THE DEVELOPMENT OF THE COMMON FRACTION CONCEPT IN GRADE THREE LEARNERS

A thesis submitted in fulfilment of the
requirements for the degree of

MASTER OF EDUCATION

of

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by

CLAIRE ANNE FRASER

November 2000

Supervisor: Ms Brenda Hayward
Co-supervisor: Professor P. Irwin

The financial assistance of National Research Foundation (NRF) towards this research is hereby acknowledged. Opinions expressed and conclusions arrived at, are those of the author and are not necessarily to be attributed to the National Research Foundation.

ABSTRACT

Over a period of nine months in 1999, a longitudinal teaching intervention was undertaken with Grade 3 learners in the Fort Beaufort district, Eastern Cape. Working in the interpretive paradigm, the intervention focussed on:

- the development of the common fraction concept,
- the relevance of the hierarchy of Murray and Olivier's Four Levels of Development in common fractions and
- whether learners' informal knowledge could be utilised in developing this concept.

Using the Problem-centred approach to teaching mathematics, problems set in real-life contexts were used as vehicles for learning. Learners were required to discuss, reflect and make sense of the mathematics they were doing.

Participant observation, completed worksheets and unstructured interviews with learners, formed the primary method of data collection. Learners' work was analysed and classified according to the method used and manner in which the solution was notated.

Results showed that learners were able to achieve a significant degree of success in developing a stable common fraction concept. Learners were afforded opportunities to construct their own ideas and to develop a deeper understanding of the concept. Many methods used were based on their informal knowledge of sharing. Learners made sense of realistic problems using drawings, and invented their own procedures.

Apart from Level One, Phase Three, all Murray and Olivier's Levels of Development could be identified during the research.

This study will provide educators with valuable information on how learners solve mathematical problems involving fractions and how informal knowledge can be used as a foundation on which to build.

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‘I can do all things through Him who gives me the strength’ (Philippians 4:13). Praise and thanks to you, O Lord, for the strength and wisdom that you have constantly provided me with throughout this study.

My debt to friends and colleagues is great. Firstly to my supervisor, Ms Brenda Hayward and co-supervisor, Professor Pat Irwin, I owe my sincere thanks for your constructive guidance and patience during this research. I wish to thank Mr Marc Schafer for his assistance during the early stages of this study. I record my gratitude for the generous assistance given by the Principals, staff and learners of the two schools where this research was conducted.

I am deeply indebted to my very dear friend, colleague and mentor, Mrs Hanlie Murray. Thank you for your wise counsel, guidance, encouragement and continuous availability, but most of all, for believing in me. My grateful thanks to my loyal friends who listened and encouraged me in my work and to Bevan Hobbs, for supplying the necessary computer back-up.

My mother and son deserve particular mention. Thank you Mom, for your support, love, prayers and constant encouragement. Thank you Stewart, for constantly encouraging me to complete my work and for all the interruptions!

Finally, to my husband Gavin, this work owes far more than any formal acknowledgement can convey. I thank you for your constant encouragement throughout, for your wise judgement and for your long-sustained labours in checking and proof-reading the thesis. This work would never have been completed without you having been at my side.

I wish to dedicate this study to my late father, Peter, my mother, Renee, my husband, Gavin and my son, Stewart.

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CHAPTER 1

INTRODUCTION

Recent research shows that some learners have great difficulty in solving problems involving common fractions (Pothier and Sawada, 1990; D'Ambrosio and Mewborn, 1994; Olivier *et al*., 1990; Piel and Green, 1994). It is a widely held view that the common fraction concept (See Section 2.5) is one of the most difficult mathematical concepts to teach in the primary school and that traditional instruction has been ineffective, as learners have failed to develop the concept of a fraction and to gain a sense of fractional number (Kamii and Clark, 1995; Strang, 1990; Kerslake, 1986 and Hunting, 1980; Kieren 1988, both cited in Steffe and Olive, 1990). Traditional instruction involved learners being taught to apply rote procedures and follow set rules when solving problems involving fractions.

Murray (pers. comm., 1999) feels that the fraction concept is not well developed and stable[1] and that the concept should be anchored in word problems and developed over a long period of time. According to Murray *et al*. (1996a) and D'Ambrosio and Mewborn (1994), misconceptions and limiting constructions (See Section 2.7 and 2.8) can occur from teaching learners rote procedures and set rules.

Many researchers (Murray *et al*., 1996a, 1998; Ball, 1993; D'Ambrosio and Mewborn, 1994; Empson, 1995; Kamii and Clark, 1995; Mack, 1990; Mack, 1995; Piel and Green, 1994; Pothier and Sawada, 1990) have been looking at alternative approaches to the teaching and learning of the common fraction concept. Understanding of common fractions is seen as an important foundation for building understanding for decimal fractions, percentages and our decimal measurement system, as well as for later algebra and algebraic manipulations, probability theory and statistics (Lebethe *et al*., 1997). It is therefore of vital importance that this concept be soundly developed and that it become stable.

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[1] Knowledge about a mathematical structure is said to be stable if the learner can solve a problem of the same structure, set in a different context and with different numbers, at a later date.

One of the crucial issues currently facing mathematics educators is the need to assist learners to develop their own meaning for mathematical symbols and procedures. As Hiebert *et al*. (1997:2) suggest:

> "…When we memorize rules for moving symbols around on paper, we may be learning something, but we are not learning mathematics. Knowing a subject means getting inside it and seeing how things work, how things are related to each other, and why they work the way they do".

The constructivist model of knowledge, which is embedded in the cognitive learning theory, emphasises that learner's need to construct and find meaning for the world around them. This study relies on the learners' understanding and making sense of the mathematics they do. It is necessary for mathematics educators to know how learners think and solve different mathematical problems. This information will enable mathematics educators to refine their thinking and identify different variables that will positively or negatively affect learners. It could influence educators and the authors of mathematics textbooks, as well as have an influence on defining and planning of curricula.

The primary aim of the study was to investigate how learners develop their own conceptual and procedural knowledge of common fractions using real-life situations. Secondly, this study aimed to investigate whether the hierarchy of the four Levels of Development, as outlined by Murray and Olivier (1989), which are applicable to whole number concept, are applicable to common fractions (See Section 2.3.2). Finally, an attempt was made to explore how the informal knowledge of learners could be used in developing the concept of common fractions with Grade 3 learners.

This study took the form of a longitudinal teaching intervention conducted with Grade 3 learners over a period of nine months. Working in the interpretive paradigm and using the Problem-centred approach (See Section 2.2) to teaching mathematics, the study focused on the development of the common fraction concept. The researcher identified two schools in the Fort Beaufort district in the Eastern Cape, South Africa in which to conduct the study. One school is situated in Fort Beaufort and the other, 20 kilometres out of town. The school in Fort Beaufort (School A) is a dual-medium, co-education primary school, with approximately 412 learners. The other school

(School B) is an English medium, co-educational, private primary school with approximately 95 learners.

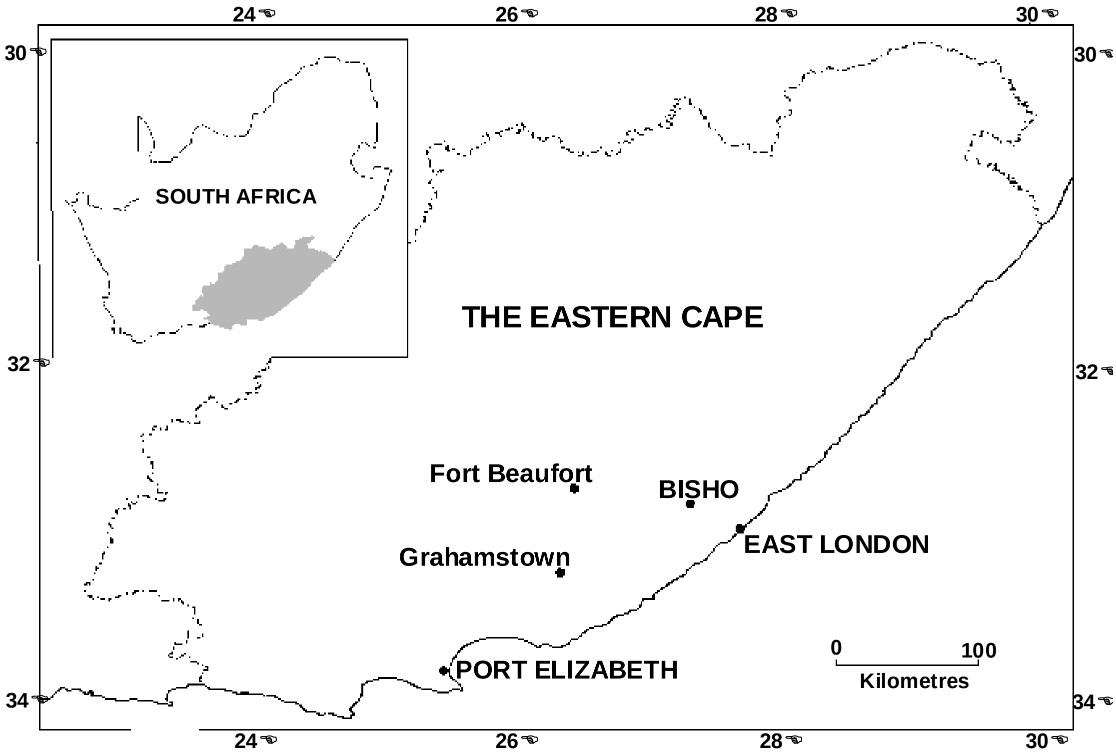


Figure 1.1 Map of Eastern Cape and South Africa

The learners in School A were from Afrikaans speaking families with the exception of one learner who had English as her home language. School B had learners with English, Afrikaans and Xhosa as their home languages.

Research (Mack, 1990) has indicated that when learners come to school they possess a rich store of informal knowledge about mathematics. One of the points of departure used in this study was the informal knowledge that learners possess. Mathematical problems set in a real-life context (See Section 2.6 and 4.2.2) were posed to the learners. These mathematical problems became the vehicles for learning. Posing problems set in a real-life context assisted learners to make sense of the mathematics they were doing, thus enabling them to understand the common fraction concept. These experiences allowed learners to invent procedures for problems that they had not previously encountered and in so doing construct their own mathematical knowledge.

Since the beginning of the researcher's teaching career, she had been involved in teaching primary mathematics. This has aroused an interest in the development of concepts and computational methods used by learners in primary school mathematics. As a mathematics educator at a teacher training college, the researcher feels that it is of utmost importance to keep abreast with latest research done in the field of primary mathematics. Having last taught in primary schools 19 years ago, the researcher was eager to use the Problem-centred approach to mathematics (See Section 2.2) with Foundation Phase learners. This would enable her to have practical, first hand knowledge about the latest methodologies and therefore be able to teach the students with more understanding on the subject.

Political transformation in South Africa, since 1994 gave rise to and still involves ongoing policy development and extensive restructuring, the education system being central among them. In March 1997, the Education Ministry launched the new curriculum under the title *Curriculum 2005*. The transformation of education is based on the adoption of an integrated system of life-long learning Hughes (1998). The Outcomes-Based Education (O.B.E.) curriculum intends to replace content-based education with outcomes-based education and the teacher-centred pedagogies with more learner-centred pedagogies (Le Grange, 2000). Eight learning areas have replaced the 42 school subjects in order to promote a more holistic approach to teaching and learning. Mathematics Literacy, Mathematics and Mathematical Sciences form one of the eight learning areas. The O.B.E. system was introduced into the Foundation Phase in South Africa in 1998.

Following Flanagan (1998), learners need a strong foundation of general education, the desire and ability to continue to learn and develop new knowledge, skills and technologies that will allow for flexibility. South Africa needs people who are able to solve problems, accept responsibility for the way in which they complete tasks and who are able to work cooperatively. Providing educational opportunities and processes should prepare learners to become the people the country needs.

This researcher attempted to carry out this research in line with the O.B.E. policy. Learners were encouraged to solve problems, develop mathematical skills, work in groups, interact with each other and accept responsibility for their learning.

Chapter 2 deals with recent literature on the development of the fraction concept. The methodology of the Problem-centred approach as well as the development of classroom culture is discussed. Piaget’s types of knowledge, Murray and Olivier’s Levels of Development and the Van Hiele Model are also outlined in this chapter. Chapter 3 describes the methodology applied in this study as well as the setting of the research.

Chapter 4 begins with a description of a typical lesson and then continues with how the researcher selected tasks aimed at developing the fraction concept. The analysis of data collected to develop the fraction concept, is written up in the chapter. Chapter 5 deals with the analysis of data for the development of Social Knowledge, while Chapter 6 deals with comparison and equivalence of fractions. Problems involving division of fractions were analysed in Chapter 7, while addition of fractions and iteration were analysed in Chapter 8. Chapter 9 deals with the analysis of data on ‘fractions as a part of a collection of objects’. The conclusion, evaluation and recommendations arising from the study are discussed in Chapter 10.

CHAPTER 2

LITERATURE SURVEY

Section 2.1 to 2.3 deals with the way learners learn and construct knowledge, the approach used during the lessons and the three models that were used in developing the materials. Section 2.4 to 2.11 deals with the present state of affairs in the teaching and learning of common fractions as well as a literature survey consisting of research done by leading mathematics educators in the field of common fractions. These educators include *inter alia* Cobb; Yackel and Wood, Murray, Olivier and Human; Empson, Pitkethly and Hunting; Steffe and Olive; Davis, Hunting and Pearn, D'Ambrosio and Mewborn and Mack.

2.1 THE THEORIES OF HOW CHILDREN LEARN

This study on the development of the common fraction concept weighs heavily on the learners' understanding of the mathematics they are doing as well as making sense of the mathematics that they are doing. Learning can be defined as a change in an individual that results from experience (Slavin, 1994). It involves the acquisition of abilities that are not innate.

Pavlov, Thorndike and Skinner have all contributed to the behavioural learning theory (Slavin, 1994). This theory is transferist and product focussed. The implications of the behaviourist theory apply especially to class management, discipline and motivation. The social learning theory on the other hand emphasises learning through the observation of others. Bandura and Meichenbaum felt that learning takes place through modelling (Slavin, 1994). Learners pay attention, retain models of behaviour, repeat these models and are motivated to repeat the behaviour.

Cognitive learning theory is learner and process focussed (Slavin, 1994). Cognitive learning theorists believe that learning is a result of attempts to try and make sense of the world. Piaget, a cognitive development and learning theorist, felt that children grow and develop when their equilibrium is disturbed.

Vygotsky believed that higher functioning usually exists in conversation and in collaboration among individuals before it exists within the individual (Slavin, 1994). He felt that learning takes place when children are working within their zone of proximal development (See Section 2.2.3). Tasks which the children have not yet learnt, but which they are capable of learning if performed with the assistance of a capable peer or adult at any given time, should be given to the learner to complete. Vygotsky believed in scaffolding, that is, supporting the learning process a great deal during the early stages and then diminishing the support and allowing the children to take on their own responsibility for learning (Slavin, 1994).

Slavin (1994) is of the opinion that the constructivist model of knowledge has its roots in the work of Piaget, Vygotsky, Bruner and Ausubel, whereas Ernest (1994) states that constructivism for mathematics education, draws on Kelly's Personal Construct Theory, Problem-solving research from the Gestalt Psychologists such as Wertheimer, Koffka and Köhler and early cognitive scientists such as Newell and Simon. This model can be summarised as: "Knowledge is constructed in the mind of the learner" (Bodner, 1986:873). Learners need to construct understanding and find meaning for the world around them. Etchberger and Shaw (1992) believe that data gathering initiates the construction of knowledge and that this step must be followed by reflection. Collaboration must occur while the learners are gathering data and reflecting. While learners are exploring, classifying, elaborating, questioning, evaluating, justifying, extending and arguing, they are making sense of and constructing knowledge. As meaning needs to be assigned, different ways of solving problems needs to be examined, tried, rejected, defended, justified and explained. Groups of learners need to reach consensus as to how and why the problem was solved. Accommodation to existing schemes occurs at this point thus enriching and expanding existing constructs. The learners then need to share their methods so other learners can benefit from them.

2.2 PROBLEM-CENTRED APPROACH

2.2.1 Constructing Mathematical Knowledge

All research mentioned in has the underlying assumption that learners' build on their previous knowledge. For educators to encourage learners to make sense and learn with understanding, it is necessary for materials to be developed that will encourage learners to engage with the materials. The type of research being undertaken requires a certain type of classroom structure that makes it necessary to establish a classroom culture and environment suitable for collaborative learning and teaching.

Contrary to a behaviourist view of teaching as the transmission of knowledge and learning as the absorption of knowledge, recent research indicates that students construct their own mathematical knowledge irrespective of how they are taught (Murray *et al*., 1993). Cobb *et al*. (1992:28) argue: "… we contend that students must necessarily construct their mathematical ways of knowing in any instructional setting whatsoever, including that of traditional direct instruction" and "The central issue is not whether students are constructing, but the nature and quality of those constructions".

Following Cobb *et al*. (1992) and Olivier *et al*. (1990), a problem-centred learning approach to mathematics teaching encourages students to construct their own knowledge and also attempts to establish individual and social procedures to monitor and improve the nature and quality of those constructions. This approach is based on the view that the construction of mathematical knowledge is, firstly, an individual and, secondly, a social activity, described by Ernest (1991:42) as follows:

> "(i) This basis of mathematical knowledge is linguistic knowledge, conventions and rule, and language is a social construction.
> (ii) Interpersonal social processes are required to turn an individual's subjective mathematical knowledge, after publication, into accepted objective mathematical knowledge.
> (iii) Objectivity itself will be understood to be social".

Murray *et al*. (1993:74) continue saying:

> "Whereas a traditional, transmission-type-teaching approach necessarily leads to subjective knowledge that is largely re-constructed objective knowledge, a problem-centred learning approach reflects the belief that subjective knowledge should be personal constructions and not re-constructed objective knowledge. This does not imply that learners are actually creating knowledge that does not already exist as objective knowledge, but learners in this approach construct their knowledge as new."

2.2.2 Five Social Components necessary in the learning environment

If one accepts that learners' learn by constructing their own knowledge, certain implications for setting up the learning environment and learning episodes need to be taken into account. Hiebert *et al*. (1997) identifies five social components that are necessary in the learning environment. They include:

1. the nature of the problems that the learners are asked to complete,
2. the role of the facilitator,
3. the social culture of the classroom,
4. the mathematical tools available, and
5. the extent to which all learners can participate fully in the mathematics community of the classroom.

2.2.2.1 The Nature of the Problems

The problems that the learners are asked to solve need to set the foundation for the system of instruction that is created. According to Romberg and Kaput (1999), different kinds of problems lead to different systems of instruction. They believe problems should:

- afford the learners opportunities to reflect and communicate about mathematics and do not require the learners to memorise rules, nor perceive that there is only one correct solution method,
- be seen as opportunities to explore mathematics and for the learners to come up with reasonable methods for solutions,
- afford the learners the need to see that there is something to work out and something to make sense of,
- be accessible, at some level, to all learners and must also offer the learners the opportunity to reflect on important mathematical ideas, and
- extend the learners by beginning with the learners' prior knowledge and gradually proceeding from their informal knowledge of ideas to more formal notions. The learners must be able to make use of knowledge and skills that they already have to begin developing a method for completing the task.

2.2.2.2 The Role of the Facilitator

The role of the facilitator is to facilitate conceptual understanding and he/she needs to be actively involved by sharing information when it is essential for solving a problem (Cobb *et al*., 1991; Hiebert *et al*., 1997; Carpenter and Lehrer, 1999). The facilitator should not be seen as the source of mathematical information, but should encourage the learners to devise their own methods. He/she expects every learner to become involved with the problem and to attempt to solve it. By selecting and posing appropriate sequences of problems the facilitator creates opportunities for learning.

The facilitator needs to establish a classroom culture that will enable the learners to work on problems individually and interactively, giving them the opportunity to discuss and reflect on their solutions and methods. The facilitator relies on the reflective and conversational problem-solving activities of the learners to drive the learning process. The learning process is supported by the facilitator participating in class discussions and sharing information with the learners being ever mindful that if he/she intervenes too much and too deeply, he/she can easily cut off the learners' initiative and creativity and remove the problematic nature of the material. It is of vital importance that a balance between allowing learners to pursue their own ways of thinking and providing important information that supports the development of understanding mathematics is achieved.

2.2.2.3 The Social Culture of the Classroom

A classroom is made up of a community of learners and the social culture of the classroom refers to the way that learners relate and interact with one another. Cobb *et al*. (1991:7) confirms that "…the development of classroom social norms (that) are crucial to the constitution of an inquiry mathematics tradition". It is important that certain norms and expectations of how learners interact with each other are established (e.g. learners may not copy from one another unless they understand the method used and can explain how the solution was obtained, not laughing at others mistakes and listening to each other's ideas, participating in the discussion).

Social interaction creates opportunities for learners to talk about their thinking and this discussion encourages reflection. Reflection is the "… the conscious examination of one's own actions and thought" and "…reflection plays an important role in solving unfamiliar problems" (Carpenter and Lehrer, 1999:22). Problem solving often involves consciously examining the relationship between one's existing knowledge and the conditions of a problem situation. Learners stand a better chance of acquiring this ability if reflection is part of the knowledge-acquisition process (Carpenter and Lehrer, 1999).

> "From a constructivist point of view, there can be no doubt that reflective ability is a major source of knowledge on all levels of mathematics…to verbalise what one is doing ensures that one is examining it. And it is precisely during such examination of mental operating that insufficiencies, contradictions, or irrelevancies are likely to be spotted". And, "…leading students to discuss their view of a problem and their own tentative approaches, raises their self-confidence and provides opportunities for them to reflect and to devise new and perhaps more viable conceptual strategies" (von Glaserfeld, 1991:xviii, xix).

Communication and reflection are essential components of classrooms where learners are expected to make sense of their mathematics. In order for the learners to have opportunities to reflect on and communicate about mathematics, the social culture of the classroom needs to be conducive to fostering open communication.

Four features of the social culture that encourage learners to treat tasks as real mathematical problems are recognised by Hiebert *et al*. (1997:9).

1. All ideas belong to all learners in the classroom. All ideas can help everyone's learning and therefore they need to be treated with respect. Learners need to appreciate and examine the methods of others.
2. Each learner has the right to solve the problem using the method that he understands and learners need to respect one another's methods. All learners have the freedom to explore and share these methods with their peers.
3. All mistakes must be seen as chances to learn. Errors can be examined and because of this, all learners can raise their levels of analysis. Mistakes need to be used constructively and not be covered up.
4. The final feature is the recognition of the correctness of the logic and method that is important and not the popularity of the presenter.

2.2.2.4 Mathematical Tools

Tools should be seen as supports for learning, however, using tools does not happen automatically, as meaning does not reside in the tools. Learners need to construct meaning for the tools they use. They need to work with tools over a period of time, try them out and watch what happens and in so doing, they will construct meaning as they use the tools.

> "A common impression is that the reform movement in mathematics instruction is mostly a recommendation to use physical materials to teach mathematics. We believe that the reform movement is about much more than using physical materials. We also believe that the discussion of mathematical tools would benefit from broadening the definition to include oral language, physical materials, written notation, and any other tools with which students can think about mathematics" (Hiebert *et al*. 1997:10).

Tools should be used to solve problems in a variety of ways. They can provide a record of something already achieved, be used to communicate effectively and/or used as an aid for thinking. Tools are likely to shape the way one thinks and different tools will influence the kinds of understanding learners develop. For example, learners think differently about numbers and because of the different tools they have used, they have developed different methods to add and subtract. Learners who have a units conception, are likely to add 34 and 51 by combining the three (tens) and five (tens) to get eight (tens) and then combining the four (ones) and one (one) to get five (ones). However, learners who have a counting conception are likely to count on from 51, perhaps 51, 61, 71, 81, 82, 83, 84, 85. Both methods are correct, but they show different understandings.

2.2.2.5 Equity and Accessibility

Every learner has the right to understand what they do in mathematics, to reflect on, and communicate about mathematics and that understanding is not just the privilege of the high achieving group (See Hiebert *et al*., 1997). "Our experience is that, given classrooms like those we describe here, girls and boys at all levels of achievement and from all backgrounds can understand what they do in mathematics" (Hiebert *et al*., 1997:11). Understanding supports understanding for all learners at all levels.

> "The essential features are intertwined and work together to create classrooms for understanding. They define a system of instruction rather than a series of individual components. It makes little sense to introduce a few features and ignore the rest; their benefits come from working together as a coherent, integrated system" (Hiebert *et al*., 1997:172).

2.2.3 Murray *et al*.s' (1993) three salient characteristics for their problem-centred classrooms

Murray *et al*. (1993) identified three salient characteristics for their problem-centred classrooms. These include:

1. Students are presented with problems that are meaningful and interesting to them, but which they cannot solve with ease using routinized procedures or drilled responses.
2. The teacher does not demonstrate a solution method, nor does he/she steer any activity in a direction that he/she had previously conceived as desirable, yet he/she expects every learner to become involved with the problem and attempt to solve it. Learners' own invented methods are expected and encouraged.
3. It is expected of learners to discuss, critique, explain and when necessary, to justify their interpretations and solutions.

The combination of characteristics one and two show why a problem-centred learning approach is incompatible with traditional transmission mathematics teaching (Murray *et al*., 1993). Characteristic one requires that the problems posed lie in the learners' zone of proximal development. Vygotsky's zone of proximal development can be defined as: "The difference between the level of the tasks that can be performed with the help of adults and the level of the tasks that can be solved with independent activity…" (Vygotsky, 1984:112, as quoted in Rosa and Montero, 1990:79). This implies that the individual can only solve this type of problem if help is available. Characteristic two means that the teacher does not supply that help even in the roundabout form of steering conversations towards a particular goal.

In order to determine the apparent conflict between characteristics one and two, characteristic three implies that opportunities for discussion and justification among peers, with the teacher as a facilitator, will provide the necessary support for learners to solve problems that lie in their zones of proximal development. Murray *et al*. (1993) circumscribes the teacher's role to that of facilitator, chairman, or if necessary, devil's advocate, and regard student's mathematical discourse the main vehicle for learning. This statement is in agreement with what Richards (1991:15) calls inquiry mathematics: "…asking mathematical questions; solving mathematical problems that are new to you; proposing conjectures; listening to mathematical arguments".

Learners need to see themselves as independent problem-solvers, accept responsibility for their own learning and also learn to respect and evaluate other points of view. Learners at times need to work individually, however, when they work co-operatively, they function better when interacting with peers of equal ability. Learners, when given the chance to choose groups, chose learners of equal ability. Murray *et al*. (1993:75) reported that:

> " … students obtain more satisfaction from constructing a solution to a problem than from simply producing the right answer, and furthermore, that they themselves believe that this process is aided by collaboration with peers of equal ability and not collaboration with more able students."

This is in contrast to Vygotsky's idea that learning should take place through collaboration with a more capable peer or adult. Murray *et al*. (1993) found that learners are able to understand and cope with learning through problem solving and social interaction with peers.

2.3 THEORIES USED IN DEVELOPING LEARNING MATERIALS

The materials used for this research were based on three models, namely, Piaget's Knowledge Types, Murray and Olivier's Levels of Development and Van Hiele's Model of how a mathematics concept can be developed. By keeping the Van Hiele and Piaget models in mind ensures that all knowledge types will be included in the study and that new concepts and social knowledge be introduced when relevant. Analysing the Levels of Development of the learners gave the researcher the ability to identify when problems of a more abstract nature could be introduced.

2.3.1 Piaget's Knowledge Types

Piaget differentiated between three different types of knowledge that aid understanding of the process by which a child learns (Bodner, 1986). As all three of these knowledge types need to be addressed in the development of the common fraction, it was necessary to use them as a guideline when compiling material for this study.

These knowledge types, according to Murray *et al*. (1994), can be applied to mathematics as follows:

1. Physical knowledge. This knowledge is gained from the learners' physical environment and his/her physical manipulation of physical objects. The learner gains knowledge when he:
   - counts real objects,
   - measures (length, mass, volume),
   - explores shapes by building,
   - folds paper and draws patterns,
   - investigates angles and the properties of figures, to name but a few.

   This type of knowledge forms the base for the learners' knowledge of number. Only by counting and handling real objects can he/she initially learn about number.
2. Social knowledge. This type of knowledge makes communication about mathematics possible. It includes terminology such as number names, notation such as numerals and conventions such as multiplying first as in this case:
   $4 + 3 \times 5$, but by adding first in this case, $(4 + 3) \times 5$, in this case.
3. Logico-mathematical knowledge. It is the type of knowledge that every learner constructs by him/herself, for him/herself by thinking beyond the knowledge that was obtained from handling physical objects or from listening to others. The construction of logico-mathematical knowledge involves mentally creating further knowledge by thinking about previously constructed knowledge and constructing further relationships and patterns, for example, the properties of numbers and operations.

It is of great importance that the learning activities and social interaction patterns through which learners learn are appropriate to the type of knowledge concerned, and specifically that they convey true messages regarding the nature of the specific items of knowledge. The facilitator therefore needs to initiate different interaction patterns according to the type of activity that he/she has designed for the pupils. The activities should aim at helping learners to construct physical knowledge, gain social knowledge as well as assisting them to construct logico-mathematical knowledge. Learners need to be assisted in acquiring fluency with numbers, estimation and computational skills. Many activities will combine more than one of the above.

2.3.2 Murray and Olivier's Levels of Development

While being exposed to physical, social and logico-mathematics, learners develop computational strategies for the basic arithmetical operations. Murray and Olivier (1989) formulated a semantic model (cf. Baroody and Ginsburg, 1986) describing the development of learners' computational strategies for the basic arithmetical operations through four increasingly abstract levels, each level associated with its prerequisite understanding of number and numeration. These Levels of Development for number sense clearly indicate, by the responses that learners make, the level of the learners. These responses act as guidelines when developing materials as abstract responses cannot be required from learners not at that particular level.

The four levels are described below:

Level One of Development

A learner who is on Level One has the ability to count a number of objects and has a knowledge of the number names and their associated numerals, without assigning meaning to the individual digits of a numeral. It is necessary for learners at this level to 'create' each number before he/she can use it. This needs to be done repeatedly. The typical computational strategy at Level One is 'counting all'. If the learner has to compute 2 + 3, he/she will first count 'one, two' and 'one, two, three' before he/she can use the number and then to find the solution, he/she will have to count, 'one, two, three, four, five'.

This level can be further divided into three phases. Phase 1 is where the learner cannot solve the problem unless the physical objects in the problem are available to be manipulated. Where the learner is able to solve the problem involving non-available objects by representing the objects by means of his fingers, bottle-tops or drawings of the objects, but still needs a full physical representation of all the objects involved, would place the learner in Phase 2. For Phase 3, the learner needs all the same actions to solve the problem, but he/she now performs them in his imagination, and does not rely on any kind of physical representation. He/she actually uses the number names to solve the problem (e.g. I have 6 biscuits and I gave 2 to Gavin. How many will I have left? I have 4 left. I gave number 5 and number 6 to Gavin. I kept one, two, three and four.).

Level Two of Development

On reaching Level Two, the learner has the ability to conceptualise a given number as an abstract unit item with a meaning independent of physical referents or counting acts (the numerosity of the number has been acquired). Level Two understanding of number is characterised by 'counting-on' computational strategies. If the learner was asked to find the total number of balls he/she was given if he/she was first given four and then another three, the learner would say: 'four;…, five, six, seven'.

Level Three of Development

At Level Three, the learner has the ability to decompose or change numbers in the task so that he can perform another task that will give him/her the same solution. (e.g. $26 + 38 \rightarrow 20 + 30 \rightarrow 50 + 6 \rightarrow 56 + 8 \rightarrow 64$). This provides the learner with the computational basis to use thinking strategies i.e. solving a computation by relating it to other known results.

Level Four of Development

On Level Four, the learner has the ability to interpret a two-digit number as consisting of groups of tens and some ones, without losing the meaning of the number as a number. Level Four understanding of a number is the prerequisite for the meaningful execution of the standard algorithms in its most sophisticated form. e.g.

$$\begin{array}{r} 34 \\ +\ \underline{28} \\ 62 \end{array}$$

2.3.3 The Van Hiele Model of Developing a Mathematical Concept

Van Hiele (1973) described the way in which understanding of a new topic may develop. His model can serve as a useful guide to educators as to how to present a new topic. It could guide the educator as to what activities and tasks to pose to the learners and how to sequence the activities. The model is most effective to help educators to know when to introduce formal notations and logical reasoning.

Van Hiele’s model (1973) described five levels of progressively more abstract understanding. Murray (pers. comm.1996) later reduced these levels to three, with the approval of Van Hiele. Although this model was initially applied to geometric concepts, the Van Hiele model is extremely powerful and applicable to the learning of other mathematical areas as well. The model can be used for the introduction and development of any new topic to learners of all ages. The researcher attempted to base the learning sequence of this study according to Van Hiele’s Model.

Level 0

At level 0, the learner should be exposed to and should grapple with situations in which the new topic, or several parts of the topic, is an intrinsic part. These activities should present authentic situations of which the topic or some of its basic concepts form a natural part. No formal terminology, definitions or methods of solutions are introduced. The learners simply experience the situations and solve the problems related to these situations by whatever means they have at their disposal.

Common fractions need “sharing out and cutting up” situations with sausages, chocolate bars and other rectangular figures as level 0 activities. The activities need not be concrete and should not be complicated stories or models, which are more difficult to understand than the concept itself. The learner may choose to use diagrams/sketches to solve the problem.

At this stage, exact terminology and symbols have not yet been introduced. However, as they are needed, formal and exact terminology and symbols should be introduced. For example: “When we share 7 chocolate bars equally among two friends, each friend gets 3 chocolate bars and a ‘piece’. But, when sharing 7 chocolate bars among three friends, the ‘piece’ will be smaller than the previous ‘piece’. So, the need arises to introduce the terminology of halves, thirds, quarters, and so on. For example, when solving an algebra problem, it becomes tedious to say “four times the length of the side. What about calling the length of the side x units, and then say $4x$?”

The learners should gradually be required to describe the concepts or algorithms they are dealing with more accurately. For example: “What is a third of a chocolate bar?” After some experience with sharing out situations, the learners should be encouraged to explain that a third is one of three equal pieces and that one of these three equal pieces, repeated three times, will make a whole chocolate bar. By doing this, the learners will be prepared for more formal definitions, symbols and logical conclusions.

<u>Level 1</u>

At level 1, the learner has constructed concepts or ideas in such a way that they can be used or thought about separately from or removed from the situations in which they feature naturally. It is however, very important that these ‘formalised’ or ‘abstract’ concepts should exist within a network of related concepts.

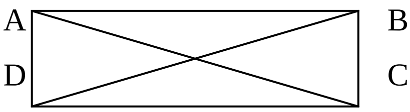
Learners can formulate definitions and accept them as binding for logical arguments and discussion in level 1. At level 0, arguments can only be based on statements of belief. At level 1, arguments can be based on logical conclusions. A level 0 argument would be: “This is a square because it looks like a square”, but, a level 1 argument would be: “This can be called a square because it has four equal sides and one interior right angle so although it may look like a rhombus, according to what is decided, it is a square”.

The Van Hiele model advocates that learners should be exposed to authentic experiences of a concept over a period of time before any formalisation is attempted, or abstract notations and definitions are introduced.

Level 2

At level 2, the learners have developed a network of related concepts that have become sufficiently stable and complete. The connections that the learners make between the concepts now develop into a new network – the level 2 network. At this level a logical system can be developed. The logic of the reasoning process is important.

For example: Prove that AC = BD.



A level 2 learner would reason that because Δ ABC and Δ BAD are congruent, AC = BD.

A level 1 learner would reason that he/she knows that AC = BD.

The researcher did not proceed beyond level 1 as learners in Grade 3 rarely reach beyond level 0.

Due to the fact that both Van Hiele and Murray and Olivier refer to ‘levels’ in their models, the researcher would like to identify between the various ‘levels’. The Van Hiele ‘levels’ are only referred to where teaching materials were being developed whereas Murray and Olivier’s ‘levels’ refer to the development of the learners.

2.4 THE PRESENT STATE OF AFFAIRS

Research has been done on the difficulty elementary school learners’ experience when learning common fractions and on teaching programmes that have been designed for teaching common fractions at different grade levels (See Pitkethly and Hunting, 1996 for a review of the recent research in the area of initial fraction concepts).

According to Davis *et al*. (1993:63), the teaching and learning of fractions in the broader scheme of things is a "dismal failure". Results documented by Hart (1981), as cited in Davis *et al*. (1993:63) on the understanding of fractions by children in the 11 – 16 age range are described as 'depressingly negative'. It was found that learners, who were solving problems involving fractions by applying rote procedures and rules, were not developing the fraction concept, nor were they gaining a sense of a fractional number. Learners were not using models to solve equivalent fractions nor were they applying fractions to problem situations (See Strang, 1990; Kerslake, 1986; Kieren, 1988; Mack 1990; Pothier and Sawada, 1990).

Steffe and Olive (1991:22) supported the fact that the above-mentioned goals were not being achieved by the way in which fractions are being taught. They suggested that in order to achieve these goals, learners need to make the connections between what is taught and their informal ways and means of operating. This resulted in a major transformation in teaching and learning of common fractions.

2.5 THE FRACTION CONCEPT

Rational numbers are involved in representing and controlling part-whole situations and relationships. They are fundamental to measuring continuous quantities as well as being involved where quantities, particularly continuous quantities, are divided. "…Physical contexts may be discrete or continuous" (Pitkethly and Hunting, 1996:9). Although discrete and continuous quantities are not the same, they are connected. In a discrete context, the fractional parts are 'countable' (e.g. $\frac{1}{4}$ of 20 sweets) and in a continuous context, the parts are 'measurable' (e.g. $\frac{1}{2}$ the length of the table). Rational numbers are involved in any quantitative comparison of two quantities.

Rational numbers consist of many interwoven strands (Kieren, 1988; Kieren, 1976, 1980 and Behr, 1983, as cited in Pitkethly and Hunting, 1996). Five sub-constructs, which according to Behr *et al*. (1993), as cited in Pitkethly and Hunting (1996:6), have "stood the test of time", were identified by Kieren (1980). Together these five sub-constructs form the basis for mature functioning for rational numbers. They are:

1. part-whole relations (e.g. $\frac{1}{4}$ is one of four equal parts of a whole and $\frac{1}{4}$ is one of four equal parts of a number of objects),
2. ratios (e.g. a concrete mix suitable for paths is 1 part cement, 2 parts sand and 3 parts gravel),
3. quotients (e.g. 25 ÷ 5 can be written as $\frac{25}{5}$),
4. measures (e.g. three fifths of a metre), and
5. operations (e.g. $\frac{3}{5}$ can simply imply "times three and divide by five").

Other researchers, namely Murray *et al*. (1998) and D'Ambrosio and Mewborn (1994) have by contrast, identified two sub-constructs:

1. the part-whole relationship between the fractional part and the unit where the unit can be a single object or a number of objects (e.g. $\frac{1}{4}$ is one of four equal parts of a whole and/or $\frac{1}{4}$ is one of four equal parts of a collection of objects), and
2. the idea that the fractional part is that part which can be iterated (repeated) a certain number of times to produce the unit (e.g. $\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=1$ or a third of a sausage taken three times will form a whole sausage).

Murray *et al*. (1998) identified that fractions can be used in different ways and have different meanings, for example, the part-whole as mentioned above, ratio (e.g. Ann earns half of what her mother earns), measure (e.g. twenty centimetres wide) and a quotient (e.g. $\frac{3}{4}$ means 3 ÷ 4). They identified five topics that needed to be addressed. These are:

a) developing the fraction concept through sharing situations
b) introducing realistic problem situations for operations involving fractions
c) comparison of fractions
d) equivalence

e) introducing the fraction symbol (e.g. $\frac{1}{4}$)

For rational number knowledge building, the initial fraction concept emphasis must be based on unit fractions (where the numerator is equal to 1). Learners need to be able to divide a unit (whole or a number of objects) into equal parts, divide a part into sub-parts and to iterate parts to form a unit. Many researchers: Pothier and Sawada, 1990; Murray *et al*., 1996a; Mack, 1990; Kieren, 1988; along with numerous others (see Pitkethly and Hunting, 1996) have emphasised the importance of partitioning schemes for establishing the fraction concept. They felt that the development of fraction number knowledge grows out of and organises the phenomenon of sharing and dividing up. Empson (1995:114) confirmed: "… using equal-sharing problems can help children learn and understand fractions" (e.g. Share three chocolate bars equally among two children so that they each get the same amount).

Experience in partitioning enables the learners to observe the relationship between the size and number of parts into which the whole has been partitioned. A flexible concept of the unit is also important for later rational number interpretations. Pitkethly and Hunting (1996) are of the opinion that a learner who is beginning to learn rational numbers is actually transforming existing knowledge through experience.

Researchers have placed the part-whole sub-construct, where the fraction is part of a single unit and where the fraction is part of a number of objects, in a special role. Behr *et al*. (1983), as cited in Pitkethly and Hunting (1996), hypothesised that the fundamental construct for rational number development is based on the ability to partition either a continuous (measurable) quantity or a set of discrete (countable) objects into equal size sub-parts. Kieren, (1988:6), identified the part-whole relationship as, "the foundation of rational number knowledge and fundamental to all later interpretations".

2.6 INFORMAL KNOWLEDGE

Informal knowledge is "…the personal, experiential information an individual brings to the instructional setting" (Piel and Green, 1994:44). Pitkethly and Hunting (1996:7) on the other hand define it as "…consisting of imagery, lived through experiences, thought tools and constructive mechanisms, all of which provide a basis for formal knowledge." Mack (1990:16) characterised informal knowledge as "…applied, real-life circumstantial knowledge constructed by the individual learner that may be either correct or incorrect and can be drawn upon in response to problems posed in the context of real-life situations which are familiar to him/her". This type of knowledge will be referred to as informal knowledge in this research.

Researchers have documented that learners possess a rich store of informal knowledge related to a variety of mathematical content domains and that they are able to draw upon this knowledge to solve everyday problems (Murray *et al*., 1996a; Mack, 1990; Empson, 1995). It is suggested that young children also possess similar informal knowledge about fractions that should be used in the same way. (Steffe and Olive, 1991; and Baroody and Hume, 1991, as cited in Murray *et al*., 1996a).

Informal knowledge plays an important role in mathematics learning and by interweaving informal and formal knowledge, meaning can be brought to the language and symbols of fractions. Instead of ignoring or suppressing learners' informal knowledge, educators should recognise, encourage and build on learners' informal knowledge (See Pitkethly and Hunting, 1996). This could be incorporated by introducing equal-sharing situations as appropriate links to young learners' intuitions (Murray *et al*., 1996a).

Mack's research (1990) found that learners came with a substantial amount of informal knowledge about fractions that enabled them to solve problems that were presented in real-life contexts (contexts that were familiar to the learners and that occur in everyday life). Both Mack (1990) and Empson (1995) found that learners were able to use informal knowledge to solve problems that involved partitioning. Mack (1990) found that learners initially disconnected their informal knowledge from their knowledge of fraction symbols, procedures and concrete representations. When learners correlated problems represented symbolically to problems in real-life situations, learners made use of their informal knowledge. However, learners encountered difficulties from knowledge of rote procedures when attempting to solve problems represented symbolically and in real-life situations. Mack (1990) also found that there was limited transfer of knowledge even in learners who were able to build on their informal knowledge in other contexts. Piel and Green (1994:44) confirmed that:

> "…a lack of connectedness between computational algorithms and what is known about computational objects (intuitive knowledge) may be one reason students conceptual understanding of fractions is so limited."

By using informal knowledge to explain their solutions, learners were able to solve numerous problems set in real-life contexts however they were not able to solve problems represented symbolically unless they were similar to the real-life problems. Solutions were often explained using incorrect social knowledge related to formal symbols and computational methods (See Mack, 1990; Piel and Green, 1994).

Contrary to other research, Mack (1990) felt that learners have very little understanding of fraction symbols (e.g. $\frac{1}{5}$) and computational methods (See Section 2.4). Her study showed that learners have a rich store of informal knowledge about fractions and that they are able to build on and so give meaning to formal symbols and methods. Learners were able to use their partitioning approach to solve many different problems in meaningful ways. Mack's results (1990:29) suggested:

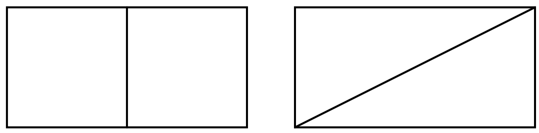
> "… despite limitations that exist, learners are able to relate fraction symbols to informal knowledge in meaningful ways provided that the connection between the informal knowledge and the fraction symbols is reasonably clear".

2.7 LIMITING CONSTRUCTIONS

Learners construct their own knowledge based on their experiences and at times their experiences only provide them with a limited view of a particular concept. This may result in learners 'closing off their minds' to other aspects of the concept. These limiting experiences may result in limiting constructions.

Potentially learners hold a large number of limiting constructions about fractions. Some of these limiting constructions originate in the learners' pre-school or outside school experiences and others, after teaching (See D'Ambrosio and Mewborn, 1994). Limiting constructions may incorporate whole number schemes, limited part-whole contexts, the knowledge of $\frac{1}{2}$ and perceptual and visual representations (Davis, 1989, as cited in Pitkethly and Hunting, 1996; and Murray *et al*. 1999).

Because of their lack of precision (e.g. paper-folding, cutting-up situations), many materials and manipulators (See Section 2.8), as well as extensive use of pre-partitioned models and pre-partitioned shapes in textbooks, could be the cause of limiting constructions (See D'Ambrosio and Mewborn, 1994; Pothier and Sawada, 1990; Empson, 1995). Teachers use pictures and manipulatives when they introduce fractions, thus teaching it mainly perceptually and figuratively. These concepts formed by learners stay figurative and learners do not learn about fractions (Kamii and Clark, 1995). (e.g. they are not able to see that two halves of a certain shape are the same even though they have different shapes.)



Emspon (1995) was of the opinion that when learners use pre-formed manipulatives to represent fractions, they may not realise the significance of all the pieces being the same size and in the context of equal-sharing situations, the learners need to realise the pieces must be of equal sizes. In other words the manipulatives dictate the type of thinking that takes place (See Section 2.8).

Paper-folding and similar activities used to develop the concept of common fractions has had limited success. Due to the imprecision of paper-folding, using this method to develop iteration of a unit fraction to produce a whole unit is 'totally ineffective and generated incorrect findings', the whole unit often being larger than the original whole unit (D'Ambrosio and Mewborn, 1994:153).

Learners often used additive reasoning skills to predict the number of pieces they would find upon opening a repeatedly folded strip. Learners would, for example, after folding a strip in half and half again to produce fourths, would be asked to predict what the result would be after another fold. Many learners would then predict that the paper would be divided into sixths rather than eighths. D'Ambrosio and Mewborn (1994:157) are of the opinion that this construction would later "…greatly limit their understanding of equivalent fractions".

Learners bring a wealth of knowledge related to whole numbers to the study of fractions (See Mack, 1995). Whole number schemes encourage the learners to interpret the fraction symbol as two separate whole numbers. Because learners perceive the symbol to be made up of two whole numbers they apply whole number strategies to fraction problems.

Where learners have had insufficient or no experience of the part-whole context, where the fraction is part of a collection of objects, limiting constructions may occur. The learners find it difficult to solve problems, where a fraction is part of a collection of objects, if this concept has not introduced within a reasonable time.

Other limiting constructions could result if only certain fractions are introduced to the learners, for example, halves and quarters and not others, for example, thirds, fifths, and so on. If learners are only introduced to selected fractions they are of the opinion that these are the only fractions there are.

Algorithmic halving, is the act of halving by subdividing at approximately the midpoint of continuous quantities that are repeatedly created by the subdividing process (Pothier and Sawada, 1983). This strategy of algorithmic halving may hinder the learners' ability to develop partitioning schemes to create fractions that have odd numbers as denominators, e.g. thirds.

Davis (1989), as cited in Pitkethly and Hunting (1996), indicated that limiting constructions are very resistant to change, whereas, Empson (1995) and Murray *et al.* (1996a) felt that appropriate programmes for learning fractions in the lower elementary grades could prevent the problems above.

2.8 MANIPULATIVES

Manipulatives refer to concrete teaching and learning aids that could be used in the development of a concept. When manipulatives were not made available to the learners, learners were encouraged to create their own representations to solve the problems and to represent their own thinking on paper in the form of diagrams/pictures (Empson, 1995; Murray *et al*., 1996a). By withholding manipulatives, the learners' thinking could dictate the situation and not the manipulatives. This enabled the learners' to develop their reasoning skills. Visual representation of the fractional part and pre-partitioned materials must not become the only point of reference for the concept and they need to be withheld until the concept of a fraction has been developed by the learner themselves (Murray *et al*., 1999; Bonotto, 1993 as cited in Pitkethly and Hunting, 1996)

2.9 TEACHING EQUIVALENT FRACTIONS

Traditional instruction for the teaching of equivalent fractions is widely held to be ineffective (See Kamii and Clark, 1995). Learners did not have the ability to call the same fractional size by different names, (e.g. $\frac{2}{6} = \frac{1}{3}$), they could not ignore or imagine partition lines nor were they able to think flexibly. Researchers (Mack, 1990; Kamii and Clark, 1995; Empson, 1995) have found that if learners solve realistic equal-sharing problems (problems that learners can identify with and make sense of), the concept of equivalence could grow from the learners' own thinking. Comparing solutions, as well as discussion and reflection with their peers, would assist learners to develop the concept of equivalence at an earlier stage. In the traditional approach, this concept was taught at a much later stage.

2.10 ADDITIONAL RESEARCH

Much research and many teaching experiments have focussed on the nature and growth of learners' knowledge of initial fraction concepts (See Section 2.5), their language and symbols. A brief summary of some of the research which has not yet been referred to will be outlined below:

Deborah Ball's research (1993) examined teaching for understanding from the teacher's point of view, but she also elaborated on the learners' thinking. The content of her research was based on the development of the initial fraction concept. She researched the contexts that would assist Grade 3 learners to construct useful and sensible understandings of fractions. She thought about what her learners should know about the meaning of fractions and what they already knew. Her findings showed that learners have a strong knowledge about one half, but that they often referred to any fraction part as a half. Learners tended to have a visual rather than formal knowledge of other familiar fractions and they tended to have little experience of making sense of the written notation. Her findings showed, as did Murray *et al*. (1999) and Davis (1989), as cited in Pitkethly and Hunting (1996), that learners applied whole number strategies to the symbols that expressed the fraction concept. Learners often believed that fractions with larger denominators were larger than those with smaller denominators. Ball (1993) felt that the context was crucial to the development and understanding of fractions and developed the idea of sharing cookies. Round cookies proved to be too difficult to divide equally and this resulted in dividing rectangular cookies. The rectangular model produced opportunities for pattern finding and deduction. One of the learners discovered that to get four pieces, you draw one less line. Ball (1993:192) concluded:

> "I realised the distance that these girls had come. Beginning with intuitive, inexplicit and visual notion of one half that they could draw, use and write, I had helped them travel to a new domain of numbers…their comprehension of fractions had evolved into principled understanding of part-whole relationships and their symbolic notation for fractional quantities".

Bonotto (1993), as cited in Pitkethly and Hunting (1996), together with a group of primary school teachers, analysed the way in which rational numbers were taught in Italian schools to learners from the age of 8 to 11 years. Their findings revealed that the idea of a fraction as part of a whole dominated rational number teaching. The team assumed that this was because activities such as cutting up and drawing, which gave fractions concrete form, were used for interpretation. This approach gave rise to limiting constructions. The learners understood a fraction as a label attached to a visual or manipulatory context. There was no understanding that entities presented in different ways can be unified within a single construct. The teachers and the learners had difficulty in understanding fractions as numbers. The continuous quantity part-whole approach was found to advance ideas such as: a fraction is always seen as a part less than a whole, that two quantities being compared often are not distinct and that one must divide first then multiply. The team created a program to overcome the limits of the part-whole presentation and which would facilitate the maturation of the fraction concept through its various interpretations and promote the concept of rational number through equivalence classes of fractions. Concept building was promoted through different contextual situations that are related to everyday life. These concepts were explained verbally and symbols were used to describe the situation. Both discrete and continuous contexts were used in the program. Because Bonotto did not want the visual representation to become the unique point of reference for the concept, she felt that the learner must quickly move away from the particular concrete situations. She felt that the context was just one contribution to the concept's formation.

Davis, Hunting and Pearn (1993) conducted a constructivist teaching experiment, over two years, involving 8 to 10 year old learners. An operator-like computer tool called Copycat was devised by Hunting, Davis and Bigelow, as cited in Davis *et al*., (1993). It was used in conjunction with a graphic tool, Superpaint. By using this tool, learners explored relationships governing inputs and outputs of discrete items. They determined the fraction responsible for observed numerical inputs and outputs for selected fractions governing the Copycat's response. At times it was necessary for the learners' to verbalise a rule or reason to the response Copycat gave. The learners were also asked to predict inputs, given outputs, or *vice versa*. The tool enabled learners to develop their understanding of the relationship between parts and wholes and to apply iterative schemes in the development of relational thought. Davis *et al*. (1993:75) felt that learners operating in the Copycat context, "…highlighted a discontinuity between unit fractions, for example $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$ and their multiples such as $\frac{2}{3}$ and $\frac{2}{5}$". $\frac{2}{3}$ does not naturally develop from its related unit fraction. Their findings showed that learners built on their intuitive understandings of the relationships between the input and output to form patterns to help in developing an understanding of fractions as relations. Learners were also able to portray fractions in reduced forms. Davis, Hunting and Pearn (1993) concluded that many learners have a model of fractions based on quantities. By using the computer environment the interconnectedness of rational number interpretations and the role of informal knowledge and constructive mechanisms in fractional learning is strengthened. The computer environment stimulated the learners and their whole number knowledge was extended by it. Learners were able to build on their existing whole number knowledge to investigate problems of order and equivalence.

Gray (1993) researched the difficulties experienced by learners in passing from whole number arithmetic to the arithmetic of fractions. Gray found differences existed between learners who treated number symbolism flexibility as process and concept and those who see fractions in terms of counting procedures. Gray cited examples of nine-year-old learners, who counted parts of a rectangle, subdivided into eight equal parts, not as eighths, but as wholes. The total was then named eighths. The meaning given to the fraction symbol was related to the meaning given to whole numbers. This concept did not aid the learners understanding of fractions as both a process and product (e.g. division of a whole) or concept (e.g. part of a whole). Gray concluded from his research that learners, who did not understand the flexible nature of the fraction symbol, would also fail to understand rational numbers at more complex levels.

Kieren, Mason and Pirie (1992) researched a Grade 3 class of 22 learners using a constructivist teaching experiment. The learners used paper-folding. This activity generalised the idea of partitioning or dividing up into equal parts in a wide variety of multiplicative tasks. The folding action involved iterating halves. The learners came up with different results because they organised their own thinking. The fraction terminology grew as the learners developed experience in folding. However, as they moved from familiar to unfamiliar quantity contexts, the learners moved from primitive knowing to formalising. They found that learners developed the ability to form and use iterable fractional units, combine fractional units and recognise fractional units. Learners did not generalise from one kind of formal knowledge to another; they reverted back to informal actions. The researchers found that growth was an active, ‘folding back’ process. Learners were able to predict the relationship between the number of folds and the number and size of the folded parts. Kieren (1993), as cited in Pitkethly and Hunting (1996), went on to say that learners fractional schemes should develop from an initial emphasis on unit fractions and that partitioning is pivotal to the development of fractional number knowledge.

Neuman (1993) investigated early conceptions of fractions using clinical interviews. She investigated learners understanding of $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$. The learners were given seven problems in both continuous and discrete contexts that were designed to show their conceptions and misconceptions of these fractions. Neuman (1993) aimed at revealing the learners' experience of fractions. She wanted to emphasise the informal knowledge that educators could use to base the construction of rational number knowledge upon. Neuman (1993) used problems including fair shares (equal-sharing), parts, form (visual images), size, ratio and one of a number of parts. She found that only a few learners were able to express understandings associated with fair shares, parts and form. She found that the size conception seemed to be related to learners' experience of whole numbers. The ratio concept seemed to exist, but more so in the younger learners. The older learners she found were convinced that thirds couldn't be sixths in both discrete and continuous contexts. Neuman (1993) concluded that teaching hampered the development of intuitively expressed 'ratio' concept of fractions by young learners. She did, however, find an early powerful functional understanding that she felt could be expanded to develop rational number knowledge. Learners were able to make use of existing knowledge in situations that were sensible to them and where they were applicable to the problem situation.

Saenz-Ludlow (1992) completed a longitudinal constructivist teaching experiment with Grade 3 learners. She analysed the different ways that learners solved the tasks. Her hypothesis was that learners would use and modify their methods of operating with positive whole numbers to generate fraction numbers. The tasks used stressed the whole as a multiple of equal sized parts and that they formed part-to-whole relations that allowed them to see the whole as part dependent. Saenz-Ludlow (1992) concluded that learners used their prior knowledge of counting numbers to form initial fraction conceptualisations. Her findings suggested that the fraction concept be introduced before fraction notation and computational algorithms.

Steffe and Olive (1990) began a longitudinal teaching experiment with Grade 3 learners using computer microworlds that the learners' manipulated to model and solve rational number problems. The aim of the experiment was to identify the change learners made to their whole number counting sequences to form fraction schemes. The problems learners had to solve included discrete units, continuous quantities and rectangular shapes that could be divided in various ways. They were structured in such a way that the learners could perform them without performing numerical operations. Two distinct fraction schemes came out of the research. Learners were able to use their existing counting scheme to form a unit fraction as an iterative part of a continuous but segmented unit and they formed their own fraction language by iterating unit fractions. Steffe and Olive (1990) referred to this as an iterative scheme. Secondly, the learners' adapted their number sequence to form a connected number sequence, the foundation of a measurement scheme. Steffe and Olive (1990) maintained learners were able to move from understanding the fraction name of the number of parts in a unit, to the name of an amount or a measure of a quantity. Steffe and Olive (1990) felt that learners' whole number knowledge does not interfere with the construction of fraction knowledge. They showed how learners used their natural language to construct their fraction knowledge using concepts and operations.

Streefland (1991, 1993) conducted a teaching experiment in which he identified the learning patterns of individual learners in the basic concepts of rational number. His aim was to develop a primary school fraction program and produce theory on the teaching and learning of fractions. Streefland (1991:19), as quoted by Pitkethly and Hunting (1996:22) stated "Starting from reality, students can cross the border to mathematics on their own, by learning to structure, arrange, symbolise, visualise, schematise and much more". He stressed the importance of learners constructing their own understanding of fractions. The problems he used involved partitioning and distribution, and provided contexts at concrete level. These problems were solved constructively. During the learning process, fractions and ratio were linked. They also emphasised connections and relations. Streefland (1991) found that errors the learners made were linked to the learners' whole number knowledge, repeated halving and persistent additive reasoning. Errors were not as apparent when the problems were in a concrete context. Considerable differences were found between the control group and the experimental group. The control group focussed on rules and algorithms and it was found that they did not have an advantage over the experimental group. However, the experimental group had an advantage over the control group whenever the problems required a solution without observable, mathematical or visual aids. Streefland concluded that a fraction program for primary learners could be designed on the basis of open, generative material, which would encourage the learners to develop their own ideas and computational methods. It would also encourage the learners to discuss, share their ideas and to reflect on the methods they employed.

Bell (1993) did three teaching experiments to study the aspects of a diagnostic teaching methodology based on identifying key conceptual points and misconceptions. The teaching was designed to provide learners with open challenges and provoked cognitive conflict by exposing misconceptions and resolving them through discussion. The experiments were designed to test an aspect of methodology. In the first experiment, the main question for study was the relation between successful learning and the intensity of discussion. The second question was whether by resolving the misconception in one context it could be transferred to other contexts. The experimental group learners based their learning on conflict, discussion and creative productions while the control group used individual booklets containing gently graded, guided discovery sequences of problems. The results showed that the booklets were not effective and that there was a good retention with the discussion and conflict method. Bell (1993) felt that the discussion and conflict method allowed for flexibility and creativity in the responses of the learners and that it could generate a situation where the class as a whole could learn more successfully.

2.11 CONCLUSION

This study on the cognitive development of the common fraction weighs heavily on the learners' understanding and making sense of the mathematics that they are doing. Various theorists have different ideas as to how learners gain knowledge. The behaviourist theory is tranferist and product focussed, the social theory emphasises learning through observation while the cognitive learning theory is learner and process focussed. The constructivist model of knowledge, which is embedded in cognitive learning theory, emphasises that learner's need to construct and find meaning for the world around them. For the purposes of this study, the researcher based her lessons on the Problem-centred approach that includes many of the intentions of the constructivist model of knowledge.

The Problem-centred approach encourages learners to build on their previous knowledge, to learn with understanding and to make sense of the mathematics that they do. It encourages learners to construct their own knowledge in an environment conducive to learning and where a suitable classroom culture has developed. The social components of the environment and classroom need to be established and addressed.

Three theories were followed in developing the materials. By following Piaget's Knowledge Types, the researcher was able to make sure that all three of these knowledge types were addressed during the intervention. Examining the learners written work according to Murray and Olivier's Levels of Development, the researcher was able to establish at what levels the learners were. This prevented the researcher from setting problems above the capabilities of the learners and from expecting solutions beyond the capabilities of the learners. The researcher was also able to follow the learners' progress and development. The Van Hiele Model of how a new topic be developed was followed. This guided the researcher as to what activities and tasks should be posed as well as the sequencing of these activities. The model was most effective in assisting the researcher to know when to introduce the formal notations and logical reasoning.

Some learners have difficulty in developing the concept of common fractions. Many educators have conducted research on this topic in order to identify problem areas and to develop appropriate programmes for improving the understanding of this seemingly troublesome topic. In Section 2.5, the researcher has outlined the sub-constructs that recent researchers feel ought to be covered while developing the concept.

Learners have a rich store of informal knowledge on whole number arithmetic and research indicates that learners should also possess similar informal knowledge about fractions. Limiting constructions were identified and programmes to rectify these were reviewed. Some researchers are of the opinion that some manipulatives can cause limiting constructions, and that learners need to allow their thinking to dictate the situation, and not the manipulatives.

CHAPTER 3

METHODOLOGY

3.1 A CHOICE OF PARADIGM

As noted in Chapter One, this research was undertaken in an attempt to understand the cognitive developmental processes of individual learners in developing the concept of common fractions. The research was designed, to emphasise firstly, the development of the conceptual knowledge and, secondly, the procedural knowledge of developing common fractions. Cognitive development can be described as consisting of two parts, namely, conceptual knowledge and procedural knowledge. Both need to be developed simultaneously in an integrated way. By solving problems, the learner should develop the concept of a fraction as well as algorithms for solving problems involving common fractions. As common fractions form the basis for many mathematical topics (See Chapter 1), it is necessary to develop stable, error-free concepts so as to develop procedural knowledge, free of misconceptions and limiting constructions. Procedural knowledge can be taught without conceptual knowledge, but this may develop into procedures being followed without understanding.

Due to the nature of the research, the researcher chose to work in the interpretive paradigm. The interpretive paradigm has been characterised "…by a concern for the individual" and it endeavours to "…understand the subjective world of human experience", by attempting to "get inside the person and to understand from within" (Cohen and Manion, 1994:36). The interpretive paradigm can further be described as being "subjective" and "complementing rather than competing with the experimental stance" (Cohen and Manion, 1994:106).

The interpretive paradigm focuses on action (Cohen and Manion, 1994). These actions can only be meaningful when clarified and shared by the learners. Cantrell (1993:84) states: " …interpretive researchers seek to understand phenomena and to interpret meaning within the social and cultural context of the natural setting". Interpretivists believe that reality is constructed and they seek to understand the subjective perceptions of individuals. The interpretive paradigm allowed the researcher to employ qualitative research methods, which according to Cantrell

(1993:87), can be used "…synonymously for a number of research approaches associated with the interpretive and critical science perspectives".

In order to understand the cognitive developmental processes used by the learners, it was necessary for the researcher to become personally involved in order to try and understand the learners' interpretations of the problems they were required to solve. Using the interpretive approach allowed the researcher the ability to work directly with the learners' experiences and to gain understanding and insight of their methods.

Every effort was made to put aside previous perspectives, personal viewpoints, preconceived assumptions and expectations and take direct experience at face value. The researcher made every attempt to see through the eyes of the learners and understand how and why they did certain things in order to understand, interpret and construct meaning of the problems they were required to solve.

The data collected did not lend itself to quantification and therefore it was necessary to analyse the data qualitatively. This allowed the researcher to approach the data without being constrained by predetermined categories, but enabled her to develop concepts, insights and understandings from patterns in the data.

3.2 SETTING OF THE RESEARCH

The researcher identified two schools in the Fort Beaufort district in the Eastern Cape in which the research could be carried out. The selection of these schools, hereafter referred to as Schools A and B (School B was 20 km out of town.), was based on convenience sampling as described in Cohen and Manion (1994) and Patton (1990). The reason convenience sampling was chosen was that there are only two schools in close proximity in which the researcher could work due to the fact that Grade 3 learners are instructed in their mother tongue and the researcher was not fluent in Xhosa, but in Afrikaans and English. Travelling distances and the concomitant cost factor were also a limiting factor.

The principals of both schools were agreeable to the research being conducted in their schools. The two Grade 3 teachers involved also agreed to the research being done with their learners. The parents' permission was also sought and they signed permission slips allowing their children to participate in the research. All parents granted permission. The names of all the children have been changed to prevent them from being identified. With the exception of two, all learners agreed to be audiotaped while being interviewed. Towards the end of the research these two learners agreed.

Identifying the classes initially posed a problem. The Grade 3 English class at School A consisted of 34 learners of whom 30 had Xhosa as their first language and English as their second language. It soon became evident that the learners in this class could not express themselves in English. They also had great difficulty in understanding the mathematical terminology in English. This posed an enormous problem due to the nature of the research to be conducted, because it was of utmost importance that the learners be able to verbalise and explain their thinking to the researcher as well as to other members of their group. To overcome this language barrier, the researcher made the decision to change from the English to the Afrikaans class at School A. The research was conducted in Afrikaans and then translated into English. At School A, there were initially 11 Grade 3 learners and a 12th learner joined the class in May. The learners in Grade 2 and 3 at School A formed a combined class.

At School B the Grade 2 and 3 learners formed a combined class of whom 12 Grade 3 learners were taken out and worked with. Initially the teacher insisted on teaching, while the researcher observed. She also insisted on including the Grade 2 learners in the research. After three interventions, the school recessed for their holiday and the teacher went on leave for the second term. Thereafter, the researcher was able to do the facilitating herself and continue the research with only the Grade 3 learners.

Unfortunately, due to the fact that at both schools, the learners formed a combined class with the Grade 2's, the researcher found it necessary to relocate the learners to other venues. At School A, the researcher conducted her research in a classroom next to the learners' own classroom. This room was used as a storeroom and on many occasions, furniture from the adjoining classroom had to be brought in. This seemed to unsettle the learners and it usually took some time to settle them before the class could begin.

At School B, the learners were initially moved to the dining room and, at a later stage, to the art classroom. Neither of these venues was conducive to learning. The dining room was noisy, the furniture was not suitable and there was no blackboard. The tables in the art classroom were made of metal and were noisy. Under these circumstances, discussion and reflection, which play a major role in the Problem-centred approach, could not be successfully carried out. The learners found it difficult to communicate quietly and they had limited means of sharing their ideas and methods with the class as there was no blackboard.

Because learners were required to move and the venues were not ideal, many discipline problems were encountered. The learners used to 'fight' over furniture and argue about where they had to sit. At School B, the boys used to engage in all sorts of antics while walking to the other venue. The folding tables at School B made group work very difficult. They were wide and prevented the learners from sitting close together. The tables were also too high for some of the learners making it difficult for them to write.

The learners at both schools were divided into groups of two or three for the duration of the research. The researcher in conjunction with the class teacher grouped learners of more or less equal mathematical ability together. Murray *et al*. (1993) found that when learners worked co-operatively, they functioned better when interacting with peers of equal ability. (See Section 2.2.3) The groups remained stable for most of the research, however, there were times when learners were absent and this required re-grouping. This was done with as little disruption to the groups as possible, but at times, it did result in learners being in mixed ability groups.

The research intervention was done on a part-time basis. The researcher had originally planned to conduct 33 lessons with learners at both schools. These 33 lessons covered the two sub-constructs identified by Murray *et al*. (1996a) and D'Ambrosio and Mewborn (1994) as well as the different ways in which fractions can be used (See Section 2.5). It was necessary that all the original 33 lessons be completed in order for learners to meet similar problems in different contexts and with different numbers over a period of time. By doing this, learners would be afforded the opportunity to develop a stable common fraction concept. The researcher had to comply with the school term as well as the activities of the school.

By September 1999, 21 lessons had been completed with learners at School A and 12 at School B. The researcher had not obtained any significantly different results from learners at School B and the continued further cost in terms of time and travelling could no longer be justified. A decision was made to concentrate on the learners at School A. After terminating the data collection at School B, the researcher was able to devote more time to the data collection at School A. By having two lessons per week, she was able to complete the necessary number of lessons with the learners at this school before the end of the academic year.

3.3 DATA COLLECTION

The research intervention consisted of 41 lessons (33 original lessons plus additional lessons that the researcher felt were necessary to include in order to clarify certain topics) that extended over a period of nine months. During these 41 lessons, the learners were required to solve problems involving common fractions and social knowledge needed for communication about common fractions was taught.

Following Cohen and Manion (1994), triangulation requires the use of two or more methods of data collection. They further argue that single methods of investigation provide only a limited view and may bias the researcher's interpretation. Using alternative methods of collecting data assisted the researcher to analyse the data from all perspectives so as to establish reliability[2] and validity[3]. Data collected by the methods listed below, will be compared and contrasted repeatedly thus providing a check on validity. These methods will hopefully diminish bias by increasing the richness of information available to the researcher.

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[2] "Data are reliable to the extent they are consistent." (Charles, 1995:27). Data is said to be reliable if there is consistency among the information provided by different data collection techniques. The researcher is of the opinion that had this study been repeated under similar circumstances, similar results would have been obtained.

[3] Validity can be generally defined as the trustworthiness of conclusions drawn from data (De Vos and Fouchè 1998). It refers to whether the data collection methods measures what it purports to measure and whether it is measured accurately.

Participant observation, completed worksheets and unstructured audiotaped interviews with the learners formed the primary method of data collection. Participant observation and the completion of worksheets were done simultaneously throughout the process. Unstructured group interviews were held with the learners and unstructured interviews were held with both Grade 3 teachers.

3.3.1 Participant observation

Participant observation was practised during the lessons. This method of data collection allowed the researcher to function both as a participant and as an observer – two different roles. As a participant, the researcher was able to participate as a group member and while participating, observe the methods used by the learners, being ever mindful of being too subjective (Cohen and Manion, 1994). This practise allowed for "direct, first-hand experiences with the phenomena under study" (Cantrell, 1993:93). Participant observation allowed the researcher to probe deeply and to intensely analyse the learners' work and thought processes so as to enhance her understanding.

Participant observation also allowed the researcher to describe the solutions to the problems the way the learners themselves experienced and solved the problems (Taylor and Bogdan, 1998). These records of comments, reflections and data perceived to be relevant by the researcher were entered into her journal.

3.3.2 Worksheets

The 33 original worksheets as well as eight additional worksheets, together with the participant observation formed the primary data collection tools (See Chapters 4 to 9). These worksheets were to be dealt with by the learners in the class. They contained problems and exercises that the learners were required to complete. The content of these worksheets covered the two sub-constructs identified D'Ambrosio and Mewborn (1994) and Murray *et al*. (1996a) as well as the different ways in which common fractions can be used (See Section 2.5). All these topics needed to be addressed in the cognitive development of common fractions with Grade 3 learners.

The topics include the development of the common fraction concept using equal-sharing situations, introducing realistic problem situations for operations involving fractions, comparison of fractions, equivalence, and introduction of the fraction symbol. The problems covering the above-mentioned aspects were used as vehicles for learning and were based on real-life situations so as to encourage learners to use and build on their informal and previous knowledge (See Section 2.6).

The Problem-centred approach, as outlined in Section 2.2, was used throughout the 41, approximately 1-hour, lessons. The purpose of the lessons that were held once a week with each class was to observe the cognitive development of the common fraction concept. The time varied according to discussion, reflection and the degree of difficulty of the problems.

The facilitator (researcher) usually set the tone for the lesson and at times found it necessary for the learners to do reflection on the work that was done during the previous lesson. The problems were read to the learners making sure that they understood both the situation and the context (See Sections 2.2.2.2 and 2.6). No methods were given to the learners to follow and they were encouraged by the facilitator to discuss the problem with the members of their group. They were encouraged to share and reflect on the methods used and the solutions obtained. During each intervention, the researcher strove to develop the classroom culture by paying special attention to the five social components that Hiebert *et al*. (1997) identified as necessary in the learning environment (See Section 2.2.2).

All worksheets were collected after each lesson. It was necessary for the researcher to decide whether the learners had grasped the concept that was addressed during the lesson before going onto the next lesson. At times, due to the fact that some concepts take longer to develop than others, it was necessary to repeat a similar problem structure type in order to develop the concept and for it to become stable.

3.3.3 Journal

Throughout the learning process, a journal, in which all the processes that could assist the researcher to understand the context, setting and activities that occurred in the classroom, was kept. All observations and impressions during the participant observation and class interventions were recorded in the journal. The transcriptions of all audio and video tapings were recorded in the journal.

3.3.4 Interviews

Bogdan (1982:135) defines an interview as "…a purposeful conversation, usually between two people in order to gather information". The researcher made use of unstructured interviews with individual learners, groups and teachers. Bogdan and Taylor (1998:135), are of the opinion that by interviewing the researcher attempts to give readers a feeling of "walking in the informants' shoes" and seeing things from the informants' point of view.

With the exception of two learners who initially refused to be audiotaped, all individual and group interviews were audiotaped. These tapes were then translated and transcribed and all transcriptions were kept in the journal.

3.3.4.1 Unstructured Interviews with Learners

To determine how the learners thought about the methods they used to solve specific problems, individual, unstructured interviews were conducted with two or three learners after each session. The unstructured interview is an open situation allowing greater flexibility and freedom. This enabled the researcher to pursue the methods used by the learners more deeply. Using unstructured interviews enabled the questions to emerge from the immediate context and they could be asked in the natural course of things. The researcher could respond to the immediate situation. The interview could therefore be built on and emerge from observations. Unstructured interviews offered the interviewer the latitude to pursue various aspects being led by the responses of the interviewee. Bogdan (1982) feels that the open-ended nature of an unstructured interview allows the subjects to answer from their own frame of reference.

3.3.4.2 Group Interviews

"Group interviews are essentially a qualitative data gathering technique, which can be directed in a structured or unstructured manner" (Denzin and Lincoln, 1994:364). Where the learners had worked together in groups and there was a need for all participants to explain the methods employed, group interviews were conducted. The various methods used by the learners in one group could then be explained. These informal group interviews were not conducted as frequently as the individual interviews, but only where deemed necessary. Learners were able to reflect and share their ideas and methods as they had done during the lesson.

3.3.4.3 Unstructured Interviews with Teachers

The researcher found it necessary to conduct unstructured interviews with the teachers of the two Grade 3 classes in order to ascertain information about the individual learners' family life as well as any social and/or learning problems that the learners might have had.

3.3.5 Video Recordings

Some of the interventions were recorded using a video camera. Colleagues of the researcher, from Cape College of Education, did these recordings. They captured the classroom culture and the interaction between the members of the groups. These recordings could then be analysed.

3.4 DATA ANALYSIS

After each lesson, comprehensive notes were made on the participant observation that had occurred during the lesson. The learners' written work was collected and analysed and all audio- and videotapes were transcribed.

Qualitative methods allowed the researcher to study selected issues in depth and detail without the constraint of predetermined categories (Patton, 1990). By using inductive analysis (See Taylor and Bogdan, 1998 and Patton, 1990) the researcher was able to identify patterns, themes and categories of analysis that emerged from the data, thus enabling her to enhance her understanding of methods used by the learners. Patton (1990:224) describes the process of analysis as "…the process of looking for patterns across units of activity". The researcher examined the individual learner's work to determine how he/she had differed over tasks and how different learners had differed over the subject. Patton (1990:376) refers to this grouping together of answers from different people to common questions as "cross case analysis".

The researcher divided the tasks that the learners had done according to the two sub-constructs identified by Murray *et al*. (1996a) and D'Ambrosio and Mewborn (1994) as well as the other topics that were dealt with during the intervention (See Section 2.5). Each learner's tasks were categorised according to the approaches used and/or the manner in which they had notated the solution. At times, depending on the problem type, it was necessary to examine the reason given for the solution in order to determine the thought processes of the learners.

A selection of individual learners' cognitive development of common fractions was traced through the topics dealt with (See Individual Studies 1 to 10 in Chapters 4 to 9). During the research the analysis of data was based on Murray and Olivier's Levels of Development for natural numbers (Murray and Olivier, 1989) (See Section 2.3.2). From this framework the analysis of common fractions was formulated for the purpose of this study (See Chapter 10). The progress of learners, of differing mathematical ability, was closely monitored.

The transcriptions of the interviews and videotape assisted the researcher to gain valuable insights as to how the learners thought and how they functioned in the class. Many of these transcriptions are documented in the sections entitled 'A Deeper Understanding of the Fraction Concept' (See Section 4.6, 6.3, 7.3, 8.3 and 9.3).

3.5 A CRITICAL EVALUATION OF THE RESEARCH METHODOLOGY

Upon reflection, the researcher felt that her choice of working in the interpretive paradigm was correct. This gave her the freedom to become involved with the learners and gain valuable information, insights and understanding of the processes used by the learners as they went about solving problems.

However, a number of difficulties were encountered. Participant observation as a method of collecting data became very difficult at times. Seeing to discipline in the class, explaining problems to the learners, dealing with learners who had completed their work quickly, as well as trying to observe what other learners were doing, became impractical at times. Due to this difficulty the researcher felt that some valuable information was lost. Simply being an observer or having colleagues observe could have alleviated some of these problems to a certain extent. On the other hand, being involved with the learners and being part of a group with them, gave the researcher valuable insight into their thought processes.

Having worksheets to analyse after each intervention gave the researcher evidence of how learners solved their problems. Even if the researcher had not managed to reach all learners, these worksheets provided useful information. Being able to refer to these constantly after the interventions and listening to tape recordings of their interviews contributed enormously to the analysis of the data. Many learners were not able to verbalise how they had solved a problem and by following the written work, each individual’s methods could be followed.

Informal interviews were conducted with learners after each intervention. This gave the researcher access to information that would otherwise have been inaccessible. Having learners explain their methods gave the researcher valuable insights into the way in which they thought. Two weaknesses of interviews are that they depend on the respondent’s ability to recall and that they can be affected by the physical and emotional state of the respondent (Patton, 1990). The researcher found that the learners were tired after working on mathematical problems for an hour and she is of the opinion that they did not always give of their best during the interview. At times, they could not remember what they had done to solve the problems. Initially, learners were nervous when they were interviewed, however, the researcher doubts that this influenced the responses made by learners Other learners found it extremely difficult to verbalise their thinking but fortunately, one could interpret their methods from the worksheets.

The freedom of working in the interpretive paradigm gave the researcher valuable access to the learners' thought processes as well as insight and understanding to their methods. Worksheets that learners completed contained rich descriptive data that could be analysed. Although participant observation and interviews did cause certain problems to occur during the data collection of this study, the researcher is of the opinion that these two data collection techniques allowed the researcher access to information and insights that were relevant to the study.

CHAPTER 4

LESSON OUTLINE, CHOICE OF PROBLEMS AND DEVELOPMENT OF THE FRACTION CONCEPT

This chapter will include an outline of a typical lesson, how the researcher chose the problems that were used in the lessons and the data analysis of the first topic, namely the development of the fraction concept.

The data collected will be analysed according to:

1. the development of the fraction concept,
2. social knowledge,
3. comparison,
4. equivalence, and
5. operations involving fractions.

Chapter Five will deal with social knowledge, Chapter Six with comparison and equivalence of fractions, while Chapter Seven will deal with operations involving fractions. Chapter Eight deals with a fraction as part of a collection of objects. This forms part of the sub-construct, namely, the part-whole relationship between the fractional part and the unit where the unit can be a single object or a number of objects.

All problems were posed to the learners in the form of word problems. Abbreviated number sentences in brackets after the figure names are used to indicate which problem the learners were dealing with.

4.1 AN OUTLINE OF A TYPICAL LESSON

At the beginning of each lesson, the researcher would set the tone of the lesson by discussing the topic on which the problems were based. For example, if the problem posed to the learners was about a netball team, the researcher would ask the learners if netball was played at their school or if any of the girls played netball. In order to include the boys in the class, they were asked if they had watched the girls playing or if maybe they had played the game themselves.

The problem would then be read to the learners. The researcher would make sure that all vocabulary and questions were understood. The learners, who were already sitting in their groups, would be handed worksheets on which the problem was typed. At times, the learners preferred to work individually, and at other times, they would discuss the problem with the members of their group.

While the learners were solving the problem, the researcher would practise participant observation being ever mindful of trying to adhere to the role of the facilitator (See Section 2.2.2.2). The learners were encouraged to reflect and discuss their solutions and methods with members of their group as well as the members of other groups.

When most of the learners had completed their solutions, entire class would have a time of reflection. The learners would then share their ideas and discuss their methods and solutions.

After each lesson, the worksheets would be collected and the researcher would conduct informal interviews, either with individuals or a group.

4.2 CHOICE OF TASKS AIMED AT DEVELOPING THE TWO SUB-CONSTRUCTS OF THE FRACTION CONCEPT, EQUIVALENCE, COMPARISON, SOCIAL KNOWLEDGE AND INITIAL OPERATIONS INVOLVING COMMON FRACTIONS

The subject matter used for this research covered the 2 sub-constructs identified by Murray *et al*. (1996a) and D'Ambrosio and Mewborn (1994) (See Section 2.5), as well as the different ways in which fractions can be used. The research material covered:

a) developing the fraction concept through sharing situations,
b) introducing realistic problem situations for operations involving fractions,
c) comparison of fractions,
d) equivalence, and
e) introducing the fraction symbol (e.g. $\frac{1}{4}$).

Fractions are used in different ways and have different meanings, part-whole, ratio, a quotient and a measure, to name but a few. The different meanings include, for example,

- a part of a whole (half of an orange),
- a part of a number of objects (half of the learners are boys) and
- a relationship (a tenth of his income is given to the church).

Limiting constructions could be formed if the problems posed do not cover these different sub-constructs and meanings within a reasonable time (See Section 2.7). For the same reason, the fractions addressed should also immediately include thirds, fifths etc., and not only halves and quarters (Murray *et al*., 1999; Empson, 1995).

As some researchers (See Section 2.7) felt that manipulatives could cause limiting constructions, the researcher made the decision to limit the number of manipulatives in order to encourage the learners to draw their solutions and to allow their thinking to dictate the situation and not the manipulatives (See Section 2.8). The learners were given blank paper and black felt pens to work with.

All worksheets used during the research can be found in Appendix A.

4.2.1 Developing the fraction concept through equal-sharing situations

In order to develop the fraction concept, the researcher made use of many experiences of fractions through word problems that made sense to the learners (See below). An attempt was made to choose problems that learners could solve based on their informal knowledge. The problems were set in a real-life context that the learners could identify with and that were related to everyday life (See Section 2.6). "Equal-sharing problems constitute a rich context in which to develop fraction concepts" (Empson, 1995,112). "…using equal-sharing problems can help children learn and understand fractions" (Empson, 1995:114). Murray *et al*. (1996a) feel that posing sharing problems with remainders that also have to be shared out encourage young learners to construct their own idea of fractions through their own actions. Behr, Lesh, Post and Silver (1983), as cited in Pitkethly and Hunting (1996:9), emphasised that, "…young children should be given broad experience in partitioning activities to enable them to observe the compensatory relationship between size and the number of parts into which the whole is partitioned". Kamii and Clark (1995:376) state that "'Teaching' starts with realistic problems and encourages children to invent their own solutions so that fractions can grow out of the children's own thinking".

As Ball (1993), as cited in Pitkethly and Hunting 1996, stated that round cookies were too difficult for learners to share, the researcher chose rectangular chocolate bars for equal-sharing problems.

Worksheet 1 included equal-sharing problems where the learners were introduced to halves, thirds and quarters. In each of these problems, a single unit remained that had to be shared. (e.g. Share three chocolate bars equally among two friends.) Worksheet 2 and 2.2 introduced learners to fifths and continued with equal-sharing problems where a single unit remained. Problem 1 of Worksheet 5 covered the same concept as the other worksheets however the remaining unit had to be divided into sixths.

Problem 2 introduced a new situation. The remainder involved two units that had to be shared. (e.g. Share 8 chocolate bars equally between 6 friends.) Worksheet 7, problem 2 involved an equal-sharing problem, where seven units remained.

Worksheet 17 introduced a new situation. The learners had to share a unit as well as a fractional part. (e.g. Share $1\frac{1}{2}$ chocolate bars equally between 2 friends.)

All the problems on Worksheet 24 involved equal-sharing situations, however, problem 1 and 2 were very different from the previous equal-sharing problems involving fractions – there were fewer objects to be shared than the number of children who needed to get shares. (e.g. Share 3 chocolate bars equally between 4 friends.)

4.2.2 Introducing realistic problem situations for operations involving fractions

In order to stimulate thinking about operations of fractions, the researcher chose to use problems that covered different meanings of fractions as well as different ways in which fractions can be used. The problems were set in real-life contexts to facilitate understanding, provide a sound basis for building fraction knowledge and for learners to build connections (See Ball, 1993 and Streefland, 1991, 1993, as cited in Pitkethly and Hunting, 1996).

Menon (1995) in his research found that students constructed questions that were based on their experience and interests. He concluded that word problems based on the learners' experience and interest must not be overlooked in the mathematics classroom. Menon (1995:32) felt that " …the role of context in word problems involving common fractions seems to add personal meaning to the mathematics involved" and "It looks as if the more different contexts in which the word problem is situated, the easier it is for students to link mathematical relationships and make abstractions and generalisations".

(Example 1: Vusi and Anna prepare soft porridge for breakfast. For each bowl, they use $\frac{1}{3}$ of a litre of milk. They have 5 litres of milk. How many bowls of porridge can they prepare? (See Chapter 7, Category 1)

(Example 2: Vuyo wants to know what she has left over after baking for a party. How much of each does she have? 3 big packets of chips, each a $\frac{1}{2}$ full, 5 containers of ice-cream, each $\frac{1}{4}$ full, and 2 jugs of milk, each $\frac{2}{3}$ full)

As iteration is closely linked with the concept of addition of fractions, these two sections have been linked together (See Section 2.5). Some of the problems included in these worksheets provide exposure to two of the multiplicative structures, namely repeated addition and rate. Worksheet 7, problem 1 introduces the very important concept of iteration (e.g. Two netball teams play a game. There are 14 children altogether. The sports teacher wants to give each child $\frac{1}{2}$ of an orange. How many oranges does she need?). Learners had to iterate halves to form wholes. This problem type leads to repeated addition. Worksheet 8 and 8.1 presented a more difficult version of the same problem. The problems also touched on the addition of fractions. The first problem on Worksheet 9 continued to reinforce the idea of iteration. The problem type was one of repeated addition, i.e. one of the multiplication types. The second problem type was a grouping problem, i.e. a division problem.

Problem 1 of Worksheet 10 continued with a grouping problem. Worksheet 11 involved the learners calculating the price for half-metres. The learners were encouraged to make use of previous solutions to obtain their current solution. Problems found on Worksheets 14 and 15 were almost identical to the second and third problems of Worksheet 9. They are grouping problems and can be formally classified as division with a fraction. According to Murray *et al*. (1996b), it is essential that learners meet this problem type at the Grade 3 level in a context that makes sense to them. Problem 1 on Worksheet 17 again posed a grouping problem to the learners.

The chains included on Worksheet 19 and 20 involved the addition of quarters, and halves and thirds respectively. These fractions were all unit fractions (See Section 2.5). The chains also provided the learners with an opportunity to get a 'feel' for the numbers. Worksheet 21 involved another problem concerning addition of fractions. The chain on Worksheet 25 involved the addition of thirds. These were once again unit fractions whereas the chain on Worksheet 26 included the addition of thirds where the numerator was two.

The tables that the learners had to complete on Worksheets 27, 28 and 29 serve as preliminary problems for the addition of fractions with unlike denominators. The chain on Worksheet 30 involved addition of quarters – unit fractions. Addition and subtraction of thirds, both unit fractions and fractions that have numerators greater than one were included in the chain of Worksheet 31. Learners were also required to fill in whether the fractional part needed to be added or subtracted to reach the following solution. Unit fifths, and fifths where the numerator was greater than 1, as well as addition and subtraction, were included in the chain on Worksheet 32. The last chain to be completed on Worksheet 33 involved learners working with thirds and fifths. The solution was filled in for learners, and learners were required to decide on the operation as well as the fractional part that needed to be added or subtracted to obtain the solution.

4.2.3 Comparison of fractional quantities and equivalence

Problems requiring the comparison of fractional quantities are a powerful vehicle for developing the understanding of equivalent fractions (Human, undated). Human believes that mechanical rules should not be taught, since it may strongly inhibit true understanding of equivalent fractions. Equivalent fractions are a crucial concept in operations involving fractions and therefore need to be introduced at an early stage of development of the fraction concept (Murray *et al.* 1996b).

The purpose of Worksheets 3 and 4 was to start the learners thinking about the equivalence and non-equivalence of fractions (See Appendix A). Worksheet 4 initiates learners thinking in the direction of equivalent fractions (See Appendix A).

Because the concept of equivalence takes a long time to develop and become stable, Worksheet 5 and 6 continued with this concept. Question 2 of Worksheet 5 involved a remainder of two units ($8 \div 6$). Two solutions could have been found for this problem. This afforded the opportunity to initiate a discussion on equivalent fractions. If the learners divided the remaining two units into sixths, the solution would have been $1\frac{2}{6}$. If the learner had divided the remaining two units into thirds, the solution would have been $1\frac{1}{3}$. The learners were confronted with the problem of deciding whether $1\frac{2}{6}$ and $1\frac{1}{3}$ were the same or not. The learners were supplied with a fraction wall to consult. They were not required to memorise the equivalent fractions. Worksheets 12 and 13 involved the equivalence of fractions and comparison between the sizes of different fractions. Learners were once again supplied with a fraction wall and the main purpose of these tasks was to have them handle and inspect fractions.

4.2.4 Social knowledge

Social knowledge that learners require for communicating about fractions includes naming of fractions and writing of fractions using fraction notation (Murray *et al*. 1996a). The researcher gradually introduced this terminology and notation, as she deemed necessary (See Section 2.3.3). Fraction names were introduced to the learners in Worksheet 3. The relevant social knowledge was written at the top of each worksheet where it was needed and before the problems were posed to the learners, the researcher would introduce the terminology, read through and speak about the social knowledge. Social knowledge was transmitted to the learners verbally, by means of posters in the classroom and drawings on the chalkboard (See Section 2.3.1). Posters displayed in the classrooms were enlarged replicas of the social knowledge written on the worksheets.

The important concept that fractions imply equal parts was also discussed during this lesson. Learners were requested to name the fractional parts on all worksheets involving fraction walls after the fraction names had been introduced to them. This was used to try and reinforce the social knowledge. Fraction notation (for unit fractions) was introduced in Worksheet 7. Although the researcher initially felt that fraction notation for fractions where the numerator was greater than one, was to be introduced at a later stage, she felt learners needed this social knowledge after Worksheet 8. Worksheet 8.1 was then completed with the learners in order to reinforce the social knowledge.

4.2.5 Fractions as part of a number of objects

Worksheets 3, 18, 22 and 23 were included so as to cover the second aspect of the first sub-construct as described by Murray *et al*. (1998) and D'Ambrosio and Mewborn (1994) namely, the part-whole relationship between the fractional part and the unit where the unit can be a number of objects (e.g. $\frac{1}{4}$ is one of four equal parts of a collection of objects) (e.g. What would you rather have, a fourth of a R1 or a third of R1?). Murray *et al*. (1996b) felt that if learners did not meet fractions used this way, they might find this concept difficult to manage at a later stage, resulting in limiting constructions.

4.3 PREVIOUS SCHOOLING ON COMMON FRACTIONS

In both schools, learners had been exposed to a minimal amount of prior learning of the common fraction concept in Grade 2. At School A, the learners had been exposed to halves and quarters. The activities that had previously been done included shading in of a fractional part and naming a fractional part that had been previously shaded. The learners had also been taught how to write halves and quarters using fractional notation. At School B, the teacher had exposed the learners to halves, thirds and quarters by involving them in cut up activities and activities that involved shading in of a fractional part. The learners were also taught how to write fractions using the fraction notation.

In both schools, common fractions had not been studied in Grade 3.

4.4 ANALYSIS OF DATA – DEVELOPMENT OF THE FRACTION CONCEPT

The main purpose of the first three interventions was to introduce the concept of the common fraction to the Grade 3 learners by posing problems that involved equal-sharing situations.

At regular intervals throughout the study, the learners were exposed to equal-sharing situations that involved different (new) concepts (See Section 2.5; 2.6; 2.9).

The following problems were posed to the learners:

<u>Worksheet 1</u>
Share 7 chocolate bars equally between 2 people.
Share 7 chocolate bars equally between 3 people.
Share 13 chocolate bars equally between 4 people.

Worksheet 2

Five friends want to share 11 chocolate bars equally. How must they do it?

Five friends want to share 21 chocolate bars equally. How must they do it?

Worksheet 2.2 (The additional worksheet was used only for School A as the researcher found that some of the learners at this school had difficulty with larger numbers.)

Five friends want to share 26 chocolate bars equally. How must they do it?

Worksheet 5

Six friends want to share 7 chocolate bars equally. How must they do it?

Six friends want to share 8 chocolate bars equally. How must they do it?

The above equal-sharing problems involved a remainder of one unit, however, the second problem in Worksheet 5 involved there being two units as a remainder. This was the first time that the learners had encountered this situation.

Worksheet 7 (the second problem)

One of the parents brings along a bag of 35 chocolate bars to share among the 14 players. How much chocolate does each player get?

Worksheet 17 (the second sum)

Bingo and Mary have $1\frac{1}{2}$ bars of chocolate. They want to share the chocolate equally. How much chocolate must each child get?

All equal-sharing problems solved by the learners up to this stage had involved sharing whole units. Problem two on worksheet 17 was the first time that the learners had been exposed to sharing a fractional portion as well as a unit.

Worksheet 24

The children have brought different things to eat and drink.

They have three chocolate bars to share equally among four children. How much chocolate does each get?
They have two litres of cooldrink to share equally among eight children. How much cooldrink must each child get?

This was the first time that the learners had been exposed to the total amount per person being less than a whole unit.

They have nine oranges to share equally among the four children. How much orange must each one get?

The researcher initially requested the learners to draw their solutions and try to notate the total amount that each person would receive.

Solution Categories

From the data collected during the lessons, the researcher was able to identify four solution categories that emerged (See Murray *et al*., 1996a). Firstly, the learners used economic sharing. They shared out the maximum number of units, divided the remaining unit into an appropriate number of fractional parts and then shared them out. They either used drawings, numbers and drawings, used numbers and no drawings or drawings and words to notate their answers. Some learners were able to solve the problem using diagrams, but used the incorrect fractional notation when writing down the answer. Murray *et al*. (1996a : 48-49) found that "…the high incidence of incorrect naming of the fractional part among third graders, although their drawings clearly show that the students are dealing with thirds and not halves or quarters". Mack (1990: 21) found that learners "…often explained their solutions using faulty knowledge related to formal symbols…".

The second category that emerged was partitioning all the units into an appropriate number of fractional pieces and then sharing out the parts from each in turn. Learners only used drawings to notate their solutions.

In Category Three, the learners shared out the maximum number of units, but divided the remaining unit into an inappropriate number of fractional parts. The last category, Category Four, was when the learners produced incorrect solutions. These included solutions where the learner had shared out the incorrect number of units and at times, the researcher could not identify the mistake that the learners had made.

Examples of the categories of solutions are as follows:

Category 1

Economic sharing, i.e. sharing out the maximum number of units then partitioning the remaining unit/s into appropriate fractional parts. The fractional parts were then shared out.

The solutions for the strategy outlined above were illustrated using four different methods:

1.1 Drawing only

For 7 ÷ 2, Nosisa gives each friend one unit. She then partitioned the remaining unit in half and gives each friend a half. Nosisa indicated that the lines drawn in the chocolate bars were to make them look pretty.

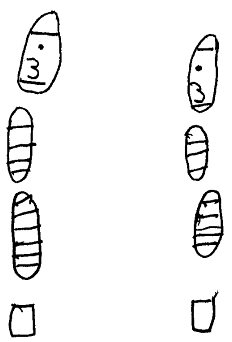


Figure 4.1 Nosisa Worksheet 1 (7 ÷ 2)

1.2 Drawing and fractional notation

For 7 ÷ 2, Caroline shares out the units and the remaining unit is divided into halves. She gives each friend a half. Caroline notated her answer using the correct fraction notation.

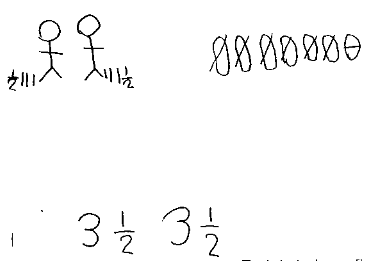


Figure 4.2 Caroline Worksheet 1 (7 ÷ 2)

1.3 Drawing and the answer was also written in words

For 7 ÷ 2, Martin shares out the units and divides the remaining unit into halves. He gives each friend a half. The total amount, he writes as 3 half.

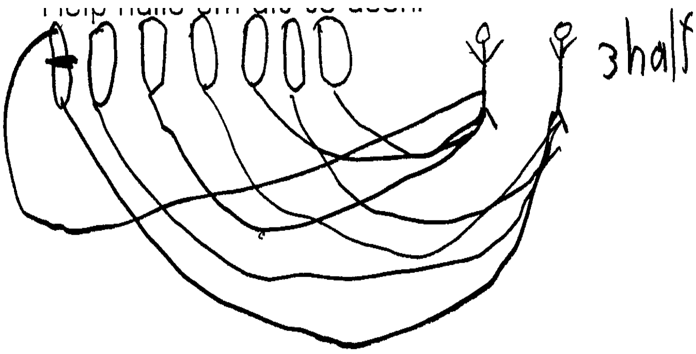


Figure 4.3 Martin Worksheet 1 (7 ÷ 2)

1.4 Numbers and fractional notation only

For 7 ÷ 2, Cheryl shared out the maximum number of units and simply gave each friend an additional $\frac{1}{2}$ without showing any drawings.

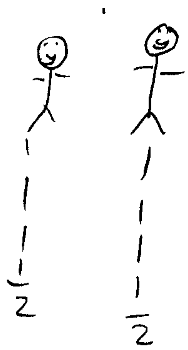


Figure 4.4 Cheryl Worksheet 1 (7 ÷ 2)

1.5 Drawing was correct, but the fractional part was named incorrectly

For 7 ÷ 3, Phindiwe shares out the maximum number of units, she divides the remaining unit into thirds and indicates using the drawing, that each friend gets an additional third. However, Phindiwe notates the fractional part as quarters (in words) and $\frac{1}{2}$ in fraction notation.

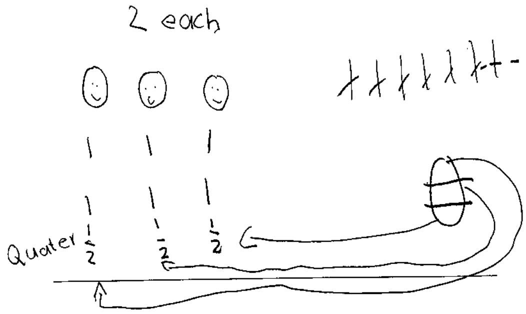


Figure 4.5 Phindiwe Worksheet 1 (7 ÷ 3)

Category 2

All the units were partitioned into fractional parts of which the denominator is the same as the divisor, then sharing out the parts from each in turn.

The solutions for strategy number two were illustrated using:

2.1 Drawings only

For 7 ÷ 3, Kate divides each unit into thirds and says that each friend receives seven pieces.

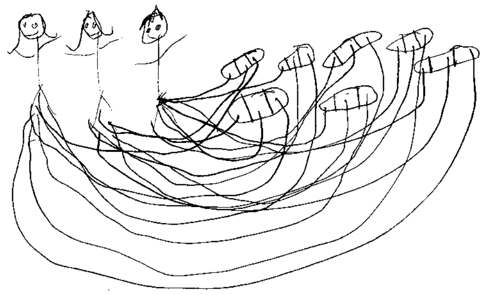


Figure 4.6 Kate Worksheet 1 (7 ÷ 3)

It should be noted that wherever dialogue is reported the *Teacher* is the researcher.

Kate explained as follows:

Teacher: *Kate, can you explain what you did here?*

Kate: *I took each chocolate and divided it into three pieces. Then I gave each child a piece. I did that with each chocolate.*

Teacher: *So Kate, how much did each child get?*

Kate: *seven*

Teacher: *seven whole chocolates?*

Kate: *No, seven pieces*

Category 3

Economic sharing, i.e. sharing out the maximum number of units and the remaining unit partitioned into inappropriate number of fractional parts. This strategy can be further divided into two sections, those who chose to ignore the remaining fractional part, and those who chose to further divide the remaining fractional part.

The solutions for strategy number three were illustrated using:

3.1 Drawings

For 7 ÷ 3, Martin shares out the maximum number of units and then divides the remaining unit into the inappropriate number of fractional parts (quarters). He gives one quarter to each of the friends, and chooses to ignore the remaining part.

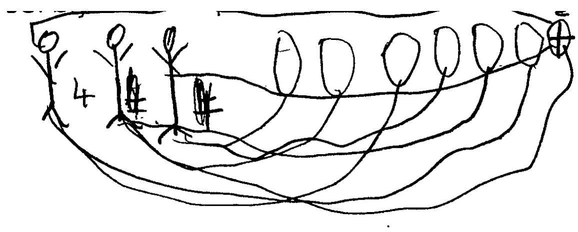


Figure 4.7 Martin Worksheet 1 (7 ÷ 3)

From the drawing one can see that Martin made two unsuccessful attempts to cut the chocolate up into an appropriate number of fractional parts.

Teacher:	*I see you have given each child two whole chocolates. Can you explain what you did to the last chocolate?*
Martin:	*Yes…….. I cut it up and then I shared it.*
Teacher:	*Did you share it all?*
Martin:	*No.*
Teacher:	*What must you do with that piece?* Pointing to the remaining fractional part.
Martin:	*I do not know.*
Teacher:	*What would you do if I gave you a knife?*
Martin:	After a considerable time. *I do not know.*
Teacher:	*Can you cut it again?*
Martin:	*No.*

When asked to explain where the four came from, he said that he could not explain. At this stage, the researcher felt it better not to pursue the interview as Martin was showing signs of becoming irritable.

3.2 Drawings and incorrect fractional notation

For 7 ÷ 3, Sipho shares out the maximum number of units and divides the remaining unit into the inappropriate number of fractional pieces (quarters). He shares out three of the four pieces and then divides the remaining piece into an appropriate number of fractional parts. He then gives each friend another piece.

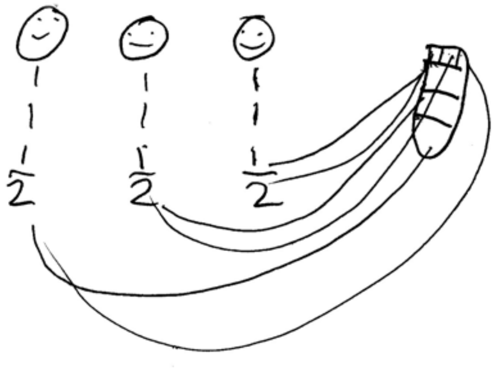


Figure 4.8 Sipho Worksheet 1 (7 ÷ 3)

Teacher: *Sipho, I see that you have given each child two and a half bars. Could you please show me how you have shared the bars?*

Sipho drew the last bar having four pieces. He drew lines to indicate who would get which piece, but landed up with an extra piece.

Teacher: *What are we going to do with the piece that is left over?*

This time Sipho cut it into three pieces– actually hesitating before he drew the lines. He shared them out by drawing lines again.

Teacher: *Can you tell me how much each child will get?*

Sipho: *Yes, two wholes, a piece and a piece.*

Category 4

Incorrect answers. Some of these incorrect answers were the result of the units being shared incorrectly.

4.1

For 11 ÷ 5, Tammy shares out the maximum number of units, but chose to ignore the remaining unit.

Figure 4.9 Tammy Worksheet 2 (11 ÷ 5)

4.2

For 11 ÷ 5, Sharon shared out more units than there were.

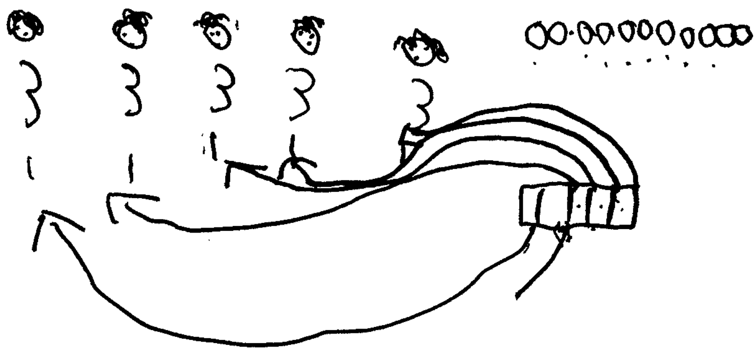


Figure 4.10 Sharon Worksheet 2 (11 ÷ 5)

4.3

For 7 ÷ 3 and 13 ÷ 4, Zelda divides the remaining unit into halves. This resulted in her sharing out more than the maximum number of units available.

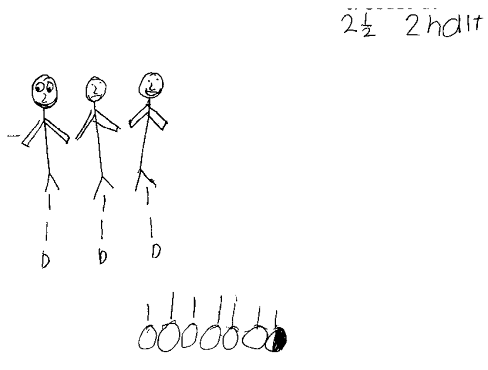


Figure 4.11 Zelda Worksheet 1 (7 ÷ 3)

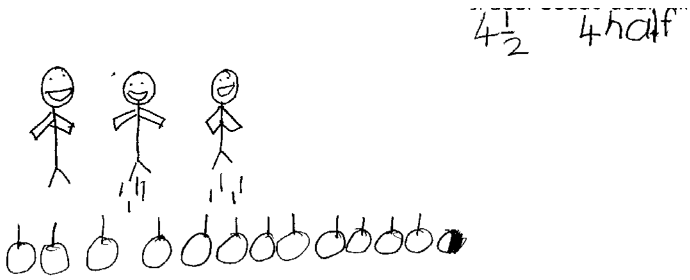


Figure 4.12 Zelda Worksheet 1 (13 ÷ 4)

Appendix B contains the grids of the solution strategies that the learners have used to solve the problems on the section of the development of the fraction concept.

4.5 COGNITIVE DEVELOPMENT OF INDIVIDUAL LEARNERS

The examples below are of an individual's solutions to the problems. These are included to show the progression of learning as well as Murray and Olivier's Levels of Development as outlined in 2.3.2.

The researcher, together with the Grade 3 teachers, identified learners of stronger, average and weaker mathematical ability and traced their development through the lessons that were done to develop the initial fraction concept.

Individual Study 1

Zane is of the stronger ability group.

In Worksheet 1, for (7 ÷ 2), Zane applied economic sharing (Category 1).

He notated by using a '1' next to each person as he shared out the chocolate bars. The remaining unit was not drawn, he simply wrote down a $\frac{1}{2}$ next to each person. His answer was written in words. (*three half*)

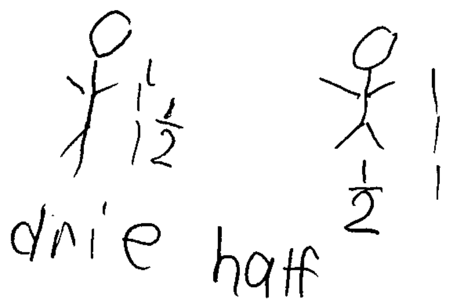


Figure 4.13 Zane Worksheet 1 (7 ÷ 2)

For 7 ÷ 3, he once again used economic sharing (Category 1), however, the sum was incorrect. He called the remaining piece 'a quarter' when it was actually a third.

Figure 4.14 Zane Worksheet 1 (7 ÷ 3)

His confusion as to what to name the fractional piece is evident in the drawing. This was probably due to a limiting construction, because in Grade 2, the learners in School A had only been exposed to halves and quarters (See Section 2.7). He corrected his mistake during reflection time. For both problems 1 and 2, Zane did not draw his answer.

For 13 ÷ 4, Zane made the use of economic sharing (Category 1).

By using drawings, he obtained the correct solution. Zane wrote the fractional part of the solution in words, '*3 quarter*', instead of three and a quarter. His method of notating the solution was incorrect. It seems as if the solution is three quarters instead of three and a quarter.

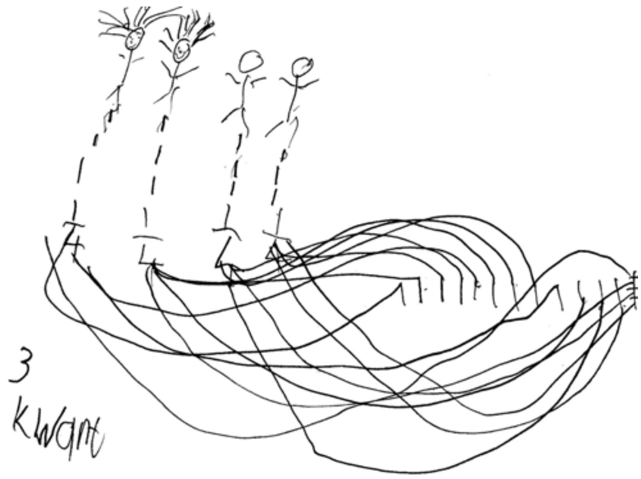


Figure 4.15 Zane Worksheet 1 (13 ÷ 4)

In Worksheet 2, Zane used economic sharing (Category 1) to solve both problems.

He drew his solution for problem 1 and chose to draw the total amount each person would receive.

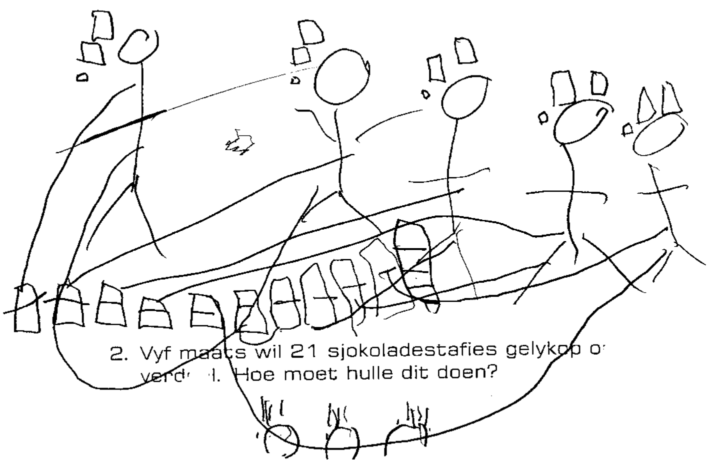


Figure 4.16 Zane Worksheet 2 (11 ÷ 5)

Zane was willing to draw the chocolate bars as bars for the first problem, but not the second. He once again reverted to 'lines' representing the chocolate bars. Both solutions are correct.

If one has to compare the Levels of Development (Murray and Olivier, 1989) between Worksheet 1 and 2, one could say that Zane had regressed. This could be as a result of the researcher requesting that the learners draw. On the other hand, Zane did stop using fractional notation incorrectly. In the initial stage of developing the fraction concept, it is necessary that the learners draw their solutions as it assists them in developing this concept.

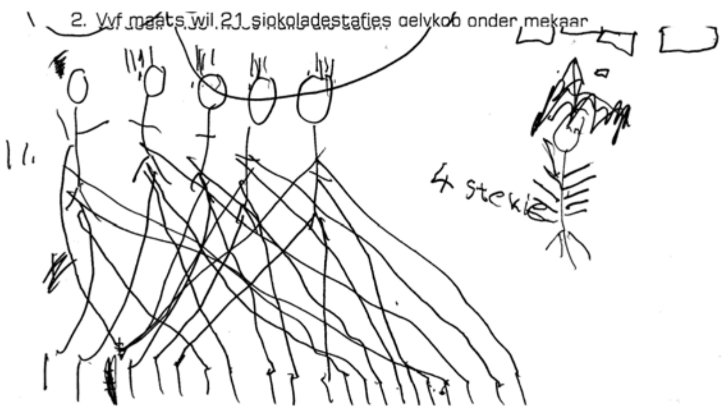


Figure 4.17 Zane Worksheet 2 (21 ÷ 5)

For 26 ÷ 6, Zane continued to use economic sharing and drawings (Category 1). He wrote the total amount in words and called the fractional part 'a piece' (5 *five and a piece*). Once again, this was probably due to the fact that a suggestion was made to the learners that they call the fractional part 'a piece', because the social knowledge of how to name a fraction had not yet been transmitted to the learners.

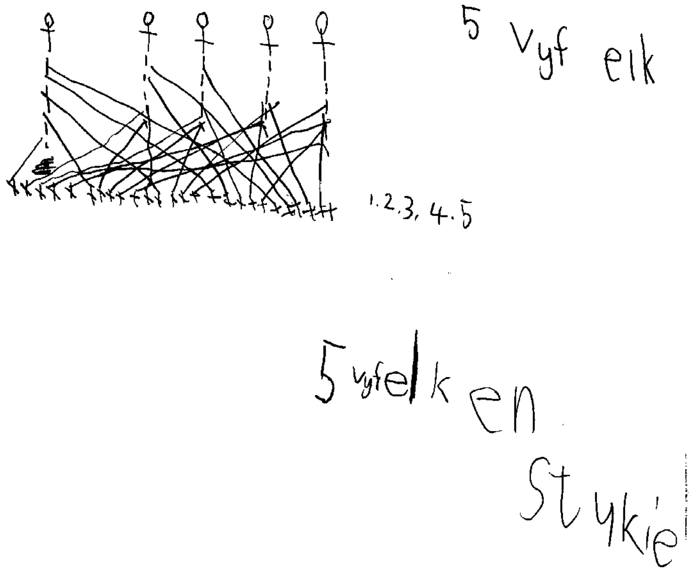


Figure 4.18 Zane Worksheet 2.2 (26 ÷ 6)

By the time the learners attempted Worksheet 5, they had been exposed to some social knowledge, namely how to name the fractional parts. For both problems in Worksheet 5, Zane used economic sharing (Category 1) and he drew the solution and notated the total amount using words – '*one and one sixth*'.

Figure 4.19 Zane Worksheet 5 (7 ÷ 6)

In Worksheet 5 (8 ÷ 6), the learners were exposed, for the first time, to the remainder being more than one unit. Zane chose to use the strategy where the maximum number of units was shared out and the remaining unit was divided into the appropriate number of fractional pieces (Category 1). His solution was written in words: '*Each one gets one and one third*'.

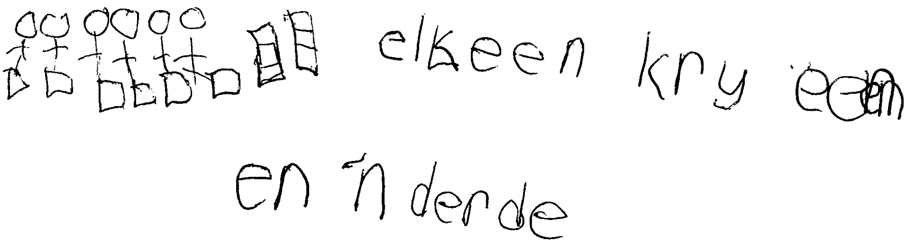


Figure 4.20 Zane Worksheet 5 (8 ÷ 6)

For 35 ÷ 14, Zane made marks for the 35 chocolates. If one looks at this method used by him, it seems to be that when the number to be shared is large, Zane needs to have a picture of all the chocolate bars that need to be shared out. He drew all the chocolates for 13 ÷ 4, 21 ÷ 5, 26 ÷ 5 and again for 35 ÷ 5. He did not use this method again. Zane crossed off the bars as he shared them out, but he did not find it necessary to draw the remaining bars showing the partitioning. Zane could write his answer using fractional notation as the learners had been exposed to this social knowledge.

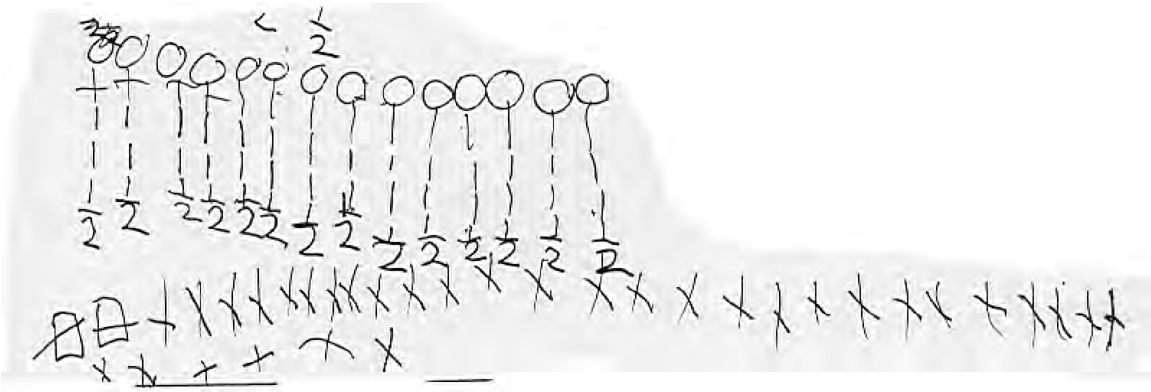


Figure 4.21 Zane Worksheet 7 (35 ÷ 14)

Zane explained as follows:

> *I drew 14 people again – I drew 35 chocolates – I shared them out and as I did so I crossed out the ones that I had shared. I wanted to give them each another one, but there was not enough for each one to get another one, so I took a half, a half …. And each one got two and a half.*

By the time Zane reached the last four problems, he was simply able to write down the answers. Zane had progressed to working at an abstract level, placing him at Level Three of Development (Murray and Olivier, 1989).

For $1\frac{1}{2} \div 2$, Zane initially started drawing and then he changed his mind and simply wrote down the answer.

This was the first time that the learners had to share a fractional part as well as a unit. He wrote: '$\frac{6}{8}$ *for each*'.

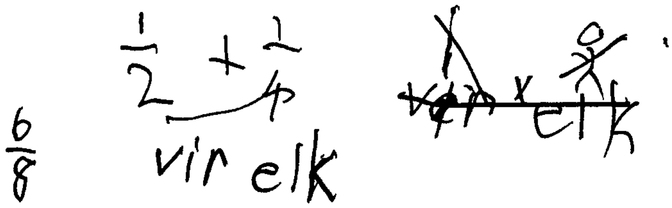


Figure 4.22 Zane Worksheet 17 ($1\frac{1}{2} \div 2$)

For 3 ÷ 4, Zane wrote down the answer and only after the researcher asked him to explain what he had done, did he included his drawings. Zane was able to work at an abstract level. (Level Three of Development, Murray and Olivier, 1989) He wrote down his solutions as follows: '*Cut two pieces in half – one in fourths and then you get* $\frac{1}{2} + \frac{1}{4}$'.

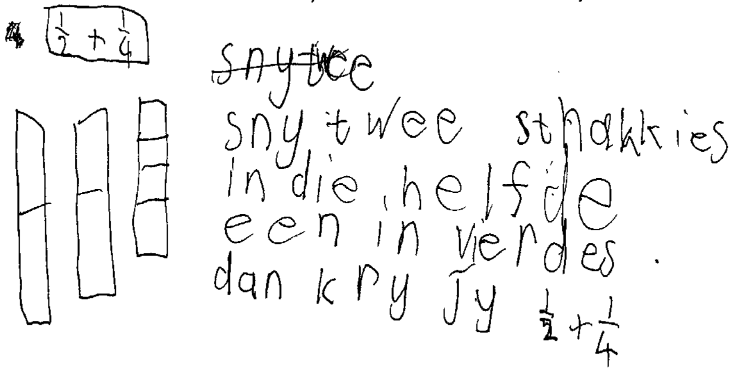


Figure 4.23 Zane Worksheet 24 (3 ÷ 4)

The researcher asked Zane to explain how he had solved the problem. His explanation was:

"Cut two pieces in half and one in quarters. Then you get $\frac{1}{2} + \frac{1}{4}$*"*.

Zane continued to write down only the solutions to the problems. For 2*l* ÷ 8, he wrote: '*each one would get a* $\frac{1}{4}$ *of 1 litre*'.

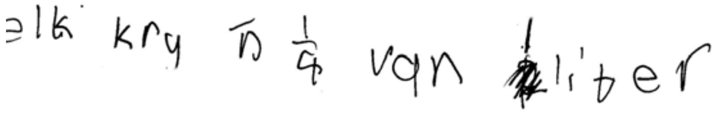


Figure 4.24 Zane Worksheet 24 (2 ÷ 8)

For 9 ÷ 4, Zane wrote down the solution and after the researcher asked him to explain, he added the drawing.

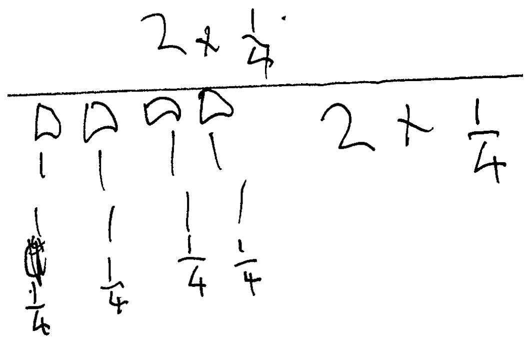


Figure 4.25 Zane Worksheet 24 (9 ÷ 4)

Zane chose, from the initial problem to employ economic sharing (Category 1). One can see from the way that he chose to solve the problems, that it was only his method of notation that changed. This changed as he was exposed to the social knowledge of how to name and write a fraction. As earlier stated, he was able to function at Level Three of Development (abstractly) from quite early on (See Section 2.3.2) (Murray and Olivier, 1989).

Zane initially did not want to be part of a group. He started off working on his own and when he was placed in a group consisting of two other boys who were not of equal ability, he covered his work and would not share his methods or ideas. As social interaction plays an important part of the learning process, it was to Zane's advantage that he found a partner he could work with (See Section 2.2.3). He would then be able to share ideas and learn from his peers. After consultation with his teacher, it was decided that Zane and Elaine should be partners as they were learners of equal ability in mathematics (See Section 2.2.3). These two learners worked well together and came up with very interesting strategies and methods for solving the problems. Towards the end of the study, they were reflecting on their work and discussing their solutions and methods.

In summary, Zane's methods represented a typical learning curve in that he developed from primitive knowledge to formalising his knowledge. Although he did not initially draw the solutions to the problems, he found that once he was unfamiliar with the notation or size of fraction, he would draw. Zane could also have "folded back" (See Section 2.10) as a result of reflection and the fact that during the first few lessons, the researcher encouraged the learners to draw their solutions. Zane was soon able to work on an abstract level (Level Three of Development; Murray and Olivier, 1989), choosing to solve all his problems using economic sharing (Category 1). It became very evident early on in the study that both Zane and Elaine needed to be challenged as they had mastered the concept of common fractions.

<u>Individual Study 2</u>

Zelda was identified as having average mathematical ability.

For all three problems on Worksheet 1 (7 ÷ 2, 7 ÷ 3 and 13 ÷ 4), Zelda employed economic sharing (Category 1) and chose to draw. She notated her answer using drawings, words and fraction notation. The first problem she solved correctly, however the second and third problems were incorrect.

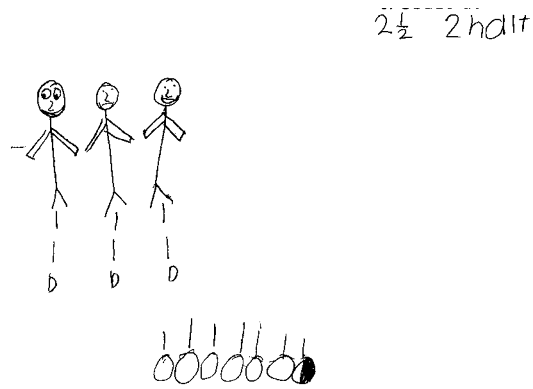


Figure 4.26 Zelda Worksheet 1 (7 ÷ 2)

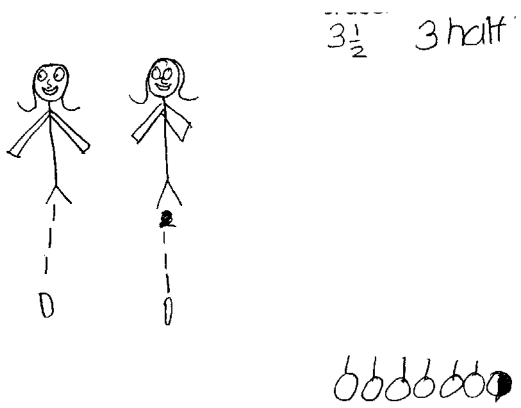


Figure 4.27 Zelda Worksheet 1 (7 ÷ 3)

In the second and third problem, the remaining unit was partitioned into halves and this resulted in Zelda sharing out more chocolate than she had!

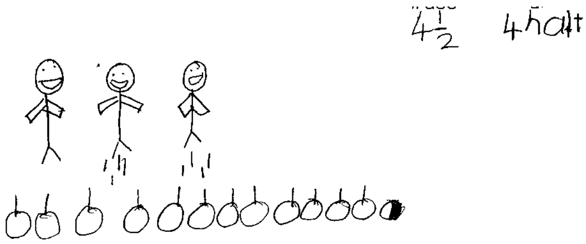


Figure 4.28 Zelda Worksheet 1 (13 ÷ 4)

From Worksheet 2, Zelda changed the layout of her solutions as well as her method of notation. While observing Zelda, the researcher saw that for every chocolate that was shared out, Zelda made a 'tick' next to the chocolate that had been shared out so that she would not become confused by what was shared and what not.

For 11 ÷ 5, Zelda drew the 11 chocolates, employed economic sharing and partitioned the remaining unit into an inappropriate number of fractional parts (Category 3). She ignored the extra fractional part and still chose to call the pieces 'halves', but her drawing of the total amount that each friend receives looks correct.

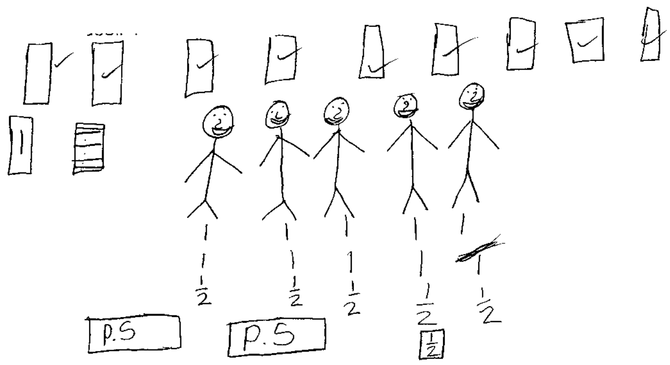


Figure 4.29 Zelda Worksheet 2 (11÷ 5)

Zelda was basing her solution on her previous limited knowledge and this resulted in her calling the fractional piece by the incorrect name. This could have been due to the fact that at School A, learners had been taught only halves and quarters in Grade 2 (See Section 2.7). Davis (1989) as cited in Pitkethly and Hunting (1996), is of the opinion that knowledge of a half inhibits the learners thinking to include other fraction interpretations.

Although she chose to divide the remaining unit into an inappropriate number of fractional parts, Zelda has progressed from her first attempts where she simply divided the remaining unit into halves.

For 21 ÷ 5, Zelda drew the 21 chocolate bars and employed economic sharing (Category 1). It is not clear from the drawing how she divided the remaining unit. She once again chose to call the fractional parts 'halves' although her drawing of the solution looks correct.

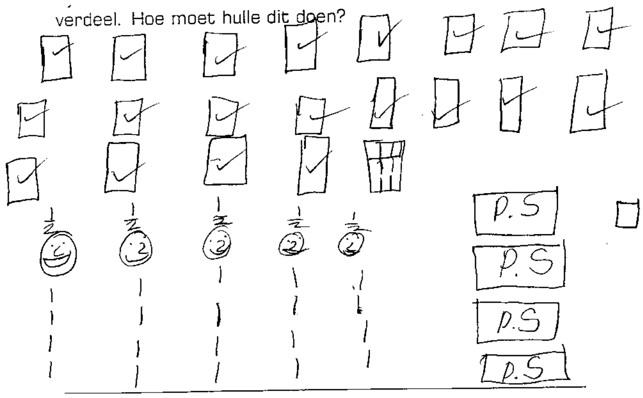


Figure 4.30 Zelda Worksheet 2 (21 ÷ 5)

For 26 ÷ 5, Zelda continued to employ economic sharing (Category 1) and still made use of drawings. However, the remaining unit was divided into an appropriate number of fractional parts (after one incorrect attempt) and she stopped referring to all fractional parts as 'halves', but has referred to the fractional part as '*a piece*'.

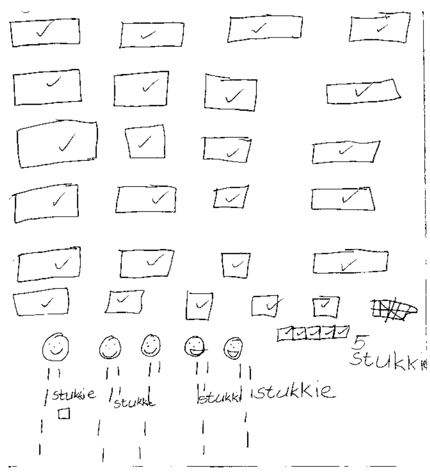


Figure 4.31 Zelda Worksheet 2.2 (26 ÷ 5)

Zelda continued to employ economic sharing for 7 ÷ 6. However, she once again partitioned the remaining unit into an inappropriate number of fractional parts and chose to ignore the additional fractional part (Category 3).

Because social knowledge had been transmitted to the learners, Zelda was able to call the fractional parts '*sixths*' – although hers were actually 'sevenths'! She wrote her solution as: '*1 and a sixth*'.

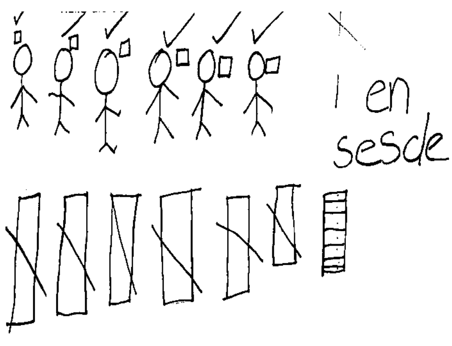


Figure 4.32 Zelda Worksheet 5 (7 ÷ 6)

When Zelda was confronted with a new situation in 8 ÷ 6 (where there were 2 remaining units), she regressed and chose to call the fractional parts '*halves*' and the two remaining units were both divided into an inappropriate number of fractional parts (Category 3). This regression could be as a result of the fractional concept not yet being stable and the social knowledge not being part of her conceptual framework yet.

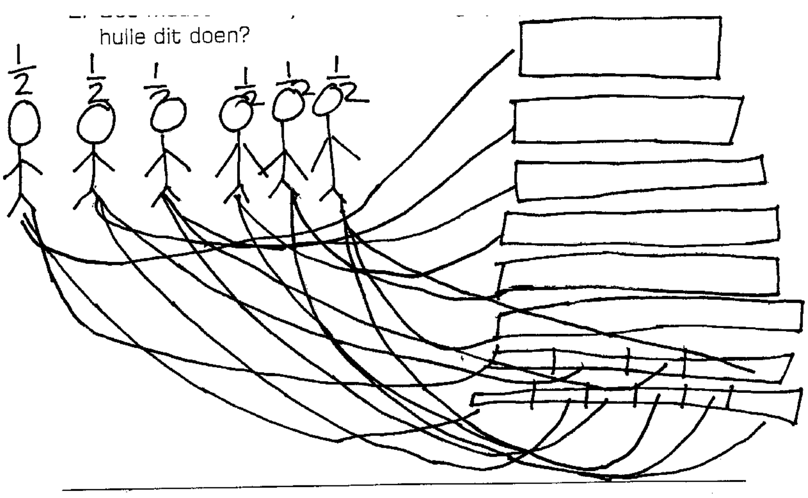


Figure 4.33 Zelda Worksheet 5 (8 ÷ 6)

In an interview with Zelda, she explained how she solved 8 ÷ 6 as follows:

Teacher: *Let us have a look at number two*

Zelda: *I drew six friends and eight chocolates. I gave each one one whole and I had two left over. I divided them into pieces.*

Teacher: *How many pieces?*

Zelda: *six*

Zelda had divided the one into sixths and the other into fourths.

Teacher: *What did you do with the pieces?*

Zelda: *I gave them all some.*

Teacher: *Show me how you divided that one up.* Pointing to the one that was divided into four.

Zelda: *four pieces*

Teacher: *And the other one?*

Zelda: *six pieces*

Teacher: *How did you share them then?*

Zelda proceeded to show me how she shared them out.

Teacher: *Do they get the same?*

Zelda: *No*

Teacher: *Will that friend be satisfied?*

Zelda: *No.*

Teacher: *What should you have done?*

Zelda: *Cut them up into smaller pieces.*

For 35 ÷ 14, Zelda obtained the answer of three chocolate bars each. She employed economic sharing, but shared out more chocolates than she had! (Category 2)

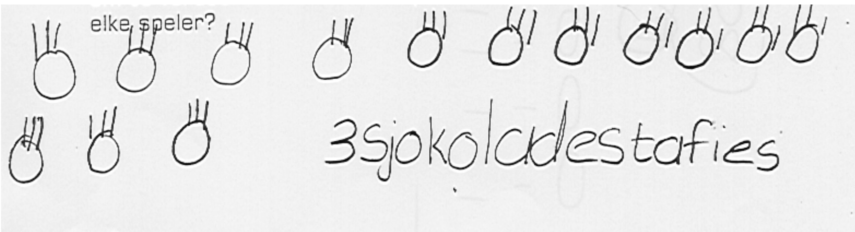


Figure 4.34 Zelda Worksheet 7 (35 ÷ 14)

It became evident that larger numbers caused a problem for some of the learners and this could have inhibited their development of the fraction concept. It must be kept in mind that the materials used were sensitive to the learners other mathematical concepts and skills and that the large numbers could have inhibited the development of the fraction concept.

Zelda and her partner Cheryl explained how they had solved the problem:

Teacher: *How much chocolate is each child going to get?*

Cheryl: *I gave each child a half and then I put the halves together and that gave me seven whole chocolates.*

Teacher: *Yes, but my question was how much chocolate is each child going to get. You divided them all out, but how much is each child going to get?*

Cheryl: *One*

Teacher: *Can you explain to me what you did from the beginning?*

Cheryl: *I made a mistake here.*

Teacher: *And then when you fixed it? Now tell me, how did you start the second time?*

Cheryl: *I drew 14 people and gave them each one but I could not go any further and then Yvonne came and she showed us. She then explained that the two halves put together give you one and the other two halves put together give you one etc. and then I got seven.*

Teacher: *But what about the other chocolates. There were 35. Zelda?*

Zelda: *I drew 14 people and then gave each three and then these five did not come out and so I gave each of them another one.*

Teacher: *Can we do the sum together? We have 35 chocolates and 14 children. Let us draw the 14 children. O.K. Can I give each child a whole chocolate?*

Zelda: *Yes.*

Teacher: *O.K. let's give them each one. How many chocolates have I given out?*

Cheryl: *14*

Teacher: *Good. Can I give these children each another chocolate?*

Zelda: *Yes.*

Teacher: *Let's give them each another one. How many chocolates have I shared out now altogether?*

Zelda: *28*

Teacher: *Good. How many are left?*

Zelda: *six*

Teacher: *Let us count together. 28,29,30,31,32,33,34,35. That gives us …seven. Can each child get another whole one?*

Zelda/Cheryl: *No*

Teacher: *So how are we going to share those that are left?*

Zelda: *Cut them into small pieces.*

Teacher: *How small must those pieces be?*

Cheryl: *half*

Teacher: *How do you know it must be a half?*

Cheryl: *Because if we put two halves together we will get one.*

Teacher: *So each child gets a half. How much does each child get altogether?*

Zelda: *Three and a half.*

Teacher: *Are you sure it is three and a half?*

Zelda: *Yes, three and a half – no, two and a half.*

Teacher: *Yes, two and a half – here's one, here's another one that gives me two and this is a …..*

Zelda: *Half.*

Teacher: *So each child gets two and a half. O.K. Thank you.*

The above transcription is a good example how reflecting on and discussing one's solutions can assist the learners find out where their mistakes are. Discussing methods employed assists the learners to consciously examine their thoughts and actions. Communication and reflection are essential components of classrooms where learners are to make sense of the mathematics that they are doing (See Sections 2.2.2.3, 2.2.3 and 2.10).

For $1\frac{1}{2} \div 2$, Zelda drew one and a half bars. She shared the whole unit into halves and the half a unit into two parts. Zelda gave each friend a $\frac{1}{2}$ and a $\frac{1}{4}$. She added what each friend would get and this totaled $\frac{3}{4}$.

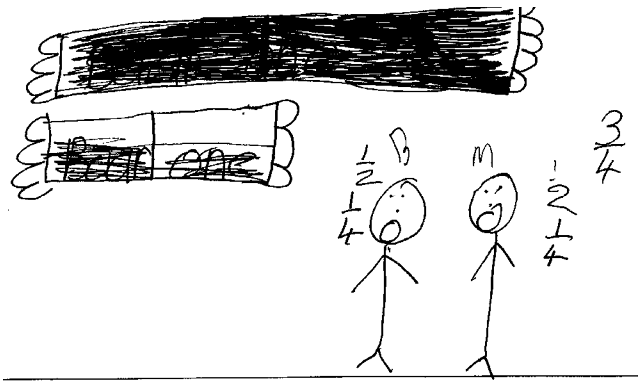


Figure 4.35 Zelda Worksheet 17 $(1\frac{1}{2} \div 2)$

For $3 \div 4$, Zelda chose to partition each unit into fractional parts of which the denominator is the same as the divisor and then shared out the parts (Category 2). Zelda once again applied the method of 'ticking off' the fractional parts as she was sharing them out. She was able to use fractional notation correctly and she chose to write the total amount in words. She wrote: '*They each get three quarters*'.

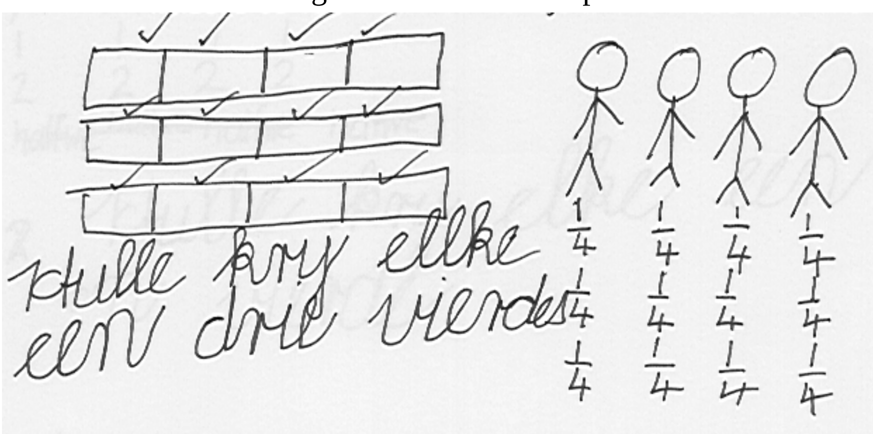


Figure 4.36 Zelda Worksheet 24 $(3 \div 4)$

Zelda, for 2*l* ÷ 8, drew the two units and partitioned them into the appropriate number fractional parts (Category 2) and shared them out correctly. Zelda drew the 2*l* 'bottle' and divided it into 2 parts. She then divided each litre into eighths and shared out the eighths. She wrote: '*Each child gets 2 eighths*'.

Figure 4.37 Zelda Worksheet 24 (2 ÷ 8)

Zelda reverted to using economic sharing and the remaining unit was partitioned into the appropriate number of fractional parts for 9 ÷ 4. She shared out the partitioned pieces correctly (Category 1). Zelda chose to write the total amount that each friend received in words. She wrote: '*They each get 2 and a fourth*'. She notated the solution correctly although it is evident that she wanted to call the fractional parts 'halves' at first.

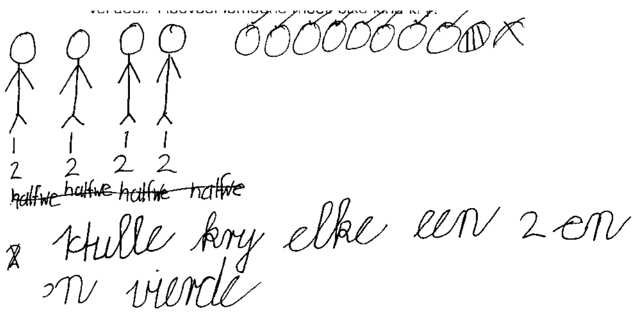


Figure 4.38 Zelda Worksheet 24 (9 ÷ 4)

Zelda is a very sensitive little girl who found it very difficult to adapt to the problem-centred approach. She was constantly seeking reassurance and she also found it very difficult to work in a group. This was her first year at School A and her teacher explained that Zelda did not adapt to the school very easily or quickly. Once Linda arrived, Zelda's work and her attitude changed completely. She asked if she could be Linda's partner and the two of them collaborated well.

Zelda definitely had a problem of calling any fractional piece a ‘half’. Ball (1993) is of the opinion that learners, who have a strong knowledge of one half, refer to any fractional part as a half (See Section 2.10). The researcher could not establish what prior learning had been done as Zelda was new to the school.

At times, Zelda worked at Level One, Phase Two of Development and at times at Level Two (Murray and Olivier, 1989). Zelda seemed to ‘hover’ between the two levels and it was noticed that she would, when uncertain, revert back to Level One, Phase Two (See Sections 2.3.2 and 2.10).

In summary, it seemed that Zelda initially found the fraction concept somewhat difficult. As mentioned earlier, it appears that Zelda had a strong knowledge of a half and that it took four problems to correct the limiting construction (See Section 2.7). Zelda struggled with the concept that fractions are equal parts and initially shared out more chocolate bars than she had. It appears that Zelda had problems ‘cutting up’ her chocolate bars. Initially she partitioned everything into halves. She went on to partition the chocolate into inappropriate number of pieces. Zelda showed signs of benefiting from reflection and discussions with her peers. However, it took her a long time to embark on reflecting and discussing (See Section 2.2.2.3). From Worksheet 17 on, Zelda’s solutions were correct. The researcher is of the opinion that for Zelda, the development of the common fraction concept took a long time to develop and become stable.

Individual Study 3

Mbulelo is of the weaker mathematical ability group.

Mbulelo, for 7 ÷ 2, used a method that led to the incorrect answer (Category 4). No pattern could be identified and the incorrect answer was given.

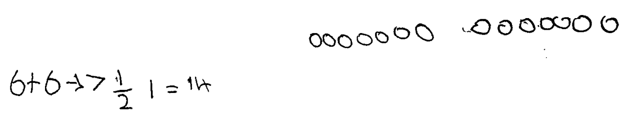


Figure 4.39 Mbulelo Worksheet 1 (7 ÷ 2)

Once again, for 7 ÷ 3, no method could be identified. The incorrect answer was given.

Figure 4.40 Mbulelo Worksheet 1 (7 ÷ 3)

It seemed as though for 13 ÷ 4, Mbulelo shared out the units using economic sharing, but he shared out more units than he had. He then partitioned the remaining unit into an inappropriate number of fractional parts and ignored the fractional parts (Category 3).

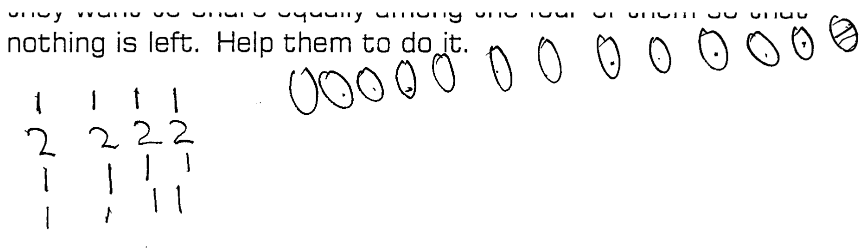


Figure 4.41 Mbulelo Worksheet 1 (13 ÷ 4)

Mbulelo once again shared out more units than were available for 11 ÷ 5 however, he did partition one unit into an inappropriate number of fractional parts and shared them out (Category 4). He recorded that each friend would receive 12.

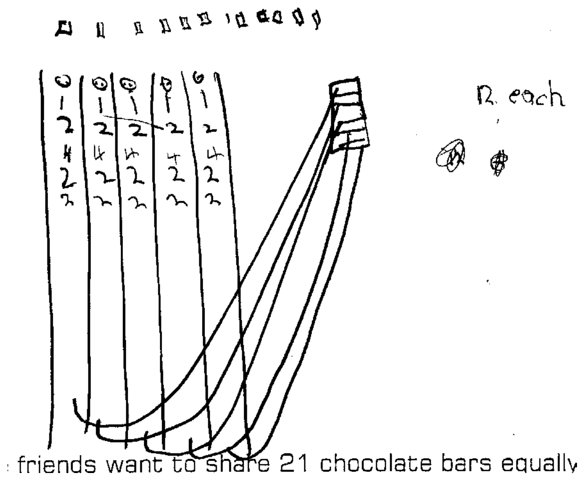


Figure 4.42 Mbulelo Worksheet 2 (11 ÷ 5)

In the next problem, Mbulelo shared out more units than were available (Category 4).

He indicated that each friend would receive five.

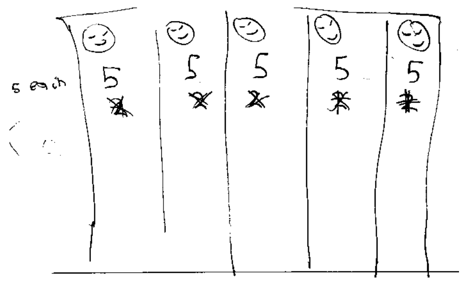


Figure 4.43 Mbulelo Worksheet 2 (21 ÷ 5)

For 7÷6, Mbulelo used economic sharing. He partitioned the remaining unit into an inappropriate number of fractional pieces and ignored the extra fractional piece (Category 3).

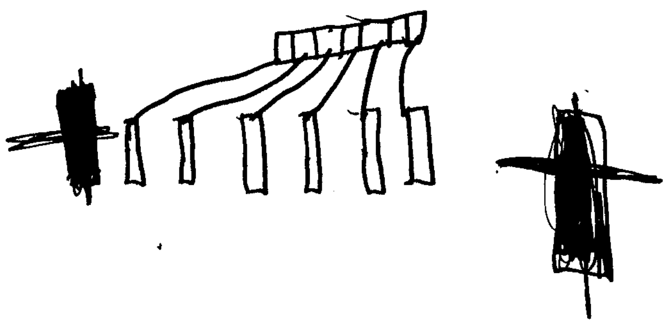


Figure 4.44 Mbulelo Worksheet 5 (7 ÷ 6)

For 8 ÷ 6, Mbulelo used economic sharing, but although he partitioned the remaining unit into an inappropriate number of fractional pieces, he acknowledged that there was one left over (Category 3).

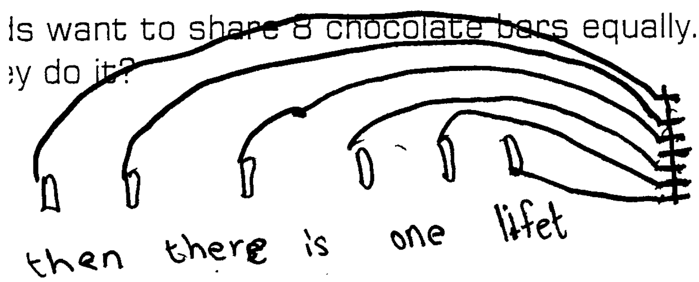


Figure 4.45 Mbulelo Worksheet 5 (8 ÷ 6)

Mbulelo did not manage to complete 35 ÷ 14. The researcher observed that he became very confused using the larger numbers.

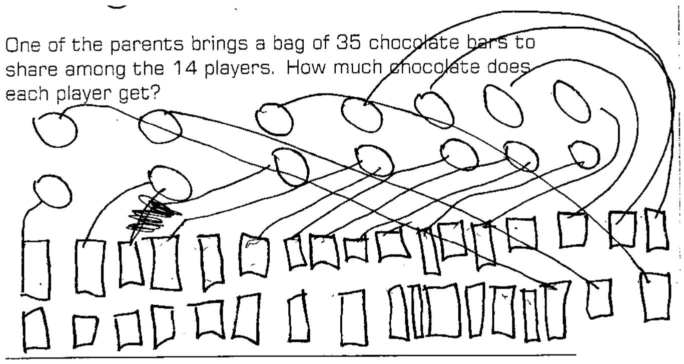


Figure 4.46 Mbulelo Worksheet 7 (35 ÷ 14)

Comparing Mbulelo's responses in Worksheets 1 and 2, to Worksheet 5, the researcher could see a definite progression. This could be due to a number of factors. When the researcher first began her research at School B, the teacher insisted on teaching the class. She did not allow the learners to discuss the problem or their solutions and reflection was done at the end of the lesson. The teacher went on leave in the second term. The researcher then facilitated the lessons and Mbulelo partnered Tammy. Mbulelo and Tammy usually engaged in discussions before either of them began solving their problems (Video no1). Tammy and Mbulelo were of equal ability and according to Murray *et al*. (1993) learners interact better with peers of equal ability (See Section 2.2.3). The discussions between Tammy and Mbulelo, drove the learning process.

Overall, Mbulelo remained on Level One, Phase Two of Development throughout the study (Murray and Olivier, 1989). Although the researcher was not able to complete all the worksheets with School B and therefore not monitor the progress of Mbulelo, she felt that there had definitely been an improvement and that Mbulelo was beginning to make sense of the common fraction concept.

4.6 SOME DEEPER UNDERSTANDINGS OF THE FRACTIONAL CONCEPT

1. After Phindiwe had completed 7 ÷ 2 and 7 ÷ 3, the researcher noticed that the fractional pieces in both solutions were referred to as halves. The interview with Phindiwe continued as follows:

Teacher: *Phindiwe, I see that you have called the pieces in number one halves and the pieces in number two halves. Are these pieces the same size?*

Phindiwe: *No. The ones in number two are bigger pieces.*

2. The problem 8 ÷ 6, resulted in two different solutions, namely $1\frac{1}{3}$ and $1\frac{2}{6}$. This caused the learners to have 'very serious' and lengthy discussions and eventually they reached consensus that they were actually the same size!

The researcher had a discussion with the class at School B about this problem:

Teacher: *Something interesting is happening in number two because….in number two the child is going to get one and two sixths……*Lucille started laughing*. No, no, no, we do not laugh. And then some of the children are telling me we are going to get one and a third. Now I want you to discuss and tell me who is correct and who is incorrect. Ok Lucille, what do you say?*

Lucille: *One and a third.*

Teacher: *Why do you say a third?*

Lucille: *Because there are two left.*

Teacher: *I am going to draw those two chocolate bars. Now what did you do with those two chocolate bars?*

Lucille: *I cut them into thirds.*

Teacher: *How many cuts must I make, Mbulelo?*

Mbulelo: *Three, no two.*

Teacher: *Two lines. How many pieces have I got?*

Class: *six.* There were two remaining units.

Teacher: *So each child can get one piece. Yes Gail.*

Gail: *Two sixths and one third are the same.*

Teacher: *Gail said two sixths and one third are the same. What do you say about that? Let us just draw. We are going to divide each one into sixths – six equal pieces. Now, let us have a look here. Gail said two sixths are equal to one third. Is she correct?*

Class: *Yes.*

Teacher:	*Some children said each child was going to get one and a third - other children said that they were going to get one and two sixths so who is correct and who is incorrect?*
Class:	*They are all right.*
Teacher:	*Both are correct. So Lucille, we must not laugh because they were actually correct. O.K.*

3. For $1\frac{1}{2} \div 2$, both Elaine and Zane simply wrote down the answer $\frac{1}{2} + \frac{1}{4}$.

The researcher interviewed Elaine after the lesson:

Teacher:	*Elaine can you tell me how you worked out that each child would get* $\frac{3}{4}$ *or* $\frac{6}{8}$ *of the chocolate that needed to be shared?*
Elaine:	*I made a picture of chocolates in my head and I saw that if a cut the half through the middle, I would get a* $\frac{1}{4}$ *and if I cut the whole one in the middle, I would get a* $\frac{1}{2}$*. I looked to see what was equal to a half and a quarter and I got* $\frac{3}{4}$ *and* $\frac{6}{8}$*.*

Elaine was referring to the fraction wall that was in the classroom.

By this stage of the study, Elaine and Zane were both on Level Three of Development (Murray and Olivier, 1989). They were working on an abstract level. Elaine was able to picture in her mind what the ‘cut up’ chocolate bar would look like. She no longer needed to physically draw the chocolate bar.

4. The sum $2l \div 8$ caused the learners once again to have ‘very serious’ group discussions about the solution. Some learners agreed that $\frac{2}{8}$ was the correct solution and while others agreed that $\frac{1}{4}$ was the correct solution. After much discussion, the learners decided that all their solutions were correct because $\frac{2}{8}$ of $1l$ is the same as $\frac{1}{4}$ of $1l$.

5. When Elaine had the problem 9 ÷ 4, posed to her she reasoned as follows:

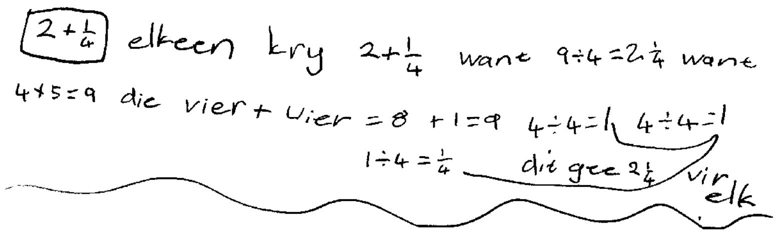


Figure 4.47 Elaine Worksheet 24 (9 ÷ 4)

2 + $\frac{1}{4}$ each one gets 2 + $\frac{1}{4}$ because 9 ÷ 4 = 2$\frac{1}{4}$ because

4 + 5 = 9 the four + four = 8 + 1 = 9

4 ÷ 4 = 1 and 4 ÷ 4 = 1 and 1 ÷ 4 = $\frac{1}{4}$. She then 'connected' the 1, 1 and $\frac{1}{4}$ and said that the answer was 2 $\frac{1}{4}$.

4.7 SUMMARY

From the grids indicating in which category the learners fell (See Appendix B), it was evident that the majority of learners from both School A and B used Category 1 (economic sharing) to solve the problems in Worksheet 1. The majority of learners notated their solutions using drawings and fractional notation. Most of the learners who had shared the units using an inappropriate fractional part (Category 3) chose to ignore the extra fractional part.

Worksheet 2: the majority of learners from both schools once again fell into Category 1 and they either chose to draw the solution or draw the solution and write the answer in words. Many of them chose to call the fractional part 'a piece'. This could be due to the fact that the researcher mentioned that if they did not know what to call the fractional part, they could refer to it as 'a piece'. Mack (1990) found learners referred to the fractional part as a 'piece' and Murray *et al*. (1996a) also found that learners referred to the fractional part as a 'piece', but not as often as they thought would be the case. Those learners, who fell into Category 3, also chose to ignore the extra fractional part.

Worksheet 5: the learners from School A, with the exception of one, fell into Category 1 and notated their answers using drawings and words. It must be mentioned once again that at this stage, the learners had been exposed to how to name a fractional part, but there were still some learners where the terminology had not become stable and mistakes were made. Although in School B the majority used Category 1, the manner in which they notated their solutions were spread over the various notation strategies.

From Worksheet 5, most learners employed Category 1 (economic sharing) to solve their problems, but when they were exposed to a remainder of two units, most of the learners in both Schools A and B chose to employ Category 2 (partitioning all). When learners are confronted with a new experience, they often simplify the problem for themselves by solving the problem on a lower Level of Development (See Section 2.10) (Murray and Olivier, 1989). Kieren *et al*. (1992) as cited in Pitkethly and Hunting (1996) refer this to as “folding back”. With the problem $8 \div 6$, some learners chose to partition the remaining units into thirds while others chose sixths. These methods are at the same level of development although it is possible to say that partitioning the units into thirds could be at a more advanced level. It is definitely a shortened version. This solution gave rise to a discussion on which solution was correct - $1\frac{2}{6}$ or $1\frac{1}{3}$. This initiates thinking about equivalent fractions (See Section 2.9).

Worksheet 7 yielded the same result with most of the learners choosing Category 1 to solve the problem. However, the large numbers involved in the problem also caused many learners to obtain the incorrect solution. As mentioned above, it must be kept in mind that the problems used were sensitive to the learners other mathematical concepts and skills that the large numbers could have inhibited the development of the fraction concept. Learners, especially at School A were not familiar to the Problem-centred approach to mathematics and their number concept development was not well developed. The learners were not used to working with larger numbers.

From Worksheet 7 onwards, there was a marked improvement in the learners' social knowledge of naming fractions and using the fractional notation correctly. This was a result of the learners being introduced to the terminology and fraction notation in Worksheets 3, 4 and 5.

Worksheet 17: This involved only School A and saw all the learners employing economic sharing – Category 1, but once again, where a new concept was introduced in Worksheet 24, some of the learners reverted back to using Category 2. (Partitioning all the units into fractional parts of which the denominator is the same as the divisor, then sharing out the parts from each in turn.) This confirms the assumption of Kieren *et al*. 1992, as cited in Pitkethly and Hunting (1996), that a learner will solve a problem at a lower level when a 'new' or difficult problem or situation confronts him/her (See Section 2.10). The last problem posed to the learners to develop the fraction concept saw all the learners using Category 1 (Economic Sharing - sharing out the maximum number of units and dividing the remaining unit (whole) into an appropriate number of fractional parts) with the majority of the learners notating their solutions using drawings and the correct fractional notation.

Learners were able to understand and solve the equal-sharing problems involving fractions. The majority of learners fully understood the equal-sharing situation however, Mbulelo showed signs of not understanding equal-sharing. A few learners did produce incorrect answers (See Appendix B). This could have been due to previous instruction being limited to halves and quarters and/or the activities that they were required to do (e.g. Shade in $\frac{1}{4}$ of the square) (See Section 2.7).

Initially, some learners had difficulty in 'cutting up' the chocolates into the correct number of pieces. If learners had to, for example, cut a bar into fifths, they needed to make four 'cuts'. The learners would then make five 'cuts' and this would result in six pieces and not five. One possible reason for this could be the fact that in everyday life, when slicing bread, if a person wants four slices, he/she will cut four times. Some learners took quite a few attempts to master this. This concurs with the findings of Ball (1993), as cited in Pitkethly and Hunting (1996) (See Section 4.6 Individual Study 2).

According to Mack (1990) and Steffe and Olive (1991), learners have a "…wealth of informal knowledge on which we can base the teaching of fractions". The researcher is of the opinion that the Grade 3 learners she was working with, had a certain amount of informal knowledge that enabled them to understand and solve sharing problems involving fractions. They did, however, have a wealth of ability to think about and solve problems.

Although some fraction notation had been taught to learners in both schools, there was a high incidence of incorrect naming of the fractional parts. It was evident in the way the learners named their fractional parts, that this social knowledge had not yet become stable. One learner called a third, 'a quarter' in words and wrote $\frac{1}{2}$ in symbols. Some learners referred to fractional parts as 'pieces' and yet others called all fractional parts 'halves'. These limiting constructions could be due to the teaching and teaching materials that had been used in Grade 2 at School B. It cannot be determined to what extent these limiting constructions could have originated in the learner's pre-school or outside school experiences.

The majority of the learners had developed a great deal from Worksheet 1 to Worksheet 24. The researcher is of the opinion that most of the learners had developed the concept of a common fraction by the end of Worksheet 24. They were able to identify the part-whole relationship between the fractional part and the unit (See Section 2.5). By solving equal-sharing problems that were set in a real-life context that they could identify with, the learners were able to make sense of the mathematics they had done.

The researcher confirms the findings of Empson (1995) who said that using equal-sharing problems could help children learn and understand fractions. For these learners, the development of fraction number knowledge grew out of sharing and dividing up situations.

Once the initial fraction concept had been developed after the first two worksheets, it was necessary to develop the other topics to form the basis for mature functioning for rational numbers (See Section 2.5). Since knowledge of fraction names and symbols (See Section 2.3.1) need to be introduced so that the learners can communicate about fractions, these names and fraction symbols were introduced to the learners as deemed necessary (See Section 2.3.3).

CHAPTER 5

SOCIAL KNOWLEDGE

Social knowledge that learners require for the common fraction concept includes the naming of fractional parts and writing of the common fraction notation (fraction symbols).

5.1 ANALYSIS OF DATA

The social knowledge was transmitted to the learners by the researcher as it was deemed necessary (See Section 2.3.3). This was done in two ways – verbally and on worksheets and once this social knowledge had been transmitted, posters were displayed in the classroom throughout the duration of the research (This could not be done at School B due to the fact that the researcher was not in a classroom of her own, but in a room that was used for different activities. The researcher did however display a poster in the learners' own classroom). The social knowledge always appeared in a rectangular frame at the beginning of the worksheet.

The learners were first exposed to the fraction names in words and at a later stage, the fraction symbols were shown to the learners. Some learners from both School A and B could write a quarter and a half using fraction symbols.

Fraction names, in the form of words, were initially transmitted and then the unit fraction symbols. When it became necessary, the learners were shown how to write a fraction where the numerator was greater than one.

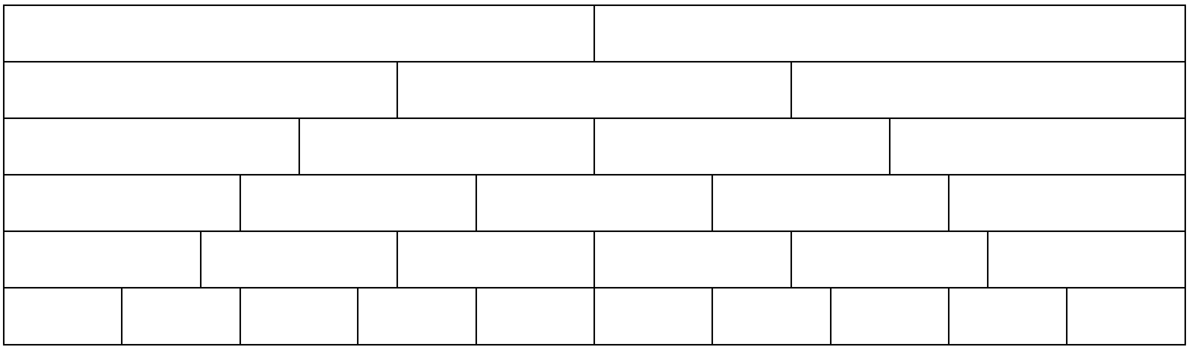
The following fraction names and symbols were shared with the learners.

Worksheet 3

When we divide something into 2 equal parts, we call these parts halves
When we divide something into 3 equal parts, we call these parts thirds
When we divide something into 4 equal parts, we call these parts fourths or quarters
When we divide something into 5 equal parts, we call these parts fifths

Worksheet 4

Look at these big chocolate bars. They have all been cut into different equal pieces. Give the pieces names (halves or thirds or whatever you think).



Worksheet 5

A chocolate bar is cut into two equal pieces. What do you call each piece?

A chocolate bar is cut into 7 equal pieces. What do you call each piece?

Worksheet 7

A short way to write a half is $\frac{1}{2}$

A short way to write a seventh is $\frac{1}{7}$

A short way to write a twentieth is $\frac{1}{20}$

Worksheet 8.1 (Worksheet 8.1 was included in the study as it became necessary for the learners to be shown how to write a fraction which has a numerator greater than one earlier than was originally planned.)

If we cut a chocolate bar into **three** equal pieces, we call these parts **thirds**.

We write a third like this: $\frac{1}{3}$

If we give two thirds away, then we write two thirds like this: $\frac{2}{3}$

If we cut a chocolate bar into **four** equal pieces, we call these parts **quarters or fourths**.

We write a quarter or fourth like this: $\frac{1}{4}$

If we give two quarters or fourths away, then we write two quarters like this: $\frac{2}{4}$

If we give three quarters or fourths away, then we write three quarters like this: $\frac{3}{4}$

Write the following using fraction symbols:

Two fifths	Three sevenths
Three fifths	Five sevenths
Five sixths	Four eighths
Two ninths	Seven tenths

Worksheet 12

A short way to write a fifth is $\frac{1}{5}$

A short way to write three fifths is $\frac{3}{5}$

Worksheet 3

The learners were merely exposed to the fraction names (halves and thirds) and were not required to implement this knowledge in any way.

Worksheet 4

Between Worksheet 3 and 4, School A had been on holiday for two weeks and School B had been on holiday for three weeks.

After the initial introduction of the fraction names in Worksheet 3, the majority of learners named these fractional parts correctly.

The learners were required to name the fractional pieces on a fraction wall and the grid (Appendix C1) indicates the success of the learners. All the learners at School A could name the halves, thirds, quarters and fifths. One learner omitted to name the sixths and 72% of the learners named the tenths correctly. At School B, the results concurred with the results from School A. All the learners could name the halves, thirds, quarters and fifths. One learner omitted to name the sixths and 83% of the learners named the tenths correctly.

All the learners could name the fractional pieces for halves, thirds, quarters, fifths and sixths except for Stewart, who left out the name for sixths. It seems that this was done unintentionally. This was the first time that the learners had come across tenths. The other fractions had been met while the learners were developing the concept of a fraction in Worksheets 1 and 2. One learner initially referred to the tenths as ‘pieces’ (See explanation below), another as ‘a whole’, and yet another as eighths. Three learners referred to tenths as sevenths. This could have been as a result of the method used to decide on the names for the fraction pieces. The learners counted the number of fractional pieces in each bar and by the time they reached the last row, they simply guessed instead of counting!

During participant observation, Sue was questioned about how she had approached naming the fractional pieces on the fraction wall. She replied:

"Well, if you count the pieces, then there are ten, so each piece must be a tenth."

In Worksheet 4, learners were asked to name the fractional parts. Caroline named them as follows: the first bar '*halves'*, the second '*thirds'*, the fourth, '*fourths'*, the fifth, '*fifths',* the sixth bar she called '*sixths'* and the last bar she first called '*pieces'* and then, after reflection, she named the pieces '*tenths'*.

For the second question where learners were required to find equivalent fractions, she wrote:

halves = 2 quarters

thirds = 2 sixths

5 of a tenth = a half

a half is = to   pieces

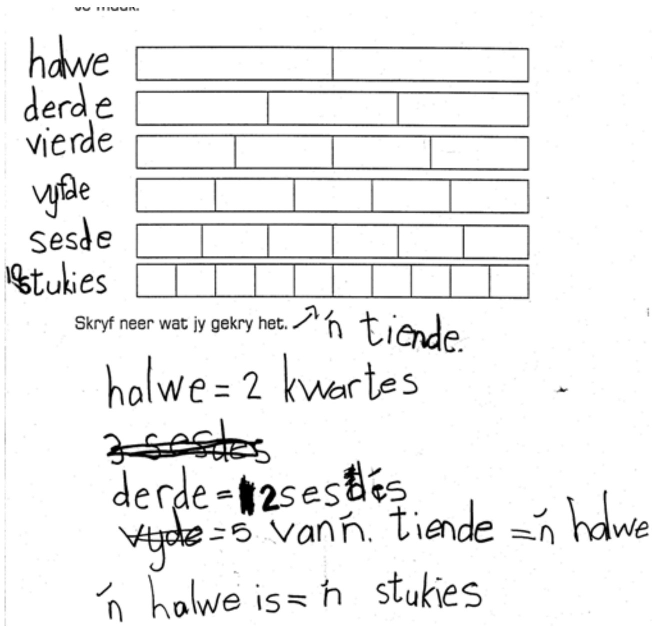


Figure 5.1 Caroline Worksheet 4 (Equivalent fractions)

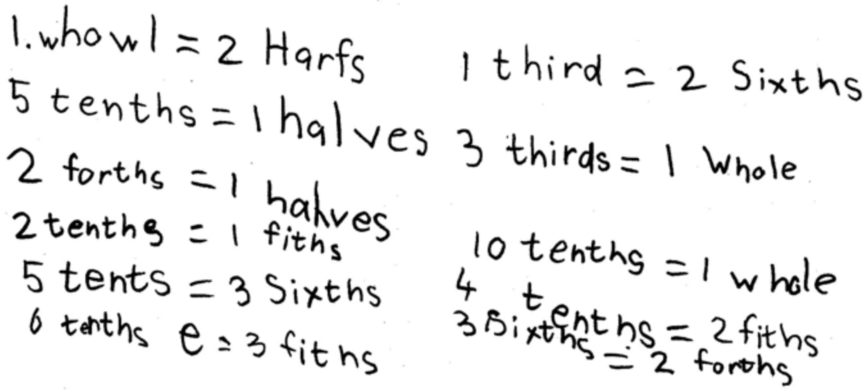


Figure 5.2 Sue Worksheet 4 no 2 (Equivalent fractions)

Worksheet 5

The learners were asked to name the fractional pieces (halves and sevenths). They were not told whether to use words or fraction symbols. The researcher identified eight approaches used by the learners. These approaches are summarised on grids (Appendix C2).

All learners were able to name the fraction pieces correctly. The majority of learners chose to write the fraction names using words (Approach 1) while others used fraction symbols (Approach 4). During prior learning the fraction symbols for halves and quarters were shown to learners at both schools in Grade 2. School B introduced the fraction symbol for thirds in Grade 2 as well.

Elaine wrote: "*one half*"

Figure 5.3 Elaine Worksheet 5 no 3 (naming fractional parts)

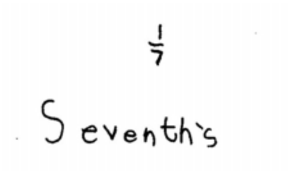


Figure 5.4 Sue Worksheet 5 no 4 (naming fractional parts)

Worksheet 7

The learners were exposed to the fraction symbols ($\frac{1}{2}$; $\frac{1}{7}$; $\frac{1}{20}$).

<u>Worksheet 8.1</u>

At this point in the study, learners were solving problems where the fractional part of the solution was greater than a unit fraction. The researcher felt it necessary to include an additional worksheet exposing the learners to writing fractions where the numerator was greater than one, using symbols ($\frac{2}{3}$; $\frac{2}{4}$; $\frac{3}{4}$).

All the learners were able to answer these questions correctly.

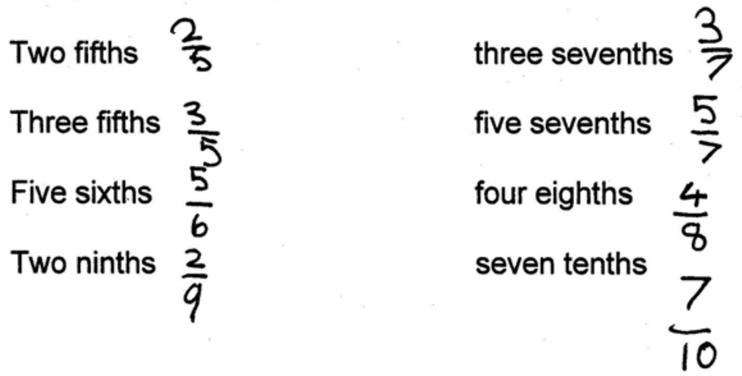


Figure 5.5 Gail Worksheet 8.1 no 5

<u>Worksheet 12</u>

The researcher identified three approaches used by the learners to name the fractional parts of a fraction wall. These approaches are summarised on grid (Appendix C3).

Learners were still employing the same method to name the fractional parts, namely, counting the number of pieces although the majority of learners were using fraction symbols to name the fractional parts on the number wall (Approach 1). The number of approaches used by the learners diminished from eight to three.

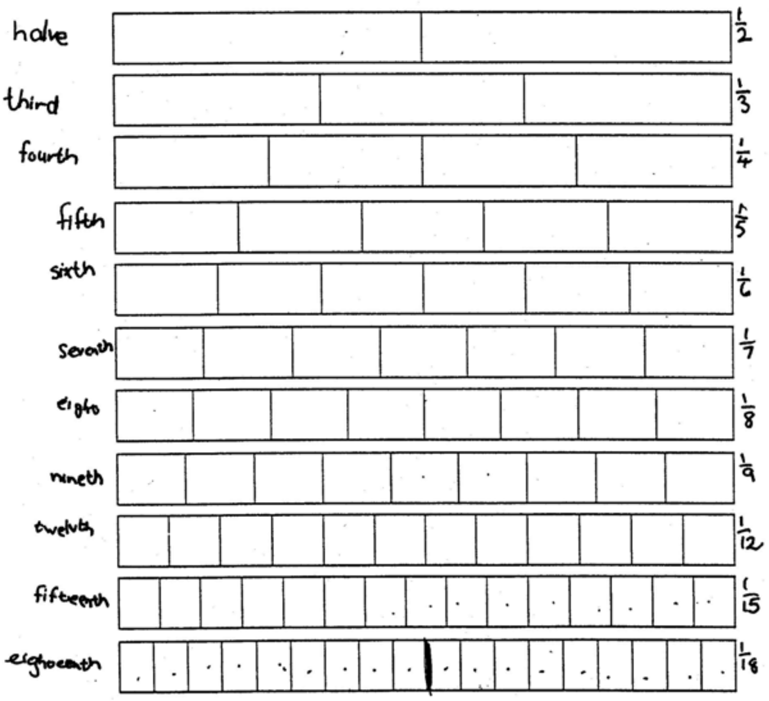


Figure 5.6 Nosisa Worksheet 12 no 6 (Approach 2)(naming fractional parts)

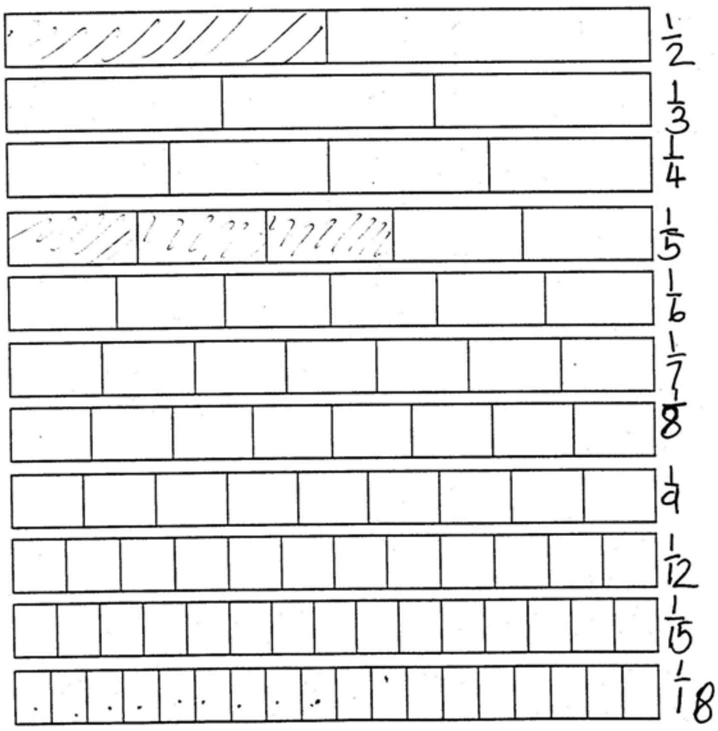


Figure 5.7 Zelda Worksheet 12 no 7 (Approach 1) (naming fractional parts)

5.2 SUMMARY

The learners did not have any difficulty in applying the social knowledge that they had been exposed to. Initially, the learners did not know what to name the various fractional parts, and once exposed to the social knowledge, they used it immediately albeit sometimes incorrectly. Some of the learners did however feel more comfortable with writing the fraction names using words, but towards the end of the research, all the learners were using fraction symbols.

Learners were able to name certain fraction parts on a fraction wall, when asked to do so, but when solving equal-sharing problems where the solution included a fraction, a few of the learners were still referring to the fractional part as a ‘piece’. This was found to be the case earlier on in the research. This could have been due to the fact that the social knowledge had not yet become conceptually stable or that the terminology of fraction names had not yet become part of their vocabulary. It could have been that the learners did not see the necessity of naming the fractional part correctly as it was ‘a piece’. They saw no reason for giving it the appropriate name and they did not use the terminology as the researcher expected them to use it.

The learners were exposed to too many tasks involving unit fractions before the symbols for fractions with numerators greater than one were introduced. Some solutions to the problems resulted in fractions where the numerator was greater than one, and this gave rise for the necessity of introducing the fraction symbols earlier than originally planned.

The social knowledge required for the fraction concept took a long time to become part of the learners’ conceptual knowledge. The learners do not make sense of or need to understand social knowledge and this could be a contributing factor. Another contributing factor could be that learners’ do not see the need for each fractional part to have a name as they see the fractional part as a ‘piece’. The researcher is of the opinion that if some tasks had been included that required learners to distinguish between the size of the different ‘pieces’ this problem could have been alleviated to some degree (e.g. Share 4 T.V. bars between 3 children so that each child gets the same amount. Share 5 T.V bars between 4 children so that each child gets the same amount. Which children are going to get the most, those who are going to have to share the T.V. bar's between 3 or those who are going to have to share the T.V. bars between 4?).

Equivalence, of which comparison of fractions forms an integral part, needs to be developed in conjunction with developing the fraction concept and social knowledge. Chapter 6 will deal with these two topics.

CHAPTER 6

EQUIVALENT FRACTIONS AND COMPARISON OF FRACTIONS

Traditionally, equivalent fractions are introduced in the fourth grade and repeated in the subsequent grades as the four arithmetic operations are taught (Kamii and Clark (1995). By using problems as the vehicles for learning (See Section 2.2.2.1), discussions around the different solutions that learners found resulted in equivalence and comparison of fractions occurring early on in the study (See Chapter 4.4 Worksheet 5).

6.1 ANALYSIS OF DATA

The researcher has grouped together all the worksheets where this concept is addressed in order to identify the methods used by the learners.

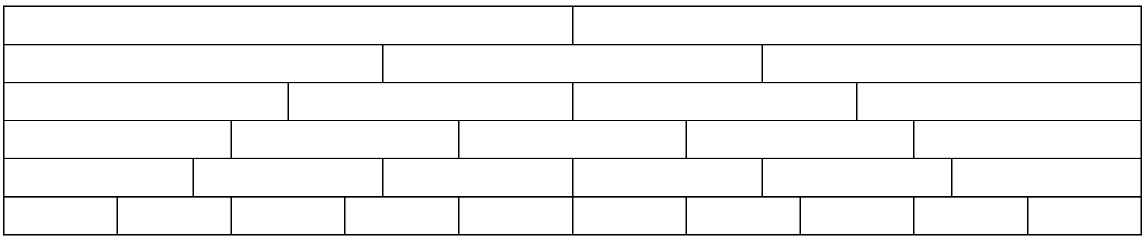
The following problems were presented to the learners:

<u>Worksheet 3</u>

1. What would you rather have, a third of a chocolate bar or a fifth of a chocolate bar? Why?
2. What would you rather have, a fourth of R1 or a third of R1? Why?
3. How many cents is a half of R1? A fourth of R1? What would you rather have, a half of R1 or two fourths of R1? Why?

<u>Worksheet 4</u>

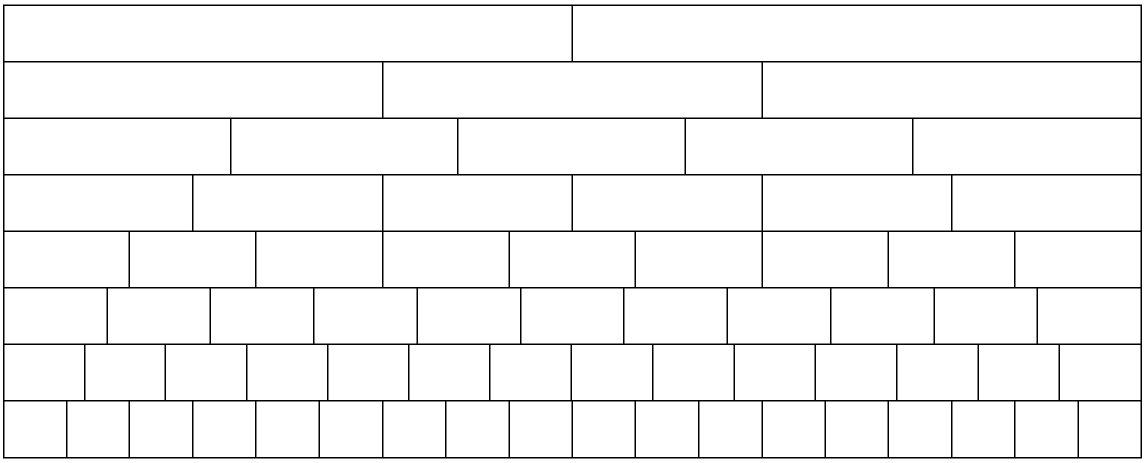
Look at these big chocolate bars. They have all been cut into different equal pieces. Give the pieces names (halves or thirds or whatever you think). Now see if you can put together some smaller pieces to form a bigger piece with another name.



Write down what you have found.

<u>Worksheet 6</u>

Look at these chocolate bars that have been cut in different ways:



1. How many sixths of a chocolate bar must you put together to make a third of a chocolate bar?

   How many sixths to make two thirds?

   How many tenths to make a fifth?

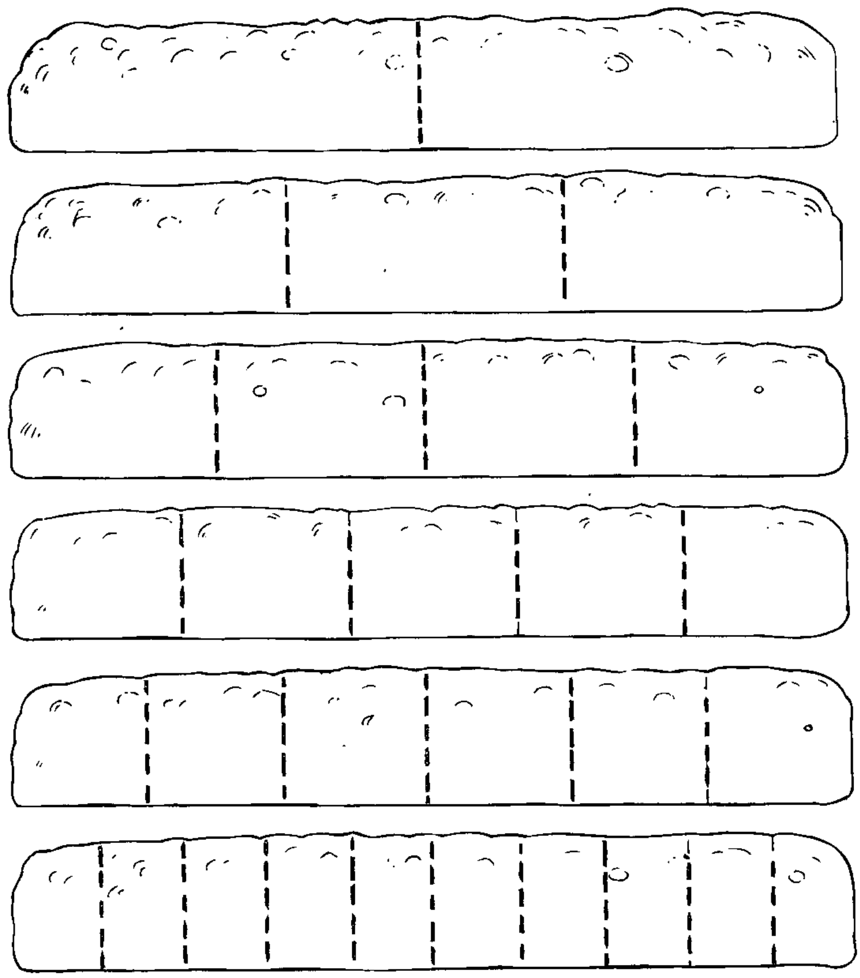
   How many tenths to make a half?

   How many ninths to make two thirds?

2. Write down all the bigger pieces you can make from

   ninths fifteenths eighteenths

<u>Worksheet 6.1</u> (The researcher felt it necessary to include an additional worksheet for School A. They had difficulty with this concept.)



Look at the different ways in which these chocolates have been divided up.

1. Give each piece a name.
2. Use your ruler to help you – slide it from left to right over your page.
3. Look at the dotted lines and check to see if there are other chocolates with a dotted line in the same place.
4. Write down all the pieces you find that are equal to other pieces.

<u>Worksheet 6.2</u> (Additional worksheet for School A)

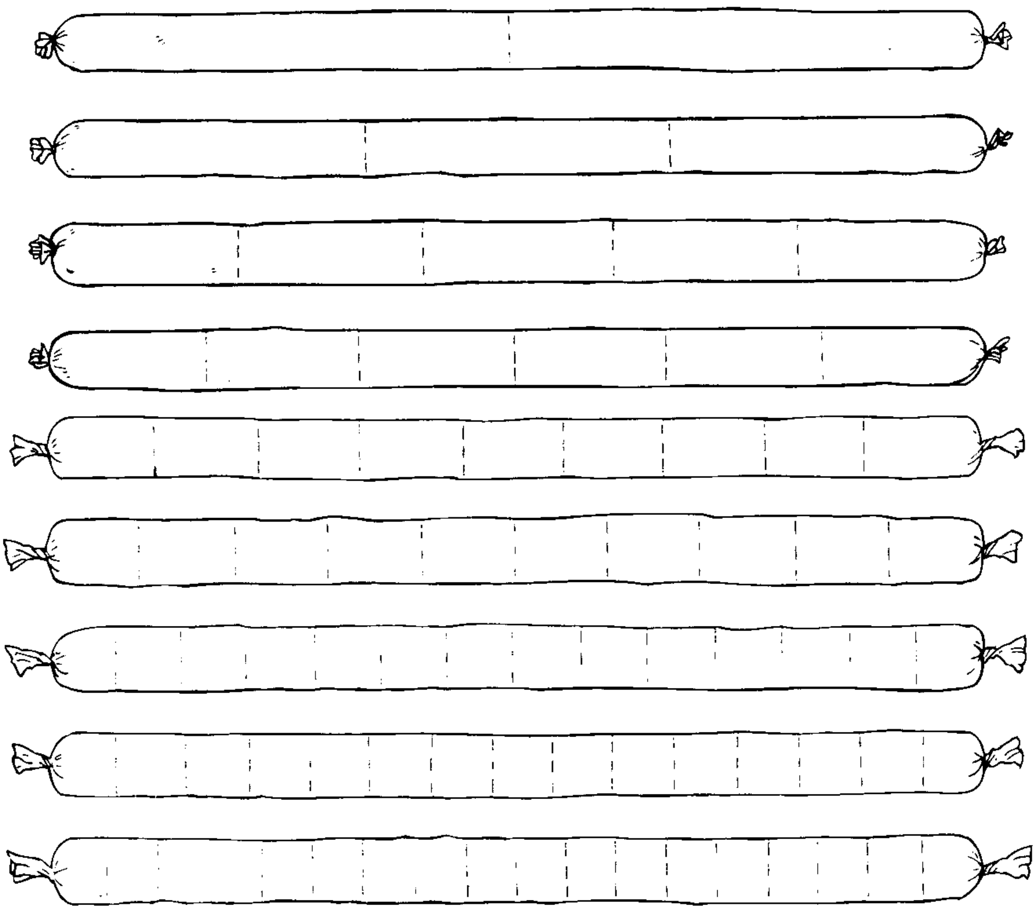
Look at the different ways in which these vienna sausages are cut up:

How many ninths of a sausage must you put together to get a third of a sausage?

How many sixths to make two thirds?

How many tenths to make a fifth?

How many tenths to make a half?

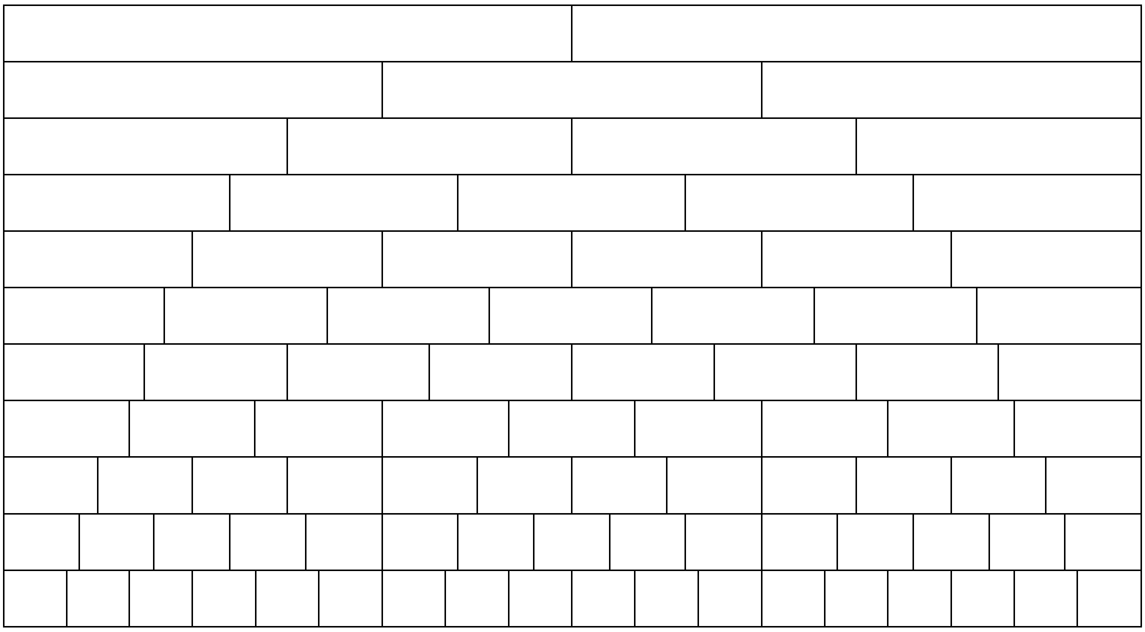


Write down all the bigger pieces you can make from:

ninths tenths eighteenths

Worksheet 12

Here are some chocolate bars that have been cut:



Which piece of chocolate bar would you rather have? Why?

$\frac{1}{3}$ or $\frac{2}{6}$

$\frac{1}{2}$ or $\frac{3}{5}$

$\frac{2}{4}$ or $\frac{3}{6}$

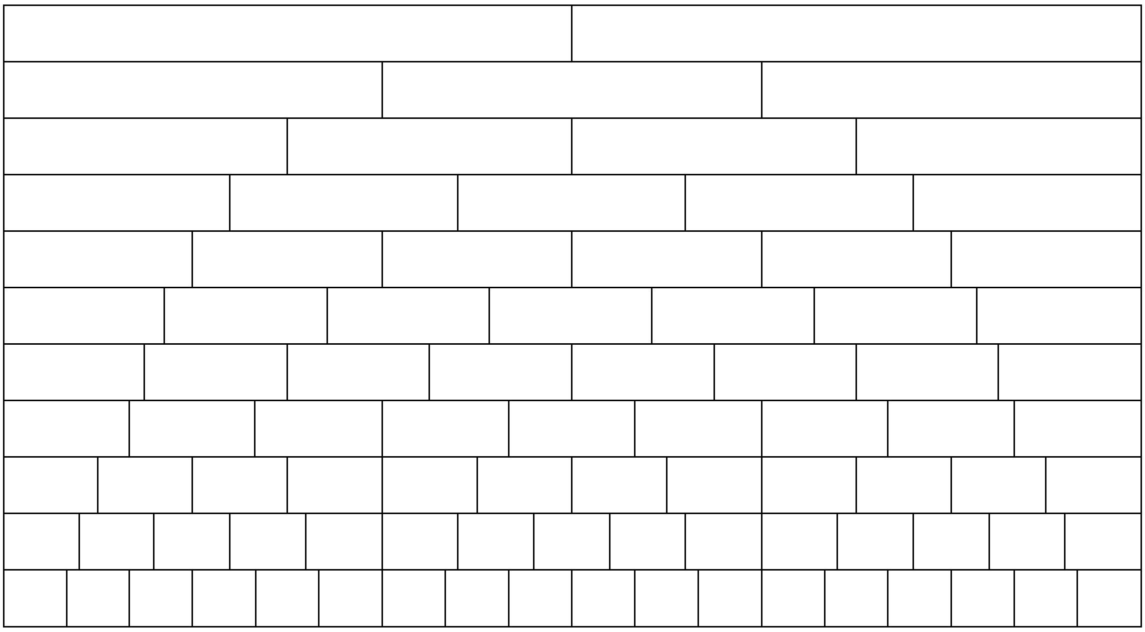
$\frac{5}{10}$ or $\frac{2}{5}$

$\frac{1}{7}$ or $\frac{1}{8}$

$\frac{7}{15}$ or $\frac{4}{8}$

$\frac{9}{18}$ or $\frac{6}{10}$

Worksheet 13



Here are some pieces of chocolate bar:

$\frac{6}{18}$ $\frac{5}{10}$ $\frac{1}{3}$ $\frac{2}{4}$ $\frac{3}{6}$

$\frac{4}{12}$ $\frac{6}{12}$ $\frac{5}{15}$ $\frac{5}{6}$ $\frac{4}{5}$

$\frac{12}{18}$ $\frac{12}{15}$ $\frac{10}{12}$ $\frac{2}{3}$ $\frac{2}{6}$

1. First say which of these pieces of chocolate do you think are the same size. Explain why you say so.
2. Then check on the drawing if you were right.
3. Now use the drawing to find all the other pieces of the same size that you might have missed on this list.

In Worksheet 3, the researcher could identify 4 solution strategies:

Solution Strategy 1

Correct solution and reason. Some learners chose the smaller fraction with an appropriate reason while others chose the larger piece with an appropriate reason. Learners were asked: Which would you rather have a third of a chocolate bar or a fifth of a chocolate bar? Why?

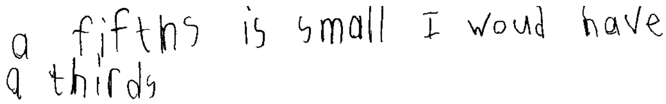

a fifths is small I woud have
a thirds

Figure 6.1 Bob Worksheet 3 no 1

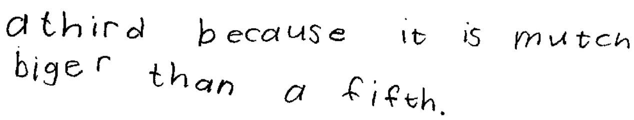

a third because it is mutch
biger than a fifth.

Figure 6.2 Lyn Worksheet 3 no 1

Elaine wrote: *"A fifth because I get a bit tired of chewing and a third will take too long".*

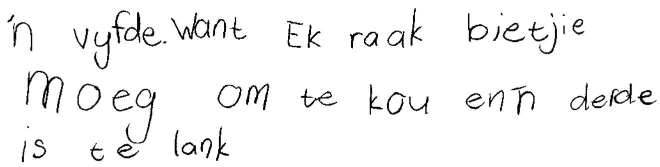

'n vyfde. Want Ek raak bietjie
moeg om te kou en 'n derde
is te lank

Figure 6.3 Elaine Worksheet 3 no 2

Yvonne wrote: *"fifth because I get nauseous from chocolates".*

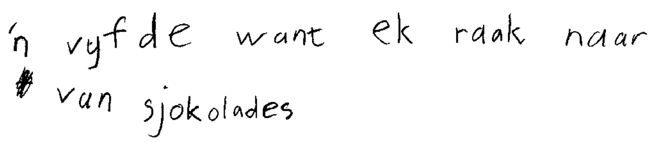

'n vyfde want ek raak naar
van sjokolades

Figure 6.4 Yvonne Worksheet 3 no 1

After completing the problem of which piece of chocolate would she choose $\frac{1}{3}$ or $\frac{1}{5}$, Sharon had the following question posed to her during reflection time:

Teacher: *If you do not like chocolate, which piece would you choose?*

Sharon: *I would choose the smallest piece.*

Teacher: *Which is the smallest?*

Sharon: *A fifth*

The researcher should have continued questioning by asking why Sharon thought that a $\frac{1}{5}$ was smaller.

For which was would you rather have $\frac{1}{2}$ of R1 or $\frac{2}{4}$ of R1, Gail gave her explanation as follows:

Half of R1 is 50 cents.
Half of 50 cents is 25 cents.
So now you see they are actually the same because 25 cents plus 25 cents is equal to 50 cents and half of R1 is 50 cents.

Although her explanation was correct, Gail wrote the incorrect answer on her worksheet.

Lucille wrote:

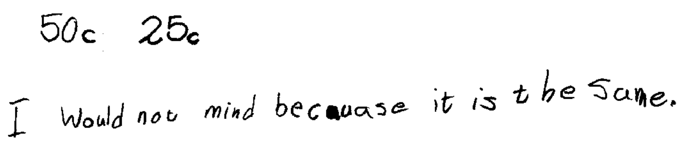


Figure 6.5 Lucille Worksheet 3 no3

During an interview, Lucille explained as follows:

Teacher: *Why are you saying 25c + 25c?*

Lucille: *It is actually the same because a fourth is equal to 25c and they asking two fourths.*
Double 20c = 40c
Double 5c = 10c
So, that gives you 50c.

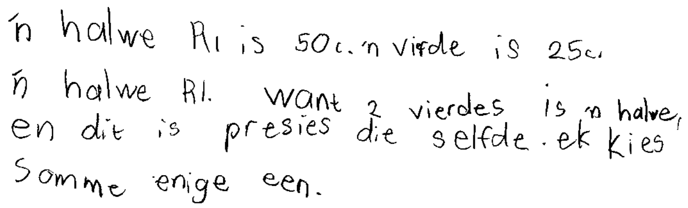


Figure 6.6 Elaine Worksheet 3 no 3.

Elaine chose $\frac{1}{2}$ of a R1. She wrote the following:

A half of R1 is 50c. A fourth is 25c.

I chose $\frac{1}{2}$ *of R1 because 2 fourths is a half and it is actually exactly the same.*

I can choose any one.

<u>Solution Strategy 2</u>

The learner chose a fractional piece and gave an inappropriate reason for his/her choice. Learners were asked: Which would you rather have a third of a chocolate bar or a fifth of a chocolate bar? Why?

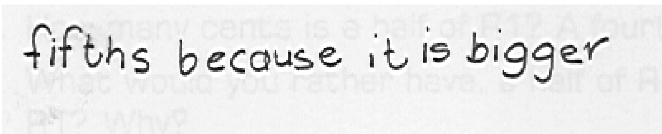


Figure 6.7 Nosisa Worksheet 3 no 1

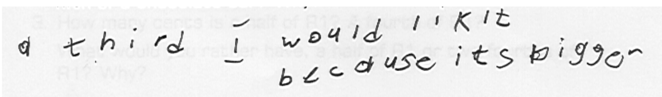


Figure 6.8 Tammy Worksheet 3 no 3

For which was would you rather have $\frac{1}{2}$ of R1 or $\frac{2}{4}$ of R1, learners answered as follows:

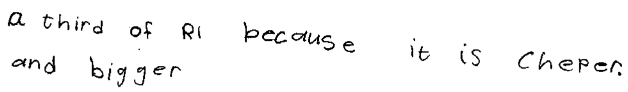


Figure 6.9 Lyn Worksheet 3 no 2

These questions were put to Lyn during the reflection time.

Teacher: *Lyn, what do you mean that it is cheaper?*

Lyn: *It is less money.*

Teacher: *Do you want less money?*

Lyn: *No.*

Teacher: *If you had to choose again?*

Lyn: *I would choose one third because it is more.*

<u>Solution Strategy 3</u>

The learner chose a fractional part and gave no reason for his/her choice. The researcher could not identify why the learner had chosen that specific fraction. Learners were asked: Which would you rather have a third of a chocolate bar or a fifth of a chocolate bar? Why?

Figure 6.10 Sue Worksheet 3 no 1

<u>Solution Strategy 4</u>

The learner did not give a conclusive answer.

Caroline wrote: "*a R1 for 2 'manch*'" (the name she gave the chocolate bar) for the following problem, 'How many cents is half of R1? A fourth of R1? What would you rather have, a half of R1 or two fourths of R1? Why?'

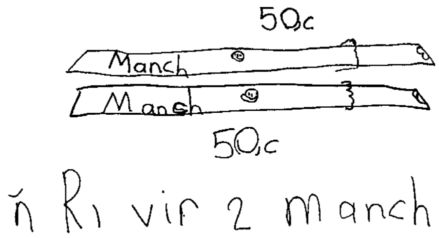


Figure 6.11 Caroline Worksheet 3 no 3

A grid indicating the summary of solution strategies used by the learners can be found in Appendix D1.

The learners solved problems 1 and 2 by comparing the size of the fractional parts. They explained that if one divides something into three pieces, those pieces must be larger than if you divide the same thing up into five pieces. However, in Question 3 the learners worked out exactly how much a half of a rand and two quarters of a rand were.

During this intervention, some of the learners from School A remarked that the problems were difficult and when discussing why they thought this to be the case, they remarked that the numbers that they had to work with were too large. It was the first time that learners had been exposed to a fraction as part of a collection of objects.

In Worksheet 4, the learners were given a fraction wall and asked to form equivalent fractions.

The learners were able to form 11 different equivalent fractions.

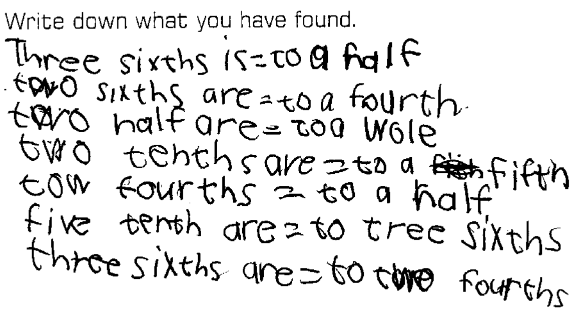


Figure 6.12 Sharon Worksheet 4 (equivalent fractions)

Mary wrote:

"two tenths is equal to a fifth

One 3^{rd} is equal to two sixths

Two 5ths is equal to tenths

Two tenths of 4 and two 5ths"

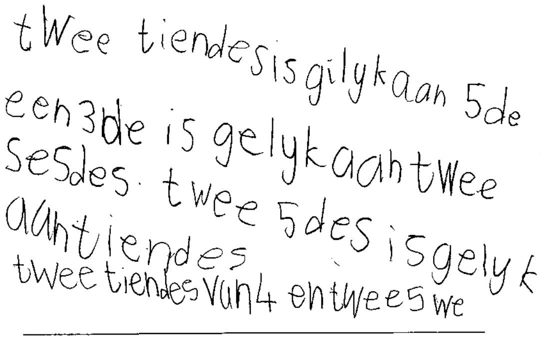
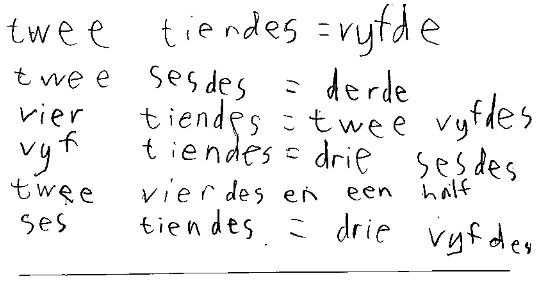


Figure 6.13 Mary Worksheet 4 (equivalent fractions)

Yvonne wrote:
two tenths = fifth
two sixths = thirds
four tenths = two fifths
five tenths = three sixths
two fourths and one half
six tenths = three fifths



Yvonne and Elaine were asked to explain how they had 'put little pieces together' to find 'a bigger piece' with a different name.

Elaine and Yvonne grouped two tenths together and drew in a line after each group of two.

Teacher:	*Elaine, can you explain to me what you have done? Can you tell me what name you could give to the one group of two tenths?*
Elaine:	*Yes, two tenths are equal to a fifth.*
Teacher:	*Can you write that down for me?*
Elaine:	*two tenths = one fifth*
Teacher:	*Good, now can you find any other smaller pieces that make a bigger piece?*

I left Elaine and Yvonne to investigate further. When I came back, she had written:

two halves = one whole

four quarters =one whole

Teacher: *Let's have a look at what you have found.*

Elaine: *If you add all the halves together, you get a whole and when you add all the quarters together you get a whole.*

I left Elaine and Yvonne to investigate further. When I came back, she had written:

Four tenths = two fifths

Five tenths = three sixths

Five tenths = half

Six tenths = three fifths

Eight tenths = four fifths

The researcher felt Yvonne and Elaine had grasped the concept of equivalent fractions and iteration of fractional parts, however, in general, learners at School A found this task of forming equivalent fractions (Worksheet 4), extremely difficult. It was necessary to explain what was required several times before the learners felt confident enough to attempt this problem.

This could have been due to the fact that learners had just returned after a two-week vacation and that it was the first time that they had been exposed to the fraction wall (pre-partitioned material – See Section 2.8). Learners found it necessary to make sense of the learning aid before attempting to solve the problems. Murray *et al*. (1999) and Bonotto, 1993, as cited in Pitkethly and Hunting (1996), are of the opinion that learners who have not yet fully developed the concept of a fraction could have difficulty working with pre-partitioned material (See Section 2.8). The researcher felt that it would be in the learners' best interest to suggest a method to the learners. She suggested that they use a ruler and move it slowly from left to right across the fraction wall.

In School A, only three of the learners managed to form more than five equivalent fractions in Worksheet 4. Most of the learners only managed to form two, three or four equivalent fractions. Six of the 11 learners also formed incorrect equivalent fractions.

In School B, nine learners were able to form five or more equivalent fractions with only three learners forming less than five. Three of the 12 learners also formed incorrect equivalent fractions. More of the learners at this school iterated the fractional parts and equated them to a whole than in School A.

Appendix D2 gives a summary of the equivalent fractions learners had mastered.

Due to the difficulties that the learners experienced, the researcher decided to approach the topic using manipulatives. To avoid the difficulties experienced in School A, the researcher approached the lesson with manipulatives with School B from the outset. These manipulatives were actual chocolate bars (Chomps) that were partitioned into the correct number of equal size pieces. The learners were instructed to make bigger fractional parts out of smaller fractional parts.

Worksheet 6 continued with the idea of forming equivalent fractions, but the learners were given specific equivalent fractions to look for and they could make use of the manipulatives supplied.

Carol picked out three sixths and measured it against one half.

Kate took $\frac{1}{6}$ *and* $\frac{1}{5}$ made it equal to $\frac{1}{3}$. Here is the discussion that followed:

Teacher: *Kate, if I take a sixth and a fifth, will it be equal to a third?*

Kate: *Yes.*

Teacher: *Let us measure.*

Kate: *No, it will not be the same.*

Teacher: *Can you correct it?*

Kate put the third back and put $\frac{2}{5}$ down.

Teacher: *Do you think they are the same size now?*

Kate: *No.*

Yvonne offered to help. Still no correct solution was found. Kate eventually reshuffled the pieces and had a fifth and a sixth in the top row and a fifth and a sixth in the bottom row. Although this was not what was actually asked, one could say that her arrangement was equal in size.

Mary took the whole and under the whole she placed two thirds and a sixth.

Teacher: *Mary, are they equal?*
Mary: *No.*
Teacher: *Can you correct it?*
Mary: *Yes.*

Mary took the sixth away and replaced it with another third.

Stewart took out a half, placed two sixths under it – took them away and took out two quarters.

To solve the problems on Worksheet 6, the learners used the same method as in the previous worksheet, namely the ruler sliding over the fraction wall. During reflection they shared their ideas and answers.

One learner explained as follows:
I put my ruler down on a third. Then I went to the row where the sixths are and counted how many sixths there were. That is how I found the answer that two sixths were equal to one third.

Elaine named the pieces as follows: *halves, thirds, fifths, sixths, ninths, tenths, fourteenths, fifteenths, eighteenths.*

Elaine numbered her fractional pieces to make it easier to work out the solution.

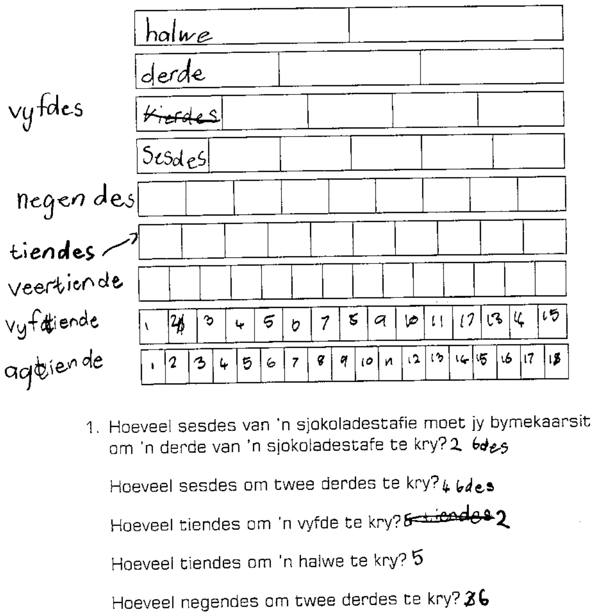


Figure 6.15 Elaine Worksheet 6 (naming fractional parts and forming equivalent fractions)

The researcher felt that the manipulatives prohibited the learners from thinking about the situation and that if they had created their own representations, it would have been more beneficial to them. These finding are in line with the findings of Empson (1995) and Murray *et al*. (1996a). The learners went back to the method of using the ruler and the pre-partitioned material (fraction wall) to solve the problems. None of the learners referred back to the pieces of chocolate once they began working out the problems on Worksheet 6 (See Sections 2.7 and 2.8). This placed the learners on Level One, Phase Two of Development (See Section 2.3.2) (Murray and Olivier, 1998).

The grid (Appendix D3) shows which equivalent fractions that learners could form.

School B seemed to manage far better with the second section of this worksheet. One of the possible reasons for the learners' difficulty could have been because there were fractions with large denominators and learners at School A were constantly complaining about the large numbers they had to work with. It also appeared that some of the learners did not understand the problems posed to them.

As previously mentioned in 4.5, the worksheets for the development of fractions were not developed in isolation. Bearing in mind that the learners other mathematical concepts and number development would also be developed through the year, the researcher felt that the numbers on the worksheets should not have been a problem for the learners.

Two additional worksheets for School A were included at this point. The first dealt with fractions where the denominator was not larger than ten and the second included fractions with denominators of fifteen as well as eighteen. The drawings on Worksheet 6.1 were also larger than the fraction wall on Worksheet 6. On the additional worksheets drawings of chocolate bars were used on Worksheet 6.1 and sausages were used on Worksheet 6.2 to form the fraction wall.

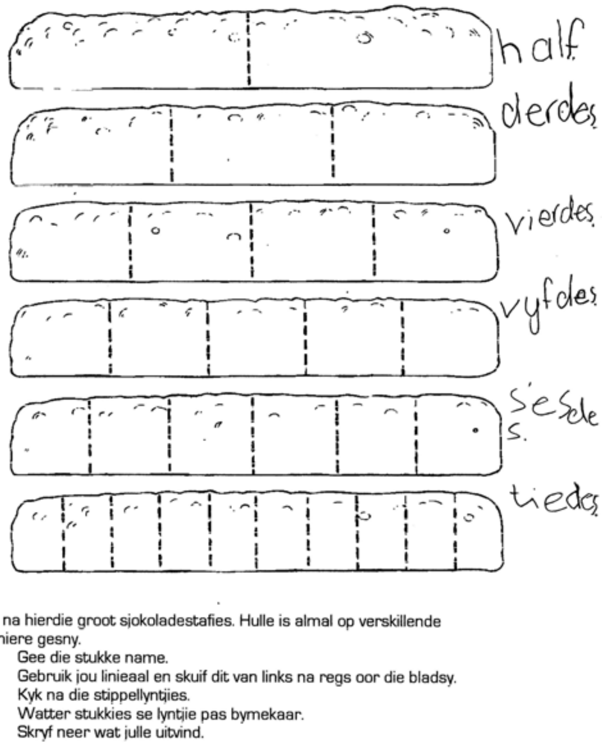
Although the learners used the same method to form equivalent fractions, the results were much improved. This could have been due to the fact that the drawings of the chocolate bars were larger than the fractional parts of the fraction wall. Using fractions where the denominator was less than ten could have increased the learners' confidence and therefore improved performance.

The grid (Appendix D4) shows the correct equivalent fractions formed by the learners as well as which fractions they used. With an exception of Elaine, all learners were able to form more equivalent fractions than in previous worksheets and there were fewer incorrect solutions.

It became evident that the concept of equivalence takes a very long time to develop and become stable. Learners need to be exposed to similar problem types over a longer period of time. By the end of the research some learners had begun to work abstractly while others had not. This also, to a large extent, depended on the Level of Development of the individual learner (See Section 2.3.2) (Murray and Olivier, 1989).

Learners were asked to name the fractional parts and to form equivalent fractions. Carol named the fractional parts as follows:

Halves, thirds, fifths, sixths, tenths.



She formed the following equivalent fractions:

2 tenths = 1 fifth

2 sixths = 1 third

4 tenths = 2 fifths

2 fourths = 1 half

6 tenths = 3 fifths

2 thirds = 4 sixths

4 fifths = 8 tenths

6 wholes

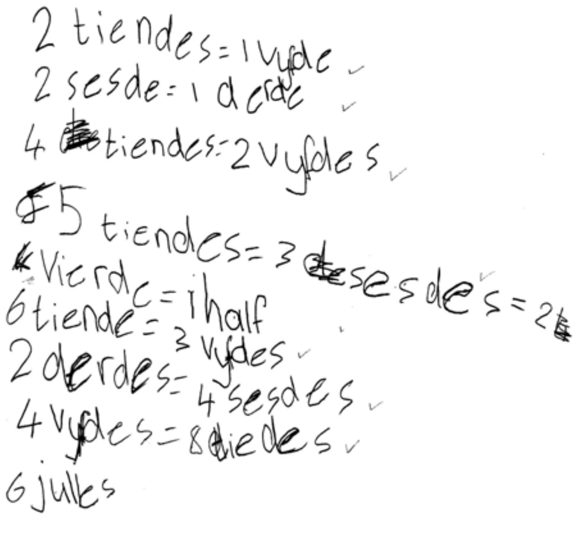


Figure 6.16 Carol Worksheet 6.1 (equivalent fractions)

During the lesson the following discussions were held:

1. With Mary and Stewart:

Mary: *Tannie (Aunty), is two thirds equal to something?*

Teacher: *What do you think Mary?*

Stewart: (his partner*) Let us have a look. Put the ruler down.*

They placed the ruler onto the dotted line that represented two thirds.

Mary and Stewart together: Yes… it is… two thirds is equal four sixths.

2. With Zelda:

Zelda: *Tannie, (Aunty) how many sixths make two thirds? Tannie, (Aunty) must I put my ruler onto two thirds?*

Teacher: *Yes.*

Zelda: *Now I see I need to count the sixths. That gives me four.*

Zelda kept on asking questions and needed to be reassured that her work was correct. She always employed the same method to solve her problem and always ended up with the correct result.

In the second additional worksheet that included ninths, fourteenths and fifteenths, although the learners did not form as many equivalent fractions as in Worksheet 6.1 they continued to show signs of improvement. Only 5 learners formed equivalent fractions with the fractions with larger denominators. This was the third consecutive worksheet where learners had to form equivalent fractions. Some of the learners showed signs of being bored (Elaine, Zane were the more able learners). This type of problem is very time consuming and some of the learners were not willing to spend time on any given problem. They had to be constantly encouraged to reflect, discuss and compare their solutions and methods (See Section 2.2.2.3). This was one of the problems that the researcher encountered throughout the research and at times, felt that it jeopardised the learning process.

Cheryl, always a little apprehensive, was asked to find larger pieces that could be made up from ninths, she immediately asked:

Cheryl: *Tannie, (Aunty) how are we going to work this out?*

Teacher: *How do you think we should work it out? You tell me what you think.*

Cheryl: *I am going to take my ruler and place it on one ninth and then I am going to see if there is another place on the ruler where there is a dotted line.*

Teacher: *Good.*

Cheryl: *Yes, I see,…… one ninth is equal to two eighteenths.*

Four approaches were identified in Worksheet 12. Learners were asked which piece of chocolate they would like and why.

Approach 1

Correct solution and reason given. Some learners chose the smaller fraction with an appropriate reason while others chose the larger piece with an appropriate reason.

Caroline wrote down her solutions:

Equal

$\frac{3}{5}$ *is greater than* $\frac{1}{2}$

equal

$\frac{5}{10}$ *is greater than* $\frac{2}{5}$

$\frac{1}{7}$ *is greater than* $\frac{1}{8}$

$\frac{6}{10}$ *is greater than* $\frac{9}{18}$

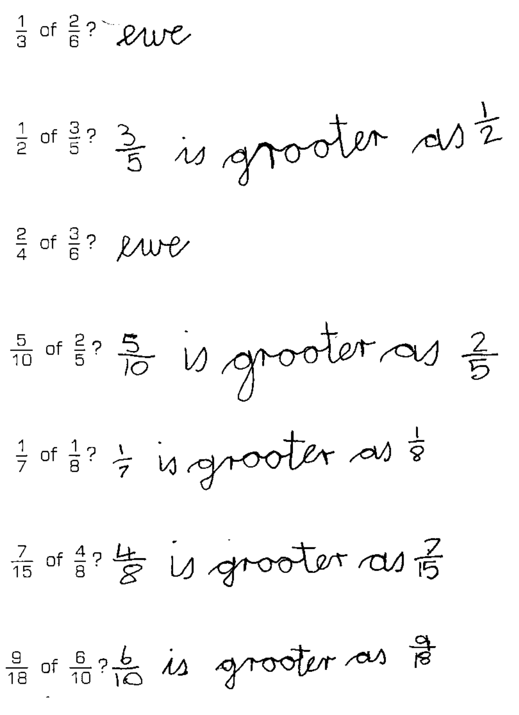


Figure 6.17 Caroline Worksheet 12 no 2 (comparing fractions)

Yvonne gave the following reason:

I do not like chocolate very much.

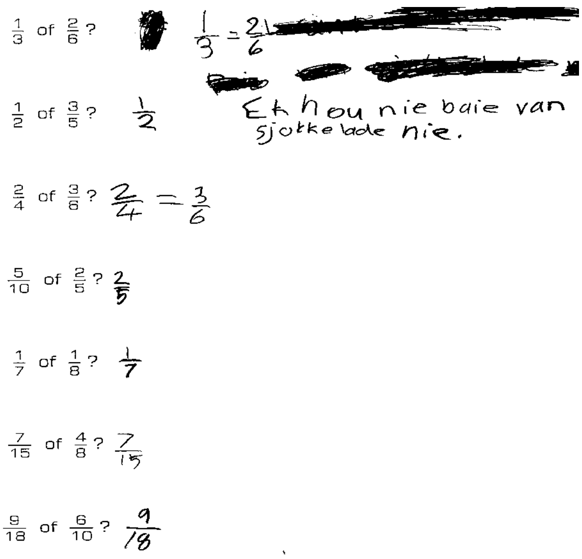


Figure 6.18 Yvonne Worksheet 12 no2 (comparing fractions)

Approach 2

The learner chose a fractional piece and gave an inappropriate reason for his/her choice. Learners were asked which piece of chocolate they would like and why?

Carol gave her reason as:

I like chocolate.

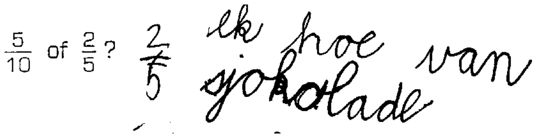


Figure 6.19 Carol Worksheet 12 no 4 (comparing fractions)

Gail wrote:

$\frac{7}{15}$ *it is much bigger.*

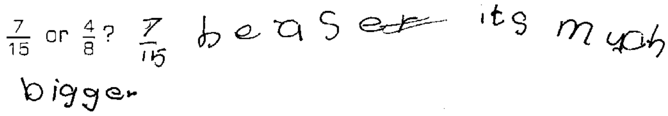


Figure 6.20 Gail Worksheet 12 no 6 (comparing fractions)

Approach 3

The learner chose a fractional piece and gave no reason for his/her choice. The researcher could not identify why the learner had chosen that specific fraction.

Kate gave the following solutions:

The $\frac{1}{3}$ *and the* $\frac{2}{6}$

The $\frac{5}{3}$*.*

The $\frac{2}{4}$ *and the* $\frac{3}{6}$

I take $\frac{2}{5}$

I take $\frac{1}{7}$

I take $\frac{4}{8}$

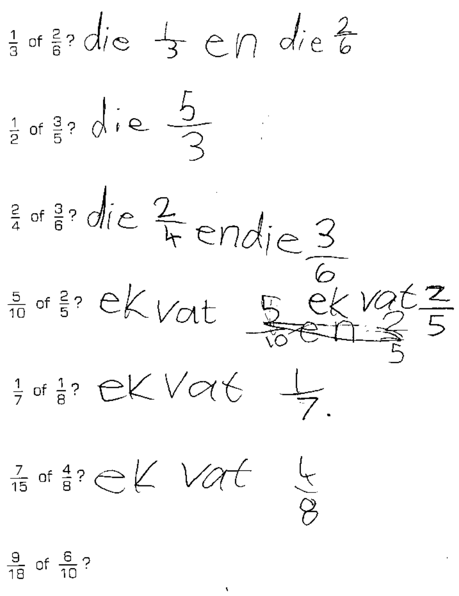


Figure 6.21 Kate Worksheet 12 page 2 (comparing fractions)

<u>Approach 4</u>

The learner did not give a conclusive solution.

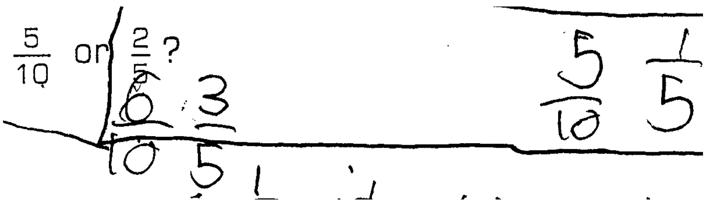


Figure 6.22 Sipho Worksheet no 4 (comparing fractions)

Appendix D5 gives an overview of the distribution of approaches used by learners.

Most of the learners were able to give the correct solution with an appropriate reason for their choice (Approach 1). Because the learners were referring to the fraction wall to form equivalent fractions, the researcher placed them on Level One, Phase Two of Development (Murray and Olivier, 1989).

Elaine was the only learner who was beginning to show signs of being able to work at the abstract level. She would reason as follows:

Which is greater $\frac{5}{10}$ or $\frac{2}{5}$?

Five tenths is equal to a half - a half is bigger than two fifths therefore five tenths is greater than two fifths.

At times it was necessary to pose questions as well as to lead the discussion in the groups.

An example is given below:

Teacher: *Show me* $\frac{2}{4}$ *on the fraction wall. Now show me* $\frac{3}{6}$. *Now can you tell me which is the bigger?*

Some of the learners found it necessary to shade in the fractional parts they were trying to compare. The learners seemed to become confused with all the rows and columns!

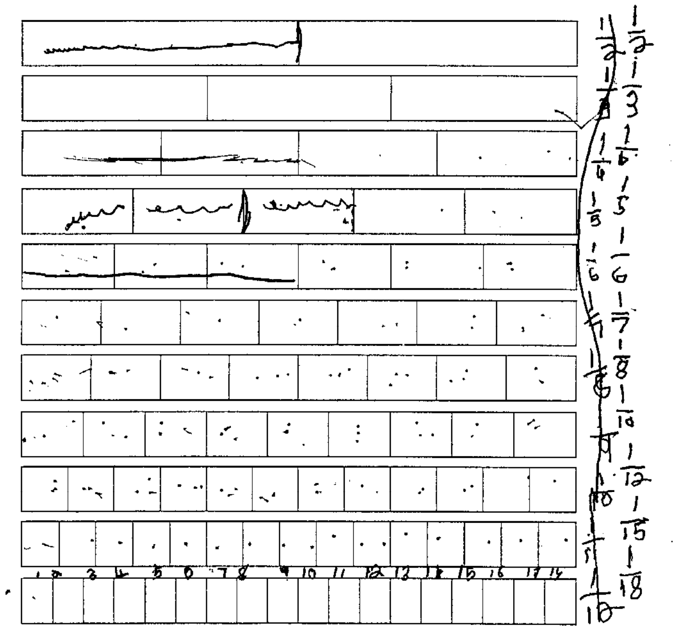


Figure 6.23 Tammy Worksheet 12 (comparing the size of fractional parts)

During participant observation with some of the learners in School A, this conversation was recorded illustrating the reasoning of the learners:

Teacher: *Which is the bigger piece:* $\frac{1}{2}$ *or* $\frac{1}{3}$ *?*

Learner: $\frac{1}{2}$ *is bigger.*

Teacher: *Why is it bigger?*

Learner: *Because the whole has just been cut into two pieces and the other three. The one with three pieces must be smaller.*

This reasoning was given for the other examples as well.

Teacher: *Which one is bigger:* $\frac{1}{2}$ *or* $\frac{2}{4}$*?*

Learner: *They are the same because if you add the two pieces together, it gives you a half.*

This conversation was held with Sipho, Tammy and Mbulelo:

The three boys did not want to be recorded while explaining their methods. Tammy said that he did not want his voice to go to the university!

Teacher: *Mbulelo, can you tell me how you worked out which one was bigger, one third or two sixths?*

Mbulelo: *Which one?*

Teacher: *The first one.*

Mbulelo: *Ah………well I, took my ruler and I actually looked over there and it was one over three.*

Teacher: *You put the ruler down showing where one third was.*

Mbulelo: *Yes and then I shaded in two blocks and three blocks under and then they were the same.*

Mbulelo showed me the size of one third. He showed me the size of two sixths and placed his ruler where the fractional parts ended. He saw that the two fractional parts ended in the same place and came to the conclusion that they were the same size.

Teacher: *OK, they were the same. Right, Tammy how did you work out which one was bigger?*

Tammy: *Which one?*

Teacher: *In number two – which is bigger – a half or three fifths?*

Tammy: *A half or three fifths?*

Teacher: *How did you work out which one was bigger?*

Tammy: *A half or three fifths? I took page number one* (had the fraction wall on it) *and it is two over four and three over five.*

Tammy counted out the fractional parts. He also used the ruler to compare the fractional parts once he had completed the counting.

Tammy: *three fifths is bigger.*

Teacher: *OK, thank you. Now Sipho, can you tell Mrs Fraser how you worked out which was bigger – one seventh or one eighth?*

Sipho: *Ummmmmm…..Um………….. I did it here.* (Pointing to the fraction wall)

Teacher: *You did it on the fractional wall. How did you do it on the fractional wall?*

Sipho: *I did it like Tammy.*

Teacher: *You did it like Tammy. Ok. Does anybody want to tell me anything else that is interesting?*

Mbulelo: *It was quite easy for me.*

Teacher: *Easy for you. OK.*

This group of boys always caused discipline problems in class. Sipho was very cheeky and always wanting to get everybody's attention, Tammy was always dreaming and Mbulelo had to be continually reminded to focus on the mathematics. The class teacher classified this group as being of the weaker ability in mathematics.

It was found that some of the learners initially had difficulty in verbalising their thoughts, however discussing and reflecting is an essential part of the learning process as it encourages learners to clarify their thoughts (See Section 2.2.2.3).

Looking at Zelda's solution, it seems as if she wanted to say that $\frac{1}{7}$ is larger than $\frac{1}{8}$. The reason she gave was seven is larger than eight. The researcher sees this as a limiting construction caused by Zelda's knowledge of whole numbers. She applied whole number strategies to the fraction problem (See Section 2.7).

Zelda gave the following reason:

7 is greater than 8

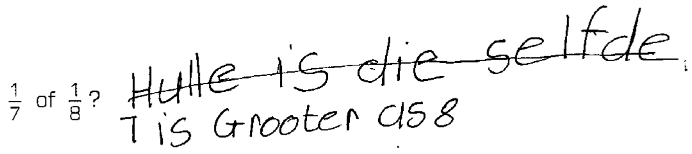


Figure 6.24 Zelda Worksheet 12 no 5 (comparing fractions)

In Worksheet 13, the learners were requested to choose pieces of the same size from a given number of fractions. Although the learners were requested to say which fractions were equivalent and then check their answers on the fraction wall, many immediately referred to the fraction wall.

Some of the learners were able to equate a 'string' of equivalent fractions while others were only able to find pairs of equivalent fractions.

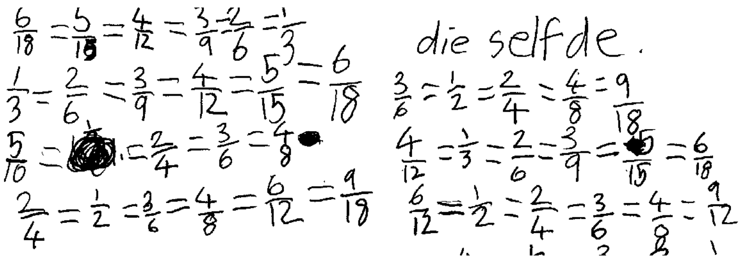


Figure 6.25 Yvonne Worksheet 13 (equivalent fractions)

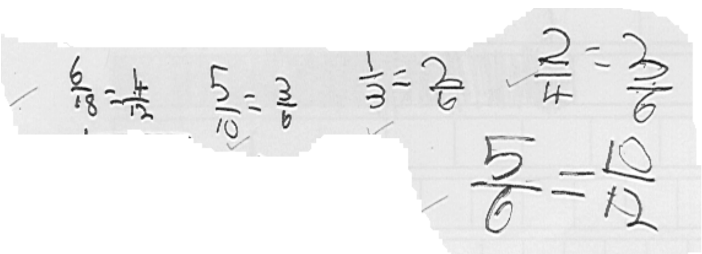


Figure 6.26 Mbulelo Worksheet 13 (equivalent fractions)

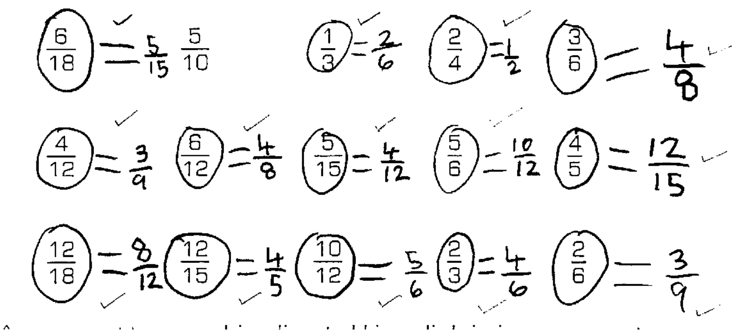


Figure 6.27 Kate Worksheet 13 (equivalent fractions)

The learners continued to use the 'ruler method' to find equivalent fractions. Elaine once again showed signs of being able work abstractly. The majority of learners were still working on Level One, Phase Two and Elaine on Level Three of Development (Murray and Olivier, 1989). Although some of the learners were able to equate a 'string' of equivalent fractions, the researcher felt that this did not place the learners on a higher Level of Development. The learners simply decided to notate them as a 'string' of equivalent fractions.

In Worksheet 12, Elaine continued to be the only learner who was able to work abstractly and she continued to show signs of being able to work at this level in Worksheet 13. Caroline and Stewart both showed signs of 'getting a feel' for the size of fractions.

By the end of Worksheet 13, the learners could tell the researcher that the denominator told them how many equal parts the whole had been divided into.

During the lesson, Caroline was asked how she had arrived at the conclusion that they fractions were equivalent. Her reply was:

Caroline: *I thought and found out that they were the same size because of what the teacher had said on the board.*

Teacher: *For no other reason?*

Caroline: *No.*

Elaine was able to find the solution without using the ruler method. She explained as follows:

Elaine: *I took the fifteenths and I saw five fifteenths and I knew it was the same as four twelfths.*

Stewart was beginning to 'get a feel' for the size of the fractional parts. He had the following to say:

Teacher: *Just now when you had to work with* $\frac{6}{12}$ *you immediately knew where to put the ruler. How did you know where to put it?*

Stewart: *I knew in my head.*

By introducing the common fraction concept using real-life equal-sharing problems, learners were able to discuss and diagrammatically represent the solutions they obtained. This led to learners meeting the equivalent fraction concept at an early stage of the study (Worksheet 5). The researcher felt that the manipulatives used with the earlier worksheets were not sufficiently beneficial to pursue the use of manipulatives.

The learners at both schools were not familiar with working with manipulatives. It took the learners a long time to get used to the manipulatives and to get over the excitement of having them in class. Having the manipulatives with which to solve the problems prevented the learners from drawing or notating what they were doing. The learners did not give much thought to what they were actually working out. Empson (1995) is of the opinion that the manipulatives dictate the situation and not the learners' thinking. When they were presented with a similar problem structure without the manipulatives, the learners could not solve the problem (See Section 2.8).

6.2 COGNITIVE DEVELOPMENT OF THREE INDIVIDUAL LEARNERS

The researcher chose to trace the development of this concept with Elaine (a learner from the stronger ability group) Sharon (of average ability) and Nosisa (from the weaker ability group).

Individual Study 4

From the outset, Elaine chose the smaller fractional piece and gave an appropriate solution (Approach 1). She said that she did not like chocolate!

For all three solutions in Worksheet 3 Elaine fell into Approach 1. For number 1, Elaine chose $\frac{1}{5}$ and not $\frac{1}{3}$ when asked which she would prefer to have. Her reason was that she gets tired of chewing and to chew a third would be a bit long!

Elaine wrote: $\frac{4}{5} = \frac{12}{15}$ *the same*

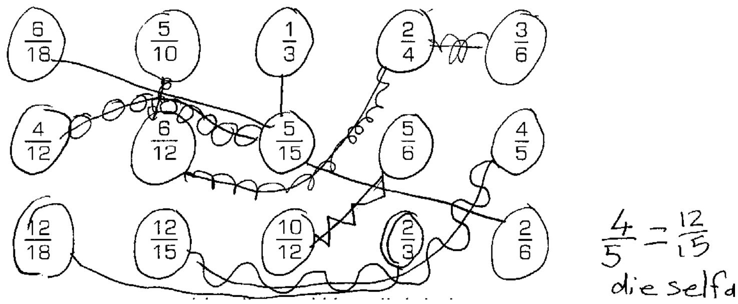


Figure 6.28 Elaine Worksheet 3 no 1 (equivalent fractions)

When asked which she would prefer, $\frac{1}{4}$ of R1 or $\frac{1}{3}$ of R1 and why, she indicated that she would chose $\frac{1}{4}$ because a third is a lot of money that could get lost!

Her explanation for which she would rather have, $\frac{1}{2}$ of R1 or $\frac{2}{4}$ of R1, Elaine chose the correct solution and gave an appropriate reason (Approach 1).

In Worksheet 4, Elaine found eight fractions that were equivalent to one another.

Elaine named the fractional parts: *Halves, thirds, quarters, sixths , tenths*

Equivalent fractions formed:

two tenths = fifth

two halves = whole

four quarters = whole

four tenths = two fifths

five tenths = half

fife tenths = three fifths

six tenths = three fifths

eight tenths = four fifths

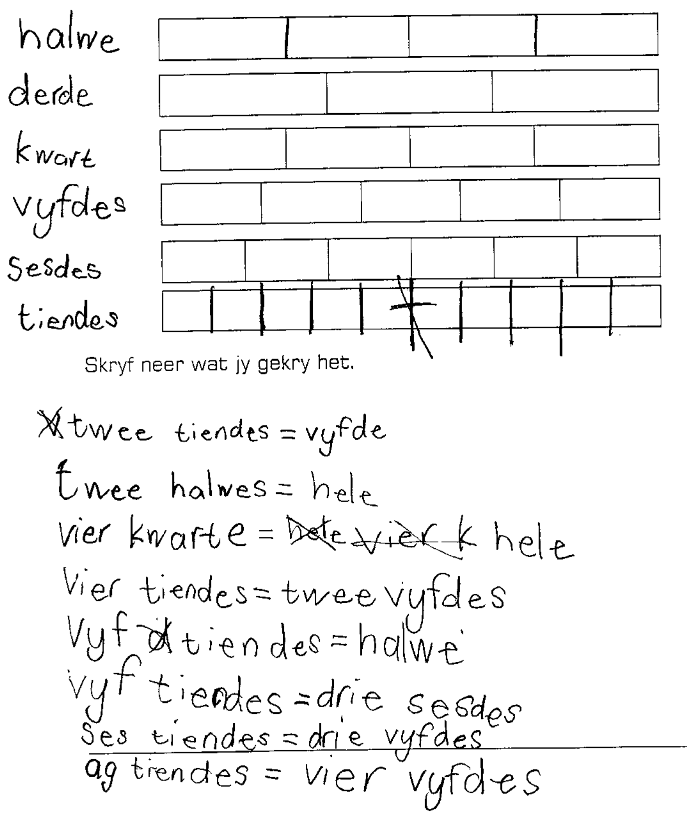


Figure 6.29 Elaine Worksheet 4 (equivalent fractions)

Elaine formed the most equivalent fractions in the class in Worksheet 6.

She wrote:

2 6ths

4 6ths

2

5

6

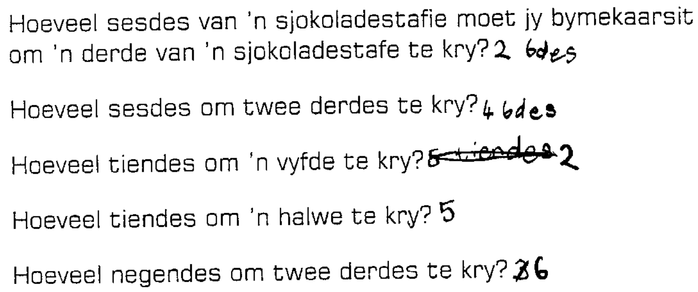


Figure 6.30 Elaine Worksheet 6 (equivalent fractions)

Although the researcher found it necessary to do additional worksheets with learners in School A, she is of the opinion that it would not have been necessary for Elaine. The researcher felt that Elaine was getting bored because she iterated a number of fractions to form wholes. In Worksheet 6.2, she did not form as many equivalent fractions as the researcher felt she was capable of doing.

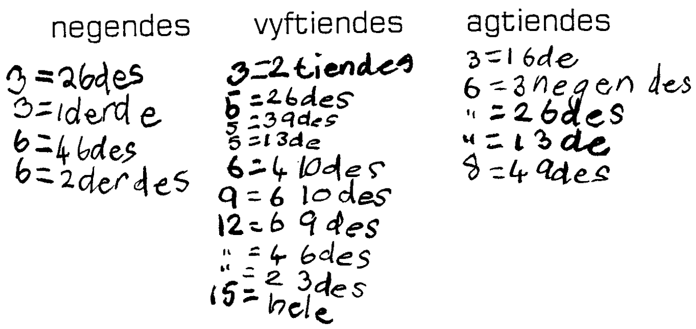


Figure 6.31 Elaine Worksheet 6.1 and 6.2 (equivalent fractions)

Elaine formed the following equivalent fractions:

ninths	*fifteenths*	*eighteenths*
3 = 2 6ths	*3 = 2 tenths*	*3 = 1 6th*
3 = 1 third	*5 = 2 6ths*	*6 = 3 ninths*
6 = 4 6ths	*5 = 3 9ths*	*6 = 2 6ths*
6 = 2 thirds	*6 = 4 10ths*	*6 = 1 third*
	9 = 6 10ths	*8 = 4 9ths*
	12 = 6 9ths	
	12 = 4 6ths	
	12 = 2 3rds	
	15 = whole	

Elaine chose the correct solution and gave an appropriate reason for the problems in Worksheet 12 (Approach 1). The solution to the problem "Which would she prefer, $\frac{1}{3}$ or $\frac{2}{6}$ of the chocolate bar?" She wrote $\frac{1}{3}$ and then crossed it out and circled $\frac{2}{6}$. Her explanation for this was that she would take $\frac{1}{6}$ and then give the other $\frac{1}{6}$ to her sister, Nelda. The others she explained as follows:

$\frac{1}{2}$ *I do not want to get fat*

$\frac{2}{4}$ *the same as the first one*

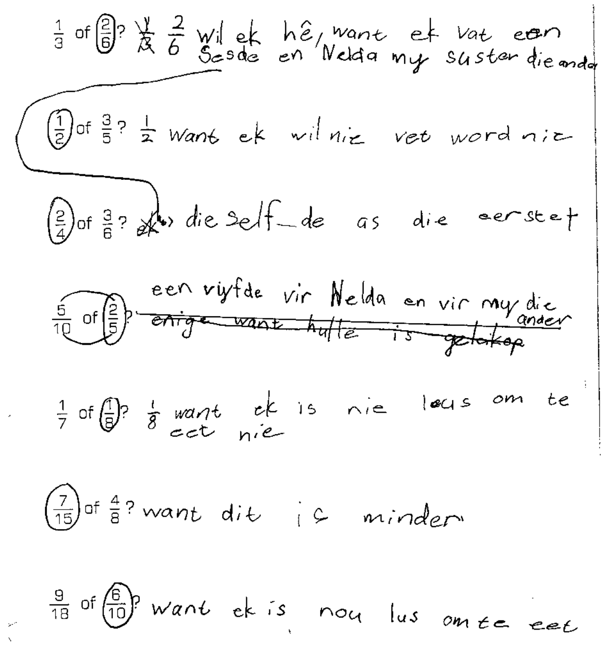


Figure 6.32 Elaine Worksheet 12 (equivalent fractions)

$\frac{2}{5}$ *one fifth for Nelda and the other for me*

$\frac{1}{8}$ *because I do not feel like eating*

$\frac{7}{15}$ *because it is less*

The researcher felt that Elaine had developed a 'feel' for the size of fractions by this stage and that she was able to 'play' with them to suit her needs! Elaine was definitely showing signs of being on Level Three of Development (Murray and Olivier, 1989) and was able to work abstractly with the fractions.

Elaine was able to write down a 'string' of equivalent fractions for Worksheet 13. Her wrote the reason for equating certain fractions as '*her ruler said so*'.

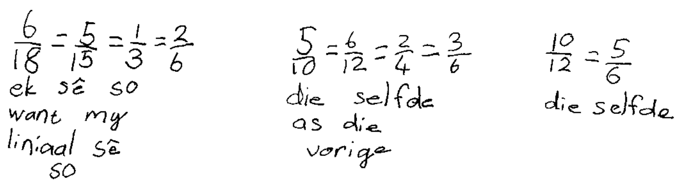


Figure 6.33 Elaine Worksheet 13 (equivalent fractions)

Individual Study 5

Sharon was absent for the first intervention involving equivalence and comparison of fractions. This did not seem to deter her in any way as she was able to form seven equivalent fractions in Worksheet 4. It could be due to the fact that her first lesson on equivalent fractions started with manipulators - the ‘cut up’ chocolate bars.

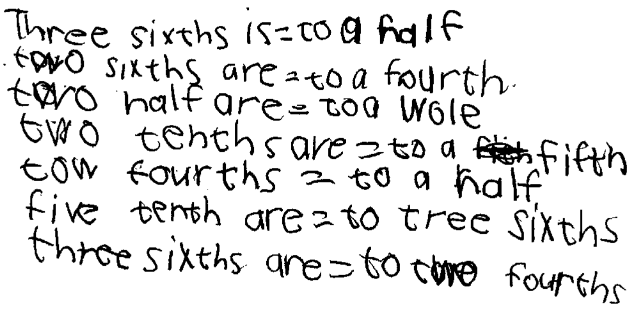


Figure 6.34 Sharon Worksheet 4 (equivalent fractions)

In Worksheet 6 she made some very interesting combinations of equivalent fractions. She made six correct combinations and one incorrect combination.

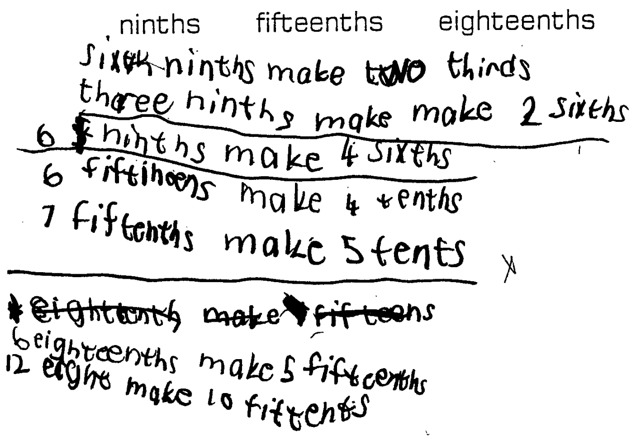


Figure 6.35 Sharon Worksheet 6 (equivalent fractions)

During Worksheet 12, the researcher noted that Sharon, Phindiwe and Lyn were working very well together. They were discussing their answers and in so doing, Sharon found that two of her answers were incorrect. She was then able to correct them. She coloured in some of the fractional parts in order to compare their sizes.

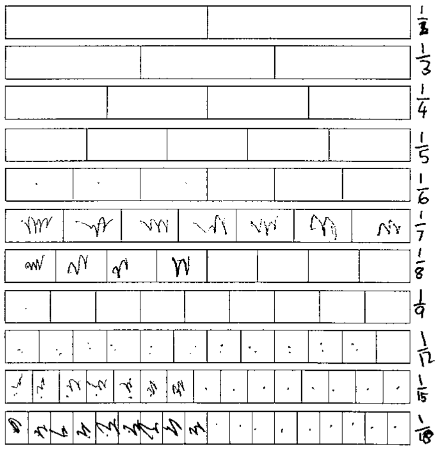
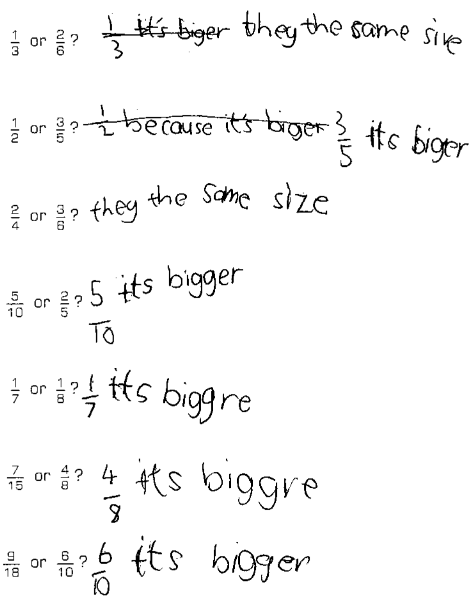


Figure 6.36 Sharon Worksheet 12 (equivalent fractions)



Worksheet 6.37 Sharon Worksheet 12 (comparison of fractions)

Sharon managed to find equivalent fractions in Worksheet 13. She could not write down a 'string' of equivalent fractions, but was able to find six correct and two incorrect solutions.

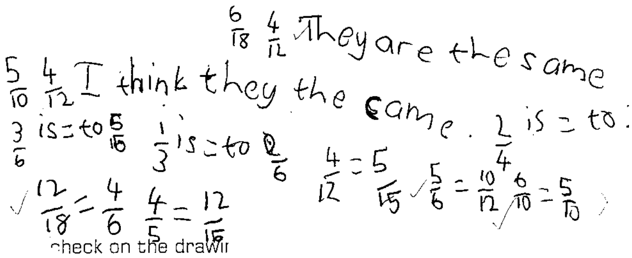


Figure 6.38 Sharon Worksheet 13 (equivalent fractions)

At this stage in the research, Sharon was on Level One, Phase Two of Development (Murray and Olivier, 1989). She was not yet able to work with equivalence on an abstract level.

<u>**Individual Study 6**</u>

In Worksheet 3, Nosisa chose $\frac{1}{5}$ of a chocolate bar because she thought that it was bigger than $\frac{1}{3}$. Nosisa chose a fractional piece, but gave an inappropriate reason for her choice (Approach 2).

Figure 6.39 Nosisa Worksheet 3 no 1 (comparison of fractions)

For the second problem on Worksheet 3, she chose a fractional piece and gave an appropriate reason for her choice (Approach 1).

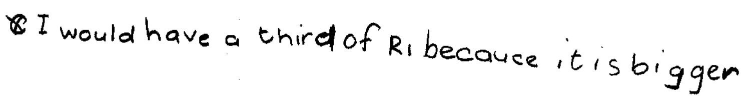


Figure 6.40 Nosisa Worksheet 3 no 2 (comparison of fractions)

For the third problem, she chose a half of a R1. Nosisa did not give a reason for her choice (Approach 3).

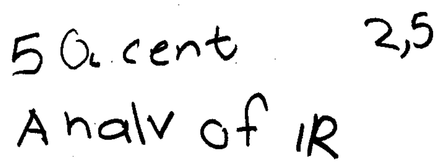


Figure 6.41 Nosisa Worksheet 3 no 3 (comparison of fractions)

Of the six solutions that Nosisa gave in Worksheet 4, she iterated the fractional parts to form wholes for five of them and $\frac{2}{10} = \frac{1}{5}$ was the only one formed where fractional parts were combined to make another fractional part which was not equal to a whole.

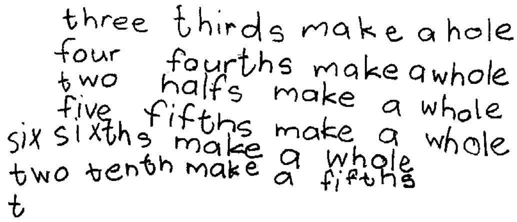


Figure 6.42 Nosisa Worksheet 4 (equivalent fractions)

Nosisa felt comfortable iterating fractional parts and making a whole rather than forming equivalent fractions where fractional parts combined formed a fraction that was not a whole. This could have been due to the fact that she had not advanced sufficiently in this stage of her of the development. She showed signs of not being prepared to accept the idea that smaller fractional parts could be joined to form a larger fractional part that was not a whole. The researcher did not feel that Nosisa had formed any misconceptions.

Nosisa seemed to manage the structured questions on Worksheet 6. The first statement she made, although not required, was correct. Nosisa did not have enough time to complete the second half of the worksheet.

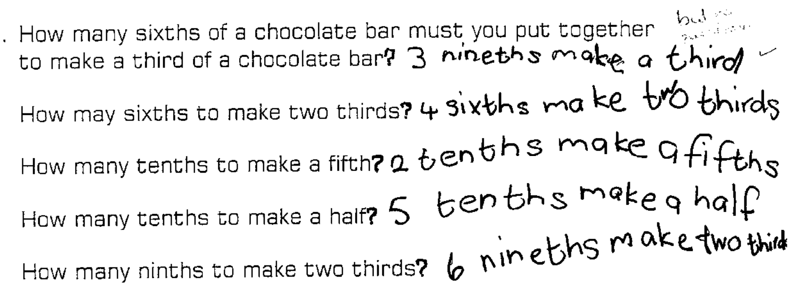


Figure 6.43 Nosisa Worksheet 6 (equivalent fractions)

Nosisa still showed signs of working at Level One, Phase Two of Development. She still needed to use the fraction wall to help her solve problems.

Nosisa had advanced a great deal by the time she attempted to compare the fractions on Worksheet 12. Although she chose a fractional piece and gave an appropriate reason for her choice (Approach 1), it is felt that her knowledge of equivalent fractions was surface knowledge only. She seemed to be able to produce the correct solutions in 'structured' questions but the researcher, due to the fact that data collection ceased in School B after Worksheet 13, was not able to test in an unstructured situation, if Nosisa had developed a deeper understanding of this concept.

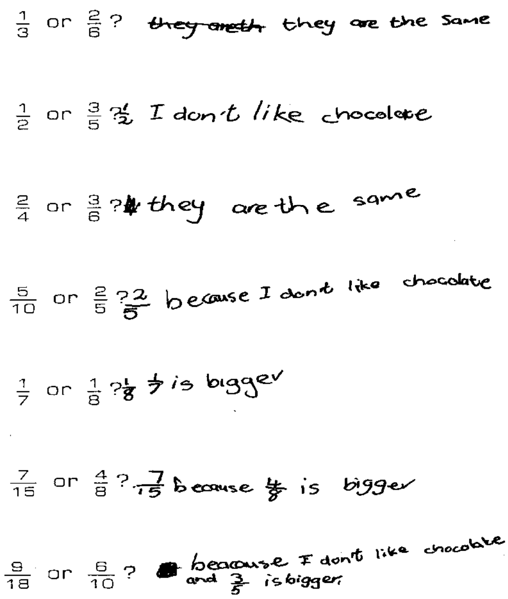


Figure 6.44 Nosisa Worksheet 12 (comparison of fractions)

In Worksheet 13, she managed to find two correct pairs of equivalent fractions.

She did not find time to answer problem 3. Nosisa did not attempt problem 1, but immediately used the fraction wall to test if the fractions were equivalent.

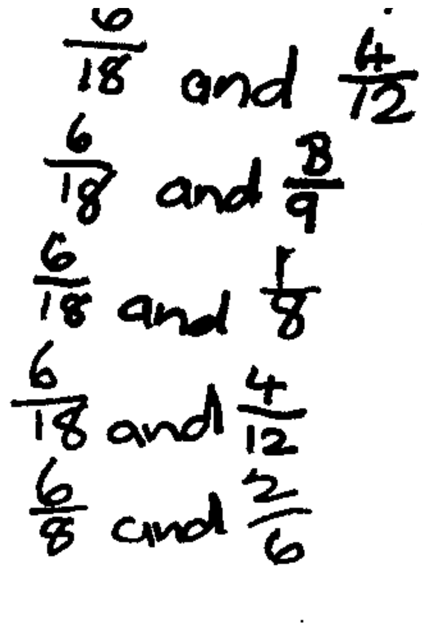


Figure 6.45 Nosisa Worksheet 13 no 1 (equivalent fractions)

6.3 SOME DEEPER UNDERSTANDINGS OF THE FRACTION CONCEPT

At the end of Worksheet 4 at School B, the learners wanted to know what the researcher was going to do with the chocolate (manipulatives). The researcher said that if they could tell her how much chocolate each should get they could have it. The problem posed to them was five chocolates and 12 children. The concept was a difficult one because learners had not yet met the situation where each person would not receive a whole chocolate. They could not manage to tell the researcher the answer, but they decided that if another chocolate was added, they would each get a half and this is what was done. One learner noticed that $\frac{1}{6} + \frac{1}{3} = \frac{1}{2}$.

The researcher asked the learners at School B which would be the bigger piece of chocolate bar: $\frac{2}{5}$ or $\frac{2}{4}$. The learners explained that $\frac{2}{4}$ would be the bigger piece because quarters are bigger than fifths and if one takes two quarters it must be bigger than two fifths.

During Worksheet 13, Phindiwe, from School B explained her method to the class. She said that if she was looking for fractions that were the same size as $\frac{6}{18}$, she would place her ruler on the $\frac{6}{18}$ 'place' and then she would look up and down and see what other fractions were the same sizes.

6.4 SUMMARY

The concept of equivalent fractions takes a very long time to develop and become stable. Even though the learners at School A were exposed to additional worksheets, some of the learners still experienced problems with this concept. With the exception of Elaine, all learners were still at Level One, Phase Two of Development at the end of Worksheet 13.

The researcher found the material relevant, but felt that for learners at School A, the fractions with the larger denominators at times caused problems. Possibly the fraction wall used at School A was too small for the learners. They seemed to cope better when drawings of chocolate bars and Vienna sausages were used. Throughout all these worksheets, learners used the 'ruler method' and at times, a few of the learners shaded in the fractions that needed to be compared.

Contrary to Pothier and Sawada (1983) and Keiren (1976), as cited in Mack (1990), the researcher found learners had a limited informal knowledge about equivalent fractions. Most learners seemed to find this concept of equivalence difficult. However, when comparing fractions they were able to do this without too much difficulty. Towards the end of the research, when solving problems where the solution could be simplified, Zane, Elaine, Yvonne and Carol were able to form equivalent fractions without the use of the fraction wall. Elaine talks about, "*Seeing it in her head*". They were able to work on an abstract level and this would have placed them on Level Three of Development (See Section 2.3.2) (Murray and Olivier, 1989).

Empson (1995) stated that the idea of equivalence is of crucial importance to all basic operations involving fractions. By posing equal-sharing problems that resulted in solutions giving rise to equivalent fractions, learners were introduced to this concept in an intrinsic way from early on in the intervention. Discussing and reflecting on these solutions, enabled learners to make sense of the concept of equivalence and begin to develop an understanding of equivalent fractions. Posing suitable problems and making fraction walls available for the learners to consult can develop this concept without giving learners a ‘rule’ to be applied by rote with little conceptual understanding.

CHAPTER 7

BASIC OPERATIONS INVOLVING FRACTIONS – DIVISION BY A FRACTION

In Grade 3, learners should be exposed to problems that will lead to the development of the four basic operations in common fractions. The earlier learners are exposed to problems involving basic operations, the sooner they will start thinking about it. Chapter 7 will deal with the operation of division by a fraction.

In Worksheets 9, 10, 14, 15 and 17 grouping problems were posed to the learners. The structure of these problems should have led the learners to the development of the basic operation of division.

7.1 ANALYSIS OF DATA

The following problems were presented to the learners:

Worksheet 9

Vusi and Anna prepare soft porridge for breakfast. For each bowl, they use $\frac{1}{3}$ of a litre of milk.

They have 5 litres of milk. How many bowls of porridge can they prepare?

They have $2\frac{1}{2}$ litres of milk. How many bowls of porridge can they prepare?

Worksheet 10

The children are making different animals and cars from wire. A car needs $2\frac{1}{2}$ metres of wire. An animal needs $1\frac{1}{2}$ metres of wire. The children have 20 metres of wire.

How many cars can they make from 20 metres of wire?

How many animals can they make from 20 metres of wire?

Worksheet 14

1. The children make a small duck from $\frac{1}{3}$ of a metre of thin wire. They have $5\frac{1}{2}$ metres of thin wire. How many small ducks can they make?

2. In a needlework class, they use $\frac{1}{4}$ metre ribbon to trim one apron. If each child is given $2\frac{1}{2}$ metres of ribbon, how many aprons can each child make?

Worksheet 15

David needs some stakes to put labels on his vegetable seedbeds. He needs stakes that are $\frac{1}{5}$ metre long. He has $1\frac{1}{2}$ metres of rod. How many stakes of $\frac{1}{5}$ metre can he cut from this rod?

Worksheet 17

Vuyo uses the juice of $1\frac{1}{2}$ oranges for a large birthday cake. She has 8 oranges. How many cakes can she bake?

The researcher identified five solution strategies:

Solution Strategy 1

The learner made use of drawings to solve the problem.

Elaine wrote: *15 plates of porridge*.

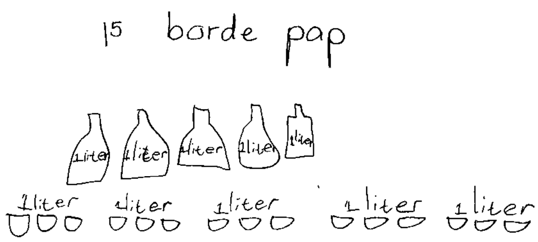


Figure 7.1 Elaine Worksheet 9 no 2 ($5 \div \frac{1}{3}$)

The researcher found Carol's drawing of number two very interesting. She drew two, two litre bottles of milk and one, one litre bottle. Next to this she put an equal sign and wrote five litres. She then divided all the bottles into thirds and divided the two litre bottles down the middle as well, thus giving her six plates of porridge from two litres of milk. She did this because she said she had drawn two litres and not one litre bottles. She then drew six plates of porridge each from the two litre bottles and three from the one litre bottle.

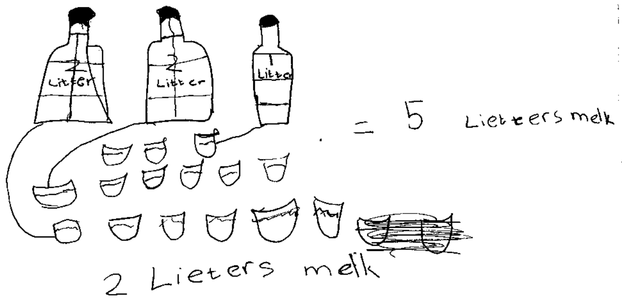


Figure 7.2 Carol Worksheet 9 no 2 ($5 \div \frac{1}{3}$)

Although Gail does not draw the two and a half litres of milk, she drew one jug and divided it into thirds, getting three plates and the half a jug, she decided she would get a plate and a half out of it.

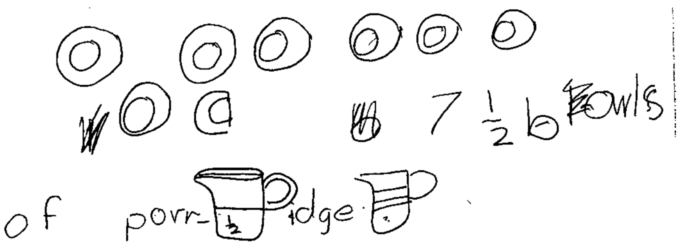


Figure 7.3 Gail Worksheet 9 no 3 ($2\frac{1}{2} \div \frac{1}{3}$)

It appears that she used abstract reasoning for the other litre.

Linda drew the eight oranges and divided every second one in half. She then linked a whole orange with a half an orange and drew a cake underneath. She wrote her solution as: '*5 cakes and there is a half left over*'.

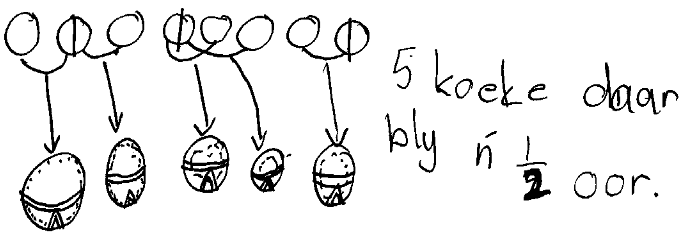


Figure 7.4 Linda Worksheet 17 no 1 ($8 \div 1\frac{1}{2}$)

Mary drew the eight oranges. She then drew an orange and a half an orange and circled them together indicating that they would make one cake. She then combined three halves to make a fifth cake. Mary did not indicate the answer in numbers.

Figure 7.5 Mary Worksheet 17 ($8 \div 1\frac{1}{2}$)

Solution Strategy 2

The learner gave the correct answer, but no method was shown.

Figure 7.6 Cheryl Worksheet 9 no 1 ($5 \div \frac{1}{3}$)

Cheryl's solution: '15 plates of porridge'

Solution Strategy 3

The problem was solved incorrectly.

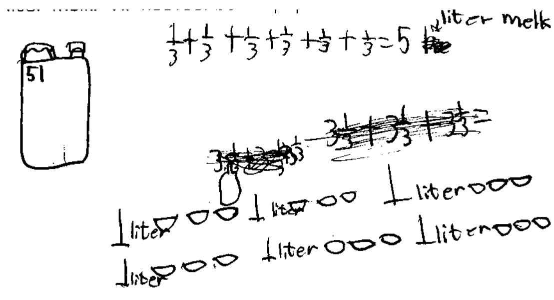


Figure 7.7 Martin Worksheet 9 no 1 ($5 \div \frac{1}{3}$)

Although Sipho worked out that there would be three plates of porridge, he neglected to say how many plates could be made altogether.

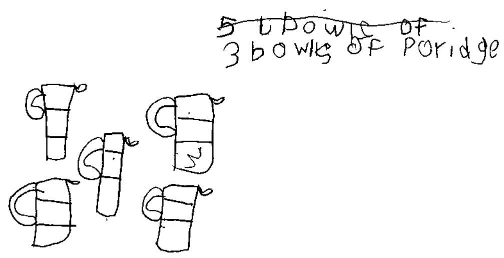


Figure 7.8 Sipho Worksheet 9 no 1

$(5 \div \frac{1}{3})$

Solution Strategy 4

The learners solved the problem using numbers – some by counting all (Level One) and others by doubling (Level Two - Murray and Olivier, 1989).

Yvonne found her solution by doubling. She found that she could make eight cars.

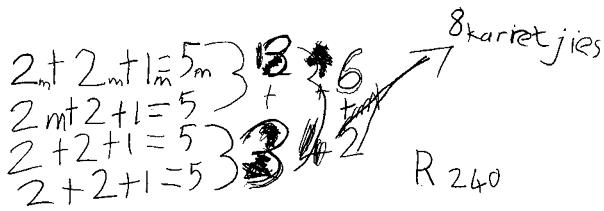


Figure 7.9 Yvonne Worksheet10 no 1

$(20 \div 2\ \frac{1}{2})$

Carol found her solution by counting all and doubling (verdubbel). She found that 13 animals could be made.

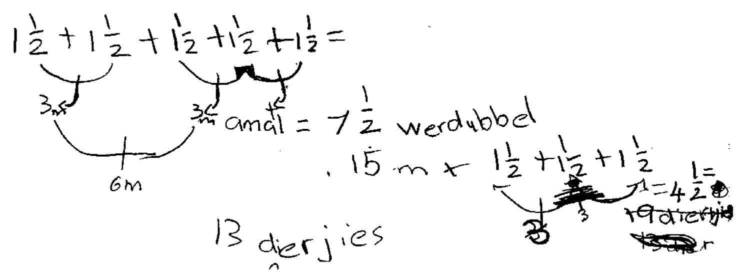


Figure 7.10 Carol Worksheet 10 no 2

$(20 \div 1\frac{1}{2})$

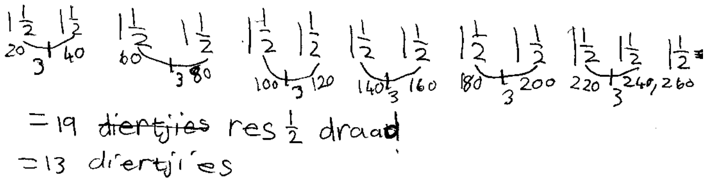


Figure 7.11 Linda Worksheet 10 no 2 ($20 \div 1\frac{1}{2}$)

Linda worked out how much wire two animals would use. She continued to link two together until she had used 18 m, she then added one more animal. The total used was $19\frac{1}{2}$ m. She indicated that she had $\frac{1}{2}$ m left and that she could make 13 animals.

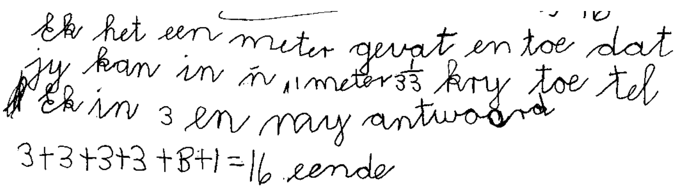


Figure 7.12 Caroline Worksheet 14 no 1 ($5\frac{1}{2} \div \frac{1}{3}$)

Caroline wrote down the following: '*I took one metre and saw that from 1 metre you can get 3-* $\frac{1}{3}$ *and so I counted in three's and got my answer. 3 + 3 + 3 + 3 + 3 + 1 = 16 ducks'* (She made another duck out of the $\frac{1}{2}$ m).

<u>Solution Strategy 5</u>

Some learners did not attempt the problem.

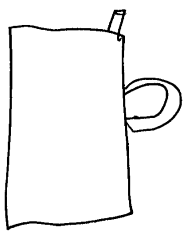


Figure 7.13 Kate Worksheet 9 no 2 ($2\frac{1}{2} \div \frac{1}{3}$)

The grids (Appendix E) show which solution strategy the learners chose to use. From these grids, it can be clearly seen that the majority of learners chose to solve the problem using drawings (Solution Strategy 1). In Worksheet 10, most of the learners used numbers to solve the problems, thus showing a tendency to work more abstractly. (Solution Strategy 4). However, in Worksheets 14, 15 and 17, the majority of learners used drawings to solve the problems. The learners perceived these problems as being more difficult. In most of these problems, the learners had to divide a mixed number by a fraction and not a whole number by a fraction. According to Kieren *et al*. (1992), as cited in Pitkethly and Hunting (1996), learners 'folded back' (reverted back) to informal actions when they perceive problems to be difficult.

7.2 COGNITIVE DEVELOPMENT OF INDIVIDUAL LEARNERS

Individual Study 7

The researcher identified Yvonne from School A as an above average learner in mathematics. Yvonne's development was followed from Worksheet 10 as she was absent for the first intervention involving grouping problems.

Yvonne gave her solution of '*8 cars*'. She solved the problem using numbers and doubling (Solution Strategy 4)

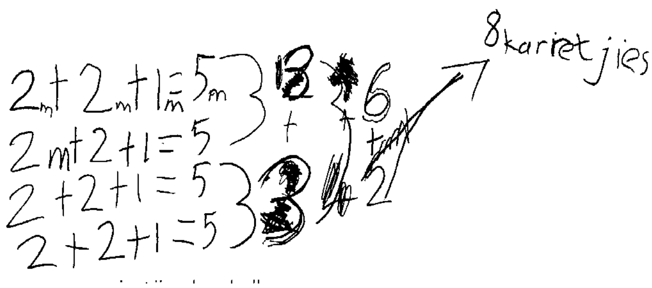


Figure 7.14 Yvonne 10 no 1($20 \div 2\frac{1}{2}$)

Yvonne solved the problem using the method employed for the previous problem. Her solution was '*13 animals*'.

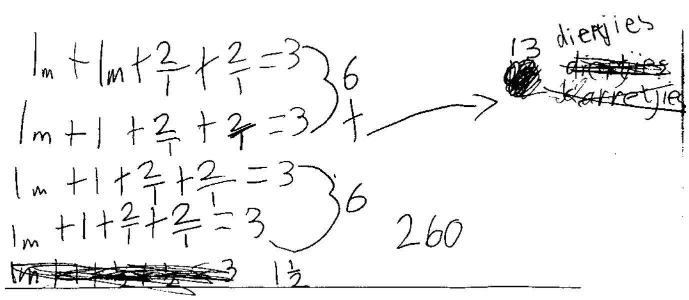


Figure 7.15 Yvonne Worksheet 10 no 2 ($20 \div 1\frac{1}{2}$)

Although Yvonne had been absent for the first intervention involving grouping problems, she showed the ability of solving the problem on Level Three of Development (Murray and Olivier, 1989). She did not find the need to draw the solution. In problem 10 no.1, Yvonne decided to 'make two cars at a time'. She added two metres and two metres and one metre to find the amount of wire that was used for two cars. She doubled $\frac{1}{2}$ to get one. This calculation was repeated four times and she found that eight cars could be made from 20 metres.

For problem 10 no. 2, Yvonne used the same method to find the solution. It is interesting to note that Yvonne notated the fraction incorrectly. She inverted the fraction symbol. Yvonne added 1 metre, 1 metre, $\frac{1}{2}$ metre and $\frac{1}{2}$ metre and obtained three metres. She repeated this calculation and decided that this would make six animals. The whole calculation was repeated once more and this gave Yvonne 12 animals. Yvonne tried to repeat the calculation once more but found that she did not have sufficient wire to complete another three animals. She decided that she would be able to make one more animal, thus giving her a total of 13. Yvonne does not state how much wire was left over.

Although Yvonne worked abstractly in the previous problems, when she solved the problems in Worksheet 14, she chose to draw her solutions (Solution Strategy 1). She drew the five and a half metres of wire and partitioned it off into metres. She then divided each metre into thirds and counted how many 'thirds' she had made. Yvonne became confused and had to re-think her strategy. One can see on the drawing that she had to make 'dashes' on top of the five metres of wire to assist her with counting.

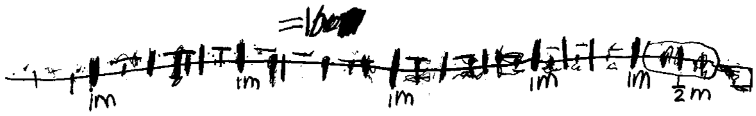


Figure 7.16 Yvonne Worksheet 14 no 1 ($5\frac{1}{2} \div \frac{1}{3}$)

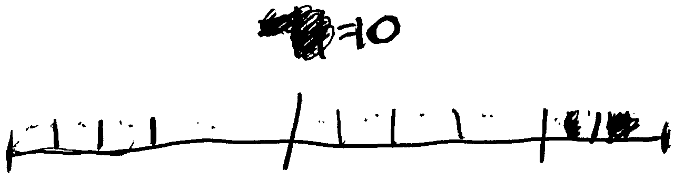


Figure 7.17 Yvonne Worksheet 14 no2 ($2\frac{1}{2} \div \frac{1}{4}$)

Yvonne used the same method to calculate Worksheet 15 as she did for Worksheet 14 (Solution Strategy 1). This method placed her on Level One, Phase Two of Development (Murray and Olivier, 1989). She drew a stick of one metre and another half its length. She proceeded to divide the one metre into five equal parts, and on the half metre, she partitioned another two fifths. She indicated that this gave her a total of $\frac{7}{5}$. She gave her solution as *' 7 sticks'*.

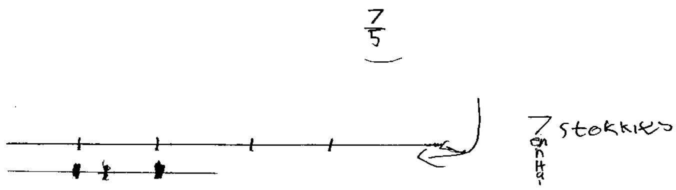


Figure 7.18 Yvonne Worksheet 15 ($1\frac{1}{2} \div 5$)

Yvonne drew eight oranges and she combined one orange with a half of the next orange to give her one cake, which she drew underneath (Solution Strategy 1).

Figure 7.19 Yvonne Worksheet 17 no 1 ($8 \div 1\frac{1}{2}$)

She continued in this manner until she had used all the oranges. Yvonne gave her solution as five and a half. She should have written five cakes and a half an orange left over.

Although Yvonne initially showed signs of being on Level Three of Development, solved all the other problems on Level One, Phase Two. Yvonne perceived the other problems as being more difficult and she reverted back to a method she could have picked up during reflection time (See Section 2.10).

7.3 SOME DEEPER UNDERSTANDINGS OF THE FRACTION CONCEPT

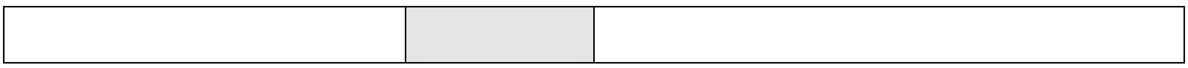
All three girls from School B, Lyn, Sharon and Phindiwe, drew the solution for the second question of Worksheet 9 (Solution Strategy 1). They all divided the jugs into thirds and counted up the thirds getting 15.

Question 3 ($2\frac{1}{2} \div \frac{1}{3}$) caused some discussion between the three girls. Interestingly though, they drew the jugs as 'bars of chocolate' in order to solve the problem. At first, they complained about not knowing what to do and they had to be encouraged to talk about the problem. This was how Lyn explained the problem to the group and then to the class during reflection time:

That one jug will give three plates of porridge.

That one jug will give three plates of porridge.

She then drew another whole and divided it into half.



She then explained that the third would fit into the half and there would be a bit left over – this bit she coloured in.

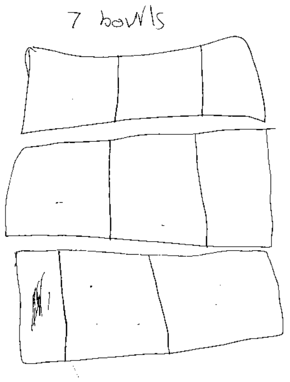


Figure 7.20 Lyn Worksheet 9 no 3 ($2\frac{1}{2} \div \frac{1}{3}$)

Sharon also explained it in this way.

For the last two problems on Worksheet 9 ($5 \div \frac{1}{3}$ and $2\frac{1}{2} \div \frac{1}{3}$), Sue and Gail explained to the researcher as follows (Sue was trying to work abstractly and Gail needed to use pictorial representation.):

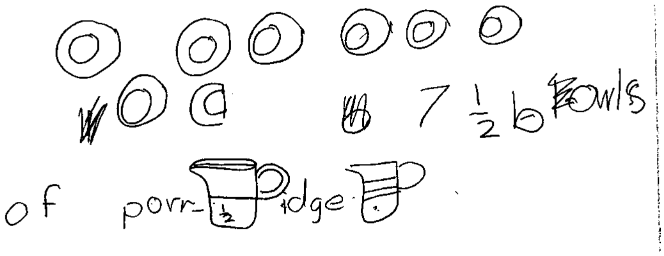


Figure 7.21 Gail Worksheet 9 no3 ($2\frac{1}{2} \div \frac{1}{3}$)

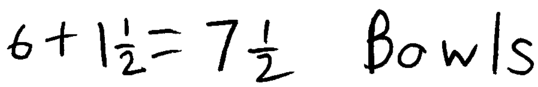


Figure 7.22 Sue Worksheet9 no 3 ($2\frac{1}{2} \div \frac{1}{3}$)

Teacher:	*Sue, your turn to explain. How did you get to five?*
Sue:	*one litre is three bowls and another litre is um………six bowls and then I said no wait and then I said three fifths equals is equal to two thirds.* There was too much background noise in order to hear what Sue said further, but she was getting herself very confused with the three fifths equalling two thirds. She had actually drawn a fraction wall and was trying to compare the sizes of the fractions. Her wall was not accurate enough.
Teacher:	*Can you tell Sue how you worked yours out Gail?*
Gail:	*Well, I drew three bowls, that is one litre and I drew another three that is two litres, and I drew another three that is three litres, and then I drew another three that is four litres and I drew another three and that is five litres.*
Sue:	*I got it wrong, I got it wrong!*
Gail:	*That gave me 15.*

Teacher:	*Sue, you made a mistake this time hey? Never mind. O.K. So we are learning from each other. Let us have a look at the last one. Bob what did you get for the last one?*
Bob:	*Nothing.*
Teacher:	*No answer. O.K.* Bob was never willing to participate in the class. All the teachers as well as the principal were having serious discipline problems with him.
Gail:	*I got seven bowls of porridge.*
Sue:	*seven and a half bowls.*
Teacher:	*Can you explain to me how you did it?*
Gail:	*I drew three bowls of porridge, that is one litre and another three bowls, that is two litres and then I drew………I drew a milk jug and divided it into half and then I drew another one and divided that one into thirds and the one of the thirds I cut in half and that gives me another one and a half.*
Teacher:	*Let me see. The one you cut into six thirds…*
Gail:	*The one I cut into thirds and the one I cut into half.* What Gail was trying to say was that she divided the jug into half and the same jug into thirds and from this she could see that she could get another bowl of porridge and a half.
Teacher:	*Oh, I see. So what could you see from that?*
Gail:	*That you can get one and a half bowls of porridge from that half.*
Teacher:	*Did you do it like that too Sue?*
Sue:	*six bowls will be equal to two litres and then there was a half a litre and then there was one and a half litres left* (I think Sue meant a half a litre) *and then I said one litre was equal to seven and then I added the half and got seven and a half.*
Teacher:	*Thank you.*

The learners were finding it very difficult to verbalise what they had done pictorially, however verbalising their methods assisted them in organising their thoughts.

Referring to Solution Strategy Four (Learners solved the problem using numbers), Carol had a very interesting way of working out her problem (Worksheet 10 no 2 [20

$\div 1\frac{1}{2}$]). By looking at her method, the researcher placed her on Level Three of Development as she is doubling and dividing her numbers into chunks (Murray and Olivier, 1989). She started off by saying two and a half plus two and a half is equal to five. She did this twice. She then added the two fives and got 10. She added the next two and a half and got an answer of $12\frac{1}{2}$. She proceeded to add three, two and a halves and got 20. She counted how many times she added up two and a half and got eight cars.

The researcher felt that for the second problem ($20 \div 1\frac{1}{2}$), Carol had improved her method. She added up as follows: One and a half plus one and a half is equal to three. She did this twice and then added the two threes together to get six. She added the wire for one more animal and got seven and a half. She then doubled her answer**.** The total was then 15 and she proceeded to add on three, times one and a half and got four and a half. This was then added to the 15 and she got an answer of $19\frac{1}{2}$. Instead of counting how many times she had added one and a half, she made the mistake of saying that there were $19\frac{1}{2}$ animals instead of wire used. The researcher intervened with relevant questions and Carol realised that she needed to count how many times she had added the one and a half to find the number of animals made.

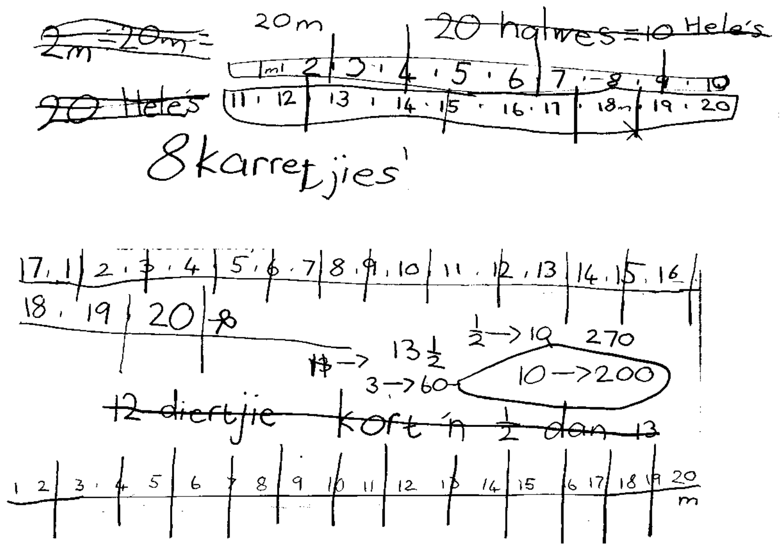


Figure 7.23 Elaine Worksheet 10 no 1 and 2 ($20 \div 2\frac{1}{2}$) ($20 \div 1\frac{1}{2}$)

Zane and Elaine struggled to get started with this activity, but eventually solved it by using drawings (Approach 1). The researcher found it interesting that they needed to discuss early on in the lesson. This was contrary to most lessons when they first worked on the problems individually and then discussed their methods. Zane and Elaine initially tried to work the problem out abstractly and tried to categorise the problem into one of the four basic operations, namely division. They also worked out how many halves made 10 wholes (20). They decided that that information was irrelevant, scratched it out and started again. According to Keiren *et al*. (1993) as cited in Pitkethly and Hunting (1996), learners' will 'fold back' (revert back) to methods that they can make sense of in situations were they are not able to solve the problem and this is exactly what Elaine and Zane did (See Section 2.10).

After they erased their initial work, they drew a piece of wire 20m long. They then proceeded to 'cut off' pieces two and a half metres long. They saw how many 'pieces' they had made and this gave them the number of cars and animals that could be made. With the second problem, it was necessary for the researcher to share social knowledge with the learners, namely that a number line or 'the piece of wire' needed to start at zero. Both learners then redrew the wire and marked it off – Elaine numbered the pieces that had been cut off.

D' Ambriosio and Mewborn (1994) feel that it is crucial to connect other topics in the mathematics curriculum in order to develop a rich understanding of fractions. They are of the opinion that linking fractions to linear measurement can aid learners in learning concepts such as the iterative unit and skills such as juxtaposition of units to measure a segment. Elaine and Zane of their own accord, linked their knowledge of linear measurement to assist them solve the above problem (Figure 7.22).

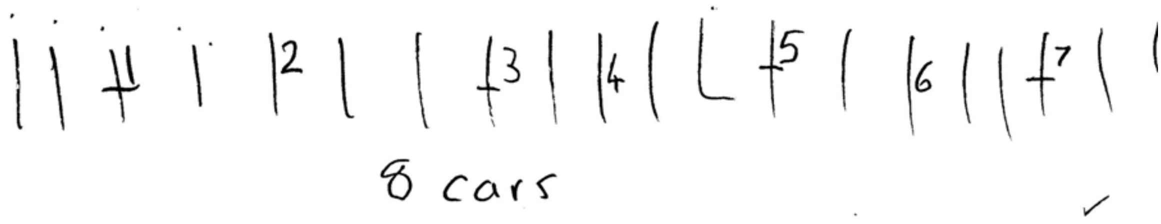


Figure 7.24 Lucille Worksheet 10 no 1 ($20 \div 2\frac{1}{2}$)

Lucille used drawings to solve the problem (Solution Strategy 1) and this is how she described her method:

Lucille: *I drew 20 lines for the cars and then I said two and a half metres and then I counted – I drew a line for two and a half metres and that was for one car and then I carried on doing the same thing until the end and then I counted all the spaces and it was eight.*

Teacher: *Let us have a careful look at what she did. She said she drew 20 metres. OK each one of those lines is one metre. Then she said for one car I need one, two and a half. Then for the next car I need – there is my half, one, two. And for my next car I need one, two and a half. And for my next car I need half, one, two. And for the next car I needed, one, two and a half. And the next car I needed half, one, two. And the next car I needed one, two and a half. And the next car I needed half, one, two.*

Lucille: *And that all adds up to eight.*

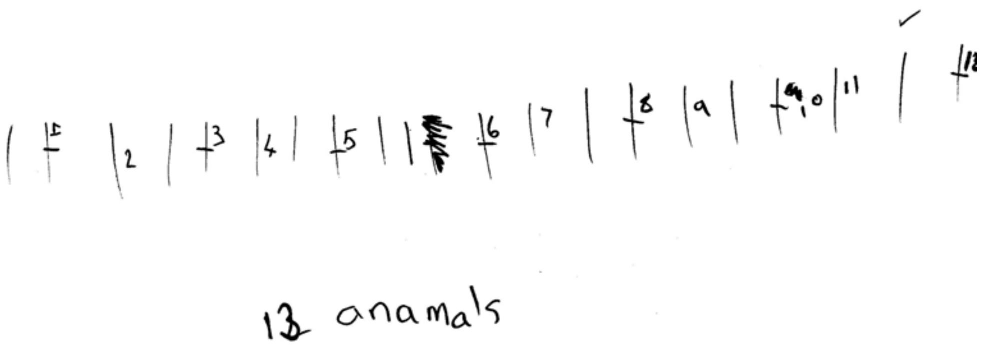


Figure 7.25 Lucille Worksheet 10 no 2 ($20 \div 1\frac{1}{2}$)

Teacher: *Lucille what did you do for number two?*

Lucille: *I did the same – I drew 20 metres and then I just did 1 and then I did a half and I did one and a half and I did it to the end and I counted how many cuts and I got 20.*

Teacher: *No, it was not 20.*

Lucille: *No, I mean 12.*

Sharon managed to do the problems correctly by using numbers (Solution Strategy 2), but when she was asked to verbalise what she had done, she became confused.

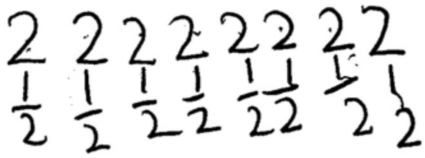


Figure 7.26 Sharon Worksheet 10 no 1 ($20 \div 2\frac{1}{2}$)

Teacher: *Who would like to tell me how they worked out the one with the cars?*

Sharon: *I would.*

Teacher: *O .K. Sharon, how did you work it out?*

Sharon: *Well, I first did two metres and a half plus one is two plus a half gives two and a half and then two – O.K. That's two, that's four + two halves is three.*

Teacher: *No, no hang on………. Two and two is four plus two halves is one – so four plus one.*

Sharon: *Is five.*

Teacher: *OK.*

Sharon: *five then six, seven, plus one = eight that is 10 + 1 =11 +2 =13 No, plus 11 is 15 + 1 =16. How did I work it out?*

Teacher: *I do not know! How did she work it out Phindiwe?*

Phindiwe: *I do not know.*

Teacher: *But she said that she helped you.*

Phindiwe: *Which one is it?*

Teacher: *First one. Add up the two's.*

Sharon: *2, 4, 6, 8, 10, 12, 14, 16.*

Teacher: *Add up the halves.*

Sharon: *16, 17, 18, 19, 20.*

Teacher: *OK. So how many cars?*

Sharon: *1, 2, 3, 4, 5, 6, 7, 8*

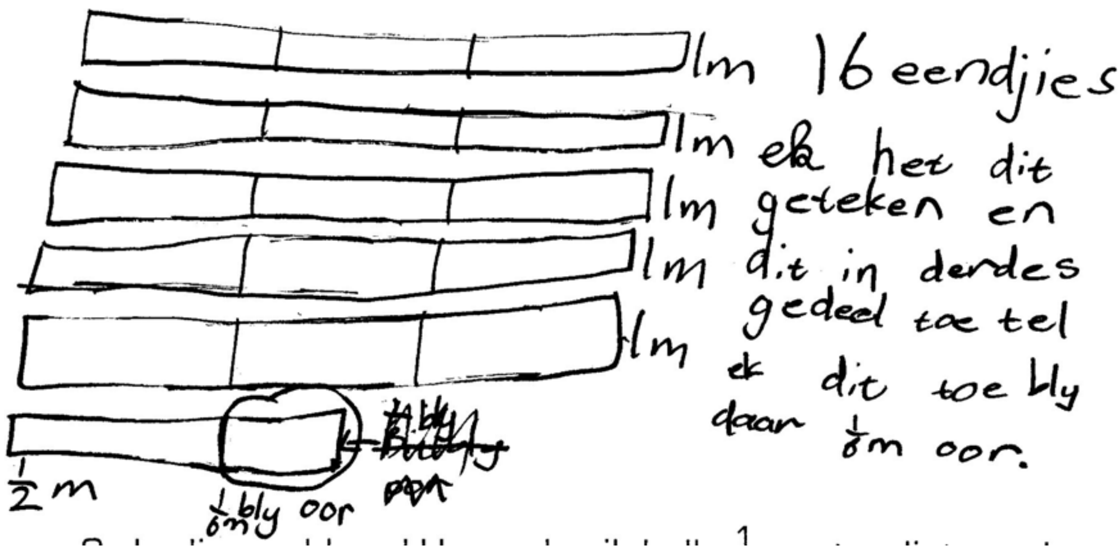


Figure 7.27 Elaine Worksheet 14 no1 ($5\frac{1}{2} \div \frac{1}{3}$)

Elaine wrote: '*16 ducks. I drew it and divided it into thirds and I counted. Then $\frac{1}{6}$ was left.*

Elaine used drawings to solve the problem (Approach 1) and explained as follows:

Teacher: *OK, Elaine, how did you work out that first sum?*

Elaine: *I drew the wire and the half a metre and I drew in the thirds and I saw that a third could fit into the half and then there was a sixth left over. Then I counted how many pieces and I got 16.*

Teacher: *How do you work out that there is a sixth left over?*

Elaine: *Because of the half and the half of the third is a sixth.*

While Elaine was working I asked her how she found that the remainder was a sixth and she told me that three sixths is equal to a half and that two sixths is equal to a third and then there must be a sixth left over.

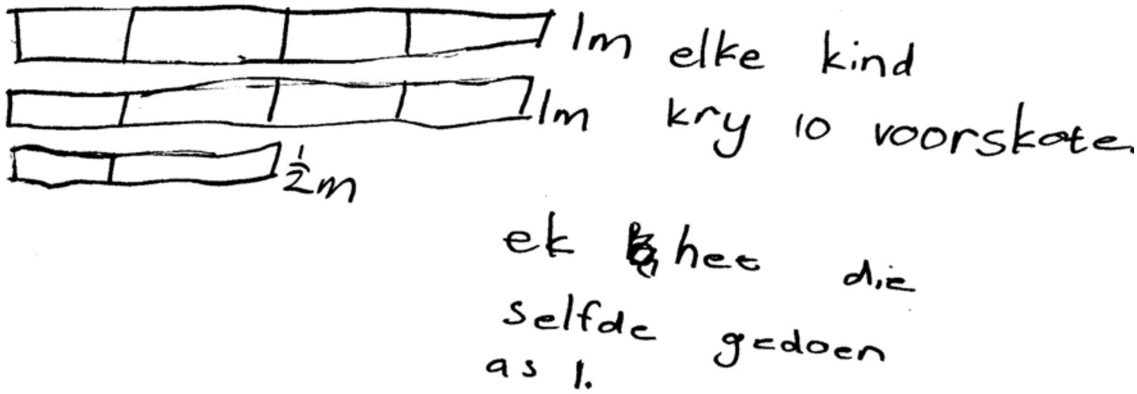


Figure 7.28 Elaine Worksheet 14 no2 $(2\frac{1}{2} \div \frac{1}{4})$

Elaine noted: '*Each child will get 10 aprons. I did the same as I did for number one*'.

Elaine continued to explain:

Teacher: *O.K. Very clever. And the next one?*

Elaine: *I again drew the wire and divided it up into quarters and the last piece I saw that it was equal to the lines I had drawn in the middle and I saw that you can make two more.*

Teacher: *Why could you only make two out of that half?*

Elaine: Silence.

Teacher: *Is there a piece left over?*

Elaine: *No.*

Teacher: *Why not?*

Elaine: *Because two quarters make a half.*

Teacher: *Thank you very much.*

Caroline explained her method in the following way:

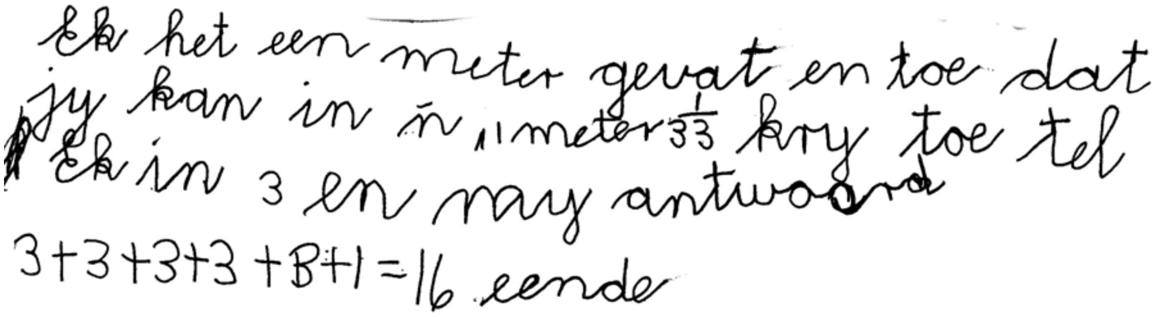


Figure 7.29 Caroline Worksheet 14 no 1 ($5\frac{1}{2} \div \frac{1}{3}$)

Caroline used numbers to solve her problem (Approach 4).

Caroline: *I took one metre and out of that one meter, you can make three and then I counted in three's and got my answer of 15. I can add another one that I make out of the half and so can make 16 ducks.*

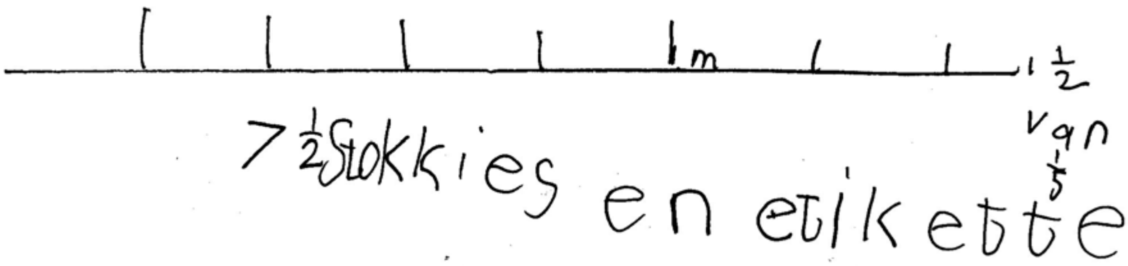


Figure 7.30 Zane Worksheet 15 ($1\frac{1}{2} \div \frac{1}{5}$)

At times, learners find it necessary to work on their own and at other times, they find it necessary to discuss their approaches. Zane and Elaine usually worked individually and then shared their solutions. However, for the above problem, they started discussing the problem first and then worked on their own. This was the second time this had occurred. When learners were not sure of how to solve a problem, or if they found the problem difficult, they usually discussed the problem first, before working on their own.

Zane and Elaine drew a line 10 cm long. They set about dividing it into fifths by marking off two centimetres 'pieces'. They got five pieces out of the whole. They drew the half a metre five centimetres long. Again they measured off the $\frac{1}{5}$ths - two centimetres long. They measured two additional pieces. They concluded that there was a remainder of $\frac{1}{2}$ of a $\frac{1}{5}$ and that they would be able to get seven pieces. The researcher felt that she should have prompted the learners to think about the size of '$\frac{1}{2}$ *of a* $\frac{1}{5}$'.

Zane explained as follows:

Zane: *Teacher, I drew a line on each one that was even. Two is even, four is even, six is even, an eight and a ten, a twelve and a fourteen and then I reached 15. Then I got seven and a half pieces.*

Teacher: *How many centimetres long did you make your metre?*

Zane: *five*

Teacher: *No, your metre? One metre was equal to how many centimetres?*

Zane: *10.* (Indicating that 10 cm represents 1 metre.)

Teacher: *O.K. And your half a metre was equal to how many centimetres?*

Zane: *five*

Teacher: *And then every fifth was equal to how many centimetres?*

Zane: *two*

Teacher: *How did you know that it had to be two centimetres and not three?*

Zane: *Because I worked it out.*

Teacher: *How did you work out that each fifth was equal to two centimetres?*

Zane: *I looked at the ruler.*

Teacher: *Is there anything else that you would like to tell me?*

Zane: *No.*

Teacher: *O.K. Thank you Zane.*

7.4 SUMMARY

Division is traditionally considered to be one of the more difficult basic operations in common fractions (See Murray *et al.,* 1999). Traditionally, learners were given rules and rote procedures to follow without understanding why certain steps were being done. This, according to Mack (1990), causes misconceptions to occur. Learners were, for example, taught to "invert the fraction and multiply". This rule would then be partially or incorrectly used (e.g. $20 \div \frac{3}{4} = \frac{1}{20} \times \frac{3}{4} = \frac{3}{80}$).

During this study, learners were able to solve division by a fraction using real-life situations. Learners relied on their informal knowledge of equal-sharing to solve the problems and used drawings in such a way that the problems became meaningful to them. This corresponds with the findings of Mack (1990) and Murray *et al*. (1999).

Du Toit (1995, ii) found that most learners conceptualised sharing problems in three ways:

> "A. Determine the total number of fractional parts of the source items and then the number of product items that can be acquired from this.
>
> B. Determine the number of product items which can be obtained from one source item and then the number of product items that can be obtained from the remainders.
>
> C. Determine the smallest group of source items from which a discrete number of product items can be obtained and then determine how many of such groups of source items are available."

In this study, most learners solved the problems using method B. They first found the total number that could be obtained from each unit and then added to find the total amount.

Initially the learners solved the problems using drawings (Level One, Phase Two – Murray and Olivier, 1989). The researcher noted that during the second lesson, the majority of the learners worked on an abstract level. This placed them on Level Three of Development (Murray and Olivier, 1989). This could be due to the fact that the learners were required to work with a more familiar fraction, namely, halves. Only two learners showed signs of being able to work abstractly during subsequent lessons.

It was felt that the material was relevant and the context of the problems familiar to the learners. Mack (1990) emphasised the necessity of posing problems set in the context of real-life situations as it allows learners to base their thoughts and solutions on their informal knowledge. Ball (1993) confirms the idea that the context is crucial to the development of the understanding of the fraction concept. If more problems of a similar structure were posed to the learners, the concept should have become more stable and the learners could have developed a more abstract method.

Many learners found it difficult to verbalise their methods of solving the problem. However, they were constantly encouraged to reflect and communicate as, according to Von Glaserfeld (1991), communication and reflection form an essential part of a classroom where learners are supposed to make sense of their mathematics (See Section 2.2.2.3).

Although most learners used drawings to solve the problems, they were able to invent procedures that assisted them in finding a solution. Mack (1990) suspects that it division by a fraction, may be the easiest of the four operations to invent operations for as long as the context makes sense to the student.

The following chapter deals with the development of addition and iteration of fractions.

CHAPTER 8

BASIC OPERATIONS OF FRACTIONS - ITERATION AND ADDITION

Iteration, the putting together of fractional parts to form a whole, is a very important sub-construct of the development of the common fraction concept. The researcher chose to include this concept with the addition of fractions, as they are closely linked to each other. Some of the problems exposed the learners to some multiplication structures, namely repeated addition and rate.

Worksheets 27, 28 and 29 form a subsection of the section on addition of fractions. These worksheets serve as preliminary problems for addition of fractions with unlike denominators.

8.1 ANALYSIS OF DATA

The following problems were used to introduce these three aspects:

Worksheet 7

Two netball teams play a game. There are 14 children all together. The sports teacher wants to give each child $\frac{1}{2}$ of an orange. How many oranges does she need?

Worksheet 8

Mrs Jeremiah bakes five cakes for the party after the netball game.

For one cake she needs:

$\frac{1}{4}$ cup of margarine	$\frac{1}{2}$ cup sugar
1 egg	$\frac{1}{2}$ cup milk
$1\frac{1}{2}$ cups flour	$2\frac{1}{2}$ teaspoons baking powder
$\frac{1}{4}$ teaspoon salt	$\frac{1}{2}$ teaspoon flavouring

Work out how much she needs for five cakes.

Additional Worksheet 8.1 (An additional worksheet was included to re-enforce this concept.)

Please will you help the baker to work out how much ingredients he needs to make six batches of rusks for the children that stay in the hostel.

For one batch he needs:

$\frac{1}{2}$ cup butter	$\frac{1}{4}$ cup sugar
$1\frac{1}{2}$ cups of flour	1 teaspoon baking powder
$\frac{1}{3}$ cup of milk	$\frac{1}{5}$ teaspoon of salt

Worksheet 9

Vusi and Anna prepare soft porridge for breakfast. For each bowl they use $\frac{1}{3}$ of a litre of milk. If they make 6 bowls of porridge, how many litres of milk do they need?

Worksheet 19 (All chains can be found in Appendix A.)

Chain - adding quarters starting at one quarter

<u>Worksheet 20</u>

Chain – adding halves and thirds starting at 5

<u>Worksheet 21</u>

Vuyo wants to know what she has left over after baking for a party. How much does she have?

3 big packets of chips, each $\frac{1}{2}$ full

5 containers of ice-cream, each $\frac{1}{4}$ full

2 jugs of milk, each $\frac{2}{3}$ full

<u>Worksheet 21.1</u> (An additional worksheet was included as some of the learners were finding it difficult to work on an abstract level.)

Chain – addition of quarters starting at one quarter

<u>Worksheet 25</u>

Chain – addition of thirds starting at one third

<u>Worksheet 26</u>

Chain – addition of ‘two thirds’ starting at two thirds

<u>Worksheet 27</u>

Complete this table:

Number of hours	Number of minutes
$\frac{1}{6}$	10
$\frac{1}{5}$	
$\frac{1}{3}$	
$\frac{1}{4}$	
$\frac{1}{2}$	
1	60
	120
$1 + \frac{1}{2}$	
$1 + \frac{1}{3}$	
$1 + \frac{1}{4} + \frac{1}{4}$	
$\frac{1}{6} + \frac{1}{6}$	
$\frac{1}{3} + \frac{1}{6}$	

Do you notice anything?

<u>Worksheet 28</u>

It takes Thandi 10 minutes to walk one kilometre.

Complete this table:

Number of Kilometres	Number of minutes
1	10
$\frac{1}{2}$	
$\frac{1}{5}$	
2	
$2\frac{1}{2}$	
$1\frac{1}{2}$	
$1\frac{1}{5}$	
3	
$3\frac{1}{2}$	
$\frac{1}{2} + \frac{1}{5}$	
$1\frac{1}{2} + \frac{1}{5}$	
$1\frac{1}{5} + \frac{1}{2}$	

Worksheet 29

There are 24 hours in a day.

Complete this table.

Number of days	Number of hours
1	24
2	
$\frac{1}{2}$	
$\frac{1}{4}$	
$\frac{1}{3}$	
$\frac{1}{6}$	4
$\frac{1}{4}+\frac{1}{4}$	
$\frac{1}{6}+\frac{1}{6}$	
$\frac{1}{3}+\frac{1}{3}$	
$\frac{1}{2}+\frac{1}{6}$	
$\frac{1}{2}+\frac{1}{4}$	
$2\frac{1}{2}$	
5	
6	

Worksheet 30

Chain – addition of quarters starting at one

<u>Worksheet 31</u>

Chain – addition and subtraction of thirds starting at two thirds

<u>Worksheet 32</u>

Chain – addition and subtraction of different amounts of fifths starting at four

<u>Worksheet 33</u>

Chain - Learners had to fill in the amount that was added or subtracted in order to obtain the next frame.

An analysis of the data shows that, three approaches were identified in Worksheet 7:

<u>Approach 1</u>

The learner wrote down the correct answer. Yvonne notated her solution as '*7 oranges*'.

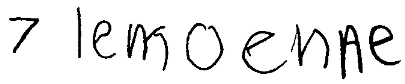


Figure 8.1 Yvonne Worksheet 7 no 1 ($\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$ - The problem is shown as iteration as this is the way that the learners solved the problem.)

<u>Approach 2</u>

Learners drew the answer. The researcher felt that not all learners in this category needed a drawing to solve the problem. Some of the learners drew the answer as a way of explaining the method they had used.

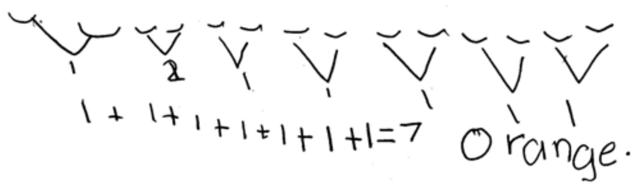


Figure 8.2 Gail Worksheet7 no 1($\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$)

Figure 8.3 Lucille Worksheet 7 no 1 ($\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$)

Approach 3

The learner worked the sum out incorrectly and/or did not manage to complete the problem.

Zelda wrote her solution as *'14 oranges'*.

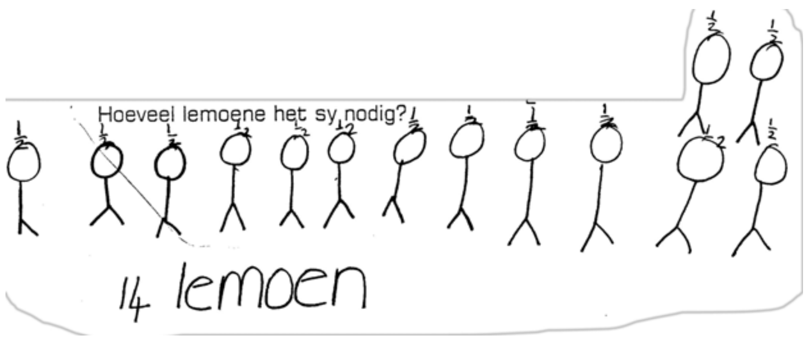


Figure 8.4 Zelda Worksheet 7 no 1 ($\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$)

Figure 8.5 Bob Worksheet 7 no 1 ($\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$)

A discussion was held with Bob and Sue after the lesson:

Teacher: *Bob and Sue, can you tell me what your answer was to number one?*

Bob:	*I got three and a half*
Sue:	*I got seven.*
Teacher:	*Can you tell us how you got your answer Bob?*
Bob:	*For number one?*
Teacher:	*Mmmm…*
Bob:	*Oopsie.*
Teacher:	*Oopsie what?*
Bob:	*I did it wrong.*
Teacher:	*What mistake did you make?*
Bob:	*I said three and a half for one team and there were 14.*
Teacher:	*And if there were 14?*
Bob:	*Seven.*
Teacher:	*Seven you said. Where did you get that seven come from?*
Bob:	Silence.
Teacher:	*If you give one team three and a half, how many do you have to give the other team?*
Bob:	*Three and a half*
Teacher:	*Three and a half. And if you add them together?*
Bob:	*Six…. Seven.*
Teacher:	*Right. That is where the seven comes from hey? So you only took the oranges for one team and not the other team. O.K.*

Appendix F1 gives a summary of the approaches the learners used to solve their problems.

In Worksheet 8, learners had to calculate the total amount of each ingredient needed for the recipe. An additional solution strategy was identified. Some learners chose to write out a number sentence and no learners drew their solutions. The researcher is of the opinion that learners did not draw because they had manipulatives to use to solve the problem.

<u>Approach 1</u>

The learner only wrote down the answer.

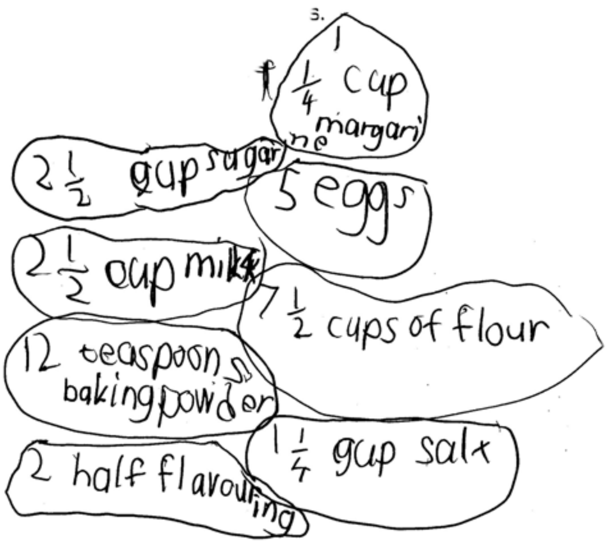


Figure 8.6 Sharon Worksheet 8

$(\frac{1}{4}\times\ 5)$; $(\frac{1}{2}\times 5)$; (1×5) ; $(\frac{1}{2}\times 5)$; $(1\frac{1}{2}\times 5)$; $(2\frac{1}{2}\times 5)$; $(\frac{1}{4}\times\ 5)$; $(\frac{1}{2}\times 5)$

Approach 2

The learners wrote down a number sentence.

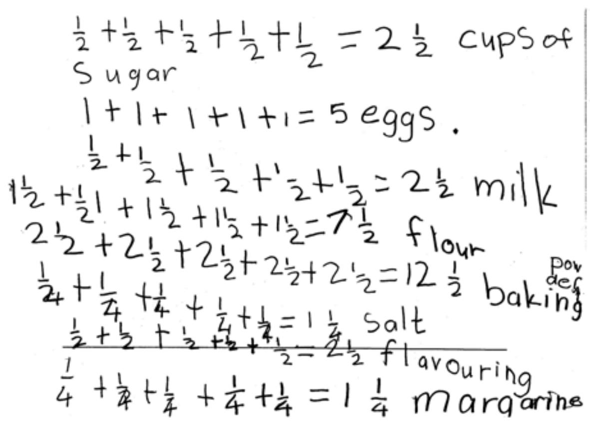


Figure 8.7 Gail Worksheet 8

$(\frac{1}{4}\times\ 5)$; $(\frac{1}{2}\times 5)$; $(1\ \times 5)$; $(\frac{1}{2}\times 5)$; $(1\frac{1}{2}\times 5)$; $(2\frac{1}{2}\times 5)$; $(\frac{1}{4}\times\ 5)$; $(\frac{1}{2}\times 5)$

Approach 3

The learners gave the incorrect answer.

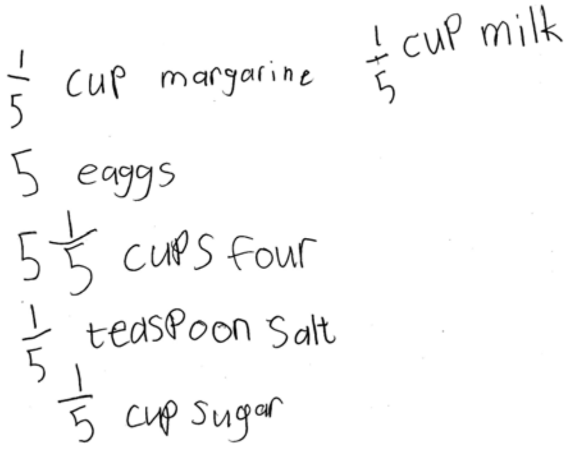


Figure 8.8 Lyn Worksheet 8

$(\frac{1}{4}\times 5)$; $(\frac{1}{2}\times 5)$; (1×5) ; $(\frac{1}{2}\times 5)$; $(1\frac{1}{2}\times 5)$; $(2\frac{1}{2}\times 5)$; $(\frac{1}{4}\times 5)$; $(\frac{1}{2}\times 5)$

Approaches that learners used to solve the problems are contained in Appendix F2.

The learners were notating the fractions using different forms e.g. some were writing the solutions in words, some in symbols and other were using a mixture of words and numbers. Some of the learners notated by writing their answer as follows:

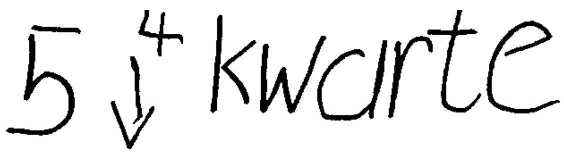


Figure 8.9 Zelda Worksheet 8 $(\frac{1}{4}\times 5)$ Zelda wrote '*5 - 4 quarters*'.

Figure 8.10 Stewart Worksheet 8 $(\frac{1}{2}\times 5)$ Stewart wrote '*Five halves'.*

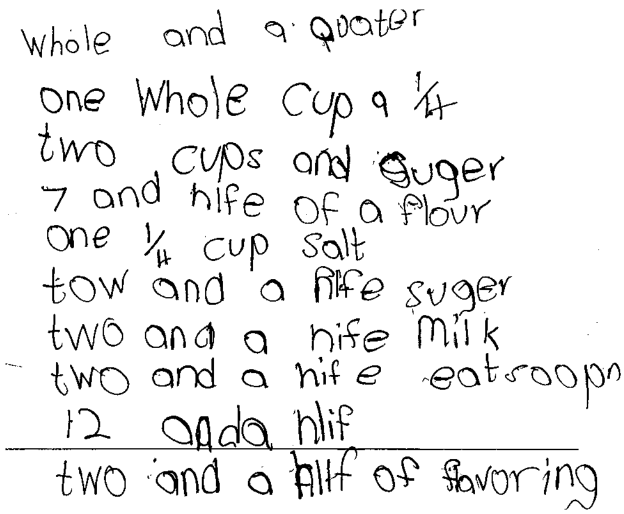


Figure 8.11 Mbulelo Worksheet 8

$(\frac{1}{4} \times 5)$; $(\frac{1}{2} \times 5)$; (1×5) ; $(\frac{1}{2} \times 5)$; $(1\frac{1}{2} \times 5)$; $(2\frac{1}{2} \times 5)$; $(\frac{1}{4} \times 5)$; $(\frac{1}{2} \times 5)$

The learners at both School A and B commented that the problem was difficult. They gave no reason why they experienced difficulty. At the time, the researcher did not inquire as to why the learners found the problem difficult. It could have been caused by the fact that this was only the second time that manipulatives had been brought to class and learners first had to familiarise themselves with the measuring cups and rice that was used to measure with. As Empson (1995) and Murray *et al.* (1996a) report, when manipulatives are used, the manipulatives dictate the situation and not the learners thinking (See Section 2.8).

Because the concept of iteration was taking a long time to develop and become stable, the researcher added an additional worksheet using the same problem structure.

Four approaches were identified in Worksheet 8.1. These are summarised in Appendix F3.

<u>Approach 1</u>

The learners wrote down answers only.

Elaine's solutions were:

Butter 3 whole

sugar $1\frac{1}{2}$

Cake flour 9 wholes

baking powder 6 wholes

Milk 2 wholes

salt $1\frac{1}{5}$

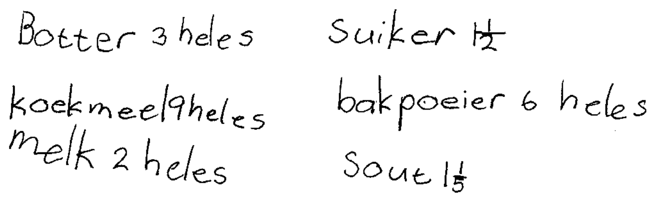


Figure 8.12 Elaine Worksheet 8.1

$(\frac{1}{2}\times 6)$; $(\frac{1}{4}\times 6)$; $(1\frac{1}{2}\times 6)$; (1×6) ; $(\frac{1}{3}\times 6)$ $(\frac{1}{5}\times 6)$

Approach 2

The learners wrote down a number sentence.

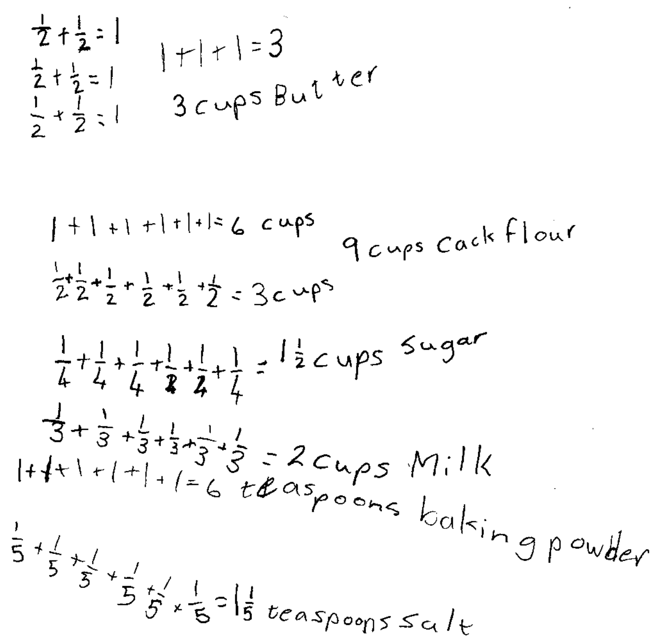


Figure 8.13 Lucille Worksheet 8.1

$(\frac{1}{2}\times 6)$; $(\frac{1}{4}\times 6)$; $(1\frac{1}{2}\times 6)$; (1×6) ; $(\frac{1}{3}\times 6)$; $(\frac{1}{5}\times 6)$

Approach 3

The learners used fractional notation and indicated the solution.

Linda wrote:

3 cups butter

9 cups flour

2 cups milk

2 cups sugar

6 teaspoons baking powder

$1\frac{1}{5}$ *teaspoons salt*

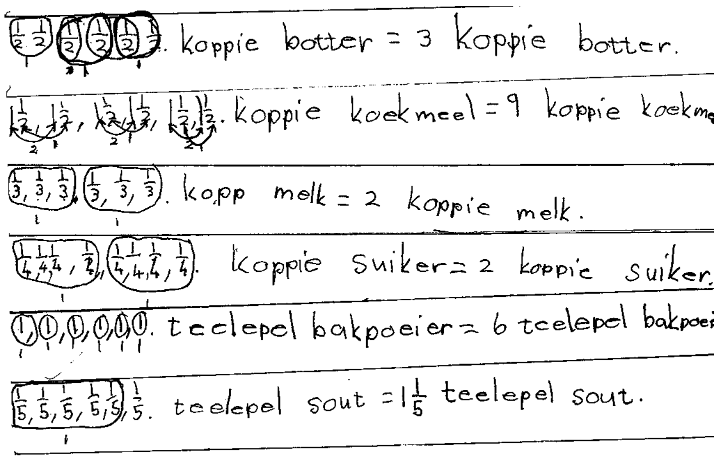


Figure 8.14 Linda Worksheet 8.1

$$(\frac{1}{2}\times 6)\,;\,(\frac{1}{4}\times 6)\,;\,(1\frac{1}{2}\times 6)\,;\,(1\times 6)\,;\,(\frac{1}{3}\times 6)\,;\,(\frac{1}{5}\times 6)$$

<u>Approach 4</u>

Learners gave the incorrect answer.

At the end of the lesson, the learners at School A once again commented that the problem was difficult. They gave no explanation why they thought this. The researcher did not ask learners why they thought this.

Comparing the approaches used in Worksheet 8 and 8.1, more learners used Approach 1 in Worksheet 8 than in Worksheet 8.1. The researcher had physical apparatus available for Worksheet 8, but not for 8.1. Without apparatus being available learners, had to construct their own method of notating the solution in order to solve the problem (See Section 2.8). The number of learners using Approach 2 was about the same for both worksheets.

School B also had more learners giving incorrect solutions to the problem. One of the learners had a very negative effect on the other learners at the beginning of the lesson. He made the following comments at the beginning of the lesson and these seemed to influence the whole class:

"I hate hard work." "I do not like thinking." "I'm going to write down any old answer." "I hate explaining."

Another factor that influenced the learners that particular day was that they were going to simulate Election Day after the lesson and they were very distracted and excitable. Another reason for the increase in incorrect solutions, could have be that learners were not yet ready to work on a more abstract level.

There had been definite progress with some of the learners at School A between Worksheet 8 and 8.1. Elaine and Zane needed to write down the solution only (Approach 1) thus working at Level Three of Development (Murray and Olivier, 1989). All the solutions were given (Approach 3) at School A were correct.

An analysis of the data shows four approaches were identified in Worksheet 9.

Approach 1

The learners used number sentences.

Carol wrote the following:

$\frac{1}{3} + \frac{1}{3} + \frac{1}{3}$ *= 1 cup milk*

$\frac{1}{3} + \frac{1}{3} + \frac{1}{3}$ *= 1 cup milk*

= 2 cups milk

Figure 8.15 Carol Worksheet 9 no 1($6 \times \frac{1}{3}$)

Approach 2

The learners drew their solutions and indicated their solution using fractional notation.

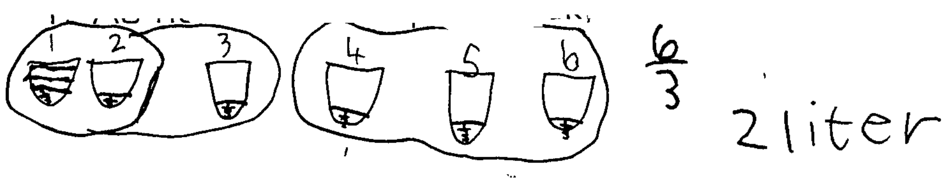


Figure 8.16 Kate Worksheet 9 no 1($6 \times \frac{1}{3}$)

Approach 3

The learners drew their solutions.

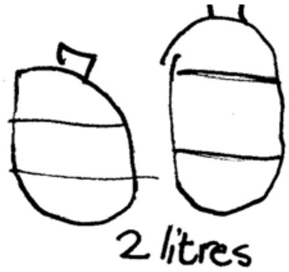


Figure 8.17 Nosisa Worksheet 9 no 1($6 \times \frac{1}{3}$)

Approach 4

Learners gave the incorrect solution.

The approaches learners used are summarised on grid as Appendix F4.

It was felt that at this stage some of the drawings were really irrelevant. They did not help the learners to solve the problem but merely were used to explain the method that the learners used. Some learners found it easier to explain by means of a drawing rather than to verbalise their thoughts. Initially the researcher requested the learners to draw their solutions, as this was beneficial to develop the initial fraction concept.

Elaine's example is a typical example of this:

She wrote:

One litre one litre

2 litres

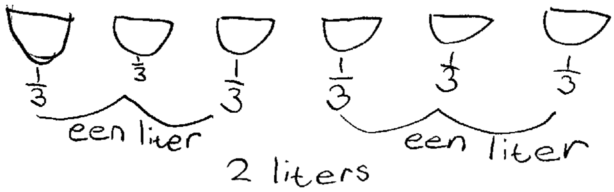


Figure 8.18 Elaine Worksheet 9 no 1($6 \times \frac{1}{3}$)

The researcher was able to identify three approaches used by the learners in Worksheet 21. All learners worked out the solution correctly. Some learners wrote down a number sentence (Approach 1) while other used drawings (Approach 3) to solve the problem. The number of learners who were able to go directly to writing down the solutions increased to four (working abstractly – Level Three of Development – Murray and Olivier, 1989).

These approaches are included on a grid as Appendix F5.

Kate wrote the solution as '*whole and a* $\frac{1}{3}$'. She used a drawing to help her find the solution (Approach 3)

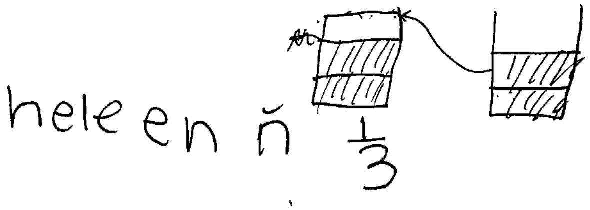


Figure 8.19 Kate Worksheet 21 no 3 ($\frac{2}{3} \times 2$)

Martin also used a drawing to assist him to find the solution (Category 3).

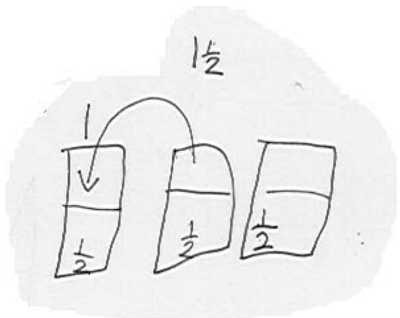


Figure 8.20 Martin Worksheet 21 no 1 ($\frac{1}{2} \times 3$)

Worksheets 19, 20, 21.1, 25, 26, 30, 31, 32 and 33 involved the learners completing chains. These chains included addition and at times a few subtraction problems.

In Worksheet 19, the researcher identified three solution approaches.

Approach 1

Learners were able to complete the chain and change an improper fraction into a mixed number, but they were not able to simplify the fraction.

Approach 2

Learners were able to complete the chain and change an improper fraction into a mixed number and simplify the fraction.

Approach 3

Learners were not able to complete the chain successfully.

The solution approaches are summarized on Appendix F6.

The chain was completed satisfactorily, the improper fractions were converted to mixed numbers, but the fractions were not simplified (Approach1).

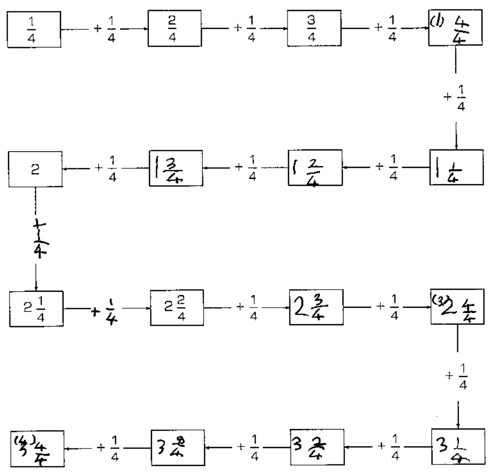


Figure 8.21 Yvonne Worksheet 19

The chain was completed satisfactorily, the improper fractions were converted to mixed numbers, and the fractions were simplified (Approach 2).

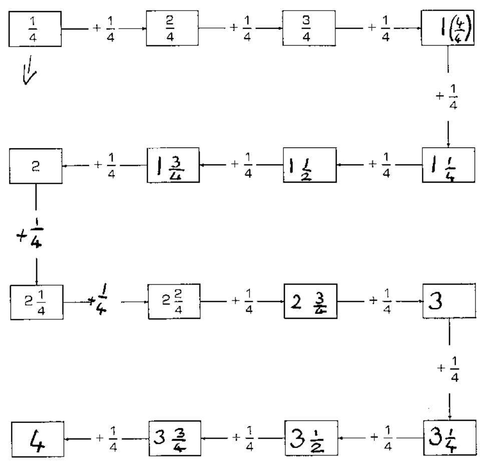


Figure 8.22 Elaine Worksheet 19

From Worksheet 20, with an exception of four and at times five learners, all learners' used Approach 1. They completed the chain and changed the improper fraction into a mixed number. No learner however, simplified the fraction (e.g. $\frac{4}{8}=\frac{1}{2}$). Kate, Mary, Zelda, Stewart and at times, Martin, all experienced problems with this concept. This could have been due to the fact that these chains involved abstract reasoning and the learners were not ready to move onto this abstract level.

It was necessary to assist these learners by using drawings of chocolate bars. The problem was put into a context. The learners had to shade in the piece of chocolate that had been eaten and then work out the total amount that had been eaten. This method helped the learners to make sense of the situation and they used this method throughout the research (See Sections 2.6 and 2.10). Mack (1990) and Piel and Green (1994) found that learners who could not solve problems abstractly were able to do so once the problems had been put into a real-life context.

This is how Linda solved the problems on the chain.

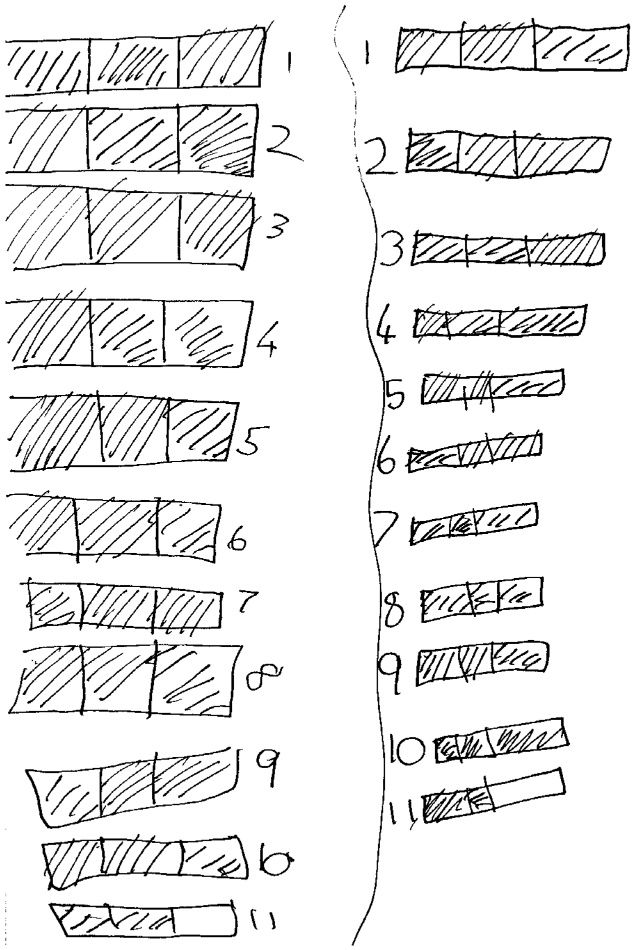


Figure 8.23 Linda Worksheet 26

Initially Kate and Mary made the following mistake:

$2\frac{2}{4}+\frac{1}{4}=3\frac{3}{4}+\frac{1}{4}=4\frac{4}{4}$ etc. The fractional part of the sum was correct, but not the whole number. They added one onto the numerator of the fraction as well as the whole number. The manner in which Kate and Mary solved these problems showed that they did not understand the symbols and they were unable to work on an abstract level. They were both showing signs of interpreting the fraction as a whole number and then applying whole number strategies to fraction problems as reported by D'Ambrosio and Mewborn (1994) and Murray *et al*. (1999) (See Section 2.7).

Kate and Mary improved slightly in Worksheet 20 although at times they still made the same mistake. Kate and Mary did not have a well-developed number concept and when the numbers were smaller, they seemed to manage better.

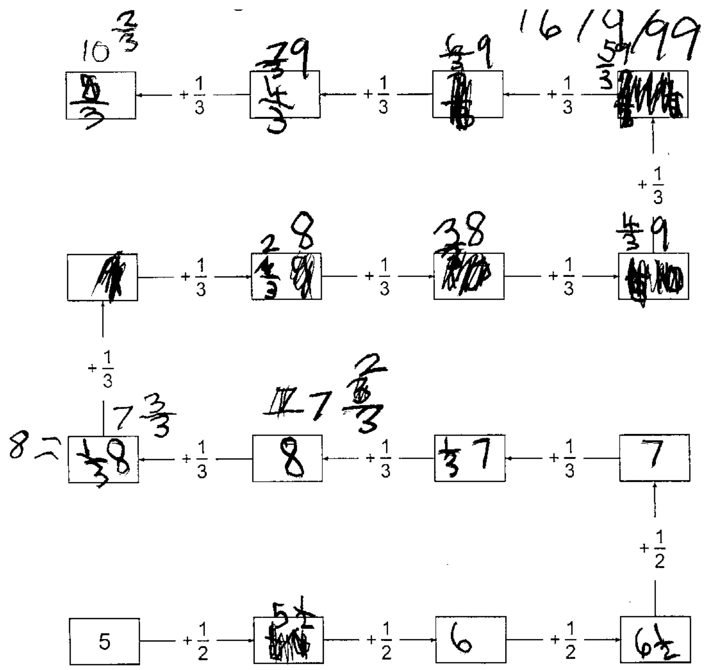


Figure 8.24 Kate Worksheet 20

Preliminary problems for addition of fractions with unlike denominators were included on Worksheets 27, 28 and 29. As some of the learners were having difficulty with problems that contained large numbers, the researcher decided to embark on Worksheet 29 where learners would be working with smaller numbers. The numbers on the other two worksheets were somewhat larger.

Three approaches could be identified in these three worksheets.

<u>Approach 1</u>

Learners wrote down the solutions only.

The headings of the two columns were: Number of kilometres and Total minutes.

Getal kilometers	Getal minute
1	10
$\frac{1}{2}$	5 ✓
$\frac{1}{5}$	2 ✓
2	20 ✓
$2\frac{1}{2}$	20 + 5 = 25 25 ✓
$1\frac{1}{2}$	10 + 5 = 15 15 ✓
$1\frac{1}{5}$	10 + 2 = 12 12 ✓
3	30
$3\frac{1}{2}$	30 + 5 = 35 35 ✓
$\frac{1}{2} + \frac{1}{5}$	5 + 2 = 7 7 ✓
$1\frac{1}{2} + \frac{1}{5}$	15 + 2 = 17 17 ✓
$1\frac{1}{5} + \frac{1}{2}$	12 + 5 = 17 17 ✓

Figure 8.25 Linda Worksheet 28

Approach 2

Learners first worked out the fractional part by sharing.

Getal ure	Getal minute
$\frac{1}{6}$	10
$\frac{1}{5}$	~~8~~ 12
$\frac{1}{3}$	20
$\frac{1}{4}$	15
$\frac{1}{2}$	30
1	60
	120
$1 + \frac{1}{2}$	90
$1 + \frac{1}{3}$	60+20 80
$1 + \frac{1}{4} + \frac{1}{4}$	60+ 15+15 90
$\frac{1}{6} + \frac{1}{6}$	10+10=20 20
$\frac{1}{3} + \frac{1}{6}$	20+10= 30

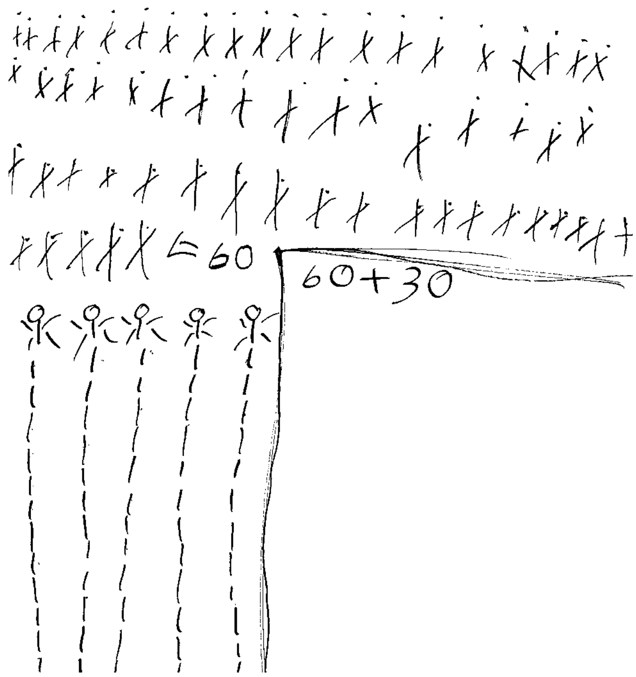


Figure 8.26 Stewart Worksheet 27

Approach 3

Learners first worked out the fractional part by accelerated counting on.

Getal ure	Getal minute
$\frac{1}{6}$	10
$\frac{1}{5}$	12 ✓
$\frac{1}{3}$	20 ✓
$\frac{1}{4}$	15 ✓
$\frac{1}{2}$	30 ✓
1	60 ✓
2 ✓	120
$1 + \frac{1}{2}$	60+30=90 90 ✓
$1 + \frac{1}{3}$	60+20=80 80 ✓
$1 + \frac{1}{4} + \frac{1}{4}$	60+15+15= 90 ✓
$\frac{1}{6} + \frac{1}{6}$	10+10= 20 ✓
$\frac{1}{3} + \frac{1}{6}$	20+10= 30 ✓

The headings of the columns are Number of hours and Number of minutes.

Figure 8.27 Zelda Worksheet 27

Grids showing the various approaches used by the learners are to be found in Appendix F7. From these grids one can see that all learners made use of previous solutions and were able to simply write down the solutions.

8.2 COGNITIVE DEVELOPMENT OF INDIVIDUAL LEARNERS

Individual Study 8

Linda is an example of a learner with an average mathematical ability. She used a drawing to solve the problem (Approach 2). She drew the 14 players and gave each one $\frac{1}{2}$ an orange. Linda attempted to write out a number sentence, but although it was not correct, her solution was.

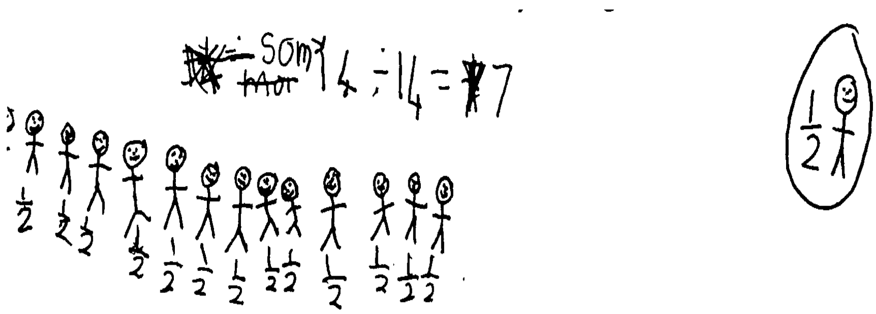


Figure 8.28 Linda Worksheet 7 no 1$(14 \times \frac{1}{2})$

Linda managed to work out most of the ingredients for the cakes. Her solution for the amount of milk and egg was incorrect. She also chose to notate her answer for the amount of margarine as '*5 quarters*' and then used the fraction symbols. Manipulatives were available for this lesson and Linda made use of them. This placed her on Level One, Phase One of Development (Murray and Olivier, 1989).

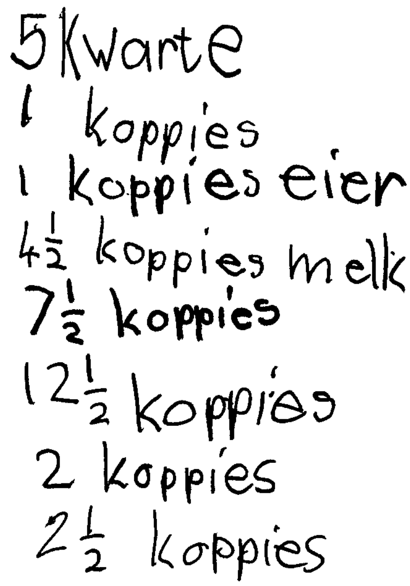


Figure 8.29 Linda Worksheet 8

$$(\frac{1}{4}\times 5)\,;\ (\frac{1}{2}\times 5)\,;\ (1\times 5)\,;\ (\frac{1}{2}\times 5)\,;\ (1\frac{1}{2}\times 5)\,;\ (2\frac{1}{2}\times 5)\,;\ (\frac{1}{4}\times 5)\,;\ (\frac{1}{2}\times 5)$$

Linda's method for working out the total quantity of each ingredient for Worksheet 8.1 was very interesting (Refer to **Figure 8.14)**. There were no manipulatives available and she made use of fraction symbols to solve her problem (Approach 3).

This placed her on Level One, Phase Two of Development (Murray and Olivier, 1989). Linda wrote down $\frac{1}{2}$ six times and then circled the two halves and wrote a one underneath the circle. Her method of working out the one and a half times six was also very interesting. She wrote out one and a half – six times. She then drew an arrow adding the two ones and put a two underneath it. She did this three times. She drew an arrow joining two halves and put a one under it. Again she did this three times. She then added the whole numbers together and got nine. Her answer for the amount of sugar was incorrect. She thought that three quarters made a whole.

For Worksheet 9, Linda wrote down her solution as follows: $\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=1$ *cup* $\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=1$ *cup. This makes 2 cups of milk.*

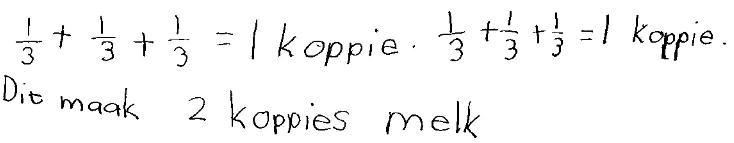


Figure 8.30 Linda Worksheet 9 no 1 ($6 \times \frac{1}{3}$)

She wrote down $\frac{1}{3}$, three times and equated it to a whole (Approach 2). She repeated this procedure and found the solution to be two cups. Although Linda was supposed to be finding how many litres, the method used would have been the same.

Linda was absent for Worksheet 21. This was a pity as it would have been interesting to see if she would have been able to work more abstractly with this problem.

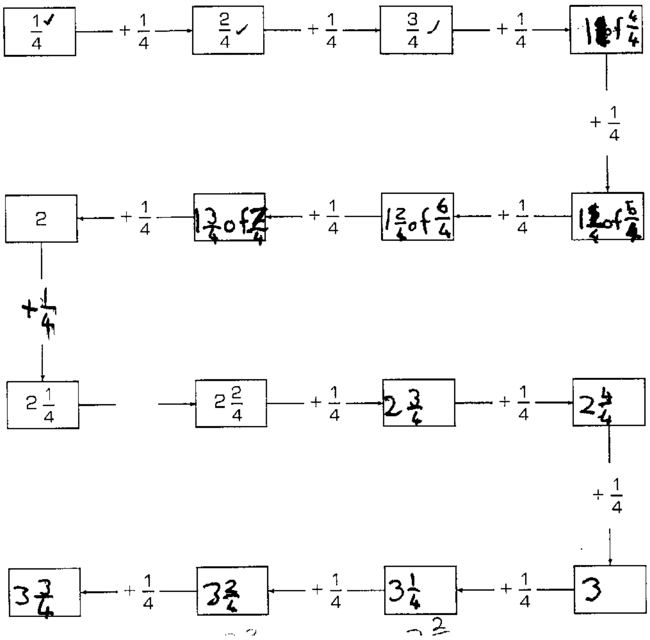


Figure 8.31 Linda Worksheet 19

Linda filled in the first three horizontal bars of the chain correctly. She wrote down both an improper fraction and the mixed number. Some of the mixed numbers were simplified and others not (Approach 1 and 2). She made a mistake when she added a quarter to two and four quarters and obtained the answer of three instead of three and one quarter. From there on, the chain was incorrect. Linda was showing signs of being able to work abstractly. She became very confused when she used concrete aids and found it easier to draw pictures of the problem and to put it into a context. She gradually developed methods for addition of fractions and iteration. She began to solve problems abstractly.

Although there are definite signs that Linda made a few errors in this chain, after reflecting with her partner, she was able to rectify them.

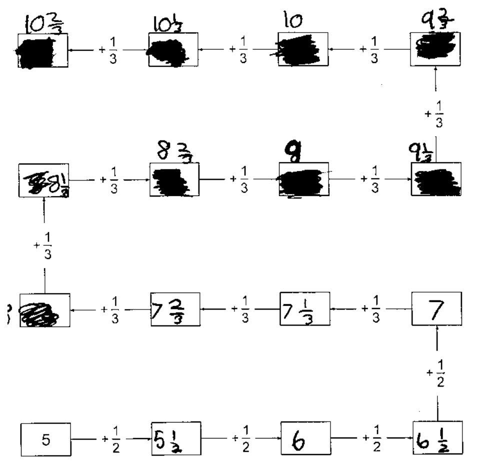


Figure 8.32 Linda Worksheet20

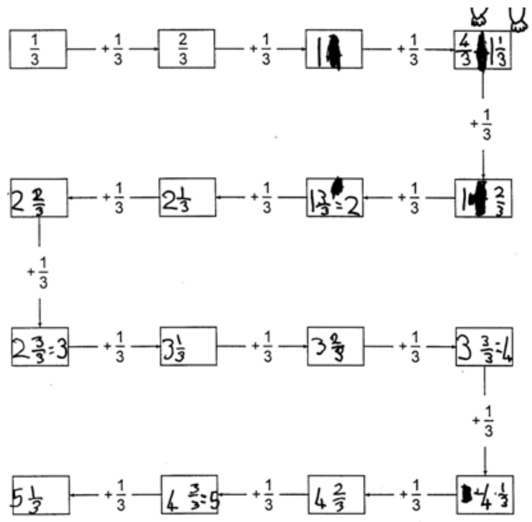


Figure 8.33 Linda Worksheet 25

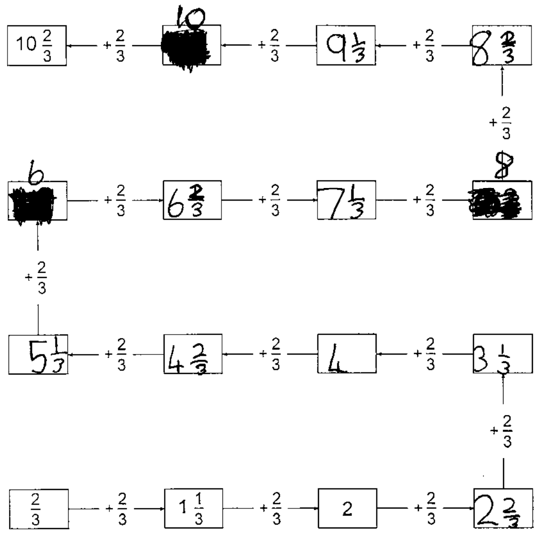


Figure 8.34 Linda Worksheet 26

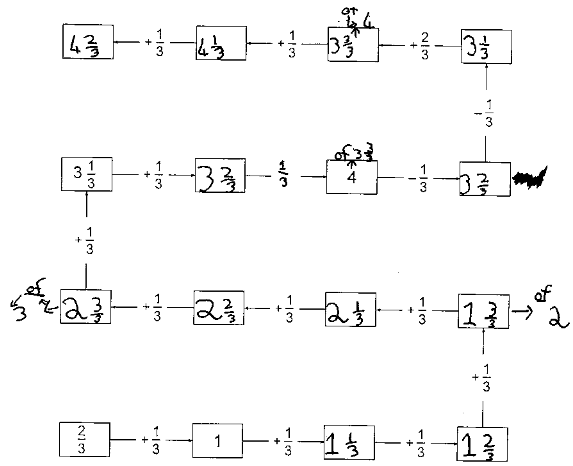


Figure 8.35 Linda Worksheet 31

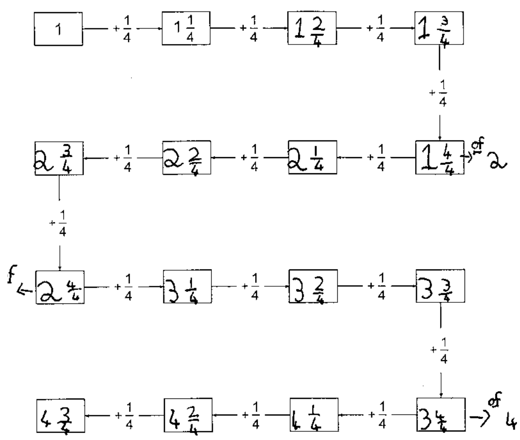


Figure 8.36 Linda Worksheet30

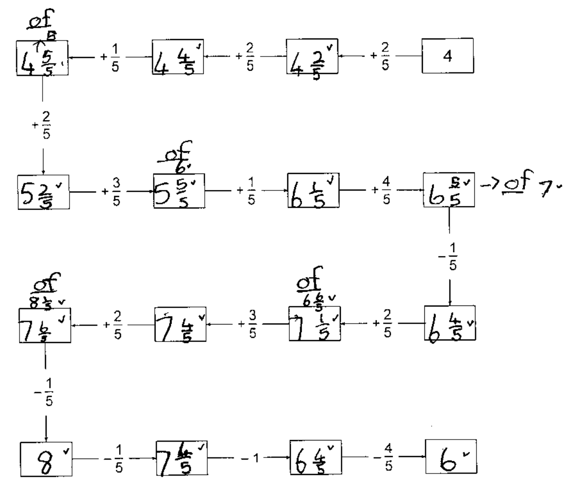


Figure 8.37 Linda Worksheet32

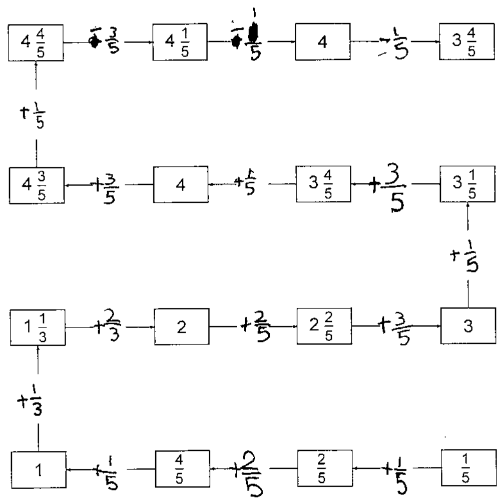


Figure 8.38 Linda Worksheet 33

Linda completed the chains successfully. She continued to write the solutions as mixed numbers and improper fractions (Approach 1 and 2). She did not simplify the fractions. (e.g. $\frac{2}{4} = \frac{1}{2}$)

Linda began solving the problems in the first part of this section (i.e. repeated addition of fractions) using drawings and fractional notation however she produced some incorrect solutions. She attempted to work abstractly, but was not quite at Level Three of Development (Murray and Olivier, 1989). Although she developed a great deal she needed to revert (‘fold back’: Kieren *et al*., 1992 as cited in Pitkethly and Hunting 1996) to work at a Level One, Phase Two Level of Development at times. The last problem, she was able to do abstractly (Level Three of Development).

The main message of the Van Hiele model (See Section 2.3.3) is that learners should be exposed to authentic, real-life experiences of a concept over a period of time before any abstract notations are attempted. At times, Linda did make mistakes when filling in the chains. The researcher felt that it would have been more beneficial for Linda to be exposed to more problems set in a real-life context before attempting the chains.

Individual Study 9

Mary was a below average learner in mathematics. She found difficulty with the concepts of addition of fractions and iteration.

Mary did not obtain the correct solution (Approach 4). She drew 14 people and 14 oranges and did not indicate a solution. After reflection, she attempted the problem again.

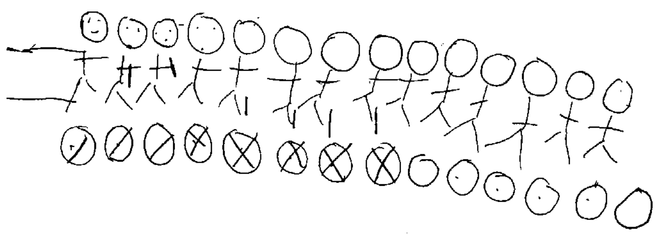


Figure 8.39 Mary Worksheet 7 no 1

($14 \times \frac{1}{2}$)

The second attempt saw her once again drawing the 14 people and 14 oranges. She divided seven oranges in half and as she shared them, she made a cross showing which had been shared and which not. Mary did not indicate how many oranges she had shared altogether.

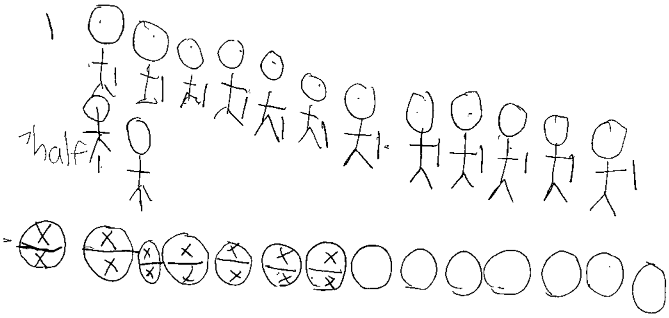
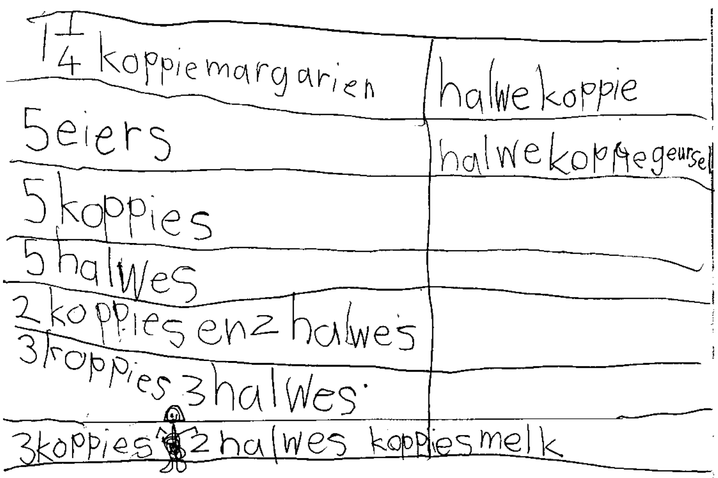


Figure 8.40 Mary Worksheet 7 no 1

$(14 \times \frac{1}{2})$

In Worksheet 8, Mary was still writing out some of the fraction names using words. The class worked out the initial solution for the margarine. From her work, it is very difficult to identify which solution is for which ingredient, but she did not manage to find the correct solutions (Approach 3). She speaks of three halves and two halves. At this stage of the research, she was not able to iterate fractions. Mary used manipulatives during this lesson.

Mary wrote the following solutions down for Worksheet 8.



$1\frac{1}{4}$ cups margarine half a cup
5 egg half a cup of flavouring
5 cups
5 halves
2 cups and 2 halves
3 cups and 3 halves
3 cups and 2 half cups of milk

Figure 8.41 Mary Worksheet 8

$(\frac{1}{4} \times 5)$; $(\frac{1}{2} \times 5)$; (1×5) ; $(\frac{1}{2} \times 5)$; $(1\frac{1}{2} \times 5)$; $(2\frac{1}{2} \times 5)$; $(\frac{1}{4} \times 5)$; $(\frac{1}{2} \times 5)$

She used drawings to illustrate her solutions (Approach 3). Her solutions were written as follows:

$\frac{6}{2} \rightarrow 3$

cups of butter = 3

cups of sugar 1$\frac{1}{2}$

cups of flour 9

teaspoons baking powder 6

cups of milk 2

teaspoons salt $\frac{6}{5}$

Half and half gives 1 whole

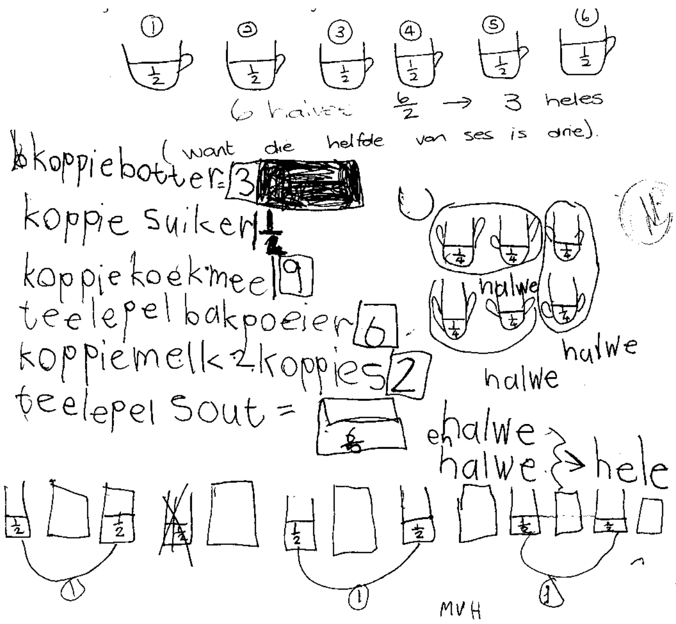


Figure 8.42 Mary Worksheet 8.1

$$(\frac{1}{2}\times 6);\ (\frac{1}{4}\times 6);\ (1\frac{1}{2}\times 6);\ (1\times 6);\ (\frac{1}{3}\times 6);\ (\frac{1}{5}\times 6)$$

The researcher made the following entry into her journal after the lesson at School A:

> Mary came to me early in the lesson with a sad tale of not understanding what to do. We went through the first part of finding six times a half a cup of butter. We drew the six cups and filled each one with a half. She then answered that there were six half cups of butter and after a few moments she decided that this was equal to three whole cups. Correct! The reason she gave was. "*Want die helfde van ses is drie"* (because half of six is three). This could have been a primitive for Mary. A primitive can be defined as previously gained knowledge that can be recalled without conscious effort (Murray *et al*., 1992).
>
> She had no problem after this while working out the sugar, baking powder, milk and flour. She also had no problem with the salt although she left the answer as six fifths where all the others she worked out the wholes. Time was beginning to be a factor so this could have been the reason.
>
> I particularly enjoyed the way she worked out the sugar: She drew six quarter cups of sugar. She then proceeded to circle two of the quarter cups together

> and decided that that was equal to a half. She then decided that she had three halves. Finally she said that a half and a half make a whole and there will be another half to which you cannot add anything.

There was a marked improvement between the solutions that Mary gave for Worksheet 8 and 8.1. This could have been attributed to the fact that Mary was using drawings to find the solutions and not relying on manipulatives. She had to make her own representations and she had to represent her thinking on paper. Mary's thinking then dictated the situation and she could go further in her thinking (See Section 2.8: Empson, 1995). A second contributing factor could have been that after Worksheet 8, the learners had done reflection of their work and Mary could have gained from this.

Mary was very easily distracted and on numerous occasions the researcher spoke to the class teacher about her. The teacher confirmed that Mary had a very short attention span. The researcher felt that the manipulatives that were available during Worksheet 8 might have distracted Mary and that had she initially drawn her solution, she would have been forced to have given the problem more thought.

Mary drew her solution (Approach 3) in Worksheet 9. She was not able to convert the six thirds into two wholes.

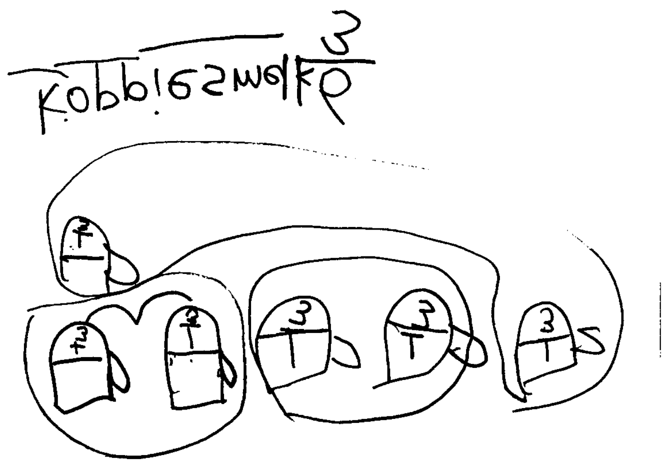


Figure 8.43 Mary Worksheet 9 ($6 \times \frac{1}{3}$)

This conversation was held with the researcher:

Teacher: *Come, let us have a look at the answer. You both got the answer six, but Kate says that the answer is six millilitres and Mary says that it is six thirds. Now, who is correct and who is incorrect?*

Mary: *I am right.*

Teacher: *Why do you think you are correct? Listen carefully to her explanation Kate.*

Mary: *I added these two together and I got two thirds and I added those two together and I got two thirds and I added these two together and I got another two thirds and altogether there are six thirds of a cup. A whole litre will make the porridge too runny.*

Teacher: *Let us have a look Kate. She says that for every plate of porridge, you use a third of a cup. She drew it over here – a third, a third, a third etc. And then she added those two thirds together and those two thirds and those two thirds and so she got six thirds. And what are you going to do with yours. You can't just write down the answer if you do not understand. Can you explain to me?*

Kate: *She added them together.*

Teacher: *What did she add together?*

Kate: *The milk.*

Teacher: *And what milk did she add together?*

Kate: *All the milk.*

Teacher: *All the milk?*

Kate: *The thirds.*

Teacher: *The thirds. O.K. Now see if you can add the thirds together.*

Kate added quietly on her own.

Teacher: *How much does it give you now?*

Kate: *A whole.*

Teacher: *No. How many thirds give you a whole?*

Kate: *Three*

Teacher: *If three thirds make a whole can you not add three of the cups together to get a whole?*

Kate: *Yes.*

Kate then drew a circle around three of the plates.

Teacher: *What does it give you now?*

Kate: *A whole.*

Teacher: *Can you do the same with the others? What do you get now?*

Kate: *A whole.*

Teacher: *How much milk do you get altogether?*

Kate: *two thirds*

Teacher: *No, not two thirds. A whole and a whole gives you….*

Kate: *two wholes.*

Teacher: *Yes, two wholes. Write down your answer. Now we have got a problem. Mary, you say six thirds and Kate says two litres. Now, who is right and who is wrong?*

Mary: *I am right.*

Teacher: *Explain to us why you think you are correct.*

Mary: Mary said something here that I could not pick up from the tape.

Teacher: *Let us have a look at the sum again. How many litres of milk are needed to make six plates of porridge. Can you explain Kate, what you did?*

Kate: *I took these three thirds and I added them together.*

Teacher: *Why did you add the three thirds together?*

Kate: *Um, because if I add them together I will get a whole.*

Teacher: *Do you hear how she explained, Mary? She said that if you add three thirds together, you would get a whole. And further, what did you do?*

Kate: *I added the other three together.*

Teacher: *Added the other three together and what did you get?*

Kate: *Another whole.*

Teacher: *How many litres does Kate have now?*

Kate: *Two*

Teacher: *She has two litres. Can you do the same with your pictures, Mary?*

Mary: *I do not know.*

Teacher: *Can you try?*

Silence.

Teacher: *I really would like to know, who is correct, you with your six thirds or Kate, with her two litres.*

Mary: *I do not know.*

Teacher: *I think you two should talk a little bit more to see who is correct and who is incorrect. Can I leave you?*

I left the two of them working and discussing and they returned to tell me that they thought they are both correct because six thirds is the same as two wholes.

By allowing the two learners to discuss and reflect on their mathematics, they were able to solve the problem for themselves. Their discussion drove the learning process. This once again highlights the importance for time for discussions and reflection (See Section 2.2.3).

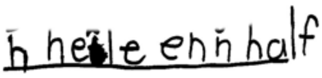


Figure 8.44 Mary Worksheet 21 no 1
$(\frac{1}{2}+\frac{1}{2}+\frac{1}{2}=1\frac{1}{2})$

Mary was able to write down the answer for the first question, 'a *whole and a half*'. (Approach 1). She was showing signs of starting to be able to work abstractly when using halves.

For number 2 and 3 Mary drew her solution (Approach 3). In number 2, she drew five containers, each containing a quarter, and then showed how she filled up the container and yet had a quarter left.

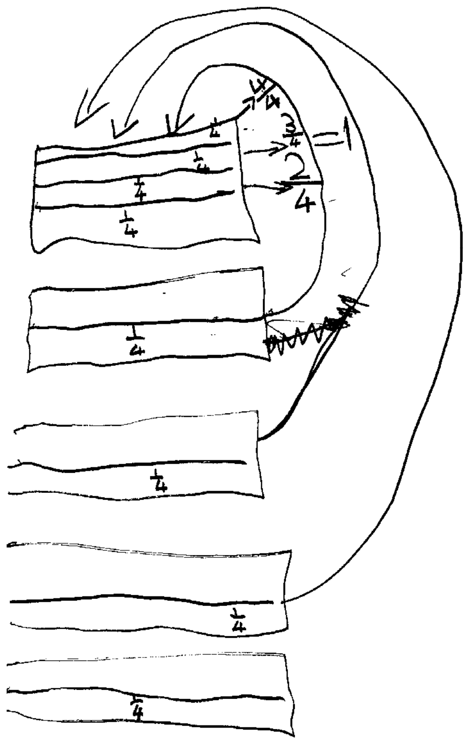


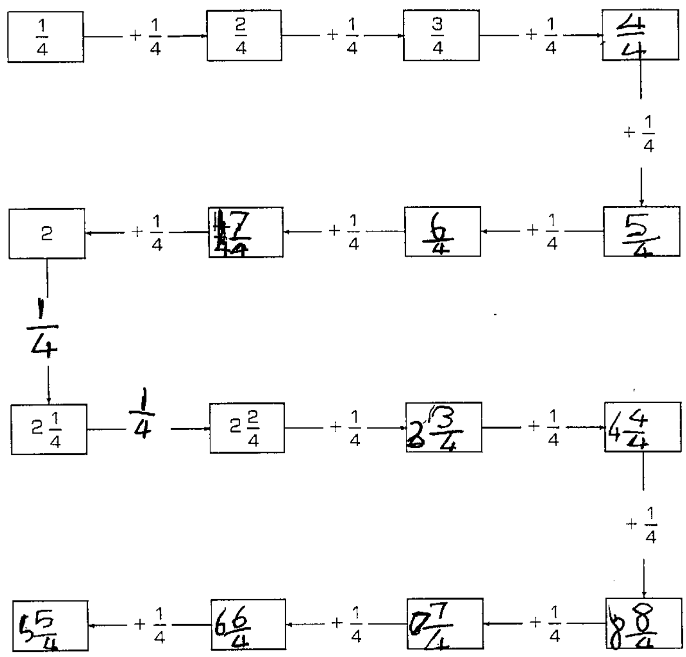
Figure 8.45 Mary Worksheet 21 no2
$(\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4})$

Figure 8.46 Mary No 3 ($\frac{2}{3} + \frac{2}{3}$)

She wrote her solution in words: '*a whole and a third*'.

She solved the problem by drawing two cups and showing the two thirds in each cup. She used an arrow to show that the one third from the one cup would be used to fill up the other cup.

Mary found it difficult to fill in the chains. Although she was showing signs of beginning to work abstractly with halves, she was not ready to work at an abstract level in all forms of fraction computations.



Worksheet 8.47 Mary Worksheet19

Mary managed to complete the first two horizontal lines of the chain. She filled in the solutions in as improper fractions. The solutions filled in on the chain were mixed numbers and from this point she started adding one to the numerator as well as to the whole number. From four and four quarters, Kate found the solution to be eight and eight quarters. When she had to add another one quarter, she wrote the solution as seven and seven quarters. She then continued subtracting one from the numerator and the whole number. Mary was again showing signs of interpreting fractions as whole numbers and using the whole number strategies to solve her problems (See Section 2.7).

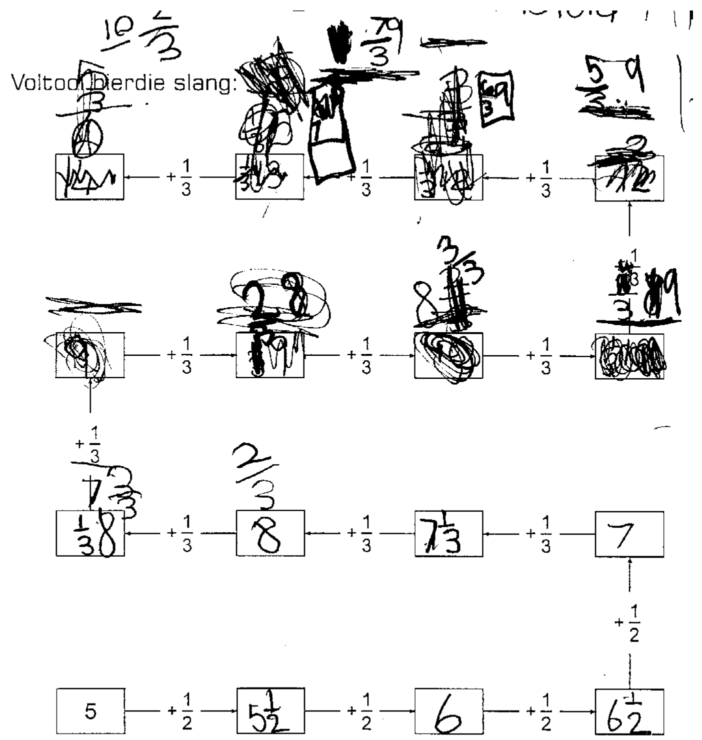


Figure 8.48 Mary Worksheet 20

Mary was able to complete the chain when halves were being added. As before, she showed signs of being able to work abstractly when dealing with halves. She managed to add the first third successfully, but from the appearance of the worksheet, one can see that Mary was very confused.

The researcher intervened at this stage and the group of learners, who were not able to work abstractly, was called aside and the problem was put into a real-life context (See Sections 2.6 and 2.10). The learners were told to think about all the fractions as pieces of chocolate that were eaten or given away and each time a piece was eaten or given away, a record had to be kept. A suggestion was made that they draw chocolate bars to help them work out the solution.

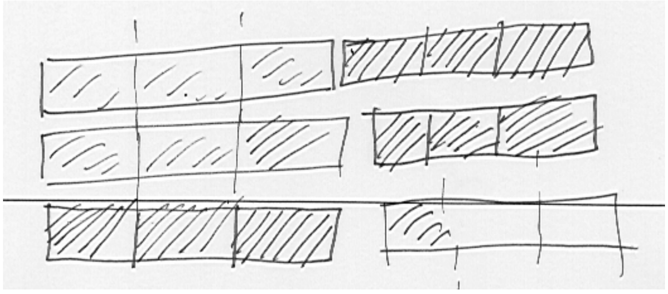


Figure 8.49 Mary Worksheet 20

Once the problems were based in a real-life context, there was a definite improvement in the number of correct solutions that were obtained.

Even though she continued to write the solutions as improper fractions in Worksheet 25, Mary demonstrated a remarkable improvement. She now indicated where the whole numbers were (Approach 1). She made one mistake and said seven thirds and not six thirds was a whole.

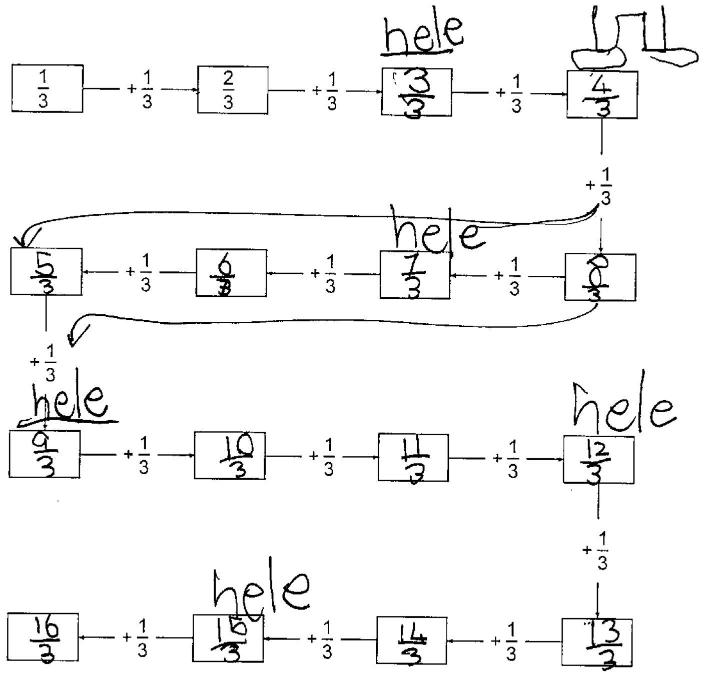


Figure 8.50 Mary Worksheet 25

Although Mary could not complete the chains abstractly she was able to complete them successfully when the problem was put into a real-life context. She showed signs of converting the improper fractions into mixed numbers (Approach 2).

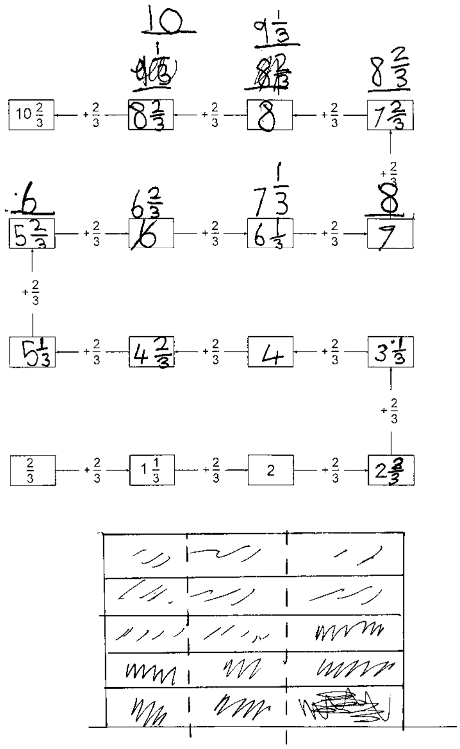


Figure 8.51 Mary Worksheet 26

Mary worked out all the solutions and initially wrote them down as improper fractions. She then converted them into mixed numbers.

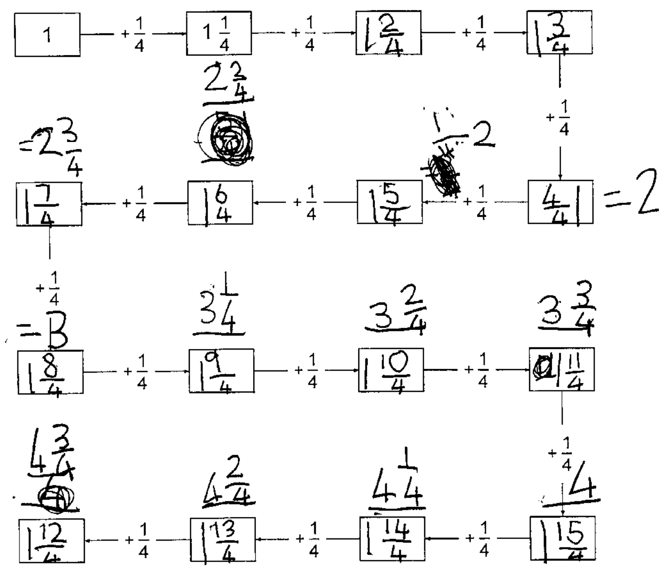


Figure 8.52 Mary Worksheet 30

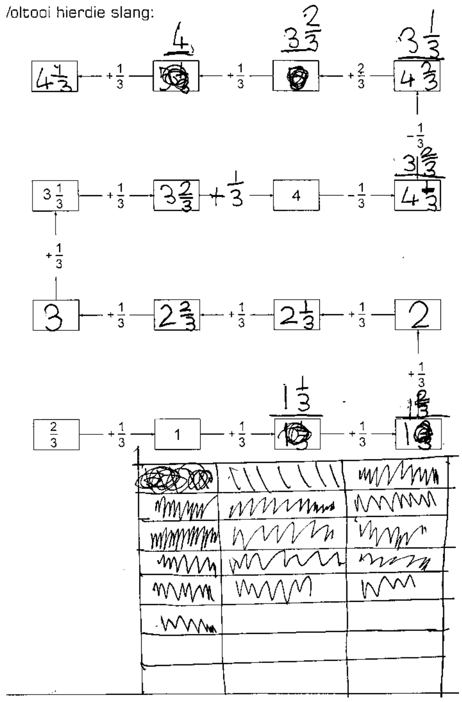


Figure 8.53 Mary Worksheet31

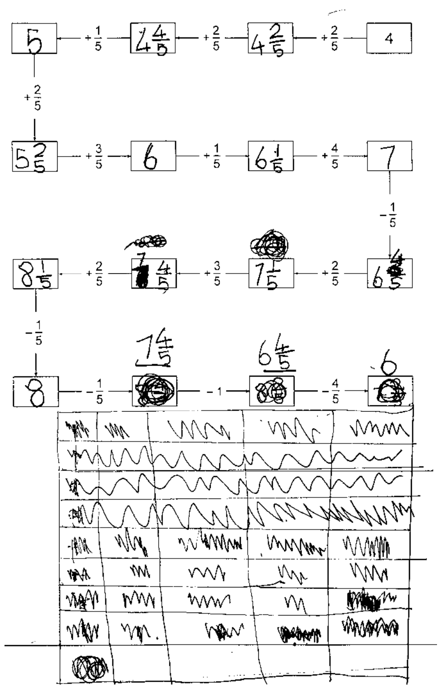


Figure 8.54 Mary Worksheet 32

Mary still needed to work in a real-life context and she continued to draw her 'chocolate bars' to assist her to obtain the solutions. She made a few errors, but this was mainly due to the fact that she did not notice that the operation had changed. She was able to correct this.

Mary was able to fill in the fractional part that needed to be added or subtracted so as to find the solution of the problem.

Although Mary showed signs of improvement she was still not able to work on a completely abstract level by the end of the research. She was still on Level One, Phase Two of Development (Murray and Olivier, 1989).

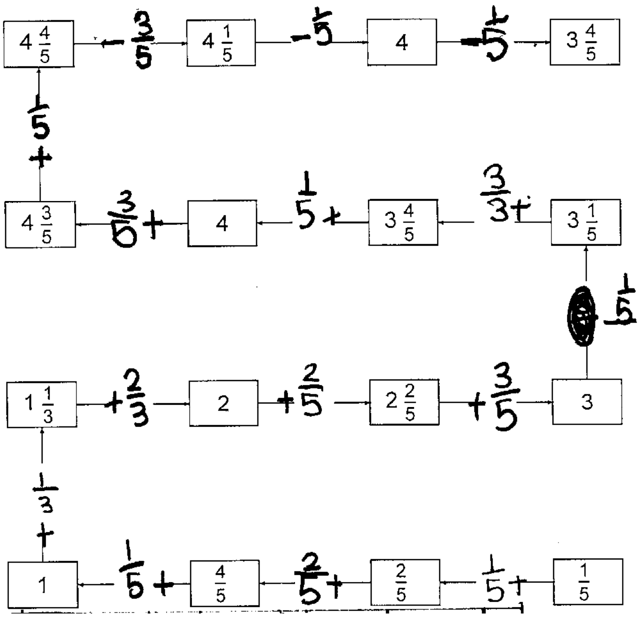
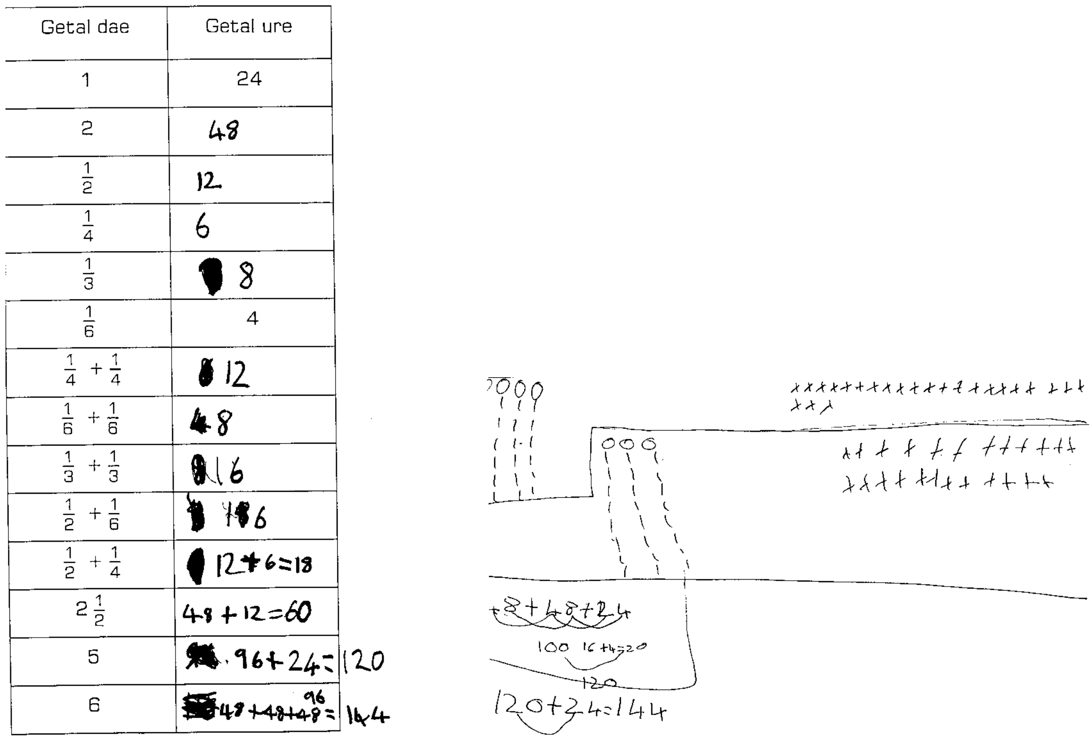


Figure 8.55 Mary Worksheet 33

Individual Study 10

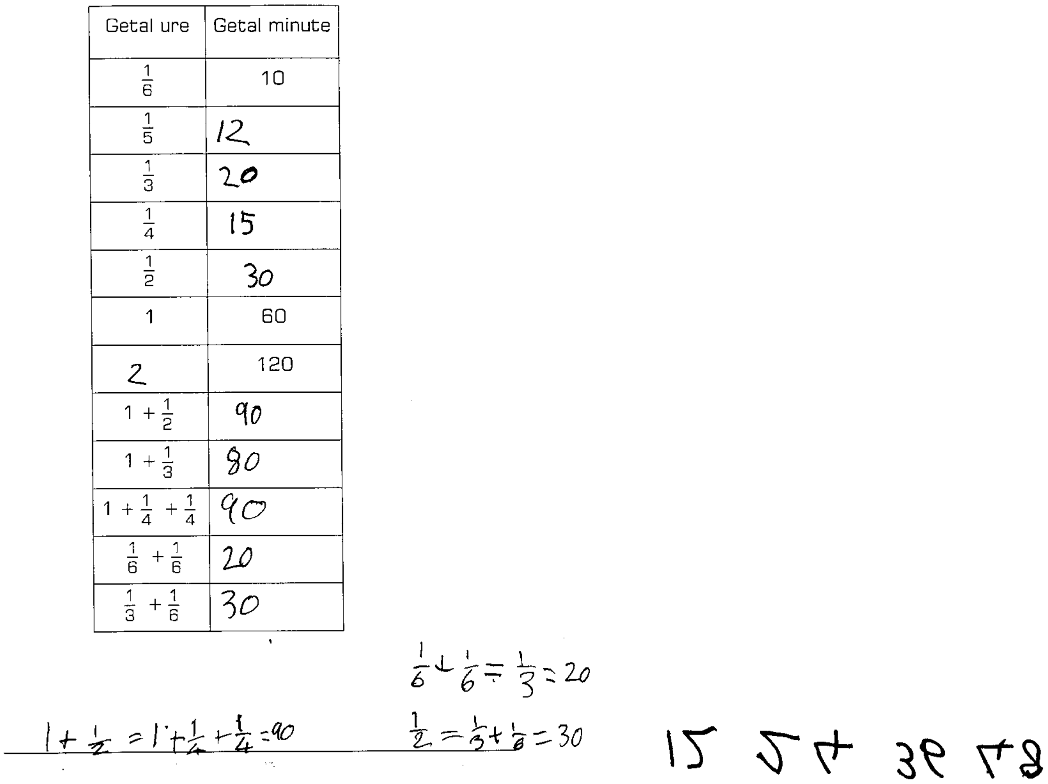
Yvonne was identified as above average learner in the mathematics class.



Getal dae	Getal ure
1	24
2	48
$\frac{1}{2}$	12
$\frac{1}{4}$	6
$\frac{1}{3}$	8
$\frac{1}{6}$	4
$\frac{1}{4}+\frac{1}{4}$	12
$\frac{1}{6}+\frac{1}{6}$	8
$\frac{1}{3}+\frac{1}{3}$	16
$\frac{1}{2}+\frac{1}{6}$	16
$\frac{1}{2}+\frac{1}{4}$	12+6=18
$2\frac{1}{2}$	48+12=60
5	96+24= 120
6	48+48+48= 144

Figure 8.56 Yvonne Worksheet 29

Yvonne was able to complete the table up to half of 24 (Approach 1). She was able to make use of her primitive knowledge that half of 24 is 12. Primitive knowledge is that knowledge that learners can recall immediately without conscious effort (Murray *et al*. 1992). To calculate a third and a quarter, Yvonne had to ‘share out’ the 24 (Approach 2). She made use of her previous solutions to assist her to complete the table.



Getal ure	Getal minute
$\frac{1}{6}$	10
$\frac{1}{5}$	12
$\frac{1}{3}$	20
$\frac{1}{4}$	15
$\frac{1}{2}$	30
1	60
2	120
$1 + \frac{1}{2}$	90
$1 + \frac{1}{3}$	80
$1 + \frac{1}{4} + \frac{1}{4}$	90
$\frac{1}{6} + \frac{1}{6}$	20
$\frac{1}{3} + \frac{1}{6}$	30

Figure 8.57 Yvonne Worksheet 27

Although the units of measure have been omitted from the calculation, it is evident that Yvonne knew what she was doing. The researcher did not insist on the units of measure being included. Had she done this, the number sentence would have been mathematically correct.

Yvonne filled in the total minutes without having to ‘share out’ the minutes. The researcher saw her writing 12, 24, 36, 48 on the back of the worksheet (Approach 3). On asking Yvonne what she was doing this for, she said that she thought a fifth was 12 and she was just making sure that if you counted 12 – five times she would get 60.

To answer the question: “Do you notice anything?” Yvonne noted:

$$1 + \frac{1}{2} = 1 + \frac{1}{4} + \frac{1}{4} = 90$$

$$\frac{1}{6} + \frac{1}{6} = \frac{1}{3} = 30$$

$$\frac{1}{2} = \frac{1}{3} + \frac{1}{6} = 30$$

Getal kilometers	Getal minute
1	10
$\frac{1}{2}$	5
$\frac{1}{5}$	2 [redacted]
2	20
$2\frac{1}{2}$	25
$1\frac{1}{2}$	15
$1\frac{1}{5}$	12 [redacted]
3	30
$3\frac{1}{2}$	35
$\frac{1}{2} + \frac{1}{5}$	7 [redacted]
$1\frac{1}{2} + \frac{1}{5}$	17 [redacted]
$1\frac{1}{5} + \frac{1}{2}$	17 [redacted]

Figure 8.58 Yvonne Worksheet 28

Yvonne completed this table abstractly (Approach 1).

8.3 SOME DEEPER UNDERSTANDING OF THE FRACTION CONCEPT

After Worksheet 7, the researcher had the following conversation with and Zane:

Teacher: *Elaine, you say the teacher must bring seven oranges. Zane, what do you say?*

Zane: *seven*

Teacher: *And what do you say Stewart?*

Stewart: *seven*

Teacher: *How did you work it out?*

Elaine: *I took 14 and divided it in two and got seven.*

Teacher: *O.K. How did you work it out Zane?*

Zane: *I drew people, and I drew half oranges and then two halves make a whole.*

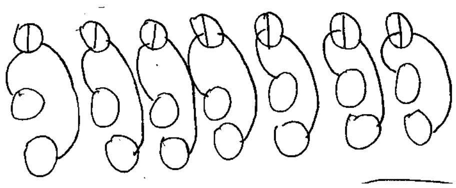
Teacher: *O.K.*

Zane: *And then there are seven.*

The researcher felt that 14 ÷ 2 was a primitive for both Zane and Elaine.

Sue was also able to solve the problem in this manner.

Sue: *I said because there are 14 players and I said that half of fourteen is seven and that is how I found my answer out.*



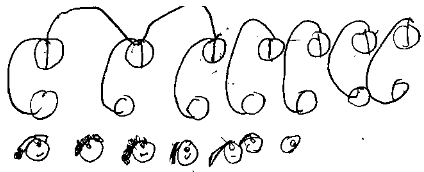


Figure 8.59 Mbulelo Worksheet 7 $(14 \div \frac{1}{2})$ **Figure 8.60** Tammy Worksheet 7

Teacher: *Mbulelo and Tammy, can you tell me how you did number one?*

Mbulelo: *Well, I drew 14*

Teacher: *14 what?*

Mbulelo: *14 players and then I brought seven oranges, then I said to myself each girl gets a half.*

Teacher: *mmmmmmm…*

Mbulelo: *Then I took the seven and cut them in half and I gave one half to all the 7 ones and there were seven halves left and I forgot to do the other seven players, so I drew the other seven and I took the other halves and I gave them to the other players.*

Teacher: *Right. Now, Mbulelo, how did you and Tammy know that the teacher had to bring seven oranges?*

Mbulelo: *It is because I know that there are seven players in each team and they each get a half so the teacher had to buy seven oranges because you actually cut a half. You do not give them the whole one.*

Teacher: *Do you agree with that Tammy?*

Tammy: *Yes.*

Teacher: *Thank you.*

The researcher asked Carol to explain how she found the total amount of baking powder for five cakes if two and a half teaspoons were needed for one cake. Carol wrote:

$$2\frac{1}{2} + 2\frac{1}{2} + 2\frac{1}{2} + 2\frac{1}{2} + 2\frac{1}{2}$$

She then took the two, two's from the first two terms and said:

2 + 2 = 4

She then took the two halves from the first two terms and said:

$$\frac{1}{2} + \frac{1}{2} = 1$$

She added: 4 + 1 = 5

She said: *And so I go on with all the others. I left the last two and a half out and added it to the 10 and so I got 12 and a half.*

Elaine and Zane explained their method as follows:

Teacher: *What did you get for your first answer, Zane?*

Zane: *One and a quarter cups of margarine.*

Elaine: *I also got one and a quarter cups.*

Teacher: *Now how did you work out that you must have one and a quarter cups?*

Zane: *Two quarters give you a half. Two quarters give you a half. A half and a half give you one and then there was another quarter.*

Teacher: *How many eggs do you need?*

Elaine: *five eggs. You need one egg for each cake that you must bake.*

Teacher: *How much cake flour do you need?*

Zane: *seven and a half.*

Teacher: *How did you work out the answer Zane?*

Zane: *Here is one and a half and there will be five.* Zane pointed to the 'ones'. He added together all the whole ones first. *There is a half and there is a half, so 2 halves give you one. And there are another two halves so that gives you seven and there is one half left over. That gives you seven and a half.*

Teacher: *Elaine, did you also work it out like that?*

Elaine: *Yes.*

Teacher: *Where are we now, at the sugar or at the salt? Salt*

Zane and Elaine: *One and a quarter*

Teacher: *How did you work it out?*

Elaine: *A quarter and a quarter gives you a half and then another quarter and quarter give you a half – that gives you one and then there is another quarter left over so altogether it is one and a quarter. It is the same as the margarine.*

Teacher: *O.K. Let us have a look at the sugar Zane.*

Zane: *Two and a half*

Teacher: *And, how did you work that out?*

Zane: *a half and a half is one – another one is two and then there is another half and that gives you two and a half.*

Teacher: *And now the milk?*

Zane: *Two and a half*

Teacher: *Elaine, did you also get that?*

Elaine: *I worked it out like the sugar.*

Teacher: *A half and a half gives you one.* And then we were interrupted. *Let us have a look at the baking powder. What did you get for the baking powder?*

Zane: *12 and a half.*

Teacher: *That's a lot of baking powder. How do we get so much?*

Elaine: *five times two is equal to 10 and then we put the halves together and get two and a half so that gives us 12 and a half.*

Teacher: *O.K. and the flavouring?*

Elaine: *two and a half*

Teacher: *Zane, how did you get that answer?*

Zane: *Just like the sugar, teacher. A half and another half is one. A half and another half is one. There is one half left over so that gives us two and a half.*

Teacher: *Thank you very much.*

Sue explained her method of working out the total amount of baking powder as follows:

For five cakes – 2, 4, 6, 8, 10 and you add a half – that is 10 and a half – add half and that is 11 – add half and that is 11 and a half – add half and that is twelve - add half and that is 12 and a half.

In Worksheet 8.1, Mary drew six-quarter cups of sugar and proceeded to circle two of the quarter cups together and called it a half. She then decided that she had three halves. She went on to say that a half and a half make a whole and there will be another half which you cannot add to anything (See Figure 8.42).

Zelda wrote down the following solutions:

3 cups of butter

6 wholes and 6 halves

2 whole cups

$\frac{6}{4}$ *cups of sugar*

6 teaspoons baking powder

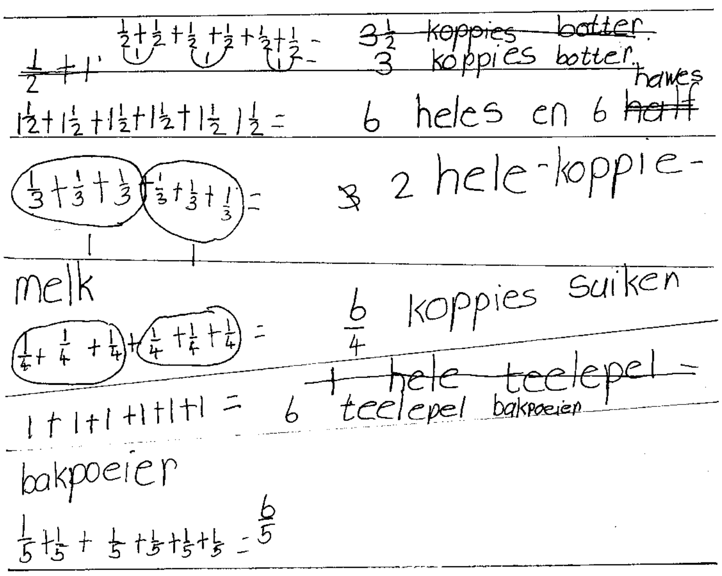


Figure 8.61 Zelda Worksheet 8.1

$$(\frac{1}{2}\times 6);\ (\frac{1}{4}\times 6);\ (1\frac{1}{2}\times 6);\ (1\times 6);\ (\frac{1}{3}\times 6);\ (\frac{1}{5}\times 6)$$

Zelda's method of solving the problems was very interesting. When working out the amount of butter, Zelda wrote down six halves. She then linked two halves together and wrote a one under each. She added up the ones and got three cups. When it came to adding up one and a half, six times, she wrote them down repeatedly showing the repetition process.

This extract was written in the researcher's journal:

> She came to me and said that she did not know what to do next. I asked her how many wholes she had and she said six. I then asked her how many halves she had and she said six. She was perfectly happy with the answer of six wholes and six halves. When I asked her if she could add the halves together she said no.

The researcher was amazed by the manner in which Sue notated her problem.

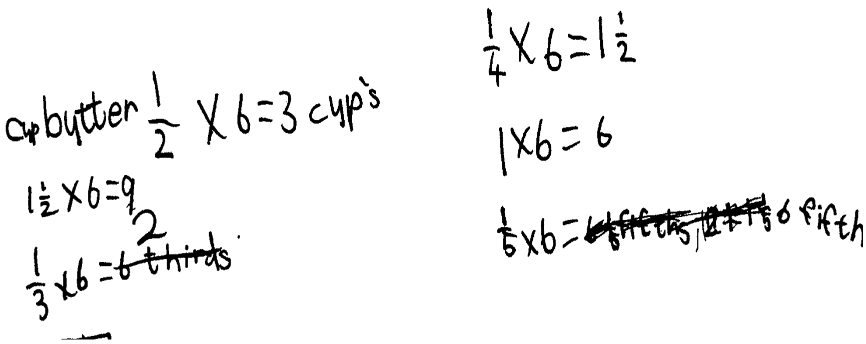


Figure 8.62 Sue Worksheet 8.1

$(\frac{1}{2}\times 6)$; $(\frac{1}{4}\times 6)$; $(1\frac{1}{2}\times 6)$; (1×6); $(\frac{1}{3}\times 6)$; $(\frac{1}{5}\times 6)$

She wrote:

$$\textit{butter} \quad \frac{1}{2} \times 6 = 3 \quad \textit{cups}$$

$$1\frac{1}{2} \times 6 = 9$$

$$\frac{1}{4} \times 6 = 1\frac{1}{2}$$

However, in a subsequent problem, the researcher asked Sue if six fifths could be written in another way and she said that she did not think so. It seems to the researcher that Sue did not understand the iterative meaning of the fraction concept

(See Section 2.5) and that her notation understanding goes beyond her conceptual understanding of the topic. Sue could have possibly worked out her solutions for the first five ingredients her whole number knowledge. She could have said that a half of six is three and that one and a half times six is equal to one times six which is six plus a half of six which is three, thus giving her nine. However, she was not able to do this for the last ingredient ($\frac{1}{5} \times 6$).

Caroline explained how she had solved the problems in Worksheet 21 during an interview with the researcher.

Teacher: *Come Caroline, let us see how you have worked your sums out today. Vuyo wants to know how much she has got left after a party – she has got three large packets of chips, each half full.*

Caroline: *I worked out that a half and a half is equal to one and there is another half left over and that is one and a half.*

Teacher: *O.K. And if she has five containers of ice cream with each having one quarter ice cream left in each, how much does she have?*

Caroline: *I drew five containers of ice cream and I drew lines to join them all and it is* one *and one quarter.*

Teacher: *Where does the one come from?*

Caroline: *The one comes from adding the first one right to the last one and that is where the one comes from.*

Teacher: *How many of the quarters do you have to put together to make that one.*

Caroline: *four*

Teacher: *Come let us have a look at the last one – two containers of milk each two thirds full?*

Caroline: *I drew two containers and I drew two thirds into the one and two thirds into the other and I took one third and put it into the other one, and that made it full, and there was one third left over. The answer is one and one third.*

Teacher: *Thank you very much.*

In Worksheet 29, when Carol and Yvonne had to solve one quarter plus one quarter they used the solution worked out above, namely that one quarter is equal to six hours, and simply added six twice to get 12 hours. They used this method to work out all the other problems. To work out the number of hours in five days, they doubled two days (48×2) and added another 24 hours. To work out six days, they added two days three times: $48 + 48 + 48$ to get 144.

At the end of the lesson, Worksheet 29, the learners were asked if they had seen anything interesting on the table. The learners noted that although the problems were different, the solutions were the same.

Once again, the learners did not indicate the units of measure, but had these been included, the number sentences would have made mathematical sense.

e.g.

$$\frac{1}{2} = 12 \quad \textit{and} \quad \frac{1}{4} + \frac{1}{4} = 12$$

$$\frac{1}{6} + \frac{1}{6} = 8 \quad \textit{and} \; \frac{1}{3} = 8$$

Zane had an interview after Worksheet 29:

Teacher: *Zane, let us have a look at how you did your work today.*

Zane: *The hours in one day, teacher is 24, and the hours in two days gives me 48 - you multiply the 24 by two because it is two days and that gives you 48. And a half a day- you divide that 24 by two and you get 12. And then one quarter is six because six times four gives me ………six times four gives me the answer of 24 and one third gives a person eight because eight times three is 24 and the one sixth - there stands a four but six times four gives me 24.*

A quarter and a quarter is the same as a half and a half is 12 and a quarter and a quarter is equal to a half and that is 12 and the sixth and one sixth is the same as one third and this is also eight hours and one *third and one third gives me 16 because six, three … three plus three gives me six……*

Teacher: *No, wait a bit. Let us do that one again. One third plus one third gives me 16 because one third is equal to ……*

Zane: *Eight and eight times two is equal to 16.*

Teacher: *O.K.*

Zane: *And one half and one sixth is the same as two thirds and this is also 16. And a half and a quarter gives me 18 and …..*

Teacher: *How do you get 18 here? One half is equal to …………how many hours?*

Zane: *12 and one quarter gives me six and six plus 12 = 18 and two and a half days : you count those two at the top.* Zane was pointing to the answer of how many hours in two days - 48 hours. That is 48.

Teacher: *mmmmm…*

Zane: *You add 12 and you get 60…*

Teacher: *Why do you add 12?*

Zane: *Because it is two and a half days : because here is the two days (*he was pointing to the 48) *and then you add a half so you add 12 on.*

Teacher: *O.K.*

Zane: *For the five days you take the two and a half days. Two times two is equal to four and one half and one half is equal to one and four plus one is equal to five so you take that 60 and you multiply it by two and you get 120. For the six days, you just put a day on that is 24 + 120 and this gives you 144.*

Teacher: *Thank you Zane.*

Linda, completed her problems on the table giving an alternative method, she noted:

$1+\frac{1}{2} = 1+\frac{1}{4}+\frac{1}{4}$ *and that* $\frac{1}{6}+\frac{1}{6}=\frac{1}{3}$ *and* $\frac{1}{2}=\frac{1}{3}+\frac{1}{6}$

During reflection time, learners explained their methods as follows:

Teacher:	*If I walk one kilometre it takes me 10 minutes, Kate, can you tell me how long it will take if you walk one half kilometre?*
Kate:	*five*
Teacher:	*And how do you get the answer of five?*
Kate:	*Because half of 10 is five.*
Teacher:	*And if you walk one fifth of that kilometre, Yvonne, how long will it take you?*
Yvonne:	*two*
Teacher:	*And how do you get the answer to being two?*
Yvonne:	*two plus two plus two plus two plus two gives me 10 and I added two five times.*

Other methods that were used here were grouping in twos and sharing.

Teacher:	*You added two together five times Ok good. And let us go further: if you walk two kilometres, Kate, how long is it going to take you?*
Kate:	*20*
Teacher:	*And how to you get that 20?*
Kate:	*10 + 10 = 20*
Teacher:	*Ok good. Come, let us have a look at two and a half. If I walk two and a half kilometre, Martin, how long is it going to take us?*
Martin:	*25 minute because 10 + 10 + 5 = 25*
Teacher:	*Where does the 10 come from?*
Martin:	*From the one kilometre.*
Teacher:	*And the five?*
Martin:	*From the half.*
Teacher:	*Linda, did you do that sum in exactly the same way?*
Linda:	*No, Teacher. I did 20 plus five.*
Teacher:	*Where does the 20 come from?*
Linda:	*From the two Teacher.*
Teacher:	*From the two kilometre. Ok. Let us go on : one and a half kilometre will take how long : Cheryl?*
Cheryl:	*15*
Teacher:	*15 and how did you get that 15?*
Cheryl:	*I took one kilometre which is 10 and a half which is five.*

Teacher: *Let us go on. One and one fifth: Carol, what is the answer there?*

Carol: *12*

Teacher: *How do you get 12?*

Carol: *I looked at the one kilometre and got 10 and looked at the one fifth and got 12 and I added them together.*

Teacher: *Elaine says that three and a half gives you 35 minutes. How did you get this?*

Elaine: *I looked at the three and saw that it was 30 and I looked at the half and it was five.*

Teacher: *Ok. Very good. One half and one fifth give you how much, Yvonne?*

Yvonne: *Seven.*

Teacher: *How do you get seven?*

Yvonne: *One half is equal to five and one fifth is equal to two and I added to get seven.*

Teacher: *One and one half plus one fifth. Stewart, what is your answer there?*

Stewart: *15*

Teacher: *No, not 15*

Stewart: *Seven*

Teacher: *Noooo……*

Stewart: *17*

Teacher: *How do you get 17?*

Stewart: *I looked up to the answer of one and a half and I saw that it was 15 and I wrote 15 down and then I went and looked up again and saw that one fifth is equal to two and I wrote down the two and I added.*

Teacher: *And the last one : one and one fifth plus a half gives us……Zelda?*

Zelda: *17*

Teacher: *How do you get 17?*

Zelda: *One and one fifth is equal to twelve and one half is equal to five and that is 17.*

Teacher: *Do you notice anything?*

Class: *Yes*

Teacher: *What do you notice?*

Class: *One and a half plus one fifth is equal to one and one fifth plus a half. It is just written the other way around.*

Teacher: *In other words one and a half plus one fifth is equal to one and one fifth plus a half. Thank you very much.*

8.4 SUMMARY

On two occasions, learners at both schools commented on how difficult the problems dealing with this concept of adding fractions with unlike denominators was. It took longer to develop than others and it was necessary for the researcher to complete additional worksheets and also to use manipulatives.

Using manipulatives introduced other difficulties. Learners at both schools were not used to having manipulatives to work with and it took them some time to get used to being 'free' to experiment with the apparatus as well as becoming familiar with the apparatus (Measuring cups, teaspoons and rice). The learners at both schools took some time to become acquainted with the whole, half, quarter and third – size cups.
All, but two learners, used the manipulatives. This placed the learners on Level One, Phase One of Development (Murray and Olivier, 1989). The researcher felt that this was not a true reflection of most of the learners' level of development. She is of the opinion that the learners felt that because manipulatives were available and something out of the ordinary, they needed to be used. The researcher did not supply manipulatives for the second intervention and there were fewer learners with the incorrect solutions at School A.

In the first worksheet where the manipulatives were used, learners simply wrote the solution to the problem. When the manipulatives were not available, learners had to draw the solution. Drawing the solutions took the learners one step closer to abstract representation through mental imagery.

Many of the learners were trying to solve the problem on Level Two/Three (abstractly), whereas some of them were still on Level One, Phase Two of Development (Murray and Olivier, 1989) and needed to draw their solutions. This

could have contributed to the fact that learners were finding the problems difficult. On the other hand, some of the learners included drawings in their solutions, but it was felt that some of these drawings were not used to solve the problem, but added in, merely as a method of explaining their reasoning.
As stated earlier, the chains were not suitable for four or five of the learners as it required them to work abstractly at Level Three of Development (Murray and Olivier, 1989). They were not able to work abstractly and needed to be exposed to more ground level activities (as outlined by Van Hiele) before attempting the chains (See Section 2.3.3).

Most learners worked at Level Two or Three of Development when working on the problems in Worksheets 27, 28 and 29 (Murray and Olivier, 1989). Some learners still found it necessary to draw their 'sharing out', placing them on Level One, Phase Two of Development. The researcher noted that most of the learners could fill in the tables using abstract reasoning when the fractional part involved was a half or a quarter. As soon as other fractional parts were required, they needed to draw and solve the problem using 'sharing out' situations.

Continuous problems with Martin and Stewart were encountered through out the research. They would hear the solution being discussed by other groups and then would simply copy down the solution without understanding the method employed. Because so many learners were absent when Worksheet 29 was being done, Zane joined these two boys. When interviewing them after the lesson, they could explain exactly how they had solved the problems. In this instance, a learner of higher ability was beneficial to learners of weaker ability.

It was noted that some learners, who were able to write down the solutions in Worksheet 29, had to 'share out' and draw pictures when solving the problems on Worksheet 27. This could be, once again, due to the problem of larger numbers.

By the end of the three worksheets, the learners had refined their methods and were able to complete the tables successfully by using previous solutions to assist them to find the subsequent solution.

It was found that, by the end of the research, the majority of learners were able to iterate fractional parts to form a whole. Learners were able to add fractional parts with like denominators. Some learners were able to function at an abstract level, while others still needed the problem to be embedded in a real-life context.

Learners used previous solutions to assist them to find subsequent solutions when they added fractions with unlike denominators. At the end of the third worksheet, all learners were able to write down the solutions only.

D' Ambrosio and Mewborn (1994) and Murray *et al*. (1996a) identified two sub-constructs needed for mature rational number functioning, the one being the part-whole relationship between the fractional part and the unit where the unit can be a single object or a number of objects (See Section 4.2). The final chapter on data collection discusses the methods learners used to solve these problems where the unit consists of a number of objects.

CHAPTER 9

FRACTIONS AS PART OF A NUMBER OF OBJECTS

Learners need to be introduced at an early stage to fractions as part of a collection of objects. This concept was initially introduced to the learners in Worksheet 3 and again in Worksheets 18, 22 and 23. The researcher has however discussed the methods used in Worksheet 3 under the section of comparison and equivalence of fractions (See Chapter 6).

9.1 ANALYSIS OF DATA

The following problems were posed to the learners:

<u>Worksheet 18</u>

See how far you can get with this table. You can do the easy ones first if you want to.

A half of R1

A third of R1

A fourth of R1

A fifth of R1

A sixth of R1

A seventh of R1

An eighth of R1

A ninth of R1

A tenth of R1

A twentieth of R1

A fiftieth of R1

A hundredth of R1

<u>Additional worksheet 18.1</u> (This worksheet was included as some of the learners were having difficulty with the large numbers in Worksheet 18)

See how far you can get with this table. You can do the easy ones first if you want to.

A half of 30c

A third of 30c

A fourth of 20c

A fifth of 20c

A sixth of 30c

A seventh of 14c

An eighth of 40c

A ninth of 27c

A tenth of 30c

<u>Worksheet 22</u> (As some of the learners were still having difficulty working with large numbers, the researcher retained the problem structure for all learners and changed the numbers substituting smaller numbers for these learners. These numbers are indicated in brackets.)

Vusi's book has 88 (44) pages. He says: "I have read more than half of the book. I am on page 41(21)." Is it true? Explain.

Vuyo has 27 sweets. She gives 10 sweets to Peter. She says: "Peter, I have given you a third of my sweets." Is it true? Explain.

<u>Worksheet 23</u>

Sam promises to give a tenth of his peaches to the school. He has 90 (30) trays of peaches. How many trays must he give the school?

Simba gives $\frac{1}{3}$ of a box of chips to Dodi's class. If there are 90 (30) packets in a box, how many packets of chips does the class get?

In Worksheet 18, of the 12 problems, Yvonne and Carol were the only two learners who were able to complete it successfully. Elaine and Zane completed eight successfully and although Martin and Stewart completed eight, they had four incorrect. Linda completed seven, Zelda six, Kate and Mary three and Caroline and Cheryl two. The learners did not attempt to draw solutions. They simply wrote down their solution.

It was difficult to identify the approaches used in all the worksheets and for this reason, each worksheet has been analysed separately. In Worksheet 18.1, the researcher could identify four approaches used.

Approach 1

The learners wrote down the solution only.

Elaine explains how she thought (one sixth of 30):

Teacher: *Elaine, can you tell me how you worked out one sixth of 30c?*

Elaine: *I thought about the times tables and I decided that if I could use the times tables to work out the answers.*

Teacher: *With the times tables? O.K. Tell me how you used the times tables to work out the answer.*

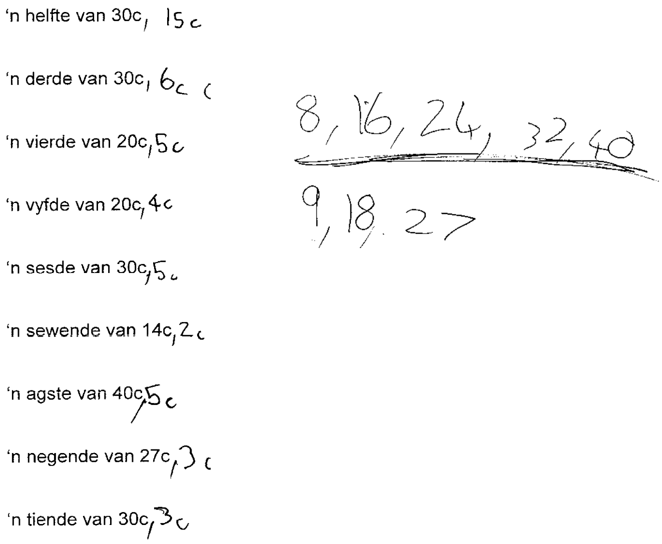
Elaine: *I stuck out my fingers and I said 6; 12; 18; 24; 30 – I used five fingers.*

Teacher: *O.K. Very good. Thank you.*

Approach 2

The learners did accelerated counting (See Section 2.3.2).

When they were uncertain of their solutions, the learners would count for example in eighths or thirds and then count how many ‘groups’ they had counted. Learners were required to find: a half of 30c; a third of 30c; a fourth of 20c; a fifth of 20c; a sixth of 30c; a seventh of 14c; an eighth of 40c; a ninth of 27c and a tenth of 30c.


'n helfte van 30c, 15c

'n derde van 30c, 6c c

'n vierde van 20c, 5c

'n vyfde van 20c, 4c

'n sesde van 30c, 5c

'n sewende van 14c, 2c

'n agste van 40c, 5c

'n negende van 27c, 3c

'n tiende van 30c, 3c

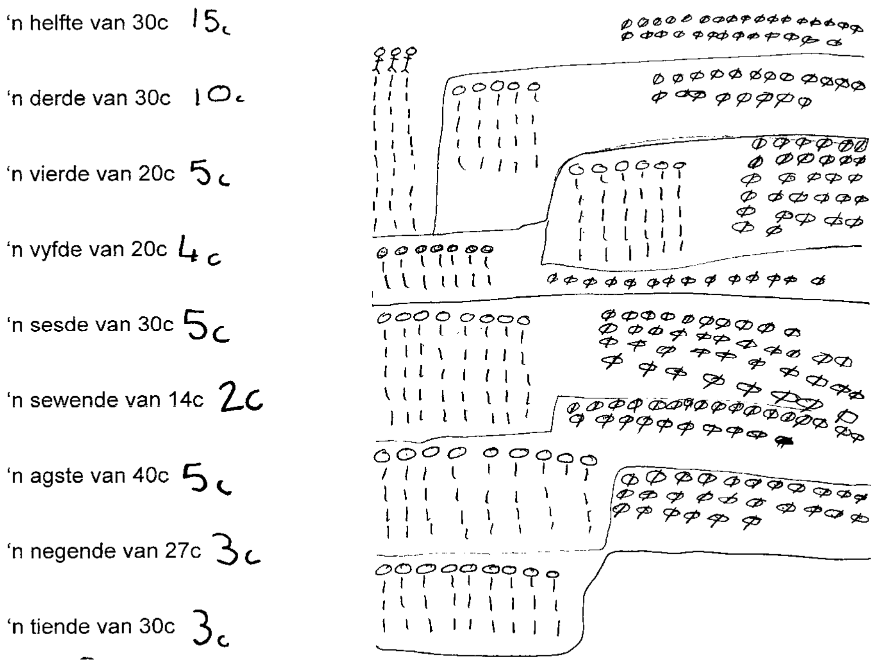
8, 16, 24, 32, 40

9, 18, 27

Figure 9.1 Linda Worksheet 18.1

Approach 3

The learners used economic sharing (shared out the maximum number of cents) and also drew their solutions. Learners were required to find: a half of 30c; a third of 30c; a fourth of 20c; a fifth of 20c; a sixth of 30c; a seventh of 14c; an eighth of 40c; a ninth of 27c and a tenth of 30c.


'n helfte van 30c 15c

'n derde van 30c 10c

'n vierde van 20c 5c

'n vyfde van 20c 4c

'n sesde van 30c 5c

'n sewende van 14c 2c

'n agste van 40c 5c

'n negende van 27c 3c

'n tiende van 30c 3c

Figure 9.2 Cheryl Worksheet 18.1

Cheryl and Caroline explained their method as follows (a seventh of 14c):

Teacher: *Cheryl and Caroline, can you tell me how you worked out the answer to one seventh of 14c?*

Cheryl: *We drew nine people.*

Teacher: *Why nine people?*

Cheryl: *Because there are nine and 14. And we drew 14c.* Confused and silence! *No, seven people.*

Teacher: *You drew seven people.*

Cheryl: *And we made 14c and then I went and shared them out and I crossed out, and shared one and crossed one out and shared one and so we went on and then we got out answer and it is five.*

Teacher: *Is your answer five?*

Cheryl: *Yes.*

Teacher: *Are you sure?*

Cheryl: *No. The answer is two.*

Teacher: *The answer is two. Come let us have a look at your drawing – every child got two. One child, two children, three, four, five, six, seven. Yes we made a mistake it is this drawing hey? Each child got……*

Cheryl and Caroline: Two

Approach 4

The learners used manipulatives to find the solution and then wrote down the solution.

Mary explained how she used the manipulatives (a quarter of 20c):

Teacher: *Can you tell me Mary how you worked out one quarter of 20c?*

Mary: *I had, I had four of those counters and I packed them – and then put four under them again until I had packed out 20. And when it was 20, I counted down the row to see how many there were in the group. I counted one, two, three, four, five. And my answer was five.*

Teacher: *Why did you count the number in one row?*

Mary: *Because if I count the other groups I am going to get the same answer.*

Teacher: *In other words, you want to work out a quarter and one group represents a …………*

Mary: *Fifth*

Teacher: *no, not a* fifth ……

Mary: *One quarter.*

Appendix G1 summarises the approaches the learners used to solve the problems.

For problem number 1 on Worksheet 22, three approaches could be identified.

Approach 1

The learners wrote down the solution.

Kate wrote down the following: ‘*No it is not the answer because the number 21 is too early in the book.’*

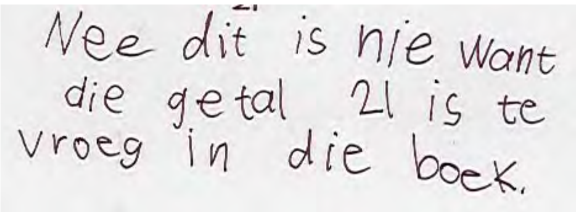


Figure 9.3 Kate Worksheet 22 no 1 ($\frac{1}{2}$ *of* 44)

Approach 2

Learners used their primitives. A primitive can be defined as previously gained knowledge that can be recalled without conscious effort (See Murray *et al*., 1992)

Yvonne explained: *22 + 22 = 44 not 21 + 22 because that is 43.*

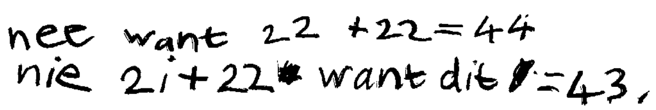


Figure 9.4 Yvonne Worksheet 22 no 1 ($\frac{1}{2}$ *of* 44)

Approach 3

The learners halved the number.

Linda solved her problem as follows:

Half of 44 is?

40 → 20

4 → 2

He is a page behind the middle.

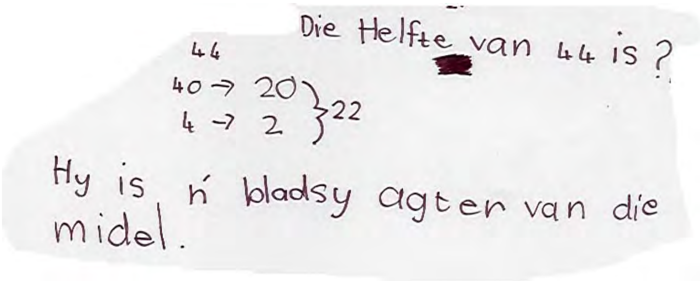


Figure 9.5 Linda Worksheet 22 no 1 ($\frac{1}{2}$ *of* 44)

Appendix G2 summarises the approaches that learners used in Worksheet 22 number 1.

For Number 2, four approaches were identified.

Approach 1

Only the solution was given.

Approach 2

The learners used their primitives (See Worksheet 22, Approach 2).

Elaine wrote down the following: '*No, not true, false, 9 is the third of 27 and not ten.*

9 × 3 = 27

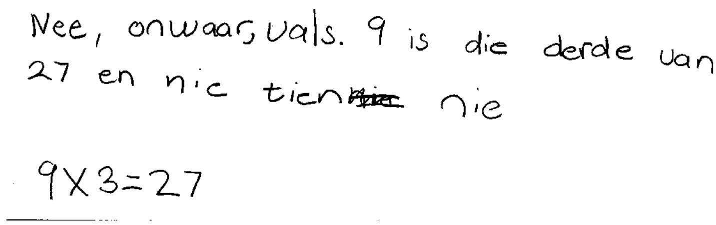


Figure 9.6 Elaine Worksheet 22 no 2 ($\frac{1}{9}$ *of* 27)

Approach 3

The learners used economic sharing (shared out the maximum number of sweets) and drew the solution.

Next to the drawing, Linda wrote: '*one third is equal to* *9'*

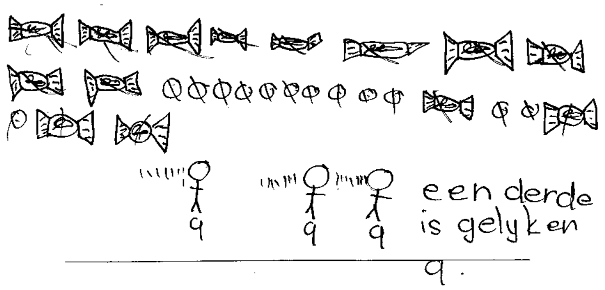


Figure 9.7 Linda Worksheet 22 no 2 ($\frac{1}{9}$ *of* 27)

Approach 4

The learners grouped the sweets and drew the solution.

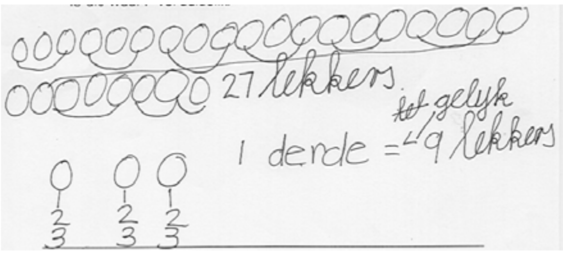


Figure 9.8 Zelda Worksheet 22 no 2 ($\frac{1}{9}$ *of* 27)

Here is Zelda's explanation of how she solved the problem:

Teacher: *Zelda, please won't you explain to me how you worked out that a third of 27 sweets is nine.*

Zelda: *I drew 27 sweets and then I drew three people. And then I gave one, one, to each group and so I went on and then I had 27 and then each group got three. And altogether the number of sweets that I gave them was three.*

Teacher: *O.K. Because you gave them a group of three sweets. Thank you.*

Zelda's explanation was not very clear. She tried to explain that each group received three groups of three and that one third of 27 sweets was nine.

The summary of the approaches the learners used can be found in Appendix G3.

The categories identified in Worksheet 23 differed slightly from Worksheet 22. No learner used primitives and two learners obtained incorrect solutions.

Approach 1

Only the solution was given.

Approach 2

The learners used economic sharing (shared out the maximum number of sweets) and drew the solution.

Caroline stated that, '*each one gets 10*'.

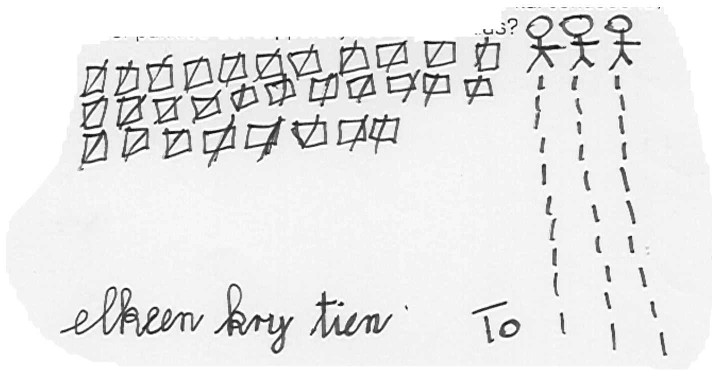


Figure 9.9 Caroline Worksheet 23 no 2 ($\frac{1}{3}$ *of* 30)

Approach 3

The learners grouped the sweets by counting in 3's (accelerated counting) and drew the solution.

Yvonne's solution was,

'*3 packets'*

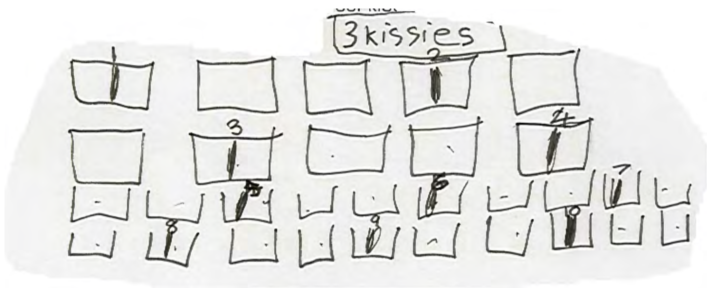


Figure 9.10 Yvonne Worksheet 23 no ($\frac{1}{10}$ *of* 30)

Yvonne explained as follows:

Teacher: *Yvonne, you worked with other numbers, how did you work out a tenth of 30?*

Yvonne: *Teacher, I said that 30 divided by three is equal to 10 and then every time I counted in groups of three.* She had drawn all 30 trays of peaches and marked them off. *I marked them off at the 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. I counted them all and got ten.*

Teacher: *Yvonne, how did you know that you must group three of the trays of peaches together?*

Yvonne: *Because three times 10 is equal to 30.*

Teacher: *And if I said to you there were 40 trays….?*

Yvonne: *Then I would have made it four.*

Teacher: *O.K. Thank you very much.*

Approach 4

Learners obtained incorrect solutions. One learner drew 13 rectangles for his solution, while the other learner drew 10. The researcher is of the opinion that the learners did not understand the context within which this problem was placed. Had the context been changed to one the learners were familiar with, the researcher is of the opinion that they would have been able to solve the problem.

The approaches the learners used are summarised on a grid in Appendix G4.

9.2 COGNITIVE DEVELOPMENT OF INDIVIDUAL LEARNERS

Individual Study 11

Zane, of the stronger ability group, was able to write down the solutions for the problems on Worksheet 18 and 18.1 (Approach 1). For number 1 on Worksheet 22, Zane was able to use a primitive of his (Approach 2). He knew that half of 88 was 44.

Zane obtained the solution of nine. He wrote down '*a third of 27*' and because he thought there were so many sweets, he wrote down, '*Sweets for Africa*'.

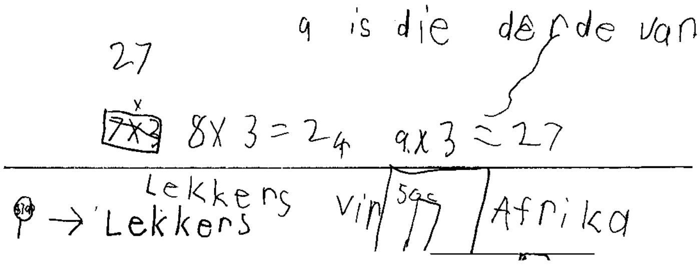


Figure 9.11 Zane Worksheet 22 no 2 ($\frac{1}{3}$ *of* 27)

In an interview with Zane, he explained his thinking:

Teacher: *Zane, let us have a look to see how you did the second sum.*

Zane: *I started with 27 sweets. She gives 10. It can't be a third. Vuyo has 27 sweets and he gives Pieter 10. She says that she has given him one third of the sweets. Is it true? No because three times eight gives me 24 and so I added three on and I got nine …………I got 27 and I saw that it is nine.*

Teacher: *So, in other words, a third of 27 sweets is how much?*

Zane: *Nine.*

Teacher: *O.K. Thank you very much see.*

For Worksheet 23, Zane again merely wrote down his solution (Approach 1). He and Elaine, his partner, used their knowledge of the multiplication tables to solve the problems.

Individual Study 12

Zelda was previously identified as having average mathematical ability. In Worksheet 18, she managed to answer 6 of the problems successfully. No method was shown (Approach 1).

Learners had to solve the following: a half of 30c; a third of 30c; a fourth of 20c; a fifth of 20c; a sixth of 30c; a seventh of 14c; an eighth of 40c; a ninth of 27c and a tenth of 30c

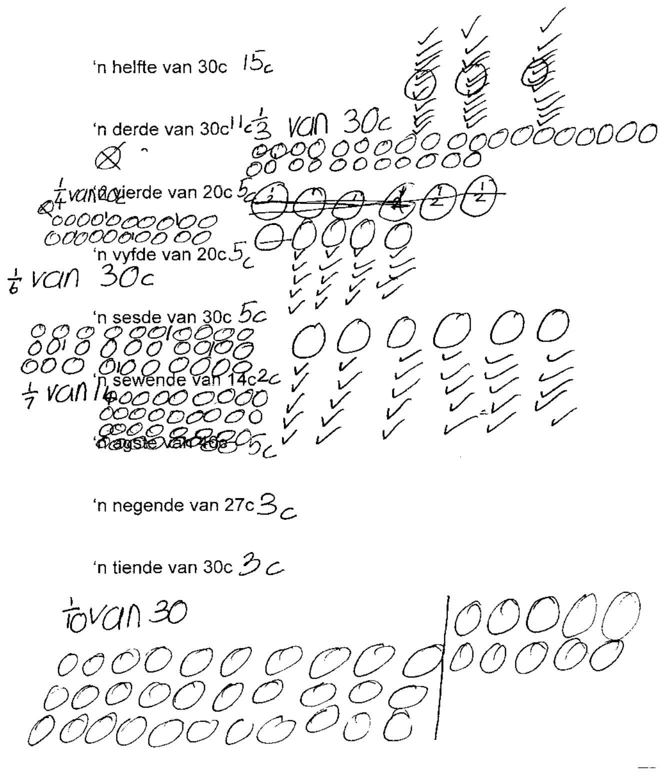


Figure 9.12 Zelda Worksheet 18.1

Zelda solved all the problems on Worksheet 18.1 using economic sharing (Approach 3). However it seems as if a half of 30 = 15 is a primitive of hers as there is no explanation for this problem (Approach 2). For the other problems, she drew her total number of cents and as she did in previous worksheets, she systematically ticked off the ones she had shared. She counted how many ticks in one group and this gave her the answer to her solution.

Zelda worked out the problem as follows:

Half of 44.

40 4

↓ ↓

20 2

22

Then he is on page 22.

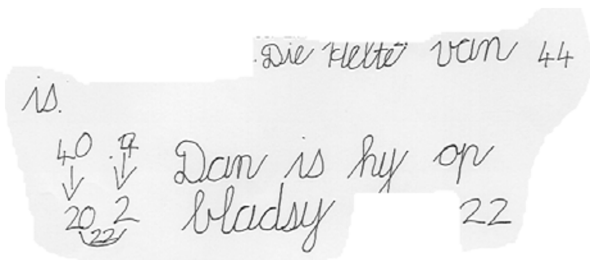


Figure 9.13 Zelda Worksheet 22 no 1 ($\frac{1}{2}$ *of* 44)

On Worksheet 22, Zelda used her whole number concept to help her solve 'a half of 44'. She halved 40 and halved four and added the solutions together (Approach 3).

For the second sum, she drew 27 sweets. She grouped them into groups of three and discovered that she had nine groups (Approach 4). She then indicated that one third was equal to nine. Her solution was '*27 sweets. 1 third = 9 sweets*'.

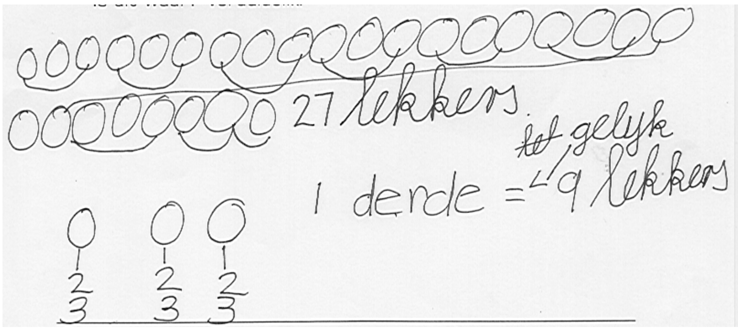


Figure 9.14 Zelda Worksheet 22 no 2 ($\frac{1}{3}$ *of* 27)

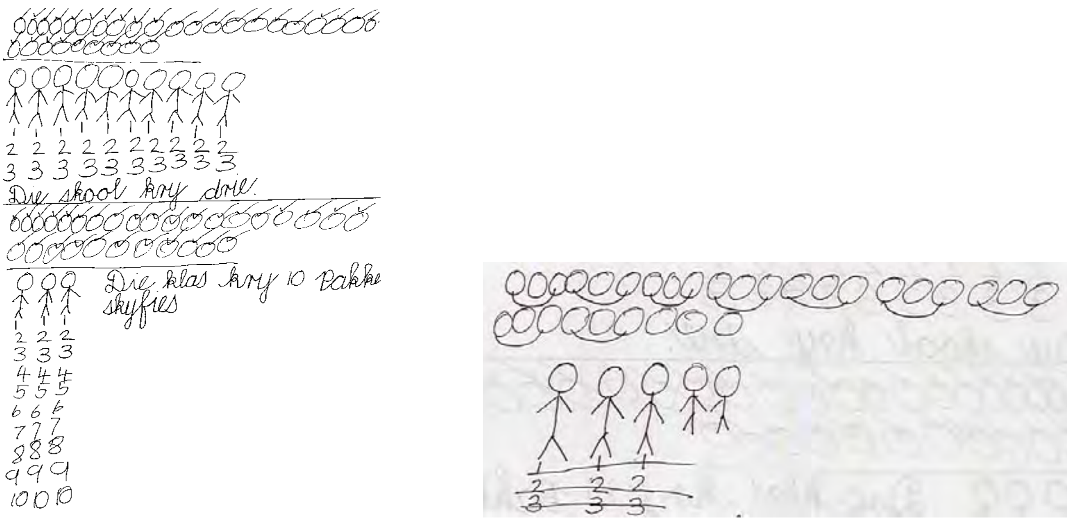


Figure 9.15 Zelda Worksheet 23 no1 ($\frac{1}{10}$ *of* 30) and ($\frac{1}{3}$ *of* 30)

For the first problem, Zelda worked out that, '*The school gets three*', and for the second problem, '*The class gets 10 packets of chips*'. Zelda attempted to solve the problem using the same method that she had used for Worksheet 22. She was working it out correctly, but changed her mind and solved it by sharing (Approach 3).

9.3 SOME DEEPER UNDERSTANDINGS OF THE FRACTION CONCEPT

Elaine shared with the class how she worked out what the value of a quarter of R1 was. She explained that a half of a half is a quarter and if you take 50c and halve it, you will get 25c, which is a quarter.

Half way through Worksheet 18, Zane commented: *"Oh, these are getting easier"*.

Elaine explained how she worked out the solution to the problems on Worksheet 23 $(\frac{1}{10}\ \textit{of}\ 90)$:

Teacher: *Come, Elaine, how did you work out how many boxes of peaches needed to be dropped off at the school?*

Elaine: *I said that we had to divide them up into tenths and I said 90 divided by 10 and we got nine. I thought 10 times nine equals 90.*

Teacher: *O.K. very good and let us have a look at how we do the second one.*

Elaine: *I took 90 and divided it by three and I got 30. I then double-checked and said that three times 30 is equal to 90.*

Teacher: *Why did you divide by three?*

Elaine: *Because I wanted to work out a third*

Teacher: *Thank you.*

9.4 SUMMARY

At the beginning of each lesson, the context of the problems was discussed with the learners. Throughout the intervention, learners could identify with the real-life contexts used. The only problem experienced was with the context in Worksheet 23, number 1. Learners were required to solve a problem about the number of trays of peaches. The learners were not familiar with trays of peaches and because of this, they asked questions such as the following: "How many peaches are in a tray?" and "What is a tray of peaches?". Being a citrus growing area, it might have been more appropriate to ask the learners how many pockets of oranges would the school get instead of peaches.

The second problem in Worksheet 23 was of a similar problem structure involving boxes of chips that needed to be shared. Because of the problems that learners had in identifying with the context, the researcher advised the learners to attempt problem two first. They had no problem solving number two and by solving this problem first, they were able to find a method that they then employed to solve the first problem. These findings concur with the findings of Mack (1990), Ball (1993), Piel and Green (1994) and Menon (1995) that state that the context plays an important role in conceptualising the problem.

Ball (1993), Menon (1995) and Neuman (1993) and have documented that the physical context in which initial fraction concepts develop is significant to the development of the fraction concept (See Sections 4.2.2 and 2.10). It is important that the learners can identify with the context of the word problem.

The environment and activities that surround the learners influences the learners' performance in class. At School A, the Foundation Phase learners' were involved in a concert and when the researcher arrived for the lesson, the learners were still practising in the school hall. When they eventually came to class, they were very distracted. Unfortunately, the concert practises continuously interrupted the development of this concept. As a result, many of the lessons were disrupted and not very successful. Another contributing factor could have been that the numbers used for the problems were too large for the learners' stage of number development.

The researcher found, time and again, that the learners' other mathematical development hindered the research. It became evident by cues from the learners that the teacher was using 'traditional mathematics' teaching methods. She indicated this during an interview that was conducted with her by the researcher. Her teaching methods included rote learning of bonds and multiplication tables. Rules and mechanical procedures were being drilled and implemented until they were mastered. Learners interacted with the teacher and not with each other. This influenced the learning process during the research.

The researcher found that the learners had not done any division with whole numbers and this influenced the success of these particular lessons. As stated in Chapter 4, Individual Study 2, the worksheets used to develop the common fraction concepts were not developed in isolation and they are dependent on the conceptual mathematical development of the whole number arithmetic.

Martin and Stewart, two of the weaker ability group, never truly managed to develop the correct culture as far as copying from others was concerned. They definitely copied the solutions to some of the problems and on many occasions could not explain their methods.

Due to the disruptions, the large numbers used and the disaster experienced with Worksheet 18, the researcher felt that it was in the best interest of the learners to do another worksheet with a similar problem structure, smaller numbers and manipulatives. This was done in the form of Worksheet 18.1.

The following lesson began with the researcher showing the learners, by using manipulatives, how to solve a problem such as a half of 20. Questions such as the following were posed to the learners: How many groups will we have if we divide something into two equal halves? Let us share out the 20. How many do we have in one group? (10) What fraction does the 10 represent? (It represents a half of the 20). These steps were repeated with one fifth of 20, a quarter of 20, one third of 15 and so on.

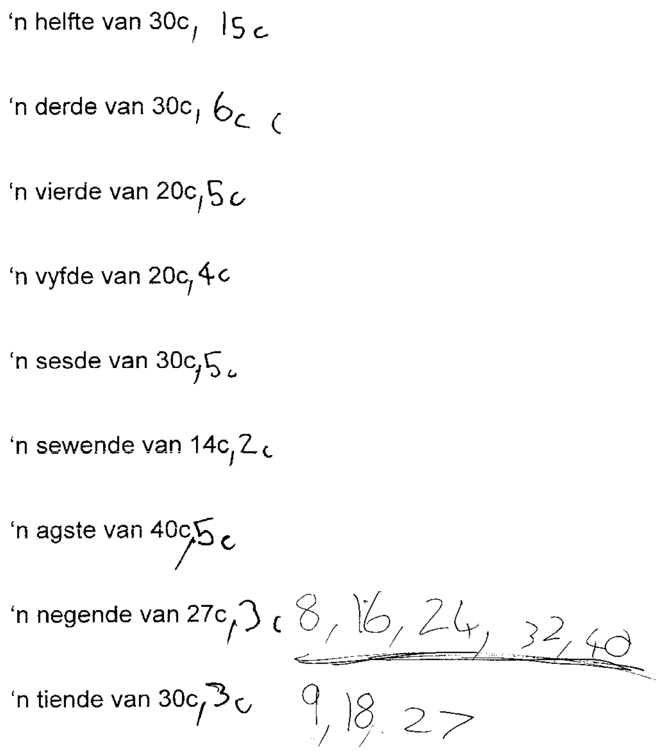
Some learners indicated that they were not too excited about doing the same kind of problems as the time before, but, once they had started their work, they all managed better than on Worksheet 18. The researcher attributed this to various factors:

- The numbers were within most of the learners' number range. Kate, Mary, Martin and Stewart still found some of the numbers to be too large (Martin could not tell me how 30 could be written in another way e.g. 10 + 20).
- It was the second time this problem structure had been met.
- Manipulatives were available if needed.
- Learners could have gained knowledge from the previous lesson and the reflection that had taken place.

The number range was obviously more suitable to the class than the previous material.

The Murray and Olivier's (1989), Levels of Development were very evident in Worksheet 18.1. Kate, Mary, Martin and Stewart needed to work with manipulatives and this placed them on Level One, Phase One. Yvonne, Carol and Linda started off with manipulatives, but later worked without them. They moved from Level One, Phase One to Level One, Phase Two during the lesson.

Linda, who started off using manipulatives (Level One, Phase One) developed to Level Two of Development in one lesson. She began using accelerated counting on to solve the problems. An example of her work is shown in the figure below.


'n helfte van 30c, 15c

'n derde van 30c, 6c

'n vierde van 20c, 5c

'n vyfde van 20c, 4c

'n sesde van 30c, 5c

'n sewende van 14c, 2c

'n agste van 40c, 5c

'n negende van 27c, 3c 8, 16, 24, 32, 40

'n tiende van 30c, 3c 9, 18, 27

Figure 9.16 Linda Worksheet 18.1

Caroline, Cheryl and Zelda (See Figure 9.12) drew their solutions thus placing them on Level One, Phase Two of Development. Zane and Elaine worked abstractly and were on Level Three.

Learners found this concept, fractions as a part of a collection of objects, somewhat difficult. As stated earlier, it could have been due to the fact that it was handled at a stage in the school year where they were more involved in the concert than in their schoolwork. The fact that no division of whole numbers had been done by the teacher in class also hindered the progress. The large numbers that they had to work with also caused a problem and once they were made smaller, the success rate improved.

Having said that, the researcher must hasten to add that each learner ‘discovered’ a method that enabled him/her to solve the problems. The researcher is of the opinion that if the learners had been frequently exposed to similar type problems, the learners would have been able to master this concept.

CHAPTER 10

CONCLUSIONS, EVALUATION AND RECOMMENDATIONS

The primary aim of the research was to investigate how learners develop their own conceptual and procedural knowledge of common fractions using real-life situations. Secondary aims were to investigate whether the hierarchy of the four Levels of Development as outlined by Murray and Olivier (1989), which are applicable to whole number concept, are applicable to common fractions (See Section 2.3.2) and to explore how the informal knowledge of learners could be used to develop the concept of common fractions with Grade 3 learners.

The outcomes of the study showed that learners were able to achieve a significant degree of success in developing a stable common fraction concept and that they had the ability to solve problems involving common fractions with confidence. The Problem-centred approach afforded learners the opportunity to construct their own ideas and to develop a deeper understanding of the concepts involved. The approach encouraged learners to make sense of realistic problems and to invent their own procedures in an atmosphere of discussion and argument.

Initially, learners based their methods on their informal knowledge of sharing. The researcher is of the opinion that the development of the fraction concept can be successfully based on this informal knowledge and recommends that this be considered as a starting point for the teaching of common fractions. Learners fully understood equal-sharing and had the ability to deal with equal-sharing situations. At the outset, a few learners had to be encouraged to share out the remaining unit (See Section 4.4). The development of the fraction concept was successfully initiated using equal-sharing situations, as learners were able to construct their own idea of fractions through their own thinking and actions and in collaboration with their peers (analysed and discussed in Chapter 4).

To begin with, knowledge of the fraction names and symbols (with the exception of halves and sometimes quarters) was generally poor. Some learners called all fractional parts ‘pieces’ while others referred to any fractional part as ‘a half’ or ‘quarter’ (See Section 4.4). This limiting construction could have been the result of previous teaching. It became evident that social knowledge relating to the common fraction concept took a long time to become stable and for the terminology to become part of the learners’ vocabulary; it needed to be repeated often. It also needed to be transmitted to the learners in a variety of ways (analysed and discussed in Chapter 5).

The findings suggest that the teaching of equivalent fractions need not be delayed to the higher grades as has been the traditional practice, but can be successfully introduced in the lower grades using the materials used in this study. The concept was developed gradually by posing many real-life practical experiences of comparing fractions and putting together fractional parts. Learners, to a lesser or greater extent, were able to grasp the idea of equivalence, albeit gradually. Most learners were able to identify equivalent fractions by the end of the intervention. They usually solved the problems either by consulting the fraction wall or by drawing the solution, and not by memorisation. By the end of the intervention, three learners were able to identify equivalent fractions without the use of the fraction wall, whereas the other nine still needed physical representation (analysed and discussed in Chapter 6).

Learners were able to compare fractions successfully. Initially they used drawings or the fraction wall to do so. At times though, learners’ drawings were not accurate enough causing solutions to be incorrect. Later during the intervention, some of the learners were able to compare unit fractions by deciding how many parts the unit had to be divided into and the more pieces there were, the smaller the pieces were (analysed and discussed in Chapter 6).

Traditional methods of teaching operations involving fractions usually result in learners applying rote procedures and rules with little or no understanding or conceptualisation (See Section 2.4). These operations were usually presented in a particular sequence (e.g. first add fractions with like denominators, then add fractions with unlike denominators but of which the one denominator is a multiple of another, and lastly teach addition of fractions with unlike denominators). However, during the intervention, learners were challenged with problem situations in an integrated manner. The problems covered the different meanings of fractions as well as the different ways in which fractions can be used. The problems were set in real-life contexts to facilitate understanding, provide a sound basis for building fraction knowledge and for learners to build connections to existing knowledge. The researcher found the problems to be powerful vehicles for preparing the way for operations involving fractions. Learners used their own diagrammatic representation of the physical situations to solve the problems. They worked co-operatively spending much time discussing, arguing and reflecting on the methods of others.

By the end of the intervention, when learners were presented with mixed problem types, most of them were able to recognise the different structures. They were able to alternate from the one problem type to the other with confidence. The learners were generally able to add fractions, both with like and unlike denominators and to subtract fractions with like denominators. They solved repeated addition and rate problems, two of the multiplication types, successfully (See Chapter 8). Division problems that are traditionally considered the most difficult basic operation were solved successfully. By making use of diagrams, the learners made sense of the situation and invented their own procedures to work out the problems accordingly. Learners were also able to construct solutions for problems where fractions were part of a collection of objects. The concept of iteration assisted learners to convert improper fractions to proper fractions and *vice versa*. (analysed and discussed in Chapters 7, 8 and 9).

One of the most difficult aspects to develop throughout the research was a classroom culture conducive to the Problem-centred approach. It should be remembered that for the most part these learners were working in classrooms where they were not encouraged to collaborate and compare their work, or to reflect on the methods they used. To a large extent, they were used to being shown a method, required to copy it, and follow rote procedures for standard algorithms.

The Problem-centred approach to mathematics requires learners to spend time discussing and reflecting on their work with their peers. Learners should be encouraged to examine their mistakes and use them as learning opportunities. Since the researcher's lessons only took place for one hour per week, and later on in School A, twice a week, learners found it very difficult to adjust to collaborating, discussing, reflecting and to sharing their solutions and methods. Learners had to be constantly reminded and encouraged to work co-operatively. As the intervention progressed, learners became more familiar with this approach and their social interaction, albeit slowly, gradually improved. The learners' degree of interaction, to a large extent, depended on the problem posed. If the problem lent itself to more discussion, the researcher found the learners would discuss and interact without prompting.

When the learners worked co-operatively and collaboratively, they generally achieved higher learning levels than they were able to achieve on their own (See Section 2.2.3). They often discovered their own mistakes when reflecting on their solutions and methods individually, or while explaining their methods to others. Learners gradually gained confidence in themselves and did not always consult the facilitator about the correctness of the solution or method. Towards the end of the intervention, they were prepared to spend time on the problems and not simply give up and say that they did not know how to find the solution.

Murray and Olivier (1989) found that learners passed through four Levels of Development when developing computational strategies for basic arithmetical operations, the researcher found that learners developing understanding of the common fraction concept also passed through three of the four Levels of Development (See Section 2.3.2). As the research did not lend itself to Level Four stage of Development, the researcher was not able to comment on this level.

Initially problems were solved on Level One, Phase Two of Development (See Section 2.2.3). As stated earlier (See Chapter 4), this could have been due to the fact that the researcher insisted on learners drawing their own solutions. Most learners remained on this level for quite some time and some never developed beyond it. Those learners, who did not develop beyond this level, did however refine their solution methods.

When learners are on Level One, Phase One of Development, they need to manipulate the physical objects used in the problem. These physical objects need to be available to them. On three occasions it was necessary to encourage learners to work at Level One, Phase One of Development so as to improve their understanding of the problem. The first was during the second lesson on the comparison of fractions, the second being the second attempt at developing the concept of iteration and the third was when two learners experienced difficulty with fractions as a part of a collection of objects (See Chapter 9).

At no stage during the research did learners function on Level One, Phase Three of Development. Learners did not develop the technique of using the fractional names to solve problems (See Section 2.3.2). It became clear that at times, learners were on Level Two of Development as they were successfully solving problems by using characteristics of Level Two of Development. The learners were conceptualising a given number as an abstract item with meaning independent of physical referents.

Towards the later half of the intervention, some learners were able to visualise the fractional parts and were able to solve problems abstractly. They merely wrote down solutions for the problems. These learners were working on an abstract level and this placed them on Level Three of Development.

As previously stated, Murray and Olivier's Levels of Development were used as a guideline for the initial analysis of data. From the analysis of the data in this study, the following Levels of Development for common fractions are suggested.

Level One

At this level, learners are able identify the part-whole relationship between the fractional part and the unit where the unit is a single object. They form the idea that the fractional part can be iterated a certain number of times to produce the unit. Learners have a knowledge of fraction names and associated fraction symbols. Most of the problems are solved using drawings. (See Figures 4.36, 5.3, and 8.19)

Level Two

Learners are able to conceptualise a given rational number as an abstract item with meaning independent of physical referents showing that they have acquired the numerosity of the rational number. Learners use fraction symbols instead of drawings when solving and notating the problem. For example, $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=1$. (See Figures 7.10 and 8.14)

<u>Level Three</u>

Learners are able to solve tasks abstractly (See Figure 4.22) and are able to transform rational numbers, and/or the problem, to make the calculation easier. Solutions indicate that learners who were at Level Three were able to 'manipulate' the rational numbers and/or problems to suit their needs. Learners are, for example, able to recognise equivalent fractions and to form equivalent fractions that will enable them to solve a task. This provides the learner with a computational basis to solve a computation by relating it to other known results. For example, $5\frac{1}{2} \div \frac{1}{3}$ - five divided by one third equals 15 remainder half. A half is equal to $\frac{3}{6}$ and a third is equal to $\frac{2}{6}$, therefore, one third will fit into one half once more resulting in a solution of 16 and $\frac{1}{6}$. (See Figure 6.32 and 7.27 and transcription from interview).

As it was not the researcher's intention to examine the learner's work on standard computational algorithms, no data was collected that would lead to the necessity of a Level Four.

Learners never remained on the highest level of development they had obtained. As soon as the learners had difficulty solving a problem and/or they met a new problem structure, they reverted ('folded back') to a lower level of development (See Section 2.10). They always functioned at a level that they felt comfortable with and used methods they understood. It became evident that learners cannot be forced to function at a level of understanding they have not yet reached and it is important for the classroom practitioner to understand that even the most able learners will almost always revert back to a previous level of development when working with a problem they perceive as being new or difficult.

Learners who initially found it difficult to verbalise their thoughts improved steadily and felt less intimidated as they progressed. Learners eagerly looked forward to the mathematics lessons and remarked to their teacher and the researcher that they enjoyed the mathematics they were doing. Some of the learners remarked that it was difficult, but were ecstatic when they were asked to share their ideas and their solutions were correct. Many learners became confident in their ability to solve problems.

On the basis of the insights and experience the researcher gained through the intervention, it became apparent that developing the correct classroom culture prior to the intervention is of great importance. Classroom culture takes a considerable time to establish and if learners are not accustomed to working in this manner, it is likely that the initial impact of carefully designed research materials will be lost because the culture is not conducive. When using the Problem-centred approach to mathematics, learners need to work collaboratively and co-operatively with peers, of preferably equal ability, as discussion and reflection need to drive the learning process. This culture could be established prior to the intervention by posing mathematical problems that are set in a suitable context and that make sense to the learners. This will afford learners the opportunity to engage in mathematics and establish the correct classroom culture.

Learners working with peers of equal ability as suggested by Murray *et al*. (1993) proved to be most successful. Learners established a working relationship with their peers and it did not appear that any one felt threatened or monopolised by others. They gained much satisfaction from contributing to the discussion, constructing a solution to a problem and obtaining the correct solution. The researcher is of the opinion that it may be harmful for learners to, under these circumstances, work in collaboration with a more capable peer or adult as Vygotsky advocates (See Section 2.2.3). By observing learners who were, at times, in mixed ability groups due to absenteeism, weaker ability learners adopted approaches advocated by the more capable peer, but during reflection time it became evident that they did not understand the methods used, as they could not explain how the solution had been arrived at.

Reflection time was essential. It was felt that, as did Ball (1993), the discussion allowed for flexibility and creativity in the responses of the learners and it generated a situation where the class, as a whole could learn more successfully (See Section 2.10). During this time, learners shared their ideas and methods and they were able to refine their methods when next confronted by a similar problem structure. Based on the success of the interaction with equal peers, the researcher would recommend that learners collaborate with peers of equal ability.

The tasks for this study were developed on the assumption that the learners other mathematical concepts and skills would be developed satisfactorily through the year. Because, however, learners were exposed to traditional teaching, their numerical development was slower than the researcher had anticipated. At times, numbers used in the problems were too large and needed to be adjusted. Careful attention needs to be given to the level of the learners' numerical competence before the tasks are finalised. The researcher is of the opinion that, wherever possible, teachers should use the Problem-centred approach to introduce and develop mathematical concepts. Learners should then ideally be able make sense of and develop a sound understanding of the mathematics they are doing.

The Grade 3 learners made great strides with respect to the development of the fraction concept without being given rote procedures to follow and rules to learn and apply. They coped relatively well in all aspects of common fractions dealt with during the research and for this reason, the researcher strongly suggests that this method of teaching fractions be tried more readily at all levels of the primary school. Many teachers may however need a re-orientation in their thinking to do it this way.

In reflecting on the research process as a whole, the researcher has gained some valuable insights as regards to the practical implementation of a research project of this nature. A critical evaluation of the research methodology has been discussed in Chapter 3 (See Section 3.5). The classes and schools where the research is to be conducted needs to be chosen carefully. The distance of the school from the researcher's place of employment/residence must be carefully considered as travelling is time-consuming and expensive. Due to the fact that discussion drove the learning process, it would perhaps have been more beneficial if the number of learners in the classes were greater as learners could have been divided into groups of four instead of into pairs or three's. The researcher is of the opinion that this could have provided greater opportunity for discussion. A useful investigation would be to see if larger groups would be more beneficial.

As the researcher needs to be accommodated into the school programme, careful planning of the number of lessons needs to be taken into account. One of the main reasons the researcher deciding to terminate the research at one school was due to the fact that the holidays and school programme had disrupted data collection to such an extent that it was impossible to complete the required number of lessons (33) at that particular school (See Section 3.2). It was necessary to conduct two lessons per week at the other school in order to complete the necessary data collection within the school calendar.

The Problem-centred approach to mathematics was not implemented at the schools where the research was conducted. It is conceivable that if this approach had been used readily, the learners would have been accustomed to working co-operatively and their mathematical knowledge would have been flexible. They would have been accustomed to using previous solutions to assist them in working out subsequent solutions. By using the Problem-centred approach, exposing learners to situations involving common fractions and the teaching materials used during the intervention, a significant degree of success in addressing the learners' concept of fractions was achieved.

From the research it is evident that the informal knowledge that learners have forms a foundation upon which to develop a sound and stable knowledge of the common fraction concept. Learners have the ability to think and construct knowledge both individually and through interacting with others. Solving real-life problems set in a context that makes sense to them, afforded learners the opportunity of making sense of and understanding the mathematics that they were doing.

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PERSONAL COMMUNICATION

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APPENDIX A

WORKSHEETS USED DURING THE STUDY

Worksheet 1

1. Vuyo and Mary have 7 chocolate bars that they want to share equally between 2 of them so that nothing is left. Help them to do it.

2. Vuyo, Mary and Bingo have 7 bars of chocolate that they want to share equally among the three of them so that nothing is left. Help them to do it.

3. Vuyo, Mary, Bingo and Vusi have 13 bars of chocolate that they want to share equally among the four of them so that nothing is left. Help them do it.

Worksheet 2

1. Five friends want to share 11 chocolate bars equally. How must they do it?

2. Five friends want to share 21 chocolate bars equally. How must they do it?

Worksheet 2.1 (For School A only)

1. Five friends want to share 26 chocolate bars equally. How must they do it?

Worksheet 3

When we divide something into 2 equal parts, we call these parts halves

When we divide something into 3 equal parts, we call these parts thirds

When we divide something into 4 equal parts, we call these parts fourths or quarters

When we divide something into 5 equal parts, we call these parts fifths

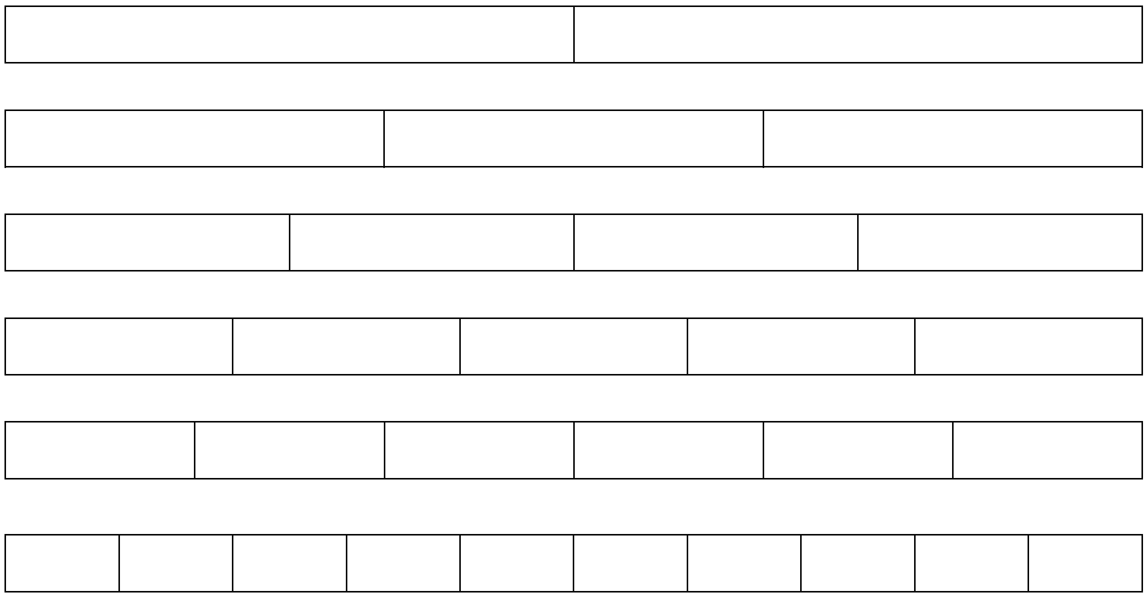
1. What would you rather have, a third of a chocolate bar or a fifth of a chocolate bar? Why?

2. What would you rather have, a fourth of R1 or a third of R1? Why?

3. How many cents is a half of R1? A fourth of R1? What would you rather have, a half of R1 or two fourths of R1? Why?

Worksheet 4

Look at these big chocolate bars. They have all been cut into different equal pieces. Give the pieces names (halves or thirds or whatever you think). Now see if you can put together some smaller pieces to form a bigger piece with another name.



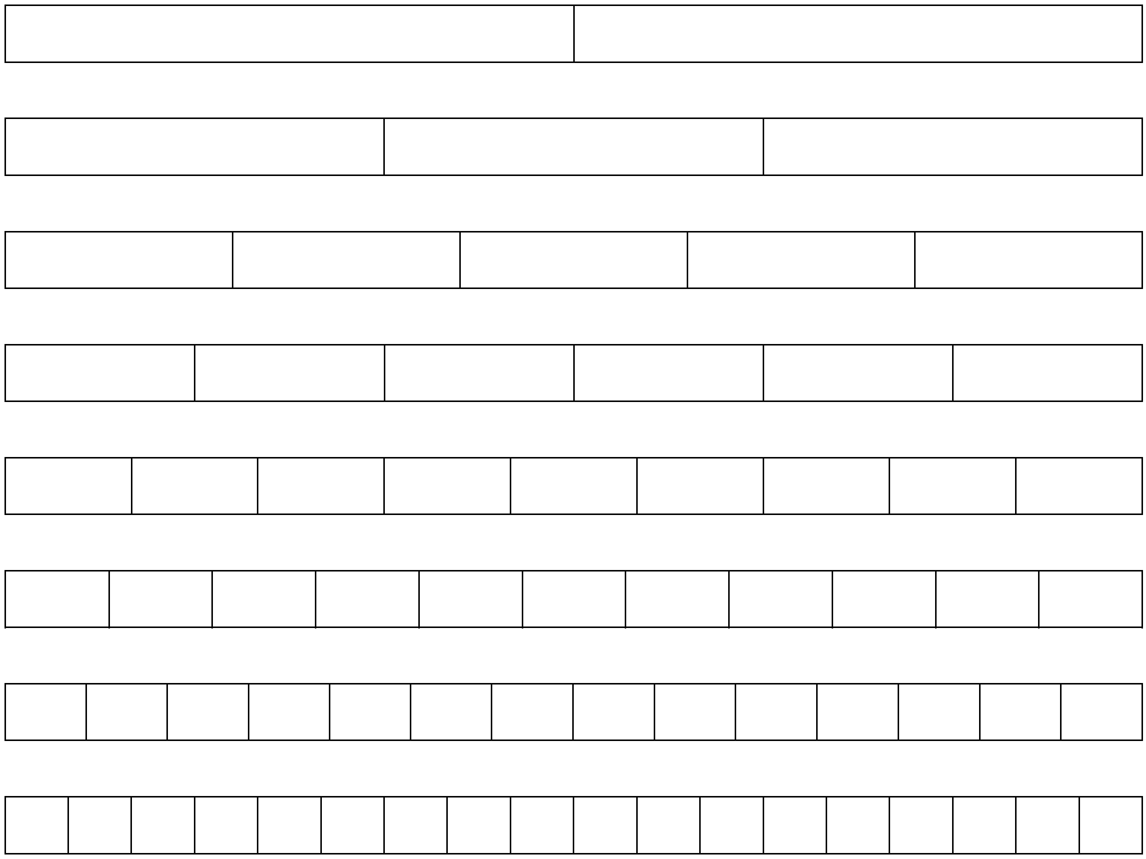
Write down what you have found.

Worksheet 5

1. Six friends want to share 7 chocolate bars equally. How must they do it?
2. Six friends want to share 8 chocolate bars equally. How must they do it?
3. A chocolate bar is cut into two equal pieces. What do you call each piece?
4. A chocolate bar is cut into 7 equal pieces. What do you call each piece?

Worksheet 6

Look at these chocolate bars which have been cut in different ways:



1. How many sixths of a chocolate bar must you put together to make a third of a chocolate bar?

   How many sixths to make two thirds?

   How many tenths to make a fifth?

   How many tenths to make a half?

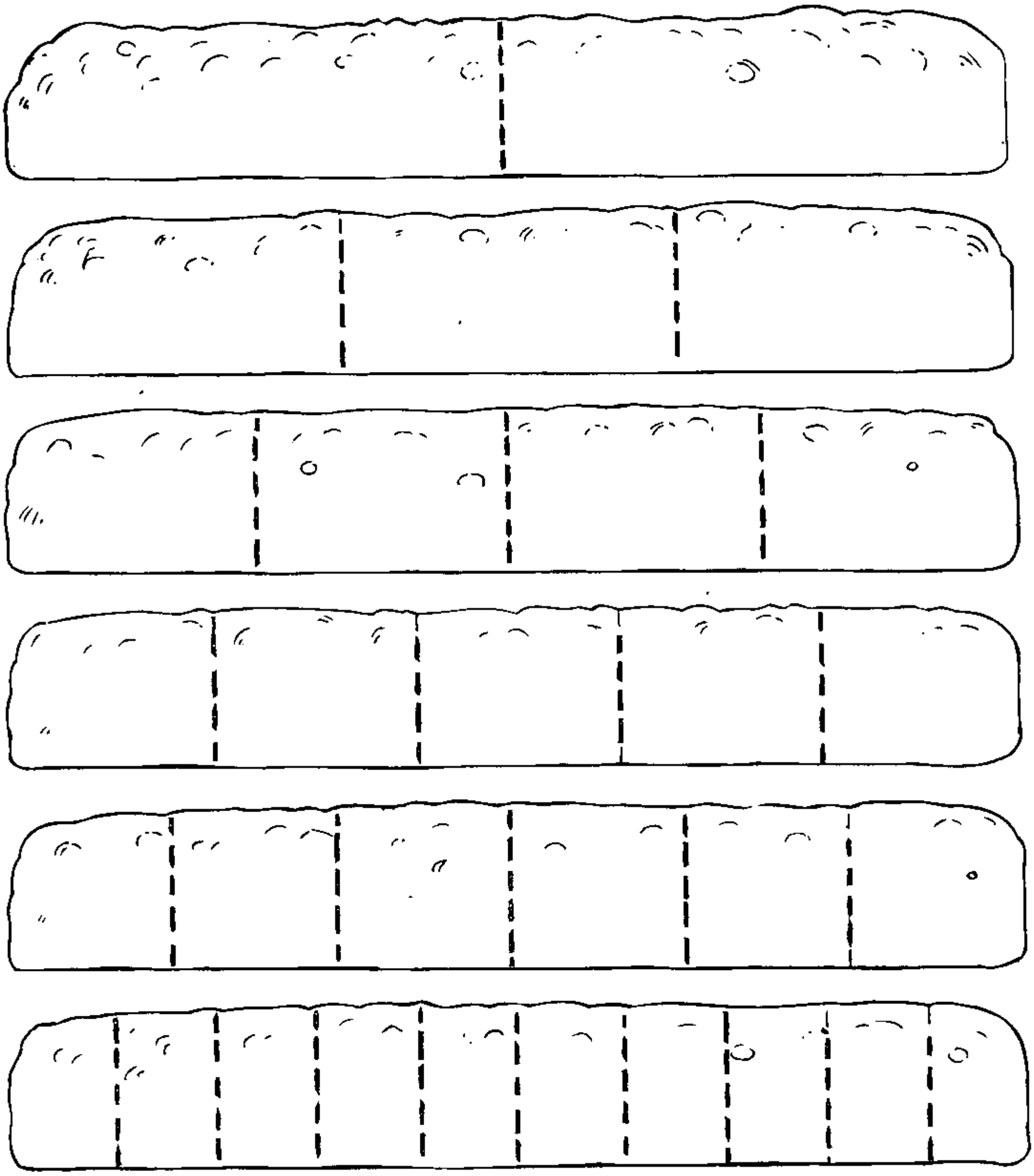
   How many ninths to make two thirds?

2. Write down all the bigger pieces you can make from

   ninths fifteenths eighteenths

<u>Worksheet 6.1</u> (additional worksheet for School A)

Look at the different ways in which these chocolates have been divided up.



1. Give each piece a name.
2. Use your ruler to help you – slide it from left to right over your page.
3. Look at the dotted lines and check to see if there are other chocolates with a dotted line in the same place.
4. Write down all the pieces you find that are equal to other pieces.

<u>Worksheet 6.2</u> (additional worksheet for School A)

Look at the different ways in which these vienna sausages are cut up:

1. How many ninths of a sausage must you put together to get a third of a sausage?
2. How many sixths to make two thirds?
3. How many tenths to make a fifth?
4. How many tenths to make a half?
5. Write down all the bigger pieces you can make from:

<u>ninths tenths eighteenths</u>

Worksheet 7

A short way to write a half is $\frac{1}{2}$
A short way to write a seventh is $\frac{1}{7}$
A short way to write a twentieth is $\frac{1}{20}$

1. Two netball teams play a game. There are 14 children all together. The sports teacher wants to give each child $\frac{1}{2}$ of an orange. How many oranges does she need?
2. One of the parents brings along a bag of 35 chocolate bars to share among the 14 players. How much chocolate does each player get?

Worksheet 8

Mrs Jeremiah bakes five cakes for the party after the netball game.
For one cake she needs:

$\frac{1}{4}$ cup of margarine	$\frac{1}{2}$ cup sugar
1 egg	$\frac{1}{2}$ cup milk
$1\frac{1}{2}$ cups flour	$2\frac{1}{2}$ teaspoons baking powder
$\frac{1}{4}$ teaspoon salt	$\frac{1}{2}$ teaspoon flavouring

Work out how much she needs for five cakes.

<u>Worksheet 8.1</u> (additional worksheet for School B)

Please will you help the baker to work out how much ingredients he needs to make six batches of rusks for the children that stay in the hostel.

For one batch he needs:

$\frac{1}{2}$ cup butter

$\frac{1}{4}$ cup sugar

$1\frac{1}{2}$ cups of flour

1 teaspoon baking powder

$\frac{1}{3}$ cup of milk

$\frac{1}{5}$ teaspoon of salt

Worksheet 8.2 (additional worksheet for School A)

If we cut a chocolate bar into **three** equal pieces, we call these parts **thirds**.

We write a third like this: $\frac{1}{3}$

If we give two thirds away, then we write two thirds like this: $\frac{2}{3}$

If we cut a chocolate bar into **four** equal pieces, we call these parts **quarters or fourths**.

We write a quarter or fourth like this: $\frac{1}{4}$

If we give two quarters or fourths away, then we write two quarters like this: $\frac{2}{4}$

If we give three quarters or fourths away, then we write three quarters like this: $\frac{3}{4}$

Write the following using fraction symbols:

Two fifths

Three fifths

Five sixths

Two ninths

Three sevenths

Five sevenths

Four eighths

Seven tenths

Worksheet 9

Vusi and Anna prepare soft porridge for breakfast. For each bowl they use $\frac{1}{3}$ of a litre of milk.

1. If they make 6 bowls of porridge, how many litres of milk do they need?
2. They have 5 litres of milk. How many bowls of porridge can they prepare?
3. They have $2\frac{1}{2}$ litres of milk. How many bowls of porridge can they prepare?

Worksheet 10

The children are making different animals and cars from wire. A car needs $2\frac{1}{2}$ metres of wire. An animal needs $1\frac{1}{2}$ metres of wire.

1. The children have 20 metres of wire.

(a) How many cars can they make from 20 metres of wire?

(b) How many animals can they make from 20 metres of wire?

2. Look at the answers you get for question 1. Look at the selling prices for the cars and animals. Now help the children decide what they must make, cars or animals, to earn the most money.

Worksheet 11

1. The wire to make animals and cars from costs six cents per metre. Complete the following table:

Length in metres	Cost in cents	Length in metres	Cost in cents
1	6	10	
$1\frac{1}{2}$		20	
2	12	30	
$2\frac{1}{2}$		40	
3	18	50	
$3\frac{1}{2}$		60	
4		70	
$4\frac{1}{2}$		80	
5		90	
$5\frac{1}{2}$		100	

2. Now show the children how to use this table to find the cost of:

$7\frac{1}{2}$ metres of wire

$11\frac{1}{2}$ metres of wire

25 metres of wire

$61\frac{1}{2}$ metres of wire

85 metres of wire

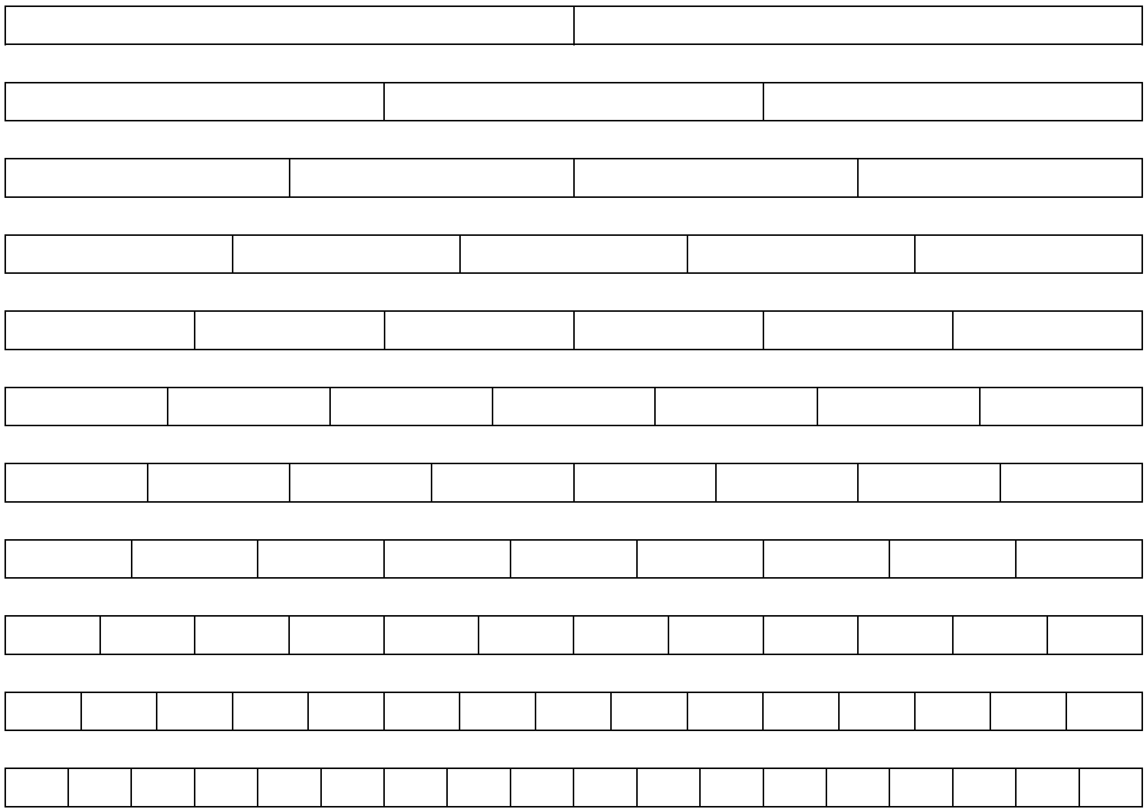
$91\frac{1}{2}$ metres of wire

Worksheet 12

A short way to write a fifth is $\frac{1}{5}$

A short way to write three fifths is $\frac{3}{5}$

Here are some chocolate bars that have been cut:



Which piece of chocolate bar would you rather have? Why?

$\frac{1}{3}$ or $\frac{2}{6}$?

$\frac{1}{2}$ or $\frac{3}{5}$?

$\frac{2}{4}$ or $\frac{3}{6}$?

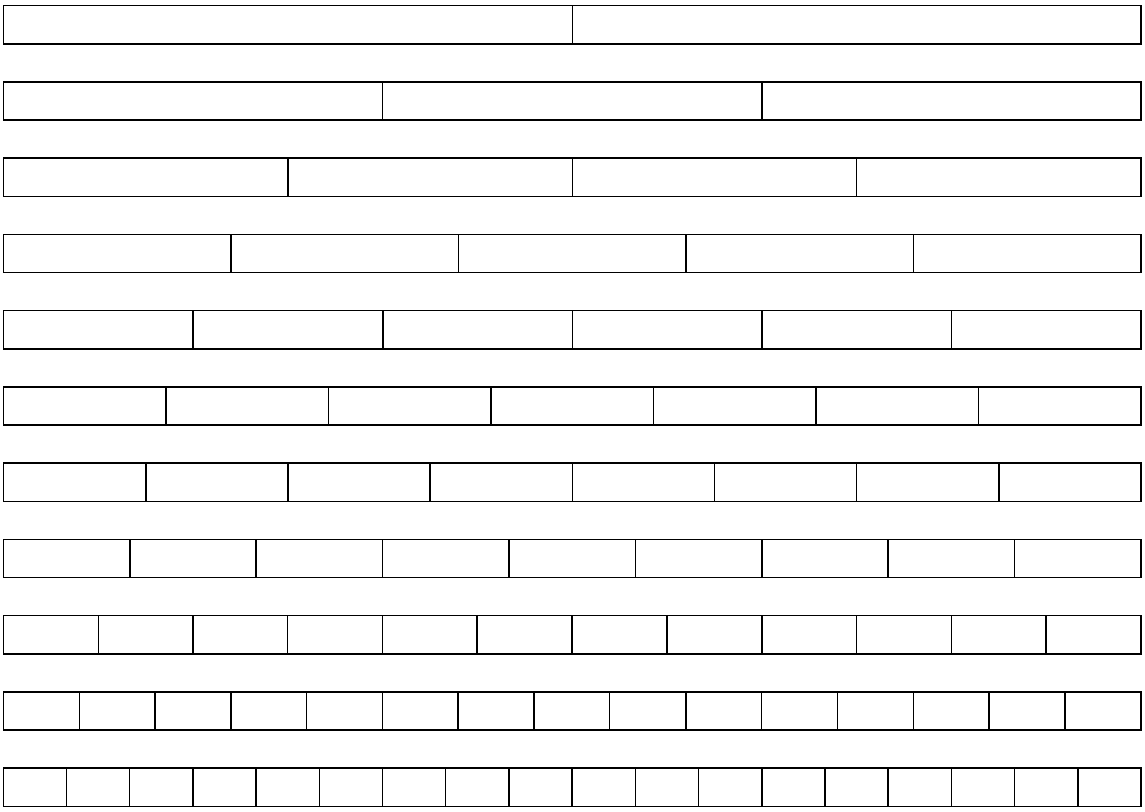
$\frac{5}{10}$ or $\frac{2}{5}$?

$\frac{1}{7}$ or $\frac{1}{8}$?

$\frac{7}{15}$ or $\frac{4}{8}$?

$\frac{9}{18}$ or $\frac{6}{10}$?

<u>Worksheet 13</u>



Here are some pieces of chocolate bar:

$\frac{6}{18}$	$\frac{5}{10}$	$\frac{1}{3}$	$\frac{2}{4}$	$\frac{3}{6}$
$\frac{4}{12}$	$\frac{6}{12}$	$\frac{5}{15}$	$\frac{5}{6}$	$\frac{4}{5}$
$\frac{12}{18}$	$\frac{12}{15}$	$\frac{10}{12}$	$\frac{2}{3}$	$\frac{2}{6}$

1. First say which of these pieces of chocolate do you think are the same size. Explain why you say so.
2. Then check on the drawing if you were right.
3. Now use the drawing to find all the other pieces of the same size that you might have missed on this list.

Worksheet 14

1. The children make a small duck from $\frac{1}{3}$ of a metre of thin wire. They have $5\frac{1}{2}$ metres of thin wire. How many small ducks can they make?

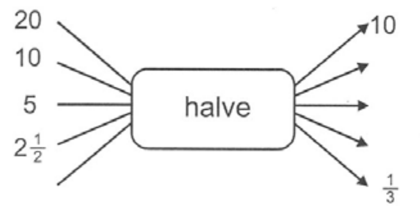
2. In a needlework class, they use $\frac{1}{4}$ metre ribbon to trim one apron. If each child is given $2\frac{1}{2}$ metres of ribbon, how many aprons can each child make?

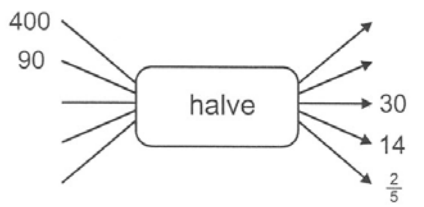
Worksheet 15

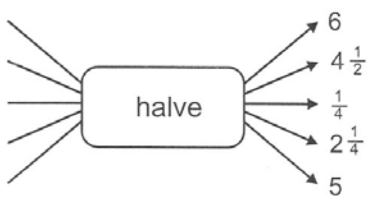
David needs some stakes to put labels on his vegetable seedbeds. He needs stakes that are $\frac{1}{5}$ metre long. He has $1\frac{1}{2}$ metres of rod. How many stakes of $\frac{1}{5}$ metre can he cut from this rod?

Worksheet 16

Complete these flow diagrams:







<u>Worksheet 17</u>

1. Vuyo uses the juice of $1\frac{1}{2}$ oranges for a large birthday cake. She has 8 oranges. How many cakes can she bake?

2. Bingo and Mary have $1\frac{1}{2}$ bars of chocolate. They want to share the chocolate equally. How much chocolate must each child get?

<u>Worksheet 18</u>

See how far you can get with this table. You can do the easy ones first if you want to.

A half of R1

A third of R1

A fourth of R1

A fifth of R1

A sixth of R1

A seventh of R1

An eighth of R1

A ninth of R1

A tenth of R1

A twentieth of R1

A fiftieth of R1

A hundredth of R1

Worksheet 18.1 (additional worksheet for School A)

See how far you can get with this table. You can do the easy ones first if you want to.

A half of 30c

A third of 30c

A fourth of 20c

A fifth of 20c

A sixth of 30c

A seventh of 14c

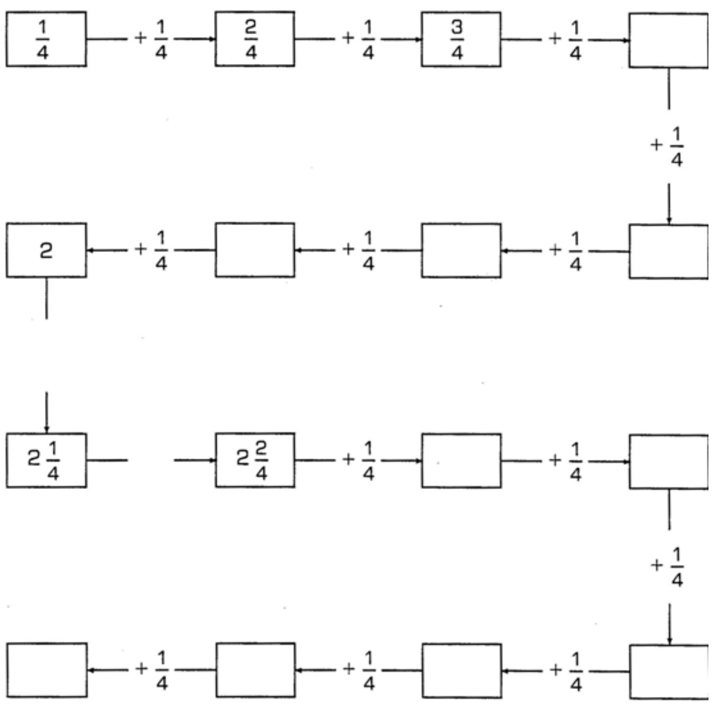
An eighth of 40c

A ninth of 27c

A tenth of 30c

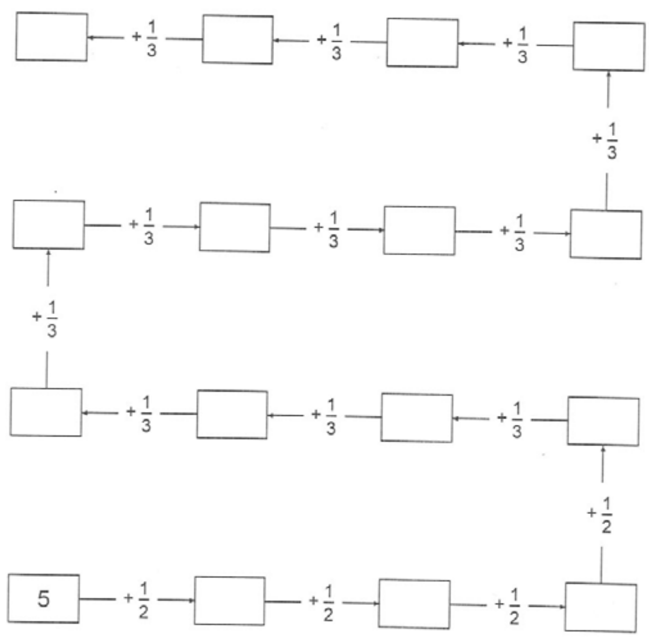
Worksheet 19

Complete this chain:



Worksheet 20

Complete this chain:



Worksheet 21

Vuyo wants to know what she has left over after baking for a party. How much does she have?

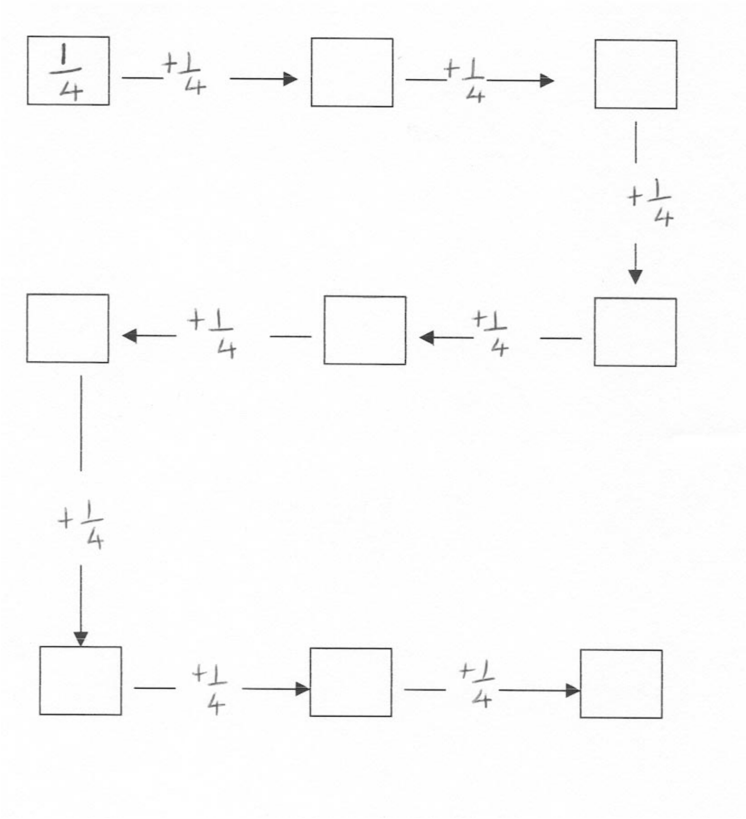
3 big packets of chips, each $\frac{1}{2}$ full

5 containers of ice-cream, each $\frac{1}{4}$ full

2 jugs of milk, each $\frac{2}{3}$ full

Worksheet 21.1 (additional worksheet for School A)

Complete this chain:



Worksheet 22

1. Vusi's book has 88 (44) pages. He says: "I have read more than half of the book. I am on page 41(21)." Is it true? Explain.

2. Vuyo has 27 sweets. She gives 10 sweets to Peter. She says: "Peter, I have given you a third of my sweets." Is it true? Explain.

Worksheet 23

1. Sam promises to give a tenth of his peaches to the school. He has 90 (30) trays of peaches. How many trays must he give the school?

2. Simba gives a $\frac{1}{3}$ of a box of chips to Dodi's class. If there are 90 (30) packets in a box, how many packets of chips does the class get?

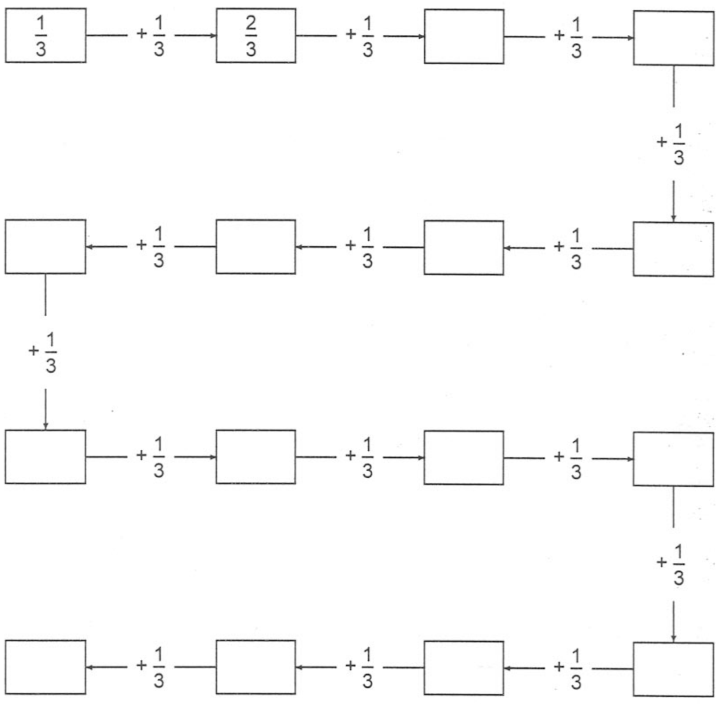
Worksheet 24

The children have brought different things to eat and drink.

1. They have three chocolate bars to share equally among four children. How much chocolate does each get?
2. They have two litres of cooldrink to share equally among eight children. How much cooldrink must each child get?
3. They have nine oranges to share equally among the four children. How much orange must each one get?

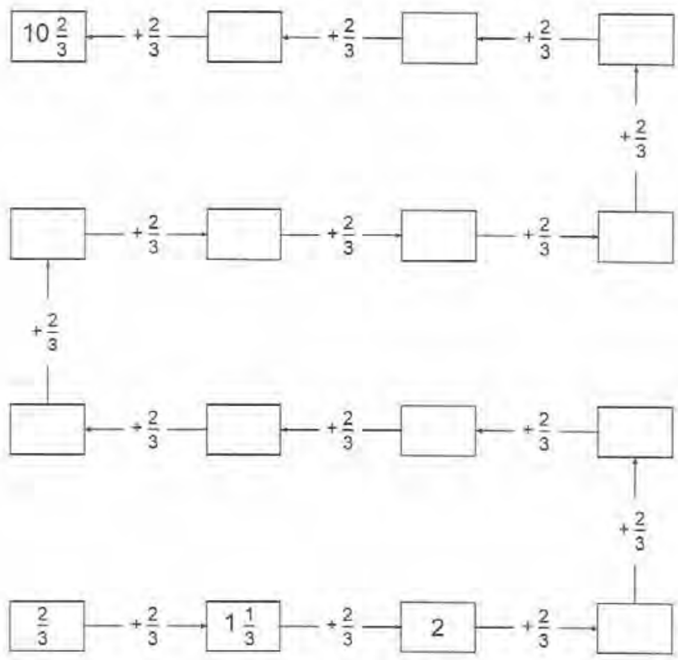
Worksheet 25

Complete this chain:



Worksheet 26

Complete this chain:



Worksheet 27

Complete this table:

Number of hours	Number of minutes
$\frac{1}{6}$	10
$\frac{1}{5}$	
$\frac{1}{3}$	
$\frac{1}{4}$	
$\frac{1}{2}$	
1	60
	120
$1 + \frac{1}{2}$	
$1 + \frac{1}{3}$	
$1 + \frac{1}{4} + \frac{1}{4}$	
$\frac{1}{6} + \frac{1}{6}$	
$\frac{1}{3} + \frac{1}{6}$	

Do you notice anything?

<u>Worksheet 28</u>

It takes Thandi 10 minutes to walk one kilometre.

Complete this table:

Number of Kilometres	Number of minutes
1	10
$\frac{1}{2}$	
$\frac{1}{5}$	
2	
$2\frac{1}{2}$	
$1\frac{1}{2}$	
$1\frac{1}{5}$	
3	
$3\frac{1}{2}$	
$\frac{1}{2} + \frac{1}{5}$	
$1\frac{1}{2} + \frac{1}{5}$	
$1\frac{1}{5} + \frac{1}{2}$	

Worksheet 29

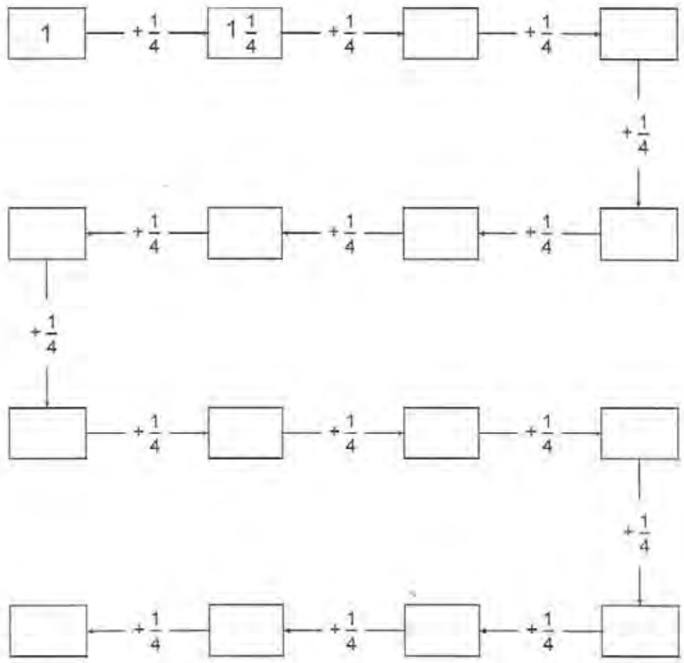
There are 24 hours in a day.

Complete this table.

Number of days	Number of hours
1	24
2	
$\frac{1}{2}$	
$\frac{1}{4}$	
$\frac{1}{3}$	
$\frac{1}{6}$	4
$\frac{1}{4} + \frac{1}{4}$	
$\frac{1}{6} + \frac{1}{6}$	
$\frac{1}{3} + \frac{1}{3}$	
$\frac{1}{2} + \frac{1}{6}$	
$\frac{1}{2} + \frac{1}{4}$	
$2\frac{1}{2}$	
5	
6	

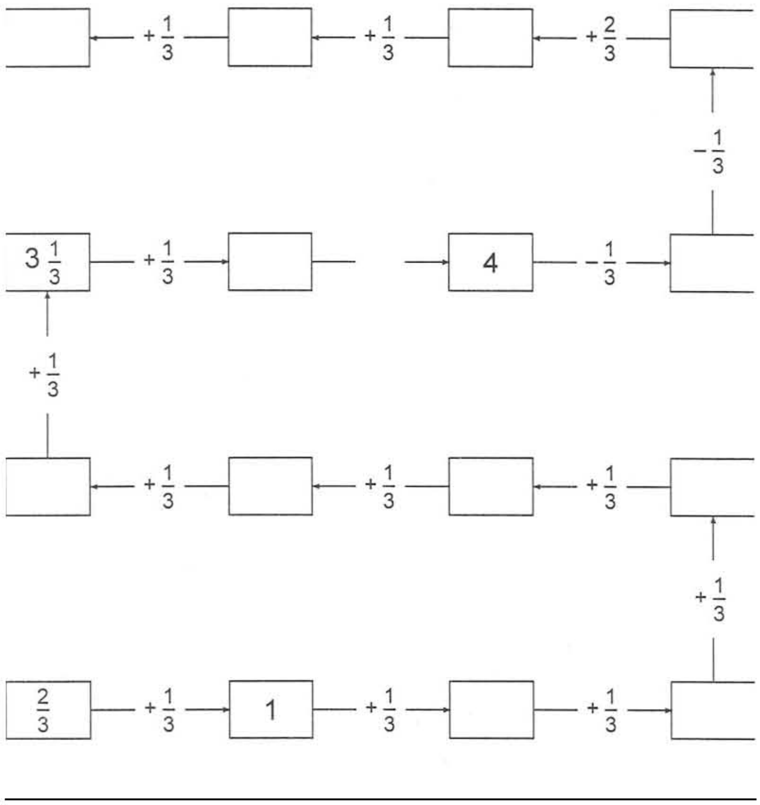
Worksheet 30

Complete this chain:



Worksheet 31

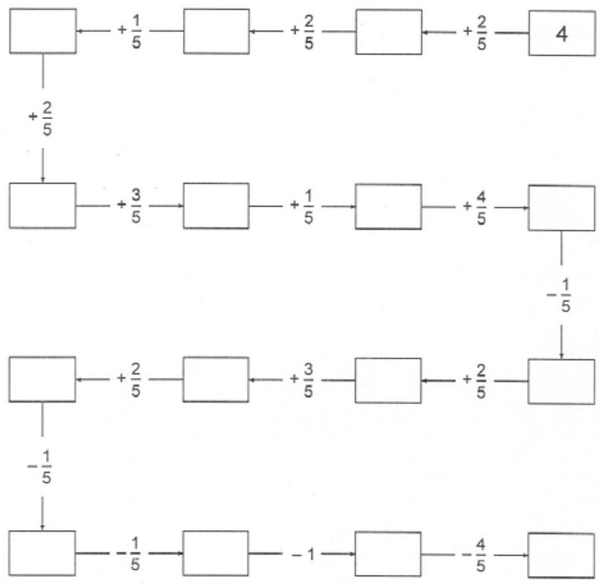
Complete this chain:



Worksheet 32

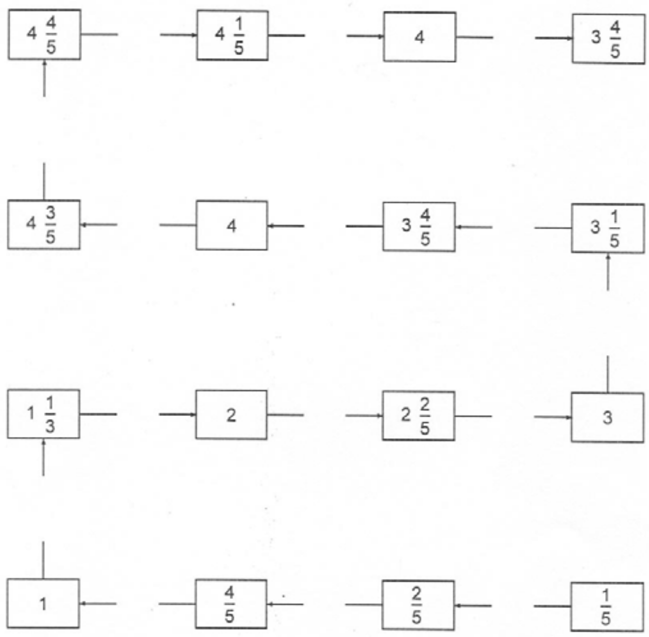
Complete this chain:

Complete this chain:



Worksheet 33

Complete this chain:



APPENDIX B

SUMMARIES OF SOLUTION STRATEGIES USED TO DEVELOP THE CONCEPT OF A COMMON FRACTION

<u>Worksheet 1</u>

School A
7 ÷ 2

Category 1					Category 2	Category 3			Category 4	
Economic Sharing - sharing out the maximum number of units and divided the remaining unit (whole) into an appropriate number of fractional parts.					Partitioning all the units into fractional parts of which the denominator is the same as the divisor, then sharing out the parts from each in turn.	Shared out the maximum number of units and remaining unit (whole) was divided into an inappropriate number of fractional pieces.			Shared units incorrectly .	Incorrect solution
Drawing	Drawing and numbers	Answer Correct but incorrect naming of fraction parts	Used numbers to solve - no drawings	Used drawings and words	Drawing only	Did not show the sharing	Ignored extra fractional part	Divided the remaining fractional part into inappropriate no of fractional parts		
	Elaine		Zane	Martin	Kate					Mary
	Carol									
	Caroline									
	Yvonne									
	Cheryl									
	Zelda									

Worksheet 1

School A

7 ÷ 3

Category 1					Category 2	Category 3			Category 4	
Economic Sharing - sharing out the maximum number of units and divided the remaining unit (whole) into an appropriate number of fractional parts.					Partitioning all the units into fractional parts of which the denominator is the same as the divisor, then sharing out the parts from each in turn.	Shared out the maximum number of units and remaining unit (whole) was divided into inappropriate number of fractional pieces.			Shared units incorrectly .	Incorrect solution
Drawing	Drawing and numbers	Answer Correct but incorrect naming of fraction parts	Used numbers to solve - no drawings	Used drawings and words	Drawing only	Did not show the sharing	Ignored extra fractional part	Divided the remaining fractional part into inappropriate no of fractional parts		
	Elaine		Zane	Martin	Kate		Martin			Mary
	Carol		Cheryl							Zelda
	Caroline									
	Yvonne									

<u>Worksheet 1</u>

School A

13 ÷ 4

Category 1					Category 2	Category 3			Category 4	
Economic Sharing - sharing out the maximum number of units and divided the remaining unit (whole) into an appropriate number of fractional parts.					Partitioning all the units into fractional parts of which the denominator is the same as the divisor, then sharing out the parts from each in turn.	Shared out the maximum number of units and remaining unit (whole) was divided into inappropriate number of fractional pieces.			Shared units incorrectly .	Incorrect solution
Drawing	Drawing and numbers	Answer Correct but incorrect naming of fraction parts	Used numbers to solve - no drawings	Used drawings and words	Drawing only	Did not show the sharing	Ignored extra fractional part	Divided the remaining fractional part into inappropriate no of fractional parts		
Martin	Zane		Yvonne	Martin	Kate				Elaine	Mary
	Carol									Cheryl
	Caroline									Zelda

<u>Worksheet 1</u>

School B

7 ÷ 2

Category 1					Category 2	Category 3			Category 4	
Economic Sharing - sharing out the maximum number of units (whole) and divided the remaining unit into an appropriate number of fractional parts.					Partitioning all the units into fractional parts of which the denominator is the same as the divisor, then sharing out the parts from each in turn.	Shared out the maximum number of units and remaining unit (whole) was divided into inappropriate number of fractional pieces.			Shared units incorrectly .	Incorrect solution.
Drawing	Drawing and numbers	Answer Correct but incorrect naming of fraction parts	Used numbers to solve - no drawings	Used drawings and words	Drawing only	Did not show the sharing	Ignored extra fractional part	Divided the remaining fractional part into inappropriate no of fractional parts		
Lucille	Lusanda	Phindiwe				Nosisa				Mbulelo
Nosisa	Lyn									
Sipho										

<u>Worksheet 1</u>

School B

7 ÷ 3

Category 1					Category 2	Category 3			Category 4	
Economic Sharing - sharing out the maximum number of units and divided the remaining unit (whole) into an appropriate number of fractional parts.					Partitioning all the units into fractional parts of which the denominator is the same as the divisor, then sharing out the parts from each in turn.	Shared out the maximum number of units and remaining unit (whole) was divided into inappropriate number of fractional pieces.			Shared units incorrectly .	Incorrect solution
Drawing	Drawing and numbers	Answer Correct but incorrect naming of fraction parts	Used numbers to solve - no drawings	Used drawings and words	Drawing only	Did not show the sharing	Ignored extra fractional part	Divided the remaining fractional part into inappropriate no of fractional parts		
Lucille		Gail				Nosisa	Lusanda	Sipho		Mbulelo
Bob		Phindiwe								Lyn

Worksheet 1

School B

13 ÷ 4

Category 1					Category 2	Category 3			Category 4	
Economic Sharing - sharing out the maximum number of units and divided the remaining unit into an appropriate number of fractional parts.					Partitioning all the units into fractional parts of which the denominator is the same as the divisor, then sharing out the parts from each in turn.	Shared out the maximum number of units and remaining unit was divided into inappropriate number of fractional pieces.			Shared units incorrectly .	Incorrect solution
Drawing	Drawing and numbers	Answer Correct but incorrect naming of fraction parts	Used numbers to solve - no drawings	Used drawings and words	Drawing only	Did not show the sharing	Ignored extra fractional part	Divided the remaining fractional part into inappropriate no of fractional parts		
	Lucille			Lusanda					Gail	Bob
	Phindiwe								Lyn	Mbulelo
	Nosisa									Sipho

<u>Worksheet 2</u>

School A

11 ÷ 5

Category 1				Category 2				Category 3		
Economic sharing – sharing out the maximum number of units and dividing the remaining unit (whole) into an appropriate number of fractional parts				Shared out he maximum number of units - remaining unit (whole) divided into an inappropriate number of fractional parts				Incorrect solutions		
Drawing only	Drawing and numbers	Drawing of solution correct - incorrect social knowledge	Drawings and words	Did not show partitioning - shared out 'halves'	Ignored extra fractional piece	Divided remaining fractional part into appropriate number of fractional parts	Unequal sharing	Ignored remaining unit	Shared units incorrectly	No method identified
Elaine		Mary	Caroline		Zelda	Carol	Martin			Cheryl
Kate			Yvonne							
Zane			Stewart							

Worksheet 2

School A

21 ÷ 5

Category 1				Category 2				Category 3		
Economic sharing - sharing out the maximum number of units and dividing the remaining unit (whole) into an appropriate number of fractional parts				Shared out the maximum number of units - remaining unit (whole) divided into an inappropriate number of fractional parts				Incorrect solutions		
Drawing only	Drawing and numbers	Drawing of solution correct - incorrect social knowledge	Drawings and words	Did not show partitioning - shared out 'halves'	Ignored extra fractional piece	Divided remaining fractional part into appropriate number of fractional parts	Unequal sharing	Ignored remaining unit	Shared units incorrectly	No method identified
Martin	Carol		Zane	Yvonne	Zelda				Caroline	
Kate			Stewart		Elaine				Mary	

<u>Worksheet 2</u>

School B

11 ÷ 5

Category 1				Category 2				Category 3		
Economic sharing - sharing out the maximum number of units and dividing the remaining unit (whole) into an appropriate number of fractional parts				Shared out he maximum number of units - remaining unit (whole) divided into an inappropriate number of fractional parts				Incorrect solutions		
Drawing only	Drawing and numbers	Drawing of solution correct - incorrect social knowledge	Drawings and words	Did not show partitioning - shared out 'halves'	Ignored extra fractional piece	Divided remaining fractional part into appropriate number of fractional parts	Unequal sharing	Ignored remaining unit	Shared units incorrectly	No method identified
Gail	Lucille		Lyn	Sipho	Phindiwe			Tammy	Sharon	Mbulelo
Nosisa			Lusanda							

<u>Worksheet 2</u>

School B

21 ÷ 5

Category 1				Category 2				Category 3		
Economic sharing - sharing out the maximum number of units and dividing the remaining unit (whole) into an appropriate number of fractional parts				Shared out he maximum number of units - remaining unit (whole) divided into an inappropriate number of fractional parts				Incorrect solutions		
Drawing only	Drawing and numbers	Drawing of solution correct - incorrect social knowledge	Drawings and words	Did not show partitioning - shared out 'halves'	Ignored extra fractional piece	Divided remaining fractional part into appropriate number of fractional parts	Unequal sharing	Ignored remaining unit	Shared units incorrectly	No method identified
Lucille			Phindiwe		Tammy				Mbulelo	
Gail			Lyn							
Nosisa			Lusanda							
Sharon										

<u>Worksheet 2.1</u>

Only School A

26 ÷ 5

Category 1				Category 2				Category 3		
Economic sharing – sharing out the maximum number of units and dividing the remaining unit (whole) into an appropriate number of fractional parts				Shared out he maximum number of units - remaining unit (whole) divided into an inappropriate number of fractional parts				Incorrect solutions		
Drawing only	Drawing and numbers	Drawing of solution correct – incorrect social knowledge	Drawings and words	Did not show partitioning - shared out 'halves'	Ignored extra fractional piece	Divided remaining fractional part into appropriate number of fractional parts	Unequal sharing	Ignored remaining unit	Shared units incorrectly	No method identified
Carol		Stewart	Mary							Cheryl
Kate			Zane							
			Caroline							
			Martin							
			Yvonne			Elaine				
			Zelda							

Worksheet 5

School A

7 ÷ 6

Category 1					Category 2	Category 3				Category 4
Economic sharing – sharing out the maximum number of units- partitioning the remaining unit (whole) into an appropriate number of fractional parts					Partitioning all	Shared out the maximum number of units – remaining unit (whole) divided into an inappropriate number of fractional parts				Incorrect solutions
Drawing	Drawing and numbers	Solution correct – incorrect social knowledge	Used numbers to solve the problem	Drawings and words		Did not show partitioning	Ignored extra fractional part	Partitioned remaining unit into inappropriate number of fractional parts	Unequal sharing	
		Kate		Yvonne			Zelda			Stewart
		Mary		Caroline						
		Cheryl		Zane						
		Martin		Carol						
				Elaine						

Worksheet 5

School A

8 ÷ 6

Category 1					Category 2		Category 3			Category 4
Economic sharing – sharing out the maximum number of units- partitioning the remaining unit (whole) into an appropriate number of fractional parts					Partitioning all		Shared out the maximum number of units – remaining unit (whole) divided into an inappropriate number of fractional parts			Incorrect solutions
Drawing	Drawing and numbers	Solution correct – incorrect social knowledge	Used numbers to solve the problem	Drawings and words	Shared into $\frac{1}{6}$	Shared into $\frac{1}{3}$	Did not show partitioning	Ignored extra fractional part	Partitioned remaining unit into inappropriate number of fractional parts	
Caroline		Martin		Zane						Zelda
		Kate		Elaine						Mary
				Yvonne				Cheryl		Stewart

<u>Worksheet 5</u>

School B

8 ÷ 6

Category 1					Category 2	Category 3				Category 4
Economic sharing – sharing out the maximum number of units- partitioning the remaining unit into an appropriate number of fractional parts					Partitioning all	Shared out the maximum number of units – remaining unit divided into an inappropriate number of fractional parts				Incorrect solutions
Drawing	Drawing and numbers	Solution correct – incorrect social knowledge	Used numbers to solve the problem	Drawings and words		Did not show partitioning	Ignored extra fractional part	Partitioned remaining unit into inappropriate number of fractional parts	Unequal sharing	
Lucille	Sipho			Lusanda			Mbulelo			Tammy
Gail	Nosisa			Lyn						
	Bob									
		Martin								

Worksheet 5

School A

$8 \div 6$

Category 1					Category 2		Category 3			Category 4
Economic sharing – sharing out the maximum number of units- partitioning the remaining unit (whole) into an appropriate number of fractional parts					Partitioning all		Shared out the maximum number of units – remaining unit (whole) divided into an inappropriate number of fractional parts			Incorrect solutions
Drawing	Drawing and numbers	Solution correct – incorrect social knowledge	Used numbers to solve the problem	Drawings and words	Shared into $\frac{1}{6}$	Shared into $\frac{1}{3}$	Did not show partitioning	Ignored extra fractional part	Partitioned remaining unit into inappropriate number of fractional parts	
Lucille				Lyn	Nosisa			Sipho		Sue
Gail								Mbulelo		Bob
Nosisa										Tammy
Lusanda										

<u>Worksheet 7</u>

School A

35 ÷ 14

Category 1				Category 2	
Economic Sharing - shared out the maximum number of units and divided the remaining unit (whole) into appropriate number of fractional parts				Incorrect solutions	
Drawing and numbers	Solution correct – incorrect social knowledge	Only used numbers to solve	Drawings and words	Shared units incorrectly	No method identified
Elaine		Zane	Yvonne	Kate	Cheryl
Caroline		Carol		Mary	
Stewart					
Linda					

<u>Worksheet 7</u>

School B

35 ÷ 14

Category 1				Category 2	
Economic Sharing - shared out the maximum number of units and divided the remaining unit (whole) into appropriate number of fractional parts				Incorrect solutions	
Drawing and numbers	Solution correct – incorrect social knowledge	Only used numbers to solve	Drawings and words	Shared units incorrectly	No method identified
Lucille					
Bob					
Nosisa					
Gail					
Lyn					
Phindiwe					

Worksheet 17

School A

$1\frac{1}{2} \div 2$

Category 1				
Economic Sharing - sharing out the maximum number of units and dividing the remaining unit (whole) into an appropriate number of fractional parts				
Drawing, fraction notation and addition	Drawing, words and addition	Numbers only	Words only	Drawing and words - no addition
Yvonne	Linda	Zane	Mary	Martin
Carol		Elaine	Kate	Stewart
Zelda				Caroline
				Cheryl

Worksheet 24

School A

3 ÷ 4

Category 1			Category 2	Category 3
Partitioned 2 units into halves and one unit into quarters and then shared			Partitioned each unit into quarters and then shared out	Partitioned each unit into an inappropriate number of fractional parts
Used drawings, fractional notation and addition	Wrote down the solution in fractional notation and addition	Drawings, fractional notation and no addition	Drawings, fractional notation and addition	Unequal sharing
Yvonne	Zane	Linda	Zelda	Stewart
Carol	Elaine	Cheryl	Kate	
		Caroline	Martin	
			Mary	

Worksheet 24

School A

2 ÷ 8

Category 1	Category 2	Category 3	Category 4
Partitioned 2 into 8 equal parts (one unit into 4 equal parts)	Partitioned each unit into an appropriate number of fractional parts	Wrote down the correct solution	Incorrect solution
Drawing and fractional notation	Drawings and words	Elaine	Martin
Yvonne	Zelda	Zane	
Carol	Linda		
Stewart	Kate		
Cheryl			
Caroline			
Mary			

Worksheet 24

School A

9 ÷ 4

Category 1			Category 2	Category 3
Economic Sharing – shared out the maximum number of units and partitioned the remaining unit into an appropriate number of fractional parts			Only solution given	Not attempted
Drawings and words	Drawings and fractional notation	Numbers only	Stewart	Kate
Yvonne	Carol	Elaine		
Zelda	Linda	Zane		
	Mary			
	Martin			
	Cheryl			
	Caroline			

APPENDIX C

C1 NAMING FRACTIONAL PARTS -GRIDS SHOWING THE ACCURACY OF THE LEARNERS' RESPONSES IN WORKSHEETS 4

Worksheet 4

School A

	Halves	Thirds	Quarters	Fifths	Sixths	Tenths
Zelda	correct	correct	correct	correct	Correct	correct
Cheryl	correct	correct	correct	correct	Correct	correct
Caroline	correct	correct	correct	correct	Correct	pieces
Carol	correct	correct	correct	correct	Correct	whole (after reflection – tenths)
Mary	correct	correct	correct	correct	Correct	correct
Kate	correct	correct	correct	correct	Correct	correct
Zane	correct	correct	correct	correct	Correct	sevenths (corrected to tenths)
Martin	correct	correct	correct	correct	Correct	correct
Stewart	correct	correct	correct	correct	left out	correct
Yvonne	correct	correct	correct	correct	Correct	correct
Elaine	correct	correct	correct	correct	Correct	correct

<u>Worksheet 4</u>

School B

	Halves	Thirds	Quarters	Fifths	Sixths	Tenths
Lucille	correct	correct	correct	correct	correct	tenths
Lusanda	correct	correct	correct	correct	correct	correct
Tammy	correct	correct	correct	correct	correct	8^{th}/tenth
Phindiwe	correct	correct	correct	correct	correct	correct
Sipho	correct	correct	correct	correct	correct	correct
Mbulelo	correct	correct	correct	correct	correct	correct
Gail	correct	correct	correct	correct	correct	sevenths (corrected to tenths)
Nosisa	correct	correct	correct	correct	correct	correct
Bob	correct	correct	correct	correct	left out	correct
Lyn	correct	correct	correct	correct	correct	correct
Sharon	correct	correct	correct	correct	correct	correct
Sue	correct	correct	correct	correct	correct	correct

C2 Approaches to naming fractional parts

<u>Worksheet 5</u>

School A

Approach 1	Approach 2	Approach 3	Approach 4	Approach 5	Approach 6	Approach 7	Approach 8
Drew the answer and wrote in words	Drew the answer and used words and symbols	Drew only	Drew and used symbols	Drew incorrectly and used symbols	Drew incorrectly and used words and symbols	Words and symbols	Words only
Yvonne	Carol $\frac{1}{2}$	Carol $\frac{1}{7}$	Mary	Martin $\frac{1}{7}$	Caroline $\frac{1}{7}$		
Zelda	Caroline		Stewart				
Elaine			Martin $\frac{1}{2}$				
Zane							
Kate							
Cheryl							

Worksheet 5

School B

Approach 1	Approach 2	Approach 3	Approach 4	Approach 5	Approach 6	Approach 7	Approach 8
Drew the answer and wrote in words	Drew the answer and used words and symbols	Drew only	Drew and used symbols	Drew incorrectly and used symbols	Drew incorrectly and used words and symbols	Words and symbols	Words only
Gail $\frac{1}{2}$			Gail $\frac{1}{7}$			Sue	Sipho
Nosisa $\frac{1}{2}$			Nosisa $\frac{1}{7}$				Lyn
							Lusanda
							Lucille
							Tammy
							Mbulelo

C3

Worksheet 12

School A

Approach 1	Approach 2	Apporach 3
Used symbols	Used words and symbols	Words only
Cheryl	Martin ($\frac{1}{2}\frac{1}{3}\frac{1}{4}\frac{1}{5}$)	Kate $\frac{1}{4}$
Caroline	Stewart	Mary
Martin		
Kate		
Elaine		
Zane		
Zelda		
Linda		
Carol		
Yvonne		

Worksheet 12

School B

Category 1	Category 2	Category 3
Used symbols	Used words and symbols	
Gail	Lusanda	
Bob	Nosisa	
Lucille	Sue	
Lyn		
Phindiwe		
Sharon		
Tammy		
Sipho		
Mbulelo		

APPENDIX D

D1 SOLUTION STRATEGIES USED FOR EQUIVALENCE AND COMPARISON OF FRACTIONS.

Worksheet 3

School A

Solution Strategy 1	Solution Strategy 2	Solution Strategy 3	Solution Strategy 4
Correct solution and reason	Inappropriate reason for fraction chosen	No reason given for choice	No conclusive answer
$\frac{1}{3}$ or $\frac{1}{5}$			
Caroline	Mary		
Martin	Kate		
Stewart	Zelda		
Elaine			
Yvonne			
Carol			
Zane			

$\frac{1}{4}$ of R1 or $\frac{1}{3}$ of R1			
Zelda		Caroline	Carol
Martin			
Stewart			
Mary			
Kate			
Zane			
Elaine			
Yvonne			
$\frac{1}{2}$ of R1 or $\frac{2}{4}$ of R1			
Elaine	Martin		Yvonne
	Stewart		Caroline
	Zane		Carol
	Zelda		
	Mary		
	Kate		

<u>Worksheet 3</u>

School B

Solution Strategy 1	Solution Strategy 2	Solution Strategy 3	Solution Strategy 4
Correct solution and reason	Inappropriate reason for fraction chosen	No reason given for choice	No conclusive answer
$\frac{1}{3}$ or $\frac{1}{5}$			
Lucille	Nosisa	Sue	
Gail	Mbulelo		
Sipho	Zelda		
Tammy			
Phindiwe			
Lusanda			
Lyn			
Bob			

$\frac{1}{4}$ of R1 or $\frac{1}{3}$ of R1			
Gail	Mbulelo	Bob	
Lucille	Lusanda		
Nosisa	Lyn		
Sipho	Phindiwe		
Sue			
Tammy			
$\frac{1}{2}$ of R1 or $\frac{2}{4}$ of R1			
Lucille	Gail	Nosisa	
Sue	Sipho	Phindiwe	
	Mbulelo	Bob	
	Tammy	Lusanda	
	Lyn		

D2 Equivalent fractions formed by learners

<u>Worksheet 4</u>

School A

$\frac{1}{2}=\frac{2}{4}$	$\frac{1}{3}=\frac{2}{6}$	$\frac{5}{10}=\frac{1}{2}$	$\frac{3}{6}=\frac{1}{2}$	$\frac{2}{10}=\frac{1}{5}$	$\frac{4}{10}=\frac{2}{5}$	$\frac{5}{10}=\frac{3}{6}$	$\frac{6}{10}=\frac{3}{5}$	$\frac{8}{10}=\frac{4}{5}$	$\frac{3}{6}=\frac{2}{4}$	'wholes'
Zelda	Zelda	Caroline	Carol	Mary	Mary	Kate	Zane	Elaine		Elaine 2
Cheryl	Cheryl	Carol	Zane	Kate	Kate	Zane	Martin			
Caroline	Caroline	Zane	Martin	Zane	Zane	Martin	Yvonne			
Carol	Carol	Elaine		Martin	Stewart	Yvonne	Elaine			
Zane	Mary			Stewart	Yvonne	Elaine				
Yvonne	Kate			Yvonne	Elaine					
	Zane			Elaine						
	Stewart									
	Yvonne									

e.g. 'wholes': $\frac{2}{2}=1$

<u>Worksheet 4</u>

School B

$\frac{1}{2}=\frac{2}{4}$	$\frac{1}{3}=\frac{2}{6}$	$\frac{5}{10}=\frac{1}{2}$	$\frac{3}{6}=\frac{1}{2}$	$\frac{2}{10}=\frac{1}{5}$	$\frac{4}{10}=\frac{2}{5}$	$\frac{5}{10}=\frac{3}{6}$	$\frac{6}{10}=\frac{3}{5}$	$\frac{8}{10}=\frac{4}{5}$	$\frac{3}{6}=\frac{2}{4}$	'wholes'
Lucille	Lucille	Lucille	Lucille	Lucille	Lusanda	Tammy	Lusanda	Gail	Sue	Lucille 6
Phindiwe	Gail	Bob	Lusanda	Lusanda	Tammy	Gail	Lyn	Lyn	Sharon	Lusanda 5
Gail	Bob	Sue	Tammy	Phindiwe	Gail	Sharon	Sue			Nosisa 5
Lyn	Sue		Sipho	Gail	Sue	Sue				Lyn 3
Sharon			Mbulelo	Nosisa						Sue 4
Sue			Gail	Lyn						Bob 2
			Lyn	Sharon						Tammy 1
			Sharon	Sue						Phindiwe 1
										Sipho 1
										Gail 1

D3 Equivalent fractions learners were able to form

Worksheet 6

School A

Question 1

$\frac{?}{6}=\frac{1}{3}$	$\frac{?}{6}=\frac{2}{3}$	$\frac{?}{10}=\frac{1}{5}$	$\frac{?}{10}=\frac{1}{2}$	$\frac{?}{9}=\frac{2}{3}$
Elaine	Elaine	Elaine	Elaine	Elaine
Yvonne	Yvonne		Yvonne	Yvonne
Caroline	Caroline		Caroline	Caroline
Zelda	Zelda	Zelda	Zelda	
Stewart	Stewart	Stewart	Stewart	Stewart
Martin			Martin	Martin
Kate	Kate	Kate	Kate	Kate
Mary	Mary	Mary	Mary	
Carol	Carol	Carol	Carol	Carol
Cheryl	Cheryl		Cheryl	Cheryl
Zane	Zane	Zane	Zane	Zane

<u>Worksheet 6</u>

School B

Question 1

$\frac{?}{6}=\frac{1}{3}$	$\frac{?}{6}=\frac{2}{3}$	$\frac{?}{10}=\frac{1}{5}$	$\frac{?}{10}=\frac{1}{2}$	$\frac{?}{9}=\frac{2}{3}$
		Sue	Sue	Sue
Nosisa	Nosisa	Nosisa	Nosisa	Nosisa
Sharon	Sharon	Sharon	Sharon	Sharon
Bob	Bob	Bob	Bob	
Gail		Gail	Gail	
		Phindiwe	Phindiwe	
Lusanda	Lusanda	Lusanda	Lusanda	Lusanda
Tammy	Tammy	Tammy		
Sipho			Sipho	
Lyn	Lyn	Lyn	Lyn	Lyn
Lucille	Lucille	Lucille	Lucille	Lucille
Mbulelo	Mbulelo	Mbulelo	Mbulelo	Mbulelo

The above grid indicates which correct solutions the learners obtained.

The following grid indicates how many equivalent fractions the learners could make for ninths, fifteenths and eighteenths.

Worksheet 6

School A

	ninths	fifteenths	eighteenths	Total
Elaine	3	10	5	18
Zane	2	1	2	5 (2 incorrect)
Yvonne	5	0	0	5
Zelda	2	0	0	2
Cheryl	2	0	0	2
Caroline				All incorrect
Stewart				All incorrect
Martin				All incorrect
Kate				All incorrect
Mary				All incorrect
Carol				All incorrect

Worksheet 6

School B

	Ninths	fifteenths	eighteenths	Total
Sue	2	3	1	6 (2 incorrect)
Nosisa	Had no time to complete this section of the worksheet.			
Sharon	3	1	2	6
Bob	2			2
Gail	2	4	5	11
Phindiwe	2	1	1	4 (1 incorrect)
Lusanda	2	1	2	5 (2 incorrect)
Tammy				All incorrect
Sipho				All incorrect
Lyn	1	3	2	6 (1 incorrect)
Lucille	3	2	2	7 (3 incorrect)
Mbulelo	1			1

D4 Correct equivalent fractions formed by learners

Worksheet 6.1

School A

	Halves	thirds	Fourths	fifths	Sixths	Tenths	Total
Carol		2		4	1		6
Yvonne		1		4	1		6 (1 incorrect)
Kate	2	2		3			7
Mary		1	1	2			4 (1 incorrect)
Martin	1	2		4	1		8 (1 incorrect)
Stewart	1	1		4	2		8
Zane	2	2	1	2	1		8 (2 incorrect)
Zelda		2	3				5 (1 incorrect)
Elaine			1	3	1		5
Cheryl	1	1		3	1		6

D5 Approaches used by learners

Worksheet 12

School A

$\frac{1}{3}$ or $\frac{2}{6}$

Approach 1	Approach 2	Approach 3	Approach 4
Correct solution and appropriate reason	Inappropriate reason given	No reason given for his/her choice	No conclusive solution
Cheryl	Martin	Elaine	
Caroline	Stewart		
Kate			
Mary			
Zane			
Zelda			
Linda			
Carol			
Yvonne			

$\frac{1}{2}$ or $\frac{3}{5}$

Approach 1	Approach 2	Approach 3	Approach 4
Correct solution and appropriate reason	Inappropriate reason given	No reason given for his/her choice	No conclusive solution
Cheryl		Stewart	
Caroline			
Mary			
Zane			
Zelda			
Linda			
Carol			
Martin			
Elaine			
Yvonne			

$\frac{2}{4}$ or $\frac{3}{6}$

Approach 1	Approach 2	Approach 3	Approach 4
Correct solution and appropriate reason	Inappropriate reason given	No reason given for his/her choice	No conclusive solution
Cheryl	Martin		
Caroline	Stewart		
Kate			
Mary			
Zane			
Zelda			
Linda			
Carol			
Yvonne			

$\frac{5}{10}$ or $\frac{2}{5}$

Approach 1	Approach 2	Approach 3	Approach 4
Correct solution and appropriate reason	Inappropriate reason given	No reason given for his/her choice	No conclusive solution
Cheryl	Carol	Stewart	
Caroline		Kate	
Martin		Mary	
Zane			
Zelda			
Linda			
Elaine			
Yvonne			

$\frac{1}{7}$ or $\frac{1}{8}$

Approach 1	Approach 2	Approach 3	Approach 4
Correct solution and appropriate reason	Inappropriate reason given	No reason given for his/her choice	No conclusive solution
Caldene	Zane	Stewart	
Caroline		Kate	
Martin		Mary	
Linda		Yvonne	
Carol			
Elaine			

$\frac{7}{15}$ or $\frac{4}{8}$

Approach 1	Approach 2	Approach 3	Approach 4
Correct solution and appropriate reason	Inappropriate reason given	No reason given for his/her choice	No conclusive solution
Cheryl	Mary	Stewart	
Caroline		Kate	
Martin			
Zane			
Zelda			
Linda			
Carol			
Elaine			
Yvonne			

$\frac{9}{18}$ or $\frac{6}{10}$

Approach 1	Approach 2	Approach 3	Approach 4
Correct solution and appropriate reason	Inappropriate reason given	No reason given for his/her choice	No conclusive solution
Cheryl	Zelda	Stewart	
Caroline			
Martin			
Elaine			
Zane			
Linda			
Carol			
Yvonne			

Worksheet 12

School B

$\frac{1}{3}$ or $\frac{2}{6}$

Approach 1	Approach 2	Approach 3	Approach 4
Correct solution and appropriate reason	Inappropriate reason given	No reason given for his/her choice	No conclusive solution
Gail	Sipho		
Bob			
Lusanda			
Lucille			
Nosisa			
Lyn			
Phindiwe			
Sharon			
Sue			
Tammy			
Mbulelo			

$\frac{1}{2}$ or $\frac{3}{5}$

Approach 1	Approach 2	Approach 3	Approach 4
Correct solution and appropriate reason	Inappropriate reason given	No reason given for his/her choice	No conclusive solution
Lusanda	Gail	Bob	
Lyn	Phindiwe		
Sharon			
Sue			
Tammy			
Sipho			
Mbulelo			
Lucille			
Nosisa			

$\frac{2}{4}$ or $\frac{3}{6}$

Approach 1	Approach 2	Approach 3	Approach 4
Correct solution and appropriate reason	Inappropriate reason given	No reason given for his/her choice	No conclusive solution
Gail			
Bob			
Lusanda			
Lucille			
Nosisa			
Lyn			
Phindiwe			
Sharon			
Sue			
Tammy			
Sipho			
Mbulelo			

$\frac{5}{10}$ or $\frac{2}{5}$

Approach 1	Approach 2	Approach 3	Approach 4
Correct solution and appropriate reason	Inappropriate reason given	No reason given for his/her choice	No conclusive solution
Gail	Lyn		Sipho
Bob			Mbulelo
Lusanda			
Phindiwe			
Sharon			
Sue			
Tammy			
Lucille			
Nosisa			

$\frac{1}{7}$ or $\frac{1}{8}$

Approach 1	Approach 2	Approach 3	Approach 4
Correct solution and appropriate reason	Inappropriate reason given	No reason given for his/her choice	No conclusive solution
Gail	Lyn		
Bob			
Lusanda			
Phindiwe			
Sharon			
Sue			
Tammy			
Sipho			
Mbulelo			
Lucille			
Nosisa			

$\frac{7}{15}$ or $\frac{4}{8}$

Approach 1	Approach 2	Approach 3	Approach 4
Correct solution and appropriate reason	Inappropriate reason given	No reason given for his/her choice	No conclusive solution
Bob	Gail		
Lusanda	Tammy		
Lyn	Mbulelo		
Phindiwe			
Sharon			
Sue			
Sipho			
Lucille			
Nosisa			

$\frac{9}{18}$ or $\frac{6}{10}$

Approach 1	Approach 2	Approach 3	Approach 4
Correct solution and appropriate reason	Inappropriate reason given	No reason given for his/her choice	No conclusive solution
Gail			Mbulelo
Bob			
Lusanda			
Lyn			
Phindiwe			
Sharon			
Sue			
Tammy			
Lucille			
Nosisa			

APPENDIX E

SOLUTION STRATEGIES USED TO SOLVE DIVISION PROBLEMS

Worksheet 9

School A

$5 \div \frac{1}{3}$

Solution Strategy 1	Solution Strategy 2	Solution Strategy 3	Solution Strategy 4	Solution Strategy 5
Drawings were used to solve the problem	Correct solution given – no method	Incorrect solution	Solved problem using numbers	Did not attempt the problem
Elaine		Mary		
Zane		Kate		
Carol		Stewart		
Linda		Martin		
Caroline				
Cheryl				

Worksheet 9

School B

$5 \div \frac{1}{3}$

Solution Strategy 1	Solution Strategy 2	Solution Strategy 3	Solution Strategy 4	Solution Strategy 5
Drawings were used to solve the problem	Correct solution given – no method	Incorrect solution	Solved problem using numbers	Did not attempt the problem
Lucille	Gail	Sipho		
Lusanda	Bob (copied)	Tammy		
Nosisa				
Lyn				
Sharon				
Phindiwe				

Worksheet 9

School A

$2\frac{1}{2} \div \frac{1}{3}$

Solution Strategy 1	Solution Strategy 2	Solution Strategy 3	Solution Strategy 4	Solution Strategy 5
Drawings were used to solve the problem	Correct solution given – no method	Incorrect solution	Solved problem using numbers	Did not attempt the problem
Elaine		Caroline		Cheryl
Zane		Linda		Kate
		Carol		Mary

Worksheet 9

School B

$2\frac{1}{2} \div \frac{1}{3}$

Solution Strategy 1	Solution Strategy 2	Solution Strategy 3	Solution Strategy 4	Solution Strategy 5
Drawings were used to solve the problem	Correct solution given – no method	Incorrect solution	Solved problem using numbers	Did not attempt the problem
Gail	Mbulelo	Lusanda	Sue	
Lucille	Tammy			
Nosisa	Sipho			
Lyn	Bob			
Phindiwe				
Sharon				

Worksheet 10

School A

$20 \div 2\frac{1}{2}$

Solution Strategy 1	Solution Strategy 2	Solution Strategy 3	Solution Strategy 4	Solution Strategy 5
Drawings were used to solve the problem	Correct solution given – no method	Incorrect solution	Solved problem using numbers	Did not attempt the problem
Elaine			Yvonne	
Zane			Carol	
			Linda	
			Zelda	
			Stewart	
			Martin	

Worksheet 10

School B

$20 \div 2\frac{1}{2}$

Solution Strategy 1	Solution Strategy 2	Solution Strategy 3	Solution Strategy 4	Solution Strategy 5
Drawings were used to solve the problem	Correct solution given – no method	Incorrect solution	Solved problem using numbers	Did not attempt the problem
Lucille	Sue	Tammy	Gail	
Phindiwe	Sipho	Mbulelo	Lusanda	
		Bob	Sharon	
		Nosisa	Lyn	

Worksheet 10

School A

$20 \div 1\frac{1}{2}$

Solution Strategy 1	Solution Strategy 2	Solution Strategy 3	Solution Strategy 4	Solution Strategy 5
Drawings were used to solve the problem	Correct solution given – no method	Incorrect solution	Solved problem using numbers	Did not attempt the problem
Elaine		Martin	Yvonne	
Zane		Stewart	Carol	
			Linda	
			Zelda	

Worksheet 10

School B

$20 \div 1\frac{1}{2}$

Solution Strategy 1	Solution Strategy 2	Solution Strategy 3	Solution Strategy 4	Solution Strategy 5
Drawings were used to solve the problem	Correct solution given – no method	Incorrect solution	Solved problem using numbers	Did not attempt the problem
Lucille		Bob	Sue	
		Nosisa	Gail	
		Sipho	Lusanda	
		Lyn	Phindiwe	
			Sharon	

Worksheet 14

School A

$5\frac{1}{2} \div \frac{1}{3}$

Solution Strategy 1	Solution Strategy 2	Solution Strategy 3	Solution Strategy 4	Solution Strategy 5
Drawings were used to solve the problem	Correct solution given – no method	Incorrect solution	Solved problem using numbers	Did not attempt the problem
Yvonne	Carol	Kate	Caroline	
Elaine	Zelda	Mary	Cheryl	
Zane				
Linda				
Martin				
Stewart				

Worksheet 14

School A

$2\frac{1}{2} \div \frac{1}{4}$

Solution Strategy 1	Solution Strategy 2	Solution Strategy 3	Solution Strategy 4	Solution Strategy 5
Drawings were used to solve the problem	Correct solution given – no method	Incorrect solution	Solved problem using numbers	Did not attempt the problem
Carol	Zelda	Mary	Caroline	
Yvonne		Kate		
Elaine		Stewart		
Zane				
Linda				
Martin				
Cheryl				

<u>Worksheet 15</u>

School A

$1\frac{1}{2} \div \frac{1}{5}$

Solution Strategy 1	Solution Strategy 2	Solution Strategy 3	Solution Strategy 4	Solution Strategy 5
Drawings were used to solve the problem	Correct solution given – no method	Incorrect solution	Solved problem using numbers	Did not attempt the problem
Carol	Martin			Cheryl
Yvonne	Stewart			
Linda				
Zelda				
Elaine				
Zane				
Mary (assist)				
Kate (assist)				
Caroline				

Worksheet 17

School A

$8 \div 1\frac{1}{2}$

Solution Strategy 1	Solution Strategy 2	Solution Strategy 3	Solution Strategy 4	Solution Strategy 5
Drawings were used to solve the problem	Correct solution given – no method	Incorrect solution	Solved problem using numbers	Did not attempt the problem
Yvonne	Stewart	Zelda		
Linda	Martin			
Elaine				
Zane				
Carol				
Mary				
Cheryl				
Caroline				
Kate (assisted)				

APPENDIX F

F1 APPROACHES LEARNERS USED TO NOTATE SOLUTIONS OF PROBLEMS THAT INVOLVED ITERATION OF FRACTIONS.

Worksheet 7

School A

Approach 1	Approach 2	Approach 3
Correct solution was given	Solution was drawn	Incorrect solution/Learners did not manage to complete the problem
Yvonne	Linda	Zelda
	Elaine	Kate
	Carol	Mary
	Zane	Stewart
	Caroline	Cheryl

Worksheet 7

School B

Approach 1	Approach 2	Approach 3
Correct solution was given	Solution was drawn	Incorrect solution/Learners did not manage to complete the problem
Lyn	Sharon	Sipho
Lusanda	Sue	Phindiwe
	Gail	Bob
	Nosisa	
	Lucille	
	Tammy	
	Mbulelo	

F2

Worksheet 8

School A

Approach 1	Approach 2	Approach 3
Correct solution was given	Wrote out a number sentence	Incorrect solution
Zelda	Zane	Mary
Cheryl	Elaine	Kate
Linda	Martin – only 2	
	Stewart – only 2	
	Caroline	
	Carol	

Worksheet 8

School B

Approach 1	Approach 2	Approach 3
Correct solution was given	Wrote out a number sentence	Incorrect solution
Mbulelo	Gail	Lyn
Phindiwe	Sue	Nosisa – only 2
Sharon		Lusanda
Tammy		
Sipho		
Lucille		
Bob		

F3

Worksheet 8.1

School A

Approach 1	Approach 2	Approach 3	Approach 4
Correct solution given	Wrote out a number sentence	Learners used fraction notation and indicated the solution	Incorrect solution given
Zane	Martin	Mary	
Elaine	Yvonne	Linda	
	Carol	Zelda	
	Cheryl	Kate	
	Caroline		

School B

Approach 1	Approach 2	Approach 3	Approach 4
Correct solution given	Wrote out a number sentence	Learners used fraction notation and indicated the solution	Incorrect solution given
Sue	Lucille	Tammy – 1 correct	Sipho
	Lusanda	Mbulelo – 1 correct	Lyn
	Gail	Phindiwe – 1 correct	Tammy majority
	Nosisa only did 1		Phindiwe majority
			Mbulelo

F4

Worksheet 9

School A

$6 \times \frac{1}{3}$

Approach 1	Approach 2	Approach 3	Approach 4
Wrote out a number sentence	Drew the solution and used fraction notation	Drew the solution	Incorrect solution
Carol	Kate	Mary	
Linda	Cheryl	Elaine	
Zane	Caroline	Stewart	
		Martin	
		Caroline	

F4

School B

$6 \times \frac{1}{3}$

Approach 1	Approach 2	Approach 3	Approach 4
Wrote out a number sentence	Drew the solution and used fraction notation	Drew the solution	Incorrect solution
Sue		Lyn	Mbulelo
Phindiwe		Nosisa	Sipho
Sharon		Lusanda	Tammy
		Lucille	Gail
			Bob

F5

Worksheet 21

School A

$\frac{2}{3} \times 2$

Approach 1	Approach 2	Approach 3
Wrote out a number sentence	Drew the solution and used fraction notation	Drew the solution
Yvonne		Zelda
Carol		Kate
Elaine		Mary
Zane		Marine
		Stewart
		Caroline
		Cheryl

F6

Worksheet 19

School A – Addition chains

Approach 1	Approach 2	Approach 3
Completed the chain and converted improper fractions into mixed numbers – did not simplify the fraction	Completed the chain and converted improper fractions into mixed numbers – simplified the fraction	Chain was not completed satisfactorily
Yvonne	Zane	Kate
Carol	Elaine	Mary
Linda		Stewart
Martin		Zelda
Caroline		
Cheryl		

F7 Approaches used to add fractions with different denominators.

Worksheet 29

School A

Approach 1	Approach 2	Approach 3
Solution only	Initially worked out fractional part by sharing	Initially worked out fractional part by accelerated counting
Carol	Carol (for a quarter)	Linda
Yvonne	Yvonne (for a quarter)	Zelda
Zane	Cheryl (quarter and thirds)	
Martin		
Cheryl		
Mary		

F7

Worksheet 27

School A

Approach 1	Approach 2	Approach 3
Solution only	Initially worked out fractional part by sharing	Initially worked out fractional part by accelerated counting
Carol	Stewart (fifths)	Linda
Stewart	Martin (thirds, fifths)	Zelda
Martin	Kate (fifths)	
Kate	Mary (thirds, quarters, fifths)	
Mary (half only)	Caroline (quarter, thirds)	
Yvonne		
Elaine	Cheryl (fifths)	
Zane		
Caroline (half only)		

F7

Worksheet 28

School A

Approach 1	Approach 2	Approach 3
Solution only	Initially worked out fractional part by sharing	Initially worked out fractional part by accelerated counting
Carol		
Linda		
Zelda		
Stewart		
Martin		
Kate		
Mary		
Yvonne		
Elaine		
Zane		
Caroline		
Cheryl		

APPENDIX G

G1 APPROACHES USED TO SOLVE FRACTIONS AS PART OF A COLLECTION OF OBJECTS

School A

Worksheet 18.1

Approach 1	Approach 2	Approach 3	Approach 4
Solutions only	Used accelerated counting	Economic Sharing-out the maximum number of units and partitioning the remaining unit into an appropriate number of fractional parts	Used manipulatives and wrote down the solution
Yvonne	Linda	Zelda	Mary
Carol		Cheryl	Kate
Elaine		Caroline	Martin
Zane			Stewart

G2

Worksheet 22 number 1

School A

Approach 1	Approach 2	Approach 3	Approach 4
Wrote down the solution only	Used primitives	Halved the number	Incorrect solution
Kate	Yvonne	Linda	Martin
Mary	Zane	Zelda	Stewart
Stewart	Carol		
Caroline	Elaine		
Cheryl			

G3

Worksheet 23

School A

Approach 1	Approach 2	Approach 3	Approach 4
Solution only	Used primitives	Economic Sharing-out the maximum number of units and partitioning the remaining unit into an appropriate number of fractional parts	Learners grouped the sweets and drew the solution
Kate	Zane	Linda	Zelda
Martin	Elaine	Caroline	
Stewart		Cheryl	
Mary			

G4

Worksheet 23

School A

Approach 1	Approach 2	Approach 3	Approach 4
Solution only	Economic Sharing-out the maximum number of units and partitioning the remaining unit into an appropriate number of fractional parts	Learners grouped the sweets and used accelerated counting	No method could be identified
Carol	Zelda	Yvonne	Martin
Elaine	Linda		Stewart
Zane	Mary		
	Kate		
	Cheryl		
	Caroline		