AN INVESTIGATION INTO THE PERCEPTIONS OF THE FIRST YEAR MATHEMATICS STUDENTS TOWARDS THE ALTERNATIVE MODE INTERVENTION: UNAM CASE STUDY

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DECLARATION OF ORIGINALITY

I, **Reginald Iipinge (Student Number: 11I3692**) declare that this thesis **an investigation into the perceptions of the first year mathematics students towards the alternative mode intervention: a UNAM case study** is my own work written in my own words. Where I have drawn on the words or ideas of others, these have been acknowledged using the reference practices according to the Rhodes University Education Department Guide to Referencing.

Reginald Iipinge	30 November 2012
(Signature)	(Date)

ABSTRACT

A number of tertiary institutions offer bridging courses and intervention programmes in order to increase the number of students performing well in first year mathematics. At the university of Namibia, the science faculty provides educational opportunities to students who have not met the requirements to proceed with MAT 3511 (Basic Math). Unfortunately, the majority of students are not able to cope with the first year modules in Mathematics and the pass rates are unacceptably low. In the interest of supporting students, the University was prompted to introduce a two mode intervention programme in first year mathematics, namely: the normal mode and the alternative mode intervention. The alternative mode intervention was designed to improve the mathematics achievement of first year students who are considered low achieving or at risk of failure. This programme involves the identification of the lowest attainers in first year mathematics, and the provision of professional and faculty trained tutors and individualized teaching to these students in order to advance them to a level at which they are likely to learn successfully in a normal mode system. This research explores the experience and perceptions of first year students on the alternative mode, in particular as it relates to mathematical strands of proficiency. A second step was to explore how teaching contributes to the learning of mathematics on the intervention programme.

The empirical investigation was done in 4 phases. A questionnaire on mathematics teaching and learning was given to the students during the first phase. During the second phase, two focus group discussions were conducted. Thereafter four interviews were carried out with lecturers, and finally tutorial and lecture observation were conducted. An analysis of these findings led to the identification of the students' experiences on the alternative mode. Analysis of the results indicate that the students identified mathematical proficiency as the central element to their learning, and pedagogical knowledge and exploratory talk were critical aspects of good teaching in the mathematical intervention programme.

Key words: mathematics proficiency, intervention, bridging-gap, transition, perceptions, exploratory talk

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ACRONYMS USED IN THIS STUDY

5YSP/4YSP	Five Year Study Programme / Four Year Study Programme
AARP	Alternative Admissions Research Project
AEC	Australian Education Council
CCK	Common Content Knowledge
DNR	Duality Necessity Repeated-reasoning
IDZ	Intermental Development Zone
KCT	Knowledge of Content and Teaching
NCTM	National Council of Teachers of Mathematics
NIED	National Institute for Educational Development
NSSC/O/H	Namibia Senior Secondary Certificate/Ordinary/Higher
NSSCO	Namibia Senior Secondary Certificate Ordinary level
PBS	Pre-University Bursary Scheme
PCK	Pedagogical Content Knowledge
PISA	Programme for International Student Assessment
RtI	Response to Intervention
SCK	Specialized Content Knowledge
SLANT	Spoken Language and New Technology
TIMSS	Trends in International Mathematics and Science Study
TMI	Targeted Mathematics Intervention

UNAM	University of Namibia
WoT/WoU	Ways of Thinking / Ways of Understanding
ZPD	Zone of Proximal Development

CHAPTER ONE

INTRODUCTION

1.1Introduction

This research study was conducted to investigate the perceptions of first year mathematics students towards an alternative mode of delivery intervention programme. In this chapter I introduce the research study, explain why the study was conducted and summarize the aspects that it will explore. This will be done by looking at the following: the background, rationale, research context, aim of the research, the problem statement, value of the research and research design. The chapter will be concluded by giving a layout of the investigation.

1.2 Background of the study

According to the University of Namibia (UNAM) Science Faculty (2010), the first year study of Mathematics at UNAM has been a general problem for the past 10 years. The pass rates are unacceptably low. This reveals that the majority of the students are not able to cope with the first year modules in mathematics. The pass rates for the three mathematics modules for 2009 are provided in the following table:

Table 1.1: Pass rates of the three Mathematics modules of 2009

	Numbers of students	Admitted to exam (%)	Pass the exam	Overall pass (%)
Basic Mathematics (MAT 3511)	831	615 (74.01%)	420	50.54%
Analytic Geometric, Matrices & Complex Numbers (MAT 3531)	475	237(49.89%)	120	25.26%
Pre-Calculus (MAT 3512)	1006	566(56.26%)	254	25.25%

The UNAM Science Faculty (2010) concluded that the figures in Table 1.1 reveal that the majority of students are not able to cope with the first year modules in mathematics. Many students fail these modules even after repeating them. According to the UNAM Science Faculty (2010) the main reasons for the poor performance were: poor high school curriculum, lack of teaching aids and poorly qualified teachers at high schools.

According to the UNAM Science Faculty (2010), the mathematical content required for first year mathematics was not taught or not adequately taught in high school. This content includes:

- **Basic Mathematics**: Polynomials, sets, partial fractions, absolute value, inequalities, functions and sequences.
- **Analytic Geometry, Complex Numbers and Matrices**: Conic sections, vectors, matrices, systems of linear equations, Cramer's rule, Gaussian elimination and complex numbers.
- **Pre-calculus**: One-to-one functions, composition of functions, inverse function, limit of a function, differentiation, increasing and decreasing functions, integration, and application of calculus.

High school Mathematics consists of three exit levels, namely the Namibia Senior Secondary Certificate Higher level (NSSCH), known as the higher level and the Namibia Senior Secondary Certificate Ordinary level (NSSCO), known as the ordinary level, which make provision for the extended and core levels respectively. These curricula are taught over two years in high school. The core and extended curricula are assessed in separate question papers at the end of the two year course.

The NSSCH Mathematics curriculum consists of 20 topics. These are:

1. Numbers and operations
2. Measures
3. Mensuration
4. Geometry
5. Algebra
6. Graphs and Functions
7. Coordinate Geometry
8. Trigonometry
9. Vectors in two dimensions and transformation
10. Statistics and probability
11. Polynomials
12. Identities, equations and inequalities
13. Vectors
14. Functions

15. Logarithmic and exponential functions
16. Absolute value (Modulus)
17. Trigonometry
18. Sequences
19. Differentiation
20. Integration

The NSSCO curriculum consists of 10 topics, namely:

1. Numbers and operations
2. Measures
3. Mensuration
4. Geometry
5. Algebra
6. Graphs and Functions
7. Coordinate Geometry
8. Trigonometry
9. Vectors in two dimensions and transformation
10. Statistics and probability

According to the National Institute for Educational Development (NIED) (2005) the NSSCO curriculum portrays learning content designed to provide guidance to teachers as to what will be assessed. The learning content is set out in three elements: topics, general objectives and specific objectives. According to NIED (2005) specific objectives are the detailed and specified content of the syllabus, which will be assessed for core and extended levels respectively.

The fact that all 400 registered students were taught in a single group was also seen as contributing to the poor performance. Historically, a number of ways had been attempted to overcome this, but none yielded improved performance. The UNAM Science faculty was thus prompted to introduce a two mode intervention programme in first year Mathematics, namely the normal mode and the alternative mode intervention. The alternative mode intervention was designed to improve the mathematics achievement of first year students who are considered low achieving or at risk of failure. This programme involves: identification of the lowest attainers in first year Mathematics, and the provision of tutorials and individualized teaching to these students in order to advance them to a level at which they are likely to learn successfully in a normal mode system. This study investigates the implementation of the alternative mode in the UNAM first year mathematics programme. The intention underlying this study is to understand, inform and to ultimately contribute to effective insights into the learning of mathematics in the alternative mode intervention programme in particular, and to possibly inform other constructive interventions in higher mathematics education.

The framework for the alternative mode was designed to slow the pace of teaching by covering two modules in the first year and one in the second year, and to provide adequate support to assist students to progress. Student performance in the basic Mathematics first test is used to channel students to respective modes. Those who score less than 40% in this test are required to enter the alternative mode and the others continue with normal mode. The two modes are equivalent in content, National Qualifications Framework (NQF) level and level of credit. The difference is that the alternative mode is taught at a slower pace, allowing time for more examples and the repetition of unmastered content. The split also results in smaller classes, which allows the lecturers to use a more learner-centered approach. In addition, students attend two tutorials a week, offered for two hours from 14h00-16h00 and so spend more time with tutors. Those who are struggling are given the opportunity to review the course content in the supportive environment of a small tutorial. A team-teaching strategy, using 8 support staff was adopted to conduct tutorials. As a final measure, students write a 15 minute test after each tutorial.

This research project was conducted at the main campus of UNAM, in Windhoek. The researcher is a high school Mathematics teacher and a former student at UNAM (2001-2004). The first year mathematics students on the alternative mode of delivery intervention are enrolled for BSc. & BEd Hons. degrees. For some students, Mathematics is a compulsory subject in the course, however for others Mathematics is a prerequisite subject. According to the UNAM Science Faculty (2010), as from 2011, all students who register in the faculty of Science sit for the first class test in mathematics after four weeks of teaching. Those who score a mark of at least 40% are admitted to the modules SMAT 3511 Basic Mathematics, SMAT 3531 Analytic Geometry, Complex Numbers and Matrices and SMAT 3512 Pre-Calculus. Those who score a mark of less than 40%, do the following modules in year 1: SMAT 3580 Basic Mathematics A and SMAT 3590 Analytic Geometry, Complex Numbers and Matrices A and in year 2: SMAT 3570 Precalculus A.

The content of SMAT 3580 Basic Mathematics A, SMAT 3590 Analytic Geometry, Complex Numbers and Matrices A and SMAT 3570 Precalculus A corresponds to SMAT 3511 Basic Mathematics, SMAT 3531 Analytic Geometry, Complex Numbers and Matrices, SMAT 3512 Pre-calculus. In a multi-cultural classroom, the students of the alternative mode of delivery intervention are taught through the medium of English of which is their first or second language respectively. Initially, SMAT 3580 and SMAT 3590 have four lectures per week for 22 weeks, two tutorials per week for 22 weeks and 16 credits respectively. However, SMAT 3570 has four lectures per week for 28 weeks, two tutorials per week for 28 weeks and 16 credits.

1.3 Rationale

The rationale of this study lies in its potential to explore the conception of the mathematics intervention programme for first year mathematics students at UNAM and so contribute to further improvement of this programme. Therefore if high school teachers share the challenges faced by first year university mathematics students and inform their learners, it could increase opportunities to attract learners to take the higher level as a preparation for university mathematics. This research emphasises the importance of coherent, group work dynamics that involve elaboration and justification, and that lead to reasoning. The mathematical skills gained on the intervention programme are essential for a successful transition from high school mathematics to university mathematics. It can also increase opportunities to facilitate a student’s transition into higher education.

1.4 Research Context

The characteristics of first year students in undergraduate university mathematics courses have recently changed drastically (Selden, 2005). This change is attributed to a number of factors. Firstly, the mathematics background of many students is not advanced enough for university entry. Fraser & Killen (2003) state that a number of students entering South African Universities come from a wide range of social and cultural backgrounds. As a result many universities are now offering mathematics intervention programmes to provide students with the required mathematics as well as to bridge the gap in mathematical content not learned in high school (Wood, 2001; Kajander & Lovric, 2005; Jennings, 2009, Harrison & Robinson, 2009, Galligan

& Taylor, 2008). Programmes that ease the transition to higher education (Kajander & Lovric, 2005) ought to benefit mathematics students to better develop mathematical proficiency (Kilpatrick, Swafford & Findell, 2001).

As a high school mathematics teacher I came to realize that a high level of mathematics is struggling to attract interest even at school level. Gordon & Nicholas (2012) conclude that failure to choose higher level mathematics courses in high school can have serious consequences both for a student's success in university mathematics and for whether a student continues with his or her mathematical studies. Leviatan (2008) argues that while school mathematics tends to concentrate on problem solving skills, tertiary mathematics is more abstract and emphasizes the inquisitive as well as the rigorous nature of mathematics.

The nature of mathematics knowledge, understanding and skills that people need today has led Killpatrick et al. (2001) to select the term 'mathematical proficiency' to capture what they think it means for anyone to learn mathematics successfully.

1.5 Aim of the research

The aim of this research is to investigate the influence of the alternative mode of delivery on students learning and development of mathematical proficiency. This will involve developing an understanding of how the teaching and learning context of the mathematics remedial mode intervention programme supports effective student learning. The focus will be on engagement with the experiences of the individuals who are part of the intervention programme and the meaningful insights they made of their learning experiences. The researcher will explore student experiences in this programme, in particular as they relate to the development of strands of mathematical proficiency.

In order to achieve this aim, the following objectives are formulated:

- To gather data of the students' perceptions in learning Mathematics through the alternative mode of delivery, in particular as it relates to the development of strands of mathematical proficiency

- To analyse comprehensively the gathered data in order to explore students' experiences in learning mathematics through the alternative mode of delivery intervention.

1.6 Problem statement

A method was been designed to answer the following research questions.

The over-arching question: What are the perceptions of the first year mathematics students toward the alternative mode intervention?

The following sub-questions with relation to the intervention programme were investigated:

1. What are the experiences of first year mathematics students in the alternative mode of delivery?
2. What influence does the alternative mode of delivery have on the student learning experience?
3. How does this student learning experience influence the development of their mathematical proficiency?
4. What are the potentials of the alternative mode intervention as a vehicle for first year mathematics teaching?
5. What effective constructs, if any, can be observed within the mathematics teaching on the alternative intervention programme?

The term 'experience' (sub-question 1) was used because it encompasses several criteria which would be required of a successful teaching outcome: mathematical proficiency as described in Chapter 2 (Section 2.5). It is crucial to note once again that experience here refers to the students' experience of all aspects of the intervention programme.

1.7 Value of the research

The important role that intervention programmes play in first year mathematics is acknowledged by most educators and researchers (Gordon & Nicholas, 2011), but it is a fact that the nature of mathematics intervention in first year mathematics is a quite crucial and decisive element (Wood, 2001, 2004, Bahr, 2008, Gordon & Nicholas, 2012).

Mathematics intervention programmes have been a part of tertiary mathematics teaching for many years. Lecturers and researchers have been seeking answers to what they can do to help first year mathematics students to bridge the gap between high school and university and make the transition to higher education mathematics. The identification of students' experiences may be of value in the following ways:

- It might lead to better mathematics proficiency
- The identification of mathematics content that is not taught or not adequately taught in high school may indicate what mathematics learning and teaching may bridge the gaps.
- Identifying ways of reducing mathematics anxiety that may enhance learning
- Identification of effective teaching strategies that may be employed to mediate the learning process.

1.8 Layout of the thesis

This study consists of six chapters, including this introductory chapter. The layout of the remainder of this thesis is as follows:

Chapter 2 gives an overview of the literature study. I explore the nature of mathematics intervention programmes. I also outline research into students' experiences of mathematics intervention programmes. Transition and bridging the gap into the higher education also receives attention. Important aspects regarding exploratory talk, scaffolding, apprenticeship, and collaborative small groups in mathematics are also included.

In Chapter 3 the empirical investigation is described. This describes the research design and methods used to collect the research data, namely questionnaires, focus group discussions, lecture interviews, lecture and tutorial observation and document analysis. I also outline how data was analysed.

Chapter 4 shows the data analysis, and representations that were gathered during the investigation. An analysis of the responses in the different data collection methods is given. The analysed data are interpreted in terms of the literature review findings. Chapter 5 discusses the important themes arising from the data analysis.

Finally, Chapter 6 concludes the study with the summaries of the findings. The limitations of this investigation are described by the researcher. The chapter is concluded with recommendations for further research in mathematics intervention and suggests possible areas for future investigation.

CHAPTER TWO

LITERATURE REVIEW

Theoretical Background to the Study of First year Mathematics Intervention Programmes

2.1 Introduction

This Chapter presents the review of literature relevant to mathematics intervention programmes at tertiary institutions. The historical development of Mathematics Intervention courses is described. The views of various authors and researchers on the nature of mathematics intervention programmes and mathematics teaching are also described. In order to strengthen the theoretical perspectives of this research, I will review literature on implementing mathematics tertiary interventions, mathematics learning, aspects of exploratory talk, scaffolding, extreme apprenticeship, as well as collaborative small groups. The significance of undertaking large class teaching of Mathematics at university level as well as students' conceptions of mathematics bridging courses will be reviewed. I will further review literature on issues in bridging between senior secondary and first year university mathematics, as these reflect my interest in enabling and constraining factors for the support of tertiary mathematics at university level. Thereafter, the nature of mathematical proficiency, in particular as it relates to mathematics teaching and learning is discussed. A pedagogical stance oriented within a theoretical framework called Duality Necessity Repeated-reasoning (DNR)-based instruction in mathematics is also discussed and the concept of Pedagogical Content Knowledge (PCK) will also be reviewed as this helped me understand mathematics learning, knowledge and teaching respectively. In order to address the research problem, the review of the literature will follow the conceptual map provided in Figure 2.1.

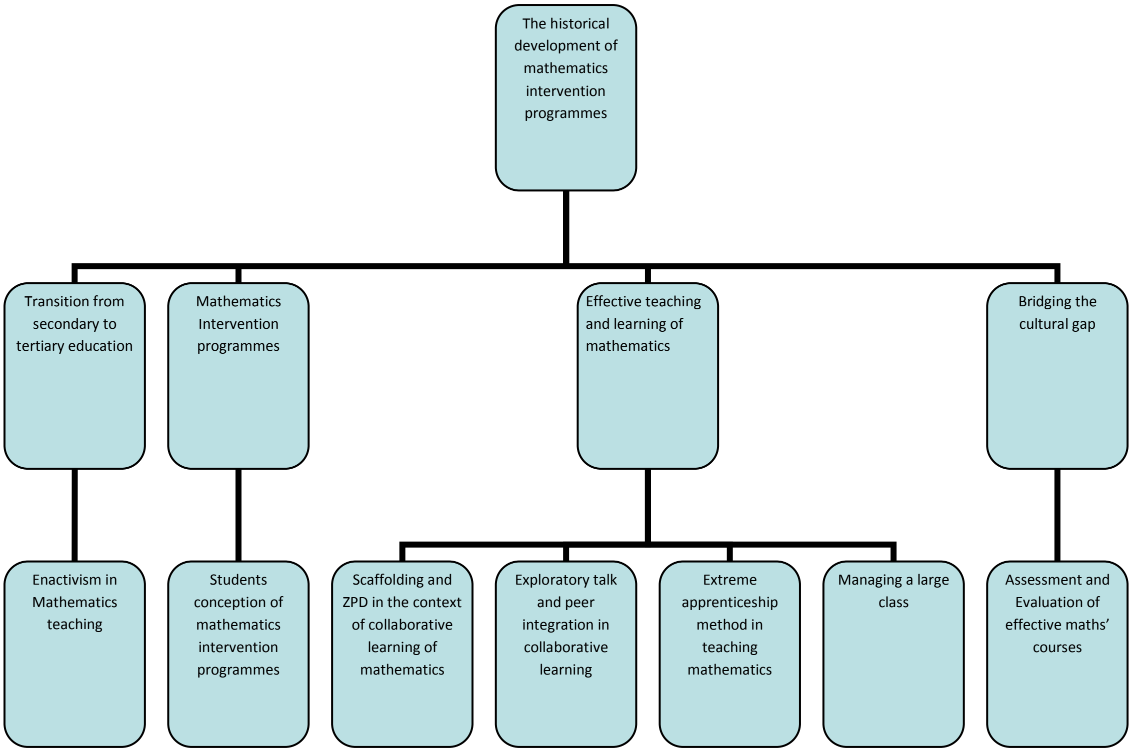

Figure 2.1 A conceptual map of the review of literature

2.2 The Historical Development of Mathematics Intervention Programmes

According to Wood (2001), Mathematics is important for university study in a wide range of disciplines and not studying mathematics can severely limit one's choice of degree programme. In recent years, research in mathematics education has highlighted the difficulty of failure in Mathematics learning. At university level, the failure rate is untenable.

Dowker (2004) mentions that the range of students in different undergraduate university mathematics courses has changed. This change relates to a number of factors. Fraser & Killen (2003) state that a number of students entering South African universities come from a wide range of social and cultural backgrounds. Zevenbergen (2001) argues that students in universities have more diverse backgrounds, both academic and cultural, than ever before.

Gordon & Nicholas (2011) accentuate the valuable opportunities that mathematics bridging courses present. They also investigate challenges in teaching and learning mathematics as students are being inducted into a university study of the displine. According to them Mathematics bridging students are diverse in educational and mathematical background, intended degree program, confidence and willingness to engage with mathematics and expectations about their study. Hence the bridging environment represents a microcosm of the diversity that is a hallmark of introductory mathematics classes in higher education (Gordon & Nicholas, 2011).

2.3 The Transition from Secondary Education to Tertiary Education

The transition from secondary education to tertiary education has been remarked as an area of particular research interest. Gordon & Nicholas (2012) stress that there has been considerable research in the area of students' transition to university. One area of investigation concerns the fit between the students' prior context of education, such as school, and university education (Gordon & Nicholas 2012). According to Selden (2005), universities in Hong Kong see four factors that make the transition more difficult: students are less well-prepared; the fast pace of university courses; expected mathematical rigour; and the examination system. They have introduced bridging courses, included more high school mathematics in their first year courses, and are contemplating using computers to make abstract mathematics more concrete.

Leviatan (2008) conjectures that there is a distinct cultural gap between school mathematics and tertiary mathematics. While school mathematics tends to concentrate on problem solving skills, tertiary mathematics is more abstract and emphasizes the inquisitive as well as the rigorous nature of mathematics. Many first year college students find it difficult to adapt to a culture where concepts are abstract yet require rigorous definitions, theorems have to be proved, and their assumptions meticulously verified before their results can be applied (Leviatan, 2008). According to Jennings (2009), numerous universities are investigating and trying to improve their students' transition.

Varsavsky (2010) argues that in order to attract more students to mathematics and mathematics-based disciplines, and to improve retention, universities have been addressing the under-preparedness in mathematics of their incoming students with bridging or remediation programmes and, more generally, with programmes that support student transition from secondary school to university mathematics study. In South Africa, there is an additional problem with the transition to mathematics at university (Wood, Carmody & Godfrey, 2006). According to them, students from disadvantaged backgrounds do not perform academically well at school due to historical under-resourcing of their schools. Frith, Frith & Conradie (2006) report that universities in South Africa have instituted a national test to assess the abilities of students to learn mathematics rather than assess their current knowledge. According to them students are given written instructional materials of unknown content, with questions that assess the students' abilities to comprehend the given instructions, work with definitions and generalize as well as make valid deductions. Frith et al., (2006) report that the test was used with students after the transition period at university as well as other students who gained university entry through normal school selection criteria.

Wood et al., (2006) conducted research to explore the issues around diagnostic testing of first year students in mathematics and they give an example of the use of such a test in a large first year subject. They compare the scores on the diagnostic test with the end of semester results for 2005 and conclude that the diagnostic test was useful in alerting those students who were seriously under-prepared for mathematics at university. According to them, a small amount of pain for the students in their first lecture can lead to a positive outcome.

According to Gordon & Nicholas (2012) various factors have been identified that ease the transition of students into university, enhance the early student experience and appear to contribute to improved rates of retention. These include activities that help students find their feet, make friends and get to know other students on their programme (Trotter & Roberts, 2006, as cited in Gordon & Nicholas, 2012); learning-to-learn programmes (Zeegers and Martin, 2001, as cited in Gordon & Nicholas, 2012) and workshops facilitating the early formation of social networks and peer groups (Peat, et al., 2001, as cited in Gordon & Nicholas, 2012).

Taylor (1999) stressed that the numerous factors researched that can reduce transition of first year students into university that are believed to overcome improved rates or retention. These include:

- A short bridging course before entry

- Person-to-person support for short periods after entry
- Changes to curriculum structure of the mainstream unit independent of mathematics support initiatives
- Short workshops or tutorials after entry
- Integration of bridging and mainstream units
- Overlay of specialized programs(e.g. supplemented instruction)

Fischer, Biehler, Hochmuth & Wassong (2007), remark that the transition between school and university studies is a difficult one. Fischer et al., (2007) carried out a study on designing and evaluating blended learning bridging courses in mathematics. They report on the project VEMA – "Virtuelles Eingangstutorium Mathematik (Virtual Entrance Tutorial for Mathematics) (http://www.mathematik.uni-kassel.de/vorkurs) started in 2003 for the development of multimedia resources primarily for supporting pre-term bridging courses, which try to bridge this gap. VEMA was initiated at the University of Kassel and was extended to the University of Darmstadt and Paderborn (Fischer et al., 2007). In this project students were given self-directed and externally regulated types of instructional formats in which content was structured into small learning units called modules. Fischer et al., (2007) reported that the finding results substantiate the acceptance and success of the course.

Leviatan (2008) conducted a study that focused on a four-year secondary school teacher training programme. According to Leviatan (2008) the first step in building a transition programme was to reorganize and compress the pre-calculus courses of the traditional mathematics programme in order to enable the addition of the new pre-calculus courses. The effectiveness of the innovative teaching methods employed in the transition programme like Project Based Learning, Questionnaire Based instruction, self-study, group study and workshops, a transition programme is a means of giving students a final polish on their knowledge base just prior to their embarking on a career in mathematics (Leviatan, 2008). Leviatan (2008) stresses that it is not trivial to measure scientifically the success of a programme aimed at achieving all the goals. He further states that interviews with students served as an indication of the programme's success in improving the preparedness of students for tertiary mathematics.

Cuoco, Goldenberg & Mark, (1996) stresses that high school students have studied something in school that has been called mathematics but has very little to do with the way mathematics is created or applied outside of school. According to him, one of the reasons for this has been a view of curriculum in which mathematics courses are seen as mechanisms for communicating established results and methods for preparing students for life after school by simply giving them a bag of facts.

Godden & Pegg (1993) identified three primary reasons why many tertiary students find themselves mathematically underprepared for their chosen studies, firstly the mathematical skills of the student are deficient, secondly, the depth of treatment (exposure) to various mathematics topics is limited, thirdly, there is an increasing reliance on mathematics techniques and concepts in subjects and courses not traditionally mathematically orientated.

Fraser & Killen (2003) acknowledge that when students are admitted to a higher education institution there is a tacit assumption that they will be capable of successfully completing the course in which they are permitted to enroll. Fraser & Killen (2003) carried out an empirical investigation at the university of Pretoria that aimed to identify and categorise the post-enrolment factors that lecturers and students see as having important influences on student success.They further state that to knowingly admit students who, for whatever reason, have no chance of academic success would be immoral. Therefore, it is necessary to have entry requirements that permit valid student selection decisions to be made. According to them, students results show that a better understanding of the mechanisms and functions of the institution would contribute to their success in the institutional programme of their choice. However, on the other hand, some lecturers point out that assisting students to become fully integrated in the social structures of the university would be an asset worth pursuing in a more effective and progressive learning environment(Fraser & Killen, 2003).

Tall (1991) describes the transition to formal thinking in mathematics, where transition problems to formal thinking have been encountered due to a more sophisticated form of embodiment and symbolism through the structure of theorems. A wider awareness of the mental structures that we are born with, of embodiment and symbolism and their subtle effects on the students' transition to formal mathematical thinking, offers the possibility of explicit discussion between mathematicians and students of the nature of the transition that occurs in the learning of formal mathematics (Tall, 1991).

Du Preez, Steyn & Owen (2008) postulate that at the University of Pretoria they have increasingly become aware of freshmen engineering students' lack of understanding of fundamental mathematical concepts. In addition, they found that another consistent problem area in the background knowledge of freshmen engineering students is a lack of competence in communication skills, especially technical (including mathematical) communication skills (Du Preez et al., 2008).

Pilgrim (2010) conducted research on measuring students' achievement in calculus where undergraduate students enrolled in calculus for natural science and engineering majors at national four year post-secondary institutions. According to Pilgrim (2010), the percentage of students completing the first semester of calculus with grade C or better has not improved much since the 1980s. It lingers around 60%, leaving 40% of students receiving a grade of D or F, dropping the course, or withdrawing from the course.

Fraser & Killen (2003) state that the practice of using school matriculation results as the sole or primary determinant for university entrance is common in many countries, such as Australia and the USA, where there is strong competition for university entrance. Graham (1991) acknowledges that the utility of the techniques of using matriculation results has been limited. Fraser & Killen (2003) remark that the fact that so many elements can be important is probably the main reason that single measures based on previous academic success, particularly at school, are not strong predictors of success at university. This is in line with the national statement (Australian Education Council., 1991) that:

> Whatever their particular needs or abilities, all students have the right to learn mathematics in a way that is personally challenging and stretches their capabilities. Achievable and satisfying tasks are an important prerequisite for success. (p.10)

Varsavsky (2010), remarks that one of the difficulties with raising the mathematics skill base is the under preparedness of students entering higher education, which results in low levels of success and engagement with university level mathematics. The increasingly weaker mathematics background of university entrants and its consequences have been reported around the world (Varsavsky, 2010). Barrington (2010) relates this to the fact that fewer Australian students study mathematics at the higher level in the senior year of secondary school.

Gordon & Nicholas (2012) emphasize that even though the study of mathematics at university is essential for a wide range of undergraduate programmes, including science, medical fields, engineering, agriculture, pharmacy, economics and business, there is some state variation. Australian students can choose to study mathematics in senior secondary school at essentially three levels: elementary, intermediate and advanced. In addition, Gordon & Nicholas (2010) ascertain that in New South Wales, seven universities run bridging courses in mathematics annually - an indication that these are considered a valuable resource for helping prepare students who have not studied the required level of mathematics prior to entering university. Hence information about teaching and learning in bridging courses is significant to the efforts to ameliorate students' difficulties with mathematics, and help reduce attrition in first year for 'at risk' students.

2.4 Mathematics Intervention Programmes

Gordon & Nicholas (2011) defined mathematics intervention courses as preparatory courses that are intensive, with 40 hours or less of instruction. According to McGillivray, as cited in Gordon & Nicholas (2010) a mathematical bridging program is any preparatory programme that enables a prospective student to obtain prerequisite or assumed knowledge in mathematics before commencing their degree programme. Dowker (2004) clarifies that it is desirable that interventions should take place at an early stage. This is not because of any critical period or rigid timescale for learning. The age of starting formal education has little impact on the final outcome of learning (Trends in International Mathematics and Science Study (TIMSS, 1996)). People who, to varying degrees, lack opportunity for, or interest in learning arithmetic in school, may learn later as adults (Evans, 2000).

It is often important to distinguish between tools of access and tools of intervention. According to Dowker (2004) tools of access are means of circumventing a difficulty which does not affect mathematics learning directly, but which may interfere with a child's benefit from standard forms of mathematics teaching or mathematical activities. Tools of intervention involve remediating, or in some cases preventing difficulties with mathematical learning itself (Dowker, 2004). Hunting & Pearn (1995) stress that a mathematics intervention programme is designed to enable children to succeed at mathematics activities. The Institute of Education Sciences (IES) (2009) clarifies that students struggling with mathematics may benefit from early interventions aimed at improving their mathematics ability and ultimately preventing subsequent failure.

Gordon & Nicholas (2012) state that a mathematics bridging course is often a student's first experience of learning in the university environment and offers the chance to meet other students and university teachers. Hence, the way in which students experience a mathematics bridging course could have a profound effect on students' performance in first year mathematics topics and could also play an important part in supporting students during a difficult period of transition, potentially helping to reduce early attrition at university.

Teacher Created Materials (TCM) (2008b) reflect that to have a mathematically literate society, the population needs to have an understanding of and proficiency in mathematics concepts and procedures, as well as the ability to apply that knowledge and use it to develop models, and apply those models to similar situations. The article emphasizes how essential it is that education systems try to meet the mathematical needs of all students before they fail. For this reason, mathematical intervention is critical. The Targeted Mathematics Intervention (TMI) is basically a research-based program that aligns with the National Council of Teachers of Mathematics (NCTM) Principles and content standards, while meeting the needs of students and teachers. The TMI has achieved teaching materials that stress not only procedural fluency but also conceptual understanding within an intervention framework for instruction.

According to Hunting & Pearn (1995) the importance of mathematics intervention programmes to students mathematically at risk cannot be over-emphasized. As stated by the national Statement Council (AEC, 1991): "whether a particular student gains the full benefit form mathematics may be influenced by a range of personal characteristics and circumstances. It will also depend on the quality of the mathematics offered" (p.8).

Zan (2000) emphasizes that we often observe the failure in Mathematics of students, who probably possess the necessary knowledge, but seem unable to use it. Zan (2000) conducted research into metacognitive intervention in Mathematics at *Universita di Pisa* in Italy. The study was aimed at improving the performance in Mathematics of a group of university students of biology who repeatedly failed the final examination of a compulsory course in mathematics. The programme was a classic curriculum of basic mathematics, and includes calculus, linear algebra and geometry. Students wrote the examination in two parts: a written test and oral examination. The intervention was successful. At the end of the course all students passed the examination that they had previously repeatedly failed. Indeed, metacognitive and affective factors can inhibit the correct utilization of knowledge that the students do, in fact, already possess. Zan's (2000) research aimed to verify the effectiveness of an instructional intervention, which explicitly involved the following metacognitive and affective features: knowledge about cognition, monitoring, beliefs, emotion and attitudes. Zan (2000) concluded that it may be possible (and necessary) to "teach learning to learn" mathematics.

Boland (2009) highlighted a bridging program with courses in mathematics, physics, chemistry and Communication at the Division of Information Technology, Engineering and the Environment at the University of South Australia. The bridging course programme goal was to provide an alternative pathway for prospective students to gain access to a science or engineering degree programme, where innovative methods were devised to try and fill some of the gaps in the students' backgrounds (*ibid*.). According to Boland (2009) mathematics students were exposed to computer software for problem solving, and utilized Matlab and Maple. The students were also exposed to the spreadsheet as a remarkable tool for both doing and illustrating mathematics. Additionally, the mathematics segment of the bridging programme proved to be a good indicator of success in mathematics courses in the degree programme (*ibid*.). Boland (2009) concluded that was also shown that the study materials developed for the course involving explaining mathematical concepts using Excel spreadsheets were rated as successful by the students (*ibid.*).

Bernstein (2003) conducted a study about an engineering bridging course success or failure at the University of Witwatersrand. The aim of the study was to evaluate the success of a one-year undergraduate bridging course in Engineering offered to educationally disadvantaged students, with special emphasis on the role of mathematics in addressing and overcoming some of the problems encountered by engineering students. These problems include the inability to relate classroom examples to the real world, and the importance of students of making approximations and estimates in the absence of calculators. The bridging students' results were compared with the results of engineering students in the same class who were not exposed to the special attention the bridging students received in their pre-university year. He concluded that the mathematics marks of the Pre-University Bursary Scheme (PBS) students were better than the marks of the students not attending the bridging course in their first year. This is because PBS students had a distinctive advantage as much of the work was familiar to them, and their pass rates were also significantly higher. Bernstein (2003) further concluded that although the students were academically disadvantaged when they started the PBS course, by the second year there was no difference between the PBS students group and the mainstream students.

Du Preez et al., (2008) conducted research to investigate first year student's preparedness for a study in calculus at the University of Pretoria. The purpose of the 5YSP (Five Year Study Programme) was to create opportunities for students who had the potential to become engineers, but who did not meet the entrance requirements to enroll for the standard 4YSP (Four Year Study Programme) and/or who were academically at risk of not coping with the high demands of engineering study. According to Du Preez et al., (2008) the results indicate that the developmental intervention had a long-term effect and that the learnt skills are transferred to subsequent (mathematics) modules. This is because the students lacked the mathematical skills they needed in relation to a solid foundation in mathematics at the calculus level, and engineering students will often find difficulty in understanding and applying the knowledge involved in the upper level of engineering classes.

Du Preez et al., (2008) affirm the necessity of rethinking the learning content and the learning facilitation strategies to purposefully focus mathematics interventions so that the process of learning is promoted and encouraged. This is in line with Zucker (2000) who states that one should keep in mind that “no style of teaching mathematics can substitute for insisting students to pick up their share of the work, unless one is willing to compromise the standards.” (p. 277)

Dray, Edwards & Monogue (2008) reflect on the Vector Calculus Bridge Project at Oregon State University which was carried out to understand the differences in perspective between mathematicians and physicists and why these differences cause transition problems for students. According to them, there are several conclusions that one can draw from the study. Firstly, when comparing the two groups of students as a whole, those using Bridge Project materials did appear to reason in a more visual way. They defined geometric reasoning broadly to mean thinking about and reasoning from geometric objects, including, but not limited to, graphs of functions and/or equations. Students using Bridge Project materials were more likely to use such geometric arguments. Secondly, the enthusiasm that students had for the laboratory activities seem important. Finally, the preparation of instructors to use the materials is important in that the Bridge Project provides a crucial step on the road to achieving fluency in both languages Dray, et al. (2008).

Cooper, Ellis & Sawyer (2000) remark on the bridging course at the University of South Australia that played a vital part in the students’ experiences and enculturation to the university environment. The aims of the bridging programme included: preparing individuals for degree entry; enabling them ‘to gauge their own capacities in a tertiary environment’; facilitating ‘the transition of disadvantaged students from their previous experience to the culture of an academic system’;, plus enhancing student confidence to acquire skill ‘pre-requisites to first year undergraduate study’ while highlighting the importance of student acclimatization to a university environment. They concluded that there were many success stories in the subsequent tertiary study of bridging students, and the aims were achieved.

Baker, Gersten & Lee (2002) did a synthesis research study on the effects of interventions to improve the mathematics achievement of students considered low achieving or at risk for failure. Low achieving students in the study were identified on the basis of their performance on standardized or informal tests or by their placement in remedial mathematics classes. According to them the results indicated that different types of interventions led to improvements in the mathematics achievement of students experiencing mathematics difficulty, including the following:

(a) Providing teachers and students with data on student performance,
(b) Using peers as tutors or instructional guides,
(c) Providing clear, specific feedback to parents on their children's mathematics success, and
(d) Using principles of explicit instruction in teaching mathematical concepts and procedures.

According to (Baker, et al., 2002) they found a burgeoning sense of the concept of mathematical proficiency, as formulated by Kilpatrick, et al. (2001) "the integrated attainment of conceptual understanding, procedural fluency, strategy competence, adaptive reasoning, and productive disposition" (p. 313). Baker, et al. (2002) concluded that their synthesis provides suggestions that can serve as initial steps in the improvement, and perhaps the ultimate transformation of the teaching of mathematics for low achievers.

Godden & Pegg (1993) conducted research on the effective evaluation of tertiary bridging mathematics programmes at the University of Central Queensland and the University of New England. They assert that one of the major strengths of bridging mathematics programs is flexibility, a student-centred approach as well as the desire and necessity to evaluate the programs. They conclude that evaluation, in the traditional sense, may be incompatible with the successful conduct of tertiary mathematics assistance programs.

2.5 The Effective Teaching and Learning of Mathematics

Gordon & Nicholas (2011) describe how the challenges of teaching an increasingly diverse cohort in higher education are felt in every discipline, but arguably more in mathematics units. They stress that Mathematics underpins many different topics in science, engineering and other disciplines. A modern view of learning is constructivism, where students are expected to be active in the learning process by participating in discussions and/or collaborative activities (Fosnot, as cited in Carpenter, 2006, p. 4). Overall, the results of recent studies concerning the effectiveness of teaching methods favour constructivist, active learning methods. According to TCM (2008a) it is crucial to effectively teach students what they need to know and support them to learn it well. A comprehensive teacher's guide offers step by step lessons, so teachers can focus on delivery of instruction rather than development of the lessons (TCM, 2008a).

The findings of a study by de Caprarriis, Barman & Magee, as cited in Carpenter (2006), suggest that lecturing leads to the ability to recall facts, but discussion produces higher level comprehension. The findings of the study by Carpenter (2006) suggest that faculties teaching large classes should attempt to include constructive, active teaching methods in their courses whenever possible. Carpenter (2006) suggested that structured, controlled collaboration (e.g. jigsaw, case study) would be most comfortable for students as opposed to uncontrolled, unstructured experiences e.g. team projects.

Effective teaching can be achieved through using time efficiently in a large class to prepare typed lecture notes for students in advance (Jungic, Kent & Menz, 2006). In contrast, research by Carpenter (2006) examined perceptions across six teaching methods: lecture/discussion, laboratory work, in-class exercises, guest speakers, applied projects, and oral presentations. Most students preferred the lecture/discussion method (Jungic, et al., 2006).

In terms of students' preferences for teaching methods, a study by Samson, Sewry & Southwood (2011) suggests that students prefer team teaching methods. Having two lecturers with different but complementary styles was cited as strengthening the class's overall confidence in the subjects. Hunt, et al., as cited in Carpenter (2006, p.14), noted favourable student attitudes towards active learning methods. Wright, Martland & Stafford (2006) indicate that any child receiving individualized teaching should make progress.

Kilpatrick, et al. (2001) suggest that teachers should play a more active instructional role in helping students build mathematical proficiency than they currently do. Bryant, Bryant, Gersten, Scammanca & Chavez (2008) stipulate that, without early identification, intervention and progress monitoring to determine students' responses to intervention, many students with mathematics difficulties may not develop a level of mathematics automaticity that is necessary for becoming proficient in mathematics. Kilpatrick, et al. (2001) articulate that in order to satisfactorily attain all aspects of expertise, competence, knowledge, and facility in mathematics, they identified mathematics proficiency as necessary for anyone to learn mathematics successfully. According to Kilpatrick, et al., (2001) mathematics proficiency has five components or strands that are not independent. Because these are interwoven, and interdependent, mathematics proficiency is not a one dimensional trait, and it cannot be achieved by focusing on just one or two of these strands. The strands of mathematics proficiency are:

- **Conceptual understanding** - a comprehension of mathematical concepts, operations, and relations.
- **Procedural fluency** - knowing, selecting and performing calculations and procedures or skills.
- **Strategic competence** - the ability to formulate, represent, and solve mathematical problems.
- **Adaptive reasoning** - the capacity for logical thought, reflection, explanation, and justification.
- **Productive disposition** – the habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy (Kilpatrick, et al., 2001,p. 5).

Knowing how to teach mathematics well to students with differing abilities seems to be much more important than having mathematics teachers who possess strong backgrounds in mathematics (Ball, Lubienski, & Newborn, as cited in Baker, 2002, p. 56). Instructors need to have a vast repertoire of effective lecture methods on hand (Jungic, et al 2006). A synthesis of empirical research on the teaching of Mathematics by Baker (2002) stipulates that using peers as tutors or guides enhances achievement. The use of the peers to provide feedback and support improves low achievers' computational abilities and holds promise as a means to enhance problem-solving abilities (Baker, 2002).

According to Fraser & Killen (2003) there is ample evidence in the literature on teaching and learning to suggest that factors such as teaching strategies (Bartz & Miller, 1991), students' motivation (Talbot, 1990), students' approach to studying (Meyer, 1990), the interaction between students and the academic and the social systems of the university (Tinto, 1975), cultural expectations (Ginsburg, 1992), psychosocial factors (McKenzie & Schweitzer, 2001) and numerous other dynamics (Watkins, 1984; Logan, 1990; Jacobi, 1991; Keef, 1992; Minnaert & Janssen,1992) which are likely to influence students' success at university.

Kilpatrick, et al. (2001) articulate that just as mathematical proficiency itself involves interwoven strands; teaching for mathematical proficiency requires similarly interrelated components. According to them the components require:

- Conceptual understanding of the core knowledge required in the practice of teaching;
- Fluency in carrying out basic instructions and solving problems that arise during instructions;
- Strategic competence in planning effective instruction and solving problems that arise during instruction;
- Adaptive reasoning in justifying and explaining one's instructional practices and in reflecting on those practices so as to improve them; and a
- Productive disposition toward mathematics, teaching, learning and the improvement of practice (Kilpatrick et al., 2001 p. 380).

Kilpatrick, et al. (2001) stress that as a goal of instruction, mathematical proficiency provides a better way to think about mathematics learning than narrower views that leave out key features of what it means to know and be able to do mathematics. This is quite similar to Harel (2008a) who asserts that mathematics consists of two complementary subsets.

> The first subset is a collection, or structure, of structures consisting of particular axioms, definition, theorems, proofs, problems, and solutions. This subset consists of all the institutionalized ways of understanding in mathematics throughout history. It is denoted by WoU (to denote 'Ways of Understanding'). The second subset consists of all the ways of thinking, which are characteristics of the mental acts whose products comprise the first set. It is denoted by WoT (to denote 'Ways of Understanding'). (p.490)

According to Harel (2008b), Mathematics grows continually as mathematicians carry out mental acts and their mathematical communities assimilate the ways of understanding and ways of thinking associated with the mathematicians’ mental acts.

Harel (2008a) accentuates that the teaching and learning of mathematics can be oriented within a theoretical framework called DNR-based instruction in mathematics (DNR, for short). DNR is a conceptual framework for learning and teaching mathematics. The initials D, N, and R stand for three leading principles in the framework - duality, necessity and repeated-reasoning.

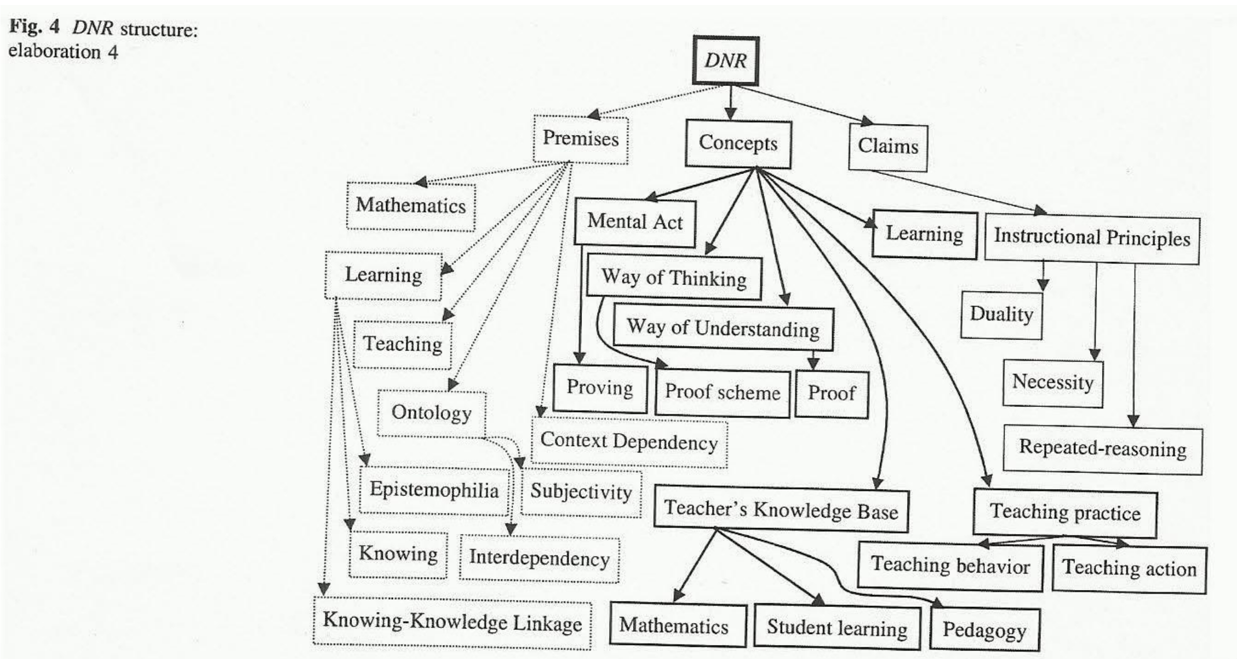

Figure 2.2 DNR structure (adapted from Harel, 2008b)

Harel (2008b) builds his theory of teaching and learning on 8 premises. He conjectures that the premises underlie DNR’s philosophy of mathematics and the learning and teaching of mathematics. DNR has eight premises which are loosely organized in four categories (*ibid.)*:

1. **Mathematics**

   According to Harel (2008b), knowledge of mathematics consists of ways of understanding and ways of thinking that have become institutionalized throughout history.

2. **Learning**

Harel (2008b) delineates that all humans possess the capacity to develop a desire to be puzzled and to learn to carry out mental acts to solve the puzzles they create. Individual differences in this capacity, though present, do not reflect innate capacities that cannot be modified through adequate experience. Harel (2008) called this Epistemophilia:

- Knowing: Knowing is a developmental process that proceeds through a continual tension between assimilation and accomodation, directed toward a (temporary) equilibrium.
- Knowing-knowledge Linkage: Any piece of knowledge humans know is an outcome of their resolution of a problematic situation.
- Context Dependency: Learning is context dependent.

3. **Teaching**

Teaching: Learning mathematics is not spontaneous. There will always be a difference between what one can do under expert guidance or in collaboration with more capable peers and what he or she can do without guidance.

4. **Ontology**

- Subjectivity: Any observations humans claim to have made are due to what their mental structure attributes to their environment.
- Interdependency: Humans' actions are induced and governed by their views of the world, and conversely, their views of the world are formed by their actions (Harel, 2008b p. 894).

There is a dynamic notion of mathematics activities in which students and teachers should engage during classroom activities (Stein & Henningsen, 1997). Students' learning is seen as the process of acquiring a mathematical point of view (Schoenfeld, 1992, 1994 as cited in Stein & Henningsen, 1997). Kilpatrick, et al. (2001) include acquiring mathematical knowledge and tools for working with and constructing knowledge. Harel (2008b) broadens this, saying that while mathematical knowledge is indispensable for quality teaching, it is not sufficient. Teachers must also know how to address students as learners. In DNR, however, the teacher's knowledge of student learning and pedagogy rests on the teacher's knowledge of mathematics, that is to say, although each of the three components of knowledge is indispensable for quality teaching, they are not symmetric: the development of the teacher's knowledge of student learning and of pedagogy depends on, and is conditioned by their knowledge of mathematics (Harel, 2008b).

Kilpatrick, et al. (2001) remark that teaching and learning mathematics is the product of interactions among the teacher, the students, and the mathematics. Harel (2008b) defines mathematics teaching and learning as composed of three components: knowledge of mathematics; knowledge of student learning; and knowledge of pedagogy. If students are to develop the capacity of high-level thinking in a variety of academic domains, then classrooms must become environments in which they have frequent opportunities to engage in dynamic mathematical activity that is grounded in rich, worthwhile mathematical tasks (NCTM, 1991; Schoenfeld, 1994).

The tasks in which students engage, provide the contexts in which they learn to think about subject matter (Stein & Henningsen, 1997). Harel (2008b) argues that students develop ways of thinking only through their construction of ways of understanding, and the ways of understanding they produce are determined by the ways of thinking they possess. Kilpatrick, et al. (2001) puts forth that:

> mathematical instruction takes place in contexts, which is a wide range of environmental and situational elements that bear on instruction-for instance, educational policies, assessments of students and teachers, school organizational structures, school leadership characteristics, the nature and organization of teacher's work, and the social matrix in which the school is embedded. (p. 315).

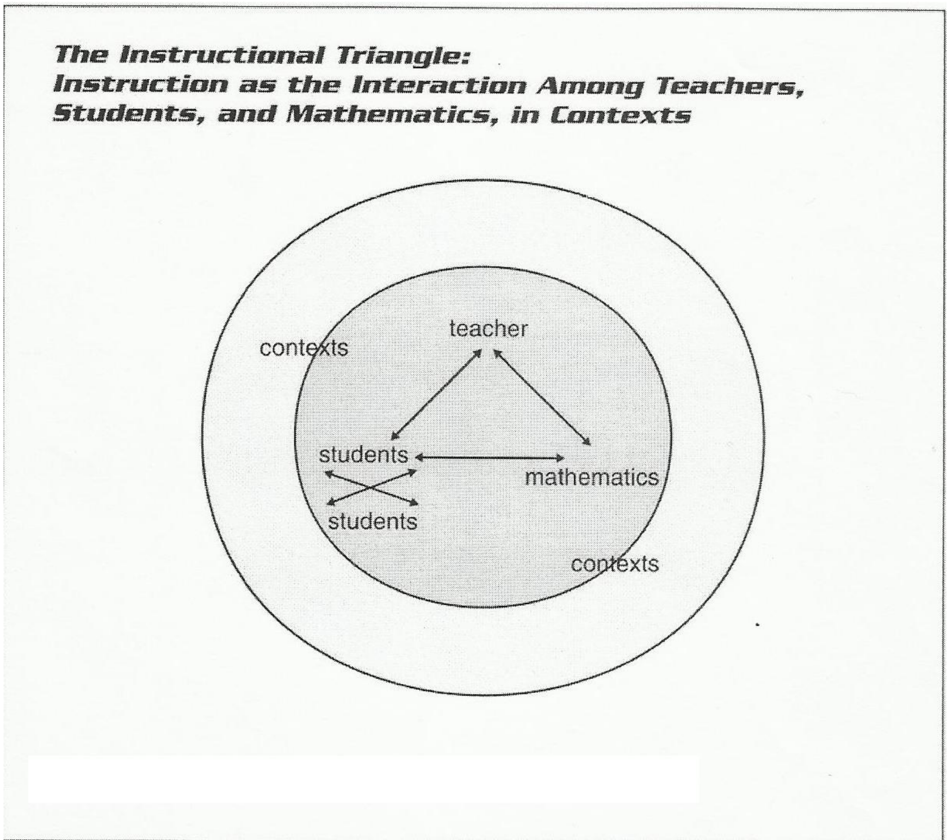

Figure 2.3 Instructional triangle (adapted from Kilpatrick, et al., 2001 p. 314)

The Kilpatrick, et al. (2001) Instructional triangle in Figure 2.3 emphasises effective teaching that fosters the development of mathematical proficiency. Instruction can best be examined from the perspective of how teachers, students, and the content interact in contexts to produce teaching and learning (Kilpatrick, et al., 2001). Cuoco, et al. (1996) accentuate that good thinking must be relearned in a variety of domains. Mental habits allow students to develop a repertoire of general heuristics and approaches that can be applied in many different situations (Cuoco, et al., 1996).

Hamilton (2006) accentuates dualistic and holistic thinking as crucial in the educative process. There are commendable pedagogical approaches used extensively in the middle years of schooling that are consistent with the emergent world, and which encourage students to:

- take action to make their world better place (Murdoch & Hornsby, 1997; Murdoch, 1998 in Halmilton, 2006, p.7)
- become active citizens (Holdworth, 2003 in Hamilton, 2006, p. 7)

- become creative and critical thinkers (De Bono, Fogarty, 1997; 2001; Costa, 2004; Pohl, 2004 in Hamilton, 2006, p.7)
- be meta cognitive (Baird & Mitchell, 1986; Fogarty, 1997; 2001; Costa, 2004; Pohl, 2004 in Hamilton, 2006, p.7)
- self-assess (Fogarty, 1997; 2001; Costa, 2004; Pohl, 2004 in Hamilton, 2006, p.7)

These approaches are dynamic in that they imply the continuous process of becoming' rather than only acquiring knowledge (Hamilton, 2006).

Cuoco, et al. (1996) remark that they would like students to think about mathematics the way mathematicians do and develop useful habits of mind such as:

- Students should be pattern sniffers
- Students should be experimenters
- Students should be describers
- Students should be tinkerers
- Students should be inventors
- Students should be visualizers
- Students should be conjectures
- Students should be guessers

Connections with what students already know and understand also play an important role in engaging students in high-level thought processes (Carpenter & Hilbert, 1992 as cited in Stein & Henningsen 1997).

Tall (1995) stresses that advanced mathematical thinking involves using cognitive structures produced by a wide range of mathematical activities to construct new ideas that build on and extend an ever-growing system of established theorems. Harel (2008b) surmises that cognitive and epistemological issues help teachers to distinguish a variable parameter of transitioning from additive reasoning to multiplicative reasoning.

Tall (1995) emphasizes that the cognitive growth from elementary to advanced mathematical thinking in the individual may therefore be hypothesized to start from the perception of, and action on objects in the external world.

Tall (1995) also stresses specific ways to think mathematically. These include:

- Recognition of patterns, similarities and differences;
- Repetition of sequences of actions until they become automatic; and
- Language to describe and refine the way we think about things.

According to Schoenfeld (1992,1994) students develop their sense of what it means to ’do mathematics’, and their actual experiences with mathematics, and their primary opportunities to experience mathematics as a discipline are seated in the classroom activities in which they engage.

Ball, Thames & Phelps (2008) give credence to Shulman’s (1986) notion of pedagogical content knowledge as a concept to investigate the nature of professionally oriented subject matter knowledge in mathematics, which develops through studying actual mathematics teaching and identifying mathematical knowledge for teaching based on analyses of mathematical problems that arise in teaching. Shulman (1986) defines pedagogical content knowledge as comprising:

> The most useful forms of representation of those ideas, the most powerful analogies, illustration, examples, explanations, and demonstrations-in a word, the most useful ways of representing and formulating the subject that make it comprehensible to others….Pedagogical content knowledge also includes an understanding of the learning of specific topics easy or difficult: the conceptions

and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons. (p. 9).

Harel (2008b) delineates that the quality of instruction is determined largely by what teachers know. Building on Shulman's (1986, 1987) work, Harel (2008b) refines the definition of three components: knowledge of mathematics, knowledge of student learning, and knowledge of pedagogy that is aligned with the definition of mathematics:

- *Knowledge of mathematics* refers to a teacher's ways of understanding and ways of thinking. It is the quality of this knowledge that is the cornerstone of teaching, for it affects both what the teachers teach and how they teach it.

- *Knowledge of student learning* refers to the teacher's understanding of fundamental psychological principles of learning, such as how students learn and the impact of their previous and existing knowledge on the acquisition of new knowledge.

- *Knowledge of pedagogy* refers to the teacher's understanding of how to teach in accordance with these principles. This includes an understanding of how to assess students' knowledge, how to utilize assessment to pose problems that stimulate students intellectual curiosity, and how to help students solidify and retain the knowledge they have acquired.

Ball, et al. (2008) hypothesize that Shulman's content knowledge could be subdivided into common content knowledge (CCK) and specialized content knowledge (SCK) and his pedagogical content knowledge (PCK) could be divided into knowledge of content and teaching. Ball, et al. (2008) define the mathematical knowledge studied as mathematical knowledge "entailed by teaching"- in other words, the mathematical knowledge needed to perform the recurrent task of teaching mathematics to students. They define the first domain, CCK as the mathematical knowledge and skill used in settings other than teaching.

Ball, et al. (2008) cogitates the second domain, specialized content knowledge (SCK), as the mathematical knowledge and skill unique to teaching. The third domain, knowledge of content and students (KCS), is knowledge that combines knowing about students and knowing about mathematics. The last domain, knowledge of content and teaching (KCT), combines knowing about teaching and knowing about mathematics (Ball, et al. 2008).

Many mathematical tasks of teaching require a mathematical knowledge of the design of instruction (Ball, et al., 2008). Hamilton (2006) gives credence to Maturana and Varela's(1992) enactivism learning theory in teaching mathematics - that it emphasizes knowing rather than knowledge and changes the emphasis from the identifiable entities of the individual and society to that of a system.

Hamilton (2006) explains s that if education re-explored learning theory in enactivist terms this would support the development of students and delineate the school as learning web by:

- Encouraging teachers and students to reflect deeply on their practice to understand the purpose of all actions;
- Encouraging teachers and students to research in their own classrooms and develop appropriate curriculum that values the development of students holistically;
- Supporting assessment procedures that value all the domains in which students operate;
- Encouraging student self-assessment;
- Encouraging processes and procedures that obviate dualistic thinking; and
- Valuing the rational (but not at the expense of other (sensual)) ways of knowing.

With this explicit encouragement and support, teacher conversations would focus on the multiplicity of ways in which students learn and how they can provide flexible pathways for their students, rather than on the 'one size fits all' mentality.

2.6 Bridging the Cultural gap

Leviatan (2008) stresses that there is a distinct cultural gap between school mathematics and tertiary mathematics. One of the major difficulties with raising the mathematics skills is the under-preparedness of students entering higher education - which results in low levels of success and engagement with university mathematics (Varsavsky, 2010). Varsavsky (2010) further stresses that in order to attract students to mathematics-based disciplines and to improve retention, universities have been addressing under-preparedness in mathematics of their incoming students with bridging or remediation programmes and, more generally, with programmes that support transition from secondary school to university. Many students find it difficult to cope with proofs and abstract concepts. There is much literature that supports and shows efforts to prepare students to bridge cultural gap between high school and tertiary mathematics (Wood, 2001; Bahr, 2008; Leviatan, 2008; Oikkonen, 2009; Tall, 2008; Clark & Lovric, 2008, Harrison & Robinson, 2009). A cultural gap between high school mathematics and tertiary mathematics is composed more of abstract concepts than formal proofs (Leviatan, 2008).

Problem-solving approaches proposed by Mason et al (1982), Schoenfeld (1992), Rogers (1988) and Alibert (1988) encourage students to think in a mathematical way at university level. In addition, Tall (1995) cogitates that advanced mathematical thinking involves formal definitions and formal deductions.

Mathematics underpins many different disciplines at university. Gordon & Nicholas (2010) emphasise that the nature of mathematical knowledge presents particular challenges as both the content and way of reasoning builds on students' previous knowledge and experiences. A transitional stage in tertiary mathematics programmes consists of a cluster of lower level courses, such as linear algebra, analytic geometry, and introductory calculus, which basically reinforces school mathematics (Leviatan, 2008). D'Souza and Wood (2007) put forth that the transition adjustment is considerable to first year students, particularly for those entering tertiary education directly after the end of their high school education.

2.7 Scaffolding and the Zone of Proximal Development (ZPD) in the Context of Collaboratively Learning Mathematics.

Drawing on both the concepts of the mathematics team teaching strategy presented in the intervention programme and the tutorial sessions, I have decided to include literature on scaffolding writing in my research on teaching mathematics. This is because scaffolding is a metaphor for the way tutors can support students through relatively difficult mathematics problems.

Bruner (1978) describes ‘scaffolding’ as cognitive support given by teachers to learners to help them solve tasks that they would not be able to solve while working on their own. In addition, Bruner (1978) describes scaffolding as a form of ‘vicarious consciousness’ in which students are taken beyond themselves through participation, in the consciousness of the teacher. Vygotsky (1978) stresses that what the learner can do today only with assistance, she will do independently tomorrow.

Leo Van Lier (1996) formulated six principles of scaffolding:

- Context support - a safe but challenging environment: errors are expected and accepted as part of the learning process.
- Continuity - repeated occurrences over time of a complex of actions, keeping a balance between routine and variation.
- Intersubjectivity - mutual engagement and support: two minds thinking as one.
- Flow - communication between participants is not forced, but flows in a natural way.
- Contingency - the scaffold assistance depends on the learner’s reactions: elements can be added, deleted, repeated etc.
- Handover - the ZPD closes when the learner is ready to undertake similar tasks without help (Van Lier, 1996, p.196).

With effective scaffolding, understanding is co-constructed during the verbal dialogue of the ZPD. It is important to note that learning in a ZPD may be effectively scaffolded by either teachers or fellow learners.

Bruner, Wood & Ross (1976) define scaffolding as a metaphor for the way an expert 'tutor' (such as a parent) can support a young child's progress and achievement through a relatively difficult task.

Bruner, et al. (1976) further describe six functions of the tutor in scaffolding the activity of the child:

1. To orientate the child's attention to the version of the task defined by the tutor.
2. To reduce the number of steps that are required to solve a problem, thus simplifying the situation in a way that the learner can handle the components of the process.
3. To maintain the activity of the child as he/she strives to achieve a specific goal, motivating her/him and directing her/his actions.
4. To highlight critical features of the task for the learner.
5. To control the frustration of the child and the risk of failure.
6. To provide the child with an idealized model of required actions.

Fernandez, Wegerif, Mercer & Rojas-Drummond (2001) show that the concept of scaffolding is closely related to Vygotsky's concept of the Zone of Proximal Development (ZPD). Vygotsky (1978) describes the ZPD as:

> The distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers (p. 86).

Edwards & Jones (2001) emphasize that in a classroom situation, the actual development level can be determined by traditional question-response evaluations and therefore described. The potential development can only be explained rather than described because it is a process observed in relation to working with others. The 'next' development level, if not achieved, can be described but the process requires explanation rather than description. Achieving the potential is usually described in relation to a 'more learned other'.

Vygotsky (1978) proposes that the ZPD can be offered to assess what an individual is capable of with the help of an adult or teacher. Rogoff (1990) considers the ZPD to be a key element in the culturally based process of learning, whereby children 'appropriate' knowledge and skills from more expert members of their society. This is a development of Vygotsky's claim that cognitive processes of society appear first at the social (intermental) level, and then internalized and transformed as individual ways of thinking (the intramental level) (Vygotsky,1978).

Wertsch (1991) states that the ZPD can be used to analyse the language interactions between learners and teachers. Mercer (2000) proposed the concept Intermental Development Zone (IDZ). Mercer further described that IDZ is not a characteristic of individual ability but rather a dialogical phenomenon created between people in interaction. According to Mercer (2000), IDZ can be applied to understand how interpersonal communication can aid learning and conceptual development. Fernandez et al. (2001) accentuate that the IDZ captures the way in which the interactive process of teaching-and-learning rests on the maintenance of a dynamic contextual framework of shared knowledge, created through language and joint action.

IDZ embodies the following claims which may be relevant to symmetrical as well as to asymmetrical teaching and learning:

- Any joint, goal-directed task must involve the creation and maintenance of a dynamic, contextual basis of shared knowledge and understanding;
- Language use during joint activity both generates and depends on the creation of this contextual framework;
- The success of any collaborative endeavour will be related to the appropriateness of the communication strategies that participants use to combine their intellectual resources.

Pea (1993, p. 47) as cited in Barnand & Campbell (2001) states “the mind rarely works alone” and writing, as a learning activity, is one that lends itself to the construction of texts by students working together.

Barnand, et al. (2001) carried out a study on how independent learning can be fostered through a process approach to the teaching of writing through university academic skills programmes. The study illustrates how scaffolding is achieved by teachers and students at the University of Waikato. The six principles of scaffolding (Van Lier, 1996) were applied throughout the course. Various forms of tutor scaffolding are outlined, and then a short sample of transcript data illustrates how students on the course worked collaboratively to co-construct texts and scaffold each other’s learning. Vygotsky (1978, p. 86) stipulates more fully that scaffolding can be accomplished by both peers and teachers.

Barnand, et al. (2001) conclude that they found a sociocultural perspective to be extremely relevant, but of course other theoretical models can also be helpful to explain key aspects of learning and teaching. Teachers need to be provided with an appropriate theoretical foundation as well as technical expertise both before embarking on, and implementing an innovative approach to teaching writing (Barnand, et al., 2001

2.8 Exploratory Talk and Peer Interaction in Small Collaborative Groups in Mathematics

Collaborative learning has been recognized as an effective promotion of cognitive abilities in mathematics learning (Johnson & Johnson, 1992; Barnes, 1998; Cobb & Bauersfeld, 1995). According to Neyland (1994), collaborative learning is one of a range of valuable approaches, based on the premise that

> each student has an individual thinking style that needs to be identified & used; individual thoughtful concentration & knowledge construction are important components of the learning and problem-solving process; the learning and problem-solving process is enhanced when individuals pool in their ideas, challenge & elaborate on each other’s thinking (p. 25.)

Nelson-LeGall (1992) captures the nature of collaborative learning when she states that

> learning and understanding are not merely individual processes supported by the social context; rather they are the result of a continuous, dynamic negotiation between the individual and the social setting in which the individual's activity takes place. Both the individual and the social context are active and constructive in producing learning and understanding (p. 52)

Many studies recognize the effective use of collaborative learning (Barnes, 1999; Forman et al., 1985; Lyle, 1996; Dalziel & Peat, 1998; Neyland, 1994; Sandberg, 1995; Edwards & Jones, 2001). In 2001, Edwards and Jones conducted a study on collaborative small groups, where they defined collaborative learning as that which is constructed amongst student peers working together in self-selected groups. The process involved in mathematical endeavour is as important a focus to the group as the end outcome.

Dekker & Elshout-Mohr (1997) articulated a model that contributes to peer interaction in mathematics classrooms. The model has four key activities in which students take part.

1. Show one's work
2. Explain one's work
3. Justify one's work
4. Reconstruct one's work

According to D'Souza and Wood (2007), collaborative learning has numerous benefits. Collaborative approaches provide opportunities for more decision making responsibilities with the students, thus giving them more autonomy & control (D'Souza & Wood, 2007). Resnick (1987) state that collaborative learning fosters innovation in teaching and classroom techniques. Slavin (1990) reveals that collaborative learning creates a conducive environment of active participatory and exploratory learning with all the learners involved. Webb (1991) articulates that collaborative learning promotes higher level thinking skills. Cooper, et al. (1984) suggest that collaborative learning fosters student-teacher interaction and familiarity, while Hertz-Lazarowitz, Kirkus & Miller (1992) state that collaborative learning encourages students to seek help and accept tutoring from their peers.

2.9 Types of Talk in Discussions about Mathematics

From the literature we can ascertain and define types of talk which arise in mathematics discussion. Mercer (1995), Wegerif & Mercer (1997a); Wegerif & Mercer (1997b), Wegerif, Mercer & Dawes (1998) and Mercer (2000) characterize three educationally significant ways of talking, and argue that the three can be considered as social ways of thinking. The three types of talk are:

a) **Disputational talk**: characterized by disagreements, individualized decision-making and short assertions and counter-assertions.

b) **Cumulative talk**: speakers build positively but uncritically on what the other has said; it is characterized by repetitions, confirmations and elaborations.

c) **Exploratory talk**: participants engage critically but constructively with each other's ideas, offering justifications and alternative hypotheses. Knowledge is made publicly accountable, reasoning is more visible in the talk, and progress results from the eventual agreements reached.

According to Wegerif and Mercer (1997b) 'disputational', 'cumulative and 'exploratory' are not meant to be descriptive categories into which all observed speech can be neatly and separately coded. According to Edwards & Jones (2001) the term 'exploratory talk' was first used by Douglas Barnes (1975) in his influential work *From Communication to Curriculum*. Edwards & Jones (2001) further clarify that Barnes defined a particular type of talk observed between peers in classrooms that, he argued, was essentially different from the type of language used in interactions with the teacher (Barnes called this 'presentational talk'). Although Barnes' definitions clearly acknowledge a social aspect to learning, his view of learning is Piagetian so that the social arena within which this exploratory talk, and subsequent learning takes place, does not impact on his view of the psychology of the learning process (Edwards & Jones, 2001).

In a study by Wegerif and Mercer (1997b), sixty British primary school children aged 9-10 years took part in an experimental teaching programme designed to improve the quality of the children's reasoning and collaborative activity by developing their awareness of language use.

The aim was to develop and evaluate a teaching programme for 'scaffolding' children's effective use of language as tool for reasoning. The findings support a sociocultural view of intellectual development and confirm the value of explicitly teaching children how to use language to reason (Wegerif and Mercer, 1997a).

Mercer (1995) researched exploratory talk where he applied a Vygotskian view of learning in which language is described as a 'social mode of thinking', to his theory-building. According to Vygotsky (1962) language has three crucial functions: as a cognitive tool which children use to process knowledge; as a social or cultural tool for sharing knowledge amongst people; and as a pedagogical tool which one person can use to provide intellectual guidance to another.

Vygotsky (1978) pioneered this work that inspired the development of sociocultural research, where he argued that 'cognitive development' results from a process of linguistic socialization. Socio-cultural theory research took place between Vygotsky and Piaget. Mercer (1995) elaborates on this. According to Wegerif and Mercer (1997b), Vygotsky's theory of development was not as different from that of Piaget's as we are often led to believe. Vygotsky (1981) claims that thought, at least 'the higher mental functions' such as logic, originates in social practices. He certainly did not claim that thought was fully embedded in language use and in social interaction. Vygotsky (1981) described language as a 'tool', or 'mediating means'. He appears to have meant by this that it has a function for facilitating the cognitive development of individual children.

Wegerif and Mercer (1997b) chronicle their recent definition of exploratory talk from the outcome of SLANT (Spoken Language and New Technology) project, which observed children engaged in computer-based joint activities in 12 British primary schools (as described in Mercer, 1996). They define exploratory talk as follows:

> Exploratory talk is that in which partners engage critically but constructively with each other's ideas. Statements and suggestions are sought and offered for joint consideration. These may be challenged and counter-challenged, but challenges are justified and alternative hypotheses are offered. In exploratory talk, knowledge is made publicly accountable and reasoning is visible in the talk (p. 53).

Wegerif and Mercer (1997b) stress that 'cognitive development' can be loosely translated into ordinary language as 'the process by which children learn to reason'. The ultimate goal of ‘cognitive development’ is abstract rationality, based on the model of logic or mathematics. Wegerif and Mercer (1997b) describe ‘logical’, ‘rational’ or ‘reasonable’ as a person that can make appropriate, clear and useful contributions to discussions, in ways that enable the achievement of solutions to shared problems.

Co-operative small group problem-solving improves student learning, according to Slavin (1990). Edwards & Jones (2001) stresses that a distinction must be made between co-operative and collaborative small group work. Damon & Phelps (1989) provide a valid description for the distinction of these two types of group activity:

> In peer collaboration, a pair of relative novices work together to solve challenging learning tasks that neither could do on their own prior to the collaborative engagement. Unlike cooperative learning, the children at all times work jointly on the same problem rather than individually on separate components of the problem. This creates an engagement rich in mutual discovery, reciprocal feedback, and frequent sharing of providing the learner with a partner in discovery it places these challenges in a context of supportive communication and assistance. (p. 13)

Edwards & Jones (2001) consider one study by Mulryan (1994) which involved interviewing students in secondary mathematics classrooms about their experience of working in co-operative groups. Mulryan (1994) found that the perceptions of high attaining students were more in line with those of their teacher than are those of low attaining students, something that might increase the gap between the higher and lower attaining students.

Webb (1982) discusses the kinds of peer interaction among students learning in small groups and describes the characteristics of students, groups, and tasks that predict different patterns of peer interaction. Webb (1982) presents a model of peer interactions in small groups that shows the possible experiences that an individual student may have in a peer-directed small group engaged in a solving a problem. Whether help received by students is effective, most likely depends on many factors (Webb, 1982). When a student does need help, the effectiveness of help received may depend on the following conditions:

- The help must be relevant to the particular misunderstanding, or lack of understanding of the target student

- It must be at a level of elaboration that corresponds to the level of help needed
- It must be given in close proximity in time to the target student's error or question
- The target student must understand the explanation
- The target student must have an opportunity to use the explanation to solve the problem (Vedder, 1985 as cited in Webb, 1982)
- The target student must use that opportunity(p. 24)

Webb (1982) conjectures that the sequences of student experiences in group work may have direct effects on their learning and achievement. The level of elaboration of students' interaction with other students is related to achievement: giving high-level elaboration to other members of the group is positively related to achievement, receiving high-level elaboration in response to errors and requests for explanations is not consistently related to achievement, and receiving a lower level of elaboration than requested is negatively related to achievement (Webb, 1982).

According to Webb (1982), analyzing only the level of elaboration, however, is not sufficient to understand students' experiences in the group that lead to increased understanding and higher achievement. To fully understand whether a particular response to a student's need for help is effective, it is necessary to know:

- The specific need of the student asking for help or making an error;
- Whether the response is appropriate to the student's need;
- Whether the student can understand the response; and
- Whether the student can and does internalize it.

2.10 Enactivism in Mathematics Teaching

Drawing on the concept of mathematical proficiency, I have decided to include literature on enactivism theory which is an emerging educational theory. This is because enactivism, according to Maturana and Varela (1992) is an effective action that operates in the domain of existence of living beings.

Maturana and Varela (1992) examine the enactivism theory of cognition, in which, instead of seeing learning as coming to know, one envisages the learner and the learned, the knower and the known, the self and the other, as co-evolving and being co-implicated. Learning mathematics involves critical thinking. Harel (2008b) emphasises the mathematics premise that teachers' knowledge of Mathematics ought to be determined by the desirable ways of understanding and ways of thinking, targeted by the curricula they are expected to teach. In enactivism, thinking and cognition are grounded in bodily actions (Begg, 2000). Tall (1995) stresses that body actions in mathematics are features such as counting. According to Begg (1999) enactivism seems to be emerging as a result of five interrelated influences such the criticisms of constructivism, the Cartesian dichotomies, phenomenology, non-cognitive knowing, and issues from Biology. According to Maturana and Varela (1992), this situational context is neither the setting for a learning activity, nor the place, where the student is literally part of the context.

2.11 Managing a Large Class

Effective management of a large class is a popular topic among teaching staff in higher education. Carbone, and Stanley & Potter, as cited in Carpenter (2006, p. 13) have produced books focusing on large class environments, offering strategies for course design, student engagement, active learning, and assessment. The advantages of large classes include decreased instructor costs, efficient use of time and talent, availability of resources, and standardization of the learning experience (McLeod, as cited in Carpenter, 2006, p.13). However, there are significant disadvantages to large classes, including strained interpersonal relations between students and the instructor, limited range of teaching methods, discomfort among instructors teaching large classes, and a perception that those faculties who teach large classes are of lower status at the institution (McLeod, as cited in Carpenter, 2006, p.13).

The outcome of undertaking large class teaching presents many obstacles: human interaction with many students, scheduling office hours, dealing with e-mails, assigning homework, recording grades on a large scale, communicating the subject material, presentation and crowd control, providing feedback to students, and creating an atmosphere conducive to learning (Jungic, et al.,2006).

Jungic, et.al. (2006) stresses that managing a large group of students requires advanced organization and detailed planning. Studies that describe the challenges of managing bridging course teaching, acknowledge that there is a need to overcome them. For example, Gordon & Nicholas (2011) find four challenges of teaching in a mathematics bridging course. Firstly, is the challenge of teaching a diverse student group in a bridging course format with its demands for skillful teaching exacerbated by a short time frame. Secondly, is the challenge of teaching complex mathematical concepts which students find difficult to understand, and to express themselves in the appropriate syntax.

Thirdly, changing students' perceptions about mathematics and themselves as learners by increasing their mathematical understanding, and by changing their attitudes towards mathematics through an increase in their confidence in doing mathematics. and Fourthly, the organizational and logistic challenges of finding staff who teach creatively, and have enough sound mathematical knowledge to explain concepts and not just teach mathematics as an exercise, and know where Mathematics is used in a variety of university subjects from engineering to economics, providing different levels and areas of bridging mathematics within the timeframe. (Gordon & Nicholas, 2011).

2.12 Students' Conceptions of Mathematics Intervention Courses

The nature of mathematical knowledge presents particular challenges as both the content and way of reasoning builds on students' previous knowledge and experiences (Gordon & Nicholas, 2011). Gordon & Nicholas (2011) investigated the conceptions of mathematics bridging courses held by students enrolled at major Australian universities. They found that on one dimension the conceptions relate to cognitive functions: the course bridges students' difficulties with mathematical concepts. On a similar note, Dray, et al. (2008) stress that bridging courses help develop strategies for learning Mathematics and extend skills in thinking and reasoning, advance personal goals and enhance self-development, ameliorate prior difficulties with Mathematics and improve approaches to learning mathematics, facilitate transition into higher education, enables students to meet each other and help realize their potential.

In a study conducted by Leviatan (2008) focusing on potential transition courses aimed at bridging the gap between the students of four-year secondary/high-school teacher training programmes, he found that students gained self-confidence, and the programme helped a lot with other courses in the regular mathematics curriculum. Students appreciated the value of the session dealing with misconceptions and common mistakes, and benefited from the accompanying projects (e.g. applying newly acquired tools to other mathematics subjects). The chance to perform non-routine mathematical work in a closely guided way and reviewing newly written mathematical material, were enjoyed, and taken very seriously Leviatan, (2008).

Dray, et al. (2008) reports on a bridging project at the Oregon State University. They found that documenting growth in students' geometric understanding and problem solving ability were essential skills for a successful transition from lower-division mathematics courses to upper division physics courses. Other observations arising from the project was students' enthusiasm for the laboratory activities as well as their engagement in mathematics if they enjoyed the work and found it applicable in their lives.

In a study conducted by Cooper, et al. (2008) students' perceptions about the bridging programme offered at the University of South Australia were noted. These include a highly positive remark about the bridging programme being a discovery about themselves and their capabilities, an exciting opportunity to escape unemployment, invaluable learning about referencing, researching, essay and report writing in communications and development of their skills i.e. being comfortable in the university setting through being made welcome, small classes, friendly lecturers and easy access to computers and library resources.

Another study of students' conceptions was presented by Samson, et al. (2011) who investigated the nature of epistemological access afforded by a first year chemistry intervention programme at Rhodes University. The outcome of the research provides useful insights about the students' experience. According to Samson et al. (2011), positive experiences included the greater individual attention students got from lecturers, more meaningful student-lecturer interactions, the potential richness emanating from the complementary roles of the different lecturers' team-teaching, a productive disposition attained which more closely resonated with students' particular strengths, greater lecturer engagement and understanding of the students as well as different strengths from lecturers that improved the interaction and understanding better. In addition, Samson, et al. (2011) emphasises that the epistemological access facilitated on the intervention programme (not only by the complementary backgrounds and experiences of the lecturers) but also by the effective smooth transition from school classroom to university lecture theatre and the associated changes in mode of delivery (Gordon & Nicholas, 2011) which serve as an advantage afforded by differing backgrounds and fill the gaps.

The Concordia University embarked on a study aimed at a better understanding of frustration in prerequisite mathematics courses (Sierpinska, 2006). Questionnaires and interviews were given to gain experience of teaching the bridging course. The Concordia University found that their students perceived little relevance of Mathematics for their future studies or professions, and students depend on teachers for the validity of their solutions (Sierpinska, 2006). According to Sierpinska (2006), 84% of students agreed with the statement that they were being forced to take a bridging mathematics course and felt unhappy about it. Students responded that mathematics is extremely discouraging when you are forced to take it as a prerequisite. 44% of students claimed that they perceived the mathematics courses as useless. Students expressed their doubts about the relevance of the mathematics courses by agreeing with 'I'll never use most of the material we covered in this course' and 70% of the students were dependent on teachers for the validity of solutions.

Another remarkable conception in research conducted by Wood et al., (2006) was that students felt really confident and fitted in well; understanding the basics enabled them to broaden the understanding of the topic, it allowed them to start the subject with confidence and gave them confidence to go through with the subject. Some students remarked that it allowed them to make a choice between Foundation and MM1 (Math Modelling 1) and it also helped them to identify their weaknesses. Students recognized that the intervention gave them insight as to how much extra work they would need to do to keep up with the subject (Wood et al., 2006).

2.13 Conclusion

There is ample evidence of students' perceptions of intervention programmes, mathematics teaching in literature, and research results which describe the ideals mathematics intervention programmes and/bridging courses. Research on students' perceptions about mathematics intervention programmes is less extensive. Research in the area of transition from secondary to tertiary education should be considerable. The gap between school and university mathematics seems to be larger than in other subjects (Tall, 1991). Failure to choose higher-level mathematics courses in high school can have serious consequences both for student success in university mathematics and on whether a student continues with his or her mathematical studies (Gordon & Nicholas, 2012).

Intervention programmes may ameliorate prior difficulties with mathematics and improve the student's approach to learning mathematics. In addition, Kilpatrick, et al. (2001) stress that in order to learn mathematics successfully one should acquire mathematics proficiency.

Students' perceptions of intervention programmes are divergent. It is clear that students accept intervention programmes and experience them differently. Further research is needed to shed more light on the following mathematics issues:

- Which teaching strategy is likely to bring about filling the gaps between secondary education and university mathematics?
- In what way can the strands of mathematics proficiency and the DNR framework improve teaching strategies and the quality of learning Mathematics successfully?

With the theoretical background of mathematics learning in mind, the aim of this investigation is to obtain the perceptions of the first year mathematics students towards the alternative mode intervention. This investigation is an attempt to add contrasting findings and/or viewpoints to mathematics intervention programmes.

CHAPTER 3

METHODOLOGY

3.1 Introduction

In this chapter I describe and justify the research methods used in this study. The aim of the investigation was to determine the perceptions of first year mathematics students to the alternative mode intervention. This involved developing an understanding of how the teaching and learning context of the mathematics remedial mode intervention programme supported effective student learning. It also engaged with the experiences of the individuals who are part of the intervention programme and the meaningful insights they acquired from their learning experiences, in particular as they relate to the development of mathematical proficiency.

In this chapter various aspects of the research design are discussed, including:

- The overall research design
- The research methods
- A description of the items as well as schedules used in lecture interviews, student focus group discussions and the observation of lecture and tutorial sessions
- The pilot testing of the questionnaire and interviews
- Lecture and tutorial notes
- Document analysis of the two-mode system
- The methods/procedures used in the data analysis

A qualitative-interpretive design was used in this research project. This is because I wanted to investigate the perceptions of the students. The investigation was conducted in three different phases.

In the first phase a questionnaire was used to gain a deeper understanding of the first year mathematics students' experiences and perceptions towards learning mathematics in the intervention programme. Their responses were recorded and analysed. A sample of 50 students was used.

In the second phase, focus group discussions were held with 12 volunteer students. These discussions were structured around the key questions of the research. These questions were:

Over-arching question: what are the perceptions of first year mathematics students towards the alternative mode intervention?

The following sub-questions, related to the intervention programme were investigated:

1. What are the experiences of first year mathematics students in the alternative mode of delivery?
2. What influence does the alternative mode of delivery have on the student learning experience?
3. How does this student learning experience influence the development of their mathematical proficiency?
4. What are the potentials of the alternative mode intervention as a vehicle for first year mathematics teaching?

During the third phase, interviews were conducted with 6 lecturers who were currently teaching on the intervention programme, to gather their experiences and to understand how they teach mathematics, relating particularly to mathematical proficiency. Their responses were recorded and analysed. Observations of lecture and tutorial sessions were conducted during this phase.

3.2 Research Survey Design

This research is a case study. In this case study research the investigator selects a sample of questionnaires and conducts interviews as well as observation to collect particular information respective to the variables of interest. The data collected are used to describe the characteristics of the specific case. Structured approaches in data collection ensure the comparability of data across the sources. This is because a case study allows for an in-depth investigation to be undertaken, enabling particular detail to be captured for interpretive analysis (Yin, 1994; Kumar, 1999).

According to Simons (2009), case studies using qualitative methods in particular, enable the experiences and complexity of programmes and policies to be studied in depth and interpreted in the precise socio-political contexts in which the programmes and policies are enacted. Creswell (2009) stresses that case studies are strategies of inquiry in which the researcher identifies the essence of human activity, explores in depth a programme, event or activity, and collects detailed information using a variety of data collection procedures over a sustained period of time.

In addition, Cohen, Manion & Morrison (2000) state that case studies have a number of distinguishing features. They are able to blend a description of events with an analysis of them; they focus on individuals or groups of people; they are able to highlight specific events that are relevant to the case; and the researcher is integrally involved in the case. Sayer (2000) refers to such research as intensive research.

3.3 Data Collection Methods

The data were collected by means of a first year student questionnaire, lecturers' interviews, lecture and tutorial session observations as well as first year student focus group discussions.

3.3.1 The Questionnaire

Murray (2011) suggests that questionnaires can range from a highly planned and structured verbal questionnaire to more unplanned and informal 'opportunistic chats'. The aim of this first year mathematics student questionnaire was to gain a deeper understanding of students' experiences and perceptions towards learning mathematics through the intervention programme. The focus was on the student learning and how teaching was conducted on the intervention programme as it related to mathematical proficiency. The researcher developed her own questionnaire with the following aims:

- To investigate students' experiences and perceptions about the first year mathematics intervention programme;
- To determine how the teaching and learning of mathematics is conducted on the intervention programme in particular, as it relates to mathematical proficiency; and
- To find out how students understand the existence of the mathematical intervention programme.

The questionnaire consisted of open ended questions. (See Appendix A).

3.3.1.1 Sampling procedures

The mathematics intervention programme had 600 students during the first semester (2012). About 62 students took part on this investigation. 50 students answered the questionnaire and 12 student focus group discussions were conducted. These students were completing their Analytic Geometric, Matrices & Complex Numbers (MAT 3590) and Basic Mathematics (MAT 3580) courses for the Science and Education Honours degree. They were a multicultural group of students who had recently matriculated on three mathematics intermediate levels: the NSSC Ordinary Core, Extended Level and Higher Level. For most of these first year Mathematics students, English was their second language.

The framework of first year mathematics intervention programme was designed to slow the pace of teaching by covering two modules in the first year and one in the second year, to provide adequate support to assist students to learn mathematics successfully and to progress. Student performance in the Basic Mathematics first test was used to channel students into respective modes. Those who scored less than 40% in this test were required to enter the alternative mode and the others continued with the normal mode.

The questionnaire was administered to the group as a whole during one of their mathematics lectures. 50% of the students were asked to voluntarily answer the questionnaires. More or less 25 minutes were needed to complete the 24 items on the questionnaire. The students were allowed to ask questions about several items but they were not allowed to communicate with each other. The researcher wanted to gather each student's own perceptions and understandings of the items on the questionnaire.

3.3.2 Focus group discussions

Cohen, et al. (2000) define a focus group as a form of group interview, though not in the sense of a back and forth interaction between the interviewer and group. Hence the participants interact with each other rather than with the interviewer, such that the views of the participants can emerge. The participants' rather than the researcher's agenda is able to predominate. Focus group discussions were conducted with the 13 selected first year mathematics students who were currently on the intervention programme. The aim of the focus group discussions was to gather insights of the experiences of the first year mathematics students about the intervention programme. The researcher wanted to investigate how students felt about this intervention programme. The respondents were asked to answer the questions as honestly as possible, to share their own opinions about the mathematics teaching on the intervention programmes and make possible suggestions where applicable.

Each student's responses to the questions were recorded and transcribed. They were then analysed according to certain aspects, namely:

- Students' experiences of the mathematics intervention programme (this includes how teaching and learning was conducted, in particular as it related to mathematical proficiency
- Students, own understandings of the existence of mathematics interventions programmes.

3.3.3 Lecturer Interviews

Barker & Johnson (1998) argue that the interview is a particular medium for enacting or displaying people's knowledge of cultural forms, as questions, far from being neutral, are couched in the cultural repertoires of all participants, indicating how people make sense of their social world and of each other. Cohen, et al. (2000) state that in a semi-structured interview a schedule is prepared that is sufficiently open-ended to enable the contents to be reordered, digressions and expansions made, new avenues to be included, and further probing to be undertaken. Interviews with four lecturers were conducted. These were lecturers who were currently teaching on the first year mathematics intervention programme. The aims of these interviews were to find out more about how the teaching of mathematics was done, in particular as it relates to mathematical proficiency. It was a way of assessing what/how lecturers understood what is meant by mathematical proficiency. This includes how they help students acquire mathematical proficiency in order for students to learn mathematics successfully.

In addition, it was a way of assessing the program implementation, with the aim of attaining effective insights into the learning of mathematics with respect to the following aspects:

- The lecturers' role of teaching the first year mathematics intervention programme;
- The lecturers' perception of mathematical proficiency and how they help students acquire mathematical proficiency;
- The lecturers' experience of managing instruction as a product of teaching and learning mathematics, to help students on the intervention develop mathematical proficiency;
- How the lecturers help students discover meanings and relationships in mathematical concepts;
- How the lecturers motivate students to engage productively in mathematical lectures and tutorial sessions; and
- How the lecturers stimulate high-level ways of thinking in their mathematical lectures to aid student ways of understanding.

Lecturer interviews were recorded and analysed. Attention was given to the expressions of the lecturers and further probing questioning was done. The lecturers' interviews were later transcribed. Cohen, et al. (2000) stress that transcribing is a crucial step in interviewing. As Kvale (1996) remarks, the transcript can become an opaque screen between the researcher and the original live interview situation. Each response was then analysed in depth to gain an understanding of the lecturers' perceptions of how the teaching and learning of first year mathematics was conducted on the intervention programme.

The aim was to engage lecturers in their own experiences of teaching mathematics, in particular as it relates to mathematical proficiency as well as their own knowledge of pedagogy. Their understandings of how to assess students' intellectual curiosity and how to help students solidify and retain the knowledge they have acquired to learn to mathematics successfully were important aspects of the lecturer interviews.

3.3.4 Observations

Observations, according to Cohen, et al. (2000), enable a researcher to gather live data from lived situations, in other words the researcher is able to look at what is going on in situations rather than from a second hand perspective. Patton (2002) stated that observation data should enable the researcher to enter and understand the situation that is being described.

The researcher developed her own lecture and tutorial session analytic protocol to gather data on interactions between the students and the lecturers, as well as one-on-one interactions between students themselves, especially in their tutorial groups. The researcher also gathered data on how students carried out tasks, and how, during the lecture, discipline was maintained to enhance evaluative listening, interpretive and hermeneutic listening as well as the deeper level listening that leads to flexibility.

Several tutorial sessions were observed to explore the nature of the ZPD. In the sessions, I focused on the talk of a group of students on the alternative mode of delivery as they attempted to solve SMAT 3580, Basic Mathematics Tutorial questions. With the help of tutors, we selected several groups of students as being representative of the range of ability in the tutorial group. As the observation was carried out, I recorded the students' interactions and took tutorial notes. I transcribed the recording of their tutorial group performance, writing down the dialogues and actions that took place for each problem tackled. All the tutorial group sessions observed were then analysed to determine whether the type of discourse could be classified as mainly exploratory, disputational or cumulative in each case, according to Mercer's categorizations (1995).

3.3.5 Document Analysis

According to Cohen, et al. (2000), documents can be classified as primary or secondary. Primary documents are those documents that have had a direct physical relationship with the events being reconstructed, while secondary sources do not bear a direct relationship with the event. Primary documents were utilized in this research. In particular, a submission document of the implementation of a two mode system at the University of Namibia Science Faculty was used. Students' lecture notes, tutorial notes, tests and assignments were also used.

The aim was to gather insights into how the instructions were conducted in the context of mathematics teaching, in particular as it relates to mathematical proficiency.

3.3.6 Analysis of transcripts

The data were analyzed in order to investigate the perceptions of the first year mathematics students towards the mathematics intervention programme and to determine how the teaching and learning of mathematics was conducted on the mathematics intervention programme, in particular as it relates to mathematical proficiency.

The transcripts were analysed, making use of a coding system. Since it was a grounded approach, analytical memos with themes and headings were utilized. Questionnaires and focus group discussion responses were organized into categories and subcategories using a comparative approach by reading across all the transcripts. From this inductive process the researcher generated analytical statements which were dominant and recurring themes emerging from the data. According to Bassey (1999), analytical statements are based on the raw data but speak directly to the research question.

Conceptualizing the data was the second step in data analysis. An abductive process was used for this purpose. This enabled the researcher to understand the learning context of the mathematical intervention programme. This included the organization of themes and statements that arose from the analysis phase. The researcher developed this understanding by examining other data sources i.e. the curriculum and results from the lectures tutorial tests and lecturers' interviews, about the mathematics intervention programme, for comments, clarification and amplification.

3.4 Research Ethics

Simons (2009) postulates that the traditional way in which informed consent is sought, is through a form participants are asked to sign prior to being interviewed or taking part in the research. Permission to carry out the research was obtained from the Dean of Science faculty, see Appendix E - Permission letters.

Anonymity, negotiation, accessibility and confidentiality were assured as far as possible and participants were told about the aim of research, the implications of the research and their right to withdraw from the research whenever they wished.

3.5 Conclusion

The phases that are incorporated during an investigation play a crucial role in pertaining the ultimate success thereof, as well as validity, ethics and reliability of the end results. House (1980) argues that validity is concerned with how you establish the warrant for your work, whether it is sound, defensible, coherent, well-grounded, and appropriate to the case, as 'worthy recognition'. Maxwell (1992) describes validity as being descriptive or interpretive. Descriptive validity refers to researchers not distorting information, in other words it is the factual accuracy of the researcher accounts. In order to help me ensure trustworthiness I decided to record the focus group discussions and provide the questionnaire transcripts. Interpretive validity is the match between the meaning attributed to participants' behaviours and the participants' actual perspectives.

This chapter discussed the research design used in the investigation of this research project. Data collection methods were comprehensively described. These were the first year mathematics students' questionnaire, lecturer interviews, lecture and tutorial sessions observations, focus group discussions as well as document analysis. The motivations of these data collection methods were discussed.

Aims for the inclusion of each of the items in the first year mathematics students' questionnaire and lecturers interviews as well as the observation analytic protocol schedule, interview schedule were given.

The sampling procedures used to determine the participants in the focus group discussions, questionnaire and lecturer interviewees were discussed. Finally, the intended methods for analyzing the data were described, clarified and amplified. The following chapter describes how data was represented, analysed and interpreted respectively.

CHAPTER 4

Data Analysis and Interpretation

4.1 Introduction

The main purpose of this chapter is to present, interpret and analyse data that emerged from the investigation. The aim of data collection was to investigate the perceptions of the first year students currently on the intervention programme, and the meaningful insights they made of their learning experiences, in particular as this relates to the development of the strands of mathematical proficiency.

The data collected from the transcribed focus group discussions and the questionnaires are presented first. Thereafter, data collected from the lecturers' transcribed interviews and lecture observations are presented. Finally data collected from the tutorial session observations are. The data are analysed and interpreted in order to reveal the perceptions of the first year students on the intervention programme.

4.2 Representation and Data Analysis from Focus Group Discussions and Questionnaires

4.2.1 Brief Background of participants and the focus group discussion

Two focus group discussions of 6 and 7 first year mathematics students currently on the intervention programme were administered. There were no disturbances during the focus group discussions. Each focus group discussion was tape recorded and lasted about 30-45 minutes. To preserve anonymity, the students were given pseudonyms. The students were informed about the focus group discussion two weeks in advance, but not the content of questions. The reason for this was that the researcher did not want the students to research the questions presented. They had to rely on their personal thoughts and express themselves about their own experiences. The questions were aimed at finding out their personal experiences related to learning and teaching mathematics on the intervention programme. This was done using some features of the framework of focus group discussion, namely: what students liked best about the intervention programme; what students liked least about the intervention programme; students' perceptions of the team teaching approach on the intervention programme; students' concerns about the team teaching strategy; students' perceptions on improving mathematics teaching on the programme; students' perceptions about mathematical proficiency and students' perceptions of the valuable aspects of the intervention programme (see Appendix B for the Focus group discussion).

4.2.2 Students' perceptions about the mathematics intervention programme

Three dimensions of students' perception about the mathematics intervention programme were explored in the focus groups. The first dimension refers to what the intervention programme meant to them. *What does it mean to learn mathematics on this intervention programme? What did the students do on the intervention programme that contributed to their mathematical learning?* The second dimension concerns students' personal goals for studying mathematics on the intervention programme. *What does the student expect to achieve at the end of the intervention programme?* The third dimension concerns students' perceptions of the team teaching strategy adopted on the intervention programme. *What does student think about the team teaching strategy? How do students propose that lecturers improve teaching mathematics on the intervention programme?* I discuss each dimension below.

4.2.3 Dimension One: What the programme meant to the students

Five categories emerged here:

- The mathematical content.
- Opportunity to explore content not covered in high school.
- Importance of perseverance for succeeding in mathematics modules.
- Slower pace.
- Improved cognitive abilities.

4.2.4 Dimension Two: The mathematics intervention programme helps students to attain personal goals

On this dimension, six categories emerged:

- Gate keeping.

- Reduction of anxiety.
- Transition from school to university.
- Self-development.
- Use of the smart board.
- Importance of tutorial sessions.

4.2.5 Dimension Three: perception of the team teaching strategy

Five categories emerged:

- Greater interaction.
- Individual attention.
- Presentation teaching styles.
- Mediation.
- Enthusiastic tutors and lecturers.

In the next section I present the extracts/excerpts from first year mathematics students' transcripts. These extracts are presented with pseudonyms chosen by the researcher.

4.2.6 Presentation/illustrations of different categories

<u>Dimension One: What the programme meant for the students</u>

(a) The mathematical content

The students reported that the mathematics intervention programme did bridge the gap between mathematics content not learned in high school and the knowledge required for their studies.

> *The Intervention programme bridges the gap in specific mathematics concepts we need to aid our understanding. There are many concepts that we didn't cover in high school as a result of three mathematics levels (i.e. higher level, extended and core).* (FG1: Gift, Line 3).

Romeo reported that the mathematics intervention programme helped him to fill all the gaps that affected his learning. He reported:

> *I have gained all my strengths needed to encounter all my numeracy problems like for instance polynomials. I was a bit lost and confused as it was the first time learning this topic. This helped to fill in all gaps in my abilities that hinder my understanding of this topic* (FG2: Romeo, Line 49)

Forty eight percent of students also agreed that the disparity between students' entry skills and the new materials presented to them during the lectures were fulfilled by lecture discussions.

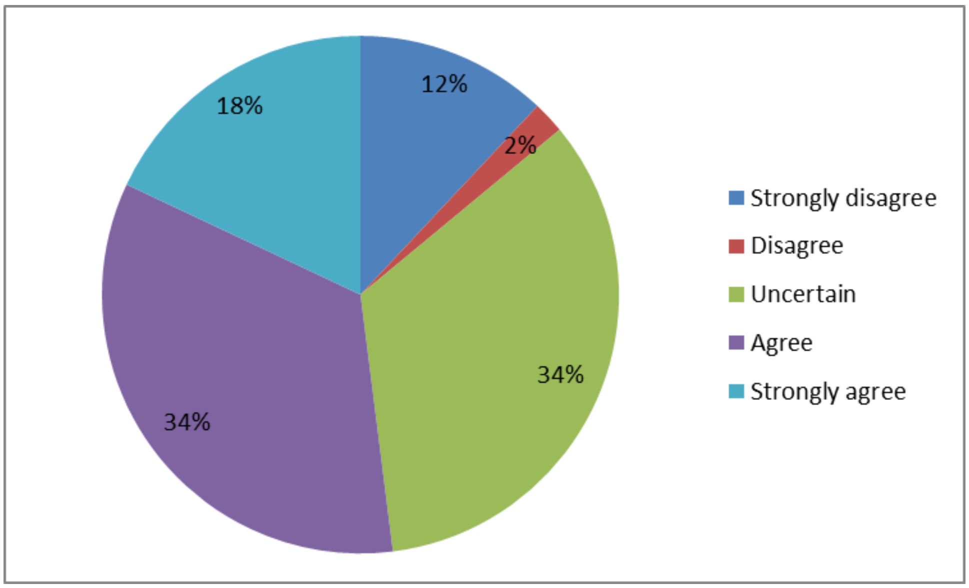

Figure 4.1: Pie chart of question 8: "The disparity between students' entry skills and the new materials presented to them during the lectures fulfilled by lecture discussions" (First year students' questionnaire).

(b) Opportunity to explore content not covered in high school

The students reported that the mathematics intervention programme provided opportunities to explore higher level content that they didn't learn in high school. Keith reported:

> *This program gives you more time to understand certain concepts and for me as a statistics students, Basic mathematics is linked to most of my modules, like I have a module of probability and some of the mathematical problems that we do in Basic Mathematics are also in probability so I like it very much (*FG2: Keith, Line 1).

Many students reported that the mathematics intervention programme was an opportunity for them to learn certain concepts not covered in high school, as so they felt that their mathematical background was not advanced enough for university entry. In the questionnaire, 22% of the students strongly disagreed that their mathematics background was advanced enough for university entry.

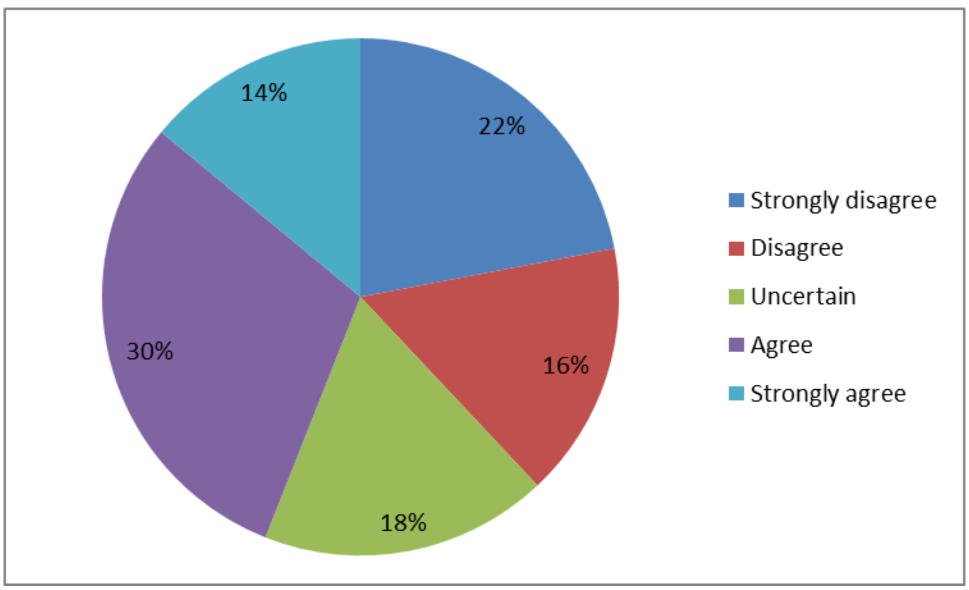

Figure 4.2: Pie chart of question 5: "I think my mathematical background is advanced enough for university entry" (First year student questionnaire)

Romeo discovered his confusion with certain mathematical concepts, that the mathematics intervention programme helped to assuage by teaching him the necessary skills to understand the topic. He reported:

> *I was a bit lost and confused as it was the first time learning this topic. This helped to fill in all gaps in my abilities that hinder my understanding of this topic* (FG2: Romeo, Line 49)

(c) Importance of perseverance for succeeding in mathematics modules

The students' perceptions of their experiences of learning mathematics on the intervention programme had a definite effect on their perseverance in succeeding mathematics modules. Gift reported:

> *I basically have done core level in high school and found most things difficult, so being on this intervention programme is really a good opportunity to discover more mathematics knowledge needed to build my foundation to continue with my second year mathematics modules*(FG1:Gift, Line 3).

In addition, Keith reported:

> *I have gained self-confidence and all motivation needed to go through this programme. This gave me extra motivation to work harder than before. I hope this will work forever* (FG2: Keith, Line 48).

(d) Slower pace

Many students' ideas about this category are based on the fact that the mathematics content was presented at a slower pace on mathematics intervention programme.

> *It gives us more time to do things like in this case we have a whole year to study for a certain module*(FG1: Jennifer, Line 4).
>
> *Intervention programme is of the duration of one year that gives us time to do pre-calculus on the following year that will give us more modules for the preceding year which can be difficult* (FG2: Candice, Line 10)
>
> *It is actually nice since are given a longer period to understand the concepts, rules, and the way you will exercise it will be useful*" (FG2: Henry, Line 33).

On the first year student questionnaire 50% agreed that the time allocated to whole-lecture discussion in mathematics is adequate.

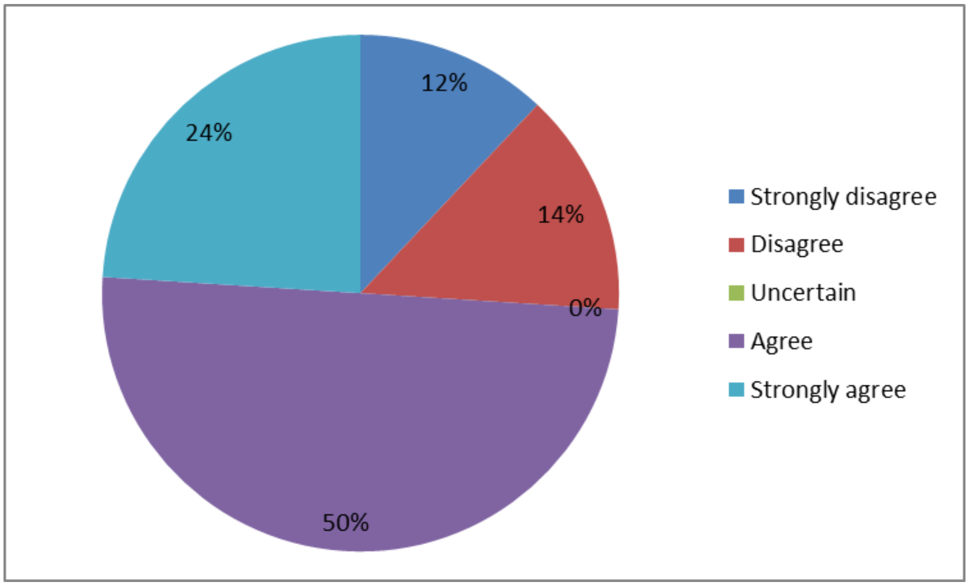

Figure 4.3: pie chart of question 13: "Time allocated to whole-lecture discussion in mathematics is adequate" (First year mathematics questionnaire)

One lecturer concurred with this idea. Ms. Volvo reported:

> *We conduct teaching at a slow pace so that students can grasp the methods and concepts* (LI: Ms. Volvo, Line 4)

(e) Improved cognitive abilities

The mathematics intervention programme helped students' ways of understanding and ways of thinking.

> *The more rules and justification learnt in tutorial as we interact with the tutors lead to more reasoning. In the end we are ready and joyful to solve new mathematical problems on our own* (FG1: Gift, Line 26).

> *I carry out my procedures efficiently and much quicker, this taught me to read questions comprehensively and think of it not just do it but I actually understand how to solve it* (FG1: Martin, Line 6).

In the questionnaire, 32% of the students reported that in the alternative mode of instruction, they have an opportunity to define concepts, understand proofs and identify their ways of understanding and thinking more successfully than in normal mathematics modes of instruction.

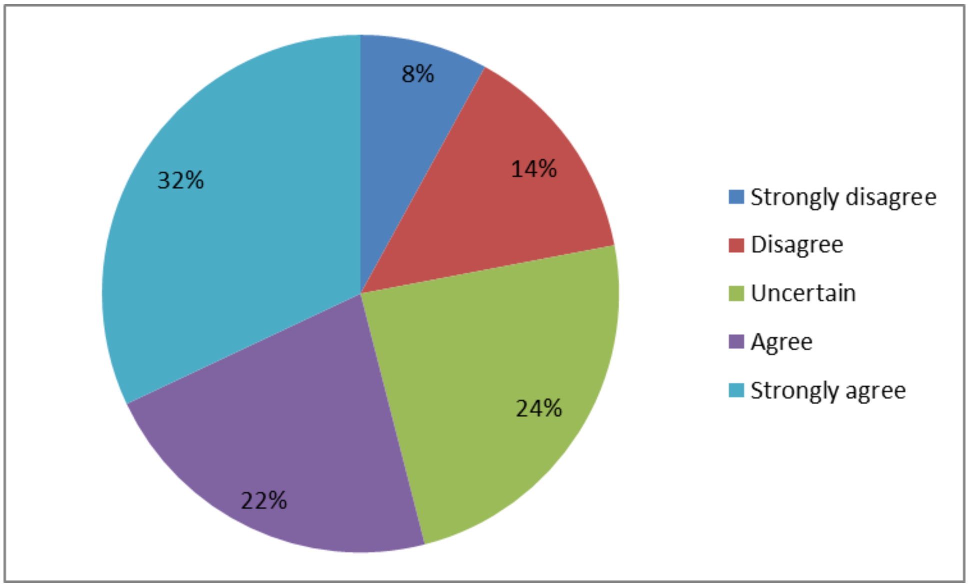

Figure 4.4: pie chart of question 21: "In the alternative mode of instruction, students have an opportunity to define concepts, understand proofs and identify their ways of understanding and thinking more successfully than in normal mathematics modes of instruction" (First year mathematics questionnaire).

Dimension Two: mathematics intervention programme helps students to attain personal goals

(a) Gate keeping

Students reported that they applaud the mathematics intervention programme as it afforded them knowledge and skills needed to pass first year mathematics. For some students, mathematics is not their major subject, so passing first year mathematics on the intervention programme solves problems of repetition of first year modules which would otherwise affect the duration of their studies. Henry applauded the mathematics intervention by saying:

> *Since first year mathematics modules are prerequisite for second year mathematics or other modules, being on this intervention programme it allows and enables students to pass and continue with other modules as they are done based on the progress of these modules. This assures me that the gate is open to proceed to the following year* (FG2: Henry, Line 47).

(b) Reduction of anxiety

> *What I like most about the intervention program is that it takes pressure off the students work and they can focus not only on mathematics but other subjects* (FG1: Sawyers, Line 7).

(c) Transition to university

Students reported that the mathematics intervention programme serves a transition from school to university.

> *I like working and interacting with people. I am happy with the way things are done. High school was totally different as we normally just listen to our teachers presenting stuff to us on the board then we get classwork and homework. On the intervention we have group discussions which are helpful. The tutors are available to help us understand. If you didn't understand something, another student can explain it in their own words*(FG1: Titus, Line 8).

(d) Self-development

Students thought that the mathematics intervention programme promoted self-development. Keith reported:

> *I have gained self-confidence and all motivation needed to go through this programme. This gave me extra motivation to work harder than before. I hope this will work forever* (FG2: Keith, Line 48).

(e) Use of the smart board

Forty four percent of the students taking part in the questionnaire agreed that the use of smart boards and the wireless network on campus, reinforced student achievement in mathematics.

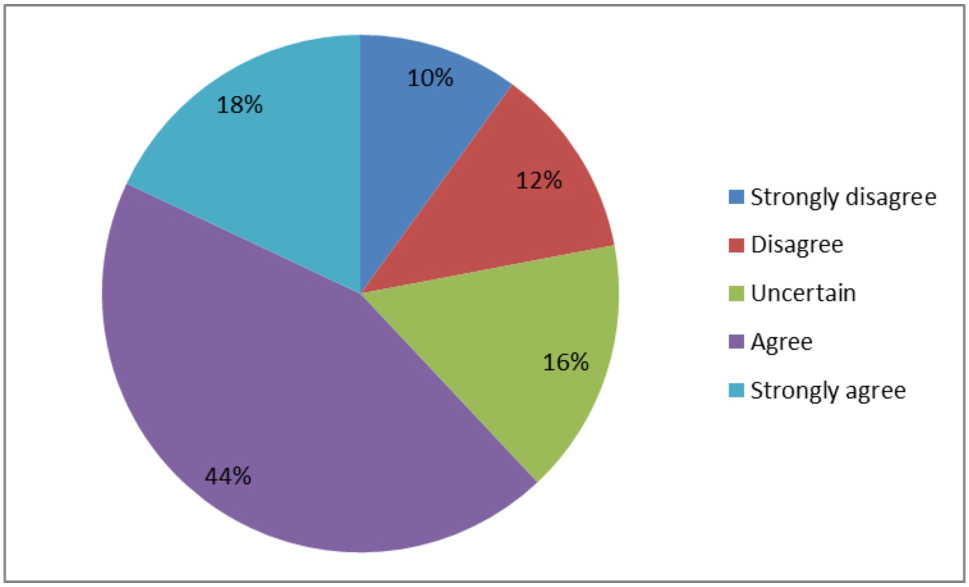

Figure 4.5: pie chart of question 9: "The use of smart boards and the wireless network on campus reinforces student achievement in mathematics"(First year mathematics questionnaire)

(f) Importance of the tutorial session

Many students reported that tutorial sessions encourage scaffolding, and an openness to group work. Exploratory talk is thus enhanced through group mediation and this leads to adaptive reasoning. Gift reported:

> *The more rules and justification learnt in tutorial as we interact with the tutors lead to more reasoning. In the end we are ready and joyful to solve new mathematical problems on our own* (FG1: Gift, Line 26).

> *I find Tutorial sessions beneficial to me as we get to interact with other students and justify our answers. Problems were solved quicker. It's more convenient and comfortable solving mathematical problems with friends. In tutorials we are free to ask further questions and clarifications as need arises* (FG1: Sacky, Line 5).

Forty eight percent of the students strongly agreed that tutorial group sessions allowed them to work on activities to increase their mathematical skills.

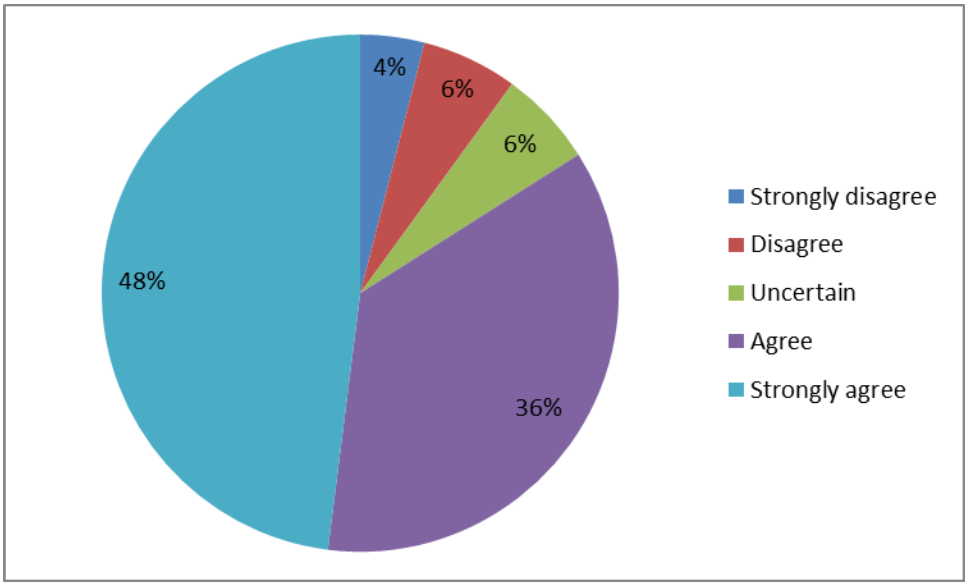

Figure 4.6: Pie chart of question 7: "Tutorial group sessions allow students to work on activities, problems and assignments that increase their mathematics skills" (first year mathematics questionnaire)

Dimension Three: perception of the team teaching strategy

(a) **Students' interactions**

Many students reported that the team teaching strategy advanced the level of interaction between the students and lecturers. Kelly reported:

> *The intervention programme helped me to do stuff on my own. After the tutor's explanation was clear to one student in our group, that student will help other students who don't understand. It allowed us to meet new friends which lead to more comfortable tutorial sessions. Having more friends that one can ask for help it's really fascinating* (FG2: Kelly, Line 35).

(b) Individual attention

In their questionnaire responses, 40% of the students agreed that lecturers avail themselves for discussions on the work done by students during lectures, and offer instructional help to deal with students' difficulties in carrying out procedures flexibly, accurately, efficiently, and appropriately.

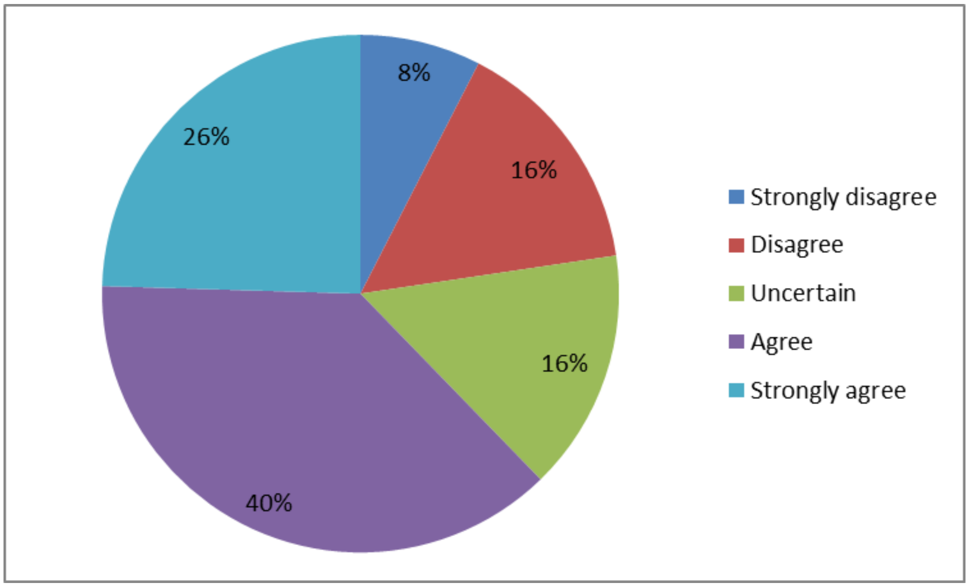

Figure 4.7: Pie chart of question 18: "Lecturers avail themselves for discussion of work done by students during lectures and offer instructional help to deal with students difficulties to carrying out procedures flexibly, accurately, efficiently, and appropriately" (First year mathematics questionnaire)

(c) Presentation styles

Students reported that the team teaching strategy afforded a well-structured presentation, however Simon also reported that:

> *because of wide range of helping hand opportunities we have on the programme like, more lecturers (FG1: Simon, Line 25)*

In the questionnaires, 38% of the students strongly agreed that lecturers initiated concept mapping in teaching.

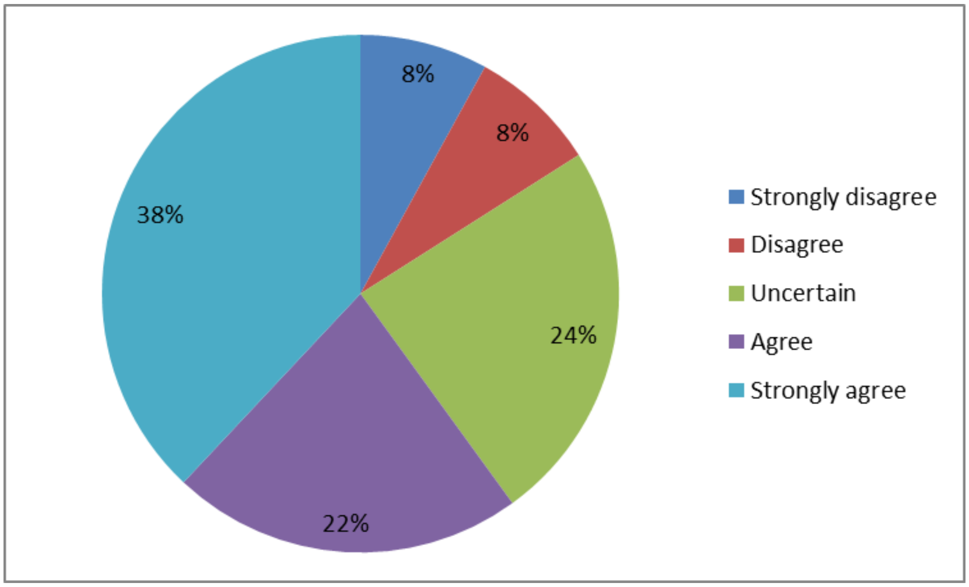

Figure 4.8: Pie chart of question 14: "Lecturers initiate teaching and learning methods such as concept mapping "(First year students' questionnaire).

Forty percent of the students strongly agreed that concept mapping helped them to visualize the abstract concepts and enhanced their understanding of mathematics.

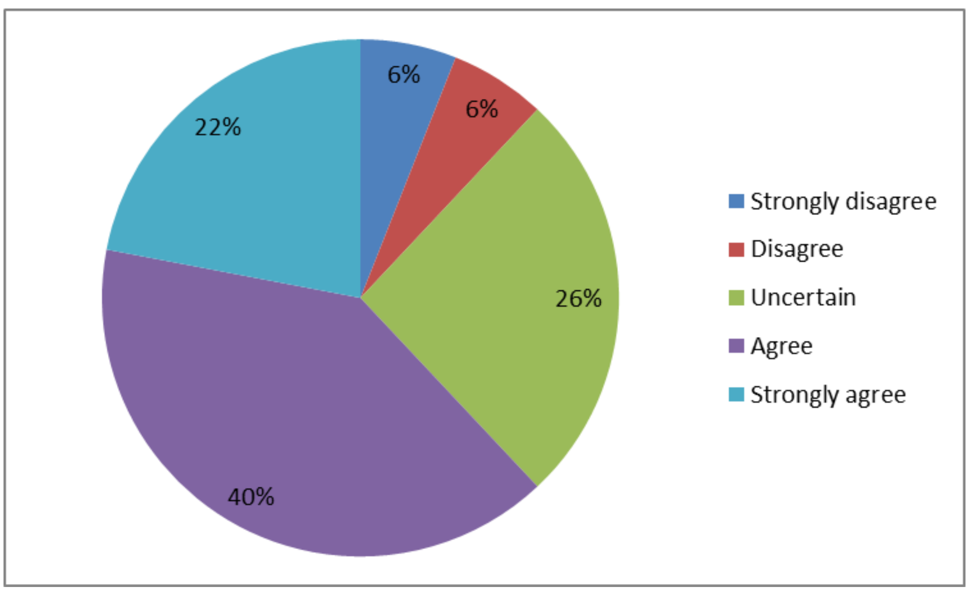

Figure 4.9: Pie chart of question 15: "Concept mapping helps students visualize the abstract concepts and enhances their understanding of mathematics" (First year student questionnaire)

(d) Mediation

Fifty percent of the students agreed that the lecturers presented and scaffolded students with instructions that focused on important mathematics content and hence developed the students' knowledge, skills, abilities and inclinations.

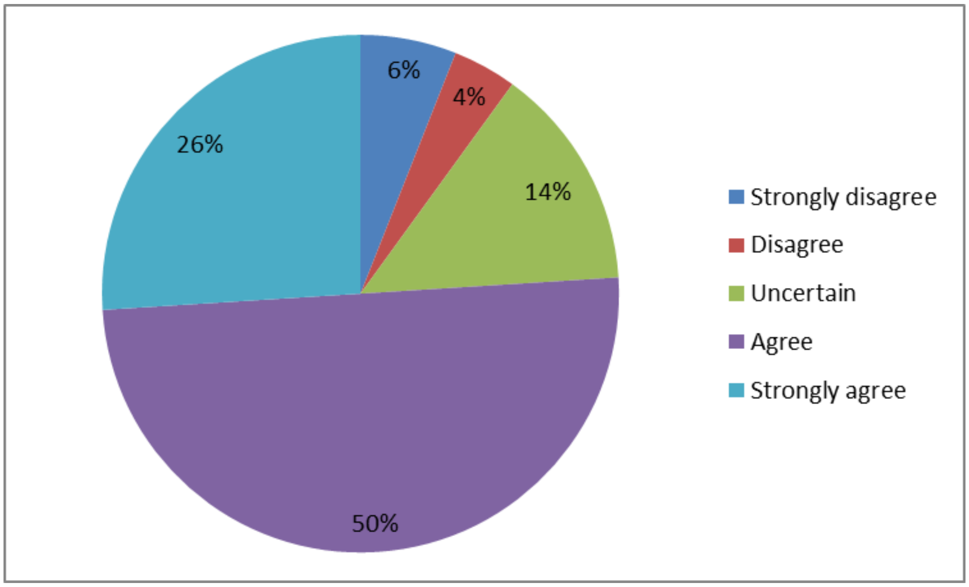

Figure 4.10: Pie chart of question 24: "Lecturers present and scaffold students with instructions that focus on important mathematics content and hence develop students knowledge, skills, abilities and inclinations" (First year student questionnaire)

(e) Enthusiastic team teaching

In many students views, the mathematics intervention ought to have an enthusiastic team.

Hanriette reported:

> *It's more on the way he teaches, he makes jokes in a way of simplifying mathematical problems to us which makes him good to us. I prefer going to his lecture than others. I really enjoy his lectures so much.*(FG2: Hanriette, Line 19)

4.3 Representation and Data analysis of Lecture Observation and Lecturers' interviews

Two categories were identified:

(a) Student engagement

(b) Conception of mathematical proficiency

(a) Conception of student engagement

Lecturers reported that students on the intervention programme needed more attention. Ms. Volvo reported*:*

> *Students on the intervention need attention. One doesn't have to teach a lot of problems per teaching session. So, let's say you can only take like teach two or three problems and then you make sure that you give enough time and chance for students to carry out the problems after you have explained the concepts and give them enough time to talk to one another to discuss, is the only way they can learn effectively* (LI1: Ms. Volvo, line 10)

Dr Calculus added that: "*Working in groups help them a lot*" (LI3: Dr Calculus, Line 12).

Lecturers articulated their pedagogy to help students learn mathematics by using the constructivist teaching approach: As Ms Volvo commented:

> *I use constructivism approach in teaching where students make meanings of mathematical concepts to their experiences with others. So we encourage them to study mathematics with their friends and peers to solve problems together to teach others. When you teach others, explain steps or procedures, how to find a solution to a problem by teaching your peer you will learn and help reinforce the concept and boost their morale and confidence in the subject. So it is about learner-centred approach. Focusing more on students in such a way that they should learn with one another and the lecturer just facilitate them.*(LI1: Ms Volvo, line 18)

Dr Theorem added:

> *We always try to help them and look forward and apply what they are learning so that they really actually make use of that what is taught to pass the exam and I encourage my students to keep the notes I am teaching them so in that way they get interested.*(LI2: Dr Theorem, line 14).

Motivational features were considered important to engage students productively in mathematics lectures. Dr Theorem reported:

> *usually I mention the historical aspects of the number of results in mathematics especially mathematicians who came up with those results from way back I have seen that it motivate the students when you mention that these theorems was discovered by a particular mathematician that he was going through these or at that point he was going through that. At the same time they get to realize that those are just people who were working hard and coming up with these ideas so if they can do it, we can also do it as well.*(LI2: Dr Theorem, line 20).

Ms Volvo added:

> *In the beginning, there are certain topics in mathematics where you can tell stories related to that topic and that stimulate or motivate them to be interested or to take part actively in class also when you give equal chances to the students to solve problems by treating them equally or you answer their questions or you get to be very friendly to them. Students will start feeling at home and them they will listen to you attentively, so the issue is providing a conducive environment for learning is what is important. One needs to vary our teaching methods, not just lecturing every day or preach like a pastor. For the students that find it difficult to attend lectures, sometimes we keep class /lecture attendance also in UNAM prospectus if is clearly stated that students require 90% attendance.*(LI1: Ms. Volvo, line 20).

Dr Experience reported:

> *What one can do is that when you teach you should crack some jokes, you must also tell them where mathematics is applicable and then you must also tell them some successful stories of people who already finished their studies you know in mathematics and where they are today, you must also define career path with regard to mathematics and also applications in other subjects and students also to get interested because you are really telling them some actual and practical things*.(LI4: Dr Experience, line 20).

Dr Calculus added that:

> *what we do to encourage them to attend tutorials is that we give them tutorial test at the end of the session. Tutorial tests motivate them because the tutorial sessions give a hint of what is to be covered in the tutorial test*." (LI3: Dr Calculus. Line 16)

(b) Conception of mathematical proficiency

Many lecturers had different views about what mathematical proficiency meant to them. Ms. Volvo reported:

> *It is all about conceptual understanding, how we comprehend mathematical concepts, operations and relationships, about the procedural fluency the skills in carrying out the steps accurately when you are solving problems, strategic competence that is the ability to formulate the solve problems, reasoning that is showing logical thinking that we should explain and able to justify* (LI1: Ms. Volvo, Line 6)

Dr Theorem reported:

> *I think it is the ability to read mathematics and to explain both to you and to the audience. If one is not able to do that then they definitely have a problem with proficiency for example, if one is not able to explain the subject of continuous function and*

communicate clearly to others and then they are able to link that than we can say they are proficient with that aspect and then to be able to realize where are those things being applied in the various areas of mathematics so proficient has a lot to do with the ability to tell what you are reading and should be able to communicate(LI2: Dr Theorem, Line 6).

Dr Calculus reported:

In my understanding you get to teach certain concepts to students in most cases for example I start with a simple idea that they already know and then I develop that idea towards a new concept that I would like to introduce. Once I have introduced the concept then I tend to give them examples because they tend to be more comfortable after seeing some examples and then I give them to solve any questions in general related to what you discussing from there they will get the idea to answer questions more (LI3: Dr Calculus, Line 6)

Dr Experience reported that:

It is basically how you understand, read and write mathematics. Writing Mathematics is actually a profession where things are expressed in letters and there are a lot of relations that one needs to understand, that's how mathematics is actually written. For example, Say for instance you say $x + y = 1$. It is one way of actually writing and one way of reading mathematics, that means once you add two numbers, x and y together the answer is 1. That's how you read it (LI4: Dr Experience, Line 6)

4.4 Representation of Data Analysis from Tutorial Sessions

Mathematics learning is a social activity. The discussion of learning mathematics moves from the individual students to collaborative small groups. As Vygotsky articulated the importance of language in learning, the communication system that the lecturer provides in lectures or tutorials shapes the role that the students can play and determine the kinds of learning that students can engage in. In my analysis of students' talk, I have used the three types of talk elaborated by Mercer (1995) and Wegerif and Mercer (1996) that I discussed in detail in Chapter Two.

1. <u>Disputational talk</u> - is the talk involving students' in unco-operative discussions. Students may disagree on certain aspects. They question the validity of ways of solving mathematical problems. Exchanges are usually brief and consist of counter-assertions. There are lots of disagreements and students make their own decisions. There are lots of interactions of the 'Yes it is', and 'No it's not'. The atmosphere prevailing during this type of talk is competitive, rather co-operative.

2. Cumulative talk - represents a building of ideas based on each other's opinions. Students argue and co-operate at a certain level. When co-operation is reached, they tend to share and build information in an uncritical way, which is aimed at providing a common consensus. Every student simply accepts and agrees with what other students say. Students use this type of talk to share knowledge, but they do so in an uncritical way. Students repeat and elaborate each other's ideas, but they do not evaluate them carefully.

3. Exploratory talk - is characterized by critical but constructive engagement with each other's opinions. Here students try out ideas to see what other students make of them. Challenges met during the discussions are justified, and alternatives are suggested. During this type of talk, students listen actively. They ask questions and share relevant information. The students' ideas may be challenged and reasons are given for challenges. All students are encouraged to contribute and there is an atmosphere of trust and sense of shared purpose. Students agree about joint decisions to find solutions to the mathematical problems.

Sequence 1: Disputational talk

In the first sequence, three first year mathematics students are trying to find the solution to polynomials in Tutorial 3 (Basic Math 3580)

Find a polynomial of degree 3, whose zeros are 2, -4 and 5. Simplify your answer.

1	Marcha	What is the degree of the polynomial?
2	Leo	Isn't the power of x?
3	Marcha	What do you mean by power of x?
4	Leo	I meant like the x^n
5	Susan	No it isn't that one (pointing to x^n)
6	Leo	Yes it is, it is like x^n

7	Susan	What if it is like p(x) = 5?
8	Leo	No, it is not because that's constant polynomial
9	Marcha	No, it is not
10	Leo	It is a constant polynomial because the degree of that polynomial is equal to zero.
11	Susan	Shuuush. You are the one to blame if we get it wrong, it isn't gonna be your fault Leo if we get it wrong.
12	Marcha	Yes, it is your fault Leo if we get it wrong.
13	Leo	Why is it my fault? You can try it as well.
14	Susan	Polynomial whose zeros are 2 is (x+ 2), yes it is
15	Marcha	It is that one (Pointing to Susan's work)
16	Leo	No it is not, look guys (pointing at his lecture notes which states: *A polynomial in x is an algebraic expression of the form* $a_nx^n + a_{n-1}x^{n-1} + \ldots + a_2x^2 + a_1x + a_0$ *where* $a_0, a_1 \ldots a_n € R$ *with* $a_n \neq 0$. *n is called the degree of the polynomial.* a_0 *is the constant term.* $a_n x^n$ *is called the leading term. If* $P(x) = a_0 + a_1x + \ldots + a_{nx}^n$ *is a polynomial, we denote its degree by deg (p(x)) or deg (p).i.e. deg (p) =n.*
17	Susan	So how will find the degree 3?
18	Marcha	Let me try (writing down) it is (x+2)(x-4)(x-5)
19	Susan	No it's not (pointing at her solution) it is (x+2)(x+4)(x-5)
20	Leo	Stop it Susan, let's go on, if we get it wrong we will call tutor for help
21	Marcha	Ooooooooh...It's your fault guys if we get it wrong….

22	Leo	He he(with the help of tutor) $P(x) = (x-2)\ (x+ 4)\ (x- 5)$ $=(x^2 + 4x - 2x-8)\ (x-5)$ $=x^3-5x^2+4x^2-20x-2x^2+10x-8x+40$ $=x^3-3x^2-18x+40$

Sequence 2: Cumulative talk

In the second sequence, another group of first year mathematics students is trying to solve a polynomial in Tutorial 3 (Basic math 3580).

Find a polynomial of degree two whose roots are 3 and -5

1	Alba	Common guys, what is the polynomial in this case?
2	Selma	Shuuush (with a funny facial expression) perhaps $(x+3) + (x-5)$
3	Sue	I think you are right (pointing at Selma's answer)
4	Alba	Oooooh but look (showing her answer) $x+3 =0$ shows that $x = -3$
5	Selma	Yes it is. I think you are right
6	Sue	Yes Alba is right, it should be the other way round
7	Alba	Look guys(pointing at her solution) $P(x) = (x-3)\ (x+5)$ $=x^2+5x-3x-15$ $=x^2+2x-15$
8	Selma	Yes, I think you are right: $x^2+2x-15$ so that means $P(3) = 3^2 + 2\ (3) - 15$

		$= 9 + 6 - 15 = 0$
9	Sue	Yes, you are we are right $P(-5) = (-5)^2 + 2(-5) - 15$ $= 25 - 10 - 15$ *which is also zero*
10	Alba	Yes, it is I agree to that

Sequence 3: Exploratory talk

In the third sequence, 3 first year mathematics students are trying to find the solution to polynomials in Tutorial 3 (Basic Math 3580)

Simplify each expression below and give your final answer without negative exponents.

(a) $\left[\frac{2x^2y^{-1}z}{z^2}\right]^{-2}$

1	Ivan	Oh guys the expression has negative exponent, how do we solve it?
2	Peter	I think we apply the laws
3	Sarry	No, it doesn't matter we can still solve it as long as the final answer have positive power
4	Ivan	Oh yeah, the negative exponent outside brackets should be applied to everything inside brackets (pointing to -2 outside brackets)
5	Peter	I think you are right, look guys (pointing at her answer) $\left[\frac{2x^2y^{-1}z}{z^2}\right]^{-2} = \frac{2x^{-4}y^2z^{-2}}{z^{-4}}$

6	Sarry	No, It's like this guys (pointing at her answer)…look -2 outside brackets should apply to everything inside brackets even 2. $\left[\frac{2x^2y^{-1}z}{z^2}\right]^{-2} = \frac{2^{-2}x^{-4}y^2z^{-2}}{z^{-4}}$ $= \frac{y^2z^4}{4x^4z^2}$ $=\frac{y^2z^2}{4x^4}$
7	Ivan	Yes guys we can also use the law $\left[\frac{a}{b}\right]^{-n} = \left[\frac{b}{a}\right]^{n}$
8	Sarry	I think you are right Ivan, we are asked to simplify and give the final answer without negative exponent….
9	Peter	Shuuuu Ivan, I think you are right
10	Ivan	If you apply this law (referring to $\left[\frac{a}{b}\right]^{-n} = \left[\frac{b}{a}\right]^{n}$ Therefore $\left[\frac{2x^2y^{-1}z}{z^2}\right]^{-2} = \frac{z^2}{2x^2y^{-1}z}$ $= \frac{z^4}{4x^4y^{-2}z^2}$ $=\frac{z^2y^2}{4x^4}$
11	Sarry	I think you are right. OK.

4.5 Conclusion

The analysis chapter shed some light on the ways in which students think about the first year mathematics intervention programme. The identified conceptions related to the strands of mathematical proficiency outlined by Kilpatrick, et al. (2001). The findings of this study show that there are mathematical disparities that ought to be bridged on the intervention programme.

Lecturers ought to become aware of gaps in their students' knowledge, and ensure that possible remedial strategies are in place. The remedial intervention programme comprises a slower pace of teaching, accommodating their needs and disparities in their mathematics background. Diagnosing the nature of mathematical proficiency, lecturers developed specific teaching strategies that afforded first year students individual attention, greater interaction, mediation and tutorial sessions. The nature of group work in mathematics enhances students to experience different types of talk in tutorial sessions, namely: disputational, cumulative and exploratory talk. Exploratory talk impacted on students' ways of understanding and ways of thinking.

CHAPTER 5

DISCUSSION

5.1 Introduction

The purpose of this chapter is to discuss the analysed data that emerged from the investigation. The discussion of data collected from the focus group discussions and students' questionnaires are presented first. Thereafter, discussion of data collected from the lecturers' interviews and lecture observations are presented. Finally, a discussion of the data collected from the tutorial session observations will be presented.

5.2 Discussion of Data Collected from the Focus Group Discussions and Questionnaires

5.2.1 Students' perceptions about the mathematics intervention programme

It was evident from their responses that first year students think that the mathematics intervention programme bridges the gap between mathematics content learned in high school and the knowledge required for their studies. This is consistent with Leviatan's (2008) finding that there is a distinct cultural gap between school mathematics and tertiary mathematics. Boland (2009) concludes that one goal of mathematics intervention programmes is to use innovative methods to try and fill some of the disparities in the students' backgrounds.

Similarly, Varsavsky (2010) found that in order to attract students to mathematics-based disciplines and to improve retention, universities have been addressing under-preparedness in mathematics with bridging or remediation programmes, and more generally, with programmes that support the transition from secondary school to university. This is not surprising, given the level of preparedness of many first year university students. Much literature shows efforts to prepare first year students to bridge the mathematics gaps between high school and tertiary mathematics (Wood, 2001; Bahr, 2008; Leviatan, 2008; Oikkonen, 2009; Tall, 2008; Clark & Lovric, 2008; Harrison & Robinson, 2009). A cultural gap between high school mathematics and tertiary mathematics is related to the use of more abstract concepts and formal proofs (Leviatan, 2008).

The data show that students recognized many benefits of studying first year mathematics on the intervention programme. Many students reports were about the slower pace of mathematics teaching and the reduction of anxiety about mathematics modules. For example, Henry expressed the view that they were given a longer period to understand the concepts and rules. Gift stressed that he had studied mathematics on core level in high school and found most things difficult, so being on the intervention programme was really a good opportunity for him to discover the mathematics knowledge needed to build on his foundation and to continue with his second year mathematics modules. This is consistent with Pargetter, McInnis, James, Evans, Peel, and Dobson (1998) who found that the transition from high school to university involves adjusting to different teaching styles, a different learning environment and different assessment systems.

Students reported that the mathematics intervention programme helped initiate them into new ways of understanding and thinking. For example, Martin stated that he could carry out his procedures efficiently and much more quickly. The programme taught him to read questions comprehensively and think, not merely do mathematical problems, but actually understand how to solve them. This is in line with Harel's (2008b) theory that students develop ways of thinking through the production of ways of understanding, and conversely, the ways of understanding they produce are impacted by the ways of thinking they possess.

Tall (1995) anticipates that ways of thinking mathematically includes repetition of sequences of actions until they become automatic. Cuoco, et al. (1996) accentuate that good thinking should be relearned in a variety of domains, to develop a repertoire of general heuristics. Dualistic and holistic thinking should be commendable pedagogical approaches that encourage students to become creative and critical thinkers (Hamilton, 2006). Advanced mathematical thinking involves a wide range of mathematical activities to construct new ideas that build on and extend an ever-growing system of established theorems (Tall, 1995).

More practically students' experiences in the mathematics intervention helped to reduce anxiety. For example, Sawyers reported that the intervention programme took pressure off the students so that they could focus not only on mathematics but on other subjects.

On a different note, many students expressed the value of their tutorial sessions. This showed the importance of collaborative small groups in mathematics learning. For example, Gift reported that they learnt more rules and justification for their answers as they interacted with tutors, and that led to more reasoning. This is supported by Dekker & Elshout-Mohr (1997) who articulate a model that contributes to peer interaction that has four key activities in which students take part: show one's work, explain one's work, justify one's work and reconstruct one's work.

Sacky articulated it as beneficial that at tutorial sessions where problems were solved more quickly, it was more convenient and comfortable to solve mathematical problems with friends, with freedom to ask further questions and clarifications. This is consistent with what Lazarowitz, et al. (1992) put forth - that collaborative learning encourages students to seek help and accept tutoring from their peers. Baker (2002) supports this idea because she stresses that using peers as tutors or guides enhances achievement. Nelson-Le Gall (1992) draws a parallel to collaborative learning when she states that

> learning and understanding are not merely individual processes supported by the social context; rather they are the result of a continuous, dynamic negotiation between the individual and the social setting in which the individual's activity takes place. Both the individual and the social context are active and constructive in producing learning and understanding (p.52)

Kelly stated that the intervention programme helped her to do more work on her own. She continued by saying that when the tutor's explanation became clear to one student in their group, that same student could help other students who did not understand. This is supported by Cohen (1994), that collaborative learning shapes weaker students to improve their performance when included in a group of higher achievers.

First year mathematics students provide some evidence in reference to the key activities in the model defined by Dekker & Elshout-Mohr (2008). Students presented their work and explanations were validated among their peers. Validations of answers were clearly demanded by the students. For example, Gift articulated that the more rules and validations learnt in tutorials, the more they led to better reasoning. This is consistent with what Kilpatrick, et al. (2001) refer to as adaptive reasoning, defined as valid reasoning from careful considerations of alternatives that include knowledge of how to justify conclusions. In addition, adaptive reasoning is used to navigate through the many facts, procedures, concepts and solution methods, and to see that they all fit together in some way, and make sense (Kilpatrick, et al., 2001). What settles disputes, disagreements and confusions in mathematics is deductive reasoning. By following a series of logical steps, students are able to arrive at the correct answers. One manifestation of adaptive reasoning is the ability to justify one's work by using proofs (Kilpatrick, et al., 2001).

Collaborative learning enhances exploratory learning (Slavin, 1990). Exploratory talk is characterized by critical but constructive engagement with each other's ideas. Challenges are justified and alternatives suggested. Joint agreement in decision-making is the end result (Mercer, 1995). This is line with one student, Gift, who stated that they discovered more strategies as they shared ideas with their peers. He further said that they learnt more rules and justifications in tutorials.

Collaborative learning gives credence to the modeling of solving mathematical problems by peers (Schuck & Hanson, 1985). Levin, Glass & Meister (1984) state that students learn more by listening to their peers than to a teacher. As students interact with each other, peers discover their weaknesses and give them a helping hand. Therefore group work provides an opportunity for students to demonstrate their acquired knowledge by helping their peers. Sacky concurred with collaborative learning, saying:

The best thing about intervention is the tutorial sessions. I find tutorial session beneficial to me as we get to interact with other students and justify our answers. We explore more mathematical strategies to tackle problems as we share ideas and information, seek additional help from tutors who are always present and ready to help us.(FG1: Sacky, Line 5)

5.2.2 Discussion of data from tutorial observations

5.2.2.1 Sequence 1: Disputational talk

In sequence 1 we observe that there are different proposals as the answer for the polynomial of degree three, whose zeros are 2, -4 and 5. Several options are initiated and most of them are followed by: " *No it isn't that one (pointing to x^n" (*TS1: Susan, Line 5). *" No it is not because that's a constant polynomial"* (TS1: Leo, Line 8). *" No it is not" (*TS1: Marcha, Line 9) without clarification of arguments.

There is a lack of clarity and justification of answers, as Marcha and Susan were not at all convinced that Leo's final answer was correct. There were no appropriate responses linked to the method. *"Shuuush. You are the one to blame if we get it wrong, it's gonna be you fault Leo if we get it wrong"* (TS1: Susan, Line 11). *"Yes, it is your fault Leo if we get it wrong"* (TS1: Marcha, Line 12). *"No, it isn't (pointing at her solution) it is (x+2) (x+4) (x-5)"* (TS1: Susan, Line 19). *"Oooooh it's your fault guys if we get it wrong"* (TS1: Marcha, Line 21).

Although the correct solution was attained, this was not built on agreement of the members of the group. Students have different ideas that led to disagreements and no consensus was obtained.

5.2.2.2 Sequence 2: Cumulative talk

In sequence 2 we observe that initiations are agreed upon without further elaboration. Students could not come up with strategies for this mathematical problem, to suit the demands of the problem and the situation in which it was posed. "*I think you are right (pointing at Selma's answer"* (TS2: Sue, Line 3). *"Yes it is. I think you are right"* (TS2: Selma, Line 5*). "Yes Alba is right, it should be the other way round"* (TS2: Sue, Line 6).

There were no mutually supportive relations between the strategies of students in addition to the development of previous ideas. "*Common guys, what is the polynomial in this case?* (TS2: Alba, Line 1). Then the answer was "*Shuuush (with a funny facial expression) perhaps (x +3) + (x – 5)" (*TS2: Selma, Line 2).

Alba attempts to find the answer by suggesting that *"Oooooh but look (showing her answer) x + 3 = 0 shows that x = -3" (*TS2: Alba, Line 4*)*. Thereafter, Selma supported: *"Yes it is. I think you are right" (*TS2: Selma, Line 5).

Students do not build positively and elaborate on the first suggested solution. It is clear that the conversation lacks strategic competence and adaptive reasoning to decide about the answer. As

Sue mentioned *"Yes Alba is right, it should be the other way round"* (TS2: Sue, Line 6). *"Yes, it is. I agree on that"* (TS2: Alba, Line 10).

5.2.2.3 Sequence 3: Exploratory talk

In sequence 3, it is evident that initiations i.e. *"I think we apply the laws" (*TS3: Peter, Line 2) are challenged by hypotheses which are collectively the building blocks of the initiations "*No, I think you are right, look guys (pointing as her answer)*

$\left[\frac{2x^2y^{-1}z}{z^2}\right]^{-2}$ $=\frac{2x^{-4}y^2z^{-2}}{z^{-4}}$*)"* (TS3: Peter, Line 5).

The solution is achieved with collaborations from all the students, whereby further justifications are made. *"Yes guys we can also use the law* $\left[\frac{a}{b}\right]^{-n}$ $=\left[\frac{b}{a}\right]^{n}$ *" (*TS3: Ivan, Line 7).

There is a continuous strategy of previous solutions being agreed upon*" I think you are right Ivan, we are asked to simplify and give the final answer without negative exponent…." (*TS3: Sarry, Line 8). This shows that as students build strategic competence in solving problems, their beliefs become positive.

From the data presented in Chapter 4, it is not possible to determine different types of talk or which types of prompts were more successful at producing certain types of talk. Therefore, I discussed the types of talk defined in Chapter 2, and in which situations they were observed in tutorial sessions. Table 5.1 gives an overview of the relationship between the three types of talk and types of prompts.

Prompt/ Talk	Disputational	Cumulative	Exploratory
Justification			X
Disagreement	X		
Representation	X	X	X

Repetition		X	
Critique	X	X	X
Elaboration		X	

Table 5.1 Relationship between types of talk with types of prompts

5.3 Discussion of data collected from lecture observation and lecturers' interviews

The lecture by Dr Calculus was observed on 25 May 2012. This lecture presented the Remainder theorem. He commenced by giving corrections of previous work on long division (polynomials). There were more than 60 students present. Table 5.2 summarizes my observations.

Table 5.2 Observation Schedule and Analytic Protocol

Lecture Observation	Strongest	Average	Weakest	Absent
1. How does mathematics instruction look in the nation's lecture?		✓		
2. To what extent does the lecture engage students intellectually with important mathematics disciplinary	✓			
3. Degree of sense-making and habitual inclination is appropriate for this lecture		✓		
4. The lecture hall management enhances quality of lesson		✓		
5.Lecturers' questioning enhances development of students ways of understanding		✓		
6.Instructional strategies are consistent with Kilpatrick et al.'s (2001) instructional framework		✓		
7.Active participation of all students is encouraged and valued appropriately		✓		
8. Students are encouraged to generate ideas, questions and conjectures	✓			
9. Students are active listeners		✓		
10. There is adequate time and structure provided for student reflection and sense-making		✓		
11. To what extent does each of the following factors shape lecture's decisions about the learning outcomes				

(a) Student self-efficacy	✓			
(b) Discipline maintained	✓			
(c) Time management		✓		
(d) Modeling students		✓		
(e) Group instruction		✓		
(f) Students notion of taking notes	✓			
12. To what extent are the following strands of Mathematical proficiency incorporated?				
(a) Conceptual understanding		✓		
(b) Procedural fluency	✓			
(c) Strategic competence	✓			
(d) Adaptive reasoning	✓			
(e) Productive disposition			✓	
13. Students are intellectually engaged with important ideas	✓			
14. Intellectual rigor, constructive criticism, and challenging of ideas are evident		✓		
15. The lecturer is apparently confident in teaching	✓			

The observation and analytic protocol developed for lecture observation was designed to assess the quality of lectures in relation to the strands of mathematical proficiency. The vision of effective mathematics teaching and learning that guided this research considers the five strands of mathematical proficiency, as well as Harel's (2008b) DNR principles for mathematics learning. To achieve the mathematical proficiency goals, not only do lectures need to provide students with opportunities to learn mathematics, but they also need to be very clear about the purposes of each lecture in relation to the specific concepts being addressed, in order to help mediate student learning.

(a) Student engagement

Lecturers reported that students on the intervention programme needed more attention. Ms. Volvo reported*:*

> *Students on the intervention need attention. One doesn't have to teach a lot of problems per teaching session. So, let's say you can only take like teach two or three problems and then you make sure that you give enough time and chance for students to carry out the problems after you have explained the concepts and give them enough time to talk to one another to discuss, is the only way they can learn effectively*.(LI1, Ms Volvo, Line 10)

Dr Calculus added that: "*Working in groups help them a lot*."(LI3, Dr Calculus, Line 12)

Lecturers articulated their pedagogy to help students learn Mathematics, by using the constructivist teaching approach. Ms. Volvo commented on her use of constructivism and her strategy of encouraging students to solve problems together with peers Dr Theorem added that:

> *We always try to help them and look forward and apply what they are learning so that they really actually make use of that what is taught to pass the exam and I encourage my students to keep the notes I am teaching them so in that way they get interested.* (LI1: Ms Volvo, line 18).

Motivational aspects were important to engage students productively in mathematics lectures. Dr Theorem reported:

> *Usually I mention the historical aspects of the number of results in mathematics especially mathematicians who came up with those results from way back I have seen that it motivate the students when you mention that these theorems was discovered by a particular mathematician that he was going through these or at that point he was going through that. At the same time they get to realize that those are just people who were working hard and coming up with these ideas so if they can do it, we can also do it as well.*(LI2: Dr Theorem, Line 20).

In addition, Ms. Volvo emphasised the issue of storytelling in mathematics to motivate students (LI1: Ms Volvo, Line 20

Dr Experience reported:

> W*hat one can do is that when you teach you should crack some jokes, you must also tell them where mathematics is applicable and then you must also tell them some successful stories of people who already finished their studies you know in mathematics and where they are today, you must also define career path with regard to mathematics and also*

> *applications in other subjects and students also to get interested because you are really telling them some actual and practical things*. (LI4: Dr Experience, Line 20)

Dr Calculus added that:

> *What we do to encourage them to attend tutorials is that we give them tutorial test at the end of the session. Tutorial tests motivate them because the tutorial sessions give a hint of what is to be covered in the tutorial test*. (LI3, Dr Calculus, Line16)

As can be seen in Table 5.1, and based on the researcher's judgments, the mathematics lecture is likely to have a positive impact on engaging students intellectually with important mathematics disciplinary content. The pedagogical challenge of teaching is to manage instruction in ways that help particular students develop mathematical proficiency. Instructions for the observed lectures were focused on the important content and developed with integrity. Dr Calculus constructed the lecture so that the students' learning paths about the remainder theorem were defined.

According to Kilpatrick, et al., (2001) such instruction is effective with a range of students and over time develops the knowledge, skills, abilities and inclinations that we term mathematical proficiency. It is not only the elements of instructional mathematical content, teacher and students alone that determine what happens, but rather it is an enactment in their mutual interdependence as instruction unfolds (Kilpatrick, et al., 2001). Harel (2008b) remarks that in DNR however, the teacher's knowledge of students' learning and pedagogy rests on the teacher's knowledge of mathematics, that is to say, each of the three components of knowledge are indispensable for quality teaching.

Dr Calculus engaged his students in groups to discuss the remainder theorem. To make consistent progress towards proficiency, Kilpatrick, et al. (2001) stress that students need to be motivated to engage productively in mathematics lesson. Webb (1982) conjectures that group work may have direct effects on students' learning and achievements. This is consistent with Dray, et al. (2008) who remark that students are more likely to engage in mathematics if they enjoy the work and if they find it applicable. The tasks in which students engage, provide the contexts in which they learn to think about subject matter (Stein & Henningsen, 1997).

Lecturers' conceptions on student engagement include encouraging students to take notes and demonstrating mathematical problems to students. This is consistent to Jungic, et al. (2006) who found out in his study that effective teaching could be achieved through using time efficiently by preparing typed lecture notes for the students in advance. The findings of the study by Carpenter (2006) suggest that staff teaching large classes should attempt to include constructive, active teaching methods such as structured, controlled collaboration (e.g. jigsaw, case study) in their courses whenever possible.

(b) Conception of mathematical proficiency

Many lecturers had different views on what mathematical proficiency meant to them. Ms. Volvo reported:

> *It is all about conceptual understanding, how we comprehend mathematical concepts, operations and relationships, about the procedural fluency the skills in carrying out the steps accurately when you are solving problems, strategic competence that is the ability to formulate the solve problems, reasoning that is showing logical thinking that we should explain and able to justify*. (LI1, Ms Volvo, Line6).

Dr Theorem reported that mathematical proficiency is the ability to read mathematics and communicate it to others (LI2: Dr Theorem, Line 6).

Dr Calculus had a different view of mathematical proficiency. He reported that mathematical proficiency was about developing ideas towards new concepts (LI3: Dr Calculus, Line 6). Dr Experience reported that:

> *It is basically how you understand, read and write mathematics. Writing Mathematics is actually a profession where things are expressed in letters and there are a lot of relations that one needs to understand, that's how mathematics is actually written. For example, Say for instance you say $x + y = 1$. It is one way of actually writing and one way of reading mathematics, that means once you add two numbers, x and y together the answer is 1. That's how you read it*. (LI4, Dr Experience, Line 6)

In this study, lecturers recognized that mathematical proficiency is the ability to read, write and understand mathematics so that you can communicate to your audience. Dr Experience reported that: "*It is basically how you understand, read and write mathematics".* In addition, Dr Theorem stated: "*I think it is the ability to read mathematics and to explain both to you and to the audience. If one is not able to do that then they definitely have a problem with proficiency.''* It is also the conceptual understanding and procedural fluency to carry out the steps in solving problems; strategic competence to express the solved problems; and reasoning, showing logical thinking to explain and justify solutions. This is consistent with Kilpatrick, et al.'s(2001) strands of mathematical proficiency.

(i) Conceptual understanding

The lecturers' conceptions of conceptual understanding was that one should teach concepts, starting with the simple idea of what students already know, and then develop the idea towards the new concept, giving examples. This is consistent with to Kilpatrick, et al. (2001) who emphasized that:

> Conceptual understanding refers to an integrated and functional grasp of mathematical ideas. Students with conceptual understanding know more than isolated facts and methods. They understand why a mathematical idea is important and the kinds of contexts in which is it useful. They have organized their knowledge into a coherent whole, which enables them to learn new ideas by connecting those ideas to what they already know (p.118).

(ii) Procedural fluency

The lecturers' conceptions of procedural fluency was that it is the skill involved in carrying out the steps accurately when solving problems. Kilpatrick, et al. (2001) defined procedural fluency as the knowledge of procedures, knowledge of when and how to use them appropriately, and the skill in performing them flexibly, accurately, and efficiently. The lecturers' conceptions of procedural fluency emphasise that one does not have to teach many problems per teaching session. Ms. Volvo (LI, p.10) stressed that one can teach two or three problems and then make sure enough time is allocated for students to carry out problems and discuss with each other. Killpatrick, et al. (2001) empasises that it is important for procedures to be efficient, to be used accurately, and to result in correct answers.

Lecturers on the intervention programme used the constructivist approach, where students made meanings of mathematical concepts in relation to their experiences with others. Students were encouraged to study mathematics with their peers. Ms. Volvo (LI, p.18) outlined that when you teach others, and explain steps or procedures by teaching your peers you will learn and help reinforce the concept. Kilpatrick, et al. (2001) surmise that both accuracy and efficiency can be improved with practice, which can also help students maintain fluency. Procedural fluency and conceptual understanding are seen to be the main concepts that need more attention in school mathematics. When students practice procedures they do not understand, there is a danger they will carry out incorrect procedures, thereby making it more difficult to learn correct ones (Kilpatrick, et al., 2001).

The lecturers' views that the teaching of concepts should relate to previous knowledge is consistent with Kilpatrick, et al., (2001) who acknowledge that when skills are learned without understanding, they are learned as isolated bits of knowledge. Learning new topics then becomes harder since students can't relate to what they know or learned before. According to Kilpatrick et al. (2001) this concern leads to a compartmentalization of steps that can become quite extreme, so that students believe that even slightly different problems require different steps (Kilpatrick, et al., 2001).

(iii) Strategic Competence

The lecturers conceptions about strategic competence refers to the ability to formulate the solved problems. Kilpatrick, et al. (2001) refer to strategic competence as the ability to formulate mathematical problems, represent them, and solve them. In formulating a problem, teachers should construct a problem model, that is a mental model of the situation described in the problem. Kilpatrick, et al. (2001) emphasized that there are mutually supportive relations between strategic competence and both conceptual understanding and procedural fluency.

(iv) Adaptive reasoning

The lecturers' views of adaptive reasoning refers to showing the logical thinking that one should explain and be able to justify. For example Ms. Volvo cogitates that she asks students questions that force them to think critically,

> *…like I ask them to explain why getting certain answers for example, justify for instance why is that vector orthogonal to the other vector? They must be able to explain why, they must say orthogonal or why two lines are parallel, they must make sure they use proofs of properties and axioms because in proofs you ask them to show their logic.* (LI1, Ms Volvo, Line 22)

Kilpatrick, et al. (2001) define adaptive reasoning as the capacity to think logically about relationships among concepts and situations. In Mathematics, adaptive reasoning is the glue that holds everything together, the lodestar that guides learning. According to Kilpatrick, et al. (2001) one manifestation of adaptive reasoning is the ability to justify one's work. "Justification" is defined as "provide sufficient reason for." Proof is a form of justification, but not all justifications are proofs (Kilpatrick, et al.,2001).

(v) Productive disposition

The lecturers' conceptions of productive disposition refers to the appreciation of grasping mathematical concepts, enjoyment of the teaching at a slow pace, more time to solve problems, helping others, peer interactions and the team teaching strategy that helped them pass Mathematics. Kilpatrick, et al. (2001) refer to the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of Mathematics.

Acquiring proficiency takes time. Students need enough time to engage in activities around a specific mathematical topic if they are to become proficient with it. Leviatan (2008) found that students find it difficult to adapt to a culture where concepts are abstract, yet require rigorous definitions. Du Preez et al., (2008) conclude that results of the developmental intervention have a long-term effect and that the learnt skills can be transferred to subsequent (mathematics) modules. Students need enough time to engage in problems if they are to become proficient. This is consistent with Harel's (2008b) second component of a mathematics definition, where he argues that the acquisition of new knowledge has an impact on students' previous and existing knowledge.. To become proficient, students need to spend sustained periods of time doing mathematics - solving problems, reasoning, developing understanding, practicing skills, and building connections between their previous knowledge and new knowledge (Kilpatrick, et al., 2001).

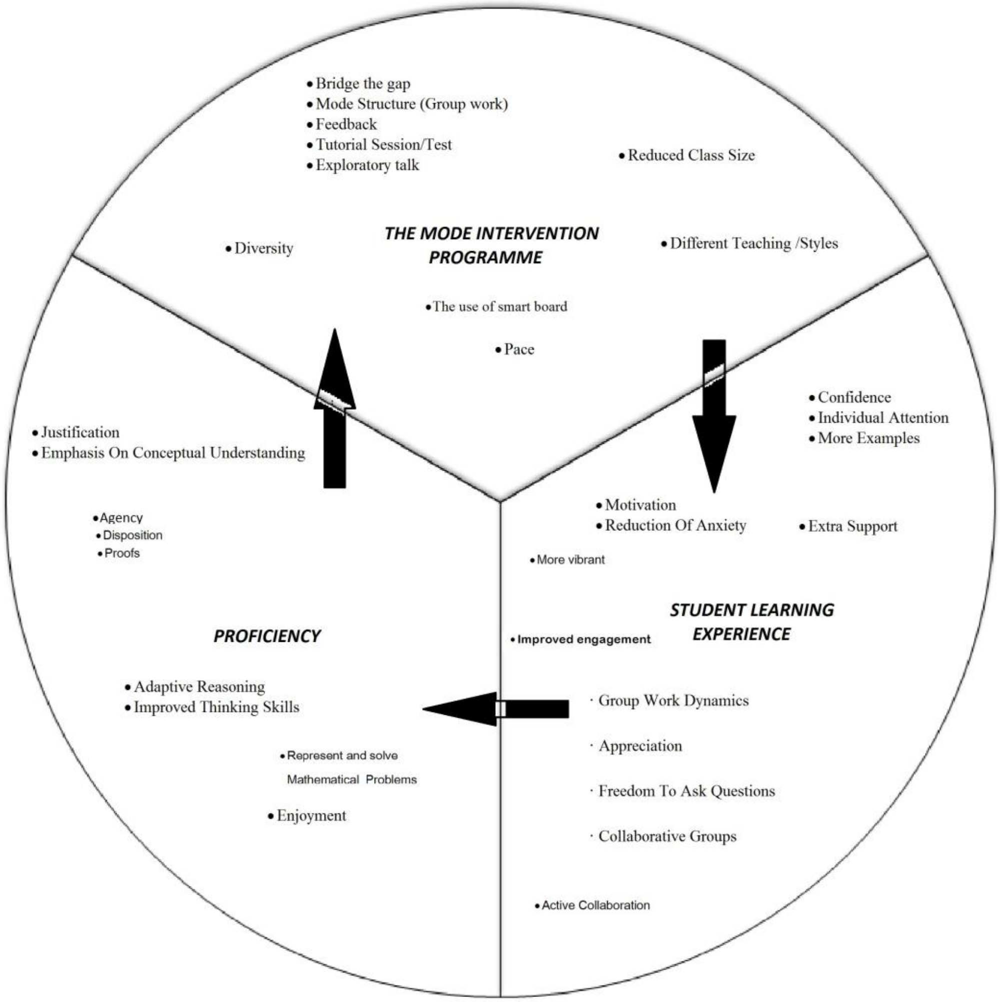

Figure 5.1: The link between the three research questions

It became apparent that the three research questions of the investigation were explicitly linked and each had an influence on the students' experience. Figure 5.1 shows how the programme experience influenced the learning experience and how this impacted on student development of proficiency. The mode intervention programme impacted on student learning. It portrays group work, where students expressed how they explored the power dynamics of group work. Students reported how they appreciate working with others in collaborative small groups. More rules and justifications of one's work were explored which subsequently led to more reasoning. Group work improved their ways of thinking and understanding. Students adopted a sense of comfort when asking questions and clarification about mathematical problems with their friends.

Different types of talk were explored in tutorial sessions, namely disputational, cumulative and exploratory talk. Exploratory talk enabled the students to justify their opinions and this led to adaptive reasoning.

The intervention programme used tutorial sessions as well as tutorial tests. Students enjoyed and appreciated the tutorial sessions. A slower pace of mathematics teaching was used, of which the majority of students found the content and pace 'just right'. The module was designed with a moderate introduction period to build student confidence, morale and help alleviate some students' maths phobias. Students enjoyed this because they had more time to focus on other mathematics modules and other subjects. Students also appreciated more feedback. The lecturers could explain more mathematical problems and had more time to answer the questions. The slower pace of mathematics teaching reduced students' anxiety about mathematics modules and changed their perceptions of what they could actually achieve, worries went from whether I could pass the maths tests, maths assignments, qualify to sit for examination to how well I would perform.

The intervention programme bridged the gap in mathematics content between high school and university. Students rationalized and believed that the results of their mathematics tests, assignments and examinations were testament to the quality of the teaching, and this gave them the confidence to complete favourably with other students on the current mode. Different teaching approaches were explored, and students had the flexibility of choosing lectures of their choice. The students could voice how they perceived learning. Students found the applications to statistics, engineering and other science courses particularly useful in aiding understanding. Students could recognize and expand the importance of mathematics within statistics, engineering and other science courses, and hence gain an appreciation of the direct relevance of the topics studied. This helped to create enthusiasm and great interest in solving mathematical problems, which was, in turn linked to the intervention structure. The issues of group work and tutorial sessions with more feedback impacted on student learning.

The mathematics intervention programme was viewed as successful in achieving its aims of improving mathematics competency, thereby increasing student confidence in studying mathematics as well as science. It is anticipated that as these students on the intervention have the necessary mathematics skills and the increased confidence to effectively engage in mathematics modules, anxiety and worries are reduced and they will continue with their second year mathematics modules.

5.4 Conclusion

From these discussions I concluded that students identified that mathematical proficiency was central to their learning and pedagogical knowledge and exploratory talk were critical aspects of good teaching in a mathematical intervention programme. It became apparent that the five strands of mathematical proficiency were intimately linked and each strand has an influence in terms of becoming proficient.

CHAPTER 6:

CONCLUSION

6.1 Introduction

The idea of mathematics intervention is viewed by many authors as the most fundamental aspect of first year mathematics modules. Varsavsky (2010) stresses that in order to attract students to mathematics-based disciplines and to improve retention, universities can address the under-preparedness in mathematics of their incoming students with bridging programmes or more generally, with programmes that support the transition from secondary school to university.

6.2 The Following Conclusions, Showing the Sequences of Students' Experiences on the Mathematics Intervention Programmes arose from this Study.

1. The mathematics intervention programme bridges the gap in mathematics content and enables students to build a strong foundation.

The first conclusion highlights the gap between school mathematics content and tertiary mathematics. The perceptions of students and lecturers currently teaching on the mathematics intervention programmes about bridging the gap, focus on the content of the course to improve transition to university mathematics. The content of the programme is taught at a slower pace which has an advantage of overcoming gate keeping. Leviatan (2008) stresses that there is a distinct cultural gap between school mathematics and tertiary mathematics. Nicholas & Goldon (2011) highlight that the nature of mathematical knowledge presents particular challenges as both the content and way of reasoning builds on students' previous knowledge and experiences.

2. The mathematics intervention programme reduces students anxiety' about mathematics learning.

The second conclusion is that the programme reduces anxiety about mathematical learning through peer interactions, the growth of self-confidence through asking questions during lectures and tutorials, self-satisfaction after solving more mathematical problems, prioritizing and scheduling one's time to do other things. The reduced class size leads to more individual attention and acquiring the necessary skills needed for succeeding modules. Keith concurred that the mathematics intervention promotes self-development by saying

> *Being on the intervention programme, I have gained self-confidence and all motivation needed to go through this programme. This gave me extra motivation to work harder than before. I hope this will work forever* (FG2: Keith, Line 48).

3. <u>The use of smart board provides strong visual representation that leads to improved visualization, student engagement and agency.</u>

(a) Use of the smart board in teaching

The Smart board provides a strong, detailed visual representation that leads to improved visualization. When you immerse students with the smart board, visualization can have an immediate impact on learning. Through the lecture and tutorial session observations, students' interactions show the improved engagement that leads to agency and a productive disposition.

(b) Relevance to specificity of programme

Smart board software tools enhance lectures to be more visual, understandable and interactive. Showing a single mathematics concept symbolically, numerically and visually can lead to a productive disposition. Student interactions during smart board use could be more vibrant because the board offers space for students to engage in active collaborations. Lecturers could present material featuring large, vibrant images. The work produced during the lecture is dynamic and contingent.

4. <u>Importance of small group sessions.</u>

(a) Openness to collaborative small groups

Collaborative small groups can be beneficial in mathematics learning. Sacky captured the benefits of collaborative small groups when he stated that

> *I find Tutorial sessions beneficial to me as we get to interact with other students and justify our answers. Problems were solved quicker. It's more convenient and comfortable*

> *solving mathematical problems with friends. In tutorials we are free to ask further questions and clarifications as need arises* (FG1: Sacky, Line 5).

Collaborative learning is defined as that which is constructed amongst student peers working together in self-selected groups (Edwards & Jones, 2001). Edwards & Jones (2001) also highlight the works of Mulryan (1994) who described the clear and compelling evidence that small group work can facilitate student achievement as well as more favourable attitudes towards peers and subject matter. Collaborative small group work can be linked with ideas such as scaffolding and the ZPD. Collaborative learning has been recognized as an effective support for cognitive abilities in mathematical learning (Johnson & Johnson, 1992; Barnes, 1998; Cobb and Bauersfeld, 1995; Wood, 2001; Barnes, 1999; Forman et al., 1993; Lyle, 1996; Dalziel & Peat, 1998; Neyland, 1994; Sandberg, 1995; Edwards & Jones, 2001).

(b) Exploratory talk

Exploratory talk is elaborated in the literature as improvisational talk in which partners engage critically but constructively with each other's ideas (Mercer, 1995). Statements and suggestions are offered for joint consideration. Many students reported that tutorial sessions encouraged scaffolding of students, and an openness to group work that enhanced exploratory talk through group mediation and that led to adaptive reasoning. Gift reported:

> *The more rules and justification learnt in tutorial as we interact with the tutors lead to more reasoning. In the end we are ready and joyful to solve new mathematical problems on our own* (FG1: Gift, Line 26).

These may be challenged and counter-challenged, but challenges are justified, and alternative hypotheses are offered (cf. Barnes & Todd, 1978). Cazden & Forman (1985) also highlight Barnes & Todd's (1978) description of exploratory talk as speaking "without answers fully intact" and as a "rehearsal of knowledge" (p. 330). The three types of talk observed in tutorial collaborative groups, namely disputational talk, cumulative talk and exploratory talk mediate mathematical learning. Mercer (1995) stresses exploratory talk in which language is described as a social mode of thinking. Vygotsky (1962) proposes that language has three crucial functions; as a cognitive tool which children use to process knowledge; a social or cultural tool for sharing knowledge amongst people; and as a pedagogical tool which one person can use to provide intellectual guidance to each other. Compared with disputational talk and cumulative talk, in exploratory talk knowledge is made more publicly accountable and reasoning is more visible in the talk.

5. The team teaching strategy advanced the level of interaction between students and lecturers as well as between the students themselves.

The effort made by the team teaching strategy provided scaffolding to students during tutorial sessions. Students' perceptions of the collaborative small groups acknowledged the effective exploratory talk as they interacted with peers to solve mathematical problems. This helped them justify their opinions, and this led to adaptive reasoning.

6. Finally, a useful recommendation for practice arising from my results, supports the Australian Mathematics Sciences' (2006) recommendation to encourage students to do more Mathematics in high school, preferably higher level mathematics and to increase the exposure of high school students to Mathematics at various levels.

6.3 Recommendations for future research

There are a number of recommendations which result from this research project. Firstly, it could be essential to do the same study for a longer period of time - for example a year - with many students on the intervention programme. This could allow for more descriptions of their experiences on the intervention programme, changes in bridging their mathematical disparities, and confidence in discussing their opinions and views as far as mathematics intervention is concerned. To further explore the use of talk in tutorial sessions, it would be useful to work with pedagogical methods using ICT.

Finally, the problem which motivated the research was introduced as one of major concern - to improve first year mathematics results. With indications that the different types of talk explored in tutorial sessions has the potential in mathematics tutorial sessions, a future step in research is to promote the effectiveness of the three types of talk in terms of justification and improving academic achievement in first year mathematics modules.

6.4 Limitations

The major limitations of this study are that the study sought not to find the differences between the two modes but rather focused specifically on the alternative mode intervention programme. However, the study did not intend to comparing the results of the students on these two modes. The input of the first year mathematics on the current mode could have contributed immensely to the findings of the study. However, something crucial was learnt from the specificity of the study being only on the alternative mode intervention. The fact that none of the first year mathematics students on the alternate mode wished to be on the current mode implies that most of them preferred the alternative intervention mode intervention.

What transpired however is that many students felt that their knowledge was not advanced enough for first year mathematics modules, hence the alternative mode intervention was the option to reduce attrition from university mathematics.

6.5 Personal Reflections

This study has contributed significantly to my personal, academic and professional growth. It was the first time I carried out research of this magnitude, hence it was a lesson of a kind. I learned how to focus and concentrate on goals, in order to achieve this. I learned how to deal with disappointments. I have learned how to schedule, no matter how tightly, because in the end, everything had to work correctly. This study gave me a chance to learn many aspects involved in academic writing and research. I enjoyed the data collection stage the most, because I had to interview UNAM lecturers and students and I was looking forward to getting their input and perceptions of the alternative mode intervention.

At one stage, I almost gave up on my studies when I lost my dearest father (2012). I was confused and in a dilemma. This experience affected my academic work negatively because I could not cope with stress. My late father was an inspiration to my studies. The data collection stage was frantic for me as I was in hospital. My health deteriorated and my blood pressure was very high. I was very excited when I heard our August contact session would be in Grahamstown. Another environment helped me to catch up significantly on my work and thoughts.

6.6 Avenues for Further Research

Firstly, if I got an opportunity to conduct further research of this kind, I would look at the significant impact of the alternative mode and compare the results of the two modes respectively. I would look at the mathematics content not learnt in high school, that seems to be the significant disparity to be bridged on the intervention programme. I would therefore investigate the practices of high school educators that have led to the persistence of these disparities in mathematics content. I would like to know: How we can assist high school learners in a way which allows them to develop a more positive attitude towards Mathematics as well as their own ability to do Mathematics? What type of motivation and assistance can benefit them in the future courses, problem solving, and basic mathematics literacy to prepare for them university mathematics? Although these questions do not directly address the issue of the first year mathematics student perceptions which I have researched, perhaps such an exploration could reveal the potential of such changes in attitudes towards their view of mathematics. Secondly, perhaps our tertiary institutions could be involved in designing curricula for high school mathematics.

Thirdly, I would look at mathematics lecturers' perceptions and views on the three Mathematics exit levels (core, extended and higher level) that their first year mathematics students matriculated from.

Fourthly, it would be interesting to investigate the national policy maker's perceptions on having higher level mathematics in Grade 12 only and abolishing the other two levels - core and extended.

6.7 Conclusion

The project was undertaken to find answers to the following research questions:

Over-arching question: What are the perceptions of the first year mathematics students towards the alternative mode intervention?

Sub-questions with relation to intervention programme:

1. What are the experiences of first year mathematics students in the alternative mode of delivery intervention programme?
2. What influence does this alternative mode of delivery have on the student learning experience?
3. How does this student learning experience influence their development of mathematical proficiency?
4. What are the potentials of the alternative mode intervention as a vehicle for first year mathematics teaching?
5. What effective constructs, if any, can be observed within the mathematics teaching?

This research questions can now be answered as follows:

The outstanding observation was that students find that the alternative mode of delivery intervention programme bridges the gaps in mathematics content, to build the strong foundation needed for understanding and applying their knowledge in order to become proficient in Mathematics. This statement is important for action as Leviatan (2008) describes the distinct cultural gaps between high school and university mathematics. While some researchers (e.g., McGillivray, 2009) stress that revising high school mathematics content at university is a good option even though the amount of mathematics able to be covered may be affected.

Many students think that the mathematics content is taught at a slower pace. They also think that the alternative mode of delivery intervention programme reduces student anxiety about mathematics learning. This explains the dynamic view of the majority of students that the intervention programme takes pressure off students to concentrate on other things. Another view of the intervention programme that the students have is the use of smart board, which provides strong visual representation that leads to improved visualization, student engagement and agency. This explains another dynamic view of production disposition.

Many students enjoyed the tutorial sessions where tutors encouraged scaffolding. In the tutorial sessions, collaborative small groups were formed, to mediate learning. Students could explain procedures and define concepts to their peers. Students could justify their answers through exploratory talk and that led to adaptive reasoning. The team teaching strategy on the intervention programme afforded the lecturers the opportunity to organize more tasks which were meant to bridge the gaps during their lectures with. More tutorial activities and tutorial tests were encouraged. It was suggested that this enhanced the lecturers' passion, which ultimately led to greater engagement and individual attention. Consequently, more strategies to solve mathematical problems were initiated by the students.

The first, second and third sub- research questions involved the influence of the students' learning experiences on the development of their mathematical proficiency. The transition to higher education (Lovric, 2005) ought to improve the acquisition of mathematics proficiency in mathematics students (Kilpatrick, et al., 2001).

Kilpatrick, et al. (2001) describe five interrelated strands of mathematical proficiency namely: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition. The alternative modes of delivery engage students around these strands of mathematical proficiency. The model of mathematical proficiency emphasizes Mathematics in an instructional triangle (described in chapter 2) of the different contexts experienced on the alternative mode of delivery intervention programme. Harel's (2008b) DNR principles ought to accentuate the teaching and learning of Mathematics. The three instructional principles, Duality, Necessity and Repeated-reasoning assert the potential effect of teaching actions on student learning.

The fourth sub question discussed the potentials of the alternative mode intervention as a vehicle for first year mathematics teaching. Three types of talk, accounting for the bulk of student talk was exploratory talk. This category is defined as talk which indicates that the students are engaging constructively with each other's ideas. The students are not entirely sure of the mathematics, but they are practicing and rehearsing, using their knowledge through talking. Therefore the reasoning is more visible in the talk. The majority of first year students were new to polynomials so they had enough time to explore their arguments, for example, when describing how they might simplify the polynomials without negative exponents. The majority of the time was spent on how they might approach these polynomials, as well as how they had already solved the problem.

Exploratory talk was used for different purposes. For example, it was used to describe a context in response to a contextualization prompt. "*I think we apply the laws*" (TS3: Peter, Line 2). Here one could see that the students referred to the laws of indices to solve the mathematical problem. There were also situations when students referred to their original terms or ideas. "*No, it doesn't matter we can still solve it as long as the final answer have positive problem"* (TS3, Sarry, Line 3). There were also situations where students referred to their original terms or ideas - *"Oh yeah, the negative exponent outside brackets should be applied to everything inside brackets pointing to - 2 outside brackets"* (TS3, Ivan, Line 4). All these prompts were classified as exploratory talk.

Disputational talk was also common during the sessions, and for some students this talk was quite important, as it shaped discussions around their frustrations with mathematics. Students blamed each other for not getting correct answers. Some students felt anxious or worried. "*Shuush, you are the one to blame if we get it wrong, it is gonna be your fault Leo if we get it wrong"* (TS1, Susan, Line 11). In the reported discussion, disputational talk played a crucial role. Disputational talk occurred in tutor-student discussion in which students responded to being asked to show their working and explain their own methods (see TS1, Leo, Line 22).

Cumulative talk was used to positively build the students' ideas by repeating what other students said. Cumulative talk was used to express students' understanding by confirming and elaborating on other students arguments. This expressed students successes in solving mathematical problems (see TS1, Selma, Line 8). Cumulative talk was used to express assurance of proof when attempting to justify different strategies (see TS1, Sue, Line 9).

Sub question five: students reported that they viewed working together collaboratively as beneficial for specific reasons. The discovery of different approaches to learning mathematics, , different ways of explaining, talking to students or teaching a concept or procedure to other students helped them to a better understanding (see FG1, Sacky, Line5). Concerning working in groups during tutorial sessions, students referred to the usefulness of engaging themselves in the learning process (FG1, Martine, Line 6). Titus also indicated that the "… *group work and its interaction work*…" in the tutorial session had an impact on his view of being successful and it cannot be compared to high school teaching (FG1, Titus, Line 8). The majority of students viewed the alternative intervention programme as learning Mathematics at a slower pace. This was especially true for some students, for example Jennifer, who expressed the view that intervention gave her more time to study as they had a whole year to study one module instead of completing a module in a semester period (FG1, Jennifer, Line 4).

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Appendix A

Questionnaire for First Year Mathematics Students.

Please take a few moments to fill in this questionnaire. All information will be treated in the strictest confidence. The questionnaire is being used for a research study. The Department of Mathematics would like to find out what it can do to help more students to pass first year mathematics in future years. The questions are designed to examine your experiences of the alternative mode intervention program.

Student Name (optional)………………………………

Age………………….

Gender…………………….

Please indicate your level of disagreement or agreement with each of the following statements by ticking the appropriate responses.

1. Students beliefs and attitudes toward mathematics influence how they approach mathematics.

Strongly Disagree	
Disagree	
Uncertain	
Agree	
Strongly Agree	

2. I enjoyed working in tutorial group sessions on problems posed during lectures.

Strongly Disagree	
Disagree	
Uncertain	
Agree	

Strongly Agree	

3. I see mathematics as sensible, useful and worthwhile.

Strongly Disagree	
Disagree	
Uncertain	
Agree	
Strongly Agree	

4. Lecturers give regular and comprehensive feedback on tests and student questions about academic progress.

Strongly Disagree	
Disagree	
Uncertain	
Agree	
Strongly Agree	

5. I think my mathematical background is well advanced for university entry.

Strongly Disagree	
Disagree	
Uncertain	
Agree	
Strongly Agree	

6. Students are shown problems in lecture that provide a template for homework exercises.

Strongly Disagree	
Disagree	
Uncertain	
Agree	
Strongly Agree	

7. Tutorial group sessions allow students to work on activities, problems and assignments that increase their mathematics achievements.

Strongly Disagree	
Disagree	
Uncertain	
Agree	
Strongly Agree	

8. The gap match between students entry skills and new materials presented to them during lecture is filled by lecture discussion.

Strongly Disagree	
Disagree	
Uncertain	
Agree	
Strongly Agree	

9. The use of smart boards and the wireless network on campus reinforces student achievement in mathematics.

Strongly Disagree	
Disagree	
Uncertain	
Agree	
Strongly Agree	

10. There is effective communication between students and Lecturers regarding practice and examination.

Strongly Disagree	
Disagree	
Uncertain	
Agree	
Strongly Agree	

11. Students have opportunities to develop their own solution methods in their small tutorial groups.

Strongly Disagree	
Disagree	
Uncertain	
Agree	
Strongly Agree	

12. Students are allowed to discuss, reflect, explain and justify these solutions to their peers and hence can able to apply mathematical knowledge in new problem situations.

Strongly Disagree	
Disagree	
Uncertain	
Agree	
Strongly Agree	

13. Time allocated to whole-lecture discussion in mathematics is adequate.

Strongly Disagree	
Disagree	
Uncertain	
Agree	
Strongly Agree	

14. Lecturers initiate teaching and learning methods such as concepts mapping.

Strongly Disagree	
Disagree	
Uncertain	
Agree	
Strongly Agree	

15. Concept mapping helps students visualize the abstract concepts and enhances their understanding of mathematics.

Strongly Disagree	
Disagree	
Uncertain	
Agree	
Strongly Agree	

16. There is regular attendance by students at lectures and tutorial group sessions.

Strongly Disagree	
Disagree	
Uncertain	
Agree	
Strongly Agree	

17. There is insufficient effort by students to study and prepare well for tests and examinations.

Strongly Disagree	
Disagree	
Uncertain	
Agree	
Strongly Agree	

18. Lecturers avail themselves for discussion of work done by students during lectures and offer instructional help to deal with students difficulties to carrying out procedures flexibly, accurately, efficiently, and appropriately.

Strongly Disagree	
Disagree	
Uncertain	
Agree	
Strongly Agree	

19. Students practice reasoning in order to solve mathematical problems.

Strongly Disagree	
Disagree	
Uncertain	
Agree	
Strongly Agree	

20. Students participate actively in making conjectures, constructing arguments to convince others, and reflect on problem-solving approaches such as heuristics during tutorials.

Strongly Disagree	
Disagree	
Uncertain	
Agree	
Strongly Agree	

21. In the alternative mode of instruction students have an opportunity to define concepts, understand proofs and identify their ways of understanding and thinking much better than in normal mathematics mode of instruction.

Strongly Disagree	
Disagree	
Uncertain	
Agree	
Strongly Agree	

22. I am confident in my mathematical knowledge and abilities.

Strongly Disagree	
Disagree	
Uncertain	
Agree	
Strongly Agree	

23. I see mathematics as reasonable, intelligible and can solve mathematical problems by working hard on them.

Strongly Disagree	
Disagree	
Uncertain	
Agree	
Strongly Agree	

24. Lecturers present and scaffold students with instructions that focus on important mathematics content and hence develop students knowledge, skills, abilities and inclinations.

Strongly Disagree	
Disagree	
Uncertain	
Agree	
Strongly Agree	

Appendix B - Focus Group Discussion

UNAM Mathematics First Year (2012)

1. What did you like best about the First year Mathematics Intervention programme?

2. What did you like least about the First year Mathematics Intervention programme?

3. How do you think the teaching strategies approach implemented by Dr Gideon, Ms. Amakutsi, Dr Shuungula and others helps to develop your mathematical understanding?

4. Do you have any concerns about this team teaching strategy? If so, what are they?

5. One student said “It deepens my level of my understanding and provides a comprehensive knowledge broadening to be proficient in Mathematics” What are your thoughts on this? Please elaborate.

6. If there was one thing your lecturers could do to improve teaching mathematics in your course, what would it be?

7. What do you think was the most positive or valuable aspect of the course?

Appendix C - <u>Consent Form</u>

<u>UNAM First Year Mathematics Lecturers Interview Guide</u>

I want to thank you for taking your crucial time to meet with me today. My name is Ms. Reginald Iipinge, Part-time student (**11I3692**) studying for a **Master's Degree in Mathematics** Education with Rhodes University of South Africa. I am also a former **B.ED** (Mathematics & Biology) student of University of Namibia and currently teaching Mathematics and biology at Jan Mohr Secondary School.

I would like to talk to you about your experiences and personal thoughts in participating in the first year Mathematics **alternative mode of delivery intervention programme** at the University of Namibia (Main Campus).

Specifically, as one of the key areas of the overall course program evaluation, we are assessing program implementation, with the aim of attaining effective insights into the learning of Mathematics, that can feed into future constructive interventions in this course and higher Mathematics education in general. The interview should take less than an hour. I will be taping the interview session because I don't want to miss your comments. Notes will be taken as well. All responses will be kept confidential. The interview responses will not be shared with anyone else apart from Rhodes University staff members involved in the supervision of this process. We will ensure that any information we include in our report does not identify you as the respondent.

Do you have any questions or comments?

1.

2.

Are you willing to participate in this interview?

Yes…..   No……………………..

…………………………….   ……………………………

Interviewee   Date

Appendix D: Lecturers interview

1. When did you start with your Mathematics teaching career at University of Namibia?
2. What is your role in teaching the First year Mathematics Intervention Program?
3. People often have difficulty grasping the concept of Mathematical Proficiency, what is mathematical proficiency to you?
4. In order to learn mathematics successfully students need to capture Mathematical Proficiency. How do you help students acquire mathematical proficiency?
5. Teaching and learning of mathematics is the product of Instruction. That is, the interaction among the lecturer and the students, around the mathematical content in contexts. How do you manage instruction in ways that help the students on the intervention, develop mathematical proficiency?
6. How do you model the performance of your students and so integrate them into a specific implicit and explicit culture of knowledge?
7. How do your cover the prescribed learning outcomes in a way that interests students and keeps them actively engaged in learning?
8. Do you think that most students find the use of tutorial sessions helpful? If so how?

(a) Presenting complex mathematical problems

(b) Helping students by giving them more individual attention……If, not, why not?

9. As Dewey expressed, knowing mathematics involves discovering meanings and relationships in mathematical concepts. How do you help students discover meanings and relationships in mathematical concepts?
10. What do you do to motivate your students to engage productively in mathematical lectures and learning activities in those lectures?
11. Do you stimulate high-level ways of thinking among your students in a mathematics lecture? Please elaborate...
12. Was there any remarkable appreciation regarding to the effective strategies by the students? Please elaborate.
13. Do you have questions or comments?

Appendix E: Observation Schedule and Analytic Protocol

Lecture Observation	Strongest	Average	Weakest	Absent
1. How does Mathematics instruction look in the nation's lecture?				
2. To what extent does the lecture engage students intellectually with important mathematics disciplinary content?				
3. Degree of sense-making and habitual inclination is appropriate for this lecture				
4. The lecture hall management enhances quality of lesson				
5.Lecturers questioning enhances development of students ways of understanding				
6.Instructional strategies are consistent with Kilpatrick instructional framework				
7.Active participation of all students is encouraged and valued appropriately				
8. Students are encouraged to generate ideas, questions and conjectures				
9. Students are active listeners				
10. There is adequate time and structure provided for student reflection and sense-making				
11. To what extent does each of the following factors shape lecture's decisions about the learning outcomes and pedagogy?				

(g) Student self-efficacy				
(h) Discipline maintained				
(i) Time management				
(j) Modeling students				
(k) Group instruction				
(l) Students notion of taking notes				
12. To what extent are the following strands of Mathematical proficiency incorporated?				
(f) Conceptual understanding				
(g) Procedural fluency				
(h) Strategic competence				
(i) Adaptive reasoning				
(j) Productive disposition				
13. Students are intellectually engaged with important ideas				
14. Intellectual rigor, constructive criticism, and challenging of ideas are evident				
15. The Lecturer is apparent confident in teaching				

Appendix F: Permission letters

Reginald Iipinge
P.O. Box 70456
Khomasdal
Windhoek
03 February 2012

The Vice-Chancellor
University of Namibia
Private Bag 13301
Windhoek

Dear Prof L. Hangula

Re: Request for permission to conduct an educational research at your institution

I do hereby request the permission from your office to conduct an educational study at your institution in the field of Mathematics. The purpose of this research is to investigate the influence of the alternative mode of delivery on the teaching and learning and development of Mathematics Proficiency. This will involve developing understanding of how teaching and learning context of the First year mathematics remedial mode intervention programme supported effective student learning. It will also engage with the experiences of the individuals who are part of the intervention programme and the meaningful insights they made of their learning experiences.

I am a part-time student studying for a Master's Degree in Mathematics Education with Rhodes University of South Africa. I am also a former B.ED (Mathematics & Biology) student of University of Namibia and currently teaching Mathematics and Biology at Jan Mohr Secondary School.

The study will take place in March-June 2012 and there are no foreseeable risks to students and lecturers who will be participating in the study. All the information gathered in this research will be kept confidential. My research proposal was approved by the Education Higher Degrees Committee and complied with the ethical clearance requirements of the Faculty of Education of Rhodes University and in case if there is any question or query about the research proposal it is ready and available.

Yours in education

…………………………..

Ms. Reginald Iipinge

Reginald Iipinge
P.O. Box 70456
Khomasdal
Windhoek

03 February 2012

The Dean of Science Faculty
University of Namibia
Private Bag 13301
Windhoek

Dear Dr Gideon

Re: Request for permission to conduct an educational research at your Science department

I do hereby request the permission from your office to conduct an educational study at your institution in the field of Mathematics. The purpose of this research is to investigate the influence of the alternative mode of delivery on the teaching and learning and development of Mathematics Proficiency. This will involve developing understanding of how teaching and learning context of the First year mathematics remedial mode intervention programme supported effective student learning. It will also engage with the experiences of the individuals who are part of the intervention programme and the meaningful insights they made of their learning experiences.

I am a part-time student studying for a Master's Degree in Mathematics Education with Rhodes University of South Africa. I am also a former B.ED (Mathematics & Biology) student of University of Namibia and currently teaching Mathematics and Biology at Jan Mohr Secondary School.

The study will take place in March-June 2012 and there are no foreseeable risks to students and lecturers who will be participating in the study. All the information gathered in this research will be kept confidential. My research proposal was approved by the Education Higher Degrees Committee and complied with the ethical clearance requirements of the Faculty of Education of Rhodes University and in case if there is any question or query about the research proposal it is ready and available.

Yours in education

…………………………..

Ms. Reginald Iipinge

Appendix G: Authorization letter to conduct research

UNIVERSITY OF NAMIBIA

(+264 61) 206 3934 / 3741
Fax: (+264 61) 206 3791
Website: www.unam.na

340 Mandume Ndemufayo Avenue
Private Bag 13301
Windhoek
NAMIBIA

Inspiring minds & shaping the future

FACULTY OF SCIENCE

Ms Reginald Iipinge

P. O. Box 70456

Khomasdal

Windhoek

12 March 2012

Dear Ms. Iipinge

RE: REQUEST FOR PERMISSION TO CONDUCT EDUCATIONAL RESEARCH

Your letter requesting permission to do research as part of the post-graduate studies for masters of Mathematics Education Degree, through the Rhodes University, Grahamstown, South Africa, has reference.

Kindly be informed that the University of Namibia recognizes your magnificent effort and the possible contribution your research initiative can make towards successful mathematics interventions implementation for education in a broader sense.

This letter grants your permission to do the required work in terms of consultations, interviews and other related interactions at the University of Namibia.

Kindly note that the University of Namibia would expect from you to deposit copies of your published work in the respective UNAM libraries and resource centres. Also ensure that your research activities do not interfere with normal school programmes.

Best wishes for success in your academic endeavours.

Yours faithfully

....

Gideon Frednard

Dean of Science Faculty

UNIVERSITY OF NAMIBIA
Inspiring minds & shaping the future
12 -03- 2012
FACULTY OF SCIENCE
OFFICE OF THE DEAN
SIGNATURE: ....

Appendix H: Transcript for UNAM First Year Mathematics: Focus Group Discussion 1
20/05/2012 Time: 8:30

1	**I**	**What did you like best about the First year Mathematics Intervention programme?**
2	Simon	We actually get taught and the lecturer takes more time to explain things to us....The intervention portrays a slower pace on how the mathematics content is presented and the lecturers makes sure that we all understand, it is really nice. If you come 5 minutes late it's like you will miss out on a lot of things because the lecture commence at 07h30. Being on the Intervention programme you are likely to fail and repeat first year mathematics modules.
3	Gift	There is something interesting about the lecture presentation. The Intervention programme bridges the gap in specific mathematics concepts we need to aid our understanding. There are many concepts that we didn't cover in high school as a result of three mathematics levels (i.e. Higher level, extended and core). I basically have done core level in high school and found most things difficult, so being on this intervention programme is really a good opportunity to discover more mathematics knowledge needed to build my foundation to continue with my second year mathematics modules.
4	Jennifer	It gives us more time to do things like in this case we have a whole year to study for a certain module. Instead of a semester this time you don't rush to do things, if you don't get help with that something you need help on at that time, you actually have a chance to know it in class, in this case with the intervention program we have enough time to ask our tutors and to help us more, so there is a wider range of helping hand to get solution to mathematical problems.

5	Sacky	The best thing about Intervention programme is the Tutorial sessions. I find Tutorial session beneficial to me as we get to interact with other students and justify our answers. We explore more mathematical strategies to tackle problems as we share ideas and information, seek additional help from tutors who are always present and ready to help us. I can compare with learning with high school mathematics teaching. In high school we usually sit in groups and our teacher gave us more group work tasks. Problems were solved quicker. It's more convenient and comfortable solving mathematical problems with friends. In tutorials we are free to ask further questions and clarifications as need arises.
6	Martin	I think Group work or team work in tutorial sessions are helpful. We work together and engage ourselves in the learning process instead of listening to a lecturer presentation. We are given more time to formulate our ideas, discuss them with our partners. I have learned so many things I haven't learnt in high school. After discussions with my peers, I carry out my procedures efficiently and much quicker, this taught me to read questions comprehensively and think of it not just do it but I actually understand how to solve it.
7	Sawyers	What I like most about the intervention program is that it takes pressure off the students work and they can focus not only on mathematics but other subjects. This is because in our tutorial sessions we share ideas with our peers. Solving mathematical problems in a group aid more understanding of concepts and rules. In the end their results will look better. Learning mathematics on the intervention programme is totally different from high school mathematics learning. We are taught in different approaches.
8	Titus	I like working and interacting with people. I am happy with the way things are done. High school was totally different as we normally just listen to our teachers presenting stuff to us on the board then we get classwork and homework. On the intervention we have group discussions which are helpful. The tutors are available to help us understand. If you didn't understand something, another

		student can explain it in their own words.
9	**I**	**What did you like least about the First year Mathematics Intervention programme?**
10	Sacky	The mathematics intervention is delaying other mathematical modules because it is of duration of one year, by the time you complete basic mathematics you will be delayed to start with other modules like pre-calculus module.
11	martin	Basically for me the only thing is, in basic mathematics they explain things too slow for me. Even on the slow mode the things like fractions you don't have to go too slow so the fact is they take too long to explain one question I know it's a year module but time is not of their essence.
12	Titus	Tutorials. Group work is only beneficial if members understand stuff. Group work might not be good on one hand because sometimes you go to tutorial sessions and get in groups and work at the question, the knowledge that we have might not be advanced if we are not guided every time. There are few tutors compared to a group of students. So a lot of time is wasted searching through books and waiting for tutors to help.
13	**I**	**How do you think the teaching strategies approach implemented by Dr Experience, Ms. Volvo, Dr Calculus and others helps to develop your mathematical understanding?**
14	Titus	I personally think that so far it has been well. Referring to Dr Calculus for example he explains very well taking time and he allows students who don't understand to ask questions that's how he develops my mathematical understanding of concepts more than I did in high school.
15	Sacky	In my own opinion I think it is very fair for example Dr Experience who gives lecturers of Analytic Geometry, he gives a problem with a solution. He doesn't

		go through the basics for example how to get to the answer, meaning you are forced to work it out at home and see if you understand how he work out the answer.
16	Sawyers	It is also about discovering different approach to solve mathematical Problems so that you can share with other students. Some lecturers were high school teachers and have good background on how to interact with students and help them discover meanings in mathematical concepts. It is also about the different styles of lecture that this team teaching strategy possess.
17	Jennifer	It's more about discovery since there is a wider range of different lecturers. Some of them have an opportunity to actually go through or to have a class with maybe 3 lecturers like for you to choose which lecturer do you think is the best for you, so I think we as students have a chance to go about if you have that chance you should go about to look through the 3 lecturers and choose one that does it good for you. With a good lecturer it's obvious that you have good grades in Basic Mathematics or Analytic
18	Martin	I think for some it varies. We have different lecturers, so some lecturers need more teaching aids than others; some have a good backgrounds and experiences which we find effective, so basically it's just for a student to see which one works out for you. Some lectures are good with theory others are good with numbers and can explain them differently.
19	Simon	For instance we have a wireless network, so I think they should create groups or net, for instance like a math group on Basic Mathematics or Analytic Geometry where one can chat with the lecturer, helping each other with the students website, chat groups, so it's one of the things we would really like to have.
20	Gift	The team teaching strategy adopted by different lecturers helped me to develop my approaches to learning mathematics. They possess different characteristics which we found interesting and helpful in the end. Having many lecturers makes

		it easier to get the individual attention we want. We have more time to approach them anytime rather waiting for the following day in lecture to ask questions.
21	I	One student said "It deepens my level of my understanding and provides a comprehensive knowledge broadening to be proficient in Mathematics" What are your thoughts on this? Please elaborate.
22	Jennifer	I would like to have an exchange student group for instance we go visit other Universities and students from other universities can visit us or something in that way we can actually learn different ways of doing and different ways of learning mathematics.
23	Titus	I think so far it deepens my understanding, for example Dr Calculus giving you a sum to do but he gives you the answer and shows you the steps procedures to get to the right answer that gives me courage to do it on my own.
24	Sacky	It all depends on the background where we are coming from. Some students completed their high school mathematics on NSSCO-Ordinary level which is no really advanced as NSSCH. So I think this is a nice platform to students that completed their grade 12 mathematics on NSSCO Ordinary level, core and extended where most topics in first year mathematics were not part of the syllabus.
25	Simon	Yes the intervention deepens my understanding. It helps me to understand every concept better because of wide range of helping hand opportunities we have on the programme like, more lecturers. If I didn't really understand what one lecturer presented than can ask the other to explain to me in another way. We have group works that are outrageous. We have more time to interact with each other and justify our work to each other.
26	Gift	It provides a comprehensive knowledge broadening to be proficient in mathematics, yes so the way the presentation is done in such a way that all new

		concepts learned are defined in an easier way. We discover more strategies as we share ideas with peers. The more rules and justification learnt in tutorial as we interact with the tutors lead to more reasoning. In the end we are ready and joyful to solve new mathematical problems on our own.
27	I	If there was one thing your lecturers could do to improve teaching mathematics in your course, what would it be?
28	Sawyers	They could slow on their teaching pace. Most lecturers teach fast and skip a lot of important steps. They could focus more on doing mathematical problems procedurally, step by step and they perhaps can include online tutorials with necessary workings and solutions. When students go home, they can see how things are done.
29	Martin	For the methods, they actually should extend the time because the time is limited. One hour is not enough for us. In an hour of class time is not enough grasp a lot of things. If we could have 2 hours we could actually have enough time to reflesh our minds like 5 minutes break and come back like for instance my attention span is 55 minutes than I am off so I have to go to the toilet and come back to the lecture.
30	Simon	Do something to attract more students to come to lecture every day. Students do not turn up regularly especially during the 07h30 lecture. Some students complained of transport problems in the morning. Some perhaps time shift to 09h30 latest instead of 07h30 will do for most students.
31	Titus	For the tutorials I don’t really think like for the past three months tutorials were not useful at all because the tutors only consult the specific people on one question instead of presenting it to everybody. Tutorial sheets should be ready two days in advance so that we have enough time to tackle them ourselves before the tutorial session, in this way the can manage time effectively and concentrate

		more on other things as well.
32	Gift	Increase the number of tutors. Bring in more female tutors because male tutors concentrate more on female students. And they should get more qualified tutors instead of 3rd and 4th year students who are not experienced in teaching. This is because sometimes they make too many mistakes during tutorials or perhaps they don't prepare well.
33	Jennifer	Initiate on key strategies that are crucial for teaching complex and new ideas to students who are weak, in a short, compact time. Lecturers can go beyond the programme if students are not challenged by the new material presented to them.
34	Sacky	When the new topic is taught the lecture must make sure that the delivery is short and concise and be in story like format that is easy to follow. Explain the mathematical symbolic language in detail and they must not assume students know them. Looking at the backgrounds of students is also good so that lecturers can have a clear picture of students that they are dealing with.
35	I	What do you think was the most positive or valuable aspect of the course?
36	Martin	The most positive thing I would like to comment on is that it gives everybody a chance to pass Basic Mathematics because sometimes when you fail, you feel down that you will not have a chance to finish this course in a year's time, giving chances to start focusing on your work than you can do better because eve at the current stream (normal mode) it doesn't matter how smart you are it doesn't always work out, so it is pretty good.
37	Titus	I think the most positive aspect is that some of us are slow students that take time to understand things so it is a valuable platform that they brought up this intervention programme to make us understand.
38	Sacky	It provides an environment or the pace in which we can understand and easily adapt and it's also where we can perform at our optimum best to get quality

		results.
39	Jennifer	It's a good thing they brought up even though there are two modes the students are further encouraged to continue with their work no to give up hope on losing their course to the preceding year so you can still make it with the intervention programme that will give a second chance.
40	Sawyers	I like the intervention programme and it should definitely continue because it takes pressure off from students and give them time to focus on their modules.
41	Gift	I give credence to the cooperative group work we have in our tutorial sessions. Every student has a chance to open up to their peers.
42	Simon	The way the course is structured is good. More lecturers and tutors
43	I	Thank you very much guys, it was my pleasure talking to you! All the best

Transcript for UNAM First Year Mathematics: Focus Group Discussion 2

20/05/2012 Time: 9:30

1	I	**What did you like best about First year Mathematics Intervention programme?**
2	Keith	This program gives you more time to understand certain concepts and for me as a statistics students, Basic mathematics is linked to most of my modules, like I have a module of probability and some of the mathematical problems that we do in Basic Mathematics are also in probability so I like it very much.
3	Kelly	As a physics student this intervention is very good because it gives me more time to understand my work and so that by the time I go write my exams I won't fail and then he will make it to the following year to complete my course.
4	Romeo	I support Keith's idea, as a mathematics student this intervention programme gives me more time since I am also slow at grasping mathematical concepts. The programme also provides me with skills and knowledge I need as well as how to keep up with time in order to become an advanced university student. I need to learn how to do things on time for example, when to hand in assignments, tutorial sessions, lecture and when to write tests. I was not used to such things because in high school it was different.
5	Henry	I think it is a good approach that can determine each respective student possible or mental capabilities in the module and it also label you as an intervened student to have more time to study that particular module. One needs to prioritize and schedule one's time outside of the university. I learned a lot about planning my assignments, revising for my test and prepare my questions to ask in tutorial sessions. So it up to me to plan for my social life so that it doesn't clash with my studies.

6	Hanriette	This course or intervention programme gives me a better opportunity to understand things more like suppose we were to have a module in a semester but due to the intervention programme we are given a longer period doing this. It also gives us more attention and we are considered well since we are few students
7	Candice	I like the intervention programme because it is an advance opportunity to meet new people. I am really lucky to have 5 good friends in Mathematics that I can count on every time I need help. If we didn't have the Intervention programme, I probably wouldn't have those friends.
8	**I**	**What did you like least about the First year Mathematics Intervention programme?**
9	Henry	The course doesn't necessarily determine student capabilities you probably be on the intervention programme but than your capabilities are just as good as the other students on the current normal mode but I don't think it is fair at times for students unless if the students probably start complaining.
10	Candice	Intervention programme is of the duration of one year that gives us time to do pre-calculus on the following year that will give us more modules for the preceding year which can be difficult.
11	Romeo	It also elongate the stay at university because of the extra year module added and other modules will be shifted to other years, because for other modules Basic Mathematics is a prerequisite for them.
12	Kelly	Time management. Attending lecturers in the morning like 07h30 is a disaster.
13	Hanriette	I basically feel the students that came from college filled up more spaces on our programme. The number of students have increased than it was supposed to be, nevertheless this is concern to slow students who don't have confidence to ask

		questions
14	Keith	Nothing so far because I am happy with the programme, I have gained knowledge in mathematics. This helped me to adopt new study methods for example constant revision that I can apply to other modules as well.
15	**I**	**How do you think the teaching strategies approach implemented by Dr Experience, Ms. Volvo, Dr Calculus and others helps to develop your mathematical understanding?**
16	Kelly	Dr Calculus is a very good lecturer since he gives more examples during lectures. For instance when he introduces a new topic, he first defines and explains the concept before he explains further, so that you can have an idea when you start working on the real work. For example when we started working on sets. He commenced the lesson with definition of sets i.e. any collection of well-defined objects and giving more examples of sets i.e. the sets of all science students at UNAM is part of the set of students at UNAM. To recall on that he gave an example of set containing numbers 1,2,3,4 and he also gave examples of subsets of sets, cardinality of sets, equivalent (equipotent) of sets and universal set.
17	Henry	To add on that, Dr Calculus also emphasized more on rules underpinned by sets in terms of the union of sets, intersection of sets, something else that do not apply to that topic. Moving on other topics like polynomials. I like Dr Zoo for instance. He first defined the term polynomials and he gave us examples of polynomials and how to factorize polynomial.
18	Romeo	To add to what Henry said, I prefer Dr Zoo either. His teaching strategy is awesome because after defining the concept, he gives the theorem underpinned by the concept followed by the proof.
19	Hanriette	It’s more on the way he teaches, he makes jokes in a way of simplifying

		mathematical problems to us which makes him good to us. I prefer going to his lecture than others. I really enjoy his lectures so much.
20	Henry	He is always efficient. He values lecture time best. He makes most of his lectures count. He uses more of his time of doing what he prepared to do in lectures. Although Hanriette argued that he is fast we get to catch upon what he presents in lectures and tutorials.
21	Keith	Ms. Volvo is an experience lecturer. She knows how to present her lecture to her best of her capabilities for the best of her students. It's fun to be in her lecture. I really enjoy her very much.
22	**I**	**Do you have any concerns about this team teaching strategy? If so, what are they?**
23	Keith	There are not really much concerns, it is just that the number of the students is too large and the way we sit in the lecture halls and their designs of sound system are not up to standard. They really need some attention. The presentation of lectures sometimes is not good, we hardly can hardly see and hear what is happening and we don't really get time to go back on these mathematical problems presented because we always have other lectures to go to afterwards.
24	Kelly	During the tutorial sessions, tutors are very limited and when one ask tutors are very limited and when one ask tutors questions, they always have different answers and sometimes it's really confusing for students. This leaves us with doubts of what to the correct answer might be.
25	Hanriette	For some students they have misconceptions about the slow mode intervention programme that means that we actually have a long way to go and this makes us lazy though.
26	Candice	Not really, the lecturers on the programme are really nice. The should just help

		us more to get used to university life and help us with study methods
27	Romeo	The team strategy is really good. Lecturers should explain more to slow students
28	Henry	No, I'm happy with everything. It's up to me to put in more effort to pass well
29	I	One student said "It deepens my level of my understanding and provides a comprehensive knowledge broadening to be proficient in Mathematics" What are your thoughts on this? Please elaborate.
30	Candice	I partially agree with that, since we are on this intervention programme over a longer period of time, we are given more attention so we have more time to go through our work and understand thoroughly on what we are doing. Ultimately we enjoy the lectures more since we understand things better.
31	Hanriette	Since our understanding is different, we get to understand what we are taught especially the basic important things of mathematics like getting to know the rules, the theorems of mathematics, concepts which improves our proficiency of understanding mathematics.
32	Romeo	I fully agree with that. Being on the intervention programme we actually have more time to study, rehearse, revise and elaborate which eventually help us in the examination.
33	Henry	It is actually nice since are given a longer period to understand the concepts, rules, and the way you will exercise it will be useful.
34	Keith	The intervention programme helped us to build a community of students. We work together in a conducive environment, this helped my approach towards mathematics.
35	Kelly	The intervention programme helped me to do stuff on my own. After the tutor's explanation was clear to one student in our group, that student will help other students who don't understand. It allowed us to meet new friends which lead to

		more comfortable tutorial sessions. Having more friends that one can ask for help it's really fascinating.
36	I	If there was one thing your lecturers could do to improve teaching mathematics in your course, what would it be?
37	Romeo	What I basically think on this regard is that the lecturers should be more on the same pace of presenting at different venues so that this whole large group could be divided into subgroups. This will help the lecturers to give attention to each respective student even though not all but most students will like it.
38	Keith	To add on what andreas stressed I think this would also help the students to be on the same level because if they are divided into certain groups all lecturers will carry out their presentation at the same/ time and this would help students to understand things much better. Students on campus might help each other especially those that are doing Basic Mathematics and Analytic Geometry and so forth.
39	Hanriette	We should have tutors that help us understand more concepts in the tutorial sessions. Before we attend the tutorials they usually hand out tutorial sheets to work them out and you ask questions where you don't understand. There are times when these tutorial sheets are not enough. This leaves a bad impression for us because we will feel left out with the work, so more tutorial sheets must be duplicated and some students take more tutorial sheets than they need while others didn't get.
40	Candice	The tutors should always prepare in advance before the tutorial sessions to save more time. They can just work out the answers so that we can work on them. In that way more students will get help from the tutors.
41	Henry	To accommodate all students to get tutorial sheets they should introduce online tutorials they should email them to students. We need more lecturers to

		accommodate our needs. The ratio of numbers of students does not correlate with the number of students. We need a mathematics society club on campus. The intervention programmes help everyone regardless of your high school background. It doesn't really matter on which level you have matriculated. The remarkable aspect is that the programme allows slow students to catch easily because of the pace it portrays.
42	Kelly	I think they should start introducing sort of campaigns so that we can meet often to help each other with more challenging mathematical problems. We don't get enough time in the tutorial sessions we only have two hours which is not enough really because we have 30 minutes to write tutorial test and that time is not really enough because not all of us have chances to ask tutors for help at their offices. I think the campaigns will do the best in this regard to help us more because we will get to meet. The lecturers should encourage us, they should also give us more questions to work on and we can meet lecturers and they can tell us whether we are right or wrong. Maybe in that way they can help us with the solutions.
43	I	What do you think was the most positive or valuable aspect of the course?
44	Keith	The intervention programmes serve as stepping stone to Tertiary level. The intervention programme polishes your mind very well and makes you understand that nothing is really easy and nothing is really difficult, all what students need to do is just to study hard and it will make us study more in the coming years. The motivation from lecturers on the programme is really positive and outstanding.
45	Hanriette	The valuable aspect of the intervention programme is that it increases the chances of broadening my proficiency in mathematics since I am on this intervention where things are presented in a simpler way or simplified this means that I can take my work very slowly and ultimately I understand to

		broaden my mathematics proficiency.
46	Candice	As Geology student it is required for me to pass all my mathematics modules, so being on this intervention where things are presented in a simpler way or simplified this means that I can take my work very slowly and ultimately understand well. Since this programme I have learnt that getting an answer is not as crucial as getting the theory or procedures behind it. It helped me to ask self-questions for example if a get a wrong answer, I would look at the solution and work out the reasons why I failed to get this solution and the strategies and procedures used that led to that answer this broaden my mathematics proficiency.
47	Henry	Since first year mathematics modules are prerequisite for second year mathematics or other modules, being on this intervention programme it allows and enables students to pass and continue with other modules as they are done based on the progress of these modules. This assures me that the gate is open to proceed to the following year.
48	Keith	Being on the intervention programme, I have gained self-confidence and all motivation needed to go through this programme. This gave me extra motivation to work harder than before. I hope this will work forever.
49	Romeo	I have gained all my strengths needed to encounter all my numeracy problems like for instance polynomials. I was a bit lost and confused as it was the first time learning this topic. This helped to fill in all gaps in my abilities that hinder my understanding of this topic.
50	Kelly	The intervention programme advanced my greatest challenge i.e. fear. I have never really understood many concepts, theories and methods of solving polynomials. Looking at things like rationalizing denominators of the expressions, how to express real numbers without using the absolute value symbols and properties or radicals. I really wanted to understand these things till

		I am more comfortable doing it self with acquired knowledge. This helped to improve my ways of thinking and reasoning on my own without getting help from others.
51	I	Thank you very much guys, it was my pleasure talking to you! All the best

Appendix I: Transcripts for the Lecturers' interview1: Ms. Volvo

26 May 2012 Time: 14h30

	I	**When did you start with your Mathematics teaching career at University of Namibia?**
2	Ms. Volvo	I started my teaching career at UNAM in 200l as a mathematics tutor. Although I am still a tutor, I help in the lecturing process, as the mathematics department has shortage of lecturers.
3	I	**What is your role in teaching the First year Mathematics Intervention Program?**
4	Ms. Volvo	What we normally do at UNAM is that we conduct teaching at a slow pace so that students can grasp the methods and concepts.
5	I	**People often have difficulty grasping the concept of Mathematical Proficiency, what is mathematical proficiency to you?**
6	Ms. Volvo	It is all about conceptual understanding, how we comprehend mathematical concepts, operations and relationships, about the procedural fluency the skills in carrying out the steps accurately when you are solving problems, strategic competence that is the ability to formulate the solve problems, reasoning that is showing logical thinking that we should explain and able to justify.

9	**I**	**Teaching and learning of mathematics is the product of Instruction. That is, the interaction among the lecturer and the students, around the mathematical content in contexts. How do you manage instruction in ways that help the students on the intervention, develop mathematical proficiency?**
10	Ms. Volvo	Students on the intervention need attention. One doesn't have to teach a lot of problems per teaching session. So, let's say you can only take like teach two or three problems and then you make sure that you give enough time and chance for students to carry out the problems after you have explained the concepts and give them enough time to talk to one another to discuss, is the only way they can learn effectively. Let them learn from one another before we can do the problems together. I apply these strategies during lecture and tutorial sessions respectively. During lecture for instance it takes only about 5 minutes to pose the problem and they discuss. As long as one knows how to manage time effectively to finish the intended curriculum at the end of the year. One doesn't need to rush as we are working with slow students.
11	**I**	**How do you model the performance of your students and so integrate them into a specific implicit and explicit culture of knowledge?**
12	Ms. Volvo	During Lecture one can give a problem that they can do at home and the next day you can do at home and the next day you can check what they have done. Sometime I call one student on the smart board to present his/her homework while others listen or I can take 2 minutes to go from one student to the other from both rows to check if homework is really done and whether it is correct. But I think calling one student

		as representative of others can save time and the rest can ask questions to clarify more how the representative student got the answer.
13	**I**	**How do your cover the prescribed learning outcomes in a way that interests students and keeps them actively engaged in learning?**
14	Ms. Volvo	At the beginning we usually relate the new topic to what they already know then you show the leak on previous knowledge and probably explain why that topic is related to everyday problem.
15	**I**	**Do you think that most students find the use of tutorial sessions helpful? If so how?** **(c) Presenting complex mathematical problems** **(d) Helping students by giving them more individual attention……If, not, why not?**
16	Ms. Volvo	Yes, they find the tutorial sessions helpful. For the past year tutorial sessions were smaller groups but this year's intake is too much that one tutorial group has up to 70-100 students. With 2-3 tutors per tutorial sessions we cannot really give individual attention to all students. The way we run tutorial session is in a way that students solve the problems and if they encounter problems they put their hands up and we go to them and they work in groups so they learn from each other but even than the classes are too big, you cannot help all students. Tutorials are good sessions for students to take notes and learn what they been taught provided with tutorial questions. For the fact they know that at the end of the

		session they will write a test of 15 minutes. Tutorial sessions are forcing and motivating them to study and attend also. If there is no tutorial test some students don’t turn up for tutorial sessions.
17	**I**	**As Dewey expressed, knowing mathematics involves discovering meanings and relationships in mathematical concepts. How do you help students discover meanings and relationships in mathematical concepts?**
18	Ms. Volvo	I use constructivism approach in teaching where students make meanings of mathematical concepts to their experiences with others. So we encourage them to study mathematics with their friends and peers to solve problems together to teach others. When you teach others, explain steps or procedures, how to find a solution to a problem by teaching your peer you will learn and help reinforce the concept and boost their morale and confidence in the subject. So it is about learner-centred approach. Focusing more on students in such a way that they should learn with one another and the lecturer just facilitate them.
19	I	**What do you do to motivate your students to engage productively in mathematical lectures and learning activities in those lectures?**
20	Ms. Volvo	In the beginning, there are certain topics in mathematics where you can tell stories related to that topic and that stimulate or motivate them to be interested or to take part actively in class also when you give equal chances to the students to solve problems by treating them equally or you answer their questions or you get to be very friendly to them. Students will start feeling at home and them they will listen to

		you attentively, so the issue is providing a conducive environment for learning is what is important. One needs to vary our teaching methods, not just lecturing every day or preach like a pastor. For the students that find it difficult to attend lectures, sometimes we keep class /lecture attendance also in UNAM prospectus if is clearly stated that students require 90% attendance.
21	**I**	**Do you stimulate high-level ways of thinking among your students in a mathematics lecture? Please elaborate...**
22	Ms. Volvo	Yes by asking questions that force them to think critically. I ask them to explain why getting certain answers for example, justify for instance why is that vector orthogonal to the other vector? They must be able to explain why, they must say why orthogonal or why two lines are parallel? They must make sure they use proofs of properties and axioms because in proofs you ask them to show their logic.
23	**I**	**Was there any remarkable appreciation regarding to the effective strategies by the students? Please elaborate.**
24	Ms. Volvo	Last year most of them were able to pass at the end of the year and this year they came back to us to say thank you because they were taught at a very slow pace, they had enough time to solve problems, they were given more attention they were able to understand and grasp the concepts and now they are helping others out.
25	**I**	**Do you have questions or comments?**

		No comments or questions but I am happy I was able to help.

Transcripts for Lecture Interview2: Dr Theorem

1	**I**	**When did you start with your Mathematics teaching career at University of Namibia?**
2	Dr Theorem	I started my teaching career in 2002 as a lecturer that time. For now I am still a lecturer and also an HOD.
3	**I**	**What is your role in teaching the First year Mathematics Intervention Program?**
4	Dr theorem	I was one of the lecturers who helped to design the intervention program in 2010 and it was something we brought in so that we assist student struggling with mathematics. We noticed that there is huge gap from high school to university so I helped the department to implement this program.
5	**I**	**People often have difficulty grasping the concept of Mathematical Proficiency, what is mathematical proficiency to you?**
6	Dr Theorem	I think it is the ability to read mathematics and to explain both to you and to the audience. If one is not able to do that then they definitely have a problem with proficiency for example, if one is not able to explain the subject of continuous function and communicate clearly to others and then they are able to link that than we can say they are proficient with that aspect and then to be able to realize where are those things being applied in the various areas of mathematics so proficient has a lot to do with the ability to tell what you are reading and should

		be able to communicate.
7	**I**	**In order to learn mathematics successfully students need to capture Mathematical Proficiency. How do you help students acquire mathematical proficiency?**
8	Dr Theorem	I think that's through of course explaining concepts in mathematics and teaching them how to read mathematical passages sort of articles and they you now showing them how things are done and also using whenever you introduce a concept use one or two examples to illustrate that concept. So in that sense students have to see and understand the concept and also teach the students all the mathematics at once, if you want the students to be able to read on their own in future and explain as well to others.
9	**I**	**Teaching and learning of mathematics is the product of Instruction. That is, the interaction among the lecturer and the students, around the mathematical content in contexts. How do you manage instruction in ways that help the students on the intervention, develop mathematical proficiency?**
10	Dr Theorem	The university has helped us already in that area of by having a system of lecturers and tutors and so our instructions involve that we want to make sure that we spend our time teaching students and then in tutorial sessions, we are spending time helping the students to solve problems on their own, so in that situation we are actually pushing students to devote time on their own under

		your supervision. Sometimes when we just give these students problems to take them home and not necessarily do these things so they time when they are committed really getting to the grasp of things and we consider that as part of our instructions as way to actually help the students to develop that mathematical proficiency
11	**I**	**How do you model the performance of your students and so integrate them into a specific implicit and explicit culture of knowledge?**
12	Dr Theorem	It is actually a difficult thing really I mean when you look at the sizes of our classes I must say we are really battling to reduce the sizes by introducing more staff so that we are able to really have a pro overview in the size of lecturers. Currently we are definitely understaff, we are still give them tests, assignments through assessing them how they are doing of course but we still feel that if we are to reduce the class size we will be more effectively. We go through the memorandum of test and assignments together; Yes we so that sometimes we display the memorandum but personally my intention is that rather do not dispose the memorandum but go through it with the students. That's normally my approach.
13	**I**	**How do your cover the prescribed learning outcomes in a way that interests students and keeps them actively engaged in learning?**
14	Dr Theorem	We always try to look at the course outline, what it is all about and then while we are covering the course outline and looking at the various topics we indicate to them what is that and these going to be used for in higher mathematics and so we always try to help them and look forward and apply what they are learning

		so that they really actually make use of that what is taught to pass the exam and I encourage my students to keep the notes I am teaching them so in that way they get interested. Beginning of the year they get a specific course outline but, yes they get the course outline, that's what we give them and of course the aim of the course. We don't as such list out the learning outcomes to them we should expect them to read them from the prospectus.
15	**I**	**Do you think that most students find the use of tutorial sessions helpful? If so how?** **(a) Presenting complex mathematical problems** **(b) Helping students by giving them more individual attention……If, not, why not?**
16	Dr Theorem	We think so yes. On one hand our tutorial sessions become more specific, when students realize or notice that there is a test, but we tend to feel that students up to a certain extend they don't know the usefulness of these tutorials. We feel that most of their time because the challenge we have that one tutorial session of 70 students have only one tutor which is difficult. The tutor can't really go around and help everyone. That is shortcomings that have been making students not interested in tutorial sessions. They get time to ask questions but them you now and then on the basis of that they do ask but the tutor can help those that he is able to help in that tutorial session unfortunately but we would want them to be in such a way that the classes are smaller where one is about 40 students in a tutorial sessions. But we are still battling with our management and human resources to get more staff.

17	**I**	**As Dewey expressed, knowing mathematics involves discovering meanings and relationships in mathematical concepts. How do you help students discover meanings and relationships in mathematical concepts?**
18	Dr Theorem	Sometimes we tend to think that the teaching of mathematics is more of the discourse you know. You lay down the alphabet, the words and the axioms and the sentences that go along with that namely definitions and theorems and you expect the students to show these theorems demonstrate how it comes about so you set certain extent how we try to, as we are showing the results about you want to emphasize more on that not memorize the demonstration of the theorem learning how being able to understand it the theorem and how it is what it is saying the better that they have actually show a demonstration of how the results work or in a certain sense we are now helping them to realize that in mathematics it's not about memorizing things but it is about to develop ideas.
19	**I**	**What do you do to motivate your students to engage productively in mathematical lectures and learning activities in those lectures?**
20	Dr Theorem	Yes usually I mention the historical aspects of the number of results in mathematics especially mathematicians who came up with those results from way back I have seen that it motivate the students when you mention that these theorems was discovered by a particular mathematician that he was going through these or at that point he was going through that. At the same time they get to realize that those are just people who were working hard and coming up

		with these ideas so if they can do it, we can also do it as well.
21	**I**	**Do you stimulate high-level ways of thinking among your students in a mathematics lecture? Please elaborate...**
22	Dr Theorem	I always emphasize on whatever statement that they use it must be mathematically correct not just that they want to get the answer. So and as I said earlier on these are mathematicians born before us we can do better and I always ask them well in terms of what are they aiming in life what do they want to become in life and especially the smaller group, I got more interested in career path that they are taking so that in that way I need to help them to focus more on their career path. I always try to encourage them to become mathematicians. Being a mathematics, to take higher level mathematical text and
23	**I**	**Was there any remarkable appreciation regarding to the effective strategies by the students? Please elaborate.**
24	Dr Theorem	Yes through university evaluation so they wrote comments about how they feel about the running the course and through those comments. I realize students indeed appreciate what we have been teaching them and some of them make negative remarks about my teaching which I have taken into account. It is something positive that will help me to change how I have been doing things, making decisions better and so forth.

25	**I**	**Do you have any questions or comments?**
26	Dr Theorem	Well, as I said earlier on some of the questions are framed into a psychological way that make one to think deeply but it's quite okay.

Transcript for Lecture Interview 3: Dr Calculus

26 May 2012 Time: 11h30

1	I	**When did you start with your Mathematics teaching career at University of Namibia?**
2	Dr Calculus	I started my teaching career in 2002, I have been a tutor for about 2 years before that and then I went for studies. I came back in 2002 as a lecturer at UNAM.
3	I	**What is your role in teaching the First year Mathematics Intervention Program?**
4	Dr calculus	At the moment, I am teaching slow mode Intervention at UNAM Windhoek campus but I am coordinating with the other 4 campuses from other regions namely; Khomasdal, Ongwediva, Rundu and Katima mulilo. When it comes to setting tutorial questions and tests I am the one in charge and sent it to the other campuses as well.
5	I	**People often have difficulty grasping the concept of Mathematical Proficiency, what is mathematical proficiency to you?**
6	Dr calculus	In my understanding you get to teach certain it's like a process whereby you get to teach certain concepts to students in most cases for example I start with a simple idea that they already know and then I develop that idea towards a new concept that I would like to introduce. Once I have introduced the

		concept then I tend to give then I tend to give them examples because they tend to be more comfortable after seeing some examples and then I give the to solve any questions in general related to what you discussing from there they will get the idea to answer questions more. Basically, it's more of that process from what they already know step by step introducing a new concept giving than a general case with the examples for example,Yes, let's say for example like today I taught the students how to solve polynomials so they are required to if given two polynomials, let's say polynomial P and polynomial D now we want to divide the P/D using long division. So we are going to get the quotient let's say Q plus Remainder let's say R, so that P/Q is equal to Q+R/D, now introducing that, I explained for that if you take 9/2= you get 4 plus Remainder ½. Furthermore I told that if it was for example 8/2 = Remainder 0 because you will get 4, that is 8/2=4 whereas 9/2=4 plus ½. From there I told them that division of the polynomials follow the same procedure of the polynomial of a bigger degree so polynomial P/D= Q plus R/D, which they can also write as P=Q*D+R,From there, they will have an idea of that they can do. I will then introduce the idea of long division involved and introducing them to the steps that are for example how do you divide when do you stop when the degree of the Remainder is smaller than the degree of the polynomial that is dividing so that at the end they have a general procedure but they understood it with the simple example shown with the new procedure they can divide any two polynomials.
7	I	**In order to learn mathematics successfully students need to capture Mathematical Proficiency. How do you help students acquire mathematical?**

8	Dr Calculus	What I usually do is, since I use transparencies I avoid writing things down before the class. If there is a kind of paragraph to be written before class just to save time but I do calculations until I am in class and I get to involve them in solving examples. I have to get answers from them by asking them how you go about solving that. I write theories and I explain it and we do it together sometimes when you write examples already and you have solved it might seem it is fully understood because they will just copy.
9	I	**Teaching and learning of mathematics is the product of Instruction. That is, the interaction among the lecturer and the students, around the mathematical content in contexts. How do you manage instruction in ways that help the students on the intervention, develop mathematical proficiency?**
10	Dr Calculus	What I normally do is that, I make use of tutorials sessions where by students are allowed to talk to each other. I give students instructions to solve mathematical problems based on the curriculum as stipulated in the course outline. I expect students to ask questions and collaborate more effectively in group work. I give them feedback at the end of each task given. At most I display the solutions to tests for them to copy, and if there are questions I help them individually.
11	I	**How do you model the performance of your students and so integrate them into a specific implicit and explicit culture of knowledge?**
12	Dr Calculus	In most cases, I model performance sometimes when I am marking on test, I tend to identify the problematic areas, some of them have problems like for

		example they are solving or required to find certain function and solve that function equated to zero and sometimes before they that to zero they can divide through by a certain number whereas you expect them only once they equate that to zero and maybe they can divide both sides and tend to identify these problematic areas and then discuss both procedures how to solve these problem, that way it also improve their performance because to realize that a person knows what he supposed to do, but might not be fluent in doing that. In tutorial, Mostly we acquire them exercise examples to solve they discuss them in groups if 2 or 4 people than they will try to solve that questions together and only among themselves nobody can really like solve it and then ask the tutors present to help them and we avoid telling them answers, rather than hints that lead to answer, while assisting others. Working in groups help them a lot. They are allowed also talk to other students because sometimes only one tutor present to help them and we avoid telling them answers, rather than hints that lead to answer, while assisting others. Working in groups help them a lot. They are allowed also to talk to other students because sometimes only one tutors present. Tutors might solve it for everyone on the board or give them instructions all at once.
13	I	**How do you cover the prescribed learning outcomes in a way that interests students and keeps them actively engaged in learning?**
14	Dr Calculus	What we try to do, in most cases we give them the course outline at the beginning and then all at the beginning of each chapter we explain the whole aim of the chapter, like what it is expected from them to learn in those chapters. In most cases at the beginning of the lecture I give them an overview what is the aim of the chapter, how far we have gone, and what

		skills, what learning outcomes they have learnt so far. It helps them when you tell them that by the end of the chapter certain learning outcomes should be mastered, and then can write it down it helps a lecturer because minds you like what are you supposed to give to these students.
15	I	**Do you think that most students find the use of tutorial sessions helpful? If so how?** **(a) Presenting complex mathematical problems** **(b) Helping students by giving them more individual attention….If not, why not?**
16	Dr Calculus	Yes, because in most cases they might think that understood from lecture and then you give them questions and they realize that they don't really quite sure how to solve some questions, so they come for tutorials. Serious students solve the tutorials questions before the sessions when they come to tutorials they only ask questions where they don't understand or they ask the lecturer to verify some works that they have already done there are students that are repeating the module, they think that they already know certain things so they avoid tutorials at all times, but what we do to encourage them to attend is that we give them tutorial test for 10 minutes at the end. Everybody attend tutorial once a week so we do it every second week. Tutorial tests motivate them because the tutorial sessions give a hint of what is to be covered in the tutorial test. For the alternative mode the pace is slower than the mainstream. Students enjoy getting individual attention from me because unfortunately when there is a test really wants that attention irrespective of the group size

		some might not get enough attention they would like to get.
17	I	**As Dewey expressed, knowing mathematics involves discovering meanings and relationships in mathematical concepts. How do you help students discover meanings and relationships in mathematical concepts?**
18	Dr Calculus	At the beginning of the semester we try to look at the course content and see how the content is related or various things are related to each other and we decide to tell them that we are going to start with this chapter until the last chapter and we design that according to the relationship between various concepts and then this makes it easy because when we teach for example this year the first chapter was on sets, things like intervals and now we are moving toward to solving things like equations and inequalities and then it will help them for example when they are solving inequality they get individual solutions but they get solutions that are in a form of intervals. We try to keep that order there so that they can see trueness of one concept with another and then we put in some practical questions in tutorials and test questions let's say for example when it comes to equations you give them practical questions like Y is your telephone bill and X is your number of units that you have used so that they can see that these things are real.
19	I	**What do you do to motivate your students to engage productively in mathematical lecturers and learning activities in those lectures?**
20	Dr calculus	Dealing with quite a very big group it's not easy. If you are giving them examples we try to analyse the level of mathematical problems to solve irrespective of their background knowledge so that you capture everybody's

		attention and so actively giving them like homework we give them examples with practical kind faced to tit rather than theoretical example i.e. geometrical sequences, you want to get from point A to point B every day you cover half the distance. Than you ask will you ever get there?
21	I	**Do you stimulate high-level ways of thinking among your students in a mathematics lecture? Please elaborate.**
22	Dr Calculus	Yes in this regard, there are simple and easy to prove theorem. They need a bit of logic. I do these theorems in lectures and get them involved them. So actually it's about proving theorem yet gets elementary results and they enjoy.
23	I	**Was there any remarkable appreciation regarding to the effective strategies by the students? Please elaborate.**
24	Dr Calculus	Yes, especially students who are not majoring in any subjects that will require them to do second year mathematics. They appreciate it because to them being in this new alternative mode of delivery is assuring them that they will pass their first year maths. It will not meant any extra year for them yet by the time they complete other modules they will finish their math's modules too other students find analytic geometry more difficult than basic mathematics and in some cases you find that the test that is separating them is only based on basic mathematics. If a student on slow mode on both basic mathematics and analytic geometry. Core students appreciate the new stream because the pace of teaching is slower accommodating all their needs and background gap

		that need to be filled in. Students with good background in mathematics like those that did higher level in higher school automatically go to normal mode.
25	I	**Do you have questions or comments?**
26	Dr Calculus	As a new lecturer on the alternative mode I didn't experience much unlike others who were part of this mode last year who made remarks that sometime students fail the first test due to some reasons and they might end up on alternative mode not really that they can't cope with the new stream, so in the end they sleep during lecture or they don't turn up at all.

Transcript for Lecture interview 4: Dr Experience

30 May 2012 Time: 17h30

1	**I**	**When did you start with your Mathematics teaching career at University of Namibia?**
2	Dr Experience	I started my teaching career at UNAM in 200l as a mathematics tutor and I became a lecturer in 2006.
3	**I**	**What is your role in teaching the First year Mathematics Intervention Program?**
4	Dr Experience	The role is basically to ensure that everyone has an opportunity to understand the concepts that we teach in Mathematics at UNAM and eventually pass.
5	**I**	**People often have difficulty grasping the concept of Mathematical Proficiency, what is mathematical proficiency to you?**
6	Dr Experience	It is basically how you understand, read and write mathematics. Writing Mathematics is actually a profession where things are expressed in letters and there is a lot of relations that one needs to understand, that's how mathematics is actually written. For example, Say for instance you say $x + y = 1$. It is one way of actually writing and one way of reading mathematics, that means once you add two numbers, x and y together the answer is 1. That's how you read it.

7	**I**	**In order to learn mathematics successfully students need to capture Mathematical Proficiency. How do you help students acquire mathematical proficiency?**
8	Dr Experience	In most cases you start actually teaching very simple things that build on more advanced things or terminologies if you want and through that you are actually building the students to advance into more advanced mathematical structures, or natures or subjects and so forth. If they don't capture anything normally what we do, we than have to go back to the simplest situations that should also form the main things that we are teaching and then slowly give them more examples just to connect issues that they might have missed during the earlier meetings or classes or education that's actually how you can intervene. But more into that you can also make them do things on their own so that one can have a fear. Ultimately to add to that slowly but surely you are building them and they can get matured into more advanced mathematics or you can give them more tutorials or extra lecturers etc…. etc., there are quite a lot of things.
9	**I**	**Teaching and learning of mathematics is the product of Instruction. That is, the interaction among the lecturer and the students, around the mathematical content in contexts. How do you manage instruction in ways that help the students on the intervention, develop mathematical proficiency?**

10	Dr Experience	What we have decided to do is that we are using some drawing which is a different language talking to the students, or we have actually decided to use various platforms like extra sessions, younger people teaching them and also we are setting up some groups where they can also learn among each other. We are also subscribing more textbooks all these things contribute actually to the strategies of building a better way of understanding mathematics proficiency, so that's how we are doing it here at UNAM.
11	**I**	**How do you model the performance of your students and so integrate them into a specific implicit and explicit culture of knowledge?**
12	Dr Experience	What is happening is this: we normally do study our class lists, after we have given students the first test. From there we can now take an average of those who passed and those failed. We also check if majority have passed or failed the test. With that analysis, we use it to influence other ways of introducing the concept again to students that can enhance the culture of learning and it can also assist somehow students and also assist lecturers to understand what they have taught was meaningful and if the objectives were met. The same analysis can assist the lecturers to evaluate the lecturing process so that the lecturer can obviously go back to the drawing table and maybe the lecturer also needs to empower students with something else not just to deal with the same way of lecturing methods or tactics that he have actually used in the process. That's how we normally do it in order to implicit the culture of learning.
13	**I**	**How do your cover the prescribed learning outcomes in a way that**

		interests students and keeps them actively engaged in learning?
14	Dr Experience	In most cases you need to demonstrate things or allow students to demonstrate things themselves. This is because in most cases especially in mathematics if a person can solve the mathematical problem, which is demonstrating. Then, that person stand a better chance to solve many other problems related to the same environment, so that's how we normally influence the learning outcomes that we have set out in the module we are teaching at the university and many other ways actually can be used as well apart from demonstrating. They do enjoy that, in fact serious students really like to do things on their own. But I mean you must understand if you are teaching more than 1400 students. More than half of that number will be very lazy, thus why it's only a head of students that you can find that are very serious while others are jumping around.
15	**I**	**Do you think that most students find the use of tutorial sessions helpful? If so how?** **(a) Presenting complex mathematical problems** **(b) Helping students by giving them more individual attention……If, not, why not?**
16	Dr Experience	Yes, because they really demand. What I have observed here at our university, they are demanding that they don't have enough tutors to assist them during tutorials so normally we have about two or three tutors to assist them during tutorials so normally we have about two or three tutors per session looking

		after 80 students and you know it is a 2 hours session, in most cases some students do not really get attention and they are complaining to us so it is a number of complaints that we receive from our students that actually indicate that they are really enjoying the tutorials and the help they receive from tutors. In terms of solving complex mathematics in the session, the strategy that we have developed on the ground is that students do most of the problems on their own and tutors only assist them where they can't bred through but otherwise also assist them to start the problem. The tutors actually never solve problems to the students.
17	**I**	**As Dewey expressed, knowing mathematics involves discovering meanings and relationships in mathematical concepts. How do you help students discover meanings and relationships in mathematical concepts?**
18	Dr Experience	By relating certain things to real situations also trying to simplify problems or situations and define them in a lame language that people can understand and also other meanings can also be found through demonstrating situations because sometimes some things don't have real meanings in mathematics you can only determine a way and you relate a meaning to it. Once you determine how you will do it or once that or what really entails and you can relates a meaning to it and that's how we normally do it to define meanings and situations that don't have meanings in mathematics as such.
19	**I**	**What do you do to motivate your students to engage productively in mathematical lectures and learning activities in those lectures?**

20	Dr Experience	I normally get very good comments from my students. what one can do is that when you teach you should crack some jokes, you must also tell them where mathematics is applicable and then you must also tell them some successful stories of people who already finished their studies you know in mathematics and where they are today, you must also define career path with regard to mathematics and also applications in other subjects and students also to get interested because you are really telling them some actual and practical things.
21	**I**	**Do you stimulate high-level ways of thinking among your students in a mathematics lecture? Please elaborate...**
22	Dr Experience	Yes because you need to ask them questions and you should catch up on what you have taught or when you start a new topic, you must ask them if they know the definition. If they can it is ok, if they can't then you assist them. Through that you are stimulating or cracking something new to them.
23	**I**	**Was there any remarkable appreciation regarding to the effective strategies by the students? Please elaborate.**
24	Dr Experience	Yes, in fact we had a student-lecture forum where a student thank the lecture who have this new adopt or devise this new strategy and you know the majority of the students who are on this intervention could have been repeating this year simply because there was an alternative so now they are actually so much saved in such a way that they are very happy that they didn't

		miss their career path and some of the career that they have taken up in the beginning require some conditions that they should pass mathematics so obviously they have appreciated quite a lot because of this strategy. As I have said before, they do that during the students-lecturer forum. It's a dialect platform where by students and lecturers have a platform to engage into each other.
25	**I**	**Do you have questions or comments?**
26	Dr Experience	No comments or questions thank you.

Appendix J: Transcript for Observed Tutorial session: 1
05/06/2012

Sequence 1: Disputational talk

In the first sequence, 3 first year mathematics students are trying to find together the solution to polynomials in the tutorial 3 (Basic Math 3580)

Find a polynomial of degree, whose zeros are 2, -4 and 5. Simplify your answer.

1	Marcha	What is the degree of the polynomial?
2	Leo	Isn't the power of x?
3	Marcha	What do you mean by power of x?
4	Leo	I meant like the x^n
5	Susan	No it isn't that one (pointing to x^n)
6	Leo	Yes it is, it is like x^n
7	Susan	What if it is like p(x) = 5?
8	Leo	No, it is not because that's constant polynomial
9	Marcha	No, it is not
10	Leo	It is a constant polynomial because the degree of that polynomial is equal to zero.
11	Susan	Shuuush. You are the one to blame if we get it wrong, it isn't gona be your fault Leo if we get it wrong.
12	Marcha	Yes, it is your fault Leo if we get it wrong.

13	Leo	Why is it my fault? You can try it as well.
14	Susan	Polynomial whose zeros are 2 is (x+ 2), yes it is
15	Marcha	It is that one (Pointing to Susan's work)
16	Leo	No it is not, look guys (pointing at his lecture notes which states: *A polynomial in x is an algebraic expression of the form* $a_nx^n + a_{n-1}x^{n-1} + \ldots + a_2x^2 + a_1x + a_0$ *where* $a_0, a_1 \ldots a_n € R$ *with* $a_n \neq 0$. *N is called the degree of the polynomial.* A_0 *is the constant term.* $A_{n\ x}^{\ n}$ *is called the leading term. If* $P (x) = a_0 + a_1x + \ldots + a_{nx}^{n}$ *is a polynomial, we denote its degree by deg (p(x)) or deg (p).i.e. deg (p) =n.*
17	Susan	So how will find the degree 3?
18	Marcha	Let me try (writing down) it is (x+2)(x-4)(x-5)
19	Susan	No it's not (pointing at her solution) it is (x+2)(x+4)(x-5)
20	Leo	Stop it Susan, let's go on, if we get it wrong we will call tutor for help
21	Marcha	Oooooooh...It's your fault guys if we get it wrong….
22	Leo	He he(with the help of tutor) $P(x) = (x-2)\ (x+ 4)\ (x- 5)$ $=(x^2 + 4x - 2x-8)\ (x-5)$ $=x^3-5x^2+4x^2-20x-2x^2+10x-8x+40$ $=x^3-3x^2-18x+40$

Transcript for Observed Tutorial session: 2
05/06/2012

Sequence 2: Cumulative talk

In the Second sequence, another group of First year Mathematics students is trying to solve together a polynomial in the tutorial 3 (Basic math 3580).

Find a polynomial of degree two whose roots are 3 and -5

1	Alba	Common guys, what is the polynomial in this case?
2	Selma	Shuuush(with a funny facial expression) perhaps $(x+3) + (x-5)$
3	Sue	I think you are right (pointing at Selma's answer)
4	Alba	Oooooh but look (showing her answer) $x+3 = 0$ shows that $x = -3$
5	Selma	Yes it is. I think you are right
6	Sue	Yes Alba is right, it should be the other way round
7	Alba	Look guys(pointing at her solution) $P(x) = (x-3)(x+5)$ $= x^2+5x-3x-15$ $= x^2+2x-15$
8	Selma	Yes, I think you are right: $x^2+2x-15$ so that means $P(3) = 3^2 + 2(3) - 15$ $= 9 + 6 - 15 = 0$
9	Sue	Yes, you are we are right $P(-5) = (-5)^2 + 2(-5) - 15$

		= 25 -10-15 which is also zero
10	Alba	Yes, it is I agree to that

Sequence 3: Exploratory talk

In the third sequence, 3 first year mathematics students are trying to find together the solution to polynomials in the tutorial 3 (Basic Math 3580)

Simplify each expression below and give your final answer without negative exponents.

(a) $\left[\frac{2x^2y^{-1}z}{z^2}\right]^{-2}$

1	Ivan	Oh guys the expression has negative exponent, how do we solve it?
2	Peter	I think we apply the laws
3	Sarry	No, it doesn't matter we can still solve it as long as the final answer have positive power
4	Ivan	Oh yeah, the negative exponent outside brackets should be applied to everything inside brackets (pointing to -2 outside brackets)
5	Peter	I think you are right, look guys (pointing as her answer) $\left[\frac{2x^2y^{-1}z}{z^2}\right]^{-2} = \frac{2x^{-4}y^2z^{-2}}{z^{-4}}$
6	Sarry	No, It's like this guys (pointing at her answer)…look -2outside brackets should apply to everything inside brackets even 2. $\left[\frac{2x^2y^{-1}z}{z^2}\right]^{-2} = \frac{2^{-2}x^{-4}y^2z^{-2}}{z^{-4}}$ $= \frac{y^2z^4}{4x^4z^2}$

		$=\frac{y^2z^2}{4x^4}$
7	Ivan	Yes guys we can also use the law $\left[\frac{a}{b}\right]^{-n} = \left[\frac{b}{a}\right]^{n}$
8	Sarry	I think you are right Ivan, we are asked to simplify and give the final answer without negative exponent….
9	Peter	Shuuuu Ivan, I think you are right
10	Ivan	If you apply this law (referring to $\left[\frac{a}{b}\right]^{-n} = \left[\frac{b}{a}\right]^{n}$ Therefore $\left[\frac{2x^2y^{-1}z}{z^2}\right]^{-2} = \frac{z^2}{2x^2y^{-1}z}$ $= \frac{z^4}{4x^4y^{-2}z^2}$ $=\frac{z^2y^2}{4x^4}$
11	Sarry	I think you are right. OK.

Appendix K:Response of students questionnaire

Students	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	Q15	Q16	Q17	Q18	Q19	Q20	Q21	Q22	Q23	Q24
Student 1	E	A	E	A	D	D	B	C	D	A	D	D	E	C	D	D	B	B	D	A	B	E	E	D
Student 2	D	D	D	D	D	C	D	A	C	B	A	D	D	A	D	B	D	C	D	B	B	E	E	D
Student 3	E	E	E	C	D	D	E	D	B	D	E	E	B	E	E	D	D	D	E	B	C	E	E	E
Student 4	D	D	E	D	E	C	E	C	D	C	E	D	E	D	E	E	E	C	D	B	D	D	E	D
Student 5	E	D	C	B	A	D	E	C	E	E	E	D	D	B	C	B	E	E	D	D	C	C	E	D
Student 6	D	C	D	A	B	B	D	C	D	B	D	E	A	C	C	C	D	E	D	D	B	C	D	D
Student 7	C	D	E	D	B	C	E	C	B	D	E	C	A	C	C	C	C	D	D	C	B	C	D	C
Student 8	E	B	D	B	B	D	C	C	C	B	C	C	B	C	C	C	C	B	C	B	C	B	C	C
Student 9	A	E	B	D	A	D	D	A	D	C	D	E	A	E	D	D	B	E	D	E	E	D	E	E
Student 10	D	C	C	E	C	D	D	C	D	D	E	D	D	E	D	D	D	C	E	D	D	D	D	D
Student 11	A	D	A	E	A	D	D	A	D	C	D	C	D	A	A	A	A	A	A	A	A	A	A	A
Student 12	E	C	D	D	C	D	E	E	D	C	C	C	D	D	D	C	D	E	D	C	C	E	E	D
Student 13	D	E		E	E	D	E	D	C	D	C	D	D	C	C	E	E	C	E	C	E	E	E	D
Student 14	D	C	E	D	C	D	E	E	D	B	D	D	D	D	D	C	D	D	D	D	D	D	D	D
Student 15	D	B	D	D	D	A	D	A	A	A	D	D	A	C	C	E	B	D	C	B	A	D	D	D
Student 16	E	C	E	E	B	D	E	D	A	D	A	A	D	D	A	B	D	D	D	D	E	E	D	D
Student 17	D	B	C	C	D	E	B	D	A	D	C	D	D	C	C	D	D	D	D	B	C	C	D	D
Student 18	D	E	D	C	C	D	D	D	D	D	D	C	D	E	E	D	B	D	D	C	D	D	E	E
Student 19	A	D	A	D	B	D	A	D	A	D	B	B	B	B	B	D	A	D	D	D	D	B	A	A
Student 20	C	D	E	E	A	E	E	D	C	A	C	C	D	C	C	E	E	D	D	C	C	D	E	D
Student 21	E	E	E	C	C	E	E	D	D	C	C	E	E	A	B	D	C	D	D	D	E	D	E	C
Student 22	D	D	D	D	B	D	D	C	C	D	C	D	D	C	C	D	C	D	D	D	C	C	C	C
Student 23	C	A	E	B	C	A	B	D	C	D	E	D	B	D	D	C	C	B	D	D	C	D	D	B
Student 24	D	D	E	D	A	D	D	C	D	E	E	E	A	E	D	C	C	D	D	D	D	A	E	D
Student 25	D	E	E	D	E	D	D	C	E	E	E	B	B	C	C	D	B	B	C	C	D	C	E	A
Student 26	D	E	D	A	D	B	E	B	B	D	D	D	B	D	D	A	A	A	D	A	B	E	E	E
Student 27	D	D	D	D	D	D	D	C	D	C	B	D	D	C	D	C	D	C	D	D	D	D	D	D
Student 28	E	B	E	D	C	D	C	D	E	B	C	D	D	E	D	B	E	B	B	C	E	C	D	E
Student 29	D	D	D	D	D	E	C	D	B	D	D	D	D	D	C	C	D	D	D	D	E	D	D	C
Student 30	D	D	E	E	D	E	E	D	D	D	E	D	D	D	D	D	B	B	E	D	D	E	E	D
Student 31	D	C	D	C	D	D	D	C	B	C	D	D	D	B	C	D	B	D	D	D	C	D	D	D
Student 32	D	E	E	D	D	D	D	C	D	D	D	D	E	D	D	E	C	E	B	B	E	D	E	D
Student 33	D	D	D	C	B	C	E	A	D	C	D	C	A	C	C	B	B	B	D	D	C	A	D	D
Student 34	E	D	E	D	C	D	E	C	E	D	D	B	B	E	D	E	B	B	C	D	A	E	D	D
Student 35	E	E	E	E	D	E	E	D	E	E	E	E	E	E	E	A	E	E	E	E	E	E	E	D
Student 36	E	E	E	D	D	E	E	C	A	E	E	E	E	E	D	A	E	C	D	D	D	E	E	E
Student 37	B	A	E	E	C	A	A	A	B	A	A	A	D	D	A	D	A	A	A	A	A	E	E	D
Student 38	D	D	E	B	A	D	D	C	C	B	C	D	D	A	D	D	D	D	D	C	C	B	D	D
Student 39	E	E	D	D	B	E	D	D	D	E	C	C	D	E	B	E	E	C	D	E	E	C	E	C
Student 40	E	A	C	D	A	C	D	D	D	D	C	E	D	E	D	D	D	C	E	C	C	C	C	C
Student 41	E	E	A	D	D	C	D	C	C	C	D	D	D	E	D	B	D	D	B	D	B	C	B	D
Student 42	E	E	E	B	A	B	E	D	D	A	E	E	D	B	D	A	D	A	E	D	B	E	B	B
Student 43	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D
Student 44	E	E	E	E	E	E	E	E	E	E	E	E	D	E	E	B	E	E	E	E	E	E	E	E
Student 45	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	B	E	E	E	E	E	E	E	E
Student 46	E	E	E	E	E	E	E	E	D	E	E	E	E	E	E	B	E	E	E	E	E	E	E	E
Student 47	D	D	E	E	E	E	E	E	D	D	D	E	E	E	E	B	E	E	E	E	E	E	E	E
Student 48	D	D	E	E	A	E	E	E	D	E	D	E	E	E	E	B	E	E	E	E	E	E	E	E
Student 49	E	E	E	E	A	E	E	E	E	D	D	E	E	E	E	E	E	E	E	E	E	E	E	E
Student50	D	D	E	E	A	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E

Key;
A-Strongly Disagree
B-Disagree
C-Uncertain
D-Agree
E-Strongly Agree

Q-Question