THE MATHEMATICAL AND PEDAGOGICAL CONTENT KNOWLEDGE THAT NAMIBIAN SENIOR PRIMARY TEACHERS DRAW ON TO DEVELOP THEIR LEARNERS' COMPUTATIONAL ESTIMATION

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by

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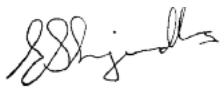
DECLARATION

I, Emilia Shigwedha, hereby declare that the work in this thesis is my own and where ideas of other authors have been used, they have been acknowledged using referencing according to the Rhodes University Guide to References. I further declare that the work in this thesis has not been submitted to any university for degree purposes.



Signature: Date: 05.03.2023

DEDICATION

This thesis is dedicated to my daughter, Etuna Nafimane Haimbodi for her love and support, despite me abandoning her sometimes during the writing of this thesis. *Mama* loves you *shiveli.* The thesis is also dedicated to my parents, for always being there for me.

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ABSTRACT

Computational estimation is important in the development of learners' number sense. It is through the process of finding an approximate (but satisfactory) that learners can check the reasonableness of their answers to calculations, develop an understanding of place value and by implication the four number operations. It is the role of teachers to develop the computational estimation skills of learners. To do this, teachers need to have a sound knowledge of computational estimation, its value and how to teach it. This study thus seeks to explore and understand Namibian senior primary teachers' mathematical and pedagogical content knowledge to develop their learners' computation estimation knowledge. The research is guided by the following question: *What mathematical and pedagogical content knowledge do senior primary mathematics teachers draw on to develop their learners' computational estimation skills?*

The research is a qualitative interpretivist case study. Eight senior primary teachers of Mathematics from the Ohangwena region in Namibia participated in the study. Data was generated through questionnaires, a focus group interview and lesson observations. The Mathematics Knowledge for Teaching (Ball et al., 2008) and the Knowledge Quartet (Rowland, 2005) frameworks were used as both analytic and explanatory tools for the study. Key findings from the research are that teachers have knowledge of and use a variety of strategies for estimation, however, they only use the 'rounding off' strategy when teaching learners computational estimation. The teachers appear to teach computational estimation by first focusing on place value before moving on to 'rounding off' to the nearest 10s, 100s, 1000s and so forth. My research recommends that the National Institute of Educational Development together with the Ministry of Education, Art and Culture in Namibia, provide teachers with professional development opportunities on how to develop learners' computational estimation. Such professional development will further develop teachers' mathematical and pedagogical content knowledge. Furthermore, the Namibian syllabus should include a variety of strategies for computational estimation.

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LIST OF ACRONYMS

B.Ed	Bachelor of Education
BETD	Basic Education Teacher Diploma
CCK	Common Content Knowledge
ETSIP	Education and Training Sector Improvement Programme
HCK	Horizon Content Knowledge
KCC	Knowledge of Content and Curriculum
KCS	Knowledge of Content and Students
KCT	Knowledge of Content and Teaching
KQ	Knowledge Quartet
MKfT	Mathematical Knowledge for Teaching
MoEAC	Ministry of Education, Arts and Culture
NIED	National Institute of Educational Development
PCK	Pedagogical Content Knowledge
SACMEQ	Southern and Eastern Africa Consortium for Monitoring Educational Quality
SATs	Standardized Achievement Tests
SCK	Specialized Content Knowledge
SMK	Subject Matter Knowledge
UNAM	University of Namibia

CHAPTER 1: RATIONALE FOR THE STUDY

Mathematics is a compulsory subject throughout the schooling system in Namibia. The Namibian government made this decision in 2012 to support the development of economically productive citizens (Laato, et al., 2019). However, many studies (National Institute for Educational Development (NIED), 2010; Courtney-Clarke, 2012; Peters, 2016; Naukushu, 2016) have highlighted several challenges with the decision to make mathematics compulsory. These include concerns with teachers' poor content and pedagogical content knowledge, teachers and learners' beliefs about mathematics, a paucity of teaching and learning materials and learners' social backgrounds. These are viewed as hindering the teaching and learning of mathematics in schools. While I acknowledge that there is a multiplicity of factors that constrain the teaching and learning of mathematics, my interest in this study is specifically related to teachers' mathematics content and pedagogical content knowledge. To this end, my research seeks to understand what mathematical content and pedagogical content knowledge senior primary mathematics teachers draw on when developing their learners' computational estimation.

1.1. BACKGROUND OF THE STUDY

The goal of basic education in Namibia is to empower learners to participate in developing a knowledge-based society where "knowledge is created, transformed, and used for innovation to improve quality of life" Ministry of Basic Education Arts and Culture [MoEAC], 2016, p. 12) and to develop an economically active citizenry. Mathematics knowledge is deemed to be core to the development of a thriving economy and improved quality of life. It is for these reasons that mathematics became a compulsory subject in schools from 2012 (Laato, et al., 2019). In support of this decision, the (Ministry of Education, Arts and Culture (MoEAC) (2016) requires learners to develop various mathematics-related skills and "understand and use mathematical language confidently and effectively" (p. 16) in a fast-growing technological and economic world. It is thus important that teachers link the mathematics they teach to "what it takes to live in today's societies and what learners learn" (Lekoko et al., 2018, p. 206).

Namibia is a member of the Southern and Eastern African Consortium for Monitoring Educational Quality (SACMEQ) which seeks to ensure quality education in Sub-Saharan Africa by providing

necessary data to monitor education quality. Since 1995 Namibia has participated in the four SACMEQ studies (SACMEQ I, SACMEQ II, SACMEQ III and SACMEQ IV). In all four of these SACMEQ studies the learners' results have remained a concern.

The SACMEQ I study (1995) was conducted with Grade 6 learners. The results raised concern about the mathematics education in Namibia because the learners were ranked second last out of the seven participating countries. Grade 5 Namibian learners participated in the SACMEQ II study (2000) and had the lowest results out of 12 countries. The SACMEQ III study (2007) indicated that there was a slight improvement in the performance of grade 5 Namibian learners as they were ranked tenth out of the fifteen participating countries. The SACMEQ IV results (2017) placed Namibian learners at ninth out of the same fifteen participating countries. While the SACMEQ IV (2017) report indicates that grade 5 and grade 7 Namibian learners have improved in mathematics since the SACMEQ I study in 1995, learner performance varies across the different regions of Namibia. In the Ohangwena region, where this research took place, there has been no improvement between 1995 and 2017. This suggests that greater emphasis needs to be placed on developing learners' mathematics competency in the Ohangwena region.

Spaull (2011) compared primary school learners' functional numeracy performance in Namibia, Botswana, Mozambique and South Africa, using the SACMEQ III results. He defines functional numeracy as an individual's ability to acquire "sufficient numeracy skills such that she/he is able to satisfactorily use those skills in everyday life" (p. 33). His study revealed that the learners in all four countries are predominantly functionally innumerate, with Namibia being the worst. 47.69% of the 6398 Namibian learners who participated in the SACMEQ III test were deemed functionally innumerate. This compares with 22% of learners in Botswana, 33% in Mozambique and 40% in South Africa.

Alarmed by the SACMEQ results, the Ministry of Education has put several measures in place to improve the learners' numeracy skills. One of these measures was the Education and Training Sector Improvement Programme (ETSIP). The ETSIP developed the Standardised Achievement Tests (SATs) to improve the quality of education in the primary school through learners' assessment on a national level (Shaakumeni & Mupupa, 2019). The SATs monitor the acquisition

of the identified competencies and skills in Mathematics, English and Science of Grades 5 and 7 learners. Since the inception of the SATs in 2009, the results indicate that senior primary school learners continue to perform below the required standards of numeracy (Mutuku, 2015). Due to the recurring poor performance of the learners in Mathematics, the National Institute for Education Development (NIED) (2010) has called on researchers to develop interventions to remedy the poor results across all the school grades countywide.

One of the factors that researchers have identified as contributing to the low level of numeracy in the Namibian learners is the teachers' poor content knowledge (Kapenda & Kasanda, 2015). The SACMEQ IV study showed that it was not only Namibian learners who were underperforming in mathematics, but also the teachers. (Shigwedha, et al., 2017). Hurrel (2013) maintains that many teachers have insufficient content knowledge, making it difficult for teachers to ensure that mathematics is accessible to the learners. Echoing similar sentiments, Kapenda and Kasanda (2015) state that without enough content knowledge, teachers are unable to deliver the required mathematics content knowledge to the learners, resulting in poor mastery and understanding of mathematics.

Research has not only raised concerns about teachers' mathematics content knowledge, but also their pedagogical practices (Courtney-Clarke, 2012; SACMEQ Report, 2017). The teaching of mathematics in primary school classrooms is still characterized by drill and practice, rote learning and the use of standard procedures that emphasise the product rather than the process of calculating. Despite the efforts made by those responsible for the development of policy and curriculum to implement changes that are in line with reform movements world-wide, that is, the move towards constructivist and learner-centered pedagogies (Courtney-Clarke, 2012), little (if any) attention is given to the development of number sense (Haimbodi, 2019).

Haimbodi (2019) argues that computational estimation is central to the development of number sense. It is through the process of computational estimation that learners are able to judge the reasonableness of the results of their calculations. Reference to computational estimation is only found in relation to whole number addition, subtraction, multiplication and division in the Grades 4 – 7 Namibian mathematics syllabus (NIED, 2015). The syllabus suggests that learners should be

able to "determine solutions by using estimation or inverse operations" (MoEAC, 2016, p. 22). Apart from the single computational estimation strategy, rounding off, there is no further elaboration to extra/more strategies that learners should be introduced to in order to estimate the reasonableness of their calculations. I elaborate on different computational estimation strategies in Chapter 3 and the computational strategies that the teachers teach learners in Chapter 5.

Computational estimation allows learners to check the reasonableness of their answers and helps them better understand place value and by implication, the four number operations (addition, subtraction, multiplication and division) (Courtney-Clarke, 2012), Courtney-Clarke (2012) indicated that proficiency in computational estimation is an indicator of number sense. She further claims that learners with number sense "naturally decompose numbers, use particular numbers as referents, solve problems using the relationships among operations and knowledge about the base-ten system, estimate a reasonable result for a problem, and have a disposition to make sense of numbers, problems and results" (National Coucil of Teachers of Mathematics, [NCTM], cited in Courtney-Clarke, 2012, p. 17). Veloo (cited in Courtney-Clarke 2012) maintains that good estimators use a wide variety of strategies for calculating. They are deemed to be proficient at calculating mentally because they have a better understanding of mathematics, particularly the basic facts. Good estimators are deemed to be self-confident and tolerant of errors (Courtney-Clarke, 2012).

It is the role of teachers to develop learners' computational estimation skills. To do this, teachers need to have sound knowledge of computational estimation and its value. It is with this in mind that this study seeks to understand the mathematical content and pedagogical content knowledge that senior primary teachers use and draw on in developing learners' computational estimation in the Ohangwena region in Namibia.

1.2. STATEMENT OF THE PROBLEM

Although mathematics is a priority subject in Namibia, the efforts embarked on by the MoEAC to improve the mathematics results seem not to have borne the desired output. Clegg and Coutney-Clarke (2009) observed that both the Bachelor of Education (B.Ed) offered by the University of Namibia (UNAM) and the Basic Education Teacher Diploma (BETD) offered by the then Colleges

of Education in Namibia produced teachers with poor mathematics content and pedagogical content knowledge. The SACMEQ IV Report demonstrates that Namibian teachers do not have sufficient content knowledge to teaching mathematics (Shigwedha, et al., 2017). Son et al. (2019) suggest that teachers lack the necessary content knowledge to develop their learners' knowledge of computational estimation. Therefore, this study seeks to explore and understand senior primary teachers' content and pedagogical content knowledge when developing their learners' computational estimation skills.

1.3. RESEARCH AIMS AND QUESTIONS

This study seeks to understand the knowledge teachers draw on when developing their learners' computational estimation skills.

The following question guides the study:

What mathematical and pedagogical content knowledge do senior primary mathematics teachers draw on to develop their learners' computational estimation skills?

The two sub-questions are:

- What mathematical content knowledge do teachers draw on to develop the learners' computational estimation skills?
- What pedagogical content knowledge do teachers draw on to develop the learners' computational estimation skills?

1.4. DELIMITATION OF THE STUDY

The study focused solely on senior primary mathematics teachers and the content investigated was restricted to the Namibian Senior Primary Mathematics syllabus. The study took place in the Ohangwena Regional Education; therefore, the results of this study cannot be generalized to other regions in Namibia.

1.5. DEFINITIONS OF TERMS

Computational estimation involves simplifying a calculation to produce a satisfactory approximate to a calculation (Courtney-Clarke, 2012).

Computational estimation strategies refer to different ways of solving computational estimation problems.

Subject Matter Knowledge refers to the content knowledge of the subject to be taught. This includes the concepts, explanations, calculation strategies, as well as, assumptions and beliefs about the content (Tatto, et al., 2012).

Pedagogical Content Knowledge refers to the teachers' knowledge of appropriate mathematics tasks, suitable strategies, a variety of pedagogical practices, knowledge of how to identify learners' misconceptions and difficulties and knowing how to address them (Kraus, et al., 2008).

1.6. THESIS OUTLINE

The thesis is divided into six chapters.

The first chapter of this thesis provides an orientation of the study by elaborating on the state of mathematics education in Namibia. The chapter discusses the problems and challenges faced by the current education system in terms of the learners' mathematics performance and teachers' content and pedagogical content knowledge. The research questions emerge from concerns about both learners' mathematics results and teachers' competence to teach mathematics.

Chapter Two highlights the evolution of the knowledge required to teach mathematics by drawing on the work of Shulman (1986, 1987), Ball et al., (2008) and Rowland (2009). Ball et al.' (2008) Mathematics Knowledge for Teaching (MKfT) and Rowland's Knowledge Quartet (KQ) are the theoretical frameworks used to explain and analyse the data generated through this study.

The literature review in Chapter Three elaborates on the key concepts of this research, that is number sense, mathematical proficiency, mental mathematics and computational estimation. The chapter reviews studies on computational estimation, and the benefits and challenges with the application of computational estimation in the international research and Namibian Curriculum.

Chapter Four argues that the research is a qualitative case study within the interpretivist paradigm. The chapter justifies the choice of the research design for the study and provides insight to the research site, sampling methods, data collection procedures, data analysis and ethical considerations undertaken by the researcher.

The presentation and analysis of the data is found in Chapter Five. Specifically, the chapter uses the MKFT and KQ frameworks to examine the mathematical and pedagogical content knowledge that teachers draw on to teach and in the process of teaching computational estimation.

Chapter Six provides a summary of the study and highlights the research findings. The implications and recommendations of the study are presented followed by a brief reflection of this study on my personal, professional and academic development.

CHAPTER 2: THEORETICAL FRAMEWORKS

2.1. INTRODUCTION

In Chapter 1, I argued that Namibian teachers do not have the required content and pedagogical content knowledge to teach mathematics in the senior primary phase (e.g., Spaull, 2011; Courtney-Clarke, 2012). In response to these findings, this study seeks to understand the content[1] and pedagogical content knowledge senior primary mathematics teachers draw on and use as they develop learners' computational estimation. Two frameworks that assist in unpacking the content and pedagogical content knowledge teachers draw on for teaching and use in the act of teaching that inform this study are Deborah Ball's Mathematics Knowledge for Teaching (MKfT) and Tim Rowland's Knowledge Quartet (KQ). I chose these two frameworks for this study because they both focus on different dimensions of teacher knowledge for teaching and in teaching.

The chapter begins with an explanation of the work of Shulman (1986, 1987) as he first alerted teacher educators that to 'carry out' their work effectively, they need to avoid separating content and pedagogical knowledge as both are important. This is followed with an explanation of Ball et al.'s (2008) MKfT, as this framework emerged from Shulman's conceptualization of Pedagogical Content Knowledge (PCK) and Rowland (2005) KQ. The KQ framework shares a close link with the MKfT and is also underpinned by Shulman's PCK. The distinction between these two frameworks is that Ball et al. (2008) focus on the knowledge required for teaching, while Rowland et al. (2005) examine both the knowledge required for teaching and knowledge used in teaching.

2.2. SHULMAN'S CONCEPTUALISATION OF PEDAGOGICAL CONTENT KNOWLEDGE

Shulman (1986) claimed that historically, teacher training programs treated subject matter knowledge and pedagogy separately and tended to focus on either one of them. He argued that subject matter knowledge and pedagogical knowledge could not be separated as both are required to enable teachers to effectively carry out their work as teachers. He attempted to address this

[1] I use the terms content knowledge and subject matter knowledge interchangeably as the three key frameworks that I draw on in this research, that is, Shulman (1986, 1987), Ball et al. (2008) and Rowland (2005, 2014), use different terms.

separation of subject matter knowledge and pedagogy by introducing a unique domain of teacher knowledge which he referred to as Pedagogical Content Knowledge (PCK). Shulman (1987) defined PCK as "that special amalgam of content and pedagogy that is uniquely the province of teachers, their special form of professional understanding" (p. 8). Shulman (1986) developed an extensive explanation of what he considered PCK. In 1986, he wrote, PCK:

> embodies the aspects of content most germane to its teachability. Within the category of pedagogical content knowledge I include, for the most regularly taught topics in one's subject area, the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations - in a word, the ways of representing and formulating the subject that make it comprehensible to others . . . [It] also includes an understanding of what makes the learning of specific concepts easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning. (p. 9)

In short, PCK, for Shulman (1986) includes subject matter knowledge, forms of representation and examples, how learners learn, learner misconceptions, and background knowledge.

Shulman (1986) lists three components of knowledge important to the process of teaching. These include subject matter knowledge, PCK and curricular knowledge. Subject matter knowledge consists of two subject matter structures. These are the substantive and the syntactic structures. *Substantive structures* refer to how concepts within the discipline and basic principles are organized. *Syntactic structures* refer to the processes through which new knowledge is produced and proved (Shulman, 1986).

Curricular knowledge encompasses knowledge of curriculum and related programmes, and learning and teaching support materials designed for teaching specific topics (Shulman, 1986). Curricular knowledge supports the formulation of connections between topics that enable learners to build upon prior knowledge. In Namibia, the National Institute of Educational Development (NIED) is responsible for educational programs, resources that are fit for every subject and designing a subject syllabus, this is regulated or governed by the Ministry of Education, Arts and Culture (MoEAC). However, the teacher education programs are responsible for educating teachers on curricula and the range of possible learning and teaching support materials. Shulman (1986) refers to two forms of curricula knowledge: lateral and vertical curriculum knowledge. Lateral curriculum knowledge concerns the teacher's ability to link what is taught in a particular

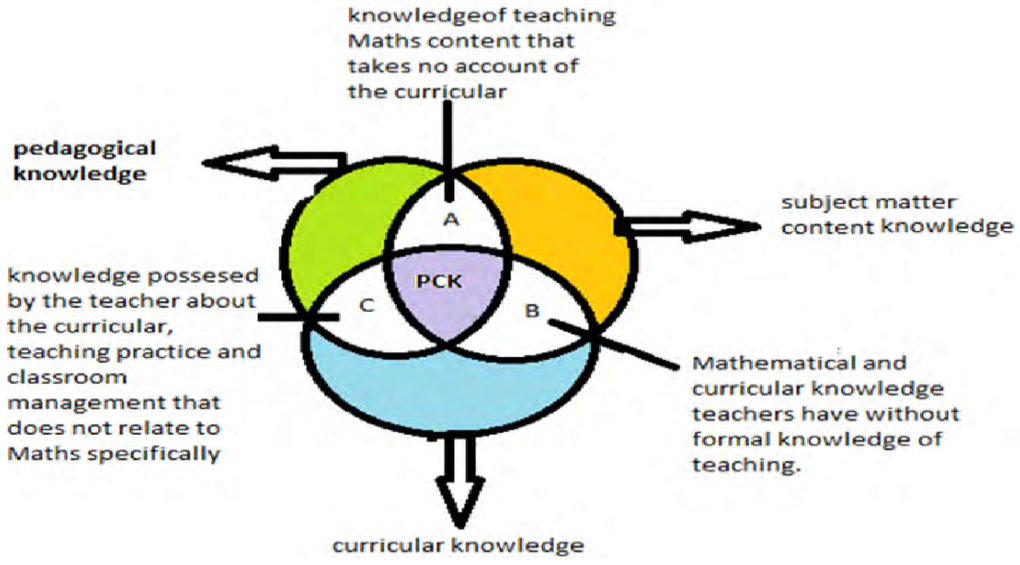
subject to other subjects that learners may be learning. Vertical curriculum knowledge is knowing the topics of the subject taught in previous and later years of schooling.

Pedagogical knowledge refers to teachers' specialized knowledge, which allows them to create an effective teaching and learning environment for all the learners they are responsible for teaching (Shulman, 1987). Hence, PCK exists at the nexus of subject matter knowledge, pedagogical knowledge and curricular knowledge.

As noted in Figure 2.1, the nexus between pedagogical knowledge and subject matter knowledge is the knowledge required for teaching mathematics content. The relationship between subject matter knowledge and curriculum knowledge refers to the knowledge that teachers have without formal pedagogical knowledge. The connection between curricula knowledge and pedagogical knowledge is knowledge that does not relate specifically to mathematics. At the center of Figure 2.1 is PCK. As noted by the colours in Figure 2.1, PCK thus includes knowledge of curriculum, subject matter knowledge and the knowledge required to teach mathematics. Numbers A, B and C in Figure 2.1 indicate that the intersection:

- A refers to knowledge that does not take curriculum into account;
- B excludes pedagogical knowledge; and
- C focuses on curriculum knowledge and general pedagogical knowledge and skills that are generic across all subjects (e.g., classroom management).

Figure 2.1 Illustration of Shulman's PCK



Note: From Chikiwa, S. (2017). *An investigation into the mathematics knowledge for teaching required to develop grade 2 learners' number sense through counting.* A thesis submitted in fulfilment of the requirements for the degree of Master of Education. Rhodes University: South Africa. p. 22.

Within the context of mathematics education, PCK is a form of knowledge that distinguishes mathematics teachers from mathematicians (Gudmundsdottir, 1987). Mathematics teachers differ from mathematicians, not necessarily in terms of their understanding of the subject matter knowledge, but in how that knowledge is organized for and used in the classroom context. In other words, a mathematics teacher's knowledge of mathematics is explicitly used to help learners understand the various mathematics concepts. On the other hand, the mathematicians' knowledge is organized from a research perspective and focuses on developing new knowledge in that field of mathematics (Cochran, 1997).

Shulman's framework helps us understand a significant aspect of pre-service teachers' learning, but it does not cover the wider range of pedagogical knowledge that might seem necessary for teachers already in the field (Meredith, 2006). Hill et al. (2005) argue that Shulman's PCK model is less likely to change, hence they suggested there is a need to adapt the model for teacher education and professional development in order to meet the particular needs for teaching mathematics. While Ball et al. (2008) were influenced by the ideas of Shulman (1986, 1987), they

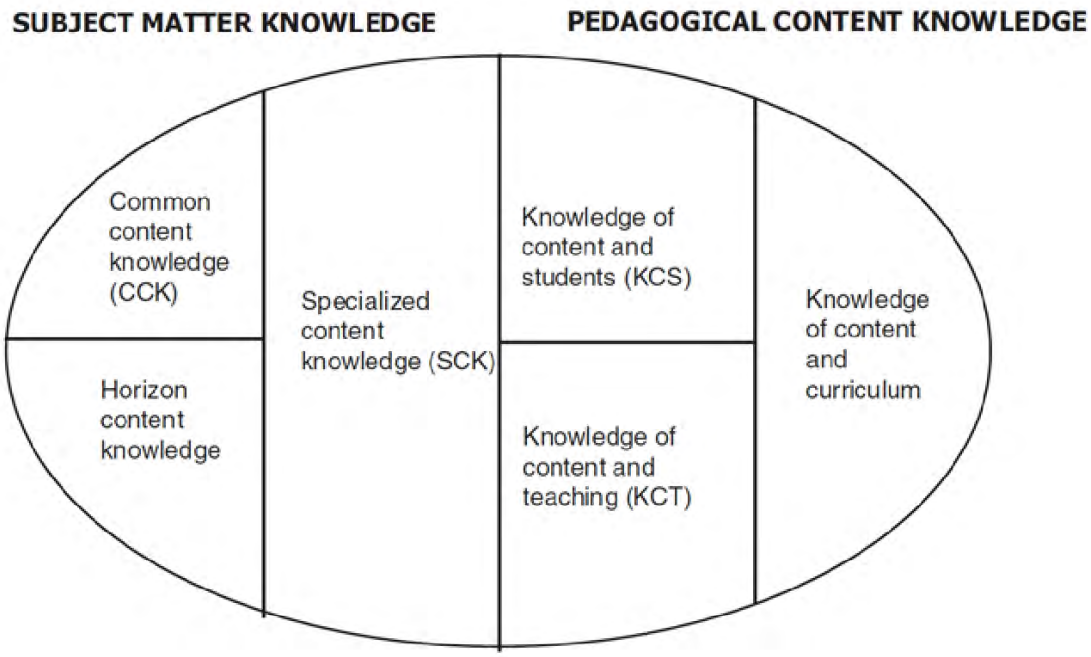
were interested specifically in primary school education. Ball et al. (2008) developed a practice-based theoretical framework referred to as Mathematics Knowledge for Teaching (MKfT) based on their research with teachers. The following section focuses on Ball et al's. (2008) conceptualization of MKfT.

2.3. MATHEMATICS KNOWLEDGE FOR TEACHING

Ball et al. (2008) elaborated on the work of Shulman and developed the Mathematics Knowledge for Teaching (MKfT) framework. The MKfT emerged from fifteen years of research in the Mathematics Teaching and Learning to Teach and the Learning Mathematics for Teaching projects. MKfT highlights the knowledge primary school teachers require to teach mathematics effectively (Ball et al., 2008). It is largely built on the three categories of knowledge that Shulman originally identified: content, curriculum, and pedagogical knowledge.

The MKfT framework includes two broad categories: Subject Matter Knowledge (SMK) and Pedagogical Content Knowledge (PCK). Each category consists of three domains, as highlighted in Figure 2.2.

Figure 2.2 The MKfT Framework



Note: From Ball, D.L., Thames, M.H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5), 389-407., p. 403

2.3.1. Subject Matter Knowledge

SMK consists of Common Content Knowledge (CCK), Horizon Content Knowledge (HCK) and Specialized Content Knowledge (SCK).

2.3.1.1. Common Content Knowledge

CCK is "the mathematical knowledge and skills used in settings other than teaching" (Ball et al., 2008, p. 399). It is the knowledge that all economically active citizens require. Hill and Ball (2009) define CCK as the knowledge that informs teaching tasks, such as knowing how to carry out a procedure and how to define concepts. CCK is the general knowledge of mathematical concepts and skills that anybody can utilize in everyday life. It is the mathematical knowledge that every educated person should possess and use, for example, knowing how to determine the offerings collected at church on Sunday. Ball et al. (2008) argued that while CCK is not specific to teaching, a teacher needs to develop this knowledge as it would be challenging for teachers to teach what they do not know (Chapter 1). CCK enables teachers to identify learners' correct and incorrect

answers as they teach. In this research CCK refers to the teacher's own knowledge of how to use estimation to judge the reasonableness of their results to different computations.

2.3.1.2. Specialized Content Knowledge

SCK is "the mathematical knowledge and skill unique to the work of teaching" (Ball et al., 2008, p. 400). This kind of mathematical knowledge allows teachers to identify the errors in their learners' work and correct such errors. Ball et al. (2008) argue that the work of teaching involves:

> a kind of unpacking of mathematics that is not needed— or even desirable—in settings other than teaching. Many of the everyday tasks of teaching are distinctive to this special work and require unique mathematical understanding and reasoning that is uncommon to other professions. (p. 400)

Ball et al. (2008) suggest that teachers should have knowledge beyond that of the learners and therefore "must hold unpacked mathematical knowledge because teaching involves making features of particular content visible and learnable by students" (p. 400). For example, teachers need to know the different computational estimation strategies that learners might use (SCK) and not only how to estimate or calculate (CCK) (Ball et al., 2008).

2.3.1.3. Horizon Content Knowledge

HCK refers to the mathematical knowledge spanning grades and the different curricula subjects (e.g., natural science). As noted above, Shulman (1986) refers to this as lateral curriculum knowledge. Lateral curriculum knowledge is an awareness of how mathematics topics relate to one another over the grades and phases. HCK relates to the extent to which the teacher understands mathematics as a whole and is able to connect it with other areas of the curriculum and other subjects. This connection with other subjects is referred to as lateral knowledge (Shulman, 1986). Ball and Bass (2009) define HCK as the knowledge that "supports a kind of awareness, sensibility [and] disposition that informs, orientates and culturally frames instructional practice" (p. 5).

Ball et al., (2008) argued that HCK provides a broad overview of mathematics and the ability to see how it connects to other topics in the mathematics curriculum that are taught in primary school and those that learners may or may not meet in the future. In this study, HCK is the knowledge required to relate computational estimation to other mathematics topics throughout the curriculum. For example, it connects computational estimation to different calculation strategies. This lateral

knowledge is evident in the Namibian syllabus as it states that, "computational estimation develops the use of approximation and rounding numbers to facilitate estimation of answers to calculations" (MoEAC, 2016, p. 11).

2.3.2. Pedagogical Content Knowledge

PCK consists of three domains in Ball et al.'s (2008) MKfT. These are the Knowledge of Content and Students (KCS), the Knowledge of Content and Teaching (KCT), and the Knowledge of Content and the Curriculum (KCC).

2.3.2.1. Knowledge of Content and Students

According to Ball et al. (2008), KCS "implies an understanding of students' thinking and what makes the learning of particular concepts easy or difficult" (p. 378). Ball et al. (2008) further described KCS as the knowledge that integrates knowing about mathematics and knowing about students, allowing the teacher to connect the learners to the content in order to enhance their learning. KCS refers to how well the mathematics teachers understand their learners and anticipates how their learners think about the subject content. This is the domain where teachers show familiarity with common errors and know which errors their learners are most likely to make. For example, teachers need to know that learners experience challenges and are likely to make errors with computational estimation for decimal fractions. KCS allows teachers to design their lessons with their learners in mind and execute lessons that the learners will find interesting and stimulating (Chua, 2018).

2.3.2.2. Knowledge of Content and Teaching

According to Ball et al. (2008), KCT connects knowledge about mathematics and teaching. Teachers are required to integrate their mathematical knowledge with instructional design. Furthermore, KCT refers to the teachers' ability to know beforehand the level of difficulty in a lesson and identify what examples may help prepare the class for the lessons' demand (Chua, 2018). According to Wilkie (2015), KCT "includes knowledge about how to choose appropriate representations and examples, how to build on students' thinking, and how to address student errors effectively" (p. 249). Ball et al. (2008) propose that teachers require a sound mathematical knowledge to design appropriate mathematical tasks. The teacher needs to know what teaching

strategies to use, what resources to use, and what representations and examples to employ so that students can learn with understanding. In other words, KCT refers to the mathematical knowledge that informs teachers' instructional practice in the classroom. It involves deciding how to go about the lesson and what to include in the lesson. It is the knowledge needed by teachers to decide what teaching methods fit the topic and the kind of instruction and procedures they are going to use so that their learners develop a better understanding of the topic at hand. Ball et al. (2008) maintain that KCT:

> includes knowledge of how to choose which examples to start with and which examples to use to take students deeper into the content. Teachers evaluate the instructional advantages and disadvantages of representations used to teach a specific idea and identify what different methods and procedures afford instructionally. (p. 401)

For example, teachers may use real-life examples to assist the learners in developing their computational estimation. KCT in this research refers to the strategies that teachers employ to assist learners in developing their computational estimation competence.

2.3.2.3. Knowledge of Content and Curriculum

Petrou and Goulding (2011) define the curriculum as the entire range of programs designed to teach a specific subject and its different topics in a grade. The curriculum includes a variety of instructional materials available, for example, the national workbooks, the prescribed textbooks and the syllabus that the Ministry of Education provides. Ball et al. (2008) referred to KCC as the knowledge teachers require to evaluate, adapt and use instructional materials in their teaching of the different mathematical concepts.

KCC is the knowledge which Shulman (1986) referred to as lateral and vertical curriculum knowledge. Lateral knowledge is the teachers' ability to know what the learners are learning in the different subjects (e.g., science). This may sound similar to HCK, but the focus here is on the teacher's knowledge of mathematics and how mathematics connects to the other subjects in the curriculum (Shulman, 1986). For example, computational estimation supports not only learners understanding of number but also measurement. In comparison, vertical curriculum knowledge would refer to teachers' knowledge of what was taught previously and what will be taught later during the learners' schooling career (Shulman, 1986). KCC may also refer to teachers' awareness

of the progression of computational estimation in the curriculum. Being able to evaluate, adapt and use a variety of instructional materials available in the teaching and learning of different mathematical concepts.

2.4. CHALLENGES WITH THE MKfT FRAMEWORK

Effective learning of mathematics requires quality teaching. Ball et al's. (2008) empirical research on the knowledge required by teachers to teach mathematics gave rise to the MKfT framework. It provides evidence of a constructive relationship between learners' learning, and teachers' PCK and SMK within mathematics education.

Hurrel (2013) highlights that the lines between the MKfT domains are too blurred, making it difficult to know where one domain starts and where the other ends. This blurring of the domains is problematic when analyzing data using the MKfT domains. Ball et al. (2008) acknowledge that:

> recognizing a wrong answer is common content knowledge (CCK), while sizing up the nature of the error may be either specialized content knowledge (SCK) or knowledge of content and students (KCS) depending on whether a teacher draws predominantly from her knowledge of mathematics and her ability to carry out a kind of mathematical analysis or instead draws from experience with students and familiarity with common student errors. Deciding how best to remediate the error may require knowledge of content and teaching (KCT). (p. 400)

Hurrel (2013) further argues that "the model does not display the possibilities of all the interactions between the domains" (p. 59). There is an issue with how the domains are represented in the MKfT diagram (Figure 2.2). The SCK appears to take up a larger area, with the possibility of using the diagram to establish the importance of one knowledge domain over the others. No research provides evidence that one domain is more important than the others (Hurrel, 2013).

Jaffer's (2020) critique of the MKfT framework highlights the challenge of ascertaining whether SCK is unique to teaching or common to all. For example, being able to tell where a learner made an error in a calculation, may not be limited to the work of teachers. She maintains that much of what Ball et al. (2008) describe as SMK could be CCK for mathematicians and not only teachers.

Since the MKfT categories overlap, operationalising the knowledge typology might be difficult (Jaffer, 2020). It is for this reason that I draw on a set of domain indicators adapted from Chikiwa (2017) to assist me in analysing my data.

Table 2.1 Ball et al's (2008) MKfT Domain Indicators

MKfT domains	MKfT domain indicators
CCK	• knowledge of how to calculate • calculate the reasonableness of a calculation • recognize when a student gives a wrong answer • recognize when a textbook gives an inaccurate answer to a calculation or gives an inaccurate definition and uses terms and notations incorrectly
HCK	• make connections across mathematics topics within a grade and across grades • articulates how the mathematics taught relates to the mathematics topics before and after the specific grade
SCK	• interpret students' emerging and incomplete ideas • evaluate the plausibility of students' claims or evaluate mathematical explanations • use appropriate mathematical notation and language and critique its use • interpret learners' mathematical calculation strategies, and those of other teachers or learning materials • evaluate mathematical explanations for common rules and procedures • evaluate estimation calculation explanations and procedures • identifying the nature of student errors in making a suitable approximation
KCT	• sequence mathematical content • present mathematical ideas • select examples to take students deeper into mathematical content • select appropriate representations to illustrate the content • ask productive mathematical questions • recognize what is involved in using a particular representation • modify tasks to be either easier or harder • use appropriate teaching strategies • respond to students' 'why questions' • choose and develop useable definitions • provide suitable examples
KCS	• anticipate what students are likely to think and do • predict what students will find interesting and motivating when choosing an example • anticipate what a student will find difficult and easy when completing a task • anticipate students' emerging and incomplete ideas • recognize and articulate misconceptions students carry about particular mathematics content
KCC	• articulate the topics in the curriculum • articulate the competencies related to each topic in the mathematics curriculum • articulate and demonstrate a familiarity with the structure of the mathematics curriculum

	• link representations to underlying ideas and to other representations • knowledgeability of available materials (e.g., textbooks) and their purposes • appraise and adapt the mathematical content of textbooks

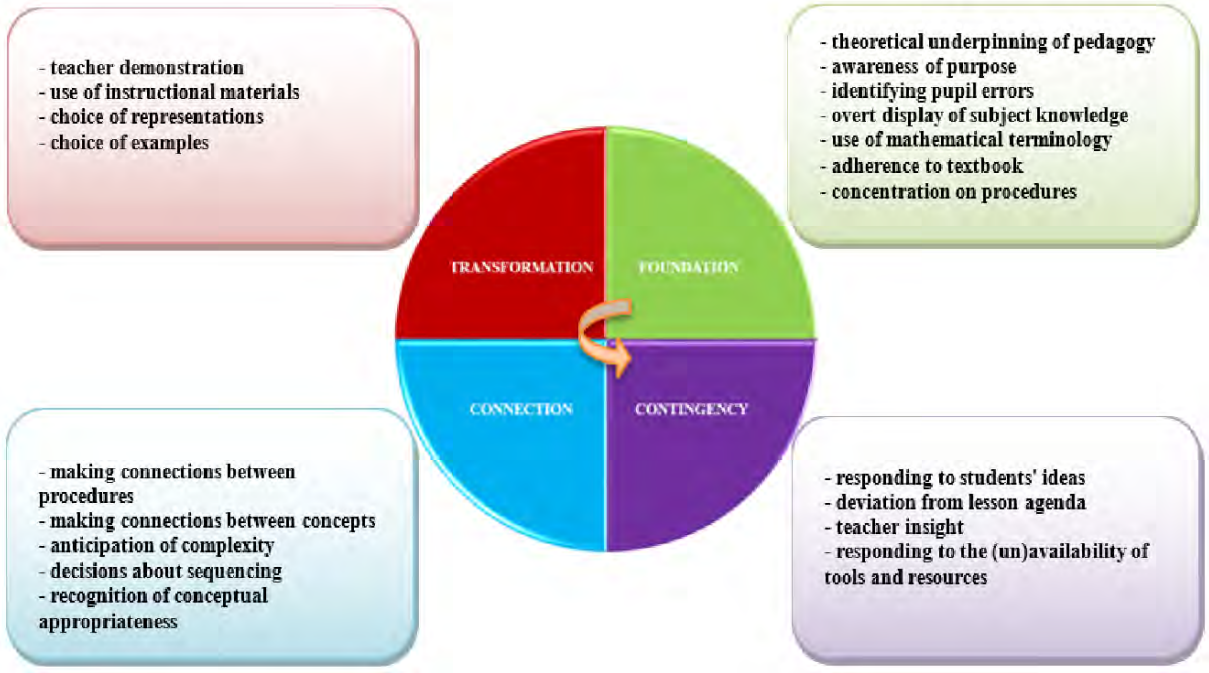
Note: Taken and adapted from: Chikiwa, S. (2017). *An investigation into the mathematics knowledge for teaching required to develop grade 2 learners' number sense through counting.* A thesis submitted in fulfilment of the requirements for the degree of Master of Education. Rhodes University: South Africa., p. 22.

Ball et al's. (2008) framework has also been criticised for its emphasis on knowledge *for* teaching and placing little emphasis on the knowledge teachers draw on *in* teaching. Based on this critique of the MKfT framework, I used the MKfT in conjunction with the KQ with a view to possibly overcoming some of the concerns mentioned above, particularly in relation to teaching-in-action. I draw on the Knowledge Quartet (KQ) of Rowland to explain both the knowledge teachers require to teach and the knowledge that they use in teaching mathematics.

2.5. CONCEPTUALISATION OF THE KNOWLEDGE QUARTET

Rowland and colleagues began to develop the KQ framework in 2002 to observe and analyse pre-service teachers' knowledge in teaching. The framework focused on pre-service teachers' mathematical content knowledge. Like Ball, Rowland (2005) did not consider the generic features of the lesson, that is, behavior management or classroom organization. Instead, they focused on happenings in mathematics lessons with reference to the content taught and the mathematics-related knowledge that teachers have and use during the act of teaching. The KQ is underpinned by Shulman (1986) and Ball et al's. (2008) distinction between subject matter knowledge (SMK) and PCK. The KQ identifies situations in which the types of knowledge identified by Shulman are made visible *in the act of teaching*. From the KQ perspective, mathematics teaching knowledge and beliefs are present in four dimensions. These are Foundation, Transformation, Connection and Contingency Knowledge (Figure 2.3).

Figure 2.3 Knowledge Quartet and Its Codes



Note: Rowland, cited in Kula-Unver, S. (2018). The knowledge quartet in the light of the literature on subject matter and pedagogical content knowledge. *Acta Didactica Napocensia, 11(2)*, 27-42., p. 28.

2.5.1. Foundation Knowledge

Foundation knowledge consists of knowledge, beliefs and understandings acquired at school, in teacher education institutions, in preparation for their classroom role, and in the classroom itself. Foundation Knowledge is modeled in what Shulman (2005) called s*ignature pedagogies*. According to Shulman (2005), signature pedagogies are forms of teaching that organize the important ways in which future teachers, that is pre-service teachers, are educated, instructed and prepared for their new profession. In preparation for the teaching profession, Shulman (2005) maintains that pre-service teachers must learn vast amounts of theory and bodies of knowledge. In other words, teachers must understand what and how they are going to teach the learners. Foundation Knowledge differs from the other three dimensions because it comprises the knowledge teachers have, regardless of whether it is used or not (Rowland, 2012).

Foundation Knowledge acknowledges the implicit structure of Shulman's signature pedagogy, that is, "the moral dimension that comprises a set of beliefs about professional attitudes, values and dispositions" (Shulman, 2005, p. 55). Such knowledge and beliefs fundamentally inform pedagogical choices and strategies.

There are three key components of Foundation Knowledge:

- knowledge and understanding of mathematics itself;
- knowledge of mathematics pedagogy; and
- beliefs about mathematics, why and how mathematics is learned, and pedagogy.

Rowland et al. (2009) highlighted the knowledge needed to teach mathematics by explaining the relationship between the theoretical structures of Foundation Knowledge and that of Shulman's SMK and PCK.

As noted above, the first category in Shulman's (1986, 1987) typology was subject matter knowledge. Subject matter knowledge consists of substantive and syntactic knowledge. Substantive knowledge refers to the knowledge of the mathematical concepts, processes and the connections within and across the subject. By contrast, syntactic knowledge refers to how the knowledge of the subject is investigated and produced. This relates closely to the work of mathematicians, that is, problem-solving, identifying patterns and generalisations, justifying one's solution strategies etc. While both aspects of subject-matter knowledge are central to Foundation Knowledge, they do not provide the whole picture. PCK is also an important component of Foundation Knowledge. Rowland et al., (2009) add that

> Teachers need to understand the ways in which pedagogical strategies relate to the mathematics they are trying to teach in order to make decisions about which strategies to use. These actual decisions would be considered to be part of the transformation dimension; however, the theoretical understanding that underpins them is part of a teacher's Foundation Knowledge (p. 153).

A summary of the Foundation Knowledge codes appears in Figure 2.3.

What sets the KQ apart from Shulman and Ball and colleagues' work is that it includes both the mathematics knowledge required *for* teaching and mathematics knowledge required *in* teaching. The next three dimensions of the KQ focus on knowledge-in-action as shown both in planning to

teach and during the process of teaching. These include transformation, connection and contingency knowledge.

2.5.2. Transformation Knowledge

The second category of the KQ, gets to the crux of what it means to teach mathematics. This dimension takes the heart of Shulman's (1987) observation that the knowledge base for teaching is distinguished by "the capacity of a teacher to transform the content knowledge he or she possesses into forms that are pedagogically powerful" (p. 15). Rowland et al. (2009) refer to Transformation Knowledge as the teachers' ability to transform what they know in ways that the learners understand. They maintained that Transformation Knowledge is about how well teachers can make what they know accessible to the learners typically through different forms of representation, such as, "analogies, illustrations, examples, explanations and demonstrations" (Shulman, 1986, p. 9). The choices and use of examples presented to the learners are to assist their sense-making. Transformation Knowledge can be summed up as how the teacher could best transform his/her Foundation Knowledge into a lesson plan and how the teacher transforms the Foundation Knowledge into the actual lesson. This is indicated by their choice of materials, examples and representations as noted in Figure 2.3.

2.5.3. Connection Knowledge

According to Rowland et al. (2009), Connection Knowledge is how well a teacher's lesson hangs together and relates to what is taught in previous lessons and the learners' knowledge. Rowland et al. (2015) maintain that Connection Knowledge includes choosing mathematical topics, the sequencing of topics for instruction, and how tasks are explained and ordered as noted in Figure 2.3. Turner (2007) echoed that the significant components of Connection Knowledge are the knowledge of complexity of a topic, making connections and acknowledgement of the cognitive demand for the learners. Liston (2015) concurred that Connection Knowledge assists learners in making links between the concepts and procedures taught. This dimension is about the coherence of planning and teaching throughout the lesson, the connectedness of mathematical content and the knowledge of structural connections within mathematics, and an awareness of the conceptual suitability of the various curriculum topics and tasks (Rowland et al. 2009). Connection Knowledge entails sequencing instructional topics during and between lessons.

For this study, connection knowledge could be observed in how well a teacher can anticipate the complexity of computational estimation and break the computational estimation strategies into steps that the learners understand. It could also be how the teacher could link various number concepts (e.g., place value and rounding off) to computational estimation.

2.5.4. Contingency Knowledge

Responding to the learners' ideas immediately in class is likely to ensure more meaningful learning (Turner, 2009). In this sense, Turner (2009) indicates that Contingency Knowledge is about how teachers respond to these unplanned and unexpected events in a lesson. Rowland et al. (2009) argue that teachers should equip themselves with greater knowledge for "fewer surprises when teaching since such knowledge enables the teacher to anticipate and plan for a greater number of pupil responses" (p. 31). Rowland et al. (2009) described this dimension as the ability to "think on one's feet" (p. 33) as it is about contingent action.

Contingency Knowledge further links to Shulman's signature pedagogies. "Signature pedagogies are pedagogies of uncertainty" (Shulman, 2005, p. 57). They offer classroom settings that are unpredictable and surprising, raising concerns for both the teachers and the learners. This unpredictable and surprising moment is similar to the one modelled in the Contingency Knowledge dimension of the KQ. Shulman (2005) maintains that signature pedagogies create environments that support risk-taking. A lesson uncertainty tends to raise emotional stakes in the classroom. Shulman's signature pedagogies acknowledge that when the "emotional content of learning is well sustained, we have the real possibility of pedagogies of formation, that is, experiences of teaching and learning that can influence the values, dispositions and characters of those who learn" (Shulman, 2005, p. 57).

To conclude, Rowland (2005) based the KQ framework on Shulman's (1987) conceptualisation of SMK and PCK. The KQ provides a way of establishing and organizing how mathematics teachers' knowledge work during teaching (Rowland & Zazkis, 2013). According to Liston (2015), mathematics teachers' knowledge could be improved using the KQ framework. The knowledge related to teaching in this study are the teachers' SMK and the PCK. The KQ framework has four

dimensions of which each dimension has different codes. As noted in Figure 2.3, Kula-Unver (2018) formed indicators for each of the dimensions. I summarize the KQ indicators in Table 2.2 below. Evidence for this may be found in both planning and teaching.

Table 2.2 Knowledge Quartet Indicators

KQ dimension	KQ code indicators
Foundation	● clear and coherent beliefs about the purposes of mathematics education ● recognize the conceptual appropriateness of mathematical ideas for the children they are teaching ● sound procedural and conceptual knowledge ● knowledge of common learner errors and misconceptions ● adapt textbooks to teaching ● use mathematical language correctly (e.g., using approximately instead of equal to in the context of estimation)
Transformation	● choose appropriate examples and representations when demonstrating or eliciting an idea ● give clear explanations of the concepts and strategies ● demonstrate clearly and accurately how to carry out procedures ● make use of interactive teaching techniques to develop and assess understanding of concepts
Connection	● make links to previous lessons ● make appropriate conceptual connections within the subject matter content ● ask questions to elicit children's understanding of connections between mathematical ideas ● anticipate what learners will find challenging ● assess learners' understanding
Contingency	● respond to learner's comments, questions and answers ● deal appropriately with learners' responses to activities ● respond appropriately to the learners' errors and misconceptions ● deviate from the lesson plan when appropriate ● respond to the strengths and limitations of the LTSM available

Note: Taken from and rearranged from Kula-Unver, S. (2018). The knowledge quartet in the light of the literature on subject matter and pedagogical content knowledge. *Acta Didactica Napocensia,* 11(2), 27-42., p. 29-33.

2.6. CRITIQUE OF THE KNOWLEDGE QUARTET

The KQ framework is based on empirical research in classrooms and consists of four dimensions encompassing a total of 20 different codes (Rowland, 2014). Given that the framework is an "analytical framework revealed through observations of practice" (Turner, 2012, p. 256), the KQ codes may be too dense to assist with focused reflection on the mathematical content taught. Also, some KQ indicators or codes may not be observed, given the nature of the mathematics topic at hand (Breen, et al., 2018).

The KQ does not address the issue of curriculum content knowledge, which Shulman regarded as important (Petrou & Goulding, 2011). Petrou and Goulding (2011) argued that if the curriculum is the key to mathematics teaching, then knowledge of the curriculum is "important in any attempt to understand what teachers need to know to teach mathematics effectively" (p. 20).

2.7. MATHEMATICS KNOWLEDGE FOR TEACHING AND THE KNOWLEDGE QUARTET

The MKfT framework differs from the KQ. Ball et al., (2008) refer to the MKfT as a practice-based and empirically grounded theory of knowledge for teaching. Rowland & Zazkis (2013) maintained that this description is also applicable to the KQ, "but while parallels can be drawn between the methods and some of the outcomes, the theories look very different" (p.22). According to Rowland & Zazkis (2013), the MKfT framework aims to clarify and unpack the formerly obscure and theoretically under-developed notions of SMK and PCK. Whereas the KQ aims to classify the situations in which mathematical knowledge surfaces during the activity of teaching, thus, giving lesser importance between different kinds of mathematical knowledge. These two theories are complementary, offering useful perspectives that complement one another (Turner & Rowland, 2009).

2.8. CHAPTER SUMMARY

In this chapter, I examine three different conceptions of the knowledge base that teachers require to do their work of teaching. The chapter begins with an explanation of Shulman's explanation of this knowledge base as inclusive of subject matter knowledge, PCK and curricular knowledge. Based on the criticisms of Shulman's knowledge base for teaching being 'static', the emphasis on

secondary schools, where teachers are by and large subject specialists, and the concern that his theory is difficult to operationalize (Hill et al., 2005), the MKfT framework of Ball and colleagues was introduced. The MKfT framework focuses explicitly on primary mathematics education. It includes two categories each with three domains as highlighted in the chapter. The MKfT framework is useful in understanding the knowledge that teachers need *for* teaching mathematics but has been critiqued (Rowland & Zazkis, 2013) for the lack of focus on the knowledge teachers use *in* teaching, that is, in the act of teaching. The Knowledge Quartet provides a framework for explaining and analysing the knowledge that teachers need and use *in* the process of teaching mathematics in the classroom.

The next chapter focuses on the key concepts informing this research, that is, the development of computational estimation broadly and computational estimation strategies specifically.

CHAPTER 3: LITERATURE REVIEW

3.1. INTRODUCTION

Driscoll (2007) maintains that primary teachers are at the forefront of mathematics education because of their responsibility in preparing the learners for their future education at secondary schools and tertiary institutions. Like Shulman (1986), Ball et al., (2008) and Rowland et al., (2009). Driscoll (2007) claims that to adequately prepare primary school teachers for this role, they must have a good knowledge of mathematics, and how to teach mathematics, to be able to make the mathematics content accessible to all learners.

As noted in Chapter Two, teachers' mathematics and pedagogical content knowledge plays a crucial role in the teaching of mathematics (Hill et al., 2008). As Ball et al. (2008) notes, teachers need as a bare minimum to know and understand the content that the learners need to know (Ball et al., 2008). It is in this vein that mathematics teacher education programmes should ensure that pre-service and in-service teachers have what Ball et al. (2008) refer to as Subject Matter Knowledge and Pedagogical Content Knowledge to teach mathematics effectively. Kapenda and Kasanda (2015) advise that teacher education programmes should make sure that they enable teachers to make the content accessible to the learners. Such training should also enhance teachers' knowledge of the mathematics that they need to teach.

Courtney-Clarke and Wessels (2014) argue that teacher education institutions need to place a special emphasis on developing pre-service teachers' number sense so that they in turn will be able to develop their learners' number sense. Central to the development of number sense is the ability to draw on a wide repertoire of calculation strategies in order to work flexibly, efficiently and accurately when performing calculations (Anghileri, 2000). In working accurately, learners need to be able to judge the reasonableness of their answers to different calculations (Anghileri, 2000). This means that learners require a variety of computational estimation strategies to assist them in determining the accuracy of their answers.

I begin this chapter by focusing on the development of computational estimation as this is a key concept informing this research.

3.2. ESTIMATION

Estimation refers to a number that is a "suitable approximation for an exact number given in a particular context" (Van de Walle, et al., 2010, p. 241). Van de Walle et al. (2010) explain that one of the important goals for estimation is that learners make sense of the context of the problem or question they are required to solve. Utilizing the context of a problem is relevant to estimation, but also mathematics more generally (Van de Walle et al., 2010). According to Sunde et al. (2022) estimation takes four forms: *measurement estimation, quantity (or numerosity) estimation, number line estimation and computational estimation.*

Measurement estimation is the ability to measure without measuring tools (Sunde et al., 2022). According to Sowder (1992), measurement estimation draws on mental referents to assist in the process of measuring, of particular importance in everyday situations (e.g., baking a cake). Sunde et al. (2022) argues that measurement estimation is poorly taught in school mathematics due to a lack of teacher confidence.

Number line estimation is an example of numerical estimation and it is necessary for developing learners' number sense (Dehaene, 1997). Number line estimation is the ability to visualise and place numbers on a number line. Children require and develop knowledge of reference points on a number line (e.g. the multiples of 10 on a number line from 0 to 100). This is particularly important when developing arithmetic competence and learning fractions (Bailey et al., 2014).

Quantity estimation concerns "the ability to discern or produce the number of objects in a set without recourse to counting" (Sunde et al., 2022, p. 628). Quantity estimation is about finding an approximate number of items in a set (e.g., subitising, that is, the ability to identify the number of objects in a collection without having to count each object). Quantity estimation creates opportunities to estimate and reflect on the accuracy of the estimation.

This chapter focuses on the fourth form of estimation as proposed by Sunde, et al. (2020), that is, *computational estimation*. Reys and Bestgen (1981) maintain that computational estimation is a necessary and basic skill which should be an integral part of every mathematics classroom. The

goal of computational estimation is to produce an approximation that will give a sense of the reasonableness of the answer.

3.3. COMPUTATIONAL ESTIMATION

Computational estimation is defined as the process of simplifying an arithmetic problem to produce an approximate but satisfactory answer to a calculation (Courtney-Clarke, 2012; Son, et al., 2019). Computational estimation is the internal thinking required to obtain a rough answer before, while or after calculating. For example, when shopping, we often estimate to get an approximate amount that we are going to pay for our items and to determine if the money we have is enough. Reys and Bestgen (cited in Tsao & Pan, 2011) argued that computational estimation is the application of number concepts and various calculation skills to obtain approximate answers rapidly through mental computation.

Like Reys and Bertgen (cited in Tsao & Pan, 2011), Lin (cited in Tsao & Pan, 2011) proposes that computational estimation is a concept of mental calculation that involves a better understanding of number concept and mathematics operations. Lin maintains that computational estimation is roughly done without any external tools to aid the computation. Computational estimation is a useful strategy to check the reasonableness of a result and thus requires an understanding of number operations and a competence in making mathematical judgments. It requires the flexible application of mathematical knowledge to develop a useful strategy to calculate Yang (cited in Tsao & Pan, 2011).

Cochran and Dugger (2013) indicated that learners make use of the following mathematical concepts when performing computational estimation: an understanding of the relative sizes of numbers, knowledge of basic number facts, an understanding of the base-10 number system, an understanding of place value, and fluency in mental computations. The development of computational estimation capabilities enables learners to make a judgment about the reasonableness of their answers and in the process develops their understanding of number and number operations (Tsao & Pan, 2011).

Computational estimation skills enable learners to use more efficient ways to calculate and find reasonable approximates. Fennell (2008) explained that, "as learners estimate, talk about numbers, compute, use mental math and judge the reasonableness of their results, they become more flexible in working with numbers" (p. 3). Computational estimation in the classroom may improve the learners' calculation skills and generally their number sense (Cochran & Dugger, 2013 ; Fennell, 2008). As such, teachers need to target skills (e.g., computational estimation) that are likely to develop and reinforce learners' number sense. Teachers should design appropriate computational estimation tasks which can improve learners' sense of how numbers are organized and how numbers work.

3.4. THE RELATIONSHIP BETWEEN COMPUTATIONAL ESTIMATION AND MATHEMATICS PROFICIENCY

Kilpatrick, et al. (2001) argue that computational estimation should incorporate all strands of mathematical proficiency. They highlight five strands of mathematical proficiency which I link directly to computational estimation. These are:

- *Conceptual understanding* refers to the "comprehension of mathematical concepts, operations, and relations" (Kilpatrick et al., 2001, p. 116). This would include the ability to represent and apply computational estimation in different ways. For example, being able to use computational estimation in a variety of different situations.
- *Procedural fluency* refers to the "skill in carrying out procedures flexibly, accurately, efficiently" (Kilpatrick et al., 2001, p. 116). This involves understanding of a variety of strategies used in computational estimation, and when and how to use such strategies. Learners should be able to calculate mentally by drawing on the basic number facts. Good estimators are flexible in their approach to computation and can use different computational estimation strategies.
- *Strategic competence* is the ability "to formulate, represent, and solve mathematical problems" (Kilpatrick et al., 2001, p. 116). In other words, learners should be familiar with different ways to estimate the reasonableness of their result for a calculation.
- *Adaptive reasoning* refers to the "capacity for logical thought, reflection, explanation and justification" (Kilpatrick et al., 2001, p. 116). In computational estimation, it is being

able to do calculations that emphasize logical reasoning, reflect on the reasonableness of the computation result, and explain the use of chosen computational estimation strategy.

- *Productive disposition* is the "habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy" (Kilpatrick et al., 2001, p. 116). It is a tendency to see the importance of computational estimation, being able to make sense of the computational estimation and recognizing the usefulness of computational estimation. It is the propensity of seeing oneself as capable of being a good estimator.

The above-mentioned strands are completely dependent on one another and need to be developed together.

3.5. CHALLENGES IN DEVELOPING COMPUTATIONAL ESTIMATION

There are a number of challenges with regard to the development of computational estimation in mathematics classrooms. The mathematics syllabus for Grades 4-7 in Namibia, places computational estimation under the topic of 'number concept', 'addition and subtraction' and 'multiplication and division' (MoEAC, 2016). Despite this, and as noted in Chapter 1, little attention is given to computational estimation. Thomsen (2017) maintains that although computational estimation is an important skill in advancing learners' number sense, daily practice in classrooms is rare. It is important for primary school teachers to engage learners in daily computational estimation activities and provide practice for mathematical situations they may come across outside of the classroom (e.g., working out the approximate cost of groceries). The Namibian mathematics syllabus does not give much attention to computational estimation, hence placing it under the number concept umbrella. This makes it difficult for mathematics teachers to integrate computational estimation skills in their daily classroom practice. The syllabus does not provide the teachers with appropriate computational estimation strategies, hence most of the teachers only limit computational estimation to rounding off (MoEAC, 2016), for example 38 + 24 is estimated as 40 + 20. Learners' understanding of computational estimation is often hindered because of the emphasis in mathematics on finding exact answers (Reys et al., 1980). Learners learn procedures for calculating and, if implemented correctly, obtain answers that are always exact.

3.6. THE CONNECTION BETWEEN COMPUTATIONAL ESTIMATION AND NUMBER SENSE

Computational estimation is an important component of number sense. Kilpatrick et al. (2001) outlined the benefits of computational estimation as a real-world skill that develops learners' understanding of place value, their number sense and knowledge of mathematical operations.

Kim (2016) defines number sense as a person's understanding of numbers and operations, competence to work flexibly, efficiently and accurately with numbers and operations, and the ability to apply a range of useful and appropriate strategies. Schneider and Thompson (2000) refer to number sense as a person's understanding of number meanings and relationships among numbers. This they argue is central to flexible thinking about numbers. The NCTM (2000) extends these two conceptions of number sense explaining that number sense includes an (a) understanding of numbers, the different ways of representing numbers and "the relationships between numbers and the number system", (b) "understanding the meaning of operations and how they relate to one another", and (c) the ability to "compute fluently and make reasonable estimates" (p. 32).

Naukushu (2016) argues that we cannot completely define number sense without incorporating computational estimation, as number sense involves a deeper understanding of counting, computation strategies and the use of benchmarks to estimate. According to Haimbodi (2019) the development of number sense has possibly occurred in curricula and some classrooms because of the vital role it plays in an individual's ability to respond flexibly to numbers. Haimbodi (2019) further alluded to the development of number sense as the main goal of primary school mathematics. Equally so, the MoEAC (2016) considers number sense important in the senior primary phase. According to the MoEAC (2016), number sense develops "gradually as a result of exposing numbers, visualizing them in a variety of concepts and relating them in ways that are not limited by traditional algorithms" (p. 11). Bana (cited in Naukushu, 2016) holds the view that number sense:

> should incorporate the ability to do one of the following or more: to approximate or estimate, make meaningful numerical magnitude comparisons, decompose numbers naturally, develop useful computational strategies, solve complex problems, use the relationships among arithmetic operations, understand the base-ten number system, use numbers and quantitative methods communicate, process, and interpret information. (p. 60)

Computational estimation and number sense develop iteratively. Therefore, computational estimation is not a content topic on its own but is rather a learning process that should be developed throughout schooling and beyond (MoEAC, 2016). This may explain why computational estimation does not feature extensively in the Namibian mathematics syllabus.

3.7. COMPUTATIONAL ESTIMATION AND MENTAL COMPUTATION

Mental mathematics is central to the development of number sense and mathematical proficiency (Westaway & Vale, 2021). In the process of calculating mentally, learners draw on a range of strategies and select those that are deemed most efficient in a particular situation (Pourdavood et al., 2020). When asked to calculate in written form, many learners tend to use the standard algorithm, which, as explained in Chapter 1, is the taught procedure in Namibia (Courtney-Clarke, 2012). Mental calculations tend to focus on the structure of numbers and operations and the use of basic facts (Rathgeb-Schnierer & Green, 2019). The distinction between taught procedures and mental mathematics, is that mental mathematics focuses on calculating with numbers rather than digits (Westaway & Vale, 2021).

Computational estimation involves mental manipulation of numbers. “Mental computation and computational estimation share a common background characterized by factors essential to a well-developed sense of numbers” (Haimbodi, 2019, p. 46). According to Case and Sowder (1990), this includes knowledge of the symbol system used to represent numbers, an understanding of place value and an ability to operate with multiples and powers of ten.

Reys (1984) asserts that “mental computation is an important component of estimation as it provides the cornerstone necessary for the diverse numeric processes used in computational estimation” (p. 548). There are many advantages of teaching mental computation. Reys (1984) pioneered five reasons for teaching mental computation, but the most essential benefit of teaching mental computation is that mental computation promotes creative and independent thinking which encourages learners to create and develop adroit ways of working with numbers. He further argues that mental computation encourages a broader understanding of the structure of numbers and their properties, and by doing so, creates a foundation for developing computational estimation.

Reys (1984) suggests that "mental computation should be a visible part of elementary mathematics" (p. 550). To support the above, senior primary teachers are expected to place an emphasis on the "mental arithmetic strategies to develop the learners' awareness of numbers and number sense" (MoEAC, 2016, p. 1). Mathematics teachers should design instruction in such a way that it elicits learners' use of a variety of mental computation strategies. The senior primary school mathematics syllabus has listed mental computation as a topic on its own, with learning objectives distributed across the grades. There are a few mental mathematics strategies in the Namibian mathematics syllabus that support learners in the development of mental computation. These include compensation, bridging the decades, changing the order of the addends, and splitting both numbers into ten and units (MoEAC, 2016). According to Haimbodi (2019), teachers should provide opportunities for the learners to develop and use a range of different mental strategies, give clear guidance "where each strategy can be used and encourage learners to use mental strategies regularly to improve their skills" (p. 59). A wide range of mental calculation strategies is required to support the development of computational estimation.

3.8. DEVELOPING COMPUTATIONAL ESTIMATION

If mathematics teachers are required to develop learners' computational estimation, then they should be familiar with and able to use a variety of strategies in efficient and flexible ways. According to Morgan (1999) there are three key cognitive processes required by efficient computational estimators. These key cognitive processes are: *Reformulation*, *Compensation*, and *Translation*.

- Reformulation

Reformulation is the "process of altering numerical data to produce a more mentally manageable form, while leaving the structure of the problem intact" (Reys et al., 1982, p. 187). Reys (1984) argues that in many school mathematics curricula, the development of computational estimation skills has been restricted to approximating numbers using *rounding off,* which is a reformulation strategy. In the Namibian mathematics syllabus for Grades 4-7, learners are required to be able to round off to the nearest multiple of 10, 100 and 1000. This is emphasised in the mental mathematics component of the curriculum. Other common reformulation strategies include the

front-end strategy, *benchmarks* and *compatible numbers* (Table 3.1). The latter three strategies are not referred to in the Namibian mathematics syllabus.

- Compensation

According to Reys (1984), compensation entails the ability to compare numbers. Compensation entails an understanding that one may compensate not only after an initial estimate has been calculated, but also during the stages of calculating and after performing the calculation. Compensation strategies include *adjusting*, *special numbers* and *truncation* (Table 3.1)

- Translation

Translation is a process of "modifying or changing the mathematical structure of a problem to a form which is easily managed mentally" (Reys et al., 1982, p. 188). Reys et al. (1982) maintain that translation is more flexible given that the learners have a wider view of the problem which is less restricted by the numbers involved. Sowder and Wheeler (1989) characterized *averaging* as the main translation strategy. Van de Walle et al. (2010) refer to averaging as clustering. In this study, averaging and clustering are used interchangeably. Averaging is mainly used when the values of the given numbers are clustered around a common value. Averaging entails finding an estimate of the average in a list of numbers by looking for a number that all other numbers seem to surround (Table 3.1). For example, the numbers in the calculation 492 + 498 + 505 are all clustered around 500.

The following table provides a summary of all the strategies deemed suitable, according to the literature that I have read, for the development of senior primary learners' computational estimation skills.

Table 3.1 Computational Estimation Strategies

Process	Strategy	Description	Example(s)
Reformulation	Rounding off	Using the closest ten to make the computation easier.	43 + 29 is estimated by calculating 40 + 30 = 70

	Benchmarks	Using knowledge of the base-ten system when rounding to make a problem simpler.	35+15 may be worked out as 30 + 10 + 5 + 5 = 40 + 10 = 50
	Compatible numbers	This refers to a pair of numbers that are easy to add, subtract, multiply or divide mentally. The compatible strategy can be used when a student realises that the numbers can be re-ordered to make calculation easier.	27 + 48 + 56 + 72 27 + 72 is almost 100; and 48 + 56 is almost 100. These two pairs of numbers are easy to work with. A learner can estimate the reasonableness of their calculation by doubling 100.
	Front-end method	Using the front-digits of each number.	34 + 45 Start with the tens: 30 + 40 = 70
Translation	Clustering/Averaging	Looking for an estimate of an average in a list of numbers by looking for a number that all other numbers seem to surround.	699 + 710 + 695 + 705 All these numbers cluster around 700. So, one could calculate 700 × 4 = 2800 to get the estimation.
Compensation	Adjusting	When one number is changed/adjusted in a problem to make it easier to solve .	35 - 18 could be calculated mentally as 35 – 20 = 15
	Special numbers	Using the numbers that are friendly to use.	9.5% of 500ml One can calculate the 10% of 500 ml which is easier, 10% being a special number: 10% of 500ml =50ml

	Truncation	Means cutting off the decimal portion of a number without rounding it. In other words, the decimal point in the number is ignored.	20.4 + 15.8 We get 20 + 15 =35

A number of research studies have been conducted to ascertain which of the above computational estimation strategies are favoured by teachers, adults and learners (Anestakis & Desli, 2014; Cochran & Dugger, 2013; Desli & Lioliou, 2020; Lemonidis & Likidis, 2021; Liu & Neber, 2012; Tsao & Pan, 2013; Ulu & Ozdemir, 2018;).

3.9. RELATED RESEARCH ON COMPUTATIONAL ESTIMATION

Several studies have been conducted on computational estimation (Anestakis & Desli, 2014; Cochram & Dugger, 2013; Desli & Lioliou 2020 ; Lemonidis & Likidis, 2021; Liu & Neber, 2012; Tsao & Pan, 2013; Ulu & Ozdemir, 2018). Cochram and Dugger (2013) worked with 26 students in Grades 6, 7 and 8 at an independent school located in a rural community in the south-eastern United States of America. In their study, they presented the participants with both written and oral problems that required computational estimation. The study found that the participants had little knowledge of the range of computational estimation strategies. The only computational estimation strategy that the learners used was rounding off. For the most, the learners gave the exact answer using a set procedure.

Desli and Lioliou (2020) explored the relationship between problem solving and computational estimation. Their study participants were a total of 72 adults and Grade 6 learners. They presented the participants with tasks which resulted in adults being more successful at estimation with the use of rounding. The learners did not estimate at all but resorted to standard algorithms instead. Based on my reading of various articles, learners were not inclined to use any estimation strategies to check the reasonableness of their answers when asked.

Liu and Neber (2012) researched the estimation strategies used by Chinese and Polish children through estimation tests, questionnaires and interviews. The study found that the dominant strategies that the Chinese learners used included rounding off and benchmarks, while the estimation strategies applied by Polish children were rounding off, benchmarks and using special

numbers. Lemonidis and Likidis (2021) investigated the computational estimation strategies used by 62 Grade 6 learners. The study collected data through computational estimation tests, before and after an intervention. Contrary to Cochram and Dugger (2013) and Desli and Lioliou (2020), they found that the learners used a wide variety of computational estimation strategies. These included compensation, translation, clustering, special numbers, compatible numbers, the front-end method, truncating and rounding off.

Ulu and Ozdemir (2018) conducted research that was aimed at determining the strategies used by 26 Grade 4 primary school learners. The research data was collected via semi-structured interviews. They concluded from their research that the Grade 4 learners used the strategies of adjusting or rearranging, rounding off and special numbers. This concurs with the studies of Lemonidis and Likidis (2019 and Ulu and Ozdemir (2018) (noted above) that learners use a variety of strategies when estimating the reasonableness of the results of their computations although some strategies were preferred by the learners over other strategies.

Tsao and Pan (2013) investigated six Grade 5 primary school teachers' knowledge and understanding of computational estimation and how they developed their learners' understanding of computational estimation. The study participants were each interviewed three times during the course of the research. They found that the strategies that the teachers used when faced with computational estimation questions included rounding, compatible numbers, front-end rounding and friendly numbers.

Anestakis and Desli (2014), conducted a study that was aimed at examining primary school pre-service teachers' views of computational estimation and its teaching in primary schools. Their study participants were a total of 113 Greek pre-service teachers. Data were gathered through questionnaires, followed by interviews with 10 pre-service teachers. Questionnaires were administered to find out how the pre-service teachers value computational estimation and their suggestions for teaching it. The interviews were done to find out teachers' understanding of computational estimation and the skills needed in developing it. The results revealed that the majority of the pre-service teachers believe computational estimation is important for both daily

life and in school. Their study proposed that research be conducted on how pre-service teachers incorporate computational estimation in the classroom context.

The commonalities in the research reviewed above lies in the estimation strategies used by pre-service and in-service teachers, adults and learners when faced with computational estimation questions. By reviewing the studies above, it can only be assumed that if teachers are to develop the learners' computational estimation skills, they should have a sound knowledge in computational estimation. Sharing similar sentiments, Anestakis and Desli (2014) are of the view that if teachers' knowledge and use of computational estimation, and the value ascribe to it, is limited, it is likely that they will experience difficulties helping children to understand the value and importance of computational estimation. As Greeno (1991) puts it, "if someone is to serve as an effective guide to newcomers in an environment, it is essential that the guide himself or herself should be a comfortable resident of the environment" (p. 55). Hence my interest in developing an understanding of the mathematics content and pedagogical content knowledge required to develop learners' computational estimation.

3.10. CHAPTER SUMMARY

This chapter discussed the literature on computational estimation and its place in the Namibian senior primary mathematics curriculum. The chapter explored number sense, the five strands of mathematics proficiency and mental arithmetic as these are all directly linked to the development of computational estimation. A number of studies were reviewed and revealed that both pre-service and in-service teachers have little knowledge of computational estimation, which is directly linked to lack of content and pedagogical content knowledge. The chapter suggests that in order for learners to be exposed to a variety of computational estimation strategies, teachers need to understand a wide variety of computational estimation strategies. The variety of strategies discussed requires the three cognitive processes of reformulation, compensation and translation. The next chapter will focus on the methodology used to carry out this study.

CHAPTER 4: METHODOLOGY

4.1. INTRODUCTION

This chapter presents the research design of my study that seeks to understand the knowledge that senior primary mathematics teachers have and draw on in order to develop their learners' computational estimation skills. The question informing this study is: *What mathematical and pedagogical content knowledge do senior primary mathematics teachers draw on to develop their learners' computational estimation skills?* I have two sub-questions to assist me in answering the main question, these are,

(1) What mathematical content knowledge do teachers draw on to develop the learners' computational estimation skills? and
(2) What pedagogical content knowledge do teachers draw on to develop the learners' computational estimation skills?

This is a qualitative case study located within the interpretivist orientation as I seek to understand the teachers' knowledge. The methods of data collection include questionnaires, lesson observations and focus group interviews. The use of various data collection methods enables me to triangulate the data. I begin the analysis with inductive reasoning, that is, identifying the codes without the overt use of my theory. Thereafter, I used deductive reasoning to examine the data based on the domains and categories from the MKfT and KQ frameworks.

4.2. RESEARCH ORIENTATION

This study is underpinned by the interpretivist orientation. Bertram and Christiansen (2014) suggest that interpretivism aims to understand people's social behaviour and how they make sense or draw meaning from their experiences. They further stress that interpretivism is based on the view that there is no one true reality. Rather, they suggest that there are multiple interpretations of events in the world. Alvermann and Mallozzi (2010) maintain that interpretivism denotes "an approach of social life with an assumption that meaning of human action is inherent in action" (p. 12). The study employs a qualitative research approach using a case study research design underpinned by an interpretive orientation (Maree, 2010). Denzin and Lincoln (2005) defined qualitative research as:

> involving an interpretative, naturalistic approach to its subject matter. This means that qualitative researchers study things in their natural settings, attempting to make sense of, or interpret, phenomena in terms of the meanings people bring to them. Qualitative research involves the studied use and collection of a variety of empirical materials – case study, personal experience, introspective, life story, interview, observational, historical, interactional, and visual texts – that describe routine and problematic moments and meanings in individuals' lives. (p. 2)

This means that the qualitative researcher studies things in their natural settings and tries to make sense of phenomena in terms of the meanings people bring to those phenomena. In this research, I used a qualitative research approach to enable me to explore and understand the knowledge that teachers use and draw on when developing their learners' computational estimation skills. A qualitative research design allowed me to study teachers' knowledge and use of computational estimation in depth. Blanche et al. (2006) argue that the researcher generates data in the form of written or spoken language (e.g., questionnaires or interviews) or in the form of observations, hence these are the main data generation tools in this study. The study adopts a qualitative research approach because the data collection was done in the natural social setting of the classroom in order to build a complex and holistic picture of how teachers develop their learners' computational estimation. My intention was to generate rich data from questionnaires, a focus group interview and observations of the participants teaching lessons.

4.3. RESEARCH DESIGN

Yin (2014) describes case study research as "an empirical inquiry that investigates a contemporary phenomenon ('the case') in-depth and within its real-world context when the boundaries between phenomenon and context may not be clear" (p. 16). Creswell (2012) adds that a case study is "a qualitative approach in which the researcher explores a particular unit or phenomenon bound by time and activity. The boundedness of the case study is significant in my research as it enables me to study the complexity of a particular phenomenon, that is Senior Primary teachers' PCK in developing learners' computational estimation, within a specific context and time.

Yin (2014) further describes a case study is also "an in-depth investigation of a given social unit, be it an organization, entity, individual or an event" (p. 30). In the context of this study, senior primary mathematics teachers in the Ohangwena region made up such a social unit. In this case

study, I attempted to capture the 'reality' of the participant's lived experiences as senior primary teachers of mathematics required to teach computational estimation. Thus, I engaged with senior primary mathematics teachers from two schools in the Ohangwena region over the period of one month in their naturalistic setting (Creswell & Cresswell, 2018), that being, the classrooms.

Case studies allow for the generation of rich, in-depth data that involves multiple sources of information (Cresswell, 2012). In this study, the data generation sources included:

- questionnaires, distributed to eight teachers;
- observations of two teachers; and
- a focus group interview with three teachers.

The choice of multiple methods of data generation assisted me in triangulating my data as will be discussed later in this chapter.

This study was an instrumental case study, which was administered to develop a broad understanding of the phenomenon, that is, an extensive understanding of how eight senior primary teachers of mathematics develop their learners' computational estimation skills, thereby also providing insights on how these eight teachers develop their learners' computational estimation skills (Stake, 1995).

4.4. RESEARCH SITE

The study took place in the north of Namibia, in the Ohangwena region. The study focused on the senior primary (Grades 4-7) teachers of mathematics. There are a variety of schools in the Ohangwena region - rural schools, farm schools and township/urban schools. The language used for teaching and learning in the senior primary in all the schools in Namibia is English. Out of the many schools in the Ohangwena region, the study focused on two public rural schools. These schools were chosen because they are close to where I live and work. One school is a primary school from Grades 0 to 7, while the other school is a combined school (i.e., it includes both primary and junior secondary learners (Grades 0 to 9)).

4.5. SELECTION OF PARTICIPANTS

Choosing the research participants is one of the most important parts of research as participants determine the quality and quantity of data that the researcher will collect (Saunders, 2012). This makes choosing the sample and sampling criteria crucial. This study used purposive and convenience sampling. Purposive sampling is "when a researcher makes a specific choice about which people, group or object to include in the sample" (Bertram & Christiansen, 2016, p. 60). The sample of this study consisted of senior primary school mathematics teachers from two schools in the Ohangwena region. The home language of the participants varied but Mathematics was taught in English. Bertram and Christiansen (2016) describe convenience sampling as a way of choosing a sample which is easy for the researcher to reach and driven by a particular purpose. The four teachers were from two schools at the roadside near a highway, which gave me easy access.

I sent questionnaires to eight teachers and based on their responses I chose four teachers to participate in a focus group interview. I purposely selected two teachers (one teacher from each of the two schools) and observed them teaching in order to ascertain how they were developing their learners' computational estimation skills in their classrooms. The two schools are referred to in this thesis as School A and School B and the teachers are referred to as Teachers A to H. I had planned to conduct a focus group interview with four teachers (Teachers A to D), but Teacher D dropped out of the study before the interview. Table 4.1 summarizes the participants' information.

Table 4.1 Participants' Years of Experience, Gender, Age and Highest Level of Qualification

Teacher name	**Teaching experience**	**Highest qualification**	**Gender**	**age (average)**
A	6 years	BEd Honours, University of Namibia (UNAM)	Female	20- 29
B	14 years	BEd Honours, UNAM	Female	30- 39
C	7 years	Basic Education Teacher	Female	30- 39

		Diploma (BETD), Ongwediva College of Education (OCE)		
D	24 years	BETD, OCE	Male	50- 60
E	9 years	BEd Honours, North West University (NWU)	Female	30- 39
F	7 years	Masters in Education Degree, Midlands State University (MSU)	Female	20- 29
G	7 years	BEd Honours, UNAM	Male	30- 39
H	5 years	BEd Honours, UNAM	Female	20- 29

As evident in Table 4.1, all of the teachers that participated in my research had five or more years of experience, in that sense, I regard them as experienced teachers. The majority of teachers (4) were between the ages of 30 to 39. One teacher was significantly older and had 24 years experience. This teacher is an outlier in the group in terms of years of experience and age. Six of the eight teachers are female, which reflects the gender of most teachers in primary schools in Namibia. The most common qualification was a Bachelor of Education Honours. It is worth pointing out that the Bachelor of Education Honours in Namibia, that is from the University of Namibia (UNAM) is a four year undergraduate qualification. As such, this qualification is not at the same level on the National Qualifications Framework as an Honours degree in South Africa. There was only one teacher that had an Honours degree from a South African University, that is North West University. Two teachers had the old Basic Education Teacher Diploma (BETD). This diploma has been phased out in preference for the Bachelor of Education Honours qualification.

One teacher had a Masters in Education qualification; she had the highest qualification of the participants in my research.

4.6. DATA GENERATION

As mentioned earlier, the following data generation methods were utilized: questionnaires, a focus group interview and classroom observations.

4.6.1. Questionnaires

Cohen et al. (2007) describe questionnaires as a widely used and useful tool for data collection that is administered without the presence of the researcher. In this study, I used a combination of closed and open-ended questions as it helped me in capturing the specificity of senior primary mathematics teachers' knowledge of computational estimation and how to develop the computational estimation skills of their learners (Appendix 8). In the questionnaire, I asked the teachers to explain computational estimation, suggest strategies for teaching computational estimation and to use computational estimation strategies to solve a number of addition calculations involving both whole and decimal numbers. I confirmed with the teachers that they were all able to complete a Google form so that I may send the questionnaire via email. Considering the on-going COVID-19 pandemic at the time of my research, the questionnaires were sent out through a Google link (on Google forms) to eight senior primary school teachers of mathematics in Ohangwena region as soon as permission to collect data was granted. I requested that the participants return the form within five days of receiving it and reminded the participants two days before it was due. All eight teachers returned their questionnaires giving a 100% return rate. Once I had coded the data from the questionnaires, I approached two teachers to observe them teaching.

4.6.2. Observations

I observed two teachers (Teachers A and B) in the classroom in order to gather ‘live data’ (Cohen et al., 2011) on how they developed learners’ computational estimation. Observation is explained “as the process of gathering first-hand information, by observing or watching the participants in order to gain information” (Gay et al., 2012, p. 21). The researcher uses all senses to examine people in natural settings or naturally occurring situations. The initial plan was to observe a total

of four lessons (i.e., 2 lessons per teacher) using a classroom observation schedule that I designed; however, I ended up observing only three lessons. One of the teachers that I interviewed chose not to be included in the observations.

I compared the observation data with the questionnaires, and the focus group data with the questionnaire and observation data and checked for consistency between their actual teaching practices and what the teachers said in the questionnaires and focus group interviews. I took field notes during classroom observation, which guided me to collect data about the mathematics and pedagogical content knowledge that the teachers draw on to develop the learners' computational estimation skills. This cross-checking made it easier to write analytic memos as there were some aspects in their teaching that were not outlined in my field notes yet proved equally valuable data to my study.

I was a non-participant observer because I did not want to be directly involved in the lessons being observed. As a non-participant observer, I did not react to any events that occurred during the lesson nor did I respond to what the learners and teachers did in the classroom. This allowed me to understand the natural environment (classroom) as lived by the participants (teachers and learners) without influencing or changing it. After the lesson observations, I approached four teachers to participate in a focus group interview. All four teachers agreed at first, but one had to drop out, bringing the number of participants down to three.

4.6.3. Focus group interview

A focus group interview was used for this study. Lloyd (2006) defined a focus group interview as a group of individuals who have some common interest or characteristics discussing the phenomena under study. These individuals are brought together by the interviewer who uses the group and their interactions for the purpose of gaining information about a specific issue. In this study the three interviewees (participants) (Teachers A, B and C) interacted and discussed how they develop the computational estimation skills of their learners. These discussions were based on the questions I generated as part of my semi-structured focus group interview so that I could ask probing questions to invite the participants to elaborate on their views. This helped me to generate data about the mathematics and pedagogical content knowledge the participants drew on

to develop their learners' computational estimation skills. It also assisted me in understanding the strategies that the teachers used to develop the learners' computational estimation skills. The focus group interview was face-to-face in the community library as this space was easily accessible to all three teachers.

4.7. DATA ANALYSIS

Koshy (2005) stated that the purpose of data analysis is to identify themes, patterns and relationships in order to present strong evidence for claims being made. Qualitative data from different sources were collected as mentioned in the previous section. Data collected was analysed following the commonly used steps in analysing and interpreting qualitative data as developed by Creswell (2012). I did not follow the steps below in an orderly manner as my data collection and analysis process was iterative. Nevertheless, these steps were useful in guiding my data analysis process.

1. *Collecting the data:* As mentioned above, I distributed questionnaires to collect my data, observed the teachers in their classrooms and conducted a focus group interview.

2. *Preparing and organizing the data for analysis:* I started by reading through the questionnaires and captured the data onto an excel spreadsheet. I watched and transcribed the videos from the lesson observations. I conducted the focus group interview and I transcribed the focus group audio-recordings onto an excel spreadsheet.

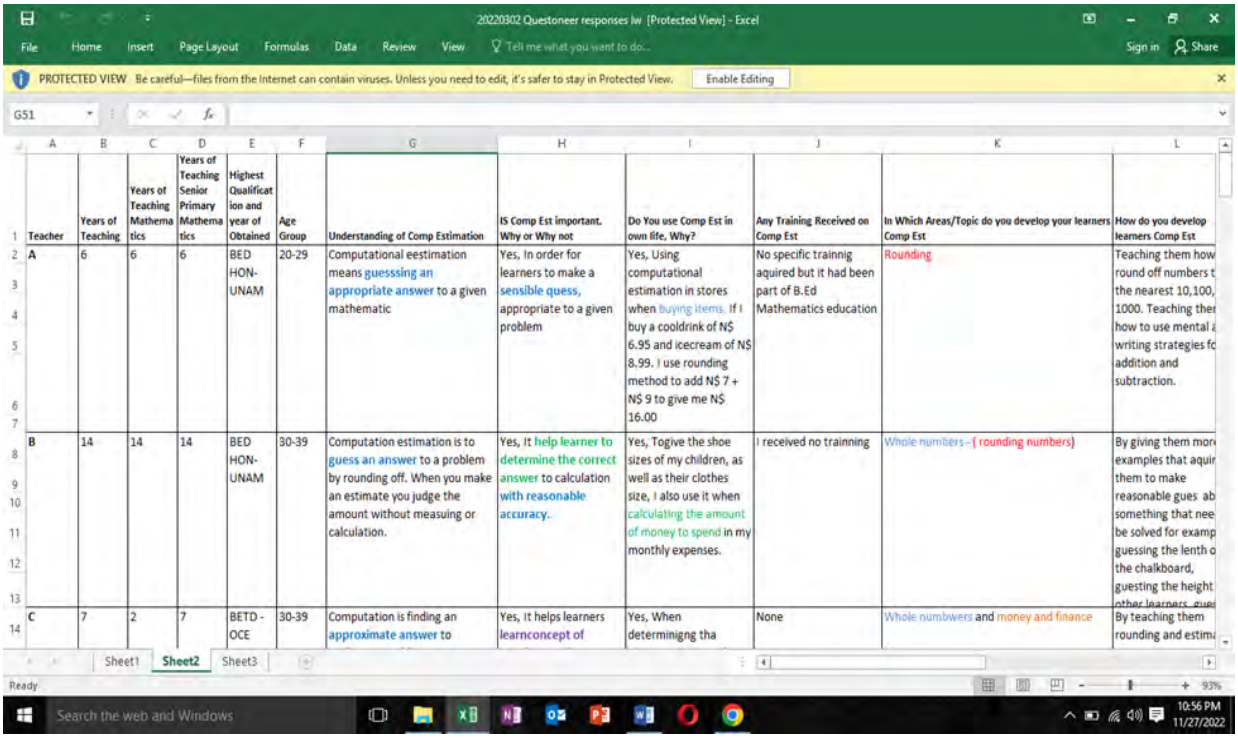
3. *Engaging in the initial exploration of the data:* I read through the data so that I could obtain a broad sense of the data and reflect on its overall meaning. It is by reading through my data that I was able to develop analytic memos that enabled me to identify places in the data that were of analytic importance.

4. *Coding the data:* After reading through the data, I assigned labels or codes to my data and organised them into initial themes. This was done inductively. The reason for doing the initial coding inductively was to see what emerges from the data and to ascertain whether there are possible categories that may not be included in the MKfT and the KQ frameworks but may be of relevance to my study. After the data was inductively coded, I then coded

my data using the six MKfT domains and the four domains of KQ. I only drew on the four KQ domains for the lesson observations as the focus of the MKfT framework was the knowledge teachers require *for* teaching as opposed to *in* teaching (Chapter 2). Figure 4.1 is an example of an excel spreadsheet that I used to assist in coding my data.

Figure 4.1 reflects how I coded my data for the questionnaire. In the first column, I have the alpha-numerical name given as pseudonyms to the teachers. Columns B to E provided general information about each of the teachers which I tabulated and reflected on earlier in this chapter. Each of the columns from G onwards relates to a particular question in the questionnaire. I transcribed each teachers' responses to the questions in the questionnaire under the relevant question. The words that appear in colour refer to the codes that emerged inductively from the data. I developed themes from the codes and using the teachers' voices, I have elaborated on each theme in Chapter 5.

Figure 4.1 An Example of My Inductive Coding of the Questionnaire Data



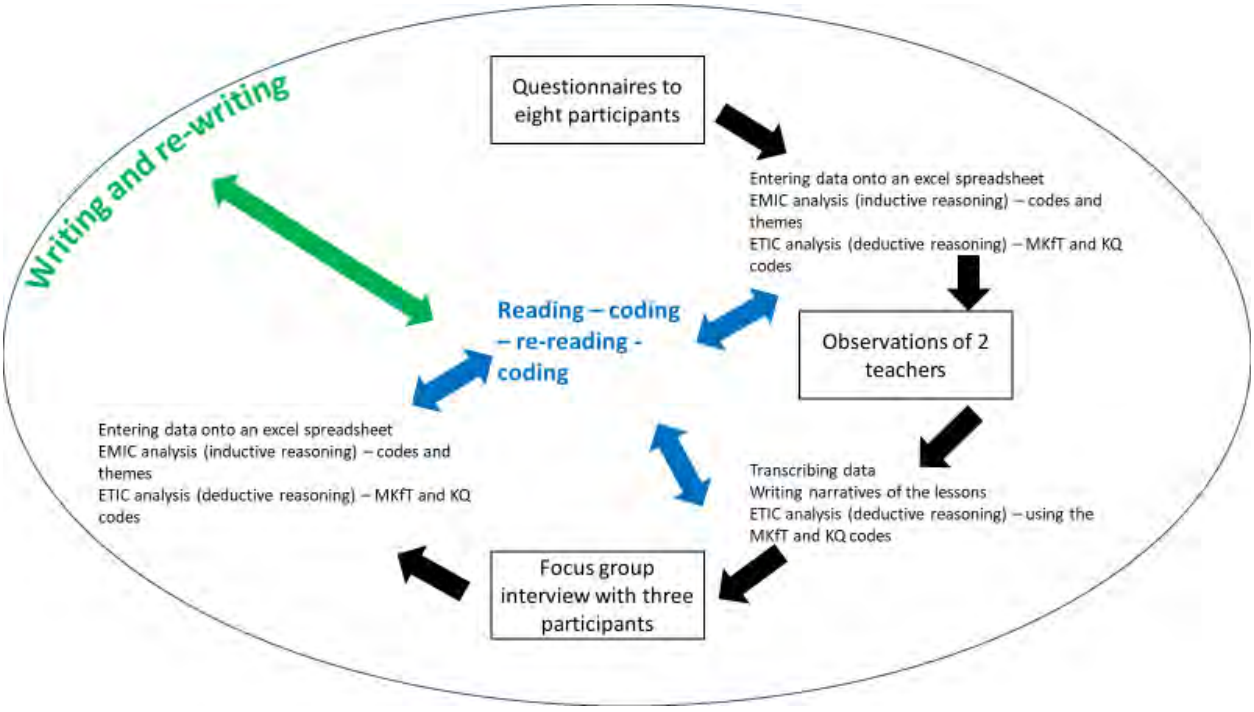
Teacher	Years of Teaching	Years of Teaching Mathematics	Years of Teaching Senior Primary Mathematics	Highest Qualification and year of Obtained	Age Group	Understanding of Comp Estimation	IS Comp Est important. Why or Why not	Do You use Comp Est in own life, Why?	Any Training Received on Comp Est	In Which Areas/Topic do you develop your learners Comp Est	How do you develop learners Comp Est
A	6	6	6	BED HON-UNAM	20-29	Computational eestimation means guesssing an appropriate answer to a given mathematic	Yes, In order for learners to make a sensible quess, appropriate to a given problem	Yes, Using computational estimation in stores when buying items. If I buy a cooldrink of N$ 6.95 and icecream of N$ 8.99. I use rounding method to add N$ 7 + N$ 9 to give me N$ 16.00	No specific trainnig aquired but it had been part of B.Ed Mathematics education	Rounding	Teaching them how round off numbers t the nearest 10,100, 1000. Teaching ther how to use mental a writing strategies fc addition and subtraction.
B	14	14	14	BED HON-UNAM	30-39	Computation estimation is to guess an answer to a problem by rounding off. When you make an estimate you judge the amount without measuing or calculation.	Yes, It help learner to determine the correct answer to calculation with reasonable accuracy.	Yes, Togive the shoe sizes of my children, as well as their clothes size, I also use it when calculating the amount of money to spend in my monthly expenses.	I received no trainning	Whole numbers -(rounding numbers)	By giving them mor examples that aquir them to make reasonable gues ab something that nee be solved for examp guessing the lenth o the chalkboard, guesting the height
C	7	2	7	BETD - OCE	30-39	Computation is finding an approximate answer to	Yes, It helps learners learnconcept of	Yes, When determinigng tha	None	Whole numbwers and money and finance	By teaching them rounding and estim

5. *Representing the findings:* Once I coded and categorised the data into themes, I reflected on the impact of the findings and examined the literature with supporting evidence from the data, and where necessary used the exact words of the participants.

6. *Validate the accuracy of the findings:* To ensure that my findings correspond well with the perspectives and personal experiences of the research participants, I validated the accuracy of the findings by triangulating the data collected from the observations, focus group interview and questionnaires to determine both consistencies and inconsistencies.

Figure 4.2 summarises the process or steps that I had followed to analyse and present the data. While this summary is linear in its appearance, as mentioned above, the data collection and analysis was an iterative process.

Figure 4.2 Iterative Data Generation, Analysis and Writing Process



4.8. VALIDITY AND TRUSTWORTHINESS

Cohen et al. (2007) stressed that key to establishing the validity of qualitative data are the notions of honesty, depth, richness and scope of the data generated, coupled with triangulation and the 'objectivity' of the researcher. In this study, I used questionnaires, conducted a focus group interview and lesson observations for the purpose of triangulating my data. By triangulating the data, I reduced "the risk of chance associations and systemic bias" (Maxwell, 1996, p. 93). Triangulating the data allowed me to rely on information collected from a diverse group of individuals (teachers) and settings (classrooms) using a variety of methods. I asked my supervisors to code sections of my data to ensure inter-rater reliability. I also further sought to ensure trustworthiness through member checking. I ensured that the research participants had an opportunity to read the transcripts of interviews and comment on the conclusions of the research. The focus group interview also provided an opportunity for the teachers to reflect on what they wrote in the questionnaire. Realising that the participants and I bring our own experiences, perceptions and understandings to the research site, I developed shared meanings through our interactions. I asked probing questions for clarification during the focus-group interviews.

4.9. POSITIONALITY

As a teacher, I am well aware of the power relationships between my participants (senior primary teachers of mathematics) and me. I had established a good working relationship with fellow teachers of mathematics through workshops and sport events in our region. However, I was aware that my colleagues might have seen me as a more knowledgeable person because of my interest in computational estimation and my registration as a MEd student at Rhodes University. As Table 4.1 highlights, only one of the teachers who participated in my research had a MEd qualification and one had an Honours degree that is on the same NQF level as my MEd degree. The teachers may also be concerned that their classroom practices would be shared with their colleagues and principal. I realised that my positionality might have influenced their decision of whether to participate or not in this research. For the reasons mentioned above, I was clear about the aims and focus of the study and explained that they may withdraw from the study at any time without any repercussions. During the research, one of the teachers, Teacher D, withdrew from my study. Unfortunately, he did not share reasons as to why he chose to withdraw from the research.

4.10. ETHICAL CONSIDERATIONS

The researcher has a responsibility to respect the rights, desires, needs and values of the people participating in the research (Creswell & Creswell, 2018). My formal ethical process took part over five stages. First, I sought permission to conduct my research from the Rhodes University Faculty of Education Higher Degrees Committee (Appendix 1). Second, I requested permission from the Ministry of Education, Arts & Culture to have access to the selected schools in the region through the Permanent Secretary of Education (Appendix 2). Third, I sought permission from the Ohangwena Regional Education office in Eenhana to access the selected schools (Appendix 3) and from the School Principals (Appendix 4). Fourth, I informed the participants that their participation was voluntary and that they were free to withdraw from the study at any time during the research. I gave each of the participants a consent form to complete before I began the process of collecting my data (Appendix 5). Fifth, letters were sent to parents for their consent and the learners signed an assent form (Appendix 6).

I promised the participants that the data which they provided would be kept confidential and that their names would not appear in my research or any subsequent publications. Hence, I made use of pseudonyms in my research. I also explained that the data collected would only be used for the purpose of my research. I told them that the information collected during my research would be kept on my personal computer and on Google Drive and protected by a password known only to me. The information would be kept for a period of 3-5 years, after which I would discard it.

4.11. CHAPTER SUMMARY

This chapter focused on the methodology used to collect data from eight senior primary teachers of mathematics in the Ohangwena Education Region. Under the methodology, the research orientation and method, samples and sampling procedures, research site, data generation instruments, data analysis, validity and ethical considerations are discussed. I summarise this chapter in Table 4.2 below.

Table 4.2 Research Summary

Research goals	This study seeks to understand the mathematical and pedagogical content knowledge teachers have and use

<table>
<tr><td colspan="2"></td><td colspan="3">when developing their learners' computational estimation skills to inform my own teaching practice and that of other mathematics educators. This study may hopefully be of potential benefit to the learners and in-service teacher education programmes in Namibia.</td></tr>
<tr><td colspan="2">Key research questions</td><td colspan="3">What mathematics and pedagogical content knowledge do senior primary mathematics teachers draw on to develop their learners' computational estimation skills?
What mathematical content knowledge do teachers draw on to develop the learners' computational estimation skills?
What pedagogical content knowledge do teachers draw on to develop the learners' computational estimation skills?</td></tr>
<tr><td colspan="2">Research design of the study</td><td colspan="3">Case study (Interpretive)</td></tr>
<tr><td colspan="2">Nature of data collected</td><td colspan="3">Qualitative</td></tr>
<tr><td rowspan="3">Action plan</td><td>Data collection instruments</td><td>Questionnaires</td><td>Lesson observations</td><td>Focus Group interview</td></tr>
<tr><td>Data source</td><td>Transcription of the questionnaires</td><td>Lesson videos and memos. Transcriptions of the lessons</td><td>Audio-recorded and transcription of interview</td></tr>
<tr><td>Data analysis</td><td colspan="2">Inductive reasoning (Emic)</td><td>Deductive reasoning (Etic)</td></tr>
<tr><td colspan="2">Ethical consideration</td><td colspan="3">University approved research, informed consent, confidentiality and anonymity.</td></tr>
</table>

The next chapter presents the analysis of my research data.

CHAPTER 5: PRESENTATION AND DISCUSSION OF THE RESULTS

5.1. INTRODUCTION

This chapter presents the findings and discussion of the knowledge senior primary mathematics teachers require and draw on to develop their learners' computational estimation skills. The presentation and discussion of the findings centre on three main data collection methods, namely,

- findings from the questionnaires,
- findings from the observations, and
- findings from the focus group interview.

The results of the study were used to answer the following research question:

What mathematical and pedagogical content knowledge do senior primary mathematics teachers draw on to develop their learners' computational estimation skills?

The two sub-questions are:

- What mathematical content knowledge do teachers draw on to develop the learners' computational estimation skills?
- What pedagogical content knowledge do teachers draw on to develop the learners' computational estimation skills?

To answer the above-mentioned questions, the results were first analysed using inductive reasoning followed by deductive reasoning, that is, based on the six MKfT domains and the four categories of the KQ respectively.

5.2. QUESTIONNAIRE RESPONSES

At the beginning of collecting the data, eight teachers answered the questionnaires (Appendix 8). Seven of the eight teachers completed the questionnaires fully and one teacher submitted an incomplete questionnaire. The eight teachers were named Teacher A to Teacher H (pseudonyms).

5.2.1. Teachers' definitions of computational estimation

Six of the eight teachers (Teachers A, C, E, F, G & H) described computational estimation as getting an approximate answer for calculations, for example, Teacher C wrote that computational estimation is "finding an approximate answer to arithmetic problems". Teacher D elaborated,

indicating that computational estimation “is estimating the number before the real calculation is done, by using the four basic operations”. This is in line with Courtney-Clarke (2012), who argues that computational estimation is defined as the process of simplifying an arithmetic problem to produce an approximate but satisfactory answer to a calculation.

Teacher B explained that computational estimation involves “guessing an answer”. In her explanation, there was no reference to the significance of approximation. Teacher B argued that estimation is not simply a guess, but rather it is a considered guess. She offered an example of a computational estimation strategy when she elaborated on her initial comment suggesting that computational estimation is to “guess an answer to a problem by rounding off". The process of rounding off means that the estimation is not simply a guess. Teacher G also mentioned rounding off as a computational strategy. She said, "I understand computation estimations as a process in which some or all the numbers in an arithmetic problem are approximated to simplify without the use of any scientific tool such as a calculator to do the calculation. It applies the idea of rounding off the base 10 numbers". While Teachers B and G specified the use of rounding off, Teacher F added that “computational estimation involves calculations where one uses different strategies that enable them to easily calculate the sums to get an estimate or approximate answer”. The dominant explanation of estimation was that it is an approximation. This was seen as a means to calculate the reasonableness of the result primarily through the process of rounding off.

5.2.2. Importance of computational estimation

All of the eight teachers agreed that computational estimation is important. Four of the teachers acknowledged that computational estimation is important because it allows the learners to find the reasonableness of their answers (Teachers B, E, F & H). Teacher E noted:

> Yes, so that learners will be able to determine whether their answers are making sense within the reasonable range of the correct answers. This is as crucial as it will reduce the high failure rate in mathematics. It will also develop a high accuracy of the calculation in mathematics and learners will stop singing the song of saying mathematics is a difficult subject.

This supports the explanation by Fennell (2008) that “as learners estimate, talk about numbers, compute, use mental math and judge the reasonableness of their results, they become more flexible in working with numbers” (p. 3). Teacher D explained the importance of computational estimation in the classroom context, "Yes, I use it when I am thinking about the sum of given two numbers, products or the difference between the two numbers". Teacher F noted that computational estimation enables the learners to apply it to real-life situations where calculation may be required without considering getting an exact answer. Hence computational estimation in the classroom may improve the learners’ calculation skills and generally their number sense (Cochran & Dugger, 2013; Fennell, 2008), particularly in relation to how their understanding of number and number operations are used in real-life settings (McIntosh et al., 1992) that require computational estimation. Teacher G’s comment concurred with this as she said, “estimation allows learners to develop their number sense" (Teacher G).

5.2.3. Computational estimation in the real world

All eight teachers acknowledged that they use computational estimation in their everyday lives. Teacher A, C and H referred to the importance of computational estimation in determining the change required when paying for groceries. Teacher A gave an example, “Yes, using computational estimation in stores when buying items. If I buy a cool drink of N$ [Namibian Dollars] 6.95 and ice cream of N$ 8.99. I use the rounding method to add N$ 7 + N$ 9 to give me N$ 16.00". Teacher B, E, F and G commented that computational estimation was important for budgeting. Teacher E elaborated ,"Yes when doing budget by filling by basket depending on the amount of money in my purse. When traveling, I also predict the time it will be, take, to drive within a specific distance". The teachers saw the value of estimation in working out budgets, change and the time required to travel. One teacher emphasised the value of estimation for calculating in the classroom.

5.2.4. Training received on computational estimation

Seven teachers referred to the training they had received on computational estimation. Of the seven teachers, Teachers B, C and E indicated that they had not received any training about computational estimation. Teachers A, F, G and H received some training on computational estimation. These four teachers all indicated that computational estimation was incorporated into

their preservice teachers' training courses. Teacher A commented that she had "no specific training acquired, but it had been part of B.Ed (Bachelor of Education) Mathematics education" course. Teacher G explained that she "had a module during my third and fourth year called mathematical education which focuses on how to develop number sense in learners". As such, she made a direct reference to the relationship between computational estimation and number sense as I have described in Chapter 3. As Fennell (2008), and Cochran and Dugger (2013) explained, computational estimation in the classroom has the potential to improve the learners' calculation skills and their number sense more broadly. The teachers commented that they had limited training in using estimation in their mathematics classrooms. When they did receive training, it was during their initial teacher education qualification.

5.2.5. Computational estimation in the curriculum

It was evident from the data that the teachers draw their knowledge of where computational estimation is taught from the Namibian mathematics syllabus. The Namibian syllabus indicated that learners are required to "apply rounding to estimate the answer to a calculation involving money" (MoEAC, 2016, p. 29). Six of the eight teachers (Teachers A, B, C, E, F and G) wrote that they developed learners' computational estimation when teaching rounding off. Teachers B, C, D, E, F and H include computational estimation when teaching whole numbers, particularly computations such as addition and subtraction. Teacher H explained that she taught computational estimation "mostly in whole numbers and computation and sometimes in money and finance". Four of the eight teachers explained that they taught computational estimation when they taught the topic of money and finance (Teachers C, F, G & H). The teachers explained that they used estimation when teaching rounding off, addition and subtraction of whole numbers and when teaching money and finance.

5.2.6. Developing learners' computational estimation skills

Each of the eight teachers develop their learners' computational estimation differently depending on the perceived needs of their learners. Despite this, all eight teachers give their learners practical work to develop their computational estimation skills. Teachers G and H provided responses that were generic. Teacher G noted that she develops the learners' computational estimation by

allowing learners to freely estimate and provide varying answers that are close to each other, and Teacher H said, "I give the learners activities for them to estimate."

Teacher C develops her learners' computational estimation "by teaching them rounding and estimation." Teacher A also mentioned rounding off. She wrote that she teaches them “how to round off numbers to the nearest 10, 100 or 1000 [and] how to use mental and writing strategies for addition and subtraction”. Teacher D also emphasised place value. "I develop learners' computation estimation by explaining the important factors that they must consider e.g., know the place value of the numbers so that they (learners) can round the numbers correctly."

Teachers E and F explained that they taught the learners different strategies for calculating. Teacher E noted, "I teach them different methods that make it possible to get the answer that is approximate to the answer. I also try my best to make learners understand how operations work to prove the correct answer". Teacher F said, "learners are given problems to solve which allows them to apply different methods of calculations to estimate answer i.e., products’ prices, rounding numbers."

Teacher B included measurement estimation (Chapter 3) in her response. She develops the learners' computational estimation “by giving them more examples that require them to make a reasonable guess about something that needs to be solved for example: guessing the length of the chalkboard, guessing the height of other learners, guessing the amount of money they spend on their cosmetics and many more."

Five of the eight teachers motivate their learners to estimate their answers before doing exact calculations or by giving them more estimating tasks to complete (Teachers A, B, C, G & H). Teacher C encouraged the learners to estimate the reasonableness of their answers every day. She said, "I give them an estimation exercise every day before I start with my daily lesson." Teacher A said she "motivates the learners to do computational estimation before they calculate" and Teacher H “always tell[s] them to estimate the answer first before exact calculations." Teacher B emphasises the “importance of estimation and how it helps in determining the answers." The importance of computational estimation is elaborated on by Teacher G who mentions that learners

need to apply computational estimation to “real-life situations, whereby learners have to approximate and round off numbers to the nearest next number”. Teacher G also noted that she encourages her learners to make “use of visual and images and at times incorporate storytelling to make connections to the real-world scenarios." The teachers mentioned that they encouraged learners to estimate when performing bare (context-free) calculations and calculations in context (word problems).

5.2.7. Computational estimation strategies

Five of the teachers use rounding off as a computational estimation strategy (Teachers B, C, D, E & H). Teacher H acknowledged that “we only use rounding off to the nearest 10, 100, 1000 or 10 000, since it's the only method in the syllabus". As noted in Chapter 3, the Namibian Senior Primary syllabus for mathematics only refers to rounding off as a computational estimation strategy. However, Teacher A, F and G include additional strategies when developing their learners’ computational estimation. Teacher A wrote that she encouraged the learners to "split both numbers into tens and units". Teacher F’s comment in the questionnaire added to this as she also included, "Rounding, Bridging the decade, Associative property". Teacher G added further strategies that the teachers had not mentioned previously. These included “compatible numbers, special numbers, and distributive properties" (Teacher G). Three of the teachers thus used a wide repertoire of strategies, many of which are explained in Chapter 3. Despite the apparent limitation of estimation to rounding off in the Senior Primary Mathematics Syllabus, the teachers taught the learners a variety of computational estimation strategies.

5.2.8. Teachers' knowledge of computational estimation

The participants were presented with computational estimation problems and they were asked to show the strategies they would use to judge the reasonableness of their calculations. In Question 9 of the questionnaire, the teachers were required to demonstrate the estimation strategies they would use to solve six addition sums involving whole and decimal numbers. The sums were as follows (Table 5.1).

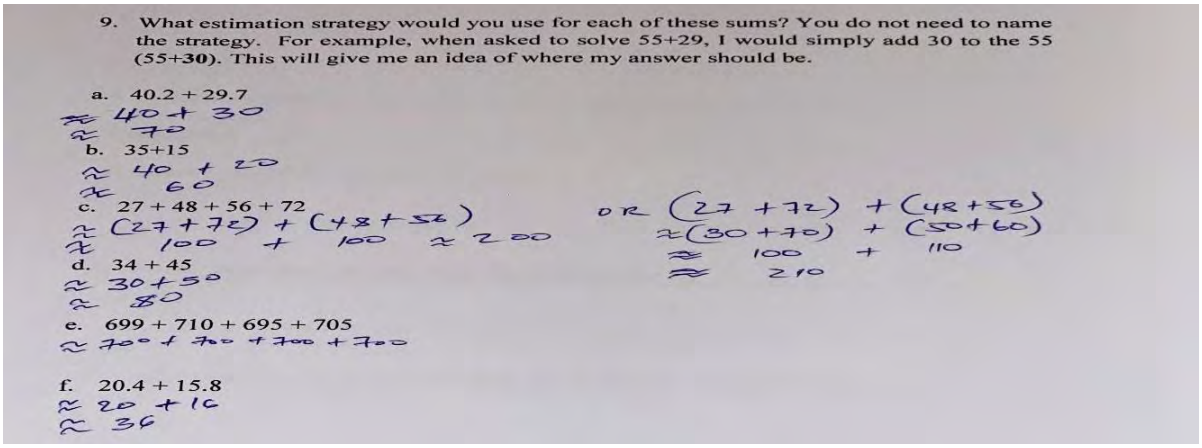
Table 5.1 Addition Sums in the Questionnaire

a	40.2 + 29.7
b	35 + 15
c	27 + 48 + 56 + 72
d	34 + 45
e	699 + 710 + 695 + 705
f	204.4 + 15.8

The teachers' knowledge of computational estimation is shown in the figures below.

As noted in Figure 5.1, Teacher A used rounding off as her computational estimation strategy for question A. For question F, she also rounded off, but not entirely. Rather she rounded off the second addend to the next whole number meaning that 20.4 + 15.8 became 20 + 16. In question E, she used clustering/ averaging as her computational strategy as she realised that all the numbers in the sum cluster around 700. In question B, Teacher A used the rounding-off method. In other words, she rounded off the tens in both numbers. For question C, Teacher A provided two possible strategies. Both strategies use compatible numbers. In both examples of question C, the numbers are re-ordered ((27+72) and (48+56)) so that each partial sum is close to 100. For question D, Teacher A used a rounding-off strategy. She rounded the 34 down to 30 and the 45 up to 50. Based on Teacher A's computational strategies, she used two of the three cognitive processes. These are reformulation (rounding off and compatible numbers) and translation (clustering) as explained in Chapter 3.

Figure 5.1 Teacher A's Computational Estimation Strategies


9. What estimation strategy would you use for each of these sums? You do not need to name the strategy. For example, when asked to solve 55+29, I would simply add 30 to the 55 (55+**30**). This will give me an idea of where my answer should be.

a. 40.2 + 29.7
≈ 40 + 30
≈ 70

b. 35+15
≈ 40 + 20
≈ 60

c. 27 + 48 + 56 + 72
≈ (27 + 72) + (48 + 56)
≈ 100 + 100 ≈ 200

OR (27 + 72) + (48 + 56)
≈ (30 + 70) + (50 + 60)
≈ 100 + 110
≈ 210

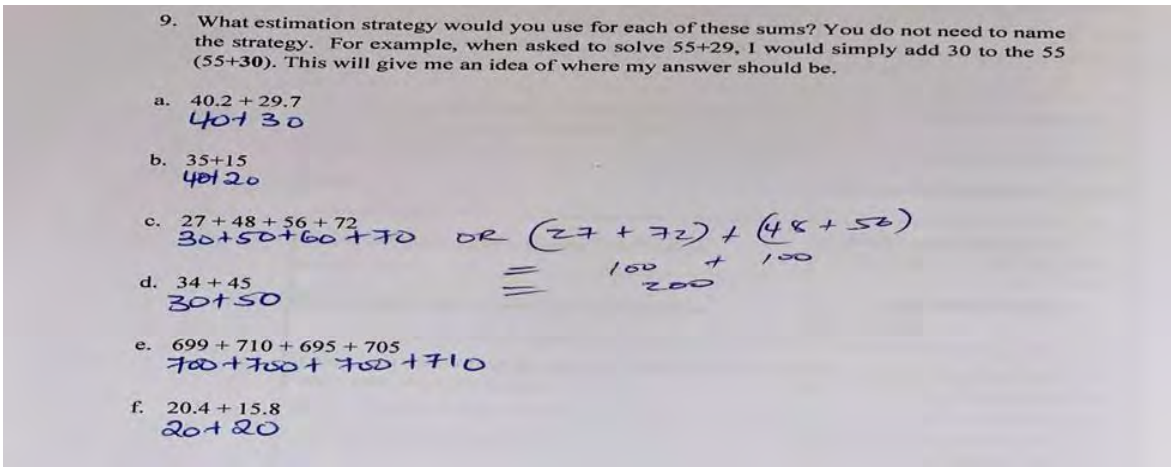
d. 34 + 45
≈ 30 + 50
≈ 80

e. 699 + 710 + 695 + 705
≈ 700 + 700 + 700 + 700

f. 20.4 + 15.8
≈ 20 + 16
≈ 36

Teacher B used rounding off, whereby she rounded the numbers to the nearest tens in questions A, B, C, D and F. In question C, she provided two methods. In the first method she rounded off the numbers to the nearest tens while in the second method she used the compatible numbers strategy as she re-ordered the numbers ((27 + 72) and (48 + 58)) so that each partial sum was close to 100. In question E, she used the clustering/averaging method whereby she realised that all the numbers cluster around 700. However, for the last addend in the calculation she rounded the number up. Like Teacher A, Teacher B used two of the computational estimation processes. The dominant cognitive process that she used was reformulation (Chapter 3) as used in rounding off five of the calculations.

Figure 5.2 Teacher B's Computational Estimation Strategies


9. What estimation strategy would you use for each of these sums? You do not need to name the strategy. For example, when asked to solve 55+29, I would simply add 30 to the 55 (55+**30**). This will give me an idea of where my answer should be.

a. 40.2 + 29.7
40 + 30

b. 35+15
40 + 20

c. 27 + 48 + 56 + 72
30 + 50 + 60 + 70

OR (27 + 72) + (48 + 58)
= 100 + 100
= 200

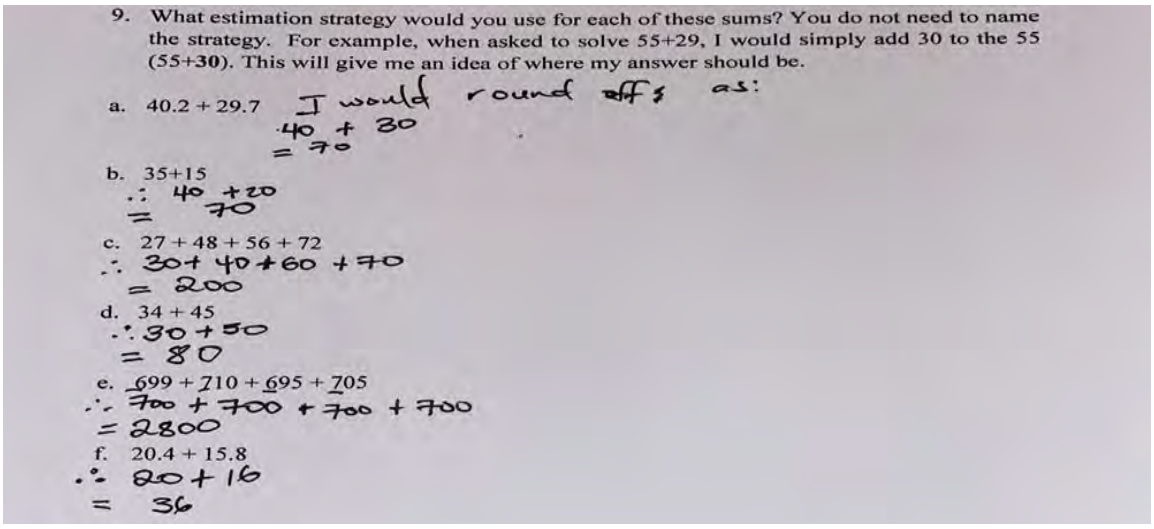
d. 34 + 45
30 + 50

e. 699 + 710 + 695 + 705
700 + 700 + 700 + 710

f. 20.4 + 15.8
20 + 20

Like Teacher A, Teacher C used rounding off to the nearest ten for question A, B and D. She also used rounding off for question F, but here she rounded off the second addend to the next whole number. In other words, 15.8 became 16. She miscalculated her estimation in B, as she wrote that 40 and 20 is equal to 70. In question C the teacher rounded 27 up to 30 and rounded 48 down to 40 instead of 50. She used a clustering/averaging method to solve question E, realising that all the numbers are clustered around 700. Teacher C used two cognitive processes, however, her dominant computational estimation process was reformulation whereby she used rounding off to solve most of the calculations.

Figure 5.3 Teacher C's Computational Estimation Strategies


9. What estimation strategy would you use for each of these sums? You do not need to name the strategy. For example, when asked to solve 55+29, I would simply add 30 to the 55 (55+30). This will give me an idea of where my answer should be.

a. 40.2 + 29.7 I would round off as: 40 + 30 = 70

b. 35+15 ∴ 40 +20 = 70

c. 27 + 48 + 56 + 72 ∴ 30+ 40+60 +70 = 200

d. 34 + 45 ∴ 30 + 50 = 80

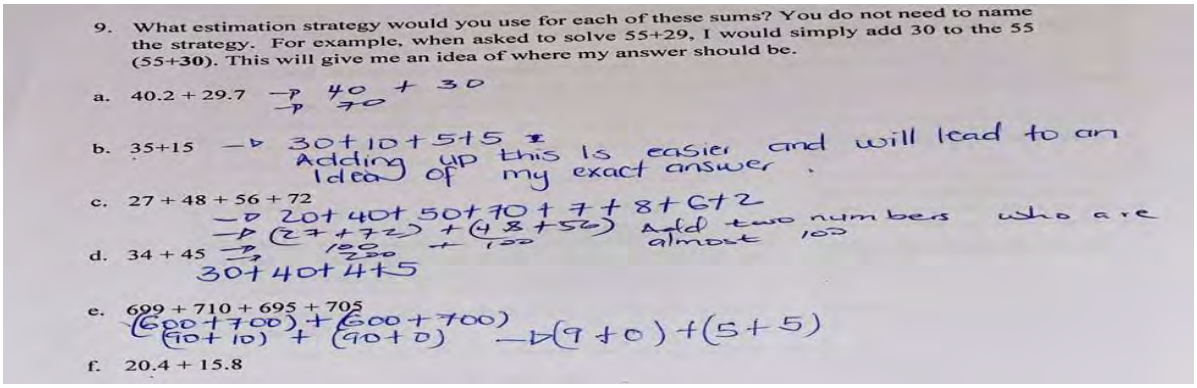
e. 699 + 710 + 695 + 705 ∴ 700 + 700 + 700 + 700 = 2800

f. 20.4 + 15.8 ∴ 20 + 16 = 36

Teacher D did not attempt to estimate the answer to question F. He only used rounding off as a computational estimation strategy to solve question A, as he rounded off the decimal numbers to the next ten. For questions B, C, D and E, he broke down all the numbers and wrote every place value separately. He expanded the numbers, for example in B, he broke down 35 as 30 + 5 and 15 as 10 + 5, and he wrote that this is an easier strategy to add numbers. As noted in Chapter 3, this is a reformulation process known as benchmarking. In other words, he used expanded notation to calculate and added both the tens and units. Even though he broke down the numbers according to their place values, it appears that his focus was on getting the correct answers rather than estimating. He notes this in his comment that "adding up this 1s easier and will lead to an idea of

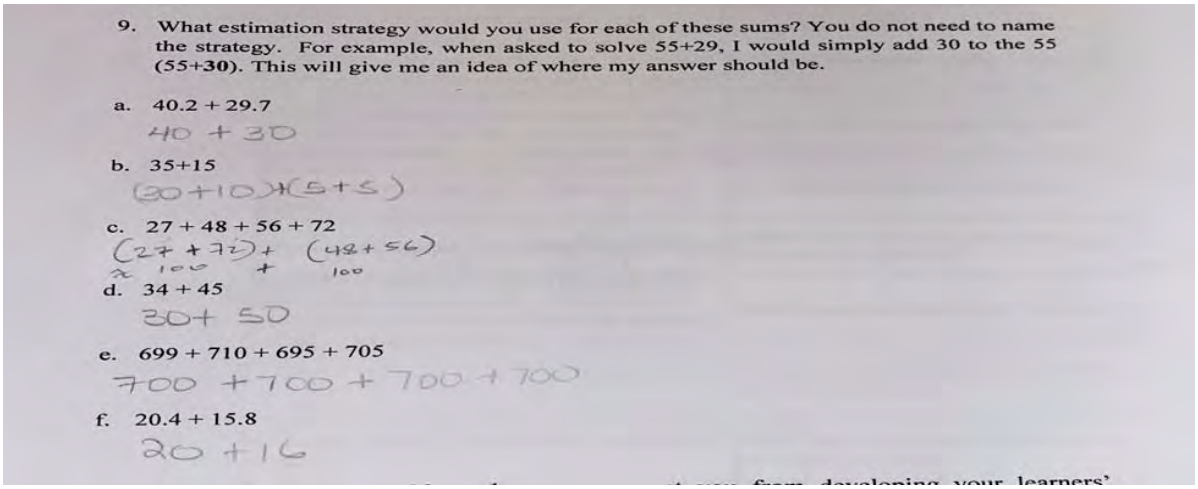
my exact answer". Teacher D only used the cognitive process of reformulation as he used two strategies, that is rounding off and benchmarking.

Figure 5.4 Teacher D's Computational Estimation Strategies



Teacher E used rounding off for A, D and F. However, instead of rounding off to the nearest tens for F, she rounded off the second addend to the next whole number. Like Teacher D, Teacher E broke down the numbers according to their place values for question B. In other words, she added 35 +15 as ((30 + 10) and (5 + 5)) which resulted in an exact answer. This strategy is known as benchmarking. In question C, Teacher E used compatible numbers as she re-ordered the numbers so that two pairs of addends were each close to 100. For question E, she used a clustering/averaging method, realising that all the numbers are clustering around 700. Teacher E used two computational estimation processes, that is reformulation (rounding off, benchmarking and compatible numbers) and translation (clustering/averaging).

Figure 5.5 Teacher E's Computational Estimation Strategies



9. What estimation strategy would you use for each of these sums? You do not need to name the strategy. For example, when asked to solve 55+29, I would simply add 30 to the 55 (55+30). This will give me an idea of where my answer should be.

a. 40.2 + 29.7
40 + 30

b. 35+15
(30+10)+(5+5)

c. 27 + 48 + 56 + 72
(27 + 73) + (48 + 56)
≈ 100 + 100

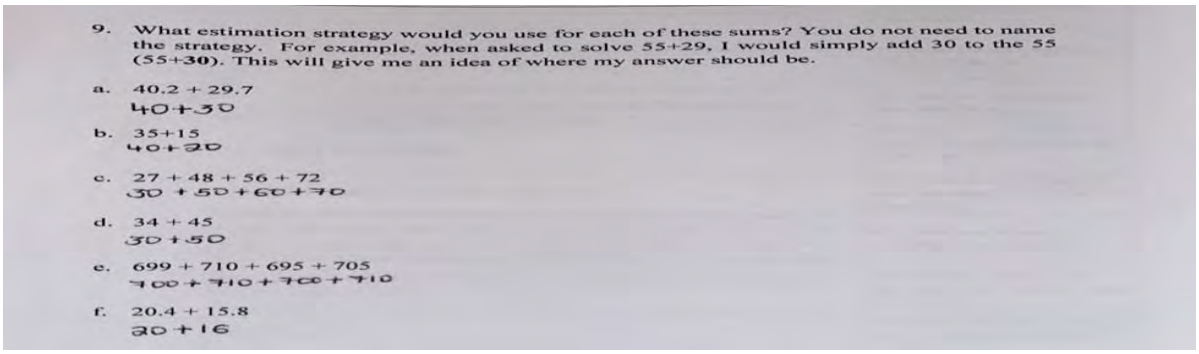
d. 34 + 45
30+ 50

e. 699 + 710 + 695 + 705
700 + 700 + 700 + 700

f. 20.4 + 15.8
20 + 16

Teacher F used rounding off for all of the calculations. However, like Teachers A, C and E, she rounded off the second addend in question f to the nearest whole number. Unlike the other teachers who used clustering/averaging for question E, Teacher F used rounding off to estimate 699 + 710 + 695 + 705. She rounded off 699 to 700, 695 to 700 and 705 to 710. The majority of the computational estimation strategies that Teacher F used were rounding off and the cognitive process of reformulation.

Figure 5.6 Teacher F's Computational Estimation Strategies.



9. What estimation strategy would you use for each of these sums? You do not need to name the strategy. For example, when asked to solve 55+29, I would simply add 30 to the 55 (55+30). This will give me an idea of where my answer should be.

a. 40.2 + 29.7
40+30

b. 35+15
40+20

c. 27 + 48 + 56 + 72
30 + 50 + 60 + 70

d. 34 + 45
30 + 50

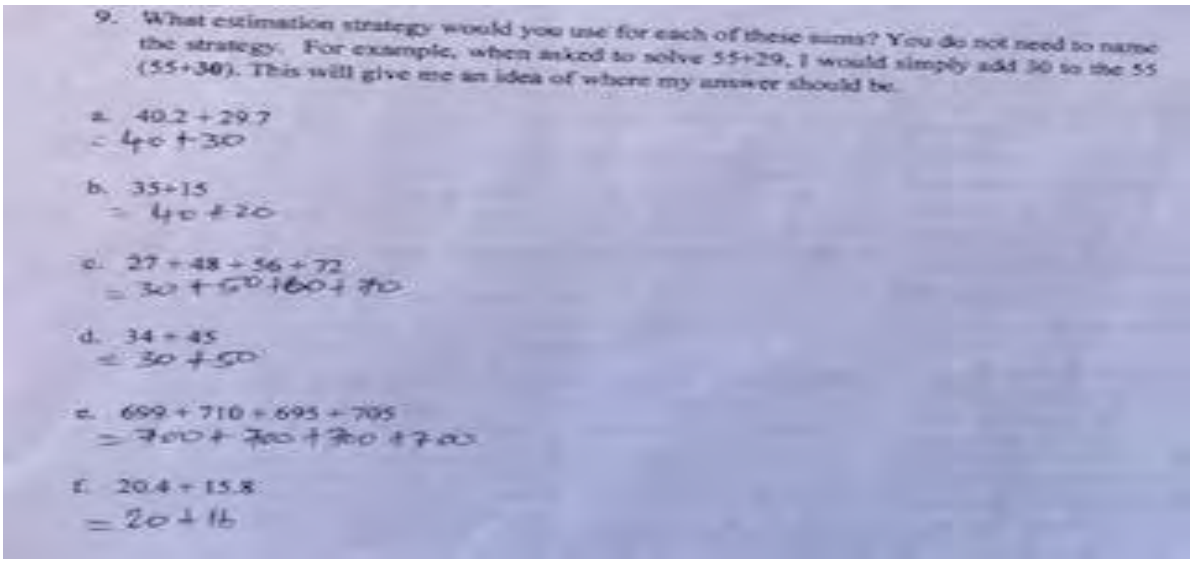
e. 699 + 710 + 695 + 705
700 + 710 + 700 + 710

f. 20.4 + 15.8
20 + 16

Teacher G used two computational estimation strategies. These were rounding off and clustering/averaging. Rounding off where the focus is on the tens, is evident in questions A, B, C and D. For question F, the second addend is rounded off to the next whole number and not the next ten. Teacher G used clustering/averaging as the computational estimation strategy to estimate the

answer to question E, as he realised that all the numbers are clustering around 700. Teacher G thus used two computational estimation processes, that is reformulation and translation.

Figure 5.7 Teacher G's Computational Estimation Strategies


9. What estimation strategy would you use for each of these sums? You do not need to name the strategy. For example, when asked to solve 55+29, I would simply add 30 to the 55 (55+30). This will give me an idea of where my answer should be.

a. 40.2 + 29.7
= 40 + 30

b. 35+15
= 40 + 20

c. 27 + 48 + 56 + 72
= 30 + 50 + 60 + 70

d. 34 + 45
= 30 + 50

e. 699 + 710 + 695 + 705
= 700 + 700 + 700 + 700

f. 20.4 + 15.8
= 20 + 16

Teacher H used three different computation estimation strategies. These were rounding off, benchmarking and compatible numbers. As with most of the teachers, Teacher H rounded off the second addend in question F to the next whole number and not the next ten. For questions B and D, she broke down the numbers (wrote the numbers in expanded notation), for example, in B he wrote, 35 + 15 as ((30 + 10) + (5 + 5)). This strategy resulted in her giving an exact answer rather than an estimate. This seems to be the case with the benchmarking strategy. In question C, she re-ordered the numbers using compatible numbers to give ((27 + 56) + (48 + 72), by writing the two pairs of numbers representing partial sums close to 100. She did not attempt to solve question E. Teacher H used only two processes of computational estimation, that is, reformulation and translation.

Figure 5.8 Teacher H's Computational Estimation Strategies

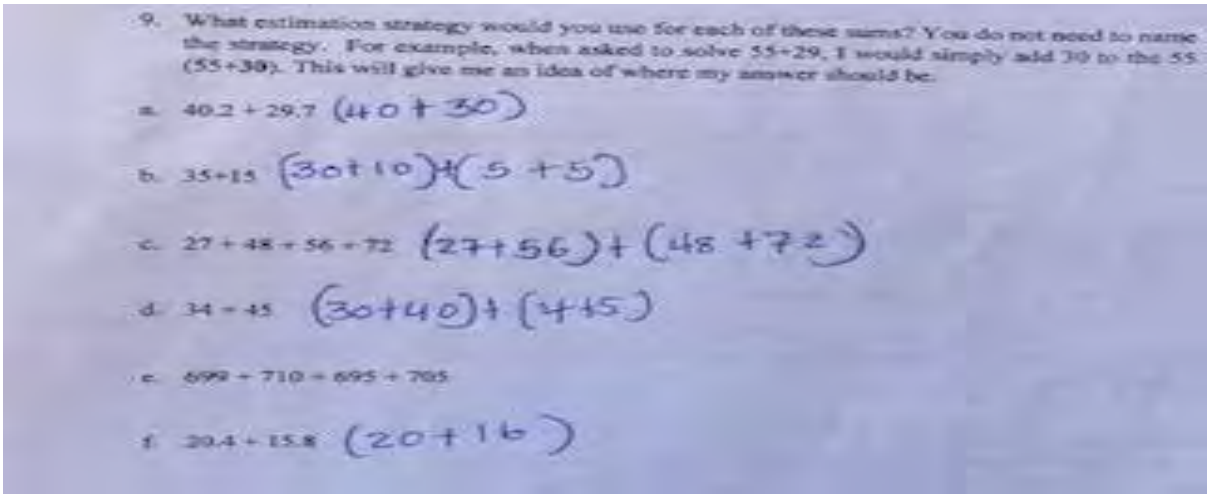


Table 5.2 provides a summary of the data of the teachers' estimation responses to the calculations presented in the questionnaire. As noted in Table 5.2, the most common strategies used for estimating the reasonableness of their calculations was rounding off and truncation. In questions A and F, the teachers changed the decimal fractions into whole numbers. In questions D and E, the teachers all rounded off to the nearest 10 or 100. There were two different responses to question B. The teachers either broke the number up into tens and ones or rounded off. Those that broke the number into tens and ones gave the correct answer rather than an estimate.

Table 5.2 Teachers' Computational Estimation Processes

Questions	**Strategies**	**Teachers**
a. 40.2 + 29.7	Rounding off	Teachers A, B, C, D, E, F, G and H
b. 35 + 15	Benchmarks	Teachers D, E and H
	Rounding off	Teachers A, B, C, F and G
c. 27 + 48 + 56 + 72	Rounding off	Teachers B, F and G
	Rounding off with error	Teacher C
	Compatible numbers	Teacher A and E

	Compatible numbers with error	Teacher H
	Benchmarking	Teacher D
d. 34 + 45	Rounding off	Teachers A, B, C, E, F and G
	Benchmarks	Teachers D and H
e. 699 + 710 + 695 + 705	Clustering	Teachers A, C, E, F and G
	Clustering with rounding	Teacher B
	Benchmarks	Teacher D
f. 20.4 + 15.8	Rounding off	Teachers B
	Rounding off to the next whole number	Teachers A, C, E, F,G and H

The teachers used a variety of computational estimation strategies to calculate and in some cases, they used two different strategies for the same calculation (Teachers A and B). The teachers used two of the three computational estimation processes, which are reformulation (rounding off, benchmarks and compatible numbers) and translation (clustering/averaging). Based on the strategies identified in the literature (Chapter 3, Table 3.1), there were a number of strategies that the teachers did not use. These included front-end, adjusting, special numbers and truncation. The last three of these strategies are compensation strategies, meaning that none of the teachers used the compensation cognitive process when estimating.

5.2.9. Factors preventing the teaching of computational estimation

The teachers identified a variety of factors that constrained the teaching of computational estimation. These included a shortage of time, learners' perception of mathematics and the limited reference to computational estimation in the Namibian Senior Primary Mathematics Syllabus. Two of the teachers, Teacher B and Teacher C, alluded that time prevents them from teaching computational estimation. Apart from time, two other teachers mentioned that the difficulty of the subject itself prevents them from teaching computational estimation. Teacher F expressed,

"[L]earners' perception toward mathematics, in general, they regard maths as a difficult subject." Teacher A expanded on this view suggesting that learners struggle with place value. Teacher A expressed that "[S]ome learners do not understand place value yet and this might make it difficult during the development of computation estimation." In other words, Teacher A maintains that the lack of knowledge of place value impacts on learners' computational estimation. For Teacher G, mathematics is often perceived by learners as being exact. This means that learners "always give exact answers, especially how many questions". One of the constraints that teachers experience as a result of policy is that "since the syllabus does not say much about estimation, it will be a problem in developing the learners' computation." (Teacher H).

The narrative above is analysed deductively using the MKfT framework, the domain indicators relate directly to Ball et al.'s work (2008) and may not relate directly to what the teachers did. As noted in Table 5.3, the domain indicators that were not evident in the questionnaires were left blank. I chose to include the indicators that were not evident from the questionnaire as this also provides insights into the teachers' knowledge. The data was later analysed using the KQ framework.

Table 5.3 The Teachers' MKfT

MKfT domains	MKfT domain indicators	Teachers' responses
CCK	● estimate the reasonableness of their calculation	When the teachers were presented with computational estimation examples, they were able to estimate the reasonableness of their calculations. They drew on a wide variety of strategies when using computational estimation, such as, rounding off, benchmarking, compatible numbers, and clustering and averaging.
	● understand the mathematics they teach – estimation and place value	The teachers' definitions of computational estimation show that they understand and know what computational estimation is. They used terms and phrases such as 'approximation' and 'a considered guess'.
	● recognize when a student gives a wrong answer	

	● recognize when a textbook is an inaccurate answer to a calculation or gives an inaccurate definition use terms and notations correctly	
HCK	● make connections across mathematics topics within a grade and across grades	The teachers maintained that computational estimation is central to learners' understanding of number sense. They agreed that computational estimation is vital to understanding of mental calculations and place value.
	● articulate how the mathematics you teach fits into the mathematics topics before and after the specific grade.	
SCK	● interpret students' emerging and incomplete ideas	
	● evaluate the plausibility of students' claims give or evaluate mathematical explanations	
	● use mathematical notation and language and critique its use	The teachers are familiar with the terms for different computational estimation strategies.
	● the ability to interpret mathematical productions by learners, other teachers or learning materials	
	● evaluate mathematical explanations for common rules and procedures	
KCT	● sequence mathematical content	The teachers articulated their knowledge of computational estimation which informs their instructional practice in the classroom. The teachers appreciate the use of the rounding off

		strategy as it develops a better understanding of computational estimation.
	● present mathematical ideas	The teachers recognize the need to use real life situations as it deepens the learners understanding of computational estimation.
	● select examples to take students deeper into mathematical content	The teachers provide suitable examples of computational estimation in a manner that they first teach place value which informs rounding off rules which is vital in computational estimation.
	● select appropriate representations to illustrate the content	
	● ask productive mathematical questions	
	● recognize what is involved in using a particular representation	
	● modify tasks to be either easier or harder	
	● use appropriate teaching strategies	The teachers use real-life situations, practical work, activities, estimating tasks, visual images and storytelling.
	● respond to students' why questions	
	● choose and develop useable definitions	
	● provide suitable examples	
KCS	● anticipate what students are likely to think and do	The teachers articulated that their learners might give exact answers when they are estimating instead of giving reasonable close answers.
	● predict what students will find interesting and motivating when choosing an example	

	● anticipate what a student will find difficult and easy when completing a task	The teachers provide suitable examples of computational estimation in a manner that they first teach place value which informs rounding off rules which is vital in computational estimation.
	● anticipate students' emerging and incomplete ideas	
	● recognize and articulate misconceptions students carry about particular mathematics content	
KCC	● articulate the topics in the curriculum	The teachers articulated the topics in the curriculum and how they linked computational estimation to other topics in the syllabus. They suggested some other topics in the syllabus need to integrate computational estimation.
	● articulate the competencies related to each topic in the mathematics curriculum	
	● articulate and demonstrate a familiarity with the structure of the mathematics curriculum	
	● link representations to underlying ideas and to other representations	
	● knowledgeability of available materials (e.g., textbooks) and their purposes	
	● appraise and adapt the mathematical content of textbooks	

Despite mentioning in Chapter 4 that the KQ framework was only used to analyse the data collected from lesson observations, the eight teachers displayed aspects of foundation (e.g., knowledge of the curriculum) in their questionnaire responses.

Table 5.4 Teachers' Knowledge Using the KQ Framework

KQ Dimension	**KQ Indicators**	**Participant responses**
Foundation	● Have a clear and coherent belief about the purposes of mathematics education and why his/her learners are compelled to learn it.	The teachers believe that computational estimation is important because it assists with real-life experiences. For example, the teachers said that computational estimation is necessary for shopping, budgeting and calculating distance.
	● Recognize the conceptual appropriateness of mathematical ideas for the children they are teaching.	
	● Have sound procedural and conceptual knowledge	The teachers demonstrated knowledge of various computation estimation strategies. These included all of the cognitive processes mentioned in Chapter 3.
	● Adapt textbooks to teaching.	
	● Show in his/her planning, knowledge of common errors and misconceptions and take steps to avoid them	The teachers adapt the content of teaching from the national syllabus hence their knowledge of rounding off as a strategy of computational estimation. The teachers appear to know how to avoid possible errors by focusing on place value prior to rounding off.
	● Have a knowledge of quick mental methods.	The teachers demonstrated knowledge of various computation estimation strategies. These included all of the cognitive processes mentioned in Chapter 3. While rounding off is the dominant strategy mentioned by the teachers, Teacher A mentioned bridging the decade and the associative property, while Teacher G referred to the compatible number, special numbers and distributive property as additional strategies.
	● Use mathematical language correctly	The teachers used the terminology for different strategies.

5.3. LESSON OBSERVATIONS

The initial plan was to observe two lessons of each teacher (Chapter 4). Teacher A expressed that she was not able to do two lessons and suggested that she do a single lesson. She told me that computational estimation is too short to plan two lessons. This seems to contradict her statement made in the questionnaire that she gives the learners "an estimation exercise every day before I start with my daily lesson." Teacher A said she "motivates the learners to do computational estimation before they calculate". I assume that she thought that she had to teach computational estimation rather than simply presenting any lesson relating to number operations in which she encourages the learners to draw on their computation estimation skills.

Teacher A taught her lesson to a Grade 6 class. By the end of the lesson, learners are expected to develop the skill to use approximation to reduce the complexity of large numbers and to "facilitate controlled estimation" (MoEAC, 2016, p. 22).

Teacher A introduced the lesson to the learners.

TA: Class, how do you understand the term estimation? What is to estimation?

LA: Umm, it means to approximate.

LB: To estimate is when we are rounding off.

TA: To estimate means having a rough calculation to a specific problem or mathematic problem, and it does not necessarily mean that you will get an exact answer, it means that you will get an approximate answer. An approximate answer is an answer that is close to the correct answer. Now, in estimation we also use an approximate sign …Now, for you to be able to estimate, what do you do first?

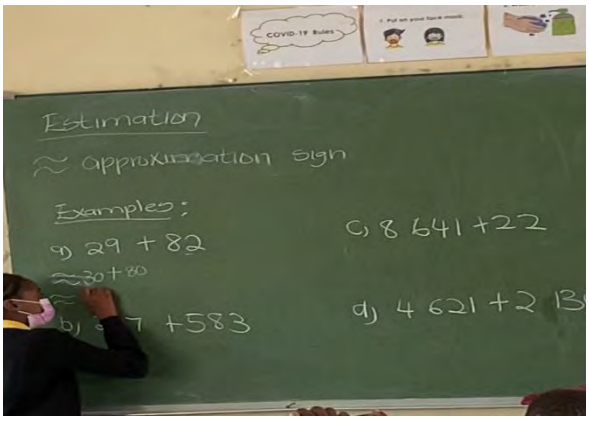
LC: We calculate.

TA: Yes, we calculate, but calculation comes after. What do we do first?

LD: We round off Ms

TA: Yes, for you to estimate, we need to round off first. Now remember when we are rounding, we round off to the nearest biggest unit provided. … If your biggest number is tens, you round off to the nearest tens. If your biggest digit is a hundred, you round off to the nearest hundred, if your biggest unit is a thousand, you round off to the nearest thousand…okay? Teacher A writes the approximation sign { ≈ } and a few examples taken from the Platinum textbook on the board and invites the learners to assist with the calculations (Figure 5.1).

Figure 5.9 Shows one of the Learners Adds 29 + 82 Using Rounding Off

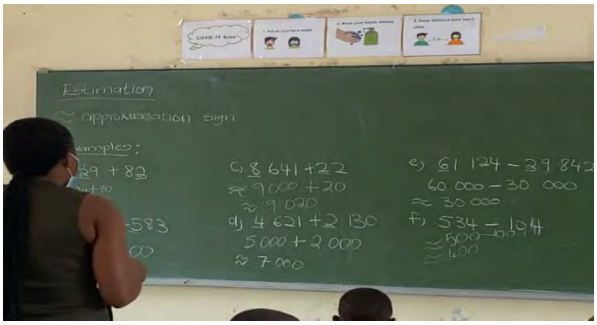


TA: How did you get 30 + 80 = 110? (Figure 5.1)

LE: I rounded 29 up and got 30, 82 I rounded it down, and got 80, thus why 30 plus 80 is 110.

TA: Very good {The teacher invites different learners to complete the all the examples on the board (Figure 5.2)}

Figure 5.10 Completed Sums on the Board



Once all the examples are completed. She tells the class “feel free to ask anything you don’t understand” to which the learners chorused “we understand Ms” Teacher A writes more examples on the board while the learners complete them in their exercise books. The teacher walks around the class marking learners’ work and assisting them where needed (Figure 5.3).

Figure 5.11 Marking Learners' Work

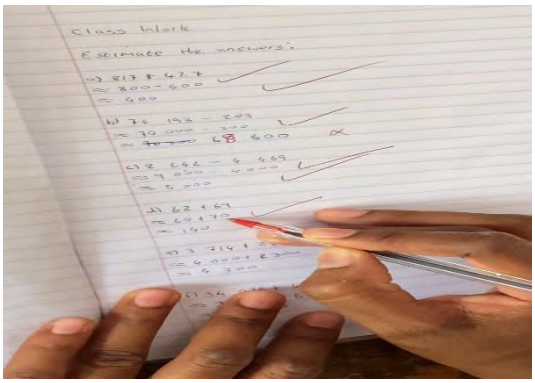


Figure 5.12 Feedback of the Marked Work

The teacher walked round the class marking the learners' books. During this time, she assisted those learners who had made mistakes. After all the learners' books were marked the teacher asked the learners to share their calculations on the board (Figure 5.12).

Teacher A explained that estimation is an approximate answer that is achieved through the process of rounding off. She introduced the learners to the symbol for approximation.

Lesson 1 (of Teacher B) was observed in a Grade 7 class. At the end of the lesson the learners are expected to apply approximation in real life situations and estimate answers to calculations with reasonable accuracy.

TB: Class, how many palm fruits do you think are in this bucket?

Figure 5.13 Estimating the Number of Palm Fruit

LA: 22 palm fruits Ms.

LB: I think they are 30 Ms.

LC: I think they are 11 Ms.

TB: Okay, let's count them and see who is correct. The teacher starts counting, one, two, three…28 palm fruits.

LEARNERS: No one is correct Ms.

TB: But there is someone whose answer is almost correct…

LEARNERS: Yes Ms., Leaner B is almost correct Ms.

TB: Yes. She is correct, (pick some fruits from the bucket and give it to learner B). What do you think is today's topic?

LD: It's about counting

TB: Yes, who else want to try?

LE: It's about guessing Ms.

TB: You are almost correct, but we call it estimation. In estimation we make an approximate guess or giving answers which are a bit closer … We use estimation in our everyday life. Who can give me an example?

LA: I can estimate the distance I walk to school …

TB: Okay, that's a great estimation. When I am in the shops shopping for my things, before I pay at the till, I always make an estimate of how much I am going to pay for the things in my trolley …What else can we estimate?
LF: Sometimes we don't have watches to see how much time it took us to walk a certain distance Ms. So, I can estimate of how long it took me to walk that distance, let me say it takes me about 20 minutes to reach to school.
TB: Very good. We have a parents' meeting on Friday, can you calculate the exact number of parents attending the meeting?
LEARNERS: YES, NO, it will be difficult Ms.
TB: Exactly. It's practically impossible. That's why we can only estimate the number of parents who are going to attend the meeting. Can someone tell me why do we estimate?
LG: We estimate when we are unable to get an exact answer.
TB: Very, good. Why else do we estimate?
LF: To get an answer fast Ms.
TB: Yes, correct. Estimation gives a quick and almost realistic answer. Hence it saves us time and we can get a better picture of things in a simpler way.

The focus of Lesson 1 in Teacher B's class was estimation in real-life contexts rather than computational estimation. She performed an activity at the start of the lesson to enable the learners to identify the topic for the lesson. The example she gave led the learners to suggest that the focus of the lesson was to guess the answer whereas in her questionnaire, she highlighted the importance of rounding off to estimate the answer to a calculation. In other words, she suggested that computational estimation is a reasoned guess.

The learners were asked to provide examples of when estimation is of value.

Lesson 2 (of Teacher B) is a continuation from lesson one.

Teacher B starts the lesson with an explanation of estimation.
TB: Estimation is to make a reasonable guess, or to make a rough calculation about values, about numbers or any quantity of something. In other words when we are estimating we use rounding off. When we are estimating, the answer can either go up, go down and sometimes it remains the same … We follow rounding off rules when we are estimating. Let's say we have 56 rounded to the nearest 10. If we are rounding off to the nearest 10, we underline the number that represents 10s which is 5, after that we look at the number on the right which is 6, if the number is 5 or more, what do we do?
LEARNERS: We round up!

TB: Yes, you round up, if the number is 4, 3, 2, 1 or 0,

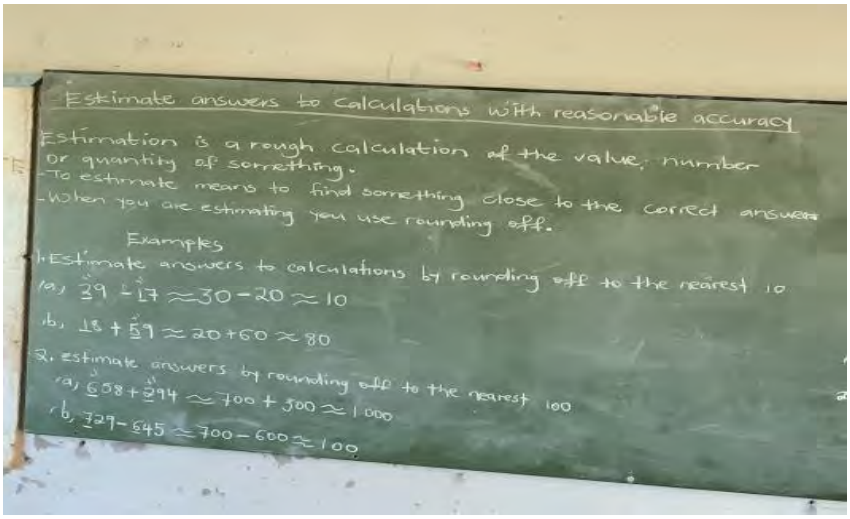
LEARNERS: We round down.

TB: Since there is 6, we round up, so 56 become?

LEARNERS: It becomes 60.

Teacher B writes a summary on the chalkboard with some exercises for the learners to complete (Figure 5.5). The exercises were taken from the Platinum textbook.

Figure 5.14 The Teacher's Explanation Written on the Chalkboard



The lesson ended with the learners sharing the answers to the examples written on the board by the teacher.

In the second lesson, Teacher B provided an explanation of estimation. She provided the learners with examples that all required rounding off. This was the computational estimation strategy that she emphasised in Lesson 1. Like Teacher A, Teacher B also taught the learners the approximation symbol.

5.3.1. Analysis of the two teachers' knowledge using the MKfT framework

As with Table 5.4, I have left the domain indicators that were not evident in the lessons in Table 5.5 as these also provide an indication of the knowledge not evident in the lessons.

Table 5.5 The Teachers' Mathematical Knowledge for Teaching: PCK

MKfT domains	MKfT domain indicators	Teachers' responses
KCT	● sequence mathematical content	The teachers articulated their knowledge of computational estimation which informs their instructional practice in the classroom. The teachers recognize the need to use real life situations as it deepens the learners understanding of computational estimation. The teachers provide suitable examples of computational estimation in a manner that they first teach place value which informs rounding off rules which is vital in computational estimation.
	● present mathematical ideas	The teachers appreciate the use of rounding off strategy as it develops a better understanding of computational estimation.
	● select examples to take students deeper into mathematical content	The lesson activities were sequenced to start with the easiest examples, moving to the difficult ones.
	● select appropriate representations to illustrate the content	The teacher (teacher B) used concrete objects (palm fruits) to illustrate the lesson content.
	● ask productive mathematical questions	Teachers asked the learners to complete examples on the chalkboard.
	● recognize what is involved in using a particular representation	
	● modify tasks to be either easier or harder	
	● use appropriate teaching strategies	The teachers used different teaching strategies to assist in the development of computational estimation (e.g., demonstration, explication and questioning).

	● respond to students' why questions	
	● choose and develop useable definitions	
	● provide suitable examples	The teachers check the learners' understanding by providing the learners with suitable examples to complete on the board and in their exercise books.
KCS	● anticipate what students are likely to think and do	When Teacher B introduced the topic of computational estimation she made use of concrete objects as examples. During the lesson observations, the teachers identified the learners' prior knowledge by questioning the learners.
	● predict what students will find interesting and motivating when choosing an example	Her view is that the learners find concrete objects interesting and motivating such that if the learners get a reasonably close answer they get one/some of the objects being used as a reward.
	● anticipate what a student will find difficult and easy when completing a task	
	● anticipate students' emerging and incomplete ideas	
	● recognize and articulate misconceptions students carry about particular mathematics content	
KCC	● articulate the topics in the curriculum	The teachers know the topics in the curriculum (e.g., the steps or rounding off procedures and using the approximation sign).
	● articulate the competencies related to each topic in the mathematics curriculum	
	● articulate and demonstrate a familiarity with the	

	structure of the mathematics curriculum	
	● link representations to underlying ideas and to other representations	
	● knowledgeability of available materials (e.g., textbooks) and their purposes	
	● appraise and adapt the mathematical content of textbooks	

I assumed the teachers had the required CCK, SCK and HCK, during the lesson observations. These domains of the MKfT framework (CCK, SCK & HCK), were not highlighted in the lesson observations because the teachers appeared to be familiar with the content required in developing the learners' computational estimation. I Soon realised that the dominant MKfT category being PCK during the lesson observations. In attempting to identify the PCK domains, it became evident that the boundaries between the MKfT framework domains are rigid. I thus turned to the KQ framework.

5.3.2. Analysis of the teachers' knowledge through the KQ theory

Drawing on the Knowledge Quartet framework (Rowland et al., 2009) I have also analyzed the data using the four knowledge categories of the KQ that the teachers drew on during the lesson observations (Chapter 4).

Table 5.6 The Teachers' Foundation, Transformation, Connection and Contingency Knowledge

KQ Dimension	KQ Indicators	Participant responses
Foundation	● Have a clear and coherent belief about the purposes of mathematics education and	The two teachers that were observed demonstrate an understanding of the knowledge of procedures used in computational estimation. Their knowledge of concepts

	why his/her learners are compelled to learn it.	around computational estimation reflects their degree of expertise in computational estimation. The teachers know the concepts related to the teaching of computational estimation and how it fits in the curriculum.
	● Recognize the conceptual appropriateness of mathematical ideas for the children they are teaching.	The teachers recognise the appropriateness of computational estimation hence making connections that computational estimation is important in real-life situations and also in classrooms.
	● Have sound procedural and conceptual knowledge.	The teachers were able to estimate the reasonableness of the calculations.
	● Adapt textbooks to teaching.	The teachers made use of the Platinum textbooks, but theyselected the exercises and examples from the textbook that they felt were appropriate.
	● Show in his/her planning, knowledge of common errors and misconceptions and take steps to avoid them.	
	● Have knowledge of quick mental methods.	
	● Use mathematical language correctly	The teachers use the mathematical language correctly. Using approximately instead of equal to in the context of estimation.
Transformation	● Choose appropriate examples when demonstrating or eliciting an idea.	The teachers choose appropriate examples when they are teaching computational estimation. Both teachers used real-life examples with Teacher B also using a concrete example at the start of her first lesson.
	● Give clear explanations of the concepts and strategies.	The teachers give clear explanations of what computational estimation is at the beginning of the lesson. The teachers also ask for the learners' opinions and ideas of what computational estimation is. The teachers explain the rounding off strategy which is essential to the teaching and learning of computational estimation.
	● Demonstrate clearly and accurately how to carry out procedures.	The teachers make explicit the rounding of rules as these are the procedures that need to be followed so we get a reasonably close answer when estimating.

	● Make use of interactive teaching techniques to develop and assess understanding of concepts.	Both participants use peer review (i.e. the learners have to review what their peers have written on the chalkboard) as an interactive teaching in their classrooms. The learners are called upon to show how they estimated the answers on the chalkboard.
Connection	● Make links to previous lessons.	
	● Make appropriate conceptual connections within the subject matter.	The teachers make connections to the topics within the subject. The teachers understand that in order for the learners to understand the computational estimation concepts, they have to make the connection between place value, rounding off and computational estimation.
	● Ask questions to elicit children's understanding of connections between mathematical ideas.	The teachers asked the learners when they estimate in real-life situations and the learners made a connection to other topics like money and finance, measurements and not only in the topic of whole numbers.
	● Introduce ideas and estimation strategies in an appropriate progressive order.	The teachers introduce ideas and estimation in an appropriate progressive order, they start with the learners' prior knowledge of computational estimation, followed by the definition of the term computational estimation, teaching the rounding-off computational estimation strategy and then giving the learners exercises to complete.
	● Assess the learners' understanding of computational estimation.	The teachers assessed the learners' computational estimation understanding through class activities which were done first as a whole class and then individually (marking their work).
Contingency	● Respond to learner's comments, questions and answers.	The teachers deal appropriately with the learners' responses to activities when they were marking each of the learners' books..
	● Cope with questions from all the learners.	The teachers respond to learners' questions by either signalling that the answer is correct or by probing further if the answer is incomplete.

	● Deal appropriately with learners' responses to activities.	
	● Respond appropriately to the learners' errors and misconceptions.	The errors made by the learners in their books were dealt with immediately as the teacher was marking their books.
	● Deviate from the lesson plan when appropriate.	

To summarize the classroom observation sessions, I observed the participants' SMK and PCK. In their daily practice, the participants perform their lessons in a rather fixed sequence. Starting off with the introduction, they both introduced the lessons by assessing the learners' prior knowledge, and then connecting the learners' prior knowledge to computational estimation. While the Teachers A and B introduced the learners to rounding off, they also introduced the learners to front-end strategy, whereby the learners focus on the front-end of the digit to enable them to round off correctly. They both explained the terms, rules and procedures involved in computational estimation and then demonstrated with examples on the board. The teachers gave the learners exercises to complete on the chalkboard, closing off their lessons with class activities or homework and marking the learners' books.

5.4. FOCUS GROUP INTERVIEW

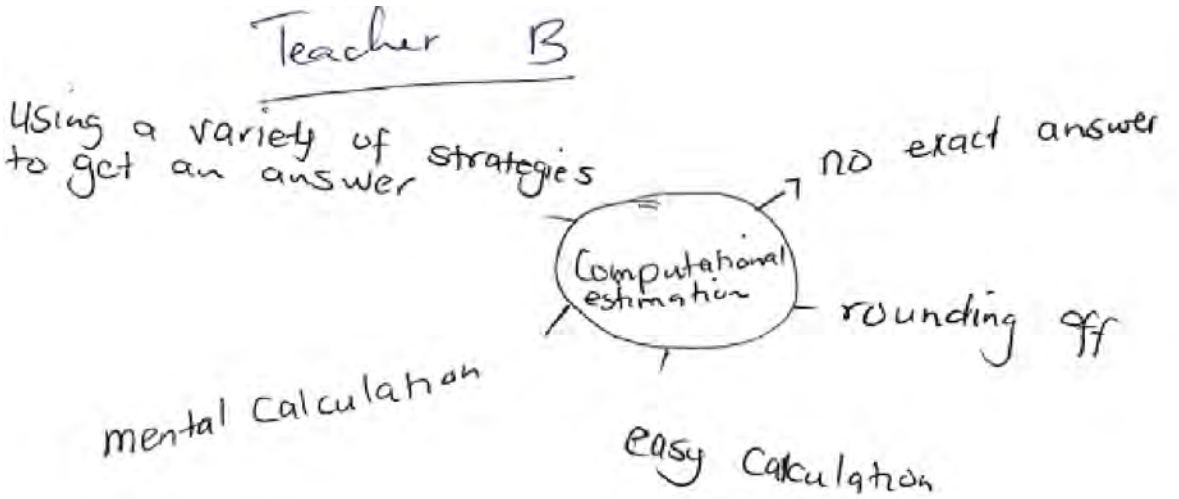
In this phase of data collection, the initial plan was to have a focus group with two teachers from each school, giving a total of four teachers. However, one teacher opted to withdraw from the study for reasons that he did not share. Three female teachers were interviewed, that is, Teachers A, B & C.

5.4.1. Conceptions of Computational Estimation

All three teachers explained computational estimation as a calculation that leads to no specific answer. As noted in Figures 5.15, 5.16 and 5.17, Teachers B and C wrote that computation estimation means that there is "no exact answer," while Teacher A wrote there is "no specific

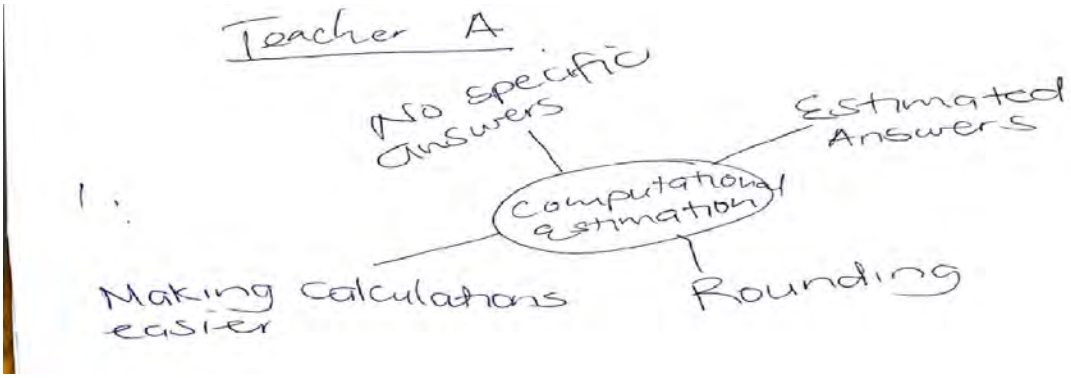
answer". Teacher B further explained "computational estimation is a way of using a variety of strategies to get an answer which is not exact." She also emphasised the role of mental calculations in computational estimation suggesting that computational estimation is a form of “mental calculation". Reys and Bestgen (cited in Tsao & Pan, 2011) argued that computational estimation is the comprehensive application of mental computation.

Figure 5.15 Teacher B’s Computational Estimation Mind Map



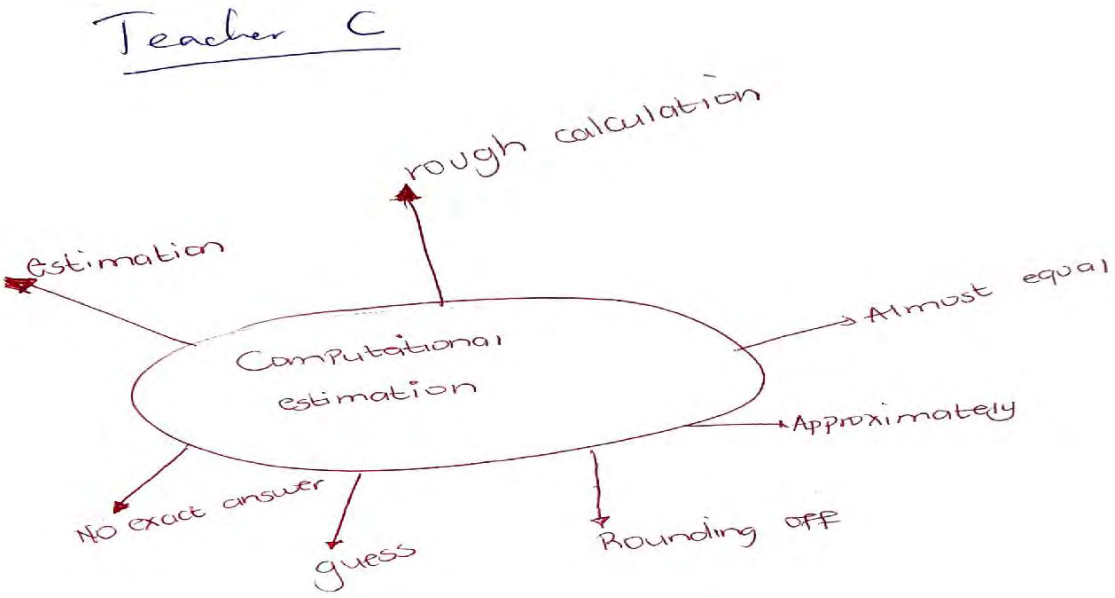
Teacher A expressed that computational estimation is "making calculations easier".

Figure 5.16 Teacher A’s Computational Estimation Mind Map



Teacher C referred to computational estimation as making a "rough calculation" to get an approximate answer or an answer that is "almost equal." Teacher C also linked computational estimation to a "guess."

Figure 5.17 Teacher C's Computational Estimation Mind Map



While Teacher B wrote that computational estimation is "using a variety of strategies to get an answer", the only computation estimation strategy the teachers specifically referred to in their mind maps (Figures 5.15, 5.16, 5.17) was 'rounding off'. All three teachers mentioned this. This is consistent with the current syllabus and the lessons of Teachers A and B which only emphasised rounding off as a strategy for computational estimation.

5.4.2. Teachers' knowledge of computational estimation strategies

When I asked the teachers about the strategies they would like to use when teaching computational estimation, the teachers provided an additional strategy that could be used for computational estimation, that is, the front-end strategy. Teacher A suggested that the compatible number strategy should be part of the curriculum. She explained compatible numbers as "a strategy which involves using friendly numbers." She gave the example, '57 + 38 + 42 + 61, so 57 and 42 are almost a 100

and 38 + 61 is almost 100, so this could be 100+100= 200'. Teacher B suggested that the benchmark strategy should be added to the curriculum, She said:

> let me say I have 35 + 47. I have to add the 30 plus 40, then that will give me 70. And then I'll add seven plus five, which will give me twelve. And then from there, I will add this seventy plus the 10; I got from 12. So, I get 80 and then I will add eighty to two. So, it will give me eighty-two.

Despite acknowledging in Figure 5.15 that computational estimation requires "no exact answer", her example provided an exact answer. As I noted earlier in this chapter, this is seemingly what the benchmark strategy encourages. Teacher C suggested the front-end strategy, which she explained as a "method of estimating an answer to a calculation problem by focusing on the front-end or the left-most digit number". She gave the example 521 + 287. "So I will look at the front-end. The left-most digit like 521 is almost close to 500 and 287 is almost close to 300, so I would say 500 + 300 = 800".

5.4.3. Computational estimation in the syllabus

The teachers suggested some topics in the syllabus should include computational estimation. The teachers suggested topics that included real-life situations, measurement, money and finance. Teacher C responded, "so, in a measurement, we are dealing with length, learners can estimate the distance between different towns, or they can estimate the amount a water tank can hold." Teacher B suggested that she would include the use of money and finance to teach computational estimation. "Computational estimation should be integrated into money and finance so that learners be estimating the prices of things when are budgeting." Teacher A suggested that "computational estimation should remain in the whole number topic since in whole numbers there is problem-solving which involves real-life situations." As she did in the questionnaire, Teacher B mentioned that estimation should be included with the measurement topic, specifically, length and capacity. The topics that the teachers thought could be used to develop learners' computational estimation were the same as those they provided during the questionnaire.

5.4.4. Knowledge of pre-service teachers

When asked what knowledge they would expect a pre-service teacher to have, all three teachers reverted to naming a single calculation strategy that is rounding off. When a student teacher is required to teach computational estimation, they need some knowledge of how they are supposed to teach it. The teachers suggested that in order for a student teacher to teach computational estimation, they should know computational estimation strategies, have knowledge about rounding off and knowledge of place value. Teacher B expressed that in order to estimate, you need to have a strategy that you will use, for example, "rounding off, and again I expect him/her to not expect an exact answer from the learners". Teacher A added that the student-teacher needs to "know rounding off rules because it entails rounding off to the nearest different place values." Teacher C concurred that the student teacher should know when to round up and when to round down. Their comments are consistent with those expressed in the questionnaire and in the lessons that I observed.

5.4.5. Teachers' Content Knowledge

In the focus group interview, the teachers were asked the following question:

A child in Mr. Clarke in Grade 7 wanted to find an estimate for the following sum:

21 014 + 2 811 + 19 112

- What estimation strategy would you use to estimate the answer?
- Which of the strategies would be most efficient for this calculation?

Teacher B answered that "I would use a benchmark as I find it easier compared to other strategies", however, she suggested that rounding off would be most efficient for this calculation. Teacher C expressed, "I would use the rounding off strategy. And I think it would be the most efficient for the calculation." While Teacher A answered, "I will stick to compatible numbers because I feel adding up or subtracting numbers which can be easily added (that is numbers that are closer together), make the calculations much easier." The teachers identified three strategies in the focus group interview that could be used to estimate the approximate answer for the above sum. These included benchmarks, rounding off and compatible numbers.

5.4.6. Support for teachers

The teachers articulated what support they needed in order to teach computational estimation. This included support from the subject heads, the senior education officers, and the curriculum. All the teachers believe that it would be helpful if they were provided with in-service training about computational estimation through the forms of workshops. Teacher A said, "I would require to have an in-service training on computational calculations and on strategies that can be used." Teacher C said, "I am also supporting the ideas of workshops and training", while Teacher B “also supporting training is needed is here."

5.4.7. Constraints teaching computational estimation

There are some difficulties that teachers encounter when teaching computational estimation. The teachers provided examples such as learners' perception of the subject, lack of knowledge, understanding of place value, and computational estimation coverage in the syllabus. Teacher A believes that the learners perceive mathematics as a difficult subject, "so by the time you introduce the topic to the learners, it's already a mess." She further suggested that teachers can only achieve better results if they make the lessons enjoyable to the learners. Teacher C concurred that the lack of knowledge and understanding about place value makes computational estimation difficult to teach as the learners are unable to round off. She gave an example "you are rounding up to the nearest hundred. If the learner they're not able to identify which digit is representing hundreds, then she will not be able to proceed". Teacher B expressed that "the syllabus makes the teaching of computational estimation difficult, because the syllabus only focuses on computational estimation in whole numbers only, and I find it difficult to integrate it in other topics within the syllabus." Apart from time, the constraints mentioned in the focus group interview are consistent with those mentioned in the questionnaire.

5.4.8. Introducing computational estimation

The teachers mentioned that in order to teach computational estimation efficiently, they need to have ways of introducing the topic for instruction. The teachers provided real-life or concrete examples to introduce computational estimation to the learners. Teacher A said, "for example, for them to estimate the time it takes them from, I mean, to walk from home to school or maybe to travel from home to school by then, they will relate to real-life situations on the lesson at hand."

Teacher B explained that she starts the lesson by defining the term computational estimation: "Which is a way of finding an approximate answer to an arithmetic problem without actually computing the exact answer. And I will give them an example in real-life situations, like by finding the distance they walk from home to school". Teacher C explained that she "comes with a bucket of apples and asks the learners to guess the number of apples in the bucket." This correlates with what Teacher B actually did during her lesson.

5.4.9. Analysis of the teachers' knowledge through the MKfT theory

The summary presented in Table 5.7 includes the indicators that were not evident in the focus group interview as this also reflects on the teachers' knowledge.

Table 5.7 Summary of the Teachers' MKfT During the Focus Group Interview

MKfT domains	MKfT domain indicators	Teachers' responses
CCK	● estimate the reasonableness of their calculation	When the teachers were presented with the single computational estimation example, they were able to estimate the reasonableness of their calculations. They drew on a wide variety of strategies, such, as rounding off, benchmarking and compatible numbers.
	● understand the mathematics they teach – estimation and place value	The way the teachers defined computational estimation shows they understand and know what computational estimation is. The teachers' description of computational estimation shows they understand the mathematics they are required to teach. Teacher C did mention that computational estimation requires one to 'guess' the answer, but this was not consistent with her explanation of computational estimation in the questionnaire.
	● recognize when a student gives a wrong answer	
	● recognize when a textbook is an inaccurate answer to a	

	calculation or gives an inaccurate definition use terms and notations correctly	
HCK	● make connections across mathematics topics within a grade and across grades	They agreed that computational estimation is vital to understanding of mental calculations and place value.
	● articulate how the mathematics you teach fits into the mathematics topics before and after the specific grade.	
SCK	● interpret students' emerging and incomplete ideas	
	● evaluate the plausibility of students' claims give or evaluate mathematical explanations	
	● use mathematical notation and language and critique its use	
	● the ability to interpret mathematical productions by learners, other teachers or learning materials	
	● evaluate mathematical explanations for common rules and procedures	
KCT	● sequence mathematical content	The teachers appreciate the use of the rounding off strategy as it develops a better understanding of computational estimation and knowledge of place values, hence the suggestion that the pre-service teacher should possess this knowledge in order to develop learners' computational estimation skills.
	● present mathematical ideas	

	● select examples to take students deeper into mathematical content	
	● select appropriate representations to illustrate the content	
	● ask productive mathematical questions	
	● recognize what is involved in using a particular representation	
	● modify tasks to be either easier or harder	
	● use appropriate teaching strategies	
	● respond to students' why questions	
	● choose and develop useable definitions	The teachers were all able to provide an explanation of computational estimation.
	● provide suitable examples	
KCS	● anticipate what students are likely to think and do	The teachers articulated that their learners might give exact answers when they are estimating instead of giving reasonably close answers.
	● predict what students will find interesting and motivating when choosing an example	The teachers explained that the learners would find real-life examples interesting.
	● anticipate what a student will find difficult and easy when completing a task	
	● anticipate students' emerging and incomplete ideas	
	● recognize and articulate misconceptions students carry about particular mathematics content	The teachers articulated that some of the learners are likely to calculate exact answers when asked to estimate.
KCC	● articulate the topics in the curriculum	The teacher's teaching of computational estimation is informed by the syllabus, hence their use of a single

		computational estimation strategy which is rounding off. The teachers articulated the topics in the curriculum and explained how they linked computational estimation to other topics in the syllabus. They suggested some other topics in the syllabus need to integrate computational estimation. They suggested that more computational estimation strategies such as benchmarking, front- end strategy, be part of the syllabus.
	● articulate the competencies related to each topic in the mathematics curriculum	
	● articulate and demonstrate a familiarity with the structure of the mathematics curriculum	The teachers know that place value and rounding off appear in the syllabus before computational estimation.
	● link representations to underlying ideas and to other representations	
	● knowledgeability of available materials (e.g., textbooks) and their purposes	
	● appraise and adapt the mathematical content of textbooks	

Drawing on the KQ framework during the focus group interview, the data has shown that only the Foundation Knowledge has emerged, confirming the views of Rowland et al. (2009), that the foundation knowledge is demonstrated in the other three dimensions (transformation, connection and contingency).

5.4.10. Analysis using the Knowledge Quartet framework

As noted in Table 5.8, the foundation knowledge evident in the focus group interview was listed too.

Table 5.8 A Summary of the Teachers' Foundation Knowledge that They Drew on During the Focus Group

KQ Dimension	KQ Indicators	Participant responses
Foundation	● Have a clear and coherent belief about the purposes of mathematics education and why his/her learners are compelled to learn it.	The teachers demonstrated an accurate understanding of the computational estimation concepts, they could unpack the syllabus, thereof knowing the strategies of computational estimation which are outlined in the syllabus. The teachers demonstrated the awareness and appropriateness of teaching and developing computational estimation of the learners.
	● Recognize the conceptual appropriateness of mathematical ideas for the children they are teaching.	
	● Have sound procedural and conceptual knowledge	The teachers demonstrated an accurate understanding of the computational estimation concepts.
	● Adapt textbooks to teaching.	
	● Show in his/her planning, knowledge of common errors and misconceptions and take steps to avoid them	
	● Have a knowledge of quick mental methods.	The teachers are able to comment on which computational estimation strategies would be more efficient for performing a particular calculation.
	● Use mathematical language correctly	The teachers are familiar with some of the terminology of the computational estimation strategies.

5.5. DISCUSSION

This study explored the mathematical knowledge for teaching that the senior primary mathematics teachers draw on to develop their learners' computational estimation skills. The exploration was carried out to understand the teachers' SMK and PCK in developing the learners' computational estimation in selected schools in the Ohangwena Region, Namibia. The data were collected from questionnaires, lesson observations and lastly, the focus group interview. The data was analysed using the MKfT framework of Ball et al. (2008) and later the KQ framework of Rowland (2005). The study's findings are discussed according to each of the frameworks, that is, Mathematics Knowledge for Teaching and the Knowledge Quartet.

5.5.1. Mathematics Knowledge for Teaching

In this section, I start with the teachers' SMK and then progress to identify their PCK.

5.5.1.2. Teachers' Subject Matter Knowledge

Ball et al. (2008), outlined that while CCK is not specific to teaching, a teacher needs to develop this knowledge as it would be challenging to teach what they do not know. They highlight that CCK is not sufficient for teaching as teachers also require subject matter knowledge that is specific to teaching (SCK) and knowledge of how the content in the curriculum links to the content required in previous and post grades. Rowland (2012) validates this claim and suggests that both pre- and in-service teachers require sound foundation knowledge to understand what they are going to teach to the learners (HCK).

The findings of this study clearly show the teachers are aware of the importance of computational estimation, can define computational estimation, and know some computational estimation strategies. This suggests that the teachers understand the mathematics they are required to teach. In explaining what computational estimation is, the teachers used terms and phrases such as 'approximation' and 'a considered guess'.

When the teachers were presented with computational estimation examples, they were able to estimate the reasonableness of their calculations. However, as evident from the focus group interview, not all of the teachers were able to identify the most efficient strategy for computational estimation. The strategies the teachers used when calculating focused on two of the computational cognitive processes, that is, reformulation and translation. The strategies that they chose included rounding off, benchmarking, compatible numbers and clustering/averaging. The most common strategy chosen by the teachers was rounding off. This is not surprising as it is the only strategy mentioned in the Namibian Senior Primary Mathematics syllabus (MoEAC, 2016).

As noted above, the teachers are familiar with the terms for some of the different computational estimation strategies and know the approximation symbol.

The teachers maintained that computational estimation is central to learners' understanding of number sense, specifically in relation to the development of mental computation. They argued that an understanding of place value was necessary in order to estimate the reasonableness of the answers to calculations as learners needed to know how to round off to the next 10, 100, 1000 and so forth. A concern was expressed by one of the teachers that there was no support in the syllabus to encourage computational estimation when dealing with fractions.

5.5.1.3. Teachers' Pedagogical Content Knowledge

According to Ball et al. (2008), teachers need to understand how to relate the content that needs to be taught to their teaching, knowledge of the learners and the curriculum.

The teachers' knowledge of computational estimation and the Namibian Senior Primary Mathematics syllabus informs their instructional practice in the classroom. To this end, they focus on the rounding off strategy. The teachers appear to sequence the development of computational estimation by first teaching place value and then teach learners the rounding off rules to the nearest 10, 100, 1000 and so forth. The lesson activities are sequenced to start with the easiest examples, moving to the difficult ones. The teachers asked the learners to complete examples on the chalkboard and they used different teaching strategies to assist in the development of

computational estimation (e.g., demonstration, explication and questioning). Through questioning the learners, observing their use of rounding off on the chalkboard and marking their books, the teachers assess the learners' understanding of computational estimation as it relates to rounding off.

The teachers recognize the need to use real life situations as it highlights the relevance of computational estimation and deepens their learners' understanding of computational estimation. The teachers mentioned that they used concrete objects, practical work, activities, estimating tasks, visual images and various examples taken from the Platinum textbook to illustrate the lesson content and motivate the learners.

The teachers recognised that there is a tendency with their learners to view mathematics as a subject that provides 'exact answers'. When Teacher B introduced the topic of computational estimation, she made use of concrete objects as examples. Her view is that the learners find concrete objects interesting and motivating such that if the learners get a reasonably close answer, they get one/some of the objects being used as a reward. The teachers identified the learners' prior knowledge by questioning the learners. They structure their teaching in such a way that the learners build an understanding of place value, followed by rounding off, and then computational estimation.

The teachers are familiar with the topics in the curriculum and how they link to computational estimation. They offered examples that also related to measurement estimation (Chapter 3). The teacher's teaching of computational estimation is informed by the syllabus, hence their use of a single computational estimation strategy which is rounding off. They were able to critique the syllabus, suggesting that more computational estimation strategies be included. They referred specifically to benchmarking, compatible numbers and the front-end strategy, be part of the syllabus.

The teachers appeared to have broader Foundation Knowledge. They believe that computational estimation is important because it assists with real-life experiences. For example, the teachers said that computational estimation is necessary for shopping, budgeting and calculating distance. The

teachers demonstrated knowledge of various computation estimation strategies. These included two of the cognitive processes mentioned in Chapter 3 (reformulation and translation). Their knowledge of concepts related to computational estimation reflects their degree of expertise in computational estimation. The teachers know the concepts related to the teaching of computational estimation and how it fits in the curriculum.

The teachers recognise the importance of computational estimation and they are able to comment on which computational estimation strategies would be more efficient for performing a particular calculation. The teachers adapt the content of teaching from the national syllabus hence their knowledge of rounding off as a strategy of computational estimation and they also make reference to Platinum textbooks where they choose which exercises to write on the board for the learner to complete. The teachers appear to know how to avoid possible errors by focusing on place value prior to rounding off and computation estimation. The teachers use the mathematical language correctly by being familiar with some of the terminology for different computational estimation strategies. The teachers also used approximately instead of equal to in the context of estimation.

5.5.2. The knowledge teachers demonstrated in the process of teaching

During the lesson observations, the teachers showed how best they could transform their Foundation Knowledge into an actual lesson, making what they know accessible to the learners. The results are in accordance with Rowland et al. (2009) who maintain that the teachers' lessons should hang well together and relate to what is taught in previous lessons and the learners' knowledge in order to reflect their connection knowledge. The results also support the claims of Liston (2015) that connection knowledge assists learners in making connections between concepts and procedures.

The two teachers were able to transform their knowledge of estimation into the classroom, albeit only emphasising the rounding off strategy. They chose appropriate examples when they were teaching computational estimation. At the beginning of their lessons, the teachers sought to ascertain the learners' prior knowledge by asking for their opinions and ideas of what computational estimation is. The teachers gave clear explanations of what computational

estimation is and used real-life examples to assist the learners in understanding the relevance of computational estimation.

The teachers explain the rounding off strategy which the Namibian Senior Primary Mathematics syllabus regards as essential to the teaching and learning of computational estimation whilst making explicit the rounding of rules as these are the procedures that need to be followed so we get a reasonably close answer when estimating. The participants use peer review (i.e. the learners have to review what their peers have written on the chalkboard) as an example of interactive teaching in their classrooms. The learners were called upon to show how they estimated the answers on the chalkboard.

The teachers made connections to the topics within the subject in order for the learners to understand computational estimation. They made the connection between place value, rounding off and computational estimation. The teachers asked the learners when they estimate in real-life situations and the learners made a connection to other topics like money and finance, measurements and not only in the topic of whole numbers. The teachers introduce ideas and estimation in an appropriate progressive order, they start with the learners' prior knowledge of computational estimation, followed by the definition of the term computational estimation, teaching the rounding-off computational estimation strategy and then giving the learners exercises to complete. The teachers assessed the learners' computational estimation understanding through class activities which were done first as a whole class and then individually (marking their work).

Even though the teachers also require contingency knowledge in developing their learners' computational estimation, this knowledge was not easy to observe, since there were seemingly no unpredictable or surprising events in the classroom settings. I reflect on possible reasons for this in Chapter 6.

Despite contingency knowledge not being easy to observe, it was evident that the teachers responded appropriately with the learners' questions and answers. They signalled if an answer was correct or probed further if the answer was incomplete. The errors made by the learners in their books were dealt with immediately as the teacher was marking their books.

5.6. CHAPTER SUMMARY

The study was conducted to understand and explore the knowledge that senior primary Mathematics teachers require and draw on to develop the learners' computational estimation skills. In this chapter, I analysed the teachers' knowledge for and in the process of teaching computational estimation by drawing on the Mathematics Knowledge for Teaching and Knowledge Quartet frameworks. The next chapter provides a summary of my research, the findings emerging from the research, and recommendations and limitations of my study.

CHAPTER 6: SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

6.1. INTRODUCTION

This chapter provides a summary of my research and thesis, together with the key findings, limitations and recommendations.

6.2. SUMMARY OF THE STUDY

The study sought to understand the knowledge teachers use when developing their learners' computational estimation skills. My interest in this topic was to inform my own teaching practice and that of other mathematics teachers whom I work with. I hoped that the study will be of potential benefit to the learners and in-service teacher education programs in Namibia.

The following question guided the study:
What mathematical and pedagogical content knowledge do senior primary mathematics teachers draw on to develop their learners' computational estimation skills?
The two sub-questions that assisted me in answering this question were:

- What mathematical content knowledge do teachers draw on to develop the learners' computational estimation skills?
- What pedagogical content knowledge do teachers draw on to develop the learners' computational estimation skills?

In Chapter 1, I provided the rationale for my research. Drawing on research from various national and international benchmarking studies, I argued that Namibian learners are underperforming in mathematics and that one of the explanations for this, is the seemingly limited content and pedagogical content knowledge of the teachers. Teaching in Namibia is still based primarily on drill and practice and using standard procedures to calculate exact answers. This is contrary to the Namibian Senior Primary Mathematics syllabus that suggests that teachers should develop learners' number sense. Central to the development of number sense, is the ability to judge the reasonableness of an answer to a calculation by using a variety of computational estimation strategies. Courtney-Clarke (2012) asserts that in order to develop learners' number sense, teachers should themselves have number sense. In relation to judging the reasonableness of one's calculations, teachers should be familiar with and use a variety of computational estimation

strategies. Based on this and my own experiences in the classroom, I argued that research is needed to understand the content and pedagogical content knowledge teachers use and draw on when teaching computational estimation.

The focus of Chapter 2 was the evolution of the knowledge required to teach mathematics by drawing on the work of Shulman's (1986, 1987) PCK, Ball et al's (2008) MKfT and Rowland et al.'s (2005) Knowledge Quartet. This chapter argues that since the MKfT framework focuses specifically on knowledge *for* teaching, the KQ was used to examine teachers' knowledge *in* teaching, that is, the knowledge teachers draw on during the actual process of teaching computational estimation in the classrooms.

Chapter 3 elaborated on the key concepts of this study. While the central concept in my research was computational estimation, I located computational estimation within the literature on number sense, mathematical proficiency, mental mathematics and estimation more broadly. The chapter argues that if teachers are required to develop the learners' computational estimation, they should be familiar with and able to use a variety of strategies to determine the reasonableness of their calculation answers in flexible and efficient ways. I provide insight into Morgan's (1999) three cognitive processes required by efficient computational estimators which are reformulation, compensation and translation and the variety of computational estimation strategies related to each of these processes.

In order to find answers to the research questions, Chapter 4 presents the methodology of my thesis. The study adopted a qualitative research approach using a case study research design underpinned by an interpretivist orientation. The study used purposive and convenience sampling design to select eight senior primary school mathematics teachers from two schools in the Ohangwena region. The study used a combination of questionnaires, observations and a focus group interview. Data was analysed both inductively and deductively. I identified codes and themes emergent from each of my data sets before using the codes from the MkfT and KQ frameworks to analyse and explain my data.

The focus of Chapter 5 was to present my analysis of the research data. I chose to present the data in the order it was collected. I found this useful in identifying (in)consistencies across the data sets. With each data set, I highlight my initial themes prior to drawing on the MKfT and KQ frameworks. The chapter concludes with a discussion of the findings that emerged from the data, specifically in relation to my two sub-questions.

6.3. FINDINGS

I present the key findings in the following order: Teachers' subject matter knowledge, teachers pedagogical content knowledge, and a reflection of the frameworks that I used.

6.3.1. Teachers' subject matter knowledge

The findings of this study clearly show the teachers are aware of the importance of computational estimation, are able to define it and use the appropriate terminology and symbols. The teachers knew and used a variety of computational estimation strategies when calculating. However, the teachers were only familiar with strategies relating to two of the computational estimation cognitive processes that Morgan (1999) refers to (Chapter 3). While teachers spontaneously use a variety of computational strategies, rounding off is the dominant strategy and the only one teachers focus on in the classroom. I thus maintain that teachers have the necessary CCK to teach computational estimation, but that their SCK should be extended. While most people use computational estimation in their daily lives (e.g., shopping), I suggest that knowing the full range of computational estimation strategies and how to use them in flexible and efficient ways is specific to the work of teachers. In this sense, teachers are influenced by the Namibian Senior Primary Mathematics syllabus. The teachers' HCK was evident in how they reflected on the relationship between computational estimation and number sense, particularly mental mathematics. The teachers mentioned the importance of making the connections between place value, rounding off and computational estimation.

6.3.2. Teachers pedagogical content knowledge

The study also highlights on the teachers' pedagogical content knowledge, which is reflected in the teachers KCT, as they appeared to sequence the development of computational estimation by first teaching place value before moving on to rounding off to the nearest 10s, 100s, 1000s and so

forth. The teachers' further reflected on their KCT by selecting appropriate representations (using concrete and real life examples) in making explicit the lesson content. The teachers' KCT seemed to be proficient in the development of computational estimation skills of their learners. However, I maintain that the teachers' KCS should be extended such that when they are to be presented with computational estimation, they do not give exact answers. The teachers KCC was evident in how they are familiar with the topics in the curriculum and how they link to computational estimation and also in being able to critique the syllabus, in that it does not provide sufficient computational estimation strategies that the teachers should use in developing their learners' computational estimation skills.

6.3.3. An unintended finding: The computational estimation framework

During the analysis of my data, it became apparent that the computational estimation framework of Morgan (1999) warranted some changes. While my research did not intend to examine Morgan's framework, I realised that there was a potential unintended finding which I thought would be worthwhile sharing.

Based on the literature that I read about computational estimation, I used a framework by Morgan (1999) to assist me in analysing the computational estimation strategies that the teachers were aware of, taught and used. Having completed my research, I think that two changes to the framework are warranted:

(1) The addition of 'rounding to the next whole number'. When working with decimal fractions, most of the teachers used a 'partial' rounding off strategy. Instead of rounding off to the 10s, they chose to round off to the next whole number. For example, they estimated the answer to 20.2 + 15.8 by rounding the 20.2 to 20 (rounding off) and 15.8 to 16 (the next whole number) rather than 20.

(2) The exclusion of the benchmark strategy. Expanded notation is a useful strategy for calculating, particularly when calculating with pen and paper. However, the benchmark strategy, in all likelihood and as evident in my research, will produce an exact answer rather than an estimation.

My adapted version of Morgan's (1999) computation estimation framework thus appears in Table 6.1.

Table 6.1 My Adapted Framework of the Computational Estimation Strategies

Process	Strategy	Description	Example(s)
Reformulation	Rounding off	Using the closest ten to make the computation easier.	43 + 29 is estimated by calculating 40 + 30 = 70
	Rounding off to the nearest whole number	Converting a number to an approximate value that is easier to compute, usually in decimal numbers.	34.2 + 15.7 may be worked out as 34 + 16
	Compatible numbers	This refers to a pair of numbers that are easy to add, subtract, multiply or divide mentally. The compatible strategy can be used when a student realises that the numbers can be re-ordered to make calculation easier.	27 + 48 + 56 + 72 27 + 72 is almost 100; and 48 + 56 is almost 100. These two pairs of numbers are easy to work with. A learner can estimate the reasonableness of their calculation by doubling 100.
	Front-end method	Using the front-digits of each number.	34 + 45 Start with the tens: 30 + 40 = 70
Translation	Clustering/Averaging	Looking for an estimate of an average in a list of numbers by looking for a number that all other numbers seem to surround.	699 + 710 + 695 + 705 All these numbers cluster around 700. So, one could calculate 700 × 4 = 2800 to get the estimation.
Compensation	Adjusting	When one number is changed/adjusted in a problem to make it easier to solve .	35 - 18 could be calculated mentally as 35 – 20 = 15

	Special numbers	Using the numbers that are friendly to use.	9.5% of 500ml One can calculate the 10% of 500 ml which is easier, 10% being a special number: 10% of 500ml =50ml
	Truncation	Means cutting off the decimal portion of a number without rounding it. In other words, the decimal point in the number is ignored.	20.4 + 15.8 We get 20 + 15 =35

6.4. LIMITATIONS OF USING MKfT AND KQ AS THEORETICAL FRAMEWORKS

Working with Ball et al. (2008)'s Mathematical Knowledge for Teachers (MKfT) was rather complex during this study. It was not easy to establish the domains of MKfT in the development of the learners' computational estimation skills because of the thin lines between the MKfT domains. Ball et al. (2008) acknowledge that "it is not always easy to discern where one of our categories divides from the next, and this affects the precision (or lack thereof) of our definitions" (p. 402). Hurrel (2013) highlights that the lines between the MKfT domains are too blurred, making it difficult to know where one domain starts and where the other ends. Given the above, it was difficult to determine what knowledge was used where and when since any given task could easily require knowledge from many or all of the other domains.

I used the MKfT framework developed by Chikiwa (2017) to analyse the knowledge the teachers required to teach mathematics. As noted by Chikiwa (2017) this was a framework that she developed from the work of Shulman, and Ball and colleagues. In using this framework, I noted that there were codes that did not relate explicitly to my research. I thus propose an adapted framework for research on teachers' subject matter and pedagogical content knowledge relating to computational estimation in the senior primary. As noted by Adler and Davis (2006), there are a variety of topics in mathematics, each with their own specialised knowledge structure. In this

sense, one could argue that understanding the subject matter and pedagogical content knowledge that teachers have and draw on would be different in the context of teaching computational estimation to other mathematics topics (e.g., Chikiwa's (2017) research on the mathematics knowledge teachers require in teaching learners to count). In addition, Adler and Davis argue that context matters in teaching mathematics. This was evident by the emphasis that the teachers in my research placed on the Namibian Senior Primary Mathematics syllabus. Despite the teachers being aware of and using a variety of computational estimation strategies in the questionnaires and focus group interviews, the two teachers whose lessons I observed only taught the rounding off strategy. In a context where curricula encourage the use of a variety of strategies, the indicators proposed in the MKfT and KQ frameworks, may differ to that which I present in Table 6.2.

Table 6.2 The Teachers' MKfT

MKfT domains	**MKfT domain indicators**	**MKfT domains related to computational estimation**
CCK	• calculate the reasonableness of a calculation • recognize when a student gives a wrong answer • recognize when a textbook gives an inaccurate answer to a calculation or gives an inaccurate definition and uses terms and notations incorrectly	• estimate the reasonableness of their calculation • understand the mathematics they teach – estimation and place value
HCK	• make connections across mathematics topics within a grade and across grades • articulates how the mathematics taught relates to the mathematics topics before and after the specific grade	• make connections across mathematics topics within a grade and across grades
SCK	• interpret students' emerging and incomplete ideas • evaluate the plausibility of students' claims or evaluate mathematical explanations • use appropriate mathematical notation and language and critique its use • interpret learners' mathematical calculation strategies, and those of other teachers or learning materials • evaluate mathematical explanations for common rules and procedures	• use mathematical notation and language and critique its use

	• evaluate estimation calculation explanations and procedures	
KCT	• sequence mathematical content • present mathematical ideas • select examples to take students deeper into mathematical content • select appropriate representations to illustrate the content • ask productive mathematical questions • recognize what is involved in using a particular representation • modify tasks to be either easier or harder • use appropriate teaching strategies • respond to students' 'why questions' • choose and develop useable definitions • provide suitable examples	• sequence mathematical content • choose and develop usable definitions • present mathematical ideas • select examples to take students deeper into mathematical content • select appropriate representations to illustrate the content • ask productive mathematical questions • use appropriate teaching strategies • provide suitable examples
KCS	• anticipate what students are likely to think and do • predict what students will find interesting and motivating when choosing an example • anticipate what a student will find difficult and easy when completing a task • anticipate students' emerging and incomplete ideas • recognize and articulate misconceptions students carry about particular mathematics content	• anticipate what students are likely to think and do • predict what students will find interesting and motivating when choosing an example • anticipate what a student will find difficult and easy when completing a task
KCC	• articulate the topics in the curriculum • articulate the competencies related to each topic in the mathematics curriculum • articulate and demonstrate a familiarity with the structure of the mathematics curriculum • link representations to underlying ideas and to other representations • knowledgeability of available materials (e.g., textbooks) and their purposes • appraise and adapt the mathematical content of textbooks	• articulate the topics in the curriculum • articulate and demonstrate a familiarity with the structure of the mathematics curriculum

With the KQ framework which is empirically grounded in classroom observations, some KQ indicators or codes were not observed, given the nature of the mathematics topic at hand (Breen et al., 2018). Contingency knowledge was not easy to observe as it is difficult to ascertain any changes teachers may make while teaching their lessons without seeing their lesson plans. Despite this, there were some aspects of contingency knowledge that I did observe and these related to how the teachers responded to the learners' questions and answers. The teachers responded to the learners' answers by indicating that they were correct or probing further if they were incomplete or incorrect.

6.5. LIMITATIONS OF THE RESEARCH

- I should have asked the two teachers who presented their lessons to provide me with a lesson plan. This would have given me information on how they adapted their lessons. Alternatively, I could have conducted a stimulated recall interview after the lesson. It would have been useful to share my transcriptions of the lesson with the teachers and to probe them on why they made certain decisions and choices in the lesson. This would have given me greater insight into their contingency knowledge.

- My questionnaire could have included questions that would enable teachers to draw on their SCK. For example, I could have developed a scenario where I provide the teachers with calculations and examples of how different learners estimated the reasonableness of their answers to the calculations. This would have enabled me to ascertain the teachers' response to the learners' computation estimation strategies, particularly identifying which would be more efficient and why.

- The research was a case study with only eight participants. While all eight responded to the questionnaire, I only observed two teachers teaching computational estimation and conducted a focus group interview with three teachers. This means that the findings are specific to the context and teachers I worked with and cannot be generalised across all senior primary mathematics teachers in Namibia and beyond.

Furthermore, the study established that the teachers face many challenges and constraints in developing the learners' computational estimation skills. It can be concluded that if the curriculum developers and policymakers do not offer the support, training and professional development to the teachers, their PCK will become weak, enabling them not to deliver the subject content needed by the learners.

6.6. RECOMMENDATIONS

Based on the study's findings, the following recommendations are proposed to strengthen the senior primary Mathematics teachers' SMK and PCK and to also for the expansion of this topic.

Recommendations to strengthen teachers' SMK and PCK towards the development of the learners' computational estimation skills:

- During my research, the teachers noted that there are several constraints that shaped the teaching of computational estimation. These included times available during class periods, time allocated to cover the syllabus topics, and the limited examples of different computational estimation exercises in the textbooks. I recommend that the MoEAC and NIED should consider curriculum reform to ensure that more computational estimation strategies are included into the syllabus as the single focus on rounding off limits the development of number sense, that is, the ability to use computational estimation strategies flexibly and in efficient ways.

- The participants also acknowledged that they required training on computational estimation through workshops or any professional development training. While some had received training during their pre-service teacher education, they said that this was insufficient. As expected, the participants explained that they require training on how to develop their learners' computational estimation skills. I recommend that NIED together with MoEAC, should implement in-service trainings or workshops that will equip, and extend the knowledge base, that's the SMK and PCK of the senior primary Mathematics teachers' in the Ohangwena region. Furthermore, teacher training institutions in Namibia should ensure that they produce well-equipped pre-service teachers in terms of the mathematical knowledge required in developing the learners' computational estimation skills.

- Senior education officers and subject heads should provide the necessary support to the teachers, not only in developing the learners' computational estimation skills but in all the subject areas.

Recommendations for further studies:

- This study was only conducted in Ohangwena Region, a larger scale study should be conducted across all the regions to explore and understand the subject matter and pedagogical content knowledge the senior primary mathematics teachers require and draw on to develop their learners' computational estimation skills.

- Policy makers and curriculum developers should conduct a study to identify both pre-service and in-service teachers' needs in terms of SMK and PCK required in developing the learners' computational estimation skills in order to develop suitable professional development programmes.

- It may be interesting to revisit Morgan's (1999) computational estimation framework and ascertain its relevance in classrooms today.

6.7. A REFLECTION ON MY RESEARCH PROCESS

This study has made an important contribution to my growth as a senior primary mathematics teacher and as a researcher. The study has made a significant improvement in strengthening my teaching strategies and most specifically the development of the computational estimation skills of my learners. Having had an opportunity to identify in the literature a variety of different computational estimation strategies, this research will no doubt influence my teaching.

My interaction with fellow senior primary mathematics teachers provided me with the opportunity to unpack their mathematical and pedagogical content knowledge that they draw on when developing the learners' computational estimation skills. In so doing, I was able to reflect on my own practice.

This research has also developed my skills as a novice researcher and has ignited in me an interest to further my academic qualifications.

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APPENDICES

Appendix 1: Ethical clearance from Rhodes University

RHODES UNIVERSITY
Where leaders learn

Rhodes University, Education Faculty
Research Ethics Committee
PO Box 94, Makhanda, 6140, South Africa
Tel: +27 (0) 46 603 8393
Fax: +27 (0) 46 603 8028
email: e.rosenberg@ru.ac.za

https://www.ru.ac.za/researchgateway/ethics/

14 August 2023

emilia shigwedha

Education Department

g19s9479@campus.ru.ac.za

Dear Emilia Shigwedha

Re: The mathematics content and pedagogical knowledge that Namibian Primary School teachers draw on to develop their learners' computational estimation

APPLICATION NUMBER: 2021-5172-6266

This letter confirms that your research ethics application has been reviewed and **APPROVED** by the Education Faculty Research Ethics Committee (EF-REC). Your permission letter(s) where applicable have been received and you are free to proceed with your study.

Approval is granted for 1 year. An annual progress report is required in order to renew approval for an additional period. You will receive an email notifying you when the progress report is due.

Should any substantive change(s) be made during the research process, that may have ethical implications, you should notify the Education Faculty REC Chair via email. This includes changes in investigators. The REC Chair will advise as to whether a new application is necessary.

Do keep this clearance letter secure and accessible throughout your study and after its completion. It will be needed when a thesis is examined and when publications are submitted to journals.

Please also submit a brief report to the REC Chair on the completion of the research. This can be done via email. The purpose of this report is to indicate whether the research was conducted successfully and whether any ethics-related matters arose that the committee should be aware of, in order to guide future studies.

Sincerely,

ERosenberg

Prof Eureta Rosenberg

Chair: Education Faculty Research Ethics Committee

Page 1 of 1

Appendix 2: Letter to Executive Director, Ministry of Education, Arts and Culture

Education Department
Rhodes University
PO Box 94, Makhanda 6140

The Executive Director
Ministry of Education, Arts & Culture
Private Bag 13186, Windhoek

27th September 2021

Dear Ms. Steenkamp

RE: REQUEST TO CONDUCT RESEARCH IN THE OHANGWENA REGION

My name is Emilia Ndilimeke Shigwedha (19s9479). I am a MEd scholar at Rhodes University, South Africa. I am writing this letter to request permission to conduct my research in two schools in the Ohangwena Region. The schools are: [redacted] and [redacted].

My research seeks to understand the mathematical content and pedagogical knowledge that teachers require and use to develop their learners' computational estimation skills. The research will involve the distribution of questionnaires to the senior primary mathematics teachers at the two schools, a focus group interview with four teachers from the participating schools and observations of two teachers' lessons. Permission will be requested from the principals, teachers and parents to audio- or video-record the interviews and lessons. All recordings will be kept safe on my laptop, which is password protected, and in the cloud.

The focus group interview will take place after school and in a neutral space. The focus group interview will be audio-recorded. Should any of the teachers feel uncomfortable with this, I will stop the recording and make hand-written notes instead. All of the school protocols, and protocols related to Covid-19 will be adhered to, to ensure the safety of all. All data will be treated confidentiality and anonymity. Pseudonyms will be used to protect the identities of the schools and teachers. No learners will be referred to.

My research proposal has been reviewed by the Education Faculty's High Education Degree Committee to ensure that it meets ethical guidelines. If you have any enquiries about my study at any stage of the research process, please do not hesitate to contact me or my supervisor using the details provided below. Should you have any ethical concerns, please contact Mr Siyanda Manqele at s.manqele@ru.ac.za.

Yours sincerely,

Emilia Ndilimeke Shigwedha (MEd Scholar, Rhodes University)
Supervisors: Dr Lise Westaway (l.westaway@ru.ac.za)
Dr Samukeliso Chikiwa (s.chikiwa@ru.ac.za)

Appendix 3: Permission from the Executive Director through the Directorate of Education, Ohangwena

REPUBLIC OF NAMIBIA

OHANGWENA REGIONAL COUNCIL

DIRECTORATE OF EDUCATION, ARTS AND CULTURE

Office of the Director
Tel. (+264) 65 290200
Fax: (+264) 65 290224
Enquiries: Mirjam Nambahu
Email:
Our Ref: 26/1/9.8

28 September 2021

Emilia N Shigwedha
PO Box 94
Makhanda

SUBJECT: REQUEST FOR PERMISSION TO CONDUCT RESEARCH IN OHANGWENA REGION

1. Receipt of your letter dated 27 September 2021 is hereby acknowledged.
2. The request has been evaluated and found to have merit.
3. Kindly be informed that permission to conduct research at [redacted] and [redacted] has been granted under the following conditions and requests:
   - The data to be collected only be used for the completion of your studies.
   - You have to liaise with the Principals concerned to make prior arrangements before the date of the research.
   - No other data should be collected other than the data stated in the request.
   - You may share the final report of your study with the directorate.
4. It is trusted that you will find this arrangement in order. In addition, we wish you all the best with your studies.

Yours Sincerely,

Ohangwena Regional Council
Directorate of Education Arts and Culture
29 SEP 2021

Isak Hamatwi 29/09/2021
REGIONAL DIRECTOR

Appendix 4: Letter to the school principals

Education Department
Rhodes University
PO Box 94
Makhanda
6140
30 September 2021

The Principal
…….. School
Ohangwena region

Dear _____

RE: REQUEST TO CONDUCT RESEARCH IN YOUR SCHOOL

My name is Emilia Ndilimeke Shigwedha (19s9479). I am a MEd scholar at Rhodes University, South Africa. I am writing this letter to request permission to conduct my research in your school.

My research seeks to understand the mathematical content and pedagogical content knowledge that teachers require and use to develop their learners' computational estimation skills. The research will involve the distribution of questionnaires to the senior primary mathematics teachers in your school, a focus group interview with four teachers and observations of two teachers' lessons.

Permission will be requested from the teachers to audio-record the focus group interview, and from the teachers and parents to video-record the lesson observations. While the focus of the research is the teacher, the learners will be in the classroom while I conduct my observations. The video will be focused on the teacher, but it is likely that images of the learners will be captured on the video.

In compliance with the ethical standards, the audio and video data will be treated with the utmost confidentiality. All recordings will be kept safe on my laptop, which is password protected, and in the cloud. The focus group interview will take place after school hours outside the school premises.

All of the school protocols, and protocols related to Covid-19 will be adhered to, to ensure the safety of all. All data will be treated confidentiality and anonymity. Pseudonyms will be used to protect the identity of the school and teachers. No learners will be referred to.

Attached please find a copy of the permission letter from the office of the Permanent Secretary of the Ministry of Education through the office of the Director of Education, Ohangwena Region.

My research proposal has been reviewed by the Education Faculty's High Education Degree Committee to ensure that it meets ethical guidelines. If you have any enquiries about my study at any stage of the research process, please do not hesitate to contact me or my supervisor using the details provided below. Should you have any ethical concerns, please contact Mr Siyanda Manqele at s.manqele@ru.ac.za.

I hope this request will receive your favourable consideration. I look forward to hearing from you.

Yours sincerely,

Emilia Ndilimeke Shigwedha
MEd Scholar, Rhodes University

Supervisors: Dr Lise Westaway (l.westaway@ru.ac.za)
Dr Samukeliso Chikiwa (s.chikiwa@ru.ac.za)

I, the undersigned _____, consent to the above research being conducted by Emilia Ndilimeke Shigwedha from Rhodes University in my school.

Signature……

Date…….....

Appendix 5: Information sheet and letter to the teachers

Education Department
Rhodes University
PO Box 94
Makhanda
6140
26 July 2021

Dear _____

RE: REQUEST FOR YOUR PARTICIPATION IN MY MED RESEARCH

My name is Emilia Ndilimeke Shigwedha (19s9479). I am a MEd scholar at Rhodes University, South Africa. I am writing this letter to request your participation in my research. The title of my research is: *The mathematics knowledge for teaching that teachers use in developing their learners' computational estimation skills.*

The research will involve the distribution of questionnaires, a focus group interview with four teachers and observations of two teachers' lessons. I would like to invite you to participate in my research as a senior primary mathematics teacher. Your teaching experience and knowledge will provide insights to the research questions. If you are willing to participate, I will ask you to complete a questionnaire, participate in a focus group interview and observe you teaching mathematics twice. With your permission, the focus group interview will be audio-recorded and the lesson observation, video-recorded.

In compliance with the ethical standards, the audio and video data will be treated with the utmost confidentiality. All recordings will be kept safe on my laptop, which is password protected, and in the cloud. The focus group interview will take place after school hours outside the school premises at a time that is convenient for you.

All of the school protocols, and protocols related to Covid-19 will be adhered to, to ensure the safety of all. Your name will not appear in the thesis. You will be given a pseudonym in order to protect your identity.

I hope this request will receive your favourable consideration. I look forward to hearing from you.

My research proposal has been reviewed by the Education Faculty's High Education Degree Committee to ensure that it meets ethical guidelines. If you have any enquiries about my study at any stage of the research process, please do not hesitate to contact me or my supervisor using the details provided below. Should you have any ethical concerns, please contact Mr Siyanda Manqele at s.manqele@ru.ac.za.

Yours sincerely,

Emilia Ndilimeke Shigwedha
MEd Scholar, Rhodes University

Supervisors: Dr Lise Westaway (l.westaway@ru.ac.za)
Dr Samukeliso Chikiwa (s.chikiwa@ru.ac.za)

I, _____ (name and surname) hereby acknowledge that I consent to participating in the research study of Ms. Emilia Ndilimeke Shigwedha on '*The mathematics knowledge for teaching that teachers use in developing their learners' computational estimation skills'.* In consenting to participating in this research, I agree to:

	YES	NO
Completing a questionnaire		
Participating in a focus group interview that will be audio-recorded		
The researcher video-recording me teaching		

I understand that:

- the results of the research will be published as a thesis and may be presented at conferences or be submitted as journal articles;
- my participation is entirely voluntary and that I may withdraw from the research at any time;
- my identity will remain anonymous (where possible) and all data will be treated

confidentially; and

- I will not receive any remuneration for participating in the research.

The study has been explained to me and I have read and understood the consent letter and form. I am satisfied that all my questions have been answered.

I, the undersigned_____, consent to take part in the above research conducted by Emilia N. Shigwedha from Rhodes University.

Signature_____

Date_____

Appendix 6: Letter to the parents

Education Department
Rhodes University
PO Box 94
Makhanda
6140
26 July 2021

Dear parents/guardians

My name is Emilia Ndilimeke Shigwedha (19s9479). I am studying at a university in South Africa, called Rhodes University. I am writing this letter to request your consent for your child to be part of my research. The title of my research is: *The mathematics knowledge for teaching that teachers use in developing their learners' computational estimation skills.*

The focus of the research is the teachers. However, I have to research the teachers in context, which means the senior primary teachers of mathematics will be in classrooms and interacting with the learners. I wish to observe the teachers on the maximum of two mathematics lessons. These lessons will be video recorded. While the video camera will be focused on the teacher, it is likely that images of the learners working will be captured on the video. However my interest is not on your child, but their teacher.

In compliance with the ethical standards, the video data will be treated with the utmost confidentiality. All recordings will be kept safe on my laptop, which is password protected, and in the cloud. Your child will not appear in the thesis. To protect the identity of the learners, no learner will be referred to.

All of the school protocols, and protocols related to Covid-19 will be adhered to, to ensure the safety of all. Although the principal and the teachers agreed to my research, I do require permission from all the parents of the learners in the classes that I am going to observe. **Please return a signed copy of the consent form attached**

My research proposal has been reviewed by the Education Faculty's High Education Degree Committee to ensure that it meets ethical guidelines. If you have any enquiries about my study at any stage of the research process, please do not hesitate to contact me or my supervisor using the details provided below. Should you have any ethical concerns, please contact Mr Siyanda Manqele at s.manqele@ru.ac.za.

I hope this request will receive your favourable consideration. I look forward to hearing from you.

Yours sincerely,

Emilia Ndilimeke Shigwedha
MEd Scholar, Rhodes University

Supervisors: Dr Lise Westaway (l.westaway@ru.ac.za)
Dr Samukeliso Chikiwa (s.chikiwa@ru.ac.za)

CONSENT FORM

I, the undersigned _____ (parent name and surname) give permission for _____ (child name) to participate in a research project conducted by Emilia Ndilimeke Shigwedha from Rhodes University.

I understand that:

1. My child is under no obligation to participate, and may withdraw from the study at any point without any penalty.
2. Anonymity will be maintained through the use of pseudonyms. The name of my child will not be reported.
3. The research will be used for academic and professional presentations and publications.
4. Participation involves video recording during the lessons by the researcher from Rhodes University.
5. There will be no remuneration involved in the study.

6. I have read and understand the explanations provided to me. I have had all my questions answered to my satisfaction, and I voluntarily agree that my child participate in this study.

NB: Signing your name below means you agree your child to be in this study. You will get a copy of this form.

NAME OF PARENT/GUARDIAN

_____	_____	_____
Print name	Signature	Date

Appendix 7: Leaners assent form

LEARNERS' ASSENT FORM

My name is *Emilia N. Shigwedha.* I am studying at a university in South Africa, called Rhodes University. One of my tasks is to do some research. I have chosen to do my research on mathematics and to do it in your classroom. I'm going to be observing your teacher, and how she/he teaches you mathematics. I will be using a video-recorder but my focus will solely be your teacher.

There will be no payment for partaking in this study. The video data will be treated with utmost confidentiality and no learner will be referred to. I kindly request you to grant me permission to observe you and your teacher by signing below.

NB: Signing your name below means you agree to you being in class, observing your teacher and video-recording your teacher.

NAME OF LEARNER

_____	_____	_____
Print name	Signature of learner	Date

Appendix 8: Questionnaire

QUESTIONNAIRE

SECTION A

Name of the teacher _____

School name _____

How many years have you been teaching?

How many years have you been teaching Mathematics?

How many years have you been teaching Senior Primary Mathematics?

What is your highest qualification?

Where did you get your highest qualification?

Please tick the age category that applies to you

18 & 19	20-29	30-39	40-49	50-60

SECTION B: Kindly answer the following question by filling in the spaces provided.

1. What is your understanding of computational estimation?

2. Do you think it is important to develop learners' computational estimation?

YES	NO

Why?

3. Do you use computational estimation in your own life?

YES	NO

If yes, provide some examples of when you use computational estimation.

4. What training have you received on strategies for developing learners' computational estimation?

The following questions are only if you encourage computational estimation in your mathematics classroom.

5. In what areas/topics of mathematics do you develop learners' computational estimation?

6. How do you develop learners' computational estimation?

7. How do you encourage your learners to use computational estimation to check their answers?

8. What computational estimation strategies do you focus on in your classroom?

9. What estimation strategy would you use for each of these sums?
   E.g. 55+29; 55+30; rounding off

   a. 40.2 + 29.7

   b. 35+15

   c. 27 + 48 + 56 + 72

d. 34 + 45

e. 699 + 710 + 695 + 705

f. 20.4 + 15.8

10. What are some of the things that may prevent you from developing your learners' computational estimation?

Appendix 9: Focus group interview items