# AN ANALYSIS OF THE NATURE OF VISUALISATION OBJECTS IN THREE NAMIBIAN GRADE 9 MATHEMATICS TEXTBOOKS: A CASE STUDY IN NAMIBIA

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# ABSTRACT

Textbooks are a universal and central element of teaching and learning mathematics (Namibia. Ministry of Education [MoE], 2008). Steenpaβ & Steinbring (2014) state that diagrams in mathematics textbooks are often used as Visualisation Objects (VOs) to enhance learning of mathematical concepts. VOs in textbooks are thus important teaching and learning tools (Fotakopoulou & Spiliotopoulou, 2008).

This Namibian interpretive case study analysed the nature of VOs used in the three approved grade 9 Namibian mathematics textbooks namely: *y=mx+c to success, Maths for Life 9* and *Discover Mathematics 9.* The VOs were analysed by using an analytical framework adapted from Fotakopoulou & Spiliotopoulou (2008). This analytic tool was specifically used to interrogate the following categories: the type of VOs, the roles of VOs, the relation of VOs to mathematical content, the relation of VOs to reality, and their properties. The 266 VOs under study were collected from the Algebra and Geometry chapters of each book. This study also included survey questionnaires with the 50 selected mathematics teachers, which sought their views and perceptions on the use of the identified VOs. In addition, the authors' rationale in selecting the identified VOs used in their textbooks was sought through interviews.

This research study is part of the "Visualisation in Namibia and Zambia" (VISNAMZA) project which seeks to research the effective use of visualisation processes in the mathematics classroom in Namibia and Zambia (Schäfer, 2015). It is hoped that this study contributes towards improving the quality of textbook evaluations, and design of suitable and more comprehensive assessment procedures in Namibia. It is also hoped that it creates a critical awareness of the roles of VOs in textbooks amongst teachers, inspiring them to help their learners interpret VOs effectively. It should also inspire potential authors to use suitable and appropriate VOs that enhance conceptual teaching and learning of mathematics.

The study discovered that most of the VOs used in the selected textbooks align well with the mathematical content. The VOs can help make abstract ideas concrete, stimulate learning, simplify and clarify written texts. In addition, VOs can also be used as a tool for reasoning and an instrument for problem solving. The findings however also indicate that some of the VOs used are not self-explanatory; they are vague, unfamiliar and confusing, leading to misinterpretations by some learners. Another interesting finding was that some of the learners found it difficult to interpret VOs on their own without the help of the teacher.

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## DEDICATIONS

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## DECLARATION OF ORIGINALITY

I Selma Ndilipomwene Nghifimule, student number g13N6665, declare that this thesis entitled: An analysis of the nature of visualization objects in three Namibian grade 9 mathematics textbooks, is my own work, written in my own words. Where I have drawn on the words or ideas of others, these have been acknowledged according to Rhodes University Education Department referencing guidelines.

………………………………………………… 22/11/2016

Selma Ndilipomwene Nghifimule (Signature) Date

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# 1. CHAPTER ONE: INTRODUCTION

This chapter introduces the research study, entitled *An Analysis of the nature of Visualisation Objects evident in three selected Namibian Grade 9 Mathematics textbooks*. The chapter begins by contextualizing this study in the Namibian context. The research goals, research methodology and research process are discussed briefly. The limitations of the study are summarized. The chapter concludes with a brief overview of the structure of the thesis.

## 1.1 BACKGROUND OF THE STUDY

Textbooks are a universal and central element of teaching and learning mathematics, as is the case for most school subjects (Nicol & Crespo, 2006; Namibia. MoE, 2008). In many developing countries where there is often a shortage of reading materials, the textbook is often the only resource text that teachers and learners can draw on for subject-specific content and information (Namibia. MoE, 2008) and thus their quality assumes special importance. Further, national and school examinations are often largely based on an ability to reproduce what is to be found in prescribed textbooks (*ibid.*).

Researchers indicated that diagrams in mathematics texts, such as textbooks, are often used as visual objects to enhance the learning of mathematical concepts (Konyalioglu 2003; Guzman 2002; Steenpaβ & Steinbring, 2014). These visual objects can illustrate an abstract idea in a concrete way and they can reinforce a mathematical procedure diagrammatically. Steinberg (2005) and Sobbeke (2005) stated that taking a visual approach to teaching and learning mathematics could be meaningful and effective. Although visual objects in textbooks are important teaching and learning tools, only recently have they attracted research attention (Fotakopoulou & Spiliotopoulou, 2008).

Many researchers recorded that the use of visualisation objects are central elements to the effective teaching and learning of mathematics (Stacey & Vincent, 2009; Gellert & Steinbring, 2014; Dimmel & Herbest, 2015). Arcavi (2003) suggested that visual representations help us to ‘see the unseen’. Likewise, Sobbëke (2005) proposed that in mathematics classrooms, visual diagrams help the learners to better see mathematical concepts and ideas. Gellert & Steinbring, (2014) affirmed that visualisation representations could be used as an epistemological tool to explore mathematics structures and bring about new meaning. Visual objects are “especially important for textbooks which aim to lead non-expert readers to an understanding of the esoteric domain of a scientific field" (Fotakopoulou & Spiliotopoulou, 2008, p. 321).

Duval (1999) suggested, "Invisible mathematical tools need other forms of representation to become communicable" (p.3). They can be presented as visualisation objects, which include drawings, pictorial illustrations/pictures, diagrams, charts and graphs (Fotakopoulou & Spiliotopoulou, 2008). Kim (2012) observed, "More pictures, illustrations, and diagrams have been used in recent textbooks than in the past" (p.175). Kim's (2012) study finds that visual representations in mathematics textbooks can serve not only as informative agents but also as "tools for thinking" which students manipulate (p.178). Many researchers deem the use of visualisation objects to be a powerful tool in learning and understanding mathematical concepts (Fotakopoulou & Spiliotopoulou, 2008; Kim, 2012; Gellert & Steinbring, 2014). Kilpatrick, Swafford & Findell (2001) asserted that learning with understanding is more powerful than simply memorizing because it improves retention, promotes fluency, and facilitates learning related material (p.113). As Levin & Mayer (1993), as cited in Kim (2012) suggested, "Since visual representations provide students with concrete and concise images of related concepts, they help improve students' understanding of the contents" (p. 177).

In contrast, Steenpaβ & Steinbring (2014) warned that mathematical visual images could be ambiguous elements that do not necessarily convey the concept effectively to students, and can thus lead to misunderstanding. Gellert & Steinbring (2014) added that "interpreting mathematical visual diagrams is a challenge faced by mathematicians as well as students" (p.16). Therefore, students require appropriate assistance and guidance from teachers and knowledgeable peers as they select, interpret, and create visual models of mathematics (Moyer & Jones, as cited in Moyer, 2014, p.3). For these reasons, it is important to understand how representations are used in mathematics textbooks because "representation is more than a process; it is a way of teaching and learning mathematics" (Fennell & Rowan, as quoted by Kim, 2012, p.1).

In 2007, a survey of schools in Namibia for Grades 5-12 revealed that the textbook to learner ratio for the core subjects of English, Mathematics and Science averaged nearly 1 textbook to 5 students (Namibia. Millennium Challenge Account [MCA], 2009). A shortage of books and other instructional materials was prevalent in Namibian primary and secondary schools in those years

(Namibia. MoE, 2008). In July 2008, the government of the Republic of Namibia and that of the United States of America (USA) reached an agreement whereby the USA government provided grant funding for public investment in Education through the Millennium Challenge Account (MCA) for five years. In the education sector, the MCA aimed to improve the quality of general education and access to and management of textbooks (Namibia. Ministry of Education and Millenium Challenge Account Namibia [MoE & MCA], 2010).

The objective of the Textbooks Activity in the Education Project of the MCA, Namibia, is "to give all learners access to a textbook for each priority subject" (Namibia. MCA, 2009, p. 3). "This project focuses on the procurement system and procedures as well as textbook management and utilization at school level" (Namibia. Ministry of Education [MoE], 2010. p. 3). The overall objective of this sub-activity in education was to achieve a textbook to learner ratio of 1:1 by 2013 for core subjects. These were in the first instance: English, Mathematics and Science at primary, junior, and secondary levels (Namibia. MOE, 2008). At the end of this project, the report (Namibia. Millennium Challenge Account [MCA], 2015) indicated a great improvement in access to textbooks, and the achievement of the goal of 1:1 learner to textbook ratio in the three core subjects was reached.

Indeed, this is evident in the school where I work as well as in other neighbouring schools that form the Oupumako cluster. The Grade 9 learners of the rural school where I work and the other schools in this cluster have one or more of the three recommended mathematics textbooks. These textbooks are typically the only books that the learners have access to (Namibia. MoE, 2008). The textbooks are used in the mathematics classrooms during the lessons, during examination preparations, and at home for homework and study purposes.

As a procedure in Namibia "all textbooks in education require the evaluation and approval of the Namibia National Institute for Educational Development (NIED)'s curriculum panels" (Namibia. MoE, 2008, p. 4). Textbook evaluators mostly base their selection on four criteria: the conformity to the subject syllabus; the appropriate coverage of the prescribed content; the language and editorial quality; the design, presentation and ease of use of the textbook (Namibia. Ministry of Education Arts and Culture [MoEAC], 2015). Although the use of visualisation is not specifically mentioned, it is partly incorporated in the last criterion, which looks at the usefulness, relevance and accuracy of graphics and illustrations, and the quality and attractiveness of illustrations.

This criterion, in my view, does not sufficiently and explicitly articulate the desired characteristics of visualisation objects that are required. I argue that the criterion is not substantial enough to assess the appropriateness and quality of visualisation objects in mathematics textbooks. This thus suggests that there is a gap in the Namibian evaluation criteria for assessing visualisation objects in mathematics. I therefore envisage that this study may *inter alia* contribute to improving the quality of textbook evaluations and design of suitable evaluation criteria in Namibia.

The textbook review policy stated that only three titles per grade system are approved and entered on the textbook catalogue (Namibia. MoEAC, 2015). In 2008, NIED's (Namibia. MoE, 2008) curriculum panel approved three mathematics textbooks for Grade 9 learners. These are the books that are central to my research project. They are called: *'y= mx+ c to Success 8-10'*, *'Mathematics for Life Grade 9' and 'Discover Mathematics Grade 9'.* For the purpose of this study, I chose to focus on the efficacy of visualisation as a product. These products will be referred to as visual objects (VOs) and they include diagrams, sketches, pictures, graphs and illustrations.

Currently, mathematics textbooks are accessible to both teachers and learners at most Namibian schools with a 1:1 learner to textbook ratio (Namibia. MCA, 2015). Thus, there is a need to take advantage of the availability of textbooks to get the best out them, particularly in relation to the use of VOs evident in mathematics textbooks. This research needs to be conducted in order to shed more light on how to improve the quality of the textbooks in terms of VOs, improve the use of VOs among teachers, and create awareness among authors to ensure that suitable and appropriate VOs, which can enhance teaching and learning of mathematics in Namibia, are used.

## 1.2 RESEARCH GOALS & THE SIGNIFICANCE OF THE STUDY

The aims of this study are threefold. Firstly, to analyse the visualisation objects used in three grade 9 Namibian mathematics textbooks in terms of their types, roles, relation to mathematical content, relation to reality and visual dimensions. Secondly to understand selected mathematics teachers' views and perceptions about VOs in terms of their use. Finally, to describe the choices that the two selected authors made in selecting the visualisation objects used in their textbooks.

It is hoped that the study contributes toward improving the quality of textbook evaluations and design of a suitable and more comprehensive assessment of the textbook evaluation process in Namibia. This can be done by revising the criteria for including illustrations. These should be precise and clear. It is hoped that this study will provide some insights into the use and nature of the visualisation objects in textbooks. It is also hoped that it creates a critical awareness of the roles of visualisation objects in textbooks amongst teachers, inspiring them to help their learners interpret the VOs effectively. With the emphasis on the importance of VOs, it should inspire potential authors to use suitable and appropriate VOs that enhance conceptual teaching and learning of mathematics.

## 1.3 RESEARCH QUESTIONS

Having the aforesaid goals in mind, the three research questions that framed this study are:

- What is the nature of different visualisation objects evident in the Namibian Grade 9 mathematics textbooks?
- How are these visualisation objects viewed in terms of their use by selected teachers in their teaching?
- What were the authors' rationales for using the identified visualisation objects in their textbooks?

## 1.4 RESEARCH METHODOLOGY

This case study research was underpinned by an interpretive paradigm using a mixed method approach, utilizing qualitative and quantitative methods (Bertram & Christiansen, 2014). The choice of this paradigm is well aligned with the purpose of this study, which is to understand how teachers make sense of visualisation objects and how textbook authors made their choices in using these objects in their mathematics textbooks. This is done through survey questionnaires and interviews respectively.

The selection of the sources of data in this study namely textbooks, teachers and authors was done using a 'purposive sampling strategy' (Bertram & Christiansen, 2014, p.60). The three Grade 9 Namibian mathematics textbooks namely: y=*mx+c to success; Maths for Life Grade 9 and Discover Mathematics Grade 9* used in this study, are approved for use in the junior secondary phase in Namibian schools. Furthermore, all these three textbooks are written based on the 2010 revised national curriculum (Namibia. MoE, 2010). In addition, the participating teachers in this study are Grade 9 mathematics teachers from different regions of Namibia. This was also purposive sampling since I selected these participants for the specific purpose of completing the survey questionnaire. The two selected authors both reside in Namibia and were thus easily reachable.

## 1.5 LIMITATION OF THE STUDY

I recognize that this study provided and presented an in-depth analysis of the nature of VOs in only three textbooks. The findings of this case study can thus not be generalized to other textbooks due to the limited scope. However, the findings can provide insights for further research.

## 1.6 THE STRUCTURE OF THE THESIS

This section provides a brief overview of the remaining chapters in the thesis.

**Chapter 2** critically reviewed the literature relevant to the study. The chapter reviewed the roles and relevancy of textbooks in teaching and learning of mathematics. A brief outline of the process of selecting mathematics textbooks for Namibian schools is given. An overview summary of relevant aspects of visualisation formed part of this chapter as well. Finally, the relevancy of literature reviewed on the theoretical framework underpinning this study, which is social constructivism, was outlined.

**Chapter 3** focused on the methodology used in the study. This chapter outlined the methodology used in this research study as well as providing reasons as to why the specific methods were chosen in this study. It also provided the rationale for the study, and described the research instruments that were employed, followed by the limitations encountered in the study. The chapter concluded with a discussion of the data analysis, as well as an interrogation of issues relating to ethics and validity.

**Chapter 4** looked at data analysis. This chapter presented and discussed the findings of the study. The summary of findings from all three sources of data was presented. The chapter concludes with a discussion of emerging themes. A conclusive summary for all the data collected from textbooks, survey questionnaires and interviews was presented in a brief summary.

**Chapter 5** of my research study presents the conclusion and recommendations. The key issues of my research findings are concisely summarized. The chapter included a summary of main themes in relation to the nature of VOs evident in three selected textbooks. Recommendations and implications arising from the study were noted. Finally, the concluding remarks and personal reflection of my journey are outlined.

# 2. CHAPTER TWO: LITERATURE REVIEW

## 2.1 INTRODUCTION

This chapter engages with the literature relevant to the study. It starts with a review of the roles and relevance of textbooks in the teaching and learning of mathematics and the attributes of a good textbook. A brief outline of the process of selecting mathematics textbooks for Namibian schools is given. It also discusses the intervention of the Millennium Challenge Account Textbooks Project. It then examines facets of visualisation under the following discussion points:

- What is visualisation
- Visualisation in mathematics
- Type and nature of VOs
- Functions and roles of VOs
- The challenges of visualisation
- Ways of helping learners to develop visualisation skills

As the focus of this study is on visualisation, I will use the term VOs to be synonymous with terms such as diagrams, drawings, pictures, sketches, charts, graphs, illustrations etc. The latter part of this chapter provides an overview of the theoretical framework underpinning this study.

## 2.2 CONTEXT

### 2.2.1 Relevance of textbooks in mathematics education

Textbooks are a worldwide element, key to the teaching and learning of school subjects in general and mathematics in particular (Nicol & Crespo, 2006; Namibia. MoE, 2008). Triantafillou, Spiliotopoulou, & Potari (2013) state, “school textbooks are textual products, usually produced by discipline specialists, with the purpose of being used in specific educational communities in order to achieve their institutional goal” (p. 1). This disciplinary embedding of a school textbook could involve variations in knowledge structures, norms of inquiry, vocabularies, representations and standards of rhetorical intimacy (Bhatia, 2010). The school textbook is also known as curriculum material that mediates school activities, while its use is situated in the social and cultural context of institutional teaching and learning (Rezat, 2006).

In many developing countries where there is often a shortage of reading materials, textbooks are often the only resource that teachers and learners can rely on for subject-specific content (Namibia. MoE, 2008). Further, external and internal examinations are often largely based on the knowledge base found in the prescribed textbooks, making it imperative for every learner to have access to their own set of books (*ibid.*).

Textbooks are an essential part of classroom life in primary and secondary schools. Pepin & Haggerty (2004) highlights that "textbooks embody a teaching philosophy and pedagogy as well as the more obvious scope and sequence" (p. 1). Textbooks often embody the full curriculum for a subject and the sequence in which topics must be presented. A good textbook typically addresses the needs of a diversity of learners *(ibid.)*. In the same way, Nicol & Crespo (2006) state, "in mathematics classrooms, textbooks are an intricate part of what is involved in doing school mathematics; they provide frameworks for what is involved in doing school mathematics; they provide frameworks for what is taught, how it might be taught, and the sequence in how it could be taught" (p. 331).

Textbooks are usually familiar and non-threatening to teachers. They are relatively inexpensive and require little or no maintenance" (Verspoor, 1989, p. 67). In agreement Neumann (1989) states, "books are among the most effective, and probably the most cost-effective, tools for teaching and learning" (p.127).

Farrell & Heyneman (1989) assert that the textbook is one of the key factors that contribute to improving the quality of education in schools. They further observe, "One of the more consistent indicators of higher student achievement is the availability of textbooks and other printed materials" (p. 17). Verspoor (1989) affirms, "Textbooks make an important contribution to improving the quality of education in all stages of educational development" (p. 66). In the same vein Heyneman, Farell & Sepulveda-Stuardo, (1981), as quoted by Gopinathan, 1989), states, "the availability of books appears to be the most consistent factor in predicting academic achievement" (p. 71). Fernig, McDougal, & Ohlman (1989) notes, "where textbooks are lacking, their absence is considered detrimental to both learning and teaching" (p. 206).

The importance of textbooks for both teachers and learners cannot be overemphasized. Femig et al. (1989) give credence to the fact that “the textbook is a versatile tool for both teacher and learner” (p. 206), and that good textbooks support teachers and students in several ways:

- they contain lesson material in a graded sequence
- they help structure and organize the learning experience of the class
- they assist the teacher in the daily task of preparing lesson plans
- they provide capitulatory material and exercises to test progress
- they can be conveniently carried from school to home and provide a summary record of each lesson
- they provide reference information
- they help students make links between what they learn in school and their comprehension of the outside world

The place of textbooks in the classroom is significant to teaching and learning mathematics (Verspoor, 1989; Nicol & Crespo 2006; MoE, 2008). In agreement, Neumann (1989) say that despite the claims of the more optimistic proponents of computers and other new teaching technologies, the evidence examined during their research suggests that for the foreseeable future, textbooks will continue to be the principal mechanism for instruction. They further state that though applications will be found for computers, they will be limited compared to the use of textbooks *(ibid.)*. Textbooks are thus still key resources for teaching and learning in the Oshana region of Namibia. Gopinathan (1989) highlights that although “the textbook has been challenged by instructional innovations – radio and television, programmed learning, multimedia packages and language laboratories, among others”, they remain the most used (p. 72). All these innovations have failed to hold their own against the textbook as the prime pedagogical tool in the classroom *(ibid.).* Similarly, Pearce (1982) elaborates that these innovations “have stimulated further the demand for textbooks rather than replacing them”. Altabach (1983) as quoted by Searle (1989, p.28) notes that nothing has ever replaced the printed word as the key element in the educational process and, as a result, textbooks have remained central to schooling at all levels.

In situations where there is a shortage of teachers and where teacher training is sometimes limited in scope, textbooks are crucial in maintaining standards of quality and giving direction to the curriculum (Searle, 1989). When teachers are not present, learners are likely to be given activities from their textbooks. Likewise Fermig et al. (1989) highlight that “the textbook is likely to retain its primacy among instructional materials” (p. 213). Nonetheless “trends are visible from both technological and educational perspectives, which publishers of textbooks need to consider carefully” *(ibid.)*. Neumann (1989) reports that the national Minister of Education in France in March 1985 said, “The book remains, in spite of the appearance of newer devices, the principal support of teaching” (p. 127).

### 2.2.2 Qualities of a good textbook

This section discusses the characteristics of good textbooks in general and mathematics textbooks in particular. The ability to select the best textbook for a curriculum can determine the difference between the success and failure of a school or teacher to meet the objectives of a course.

Good textbooks must in general take into account the specific intended functions and also context in which they will be used (Nicol & Crespo, 2006). The design of textbooks is critical to their users, i.e. to both teachers and learners *(ibid.)*. In designing a quality mathematics textbook, the following elements should be considered:

- It should align with the curriculum (Gu, Huang, & Marton, 2004). The textbook should be written according to the prescribed syllabus and every aspect of the syllabus should be adequately covered *(ibid.).* In the same vein, Pepin & Haggerty (2004) states, “quality texts should align with the frameworks or syllabi they represent”.
- The approach that the textbook adopts and promotes needs to be stimulating and creative (Gu et al., 2004).
- Textbooks need to draw on mathematical history, and be driven by research on teaching and learning mathematics. Textbooks should be written by qualified, experienced and competent practitioners of mathematics or a committee of experts (Gu et al., 2004).
- The textbook should use style and vocabulary suitable for the age group of students for whom the book is written, and should provide for individual differences. It should meet the needs of students of varying abilities, interests and attitudes. The language used in the textbook should be simple and easily understandable and within the grasp of the pupils (Gu et al., 2004).

- There should be sufficient provision for revision, practice and review. The textbook must have an appropriate number of exercises that are aligned with the relevant assessment policy. It should satisfy the demands of examination (Gu et al., 2004).
- The content should be organized in order of increasing difficulty. (Gu et al., 2004).
- The textbook should contain some difficult problems or exercises to challenge the mathematically gifted students (Gu et al., 2004).
- The textbook should stimulate the initiative and originality of the students, and cultural and social aspects of learners need to be considered (Gu et al., 2004).
- The quality of visuals objects such as illustrations and graphics are critical elements that need to be considered (Namibia. MoEAC, 2015). Likewise Tyson-Bernestein (1989) states that, in analyzing the quality of the textbook graphics, which include illustrations, charts, maps and graphs, clarity and meaning should be the criteria; photographs and pictures help clarify the text (p. 92). Similarly, Neumann (1989) suggests some major criteria for evaluation, which include quality of presentations and attractiveness of illustration. Illustrations help motivate students to read and stimulate class discussion, and help students in thinking and problem solving (Tyson-Bernestein, 1989).
- The terms and symbols used must be those, which are current and internationally accepted. All the terms, concepts and principles used in the text should be clearly and accurately stated and defined (Pepin & Haggerty, 2004).
- The diagrams used in the textbook should be easily recognizable, and geometric constructions should be in proportion to the measurements prescribed by the problem (Gu et al., 2004).
- Searle (1989) suggests, “All textbooks for use in classrooms should be reviewed by experts and by teachers and should be field-tested in classrooms” (p. 37). He further contends that the purpose of the field-testing is to determine whether the books are suitable for the children who will use them – whether the language, the illustrations, and the level of difficulty are appropriate *(ibid.).*
- The textbook should offer suggestions to improve study habits (Gu et al., 2004).

Although the study focus of this study takes cognizance of the qualities listed above, its analytical framework is guided by the nature of VOs and not the quality thereof.

### 2.2.3 Outline of the Namibian process of selecting mathematics textbooks and the selected Grade 9 textbooks

Before examining the literature on visualisation and VOs, a brief outline of the Namibian selection process of mathematics textbooks is provided. It is a requirement in Namibia that all textbooks to be used in schools need to be evaluated and approved by the curriculum panels of Namibia National Institute for Education Development (NIED) (Namibia. MoE, 2008). The Namibian curriculum and learning support materials review policy and review cycle of 2015 lists four criteria, for content subjects such as mathematics, on which textbook evaluators mostly base their selection (Namibia. MoEAC, 2015):

- conformity to the subject syllabus
- appropriate coverage of the prescribed content
- language and editorial quality
- design, presentation and ease of use of the textbook

Although the use of visualisation is not specifically mentioned, it is partly incorporated in the last criterion, which looks at the usefulness, relevance and accuracy of graphics and illustrations as well as quality and attractiveness of illustrations. However, this criterion, in my view, insufficiently articulates the desired characteristics of VOs that are required. I argue that the criterion is not substantial enough to assess the appropriateness and quality of VOs in mathematics textbooks. This thus suggests that there is a gap in the Namibian evaluation criteria for assessing VOs in mathematics. I therefore envisage that this study may *inter alia* contribute to improving the quality of textbook evaluations and design of suitable evaluation criteria in Namibia.

The Namibia National Institute for Educational Development (NIED)'s curriculum panel is responsible for setting up the national curriculum for education in the country. This results in 157 different syllabuses for different subjects (Namibia. MoE, 2010). Textbooks are then developed by publishing houses in the private sector, where an attempt is made to align their content with the curriculum and syllabi developed by NIED (Namibia. MoE, 2008).

According to the policy (Namibia. MoEAC, 2015), "a Namibian textbook's lifespan is at least five years, as the curriculum will be reviewed in a five-year cycle" (p. 4). Further, "All textbooks in education require the evaluation and approval of the National Institute for Educational Development (NIED)'s curriculum panel" (Namibia. MoE, 2008, p. 4). The NIED curriculum panel approves three textbook titles for each subject per grade. The approved textbook titles are then listed in an official catalogue, which is sent to schools annually. The schools select and order their textbooks from this approved catalogue. Each school is given a budget of how much they may spend on textbooks per phase (for example Junior Primary Phase, Senior Primary Phase, Junior Secondary Phase and Senior Secondary Phase). Subject teachers are then advised to order subject textbooks of their choice. Teachers are allowed to order the textbooks that they deem suitable for their learners considering their school context, and they are not restricted to ordering only one textbook title. They may order all three textbooks, provided they remain within the given budget. All orders go through the Inspector of Education to the contracted distributors. They are returned to Regional Education Offices (REOs) who check them against the school textbook budget and finally submit the requests to the directorate of general services of the Ministry of Education (MoE) for procurement. The textbooks are then issued to teachers who use them in their classrooms. At schools where there is an adequate supply of textbooks, teachers may issue the books to their learners to take home for study and homework.

The textbook review policy states that only three titles per grade may feature in the textbook catalogue (Namibia. MoEAC, 2015). In 2008, NIED's curriculum panel approved three mathematics textbooks for grade 9 (Namibia. MoE, 2008). Two are written by Namibian authors and the third by a South African author. These are the books that are central to my research project. Their titles are *'y = mx+ c To Success', 'Maths for Life Grade 9' and 'Discover Mathematics Grade 9'*. All three cover the seven topics that are stipulated in the Grade 9 Namibian mathematics syllabus. Some of them have expanded the topics by including subtopics in their chapters. Table 2.1 illustrates the topics presented in the three textbooks and the number of VOs used in the chapters to be studied. The shaded topics are those relevant to my study.

Table 2.2.1 Topics represented in the three Namibian mathematics grade 9 textbooks and the number of VO evident in the chapters to be studied

| Topics stipulated in the grade 9 Syllabus | y = mx+ c | Number of VOs | Maths for Life 9 | Number of VOs | Discover Maths 9 | Number Of VOs | Total |
|---|---|---|---|---|---|---|---|
| Numbers and Operations | ✓ | | ✓ | | ✓ | | |
| Money and Finance | ✓ | | ✓ | | ✓ | | |
| Algebra | ✓ | 15 | ✓ | 9 | ✓ | 17 | 41 |
| Geometry | ✓ | 89 | ✓ | 76 | ✓ | 60 | 225 |
| Graphs | ✓ | | ✓ | | ✓ | | |
| Mensuration | ✓ | | ✓ | | ✓ | | |
| Statistics and Probability | ✓ | | ✓ | | ✓ | | |

### 2.2.4 Millennium Challenge Account (MCA) textbooks project

In 2007, a survey of 13 schools in Namibia, for Grades 5 to 12, revealed that textbook to learner ratios for the core subjects of English, Mathematics and Science averaged nearly 1 textbook to 5 students (Namibia. MCA, 2009). A shortage of books and other instructional materials was prevalent in Namibian primary and secondary schools (Namibia. MoE, 2008).

In July 2008, the government of the Republic of Namibia and that of the United States of America (USA) reached an agreement under which the USA government provided grant funding for public investment in education through the Millennium Challenge Account (MCA) for five years. In the education sector, the MCA aimed to improve the quality of general education and access to, and management of, textbooks (Namibia. MoE & MCA, 2010).
The objective of the Textbooks Activity in the Education Project of the MCA Namibia is “To give all learners access to a textbook for each priority subject” (Namibia. MoE, 2009, p. 3). “This project focuses on the procurement system and procedures as well as textbook management and utilization at school level” (Namibia. MoE, 2010, p.3). The overall objective of this sub-activity in education was to achieve a textbook to learner ratio of 1:1 by 2013 for core/key

subjects, which were in the first instance: English, Mathematics and Science at primary, junior and secondary levels (Namibia. MOE, 2008).

Millions of textbooks were distributed across the country and schoolchildren who would have gone without textbooks now have access to books in major subjects. At the end of the project, the report (Namibia. MCA, 2015) indicated a great improvement in the access to textbooks. The goal of a 1:1 learner to textbook ratio in the three core subjects was reached. This is evident in schools across Namibia in general and in our region, Oshana, in particular.

## 2.3 VISUALISATION

### 2.3.1 Mathematical Visualisation

Visualisation is not a recent invention in mathematics education (Zimmerman & Cunningham, 1991). Nevertheless, visual objects in textbooks have only recently attracted research attention and Fotakopoulou & Spiliotopoulou (2008) deem visual objects in textbooks to be important teaching and learning tools in mathematics. They further argue that in mathematical visualisation, the interest is on the student's ability to draw an appropriate diagram (with a pencil or pen, or in some cases, with a computer) to represent a mathematical concept or problem and to use the diagram to achieve understanding, and as an aid in problem solving.

In mathematics, visualisation is not an end in itself but a means toward an end, which is to gain understanding (Fotakopoulou & Spiliotopoulou, 2008). In the same spirit, Konyalioglu (2003) states that by using a visualisation approach many mathematical concepts can become concrete and clear for students to understand. The meaningful transformations of representations are at the core of understanding human information processing (Mckendree, Monaghan, Conlon, Lee, & Small, 2000).

The term visualisation is defined in different ways by various mathematics researchers. According to Zimmermann & Cunningham (1991, p. 3), "mathematical visualisation is the process of forming images and using such images effectively for mathematical discovery and understanding". Similarly, Lohse, Biolsi, Walker, & Rueler (1994) define visualisation as "the study of mechanisms in computers and in humans, which allow them in concert to perceive, use, and communicate visual information" (p. 37). Likewise, in his discussion on the roles of visualisation, Arcavi (2003) defines visualisation as:

“The ability, the process and the **product** of creation, interpretation, use of and reflection upon **pictures, images, diagrams**, in our minds, **on paper** or with technological tools, with the purpose of depicting and communicating information, thinking about and **developing previously unknown ideas, and advancing understandings**”.

This definition resonates well with my study, as it focuses on VOs, which are “the product of creation”. On the other hand, Zazkis, Dubinsky & Dautermann (1996), defines visualisation as an act in which an individual establishes a strong connection between an internal construct and something to which access is gained through the senses. They further elucidate that such a connection can be made in two directions. An act of visualisation may consist of any mental construction of object or processes that an individual associates with objects or events perceived by an external source. Alternatively, it may consist of the construction, on some external medium *(ibid.).*

Thus, visualisation can be used as a powerful tool for teaching and learning mathematics (Konyalioglu, 2003; Arcavi, 2003; Fotakopoulou & Spiliotopoulou, 2008). According to Steinberg (2005) and Sobbeke (2005), taking a visual approach to teaching and learning mathematics can be meaningful and effective. Although visual objects in textbooks are important teaching and learning tools, only recently have they attracted research attention (Fotakopoulou & Spiliotopoulou, 2008).

### 2.3.2 Nature and Types of VOs

A VO refers to any configuration of characters, images, concrete objects etc., that can symbolize or “represent” something else (Kaput 1985; Goldin, 1998; DeWindt-King & Goldin, 2003). Similarly, Diezmann & English (2001) define a diagram as a “visual representation that displays information in a spatial layout” (p. 77). McKendree et al. (2000) define a representation as a structure that stands for something else: a word for an object, a sentence for a state-of-affairs, a diagram for an arrangement of things, a picture for a scene (p. 59). On the same note, Moyer in his 2001 study remarks, “Specific mathematical apparatus, or manipulatives, are ‘objects designed to represent explicitly and concretely mathematical ideas that are abstract” (p. 176). In similar fashion, representations such as numerals, algebraic equations, graphs, tables, diagrams, and charts are *external* manifestations of mathematical concepts that “act as stimuli on the senses” and help us understand these concepts (Janvier, Girardon, & Morand, 1993, p. 81).

On the other hand, Pape & Tchoshanov (2001) explain, “Representations may be thought of as *internal*—abstractions of mathematical ideas or cognitive schemata that are developed by a learner through experience” (p. 119). Similarly Presmeg (1989) describes a visual image as a “mental scheme depicting visual or spatial information” (p. 42) and “visual representations are data structures for expressing knowledge” (Lohse et al., 1994).

Duval (1999) suggests, “Invisible mathematical tools need other forms of representation to become communicable” (p.3). They can be presented as VOs, which include drawings, pictorial illustrations/pictures, diagrams, charts and graphs (Fotakopoulou & Spiliotopoulou, 2008). Kim (2012) observes, “More pictures, illustrations, and diagrams have been used in recent textbooks than in the past” (p. 175). Kim’s study finds that visual representations in mathematics textbooks can serve not only as informative agents but also as “tools for thinking” which students manipulate (p. 178). Many researchers deem the use of VOs to be a powerful tool in learning and understanding mathematical concepts (Fotakopoulou & Spiliotopoulou, 2008; Kim, 2012; Gellert & Steinbring, 2014).

Visual objects can be presented in different forms. Monoyiou, Papageorgiou, & Gagatsis (2007) identify two forms of VOs in mathematics: the iconic and the linguistic. They further expound that if an object is not an icon/picture then it is a linguistic property and depends on the language *(ibid.).* Likewise, “Visual objects belong to different classes of representations; they are depictive representations” (Schnotz & Bannert, 2003). “Depictive representations include iconic signs that are associated with the content they represent through common structural features on either a concrete or more abstract level” (Monoyiou et al., 2007, p. 2). Schnotz (2002) makes a similar point when emphasizing the distinction between descriptive (symbolic) and depictive (iconic) representations. Thus, depictive representations are most useful in providing concrete information and are often efficient as specific information can just be read off. Alternatively, descriptive representations can more easily express abstract information as well as more general negations and disjunctions *(ibid.).*

There are many frameworks proposed for categorizing VOs into different types (Cox & Brna, 1995; Lohse et al., 1994). Although there are many classifications, no single classification is universally accepted. For example, Lohse et al. (1994) propose a framework where categorization can be characterized as either functional or structural. Functional categorization focuses on the intended use and purpose of the object, while a structural category focuses on the form of the image rather than its content *(ibid.)*; they offer an example of a structural classification with 10 major clusters/categories:

- graphs
- numerical and graphical tables
- time charts
- cartograms
- icons
- pictures
- networks
- structure diagrams
- process diagrams
- map clusters.

These objects may take a variety of forms, from diagrams and models to graphs and symbolic expressions (Konyalioglu, 2003). Moyer (2001) stresses the variety of representations: pictures, invented symbols, combination charts, tables, and algebraic symbols (p. 239). On the same note, Konyalioglu in his 2003 study categorized the visual objects such as graphs, diagrams, pictures and geometrical shapes or models as a tool for visualisation of an abstract concept in mathematics. Similarly in her study Ainsworth (2006) discusses the “different types of representations e.g., histogram, equation, table, line graph, narrative text, picture” (p. 90). She further states that some visual representations are more efficient than others for conveying information *(ibid.).*

On the other hand, Guzman (2002) in his study categorizes visualisation objects into four different types according to their roles in teaching and learning mathematics: isomorphic, homeomorphic, analogical and diagrammatic visualisation. He describes Isomorphic visualisation as objects that may have an "exact" correspondence with the representations we make of them. Homeomorphic visualisation is a kind of VO where some of the elements have certain mutual relations that imitate sufficiently well the relationships between the abstract objects, and can thus provide us with support, sometimes very important, to guide our imagination in the mathematical processes of conjecturing, searching, and proving. With analogical visualisation, we mentally substitute the objects we are working with by others that relate in an analogous way and whose behavior is better known or perhaps easier to handle, because they have already been explored. Diagrammatic visualisation is a type of visualisation where mental objects and their mutual relationships in aspects that are of interest to us, are merely represented by diagrams that enhance or aid our thinking processes. One could say that in many cases such diagrams are similar to mnemotechnicrules e.g. tree diagrams *(ibid.).*

Although many different types of VOs are discussed below, due to the limited scope of this thesis, only a few are considered. These become apparent as the narrative unfold.

### 2.3.3 Functions and roles of VOs in teaching and learning mathematics

Many researchers record that the use of VOs are central elements in the effective teaching and learning of mathematics (Stacey & Vincent, 2009; Gellert & Steinbring, 2014; Dimmel & Herbest, 2015). Arcavi (2003) suggests that visual representations help us to 'see the unseen'. Likewise, Sobbëke (2005) proposes that in mathematics classrooms, visual diagrams help the learners to see mathematical concepts and ideas more clearly. Gellert & Steinbring (2014) affirm that visualisation representations can be used as an epistemological tool to explore mathematical structures and bring about new meaning. Visual objects are "especially important in textbooks, which aim to lead non-expert readers to an understanding of the esoteric domain of a scientific field" (Fotakopoulou & Spiliotopoulou, 2008, p. 321). Woodward (1989) describes the pictorial as the "dominant and defining" element in books (p. 101).

The roles of visualisation in mathematics teaching and learning cannot be over emphasized. Guzman (2002) affirms that visualisation is extraordinarily useful in the context of the initial process of 'mathematization' as well as in that of teaching and learning mathematics (p.11). Pictorial representations clarify and enhance student learning (Evans, Watson & Willow, 1987), and other studies have claimed that visual information is recognized and remembered for longer durations than verbal information alone (Levie, 1987; Mayer, 1989; McDaniel & Pressley, 1989; Peeck, 1987). In agreement, Sfard (1991) reveals that visualisation makes abstract ideas more tangible, and encourages treating them almost as if they were material entities (p. 6). Larkin & Simon (1987) says that diagrammatic representation preserves information explicitly (p. 66). Studies have shown a multitude of findings about visual images in texts. Still others have demonstrated that illustrations improve reader comprehension (Holliday & Harvey, 1976), and that they are especially beneficial in helping poor readers to better understand the written word (Fleming & Levie, 1993).

Teaching and learning of mathematics benefit from effective use of visual and practical aids (Drew, 2007). Ainsworth (2006) in her research on learning with multiple representations has shown that "when learners can interact with an appropriate representation their performance is enhanced" (p. 183). She also expresses that the concern is not regarding the effectiveness of visual objects but rather the circumstances that influence the effectiveness of objects. She further suggests that matching the type of representation to the learning demands of the situation can significantly improve performance and understanding *(ibid.).*

VOs that are used in textbooks are of great significance and play a great role in learning and teaching mathematics. "It is now well accepted that the use of particular modes of visual objects leads to improvement of students' mathematical abilities and development of their advanced problem solving and reasoning skills" (Krutetskii, 1976; Yakimanskaya, 1991; Presmeg, 1999; Pape & Tshochanov 2001). In addition, Lohse et al. (1994) says, "visual objects can facilitate problem-solving and discovery by providing an efficient structure for expressing the data" (p. 37). In agreement, Diezmann & English (2001) suggest, "In problem solving, a diagram can serve to unpack the structure of a problem and lay the foundation for its solution, and can also serve as a tool of mathematical learning" (p. 77). Likewise, Ho in his 2010 study reflects that visualisation is at the heart of mathematical problem solving and "it can be a powerful cognitive tool in problem solving" (p. 1). He further identifies seven roles/functions of visualisation when it comes to problem solving. Visual objects help learners to understand the problem, to simplify the problem, to see connections to a related problem; they cater for individual learning styles, serves as a substitute for computation, as a tool to check the solution, and can transform the problem into a mathematical form *(ibid.)*.

VOs are a type of external representation that is used extensively in mathematics textbooks and are considered to enhance problem solving in all the phases (Larkin & Simon, 1987; Monoyiou et al. (2007). Many researchers consider imagistic representations as a fundamental cognitive system for mathematical learning and problem solving (De Windt-King & Goldin, 2003), while expert mathematicians as well as mathematics students perceive visual representations as a useful tool in mathematical problem solving and frequently attempt to use them. Pape & Tchoshanov (2001) states, "representations allow individuals to track intermediary results, ideas, and inferences" (p. 124). Since VOs embody the important relationships presented in data or a word problem, they lighten the cognitive load of the individual and serve to organize the individual's further work on a problem. Given the representation, the learner may work on alternative parts of the problem. Further representations then may be used to facilitate an argument and support conclusions *(ibid.).*

Recent researchers in their findings stipulate that visual objects can be used as a tool for reasoning. Pape and Tchoshanov (2001) suggest, "Representations must be thought of as tools for thinking, explaining, and justifying" (p. 126). Furthermore, "Pictures, graphs and signs, and other spatial representations play an important role in human communication and reasoning, in part because we are able to interpret or infer meanings from these representations without specific instruction in how to do so" (Monoyiou et al., 2007, p. 2). On the same note Triantafillou et al. (2013) highlight that in mathematics "VOs seem to undertake more significant roles, acting as starting points or as the fundamental tools in reasoning" (p. 1).

Diagrams in mathematics texts, such as textbooks, are often used as visual objects to enhance the learning of mathematical concepts (Konyalioglu 2003; Guzman 2002; Steenpaβ & Steinbring, 2014). These visual objects can illustrate an abstract idea in a concrete way or they can reinforce a mathematical procedure diagrammatically. Levin & Mayer, as cited in Kim (2012), suggest that "since visual representations provide students with concrete and concise images of related concepts, they help improve students' understanding of the contents" (p. 177). Kilpatrick et al. (2001) define "conceptual understanding as a comprehension of mathematical concepts, operations, and relations" (p. 116). The use of visualisation objects is integral to the process of understanding mathematics.

Past researchers note that instructional materials and textbooks today include more pictures, diagrams and graphs than ever before (Schnotz, 2002; Carney & Levin, 2002). These VOs are an "essential part of mathematical concepts, a tool for manipulating them during instruction and a means of evaluating understanding in mathematics" (Monoyiou et al., 2007, p. 1). Each VO provides information about certain aspects of the concept (Watkins, Miller, and Brubaker, 2004). According to Woodward (1989), readers are attracted to the illustration first and subsequently decide whether to read the accompanying explanation to further their understanding.

In addition, "representations can be used by children in distinctly different ways to support mathematical learning and communication" (Woleck, 2001, p.216). Sfard (1991) highlights that “visual representation is holistic in its nature and various aspects of the mathematical construct may be extracted from it by ‘random access’" (p. 7). VOs are not static products but rather, they capture the process of constructing a mathematical concept or relationship, so that they may allow mathematicians to record, reflect on, and later recall their process and thinking *(ibid.).* Monoyiou et al. (2007) in their study show that visual representations play an important role as an aid for supporting reflection and as a means in communicating mathematical ideas. Reading pictures is not like reading written language in which the connections between symbol and referent are arbitrary *(ibid.).* In their study they discovered that “text information is remembered better when pictures illustrate it, rather than when there is no illustration” (Levie & Lentz, 1982).

Guzman in his 2002 study says the image, as we have seen, has very important uses in many different types of mathematical activity. It is a stimulating influence for the rise of interesting problems in different ways. It is also a rapid vehicle for the communication of ideas *(ibid.).* Bosch, Chevallard, & Gascon in their 2006 study describes that a picture is worth a thousand words, a picture gives a meaning better than the written text. As regards the effects of visual representations, in some cases the presence of visual representations was found to have a helpful role on students’ performance (Monoyiou et al., 2007).

According to Duchastel and Waller (1979), VOs are found in seven functional categories, which are classified according to their roles. These seven categories include:

- expressive
- descriptive
- constructional
- functional
- logico-mathematical
- algorithmic
- data display

On the same note Carney and Levin (2002) proposes five functions that VOs serve in text learning, which is defined as “the processing—which includes perceiving, understanding, and remembering—of text Information” (p. 87), which are:

- Decorative pictures simply decorate the page bearing little or no relationship to the text content.

- Representational pictures mirror part or all of the text content and are by far the most commonly used type of illustration.
- Organizational pictures provide a useful structural framework for the text content.
- Interpretational pictures help to clarify difficult text.
- Transformational pictures include systematic mnemonic components that are designed to improve a reader's recall of text information.

Monoyiou et al. (2007) proposes a similar categorization for the functions of pictures in solving mathematical problems. Specifically, they suggested that pictures have the following four functions in mathematical problem solving:

- Decorative Pictures - do not provide any information to the students for the solution of the problem, but simply decorate the page bearing little or no relationship to the problem content.

- Auxiliary-representational pictures - mirror part or all of the problem content, but are not necessary in order to solve the problem.

- Auxiliary-organizational pictures - help the students solve the problem by guiding them to organize the given statements of the problem, e.g. by drawing a picture that leads to the solution of the problem. Auxiliary-organizational pictures do not necessarily need to be used in order to solve the problem.

- Informational - the last function that pictures serve is informational. Informational pictures provide information that is necessary in order to solve the problem; i.e. the problem is based on the picture.

Visualisation as a method of 'seeing the unseen' in images (Arcavi, 2003, p. 216), is being recognized as a key component of reasoning *(ibid.).* In agreement, Triantafillou, Spiliotopoulou & Potari (2013, p. 1) acknowledge that "in mathematics, visualisation representations seem to undertake a more significant role acting as starting points or as the fundamental tools in reasoning". They further identified the roles VOs play:

- Decorative (there is no caption and there is no reference from the main text to the photograph)

- Illustrative (include a caption that describes what the reader is to see in the photograph but the caption does not provide additional information to the main text)
- Explanatory (captions provide an explanation or a classification of what is represented in the photographs)
- Complementary (captions add new information about the subject matter treated in the main text). On the same note, Monoyiou et al. (2007) elaborates that in the complementary category, the content of the VO complements the reasoning by adding new information not mentioned before in the main text.

In order to better understand VOs in Grade 9 mathematics textbooks, which are under study, the adopted and modified framework illustrated in Figure 2.1 will be used. This framework is adopted from the study conducted on analysis of visualisation representations found in secondary school textbooks of Economics in Greek secondary schools. I found this framework very helpful as it enabled me to analyze and classify VOs of my selected textbooks. I thus adapted the framework to suit the purposes of my study by ensuring a mathematical context. For example I dispensed with the original 'conceptual map' and replaced it with 'a distribution diagram' to suit the mathematical context.

VOs can be categorized into many groups. For the purpose of this study they will be categorized according to their type, relation to the content, relation to reality, function and dimension (Fotakopoulou & Spiliotopolou, 2008), as in Figure 2.1 below.

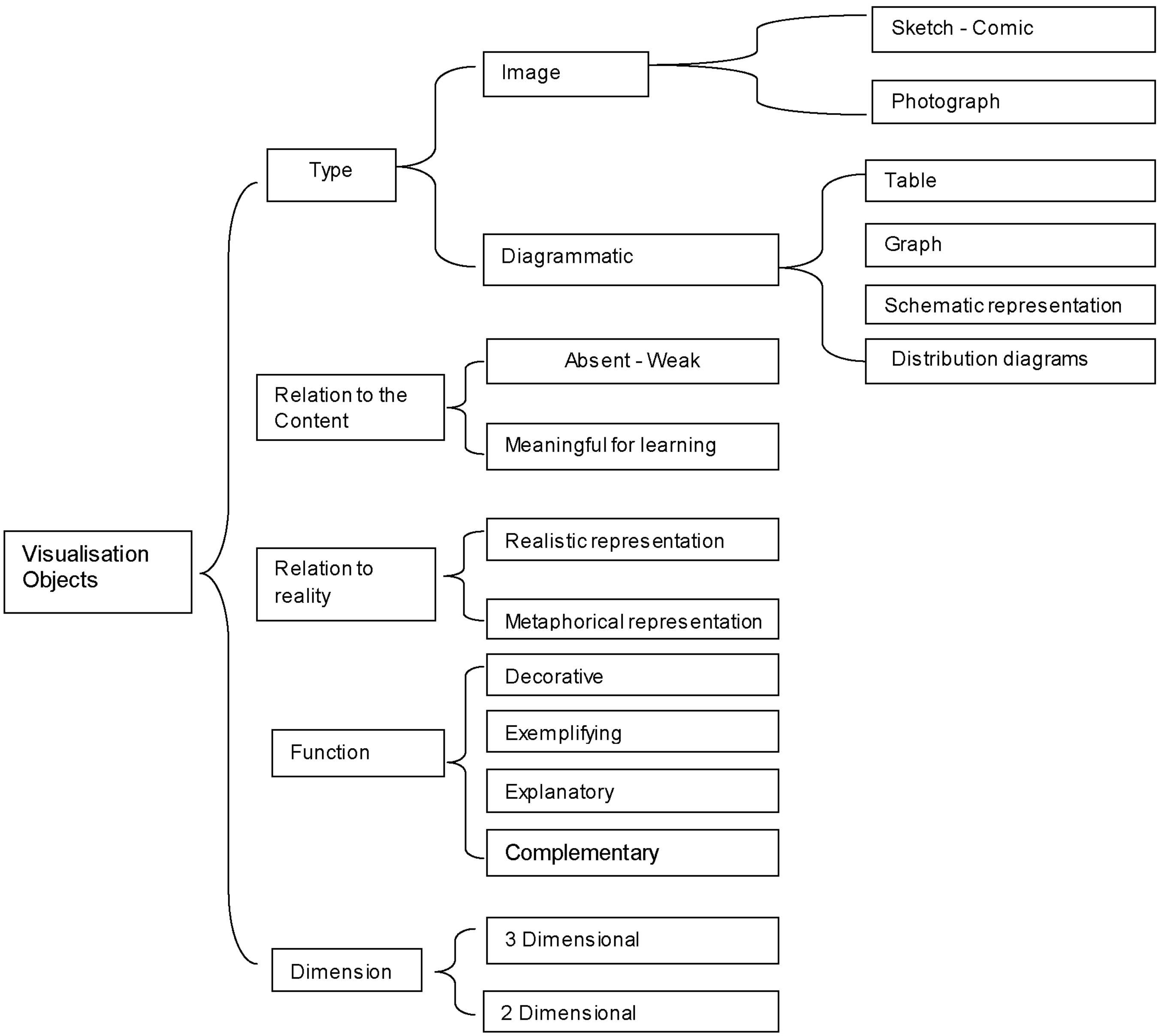



Figure 2.1The Analytical tool used to analyse VOs

(Source: Adapted from Fotakopoulou & Spiliotopoulou, 2008)

In terms of their type, the objects can be categorised into two groups (Fotakopoulou & Spiliotopoulou, 2008) viz. as an image or as a diagrammatic:

- An image refers to any sketch, picture, comic or photograph
- Diagrammatic visuals are like tables, graphs or schematic representations. In terms of relation to the content, the VOs can be either meaningful or irrelevant to the content.

In terms of their relationship to reality, VOs used can be classified as:

- Realistic - representing something in the real world or
- Metaphoric - something that has no real existence, but only illustrate reality.

In terms of the function of visualisation objects, they can be used:

- For decoration i.e. to make the page look attractive and interesting.
- To exemplify an idea or explain a concept
- To complement a definition or explanation

Finally, a visual object can be represented as either 2-dimensional or 3-dimensional.

### 2.3.4 Challenges of visualisation

Past researchers have discovered some limitations of using visual objects that teachers need to take into consideration (Ho. 2010; Pape & Tchoshanov 2001; Gagatsis & Elia 2004; Stylianou 2001; Drew 2007). Steenpaβ & Steinbring (2014) warn that mathematical visual images can be ambiguous elements that do not necessarily convey the concept effectively to students, and can thus lead to misunderstanding. In addition, Gellert & Steinbring (2014) state that "interpreting mathematical visual diagrams is a challenge faced by mathematicians as well as students" (p.16). Therefore, students require appropriate assistance and guidance from teachers and knowledgeable peers as they select, interpret, and create visual models of mathematics (Moyer & Jones, as cited in Moyer, 2014, p.3). For these reasons, it is important to understand how representations are used in mathematics textbooks because "representation is more than a process; it is a way of teaching and learning mathematics" (Fennell & Rowan, as quoted by Kim, 2012, p.1).

Furthermore Monoyiou et al. (2007) highlight that “the use of visual representations in mathematical teaching and learning is a multidimensional and complicated process and should be conducted with great attention” (p. 8). Guzman in his study points out that “visualisation leads to errors" (2002, p. 12). In agreement Pape & Tchoshanov in their study discover that “students have great difficulty developing fluency in using representational forms in mathematics” (2001, p. 123). In the same vein Drew (2007) accentuates that “the selection and effective use of appropriate mathematical resources requires careful consideration and planning on the part of the teacher” (p. 21). The appropriateness of the resource should be “judged by the extent to which the mental images that children form as a result are likely to be helpful or unhelpful in structuring their thinking” (Bottle, 2005, p. 84). It is noted that reading and using images constitute skills that should not be left to chance, but should be taught systematically (Dreyfus & Eisenberg, 1990).

Learners should understand the form of representations because diagrams need to be “read” (Zodik & Zaslavsky, 2007, p. 266). Cognitive processes are needed in order to make sense of and understand diagrams (Eisenberg & Dreyfus, 1991). Gagatsis & Elia (2004) say that students do not have adequate training associated with the use of visual representations and lack that particular skill (p. 453). In different contexts, the “same visual objects may have different meaning even for experts” (Arcavi, 2003, p. 232). Ainsworth (2006) records that “learners must know how a representation encodes and presents information and they need to understand the relation between the representation and the domain it represents” (p. 186). Further, Watkins, Miller, & Brubaker (2004) suggest that young learners must be better prepared to interpret and utilize the visual information presented to them in elementary books (p. 23). They further point out that illustrations may not promote student understanding, but may, in fact, encourage misconceptions. Thus, illustrations may not foster an accurate understanding and may contribute to misunderstanding, alone or embedded within a text *(ibid.).*

### 2.3.5 Helping learners to develop visualisation skills

In his Ho, 2010 study, Ho suggests that in order to help our learners develop visualisation skills, teachers and curriculum designers need to be mindful of the processes and roles that visualisation plays in mathematical problem solving. Although there is much evidence to suggest that illustrations promote learning and understanding of text, other educators insist that illustrations, if misused or misinterpreted, may in fact interfere with learning (Downey, 1980;

Weidenmann, 1987; Waddell, McDaniel & Einstein, 1988; Pena & Quilez, 2001). Downey (1980) has argued that the effectiveness of visual representations is dependent on the learner's ability to independently and accurately interpret them. Thus, the issue of pictorial utilization manifests itself. The creator has the responsibility of developing visual images that are age-appropriate, and the learner has the task of realizing their full meaning and significance *(ibid.).*

Because of the widespread and growing use of illustrations in books, learners must develop visual literacy skills, which include the ability to examine different pictorial representations in a variety of contexts and make meaning from the images (Watkins et al. 2004; Glasgow, 1994; Moline, 1995; Stonehill, 1998; Morse, 1994). According to Glasgow (1994), for learners to become visually literate, they must be schooled, just as they are schooled in reading. She wrote, "Since our goal is to educate them (students) to make text their own, we must provide them with skills to interpret the content of visual images as well as print" (p. 499). This is particularly true in children's publications, where illustrated books have become the most prominent literary form (Pressley & Miller, 1987).

In agreement, Diezmann & English (2001) suggests that students who lack knowledge about the utility of diagrams “need help to use the diagrams as an effective mathematical tool” (p. 77). This can be done through diagram literacy. The term diagram literacy refers to knowing how to use diagrams and being able to use that knowledge appropriately *(ibid.).* Similarly, Glasgow (1994) says that the ability to use diagrams effectively is integral to mathematical thinking and learning. Likewise, diagram literacy is described as a component of visual literacy, which is “the ability to understand (read) and use (write) and to think and learn in terms of images” (Horton 1994, p. 25).

Consequently Watkins et al. (2004) accentuates that it is important that authors, publishers, teachers, and others who are seeking to improve student learning through the use of visual representations, be cognizant of the manner in which learners construct in their minds cognitive structures which constitute the meaning and understanding of their world. They and other researchers further suggest some possible strategies for educating students about visual literacy, which include:

- Instruction by teachers on the nature of illustration and how to observe and interpret the pictorial representation alone and in conjunction with the written word (Watkins et al., 2004).
- "Teachers need to spend time and effort talking through the meaning of the images..." with their students (Stylianidous, Ormerod & Ogborn, 2002, p. 257).
- Use of simple captions, set apart from the illustration by color or border, which would be aimed at attracting reader interest to encourage them to read and verify their preconceived notions regarding the visual representation (Watkins et al., 2004, p.36).

## 2.4 THEORETICAL FRAMEWORK

### 2.4.1 SOCIAL CONSTRUCTIVISM & VISUALISATION

From a constructivist point of view, "all knowledge is constructed" (Noddings, 1990, p.7). Constructivism proposes, "We construct our knowledge of our world from our perceptions and experiences, which are themselves mediated through our previous knowledge" (Simon, 1995, p. 115).

A constructivist perspective maintains that individuals' views and understanding of the world around them are based on an ongoing lifelong process of building and constructing knowledge (Saunders, 1992; Kelly, 2000; Slater, Carpenter & Safko, 1996; Watkins et al., 2004). Constructivism also emphasizes the importance of a learner being actively involved in the learning process. A social constructivists' perspective places the learner within a social setting, observing the individual as a social participant not just as an individual (Smith, 1999). Social constructivists place special emphasis on the cultural and sociological processes through which knowledge is formulated and constructed (Ernest, 1991 as cited by Goldin & Shteingold, 2001). Smith (1999) states that it is the "Vygotskian tradition which emphasizes the situated nature of learning within social and cultural settings" (p.411).

Yackel (2001), a constructivist, indicates, "Students construct their own meaning from the words or visual images they see or hear" (p.41). Hence, students need various resources, such as books, technological devices and media with VOs to construct knowledge. Textbooks are traditionally a key resource for teaching and learning mathematics worldwide. In most cases the textbooks are the most readily available and accessible resource for teachers and learners in the classroom. Thus, textbooks are a necessary and critical means for knowledge construction.

By interpreting VOs used in textbooks, learners become more fully involved in learning. However, VOs presented in textbooks need interpretation by learners who are trained in diagram literacy. Pape & Tchoshanov (2001) state that visualisation is not an immediate vision of the relationships, but rather an interpretation of what is presented for our contemplation, which we can only do when we have learned to appropriately read the type of communication it offers us (p. 123). In the constructivist tradition, they can be assisted in this process by teachers, experts, and peers in a social setting such as a classroom (Wertsch, 1997).

From a constructivist perspective, Pape & Tchoshanov (2001) highlights that the "necessary mapping between the concrete materials and the arithmetic algorithm (procedure) requires intensive social co-construction of meanings" (p. 123). Teachers and students co-construct their understanding of the steps of the mathematical operation while manipulating the materials *(ibid.)*. This happens as learners and teachers are engaged in interpretations of objects found in textbooks. In agreement, Graven, Stott, Mofu, & Ndongeni (2015) note that "learning mathematics is an active process, each child constructs their own mathematical knowledge and they develop mathematical concepts as they engage in sense-making, mathematical activity" (p. 71). They further clarify that this takes place alongside a teacher or other adult.

Prawat & Floden (1994) argues that without social interaction with other more knowledgeable people, it is impossible for a learner to acquire social meanings of important symbol systems and to learn how to utilize them. Moreover, Kukla (2000) indicates that constructivism also emphasizes the importance of meaningful discussions in the learning process that takes place in classes. Graven et al. (2015) point out the two-way learning as well as the learning through interaction between learners and teachers. They further clarified that learning is not just from teachers, it can also be from and between peers.

In line with the social constructivists' perspective, VOs need to be actively interpreted, embedded in a social cultural milieu (Steenpaß & Steinbring, 2014). Learning mathematics can be conceived as a "process of interpreting visual diagrams in a more and more complex way" (Steinberg, 2005, p. 13). As learners interpret VOs in textbooks, comprehension of mathematics concepts, operations and relations is enhanced. Likewise, Kilpatrick et al. (2001) state that interpretation of visualisation objects is integral to the process of understanding mathematics. The use of visual diagrams helps the learners to better see mathematical concepts and ideas

(Sobbëke, 2005). It also helps learners to explore mathematics structures and enhance their meaning (Gellert & Steinbring, 2014).

Pape & Tchoshanov (2001) state that practicing representations must be part of a social environment (p.124), where teachers educate and help learners to develop visualisation skills to enable them to interpret VOs presented in textbooks (Ho, 2010). "Learning to construct and interpret representations involves learning to participate in the complex practices of communication and reasoning in which the representations are used" (Greeno & Hall, 1997, p. 361). While practicing the construction and use of representational forms, students negotiate the meanings of the forms they have produced as well as the meanings of standard representational forms (DiSessa, Hammer, Sherin, & Kolpakowski, 1991; Greeno & Hall, 1997).

Thus, when the goal of instruction is to learn to represent mathematical concepts or to solve problems involving mathematical representations, students must be given the opportunity to interact with one another and the teacher. Through this interaction within problem-solving situations, knowledge of mathematical representations and mathematical understanding emerges and develops (Pape & Tchoshanov, 2001, p. 124).

To understand how learners learn, Bruner (1990) developed three stages of representation: the enactive mode, the iconic mode and the symbolic mode. "These three modes of representing our experience are considered important to the development of children's understanding" (Drew, 2007, p. 20):

- The enactive mode involves representation of ideas through taking some form of action such as manipulating physical objects and performing some physical task.
- The iconic mode, which is significant for this study, involves representing those ideas using pictures or images (Drew, 2007). In the same manner Post, Post, Van Walsum, & Silver (1995) states that the general meaning of the term 'icon' is a conventional pictorial representation or a sign sharing a property with the thing it represents (p. 2).
- The symbolic mode involves ideas represented through language or symbols usage, whereby learners are expected to judge and think critically (Bruner, 1990).

Bruner (1966, as quoted in Drew, 2007) proposes that the use of physical resources, models and images in mathematics teaching and learning relates well to the enactive and iconic modes of representation. VOs such as images, pictures, graphs, diagrams and photos that are used in textbooks are all capsuled in the second level of engagement. Thus, the iconic mode resonates well with the purpose of my study, which is focusing on VOs used in different textbooks.

As learners learn in the second level, they start making a network of connections between concrete experiences, pictures, language and symbols that could be significant to the understanding of a mathematical concept (Haylock & Cockburn, 2003). However, (Cobb, Yackel, & Wood, 1992) contest this formulation as problematic because instructional materials said to embody mathematical concepts are developed by experts, embody experts' conceptions of mathematical ideas, and may not be readily available or understandable to the novice. Glasgow (1994), suggest that for learners to become visually literate and not remain novice, they must be schooled, just as they are schooled in reading.

Students who lack knowledge about the utility of diagrams "need help to use the diagrams as an effective mathematical tool" from teachers, other students and experts (Diezmann & English, 2001, p. 77). It is "only when the use of representations is built up in the classroom as a cultural activity that students are able to come to an understanding of the meanings of the concrete materials and the associated symbolism" (Pape & Tchoshanov, 2001, p.123). That is, in order for the connection to be made between external representations and the mathematical concept they represent, and between the mathematics and the children's experiences, representations must be viewed as vehicles for exploration within social contexts that allow for multiple understandings of mathematical content (Seeger, 1998).

### 2.4.2 Constructivists' views on teaching

Constructivism does not define a particular way of teaching (Simon, 1995). It does not determine the appropriateness or inappropriateness of teaching strategies *(ibid.).* However, in a constructivist mathematics classroom it is acknowledged that "social interaction plays an important role in students' mathematical learning" (Cobb et al., 1992, p.5). With VOs in textbooks, "teachers need to spend time and effort talking through the meaning of the images..." with their students (Stylianidous et al., 2002, p. 257). This is called diagram literacy.

In order for the teacher to teach well, they need to know what their students are thinking, how they produce the chain of little marks we see on their papers and what they can do with the materials we present to them (Noddings, 1990). Similarly in mathematical visualisation, the interest is in the student's ability to draw an appropriate diagram (with a pencil or a pen, or in some cases, with a computer) to represent a mathematical concept or problem and to use the diagram to achieve understanding, and as an aid in problem solving (Spiliotopoulou, Karatrantou, Panagiotakopoulos, & Koustourakis, 2009).

Cobb et al. (1992) points out that "the teacher has a role of fostering the development of conceptual knowledge among her/his students and facilitating the constitution of shared knowledge in the classroom community" (p. 1). Cobb (1988) states that "the teacher's primary responsibility is to facilitate profound cognitive restructuring and conceptual reorganizations" p.89). Simon (1995) alludes, "understanding learning as a process of individual and social construction gives teachers a conceptual framework with which to understand the learning of their students" (p.117). Students actively construct their mathematical ways of knowing as they are initiated into the taken-as-shared mathematical practices of wider society by the teacher" (Cobb et al., 1992). Cobb (1988) states, "teachers and students are viewed as active meaning makers who continually give contextually based meaning to each other's words and actions as they interact" (p.88). This happens as the teacher helps learners to interpret the VOs given in either textbooks or elsewhere. Therefore, it is also the teacher's responsibility to help learners learn how to interpret VOs used in textbooks, which can enhance conceptual understanding and comprehension of mathematical concepts (Diezmann and English 2001).

## 2.5 CONCLUSION

The purpose of this chapter was to provide a background to this study. The relevance of textbooks in teaching and learning of mathematics was discussed, the qualities of a good textbook were presented, and a brief outline of the process of selecting mathematics textbooks for Namibian schools was given. The intervention of the Millennium Challenge Account Textbooks Projects (MCA) was explained. An overview of relevant aspects of visualisation was presented. Visualisation was defined, and the nature, types, roles, and functions of visualisation were described. Challenges of visualisation and ways of helping learners to develop visualisation skills were also presented. Finally, this chapter also reviewed literature on the theoretical framework underpinning this study of social constructivism. These points prepared the way for the next chapter, where the research methodology is described.

# 3. CHAPTER THREE: METHODOLOGY

## 3.1 INTRODUCTION

The focus of this study was to investigate and analyze the nature of VOs that are evident in three grade 9 mathematics textbooks. This chapter outlines and reports on the methodologies used in this research study. It also provides reasons as to why the specific methods were chosen in this study. The structure of this chapter includes the following: research goals, research methodology, research orientation, sampling, data collection tools and data analysis. The issues pertaining to ethics and limitations of the study are also discussed in this chapter.

## 3.2 RESEARCH GOALS

The aims of this study are threefold. Firstly, to analyse the VOs used in three grade 9 Namibian mathematics textbooks in terms of their types, roles, relation to mathematical contents, and relation to reality and visual dimensions. Secondly, to understand selected mathematics teachers' views and perceptions about VOs in terms of their use. Finally, to describe the choices that the two selected authors made in selecting the VOs used in their textbooks. The three research questions that framed this study were:

- What is the nature of different VOs evident in the Namibian grade 9 mathematics textbooks?
- How are these VOs viewed in terms of their use by selected teachers in their teaching?
- What were the authors' rationales for using the identified VOs in their textbooks?

It is hoped that this study will contribute towards improving the quality of textbook evaluations, and the design of suitable and more comprehensive assessment procedures in Namibia. It is further hoped that this study will provide some insights into the use and nature of VOs in textbooks and that it creates a critical awareness of the role of VOs in textbooks amongst teachers, inspiring them to help their learners interpret the VOs effectively. Given the importance of VOs, it will hopefully inspire potential authors to use suitable and appropriate VOs that enhance conceptual teaching and learning of mathematics.

## 3.3 RESEARCH ORIENTATION

This case study research was underpinned by an interpretive paradigm using a mixed method approach, utilizing qualitative and quantitative methods (Bertram & Christiansen, 2014). In an interpretive paradigm, the researchers "do not predict what people will do, but rather describe and understand how people make sense of their world and how they make meaning of their particular actions" (p. 26). A case study is well suited to "make sense of feelings, experiences, social situations, or phenomena as they occur in the real world, and therefore study them in their natural settings" (Rule & John, 2011, p. 60).

Using mixed methods helps the researcher to gain complete understanding and comprehension of results, discover new perspectives, or develop new measurement tools (Tashakkori & Teddlie, 2003). It involves collecting both quantitative and qualitative data. Creswell (2003) defined qualitative research as that which involves interaction and collaboration between researcher and participants. In agreement, Newby (2010) explained that in qualitative research the main intention is to discover the reasons why people behave the way they do. The choice of this paradigm is well aligned with the purpose of this study, which is to understand how teachers make sense of VOs and how textbook authors made their choices in using these VOs in their mathematics textbooks. This is done through survey questionnaires and interviews respectively.

On the other hand, Creswell (2003) described a quantitative research as an approach used for testing objectives, theories and models by examining the relationship among variables whereby the data is analyzed using statistical procedures. For this study, I collected VOs from three textbooks and analyzed them by using descriptive statistics.

## 3.4 RESEARCH METHODS

This study took the form of a case study, which I regarded appropriate for my research project because I was looking at a specific, real life phenomenon in a specific context (Rule & John, 2011). The case in this study consisted of three grade 9 Namibian mathematics textbooks that are approved by the NIED Curriculum panellists, fifty grade 9 mathematics teachers from different regions in Namibia, and two authors. The unit of analysis was threefold, namely the nature and roles of VOs in three mathematics textbooks, the perceptions of teachers and the rationale as to the choices of visualisation objects by authors.

A case study is also defined as a "systematic inquiry into an event or a set of related events which aims to describe and explain the phenomenon of interest" (Bromley, 1991, p.302). In addition, Schumacher & McMillan (1989), as quoted in Pieterson & Maree (2011), state that a case study does not necessarily mean that there is only one site studied. Furthermore, a case study is said to be valuable, because it allows researchers to develop and present an in-depth view of a particular situation, event or entity (Rule & John (2011, p. 77). Nibest & Watt (1984) as quoted in Cohen, Manion & Morrison (2007, p. 256) revealed some strengths of a case study, that results are more easily understood and are self-explanatory. They further explain that the case study can be undertaken by a single researcher without needing a full research team and they provide insights into other similar situations and cases. In addition, Cohen, Manion, & Morrison (2007, p. 249) states, "a case study gives a rich and vivid description of events relevant to the case, and it highlights specific events that are relevant to the case".

Despite the benefit of a case study, there are also some weaknesses associated with the use of a case study. Nisbet & Watt (1984, as cited in Cohen et al., 2007) state that in a case study "the results may not be generalizable except where other readers see their application, they are not easily open to cross checking, they may be selective and are prone to problems of observer bias, despite attempts made to address reflexivity" (p. 256). This aspect is addressed under the section on validity.

## 3.5 RESEARCH DESIGN

This study made use of a variety of data collection techniques and methods i.e. survey questionnaires with 50 teachers and interviews with 2 authors. I have also generated data from several textbooks. The data generation unfolded in four phases that are explained below.

## PHASE 1

*Analytical tool and Survey Questionnaires*

The aim of this phase was to develop the analytical tool and survey questionnaires. I adapted my analytical tool from Fotakopoulou & Spiliotopoulou (2008) – see details of this tool in the section on analysis. I piloted this analytical tool with a few VOs from *Maths for Life* grade 9 textbook. This enabled me to test the reliability of the tool and identify some of the shortcomings. I then set about modifying and refining the tool to fit the purpose of answering research question 1. At the same time, I developed and piloted the survey questionnaire with a few of my colleagues, which I then refined. I then sent it out to all of the 54 grade 9 mathematics teachers in the Oshana Region via the five circuit offices in the region. These were the Onamutai, Oluno, Ompundja, Eheke and Oshakati circuits.

## PHASE 2

*Analysis of VOs and survey questionnaires*

This phase aimed to analyse the VOs and the survey questionnaires. In this phase, I collected all the VOs from the three mathematics textbooks relevant to my study. I then went about analysing them by subjecting each of the objects to my refined analytical instrument. I also collected and collated the completed survey questionnaires from the schools as well as from the circuit offices. Surprisingly, only a few survey questionnaires were returned from schools and some were only partly completed. I then decided to give out more questionnaires to the fifty grade 9 mathematics teachers from different regions that attended the Mathematics National Congress in Swakopmund in May 2016. Forty-eight of those survey questionnaires were returned. I then analysed the survey questionnaires using both descriptive statistics and inductive analysis. This gave me an overall picture of teachers' views and understanding of the importance and functionality of VOs used in the three textbooks. This process helped me to answer research questions 1 and 2.

**PHASE 3**

*Interviews with authors*

In this phase, I conducted one-on-one interviews with two of the three authors of the identified textbooks. These interviews were guided by a series of ten questions relating to the rationale and choices they had made on selecting VOs evident in their textbooks. Pertinent issues that were probed included aspects of the relevance and usefulness of VOs in the textbooks and how they, in their opinion, had contributed to the teaching and learning of mathematics. Further questions probed other areas of interest that opened up during the course of the interview. All interview discussions were voice recorded. This aided the analysis process by allowing for multiple hearings. “Video recording can be a great help as it allows the same observation to be reviewed many times, with each viewing having the potential to elicit additional information” (Simpson & Tuson, 2003, p. 48). This process helped me to answer research question 3.

**PHASE 4**

*Analysis of transcribed interviews*

In this phase, the interviews were transcribed and then analyzed using inductive analysis. Inductive analysis is the process whereby a researcher “starts with raw data that she/he has collected and then begins to look for patterns to formulate some themes that can be explored” (Bertram & Christiansen, 2014, p. 204). Table 3.1 summarizes the entire research process.

Table 3.3.1 Summary of the Research process

| Phases | Instrument | Purpose | Data | Analysis |
|---|---|---|---|---|
| Phase 1 | Consent letters | To seek permission from the Director of Education Oshana region, and the permanent secretary of Ministry of Education | Not applicable | Not applicable |
| | Analytical tool | The development of the analytical tool to analyze VOs | Not applicable | Not applicable |
| | Select sample VOs from one text book and pilot the analytical tool | To analyze them and to ensure validity | The visual objects: nature, function, properties, relation to reality, relation to content. | Analytic tool that was developed |
| | Survey questionnaires | Development of the survey questionnaire for the Oshana region teachers and send them to teachers | Not applicable | Not applicable |
| Phase 2 | Analytical tool | Analyze and categorize the VOs of the selected textbooks | The nature, type, function, relation to reality and relation to the content of those objects | Analysis of VOs using analytical tool that had been developed |
| | Survey questionnaires | Collect and gather completed survey questionnaires | The views of teachers on the VOs used in the three textbooks | Quantitative descriptive statistics |
| Phase 3 | Interview | Design the interview questions and conduct it with two authors. | Authors' rationale on the selection of the VOs used. | Qualitative analysis using inductive approach |

## 3.6 SELECTION OF PARTICIPANTS

### 3.6.1 Textbooks

The selection of the textbooks for this study was done by using a 'purposive sampling strategy' (Bertram & Christiansen, 2014, p.60). In purposive sampling, the researcher makes specific choices about the objects or people to be included in the sampling process for validity. I chose to use all three of the approved mathematics textbooks for the junior secondary phase, specifically grade 9. These three textbooks are based on the 2010 revised national curriculum (Namibia. MoE, 2010). The three textbooks under study are y=*mx+c to success, Maths for Life* and *Discover Mathematics 9.* The editions of these books were published in 2008, 2009 and 2008 respectively, whilst the first two are published in Namibia, and *Discover Mathematics* 9 is published in South Africa.

Due to the scope of this study only two chapters from each book were analyzed, namely the algebra and geometry chapters. From my teaching experience, as well as the mathematics congresses that I have attended, these two chapters are perceived to be particularly challenging, poorly performed and not well understood by both teachers and learners. This is one of the main motives for selecting these chapters.

### 3.6.2 Teachers

The participating teachers in this study are Grade 9 mathematics teachers from different regions of Namibia. Although the initial target was Grade 9 mathematics teachers from Oshana region, the focus changed due to the poor return from the survey questionnaires and data collected during my fieldwork. Initially 54 survey questionnaires were sent out to all grade 9 mathematics teachers in Oshana region through circuit offices. The response from my region was only 18, which I felt did not provide adequate and sufficient information for this study. I coincidentally attended a National Mathematics Congress in Swakopmund that took place in May 2016. The congress was attended by mathematics teachers from all fourteen regions in the country, from primary, junior primary, junior secondary and senior secondary levels. Since I was one of the presenters at the congress with the junior secondary teachers, I seized the opportunity and asked 50 other grade 9 teachers who participated in the congress whether they were willing to complete the questionnaire. Their participation in completing the questionnaire was entirely voluntary. Once I explained the rationale and goals to the teachers, those that were so inclined completed the questionnaire.

In the end, I was able to use a total of 50 survey questionnaires for this study. The participating teachers were coded T1 for teacher 1 etc. When quoting a specific answer to the questionnaire, I used T1Q3 meaning teacher 1, answer to question 3.

### 3.6.3 Authors

The two participating authors are both Namibians who are well known for authoring mathematics textbooks for junior and secondary level (Grades 8 - 12). They were both former Grade 8 - 12 mathematics teachers at secondary schools in Namibia. Author 1's book is her second published textbook, while author 2's is her third textbook. The selected authors reside in Namibia thus making them easily accessible to me.
They were coded author 1 as A1, author 2 as A2, and if the statement quoted is from line 20 of the interview, I quote it as A1L20.

## 3.7 METHODS OF COLLECTING DATA

For the purpose of this research study, a number of data collection tools were used. These included collecting VOs from the textbooks, survey questionnaires and interviews. Bertram & Christiansen (2014) suggests that a researcher needs to use a "fitness for purpose approach" whereby data collection methods need to match the kind of data he/she wants to collect (p. 41).

### 3.7.1 VOs

In this case a collection of selected VOs from the three grade 9 mathematics textbooks, *y= mx + c, Maths for Life 9* and *Discover Mathematics 9* in the specified chapters of algebra and geometry was done by copying the visual objects and pasting them into organised documents.

### 3.7.2 Survey Questionnaires

The survey questionnaire is divided into six parts. Part 1 consisted of questions 1 and 2 which sought teachers' views on the type of VOs teachers use in class. Part 2 consisted of question 3, which sought the teachers' general views on the visualisation objects presented in the selected textbooks. Part 3 consisted of questions 4 and 5 which sought the teachers' opinions on the role of these VOs in teaching and learning mathematics. Part 4 consisted of question 6, which sought teachers' understanding on the appropriateness of VOs and the relationship of VOs to reality. Part 5, which consisted of question 7, addressed the relation of VOs to mathematical contents. Part 6 consisted of questions 8 and 9, which encapsulated the limitations and challenges that teachers have experienced using those VOs, and questions 11 and 12, in which I selected a sample of pictures from the textbooks and set questions based on these objects to ascertain how they are used by the teachers. Find the attached sample of the survey questionnaire in appendix 3, p. 104.

### 3.7.3 Interviews

Bertram & Christiansen (2014) emphasized that an interview is categorized to be a useful data collection method as it allows the researcher to ask more probing and clarifying questions. I conducted interviews with the two authors of the textbooks in question to obtain their views and explanations of the choices of VOs that they used (or included) in their respective textbooks. The one-on-one, semi-structured interview with the authors consisted of 10 main questions. I wished to elicit from the authors what they thought the roles of the visualisation objects they chose served, why they chose those specific visualisation objects and what the relevance of the selected objects were in relation to teaching and learning. Find the attached list of interview questionnaire in appendix 4, p. 108.

The first two questions sought the authors' general views and perspectives on VOs. Questions 3, 4 and 5 were on the choices of VOs they used in their textbooks. Question 6 was on the type of VOs used, while question 7 addressed the roles of VOs. Question 8 and 9 were on the relation of VOs to mathematical content and context, respectively. The last question addressed the limitations they experienced in choosing VOs to be used in their textbook.

## 3.8 DATA ANALYSIS

My research study involved both quantitative and qualitative analysis, as described below.

### Quantitative approaches

In this study, the quantitative analysis consisted of statistically analyzing VOs by using the analytical framework illustrated in table 3.3 page 52. This enabled me to answer research question one. This analytical tool is adapted from Fotakopoulou & Spiliotopoulou (2008). It is modified to suitably analyse and classify 266 different VOs collected from two chapters of the three selected textbooks under study. This analytical tool is divided into five categories: the types of VOs, the roles of VOs, the relation of VOs to mathematical contents, the relation of VOs to reality, and the properties of VOs. See the attached sample of the coding scheme of two VOs that are analysed in appendix 6, p. 110.

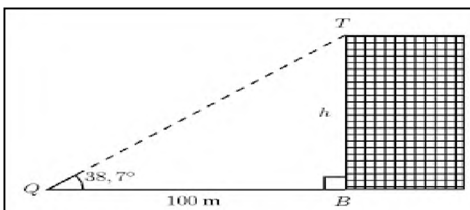
The first category enabled me to analyse the type of the VO i.e. whether it is an image, a sketch, a picture, a comic, a distribution diagram, a photograph or a diagrammatic table, graph or schematic representation. The second category focused on the roles of the visualisation objects, namely: whether the VO was decorative, exemplifying, explanatory or complementary. The third category distinguishes the visualisation object as either being realistic or metaphorical. The fourth category differentiates a visualisation object as being a 2-dimensional or 3-dimensional object. The final category is on content – how do VOs align with the content? For a detailed explanation, see table 3.3.3

I coded each category and sub-category using table 3.3.3.

Table 3.3.2 Coding scheme for analysis of the visual objects

| Score | Description of evidence of the indicator |
|---|---|
| n/a | Not applicable |
| 0 | Weak evidence |
| 1 | Average evidence |
| 2 | Strong evidence |

To illustrate the use of my analytical tool and coding, I provide a hypothetical example.




Problem: Use the trigonometric ratios to find

Figure 3.1 VO, sourced from y=mx+c p.67

Table 3.2.3 Coding template to analyze each VO

| Category | Subtype | Definition | Score A |
|---|---|---|---|
| Type | Picture, Sketch/ drawing | A simple drawings/shape which does not have many details | 2 |
| | Distribution | Histogram; Point Graph; Stem-and-Leaf Plot; Tally Graph | n/a |
| | Photograph | A picture produced using a camera | n/a |
| | Table, matrix, text chart | A table of different sizes and forms | n/a |
| | Graph | A graphical representation of data, relations and processes in bar, column, pie, spider graphs, graphic timetable form. | n/a |
| | Schematic | A representation of an idea or theory for easy understanding. | 2 |
| Roles | Decorative | Made to look more attractive or ornamental – a decoration | 1 |
| | Exemplifying | Diagrams that exemplify the written text. They make the text and the mathematical idea clearer. | 2 |
| | Explanatory | Notes accompanying the diagram, which assist the text and provide new aspects of information necessary to make the idea under consideration clearer. The diagram is used to explain the main mathematical idea. | 2 |
| | Complementary | Diagrams that provide information not included in the text nor described explicitly in the written form. The diagram complements and reinforces the written text | 2 |
| Reality | Realistic | Realistically represents the concept true to real life situation | 2 |
| | Metaphorical | Not having real existence | 0 |
| Properties | 3D | A three-dimensional VO | 0 |
| | 2D | A two-dimensional VO | 2 |
| Content | Absent/weak | Having a weak connection to the content | 0 |
| | Meaningful to the content | Aligns with the content | 2 |

The scores in the fourth column are then counted and analysed using descriptive statistics. Pietersen & Maree (2011) defines descriptive statistics as “a collective name for a number of statistical methods that are used to organize and summarise data in a meaningful way” (p.183). They further state that descriptive statistics enhance the understanding of the properties of the data and that data can be represented in a graphical or numerical way (*ibid.*). In the coding template that was used to analyze each VO, the score for each type reflected in the last column of the template added and an average was calculated.

**Qualitative approaches**

During the qualitative analysis, words, text or images are combined into categories of information that are presented to show the variety of ideas that were gathered during the data collection process (Cresswell & Clark, 2007). During my research analysis, the responses to the survey questionnaires and the transcribed interviews of the authors were analysed using inductive analysis. Maxwell (1996), as cited in Fotakopoulou & Spiliotopoulou (2008), stated that there are three main categorizing strategies of qualitative analysis where the categories, themes and patterns emerge from data. I adopted these strategies for my interviews' and survey questionnaires' analysis. These categorizing strategies were organizing data into categories, identifying patterns and generating themes and concepts. The themes that emerged from the collected data were the interpretation of VOs by learners, the types of VOs chosen by the authors, the role of VOs for authors and teachers, the relation of VOs to mathematical content, and the VOs' relation to reality. In the survey questionnaires, key words were identified, their frequency counted and an average for each keyword was calculated. For example many teacher identified VOs as tools for reasoning. 'Reasoning' thus became a key word.

## 3.9 VALIDITY

To ensure validity of this study, I firstly piloted the analytical tool on ten VOs from the textbook '*Maths for life*' to help me find out if this analytic tool is appropriate and effective in terms of answering my research questions. In addition, I also had my questionnaire reviewed by my supervisor before piloting it with two mathematics teachers at our school before sending it out to the selected mathematics teachers. Furthermore, to enhance credibility during the interviews, I made use of an audio-recording device to gather a full record of the interviews. I explicitly explained the aims and objectives of this study to all participants to enhance their understanding of this project and thus ensure alignment of their responses to the goals of the study. In this study, I collected data using a variety of methods i.e. document analysis, questionnaires and interviews, and obtaining information from many different participants.

## 3.10 ETHICS

Ethics is an important consideration in this study. The ethical considerations are explained below.

### 3.10.1 Respect and Dignity

The participants were well informed about their voluntarily participation in the study as well as their freedom to withdraw at any time. Participants' identity remained anonymous. As the textbooks are in the public domain it is however impossible to keep the name of the authors anonymous. They were made aware of this before the interviews. I have ensured mutual respect. All the data remained and will remain confidential between my supervisor and me. Consent to use a voice recorder during the interviews was obtained from both authors. All the planned research activities took place after formal classes as agreed upon with the permanent secretary of education in the Ministry of Education, see appendix A.

### 3.10.2 Transparency and honesty

Formal consents were sought from the Director of Education in the Oshana region, authors and the permanent secretary of Ministry of Education, see appendix A. The nature, purpose and content of this study was clearly explained to all participants. In terms of the authors, I specifically clarified that the research was not about the comparison of textbooks; rather it is an analysis of the nature of the diagrams and about how teachers use and perceive these in the respective textbooks.

### 3.10.3 Accountability and Responsibility

All the data were treated with sensitivity and securely stored. Being aware of my position as a mathematics facilitator in the region, I assured the participants that my position did not compromise any of their responses.

### 3.10.4 Integrity and Academic professionalism

This research is my own work, using my own words. Where I draw on the works or ideas of others, I have appropriately acknowledged and referenced them. In addition, collected data were not manipulated, compromised, fabricated or misreported; rather they were presented as they are.

## 3.11 LIMITATIONS AND CHALLENGES

Getting the survey questionnaires back from grade 9 mathematics teachers of Oshana region was a challenge. Some of the teachers complained that they did not get the questionnaires on time from the circuit office, while some of them did not get them at all. Some further claimed that they gave the completed questionnaires back to their principals to take along to the circuit offices, but I was unable to find them. Furthermore, some questionnaires that I got back at the mathematics congress were partly completed and in some cases, the answers were not elaborated upon. In addition, one of the authors kept postponing the interview and was only able to conduct it towards the end of August. At first, the authors were a little concerned about revealing their identity. After discussing with them the public nature of their textbooks and the notion that teachers would find it very meaningful to read about why the authors chose the visual objects they did, they were comfortable for their identity to be revealed.

Despite this study providing and presenting an in-depth analysis of the nature of VOs in three textbooks, the findings of this case study cannot be generalized to other textbooks. However, the findings can provide insights for further research.

## 3.12 CONCLUSION

This chapter provided a brief description of the rationale for the study, the sampling strategies and the targeted population of this study. It described the research instruments that were employed. It further showed how data was collected and analyzed and looked at the ethical considerations that were taken into consideration. It also addressed the limitations and challenges encountered in the study. This research study was a case study, grounded in an interpretive paradigm. The chapter concluded with a discussion of the data analysis process, as well as an interrogation of issues relating to ethics and validity. In the next chapter, the data is presented and discussed in detail.

# 4. CHAPTER FOUR: DATA ANALYSIS

## 4.1 INTRODUCTION

The purpose of this chapter is to present and discuss the results of the study. The chapter begins with the data presentation on the nature of the VOs that are evident in three selected grade 9 mathematics textbooks. They are presented under the five categories of the analytical tool as discussed in the methodology chapter page 52. Secondly, the research results from the survey questionnaires are presented, followed by the presentation of research results from the interviews with the authors. The summary of findings from the three sources of data is then presented.

During the course of the data analysis of interviews, survey questionnaires and the VOs, several themes emerged that framed the ensuing narrative. The chapter concludes with a discussion of each of these emerging themes, namely: the interpretation of VOs by learners, the choices on the type of VOs used by authors, the role of VOs as understood by the authors and teachers, the relation of VOs to mathematical content, and the VOs' relation to reality. Although these themes are discussed individually, it is acknowledged that they are intertwined and interrelated.

## 4.2 DATA PRESENTATION OF THE NATURE OF THE VOS EVIDENT IN THREE SELECTED MATHEMATICS TEXTBOOKS FOR GRADE 9

The data on the nature of VOs from each of the three Namibian mathematics grade 9 textbooks is presented in Figure 4.1. These VOs are derived from two chapters namely, Algebra and Geometry. Each VO is analyzed by using the analytical tool as discussed in the methodology chapter, page 45. The sample of the analyzed VOs appears in appendix 6.

VOs were counted based on their frequencies. Figure 4.1 below specifically illustrates the topics under scrutiny in the three textbooks and the number of VOs used in each topic.

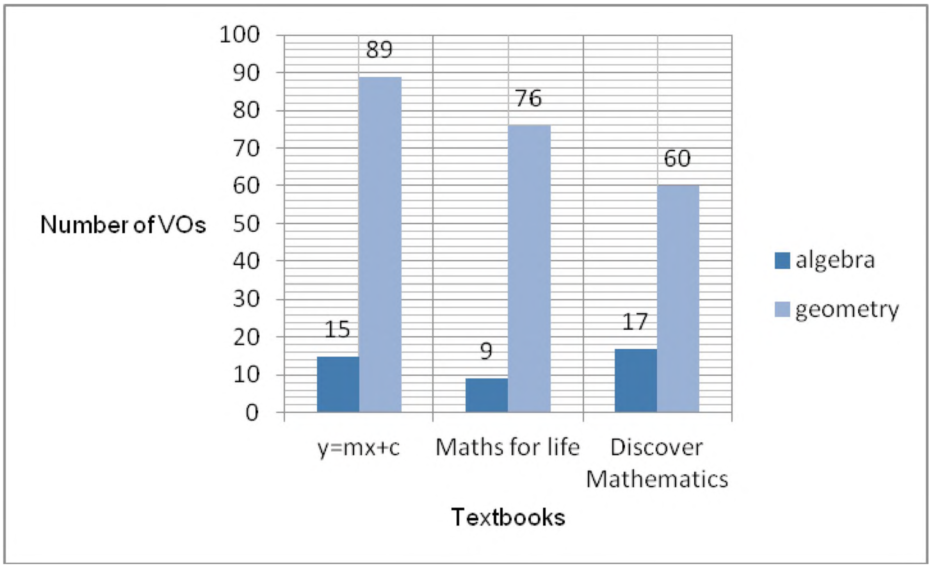



Figure 4.1The number of VOs under study

As anticipated, Figure 4.1 shows that there are more VOs used in Geometry in comparison to Algebra in all three grade 9 textbooks selected in the study.

For the discussion below I used the same headings of the categories as in my analytical tool (see chapter 3, p. 48).

### 4.2.1 Types of VOs

The VOs are divided into the following categories:

- Pictures, sketches and drawings; this refers to simple drawings/shapes with few details
- Distribution diagrams; this refers to a histogram, point graph, stem-and-leaf plot, or tally graph
- Photograph; this refers to a picture produced using a camera
- Table, matrix, text chart; this refers to a table of different sizes and forms
- Graphs; this refers to any graphical representation of data, relations and processes in bar; column, pie, spider graphs, or graphic timetable form
- Schematics; this refers to a representation of an idea or theory for easy understanding.

The number of different types of VOs used in Algebra and Geometry from the three textbooks is presented separately below.

#### 4.2.1.1 Algebra

Figure 4.2 displays the number of different types of VOs evident in the three textbooks in the Algebra chapters.

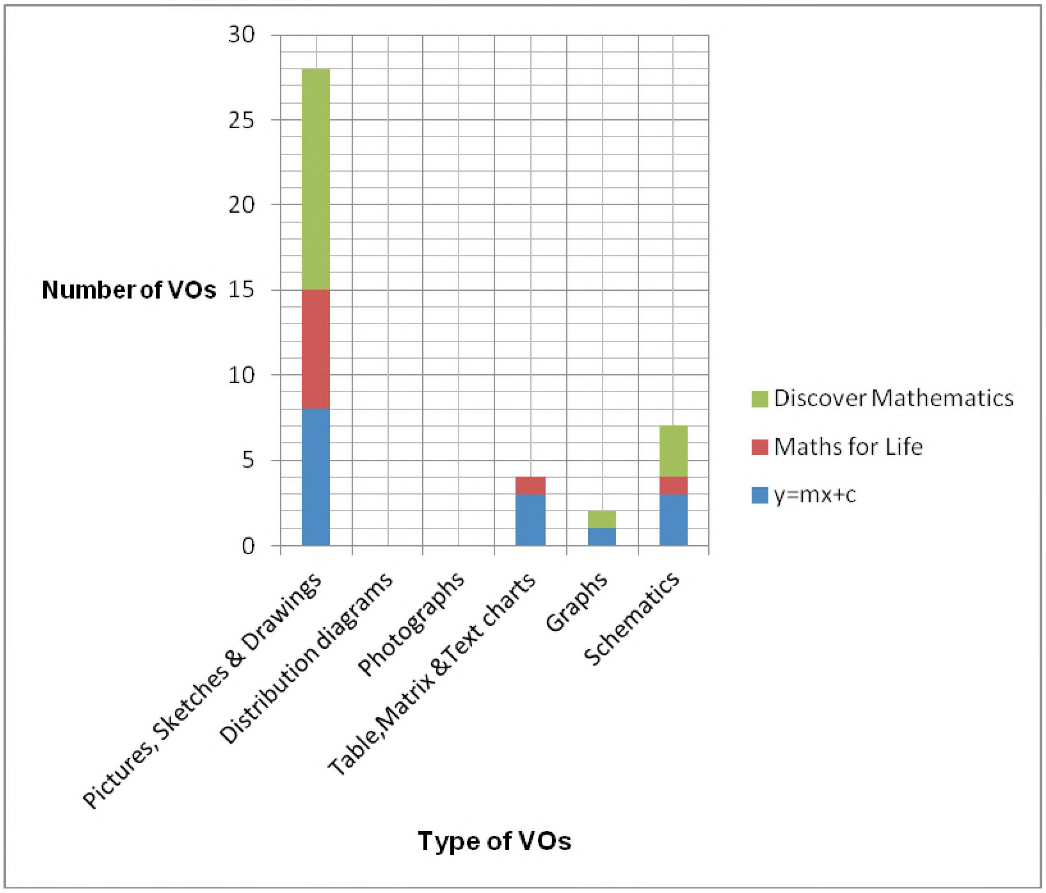



Figure 4.2 Types of VOs used in the Algebra chapter

It is interesting to note that for the Algebra chapter mostly pictures, sketches and drawings were used as opposed to photographs. It is notable that neither distribution diagrams nor photographs were used in any of the three textbooks in Algebra.

#### 4.2.1.2 Geometry

Figure 4.3 below indicates the number of different types of VOs evident in the three textbooks in the Geometry chapter.

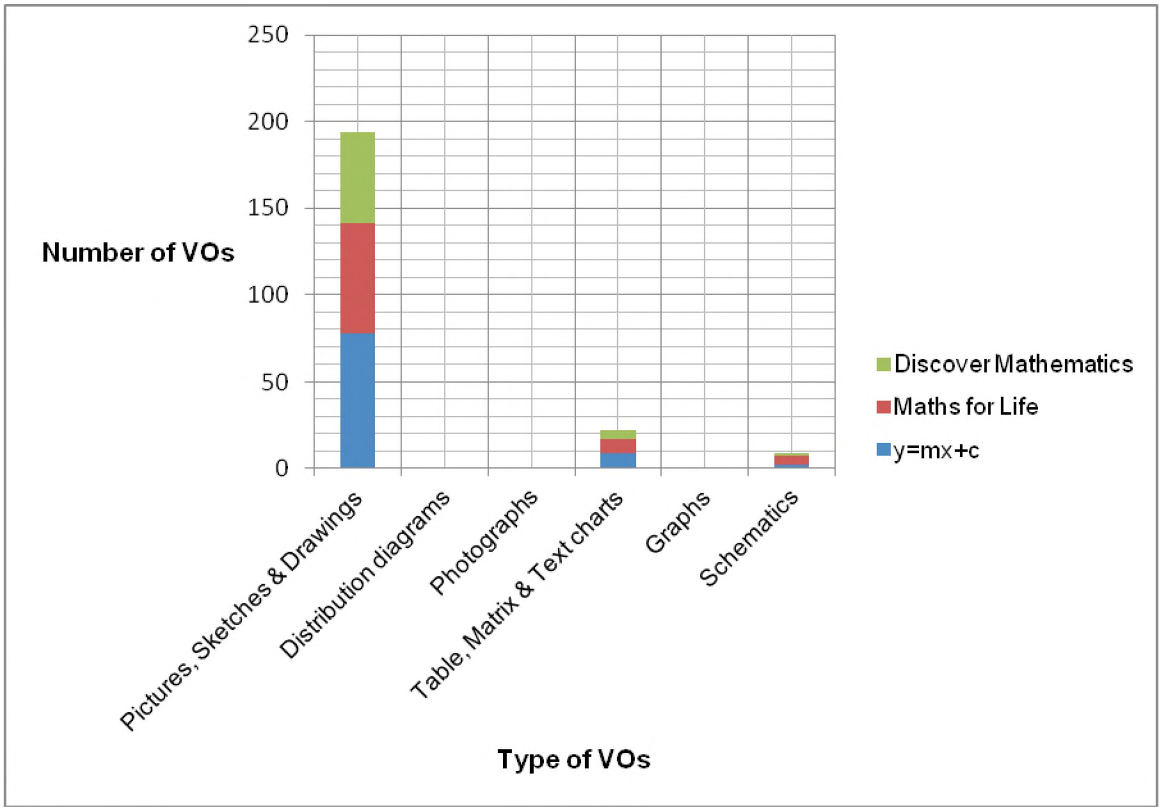



Figure 4.3 Types of VOs used in Geometry chapter

Notably more than 80% of VOs used in all three textbooks represent pictures, sketches and drawings.

### 4.2.2 Relation to reality

The category on the relation of the VOs to reality is divided into two sub-categories. A VO can either be realistic, representing the concept true to real life situation, or metaphoric, not having real existence. See methodology chapter, page 55 for details. Table 4.1 presents the number of VOs from two chapters in each of the three textbooks and their relation to reality.

| | *y=mx+c* | | *Maths for Life* | | *Discover Mathematics* | | *Total* |
|---|---|---|---|---|---|---|---|
| | realistic | metaphoric | realistic | metaphoric | realistic | metaphoric | |
| Algebra | 15 | 0 | 9 | 0 | 17 | 0 | 41 |
| Geometry | 89 | 0 | 76 | 0 | 60 | 0 | 225 |
| Total | 104 | 0 | 85 | 0 | 77 | 0 | 266 |

Table 4.4.1 The number of VOs in relation to reality in Algebra and Geometry

Significantly, none of the VOs used in any of the textbooks in either the Algebra or Geometry chapters are metaphoric. Thus, all represent the concept true to real life. For instance, the VO illustrated in Figure 4.4 below represents a concept that is true to a real life situation. Goats are animals that exist in real life and are familiar to most learners as they are one of the domesticated species in Namibia.

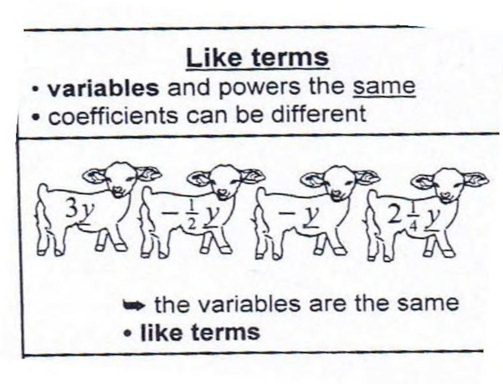



Figure 4.4 VO, from y=mx+c p. 54

### 4.2.3 Relation to mathematical content

In this category, the VO either can align with the mathematical content or has no connection to the content. The pie charts below, Figures 4.5 and 4.6, indicate the percentages of VOs evident in Algebra and Geometry in relation to content.

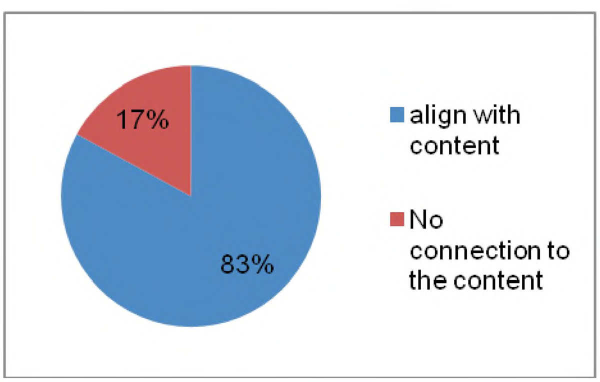



Figure 4.5 % of VOs used in Algebra in relation to content

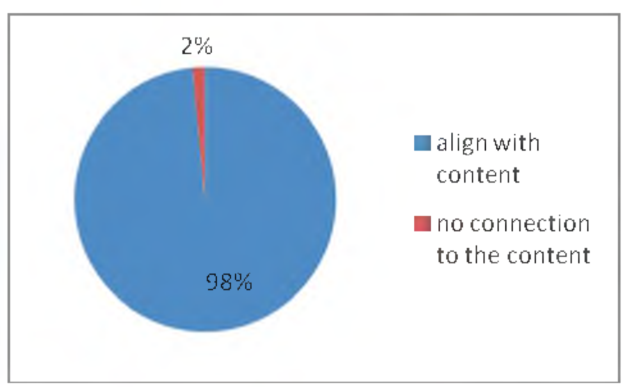



Figure 4.6 % of VOs used in Geometry in relation to content

Figures 4.5 and figure 4.6 above show that more than 80% of the VOs used in all three textbooks in the two chapters align well with the content. For example, Figure 4.7 below aligns well with the content, because that VO encompasses the details on the center of rotation, which is the topic under study.

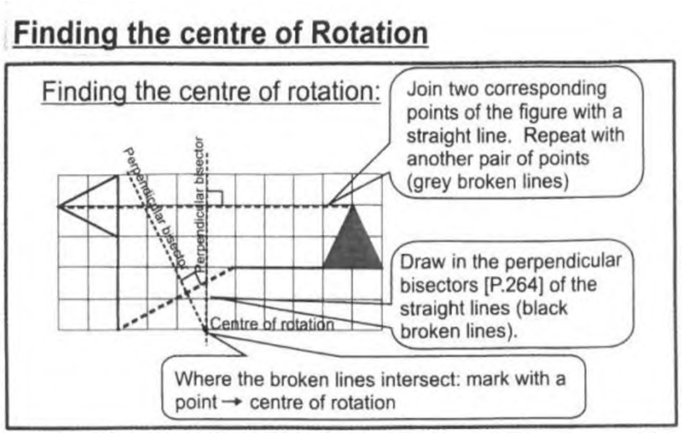



Figure 4.7 VO from y=mx+c to success p.249 topic, transformation geometry, center of rotation

### 4.2.4 Properties

The VOs can be either a two-dimensional or a three-dimensional shape. Table 4.2 shows the number of VOs used in all three books in relation to their properties.

| | Algebra | | Geometry | | **Total** |
|---|---|---|---|---|---|
| | 2 dimensions | 3 dimensions | 2 dimensions | 3 dimensions | |
| *Y=mx+c* | 15 | 0 | 60 | 29 | **104** |
| *Maths for Life* | 9 | 0 | 57 | 19 | **85** |
| *Discover Mathematics* | 17 | 0 | 47 | 13 | **77** |
| **Total** | **41** | **0** | **164** | **61** | **266** |

Table 4.4.2 Number of VOs in relation to their properties

It is noted that in Algebra across all three textbooks none of the VOs are in three dimensions. As expected, a number of three-dimensional figures are observed in Geometry across all three textbooks although they are still few in number in comparison to the two- dimensional figures evident in Geometry.

### 4.2.5 Roles of VOs

Roles of VOs are divided into four subcategories viz:

- Decorative; when the VO is made to look more attractive or ornamental – a decoration
- Exemplifying; this refers to VOs that make the text and the mathematical idea clearer
- Explanatory; this refers to VOs with accompanying notes, which clarify the text and provide new aspects of information necessary to make the idea under consideration clearer
- Complementary; this refers to diagrams that provide information not included in the text nor described explicitly in the written form.

See the methodology chapter, page 54 for details. The number of VOs used for these roles in both Algebra and Geometry are presented in the graphs below.

#### 4.2.5.1 Algebra

Figure 4.8 shows the number of VOs used for different roles in the three textbooks.

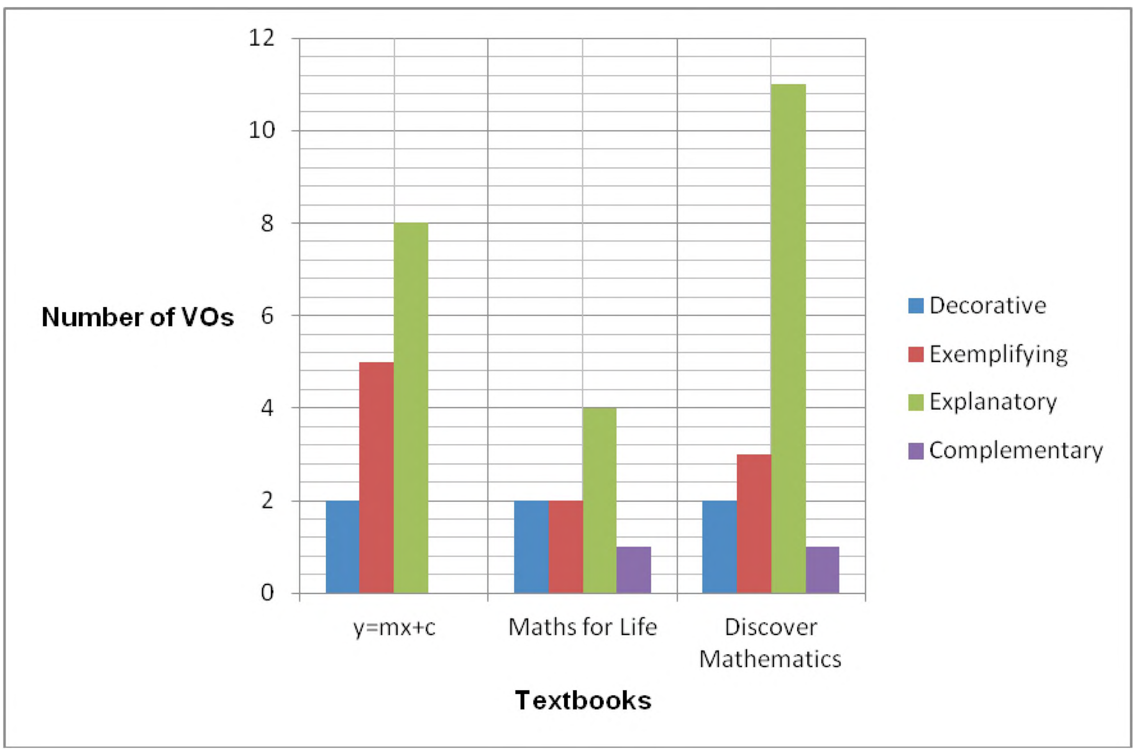



Figure 4.8 Number of VOs used in their roles in Algebra

It is clear that VOs used for explanatory purposes are more commonly used in Algebra than the other categories in all three textbooks. It is noted that VOs used for complementing are seldom used in all the textbooks.

Specifically Figure 4.9 below is an example of a VO used as an explanatory, the notes attached explain what the diagram is illustrating, making the idea clearer.

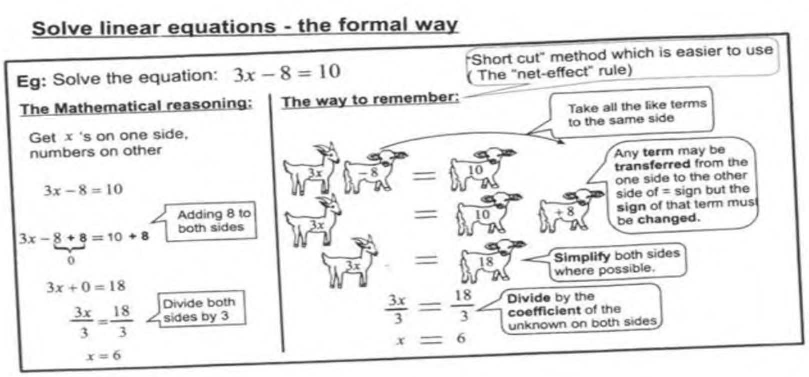



Figure 4.9 VO from y=mx+c p.100, topic: Algebra, solving linear equations

#### 4.2.5.2 Geometry

Figure 4.10 below shows the number of VOs evident in the Geometry chapters of the textbooks in relation to their roles.

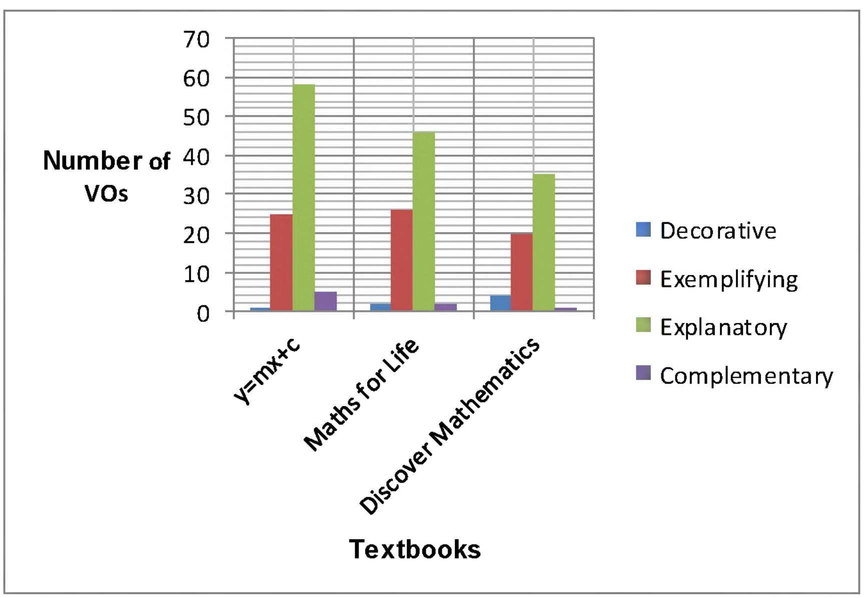



Figure 4.10 Number of VOs used in Geometry and their roles

The graph indicates that more than 50% of VOs in Geometry in all the textbooks are used for the purpose of explanation. It further shows that the decorative and complementary VOs were hardly used in all three textbooks.

For instance, Figure 4.11 is an example of a VO used to exemplify. The diagram makes the written text clearer, by showing the mirror line, the distance from the mirror line to the object and to the reflected image.

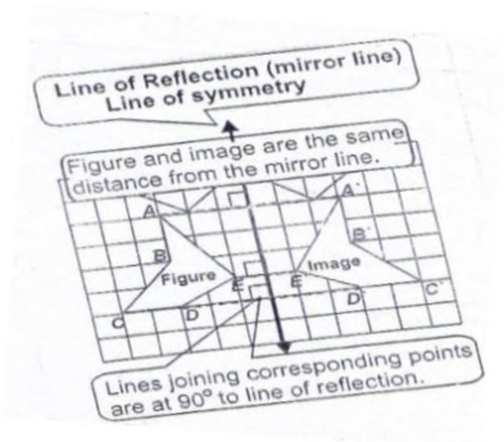



Figure 4.11 VO from Maths for Life page 168, Topic: Transformation Geometry, reflection

Nonetheless, a third of the VOs only partially illustrate or make clear the idea given. They do not really exemplify the text. For example, Figure 4.12 below is used to illustrate Pythagoras' theorem, which states, "In a right-angled triangle, the square on the hypotenuse equals the sum of the squares on the other two sides".

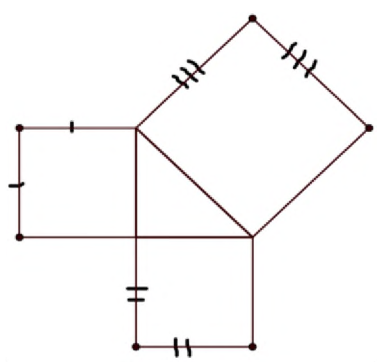


Figure 4.12 VO from y=mx+c p. 210

This VO only partially illustrates or makes clearer the idea given in the text above. It does not fully exemplify the text, i.e. Pythagoras' theorem, because it does not fully show in a visual sense what the text has stated, as it does not illustrate that the square on the hypotenuse is the sum of the squares on the other two sides. If, for example, the diagram had included 5:4:3 ratios on the sides then it would exemplify the text statement more fully.

### 4.2.6 Summary

In conclusion, it is observed that as expected more VOs are evident in Geometry as compared to Algebra in all textbooks. Furthermore, in the two chapters of all three textbooks, mostly sketches, drawings and pictures were used in comparison to graphs, tables and schematics. Surprisingly no distribution diagrams and photographs were used in either chapter of the three textbooks. Remarkably, all VOs used represent the concept true to real life in both algebra and geometry. Additionally, more than 80% of the VOs used align well with the content. VOs are mostly used for the purpose of explanation in comparison to exemplifying, decorating and complementing.

## 4.3 PRESENTATION OF RESEARCH RESULTS FROM SURVEY QUESTIONNAIRES

The results based on the responses from the survey questionnaires of the 50 teachers are analyzed and presented as follows. Key words were identified and counted, and converted to percentages.

### 4.3.1 The roles of VOs

All the teachers revealed that VOs are central and imperative to the teaching and learning of mathematics as a subject. Therefore, the roles of VOs cannot be overemphasized. The majority of the teachers' responses stated clearly that VOs make abstract concepts concrete. VOs also clarify mathematical ideas whose meanings are difficult to comprehend. In addition, the teachers further asserted that the VOs stimulate learners' interest in mathematics, and serve as the instrument for learning. The teachers further ratified that VOs have different roles as far as the teaching and learning of mathematics is concerned. The roles and functions are discussed below.

#### 4.3.1.1 VOs attract learners' attention and stimulate learners' interest

Seventy percent of the teachers claimed that VOs attract learners' attention and stimulate learners' interest in learning mathematics. They further argued that VOs also make mathematics fun and practical. In support, T4Q4 stated that VOs *"attract learners' attention and help learners to develop their love of the subject"*. T23Q4 shared the same sentiment, that visualisation objects *"make maths more fun and interesting"*. She further noted that VOs *"provoke the learners' interest to learn and apply knowledge in mathematics".* In the same vein T46Q4 articulated that VOs *"make maths fun and interesting, and learners are more curious when they see pictures than texts alone"*. In support of the above statements, T45Q4 also acknowledged that *"diagrams make the lesson more fun and interesting, making the lesson alive"*.

#### 4.3.1.2 VOs as a tool for conceptual understanding

Eighty percent of the teachers suggested that VOs enhance deep conceptual understanding. They further highlighted that VOs help learners to grasp mathematical concepts without difficulty as they learn better by seeing visuals than texts. In the light of this statement, T9Q7 indicated that *"learners learn best when they see or draw the diagrams".* She further emphasized that *"teaching mathematics without VOs will make it difficult for the learner to grasp the subject content concept, because mathematics is not about theories only but the learners need to do practical exercise through homework, class activities and projects and real objects and diagrams are needed to enhance learners basic competencies"*. In addition, T15Q6 pointed out that VOs are "*to make learning easier for the learners to understand a given topic"*. The teachers also emphasized that VOs make the content clearer, as abstract content is made real to their eyes. In line with this, T43Q7 mentioned, *"VOs outline what is being implied in the word problem and they map out a way of calculating and finding the solutions".*

#### 4.3.1.3 VOs help learners to recall the concepts easily

Notably 64% of teachers explained that VOs play a significant role in mathematics as they aid learners to recall the concept discussed during the lesson. T4Q4 similarly outlined that *"learners tend to remember what they have seen faster than what they have heard".* In agreement with the above statements, T3Q4 mentions, *"VOs help learners to remember easily what they were taught"*. In addition, T9Q4 revealed that VOs *"assist learners to recall what is said and to remember easily during examination"*. The VOs also *"make it easy for a learner to remember with understanding because with visualisation, learners can easily remember the content taught"* (T40Q5).

Moreover, teachers highlighted that VOs can be used to reinforce the key mathematical concepts taught. Correspondingly, T10Q4 highlighted that *"visualisations really help, as this is a way to visually remind the learners of the key concepts and to reinforce what they learned so that it sticks in their mind"*. In addition, T14Q4 noted, *"the methodology helps learners remember shapes easily"*. Likewise, T15Q4 stated that *"VOs are very useful because learners remember more what they see than what they hear; as a result it helps to improve learners' performance".* In the same vein, T28Q4 explained that VOs *"are very important, because learners will never forget what they see as compared to their own imagination"*.

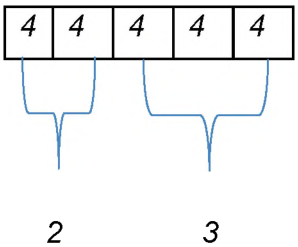
#### 4.3.1.4 Simplify and clarify the concepts

A number of teachers wrote that VOs also simplify and clarify the abstract mathematical concepts. In support, T5Q5 explicated, *"through the use of visualisation objects, learners get a clear picture of what is meant by a certain mathematical concept, e.g. a cube; showing them a drawing of a cube helps them understand better, the faces, the vertex, and also makes it easier to calculate the total surface area"*. In the same fashion, T6Q4 shows that VOs *"clarify and simplify complex or abstract mathematical problems".* Likewise, T7Q4 articulated that VOs *"simplify the topic being taught, because it brings in the real sense of the object under discussion".* In accordance, T37Q5 stated, *"diagrams help to simplify the work, meaning simplify mathematical problems as they learn".*

#### 4.3.1.5 VOs as a tool for reasoning

It was noted that 40% of the teachers' responses collected, emphasized that VOs can also be used as a tool for reasoning. In support, T13Q5 affirmed, *"diagrams help learners to develop critical thinking and reasoning".* In like manner, T36Q6 mentioned that VOs *"open learners' minds and broaden their thinking towards desired answers as they play around with objects".* In addition, T34Q5 stated, *"as learners get exposed to different visuals, it helps them think clearly and reason logically".* Similarly, T13Q5 explicitly explained, *"visuals instigate a mental process, especially for learners whose English is their 2nd language. For instance, ratios using the Singapore Bar method helped the grade 9 learners to love and excel in ratios. They even applied SBM in exams. E.g., share 20 kg in the ratio of 2:3*

*2 + 3 = 5 total parts* $\frac{20}{5}$ *= 4kg/part*




*Hence: 8kg and 12 kg - this is visualisation".*

#### 4.3.1.6 VOs as a tool for problem solving

The majority (90%) of teachers indicated that VOs are central to problem solving in mathematics. VOs also help learners to solve real life problems by relating them to mathematical problems. On the same note, T6Q5 pointed out that VOs *"are useful in a way that learners are afforded an opportunity to approach problems in different ways"*. In addition, T20Q5 showed that a VO *"increases the understanding of the problem, to get a better understanding of a problem"*. Likewise, T21Q5 highlighted that *"diagrams make learners solve problems easily, as one can get many ways of doing it"*. In agreement with the above statements, T41Q5 explained that VOs *"open up a learner's mind (picture-wise) to see what is happening in order to calculate and solve maths problems".* Similarly, T3Q4 stated that VOs *"develop learners' problem solving skills".* She further explained, *"Visualisation helps learners to understand the concept and helps them to connect it to real life situations and encourages learners to do mathematics and it helps them learn to solve real life problems"*.

#### 4.3.1.7 VOs as a tool for easy interpretation

Half of the teachers' responses collected emphasized that VOs are more easily interpreted than concepts presented in words. In support of the above assertion, T26Q5 acknowledged that VOs *"make concepts/questions to be more practical and easy to interpret and this make it easier to answer them".* Furthermore, T8Q5 added, *"learners get an opportunity to better interpret the concepts on their own when using VOs".* T24Q5 explained that VOs are vital "*because they make it easier for learners to interpret questions in a practical way and help them answer with ease and they are easy to interpret compared to concepts presented in words"*. In addition, T37Q7 revealed, *"I use them when converting word problems to algebraic expressions. I also use them when finding the unknown length if given sufficient information".*

#### 4.3.1.8 VOs link mathematics to reality

The teachers acknowledged that VOs enable learners to link and connect the mathematical concepts learnt in class to real life situations. They further highlighted that VOs enable the learners to see the unseen. In similar fashion, T22Q5 stated, *"visualisations in textbooks are linked to the content, which is good because it helps learners to link maths to the real life situation/world. This helps them know that what they learn is not just for the class usage but is applicable in a real life situation".* To support the above-mentioned claims, T44Q7 highlighted that *"VOs used in the textbooks are linked to reality and help to represent real life situations, which makes learners understand things easily and also helps learners to see and reflect reality".*

T28Q7 indicated that VOs *"help learners to connect classroom learning experience to the real world problems".* In addition, T29Q7 argued, *"nowadays learners work more with pictures they see every day on TV and on internet so it's best to use diagrams/pictures. It must be noted that this type of teaching/learning approach has developed a new teaching mode - that of a learner-centered paradigm whereby learners are expected to discover answers for themselves".* T4Q7 said, *"Diagrams help learners to make connections with what they learn theoretically and with what they can see".* In support, T22Q6 explained, *"pictures help learners understand the reality and practicality of the situation. It also helps them to link what they are learning in class to their prior knowledge and experience".* Similarly, T16Q7 stated that VOs *"create a link toward learners' prior knowledge with what they are learning in the classroom situation".*

#### 4.3.1.9 Relation to content

The majority of teachers, in fact 90%, stated that most of the VOs that are evident in all the three textbooks align well with the content where they are presented. In support, T3Q8 stipulated, *"diagrams have a strong relationship to content because they match/link well with basic competencies".* Similarly, T10Q8 elaborated, *"pictures in textbooks are relevant and useful, but I think in Namibia we teachers depend too much on the textbooks rather than coming up with our own ideas".* Moreover, T16Q9 explained, *"the diagrams are relevant to our school curriculum. They are created in accordance to the goals of our education and most of them are* *personified according to the learners' experience"*. Likewise, T44Q7 states that VOs *"are related to the content because they show the real pictures of the content they serve"*.

Additionally, T12Q8 specifically outlined, *"Maths for life provides many relevant visualisation objects in comparison to other books. The book is useful and is in line with the syllabus content"*. On the same note T13Q8 revealed that *"I only use Maths for life thus I can't talk for other books"*. She further explained, *"The author of this book uses a variety of VOs and they are useful in that they require thinking when solving problems. The story problems used in the book also calls for creative thinking"*. Conversely T33Q9 emphasized that *"y=mx+c is the book that I prefer because it's well to the point with the content"*. In agreement, T43Q8 said, *"the diagrams are in conjunction with the topics in which they are used"*. Likewise, T10Q9 added, *"visualisations in the textbooks are clear; it just depends on how the teachers interpret them"*. On the same note, T16Q9 added that VOs *"are appropriate especially in the y=mx+c textbook. Learners can even do the activities and study on their own - the book is learner-friendly"*. T9Q9 highlighted the same sentiments, that *"y=mx+c is the book I prefer and the visual objects in the book help me as a teacher to explain the topic or problem to the learners in a clear and understandable way"*.

On the other hand, other teachers argued that some VOs used are inappropriate, unsuitable, and are not in line with the content. Furthermore, some VOs have weaker connections to the content presented, as they do not give sufficient information. In addition, T5Q8 affirmed, *"though some diagrams are appropriate or relevant to the content or concept taught, they are hardly demonstrating what they are supposed to demonstrate or they are not conveying any message"*. In the light of this, T9Q9 showed that *"some of the VOs are relevant to the mathematics curriculum and some of them are not. As a teacher I have to choose the book which is relevant to my context"*. In the same way, T18Q8 echoed, *"some are not connected to the content and are irrelevant to the content/subject content where they are presented and some are not appropriate"*. Likewise, T15Q8 reported, *"some diagrams are not relevant and useful because they are not appropriate for some topics"*. To complement what T15 highlighted in Q8, T5Q9 stipulates clearly, *"some diagrams are relevant as they are demonstrating clearly the maths concept but some are not relevant as they are far from demonstrating the specific concept"*. In particular, T33Q9 pointed out *"Maths for life – the book content is difficult, even learners could not link it to content without help"*.

#### 4.3.1.10 Relation to context

Some teachers claimed that some VOs used in textbooks are not familiar to the learners. This makes it challenging for learners to comprehend them and understand the message they display. In support of the above statements, T48Q7 noted, *"most of the diagrams are complex and strange to the learners, context or duplicate of what is in all these books".* In agreement, T14Q7 stressed, *"Some diagrams are too far from real life situations, only a few make sense of mathematics".* Additionally, T35Q8 highlighted that *"not all pictures are useful because some do not fit the geographical location of the learners e.g. in the rural area".*

#### 4.3.1.11 Topics in which teachers use VOs

Three-quarters of the teachers who participated in the study revealed that they use VOs in all chapters, as they strongly believe that learners grasp the content fast and effortlessly with VOs. In particular, T4Q10 argued, *"I use diagrams in all chapters, because in every topic a teacher needs to use visualisation to facilitate learning faster and effectively".* To support the above-mentioned assertion, T6Q10 stated, *"I use VOs in all chapters and themes because visualisation can be imaginary and not only the use of objects".* In agreement, T13Q10 explains, *"I use visualisation in all chapters".* T13 further alluded that *"I cannot teach any concept/topic without employing mental imagery or kinesthetic imagery; people mostly think that visualisation is physical".* Besides that, T20Q10 stated, *"I use it in all chapters, all mathematical problems are based on real life situations, and therefore require visualisations".* T44Q10 reported, *"I use VOs for learners to see and interpret in the correct manner and I use it in all chapters to make lessons interesting and for learners to learn the real life situation".*

Nevertheless, some teachers revealed that they only use VOs in some topics i.e. geometry, mensuration, algebra and others. They highlighted that they lack skills on how to use VOs in other chapters. They therefore demanded that they need training on how to use VOs especially in algebra. On the same note, they further elucidated that the use of VOs entirely depends on the topic and the basic competency under discussion. In the same way, T30Q10 explained, *"When to use diagrams very often depends on the topic and learners' level of understanding. Some topics are too abstract and therefore visualising is more important".* In the light of this T31Q41 showed that *"I only use it in graphs, mensuration, geometry, and trigonometry to induce the learner's interest to aspire to learn with understanding".* In concord, T10Q10 explained, *"I* *use pictures in mensuration and geometry only, as learners need to see pictures of how the shapes they are learning look like in real life".*

#### 4.3.1.12 Types of visualisation teachers uses

Most of the teachers noted that they do not have preferences in terms of types of VOs to use. They stated that it all depends on the topic under study and the VOs available at that moment either in textbooks or in other sources. In the same vein, T7Q1 noted, *"I prefer diagrams in textbooks, because they are readily available".* Additionally, T8Q1 said, *"It depends on the topic I am teaching. Diagrams, drawings, sketches, pictures, charts, graphs and illustrations could all be useful".*

Some of the teachers specifically indicated that they prefer sketches, drawings and diagrams because they are easily drawn and interpreted. In the same manner, T13Q1 stated, *"diagrams, drawings, sketches, pictures, charts, graphs, and illustrations depending on the topic under study. Using VOs makes it easier to explain concepts in a more practical way and also helps with slower learners".* Likewise, T16Q1 mentioned, *"I prefer diagrams and drawings because they can easily allow learners to recall what they know about the object, the common features of the object".* T26Q1 highlighted that *"my choice of which VO to use depends on the topic at hand, but if I have to generalize I use sketches because they are easy to draw and simple to interpret in most cases".*

#### 4.3.1.13 Limitations on using VOs

Most teachers revealed that although they find VOs to be essential in teaching and learning mathematics, there are some challenges coupled with the use of VOs, especially in their interpretation. Some of the VOs lack sufficient information; they are ambiguous and are not self-explanatory.

The teachers suggested that most of the learners could not visualise on their own without the help of the teacher. In support of the statement, the samples of teachers' quotes are as follows: T33Q8 noted, *"Some of the VOs are difficult for the learners to interpret on their own, they need the help of the teacher, but if the learner studies on their own, they might miss the point".* In agreement, T41 highlighted that *"the challenge is for the learners to visualise objects on their own".* T19Q10 said that *"some learners (few) particularly who are not good at visualisation fail to find the visual objects useful in mathematics. Thus these learners prefer algorithm methods".*

T30Q6 also indicated, *"Some objects do not really explain to the maximum how they are used/ how they are part of the questions hence some diagrams are difficult to understand"*.

The teachers further reported that the language used on notes accompanying the diagram in some instances is not at the level of the learners; the English was too high a level for the learners. T19Q6 said, *"The languages used on VOs is not clear to the learners and makes it a bit tough particularly in the book Maths for life*". Some teachers highlighted that VOs lack sufficient information. T7Q9 lamented, *"some lack details and some are not clear"*. Moreover, T11Q8 pointed out, *"Some pictures are not so useful, because they do not have sufficient information"*.

Teachers also revealed that, learners misinterpreted some of the VOs used in textbooks. T33Q10 mentioned, *"Some learners do not know how to interpret the visual objects; some visual objects are not self-explanatory"*. In addition to the above, T12Q10 noted, *"most of the challenges can be found once VOs are wrongly interpreted or not clearly visible"*. T8Q10 also said, *"Learners need help from the teachers to interpret the VOs"*. T13Q10 explained, *"Learners sometimes misinterpret the diagrams and get discouraged to read instructions: they mostly begin to answer the questions with little information as they omit the instructions while studying the drawing inappropriately"*. T15Q10 suggested, *"Some learners interpret them differently and it requires extra time to explain some objects that are not drawn to scale"*. Furthermore, T44Q10 acknowledged, *"people interpret VOs differently and it causes a lot of argument and takes up a lot of time"*. In addition, T47Q10 declared, *"Learners think or visualise differently so if I tried just using one way then some learners get lost and don't understand"*.

Ninety percent of the teachers highlighted that when VOs used are not familiar to learners it becomes challenging for the learners to understand them on their own, without the assistance of the teacher. In like manner, T46Q10 wrote, *"sometimes when the used pictures are not known or foreign to learners it is difficult for them to interpret them and difficult to connect theory to real life problems"*. Likewise, T5Q10 stated, *"When the objects are representing something that learners are not familiar with or something that they have never seen in real life it is a challenge to use them to solve problems"*. T18Q10 said that some VOs *"do not suit the context of the learners"*. T1Q10 highlighted that *"the challenge is when learners focus only on a diagram in a textbook and ignore that it is the real world"*.

Twenty-five percent of teachers contended that in the extreme case, when VOs are excessively used in textbooks, it makes learners dependent on VOs whereby they will not be able to solve problems without the pictures. In support, T36Q10 stated, *"If you get learners used to too many pictures in textbooks, it will be a problem, because examiners use a limited number of VOs which fail learners who are used to solving problems when the diagram is provided".*

### 4.3.2 Recommendations

Generally, the teachers recommended that there are changes that need to be made to some VOs that are used in all three mathematics textbooks. T20Q10 outlines, *"some diagrams need to be re-designed to fit the situations and correspond to the industrial labor market and the school curriculum".* Some teachers indicated that some VOs used in textbooks are not clearly explained. In some instances, it then becomes difficult for the teachers and learners to interpret them. Thus, teachers' induction and in-service training are needed in this regard. Regarding the learners, activities that are more practical should be included in the textbook with familiar objects. Thus, authors need to use diagrams that correspond to real life mathematical approaches and methodologies. The examiners should use at least 50% of VOs in the examination to cater for all learners, unlike the current situation, where few are used.

### 4.3.3 Summary

All teachers noted that VOs are central and imperative to the teaching and learning of mathematics as a subject. Therefore, the roles of VOs used in mathematics textbooks cannot be overemphasized. The majority of the teachers' responses stated clearly that VOs make abstract concepts concrete and clarify mathematical ideas whose meanings are difficult to comprehend. In addition, the teachers further asserted that VOs attract learners' attention and stimulate learners' interest to learn mathematics. They further argued that VOs also makes mathematics fun and practical. VOs enhance deep conceptual understanding.

They further highlighted that VOs help learners to grasp mathematical concepts without difficulty as they learn better by seeing visuals rather than texts. VOs play a significant role in mathematics as they aid learners to recall the concept discussed during the lesson. It was noted that 40% of the teachers' responses collected, emphasized that VOs can also be used as a tool for reasoning. The majority (90%) of teachers indicated that VOs are central to problem solving in mathematics. VOs also help learners to solve real life problems by relating them to mathematical problems, thus enabling learners to link and connect the mathematical concepts learnt in class to real life situations.

The majority of teachers (90%) stated that most of the VOs which are evident in all the three textbooks align well with the content where they are presented, despite arguing that some VOs used are inappropriate, unsuitable, and do not align well with the content. In addition, some teachers claimed that several VOs used in textbooks are not familiar to the learners. This makes it challenging for learners to comprehend them and understand the message they display without the assistance of the teacher.

Three-quarters of the teachers who participated in the study revealed that they use VOs in all chapters, as they strongly believe that learners grasp the content fast and effortlessly with VOs. Nevertheless, a few teachers revealed that they only use VOs in some topics i.e. geometry, mensuration, algebra and others. They highlighted that they lack skills on how to use VOs in other chapters. They therefore demanded that they needed training on how to use VOs, especially in algebra.

Most teachers revealed that although they find VOs to be essential to teaching and learning mathematics, there are some challenges coupled with the use of VOs, especially in their interpretation. Some of the VOs lack information; they are ambiguous and are not self-explanatory. The teachers noted that most of the learners could not visualise on their own without the help of the teacher. They further reported that the language used on notes accompanying the diagrams in some instances, is not at the level of the learners; that the level of English is too high for the learners.

Because of these points, they requested authors to use VOs that are familiar to learners and straightforward. On the same note, authors should use diagrams that respond to real life mathematical approaches and methodologies. In addition, the VOs should also align well with the content. Since they also experienced challenges with the interpretation, they suggested that teachers' induction and in-service training are needed in this regard.

## 4.4 RESEARCH RESULTS FROM INTERVIEWS WITH AUTHORS

By reading and analyzing the responses of the two authors interviewed, a sense of similarity and coherence is evident in both the authors' remarks and responses to the questions asked. Throughout the interview process, both authors were cooperative and confident. Both authors displayed their individual abilities and possessed broad and vast experience in the subject matters. The two presented in-depth and detailed responses on the questions posed to them, with practical examples that were relevant to the study. See appendix 4 for the authors' interview questions, and appendix 5 for the links of questions to themes.

### 4.4.1 The choices of the type of VOs used

Both authors indicated that they are aware of different VOs that can be used in mathematics textbooks namely, graphs, sketches, pictures, tables and many more. They pointed out that there are no criteria or conditions on the type of VOs to be included in textbooks; it is a choice that one has to make. Nevertheless, they indicated the relevance and significance of choosing the VOs wisely. In their case, they stipulated some aspects that they considered, for instance, the content, the context and the level of intellect for learners; gifted, average or slow.

Both authors contended that it is not about the variety of VOs used; rather it is about using VOs that fit the purpose. In support of this statement, A2L3 noted, *"the main objective is to use figures that are well and easily understood by learners and above all that make the text clear".* Additionally, A1L5 highlighted, *"when I choose the diagrams I always consider the slow or weak students, whereby I use simple diagrams and sketches which will enable all the students to understand and comprehend the content of the subject".* A2L6 noted, *"I design the book to suit the need of the learners, to help them understand the subject contents".* A1L5 further echoed the same thoughts that *"though the book is designed to suit the needs of the learners, it must be understood by both the teacher and learners alike on their own without a teacher especially when they are working on their own doing homework and practical activities".*

On the same note, they both argued that the VOs used in textbooks needed to explain the content explicitly, be familiar to the readers and needed to simplify the writings/texts. They both affirmed that VOs needed to be easily interpreted by learners alone without help from the teacher or peers. In support, A2L3 argues, *"VOs are of great importance in teaching and learning mathematics; that is why I chose the diagrams carefully when I wrote my book".* She further stated, *"I consider the things that I think learners will be familiar with, for example the simple drawings that can help a learner understand better on their own".* In accordance with the above statements, A1L2 states, *"it is very important to choose the right type of visualisation objects to be used in a textbook. Thus I opted for diagrams that are easy to interpret and explain the content well".*

#### 4.4.1.1 The choices of VOs used for decoration purpose

Decorative VOs in this study refer to figures that are made to look more attractive. The two authors expressed that they both use VOs for decoration purposes in their textbooks. They further acknowledged that VOs used for decoration purposes help to stimulates learners' interest in learning mathematics, as they are attracted to colourful drawings and sketches. In the light of the statement, A2L7 explained that she has tried to make the drawings interesting and attractive to the reader by putting them in colour. She also highlighted that *"this stimulates the learner's interest to study on their own".* She further noted, *"I also use real life teaching objects that children like most, e.g. cars, shoes, TV and many more"*. In agreement, A1L5 indicated that she used many drawings to attract learners' attention through the VOs. A1L5 further noted, *"I also used drawings that are familiar to the learners to make mathematics an easy and enjoyable subject".*

In her case particularly, A1L5 argued that diagrams on their own are enough to stimulate the learners' interest and make mathematics fun and enjoyable without being in color. A1L6 further acknowledged that she does not use colour in all her books for specific reasons. A1L7 explicitly explains that she does not use colour in her books, as learners in grade 8-10 are grown-up and are no longer attracted much to colours and she does not consider colour drawings due to the fact that it is too costly. In addition, she anticipated that *"without the colour the publication of the book will be cheaper, and everyone can afford a copy, but if one adds colour, it will be expensive and unaffordable. A textbook is a basic teaching and learning aid that everyone needs to have thus making it affordable will be wise".*

#### 4.4.1.2 The choices of VOs used for explanation purpose

Both authors acknowledged that it is vital for the diagrams to be accompanied by written notes. They explained that at times the VOs are not self-explanatory; they require additional notes to explain what the figure entails. The authors further emphasized that notes help learners to understand what the VOs portray and offer clear understanding of mathematical concepts. A1L12 confirmed that her book *"was designed for learners who are not yet expert in mathematics and they need the help of extra notes to understand mathematics concepts better"*. Thus, *"the notes accompanying the diagrams are meant to help the learners to understand the picture on their own, even without the help of the teacher or parent".* In support, A2L15 clarified, *"if one drew a diagram without any notes, the learners might not know what you are trying to* *portray, that is why I emphasize that most of the diagrams I used have notes for clarity and better understanding, and to ensure that learners are able to understand it on their own even when they are at home studying".*

### 4.4.2 Roles of VOs

Both authors strongly believe that the VOs serve various purposes in mathematics textbooks. The authors' views on roles of VOs are discussed below.

#### 4.4.2.1 VOs stimulate the learners' interest to learn mathematics

The two authors revealed that the VOs can be used a tool to stimulate the learners interest to learn mathematics. In support, A1L22 explained, *"for the learners who do not like mathematics, using diagrams is one way of getting them to like the subject as you get their attention through pictures".* She further denotes that *"learners do not like reading but when you bring in a picture they will get attracted and tempted to find out what the picture is all about.* A2L22 indicated, *"Learners like diagrams and for you to get their attention and get them interested give them what they like then they will get involved".*

#### 4.4.2.2 VOs simplify and clarify the mathematical concepts

The authors agreed that VOs help to simplify and clarify the written text making it easy for the learners to grasp the mathematical concepts. In the light of the above statement A2L20 explained, *"Diagrams simplify the content of the text, give it meaning by making it easy to understand, and further makes mathematic interesting to study and do".* In addition, she highlighted, *"diagrams make mathematics easy to do, because it becomes practical because if one does not understand the written text then one can't get help from the diagram"*. In agreement, A1L26 stated, *"sometimes one will discover that in the textbook, the instructions given are not clear and some information is missing, but with the help of the diagram the learner will be able to relate to and comprehend the diagrams better*". A1L21 further indicated, *"Learners understand diagrams better than written words, because they are lazy to read".*

#### 4.4.2.3 VOs help learners to recall the mathematical concepts

Both authors believe that VOs help learners recall the mathematical concepts. In support, A2L20 mentioned, *"diagrams help learners to remember easily what was taught and thus* *diagrams enable the learners to remember well when they see real life objects, unlike when they just read without diagrams".* Additionally A1L24 explained, "*Diagrams help you remember, you do not easily forget what you have seen, and when you see the pictures it sticks in your mind rather than just hearing about something".*

#### 4.4.2.4VOs as a tool for problem solving

Both authors affirmed that VOs could be used as an instrument to solve mathematical problems. VOs simplify the problem as it clearly portrays what the written texts signify and provide extra information, which allows learners to understand the problem better. A1L26 asserted, *"Diagrams in the textbooks give one a picture of what the texts explain and give one extra information that can help one understand the problem and hence solve it easily, especially in algebra and geometry".* She further gave an example, "on *page 200 of y=mx+c one will see that I gave a statement about the garden and then drew it; this makes it easy to understand and solve the problem easily".* Correspondingly, A2L25 noted, *"pictures play a big role in solving mathematical problems, pictures make abstract ideas concrete, and it is easy to play around with something you see to get an answer".*

### 4.4.3 VOs in relation to reality

Both authors indicated that it is worthwhile to use VOs that represent the concept true to real life situation. They explained that this helps learners recognize that mathematics is not confined to the classroom; it is practical in real life. Thus inspiring learners to study mathematics. In support A1L24 stated that *"mathematics is a practical subject, that is why it is very important to use diagrams relating to real life things or things that exist in real life, for instance relating a dam, to a cylinder, a box of candy to a cuboid and many more".* In the same accord, A2L21 highlighted, *"I use real life examples in the textbooks to arouse learners' interest to learn and practice mathematics, as they will be able to use what they learn at their homes and in the outside world".*

### 4.4.4 VOs in relation to context

Both authors acknowledged that it is significant to use VOs that are familiar to learners and that connected to their background and experiences. In spite of that, they argued that it is challenging to use VOs that are suitable and familiar to learners from all 14 regions since they are from different backgrounds: rural, urban and different ethnic groups. In addition, the two revealed that they selected VOs on the assumption that most of the learners are exposed to different pictures via social media, TV and newspapers. In support, A2L20 explained, *"I am aware that there are learners who stay in rural areas only, but nowadays learners, even those in the rural areas have access to TV, internet and newspapers. Therefore when it comes to the selection of diagrams which are contextualized and realistic, I use examples of local materials, assuming that our learners are exposed to different things via social media and other ways".* A1L22 put forth her argument that she integrates examples from science to make it more interesting. Although she has included many Namibian things, for example, mountains, the sea etc., she is aware that there are learners who have not seen these things, however, it is a motivation for them to study and see those things in the future. However, A1L23 argued, *"it is vital to leave some things out of mathematics; much as I would like to contextualize the diagrams that are used nationally within the 14 cultural regions, it is close to impossible; I also wanted to keep the book free of cultural orientation, so that in 100 years it will still work in our country, as well as in other countries"*.

### 4.4.5 VOs in relation to content

Both authors claimed that VOs are entirely dependent on the content, thus the content pre-exists the VOs. They further clarified that content is one of the aspects they consider when selecting the VOs, therefore one cannot choose the VOs before studying the content. This implies that VOs should always be in line with the content. In support, A1L26 emphasized, *"This is very important; the drawings need to be in line with the content because one cannot just use anything that does not have any connection to what one is talking about*". She strongly believes that *"if there is one in my textbook it is an error because that is one aspect that I pay attention to, not to mislead the learners".* In addition, A2L22 recorded that *"one needs to understand the content before selecting pictures; this is why all diagrams have to be in line with the content, as this will enhance the understanding of the concept under study".* She further argues, *"One cannot use just any figure anywhere, as an author one has to be careful when choosing these visuals otherwise one can easily mislead learners and misrepresent the concepts under study. That is why editing the book before publishing is vital".*

### 4.4.6 Limitations in the use of VOs

The two authors acknowledged that there are numerous limitations associated with using suitable VOs.

A1L27 indicated that the program she used to draw or insert the VOs was a challenge, as she was unfamiliar with it and the program did not have all the VOs she preferred using. A1L28 argued, *"The program that I used when I wrote this book posed a great challenge because I could not master it in time to use it to draw as many diagrams as I wished".* She further indicated that it requires time to insert and draw the desired VOs. In support, A2L26 reported, *"it requires time to insert and draw the preferred pictures".*

The authors further showed that using VOs that are familiar to all learners and teachers in the country seems impossible as learners originate from different settings and locations in the country. In support, A1L29 asserted, *"one cannot contextualize it as much as one wants to as learners are from different cultural groups".* Moreover, A2L31 demonstrated that *"trying to use pictures that fit all learners seems to be a great challenge, not all learners are from town or village; if one uses pictures of things from the village others are left out and vice versa".*

### 4.4.7 Recommendations

Both authors invited teachers to make use of the platforms available, for example to participate in feedback sessions to give their inputs and contributions to the textbooks. They clarified that before the final publication, copies of a textbook are sent out to schools, so that teachers can give their suggestions and counsel about the books. They further called on teachers to take part in the improvement of the quality of the textbooks. In support, A1L32 said, *"let the teachers talk to us, advise us, my details are on social media and I also send out a copy of my textbooks to schools before the final publication, they must take it seriously and give me their inputs".*

They further stressed that teachers should continuously assist their learners with the interpretation of the VOs, and bear in mind that learners are unskilled, inexperienced and still in need of help. In support, A2L30 called on teachers to *"help their learners to interpret these diagrams because learners are not as skilled nor exposed to many diagrams as teachers are".*

### 4.4.8 Summary

In summary, both authors indicated that the textbooks are central to teaching and learning of mathematics, thus the importance of VOs as used in textbooks cannot be overemphasized. They pointed out that though there are no criteria or conditions on the type of VOs to be included in textbooks; it is a choice that one has to make. In their case, they stipulated several aspects that they considered; for instance, the content, the context and the level of intellect for learners; gifted, average or slow. Nevertheless, they indicated the relevance and significance of choosing the VOs wisely to ensure that suitable and appropriate VOs are used. For this reason, if the VOs are not carefully selected they can mislead learners and cause misconceptions.

On the same note, they both affirmed that VOs need to be easily interpreted by learners alone without help from the teacher or peers. Both authors acknowledged that it is vital for the diagrams to be accompanied by written notes. They explained that at times the VOs are not self-explanatory; they require additional notes to explain what the figure entails. The authors further emphasized that notes help learners to understand what the VOs portray and promote clear understanding of mathematical concepts. Additionally, the two authors expressed that they both use VOs for decoration purposes in their textbooks. They further acknowledged that VOs used for decoration purposes help to stimulates learners' interest in learning mathematics, as they are attracted to colourful drawings and sketches.

Nevertheless, one of the authors did not have the experience to fully utilize the computer technology in drawing diagrams. It is thus important that authors have the necessary skills to capitalize on the drawing capacity of the technologies at their disposal. There is a need for them to do research and acquaint themselves with what learners know and what is trending so that they can use familiar VOs to attract learners' interest in mathematics.

Both authors strongly believe that VOs serve various purposes in mathematics textbooks. The two authors revealed that the VOs could be used as a tool to stimulate the learners' interest in learning mathematics. The authors agreed that VOs help to simplify and clarify the written text making it easy for the learners to grasp the mathematical concepts. Both authors believe that VOs help learners recall the mathematical concepts as the visual information is more easily remembered than text information.  Both authors affirmed that VOs could be used as an instrument to solve mathematical problems. VOs simplify the problem as they clearly portray what the written texts signify and provide extra information that allows learners to understand the problem better.

Both authors indicated that it is worthwhile to use VOs that represent the concept true to real life situation. They explained that this helps learners recognize that mathematics is not confined to the classroom; rather it is practical and has a place in real life, thus inspiring learners to study mathematics. Both authors argued that the VOs used in textbooks need to explain the content explicitly. They further claimed that VOs are entirely dependent on the content, thus the content pre-exists the VOs. They further clarified that content is one of the aspects they consider when selecting the VOs, therefore one cannot choose the VOs before studying the content. This implies that VOs should always be in line with the content.

Both authors acknowledged that it is significant to use VOs that are familiar to learners and connected to their background and experiences. In spite of that, they argued that it is challenging to use VOs that are suitable and familiar to learners from all 14 regions since they come from different backgrounds: rural, urban and different ethnic groups. In addition, the two revealed that they select VOs on an assumption that most of the learners are exposed to different pictures via social media, TV and newspapers.

In the next section, findings on various themes that emerged from the analysis of the VOs, the survey questionnaires from teachers, and the authors' interviews are discussed.

## 4.5 SUMMARY OF FINDINGS

The main messages from the three sources of data are as follows:

It was observed that more VOs were used in Geometry than in Algebra in all textbooks, as would be expected. Furthermore, sketches, drawings and pictures were used more often than graphs, tables and schematics. Remarkably, all VOs used represent the concept true to real life in both algebra and geometry. Additionally, more than 80% of the VOs used align well with the content. VOs are mostly used as explanatory, as opposed to exemplifying, decorating and complementing.

Teachers noted that VOs serve as tools for learning. They stressed that VOs can only be beneficial and useful if they are clearly presented in textbooks and self–explanatory. They revealed that some of the VOs used are inappropriate; they do not align with the content and are unfamiliar to learners. Teachers further indicated that they lack knowledge on how to interpret and use some VOs, specifically in chapters such as algebra. They therefore requested to be given training in this aspect.

Both authors indicated that the importance of VOs used in textbooks could not be overstressed. They revealed some of the factors they consider in selecting VOs namely, content, context and level of intellect of learners among others. They further highlighted the significance of using suitable and appropriate VOs in textbooks, to avoid misconceptions and misinterpretations. Despite that, they pointed out some of the limitations on the use of VOs namely, the use of computer technology, and finding VOs that are familiar to all learners. They called on teachers to take part in field-tests of textbooks to give their inputs.

## 4.6 EMERGED THEMES

The data analysis process involved the continual engagement with data by studying the interview transcripts and survey questionnaires repeatedly. During this analysis process, similarities were coded and categorized to bring about my five major themes. The themes that emerged from the collected data are the interpretation of VOs by learners, the choices on the type of VOs by authors, the roles of VOs by authors and teachers, the relation of VOs to mathematical content, and the VOs relation to reality. Although they are discussed separately, it is noted that they are interrelated and intertwined.

### 4.6.1 Interpretation of VOs

Interpreting mathematical visual diagrams is a challenge faced by mathematicians as well as learners (Gellert & Steinbring, 2014). In agreement with the literature, the findings from teachers indicated that most of the learners could not visualise VOs on their own without the help of the teacher. Moyer (2014) pointed out that “students require appropriate assistance and guidance from teachers and knowledgeable peers as they select, interpret, and create visual models of mathematics” (p.3). Authors also pointed out that teachers should continuously assist their learners with the interpretation of the VOs given that learners are unskilled and inexperienced and still need help. A2L30 called on teachers to *“help their learners to interpret these diagrams because learners are not so skilled nor exposed to as many diagrams as teachers are”.* From the literature, it is noted that reading and using images constitute skills that should not be left to chance, but should be taught systematically (Dreyfus & Eisenberg, 1990). Thus, learners should be better prepared to interpret and utilize the information presented in the books (Watkins et al., 2004). This refers to diagram literacy (Diezmann & English, 2001).

In spite of the findings from both authors claiming that they used suitable, self-explanatory and straightforward VOs in their textbooks, the findings from the teachers indicated that some diagrams are not self-explanatory, and are vague or confusing, leading to misinterpretation by learners. T33Q10 mentioned, *"Some visual objects are not self-explanatory".* T12Q10 noted, *"Most of the challenges occur once a diagram is wrongly presented or not clearly visible; it is difficult for them to interpret them".* This matched with what Monoyiou et al. (2007) stated, "The selection and effective use of appropriate mathematical resources requires careful consideration and planning" (p.21). For this reason, if VOs are misused or misinterpreted, it may in fact interfere with learning (Pena & Quilez, 2001) and lead to misunderstanding (Steenpaβ & Steinbring, 2014). Thus, the authors need to be mindful of the roles that VOs play in mathematics textbooks and use VOs that are appropriate and suitable for learners. They should use VOs that are clear and meaningful. Their pictures and photographs should help clarify their text (Tyson-Bernestein, 1989).

Towards achieving this end, both authors urged teachers to make use of the platforms available, for example to participate in feedback sessions to give their inputs and suggestions for improvements of the textbooks. In addition, they clarified that before the final publication, the textbook is sent out to schools, so that teachers can give their suggestions and advice about the books. They further called on teachers to take part in the improvement of the quality of the textbooks. A1L32 said, *"Let the teachers talk to us, advise us, my details are on social media and I also send out a copy of my textbooks to schools before the final publication, they must take it seriously and give me their inputs".* In agreement, Searle (1989) recommended, "all textbooks for use in classrooms should be reviewed by experts and by teachers and should be field-tested in classrooms" (p. 37). He further contended that the purpose of the field-testing is to determine whether the books are suitable for the children who will use them – whether the language, the illustrations, and the level of difficulty are appropriate *(ibid.).* In my experience although textbooks are sent to schools before final printing/publication, most teachers are not well informed on what to do with the textbooks, or what is required of them. Thus, teachers need to be well informed on what to do and ensure that the feedback reaches the authors on time.

### 4.6.2 Choices on the type of VOs

According to the findings, both authors specified that the selection of VOs was crucial and should be done wisely. They further indicated that it is not about variety of VOs used; rather it is about using VOs that fit the purpose. In support, literature points out that the quality of VOs such as illustrations and graphics are critical elements that need to be considered when selecting the VOs to be included in a textbook (Namibia. MoEAC, 2015). Neumann (1989) suggests some major criteria for evaluation, which includes quality of presentations and attractiveness of illustration for selecting textbooks. In agreement, both authors indicated that they used VOs that are attractive to learners to stimulate learners' interest in mathematics, as learners are attracted to drawings and sketches. Furthermore, some teachers indicated that most of the VOs are in colour and look attractive while a few are the opposite.

On a different note, A1L5 acknowledged that she does not use colour in all her books for various reasons. A1L7 explicitly explains that she does not use colour in her books as learners in Grades 8-10 are grownup and are no longer attracted much to colours. Further, she does not consider colour drawings because it is too costly. In addition, she said, *"without the colour the publication of the book will be cheaper, and everyone can afford a copy, but if one adds colour it will be expensive and unaffordable. A textbook is a basic teaching and learning aid that everyone needs to have thus it has to be made affordable".* Verspoor (1989) observed that textbooks "are relatively inexpensive and require little or no maintenance" (p. 67). This is important to keep the cost of textbooks low. Neumann (1989) stated, "Books are among the most effective, and probably the most cost-effective, tools for teaching and learning" (p.127). In my view it is however important that the quality of the content of a textbook is not compromised.

In my experience it is a well-known syndrome that textbooks in developing countries are usually bland and of poor quality. I personally disagree with A1's assertion that Grades 8-10 learners are not attracted to colours. In my experience colour adds appeal and interest to any product that young (and old) come into contact with. One only needs to observe the use of vivid colours in any advertising object. As much as the economic imperatives cannot be ignored, the attractiveness of a textbook and its VOs is important to ensure that it is stimulating to young people.

Tyson-Bernstein (1989) indicated that the criteria for analyzing and evaluating quality textbook graphics should be clear and meaningful. In the Namibian textbook policy of 2015, the last criterion provides guidelines on design, presentation and the ease of use of textbooks (Namibia. MoEAC, 2015). However, both authors indicated that they are not aware of these criteria. Though these criteria do not specifically mention VOs, it is implied that VOs need to be accurate, attractive and useful. The above suggests that there is a need for more clarity on these criteria and explicit guidelines need to be spelled out to potential authors about the use of VOs.

### 4.6.3 Relation to content

A quality mathematics textbook should align with the curriculum (Gu et al., 2004). The textbook should be written according to the prescribed syllabus and every aspect of the syllabus should be adequately covered *(ibid.).* In agreement, both authors claimed that VOs are entirely dependent on the content, thus the content pre-exists the VOs. They further clarified that content is an important aspect they consider when selecting the VOs. They said that one could not choose the VOs before studying the content. It is important that VOs should always align well with the content. In the same vein, Pepin & Haggerty (2004) stated that quality representations should align with the frameworks or syllabi they represent. In spite of the authors' statements, teachers argued that some VOs used in the three textbooks are inappropriate, unsuitable, and not in line with the content. Furthermore, some VOs have weaker connections to the content presented, as they do not provide sufficient information.

### 4.6.4 Relation to context

Textbooks should inspire and initiate learners. They should however take into account the cultural and social background of learners (Gu et al., 2004). From the findings, authors have acknowledged that they are aware of the fact that VOs to be used in textbooks ought to be familiar to learners and connected to their background and experiences. However, they admitted that it is challenging to use VOs that are suitable and familiar to learners from all 14 regions since they are from different backgrounds: rural, urban and different ethnic groups. In addition, the two authors revealed that they select VOs on the assumption that most of the learners are exposed to different pictures via social media, TV and newspapers. In contrast, teachers indicated that some VOs used in textbooks are not familiar to the learners. This makes it challenging learners to comprehend them and understand the message they display. Seeger (1998) states that, in order for a connection to be made between external representations and the mathematical concept they represent, and between the mathematics and the children's experiences, VOs must be viewed as vehicles for exploration within social contexts that allow for multiple understandings of mathematical content.

### 4.6.5 Roles of VOs

The use of VOs are central elements to the effective teaching and learning of mathematics (Gellert & Steinbring, 2014). Thus, the roles of visualisation in mathematics teaching and learning cannot be over-emphasized.

Both authors and 70% of the teachers specified that VOs attract learners' attention and stimulate learners' interest in learning mathematics. They further argued that VOs also make mathematics fun, enjoyable and practical. A1L22 confidently explained, *"For the learners who do not like mathematics, using diagrams is one way of getting them to like the subject as you get their attention through pictures".* She further denoted, *"learners do not like reading but when you bring in a picture they will get attracted and tempted to find out what the picture is all about".* In accordance, Tyson-Bernestein (1989) recorded that illustrations help motivate students' reading and stimulate class discussion.

Levie & Lentz (1982) highlighted that text information is remembered well when pictures illustrate it, rather than when there are no illustrations. In agreement, the findings from both authors and teachers indicated that VOs play a significant role in mathematics as they aid learners to remember the concept discussed during the lesson for a longer period. Similarly, visual information is recognized and remembered for longer durations than verbal information alone (Mayer, 1989).

Sobbëke (2005) proposes that in mathematics classrooms, visual diagrams help the learners to better see mathematical concepts and ideas. In agreement, the findings from a number of the teachers and both the authors suggested that VOs help to simplify and clarify the written text, making it easier for the learners to grasp the abstract mathematical concepts easily. A picture gives a meaning better than the written text (Bosch et al., 2006). T5Q5 explicated, *"through the use of visualisation objects, it gives learners a clear picture of what is meant by a certain mathematical concept, e.g. a cube, showing them a drawing of a net of a cube helps them understand better, the faces, the vertex, and make it easier to calculate the total surface area"*. Mathematical representations are designed to represent explicitly and concretely mathematical ideas that are abstract (Moyer, 2001).

In the same light, teachers indicated that VOs could also be used as a tool for reasoning. “Pictures, graphs and signs, and other spatial representations play an important role in reasoning, in part because we are able to interpret or infer meanings from these representations without specific instruction in how to do so” (Monoyiou et al., 2007, p. 2).

Ho (2010) in his study reflects that visualisation is at the heart of mathematical problem solving and “it can be a powerful cognitive tool in problem solving” (p. 1). The finding from the majority (90%) of teachers and both authors indicated that VOs are central to problem solving in mathematics. VOs simplify the problem as they clearly portray what the written texts signify and provide extra information that allows learners to understand the problem better. In addition, Lohse et al. (1994) says, “visualisation objects can facilitate problem-solving and discovery by providing an efficient structure for expressing the data” (p.37). In agreement Diezmann & English (2001) suggest, “In problem solving a diagram can serve to unpack the structure of a problem and lay that foundation for its solution and it can serve as a tool of mathematical learning” (p. 77).

## 4.7 CONCLUSION

In this chapter, I have presented and discussed the data collected from textbooks, survey questionnaires and interviews. Each section was concluded with a brief summary. The data stemming from the survey questionnaires and interviews was based on the views, perceptions and experiences of the participating mathematics teachers as well as the authors. The themes used to categorize and discuss data came from the analytical tool and some emerged from the survey questionnaires and interviews.

Both teachers and authors acknowledged that VOs are vital to teaching and learning and can be used as a tool for learning. VOs can also be used as a tool for conceptual understanding to stimulate learners' interest in learning mathematics, to assist with reasoning, and to help them recall mathematical concept. It is important that authors choose VOs wisely. The VOs need to be related to the learners' contexts, be suitably aligned to the mathematics content, and be self-explanatory and non-ambiguous. Teachers should support learners in using VOs, to enrich their learning experience. Teachers are aware that learners need support and scaffolding in using VOs appropriately. How to use VOs appropriately and strategically should be central to any teacher training. Some findings, and conclusions drawn, together with some recommendations will be extensively discussed in the following chapter.

# 5. CHAPTER FIVE: CONCLUSION AND RECOMMENDATION

*The effectiveness of visual representations is dependent on the learner's ability to independently and accurately interpret them* (Downey, 1980).

## 5.1 INTRODUCTION

This chapter serves as a conclusion to my research study and concisely summarizes the key research findings. The chapter includes a summary of main emerging themes in relation to the nature of VOs evident in Grade 9 Namibia mathematics textbooks, a discussion of the limitations of the study, the significance of the study, and recommendations and implications arising from the study. Lastly, suggestions for future research, the concluding remarks and personal reflections are presented.

## 5.2 SUMMARY OF EMERGING THEMES

The study sought to analyze the nature of VOs evident in the three grade 9 mathematics textbook namely *y=mx+c to success, Maths for Life* and *Discover Mathematics 9*. I was particularly looking at the types of VOs, roles of VOs, relation of VOs to mathematical content, relation of VOs to reality and properties as outlined in the analytical tool discussed earlier in methodology chapter page 52. The study also tried to understand the mathematics teachers' views and perception on the use of VOs and the authors' rationale on selecting VOs. This was done via survey questionnaires and interviews respectively.

The findings were clustered into the following main themes: the interpretation of VOs by learners, the choices on the type of VOs by author, the roles of VOs by authors and teachers, the relation of VOs to mathematical content, and the VOs relation to reality. Although they are discussed separately, it is noted that they are interrelated.

### 5.2.1 Interpretation of VOs by learners

The findings from both teachers and authors indicated that interpreting mathematical VOs is a challenge encountered by learners. The findings from teachers indicated that most of the learners could not interpret VOs on their own without the help of the teacher. Similarly, authors also pointed out that teachers should continuously assist their learners with the interpretation of the VOs, mindful of the fact that learners are unskilled and inexperienced. In using VOs, learners should be better prepared to interpret information and diagrams in the books (Watkin et al., 2004).

In spite of the findings from both authors claiming that they used suitable, self-explanatory and straightforward VOs in their textbooks, the findings from the teachers indicated that some diagrams are not self-explanatory. They are vague and confusing, leading to misinterpretations by learners.

On a different note, both authors urged teachers to participate in field-testing of the textbooks to give their input.

### 5.2.2 Choices on the type of VOs

From the findings, both authors specified that the selection of VOs was crucial and should be done wisely. They further indicated that it is not about the variety of VOs used; rather it is about using VOs that fit the purpose.

One author acknowledged that she does not use colour in her books for various reasons. She believes learners in Grades 8-10 are grownup and are no longer attracted much to colours. In contrast, in my experience colour adds appeal and interest to any product that young (and old) people come into contact with. She further added that she does not consider colour drawings because it is too costly, and that it was important to keep the cost of textbooks low. I argue that it is however more important that the quality of the content of a textbook is not compromised.

Both authors indicated that they are not aware of any prescribed criteria they should consider on the choices of the VOs to be used in textbooks. However, criteria do exist, although they are not specific enough. This suggests that there is a need for more clarity on these criteria and explicit guidelines need to be spelled out to potential authors about including VOs in their works.

### 5.2.3 Relation to content

In relation to content, both authors claimed that VOs are entirely dependent on the content, thus the content pre-exists the VOs. They said that one could not choose the VOs before studying the content. Gu et al (2004) indicated that the textbook should be written according to the prescribed syllabus and every aspect of the syllabus should be adequately covered. Despite the authors' claim, teachers argued that some VOs used in the three textbooks are inappropriate, unsuitable, and not in line with the content.

### 5.2.4 Relation to context

From the findings, authors acknowledged that they are aware of the fact that VOs to be used in textbooks ought to be familiar to learners and connected to their background and experiences. However, they admitted that it is challenging to use VOs that are suitable and familiar to learners from all 14 regions of Namibia. In addition, the two authors revealed that they selected VOs on the assumption that most of the learners are exposed to different pictures via social media. Similarly, teachers confirmed that some VOs used in textbooks are not familiar to the learners. This makes it challenging for learners to comprehend them and understand the message they display.

### 5.2.5 Roles of VOs

Both authors and teachers admitted that VOs are central elements to the effective teaching and learning of mathematics**.** Thus, the roles of visualisation in mathematics teaching and learning cannot be over-emphasized.

The findings from both authors and teachers showed that VOs attract learners' attention and stimulate learners' interest in mathematics. They further argued that VOs also make mathematics fun, enjoyable and practical. In accordance, Tyson-Bernestein (1989) recorded that illustrations help motivate students' reading and stimulate class discussion. They further indicated that VOs play a significant role in mathematics as they aid learners to recall the concept discussed during the lesson. In agreement, Levie & Lentz (1982) highlighted that text information is remembered well when pictures illustrate them, rather than when there are no illustrations.

Sobbëke (2005) proposes that in mathematics classrooms, visual diagrams help the learners to better see mathematical concepts and ideas. In agreement, the findings from a number of teachers and the authors suggested that VOs help to simplify and clarify the written text, making it easier for the learners to grasp the abstract mathematical concepts more easily. In the same light, teachers indicated that VOs could also be used as a tool for reasoning. "Pictures, graphs and signs, and other spatial representations play an important role in reasoning, in part because we are able to interpret or infer meanings from these representations without specific instruction in how to do so" (Monoyiou et al., 2007, p. 2).

The findings from the majority (90%) of teachers and both authors indicated that VOs are central to problem solving in mathematics. VOs simplify the problem as they clearly portray what the written texts signify and provide extra information that allows learners to understand the problem better. In agreement, Ho (2010) in his study reflects that visualisation is at the heart of mathematical problem solving.

## 5.3 LIMITATIONS

In spite of this study providing and presenting an in-depth analysis of the nature of VOs in three textbooks, the findings of this case study cannot be generalized to other textbooks. However, the findings can provide insights for further research.

## 5.4 SIGNIFICANCE OF THE STUDY

The study contributes towards improving the quality of textbook evaluation and design of a suitable and more comprehensive assessment of the textbook evaluation process in Namibia. This can be done by revising the criteria for including illustrations. These should be precise and clear. It is hoped that this study will provide some insights into the use and nature of the VOs in textbooks. It is also hoped that it creates a critical awareness among teachers of the role of VOs in textbooks, inspiring them to help their learners interpret the VOs effectively. The importance of VOs should inspire potential authors to use suitable and appropriate VOs that enhance conceptual teaching and learning of mathematics.

## 5.5 RECOMMENDATIONS AND IMPLICATIONS

Textbooks are a universal and central element of teaching and learning mathematics. This study showed that VOs in mathematics textbooks are important teaching and learning tools that are often used to enhance learning of mathematical concepts (Steenpaβ & Steinbring, 2014). Mathematics textbooks are accessible to both teachers and learners at most Namibian schools with a 1:1 learner to textbook ratio (Namibia. MCA, 2015). Thus, there is a need to take advantage of the availability of textbooks to get the best out them, particularly in relation to the use of VOs evident in mathematics textbooks. Recommendations that could be implemented to: improve the quality of the textbook in terms of VOs, improve the use of VOs among teachers, create awareness among authors to ensure that suitable and appropriate VOs are used, which can be of benefits to our learners, are hereby made:

1. Authors should acquaint and familiarize themselves with computer technology, so that they can utilize and capitalize on the drawing capacity of the technologies at their disposal.
2. There is a need for authors to do research and acquaint themselves with what learners know and what is trending, so that they can use familiar VOs to attract learners' interest to learn mathematics.
3. Teachers should be encouraged to teach and help learners how to read and interpret diagrams in mathematics.
4. Teachers should be encouraged to participate in field-test feedback sessions to give their inputs and contributions to authors to help improve the quality of the textbooks
5. Formal platforms should be created whereby teachers and authors engage and review collaboratively the textbooks to determine whether the books are suitable for the children who will use them.
6. The Ministry of Education should subsidize the printing of textbooks to ensure that the quality of VOs is not compromised.
7. The curriculum planners and textbooks evaluators need to revise the criteria for the evaluation of VOs used in mathematics textbooks and give explicit guidelines to authors.
8. Teachers' induction and in-service training should include diagram interpretation i.e. training on diagram literacy.
9. Authors should provide teachers with teachers' guides, where explanations for the different VOs are articulated.

Despite much evidence to suggest that VOs used in textbooks promote learning and understanding of texts, most of the learners do not have adequate skills to interpret the VOs. Therefore, learners require appropriate assistance and guidance from teachers and knowledgeable, competent peers to interpret VOs. This is in agreement with Dreyfus & Eisenberg (1990) who stated that reading and using images constitute skills that should not be left to chance, but should be taught systematically. Nevertheless, it is not clearly stipulated in the curriculum how this can be done.

## 5.6 FUTURE RESEARCH

This study was carried out on two chapters from three grade 9 mathematics textbooks only and it would be interesting to expand this study. The following are proposed possible areas for further research:

- conduct using all chapters
- use mathematics textbooks of different grades at primary, junior and secondary level
- investigate how learners interpret VOs evident in mathematics textbooks
- general practices of how teachers help learners interpret VOs evident in textbooks.

Even though it is not clearly stipulated on how diagram literacy can be provided, it would be interesting to conduct a research project that focuses on how teachers teach learners to read and interpret diagrams in mathematics at primary, junior and secondary phase.

## 5.7 CONCLUDING COMMENT

From this study, it is clear that textbooks are key resources for the teaching and learning of mathematics. They make important contributions to improving the quality of education in all stages of educational development (Verspoor, 1989). Besides that, I strongly believe that if the quality of textbooks improves, particularly in terms of VOs used, this could have a positive impact on the teaching and learning of Mathematics particularly in Namibia. From my experience, teachers are continuously looking out for new practices on how to help their learners learn mathematics better and improve performance. I believe that when teachers are aware of the roles and importance of VOs in textbooks, they are likely to be inspired to help their learners interpret the VOs effectively. For the same reason, noting the importance of VOs, it should inspire potential authors to use suitable and appropriate VOs.

## 5.8 PERSONAL REFLECTIONS

It was a pleasure to have walked this journey. As a beginner researcher, I came to realize that research is a lengthy journey that requires concentration and a massive amount of time. It is a journey where you are required to devote time to critically read, analyse and make sense of other writing. This has broadened my knowledge and understanding of visualisation in mathematics and in education matters in general. In addition, my writing skills have improved.

This research study also allowed me to reflect personally on how I neglected the VOs used in textbooks and how learners interpreted them. I will definitely support and encourage the use of suitable VOs in textbooks and help learners to interpret the VOs appropriately.

The VIZNAMZA project afforded me my first opportunity to present at a national level. I was able to present my proposal at the National Mathematics Congress that took place in May 2016 in Swakopmund, Namibia. This has encouraged me to participate in other Congresses of the same nature. In the same vein, I have registered to present a short paper on the findings of my study at Southern African Association for Research in Mathematics, Science and Technology Education (SAARMSTE) annual conference that will be taking place in January 2017 in Bloemfontein, South Africa. This will be an achievement for me.

This was a journey worth taking and it has enriched me both professionally and academically.

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## APPENDICES

## Appendix1: Approval letter from the permanent secretary of Education

REPUBLIC OF NAMIBIA

**MINISTRY OF EDUCATION, ARTS AND CULTURE**

*Tel: +264 61 -2933200*
*Fax: +264 61- 2933922*
*Enquiries: C. Muchila*
*Email: Cavin.Muchila@moe.gov.na*

*Luther Street, Govt. Office Park*
*Private Bag 13186*
*Windhoek*
*Namibia*

File no: 11/1/1

Ms Selma N. Nghifimule
P/Box 443
Student Number: 200027425
Ondangwa

Dear Ms Selma N. Nghifimule

**SUBJECT: PERMISSION TO CONDUCT RESEARCH IN OSHANA REGION**

Kindly be informed that permission to conduct research for your Master's Degree in Oshana region is herewith granted. You are further requested to present the letter of approval to the Regional Director to ensure that research ethics are adhered to and disruption of curriculum delivery is avoided.

Furthermore, we humbly request you to share your research finds with the ministry. You may contact Mr C. Muchila at the Directorate: Programmes and Quality Assurance (PQA) for provision of summary of your research findings.

I wish you the best in conducting your research and I look forward to hearing from you soon.

Sincerely yours

SANET L. STEENKAMP
**PERMANENT SECRETARY**

08.3.16.
Date

***All official correspondences must be addressed to the Permanent Secretary***

## Appendix 2: Consent letters from the participating authors

Oshakati
Email: tilenikalunga@hotmail.com
Cell No.: +264812263918

30 March 2016

Attention to:
[redacted]
Author

Dear Mrs [redacted]

**Re: Request to participate in a research project by being interviewed**

I am Selma Ndilipomwene Nghifimule, currently pursuing a Master's degree in Education (Mathematics Education) at Rhodes University. The study aims to analyse the nature of visualization objects in three Namibian Grade 9 mathematics textbooks. Visualization objects refer to, diagrams, drawings, sketches, pictures, charts, graphs and illustrations.

It is hoped that this study will contribute to improving the quality of textbooks evaluations and design of suitable textbook evaluation criteria in Namibia, and create a critical awareness of the role of visualization objects in textbooks amongst teachers, policy makers and potential authors. It is on this basis that I seek permission to interview you on your rationales and choices for using the identified visualization objects in your mathematics textbooks ([redacted]). The interview can take place on the date, time and place of your choice.

I also offer you the opportunity to read through the thesis once it has been completed. You are free to refuse to participate in this research and you may also withdraw from participation at any time.

I look forward to hear from you.

Yours faithfully

Selma Ndilipomwene Nghifimule

**PARTICIPANT RESPONSE:**

I [redacted] have read the above form, understand its content, and I agree to participate in the above research. I understand that participation is voluntary and that I may withdraw at any stage.

Signature of Participant:

Date: 04 May 2016

APPENDIX B: CONSENT

P.O.Box 443
Oshakati
Email: tilenikalunga@hotmail.com
Cell No.: +264812263918

30 March 2016

Attention to:
[redacted]
Author

Dear Mrs [redacted]

**Re: Request to participate in a research project by being interviewed**

I am Selma Ndilipomwene Nghifimule, currently pursuing a Master's degree in Education (Mathematics Education) at Rhodes University. The study aims to analyse the nature of visualization objects in three Namibian Grade 9 mathematics textbooks. Visualization objects refer to, diagrams, drawings, sketches, pictures, charts, graphs and illustrations.

It is hoped that this study will contribute to improving the quality of textbooks evaluations and design of suitable textbook evaluation criteria in Namibia, and create a critical awareness of the role of visualization objects in textbooks amongst teachers, policy makers and potential authors. It is on this basis that I seek permission to interview you on your rationales and choices for using the identified visualization objects in your mathematics textbooks [redacted]. The interview can take place on the date, time and place of your choice.

I also offer you the opportunity to read through the thesis once it has been completed. You are free to refuse to participate in this research and you may also withdraw from participation at any time.

I look forward to hear from you.

Yours faithfully

Selma Ndilipomwene Nghifimule

**PARTICIPANT RESPONSE:**

I [redacted] have read the above form, understand its content, and I agree to participate in the above research. I understand that participation is voluntary and that I may withdraw at any stage.

Signature of Participant: ____________________

Date: 30.06.2016

## Appendix 3: Sample of the survey questionnaire

**Visualization is viewed as an important teaching and learning tool, whereby using of visualization objects is to enhance learning of mathematical concepts.** With this questionnaire I am dedicated to obtain mathematics teachers' views and opinions on their perception and use of **(identified)** visualization objects in their teaching from the three grade 9 textbooks: y=mx+c, Discover mathematics and Maths for life. For the purpose of this study, visualization objects refer to, diagrams, drawings, sketches, pictures, charts, graphs and illustrations.

Please take a few moments to complete the following questions. Your responses/input is essential to provide some insights into the use of visualization Objects and create a critical awareness of the role of visualization objects used in textbooks amongst teachers. Your responses to this form are completely anonymous. Thank you very much for your time and answers.

**1.** Which visualization object do you prefer using in your teaching? Why?
________________________________________________________________
________________________________________________________________
________________________________________________________________
________________________________________________________________
________________________

**2.** Which visualization objects do you prefer using (the one portrayed in textbooks or do you use your own)? Elaborate
________________________________________________________________
________________________________________________________________
________________________________________________________________
________________________________________________________________
________________________________________________________________
________________________________________________________________

**3.** How often do you use visualization objects in your teaching?
________________________________________________________________
________________________________________________________________
________________________________________________________________
________________________________________________________________
________________________________________________________________

**4.** During which chapters do you use visualization objects and why? / Do you use visualization objects in all chapters? If yes explain, if no specify?

______________________________________________________________________
______________________________________________________________________
______________________________________________________________________
______________________________________________________________________

**5.** In your own perspective, what purposes do visualization objects serve in mathematics teaching?

______________________________________________________________________
______________________________________________________________________
______________________________________________________________________
______________________________________________________________________
______________________________________________________________________

**6.** Do you find using visualization objects useful in helping learners to learn mathematics? If yes in what way explain. If not what are the challenges?

______________________________________________________________________
______________________________________________________________________
______________________________________________________________________

**7.** Do you find those visualization objects used in those three text books relevant and useful? Yes/ No, elaborate

______________________________________________________________________
______________________________________________________________________
______________________________________________________________________
______________________________________________________________________
______________________________________________________________________

**8.** How appropriate are these visualization objects in relation to content?

______________________________________________________________________
______________________________________________________________________
______________________________________________________________________
______________________________________________________________________

**9.** Do they need improvement? If yes what improvement?

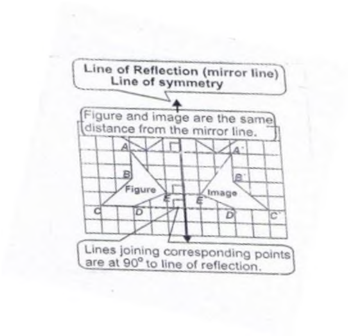
______________________________________________________________________
______________________________________________________________________
______________________________________________________________________
______________________________________________________________________
______________________________________________________________________
______________________________________________________________________

**10.** What challenges have you experienced using visualization objects during your teaching? Elaborate.

______________________________________________________________________
______________________________________________________________________
______________________________________________________________________

**11.** Study figure A and B from the grade 9 textbooks and answer the following questions.

**Figure A**




(i) In your views is this picture relevant to content? In what ways?

(ii) Does it represent reality? Elaborate.

(iii) What functions does this picture serve in this topic?

(iv) How do you use this diagram during your teaching?

**Figure B**


(i) In your views is this picture relevant to content? In what ways?

(ii) Does it represent reality? Elaborate.

(iii) What functions does this picture serve in this topic?

(iv) How do you use this diagram during your teaching?

**Thank you for sharing your thoughts with me. Please return it to the circuit office before the 22 April 2016. For enquiries contact Selma 0812263918/0817445766.**

## Appendix 4: Sample of interview questions with the authors

1. How does it feel to be an author of a mathematics textbook that is widely/commonly used in Namibian school?
2. What was your motive to write textbooks?
3. Which visualization objects do you prefer using in your textbooks? Elaborate.
4. How did you decide as which visualization objects to be include and which one not to be included in your textbook?
5. Do you use colour diagrams? Why/ why not? Briefly elaborate you answer.
6. I have discovered that in most cases you have attached explanatory notes on the diagrams. May you please shed more light on that?
7. Why do you prefer using a lot of visualization objects?
8. How do you make your choices to ensure that these diagrams are aligned with the content?
9. When you are choosing diagrams and pictures, do you consider the context of learners (social and cultural background)? Elaborate.
10. What purposes do visualization objects (drawings, diagrams and sketches) serve in mathematics textbook (y=mx+c/ Math's for life) in relation to teaching and learning mathematics?
11. What challenges have you experienced by selecting visualisation objects in your textbook? Elaborate.
12. What are some of the improvements you will consider making in your next textbook concerning visualization objects?

## Appendix 5: Linking questions to themes that emerged as discussed in chapter 4 & 5

1. **Survey questionnaire**

   The questions used in the survey questionnaire of the teachers are hereby related to the themes used in chapter 4.

| Themes | Question No. |
|---|---|
| The roles of VOs | Q3,Q5, Q6 & Q4 |
| Relation of mathematics to reality | Q5, Q7, |
| Relation to content | Q8 & Q9 |
| Relation to context | Q7 |
| Type of VOs teachers use | Q2, Q1 |
| Limitations on using VOs | Q7, Q10, Q11 & Q12 |

2. **Interview questions**

   The questions used in the interviews with the authors are hereby related to the themes used in the analysis chapter.

| Themes | Question No. |
|---|---|
| The choices of the type of VOs used | Q3 & Q4 |
| The roles of VOs | Q5, Q6, Q7 & Q10 |
| VOs in relation to reality | Q9 |
| VOs in relation to context | Q9 |
| VOs in relation to content | Q8 |
| Limitations in the use of VOs | Q11 & Q12 |

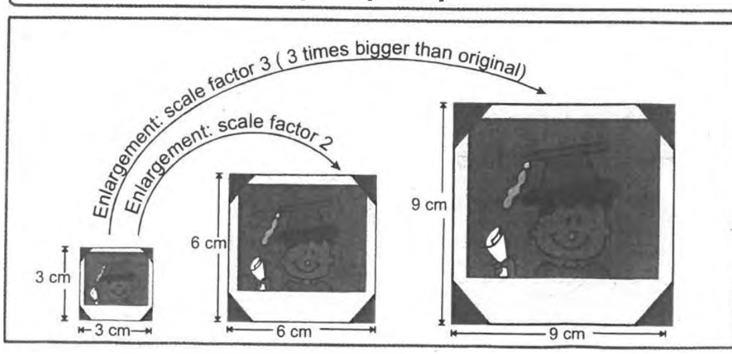
## Appendix 6:

Coding template to analyze each VO

| Category | Subtype | Definition | Score A |
|---|---|---|---|
| Type | Picture, Sketch/ drawing | A simple drawings/shape which does not have many details | 2 |
| | Distribution | Histogram; Point Graph; Stem-and-Leaf Plot; Tally Graph | 0 |
| | Photograph | A picture produced using a camera | 0 |
| | Table, matrix, text chart | A table of different sizes and forms | 0 |
| | Graph | A graphical representation of data, relations and processes in bar, column, pie, spider graphs, graphic timetable form. | 0 |
| | Schematic | A representation of an idea or theory for easy understanding. | 1 |
| Roles | Decorative | Made to look more attractive or ornamental – a decoration | 1 |
| | Exemplifying | Diagrams that exemplify the written text. They make the text and the mathematical idea clearer. | 2 |
| | Explanatory | Notes accompanying the diagram, which assist the text and provide new aspects of information necessary to make the idea under consideration clearer. The diagram is used to explain the main mathematical idea. | 2 |
| | Complementary | Diagrams that provide information not included in the text nor described explicitly in the written form. The diagram complements and reinforces the written text | 1 |
| Reality | Realistic | Realistically represents the concept true to real life situation | 2 |
| | Metaphorical | Not having real existence | 0 |
| Properties | 3D | A three-dimensional VO | 0 |
| | 2D | A two-dimensional VO | 2 |
| Content | Absent/weak | Having a weak connection to the content | 0 |
| | Meaningful to the content | Aligns with the content | 2 |

**Enlargement**

An enlargement changes a figure into one with the same **shape** but a different **size**. Like similar figures. [P.194 ]

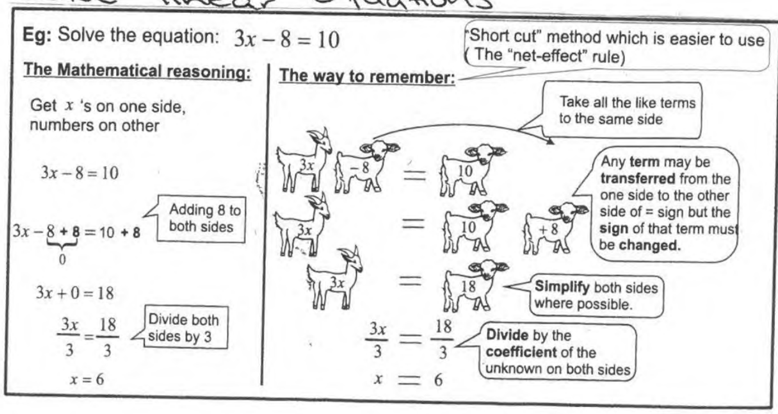



Geometry
y = mx + c
Page 250
on exemplifying
* The diagram show how the figure was enlarged however it did not show the dimensions.

Coding template to analyze each VO

| Category | Subtype | Definition | Score A |
|---|---|---|---|
| Type | Picture, Sketch/ drawing | A simple drawings/shape which does not have many details | 1 |
| | Distribution | Histogram; Point Graph; Stem-and-Leaf Plot; Tally Graph | 0 |
| | Photograph | A picture produced using a camera | 0 |
| | Table, matrix, text chart | A table of different sizes and forms | 0 |
| | Graph | A graphical representation of data, relations and processes in bar, column, pie, spider graphs, graphic timetable form. | 0 |
| | Schematic | A representation of an idea or theory for easy understanding. | 1 |
| Roles | Decorative | Made to look more attractive or ornamental – a decoration | 1 |
| | Exemplifying | Diagrams that exemplify the written text. They make the text and the mathematical idea clearer. | 2 |
| | Explanatory | Notes accompanying the diagram, which assist the text and provide new aspects of information necessary to make the idea under consideration clearer. The diagram is used to explain the main mathematical idea. | 2 |
| | Complementary | Diagrams that provide information not included in the text nor described explicitly in the written form. The diagram complements and reinforces the written text | 2 |
| Reality | Realistic | Realistically represents the concept true to real life situation | 2 |
| | Metaphorical | Not having real existence | 0 |
| Properties | 3D | A three-dimensional VO | 2 |
| | 2D | A two-dimensional VO | 0 |
| Content | Absent/weak | Having a weak connection to the content | 0 |
| | Meaningful to the content | Aligns with the content | 2 |

Solve linear equations




Algebra
$y = mx + c$
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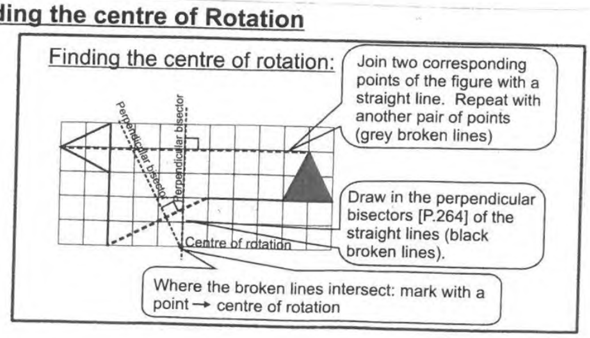
* The diagram make the text clear on how to solve the linear equation
→ changing the sign when the number

Coding template to analyze each VO

| Category | Subtype | Definition | Score A |
|---|---|---|---|
| Type | Picture, Sketch/ drawing | A simple drawings/shape which does not have many details | 2 |
| | Distribution | Histogram; Point Graph; Stem-and-Leaf Plot; Tally Graph | 0 |
| | Photograph | A picture produced using a camera | 2 |
| | Table, matrix, text chart | A table of different sizes and forms | 0 |
| | Graph | A graphical representation of data, relations and processes in bar, column, pie, spider graphs, graphic timetable form. | 0 |
| | Schematic | A representation of an idea or theory for easy understanding. | 2 |
| Roles | Decorative | Made to look more attractive or ornamental – a decoration | 1 |
| | Exemplifying | Diagrams that exemplify the written text. They make the text and the mathematical idea clearer. | 1 |
| | Explanatory | Notes accompanying the diagram, which assist the text and provide new aspects of information necessary to make the idea under consideration clearer. The diagram is used to explain the main mathematical idea. | 2 |
| | Complementary | Diagrams that provide information not included in the text nor described explicitly in the written form. The diagram complements and reinforces the written text | 2 |
| Reality | Realistic | Realistically represents the concept true to real life situation | 1 |
| | Metaphorical | Not having real existence | 0 |
| Properties | 3D | A three-dimensional VO | 0 |
| | 2D | A two-dimensional VO | 2 |
| Content | Absent/weak | Having a weak connection to the content | 0 |
| | Meaningful to the content | Aligns with the content | 2 |

**Finding the centre of Rotation**




Geometry
y=mx+c
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