AN INVESTIGATION INTO THE NATURE OF MATHEMATICS CONNECTIONS USED BY SELECTED GRADE 11 TEACHERS WHEN TEACHING ALGEBRA: A CASE STUDY

A thesis submitted in partial fulfilment of the requirements for the degree of Master of Education (Mathematics Education)

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BY

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DECLARATION OF ORIGINALITY

I, ESTER NDAHEKOMWENYO KANYANDA (Student number: 611K7081) declare that this thesis: **an investigation into the nature of mathematics connections used by selected grade 11 teachers when teaching algebra: a case study** is my own work written in my own words. Where I have drawn on the words or ideas of others, these have been acknowledged using the reference practices according to the Rhodes University Education Department Guide to Referencing.

E.N. KANYANDA **30 November 2014**

(Signature) (Date)

ABSTRACT

The purpose of this study was to investigate the nature of mathematical connections used by selected teachers when teaching the topic of algebra and to investigate their perceptions of their use of connections. The participants were selected on the basis of teaching experience as well as their willingness to share their ideas. An interpretive paradigm was used to collect and analyse data.

The data was collected from three participating teachers. These participants were selected from the three secondary schools in the town of Tsumeb in Namibia. I used video recordings of two lessons per teacher as well as semi-structured interviews as my tools to gather data. After the two lessons were video recorded, I conducted a workshop with the teachers to introduce them to the 5 types of mathematical connections pertinent to this study. We analysed the videos together using Businskas' framework as a basis for analysis. This then formed part of the stimulated recall interviews.

It was found that, even though teachers were not aware of the concept of mathematical connections before our interactions, there was strong evidence of connections being made and used in their lessons. The two types of connections that were used most frequently (24.1% each) were procedural and instruction-oriented connections respectively. Part-whole relationships connections were used the least with a frequency of 12%. All three teachers agreed that they needed to make more connections when teaching and that they would think more about connections in future, particularly when preparing their lessons. The study makes recommendations to encourage the continuous use of connections in teaching mathematics.

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DEDICATION

I dedicate this research to my wonderful mother, Meme PENEYAMBEKO NGHISHIKUSHITYA–KANYANDA.

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CHAPTER ONE

INTRODUCTION

1.1 INTRODUCTION

In this chapter, I briefly describe the context of my study, the research site as well as state the research question that this study aims to answer. I also explain the significance of my study and then provide an outline of the research project.

1.2 CONTEXT AND RATIONALE

When the Namibian curriculum was recently revised and stipulated that mathematics should be a compulsory subject in the Grade 11-12 phase as of 2012 (Namibia. Ministry of Education [MoE], 2008, p. 3), Grade 10 learners, including those who had never anticipated taking mathematics further, were then faced with the challenge of mathematics as a compulsory subject.

Mathematics as a subject is not performed well in Namibian schools and from my experience as a mathematics teacher; learners tend to perform particularly badly in algebraic topics. This poor performance can be clearly seen in Figure 1.1 below showing the August Examination 2012 results of a particular school in my region. The performance of this school is not out of the ordinary. In my experience many of the schools in my region reflect a similar performance profile.

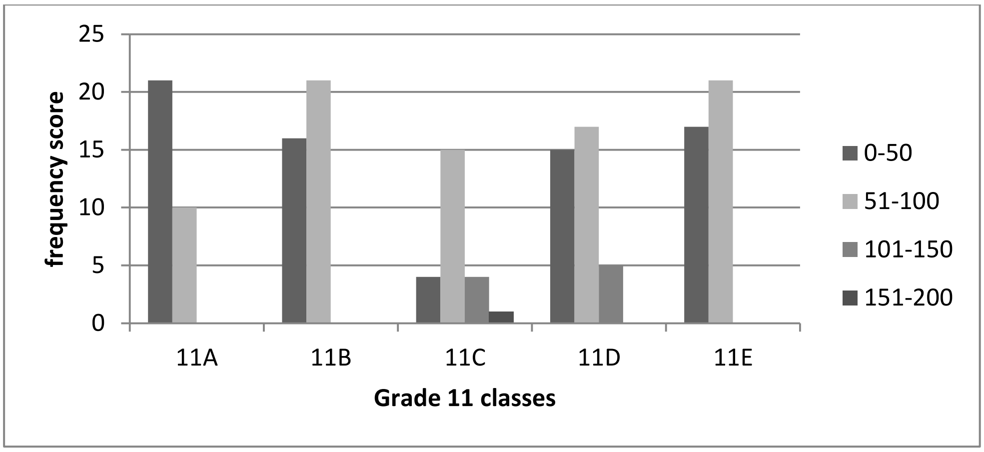

Figure 1.1 Mathematics performances in Grade 11 classes of a school in my region

Kilpatrick, Swafford & Findell (2001) proposed five interconnected strands of mathematical proficiency, of which one is conceptual understanding. Conceptual understanding is defined as "an integrated and functional grasp of mathematical ideas" (p. 118). They further stated that conceptual understanding refers to the ability to represent mathematical situations in different ways as well as knowing how different representations can be useful for different purposes. "When students have acquired conceptual understanding in an area of Mathematics, they see the connections among concepts and procedures and can provide arguments to explain why some facts are consequences of others" (p. 119).

Kilpatrick et al. (2001) further assert that making mathematical connections enables learners to better understand and appreciate mathematics. Mathematical connection is broadly defined by Businskas (2008), as quoted by Mhlolo, Venkat & Schafer, (2012, p. 1) as:

- A relationship between ideas or processes that one can use to link topics in mathematics;
- A process of making or recognising links between mathematical ideas;
- An association a learner might make between two or more mathematical ideas;
- A causal or logical relationship or interdependence between two mathematical entities.

Different researchers have explored connections in mathematics using the definitions mentioned above. Mhlolo et al. (2012) for example, focused their research on making connections from the perspective of using different representations. They defined different representations as a mathematical connection used by teachers when they present mathematical ideas from multiple perspectives and referring to different contexts, using a multitude of approaches. Cleaves (2008) stated that "the ability to examine problems using varied approaches is one of the most important characteristics of a good problem solver" (2008, p. 1). The varied ways in which learners can examine or solve problems, is another way of using multiple representations.

Several authors (Mhlolo et al., 2012; Hanes & Hoffman, 2010; Cleaves, 2008) have highlighted the notion that teachers play a crucial role in enabling leaners to recognise and interpret mathematical connections. They stated that the teacher is the most important person to observe if you want to study mathematical connections. Sawyer (2008) echoed these sentiments by saying that "helping learners make connections is integral to effective teaching and learning" (2008, p. 2). This study focusses specifically on the connections that three selected Grade 11 teachers made whilst teaching different aspects of algebra.

1.3 RESEARCH SITE

This research was conducted at all three Senior Secondary Schools in the town of Tsumeb in Oshivelo Circuit of the Oshikoto Region in Namibia. The Oshikoto Region is one of the 14 political regions of Namibia. The Oshikoto Region has 11 educational circuits. With most of its towns and schools mainly in the northern part of Namibia, Tsumeb is furthest south in this region. It is however centrally located in relation to the rest of the country.

1.4 RESEARCH QUESTION

The aim of this research was to explore the types of connections used by teachers when teaching mathematics. The research questions are:

- What type of connections do selected Grade 11 teachers make when teaching algebra with regard to Businskas' framework?
- What are the selected teachers' perceptions of making use of connections when teaching algebra?

1.5 METHODOLOGY

This study is a qualitative investigation and was conducted within an interpretive paradigm. Cohen, Manion & Morrison (2011) describe an interpretive paradigm as one that gives the researcher an opportunity to understand and interpret the world around them. I found this paradigm to be suitable for my research since my interest was in understanding the type of connections that teachers make when teaching. My focus was thus on teachers and how they teach algebra.

This study was a case study bounded specifically by the practices of my three selected Grade 11 teachers in their classroom environments. In this study, my case was the use and the nature of mathematics connections employed by Grade 11 teachers. My unit of analysis was thus the nature of the connections that the teachers made when teaching algebra, as well as the teachers perceptions of making use of these connections.

My participants included three teachers from three different secondary schools in Tsumeb in the Oshikoto region of Namibia. Since I work in Tsumeb, it was convenient to choose schools from Tsumeb (my town) as it is close to my workstation in order to avoid or minimise travel expenses. This also helped with understanding the context due to my own prior knowledge and insights which I have of the participating schools. I chose Grade 11 teachers because algebra forms an important part of the Grade 11 curriculum.

For the purpose of "triangulation" (Cohen et al., 2011, p. 195), I collected my data using a variety of methods such as classroom observation, workshop as well as semi-structured interviews. As mentioned above, the observations and the interviews were recorded and transcribed. I then used a qualitative approach to analyse the data which was framed by Businskas' themes of making connections.

1.6 SIGNIFICANCE OF THE STUDY

Although much research has been done on teaching and learning of algebra in mathematics, relatively little has been done on identifying specific connections when teaching algebra. This study focuses particularly on the teaching of algebra, with an emphasis on making connections when teaching.

The area of researching connections in mathematics is relatively new and I am not aware of any research done in Namibia on this topic. I thus hope that this study will shed some interesting light on the teaching of algebra in Namibia.

1.7 LIMITATIONS

In order to carry out a more comprehensive study nationwide, I would have required a considerable amount of time, human capacity as well as financial resources. As I had none of these at my disposal, coupled with the long distances between schools throughout the country it proved impossible to carry out a more widespread and in-depth nation-wide study.

1.8 OVERVIEW OF THE STUDY

The thesis consists of five chapters themed as follows:

Chapter one is the introduction chapter. It provides a brief description of the context of the research. It also highlights the research site as well as the research questions for the study.

Chapter two gives a brief review of the relevant literature for this research. It reviews texts about the understanding and teaching of algebra and provides a discussion on Businskas' (2008) conceptual framework about connections.

Chapter three describes the research methodology and design for the study. It also describes the research tools used to collect the data as well as an overview of how the data was analysed.

Chapter four provides an analysis of the data collected. In this chapter, an attempt is made to answer the stated research questions.

Chapter five is the concluding chapter. In this chapter, I attempt to weave the literature together with the analysis of the data. It also provides some recommendations and concluding remarks.

CHAPTER TWO

LITERATURE REVIEW

2.1 INTRODUCTION

Making connections is crucial to learning mathematics with understanding and as such it is seen as a very important part of mathematics education research. Much has been written about mathematics connections, particularly about mathematics connections in the real world (Businskas, 2008), but in my experience there seems to be relatively little written about exploring connections or interconnectedness within mathematics itself in the context of the classroom.

This chapter thus reviews the literature specifically with regard to connections within mathematics education. My main argument in this research project is that in order for learners to perform successfully in mathematics in Namibia, making connections should be an important part of their learning. For this to happen, teachers need to enable learners to make these connections by themselves. The teachers themselves also need to make these connections when teaching. The focus of this study is to investigate the connections that selected Grade 11 teachers make when teaching the topic of algebra.

This chapter begins by describing connections and discusses the nature of these connections in mathematics. I proceed to discuss making connections when teaching in general, as well as when teaching the topic of algebra specifically. These discussions are framed within a proficiency discourse in the Namibian context.

2.2 THE NATURE OF CONNECTIONS

A connection is defined by the Oxford English Dictionary as “a causal or logical relationship or association; an interdependence” (Brown, 1993, p. 481). It was from this definition that Businskas (2008) came up with a comprehensive definition of mathematical connection which reads as follows:

- A relationship between ideas or processes that one can use to link topics in mathematics;
- a process of making or recognising links between mathematical ideas;
- an association a person might make between two or more mathematical ideas;

- a causal or logical relationship or interdependence between two mathematical entities.

Businskas emphasised that it was important for teachers to create opportunities for learners to acquire and make mathematical connections that can be used in problem solving. She suggested that mathematical understanding can be measured through:

- Connections made between different mathematical ideas;
- Different representations of mathematical ideas;
- Reasoning between different mathematical ideas.

A useful framework for analysing the use of connections is a model developed by Businskas (2008) and modified by Mhlolo et al. (2012). This framework or model identifies five types of mathematical connections that teachers use. These are as follows:

- Multiple or different representations as a form of mathematical connection;
- part-whole relationship connections (*hierarchical nature of concepts*);
- connections where **A** implies **B** (*logical reasoning*);
- connections which show **A** is a procedure for doing **B** (*algorithms*);
- instructional-oriented connections (*building on student's prior knowledge*).

This model is very useful and helpful for my particular study because it deals with working with relationships, mostly between variables, which is an essential part of Grade 11 Algebra. I discuss each of these below:

2.2.1 Connections through different representations

The aims of the Mathematics Ordinary Level for the Namibia Senior Secondary Certificate (NSSC) examinations include some of the following:

a) Read mathematics, and write and talk about the subject in a variety of ways;
b) Appreciate patterns and relationships in mathematics;
c) Recognise when and how a situation may be represented mathematically (Namibia. Ministry of Education, [MoE], 2005, p. 2).

Learners are thus expected to communicate appropriately in a variety of ways by recognising when and how situations may be represented mathematically. This suggests that learners should be guided on how to establish these links within mathematics and teachers should align their mathematics teaching in order for these aims to be realised.

Multiple representations are defined as tools that support and extend mathematical reasoning and come in a variety of forms including numbers, algebraic symbolism, graphs, diagrams, models, equations, notations, images, analogies, metaphors, stories or games (Businskas, 2008). Goldin and Shteingold (2001) assert that multiple representations provide the same information in more than one form of mathematical representation. Businskas (2008) identifies two specific types of multiple representations:

(a) Alternate representation: this is when two representations are from different modes;
(b) Equivalent representation: concepts that are represented in different ways within the same form of representation. (Businskas, 2008)

Businskas maintains that alternate representation can be referred to when two representations A and B are described in different modes. For example an alternate representation could be a representation from a graphic mode to a symbolic mode or from a pictorial mode to a manipulative mode and vice versa. A specific example could be a representation of a straight line graph having a y-intercept of 3 and a gradient of 2 which can be represented alternately as $y=2x+3$. On the other hand A as an equivalent representation of B refers to concepts that are represented differently within the same form of representation. For example 5+2 is equivalent to 2+5 and also equivalent to 7. All these representations consist of symbols or numerals written in different forms of seven.

Leikin and Levav-Waynberg (2007) describe a multiple-solution connecting task and defined it as "a task that may be attributed to different topics or to different concepts within a topic of the mathematics curriculum, and therefore may be solved in different ways" (2007, p. 1). For example a teacher may ask the learners, given an equation $2x^2+7x+5=0$, to sketch the curve of $y = 2x^2+7x+5$. Such a task has multiple steps as it requires learners to work out the turning point of the curve, the roots of the equations and the intercepts. A learner will have to use many topics attributed to different concepts to solve this problem, i.e. you can use the topic of differentiation or use the equation of axis of symmetry to find the turning point. This task asks the learner to connect the equation $2x^2+7x+5=0$ to its graphical representation as well as its parameters. These types of connecting tasks can be seen as tasks that require multiple representations, whether in the way it is asked or in the way it is worked out.

2.2.2 Connections through part-whole relationships (*hierarchical nature of concepts*)

The second form of connection is part-whole relationships or the hierarchical nature of concepts.

Businskas' (2008) framework has two forms of connection referred to as Generalisation and Inclusion instead of part-whole relationships. She said for two concepts A and B, A is a generalisation of B. Inclusion is defined as a hierarchical relationship between two concepts for instance that A is included in B. However Mhlolo (2012) combined the two types of connections, implication and generalisation, and called them part-whole relationships. He defined part-whole relationships as relationships of the form A is a generalisation of B, where B is a specific instance (or example) of A. This generalisation, he says, is derived from the practical examples that teachers use when teaching. For instance, an equation $y = 2x+3$ is a generalisation of $y = mx+c$. $y= 2x+4$ is one of many examples that a teacher may use to explain straight line graphs.

Bills and Bills (n.d.) stress that the language of examples used in school is sometimes ambiguous. Sometimes teachers use two types of examples interchangeably which are not necessarily the same. These are examples of a concept and examples of the application of a procedure. An example of a concept, for instance of triangles, will include the different types of triangles; right angled, equilateral and so on. On the other hand, an example of an application, for instance calculating the area of a rectangle, will include the application of the formula of a rectangle. One aspect of a good example, says Mhlolo (2012) is the extent to which generalisations can be drawn from them. This is to say that examples used in the classroom are part of generalisations that a teacher is able to make and these generalisations fall under the connection of part-whole relationships or hierarchical nature of concepts.
Hart (1981) described the word hierarchy as follows:

a) A learning sequence or sequence of understanding;
b) A teaching sequence which the teacher uses;
c) A logical sequence which is inherent in the mathematical topic. (1981, p. 1)

From the above definition, it can be concluded that part-whole relationships or the hierarchical nature of concepts also deal with sequencing of the content that is being learnt. It can be the sequence of the learners' understanding within themselves or it can be the sequence that the teacher uses when presenting his content. This sequence can also refer to the logical sequence in the topic which could be in the syllabus or in the textbook outline. Hart further stated that a hierarchy is that of "a sequence of instructional content". (1981, p.

6). He also affirms that "hierarchy" implies a string of skills, levels, stages or concepts that are ordered from simple to complex which then form a sequence as described above.

In summary, part-whole relationships or hierarchical relationships are those where there is a logical link or relationship between any two concepts.

2.2.3 Connections where A implies B

Businskas (2008) calls this type of connection Implication connections. She defines it as a connection that indicates a dependence of a concept on another in a logical way. Mhlolo (2012) calls these the 'if-then' types of connections. He says these are the types resulting from logical reasoning in mathematics and are closely related to generalisation even though not exclusively so. These types of connections, he adds, include but are not limited to the following: seeing the links between the hypothesis and conclusion in a deduction; testing a general assertion with examples; identifying logical errors in chains of reasoning involving more than one step. Learners and teachers make these connections when they know what it means to use sentences involving "and", "or", "not", "if-then", "some" and "all" to be true or false. An example of such a connection is being able to conclude that if $2x + 5=9$ then $2x$ is four and hence x is 2.

One aim of the Mathematics Ordinary Level for the Namibia Senior Secondary Certificate (NSSC) examinations states specifically that learners should 'develop the abilities to reason logically, to classify, to generalise and to prove' Namibia. MoE, (2005, p. 2). It is thus important to make mathematical connections in the Namibian mathematics classroom as generalising and proving are examples of Implication connections.

2.2.4 Procedural connections

Mhlolo (2012) defined an algorithm as a procedure involving prescribed steps that lead to a specific outcome, which is often the calculation of something. An algorithm in this case refers to procedures designed to solve a specific problem. He further states that when procedural connections are made, one should make references to efficient and accurate procedures for working out a mathematical task. One makes a connection when he/she knows that A is a procedure for doing B.

Hiebert and Lefevre (1986) defined procedural fluency as follows:

> One kind of procedural knowledge is a familiarity with the individual symbols of the system and with the syntactic conventions for acceptable configurations of symbols. The second kind of procedural knowledge consists of rules or procedures for solving mathematical problems. Many of the procedures that students possess probably are chains of prescriptions for manipulating symbols. (p. 3)

Kilpatrick et al. (2001) defines procedural fluency as the skill in carrying out procedures flexibly, accurately, efficiently, and appropriately.

In summary, procedural fluency is about working with an algorithm as a procedure to solving mathematical problems. It is also about being familiar with the individual symbols of mathematics. A teacher who makes a procedural connection when solving an equation $2x^2+7x=5$ will know that he or she can either solve this equation by factorising or completing the square. In order to save time the method of choice may be that of factorising.

2.2.5 Instruction oriented connections

This type of connection relates to organisation, planning and teaching for differently abled learners. Of all the other types of connections, this is the one type that deals specifically with teachers and how they prepare to teach as well as how they teach. Mhlolo (2012) introduced two types of instructional oriented connections; the first refers to the hierarchical nature of mathematical concepts. In the hierarchical structure for example **A** and **B** are prerequisites concepts or skills that should be known in order to learn **C**. The second type of instruction oriented connections include an extension of what learners already know by linking prior knowledge to new concepts.

A teacher who makes an instruction oriented connection arranges his or her content in hierarchies of concepts as well as arranging the content in ways that makes sense to the learners. For that to happen, the teacher needs to know his or her learners' preferred way of learning according to their abilities, their immediate environments as well as their aspirations.

2.3 MAKING CONNECTIONS IN TEACHING

2.3.1 Connections as indicative for effective teaching

Several authors (Mhlolo et al., 2012; Haines & Hoffman, 2010; Cleaves, 2008) highlight the notion that teachers play a crucial role in enabling leaners to recognise and interpret mathematical connections. They stated that the most important person to observe if you want to study and research mathematical connections must be the teacher. Sawyer (2008) echoed these sentiments by saying that "Helping learners make connections is integral to effective teaching and learning" (200, p. 2).

Ma's (1999) notion of "profound understanding of fundamental mathematics (PUFM)", describes the type of knowledge that a teacher should hold. It involves both skill in mathematics and an understanding of how to communicate with students. She suggests that teachers who possess PUFM should be able to make connections between mathematical concepts and procedures from the simple to the more complex. She further states that such a teacher should have the knowledge of the whole mathematics curriculum and not just that of the grade he or she is teaching. She referred to it as longitudinal coherence.

Gagatsis and Elia (2004) as cited in Mhlolo et al. (2012) are of the view that recognising and producing alternate representation is a particularly fruitful way of conceptualising what mathematics connections are.

2.3.2 Teacher perspectives and practises

Sawyer (2008) is of the view that teachers believe that connectedness and making connections is fundamental to mathematics education. He further says that teachers also believe that making connections equips learners to see how mathematics is related to everyday tasks. There is however a view among teachers that using multiple representations to teach mathematics can be time consuming even though research has proven otherwise.

Since teachers play an integral part in making connections, the way they present their lessons might promote or hinder learners' chances of making or realising these connections. For example Venkat and Adler (2012) spoke about a transformation activity. They said that a transformation activity involves the transfer of information between the learners and the teacher through the given accompanying explanations. They further defined a transformation activity as an activity or action through which the teacher passes the information to the learners. This can for instance be the examples that teachers use when teaching. Examples that teachers use are connections of part-whole relationships. There is however criticism of the ways in which these activities take place in the classroom. Sometimes leaners do not know or are not made aware of which needs are met by the mathematics that is being taught. Learners are not aware of how relevant the mathematics is to their daily lives. They also looked at whether the teacher's explanation is coherent and connected to the way the solution is given.

Teachers can provide opportunities for enabling connections by not dictating what procedures learners must use (Sawyer, 2008). This is in agreement with Cleave's (2008) idea that allowing learners to use varying representations to answer questions deepens their understanding of the concepts. However in some cases, teachers are disabling learners' use of connections in their classes as sometimes the action given does not cohere with the method used to solve it. Venkat and Adler (2012) provide an example of how a teacher disrupted a transformation activity. In this particular instance, the teacher asked the learners to find the missing addend in 3+□ =7, in other words asked the learners to find which number can be added to 3 to give 7. This teacher ended up showing the learners (using fingers) 7 fingers and hiding 3 and asked how many were left, instead of starting with 3 fingers and adding until he reached 7. Even though the answer given was four, this teacher's method of getting the answer did not cohere with the question given.

2.3.3 Connections as indicative of mathematical understanding

It has been argued (De Jong & Van der Meij, 2003; Skemp, 1976; Carpenter & Lehrer, 1999; Kilpatrick et al., 2001) that making use of connections leads to deep understanding of mathematics. Hiebert and Carpenter (1992) defined mathematical understanding in terms of how knowledge was structured from the following relationship:

> A mathematical idea or procedure or fact is understood if it is part of an internal network. More specifically, the mathematics is understood if its mental representation is part of a network of representations. The degree of understanding is determined by the number and strength of the connections. A mathematical idea, procedure, or fact is understood if it is linked to existing networks with stronger or more numerous connections. (p. 67)

It is clear from this quote that making connections has a huge effect in determining the mathematical understanding of the learners. The strength of the connections that learners are able to make leads to a certain degree of understanding. Stylinides and Stylinides (2007) concluded that learning mathematics with understanding encompasses making connections among ideas and these connections facilitate the transfer of prior knowledge to new situations.

“When students acquire knowledge with understanding, they can apply that knowledge to learn new topics and solve new and unfamiliar problems”, say Carpenter and Lehrer (1999, p. 19). They proposed “five forms of mental activity from which mathematical understanding emerges: constructing relationships, extending and applying mathematical knowledge, reflecting about experiences, articulating what one knows and making mathematical knowledge one’s own” (1999, p. 20). These forms of mental activity go hand in hand with the types of connections introduced by Businskas (2008). The mental activity of constructing relationships can be found as an example of the type of the if-then/implication (I) connection which shows how concepts are dependent on one another. Equally, reflecting about experiences as well as transferring of prior knowledge are clear examples of the instruction-oriented connection (IOC) where a teacher recognises and is aware of learners’ prior knowledge.

Carpenter and Lehrer (1999) warn that understanding is not universal for all students, but varies from case to case. They spoke of critical dimensions of classrooms that promote understanding, one of which is the tools to represent mathematical ideas used in the class. Included in this category of tools was pen, pencil, books, manipulatives, computers and symbols amongst others. These are the tools needed by the learner in order for learning to take place. They provided an example of representing numbers with counters or blocks in appropriate ways. They added that connections with representations that have intuitive meaning for learners can help them give meaning to mathematical symbols. This is a clear form of multiple representations as a form of a mathematical connection.

In the same vein, Skemp (1976) identified two types of understanding; instrumental and relational understanding. He described instrumental understanding as the learning of ‘how to’, including learning by rote, memorising facts and rules. In contrast, he described relational learning as learning of ‘why to’. Skemp (1976) later extended the types of knowledge to three, adding logical understanding which he described as follows:

> Logical understanding is evidenced by the ability to demonstrate that what has been stated follows of logical necessity, by a chain of inferences, from
> a) the given premises, together with;
> b) suitably chosen items from what is accepted as established mathematical knowledge (axioms and theorems). (p. 47)

The three types of understanding described above can be related to the types of connections outlined in the previous section. The type of learning "how to" can be referred to as procedural knowledge/connections. The understanding of "why to" and logical understanding is an example of Implication connections. For this type of understanding, teachers and learners show the knowledge of logical reasoning, testing hypotheses and justifying solutions. For learners to perform better and show understanding of algebra, and mathematics in general, teachers also need to portray good understanding of the subject.

2.3.4 Teaching for mathematical proficiency

Kilpatrick et al. (2001) proposed a model of teaching for mathematical proficiency which emphasized the relationships between the teaching and learning of mathematics. Kilpatrick et al. (2001) described mathematical proficiency as "the integrated attainment of conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition" (2001, p. 313). A brief description of each strand is as follows:

- *conceptual understanding*- comprehension of mathematical concepts and their connections
- *procedural fluency*- skills in carrying out procedures flexibly, accurately, efficiently
- *strategic competence*- ability to formulate, represent, and solve mathematical problems
- *adaptive reasoning*- capacity for logical thought, reflection, explanation and justification
- *productive disposition*- habitual inclination to see mathematics as sensible, useful. (2001, p. 16).

The first strand, conceptual understanding, is defined further by Hiebert and Lefevre (1986) as:

> The knowledge that is rich in relationships. It can be thought of as a connected web of knowledge, a network in which the linking relationships are as prominent as the discrete pieces of information. Relationships pervade the individual facts and propositions so that all pieces of information are linked to some network. (p. 3)

One can clearly see from this definition that conceptual understanding involves the understanding that results from making connections in mathematics. It refers to the richness in relationships, connected web of knowledge and linking relationships. This is a clear indication of mathematical connections.

Teaching for proficiency was defined in the same report as teaching that promotes learning over time so that it yields mathematical proficiency. Teaching for mathematical proficiency also has the same five (5) interwoven strands and are explained in terms of what the teacher would do, as outlined below:

- *conceptual understanding* of the core knowledge required in the practice of teaching;
- *procedural fluency* in carrying out basic instructional routines;
- *strategic competence* in planning effective instruction and solving problems;
- *adaptive reasoning* in justifying and explaining one's instructional practices;
- *productive disposition* toward mathematics, teaching, learning, and to improving practice (2001, p. 380).

Kilpatrick et al. (2001) affirm that a teacher who teaches for mathematical proficiency makes connections in his or her teaching of mathematics. They continue to say that teaching is not only about how teachers teach but also about the interactions and connections the teachers and students make about content. They refer to the "instructional triangle" as "the product of interactions among the teacher, the students and the mathematics" (p. 314) in different contexts. By contexts, Kilpatrick et al. (2001) refer to the wide range of environmental and situational elements that bear on instruction, like educational policies, assessments, school leadership structures and the like. The quality of instruction in these interactions depends on the teacher's knowledge and use of mathematical content, the teacher's attention to and handling of students, and the students' engagement in and use of mathematical tasks. Similarly, Schoenfeld and Kilpatrick (2008) define proficient teachers as teachers who have multiple ways of conceptualizing the current grade-level content, can represent it in a variety of ways, understand the key aspects of each topic, and see connections to other topics at the same level.

The ability to make connections when teaching is one way of looking at the teacher's knowledge and use of mathematical content as alluded to in the previous sections of this chapter. Kilpatrick et al. (2001) agree by saying that "effective instruction depends on coherent connection over time among lessons designed collectively to achieve important mathematical goals" (2001, p. 316). It is the teachers with conceptual understanding that will be able to engage their learners in fruitful conversations about multiple ways of solving mathematical problems. Multiple ways of solving mathematical problems is a form of a connection. Teachers with weak conceptual knowledge of mathematics have a tendency to only demonstrate procedures to learners and then providing them with repetitive opportunities to practice the procedures (Kilpatrick et al., 2001).

2.3.5 Pedagogical Content Knowledge

Some mathematics education researchers (Shulman, 1986; Ball, 2003) argue that there exists a certain kind of content knowledge that only subject teachers can possess. This type of knowledge was coined by Shulman (1986) as Pedagogical Content Knowledge. Shulman (1986) describes Pedagogical Content Knowledge (PCK) as a kind of knowledge which goes beyond knowledge of subject matter and stretches to knowledge of subject matter for teaching. PCK is "the particular form of content knowledge that embodies the aspects of content most germane to its teachability" (Shulman, 1986, p. 9). He further states that pedagogical knowledge also includes an understanding of what makes the learning of a certain topic easy or difficult. Teachers must have an understanding of what their learners bring to class and be able clear up any misconceptions. This is in agreement with Kilpatrick et al.'s (2001) teaching for proficiency. For effective teaching to take place, according to Kilpatrick et al. (2001), a teacher needs to possess a knowledge of how teach mathematics and when to teach it. They further stated that mathematical proficiency is best understood through an examination of how teachers, students and content interact in contexts to produce teaching and learning.

In support of Shulman's idea of pedagogical content knowledge, Ball, Thames & Phelps (2008) add that knowledge for teaching should be detailed in ways unnecessary for everyday functioning, " a teacher needs to know more, and different mathematics not less" (p. 396). They further argue that teaching involves more than identifying an incorrect answer but requires being able to identify the source of a mathematical error. Adler (2005) adds that the mathematical roots of these errors are quite different and that a teacher who is faced with these in class, needs to do on the spot analysis of the nature of the error, and its mathematical entailments, as well as what it means to engage learners productively to shift their thinking. As discussed previously, Instruction oriented connection (IOC) is a type of connection whereby teachers prepare their content in such a way that it connects with the learners' sense making ways in order to mitigate the effects of various barriers. For a teacher to make instruction oriented connections, he or she must have the knowledge of what makes sense to the learners and what will possibly cause confusion when teaching.

Adler (2005) says that such teachers must have the ability to work with definitions, relative to the community he or she is working with and should also have the ability to use language carefully to impart useful mathematical explanations. In agreement with these sentiments McCrory, Floden, Ferrini-Mundy, Reckase and Senk (2012) state that for teachers to address the difficulties that leaners experience, they need to examine their own knowledge in order to be able to understand what is difficult for students and then be able to present the explanation in a way that addresses students' thinking.

Ball and Bass (2000) spoke about mathematical knowledge for teaching elementary school mathematics. They described mathematical knowledge for teaching as a "kind of understanding [that] is not something a mathematician would have, but neither would be part of a high school social studies' teacher's knowledge" (p. 87). A mathematics teacher needs a special kind of knowledge different from others as indicated by Shulman (1986). Similarly, McCrory et al. (2012) stated that secondary school mathematics teachers need a deep knowledge of more advanced mathematics as "this gives them a better understanding both of mathematics as a discipline and the trajectory of a secondary school course in more advanced mathematics" (p. 85).

Ma's (1999) notion of Profound Understanding of Fundamental Mathematics (PUFM) coheres with these requirements of PCK whereby teachers need to have longitudinal coherence in the subject. This means that teachers should know more than the mathematics level they are teaching and should also have a clear understanding of the whole curriculum. For instance a grade 12 mathematics teacher needs to have the knowledge of the primary education curriculum. These sentiments are in agreement with the instruction-oriented type of connections (IOC) where teachers are expected to be aware of their learners' prior knowledge as well as be able to extend what they already know.

Ball and Bass (2000) and Usiskin (2001) have come up with numerous types of knowledge they deemed necessary for teaching mathematics. Usiskin (2001) as quoted by McCrory et al. (2012) introduced his framework with three categories, "*concept analysis* - the phenomenology of mathematical concepts; *problem analysis* - the extend analysis of related problems; and *the connections and generalizations* within and among the diverse branches of mathematics" (p. 587). It is evident that making connections forms a key part in the teaching and learning of mathematics. A third category of *connections and generalisations* mentioned above is proof to that. Generalisations are examples of a part-whole relationship, a connection as defined in Businskas (2008) framework.

2.4 MAKING CONNECTIONS WHEN TEACHING ALGEBRA

Kaput (1999) asserts that school algebra has traditionally been taught and learned as a set of procedures disconnected both from other mathematical knowledge and from the students' real world. He further argues that even though algebra has in the past served as a gateway to higher mathematics; the gateway has been closed for many students in the United States, for example who are shunted into academic and career dead ends as a result. Even though Kaput's views were expressed in the context of the United States education system, I am of the opinion that a similar scenario exists here in Namibia. Kaput (1999) continues by saying that algebraic reasoning in its many forms, and the use of algebraic representations such as "graphs, tables, spreadsheets and traditional formulas, are among the most powerful intellectual tools that our civilization has developed" (p. 3).

It can be concluded from the above paragraph that algebra plays a major role in our learning of mathematics. Algebra involves high level calculations. It is thus important that the teaching and learning process of algebra is not compromised if we want our learners to be proficient mathematicians. New ways of teaching algebra are required if Namibia is to realise its' goal of vision 2030 of "being a prosperous and industrialised Namibia, developed by her human resources, enjoying peace, harmony and political stability" in the year 2030, (Namibia. MoE. 2009, p. 2).

McCrory et al. (2012) concluded in their paper that teachers must be able to reinterpret or reorganize their mathematical knowledge to fit the curriculum they are required to use as well as making connections across different ways of knowing the same mathematics. They said teachers should be able to understand the content in the text books they are using and must be able to rearrange this content into something that their learners can comprehend. Teachers must be able to make the connections between the mathematics required in the curriculum and the mathematics they know.

For effective teaching and learning of algebra, Kaput (1999) suggested the following:

- begin early (in part, by building on students' informal knowledge);
- integrate the learning of algebra with the learning of other subject matter (by extending and applying mathematical knowledge);
- include the several different forms of algebraic thinking (by applying mathematical knowledge);
- build on students' naturally occurring linguistic and cognitive powers (encouraging them at the same time to reflect on what they learn and to articulate what they know) and
- encourage active learning (and the construction of relationships) that puts a premium on sense-making and understanding. (p. 4)

All the changes outlined above go hand in hand with the 5 types of connections discussed earlier. Integrating algebra with other subject matter is in essence a connection between different concepts, which is an example of multiple representations. Including several different forms of algebraic thinking is also an example of multiple representations. Teachers should allow the learners to think outside the box and apply their knowledge of algebra in different and multiple forms. He also suggested that teachers build on students' prior knowledge as well as allowing them to reflect on this. This is a clear indication of encouraging connections, specifically instruction oriented connections. Lastly, encouraging active learning for sense-making and understanding is also an example of instruction oriented connection.

In the same vein, Kozma (2003) as cited by van der Meij & de Jong (2003), suggest three principles to increase connections made between representations and those which support student domain understanding:

- providing at least one representational system that has features corresponding explicitly to the entities that underlie the phenomenon being taught;
- have students use multiple, linked representations in the context of experiments;
- engage students in collaborative activities in which they generate representations and coordinate the features of representations to confirm and explain the findings of their investigations. (p. 4)

Even though these points were initially written for science students, I find them applicable to mathematics, specifically to algebra as they emphasize the use of representations that explicitly represent the concept being learned as well as allowing students to use multiple representations as a form of connection. Investigations here can also mean mathematical investigations where students test and conclude hypotheses, which is an example of the implication (if-then) connection.

2.5 MATHEMATICS EDUCATION IN THE NAMIBIAN CONTEXT

According to the Namibian Mathematics Policy document (Namibia. MoE. 2005), the purpose of learning mathematics in schools is:

- to develop their mathematical knowledge and oral, written and practical skills in a way which encourages confidence and provides satisfaction and enjoyment;
- to apply mathematics in every situation and develop an understanding of the part which mathematics plays in the world around them;
- to recognize when and how a situation may be represented mathematically, to identify and interpret relevant factors and, where necessary, to select an appropriate mathematical method to solve the problem;
- to acquire a foundation appropriate to further study of mathematics and of other disciplines. (p. 2)

These points align well with what a teacher using mathematical connections as well as a teacher teaching for mathematical proficiency aims to achieve. A teacher who is using instruction oriented connections will ensure that his/her students appreciate the role that mathematics plays in the world. Similarly, a teacher using multiple/different representations in his/her teaching will enable learners to recognise when and how a situation may be represented mathematically as well as to select an appropriate mathematical method to solve the problem.

The Mathematics Namibia Senior Secondary Certificate Ordinary (NSSCO) level syllabus for Namibian schools (Namibia. MoE, 2005) has ten topics that should be covered in Grades 11-12, namely:

Numbers and Operations, Measures, Mensuration, Geometry, Algebra, Graphs and Functions, Coordinate Geometry, Trigonometry, Vectors in two dimensions and

Transformations and Statistics and Probability. The chapters are divided into two levels, core and extended. Learners have a choice about which level they want to enrol for at the end of their Grade 12 academic year. Of interest to this study is the topic of algebra and how it is taught. Algebra is divided into the following subtopics:

- algebraic representation and formulae;
- algebraic manipulation;
- polynomials;
- equations and inequalities;
- sequences;
- indices and
- logarithms.

After Namibia gained independence from South Africa in 1990, a new educational policy called learner centred education was formulated and chosen as the basic philosophy for the reform. The Ministry of Education (MoE) adopted a learner centred approach (LCE) replacing the previous teacher centred approach. Learner centred education (LCE) is defined as "an approach that means that teachers put the needs of the learner at the centre of what they do in the classroom, rather than the learner being made to fit whatever needs the teacher has decided upon" MoE,(1999. P. 5)

The benefits of this change, according to Thekwane (2001) were to encourage a shift:

- toward classrooms as mathematical communities - away from classrooms as simply a collection of individuals;
- toward using logic and mathematical evidence - away from the teacher as the sole authority for right answers;
- toward mathematical reasoning - away from merely memorising procedures;
- toward conjecturing, inventing and problem solving - away from an emphasis on answer -finding; and
- toward connecting mathematics, its ideas, and its applications - away from treating mathematics as a body of isolated concepts and procedures. (p. 4)

The learner centred approach in mathematics aligns well with the concept of making connections. The points mentioned above, the shift to a new approach, towards mathematics reasoning, towards conjecturing and towards connecting mathematics tell us that mathematics should be seen as a coherent whole and not a body of isolated concepts. Teachers should be able to show their leaners that mathematics is made up of related and connected concepts by using mathematical connections in their practice. A conducive learning environment is required for this to happen. Thekwane (2001) outlined the following requirements for enabling an environment conducive to the learner centred approach:

- encouraging learners to explore sound mathematics and grapple with significant ideas and problems;
- helping learners to verbalise their mathematical ideas;
- showing learners that many mathematical questions have more than one right answer;
- teaching learners through experience the importance of careful reasoning and disciplined understanding;
- building confidence in all learners that they can learn mathematics;
- using the physical space and materials in ways that facilitate learners' learning of mathematics;
- providing a context that encourages the development of the mathematical skills and proficiency. (p. 6).

This learning approach aligns well with the mathematical connections mentioned above in the sense that most of these teacher requirements are in fact connections on their own. Teachers should "help learners to verbalise their mathematical ideas" and "show learners that many mathematical questions have more than one right answer". This means that teachers should help and encourage multiple solutions and ideas from their learners, which is a connection of multiple or different representations according to Businskas' framework (2008). Similarly, using physical space and materials in ways that facilitate the learning of mathematics can be interpreted in terms of using physical manipulatives to present a certain concept; this in itself is an example of alternate representations. The LCE approach, if interpreted and implemented well can help enforce and strengthen the use of mathematical connections when teaching which will then lead to deeper understanding of the mathematics concepts.

2.6 CONSTRUCTIVISM AS THE UNDERPINING THEORY

2.6.1 Social constructivism as a theory of learning

This study is broadly underpinned by social constructivism.

Some brief definitions of what constructivism and social constructivism consist of follows:

> The central principles of this approach are that learners can only make sense of new situations in terms of their understanding. Learning involves an active process in which learners construct meaning by linking new ideas with their existing knowledge. (Naylor & Keogh, 1999, p. 32)

According to Vygotsky (1978), social constructivism is a theory of learning and an approach to education that emphasises the ways that people and specifically learners create meaning of the world through a series of individual constructs within a social environment. In a constructivist environment, teachers are aware of the role of prior knowledge in student's learning, recognising that students are not blank slates waiting to be filled with knowledge.

He further states that these constructs are the different types of filters learners choose to place over their realities to change their reality from chaos to order. This means that individuals learn by making sense of (filtering) or rearranging whatever information (chaos) is provided to them. This happens through the process of what Piaget calls accommodation and assimilation.

Von Glaserfeld (1989) also describes constructivism as a theory of knowledge. He says learning is a process which allows a student to experience an environment first-hand, thereby giving the student reliable trustworthy knowledge. The student is then required to act upon the environment to both acquire and test new knowledge. The environment, according to constructivism plays a very important role in learning. It is in these environments that learners interact with each other. These environments may not necessarily be in the classroom as learning takes place anywhere. In his later publication, Von Glaserfeld (1992) then introduced the term "viability", to say that knowledge is viable in that it changes relative to a context of goals and purpose.

Noddings (1990) provided the following summary of constructivism:

- All knowledge is constructed. Mathematical knowledge is constructed, at least in part, through a process of reflective abstraction.
- There exist cognitive structures that are activated in the processes of construction. These structures account for the construction; that is, they explain the result of cognitive activity in roughly the way a computer program accounts for the output of a computer.

- Cognitive structures are under continual development. Purposive activity induces transformation of existing structures. The environment presses the organism to adapt.
- Acknowledgement of constructivism as a cognitive position leads to the adoption of methodological constructivism. (p. 10)

Vygotsky's (1978) work implies that learners who experience the processes involved in thinking together will come to experience the self-actualization of the processes involved in their own thinking. If learning from a social and interactive experience is important, then much needs to be changed. According to Perry, Geoghegan, Owens and Howe (1995), constructivism requires of us that we provide our students and teachers with the appropriate forms of experiences. They further stated that students should be afforded the opportunity to interactively constitute their understanding through interpersonal and intrapersonal communication, small and large group discussion, and a cooperative environment where teachers do not provide answers but more importantly encourage and mediate discussion. Vygotsky (1978) later states that:

> The greatest change in children's capacity to use language as a problem solving tool takes place somewhat later in their development, when socialized speech (which has previously been used to address an adult) is turned inward. Instead appealing to the adult, children appeal to themselves; language thus takes on an intrapersonal function in addition to its interpersonal use. (p. 27)

2.6.2 Constructivism in educational practices

The principles of LCE discussed in the previous sections of this chapter are deeply rooted in the paradigm of social constructivism. When learners are involved in group discussions, they have a chance to present their discussions to the whole class which would lead to multiple solutions of these tasks, which is an example of multiple representations as a connection according to Businskas' (2008) framework. Social constructivism tells us that learning takes place as a results of these interactions with others and that a teacher will act as a more knowledgeable other (MKO), in other words in LCE, the teacher acts merely as a facilitator.

According to Cobb (1988), constructivists clearly agree that the teacher's actions and instructional activities are of crucial importance in that they are potential sources of challenging situations for learners. He further states that the constructivists' analysis should include the teacher's and leaners' beliefs about the nature of mathematics, their beliefs about their own and each other's role as well as their forms of motivation while doing and talking about mathematics. When a teacher does not possess that special knowledge for teaching, PCK (Shulman, 1986) or does not teach for mathematical proficiency (Kilpatrick et al., 2001), then they will not be able to provide the required environment for learning to take place in the learner centred or constructivist context.

2.6.3 Constructivism and making connections

As discussed in the above paragraphs, social constructivism is a theory of learning whereby learners construct meaning of the world around them. According to Vygotsky (1962), in a constructivist environment, teachers are aware of the role prior knowledge plays in students learning. Recognition of prior knowledge according to Businskas' (2008) framework for connections is one indicator for a instruction-oriented connection. A student in a constructivism environment is required to act upon the environment to acquire and test new knowledge. Acting on the environment can be made through the use of different representation connections, whereby the teachers and learners make use of diagrams, models, pictures and so forth to present a certain concept in mathematics.

Since this research aims to probe the nature of the connections that teachers use when teaching algebra, I will engage more with these literatures as I answer my research questions. The data will be collected using classroom observations and interviews. A well-crafted observation schedule that emanated from Businskas' (2008) framework will be used to analyse the data.

2.7 CONCLUSION

In this chapter I critically analysed and reviewed the literature that shapes, enlightens and provides the foundation and framework for my study. I began by giving a brief overview of the nature of mathematical connections. This was followed by a discussion of what makes an effective teacher of mathematics and an effective teacher of algebra respectively with regard to making connections. I then spoke about a framework for teaching for mathematical proficiency as well as pedagogical content knowledge and highlighted some common themes that emanated from these. Lastly I talked about constructivism as an educational theory that underpins my research. I also placed this study in the Namibian context.

The next chapter examines the methodology used to carry out this study.

CHAPTER 3

RESEARCH METHODOLOGY

3.1 INTRODUCTION

In this chapter, I describe and justify the methodology used in this research project. I start by explaining the design as well as the approach used in the study. I also describe the sampling procedures used for selecting the three participants, the data collection methods and finally, I explain how I analysed my data.

3.2 RESEARCH GOAL AND QUESTIONS

This research aims to investigate the nature of mathematics connections that selected Grade 11 teachers make when teaching algebra. To achieve this goal, I pose the following research questions:

- What type of connections do selected Grade 11 teachers make when teaching algebra with regard to Businskas' framework?
- What are the selected teachers' perceptions of making use of connections when teaching algebra?

3.3 RESEARCH ORIENTATION

This research was conducted within an interpretive paradigm. Cohen et al. (2011) describe an interpretive paradigm as one that gives the researcher an opportunity to understand and interpret the world around them. An interpretive paradigm is characterised by its concern for an individual and it focuses on action. In this regard, I am hoping to understand and interpret the teaching of mathematics by the selected Grade 11 teachers. I find this paradigm to be suitable for my research since my interest is in understanding the type of connections that teachers make when teaching, as well as in their perceptions towards those connections.

3.4 RESEARCH METHODOLOGY

This study is a case study, bounded specifically by the practices of my three selected Grade 11 teachers in their classroom environments. A case study, according to Hamilton (2011), provides "rich data because it gives the researcher in-depth insights into participants' lived experiences within this particular context" (p. 1). Cohen et al. (2011) agreed by stating that case studies afford unique instances of real people operating in real situations.

In this study, my case is the nature of mathematics connections and their use by Grade 11 teachers. My unit of analysis is thus the nature of these connections that the teachers make when teaching algebra, as well as the teachers perceptions of making use of these connections.

3.5 RESEARCH DESIGN

This study was designed to take place in three phases as follows:

Phase 1-Video recording of 2 lessons per participating teacher. The six lessons were video recorded at mutually convenient times when the participating teachers were teaching algebra. For each teacher, the two lessons were of different classes and were not recorded on the same day.

Phase 2-A workshop with the three participating teachers was held to discuss the nature of connections. In particular we wanted to reach consensus about the definitions of connections and reach a common understanding of Businskas' framework. This involved carefully working through Businskas' framework with the teachers. It was important that Phase 1 occurred before Phase 2 as I did not wish the teachers to specifically prepare the lessons according to Businskas' framework. I merely wished to use Businskas' framework to collaboratively analyse their lessons.

Phase 3-Stage 1: In this stage the respective teachers and I analysed their two lessons and identified and classified the connections made by the specific teacher according to Businskas' framework. We made use of an observation schedule (see below) to identify and classify the different connections that the teacher made throughout the two recorded lessons.

-Stage 2: In this stage I conducted semi-structured interviews with each of the three teachers about the connections they made in their lessons.

3.6 DATA COLLECTING METHODS

For the purpose of "triangulation" (Cohen et al., 2011, p. 195), my data collection included a variety of methods. These were classroom observations, workshops and semi-structured interviews.

3.6.1 Observations

As previously discussed, six classroom observations were done by video recording. Cohen et al. (2011) are of the opinion that observations in case studies provide an opportunity for the researcher to develop a more intimate and informal relationship with their participants. Whilst I video recorded the six lessons I also took field notes.

3.6.2 Workshop

After both recordings for each teacher was carried out I ran a workshop as discussed above. Apart from reaching consensus about connections, the workshop also greatly enriched the teachers' knowledge about their own teaching and expanded their horizons about Mathematics, as was anticipated.

The workshop also constituted the beginning of the analysis process of the lessons.

3.6.3 Semi-structured interview/ stimulated recall interviews

Stimulated recall, according to Lyle (2003) is a procedure in which videotaped films are used for collaborative analysis and for helping the subject to reflect on his/her practice during the replayed episode. The semi-structured interview with each teacher was coupled together with the stimulated recall of viewing the videos.

The semi-structured interviews afforded me the opportunity to interact deeply with my respondents. It gave the teachers a chance to reflect on their lessons as we looked at the recordings for evidence of connections. Teachers were quick to identify connections that they made, as per consensus of the workshop and our discussions were centred on those connections. Referring to their use of connections, I asked them questions like "do you mostly (always) do that?", "why did you do that?" or "were you aware that you were making a connection here?" Their responses then directed our discussions.

This method provided me with rich data. It was very interesting to hear the individual reactions and comments about their use of connections. This platform also created reflective opportunities for the teachers to deepen their own knowledge about their practice. Some misconceptions that some teachers had were cleared up here. It was also a learning experience for both me and the teachers as we talked about certain areas that the teachers covered exceptionally well.

3.7 PARTICIPANTS

My participants consisted of three teachers from three different secondary schools in Tsumeb in the Oshikoto region of Namibia. One teacher was selected from each of the three Secondary Schools in the town. Since I work in Tsumeb, it was convenient to choose schools close to my workstation to minimise travel expenses. This also helped me with understanding the context due to my own prior knowledge and insights which I had of the participating schools.

I purposively selected three teachers from the abovementioned schools on the basis that they had taught Grade 11 for more than four years, which I believed was sufficient experience to enable them to talk about their practice with depth and insight. I chose Grade 11 teachers because the topic of algebra forms an important part of the Grade 11 curriculum. The three teachers were asked to participate on a voluntary basis after the data collection process was explained to them.

3.8 DATA ANALYSIS

The recorded lessons and the discussions were transcribed and a qualitative approach was used to analyse them.

In order to capture the frequency and nature of the connections, I used the instrument illustrated in Table 3.1 below to understand the descriptions of these connections.

Table 3.1 Observation instrument

Type of Connection	Coding	Description	Instances/examples	Frequency
Different /Multiple representation	DR	**Alternate Representation (AR)** The teacher: • uses representations of different modes of the same concepts. e.g. Straight line graph used as an alternate representation of y=mx+c, same concept of graphs represented differently by a graph/line and by an equation. Another example is a graph of a parabola is an alternate representation of $y=ax^2 + bx+c$, algebraic to graphic. • Allows learners to use varying representations to answer questions • Allows learners to see the differences in different methods of calculations **Equivalent Representation (ER)** The teacher • uses concepts that are represented differently within the same form of representation. e.g. $f(x)= ax^2+bx+c$ used as equivalent to $f(x)=a(x-p)^2+q$, two different expressions of quadratic represented in the same form of an equation. Another example could be that 3x+2x is equivalent to 2x+3x which is equivalent to 5x • shows and highlights the relationships between equivalent representations		
		Multiple representations could entail algebraic symbolism, graphs, diagrams, models, equations, notations, images, analogies, metaphors, stories or games		
		Other		
Part-whole	PWR	The teacher:		

Relationships		• makes connections between the general and the specific through some particular example • presents the lesson in some logical/ordered way • uses practical examples to move to the abstract • makes use of particular examples, ideas, concepts and/or techniques • uses examples that students can relate to through experience • makes use of generalisation of concepts or actions. $ax^2+bx+c=0$ is a generalisation of quadratic equations and $2x^2+7x+3=0$ is a particular case for quadratic equations.		
If-then/Implication	IM	The teacher : • shows how one concept is dependent on another in some logical way. • is able to make conjectures, proving or justifying them • has problem solving skills and tests hypotheses, • uses logical reasoning, draws conclusions from premises, • shows links between hypothesis and conclusion in a deduction, • tests general assertion with examples, • asks learners to justify answers and why their procedures make sense • uses clear distinction for sentences involving 'and', or, not', if-then', some' and 'all'. e.g. If $2x=12$, then $x=6$; $x^2+5x+6=0$ will have at most two roots as implied by the degree of the equation.		
Procedural	P	The teacher: • shows fluency in computational skills • knows and uses efficient and accurate procedures for working out tasks • uses the calculator appropriately and only allows learners to use calculators		

		after they have learned how to do the relevant mathematics without calculators • does not dictate to learners what procedures to use • realises that there are many ways to solving a problem e.g. using a quadratic formula to solve a quadratic equation, knowing that an equation cannot be factorised • allows learners to communicate their thoughts in a commonly acceptable language		
Instruction-oriented connection	IOC	The teacher: • shows the hierarchical nature of concepts by highlighting the prerequisites, for example the fact that factors and multiples are prerequisites for understanding fractions • understands how students are making sense of the tasks but also aligns their disparate ideas with the subject • knows the learners' backgrounds • connects teaching to learners' sense making ways • recognises and is aware of learners' prior knowledge and links this prior knowledge to new concepts • has an awareness of his/her students' thinking and ideas • uses the content that caters for learners' different abilities • encourages learners to borrow ideas from other disciplines or own experiences • makes connections to different branches of mathematics		

After thorough deliberation with the instrument, I then analysed the lessons with regard to the above definitions. I used these descriptions as the themes to look for in my analysis. I used colour coding on the transcripts to highlight each type of connection used. I then summarised the transcripts using table 3.1. The table was divided into 3 minute intervals (see appendix).

Secondly I analysed what the teachers said in our discussions about connections. I paid attention to the teachers' comments and reactions and then recorded their perceptions about connections. Since the themes were already defined, as per Businskas' framework, this was merely for consolidation purpose as well as highlighting their perceptions.

3.9 ETHICAL CONSIDERATION

Access to the schools and teachers was negotiated with the principals concerned. I asked the schools to provide consent in writing. Letters of consent were also written and sent to the participating teachers. Ethical practices of confidentiality and anonymity were strictly adhered to. The purpose of the research and its implications were explained to the teachers concerned well in advance of the data collection process. Their right to withdraw from the study whenever they wished to was also communicated to them. The respective participants were informed that the outcomes of the study would be shared with them if they so wished.

3.10 VALIDITY

Cohen et al. (2011) argued that validity can be improved through careful sampling and through appropriate instrumentation. They also remind us that research can never be 100 % valid. Hence, to validate my data as much as possible, I carefully selected my sample of the participating teachers as well as selecting a number of appropriate data collection instruments.

For the purpose of "triangulation" (Cohen et al., 2011, p. 195), I used two methods of data collection, namely observations and interviews respectively.

3.11 LIMITATIONS AND CHALLENGES

Due to the small size of this research sample, the results cannot be generalised but can only be applied within the context of the participating schools. The findings and deliberations in this study could however form a platform for similar research in different contexts and on a larger scale.

The fact that the teachers involved are from different schools with different afternoon programs proved a challenge, especially in finding a convenient time to hold the workshop. As a result I ended up conducting workshops at different times to accommodate everyone.

3.12 CONCLUSION

In this chapter, I described and justified the methodology that I used in this research project. I explained the design of my research as well as the approach I used. I then described the sampling procedure, the data collection methods as well as the data analysis. Lastly, I spoke about how I validated my research and also the limitations and challenges that I experienced.

The next chapter deals with the in depth analysis of my data.

CHAPTER 4

RESEARCH FINDINGS AND DISCUSSIONS

4.1 INTRODUCTION

This chapter presents and discusses the data collected in this research. The goal of the study was to understand the nature of mathematical connections that teachers make when teaching algebra as well as their perceptions of these connections. Firstly, I present the connections that were identified from the analysis of the video recorded lessons of the three Grade 11 teachers. I then present the data collected from the semi-structured interviews. Lastly, I consolidate the two parts of the data analysis.

As already mentioned in Chapter 3, the five connections identified for the purpose of this study were:

Table 4.1 Type of connections

TYPE OF CONNECTION	**CODING**
Different/Multiple representations	DR/MR
Part-whole connections	PWC
If-then/ Implication	IM
Procedural	P
Instruction-oriented connection	IOC

For the purpose of this research, this coding will be used to indicate the type of connections identified.

4.2 LESSON ANALYSIS AND DISCUSSION

4.2.1 Teacher E; Lesson 1; Grade 11B

This lesson was about factorising using the difference between squares. It also focussed on factorising quadratic trinomials. It was a continuation lesson from previous lessons on factorising quadratic trinomials. This class consisted of both core and extended learners, so the teacher alternated between the two levels. The analysis was done in three (3) minute intervals. The transcripts of the lessons are included in Appendix 8.

0-3 minutes

The teacher started off by making an instruction oriented connection (**IOC**) by asking what the class did last time and what the homework was. This was classified as **IOC** because the teacher was making an extension of what the learners already knew by linking their prior knowledge with the new concepts. The teacher then proceeded to do the correction of the homework, and in so doing, she made the following connections: procedural (**P**) connections when she said the following: *" when factorising by difference between the squares, you find the square roots of the coefficients and for the index you divide, then have your two brackets".* This was seen as a procedure for carrying out factorisation by difference between squares and hence a procedural connection. The teacher then made an implication (**IM**) connection, *"If one bracket has (a+12) the other bracket will have what"?* Learners should know that when factorising the difference between squares one of the two brackets will contain a plus and the other a minus sign. So if the one bracket given is (a+12) then by implication the other bracket should be (a-12). This is thus an implication connection.

3-6 minutes

In this interval, the teacher introduced a new form of factorising. Before she did that she said the following: *"If you are given an expression to factorise you either use common factors or use factorisation by grouping or use the difference between squares right? Then there is a fourth method which is factorising by quadratic trinomials".* In this statement, the teacher was referring the learners to something that was already known or done previously therefore it was an extension of previous lessons. It shows how the content is sequenced in a hierarchy. It is thus classified as an instruction oriented connection. This statement however falls short of being a multiple representation (**MR**) type of connection since the teacher was not telling the class that there are four types of factorising but was rather looking for affirmation of something that they already knew. Next, the teacher made a part-whole relationship (**PWR**) when she gave $x^2 + 5x + 6$ as a quadratic expression to factorise. $x^2 + 5x + 6$ was given as a particular case of a generalisation of quadratic equations. She then made an instruction oriented connection (**IOC**) by saying that you can write $x^2 + 5x + 6$ as $ax^2 + bx + c$, and then she asked: *"do you still remember that expression, where a is the coefficient of x^2, b is for x and c is a constant"?* In this instance the teacher reminded the learners of an expression that they already knew as well as reminded them of what each letter represented. Such a statement also made the learners to think of what a coefficient or constant was, something that is normally introduced in Grade 8 Mathematics.

6-9 minutes

During the next three minutes, the teacher used mostly the **P** type of connection as she presented the procedures for factorising quadratic trinomials. In her procedures, she mentioned that learners should look for factors of 6 that can give 5 when added. This means that to factorise using this method, learners need to know the factors of the numbers given as well as factors in general. This means that the knowledge of factors should be a pre-requisite for factorising. This was then identified as an **IOC** as it showed the hierarchical nature of concepts. The teacher continued to factorise the given example and said: *"we are no longer going to write* $5x$*, so instead of* $5x$ *we are going to write* $2x$ *and* $3x$*".* $5x$ is equivalent to $2x$ and $3x$, so $2x$ and $3x$ is another way of writing $5x$, and hence this was identified as an equivalent representation which is a type of different representation (**DR**) connection.

9-12 minutes

The teacher proceeded by showing the procedures for factorising, and then she said: *"when dividing* x^2 *by* x *we say it is 2 minus one, using the indices".* The teacher referred to the laws of indices that are normally introduced in Grade 9. This was identified as an **IOC**. After asking the core students to solve $a + 2(2 - a) = 8$, she solved it on the board and got $-a = 4$. Then she said if $-a = 4$, then $a = -4$. This is an if–then type of relationship, so it was identified as an implication connection (**IM**). After going back to the extended students, she asked them to factorise $2x^2 - 3x + 1$. She asked the learners to find the factors of 2 that would give -3 when added. She concluded that both factors must be negative since c is positive. Even though the teacher did not elaborate on this, it shows the use of an **IM** connection. This was because the teacher tried to say that for the two numbers to give a positive product and a negative sum, means that both numbers are negative, because if they were both positive then it is impossible to get a negative sum. Lastly, the teacher asked the class to factorise $2x^2 + 14x + 20$ using quadratic trinomials. She then told the class: *"if you can see there is a common factor you can take it out, you know how to factorise by the first method of common factor"?* Here the teacher made the learners recall a method that was learnt or taught previously, so she was referring to their prior knowledge. It was identified as **IOC**.

No connections were made after the 9th minute as the learners were engaged in the tasks she set for the remainder of the lesson.

4.2.2 Teacher E; lesson 2; Grade 11E

This lesson was about solving simultaneous equations

0-3 minutes

The teacher started off by doing homework corrections. She asked the class to solve a pair of simultaneous equations and said: *"we have 3 methods of solving for c and d".* This was identified as different methods of solving a problem which is found in a **DR** type of connection. She made the learners aware that there is more than one way of solving these simultaneous equations and thus encouraging them to use any method they deemed to be appropriate. She then asked the class which method they should use together to solve. After one learner replied that they should use elimination, the teacher asked this learner: *"elimination how"*. Here the teacher wanted the learner to justify his answer as well as justify why the elimination method would work. By asking the learner to justify his answer, the teacher used the **IM** type of connection. The teacher reiterated the fact that they were solving for c and d, after one learner asked what to solve for. *"Solve for c and d, use either elimination or substitution. Now which one do you want to use"*. She showed there were multiple ways of solving a problem, so it is **DR**. After solving the problem on the chalkboard and involving the class in the process, the teacher wrote $c = 4 + 1$, and said $4 + 1$ is what? In this instance, even though it was not clearly stated, the teacher was making equivalent representation of a value 5. So this is a **DR** type of connection.

3-6 minutes

The teacher continued to ask the class to solve and said: *"if you want to use the elimination method then the coefficients of the letter you are eliminating must be the same".* In this instance the teacher highlighted the differences in various methods, so it is a **DR** type of connection. For most of this period learners were busy working out the tasks on their own.

6-9 minutes

Next, the teacher continued to solve by elimination making **P** connections as she solved. Then she asked the class: *"any number multiplied by zero is what? So this is zero"?* In this question, the teacher made the learners aware of the result of multiplying by zero through the **IM** type of connection. If anything multiplied by zero is zero, then it is implied that multiplying by zero should give an answer zero. One boy asked the teacher how the teacher knew which method to use and she replied: *"ok that's why I am giving you two methods, if you forgot how to use elimination then you can use the substitution method".* This answer was a clear indication that the teacher did not expect the learners to use one specific method but encouraged the use of other methods. She did not dictate what method learners should use, so this is a **P** connection. The fact that that the teacher acknowledged that there is more than one way of solving simultaneous equations that is a **DR** connection. She continued with another **DR** connection when one learner asked why she did not use the second equation when substituting. To this the teacher replied: "*ok use the second equation to substitute and see if you will get the same answer".*

9-12 minutes

During the next three minutes, the teacher gave learners simultaneous equations to solve and asked the learners to use the method of their choice. *"I am not saying use elimination or substitution but as long as you solve the two equations. Use the method of your choice".* Here the teacher made learners aware that there are many methods of solving a problem. This type of connection was again identified as both **P** and **DR** types of connections. It is procedural in that the teacher allows learners to choose the method they prefer and it is also different representation because the learners were made aware of the different ways they could use to represent their answers.

12-15 minutes

Next the teacher did the corrections on the work the learners were doing, solving $2x - 3y = 16\,; x + 2y = 15$. She made two types of connections in her explanations. She said she would use both methods but first would use substitution. She numbered her equations 1 and 2. When she was solving the equation by substitution, she said the following: "*and because I have the coefficients of x as one, I will make x the subject of the formula. So I have $x = 15 - 2y$, are we together"?* In this instance, the teacher used the **IM** method to justify the use of making x the subject of the formula. It was also a **P** connection in the way that the teacher chose an efficient procedure by making x the subject of the formula and not y, which would take more time. She continued to make a **P** connection when solving: *"so I have* $x = 15 -$

$2y$*, so in the first equation I put* $15 - 2y$ *where there is x?* This step showed the procedure for solving simultaneous equations by substitution, which is also a **P** type of connection. It also showed $15 - 2y$ as an equivalent way of representing x, so this falls under the **DR** type of connection.

15-18 minutes

In the last three minutes, the teacher made an **IM** connection by saying: *"so I have 2x = 16+6, what is 16 plus 6?, so x is 11".* The teacher got an answer from the learners that *16+6* is *22*, and by implication she concluded that if *2x=22*, then *x* should be *11*.

In summary, Table 4.2 illustrates the connections that Teacher E made in the two observed lessons.

Table 4.2 Summary of connections that Teacher E made

Type of connection	**Frequency**	
	Lesson 1	**Lesson 2**
DR	1	8
PWR	1	0
IM	2	4
P	2	6
IOC	5	0

4.2.3 Teacher T; Lesson 1; Grade 11C

This lesson was an introduction to algebraic fractions. This class was mixed with both core and extended learners. Since algebraic fractions is for extended learners only, only the extended learners were involved in the lesson while the core learners were busy with their own work.

0-3 minutes

The teacher started by writing the topic algebra on the chalkboard and then said to the class: "*we all know what algebra is about…we know it is a discipline that has to do with numbers and letters".* In this statement, the teacher made a **DR** type of connection. He described algebra as a discipline that can be represented differently by numbers as well as letters. He then continued to say "*why don't we take a look at that tree outside, can you count the number of leaves? If you count from this direction 1,2,3,4…"*. Here, the teacher made use of a tree, which is a physical object. The tree was used as a representation for telling a story, in other words as a teaching aid. This was identified as a **DR** type of connection. He used the number to quantify the number of leaves as he started counting them from one side. This showed a relationship between the leaves (image or picture) and the numerals.

3-6 minutes

The teacher continued: *"we can use a letter to represent the number of leaves, say x"*. This was identified as **DR** as the number of leaves was represented in another form which is algebraic symbolism. The teacher continued: *"we have to use a variable for something that is impractical to achieve or unknown"*. In this case, the teacher introduced generalisation of concepts. He made a connection between the general and the specific through a particular example. This was thus identified as a **PWR** type of connection. The two statements above showed that the teacher was moving from the trees and leaves to introducing new notions of variables. Therefore the teacher made an **IOC** in the sense that the concepts used were hierarchical. The teacher continued by introducing another tree that was "*genetically engineered"*. The teacher borrowed the term genetically engineered from a discipline of life/ human sciences. This was evidence of an **IOC**. It was not clear however whether the learners knew what genetic engineering was. The teacher then explained, "*genetically engineered which means the number of leaves will be the same as on the other tree because they are the same"*. That statement carried two types of connections, first the teacher made an **IM** connection by concluding that since the two trees are genetically engineered then they will have the same number of leaves. Secondly, it provided a practical example of moving to the abstract of algebraic concepts. It was again identified as a **PWR** type of connection. He continued explaining: *"let us say we have another tree, different species with different number of leaves, we cannot use x again because it is not like the other one"*. This was another case of a practical example being used to move to the abstract of like terms in algebra, so it was identified as a **PWR**. It also included an **IM** connection in that the teacher used the fact that the trees are of different species and hence will have different number of leaves, which cannot all be represented by *x*.

6-9 minutes

The teacher continued with the example of a tree, "*if we mobilise the whole school to count the number of leaves on that tree, let us say we find out that the number of leaves is roundabout 400 000 leaves, so x is 400 000".* Here, the teacher used logical reasoning to conclude that the number of leaves are 400 000. He realised that it would not make sense to have 100 leaves for example, so he thought carefully what the sensible number of leaves should be, by making a calculated guess. This was classified as an **IM** type of connection. In concluding his example, the teacher said *"numbers are now called variables, x and y are different species with different number of leaves so they are unlike terms"*. The example that the teacher used led to the adding of like and unlike terms which leads to adding and subtraction of algebraic fractions. The examples that he used acted as a connection between general and specific concepts. This is clear indication of a **PWR**.

9-12 minutes

In this interval, the teacher started with adding and subtraction of fraction. *" When adding and subtracting fractions, and if the denominators are not the same, then we look for the lowest common multiples, which means you must know the multiples of these*". Here the teacher outlined the procedures for simplifying fractions, which included making the denominators the same. It was identified as a **P** type of connection. Another type of connection was identified from the above statement. The teacher was referring to a prerequisite of making the denominators the same. This includes knowing the multiples of the denominators given. The teacher highlighted the fact that factors and multiples are pre-requisites for understanding fractions. This is an indication of **IOC**.

12-15 minutes

The teacher wrote a fraction on the chalkboard, $\frac{7}{a} - \frac{1}{2a}$, "to *get 2a as my denominator, with what should I multiply a",* after one boy replied that we should multiply with a. He then asked: "*multiply by a? But I want to get 2a".* In this case the teacher was trying to make sense of how this learner understood the question. He knew that the learner's answer was wrong but he was looking to understand why the learner thought multiplying *a* by *a* would give *2a*. This was identified as an **IOC**. When working out another example $\frac{c}{13} + \frac{d}{13}$, the teacher asked the class: *"can we add these numbers of leaves together? Why not"?* Since the teacher had told the story of leaves, he used that to get a justification on whether the two terms are like or unlike, and whether they could be added together or not. This is an **IM** type of connection. The teacher wrote another fraction on the board and asked the learners to work it out $\frac{p}{3} - \frac{q}{15}$, and then he said: *"make sure the denominators are the same"*. Here the teacher was connecting to the procedures used for simplifying fractions, hence it was identified as a **P** type of connection.

15-18 minutes

In solving the previous problem, the teacher asked "*with what should you multiply to 3 and get 15"?* By referring the learners to the procedure of making the denominators the same, the teacher was also testing their prerequisite knowledge of multiples and factors. This was identified as an **IOC**. In the last step of solving the problem, the teacher had *x+3x+y* as the numerator and asked the class: "*can I add these together? Are they the same number of species? Why? What will our final answer be?* In all these questions, the teacher was looking for a justification so check whether the learners could see which terms were like terms to be added together and which were not. This was an **IM** connection.

4.2.4 Teacher T; Lesson 2; Grade 11C

This lesson was a continuation of simplifying algebraic fractions presented in lesson 1 above. As mentioned earlier, the section on algebraic fractions is only for the extended learners.

0-3 minutes

The teacher started off the lesson by making an **IOC** when he said: "*continuing from where we stopped last time, I hope we remember algebra, where letters are coming from and letters representing numbers. Algebraic fractions. Let's say I have* $\frac{1}{x} + \frac{1}{2x}$*, ok so we remember adding fractions, we know about the lcd [lowest common denominator]"?* Here the teacher was fully aware of and recognised the learner's prior knowledge. He is making an extension of what they already knew by linking prior knowledge to new concepts. He decided to refresh their minds with simple algebraic expressions before moving to the more complex ones.

3-6 minutes

He showed the procedures for simplifying, involving the learners as he did so. He then asked, *"with what should I multiply this x to get 2x"?* When one learner replied that we cross-multiply, the teacher replied: *"well we only cross multiply when we have an equal sign or do* *you mean multiply horizontally?, do you mean I multiply x with 2x or x with 2"?* The learner replied that we multiply x with 2, then the teacher asked again: *"now do you mean 2 with this 2 (pointing to 2x at denominator), what if there was a 3? Will it make the denominators the same"?* In this case the teacher kept probing the learner to communicate his thoughts in a commonly acceptable language. Both the teacher and learner might have been saying the same thing about multiplying to get the same denominators, but since the learner used the term "cross-multiplication", the teacher cleared up this misconception. This was identified as a **P** type of connection. The teacher then continued to work out the solution and at the last step he said: *"I have seen some people adding the denominators together, which is wrong. The reason we are looking for the LCM [lowest common multiple] is so that we only add the numerators".* In this instance, the teacher made an **IM** connection. He used logical reasoning to imply that since the denominators are the same then they need to only work out the numerators.

6-9 minutes

After the teacher finished with his refresher example, he moved on and said: "*ok, now the complex ones, you do the same".* He wrote $\frac{2}{x+1}+\frac{3}{x+2}$ on the chalkboard. The teacher organised his content in such a way that he started with the easier tasks and then moved to the more complex ones. He prepared his learner for this shift. The moment he mentioned that they were moving to the complex tasks the learners became aware that more abstract concepts were going to be part of the lesson. In this case the teacher made an **IOC**. After the first step of making the denominators the same, the teacher asked the class: *"do we still know how to expand, what we do"?* Here the teacher was simply testing the learners' prior knowledge on expanding brackets. Expanding brackets in Grade 11 is usually taught before algebraic fractions. In this case, expanding brackets is a prerequisite of simplifying algebraic fractions. This was identified as an **IOC**. The teacher continued to simplify the fraction on the chalkboard; he paused and asked the class: *"do we still remember something about like and unlike terms? Is there something we can simplify here? Which ones are like terms"?* Again, this was identified as an **IOC** because he was referring to something the learners knew and had already done. Like and unlike terms are introduced in Grade 8 algebra and for this specific class, had been discussed the previous day in terms of different species of leaves. In this interval, the teacher made a **P** type of connection as he explained simplifying complex fractions.

9-12 minutes

After the learners identified the like terms, the teacher then said: *"the like terms of x, 2x and 3x gives you 5x".* This was identified as a **DR** type of connection. *5x* is an equivalent representation of *x*, *2x*, and *3x* added together. Even though the teacher did not use these exact words, it was concluded that he meant it in that way.

***12 -15* minutes**

"Ok now, let us try a different one $\frac{5}{p+3}-\frac{3}{p-5}$ *there is not any other constant to multiply with anymore, the previous example there was a constant to multiply to get the lowest common denominator. So what happens is, the previous example we multiplied (x+2) by (x+1) and (x+1) by (x+2) so that we got the same denominator. Now what do I do here, so I multiply this one by that one and this one by that one (pointing to the two denominators)".* The teacher showed how one task is related to the other, showing the hierarchical nature of the tasks given. Even though it was not clear to which previous task the teacher was referring to, it was still identified as an **IOC**.

15-18 minutes

In this interval, the teacher continued to simplify the fraction on the chalkboard thereby making a **P** connection. In the last step he said: *"so we get 5p-25-3p-9, so which ones are like terms and what do we do? 5p and 3p are like terms, ok what is the result? So maybe I write them together, so I have 5p-3p-25-9 over our two brackets. What happens now"?* The teacher asked the class what 5p-3p is, the class gave the answer *2p*. Then he continued to ask what *-25-9* is equal to, one learner gave the answer of -38 while another one gave 16. The teacher then asked "*how did you come to 16"?* In the first case, the teacher used a **P** type of connection as he followed the procedures of collecting like terms. In the last case, he was looking for justification. Even though the answers given were wrong, he still wanted to know how the learner arrived at their answer. This was categorised as an **IM** connection.

In the last few intervals of the lesson, the teacher gave learners some tasks to work out while he was walking around to see their work.

In summary, Table 4.3 illustrates the connections that Teacher T made in the two observed lessons.

Table 4.3 Summary of connections that Teacher T made

Type of connection	**Frequency**	
	Lesson 1	**Lesson 2**
DR	3	1
PWR	4	0
IM	5	2
P	2	3
IOC	5	5

4.2.5 Teacher O; Lesson 1; Grade 11E

This lesson was an introduction to solving quadratic equations by the quadratic formula.

0-3 minutes

The teacher started off by making an **IOC** by saying: *"last meeting we learnt how to solve quadratic equations by factorisation".* This was identified as such because the teacher was referring to what the class had covered in the previous lesson.

3-6 minutes

No connections were made in this interval as the teacher waited for the learners to finish copying the summaries from the chalkboard.

6-9 minutes

He continued with another **IOC**: *"as you can see there, it is still the quadratic equation resurfacing there from what we did at factorisation".* The teacher connected what they had done previously to the new content. His concepts were presented in a hierarchical form. As he explained, he made a **PWR** type of connection by generalising quadratic equations as follows: *"we are saying that for a quadratic equation, the general formula is* $ax^2+bx+c=0$*, a, b and c are integers. We all know what integers are right? What are integers"?* In the second part of this sentence, the teacher referred the learners to integers. Integers are first taught in

Grade 8, so in this case he referred them to earlier content. Understanding integers is also a pre-requisite for working with the quadratic formula. This was categorised as **IOC**. He continued to say that the variable they were solving for was not always *x* but could be *y* or any other letter that the examiner used. This was evidence of a **DR** connection because it showed that the same concept of a quadratic equation can be represented by different symbols in the same form.

9-12 minutes

The teacher then gave an example: *"let us look at the following example* $x^2 + 7x + 3 = 0$*, this is a quadratic equation. Why do we say this is a quadratic equation"?* In this instance, the teacher gave a specific example of a quadratic equation. That was identified as a **PWC**. He then asked learners to justify why it could be concluded that $x^2 + 7x + 3 = 0$ is a quadratic equation. This is an indication of an **IM** type of connection. One learner answered that it was because of the *x* that is squared. He replied by giving his justification: "*ok because you have your x squared then that is the indicator of a quadratic equation".* Then he said: *"now we can connect this equation to the generalised form and we identify our integers".* This was a straight forward **PWR** kind of connection. The teacher highlighted the fact that there was a relationship between the specific example and the generalised form of equation. It was interesting to note that the teacher used the exact word *connection.*

12-15 minutes

He continued: "*I am trying to connect the generalised formula to the defined one we are given. We are now equating those values to the letters and plug them in the formula".* This was an indication of a **PWR** type of connection. The teacher created a connection between the specific example and the generalised form. When one learner asked where the formula came from, he replied that it was just like any other formula just like that of an area of a circle. This was identified as an **IOC** because the teacher referred to the area of a circle which they already knew and compared that to the quadratic formula.

15-18 minutes

He then proceeded to make a **P** type of connection as he showed the procedure for solving using the formula. He proceeded to say: *"we are now going to simplify what is in the square root, work out the square root then divide".*

18-21 minutes

He continued with the **P** connection by explaining how they should give their answer. *"Because of these decimals, you are expected to round your answer to 3 significant figures".* One learner asked the teacher to check why his calculations did not give the same results.

21-24 minutes

After some checking, the teacher said: *"ok I think the problem is calculator skills. You must first work out what is up and then you divide".* The teacher was able to detect the learner's incorrect use of a calculator. This was also categorised as a **P** connection.

4.2.6 Teacher O; Lesson 2; Grade 11E

This lesson was about solving quadratic equations by completing the square. It was not a continuation of the lesson analysed above.

0-3 minutes

The teacher started with an **IOC** by saying: *"we have been doing two methods of solving quadratic equations. The first one was by factorisation and the second was the formula".* In this instance, the teacher referred to what was previously covered before he introduced the topic of the day. He continued: "*I was saying we have two methods of solving quadratic equations, but at times you find that there are quadratic equations that will never factorise no matter how hard you try"*. The teacher explained that there are various ways to solve quadratic equations. He made the learners aware that when one method does not work, they should go for the next possible one. This is a **P** type of connection. He assured the learners with the following **IM** connection. "*If it will never factorise and you are not told to use the formula, then it is obvious you will only opt for the completing the square method".* In his explanation of how to complete the square, he said: "*this means you have to express that equation as a perfect square".* He then showed the connection by writing $x^2 - 4x + 4 = (x-2)^2$. This is a connection between an expanded expression with its factorised version, which is an equivalent representation of **DR**. It shows the same expression in different forms of representations.

3-6 minutes

The teacher continued to give a specific example of a quadratic equation as $x^2+5x-7=0$, this was an instance of a **PWR** connection. The teacher then proceeded with a **P** connection by saying: "*if I want to solve for x, there are steps to follow, move 7 to the other side. Next you add your* $(b/2)^2$ *to both sides"*. The teacher said this as he worked out the equation by completing the square. He was explaining as well as demonstrating the procedure for working out that specific equation. He then asked the class: *"what is b here"?* This can be seen as relation of a specific example $x^2+5x-7=0$ to the general form $ax^2+bx+c=0$. This is a case of a **DR** type of connection.

6-9 minutes

In this time interval, the teacher continued by saying: *"now I am going to factorise this and make it in this form. This part will give me my left part if I expand it".* He showed the relationship between an expanded form with its factorised form. This is an indication of a **DR**. He continued with an explanation of the steps to use in order to solve the problem,

9-12 minutes

The teacher continued: *"now I have factorised what I have here, when I expand it will bring me to this form".* Again, this statement was identified as a **DR** because the teacher showed the relation between two different representations of a quadratic equation. No other connections were made in this time interval.

12-15 minutes

As the teacher continued with the **P** connection of solving by completing the square, he said the following: *"and because of this two here, because of this index here we are going to square root both sides not"?* The teacher revealed by implication that to dispense with the square you must square root; this was a case of **IM** connection.

15-18 minutes

At the very end when he finished solving, the teacher cautioned the class to give their answers to 3 significant figures or 2 decimal places. He added that the learners should substitute their *x* values in the equation and see if they would get zero as the answer. In this interval the teacher reaffirmed the procedures to follow when solving, so it was a **P** connection.

In summary, Table 4.4 illustrates the connections that Teacher O made in the two observed lessons.

Table 4.4 Summary of connections that Teacher O made

Type of connection	**Frequency**	
	Lesson 1	**Lesson 2**
DR	1	3
PWR	4	1
IM	1	2
P	3	4
IOC	4	1

4.2.7 Summary

All three teachers used connections in their own way, depending on the topic that they taught. Teacher E made the most use of the **DR** connections - 8 times in lesson 2 and only once in lesson 1. This could be because the topic was simultaneous equations and since she talked about two methods of solving, she alternated between these two. She also did not dictate what method learners should use in most cases. In the same second lesson Teacher E did not make any **PWR** or **IOC**.

Teacher T made 5 connections each of **IM** and **IOC** as well as **IOC** in the second lesson. This could be partly attributed to the fact that the two lessons were presented after each other. So the second lesson was a continuation of the first one, the class groups were however different. This made it easier for the teacher to refer to the previous lesson in most cases. **PWR** and **DR** were the least made connections in the second lesson, with no **PWR** connection made at all.

The highest number of connections that Teacher O made was 4 in lesson 1 of both **PWR** and **IOC**. The least number of connections made were **DR** and **IM** with one of each in the first lesson as well as one each of **PWR** and **IOC** in the second lesson. A further discussion on the nature of connections used follows in 4.4.1.

4.3 INTERVIEW ANALYSIS

The three teachers were interviewed by way of stimulated recall. As we viewed the video recorded lessons together, we identified and talked about the connections made at different intervals. We also talked about what the teachers learnt and could use in future lessons. This was done in agreement with the teachers and was influenced by the stimulated recall as they reflected on their own practice. Since the interview was mainly guided by the connections made in the video, there were no pre-prepared interview questions.

The discussions will be presented in the same order as the lesson analysis.

4.3.1 Discussions with Teacher E

Teacher E is a 29 year old female who graduated from the University of Namibia (UNAM) with a Bachelor's degree in Education. She specialised in both Mathematics and Physical Science. She has 4 years' teaching experience of Mathematics at both Core and Extended levels. The two lessons analysed were: Factorising and Solving Simultaneous equations.
Teacher E made 5 I**OCs**, 2 each of **I** and **P** and 1 each of **DR** and **PWR** in the first lesson. In the second lesson 8 of **DR**, 4 of **I**, 6 of **P** and no **IOC** or **P** were made.

Lesson 1

Teacher E started by identifying a **DR** connection by comparing x^2+5x+6 to $ax^2 + bx + c$. She agreed that x^2+5x+6 was another way of writing $ax^2 + bx + c$. Even though I don't think this identification is wrong, I had earlier identified it as **PWR** since the specific expression given was simply an example of a bigger class of quadratic expressions.

I then referred her to something she said earlier in the video: *'you can have $ax^2 + bx + c$ or $ax^2 + bx - c$".* I asked her to explain the significance of mentioning that. She then replied: *"the positive sign in front of c and a negative sign in front of c? Ok maybe I was supposed to relate the two and maybe give examples".* We agreed that had she gone further and maybe shown the difference between the two factors for each type of expression, she would have made a connection.

The teacher then identified an **IOC** when she asked the learners if they could still remember the general expression $ax^2 + bx + c$. She also identified **IOC** from the prior knowledge of knowing what coefficients are. The next was **P** and then **IOC** for prior knowledge of

multiples and factors. She identified *5x* as alternate representation of *2x* and *3x*. After she wrote *5x* as *3x+2x*, she said: "*now you can divide your expression into two and factorise"*. I asked her what she meant by dividing the fraction. *"Yes I think the word divide brings in confusion, maybe I should have used group since we are actually factorising by grouping".*
In the next example $2x^2 - 3x+1$, she told the class that when factoring: "*both numbers should be negative".* I asked her if she thought her learners knew why both numbers should be negative. *"Yes because as directed numbers, we dealt with adding and subtracting negative numbers and I already taught them that".* I referred her to my previous question of the difference between the two expressions: ax^2+bx+c and ax^2+bx-c. We agreed that if she had compared the two factors of when c is positive and when c is negative, she would have referred to the c being positive and b being negative.

For the last interval of the lesson, with expression $2x^2+14x+20$, the teacher identified an **IOC** from the following statement: *"if you are given an expression and you can take out a common factor, then you should do that".*

Lesson 2

As we continued to view the second lesson, we came across the following statement: *"and because the coefficient of x is one, I will make x the subject of the formula".* She realised that that was a connection but was not quite sure which connection it was. I asked her why she chose to make x the subject of the formula because the coefficient is one, I asked her if it is easier, *"Yes it is easier when you are using the substitution method because you are avoiding the fractions".* She explained: *"we did it previously at the beginning of the chapter that use substitution [method] only when your coefficient is one".* We agreed that this should be the **I** connection.

The next connection identified was the **DR** where the teacher told the learners to use any method to solve. She thought that since she did not dictate to the class what method to use, learners would end up using a method they are mostly comfortable with. She also identified the next connection to be **DR**, *"when we make y the subject of the formula and replace it with 2 where there is y".* In the next step, we agreed that she came short of using the I connection when she told the class: *"If I want to eliminate x I have to multiply this equation by 2 and this one by 1 so that I have the same coefficient".* If she asked them to give out the numbers that can be multiplied to the two equations to make the coefficients the same, she would have made a connection. She also identified an **I** connection from the conclusion that multiply the equation by one will not change that equation.

In the next interval, referring to this statement that she made in the class, *"people know how to use directed numbers"* she correctly identified as **IOC** as she referred to their prior knowledge.

As we reached the end of the viewing, I asked her to reflect critically on the connections she uses in her teaching and how often. Her honest response was: *"I think I use mostly multiple representations and 90% of this was procedural and I think I need to focus mostly on the last one,* ***IOC****. I don't really understand it"*. After I explained it again to her, she realised that she actually made those connections since her teaching always follows a tangible hierarchy. She added: *"I think I need to know more of my learners' way of understanding as well as knowing how they will make sense of the questions".* In her conclusion, Teacher E admitted that she had learnt a lot about connections and would think about how to use them in future: *"now I have learnt a lot, because I did not know about these types of connections and I will definitely be using them more in the future. I think I used more of teacher centred than learner centred. But with this class one has no choice because they want you to do everything"*.

4.3.2 Discussions with Teacher T

Teacher T is a 34 years old male who graduated from the University of Namibia. He specialises in Mathematics and Physical Science Grades 8-12. He has been teaching at Grade 11 -12 levels for over 8 years. Mr T has been teaching Mathematics at both Core and Extended levels from the start.

The two lessons analysed were both about algebraic fractions. The first one was the introduction of algebraic fractions while the second one was continuation, even though not to the same class.

Since this was the last analysis I did, teacher T admitted that the workshop was done a very long time ago and he did not fully recall what was discussed. I started off by refreshing teacher T with the observation instrument, explaining exactly what to look for. It was thus not easy for him to identify connections.

Lesson 1

The teacher started off by introducing the topic of algebra as a "*a discipline to do with numbers and letters",* so we identified it as his first **DR** connection. We again identified the next one as a **DR** as he referred to the tree outside of the classroom. We agreed on that as he used a physical object outside, which is an example of a multiple representation. Next, he used the letter to represent the number of leaves on that specific tree. He said he used that specific example since learners tend to see algebra as *"a thing that is very far from their lives"* and so he tried to show that it was actually part of their *"everyday lives"*. We categorised that as **DR**.

When I asked him which type of connection he used next, he replied: *"I am not sure but I am thinking that my next example that I used maybe should be some type of connection"?* As I alluded to earlier, Teacher T did not find it easy to identify the connections. I referred him back to what he said in the video: "*we have to use a variable for something that is impractical to achieve or unknown".* After we agreed that this was a generalisation, he then categorised it correctly as a **PWR**. I initially thought this was a **DR** connection but later agreed with him.

In his next example, Teacher T used another tree that is "*genetically engineered".* That is an **IOC**. I asked him if the learners knew what "*genetically engineered"* meant. He replied that it is concept from Life Sciences or Biology and that he assumed that since they did Life Science in Grade 10, they should know it. I asked him if he thought that using that term helped him to explain the concept of like and unlike terms. He answered that since most learners think that algebra is something distant from them, *"I decided to use something that is closer to them and just by looking through the window I saw a tree and so the idea came to me".*

In the next interval when the teacher referred to genetically engineered trees with the same number of leaves, we categorised it as an **IM** connection because the two trees are identical hence the number of leaves must be the same. He also categorised it as a **PWR** since it is a practical example. In the next statement, he guessed the number of leaves on the tree outside as 400 000, I then asked him why he used that number and he said: "…*since it was a big tree, it was not practical to say maybe 2000 leaves so I just used a number that I thought makes* *more sense".* His response implies logical reasoning, so we categorised it as an IM connection even though he admitted that he did not think at that time that it was a connection. As the lesson continued, teacher T used **P** connections as he showed procedures to simplifying fractions. He also referred to pre-requisite knowledge when he told the class that they should know the multiples of the numbers in the denominators. Since multiples were highlighted as pre-requisites to simplifying fractions, this was easily classified as an **IOC**. The next sentence also contained **IOC** of multiples and factors as he continued to simplify and asked them: *"to get 2a as my denominator, with what should I multiply to a"?* We identified **IM** connection as he asked whether they could add that number of leaves together. He said he wanted them to tell whether they were like or unlike terms, and whether they could be added together or not.

Next, with the fraction of $\frac{p}{3}$ - $\frac{p}{15}$, he asked: *"with what should you multiply to 3 to get 15"?* We agreed that he was looking for their knowledge of factors and multiples, which are pre-requisites of simplifying fractions. We agreed that it is **IOC.**

Lesson 2

In the second lesson, we started by identifying the first connection he made in his introduction. He identified it as **IOC** since he was linking new concepts to prior knowledge when he asked the learners to recall what was done previously which was adding fractions. Teacher T was not sure what his next connection was when one learner talked about cross-multiplication. Since he was clearing up misconceptions in the method of simplifying fractions, we thought it was just procedural connection **P**. He was of the opinion that some learners misuse the term cross-multiplication. Since I also experience this with my learners, we agreed that we should only use that term when we have two fractions with an equal sign between them. We also realised that learners still don't understand the difference between solving an equation and just simplifying a fraction.

The next connection was **IOC** as the teacher made a connection from the simpler fraction to the more complex example. I asked why he saw it as important to say that the next example of fractions would be complex, to that he said: *"I wanted them to know that not all algebraic fractions are in the form of* $\frac{1}{x} + \frac{1}{2x}$*, I wanted them to know right away that there will be easier* *fractions and complex ones[at their level]".* He also said it makes more sense if you start with easier examples.

He then tested their prior knowledge by asking whether they still knew how to expand; it was also identified as **IOC**. He asked if they could still recall like and unlike terms. He identified the connection of working out the procedures of simplifying fractions as **P**. I told him to look at the statement that he made and say whether it was a connection or not: *"the like terms of x, 2x and 3x gives you 5x".* He replied that it was a connection, after going through the instrument again, he came across an example of 3+2 being an equivalent representation of 5. Hence, he concluded that this should also mean *2x* and *3x* are equivalent to *5x*, then it is **DR**.

He identified a hierarchical connection as he referred to the previous example, **IOC**. He then used a **P** connection as he showed the procedures for simplifying fractions. When he asked one learner to justify how he got 16, knowing that 16 was a wrong answer, it was identified as **IM**.

In conclusion, teacher T said the concept of connections was new to him and had opened his eyes to the way he teaches. He realised that he always made use of connections in his teaching, especially **PWR** because he used specific examples. When asked about the use of connections in future, he admitted that he used mostly **DR** and **P** connections but would like to make more of **IM** and **IOC**. He also said he would *"think about connections"* when he prepared his lessons.

He appreciated the chance he was given to reflect on his lessons as it helped him to identify his weaknesses and strengths in teaching mathematics.

4.3.3 Discussions with Teacher O

Teacher O is a 31 years old male with just over 3 years teaching experience in mathematics. He graduated from the University of Belvedere in Zimbabwe. His speciality is Mathematics for Grades 11-12. Teacher O admitted from the beginning that he finds teaching Mathematics challenging.

Teacher O's two lessons that were observed and analysed dealt with solving quadratic equations by completing the squares and by the quadratic formula respectively. Since he only taught one Grade 11 class, I observed the same class but not on the same day.

After we had gone through our analysing instrument, we started identifying the connections made in the first lesson. Teacher O started his lesson by referring to what was done previously so we identified it as **IOC**. As he extended the previous method of solving to the new method, he realised that it was an extension of the arranged content, so it was still **IOC**. Even though Teacher O did not really say much during our discussions, he showed an understanding of the nature of connections that he used. The next connection identified was when he wrote the equation $ax^2+bx+c = 0$ on the board and referred it to it as the general equation for all the quadratic equations. Even though teacher O was aware that that was a connection, he couldn't pinpoint exactly which one it was as he said: *"I think that is a connection because you are connecting a general equation to all the quadratic equations but just not sure which one".* We then agreed that it would be **PWR**, because it was a generalisation.

He then referred to his use of integers in his explanation and identified it as a connection. It was categorised it as **IOC**. I then brought it to our attention that when he asked one learner to say what integers were, the child just said that integers are positive and negative numbers. The fact that they are whole numbers was not mentioned. After we viewed the clip again, as per his request, he realised it was true and said: *"I guess I just never really paid much attention to it because I just expected them to know it from Grade 8".*

Next we identified a **DR** after he said a variable of an equation is not always x but can be any number. I then suggested how it would have been better had he given some examples of quadratic equations with variables other than *x*. Then he replied: *"Og it is true, in order for them to get used to it. Because if you always use x the moment they see let me say* $t^2+5t+6=0$ *they will be confused at first. But I just thought at a time mentioning it was enough".*

After Teacher O gave $2x^2+7x+3=0$ as an example of a quadratic equation to be solved, he realised that it was a connection because it was a specific example of quadratic equations. It was a **PWR** connection. He also asked the learners why that could be a quadratic equation. We agreed that it is an **I** connection.

In the next segment, Teacher O used these exact words in his lesson: "*now we can connect the generalised formula to the defined one we are given and identify our integers"?* In our discussion he said it was actually funny since he used the word *"connect"*. So he concluded that it was a connection but was not sure which one. I then suggested that it should be **PWR** since it is a connection between the general and the specific. In the next identified connection, he again used the exact word *"connect"* and this was also classified as **PWR**.

In the last part of the lesson, Teacher O use **P** connections as he showed the procedures to solving equation $2x^2+7x+3=0$.

Lesson 2

As we continued to view the second lesson, I could tell that Teacher O had learnt a lot about connections from our viewing and analysis of lesson 1. I asked him if there were any connections in the first clip we viewed. He replied by: "*I think there is this one, when I told them that we have been doing two methods of solving quadratic equations. It is showing the recognition of prior knowledge".* We categorised it as **IOC**.

He identified another connection: "*there is this one also, at times there are quadratic equations that will never factorise no matter how hard you try, if it never factorise then you can opt for completing the square".* Again I asked him what connection he thought that was. After some thought, he replied: *"is it not this one, realising that there are many ways to solving a problem? So I think it is P"*. I replied that it was **P** since he was making the learners aware that there is more than one way of solving quadratic equations.

We identified $x^2-4x+4=0$ as a specific example, which is **PWR**. After he wrote $x^2-4x+4=0$ as a perfect square, $(x-2)^2=0$, as a **DR** type of connection. What followed were **P** connections as he was working out the steps to solving $(x-2)^2=0$. He identified the instance when he asked the learners to identify *b* from $x^2-4x+4=0$ as **DR**. We then agreed that it should be **PWR** since the connection was between a specific example and the general.

I referred him to the section where one learner asked why it was necessary to divide *b* by 2. He told this learner that he would find out why we have to divide by two. I referred him to the fact that the opposite of factorising is expanding. We worked out an expansion of $(x+b)^2$, and since the middle term from $(x+b)^2$ would be $2bx$, the reverse of this should then be $\frac{b}{2}$. He said he never really paid much attention to it. We then identified a **DR** connection as he compared an expanded form of equation to its factorised form.

We discussed how he could have made another connection if he had connected the equation to the curve and then the solution would be the x-values where the curve cuts the *x-axis.* After this he said: *"teaching is not easy. Ok now I know for next time. But maybe also because we did not do sketching and drawing of curves yet".*

In the next episodes, we identified mostly **P** connections as he solved the equation. Since the answers that they got in the class were wrong, I then asked if they worked it out again correctly. Since he said he could not remember, we worked it out together and realised that the answers that the learners were arriving at in the steps were wrong. So he said: *"and I did not have time to check whether the answers they were giving me are wrong, I just accepted them. Teaching mathematics is challenging".*

In conclusion, I asked him his impressions of connections in general and he said: *"Well, I think it is just the term and the grouping that is new because I have been making them in my teaching without really know exactly what type of connection they were. So now I am familiar with the types so I will think about them more in my teaching, especially when I prepare".*

4.4 CONSOLIDATION

4.4.1 The nature of connections – research question 1

The following table summarises the type and frequencies of connections made by the three teachers in their two lessons combined.

Table 4.5 Summary of total connections made

	Teacher E	**Teacher T**	**Teacher O**	**Total**	**% of Total**
DR	9	4	4	17	20.5
PWR	1	4	5	10	12.0
IM	6	7	3	16	19.3
P	8	5	7	20	24.1
IOC	5	10	5	20	24.1
Total	29	30	24	83	100

As evident in the above table, the highest number of connections made was 10 **IOC**s by teacher T, followed by 9 **DR** by teacher E and then again 8 **P** connections by Teacher E. The least number of connections were as follows: 1 **PWR** connection by Teacher E, 4 each of **DR** and **PWR** by Teacher T and then 3 **IM** connections by Teacher O. In the two lessons observed, Teacher T made the highest number of overall connections with 30, followed by Teacher E with 29 and then Teacher O with 24. It is interesting to note that Teacher T is the most experienced teacher with Teacher O the least with 3 years' experience. This leads one to conjecture that the use of connections is directly linked to teaching experience.

Overall the most prevalent connections made were the **P** and **IOC**. They were both used 24.1% of the time by all three teachers. This could be because teachers spent most of the time demonstrating the procedures to carrying out certain tasks as well as clearing up misconceptions in those procedures. Mhlolo (2012) stated that when **P** connections are made, teachers should make reference to efficient and accurate procedures for working out a mathematical task. Most of the teachers also acknowledged their learners prior knowledge. Kaput (1999) suggested one route to effective teaching is that teachers should build on students' prior knowledge as well as allowing them to reflect on it. All three teachers showed considerable evidence of this.

4.4.2 Teacher's perceptions about connections – research question 2

Even though all three teachers admitted that teaching mathematics can be challenging, they welcomed the notion that making connections might help alleviate that challenge. This was in agreement with Sawyer (2008) who is of the opinion that the teachers he interviewed thought that connectedness and making connections is fundamental to mathematics education. He continued to say that teachers believe that making connections equips learners to see how mathematics is related to everyday life. This could be seen from Teacher T's interview when he said he used an example of a tree outside because learners thought algebra was something remote.

Teacher E said she has learnt a lot as she did not know about these different types of connections and would definitely be using them in future. She felt that she used mostly **DR** connections as well as procedural **P** connections and realised that she needed to know more about **IOCs**. Teacher E was one of the teachers who did not dictate what methods learners should use when solving simultaneous equations. This she said was because she covered both methods and wanted to see which one the learners were comfortable with. That statement affirms what Cleave (2008) said about not dictating what method to use as it deepens the learners' understanding of concepts. After we discussed **IOC** again, she reaffirmed that she needed to know more about how her learners made sense of her questions as well as their way of understanding.

Teacher T said he appreciated the fact that he was given a chance to reflect on his own teaching practice. He said the concept of connections was new to him and helped open his eyes to the way he teaches. He said that he always uses connections when teaching, but did not know about the different types until he attended the workshop. He said he would definitely think about connections when preparing his lessons. Teacher T realised that he needed to make more of **IM** and **IOC**. **IOC** was however the most prevalent type of connection he made.

Teacher O admitted that the term connection was new to him as well as their different categories. He said he had been making connections in his teaching, without really knowing what type they were or even whether they were connections. Teacher O also said he would think more about connections, especially when he is preparing lessons. Teacher O realised that there were several occasions where he could have made more connections but fell short because he assumed that his learners already knew what he was talking about.

4.5 CONCLUSION

In this chapter, the collected data was analysed in accordance with Businskas' (2008) framework. The nature of the mathematical connections was presented for the three teachers as well as the frequencies of use. The three teachers' perceptions towards the use of mathematical connections were also analysed. All three teachers undertook to think more about connections when preparing as well as when teaching. It is interesting to note however, that often connections are made spontaneously without necessarily explicitly preparing for them. For example Teacher T made spontaneous reference to a tree that he saw outside through the window.

The next chapter summarises, concludes and attempts to provide some recommendations on the use of mathematical connections.

CHAPTER FIVE

SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.1 INTRODUCTION

This chapter gives a brief summary of the study, followed by a discussion of the limitations and challenges experienced in this research. It also provides some recommendations for further research as suggested by the findings. It ends with some personal reflections and a conclusion.

5.2 SUMMARY OF FINDINGS

A connection as defined by the Oxford English Dictionary is a causal or logical relationship. Businskas (2008) adopted a similar definition when she developed her framework for analysing connections.

As mentioned earlier, this study attempted to answer the following two questions:

- What type of connections do selected Grade 11 teachers make when teaching algebra with regard to Businskas' framework?
- What are the selected teachers' perceptions of making use of connections when teaching algebra?

The findings pertaining to the first question are summarised in Table 5.1 which shows the frequency of the different types of connections used by the participants in the observed lessons.

Table 5.1 Summary of total connections made

	Teacher E	**Teacher T**	**Teacher O**	**Total**	**Total %**
DR	9	4	4	17	20.5
PWR	1	4	5	10	12.0
IM	6	7	3	16	19.3
P	8	5	7	20	24.1
IOC	5	10	5	20	24.1
Total	29	30	24	83	100

I discovered that the most frequent connections made by the teachers in their two lessons were procedural (**P**) and instruction-oriented (**IOC**) respectively. This was mainly because teachers were demonstrating and carrying out the procedures on the chalkboard most of the time. Teachers also reminded the learners what mistakes to avoid when working with a certain procedure. For instance, Teacher T told his class that they should avoid adding the numerators together when adding fractions.

All three teachers referred to their learners' prior knowledge. This was either in the form of recall to the previous lesson or highlighting the pre-requisite knowledge in order to understand the new concepts. This was done, for instance by Teacher O when he asked the class what integers were when he introduced solving quadratic equations by the quadratic formula. Teacher E also referred to directed numbers when she was teaching how to solve simultaneous equations.

The next most frequently used connections were different representations (**DR**) and implications connections (**IM**) respectively. All three teachers occasionally used different representation in the observed lessons. The connections that they made in **DR** were mostly for equivalent representation (ER) which is part of a **DR** connection. For example Teacher T told the learners that $3x$ and $2x$ becomes $5x$, even though he did not use the phrase equivalent to, it was still categorised as an equivalent representation.

Part-whole relationships were used the least as compared with the others, with only 10 instances out of the 83 total connections made. Since this type of connection is about generalisations, in most cases teachers did not make any generalisations. Instead they made use of pure arithmetic examples and questions to arrive at the solution. Most questions given were taken directly from the textbook, so those questions or examples served as instances of PWR connections in their lessons.

In terms of the participating teachers' perceptions, they all said that teaching mathematics can at times be challenging and thus making connections might help in reducing the problems. All three teachers said taking part in this research had alerted them to a new concept of mathematical connections. They all promised to make more use of connections in future. They added that they would think of connections particularly when preparing their lessons just to make sure that they have made a significant number of connections.

Teacher E said she needed to use more IOCs as she realised that the majority of connections that she made were the P type. She also realised that she used mainly a teacher-centred approach in one of her lesson. She however justified it by saying that with that specific class the only way she could achieve something was if she controlled the lesson because that class did not like participating.

Teacher T saw this as an opportunity for him to reflect on his teaching practice. He shared why he made the connections he made because learners tend to fear algebra. He said making those connections made his learners feel that even if algebra contained letters and numbers, they should be able to come up with these letters and numbers as expressions or even equations. Teacher T was the one with the most generalisations in his two lessons.

Teacher O made the lowest number of connections. He is also the least experienced of the participating teachers. He said he had been making connections in his teaching without knowing what they were called. He missed some opportunities of making connections. One instance was when he told the class that the variable in quadratic equations is not always x, but ended up using only equations of the variable x. He acknowledged this and said he takes things for granted sometimes and assumes that the learners know what he is talking about.

5.3 LIMITATIONS

A limitation of this research is the very small sample size. The results of this research can therefore not be generalised. Another limitation is that the time frame was too short to test the responses and perceptions further.

5.4 CHALLENGES

A challenge that I encountered was the varied availability of teachers to work with in the afternoons or during weekends. The first term is a very busy and a short term for the teachers, so after I recorded the observations and analysed them, it was not easy to find mutually convenient times to get together for the planned workshop and interviews.

5.5 RECOMMENDATIONS FOR FURTHER RESEARCH

As the scope of this study was relatively small, and also the fact that this study was perhaps the first one of its kind done in Namibia, there is a need to continue with this research. Namibia is relatively limited when it comes to mathematics education research. I recommend more research should be carried out in this area, particularly in the teaching of algebra. It should be done on a larger scale, most likely in all the 14 regions of our country. For the above recommendation to come to life funds need to be made available.

I would recommend that we make use of the experienced teachers that we have in the system, to share with others what connections they make in their teaching. They are ideally placed to mentor younger teachers in becoming aware of the importance of explicitly using connections in their teaching. Hence, they should be used as facilitators and mentors to help the novice teachers.

Schools are advised to create a more conducive learning environment for their learners in order for learning to take place and for teachers to make connections. Teachers should make mathematics more real, use examples that are relevant and that make sense to their learners. For that to happen, teachers need to know their learners and think carefully about the connections they make whilst teaching. They should know their learners' preferred way of learning and should realise that every child learns differently and at different rates. These considerations are all part and parcel of making appropriate connections.

All teachers should harness the idea of reflecting on their practice, particularly on the appropriate use of connections. This means they should all be encouraged to reflect on their teaching practices, see what works and what does not work. Using the experience gained from this reflection, expertise should be shared with novice teachers. Conversely experienced teachers could also learn new ideas from novice teachers.

5.6 PERSONAL REFLECTIONS

When I registered for my master's course I was most afraid of the writing process of the research project. I dreaded the idea of having to write about one hundred pages. But I also knew from day one the area I wanted to research. This was due to my love for and experience with teaching algebra. I have taught algebra for many years and enjoyed learning it when I was at school, even when everyone in class hated it. I thought deeply about my research participants to make sure that I would get rich data from them. I enjoyed interacting with them and I learnt a lot from them. As a first time researcher I would say my whole research experience was a worthwhile and an enriching journey. I urge all interested researchers to carry out research in this new area of mathematics connections.

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APPENDICES

7.1 APPENDIX A: LESSON OBSERVATION SCHEDULES

Teacher E Lesson 1

Video E1 Factorising		Teacher Code: E		
Time	**Coding**	**Details of Connection**	**Comments**	
00:00	IOC	Asking the class what the homework was and giving the correction	Making an extension of what is already known	
	P	When factorising by difference between the squares, you find the square roots of the coefficients and for the index you divide	Procedure of carrying out factorisation a^2-144= $(a)^2 - (12)^2$	
	IMP	If one bracket has (a+12) then the other one will have what?	By implication, if one bracket has a plus sign, then the other will have a minus sign.	
03:00	IOC	If you are given an expression to factorise you either use common factor or use factorisation by grouping or use the difference between squares right? Then there is factorising by quadratic trinomials	The methods of factorising are sequenced or hierarchical	
	DR/PWR IOC	Given x^2+5x+6 to factorise, I can write this as ax^2+bx+c, Remember that expression? So a is the coefficient of x^2, b for x and c a constant	Different representation, Generalisation of quadratics Refers the learners to previous knowledge	

06:00	P IOC	Ok to factorise this, first you have to multiply the first number with the last, the coefficient of x^2 by the constant. From there look for the factors of six that can give you five when you add them	Mainly procedures to factorising quadratic trinomials Factors as pre-requisites for factorising	
	ER	We are no longer going to write 5x, instead we write 2x+3x	2x + 3x as equivalent representation of 5x.	
09:00	P IM IM IOC	When dividing X^2 by x we say its 2 minus one If –a =4, then a =-4 because you want a +a Factorise $2x^2$- 3x+1, factors of two that will give you -3 if you add them, so both must be negative since c is positive Factorise $2x^2$+14x+20, if you can see there is a common factor you can take it out. You know how to factorise the, the first method of common factor?	The procedure for dividing indices of the same base. By implication, if –a = 4,then a = -4. By implication, since c is positive and b is negative, then both factors must be negative. Recalling the first method of factorising	

Lesson 2

Video E2 Simultaneous equations			Teacher Code: E	
Time	**Coding**	**Details of Connection**	**Comments**	
00:00	DR/P IM DR/P ER	Solve the two equations, we have 3 methods of solving for c and d Teacher asks a learner, we use elimination how? We are solving for c and d, use either elimination or substitution. Now which one do you want to use? C= 4+1 which is equal to? 4+1 is 5	Realising that there are more than ways of solving Justification Multiple ways of solving 4+1 is equivalent to 5	
03:00	DR	If you want to use elimination then the coefficient of the letter you are eliminating should be the same	Differences in different methods	
06:00	IM DR/P DR/P	Any number multiplied by zero is what? So this is zero That's why I am giving you two methods, if you forgot how to use elimination then you can use substitution. You can choose the one you want to use You mean why didn't I use the second equation? Ok, use it and see if you get the same answer	Logical reasoning Not dictating what method to use Using multiple ways to solve	
09:00	DR/P	Use the method that you want to use, I am not saying use elimination or substitution but as long as you use solve the two equations Use the method of your choice	Realises that there are many ways to solving a problem	

12:00	IM/P P	And because I have the coefficient of x as one, I will make x the subject of formula So I have x=15-2y, so in the first equation I put 15-2y where there is x, nee?	Conclusions/ justification and chooses efficient procedures Procedures for substitution	
15:00	IM	So I have 2x =16+6, what is 16+6? So x is 11	Logical conclusion	

Teacher T Lesson 1

Video T1	Algebraic fractions		Teacher Code: T	
Time	**Coding**	**Details of Connection**	**Comments**	
00:00	DR MR	Algebra is a discipline that has to numbers and letters. Why don't we look at that tree outside, can you count the number of leaves If you count from this direction 1,2,3,4…	Different representations of Algebra An image as a representation	
03:00	MR/DR MR/PWR IOC IM/PWR PWR	We can use a letter to represent the number of leaves We have to use a variable for something that is impractical to achieve or unknown Another tree that is genetically engineered Genetically engineered which means the number of leaves will be the same as on the other tree because they are the same Let us say we have another tree, different species with different number of leaves, we cannot use x again because it is not like the other one	Algebraic symbolism Makes generalisation Borrows from other disciplines Drawing conclusions/ using practical examples Uses practical to move to the abstract of unlike terms	
06:00	PWR	Numbers called now variables X and y different species, different number of leaves so they are unlike terms	Practical example leading to algebraic fraction, sequencing	

09:00	IOC/P P	Adding and subtracting fractions, if the denominators are not the same, then we look for the lowest common multiples , which means you must know the multiples of these When making the denominators the same, what you do down, you do on top	Procedures for working with fractions, referring to prerequisites of understanding factions procedures for simplifying fractions	
12:00	IOC IM P	To get 2a as my denominator, with what should I multiply to a? Multiply by a? But I want to get 2a? Can we add these numbers of leaves together? Why not? Make sure the denominators are the same	Understanding how students make sense of the question Asks for justification Procedures for simplifying	
15:00	IOC IM	With what should I multiply to 3 and get 15? Can I add these together, are they like terms?	Pre-requisite knowledge of factors Justification	

Lesson 2

Video T2 Algebraic fractions Teacher Code: T				
Time	**Coding**	**Details of Connection**	**Comments**	
00:00	IOC	Continuing from where we stopped last time, So we remember adding fractions?	Linking of prior knowledge	
03:00	P IM	With what should I multiply this x to get 2x? *We cross multiply* We only cross multiply when there is an equal sign The reason we are looking for the LCM is so that we only add the numerators	Clears up misconception, learner to communicate his/her thoughts in a commonly acceptable language. Logical reasoning	

06:00	IOC	Ok, now the more complex one, you do the same Do we still know how to expand? What do we do? Ok do you still remember something about like and unlike terms? Is there something we can simplify here? Which ones are like terms?	Hierarchical Testing Prior knowledge Sequencing of prior knowledge	
09:00	AR P	The like terms of x,2x and 3x gives you 5x Making the denominators the same	Equivalent representations of 5x Procedures for simplifying	
12:00	IOC	Ok now, let us try a different one The previous example we multiplied to get lcd	hierarchical	
15:00	P IM	Which ones are like terms? what do we do? What is -25-9? Is it 16,-16 or -36? Are you sure?, how did you get that?	Procedures for simplifying Asks for justification	

Teacher O Lesson 1

Video O1 Solving quadratic equations by the quadratic formula Teacher Code: O				
Time	**Coding**	**Details of Connection**	**Comments**	
00:00	IOC	Last meeting we learnt how to solve quadratic equations by factorisation	Referring to previous knowledge	
06:00	IOC PWR IOC	As you can see there, it is still the quadratic equation resurfacing there from what we did at factorisation We are saying that for a quadratic equation, the general formula is $ax^2+bx+c=o$ A,b,c are integers, we all know what integers are right?, what are integers?	Referring to previous method of solving Generalisation of quadratic formula Referring to previously covered content of integers	

	MR/DR	The variable is not always x, it can be y or any other letter that the examiner used	Different simples of the variable/unknown	
09:00	IM/PWC PWC	Let us look at the following example x^2+7x+3, this is a quadratic equation. Why do we say it is a quadratic equation? Now we can connect this equation to the generalised form and identify our integers	Asks learners for justification and conclusion/The use of a specific example Connection between general and specific	
12:00	DR/PWR	I am trying to connect the generalised formula to the defined one we are given. We are equating those values to the letters and plug them in the formula	Makes generalisations	
15:00	P	We are now going to simplify what is in the square root,… work out the square root then divide	Procedures for working with square roots	
18:00	P	Because of these decimals, you are expected to round your answer to 3sf	Procedures for rounding	
21:00	P	Ok I think the problem is calculator skills, you must first work out what is up and then divide	Detecting wrong use of calculator	

Lesson 2

Video: 2	Solving by Completing the square		Teacher Code: O	
Time	**Coding**	**Details of Connection**	**Comments**	
00:00	P/IOC	We have been doing two methods of solving quadratic equations. The first one was by factorisation and the second was the formula	Hierarchical, links prior knowledge	
	P	I was saying we have two methods of solving quadratic equations, but at times you find that there are quadratic equations that will never factorise no matter how hard you try	Realises that there are many ways to solving	
	IM/P DR	If it will never factorise and you are not told to use the formula, then it is obvious you will only opt for the completing the square method This means you have to express that equation as a perfect square.	Knowing what method to use when as well as logical reasoning Equivalent representation $x^2 - 4x + 4 = (x-2)^2$	
03:00	PWR P DR	For example, $x^2 + 5x - 7 = 0$, If I want to solve for x, there are steps to follow, move 7 to the other side Next add your $\left(\frac{b}{2}\right)^2$ to both sides What is your b here?	Use of specific example Procedures in carrying out task $ax^2 + bx + c = 0$	
06:00	DR	Now I am going to factorise this and make it ,this part will give me my left if I expand it	Expanded and factorised form of	
09:00	DR	I have factorised what I have here, when I expand it will still bring me to here	Equivalent representation $x^2 - 4x + 4 = (x-2)^2$	
12:00	IM/P	And because of this 2 here, because of this index here, we are going to square root both side nee?	Indication of a squared value	

15:00	P	Give your answers to 3 significant figures or 2dp. If your values are correct , when you substitute them in the equation you should get zero	Procedures of writing final answers	
18:00	P	Yes that is what I am getting also, it means we must go back and see again	Procedures of testing answers	

7.2 APPENDIX B: INTERVIEW TRANSCRIBTIONS

Teacher E

Stimulated recall interviews with Teacher E

Me: Did you find anything?

TE: Uhh, we have multiple representations

Me: Which ones?

TE: x^2+5x+6 is related to ax^2+bx+c

Me: Ok, and then you said you can be given ax^2+bx+c or ax^2-bx+c, and then you just stopped there, you didn't go further. What is the difference between the two?

TE: Ok one you find the difference and one you find the sum I think, but mmhh I think something is wrong is here. I think it is supposed to be –c?

Me: Oh one is minus c and the other is plus c, Ok

TE: Ok the positive sign in front of c and the negative sign of c

Me: I thought you would go further with that,

TE: Yaa cos I was supposed to maybe relate the two and even give the examples of both positive and negative c, but I only gave one of a positive c.

Me: you only said it and then you stop. It would've been good for them to see that if there is a negative then one number is positive while the other is negative and if there is a positive both are positive or both negative

TE: Oh yaa

Me: Ok what else do we have here? here you were asking them if they remember that expression, which means you were referring to something done previously. So that is instruction oriented. It is good that you said that because if you just write it down

TE: Some will think they are seeing it for the first time

Me: Mmhh….

TE: There is also this one of a is a coefficient of x^2 and b is a coefficient of x

Me: Yes if they know what coefficients are, and they are supposed to know yes

TE: The next one I think is just procedural on how to factorise, find the factors of 6

Me: Yes, and there is also prior knowledge of what factors and multiples are

TE: Then there is this one, we are no longer going to write 5x but will write 2x and 3x. I think this is alternate representation of 5x.

Me: Yes, but there is something I wanted to ask, when you say we divide our expression into two and factorise.

TE: Yes, the word divide brings in some confusion I agree.

Me: Ok, maybe use grouping

TE: Yes because we are actually factorising by grouping.

Me: The next one, $2x^2 - 3x+1$

TE: Ok,

Me: Do you think learners understood when you said both numbers must be negative?

TE: Yes because at directed numbers we dealt with adding and subtracting negative numbers. I already explained that

Me: If you go back to your introductions of your constants, when *c* is negative and when *c* is positive, you should have brought that in. C is positive and b is negative

TE: Yes, uhhmmm,

Me: Just to highlight that, if c is positive but b is negative what does that mean about the two factors?

TE: Uhm it means both numbers must be negative.

Me: So this is implication then, *the factors are 2 and 1 so both numbers must be negative.*

Me: The next question was $2x^2+14x+20$, *"if you are given an expression and you can take out a common factor, then you should do that",* so that was?

TE: I think IOC

ME: Yes, and you can still factorise out this without dividing the two out, so let us try it $2x^2+14x+20=2x^2+1ox+4x+20=2x(x+5)+4(x+5)= (2x+4)(x+5)$. Can you see that the two answers are the same, it is only that here you have 3 terms multiplying each other while here you have only two terms.

TE: Uhm, ok.

ME: What do we have in this section?

TE: I think it is IOC, calls the learner by name.

Me: Yes ok.

Lesson 2

Me: " *and because the coefficient of x is one, I will make x the subject of the formula"*

TE: What connection is that now?

Me: I don't know, maybe it is implication. I don't know if it is easier when the coefficient is one.

TE: Yaa it is easier when you are using the substitution method because you are avoiding the fractions.

Me: Ok maybe you should have highlighted that relation between the coefficient of 1 and the subject of formula.

TE: It is because we did it previously at the beginning of the chapter, that use substitution only when your coefficient is one.

Me: Ok the next one is just procedural

Me: Ok so this will be multiple representations when you said "*solve using any method"*

TE: Yes because I am not saying use which method, so they use the one they are comfortable with.

TE: Ok here we have multiple representations

Me: Why is it multiple representations?

TE: When we make y the subject of the formula and substitute or replace it with 2 where there is y. We also have procedural.

Me: Ok we also have this one, '*if I want to eliminate x I have to multiply this equation by 2 and this one by 1 so that I have the same coefficient'. But* you could have asked them to tell you what to multiply with, and then it would be implication. *Multiplying by one will not change because I multiplied by one.*

TE: That would be implication

Me: The next one, "*people know how to use your directed numbers"*

TE: That is prior knowledge, so it is IOC.

Me: Yes the moment you say directed numbers then immediately they think of grade 9 where directed numbers are taught.

Me: Ok tell me, do you think you use connections in your teaching? Maybe, all the time? most of the time? and how?

TE: I think only multiple representations, and 90% of this is just procedural. And I think I need to focus mostly on the last one, IOC. I don't really understand.

Me: This one is about content hierarchy, about how you present your content, paying attention to what the learners know already before you introduce new concepts. For example factors and multiples are prerequisites of understanding fractions.

TE: Ok like, so this one has to do with hierarchy. Like you do this and then next you do this. I think I did some of those. I think I need to know more of my learners understanding as well as knowing how they will make sense of the questions.

Me: Yes even when you are setting a test, you need to think of how they will make sense of the questions.

Me: Ok procedural is also good but we need to be able to balance it with other forms. And just one last part, if for example you have 2x, avoid saying divide by two but rather ask what x will be. That way you are giving them room to think instead of just following routine.

TE: Yaa it is true, so instead of 4x I just say 4 times what will give me 8.

Me: On what method to use, it works only maybe in the examination once you have given them all possible methods to solve, but if you are only teaching them factorising by quadratic trinomials then that is what you will ask them to use.

TE: Yes and I also tell them when to use which method, because for example if you have 3 terms then you use quadratic trinomials and if you have four terms you use common factors or grouping.

Me: Yes, it is up to them to see which method to use

TE: Now I have learnt a lot, because I did not know about these types of connections and I will definitely be using them more in future and more correctly. I think I used more of teacher centred than learner centred. But I know with this class one has to use teacher centred cos they will just be looking to you.

Me: Teaching Maths is not easy, but it helps to go home and reflect on your teaching on what went well and what could be improved.

Teacher T

Stimulated recall interview with Teacher T

Me: Ok since know we are clear on what to look for, let us see what we will find.

TT: Ok here we go.

Me: The first thing that you said when introducing algebra as a discipline to do with numbers and letters, that was your first connection. So it is a DR because you are saying that algebra can be about both numbers and letters.

TT: mmhh.. Ok I see

ME: Let us see what is next?

TT: Maybe the use of a tree as an example?

Me: Yes, again that is DR because you are using an image of a physical tree that you are referring to.

Me: Ok what do we have next?

Me: In this case you mentioned the use of a letter to represent the number of leaves

TT: Yes because these learners see Algebra as a "thing" that is very far from their lives and so I was trying to make them see that Algebra is part of their lives

Me: Very good, that is a clear indication of a connection. So we categorise it as Dr, since it is a form of algebraic symbolism.

Me: Can you tell me what your next connection was?

TT: I am not sure but I am thinking that my next example that I used maybe should be some type of connection?

Me: Yaa, but before the example you said this: *we have to use a variable for something that is impractical to achieve or unknown,* in this case you were referring to the use of variables as generalisations in algebra. Can you check what connection is that?

TT: Mmhh ok, I think it is here, PWR, making generalisations.

Me: Ok, I actually thought it is DR but you are right it should be PWR

Me: And then now your next example of another tree that is genetically engineered should be IOC, since you are referring to another discipline of life science. But tell me, do you think they knew what *genetically engineered* meant?

TT: Genetically engineering is from biology or life science, so I just assumed that they must have done it in life science grade 10 and some of them are doing biology in grade 11 so I think they knew.

Me: So why do you think using that term helped you to explain the concept of like and unlike terms?

TT: Look, like i said earlier about these learners thinking that Algebra is something far from them, I decided to use something that is closer to them and just by looking through the window I saw a tree and so the idea came to me.

Me: Okay let us see what is next

Me: Again by using genetically engineering here, you explained that it means same number of leaves. I see IM connection there I think.

TT: But it is a practical example so it is PWR then?

Me: Yes that one too. Good let us move on

Me: I like this one, when you guessed the number of leaves to be around 400 000, why that number?

TT: Huh since it was a big tree, it was not practical to say maybe 2000 leaves so I just used a number that I thought makes more sense.

Me: I will say that is an IM connection because you did not just guess any number but thought about it logically.

TT: Oh ok, I did not think that was a connection

Me: I would say it was since you are referring to number of leaves and also practical number of leaves for a big tree. Yaa but it is interesting.

TT: In this case maybe it was just procedures of how to simplify fractions?

Me: Yes but there is also pre-requisite knowledge of multiples and factors so there is also IOC.

TT: I think it is still procedures here.

Me: How about when you asked if we can add these number of leaves together and why?

TT: Is that not procedures?

Me: A bit but you asked why, which means you were looking for justification?

TT: Oh ya, so it is Implication? Because I wanted them to tell whether they are like terms or not.

ME: In the last part?

TT: Again I was testing for their knowledge of multiples and factors, which one is for pre-requisite knowledge?

Me: That would be IOC.

TT: But this lesson was short or what, was it the whole lesson like that?

Me: Yes, maybe it started late and also since the learners were busy working out you did not say much.

Lesson 2

Me: Any connection there?

TT: May be in my introduction when I asked what we did last year

Me: What connection would that be?

TT: Mmhh I don't know, maybe the one of linking knowledge or is it prior knowledge?

Me: Yes it is IOC, you asked if they still remember adding fractions that was done previously

TT: I am not sure what this next one is

Me: I would say P because you were clearing up misconceptions in the method of simplifying and cross multiplication?

TT: Oh these kids, they were all saying cross-multiplication. They are misusing that concept; they should know that we only use it when we have an equal sign.

Me: Yes and it is good that you told them that some people add denominators together, so there was an I connection also.

TT: Is it may be the reason for finding the LCM?

Me: Yes you said that the reason for looking for the lowest common multiple was so that you only add the numerators. You could've maybe said more but I still think it is an IM connection.

Me: Ok let us look to the next one

TT: I see testing for prior knowledge there, when I asked if they still know how to expand?

ME: Yes, and you continued to ask how do we expand. Sometimes we just ask if they know how to do something and don't really ask them if they remember how to do it and when you give them work to do, you will find they are struggling.

Me: It was interesting to note that in this case you told them that the next question or example will be more complex, why did you say that?

TT: You see, I wanted them to know that not all algebraic fractions are in the form of $\frac{1}{x}+\frac{1}{2x}$, I wanted them to know right away that there will be easier fractions and complex ones.

Me: That is good, then that will be an indication of Hierarchical nature of connections, which is under IOC. Do you remember hierarchical or sequencing?, the way your content is arranged?

TT: Ok and it just makes sense to start with the easier ones and gradually go into complex ones.

ME: Let us look at this one, the like terms of *x, 2x* and *3x* gives you *5x.* Do you think that is a connection?

TE: I think maybe yes, because if I look at this instrument here you wrote that 3+2 is equivalent to 5 so it must be ER also.

ME: We can say *2x+3x* is equivalent to *5x*

ME: Is that one hierarchical also?

TT: Yes cos I referred to the previous and then the next example

ME: So it is again the sequencing of the examples

ME: What do we see in the last clip?

TT: It was just simplifying the numerator, so there is something about like and unlike terms.

ME: So we can say it is P connection, you were just showing the procedures involved. But maybe there is something else in the last part when you asked what -25-9 will give.

TT: Is that still not P?

ME: It is P but when you continued to ask if he was sure that the answer is 16, you asked him how he came to 16, can we say that you were looking for justification of the answer being 16 even though you know it was wrong?

TT: Oh ok, so it is mmmhh..what is it, implication?

ME: Yes, I would think so. But we must also remember that all the examples of algebraic fractions that you give are actually PWR connections, so every time you give a specific example you are making that connection.

TT: It is quite interesting.

ME: Now tell me, what do you think about these connections that we spoke about today?

TT: Like I said it is a new concept to me and something very interesting. It has opened my eyes to the way I teach, more especially what I say when I teach.

ME: Now that you are familiar with the concept, how often do you make these connections in your lessons?

TT: Well, I said it is a new concept but it is just a way of looking at it that is new, I always teach like that so I can say I always make connections. May be not all of them but I do. And you just said every specific example I give is a connection, so I give a lot of those.

ME: Let's see, which ones have you used most do you think?

TT: I have seen a lot of DR and P connections but I would like to use more of IM and IOCs.

ME: What are you going to do differently from now on, with regard to your teaching I mean?

TT: I think I am not doing so badly when it comes to these connections but like I said before I would like to use more of the others. But I am going to think about connections when I prepare my lessons in future because I know it helps when you visualise them in the lesson.

Me: Yes and sometimes it just happens, without even you knowing that that is a connection. Only in cases like these when you look back and reflect on your lesson you realise that it was a connection.

TT: And it is very important for one to reflect always, it helps you to identify your weak and strong areas.

Teacher O

Stimulated recall interview with Teacher O

ME: Let us look at your first connection; when you referred to what you did last time, you are about to extend on what they know already. So that would be IOC?

TO: Yaa I think so.

ME: Ok let's look to the next connection.

TO: Maybe this one, I want us to look at the next possible method of solving quadratics equations, which is the quadratic formula.

ME: Yes, this is the continuation of the last connection. So you are referring to what was done previously so that you build on to introduce the new concept.

TO: So that would still be IOC?

ME: Yes, it shows a hierarchy of concepts.

TO: So next I think is this one, it is still the general quadratic equations form resurfacing there from what we have done on factorisation.

ME: Yes, again you are showing that there is connection between what was done already to what is being covered now.

TO: So it is IOC still.

ME: Then there is this one; we are saying that for a quadratic equation, the general formula is $ax^2+bx+c = 0$. Just up to this part. What do you think that it is?

TO: I think that is a connection because you are connecting a general equation to all the quadratic equations but just no sure which one

ME: This is a clear case of PWR since you are making use of a generalisation of quadratic equations.

TO: Ok, and the next part is a connection I think, when I referred to a, b and c as integers and then asked what integers are.

ME: Yes, you have recognised their prior knowledge of integers. Again it is IOC. But just one thing, did you notice that when the boy gave a definition of what integers are he did not mention the fact that they are whole numbers.

TO: Really? Let us view it again.

TO: I guess I just never really paid much attention to it because i just expected them to know it from grade 8.

ME: Ok let us move to the next one. Here you talked about the variable not always being *x*, and that it can be y or any other letter the examiner decides to use. That is DR because you mean even if it is represented differently it still means it is a quadratic equation. But you know what I thought you would say or do after that?

TO: Mmhh…what?

Me: I just thought, if you are telling them that a variable can be any number then why don't you give them examples of equations with different variables. Then they visualise it and get used to it.

TO: Og it is true, in order for them to get used to it. Because if you always give x the moment they see let me say $t^2+5t+6=0$ they will be confused at first. But i just thought at a time mentioning it was enough.

Me: So next you asked them to look at the quadratic equation $2x^2+7x+3=0$, and then you asked why that is a quadratic equation. I see two connections in that statement.

TO: There is one of a specific example of a quadratic equation?

Me: Yes so that will be a case of PWR. We can also say that the question why it is a quadratic is making an I connection because you are asking them what makes them imply that it is a quadratic equation.

TO: Ok, I see now

Me: What about when you said this: *now we can connect the generalised formula to the defined one we are given and identify our integers?*

TO: It is funny because the exact word of connecting was used. Which connection is that?

ME: Yes It is actually interesting that you used the word connect. That would be PWR because you are making a connection between the general and the specific.

TO: And the word connect is appearing again here. So it is still PWR.

ME: Yes but you made another one, by saying they should equate the values to the letters and plug them in the formula.

TO: The next one was just procedural I think.

Me: Yes because I think you were just showing the steps to solving that specific equation.

Lesson 2

ME: What connections were there?

TO: I think there is this one, we have been doing two methods of solving quadratic equations. It is showing the recognition of prior knowledge.

ME: Yes, that would be IOC.

TO: There is this one also, at times there are quadratic equations that will never factorise no matter how hard you try, if it never factorise then you can opt for completing the square.

ME: What connection would that be?

TO: mmhh.. is it not this one, realising that there are many ways to solving a problem? So I think it is P.

ME: Yes, it is P because you are letting them know that there will be situations where factoring will not work, so they must think of another method. Good.

ME: Ok, next we have this example, $x^2-4x+4=0$, so that is a specific example of a quadratic equation, so that will be PWR.

TO: Ok, what about this one? We are saying that if I have to express $x^2-4x+4=0$ as a perfect square, then I will get $(x-2)^2=0$.

ME: Yes, in this case you are showing the relation between your expanded example,

$x^2-4x+4=0$ and it factorised form, $(x-2)^2=0$. So this is equivalent relationship, it is DR.

TO: So this next example, $x^2+5x-7=0$ is also a PWR?

ME: Yes, because you are still using a specific example to explain a concept of completing the square.

TO: Yaaa

ME: This will be procedural because you are explaining the steps to solving by completing the square.

TO: Ok,

TO: I think this is a DR, you must add your b part of the equation to both sides, what is your b here?

ME: Maybe because you have one specific equation and you are comparing it to the generalised equation, $ax^2+bx+c=0$, then it should be PWR. But it would have been nice if you had written that equation next to your specific example.

TO: Yes because I just assumed that they have that image in their heads, you are right.

ME: This one should be P, since you were just doing the steps. But tell me about this question asked by one learner why we have to divide by 2?

TO: Yes, I told him I will find out the reason as I didn’t have it that moment.

ME: Ok the reason is, if you have to expand $(x+b)^2$ you will get $x^2+2bx+b^2$. So we can see that the middle term is the 2 times the b, so to reverse expansion we factorise. And so to reverse multiplying by 2 we divide by 2.

TO: Oh Sure?, Ok I never really paid much attention to it.

ME: Ok here you show the link between the factorised form and the expanded form. So this is DR. I just wanted to ask what you meant by saying both the right side and the left side are now perfect squares?

TO: Right, I know that was maybe not correct to say because I meant only the part that is added to both sides, (25/4) is a square number.

ME: Ok

ME: What connections did we get from that?

TO: mmmh...let me see

ME: I think this can be a connection, because of index 2 we will square root.

TO: So it is implication?

ME: Yes, maybe it can also be IOC since you referred to indices.

ME: I think this was supposed to be a nice connection if you referred the solutions of the quadratic equation to the values of x where the curve will cut the x axis. Because if you sketch the curve x^2+5x-7, it will cross the x-axis at those points. That would have been alternate representation, so it would be DR. Instead you said you will have two answers with one positive and one negative.

TO: Uuhh, teaching is not easy. Ok now I know for next time. But maybe also because we did not do sketching and drawing of curves yet.

ME: What connections are there?

TO: May be just the procedures of solving

ME: Yaa, and telling them to only round off at the last answer. Most candidates loose marks because they round off too soon

TO: ..and when they come to the last answer, it will be wrong.

ME: ..and telling them to substitute their answers also, to check whether they are getting zero. But the whole class did not get zero?

TO: Yes I think they values were wrong so we corrected them the next day

ME: Let us just go through to see where it was wrong; the value from the square root was not correct. They did not add their fraction properly.

TO: And I did not have time to check whether the answers they were giving me are wrong, I just accepted them. Teaching Mathematics is challenging.

ME: Ok so now what is your general impression to making connections?

TO: Well, I think it is just the term and the grouping that is new because I have been making them in my teaching without really know exactly what type of connection they were. So now I am familiar with the types so I will think about them more in my teaching, especially when I prepare.

7.3 APPENDIX C: LETTERS OF CONSENT

To whom it may concern

I,……………………………………………, in my capacity as Principal of ………………………………hereby give written consent for Ester N Kanyanda in her capacity as a masters student at Rhodes University , to conduct research at our school. Both parties understand that this consent can be revoked at any time if they so wish.

Signed:……………………………………………

Date: 03. 02. 2014

MINISTRY OF EDUCATION

03 FEB 2014

[illegible]

Tel: [illegible]

OSHIKOTO REGION

To whom it may concern

I,..., in my capacity as inspector of education for Oshivelo circuit hereby give written consent for Ester N Kanyanda in her capacity as a masters student at Rhodes University , to conduct research at the above mentioned schools.

Signed: ....

Date: 24/01/2014

MINISTRY OF EDUCATION
OSHIVELO CIRCUIT OSHIKOTO REGION
2014 -01- 24
TEL / FAX 067 - 230084
P.O Box 9023 - OSHIVELO
REPUBLIC OF NAMIBIA

To whom it may concern

I ,... in my capacity as Teacher at ....School, hereby accept to participate in this research. Both parties understand that this consent can be revoked without explanation at any time.

Signed:...

Date: 27.02.2014

RHODES UNIVERSITY

Enq: Ms Kanyanda

0812878690

The Principal

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RE: PERMISION TO CONDUCT RESEARCH AT SCHOOL

I am currently a student with Rhodes University, doing my masters in Mathematics Education. As part of my studies, I am required to carry out a research study in teacher practice and I chose your school to be part of this wonderful study. My supervisor is Professor Marc Schafer from Rhodes University.

My research is about the nature of Mathematics connections used by selected Grade 11 teachers, when teaching Algebra. I will thus need your permission to use one of your teachers (as part of my study. My data collection methods will involve the following: observing and video recording of two lessons per teacher, one workshop and one interview. These will cause minimal disruption to the school and I will carefully negotiate convenient times with the teacher. I have already sought permission by the teacher concerned.

I guarantee that your school and the participating teacher will remain anonymous. I will not reveal anything of a personal or comprising nature. I also undertake to provide you with a copy of the final thesis if you so wish.

I would be grateful if you granted me permission to conduct this research in your school by signing the form below.

Thank you

Ester N Kanyanda

RHODES UNIVERSITY

Grahamstown • South Africa

Enq: Ms Kanyanda

0812878690

....

....

'Making connections when teaching Algebra'

Dear Colleagues,

I am currently pursuing my Master's degree with Rhodes University and for that I am carrying out a study/research on teacher practice. I am going to investigate the nature of Mathematics connections that teachers make when teaching Algebra. The study will be based in three Grade 11 mathematics lessons. This research aims to inform the practices of the participating teachers and provide some useful insights. My supervisor is Professor Marc Schafer from Rhodes University.

I would be very grateful if you agree to participate in this research study by signing the attached form. I wish to observe and record 2 lessons where you teach grade 11 Algebra. After the lessons observed, I would like you to participate in a workshop (at a mutually convenient time) where we will discuss and explore what we mean by making connections. We will then together analyse the video clips of your teaching and identify the various connections that you make (either consciously or not). This is not an evaluation of your lesson, but a description of your practice in terms of the connections that you employ. I guarantee total confidentiality and promise not to reveal your name or that of your school in the thesis. I also undertake to provide you with a copy of the final thesis if you so wish.

Thank you,

EKanyanda

Ester Kanyanda

__To whom it may concern__

I .... in my capacity as Teacher at ....School, hereby accept to participate in this research. Both parties understand that this consent can be revoked without explanation at any time.

Signed:...

Date:....26/02/2014......

To whom it may concern

I,.., in my capacity as Principal of ....hereby give written consent for Ester N Kanyanda in her capacity as a masters student at Rhodes University , to conduct research at our school. Both parties understand that this consent can be revoked at any time if they so wish.

Signed: ....

Date: 26|02|2014

<u>To whom it may concern</u>

I .... in my capacity as Teacher at ....School, hereby accept to participate in this research. Both parties understand that this consent can be revoked without explanation at any time.

Signed:...

Date: 25. 02. 2014