AN INVESTIGATION INTO THE TYPES OF CLASSROOM TASKS SENIOR SECONDARY SCHOOL (GRADE 11 AND 12) MATHEMATICS TEACHERS GIVE TO THEIR LEARNERS: A CASE STUDY.

A thesis submitted in partial fulfilment of the requirements for the degree of

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DECLARATION OF ORIGINALITY

I, SHAPANGE ISMAEL (Student number: 611S7092) declare that this thesis "*An investigation into the types of classroom tasks senior secondary school (Grade 11 and 12) mathematics teachers give to their learners: A Case study*" is my own work written in my own words. Where I have drawn on the words or ideas of others, these have been acknowledged using the reference practices according to the Rhodes University Education Department Guide to Referencing.

______________ 25 November 2014

(Signature) (Date)

ABSTRACT

This interpretive case study was undertaken to investigate the nature of classroom tasks that teachers give to their learners and to gain insights into how they choose these tasks. It was carried out at two secondary schools in the Okahao circuit of the Omusati region in Namibia. The schools range from Grade 8 to Grade 12. This study focused mainly on teachers who teach Mathematics at senior secondary phase (Grade 11 and 12). The research participants were of four mathematics teachers – two from each school. The study was designed around two phases. Phase one consisted of video-recording of lessons, and phase two consisted of interviewing the teachers. The main purpose of the study was to ascertain the types and nature of tasks that teachers give to their learners and to gain insights into how they chose these tasks. The research adopted a combination of both qualitative and quantitative approaches.

The study revealed that the participating teachers provided a near equal amount of lower level cognitive tasks and higher level cognitive tasks. It further revealed several factors that influence teachers when selecting the types of tasks they give to their learners. These factors include curriculum requirements, types of learners and their experiences, and learners' contexts.

DEDICATION

I dedicate this thesis to my son Ekonia and to my parents and siblings for the love and support that they have shown me throughout my study. I also dedicate this to my school principal for his understanding of all the challenges I encountered during my study. He inspired and encouraged me to persevere.

ACKNOWLEDGEMENT

First and foremost I would like to thank GOD for being there for me during all the challenges and difficulties I faced throughout this study. I thank GOD for giving me strength, courage and time to carry out my study.

Secondly, my thanks go to many people who supported me to complete this research project. To begin with, I thank my supervisor Prof Schafer for his advice and guidance in identifying a topic that was relevant and researchable. He provided very helpful guidelines on how to go about it. I'm grateful for all the generous support that he provided throughout the entire process of writing up this document.

I am also very grateful to my research participants who allowed me to observe their lessons and to familiarise myself with the different classroom tasks they set. I am thankful to them for availing me their precious time to attend the interview sessions. Their generous responses enriched my understanding, opening up my mind to what happens in a classroom.

I also register my gratitude to the principals of the schools where this study was conducted. Thank you for allowing me the opportunity to be in your schools.

Further, I would like to extend my gratitude to the inspector of education of the Okahao circuit for giving me a green light to conduct this research in her circuit.

All in all I am very thankful to everyone for their inspiration and encouragement.

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CHAPTER ONE
INTRODUCTION

1.1 INTRODUCTION

This research project was undertaken to investigate the nature of classroom tasks that teachers give to their learners and to gain insights into how they choose these tasks. This chapter provides a short background and rationale for the study. It articulates the research questions and highlights the scope of the study, where the study was conducted as well as the research methodology used. It ends with an overview of the study.

1.2 CONTEXT/BACKGROUND

Kilpatrick, Swafford and Findell (2001, p. 335) emphasised that the quality of mathematical tasks that teachers select for their teaching is central to students' learning because tasks convey messages about what Mathematics is and what doing Mathematics entails. Moreover, they observed that, generally, the tasks assigned to students in many classrooms make only minimal demands on their thinking, because the tasks those students are often given primarily demand only memorization or the use of procedures without making connections to concepts. As a mathematics teacher I experience first-hand that many teachers, I included, mostly pick questions and tasks uncritically from textbooks or old question papers. This mostly leads to the perpetuation of a routine of selecting tasks of mediocre quality and low cognitive demand.

Mathematics education researchers such as Stein, Grover and Henningsen (1996, p. 456) indicated that one of the greatest challenges encountered by mathematics teachers is the identification and use of appropriate tasks that challenge their pupils. It is against this background that I undertook this study to investigate the type of tasks that teachers select and on what basis they select these tasks to give to their learners. Lester (2007, p. 346), defines mathematical tasks as “classroom activities the purpose of which is to focus students’ attention on particular mathematical ideas”. In my experience and in my work environment I find that curricular materials and textbooks are the main source of mathematical tasks used by teachers for classroom instruction. Curricular materials vary considerably regarding the nature and quality of mathematical tasks that are found within them. It is thus important that teachers select tasks critically, strategically and selectively. Lester (2007) also indicated that making a distinction between tasks is important because not all tasks provide the same opportunities for students' thinking and learning. She further claims that giving tasks that only require students to perform memorized procedures in a routine manner leads to only limited types of opportunities for students' thinking. Tasks that demand engagement with concepts and that stimulate students to make connections lead to richer set of opportunities for student thinking. These opportunities are characterised by rich contexts and learning processes.

1.3 THE PURPOSE OF THE STUDY

The purpose of this study was to critically analyse task selection and to foreground the importance of selecting appropriate tasks. This study aligns with Stein's, et al. (1996, p. 526) notion that "the nature of task can influence and structure the way students think - hence the nature of task instructions surrounding the mathematical task as given by the teacher equally influence the way students organise their thinking". A further purpose was to investigate the types of tasks that are used by selected Grade 11 and 12 mathematics teachers in their teaching in the Omusati region of Namibia and to ascertain on what basis they selected these tasks.

1.4 RESEARCH QUESTIONS

The research questions that framed this study were:

1. What types of tasks do selected Grade 11 and 12 teachers select for their students in their mathematics lesson according to Stein's framework?
2. On what basis do the participating mathematics teachers select their classroom tasks they give to their learners?

1.5 METHODOLOGY

This study was orientated in the interpretative paradigm, where the researcher not only explored the types and nature of tasks that teachers gave to their learners, but also hoped to gain insight into their subjective reasoning as to the reasons why they selected these tasks. O'Leary (2004, p. 10) defines interpretive research as explorations of cultural and historical significance in the social world. The study used a mixed methods (qualitative and quantitative) approach. A quantitative methodology was used to analyse identified tasks during lesson observations and documents such as tests. The tasks were classified according to Stein's, et al. (2000) framework of cognitive demand. A qualitative approach was used to analyse interviews with the participating teachers.

This study consisted of two phases:
Phase 1 consisted of video-recordings of two Grade 11 and two Grade 12 classes' lessons. These lessons were analysed using descriptive statistics. I used Stein's, et al. (2000) conceptual framework where my unit of analysis was the type of mathematical tasks given by each of the four participating teachers. Tasks were counted and grouped according to their cognitive demands as defined in Stein's framework. In parallel I also scrutinised all the tests that the participating teachers set in the course of the term to ascertain the Stein cognitive levels of the individual items of the tests.

Phase 2 of the study consisted of interviews. The data consisted of interview transcripts of one interview per participating teacher. In the interviews I probed for the reasons as to why the teachers selected the tasks they chose. I used the data from Phase 1 to provoke discussion on the rationale for selecting each of the tasks evident in the videos. My unit of analysis was the teachers' responses to my probing about the basis of their selection of the tasks, their lessons and the tests and examinations they set.

Overall, this research project was designed as a case study - a single instance of a bounded system (Creswell, 1994, p.15).

1.6 ANALYSIS

As mentioned above, Stein's, et al. (2000) framework of cognitive demands was used to analyse the types and nature of tasks used by the teachers. The interviews were analysed according to the interview schedule that I used.

1.7 SIGNIFICANCE OF THE STUDY

It is hoped that this study, despite the small sample that was used, would contribute some insights into the dilemma that faces some teachers when selecting appropriate tasks for their students. It is well documented by Lester (2007, p. 347) that teachers often select tasks that are not at the appropriate level of their students and are of inadequate quality. This study, hopefully, contributes to enhancing awareness of the importance of selecting tasks with the appropriate levels of cognitive demands. This study aligns with Stein's et al. (1996, p. 526) notion that "the nature of task can influence and structure the way students think hence the nature of task instructions surrounding the mathematical task as given by the teacher equally influence the way students organise their thinking". The notion is equally supported by Kilpatrick, et al. (2001, p. 335), who emphasised that the quality of mathematical tasks that teachers select for their teaching are central to students' learning because tasks convey messages about what mathematics is and what doing mathematics entails.

1.8 OVERVIEW OF THE THESIS

This thesis is divided into five chapters.

Chapter One provides a brief introduction to the thesis.

Chapter Two reviews literature relevant to the study. The chapter discusses the Stein framework of cognitive demands that forms the basis of the analysis in Chapter four. It also emphasise the importance of teachers selecting appropriate tasks for their learners.

Chapter Three describes the research design and methodology. This chapter gives a detailed description of the research process, including the research context, the research tools used to collect data, how the tools were used and why they were used.

Chapter Four analyses and discusses the data.

Chapter Five synthesizes the findings, provides a summary of the findings, puts forward some recommendations, and discusses the limitations and challenges encountered in the research. It ends with some reflections and suggestions for further research.

CHAPTER TWO

LITERATURE REVIEW

2.1 INTRODUCTION

This chapter reviews the literature relevant to my study, which aims to investigate the type of mathematical tasks and their selection criteria which senior secondary school mathematics teacher employ when setting mathematical tasks for learners in their classrooms, specifically in Algebra and Mensuration.

Kilpatrick, et al. (2001, p. 335) emphasised that the quality of mathematical tasks that teachers select for their teaching is central to student learning because tasks convey messages about what mathematics is and what doing mathematics entails. Moreover, they observe that, generally, the tasks assigned to students in many classrooms make only minimal demands on their thinking, because the tasks those students are often given primarily demand only memorization or the use of procedures without making connections to concepts. As a mathematics teacher I experience first-hand that many teachers, I included, mostly pick questions and tasks uncritically from textbooks or old question papers. This mostly leads to the perpetuation of a routine of selecting tasks of mediocre quality and low cognitive demand.

Mathematics education researchers Stein, et al. (1996, p. 456) indicate that one of the greatest challenges encountered by mathematics teachers is the identification and use of appropriate tasks that challenge their pupils accordingly. It is against this background that I would like to investigate the type of tasks that teachers select and on what basis they select these tasks to give to their learners. Lester (2007, p. 346), defines mathematical tasks as “classroom activities the purpose of which is to focus students’ attention on particular mathematical ideas”.

In my experience and in my work environment I find that curricular materials and textbooks are the main source of mathematical tasks used by teachers for classroom instruction. Curricular materials vary considerably with respect to the nature and quality of mathematical tasks that are found within them. Senk and Thompson (2003), as cited by Lester (2007, p. 347) suggested that typically, not all tasks provide the same opportunities for student thinking and learning. They argued that in “conventional-based curricula” where content is prescribed, teachers mostly teach students the skills and procedures that are inherent in the goals of such a curriculum. In “standard-based curricula” however, teachers are encouraged to embody an approach to teaching and learning that focuses on the students active construction of important ideas and concepts. In standard-based curricula, tasks typically are more contextually based and can be solved by using a range of solution strategies. Lester (2007, p. 347) claimed that giving tasks that only require students to perform memorized procedures in a routine manner leads to only limited types of opportunities for student thinking. Tasks that demand engagement with concepts stimulate student thinking and allow them to make connections that lead to richer opportunities for mathematical understanding. These opportunities are characterised by rich contexts and learning processes.

2.2 DEFINITION OF MATHEMATICAL TASKS

As mentioned above mathematical tasks are defined as ‘classroom activities the purpose of which is to focus students’ attention on a particular mathematical idea’ Lester (2007, p. 347). Doyle 1983 (as cited in Stein, et al., 1996, p. 459) defined mathematical tasks as ‘the products that students are expected to produce, the operations that students are expected to use to generate those products and the resources available to students while they are generating these products’. In addition to that definition Stein (1996, p. 456) posited that tasks include expectations regarding what students are expected to produce, how they are expected to produce them, and the resources available. To summarise, mathematical tasks entail all activities that learners interact with in order to learn mathematics as well as to solve problems that require mathematical understanding.

2.3 NATURE OF MATHEMATICAL TASKS

Some tasks lead learners to a routine way of thinking while others stimulate students to make wider connections with mathematical ideas. To review the nature of mathematical tasks more deeply, I will focus on Clark and Roche's Task Type and Mathematics Learning framework (TTML) (Clark and Roche 2012, p. 156), Stein's level of cognitive demands framework (Stein, et al., 1996, p. 526) and Kilpatrick's Mathematical proficiency framework (Kilpatrick, et al., 2001, p. 5).

Clark and Roche (2012, p. 154) carried out a project on Task Type and Mathematics Learning (TTML) in Australia in 2012. The project investigated appropriate ways for teachers to use different types of mathematical tasks, particularly in Grades 5 to 8. The project focused on task selections and suggested three types of mathematical tasks that lead to students' mathematical conceptual understanding and development.

These types are:

Type 1: The teacher uses a model, example, or explanation that elaborates or exemplifies the mathematical content at hand.

Type 2: The teacher situates mathematics within a contextualised practical problem to engage the students, but the motive is explicitly mathematical.

Type 3: The teacher poses open-ended tasks that allow students to investigate specific mathematical content (Clark and Roche, 2012, p. 154).

According to Clark and Roche (2012, p. 154) tasks that are grouped as *type 1,* are associated with good traditional mathematics teaching, where mathematical purpose is clearly stated and the representations are linked directly and explicitly. An example is a teacher who uses a fraction wall to provide a linear model of fractions, and poses tasks that require students to compare fractions, to determine equivalences, and to solve fractional equations. To relate this example to

Stein's framework, see details below. The tasks involve procedure without a connection, where students are expected to follow the procedure that leads to the correct answer without necessarily making a connection to the meaning that underlies the procedure being used.

In tasks that are grouped as *type 2*, the context serves the twin purposes of showing how mathematics is used to make sense of the world and to motivate students to solve the task. Clark and Roche (2012, p. 154) gave a very good example where the teacher starts with a stimulating question within the learners' own environment and then generates a mathematics task around it, as follows:

> How many people can stand in your classroom? In this case, the task is "Imagine we have the opportunity to put on a concert in this classroom with a local band to raise funds for more school computers. How many tickets should we sell?"

Here, the context provides a motivation for what follows and dictates the mathematical decisions that the students make. This can be referred to in Stein's framework as a procedure with connection task. These tasks are usually represented in multiple ways (e.g. visual diagram, symbols, manipulative and problem solving) to focus students' attention on the use of procedures for developing a deeper level of understanding of concepts.

Clark and Roche (2012, p. 154) defined type 3 tasks as open-ended tasks that have multiple possible answers and prompt insights into specific mathematics through students seeing and discussing the range of possible answers. They provided a very good example that I would like to use as illustration. It reads: "A group of 7 people went fishing. The mean number of fish caught was 7, the median was 6 and the mode was 5. How many fish might each of the people have caught?" Clark and Roche (2012, p. 155). This can be categorized as higher-level demand task in Stein's framework. It involves doing mathematics, where learners are required to explore and understand the nature of mathematical concepts, processes or relationships. In such cases each learner could have a different approach to the question, depending on how he/she understands it; hence, multiple answers are expected based on individual interpretation. In addition learners are expected to employ different procedures that help them to reach the final answer.

In my view this framework shares similar sentiments as Stein, et al., (1996)'s framework of levels of cognitive demand tasks. Stein, et al., (1996, p. 526) suggested that "the nature of tasks can influence and structure the way students think, hence the nature of task instructions surrounding the mathematical task as given by the teacher equally influences the way students organise their thinking". The Table 2.1 below shows Stein, et al. (1996)'s framework of levels of cognitive demand tasks.

Table 2.1 Stein and Henningsen's level of demand tasks

Lower-level Demands	**Higher-level Demands**
Memorisation Tasks • Involve reproduction of previously learned facts, rules, formulae or definitions. • Tasks involve exact reproduction of previously seen material and what is to be produced is clearly and directly stated.	**Procedures with connections Tasks** • Focus students' attention on the use of procedures for developing a deeper level of understanding of concepts. • Tasks are usually represented in multiple ways (e.g. visual diagram, symbols, manipulative and problem solving).
Procedures without connections Tasks • Use of procedure is either called for or its use is evident based on placement of the task. • Require limited cognitive demand for successful completion. There is little ambiguity about what to be done and how to do it. • Have no connection to the concepts or meaning that underlie the procedure being used. • Focused on producing the correct answers rather than developing mathematical understanding.	**Doing mathematics Tasks** • Require complex non-algorithmic thinking (i.e. Task instruction or worked out examples). • Require students to explore and understand the nature of mathematical concepts, processes or relationships. • Require student to analyse the task and actively examine the task constraints that may limit possible strategies and solutions. • Focus on justification of mathematical ideas to develop mathematical understanding.

Source: Rasheed Sanni (2012) adapted from Stein et al (2000).

Explanation of Table 2.1

Mathematical tasks can be divided into two main categories in terms of their cognitive demands. The first category consists of lower-level demand tasks and the second category of higher-level demand tasks. Stein, Smith, Henningsen and Silver (2000, p. 1) examined mathematical instructional tasks in terms of their cognitive demands. With cognitive demands they examined the kind and level of thinking required of students in order to successfully engage with and solve a given task.

Lower-demand tasks

Stein, et al., (2000, p. 1) observed that these lower-level tasks are classified as *memorization* and procedures *without connections,* respectively. They further emphasised that each of these ways seek a different level of cognitive demand of students.

- **Memorisation**

Figure 2.1 below provides an example of a memorisation task as outlined by Stein, et al., (2000, p. 1). They use an example of fractional quantities to illustrate four ways in which students can be asked to think about the relationships among different representations. As shown in Figure 2.1 below, tasks with lower level demands would consist of memorizing the equivalent forms of specific fractional quantities (e.g., 1/2= 0.5 = 50%)

Memorisation

What are the decimal and percent equivalents for the fractions $\frac{1}{2}$ and $\frac{1}{4}$?

Expected Student Response

$$\frac{1}{2} = 0.5 = 50\%$$

$$\frac{1}{4} = 0.25 = 25\%$$

Figure 2.1 Stein and Henningsen's memorisation tasks

- **Procedures without connections**

Figure 2.2 below shows a procedure without connections task as outlined by Stein, et al., (2000, p. 1). It involves the conversions of fractions to percentages or decimals using standard conversion algorithms in the absence of additional context or meaning, (e.g. convert the fraction 3/8 to a decimal by dividing the numerator by the denominator to get 0.375; change 0.375 to a percent by moving the decimal point two places to the right to get 37.5%).

Procedures Without Connections

Convert the fraction $\frac{3}{8}$ to a decimal and a percent.

Expected Student Response

Fraction	Decimal	Percent
$\frac{3}{8}$	$\begin{array}{r} 0.375 \\ 8\overline{)3.00} \\ 24 \\ \hline 60 \\ 56 \\ \hline 40 \\ 40 \\ \hline \end{array}$	0.375 = 37.5%

Fig. 2.2 Stein and Henningsen's procedure without connections tasks

Higher-level demand tasks

Stein, et al., (2000, p. 2) suggested higher-level tasks can also be classified into two groups, namely Procedure with connections tasks and Doing mathematics tasks.

- **Procedures with Connections**

For example, as shown in Figure 2.3 below, students might be asked to use a 10 x 10 grid to illustrate how the fraction represents the same quantity as the decimal 0.6 or 60%. Students would also be asked to record their results on a chart containing the decimal, fraction, percent, and pictorial representations, thereby allowing them to make connections among the various representations and to attach meaning to their work by referring to the pictorial representation of the quantity every step of the way.

Procedure With Connection

Using a 10×10 grid, identify the decimal and percent equivalents of $\frac{3}{5}$.

Expected Student Response

Pictorial	Fraction	Decimal	Percent
	$\frac{60}{100} = \frac{3}{5}$	$\frac{60}{100} = 0.6$	$0.60 = 60\%$

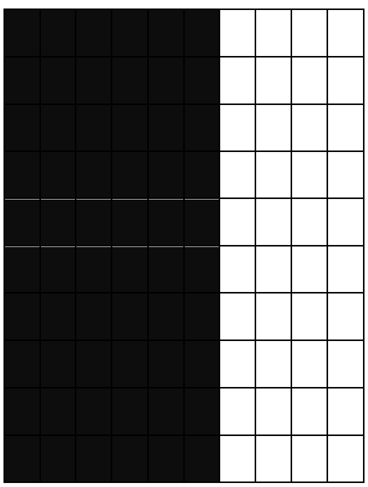


Fig. 2.3 Stein and Henningsen's procedure with connections tasks

This task is classified as procedures *with connections to understanding, meaning, or concepts* (hereafter referred to simply as *procedures with connections).*

- **Doing Mathematics**

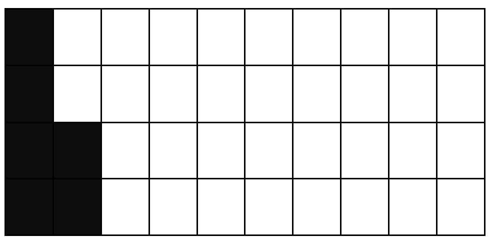
Another high-level task (classified as *doing mathematics)* would entail asking students to explore the relationships among the various ways of representing fractional quantities. Students once again use grids, but this time grids of varying sizes (not just 10 x 10) would be used. As shown in

Figure 2.4 below, students could be asked to shade six squares of a 4 x 10 rectangle and to represent the shaded area as a percent, a decimal, and a fraction.

Doing Mathematics

Shade 6 small squares in a 4×10 rectangle. Using the rectangle, explain how to determine each of the following: (a) the percent of area that is shaded, (b) the decimal part area that is shaded and (c) the fractional part of that area shaded.

One possible Student Response



(a) One column will be 10% since there are 10 columns. So four squares is 10%. Then 2 squares are half a column and half of 10% which is 5%. So the 6 shaded blocks equal 10% plus 5% or 15%.

(b) One column will be 0.10 since there are 10 columns. The second column has only 2 squares shaded so that would be one half of 0. 10 which is 0.05. So the 6 shaded blocks equal 0.1 plus 0.05 which equals 0.15.

(c) Six shaded squares out of 40 squares are $\frac{6}{40}$ which reduces to $\frac{3}{20}$.

Fig. 2.4 Stein and Henningsen's doing Mathematics tasks

Stein and Henningsen state that when students use the visual diagram to solve this problem, they are challenged to apply their understanding of the fraction on decimal and percent concepts in novel ways. For example, once a student has shaded the six squares, he or she must determine how six squares relate to the total number of squares in the rectangle. In Figure 2.4, one can see an example of a student's response to this task that illustrates the kind of mathematical reasoning used to come up with an answer that makes sense and that can be justified.

2.4 TEACHING FOR MATHEMATICAL PROFICIENCY

Kilpatrick, Swafford, and Findell (2001) adopted a useful framework of understanding proficiency in terms of mathematical proficiency and teaching proficiency.

2.4.1 Conceptual understanding

According to Kilpatrick, et al., (2001, p. 5) conceptual understanding refers to the comprehension of mathematical concepts, operations, and relations. They support the idea of McCormmick (1997, p. 142), who states that conceptual knowledge is concerned with relationships in such a way that students identify links among items of knowledge and tasks.

Star (2002), as cited by Maciejewski, Mgombelo and Savard (2011, p. 39) indicated that 'conceptual knowledge has to do with quality of learning which is associated with deep comprehension, abstraction and critical judgement'. It is important that students should be given tasks that focus on conceptual understanding of the concept, whereby an underlying meaning of the concept is considered. Maciejewski et al., (2011, p. 39) have outlined that it is more important to ask questions that allow students to develop the meaning underlying the concepts in order to assist them to model multiple ways of solving problems. They further argue that children with only procedural fluency lack the ability to deconstruct and reconstruct procedure in way that they can understand.

2.4.2 Procedural fluency

According to Kilpatrick, et al., (2001, p. 5) procedural fluency is a skill in carrying out mathematical procedures flexibly, accurately, efficiently, and appropriately. Schwartz (2004, online) defined procedural fluency as 'algorithms that enable you to find answers to problems according to set rules'. He says, for example, that actions such as divide, multiply, subtract and bring down are some of the learned procedures of long division, while cross multiplication is a learned procedure of solving mathematical proportions. He emphasises that mathematical procedures are much like recipes that have been developed to enable people to solve mathematical problems. Tasks that involve procedures help students to understand that one can develop an organised strategy to find a solution to a given mathematical problem. Kilpatrick, et al., (2001, p. 122) stated that some procedures help to strengthen and develop skills of learning some mathematical concepts with understanding. Students need to be given tasks that involve procedures to enable them to come up with sequences of actions when solving real life problems mathematically.

It is, however, asserted that 'procedural fluency is often associated with surface learning which consists of reproduction of knowledge, rote learning and lack of critical judgement', Star (2002), as cited by Maciejewski, et al., (2011, p. 39). To expand on this statement one can look at this question, given that 2300 is rounded to 2sf, find the upper and lower bound of 2300.

2.4.3 Strategic competence

Kilpatrick, et al. (2001, p. 124) defined strategic competence as the ability to formulate, represent and solve mathematical problems. To couple this with mathematical tasks, it is essential that students should be given tasks that allow them to reformulate the problem in a way that makes sense to them. Students need to develop multiple ways of tackling mathematical problems rather than focussing on a single routine way of solving them. Research shows that student can develop different ways of approaching mathematical problems if they address their everyday situations, where they can easily understand the key features of the problems presented to them. This is supported by Kilpatrick, et al., (2001, p. 125) who indicated that students can develop different strategies in solving problems that are contextualised.

2.4.4 Adaptive reasoning

Kilpatrick, et al. (2001, p. 129), defined adaptive reasoning as the capacity to think logically about relationships within concepts and situations. They further explain that adaptive reasoning is the tool that students use to navigate through facts, procedures, concepts and solutions to see if they all fit together in a way that makes sense to them. Tasks should give students opportunities to explore a variety of approaches to solve mathematical problems in ways that are more sensible to their thinking rather than being forced to follow a specific formalised method.

2.5 TASKS AS REPRESENTED IN THE CURRICULUM

A curriculum is essential to an education system of any state. Lester (2007, p. 321) defined curriculum as ‘the substance or content for teaching and learning’. It is an outline of what is expected of teachers to be taught and learners to be learnt. Senk and Thompson (2003), as cited by Lester (200, p. 347) indicated how curriculum plays a major role in the selection and implementation of classroom tasks. According to Lester (2007, p. 321), Senk and Thompson (2003) argue that in “conventional-based curricula” where content is prescribed, teachers mostly teach students the skills and procedures that are inherent in the goals of such a curriculum. In “standard-based curricula” however, teachers are encouraged to embody an approach to teaching and learning that focuses on the students’ active construction of important ideas and concepts. In standard-based curricula, tasks typically are more contextually based, and they can be solved by using a range of solution strategies. Lester further indicated that curriculum can be defined differently in different environments. Those who study curriculum frequently refer to it as the planned and unplanned components of what is taught or experienced in the classroom.

Lester (2007, p. 346) showed that curricular materials and textbooks are the main source of mathematical tasks used by teachers for classroom instruction. Grouws, et al. (2004), as cited in Lester (2007, p. 346) indicated that many teachers use textbooks as their main source of mathematical tasks in their classrooms. They further cautioned that teachers have to be very careful in their choice of tasks because curricular materials vary considerably with respect to the nature and quality of the mathematical tasks that are found within them. Some tasks are of low cognitive demand whereby students are asked to perform memorised procedures in a routine manner, whereas some are of high cognitive demand that allow students to engage critically with concepts.

Lester (2007, p. 321) indicated that research on teaching and curriculum revealed that there is a difference between the curriculum as represented in instructional materials and the curriculum as enacted in the classroom by teachers and students. Teachers bring their prior understanding, beliefs and goals to bear on the written curriculum and in process influence what they believe is workable in the classroom. A teacher may set and interpret the task in a way that he/she believe his/her learners can understand and are able to engage with the task. It is against this background that different learners at the same level who are taught by different teachers can find they engage with the same task differently. This is due to the different briefs and goals that each teacher has in mind about the task.

2.6 TEACHERS' ROLE IN SELECTING TASKS

The knowledge, beliefs, decisions, and actions of teachers affect what is taught and ultimately learned. The teachers' selection of tasks is thus a key factor in the teaching and learning process.

2.6.1 Teachers' subject content knowledge

Hammond and Ball (1997, p. 2) indicated that teacher expertise (what teachers know and can do) affects the selection of tasks for teaching. They further elaborated that what teachers understand, both about content and students, shapes how judiciously they select tasks from texts and other materials and how effectively they present materials in class. Their skill in assessing their students' progress depends also on how deeply they themselves know the content, and how well they can understand and interpret students' talk and written work.

2.6.2 Setting and implementing of mathematical tasks

Stein and Henningsen (1997) provide a framework on levels of cognitive demand tasks based on the type and level of thinking required to solve them. In addition to their taxonomy for classifying mathematical tasks, Stein and Henningson (1997) also provide a framework for tracking the cognitive demands of mathematical tasks as they are used during instruction in order to analyse the connections between instruction and student learning.

Stein and a colleague classify tasks into two broad categories: Category 1 consists of Lower-level demand tasks which are divided into memorisation and procedures-without-connection tasks. Category 2 tasks consist of Higher –level demand tasks which can be divided into procedures-with connection and doing mathematics tasks. In terms of classroom instruction the framework further suggests three types of mathematical tasks: Type 1 are tasks from curricular or instructional materials; Type 2 are tasks that are crafted, sourced and set up by the teacher; Type 3 are those tasks that are implemented by the students. See figure 2.5 below.

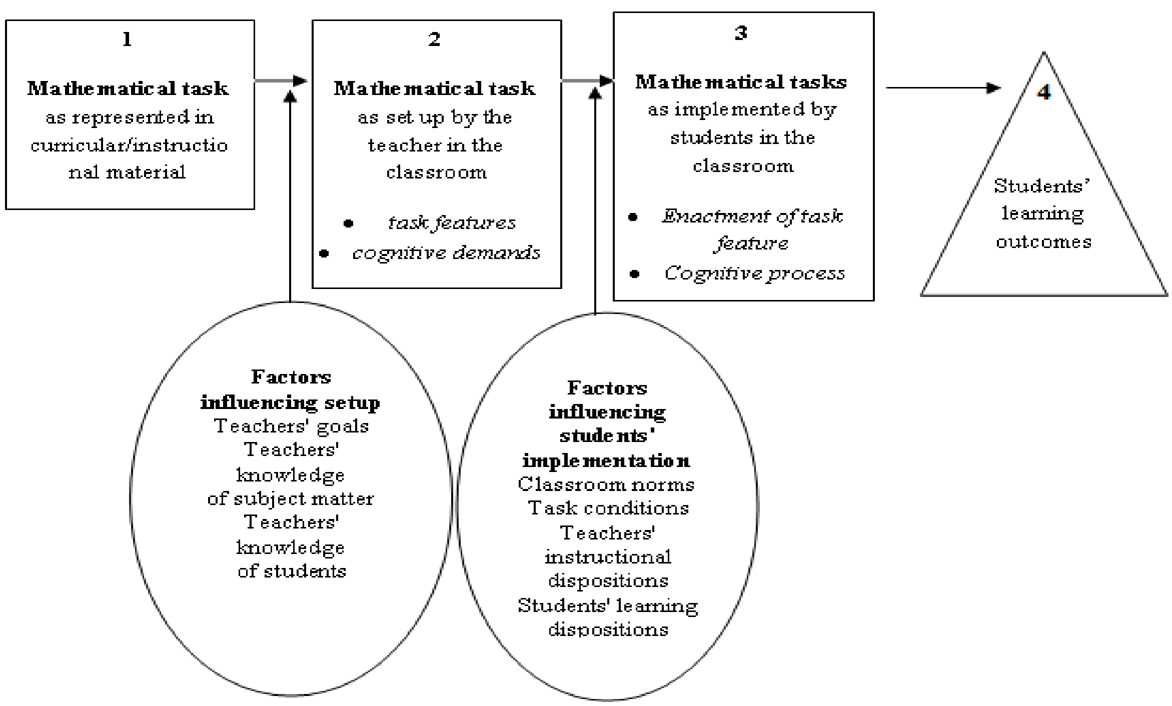


Figure 2.5 Relationships among various task-related variables and students' learning, Source: Stein and Hennigsen 1997

In this framework mathematical tasks pass through *three* phases (represented by the rectangular boxes in Figure 1): (a) as written by curriculum developers, (b) as set up by the teacher in the classroom, and (c) as implemented by students during the lesson. The framework further specifies two dimensions of mathematical tasks. The first dimension is *task features.* Task features refer to aspects of tasks that mathematics educators have identified as important considerations for the development of mathematical understanding, reasoning, and sense making. They further outlined that these features include multiple solution strategies, multiple representations, and mathematical communication. In addition they indicated that during the set-up phase, these features refer to the extent to which the task as announced by the teacher encourages students to use more than one strategy, to use multiple representations, and to supply explanations and justifications. During the implementation phase, these features refer to the extent to which students use the features.

The second dimension, *cognitive demands,* refers to the kind of thinking processes entailed in solving the task as announced by the teacher (during the setup phase) and the thinking processes in which students engage (during the implementation phase). These thinking processes can range from memorization to the use of procedures and algorithms (with or without attention to concepts, understanding, or meaning) to complex thinking and reasoning strategies that would be typical of "doing mathematics" (e.g., conjecturing, justifying, or interpreting). My study focuses on this second dimension of cognitive demands.

According to the framework that Stein and Henningson (1997, 529) outlined, the features and cognitive demands of tasks can be transformed between any two successive phases. For example, a task could be set up to require high-level cognitive activity by students, but during the implementation phase it could be transformed in such a way that students' thinking focuses only on procedures, with no conceptual connections.

The circle on the right in Figure 2.5 represents the classroom-based factors that influence the ways in which students' thinking unfolds during the task-implementation phase. These factors include classroom norms, task conditions, and teachers' and students' dispositions. According to Stein and Henningson (1997, 529) *classroom norms* refer to the established expectations regarding how academic work gets done, by whom, and with what degree of quality and accountability. *Task conditions* refers ‘to attributes of tasks as they relate to a particular set of students (e.g., the extent to which tasks build on students' prior knowledge and the appropriateness of the amount of time that is provided for students to complete tasks)’ Stein and

Henningson (1997, 529). *Teacher and student dispositions* refers 'to relatively enduring features of pedagogical and learning behaviours that tend to influence how teachers and students approach classroom event' Stein and Henningson (1997, 529). Examples include the extent to which a teacher is willing to let a student struggle with a difficult problem, the kinds of assistance that teachers typically provide students who are having difficulties, and the extent to which students are willing to persevere in their struggle to solve difficult problems.

Stein and Henningsen (1997) indicated that the nature of tasks can potentially influence and structure the way students think and can serve to limit or to broaden their views of the subject matter with which they are engaged. They concluded that tasks in which students engage provide the contexts in which they learn to think about the subject matter, and different tasks may place differing cognitive demands on students. They are in agreement with Schoenfeld (1988) who suggests that students develop their sense of what it means to "do mathematics" from their actual experiences with mathematics, and their primary opportunities to experience mathematics as disciplines are seated in the classroom activities and tasks in which they engage, cited in Stein and Henningsen (1997, p. 525). Tasks should give students opportunities to explore a variety of approaches to mathematical problems in ways that are more sensitive to their thinking rather than being forced to follow a specific formalised method. This is supported by Manson, Stephens and Watson (2009, p. 27) who suggest that any sensible approach to teaching combines work on understanding concepts with work on mastering procedures, and combines tasks designed to stimulate learners to express their own thinking using technical terms with tasks designed to highlight the use of important routines. They further outline that keeping the notion of mathematical structure in mind, together with seeking the structures that underpin any proposed tasks, provides ways for students to meaningfully experience mathematical elements.

Kilpatrick, et al., (2001, p. 8) emphasized that effective teaching and learning of mathematics can best be examined from the perspective of how teachers, students, and content interact in contexts to produce teaching and learning. It is of great importance that, when selecting classroom tasks the teacher should choose tasks that build on student prior knowledge for them to be actively involved.

2.6.3 Teachers' and learners' expectations

Kilpatrick (2001, p. 9) showed that students learn best when they are presented with academically challenging tasks that focus on sense making and problem solving. Stein, et al., (1996, p. 462) suggested some characteristics of good tasks that promote quality acquisition of mathematical knowledge and which teachers should consider when setting classroom tasks for their learners. According to them, tasks that promote acquisition of mathematical knowledge are those that:

- **Encourage problem solving**

When mathematical problem solving is to be assessed, tasks should make these requirements of students:

- Formulate an approach to a problem;
- Select the mathematical procedures, concept and strategies necessary, and then deploy these when implementing a solution; and
- Draw conclusions.

It is emphasised that problem solving tasks should make procedural and conceptual demands on students. Worthwhile problem solving tasks can assess the way in which students use skills and concepts that they have fully absorbed. Example, *a drinking trough has a trapezium cross section which measures 50cm at the base and 100cm at the top. If the trough is 40cm deep and 12m long, work out the amount of water the trough can hold when full*. This problem requires students to make high-level use of skills and concepts that lead to the link between volume and capacity. It also requires some technical thinking in terms of the unit of measurement used.

- **Connect with students' situations and experiences**

The teacher is expected to starts the lesson with a stimulating questions within the learners' environment and then generates a mathematics task around it. This helps learners to think critical around any mathematical problem given to them because they turn to see the usefulness of mathematical ideas within their own environment.

- **Engage students' thinking about important mathematical ideas**

When the assessment target is a mathematical skill, then it involve tasks that assess students' knowledge of important facts and routine or algorithms are needed. For example, *the school water tank measures 36m by 14m by 16m. Find the volume of the tank*. This task measures procedural skills since to calculate volume students must know how to use the formula for calculating the volume of a rectangular prism.

- **Require more than recall of facts or reproduction of a skill.**

When tasks focus on conceptual understanding, they require students to use an idea, reformulate it and express it in their own terms. Conceptual understanding tasks require that students represent, use or explain a concept. For example, *the sides of a triangle have length 8cm, 9c and 12cm. Describe if this is a right angle triangle or not. Give reasons to support your decision.* This question is not procedural, since students decide on ways of approaching the problem.

Stein, et al., (1996, p. 462) further emphasised that the above-mentioned characteristics should guide teachers in the selection of tasks for mathematics instructions and how they can be used in the classrooms. It is suggested that the teacher should play a major role in planning activities that help students to solve problems that may arise in their everyday living.

2.6.4 Teachers' understanding the goal of instructions

Kilpatrick, et al., (2001, p. 9) observed that the quality of instruction often depends on how students engage with learning tasks. The way students engage with learning tasks thus shapes the nature of the tasks that the teacher selects. Even if what is actually taught in classrooms is strongly influenced by the available textbooks because most teachers use textbooks as their primary instructional materials, it is of great importance that, when selecting classroom tasks the teacher should choose tasks that build on student prior knowledge for them to be actively involved. Kilpatrick, et al., (2001, p. 9) further emphasized that for effective teaching and learning to take place all elements of curriculum, instruction, materials, and assessment should be aligned toward common learning goals.

2.7 OPPORTUNITY TO LEARN

Kilpatrick, et al., (2001, p. 333) defined opportunity to learn as any circumstance that allows students to engage in and spend time on mathematical tasks such as working on problems, exploring situations and gathering information, listening to explanations, reading texts and justifying results. They further indicated that students' opportunity to learn affects their achievement. Opportunities to learn refer to equitable conditions or circumstances within the school or classroom that promote learning for all students. It includes the provision of curricula, learning materials, facilities, teachers, and instructional experiences that enable students to achieve high standards. Opportunities to learn can be influenced by individual students, their teachers, their school or country's educational system.

Jacobsen, Eggen, Kauchak and Dulaney (1985, p. 155), stated that the best way to assist learners to learn better, is to guide them by employing tasks that allow them to think critically. They further added that the type of questions that teachers should ask in the lessons must give learners opportunities to use their own approaches in solving them, as long as they are in a position to explain why they choose the approach they have used and how it yields the solution obtained. They feel that this technique aids students to respond successfully and helps students to go into more depth in their ideas. Kilpatrick, et al., (2001, p. 333) indicated that 'although children bring important mathematical knowledge with them to class, most of the mathematics they know is learned in school and depends on those who teach it to them'. Even if teachers are not involved in designing the curriculum, they have a considerable influence over their students' opportunity to learn through the type of classroom activities they set.

2.8 ASSESSMENT

Since this study analyses tests and exercises that selected grade 11 and 12 teachers give to their learners, I would like to discuss the assessment criteria as outlined in the official Namibian assessment guide. 'Assessment refers to all types of information gathered about students by the teacher' (Parson, Hinson, and Sardo-Brown (2001, p. 450). They agree that assessment is a process by which teachers collect data about their students, analyse and synthesise it and interpret and use it to make decisions in their classrooms. This process allows the teacher to gather information on what learners know or have achieved with respect to a set standard about a certain mathematical topic.

The Namibian assessment guide (1998, p. 16) has outlined that some assessment types allow teachers to determine what learners know and can demonstrate this better than other assessment methods. It further suggested that the nature of the learning task should determine which assessment type should be most suitable to be used. The Namibian assessment guide (1998, p. 16) suggested that for each topic, learners should be: assessed to recall the facts and rules; able to apply skills they are required to learn; able to reason around the concepts, and able to discuss and justify their understanding. The guide emphasized that tasks require students to demonstrate their thinking and justify their solutions; therefore the students' responses to the questions are the focus of the assessment process.

2.9 THE NAMIBIAN CONTEXT

In Namibia there are four main policy documents that guide what teachers should do in the classroom. These are: the syllabus, scheme of work, learner centred policy and subject guide. These documents work as follows:

The Syllabus is a policy document 'that outlines what mathematical topics should be taught in a particular grade' NIED (2010, p. 2). It also specifies the set of objectives to be attained by learners. The Namibian Senior Secondary Certificate Ordinary (NSSCO) mathematics syllabus recognises that learning mathematics involves conceptual structures and general strategies of solving problems. It further indicates that mathematics is a dynamic, living and cultural product. It is against this background that the syllabus is organised to:

- *Develop learners mathematical knowledge and oral, written and practical skills.*
- *Apply mathematics in everyday situation and develop an understanding of the part which mathematics plays in the world around them.*
- *Carry out calculations and understand the significance of the results obtained.*
- *Solve problems and present solutions clearly*
- *Develop an understanding of mathematical principles.*
- *Develop the abilities to apply mathematics in other subjects, particularly science and technology.*
- *Develop abilities to reason logically.* NIED (2010, p. 2).

The above mentioned goals should guide teachers' decisions in selecting mathematical tasks that are aligned to the above goals.

A ***Scheme of work*** is an official policy document 'that outlines when a certain topic is to be taught and the duration it should be taught' NIED (2010, p. 3). It is aligned to the syllabus and guides the teacher what he/she should teach in a specific term. It also suggests when to give a topic test based on basic competences that are covered. This document emphasises what specific content of the syllabus will be assessed. It specifies that teachers should ask questions that are aligned to the type of questions asked in the national examinations. It gives details of what knowledge learners should acquire at the end of the lesson taught. For example, learners should be able to use geometrical properties to calculate the size of an unknown angle in any given figure.

Learner centred policy is an additional official policy document that suggests how the teacher and learners could interact in the classroom (Ministry of Basic Education and Culture, 1996, p. 3). Since independence in Namibia in 1990 there has been a change in teaching approach from

teacher-centred to learner-centred education which places a learner at the centre of the teaching learning process. This teaching approach is known as learner-centred teaching.
It states that:

- *The starting point at each stage of learning process is the learner's existing knowledge, skills, interest and understanding, derived from previous experience in and out of school.*
- *The natural curiosity and eagerness of young people to learn to investigate and make sense of a widening world must be catered for by a variety of challenging and meaningful tasks.*
- *The learners' perspective must be appreciated and taken into consideration in the work of the school.*
- *The learner should be empowered to think and take responsibility not only for their own, but also for each other's learning and development.*
- *The learners should be involved as partners in, rather than receivers of education.* Ministry of Basic Education and Culture (1996, p. 3).

The policy has made a provision that learners should be given academic activities, tasks and experiences that stimulate them to learn. Learner-centred education can help learners to apply their own knowledge and skills in solving mathematical problems. It also helps learners to investigate mathematical ideas. It emphasises that learners should actively construct their own knowledge, hence they should be allowed to explore mathematics ideas. In this approach, teachers are required to select mathematical tasks that build on learners' prior knowledge and experiences. It does not mean the teacher can sit back and simply hand over authority to the learners, but must be more innovative since learner-centred teaching depends on a variety of activities leading to active learning. It emphasises that the teacher should give learners activities that require them to think critically, draw conclusions and summarise ideas through a well facilitated learning process.

Lastly, there is the ***subject guide*** which consists of samples of expected questions to be asked per specific topic and outlines the set standard of allocating marks to specific questions. It also provides a suggested marking scheme that guide teachers on how to give marks to learners' work. The national curriculum emphasises the importance of good task selection in order to achieve its goals.

2.10 THEORETICAL CONSIDERATIONS

In this study, the analysis of tasks that teachers select is framed by the constructivist learning theory. ‘Constructivism is a theory of learning that emphasised that knowledge is not discovered but constructed by individuals based on experiences’ Yilmaz (2008, p. 162). Constructivism outlines that teachers should give tasks that encourage students to discover principles for themselves and to construct knowledge by working to solve realistic problems. Manson, et al., (2009, p. 27) suggest that any sensible approach to teaching combines work on understanding concepts with work on mastering procedures, and combines tasks designed to stimulate learners to express their own thinking using technical terms with tasks designed to highlight the use of important routines. They further outline that keeping the notion of mathematical structure in mind, together with seeking the structures that underpin any proposed tasks, provides ways for students to meaningfully experience mathematical elements. It is based on these notions that tasks are set to give students opportunities to explore a variety of approaches to solve mathematical problems in ways that are more sensible to their thinking, rather than being forced to follow a specific formalised method.

Stein and Henningsen (1997, p. 462) indicated that the nature of tasks can potentially influence and structure the way students think and can serve to limit or to broaden their views of the subject matter with which they are engaged. One way of eliciting students’ ideas is by implementing tasks that are appropriate and challenging. These tasks should enable students to make sense of the material before them and improve their mathematical reasoning. Sanni (2012, p. 130) supported the reasoning that tasks provide the intellectual contexts for students’ mathematical development, hence using tasks are fundamental ways of teaching learners to understand mathematical concepts. He observed that curriculum reforms generally call on teachers to generate inspiring and interesting ways of eliciting mathematical ideas from their students.

2.11 CONCLUSION

This chapter highlights some literature which relates to the research topic and provides a valuable framing for my own thinking in the design of the data collection and also in my later analysis.

CHAPTER 3

RESEARCH METHODOLOGY

3.1 INTRODUCTION

This chapter discusses the research orientation that framed this study. It covers these components: the place where the research was done, the participants in the study, the sampling procedures used and the research instruments that were used. This chapter also makes some comments on how the respondents responded to the questions. The chapter also highlights how the types of tasks were selected by the teachers, and how they were used in their classrooms. The factors that influenced the participating teachers in selecting the investigated tasks will also be discussed in the next chapter. The chapter ends with a discussion on the ethics and limitations of the study.

3.2 RESEARCH GOALS

The study aimed to investigate the nature of tasks that are used by selected Grade 11 and 12 mathematics teachers in their teaching in the Omusati region of Namibia. This was done by answering the following research questions:

1. What types of tasks do selected Grade 11 and 12 teachers select for their students in their mathematics lesson, according to Stein's framework?

2. On what basis do the participating mathematics teachers select their classroom tasks they give to their learners?

Mathematics education researchers Stein, et al., (1996, p. 456) indicated that one of the greatest challenges encountered by mathematics teachers is the identification and use of appropriate tasks that challenge their pupils appropriately. It is against this background that this study was undertaken to investigate the type of tasks that teachers selected and on what basis they selected these tasks to give to their learners. Lester (2007, p. 346), defines mathematical tasks as "classroom activities the purpose of which is to focus students' attention on particular mathematical ideas". In my experience, and in my work environment it is evident that curricular materials and textbooks are the main source of mathematical tasks used by teachers for classroom instruction. Kilpatrick, et al., (2001, p. 335) emphasised that the quality of mathematical tasks that teachers select for their teaching are central to students' learning because tasks convey messages about what mathematics is and what doing mathematics entails. Moreover, they observe that, generally, the tasks assigned to students in many classrooms make only minimal demands on their thinking, because the tasks those students are often given demand primarily only memorization or the use of procedures without making connections to concepts.

3.3 RESEARCH ORIENTATION

This study is located in the interpretative paradigm. O'Leary (2004, p. 10) defined interpretive research as explorations of cultural and historical significance in the social world. The study used a mixed methods (qualitative and quantitative) approach. According to Cohen, Manion and Morrison (2011, p. 537) qualitative analysis involves organizing, accounting for and explaining the data, and making sense of data in terms of the participants' experiences of the situation. A qualitative approach is used to analyse interviews with the participating teachers.

A quantitative approach, on the other hand, involves numerical data. In this study a quantitative methodology is used to analyse lesson observations and documents (tests papers, exercises and homework books) with the specific purpose to quantify the different types of tasks the participants used in their lessons.

This research is essentially descriptive in nature. According to Parson, Hinson and Sardo-Brown (2001, p. 16), descriptive research is "research which employs systematic observation and recording of data without manipulation of the observed phenomenon". This means that no inference or predictions are made in this study. The main aim of this project was to explore the types of classroom tasks selected teachers give to their learners and to find out the main reasons for selecting these classroom tasks.

3.4 METHODOLOGY

This study is a case study involving four Grade 12 teachers. "A case study is an intensive investigation of a single unit" Yin (1994, p.10). In this study the single unit is a cohort of 4 teachers and the mathematical tasks they select in their teaching. The reason for using a case study was that it provided an ideal opportunity for one aspect of a problem (mathematical tasks) to be studied in depth within a relatively short period of time. The unit of analysis was the type and nature of tasks that the participating teachers selected for their teaching, and the teachers' responses as to the reasons why they selected these tasks.

3.5 RESEARCH DESIGN

This study consisted of two phases:

Phase 1 consisted of video-recording two Grade 11 and two Grade 12 classes' Algebra and Mensuration lessons. I focused on Algebra and Mensuration because learners usually perform poorly in examinations when it comes to these topics, according to the examiner's report of 2011 and 2012 as outlined in Table 3.1 next page.

Table 3.1: Comments from examiner's reports

Topic	Task	Year	Comments
Algebra	Factorise $6x^3y - 12x^2y^2$ Simplify a-5+10a+8	2011	Few candidates failed to identify common factors while majority subtracted instead of factorising Operations with algebraic terms were a big concern. Students added unlike terms, while some attempted to solve for a.
Mensuration	Work out the Circumference of a circle whose diameter is 9cm Work out the surface area of a cylinder whose diameter is 9cm and perpendicular height of 22cm.	2011	A clear distinction should be made between area and the circumference of a circle. C=$2\pi r$ or C=πd Some learners calculated volume instead of surface area.
Algebra	Solve the equation 4(2x - 3) = 8	2012	Students often multiply out the first term in the bracket with the factor outside while some struggle to differentiate between expression and equation.

Source: Examiner's report (2011 – 2012) NSSC

The aim of these video recordings was to observe and record the mathematical tasks that the teachers used in the lessons. The lessons were analysed using descriptive statistics. I used Stein's conceptual framework to classify each of the mathematical tasks given to the class by each of the four participating teachers. I counted the number of tasks and grouped them according to their cognitive demands as defined in Stein's framework. In addition, I also analysed all the tests and exercises in Algebra and Mensuration that the participating teachers set in 2013 to ascertain the Stein cognitive levels of the individual items of these tests and the exercises.

Phase 2 of the study consisted of a series of interviews. The data consisted of interview transcripts of one interview per participating teacher. In the interviews I probed for the reasons as to why the teachers selected the tasks they used. I used the data from Phase 1 to provoke discussion on the rationale for selecting each of the tasks evident in the videos. My unit of analysis in this case was the teachers' responses to my probing about the basis of their selection of the tasks and the test items they set.

3.6 DATA COLLECTION TOOLS

I observed two lessons per participating teacher from two schools. Parallel to this I asked each teacher for copies of tests and exercise work sheets that they set in the course of 2013 for Algebra and Mensuration. I also interviewed each teacher as articulated above.

3.6.1 Class observation

The two lessons of each of four teachers were video recorded during class time. Teachers of school A were observed on 14th March 2014, in the first trimester of that academic year. I operated the video camera from the back of the classroom. At school B, the observations were conducted on 27th May 2014, in the second trimester of the academic year. The purpose of the video recordings was to observe and record the type of classroom tasks selected teachers use in their classrooms. The tasks that were observed in each video were then analysed according to Stein's framework.

3.6.2 Interviews

After the video recordings I interviewed each of the four participating teachers. The purpose of the interviews was to find out the reasons why the teachers selected the specific classroom tasks that were evident in the video recordings. The interviews were semi-structured. According to Johnson & Christensen (2004, p.183), semi-structured interviews are types of interviews where the researcher and participants engage individually on a one to one basis. A mutually agreed time was negotiated. The interview schedule that formed the structure of the interview is outlined in Table 3.2 next page.

Table 3.2 Interview schedule

INTERVIEW SCHEDULE FRAMEWORK

1. What type of tasks do you mostly give to your learners?
2. Where do you normally find the tasks that you give to your learners? (What is the source of the classroom tasks that you use?)
3. On what basis do you select the tasks you give to your learners?
4. How do you determine the levels of difficulty of the tasks that you choose?
5. When you select a particular task from the textbook how do you present it to your learners?
6. How do you know that a particular task will help your learners to understand what you have taught?

3.7 SAMPLE AND RESEARCH SITE

The criteria for selecting the schools and the participants are discussed below.

3.7.1 Research site

The research project was conducted at two Secondary Schools in the Okahao circuit of the Omusati region in northern Namibia. For convenience purposes I selected my own school and the neighbouring school, which are one kilometre apart. The schools are coded school A and School B respectively

3.7.2 Sampling procedure

Two teachers from each school were selected purposefully. At each school I asked for volunteers from the mathematics teachers who taught Grade 12 classes. One male and one female teacher volunteered from each school. The participating teachers were coded as follow: those from school A were coded TA1 and TA2 and those from school B were coded TB1 and TB2.

In sum, sampling for this study was both purposive and convenient. It was purposive because I only needed to work with a few participants in order to acquire in-depth information about selecting and setting of classroom tasks. Ball (1990) as cited in Cohen, et al., (2011, p. 157) defined purposive sampling as a sampling method used in order to access knowledgeable people who have in-depth knowledge about a particular issue, maybe by virtue of their professional role, power, expertise or experience. It was convenient in the sense that I selected schools that are located within manageable distances from my place of work.

3.8 ANALYSIS

I used Stein's conceptual framework of cognitive demands (see Table 3.3) to analyse the tasks.

Table 3.3: The task analysis guide

Lower-level demands tasks criterion	Higher-level demands tasks criterion
Memorisation tasks • Involve reproduction of previously learned facts, rules, formulae or definitions. • Tasks involve exact reproduction of previously seen material and what is to be produced is clearly and directly stated.	**Procedures with connections tasks** • Focus students' attention on the use of procedures for developing a deeper level of understanding of concepts. • Tasks are usually represented in multiple ways (e.g. visual diagram, symbols, manipulative and problem solving).
Procedures without connections tasks • Use of procedure is either called for or its use is evident based on placement of the task. • Require limited cognitive demand for successful completion. There is little ambiguity about what to be done and how to do it. • Have no connection to the concepts or meaning that underlies the procedure being used. • Focused on producing the correct answers rather than developing mathematical understanding.	**Doing mathematics tasks** • Require complex non-algorithmic thinking (i.e. Task instruction or worked out examples). • Require students to explore and understand the nature of mathematical concepts, processes or relationships. • Require students to analyse the task and actively examine the task constraints that may limit possible strategies and solutions. • Focus on justification of mathematical ideas to develop mathematical understanding.

Source: Stein, Smith, Silver and Henningsen (2000, p. 16).

Stein and Henningsen (1997) provided an interesting taxonomy of mathematical tasks based on the type and level of thinking required to solve them. From their perspective mathematical tasks, in general, can be classified as tasks with *high-level* cognitive demand. These tasks engage students in a process of inquiry and validation. However, tasks that encourage students to use only procedures, formulas or algorithms in a way that are not actively linked to meaning, or that consist primarily of memorisation or reproduction of previously memorised facts are classified as tasks with *lower-level* cognitive demand. In addition to this taxonomy for classifying mathematical tasks, Stein and Henningson (1997) also provided a framework for tracking the cognitive demands of mathematical tasks as they are used during instruction in order to analyse the connections between instruction and student learning. The framework suggests three types of mathematical tasks. Type 1 are tasks from curricular or instructional materials, Type 2 are tasks that are crafted, sourced and set up by the teacher, and Type 3 are those tasks that are implemented by the students.

Each task in the tests, exercises and observed lessons was evaluated against each category criterion in Table 3.3 on a scale from 0 – 2, where: ***0:*** *this criterion is not evident in the task****, 1:*** *this criterion is a component of the task* and ***2:*** *this criterion is very evident in the task.*

The responses to the interviews were colour coded and organised according to themes which align with Stein's framework. According to Stein et al. (1996, p. 462) some characteristics of tasks that promote quality acquisition of mathematical knowledge are:

- tasks that encourage problem solving,
- tasks that connect with students' situations and experiences,
- tasks that engage students' thinking about important mathematical ideas and
- tasks that require more than recall of facts or reproduction of a skill.

3.9 SUMMARY OF THE RESEARCH DESIGN

Table 3.4

Phase	Tools	Approach	Reasons	Data
1	Observation	Quantitative results. Classifying of tasks according to Stein's level of cognitive demands.	To explore types of tasks given in the classrooms.	Numeric and statistical data
2	Interview	Qualitative results. A description of how and why selected teachers selected tasks they gave to the learners.	Exploring what influence teachers to select tasks they give in the classroom.	Interview transcripts

3.10 ETHICAL CONSIDERATIONS

According to Cohen, et al., (2011, p. 228), prior to the conducting of research, issues such as confidentiality, anonymity and informed consent need to be addressed. Official permission to conduct this research study in my circuit was obtained from the circuit inspector after a discussion of the purpose of the study between her and I (see Appendix 1). At the schools, permission was obtained from the school principal of each school where the study was conducted after the purpose of the study was successfully discussed. The participating teachers that volunteered were also briefed about the aim of the study and were asked to sign a permission letter (see Appendix 2). The teachers were informed about the process of the research and assured that they could withdraw at any stage of this process. They were also assured about confidentiality in handling the information and that their names would not be revealed.

3.11 VALIDITY

To ensure that the interview questions were consistent with the research questions and unambiguous, the interview was piloted with one Grade 11 teacher. The validity of Stein's framework is well established as it has been used in many international contexts. For example, the framework was used by Sanni (2012) in his PhD study on task practices and teachers' knowledge. Kilpatrick, et al. (2001) have also cited Stein's framework in exploring task selection in classrooms. Clark and Roche (2012) also used the framework when carrying out a project on Task Type and Mathematics Learning (TTML) in Australia.

3.12 LIMITATIONS AND CHALLENGES

At first some teachers were a little reluctant for their lessons to be videotaped and some were afraid to be interviewed, but in time they became accustomed to the presence of the camera and were comfortable to share their practice with me.

The biggest challenge in the study was audio recording my interview sessions with a cellular phone. The teachers declined to be recorded because they felt that if recorded the anonymity and confidentiality promised to them would no longer prevail. They were comfortable for me to manually write down their responses as best I could.

3.13 CONCLUSION

This chapter started off with the identification of the research orientation, which I used to frame the entire research design to answer my research questions. This was followed by a description of the research design and process, including a discussion on the sampling procedures and how I analysed the data. It then rounded off with a brief discussion on ethical matters and limitations encountered.

CHAPTER 4

ANALYSIS OF DATA AND DISCUSSIONS

4.1 INTRODUCTION

This chapter presents the results and findings of the investigation. The data is based on classroom observations, interviews and test analyses as described in the previous chapter.

4.2 CLASSROOM OBSERVATIONS

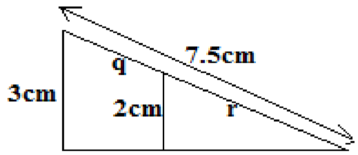
Phase 1 consisted of video-taping two Grade 11 and two Grade 12 classes. The original aim was to focus only on Algebra and Mensuration lessons, but unfortunately teachers were also busy with other topics. The observation took place in the first term at one of the participating schools and in the second term at the other school. I went ahead with the project regardless of what topic was being taught. As it happened when teacher 1 was busy with Locus in Grade 12, she was also teaching Mensuration in her Grade 11 class, the topic under investigation in this study. A similar scenario happened in the other school where Ratio and Proportion at Grade 11 coincided with Algebra at Grade 12. I was thus able to observe the topics I set out to observe.

The observed lessons were analysed by looking at each task that each participating teacher gave to the class during the filmed lesson. Each task was categorised in one of Stein's cognitive levels. For each lesson observed I provide:

- the task
- the time of the lesson when the task was given eg (07:48)
- the coding according to Stein's framework
- my justification for the coding.

4.2.1 School A Teacher 1 Lesson 1 Grade 12J

Task 1: (07:48)



The figure above consists of two similar triangles. Study the diagram carefully and find the lengths of q and r.

Coding: 3

Justifications and comments

I categorised this task as a higher cognitive demand task. It consists of elements of both procedures with connection and doing mathematics. The task is represented by a diagram to visualise the problem as well as, in words, to assist learners to create a link between the concepts and procedures. In addition the task given also allowed learners to analyse mathematical ideas such as using corresponding sides to find the ratio or scale factor that can be used to find *r*, where $r = \frac{7.5}{sf}$. Subsequently, learners realised that the sum of *r* and *q* is 7.5cm. Since they had already worked out the value of *r*, they saw that $q = 7.5 - r$, without being told what steps to follow.

Task 2: (08:03)

A rectangle has dimensions of 3m by 2m. What happens to its area when its dimensions are doubled?

Coding: 3

Justifications and comments

Some learners were quick to conclude that when dimensions are doubled, the area will be doubled as well. The teacher was also quick to caution them not to give conclusions which are not supported by calculations. Learners calculated the area of the rectangle with the given dimensions and then doubled the dimensions and worked out the area of the second rectangle. The teacher then asked them to take any dimensions of their choice and repeat the same process. The learners compared their answers and discovered that, when the dimensions are doubled, the area of the previous rectangle is multiplied by four to give the area of the bigger rectangle. Unfortunately no one figured out that four is a square of two, which is the linear factor. The teacher helped them by asking them to figure out the relationship between two and four. Learners knew that four is a multiple of two, where 2x2=4. The teacher further asked them to write 2x2 in a different way that can still yield 4. Few learners realised that the teacher wanted them to write 2x2 as 2^2. The teacher then referred the learners back to the area and volume of similar figures where, area factor = (linear factor)^2. I concluded that the question qualified to be categorised as doing a mathematics task.

I thus categorised this task as a higher cognitive demand task. It consists of elements of doing mathematics, where learners are allowed to explore the relationship between mathematical concepts, for example the link between dimensions and area. The question is relatively open-ended where learners are not told what to do in order to arrive at the correct answer.

Task 3: (08:11)

Two table tops are similar in shape. The large one has a diameter of 120cm and the small one has a diameter of 80cm. If the area of the small one is 4cm^2, what is the area of the large one?

Coding: 1

Justifications and comments

The task involves the exact reproduction of previously learnt rules and definitions. Since learners already knew what was expected of them when told that two figures are similar, I categorised this task as a low level cognitive demand task. In this case learners could use the dimensions given to find the linear factor and use the factor to calculate the area asked without making any connection to any other learnt concepts. They only followed the procedure as previously used. If the teacher had given the area of the big table and asked the area of the small table, then the problem would address the concept of division and multiplication and learners would see when to do which of the two calculations. This would then have raised the cognitive level of the task

4.2.2 School A Teacher 1 Lesson 2 Grade 11G

Task 1: (12:45)

Express each ratio in its simplest form.

a) $3:\frac{4}{3}$

b) 0.75:0.8

Coding: 2

Justifications and comments

Both tasks focused on a particular procedure to produce correct answers without any link between the procedure and the concept. Learners knew what was expected of them when told to write the ratio in the simplest form, for example in (a) learners multiplied both sides by 3 just to get rid of the fraction ($3x3:\frac{4}{3}x3$) to get 9:4 but no connection of sharing was addressed. In (b), learners knew that they could simplify more easily with whole numbers other than decimals, hence they multiplied both sides by 10 just to get rid of decimals (0.75x10:0.8x10) to get 75:80. They then divided both sides by 5 because they knew 5 can divide into any number which ends with a zero or five, to get a final answer as 15:16. I thus categorised this task as a low level cognitive demand task.

Task 2: (12:55)

A man earns N$ 5 500 and pays income tax of N$ 860. What is the ratio of tax to earnings?

Coding: 1

Justifications and comments

This task focused on a particular procedure to produce a correct answer without any link between the procedure and the concept. Learners knew what was expected of them when told to write the ratio in its simplest form. Many learners divided both sides by 10 as a way of making numbers as small as possible. For example 5500:860 was reduced to 550:86 and then both sides were further divided by 2 to give 275:16.

Task 3: (13:10)

Divide in given ratios

a) Divide 48 into two parts in the ratio of 5:7.
b) Share 45 sweets among three children in the ratio of 3:4:8
c) The angles of a triangle are in the ratio of 3:5:7. Find the size of each angle.

Coding: 3

Justifications and comments

In (a) and (b) tasks were drawn from learners' experience (e.g. sharing). At the same time they allowed learners to draw general conclusions and justify mathematical ideas based on their real life experiences. In both questions learners added the ratios together because they knew that each number is regarded as a part of the total. In this case there was a link made between the ratio and common fractions where learners knew that when something is not a whole then it is a part of something. Schoenfeld (1988, p. 15) suggested that students develop their sense of what it means to "do mathematics" from their actual experiences with mathematics, and their primary opportunities to experience mathematics as disciplines are seated in the classroom activities and tasks in which they engage. It is against this background that learners realised that the total ratio is equivalent to the total of what is being divided. In (c) the task involved a procedure with connection because it created a link between the ratio and angle properties in the triangle. The sum of angles inside the triangle equals a total of 180^0.

4.2.3 School A Teacher 2 Lesson 1 Grade 12I

Task 1: (08:35)

A port has two radar stations at **P** and **Q** which are 20km apart. The radar at **P** is set to a range of 20km while radar at **Q** is set to a range of 15km.

a) Draw a scaled diagram to show the above information.
b) Shade the region in which a ship must be sailing if it is only picked up by radar at P. Label the region **a.**
c) Identify the region in which a ship must be sailing if it is to be picked up by both radars. Label the region **b.**

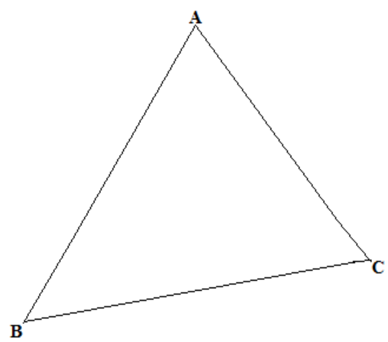
Coding: 4

Justifications and comments

In my assessment the task allows learners to analyse mathematical ideas that may lead to the solution. For example, they had to draw a diagram to a reasonable scale that leads to an accurate solution The question did not restrict or prescribe to the learners on how to draw their diagrams. Learners were given enough space to demonstrate their initiative to draw a scaled diagram. In addition the task allowed learners to draw general conclusions and justify how mathematical ideas are used in real life situations. They saw how real life distances can be represented on a map.

Task 2: (09:00)

The teacher gave a hand-out to the learners and asked them to do question 6. The question read as follows:



a) Draw accurately the locus of points inside the triangle which are:

(i) 5cm from B

(ii) Equidistance from AC and BC

b) Shade the region inside the triangle which is more than 5cm from B **and** nearer to BC than AC.

Coding: 3

Justifications and comments

The given task allowed learners to analyse a variety of mathematical ideas, which could lead to the solution. For example, learners employed knowledge of an angle bisector to find the locus of points which are equidistance from two given lines. They also applied knowledge of the locus of points which are at a fixed distance from a given point. I categorised this task as procedures with connections because of its variety and demands of learners an understanding of how concepts are related to each other within a given space. In addition it required learners to use mathematical instruments accurately and appropriately.

4.2.4 School B Teacher 1 Lesson 1 Grade 12H

Task 1: (07:42)

1. Calculate the perimeter of a rectangle whose length is 7cm and width is 4cm.
2. Calculate the circumference of a circle with a diameter of 10cm.

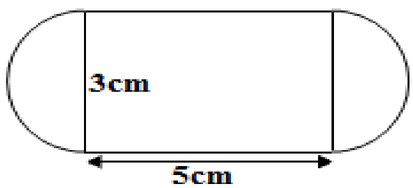
Coding: 1

Justifications and comments

This task focused on a particular procedure and formula to produce the correct answer. In terms of a rectangle some learners added 4 to 7 to yield 11, which is incorrect. In this case learners knew that when asked to calculate the perimeter they add lengths of the sides together, but failed to recognise the number of sides of a rectangle. They also did not understand the fundamental concept of perimeter. Some learners did recall the formula *p=2l+2b*. They arrived at the correct answer without necessarily understanding the role of 2 in the formula. The circle task merely involved the reproduction of the previously memorised formula $C = \pi d$.

Task 2: (07:48)

For the shape below, calculate the perimeter and the area.



Coding: 3

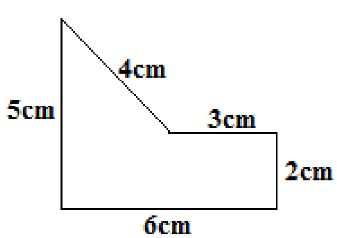
Justifications and comments

I categorised this task as a higher cognitive demand task as it consisted of elements of procedures with connections. The task allowed learners to explore the relationship between mathematical concepts, in this case the combination of shapes and the link between perimeter and circumference. The task was provided as a diagram, which assisted learners to visualise the problem and make appropriate connections between the concept of perimeter and area of different shapes and procedures in order to calculate them.

Task 3: (08:04)

For the shape below work out:

a) the perimeter
b) the area



(a)

Coding: 2

Justifications and comments

The task focused on only one particular procedure. To calculate the perimeter of the shape the learners simply had to add together all the lengths of its sides. I coded it as a level 2 (and not level 1) as the shape may have been unfamiliar and relatively complex. It is not a standard familiar 'single' shape like a triangle or rectangle.

(b)

Coding: 3

Justifications and comments

The task was represented in the form of a diagram, which assisted the learners to visualise the problem. It encouraged the learners to employ their own strategies in answering the question by suggesting that the shape needed to be divided into known shapes. Very interestingly, the majority of learners arrived at the correct answer, but they used different approaches. Some learners cut the shape into a rectangle and a triangle; some cut it into a rectangle and a trapezium while others took a rectangle of 5 by 6 and subtracted the trapezium of base 6 and height of 6. The solutions are presented in the table below:

Group 1	Group 2	Group 3
$\boldsymbol{A = \frac{1}{2}(2+5)\times 3 + 3\times 2}$ $\boldsymbol{= 10.5 + 6}$ $\boldsymbol{= 16.5cm^2}$	$\boldsymbol{A = \frac{1}{2}\times 3\times 3 + 6\times 2}$ $\boldsymbol{= 4.5 + 12}$ $\boldsymbol{= 16.5cm^2}$	$\boldsymbol{A = 6\times 5 - \frac{1}{2}(3+6)\times 3}$ = 30 - 13.5 $\boldsymbol{= 16.5cm^2}$

I thus concluded that learners were engaging in mathematics by making connections; hence the task is of relatively high cognitive demand.

4.2.5 School B Teacher 2 Lesson 1 Grade 11H

Task 1: (09:43)

Solve for x

a) $2x - 3 = 5$

b) $2x + 7 = 33$

Coding: 1

Justifications and comments

The tasks simply asked for one solution to the problem. In both questions, the learners already knew what procedures/steps to follow in order to arrive at the correct answers. For example, in these tasks, learners are expected to collect like terms on one side and then add the like terms together or subtract like terms from one another and then divide both sides by 2 to get the value of x.

Task 2: (09:51)

Find the value of each letter in each of the following equations.

a) $1 + 2a = 13$

b) $9 - 3x = 7$

c) $12x - 48 = 7x - 13$

d) $-8y - 6 + 13 = 15y$

Coding: 2

Justifications and comments

For all the questions, learners already knew the algorithm to follow to arrive at the correct answers. I coded the task as 2 as the last two questions involved a slightly more complex calculation than task 1.

Task 2: (10:09)

Solve the following equations

a) A + 2(2 – a) = 8
b) 5 – (4 – x) = 2x
c) 3(2c – 1) – 2(c – 1) = 0

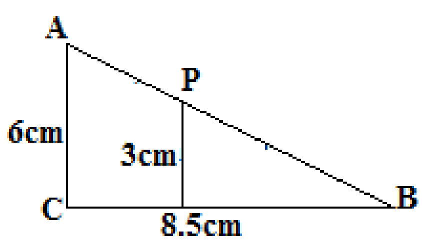
Coding: 1

Justifications and comments

This task only involved opening brackets and collecting like terms and then solving for the appropriate unknown. The learners were very familiar with the procedures to arrive at the correct answers.

4.2.6 School B Teacher 2 Lesson 1 Grade 12D

Task 1: (08:23)



In the figure above, AC = 6cm, CB = 8.5cm and the line drawn from the base to point P is 3cm.

a) Use Pythagoras' theorem to calculate the length of side AB.
b) Work out the length of AP.

Coding (a): 1

Justifications and comments

This task simply required the learners to apply Pythagoras' theorem by substituting the appropriate lengths into the Pythagorean formula

(b): 3

Justification and comments

This task required a number of inter-leading steps to arrive at the correct solution. Learners had to explore the link between the lengths of sides of the two ideas and thus make appropriate connections.

Task 2: (08:40)

The side of a cube measures 2m. What happens to its volume when the lengths of its sides are multiplied by 2? *(Hint: take a length of your choice and compare your answer to that of your colleague).*

Coding: 4

Justifications and comments

I categorised this task as a higher cognitive demand task as it required learners to do mathematics. The learners were not familiar with any formula or procedure to solve this task. They had to explore the relationships between different lengths of a cube and its resulting volume. Each learner drew rough cubes with dimensions of their choice. They then doubled the sides and calculated the new volumes. They themselves discovered that the volume changed by a factor of eight. The teacher asked them to express 8 as a product of its prime factors. All learners expressed it as 2x2x2=8. The teacher then asked them to write 2x2x2 in exponential form. The teacher then referred the learners back to the volume and the result for similar figures where the volume factor = (linear factor)^3. I concluded that the question qualified to be categorised as doing a mathematics task even if the teacher assisted learners' thinking by giving them some hints.

Task 2: (08:50)

The masses of two similar objects are 24kg and 81kg respectively. If the surface area of the large object is 540cm^2, find the surface area of the smaller object. *[Hint: the ratio of masses is the same as the ratio of volumes]*

Coding: 3

Justifications and comments

The hint given to the learners reduces the cognitive level of demand somewhat. It reduces the problem to one of procedure with connection. Further, learners have to work out the linear factor, which they then they used to find the area factor to obtain the surface area of the small object.

4.2.7 School B Teacher 2 Lesson 2 Grade 11D

Task 1: (09:02)

A number is doubled then four added to the result to give ten. Find the number.

Coding: 1

Justifications and comments

The task requires a particular known procedure to produce the correct answer. The problem is presented in a routine way without being contextualised with a specific scenario. The teacher could, for example ask: "Mary bought some sweets. If the number of sweets she has is doubled she will have ten sweets How many sweets did she buy?" If the question was rephrased this way, then it could be categorised as a higher level of cognitive demand task as the learners would have to deconstruct a real life situation into its mathematical components.

Task 2: (09:08)

Mary and three other girls gathered some nuts. Mary counted them and took her share home. When she got home she threw three nuts away because they were bad. She then was left with nine nuts. Work out how many nuts were gathered altogether.

Coding: 4

Justifications and comments

This task is drawn from learners' experiences such as sharing. It also allowed learners to draw general conclusions and justify mathematical ideas based on their real life experiences. This task analyses the role of rational numbers in decision making. Mathematically, sharing involves dividing the number of items being shared by the number of people involved. For example, some learners used their algebraic knowledge that any unknown number can be represented with the letter *x*. This prompted learners to divide *x* by 4 since there were four girls. Learners realised that each girl had the same number of nuts as Mary. Mary threw three nuts away to get nine. They formed an equation that yielded a result. I concluded that the question qualified to be categorised as doing a mathematics task, since the learners went about the problem with their own understanding. Some learners formulated an equation $\frac{x}{4} - 3 = 9$ and then solved the value for x, while others added three to nine and multiplied by four.

Task 3: (09:18)

Find three consecutive odd numbers whose sum is 39.

Coding: 4

Justifications and comments

This task required learners to analyse and solve a mathematical idea. Learners should know the meaning of the terms 'odd numbers' and 'consecutive'. The task gave enough space for learners to employ their own strategies to arrive at a solution. Some, for example, formed an equation that best fitted the given problem, while others solved it by trial and error.

Task 4: (09:26)

Jean is twice Pierre's age and Pierre is 3 years older than Allan. If the sum of their age is 17, how old is each of them?

Coding: 4

Justifications and comments

Similar to task 3 this task avoided a prescribed method of solving. Learners were encouraged to use their own methods to find the correct solution. I coded this task 4 because it gave enough space for learners to employ their own strategies to solve the problem based on individual understanding. Learners tried to construct equations that fitted the given problem and then solved the problem.

4.3 TESTS

Similar to the observed lessons each task in the tests were analysed according to Stein's framework of cognitive demand. I only analysed those tasks which pertained to the mathematical domain of my study. In total I analysed 4 tests.

For each test I provided:

- the task
- the date that the test was written
- the coding according to Stein's framework
- my justification for the coding.

4.3.1 School A TA1 and TA2 Test 1 1st July 2014 Grade 11

Task 1

Mr. Shilongo plans to paint a picture. He considers making it 48cm long. He then decides to increase the length of 48cm in the ratio of 5:4. Find the new length.

Coding: 3

Justifications and comments

This task required the learners to not only manipulate the given length, but also to 'play' with the given ratio. The task was contextualised in a real life situation and encouraged exploration. It also allowed learners to draw general conclusions and justify mathematical ideas based on their real life experiences.

4.3.2 School A TA1 and TA2 Test 2 1st July 2014 Grade 12

Task 1

Factorise the following expressions

a) $x^2 - 5x$

b) $2x^2 - 11x + 5$

a) Coding: 1

Justifications and comments

This task expects learners to follow the factorisation procedure learnt in class. They were able to do this without making any connections by simply following the appropriate procedure. Learners saw that x is the common factor, and then factored it outside the bracket. It was observed that learners simply followed the factorisation procedure in a routine way.

b) Coding: 2

Justifications and comments

This task expected learners to follow the factorisation procedure learnt in class. Learners could perform this task without making any connections and simply following the appropriate procedure. Learners only needed to identify that the expression is a trinomial quadratic and then factorise according to the learnt procedures.

Task 2

Solve the following equations

a) $logx + log5 = \log(x + 5)$

b) $y + 4 = x$ and $y^2 + 17 = 2x^2$

Coding: 2

Justifications and comments

Just as in task 1, these tasks can be grouped as tasks without any connection hence they expected learners to follow the procedure learnt in class as well. Learners could perform these tasks without making any connections and simply following the appropriate procedure. In a), learners only need to follow logarithm laws while in b) they only needed to substitute one equation into the other and then use the quadratic formula to solve the equation.

Task 3

Simplify

$$\frac{x^2 - 2x}{x^2 + x - 6}$$

Coding: 2

Justifications and comments

This task expected learners to follow the factorisation previously procedure learnt in class. Learners could perform this task without making any connections and simply following the appropriate procedure. Learners only needed only to identify common factors in the numerator and to see that the denominator is a trinomial quadratic and then factorise according to the learnt procedures.

4.3.3 School B TB1 and TB2 Test 1 26th June 2014 Grade 11

Task 1

Factorise completely

a) $9a + 3ab$

b) $p^3 + 5p^4 - 7p^6$

Coding: 2

Justifications and comments

All the tasks simply expected learners to use procedures that were learnt in class. For example learners were expected to factor out the common factors first and then complete the factorisation. Learners performed this task without making any real connections.

Task 2

Solve the following equations

a) $x - 1 = -4$

b) $5 + x = 3x + 4$

c) Peter has x goats. His brother has twice as many goats. They have 48 goats altogether. How many goats does Peter have?

Coding: 2

Justifications and comments

As with the previous tasks, all tasks emphasised exactly what learners were expected to do in class. In (a) and (b) learners identified like terms, grouped them and then added or subtracted as the procedure dictated. (c) Was supposed to be a high cognitive demand task but it was changed to a low cognitive demand task by alerting the learners that the number of unknown goats can be represented by *x*.

4.3.4 School B TB1 and TB2 Test 2 26th June 2014 Grade 12

Task 1

Expand: $4x(4c^2 + 2x - 5)$

Simplify: $2t - 5m - 7t + 9m$

Solve for a: $3(2a - 6) + 4(2 - 5a) = 18$

Factorise completely: $15p^2q - 30pq^2$

Coding: 2

Justifications and comments

Once again these tasks expected learners to follow the procedures learnt in class. For example, in two of the tasks learners were expected to open brackets and collect like terms and then add or subtract. In the factorisation tasks learners were expected to factor out the common factor.

Task 2

In a shop, pencils cost *p* cents each and erasers cost *e* cents each. Njaai buys 6 pencils and 3 erasers. Write down the total costs, in terms of *P* and *e*.

Coding: 4

Justifications and comments

This task encouraged learners to convert a real life situation into a mathematical model. This task was open-ended in the solutions strategies that the learners were encouraged to use.

4.4 SUMMARY OF TASKS GIVEN

All tasks given are summarised in frequency tables provided next page.

4.4.1 Tasks in class

The frequency table 4.1 provides a summary of the types of tasks that the participating teachers gave to their learners during the observed lessons.

Table 4.1 Frequency of tasks provided during the lessons

Type of tasks	Frequency
Low-level demand tasks	**10**
Memorisation tasks	7
Procedures without connections tasks	3
Higher-level demand tasks	**13**
Procedures with connections tasks	8
Doing mathematics tasks	5

4.4.2 Tasks in tests

The frequency table 4.2 provides a summary of the types of tasks that the participating teachers gave to their learners in the analysed tests.

Table 4.2 Frequency of tasks asked in tests

Type of tasks	Frequency
Low-level demand tasks	**7**
Memorisation tasks	1
Procedures without connections tasks	6
Higher-level demand tasks	**2**
Procedures with connections tasks	1
Doing mathematics tasks	1

4.5 SYNTHESIS OF THE TASKS

From the above it is evident that in the observed lessons the participating teachers provide a near equal amount of lower level cognitive tasks to higher cognitive level tasks. In total the teachers provided 10 of the lower level cognitive tasks and 13 of the higher cognitive tasks. This is affirmed by the literature, which says learners should be provided with different type of tasks ranging from lower cognitive demands to higher cognitive tasks, because different tasks provide different learning opportunities for learners. It is affirmed that some tasks lead learners to a routine way of thinking, while others stimulate students to make wider connections with mathematical ideas. This can be supported by Lester (2007, p. 347) who claims that giving tasks that only require students to perform memorized procedures in a routine manner leads to only limited types of opportunities for student thinking, while tasks that demand engagement with concepts stimulate students' thinking and allow them to make connections that lead to richer opportunities for mathematical understanding.

Classroom tasks should give students opportunities to explore a variety of approaches to mathematical problems in a way that is more sensitive to their thinking rather than forced to follow a specific formalised method. This is supported by Jacobsen, et al., (1985, p. 155), who stated that the best way to assist learners to learn better is to guide them by employing tasks that allow them to think critically. According to them, the type of questions that teachers ask in the lessons must give learners opportunities to use their own approaches in solving them, as long as they are in a position to explain why they choose the approach they have used and how it yields the solution obtained. Mason, Stephens and Watson (2009, p. 27) have also agreed with the idea by suggesting that any sensible approach to teaching combines work on understanding concepts with work on mastering procedures, and combines tasks designed to stimulate learners to express their own thinking using technical terms with tasks designed to highlight the use of important routines.

4.6 THE INTERVIEWS

Firstly, I interviewed two teachers from school A and then the other two teachers from school B. The interviews were conducted in a one-on-one format. I transcribed the interviews and below I present extracts of the interviews and discuss the responses of the four teacher to each question asked. The codes are as follow: IS (Ismael Shapange) the researcher, TA1, TA2 and TB1, TB2 are the codes for the four participated teachers.

IS: What type of tasks do you mostly give to your learners?
TA1*: Classroom activities, tests and home works and sometimes peer assessment tasks.*
IS (fq): What is peer assessment and what impact does it have on learning?
TA1: *Peer assessment is when learners mark other learners' work. Learners usually mark and judge the work of others and I use to ask them to justify their judgements in terms of the ticks or crosses they give. In that process I turn to understand in deeper sense what learners have grasped and what they turn not to understand based on the reasons they give.*
TA2: *Exercises (class works), Home works and tests*
TB1: *Class works, Home works and tests*
TB2: *Exercises, investigations, practical observations, tests and home works.*

Discussion

It is clear that the participating teachers give classroom activities in the form of homework tasks, class tasks, investigations and tests. TA1 indicated the importance of peer assessment, where learners mark each other's work. TA1 feels that peer assessment is a good indicator of what the learners have grasped and understood.

IS: Where do you normally find the tasks that you give to your learners? (What is the source of the classroom tasks that you use?)
TA1: *Textbooks and other printed materials, learners' participation (their contributions or when asking questions on areas they seems not to understand) and sometimes on the Internet.*

TA2: *I normally take tasks from past examinations question papers, textbooks as well as from the internet and other materials such as NAMCOL study guides.*

TB1: *I normally take tasks from past examinations question papers, textbooks as well as from the internet (www.xtremepaper.net).*

TB2: *I takes them from the prescribed textbooks, past question papers and from internet (www.xtremepaper.net).*

Discussion

It is evident from the teachers' responses above, that curricular materials and textbooks are the main source of mathematical tasks used by teachers for classroom instruction. This is supported by Grouws, et al., (2004) as cited in Lester (2007, p. 346) who indicated that many teachers are using textbooks and other printed materials as the main source of mathematical tasks in their classrooms. The textbooks play a major role in providing classroom tasks. Every participating teacher indicated that the textbook is one of the main sources, supplemented by other printed material and the internet.

IS: On what basis do you select the tasks you give to your learners?

TA1: *It is based on various reasons such as: According to the content taught, on how learners ask questions (depend on learners' level of mistakes and misconceptions) and mostly based on the basic competencies as stipulated in the syllabus.*

TA2: *I base my tasks to the basic competencies of the topic as outlined in the curriculum. In addition to that, the type of learners in the class dictates what type of tasks to give.*

IS (fq): How does the type of learners dictate the type of tasks you select?

TA2: *Some learners are very slow in catching up with what is being taught and some are fast. Slow learners need tasks that are easy in order not to discourage them to study mathematics, while fast learners need challenging tasks to keep them busy and enjoy mathematics.*

TB1: *I usually refer back to the basic competencies of the topic I am dealing with. In addition to that, the level of my learners plays a major role in selecting type of tasks to give.*

IS (fq): How does the level of learners play a major role in selecting your tasks?

TB1: *Some learners are very slow in grasping what is being taught, some are moderate and others are fast learners. Slow learners need tasks that are easy in order not to discourage them to study mathematics, while fast learners need challenging tasks to keep them busy and enjoy mathematics.*

TA2: *The level of study for example learners who are doing extended level will be given a question that require more thinking in comparison to those who are doing core. And again how fast or slow the learners are at both levels determine how challenging the question should be.*

TB2: *I base it on topics objectives as stipulated in the syllabus. The type of learners I am working with determines the task. Some learners are below average and some are above average, hence the weight of the questions uses to address this issue. As a teacher I need to know my learners in order to balance between what they knew and what they need to learn.*

Discussion

Form the above it is evident that teachers take their learners 'capabilities' into account when they select tasks. TB1 suggests that 'slow' learners need easy tasks in order not to discourage them. In the same vein TA2 indicated that how fast or slow the learners are at both levels determines how challenging the tasks should be. Kilpatrick, et al., (2001, p. 9) indicated that teacher's expectations about students often influences what tasks they give to the learners and what type of questions they can ask. Although the above stance is understandable, teachers need to ensure that the tasks they set for their learners are challenging. Even if this means that some tasks go beyond the comfort zones of the learners so-called abilities and levels.

IS: How do you determine the levels of difficulty of the tasks that you choose?

TA1: *The category of the question asked.*

IS (fq): May you elaborate more on what you mean by category of the question asked.

TA1: *As you aware that some questions require learners to use formulae to produce a required solution and some requires learners to explore their mathematical knowledge to arrive at required solutions. I usually tell my learners to pay attention to the key word used since it is one that gives the direction to what is required to be done.*

TA2: *I normally look at type of learners I have in the class and level they are doing because the subject is divided into two levels (core and extended).*

TB1: *The level of study for example learners who are doing higher level will be given a question based on higher thinking in comparison to those who are doing extended or core. And again the type of learners I have in the class determines how challenging the question should be or how easy the question should be rephrased.*

TB2: *It all depends on actual work required by the question as well as the precise ways of working out what is asked. How learners should answer questions is well outlined in the syllabus and the key words that guide them on what to do are well stressed too, hence all those can determines the level of difficulty or easiness of the task.*

Discussion

It is evident that the participating teachers take their cues from their perceptions of the learners when selecting the type of tasks that the learners should engage with. TB2 says that the type of work that needs to be covered determines his choice of tasks. This is usually determined by the syllabus. The keywords in the syllabus provide him with the direction of what tasks to give to the learners.

IS: When you select a particular task from the textbook, how do you present it to your learners?

TA1: *I usually photocopy the question and give the handout to my learners without changing the question if I saw that it is of my learners' level of understanding. If the question is testing the basic competencies as they are outlined in the curriculum and it seems not to be familiar to them in terms of finding the direction of where to start I use to give the guide lines.*

IS (fq): What type of guidelines do you give?

TA1: *I can give a hint of what learners knew especially the theorems that can be applied or by breaking the question into sub questions.*

TA2: *If they do not have the textbook, I photocopy the questions and give them hand outs but if they have a copy of the textbook, I give them a page number and a number of question(s) I want them to answer.*

TB1: *I usually photocopy the question and give it to my learners to try it before I intervene, but the question should address the basic competencies as they are outlined in the curriculum.*

IS (fq): You have indicated that you pick some tasks from past questions papers, how do you present such questions to your learners?

TB1: *Since learners use to have past questions papers with solutions, I usually change the question for example if the question require learners to work out the volume of the prism, I will ask them to work out the length of one of its side.*

TB2: *I usually change the question in order for my learners to see that questions are not always similar. I usually manipulate the question in terms of the digits used or by giving information of what was being asked in the original question and ask what was given.*

IS: May you give me an example?

TB2: *Yes, for example if learners were required to calculate the interest earned by investing certain amount of money, I will give the interest and then ask them to work out how much was invested. Another example if the question wanted them to factorise, I will ask them to expand the expression.*

Discussion

It is evident from above that majority of the participated teachers photocopy copy tasks for their learners. TB1 says that he makes sure that the task that he selects aligns with the 'competencies' of the students. This implies that he will not select tasks that are challenging and that are beyond the learners' 'competencies' In my view this process can limit learners' thinking ability because they become accustomed to the questions and their level of engagement becomes routine. They are not challenged sufficiently. It is important that tasks are selected that demand a high level of cognitive demand. It is important also that, when selecting classroom tasks the teacher presents the task in different ways. This encourages learners to think at different levels of cognitive demands. Kilpatrick, et al., (2001, p. 9) indicated that the quality of instruction depends on how students engage with learning tasks. The way students engage with learning tasks should influence how a teacher selects what type of task to give, rather than just copying a question from the learning source without any informed choice.

It is interesting to note that TB2 often changes the question from his sources. This ensures his tasks are not always the same even if it means just 'manipulating the question in terms of digits'

IS: How do you know that a particular task will help your learners to understand what you have taught?

TA1: *I use check if the questions consists the learning objectives covered. I also assess if it address my learners' experience.*

TA2: *I see to it if the task is addressing basic competency. If the learners answer the question correctly, that means that particular learner understand the topic basic competencies.*

TB1: *It is all depend on the basic competency the task is addressing, for example to factorise, solve the unknown given a mathematical equation construct an equations from word problems.*

TB2: *If the task requires the skills indicated in the syllabus.*

IS (fq): May you elaborate more on your statement?

TB2: *I mean the key words that tell learners to perform their mathematical skills outlined in the basic competencies. For example to factorise, solve the unknown given a mathematical equations construct an equations from word problems etc.*

Discussion

Once again the participating teachers use the 'competencies' of their learners as the basis of whether they think the learners will be able to perform a given task. In addition TA1, for example, pointed out that he/she checks if the task addresses the learning objectives and is aligned to the syllabus. TA1 also takes into account his/her learners' experience when selecting tasks. This is in agreement with Stein, et al., (1996, p. 462) who suggested some characteristics of good tasks that promote quality acquisition of mathematical knowledge, which teachers should consider when setting classroom tasks for their learners should include: encouraging problem solving, connecting with students' situations and experiences, engaging students' thinking about important mathematical ideas and requiring more than recall of facts or reproduction of a skill.

4.7 SUMMARY/SYNTHESIS OF THE INTERVIEWS

It is observed that teachers say that they use textbooks as the main source of tasks they give to the learners. In addition, some teachers make use of past examinations question papers as reference sources. Since some schools are connected to the internet, a number of teachers also source tasks from the internet. Based on how teachers select their tasks, they indicate two factors. Firstly, all of them said that the tasks they selected needed to align with the syllabus and meet the official objectives. Secondly, the teachers said that the tasks should also align with the students' capabilities. They suggested that 'slow' learners should be given 'easy' tasks that require low cognitive demand, and those who are more able should be given tasks that require higher cognitive demand.

It is interesting to note that none of the teachers suggested that the level of tasks they came across determined their teaching strategy. It was the teaching of learner context that determined their choice of tasks.

4.8 CONCLUSION

In this chapter, I presented and discussed the findings of the research project. Firstly, I analysed the tasks that the teachers gave to their learners in class. Secondly, I analysed the tasks that the teachers set in their tests. Finally, I presented extracts of the interviews and discussed the responses of the four teachers to the questions asked.

CHAPTER FIVE

CONCLUSION

5.1 INTRODUCTION

This chapter serves to conclude my study by providing a summary of the research findings, followed by a brief discussion of recommendations and suggestions for further research. Thereafter, the potential value of my research will be discussed and finally, I will present the limitations and challenges encountered in this research project as well as some reflections on the whole journey of the project.

5.2 SUMMARY OF FINDINGS

Mathematics education researchers such as Stein, et al., (1996, p. 456) indicated that one of the greatest challenges encountered by mathematics teachers is the identification and use of appropriate tasks that challenge their pupils accordingly. My research confirmed the observation that many teachers source their classroom tasks from available curricular materials and prescribed textbooks. This aligns with Lester (2007, p. 346) who stressed that curricular materials and textbooks are the main source of mathematical tasks used by teachers for classroom instruction. These tasks range from higher cognitive demand tasks to lower cognitive demand tasks. From the eight observed lessons, it was found that, out of 23 analysed tasks, a total of 13 tasks are of higher cognitive demand tasks and 10 are of lower cognitive demand tasks. Jacobsen, et al., (1985, p. 155) stated that the best way to assist learners to learn better is to guide them by employing tasks that allow them to think critically. They further added that the type of questions that teachers should ask in the lessons must give learners opportunities to use their own approaches in solving them, as long as they are in a position to explain why they choose the approach they have used and how it yields the solution obtained. Since some of the tasks were directly copied as they appear in the textbook, or from other material selected, an important question to answer is on what basis the participating teachers selected these tasks.

The participating teachers indicated that they selected tasks based on various factors. These factors include curriculum requirements, types of learners and their experiences. Some teachers indicated that they select tasks based on the type of learners they teach. This is determined by how slow or fast the learners are. This often corresponds with the expectations that the teachers have of their learners. If teachers have low expectations of learners then they would give low level demand tasks and similarly for high expectations, high demand level tasks would be given. Sometimes the teachers decided not to give a certain task simply because he/she believed the learners could not do the task. Other teachers put more emphasis on the curriculum. One of the teachers indicated that the keywords in the syllabus pointed him in the direction of what tasks to give to the learners. Lester (2007, p. 321) define curriculum as 'the substance or content for teaching and learning'. It is an outline of what is expected of teachers to be taught and learners to be learnt. Lester further indicated that curriculum can be defined differently in different environments. It is against this background that some teachers indicated that they selected tasks to align with the curriculum expectations.

From the interviews there was evidence that some teachers selected particular tasks based on the learners experiences. Stein, et al., (1996, p. 462) suggested one characteristic of a good task that promotes the quality acquisition of mathematical knowledge, which teachers should consider when setting classroom tasks for their learners, is to start the lesson with a stimulating question that relates to the learners' environment, and then generate a mathematics task around it. This enables learners to think critically about any mathematical problem given to them because they can then see the usefulness of mathematical ideas within their own environment.

To conclude, teachers set tasks that they believe their learners can understand and are able to engage in. This is influenced by the expectations that they have of their learners and their mathematical experiences as well as by the national curriculum requirements. Even though the literature emphasised the learners' context as one of the factors that determine the type of tasks to be given, this was not very evident in this study.

5.3 RECOMMENDATIONS AND SUGGESTIONS FOR FURTHER RESEARCH

As this was a half thesis the sample size was very small. It would thus be interesting to conduct a similar study with a bigger sample across more grades and across more geographical regions. I also propose that similar research should be conducted with different mathematical topics.

5.4 SIGNIFICANCE

This study is significant because firstly, it encourages teachers to understand the importance of their rationale for selecting classroom tasks. This is also emphasised by Stein and Henningsen (1996, p. 526) who indicated that “the nature of task can influence and structure the way students think hence the nature of task instructions surrounding the mathematical task as given by the teacher equally influence the way students organise their thinking”. Secondly, this study is significant because it highlights the fact that tasks are of different levels of cognitive demand and that teachers thus need to be strategic and critical when selecting appropriate tasks.

5.5 LIMITATIONS AND CHALLENGES ENCOUNTERED

If I had to do this research over again, there are two major aspects that I would take into consideration. Firstly, I would try by all means available to convince teachers that my research is not to expose them in any way. I realised that many teachers are sceptical of researchers and are worried that researchers would expose their weaknesses. They are thus reluctant to participate. Secondly, I would negotiate the time for the interviews more carefully. I would conduct the interviews in a less formal place than a classroom, such as around a dinner table. This would create a more relaxed atmosphere. I learnt that some teachers do not like to sacrifice their time for nothing. I found it difficult to get teachers to commit to agreed times for the interviews.

Another challenge that I encountered was with the recording of the interview sessions with a digital camera. Most of the respondents declined to be recorded. They insisted that I record the session by hand. This proved to be difficult as I could not write down every word they said.

It was a pity that the teachers were not able to provide me with all the tests they set in the past two years. They either could not find them or they had misplaced them. I thus had to do with fewer tests than intended.

5.6 SOME LAST REFLECTIONS

Despite some of the challenges that I highlighted above, I found the research journey a very satisfying one. I learnt a lot about what types of tasks teachers give to their learners and how they select such tasks. At first I found the transcribed interviews and tasks described difficult to analyse, but as I became more and more familiar with my instruments and with the assistance and guidance from my supervisor it became easier and easier. I hope that this thesis will inspire others to engage in research as it was a valuable learning experience for me. I particularly enjoyed engaging with my participants, their classrooms and their interviews.

To conclude this thesis, as a novice and beginner researcher, I wish to encourage interested researchers to conduct more research into the types of tasks that teachers select. This will bring much needed awareness to teachers who continue simply to photocopy tasks and questions from old question papers and textbooks without any proper and critical scrutiny.

5.7 CONCLUSION

In this chapter I concluded my study by providing a summary of the research findings, followed by some recommendations and suggestions for further research. Thereafter, the potential value of my research was discussed. I also provided a brief discussion about limitations and challenges encountered. Finally, I reflected on my journey by looking back at the entire process of how I have grown as a beginner researcher.

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APPENDICES

Appendix A: Permission letter

RHODES UNIVERSITY
Grahamstown • 6140 • South Africa

Education Department

Mathematics Education Masters Research Project

INFORMATION TO RESEARCH PARTICIPANTS

Introduction

I am a part-time MEd student in the Education Department of Rhodes University. My research focus is on identifying the nature of classroom tasks that selected senior secondary school mathematics teachers give to their learners. Your principal has granted permission for me to conduct the proposed research in your school, and you have been selected as a potential research participant for the pilot study. This letter provides important information regarding the study, and formally invites you to take part in the research project.

Description of involvement

The pilot phase will take place over the first term of 2014 academic year. The data collection process involves audio recording individual research participants while they are engaged in their daily teaching as well as analyzing tests you have set. In addition, field-notes will be taken by the researcher. This phase of the data collection process will be conducted during normal academic hours at times convenient to each individual research participant.

Risks and benefits

There are no foreseeable risks involved in participating in the study. Your participation in the study will also further your own understanding on the nature of mathematics task and will ultimately help in selecting appropriate classroom tasks for your learners.

Participants' rights

Your participation in this study is strictly voluntary, and at your own personal discretion. Should you agree to take part in the study you retain the right to withdraw at any point without an explanation. Anonymity and confidentiality are guaranteed at all times, both during the research process itself and in the final written thesis. Pseudonyms will be used to protect your identity in the final thesis.

Video recordings

The audio-visual recordings are necessary for detailed analysis of your strategies when implementing the tasks and learners' engagement.

Consent

After you have read and understood this letter, and made any necessary clarifications about your involvement in the research process, please sign the attached form.

Sincerely yours

Ismael Shapange

MEd student, Rhodes University

Appendix B: Consent forms

CONSENT FORM

I [redacted] in my capacity as the principal, [redacted] secondary school hereby give written consent for Ismael Shapange, in his capacity as a MEd student at Rhodes University, to conduct research at Shaanika Nashilongo secondary school as outlined in the above document. Both parties understand that this consent can be revoked, without explanation, at any time.

SIGNATURE: [signature] DATE: 20/01/14

[redacted]

THE PRINCIPAL

[redacted]

MINISTRY OF EDUCATION
The Principal
[redacted] SSS
Omusati Region
OFFICIAL
2014 -01- 20
P/Bag 507 Okahao
Tel/Fax: [illegible]
Email: shaanikass@hotmail.com

As witness: SIGNED: [signature] DATE: 20/01/2014

MED STUDENT

CONSENT FORM

I [redacted] in my capacity as the principal, [redacted] secondary school hereby give written consent for Ismael Shapange, in his capacity as a MEd student at Rhodes University, to conduct research at Etalaleko secondary school as outlined in the above document. Both parties understand that this consent can be revoked, without explanation, at any time.

SIGNATURE: ____________ DATE: 25/1/2014

[redacted]

THE PRINCIPAL

[redacted]

MINISTRY OF EDUCATION
[redacted]
2014 -01- 2 0
PRIVATE BAG 503
OKAHAO
TEL/FAX: 065-252003

As witness: SIGNED: ____________ DATE: 20/01/14

MED STUDENT

CONSENT FORM

I [redacted] in my capacity as the head of department, mathematics and science at [redacted] secondary school hereby give written consent for Ismael Shapange, in his capacity as a MEd student at Rhodes University, to conduct research at Shaanika Nashilongo secondary school as outlined in the above document. Both parties understand that this consent can be revoked, without explanation, at any time.

SIGNATURE: ______________ DATE: 20.01.2014

[redacted]

HEAD OF DEPARTMENT

MATHEMATICS AND SCIENCE

MINISTRY OF EDUCATION
[redacted] SSS
Omusati Region
Natural Science Department
For Internal use
2014 -01- 20
P/Bag 501 Okahao
Tel/Fax: 085-252250
Email: shaanikass@hotmail.com

As witness: SIGNED: ______________ DATE: 20.01.2014

MED STUDENT

CONSENT FORM

I [redacted] in my capacity as the head of department, mathematics and science at [redacted] secondary school hereby give written consent for Ismael Shapange, in his capacity as a MEd student at Rhodes University, to conduct research at Etalaleko secondary school as outlined in the above document. Both parties understand that this consent can be revoked, without explanation, at any time.

SIGNATURE: ____________ DATE: 20.01.2014

[redacted]

HEAD OF DEPARTMENT

MATHEMATICS AND SCIENCE

As witness: SIGNED: ____________ DATE: 20.01.2014

MED STUDENT

MINISTRY OF EDUCATION
MATHS AND SCIENCE DEPARTMENT
[redacted] Secondary School
2014 -01- 2 0
Private Bag 503, Ongandjera
Tel/Fax: 065-252003
REPUBLIC OF NAMIBIA

CONSENT FORM

I [redacted] in my capacity as a Mathematics teacher at [redacted] secondary school hereby give written consent for Ismael Shapange, in his capacity as a MEd student at Rhodes University, to conduct research by observing my lessons and work through my set tests as outlined in the above document. Both parties understand that this consent can be revoked, without explanation, at any time.

SIGNATURE: ____________ DATE: 20/01/2014

As witness: SIGNED: ____________ DATE: 20/01/2014

MEd STUDENT

CONSENT FORM

I [redacted] in my capacity as a Mathematics teacher at [redacted] secondary school hereby give written consent for Ismael Shapange, in his capacity as a MEd student at Rhodes University, to conduct research by observing my lessons and work through my set tests as outlined in the above document. Both parties understand that this consent can be revoked, without explanation, at any time.

SIGNATURE: ______________ DATE: 20/01/2014

As witness: SIGNED: ______________ DATE: 20/01/2014

MEd STUDENT

CONSENT FORM

I [redacted] in my capacity as a Mathematics teacher at [redacted] secondary school hereby give written consent for Ismael Shapange, in his capacity as a MEd student at Rhodes University, to conduct research by observing my lessons and work through my set tests as outlined in the above document. Both parties understand that this consent can be revoked, without explanation, at any time.

SIGNATURE: ____________ DATE: 31.01.2014

As witness: SIGNED: ____________ DATE: 31.01.2014

MEd STUDENT

CONSENT FORM

I [redacted] in my capacity as a Mathematics teacher at [redacted] secondary school hereby give written consent for Ismael Shapange, in his capacity as a MEd student at Rhodes University, to conduct research by observing my lessons and work through my set tests as outlined in the above document. Both parties understand that this consent can be revoked, without explanation, at any time.

SIGNATURE: ____________ DATE: 31.01.2014

As witness: SIGNED: ____________ DATE: 31.01.2014

MEd STUDENT

Appendix C: Timetable

TAI

[redacted]

observed day

	D1	D2	D3	D4	D5	D6	D7
07:00-07:45		12 J H-LEVEL MATH 12 J	8A MATH 12 J	8A MATH 12 J		8A MATH 12 J	
07:45-08:30	12B MATH 12 J	12A MATH 12 J	12 J H-LEVEL MATH 12 J			12A MATH 12 J	12A MATH 12 J
08:30-09:15			11G H-LEVEL MATH 12 J	12A MATH 12 J	12A MATH 12 J		12B MATH 12 J
09:15-10:00	12A MATH 12 J	8A MATH 12 J		12 J H-LEVEL MATH 12 J			
10:00-10:45	11G H-LEVEL MATH 12 J	11G H-LEVEL MATH 12 J		12B MATH 12 J	12 J H-LEVEL MATH 12 J	12 J H-LEVEL MATH 12 J	
11:15-12:00	8A MATH 12 J				12B MATH 12 J	12B MATH 12 J	
12:00-12:45			12B MATH 12 J	11G H-LEVEL MATH 12 J	8A MATH 12 J		12 J H-LEVEL MATH 12 J
12:45-13:30					11G H-LEVEL MATH 12 J	11G H-LEVEL MATH 12 J	

TA2

Observed day

<table>
<tr><td colspan="8">[redacted]</td></tr>
<tr><td></td><td>D1</td><td>D2</td><td>D3</td><td>D4</td><td>D5</td><td>D6</td><td>D7</td></tr>
<tr><td>07:00-07:45</td><td></td><td></td><td>9B
MATH
9A</td><td>12 I
MATH
9A</td><td>9B
MATH
9A</td><td></td><td></td></tr>
<tr><td>07:45-08:30</td><td></td><td>12 I
MATH
9A</td><td>9A
MATH
9A</td><td></td><td>12G
MATH
9A</td><td></td><td></td></tr>
<tr><td>08:30-09:15</td><td>12D
P-SC
9A</td><td>12D
P-SC
9A</td><td></td><td>9A
MATH
9A</td><td></td><td>12 I
MATH
9A</td><td>9A
MATH
9A</td></tr>
<tr><td>09:15-10:00</td><td></td><td></td><td></td><td>9B
MATH
9A</td><td></td><td>9B
MATH
9A</td><td>12G
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9A</td></tr>
<tr><td>10:00-10:45</td><td>12G
MATH
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9A</td></tr>
<tr><td>11:15-12:00</td><td></td><td rowspan="2">12G
MATH
9A</td><td></td><td>12G
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<tr><td>12:00-12:45</td><td>9A
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<tr><td>12:45-13:30</td><td></td><td></td><td></td><td></td><td>12 I
MATH
9A</td><td>12D
P-SC
9A</td><td>9B
MATH
9A</td></tr>
</table>

Teacher [redacted] TB1

Etalaleko Sec. School, Okahao

	1 7:00 - 7:40	2 7:40 - 8:20	3 8:20 - 9:00	4 9:00 - 9:40	5 9:40 - 10:20	6 11:00 - 11:40	7 11:40 - 12:20	8 12:20 - 13:00	9 13:00 - 13:40
Mo	Maths 11H			Maths 11K	Maths 12H	PSc 12H	Maths 12H		
Tu	ICT 11F	PSc 12H	Maths 12H		Maths 11K	Maths 11H	PSc 12H	ICT 11B	
We		Maths 11K	L Sk 11B	PSc 12H	Maths 12H		Maths 11H	Maths 11K	
Th	ICT 11B	Maths 12H		PSc 12H	Maths 11H		Maths 11H	Maths 11K	
Fr	ICT 11F	Maths 11H			Maths 12H	Maths 11K	PSc 12H	L Sk 11B	

Timetable generated: 2/20/2014

aSc Timetables

Teacher [redacted] T32

Etalaleko Sec. School, Okahao

	1 7:00 - 7:40	2 7:40 - 8:20	3 8:20 - 9:00	4 9:00 - 9:40	5 9:40 - 10:20	6 11:00 - 11:40	7 11:40 - 12:20	8 12:20 - 13:00	9 13:00 - 13:40
Mo		Maths 12G		Maths 9A		Maths 10B	Maths 11D	Maths 12D	
Tu	Maths 12G		Maths 10B	Maths 11D	Maths 12D	Maths 12D	Maths 9A		
We	Maths 11D	Maths 10B	Maths 9A	Maths 12D	Maths 12G	Maths 12G	ICT 10B		
Th	Maths 10B	Maths 11D	Maths 12D	Maths 11D		Maths 9A	Maths 12G		✓
Fr	Maths 9A		Maths 12D	Maths 10B	Maths 11D		Maths 12G		

Timetable generated: 2/20/2014

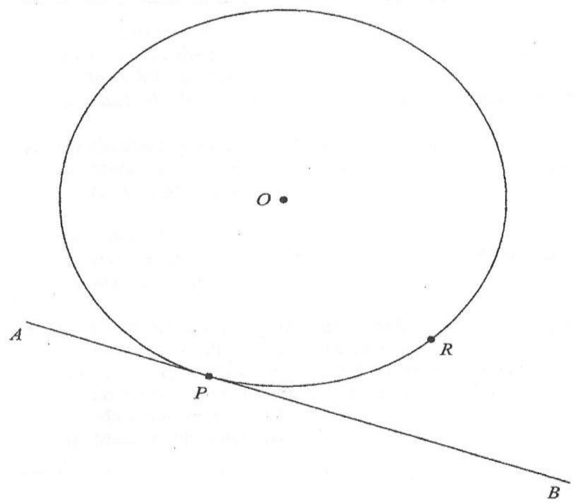
aSc Timetables

Appendix D: Source of tasks given

TA2

Name ______________ Date ______ Grade 12 ___

For Examiner's Use



The diagram shows a circular garden, centre *O*. A straight path *AB* touches the circle at *P*.

(a) **(i)** Draw on the diagram the **diameter** *PQ* and label the point *Q*. [1]

(ii) Without measuring, write down the size of angle *APQ*.

Answer(a)(ii) *Angle APQ*= .... [1]

(iii) The point *R* is marked on the circumference of the circle. Draw the lines *PR* and *QR*. [1]

(iv) Write down the reason why the angle *PRQ* is 90°.

Answer(a)(iv) .... [1]

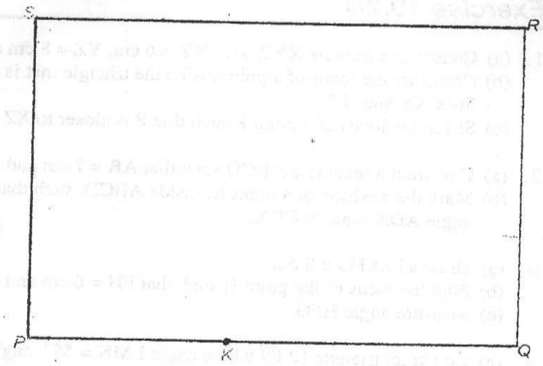
(b) Showing all your construction lines, use a straight edge and compasses only to construct

(i) the perpendicular bisector of *QR*, [2]

(ii) the bisector of angle *PRQ*. [2]

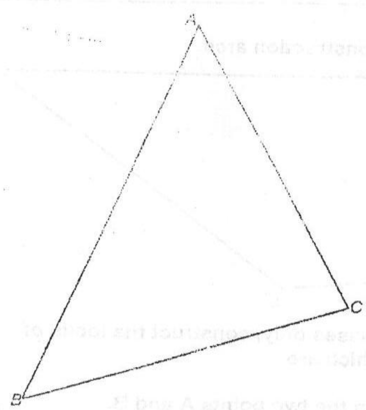
(c) Shade the region of the garden between *PQ* and *QR* which is closer to *R* than to *Q* and closer to *RQ* than to *RP*. [2]

Question 9 is on the next page.



(a) 3 cm from *P*,
(b) equidistance from *K* and *S*,
(c) equidistance from *PQ* and *QR*.

Ex.6.



(a) Draw accurately the locus of points inside the triangle which are

(i) 5 cm from *B*,

(ii) equidistance from *AC* and *BC*.

(b) Shade the region inside the triangle which is more than 5 cm from *B* **and** nearer to *BC* than to *AC*.

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TB1

Exercise 3

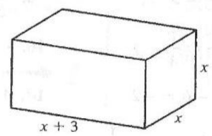
1. Write in a more simple form.
   (a) $2 \times 3x$ (b) $3a \times 5$ (c) $5t \times (-2)$ (d) $100 \times 10a$
   (e) $x \times 2x$ (f) $x \times 4x$ (g) $x \times 2x^2$ (h) $4t \times 5t$

2. Remove the brackets.
   (a) $2(2x+1)$ (b) $5(2x+3)$ (c) $a(a-3)$ (d) $2n(n+1)$
   (e) $-2(2x+3)$ (f) $2x(x+y)$ (g) $-3(a-2)$ (h) $p(q-p)$
   (i) $a(b+a)$ (j) $2b(b-1)$ (k) $2n(n+2)$ (l) $3x(2x+3)$

3. Find four pairs of equivalent expressions or terms.
   A $2a \times 3a^2$ B $6 \times a \times a$ C $4a^2$ D $6a^2$
   E $(2a)^2$ F $3a \times 3a$ G $6a^3$ H $9a^2$

4. Multiply these terms.
   (a) $2a \times 3b$ (b) $x \times 3x^3$ (c) $y \times 2x$ (d) $2p \times 5q$
   (e) $3x \times 5y$ (f) $6x \times 3x^2$ (g) $3a \times 8a^3$ (h) $ab \times 2a$
   (i) $xy \times 3y$ (j) $cd \times 5c$ (k) $ab \times ab$ (l) $2xy \times xy$
   (m) $3d \times d \times d$ (n) $5(2x \times xy)$ (o) $3a \times (-2a)$ (p) $-a \times (-2ab)$

5. A solid rectangular block measures x cm by x cm by $(x+3)$ cm.
   Find a simplified expression for its surface area in cm^2.



6. Remove the brackets and simplify.
   (a) $3(x+2)+4(x+1)$ (b) $5(x-2)+3(x+4)$
   (c) $2(a-3)+3(a+1)$ (d) $5(a+1)+6(a+2)$
   (e) $7(a-2)+(a+4)$ (f) $3(t-2)+5(2+t)$
   (g) $3(x+2)-2(x+1)$ (h) $4(x+3)-3(x+2)$

Exercise 4

Remove the brackets and simplify.

1. $(x+1)(x+3)$
2. $(x+3)(x+2)$
3. $(y+4)(y+5)$
4. $(x-3)(x+4)$
5. $(x+5)(x-2)$
6. $(x-3)(x-2)$
7. $(a-7)(a+5)$
8. $(z+9)(z-2)$
9. $(x-3)(x+3)$
10. $(k-11)(k+11)$
11. $(2x+1)(x-3)$
12. $(3x+4)(x-2)$
13. $(2y-3)(y+1)$
14. $(7y-1)(7y+1)$
15. $(x+4)^2$
16. $(x+2)^2$
17. $(x-2)^2$
18. $(2x+1)^2$
19. $(x+1)^2+(x+2)^2$
20. $(x-2)^2+(x+3)^2$

Exercise 5

Factorise the following expressions completely.

1. x^2+5x
2. x^2-6x
3. $7x-x^2$
4. y^2+8y
5. $2y^2+3y$
6. $6y^2-4y$
7. $3x^2-21x$
8. $16a-2a^2$
9. $6c^2-21c$
10. $15x-9x^2$
11. $56y-21y^2$
12. $ax+bx+2cx$
13. $x^2+xy+3xz$
14. $x^2y+y^3+z^2y$
15. $3a^2b+2ab^2$
16. x^2y+xy^2
17. $6a^2+4ab+2ac$
18. $ma+2bm+m^2$
19. $2kx+6ky+4kz$
20. $ax^2+ay+2ah$
21. $7x^2+x$
22. $4y^2-4y$
23. p^2-2p
24. $6a^2+2a$
25. $4-8x^2$
26. $5x-10x^3$
27. $4\pi r+\pi h$
28. $\pi r^2+2\pi r$
29. $3\pi r^2+\pi rh$
30. $3xy+2x$
31. x^2k+xk^2
32. a^3b+2ab^2
33. $abc-3b^2c$
34. $2a^2e-5ae^2$
35. a^3b+ab^3
36. $x^3y+x^2y^2$
37. $6xy^2-4x^2y$
38. $3ab^3-3a^3b$
39. $2a^3b+5a^2b^2$
40. ax^2y-2ax^2z
41. $2abx+2ab^2+2a^2b$
42. $ayx+yx^3-2y^2x^2$

Appendix E: Analytic tool for analysing classroom tasks

Low-level demand tasks	**Coding**
Memorisation tasks • Task involves exactly reproduction of previously learnt rules, formulae and definitions (e.g. the teacher give examples to be followed before the task). • The hint is given to alert learners what procedure to be followed. • Task focuses on one exact solution to the problem.	**1**
Procedures without connections tasks • Task focuses on a particular procedure to produce correct answers, but no mathematical understanding is developed. • Task emphasises exactly what learners are expected to do and how to go about what is to be done.	**2**
Higher-level demand tasks	**Coding**
Procedures with connections tasks • Tasks are represented in different ways (e.g. diagrams, symbols or words) to assist learners to create a link between the concepts and procedures. • Task involves solving real life problems based on learners' environment. • Task is drawn from learners' experience (e.g. buying, building, fencing or sharing).	**3**
Doing mathematics tasks • Tasks that allow learners to explore the relationship among mathematical concepts. • Task allows learners to analyse mathematical ideas that may lead or may not lead to the solution. • Tasks allow learners to draw general conclusions and justify mathematical ideas based on their real life experiences.	**4**

Adopted from Stein 2000

Appendix F: Lesson observation schedule

Teacher: .... Date of the lesson: .... Grade:			
Time	**Task**	**Coding**	**Comments**

Appendix G: Tests analytic tool

Teacher: .... Date of the test: .... Topic:			
Test	**Task**	**Coding**	**Comments**