) (*UVM: OIA,OIB*) while emphasizing that learners should put the arrows at the end to show that the number line is infinite (*BPFCU; OIA, OIB*) ‘’ We said we always count from where? ‘’zero’’, learners. ‘We start here at zero, remember this direction, steps, direction steps (showing at (-4) -3. (*BPFCU; OIA, OIB, OIC*)‘’Ok from there we must go where? ‘’to the negatives’’, learners. How many steps? ‘’-7,’’ the whole class. ‘’Is that what you got from your neighbour?’’ ‘’Yeees & noooo’’, learners.(*EUEST; OIA, OIB*)(*PPQ; OIA*) Ok how will you use the bottle cap? ‘’we have this -4 plus -3,gives -7’’. Is it clear? ‘’Yeess’’, learners. (*UVM; OIA, OIB) (BPFCU; OIA, OIB)*

“ Can I put the complicated me? “ Yes”, learners. Ok !! (d) 7-3! “ 7 is positive? “1,2,3....7(counting black caps). Ok, there is 3 positive? The teacher rewrote 7-3 as (7) – (3) using brackets to explain the middle sign as an action operation. Is it clear? “Yes”, learners. can I go to the next one? “Yes”, learners. (*UVM; OIA, OIB) (BPFCU; OIA, OIB, OIC*)(*ITPRPS; OIA, OIC*)

The next one is -7 -3.(*ITPRPS; OIA, OIC*) Remember we have 3 things -7 -3. Remember we have 3 things -7 – 3. That is 7 reds, the middle sign for achon" and then 3 which are black. (*BPFCU; OIA, OIB, OIC*) The teacher put 7 red caps the asked" can we take 3 here in 7 red? " No learners. "If now black, what can we do? we just add zero! Remember zero can not change anything! "How many zero? "3 zero, because we want to subtract 3 black learners. Ok! Like this, Like no we will take the 3 blacks because we have them! What is the answer? (*UVM; OIA, OIB) (BPFCU; OIA, OIB,OIC) (PPQ; OIA, OIB*) is the answer positive or negative? "negative" Learners 1,2,3........10, negative 10! is it clear? Can I give you the class work now? "yes "learners (*FMMD; OIA, OIB, OIC) (PPQ; OIA, OIB*)

We are only going to use bottle tops!! If you are struggling, call me please!! The teacher gave a 12+18 " Help one another please"!! "Our black are not enough", Learners. (*BPFCU; OIA, OIB,OIC) (UVM; OIA, OIB) (FMMD; OIA, OIB)* I will reduce it to 2+8 then!! After the teacher moved around, "I am seeing some learners touching red caps, do we really need to touch them in 2+8? "nooo" there leave them out! (*PPQ; OIA, OIB) (SPSL; OIA*) The teacher moved to group one and asked, I want to see that expression of caps here? After moving around confirming the expression, the teacher asked, " What is the answer"?"10",the whole class. "positive or negative? "positive, learners" positive because all the caps are black"!!say the teacher. (*SPSL; OIA, OIB) (PPQ; OIA, OIB*)

The next example, 8-6. Learners started working in groups while the teacher is watching. Later the teacher instructed them that "I will come group by group, choose a speaker for the group who will tell me what you did"! I will start here, there (while pointing)then and here (FMMD; OIA, OIB) (ITPRPS; OIA, OIC)

The teacher went to one group and found 14 black caps! How did you get 14?"we went to get the answer "confused" learner. But you have 14 black caps?" But, the expression says here 8 black! And what is this one for? ok! discuss, discuss and get the answer" ! I will come back! (*EUEST; OIA, OIB) (PPQ; OIA, OIB) (FMMD; OIA, OIB) (UVM; OIA, OIB*)

The teacher went to the other group ! What do you have here? "positive 8 minus 6", a learner. "No just say 8 minus 6! ehe! that is the answer? "correct, very good !!. (*ITPRPS; OID) (FMMD; OIA, OIB, OIC)*

The teacher went to another group! And here? Can I have the expression first? Leaners started putting caps together, to model a representative of 8-9. Ok! And you subtract? a learner, moved 2 caps instead of six. Then the teacher asked "which one do you subtract? This is (6 caps) or this (2 caps)? "This one, 2 caps, say a learner. Thank you!! (*EUEST; OIA, OIB) (PPQ; OIA, OIB)(SPSL; OIA, OIB)*

Another group. Then this group? It was like this (8 caps), we subtracted 6 caps (black) then the answer is 2. Very good. Can we go to the next one? "Yecees", (*SPSL; OIA*)

The teacher gave the example 8-(6) and kept moving around the class "Any group which is done"? Yeces", learners. Can I see? OK! you have 8: 1, 2, 3, 4, 5, 6, 7, 8. The learner explain that became we have no red cap, we added 6 zeros to be able to take 6 red caps ! OK very good!! *(BPFCU; OIA, OIB) (SPSL; OIA*)

The teacher moved to another group, but they were struggling. They were holding red and black caps to subtract 6 instead of 6 red only. The teacher asked, "remember 6 is red only , so you must only take who? reds only", learner. The learner moved 6 red leaving 14 black caps. *(PPQ; OIA, OIB) (SPSL; OIA, OIB*)

The third group was struggling too. The teacher refers them to the expression! "We have 8, where is 8 black? This one! OK, remove this red then! (guiding them). Ok now, from these 8 blackcaps, subtract 6 red! "There are no red", learner. OK, bring in zero because zero cannot change anything! Bring zero, bring zeros", learner. Ok, now we are going to subtract how many red caps? "six", learner. 1, 2, 3, 4, 5, 6! Is the answer positive or negative ? "positive", learner. Positive what? "14", learner. Good !! The teacher further confirmed how they pressed 8-6 on the calculator "oooooo!, say leaners while laughing!!. *(SPSL, OIA, OIB) (UVM; OIA, OIB) (PPQ, OIA, OIB) (BPFCU: OIA, OIB, OIC)*

The last group could not get it. The teacher started quidding them. "We have 8 black caps. Now we went to subtract -6 (red caps) but there is no way we can take red from blacks! We have to bring them in the form of zero because any number plus zero is that number. How many negatives do you want to subtract ? "6" learners. Ok, you will bring in 6 zero! are zero there, another zero there ....! now you can take these 6 red caps! what is left now? "positive 14", learners. Ok, use your calculator to confirm your answer. *(SPSL; OIA, OIB) (UVM, OIA, OIB) (PPQ; OIA, OIB)*

Lastly the teacher asked learners to take their books and write the following examples (d) 12 + - 8, (e) -12 + 8 (f) -12 + 8 (g) -12+-8. They will work them out by the head, not using a calculator, number line, or bottle caps. "Once you finish just "top top top", a teacher using figures making sand. We will award ....I left the sweets .... we will award each group member with a sweet. (*EUEST; OIA, OIB) (UVM; OIA, OIB) (FMMD; OIA*)

Lesson 2:

Topic: Removing brackets (Algebra)

The teacher entered the class, and then greeted the learners. Thereafter he introduces the topic of the day; Algebra, removing brackets. (***LGO; OIA***) He gave the first example $2(x + 3)$. Then he asked how do we do this "? The learners replied "we multiply x by 2 and then 3 by 2. "The meaning of brackets means multiply, "teacher." ***(ITPRPS; OIB) (FMMD; OIA, OIB, OIC)*** (***BPFCU; OIA***) And then multiply by who? "2 with everything inside the bracket." So, you multiply 2 by x and get $2x$ and later 2 by 3 and get 6.

The teacher further gave $2(x - 3)$ and work it out to get $2x - 6$. (***ITPSPS; OIA***) ***(UVM; OIA***)

Ok! "There is another way of solving that thing Algebraic tiles," says the teacher. "What are tiles? These are what we call tiles (while showing the class). The teacher added that tiles can be made from boxes or plastic materials.

The teacher introduced the tiles one by one and their color. This tile is brown and if you turn it, it is red, which means negative. He was referring to the "x" tiles. He continued to introduce the unit tile. Later he draw diagrams of tiles he was referring to because some were too small to be seen by learners at the back. ***(UVM; OIA***) (***ITPRPS; OIB***)

"Like in this example $2(x + 3)$, how will I do it using tiles? "you take your tiles, how many "x" do we have ? "one", learners! Ok, you put it there. How many units do you have? "3" units, learners. The teacher starts putting tiles below the expression $2(x + 3)$ as follows "1,2,3 tiles "! Now, you are multiplying those tiles by who? "by 2", learners. If you multiply those tiles by who? " by 2", learners, how many tiles will you have? (pointing at unit tiles), "6", learner the teacher was adding tiles at the board! "At the end what do you have? "$2x + 6$ units! That is how you get the answer. (***UVM; OIA, OIB) (BPFCU; OIA, OIB, OIC) (FMMD; OIA)***

"The next one is $2(x - 3)$". The teacher changed $2(x - 3)$ to $3(x - 3)$". He started putting tiles below the expression as a representation. "This is a representation of this (pointing). Remember this is red, meaning negative. Now 3 times the whole thing will be what? Let me put the tiles (putting tiles below the expression). 3 times of this will be what? "$3x + 9$", learners counting the tiles! (***UVM, OIA, OIB) (BPFCU; OIA, OIB, OIC***) (***LGO; OIA, OIB***) (***EUEST; OIB***)

The teacher asked “any other example so we can do it”? “any other”? non of the learners gave ok! (***FMMD; OIA, OIB***)

Let us take even this one $2(2x-3)$. How do you model that? The teacher model it . Ok! If you twice this one is like double. In the end, you will have what? If you are to write your answer, it will be $4x-6$ (***UVM; OIA, OIB) (FMMD; OIA, OIB, OIC) (BPFCU; OIA, OIB, OIC***) ***(ITPRPS; OIA, OIC***)

I am going to give you this and you play around with them. I will give you two ‘’problems’ ‘you work them out using tiles only and not that ordinal way of multiplying. The learners got in groups and the teacher stressed that ‘’once I give you the 1st example, I want you to model it.’’ We are not going to write, just using tiles. ‘’At the end, you tell me the answer. The teacher gave (a) $2(-x-3)$. ‘’Show me $-x$ minus 3’’. The teacher moved around the class checking how the learners are modeling! He went to one group. Ok! Correct!! ***(UVM; OIA, OIB) (SPSL; OIA) (BPFCU; OIA, OIB, OIC) (ITPRPS; OIA, OIC)(LGO; OIA, OIB)(EUEST; OIA)***

He went to another group, ‘’Is that negative x? ‘’How many x do you have there?’’ one or $2x$? Later he went to do the correction on the board. Look here, Look here! I know the majority of you have a problem with signs, when you multiply, you target about signs. But this one is showing you clearly! By the look of things, all the signs will be what? ‘’negatives’’ because they are all red’’. Ok. If you double $(-x)$, you will be having 2 reds, if you double this one? ‘’you will be having six reds’’. And the answer will be ?’’$-2x-6$. Clear!! (***PPQ; OIA, OIB) (SPSL; OIA, OIB) (UVM; OIA, OIB) (FMMD; OIA, OIC) (ITPRPS; OIA, OIB)***

Do this one for me $5(-x+1)$. The teacher moved around the class demanding learners to model the expression. What Is the answer? ‘’Negative 5 plus….’’ ‘’Be careful, if you say -5 only without x it's like you are referring to this small one, units! So, it is negative $-5x+5$. Thank you so much, that is one way to remove brackets. (***BPFCU; OIA, OIB, OIC) (FMMD; OIA, OIB, OIC) (ITPRPS; OIA, OIC, OID) (EUEST; OIA, OIB)***