EXAMINING THE NATURE OF THE RELATIONSHIP BETWEEN LEARNERS' CONCEPTUAL UNDERSTANDING AND THEIR MATHEMATICAL DISPOSITIONS IN THE CONTEXT OF MULTIPLICATION

Thesis

Presented to the Faculty of Education Rhodes University

in Partial Fulfillment of the Requirements for the

MASTER OF EDUCATION

(Mathematics Education)

Of

RHODES UNIVERSITY

by

SIVIWE LUNGELWA NDONGENI

November 2013

DECLARATION

I declare that this Research Project represents my original work. It is being submitted for the degree of Master of Mathematics Education at Rhodes University, Grahamstown. It has not been submitted for any degree or examination at any other university.

SIGNED: DATE:

APPROVED BY:

Supervisor:

(Name typed under line, omitting Ph.D. or Dr.)

(Name typed under line, omitting Ph.D. or Dr.)

DEDICATION

This thesis is dedicated to my late parents, Mr Bhanise Barnabas Mapekula and Mrs Nompumelelo Henrietta Mapekula for teaching me and my siblings the importance of education, discipline and determination

I would also like to dedicate to my wonderful husband Mr Themba Michael Ndongeni for his unwavering support and encouragement, and to my beautiful children, Noxolo, Phumlani, Daluxolo, Nompumelelo, Phumelela and my granddaughter, Sibulele for their understanding and encouragement

ACKNOWLEDGEMENTS

Words are not enough to express my appreciation for my Supervisor, Professor Mellony Graven. She did not only support me in writing this report, but she motivated and helped me to believe in myself. I also appreciated the fact that I could do my research in my school, which was convenient in the collection of my data. For that I would like to thank the community of John Masiza Primary, the staff, parents and the learners who participated in my research. I would also like to thank the Rhodes University Chair for their financial assistance in doing my Masters Degree.

ABSTRACT

The focus of this study is to explore three key aspects of learners' multiplicative proficiency: the nature of learners' conceptual understanding of multiplication, the nature of learners' numeracy dispositions (in the context of learning multiplication), and the relationship between conceptual understanding and productive dispositions in the context of multiplication. The study used a qualitative case study approach to gather rich data in relation to these. In the study a purposively selected sample of six Grade 4 learners was used from the same school: two high, two average, and two low performers.

Kilpatrick, Swafford, and Findell (2001) define conceptual understanding as a functional grasp of mathematical ideas and its significant indicator is being able to represent mathematical situations in different ways and knowing how different representations can be useful for different purposes. They then refer to productive disposition as the 'tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics' (p.131).

Individual interviews were conducted using Wright, et al.'s (2006) instrument for exploring the nature of students' conceptual understanding of multiplication. Wright, et al. (2006) argue that the topics of multiplication and division build on the students' knowledge of addition and subtraction, and also multiplication and division provide foundational knowledge for topics such as fractions, ratios, proportion and percentage, all of which are core and essential areas of mathematical learning typically addressed in the primary or elementary grades. Researchers agree that learners have to be exposed to various strategies so that they are able to see that there is a difference between additive reasoning and multiplicative reasoning.

In order to classify learners' conceptual understanding of multiplication an analysis of the data was done and learners were allocated levels according to the Wright, et al. (2006) levels of achievement.

For the classification of learner dispositions, the data was analysed in terms of the elements of productive disposition as defined by Kilpatrick, et al. (2001) and Carr and Claxton (2002). The key findings of the study indicate that for conceptual understanding most of the learners depended on using concrete materials in solving multiplication and they also used basic strategies and methods. The findings for productive dispositions were that most of the learners saw themselves as competent in doing multiplication but the aspect of sense making and steady effort was less developed. The findings for the relationship between conceptual understanding and productive disposition were that both strands have a mutual relationship in which one helped the other to develop.

Table of Contents

List of figures

List of Tables

APPENDICES

CHAPTER 1: INTRODUCTION AND CONTEXT OF STUDY

1.1Introduction

As I have been a mathematics teacher in the Senior Phase for several years, I have noticed over that period that most of the Grade 7s in my school still rely on unitary counting and written tallies when dealing with multiplication problems (and division as the inverse operation). These learners still depend on the concrete way of solving multiplication rather than the abstract, their methods are effective but time consuming. This poses problems because they are supposed to do so much work in preparation for high school.

1.2 SOUTH AFRICAN BACKGROUND

South Africa is facing a huge problem in mathematics education. A significant body of evidence points to a 'crisis' in primary mathematics education (Ensor, et al. 2009; Fleisch, 2008). Recently several researchers have found that concrete methods to solve problems, such as tally counting, are dominant in the schools they have investigated through classroom-based research. They also argue that the poor mathematical results in South Africa are the result of inefficient moves made by learners from counting to calculating. In their study, many students failed to move their thinking sufficiently forward from concrete counting actions to abstract thinking.

The Third International Mathematics and Science Study Repeat Survey (TIMSS-R) of worldwide trends in respect of scholastic performance mathematics has also confirmed that South African mathematics learners' performance was significantly poorer than the vast majority of other participating countries in tests that measured basic mathematical skills (Howie, 2001: 18). In the TIMSS (1999), South Africa was among 38 countries which participated and the results revealed that South Africa achieved the lowest average scale score of 275, far lower than the International average scale score of 487. The condition of mathematics education has often been described as the worst among other learning areas. These have been linked to poor quality of mathematics teaching, which has resulted in low learner achievement when compared to other countries around the world.

Responding to the crisis the Government and the Department of Education launched an initiative with the view to strengthen the foundational skills of Literacy and Numeracy among South African learners, and as a result, the Annual National Assessment (ANA) was launched in 2010.

Annual National Assessment (ANA)

In the light of the above mentioned problems the South African Government and the Department of Basic Education saw it proper to introduce the Annual National Assessment (ANA) to strengthen the foundational skills of Literacy and Numeracy among South African learners.

The 2011 ANA Report focuses on aggregate trends only and does not include an analysis of items or test questions. In other words, the 2011 ANA report does not give an indication of how learners performed in multiplication specifically but rather of how learners did in mathematics in general. The South African Department of Basic Education (DBE) is currently conducting this latter analysis and the results from this will provide an indication of whether upward or downward trends over time can be observed with respect to specific skills, such as multiplication or fractions in the case of mathematics (DBE:ANA Report, 2011a).

Comparing 2011 and 2012 in Mathematics, the table below shows the summary for the national average:

Table1.1 Comparing ANA Mathematics National Results for 2011 and 2012

GRADE	**2012**	**2011**
1	68	63
2	57	55
3	41	28
4	37	28
5	30	28
6	27	30
9	13	*

Looking at the 2012 ANA Report the results point towards a general improvement in the performance of learners in the ANA tests compared to the 2011 results.

The report also gives a more specific analysis in which questions are analyzed individually, in the analysis the multiplication question in Grade 3 had a pass rate of 52% and the division question had a pass rate of 44%. The areas of non-performance as specified in the District Improvement Plan were on computational skills and number operations and specifically multiplication of 2 digit number by 1 digit and division of 2 digit number by 1 digit number. The remedial measures mentioned in the plan are that the learners must be taught all the 4 basic operations and they must be able to read the mathematical signs (×,÷,+,-).

The ANA's are aimed at improving the learners' computational skills but I wish this could start with conceptual understanding so that learners can see sense in mathematics. At the moment the teachers are given the ANA exemplars so that they can train their learners on the types of questions asked. The results do not really give the true reflection of what is happening in the classes.

The South African Numeracy Chair Project

The South African Numeracy Chair at Rhodes University is one of two national numeracy Chairs. Within this Chair project, there is a growing research community of both full time and part time doctoral and masters students. I am a part time Masters student within that community.

The main focus of the Chair is on the dialectical relationship between research and development. The four inter-related objectives of the numeracy Chairs are:

- To improve the quality of teaching of in-service teachers at the primary school level.
- To improve learner performance in primary schools as a result of quality teaching and learning.
- To research sustainable and practical solutions to the challenges of improving numeracy in schools; and
- To provide leadership in numeracy education and increase dialogue around solutions for the mathematics crisis.

My research study coheres with these aims as is indicated and elaborated below. The purpose of the study is to investigate the relationship between conceptual understanding and productive dispositions in the context of multiplicative reasoning, and thus the research questions are as follows:

Research Questions

1. **What is the nature of the learners' conceptual understanding of multiplicative**

reasoning?

2. **What is the nature of the learners' numeracy dispositions (in the context of learning multiplication)?**
3. **What is the relationship between learners' conceptual understanding and their mathematical dispositions in the context of multiplicative reasoning?**

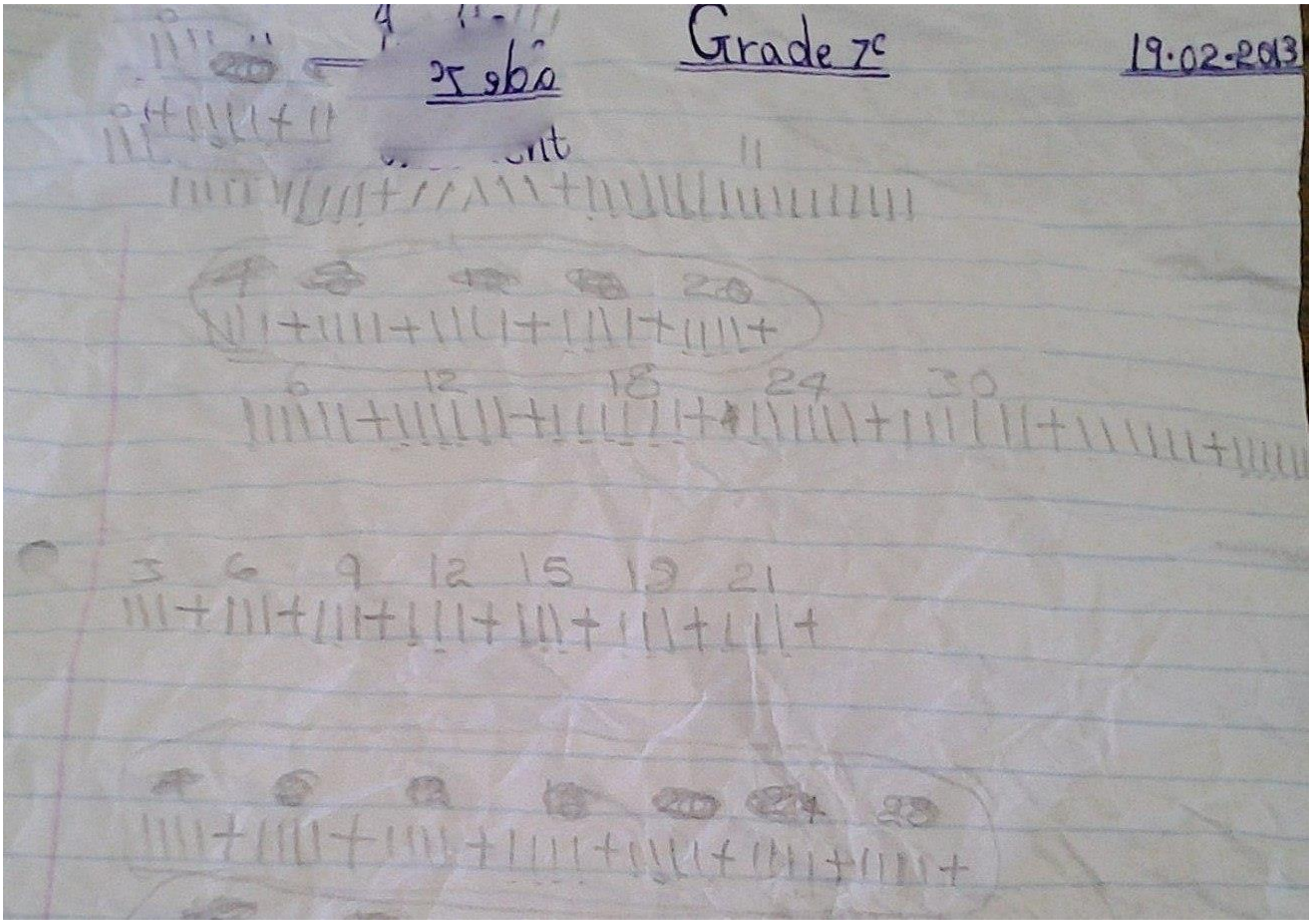
The aim of this study is to better understand the relationship between conceptual understanding and productive disposition so as to be able to strengthen each to the mutual benefit of the other and thus shift teaching for this purpose.

1.3 RATIONALE OF THE STUDY

The focus of my research is on multiplication and multiplicative reasoning. Specifically, the focus is on the relationship between conceptual understanding and productive dispositions in the context of multiplication. I have noticed over a period of years that the Grade 7s in my school still rely on unitary counting and written tallies when dealing with multiplication and division problems. Ensor, et al. (2009) and Schollar (2009) have argued that there is a lack of shift from concrete counting-based to more abstract calculation-based strategies. According to many studies conducted over a period of time, it is clear that the overall mathematics performance of learners is poor and that for children, the shift from concrete counting to calculating is problematic in South Africa (e.g. Ensor et al., 2009; Schollar, 2009; Graven, Venkat, Westaway & Tshesane, 2013). One of the studies involved a nation-wide sampling of Grade 4 learners in literacy numeracy and life skills. South Africa took part together with other African countries. The mathematics study focused on four areas: numeracy; measurement; geometry; and everyday statistics. In the numeracy tests, learners were assessed on counting knowledge and abilities; writing numbers in words; four operations; fractions and decimals; and word problems (Fleisch, 2008). Fleisch highlights, within his summary, a reliance on unit counting (using fingers or written tallies) without mathematical procedures relying on some abstraction of number.

In my own experience of working as a teacher in my school in Port Elizabeth, I too have come across a predominance of concrete methods sometimes continuing all the way up to Grade 7. The picture below indicates the working of a Grade 7 learner solving multiplication problems involving multiplication by 6 and by 3 in one of my classes.

Figure 1.1 Showing the work of a Grade 7 learner using serial counting to solve multiplication problems.



While the representation above is perhaps appropriate for an early multiplicative learner, at Grade 7 level greater fluency and abstract strategy is needed.

In the context of multiplication, I would also like to explore the nature of the relationship between learners' conceptual understanding and their mathematical dispositions. I think if we can better understand this relationship, we might be able to explore ways to strengthen each to the mutual benefit of the other and thus shift teaching for this purpose.

For this reason, I am doing the research with the Grade 4s who are supposed to be gradually ushered into the abstract strategies so that by time they reach the Senior Phase (Grades 7 to 9), they are fairly if not comfortably proficient in multiplication.

Concluding remarks

Of all the research indicated above there is little of how educators have been trained to teach mathematics and multiplication in context where one to one counting persists. In my experience most mathematics teachers in the primary schools are not trained for this learning area as a result they tend to concentrate on sections that are comfortable for them. Teachers play a vital role in the teaching and learning of mathematics so if they have a limited understanding of the learning area, they will not do justice to it. This is a challenge facing the Department of Education and the schools themselves to make sure that mathematics teachers receive proper training.

Overview of the different chapters

In the following chapters different topics are discussed and these include:-

- The literature review focuses on what different researchers say about the research questions of this study, how they define and explain the topics pertinent to this study.
- The methodology focuses on sampling and how the data was collected
- The data analysis and presentation focuses on the methods used to present and analyze the data collected.
- The discussions of the different findings and the conclusions drawn from the whole process and finally the recommendations are given.

CHAPTER 2: LITERATURE REVIEW AND THEORETICAL PERSPECTIVE

2.1 Introduction

The study deals with a number of issues. So the purpose of this chapter is to give informed explanations about these issues through the eyes of the researcher. The issues discussed involve the topics of multiplication and multiplicative reasoning, conceptual understanding and productive disposition.

2.2 Knowledge of basic mathematical operations (+, -, ×,÷)

McIntosh, et al. (1992) argue that much of school mathematics is dedicated to understanding operations and further argue that there are three things that a child has to understand when dealing with operations. Firstly, children need to understand the effect of operations. The authors include the understanding of operations on whole numbers and on fractions/ decimal numbers. The second concern is that children need to understand the mathematical properties of operations, and the last concern is that children need to know the relationship between the operations.

The point made above is important, a child has to understand that if I multiply whole numbers together the answer should be bigger so if they get an unexpected answer, they should question their working. If the child does not understand the effects of an operation, they will not be able to make that important connection. Again if they understand the relationship between the operations, when faced with a multiplication problem they should be able to connect multiplication and repeated addition. As my research is based on multiplication, I therefore look at how learners understand the connections between multiplication concepts and other operations.

2.3 What is multiplicative reasoning?

Multiplication is a mathematical operation that demands a special kind of understanding and this therefore calls for careful planning from the teacher. Some researchers have argued that multiplication requires the construction of new, higher-order numbers out of addition (Kamii & Clark (1996); Park & Nunes (2001). In other words, the progress from repeated addition to multiplication requires the construction of new elements, through reflective abstraction, rather than the mere reorganization of elements that already exist.

Multiplication develops from repeated addition, so this means for learners to understand the basics of multiplication, they have to have a firm foundation of addition. Kamii and Clark (1996) define multiplication as a more complex operation that is constructed out of addition at a higher level of abstraction. Agreeing with the this notion are Park and Nunes (2001) who also argue that multiplication is grounded on the understanding of repeated addition, but the repeated addition is only a calculation procedure and that the understanding of multiplication has its roots in the schema of correspondence.

Wright, Martland and Stafford (2006) argue that the topics of multiplication and division build on students' knowledge of addition and subtraction, and also multiplication and division provide foundational knowledge for topics such as fractions, ratios, proportion and percentage, all of which are core and essential areas of mathematical learning typically addressed in the primary or elementary grades. Many researchers agree that it is not enough that learners can do repeated addition, they have to be exposed to other strategies so that they are able to see that there is a difference between additive reasoning and multiplicative reasoning. Jacob and Willis (2001) also argue that multiplication is more than repeated addition however, and its learning more complicated. They further suggest that unless teachers consciously help children develop multiplicative thinking, which goes well beyond repeated addition, it may not happen for many children.

2.4 Understanding multiplication

It is clear that failure to develop multiplication structures in the early years impedes the general mathematical development of students into the secondary school, for example in using algebra, functions and graphs (Mulligan, 2001). Ausubel (1968) as cited by Ernest, 2006) argues that the most important single factor influencing learning is what the learner already knows.

Understanding multiplication has been proven to be a complex part of mathematics as a result most of the learners find comfort in using concrete objects rather than using abstract strategies in solving multiplication or division problems.

Baroody (2006) discusses the three phases through which children typically progress in mastering the basic number combinations- the single-digit addition and multiplication combinations and their complementary subtraction and division combinations:

Phase 1: Counting strategies- using object counting (e.g. with blocks, fingers, marks) or verbal counting to determine the answer.

Phase 2: Reasoning strategies- using known information (e.g. known facts and relationships) to logically determine (deduce) the answer of an unknown combination.

Phase 3: Mastery- efficient (fast and accurate) production of answers.

This means that if a learner has not developed one of the phases, for example, phase 2, he will have difficulty in performing at phase 3. This is what many learners are experiencing, they are still stuck at phase 1 because their reasoning strategies are not well developed.

Anghileri (1989) argues that understanding multiplication comes from the 'unification' of many schemas so that the child may recognize a mathematical operation whose application is appropriate for solving and presenting a diverse range of tasks. Park and Nunes (2001) argue that the origin of children's understanding of multiplicative relations is in their schema of one-to-many correspondence. The link between the structure of a multiplication task and solution strategy selected by an individual child may furnish further information about the individual's developing understanding.

Barmby, Harries, and Higgins, (2009) state that the important aspects of multiplication, in relation to whole numbers which primary pupils need to understand and which are discussed by Anghileri (2006 as cited by Barmby, et al. 2009) amongst others, are :

i. Replication (rather than joining in addition /subtraction);

ii. The binary rather than the unary nature of multiplication, and the notion of two distinct and different inputs;

iii. Commutative nature of multiplication but not for division; and

iv. Distributive nature.

Children use different strategies when solving multiplication or division problems and it is through the strategies they use that researchers can classify the learners according to their levels of understanding.

Wright, et al. (2006) offer a framework for assessing and exploring learners' number sense including multiplicative reasoning. They provide a stage model, based on a constructivist perspective, which indicates levels that learners may or may not have reached depending on the way in which they respond to certain questions. In order to assess a stage that a learner is at, they provide detailed and structured individually administered oral interviews. These are the descriptions for the levels of development in multiplication and division (Wright, et al. 2006, p.183).

- **Level 1: Forming equal groups or Initial Grouping** – At this stage the child uses perceptual counting and sharing because he does not see the groups as composite units and thus counts each item by ones instead of multiples.

- **Level 2: Perceptual Counting in Multiples** – At this stage the child uses multiplicative counting strategy to count visible items arranged in equal groups. The child cannot count screened groups and mainly uses perceptual strategies because he is reliant on seeing the items.

- **Level 3: Figurative Composite Grouping** – At this stage the child uses multiplicative counting strategy to count items arranged in equal groups where individual items are no visible. The child is not reliant on direct sensory experience and therefore does not rely on counting in ones.

- **Level 4: Repeated Abstract Composite Grouping** – At this stage the child counts composite units in repeated addition and subtraction, and mostly uses double counting strategy.

- **Level 5: Multiplication as Operation-** At this stage the child can immediately recall or quickly derive many of the basic facts for multiplication. The child also possesses the commutative principle of multiplication and sees the inverse relationship of multiplication and division. (Wright, et al. 2006, p.183).

2.5 Concrete strategies used by children

When one observes learners using calculation strategies, it becomes clear that they use those strategies that make sense to them and that gives an idea of the stage at which that particular child is. Hiebert (1984) raised the concern that children begin school with simple but sound problem-solving procedures and then as they receive formal mathematics instruction replace those procedures with shallow sometimes meaningless ones. He further suggests that it would be wise to help the learners improve on their existing strategies rather than making them abandon those strategies. Children use simple multiplication strategies as they grow. It is therefore up to the teachers to expose them to more comprehensive and abstract ones so that they move to the next level of understanding used by children in higher grades. This is because the concrete-based strategies are time-consuming and an upper grade learner usually has more advanced work to do.

Anghileri (1989) argues that there is a variety of stages of development that children go through on their multiplicative journey. Learners start with *unitary counting* procedures where counting is done in ones whether by using counters or other concrete objects, and it is mainly observed among the youngest children but because this strategy is "tedious" as (Fosnot & Dolk, 2001) puts it, children construct better ways to keep track. The next stage observed is the *rhythmic counting* in groups where the child counts in groups. Steffe (1986 as cited by Anghileri, 1989) states that the development from uniformly counting in ones to rhythmic counting in groups illustrates the emergence of some recognition of the *composite* nature of number as well as the *component* nature. Later on, the interim numbers in a count were internalized to produce a *number pattern* which does not rely upon a count of the total number of objects in any product set but upon acknowledged number facts that relate to the cardinality of the union of sets (Anghileri, 1989).

Fosnot and Dolk (2001) also give similar stages although termed differently, *counting-by-ones* as the initial strategy and because of its inefficiency another strategy emerges and they call it the *skip counting* strategy where the child counts in groups similar to the number pattern stage, e.g. 4×5 can either be counted as four fives or five fours. Another strategy used by children is *repeated addition* strategy where the child would add the five fours or the four fives (e.g. 4+4+4+4+4 or 5+5+5+5). Another innovative strategy observed is the *doubling strategy,* where the child counts the numbers in doubles, e.g. 5+5+5+5 = 10+10= 20.

2.6 Abstract strategies

When a child reaches the stage where he uses multiplication facts with ease, it gives so much satisfaction to the teacher because the ultimate idea is for them to be able to use their multiplicative reasoning in more complex situations. This stage for different reasons, will not be reached by the learners at the same time and some may not reach it at all. This is because of the complexity of the multiplication operation and fundamentally the fact that learners have different intellectual levels. Vergnaud (1983 as cited by Clark and Kamii, 1996) conceived of multiplicative structures as a conceptual field involving many interconnected concepts. If the child misses the opportunities of making these most needed connections, multiplication may remain a mystery for him.

Fosnot and Dolk (2001) use the term of 'big ideas' to illustrate some of the more advanced strategies used by children in multiplication problems posed to them. They call them big ideas because they involve structuring- understanding the structure and/or relationship between the parts and the whole. They talk about *unitizing,* which requires that children use number to count not only objects but also groups. They also argue that unitizing is a central organizing idea in mathematics because it underlies the understanding of place value: ten objects are one ten. And it is at the heart of the idea of exponents.

Another strategy is that of understanding the *distributive property,* by realizing that, e.g. 8×3 can be solved by adding 5×3 and 3×3 or by adding 4×3 and 4×3 or 6×3 and 2×3. They further argue that the distributive property is also a central organizing idea in mathematics: it is the basis for the multiplication algorithm with whole numbers. Another big idea is that of understanding the associative property (e.g.

$3\times(6\times7) = (3\times6)\times7$) and also the commutative property $5\times6 = 6\times5$.

2.7 How learners move from concrete to abstract strategies

Mathematics educators have a common goal, that of seeing their learners doing mathematics proficiently with increasingly efficient strategies to solve the tasks. For them to achieve this goal they should encourage and develop the learners' mathematical abstract thinking. Sfard (2008) provides concepts relating to viewing mathematical learning in terms of shifts from discourses about concrete to abstract objects. She states that, "the term abstracting is commonly used with reference to the activity of creating concepts that do not refer to tangible, concrete objects" (2008, p.111).

Anghileri (1989) in her discussion of counting procedures for multiplication argues that learners move from the unitary counting and develop rhythmic counting and then later the interim numbers are progressively internalized to produce a number pattern that is used as a count in its own right. At the last advanced stage the learners apply known multiplication facts to solve multiplication tasks. This represents a substantial step in the development of an abstraction from these procedures. Anghileri (2006) adds on arguing that the reform of mathematics has led to a shift from teaching procedures to facilitating the detection of patterns and relations, so that learners develop an insight into numbers. The realization of patterns and counting in groups are the start of the operation of multiplication. When the child can see patterns and relations between numbers, she starts to develop an abstract understanding of number.

Wright, et al. (2006) argue that an initial focus for multiplication and division is multiplication in the range of 1 to 100, and more specifically multiplication involving two 1 digit factors. This is referred to as learning basic facts for multiplication and is widely known as the times tables. They further argue that this sub-domain is regarded as the major goal of instruction, and it is a topic that traditionally 'has taken up a considerable amount of time in primary school' (Anghileri, 2006: 94). Since learning the basic facts has traditionally been a task of memorization and habituation, successful memorization of the basic facts for multiplication is commonly viewed as a major hallmark of progress in arithmetic. They further suggest that in the case of multiplication and division, students should not be pressed to memorize initially; rather, they need extensive opportunities to develop multiplicative non-counting strategies before developing automaticity.

2.8 Mathematical Proficiency as a framework for considering multiplicative proficiency

Kilpatrick, Swafford, and Findell (2001) provide the theoretical framework for proficiency that informs the study and which discusses mathematical proficiency as a group of five interwoven strands namely conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive dispositions. I provide Kilpatrick, Swafford and Findell's (2001), p. the definitions to each of these below:

(i) **Conceptual understanding** 'refers to an integrated and functional grasp of mathematical ideas' (p.118). A significant indicator of conceptual understanding is being able to represent mathematical situations in different ways and knowing how different representations can be useful for different purposes (p.119).

(ii) **Procedural fluency** 'refers to the knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently' (p.121). Without sufficient procedural fluency, students have trouble deepening their understanding of mathematical ideas or solving mathematics problems (p.122).

(iii) **Strategic competence** 'refers to the ability to formulate mathematical problems, represent them, and solve them. This strand is similar to what has been called problem solving and problem formulation' (p.124). 'Students develop procedural fluency as they use their strategic competence to choose among effective procedures' (p. 129).

(iv) **Adaptive reasoning** 'refers to the capacity to think logically about the relationships among concepts and situations. In mathematics, adaptive reasoning is the glue that holds everything together, the lodestar that guides learning' (p. 129).

(v) **Productive disposition** 'refers to the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics' (p.131). 'Students who have developed a productive disposition are confident in their knowledge and ability' (p.133).

This notion of proficiency will be used as the basis for the notion of what constitutes multiplicative proficiency. That is, each of these five strands should be present and integrally connected for a learner to be considered proficient when working with multiplicative problems. However, I focus on researching and thus paying particular attention to the two strands of conceptual understanding and productive disposition in the context of multiplication. I thus expand further on each of these strands below (and draw on other literature to support my research).

2.9 What constitutes conceptual understanding in the context of multiplication?

Anghileri (1989) argues that much research has shown that children possess considerable mathematical understanding prior to any formal instruction and that this understanding is derived from everyday situations to which the children have been exposed. Mulligan and Mitchelmore (1997) agree that students solve a variety of multiplicative problems long before formal instruction on the operations of multiplication and division. Hiebert (1984) raised a concern about why is it that many children begin school with simple but sound problem-solving procedures and then as they receive formal mathematics instruction replace them with shallow sometimes meaningless procedures. He further suggests that it may be wise to help them improve their existing strategies rather that making them abandon the strategies.

Sierpinska (1994 as cited by Tony, Barmby, 2007) clarified three different ways of looking at understanding: Firstly, there is the 'act of understanding' which is the mental experience associated with linking what is to be understood with the 'basis' for that understanding. Secondly, there is 'understanding' which is acquired as a result of the acts of understanding. Thirdly, there are 'the processes of understanding' which involve links being made between acts of understanding through reasoning processes, including developing explanations, learning by example, linking to previous knowledge, linking to figures of speech and carrying out practical and intellectual activities.

This shows that understanding involves many aspects such as showing through actions that one understands and then applying the understanding in other settings by linking present knowledge to previous knowledge. This process requires reasoning and also depends on the depth and strength of the connections that one has made on a particular domain.

Kilpatrick, Swafford and Findell (2001) argue that conceptual understanding is the functional grasp of mathematical ideas and the significant indicator of conceptual understanding is being able to represent mathematical situations in different ways and knowing how different representations can be useful for different purposes. On the other side, Skemp (1976) argues for two kinds of understanding, namely, instrumental and relational understanding. He refers to instrumental understanding as that which one uses 'rules without reasons' and it is described as mathematics that is usually easier to learn, the rewards are more immediate and because less knowledge is involved, one can often get the correct answer quickly and reliably. He refers to relational understanding as that which one 'knows both what to do and why' and this understanding is more adaptable to new tasks because by knowing not only what method works but also why it works, the learner is able to relate the method to the problem and possibly adapt the method to new problems. Skemp (1976) further argues that knowing how methods are interrelated enables the learner to remember them as parts of a connected whole. So, the focus of the teacher should be on giving learners enough time to make connections so that learners of different abilities are accommodated.

Similarly, Kilpatrick, Swafford and Findell (2001) state that the degree of students' conceptual understanding is related to the richness and extent of the connections they have made. When students have acquired conceptual understanding in an area of mathematics, they see the connections among concepts and procedures and give arguments to explain why some facts are consequences of others and they gain confidence, which then provides a base from which they move to another level of understanding.

Hiebert (1984) argues that significant learning in mathematics can be characterized as a process of linking a symbol or written procedure with a related understanding, because mathematics takes meaning when and where connections are made. Hiebert and Carpenter (1992 as cited by Barmby et al, 2009) support this notion by arguing that mathematics is understood if its mental representation is part of a network of representations. The degree of understanding is determined by the number and strength of its connections. A mathematical idea, procedure, or fact is understood thoroughly if it is linked to existing networks with stronger or more numerous connections (p.67).

In agreement with the above statement, Barmby, Harries and Higgins (2009) argue that understanding is developed through connections made between different internal representations and being able to reason between these (p.223). So, the critical instructional problem is not of teaching additional information but rather one of helping students see connections between pieces of information that they already possess. They further explain the relationship between understanding and reasoning as follows: to understand mathematics is to make connections between mental (internal) representations of a mathematical concept, and to reason is to make connections between different representations (internal or external) of a mathematical concept, through formal (logic, proof) or informal (through examples) processes (Barmby, et al., 2009)

2.10 Elaborating on productive disposition for the purposes of this study

Kilpatrick, Swafford, and Findell (2001) define productive disposition as 'the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off and to see oneself as an effective learner and doer of mathematics' (p.131).

The definition is however limited as it does not give an indication of how a learner will act when they see sense in mathematics or when they perceive it to be useful and worthwhile, thus they fail to expand on indicators for this strand. Carr and Claxton (2002) propose a matrix of three indicators by which one can observe learners 'in action'. These indicators are resilience, playfulness and reciprocity and are proposed as a way of assessing learning dispositions and thus tracking their development. These observable indicators will make it possible for one to observe learners and make deductions about their developing or non-developing mathematical dispositions. Although again the indicators are broad and they say teachers are left to find the more specific indicators of these. This work coheres with my study as I am specifically interested in the relationship between conceptual understanding and productive disposition. Kilpatrick, et al. (2001) sum it up by stating that a productive disposition develops when the other strands do and helps each of them develop.

Carr and Claxton (2002) further state that the word 'disposition' points usefully at a domain of human attributes that are clearly different from 'knowledge, skill and understanding'. Carr and Claxton (2002) further argue that the real-life 'learning power' consists of two inter-related facets: capabilities and dispositions. While capabilities are skills, strategies, and abilities which learning requires, 'dispositions are a very different type of learning from skills and knowledge, they can be thought of as habits of mind, tendencies to respond to situations in certain ways' (p.30). Learning dispositions can be construed as default responses in the presence of uncertain learning opportunities and circumstances.

Carr and Claxton (2002) state that it is argued that the fundamental purpose of education for the 21st century, is not so much the transmission of particular bodies of knowledge, skill and understanding as facilitating the development of the capacity and the confidence to engage in lifelong learning, central to this enterprise is the development of positive learning dispositions. Sadler (2002) similarly emphasizes that to understand and develop fully a person's lifelong learning potential, requires a matching level of attention to learning dispositions and as learners engage in tasks, any successes that they experience tend to set up positive feedback loops that subsequently help to improve the development of future dispositions to learn. Carr and Claxton (2002) further argue that 'education for lifelong learning has, therefore, to attend to the cultivation of positive learning dispositions, as well as of effective learning skills' (p.30).

Gresalfi (2009) argues that the social, affective, and motivational factors are not simply *influences* on learning but are, instead, central to and inseparable from the learning process" (p.328). Cobb and Gresalfi (2006 as cited by Gresalfi, 2009) argue that dispositions capture not only to what one knows but how he or she knows it, and not only the skills one has acquired, but how those skills are leveraged. Cobb and Gresalfi (2006 cited by Gresalfi, 2009) further argue that:-

learning is a process of developing dispositions, that is, ways of being in the world that involve ideas about, perspectives on, and engagement with information that can be seen both in moments of interaction and in more enduring patterns overtime (p.329).

Gresalfi and Ingram-Goble (2003) conceptualize dispositions at three levels: '(i) engaging information in local events (taking up intentionally designed opportunities to learn in an activity or assignment), (ii) ways of approaching information in a particular subject area or setting (emergent continuities in the opportunities that are taken up), and (iii) ways of seeing and engaging the world across settings (lens or perspective on the world that impacts decisions about what to work on and how long to persist)' (p.2).

Carr and Claxton (2002) in their latest work proposed a matrix of three indicators (resilience, playfulness, and reciprocity) which are learning dispositions; and on one axis sophistication and robustness which are dimensions of strength used in assessing learning dispositions and thus tracking their development. They describe these dimensions as follows:

- **Resilience** – sticking with a difficult learning task, having a relatively high tolerance for frustration without getting upset and being able to recover from setbacks relatively quickly;

- **Playfulness** – being willing and able to perceive or construct variations on learning situations and thus be more creative in interpreting and reacting to problems; and

- **Reciprocity** – willingness to engage in joint learning tasks, to express uncertainties and ask questions, to take a variety of roles in joint learning enterprises and take others' purposes and perspectives into account (Carr and Claxton, 2002, p. 16)

On the other axis, Claxton and Carr (2004) argue that 'progress' is represented as a change in the 3 dimensions of:

- ✓ **Robustness-** ‘the tendency to respond in a learning-positive way even when the conditions are not as conducive or supportive as perhaps they once needed to or appeared to be’ (p.89);
- ✓ **Breadth** – development in terms ‘of the number, range or variety of different domains in which they appear’ (p.89); and
- ✓ **Richness** – ‘Children’s persisting, questioning or collaborating can develop in flexibility and sophistication’ (p.90).

These dimensions are related in this thesis to the indicators within the definition of Kilpatrick, et al. (2001). Thus playfulness and resourcefulness are related to seeing sense in mathematics; resilience is related to believing that steady effort pays off; resourcefulness and resilience again relate to seeing oneself as an effective learner and doer of mathematics. However I point out that Carr and Claxton’s reciprocity is not noted by Kilpatrick, et al. (2001) in their definition although within their framework a willingness to engage with others about one’s methods and thinking would be taken as a resource for developing conceptual understanding. In this way I have brought together two dispositional frameworks and illuminated how methodologically these enable a richer understanding of learning dispositions. For learner mathematical dispositions, an instrument from Graven (2012) was adapted to be in line with my interest in multiplication. The questions asked were structured in order to elicit the presence or absence of productive disposition in the context of multiplication.

Graven, Hewana and Stott (2013) argue for the importance of understanding young learner mathematical learning dispositions in order to inform ways in which to support learning. Adding on, they state that mathematical learning dispositions are taken to include what learners say about learning and how they act when they learn but they also agree that it is not easy.

2.11 What is the relationship between conceptual understanding and productive dispositions?

The aim of this study is to better understand the relationship between multiplicative conceptual understanding and productive disposition so that we might be able to explore ways to strengthen each to the mutual benefit of the other and thus shift our teaching for this purpose.

Kilpatrick, Swafford, and Findell (2001) argue that some children enter school viewing mathematics as important and themselves as being competent to master it, so they suggest that parents and educators have a challenge to help children maintain this productive disposition because students who have a developed productive disposition are confident in their knowledge and ability. They further suggest that students' dispositions towards mathematics are a major factor in determining their educational success.

Carr and Claxton (2002) have argued that education for the 21st century must aim at developing young people's ability to be skilful and confident when facing complex predicaments of all kinds. While it is important to present students with valuable and engaging topics, this 'content curriculum' ought to be accompanied by attention to the attitudes, values and habits towards learning in general which are being strengthened (or weakened) in the process. Adding to the notion, Sadler (2002) argues that when learners perceive themselves to be making progress towards an end that they themselves value highly, a strong disposition towards learning, which can ultimately carry forward into lifelong learning, is likely to follow.

Mathematical understandings depend on the learner's ability to make necessary connections so that a coherent whole of concepts and procedures is achieved. If there is a lack in the connections, then the learner will have difficulty in seeing sense in mathematics.

Schoenfeld (1992) argues that students develop their sense of mathematics- and thus how they use mathematics from their experiences with learning mathematics largely in the classroom. Adding to that notion, Kilpatrick, et al. (2001) argue that teachers and students inevitably negotiate among themselves the norms of conduct in the class, and when those norms allow students to be comfortable in doing mathematics and sharing their ideas with others, they see themselves as capable of understanding. They see that mathematics is both reasonable and intelligible and believe that, with appropriate effort and experience, they can learn.

Kilpatrick, Swafford, and Findell, (2001) further argue that a significant indicator of conceptual understanding is being able to represent mathematical situations in different ways and knowing how different representations can be *useful* for different purposes. This is because facts and methods learned with understanding are connected and they are easier to remember and use and they are reconstructed when forgotten. In addition, they suggest that developing a productive disposition requires frequent opportunities for making sense of mathematics, to recognize the benefits of perseverance, and to experience the rewards of sense making in mathematics. The more mathematical concepts they understand, the more sensible mathematics becomes.

It is argued that learners' "habits and dispositions" are shaped by classroom experiences, and have great weight in shaping individual behavior. Sadler (2002) argues that capabilities are 'skills, strategies and abilities which learning requires … the "toolkit" of learning', whereas a disposition to learn is an inclination towards learning, i.e. being 'ready and willing' as a volitional activity (p. 10). Capabilities and dispositions not only interact but also frequently reinforce each other. This to some extent proves that there is a relationship between productive disposition and conceptual understanding.

Gresalfi and Ingram-Goble (2008) argue that to foster emergent dispositions the focus should be on designing for students' engagement with content and then differentiates between the types of engagements as follows:

- *Procedural engagements* – involves using procedures accurately;
- *Conceptual engagements* – involves knowing what to do, when to do it, and why it makes sense;
- *Critical engagements* – requires interrogating the usefulness, impact, or consequentiality of the selection of particular tools on outcomes.

Specifically, aspects of engagement such as approaching novel problems, persisting in the face of challenge, and interacting with others, are merely important to attend to but central to what students come to know and do (Boaler, 1997; Greeno, 1991; Schoenfeld, 1988).

They further argue that the different ways of engaging are not separable but cumulative- conceptual engagement cannot occur without a robust understanding of procedures; likewise, one cannot critically engage without having a conceptual understanding of content. So, the purpose of attending to the levels of engagement is to provoke dispositions:-

- Procedural dispositions involve using procedures accurately and viewing justification and evidence as crucial components convincing arguments;
- Conceptual dispositions involve viewing representations as reflecting real phenomena, as opposed to simply numbers without meaning; and
- Critical dispositions involve viewing operations and representations as tools that can be intentionally leveraged to develop and support particular arguments.

2.12 Analyzing multiplicative reasoning within the South African Curriculum

Looking at the *isiXhosa* version of the policy document, one cannot help but wonder if teachers will be able to keep up with the code switching involved. This is because mathematics has a lot of scientific terms that when translated into the learners' home language will probably confuse learners, e.g. multiply is translated to *phinda-phinda*, divide is *hlula-hlula*, twenty eight- *amashumi amabini anesibhozo*. In this era, without sounding like I am diminishing the HL in this case isiXhosa, the learners are exposed to scientific terminology and if they are to be compared to their counterparts in schools where the medium of instruction is English, then the learners who are solely being taught in isiXhosa as the DBE demands, will have two big problems facing them. Firstly, there is translating and then understanding the meaning of the terms, and secondly focus on the content itself. So, I decided to look at the English version of the policy document in my analysis.

In this study and thus in the table below, I will only look at the content area of Numbers, Operations, and Relationships because my focus is on multiplication and division and relationship between conceptual understanding and productive dispositions in the context of multiplication. In the CAPS document, and the specific aim pertinent to this research, it is written: "The teaching and learning of Mathematics aims to develop deep conceptual understanding in order to make sense of Mathematics". Furthermore, the Specific Skills to be developed are that the learner should, "develop number vocabulary, number concept and calculation and application skills; learn to listen, communicate, think, reason logically and apply mathematical knowledge gained" (South Africa. Department of Basic Education. Curriculum Assessment Policy Statement [CAPS], p.8).

The general content focus is on the development of number sense that includes the effects of operating with numbers and the specific content is that learners should build an understanding of the basic operations of addition, subtraction, multiplication and division.

I therefore provide a brief analysis of the ways in which the CAPS curriculum provides advice to teachers on how the deep conceptual understanding can be developed in order for learners to make sense of Mathematics. I will extract a portion of the document and interact with the topics pertaining to this research:

Table 2.3.1 – An extract from the CAPS document. (p. 18)

Topics	**Grade 1**	**Grade 2**	**Grade 3**
1.2 Count forwards and backwards	Count forwards and backwards in ones from any number between 0 and 100. Count forwards in :	Count forwards and backwards in: • **1s from any number between 0 and 200**	Count forwards and backwards in: • **1s from any number between 0 and 1000**

	• **10s from any multiple of 10 between 0 and 100** • **5s from any multiple of 5 between 5 and 100** • **2s from any multiple of 2 between 0 and 100**	• **10s from any multiple of 10 between 0 and 200** • **5s from any multiple of 5 between 0 and 200** • **2s from any multiple of 2 between 0 and 200** • **3s from any multiple of 3 between 0 and 200** • **4s form any multiple of 4 between 0 and 200**	• **10s from any multiple of 10 between 0 and 1000** • **5s from any multiple of 5 between 0 and 1000** • **2s from any multiple of 2 between 0 and 1000** • **3s from any multiple of 3 between 0 and 1000** • **4s from any multiple of 4 between 0 and 1000** • **20s, 25s, 50s, 100s to at least 1000**
1.5 Place value	Begin to recognize the place value of at least two digit numbers to 20 • Decompose two-digit numbers into multiples of 10 and ones/units	Recognize the place value of at least two-digit numbers to 99: • Decompose two-digit numbers up to 99 into multiples of 10 and ones/units • Identify and state the value of each digit	Recognize the place value of three digit numbers to 999: • Decompose three-digit numbers up to 999 into multiples of 100,multiples of 10 and ones/units • Identify and state the value of each digit
1.6 Problem-solving	Use the following techniques when solving problems and explain solutions to problems: • Concrete apparatus eg. counters • Pictures to draw the story sum • Building up and breaking down numbers	Use the following techniques when solving problems and explain solutions to problems: • Drawings or concrete apparatus eg. counters • Building up and breaking down of numbers • Doubling and halving	Use the following techniques when solving problems and explain solutions to problems: • Building up and breaking down numbers • Doubling and halving • Number lines • Rounding off in tens

	• Doubling and halving • Number lines supported by concrete apparatus	• Number lines	
Topics	**Grade 1**	**Grade 2**	**Grade 3**
1.7 Addition and subtraction	Solve word problems in context and explain own solution to problems involving addition and subtraction with answers up to 20	Solve word problems in context and explain own solution to problems involving addition and subtraction with answers up to 99	Solve word problems in context and explain own solution to problems involving addition and subtraction leading answers up to 999
1.8 Repeated addition leading to multiplication	**Solve word problems in context and explain own solution to problems involving repeated addition with answers up to 20**	**Solve word problems in context and explain own solution to problem using repeated addition and multiplication with answers up to 50.**	**Solve word problems in context and explain own solution to problems using multiplication with answers up to 100**
1.9 Grouping and sharing leading to division	Solve and explain solutions to practical problems involving equal sharing and grouping with whole numbers up to 20 and with answers that may include remainders	Solve and explain solutions to practical problems that involve equal sharing and grouping up to 50 with answers that may include remainders.	Solve and explain solutions to practical problems that involve equal sharing and grouping up to 100 with answers that may include remainders.
1.10 Sharing leading to fractions		Solve and explain solutions to practical problems that involve equal sharing leading to solutions that include unitary fractions.	Solve and explain solutions to practical problems that involve equal sharing leading to solutions that include unitary and non-unitary.
1.11 Money	• Recognize and identify the South African coins 5c, 10c, 20c, 50c, R1, R2, R5) and bank notes R10 and R20 • Solve money problems involving totals and change to R20 and in cents up to 20c	• Recognize and identify the South African coins 5c, 10c, 20c, 50c, R1, R2, R5) and bank notes R10 and R20, R50 • Solve money problems involving totals and change to 99 and in cents up to 90c	• Recognize and identify all the South African coins and bank notes • Solve money problems involving totals and change in rands or cents • Convert between rands and cents

1.13 Addition and subtraction	• Add to 20 • Subtract from 20 • Use appropriate symbols (+, -, =, □) • Practice number bonds to 10	• Add to 99 • Subtract from 99 • Use appropriate symbols(+, -, =, □) • Practice number bonds to 20	• Add to 999 • Subtract from 999 • Use appropriate symbols (+, -, =, □) • Practice number bonds to 30
Topics	**Grade 1**	**Grade 2**	**Grade 3**
1.14 Repeated addition leading to multiplication	• **Add the same number repeatedly to 20** • **Use appropriate symbols (+, =,□)**	• **Multiply numbers 1 to 10 by 2, 5, 3, and 4 to a total of 50** • **Use appropriate symbols (+, ×, =, □)**	• **Multiply any number by 2, 3,4, 5, 10 to a total of 100** • **Use appropriate symbols (×, □)**
1.15 Division			• Divide numbers up to 100 by 2, 3, 4, 5, 10 • Use appropriate symbols (÷, =, □)
1.16 Mental mathematics	Rapidly recall : • Addition and subtraction facts to 10 Calculation strategies Use calculation strategies to add and subtract efficiently: • Put the larger number first in order to count on or count back • Number line • Doubling and halving • Building and breaking down	Rapidly recall: • Addition and subtraction facts to 20 • Add or subtract multiples of 10 from 0 to 100 Calculation strategies Use calculation strategies to add and subtract efficiently: • Put the larger number first in order to count on or back • Number line • Doubling and halving • Building up and breaking down • Use the relationship	Rapidly recall: • Recall addition and subtraction facts to 20 • Add or subtract multiples of 10 from 0 to 100 • **Multiplication facts for the** - **2 times table with answers up to 20** - **10 times table with answers up to 100** • Division facts for numbers: - up to 20 divisible by 2 - up to 100 divisible by 10 Calculation strategies

		between addition and subtraction.	Use the following calculation strategies • Put the larger number first in order to count on or count back • Number line • Doubling and halving • Building up and breaking down • Use the relationship between addition and subtraction • Use the relationship between multiplication and division

My analysis of the CAPS policy document for Foundation Phase is based on the specific aim that is written as follows, “The teaching and learning of Mathematics aims to develop deep conceptual understanding in order to make sense of Mathematics”.

For the learners to have a good foundation in all the topics in mathematics, they need to have ample opportunities where the number sense is developed as much as possible and for the Foundation Phase learner more time is needed. McIntosh et al state that the “… acquisition of number sense is a gradual, evolutionary process, beginning long before formal schooling” (1992, p.3). Adding to this notion, Hiebert (1984) suggests that teachers must allocate substantial time and effort to assist students in drawing connections between symbols and their referents, because the solution to a problem represented symbolically can be connected with the notion of how this problem would have been solved in a real world or concrete context. So, for this reason, quality time has to be given for the content area on ‘Numbers, Operations and Relationships’. Looking at the percentages allocated for each grade- Grade 1-65%; Grade 2-60% and Grade 3- 58%, these percentages need to be increased because I noticed that as I was working with the Grade 4 learners a lot is to be done on number sense and developing them towards making the much needed rich connections.

Counting is the basic start for the development of the conceptual understanding in multiplication, a child who is capable of using the counting strategies to solve multiplicative problems is at a higher level of understanding multiplication. The CAPS policy document supports this notion because the first topic discussed is on counting in multiples of 1, 2, 3, 4, 5, and 10 to a specified number. Then repeated addition follows with the intention for it to lead to multiplication, a child needs to connect counting in multiples and repeated addition in order to move to another level of understanding. Wallace and Gurganus (2005) state that students taught isolated facts may not realize that multiplication has multiple meanings. So, this means teachers need to introduce a variety of strategies so that learners use whichever makes sense to them. A child who has not made that connection will be stuck in the repeated addition stage as seen in the work of a Grade 7 learner above, who is still dependent on working with tallies and repeated addition. In the next topic on multiplication, the repeated addition is replaced by the multiplication sign for the Grade 2s and 3s but Grade 1 carries on with repeated addition. The transition needs time and CAPS document gives it enough time. Carpenter, et al. (1999) argue that the process from concrete thinking to abstract understanding cannot be "given" to a child. The direct modeling strategies and the more efficient counting strategies are strategies that children have to appropriate for themselves.

Multiplication facts depend on the rich connections that the learner has made in prior work. These facts include understanding the meaning of the symbol, how it can be translated into daily life and its commutative nature. The document gradually allows the learner to develop in this regard because in Grade 3 they start by discussing the multiplication facts by 2 and 10. The final topic in the Foundation Phase is the discussion of the relationship between multiplication and division in Grade 3. Mulligan and Mitchelmore (1997) when suggesting the implications for teaching multiplication and division highlight that multiplication is usually introduced before division and separated from it, whereas children spontaneously relate them. This was clear with my research learners, most of them did not understand the relationship as a result when they were given the signs: 8, 7, =, 56, and ÷ to a division problem, they answered 7 ÷ 8 =56.

2.13 THEORIES INFORMING THIS STUDY

2.13.1 Constructivism

Constructivism is a theory by which learners construct knowledge on their own, this implies that learners need to be given time to make sense of their learning. The focus is no longer on how the teacher delivers his lesson but on how learners make the necessary connections in order to make sense of their learning. Ernest (2006) argues that constructivism takes many forms but what binds these different forms together is the metaphor of construction. The metaphor is about building up of structures from pre-existing pieces, possibly specially shaped for the task and it describes understanding as the building of mental structures. 'Simple constructivism' recognizes that knowing is active, that it is individual and personal, and that it is based on previously constructed knowledge.

Lerman (1989), and Kilpatrick (1987) (as cited by Matthews, 1992) argue that the core epistemological theses of constructivism are : (i) Knowledge is actively constructed by the cognizing subject, not passively received from the environment, (ii) Coming to know is adaptive process that organizes one's experiential world, it does not discover an independent, pre-existing world outside the mind of the knower. Constructivists are interested in the individual's construal of knowledge, that is, how a person comes to learn (Proulx, 2006). Driver and Oldham (1986) endorse that constructivism has at its center the importance of meaning as constructed by individuals in their attempt to make sense of the world. Thus, the sense made of any event is seen to be dependent not only of the situation itself but also on the individual's purposes and active construction of meaning. Boaler (2002) argues that the act of observing relationships and drawing connections, whether between different functional representations or mathematical areas, is a key aspect of mathematical work, in itself. Hiebert (1984) agrees with this notion by stating that significant learning in mathematics can be characterized as a process of linking a symbol or written procedure with a related understanding, learning in mathematics takes on meaning when and where the connections are made.

Multiplication demands that the learner makes connections and decides on the strategies that make sense to him, researchers advocate for the importance of making links and connections so that students are mathematically proficient. Constructivism has been chosen because it coheres with Kilpatrick, et al.'s (2001) framework of mathematical proficiency which is itself developed on constructivist assumptions. It also coheres with Wright, et al.'s (2006) work on assessing multiplicative reasoning which is based on constructivist assumptions. Wright's (2013) stages of development of early number sense relate closely to Piaget's stages which progress from dependency on concrete objects for reasoning towards more abstract formal logical reasoning. This is carried throughout all sections of his assessment including the assessment of multiplicative reasoning.

2.13.2 Socio-constructivism

Socio-constructivism has been proven to be beneficial to learners in any learning environment because it is through conversations and interactions that people get to learn more from one another. Vygotsky

(1978 as cited by Ernest, 2006) argues that social constructivism regards individual learners and the realm of the social as indissolubly interconnected. Ernest (1991) argues that the central focus of socio-constructivism is the genesis of mathematical knowledge, rather than its justification and further adds that socio-constructivism views mathematics as a social construction- it draws on conventionalism, in accepting that human language, rules and agreement play a key role in establishing and justifying the truths of mathematics. He further emphasizes that the active construction of knowledge, typically concepts and hypotheses, on the basis of experiences and previous knowledge – these provide the basis for understanding and serve the purpose of guiding actions.

Sfard (1998) agrees with this notion, in her discussion of the participation metaphor she argues that it entails being able to communicate in the language of the community and act according to its particular norms. The teacher and the learners are viewed as a community whose goal is to share knowledge and therefore empower the participants in the process. This means whilst the learners are learning in this community, the teacher could also be learning different strategies and approaches that would work for the learners in a particular group for optimum learning. This implies that learning is not a static process where one stops learning but it is an ongoing process.

Multiplication is a complex concept which unfortunately some learners get to higher grades with little understanding. It is this reason that I have drawn on socio-constructivism because it allows learners to talk and interact and it is through these interactions that they get to understand it better.

Concluding remarks

As indicated, above multiplication is more complex than repeated addition. So, learners need to be moved to a higher level of understanding so that they are able to see sense and move towards using more abstract procedures rather than the concrete ones. This once again puts pressure on the teacher who is supposed to guide learners by showing them different procedures and methods to solve concepts involved in multiplication.

CHAPTER 3: Methodology

3.1 Introduction

This chapter outlines the journey travelled whilst collecting the data that had to be analyzed in order to illicit all that is needed to understand the important issues pertaining to the study. The data collected was structured so as to allow the analysis of the main questions of this study.

3.2 Methods and sampling

The research was conducted as a qualitative case study because this type of approach is about learning how individuals experience and interact with their social world and the meaning it has for them. In this approach the researcher is the primary actor for data collection and data analysis, the process is inductive in that the researcher gathers data to build concepts; and the product of a qualitative inquiry is richly descriptive with words and pictures rather than numbers (Merriam, 2002). Hitchcock and Hughes (1995, p.322) further suggest that the case study approach is particularly valuable when the researcher has little control over events. They consider that a case study has several hallmarks:

- "It is concerned with a rich and vivid description of events relevant to the case;
- It provides a chronological narrative of events relevant to the case;
- It blends a description of events with the analysis of them;
- It focuses on individual actors or groups of actors, and seeks to understand their perceptions of events;
- It highlights specific events that are relevant to the case;
- The researcher is integrally involved in the case;
- An attempt is made to portray the richness of the case in writing up the report" (p.317).

Since I was interested in understanding the relationship between conceptual understanding and productive disposition, a qualitative case study seemed most appropriate.

I chose my school as the empirical field of my study. The principal, teachers and parents were consulted and permission was granted from each of these for the research. The Grade 4 class was chosen because it is the start of a new phase for the learners and for them multiplication is not a new topic. In my school the learners in Grade 4 are not streamed so the mathematics teacher agreed to make the selection from one class. The sample of six learners was purposively selected and oral assessments were administered to the individual learners. The six learners were selected according to their abilities: two below average, two average and two above average. The assessments were based on the questions of the research which were: finding out the nature of learners' conceptual understanding of multiplicative reasoning and their numeracy dispositions; and then finding out if there is a relationship between conceptual understanding and productive disposition in the context of multiplicative reasoning.

3.3 Ethical considerations

Research informed consent is a standard feature required to meet ethical standards (McNamee and Bridges, 2002). Cohen, Manion and Morrison (2000) argue that an informed consent is the 'implicit contractual relationship between the researcher and the researched and will serve as a foundation on which subsequent ethical considerations can be structured' (p.52). Diener and Crandall (1978) define informed consent as 'the procedures in which individuals choose whether to participate in an investigation after being informed of facts that would be likely to influence their decisions' (p. 51).

Furthermore, anonymity is often essential. The learners were promised anonymity and their parents were called to a special meeting where all was explained to them. They were assured that the learners' tuition would not be disturbed during the course of the research and they were at liberty to withdraw their children from the process at any point without any prejudice. After their agreement, consent forms were signed (an example of a consent form is included in Appendix 2). It was translated into isiXhosa because that is the home language of the learners in my school. Learners were given the option to withdraw at any time and their participation was voluntary. University ethical permission was received from the Education HDC in 2012. Permission from the Eastern Cape Department of Education to conduct research in Chair project related to schools was also obtained in 2012.

3.4 Data collection methods

Two key sources of data collection were used:

- For multiplicative proficiency, individual interviews were conducted using the Wright, et al. (2006) instrument (see Appendix 3).

"Interviews enable participants-be they interviewers or interviewees – to discuss their interpretations of the world in which they live, and to express how they regard situations from their point of view. In these senses the interview is not simply concerned with collecting data about life: it is part of life itself, its human embeddedness is inescapable" (Cohen, Manion & Morrison, 2000, p.267).

When the individual interviews were conducted other participants were not allowed in the classroom to prevent them from hearing the questions. The video recorder was placed in a position where the hands of the learner could be seen. This was because it was important to see what the learner was doing with his fingers when asked a question. Most of them modelled with their fingers especially in the advanced section of the instrument. It was also important to see the participant's face when thinking about the questions asked, they either showed that they were thinking or they had given up.

- For mathematical dispositions, an orally administered- written response questionnaire was used. Orally administering the questions allowed me to mediate the questions, repeat them and translate them if learners did not understand. The written response format was convenient as it allowed me to gather data from all the learners at the same time.

Administering the instrument was not easy because learners in Grade 4 are not confident in writing English as a result I had to stand by them individually and asked them in isiXhosa first what they wanted to say and then translated for them in simple English what they must write. This resulted in most of them giving one word answers which did not give much information on their dispositions.

Interviews allow for quality and rich data to be gained and so semi-structured interviews were conducted in which an instrument from Wright, et al. (2006) was used. The instrument was designed to assess conceptual understanding of multiplication (and division as the reverse operation) at a basic level and then at an advanced level (discussed above in the literature review). Observations were done on how learners answered the different questions and how they handled the materials given to them to solve the tasks and thereafter levels were assigned according to Wright's year level indicators. Some examples of Wright, et al.'s (2006) task are shown below. See Appendix 3 for an expanded set of interview items and recovery items.

EQUAL GROUPS

1.1 Making equal groups

Learners are given 12 counters and are asked to make three groups with four in each group.

How many groups? / How many in each group?

1.2 Describing equal groups

Counters/ unifix cubes/ containers

Individuals

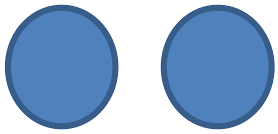
Each learner is given 10 counters and two containers, ask them to share it equally into the containers, how many counters in each containers?

Use the same activity but increase the number range

1.3 Combining and counting equal groups

Place out ten 2-dot cards, put each 2-dot card and let the learner count putting each card after the other.

Similar with three dot, four dot and five dot cards



1.4 Determining the number in an equal share

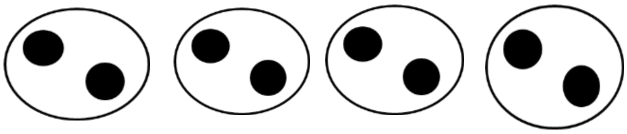
Resources: dots cards/ unifix cubes

Learners are given six counters. Share them amongst three people.

How many will each get?

Use 10 and 2, 12 and 6, 18 and 3

1.5 Determining the number of equal groups



Resources: dots cards/ unifix cubes

Place out four 2-counter cards:

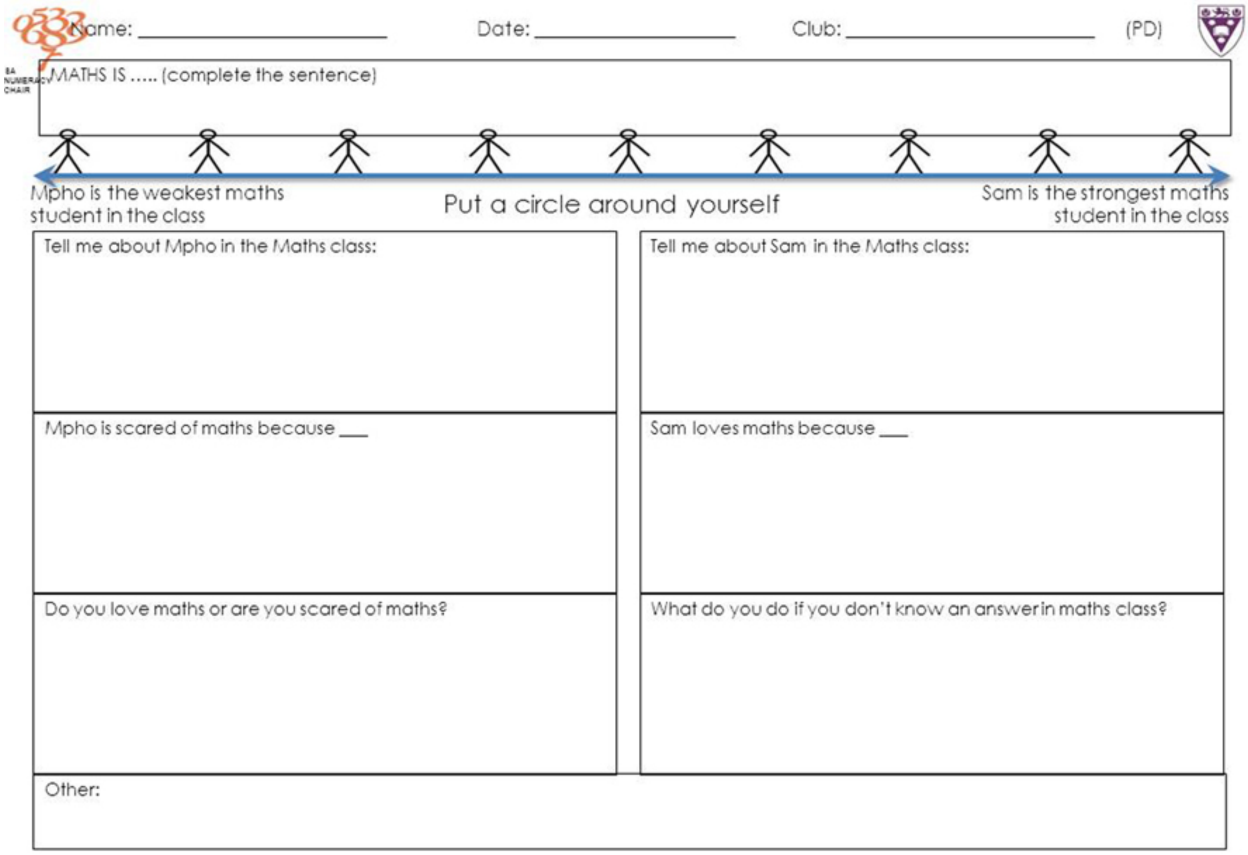
How many counters are there on each card?

How many cards are there?

How many counters are there altogether?

Appendix 1: Example of the Wright, et al.'s (2006) instrument

To assess their dispositions towards multiplication an instrument adapted from Graven (2012) was used. Graven's (2012) productive disposition instrument is given below:


Name: ______ Date: ______ Club: ______ (PD)

MATHS IS ..... (complete the sentence)

Mpho is the weakest maths student in the class

Put a circle around yourself

Sam is the strongest maths student in the class

Tell me about Mpho in the Maths class:

Tell me about Sam in the Maths class:

Mpho is scared of maths because ___

Sam loves maths because ___

Do you love maths or are you scared of maths?

What do you do if you don't know an answer in maths class?

Other:

Appendix 2: Revised Mathematics Learning Disposition Instrument (Graven, 2012, 55).

Graven (2012) admitted that the instrument is limited in that it is based on what learners say about their mathematical learning dispositions rather than observing how they are disposed to act in a given learning situation. However, it still provides data on what learners perceive a good mathematic learning disposition to be and I thus adapted it as an orally administered but written response instrument specific to multiplication. The instrument I designed is given below:

Table 4.2.4: Instrument for assessing learners' productive disposition in multiplication.

Name :___________________________________

1. Multiplication is ____
2. Give some examples of multiplication problems you know______________________________

Put a circle around yourself 1__☺_☺_☺_☺_☺_☺_☺_☺_☺_9

3. Why did you put yourself there?____
4. Why does Kuhle struggle with multiplication?_______________________________________
5. Why is Sine good at multiplication?____
6. What do you do when you get a multiplication problem that you do not know the answer to?____
7. Which do you enjoy more, addition or multiplication problems?_______________________

Why?____

3.5 Analytic Framework

After the Wright's interviews were conducted with each of the six learners the data was analyzed. For the assessment of conceptual understanding the learners' methods and performance were analyzed in terms of the Wright, et al.'s (2006) explanation of levels of development in multiplication. For mathematical dispositions, the learners' utterances were analyzed in terms of the elements of productive disposition as discussed by Kilpatrick, Swafford and Findell (2001) and Carr and Claxton (2002). That is, the extent to which the following indicators were visible:

- see sense in multiplication (resourcefulness);
- steady effort (resilience);
- reciprocity (willingness to engage with others); and
- self-belief in multiplication ability (discussed above in the literature review and used in Chapter 4) below.

The analysis of conceptual understanding was conducted so as to have a better understanding of the learners' level of development with respect to multiplication and division. The table below gives a summary of the indicators for each level of development:

Table 3.1.1 The Wright's stages of development in multiplication (and division)

LEVELS OF CONCEPTUAL UNDERSTANDING	WRIGHT, ET AL.'S INDICATORS OF THAT PARTICULAR LEVEL
Level 1 : Forming Equal Groups or Initial Grouping	The child uses perceptual counting and sharing (that is by ones) to form or make groups of specific sizes: to share items into groups of a given size (quotitive sharing); to share items into a given number of groups (partitive sharing). The child does not see the groups as composite units and thus counts each item by ones instead of multiples. (Similar to Stage 1 in SEAL)
Level 2 : Perceptual Counting in Multiples	The child uses a multiplicative counting strategy to count visible items arranged in equal groups. The child cannot count screened groups. Strategies used can be rhythmic, skip counting and double counting. All are called perceptual because the child is reliant on seeing items.
Level 3 : Figurative Composite Grouping	The child uses a multiplicative counting strategy to count visible items arranged in equal groups where the individual items are no visible. The child is not dependent upon direct sensory experience where he or she relies on counting by ones. (Equivalent to Stage 3- Counting on , in SEAL)
Level 4 : Repeated Abstract Composite Grouping	The child counts composite units in repeated addition or subtraction tasks, that is, he or she uses the composite a specified number of times. Double counting is a common strategy at this stage.
Level 5 : Multiplication and Division as Operations	The child can regard both the number in each group and the number of groups as composite unit. The child can immediately recall or quickly derive many of the basic facts for multiplication and division, for example, three sixes. A child at this level may possess commutative

	principle of multiplication (5x3 =3x5) and/ or see the inverse relationship of multiplication and division.

For the analysis of the mathematical dispositions the similarities between the elements of productive disposition as discussed by Kilpatrick, et al. (2001) and those discussed by Carr and Claxton (2002) were fused. Below is the summary of indicators used in the analysis that follows and how they connect with the questions asked in the instrument:

Table 3.1.2 Analysis of the elements of productive disposition

Elements of productive dispositions- Kilpatrick et al (2001) & Carr and Claxton (2002)	**Question asked**	**Exemplars**
Sense making –resourcefulness and playfulness	What is multiplication and give an example?	"27 × 10 = 270"
Seeing multiplication as useful and worthwhile	What is multiplication?	" It helps to do mathematics"
Seeing oneself as an effective learner and doer of multiplication	Rate yourself between 1 and 9 (where 1 stands for being the weakest and 9 stands for being the strongest)	Placed herself at 5 and said "I'm not good in multiplication"
Belief that steady effort pays off - resilience	What do you do when you do not know the answer to a multiplication problem?	"I take a paper and count until I get it" "I take it home for help"

3.6 Validity and reliability

Cohen, et al. (2000) argue that 'earlier versions of validity were based on the view that it was essentially a demonstration that a particular instrument in fact measures what it purports to measure' (p.105). 'Reliability is essentially a synonym for consistency and replicability over time, over instruments and over groups of respondents. It is concerned with precision and accuracy' (Cohen, et al. 2002, p. 117).

Many researchers have argued that validity and reliability are two factors which any qualitative researcher should be concerned about while designing a study, analyzing results and judging the quality of the study. Morse, et al. (2008) argued that reliability and validity remain appropriate concepts for attaining rigor in qualitative research. They argued that without rigor, research is worthless, becomes fiction and loses utility.

The Wright, et al.'s (2006) instrument has been widely used across various countries and the validity of the instrument and its analytical framework has been established. The one-on-one assessment interviews by Wright, et al. (2006) through the video recordings enabled a detailed transcription and in-depth analysis of learner responses. Video recordings enabled repeated visiting of data to support confidence in the level allocated to a learner. In this way rich descriptions were enabled. As Maxwell (2004) argued that one way of addressing validity threats is through gathering 'rich data' which can enable thick description. "Validity is the touch stone of all types of educational research" (Cohen et al, 2002, p.106). In this study through learner stories I have endeavored to provide thick descriptions with evidence of learners' own words in the form of learner quotes. In the interpretive qualitative approach the researcher is the primary actor for data collection, the data collected would be verified by the supervisor.

3.7 Concluding remarks

The instruments I used in collecting the data gave an idea of where the learners are at in terms of their conceptual understanding of multiplication. This information is going to be helpful in the recovery program so that the learners are not treated as having the same level of understanding. Again the instrument for productive disposition, showed how the learners feel about multiplication: one of the learners said, "I prefer to do addition than doing multiplication". This immediately showed the learner's dispositions towards multiplication. As a result his performance in the instrument for conceptual understanding gave an idea of why he was not performing well, he simply did not like multiplication and that automatically affected his performance.

This poses a challenge to the teacher before teaching a topic, they need to find out what the learners know about that particular topic and why they think it is important for them to be taught that topic. This would make the learners understand and therefore change their attitudes towards that particular topic, this would hopefully make them understand the topic better because their attitudes are no longer negative towards that topic. Again using different strategies would help learners choose those that make sense to them so that they do not feel like they have to always follow the teacher's strategies to be correct.

CHAPTER 4: DATA PRESENTATION, ANALYSIS AND DISCUSSION

4.1 Introduction

This study is about investigating learners' mathematical proficiency in multiplication and also about exploring the relationship between conceptual understanding and productive dispositions. This topic required me to observe learners individually so as to see how they solve the multiplication problems posed to them. Following interviews and repeated observation of learner methods, each learner was allocated a level in terms of the Wright et al (2006) scale of descriptors. My data analysis is done in two main sections:

- In section 4.2, I analyze learners' multiplicative proficiency as indicated by the nature of their responses to a range of multiplicative tasks in the Wright, et al.'s (2006) interviews.
- In section 4.3, I analyze learners' multiplicative dispositions as indicated by their responses in the adapted instrument assessing learner dispositions in the context of multiplication.

The learners were interviewed individually with the Wright, et al.'s (2006) instrument, as described in the previous chapter, for multiplication (and division as reverse operation). I will organize the reporting of the findings in relation to the various parts of the instrument. That is: A) the early multiplication and division, and B) the advanced multiplication and division.

4.2 Analysis of conceptual understanding and multiplicative proficiency levels of learners

Similarly, I will discuss my findings in each of these in relation to the groups of tasks within the instrument. That is, in early multiplication and division, (i) Forming equal groups, (ii) FNWS of multiples, (iii) Visible items arranged in rows or arrays (iv) Equal groups of visible items, and (v) Screened items. In the advanced multiplication and division: (i) Tasks presented without visible or screened items, (ii) Commutativity and Inverse relationship, and (iii) Area multiplication. As indicated in the methodology chapter a video recorder was used to tape all the interviews which enabled revisiting of learner methods in order to support the validity of the interpretations of learner understanding and level of performance as discussed below. As discussed in the ethical considerations above, pseudonyms have been used for all learner stories.

The codes I have used are taken from Wright, et al. (2006) and Askew, Bibby and Brown (2001). This is because I found them complementing one another in the way they describe the learners' actions. For instance, learners use mainly their fingers when they are solving multiplication and division problems, Askew, Bibby and Brown (2001) describe that as ‘modelling’, while Wright, et al. (2006) do not cater for this kind of action in their descriptors. Below I offer the different codes used by both groups of researchers and I also offer the descriptors I added:

Table 4.1 Codes for describing learner responses

Wright, et al. (2006) : The Mathematics Recovery coding schedule	**Askew, Bibby and Brown (2001) : Coding System**
√ - correct √√ - correct and with certitude ?? – needs time to think ?√ - needs some time, then correct x√ - initially incorrect, then correct SC - child self-corrects TTA - ‘Try that again’ ∧ - omission of a number in FNWS or BNWS IDK - child says ‘I don’t know’ ‘.. ‘ - indicates the words used Rev - assessor revisits an item Red - Teacher redirects or teacher prompt C. from 1 - child counts from one CO - child counts on CDF - child counts on from CDT - child counts down to	NU - not understood Modeling (M) - using physical objects including fingers Counting (Co)-using counting method whether backwards/forwards no physical objects Place Value (PV) – child used knowledge of place value and base 10 blocks to answer Known Fact (KF) –When a pupil answers rapidly to have used a calculating strategy, and indicates that she simply knew the answer Derived Fact (DF) – a pupil drew on their bank of known facts to deduce another fact.
My own disposition indicators: NT-not trying TR-tries repeatedly G-guessing SM-sense making	

The above-mentioned codes were used on each sheet of the detailed summary table of each learner’s interview (see Appendix 3). The last row of the table shows my own addition of dispositional indicators which related to my second research question of dispositions. These were only noted where there were clear indicators of these and so they do not occur for each question. Of interest dispositional indicators were mostly evident in the more advanced section where greater perseverance and sense-making was required.

As described in the methodology, the Wright, et al. (2006) assessment has two sections. Firstly, there is part 1 where early multiplication and division are assessed. Secondly, in part 2 the advanced multiplication and division are assessed. I will discuss these sections separately. The assessment is intended to find out about the learners' conceptual understanding of multiplication and to enable the placement of learners' understanding at particular levels.

Below I provide a tabularized summary of the 6 learners' responses to the tasks in the individual assessment interviews. The learner names used are pseudonyms.

Table 4.2.1 Descriptive summary of how the learners performed in Part 1 of the Wright et al (2006) individual assessment for multiplication (and division as the inverse operation)

TASKS	ANDILE	VIWE	NAKO	ANDA	LULU	SINDY
1. Forming equal groups **Learner is given 15 counters, make 3 groups with 4 in each group. How many counters did you use?**	X √	NU-Not Understood X√-initially wrong and then correct)	M-modelling with fingers ? √ (needs some time and then correct)	C from 1-counts from 1	NU SC-self corrects	X √
2. FNWS (Forward Number Word Sequences) with multiples of : Counting in : **a) 2's**	ʌ/SC-omitted no'.s & self-corrected.	√√ - correct with certitude	√√	√√	√√	√√
b) 10's	?√ -needs time to think	√- correct	√- correct	TR- tries repeatedly	?√	√- correct
c) 5's	√√	√√	√√	√√	?√	√√
d) 3's	ʌ-omits numbers ??-needs time to think	CO-counts on M /X√ modeling with fingers	?√-needs some time and then correct	√-correct	ʌ-omits numbers X√	√-correct

3.Visible items arranged in arrays **How many dots altogether** **a) 10×2 array**	C from 1 – counts from 1	√/CO-counted in 2's	X√	√√- looked at array and gave correct answer.	√/CO-counted in 2's	√/CO-counted in 2's
TASKS	**ANDILE**	**VIWE**	**NAKO**	**ANDA**	**LULU**	**SINDY**
b) 5×3 array	C from 1	?√-needs time then correct Counted in 2's	?√-needs time. counted in 1's	X√/SC-self corrects Counted in 3's	√√-counted in 3's	√√-counted in 3's
c) 4×5 array	C from 1	??-needs time to think	√-counted in 2's	?√-briefly thought gave Correct answer	√√-counted in 5's	√√-counted in 5's
d) Turning the 4×5 array through 90^0	C from 1 again	C from 1 again	?√-counted again in 2's	?√-briefly looks at the card and gives answer	√√-answered without counting SM-sense making	√√/ KF-Known Fact SM-sense making
4(a) Equal grouping of visible items **Placing 4 circles and putting 3 counters on each circle**	√-counted in 1's	?√-counted in 3's	√√-counted in 3's	√√-counted in 3's	√√-quick answer given	√√-counted in 3's
4(b) Partition division **Sharing 15 counters among 3 children**	?√-distributed in 3's and the last ones in 1's	?√-counted in 2's to make groups with 5	√√-distributed in 1's	?√-distributed in 2's and the last 3 in 1's	?√-distributed counters in 1's	CO-counted on-Grouped in 3's and added on in 1's to make 5.

Sharing 12 counters so that each child gets 4	√-Asked how many kids and then made groups with 4 counters	?√-made groups with 4 counters	√√- paused and gave answer without using the counters	NU/?√-distributed in 1's	?√-made 3 groups with 4 counters	√-made groups with 4 counters
4(c) Partition Division with Redistribution **Sharing 24 counters among 3 children**	X√-initially wrong and then correct	NU-not understood X√-initially wrong and then correct	X√/SC-initially wrong & self-corrected	√-distributed in 2's	√-distributed counters in 1's	√-first distributed in 1's and then the last 6 in 2's
Sharing 24 counters among 4 children	?√-First distributed in 2's and then in 1's rechecking his groups	X√/SC-Made mistakes during his distribution but self-corrected	?√-Started by making groups with 4 counters and checking the equality along the way	X√/SC-Made mistakes during his distribution but self-corrected	X√-distributed in 1's but gave wrong answer and then recounted to get correct answer	?√-First distributed in 1's and then in 2's
5.Tasks involving screened items **Multiplication with equal groups** **a)Place 4 screens with 3 counters under each**	M-counted in 1's with fingers	√/M/CO-M with head counting in 3's	√√-correct with certitude counted in 3's."	√-counted in 3's with fingers	√√-gave correct answer	√√-counted quietly in 3's
b) Sharing 12 counters for 3 containers	?√-Put 3 counters in each	?√-made groups outside	√√-distributed counters in 4's	X√-distributed	?√-distributed counters in 1's	√-distributed counters in 1's

				counters in 1's		
c) 30 counters and 7 containers. **Use 20 counters so that each container has 5 counters.**	??-could not do the task	?√-needed time then correct	√-managed the task	X√-initially wrong, then correct	√	√√-did very well in the task
5d) Multiplication with an array **Using a 5×3 array, screen the 2 upper rows and then the 3 lower rows**	√-briefly thought, gave the correct answer.	?√- visualized rows and counted 1's	??-needed more time to think.	√-modelled with fingers counting in 1's	√-counted in 3's to get to 15.	??-gave the wrong answer twice
Using the 6×2 array, screen 5 rows	??-needed more time to think.	X√/M- initially wrong modeled with fingers.	NT-not trying	TR-tried repeatedly	?√-needed some time to think.	√√-correct with certitude.

Discussion: From the above table of coded responses, the nature and level of learners' multiplicative conceptual understanding is discussed below-

In the first task, the learners had a general problem of not understanding what they were expected to do and I had to use code-switching in order to clarify that learners should use the counters to make three groups with four in each group. In the FNWS (Forward Number Word Sequences), they all did fairly well in the counting by 2s, 5s and 10s but had problems in counting by 3s - Andile performed worst because he said, "3, 12, 17, 18, 19…". Some of the learners used counting on strategies to get to the next number and at times omitted numbers. Sindy was the only one who counted correctly in 2s, 3s, 5s and 10s though a little bit hesitantly in some. In the task where visible items were arranged in arrays, they all did well using different counting strategies. Andile was the only one who used counting in 1's in everything that had to be counted even in the 4x5 array where they had to count the dots and then it was turned through 90^{o} and they were asked again 'how many dots?', Viwe and Andile counted it again in 1s. Lulu and Sindy knew that there was no need to count the dots again as they are still the same number- i.e. changing the direction of the card will not affect the number of dots- Sindy even laughed at the question. Nako and Anda had to briefly think before answering and Nako recounted the dots in 2s while Anda eventually gave the answer without recounting.

In task 4, 'Equal grouping of visible items', most of them counted in 3s except for Andile. The quickest to give the answer was Lulu. When I asked her how she did it she said, "I counted whilst you were asking the question." In 'partition division' (Question 4b sharing counters), they all did fairly well differing in the way they distributed the counters- i.e. they either distributed in 2s or 3s and then topped up in 1s to get to 5 counters per group. In task 5- involving screened items, they did well in the task and all counted in 3s except for Andile who counted in 1's while he could not see the counters he tapped his finger on each card 3 times counting 1, 2, 3 – 4, 5, 6 - 7, 8, 9. In the task where they had to share counters into containers, they mostly distributed counters in 1s except for Nako who distributed them in 4s. In the second part of the task (30 counters and 7 containers), most of the learners had a problem in following the instructions, "use 20 counters so that each container has 5 counters". They either used all the counters or all the containers. Sindy was the only one who counted the 20 counters first and distributed them in 5s. Andile could not do it and so after rephrasing we moved onto the next question. In the last task-Multiplication with an array (where some of the rows were screened), learners mostly needed some time to think before giving the answer. Nako was the only one who could not answer both problems even after time was given to her. Lulu got both of them correct.

I now move on to analyzing learner proficiency as indicated by responses to items in the advanced section of the interview.

Below I provide the summary table of the advanced section:-

Table 4.2.2 Summary table of Part 2 (advanced section) of the Wright et al instrument

TASKS	ANDILE	VIWE	NAKO	ANDA	LULU	SINDY
PART 2: **ADVANCED MULTIPLICATION AND DIVISION** **1.Verbal Tasks Without Materials** **a) Multiplication** **6 children have 5marbles each. How many marbles altogether?**	G-guessing	M/TR-modelled and tried repeatedly	?√-needed some time to think	?√-needed time to think	TR-tried repeatedly	√√-quietly counted in 5's to get the answer
b) Quotient division **There are 12 bananas and each child is given 2 bananas. How many children are there?**	NT- not trying	M/√-modelled and gave correct answer.	M/√-modelled with fingers and got incorrect answer.	??-needed more time to think.	X√-initially incorrect, then correct.	√√-gave correct answer.
c) Division **If we shared 18 apples amongst 3 children, how many apples would each child get?**	NT/G-did not try just guessed the answer	NT- not trying	??-needed more time.	TR-tried repeatedly	??-needed more time.	√√/KF-known fact counted in 6's. SM-sense making

d)Quotient division with remainder **There are 17 flowers. Each person is given 5 flowers. How many people are there? How many flowers are left over?**	G-guessed the answer	M/√X-modelled correct answer but incorrect remainder.	??-needed more time	M/?√-modelled and got all the correct answers.	√√/KF- counted in 5's, correct with certitude	√√/KF-known fact
e)Partition division with remainder **We shared 14 cookies equally amongst 4 children. How many cookies would each child get?** **How many cookies are left?**	Guessed the answer	M/TR-modeled trying repeatedly	√X-got the correct answer but the wrong remainder.	M/?√-modelled needed time then correct	Co/X√-counted in 4's, initially incorrect then correct	√√-quietly thinks and give the correct answers
2.Commutativity & Inverse relationship. **a)Using the 3×7 and the 7×3 cards.** **Show the cards individually and they have to say what they mean.** **Say something about the two problems.**	??-did not understand the cards and they are not the same and the answers too.	??/√-did not understand the meaning of the cards but knew that the answers are the same.	??/√-could not explain the meaning of the cards but knew the answers to the cards are the same.	√√-gave clear meanings of the cards	√√-gave clear meanings of the cards	√√-gave clear meanings of the cards
b)Using the 8×4 and 32÷4 cards. **Give answer to 8×4 card**	G-guessed the answer.	??-needed more time	??-needed more time	??-needed more time	Co- counted in 4's using the same four fingers but got	√√- counted in 4's and got correct answer

Show the 32÷4 card, can the first card help you to do the problem?					incorrect answer.	
c)Using the 8×7= 56 card and individual cards with 8, 7, 56, =, ÷ **Show the 8×7=56 card** **Give them individual cards and they must make a division problem using the individual cards.**	G-guessed the answer	G-guessed the answer	G-guessed the answer	G-guessed the answer	G-guessed the answer	DF-drew from the bank of known facts. √√-correct with certitude.
3.Area Multiplication **Use a square unit and the 8×3 rectangle.** **How many squares like this would you need to cover the rectangle completely?** **Can you draw what the squares would look like?**	X√-moved the tile and got incorrect answer. Drew the area and got correct answer.	X√-moved the tile and got incorrect answer. Drew the area and got correct answer.	X√-moved the tile and got incorrect answer. Drew the area and got correct answer.	X√-moved the tile and got incorrect answer. Drew the area and got correct answer.	X√-moved the tile and got incorrect answer. Drew the area and got correct answer.	√√-moved the square unit and got the correct answer. Drew the area and got the same answer.

Discussion: In the advanced section of the assessment instrument, the tasks are again given verbally but learners have to work without materials (e.g. counters). Thus, they have to mainly visualize the questions so as to be able to solve them. There is an indication that Andile and Viwe have low conceptual understanding of multiplication and they are dependent on the counting materials. On the other hand Nako and Anda seem to be developing some basic level of conceptual understanding. Finally, Lulu and Sindy seemed to display some conceptual understanding of multiplication and division.

In all the tasks, Andile was the only one who got all answers incorrect. In the tasks without materials, most of the learners seemed to be having problems in division (as the inverse relationship of multiplication). That is for example (sharing 18 apples amongst 3 children) $\frac{4}{6}$ of the learners got incorrect answers. In the items where the answers had remainders, Sindy was the only one who managed the problems correctly. Half $\left(\frac{3}{6}\right)$ of the learners got the answers correct but did not manage remainders the correctly. The other two learners could not find a way to solve the problem.

On the items where commutativity and inverse relationships are assessed. Most of the learners seemed to understand that multiplication has the commutative property. Half $\left(\frac{3}{6}\right)$ of the learners were able to give clear meanings of the cards (3x7and 7x3) and $\frac{5}{6}$ of the learners understood that although the numbers are not written in the same order they still give the same answer when multiplied. Andile was the only learner who did not see the commutative relationship between the two cards. By contrast, in the other hand in the 8x4 and 32÷4 cards, Sindy was the only one who solved it correctly and was able to see the inverse relationship between the two cards. The other 5 learners could not solve these and could not see how the first card could be helpful in solving the second. They did not see the relationship between multiplication and division. In the item where the learners were shown the 8x7=56 card and then given the symbols (56, 7, 8, =, ÷) and asked to make a division problem, Sindy was the only learner who comfortably managed the problem. The others constructed for example 7÷8=56 or, the other way around. In the last item of area multiplication, all the learners except Sindy could not use the square unit tile effectively. They only managed to work out how many tiles after they had drawn the whole grid and counted them one by one.

From the above discussions of the learners' performances on both Part 1 and Part 2 of the interview, I provide a summary of my allocation of the levels to the learners as provided by Wright, et al. (2006).

Table 4.2.3 Summary by levels in the development of multiplication

LEARNER	LEVEL ALLOCATED	DESCRIPTORS OF THAT PARTICULAR LEVEL
Andile	Level 1: Forming Equal Groups or Initial Grouping.	The child did not see groups as composite units and thus counted items mainly in ones instead of multiples. Thus, he mainly used perceptual counting and sharing.
Viwe	Borderline between Level 1 and 2	The child used counting in 1s and in some instances in 2s but he still used perceptual counting because he was reliant on seeing items.
Nako	Level 2 : Perceptual Counting in Multiples	The child used multiplicative counting strategies to count visible items arranged in equal groups but had difficulty in solving items where groups were screened.
Anda	Level 3 : Figurative Composite Grouping	The child used multiplicative counting strategy to count items arranged in equal groups where individual items are not visible. So, she was not dependent upon direct sensory experience. She did not use the composite unit a specified number of times.
Lulu	Borderline between 4 and 5	The child was able to use the composite unit a number of times and was not dependent upon direct sensory experience. She fell short of the next level because she did not see the inverse relationship of multiplication and division.
Sindy	Level 5 : Multiplication and Division as Operations	The child was able to immediately recall or quickly derive many of the basic facts for multiplication and division. She was also able to see and the inverse relationship of multiplication and division.

Below I provide individual learner stories in order to illuminate the findings provided in the table above and enable connection with dispositional aspects evident in the interviews.

For Carr and Claxton (2002), learning stories are ‘structured observations in everyday or ‘authentic’ settings, designed to provide a cumulative series of qualitative ‘snapshots’ or written vignettes of individual children displaying one or more of the target learning dispositions’ (p. 22). Salomon (1991, p.16 cited by Carr and Claxton, 2002) also points out that ‘learning stories, unlike experimental challenges, can retain the ‘richness, complexity, and interdependence of events and actions in the real classroom’ (p.23).

4.3.1 Andile

Andile came across as a confident boy. This was indicated by the way he quickly answered the questions but as the interview progressed and challenges started to emerge his confidence receded and signs of stress were noted. (For example, In the advanced section when he could not solve the first item, he resorted to guessing and did that in all the other items). In all the tasks where counting was possible he did not use the groups as composite units as a result he counted the items in ones instead of multiples. In the FNWS of multiples, he managed to count comfortably in 5s and 10s but had problems in counting in 2s and especially counting in 3s. When shown the 4×5 card, he again reverted to concrete counting and counted the dots in 1s from 1 to 20. When the same card was turned through 90º and asked how many dots, he counted them all again in 1s. In summary, he got almost all the answers in the initial section correct but took a long time because of the dependence on concrete one to one counting. In the advanced multiplication and division section, where learners are expected to work without concrete materials (i.e. Counters), Andile did not seem to give himself time to think before giving answers. For instance, when he was shown two cards with 3×7 and 7×3:

Teacher: “What can you tell me about these two cards?” (He raises his shoulders symbolizing that he does not know the answer) (I do code switching and ask the question in isiXhosa)

Teacher: “Are the cards the same or have equal answers?”

Andile: “No the cards are not the same and the answers are not equal.”

For all of section B he continued answering almost immediately, seemingly guessing answers which were all incorrect. Thus, rather than try to make sense of the questions or to steadily try different ways to solve, he reverted to guessing indicating a low disposition in terms of sense making, resourcefulness and resilience.

In terms of conceptual understanding, which is an integrated and functional grasp of mathematical ideas, Andile seemed to have no abstract concepts and no fluency because he was stuck on using the least efficient methods, i.e. counting in 1s even where he could have done it in 2s or 3s. Furthermore he depended on seeing the items counted which likely affected his performance in the advanced section. In Wright, et al.’s (2006) terms he would be at level 1- Forming equal groups or Initial grouping, because he did not see groups as composite units hence the counting in 1s instead of multiples.

4.3.2 Viwe

Viwe presented himself as a learner who was easily distracted and one had to remind him to concentrate on the question asked. In the early multiplication and division tasks, he mainly used counting in twos and occasionally in ones. He used a lot of modeling using his fingers and his head to count. In the FNWS of multiples, he managed to comfortably count in 2s, 5s and 10s. He struggled however with the counting in 3's. For this he could only count in 3s up to 12 and then after that used his fingers to count on to the next multiple 15, 18 etc. and got muddled up because he was not sure from which number to start and carried on like that until I stopped him. In the basic multiplication and division tasks, he had difficulty in solving most of the problems because he depended on perceptual counting where items counted had to be visible as a result he mostly used his fingers and his head to model where counting was demanded. In the item where 12 counters were hidden under 4 screens:

Teacher: "Underneath these 4 screens, there are 3 counters under each screen. How many counters are there altogether?"

Viwe: (He looks at them for a while and finally uses his head counting them in ones and answers…)

"There are 12 counters altogether."

In solving the 3×7 and 7×3 cards, he said the problems are different but the answers are the same indicating some awareness of the commutative property of multiplication.

In terms of conceptual understanding, there was an indication that this was minimal but developing because he mainly used counting in 2s in almost all the tasks but substituted it with counting in 1s where necessary. He seemed to possess an indicator of conceptual understanding to a certain extent, that of being able to represent mathematical situations in different ways. In Wright, et al.'s (2006) terms he would be between level 1 and 2 because in some instances he used counting in 1's but mainly used multiples of 2. When solving tasks where items were screened, he had difficulty to an extent that he sometimes used his head to count or his fingers. This then places him in the perceptual counting level because he is reliant on seeing the items. In his case there are two possibilities, one could be that because he is easily distracted his resilience towards solving problems is low. The other could be that because his conceptual understanding is low, he is easily distracted and lacks resilience. Both of these could be possible since disposition and conceptual understanding are complexly intertwined. For this reason being low in both can result in a self-fulfilling prophesy.

4.3.3 Nako

Nako presented herself as a shy girl who did not like to have eye contact with the interviewer. In early multiplication and division tasks, she mixed her counting strategies between counting in 1s, 2s and 3s. She had difficulty in tasks where items were screened because she still relied on seeing the items, for instance, in the tasks where she had to divide counters and put them in containers, she then divided them on the table first and then took them in their groups and put them in the containers. In the advanced multiplication and division tasks, she struggled because the use of counting materials is not allowed. In most of the tasks she modeled with her fingers but got incorrect answers.

In terms of conceptual understanding, she seemed to be doing well in the tasks where she could see and touch the materials but in the tasks where items were screened she struggled. This pointed towards her being in the concrete stage of multiplicative reasoning. In terms of Wright, et al.'s (2006) framework, she seemed to be at level 2 (i.e. Perceptual counting in multiples), because she had difficulty in solving items where groups were screened. The fact that she did not make eye contact with me as interviewer and her abbreviated answers indicated low reciprocity (i.e. A lack in the willingness to engage with others) which could be linked to low levels of sense making and low conceptual understanding.

4.3.4 Anda

Anda presented herself as a soft spoken person but seemed confident and showed steady effort in that she took her time in solving tasks and was reluctant to use the counters in solving some of the problems. In the early multiplication and division tasks, she saw groups as composite units and mainly used counting in multiples of 3, 4, and 5. She was not dependent upon direct sensory experience. As result in most of the problems she did not touch the counters or the cards, she just looked at them and thought for a while and then came up with a correct answer. In the FNWS of multiples, she comfortably did her counting in 2s, 3s, 5s and 10s. When she was shown the 4×5 card, she briefly looked at it and gave the correct answer. When the same card was turned through 90º, she looked at it and gave the correct answer 20. In the advanced multiplication and division tasks, she solved most of the problems by just looking out of the window briefly and quietly and then gave the correct answers. When shown the 3×7 and 7×3 cards, she was able to explain the meaning of each card. That is three sevens or seven threes and then said that they are the same and the answers are the same.

In terms of conceptual understanding, she seemed to possess this to some extent because she was able to describe the meaning of 3×7 and 7×3 and also showed that she understood the associative property of multiplication. In Wright, et al.'s (2006) terms she would be on level 3 (i.e. Figurative composite grouping, because she used a multiplicative counting strategy and was not dependent on seeing the items. She showed confidence which could be linked to her belief in her own ability as indicated in her disposition interview she rated herself at a 9 at the top of her class. The way that she took time in solving problems indicated perseverance and resilience and also sense making.

4.3.5 Lulu

Lulu presented herself as a vivacious confident girl who seemed to enjoy the interview. Her body language was suggesting enjoyment as she was leaning forward and smiling in most of the tasks. In the early multiplication and division tasks, she used counting in 2s, 3s, 4s and 5s in the different tasks. She did well in the tasks where items were screened because she seemed to be able to visualize them with ease. In the task where 12 items were equally divided for four screens, she gave a quick answer and when I asked her how she did it she said, "I was counting whilst you were asking the question." I repeated the question but the second time I made her close her eyes again and then put 4 counters under each screen. She counted in 4s for the first two i.e. (4; 8) then counted on from 8 to 12 (i.e. 9, 10, 11, 12) and then to 16. She did this counting fast and with confidence. In the advanced multiplication and division tasks, she did fairly well in most of the tasks because she did not rely on seeing the items.

In terms of conceptual understanding, she showed that she understands what multiplication means. When I showed her the 3×7 and 7×3 cards, she was able to explain their meanings with ease (three sevens and seven threes). In terms of Wright, et al.'s (2006) levels, she could be in level 4 (i.e. Repeated abstract composite grouping), because she is able to count composite units in repeated addition tasks and she used the composite unit a specified number of times. She also showed some indication that level 5 was emerging because she understood the commutative principle of multiplication that 3×7=7×3. The enjoyment she displayed indicated that she was willing to engage (I got a strong sense of her willingness to engage i.e. her reciprocity) and there was clear evidence of her sense making and her resilience.

4.3.6 Sindy

Sindy came across as soft spoken but very confident in her knowledge of multiplication because she smiled when certain tasks were asked as if she was suggesting how easy they were. In early multiplication and division, she used multiples of 3, 4 and 5 in solving most of the tasks. In the FNWS of multiples she was able to count in 2s, 3s, 5s and 10s comfortably and without hesitation. In some of the tasks she did not need to use the counters to get the correct answers, she just waited for me to finish the question and she gave the correct answer. When she was solving an item where a 4 by 5 array was shown to her:

Teacher: “How many rows are there?”

Sindy: “5” (I asked her to show me the rows and columns)

Teacher: “ and how many columns?”

Sindy: “4”

Teacher: “How many dots altogether?” *(Answers immediately)*

Sindy: “20”

Teacher: “How did you count the dots?”

Sindy: “I counted in 5’s” (*and she adds on*) “I could also count in 4’s)

(I turn the card through 90⁰ as per the instruction of the Wright instrument)

Teacher: “How many dots now?” (she laughs at me and answers)

Sindy: “..still 20”

In the advanced multiplication and division tasks, she was able to answer most of the questions correctly and where there were mistakes she was able to self-correct herself because she was able to immediately recall the basic facts of multiplication and division. She was the only learner in the group who managed to get the task on the inverse nature of multiplication and division. That is, in the task where the learner is shown a card with 8×7=56 and then she is given five separated cards 8, 7, 56, =, ÷ and the learner had to make a division problem using the five cards. Sindy came up with two problems: 56 ÷ 8 =7 and 56 ÷7 = 8. This showed that the learner possessed the understanding of commutative principle of multiplication and also was able to see the inverse relationship of multiplication and division.

In terms of conceptual understanding, she showed that she was able to see connections among concepts and procedures. Through this interview the degree of a learner’s conceptual understanding and its relation to the richness and extent of the connections they have made (Kilpatrick, et al. 2001) was particularly visible. In terms of Wright, et al.’s (2006) levels, she seemed to be in level 5 (i.e. Working with multiplication and division as operation) because she could immediately recall and quickly manage basic facts for multiplication and division. She also showed the understanding of the commutative principle of multiplication and the inverse relationship of multiplication and division. She showed signs of resilience because in the more demanding tasks she took time and gave steady thought to answers. For example when shown the 8x4 card, she looked at it for a while and answered 32 and said that she counted in 4's. She also showed signs of sense making also linked to resourcefulness in that she seemed to manage to draw on different resources (methods and representations) for problems.

4.3 Discussion across the stories

What was noticeable was that although many learners knew how to count in multiples of the basic numbers (2, 3, 5, 10), when they had to apply the counting strategies in solving problems they resorted to counting by ones. So, it seems as if learners did not understand the connection that counting can also help them in solving multiplication problems (and its inverse operation division) i.e. many did not understand multiplication as repeated addition of multiples. Another observation was that learners seemed to develop at different rates. Whilst others still depended on seeing the items in order for them to solve them, others managed to solve them without seeing or using counters. This difference in development should be accommodated in classrooms so that all learners can be catered for. For those who still need materials to solve multiplication items, they should be allowed to do so and through more exposure they should be encouraged and pushed to see sense in multiplication and to develop more fluent abstract methods.

From the analysis of the 6 stories above a few points emerged supporting the arguments of Kilpatrick, Swafford and Findell (2001) and Skemp (1976). Students who have acquired conceptual understanding in an area of mathematics, see the connections among concepts and procedures and are able to give arguments to explain why some facts are consequences of others (Kilpatrick, et al. 2001). Skemp (1976) concurs by stating that knowing how methods are interrelated enables the learner to remember them as parts of a connected whole. This was clear with Sindy who saw the connection between multiplication and division and was able to comfortably solve the problem where she was given a multiplication card (8x7=56) and she had to make two division problems from that card. Skemp (1976) argues that knowing how methods are interrelated enables the learner to remember them as parts of a connected whole. Kilpatrick, et al. (2001) further state that a significant indicator of conceptual understanding is being able to represent mathematical situations in different ways and knowing how different representations can be useful for different purposes.

Conclusion and recommendations

From the analysis above, there is an indication that most of the learners interviewed had a minimal knowledge of multiplication which is way below the expected level. One of the factors that could be influencing their performance is the language issue because there were many instances where one had to translate the meaning of the questions in order for them to answer them effectively. However, the language issue does not explain the dependence on one-to-counting. At the beginning of the assessment the learners were required to count in stipulated numbers and most of them managed to do the counting but when they had to apply the counting in solving the tasks, they resorted to unitary counting. This shows that they had not established the necessary connection between repeated counting in multiples and multiplication.

In the section below I discuss the second research question of this study which concerns the nature of the learners' numeracy dispositions in the context of learning multiplication.

4.4 ANALYSIS OF LEARNER DISPOSITIONS

Kilpatrick, Swafford and Findell (2001, 131) define productive disposition as ' the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off and to see oneself as an effective learner and doer of mathematics.' So, in terms of Kilpatrick, et al. (2001) a productive disposition in the context of multiplication would involve confidence in their multiplication knowledge and ability, to perceive it as useful, sensible and worthwhile and to see oneself as an effective learner and doer of multiplication (i.e. someone who is able to make sense of it and progress in one's understanding).

As discussed and provided above, Graven (2012) designed an instrument for assessing learner productive dispositions. This instrument has mostly been administered as a written instrument and is limited in this sense because of learner literacy levels. Her instrument refers to dispositions in general in numeracy and I adapted the instrument to gather learner dispositions relating specifically to multiplication. See Appendix 1, for the copy of the adapted instrument which I used as a basis for orally administered but written response questionnaires.

The learners were allowed to answer in isiXhosa, their home language to make them comfortable and to afford them the opportunity to communicate effectively. I speak isiXhosa fluently and so no translator was needed.

As discussed in the methodology chapter the analysis will be looked at in terms of how Kilpatrick, et al. (2001) and Carr and Claxton (2002) define productive dispositions. Below I outline the relationship between certain aspects of their definitions:

- Seeing sense in mathematics and seeing it as useful and worthwhile (Kilpatrick, et al.) is related to resourcefulness (Carr & Claxton). This is because when a child sees sense in mathematics, he is able to be resourceful by using strategies from the bank of methods or representations that he possesses.

- Having a belief that steady effort pays off (Kilpatrick, et al.) is related to resilience (Carr & Claxton). This is because if a child understands that if he is resilient and patient in solving mathematical problems, there will be a pay off at the end, that of being successful in mathematics.

- Seeing oneself as an effective learner and doer of mathematics (Kilpatrick) is not directly related to resourcefulness or resilience (Carr & Claxton) but it can be linked to both of them. This is because a child who is confident about his mathematics knowledge is likely to be more resilient and resourceful when faced with a mathematics problem.

- Of note reciprocity (willingness to engage with others) which is a Carr and Claxton (2002) indicator is absent from the Kilpatrick, et al.'s (2001) definition.

Below I provide a summary table of the learner responses in the orally administered written response disposition instrument in the context of multiplication. See Appendix 1 for the instrument.

Table 4.2.5 Learner responses to the productive disposition instrument (x denotes multiplication)

Questions	**Multiplication is…**	**Give an example of a x problem**	**Why did you position yourself there?**	**Put a circle around yourself**	**Why does Kuhle struggle with x ?**	**Why is Sine good in x ?**	**What do you do when you do not know the answer?**	**Which do you enjoy more, addition or x and why?**
Andile	Mathematics	5x5 = 25	I can count	9	He cannot count	She can count	I go to the teacher	Multiplication, I can count
Viwe	Something that counts	2x2 =4	I do not know multiplication	1	Kuhle cannot count	She can count	I take it home for help	Addition, I like addition
Nako	It makes me good	2x2 = 4	I can calculate	9	He does not	She is clever	I take a paper and	Multiplication, I like counting

					concentrat e		count until I get it	
Anda	Mathematics	5x5 = 25	I can calculate multiplicatio n	9	He cannot count well	She can count	I ask for an explanatio n from the teacher and count it	Multiplication, I can count
Lulu	Counting	2x2 = 4	I can count	9	He cannot count	She can count	I use my hands to count	Multiplication, it helps to do mathematics
Sindy	You multiply numbers	27 x 10 = 270	I'm not good in multiplicatio n	5	He does not count well	She counts well	I count with my fingers	Multiplication, I love counting

In the next section I analyze the responses of the learners for the various questions in relation to whether they indicate aspects of productive disposition. The analysis is based on the descriptors given above by Kilpatrick, et al. (2001) and how they relate to the Carr and Claxton (2002) descriptors.

Question: What is multiplication? Give an example of a multiplication problem. (Columns 1 and 2 above)

This question was aimed at exploring whether learners understand what multiplication is and the analysis of learner responses for the first part will be denoted by (X). Again asking them to give examples was aimed at looking at how advanced their understanding of multiplication is and the analysis will be denoted by (●).

The table below shows the analysis of their responses in terms of their dispositions towards multiplication:

Productive Disposition Indicator: Seeing sense in mathematics, seeing it as useful and resourcefulness

Table 4.2.5.1 Analysis of their responses: Seeing sense in mathematics

	Non-answer*	Restricted/Ritualized answer	Explorative/ resourceful answer	Answer indicating useful and sense-making
Andile	X	●		
Viwe	X	●		

Nako		●		X
Anda	X	●		
Lulu	X	●		
Sindy	X		●	

The table above shows that for the first part of the question, all the learners did not give answers that made sense of what multiplication is. Rather they gave answers like, '…it is math', 'it is counting', 'you multiply numbers', 'something that counts'. One of them said, 'it makes me good' although she did not give clarity as to why multiplication makes her good, she showed that it is useful for something. For the second part, 5 out of 6 of the learners gave ritualized answers where they multiplied the same numbers, some used '2x2=4 or 5x5=25'. Only one of the learners showed some resourcefulness by using the pattern of multiplying by 10, '27x10=270'. All of their responses showed that their knowledge of multiplication is limited in terms of their articulation of it. This coheres however with their performance on the Wright, et al. (2006) interview

In the next analysis the focus is on another element of productive disposition- 'belief that steady effort pays off' Kilpatrick, et al. (2001,p 131) and what, Carr and Claxton (2006) define it as resilience.

Question: What do you do when you do not know an answer to a multiplication problem? (Column 7)

Productive Disposition Indicator: Believing that steady effort pays off/resilience

The question was intended to see how learners react when they are faced with a problem they do not know how to solve. The categorization this will be denoted by a (○)

Table 4.2.5.2 Analysis of their responses: Believing that steady effort pays off

	Non-answer/ skipped the problem	Ritualized response (give answer even if it is wrong)	Ask for clarity/ask for help and try again	Steady effort/resilience
Andile			○	
Viwe			○	
Nako				○
Anda			○	

Lulu				○
Sindy				○

The table above shows that some of the learners indicate they are dependent on the assistance of either the teacher or the parent, ⅔ of those do not indicate if after receiving the assistance they try to solve the problem, Anda is the only one who says, 'I ask for an explanation from the teacher and count it'. The other three learners show some resilience, ⅔ of them say that they use their fingers to count so as to get the answer and the last one says, 'I take a paper and count until I get it'. This shows that the learner is resilient as she says she does not stop until she gets the answer. Overall, the learners indicated some degree of steady effort because none of them said I skip the question if I do not do the problem.

Below I analyze another element of productive disposition which is, 'seeing oneself as an effective learner and doer of mathematics'. This element is important because if a learner does not see himself as an effective learner and doer of mathematics, he will likely lack confidence in solving mathematical problems even if they are not difficult.

The question below was intended to find out how the learners view themselves in solving multiplication problems and they also had to give a reason why they view themselves that way. The analysis will also include the levels allocated to them in terms of the Wright, et al. (2006) instrument as this enables comparison between their assessed level and their perceived performance level. The responses to the first part of the question will be denoted by the rate that the learner has given him or herself (a number between 1 and 9), the second part will be denoted by a (○)

Question: Rate yourself between 1 and 9. Why did you rate yourself as such? (Columns 3 and 4)

Productive Disposition Indicator: Seeing oneself as an effective learner and doer of mathematics.

Table 4.2.5.3 Analysis of their responses: Seeing oneself as an effective learner and doer of multiplication.

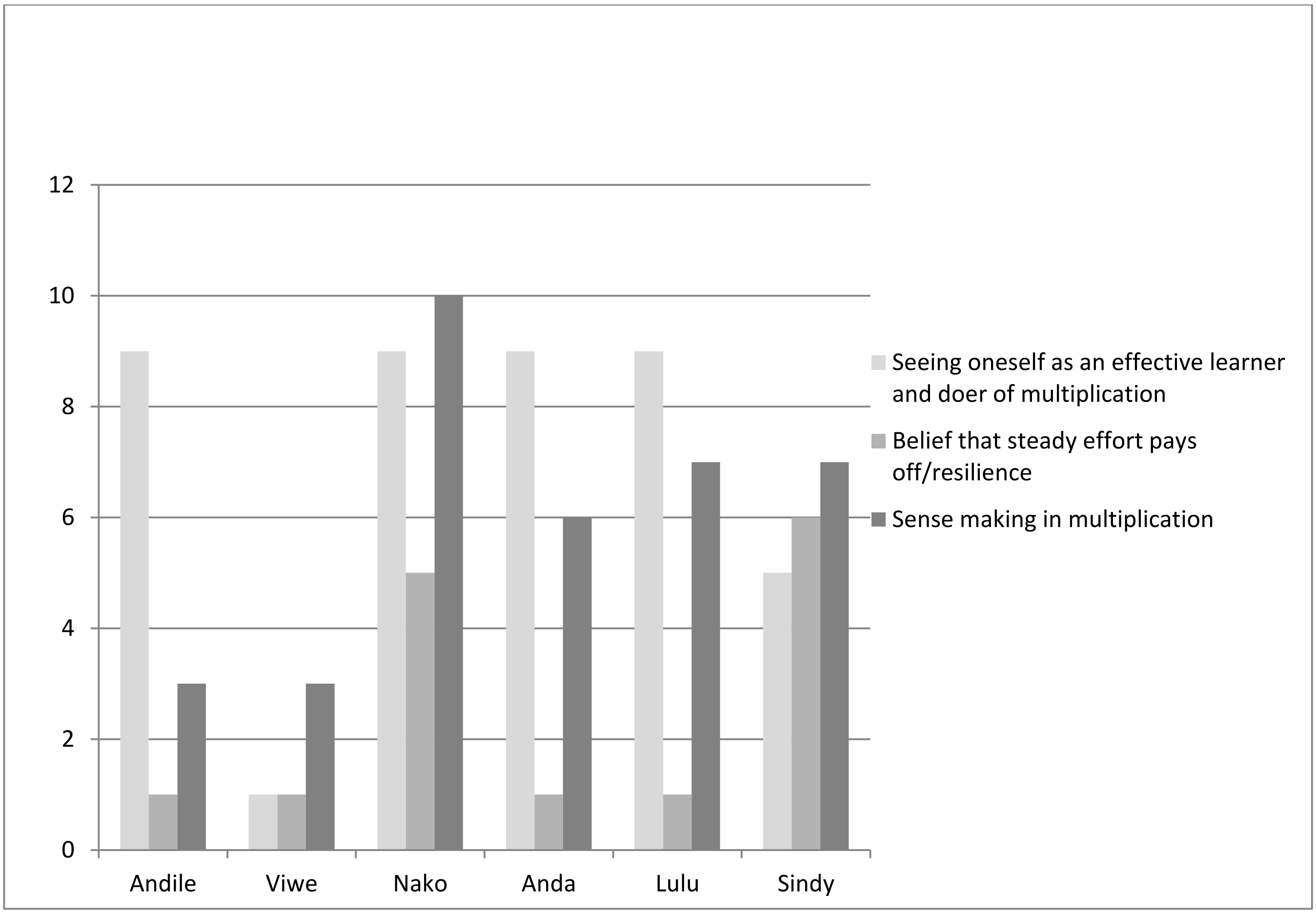
	Level allocated to the learner	**Negative/no confidence**	**Less confident**	**Confident**	**Very confident**
Andile	1				9 ○
Viwe	1-2	1 ○			
Nako	2				9 ○
Anda	2-3				9 ○
Lulu	3-4				9 ○
Sindy	5		5 ○		

As indicated in the table above, $\frac{4}{6}$ of the learners placed themselves at number 9 which is the top of the scale denoted that they see themselves as good at multiplication and they confidently said that they can count or calculate multiplication. Viwe is the only learner who rated himself as a 1 and admitted that he did not know multiplication. Sindy rated herself as a 5 and added that she is not good at multiplication. Of note the ratings given by the learners above do not necessarily reflect their performances but rather they show their dispositions towards multiplication in terms of their belief in their ability to do it.

Looking at the levels of conceptual understanding allocated to the learners and comparing them to their dispositions towards multiplication, it is noticeable that Andile rated himself as a 9 meanwhile his performance level of understanding multiplication is at the lowest level1 (i.e. perceptual counting by 1s). Tirosh, et al. (2013) cite a range of research where young learners incorrectly associate effort with competency. Within the data of this study, it seems that Andile associated multiplicative competence with giving answers for every question even if they were all incorrect. He came out as the learner who depended on the teacher when he had problems and did not make an effort to solve them by himself. Nako also associated effort with competence. She admitted that 'I take a paper and count until I get the answer'. This resilience and steady effort did not however automatically translate to competency as shown above she was allocated a level 2 (perceptual counting) in conceptual understanding of multiplication. Andile was the only learner whose dispositions and conceptual understanding were somehow matched. He placed himself at 1 on the dispositions instrument and when asked which operation he preferred between addition and multiplication, he stated that he liked addition more. This was matched with the fact that he was allocated a borderline level between 1 and 2 in the test for conceptual understanding (indicating that he was at a perceptual level). On the other, hand Sindy rated herself at 5 which indicated that she perceived herself as an average learner of multiplication but the test for conceptual understanding gave a stronger picture of her competence. Compared to others she was the best of all the learners as she was allocated a level 5 (the highest Wright, et al. level).

The graph below summarizes the learner dispositions in terms of the elements of productive disposition present in the questionnaire

Figure 4.1: Summary of learner responses according to 3 elements of productive disposition.



Of interest the indicator of sense-making matches most closely with the Wright, et al. (2006) levels of learners- i.e. the higher the level, the greater the indicator of sense-making (i.e. Lulu, Sindi, Nako and Anda higher that Andile and Viwe in both).

The analysis in the graph shows how the learners responded to the questions asked in terms of the elements of productive disposition. The purpose of the questions was to elicit their dispositions towards multiplication. The graph shows that $\frac{4}{6}$ of the learners did not provide responses indicating seeing sense in multiplication. Most of them said multiplication is ‘mathematics’ and when they were asked to give examples of multiplication problems, $\frac{5}{6}$ of them gave ritualized simple examples which did not give an indication that they have a sense of what multiplication is all about. Their confidence though showed a different story. When they were asked to rate themselves between 1 and 9 (where 1 represented the weakest and 9 the highest), $\frac{4}{6}$ of the learners rated themselves at 9 which showed that their perception of their multiplicative knowledge was high contrary to the fact that they could not give clear explanations of what multiplication is and that the Wright, et al. (2006) levels were mostly low. In terms of their beliefs that steady effort, patience and resilience usually pay off, half of the learners showed that they do all they can to make sure that they do not give up until they got the answer. Some of this steady effort was evident when answering the Wright, et al. interview.

In conclusion, other questions that were asked did not have a close association with the elements of productive disposition but they somehow showed that the learners associate multiplication with being able to count and being clever. They also associated being good in multiplication with concentrating in class and listening to the teacher.

4.5 ANALYSIS OF THE RELATIONSHIP BETWEEN CONCEPTUAL UNDERSTANDING AND PRODUCTIVE DISPOSITION

In the first part of the study multiplicative proficiency interviews were conducted so as to find out the learners’ level of understanding in the context of multiplication (and division as its inverse operation). The learners’ responses gave an indication of how much the learners understood about multiplication and division and at the end of the interviews and analysis levels were allocated to the learners based on their answers and performance. In the second part learners were given an instrument aimed at exploring their dispositions are towards multiplication. Their written responses indicated aspects of sense-making in multiplication, enjoyment of multiplication problems and also resilience towards multiplication problems. In the next section I focus on the relationship between conceptual understanding and productive disposition. (That is, my third research question).

A key aim of this study is to better understand the relationship between multiplicative conceptual understanding and productive disposition so that we might be able to explore ways to strengthen each to the mutual benefit of the other and thus shift our teaching for this purpose. Kilpatrick, Swafford, and Findell (2001) argue that the strands of mathematical proficiency are interwoven and as a result productive disposition develops when the other strands do and helps each of them develop. For most of the learners in my study their data concurs, that is, the data in the conceptual understanding and productive disposition interviews illuminate this relationship but there were two extreme cases, Andile and Sindy suggest that this is not the case across all learners nor across all dispositional indicators.

For both of the learners the data collected provided different stories about the relationship of conceptual understanding and productive disposition. Andile presented himself as a confident learner who had positive disposition in terms of self-efficacy beliefs and steady effort towards multiplication but when he was assessed for conceptual understanding a different picture emerged and he was allocated a level 1 because of the basic methods he used to solve multiplication problems and a lack in using more efficient strategies. His case suggested that because there can be a big gap between conceptual understanding and a learner's sense of self-efficacy, the relationship seemed to be weak between productive disposition and conceptual understanding.

Perhaps for the relationship to be fruitful, there is a need to bridge the gap by developing his conceptual understanding in multiplication so that it is matched with his dispositions. Sindy was allocated level 5 in the conceptual understanding interview because of the strategies she used in solving multiplication and division problems. When the dispositional interview was administered to her she put herself at 5 when asked to place a circle around a number that best describes her knowledge of multiplication and when she was asked why she placed herself as such she said, "I'm not good in multiplication". Sindy is not confident in her knowledge of multiplication probably because she does not have a developed productive disposition even while she shows sense-making. On the other hand, Andile rated himself as a 9 which means that he perceives himself as one of the best learners in multiplication but the assessment painted another picture about level of conceptual understanding. For Sindy, there is a need to develop her dispositions so that they are matched with her conceptual understanding, and for Andile, there is a need to develop his conceptual understanding so that there is a match with his dispositions. In this way these two strands could better work together to reinforce each other.

Viwe rated himself the lowest in the test for dispositions because he rated himself at a 1 and even stated that he was not good in multiplication. As a result he said he preferred addition to multiplication. Viwe's story shows the relationship between the two strands, and that if the conceptual understanding is low then the dispositions will be low. There seems to be a need to develop his conceptual understanding so that there is a development in his dispositions. Kilpatrick, et al. (2001) suggest that parents and educators have a challenge to help children maintain a productive disposition towards mathematics as they develop the other strands of mathematical proficiency.

Nako and Anda presented themselves as confident learners with positive dispositions. They rated themselves as the best in the context of multiplication but after the proficiency interviews were conducted different stories emerged. Nako was allocated at level 2 and Anda a borderline level between 2 and 3 for their conceptual understanding. This is because there were signs that they still needed some development in the topic of multiplication and division. In their case the gap between conceptual understanding and productive disposition is not too big. With a little bit of development of their conceptual understanding, the relationship could possibly work positively for both of them.

Lulu's story is similar to the two girls above except that there is not a big gap between her conceptual understanding and dispositions. She rated herself at a 9 in the dispositional interview and in the assessment for conceptual understanding she was rated as a borderline case between levels 4 and 5. In her case also her conceptual understanding and productive disposition seem to work well to influence each other and to sustain and develop for multiplicative reasoning.

DISCUSSION

The interviews provided rich data and stories that illuminate the interrelationship between the two strands of conceptual understanding and productive disposition. Productive disposition is especially related to indicators of sense making, steady effort and resilience. However, Kilpatrick, et al.'s (2001) indicator of belief in own ability which is not a Carr and Claxton (2002) indicator, showed a less predictable relation where some learners with weak conceptual understanding perceived themselves as good in multiplication.

Many important issues informed this study: when students have acquired conceptual understanding in an area of mathematics, they see the connections among concepts and procedures. They are then able to give arguments that explain why some facts are consequences of others because they gain confidence. This then provides the base from which they move to another level of understanding (Kilpatrick, Swafford, & Findell, 2001). Another issue which concurs with Kilpatrick, et al. (2001) is that students who have developed a productive disposition are confident in their knowledge and ability. They also see that mathematics is both reasonable and intelligible and believe that with appropriate effort and experience they can learn.

What was noticeable was that although most of the participants had positive dispositions towards their knowledge of multiplication, it did not necessarily translate to high conceptual understanding. For learners to see sense in any area of mathematics, they need to have conceptual understanding. This therefore means that learners need more opportunities to develop their conceptual understanding so that they can eventually see sense in that particular area of mathematics. Seeing sense is not something that can be taught to the learners but as they get more understanding, they are indirectly developing their dispositions.

Concluding Remarks

Mathematical proficiency is about developing all the five strands so that our learners are able to compete with their peers from other countries. If teachers could work towards developing these strands not only on the multiplication topic but on all mathematical sections, there would be an improvement in the learners' achievement.

CHAPTER 5: CONCLUSION AND RECOMMENDATIONS

5.1 SUMMARY DISCUSSION OF THE FINDINGS

Mathematical proficiency can be looked at as a group of five intertwined strands: procedural fluency, conceptual understanding, strategic competence, adaptive reasoning and productive disposition (Kilpatrick, et al. 2001). This study focused on two of those strands, conceptual understanding and productive disposition. Kilpatrick, et al. (2001) have argued that conceptual understanding is a wise investment that pays off for students in many ways and they further endorsed that, "students' dispositions towards mathematics are a major factor in determining their educational success" (p.131). This study has illuminated the importance of these two and the extent to which they are intertwined and the nature of this intertwined relationship. It has also illuminated the need for further development of these two strands among the learners in the class as only 4 of the 6 learners showed competence across levels and with a productive learning disposition towards multiplication.

The study showed that there is a lack in conceptual understanding which linked with learners being less likely to see sense in multiplication and thus learners tended to use the most basic (and often inefficient) strategies in solving mathematical problems. This use of basic strategies shows a lack in progress in multiplicative reasoning since in most cases strategies used were concrete and were time consuming. For example, most learners counted dots in ones even when they were arranged in fives and therefore spent much more time than would have been the case had they counted in fives or seen the relationship of x rows of fives is x times five. This implied that although they have been taught counting from the lower grades, they do not apply more efficient grouped counting skills even when moving to multiplicative arrays which encourage such counting. This raises concern since Kilpatrick, et al. (2001) have argued that mathematical understandings depend on the learner's ability to make necessary connections so that a coherent whole of concepts and procedures is achieved. Hiebert (1984) agrees by stating that the critical instructional problem is not of teaching additional information, but rather one of helping students see connections between the pieces of information that they already possess.

Another aspect that can be related to the findings is that the case study learners, as with most Grade 4 learners in Eastern Cape schools are faced with a difficult task of changing from being taught in isiXhosa to English in Grade 4. This poses problems because they often do not understand the English language let alone the mathematical language in English. In this study I found that one had to translate all the questions for most of the participants. This raises an important question for the ANA assessments where learners write in English and no translation is allowed. See also Graven and Venkat (2013) who raise concern with this based on a sample of Eastern Cape and Gauteng teachers responses to the [ANAs]. These learners could be performing below their level because of a lack in understanding the language meanwhile they are compared to learners whose home language is English. The importance of the mother tongue in the learning process is emphasized in the South Africa. Department of Education [DoE], 1996), South African Schools Act of 84. The document refers to its importance in the support of developing what is termed 'the language of academic efficiency'. The reasons stipulated about the importance of the home language instruction are that quality education can be assured through this instruction and that it facilitates the acquisition of knowledge and concepts and the development of cognitive, effective and social skills according to the learners' potential. The learners' level of articulation of written language for the productive disposition instrument would have likely had an influence on what learners were able to answer. This is a limitation of researching young learner dispositions in terms of what they say or write as their levels of articulation influence what is able to be said. For this reason I supplemented the data on what they said to their ways of working and answering questions during the one to one interviews.

Another finding in this study is that learners in Grade 4 are still very dependent on seeing and using counters and arrays when solving multiplication problems. When the counters were removed during the course of the interview, their problems started to surface. Some resorted to guessing the answers without thinking and others used modeling with their fingers. This showed that they still needed the counting materials in order to solve most of the problems. This revealed that they have not moved from the concrete strategies of multiplicative reasoning to the abstract strategies. This concurs with several researchers who have found that concrete methods to solve problems, such as tally counting, are dominant in the schools they have investigated through classroom-based research. They also argue that the poor mathematical results in South Africa are the result of inefficient moves made by learners from counting to calculating. In their study, many students failed to move their thinking sufficiently forward from concrete counting actions to abstract thinking.

This study is based on two strands: conceptual understanding and productive dispositions. Above I have elaborated on findings related to the first of these and I now discuss the findings pertaining to the latter strands. Most of the participants gave an indication that they were confident in their knowledge of multiplication, four out of six gave themselves the highest rating of 9. This showed that they saw themselves as effective learners and doers of multiplication which could have well in their favor had these been matched with conceptual understanding, unfortunately their confidence was not underpinned by good conceptual understanding. Some of the learners who stated more positive dispositions showed low conceptual understanding and one learner who was high on conceptual understanding rated herself at only 5 in the dispositional assessment. Only two of the learners were well matched in this respect. Viwe rated himself at 1 in the assessment on dispositions and even stated that he does not like multiplication and prefers addition to it. In the assessment for conceptual understanding he was allocated between level 1 and 2 (i.e. a borderline level). This could mean that he has not developed multiplicative reasoning (beyond the basics) possibly causing him to have a negative attitude towards multiplication and low confidence in his multiplicative ability. Lulu was also well matched because she rated herself at 9 in the dispositional assessment and in the instrument exploring conceptual understanding she was allocated between level 4 and 5.

Several elements of productive disposition as discussed by Kilpatrick, et al. (2001) and Carr and Claxton (2002) were displayed by the learners. When they were asked what they do when they do not know the answer to a multiplication problem, most of them stated that they had some resilience because they indicated that they persevered until they got the answer using different methods. The ones who were low on conceptual understanding gave an indication that they either go to the teacher or take it home for help. They tended to state that they put in less steady effort (resilience) possibly because they cannot easily access other ways to try since they do not have conceptual understanding to support this. Some of the learners displayed confidence in multiplication in that when asked certain questions, they laughed before giving the correct answers as if they were indicating that the question asked was too simple or obvious. Again when asked how they got the answer in some of the questions, some of them were able to display resourcefulness by showing more ways of calculating that particular problem. The data also concurs with Kilpatrick, et al. (2001) who have argued that students who have developed a productive disposition are confident in their knowledge and ability.

5.2 Implications for practice

Mulligan and Mitchelmore (1997) argued that the standard curriculum takes no advantage of the informal understanding of multiplicative situations that many students have developed well before Grade 3. They further add that multiplication is usually introduced before division and separated from it, whereas children spontaneously relate them and do not necessarily find division more difficult than multiplication. Their advice is that children would benefit if teachers provided them with opportunities to solve multiplicative word problems as early as the first year of schooling and if they linked multiplication and division much more closely.

It is not enough that our learners can immediately give answers when asked about a mathematical problem, they must also understand what the different concepts of that particular problem are and this will develop them mathematically. So, the implications for practice are that procedural fluency cannot be seen in isolation as the primary indicator for mathematical proficiency. All the other strands have to be developed so that our learners are mathematically proficient. Seeing sense in mathematics is not something that can be given to the learners. More exposure, and allowing them opportunities to see mathematical concepts from different angles, will benefit the learners and make them see sense and thus gain confidence in their ability to be effective doers and learners of mathematics.

Looking at the two strands discussed in this study: conceptual understanding and productive disposition, there is a mutually supportive relationship between the two. The relationship is reciprocal as indicated by Kilpatrick, et al. (2001) that productive disposition develops when the other strands do and helps each of them develop. If a learner possesses conceptual understanding in the concepts of multiplication, they are able to see sense and give sensible explanations for getting particular answers. This was illuminated in the data of learners in this study.

The implications for teaching multiplication are that conceptual understanding should be developed in parallel. This is because mathematical understandings depend on the learner's ability to make the necessary connections between concepts and procedures so that a coherent whole is achieved. Kilpatrick, et al. (2001) argue that because facts and methods learned with understanding are connected, they are easier to remember and use, and they are reconstructed when forgotten. Furthermore, they argue that a significant indicator of conceptual understanding is being able to represent mathematical situations in different ways and knowing how different representations can be useful for different purposes. If this happens, learners will gain confidence in their knowledge and ability, and subsequently develop their productive disposition.

Carr and Claxton (2002) define dispositions as a 'very different type of learning from skills and knowledge, and can be thought of as habits of mind or tendencies to respond to situations in certain ways' (p.30). This means that dispositions cannot be taught to the learners. The challenge is to expose them to different experiences that support positive learning dispositions such as encouraging sense making and steady effort and a key way to do this as this research has shown is to expose them to different multiplicative strategies and procedures so that they are able to choose the ones that make more sense to them. Schoenfeld (1992) argues that students develop their sense of mathematics- and thus how they use mathematics from their experiences with learning mathematics largely in the classroom. Furthermore, he argues that learners' "habits and dispositions" are shaped by classroom experiences, and have a great weight in shaping individual behavior. Kilpatrick, et al. (2001) concur by suggesting that developing a productive disposition requires frequent opportunities for making sense of mathematics, to recognize the benefits of perseverance, and to experience the rewards of sense making in mathematics.

The most important task of the teacher is to "assist learners to widen their repertoire of calculation strategies" (Mulligan & Mitchelmore, 1997, p.328). Wright, et al. (2006) provides an instrument which allows an opportunity for the educator to know the level where each learner is at. If we know the level where the learners are at, then we are better able to move them to the next level as Wright, et al. (2006) are suggesting in their recovery program. An implication of this is to think about where learners are at because one cannot move the whole class by level since learners are at differing levels as their point where they must move from. In South Africa, however, it is difficult because the learner materials and department books are a one size fits all. This therefore calls for communication amongst educators of different phases, this will give an indication of where educators in the next grade should start so that there is continuity in the learners' work.

5.2.1Limitations and opportunities for further research

In this case study a small sample was used and therefore my findings are limited and not generalizable to the general population. The use of the disposition instrument as an orally administered but written response instrument, while saving time, was constrained by the learners' levels of writing. Further research of a larger number of learners, across contexts, with interviews and follow up interview questions could allow a deeper exploration of the relationship between self-efficacy, sense-making and other dispositional indicators and conceptual understanding that might lead to generalizable findings.

5.2.2 Reflecting on my own journey through this research study

As a masters student this was my first experience of conducting original research over an extended period of time. I learnt an enormous amount about the topic itself in relation to how learners learn multiplication and the levels of development in that context. The use of the Wright, et al. instrument in this study has shown how important it is to know where the learners are in their development of multiplication so that as the educator you are able to group them accordingly and move them to the next level of understanding in their groups. I also learnt how important it is to understand the learners' mathematical dispositions towards a particular topic because if they are low they have a negative attitude towards that topic that impedes their learning of it.

The interviews I conducted with the six learners have shown how important it is for educators to work towards introducing abstract strategies earlier but gradually and with understanding so that learners can see the advantages of working in the abstract than concretely for themselves. Learners should also be afforded opportunities to discover many strategies of solving problems so that they are able to choose those strategies that make sense to them. This will help our learners to have a developed productive disposition and thus become confident in their knowledge. All of this has sparked my interest in researching further and continuing my learning trajectory into further doctoral research.

As a teacher I have realized from my own practice that it is not enough that learners are able to give the correct answers in a mathematical problem, they must also understand the concepts involved in that particular problem and also be able to explain the procedures they have used and why. This will give an indication that they understand what they are doing and if it makes sense to them. I have also realized that it is important to let learners talk amongst themselves and with the teacher about the mathematical problems and how they have come up with their answers. This will indicate to the teacher where the misunderstandings are and where more explanation and probing are needed.

The aim of my study was to better understand how learners learn about multiplication and the various levels learners must progress through, this has greatly enriched me. I plan to share my findings with my colleagues at my school and other schools firstly through a presentation of my research and findings and secondly through supporting teachers interested to conduct Wright et al interviews with learners in their classrooms as a teacher development tool. As Wright, et al. (2006) suggest, the topics of multiplication and division build on students' knowledge of addition and subtraction, and also multiplication and division. Their assessment instrument and recovery program provide foundational knowledge for topics such as fractions, ratios, proportion and percentage, all of which are core and essential areas of mathematical learning typically addressed in the primary or elementary grades. Many researchers agree that it is not enough that learners can do repeated addition, they have to be exposed to other strategies so that they are able to see that there is a difference between additive reasoning and multiplicative reasoning.

If I were to do this study again, I would increase my sample to at least 10 learners so as to get more data and insight on the topics at hand and also more information on the relationship between the two topics. I would also use Grade 5 learners instead of the Grade 4's that I used. This is because learners in Grade

4 are in their second year of learning multiplication as a result most of them were not confident in their knowledge. They are starting to use English as a language of teaching and instruction and as a result they struggled to understand the questions and one had to use code switching many times so that they could understand what was asked of them. This was not only time consuming, it also placed the researcher in an awkward position where the learners did not understand the question and some of them would drift away but as a researcher you felt like you were forcing them to give you answers.

REFERENCES

Anghileri, J. (1989). An Investigation of Young Children's Understanding of Multiplication. *Educational Studies in Mathematics*, *20,* 367–385.

Anghileri, J. (2006). *Teaching Number Sense.* London: Anthony Rowe.

Askew, M., Bibby, T., & Brown, M. (2001). *Raising attainment in primary number sense: from counting to strategy*. London: BEAM Education.

Barmby, P., Harries, T., Higgins, S., & Suggate, J. (2009). The array representation and primary children's understanding and reasoning in multiplication. *Educational studies in mathematics, 70 (3),* 217–241.

Baroody, A. J. (2006). Mastering the Basic Number Combinations. *Teaching Children Mathematics*, 23.

Boaler, J. (1997). Setting, social class and survival of the Quickest. *British Educational Research Journal*, *23*(5), 575–595.

Boaler, J. (2002). Exploring the nature of Mathematics Activity: Using Theory, Research and "Working Hypotheses" To Broaden Conceptions of Mathematics Knowing. *Educational Studies in Mathematics*, *51*(1-2), 3–21.

Carr, M., & Claxton, G. (2002). Tracking the Development of Learning Dispositions. *Assessment in Education: Principles, Policy & Practice, 9(1),* 9–37.

Claxton, G., & Carr, M. (2004). A framework for teaching learning: the dynamics of disposition. *Early Years*, *24*(1), 87-97.

Carpenter, T.P., Fennema, E., France, M.L., Levi, L., & Empson, S.B. (1999). *Children's Mathematics: Cognitively guided instruction*. Portsmouth: Heinemann.

Clark, F.B., & Kamii, C. (1996). Identification of Multiplicative Thinking in Children in Grades 1-5. *Journal for Research in Mathematics Education*, *27*(1), 41–51.

Cobb, P., Gresalfi, M., & Hodge, L. L. (2009). An interpretive scheme for analysing the identities that students develop in mathematics classrooms. *Journal for Research in Mathematics Education*, 40-68.

Cohen, L., Manion, L., & Morrison, K.R.B. (2000). *Research Methods in Education* (5th ed.). London: Routledge Falmer.

Diener, E. & Crandall, R. (1978). *Ethics in social and behavioural Research.* Chicago: University of Chicago Press.

Driver, R., and Oldham, V. (1986). A constructivist approach to curriculum development. *Studies in Science Education*, 13, 105-122.

Ensor, P., Hoadley, U., Jacklin, H., Kuhne, C., Schmitt, E., Lombard, A., et al. (2009). Specialising pedagogic text and time in Foundation Phase numeracy classrooms. *Journal of Education*, *23*(1), 5–29

Ernest, P. (1991) The Philosophy of Mathematics Education. London: Falmer.

Ernest, P. (2006). Reflections on theories of learning. *Zentralblatt für Didaktik der Mathematik*, *38*(1), 3-7.

Fleisch, B. (2008). Primary education in crisis: Why South African schoolchildren underachieve in reading and mathematics. Johannesburg: Juta.

Fosnot, C.T., & Dolk, M. (2001). *Young Mathematicians at work 2: Constructing Multiplication and Division*. Portsmouth, NH: Heinemann

Graven, M. (2012). Accessing and assessing young learners' mathematical dispositions. *South African Journal of Childhood Education*, *2(1)*, 49–62.

Graven, M. (2011). Creating New Mathematical Stories: Exploring opportunities within mathematics Clubs: Proceedings of the 17th National Congress of the Association of South Africa (AMESA, pp. 166-170). Johannesburg: University of Witwatersrand.

Graven, M., Hewana, D., & Stott, D., (2013). The Evolution of an Instrument for Researching Young Mathematical Dispositions. *African Journal of Research in Mathematics, Science and Technology Education*, 17(1-2).

Graven, M. & Venkatakrishnan, H., (2013). ANAs: ANAs Possibilities and Constraints for Mathematical Learning. *Learning and Teaching Mathematics,* (14),12–16.

Graven, M., Venkat, H., Westaway, L. & Tshesane, H. (2013) Place value without number sense.

Exploring the need for mental mathematical skills assessment within the Annual National Assessments. *South African Journal of Childhood Education*, 3, (2), 133 – 143.

Greeno, J. G. (1991). Number sense as Situated Knowing in a Conceptual Domain. *Journal For Research in Mathematics Education*, *22*(3), 170–218.

Gresalfi, M., & Ingram-Goble, A. (2008, June). Designing for dispositions. In *Proceedings of the 8th international conference on International conference for the learning sciences-Volume 1* (pp. 297-304). International Society of the Learning Sciences.

Gresalfi, M. S., & Cobb, P. (2006*).* Cultivating students' discipline-specific-dispositions as a critical goal for pedagogy and equity. *Pedagogies, 1, 49-58.*

Gresalfi, M. S. (2009). Taking up opportunities to learn: Constructing dispositions in mathematics classrooms. *The Journal of the Learning Sciences*, *18*(3), 327-369.

Hiebert, J. (1984). Children's mathematics learning: The struggle to link form and understanding. *The Elementary School Journal*. 84 (5) 497-513

Hitchcock, G. & Hughes, D. (1995). *Research and the Teacher*. London: Routledge

Howie, S. J. (2001). Mathematics and science performance in grade 8 in South Africa 1998/ 1999: TIMMS-R 1999 South African. Human Sciences Research Council.

Jacob, L., & Willis, S. (2001). Recognising the difference between additive and multiplicative thinking in young children. In: 24th Annual Conference of the Mathematics Education Research Group of Australasia, 30 June - 4 July 2001, University of Sydney, N.S.W.

Kilpatrick, J., Swafford, J. & Findell, B. (Eds.) (2001). *Adding it up: Helping children to learn mathematics*. Washington: National Academy Press.

Matthews, M. R. (1992). Constructivism and empiricism: An incomplete divorce. *Research in science education*, *22*(1), 299-307.

Maxwell, J. A. (2004). A Qualitative research design: An interactive approach. USA (Sage Publications).

McIntosh, A., Reys, B. J., & Reys, R. E. (1992). A proposed framework for examining basic number sense. *For the Learning of Mathematics*, *12*(3), 2-44.

McNamee, M. & Bridges, D. (2002). *The Ethics of Educational Research*. UK: Blackwell Publishers.

Merriam, S. B. (2002). Qualitative research in practice: Examples for discussion and analysis. San Francisco: Jossey-Bass.

Morse, J. M., Barrett, M., Mayan, M., Olson, K., & Spiers, J. (2008). Verification strategies for establishing reliability and validity in qualitative research. *International Journal of Qualitative Methods*, *1*(2), 13-22.

Mulligan, J. T., & Mitchelmore, M. C. (1997). Young children's intuitive models of multiplication and division. *Journal for Research in Mathematics Education*, 309-330.

Mulligan, J. T. (2002). The role of structure in children’s development of multiplicative reasoning. In *Proceedings of the 25th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 497-503).

Park, J. H., & Nunes, T. (2001). The development of the concept of multiplication. *Cognitive development*, *16*(3), 763-773.

Proulx, J. (2006). Constructivism: A re-equilibration and clarification of the concepts, and some potential implications for teaching and pedagogy. *Radical Pedagogy*, *8*(1), 65-85.

Sadler, D. R. (2002). Learning dispositions: can we really assess them?. *Assessment in Education: Principles, Policy & Practice*, *9*(1), 45-51.

Schoenfeld, A. H. (1988). When good teaching leads to bad results: The disasters of' well-taught mathematics courses. *Educational psychologist*, *23*(2), 145-166.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense-making in mathematics. In D. Grouws (Ed.), H*andbook for Research on Mathematics Teaching and Learning*,334-370. New York: MacMillan.

Schollar, E. (2009). *Towards Evidence-Based Educational Development in South Africa. (0-102).* Pretoria: Department of Basic Education, South Africa.

Sfard, A. (1998). On two metaphors for learning and the dangers of choosing just one. *Educational Researcher*, *27*(2), 4-13.

Sfard, A. (2008). Thinking as communicating: Human development, the growth of discourses, and mathematizing. Cambridge University Press.

Skemp, R. R. (1976). Relational Understanding and Instrumental Understanding. *Mathematics Teaching*, *77*, 20-26.

South Africa. Department_of_ Basic_Education. [DBE], (2011). *Curriculum and Assessment Policy Statement Grades 1-3: Mathematics. Policy* (0-102). Pretoria:

South Africa. Department_of_Basic_Education. [DBE], (2012). *Report on the Annual National Assessments 2011/2012: Grades 1 to 9*. Pretoria

South Africa.Department_of_Education.[DoE], South African Schools Act (84/1996)

Tirosh, D., Tsamir, P., Levenson, E., Tabach, M., & Barkai, R. (2013). Exploring young children's self-efficacy beliefs related to mathematical and nonmathematical tasks performed in kindergarten: abused and neglected children and their peers. *Educational Studies in Mathematics*, *83*(2), 309–322.

Wallace, A. H., & Gurganus, S. P. (2005). Teaching for mastery of multiplication. *Teaching Children Mathematics*, *12*(1), 26.

Wright, R. J., Martland, J., & Stafford, A. K. (2006). *Early numeracy: Assessment for teaching and intervention*. Los Angeles: Sage..

Wright, R. J. (2013) Assessing Early numeracy: Significance, trends, nomenclature, context, key topics, learning framework and assessment tasks. *South African Journal of Childhood Education*, 3, (2), 21-40.

APPENDICES

Appendix 1: Instrument for assessing learners' productive disposition in multiplication.

Name :___________________________________

8. Multiplication is ____

9. Give some examples of multiplication problems you know______________________________

Put a circle around yourself 1__☺_☺_☺_☺_☺_☺_☺_☺_☺_9

10. Why did you put yourself there?____

11. Why does Kuhle struggle with multiplication?____

12. Why is Sine good at multiplication?____

13. What do you do when you get a multiplication problem that you do not know the answer to?____

14. Which do you enjoy more, addition or multiplication problems?_______________________

Why?____

Appendix 2: Example of parental consent form provided after discussion with parents

John Masiza Intermediate School

2 Mtyobo Street
Walmer Township
PORT ELIZABETH
6070

Tel: (041) 581-1470
Fax: (041) 581-1470

Mna ________ndingumzali womntwana ongu ________________________________ ofunda ku Grade 4 e John Masiza Primary,ndiyavuma ukuba umntwana wam athathe inxaxheba kwiresearch ye Mathematics ezawube isenziwa esikolweni ukuphuma kwesikolo ngeentsuku ezithile. Ndazisiwe ukuba xa kwenzekile ndafuna ukumkhupha umntwana wam kule research, ndinelungelo lokwenza njalo kwaye loo nto ayisayi kwenza ukuba umntwana wam aphatheke kakubi.

Parent's signature : _

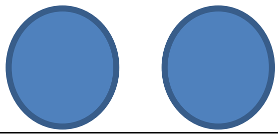
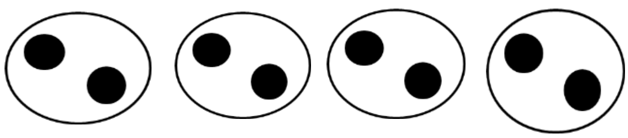
Researcher's signature : ________________________

Date : 21. 02 2013

Appendix 3

SAMPLE OF WRIGHT ET AL (2006) TASKS

Sample of intervention tasks

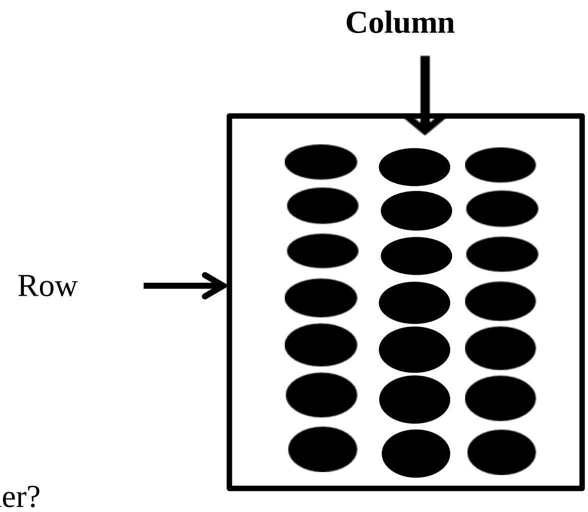
EQUAL GROUPS
1.1 Making equal groups Learners are given 12 counters and are asked to make three groups with four in each group. *How many groups? / How many in each group?*
1.2 Describing equal groups **Counters/ unifix cubes/ containers** **Individuals** Each learner is given 10 counters and two containers, ask them to share it equally into the containers, how many counters in each containers? Use the same activity but increase the number range
1.3 Combining and counting equal groups Place out ten 2 – dot cards, put each 2- dot card and let the learner count putting each card after the other. Similar with 3 dots, four dots, five dots 
1.4 Determining the number in an equal share **Resources: dots cards/ unifix cubes** Learners are given six counters. Share them amongst three people. How many will each get? Use 10 and 2, 12 and 6, 18 and 3
1.5 Determining the number of equal groups **Resources: dots cards/ unifix cubes** Place out four 2-counter cards: How many counters are there on each card? How many cards are there? How many counters are there altogether? 

1.6 Describing visible arrays

Resources: Arrays

Arrays

The arrays are explained to the learners - that it has rows and column. The learners were showed the rows and columns.



How many rows?

How many dots are altogether?

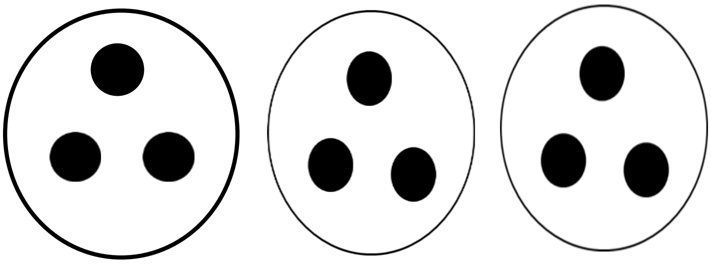
1.7 Developing counting in groups of 3's/ 4's/ 5's using screen items

Resources: two dots / 4 dots/ 5 dots cards

Place out a plate each containing three dots in it. Tell the learners that one plate has three dots, place another plate then and ask the learners how many dots are there altogether in two plates. Put more plates under a screen and ask how many dots are altogether

1.8 USE DOT UNDER THE CARD A SCREEN.

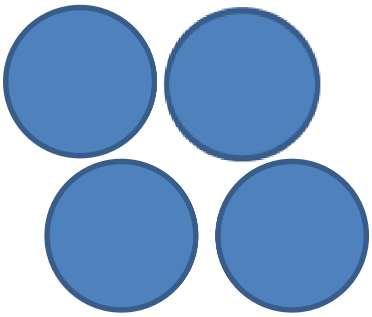
Resources: dot cards



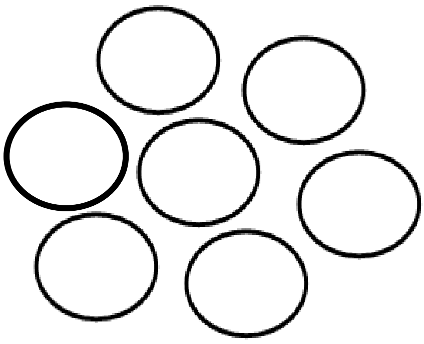
There are three dot cards, under each card there are three dots.

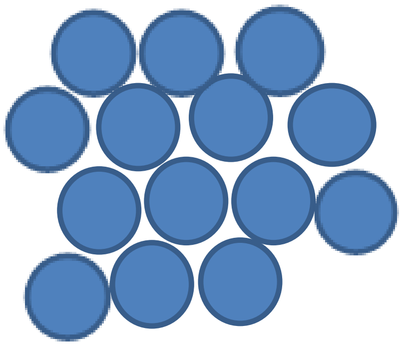
How many dots are there altogether?

The same activity was used but increases the number of dots under each card.



The teacher tells the learner that there are seven cards with 14 dots altogether. Ask how many dots in each card? Learners will not be allowed to touch the dots.

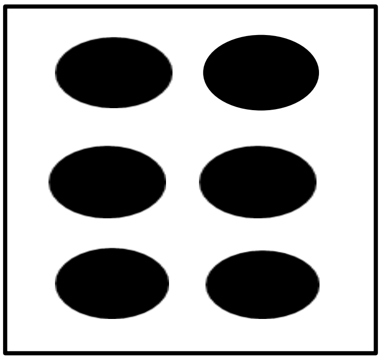




The same activity was used but increases the number range on the dot card

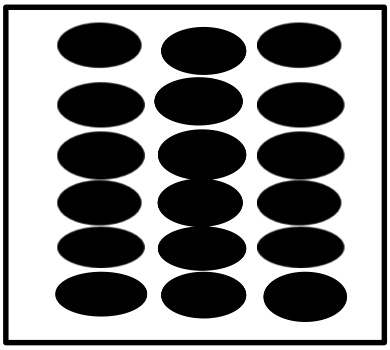
1.9 Resources: Arrays

Display the first row for a second while other rows are covered. Let the learners look at the first row and then show the others for another second or two, ask learners as to how many dots are altogether.



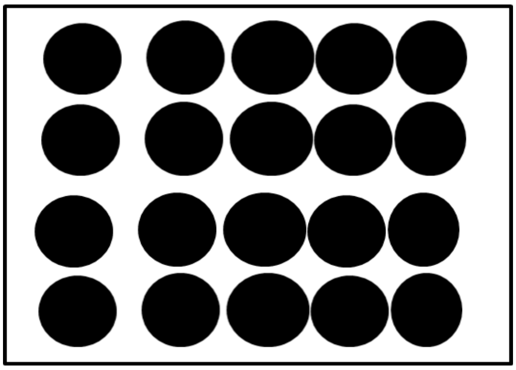
The same activity was used with different arrays

- The teacher unscreens the first row and screens the rest and tells the learners that there are six rows altogether, how many dots are there altogether.



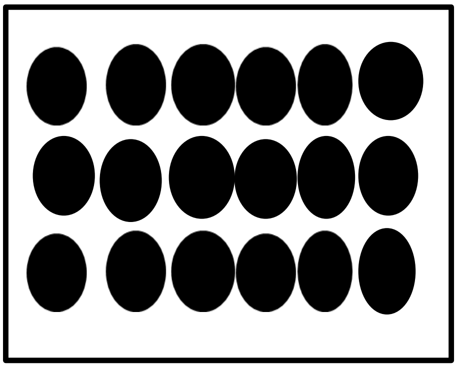
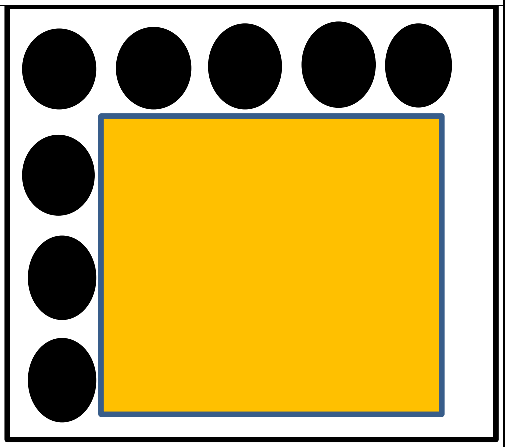
The same activity was used with different arrays. Place a 4 x 5 array and covers one row,

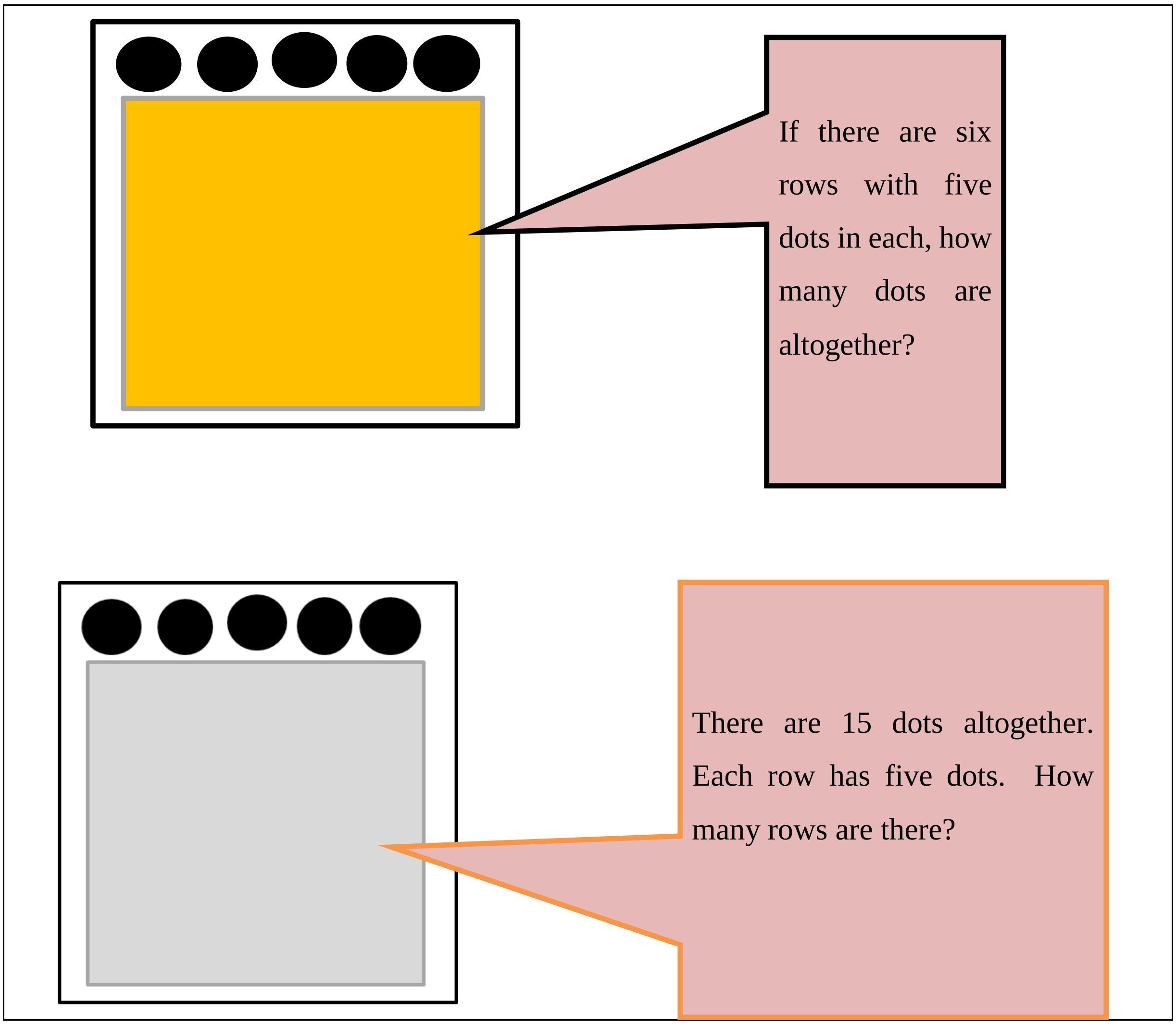
- How many dots are altogether?
- How many rows are there?
- How many columns?



The teacher turns the array at 90^0

- How many dots are altogether?
- How many rows are there?
- How many columns?

The same activity was used with different arrays
There are 18 dots altogether and there are 3 rows, how many dots in a row.  ***The same activity was used but increases the number range on the dot cards.***
1.10 USE OF A COVERED ARRAY • How many dots are there altogether? • Explain how did you get the answer?  The same activity was used but increase the number range.
Use the following array to answer the following question


If there are six rows with five dots in each, how many dots are altogether?
There are 15 dots altogether. Each row has five dots. How many rows are there?

Word problems: Multiplication and division

- There are four children and they each have three books. How many books are there altogether

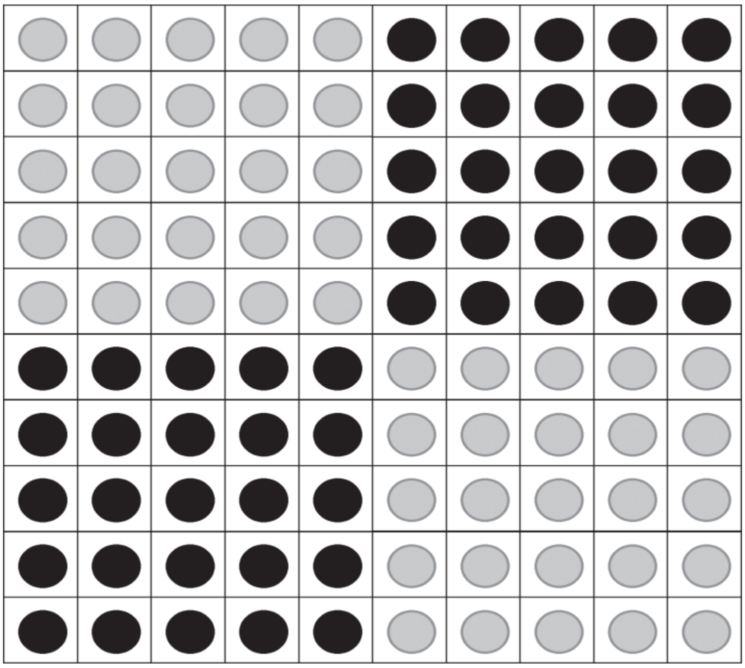
The learner must keep the track of the number of ones in each three, the number of threes and the total number of counts

- Fourteen pens are put into groups of two's, how many groups of two are there?

The learner count in multiples and keep track of the number of multiples

Multiplication Facts

Use an array in assisting the learning of multiplication facts 10 x 10 dot array



Learners are asked to come to show what 5 x 8 look like and are showed 5 x 8 array about turns it to demonstrate 8 x 5

List of abbreviations and acronyms

ANA - Annual national Assessment

CAPS - Curriculum Assessment Policy Statement

DBE – Department of Basic Education

TIMSS – The Third International Mathematics and Science Study

FNWS - Forward Number Word Sequences