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ZUSAMMENHANGEN AND LOGICAL ATOMISM
IN WITTGENSTEIN'S TRACTATUS

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The problem that appears as a tension between atomism (individuality) and hanging together (Zusammenhangen) is one which has concerned me, in one way or another, since I can remember. In this particular form, i.e. as a problem of reconciliation between Zusammenhangen and logical atomism in the Tractatus, it was directly suggested to me by my supervisor, Dr S.T. Sommerville. I hereby record my general indebtedness to him for his supervision and friendship.

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WELSH INCIDENT

'But that was nothing to what things came out
From the sea-caves of Criccieth yonder.'

'What were they? Mermaids? dragons? ghosts?'

'Nothing at all of any things like that.'

'What were they then?'

'All sorts of queer things,
Things never seen or heard or written about,
Very strange, un-Welsh, utterly peculiar
Things. Oh, solid enough they seemed to touch,
Had anyone dared it. Marvellous creation,
All various shapes and sizes, and no sizes,
All new, each perfectly unlike his neighbour,
Though all came moving slowly out together.'

'Describe just one of them.'

'I am unable.'

'What were their colours?'

'Mostly nameless colours,
Colours you'd like to see; but one was puce
Or perhaps more like crimson, but not purplish.
Some had no colour.'

'Tell me, had they legs?'

'Not a leg nor foot among them that I saw.'

'But did these things come out in any order?'

What o'clock was it? What was the day of the week?

Who else was present? How was the weather?'

'I was coming to that. It was half-past three

On Easter Tuesday last. The sun was shining.

The Harlech Silver Band played Marchog Jesu

On thirty-seven shimmering instruments

Collecting for Caernarvon's (Fever) Hospital Fund.

The populations of Pwllheli, Criccieth,

Portmadoc, Borth, Tremadoc, Penrhyndeudraeth,

Were all assembled. Criccieth's mayor addressed them

First in good Welsh and then in fluent English,

Twisting his fingers in his chain of office,

Welcoming the things. They came out on the sand,

Not keeping time to the band, moving seaward

Silently at a snail's pace. But at last

The most odd, indescribable thing of all,

Which hardly one man there could see for wonder,

Did something recognizably a something.'

'Well, what?'

'It made a noise.'

'A frightening noise?'

'No, no.'

'A musical noise? A noise of scuffling?'

'No, but a very loud, respectable noise -

Like groaning to oneself on Sunday morning

In Chapel, close before the second psalm.'

'What did the mayor do?'

'I was coming to that'.

I. INTRODUCTION

The argument presented in this thesis is that Wittgenstein's answer to the question - as to how one proposition can be generated out of another - can show a way in which the reconciliation between logical atomism and Zusammenhangen becomes obvious.

In the Preliminaries an exposition of logical atomism and the Zusammenhangen thesis is given. The way in which the problem appears is then briefly exposed.

The way towards a solution begins with the elucidation of Wittgenstein's picture theory of meaning and the truth-functional analysis of a proposition. The problem is clarified in the question which now arises: i.e., given the truth-functional analysis of a proposition, how can one proposition be generated out of another?

The problem is then shown to vanish in the logical construction of a proposition and of propositions.

II. PRELIMINARIES

- i. The Atomistic Ontology and Logical Atomism: a formal/
material distinction in the Tractatus.¹

In this introductory section I give a brief exposition of an atomistic ontology, as it arises out of the Tractatus, showing that what is crucial to it is the independence criterion. Then I argue that while there might be some difficulty in deciding exactly what the atoms of the Tractatus are, this question/decision is anyway not essential. Given the role of the atomistic ontology in the process of analysis it is simply essential that atoms should be understood logically. (Hence Wittgenstein's atomism is to be understood as a logical atomism.) The formal/material distinction is then elucidated as distinguishing between logic and ontology.

The following propositions and the remarks clarifying them present a clear account of the atomistic ontology:

The world is all that is the case (1).

The world divides into facts (1.2).

A fact is the existence of states of affairs (2).

States of affairs are independent of one another (2.061).

Wittgenstein clarifies the independence criterion by remarking that:

From the existence or non-existence of one state of affairs it is impossible to infer the existence or non-existence of another (2.062).

If philosophy is an activity that aims at the logical clarification of thoughts (4.112), and a thought is a proposition with a sense (4) that reaches out to reality (2.1511; 3.02, 3.13), then it is primarily with respect to thought that doctrines in the *Tractatus* must apply. And so it is with the atomistic ontology since an elementary proposition, the simplest kind of proposition, asserts the existence of a state of affairs (4.21). Given 2.061, elementary propositions must therefore be independent of one another (Black has made this line of argument clear²).

The independence criterion with respect to elementary propositions requires that the sense of one elementary proposition cannot depend on the truth or falsity of another, - i.e. THAN AN ELEMENTARY PROPOSITION HAS SENSE CANNOT DEPEND ON WHETHER ANOTHER IS TRUE OR NOT.

This criterion is crucial: since, without it, sense could not be determinate and analysis could have no end. If the sense of one proposition depends on the truth or falsity of another, then the proposition as a whole cannot have been fully analysed and its sense will not be determinate (3.23).

Atoms are the end of a process of analysis - the analysis of a complex (the world) into simples which constitute the existence of states of affairs. (Here it is merely to be noted that it is OBJECTS that are simple, for Wittgenstein, and these are the end of a process of analysing the constituents of reality - but reality is not ontological, i.e. in the sense of being composed of what exists. This is made clearer in exposition of the world/reality distinction below, pages 8-9).

A process of analysis can begin only if what is to be analysed is assumed to be complex. It can end only if simplicity is realisable. So, complexity is the requirement that analysis can begin - the world must be understood 'to be complex if we are to analyse it at all; simplicity is the requirement that analysis can terminate - unless an analysis of the world terminates in simples, independent and indefinable, the process cannot end and sense will not be determinate.

Realising the independence criterion as the requirement that analysis can terminate, that sense can be determined, merely stipulates what must be in order that anything can be said to be

the case. (This is not, however, a sufficient condition for making sense - see below, section II.ii and section V, page 53.)

What, then, are the atoms of the Tractatus? Are they objects, simple signs standing for objects, i.e. names; states of affairs or elementary propositions asserting the existence of states of affairs; facts or propositions representing reality?

Wittgenstein does not require that any of these candidates be established as the atoms of Tractatus Logico-Philosophicus. Indeed, he nowhere uses the word "atom" in the Tractatus, and, even espousing his initial position as a logical atomism is, at least historically, problematic. The phrase "logical atomism" originates with his friend and former teacher, Bertrand Russell, who certainly used it differently from the way in which it is here being used to characterize Wittgenstein's initial position.

That there are atoms (cf. "there are objects" 4.1272), understood logically, is a formal requirement that analysis can terminate. What atoms there are is accidental - a matter of indifference to a purely logical analysis.

If, however, atoms in the Tractatus are regarded as elementary propositions, then they can clearly be understood as setting limits to the expression of thoughts rather than as describing particular thoughts. It is thus that Wittgenstein's atoms are

regarded logically, rather than ontologically. What propositions are to be regarded as elementary, specifically where analysis does terminate in any particular process of analysis, is not here at issue - is merely accidental: rather, it is that there are elementary propositions - in order that any process of analysis (of language) can terminate - that is essential.

The formal/material distinction

The first proposition in the Tractatus is ontological:

The world is all that is the case (1).

The second major proposition elucidates:

What is the case - a fact /die Tatsache/ - is the existence of states of affairs /Sachverhalten/(2).

In response to a question of Russell's: "What is the difference between a Tatsache /fact/ and a Sachverhalt /state of affairs/?"³

Wittgenstein replies:

Sachverhalt is what corresponds to an Elementarsatz if it is true. Tatsache is what corresponds to the logical product of elementary propositions when this product is true. The reason why I introduce Tatsache before Sachverhalt would want a long explanation.

In order to see how he introduces Sachverhalt consider Wittgenstein's

first comment on 2:

Die Sachverhalt ist eine Verbindung von
Gegenständen (Sachen, Dingen) (2.01).

A state of affairs (a state of things) is a
combination of objects (things) (2.01).

"State of affairs" (Sachverhalt) here, is used in two different
ways:

Sachverhalt I

A state of affairs is a combination of OBJECTS,

- what corresponds to an Elementarsatz if it is possible.

Sachverhalt II

A state of affairs is a combination of THINGS,

- what corresponds to an Elementarsatz if it is actual.

The formal/material distinction appears in this dual use of
Sachverhalt.

Sachverhalt I is a combination of objects and, as such, must
subsist independently of what is the case (2.024). So,
Sachverhalt I neither exists nor does not exist: Sachverhalt I
is substantial; that is, real - what is real, in so far as it
subsists, is purely formal.

Sachverhalt II is a combination of things and, as such, is what
is the case or, at least, what appears to be the case (2.011-2.0122).
So, Sachverhalt II either exists or does not exist (exclusive
sense of "or"): if Sachverhalt II exists, then the Elementarsatz

with which it corresponds is true; if Sachverhalt II does not exist, then the corresponding Elementarsatz is false. Sachverhalt II is ontological; that is, actual - what is actual, in so far as it exists (and the possibility that it does not exist must, of course, also be admitted), is material.

Clearly then, what is real, for Wittgenstein, is not the same as what exists. Roughly, the world stands to reality as actual to possible.

The formal/material distinction can be taken as distinguishing between logic (philosophy) - thinking all the possibilities of objects occurring in states of affairs; and, ontology (science/mythology) - asserting that things stand somehow (thus) in the formation of existing states of affairs. Logic is prior to ontology, since the possibilities of objects combining in formation of states of affairs must be known a priori in order that actual occurrences of things in formation of existing states of affairs can be asserted.

So we have,

(i) Logic - the exploration of that which is formal - possibilities of objects combining in formation to constitute possible states of affairs, Sachverhalt I.

(Then, possible states of affairs can be realised as combining in formation to constitute a possible constituent - neither actual nor non-actual - of REALITY as it can be understood.)

(ii) Ontology - the investigation of that which is material - appearances of things combining in formation to constitute existing states of affairs, Sachverhalt II.

(Then, existing states of affairs can be asserted as combining in formation to constitute an actual constituent - which could have been non-actual - of THIS WORLD as it is understood.)

The Tractatus essentially presents the exploration of logic.

Recalling the preliminary discussion of atomism, then, given the formal/material distinction it is clear that WHAT the atoms of the Tractatus are (objects, states of affairs, propositions, facts, etc.) is not an essential issue. It is that there are atoms (logically independent of one another) that is essential, in order that the process of analysis can terminate.

With respect to propositional analysis it is clearly elementary propositions that are atomic.

Definitional analysis would terminate in names, simple signs.

Graphic, or calligraphic, analysis would terminate in letters, simple strokes. Etc. - In every case the atoms can be understood purely logically - what they are is coincidental, that they are is essential.

ii. Zusammenhagen: the various formulations of a thesis.

Having recognised the formal/material distinction that arises out of Wittgenstein's two ways of using the term Sachverhalt we can now return to the difference between Sachverhalt and

Tatsache and the question as to why the latter is introduced before the former. The importance of this question can be realised in reflection on the direction of analysis - from complex to simple (see page 4 above). Then, once the terminus of analysis has been reached in atoms (the atomistic ontology), the Zusammenhagen thesis is required in order that sense can be made.

Tatsache

A fact - what is the case - is the existence of states of affairs (2, note the plural as emphasised). In other words, a fact is a combination of existing states of affairs ("state of affairs", here, is obviously being used in the second way elucidated above).

"Tatsache is what corresponds to the logical product of elementary propositions when this product is true" (see R37, quoted on page 6 above) - that is, a Tatsache is what corresponds to the logical product of elementary propositions standing for existing states of affairs (Sachverhalten).

The difference between Tatsache and Sachverhalt, then, is clearly the difference between a fact and a constituent of a fact, a state of affairs.

But why now is Tatsache introduced before Sachverhalt, why does Wittgenstein introduce a fact before introducing its constituents?

The answer lies in what has been called the Zusammenhagen thesis⁴.

In considering why Tatsache is introduced before Sachverhalt, it should also be considered why Sachverhalt is introduced before Gegenstand (object), and why Satz (proposition) is introduced before Elementarsatz (elementary proposition); i.e. why the complex is introduced before the simple.

In 2.0121 we are told,

If I can imagine objects combined in states of affairs, I cannot imagine them excluded from the possibility of such combinations.

So objects can only be thought with respect to possible states of affairs they can combine to form. Clearly then, Sachverhalt is introduced before Gegenstand because it is only with respect to Sachverhalt that Gegenstand can be thought at all.

(Simplicity can be realised only in virtue of a recognition of complexity.)

Now, in 3.203, we are told that a name means an object and, from 3.202, we can gather that a name is a simple sign employed in (the formation of) a proposition. Then, in 3.3, a formulation of the Zusammenhagen thesis is given:

only in the nexus [Zusammenhange] of a proposition does a name have meaning.

This is more generally formulated in 3.314:

An expression has meaning only in a proposition.
 All variables can be construed as propositional
 variables.
 (Even variable names.)

This is a more general formulation than that given in 3.3 because an expression is a propositional variable that can itself be either a proposition (3.31) or a name (3.314). It is merely the manner of combination (into a proposition) that is different in either case - i.e. subject to propositional analysis or not.

(A combination can, of course, always be analysed some how. But then the particular form of analysis undertaken must also always have some terminus, or it would be quite senseless, see pages 4-5 and 9 above. It is logical analysis that is primary in the Tractatus. I have used propositional analysis as an illustration of this process, but any other form of analysis would also do to illustrate the logic of the process. I.e. the logic of analysis expresses itself in every form of analysis.)

At first glance it appears that a proposition and a name are quite different. But if they are understood as expressions - "Everything essential to their sense that propositions have in common with one another is an expression (3.31)" - i.e. as propositional variables, then plainly the following holds: if an expression - a propositional variable - is understood as a proposition, then the proposition it

forms together with other propositional variables understood as propositions is subject to propositional analysis; if it is understood as a name, then the proposition it forms together with other propositional variables understood as names is not subject to propositional analysis (but would, of course, be subject to definitional analysis).

(For example, compare: 'If it is peculiar and it is quaint, then it is peculiar' - ' $(p.q) \supset p$ ', with 'John hated Richard' - ' jHr '.)

Whatever the particular form of analysis, the process can only begin if what is to be analysed is acknowledged to be a complex. So, the reason why Tatsache is introduced before Sachverhalt is analogous to the reason why Sachverhalt is introduced before Gegenstand. For, just as I cannot imagine objects excluded from the possibility of their combining in states of affairs, so too I cannot imagine states of affairs excluded from the possibility of their combining in facts.

The Zusammenhagen thesis applies firstly to names standing for objects and more generally to propositions which, of course, are themselves facts, as well as pictures of facts.

Finally, Wittgenstein's introduction of Sachverhalt before Gegenstand, Tatsache before Sachverhalt, Satz before Elementarsatz, (the latter to be explored in detail below) is not at all surprising if it is understood that it is the elucidation of a process of analysis that he is embarking upon. For, as has already been remarked,

complexity - that which is composed (of constituents, elements) - is the requirement that analysis can begin. So complexity must first be admitted if a process of analysis is to be embarked upon, before simplicity can be realised and the process can thus terminate.

Even if the world is infinitely complex $\sqrt{\text{komplex}}$, so that every fact consists of infinitely many states of affairs and every state of affairs is composed of infinitely many objects, there would still have to be objects and states of affairs. (4.2211)

However, I use "complexity", here, as a translation of Zusammengesetztheit - compositeness (5.47).

Of course, the complexity Wittgenstein first introduces is all that is the case, the world - the totality of facts (Gesamtheit der Tatsachen)(1,1.1). It is then the world that is to be analysed into existing states of affairs. But, of course, reality is analysed into existing and non-existing states of affairs.

(See above, pages 8-9.)

In 5.47 "complexity" as Zusammengesetztheit is understood with respect to the general form of a proposition:

It is clear that whatever we can say in advance about the form of all propositions, we must be able to say all at once.
An elementary proposition really contains all logical operations in itself. For 'fa' says the same thing as

$$'(\exists x).fx.x=a'$$

Wherever there is compositeness $\sqrt{\text{Zusammengesetztheit}}$, argument and function are present, and where these are present, we already have all the logical constants.

One could say that the sole logical constant was what all propositions, by their very nature, had in common with one another.
But that is the general propositional form.

This link between complexity and the general propositional form is examined in detail below.

For the present let us remember that things (the world, etc.) must already hang together somehow before any process of analysis begins. Once the process of analysis has terminated in atoms (logically independent of one another) then sense has yet to be made and this requires some interconnectedness between the atoms (Zusammenhang thesis). The Zusammenhang thesis then, requires that the atoms of the Tractatus - propositional variables, elementary propositions, states of affairs, whatever, - hang together somehow: in order that they can have meaning and thus be understood⁵.

III. THE PROBLEM APPEARS

The problem now arises as to how the atomistic ontology can be reconciled with the Zusammenhagen thesis.

The atomistic ontology realises the requirement that analysis must terminate in order that sense can be determinate, i.e. that the analysans must be logically independent of one another.

The Zusammenhagen thesis realises the requirement that it is only within that which is composed (composite, complex) that analytic simples can make any sense. In other words, Zusammenhagen acknowledges that sense can be determined only within a complex, i.e. that analytic simples have meaning only in the formation of a complex.

Now how is it, then, that atoms (individual variables) can be logically INDEPENDENT of one another and yet somehow HANG TOGETHER? Can what is independent of any other hang together with it?

This last question can hardly, if at all, be understood - and so it is perhaps better put: WHAT can relate independent atoms together in the context of a proposition or fact?

Today the difference between a good and a poor architect is that the poor architect succumbs to every temptation and the good one resists... (Vermischte Bemerkungen, p.3e.)

Der Philosoph behandelt eine Frage; wie eine Krankheit.

A philosopher handles a question as a disease. (PI-255, my translation.)

I slip these remarks in, here, to indicate a philosophical attitude to the questions raised in connection with the problem as it appears. The consequences of their (the quotations') not belonging to the Tractatus might be fruitfully explored in some other work, but are not strictly germane to the present one. The temptation to be resisted in the present work is to say that something must relate independent atoms together in the nexus of a proposition or a fact. For it would be a violation of one's understanding of a proposition to suppose that some thing connects the atoms together (3.1432).

IV. TOWARDS A SOLUTION

i. Wittgenstein's Picture Theory of Meaning

This section begins with an exegesis of the principle that objects have signs as their representatives. It shows how, using the formal/material distinction, the picture theory of meaning is founded and built upon this principle. Wittgenstein's remarks on 'logical constants' are then introduced, and the role they play in the *Tractatus* is indicated.

... objects have signs as their representatives...
(4.0312, see also 3.221)

The following texts from the *Tractatus* illustrate how this principle connects with the picture theory of meaning.

We picture facts to ourselves (2.1).

In a picture the elements of the picture are the representatives of objects (2.131).

In a proposition a thought can be expressed in such a way that elements of the propositional sign correspond to the objects of the thought (3.2).

One name stands for one thing, another for another thing.... the whole group - like a tableau vivant - presents a state of affairs (4.0311).

... My fundamental idea is that... there can be no representatives of the logic of facts (4.0312).

If there are primitive logical signs, then any logic that fails to show clearly how they are placed relatively to one another... will be incorrect. The construction of logic out of its primitive signs must be made clear.

If logic has primitive ideas, they must be independent of one another. If a primitive idea has been introduced, it must have been introduced in all the combinations in which it ever occurs... (5.45).

Only propositions have sense; only in the nexus of a proposition does a name have meaning (3.3, also cf. 3.263 and, finally, 6.54).

Now, for ease of exposition, I include a gloss on some of the most important of the terms used here to develop the picture theory.

- OBJECT - a possible (formal) constituent of a state of affairs (cf. 2.027).
- THING - an actual (material) constituent of a state of affairs.
- SIGN - what can be perceived of a symbol (3.32).
- SIMPLE SIGN - an element in an expression which corresponds to an object in a state of affairs (3.2, 3.201); a name which can be used in a proposition; a primitive name (3.202, 3.26).
- PROPOSITIONAL SIGN - a sign used to project a possible situation (3.11); a sign used to express a thought (3.12); a fact (3.14).
- SYMBOL - that part of a proposition which expresses (or is used to express) its sense; the mark of a form and a content (3.31); a propositional sign in its projective relation to the world; a sign-in-use (3.326, cp. 3.328); a proposition (3.12).

- NAME - an element in a propositional sign which corresponds with an object (3.2);
 a simple sign (3.202);
 a primitive sign (3.26);
 that which means an object (3.203);
 the representative of an object (3.22).
- REPRESENTATIVE - an element in a picture (2.131).
- PICTURE - a model of reality (2.12);
 a fact (2.141);
 a proposition (4.01).

Pictures are first explicitly mentioned at 2.0212 and then receive considerable attention right up into the 4's. The principle behind the picture theory of meaning is presented in 4.0312:

The possibility of propositions is based on the principle that objects have signs as their representatives.

where Wittgenstein goes on to announce his "fundamental idea":

that the logical constants are not representatives;
 that there can be no representatives of the logic
 of facts.

Together, the above present Wittgenstein's elucidation of what, in a letter to Russell⁶, he called "the cardinal problem of philosophy": the theory of what can be expressed (thought), and of what cannot be expressed by propositions, but only SHOWN⁷.

In a picture objects have the elements of the picture corresponding to them (2.13).

The pictorial relationship consists in a correspondence between the

elements of a picture and things (2.1514). But "there are, indeed, things that cannot be put into words. They make themselves manifest..." (6.522).

A picture stands within (its) logical form - logical form is what all pictures have in common - this is the key to translatability: it is this that enables one picture to be translated into another (see 3.343, this is not a definition, but a precondition of any correct sign-language - that it must be translatable into any other in accordance with some definitions; cp. 3.334). However, a picture's standpoint is its representational form (2.173 and pages 25-30, below).

A picture presents reality: a situation in logical space: the existence and non-existence of states of affairs (2.11, cp. 4.121). A picture represents its sense: a possible situation in logical space: a possibility of existence and non-existence of states of affairs (2.201, 2.202, 2.221; cp. SHOWS 4.022, 4.121, 4.2)..

As a presentation of reality a picture SUBSISTS - it presents a situation in logical space.

But it is only as a RE-presentation that a picture is a picture of reality. It is only when a sign is used to project a possible situation that a picture represents. A picture represents its sense: its agreement or disagreement with reality or, at least, with a possible world.

So, the elements of a picture, signs, are used to project a possible situation, a model of reality.

The way in which the picture theory of meaning is built upon the principle that objects have signs as their representatives, i.e. using the formal/material distinction elucidated above, can now be made explicit with the aid of a short extract from S.T. Sommerville's commentary in his thesis entitled Types, Categories and Significance, (p.256):

Predications which express formal concepts are called formal, by Wittgenstein, not because they assert anything true of the forms of sentences, or any factual information that sentences ordinarily convey. Instead, formal predicates pertain to what is essential to a language - its form - in virtue of which its sentences can convey information. Form is what is shown by the propositions of a language (when analysed) and this cannot belong to what can be stated in the language, because it is presupposed by the language's being used to state anything:

Propositions can represent the whole of reality, but they cannot represent what they must have in common with reality in order to be able to represent it ... logical form
 Propositions cannot represent logical form; it is mirrored in them
 What expresses itself in language, we cannot express by means of language.
 Propositions show the logical form of reality.
 (Tractatus, 4.12-4.121)

We may now add to this,

material predicates pertain to what is accidental to a language - its content - in virtue of which its sentences do convey information. Content is what is expressed by the propositions of a language when used.

What is said by the propositions of a language, what we express by means of language, is what is represented. The principle that

objects have signs as their representatives shows how the link between language and reality is made. This changes, as the case may be.

What makes the link possible, what propositions must have in common with reality in order to be able to represent it, expresses itself in language - i.e. logical form.

The position is clarified by 3.34:

A proposition possesses essential and accidental features.
 Accidental features are those that result from the particular way in which the propositional sign is produced. Essential features are those without which the proposition could not express its sense.

Recalling 4.0312:

THE POSSIBILITY of propositions is based on the principle that objects have signs as their representatives. (Emphasis added).

But the expression of this principle is purely formal, i.e. quite nonsensical. It tries to express what cannot be said by using language but can only express itself in language: the nonsensical realisation of the possibility of propositions. That there are propositions (the proposition, 'There are propositions') is nonsense. It is just as nonsensical to say, 'There are individuals' (see 4.1272 and page 5 above). Nonetheless, the proposition that this is an individual ('This is an individual') could not express

any sense without the nonsensical realisation that there are individuals. (The proposition, 'There are individuals' is purely formal.)

Before going on to discuss Wittgenstein's "fundamental idea" it is worthwhile to consider the distinction between a sign and a symbol as consisting in the difference between that which CAN be used and that which is actually used to project or represent a possible situation or thought. (Again Sommerville's exegesis is helpful and is freely drawn upon in what follows.)

A sign is what can be perceived of a symbol - so, what mediates between signs and symbols is perception. 'A word or sentence considered in terms of its perceptible qualities - spoken or written - is a sign; a sentence, construed thus, is a "propositional sign"' (Sommerville, *ibid.* p.265). Clearly, without the mediation of some perception, no communication is possible.

But what is it now that can not be perceived of a symbol?

3.32,3.326-8 and 4.01 provide the answer to this question: ITS USE: thinking how a sign CAN BE used with sense in order that it might become a symbol - i.e. projected - and so express a thought.

Before thinking how a sign is actually used with sense, the possibility that a sign can be used with sense must already be known. Then, thinking how a sign can possibly be used, i.e. thinking whether or not it is possible to use a sign in some way

or other in order to express a thought, is the a priori activity Wittgenstein calls "philosophy" (see 4.112).

(How a sign is actually used to express a thought is in each particular case accidental.)

So,

The 'experience' that we need in order to understand logic is not that something or other is the state of things, but that something is: that, however, is not an experience.

Logic is prior to every experience - that something is so.

It is prior to the question 'How?', not prior to the question 'What?' (5.552)

And we are reminded, in 5.551,

... that whenever a question can be decided by logic at all it must be possible to decide it without more ado if we get into a position where we have to look at the world for an answer to such a problem, that shows that we are on a completely wrong track....

Then, finally, "once we know how each individual sign signifies, the rules of logical syntax need, but Wittgenstein says must go without saying" (3.334, emphasis added and order of phrasing changed).

It is necessary, however, to acknowledge that a sign is a representative, an element in a picture that is an element in an expression, only when it is used somehow to represent something

else, an object.

For A to depict B, A must be both like B and unlike B. A is like B in respect of what A and B have in common - which Wittgenstein calls "pictorial (iconic) form" (Form der Abbildung: 2.16). What distinguishes A from B is what makes A a picture of B, rather than a reduplication of B - which Wittgenstein calls "representational (symbolic) form" (Form der Darstellung: 2.173-4).....

... Considered in use as expressing something meaningful - together with the conditions for its being meaningful - an expression is a symbol: 3.326: "In order to recognise a symbol by its sign we must observe /pay attention to, reflect upon, achten/ how it is used with a /sic/ sense". And a sign does not have ... sense unless "taken together with its logico-syntactical employment" 3.327. (Sommerville, *ibid.* pp.258 and 265.)

And then 3.334 applies (and see also 3.33).

Wittgenstein's picture theory of meaning, his proper theory of symbolism⁸, is based on the principle that objects have signs as their representatives. The signs in a picture that represent objects are called 'names'. The signs are, of course, perceptible, but it is just the fact that they are used in SOME combination ("logico-syntactical employment") that produces a symbol and so expresses a communicable thought.

Signs could not be used if their capacity for meaning (that they can represent objects) were not already known (see 3.263).

Knowledge is prior to expression: we must know what can be expressed before we can express it.

Obviously, this knowledge cannot be articulated; otherwise it would not be prior to expression.

A name means an object - so, what mediates between names and objects is meaning (or referring, Wittgenstein uses "Bedeutung" sometimes meaning "meaning", sometimes "reference").

Meaning requires a reaching out to reality (2.1511): a method of projection.

Now a legitimate question (one which can be answered, see 6.5) is: how do names mean objects? Wittgenstein's answer is given in 3.3: "... only in the nexus of a proposition ..." (which is an annunciation of the Zusammenhagen thesis taken from Frege's Grundlagen der Arithmetik, section 60-1; see also 4.0311, 4.22ff, especially 4.466 and cf. 3.142).

This answer is correlated with Wittgenstein's notion of SENSE - the agreement or disagreement with the possibilities of the existence and non-existence of states of affairs - so that only propositions can express such agreement or disagreement. And, in so far as they express sense, propositions are symbols. The sense of a proposition is expressed by the particular concatenation of names which, in immediate combination (Verbindung), form the proposition.

This combination itself is not what is expressed but what is SHOWN by the expression of its constituents arranged in a particular way, i.e. according to a particular method of projection. A particular method of projection, mode of signification, requires the prior adoption of a standpoint and stipulation of values. Then the proposition expressed in this way, i.e. from this standpoint, represents its sense correctly or incorrectly.

Contrast - a purely logical picture: it makes sense or it does not make sense; but also, it neither makes sense nor does it not make sense.

The contrast between a purely logical picture and one that makes sense is elaborated in detail below. Here it is simply pertinent to note the preconditions for making any sense at all: that the limits of the possibilities of making sense are senseless; that thinking the possibility of the existence of states of affairs presupposes thinking the possibility of the non-existence of states of affairs - in other words, the possibility of expressing a true proposition presupposes the possibility of expressing a false one.

A picture conveys meaning through its representational form. That a picture has a representational form is what it has in common with every other picture. Representational form is what makes a picture a picture rather than merely a reduplication (see Sommerville quoted page 26 above). So, that a picture has

a representational form is essential. What this representational form is, i.e. how its representational form is shown, is accidental. ("How the description of the propositions is produced is not essential." (3.317) The way in which the proposition is produced is perhaps, anyway, preferably regarded as being coincidental with the adoption of a particular standpoint.)

So, THAT a picture has a representational form is purely logical. The presentation of this representational form, its projection, occupies a situation in logical space. Its presentation is what it has in common with reality, logical form.

WHAT a picture represents, and HOW it is produced (projected), depends upon the particular mode of expression used. Logic, for Wittgenstein, is prior to the question as to HOW a picture is produced (5.552).

In summary, then, what every picture has in common with every other is that it is a possible way of signifying, i.e. that it has a representational form: what every picture has in common with reality is logical form, the form of reality. Logico-pictorial form is what every picture, of whatever representational form, must have in common with reality in order to be able to depict it (2.151, 2.18, 2.182).

A picture stands in a projective relation to reality. Wittgenstein's method of projection is to think the sense of a proposition, i.e. to agree or disagree with possibilities of existence and non-

existence of states of affairs. It is just the expression of this agreement or disagreement that is meaningful. How it is meaningful depends upon the particular way in which it is expressed, its mode of signification.

Now, Wittgenstein's fundamental idea that logical constants are not representatives should be considered in its historical juxtaposition with Frege's and Russell's sense of 'logical objects' or 'logical constants'. (See also the remarks leading up to 5.4 and then 5.46 and 5.47ff.) However, it is clear that for Wittgenstein at least⁹, the 'logical constants' do not represent any thing (or object) at all. This they cannot do because they have no meaning. Nevertheless, that logical constants are signs of some sort is obvious. But equally obvious is that they are not elements in an expression which correspond with objects. Precisely, they are elements which PRESENT formal relations between objects, but which, in doing so, actually mean nothing (cf. 5.43, 6.121). They simply show how things can stand in relation to one another, i.e. they cast a net. Put another way, they present a particular formal mode of signification through which content can become meaningful, can be significantly communicated (see, for eg. Engelmann, p.105), i.e. a logical syntax.

For Wittgenstein the logical constants are punctuation marks signifying that logical OPERATIONS can be performed (5.4611). An operation is a mark of a difference between forms (5.2341, cp. 5.47, 5.473); i.e. of a difference between possibilities of

objects combining in states of affairs.

A proposition (picture) represents agreement or disagreement with the possibilities of existence and non-existence of states of affairs. The analysis of a proposition must show how all possibilities of the existence and non-existence of states of affairs can be represented (pictured) - by, eg., agreeing and disagreeing with the possibilities of existence and non-existence of states of affairs - as well as showing the limits of representation.

It is with the limits of representation, the expression of thoughts, that Wittgenstein is primarily concerned.

Wittgenstein does not give an analysis for any specific proposition (picture). Rather he presents the form for any such analysis.

Thus his concern is to show that and how analysis requires limits in order that sense can be determinate.

The analysis of propositions which shows THAT a proposition agrees and disagrees with (and hence is true or false of) existing and non-existing states of affairs, Wittgenstein calls the 'truth-functional analysis' of a proposition. Thus, if we are to understand the requirement of Wittgenstein's logical atomism, that this analysis must terminate in independent elementary propositions, the process of truth-functional analysis must be described. It is the task of the next section to do this.

ii. The Truth-functional Analysis of a Proposition.

The truth-functional analysis and description of a proposition is produced below. What is essential to this account of a proposition is that it is MERELY A DESCRIPTION OF SYMBOLS that lays everything - i.e. every truth-possibility - open to view, and STATES NOTHING ABOUT WHAT IS SIGNIFIED (see 3.317 and PI-126).

The truth functional analysis of a proposition must show all the possible truth-functional ways in which a proposition can be used to make sense, as well as showing the limits of these possibilities of representation. That is, it must show the complete truth-functional description of a proposition's agreement and disagreement with the possibilities of existence and non-existence of states of affairs.

This section, then, begins with Wittgenstein's elucidation of a proposition. It argues that what is essential for the truth-functional analysis of a proposition is that there be a clear and acknowledged terminus to it. Elementary propositions are formally understood to fill this berth. Then the limits of representation are realised in tautology and contradiction.

A proposition is a truth-function of elementary propositions.
 (An elementary proposition is a truth-function of itself.) (5)

The truth-functional analysis of a proposition has as its bases elementary propositions. (A proposition is the result of a truth-operation on its bases - elementary propositions, or propositions whose bases are elementary propositions, and so on.)

A proposition is an expression of agreement and/or disagreement with truth-possibilities of elementary propositions (4.4).

Truth-possibilities of elementary propositions mean possibilities of existence and non-existence of states of affairs (4.3 -i.e. of the states of affairs for which the elementary propositions stand).

Elementary propositions consist of names in immediate combinations (4.221,5.55).

Any analysis of a proposition requires a clear and acknowledged terminus. However, the clarity of any "acknowledged" terminus can always be called into question. The usual retort seems paradoxically founded in obscurity, viz., INDEFINABILITY. (Wittgenstein would perhaps better be understood as having recourse to SIMPLICITY - see above page 4 and, for eg., J. Griffin (1964), pp.8-12.)

In the truth-functional analysis of a proposition the analysis terminates when the constituents (propositions) of the complex (proposition) are analysed into some IMMEDIATE combinations of simple signs, i.e. concatenations of names.

However, granting that the analysis of propositions must bring us to elementary propositions which consist of names in (some) immediate combination is obvious, Wittgenstein admits that the question - as to how such combinations of NAMES into elementary propositions comes about - is raised (4.221). This seems clearly a question which cannot be decided without more ado, and so it is a question that cannot be decided by logic ALONE (5.551-2). Also it is not logic but the application of logic that decides what elementary propositions there are (5.557)¹⁰.

But Wittgenstein gives the precondition for answering this question with a formulation of the Zusammenhagen thesis at 4.23. (My exegesis works towards elucidating this requirement in juxtaposition with the requirement of logical atomism.)

What we can say a priori is that elementary propositions consist of simple signs (names standing for objects) in immediate combinations. In other words, only that some immediate combination of names in the formation of an elementary proposition be realisable is essential, in order that the truth-functional analysis of a proposition can have a clear and acknowledged terminus.

What is further a priori is the form these combinations can take. These must be determinable prior to the question 'How?' (see 5.552, quoted above on page 23).

THAT names stand in immediate combinations to constitute elementary propositions is a priori. HOW names stand in immediate combinations to constitute elementary propositions can never be decided a priori. (It depends on what form the immediate combination does take with respect to a representational form and the adoption of a particular

standpoint.)

Names can stand ANYhow in combination to form elementary propositions - so long as that is somehow IMMEDIATE.

But as we are unable to give the number of names with different meanings, so we are unable to give the SPECIFIC composition of elementary propositions (5.555,5.55). We can know that they have a composition and that it consists in an immediate combination of simple signs, but what these simple signs are cannot be said a priori - even though their meanings must already be known (3.263); and to give any specific form of their immediate combination would be completely arbitrary (5.554). (It would also be to adopt a representational standpoint and thus be the beginning of making sense.)

Therefore, the specific composition of elementary propositions is strictly irrelevant to an exploration of logic - which deals with what is essential, and this sets the limits to what is accidental. In this regard then, what is necessary for the truth-functional analysis of a proposition is that there be a clear and acknowledged terminus to it. This terminus consists in what Wittgenstein calls "elementary propositions".

A summary exegesis of the logical atomism elaborated in the Preliminaries above should suffice to recall the independence criterion, and thus to show that the truth-functional analysis of a proposition must terminate in INDEPENDENT elementary propositions.

An elementary proposition, the simplest kind of proposition, asserts the existence of a state of affairs. From the existence or non-existence of one state of affairs it is impossible to infer the existence or non-existence of another. States of affairs are independent of one another. Therefore, elementary propositions must be independent of one another, and one elementary proposition cannot be deduced from another. More strongly, that an elementary proposition has sense cannot depend upon whether another is true or not.

Clearly then, the truth-functional analysis of a proposition must terminate in elementary propositions, and these elementary propositions must be INDEPENDENT of one another.

Consider now Wittgenstein's elucidation of a proposition as a truth-function of elementary propositions. This elucidation consists in the truth-functional analysis of a proposition into its constituent elementary propositions. To attend to this analysis the truth-possibilities of an elementary proposition must first be described. These are:

1. True, when the state of affairs it represents exists;
2. False, when the state of affairs it represents does not exist (4.25, and see 4.28).

These truth-possibilities can be represented in a schema of the following kind (where 'T' means 'true', 'F' means 'false', and

'p' stands for an elementary proposition):

P
T
F

(see 4.31)

Now, if a proposition is an expression of agreement and/or disagreement with truth-possibilities of elementary propositions, then there are exactly four groups of truth-conditions the expression of which are propositions that are obtainable from the truth-possibilities of a single elementary proposition (see 4.42, 4.431, 4.45).

So, given the truth-possibilities of an elementary proposition, shown for example in the schema $\begin{matrix} P \\ T \\ F \end{matrix}$, if agreement with truth-possibilities is expressed by correlating the mark 'T' (true) with them in the schema and disagreement is expressed by correlating the mark 'F' (false) (4.43), then the following groups of truth-conditions (i.e. truth-functions) are obtainable:

| | <u>Truth-conditions of the proposition/truth-function</u> | <u>Truth-possibilities of the elementary proposition</u> |
|-----|--|--|
| I | T the proposition is 'T' when the elementary proposition is 'T', F the proposition is 'F' when the elementary proposition is 'F'. | |
| II | F the proposition is 'F' when the elementary proposition is 'T', T the proposition is 'T' when the elementary proposition is 'F'. | |
| III | T the proposition is 'T' when the elementary proposition is 'T', T the proposition is 'T' when the elementary proposition is 'F'. | |
| IV | F the proposition is 'F' when the elementary proposition is 'T', F the proposition is 'F' when the elementary proposition is 'F'. | |

(Proposition I truth-functionally analysed yields a function of an elementary proposition such that: when the elementary proposition

is true, I is true; when the elementary proposition is false, I is false.

The truth-functional analysis of proposition II yields a function of an elementary proposition such that: when the elementary proposition is true and the state of affairs it represents exists, the proposition II - the elementary proposition as a function of itself - is false; when the elementary proposition is false, proposition II is true.

The truth-functional analysis of proposition III yields a function of an elementary proposition such that: when the elementary proposition is true and the state of affairs it represents exists, the proposition III - the elementary proposition as a function of itself - is true; when the elementary proposition is false and the state of affairs it represents does not exist, proposition III is true.

Proposition IV yields a function of an elementary proposition such that: when the elementary proposition is T, IV is F; when the elementary proposition is F, IV is F.)

From the above we can see that there is a group of truth-conditions obtainable from the truth-possibilities of an elementary proposition which is the expression of those truth-possibilities themselves. That is, there is a proposition (truth-function) which is the expression of the elementary proposition constituting it. (This proposition is contingently true, i.e. true, in accordance with the truth-possibility that is true when the state of affairs its constituent elementary proposition represents exists; false, when

the state of affairs does not exist.)

The truth-conditions obtainable from truth-possibilities express ways of agreeing and disagreeing with them. These ways of agreeing and disagreeing can produce propositions that make sense - in which case the propositions they express are contingent; or they can produce propositions that make no sense (lack sense) - in which case the propositions they express say nothing, are tautologies or contradictions. (This point is elaborated below.)

The truth-possibilities of two elementary propositions, say 'p' and 'q', can be represented by the following schema:

| | |
|---|---|
| p | q |
| T | T |
| T | F |
| F | T |
| F | F |

(see 4.31)

The following truth-functions are then obtainable:

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| T | F | T | T | T | F | F | F | T | T | T | F | F | F | T | F |
| T | T | F | T | T | F | T | T | F | F | T | F | F | T | F | F |
| T | T | T | F | T | T | F | T | F | T | F | F | T | F | F | F |
| T | T | T | T | F | T | T | F | T | F | F | T | F | F | F | F |

(see 4.45, 5.101)

Clearly, for n elementary propositions (standing for states of affairs) there are 2^n truth-possibilities, i.e. possibilities of existence and non-existence of the states of affairs for which they stand (see 4.27); there are 2^{2^n} possible groups of truth-conditions (truth-functions/propositions) (see 4.42, 4.45, and cp. Black (1964), p.222 - with which this formula agrees: and Kenny (1973), p.88 - with which it disagrees¹¹.)

So, a simple table could be drawn up as follows:

| | | | | | | |
|---|---------------|-------|-------|----------|----------|---|
| no. of elementary props. (n) | n=1 | 2 | 3 | 4 | 5 | . |
| no. of truth-possibilities (K_n) | $2^n=2$ | 4 | 8 | 16 | 32 | . |
| no. of truth-functions (L_n) | $2^{2^n}=2^2$ | 2^4 | 2^8 | 2^{16} | 2^{32} | . |

For Wittgenstein, "the world is completely described by giving all elementary propositions, and adding which of them are true and which false" (4.26). Having added which of them are true and which false one can produce all possible expressions of agreement and/or disagreement with these truth-possibilities.

If all elementary propositions are given then all their truth-possibilities can be stipulated and consequently all the possible groups of truth-conditions obtainable from these truth-possibilities can be given (see 4.51).

Among the possible groups of truth-conditions for n elementary propositions, there are always two extreme cases: the proposition called a "tautology", and the proposition called a "contradiction" (truth-conditions: TT, FF; TTTT, FFFF; etc. (see 4.46)). Neither can determine reality in any way (4.463); i.e. tautology and contradiction do not stand in any representational relation to reality (4.462).

However, although they signify nothing (show that they say nothing, 4.461) they are nonetheless part of the symbolism - that part which limits it (4.4611).

The description of the truth-functional analysis of a proposition given above shows all the possible ways in which a proposition can agree and disagree with the possibilities of existence and non-existence of states of affairs, i.e. all the possible ways in which a proposition can represent its sense as well as the LIMITS of a proposition's representing its sense.

These limits Wittgenstein calls "tautology" and "contradiction".

So, having described the truth-functional analysis of a proposition in such a way that all the possibilities of representation are shown, the limits of these possibilities are also shown, viz., in tautology and contradiction. And it is just these propositions, tautology and contradiction, which are purely logical.

Tautology and contradiction are the limiting cases - indeed the disintegration - of the combination of signs (4.466, emphasis added).

They are the extreme cases among the possible groups of truth-conditions that are obtainable from the truth-possibilities of a given number of elementary propositions that can be arranged in a series (4.45, 4.46).

The truth-conditions of a proposition determine the range that it leaves open to the facts (4.463).

A tautology leaves open to reality the whole of logical space - expressing agreement with all truth-possibilities, it must be

true on all conditions. A contradiction leaves no point in logical space open for reality - expressing disagreement with all truth-possibilities, it must be true on no conditions. So, neither can determine (or represent) reality in any way. Neither can express any sense or stand in any representational relation whatever.

Tautologies and contradictions lack sense (they are SINNLÖS) but they are not nonsensical (UNSINNIG) since they already belong to a system of symbolism (symbolic notation) in which propositions show what they say.

Tautologies and contradictions show that they say nothing.

All other propositions in the notation show that they say something (that they represent some sense). They show that and how they agree and/or disagree with the possibilities of existence and non-existence of the states of affairs represented by the elementary propositions constituting them.

Tautology and contradiction also show that there can be no specific relation that must obtain (i.e. no logically necessary relation) between elementary propositions.

For, in a tautology, as well as in a contradiction, the logical combination of signs is not actually a combination at all.

What corresponds to a determinate logical combination of signs is a determinate logical combination of their meanings. It is only to the uncombined signs that absolutely any combination corresponds.

In other words, propositions that are true or false for every situation cannot be combinations of signs at all, since, if they were, only determinate combinations of objects could correspond to them.... Tautology and contradiction are the limiting cases - indeed the disintegration - of the combination of signs. (4.466)

That there can be no logically necessary relationship between elementary propositions realises the independence criterion of logical atomism: that from the truth or falsity of one elementary proposition the truth or falsity of another cannot be inferred. Moreover, it is only in virtue of the limiting cases of the combination of signs in a proposition that the independence criterion can be realised. This can be shown as follows.

Logic limits the range of possible relations between elementary propositions to their truth-possibilities and the groups of truth-conditions (truth-functions) obtainable from them (4.463, see also 6.124, 4.51).

The limiting cases - the disintegration of the combination of signs, i.e. where there is complete agreement or complete disagreement, no conditions attached - are tautology and contradiction. So, the limiting cases of the combination of signs are not themselves any actual combination of signs but the disintegration of any such combination.

However, really,

the signs are still combined with one another even in tautologies and contradictions - i.e. they stand in certain relations to one another: but these relations have no meaning, they are not essential to the symbol (4.4661).

Tautologies and contradictions - the limiting cases of the combination of signs - are the propositions of logic.

In all other propositions that there is some relation between the constituent elementary propositions is a logical requirement in order that the proposition formed can make some sense: what this relation is is circumstantially dependent upon the (logically possible) way in which the elementary propositions happen to stand together to form the proposition.

That there must be some relation or other between independent elementary propositions realises the Zusammenhagen thesis.

There appears now to be a tension between the realisation of the independence criterion of logical atomism: that there can be no relation of logical necessity between elementary propositions, and the realisation of the Zusammenhagen thesis: that there must be some relation or other between independent elementary propositions.

Does this tension dissolve in the consideration that any relation there can be between elementary propositions is not logically necessary - that there is some relation (even in the disintegration of the combination of signs) is necessary, what that relation is is not necessary?

Consider further: the world is analysable into INDEPENDENT individuals (objects, elementary propositions, atoms, plants,

people, stones, planets, solar systems, stars, finite centres, etc.). These individuals are logically independent of one another. That is, there is no (one particular) essential connection between them. However, that there must be some connection between them is the requirement referred to above as the Zusammenhagen thesis. Even in tautologies and contradictions the signs - whatever they stand for - are still combined with one another: but these relations have no meaning, they are not essential to the symbol. In other words, in tautologies and contradictions the propositional sign is not being used in any particular way. But obviously the propositional sign can always be used in some way or another - provided that the limiting cases of its usage be acknowledged.

What the connection between logical atoms is, HOW independent individuals do in fact combine, is purely coincidental.

What the particular connection can be, is a matter for logic to decide. And logic cannot deny any possible connection a priori. So, ANYTHING GOES in reflection upon the great mirror that is logic.

Nonetheless, it is not as if EVERYTHING goes (i.e. as if logic is arbitrary or merely accidental), for

The exploration of logic means the exploration of everything that is subject to law (6.3).

But the propositions of logic do not purport to say anything at

all. They are the limiting cases of what can be said. One reaches them through what can be said - untying, as it were, unravelling, until one can say nothing at all (4.115)¹².

An elucidation of non-assertion is given in 6.2322 (This remark primarily concerns identity, but the point it makes can be generalised to any combination of expressions):

It is impossible to assert the identity of meaning of two expressions. For in order to be able to assert anything about their meaning, I must know their meaning, and I cannot know their meaning without knowing whether what they mean is the same or different.

This may be glossed with Wittgenstein's remark in 3.263:

The meanings of primitive signs can be explained by means of elucidations. Elucidations are propositions that contain the primitive signs. So they can only be understood if the meanings of those signs are already known.

(Problems of interpretation involved in 3.263 are well known. But by recourse to this remark here I wish only to indicate how that there are names with meaning - i.e. that some combinations of names can issue in independent elementary propositions - is a logical prerequisite, whilst recognising as well that no specific way of combining names is prescribed by logic (see pages 34-5, above). Thus (one might say) Wittgenstein's point is a transcendental one.)

Remembering now that "what interests logic are only the unasserted propositions" (Notebooks, Appendix I, p.96, First Ed.) in 6.113 we

are told that "it is the peculiar mark of logical propositions that one can recognise that they are true from the symbol alone...." (emphasis added). Then 6.124,

The propositions of logic ... presuppose that names have meaning and elementary propositions sense; and that is their connection with the world. It is clear that something about the world must be indicated by the fact that certain combinations of symbols - whose essence involves the possession of a determinate character - are tautologies. This contains the decisive point. We have said that some things are arbitrary in the symbols that we use and that some things are not. In logic it is only the latter that express: but that means that logic is not a field in which we express what we wish with the help of signs, but rather one in which the nature of the absolutely necessary sign speaks for itself ... (first emphasis added).

This extensive quotation from 6.144 should be considered in the light of 6.53-4, as well as that of 7. At this point it is apposite to recall an example of non-assertion given in the "Notes dictated to G.E. Moore in Norway", Notebooks, Appendix II. Consider an instance of modus ponens (i.e. a tautology) in the symbolic notation of Principia Mathematica,

$$\phi a. \phi \supset \psi a : \supset : \psi a$$

which could, of course, as well have been written:

$$\overline{p} \cdot (p \supset q) \supset q \quad \text{or} \quad \overline{(p \supset q)} \cdot p \supset q.$$

But this is NOT a proposition, says Wittgenstein. The obvious conclusion therefore appears to be that a logical proposition is NOT a proposition. I can understand this only if I recognise

that a logical proposition says nothing: it is not a proposition which says anything. A logical proposition does not even purport to say anything.

It does, however, SHOW what cannot be said - i.e. "the form of a proof" (6.1264, emphasis added). By merely looking at the three groups of signs: ϕa , $\phi a \supset \psi a$, ψa , I can "see", says Wittgenstein, "what is called the truth of a logical proposition, namely, of the proposition $\phi a \cdot \phi a \supset \psi a : \supset: \psi a$ " (ibid., p.107 emphasis added). This proposition says nothing. It does, however, show the form of a proof in logic. There are no conditions attached to the truth of this proposition. It is purely formal and says nothing about the meaning of the signs that constitute it.

Now, the essential (logical) description of the symbol consists in the stipulation of the values of its variables without reference to their meaning (3.317, 6.126). As J. Griffin puts it (1964, p.134), "We stipulate the values of a variable without reference to the meaning of any symbol". And, as Sommerville has added (private communication, 1981), "the values of a variable are all the possible meanings (references) that a sign can have - not any particular one". "The stipulation of values IS the variable" (3.316, emphasis added). Moreover, the stipulation of values itself can vary. But this is not to say that the forms of objects, possibilities of their occurring in states of affairs (2.0141), can change : rather, it is to say that there are different forms (2.0271). What is unalterable are the possibilities of objects entering into states of affairs. In other words, that there are objects somehow

occurring in states of affairs (a formal proposition) is unalterable. All the possible ways in which objects can occur in states of affairs should be recognisable in the general form of a proposition, i.e. the general form of the stipulation of values. (See section V below.) (The general form of a proposition/truth-function is given first, in words, in 4.5: THIS IS HOW THINGS STAND; and then, in signs, in 6: $\sqrt{p, \xi, N(\xi)}$. In 5.47 this is understood as "the sole logical constant" ... what all propositions have in common with one another.)

So, the question as to how one can recognise that tautologies are true from the symbol alone can only be fully answered in the description of the general propositional form (below). However, if we try to state the consequences of Wittgenstein's "transcendental" point - that something must be SHOWN by the fact that certain combinations of symbols are tautologies - then we are apt to commit ourselves to nonsense. Yet it is clear enough that, in order for there to be tautologies (and contradictions), certain combinations of symbols must have a determinate character (6.124 - this is what I've called "the transcendental point"), but no specific combinations (no particular relation between signs) is necessary and this, as my exposition began, is based on the independence criterion of logical atomism: that a particular combination of symbols has meaning cannot depend on the sense of any other. But that signs be used in some IMMEDIATE combinations to project independent states of affairs is based on the requirement that objects have signs as their representatives only WHEN they combine in states of affairs (the Zusammenhagen thesis).

Thus we are left with the question: how can independent combinations of signs-in-use (i.e. elementary propositions) hang together in the formation of (non-elementary) propositions?

So now we have to consider "the point at which the logical construction of propositions begins" (5.233). The lead we are given by Wittgenstein (following, as he says, Frege and Russell) is the construal of a proposition as "a function of the expressions contained in it" (3.318).

So far the truth-functional description of a proposition has been given and it has been argued that what is essential to the truth-functional analysis of a proposition is that it should terminate in independent elementary propositions. Then, having described the truth-functional analysis of a proposition in such a way that all the possibilities of representation are shown, the limits of these possibilities have also been shown, viz., in tautology and contradiction.

However, the ways in which the truth-functional description of a proposition can be produced remain unclear. To clarify them, that point at which one proposition is, or rather can be, produced out of another in a logically meaningful way must be elucidated. That there is a logically meaningful (ordered, lawful) way in which one proposition can be generated out of another must be shown, as well as that there is more than one.

The process that articulates this demonstration is a truth-operation. The task of this next section, then, is to show how one proposition can be produced out of another.

V. THE LOGICAL CONSTRUCTION OF A PROPOSITION,
 in which the problem vanishes.

Logic must look after itself (5.473, Notebooks, first
 entry, dated 22.8.14).

Now it is clear that for n elementary propositions there are 2^n possibilities of existence and non-existence of states of affairs (i.e. 2^n truth-possibilities) and that there are 2^{2^n} groups of truth-conditions that are obtainable from the truth-possibilities of n elementary propositions. The question as to HOW one proposition can be generated out of another (or others) then arises (see 5.233). That the question is a meaningful one, i.e. that it can be answered, and that it can be answered in a number of ways will be shown. However, before it can be answered, it must be shown that it is possible for a proposition to be constructed out of its bases, elementary propositions.

A precondition for this demonstration is formulation of the Zusammenhang thesis.

The problem then arises as to how the atomistic ontology, the end of analysis, can be reconciled with the Zusammenhagen thesis, the precondition for logical construction. The problem vanishes in the demonstration that there are ways in which a proposition can be logically constructed out of independent elementary propositions. The account which follows is supposed to dissolve any tension that might be felt between logical atomism and Zusammenhagen in the Tractatus. Instead it shows that they are both simply reflections of one another. Thus the problem of reconciliation vanishes, - but it would reappear again if an atomistic ontology were to be asserted. (But this reappearance of the problem is not strictly germane to my argument since it is with logical atomism that I am concerned.)

A proposition is a truth-function of elementary propositions.
(An elementary proposition is a truth-function of itself.)(5)

This remark elucidates, amongst others, that made in 3.318:

Like Frege and Russell I construe a proposition as a function of the expressions contained in it.

Not wishing to digress here, into a comparative criticism, I take 5 simply as a clarification (Erklärung) of the way in which Wittgenstein construes a proposition.

The central position of a proposition in the Tractatus is obvious: it is that which can make sense. It is that which can be used to make

sense, or even nonsense. It is that which represents reality so that we can make sense of it. It is that whereby we can make ourselves understood.

The move from analysis to construction is critical here.

The requirement of analysis in general is that sense be determinable. The requirement of any specific analysis is that sense be determinate. (See pages 3-5, 9-15, above.)

That sense be determinate requires that simple signs be possible (3.23). In other words, analysis must terminate in indefinables. For, signs can be understood as simples only when the point has been reached at which they cannot be analysed or defined further.

An indefinable is indefinable, a simple is simple, only if it is logically independent of any other indefinable or simple, i.e. only if it cannot be further analysed ("dissected" 3.26, "anatomised" 3.261). An indefinable, then, is logically independent in so far as its meaning does not depend upon the meaning of any other. This requirement of analysis establishes the atomistic ontology in the Tractatus.

But, as has been argued above, the atomistic ontology in the Tractatus is better understood as a logical atomism. What the atoms are is not essential. In this thesis it is elementary propositions standing for states of affairs that are regarded as the logical atoms of the Tractatus. (Both atomism and Zusammenhangen apply to ATOMS, however they are pictured - see 3.314 quoted below.)

After an interval of seven numbered remarks from 3.23 we reach:

Only propositions have sense; only in the nexus
(Zusammenhange) of a proposition does a name have
meaning (3.3).

This Zusammenhangen thesis is consolidated in:

An expression has meaning only in a proposition.
All variables can be construed as propositional
variables.
(Even variable names.) (3.314)

The crux of the Zusammenhangen thesis is that we can only express
our thoughts in propositions. That is, in order to make ourselves
understood at all, we have to use expressions in a propositional
form.

Once the terminus of analysis has been reached and the world is
realised as consisting of independent states of affairs representable
by independent elementary propositions, no sense can be made of it
until these INDEPENDENT elementary propositions can be understood as
standing somehow or other together to form a proposition, i.e. to
constitute one fact representing another. The fact that they
constitute, the proposition that they form in order to represent the
existence of states of affairs, a fact, is obviously dependent upon
the way in which they stand together to form it. This depends upon
the ways in which they can stand together to form it. These ways can
be logically understood (and determined) as a result of the stipulation
of the values of the independent variables involved (or used).

And, logically, one can stipulate the value of any variable independently of any other. But having done this one can now make sense of this stipulation of values only in so far as they (the values of the variables) are specifically used to project a possible situation. What possible situation they can be (are) used to project is dependent upon the values stipulated AND the way in which these values hang together in formation of the proposition, i.e. their agreement and disagreement with the truth-possibilities of the constituent elementary propositions (4.26). It is only thus that the elements of the propositions under analysis can have meaning.

- But they must nonetheless be logically independent of one another.

The problem now seems to arise as to how it is that independent elementary propositions standing for independent states of affairs can be used, indeed can only make sense, in the formation of a proposition representing a fact (i.e. in the presentation of a picture of reality). For it seems to run counter to the independence criterion if elementary propositions must stand (or hang) together in the formation of a proposition. This necessity obviously makes the formation of a proposition dependent upon elementary propositions standing (somehow) together, but does it not also require that there be some relation between the elementary propositions themselves so that they can stand somehow together? And if this is the case, if there must be some relation between elementary propositions in order that it (the proposition formed) can make sense, then how can they be logically independent of one another?

The "answer" lies within the realm subject to the law: Simplex sigillum veri (5.4541). We must simply realise the requirement of the end of analysis that elementary propositions standing together in formation of a proposition are logically independent of one another. This realisation is shown by using elementary propositions like this. Simplex sigillum veri, - the demonstration of a "solution" to a "problem of logic". (The context of logic must be essentially simple, then its application can be as difficult as one cares to make it.)

As for the question as to how elementary propositions standing together in formation of a proposition can be independent of one another: it doesn't itself actually make any sense at all. It is simply a realisation of the requirement of (the end of) analysis. Saying this one doesn't make any sense, but showing it one shows that and how sense can be determined. The independence criterion fulfils the requirement of the determinacy of sense. But, fulfilling the requirement that sense be determinate does not itself make sense (- it is a necessary but not a sufficient condition for making sense). In order to make sense now, in accordance with the independence criterion, a formulation of the Zusammenhagen thesis is necessary. The point here is that the independence criterion ON ITS OWN does not allow any sense to be made: yet sense can only be made in virtue of it - TOGETHER WITH the Zusammenhagen thesis. I.e., sense is made determinable by understanding the independence criterion together with the Zusammenhagen thesis.

The way towards the reconciliation between Zusammenhagen and logical atomism in the Tractatus is pointed out in the following remarks:

Truth-functions of elementary propositions are results of operations with elementary propositions as bases. (These operations I call truth-operations.) (5.234).

The concept of successive applications of an operation is equivalent to the concept 'and so on' (5.2523).

All propositions are the results of truth-operations on elementary propositions.
 A truth-operation is the way in which a truth-function is produced out of elementary propositions. It is the essence of truth-operations that, just as elementary propositions yield a truth-function of themselves, so too in the same way truth-functions yield a further truth-function. When a truth-operation is applied to truth-functions of elementary propositions, it always generates another truth-function of elementary propositions, another proposition. When a truth-operation is applied to the results of truth-operations of elementary propositions, there is always a single operation on elementary propositions that has the same result (5.3, first two emphases added).

Just as the question as to how one proposition is generated out of another arises from 5.233, so the question as to how elementary propositions yield a truth-function of themselves arises from 5.3.

The atomistic ontology can be expressed by remarking that elementary propositions must be independent of one another; and, that one elementary proposition cannot be deduced from another, nor can the sense of one elementary proposition depend on whether or not another is true.

However, one proposition can be deduced from another, and a proposition is composed of, is made up of, elementary propositions - i.e. the sense of a proposition does depend upon the truth or falsity of its constituent elementary propositions.

How then, is it possible that the logical construction of a proposition can begin? How can a proposition be constructed out of independent elementary propositions? How can one proposition be deduced from another? Can an elementary proposition on its independent own, by itself, make any sense?

The point at which the logical construction of propositions can begin is the point at which the process of analysis of propositions has been completed and all elementary propositions have been given. Then the logical construction of a proposition can begin by producing expressions of agreement and/or disagreement with the truth-possibilities of elementary propositions.

Given elementary propositions we can assign truth-values to them independently of one another.

The number of truth-possibilities of each elementary proposition on its own is just two (and four propositions are constructible from each independent elementary proposition). But the number of truth-possibilities assignable to elementary propositions when they stand together in order to constitute (Zusammengestellt) a proposition is obviously dependent upon the number of elementary propositions so standing together. (See below.)

Given an elementary proposition, truth-values are assignable to it in accordance with its truth-possibilities (see pages 36-38, above).

In order to make sense by means of the truth-possibilities of an elementary proposition we must agree or disagree with them, i.e. we must reach out to reality, towards the world for which the elementary proposition stands, the state of affairs it represents. Thus we take up a position in accordance with it - i.e. in agreement or disagreement. In order to take up any such position at all, we must have understood - rightly or wrongly - how things stand. That is, we must have understood that there could be some things standing immediately together in formation of the state of affairs represented by this elementary proposition. WHAT is prior to the question as to HOW these things stand is the logical exploration of all possibilities and their limits.

A truth-operation shows how one can move - with respect to the truth-possibilities of elementary propositions - from one representation to another, as well as to the limits of representation.

Now, however many elementary propositions are used to constitute a proposition - and in whatever way truth-values are independently assignable to the essentially independent elementary propositions - they nonetheless, in so far as they can be used to constitute a particular proposition at all, must stand in some way (or another) together in order to form (Zusammengestellt) that proposition. How they do in this fact happen to stand together in formation of

this proposition is coincidental, but it is a coincidence logically limited by how (the ways in which) they can stand together. And these ways comprise the logical forms of propositions - tautologies and contradictions, the limiting cases described in section IV, above.

The logical limits of the coincidental formation of propositions are set in accordance with the given (number of) elementary propositions. Then all the propositions constructible can be laid open to view and the fact that all propositions are thus given fixes their limits (4.51).

Now a truth-function can be clearly understood as a group of truth-conditions obtainable from the truth-possibilities of elementary propositions by being in agreement or disagreement with the states of affairs they represent. Then the truth-function yielded is an expression of this agreement and/or disagreement. Starting with a single elementary proposition, expressions of agreement and disagreement with its truth-possibilities yield truth-functions. (And then a proposition can be understood as the expression of a group of truth-conditions, 4.431.¹³.)

5.3 begins to elucidate the way in which a truth-function is produced out of elementary propositions as a TRUTH-OPERATION (see also 5.6). A logical operation is what effects the generation of one proposition out of another, or others. The result of such an operation, together with its bases and the operation, is a proposition, a propositional sign in its projective relation to the world. (The starting point

is an elementary proposition, i.e. the assertion of the existence of a state of affairs.) The fundamental claim (in 5.3) that "when a truth-operation is applied to the results of truth-operations on elementary propositions, there is always a single operation on elementary propositions that has the same result" will first be discussed in general and then explicated in detail.

The claim, of course, rests on the essence of a truth-operation understood as a process of producing one proposition out of another - the expression of agreement and disagreement with the truth-possibilities of elementary propositions that produces propositions with sense or lacking sense. (By "lacking sense" I mean without sense: that cannot be used to make sense, and which can assert nothing.) But the embracing context within which the claim is to be understood is provided in the following remarks (et passim¹⁴):

An elementary proposition really contains all logical operations in itself (5.47).
 ... one could derive logic from a single primitive proposition (6.1271).
 ... the sole logical constant /is/ what all propositions, by their very nature, /have/ in common with one another (5.47).
 This is how things stand (4.5).

Returning to 3.3441, we may remember that:

For instance, we can express what is common to all NOTATIONS FOR TRUTH-FUNCTIONS in the following way: they have in common that, for example, the notation that uses '-p' ('not p') and 'pvq' ('p or q') can be substituted for any of them. (Capitals added.)

A problem seems to arise now with respect to the compositeness (Zusammengesetztheit) of elementary propositions and the requirement that they be unanalysable and logically independent of one another. The problem relates back to 5.47 (and could be traced further back through 3.263 to 2.0123-4):

It is clear that whatever we can say in advance about the form of all propositions, we must be able to say all at once.
 An elementary proposition really contains all logical operations in itself
 Wherever there is compositeness [Zusammengesetztheit], argument and function are present, and where these are present, we already have all the logical constants.

It is clearly articulated in 5.515:

If the sign 'p' in 'pvq' does not stand for a complex sign, then it cannot have sense by itself: but in that case the signs 'pvp', 'p.p', etc., which have the same sense as p, must also lack sense. But if 'pvp' has no sense, then 'pvq' cannot have a sense either.

The problem might be resolved simply by continuing:- so 'pvp' must have sense - i.e. the same sense as 'p'. Thus 'p' has the complexity of 'pvp' in it already - as well as the complexity of '-p'. (And so on.)

(Here we have an elucidation of 5.47.)

But must we now not conclude that 'p' must stand for a complex sign? And then are we not forced to admit that the sign for an elementary proposition must be complex?

Well, whatever complexity there can be in the sign for an elementary proposition it is only with respect to its truth-possibilities and, accordingly, to its SENSE - not with respect to its COMPOSITION, which must, anyway consist of names in immediate combination.

Leaving aside now the problem raised by 5.515, if we turn to 5.5, 5.502, 5.503 and 5.51, we can follow how the fundamental claim presented above can be worked out in detail.

In section IV.ii above the construal of a proposition as a function of the expressions contained in it was laid open to view. On page 37 the truth-functions obtainable from the truth-possibilities of an elementary proposition were shown. Now it remains to show how a truth-operation can be used to arrange truth-functions obtainable from the truth-possibilities of elementary propositions into an ordered series as, for example, the one given in 5.101 (represented on page 39 above).

An operation marks - gives expression to (5.24) - a difference between forms (5.241).

If there are exactly four different truth-functions (ways in which a proposition can agree and disagree with the truth-possibilities of the elementary propositions composing it, see 4.42 and 5) of an elementary proposition, then there must be a truth-operation that can mark the difference between their forms in such a (logically meaningful) way that one can be shown to be generatable out of an other.

In 5.5 Wittgenstein claims that: (Capitals added into text):

Every truth-function is a result of successive applications to elementary propositions of THE OPERATION

'(----T)(ξ ,.....)'

In 5.502 we are told that we can rewrite '(----T)(ξ ,.....)' as 'N(ξ)'. And now, N(ξ) is the NEGATION of all the values of the propositional variable ξ .

5.503 remarks that:

It is obvious that we can easily express how propositions may be constructed with this operation, and how they may not be constructed with it ; so it must be possible to find an exact expression for this.

Wittgenstein's "exact expression for this" is N(ξ), or, more properly and fully, $\sqrt{p, \xi, N(\xi)}$ (6). He indicates how the use of this expression is to be worked out in 5.51:

If ξ has only one value, then N(ξ) = -p (not p);
if it has two values, then N(ξ) = -p.-q (neither p nor q).

Recalling 5.1311 the articulation of an exact expression for the logical construction of a proposition using a truth-operation becomes obvious. That is, "the inner connection (Zusammenhang) becomes obvious " (5.1311).

The question is: how can the truth-operation $N(\xi)$ be used to produce every truth-function?

If ξ has two values, 'p' and 'q', then $N(\xi) = \neg p \cdot \neg q$, i.e. $N(\xi) = p/q$, can be used to generate all the ways in which a proposition can be formed by agreeing and disagreeing with truth-possibilities of the constituent elementary propositions in the following manner:

| | | | |
|-----------|-----------|-------------|-----------|
| 1 | 2 | 3 | 4 |
| - p . - q | - 1 . - 1 | - I . - 2 | - q . - I |
| F T F F T | T F T T F | T F F F T | F T F T F |
| F T F T F | T F T T F | T F F F T | T F T T F |
| T F F F T | T F T T F | T F F F T | F T F T F |
| T F T T F | F T F F T | F T F T F | T F F F T |
| 5 | 6 | 7 | 8 |
| - q . - 3 | - p . - I | - p . - 3 | - 4 . - 6 |
| F T F T F | F T F T F | F T F T F | T F T T F |
| T F T T F | F T F T F | F T F T F | F T F T F |
| F T F T F | T F T T F | T F T T F | T F F F T |
| T F T T F | T F F F T | T F T T F | T F T T F |
| 9 | 10 | 11 | 12 |
| - 8 . - 8 | - 5 . - 6 | - 10 . - 10 | - 7 . - 7 |
| F T F F T | T F T T F | F T F F T | T F T T F |
| T F T T F | F T F T F | T F T T F | T F T T F |
| T F T T F | T F F F T | T F T T F | F T F F T |
| F T F F T | F T F T F | T F T T F | F T F F T |
| 13 | 14 | 15 | 16 |
| - 3 . - 3 | - 4 . - 4 | - 5 . - 5 | - 6 . - 6 |
| T F T T F | T F T T F | T F T T F | T F T T F |
| T F T T F | F T F F T | F T F F T | T F T T F |
| T F T T F | T F T T F | T F T T F | F T F F T |
| T F T T F | T F T T F | F T F F T | T F T T F |

Using '/' we would have, for example:

$$\begin{array}{ccc} 1 & 2 & 3 \\ p/q & p/q \cdot / \cdot p/q & p/q \cdot / : p/q \cdot / \cdot p/q \end{array}$$

Or, we could have generated the truth-functions as follows:

$$\begin{array}{cccccccc} 1' & 2' & 3' & 4' & 5' & 6' & 7' & 8' \\ 2 / 2 & 1 / 1 & 13 / 13 & 14 / 14 & 15 / 15 & 16 / 16 & 12 / 12 & 9 / 9 \\ \\ 9' & 10' & 11' & 12' & 13' & 14' & 15' & 16' \\ 8 / 8 & 11 / 11 & 10 / 10 & 7 / 7 & 3 / 3 & 4 / 4 & 5 / 5 & 6 / 6 \end{array}$$

The abbreviations used above can be worked out, for example, as follows:

$$\begin{array}{cc} 2 & 14 \\ -(-p.-q). -(-p.-q) & -(q.-(-p.-q)). -(q.-(-p.-q)) \\ T F T F F T T T F T F F T & T F T F T F T F F T T T \\ T F T F T F T T F T F F & F T F T T F T F T F F F \\ T T F F F T T T T F F F T & T F T F T T F F F T T T \\ F T F T T F F F T F T T F & T T F F F T F T T F T T \end{array}$$

A confusion to be avoided is the temptation to abbreviate Negation to Verneinung. So, operating on 1 above, we may compare:

$$\begin{array}{ccc} -(-p.-q) & \text{with} & -(-p.-q). -(-p.-q) \\ \underline{\text{Verneinung}} & & \underline{\text{Negation}} \end{array}$$

Finch (to whom this exposition is basically indebted: see 1971, pp.121-3) "abbreviates" his explication of the operation Negation by citing (on p.122) the Verneinung above as the second application of the operation. The third application he cites is also strictly incorrect, i.e. it is a Verneinung on the conjunction of the results

of the first and second applications, viz., $\neg(\neg p \cdot \neg q) \cdot (p \vee q)$);
 whereas it should have been given as the Negation: $\neg(\neg p \cdot \neg q) \cdot \neg(p \vee q)$:
 or, more fully, $\neg(\neg p \cdot \neg q) \cdot \neg(\neg(\neg p \cdot \neg q) \cdot \neg(p \cdot \neg q))$.

A minor deviation from Finch in the above is that I have followed the convention of assigning the values, in order, TTF and TTF to p and q respectively, rather than TTF and TTF to p and q respectively as does Finch (and, incidentally, Wittgenstein, 4.31). The importance of a change in such a convention is that it demonstrates that the use of a particular truth-operation in the logical construction of propositions - rather than any other, or combination of others - and the sign for that operation, is merely conventional. By the above I have simply shown a way of producing all the possible groups of truth-conditions obtainable from the propositional variable, ξ , having two values, 'p' and 'q' (understood as elementary propositions) using Wittgenstein's logical operation Negation.

In contrast to 5.101 then (N.B. 5.46), my schema would read, less elegantly:

- (FFFT) 1 (p,q) In words: Neither p nor q. ($\neg p \cdot \neg q$)
 (TTF) 2 (p,q) " " : p or q. ($p \vee q$)
 (FFFF) 3 (p,q) Contradiction (p and not p, and q and not q) ($p \cdot \neg p \cdot q \cdot \neg q$)
 (FTFF) 4 (p,q) In words: p and not q. ($p \cdot \neg q$)
 (FTTF) 5 (p,q) " " : Not q. ($\neg q$)
 (FTTF) 6 (p,q) " " : q and not p. ($q \cdot \neg p$)
 (FTTF) 7 (p,q) " " : Not p. ($\neg p$)
 (TFFT) 8 (p,q) " " : If p then q, and if q then p. ($p \supset q$)
 (FTTF) 9 (p,q) " " : p or q, but not both. ($p \vee q \cdot \neg(p \cdot q)$)
 (TTTT) 10 (p,q) " " : p and q. ($p \cdot q$)
 (FTTF) 11 (p,q) " " : Not both p and q. ($\neg(p \cdot q)$)
 (TTTT) 12 (p,q) " " : p
 (TTTT) 13 (p,q) Tautology (If p then p, and if q then q) ($p \supset p \cdot q \supset q$)
 (TFFT) 14 (p,q) In words: If p then q. ($p \supset q$)
 (FTTF) 15 (p,q) " " : q
 (TFFT) 16 (p,q) " " : If q then p. ($q \supset p$)

Notice, each truth-function (group of truth-conditions obtainable from the truth-possibilities of elementary propositions) could be taken as defining a different logical operation.

So, for example,

(FFFT) (p,q) In words : Neither p nor q. (p/q)
 (FFTF) (p,q) " " : Neither p nor not q. ($p\text{\$}q$)
 (FTTT) (p,q) " " : Not both p and q. ($p\downarrow q$)
 (TTFT) (p,q) " " : Either p or not q. ($p\text{\$}q$)

And then any one of these could be used as the single logical operation on elementary propositions that has the same result as any other, or others, applied to the results of truth-operations on elementary propositions (see 5.3). Wittgenstein uses (FFFT)(p/q), which he calls Negation.

If ξ has only one value, 'p', can $N(\xi) = -p$ be used to generate all the ways in which a proposition can agree and disagree with the truth-possibilities of 'p'? At first sight this question may appear more difficult to answer than it is. But this is only until we remember that $N(\xi) = -p$ is an abbreviation for $N(\xi) = p/p$, i.e. $N(\xi) = -p.-p$.

Negation can now be used to mark the difference between all the propositions obtainable from the truth-possibilities of a single elementary proposition:

| | 1 | 2 | 3 | 4 |
|---|-------|----------|---------|-------------------|
| p | p / p | p/p./p/p | p/p./ p | p/p./p: / :p/p./p |
| T | T F T | F T F | F F T | F T F |
| F | F T F | T F T | T F T | F T F |

(Here '.' and ':' are used in lieu of brackets.)

Using 'alternative denial', where $p \downarrow p = \neg p$, the same result could be obtained:

| | 1 | 2 | 3 | 4 |
|---------------|-------------------------------|--|--|-------------------------------------|
| p | $p \downarrow p$ | $p \downarrow p \cdot \downarrow \cdot p \downarrow p$ | $p \downarrow p \cdot \downarrow p : \downarrow : p \downarrow p \cdot \downarrow p$ | $p \downarrow p \cdot \downarrow p$ |
| $\frac{T}{F}$ | $\frac{T \ F \ T}{F \ T \ F}$ | $\frac{F \ T \ F}{T \ F \ T}$ | $\frac{T \ F \ T}{T \ F \ T}$ | $\frac{F \ T \ T}{T \ T \ F}$ |

The difference between the way in which one uses 'joint denial' and the way in which one uses 'alternative denial' is merely conventional.

It is clear that one can obtain the same result (achieve the same purpose) either way (cf. 6.1202). Alternative denial can be obtained from joint denial in the following manner:

| $p/p \cdot / \cdot q/q$ | $:/p/p \cdot / \cdot q/q$ |
|-------------------------|---------------------------|
| F | T |
| F | F |
| T | T |
| T | F |

, and vice versa (see eg., Copi, Symbolic Logic, p.237, 3rd Ed.).

Elementary propositions are the bases out of which propositions can be produced. The way in which this can be done is through the expression of a relation between the truth-functional structures of their bases, the elementary propositions constituting them. The expression of this internal relation is a truth-operation (5.2, 5.22, 5.232). Given 5.54 it is clear that the way in which a proposition is constructed must be a result of the way in which elementary propositions are correlated with one another. (Logically this must

not be a way which makes the truth-value of one elementary proposition dependent upon what truth-value another has.)

Then the expression of the relation between the way in which elementary propositions are correlated and the truth-functional structure of the proposition is the expression of the truth-operation which produces the proposition out of the elementary propositions. The operation can then, of course, be reapplied to produce another proposition out of this one. And so on ... until every truth-function is shown.

Thus using a truth-operation a proposition can be shown to mirror the truth-possibilities of its constituent elementary proposition(s).

This mirroring process, effected by logical operations, lays everything, i.e. every truth-possibility, open to view. It is the process whereby anything, as well as nothing, can be said by agreeing and disagreeing with possibilities of existence and non-existence of states of affairs. It is the process whereby all proposition (results of truth-operations on elementary propositions) can be produced. This fact - that all propositions can be produced - fixes their limits (see 4.51, and 6.124).

Negation can be understood as applying the operation Verneinung to the TOTALITY of disjoint (independent) elementary propositions thus:

$$-(pvqv\dots)$$

Or, Negation can be understood as applying the operation Verneinung "jointly" to each independent elementary proposition thus:

$$-p.-q.-\dots$$

Understood in this way the logical operation Negation clearly reverses the sense of the logical sum totality of independent ATOMISTIC elementary propositions (symbolising the atomistic ontology in the Tractatus) such that its mirror image, the logical product of all negated elementary propositions or, which would amount to the same, the logical product of all elementary propositions - IN CONNECTION with one another (showing Zusammenhagen in the Tractatus), is presented.

When both the atomistic ontology and its reflection have been laid open to view then we must have a complete (i.e. whole) symbolic representation of reality - the existence and non-existence of states of affairs and the various ways of agreeing and disagreeing with these. We must also, therefore, have a complete logical picture of facts, a mirror-image of the world (3, 6.13).

Thus, the complete analysis and description of a proposition which symbolises the atomism in the Tractatus TOGETHER WITH the logical construction of a proposition (via truth-operations) which shows **how** the Zusammenhagen thesis works in the Tractatus reveal reality reflected in logic. There is nothing left to be reconciled. The problem has vanished. The problem vanishes in logic - "the great mirror" (5.511) of reality, only to reappear in its application to the world - the bother about HOW WHAT is pictured rather than the realisation THAT it is reflected.

VI. CONCLUSION

The argument presented in the foregoing can now be summed up thus: the demonstration that and how one proposition can be produced out of another shows the way in which logical atomism can be reconciled with Zusammenhagen.

The problem vanishes in this demonstration:- i.e. the problem which appears in the tension between logical atomism and Zusammenhagen - when the exposition of the former reveals its independence criterion for the determinancy of sense, and the exposition of the latter reveals its requirement that things hang together somehow in order that they can be understood - disappears in the demonstration that there are ways in which a proposition can be logically constructed out of independent elementary propositions, and thus that one proposition can be generated out of another.

However, the problem reappears when an atomistic ontology is asserted: because, whenever atoms are understood in some particular way - rather than purely logically (formally) - then, although that they are independent of one another is (always) a logical requirement that the analysis undertaken can end, when the atoms are stipulated to be just these individuals, i.e. when just these values are given to the atomic variables, the problem reappears as to how just these individuals can be understood TOGETHER in order now somehow to make sense (or be made sense of) - with respect to the particular stand-

point adopted (see above section II.i et passim).

It has been argued above that Zusammenhagen and logical atomism are the preconditions for making any sense. The problem appears in the tension between these preconditions. It vanishes when they are shown to be reconciled in the logical construction of a proposition.

What has been shown then, (with respect to propositional analysis), is that and how independent elementary propositions - the atoms of the Tractatus - can hang together in the formation of a proposition. What it is that can relate independent atoms together in the context of a proposition or a fact cannot be said. It can however, of course, be shown.

Finally, Wittgenstein's propositions in the Tractatus - which act as elucidations of the preconditions for making sense - must be understood as nonsensical. It is this that one can say nothing about.

FOOTNOTES AND REFERENCES:

1. Ludwig Wittgenstein, Tractatus Logico - Philosophicus. (See p.7 fn. for an explanation of the Tractarian numbering system.)
2. Max Black, A Companion to Wittgenstein's 'Tractatus'.
3. R37, - i.e. letter from Wittgenstein to Russell dated, 19.8.19, reprinted in: Ludwig Wittgenstein, Letters to Russell, Keynes and Moore.
4. See, for example, David Bell, Frege's Theory of Judgement. Others, eg. P.M.S. Hacker, have referred to it as the thesis of "semantic holism".
5. A good perspective on this paragraph is provided in a later work by Wittgenstein, published posthumously under the title, Philosophical Investigations. (References to this work are abbreviated to part and paragraph numbers as below):

One might also give the name "philosophy" to what is possible before all new discoveries and inventions (PI-126, see also PI-127).

6. R37.
7. Wittgenstein's fundamental idea is foreshadowed in his letters to Russell numbered R2, R12, R13 and dated respectively 22.6.12, June 1913, letter dated by Russell⁷, 22.7.13.
Together with the Notebooks 1914-16 and its appendices, these letters provide an insight into the processes of Wittgenstein's thought during his working on the Tractatus. However, I have restricted attention in this thesis almost exclusively to the text of the Tractatus itself.
8. See R9, dated January 1913.
9. The historiographic problem of contrasting Wittgenstein's account of logical operators with Frege's in the Begriffsschrift and Grundgesetze der Arithmetik, and with Russell's in The Principles of Mathematics and Principia Mathematica would require a thesis in its own right. See P.M.S. Hacker, "Semantic Holism: Frege and Wittgenstein"; S.T. Sommerville, "Wittgenstein to Russell (July 1913)"; N. Griffin, "Russell on the Nature of Logic (1903-1913)".
10. For a lucid critique of Wittgenstein's treatment of this problem in the Philosophical Investigations, see Robert Arrington's essay "Mechanism and Calculus".
11. Kenny gives the formula for L_n as 2^n . Hence, according to him, where $n=1$, $L_n=2$; $n=2$, $L_n=2^4$; $n=3$, $L_n=2^{27}$; $n=4$, $L_n=2^{256}$; $n=5$, $L_n=2^{3125}$; and so on.

12. Cf. the simile: "unravelling a knot" in Wittgenstein and the Vienna Circle p.37e; and also, Wittgenstein's remark on the nature of ethics, ibid pp.68-9.

Wittgenstein's remarks in 6.3-4 indicate clearly that the laws that he is primarily interested in are logical laws. He is careful to distinguish between these and scientific laws (as also, he is careful to distinguish between physics and metaphysics, see 6.53). The laws of logic are purely formal and without sense (Sinnlos) and thus are not like for example, the laws of zoology which do have content and do say something about the world.

(Contrast Russell's position outlined in his Introduction to Mathematical Philosophy. FIRST EDITION, 1919, where he claims that the laws of logic are like the laws of zoology (p.169). In his essay entitled "Logical Atomism" (1924), reprinted in Logic and Knowledge, Russell advises (p.339),

... we shall be wise to build our philosophy upon science ...

One quotation from Wittgenstein's Vermischte Bemerkungen ("Miscellaneous Remarks") must suffice to illustrate Wittgenstein's considerably different understanding:

I am not interested in constructing a building, so much as in having a perspicuous view of the foundations of possible buildings. So I am not aiming at the same target as the scientists and my way of thinking is different from theirs. (p.7e, dated 1930).

13. Pears and McGuinness translate ,,Die Wirklichkeit muss durch den Satz auf ja oder nein fixiert sein." (4.023) as: "A proposition must restrict reality to two alternatives: yes or no." Wittgenstein's comment on the Ogden translation (see Letters to C.K. Ogden. pp.27 and 41) is: "... a proposition determines reality such that by merely affirming or denying it one can make it agree with reality /sic, the world/." So, I understand it like this: 'A proposition fixes two alternatives mirroring reality: the one affirms the world, the other denies it.' Or: 'Reality is fixed in a proposition by affirmation and denial.'
14. Further remarks that could be followed up here would be:

The number of fundamental operations that are necessary depends solely on our notation (5.474, cp. 5.452).

The general propositional form is the essence of a proposition (5.471; a statement of the logic of depiction).

If I cannot say a priori what elementary propositions there are, then the attempt to do so must lead to obvious nonsense (5.5571, emphasis added. Cp. scientific propositions.).

Logic is transcendental (6.13).

One must say nothing about what cannot be said (7, my translation).

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