INVESTIGATING A MATHEMATICS RECOVERY PROGRAM FOR ASSESSMENT AND INTERVENTION WITH GROUPS OF GRADE 4 LEARNERS

Anelia Wasserman

(14w8315)

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Education Department, Rhodes University, Grahamstown

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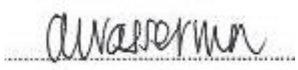
DECLARATION OF ORIGINAL AUTHORSHIP

The work contained in this thesis has not been previously submitted to meet requirements for an award at this or any other higher education institution. To the best of my knowledge and belief, the thesis contains no material previously published or written by another person except where due reference is made.



Anelia Wasserman

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ABSTRACT

This study reports on the findings of my research, which was based on an intervention focused on recovery of early arithmetic strategies with one Grade 4 class of learners in a township school in Port Elizabeth in the Eastern Cape. Learners came from poor socio-economic backgrounds and initial evaluations showed that the majority of learners still relied on concrete methods, like tally counting, to perform addition and subtraction calculations even with numbers less than 10. This is not uncommon in the South African context especially with learners in low Socio-economic Status (SES) schools. The results of numerous assessments including the Department of Education's Annual National Assessments point to a crisis in primary mathematics education where intermediate phase learners are generally operating several grade levels below the grade they are in. A large drop in mathematics performance is seen in the ANA results in grade 4 learners (the first grade of the transition from foundation phase to intermediate phase). Within this context, and my background in learning support for students, my research aimed to understand the possibilities and constraints of the implementation of a recovery program adapted from the widely implemented work of Wright et al. (2006, 2012).

The primary adaptation made to the MR program involved administering the assessments and intervention with groups of (rather than individual) learners. Within the context of the many low SES under-resourced schools in SA, individualised interview based assessments and recovery is not seen as a possible remediation strategy. Drawing on a socio- constructivist perspective, my study used action research with one class of 23 learners and found that adaptation of the MR program for a group, based on eight recovery sessions, was useful for enabling some progress for all learners in terms of their early arithmetic strategies and conceptual place value. Although the need for a longer recovery period is acknowledged, the adapted program enabled some progress in levels and stages of conceptual knowledge (as conceptualized by Wright et al.'s (2006) Learning Framework in Number) for these two domains. The study concludes with some reflections and recommendations for the future.

ACKNOWLEDGEMENT AND THANKS

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Thank you to my husband, Lukas, for encouraging me and believing that I can get this done. To my three kids, Herman, Arend and Lia, thank you for kindly sharing your mom with 23 other kids, piles of paper and a computer screen.

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ACRONYMS AND TERMS USED IN THE THESIS

Acronym	Description
ANAs	Annual National Assessment(s) (South Africa)
CAPS	Curriculum and Assessment Policy Statement (South Africa)
CPV	Conceptual Place Value
DBE	Department of Basic Education
LFIN	Learning Framework in Number
LoLT	Language of Learning and Teaching
MR	Mathematics Recovery
SACMEQ	Southern and Eastern African Consortium for Monitoring Educational Quality
SANC	South African Numeracy Chair (at Rhodes University)
SANCP	South African Numeracy Chair Project
SEAL	Stages of Early Arithmetical Learning
SES	Socio-economic status
TIMMS	Trends in International Mathematics and Science Study
TRAC	Talking, Reasoning and Computers

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1.1 Contextual rationale of the study

I am involved with the work of the South African Numeracy Chair (SANC) as a part-time Masters in Education student at Rhodes University. The SANC is one of six mathematics education Chairs jointly funded by the FirstRand Foundation, Anglo American, Rand Merchant Bank, the Department of Science and Technology and is administered by the National Research Foundation. The joint aims of the SANC Project are:

- *To improve the quality of teaching of in-service teachers at the primary school level*
- *To improve learner performance in primary schools as a result of quality teaching and learning*
- *To research sustainable and practical solutions to the challenges of improving numeracy in schools*
- *To provide leadership in numeracy education and increase dialogue around solutions for the mathematics education crisis (SANC homepage, 2014***)**

Within the aims of the Chair, my initial focus was to establish an after-school mathematics club at a township school in Port Elizabeth in the Eastern Cape in order to begin an intervention program for mathematics learners and investigate the possibility of researching remediation. As a SANC student I also wanted to meet the objectives of working "at the interface of development and research" (Graven, 2014a, p.25). As a learning support teacher I have experience working with learners with special educational needs as well as doing mathematics and language recovery. I thus aimed to apply my learning support observational experience to inform numeracy remediation in an action research study focused on learners from predominantly disadvantaged backgrounds.

This specific township school I approached for my study had 288 learners from Grade R to Grade 7 in 2014. There are 8 teachers. This school is in an area of high poverty and high unemployment and therefore a non-fee paying school like others in quantile 1 to 3 on the poverty index of the Department of Basic Education (Reddy, 2015).

Most of the learners live in the squatter camp bordering the school premises. According to the principal about 90% of the parents are unemployed and families rely on the government grant of R330 (about $30) per month per child for survival. The learners receive two meals a day at school. One meal is funded by the Department of Education and the other by a private company. Parents often wait at the fence to share in the food their children receive during break time. The principal relies greatly on volunteers and private donations, not only for teaching support, but also to provide the families with food parcels and second hand clothing.

During an informal preliminary visit to the school, aimed at exploring intended development and research possibilities in the third term, I came across a group of 23 Grade 4 learners left without a teacher after their teacher fell ill and could not be replaced due to lack of funding. Being confronted with this harsh reality resulted in a moral obligation to try and address some of the problems faced by these learners. Therefore, and at the principal's request, I became involved with the class on a weekly basis in March 2014.

In order to get a sense of learners' existing mathematical capabilities and to optimise classroom time, a SANC Project's numeracy baseline assessment was done with the class. It was noted that most learners successfully and unsuccessfully relied on methods of finger counting or tally counting no matter the size of the numbers being added or subtracted. There was little evidence of use of more efficient or abstract methods. The majority of learners were thus not able to perform calculations on large numbers (two or three digit numbers). In terms of mathematical progression these early arithmetical strategies would need to be addressed before multiplicative reasoning could develop as multiplication results from cognitive reorganization of counting, addition and subtraction strategies (Wright, Martland & Stafford, 2006, p.119).

The predominance of concrete methods and failure of many students to abstract from concrete representations, has been identified as a significant contributor to poor mathematical achievements of students in South African schools (Ensor et al., 2009, p.8). This is underlined by Schollar (2008, p. 6) stating that "79,5% of grade 5 and 60,3% of grade 7 children still rely on simple unit counting to solve problems". According to Schollar the majority of South African learners are "not developing any kind of understanding of the base-10 number system and the associated critical understanding of place value. They cannot (…) manipulate numbers (…) and cannot use the skills upon which all more complex calculations depend (2008, p.6)."

The initial assessment of learners in this Grade 4 class mirrors Schollar's (2008) findings as well as the results of the *Report of the Annual National Assessment of 2013* (DBE, 2013), which states that only 20,9% of Grade 4 learners in the Eastern Cape managed to score more than the established acceptable achievement of 50% for mathematics with a provincial average of 32,6% for Grade 4 mathematics (The national average for Grade 4 was 36,8%) (Department of Basic Education (DBE), 2013).

1.2 Purpose of the study

The purpose of the study was context driven. The aim was to help learners develop their levels of mathematical proficiency and research opportunities for remediation. On average Grade 3 learners in the Eastern Cape are already 1,8 years behind the benchmark (Spaull, 2013, p.6) so I was expecting a backlog in the mathematical development of the class. I therefore needed an effective way to establish the current early number knowledge of the learners as well as a way to facilitate instructional procedures that would result in the construction of knowledge. For these reasons Wright et al.'s (2006, 2012) Mathematics Recovery (MR) program was particularly appealing for my research and intervention with these learners.

Thus in relation to the context described above, and my experiences of interacting with the particular case study school, the following research questions emerged:

1. How might Wright's et al.'s (2006; 2012) individual interview for assessing conceptual place value and early arithmetic strategies be adapted and implemented with groups of Grade 4 South African learners? How effective is this adapted framework in assessing learners' levels of mathematical knowledge?

2. How might Wright et al.'s individually administered MR program be adapted for remediation of conceptual place value and early arithmetic strategies in the context of working with learners in groups within a South African classroom context where the majority of learners require remediation? What advantages/difficulties emerge from the adaptation of the recovery program for use in groups?

The MR program, as developed by Robert Wright and his colleagues (2006), uses an interview-based assessment to determine a learner's current knowledge and follows with intensive, individualized instruction (Wright, 2003a). They claimed that "arithmetical difficulties are highly susceptible to intervention" (Wright et al., 2006, p. 3) and thus they encourage early intervention with learners. The MR program has been widely used in the USA, UK, Canada, Ireland, Australia and New Zealand (Wright, 2013). In May 2014 Robert Wright received the US Math Recovery Council Pioneer Award for his outstanding contribution to the field of numeracy and for transforming the world of mathematics through his extensive research of assessing and understanding children's numerical knowledge and strategies (Olijnek, 2014). The apparent success of this program in various contexts made it particularly appealing as a basis for developing remediation programs for mathematics learners in the South African context. Additionally the program's progressive mathematisation and the development of number knowledge support non-count-by-one strategies and move away from dependence from materials (Wright, Ellemor-Collins & Lewis, 2007, p.844). Furthermore, in South Africa there is an increasingly growing community of knowledge surrounding the use of Wright et al.'s assessment framework and the MR (e.g. Weitz (2012), Mofu (2013), Ndongeni (2013) and Stott (2014)) and thus my work would be able to build on and extend this emerging body of work. For all these reasons I was particularly attracted to MR for my research.

Within Wright et al.'s (2006) levels in the Learning Framework in Number (LFIN), early number strategies require development before one moves onto the development of multiplicative reasoning. Since my early work with learners revealed a lack of progression across the stages of early arithmetic learning (as identified by Wright et al. 2006) and Wright, Ellemor-Collins & Tabor (2012) by the majority of learners in this class, I chose to focus my study on conceptual place value (CPV) and stages of early number strategies/addition and subtraction (also referred to as SEAL in Wright et al., 2006).

Although the program was originally developed for individual use, I wanted to establish whether it was possible to adapt the conceptual place value (CPV) and early number strategies (SEAL) in the interview assessment and the related MR for the recovery/remediation of early number strategies for a group/classroom situation. This was conducted within the South African context of a classroom that reflects the challenges of mathematical performance as indicated in the national Grade 4 ANA results as the majority of learners were way below expected levels of competence for the grade. Because individual recovery is an unrealistic luxury in most South African schools, I thought it would be valuable to investigate and then share possible ways of adapting Wright et al.'s (2006; 2012) program for a group/classroom situation in which the majority of the learners require remediation of conceptual place value and early number strategies.

This study additionally contributes to the ongoing work of the SANC Project with teachers in the broader Grahamstown area where the need for remediation of learners' early number sense is identified as a key priority for intermediate phase teacher education. Baseline data across 15 participating schools in the SANC Project teacher development program showed that most learners in Grade 3 and Grade 4 lacked the foundational concepts expected to be achieved in earlier grades (Graven, 2012). It was thus anticipated that this adaptation of the program could also be useful for other intermediate phase teachers searching for ways to remediate their learners' basic number sense in the upper primary grades. Furthermore, even within the Australian implementation context, as noted by Prof. Mellony Graven (personal correspondence, 2015) teachers using MR expressed to her during a visit to Melbourne in December 2014, that they too are looking for ways to conduct assessments in groups rather than individually.

1.3 The South African context

Although child poverty is difficult to measure, research by Proudlock et al. conducted in 2008 (as quoted by Shalem & Hoadley, 2009, p. 121) indicated that 68% of South African children grew up in families with a monthly income of less than R1200 (about $100). Taylor and Yu (2009) examined the influence of socio-economic status (SES) on educational outcomes and found a strong correlation between the school the learner attended (which is determined by the learner's SES) and the performance on, amongst others, the Trends in International Mathematics and Science Survey (TIMMS). They also identified various aspects associated with low SES that have an influence on educational outcomes, such as home support and parental education. Furthermore growing up in poor neighbourhoods could lead to a general sense of hopelessness and disruptive behavioural patterns. Schools in low SES areas also experience resource shortages caused by unequal government spending in the past. Learners from low SES communities are also subject to various health issues (e.g. malnutrition, HIV/AIDS, fetal alcohol syndrome etc.), lack of quality teaching and low expectations from teachers (Graven, 2014b).

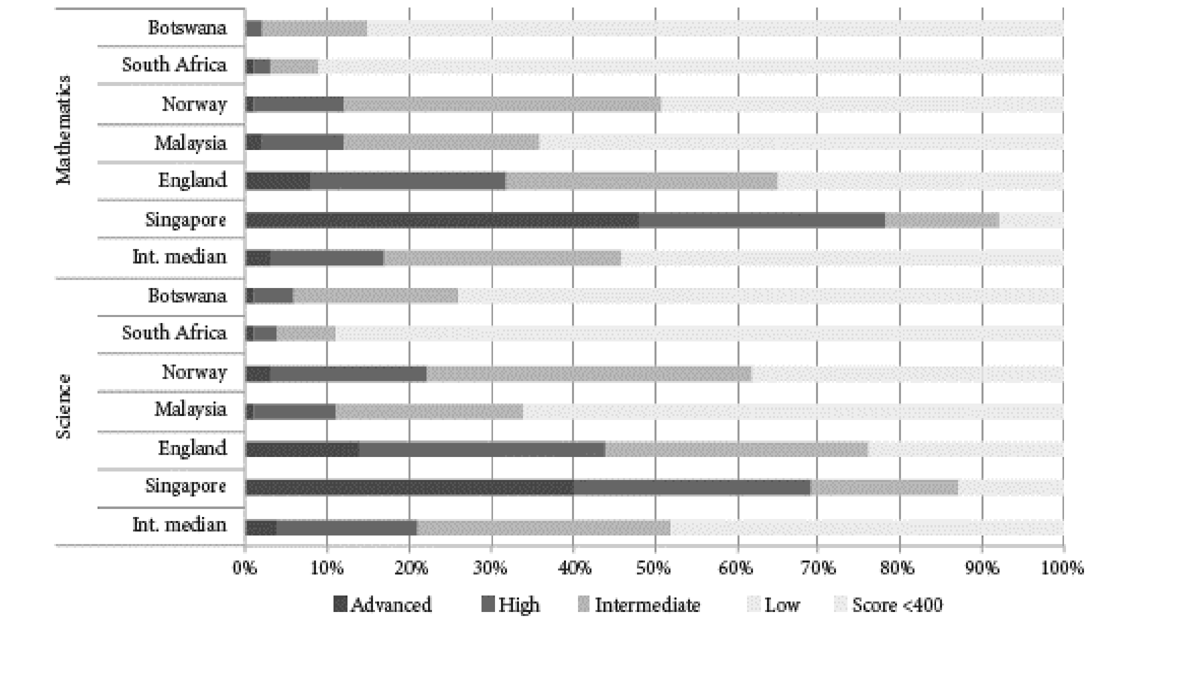
Fleisch (2008) and Spaull (2013) identify two education systems in South Africa. The first system is well resourced and represents 20-25% of learners. The second school system enrolls the vast majority of children from lower income households, bringing with them the challenges of their communities. This could be seen as a dual economy of "schools for the poor" and "schools for the rich" (Shalem & Hoadley, 2009, p.123). Motala (as cited by Shalem & Hoadley, 2009, p.123) states that "parents in the richest schools spend 570 times more on added private support to the school than in poorer schools". Unfortunately, as Spaull & Kotze (2015, p. 23) put it "poor children in South Africa (…) are starting behind and staying behind." My research focused on the assessment and remediation of early number sense in a school in the second system with the majority of learners living in poverty or borderline poverty conditions. Unfortunately the majority of learners in this second system cannot read for meaning in any language and are not numerically competent (Spaull, 2013, p. 39).

Spaull (2013) and Spaull & Kotze (2015) provided an empirical overview of the quality of education in South Africa since the transition to democracy by looking at the results of the following datasets: the 2011 and 2012 ANAs (Annual National Assessment); SACMEQ II (2000) and SACMEQ III (2007) (Southern and Eastern African Consortium for Monitoring Educational Quality), and the TIMSS 1995, 1999, 2002, 2011 (Trends in International Mathematics and Science Study (Grade 8/9 Mathematics and Science)). The analysis of pupils in the Eastern Cape showed that "while pupils are already 1,8 years behind the benchmark by Grade 3, this grows to 2,8 years behind the benchmark by Grade 9, making effective remediation at this higher grade improbable" (Spaull, 2013, p. 6). These findings are mirrored by the national average percentage marks for mathematics in the 2013 and 2014 ANAs. In 2013, for example, the Grade 3's scored 53% on average decreasing to 37% in the following year in Grade 4. When looking at children's progress through the foundation phase the Children's Institute at the University of Cape Town states that in 2012 85% of all children aged 10 and 11 were reported to have completed Grade 3, but the Eastern Cape lagged behind with only 78% of this group having completed the foundation phase. By Grade 9 only 48% of children in the Eastern Cape completed the grade by the expected age (Hall, 2014, p. 108).

The SACMEQ study found that, in Grade 6, 52% of learners achieved scores at the Grade 3 level for mathematics (Schollar, 2008, p. 4). Schollar continues by saying that "virtually all classes have become, in effect, multi graded classes in which many learners are two, three or even four grades below their required standards" (p.8). Fleisch (2008) stressed that it is imperative to identify and remediate learning gaps early on, before they become insurmountable learning deficits and lead to almost certain failure and drop-out in higher grades. Spaull and Kotze (2015, p. 21) concluded from their analysis of the datasets mentioned above as well as the Systemic Evaluation 2007 (Grade 3) and the National School Effectiveness Study 2007/2008/2009 (Grades 3, 4 and 5) that "intervening early to correct and prevent learning deficits is the only sustainable approach to raising average achievement in under-performing schools."

Furthermore in Reddy et al.'s recent analysis of the South African Trends in International Maths and Science Survey (TIMMS) performance over the past 20 years, they found that "three quarters of South African learners had not acquired even the minimum set of mathematical or science skills by Grade 9" (2015, p. 9). Only 1% of learners scored in the advanced level. Table 1 shows the South African TIMMS 2011 performance at international benchmarks for mathematics and science in a selection of countries.

Table 1: South African TIMMS 2011 performance at international benchmarks for mathematics and science in a selection of countries (Reddy et al., 2015, p. 6)



The following recommendation made by Reddy et al. (2015), to partly address the bleak picture represented in Table 1, resonates with the purpose of my study:

> One approach would be to focus on learners who have already grasped the foundations in mathematics and science and to develop their skills further. An alternative and perhaps more demanding strategy would involve shifting learners out of the bottom end of the performance spectrum. (Reddy et al., 2015, p. 38)

Chapter 2: Literature Review and theoretical framing

2.1 Theoretical assumptions informing the research

My study is based on the assumption that knowledge does not come from the subject nor the object, but from the unity of the two (Piaget & Inhelder, as cited by Brooks and Brooks, 1993, p.5). We can understand reality by assimilating it into previously constructed cognitive structures. Reality, therefore, can have different meanings. When confronted with discrepancies, a different understanding should be constructed or the original understanding should be retained (Brookes & Brookes, 1993, pp.4-5). Learners are encouraged to "construct solutions that they find acceptable, given their current ways of knowing" (Yackel, Cobb & Wood, 1991, p.395). The challenge is then to find activities that are likely to be problematic for the learners (p. 396).

It was Ernst von Glasersfeld who brought forward Piaget's constructivism in research in mathematics education in his publications in 1984 and 1987. "(His) constructivist teaching experiment was an attempt to bridge the gap between research and practice and was a hybrid of Piaget's clinical method and mathematics teaching" (Steffe & Kieran, 1994, p.716).

Von Glasersfeld theory of cognitive constructivism links to Piaget's theory of cognitive development. This forms the basic orientation of MR: it includes exploring children's construction of arithmetical strategies with a focus on finding optimal instruction activities that support the construction of arithmetical knowledge (Wright, 2003a). Because I chose to work with the MR program, and since my own learning assumptions cohere with those listed above, the constructivist framework underpinned my research. Within this context knowledge is perceived as something that is constructed through interpretation and organization of information (Adams, 2007) and not something that is merely transferred from a more knowledgeable source to a learner. Von Glasersfeld emphasized that constructivism is a "theory of knowing" rather than a "theory of knowledge" (Von Glasersfeld, 1990, p.19).

According to Piaget our interaction with the environment and with new situations depends on the level of development of our previous understanding (Kamii, 1973, pp.216-230). Cobb, Wood & Yackel (1991) describe constructivism as a framework to approach and transform uncertain and complex situations into solvable problems. Von Glasersfeld's "knowing" is not only a product of interaction between the object being studied and the researcher/learner, but it also has a social nature. The construction of knowledge occurs through dialogue and is a product of social interaction. It is an intrapersonal creation (Adams, 2007). "Knowledge constructs are formed first on an inter-psychological level (between people) before becoming internalized or existing intra-psychological" (Daniels, as cited by Adams, 2007, p. 246).

Hickey (1997) ascribed the incorporation of the impact of social collaboration mainly to the work of Vygotsky. According to Vygotsky (as cited by Steele, 2001, p. 412) "Human learning presupposes a specific social nature and a process by which children grow into the intellectual life of those around them." He saw communication as a cultural tool to make meaning by explaining and justifying your thinking to others. Socio-constructivism prioritizes social or assisted learning since with the assistance of more knowledgeable people (peers or teachers), an individual can reach a level of understanding beyond what he is capable of reaching individually. Previous investigations cited by Cobb (1995, p. 26) indicate that "small-group interactions can give rise to learning opportunities that do not typically arise in traditional classroom interaction". There is an intrinsically social aspect to learners' constructions. Reconceptualising your own cognitive construction while attempting to make sense of a partner's explanation could lead to the construction of a framework for another solution (Yackel et al., 1991, pp. 401-402).

2.2 Theoretical orientation of the intervention

Socio-constructivism not only informed my research decisions, but was an integral part of my teaching practice throughout the process. Elements of the constructivist classroom (as defined by Brooks & Brooks, 1993) resonate strongly with the structure and approach of Wright et al.'s (2006, 2012) MR and with my adapted implementation of the intervention. Because of the interview based assessment done prior to the recovery phase, lessons were shaped by learners' present conceptions and suppositions and ongoing assessment was interwoven with teaching. My implementation of the principles of action research further enabled me to allow learner responses to shift instructional strategies and alter the content of lessons. The focus on group work and dialogue supported my vision to apply the recovery program to a group/classroom setting.

The discursive nature of socio-constructivism implies that the teacher and learner co-construct knowledge with the teacher assuming the role of mediator. The idea of co-construction should, however, also include learner-learner interaction (Adams, 2007) and thus in adapting the assessment recording methods and MR activities for groups I considered aspects of learner to learner interactions and found ways to capitalize on this for learning. I reflect on this adaptation and its consequences for peer learning in the data analysis chapter. Unfortunately the South African school effectiveness and school improvement studies as well as small-scale studies cited by Hoadley (2012, pp. 193 – 199) do not reflect the favourable elements of the constructivist classroom. Descriptive features of the South African classroom include, amongst others:

- Weak forms of assessment and lack of feedback on student's responses
- Classroom interaction that privilege the collective (chorusing)
- Low levels of cognitive demand
- Slow pacing
- Dominance of concrete over abstract meanings

The MR enabled me to address these issues within the classroom.

2.3 Key concepts: progression and number sense

Askew (2013, p.3) identifies three approaches to answering single digit addition and subtraction calculations:

1. Counting – either counting all, counting on from one of the numbers or counting back from one of the numbers
2. Decomposition – splitting one or both of the numbers to make retrievable number facts e.g. 5 + 6 → 5 + 5 + 1
3. Retrieval – an answer can be recalled within 3 seconds

He notes two views regarding the relationship between these that are evident (p. 4):

1. The progression view – Learners progress from counting strategies, to

decomposition strategies to retrieval strategies

2. The number sense view – Proficiency implies selecting an efficient strategy

The progressive nature of mathematics learning is assumed within this study. If learners do not have solid foundations on which to construct new knowledge learning is hampered. Schollar (2008, p.5) puts it as follows:

> Mathematics (…) is a hierarchical subject in which the development of increasingly complex cognitive abilities at each succeeding level is dependent on the progressive and cumulative mastery of its conceptual frameworks, starting with the absolutely fundamental basics of place value (…) and the four operations (calculations).

In terms of developing addition and subtraction concepts Steffe and Cobb (2008, as cited by Dineen, 2014, pp.32-33), drawing on a constructivist perspective, delineate five progressive stages with the first four relating to counting based strategies:

1. perceptual counting – items can be physically touched
2. figurative counting – items can be visualised to re-produce the count
3. initial number sequence – more advanced counting strategies like counting-on-from is implied
4. a tacitly/implied nested number sequence – solutions to facilitate addition and subtraction tasks are used e.g. count-up-to strategy
5. explicitly nested number sequence – the use of non-counting based strategies with an understanding of subtraction as the inverse of addition.

In the South African context however, as indicated in Chapter 1, a focus on such progression is often absent from primary mathematics lessons. Venkat & Naidoo (2012), drawing on classroom research on teaching of numeracy within a South African Grade 2 classroom, argue that lack of coherent presentation of number concepts impairs possibility for learners to make connections. Similarly Askew, Venkat & Mathews (2012) illustrate how incoherent presentation and mediation of activities result in the mathematical object not coming "into being" for many of the learners (p.33). Such research points to the need for structured intervention programs that foreground coherent progression.

The MR program and particularly SEAL is informed by the work of Steffe and Cobb (1988). In the first four stages of SEAL counting strategies are prevalent, but in Stage 5 grouping strategies are also featured. Steffe and Cobb's basis of progressively more sophisticated counting strategies is echoed in SEAL's progression from perceptual counting (Stage 1) to the use of grouping strategies in Stage 5 (Dineen, 2014, p. 33).

The progression view aligns with Piaget's stages or development view. Piaget's proposal that the processes of accommodation and assimilation lead to the active construction of meaning has had a significant impact on mathematics education (Zevenberger, Dole & Wright, 2004, p. 22). Although Piaget's stage theory was criticized because of its tendency to highlight what learners cannot do, his work contributed to the understanding of the development of number concepts and his theories formed the main basis for the development of constructivist theories in education. The impact of Piaget's work is relevant because of its impact on the development of the various forms of constructivism (Zevenberger, Dole & Wright, 2004). "Constructivism recognises that mathematics must make sense to students if they are to retain and learn mathematics" (Zevenberger, Dole & Wright, 2004, p.24).

2.4 Literature review of the MR program and related research

The Primary Mathematics Research Project looked at over 7000 learners from 154 schools in South Africa. Schollar (2008, p.1) summarises the findings as follows:

> Phase I concluded that the fundamental cause of poor learner performance across our education system was a failure to extend the ability of learners from counting to true calculating in their primary schooling. All more complex mathematics depends, in the first instance, on an instinctive understanding of place value within the base-10 number system, combined with an ability to readily perform basic calculations and see numeric relationships.

Because the MR Program of Robert Wright and his colleagues has proven to explicitly address the issues highlighted by Schollar, I decided to implement the program in my classroom and to focus my research on this intervention. I thus expand on this work in the following section.

2.4.1 The origins of the Mathematics Recovery (MR) program

As mentioned above, the MR program is informed by the work of Steffe and Cobb (1988). This, in turn, is based on Von Glasersfeld's constructivist epistemology with roots in the work of Piaget (Wright, 2003a, 2003b). Since the mid 1980's research on early arithmetical learning started to integrate research of learning with research of teaching. Where Steffe and Cobb's earlier research focused on larger numbers of learners and quantitative based methods, their constructivist research documented "children's conceptual progress over time by describing children's current arithmetical strategies and ways in which these strategies were reorganized in the course of solving arithmetical problems" (Wright, 2003b, p.139). The strategies used by learners to solve problems (Steffe, 1991, as cited by Wright (2003b, p. 7) called it "schemes", a Piagetian label) and how these strategies developed were of particular interest. Wright and his colleagues also found that the methods and results of Steffe's (1991) research were "particularly suited for application to a recovery program" (Wright, 2003b, p.139). Like MR, this program included individualised problem based teaching of learners aged 6-8 years old who were less advanced in their number learning than their peers. A fundamental question for both Steffe and Wright is "What kind of instruction supports students' construction of arithmetic knowledge?" (Wright, 2003a, p. 8).

2.4.2 The Mathematics Recovery (MR) program

Robert Wright and his colleagues have researched the assessment and teaching of number and early arithmetic over the last 25 years (Wright, 2003a; Wright, 2003b; Wright, 2013; Wright et al., 2006, 2012). Drawing on a substantial body of research (discussed above) as well as practical application across several international contexts (including Australia, USA, UK, Canada, Ireland and New Zealand), they developed and implemented the Mathematics Recovery Program (MR). MR is a program of intervention in early number learning based on individual interview-based assessments and involves intensive, individualized teaching.

The interview-based assessment is also referred to as the Mathematics Recovery Assessment Interview and is based on profiling learners against progressive levels of competence in the Learning Framework in Number (LFIN). This is what Adams (2007, p.252) would refer to as "assessment for learning" instead of "assessment of learning". The individual LFIN profile obtained from the interview provides rich information regarding a learner's current early number competence. A levelled profile is formed to describe the current knowledge and most advanced numerical strategies (Wright et al., 2006). Stott (2014, p. 114) combines the key aspects of the LFIN from Wright et al.'s 2006 and 2012 works as follows:

A. Structuring numbers 1 to 20
B. Number words and numerals (including forward and backward sequences)
C. Conceptual place value knowledge (ability to reason in terms of tens and ones)
D. Strategies for Early Arithmetical Learning (strategies for counting and solving simple addition and subtraction tasks from 1-100)
E. Early multiplication and division

Each of the aspects is subdivided into a progression of 3 to 6 additional stages or levels (as shown for aspect C and D in table 2 and 3 below).

The results of the baseline assessment administered in March 2014, as well as class/group sessions already conducted with the class prior to the research indicated that concepts relating to 'Structuring numbers 1-20' and 'Number words and numerals' were in place already. For the purpose of the study I therefore chose to focus on conceptual place value knowledge, CPV (also called *Base-ten arithmetical strategies* in Wright et al., (2006)) and Strategies for early arithmetical learning (SEAL).

According to the South African Curriculum and Assessment Policy Statement (CAPS) for Grade 4, learners are required to use a variety of techniques to do addition and subtraction of whole numbers of at least 4 digits. Apart from changes in calculation techniques, CAPS also outlines progression by increasing the number range and introduction of different kinds of numbers. Adequate sense of place value and understanding of the properties of numbers and operations are stipulated as prerequisites for efficient calculation strategies (DBE, 2011).

The MR SEAL model consists of a progression of strategies used in early numeracy and corresponds well with the expectations and progression set out in the CAPS curriculum. In the next two sections I summarise the SEAL and CPV stages and levels and related interventions as these were the basis of my intervention.

Stages of Early Arithmetic Learning (SEAL)

> Developing facile mental strategies for addition and subtraction involving two 2-digit numbers is a critically important goal of arithmetic learning in the first three or four years of school (Wright et al., 2007, p. 849).

The MR SEAL model consists of a progression of mental strategies used in early numeracy. The stages of strategy progression are summarized in Table 2. The ideal would be to get all learners to perform on a Stage 5 level: being able to use a range of non-count-by-one strategies (e.g. compensation, using a known result etc.).

Table 2: SEAL stages - Addition and Subtraction (Wright et al., 2006, p. 22)

Stage 0	**Emergent counting**	Cannot count visible items
Stage 1	**Perceptual counting**	Cannot count screened/concealed items
Stage 2	**Figurative counting**	Can count screened/concealed items, but counts from one
Stage 3	**Initial number sequence**	Counts on for addition and counts down for subtraction
Stage 4	**Intermediate number sequence**	Solve missing subtrahend by counting down
Stage 5	**Facile number sequence**	Uses procedures other than counting by ones e.g. compensation

> Developing facile mental strategies for addition and subtraction involving 2-digit numbers is a critically important goal of arithmetic learning (…). This lays a strong foundation for all further learning of arithmetic (Wright et al., 2007, p. 849).

SEAL also provides a framework for selecting MR instruction activities that aim to lead to learners' construction of effective numerical strategies (Wright et al., 2006).

The three strategies used most frequently during the recovery period of my study were the following (Wright et al., 2006, 2012):

Split: Split tens and ones, add/subtract them separately, then recombine

e.g. 37 + 22
30 + 20 = 50
7 + 2 = 9
50 + 9 = 59

Jump: Begin from one number, jump tens then jump ones (or ones then tens)

e.g. 37 + 22
37 + 20 = 57
57 + 2 = 59

Jump to the decuple (Jump to the 10):

Begin from one number, jump to the nearest decuple, jump the tens, then jump the remaining ones

e.g. 37 + 25 → 37 + 3 → 40 + 10 → 50 + 10 → 60 + 2 → 62

"Transforming" is also mentioned as possible solutions during the LFIN assessment interviews:

Transforming: Change both numbers while preserving the result, then add/subtract.

e.g. 37 + 22 = 40 + 19
40 + 19 = 59

The three overlapping instruction phases of SEAL are summarized in Table 3 below.

Table 3: SEAL instruction phases - Addition and Subtraction **(Wright et al., 2012, p.111-120)**

Phase 1	**Developing foundational knowledge**	Higher decade addition and subtraction	A. Jump within a decade B. Jump forward from a decuple C. Jump back to the decuple D. Jump forward to the decuple E. Jump back from the decuple F. Jump across a decuple
		Extending to 2-digit tasks	
Phase 2	**Consolidating early strategies**	Notate and label strategies Learning to jump and learning to split Compare strategies/seek better strategies	
Phase 3	**Refining strategies and extending tasks**	Refine strategies Formalizing the tasks and notation Increasing the complexity of tasks Extending the range of numbers	

According to Wright et al. (2006) SEAL is the most important aspect of the LFIN. Effective addition and subtraction strategies become the foundation for subsequent arithmetic learning and support the learning of multiplication and division and is a prerequisite for measurement, algebra and data handling (Wright et al., 2012). This is mirrored by the introduction to algebraic expression and the corresponding associative and commutative properties of addition, for example, described in the CAPS (DBE, 2011, p.41). The CAPS stipulates the "use of a range of techniques to perform and check written and mental calculations of whole numbers” (DBE, 2011, p.100). This includes “rounding off and compensating”, “building up and breaking down of numbers” (referred to as “split” in MR) and “using a number line”. The strategies included in the MR program therefore cohere well with the guidelines set by the South African curriculum.

Once learners attain Stage 4 or 5 of SEAL, the development of the tens and ones structure becomes increasingly important (Wright et al., 2006, p.21). Being able to flexibly count by ones, tens and later hundreds, is critical in developing facile mental computation strategies (Wright et al., 2012). This resonates well with the CAPS which stipulates for Grade 4 learners that they should: “Count forwards and backwards in 2s, 3s, 5s, 10s, 25s, 50s, 100s between 0 and at least 1000” (DBE, 2011, p. 104).

Conceptual Place Value (CPV)

> Conceptual Place Value encompasses instructional sequences that develop knowledge of the structure of multidigit numbers, as a foundation for mental computation. The main instructional sequence involves flexibly incrementing and decrementing by ones and tens, and later hundreds and thousands, in the context of base-ten materials (Wright et al., 2007, p. 848).

The MR model for the development of CPV levels can be summarised by the levels described in Table 4.

Table 4: Development of CPV levels *(Wright et al., 2006, p. 22)*

Level 1	**Initial concept of 10**	Does not see ten as a unit of ten ones. Solves tasks using a counting-on and counting-back strategy by counting in ones.
Level 2	**Intermediate concept of 10**	Sees ten as a unit composed of ten ones. Needs representations of units of ten (like open hands or hidden ten-strips). Cannot solve addition and subtraction tasks involving tens and ones when presented as written number sentences.
Level 3	**Facile concept of 10**	Tens and ones and flexibly regrouped without using materials or representations. Can solve written number sentences.

MR provides various assessment tasks and instructional activities to develop conceptual place value knowledge. Initial tasks were developed along three dimensions to be pursued simultaneously as summarized in Table 5 below. It is recommended by Wright et al. (2006) that the tasks described within every dimension should be done consecutively.

Table 5: CPV instructional dimensions (Wright et al., 2012, p.80-83)

Dimension A	**Extending the range of numbers**	Begin in the range of 1-100. Extend to 200. Introduce hundreds materials and extend the range to 1000 Extend across 1000 and 1100 Later extend to 2000 and beyond
Dimension B	**Making the increments and decrements more complex**	Make increments and decrements of multiple tens or hundreds. Switch from increments and decrements of tens, to increments and decrements of ones or hundreds. Make increments and decrements of combinations of ones, tens and hundreds. Later tasks can involve determining unknown increments and decrements.
Dimension C	**Distancing the setting**	Materials are visible Materials are screened, but increments and decrements are shown briefly. Materials are screened, but increments and decrements are verbally posed. The first number is given as a numeral and increments and decrements are posed verbally.

Within Wright et al. (2006, 2012) a range of strategies and terms are introduced that are useful for teaching and research purposes as mentioned above (e.g. “split” and “jump” strategy).

The aim of this study was to adapt both the recovery program assessment and instruction phases for SEAL and CPV as described by Wright et al. (2006; 2012) for application in a group/classroom setup instead of the individual interview-based format it was developed for.

In the next section I review some international and local literature that has drawn on the work of Wright et al. (2006, 2012).

2.4.3 Review of international and local literature drawing on MR assessments

In Australia Dineen (2014) (a doctoral student, Southern Cross University in New South Wales) conducted a teaching experiment to study the use of grouping (non-counting-by-ones) strategies to solve addition tasks in the range 1-20 in an Australian context. Dineen notes that it is important to keep in mind that prior to the "advent of constructivist approaches" (Dineen, 2014, p.1) in the 1980's, direct instruction and rote learning were prevalent. She pointed out that research into the teaching and learning of addition strategies was mainly focused on "counting-based strategies". Only recently the use of grouping strategies were seen as an "alternative or a complementary approach" (Dineen, 2014, pp. 2-3). It is within the context of counting strategies that Dineen identified SEAL (as discussed above) as a theoretical framework to inform the "advancement in complexity of students' use of counting strategies to solve addition tasks" (Dineen, 2014, p. 31):

> SEAL can account for students' progression from perceptual counting (Stage 1) (…) to using grouping strategies to solve simple addition and subtraction tasks (Stage 5) (Dineen, 2014, p.33).

Throughout her study Dineen made use of the principles and methods used by Wright and his colleagues. For example, the assessment questions used reflected the "increasing levels of mathematical sophistication" (Dineen, 2014, p.104), and video recordings of assessments and lessons were analysed according to Wright et al.'s guidelines. Elements and task groups from the MR program were also incorporated into her teaching experiment.

Dineen (2014) also contrasted two progression trajectories that students might follow to solve addition tasks, namely counting strategies (based on SEAL) and grouping strategies (based on Phases in Early Grouping Strategies (PEGS)). She concluded that results from her post-teaching assessment highlighted that, in contrast with SEAL, learners "may not need to master count-by-ones from one as a prerequisite skill to using efficient grouping strategies to solve simple addition tasks" (Dineen, 2014, p. 292).

Recently there has been growing use of MR in South Africa across the two SA Numeracy Chair research teams. Graven, Stott, Mofu and Ndongeni (2015) reported on the application of Wright et al.'s numeracy assessment and recovery framework (LFIN) in the context of four research projects in after school mathematics clubs in the Eastern Cape. They stated that "LFIN enabled our research and our analysis" and also "enabled the developmental aspects of planning for future club activities and teacher development" (Graven et. al, 2015, p.74). Below I briefly summarise some of the key findings of local studies that have applied the MR framework in their research and development work.

Stott (2015) (a doctoral student in SANCP, Rhodes University), for example, used the LFIN assessments individually when researching learners' numeracy progression and the role of mediation in the context of two mathematics clubs. In order to balance the strengths and weaknesses of the group with those of individual learners, she generated quantifiable scores from the LFIN levels and stages. Although Wright (2003a) stated that the LFIN interview assessments "do(es) not result in a score" (p. 8), Stott's extension of LFIN to percentages was a valuable contribution. Stott therefore could not only use the LFIN assessments to make comparisons over time for individual learners, but also to give a broad picture of each of her mathematics club's progress and the development of mathematical proficiency for each club as a whole. The overall percentages could also show the similarity in improvements across LFIN aspects across the two clubs involved in her study.

Mofu (2013) (a Masters student, Rhodes University) examined the effectiveness of the MR program to remediate multiplicative reasoning in an after school intervention program with six learners. She also used the LFIN to profile learners using pre and post intervention interview data. Although the MR intervention was relatively short (4 sessions), all learners progressed at least one LFIN level. Constrained methods used before the intervention disappeared and learners used more efficient methods to solve multiplication tasks. The MR program therefore made it possible for learners to progress in multiplicative reasoning and held potential for developing multiplicative reasoning in a classroom context. Mofu, however, found that the time consuming and labour intensive nature of the LFIN assessments made it unfeasible to assess all learners individually.

Ndongeni (2013) (a Masters student, Rhodes University) "drew on the LFIN to establish learner levels of conceptual understanding in multiplication" (Graven et. al, 2015, p.79). She conducted individual interview assessments with six Grade 4 learners using the Wright et.al (2006) instrument. She then compared the levels of numeracy reasoning and conceptual understanding with productive disposition as described by "Kilpatrick, Swafford & Findell's (2011) five-stranded framework of mathematical proficiency" (Graven et. al, 2015, p. 79). The study "pointed to (…) the usefulness of the LFIN for assessing learner levels of conceptual understanding" (p. 81).

Similarly Weitz (2012) (a Masters student of the SANC, Wits University) used the individual LFIN assessment but did so in conjunction with the ANAs to get an understanding of the number strategies used by Grade 2 learners. The LFIN framework was used to analyse strategies seen in learner responses and used as an analytical tool. This enabled Weitz to establish the level of a learner's operational and structural thinking. She then compared this to the learner's results in the DBE ANAs. Whereas the ANAs are only summative with a focus on an answer and not a process, the LFIN assessment provided richer information. Like Stott, Weitz also produced a quantitate overview for each learner. However, because the individual assessments took on average 1½ hour per learner, Weitz could not personally conduct all interviews and completed it with help from the Wits Math Connect (WMC) Primary Project team which is part of the SANCP at Wits. She concluded that "the key disadvantage of the LFIN is that it is labour intensive and time consuming to administer" (p. 67).

Across these studies the LFIN is seen as a useful tool to assess and analyse learners' levels of understanding and to plan subsequent interventions, but the time consuming nature of the individual interview assessments are mentioned as limiting, especially in the resource strained South African context. I therefore aimed to use the LFIN assessments and the recovery in a group context to see whether it is possible to find a less labour and time intensive way to assess and analyse the mathematical strategies used by learners and to plan efficient group activities to address the identified strengths and weaknesses.

Chapter 3: Action research and Research Methodology

3.1 Action research

In this study I adapted an individually administered mathematics recovery program for implementation in a group/classroom context in an under resourced South African school. This entailed a process of adapting, implementing, reflecting and readapting of material and strategies for the implementation of an intervention for one case study class of learners. My methodology essentially embraced many action research principles. It was a "constructive enquiry" during which knowledge was constructed as part of a continuous learning process (Koshy, 2005, p.9). According to Opie (2004, p. 79) the essence of action research is that it "enables a reflective cyclic process to be brought to bear on the understanding of the problem at hand". The situation could concern people or procedures, as in my case.

My research cycle began with analysis of the problems I identified followed by a small scale intervention aimed at addressing these problems in the form of an adapted version of the MR program. The intervention was implemented and evaluated using interviews, observations and journal reflection. The findings were analysed to inform further modification of the original intervention (Opie, 2004). My research thus followed the action research cyclic process as illustrated by O'Leary (as cited by Koshy, 2005, pp. 5-7) in Fig. 1 as follows:

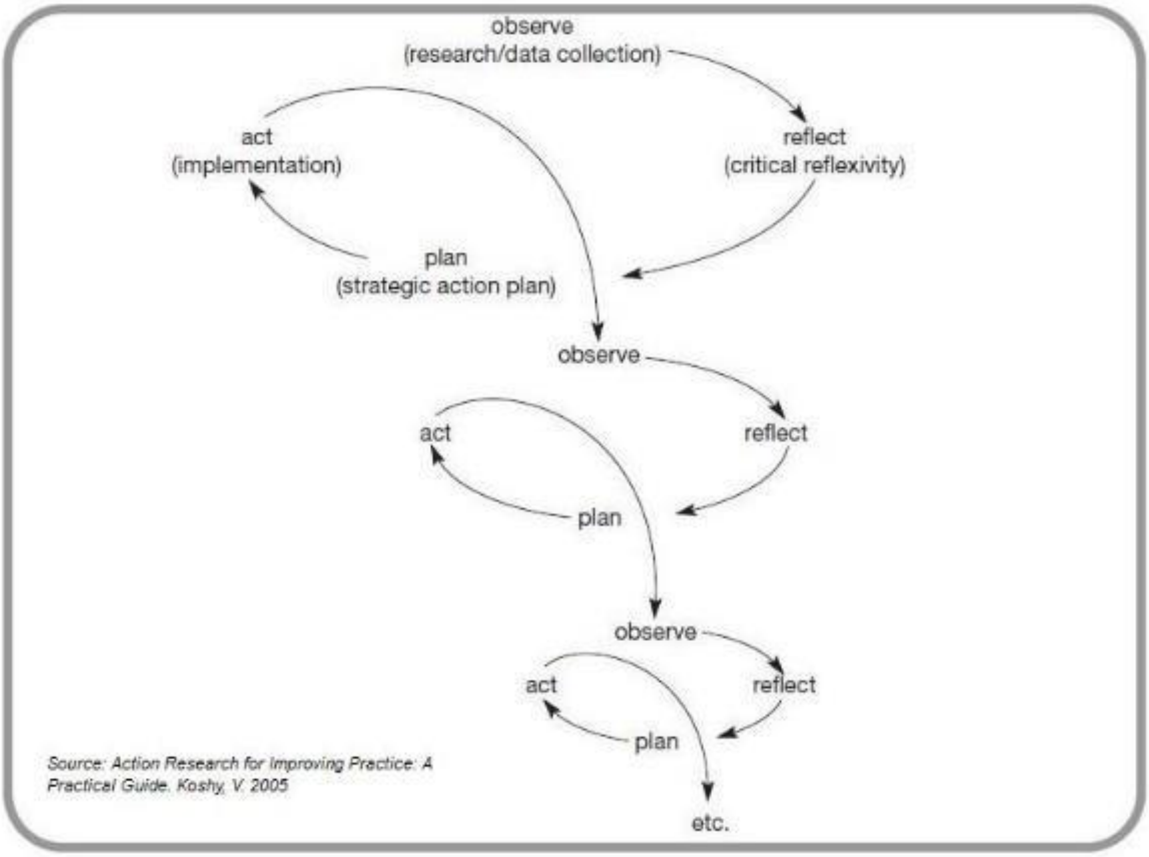


Fig. 1: O'Leary's cycles of research (Koshy, 2005, p. 7)

I find Gravemeijer's (as quoted by Dineen, 2014, p. 89) travel metaphor used in the context of local instruction theory useful to illustrate the cyclic reflective process in the sense that "the departure point and the destination are known, (although) the exact route is not finalised". In the case of my study the departure point was indicated by the snapshot of strategies and constructed knowledge provided by the LFIN profiles and the destination was the achievement of high SEAL stages and CPV levels for every learner. The route was determined by the events, strategies and outcomes of every session.

An advantage of this research, like socio-constructivist research, is the fact that as researcher I could be immediately adaptive, responsive and interpretive. I could evaluate the effectiveness of instructional activities after every session and consider the implications thereof during the adaptation of the MR for future lessons. Understanding could be expanded by means of verbal and nonverbal communication. Additionally in such research the data can be richly descriptive (Merriam, 2002, p.5). Within this paradigm action research is a powerful tool to support the construction of knowledge.

"That knowledge is derived from practice, and practice informed by knowledge, in an ongoing process, is a cornerstone of action research" (O'Brien, 1998, no page). Action research does not only set out to understand and explain a situation, but also aims to change it. O'Brien (1998, no page) refers to "the art of acting upon the conditions one faces in order to change them" as praxis. In my research I wore the hat of researcher as well as that of "teacher" – describing and explaining a situation as well as changing it. Like Graven (2005, p. 207), I experienced this duality to be "powerful praxis". In her role as both co-ordinator and researcher in a Program for Leader Educators in Senior-phase Mathematics Education (PLESME) project, it enhanced and enabled action-reflection practice and the ongoing reflection, stimulated by the research, which then maximised her own learning. As researcher my aim was to adapt and implement the MR in a group context, but as teacher my desire was to do it successfully to impact on the learning of the learners in my class.

The aim of the study was not to make generalisations, but to focus on developing depth of understanding through my ongoing engagement with one class of learners. Thus my study (in line with other teacher action research) focused on only one class of learners resulting in a relatively small number of learners. As indicated earlier, in the rationale of the study, my findings could however have the potential to be logically applied by mathematics teachers facing similar challenges in similar contexts.

3.2 Relationships and ethical issues influencing my research methodology

Since my intention was to conduct research while simultaneously developing learners' mathematical competence, in a township school context, I approached a school that I had knowledge of through a friend who was supporting teachers in the area with language education as part of a corporate social responsibility initiative. The medium of instruction at the school was isiXhosa and in Grade 4 this officially switched to English. When establishing contact with the school I was pointed by the principal to the challenge of a Grade 4 class without a teacher. Because all of the learners in this class were without a mathematics teacher they *all* needed urgent support. Thus my research and action research intervention needed to find a way to assist all 23 learners in this class.

3.2.1 Pre-research phase – Establishing relationships

I began visiting the school and learners informally from the first term of the year in the form of whole class sessions once or twice a week. After doing the SANC baseline assessment in March, I started working with four groups of 5-6 learners each once a week. At this stage my research proposal was not completed and thus this work was largely exploratory and provided the basis on which the research proposal developed. The first two terms were therefore a pre-research phase in which relationships and permission for the research were established. Although it was originally planned to focus the research on a group of six learners, the reality of a class of 23 learners without a teacher compelled me to work with the class as a whole and not exclude any learners.

3.2.2 Proposal, permission and ethical considerations

Shortly after the approval of my research proposal at the end of June 2014, the class teacher returned. (This was the beginning of the third school term.) At this stage relationships were already established with the learners as well as with the principal. Both the principal and the teacher supported the idea that research should be conducted with the whole class and assessment interviews were done accordingly.

I applied for permission from the parents of all the learners to research and implement a mathematics recovery program and to gather a range of data related to the process of the implementation and reflection on the effectiveness or lack thereof of the intervention. All the parents were informed in writing about the purpose of the study and the fact that participation is voluntary and that a learner can withdraw from the study at any time. Written consent was obtained from the parents to use recording devices and photographs. The fact that learners would remain anonymous in all reporting of data was also emphasized. The information/consent letters were written in both English and isiXhosa. (See Addendum A for a copy of the parent information letter and consent form).

Although I was available telephonically and for meetings with parents if they required further clarity or if they wished to discuss any aspect of the research involving their children, the only contact with parents was messages relayed to me by the class teacher.

Formal permission was obtained from the principal, who strongly supported the idea of a whole class intervention, and from the Department of Education. All raw data was stored securely and accessed only by my supervisor and myself. The names used for the purpose of the study are pseudonyms. Where photographs or screen shots of video recordings are used in this study, care was taken to use photos in such a way that learner identities would not be recognisable.

3.2.3 My relationship with the teacher, reflection on my positionality and ethical commitments to post-research support

The relationship with the teacher was carefully managed to enhance a situation of mutual benefit and collaboration and not one of dependence or superiority. The teacher attended some of the group sessions and weekly feedback was given to keep her up to date with the aspects I focused on during MR sessions. The teacher said that she sometimes attempted to consolidate strategies in class and by giving homework problems similar to the ones done in groups. The teacher was also willing and able to supply additional background information regarding contextual or social challenges facing specific learners. We also consulted each other to establish the best way to explain certain concepts in class or in the group context.

Before the return of the teacher, my MR work with groups of learners occurred during school hours. Each group of learners were taken out of the class for 35-40 minutes once per week. After the teacher returned I proposed to move the group contact to after school hours. Because the availability of learners after school hours could be challenging the principal and teacher requested that the program continue during school time. The time slots were arranged in consultation with the teacher to overlap with the times she was out of the Grade 4 classroom. This was as a result of her leaving to teach the Grade 7 class who was without a mathematics teacher.

During an informal meeting with the principal and teacher after the final learner interviews at the end of the school year, it was decided that I would continue to be involved at the school in the following year. At the teacher's request, in order to empower her through participation in the program, I conducted a whole class intervention with the 2015 Grade 4 class. This was done in the classroom in collaboration with the teacher and not in a group context outside the classroom. This will hopefully ensure continuity and impact the teaching and learning of future Grade 4 mathematics classrooms and establish a longer standing relationship with the teacher and school beyond the research context.

While not a part of my research data I was pleased to note that during my extended involvement in the school in 2015 the teacher regularly applied the split strategy to the current curriculum work and devised her own notation. She told me that she learned from a method she observed in one of the MR group sessions. Fig. 2 is an example given to the class by the class teacher on the 29th of April 2015.

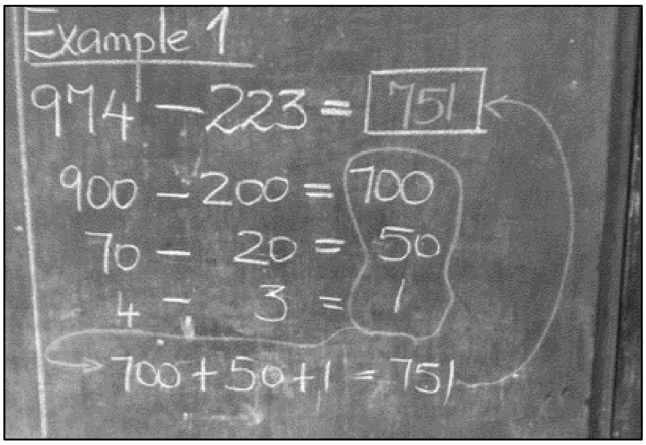


Fig. 2: An example of the split strategy used by the class teacher

3.2.4 Inclusion and group selection

All 23 learners in this class were in need of urgent support. My initial intention to purposefully select a small group of learners from one class was therefore not ethically viable. Working with the class as a whole was not only an opportunity for me as researcher, but also for the learners to engage in mathematical engagement amidst the extended absence of their teacher and thereafter.

After the initial adapted Mathematics Recovery interview assessments (discussed below and included in Appendix B1 and C1) had been completed (and learners had been assessed according to various LFIN levels) the class of 23 learners (11 boys and 12 girls) were divided into four smaller groups. The composition of the mixed ability groups changed throughout the recovery period. The way the groups changed was informed by logistical issues and the purposeful selection of learners to avoid interpersonal conflict and to enhance good collaborative working relationships.

Data of all 23 learners was recorded throughout the course of the study. Although three learners were repeatedly absent during the respective interview phases and their data therefore not included in assessment analysis, they were not excluded from groups, the recovery program or assessments on days they were present.

Two learners are discussed as case studies in Chapter 5. I, however, chose the learners for the case study after completion of both sets of interviews and the recovery program. I therefore collected the same amount of data for all 23 learners. Reasons for providing richer and more textured data of the learning process for these two learners are detailed in Chapter 5.

3.2.5 Language considerations

All the learners in the class are isiXhosa speaking and were educated in isiXhosa until the end of Grade 3. At the beginning of Grade 4 the language of instruction switched to English. At the time of my initial contact with the class they had very little exposure to English. This is a widespread South African phenomenon where the majority of South African learners switch to learning in English in Grade 4 even while the majority of South Africans are not first language English speakers. Robertson & Graven (in press) show that "Grade 4 marks a major point of transition from mother tongue to English as LoLT" (Language of Learning and Teaching) (unpaged). Although only 6,9% of learners are actual native speakers of English, by Grade 4 79,1% of learners have English as LoLT compared to 27,7% in Grade 3.

Setati (2005, p.463) quotes research Gutstein conducted in the USA in 2003 stating that a learner's home language(s) must be regarded as a legitimate language of interaction and that they be used in a range of mathematical discourses and in assessment. Therefore, due to the fact that all the learners in my class were not first language English speaking and thus most struggled with communicating in English, the initial and follow up interviews were conducted in both isiXhosa and English. "Code switching" (Setati, 2005, p.462), that is the switching between languages, was also encouraged for the recovery phase sessions and the learners were thus encouraged to simultaneously develop proficiency in English and mathematics.

Since I am able to speak isiXhosa this greatly enhanced my ability to connect with and work with the learners and their ideas. Although not fully fluent at the start, having not spoken isiXhosa for several years while abroad, I soon became more comfortable communicating in isiXhosa. During the follow up interviews the learners' improved understanding of English resulted in the use of less isiXhosa, nevertheless understanding of instructions were always clarified in interviews. Learners were, for example, able to translate instructions into isiXhosa themselves at the time of the second interviews. I realised that my limited knowledge of isiXhosa was very advantageous during the course of the study. Data collection would have been more limited and challenging without any ability to speak or understand isiXhosa.

3.3 Validity and Reliability

The assessment and teaching instrument for number and early arithmetic developed by Wright et al. (2006, 2012) has been developed through research and international implementation over a period of 25 years in collaboration with various teachers and school systems (Wright, 2013). All the assessments and teaching activities have been empirically tested and proven to be effective for the identification and analysis of arithmetic skills and the design and implementation of teaching intervention (Wright et al., 2012). Reliability and validity of assessment items used in the program have been widely tested over a number of years and across international contexts.

To ensure that conclusions made by means of qualitative research are as accurate as possible, I was aware of reactivity – the possible distortion caused by my own conceptions and values, as well as the effect that I may have had on the individuals and setting being studied (Maxwell, 2003). Although the researcher as "human instrument" brings unique characteristics to the data collection process, the potential of bias and subjectivities should be identified and monitored (Merriam, 2002, p.5).

Maxwell (2003) identified the strategies of "intensive long term involvement" and "triangulation" (pp.244-245), among others, to avoid validity threats that I incorporated in my study:

- "Intensive long-term involvement" – My involvement with the group stretched over a time frame of nine months, from March 2014 to November 2014. This enabled me to make repeated observations and avoid premature conclusions by collecting "rich" data in various forms over an extended period of time. I did not only rely on note taking during assessments/interviews/teaching, but also made use of video recordings and photographs. It was also a useful way of sharing data with my supervisor to get a broader perspective and critical input into the evaluation, recovery implementation and research process.

- "Triangulation" - I made use of triangulation (Koshy, 2005) by collecting data using a variety of methods as discussed below. I then compared and collated the data from across these various sources to check for coherence and possible disconnects. Such triangulation also allowed for thicker description of the progression of learners.

3.4 Data collection

In my study, as with other qualitative studies, data was typically gathered through interviews, observation and document analysis (Merriam, 2002, p. 6). Data analysis was simultaneous with data collection so that I could make adjustments along the way, even to the point of what Merriam (2002, p. 14) refers to as "redirecting data collection".

The following methods of data collection and instruments were used and are discussed below:

3.4.1 The SANC baseline assessment for Grade 3

3.4.2 Wright et al.'s (2006, 2012) MR assessment for conceptual place value (CPV) and early arithmetic strategies (SEAL). This assessment was done twice: before the start of the recovery period as well as after.

3.4.3 A LFIN profile summary page for each learner

3.4.4 My research journal

3.4.5 Observation sheets of learners' behaviour and strategies used compiled throughout the assessment and recovery phases

3.4.6 Video recordings and photographs

3.4.7 Written examples of learners' work done in class before and during the study period

3.4.8 The class's 2014 ANA results

3.4.9 Informal interviews with the principal and teachers

3.4.1 The SANC baseline assessment for Grade 3

Before the commencement of the study the SANC baseline assessment for Grade 3 was administered (March 2014). This assisted me in my role as a support teacher as it provided baseline contextual background of the individual learners' mathematical levels and ways of working and enabled me to identify the foci (i.e. SEAL and CPV) of my intervention.

The baseline instrument was adapted by SANC from the Brombacher & Associates' US AID test based on the Early Grade Mathematics Assessment (EGMA). It consists of 20 tasks assessing the four basic operations by means of 5 problems for each of the four operations (+ - x ÷) with progressively bigger numbers. The assessment was administered as a written test in the classroom. Although the recommended duration for this test is 30 minutes, the class took 50 minutes to complete it (partly due to the predominance of one to one methods of calculation).

See Table 6 in Chapter 5 for the questions in this assessment.

3.4.2 Wright et al.'s (2006, 2012) MR assessment for conceptual place value (CPV) and early arithmetic strategies (SEAL)

The Wright et al.'s (2006, 2012) individual MR task based assessment interviews for conceptual place value and early arithmetic strategies (SEAL) were designed to obtain information regarding a learner's most advanced numerical strategies and richness of the child's numerical knowledge. It also determines the learner's stage of SEAL and level of CPV (Wright et al., 2006, pp.33-35). This is referred to as the Learning Framework in Number (LFIN). The interview is intended to be individually administered for a few learners in a class that are perceived to need some level of mathematical recovery.

However, due to my experience of learner methods in the baseline assessment, I realized that it was necessary to assess all learners' levels of CPV and SEAL as, unlike the Australian context, here the vast majority of learners in the class required recovery. I therefore adapted the assessment to be administered in a group situation. I originally worked with six learners per group, but after the first assessment I adjusted the group size to four. The assessment was administered orally to all 23 learners (five groups of four and one group of three). Learner responses were noted on an assessment schedule. The assessment and recording sheets are included in Appendix B 1 (SEAL) and Appendix C 1 (CPV). During some tasks learners had to answer in writing on an answer sheet. All assessments were recorded on video for observation and more intensive analysis of responses. Copies of the answer sheets are included in Appendix B 2 (SEAL) and Appendix C 2 (CPV).

Fig. 3 shows the original assessment setup for a group of 6 learners as well as the position of the video camera.

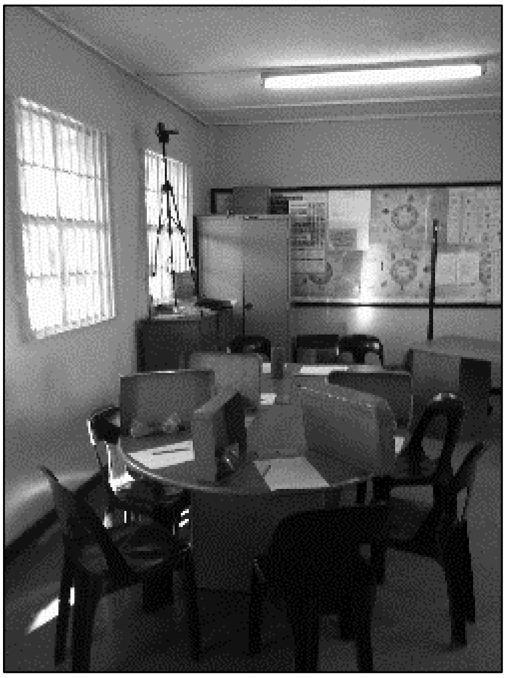


Fig. 3: The original assessment setup as well as the position of the video camera

The adapted group interview assessment was redone for all 23 learners at the end of the intervention period. All assessments were translated and administered in both English and isiXhosa, the mother tongue of the group.

3.4.3 A LFIN profile summary page for each learner

Every learner's LFIN profile, the stage of SEAL and level of CPV, as derived from the interviews, was recorded on a LFIN checklist sheet. I devised this sheet for the purpose of determining the relevant level or stage, to compare results of the two interview assessments and to record observations.

The LFIN profile summary is included in Chapter 4 below and in Appendix D for convenience. Implementation of Wright et al.'s 2012 MR program for SEAL and CPV, particularly the instruction phases and assessment tasks, was administered with the groups once a week for eight weeks in August, September and November 2014. Each of the 8 sessions was 35 – 40 minutes in duration. The adapted program was implemented, evaluated, readjusted and re-implemented as part of the reflective cyclic process of action research.

3.4.4 My research journal

I regularly kept a dated journal throughout the duration of the project to document the critical reflective process, observations, challenges, learner attitudes etc. Capturing these is key to action research. Gaye and Gaye's framework for evaluation (as cited by McAteer, 2013, p.26) was useful in this regard. They suggest that reflection on practice should be:

1. *Descriptive, in that it is personal and retrospective.*
2. *Perceptive, in that it has an emotional aspect.*
3. *Receptive, in that it relates personal views to those of others.*
4. *Interactive, in that it links learning to future action.*
5. *Critical, in that it places the individual teacher within a broader 'system'.*

> "In this way the diary can simultaneously provide a history of the project, the initial and later analyses, and the questions that arise for the researcher along the way" (McAteer, 2013, p.26).

I found the journal helpful not only to record activities done during a particular session, but also to reflect on the success/lack thereof of different tasks and activities. I also used it to make a summary of the observations jotted down during sessions. It was also used to make notes regarding changes that needed to be made and important issues to consider during the planning of further lessons. The emotional aspects associated with the socio-economical context (Taylor & Yu, 2009) was also reflected and personal notes regarding my role and positionality as researcher/teacher were made. The recording of humorous incidents helped me to keep motivated as well. I found the journal to be an important tool in my action research cycle.

Two consecutive journal entries, following two assessment interview sessions on the 5th and 8th of August are provided as examples in Appendix E. Names used are pseudonyms.

3.4.5 Observation sheets of learners' behaviour and strategies compiled throughout the assessment and recovery phases

Observation sheets were used to enable revisiting qualitative observations of learners' behaviour, habits and strategies to deepen understanding of the learning process in relation to the assessment and recovery activities trialled and implemented (see Appendix F for one example of an observation sheet). Observations were briefly jotted down either next to a specific learner's name on a printed sheet or on the lesson plan used. During reflection after group sessions the observations were summarized in the lesson reflection notes as part of the journal. Observations were supplemented by video recordings and photographs. The photographs assisted in capturing strategies used (e.g. organisation of bundling sticks), group interaction, informal notations and practical issues regarding the management of the material or the setup, as seen in Fig. 4 and Fig. 5 below:

Fig. 4: The use of fingers to do 67-52

Fig. 5: The organisation of bundling sticks

3.4.6. Video recordings and photographs

All assessment sessions were video recorded. During the recovery sessions recordings were made occasionally. I took photographs during nearly every session and found it very helpful to reflect on activities and further illustrate strategies used or group dynamics that emerged. The learners initially tended to perform for the video camera, but got used to the cell phone camera quickly. After the first week of use it did not seem to impede on their behaviour.

More examples of photographs and screen shots of video recordings are included in Chapter 4 and Chapter 5. These enhanced data gathered during the assessments and recovery period.

3.4.7 Written examples of learners' work done in class before and during the recovery session period

Although the focus of the recovery program was not to produce a lot of written work, photographs were taken of, for example, the written notation strategies of learners as these provided indicators of learner levels of mathematical progress. Written numerical productions were occasionally taken in. Examples of such written records can be seen in Chapter 4 and the case studies in Chapter 5.

3.4.8 The class's 2014 ANA results

The class's ANA answer sheets were made available to me by the class teacher. I recorded the scores of all learners and made notes and took photographs of some of the tasks and learners' answers which showed strategies used. It shed a light on where the learners fitted in in the bigger scheme of departmental expectations. It also raised for me a tension between procedural versus conceptual knowledge. I noted that while the focus of MR is more on conceptual knowledge and efficient strategies, the focus of the ANAs is more on procedural knowledge than assessing for number sense. This point has also been noted by Graven et al. (2013) and Graven & Venkat (2014).

The ANAs do not include a mental mathematics component. Graven et al. (2013, p. 131) point out that "assessment often drives teaching". They propose in this regard:

> We thus conclude that by suggesting that research be conducted into the viability and appropriateness of the inclusion of an orally administered mental mathematics assessment component in the Grade 4 – Grade 6 ANAs as a way of maintaining a focus on number sense and mental agility through the intermediate phase (2013, p.141).

Graven & Venkat (2014) agree this absence of the assessment of mental strategies in the ANAs could influence the teaching practices relating to mental strategies (as emphasized within the CAPS document) as "the extent of the influence of national assessment on teaching time and the nature of teaching should not be underestimated" (p.9).

Examples of learner work taken from the ANAs will be given in Chapter 5 during the discussion of the two case study learners.

3.4.9 Informal discussions with the principal and teachers

While I was originally concerned that my positioning as a research student might create some tensions with the class teacher an open and engaging relationship developed over time between us. I was mindful to always share with her what I was planning to do and to draw her views on this. She was always welcome to attend group sessions and often did so.

Thus I was particularly grateful that the class teacher, the teacher who taught the class during the previous year (in Grade 3) and the principal of the school were always open and readily available to shed light on issues regarding specific learners. My concerns regarding learners were always discussed and any questions I had were answered.

3.5 Data analysis

Data analysis was conducted simultaneously with data collection following the prescribed assessment process and video analysis as stipulated by Wright et al. (2006, 2012). Data analysis was an integral part of the ongoing adaptation and reflection associated with the action research process. Without this regular data analysis reflection informed adaptation of the intervention sessions would not have been possible.

The aim of the MR assessment interview is to determine the learner's stage of early arithmetic learning (as summarised in Table 2 in Chapter 2 above) as well as his/her level of conceptual place value knowledge (as summarised in Table 4 in Chapter 2 above). The learner is judged on the basis of the most advanced strategy they used. For example, a learner who 'counts-down-to' to solve missing subtrahend tasks, is judged to be at Stage 4 at least (Wright et al., 2006, pp. 73-74). Video material, answer sheets and interview schedules were therefore carefully analysed to determine the most advanced problem-solving strategy employed. This was recorded on the LFIN profile sheet (Appendix D) as mentioned before.

Once a learner's stage of arithmetical competency was determined, the goal was to raise the learner's level of performance by focusing on the learning objectives of the next level. Informal, ongoing assessment and careful observation directed the adjusting and selection of teaching procedures from the prescribed bank of possibilities (Wright et al., 2006). The teaching possibilities described in Wright et al. (2012) were used and the second assessments were analysed in the same way as the previous ones to determine whether the adjusted program impacted positively on the group's mathematical proficiency. Results are presented in various forms in Chapter 5, including graphs to show pre- and post-test results.

Apart from analysing the results of the interview assessments, data analysis also included carefully reading and analysing my research journal as I continuously aimed to present a list of significant issues and analysis of different strategies. I also analysed observations made on observations sheets regarding individual progress, processes and experiences as recorded throughout the implementation phase in a similar way. Individual observations were supplemented by work done before the start of the study as well as classroom observations made outside the research period.

In the following chapter I explore findings of the study in terms of experiences during the adaptation of the interviews and MR program. This provides the methodological contribution of the study.

Chapter 4: The adaptation of the assessment and recovery from an individually administered format to group format – experiences and findings from the action research process

4.1 The adaptation of the LFIN interviews

Led by the specificities of the South African education context (discussed in Chapter 1), I adapted Wright et al. (2006)'s assessment interviews for implementation in a group setting. The original assessments are interview based for individual use. Adaptation for a group therefore meant finding ways of assessing various aspects of SEAL and CPV with groups of learners. The need to assess groups of learners comes from the resource limited context of South African classrooms where individual assessments and recovery programs are unrealistic as discussed earlier. The process of finding an effective and efficient way to conduct the interviews in groups is described below.

4.1.1 Evolving adaptations of the interview setup for administering to a group of learners

I had to adapt the interview questions for a group while keeping in mind the physical space and furniture available for the assessment. I was not only faced with physical challenges, but also had to consider behavioural issues as well as the importance of maintaining the reliability of the assessment. Originally I planned to have six learners seated around the rounded end of the table with me sitting at the other end. Dividers (beer boxes with sand bags) were put up between learners to prevent them from copying each other's answers as seen in Fig. 6 below.

Fig. 6: Initial interview setup

On the first day of interviews, however, Nothemba (discussed as a case study in Chapter 5), amongst others, would repeatedly stand up to deliberately look over the divider at the neighbouring learners' work. The dividers also obscured my view and resulted in areas (Seat no. 4) not visible on the video recordings. The initial setup also made it difficult to place individual written tasks or counters in front of learners without necessitating me to move around constantly. The reflective and adaptive nature of action research allowed me to reconsider and change the setup in order to minimize the negative impact that a group setup could have on the assessment situation. Consequently, due to a lack of discipline in some cases, the group size was reduced to four and upright dividers, used to limit copying between learners, were replaced with sheets of cardboard to cover answer as seen in Fig. 7 and Fig. 8:

Fig. 7 and Fig. 8: The adapted interview setup

Learners used the loose sheets of cardboard to cover up their answers as they were writing to make sure that other learners could not copy their answers. It also enabled me to see all the learners more easily which resulted in more careful observations. Additionally this did not obstruct the view of the camera for recording purposes.

Fig. 7 and Fig. 8 above also show my position in between seat no. 2 and seat no. 3 and my materials placed on a lower side table. From this position I had better vision of all the learners and their work and more control over every learner. I also moved from a position in front of the learners to a seat with two learners on either side. I could now distribute materials more easily and could make more accurate observations and video recordings during the interviews. As I had hoped, the change in group size and the setup had a positive effect on the interview situation.

Each seat was numbered (1 – 4) with every number printed on a different cardboard colour. I wrote down every learner's name next to his number on my response schedule. All answers and strategies and observations regarding a specific learner were then recorded in the column corresponding with his/her number. Since some assessment items required each learner in the group to be given a different question/task (as discussed below) I had to carefully administer the materials. I printed the different questions/tasks for every learner on the same cardboard colour as the number at the specific seat. I also highlighted the corresponding column on my response schedule in the same colour. For example, the number at seat no. 1 was printed on orange cardboard, all individualised tasks for the learner at seat no. 1 were printed on orange cardboard and the column marked 1 on my response schedule was shaded orange as well. Next to the columns were tick boxes with possible strategies/comments to minimize writing. Additional space was provided for "other" methods as well.

This meticulous organization of the assessment set-up eased administering the individual questions, saved time, limited writing and enabled me to focus on learners and not on administering the materials used. See Appendix B1 and C1 for copies of the assessment/response schedules.

During the SEAL interviews learners were asked to close their eyes while a serpentine of counters was packed out in front of them (Question 2). Vuyo, a boy discussed as a case study in Chapter 5, gave the answer before he even opened his eyes. I then realized that he was able to hear when counters were put out in front of him and counted it quietly. Initially milk bottle tops were used as counters (because of their size and the fact that they could be seen by all group members during assessments). After Vuyo's answer the counters were changed to smaller counters which could be put out quietly on a piece of paper and were easier to hold in one hand. Manipulating smaller counters saved a lot of time during the next group interview.

All of the above was recorded in my journal. For an example of my post interview journal reflections after the first two days of interviews, see Appendix E.

Below I describe the interview protocol that I settled on with examples of how several items were administered in different ways.

4.1.2 The group interview protocol

As described above, to limit the amount of writing on my (as the researcher) response schedule, the learners' seats were numbered from 1 – 4 in four different colours. My response schedule had numbered columns with a check list of possibilities colour coded in correspondence with the colours of the seat numbers. Several of the Wright et al. (2006) questions could be addressed to the whole group while others necessitated individualised tasks or separate questions for each learner in the group to avoid copying. This was printed on coloured cardboard to correspond with the colour of the learner's number as well as the colour on the researcher's response schedule. This made organization of test material easier and faster. All instructions were translated in isiXhosa and printed in a different colour on the assessment schedule. Full copies of the assessment questions, response schedules and answer sheets for both SEAL and CPV are included as Appendix B1 and B2 and Appendix C1 and C2 respectively. Examples are highlighted to provide clarity.

Below I provide examples of both SEAL and CPV question items addressed to the whole group with an indication of how learner responses were recorded individually and then noted on my response schedule for each learner. Thereafter I provide an example of individualised question items and explain how these differential items were given simultaneously to learners sitting in the group and show how the differential responses of the learners in the group were recorded on my response schedule.

4.2 Administering SEAL interview questions

4.2.1 Addition tasks (Question 1) – an example of same tasks given simultaneously to the whole group

The purpose of the additive tasks are to determine the learner's solution strategies which then enable one to assess the level at which they are working.

Two groups of different coloured counters have to be added. Questions were posed to all four learners in the group simultaneously and learners had to write down their answers on an answer sheet. I also recorded their methods and relevant observations on my response sheet. For the first 3 tasks both sets of counters are screened and for the next two only one is screened. Screened counters were indicated by shading it on the interview schedule to remind me when and where to use the screen. For example:

> 5 + 2 (only the first set of counters screened).
>
> Interviewer: I am placing 5 counters under here (5 hidden under a screen). I add these two counters (2 counters visible next to screen). How many counters do I have altogether?
>
> 9 + 6 (both sets of counters screened)
>
> Interviewer: I am placing 9 counters under here and another 6 counters under here. How many counters are there altogether?

Similarly shaded numbers were used throughout the interview schedule to indicate to me when screened counters had to be used.

Throughout both assessments all instructions were printed on the interview schedule in English as well as isiXhosa. The isiXhosa instructions were printed in a different colour to be distinguished with ease. Instructions were posed by me in English and isiXhosa. Possible strategies (e.g. counting from one/counting on fingers etc.) were listed on the response schedule next to tick boxes. Because only single digit numbers were added, most learners either added the digits mentally or on their fingers, so few other strategies were observed.

Fig. 9 below shows an example of my recording of learner strategies and answers on my response sheet for all four learners. Every numbered column represents a different learner. The isiXhosa instructions are printed in a different colour.

		1	2	3	4
1. b 9 + 6 There are 9 red counters under here and 6 blue ones under here. Kukho izinto ezibomvu ezilithoba zokubala ngaphantsi, kubekho ezintandathu ngaphantsi apha. How many are there all together? Zingaphi izinto ezilapha zizonke?	Mental / immediate answer	✓		✓	
	Counting on				
	Counting from 1				
	Counting on fingers/by pointing				✓
	Other (specify)		9+1+5 (Xhosa)		

Fig. 9: An example of learner strategies recorded on the response schedule for SEAL (1.b)

4.2.2 Missing addends tasks (Question 3) – another example of same tasks given to the whole group

A given number of counters are shown to the learners and then screened. Learners were asked to look away and while they were looking away a certain number of counters were added underneath the screen and the resulting total disclosed. Learners were asked to identify the number of counters that were added.

Once again the task of the interviewer is to find the strategy learners used to complete the task. The two most likely strategies are counting-on-from and subtraction. This is indicated on the interview schedule as tick boxes. During the first group assessment I found that requesting learners to look away while I was adding the counters was too difficult to monitor and therefore just made the learners close their eyes while I was adding the counters. Learners were asked to write the missing addends on their answer sheets.

4.2.3 Missing subtrahend and removed counter tasks

Missing subtrahend and removed counter items (see Wright et al.'s (2006) interview schedule Questions 5 and 6 in Appendix B1) were similarly administered and recorded. Thus the missing subtrahend task involved displaying a selection of counters, screening it and then removing a certain number of counters while the learners are looking away (closing their eyes, in my case). When told how many counters are left, learners were asked to determine how many were taken away. Learners had to answer on their answer sheets. The interview schedule had tick boxes for the strategies counting-down-to and subtraction and other strategies.

Similarly the removed counter items task was relatively easy to administer in groups as a given amount of counters was placed in front of learners and then screened. A certain number of counters was then taken away, briefly shown and then screened with a second screen. Learners then wrote down the answer on their answer sheets while strategies like counting-down-from were noted on the response schedule. Where necessary learners were asked to describe the strategy used. This was recorded on the response schedule as well.

4.2.4 **Perceptual counting task (Question 2)** – an example of differential tasks given simultaneously to each of the four learners

Each learner was given a different and separate serpentine of counters to count. This meant I was able to observe (and listen to) how learners were counting. While learners could hear each other mumbling their counting, the different amounts they were counting meant that they could not copy from each other's answers. The response schedule had check boxes for count in ones/count in two's/other to limit writing time.

Fig. 10 below shows an example of the notation on the response schedule for all four learners for counting a serpentine of numbers.

2. *Serpentine of counters* *(different number for each child)* (p.47; p.162) Count to see how many counters there are all together. Bala ubone ukuba zingaphi zizonke. Tell me your answer. Xelela impendulo yakho.		13	15	16	12
	Count in 1's	✓			
	Count in 2's		✓	✓	
	Other (specify)	nods head with count	No fingers	fingers	5+5+2 ✓

Fig. 10: A response schedule example for SEAL (Question 2) – Serpentine of counters

4.2.5 **Subtractive tasks (Question 4)** - an example of differential tasks given simultaneously to each of the four learners

The first subtractive tasks were subtraction sentences. These are presented as a written number sentence with the questions "What does it say?" and "Do you have a way to work out what the answer is?"

Here I presented every learner with a separate written subtraction sentence though each was of a similar level of difficulty. The tasks were printed on coloured cardboard to correspond with the coloured numbers on the learners' seats. The tasks in the Wright et al. (2006) assessment were within the number range 12 to 16 (e.g. 16-12). I kept the adapted numbers in the sentences within the 10 – 20 number range without crossing the 10, e.g. 15 - 11/17 - 13.

Written tasks were placed in front of learners individually. Learners' strategies were observed and noted on my response schedule with tick boxes and space to write down other strategies observed. Fig. 11 shows an example of the notation on the response schedule for all four learners on a subtraction task.

		1 ✓✓	2 "13" ✗	3 ✓✓	4
4. *Written task (on card)* *Different for each child* (p.49; p.163) How did you get your answer? Ufemene impendelo kanjani?		*16-12*	*17-14*	*15-11* 4	*16-13*
	What does it say? *Ithini?*	✓Xhora	XKora	✓	✓
	Do you have a way to work it out? (Specify) *Unoyo indlela yokufumana impendulo?*	4✓✓ 10-10=0 6-2=4	17 - thumbs loky 17-14 = 13		
	Counting-on				
	Mental strategy			✓	✓
	Other (specify)				

Fig. 11: A response schedule example for SEAL (Question 4) – Written tasks (subtraction)

4.3 Administering CPV interview questions

4.3.1 **Addition tasks/incrementing in tens (CPV Question 1)** – adapted to being printed on individual learner question sheets rather than demonstrated by the interviewer

During the Wright et al. (2006) individual interview a learner is asked to say how many dots are on a ten strip. The interviewer then keeps adding extra ten dot strips (i.e. to increment by 10 each time) each time pausing to ask "Now how many dots?" to determine how the learners calculate the number of dots. This indicates whether the learner regards the ten strip both as a unit of 10 or as a composite of ones.

Because of the fact that learners could hear each other's counting and answers during the group assessment interview format and alternative numbers of strips could not be offered to determine the required outcome I had to come up with another strategy. I printed the incrementing ten strips on answer sheets for the learners. I deliberately used blank dots (instead of the solid ones used by Wright et.al, 2006) and pencils. If they were counting in ones they used the pencils to count the different dots and consequently left a mark on the blank dot. It was assumed that further blank dots (without marks) indicated counting in tens. For example, in Vuyo's case his answer sheet looked like this (Fig. 12):

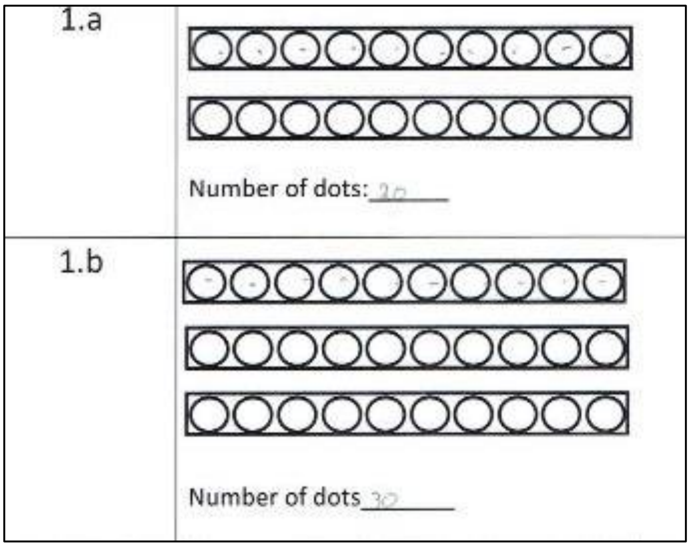


Fig. 12: An example from Vuyo's CPV answer sheet (Question 1)

Based on my observations during the interview, and from my observation of their answer sheets, I was able to record how learners counted the dots. Thus for example from the above, and my observation of Vuyo, I could tell that Vuyo counted the first strip of every question in ones and the consecutive strips as a unit of 10.

This was confirmed by the video recording. In the screenshot below (Fig. 13) Vuyo can be seen in the left bottom corner counting in ones for the first strip and then counting in tens for the remaining strips:

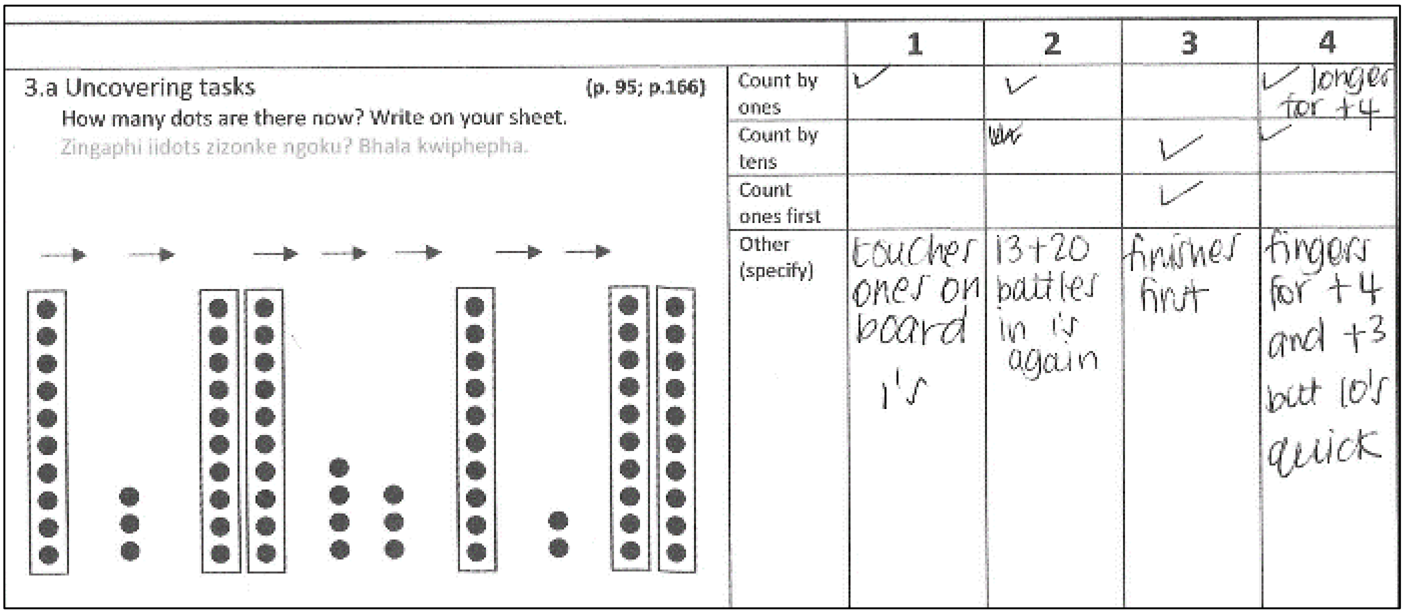
Fig. 13: A screenshot of Vuyo (in left bottom corner) counting the blank dots (CPV Question 1)

4.3.2 Uncovering tasks (CPV Question 3) – Given simultaneously to the whole group

For the two questions involving uncovering tasks (3a and 3b), two A5 sized boards with columns of 10 dots and columns with less than 10 were used (See Fig.14 below). Increasingly complex ways of incrementing by tens, ones and combinations of tens and ones are uncovered by moving screens (the movement of the screens are indicated by the arrows (→) in Fig.14 below).

Learners wrote down the total after every increment on their answer sheets next to the number I gave them orally before every move of the screen. Strategies used were noted on the assessment schedule with tick boxes labelled "count by ones", "count by tens", "count ones first" and "other". Where needed specific learners were asked to clarify the strategy used and that was written down as well.

Fig. 14 below shows an example of the notation on the response schedule for all four learners.



3.a Uncovering tasks (p. 95; p.166) How many dots are there now? Write on your sheet. Zingaphi iidots zizonke ngoku? Bhala kwiphepha.		1	2	3	4
	Count by ones	✓	✓		✓ longer for +4
	Count by tens			✓	✓
	Count ones first			✓	
	Other (specify)	touches ones on board 1's	13+20 battles in 1's again	finishes first	fingers for +4 and +3 but 10's quick

Fig. 14: A response schedule for CPV Question 3a. (Uncovering tasks)

4.3.3 Incrementing by tens off the decade tasks (Question 2)

During the Wright et al. (2006) individual interview a four strip (a strip with only four dots) is placed in front of a learner. Ten strips are added with the increments resulting in 4, 14, 24 etc. In order to adapt this to be done for each learner in the group I addressed each learner by turn (individually and orally) with every learner starting with a different number of dots (e.g. 7, 17, 27 etc. or 3, 13, 23 etc.). Fig.15 shows an example of the notation on the response schedule for all four learners.

	1	2	3	4
2. Incrementing by ten (p. 94; p.166)	4	7	3	6
Individual question for every child	14	17	13	16 1's
How many dots are there? (e.g. 4 / 7/ 3) Zingaphi iidots zizonke?	24	27	23	26
	34	37	33	36
Place a ten strip to the right of the first strip.	44	47	43	46
How many dots are there now? Zingaphi zizonke ngoku?	54	57	53	56 10's
	64	67	63	66
Continue to do with e.g. 24 / 34 / 44 / 54 / 64 / 74	74	77	73	76
	all in 1's fingers not auto	✓✓ quick	✓✓	

Fig 15. A response schedule example for CPV Question 2 (Incrementing by 10)

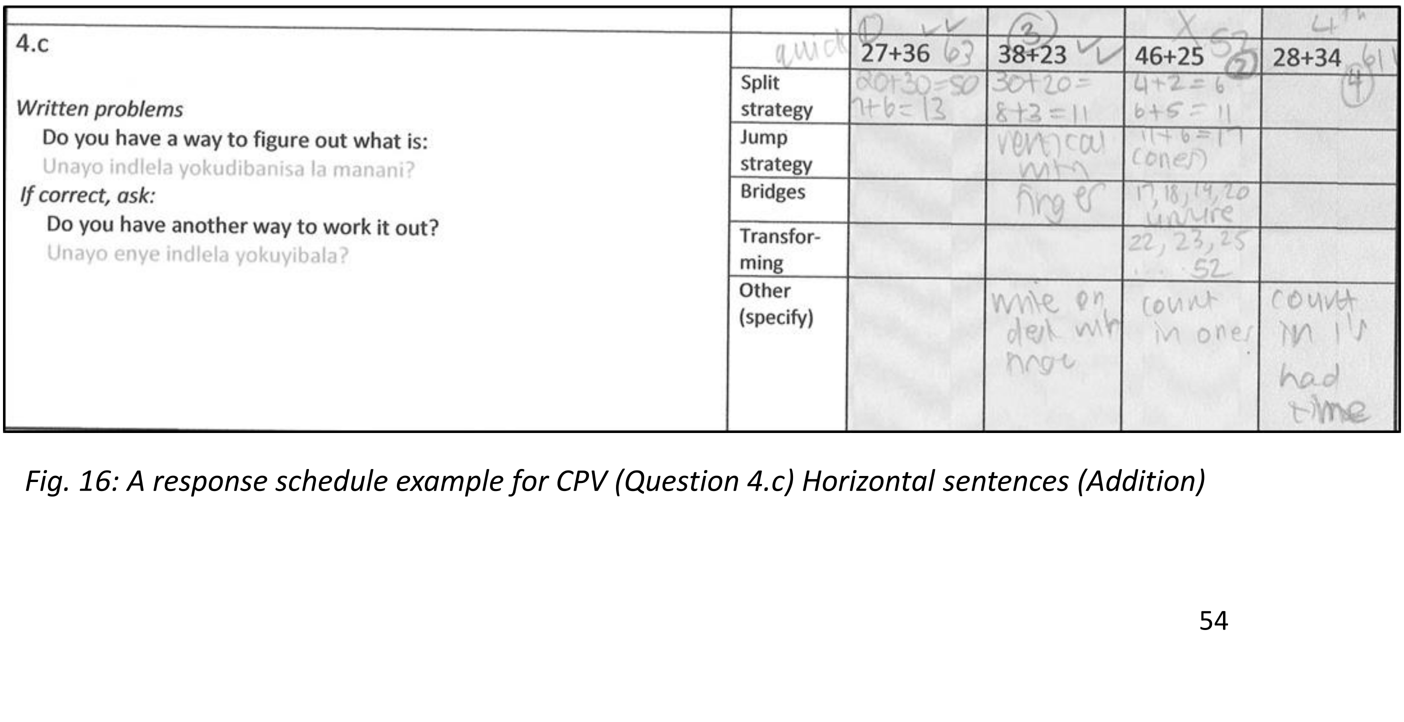
4.3.4 Horizontal sentences tasks (Question 4)

The Wright et al. (2006) individual interview tasks involve learners presented with written two-digit addition problems and written two-digit subtraction problems. They are then asked how they can figure out the answer. When an answer is given, they are asked whether they have another way to solve it.

During the group assessment learners were given individual written problems (Once again printed on colours to correspond with their numbers to make administration easier). Care was taken to ensure that the differential number problems for each of the four learners were similar to the ones used by Wright et al. (2006). For example 42 + 23 / 33 + 25 (no regrouping) and 27 + 36 / 38 + 23 (regrouping). Strategies were noted on the assessment schedule with tick boxes for split, jump, transforming and other strategies used. (These strategies are discussed in Chapter 2).

During the assessment of the first group I realized that reflection was needed and that some changes had to be made. Because a mental strategy is assessed, learners should not be using pencils and paper. Although there was no allocation for these answers on the answer sheets, learners attempted to use the pencils and answer sheets to solve the tasks. During the following group assessments I took in the paper and pencils after Question 3 was completed.

This task group was the most time consuming and the most difficult for learners to complete. Learners were given separate written tasks and some learners took a long time to come up with a solution while the others in the group were left waiting. To prevent learners from getting restless and disruptive I continued by handing out the colour coded written tasks to all the learners at the same time and getting feedback individually. Because the learners had different response times I could often get feedback from individual learners while others were still working on their questions. On the response schedule I recorded the order in which learners were able to give feedback and indicated whether a learner answered relatively quickly or whether they needed more time than the others. The recording of the order of answers/feedback given also helped me to monitor the influence that one learner's answer could have on an answer given later by a different learner. Fig. 16 shows an example of the notation on the response schedule for all four learners.



4.c *Written problems* Do you have a way to figure out what is: Unayo indlela yokudibanisa la manani? *If correct, ask:* Do you have another way to work it out? Unayo enye indlela yokuyibala?	quick	27+36	38+23	46+25	28+34
	Split strategy	20+30=50 1+6=13	30+20= 8+3=11	4+2=6 6+5=11	
	Jump strategy		vertical with	11+6=17 (cones)	
	Bridges		finger	17, 18, 19, 20 unsure	
	Transfor-ming			22, 23, 25 ... 52	
	Other (specify)		write on desk with finger	count in ones	count in 1's had time

Fig. 16: A response schedule example for CPV (Question 4.c) Horizontal sentences (Addition)

The influence of the feedback given in the group context is discussed below.

4.4 Reflections on the influence of the group setup on the assessment interview

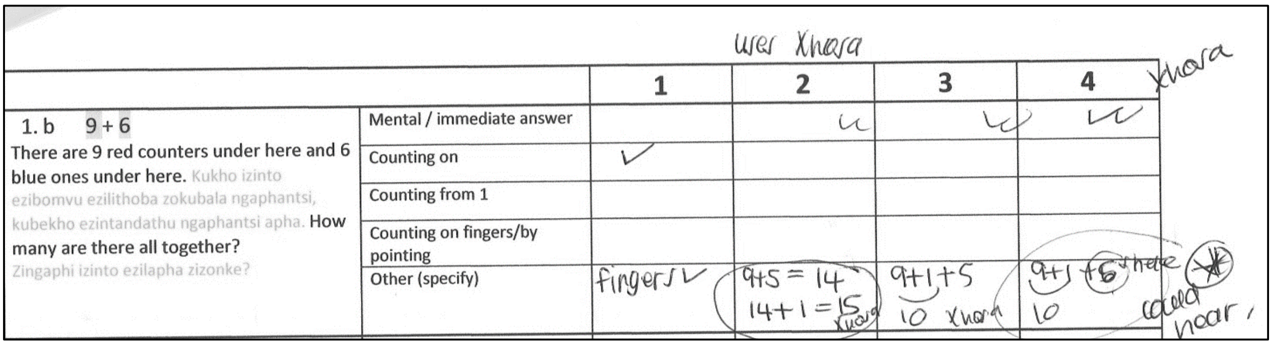
All challenges associated with the group interview assessment format could unfortunately not be eliminated. Some learners were getting restless and disruptive while waiting for others to answer individualised questions. Learners could also hear each other's answers and in some cases a learner would laugh at another. In general I found that the learners' behaviour and self-control were better during the second round of interviews.

Because one learner was absent once when the interviews were conducted one of the SEAL interviews was conducted individually. This took 25 minutes to complete. The group interviews were thus much more economical time wise. The average duration of the first round of SEAL interviews (August 2014) was 39 minutes per four learners compared to 25 minutes on average for the second round (November 2014). The average duration of the first round of CPV interviews per four learner was 36 minutes compared to 31 minutes on average for the second round. In my journal and reflection I attributed the shorter interview times for the second round to the following factors:

- Changes were made to the initial interview setup
- I was more familiar with the interview material
- The learners were more relaxed and needed less prompting
- The learners were more familiar with the interview material
- The learners could solve various problems faster than before
- Learners were accustomed to explaining their strategies
- Learners were more familiar with the vocabulary used
- I was familiar with more of the isiXhosa words used as part of their answers
- It was not necessary to repeat all instructions in isiXhosa for all groups

Apart from the obvious advantage of time, an element of competition seemingly enhanced learning experiences during interviews. For example, during the first CPV interviews learners had to write down the total number of dots on 10 strips. A few learners started out by counting in ones. Others in the same group were counting the number of strips in tens. The moment they shouted “finished!” the others realized that there was an easier way of doing things and figured out that they should count in tens too. The competition element similarly emerged during the second round of interviews when learners, now familiar with ten dot strips, did not count in ones or tens at all and merely wrote down 20 and added 10 to the previous answer every time to be able to finish first.

Another example of how learners learnt from one another during the interview assessments is the following: While asking learners about the strategy used to add the covered counters 9 and 6 (SEAL, 1.b, Interview II), the responses were recorded as follows on the researcher’s sheet (Fig. 17):


user Xhosa

		1	2	3	4	Xhosa
1. b 9 + 6 There are 9 red counters under here and 6 blue ones under here. Kukho izinto ezibomvu ezilithoba zokubala ngaphantsi, kubekho ezintandathu ngaphantsi apha. How many are there all together? Zingaphi izinto ezilapha zizonke?	Mental / immediate answer		✓✓	✓✓	✓✓	
	Counting on	✓				
	Counting from 1					
	Counting on fingers/by pointing					
	Other (specify)	fingers ✓✓	9+5 = 14 14+1 = 15 Xhosa	9+1+5 10 Xhosa	9+1+6 whole 10	could hear

Fig.17: A response schedule example for SEAL (Question 1.b) Addition of covered counters

The first learner said he counted on his fingers, the second used a known fact (9 + 5 = 14; so 9 + 6 = 14 + 1). The third learner described a strategy, jump to the ten, introduced during recovery, but not used spontaneously by any of the learners before. He said “9 + 1 = 10; 10 + 5 = 15”. Learner 4 could hear this explanation and then copied the strategy by saying “9 + 1 = 10 ‘kushiyeka’ (leaves) 5. 15”. Both used the same strategy to solve the next problem as well. It is possible that Learner 4 used a peer’s answer as a learning opportunity for himself.

It looked like the Interview situation was also a learning opportunity for Themba.

In SEAL, 4.a (Interview II) he described his strategy for **14 + 10** as:

4 + 0 = 4
1 + 1 = 2
24 (the correct answer)

He did not “echo” the quantity underlying the digits (Graven et al., 2013, p.138).

For the next question 4.b Zola did the following:

42 + 23
20 + 40 = 60
2 + 3 = 5
60 + 5 = 65

I purposefully repeated Zola’s correct phrasing of place value (i.e. “twenty plus forty” rather than “two plus four”). Themba was listening to her explanation and then began phrasing his calculations as follows:

33 + 25
30 + 20 = 50
3 + 5 = 8
50 + 8 = 58

Thus in subsequent calculations he noted the quantity underlying the tens and ones and used this in the phrasing of his method.

4.5 Post assessment considerations informing recovery intervention

After the completion of the assessment interviews I reflected on the way ahead and the most appropriate way to do recovery with a class of 23 learners and simultaneously keep track of the individual progress and the class's development. I found an online generated mind map useful to structure my thoughts and planning. (See Appendix G for a copy of the mind map). Additionally it provided a good way to share considerations and key issues that I had identified as needing attention and discussion with my supervisor.

4.5.1 Generating summaries of individual learner stages and levels from interviews

A key challenge following the interviews was to find a manageable way to determine the SEAL stage and CPV level of every learner based on my recording of learner responses in the interviews, the video recordings and the answer sheets completed by learners. After revisiting the criteria set out in Wright et al. (2006) I compiled a shortened LFIN profile checklist for every learner with columns for recording levels after the first and second interview assessments respectively. Using the assessment response schedule, the learners' answer sheets and the video recordings of the assessment, I could complete a summary sheet LFIN profile for every learner. The summary sheet for both interview assessments is provided below (as well as in Appendix D, as mentioned in Chapter 3):

LFIN profile: SEAL and Conceptual Place Value (Wright et al., 2006)

Learner: ______________________________________ **DoB** ______________

Summary	Date	Date
SEAL Stage		
CPV Level		

	Date	**Date**
SEAL STAGES		
Stage 0 – Emergent counting (p. 22)		
• Can not count visible items		
Stage 1 – Perceptual counting (p.22)		
• Can count counters	ones/two's/threes	ones/two's/threes
• Can not count screened items		
Stage 2 – Figurative counting (pp.22, 60-62, 91))		
• Can count screened items by counting from 1		
• Battles with missing addend and missing subtrahend		
• One screened and one unscreened item >> count unscreened first and then keep track of screened collection		
Stage 3 – Individual number sequence (pp. 22, 64)		
• Counting-on/counting-up-to rather than count from one		
• Count-down-from		
• NOT count-down-from for missing subtrahend (e.g. 17 -__ = 14 >> NOT 17 – 14)		
• May keep track of number of counts by using fingers		
• Misinterpret missing addend task as additive task		
• Takes long time for task >> Could mean counting by ones		
Stage 4 – Intermediate number sequence (pp.22, 67-69, 89)		
• Counts-down-to for missing subtrahend		
• Chooses between count-down-to and count-down-from		
• May use fingers to keep track of counting-down-to		

• Quick addition >> could mean non-count-by-one		
Stage 5 – Facile number sequence (pp.22, 70, 88,102)		
• Range of non-count-by-one strategies (at least 3 instances) e.g. compensation (15-3 > 5 – 3 = 2 > 15 -2 = 12) adding to ten subtraction as inverse of addition using a known result commutativity (2 + 6 + 8 + 4 = 2 + 8 + 6 + 4)		
• Some counting-by-one		
	Date	**Date**
CONCEPTUAL PLACE VALUE		
Level 1 – Initial concept of 10 (pp.22, 93)		
• At least Stage 3 on SEAL!		
• Does not see 10 as a unit		
• Focus on individual items in 10		
• Count backwards and forward by 1		
Level 2 – Intermediate concept of 10 (pp.22, 93)		
• 10 = unit of ten ones		
• Depend on representations of ten (ten strips/fingers)		
• Does + and – with materials		
• NOT written number sentences with tens and ones		
Level 3 – Facile concept of 10 (pp.22, 93)		
• + and – with tens and ones without materials/representations		
• Can + and - written number sentences of tens and ones		

Comments Date ___________________________	Comments Date ___________________________

The LFIN profiles provided me with details regarding the current knowledge and strategies used by the individual learners. The profiles from Interview I informed the focus of recovery. Once the profiles of Interview II were recorded progress made by each learner could be gauged from these LFIN sheets.

4.5.2 Planning for recovery across learner stages and levels

After careful consideration of the summaries above and the recovery materials that are suggested by Wright et al. (2012) for learners at particular stages and levels I planned a range of activities for targeted intervention. I drew up a checklist for myself so as to record and monitor areas addressed during every recovery session. This checklist included the mental strategies for addition and the mental strategies for subtraction (SEAL) as well as the dimensions for instruction of CPV as discussed below. The checklist had a column for every day and all aspects addressed during that day's session could easily be checked. See Appendix H for a copy of the recovery checklist.

Since the interviews indicated that learners were at different SEAL stages and CPV levels I also had to decide how to place learners into smaller groups to enable more targeted recovery at the level and stages required. This involved a long process of thinking through the pros and cons of mixed ability groups and ability grouping. However, mixed ability groups are recommended by much research (e.g. Boaler, 2009) to enhance mathematics learning. Thus a dilemma emerged as to grouping learners together according to similar levels or whether to mix learners at different levels within a group.

My decision to go with mixed ability grouping was largely influenced by the work of Boaler. Boaler (2009) argues that 88% of children put into low sets/groups at a young age will stay there until they leave school and do not develop their potential because they are given less challenging work and are excluded from collaboration with their peers who could stimulate their thinking. A British review of research on grouping strategies in primary schools was conducted in 2008 and the research showed "that ability grouping in primary schools had no academic benefits and severe negative consequences for children's development" (Boaler, 2009, p. 97). From Boaler's own research in England it was surprising that learners placed in higher groups were also disadvantaged by the arrangement. They experienced too much pressure and were reluctant to ask for more time or assistance. According to Steve Olsen (as quoted by Boaler, 2009, p. 111) high achievement in Japanese classrooms are also promoted by grouping students with different ability levels together. The differences are seen "as a resource that can broaden the discussion of how to solve problems".

My sense was that even while it might be easier in terms of my targeted recovery teaching to group learners of similar levels and stages together, I felt it was ethically important to not contribute to grouping learners in ways that might be read as ability grouping and additionally wished to enable learners to learn from one another's methods. Groups were therefore set up with learners from different SEAL stages and CPV levels. Behaviour and focus of learners were also taken into consideration to separate learners with concentration/behavioural challenges from each other.

4.6 Adapting and implementing the Wright et al. recovery sessions

Wright et al. (2012, p.8) recommend an individual intervention program of daily or near daily sessions for at least 10 weeks. The recovery program constructed and implemented for the use of this study was limited by time available to me and contextual constraints. Recovery was done in groups of 5 or 6 learners. Group sessions (for every group) took place once a week for a period of 8 weeks between August and November 2014. Every session lasted 35 to 40 minutes. Some activities were done with the whole group and others in pairs. While the end of the academic year in November necessitated the end of the intervention I recognised the need for longer and more intensive recovery than was conducted in this study. I thus continued to work with learners in the school post research in 2015.

In terms of the session planning, I aimed to follow the recommendations and activities used by Wright et al. (2012) as closely as possible. I also tried to build in opportunities for extension of stronger learners to keep them motivated and challenged. According to their recommendation the two to four domains (e.g. number word and numerals, CPV and multiplication and division) selected to target should be addressed in every session. In my case recovery sessions were therefore designed to focus on both SEAL and CPV during every session.

As part of the action research cycle every session was analysed and reflected upon to inform decisions regarding the focus and construction of the next session. After every recovery session I used my research journal to reflect on the activities, the responses and behaviour of the learners and aspects that needed to be adapted before the next session. I also added observation notes made during the lesson to my journal.

4.6.1 The recovery intervention program

Wright et al. (2012) view mathematics instruction in terms of progressive mathematization, described as the "development of mathematical sophistication over time" (p. 15). Progressive mathematization is characterized into 8 themes as applicable to the domains of SEAL and CPV. These themes are summarised below:

Theme A: Structuring numbers

The domains of Conceptual Place Value (CPV) and Addition and Subtraction to 100 (SEAL), structure the additive relations between numbers 1 – 100, specifically organised around ones and tens.

Theme B: Extending the range of numbers

Within each domain there is a progression in the range of numbers. As students become fluent in smaller numbers bigger numbers are immediately introduced for the specific task.

Numbers used during lessons were adapted according to the response from and level indicated by the learners. Extending the number range was an effective way to supply more advanced learners with mathematical challenges.

Theme C: Decimalizing towards base-ten thinking

Numbers are organized into ones, tens, hundreds etc. in order to develop a skilful habit of organizing numbers and calculations. The focus within SEAL and CPV is to develop flexible addition and subtraction techniques of tens and hundreds and mental strategies based on base-ten thinking.

Theme D: Unitizing and not counting by ones

The aim of both the chosen domains (SEAL and CPV) is to develop non-counting strategies through developing understanding of ten as a special unit. Although various strategies are recommended, the number of strategies used during the recovery phase of this study was limited because of the relatively short recovery period (exams started in November followed by extended summer vacation) and the fact that it was clear early on that most learners had difficulty grasping the basic strategies. Additional strategies were therefore only suggested to stronger learners.

As mentioned in Chapter 2, the following strategies used by Wright et al. (2012, pp. 99-107) were addressed during the recovery phase: Split, Jump and Jump to the decuple (Jump to the 10).

Jump strategy: Begin from one number, jump the tens and then jump the ones (or ones first and then tens)

e.g. Addition: 37 + 22 → 37 + 10 → 47 + 10 → 57 + 2 → 59

Subtraction: 53 -11 → 53 – 10 → 43 – 1 → 42

Split strategy: The tens and ones are split, then added/subtracted separately and then recombined

e.g. Addition: 37 + 22 →
30 + 20 = 50
7 + 2 = 9 →
50 + 9 = 59

Subtraction: 53 -11 →
50 – 10 = 40
3 – 1 = 2 →
40 + 2 = 42

Jump to the decuple (Jump to the 10): Begin from one number, jump to the nearest decuple, jump the tens, then jump the remaining ones

e.g. Addition: 37 + 25 → 37 + 3 → 40 + 10 → 50 + 10 → 60 + 2 → 62

Subtraction: 53 – 19 → 53 – 3 → 50 – 10 → 40 – 6 → 34

"Research suggests that most successful students use jump strategies, whereas most low-attainers use split strategies" (Wright et al., 2007, p.849). This finding by Wright et al. is in line with my findings as noted in Chapter 5.

Theme E: Distancing a setting of materials

Distancing the setting is considered an important recovery strategy in Wright et al.'s work. The setting (e.g. the use of bundling sticks) can be progressively distanced in the following stages:

1. The bundling sticks are visible
2. The bundling sticks are shown briefly then screened
3. The task is posed verbally with bundling sticks screened
4. The task is posed verbally with no bundling sticks

I found that it was possible within the group setup to distance the setting for those learners who were ready to progress and, at the same time, supply materials for other learners where needed. The use of materials, or absence thereof, was an effective way of enabling differentiation of support provided to individual learners within each group.

Theme F: Notating

Wright et al. (2012) support the supposition that mathematical notation and mathematical concepts are learned in tandem. Learners are therefore encouraged during the recovery program to use notation to express their thinking. I noted in my research journal that some learners found the notation strategies confusing. Fig. 18, for example, shows one pair of learners' attempt at using a number line to notate 54 – 37. They preferred the informal line/arrow notation illustrated in Fig. 19 below.

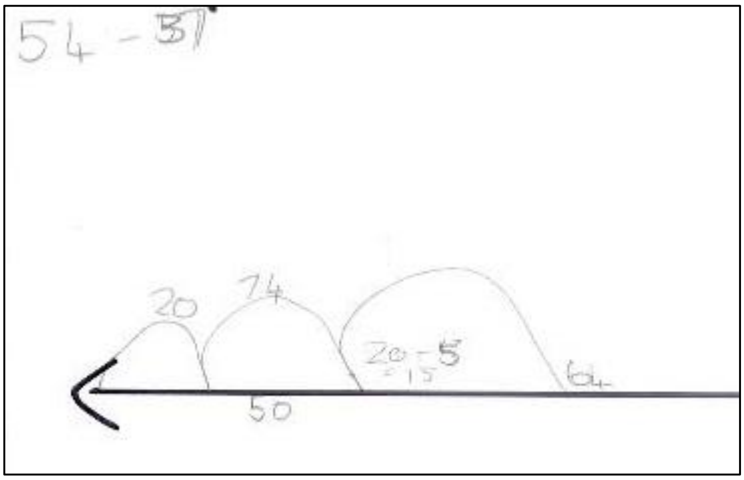


Fig. 18: An example of a number line used by a pair of learners for notational purposes

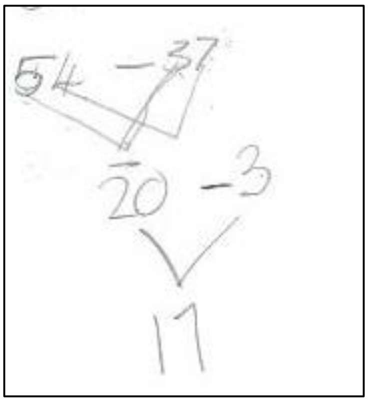


Fig. 19: An example of informal line/arrow notation to show the calculation of 54 - 37

Although various notation strategies were used (e.g. number lines, arrow charts etc.) I realised that the complexity thereof should be monitored for individual learners. Some learners preferred the use of arrow charts and others favoured number lines. It was also important to keep in mind that “notation is used to record the mental strategy rather than providing a means of solving the task” (Wright et al., 2007, p.849).

Theme G: Formalizing

Formalizing entails a progression in notation from informal to the standardised use of symbols, columns and signs. Within the short period of recovery most of the learners were not yet ready for formalization of notational strategies. During group sessions we therefore only used the two informal strategies (number lines and line/arrow charts) illustrated in Fig. 18 and Fig. 19 above. We did not address formalizing of strategies.

Theme H: Generalizing

After particular example tasks are established, Wright et al. (2012) illustrate that other tasks could be solved in similar ways. So for example in the recovery I would deliberately choose numbers to allow learners to extend the range of numbers used in various ways, e.g. 34 + 10 = 44 and 44 + 10 = 54 can be extended to 134 + 10 = 144 and even to 340 + 100 = 440. This was done by using materials like bundling sticks or simply by mental exercises during games and the use of flash cards (See Appendices I to P for all eight the recovery session plans in this regard).

4.6.2 Reflecting on group/peer interaction

In my study Wright et al.'s (2006, 2012) MR program was adapted for administering in the group set-up rather than individually. Both MR assessment tasks and instructional tasks were used. In some cases the recommended use of materials was changed (e.g. I used bundling sticks instead of ten strips as this was easier to manipulate, proven to be more durable and it is something that the learners or teacher could supply themselves in future by even using bundled matches).

Since MR was not developed for group intervention, I had to devise ways of tapping into the possibilities for the peer learning opportunities and co-construction of knowledge within a group setting. Brooks and Brooks (1993) identify social discourse as a powerful way to change or reinforce conceptions within the socio-constructive classroom. Cobb (1995) differentiates between univocal and multivocal interaction within paired activity. The term univocal is used to emphasize that the perspective of one child dominates while, in the case of multivocal interactions, both learners attempt to verify his/her own thinking and challenge that of the other. As univocal interactions are not particularly beneficial for either learner, I had to attempt to manage interactions in a way that could result in multivocal interaction.

I found the five essential components of number talks (Parrish, 2011; Boaler, 2014) to be useful and productive in this regard. (These are also used in the SANC teacher development project). The focus of a number talk is classroom conversation with the aim of making sense of mathematics. Learners present and justify their solutions to mentally solved problems.

According to Parrish (2011) "(t)hese exchanges lead to the development of more accurate, efficient, and flexible strategies" (p.199). The essential components to keep in mind are:

1. The classroom should be a safe, risk-free environment where learners are comfortable to respond and investigate new strategies.
2. Discussion is crucial - a problem is written on the board and feedback from learners is given only once nearly everybody indicated the number of solutions they have by quietly putting up a finger(s). All answers, both correct and incorrect, are recorded and discussed.
3. Teachers should not be the figure of authority, but should take on the role of facilitator, listener, learner and questioner.
4. The focus is on mental computation, and not paper-and-pencil strategies.
5. Problems should be carefully planned and well crafted to develop the required strategies (Parish, 2011, pp. 202 – 206).

I realised, however, that both the learners' lack of prior experience of group work and discussion, coupled with language barriers, would be challenges hampering the success of the use of number talks in my group context. I therefore had to pay extra attention, in my role as facilitator, to create a risk-free environment where the focus of discussion would be on the relative value of mental strategies contributed by learners rather than discussion that judged learners in terms of their contributions or explanations of their strategies. Fortunately the problems posed within the MR program were all well crafted and purposeful thus enabling both a focus on strategies and rich discussion of strategies used.

4.7 A description of the recovery sessions and adaptations made

A summary of all eight recovery session plans is included in Appendices I to P. However some significant adaptations and observations across each of the eight sessions were made during the sessions and these insights and adaptations are discussed below.

4.7.1 Session 1

See Appendix I for a full session outline.

Focus:

- Incrementing and decrementing in tens and ones (CPV)
- Addition and subtraction with the use of bundling sticks (verbal instruction) (CPV)
- Practising the jump and split strategy for addition and subtraction with bundling sticks (written instruction) (SEAL)

Description and reflections:

During the first session only unscreened material was used and bundling sticks were introduced. Other materials, like arrow cards and base ten strips were gradually introduced during follow up sessions. Certain tasks were earmarked for extension where applicable.

Initially some learners found it hard to pack out a number like 15 and 23 without counting out sticks individually. When tasked with something like 47 – 18 breaking up a bundle into ones did not occur to them as a solution. Some learners would even “boleka” (borrow) a loose stick from somewhere else rather than go to the tens. A few learners would still write with their fingers on the desk drawing out the vertical algorithm instead of counting / grouping the sticks when doing addition.

Fig. 20 shows an example of manipulation of materials (counting out bundling sticks)

Fig. 20: An example of manipulation of materials (counting out bundling sticks)

4.7.2 Session 2

See Appendix J for a full session outline.

Focus:

- Incrementing and decrementing in tens and ones off the decuple with screened bundling sticks (CPV)
- Practising the jump and split strategy for addition and subtraction with bundling sticks (written instruction) (CPV)
- Two digit addition and subtraction without regrouping (written notation; bundling sticks optional) (SEAL)

Description and reflections:

The setting was distanced by using screened materials and the numbers were extended beyond 100. Split and jump strategies were introduced. SEAL activity 3 was posed in notation and strategies were notated on a number line. The focus moved away from manipulating materials (Wright et al., 2012, p.114).

Because this was the first session where learners could not manipulate materials, no tasks with addition or subtraction involving jumps across the decuple/regrouping were posed.

Only three learners wanted to touch the cloth to count the bundles underneath as seen in Fig. 21.

Fig. 21: Screened materials

Learners progressed from the previous week by removing an elastic band from a bundle of tens to use the ones and that they could group ten loose sticks to make a bundle.

Although learners still relied on their fingers, they progressed to counting in tens on their fingers and then in ones.

Notation of strategies was introduced by using a number line. “Jumping” in tens and on the number line was a good practice for counting in tens off the decuple. I also introduced The terms “split” and “jump” which were also used during notation.

4.7.3 Session 3:

See Appendix K for a full session outline.

Focus:

- Incrementing and decrementing in tens and ones (task board with dot strips) (CPV)
- Incrementing and decrementing in tens and hundreds using screened dot strips and 100 blocks (CPV)
- Practising the jump strategy without materials with notation (SEAL)

Description and reflections:

More complex increments and decrements by tens and ones were reinforced by using a dot-strip task board to be uncovered by using two screens (Wright et al., 2012, p.86).

Ten strips and 100-dot squares were also used. Incrementing and decrementing by hundreds beyond 900 were practiced. Materials were screened (Wright et al., 2012, p.87). Wright et al.'s (2012) tasks were adapted to not only go across a 1000 in tens (like the original task), but also to increment and decrement by hundreds. The nature of the activity allowed for differentiated questions according to learners' levels e.g. incrementing by 10 or by 40 and crossing hundreds. This allowed for some extension and challenges.

Two groups indicated the addition of a hundred after 1000 as 2000 instead of 1100. One learner said "ten hundred" instead of 1000. Groups found it challenging to cross the 1000 in increments and decrements of tens (20's/30's included). All four groups battled with 994 + 10 (posed with screened dot strips with the 10 briefly shown) and 1094 + 10. They needed the use of written notation of the task (not just the dot strips) to be able to solve it.

Wright et al. (2012, p.123) recommend the use of jump-based strategies for tasks involving regrouping as it requires less organization. The tasks were posed in written format and practiced together as a group before being posed in pairs. Most learners reverted to the ritual habit of "smaller-from-larger" and a few could not use a strategy other than counting on their fingers or writing on the desk with their fingers attempting to write out the vertical algorithm. When told in advance that, for example, 83 – 26 *is not 63*, many learners were confused. This caused a cognitive conflict and challenged their conceptual knowledge.

It was clear that more time should be spent on this. Because bonds of ten were not automatic knowledge for these learners, delays in the use of the strategy to complete the ten and add the rest of the units afterwards resulted. For example:

e.g. **58 + 35**
58 + 30 = 88
88 + 2 = 90 (learners found it difficult to identify "2" as the number to add)
90 + 3 = 93

4.7.4 Session 4

See Appendix L for a full session outline.

Focus:

- Activities to enhance bonds of 10/addition and subtraction
- Place value activity (CPV)

Description and reflections:

To speed up the use of the jump strategy by avoiding the need to count on fingers, the approach of doing both CPV and SEAL activities was interrupted for this session by reinforcing bonds of 10 and 20 and place value by means of games.

Tshesane (2014) also noted this as he found development of the underlying skills for application of jump strategies in the context of using a number lines important.

Learners exhibited a surprising ability to keep track of the auditory input. It was interesting that for a number like 543 Amahle and Kungawo (in different groups) both said "five hundred and four hundred and 3".

I noted in my journal that the challenge is for me to speak less / repeat less and wait longer for them to respond (Journal entry, 16 September 2014).

4.7.5 Session 5

See Appendix M for a full session outline.

Focus:

- Incrementing and decrementing in tens and hundreds (with printed flash cards) (CPV)
- Practise jumping to the next decuple (spinner game) (SEAL)
- Practising jumping to the next decuple with addition (SEAL)

Description and reflections:

For the described CPV activity in this session (incrementing and decrementing in tens and hundreds) no materials were provided. Numbers were posed as a written numeral. Increments and decrements were on the decuple, off the decuple, in hundreds and 'hurdling' 1000.

It was noted that receiving counters as rewards for coming up with the correct answer was highly motivational for most learners and the group set-up inspired even the more quiet learners to make contributions. Although an element of competition was not always employed it worked well in this instance and in the context of the game (Journal entry, 30 September 2014).

For the SEAL activity a "Jumping to 50 game" was played to extend strategies practiced.

4.7.6 Session 6

See Appendix N for a full session outline.

Focus:

- Repetition and practice of decrementing and incrementing in tens and hundreds (CPV)

Description and reflections:

The previous CPV activity was repeated because it worked well and enabled learners to practise various skills supporting their fluency and confidence. In order to progress the learners somewhat, even while focusing on revision, a different set of numbers was used and no materials were provided. Increments and decrements were on the decuple, off the decuple, in the hundreds and hurdling 1000 and 2000. Two of the four groups had difficulty crossing the 100 or 1000. Especially with "10 less" and "100 less". So for example when asked, "What is 100 more than 994?" or "What is 10 less than 2009?" it was difficult for learners to answer. They had to be reminded of the pattern that the last two digits stay the same when adding or subtracting 100. To reinforce what comes just before 200 or 1 300 etc. I randomly asked questions such as "What is just before 200?" etc. The use of a number line was helpful to explain '10 less' and '10 more' questions. As a result of this revision and consolidation session there was no time left to do the planned SEAL activity which was then incorporated into Session 7.

4.7.7 Session 7

See Appendix O for a full session outline.

Focus:

- Addition and subtraction with regrouping (Jump, Split, Jump to the decuple strategies) (SEAL)

Description and reflections:

Learners worked in pairs with written number problems in this session. Materials were available if needed (bundling sticks, counters, flard cards/number lines etc.). The instruction was to come up with three ways of solving every problem. This was followed by feedback and a 'number talk'. The original lesson plan had to be adapted at the beginning of the first group session when it was clear that utilizing more than one strategy is too confusing for most pairs. Thus more strategies were not introduced.

Therefore a written problem (namely 37 + 15) was posed to the group as a whole. On three small posters "Jump", "Jump to the ten" and "Split" were written respectively. Learners were asked to give the answer to the problem. They were asked to explain their strategy and then the group decided which of the three strategies was used by the learner. The specific strategy was then notated on the poster. This was done for all three strategies. Posters were then displayed to refer back to. One set of posters was given to the class teacher to put up in class. She reinforced the strategies in class and with a homework assignment she marked herself.

Pairs of learners were then asked to solve subsequent problems by using all three strategies and notating their strategies. Some pairs had to be reminded that they can talk to each other and work together. When given two pencils per pair, they tended to work separately. Supplying only one pencil per pair forced them to work together better. Occasionally some pairs would have one passive partner and one doing the work, so they needed to be encouraged to both be actively involved. I noted that pairs had to be given different tasks otherwise they strongly felt the need to cover up their work or feel stressed to finish at the same time as other pairs.

Fig. 22 and Fig. 23 captures the difference in group dynamics between a pair with two pencils and a pair with only one pencil.

Fig. 22: A pair with two pencils

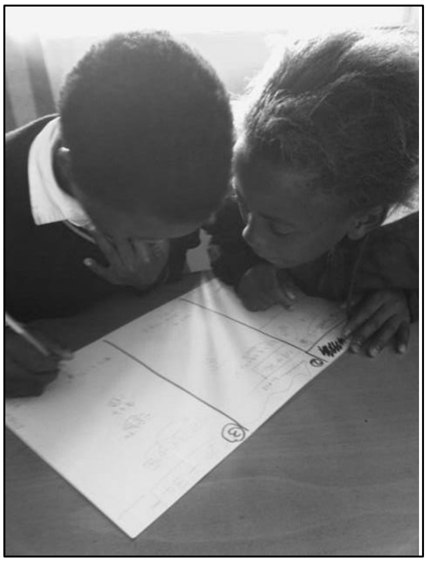


Fig. 23: A pair with one pencil

I noted that almost all pairs could successfully label the strategies. The notation of strategies as suggested by Wright et al. confused the majority of learners. Fig. 24 below, for example, shows a notation where the ones and tens are split and then just put back together again indicating that when learners focus on this split notation in written form they seemingly forget the purpose of the split strategy (i.e. to arrive at the answer of the addition).

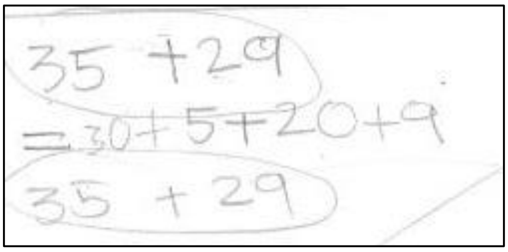


Fig. 24: An example of a confusing notation for the split strategy

The more informal notation of the split strategy seemed to be easier to master. It was however important that I reinforced the correct use of + or - to indicate whether the units subtracted were more or less than the units it was subtracted from. Fig. 25 is an example of a pair's notation of 54 - 37.

Fig. 25: Notation of the split strategy

The duration and time limit of the study did not allow spending enough time on what Wright et al. (2012, pp. 114-116) refer to as “Instruction Phase 2: Consolidating Early Strategies” as mentioned in Chapter 2. Ideally more time should be spent on notating strategies with numbers lines and arrows before formalizing it as suggested for “Instruction Phase 3: Refining strategies and extending tasks” (Wright et.al, 2012, pp.116-120). The formalized notation could be seen as procedural practise instead of as a tool of expressing the construction of conceptual understanding. Wright et al. (2012) ideally want learners to experience notation as “an invitation to partake of shared tools, which can illuminate their own thinking, rather than as an imposition of someone else’s way, which obscures their own thinking” (p.18). In future work with learners I would introduce the more informal ways of notating (e.g. Fig. 25 above and number lines) and encourage learners to develop their own informal ways of notation before introducing more formal notational strategies.

4.7.8 Session 8

See Appendix P for a full session outline.

Focus:

- Subtraction with regrouping (Jump, Split, Jump to the decuple strategies) (SEAL)

Description and reflections:

After doing a few mental CPV activities, the SEAL activity done in Lesson 7 was repeated in the same way, but this time the focus was on subtraction. Bundling sticks were used to demonstrate the different strategies before notating it together. The difference between "split" with regrouping and without (e.g. 43 – 18 and 49 – 18) was reinforced. Some pairs could master only one strategy while others managed more.

4.8 Reflecting on the advantages of mixed ability groups

Before the start of the study, when I was working informally with the class prior to my data collection, learners were grouped according to their performance on the SANC baseline assessment. This allowed me to design extension tasks for certain groups and to accommodate a slower pace for other groups. However, as discussed above, after reading the research conducted by Boaler and others on the dangers of "ability grouping" I was challenged to reconsider my assumptions about the advantages of grouping learners.

I thus decided to opt for mixed ability groups for the entire recovery period. One learner, Sipho, however, was a concern as he consistently tended to rely on counting in ones rather than tens irrespective of the question or number range in questions. (The first interview indicated he was at SEAL stage 2/3 and CPV level 1). Wright et al. (2012) are of the opinion that learners like Sipho are not likely to have success in the CPV tasks until the ability to increment and decrement in tens is established. Because the recovery was not focused on individual instruction his participation in a group exposed him to both SEAL and CPV tasks simultaneously. During the first recovery session he had to add 15 sticks to 33 sticks. For this task he counted the 15 sticks out individually, but he was also looking at what the others were doing. When counting out the 33 sticks he quickly changed his method towards counting in groups (i.e. each bundle as 10) as he had noted from his peers. Thus, in the next task, requiring him to count out more sticks, he seemed very proud to be able to pack out 48 sticks in quicker time by using 4 bundles and 8 loose sticks. In Lesson 2 he also illuminated for me how a mixed ability group enhances the learning of individuals by achieving more than he would have on his own. He counted in tens during Lesson 3 and continued to grow in confidence and ability. After Interview II (8 sessions later) he was placed at SEAL stage 3/4 and CPV level 2.

The influence of mixed ability groups will also be further addressed in the case studies discussed in Chapter 5 below.

Chapter 5: Analysis of data from the two interview assessments – reflecting on the successes and weaknesses of the adapted recovery intervention

Recall from Chapter 3 that although there were 23 learners in this class of Grade 4 learners, the data presented includes only 20 learners' data as 3 learners were absent on more than one day during the assessment periods. The data for all 23 learners were however recorded for the initial SANC four operations baseline assessment (March 2014) as all were present on the day of that assessment.

At the time of the baseline assessment the age of the learners ranged between 8 years and 3 months and 12 years and 10 months. The official age for Grade 4 learners in South Africa is that they should be turning 10 years old through the course of the calendar year (Setati, 2005, p. 453). In this class about a third of the learners (8 in total) were the official average age for the grade, with 9 older than the official average and 6 younger than it. (There were six learners turning 9, eight turning 10, eight turning 11 and one turning 13.) The school does not have a Grade 00 class and this led to some learners enrolling in Grade R when they should have been enrolled in Grade 00. They were then promoted to Grade 1 resulting in several learners (six in this class) younger than the official age for the grade.

5.1 Learner performance on the SANC four operations baseline assessment

Table 6 below summarises the problems posed as well as the frequency of different types of answers given by the class after 50 minutes of writing the four operations assessment (the normal duration of the test is 30 min). The assessment consists of five sums for every operation which become progressively harder from the first to the fifth across each of the four operations (i.e. 20 sums altogether). Space is provided for working and answers.

Table 6: Results for the SANC baseline assessment Total no. of learners: 23

Problem	Frequency of correct answers without written calculation	Frequency of correct answer with written non-tally calculation	Frequency of correct answer by using tallies to count	Frequency of wrong answer without written calculation	Frequency of wrong answer with written calculation	Frequency of wrong answer by using tallies to count	Frequency of not answered	Total number of correct answers	Total number of learners using tallies
3+4	15	8	0	0	0	0	0	23	0
8+6	12	10	0	1	0	0	0	22	0
23+18	6	12	1	3	1	0	0	19	1
55+67	1	2	1	3	11	5	0	4	6
104+97	3	0	0	2	14	4	0	3	4
8 - 2	9	7	5	0	0	1	1	20	6
12-5	4	6	11	0	0	0	2	20	11
23-18	2	0	10	3	5	2	1	12	12
467-43	0	4	2	3	5	6	3	6	8
305-97	0	0	0	3	9	9	2	0	9
2x4	9	1	3	0	1	8	1	13	11
5x3	6	1	3	3	3	6	1	10	9
12x4	0	1	5	9	1	6	1	6	11
24x6	0	0	0	8	2	10	3	0	10
120x5	0	0	0	6	4	11	2	0	11
6÷3	4	2	3	7	2	4	1	9	7
18÷2	3	1	5	4	1	7	2	9	12
24÷3	2	1	6	4	2	6	2	9	12
75÷3	0	1	2	4	1	11	4	3	13
120÷15	0	0	3	4	4	4	8	3	12

- The use of fingers and counting out loud were recorded on observation sheets as far as possible.
- The results in the table reflect only the work done on answer sheets.
- If they used both a written calculation as well as tallies, it was recorded as using tallies.

The learner results of this assessment provided me with a bench mark against which to make decisions regarding the level and focus of my research and the recovery intervention. It was clear from the poor performance on both addition and subtraction (and the predominance of one to one tally counting for all these calculations except the basic addition of single digit numbers) that recovery should be aimed at developing addition and subtraction strategies before multiplication and division could be addressed. Additive and subtractive reasoning is a basis of multiplicative reasoning. Because the purpose of the assessment was to get a baseline sense of mathematical competence of the class, in order to inform the focus of both research and development, this assessment was not redone at the end of the recovery period.

From Table 6 above it is clear that the majority of learners could not successfully complete the addition and subtraction calculations beyond the first two basic problems in each operation. Furthermore in terms of methods used:

- Approximately a quarter of learners used some sort of tally counting to solve 55 + 67 and to solve 8 - 2.
- Approximately half the learners used tallies to calculate 12 - 5 and 23 - 18.
- Some learners did not attempt the calculations with bigger numbers. About two fifths of the learners used tally methods for calculating 305 - 97 and 467 - 43 although unsuccessfully.
- The use of tallying tends to increase with the difficulty of complexity of the problem.

Below are some examples of learners attempting to solve 467 – 43 with tally counting (Fig. 26.1 and 26.2):

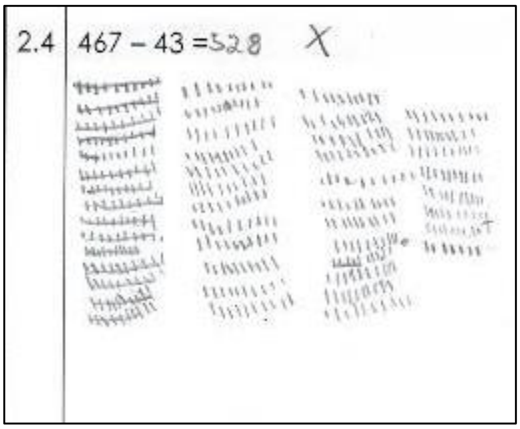


Fig. 26.1 An example of tally counting (SANC baseline assessment)

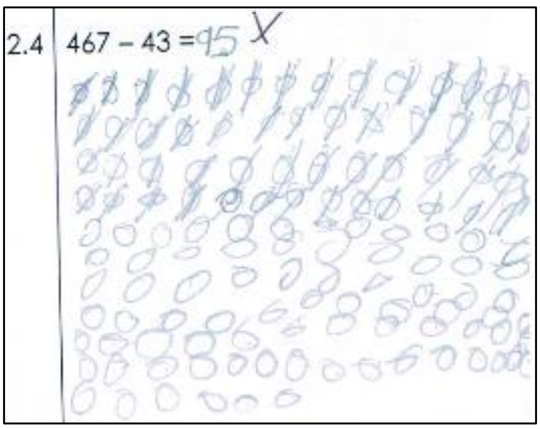


Fig. 26.2 An example of tally counting (SANC baseline assessment)

Fig. 26.1 highlights the phenomenon that when learners use elaborated finger-based counting or tallies to calculate multi-digit numbers, their finger patterns or inscriptions portray a base ten structure but they do not use incrementing or decrementing by ten.

The prevalence of such tally methods, as shown above, links to the national findings of Primary Mathematic Research Project (PMRP) as reported by Schollar (2008, p.7). His research indicated that 38,1% of Grade 5 learners rely exclusively on tally counting. Schollar attributes the excessive use of tallies to the fact that South African learners are not developing an adequate understanding of the base-10 system and place value. Due to the progressive nature of mathematics as discussed in Chapter 2, a lack of understanding of the number system and the mastering of arithmetic operations, beyond tally counting, makes it impossible for learners to develop mathematical proficiency. Because the "development of increasingly complex cognitive abilities is dependent on the construction of conceptual frameworks" (Schollar, 2008, p.5), it is essential to start recovery with the fundamental basics of place value and basic operations.

5.2 Learner performance in the SEAL Interviews

Table 7 gives a summary of shifts in learner performance that occurred between the SEAL interviews conducted in August 2014 and then again conducted in November 2014. The table below shows the frequency of learners who correctly or incorrectly answered each interview task as administered in the way described above. Recall due to absenteeism 20 of the 23 learners participated in these interviews.

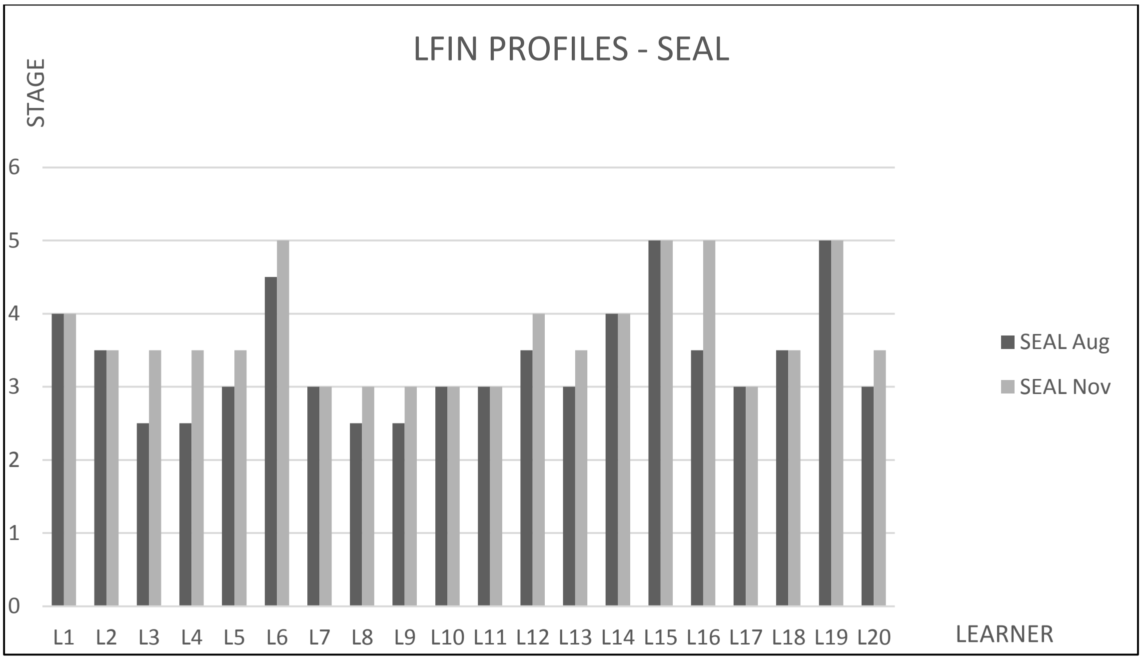
Table 7: Number of learners correct on SEAL tasks across August and November

Task	August	November
1.a. 5 + 4 = _	18	20
1.b 9 + 6 = _	18	20
1.c 8 + 5 = _	18	19
1.d 5 + 2 = _	20	20
1.e 7 + 3 = _	20	19
2. Counters	17	20
3.a 7 + __ = 10	14	18
3.b 12 + __ = 15	13	17
4. 16 – 12 /17 – 14 / 15 – 11 / 16 – 13	13	18
5.a 10 - __ = 6	17	18
5.b 12 - __ = 9	17	16
5.c 15 - __ = 11	18	17
6.a 6 – 2 = _	16	19
6.b 9 – 4 = _	18	19
6.c 15 - 3 = _	17	18
6. d 27 – 4 =	16	16
TOTAL	**270**	**294**
Number of students	**20**	

Table 7 above points to improvement in the accuracy of answers over the three month period across almost all items except item 5b (12 -__ = 9). During the first interviews 17 learners answered this item correctly and during the second interview only 16 learners answered correctly. The results for item 6d (27 – 4) also showed no improvement and stayed the same for both interviews.

The shifts in stages between August and November were noted on LFIN stage profiles and are shown across 20 learners in Table 8 below:

Table 8: LFIN Profiles for August and November – SEAL



From the table we note improvement for 10 out of 20 learners (50%) and stable SEAL stages for 10 out of 20 learners (50%). Of the 10 learners who progressed 7 progressed by a ½ level, 2 progressed by 1 level and 1 progressed by 1½ levels.

Additionally a few interesting strategies were visible during Interview II which were not visible in the first interview. Some examples of these are given below.

Item 1b involved two sets of screened counters (9 and 6 respectively). Learners had to write down on their answer sheets how many there were altogether. At the time of interview I 18/20 learners wrote down the correct answer and during interview II all 20 learners answered this correctly. A few examples of strategies used in Interview II are noted below:

- 9 + 1 + 5 and then said 9 + 1 makes 10 + 5 = 15 (Yonela and Philela)
- 9 + 3 + 3 = 15 (Avukile counting in 3's)
- 9 + 5 = 14 and 14 + 1 = 15 (Limyoli using a known fact)

The same learners applied similar strategies to answer the next question (i.e. 1c. 8 + 5 which also involved two screened sets of counters):

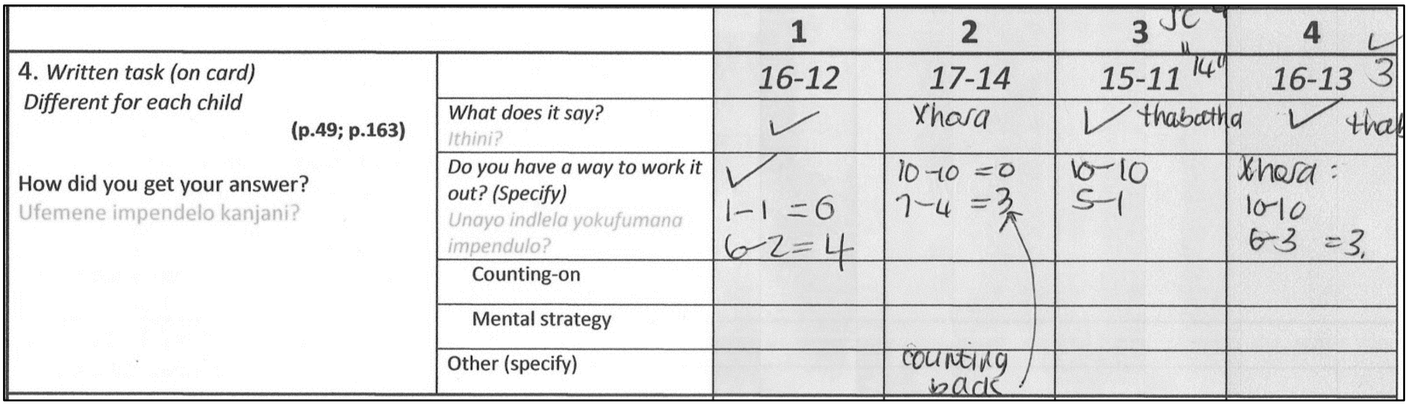
- 8 + 2 + 3 = 10 + 3 = 13 (Yonela and Philela)
- Avukile gave an immediate/mental answer
- 8 + 4 = 12 and 12 + 1 = 13 (Limyoli using a known fact again)

(The fact that Philela could hear the strategy described by Yonela is mentioned in Chapter 4 where they are referred to as Learner 3 and Learner 4 in terms of the response schedule. It is possible that Philela copied Yonela's response, but if so, it is interesting and pleasing that the same strategy was used again by both learners.)

There were only two individually posed questions in my adapted SEAL interview. For Question 2 learners had to count a serpentine of counters packed out in front them. Every learner had to count a different number of counters. During the first interview only three learners did not count their serpentine of counters correctly. All three of them counted by only looking (i.e. they did not use their fingers to count). During the second interview all 20 learners counted correctly by either counting by looking or touching with fingers.

Question 4 was the only other individually posed question in the SEAL interview. Different written number sentences were given to each of the four learners (i.e. 16 – 12 / 17 – 14 / 15 – 11 / 16 – 13). 13 learners answered successfully in Interview I and 18 in Interview II.

Once again learners were asked to explain the strategies used. Fig. 27 below is an example of the way it was recorded for all four learners during a specific interview:



		1	2	3	4
4. *Written task (on card)* *Different for each child* (p.49; p.163)		*16-12*	*17-14*	*15-11*	*16-13*
	What does it say? *Ithini?*	✓	Xhosa	✓ thabatha	✓ tha
How did you get your answer? Ufemene impendelo kanjani?	*Do you have a way to work it out? (Specify)* *Unayo indlela yokufumana impendulo?*	✓ 1-1 =6 6-2=4	10-10 =0 7-4 =3	10-10 5-1	Xhosa: 10-10 6-3 =3
	Counting-on				
	Mental strategy				
	Other (specify)		counting back		

Fig 27: A response schedule example for SEAL (Question 4) – Written tasks (Subtraction)

The strategies recorded for all 20 learners during both sets of interviews are summarized in Table 9 below:

Table 9: Number of learners using each strategy to answer individually posed written subtraction sentences

Strategy	**Interview I (Aug)**	**Interview II (Nov)**
Counting on	0	1
Counting backwards	2	2
Mental calculation	1	3
Finger counting	9	3
Split strategy	5 (3 used fingers too)	11 (2 used fingers too)
Learner unable to identify strategy	3	0

In summary, as indicated by Table 9, all learners were able to identify the strategy they used during the second interview. Six fewer learners used finger counting during Interview II than Interview I and six more used the split strategy than during the first interview. Two learners, however, made calculation errors while using the split strategy. These calculation errors are shown below:

17 – 14

1 – 1 = 0 (not signifying place value, but correct answer)
7 – 4 = 5 (incorrect answer)
So 17 – 14 = 5

And **15 – 11**

10 – 10 = 10 (incorrect answer)
5 – 1 = 4
So 15 – 11 = 14

5.3 Learner performance in the CPV Interviews

The changes/shifts between Interview I and Interview II were somewhat bigger for CPV than for SEAL. This could be attributed to the fact that the increments as part of Wright et al.'s (2006) SEAL assessment were perhaps too easy for this group of Grade 4 learners. Thus most problems could be solved mentally by these learners and that made it difficult to determine strategies used or to see shifts in mathematical strategies used. The two digit addition and subtraction tasks of the CPV interview (Question 4b – 4e) of which some involved regrouping, necessitated learners to employ strategies. These questions were posed individually and learners were asked to explain the strategy they used. All pencils and answer sheets were taken in after the uncovering tasks in order to keep the focus on mental calculation for subsequent activities.

Table 10 below summarizes the results for the CPV Interviews.

Table 10: Assessment result summary: CPV

Task	Interview I Aug		Interview II Nov	
	✓	✗	✓	✗
1.**Counting in tens** ones then tens	12	0	8	0
tens only	8	0	12	0
2. Incrementing by 10	17	3	20	0
3.a **Uncovering** 1. 10	20	0	20	0
2. + 3	18	2	20	0
3. + 20	17	3	19	1
4. + 4	15	5	19	1
5. + 3	16	4	19	1
6. + 10	15	5	17	3
7. + 2	14	6	17	3
8. + 20	15	5	16	4
3.b **Uncovering** 1. 4	19	1	20	0
2. + 10	19	1	20	0
3. + 20	14	6	17	3
4. + 12	14	6	15	5
5. + 25	7	13	10	10
4.a **incrementing by 10** 16 + 10 / 14 + 10 / 15 + 10 / 13 + 10	19	1	20	0
16 + 9 / 14 + 9 / 15 + 11 / 13 + 9	17	3	19	1
4.b **addition without regrouping** 42 + 23 / 33 + 25 / 51 + 24 / 44 + 32	16	4	19	1
4.c **addition with regrouping** 27 + 36 / 38 + 23 / 46 + 25 / 28 + 34	15	5	15	5

4.d **subtraction without regrouping** 67 -52 / 48 – 36 / 56 – 23 / 49 – 24	13	7	18	2
4.e **subtraction with regrouping** 34 – 16 / 54 – 28 / 43 – 15 / 35 - 17	2	18	4	16
TOTAL	**322**	**98**	**363**	**57**
Number of responses	**420**		**420**	
Number of students	**20**		**20**	

Table 10 above points to improvement in the accuracy of answers across all items apart from addition with regrouping (27 + 36 / 38 + 23 / 46 + 25 / 28 + 34) that stayed the same.

During the CPV Interview I about a quarter of the learners relied on finger counting to do addition items like 42 + 35 etc. It was also not possible to record the strategy used by every learner because nearly half of the learners were unable to identify or describe any strategy other than "ngenqondo" (with my brain) or "ndicingile" (I thought). The only strategy recorded, other than splitting the tens and ones and adding/subtracting them separately and adding the totals, was a combination of the split and jump strategy. This involved counting in tens and then adding the ones as shown below.

44 + 32
40, 50, 60, 70 + 4 + 1 + 1 = 76

During Interview II 14 of the 20 learners attempted a strategy like split and could describe the method used. Some even used the terms "split" or "jump".

Strategies, other than split, were recorded during Interview II. For example, a combination of strategies was used by Khanya:

38 + 23
30 + 20 = 50
8 + 3 = 11
50 + 10 = 60
60 + 1 = 61

and Kungawo:

27 + 36
20 + 30 = 50
+ 7 = 57
+ 3 = 60
+ 3 = 63

Other notable addition strategies included the ones used by Luvuse and Zola:

46 + 25
40 + 20 = 60
6 + 5 = 5 + 5 + 1 = 11
60 + 11 = 71

and

38 + 23
30 + 20 = 50
8 + 3 = 11
50 + 10 = 60
60 + 1 = 61

Litha described her strategy for **28 + 34** with the word “split” and said:
“20 and 30 is 50
8 and 4 is 10 and 2
is 62”

Across the class of 20 learners a lack of correct decomposition into place value (split strategy) was seen in about a quarter of the learners. In Ntombi’s case, for example, the split strategy caused confusion:

46 + 25 (Interview II)
4 + 2 = 6
6 + 5 = 11
6 + 11 = 17

Similarly Lutho had difficulties with the strategy as he added tens and ones together:

38 + 23
3 + 3 = 6
8 + 2 = 10
6 + 10 = 16

Graven et al. (2013) also noted this "lack of number sense underlying children's attempts to use taught methods" in their analysis of research done in more than 25 primary schools in the broader Grahamstown and Johannesburg areas. According to them there is no "'echo' of the quantity underlying the digit in the enactment of the algorithm" (p.138).

Table 11 shows a breakdown of strategies used, alongside the success or unsuccessfulness of each strategy, to solve subtraction tasks:

Table 11: Strategies used in CPV subtraction tasks in August and November

	Interview 1 Aug		**Interview II Nov**	
Subtraction (not requiring decomposing of the tens) 67 – 52 / 48 – 36 / 56 – 23 / 49 - 24	✓	✗	✓	✗
Counting in ones/fingers	4	2	1	0
Split	7	4	17	2
Jump	1	0	0	0
Writing on desk with finger	1	0	0	0
Confuse + and -	0	1	0	0
Total	**13**	**7**	**18**	**2**
	20		**20**	
Subtraction (requiring decomposing the tens) 34 – 16 / 54 – 28 / 43 – 15 / 35 - 17	✓	✗	✓	✗
Counting in ones /fingers	1	1	1	1
Split	0	14	1	15
Split/Jump mix	0	0	2	0
Use known fact	1	0	0	0
No attempt	0	1	0	0
Confuse + and -	0	1	0	0
TOTAL	**2**	**18**	**4**	**16**
Number of students	**20**		**20**	

The table above shows that the biggest shift occurred in learners being able to use the split strategy (that is splitting the tens and units) successfully for subtracting numbers that do not require decomposing the tens (i.e. regrouping). The use of the split strategy was less successful when learners were further required to decompose the tens (e.g. 43 - 15 requires decomposing 10 in order to enable 3 - 5 to become 13 - 5). On the other hand two learners successfully managed the split jump mix in the second interview.

In my study learners particularly battled to use the split strategy for problems involving regrouping. 13 out of the 20 learners attempted the subtraction with regrouping problems by trying to subtract the smaller number from the bigger one, for example learners explained this method:

54 - 28
50 – 20 = 30 / 5 – 2 = 3
8 – 4 = 4 (instead of 4 – 8)
54 – 28 = 34

or explained it like this:

34 - 16
3 – 1 = 2
4 – 6 = 2 (instead of - 2)
34 – 16 = 22

This tendency to simply subtract smaller numbers from bigger numbers instead of decomposing of the tens was also noted across several classes within the SANC teacher development community (NICLE) (Graven, 2014, p.14).

One learner in my study counted down unsuccessfully, another could not identify her strategy, and two confused the place value of the various digits as shown below:

43 - 15
3 – 1 = 2
5 – 4 = 1
"So 21"

and **54 - 28**
5 – 2 = 3
4 – 8 = 0
"So the answer is 30"

On the other hand Luvuse's method (Interview II) showed independent thinking in that she came up with her own strategy:

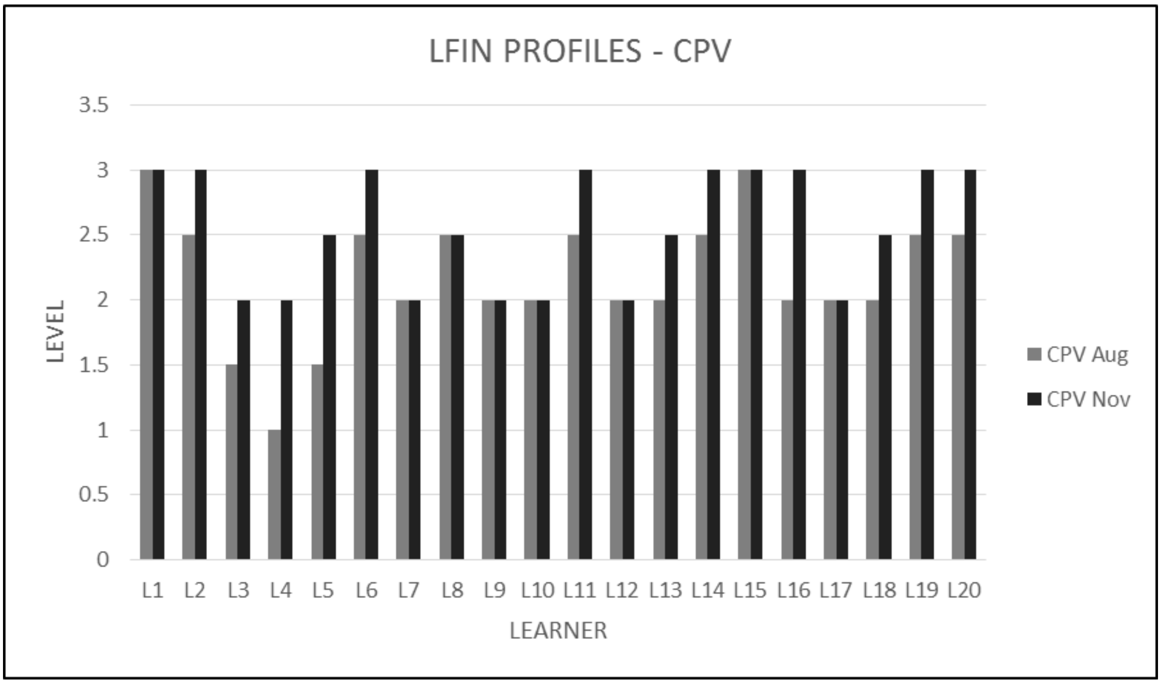
34 - 16
30 – 10 is 20
4 – 6 is 2 less
20 – 2 is 18

And Vuyo, discussed as a case study below, similarly came up with a strategy never used during the recovery period:

34 - 16
30 – 16 = 14
14 + 4 = 18

The CPV LFIN profiles for Interview I and Interview II for all learners are summarised in Table 12 below.

Table 12: LFIN Profiles for August and November – CPV



From Table 12 we note progress in CPV levels by at least half a level for 12 out of 20 learners (i.e. 60%) while levels remained stable for 8 out of 20 learners (i.e. 40%). Of the 12 learners who progressed 9 progressed by a ½ level and 3 progressed by 1 level.

5.4 Summary of findings from SEAL and CPV tables and graphs above

Following the analysis above the following can be noted. The CPV progress was greater than the SEAL progress. Although there were not huge jumps in the SEAL stages or CPV levels after eight group based recovery sessions (most jumps were ½ stage/level), there is a pleasing general movement away from tally/counting by ones. Learners also started to use more strategies like the split strategy and even strategies they came up with themselves.

Although both split and jump strategies were practised during recovery, nearly all learners favoured the split strategy although most learners could not successfully solve regrouping tasks using the split strategy. Wright et al. point out that learners do not develop jump strategies without instruction, but when they do use jump they tend to make fewer errors than when they use split. “Split appears easy to start but hard to master, whereas jump is harder to start but easier to master” (2012, p.116).

It is interesting to note that the move away from tally/counting by ones relates to the results of Annual National Assessment (ANAs) written by this class in September 2014. On analysis of learner scripts I noted that only Thandi, the learner who was absent repeatedly, was using tally counting in the ANAs as shown in Fig. 28 below:

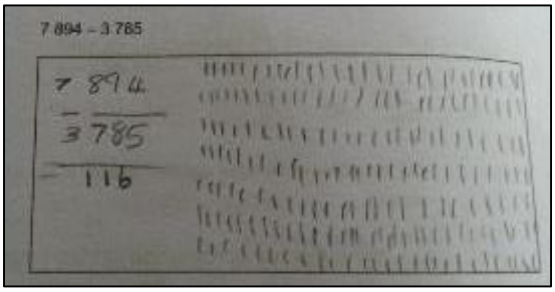


Fig. 28: An example of Thandi using tally counting in the ANAs (September 2014)

The average ANA mark for this class was 23/50 (46%) which compared favourably to the provincial average for Grade 4 in the Eastern Cape in 2014 (32%) (DBE, 2014).

Above I have summarized the shifts of all twenty learners. However in order to enrich the description of the process of assessment and recovery followed during my study, I provide case study vignettes of two learners.

5.5 Case study vignettes of two learners

Although data of all 23 learners were recorded throughout the study, vignettes of two case learners will illuminate more nuanced and important aspects of the research process and findings. The advancement of two learners from the pre-research stage to the post-research interview in terms of progress, strategies used, observations, challenges and group dynamics will be shared for this purpose.

The two learner case studies were chosen from differing levels of mathematical knowledge and for differing behavioural patterns and genders. Vuyo (a pseudonym) is a boy selected from students from the higher range of competency. Nothemba (a pseudonym) is a girl selected from students from a lower range of competency. The case study vignettes draw on a wide range of data that enables rich and thick (Maxwell, 2003) description. Because of my involvement for numerous months stretching from March 2014 to November 2014, repeated observations could be made which were accompanied by notes taken during assessments/interviews/recovery sessions. These were triangulated with video recordings and results of assessments (written baseline assessment, LFIN interviews, ANA results) and written examples of students' work. This process of triangulation allowed me to contrast different sets of data in order to give a sense of authenticity to a potentially subjective observation (Koshy, 2005).

5.5.1 The case of Vuyo

At the time of the baseline assessment, Vuyo was 10 years and three months old.

5.5.1.1 Vuyo's baseline assessment

Background data regarding Vuyo was gathered by speaking to Vuyo's class teacher as well as his Grade 3 teacher. This data indicated that Vuyo was facing various challenges in his impoverished life. Not only did he suffer from frequent headaches, but he also lived with his aunt who was still at school while his father worked on the mines in Gauteng and his mother cared for her elderly mother in the rural areas of the province, several hours away.

During the original written baseline (four operations) assessment Vuyo scored 14 marks out of a possible 20. The average for the class was 8,7 out of 20. Vuyo thus scored way above the average for his class.

He answered addition problems without any written calculation as shown by the examples from his written scripts of the baseline assessment (Fig. 29):

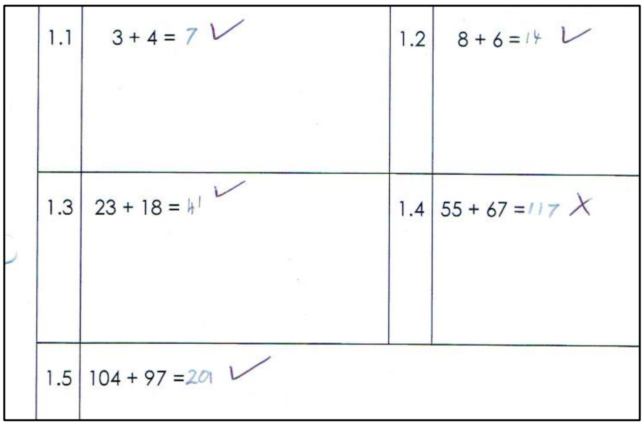


Fig. 29: Examples from Vuyo's baseline assessment (Addition)

Despite the fact that Vuyo achieved the highest score in the class, and his seeming proficiency in addition, he made use of tally counting to solve the subtraction problems 467 – 43 and 305 – 97. It is interesting to note that, when counting the tallies for 467 – 43, his answer was 427 which is relatively close to the correct answer of 424. When calculating 305 – 97 he rubbed out the tally lines and came to the answer of 145 instead of 208. Here he was also observed counting out loud (Observation sheets, 19 March 2014).

Fig. 30 below shows Vuyo's responses to these subtraction items.

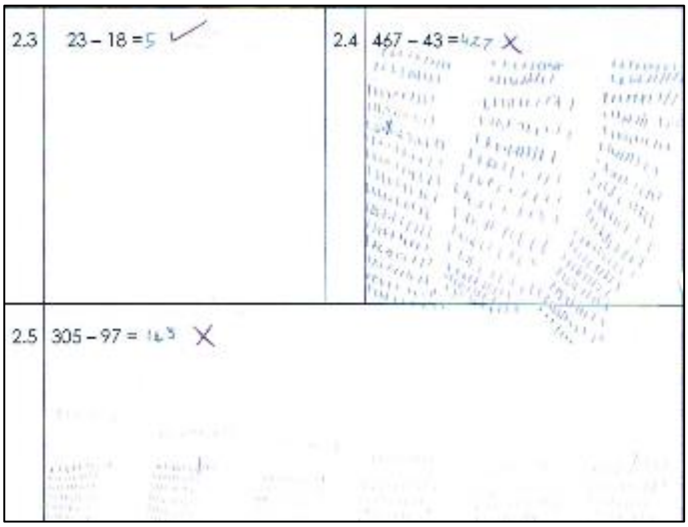


Fig. 30: Examples from Vuyo's baseline assessment (Subtraction)

Vuyo's work here indicates a lack of understanding of the base-10 system as a unit that can be used for calculation even while he marks his units in groups of 10 (i.e. 10 tally lines in each group of lines).

For multiplication Vuyo answered 2 x 4 and 5 x 3 accurately mentally. He used tally counting, however, to correctly answer 12 x 4 by drawing twelve groups of four tally lines as shown in Fig. 31 below:

Fig. 31: An example from Vuyo's baseline assessment (Multiplication)

Similarly he used tally marks to calculate 24 x 6 and 120 x 15, although he was not successful in doing so with larger numbers. On the other hand Vuyo was more successful in his attempt to use tally counting with division as shown in Fig. 32 below.

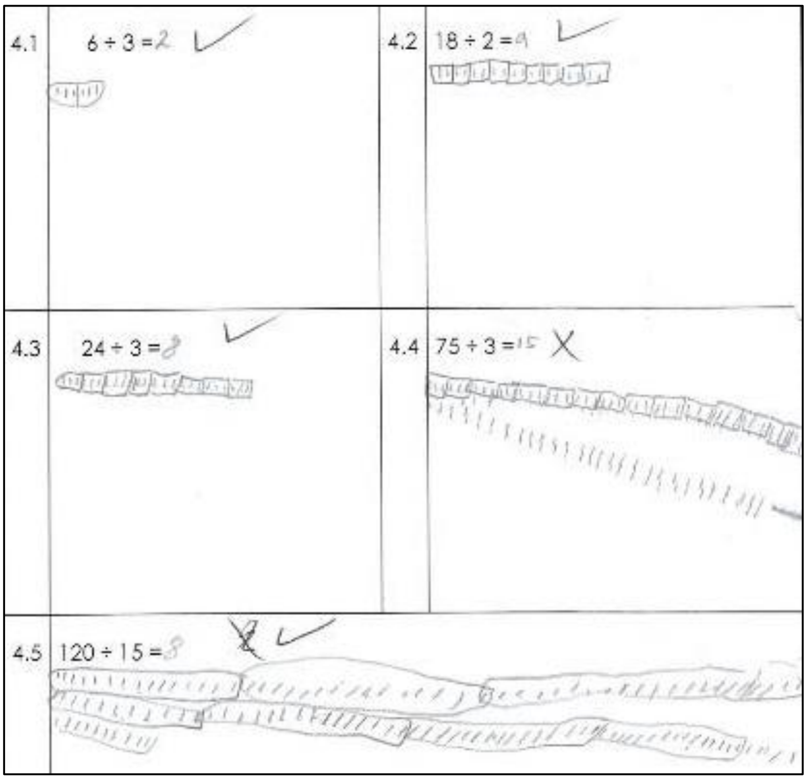


Fig. 32: Examples from Vuyo's baseline assessment (Division)

Vuyo's method for division involved concretely representing the first number (dividend) and then using circling to group the tallies based on the divisor and then counting how many groups of the divisor can be made to give the answer. He consistently applied division in a quotative sense. Thus, from the above, it is evident that Vuyo has a relatively good understanding of the concepts of multiplication and division (and the use of grouping strategies for these), however, his level of mathematical working was still dependent on concrete representation. Based on his score in this assessment Vuyo was originally placed in the group with the top six learners identified by this assessment. During group activities he displayed good mental math strategies for calculations with smaller numbers.

5.5.1.2 Vuyo's LFIN interview I

SEAL - Interview I

Vuyo was able to supply mental/immediate answers for screened addition questions, missing subtrahend questions, screened subtraction problems and missing addend questions (Questions 1, 3, 5 and 6). He correctly solved the subtractive task 17 -14 by counting down from 17 to 14.

As a result of Vuyo's responses, he was placed at SEAL Stage 3/4. This indicates a combination of *initial number sequence* (Stage 3) and *intermediate number sequence* (Stage 4). Although Vuyo was able to do quick additions and seemed to use non-count-by-one strategies and he could count-down-from, he relied on his fingers to keep track of counting when adding the two screened sets of counters 9 and 6.

CPV – Interview I

For Question 1 (counting by tens with strips) Vuyo each time counted the first strip of the first three questions in ones and then continued by counting the number of strips in tens. As mentioned in Chapter 4 and illustrated by Fig. 12 in Chapter 4, Vuyo's strategy was determined by the pencil markings on the blank dots on his answer sheet. (A pencil mark in every blank dot indicated counting in ones and empty dots indicated counting in tens). This assumption was confirmed by the video recording as shown by the screen shot in Chapter 4 (Fig. 13).

Vuyo did the first Incrementing by 10 task (Question 2) easily and fast. Similarly Vuyo completed the first uncovering tasks (Question 3a) by accurately counting in ones and tens. This task, like 3b, involved a board with columns containing ten dots and columns containing less than ten dots. Two screens are used to cover the boards. Sections indicated by arrows are revealed progressively while the previous section is re-screened and the learners had to write down the total number of revealed dots. Fig. 33 below shows the board:

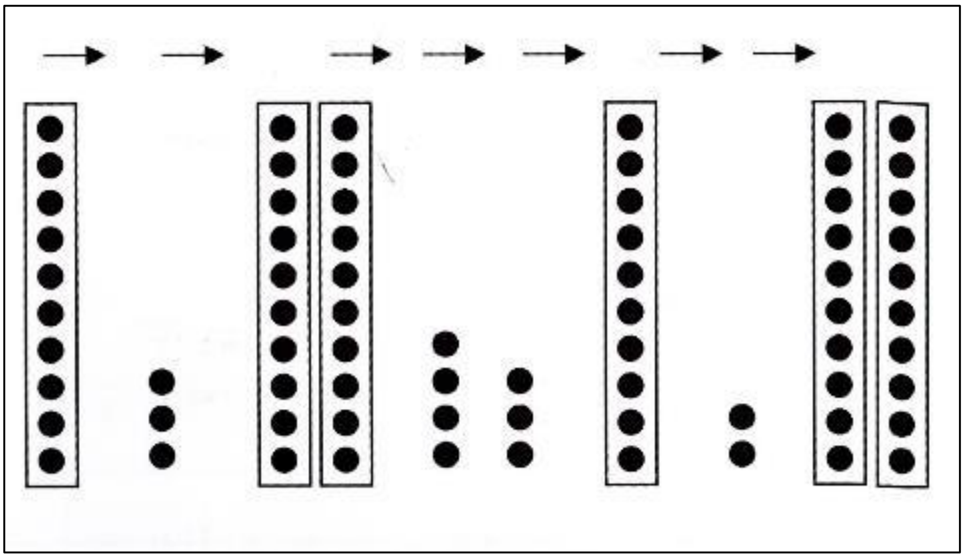


Fig. 33: Uncovering task board CPV (Question 3a)

Although Vuyo counted in ones and tens during Question 3b as well, he counted the first 14 dots as 13 and that caused further calculations to be incorrect (though correctly followed on). He also added 13 instead of 12 and then 20 instead of 25.
Fig. 34 shows the board:

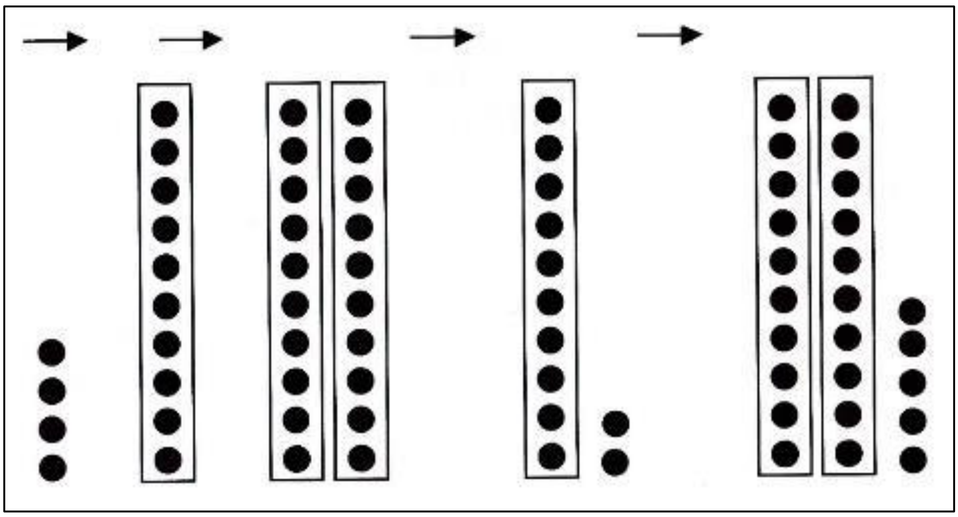


Fig. 34: Uncovering task board CPV (Question 3b)

Question 4 presented the learners with written problems involving two-digit addition and subtraction. The tasks were printed on individual cards and no pencils or paper were available for calculation. Each learner in the group had to solve a different task. The tasks posed were:

4a	16+10	14+10	15+10	13+10
4b	42+23	33+25	51+24	44+32
4c	27+36	38+23	46+25	28+34
4d	67-52	48-36	56-23	49-24
4e	34-16	54-28	43-15	35-17

The first column of tasks are the ones used by Wright et al. (2006) and the following three columns show the additional task I used for every group member. They were all set within the same number range of the original questions. Vuyo was given the tasks in the first column.

Vuyo could use the split strategy to solve 4b. That is:

42 + 23
40 + 20 = 60
2 + 3 = 5
60 + 5 = 65.

However 4c 27 + 36 was more challenging. He used the split strategy again and first said "53" as an answer and then used his fingers for 7 + 6, self-corrected and gave the correct answer of 63.

He unsuccessfully attempted a split strategy to solve 4d. 67 – 52 (He said "60 – 50 equals 10 and 7 + 2 is 9, so 19"). He said 4e. 34 – 16 was 16, but then self-corrected and said 18. When asked how he knew that, he said he knew 16 + 16 was 32. He thus successfully used a known fact to arrive at the correct answer.

He was one of only two learners in his class who were able to answer the subtraction with regrouping question correctly. The other learner who answered correctly was counting down.

Vuyo was placed at CPV Level 2 (Intermediate concept of 10) indicating his use of 10 as a unit of 10 ones.

5.5.1.3 Vuyo's participation in the recovery sessions

Vuyo was placed in a mixed ability group for the recovery period. Because of absenteeism and other factors within the classroom the composition of the groups varied from week to week. It was interesting to note that Vuyo's contribution to the group varied as well. Some weeks he was really involved, motivated and enthusiastic, which was visible in the way in which he contributed his ideas and partook in activities. Other weeks he was withdrawn and quiet. Apart from severe headaches he suffered (as communicated to me by him and his teacher), it was clear that the presence of another high functioning, but more outspoken boy in the group had an inhibiting effect on Vuyo's motivation to contribute and volunteer strategies and solutions. The more outspoken learner was absent for Session 6 and this seemingly allowed Vuyo to regain his confidence and volunteer answers again. When this became apparent I ensured that group allocation was more deliberate to avoid this negative effect of this particular dynamic on this pair of learners.

When given the opportunity to work without a partner on more complex tasks than the ones posed to the rest of the group, Vuyo accepted the challenge with quiet confidence and success.

Vuyo furthermore managed to master more than one non-counting strategy.
For example to solve 54 – 37 he could use the jump strategy and notate it on a number line as shown below in Fig. 35:

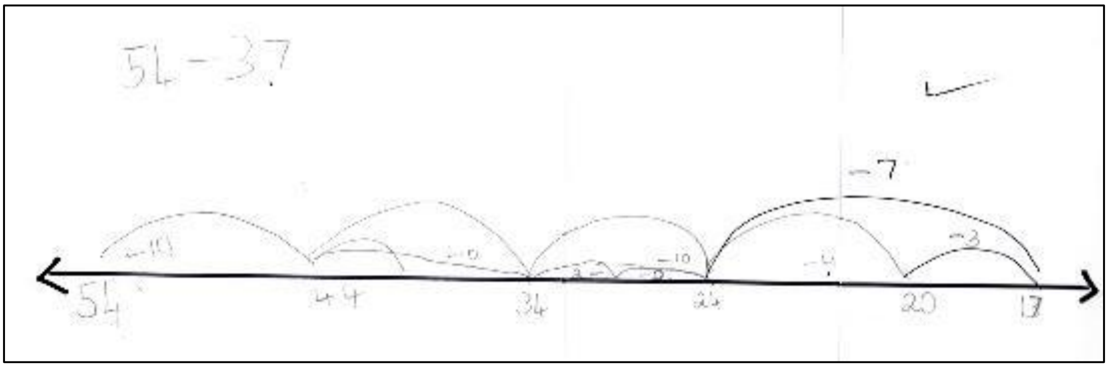


Fig. 35: Vuyo's numberline notation of 54 -37

Additionally he could use and notate the split strategy to do subtraction with regrouping as well as shown in Fig. 36 below:

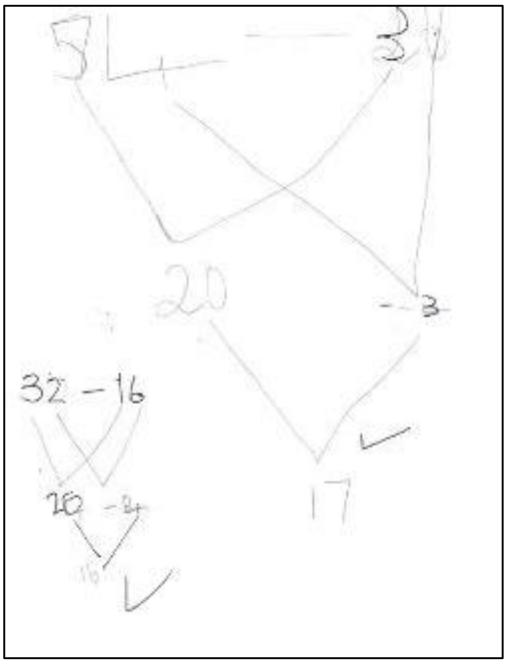


Fig. 36: Vuyo's line notation of 54 – 37 and 32 – 16

After attending one of the recovery group sessions I ran, the class teacher decided to practice the split and jump strategy with the class as well. She marked the learners' written work herself. In his written work in this lesson with his class teacher Vuyo used yet another way of notating his use of the split strategy as shown below in Fig. 37 (the markings are those of his teacher):

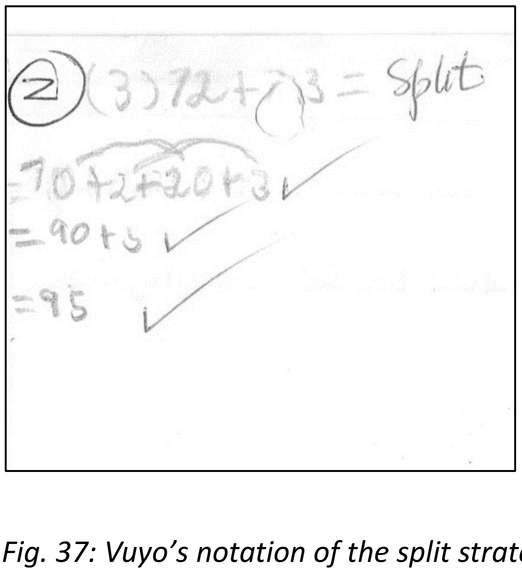


Fig. 37: Vuyo's notation of the split strategy for 72 + 23 according to a task set by his teacher

In the same lesson, Vuyo also attempted the same task in a different and interesting way, although it was not recognised as a possible solution by the teacher. In this working he used transformation to change the original numbers from 72 + 23 to 68 + 27 (even though it would likely have made more sense to change it to 70 + 25). He then split the numbers and calculated 60 + 8 + 20 + 7 mentally to get 90 + 5 = 95 as shown below in Fig. 38:

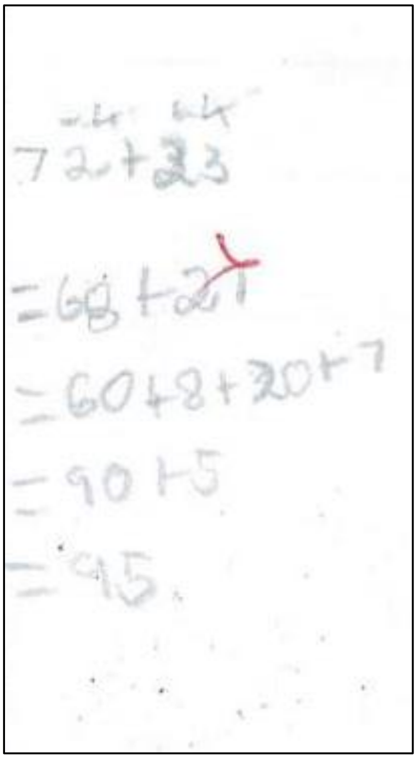


Fig. 38: Vuyo's notation for his alternative strategy to solve 72 + 23

Vuyo's tendency to increasingly find alternative strategies (and in most cases more efficient strategies) appeared to be a strong motivating and challenging factor for other learners in the mixed group. In many cases this resulted in learners with lower levels of mathematical competency arriving at solutions they would normally not be able to manage. During Session 6, for example, numbers were posed as a written numeral on flashcards. Before flashing a number, I instructed them what to do, for instance "Add 10" or "Tell me what is 100 less". Increments and decrements were on the decuple, off the decuple, in the hundreds and hurdling 1000. Sometimes I posed flashcards to everybody separately and sometimes I posed them to the group as a whole. As soon as Vuyo started to answer quickly and often even jumping out of his chair to answer first with enthusiasm, Lutho began to try to answer first as well. I noted in my journal reflection on 14 October 2014 that Vuyo "was on fire" and "on the roll". Vuyo inspired quicker responses from several learners. I had to later ask some of these learners to stop answering in order to give quieter group members a chance. I also wrote in my journal "great that that was a problem!"

5.5.1.4 Vuyo's LFIN Interview II

SEAL – Interview II

During the second SEAL interview Vuyo spoke quietly and said he had a headache. He was, however, able to supply mental answers for all tasks posed.

Vuyo was placed at SEAL Stage 5 (Facile number sequence) compared to 3/4 after the first interview. This was as a result of the fact that he demonstrated a range of non-count-by-one strategies and was able to use known results in calculations. For example during Interview II he did not use his fingers to add the screened counters 9 and 6 as he did during Interview I, instead he answered immediately. He also counted the serpentine in two's compared to counting in ones during Interview I. He used counting back to solve 17 – 4 in Interview I and instead gave a mental/immediate answer in Interview II.

Conceptual Place Value – Interview II

Although there were four different sets of colour coded individual questions for every group, Vuyo ended up getting the same questions as during the first interview. This enabled a direct comparison. For Question 1 (Counting by tens with strips) Vuyo did not count in ones at all. He counted the number of strips and multiplied by 10. Incrementing by 10 (Question 2) was also done easily and fast. Uncovering tasks (Question 3) were done fast and accurately.

Once again he could use the split strategy to solve 4b. 42 + 23, but also managed to give an alternative method of jump:

42 + 23
42 + 20 + 3 = 65.

During the first interview Vuyo used fingers to add 7 and 6 while solving 4c. 27 + 36. As indicated earlier, the short recovery period and the challenges of grasping strategies, resulted in a focus on mainly the split and jump strategy during group sessions. Vuyo, by the time of the second interview came up with additional strategies like split-jump (Wright et al., 2012, p. 100) as shown in the example from his Interview II below.

27 + 36
20 + 30 = 50
50 + 7 = 57
57 + 3 = 60
60 + 3 = 63

During the first interview he could not solve the task 4d 67 – 52 while during the second interview he accurately used the split strategy as shown below:

67 – 52
60 – 50 = 10
7 – 5 = 2
10 + 2 = 12

During the first interview Vuyo used the known fact 16 + 16 to solve 4e. 34 – 16 while during the second interview he said:

34 – 16
30 – 16 = 14
14 + 4 = 18

In this case he used the strategy Wright et al. (2012) refer to as compensation. This is interesting because this was not a strategy that was ever discussed during the recovery and group sessions.

Vuyo was placed at CPV Level 3 (Facile concept of 10) compared to Level 2 (Intermediate concept of 10) after the previous interview. He was now able to do addition and subtraction with tens and ones without the use of materials and he could add and subtract written number sentences with tens and ones.

While the ANAs do not lend themselves easily to noting SEAL and CPV levels Vuyo did perform well in the 2014 ANAs on 19 September 2014 (i.e. after four recovery sessions). Vuyo scored 64% on the Annual National Assessment (ANA) which was 22% above the average of 46% obtained by his class. The Eastern Cape provincial average for Grade 4 Mathematics in the

2014 ANAs was 34,8% (DBE), 2014.

5.5.2 The case of Nothemba

At the time of the baseline assessment Nothemba was 10 years and 8 months old.

5.5.2.1 Nothemba's baseline assessment

During the original written baseline assessment Nothemba scored 4 marks out of a possible 20. These marks were obtained from correctly solving 3 + 4; 8 + 6; 8 – 2; and 12 – 5. This was the lowest score in the class. The average for the class was 8,7 out of 20.

As described in the methodology section this four operations written assessment was done in the classroom context in March 2014. During the assessment Nothemba used tally counting on a loose piece of paper she had hidden under her desk. She also tried to see what other learners were writing and even asked others for answers. This was noted in my observation notes made on 19 March 2014 during the baseline assessment as follows:

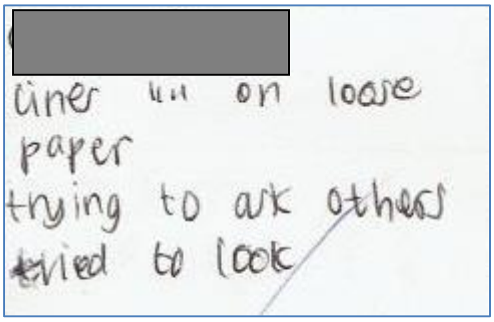


Fig. 39: Example of observation notes made during baseline assessment (Real name covered with text box).

Because I did not know the names of all 23 learners at this stage, I gave the learners name tags to wear to enable me to address them directly by name and to make individual observation notes like the ones showed above.

On her written answer script a range of tally counting was visible across several questions. Thus for example, she tally counted 1.4   55 + 67 as shown in Fig. 40:

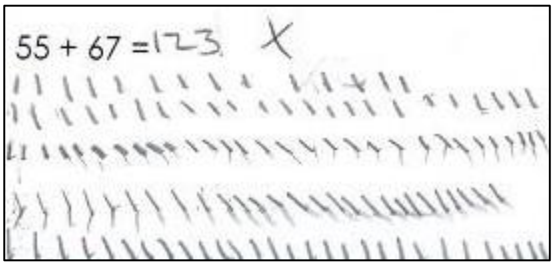


Fig. 40: An example from Nothemba's baseline assessment (Addition)

Of interest is to note that her answer was only off by one.

On the other hand she attempted to do 305 – 97 by using the vertical algorithm but copied the problem incorrectly (subtracting 997 instead of 97) and also switched the order of the unit subtraction to subtract the smaller unit number from the bigger unit number (i.e. 5 from 7) as shown in Fig. 41:

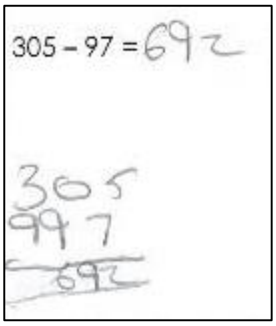


Fig. 41: An example from Nothemba's baseline assessment (Subtraction)

Although Nothemba used tally counting to solve multiplication and division problems, she did not manage to answer any of the 10 problems correctly and showed little understanding in her answers and representations, as shown below in Fig. 42 and Fig. 43:

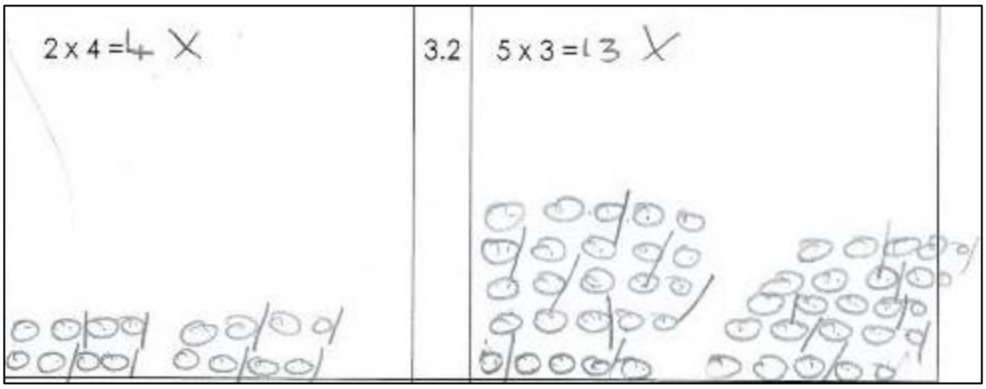


Fig. 42: An example from Nothemba's baseline assessment (Multiplication)

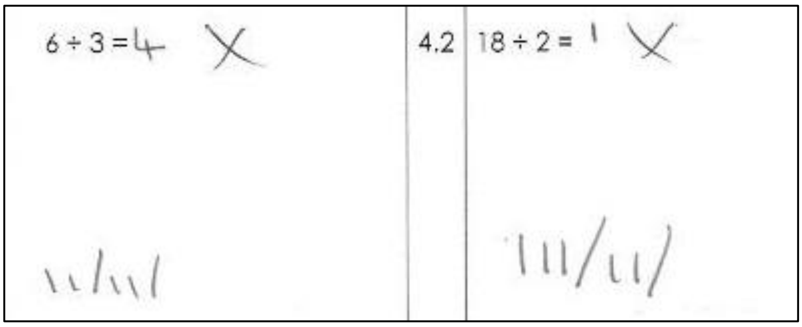


Fig. 43: An example from Nothemba's baseline assessment (Division)

While there is some evidence of appropriate grouping in the multiplication problems (i.e. we can see vertical lines separating groups of 2 and 3 in 2 x 4 and 5 x 3 respectively) she did not seem to know how many of these would be relevant to the answer nor how to derive an answer from her graphical representations. The division problems' tallies show little evidence of any correspondence with the questions indicating little understanding of what the operation means even in concrete terms.

5.5.2.2 Nothemba's LFIN Interview I

SEAL – Interview I

Nothemba's behaviour in the group assessment situation compelled me to make some changes to the assessment set-up as discussed in Chapter 4. She was, for example, trying to look at the answer sheets of other learners. She was also inclined to shout out answers and would sometimes laugh at the answers of others. She did not cope well with the group assessment situation as she was easily distracted. Fig. 44 shows Nothemba during the first interview. She can be seen standing and counting on her fingers. Fig. 44 also shows the beer boxes used as dividers which limited not only my view, but also that of the camera. Because seat no. 5 could not be seen on the video recording, the group size was subsequently reduced to 5 to exclude seat no. 5. After this assessment the setup was changed to four learners using sheets of cardboard to cover up their answers. The learners were also then sitting on both sides of me as mentioned in Chapter 4.

Fig 44: Nothemba standing and counting on her fingers during SEAL Interview I

Nothemba used her fingers to calculate problems like 9 + 6 and 8 + 5 (Question 1) as seen in Fig. 44 above. She counted from one to correctly find the missing addend in 12 + [ ] = 15 (Question 3). When posed with the written task 17 – 13 (Question 4) she started counting backwards from 13 (“13, 12, 11…”) and then realised that she will not get an answer. She then kept quiet for a while and then surprisingly gave the correct answer of 4. When prompted to describe the strategy used, she simply said “Ngengqondo” (With my brain).

In relation to the above Nothemba was placed at SEAL Stage 2/3, at the borderline between figurative counting and initial number sequence.

Conceptual Place Value – Interview I

Because of behavioural issues during the previous interview, Nothemba was interviewed in a group of only 3 learners and she was placed on her own on my side with the other two learners on my other side. The screenshot taken from the video (Fig. 45) shows Nothemba during the interview. She is busy with Question 2 (mentioned below).

Fig 45: Nothemba (seated on left) during CPV Interview I

On this recording Nothemba is often heard trying to give answers to other learners or giggling when others are giving answers. I also often look at her and say "Shhh" or "Thula" (keep quiet).

For Question 1 (Counting by tens with strips) Nothemba counted all the dots in ones for the first three tasks (1a, b and c) and then continued by counting the number of strips in tens for 1d (see Fig. 46 below for 1c as unit counting and 1d in groups of 10).

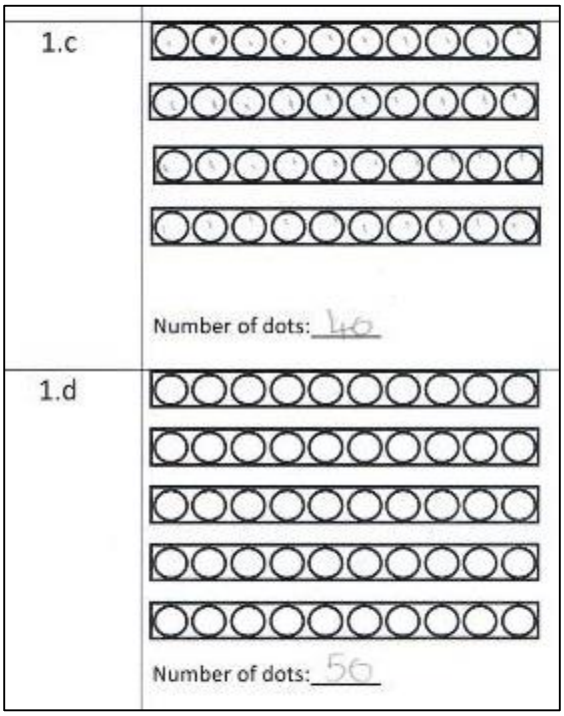


Fig. 46: An example from Nothemba's CPV answer sheet showing unit counting and counting in tens

Incrementing by 10 (Question 2) was partly done correctly although she corrected her answer once after being prompted by another learner. This was the only time she was able to count in tens and I had to keep in mind that she was not the first learner to answer. She could therefore hear the correct answers given by others. When she said "4, 14, 24, 34, 44, 45" one of the other learners the said "No!" and then Nothemba continued by saying "54, 64, 74". She was thus able to self-correct following the other learners' prompting that she made an error.

The uncovering tasks (Question 3) were done by counting in ones and tens and using her fingers. She was successful in most cases but could not solve problems like 13 + 20 and adding 24.

4b. 33 + 25 was successfully done by writing the vertical algorithm with her finger on the desk and using a split strategy. When posed with 4c. 38 + 23 she requested a pencil and when she was not given one, she was not able to supply an answer at all. She kept saying "Ndisacingela" (I am still thinking). She was not able to solve any of the subtraction tasks correctly and her strategies were unclear.

Nothemba was placed at CPV Level 1/2 (Initial concept of 10/Intermediate concept of 10). She did not see 10 as a unit of 10 ones in all tasks and was counting forwards and backwards by 1.

5.5.2.3 Nothemba's participation in the recovery sessions

Nothemba was placed in a mixed ability group for the recovery program. She was still relying on tally counting as a strategy or attempted to write on the desk with her finger. She was not always paying attention and her behaviour was challenging at times in that she was shouting out answers, moving around from her chair and was talking while instructions were given. Observational notes included comments like: "Not paying attention" (26 August 2014) and "Behaviour!" (16 September 2014). She was, however, always eager to attend group sessions and contribute.

During mental exercises involving adding increments of tens and hundreds (Sessions 3 and 4) it was clear that she could not hurdle 1000 with ease. She would, for example, say that 900 + 200 = 2 000. I noted (Session 3) that the learners in general found it easier to cross 1000 in hundreds than in tens. The nature of the activity allowed for differentiation to challenge stronger learners to work with bigger increments and hurdle thousands and allow others, like Nothemba, to work in smaller increments at first. It was also noted that the task 994 + 10 (posed with screened dot strips with the 10 briefly shown) was confusing for all four groups. Learners could, however, come up with a solution once the task was notated/written down.

Through the course of Session 2 and 3 it became clear that learners like Nothemba could not use the jump strategy successfully without using their fingers. For example:

58 + 35 could be done as:

58 + 30 = 88
88 + 2 = 90
90 + 3 = 93

However, due to the fact that Nothemba and the majority of other learners did not automatically know that 88 + 2 = 90, they preferred to count 88 + 5 on their fingers. To enable them to speed up the process, bonds of 10 were practised in Session 4 by means of various games (See Appendix I) like Pop/Fizz and making pairs of 10 with playing cards. Nothemba relied on her fingers to find bonds of 10.

Because of absenteeism and arrangements with the class teacher, Nothemba was in a different group than normal for Session 5. It was interesting that the specific change in group selection seemingly had a demotivating effect on her. I therefore noted that for future sessions I should not repeat the specific group combination (Journal entry, 30 September 2014).

Nothemba's group, in Session 5, found it challenging to jump to the next decuple by adding a single digit number to a double digit number for problems such as 46 + 7 or 67 + 6. The desired strategy would be to say 46 + 4 = 50 and 50 + 3 = 53 (SEAL Activity 6). Ideally learners should be able to jump to 50 by adding on one digit numbers for problems such as 42 + __ = 50 (SEAL Activity 5) For 45 + ___ = 50, for example, Nothemba shouted out guesses like "5!" or "10!".

A note from my journal reflection on Session 6 reads "It doesn't matter how I change the groups, I am afraid the one with Nothemba is always the most challenging of the day!" (Journal entry, 14 October 2014). Nothemba was, however, always eager to participate, but she did not have the self-control to wait for her turn and not to shout out answers when other learners were directly addressed. She also struggled to stay seated in her chair.

Session 7 entailed the practise of the strategies jump, jump to the 10 and split. Learners were working in pairs to discuss and practise the various strategies with written tasks like 37 + 15. For this activity Nothemba was paired with a girl, Thandi, generally weaker than herself who had been absent often due to personal issues at home (personal communication with her teacher). It was clear that this particular pair of learners was confused by the different strategies and mixed up the various parts of written examples.

They could not master the jump strategy and a number line did not help them to understand the process involved in jump as illustrated by Fig. 47 below:

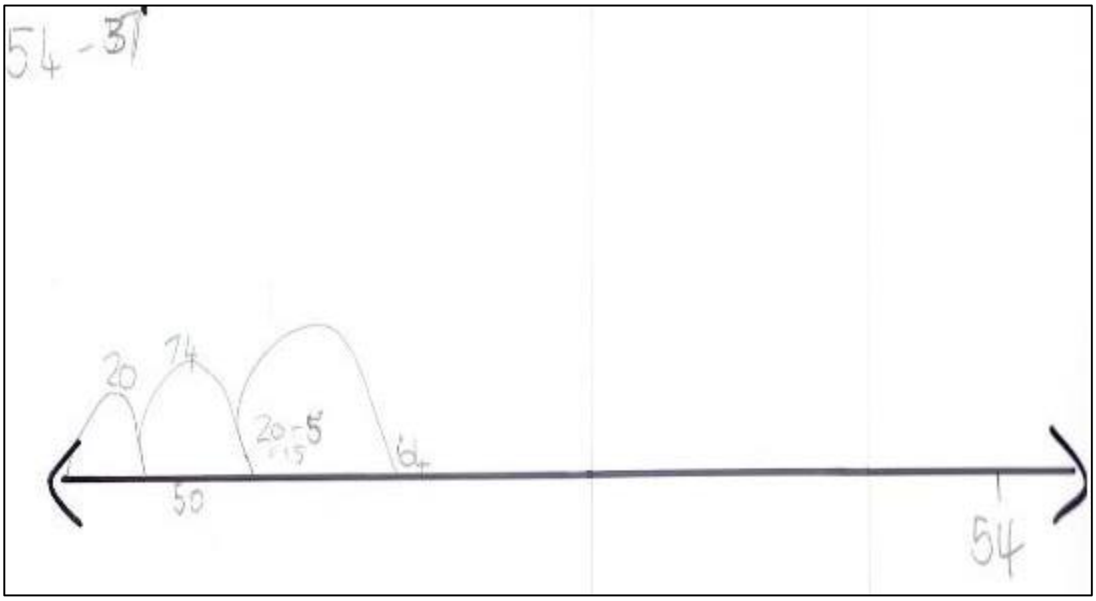


Fig. 47: Nothemba and Thandi's number line notation for 54 – 37

I then suggested that they practise only the split strategy. Fig. 48 shows their attempt to solve 54 – 37. They notated their strategy with an informal line/arrow chart. Note that they copied it as 54 – 73 at first. This example was completed with help from me. They could, however, use the strategy with bundling sticks and without notation.

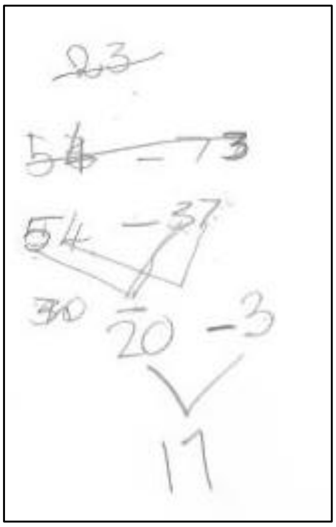


Fig. 48: Nothemba and Thandi's line notation for 54 – 37

Although Nothemba and Thandi needed bundling sticks as concrete materials to grasp the task at hand, Nothemba enjoyed being the stronger learner of the two. Thandi, for example, tended to add the tens, add the ones and then would forget to add the two answers together. This pair would have benefited from spending more time on what Wright et al. (2012, pp.

114–116) refer to as “Instruction Phase 2: Consolidating Early Strategies” by focusing on notating strategies with number lines and arrows before formalising it as suggested for “Instruction Phase 3: Refining strategies and extending tasks” (Wright et al., 2012, pp. 116 – 120) as mentioned in Chapter 2.

Because this was the session in which Nothemba cooperated the most she was paired with Thandi again in the next session (Session 8).

During the exercise posed and marked by the teacher (72 + 23), as mentioned in Vuyo’s case, it was clear that Nothemba did not know the place value of the digits (as indicated by the teacher’s markings on Nothemba’s script). She also added the 3 to the 700 and 200 to get 903. She then added the 3 again as 30 to the 2 (used as a 20) to get 50. She did not add the two totals of 903 and 50 (Fig. 49).

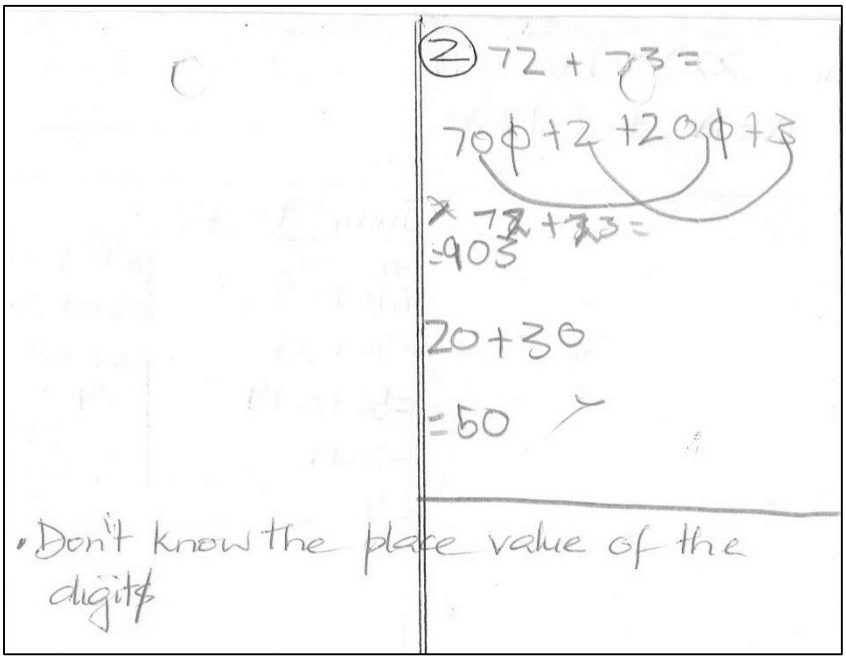


Fig. 49: Nothemba’s notation of 72 + 23 as marked by the class teacher

5.5.2.4 Nothemba's LFIN Interview II

SEAL – Interview II

For this interview Nothemba was placed in a group of three compared to a group of five as was the case for the first interview. The smaller group and changes made to the setup had a positive influence on her interview behaviour. The seating in the room was limited and there was only one chair available that was not a swivel chair. It was soon clear that Nothemba would better be seated on the stationary chair.

During the second round of interviews instructions were only repeated in isiXhosa once and Nothemba did not ask for repeated explanations in isiXhosa although she still answered by using isiXhosa vocabulary in conjunction with English numbers.

Nothemba managed to answer most problems with mental strategies and by counting-up to instead of relying on counting on her fingers. Her strategy was not always clear. However, when posed with 15 - 11 in written format (Question 4) Nothemba read the problem in isiXhosa, counted on her fingers and her lips were moving while she was quietly counting down. She came up with the incorrect answer of '2' and could not explain why.

Because Nothemba was now able to choose between count-down-to and count-down-from strategies and demonstrated quick addition she was placed at SEAL Stage 3/4 (individual number sequence/intermediate number sequence) compared to Stage 2/3 after the first interview.

Conceptual Place Value – Interview II

During the first interview Nothemba counted all the dots individually for the first three questions (1a, 1b and 1c) before starting to count in tens (for 1d). This time in Interview II she only counted the dots for the first ten strip individually and then counted the number of ten strips in tens as seen in Fig. 50:

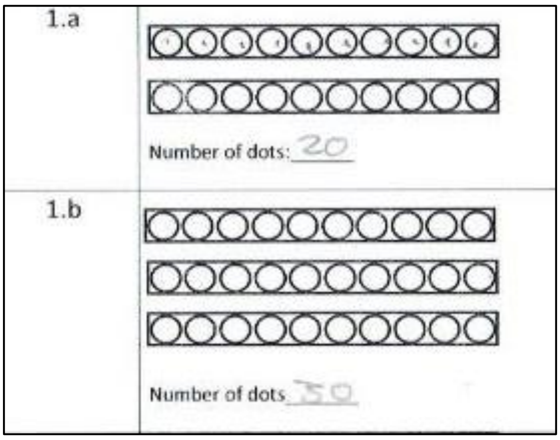


Fig. 50: An example from Nothemba's CPV answer sheet showing unit counting and counting in tens

Incrementing tasks 1c and 1d were similarly done accurately through counting in tens.

Nothemba attempted to use the split strategy to solve 46 + 25 as follows:

46 + 25
4 + 2 = 6
6 + 5 = 11
6 + 11 = 17

This showed an absence of paying attention to the underlying place value of the digits 4 and 2 in the question. At this point in the interview she was losing concentration and was looking confused. During group interviews learners often benefited from listening to the strategy used by their peers and copying it for their own task. However Nothemba did not pay attention to the other learners' strategies and was getting restless. With the written subtraction tasks she made several errors as indicated below:

56 – 23
5 – 3 = 4
6 – 2 = 3
56 – 23 = 43

As a result of the fact that Nothemba managed to do some addition tasks (but not subtraction tasks) with tens and ones without materials, she was now placed at CPV Level 2 compared to Level 1/2 after the first interview.

In September 2014 Nothemba scored 40% on the Annual National Assessment (ANA) in relation to the average of 46% obtained by her class and the average of 34,8% obtained by other Grade 4 learners in her province (DBE, 2014).

In summary, both Vuyo and Nothemba progressed during the recovery period (Vuyo progressed from SEAL stage 3/4 to stage 5 and from CPV level 2 to level 3 and Nothemba progressed from SEAL stage 2/3 to stage 3/4 and from CPV level 1/2 to level 2). Reflecting on the stories of Vuyo and Nothemba and the differences illuminated by the vignettes, it is clear that the experience of MR and the extent of progress is different for different learners.

Chapter 6 – Recommendations, limitations and conclusions

The focus of this study was an assessment and teaching experiment with a class of 23 Grade 4 learners aimed at answering the following research questions:

1. How might Wright's et al.'s (2006; 2012) individual interview for assessing conceptual place value and early arithmetic strategies be adapted and implemented with groups of Grade 4 South African learners? How effective is this adapted framework in assessing learners' levels of mathematical knowledge?
2. How might Wright et al.'s individually administered MR program be adapted for remediation of conceptual place value and early arithmetic strategies in the context of working with learners in groups within a South African classroom context where the majority of learners require remediation? What advantages/difficulties emerge from the adaptation of the recovery program for use in groups?

Chapter 4 documented the cyclic action research process of adapting, implementing and readapting Wright et al.'s individual interviews and recovery program for administration in a group context. Chapter 5 focused on the learners' performance during the pre- and post-recovery interviews with specific reference to two case studies to shed further light on the process involved and to highlight some differences in the way individual learners benefitted from the process.

It was possible to identify learners' SEAL stages and CPV levels during the administered group interviews. There were positive shifts in these stages and levels during the recovery period as demonstrated by the changed responses from the first to the second set of interviews. Despite the fact that a methodological contribution was made in this regard, continued reflection and evaluation during the course of the study resulted in the identification of some shortcomings and challenges with group administration of interviews. This affected the conclusions that can be made from the results and I therefore make recommendations for future use of the group assessment and recovery program as well as further studies. I conclude the chapter with reflection on my personal and emotional involvement in the process.

In the next two sections I summarise my findings in terms of my two research questions.

6.1 Reflections and challenges related to the adaptation of the Wright et al. (2006, 2012) interviews and recovery program

I adapted the Wright et al. (2006) interviews for a group context by combining questions posed to the group as a whole (answered on individual answer sheets) with individualized questions posed to every learner separately as described in Chapter 4. Observations, learner answers and strategies were noted on my response schedule. This schedule was designed to minimize writing by providing tick boxes for different strategies and numbered columns to correspond with the number at every learner's seat. Individual task cards were also colour coded to correspond with the colour at the different seats and the columns on the schedule. Examples of the response schedules are included in Appendix B1 and B2 and C1 and C2.

As mentioned in Chapter 4, and illustrated in the example of reflection of the setup in Appendix E, the adaptation of the interviews did not stop with the completion of a response schedule and answer sheets. While administering the interviews contextual and setup challenges emerged. Some of these could be solved after reflection between different group sessions. For example, the group size was reduced from 6 to 4 and the seating plan was changed to ensure that everyone could be observed better and that everyone was within the field of the video camera. Boxes used as screens were replaced with card board used by learners to cover up their answers. These changes reduced disciplinary issues, allowed me to more easily administer individual questions, to make better observations and to present questions to the group (like the uncovering tasks) within clear view of everyone.

Some challenges, however, could not be addressed so easily. The fact that learners shouted out answers instead of writing them down on their answer sheets improved with time and with repetition of my instruction "Bhalani kwiphepha" (Write on the paper) given with nearly every question. Learners could also hear what others were saying when explaining the strategy they used to solve a task. As mentioned in Nothemba's case study vignette in Chapter 5 another learner said "No" resulting in Nothemba correcting her answer. Although one could perceive the fact that learners could hear each other's strategies as a negative aspect of the interview, it could also be seen as a learning experience. Some learners did not pay attention to what the learner before them said while others copied the strategy used. I feel that if the learners could apply the previous strategy accurately to their individualized question, they actually gained knowledge from the group interview setup.

Although I experienced many challenges all could be addressed during the group implementation of the recovery program. Although progress and change were not addressed on a one to one basis, learners had the benefit of co-construction of knowledge through dialogue and social interaction. Learners could engage in learning opportunities arising from social learning as mentioned in Chapter 2.

6.2 Considerations for future implementation of Wright et al. (2006, 2012) in a group context

If I were to use Wright et al.'s (2006 and 2012) interviews and recovery program again in a group context, I would consider making the following changes:

1. Ideally the recovery program should be done in Grade 2 to bridge the gaps sooner to be able to establish conceptual understanding in Grade 4 up to the required 4 digit numbers (DBE, 2011).
2. A focus on the domains of structuring numbers to ten and to twenty should be considered before attempting two digit addition and subtraction as it constitutes an important basis for the development of advanced strategies such as jumping and splitting.
3. I would have a longer intervention/recovery period (20 sessions over 10 weeks/one school term could work well). The time frame of this study was limited by the approval of my research proposal and the end of the academic school year at the end of November. This limited to recovery period to eight sessions of 35-40 min each.
4. I will carefully consider the value of the LFIN interviews and profiles for a group context. It is time consuming in terms of administering the interviews, analysing the video material and other data and plotting learners on the LFIN

profiles. During the recovery period I incorporated both the Wright et al.'s (2012) assessment task groups and the recovery activities into the session plans. I found the assessment tasks to be valuable in providing information regarding the mathematical proficiency of learners. In future I will consider doing the assessment task groups and keeping record of all data gained in the process to be able to understand every learner's level of understanding and proficiency. This proposed approach could open up future research areas.

5. I would not introduce formalized notation too soon as this could be confusing to learners. I would rather allow learners to come up with their own ways of notating strategies before formalizing it. As the paucity of learner writing is a factor to consider in the South African context, further adaptations to MR should carefully explore the possibility of introducing more opportunity for writing (even if it is informal jotting down of interim solutions or counts).
6. I would limit the strategies discussed and practiced in each session to avoid confusing struggling learners. Even now, during my continued involvement at the school, I tend to spend more time on the jump strategy than the split strategy. Although it is more difficult to grasp, less mistakes are made using jump than using split (Wright et al, 2012).
7. I would spend some time prior to the recovery period to practice and instil group work skills or social norms. Wood and Yackel (as cited by Cobb, 1995) pointed out that teacher intervention was necessary in small group collaboration in order to establish certain norms. The prior development of the following norms would have been valuable during group sessions in my study:
    - Persisting to solve challenging problems
    - Explaining personal strategies to your partner
    - Listening to your partner and trying to make sense of what he is saying
    - Attempting to seek consensus about an answer
    - Finding a solution (p.27).

    The absence of these norms could potentially be addressed by implementing the TRAC (Talk, Reasoning and Computers) program developed by Mercer, Wegerif & Dawes (1999) in order to develop awareness for the use of talk in joint activities. A series of sessions guide learners through the ground rules of constructive exploratory talk:

- All relevant information is shared
- The groups seek to reach agreement
- The group takes responsibility for decisions
- Reasons are expected
- Challenges are accepted
- Alternatives are discussed before a decision is taken
- All the members in the group are encouraged to speak by other group members (pp.98-99).

Implementing the TRAC program, or similar pre-study interventions, could also greatly contribute to the creation of a safe, risk-free classroom environment ideal for the purpose of number talk mentioned in Chapter 4.

6.3 Contextual challenges

6.3.1 Language

The first obvious obstacle I faced was the English language proficiency levels of the learners. Although English is officially the language of learning and teaching in Grade 4, most learners in this class were not proficient enough in English to understand everything I said or to express their own thoughts and questions in English. Because I have some ability to speak and understand isiXhosa, it was helpful to be able to give interview instructions in both English and isiXhosa and to understand some of the answers given by learners in isiXhosa. During the recovery period, however, the language barrier was frustrating and limiting for both me as teacher/researcher and the learners. It was encouraging to see that the learners' ability to understand and speak English improved through the course of the year. During the second set of interviews in November (conducted three months after the first) less repetition of instructions in isiXhosa was needed and learners were able to describe the strategies used a bit better than was the case in August. Developing learner fluency in speaking mathematics in English was important as all departmentally provided resources to intermediate phase learners in this school (i.e. mathematics text- and workbooks and the ANAs) are in English.

Since I am Afrikaans speaking, and had to find a way to communicate with a class of isiXhosa learners in English, English was a second language for all of us. This is just one of many challenges South African learners face as explained within the context of my study.

It was encouraging to see, however, that the interaction during the pre-study phase, assessments and group sessions encouraged mathematical discourse and therefore contributed to improvements in the learners' mathematical language.

6.3.2 Learning culture, discipline and grouping of learners

I found the general learning culture and classroom discipline challenging. Because of the prolonged absenteeism of the class teacher (since the start of the academic year at the end of January to the end of June 2014) and the lack of a replacement teacher, the learners had been without a teacher for about two months by the time I started to get involved with the class in March. There was thus no established classroom culture of listening, one person talking at a time or even staying in your seat while a teacher/adult is addressing the class. Added to this was the learners' general excitement to be involved in a classroom/teaching activity during a period where they were normally given work to do on their own. I did not however want to be distracted by disciplinary issues, but unfortunately out of necessity some time was spent on managing behaviour every week.

Because of these behavioural challenges, and the fact that I soon realized during my pre-study visits to the class that the general level of proficiency required more individualized and focused attention, I decided to conduct the MR sessions with 3 groups of 6 children and 1 group of 5 children every week instead of working with the class as a whole. Apart from the fact that this was more time consuming it allowed me to minimize disciplinary issues and to establish whether learners understood instructions and explanations and to monitor the work done by learners more closely. This then enabled me to support the learners more efficiently. It was also a good way of getting to know the learners better. The learners who tended to be very quiet in the classroom context were 'forced' by the nature of the smaller groups to be involved and to contribute in the smaller group. I noted in my journal on May 20 for example, "Groups are working well. It is a lot better to be able to spend individual time with various learners". Although learners were initially grouped according to their performance on the SANC baseline assessment during the pre-study period, this changed to mixed ability groups during the recovery period of the study. This was because I agreed with Olsen (as quoted by Boaler (2009) and discussed in Chapter 4) that the differences in "ability" within groups can be seen "as a resource that can broaden the discussion of how to solve problems" (p. 111).

6.3.3 Socio-economic factors

Reddy et al. (2015) identified various aspects of the socio-economic environment of the school and learners that could have a detrimental influence on academic development. This includes parental education and involvement, home resources, home language, school infrastructure and facilities etc. The influence of these factors was almost tangible within the context of my study as well.

Although I managed to get permission slips (See Chapter 3) back from all the parents (in some cases three letters had to be sent home before a signed slip was returned) I did not have any contact with any of the parents during the period of involvement with the class despite the fact that I made myself available for telephone or personal interviews. I was also surprised to find, during my first session with the class, that only about half of the class had pencils to write with. I then bought a set of pencils which I took to class every week and took back at the end of each session to ensure that there would be enough pencils for everybody in every session. Although I tried to make use of items such as bottle tops for counters and ice cream sticks as bundling sticks I was constantly aware of the fact that some of the materials used were foreign to the learners. When I used a small white board to informally notate strategies, it was clear that nobody had seen or used one before. The excitement of having a wipe-able board initially distracted learners from the focus on notation. I noted in my journal (2 September 2015) "In one group we had to stop everything to let everybody wipe something off to get them to focus on the notation and not on the white board".

6.3.4 Ethical influences on group size

I was ethically obliged to support all 23 learners in this class because when I initially made contact with them they were without a teacher. I did not want to exclude any learners from the recovery program and I did not want to choose learners as a case study prior to the study because I wanted to treat everyone equally with the same amount of input and attention. I therefore began working informally with all 23 learners in March although my action research study only formally started in August with the first interviews. It also meant that I had 23 learners to interview and four groups of learners to meet with on a weekly basis. In this respect my involvement with learners and time spent with them as well as the amount of data was greater than expected when first deciding to do my Masters. Despite the amount of data created in the process I would have chosen to work with the whole class again as it makes sense to me from an ethical point of view in this case.

6.3.5 Relationship with the teacher

During the pre-study period the class did not have a teacher. With the return of the teacher shortly after the approval of my research proposal, in June 2014, I realized that my study could potentially disrupt the class if I continued to see learners during school time. I suggested that I could offer the recovery program after school, but both the principal and class teacher agreed that I should continue during school hours because learners would not be easily able to attend after school. Additionally my presence freed up the teacher to support the Grade 7 class who was without a mathematics teacher at that point.

I noted in my journal “I have to manage this new relationship carefully. I want a mutual relationship to evolve. Not one of dependency or one where I am in a position of power or expertise. I should be mindful to create an atmosphere of different expertise!” (24 June 2015). Because groups were taken out of the classroom I invited the teacher to attend any of the group sessions. She often popped in during sessions, asked questions and sometimes even partook in activities. After the first time she joined a group I journaled: “She helped me with supporting the kids and joined in the game of bingo (addition and subtraction) at the end. She clearly wanted to win! I had to tweak the questions a bit to make her win – worth teaming up with her” (24 June 2014).

I made a point of meeting with the teacher every week to show what we did during the session and to ask her opinion about what I was planning for the next session. She often questioned me on strategies and concepts and it was encouraging to see that she tried to practice some of the strategies with the class in my absence as well. She mentioned that she was not computer literate and where possible I tried to provide her with materials (for example some of the SANC resources) for areas of mathematics she had questions about. The teacher expressed her disappointment in the lack of understanding of learners of the concepts that should have been established in earlier grades.

The class teacher was absent for the first two terms of the year. When looking at the ANA answer scripts it was clear at which point she came back, because the work she had time to address with them corresponded with the areas the learners were getting the most correct answers. This highlighted the negative impact of the absence of a teacher.

After the completion of my study the teacher requested that I continue to be involved with her new Grade 4 mathematics class in the new school year. This is something I am currently doing.

6.4 Personal challenges

The action research cycle was apt and valuable for the adaptation and implementation of the Wright et al. (2006, 2012) materials. It also provided a suitable platform for action based on reflection. But reflection also involved personal praxis (as mentioned in Chapter 3) and interaction between my role of researcher, teacher and humanitarian. I had to constantly be mindful of my research commitment, teaching commitments and my ethical/citizenship commitments.

Within the South African reality of a high poverty context, as seen in my study, I found it difficult to focus only on being a researcher or teacher. I was acutely aware of the limiting circumstances surrounding the school and the learners. I had to deliberately limit my involvement at the school to the Grade 4 class I was working with in order to not be too side-tracked. I was tempted to, and in some cases did, get involved with a potential library, to establish a reading program and to organize second hand school clothes and eye tests for learners. At times I thus found it challenging to focus my attention on my research.

However, one of my greatest realisations, as a result of my research process and interaction with this school is that relatively small efforts can make an enormous difference. I share one anecdote of a girl in the 2015 Grade 4 class, Bonani, who was visually impaired, to exemplify this realisation. Bonani had to hold all materials and books about 10cm from her eyes to be able to see anything. She also had visible clouding of the eyes. Because Bonani had not seen a doctor and her parents were not able to arrange an eye examination I arranged for her to be seen by an ophthalmologist at the local government hospital and transported her and her father to the doctor. She was diagnosed with cataracts on both eyes, something that could be rectified by two relatively small operations. After only the first eye was operated on I was astounded to see the difference in Bonani's workbook in class. Fig. 51 below shows an example from her workbook prior to the operation and Fig. 52 shows an example from her workbook after one eye was operated on (the grey boxes were used to cover her real name).

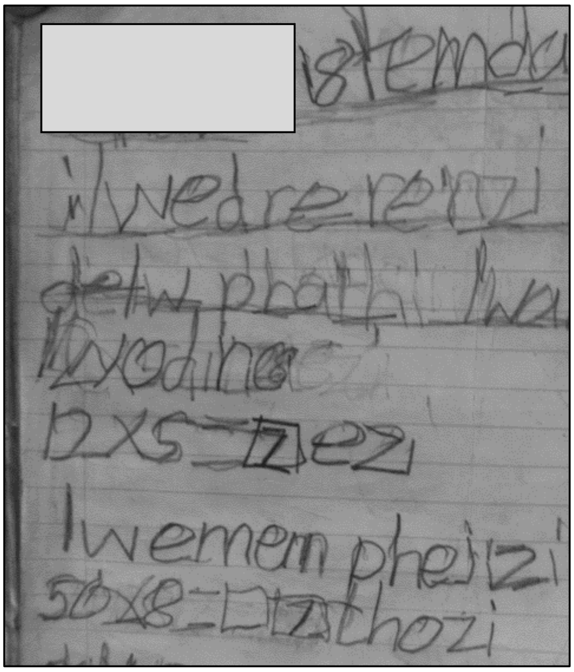


Fig. 51 Bonani's workbook before the first operation (18 February 2015)

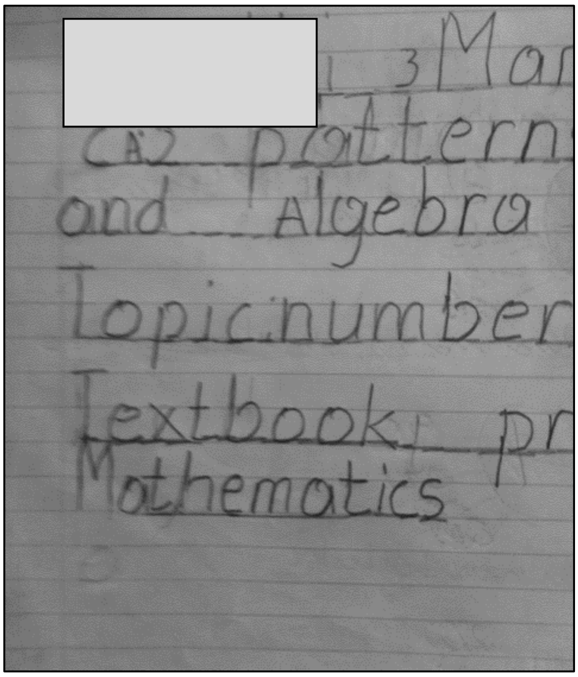


Fig. 52 Bonani's workbook after the first operation (3 March 2015)

The impact of such a small intervention highlights the need for the system to find a way to address these needs through systemic interventions such as eye tests for all.

The analysis of the video materials were also key in the process of reflecting on my teaching habits. For example, by watching the recordings I realized that I must speak slower and not repeat questions if learners were taking a long time to answer. It was also clear that organization of assessment materials was key to effective interviews. The colour coded system described in Chapter 4 worked well, but there were cases where I mixed up instructions on the first day of the interviews and that could have been confusing for the learners.

Furthermore since I did not want to take a position of disciplinarian, but rather that of facilitator and co-constructor of knowledge, I had to be mindful to find a balance between maintaining a productive class atmosphere for all learners and not being too authoritarian in my teaching style also keeping in mind Parrish (2011)'s essential characteristics of a teacher/facilitator during number talk mentioned in Chapter 4.

All of the above tensions and grappling greatly enhanced my research learning experience as well as my understanding of the complexities of working at the interface of research and development in the South African context. I am thankful to all participants in this process for my growth and learning that was enabled through my interactions with them. I am now increasingly aware that conducting research in South Africa is both a privilege and an ethical responsibility.

Appendix A: Parent Consent Letter

Information Letter and Consent Form for Parents or Guardians
Permission for Research with Children

SA NUMERACY CHAIR

RHODES UNIVERSITY
Where leaders learn

Dear Parent(s) or Guardian(s) of

____

I am a teacher and a student from the Rhodes University in Grahamstown. I am going to be teaching mathematics in the Grade 4 class at [redacted] every Wednesday morning. I am working with the consent of [redacted], the principal.

During some of the lessons I will look at ways to improve certain numeracy skills for example adding and subtraction as part of my research project. I am writing to ask your permission for your child to participate in this project. All personal information about your child and even their names will not be used in my research. I might also have to take photographs and video recordings during some of the lessons.

Your may withdraw your permission at any time without any consequences for your child.

I would appreciate it if you would permit your child to participate in this project. Please complete the attached permission form, whether or not you give permission for your child to participate, and return it to the school by Monday.

Thank you

Anelia Wasserman
Researcher/Teacher

Mzali obekekileyo

Ndingu titshala ongumfundi eRhodes University eRini. Ndiza kuba ngutitshala wezibalo kwibanga Grade 4 [redacted] *Primary School qho ngoLwezithathu kusasa, ngemvume yenqununu yesikolo* [redacted].

Kwezinye izifundo ndiza kuziphulisa ndenze indlela enika umdla nokuzimisela ebantwaneni kwizifundo zabo zonke. Ndibhala lencwadi ukucela imvume ukuba umntwana wakho angene kule nkqubo. Ndiza kuba fota (photographs) ndibashicilele kwivideo ngexesha lezifundo (video recordings).

Unarhoxa naninina kwisivumelwano xa ufuna.

Ndingavuya ukuba singamkeleka isicelo sam kweliphulo. Ndicela uzalise (iform) uxwebhu lwesivumelwano ukuba uyavuma na umntwana angene kule nkqubo. Ndicela impendulo ngoMvulo.

Enkosi

Anelia Wasserman
Researcher/Titshala

Igama lomntwana/Child's name ____

Igama lomzani/Parent/Guardian's name _________________________________

Sayina apha/Parent/Guardian's signature _________________________________

Umntwana wam angathatha inxaxheba kwiphulo lezibalo.
My child may take part in the mathematics project: EWE/YES _____ HAYI/NO ______

Appendix B 1: Assessment schedule – SEAL

Revised Wright et al. (2006)
Duration: ___________________

Assessment Interview Schedule - Addition and subtraction

Date: ________________________

1.	**2.**
3	**4.**

		1	**2**	**3**	**4**
1. Introductory task **(pp.46-47; pp.161-162)** *Different coloured counters, shaded = covered* 3 + 1 There are 3 red counters under here and one blue one under here. Kukho izinto ezibomvu ezintathu zokubala ngaphantsi, kubekho enye ezuba (blue) ngaphantsi apha. How many are there all together? Zingaphi izinto ezilapha zizonke? Tell me the answer. Xelela impendulo. Write the answer on your sheet for the following problems. Bhala impendulo kwiphepha lakho kumbuzo olandelayo.					
1.a 5 +4 There are 5 red counters under here and 4 blue ones under here. Kukho izinto ezibomvu ezintlanu zokubala ngaphantsi, kubekho ezine ngaphantsi apha. How many are there all together? Zingaphi izinto ezilapha zizonke?	Mental / immediate answer				
	Counting on				
	Counting from 1				
	Counting on fingers/by pointing				
	Other (specify)				

		1	2	3	4
1. b 9 + 6 There are 9 red counters under here and 6 blue ones under here. Kukho izinto ezibomvu ezilithoba zokubala ngaphantsi, kubekho ezintandathu ngaphantsi apha. How many are there all together? Zingaphi izinto ezilapha zizonke?	Mental / immediate answer				
	Counting on				
	Counting from 1				
	Counting on fingers/by pointing				
	Other (specify)				
1.c 8+ 5 There are 8 red counters under here and 5 blue ones under here. Kukho izinto ezibomvu ezisibhozo zokubala ngaphantsi, kubekho ezintlanu ngaphantsi apha. How many are there all together? Zingaphi izinto ezilapha zizonke?	Mental / immediate answer				
	Counting on				
	Counting from 1				
	Counting on fingers/by pointing				
	Other (specify)				
1.d 5 + 2 There are 5 red counters under here and 2 blue ones here. Kukho izinto ezibomvu ezintlanu zokubala ngaphantsi, kubekho ezimbini apha. How many are there all together? Zingaphi izinto ezilapha zizonke?	Mental / immediate answer				
	Counting on				
	Counting from 1				
	Counting on fingers/by pointing				
	Other (specify)				

<table>
<tr><th colspan="2"></th><th>1</th><th>2</th><th>3</th><th>4</th></tr>
<tr><td rowspan="5">1.e 7+ 3
There are 7 red counters under here and 3 blue ones here. Kukho izinto ezibomvu ezisixhenxe zokubala ngaphantsi, kubekho ezintathu apha. How many are there all together? Zingaphi izinto ezilapha zizonke?</td><td>Mental / immediate answer</td><td></td><td></td><td></td><td></td></tr>
<tr><td>Counting on</td><td></td><td></td><td></td><td></td></tr>
<tr><td>Counting from 1</td><td></td><td></td><td></td><td></td></tr>
<tr><td>Counting on fingers/by pointing</td><td></td><td></td><td></td><td></td></tr>
<tr><td>Other (specify)</td><td></td><td></td><td></td><td></td></tr>
<tr><td rowspan="4">2. Serpentine of counters
(different number for each child)
(p.47; p.162)
Count to see how many counters there are all together.
Bala ubone ukuba zingaphi zizonke.
Tell me your answer. Xelela impendulo yakho.</td><td></td><td>13</td><td>15</td><td>16</td><td>12</td></tr>
<tr><td>Count in 1’s</td><td></td><td></td><td></td><td></td></tr>
<tr><td>Count in 2’s</td><td></td><td></td><td></td><td></td></tr>
<tr><td>Other (specify)</td><td></td><td></td><td></td><td></td></tr>
<tr><td colspan="2">3. Missing addends: Introductory task (p.48; p.163)
Different coloured counters, one cover, child looks away when adding
4 + [ ] = 6
Here are 4 red counters. Nazi izinto ezine ezibomvu zokubala. Now close your eyes. Cimela ngoku. While you were closing your eyes I added some blue counters under here. Ucimeleyo ndiza kubeka ezinye ezizuba (blue) ngaphantsi apha. Now there are 6 counters altogether. How many did I add? Ngoku zintandathu zizonke, ndidibanise zingaphi ukuze zenze zibentandathu?
Tell me your answer. Xelela impendulo yakho.
Write the answer on your sheet for the following problems.
Bhala impendulo kwiphepha lakho kumbuzo olandelayo.</td><td></td><td></td><td></td><td></td></tr>
</table>

<table>
<tr><th colspan="2"></th><th>1</th><th>2</th><th>3</th><th>4</th></tr>
<tr><td rowspan="3">3. a 7 + [ ] = 10

Here are 7 red counters. Nazi izinto ezisixhenxe ezibomvu zokubala. Now close your eyes. Cimela ngoku. While you were closing your eyes I added some blue counters under here. Ucimeleyo ndiza kubeka ezinye ezizuba (blue) ngaphantsi apha. Now there are 10 counters altogether. How many did I add? Ngoku zintandathu zizonke, ndidibanise zingaphi ukuze zenze ezilishumi? Write the answer.
Bhala impendulo kwiphepha lakho
How did you get your answer?
Ufemene impendelo kanjani?</td><td>Counting on / counting-up-to</td><td></td><td></td><td></td><td></td></tr>
<tr><td>Subtraction</td><td></td><td></td><td></td><td></td></tr>
<tr><td>Other (specify)</td><td></td><td></td><td></td><td></td></tr>
<tr><td rowspan="3">3.b 12 + [ ] = 15

Here are 12 red counters. Nazi izinto ezilishumi elinambini ezibomvu zokubala. Now close your eyes. Cimela ngoku. While you were closing your eyes I added some blue counters under here. Ucimeleyo ndiza kubeka ezinye ezizuba (blue) ngaphantsi apha. Now there are 15 counters altogether. How many did I add? Ngoku zintandathu zizonke, ndidibanise zingaphi ukuze zenze ezilishumi elinantlanu? Write the answer.
Bhala impendulo kwiphepha lakho.
How did you get your answer?
Ufemene impendelo kanjani?</td><td>Counting on / counting-up-to</td><td></td><td></td><td></td><td></td></tr>
<tr><td>Subtraction</td><td></td><td></td><td></td><td></td></tr>
<tr><td>Other (specify)</td><td></td><td></td><td></td><td></td></tr>
</table>

		1	2	3	4
4. *Written task (on card)* *Different for each child* **(p.49; p.163)** How did you get your answer? Ufemene impendelo kanjani?		*16-12*	*17-14*	*15-11*	*16-13*
	What does it say? *Ithini?*				
	Do you have a way to work it out? (Specify) *Unayo indlela yokufumana impendulo?*				
	Counting-on				
	Mental strategy				
	Other (specify)				
5. Missing subtrahend: Introductory task **(p.50; p.163)** *Counters of one colour only!* 5 – [ ] = 3 Here are 5 counters. Nazi izinto ezintlanu zokubala. Close your eyes. Cimela. *(Remove and screen 2 counters)* There were 5 counters. While you were closing your eyes I took some away. Now there are only 3. How many did I take away? Ndithathe zingaphi xa kushiyeke ezintathu? Tell me the answer. Xelela impendulo. Write the answer on your sheet for the following problems. Bhala impendulo kwiphepha lakho kumbuzo olandelayo.					

		1	2	3	4
5.a 10 – [ ] = 6 Here are 10 counters. Nazi izinto ezilishumi zokubala. Close your eyes. Cimela. While you were closing your eyes I took some away. Now there are only 6. How many did I take away? Ndithathe zingaphi xa kushiyeke ezintandathu? Write your answer. Bhala impendulo kwiphepha lakho.	Counting-down-to				
	Subtraction				
5.b 12 – [ ] = 9 Here are 12 counters. Nazi izinto ezilishumi elinambini zokubala. Close your eyes. Cimela. While you were closing your eyes I took some away. Now there are only 9. How many did I take away? Ndithathe zingaphi xa kushiyeke ezilithoba? Write your answer. Bhala impendulo kwiphepha lakho.	Counting-down-to				
	Subtraction				
5.c 15 – [ ] = 11 Here are 15 counters. Nazi izinto ezilishumi elinantlanu zokubala. Close your eyes. Cimela. While you were closing your eyes I took some away. Now there are only 11. How many did I take away? Ndithathe zingaphi xa kushiyeke ezilishumi elinanye? Write your answer. Bhala impendulo kwiphepha lakho.	Counting-down-to				
	Subtraction				

<table>
<tr><th colspan="2"></th><th>1</th><th>2</th><th>3</th><th>4</th></tr>
<tr><td colspan="2">6. Removed items: Introductory task (p.50; p.163)

3 – 1 = [ ]

Here are 3 counters. Kukho izinto zokubala ezintathu.
(Briefly display, then screen)
If I take 1 away (Remove 1, display briefly, rescreen with 2nd screen)
How many are left under here? (first screen) Ukuba ndithathe inye kushiyeke zingaphi?
Tell me the answer. Xelela impendulo.
Write the answer on your sheet for the following problems.
Bhala impendulo kwiphepha lakho kumbuzo olandelayo.</td><td></td><td></td><td></td><td></td></tr>
<tr><td rowspan="3">6.a 6 – 2 = [ ]
Here are 6 counters. Kukho izinto zokubala ezintandathu. If I take 2 away, how many are left under here? Ukuba ndithathe ezimbini kushiyeke zingaphi?
Write your answer.
Bhala impendulo kwiphepha lakho.</td><td>Counting-down-from</td><td></td><td></td><td></td><td></td></tr>
<tr><td>Fingers</td><td></td><td></td><td></td><td></td></tr>
<tr><td>Other (specify)</td><td></td><td></td><td></td><td></td></tr>
<tr><td rowspan="3">6.b 9 – 4 = [ ]
Here are 9 counters. Kukho izinto zokubala ezintathu. If I take 4 away, how many are left under here? Ukuba ndithathe yanye kushiyeke zingaphi?
Write your answer.
Bhala impendulo kwiphepha lakho.</td><td>Counting-down-from</td><td></td><td></td><td></td><td></td></tr>
<tr><td>Fingers</td><td></td><td></td><td></td><td></td></tr>
<tr><td>Other (specify)</td><td></td><td></td><td></td><td></td></tr>
</table>

		1	2	3	4
6.c 15 – 3 = [ ] Here are 15 counters. Kukho izinto zokubala ezilishumi elinantlanu. If I take 3 away, how many are left under here? Ukuba ndithathe ezintathu kushiyeke zingaphi? Write your answer. Bhala impendulo kwiphepha lakho.	Counting-down-from				
	Fingers				
	Other (specify)				
6.d 27 – 4 = [ ] Here are 27 counters. Kukho izinto zokubala ngamashumi amabini anesixhexe. If I take 4 away, how many are left under here? Ukuba ndithathe ezine kushiyeke zingaphi? Write your answer. Bhala impendulo kwiphepha lakho.	Counting-down-from				
	Fingers				
	Other (specify)				

Appendix B 2: Learner Answer Script- SEAL

Revised Wright et al. (2006)

Assessment Interview Schedule - Answer sheet (SEAL)

Name	Date

1.a	
1.b	
1.c	
1.d	
1.e	
3.a	
3.b	
5.a	
5.b	
5.c	
6.a	
6.b	
6.c	
6.d	

Appendix C 1: Assessment schedule - CPV

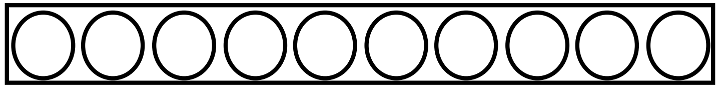
Revised Wright et al. (2006)

Duration: ________________________

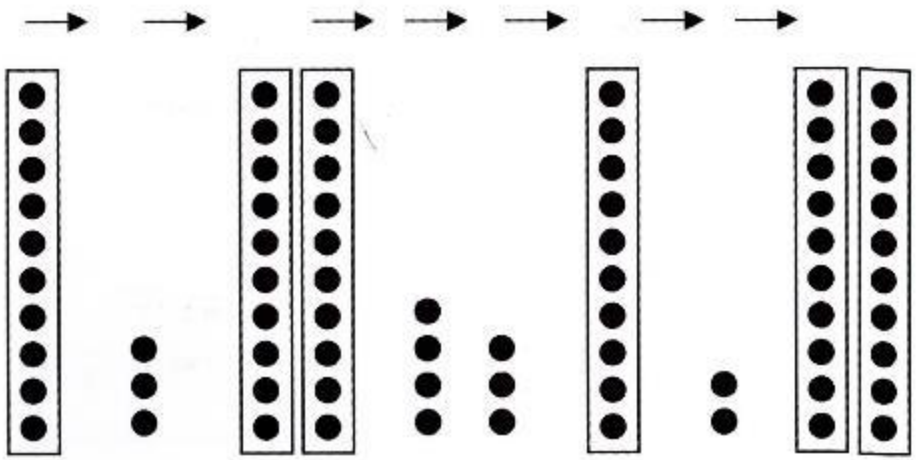
Assessment Interview Schedule - Conceptual place value

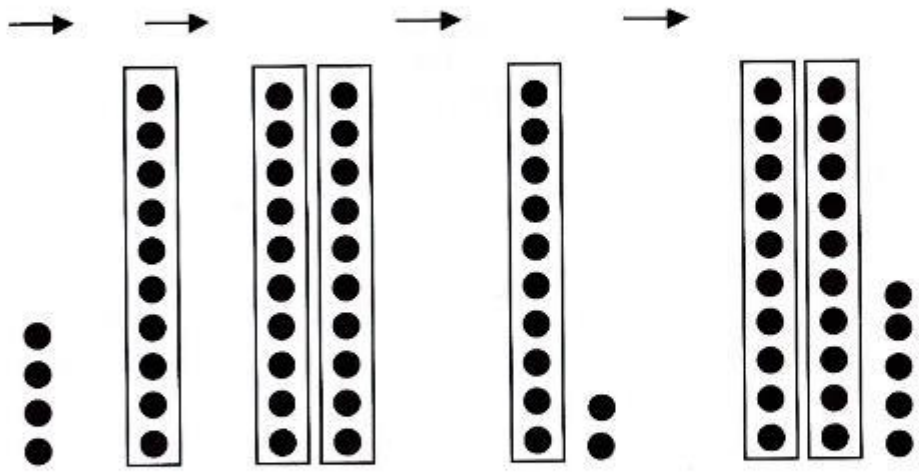
Date: ______________________

1.	**2.**
3.	**4.**

	1	**2**	**3**	**4**
1. Counting by tens with strips **(p. 94; p.166)** *Put down a 10-strip.* How many do we have? *If child says "One" ask* "How many dots are there?"  **1.a – 1.g** Please write down the numbers of dots. Bhala phantsi inani ledots kwiphepha. Zingaphi idots esinazo? ***Put down one strip at a time up to 8 strips.*** *Pick up all the strips.* How many strips are there? Zingaphi izistrips esinazo? How many dots all together? Zingaphi iidots zizonke?				

	1	**2**	**3**	**4**
2. Incrementing by ten **(p. 94; p.166)** ***Individual question for every child*** How many dots are there? (e.g. 4 / 7/ 3) Zingaphi iidots zizonke? *Place a ten strip to the right of the first strip.* How many dots are there now? Zingaphi zizonke ngoku? *Continue to do with e.g. 24 / 34 / 44 / 54 / 64 / 74*	4 14 24 34 44 54 64 74	7 17 27 37 47 57 67 77	3 13 23 33 43 53 63 73	6 16 26 36 46 56 66 76

		1	2	3	4
3.a Uncovering tasks **(p. 95; p.166)** How many dots are there now? Write on your sheet. Zingaphi iidots zizonke ngoku? Bhala kwiphepha. 	Count by ones				
	Count by tens				
	Count ones first				
	Other (specify)				

		1	2	3	4
3.b Uncovering tasks **(p. 96; p.167)** How many dots are there now? Write on your sheet. Zingaphi iidots zizonke ngoku? Bhala kwiphepha. 	Count by ones				
	Count by tens				
	Count ones first				
	Other (specify)				
4.a Horizontal sentences **(pp.97-98; p.167)** *Written problems* Do you have a way to figure out what is: Unayo indlela yokudibanisa la manani?		16+10	14+10	15+10	13+10
	Split strategy				
	Jump strategy				
	Transfor-ming				
	Other (specify)				
So what is: Ithini impendulo yalena?		16+9	14+9	15+11	13+9

		1	2	3	4
4.b *Written problems* Do you have a way to figure out what is: Unayo indlela yokudibanisa la manani? *If correct, ask:* Do you have another way to work it out? Unayo enye indlela yokuyibala?		42+23	33+25	51+24	44+32
	Split strategy				
	Jump strategy				
	Transfor-ming				
	Other (specify)				
4.c *Written problems* Do you have a way to figure out what is: Unayo indlela yokudibanisa la manani? *If correct, ask:* Do you have another way to work it out? Unayo enye indlela yokuyibala?		27+36	38+23	46+25	28+34
	Split strategy				
	Jump strategy				
	Transfor-ming				
	Other (specify)				

		1	2	3	4
4.d *Written problems* Do you have a way to figure out what is: Unayo indlela yokudibanisa la manani? *If correct, ask:* Do you have another way to work it out? Unayo enye indlela yokuyibala?		67-52	48-36	56-23	49-24
	Split strategy				
	Split/Jump mix				
	How many units short				
	Transfor-ming				
	Other (specify)				
4.e *Written problems* Do you have a way to figure out what is: Unayo indlela yokudibanisa la manani? *If correct, ask:* Do you have another way to work it out? Unayo enye indlela yokuyibala?		34-16	54-28	43-15	35-17
	Split strategy				
	Split/jump mix				
	How many units short				
	Transfor-ming				
	Other (specify)				

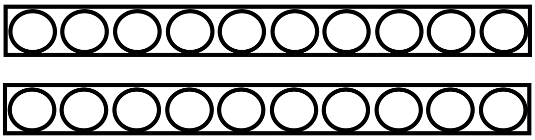
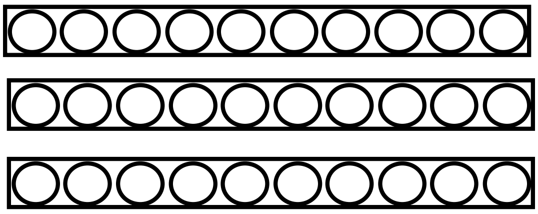
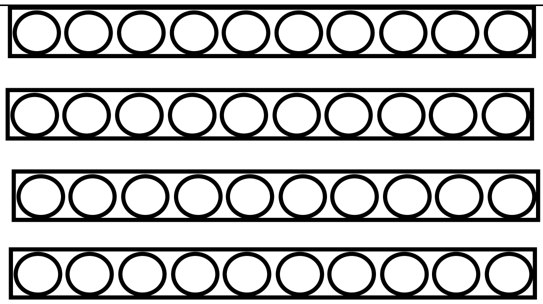
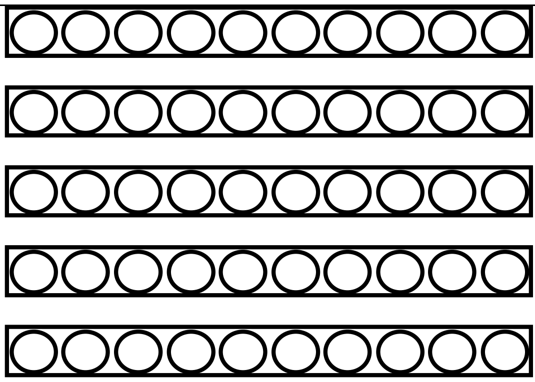
Appendix C 2: Learner Answer Script - CPV

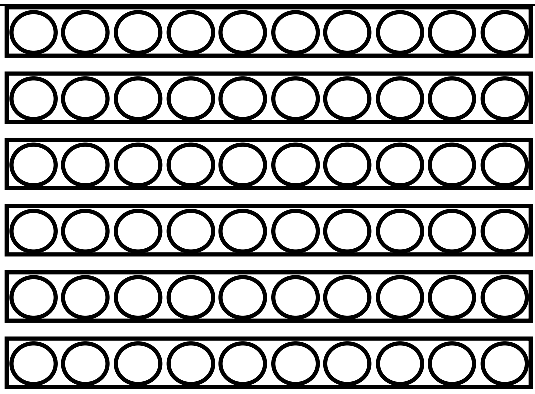
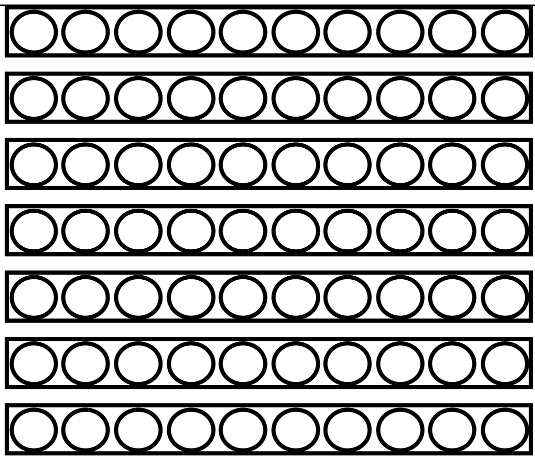
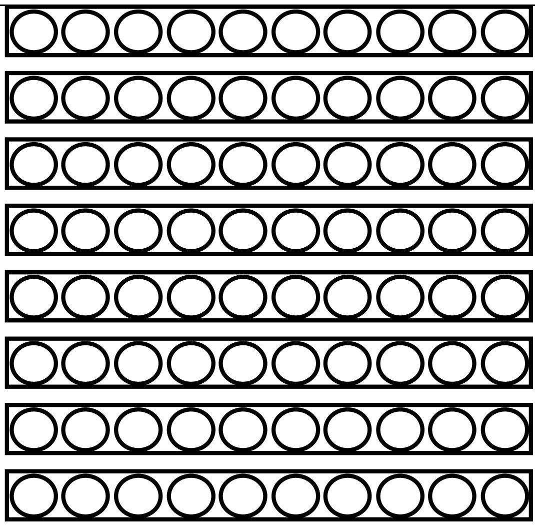
Revised Wright et al. (2006)

Assessment Interview Schedule - Answer sheet

(Conceptual place value)

Name	Date

1.a	 Number of dots:______
1.b	 Number of dots_______
1.c	 Number of dots:______
1.d	 Number of dots:______

1.e	 Number of dots:_______
1.f	 Number of dots:_______
1.g	 Number of dots:_______

3.a	1_________ 2_________ 3_________ 4_________ 5_________ 6_________ 7_________ 8_________
3.b	1_________ 2_________ 3_________ 4_________ 5_________

Appendix D: LFIN Profile Summary

LFIN profile: SEAL and Conceptual Place Value

(Wright et.al., 2006)

Learner: ____ **DoB** ______________

Summary	Date	Date
SEAL Stage		
CPV Level		

	Date	**Date**
SEAL STAGES		
Stage 0 – Emergent counting (p. 22)		
• Can not count visible items		
Stage 1 – Perceptual counting (p.22)		
• Can count counters	1's/2's/3's	1's/2's/3's
• Can not count screened items		
Stage 2 – Figurative counting (pp.22, 60-62, 91))		
• Can count screened items by counting from 1		
• Battles with missing addend and missing subtrahend		
• One screened and one unscreened item >> count unscreened first and then keep track of screened collection		
Stage 3 – Individual number sequence (pp. 22, 64)		
• Counting-on/counting-up-to rather than count from one		
• Count-down-from		
• NOT count-down-from for missing subtrahend (e.g. 17 -__ = 14 >> NOT 17 – 14)		

• May keep track of number of counts by using fingers		
• Misinterpret missing addend task as additive task		
• Takes long time for task >> Could mean counting by ones		
Stage 4 – Intermediate number sequence (pp.22, 67-69, 89)		
• Counts-down-to for missing subtrahend		
• Chooses between count-down-to and count-down-from		
• May use fingers to keep track of counting-down-to		
• Quick addition >> could mean non-count-by-one		
Stage 5 – Facile number sequence (pp.22, 70, 88,102)		
• Range of non-count-by-one strategies (at least 3 instances) e.g. compensation (15-3 > 5 – 3 = 2 > 15 -2 = 12) adding to ten subtraction as inverse of addition using a known result commutativity (2 + 6 + 8 + 4 = 2 + 8 + 6 + 4)		
• Some counting-by-one		
	Date	**Date**
CONCEPTUAL PLACE VALUE		
Level 1 – Initial concept of 10 (pp.22, 93)		
• At least Stage 3 on SEAL!		
• Does not see 10 as a unit		
• Focus on individual items in 10		
• Count backwards and forward by 1		
Level 2 – Intermediate concept of 10 (pp.22, 93)		
• 10 = unit of ten ones		
• Depend on representations of ten (ten strips/fingers)		
• Does + and – with materials		
• NOT written number sentences with 10’s and 1’s		
Level 3 – Facile concept of 10 (pp.22, 93)		
• + and – with 10’s and 1’s without materials/representations		

• Can + and - written number sentences of 10's and 1's		

Comments Date ____________________________	Comments Date ____________________________

Appendix E: Example of journal inscription

(Note that names in the journal have been changed)

Group interview – Addition and Subtraction

5 August 2014

Set up

6 numbered seats

Divided by beer trays with sand bags as weights

Colour coded individual question cards to match with colour of number to make administration easier

Problems:

- Video can not pick up all six seats. Seat nr 4 difficult to see.
- Seat nr 2 and 5 can not see counters/covers when seated.
- Can not see all the answer sheets from the front at the same time.

Language

I posed every introductory question in English and Xhosa and did not repeat Xhosa for all the other questions in that section if I could see they understand the question.

Problems:

Generally they are not disciplined enough to do an assessment effectively.

They shout out some answers

They are unsure where to write on answer sheet and mix up answers

Nomsi saw her individual subtraction in advance (the colour coded card was in my hand) and quickly did a "line" tally on the side of the box before I got to her! I gave her another one! (Note: Hide individual questions!)

They tend to tell each other the answers.

They laugh at each other when the can not answer / give way of solving problem.

When asked how they solved a problem they tend to just say "Nenqondo" (in my head) or "Ndicingile" (I thought).

They started to count the numbers in the serpentine of counters while I was putting it out – I could not accurately observe what they were doing.

I made the following adjustments during the assessment with Group 3

- I continuously checked that the kids knew where to write on the answer sheet.
- I made sure they can not see the individual questions.
- I made them close their eyes while I put out the serpentine
- I repeated even more that they should not talk, but write.

Problems:

They still said answers out loud and whispered answers to each other

Solutions??

- Set up

Seat in two rows of 3? One row of 6? How will this influence DVD?

But then presentation of questions can not be done on table. Could use board – but then back will be to group at times. Will limit observations.

Reduce group size to 4?

How to adapt questionnaire???

There are a lot of individual questions needed anyway!

- General:

Hide individual questions!

Use smaller counters for individual serpentine (milk jug lids were too big to count and pack quickly).

Make sure I give instructions clearly and accurately.

Do individual assessments for everybody and do recovery in a group??? For my purposes also, but especially for future use:

Pro's	**Cons**
Do not hear each other's answers / strategies	More time consuming on test day
Richer data	
Less time spent watching DVD's	
Easier set up for DVD / better DVD data	

Group assessment – Addition and Subtraction

8 August 2014

I made a few adjustments after the first set of assessments for Addition and Subtraction

Set up

I decided to reduce the size of the group from 6 to 4.

I initially thought I must put the kids in one row of 4 or two rows of 2 and do the assessment on the board, but I decided against it because:

- It is difficult to put counters and covers on the board (even magnetic ones)
- I will have my back to the kids often
- The DVD will not be able to pick up the kids and board effectively

So I went for 4 numbered seats

Dividers replaced with cardboard to cover up work.

Colour coded individual question cards to match with colour of number to make administration easier (that worked well before). Individual questions are also colour coded on my interview sheets with corresponding colours.

Problems solved:

- Video can pick up all seats.
- Counters can be put out in front of everybody.
- I have a better view of all the learners.
- I don't have to get up to pack out counters for individual serpentine.
- Less time is spend on individual leaners with rest getting distracted.
- I used smaller counters (not milk jug lids) and it made it easier to handle and to move and pack out individual serpentine. Saved a bit of time there!

Language

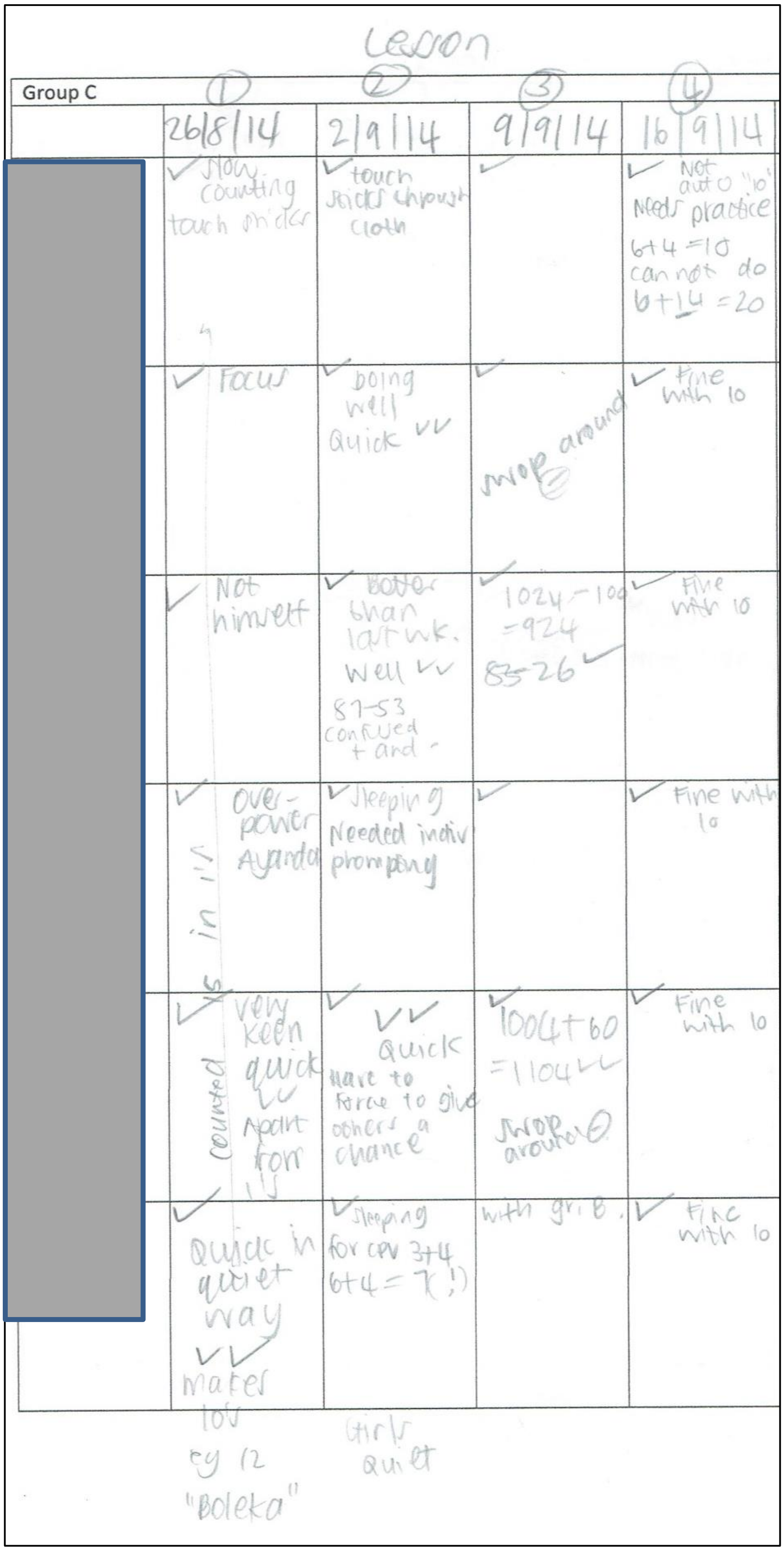
I posed every introductory question in English and Xhosa and did not repeat Xhosa for all the other questions in that section if I could see they understand the question.

Groups

The smaller group worked better in terms of time, discipline and observations.
Duration: The duration of these assessments were a shorter than during the first session (37 min, 35 min) and 23 min for the strongest group.

Appendix F: Example of an observation sheet

(Real names covered with shaded block)



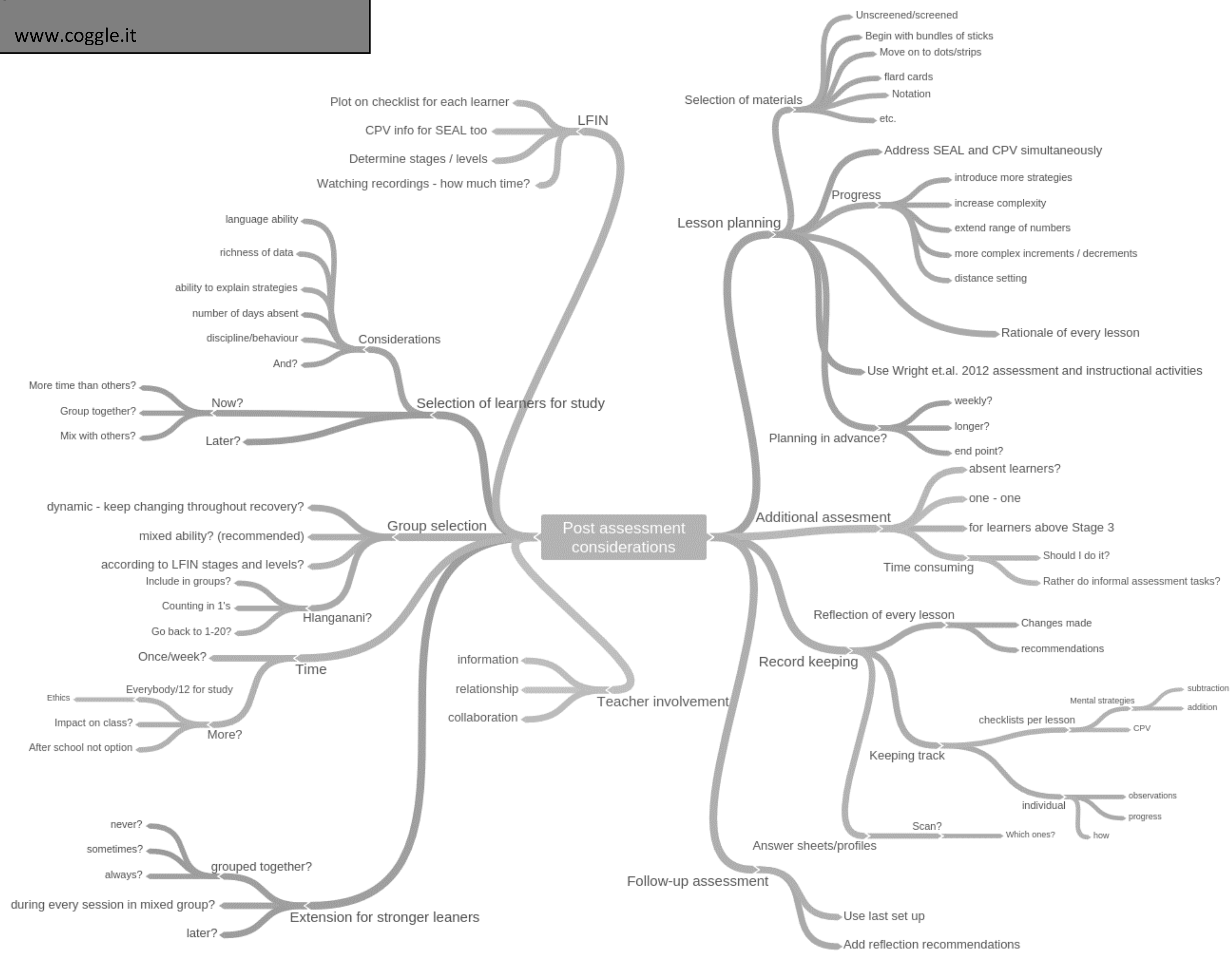
Lesson

Group C	① 26/8/14	② 2/9/14	③ 9/9/14	④ 16/9/14
	✓ slow counting touch sticks	✓ touch sticks through cloth	✓	✓ Not auto "10" Needs practice 6+4=10 can not do 6+14=20
	✓ Focus	✓ Doing well Quick ✓✓	✓ swop around	✓ Fine with 10
	✓ Not himself	✓ Better than last wk. Well ✓✓ 87-53 confused + and -	✓ 1024-100 =924 85-26 ✓	✓ Fine with 10
	✓ Over-power Ayanda; counted in 1's	✓ Sleeping Needed indiv prompting	✓	✓ Fine with 10
	✓ very keen quick ✓✓ Apart from 1's	✓ ✓✓ Quick Have to force to give others a chance	✓ 1004+60 =1104 ✓✓ swop around	✓ Fine with 10
	✓ Quick in quiet way ✓✓ makes lots eg 12 "Boleka"	✓ Sleeping for cpv 3+4 6+4=7(!)	with gr. B.	✓ Fine with 10

Girls quiet

Appendix G: Mindmap: Post assessment considerations

www.coggle.it



Appendix H: Recovery checklist

Checklist: Mental strategies for addition (Wright et.al. 2012, p.100)

	Date	Date	Date	Date	Date	Date	Date	Date	Date	Date
Addition with non-regrouping										
Jump										
Split										
Split-jump										
Addition with regrouping										
Jump										
Jump to the decuple										
Split										
Split-jump										
Compensation										
Transformation										
Notation of strategy										
Labelling of strategy										

Checklist: Mental strategies for subtraction (Wright et.al. 2012, p. 101)

	Date	Date	Date	Date	Date	Date	Date	Date	Date	Date
Subtraction with non-regrouping										
Jump										
Split										
Subtraction with regrouping										
Jump										
Split										
Split- jump										
Over-jump										
Jump to the decuple										
Compensation										
Transformation										
Notation of strategy										
Labelling of strategy										

Checklist: CPV – Dimensions of instruction (Wright et.al. 2012, pp. 80- 81)

	Date	Date	Date	Date	Date	Date	Date	Date	Date	Date
A. Extend range of numbers										
0 -100										
0 -200										
Introduce all 100's										
Extend to 1 000										
Extend across 1 000 and 1 100										
Extend to 2 000 and beyond										
B. Making increments and decrements more complex										
Multiples of 10 or 100										
Switch between 10's, 100's and 1's										
Combinations of 1's and 10's										
Combinations of 1's, 10's and 100's										
Determine unknown increments										
C. Distance the setting										

Materials are visible										
Materials are screened but increments and decrements are shown briefly										
Materials are screened and increments and decrements are posed verbally										
First number is given in a numeral and increments are decrements are posed verbally (no materials)										

Appendix I: Recovery session 1

Materials: bundling sticks (at least 12 groups of ten for each pair) Work in groups of two. Let them familiarise themselves with the sticks (e.g. count number of sticks in a bundle)		
CPV Activity 1 (Wright et.al. 2012, pp. 79-80) Decrementing and incrementing in 10's and 1's		
	Teacher	Learners
1.	X X X X X I I I I X X How many sticks? Mangaphi amakhuni?	74
2.	X X X X X I I I I X X X How many sticks?	84
3.	X X X X X I I I I X X X X X How many sticks?	104
4.	X X X X X I I I I X X X X How many sticks?	94
5.	X X X X X I I I X X X X How many sticks?	93
CPV Activity 2 Decrementing and incrementing in 10's and 1's		
1.	7	I I I I I I I
2.	17	X I I I I I I I
3.	47	X X X X I I I I I I I
4.	44	X X X X I I I I
5.	24	X X I I I I
6.	66	X X X X X I I I I I X I
SEAL Activity 1 (Adapted from Wright et.al. 2012, pp. 111-113) Higher decade addition and subtraction		

Teacher and learner pack out bundling sticks as follows: (Instructions are posed verbally/not in notation)		
1.	37 – 5 = [32]	Jump within decade
2.	How many more to get 40? 32 + [8] = 40	Jump forward to decuple
3.	If you take away 7, how many are left? 40 – 7 = [33]	Jump back from a decuple
4.	Now add 15 33 + 15 = [48]	Jump across decuple
5.	How many do I need to take away to get 40? 48 – [8] = 40	Jump back to the decuple
6.	Add 27 to 40 40 + 27 = [67]	Jump forward from decuple
* 7. (extension)	Take away 18 67 – 18 =[49]	Jump back across decuple
*8. (extension)	Add 23 49 + 23 = [72]	Jumping across decuple
SEAL Activity 2 (Adapted from Wright et.al. 2012, p. 116) Jump and Split Strategy		
Pairs must use their bundling sticks to try and find at least two ways to solve the following: (Questions are posed in notation/written format)		
	Split strategy	Jump strategy
1. 33 + 15	30 + 10 = 40 3 + 5 = 8 40 + 8 = 48	33 + 10 = 43 43 + 5 = 48
2. 48 + 21	40 + 20 = 60 8 + 1 = 9 60 + 9 = 69	48 + 40 = 68 68 + 1 = 69
*3. 69 + 18	60 + 10 = 70 9 + 8 = 17 70 + 17 = 87	69 + 10 = 79 79 + 8 = 87
4. 87 - 36	80 – 30 = 50 7 – 6 = 1 50 + 1 = 51	87 – 30 = 57 57 – 6 = 51
*5. 51 - 17	50 – 10 = 40 ???	51 – 10 = 41 41 – 7 = 34

Appendix J: Recovery session 2

Materials: Bundling sticks Screen White board and marker Empty number line		
CPV Activity 3 (Wright et.al. 2012, pp. 85, 89 – 90, A5.3, IA5.1) Decrementing and incrementing in **10's** off the decuple with **screened** bundling sticks		
	Teacher	Learners
1.	X X X X X X X I I I I I X X X X X I I I How many sticks? Mangaphi amakhuni?	128
2.	X X X X X I I I I I X X X X X I I I How many sticks?	108
3.	X X X X X I I I I I X X I I I How many sticks?	78
4.	X X X X X I I I I I X X X I I I How many sticks? How many bundles?	88
5.	X X X I I I I I I I I How many sticks?	38
6.	X X X X X I I I I I I I I	58
7.	I I I I I I I I	8

CPV Activity 4 Decrementing and incrementing in **10's and 1's** off the decuple with **screened** bundling sticks		
	Teacher	Learners
1.	X X X X X X X X X X X X	120
2.	X X X X X X X X X X X	110
3.	X X X X X X I I I I I X X X X X I I I	117
4.	X X X X X I I I I I X X X X X I I I	107
5.	X X X X X I I I I I X X X X I I I	97
*6.	X X X X X I I I I I (- 32) X	65
*7.	X X X X X I I I (+ 18) X X X	83
*8.	X X X X X I I I I I (- 22)	41
* extension (10's and 1's simultaneously)		

SEAL Activity 2 (Adapted from Wright et.al. 2012, p. 116) Jump and Split Strategy With manipulation of setting		
Questions are posed in notation/written format Pairs must use their bundling sticks to try and find at least two ways to solve the following:		
	Split strategy	Jump strategy
1. 33 + 15	30 + 10 = 40 3 + 5 = 8 40 + 8 = 48	33 + 10 = 43 43 + 5 = 48
2. 48 + 21	40 + 20 = 60 8 + 1 = 9 60 + 9 = 69	48 + 40 = 68 68 + 1 = 69
*3. 69 + 18	60 + 10 = 70 9 + 8 = 17 70 + 17 = 87	69 + 10 = 79 79 + 8 = 87
4. 87 - 36	80 – 30 = 50 7 – 6 = 1 50 + 1 = 51	87 – 30 = 57 57 – 6 = 51
*5. 51 - 17	50 – 10 = 40 ???	51 – 10 = 41 41 – 7 = 34
* extension		

SEAL Activity 3 (Adapted from Wright et.al. 2012, p.122, A6.6)
2-digit addition and subtraction without regrouping

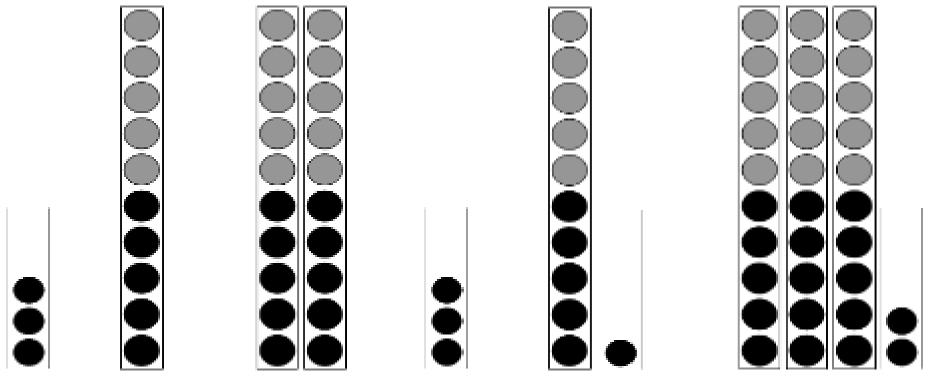
Questions are posed in notation/written format.
Wait for answer from groups.
(Pack out bundling sticks if needed. Check sticks/View and manipulate setting where needed.)
Notate strategies on white board and/or empty number line (ENL). (Split/jump or other strategies can be used.)

	Split strategy	Jump strategy
1. 52 + 35	50 + 30 = 80 2 + 5 = 7 80 + 7 = 87	52 + 30 = 82 82 + 5 = 87
2. 76 - 42	70 – 40 = 30 6 – 2 = 4 30 + 4 = 34	76 – 40 = 36 36 – 2 = 34
3. 63 + 46	60 + 40 = 100 3 + 6 = 9 100 + 9 = 109	63 + 40 = 103 103 + 6 = 109
4. 87 - 53	80 – 50 = 30 7 – 3 = 4 30 + 4 = 34	87 – 50 = 37 37 – 3 = 34
5. 95 - 32	90 – 30 = 60 5 – 2 = 3 60 + 3 = 63	95 – 30 = 65 65 – 2 = 63

Appendix K: Recovery session 3

Materials: dot-strip task board
10 strips, 100-dot squares
Two screens
White board and marker
Empty number line
Poster and permanent marker

CPV Activity 5 (Wright et.al. 2012, pp. 86-87, A5.4)
Decrementing and incrementing in **10's** and **1's**



The task board is gradually uncovered by using two screens. Only one increment is shown at a given time.
Teacher keep asking: How many dots are there altogether?
The following increments are used:

3
10 (total 13)
20 (total 33)
3 (total 36)
11 (total 47)
32 (total 79)

CPV Activity 6 Decrementing and incrementing in **10's and 100's** beyond 900 using screened dot strips/blocks (Wright et.al., 2012, p.87, A5.5, A5.6) + extension (*)
Introductory task: Show a 100 dot square (confirm that it is a 100). Put it under a screen. Keep adding one or two squares up to 13 squares. Keep asking: How many dots are there?
Activity Place out materials for 874. How many dots are there? Screen materials. Continue to add one or two ten strips up to 1124. * Add 100 squares up to 1424 * Decrement by 100's to 824

SEAL Activity 4 (Adapted from Wright et.al. 2012, p. 124) Jump Strategy with regrouping Give problem in written format Attempt to do without materials first Notate strategy	
Questions are posed in notation/written format Notation of the strategy is done on an empty number line Notation is also written on a poster to put up in the classroom for reinforcement purposes.	
	Jump strategy
1. 58 + 35	58 + 30 = 88 88 + 5 = 93
2. 53 + 19	53 + 10 = 63 63 + 9 = 72
3. 36 + 17	36 + 10 = 46 36 + 7 = 53
4. 71 - 24	71 - 20 = 51 or 71 – 20 = 51 51 – 4 = 47 51 – 1 = 50 50 – 3 = 47
5. 76 - 68	76 – 60 = 16 or 76 – 60 = 16 16 – 8 = 8 16 – 6 = 10 10 – 2 = 8
6. 83 - 26	83 – 20 = 63 or 83 – 20 = 63 63 – 6 = 57 63 – 3 = 60 60 – 3 = 57

Appendix L: Recovery session 4

Activities to enhance bonds of 10/addition and subtraction:

Fingers:

Use their fingers to practise bonds of 10.

E.g. 2 fingers up – 8 down
6 fingers up – 4 down

Fiz Pop – Bonds of 10

Teacher	Learners
Fiz	Pop
3	7
Fiz	Pop
6	4

Etc.

Playing cards:

Ten!

1 pack of playing cards / pair (remove picture cards and jokers)

1. Place 12 cards face up in 3 rows of 4
2. Take turns choosing cards which add to 10.
3. Fill in the spaces with new cards.
4. Play continue until now more sets of ten can be formed.
5. The winner is the one with the most cards at the end.

Dice Game

1. Throw 2 dice.
2. Add the numbers together.
3. Say how many more you need to make 20.

CPV activity: Stomp - Tap – Clap – Snap

(as described by Debbie Stott)

Snap of the fingers means “1”

Clap means “10”

Tap means “100”

Stomp means “1 000”

<u>Identify the following numbers:</u>

Clap, snap, snap, snap (13)

Clap, clap, clap, snap, snap, snap, snap, snap, snap (36)
Tap, tap, clap, clap, clap, clap, snap, snap, snap (243)

<u>Clap the following numbers:</u>
6
24
457

<u>Practice in pairs:</u>
One tap/clap/snap and the other one identifies the number.
Swop around.

* Can extend the activity to adding 10 or 100 or subtract 10 / 100 from the sequence.

(Games from The SANC booklet, 2012 and described by Debbie Stott)

Appendix M: Recovery session 5

Materials:	Numeral cards for CPV Activity 7 Adding to 50 game boards and spinner Counters Numeral cards with addition and subtraction problems White board/cardboard to notate strategies

CPV Activity 7 (Wright et.al. 2012, pp. 88, A5.7 and A5.8 combined)
Decrementing and incrementing in **10's** and **100's**

Use the following numbers printed on numeral cards.
Say: Read this number please
What is ten more?

Go through the list and repeat with:
What is ten less?
What is 100 more?
What is 100 less? (not for numbers smaller than a hundred!)

50
90
62
273
304
495
996
1007
Additional numbers:
70
110
84
582
709
897
994
2003

For variation learners could "Stomp, Tap, Clap, Snap" answers!

SEAL Activity 5 (Adapted from Wright et.al. 2012, pp. 126 - 127) Find the jump to the next decuple
Spin the spinner. Every learner gets a board. The learners must place a counter on the number needed to add to 50. Can also be done in pairs working on the same board. The first learner with 3 counters in a row, wins. Introduce the task with four ten frames and the ones on the spinner if needed.

SEAL Activity 6 Practise Jump to the decuple strategy with addition
Give the number problems in written format to groups. Ask them to solve them with "jump to the decuple" strategy. Get feedback. 46 + 7 (Strategy: 46 + 4 = 50 (7 = 4 + 3) 50 + 3 = 53) Notate strategy together. Now practise the following in pairs. Give feedback and notate the strategy. 73 + 8 67 + 6 25 + 18 38 + 24 56 + 29

Appendix N: Recovery session 6

Materials:	Numeral cards for CPV Activity 7 Number line Counters Numeral cards with addition and subtraction problems White board/cardboard to notate strategies

CPV Activity 7 (Wright et.al. 2012, pp. 88, A5.7 and A5.8 combined)
Decrementing and incrementing in **10's** and **100's**

Use the following numbers printed on numeral cards.
Say: Read this number please
What is ten more?

Go through the list and repeat with:
What is ten less?
What is 100 more?
What is 100 less? (not for numbers smaller than a hundred!)

60
90
83
385
602
291
994
2009
50
230
26
793
307
394
1991
2008

For variation learners could "Stomp, Tap, Clap, Snap" answers!

(Because of time constraints, we are going to do this in Lesson 7)

SEAL Activity 7

Addition and subtraction with regrouping: Jump, Split, Jump to the decuple

Materials: Printed number problems (one / pair)
Bundling sticks/counters/flard cards/number lines (only used if needed)

Work in pairs.
Every pair get a printed number problem.
They need to try to solve the problem in 3 different ways (hopefully using jump, split and jump to the decuple strategies)
Get group feedback after the first problem and reinforce all three strategies
Notate the strategies on a poster to be able to refer back to it.
The poster can be put up in class for further reinforcement.

37 + 5
26 + 28
64 - 6
75 - 8
34 - 27

Appendix O: Recovery session 7

Materials:	Number line Counters Numeral cards with addition and subtraction problems White board/cardboard to notate strategies

CPV – Quickly add a few questions like

What comes just before 500 / 300 / 1300 etc. to reinforce that.

SEAL Activity 7 Addition with regrouping: Jump, Split, Jump to the decuple
Materials: Printed number problems (one / pair) Bundling sticks/counters/flard cards/number lines (only used if needed) Cardboard to notate strategies
Work in pairs. Every pair get a printed number problem. They need to try to solve the problem in 3 different ways (hopefully using jump, split and jump to the decuple strategies) Get group feedback after the first problem and reinforce all three strategies Notate the strategies on a poster to be able to refer back to it. The poster can be put up in class for further reinforcement.

37 + 15
26 + 28

Appendix P: Recovery session 8

Materials:	Number line Counters Numeral cards with addition and subtraction problems White board/cardboard to notate strategies

CPV – Quickly add a few questions like

What comes just before 500 / 300 / 1300 etc. to reinforce that.

SEAL Activity 8 Subtraction with regrouping: Jump, Split, Jump to the decuple
Materials: Printed number problems (one / pair) Bundling sticks/counters/flard cards/number lines (only used if needed) Cardboard to notate strategies

Do 74 – 26 together.

Give the problem in written format. Ask kids to give answer. Have three posters ready with the three options for strategies "Split", "Jump", "Jump to the 10".

Ask kids to identify the strategy used (if another strategy was used, add that)

Notate the strategy. Do that for all three strategies. Try to use arrows and number lines to avoid formalizing it too much.

Notate the strategies on a poster to be able to refer back to it.

The poster can be put up in class for further reinforcement.

Now work in pairs.

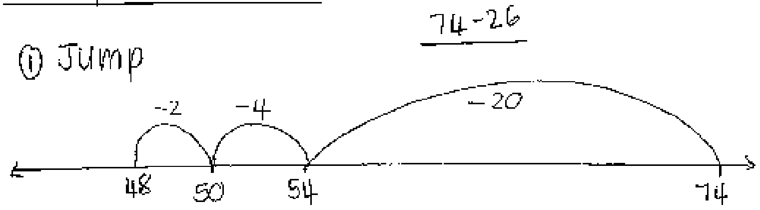
Every pair gets a printed number problem.

They need to try to solve the problem in 3 different ways (hopefully using jump, split and jump to the decuple strategies). Extra examples can be given to individual pairs working well. If needed, a pair can only work on one strategy to reinforce it.

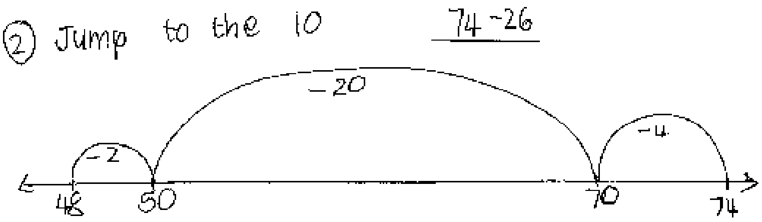
74 - 26
54 – 37
36 - 28

Example: 74 − 26

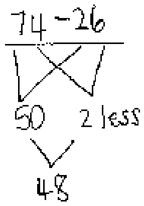
① Jump



② Jump to the 10



③ Split



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