How do teachers characterise their teaching for conceptual understanding and procedural fluency? A case study of two teachers.

A thesis submitted in partial fulfillment of the

requirements for the degree of

MASTER OF EDUCATION

(Mathematics Education)

of

RHODES UNIVERSITY

by

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December 2012

ABSTRACT

Over many years the practice or art of teaching Mathematics posed itself as, not only being different from the practice of teaching any other subject, but to have many challenges and opportunities that ask for exploration and understanding.

Just after independence in 1990, Namibia has embarked upon a reform process for the entire education system. Many changes were brought about to create a uniform, equal system for all. However, many challenges still remain to be addressed. Mathematics education remains one of the key areas where Namibian teachers can contribute towards the improvement of the subject. Unsatisfactory results, under-qualified teachers, and a negative disposition towards Mathematics are some of the challenges. These challenges are not unique to Namibia. Across the globe psychologists, philosophers and educators continue to engage in debates and research projects in search of answers and solutions for the improvement of Mathematics education. Despite encountering numerous obstacles, many teachers are dedicated and achieve outstanding results with their learners. This thesis reports on a research project that focused on the Mathematics teaching practice of two teachers whose experiences can make a positive contribution to the improvement of Mathematics teaching in Namibia.

Furthermore, this case study investigated and attempted to understand the Mathematics teaching practices of two proficient teachers who each claimed to have a specific and unique approach to teaching Mathematics. The one claimed to be mainly procedural in her Mathematics teaching, while the other one claimed to teach mainly in a conceptual manner. Both achieve very good results with their classes and attribute their own teaching orientations to a process of several experiences they went through as students and in their careers.

The study revealed that both claims are substantiated and that each teacher was consistent in her claimed approach. Many challenges and constraints were encountered by both teachers, but in their unique and specific ways each teacher's chosen teaching approach supported them to overcome these.

It was evident from the findings that each teacher's practice came about as an evolutionary process over an extended period of time. As many challenges and limitations are universal, it is believed that in sharing experiences, teachers can benefit from each other by improving their practice. It was clearly stated by both participants that the re-thinking of and reflecting on their own practices provided them with new insights and motivation. Peer support and sharing of practices contribute positively towards the improvement of the teachers' classroom practices.

ACKNOWLEDGEMENT

I would like to acknowledge and thank the following people who helped me to complete and submit this research project.

Firstly, the Rector and colleagues of Windhoek Gymnasium Private School for allowing me to undertake the research project at the school and for the assistance I received during the whole research period. Specifically, I would like to acknowledge Salomè Davin and Adri Botes for their participation in the research project. They were always ready to help and to share their experiences to make the research possible.

Secondly, all the lecturers and staff of the Mathematics Department at the Education Faculty of Rhodes University who assisted me in any way during this study period. I would like to specifically acknowledge the outstanding guidance and assistance I received from my supervisor Prof. Marc Schafer.

Thirdly, I acknowledge the constant support patience, and understanding of my wife Erina Junius throughout the entire study period.

DECLARATION OF ORIGINALITY

I, Daniel F. Junias, declare that this project is my own work written in my own words. Where I have drawn on the words or ideas of others, these have been acknowledged using complete references according to the departmental guidelines.

(Signed and date) 05/12/2012

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TABLE OF CONTENTS

APPENDICES

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS AND ACRONYMS

DNEA	Directorate of National Examination and Assessment
IGCSE	International General Certificate of Secondary Education
JS	Junior Secondary
JSC	Junior Secondary Certificate (Grade 10)
LCE	Learner Centered Education
MBESC	Ministry of Basic Education, Sport and Culture
MEC	Ministry of Education
NIED	National Institute for Educational Development
NSSC	Namibian Senior Secondary Certificate
MoE	Ministry of Education
SACMEQ	Southern African Consortium for Monitoring Educational Quality
SS	Senior Secondary

CHAPTER 1
INTRODUCTION

1.1 INTRODUCTION

Every person is unique and in Mathematics teaching every teacher has a unique and different approach to teaching the subject matter to learners. When two very different approaches are followed, but in both cases the results are excellent, it is worthwhile to make a study of each approach. In this chapter a brief introduction will be given to the content of this study and why it was interesting to do the investigation.

1.2 RATIONALE

The rationale to undertake this study lies in the potential to gain understanding about the successful teaching of Mathematics. It is worthwhile to take a critical look at methods applied by teachers and to identify any possibilities that may support other Mathematics teachers in their efforts to unlock the potential of Mathematics learners. Namibia encounters many challenges with school Mathematics. It is, however, not the aim of this study to focus on the failures of education and to remain static, but to analyse and learn from success stories, and to possibly apply these lessons to assist efforts of teaching Mathematics more effectively. It is important that research into teacher success stories is shared as it challenges others to reflect on their own practice.

1.3 RESEARCH CONTEXT

I am a Mathematics teacher at a resourceful and fully staffed secondary school in Windhoek, which has a policy not to make class groups bigger than 25. Teachers are not constrained to follow a particular teaching approach. They have all the necessary physical resources to support their own choice of teaching styles. Mathematics is compulsory for all learners at our school. We have multiple Mathematics classes for each grade, all randomly assembled. The Mathematics department at the school consists of seven experienced and well-qualified teachers. Furthermore, the Mathematics teachers all fall under a Mathematics head of department, also an experienced Mathematics teacher. During a recent tearoom discussion at my school, my colleagues enquired about the content of the lectures I attended for my M.Ed. course in Mathematics Education. I told them about the work of Kilpatrick and his team on teaching mathematical proficiency. The issue of conceptual and procedural teaching approaches came under discussion. During the conversation two Grade-10 teachers claimed that they could relate their own teaching approach directly to one of the mentioned teaching approaches. One said she focuses only on "*methods and rules*" when teaching Mathematics. The other one immediately responded that she uses every "*teaching opportunity to teach for understanding.*" Their excellent relationships with learners and constant outstanding results posed an ideal opportunity to do a case study of the manifestations of their claimed teaching approaches.

The notion of conceptual understanding and procedural fluency remains a key element of mathematical proficiency, in terms of learning. However, while these strands are defined and conceptualised by Kilpatrick, Swafford and Findell (2001) as learner proficiencies, it is not clear what teaching characteristics teachers might adopt when teaching towards these proficiencies. This study, with its focus on teaching will specifically explore two teachers' conceptualisations of their teaching practice in terms of their own stated aims of **teaching for** these two strands of mathematical proficiency. While such a case study cannot be generalized, it is likely to yield interesting insights into the way in which teachers connect Kilpatrick's strands of proficiency (conceptual understanding and procedural fluency) to the way they teach – or to specific characteristics of their teaching. Such a study will contribute to our understanding of the complex relationship of teaching Mathematics and the learner's mathematical proficiency (in terms of conceptual and procedural teaching).

1.4 RESEARCH GOALS

The goal of this study was to analyse and understand the teaching approaches of two teachers that claim that their Mathematics teaching is characterised by a procedural and a conceptual approach respectively.

1.5 RESEARCH SITE AND PARTICIPANTS

At Windhoek Gymnasium Private School two teachers often teach the same level of a subject in two or more class groups of the same grade. In this case study, this situation helped to create an ideal opportunity to study the teaching practice of two teachers simultaneously. Currently the two participants of this study have divided the four grade-ten Mathematics classes between them, each teaching two classes. Both teachers are highly acclaimed by the pupils of the school and achieve excellent results, but claim to adopt distinctively different approaches to teaching. The one openly claims that her method of teaching is mainly procedural, while the other claims that she experiences the most success by adopting a mainly conceptual teaching approach. The study interrogated their understanding of a "conceptual" and "procedural" approach, and investigated how they applied these approaches in their Mathematics teaching practice.

The two teachers studied at different institutions but have been colleagues for many years. Despite the different teaching approaches, they often share resources and make use of the same question database in the school for their teaching. Each teacher applies the same learning material and resources differently in their classes, but both attribute their excellent results to their own approach of teaching Mathematics.

The selection of the two participants was purposeful (Cohen, Manion, Morrison., 2000, p21 – 23). The two teachers, Adri Botes and Salomé Davin, willingly agreed to participate in this study and an informal meeting was held where the nature and purpose of the research study was fully explained. Both agreed to fully participate and they showed a willingness to share their experiences with me. They were eager to share their practice particularly for the purpose of the future improvement of their own teaching. They did not want to remain anonymous, as both wished to share any experience that could make a positive contribution towards the betterment of Mathematics teaching in Namibia.

These two teachers thus provided an ideal opportunity for me to explore how each of them characterised their teaching in relation to Kilpatrick's first two strands of teaching for proficiency namely: conceptual understanding and procedural fluency, and to explore the relationship between the two.

The investigation was undertaken at Windhoek Gymnasium Private School, and each teacher was observed during their normal, daily teaching without any interruptions or interference to their lessons. Interviews were done on a regular basis, sometimes individually with one teacher at a time and sometimes with both teachers simultaneously. It was agreed that the meetings would be structured to last no longer than one period and that all participants would always have access to all the data and information gathered. A time frame was provided to the participants to structure the data collection and to accommodate the research process in their own term planning.

1.6 RESEARCH OVERVIEW

This research is situated in the interpretive paradigm. Interpretivism examines how people make sense of their lives, how they define their actions and situations. This paradigm aligns well with this case study as the goal of this study was to analyse and understand the teaching approaches of the two teachers.

The research process was documented in five chapters. The first chapter introduces the study, defines the research goals, locates the study and identifies the participants and places the study in its context.

In the second chapter a detailed study was done of the literature related to the study. Moreover, underlying philosophies are investigated and coherent definitions for conceptual and procedural teaching methods are compiled. The literature also provided guidance for the steering of the research process.

The third chapter refers to the methods, techniques and procedures employed to collect and analyse data. Data was collected through semi-structured interviews, structured interviews and observations. All interviews were recorded and for observation purposes, video recordings of two lessons for each teacher were made. Different methods of data collection allowed for the triangulation of the data.

In the fourth chapter a qualitative approach was followed to analyse all collected data. In this chapter all the data was coded and it presents the hypothesis of each teacher's claimed teaching approach. The findings were discussed and contextualized in terms of the theoretical consideration of the literature review.

Finally, in the fifth chapter a summary of the main findings is presented. It also highlights the significance of the study, presents some limitations of the research and suggests some recommendations and avenues for further research. The chapter ends with a personal reflection on the entire research project.

1.7 CONCLUSION

I am convinced that the two teachers situated at the same school and working under similar conditions, but with different teaching approaches, presented a worthwhile opportunity for research. Several factors that could challenge the validity of the data were eliminated. The results not contain academic value, but also practical implications for the teaching of school Mathematics.

Many concerns about the state of school mathematics, especially in Africa, are often raised, but a positive engagement by all with Mathematics at heart is now needed. That is exactly what this research project aims to achieve.

CHAPTER 2
LITERATURE REVIEW

2.1 INTRODUCTION

The area of interest of this study is conceptual and procedural teaching of Mathematics. Over the past century numerous theories in Mathematics teaching have addressed different approaches to teaching Mathematics with different types of students acquiring the necessary skills through these teaching methods.

Over many years the debate about the learning of Mathematics has been going on. Already in 1923, Thorndike wrote an article about the Psychology of learning and identified several "laws" that direct learning. His primary focus was that learning could range from being rote or habitual to analytical (Thorndike, 1923). Over the years many new terms appeared like meaningful learning, relational understanding and instrumental understanding. Shulman introduced learning with understanding (Shulman, 1986) while Hiebert started to talk about conceptual knowledge versus procedural knowledge (Hiebert, 1986). As recent as 2001 serious research was undertaken by the National Research Council in the United States that accumulated in the identification by Kilpatrick, Swafford and Findell (2001) of five strands of understanding Mathematics. The debate over the importance of teaching conceptually or procedurally, is as relevant today as it was decades ago.

2.2 THE STRANDS OF MATHEMATICAL PROFICIENCY

Several theories of learning and cognition posit that our behaviour is shaped by at least two different kinds of knowledge: one providing an abstract understanding of the principles and relations between pieces of knowledge in a certain domain, and another one enabling us to quickly and efficiently solve problems. "In recent empirical research on mathematical learning, the former is frequently named conceptual *knowledge,* while the latter is labelled *procedural knowledge.*" (Schneider & Stern, 2005, p. 2).

It is further asserted by Schneider and Stern (2005, p. 3) that there are generally two theories on the procedural and conceptual teaching of Mathematics, namely the *concepts-first* theory and then the *procedures-first* theory. According to the concept-first theories, children will listen to a lesson with explanations, they will practise the theory and eventually gain procedural knowledge from it. Procedures-first theories, hypothesize that children will through trial and error acquire procedural knowledge and eventually by reflecting on their own work gradually gain abstract conceptual knowledge from it.

The inter-relationship between conceptual and procedural knowledge is complex and Schneider and Stern (2005) say that it is problematic to measure conceptual and procedural knowledge independently of each other with a sufficient degree of validity. Cognitive procedures are always present in conceptual knowledge representations. It remains difficult to place any action as rooted in conceptual or in procedural knowledge or, to different degrees in both?

According to Hiebert (1986), many children begin school with sound problem-solving skills, but after receiving formal instruction in Mathematics they often replace them with shallow and meaningless procedures. This suggests that teaching procedural fluency without conceptual understanding can lead to a mechanistic way of doing Mathematics without any deep understanding of the subject matter.

The notion of conceptual understanding and procedural fluency in both teaching and learning processes was a central theme in the research conducted by Kilpatrick et al. (2001) when they reviewed the state of Mathematics education in the USA. Their work led to the development of a framework that enabled the analysis of "what is meant by successful Mathematics learning and teaching in the elementary school and middle school years." (Kilpatrick et al., 2001).

The terms "conceptual and procedural understanding or fluency" was widely used in the psychology and Mathematics education but it was only during 2001 when the study, undertaken by Kilpatrick and his group of researchers, called "The National Assessment of Educational Progress, (NAEP)" was published, that the concepts of "conceptual understanding" and "procedural fluency" were "linked" to five identifiable strands of mathematical fluency. All these strands were regarded as prerequisites for what they called mathematical fluency (Kilpatrick, 2001). This has consequences for this study because, although both the teachers claim to have adopted either a conceptual or a procedural approach, neither of them had any knowledge of the work by Kilpatrick (2001).

Only in the last fifty or sixty years has the shift in Mathematics teaching moved away from emphasizing the use of "computational procedures of arithmetic" to "learning in terms of understanding" (Shulman, 1986). Current views of mathematical proficiency embrace a far more comprehensive notion of competence than mere skilful application of rules and algorithms. According to Kilpatrick et al. (2001), the term "*mathematical proficiency*" is used to capture "what we believe is necessary for anyone to learn Mathematics successfully." They recognise that *no term* captures completely all aspects of expertise, competence, knowledge, and facility in Mathematics.

2.2.1 Conceptual and Procedural, from where?

In 1962 the United Nations Educational, Scientific and Cultural Organization held a "Research Symposium on school Mathematics" with the theme, "The modernization of Mathematics teaching: Aspects and problems." In a report on the symposium, Gilbert Walusinsky mentioned that the symposium presented two methods of teaching Mathematics. Firstly, the development of axioms and the value they acquire when combined in simple structures. This was a concept of Mathematics teaching which they decided to call the 'axiomatic method' (Walusinsky, 1962, p.7).

Secondly, the symposium identified a method of teaching called the *activity method*. "Our second aim is a change in the resource to concepts which have hitherto had no place in our secondary school syllabuses… an understanding of the nature of Mathematics and a profound grasp of essential concepts… the necessity of acquiring a sound knowledge of how to use this method so that everyone can create the Mathematics he needs and be able to make effective use of existing mathematical techniques, in other words, teaching should direct learners towards *mathematical activity"*(Walusinsky, 1962, p.10). One clearly gets the feeling that this early identification of these two "modern methods of Mathematics teaching," refers to procedural and conceptual methods of teaching Mathematics.

In a paper presented at the ICMI Regional Congress, University of the Witwatersrand Caroline Long, a teacher trainer, referred to her own teacher training when she said: "*My teaching focus has generally been on conceptual knowledge, without the procedural scaffolding.*" (Long, 2005, p. 64). She admitted that the process remained complex and added: "*However, in some cases algorithms support conceptual understanding and at a more complex level form the building blocks to understand the concepts*" (Long, 2005, p. 64).

Long cited several participants to the debate. She referred to the work by Hiebert and Lefevre (1986), who identified two kinds of knowledge, conceptual knowledge and procedural knowledge, that could be identified as distinct, but which are related in complex ways. Skemp (1976) used the terms 'relational' and 'instrumental understanding' to describe similar constructs. He said relational understanding is the ability to deduce specific rules and procedures and, instrumental understanding as the ability to apply a rule to the solution of a problem without understanding how it works. On the other hand Kilpatrick, et al. (2001) included conceptual understanding and procedural fluency, similar in essence to the terms used by Hiebert and Lefevres (1986) as being only two of the five known strands of mathematical proficiency as identified during their research (Long, 2005).

2.3 MATHEMATICAL PROFICIENCY ACCORDING TO KILPATRCK

The five stands of mathematical proficiency that characterise someone who is proficient in Mathematics identified by Kilpatrick et al. (2001, page 116) are

- *conceptual understanding* – comprehension of mathematical concepts, operations, and relations;
- *procedural fluency* – skill in carrying out procedures flexibly, accurately, efficiently and appropriately;
- *strategic competence* – ability to formulate, represent and solve mathematical problems;
- *adaptive reasoning* – capacity for logical thought reflection, explanation and justification and
- *productive disposition* – habitual inclination to see Mathematics as sensible, useful and worthwhile, coupled with a belief in diligence and one's own efficacy."

The notion of conceptual understanding and procedural fluency remains the key element of mathematical proficiency, in terms of learning. However, while these strands are defined and conceptualised by Kilpatrick et al. (2001) as learner proficiencies, it is not clear what teaching characteristics teachers might adopt when teaching towards these proficiencies. This study, with its focus on teaching, specifically explored two teachers' conceptualisations of their teaching practice in terms of their stated aims of **teaching for** these two strands of mathematical proficiency.

The National Assessment of Educational Progress (NAEP) according to Kilpatrick et al. (2001) identified conceptual understanding and procedural knowledge as key ingredients in describing mathematical proficiency. Even as an experienced Mathematics teacher, it is sometimes difficult to distinguish between conceptual understanding and procedural fluency. A professional Mathematics teacher, I believe, should however have the capacity to create a balance between how much a learner should understand and relate to the world around him, and how much should be done in terms of developing procedural skills. For the sake of this case study, it is useful to take a detailed look at each strand of mathematical proficiency, as outlined by Kilpatrick et al. (2001).

2.3.1 Conceptual Understanding

According to Kilpatrick et al. (2001), "*Conceptual understanding* refers to an integrated and functional grasp of mathematical ideas." Mathematics relies on many concepts or ideas that are often abstract and intertwined. To solve a problem a learner can either execute an algorithm according to a set of rules that the learner has memorized, or see the mathematical concept in its context and apply it with understanding and insight. The latter enables learners to come up with new ideas of their own by connecting their ideas to what they already know. Conceptual understanding improves a learner's ability for retention. Once a concept is grasped and applied, it becomes part of a learner's knowledge base, but when a learner memorizes a method to apply to a certain problem the method is easily forgotten. Conceptual understanding implies learning with understanding and helps the learner to make sense of the problem and apply knowledge effectively and appropriately. A learner with conceptual understanding is able to solve a problem because he has an assimilated and integrated understanding of mathematical concepts. He is able to reason adaptively and strategically (Kilpatrick et al., 2001).

"Conceptual understanding frequently results in students having less to learn because they can see the deeper similarities between superficially unrelated situations," (Rittle-Johnson.et al., 1998, p. 181).

When learners grasp and understand that Mathematics is based on logical rules that apply in a universal sense, the subject becomes more of an adventure and learners show more interest to learn the subject.

A different perspective was placed on conceptual understanding by Shulman (1986) when he developed a model based on the idea of the "structure of knowledge" by Burner (as cited in Shulman, 1992). Shulman developed his ideas of understanding by first looking at the knowledge requirements of the teachers. Shulman (1986, 1987, and 1992) created a Model of Pedagogical Reasoning which comprises of a cycle of several activities that a teacher should complete for teaching conceptual understanding: comprehension, transformation, instruction, evaluation, reflection, and new comprehension.

To **teach** learners conceptually, teachers need to understand the subject matter deeply and flexibly so they can help learners create useful cognitive maps, relate one idea to another, and address misconceptions. Teachers need to see how ideas connect across fields and to everyday life. This kind of understanding provides a foundation for pedagogical content knowledge that enables teachers to make conceptual ideas accessible to others (Shulman, 1987).

In Shulman's theoretical framework teachers need to master two types of knowledge: (a) content, also known as "deep" knowledge of the subject itself, and (b) knowledge of the curricular development. Especially important is content knowledge that deals with the teaching process, including the most useful forms of representing and communicating content and how students best learn the specific concepts and topics of a subject.

To help all students learn, teachers need several kinds of knowledge about learning. They need to think about what it means to learn different kinds of material for different purposes and how to decide which kinds of learning are most necessary in different contexts. Teachers should learn how to best teach by studying, doing and reflecting, collaborating with other teachers, looking closely at students and their work, and by sharing what they see.

2.3.2 Procedural Fluency

According to Kilpatrick et al. (2001), "*Procedural fluency* refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently." Rittle-Johnson & Alibali (1998, p. 175), simply define it as "action sequences for solving problems." Conceptual knowledge and procedural knowledge lie on a continuum and cannot always be separated; however, the two ends of the continuum represent two different types of knowledge. It is likely that children's conceptual understanding influences the procedures they use and vice versa.

For Kilpatrick et al. (2001), procedural fluency is also connected with the ability to estimate the result of a procedure. They refer to it as the need for speed or efficiency in the calculations required in problem solving. In addition to providing tools for computing, some algorithms are important as concepts in their own right, which again illustrates the link between conceptual understanding and procedural fluency.

It is important for computational procedures to be efficient, to be used accurately, and to result in correct answers. Both accuracy and efficiency can be improved with practice, which can also help students maintain fluency. Hiebert and Lefevre (1986), regard procedural knowledge as having two distinct parts. "One part is composed of the formal language, or symbol representation system, of Mathematics. The other part consists of the algorithms or rules, for completing mathematical tasks."

To **teach** procedurally, teachers emphasise routine procedures and the development of algorithms. Some authors see procedural fluency in teaching as merely the mechanical execution of an algorithm to get a solution for a problem, while others describe it as having the knowledge and insight to arrive at a solution for a problem.

For the purpose of this study, the conceptual and procedural teaching will initially be looked at separately. I will use the definitions by Shulman (1996) and by Kilpatrick et al. (2001), as points of departure, but will also engage with the views of the two teachers being studied. An interesting outcome of the study will be to find if it is possible to focus only on one concept without involving the other one. It is expected that the relationship between the two concepts is complex and intertwined and may be inseparable.

2.3.3 Strategic competence

According to Kilpatrick et al. (2001), strategic competence refers to the ability to construct mathematical problems, represent them, and solve them. Carpenter (1988) stated that teaching is a problem-solving issue which means teachers have to be able to construct problems, whether mathematical or otherwise of any other kind in the teaching environment, represent them and solve them. Teachers are confronted with different situations where they first have to discover what the problem is and then find ways to solve the problem. For example, Kilpatrick et al. (2001) suggested that for teachers to be able to give appropriate solutions to students' questions and situations, they need to figure out what the students know. To represent a problem accurately, teachers must first understand the situation and they need a conceptual understanding of the knowledge involved, whether it is mathematical subject knowledge, knowledge of students or pedagogical knowledge.

2.3.4 Adaptive reasoning

Kilpatrick et al. (2001) define adaptive reasoning as the "capacity to think logically about the relationships among concepts and situations (p. 129)". They further describe adaptive reasoning in Mathematics as "the glue that holds everything together (p.129)". Adaptive reasoning for a teacher includes the ability to reflect and analyze his/her practice and use the reflection to improve his/her practice. A teacher who disagrees with a student's response to a mathematical question, for example, needs to analyses the situation and find out why they think a student made such a mistake and how such a mistake can be avoided. Adaptive reasoning refers to the ability to think reasonably about the interaction among concepts and situations (Ball, 2003; Kilpatrick et al., 2001). In the past, many concepts of mathematical logic have been limited to formal proof and other forms of deductive interpretation (Ernest, 1991). Today the concept of reasoning also includes other processes, including not only informal explanation and validation but also spontaneous and inductive reasoning based on example, similarity and metaphor (English, 1997).

Adaptive reasoning does not stand alone; it interacts with the other strands of mathematical proficiency, mostly during problem solving. Teachers should draw on their strategic competence to prepare and represent a problem using heuristic approaches that may provide a solution strategy. They also depend on their conceptual understanding to enhance their ability to analyze a situation.

2.3.5 Productive disposition

Kilpatrick et al. (2001) describe productive disposition as a tendency to see sense in Mathematics; to perceive it as both useful and worthwhile; to believe that steady effort in learning Mathematics pays off and to see oneself as an effective learner and active person of Mathematics. For teachers and learners to develop the abilities of all other four strands of mathematical proficiency; *conceptual understanding, procedural fluency, strategic competence, and adaptive reasoning*, they must believe that Mathematics is not arbitrary and that it is not impossible for them to understand it. Teachers, just like students, must have a sense of belonging to the mathematical practice for them to develop all other abilities in the other strands of mathematical proficiency (Ball, 2003; Kilpatrick et al., 2001). Teachers' disposition toward Mathematics is a major factor in shaping their practice success. Ball (1989) stated that teachers, who view their mathematical ability as unchanging, see all challenges in their practice as measuring their ability rather than providing opportunities to learn and are likely to avoid challenging tasks in their practice.

2.4 THE TEACHER'S UNDERSTANDING OF MATHEMATICS

In order to adopt a specific approach (conceptual or procedural) to teaching Mathematics, it is important that a teacher has both a deep conceptual and procedural understanding of Mathematics. It is expected that the teacher should process and present the subject matter to the learners in such a way that it will develop a high level of conceptual and procedural understanding in the learners.

2.4.1 Content knowledge

Shulman (1986) describes three categories of content knowledge that a teacher should have: (a) subject matter content knowledge, (b) pedagogical content knowledge, and (c) curricular knowledge. This could be translated into: (a) knowledge of the Mathematics that should be taught, (b) the method chosen to present the subject matter effectively and meaningfully to the learners and (c) knowledge of what is prescribed by the syllabus.

2.4.2 Teaching conceptually or procedurally

The approach to relate the subject content to the learners in teaching is a matter of choice for a teacher. In a study by Thompson and Thompson (1994) involving two instructional sessions between one teacher and one learner on the concept of rate, they found the teacher's language choice was a challenge for the learner as eventually he displayed a mostly procedural understanding of rate.

In a second study, Thompson and Thompson (1996) stated that a teacher with a conceptual orientation for teaching Mathematics is driven by: "an image of a system of ideas and ways of thinking that he or she intends the students to develop, an image of how these ideas and ways of thinking can develop, ideas about features of materials, activities, expositions, and students' engagement with them that can orient students' attention in productive ways, and an expectation and insistence that students be intellectually engaged in tasks and activities." (Thompsons & Thompson, 1996, p. 20-21)

Teachers with a conceptual approach focus student attention away from thoughtless procedures and towards situations, ideas and relationships among ideas. Thompson and Thompson recommended additional research to understand how teachers come to understand Mathematics and Mathematics teaching in order to be able to teach conceptually.

2.5 MATHEMATICAL PROFICIENCY – THE OUTCOME

When asked what the expected outcome should be when a learner is taught Mathematics, the answers are usually, "to understand Mathematics," or "to be able to do Mathematics" or "to be proficient in Mathematics." To get clarity on this matter, Schoenfeld (2004, p. 59) asked the question: What does it mean for a student to be proficient in Mathematics? (What should students be learning?) The "cognitive revolution" (Gardner, 1985), produced a significant reconceptualization of what it means to understand subject matter in different domains (NRC 2000). There was a fundamental shift from an exclusive emphasis on knowledge—what does the student *know*?—to a focus on what students know and can do with their knowledge. The idea was not that knowledge is unimportant. In fact, the more one knows, the greater the potential for that knowledge to be used. Rather, the idea was that having the knowledge was not enough; being able to use it in the appropriate circumstances is an essential component of proficiency.

The knowledge base remains important. It goes without saying that anyone who lacks a solid grasp of facts, procedures, definitions and concepts is significantly handicapped in Mathematics. However, there is much more to mathematical proficiency than being able to reproduce standard content on demand. A mathematician's job consists of at least one of the following: extending known results, finding new results and applying known mathematical results in new contexts.

Mathematicians possess other things too. Good problem solvers are flexible and resourceful. They have many ways to think about problems— alternative approaches if they get stuck, ways of making progress when they hit roadblocks, of being efficient with (and making use of) what they know. They also have a certain positive mathematical disposition—a willingness to pit themselves against difficult mathematical challenges under the assumption that they will be able to make progress on them, and the tenacity to keep at the task when others have given up.

According to Schoenfeld (2004), there are many different interpretations of what it means to know mathematical content. A major source of controversy over the past decade has involved not only the level of procedural skills expected of students, but also what is meant by "understanding." For example, what does it mean for an elementary school student to understand base-ten subtraction? For some people, understanding a concept means being able to compute the answers to exercises that employ that concept. Although the term "problem solving" is used, it refers to computational proficiency.

A second important aspect of mathematical proficiency is called “strategies” by Schoenfeld. It goes without saying that “knowing” Mathematics, in the sense of being able to produce facts and definitions, and executing procedures on command, is not enough. Students should be able to apply the mathematical knowledge they have. Schoenfeld (2004) refers to the work of George Pòlya, published in 1945 as “*The starting place for any discussion of problem solving strategies”*, with the pioneering first edition of *How to Solve It*. “The aim of heuristics is to study the methods and rules of discovery and invention. The book is an attempt to revive heuristics in a modern and modest form.” (Pòlya 1945, pp. 112–113) “Modern Heuristic endeavors to understand the process of solving problems, especially the *mental operations typically useful* in this process” (Pòlya 1945, pp. 129–130). Pòlya (1945) describes powerful problem-solving strategies such as making use of analogy, making generalizations, re-stating or re-formulating a problem, exploiting the solution of related problems, exploiting symmetry, and working backwards.

The heuristic strategies Pòlya (1945) describes are more complex than they appear. Consider, for example, a strategy such as “if you cannot solve the proposed problem . . . could you imagine a more accessible related problem?” (Pòlya 1945, p. 114). The idea is that although the problem you are trying to solve may be too difficult for now, you might be able to solve a simpler version of it. You might then use the result, or the idea that led to the solution of the simpler problem, to solve the original problem (Schoenfeld, 2007, pp. 64-65).

For Schoenfeld (2004), mathematical proficiency has a third aspect, namely “Metacognition” (Using what you know effectively). Very often students fail in solving a problem, just because they never stop to reconsider their strategies for solving it. Very often the student, who previously solved a similar problem, embarks on a wrong approach, but never reconsiders the current strategy, ending in failure to solve it.

As it happens, if one direction turns out to be unfruitful, one should have the ability to take stock, change directions, and go on to solve the problem. What makes the student effective is not simply that he has the knowledge to enable him to solve the problem, it is the fact that he gives himself the opportunity to use that knowledge, by truncating attempts that turned out to be profitable.

A fourth aspect of mathematical competency is called "disposition and beliefs" by Schoenfeld (2004). Everybody has an orientation towards doing Mathematics, either seeing it as useful and pleasant or seeing it as a negative, challenging experience. Compare this with "*productive disposition*" by Kilpatrick et al. (2001), to see Mathematics sensible, useful and worthwhile.

2.6 PROCEDURAL AND/OR CONCEPTUAL – A RELATIONSHIP

In their book on conceptual and procedural knowledge in Mathematics, Hiebert and Lefevre (1986) define conceptual knowledge by saying: "Conceptual knowledge is characterized most clearly as knowledge that is rich in relationships. It can be thought of as a connected web of knowledge, a network in which the linking relationships are as prominent as the discrete pieces of information." (Hiebert, 1986, pp. 3–4). They define procedural knowledge as, "made up of two distinct parts. One part is composed of the formal language, or symbol representation system of Mathematics. The other part consists of the algorithms or rules, for completing mathematical tasks" (Hiebert, 1986, p. 6). They clarify it by saying that knowledge and recognition of the symbols and structures used in Mathematics are indications of procedural knowledge. Even at a higher level can the knowledge of the syntactic configurations of formal proofs in Mathematics be seen as procedural, and not as conceptual knowledge. This excludes the logic of proofs, but points to the style in which a proof statement is written (Hiebert, 1986).

"Mathematical knowledge, in its fullest sense, includes significant, fundamental relationships between conceptual and procedural knowledge. Students are not fully competent in Mathematics if either kind of knowledge is deficient or if they both have been acquired but remain separate entities" (Hiebert, 1986, p. 9).

While most Mathematics educators would agree on both aspects of knowledge acquisition being important the "chicken first or the egg first" debate about conceptual and procedural knowledge continues. A Western Cape Departmental directive from the late 1980s and early '90s discouraged teachers from teaching procedures. It claims that with sound conceptual understanding, children will develop their own algorithms (James, 1995). The approach that encourages the development of own algorithms based on conceptual understanding has value and elicits varied responses, often insightful, from learners who have grasped the concept

(Lampert, 2001; Ball, Lubienski & Mewborn, 2001). This is in some cases the logical starting point for learning the more compacted algorithms. There is also evidence that a poor understanding on the part of teachers regarding the constructivist approach has led to learners having neither conceptual nor procedural knowledge (Schollar, 2004). The notion that there are stages in mathematical development and that learners typically go through a procedurally oriented phase before they can effectively integrate their conceptual knowledge is put forward by Davis, Gray, Simpson, Tall and Thomas (2000), who focus on high school Mathematics.

In this case study I revisited the theoretical distinction and complex relationship between these two aspects of knowledge. I apply this distinction to the chosen approaches by the two teachers namely, one mainly procedural and one mainly conceptual. The complexity of each of the approaches and their relationship raises some questions from both a theoretical and a practical perspective.

2.7 THE ISSUES

In a paper that was presented at the ICMI Regional Congress, University of the Witwatersrand, June, 2005, Caroline Long, a Mathematics teacher educator, concluded about conceptual and procedural understanding that; "While I have found the distinction between conceptual and procedural knowledge useful, the pedagogical implications are far from clear." For Long (2005), three issues are central to promoting a closer relationship between the procedural and the conceptual.

2.7.1 Conceptual first or procedural first

According to Long (2005, 64) a foundation phase teacher educator using "good constructivist methodology" may say categorically that teaching the concept must come first and thereafter children will be able to invent their own algorithms. In this case the execution of standard algorithms is perceived as indicating a lack of conceptual understanding and is aligned with rote learning, whereas the use of self-generated algorithms is perceived as conceptually rich. The view is that the understanding of concepts before the learning of procedures makes some sense at a foundation phase level, though the research done by Rittleston-Johnson and Siegler

(1998), as cited by Long (2005), indicates that even at foundation phase the conceptual/procedural distinction has a complex, sometimes iterative relationship.

More findings by Rittleston-Johnson and Siegler (1998, 109), indicate that there is a ”positive correlation between children’s understanding of mathematical concepts and their ability to execute procedures.” On the issue of concepts first or procedures first they concluded that in some domains conceptual understanding precedes procedural competence, and in other domains the order is reversed. In other domains the order of acquisition can vary within the same domain (Rittleston-Johnson & Siegler, 1998: 106). The general principles, which predict which comes first depends on timing and frequency of exposure. The more likely relation between conceptual and procedural knowledge, is an iterative one and both play a role in establishing proficiency (Long, 2005). This means that certain topics could have a procedural underpinning and others a conceptual underpinning.

2.7.2 Procedural understanding is rote learning.

Rittleston-Johnson and Siegler (1998) define rote learning as learning that is “habitual repetition and devoid of conceptual understanding.” The implication is that rote learning does not create a building block on which knowledge can be built, and does not provide a skill or knowledge that can be connected with any other skill or knowledge. According to Long (2005) the view that the execution of standard algorithms is devoid of conceptual understanding and, therefore, nothing but rote learning is questionable. The findings of Rittleston-Johnson and Siegler (1998) indicate that learning is a complex process in which the conceptual understanding underpins the skills of procedures and both play a part in establishing proficiency.

2.7.3 The undervaluing of algorithms

To successfully execute algorithms represents an aspect of procedural fluency, but algorithms in themselves are rich in mathematical concepts. They represent compressed conceptual understanding, in other words mathematical concepts at a high level of abstraction. According to Long (2005): “The answer at any level to the question of teaching algorithms and procedures is to analyze the concepts underpinning the component parts and subsequently to enable more complex mathematical thinking."

2.8 No final outcome

"Kilpatrick et al. (2001) *circumvent* a dichotomy between the two strands by saying that "In the domain of number, procedural fluency is especially needed to support conceptual understanding of place value…" (Kilpatrick, 2001, p. 121). They regard Algorithms as procedures that are powerful and could form concepts in themselves. Hiebert and Lefevre (1986) conclude that it is the relationship between conceptual and procedural knowledge "that holds the key" to improved mathematical understanding: although it is possible to consider procedures without concepts. Conceptual knowledge is seldom not linked with some procedures. "*This is due, in part, to the fact that procedures translate conceptual knowledge into something observable. Without procedures to access and act on the knowledge, we would not know it was there*" (Hiebert, 1986, p. 9).

With regard to teacher training no clear line of agreement regarding conceptual or procedural understanding could be found. In a paper delivered, Doctor Caroline Long (2001, 64), a teacher trainer reflected on her teaching and said:

> My teaching focus has generally been on conceptual knowledge, without the procedural scaffolding. In some cases students have insisted on being told "how to" find a solution to a problem, circumventing the particular concept required to understand the problem. The students might be able to apply the procedure to similar problems but when the concept is changed which in this case could be changing from base 5 to base 2; they are once more confronted with the need for understanding the concept. However, in some cases, algorithms support conceptual understanding and at a more complex level form the building blocks to understand the concepts. It is my pragmatic view… that different aspects of the debate apply differently to particular mathematical concepts, to different stages of mathematical development and to different learning styles. I am also convinced that prospective teachers' access to these constructs and the related research allows them to make

> informed choices as to when to focus on different aspects of mathematical proficiency.

According to Long (2005), it is very difficult to distinguish concepts from procedures because *understanding* and *doing* are connected in complex ways. Also Brodie (2004, pp. 72-73) refers to the complex relationship between mathematical knowledge, practices and mathematical teaching practices.

2.9 MATHEMAICS TEACHING IN AFRICA

In an effort to understand the current state of mathematics in Africa and to seek new opportunities to support mathematical development the International Mathematics Union and the Developing Countries Strategies Group compiled a study group to prepare a report on *Mathematics in Africa: Challenges and Opportunities.* The report was completed in February 2009 on request of the John Templeton Foundation.

The report draws a gloomy picture of the state of Mathematics all over the continent and in terms of school mathematics they attributed the weakness in primary schools to mandatory education laws that caused enrollments to increase dramatically. They stated that: "Secondary school education suffers from a lack of teachers with mathematics training and low student participation." (DCSG & IMU report, 2009, p.5).

With reference to procedural and conceptual teaching the report contributed the dismal situation of Mathematics to unqualified teachers without professional and didactical qualifications (DCSG & IMU report, 2009, p. 19).

> Most high school education places emphasis on algorithmic procedures rather than an understanding of basic principles. According to Prof. Banasiak, "It borders on impossibility to force students to embark on independent thinking. On average therefore the preparation of students entering university is weak.

On the contrary, an experiment was done by staff of the University of Pretoria to compare undergraduate students' performance and confidence in procedural and conceptual mathematics. The general perception was that high school teaching of Mathematics tends to be fairly procedural in general and that students who enter university will be better at solving procedural than conceptual problems. Subtle differences exist in defining conceptual understanding and procedural fluency, so for the purpose of their study the researchers adopted the distinguishes between conceptual fluency and procedural understanding as given in "Learning standard for Mathematics" of the New York State Education Department (2005). Engelbrecht et al. (2005, p. 701), used the definition

> Conceptual understanding consists of those relationships constructed internally and connected to already existing ideas. It involves the understanding of mathematical ideas and procedures and includes the knowledge of basic arithmetic facts. Students use conceptual understanding of Mathematics when they identify and apply principles, know and apply facts and definitions, and compare and contrast related concepts. *Procedural fluency* is the skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. It includes, but is not limited to, algorithms.

Contrary to the belief that students will be more confident in procedural Mathematics the research found that students are equally skilled in solving conceptual and procedural problems. It was also found that they are more confident of their ability to solve conceptual problems than to handle procedural problems (Engelbrecht et al., 2005).

2.10 PROCEDURAL AND CONCEPTUAL: A NAMIBIAN PERSPECTIVE

2.10.1 Historical background

Before Namibia's independence in 1990, the nation was segregated along the lines of ethnic groups. Education was for the privileged few. In particular, Mathematics and science education was mainly for white males (Tjikuua, 2000). This was not only politically driven, but also by some cultural ideas. Education was not meant for women, as they were expected to attend to domestic chores and bringing up children. As a result, after independence very few Namibians, especially women, were educated in Mathematics and science. This forced the new government to obtain skilled educators, especially Mathematics teachers from different countries, often well trained but lacking communication skills in English.

The Namibian government regarded the education reform process as a matter of urgency (MEC, 1993). Two of the major changes brought about by the education reform process in Namibia were that all children were given access to schools and that the subjects of Mathematics and science were made compulsory from grade 1 to 10 (Tjikuua, 2000). The other challenge to teachers was to change the teaching approach from a teacher-centered to a learner-centered approach. Since Mathematics became compulsory in all junior secondary schools and the increase in access to schools for most Namibians has increased, the need for training Mathematics teachers at all levels of basic education became a burning issue. Therefore, Mathematics teacher education in Namibia has received considerable attention over the past years. The government has developed teacher educational programmes to address the needs of teachers. Different Colleges of Education in the country trained new primary school and junior secondary school teachers and the University of Namibia was tasked to train senior secondary school teachers (MEC, 1993). Since 2011, all training of teachers is done by the University of Namibia. In addition, there was also a greater need to retrain old teachers to help them understand the newly introduced philosophies of learner centeredness. The major focus of these initiatives has been to train teachers on how to help learners to construct mathematical knowledge through a learner-centered paradigm, which involves using a range of learning activities and quality instruction (MoE, 2006).

2.10.2 A Learner Centered Approach

Before independence in 1990, the dominant approach to teaching in most Namibian schools was teacher-centered (MEC, 1993). The teacher would present the learning material on a blackboard to the learners and it was expected from them to copy from it and study the work. A general lack of textbooks contributed towards the previously unhealthy situation.

Now, the philosophy of the Ministry of Education is that a learner-centered approach shall be followed in all schools. "LCE is an approach to teaching and learning that comes directly from the National Goals of *equity* (fairness) and *democracy* (participation). It is an approach that means that the teacher put the needs of the learner at the center of what they do in the classroom." (MBEC, 1999, p. 60). Every learner is unique in his or her composition of personality and learning abilities. Schools typically have learners from different socio-economic backgrounds and it is expected from the teacher to be sensitive to the needs of every learner. A teacher should allow learners to progress at a pace that will allow maximum learning to take place. The focus should be more on the individual learner than the learning material. Every learner is allowed to progress at his/her own tempo to grasp the learning material and to assimilate the amount of knowledge to his/her own ability. It is expected from the teacher to gain knowledge about each learner's strong and weak points in terms of learning tempo, talents and understanding capacity. This should then be consolidated to manage the learning process for each individual (MBEC, NIED, 1999).

According to a publication by the National Institute for Educational Development, *How learner-centered are you,* (MBEC, NIED, 1999) teachers who follow a learner-centered approach will engage in activities such as:

- getting learners actively involved.
- encouraging trial-and-error learning. Learners are allowed to learn by their mistakes.
- giving clear guidelines on objectives and aims of learning exercises.
- being flexible according to new challenges and requirements in a lesson
- emphasizing problem-solving skills
- practising decision-making skills
- using technology and teaching aids to enhance learner activities
- using continuous assessment to monitor learning
- making the learning experience relevant
- teaching the "whole" child (a holistic approach)
- starting with what learners know and build on it
- teaching *concepts* and not unrelated facts. Allow learners to understand.
- allowing time to do tasks
- encouraging free choice
- allowing peer teaching

Conventional instruction methods still have a place in the classroom and a teacher should be able to determine when conventional methods should be applied and when a learner-centered approach should be followed (MBEC, 1999).

Since 1990, many workshops and re-training sessions were held to teach teachers to adopt a learner-centered approach in their own teaching. No literature could be found to proof that the outcome of the program for the re-orientation of teaching methods was successful. It is difficult "to change old habits" and in Namibia the right of choice of the individual is valued very highly. Although both teachers in this study attended several workshops on learner-centered approach, both stuck to their chosen method of instruction. Despite philosophical ideas about the learner-centered approach, learners still write an external examination, with results published nationally and comparisons made among the results of different schools. The excellent results that their learners achieved in the external examinations convinced both teachers, to stick to their chosen methods of teaching.

Unfortunately, in reality it happens that, due to limited resources, financially and in terms of teaching staff, class groups could be up to fifty learners per teacher in a Mathematics class room. Teachers not qualified in Mathematics teaching are expected to teach Mathematics up to grade-12 level, as is the case in this study. To succeed is possible and their stories should be told.

2.10.3 The Grade 10 syllabus

To accommodate all levels of learners in Mathematics, the subject is taught on two levels in Grade 10, namely Mathematics and Additional Mathematics. This allows the teacher to reach all learners, and the philosophy states that the ideal situation would be to teach each level in a separate class group. The curriculum clearly states that: "In the mathematical area of learning, learners understand and master a variety of mathematical skills, knowledge, concepts and processes, in order to investigate and interpret numerical and spatial relationships and patterns that exist in the world Mathematics helps learners to develop accuracy as well as logical and analytical thinking." (MOE, NIED, 2010, p. 1).

The syllabus, currently taught in Namibia, was locally developed and emphasizes a cross-curricular approach. The Mathematics syllabus is topic based and although all the classical topics, such as algebra, and geometry, are present; it relates each topic to the Namibian society in a practical manner. Emphasis is placed on learner-centered teaching, which could be read as “conceptual understanding.” (MOE, NIED, 2010). The expected approach to teaching Mathematics is clearly laid out in the curriculum as based on a paradigm of learner-centered education. “The aim is to develop learning with understanding and the skills and attitudes to contribute to the development of society…. The teacher must decide, in relation to the learning objectives and competencies to be achieved when it is best to convey content directly; when it is best to let learners discover or explore information themselves; when they need directed learning; when they need reinforcement or enrichment learning; when there is a particular progression of skills or information that needs to be followed; or when the learners can be allowed to find their own way through a topic or area of content.” (MOE, NIED, pp. 4 – 5), in fact, the teacher can follow a range of approaches to teach the curriculum content. It remains up to the teacher to decide on a specific approach for any specific syllabus topic.

2.10.4 Assessment of the Grade 10 syllabus

Webb (1992) defined assessment as the "comprehensive accounting of a student's or a group of students' knowledge (p.1)". In this context, Webb is of the opinion that assessment is a tool used by schools and teachers to measure their accountability and effectiveness.

The assessment and evaluation of education in Namibia was also reformed. In the past, examination was mainly used as a measure of success for individuals and programs (MEC, 1993). In the reform, the role of examinations was re-thought and a shift of emphasis from “success versus failure to an orientation that focused on encouraging and recording achievement” (MEC, 1993. p.124) was made. This demanded the inclusion of assessment and examinations in the teachers’ instructional practices. The new form of criterion-based assessment was introduced. The Ministry of Education also introduced two national examinations (MEC, 1993)

• The Junior Secondary Certificate (JSC) in grade 10

• The International General Certificate of Secondary Education (IGCSE) administered by the University of Cambridge in grade 12. (The IGCSE has now been localized as the Namibian Senior Secondary Certificate (NSSC)).

The implementation of all these changes demanded well-trained teachers in content of specific subjects, especially in Mathematics. It also demanded the Namibian government to provide adequate resources to all schools, especially to (a) previously disadvantaged school and/or schools with learners from poor socio-economic backgrounds. According to Battista (1994), teachers are viewed as essential agents of change in any education reform effort presently under way in education around the world. Therefore, they are expected to play a major role in changing Mathematics teaching and learning and more general instructional practices. However, despite a lot of effort by the Namibian government in developing mathematical content knowledge for teachers, it has been revealed by the Southern African Consortium for Monitoring Educational Quality (SACMEQ) reports of 1992, 1997 and 2003 as cited in MoE, (2006) that Namibia still trails behind other Southern and Eastern African countries in terms of teachers' and learners' mathematical proficiency.

The National Examination is an important component of assessment, which is also used for several purposes. In Namibia, like many other countries in the world, national examination results are used for certification and selection purposes (MEC, 1993; Wagg, 2001; MoE, 2006). For example, learners can only proceed from grade 10 to grade 11 in Namibia, if they performed well in their grade-10 national examinations. Learners who do not qualify for selection to grade 11 are certified to compete in the job market. Therefore, as Webb (1992) stated, the national examinations are putting a lot of pressure on both learners and teachers to better their performance - now more than ever before, because Namibia embarked on a road of economic independence.

As mentioned before Wragg (2001) emphasized the importance of national examinations by stating that they also provide information on which learners, or schools, or even the country's education system can be judged. Additionally, national examination results inform policy makers in government and other stakeholders in education systems around the world whether or not education programmes are effective (Webb, 1992). National examination results should therefore be accurate*, valid, reliable* and of high quality (Wragg, 2001).

With the guidelines for assessment as set out in the Namibian Junior Certificate of Education assessment should be done on three levels. On the first level, knowledge and technical skills should be assessed; on the second level, analyzing and synthesizing skills should be assessed and on the third level, presentation skills should be assessed. The assessment on the first level contains descriptors like *recognizing and representing.* The required questioning for the second and third levels should include *analysis skills, logical deductions, connections to concepts, abstraction skills* and the *judging of outcomes* (MoE, NIED, 2010, p.25).

To relate to the learner-centered approach, continuous assessment attributes up to 35% of a learner's final mark during assessment. Compulsory continuous assessment should include projects, topic tasks and weighted written tests (MOE, NIED, 2010, p. 27).

2.10.5 Textbooks

It is not possible to look at teaching practice in isolation without an in-depth look at the role textbooks can play in teacher practice. Three textbooks were approved for use by Namibian grade-ten Mathematics teachers. It is policy that textbooks are supplied free of charge to all learners in Namibia. Each school has the right to prescribe one book for use by the learners and teachers have access to copies of all the prescribed books.

"In school Mathematics, the traditional textbook has long been a key reference for teacher curriculum decision-making and the primary resource for student practice of mathematical techniques" (Shield, 2006, p. 680). The relationship between the teacher, the text book and the curriculum is very complex, but text books play a very important role in a teacher's practice of Mathematics teaching. A study in Austria found that only 5% of teachers do not use a text book at all, but that more than 50% of all grade-8 teachers used the textbook as main lesson resource. (Thomson & Fleming. 2004). According to Shield (2006), it is challenging to evaluate text books, but a good text book should follow the principles stated in the curriculum it tries to follow.

2.11 CONCLUSION

"No sooner than we propose definitions for conceptual and procedural knowledge and attempt to clarify them, we must back up and acknowledge that the definitions we have given and the impressions they convey will be flawed in some way. … Furthermore, it is difficult to imagine someone processing conceptual and procedural knowledge as *entirely* independent systems. Some connections are inevitable." (Hiebert & Lefevre, 1996, p. 9). Hiebert and Lefevre noted that almost every author on the issue came up with different definitions for conceptual and procedural knowledge. What is significant is that mathematical competency requires a fundamental relationship between conceptual and procedural knowledge. Competency cannot be achieved if either kind of knowledge is deficient or if they remain separate entities (Hiebert, 1996).

Baroody and Ginsberg (1986), found that for many children the development of procedures used to solve addition and subtraction problems does not always lead to the development of conceptual knowledge, even in cases in which procedures appear to be based directly on conceptual knowledge. They argue that in many cases the development of conceptual knowledge is neither "necessary nor sufficient" to ensure the acquisition of related procedures (Hiebert, 1986). Being competent in Mathematics involves knowing concepts, symbols, procedures and their relationships, but learners do not connect procedural and conceptual knowledge persistently in the same way (Hiebert 1986). Examining these complex relationships remains relevant and part of the convergence of knowledge to understand learning Mathematics.

The debate about procedural and conceptual understanding and procedural and conceptual instruction approaches is far from over. This case study enlightens merely one more window of the issue and underlines once again the complexity of the search for understanding the learning of Mathematics.

CHAPTER 3
METHODOLOGY

3.1 INTRODUCTION

This chapter provides an overview of the paradigm and research design that framed the study, and techniques, strategies and procedures that were employed to execute the research plan.

3.2 RESEARCH GOAL

The research methodology that a researcher adopts depends on the goals and should thus be aligned to these goals. A goal is anything that includes motives, desires and a purpose that can lead to an outcome. It includes anything that can take someone to ask questions, do a study or to engage in research (Patton, 1990).

With this research, I wished to gain insight into two teachers' different approaches to teaching Mathematics to Grade-10 learners.

In particular, the goal of this study was to analyze and understand the teaching approaches of two teachers that claimed that their Mathematics teaching was characterised by a conceptual and a procedural approach respectively.

3.3 THE INTERPRETIVIST PARADIGM

Many underlying paradigms can shape a research study. These paradigms have their own ontology (the nature of existence) and epistemology (what is known to be the truth) and thus directly influence the orientation of the research process (Janse van Rensburg, 2001). This research study adopts an interpretive case study approach.

In order to obtain a deep understanding of phenomena within a natural situation, situating one's research in the interpretive paradigm is appropriate. In the interpretivist paradigm, as stated by Cohen, Manion and Morrison (2000), the researcher tries to understand the subjective world of people's experiences and interactions: how people make sense of their lives, how they define their own situation and how their sense of self interacts with others. This aligns well with the goal of this research, namely to look at and understand two approaches adopted by two Mathematics teachers, teaching Mathematics at the same levels.

This case study focused on the teaching practice of two teachers who developed their practice over a period of many years, each one with a unique approach to Mathematics teaching. One claims to have a mainly procedural approach and the other one a mainly conceptual approach. It is significant to research their case as both constantly achieved excellent results with their learners during the external examinations as illustrated in tables 3.1.

GRADE 10 EXTERNAL EXAMINATIONS: WINDHOEK GYMNASIUM		
NATIONAL	2011	4th place out of 594 schools
	2010	3rd place out of 584 schools
	2009	6th place out of 570 schools
REGIONAL	2011	1st place out of 46 schools
	2010	1st place out of 46 schools
	2009	2nd place out of 46 schools

TABLE 3.1: Results Grade 10 National Examinations

3.3.1 Qualitative approach

Efforts were made to observe and record the two participating teachers within their own natural setting. Their teaching was video recorded and analysed. This was done with as little interference as possible of the normal operations of the classroom activities. This allowed me to take an objective look at the teachers' activities. The videos were then analysed with the teachers and this allowed for obtaining deep insight into their own understanding of their teaching approaches.

To gain a deeper understanding of each teacher's approach and to create a chain of evidence, a series of interviews were planned. An initial workshop was held to explain the process to the teachers and to ensure that the research process could be completed according to the planned phases. During the initial workshop, emphasis was placed on the orientation of this research as being a case study and not a comparative study. Insight gained from the study was not used to guide or criticize any of the individual teachers, but to learn from their teaching for the betterment of teaching Mathematics in Namibia. This ensured that the results of this case study had significance.

Qualitative data was collected by video recording of at least two lessons of each of the two participating teachers. Every time a video recording of the two teachers was made, both teachers were covering the same learning material. To ascertain their ‘theories’ of their own teaching methods, these videos were analyzed collaboratively with each of the teachers encouraging each one to comment and share their views of the unfolding of their lessons. More qualitative data was obtained through a series of interviews. In some cases I used direct questioning and in other cases a semi-structured approach was followed. Finally informal open-ended discussions were held with the two teachers, mostly at their request.

Locating this study in an interpretive paradigm enabled me to critically analyze the data collected and synthesize the study of the two approaches and to draw inferences about the different teaching approaches.

3.3.2 Underlying methodology: A case study

The path to this research project was inspired by working with two colleagues who maintain that they each adopt a specific teaching strategy. Both claim openly that their approach to teaching Mathematics is different. Both also say that their approach is the major contributor to the good results that their Grade-10 learners achieve in the external Grade-10 examinations. This situation posed itself to me as being an exciting topic for research which could contribute to the improvement of Mathematics teaching in Namibia.

I deemed the case study approach to be appropriate as it allowed me to investigate in such a way as “ … to retain the holistic and meaningful characteristics of real-life events – such as individual life cycles, small group behavior and managerial processes.” (Yin, 2006, p.4). My unit of analysis was the *lessons* of the two teachers. In particular the focus was on the teaching approaches of the two teachers in terms of Kilpatrick’s conceptual and procedural strands. “If the propositions are at the core of a well-known debate in the literature – or reflect major differences in public beliefs – the case study is likely to be significant.” (Yin, 2006, p.186)

3.4 RESEARCH DESIGN

The aim of the case study was to study and understand a real-life situation. In order to create and maintain a chain of evidence that could be presented neutrally with both supporting and challenging data, the research was structured in six stages, namely:

Step 1: Initial workshop
Step 2: Semi-structured interviews
Step 3: Video recorded lessons
Step 4: Open-ended interviews
Step 5: Document study
Step 6: Data analysis

Details of the stages are provided in the Summary Table at the end of this chapter. See page 46.

3.5 DATA COLLECTING TECHNIQUES

3.5.1 Interviews

In order to develop a deeper understanding of the two teachers’ chosen approaches and to explore possible influences on their teaching methods, both teachers were interviewed, firstly together and later individually. After the initial data collecting process was completed, both teachers expressed a desire for further discussions and the opportunity to provide more information on their teaching approaches. During the interviews, the teachers had the opportunity to express how they felt about their chosen teaching methodology and explain other factors that influenced and shaped their teaching approaches.

A four-pronged approach was followed with the interview and discussion processes. Firstly, a semi-structured approach was followed in a combined interview. A set of questions directed at their teaching methods was set, with additional opportunity for further probing. See Addendum 1. Secondly, during the individual video discussions, opportunity was given to each teacher to express her views on her own teaching practice. The video recordings were viewed in the presence of the two teachers and for the case study an in-depth interview approach was followed with each teacher. Key questions were asked and the teachers proposed their own insights into certain occurrences during the lessons given by them (Yin, 2009). A grid designed to structure and assist in the identification of conceptual or procedural actions during every phase of the recorded lessons was completed by each teacher during the screening of their second recorded lesson. Each teacher also had the opportunity to classify the steps for each other's lesson with the researcher. See Addendum 2. Thirdly, formal interviews without any probing were held with a fixed line of enquiry. See Addendum 3. This was done to ensure reliability and validity of the interviews. Certain matters were raised during the first two interview opportunities and the researcher restructured them into questions for a formal interview. Both teachers had the opportunity to elaborate and confirm their views. Fourthly, valuable data was collected during further informal interviews that arose from a desire from their side to discuss their own teaching practices.

All interviews were recorded and transcribed for data analyses purposes.

3.5.2 Observation

"Observation is more than just looking. It is looking and noting systematically people events, behaviors, settings artifacts routines and more. The distinctive feature of observation as a research process is that it offers an investigator the opportunity to gather 'live' data from naturally occurring social situations" (Cohen, Manion &, Morrison, 2011, p. 456). In order to gain a deeper understanding of the teaching approaches that the two teachers adopted, two lessons of each teacher were video recorded. It was arranged with the two teachers to cover the same topic on the day that a recording was done. Consent was given that video recording could be done with as little intervention with the normal class activities as possible.

I am a senior colleague of the two teachers and in order to create a natural setting without them being disturbed by my presence, it was decided that a professional cinematographer would film the lessons. Cameras were set up in a way that would cause the minimum interference with the class activities.

Video recorded lessons were screened in the presence of each teacher and a discussion of each step of the lesson was done with the individual teacher. Each teacher had adequate opportunity to comment on the lessons, to explain actions and to add information on the lessons. A grid was designed to evaluate each step of the lessons in terms of the procedural and conceptual actions applied. See Table 3.2 below.

This system was developed with the aim of rigorously classifying the data as supporting, being neutral or contradicting what the teachers initially claimed about their teaching approaches (i.e. either conceptually or procedurally). The same framework was used to code all the data collected during all the interviews, as well as the recorded video material. The video material was analysed with the teachers that presented the lesson. Additionally, a profile of each of the two teachers involved was created in order to facilitate a better understanding of who the participants are and to provide a contextual backdrop of both individuals.

Table 3.2. LESSON ANALYSES GRID

Procedurally and Conceptually Orientated Teaching Methods

Procedurally Orientated Teaching		Conceptually Orientated Teaching	
Teaching Practice		Teaching Practice	
1a. Explaining how to… [step-by-step algorithm]		1b. Ask "why", asking why not? [Why this step?]	
2a. Teaching definitions and symbols		2b. Explore possible definitions, symbols	
3a. Practise steps		3b. Design or discover own steps	
4a. Calculator: Teach steps to perform an operation.		4b. Solve a problem, try to use the calculator	
5a. Only one "right" way to solve a problem. [the right answer is everything]		5b. Multiple strategies to solve a problem. [the process is more important than the answer]	
6a. Answer of a problem is final.		6b. Answers pose more opportunity for learning.	
7a. Skill teaching is important. Focus on a single skill to arrive at an answer.		7b. Connecting ideas and concepts in Mathematics.	
8a. Answers in isolation		8b. Relate to the real world	
9a. Word problems directly based on the required skill.		9b. Posing a problem, develop skill through reasoning.	
10a. Algorithm is everything		10b. Algorithm one form of representation a solution.	
11a. Wrong answer is absolute.		11b. Wrong answer provides investigation opportunity into understanding of the problem	
12a. Level one questioning. Typically expecting only an answer on a question.		12b. Questioning that requires adaptive reasoning, and the consideration of alternatives	
13a. Teacher demonstrates. Teacher centered approach. Instructive		13b. Teacher facilitates. Learner centered approach. Investigative.	
14a. Problems are only computational. Follow prescribed steps		14b. Problems open ended	
15a. Focus on procedures only.		15b. Focus on concepts to develop procedures.	
16a. Body language: In front of the class – one way communication. Questioning – only expect the right answers		16b. Body language: Move between learners, constantly prompting learners for responses and comments. Leading learners in reasoning.	
17a. Homework control: Mark work right or wrong		17b. Homework control: Discuss answers – right and wrong; Add comments like; Why? Or Explain.	

3.6 DATA ANALYSES

A qualitative approach was adopted to do the data analysis. The process of qualitative data analysis is not a linear process, but a process of noticing, collecting and thinking (Seidel, 1998).

> Qualitative Data Analysis (QDA) is the range of processes and procedures whereby we move from the qualitative data that have been collected into some form of explanation, understanding or interpretation of the people and situations we are investigating. Qualitative Data Analyses is usually based on an interpretative philosophy (Lewis, et al, 2005, p.1).

Seidel used a graphical model to graphically explain the process of qualitative data analyses. See Figure 3.1.

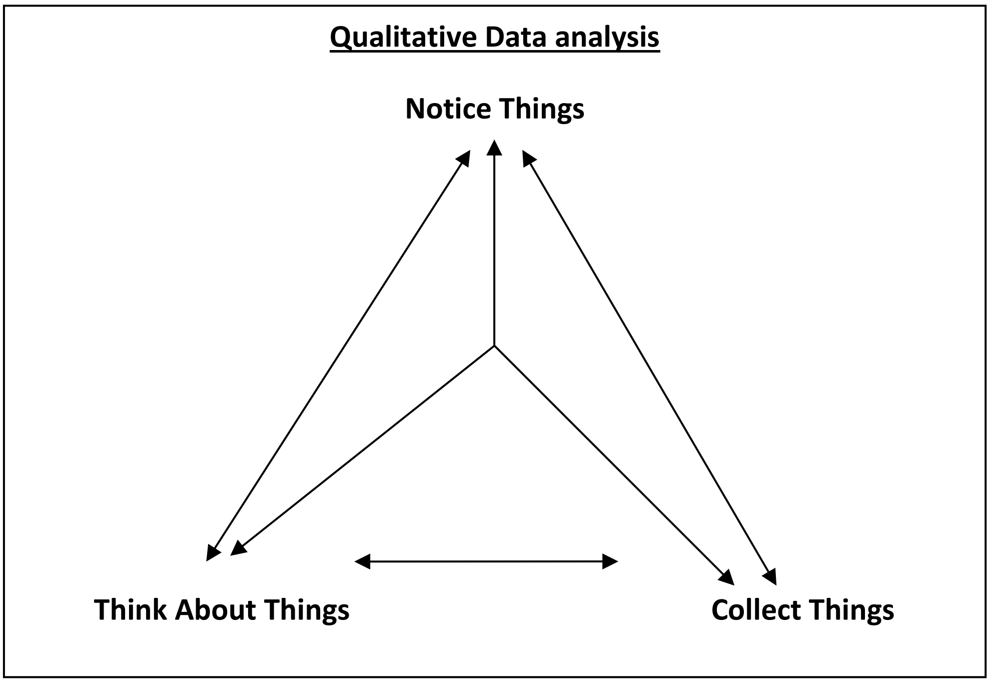

FIGURE 3.1. Seidel's model of qualitative data analyses.

By applying Seidel's model of noticing, collecting and thinking, a cycle of steps was developed for the analysis and interpretation of the data (Seidel, 1998). It is important that the researcher will apply Seidel's mode repetitively. When thinking about data, things are noticed, but when re-thinking the data, new things are noticed and even more vital discoveries are made. Figure 3.2 gives another detailed explanation of the process the researcher should adhere to.

UNDERSTANDING THE DATA

(Transcript of interviews -> Re-read -> re-listen)

(View videos -> discuss -> analysis)

⇕

FOCUS THE ANALYSIS

(Ask specific questions)

(Focus interviews -> direct questioning)

⇕

FOCUS DATA BY CONSISTENCIES AND DIFFERENCES

(Identify Conceptual or Procedural)

⇕

IDENTIFY THEMES OR PATTERNS

(**Organize them in coherent Categories**)

(Apply Coding)

⇕

INTERPRETATION

WRITE DOWN

(Re-visit steps)

FIGURE 3.2 Handling of large volumes of different data

3.7 RESEARCH PARTICIPANTS AND SITE

3.7.1 Research Participants

While teaching at Windhoek Gymnasium Private School, I noticed the consistently good results achieved by two Mathematics teachers during the external National Examinations for Grade 10 over the past years. Both claimed that they could attribute their respective successes to their chosen approaches to teaching Mathematics. During further conversations, it surfaced that the first teacher, Salomé Davin claimed a mainly procedural approach to teaching Mathematics and the other one, Adri Botes claimed a mainly conceptual approach when teaching Mathematics.

Davin revealed during an interview that she did not receive any tertiary training in Mathematics, neither did she get any didactical background for Mathematics teaching during her teacher training. She said, she always had a passion for Mathematics and when the opportunity came to teach Mathematics, she took on the challenge. For the past twenty years, she has been teaching Mathematics and developed her own methodology of teaching Mathematics which she named a "show them how to do the problems approach." This was the method used when she did Mathematics at school and this procedural approach was the only tool she knew to teach Mathematics to learners.

The second teacher that formed part of this case study is Adri Botes, who did a B. Com. Degree with Accounting and Mathematics as majors for the Higher Education Diploma. She was unaware of the formal distinction between conceptual and procedural teaching as defined by Kilpatrick et al. (2001). She claimed, however, that during her teacher training emphasis was placed on "Learning with understanding," so much so that she became driven by the idea that learners should understand the underlying concepts of any topic in Mathematics to be able to "*do Mathematics and solve problems.*" She was encouraged by her results of the external examinations and established her own approach that worked for her.

The two teachers collaborate when teaching grade-ten Mathematics. Often they share resources for homework exercises and take turns to set internal examination papers. They each teach two Grade-10 Mathematics groups, randomly compiled out of all the Grade-10 class groups. Although grade-10 Mathematics is offered on two levels, namely Core and Extended, all Grade-10 learners are initially taught at one level. It is only in the last trimester that the Additional Level learners are separated from the Mathematics ordinary level learners. This research project was completed before the learners were sorted into different levels. The timing of this was to the advantage for the investigation as learners were randomly placed with each teacher. This also meant that each teacher was teaching to a complete bouquet of weaker and stronger learners.

When I approached the two teachers to ask for their willingness to participate in this study, they immediately agreed. The selection of the participants was thus purposeful (Cohen, Manion & Morrison, 2011). They understood that their participation was entirely voluntary and that they could withdraw from the study at any time.

3.7.2 Research Setting

The whole research process was undertaken at Windhoek Gymnasium, the school where the two teachers are employed. The video recordings were done in their own classes, during normal teaching periods with their respective Grade-10 class groups. The video recordings were made with the minimum interruptions to any of their normal classroom practices. The camera was located as discreetly as possible in one corner of the classroom.

All interviews and conversations were done at the school, in a familiar environment where the participants felt at ease. Interviews were conducted during free periods.

3.8 VALIDITY AND ETHICAL ISSUES

In a case study research, the researcher should always ensure validity and reliability. This whole research was guided by the "Canons" of validity as set out by Cohen, Manion and Morrison.

1. Construct validity – employ acceptable definitions and constructions
   All definitions and constructs were widely supported in the literature.

.

2. Apply concurrent validity – use multiple sources of evidence.
   Evidence was collected through interviews, objective video recordings, plus an intense study of documents.

3. Ensure reliability – study must be replicable and internally consistent.
   The choice of participants and their claims allow the study to be replicated. The extensive period of data collected supported consistency.

4. Avoid bias – the case study cannot be an embodiment of the researcher's prejudices. The case study presented itself to the researcher as an opportunity to gain understanding of a phenomenon. Bias could not play a role as no value judgment was ever made of either the participants or their practice.

5. Triangulation (Cohen, Manion, Morrison, 2011).

3.8.1 Data triangulation

"Data triangulation may be defined as the use of two or more methods of data collection in the study of human behavior". (Cohen, Manion & Morrison, 2011, p. 195) Broadly triangulation can be seen as the use of a multi-method approach in order to increase the validity of a study. The case study of the two teachers allowed the opportunity to collect data through semi-structured interviews, informal interviews, video recording, and the objective study of several different documents.

> The use of multiple sources of evidence in case studies allows an investigator to address a broader range of historical and behavioural issues. However the most important advantage presented by using multiple sources of evidence is the development of *converging lines of inquiry,* a process of triangulation and corroboration. (Yin, 2009, p. 116-117).

The triangulation of my data can be graphically represented based on the adopted structures by R. K. Yin (2009, p. 117). With a qualitative process of data analysis, different sources of data converge through a process of analysing the data, thinking about the data, analysing again and re-thinking of the evidence eventually to emerging findings. This is illustrated in Figure 3.3 below.

Convergence of Evidence

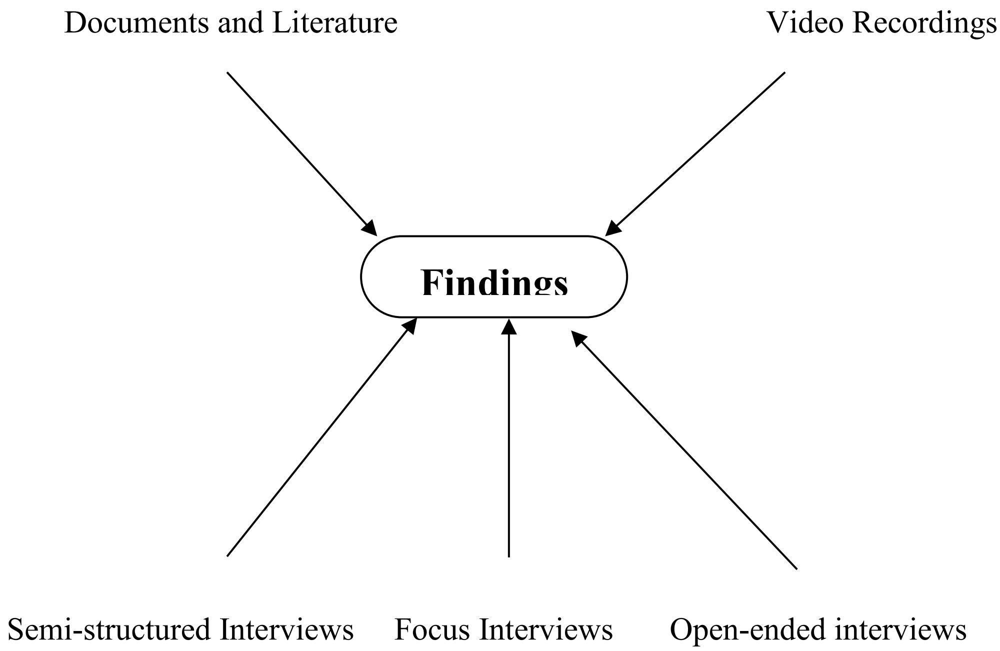

FIGURE 3.3 Process of Data Triangulation

3.8.2 Ethical Issues

To do the research with the two teachers, written permission was obtained from the rector of the school.

Both teachers gave written consent to participate in the research project and expressed a wish to have access to the final results of the research. They both undertook to be committed to the process and to be available for any of the interviews required.

They were informed in writing

- about the purpose of the research;
- about procedures that will be followed during data collection;
- that data collected will be handled confidentially;
- that the results will be available to both;

- about their rights not to disclose information they are not comfortable with and
- that anyone can withdraw at any stage of the process.

The process of consent "protects and respects the right of self-determination and places some of the responsibilities on the participants." (Cohen, Manion & Morrison, 2000, p. 51). Both participants did not wish to remain anonymous. As experienced, both Mathematics teachers saw the potential of the research process to contribute to the improvement of Mathematics education on a broader level.

The research participants were colleagues of mine and they were assured that the research would be done without any bias, and that no value judgment about any one or their teaching practices would be done.

3.9 CHALLENGES AND LIMITATIONS

The first challenge was to ensure that the participants would remain co-operative over the entire research period and that the data collected at different time intervals would be consistent. This challenge was overcome, when both decided not to remain anonymous but to have their names mentioned in the writing up of the research. They also realized that the research results would be available to them. They recognized that the research process would be a worthwhile learning experience curve for them, and that their participation would contribute to Mathematics teaching nationally.

The third challenge was to record the lessons in as natural a setting as possible. My presence in their class during video recordings would have created a contrived situation, with them teaching to my expectations and with the learners behaving differently than they would have without my presence. This challenge was overcome by using a cinematographer who was a neutral figure.

Fourthly, the time available to record and interview the participants became a huge challenge. Despite their co-operation and willingness to participate, we struggled to find convenient time slots for data collection. Very high workloads and full extra-curricular programmes were big challenges to overcome. Eventually relief staff assisted us during examination sessions to have free time together, so that the interviews could be conducted.

Fifthly, the I had very limited time to do thorough research. For a long time it has been a dream to participate in a Masters of Education like this one, but I always postponed it, because I believed that the work load will be less once an experienced Mathematics teacher with all lessons planned. Just the opposite seems to be true. Seniority brings even more responsibilities and an ever higher workload. The time available to me to do a thorough literature study and to skillfully drive the research process was extremely limited. Limited time was available to gain knowledge, to understand the process and to think about, and to re-think about the data collected. Long lines of thought had to be drawn through to acquire data on the consistency of each teacher's practice. The fragmented interaction with both participants made it extremely difficult. Eventually I had to make a choice and focus on the important matters. It was decided that a successful study could contribute over a wider spectrum and should thus receive priority.

All research has time constraints. The video recordings of the lessons are thus mere vignettes of the practice of the teachers. They, however, yielded adequate data to address my research goals.

TABLE 3.3 SUMMARY OF RESEARCH PROCESS

Stage	Method of data collection	Aim	Data Collected
Stage 1	Workshop	• To introduce the research process to the teachers. • Consensus about our understandings of conceptual and procedural approaches to teaching; • For the researcher and two teachers to familiarise themselves with each other; • Agreement on the objectives of the project. • To reach an agreement on a suitable timeline; • To discuss ethical issues; • To address any of their concerns. • To get full cooperation for the period of research.	Data on the participants
Stage 2	Semi structured interviews with individual teachers.	• To regularly engage with each teacher regarding her own teaching method. • To be up-dated on a regular basis regarding their experiences, teaching practices and beliefs • To discuss lessons in detail, • With special reference to videotaped lessons.	Qualitative data of a personal nature from each teacher. Data on the methods the participants applied in their teaching
Stage 3	Videotaped lessons Two lessons of each teacher on two topics, where both teachers cover the same subject matter each time	• To observe the application of each teacher's chosen method	Qualitative data on the following: Revision of the previous lesson; topic for the lesson introduced; methods of teaching; participation of learners; the activities used in the classroom mostly procedural or conceptual, or both: lesson revised and/or summarized; homework, the nature of the homework; strengths and weaknesses .
Stage 4	Open ended interviews	• To discuss lessons • To follow up on emerging issues • To re-visit unclear matters	Data on teaching approaches during lessons. Consistency with interviews and recorded data
Stage 5	Document Study. (1) JSE Grade 10 syllabus (2) Preparation files and lesson plans of each teacher. (3) Homework books and tests of learners. (4) Text books	To verify the detail and depth each topic to be covered. To find evidence of prescribed approaches towards any topic in the syllabus. A lesson plan clearly identifies the planned approach for a specific lesson. Whether it will be procedural or conceptual. Through a continues study of lesson plans – one can establish the line of thinking for each planned lesson To get an understanding to what level a topic was grasped and understood by learners. To get a timeframe that it took learners to achieve a certain level of competence in a certain topic. To verify the approaches used in the text books pre-scribed to the learners	Information that needed for data validation purposes. A large amount of empirical data can be collected from the lesson plans and the teacher's personal file. This includes – the approach to introduce a topic, methods of assessing and revision. Level of achievement and understanding of a syllabus topic. data regarding the period of time it takes with each method to reach a certain level of understanding. To what extend is conceptual or procedural teaching supported by the text books.
Stage 6		Anelyse data according to interview grid and lesson dissertion plans	

3.10 CONCLUSION

Structured planning and a solid theoretical framework to drive research are important components for successful research. A summary of the research process is given in Table 3.2 above. By placing the research process in a framework, as displayed above, the whole process was structured and completed within the window period as envisaged for each step.

CHAPTER 4

ANALYSIS OF DATA AND RESEARCH FINDINGS

4.1 INTRODUCTION

In this chapter an analysis of the data collected is presented. The goal of this study was to analyze and understand two teaching approaches that are characterised by the two participating teachers themselves as conceptual and procedural approaches to Mathematics teaching respectively. Firstly, an explanation of the method of data collection is presented, secondly the coding and processing was done, and thirdly the data, findings and discussion are presented.

Every teacher has individual characteristics and these are reflected in the classroom setup, teaching strategies and even during interactions with learners. Research has shown that teachers use teaching strategies that include varying degrees of conceptual and procedural methods (Ma, 1999). These contribute to the unique teaching style individual teachers develop over years of teaching. As discussed in Chapter 2, procedural knowledge was described as the "formal language, or symbol representation, system of Mathematics" (Hiebert, 1986, p. 6), and the "rules, algorithms, or procedures used to solve mathematical tasks" (Hiebert, 1986, p. 6) or as the "ability to execute action sequences to solve problems" (Rittle-Johnson, Siegler & Alibali, 2001, p. 346). Conceptual knowledge was described as "knowledge that is rich in relationships" (Hiebert, 1986, p. 3) or as flexible "implicit or explicit understanding of the principles that govern a domain and or the interrelations between units of knowledge in a domain" (Rittle-Johonson, Siegler & Alibali, 2001, p. 346 – 7). Kilpatrick, et al describe conceptual understanding as an integrated and functional grasp of mathematical ideas, while procedural fluency refers to knowledge of procedures, a skill in methods of problem solving (Kilpatrick, et al. 2001). Based on the literature (see Chapter 2) *procedurally orientated teaching* is defined as using methods that focus on the development of learners' procedural understanding of Mathematics. It is often characterized by the use of rote memorization of procedures, facts and the use of formulae for solving problems. It includes the presentation of definitions with little exploration of the definition itself. *Conceptually orientated teaching,* on the other hand, is defined as the use of teaching methods that focuses on the development of the learners' conceptual understanding of

Mathematics. This orientation can include questioning learners to suggest strategies for problem-solving, to avoid memorization by devising their own methodology to arrive at the correct answers, to do comparisons and make their own deductions from given mathematical situations.

Based on the above definitions and the discussion in Chapter 2, I designed a grid to code the data collected about the two teachers involved in the case study. See Table 3.2 on page 37.

4.2 RESEARCH FINDINGS

4.2.1 Profiles of Research Participants

This research came about when two colleagues, both teaching Mathematics at our school showed interest in the study material I used during my studies. It transpired that Davin never received any formal training as a Mathematics teacher but due to the acute shortage of Mathematics teachers, literally volunteered to start teaching the subject. She first taught as a grade-7 teacher and gradually, as she gained confidence, advanced to grade ten. At the moment, she confidently teaches Mathematics up to grade 12. She explained, "*Everybody asked me to teach the subject, but nobody ever showed me how. I just believed it could be taught in the same manner as History or Geography.*" Her progressive advance towards teaching grade-12 Mathematics provided her with the opportunity to gain the required content knowledge, mostly by studying the prescribed text books on her own. Her honest revelation of her struggles with learning and teaching the subject gave birth to this particular case study. "*When I prepared for my lessons, I studied the content and memorized the steps to solve the example problems, then I taught it to my classes the next day.*" Her perseverance and hard work proved to be fruitful over the years, and she is continuously rewarded with excellent external results. Nobody ever spoke to her about any method of teaching Mathematics and as she explained later during the interviews, "*I developed my own method of teaching Mathematics, which I call a method based system. I teach the steps to the learners, they memorize it exactly as I wrote them down on the board and then they apply it.*" The outstanding results of her grade-10 learners during the external examinations affirmed and reinforced her beliefs in her own methodology. In terms of the definitions for teaching procedurally and as showcased by her own recorded lessons, she approaches every lesson in a characteristically procedural manner.

The other colleague is Adri Botes. She is a friend of Davin and teaches in the adjoined classroom. She has taught grade-10 Mathematics for many years with Davin. She immediately claimed that she teaches conceptually, or as she described her methodology, "*I teach children to understand and not to study like parrots.*" Like Davin, she also achieves consistently excellent results in the external examinations. She ascribes this to her ability to "*reveal the workings of Mathematics to learners and to teach them to develop their own plans to solve problems.*" – a typically conceptual approach to teaching (See Grid 9b and 15b).

Botes did the subject Didactics for Mathematics, as part of her post graduate Higher Education Diploma. She was taught different strategies to approach different concepts in school Mathematics, all clearly, as she later explained, in the area of conceptual teaching. "W*e were taught to teach the children to understand and re-discovery learning methods and to design their own methods to solve problems. This is how I teach today still.*"

Both teachers completed their studies long before the notions of conceptual teaching and procedural teaching were clearly defined to them, but it was clear that each one adopted a unique and specific teaching approach. To them, their approach provided them with their own teaching tools within their own paradigm. An informed outsider would merely name it *teaching procedurally*, in the case of Davin and *teaching conceptually,* in the case of Botes.

4.2.2 Documents and Literature

4.2.2.1 Participants' responses to the Syllabus for Mathematics

During the reform process of the Namibian education system since the early 1990s, great emphasis was placed on a learner-centered approach, implying a more conceptual approach to teaching. This was, however, never articulated explicitly in any of the policy documents. The Namibian Junior Secondary Phase Mathematics Syllabus states in its rationale and aims that the learning process in Mathematics should enable students to: "develop their mathematical knowledge and skills in a way which encourages confidence and provides satisfaction and joy; develop their ability to analyze problems logically, recognize when and how a situation may be represented mathematically, identify and interpret relevant factors and, where necessary, select an appropriate mathematical method to solve the problem" (NIED, 2006, p. 2). Under the heading "Approach to teaching and learning", it is spelled out that teachers should develop learning with understanding. Teachers are advised to encourage learners to work in groups, in pairs and individually. Problems should be exemplified in a context that is meaningful to the learners. Mathematics should relate to the world the learner lives in. Finally, it is emphasized that all learners should achieve the basic competencies (NIED, 2006).

During the reform process both my participants attended several workshops which focused on the transformation process from a teacher-centered paradigm to a learner-centered paradigm. Both teachers expressed their general disappointment with the workshops because Mathematics, in their view was never treated as a special case in the sense that the presentations were so general that they never improved their skills to present the different mathematic topics, like algebra conceptually. As Botes explained, "*We thought that we would be taught skills to teach the new syllabus, but all I experienced during the workshops was that we as experienced teachers must give our examination papers to less experienced teachers so that they can use it ... Everybody had a lot to say about learner centered teaching, but nobody ever showed me how to use it in Mathematics."* Davin supported the view and added that, "*we received a lot of syllabus content and all the new topics were discussed, over and over again, but not a single person showed me how to teach them. I did not even think that different methods of teaching a new topic exist."* The workshops revolved a lot around a learner-centered approach, and content knowledge, but never focused on the **teaching** of content to learners. Already during the first combined interview Davin stated: *"I never knew that Mathematics could have other teaching methods than any other subject. It did not even come to my mind, as it was never ever mentioned."*

Secondly, they found that the focus of each reform workshop was to teach the newly introduced topics, for example statistics and possibility theories to the teachers. The content of the workshops were structured around the subject content and not around the processes of teaching and dealing with the learners. This, in my view, left the two teachers somewhat stranded. Davin was left to her own devices and she had little option but to rely on her own proven method of teaching that she had developed over the years. This, according to her, was to continue with a very procedural approach: "*After every workshop, I felt that I was doing things right. In fact after listening to other teachers complaining, I felt good about my own teaching."*

Botes admitted that she also found it difficult to apply what was taught in the workshops and thus continued to teach using what she believed was a conceptual approach. She said; "*I tried to work in groups and to allow learners to participate more, but at that stage I was teaching at ……. and had up to forty five learners in a Mathematics class. If you just try anything different or funny………you will just lose it."*

It is interesting to note that both participants pointed out that although they are familiar with the aims and rationale of the syllabus, all the learning material is set out in the syllabus in a tabulated format stating only topics with expected basic competencies for every grade from grade 8 to grade 10. Davin came back to me and said that she had the opportunity to rethink one of the previous discussions we had and she would like to make a few additional comments: *"You know what ..... All I can find in the syllabus is that the children must* ***know how to do*** *things and when they talk about* **understanding,** *they say immediately the children must be able to* **do** *sums with it."* To her that implied a very procedural approach to teaching Mathematics, despite the stated rationale and aims. An example from the syllabus is given in table 4.1 below.

Table 4.1 Learning Content: Grade 10 (NIED, 2006, p18, 19).

THEMES AND TOPICS	LEARNING OBJECTIVES Learners will:	BASIC COMPETENCIES Learners should be able to:
1. Numbers		**Solve problems involving direct and indirect proportion**
(a) Calculator skills	**Be able to use the more advanced functions of the scientific calculator**	**Use the calculator to represent numbers in standard form. Translate the calculator display of standard form into the appropriate notation Find the values of trigonometric functions for given angles and find the angle if the value of the trigonometric function is given.**
3. Mensuration		
Volume and surface area	**Know and apply the concepts of volume and surface area of cuboids and cylinders.**	**Calculate the volumes and surface areas of cuboids and cylinders using appropriate formulae. Calculate unknown dimensions of cuboids and cylinders, if sufficient other information is given.**
5. Algebra		
Equations and inequalities	**Understand how to transform linear equations to find the solution Realise the importance of algebraic equations to solve problems**	**Solve linear equations with brackets Solve problems by translating them into linear equations Solve linear equations with fractions Solve simultaneous linear equations in two unknowns Solve quadratic equations by factorization.**

It could be argued that every teacher has the freedom to choose his/her own methodological approach for every topic to be taught to learners. Unfortunately, and it is clearly spelled out by the two participating teachers, it is very difficult to teach Mathematics conceptually within the learner-centered paradigm. Davin said this is particularly the case when the teacher did not receive any training in the appropriate methodology of teaching Mathematics: "*You know now that I never did any Mathematics after school, so I had only the syllabus and a text book to guide me in the beginning. I used to write the facts down on the board for the learners so that they can study it. This is what I am doing now in Mathematics and it works. I tell you it works when they can see step by step how to solve a problem. I always write the steps or the rules on the one side of the board."* (Compare Grid 13a and 14 a)

At this stage of the interview, she was interrupted by Botes who said; "I *support her and want to agree with her, my feeling is that the syllabus only spells out what learners must be able to*

do. *It was my own choice to* ***teach with understanding methods, that is when I have the time,*** *but remember I also have to prepare the children for the external exams."*

The syllabus and the policy documents discussed in the literature review clearly spells out what learner-centered means (see page 2. NIED). It is aligned with this study's understanding of conceptual teaching with the emphasis on learning with understanding. Both the syllabus and the policy, however, lack the *how* aspect. A need was expressed by both teachers that, despite their own experiences they sometimes feel a need for assistance with the presentation of certain topics to the grade-10 learners. This was illustrated by a remark made by Botes: "*I like to make plans to help the learners understand a new topic, but sometimes I just don't have a plan and there is really nobody that I know who would have time to help me."* For both teachers the question is no longer; what conceptual teaching is, but **how** one teaches conceptually, specifically for abstract topics like solving two simultaneous equations. Davin provided evidence of the need for more guidance when she said: "*Whenever I am uncertain about any topic, I divert to the methods in the prescribed textbook. I just copy their method of explaining onto the board."*

Another aspect of the syllabus that came under discussion and that provided data to understand why each teacher adopted her own approach to teaching Mathematics was the spiralling way in which the different topics are handled by the Junior Certificate Syllabus. Most topics are introduced in grade 8 and repeated in grade 9 and 10, in a spiralling manner. This means that for every next grade an increase in content and a higher level of difficulty is expected. During the interviews, Davin pointed out that the syllabus in itself had serious consequences for her method of teaching. She explained that two aspects regarding the syllabus played a role in her decision to teach procedurally. She said, *"I have no problem with the syllabus, it is a good syllabus, but it is full, very full: too full to complete the whole syllabus for every grade in one academic year."*

The grade-ten learners write an external examination which means that the syllabus has to be completed before the examination commences, usually already by the beginning of October every year. Davin explained that she found that she made the best progress through the syllabus by using her own "*method-based*" approach. "*I just have to teach the learners how* *to do the sums. I don't have time to spend on enrichment or group work or to let learners work in pairs*."

This view was partly supported by Botes when she explained that a teacher sometimes "inherits" a class group that was taught by another teacher for grade 8 and 9 and then finds that certain topics were not covered during grade 8 and 9. This could be that the previous teacher knew that the topics would be covered again in grade ten or merely due to a lack of time. This placed her as teacher in a position where extra time had to be spent on extra classes to bring those learners on par with the rest of the grade-ten learners. This, thus, forced her to adopt a more procedural approach in this situation.

4.2.2.2 Work Schedules and lesson planning of teachers

At Windhoek Gymnasium every teacher has a Scheme of Work file. In these files, it is expected that they do term planning with target dates for the completion of every syllabus topic. Lesson periods are allocated to every topic and the ideal situation is that some time is allocated at the end of every term to revisit some topics in order to revise the work with the learners. It is also expected from every teacher that brief lesson plans for every day are done. These lesson plans consist of stating the topic, the lesson content, the planned approach to be followed, objectives to be reached and details of the homework set.

Little supportive data in terms of teaching approaches was found in these lesson plans as they are written in a very cryptic manner, but for some lessons Botes attached material that she used to introduce new topics to learners. This material included tasks like the following:

Example 1.

"*Hand out graph paper and let the learners draw two straight lines on the paper, namely:*

$y = 2x + 5$ *and* $3x = 2 - y$.

Ask them to see what they observe.

Let them read the intersection point from the graph.

Ask the learners to try to devise an algebraic method to find the coordinates of the point where the two lines cross.

Introduce the solving of simultaneous linear equations."

Example 2.

"*Hand out envelopes with variety of cut out polygons.*"

Let every group measure the interior angles and calculate the sum of interior angles.

Allow learners to discover that Sum of interior angles = (n-2) X 180^0."

Most of the learner activity examples attached to Botes's lesson plans contained convincing evidence of a conceptual approach to teaching Mathematics (Compare with Grid 3b, 7b, 9b, and 12b). Despite her previous claim that she constantly applies conceptual methods when teaching grade-ten Mathematics, she did say that at times certain constraints are preventing her from being persistently conceptual. "*I agree with Salomè that the time constraints we have to complete syllabus force us to rush through certain topics and then I cannot be my creative self.*"

A matter that transpired from the syllabus discussion was the volume of work to be covered per annum as prescribed by the syllabus. Both teachers felt that the large volumes of work to be covered forced them to revert to a more procedural approach just to complete the syllabus in the prescribed period of time. Support for this notion was found by the fact that none of the two teachers allowed any time for revision or for any enrichment programmes. This was clearly reflected in the daily lesson planning files of both teachers. It also transpired that a minimal number of lesson periods are available to complete a topic. Nowhere in the planning could any evidence be found that time is allocated to different approaches, group work sessions or discovery learning. In the lesson plans provided by Davin she consistently wrote under *methodology;* "*Show them the steps to…*" This supports her claim that she uses a procedural approach when teaching Mathematics.

Botes saved all her lesson plans electronically on the interactive Smart Board. Over a period of time, she developed a bank of interactive lessons that allowed her to let learners work on the board and do some discovery learning for themselves. These also included activities where they interacted with questions posed on the Smart Board that required reasoning, arriving at their own conclusions and proposing different solutions for mathematical problems. Each lesson plan included a mix of procedural and conceptual elements. Several actions for learners clearly required procedural skills whereas others required independent work of a conceptual nature. A very good example of Botes' Smart Board lessons with a mixed approach was a lesson about similarity. The board presents a randomly scattered collection of different triangles. By moving the different triangles over the surface of the board the learners had to fit all the triangles that are similar into each other. On the successful completion of this task, the board subsequently presented the learner with different requirements for triangles to be similar. Through a process of elimination, the learner is left with the final conclusion that if corresponding angles are equal, it follows that the corresponding sides are in the same ratio.

4.2.2.3 Participants' responses to the prescribed textbooks

For the purpose of this study, I do not wish to evaluate textbooks, but during the interviews and video recordings references ware made to the prescribed textbooks available to the teachers and the learners. As discussed in the literature review, every textbook is written for a specific audience. School Mathematics textbooks are usually written with the learners in mind. All the approved textbooks, covering the Namibian JSE Mathematics syllabus, focus primarily on the learner. They do not contain any guidelines for teachers. Each learner at the school receives the approved textbooks free of charge. Each of the participants was requested to indicate which text book she prefers to use and both indicated the same text book. When requested to motivate their choices, both regarded the chosen book as "*being the best one, because it explains the work better than the other ones.*" On inspection the preferred text book appeared to follow a similar pattern as the other grade-10 Mathematics text books, namely stating a new topic, followed by the objectives as outlined by the syllabus. Then the governing rules for the topic are placed in a coloured box, followed directly by a few examples of problems based on the topic and finally the topic is rounded off by a series of exercises progressing from easy to an advanced level. Only in a few places in the book were investigations and project ideas placed to stimulate further investigations by learners. After viewing the recorded lessons, the topics covered during the lessons were located in each of the textbooks and reviewed.

Both teachers also make use of another textbook, written purely for revision purposes. It is mainly procedural in its approach. At the beginning of every new topic, the rules governing that specific topic are stated, then a step-by-step algorithm is stated for the solving of similar problems and numerous exercises regarding the topic are given. It is assumed that the textbook was written purely for revision purposes and this is supported by the title of the book, supporting the idea that the book is written for learners to "*pass Mathematics*".

The third textbook used by the participants is more elaborate and includes more examples about the recorded lessons, but it was soon clear that although effort was made to relate to the learner's world the layout remained very procedural. Different colours are used to highlight important sectors and all examples are coded in one colour while another colour is used to highlight sections that should be memorized by the learners. This can be seen as an effort to be conceptual, but as was underlined by Botes, "*The textbook is the only tool the learners have to assist them when preparing for the examinations.*"

Both participants covered the topic; *Solving of two simultaneous linear equations.* In terms of methodology all the textbooks under discussion suggested two methods, firstly by isolating one unknown and then substituting it into the second equation and secondly the graphical solving of the two equations, by reading off the intersection of the two lines from a graph.

Only the suggested methods of the textbooks were used by the participants in the presenting of their lessons on the specific topic. Neither the syllabus nor any Namibian text book mentioned any other methods for the solving of two simultaneous linear equations. It can be said that the methods used are adequate for the purposes of the Namibian syllabus, but other methods for the solving of two simultaneous linear equations are also available for the purposes of explaining, for example, the method of *addition of the two equations to eliminate one variable* or *Cramer's rule, using determinants* to solve the two equations.

Authors of textbooks only have limited space available to give examples of alternative problems and alternative problem-solving strategies. Further inquiry into both textbooks revealed a repetitive pattern of providing rules, examples and subsequent problems based on the examples. Every time the problems to solve are structured from very simple to the most challenging. A learner receiving homework exercises based on any of the books will know immediately that the first problem will be on an elementary level and the last one should be difficult to solve.

Davin used very similar examples to those used in the textbook to explain the lesson content to her learners. In the follow-up discussion on her lessons she made very revealing remarks regarding the role of textbooks in her teaching practice: "*I chose the examples from the text book, specifically because I know that the learners have to study from the text book. I know that when they do their homework they will have exactly the example that I used in the class room to explain the work to them."* She further explained that the learners can, in some way, relate the lesson in the class to the text book and will have a second opportunity to revisit the class room example (Compare with Grid 7a and 10a)

Further, Davin made a very important and revealing remark regarding her teaching practise: "*You must remember that textbooks were the only assistance I got when I started teaching Mathematics. I got my knowledge from textbooks and from nobody else."* The guidelines followed by textbooks often act as the templates used by inexperienced teachers to structure their practice. She supported it by saying; "*What did I know? I had to learn the Mathematics myself and then I had to teach it to the learners. Many people supervised me, but only to ensure that I produce results and never, but never did anybody say anything about the teaching methods I used."*

The remarks by Botes were in contrast to Davin's views. She referred back to her studies and the opportunity she had to do the methodology of Mathematics teaching. "*I am now thankful for the opportunity I had to do the didactics of Mathematics. I think when you are young you just use the methods you were taught at university. Maybe even without thinking about other possible methods."*

4.2.3 Semi-Structured Interview Responses

It was arranged with both participants to conduct the interview at an agreed time and place where both were relaxed and had no other obligations. Both teachers were very co-operative and eager to look introspectively at their own teaching practices. The two teachers work very closely together and are thus very relaxed in each other's presence.

Before the interview, I asked them whether they wanted to be interviewed individually or together. They both preferred that the other colleague be present and both expressed the expectation that they would like to *learn from the other colleague.* Initially this eagerness was not seen as significant but as the process of discussing the interviews unfolded. It became clear that the general lack of guidance and peer learning amongst colleagues is an important contributing factor why Mathematics teachers often do not think about their practice and remain stagnant in their teaching. It is important that teachers support, challenge and learn from each other. Davin summarized her experience after the interview as positive and said: "*I really enjoyed talking about our teaching.*"

The main purpose of the interview was, firstly to establish a picture of each teacher's story and how it came about that each one had her own paradigm and method of teaching, and secondly, why they so outspokenly contributed their respective successes to these different paradigms and methods of teaching.

4.2.3.1 Salome Davin

Davin studied at the local university but never did any Mathematics, except for Statistical methods in her first year. This, for her, was enough to convince her that she has a love for Mathematics; "*I was always interested in Mathematics and took Statistics in my first year and it changed my life.*" Initially she taught different subjects, such as history until a serious lack of Mathematics teachers created an opportunity for her to teach Mathematics for grade seven. It is significant to note that she did not have any didactical background in Mathematics education, nor did she have any in-depth content knowledge of the subject. By teaching Mathematics for a few years at grade-seven level, she said that she gained the required content knowledge and confidence to allow the school to let her progress with the subject to higher grades and eventually to teaching grade 12. She explained: "*I taught myself ... I just taught myself. Every year I took it one step up and eventually later on I ended with Grade 12 Mathematics. But Grade 10 remains a pleasure to teach. It is a very interesting year – developing year for the child.*"

She asserts that a lesson can be learnt from her gradual progress through the grades. Principals could make use of this model to train unqualified Mathematics teachers and thus address the serious shortage of Mathematics teachers. Her own experience is an example of how knowledge and confidence to teach Mathematics can be accumulated by gradually moving up the grades. Her own witness underlines how someone can develop into an outstanding Mathematics teacher: "*I still taught history for a few years. ... Now I am unbelievably passionate about teaching Mathematics. I love it, love it, love it!*"

She explained that she never received any didactical directions from any colleague or official courses she attended. "*I am now teaching for 22 years and up to today nobody told me there is something different about teaching Mathematics.*" After making the revealing remark, she interrupted herself and addressed Botes: "*You know Adri, you are my neighbour and every single day we stand together outside our classes, talking to each other, and ask each other* ***what*** *we are going to teach the next period, but* **never** *did we ask each other* **how** *we are going to teach it*"

Both teachers were outspokenly critical about official workshops and courses. Financial constraints are probably the main reason why official courses are attended by too large groups of Mathematics teachers and why no time is allocated to didactical matters. Vague general guidelines are given while the focus of all the workshops remained subject content and assessment.

Davin said that meetings were often held and all her colleagues willingly shared subject matter issues with her, but, never discussed teaching approaches for Mathematics with her. During the focus interview, she explained that all the official workshops that she attended also focused only on content knowledge rather than on teaching practices. Davin's data showed that her preferred method of teaching Mathematics was the product of numerous contributing factors. She literally grew into a teacher with a procedural approach. During the final structured interview with Davin, she reflected on the whole research process and made the remark: "*No, no, don't thank me. I have to thank you. For the first time I had the opportunity to rethink my own teaching of Mathematics. I always believed that I just chose to teach in a methodological manner, but the more we talked about it, I recognised how much I grew into the teacher I am today.*"

As progress was made with the collecting of data Davin progressively identified several factors that contributed to her adopting a procedural approach to teaching Mathematics namely:

a) **The time constraints placed on her**. In her own words, "*the syllabus is full, full, too full.*" Teachers have to prepare learners for an external examination which requires that **all topics** in the syllabus must be examined. There is no opportunity for any shortcuts and no possibility not to cover the complete syllabus. It needs to be mentioned that over the past few years, the ministry of education, under severe public pressure to release results earlier, pushed the commencement dates for the grade-ten examinations forward in order to allow enough time for marking and mark processing. Davin omits no work from the syllabus.

Every year, despite excellent planning, a number of school days are lost due to unforeseen reasons. These could be illness of a teacher, official visits to the schools stretching into teaching time, large numbers of learners in a class group representing regional or national teams on sport tours, dignitaries giving children *"a day off,"* to mention but a few. To overcome this and stay on track, Davin says that the only option remaining is to teach procedurally.

b) **The limited formal training in the didactical approaches of Mathematics teaching.** As discussed earlier and mentioned by Davin, no template for teaching Mathematics other than her own ideas of teaching and the text book's procedural approach was available.

c) **Weaker learners relating better to a procedural approach**. According to Davin, learners relate well to being taught procedures and produce better results through this approach. *"I think one reason why I teach this way is because the struggling learners show better progress with this method. Maybe, because they can write the method down and learn it …. and then they can do the sums in the examination."*

d) **No knowledge and experience of teaching philosophy.** Davin was never confronted with any of the different underlying philosophies of teaching Mathematics. In her own words, "*in 22 years of teaching did it never appear to me that Mathematics should be taught differently from history."*

e) **Limited interaction with other teachers.** Davin said that never in her whole teaching career did colleagues ever discuss any didactical issues with her. She confirmed this statement when she explained during the focus interview that often between classes the Mathematics teachers would discuss *what* they are going to do the next period, but never discussed *how* they are going to do it.

During the interview Davin repeatedly said, "*I am very much focussed on the methods.*" She repeated three times during one interview that "method" is for her the most important aspect of her teaching Mathematics. She implied here the mathematical method of teaching. Later on during the interview, she described the typical formats that characterise her lessons as:

- the division of the content into learnable facts;
- writing down the steps that learners must memorize to solve problems;
- the application of the rules and
- repeatedly doing exercises.

She highlighted this pattern again during her focus interview and it was also apparent from the video recordings that were made of her lessons.

Although she says that her approach is procedural, there is evidence that it is interspersed by conceptual dimensions. For example, during the interview she said, "*It is for me important that he [the learner] can show the insight how he arrived at the answer*". This can be interpreted as a conceptual feature. On further probing, however, she explained that this insight is restricted to the required steps that the learner is expected to follow correctly to reach a solution. She illustrated this by saying, "*If you have the answer wrong – one can go back and then you should be able to show me the method. By doing so you can earn marks without having the answer right.*" Later, during the interview, she said that many children have the ability and insight to simply analyse a sum to get to an answer.

With probing and during the focus interview, it transpired that Davin makes a clear distinction between **her** method of teaching and the expected outcome for the learners as articulated in the policy documents. "*As I have said, I am a method-based teacher.'* During an informal interview after her recorded classes, she made it clear that her method of teaching is purely procedural. She emphasises that she teaches learners how to *do* the sums. Her method of teaching can, however, also lead the learner into a process of adaptive reasoning. She explained that the approach is aimed at the more talented ones. In her own words: "*Yes, sometimes I will ask a learner to suggest how he will solve a problem. I will allow the learner to guide me through the problem while I write the solution down."* This is consistent with a more conceptual approach to understanding of Mathematics. If the learners are not willing to be led, she says, they will still possess the ability to mechanically complete a problem using algorithmic procedures. This was confirmed when she replied to a question that, "*the methods opened something up for him. By the way, what is understanding*? *If the child can* **do** *the sum correctly, who am I to say he doesn't understand?"*

It emerged in all the interviews with both teachers that they believed that the weaker learners will gain more by a procedural than a conceptual approach. Both referred to the ability of certain learners to grasp a concept immediately, while other learners need numerous efforts to solve a problem. These will invariably revert to a method of applying an algorithm to solve a problem. Despite the intentions of a learner-centred policy, the Namibian Education System, in my view, still endorses a teacher-centred philosophy as it places so much emphasises on the results of external examinations. This has significant implications on a teacher with a purely conceptual approach, particularly when she prepares learners for an external examination. This is why it is important to also look at the case of Botes.

4.2.3.2 Adri Botes

To substantiate the claim by Adri Botes that she is purely conceptual in her approach proved to be more difficult than anticipated. With reference to the characteristics set out in table 4.1, she defined conceptual teaching more as a skill to lead learners to an in-depth understanding of mathematical subject matter as well as an ability to transform learners into people with adaptive reasoning abilities. In her own words, she described her teaching as, "*always having a magic plan for every new concept or idea I have to teach. I would like to describe my 'teaching practise', as you call it as* ***teaching with a*** *plan so that learners can understand the work, rather than being able to* **do** *the sums."*

Botes studied for a commercial degree at a South African university. Her degree contained Mathematics and a variety of commercial subjects. She did the post graduate Higher Education Diploma and one of her methodology subjects was Mathematics. It transpired during the focus interview that when she studied, the notion of conceptual or procedural teaching approaches was never explicitly touched on. Instead, emphasis was placed on *learning with understanding and re-discovery or reconstructive methods* of learning. This implied, that although never formally defined in the same terms as in this study, to her the process of teaching for conceptual understanding and procedural fluency (Kilpatrick, 2001) was familiar. During the focus group interview, she remarked that as a student she was never critical about her teaching methodology. Practice taught her that the methodology training she received made sense, and was adequate. She explained: "*When learners tell you after a lesson that they 'understand nothing,' you realize that you will have to revert to another* *approach."* From the outset she started to teach for "*understanding*", "*I like it when a child can say, "AHA, I understand".*"

During the interview Botes substantiated her claim of being conceptual in her teaching several times. For example, she says that learners can mechanically simplify a fraction with the press of a button, but to her it is important that the learner should **understand** the concepts involved in the simplifying fractions with the calculator. "*You know, the new calculators have so many functions that it takes only the press of one or two buttons to simplify a fraction.* The calculator can easily be used to solve several problems in the grade-ten syllabus by mechanically applying a series of algorithms. To her, however, the learner needs to understand the process in order to solve the problem independently from the calculator.

When asked to explain the process involved when introducing new learning material to her learners, she explained that one of the reasons why Mathematics remains her favourite subject was because of the many options available in teaching the subject. She uses several approaches to introduce new concepts to learners; one that works very well is using a little suspense in the process. She leads the learners to comprehension by constantly probing them to suggest their own possible solutions to a problem presented. She explained: "*I can give you an example of how I use my techniques with learners. When I introduce the solving of linear equations to learners I like to play the old little game with them. Tell the class you can read their minds. Let everybody in the class think of a number between 1 and 10. Then I ask them to multiply the number with 2 in their minds. Then add 3 to the answer. Multiply this answer again with 4. Then by asking a few their answers I can give each one the number he or she selected. It amazes them, but all I did was solving the equation y = 4(2x + 3) in my mind. They provide me with the value of y, and of course through simple substitution, I could produce the value of x. Then I ask them to provide me with possible solutions and yes many understand immediately what's going on and are able to very quickly come up with equation."*

It was found that many times during the interview Botes used the phrase; "*they should understand.*" When probed to elaborate on the term understanding, she explained that the learners should "*click*", "*that the lights should go on for them*" or that they should experience a moment where they can with confidence attack any problem based on the concept covered.

This, to me, illustrates a strong conceptual approach to teaching (Compare Grid 3b, 7b and 9b).

She made some challenging remarks by saying that it is extremely important that the learners should know the theory, but that they should also memorize. Initially this was regarded as being inconsistent with a conceptual approach, but seen in the context of Botes' teaching, it is part and parcel of teaching for understanding. Mathematics is grounded in many rules and axioms that learners simply need to study, know and memorise. Her explanations placed the comments in context; "*Take for example circle geometry. I will let them measure the angles in a few semi-circles and they will discover that the angle in a semi-circle is always a right angle. When they start to solve problems where they have to supply reasons for an answer, they just cannot remember that the angle in a semi-circle is 90^0. After we are done with all these rules I will type them out for them, each one with a picture and then they must* ***memorize*** *them.* It is true that Mathematics is governed by many rules and I support her view that there comes a time when a learner has to support an argument with these rules and then these rules should be available to prove a claim, as is the matter in geometry.

It is of significance to note that she mentioned that slower learners often force her to divert to a more procedural approach to allow them to at least have a mechanical knowledge of certain topics in the syllabus. During the focus group interview she supported her colleague and said: "*I support Salomè. I also found it better to explain certain topics step by step to the slower learner. After you tried to explain for two or more times to certain learners, you just have to write down the steps and tell them to learn it.*" Botes, like Davin, believes that in a class comprising mixed abilities learners, it is extremely important that the teacher develops skills to know when to apply procedural methods to keep the slower learners on the same level as the rest of the class: "*Remember I have to complete every topic within a certain period of time and you have to make sure everybody is … as we say in Afrikaans, 'Almal bly by'.*" Both teachers supported each other when Botes remarked that from experience she found that certain learners **prefer** to be taught in a procedural way: "*You know I think some slower learners prefer to have a recipe written down for them, maybe it make them feel safe if they have something that they can study.*"

Botes claims that it is very difficult for a teacher to sustain the momentum to constantly teach conceptually. Although a learner-centred approach requires that a learner should be allowed to learn at his or her own pace, It is at times not feasible to sustain the same level of motivation and interest if the learners are at different stages of progress with mastering the learning material. Another aspect that plays a role why teachers divert to, as she called it, "*short cuts*" is the fact that faster learners become impatient when the teacher constantly has to switch methods to lead slower learners to understanding. This is particularly true when they are intrigued by a new topic and are eager to tackle new challenges posed by the topic. Botes made an important observation: "*You know you have to be very careful when teaching learners on different levels in the same class. When you start with new work, the fast learners are very much interested, but you have to stop and go back to help the slower ones to catch up. The fast ones can even become irritated with the slower ones, but on the other hand, when you lost the slower ones – they will never catch up again.*"

With enthusiasm and through several concrete examples Botes is convinced that her approach to teaching Mathematics is characterized by a mostly conceptual orientation and that she consistently applies these methods when teaching grade-ten Mathematics. During probing to establish how consistently Botes applied conceptual methods when teaching Mathematics, she was asked how she would introduce Trigonometry to her learners. Her answer was revealing and very supportive of her approach: " *Ah!, I like that one. You know every boy wants to become a pilot. So I have this little problem for them. 'An aircraft should approach the runway at exactly* $3^{0.}$ O*n its way to the runway, it flies over certain radio beacons at specific distances from the runway and of course the pilot can read his altitude from his instruments inside the aircraft. So, if the aircraft is pointing its nose towards the runway when it is still 2 km away from the runway and at a height of 6 km, the pilot knows the approach is correct at exactly* 3^0*. How do I know that? The answer is simple if you know trigonometry; tan x =* $\frac{6}{2}$ *=* 3^0*. I promise you after one or two challenges, they work it out and never forget that the tangent of an angle in a right-angled triangle is the ratio, tan A* $= \frac{\textbf{opposite side}}{\textbf{adjacent side}}$*. Over time I collected these strategies, to help to teach them for understanding.*" (Compare with Grid 5b, 7b, 8b, 12b, 15b).

One interesting challenge that she mentioned is the amount of creativity it takes from a teacher to remain conceptual when teaching Mathematics: "*You must know that it was never easy to just teach for understanding and to use techniques to stimulate the learners to think about problems. These tricks I mentioned to you have I collected over many years of* *teaching. Sometimes a plan works and sometimes not, then you have to be ready with another plan. The* **same** *technique might work for one class and not at all for just the next class. I am constantly making plans for lessons and sometimes there is just nothing. "*

From the evidence that emerged from the interviews and interactions with Botes, it is convincing that she applies a conceptual approach on a sustained basis when teaching grad-ten Mathematics. The data reveals that she reverts to procedural methods at times, mostly as she explained herself, to bring slower learners in line with the rest of the class. It appears that slower learners often relate better to a procedural approach. The same reason for this is contributed by both participants to the fact that it provides the slower learner with some *assurance* that they have some *tools* to achieve the required results when assessed. They can, through rote learning apply an algorithm that will lead to the desired outcome.

4.2.4 Summary of the interviews

4.2.4.1 Open-ended combined interview

This interview was conducted at the beginning of the research process to construct an initial understanding of the two participants' own chosen methods of teaching. The interviews allowed them to provide verbal evidence of their teaching practice. They also enabled me to create a profile of each participating teacher. Both teachers were present at this interview and it paved the way for an excellent working relationship. It also facilitated a process whereby the participants were able to confide in me.

The findings of the combined interview can be summarized as follows:

- mutual agreement was reached on the understanding of conceptual and procedural teaching;
- both teachers confirmed their chosen teaching approach;
- information on training and teaching careers of participants was collected;
- illustrative examples were collected from the participants;
- participants provided their views on documents studied;
- supporting evidence from lesson plans were investigated and
- time constraints in completion of the syllabus were identified.

4.2.4.2 Structured interviews with individual teachers

Initially it was planned to have one structured interview with the participants but during the initial data processing, it transpired that further interviews would be necessary. An example of an emerging matter was the role textbooks played in their teaching practices. It was not envisaged that both teachers would refer to the significance that textbooks played in their daily lessons. Another structured interview was conducted with each teacher to collect concrete examples of *how* they would handle different topics in Mathematics. This was done especially for triangulation purposes, to establish each one's objectives when teaching Mathematics and to formally conclude the research.

The findings of the structured interviews can be summarized as follows:

- teachers confirmed their teaching approaches with concrete examples;
- examples from different topics were collected;
- the role text books play in their teaching practice was established and
- objectives of every teacher when teaching Mathematics were established.

4.2.4.3 Informal interviews

The research process provided the two participants with the opportunity to reflect on their own teaching, and both participants expressed the wish that we have additional informal discussions. Sometimes they wanted to elaborate on a point raised during an interview or to motivate a certain viewpoint. For record-keeping purposes it was decided to also record these informal discussions and interviews, or to provide the answers in an electronic format to the researcher. This was done by sending an e-mail with questions and written answers to the researcher.

The findings of the informal interviews can be summarized as follows:

- further evidence of each one's chosen teaching approach;
- need of each one for academic discussions on a wider platform;
- peer learning opportunities for colleagues and
- illustrative examples of challenges faced by participants.

4.2.5 Classroom observations

An excellent opportunity to collect data and to gain an understanding of the teaching practice of a teacher is through classroom observation. It provides an objective window to observe the teacher in action applying the proclaimed teaching approach. It was agreed that two lessons of each teacher would be video recorded during the normal course of the school term. To ensure that the setting remained as natural as possible, we decided that the recording would be done by a photographer from outside. The only requirement set for the two lessons was that the first one should be an introduction of a new topic or sub-topic, and that the second one should be a follow-up lesson a few days later. It was argued that this would provide the widest spectrum of data possible within the limited sample. Both teachers agreed that they would be ready for the recordings at a time that suited both parties. I regarded these steps taken as adequate to ensure the validity of the data.

Initially the videos were screened in the presence of both teachers and despite the undertaking that the purpose of the whole research process was not to compare the two teachers or to evaluate any of their actions, the screening session was not successful, and had to be abandoned. When they saw themselves in action for the first time, both teachers were very critical of their own actions and became very defensive of their actions. It was then decided that the video material would be watched with each teacher individually. Each of the teachers was also provided with copies of their own lessons. Appointments were made to analyse the videos with the individual teachers as per the grid set out in table 4.1. This proved to be very valuable and provided concrete data about each teacher's approach.

4.2.5.1 Salome Davin

With the first recorded lesson of Davin the topic, '*Simultaneous Equations'* was already written on the Smart Board when the grade-10 class entered the classroom. The first part of the lesson consisted of revision of the straight line. She reminded them of the direction of a line $y = mx + c$ where the value of the gradient m is positive, negative, zero and undefined. Then Davin announced: "*Today we are going to use the straight line in another topic namely to solve two simultaneous equations."* This was followed by a summary of the steps to draw a straight line, and a reminder about the dependent and the independent variables.

It appeared that the learners had some prior knowledge about simultaneous equations by solving them through elimination. (She wrote the word on the Smart Board). As a next step in the lesson, the teacher wrote down the *rules for simultaneous equations.* There must be *two unknown values and two equations.* Two linear equations were given as examples and Davin started to complete a grid to find the dependent variable through substitution. The learners were requested to assist with the substitution process. The teacher drew the graphs of the two straight lines on the Smart Board and then requested the learners to point out: "*Where can I find the solution to the two equations on this graph?"* A suggestion was made by a learner that it could only be at the intersection of the two lines. The example was completed by the teacher and another example was explained to the learners. Thereafter, the teacher mentioned exceptions to the learners where a solution could not be found. Finally, she revisited the rules to apply and gave homework tasks.

On a constant basis, the learners provided some "*filling-in answers".* I assumed that this was part of a familiar routine to them and they knew that this is the way learning takes place in her classroom. Questions were asked constantly on the basis that learners could "assist" the teacher to complete some steps for solving the example on the Smart Board. If, however, the answers were not provided immediately by the learners, the teacher would complete the phrase herself. With reference to the definition for teaching procedurally most of the lesson was conducted in a procedural manner, except for the moment when one learner had the opportunity to suggest that the intersection point of the two lines could be the solution to the posed problem. In terms of the grid in table 4.1 the following elements of procedural teaching were clearly visible: 3a, 6a, 7a, 9a, and 10a. None of the elements in the conceptual column (b column) was visible.

The second recorded lesson, which followed a few days later, was a continuation of the same topic namely, simultaneous equations. For the second lesson, the class progressed to the algebraic solution of simultaneous equations. Davin announced the topic of her second lesson with the words: "*We are going to do the* **solving of simultaneous equations**. *Let us summarize, there are three ways that you can deal with simultaneous equations…. Firstly…."*

In the classroom all the benches were placed in a 5 x 5 matrix format and the teacher spent the whole period in front of the class. Communication was very much one-way and learners were introduced step by step to what the lesson and the covering of the rest of the topic would entail. She referred to their prior knowledge by using the example of two lines crossing each other in one point. That point is the solution to two simultaneous linear equations. Right from the start of the lesson, she posed several questions to the learners. Questioning took place in two forms. Firstly, and that was the case with most questions, the learners just had to complete a sentence or fill in a single word into a phrase. For example:" *What is the purpose of substitution? You have to get?………, yes* ***one*** *variable alone."* A second example of questioning: "*I will always give it in the form of a……"* and whole class answered: "*Word problem."*

The next step of the lesson was to place a problem on the Smart Board: "*JP sells 27 concert tickets for the school and receives a total of N$ 388.00. If concert tickets cost N$ 20.00 for adults and N$ 12.00 for children. How many adults and how many children bought tickets from JP."* The next step of the lessons was **very** significant. Davin immediately started by saying: *"Now you have to remember that when you want to solve two simultaneous equations you must always remember that there are two variables. You will not be told that the problem is a simultaneous equation problem. You must look for two variables from the word problem."* So far in the lesson there were no expectations from the learner's side to suggest any solutions or participate.

In a next step, the teacher posed the sentence. '*JP sells 27 concert tickets'* in brackets and asked the class to produce an equation with two variables; and then she wrote the equation $x + y = 27$ onto the Smart Board. She told the class that one can make another equation by looking at money matters and then she wrote the equation $20x + 12y = 388$ onto the Smart Board. She allowed the class to lead her step by step through the solving of the two equations, while she wrote each step onto the Smart Board. To conclude the discussed example, the teacher stipulated a few rules that the learners should adhere to. Firstly, to identify two variables, they should remain with *x and y* and not use other letters and finally they must remember to test their answers by substituting the values back into the equations. This will allow them to test their answers.

A second example was then given using a rectangular figure, and the variables to be solved were the length and the breadth of the figure. Exactly the same pattern as the first example was followed. The teacher remained in front of the class and led them step by step through the process to create the two equations. There was constant interaction between the teacher and the learners, but the teacher constantly disclosed every step to follow and the learners assisted by trying to complete a suggested step. Davin interacted constantly with the learners in the class by naming individuals to help her to complete a step. Humour played an important role during the lesson and although the teacher acted from the front of the class, she made remarks like: "*John so far I have not heard a single word from you.*" She was the only one in front of the Smart Board and neatly wrote down the solution to the problem on the Smart Board. The lesson was concluded by giving hints to the learners how this topic will be assessed.

Finally Salomè Davin referred the learners to the textbook and gave them the exact page number from where she took the examples down on the Smart Board. She subsequently made the remark: "*For those of you who didn't follow me, I used the example on page 179. You are more than welcome to go through that example.*" Homework was given from the textbook and the learners were allowed to commence with their homework.

In both recorded lessons, Davin followed a fixed pattern. She explicitly *explained the learning* material to the learners. She did not leave any room for the learners to individually interrogate the material. In none of the lessons did any learner put up a hand to ask a question. All the information was provided. Problems to solve were placed on the board and the teacher solved them step by step for the class, constantly reminding them of each action needed to reach the solution. The teacher was very prescriptive and steered the class by her predetermined set of actions. She followed a clear template for both lessons observed.

As mentioned above, the only action on the side of the learners occurred when they were asked to answer a direct question. They put up one hand and then were asked by the teacher to respond. The atmosphere in the classroom was formal, yet relaxed. This could be observed by the manner in which some learners laid back in their chairs and observed the lesson in front. Many of their responses were tangibly positive with smiling faces. The rapport between the teacher and the learners appeared to be amicable. Through her actions, the teacher kept the class together and ensured that every learner was on board for the entire lesson period. This goal she reached by the large amount of short-answer questions that she constantly asked.

From the actions of the learners it could be seen that they felt *safe* with this approach. During one lesson, a learner was requested to answer a question and despite the high tempo at which they had to produce the answer, he had the confidence to tell the teacher, "*Wait just wait, I will get the answer now.*"

When homework tasks were given, everybody immediately proceeded with the tasks. The homework tasks were set to reinforce and apply what was learnt in class. The learners had the opportunity in the class to observe the method of solving similar problems. They were provided with guidelines and steps to go home and apply them.

4.2.5.2 Adri Botes

Botes used the Smart Board to introduce her lesson to the class. She asked some questions related to the solving of linear equations and used the equation of the straight line namely $y = mx + c$ to introduce the topic. Through questioning, she revisited the incline of a straight line, firstly with a positive gradient and secondly with a negative gradient. The class was then asked to suggest how the equations for two lines that crossed in one point would look. Finally, the topic of the intended lesson emerged by itself, namely the solving of two simultaneous linear equations. The approach of moving from the pre-knowledge the children had, to a new concept fitted all requirements for conceptual teaching as set out in the derived definition stated at the beginning of this chapter.

Although the arrangement of the class was also a 5 X 5 matrix, the atmosphere was more informal than in Davin's class. Questions were posed and before the teacher appointed somebody to answer, someone else from the class provided the answer. Immediately after the lesson commenced a learner took the opportunity to ask the teacher a question. The questioning by the teacher was often open-ended and included questions like: "*Can you* ***explain*** *to me what simultaneous means?*" and "*Where do* **you think** *will we find the solution?*"

During the second phase of the lesson the learners were handed an exercise with the equations of two lines and were requested to design a method to find out when the two lines will cross equal points within the Cartesian plane. After a while and before any solutions were put on the table, the teacher intervened and started to complete the exercise on the board in a procedural manner.

During the discussion of the video, Botes pointed out that the lesson had to be completed within a set period of time and in order to progress to the intended outcome planned for the lesson, she had to deviate to a procedural approach. She argued: "*I wanted to complete the lesson by allowing the learners to suggest the solution, as I always do, but you never know how long it will take before a workable suggestion, if any, comes from the class.*" This is in line with the argument by both participants that they constantly operate under immense time pressure to stay on track with the syllabus.

As the lesson progressed, the teacher used another example to lead the learners to understand how to solve simultaneous linear equations. She started to hand out sheets of paper with two linear equations on it, namely $y = -x - 2$ and $y = 3x - 6$. She then gave the command: "*Now the next one I want you to do on your own.*" Before the learners attempted the problem, she gave a few hints: "*Remember this is a negative gradient and this one has a positive gradient. So they will cross each other. Now start the problem.*" The learners were allowed to complete the problem and finally the teacher did the example for the learners on the Smart Board. During the screening of the recorded lesson, Botes argued that she would have preferred that a learner would do the problem on the Smart Board, but she wanted to conclude her lesson and hand out homework exercises to the learners. Homework was given and a selection of problems was given from the exercises in the text book. When the norms identified in the grid were applied, the following elements could clearly be identified. Firstly, in terms of conceptual teaching; 1b, 3b, 7d, 8b, 12b, 16b; secondly, in terms of procedural teaching the following elements were present, 7a and 12 a.

The second lesson was recorded a few days after the first lesson. The same topic was covered by both teachers, namely the algebraic solving of two simultaneous, linear equations. Botes used exactly the same examples from the text book as Davin did. No comparisons were made between the two teachers, the focus remained on the teaching practice of each teacher.

The video material for the second recorded lesson followed a similar pattern and it became clear that Botes made plans to intrigue the learners, to keep them involved in the development of the lesson and to allow them to creatively suggest ideas towards the finding of workable solutions to problems. She posed the following problem to the learners on the Smart Board: "*The length of a rectangle is three more than the breath and the perimeter of the rectangle is 34 cm. Let the length of the rectangle be equal to x cm and the breadth be equal to y cm.*" In the textbook, the example poses the first question namely: "*Write down two equations for x and y.*" Botes approached the problem by asking the learners to give suggestions how they would approach a word problem. One suggestion came from the class to look at the questions asked. Her response that there are no questions asked yet, prompted another one to propose that one should write down what facts are given. She pointed out to the class that the '*and'* should suggest to them that more than one fact is given. Again she asked the class to give suggestions. The class made several suggestions like: "*Draw a rectangle."* Another person suggested: "*Write down what you know."*

At this stage, she revealed the first question from the exercise namely to write down two equations in terms of x and y. The learners struggled to arrive at an equation. Botes then underlined the word **more** on the Smart Board. One proposed the equation $x = y + 3$. Other learners thought along the lines of calculating the perimeter of the rectangle, and proposed the equation $2x + 2y = 34$. The teacher requested the class to follow her while doing the substitution on the Smart Board. She constantly asked the learners to direct her on **how** to substitute $x = y + 3$ into the second equation.

After solving the two equations, the learners were asked what they would do next. They proposed that the values for x and y should be substituted into the equations to test the correctness of the algebraic manipulation.

A second problem was revealed on the Smart Board and exactly the same approach was followed to arrive at two equations. She was interrupted by a learner who asked her to explain again as he did not understand the steps. By questioning, she led him through the steps again and he confirmed that he understood up to that point.

A third problem was revealed on the Smart Board, unfortunately obscured to the researcher. This time the learners were requested to attempt the problem on their own. While the learners were busy with the problem, Botes moved among them, stopped at a few and gave a few hints to different learners. This was the end of the lesson period and no further homework exercises were given.

The lessons were viewed and discussed with Botes. After the step-by-step analysis of both lessons and taking into consideration the arguments posed by Botes, it can be concluded that she had an overwhelming tendency to teach conceptually. For the procedural actions, the participant provided evidence that it was by choice; firstly to complete the lesson within the pre-scribed lesson period and secondly to stay on track with the year planning.

From the examples gathered during the recorded lessons and the explanations provided during all the interviews, I found that Botes had her own a specific notion of how she defined a conceptual approach to teaching Mathematics. "*I try to teach through a process that includes plans to switch on the lights in the child's brain*". As mentioned earlier, she understands teaching conceptually as being creative in one's approach. It is more than being just learner-centred, but includes strategies and plans that will "*open up*" the Mathematics for the learner. Unfortunately, as was pointed out by both teachers, there is a serious time constraint to complete the prescribed syllabus, especially for grade ten.

4.2.6 Using the lesson analysis grid (page 37) to classify lessons

The lesson analysis grid (page 37) was handed out to both teachers. The second lesson of each teacher was screened again with both teachers. No discussions took place, but the teachers and I marked onto the grid which actions he or she regarded as purely conceptual or purely procedural. The grids were combined into one combined grid. Three ticks next to an action indicate that all three viewers agreed that an action is clearly either procedural or conceptual. The results were overwhelmingly supportive of the notion that Davin teaches predominantly procedurally. For Botes, the results support her claim that she is mainly conceptual when teaching Mathematics. Clear evidence is, however, also apparent that she would often revert to procedural methods in her lessons. See tables 4.3 and 4.4.

CODES USED:

- Viewer awards an *, if convinced that the observable actions/methods were purely procedural or conceptual respectively.
- Viewer awards a ?, if uncertain or if observable actions/methods appear to be conflicting.
- Viewer leaves blank, if there are no apparent observable actions/methods.

Table 4.2. LESSON ANALYSIS GRID: Salomè Davin

Procedurally and Conceptually Orientated Teaching Methods

Procedurally Orientated Teaching		Conceptually Orientated Teaching	
Teaching Practice		**Teaching Practice**	
1a. Explaining how to… [step by step algorithm]	***	1b. Ask "why", asking why not? [Why this step?]	
2a. Teaching definitions and symbols	***	2b. Explore possible definitions, symbols	
3a. Practice steps	***	3b. Design or discover own steps	
4a. Calculator: Teach steps to perform an operation.		4b. Solve a problem, try to use the calculator	
5a. Only one "right" way to solve a problem. [the right answer is everything]	**	5b. Multiple strategies to solve a problem. [the process is more important than the answer]	*
6a. Answer of a problem is final.	***	6b. Answers pose more opportunity for learning.	
7a. Skill teaching is important. Focus on a single skill to arrive at an answer.	***	7b. Connecting ideas and concepts in Mathematics.	
8a. Answers in isolation	*	8b. Relate to the real world	**
9a. Word problems directly based on the required skill.	**	9b. Posing a problem, develop skill through reasoning.	*
10a. Algorithm is everything	***	10b. Algorithm one form of representation a solution.	
11a. Wrong answer is absolute.	*??	11b. Wrong answer provides investigation opportunity into understanding of the problem	
12a. Level one questioning. Typically expecting only an answer on a question.	**	12b. Questioning that requires adaptive reasoning, and the consideration of alternatives	*
13a. Teacher demonstrates. Teacher centered approach. Instructive	***	13b. Teacher facilitates. Learner centered approach. Investigative.	
14a. Problems are only computational. Follow prescribed steps	**	14b. Problems open ended	*
15a. Focus on procedures only.	*?	15b. Focus on concepts to develop procedures.	*
16a. Body language: In front of the class – one way communication. Questioning – only expect the right answers	***	16b. Body language: Move between learners, constantly prompting learners for responses and comments. Leading learners in reasoning.	
17a. Homework control: Mark work right or wrong	???	17b. Homework control: Discuss answers – right and wrong; Add comments like; Why? Or Explain.	

Table 4.3. LESSON ANALYSES GRID FOR Adri Botes.

Procedurally and Conceptually Orientated Teaching Methods

Procedurally Orientated Teaching		Conceptually Orientated Teaching	
Teaching Practice		**Teaching Practice**	
1a. Explaining how to… [step by step algorithm]		1b. Ask "why", asking why not? [why this step?]	***
2a. Teaching definitions and symbols	**	2b. Explore possible definitions, symbols	*
3a. Practice steps	*	3b. Design or discover own steps	**
4a. Calculator: Teach steps to perform an operation.		4b. Solve a problem, try to use the calculator	
5a. Only one "right" way to solve a problem. [the right answer is everything]		5b. Multiple strategies to solve a problem. [the process is more important than the answer]	***
6a. Answer of a problem is final.	*	6b. Answers pose more opportunity for learning.	**
7a. Skill teaching is important. Focus on a single skill to arrive at an answer.	*	7b. Connecting ideas and concepts in Mathematics.	**
8a. Answers in isolation		8b. Relate to the real world	***
9a. Word problems directly based on the required skill.		9b. Posing a problem, develop skill through reasoning.	***
10a. Algorithm is everything		10b. Algorithm one form of representation a solution.	***
11a. Wrong answer is absolute.	**	11b. Wrong answer provides investigation opportunity into understanding of the problem	?
12a. Level one questioning. Typically expecting only an answer on a question.	??	12b. Questioning that requires adaptive reasoning, and the consideration of alternatives.	*
13a. Teacher demonstrates. Teacher centered approach. Instructive		13b. Teacher facilitates. Learner centered approach. Investigative.	***
14a. Problems are only computational. Follow prescribed steps	**	14b. Problems open ended	?
15a. Focus on procedures only.	??	15b. Focus on concepts to develop procedures.	*
16a. Body language: In front of the class – one way communication. Questioning – only expect the right answers		16b. Body language: Move between learners, constantly prompting learners for responses and comments. Leading learners in reasoning.	***
17a. Homework control: Mark work right or wrong		17b. Homework control: Discuss answers – right and wrong; Add comments like; Why? Or Explain.	?**

During the screening process, the researcher and both participants observed the lesson, but did not discuss any aspect of the observed lesson during completion of the grid form. For Davin all three observers awarded stars to mainly procedural actions, while for Botes the most stars were awarded for mainly conceptual actions, but to the same extend as for Davin. The results obtained with both grids are very conclusive that Davin was mainly procedural in her approach to teaching mathematics, while from the grid for Botes was also evident that she often diverted to procedural methods during lessons.

The exercise with the grid convincingly showed that both participants had a good understanding of what it means to teach conceptually or procedurally. After the exercise both regarded the exercise as a learning experience and it provided them the opportunity to objectively look at their own teaching practice.

4.2.7 Summary of observations

The recording of two lessons of each participant allowed for triangulation that the teachers are consistent in their teaching practices as claimed by them. Although unplanned, it was very fortunate that the two lessons of each teacher were related to each other. The one lesson was about graphical solving of two linear equations, and the other one about the algebraic solving of two linear equations. The approach of each teacher typified each of their claims..

That both teachers used exactly the same examples, (they did not discuss this with each other beforehand) could be attributed to their experience as Mathematics teachers and **their ability to identify good learning material.** It was interesting to see how the same learning material can be dealt with by one teacher in a procedural manner and by another in a conceptual manner. During the lesson discussions, both teachers expressed their satisfaction with the outcome of the lessons and both motivated their success by pointing out that a high standard of correctness was achieved by the learners in their homework exercises.

To summarize, Davin approached both lessons in a predominantly procedural manner. She explained the learning material to the learners and stated several times the rules to adhere to when solving problems. Questioning consistently required one-word answers to complete a phrase that she posed to the class. There were no discussions and in none of the two lessons did any learner asked any questions. It was interesting to notice that she stayed the entire lesson period in front of the Smart Board and never moved between the learners. In terms of teaching methodology, both lessons were well-structured in terms of introduction, unfolding of the lesson, summarizing the learning material and the conclusion of the lesson. The atmosphere in the class room could be described as respectful of the teacher, but relaxed.

Botes's approach of both lessons was conceptual. By requesting learners to provide solutions before she revealed any further steps on the Smart Board, she managed to keep the learner cognitively engage in the unfolding of the lessons. The learners suggested solutions to problems and designed their own methods to solve the example problems. After the initial guidance, she handed out problems that each learner could try to solve on their own. Learners were free to ask questions and to speak out in the class when they made proposals for solutions. The questioning suggested that learners should provide their own plans to devise a solution for a problem.

The moments that Botes did divert to procedural methods were motivated by her as being the result of the time constraints and to keep the class on the same level of understanding. The lessons were well-structured in terms of normal classroom practice. During both lessons, it was clear that a special relationship exists between the teacher and her learners. The learners were disciplined, but had the freedom to answer questions freely. Her body language indicated a warm approach towards the learners and the teacher often moved between the desks of the learners and bent over to look at the progress they made with the solving of a given problem. From time to time, she would engage in a conversation with individual learners. On the whole, there was enough evidence to conclude that she predominantly teaches in a conceptual manner. One should consider the challenges a teacher faces when operating within limited time frames and under the pressure of external examinations.

4.3 INFORMAL INTERVIEWS AND DISCUSSIONS

During the course of the research process, the two participants informed me that the whole process of research, despite minimal intervention into their normal teaching careers, indeed ignited their interest. The engagement inspired them and both enjoyed to be confronted about their teaching practice. It allowed them to rethink and reflect on their own practices. Both expressed a desire to engage in more discussions about the topic. They wanted to share more experiences and elaborate on new thoughts that occurred to them. We, thus, had a few informal interviews that were recorded for data collecting purposes. Very rich data about their own experiences and practices surfaced.

Botes said that to teach conceptually is very time-consuming. Despite accurate planning, the time available for the completion of the syllabus places many restrictions on the methodology that a teacher can use in the classroom. The first restriction is that when one teaches conceptually, time is needed to allow learners to suggest their own solutions for a problem, to test them, to modify them and to reach the best possible solution for a problem. Secondly, by the very nature of conceptual teaching, a conceptually orientated teacher would suggest multiple ways of adaptive reasoning to find a solution for a problem. This requires time and energy. Especially when a teacher allows learners to participate in the reasoning process, it takes time – time that they don't have during the normal course of a 40-minute lesson period, and also in the longer run with the completion of the years' work in order to write an external examination.

Davin regards herself as a product of a definite need due to the shortage of Mathematics teachers. Despite her love for the subject, her Mathematics teaching career followed a path that modelled her into the teacher that she became. Not only was she transformed by many external factors, but many valuable lessons can be learnt from her journey as a teacher. Her story provides possible solutions for the challenges facing the Namibian Mathematics teaching profession. Despite numerous discussions with colleagues and the supervision of her seniors, it *never happened* that anybody introduced her to the practice of teaching Mathematics. During the focus interview she explained: "*I will never regret how I am teaching, therefore my results are too good, but I am so happy for the opportunity to look at myself and to know there are other ways to reach the same goal. Yes I will try some.*"

Her procedural approach to teaching Mathematics was the product of her own beliefs that Mathematics was a very factual subject. She explained: "*Initially I thought that you could teach Mathematics like history. Just state the facts and let the children learn them. I still write the rules on the Smart Board for my children to study*." Her lack of teaching experience, proper guidance and exposure to a conceptual approach led to her adoption of a procedural approach. For her, facts are stated and questioning is based on the stated facts. Her attitude towards Mathematics was one of stating the rules or steps necessary to solve the problem and applying them to the appropriate problems.

Davin received positive reinforcement for her teaching approach through the system of external examinations for grade ten. Not only did she achieve well on the overall rankings, but she also achieved an excellent pass rate for all her learners.

Botes revealed during the focus interviews, and it was also supported later on during an informal interview with Davin, that the research process, although never intended in any way, allowed her to be self-critical and to revisit her own teaching methods. Both teachers expressed the desire to attend the classes of colleagues and that they sometimes experienced the need to share an excellent teaching practice with other colleagues.

Both teachers identified a need for a forum where professional Mathematics teachers can be informed about research in Mathematics education and kept up to date with the subject. It would also assist in remaining professional.

4.4 CONCLUSION

The analyses done above revealed a unique story or path for each teacher. The stories described why each one adopted a specific approach to teaching Mathematics. It surfaced repeatedly that many factors over long periods of time shaped each teacher into being a professional in her own right.

For the Namibian situation, where negative challenges are often pushed to the front, the two teachers in my research told stories of hope. One can be an excellent Mathematics teacher, even without receiving guidance about the philosophies of teaching Mathematics, and even without formal Mathematics training.

Both teachers were encouraged and inspired by participating in this study. They revisited and reflected on their own practices. They concluded the research process by sending me a note saying: "*What we both do is to prepare learners to excel when an external objective tool is used. We prepare learners to go off and pursuit a career of choice and are successful. That is why we are Mathematics teachers.*"

CHAPTER 5

CONCLUSION

5.1 INTRODUCTION

The goal of this study was to analyse and understand the teaching approaches of two teachers that claimed that their Mathematics teaching was characterized by a procedural and a conceptual approach respectively. In this chapter, the study will be concluded by reflecting on the findings, a discussion of the limitations and challenges encountered and finally, I will look at emerging avenues for further research.

5.2 FINDINGS AND RECOMMENDATIONS

The research process spanned over a fairly lengthy period of time and both participating teachers consistently kept to their teaching practice as claimed by them. The data analysis revealed that it was not possible and realistic to strictly characterise each teacher's approach as either purely procedural or purely conceptual. Each of their stories revealed many opportunities that could be considered for the improvement of Mathematics teaching in Namibia. The stories also opened up spaces for further research. The consistency of the data and the confirmation, through triangulation, showed that both teachers were mostly consistent in their teaching approaches as claimed by each one respectively. Davin consistently applied her chosen approach of teaching (procedural), and whenever she did divert from it, she motivated it as; "*just another way to keep all the sheep in the same kraal.*" A similar conclusion can be made about Botes' conceptual approach. The findings showed, however, that she was less persistent than Davin in consistently applying her approach to teaching.

The study revealed that each of the two teachers had a fairly comprehensive understanding of the principles of conceptual and procedural teaching – perhaps they did not articulate these in the same language as the academic discourse, but their utterances were certainly conceptually in line with the definitions that I derived from the literature. Different terminology and language was used in conversation by each teacher, but in their teaching practices the notions of procedural and/or conceptual approaches clearly surfaced.

During my interactions with the participating teachers, the importance of communication surfaced regularly. Both teachers felt very strongly that there is a serious lack of regular professional communication among Mathematics teachers.

Irrespective of the approach that a teacher adopts in her teaching, the diverse cognitive levels amongst learners in the class affects how one teaches. It is simply not possible to have a homogeneous group of learners (all learners on the same cognitive level) within the same class group. As it emerged in the data analysis, both teachers raised the point that it remains a challenge for a motivated teacher to accommodate these different levels of mathematical ability in one class group. This is particularly apparent when one group in the class is talented, diligent and motivated, while another group reveals just the opposite attitude. In the words of Botes: "*It is an art to teach Mathematics. Learners can very easily say that you are unable to explain the work to them. This happens especially when I try to allow them to arrive on their own at a wonderful mathematical truth. On the other hand you will get learners, the bright ones, of course, who want to be challenged. They get motivated when they are confronted with a challenging problem to solve. They will go home and try different approaches and even design their own solving tools. On the other hand there are learners who will not even attempt a challenging problem but will immediately down tools and say that the teacher's expectations for them are too high.*"

Every four years all the Namibian syllabi are revised. This process is usually done under the supervision of the National Institute of Educational Development (NIED). At NIED an Educational Officer is appointed for every subject to manage all matters relating to the specific subject. A subject panel, consisting of experienced subject teachers and specialists is appointed to advise on subject-related matters, including the revision of syllabi. The research clearly revealed that, if one adopts a mainly conceptual approach to teaching, there is not sufficient time to complete the grade-ten syllabus on time.

The analysis revealed that experienced Mathematics teachers should capitalise on the opportunity to become more involved in subject panel discussions and the revision processes of the syllabus. Davin asked: "*Who did the curriculum planning for us? Who decided what we should teach on grade-ten level? Who has the qualifications to decide how many periods I* *should spend on a topic?"* The answers to these questions lie in involvement of teachers in issues of curriculum and policy development.

As discussed in the data analysis, both teachers suggested a need to share positive experiences with colleagues and other teachers. The teachers would welcome constructive advice for the different academic challenges they face in their classrooms. Both enjoyed and valued the research process very much, because they had the opportunity to share successes and failures, to reflect on their own teaching and to empower themselves with knowledge from another person. No forum exists in Namibia for professional Mathematics educators, similar to the Association for Mathematics Education of South Africa. There is a need for such a forum, and it is my intention to drive an initiative to start such a forum.

The importance of sharing one's experiences was underlined by a remark Botes made: "*If you are blind – you are unaware of the wonder of being able to see. All the time we were unaware of the wealth that lies in sharing with each other and how motivational a discussion with colleagues that share our passion can be. Everyone wants to discover the wheel again and we are sure there are teachers that designed wonderful plans to overcome teaching obstacles and who are willing to share it with other teachers, but we have no opportunity to do it."*

In 2004 the President of the Republic of Namibia, Dr. Sam Nujoma, through the National Planning Commission of Namibia launched an inclusive development plan for the country called Vision 2030. This development plan outlined vast developmental plans for education in Namibia. Some of the objectives for Mathematics education in Namibia included, access to quality Mathematics education for all learners, the eradication of under-qualified teachers in sciences and Mathematics, and to encourage the development of lifelong learning in Namibia through institutional and human resource development (NPC, Vision 2030. 2004, p. 91). The story of Davin as it emerged from the research finding, is a story of hope and possibilities. An unqualified person can be transformed into a very effective Mathematics teacher. It aligns well with the objectives of Vision 2030 to drastically increase the number of Mathematics teachers and to ensure quality education in Mathematics. One should caution that no official structures or any support basis exists to assist any teacher who would like to get involved in Mathematics teaching. This provides ground for further research.

5.3 LIMITATIONS AND CONSTRAINTS

One of the limitations of this case study was the small sample investigated. Ideal conditions existed for both participants to perform optimally, but there are teachers performing extremely well under very difficult conditions. An expanded case study would, therefore, provide even more valuable information about the approaches teachers adopt when they teach. It would be important and very interesting to investigate the practices of teachers with very limited resources and teaching in class groups exceeding 40. Ground work has been done with this study and, despite the limitation outlined above, it has provided an interesting frame of reference for further research.

Time to spend with the participants was limited. Although adequate for the purposes of this study, I would have liked to record more lessons including other challenging topics like trigonometry and geometry.

A video recorded lesson provides the researcher with only a narrow view of the lesson and what went on in that lesson. This narrow view is limited by what the camera captures. During the screening of the videos it was difficult for example, to fully follow the interactions between the teacher and the learners. Some words or actions by the teacher would cause a reaction from the learners, but this was lost because the camera remained focused on the teacher. Poor sound quality in the recordings was also another limitation. Responses from learners could not always be heard.

5.4 FINAL REMARKS AND A PERSONAL REFLECTION

Initially in the data collection period, I had an intuitive feeling that teaching mainly in one paradigm, namely either conceptually or procedurally, would not lead to a satisfactory outcome. As my project unfolded, and as I observed and listened to how my participating teachers consistently remained true to their adopted approach, I had to adapt my own intuition. My interviews and observations revealed that being consistent, dedicated, diligent and sustaining a positive relationships with learners and with one's conviction of a particular teaching approach, my two participating teachers illustrated that they both were very effective in their teaching..

Davin is aware that she is not wrong or inferior in her approach to teaching Mathematics, but that she is following a legitimate approach to teaching Mathematics - a method that over a long time has harvested excellent fruits for her and her learners. The research process, however, taught her that there are other underlying factors in the learning and teaching of Mathematics that are worth considering. She became aware that conceptual fluency, for example, could contribute to assimilate mathematical knowledge in a way that allows a learner to not only reproduce knowledge but also create new knowledge.

The research process affirmed Botes' approach. She explained that she received positive reinforcement that she was on the right track with her methodology for Mathematics. The research, however, also revealed that. Due to contextual circumstances, she was not always able to stay true to her chosen approach.

Both participants expressed their gratitude with the whole research process. All efforts were made to do the whole research process in a non-invasive manner. By just being able to talk about their own Mathematics teaching experiences and to see themselves perform in front of learners, broadened their own views and insights. Both admitted that they had the opportunity to pause, take an objective look at themselves, and re-think their own acts and to develop further. Upon reflection, both teachers were asked if they still enjoyed teaching Mathematics. Davin said: "*Yes ... yes ...yes. I love it, I love it.*" And Botes said: *"Oh yes, it's only in Mathematics where I can experience those 'Eureka!' moments with children when they understand something."*

Very little professional support exists for Mathematics teachers. This can be attributed to financial constraints that limit the official support basis for teachers. Where support does happen, it is a pity that the focus is so much on the poor results at the expense of sharing positive experiences. As a Mathematics teacher myself, I concur strongly with my participants that better communication with and amongst fellow Mathematics teachers should be developed.

A personal mission that became more important to me as the research process unfolded, is to get involved and take the initiative for the creation of a Namibian Association of Mathematics Educators. The study was a very positive journey for me. The entire process reminded me not to become stagnant in my teaching. An experienced Mathematics teacher may well boast of having excellent content knowledge, but without engagement in the academic and philosophical developments of Mathematics, one will remain static in one's own teaching practice.

5.5 CONCLUSION

The study was significant in several ways. Apart from the transpired and discussed results, I it brought about change in both participants. The process was extremely motivational for both teachers. Their discussions with other Mathematics teachers have sparked a new debate amongst teachers in my school. It is very easy to become complacent - a new awareness that Mathematics teaching is not merely a repetitive, mechanical action is leading to positive changes at my school.

The study led to much personal growth for me. An immense amount of new knowledge was assimilated. I am motivated to rethink my practice, to share experiences with colleagues and to get involved on a broader level with other Mathematics teachers and to assist inexperienced Mathematics teachers.

What a privilege to be a Mathematics teacher, through skilful teaching we can unlock the beauty of mathematics to our learners and through sharing we as teachers can motivate and inspire each other to remain dedicated in our task with precious children.

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APPENDIX A: Coded Transcript of interview

Transcript of interview with Salomé Davin and Adri Botes:

Conceptual and Procedural teaching of Mathematics.

CODING:

…………… RED: Supportive of procedural teaching: Salomè Davin

…………… PINK: Supportive of conceptual teaching: Adri Botes

…………… GREEN: Challenging data of claim by participant Salomè Davin

…………… YELLOW: Challenging data of claim by participant Adri Botes

…………… BLUE: Emerging additional data

KEY:

Interviewer, Danie Junius – abbreviated as DJ

Interviewed 1: Salome Davin – abbreviated SD

Interviewed 2: Adri Botes – abbreviated as AB

TRANSCRIPT

DJ: Thank you that you two are willing to talk to me about Mathematics, specifically about grade 10 Mathematics. I would like to tell you that our school's success with Mathematics is the main reason for this research and especially you're two's excellent results during the external examinations. Salome I would like to start with you and ask you where did you study and what did you study?

SD: I studied History and Psychology uhm…. I completed my degree at UNAM. Because I was so interested in Mathematics did I also take Statistics in my first year.

DJ: Adri and you?

AB: I studied at Potchefstroom University and did a B.Com degree that included at that stage Mathematics. I also did my HED at that university and I took for my majors Mathematics and Accounting.

DJ: In other words you did the methodology of Mathematics?

AB: Yes definitely the methodology of Mathematics and then also the methodology of Accounting. So that you can teach a child how to learn.

DJ: Salomé how did it happen that you ended up in Mathematics teaching?

SD: It happened that we had a shortage of Mathematics teachers in standard five then or Grade 7 nowadays, did I volunteered to try it for one year because I did Statistics in my first year. And it turned my whole life around. I still gave history for a few years and from there on it was full out only Mathematics

DJ: Adri did you give Mathematics since you started teaching?

AB: I gave Mathematics together with my commercial subjects, Accounting, Economics and Business Studies, but gradually Mathematics became the more enjoyable subject for me to give. To learn with a child and to struggle with a child. It could also be the people who were with me, and took me by the hand in my subject area – and I could relate with. At the moment I don't want to teach anything else.

DJ: Good. Salomé, how did you end up with grade 10 Mathematics?

SD: I taught myself, and as Adri said, the people that work with you play a very large role, and I just taught myself. Every year I took it one step up and eventually later on I ended with Grade 12 Mathematics. But Grade 10 remains a pleasure to teach. It is a very interesting year – developing year for the child.

DJ And you how long do you teach Grade 10?

AB Shoe! Definitely close to 16 to 18 years. U start under, I remember those years, I had four grade 10 classes so it was bad, exhausting but…

DJ And how long are you with grade 10?

SD I would say 14, 15 years, because I am teaching for 22 years now. Initially it was three years of history and then I started with the Mathematics.

DJ I would like to get your answers separately, but I would like to ask you both. Are you still enthusiastic about teaching Mathematics?

SD If you saw me today in the class you would say, yes. Yes it is unbelievable if you see a child and how you lead him to learn Mathematics. No I am unbelievably passionate about teaching Mathematics.

AB I am passionate about the teaching profession and specifically the Mathematics, because there is always a challenge. There is always a different way you can say something, you can use different numbers and there is always a different method that you can use. So it is definitely not a boring (monotonous?) subject. It makes one enthusiastic when one can say "shoe" now they click; now they understand.

DJ Your enthusiasm is contagious. Salome you have a specific method that you use to teach Mathematics, how you approach each lesson. Explain it to me.

SD I am very much focused on the methods. Facts play an important role for me. If you know the method, I can throw you with anything. If you can pin down the method, one point two, one point three, point four. And if I can make the child aware of it and say let us apply it. Each topic that I teach points for me to a method. I help them to figure out, how must I do this thing and then we get the story right.

DJ Adri you also said that you have a specific method how you teach your classes. We talked about conceptual and procedural approaches and you said that you are much more conceptual. Explain to me what you mean by that.

AB I try to make sure that the child understands what he has to do. Often it goes in hand with pictures and other different methods to make sure that the child understands what he has to do. Usually I start with an easier one to lead them to a next one or a next more difficult sum. Of course he should know his theory, it is important. If you don't know the theory, you will in any case be unable to do anything. But I try to lay emphasis that he can understand why we should use this method or this formula.

DJ Do you think if a child a child understands something, he will be able to solve a problem about it?

AB I think so, because he would be able to make the connection immediately. I you show them the formula of a strait line and if you tell them this is a hyperbole, they will know why they may not divide by zero, and therefore we get asymptotes.

DJ Is it important for you, that when you confront a child with a problem, should arrive at a solution, or do you expect from him to solve it correctly step by step?

AB Do you mean does it hinder me what method a child will follow……?

DJ Yes.

AB No, if a child can arrive at an answer and it is mathematically correct and not written off from his neighbor I accept anything. This is where the enjoyment lies. There are different methods to get an answer.

DJ Is it for you important that a child arrives step by step at an answer?

SD No but……… It is for me important; it is for me important that he can show the insight how he arrived at the answer. Because sometimes it is just trail and error and then he arrives at the right answer. So it is for me important that he can show me, and because there are in

Mathematics so many method marks (used English term) I emphasize for them that it is not all about the answer. If you have the answer wrong – one can go back and then you should be able to show me the method. By doing so you can earn marks without having the answer correct.

DJ Say…

AB I think now specifically about fractions. Today's calculators are extremely clever and can do a lot of steps all in one. They can simplify a large fraction by the press of a button, but they should know (understand) how to work it out. Then the calculator will be useless, so yes it is in that sense important to show a method but ……… eh………

DJ But he should understand to solve it???

AB Yes, but up to grade 10 level … and what we are talking about...is the method more important than when you talk about the matrics. They can at higher level skip a lot of steps in his head, but in grade 10 they should write down a lot of steps…….so it is true what Salomé says…but …sorry…and specifically when a questions says, show all your workings. Then you cannot just write down an answer for three marks. So you let the numbers of marks lead you……

DJ Salomé, what topic in the grade 10 syllabus is you favorite topic?

SD Oh I like exponents….. There are so many rules and regulations (laugh). I did it today with children from different schools, because at this stage we are busy with a spring school. We just wrote down the methods on the one side (of the board) and then I showed them there are so many methods that you can ask it, but if you know the basic rules….wow… there you go. All of a sudden everything opened up for them. I like it so Gush I like it so. If they know the basics… It was so unbelievable for me today. I like it so much when they can see … if I know this, Miss can ask me anything about...this about algebra.

DJ I don't want to lead you, but I get the feeling that you are saying that the methods lead to understanding. Do you think there are children that study the method and do mechanically the sums?

SD You get a child; you get different children that can with insight analyze a sum to get to an answer without having a method. There are children that can swing a sum, without a rigid method, but I also found that when I assisted children who haven't done the work yet, that when I helped him with the methods it opened something up for him. Especially a child who has to learn more rules, that what is needed by a highly intelligent child with insight who can reasons it out for himself, and he understands without me having to explain it to him.

DJ When a child understands his mathematics conceptually, would you say it is a method that works better for more clever learners or learners who are better in Mathematics?

AD No, no for a stronger learner the understanding will be easier, while the weaker learner will understand better through step by step, the method he has to understand. If for example I have to teach a learner the inverse, I would always just swop the x and y values, and later on I will make y the subject of the formula. Where brighter learner will know what to do. He has to say for himself; I see this so now I have to do that. Especially in Algebra like factorization, there are four types of factorization, so I try to tell them there are only four types, start always with this one if it is not the one, try the next one. While the cleverer learner will have immediate insight into right this is a quadratic equation. Unfortunately are there differences between faster and slower learners and you have to make provision for both.

DJ Do you both have the whole spectrum of learner abilities in your classes?

SD Well I do not have a spectrum of learners, but I do have a slower class who only takes the Normal Math and the approach that I have…. And I must say that, that where they have to write out the method in words, literally they write the method in words, and I must say… I help them really it helps the slower learner. In general we have a wide spectrum of learners in our classes.

DJ Adri what is your most favorite topic in the grade 10 syllabus? What do you like the most?

AB I would like to say firstly Algebra, although at that stage they should have mastered the Algebra but if you take a look at the grade 8, at that stage it is a great challenge to teach a learner what an x is and what is a y. So however you look at it will it be Algebra for me, because you can take it through to graphs and even word problems and…yes.

DJ Good, to conclude I would like to ask you both a little controversial question, Salome how do you feel about the grade 10 syllabus if you have to give an opinion about it?

SD Very full, although every year is very full, but there is an unbelievable amount of new information that they have to receive. Sometimes I am afraid that they do not get enough time for mastering a topic. That one does not exercise enough to master a certain topic. But further I believe the variation is very large, it spans over a wide area, but I would have wanted more time to really let them master the work so that I can have them ready for grades 11 and 12…..

DJ When you say master, do you mean you wanted your learners to do more exercises about a topic?

SD Gush... more exercise, this is all how you are successful, exercise exercise exercise…

DJ Adri, how do you feel about the grade 10 syllabus?

AB I can now relate to Salome, the positive side of it is that we are now going to withdraw from the external examination which will allow us more time for mastering. We gave it a thought and in other subjects they want to bring down some of the grade 11 and 12 topics to grade 10, but we felt that we might bring down a small amount down to grade ten, but we would rather use the extra time to make sure that they know – how to solve or how to draw a graph. We are going to try it this year, so if you ask us the same question next year we might give you another answer.

DJ How do you feel about the external examination for Mathematics?

SD I think it was excellent exposure, I'm talking now about Additional Mathematics, the normal Math I think, was not really a challenge for the children, but the Additional Mathematics was really a challenge, but hum…. But as I said, it was a race. It was really a race. It has its advantages and disadvantages. The advantage is that you now where you stand with your children; the disadvantage is that they fall in Grade 11, because they are not really prepared for grade 11.

AB How do you feel?

AB I was one of the people who were against the discarding of the external examination, especially because of the Additional Mathematics paper that they wrote. Because it was on Standard and it was much more than a challenge, something that the normal Math don't do, but if you look at the bigger picture and if you look at other subjects, than one understands the decision………. If you look at the country as a whole... it is good that they write an external examination, because at least they will have a certificate to show, but the Ministry wants to change that again…

DJ Yes grade 11 will now be the outcome… could be very bad…

DJ Salome I would like to ask you a final question: When you plan a lesson, I know after 22 years doesn't one do it anymore, but how do you prepare yourself. Say for example you have to start with a new topic. How do you do your preparation?

SD Agggg the old story, again the methods. Firstly an outline, and where we are going to. We are going to start now with graphs, so firstly where we are going to move to, and then I outline what we are going to do. Beforehand they receive a complete planning of how we are going to do it. I try to prepare him what he should achieve at the end of the topic.

AB In that way I am a bad teacher….I start slowly and I lead him towards the end, nine out of the ten times I don't tell them the whole topic, I will not blurt out the whole syllabus…. We actually rediscover it again.

DJ Concept by concept … if I can lead you

AB Yes thank you.

DJ Is there anything else you would like to talk about with me?

AB It is again a pleasure to talk about Mathematics as we do not get time under normal situations to talk about Mathematics.

DJ I really would like to talk more and more about Mathematics with you.

Thank you. I will stop the recording now.

APPENDIX B: PERMISSION

WINDHOEK GYMNASIUM

Private School – Privaatskool

Teaching a new lifestyle- Building a better future

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2012 – 04 -11

Dear Mr. Junius

<u>RE: Permission to conduct Research at Windhoek Gymnasium</u>

Your letter refers.

Herewith permission is granted for you to conduct research at Windhoek Gymnasium Private School adhering to the conditions as set out by yourself.

The two teachers from the Mathematics Department gave their consent that you may observe their classes and permission is granted that you may have access to their personal files and lesson plans. It is in order that you make video recordings of some lessons as requested.

Our best wishes accompany you.

MM du Plessis

MS M M DU PLESSIS
HOD – MATHEMATICS
(081 246 7727)

ADDENDUM C: AGREEMENT WITH TEACHERS

Windhoek Gymnasium Private School
15 April 2012

AGREEMENT TO PARTICIPATE IN RESEARCH PROJECT

Herewith I, …………………………………………. agree to participate in the research project currently conducted by Mr. Danie Junius at Windhoek Gymnasium Private School.

I agree to the following:

- The purpose of the research is to investigate my teaching practice. I know no value judgment will be made.
- The procedures that will be followed during data collection include observations a study of my personal file and lesson plans, interviews over a period of time, and video recordings of some of my lessons.
- That the data collected will be handled confidentially;
- That the results will be available to both teachers.
- That I have the right not to disclose information I am not comfortable with and
- that I can withdraw at any stage of the process.

Signed on this day……………… of ………………………….. at Windhoek.

Signed:………………………………………………….