Observing and evaluating creative mathematical reasoning through selected VITALmaths video clips and collaborative argumentation

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ABSTRACT

Creative mathematical reasoning is a definition that the NCS policies allude to when they indicate the necessity for students to, "identify and solve problems and make decisions using critical and creative thinking."(NCS, 2011: 9). Silver (1997) and Lithner (2008) focus on creativity of reasoning in terms of the flexibility, fluency and novelty in which one approaches a mathematical problem. Learners who can creatively select appropriate strategies that are mathematically founded, and justify their answers use creative mathematical reasoning.

This research uses Visual Technology for the Autonomous Learning of Mathematics (VITALmaths) video clips that pose mathematics problems to stimulate articulated reasoning among small multi-age, multi-ability Grade 9 peer groups. Using VITALmaths clips that pose visual and open-ended task, set the stage for collaborative argumentation between peers.

This study observes creative mathematical reasoning in two ways: Firstly by observing the interaction between peers in the *process* of arriving at an answer, and secondly by examining the end *product* of the peer group's justification of their solution. (Ball & Bass, 2003)

Six grade 8 and 9 learners from no-fee public schools in the township of Grahamstown, South Africa were selected for this case study. Participants were a mixed ability, mixed gendered, sample group from an after-school programme which focused on creating a space for autonomous learning. The six participants were split into two groups and audio and video recorded as they solved selected VITALmaths tasks and presented their evidence and solutions to the tasks.

Audio and video recordings and written work were used to translate, transcribe, and code participant interactions according to a framework adapted from Krummheuer (2007) and Lithner (2008) and Silver (1997) and Toulmin (1954). This constituted the analysis of the *process* of creative mathematical reasoning.

Group presentations of evidence and solutions to the VITALmaths tasks, were used in conjunction with an evaluation framework adapted from Lithner (2008) and Campos (2010). This was the *product* analysis of creative mathematical reasoning.

This research found that there was significant evidence of creative mathematical reasoning in the *process* and *product* evaluation of group interactions and solutions. Process analysis showed that participants were very active, engaged, and creative in their participation, but struggled to integrate and implement ideas cohesively. Product analysis similarly showed that depth and concentration of strategies implemented are key to correct and exhaustive mathematically grounded solutions.

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DECLARATION OF ORIGINALITY

I, Matthew Kellen (Student number 14K7255) declare that this thesis entitled: “Observing and evaluating creative mathematical reasoning through selected VITALmaths video clips and collaborative argumentation”, is my own work and written in my own words. Where I have drawn upon the words or ideas of others, I have acknowledged the author/s by using the reference practices as set out by Rhodes University Education Guide to referencing.

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Matthew Kellen (Signature) Date

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CHAPTER ONE: INTRODUCTION AND CONTEXT

1.1 INTRODUCTION

"Knowledge emerges only through the invention and re-invention, through the restless, impatient, continuing hopeful inquiry human beings pursue in the world, with the world, and with each other."

Paulo Freire (1970 p.72)

The words of Paulo Freire (1970) poetically emphasize the critical importance of inquiry and pursuit of understanding to gain knowledge. One of the challenges of teaching and learning is being able to engage learners in a way that encourages this hopeful inquiry about the world. Guiding learners in the process of inquiry in a way that leads to knowledge supported by sound reasoning is no simple task. There are many terms used to describe this inquiry Freire (1970) speaks of, such as critical thinking, problem solving, reasoning, but these terms are often ill-defined or clichéd. As a researcher, further questions develop around what hopeful inquiry is? How can it be observed to ensure learners are pursuing understanding in a meaningful way so that their ideas of the world are sound and valid?

This case study attempts to answer some of these questions within the context of mathematics education. Piaget (1969) suggests that the fundamental knowledge for mathematical understanding is logic and reasoning, and that they should not be separated. Even though there is a clear connection between being able to solve problems logically, and the learning of mathematics, it is not found in most textbooks (Beida, Ji, Drwencke, Picard, 2014), nor is it expressly addressed in the South African National Curriculum Standards (2011). Theorists who engage with the notion of mathematical reasoning do not often agree on what it is or how it should be observed or evaluated (Yackel & Hanna, 2003).

For the purposes of this research, it became very important to find a definitive term to describe reasoning skills involved in solving problems. Creative mathematical reasoning is the most clearly defined term that best describes the self-driven inquiry involved in solving mathematical tasks. Lithner (2008) and Silver (1997) view creative mathematical reasoning as the ability to creatively select and implement strategies that justify mathematically grounded solutions. With this definitive term of creative mathematical reasoning, this research is able to explore how it can be observed and evaluated. With the right tools for observation and analysis, this research can attempt to clearly identify how mathematical learners inquire as their constructed understanding of the world of mathematics evolves.

Visual Technology for the Autonomous Learning of Mathematics (VITALmaths) video clips (www.vitalmaths.com) were developed and used to pose visual and open-ended tasks that allowed learners to use their creative mathematical reasoning. VITALmaths clips are short 3-minute stop-motion video clips that stimulate conversation and reasoning among small multi-age, multi-ability peer groups (Linneweber-Lammerskitten, Schafer & Samson 2010). For the purpose of this research the tasks only required fundamental mathematical knowledge and intellectual demand to be able to focus on the creative mathematical reasoning abilities rather than mathematical content. Cowley (2015) analyzed a similar population of learners as those in this case study, and focused her research on spatial reasoning abilities. The VITALmaths clip content focused on tasks that exercised spatial reasoning abilities specifically around the concepts of vertical, horizontal, similarity, perpendicular, degree of angles, similarity and symmetry.

Reasoning is difficult to observe in isolation, as it is primarily an internal process. This internal processes can be observed however through observing the way learners interact with mathematical tools, self-talk, through interviews, or through social interactions (Yackel & Hanna, 2003). This research focused on social interactions. The social interactions were observed through collaborative argumentation using tools from Toulmin (1964), and Krummheuer (2007). Collaborative argument is what Golanics & Nussbaum (2008) refer to as "a social process in which individuals work together to construct and critique arguments."

Argument can be seen as both a social *process* of debate and discussion as well as a *product* of propositions that support a final conclusion (Kuhn & Udell, 2003). Similarly, mathematical reasoning can also be viewed as an ends and a means worth analyzing (Ball & Bass, 2003). This research analyzed collaborative argument in terms of these two aspects of process and product.

1.2 CONTEXT

This case study was situated in the Eastern Cape, of South Africa, in the township of Grahamstown. Participants of the case study were selected learners from under-privileged no-fee public schools who attended the Inkululeko project, a small nonprofit organization with a focus on building autonomous learning (Torreano & Kellen, 2016). Learners were identified in grade 8 based on commitment to their education and development. Autonomous learning in this context are those who can learn with limited teacher input and resources and take ownership of their learning (Kamii, 1984; Chan, 2001). The aim of the Inkululeko project is to fill the educational gaps where school and home lives may not be able to. The after school project provides a safe space where learners can pursue their learning goals autonomously (Kellen et al. 2016). They do this by using a positive youth development approach that helps learners navigate the complex systems in place that sometimes work against their learning potential (Durlak et al., 2007). When learners entered Inkululeko in grade 8, they were given a grade 4 mathematiics exam. Not one learner passed the exam. This is an indicator of the educational debt that has accrued by the time they reached Grade 9 and 10. Educational debt is what Ladson-Billings (2006) refers to as the accumulation of lost educational opportunities over the years of a learners academic career. The challenge has been about identifying the missed mathematical concepts in a way develops confidence and independence of learners.

The Trends in Mathematics and Science Study (TIMSS) 2015, an international study, assessed grade 9 learners in 40 countries. While South Africa has shown improvements in mathematics scores in the last 8 years, it is still ranked 39th of 40 countries. Of the provinces in South Africa that participated, the Eastern Cape performed the worst with only 24% of learners in the assessment achieving a passing score (Reddy et al, 2016).

This is not to negate the progress that has been made over the past 8 years, but sets the stage for understanding the challenges learners face towards building a confidence in mathematics. There are a variety of factors that impact these low marks including access to resources, parents' level of education, and teacher qualifications (Reddy, et al, 2016, Mc Carath et al., 2013; SACMEQ, 2010).

Within this context, it is important to determine how, with limited access to quality education, does one intervene to best support learners in a system that is failing them. The focus of this research is on what reasoning skills educators can focus on that leads learners to using creative mathematical reasoning to solve mathematics problems.

1.3 OBJECTIVES OF THIS RESEARCH

The purpose of this research was to observe and evaluate creative mathematical reasoning of Grade 9 and 10 learners from underprivileged public schools in the Eastern Cape of South Africa participating in an after-school project that provides an educational space for autonomous learning. To do this, VITALmaths video clips and evaluation frameworks of the process and product of collaborative argument needed to be created, to observe and analyze creative mathematical reasoning.

The underlying goals of this research were to explore the following:

1. Develop VITALmaths clips, and through an interactive process create supplemental worksheets that support collaborative argumentation.
2. Develop and implement analysis tools that help gain insight into the creative mathematical reasoning of selected Grades 9 and 10 learners.

Given the above goals, this research aims to answer the following questions:

A. Do learners show creative mathematical reasoning abilities in interaction with peers (*process)?*
B. Do learners show creative mathematical reasoning abilities as they justify their claims (*product)?*

1.4 METHODOLOGY

This research was designed as a case study for several reasons. Firstly, the observation and analysis of creative mathematical reasoning was very detailed, and required a context specific interpretation of participant interactions (Yin, 2009). This case study allowed for a depth of knowledge with the privileged insight of the researcher who had over a year of experience working with the participants, thus creating a safe environment where participants could interact with each other without inhibition.

A mixed method approach was used which allowed for a unique vantage point to view what was occurring with interactions and solutions to the VITALmaths tasks. The analysis of creative reasoning required a significant amount of interpretation in being able to determine participants' reasoning and interactions. Understanding interactions and their impact on learning is complex (Sfard, 2001). Definitive observable indicators were identified and adapted from Krummheuer (2007), Lithner (2008) and Silver (1997) Toulmin (1954) which allowed for accuracy of quantitative measurement.

Qualitative measurement of the strategies employed to solve the tasks, and their novelty added depth to the analysis of creative mathematical reasoning. Creativity is not an ability that can be objectively observed. Qualitative measures were necessary to view the creativity of groups by subjective analysis of the depth of justifications and the uniqueness of strategies selected and implemented.

Two groups comprising of three participants of mixed gender and ability were video and audio recorded solving six VITALmaths tasks. They were given worksheets that prompted meta-cognitive questions that had them express their thinking processes while solving the tasks. When the groups completed the tasks, they were asked to present their findings. Learners explained, what the problem was, how they solved it, presented their evidence and justified their solutions.

To analyze *process*, the audio and video recordings and written work were translated, transcribed, and coded according to an evaluation framework. To analyze *product,* presentations of solutions were analyzed. Initially groups were evaluated for each VITALmaths task in a vertical analysis. A comparative horizontal analysis across all 6 VITALmaths tasks was undertaken.

1.5 LIMITATIONS

This case study was context specific and so there were limitations to consider in the research methods, design and analysis.

Firstly, language played a factor in the translation and interpretation of learner abilities. In pilot sessions prior to the intervention, learners preferred to have the clips presented in English rather than their home language of isiXhosa. Allowing participants to voice their thoughts in the language they understand is important for learners to gain greater understanding, (Enyedy, Rubel, Castellon, Mukhopadhyay, Shiuli, Esmonde & Secada, 2008). While the VITALmaths clips were presented in English, participants preferred to discuss the tasks in isiXhosa and then present their work in English. A significant amount of work was done by the researcher and his assistant from the after-school project to accurately translate learner interactions and their intended meanings.

The VITALmaths tasks required learners to use physical movement manipulation of objects, like wooden blocks, match sticks, marbles, and cut out angles. While these movements were referenced in the transcription of audio and video recordings, they were not explicitly analyzed.

Due to the depth of analysis done across six VITALmaths tasks, a small sample size was selected. This limited the opportunity to generalize findings on a larger scale.

1.6 THESIS OVERVIEW

1.6.1 Chapter Two Literature Review

Chapter Two gives the foundation of research and theories that make a case for creative mathematical reasoning as a definitive term and clarifies its connection to Collaborative Argumentation. The chapter goes on to support the tools used to observe and evaluate the process and product of creative mathematical reasoning. To conclude the chapter, the theoretical foundation of Social Constructivism that frames this research is explained.

1.6.2 Chapter Three Methodology

Chapter Three identifies the research goals of this research, and the methodology of research practices of a case study with qualitative and quantitative data analysis. The design of the research is then explained, and the analysis and evaluation tools is clarified. To conclude the chapter, the reliability and validity of the research is confirmed.

1.6.3 Chapter Four Data Analysis Part A

Chapter Four is a detailed vertical analysis of the process and product evaluation of creative mathematical reasoning. Two groups of three participants were analyzed across six VITALmaths tasks.

1.6.4 Chapter Five Data Analysis Part B

Chapter Five was a horizontal analysis that analyzed trends across all six VITALmaths task for the two Groups. It compares trends across clips and between the two groups in the research.

1.6.5 Chapter Six Conclusion

Chapter Six shares the summarizes the most important findings of this research. The findings discuss the process evaluations of the to two participating groups and looks at the consistent trends in how the groups worked together to solve the tasks. The product evaluations of the two groups solutions to the tasks and how they gave insight into creative mathematical reasoning. Significant findings are then suggested. The limitations of the study then leads to suggestions for further research.

CHAPTER TWO: LITERATURE REVIEW

A CRITICAL REVIEW OF CREATIVE MATHEMATICAL REASONING AND HOW IT CAN BE OBSERVED THROUGH COLLABORATIVE ARGUMENTATION

2.1 INTRODUCTION

> "[Mathematics] helps to develop mental processes that enhance logical and critical thinking, accuracy and problem solving that will contribute in decision making. Mathematical problem solving enables us to understand the world (physical, social, and economic) around us, and, most of all, to teach us to think creatively."
>
> *Further Education and Training (FET) Phase Mathematics Curriculum and Assessment Policy Statement (CAPS) South Africa (2011 p.13)*

Logic, critical thinking, problem solving, creativity, reasoning; these are all very elusive words and concepts that the South African Curriculum Assessment Policy Statement (CAPS) uses to help define mathematics proficiency. What is creative thinking? What is mathematical problem solving? What is critical thinking? How can it be observed? How can it be evaluated? This critical review intends to craft a lens through which these nuanced words can be more clearly defined and observed. The definitive term this research uses to describe aspects of problem solving and critical thinking is creative mathematical reasoning. The tool for observing creative mathematical reasoning in this research is collaborative argumentation.

The term *reasoning* itself is a very nuanced and ubiquitous word. von Glasersfeld (1995) states that reasoning is the *process* through which someone learns. While most theorists, researchers and educators would concede to this simple definition, conflicting views arise from further inquiry into how we observe and evaluate the *process* of learning (Yackel &Hanna, 2003). How does one determine if the process of learning is occurring in the mind of a learner? An educator can only observe the reasoning of learners by what they say and do. If a learner struggles with the language of instruction, or has illegible handwriting, one must make considerable assumptions to evaluate a learner's reasoning. There are also many perspectives from which to interpret a learner's thoughts. These different interpretations of what is manifested on paper or in discourse reveals different insights into the reasoning that is occurring (Gellert, 2008).

This is particularly the case in the Eastern Cape of South Africa, where learners speak isiXhosa as a home language, while their textbooks are written in English. Learners are expected to write their answers in English, which is their second language. This is a challenge when their teachers also speak English as a second language, and must interpret a learner's English. Room for misinterpretation can abound.

By observing how learners reason, one can begin to see what a learner can understand (see Figure 2.1). If a learner's reasoning can be observed through how they solve problems with their peers, one has a lens through which to view a learner's mathematical understanding.

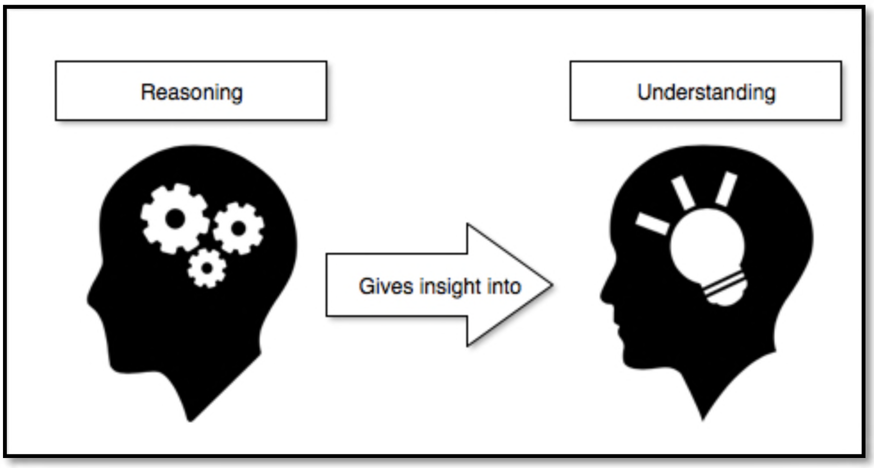


Figure 2.1 Reasoning gives insight into understanding

2.2 MATHEMATICAL UNDERSTANDING

With the broad notion of reasoning as the process of learning or understanding (von Glasersfeld, 1995), it is important within the context of mathematics education to further clarify what it means to understand mathematics. It is difficult to observe the process of mathematics without clearly identifying what should be seen as the result. (Lithner, 2008; Yackel & Hanna, 2003; Skemp, 1978). Skemp (1978) delineated two types of understanding in mathematics: instrumental and relational understanding. Understanding is the outcome of the process of reasoning, so if a learner has a different objective for understanding, the reasoning processes will be different.

Skemp (1978) states that the meaning of understanding in mathematics can often have two completely different outcomes. In one classroom, a teacher would be satisfied with the completion of a pen and paper exam at the end of a term as an indication of a learner's ability to "do maths". In another classroom, a teacher would be satisfied with a learner's ability based on his interactions with learners as they solve mathematical tasks. Both are important, but are indications of two entirely different types of understanding.

Instrumental understanding, according to Skemp (1978), is a mathematical understanding grounded in being able to follow algorithms and set procedures. He calls these "Rules without reasons." For instance, a learner is taught that 4 x 3 = 12. If every time a learner is required to multiply 4 x 3, they answer 12, they would have an instrumental understanding of 4 x 3. Instrumental understanding is the ability to compute without figuring out the why and the how. Kilpatrick, Swafford and Findell (2001) similarly refer to this understanding as procedural fluency. They define procedural fluency as "carrying out procedures flexibly, accurately, efficiently and appropriately." (Kilpatrick et. al., 2001 p. 27)

In contrast, relational understanding is knowing both what to do and why (Skemp, 1978). Using the same example of 4 x 3 = 12, a learner who knows that 4 x 3 is the same as creating 4 equal groups of 3, and knows that if you multiply 3 x 4 you will arrive at the same answer, has a relational understanding of the mechanics of multiplication, and its properties across a variety of contexts. Referred to as conceptual understanding, Kilpatrick et al. (2001) defines relational understanding as "comprehension of mathematical concepts, operations and relations." (Kilpatrick et. al., 2001 p. 27).

While both procedural fluency and conceptual understanding are important outcomes of understanding, it is important to recognize that the reasoning processes involved in arriving at this understanding are very different. (see Figure 2.2)

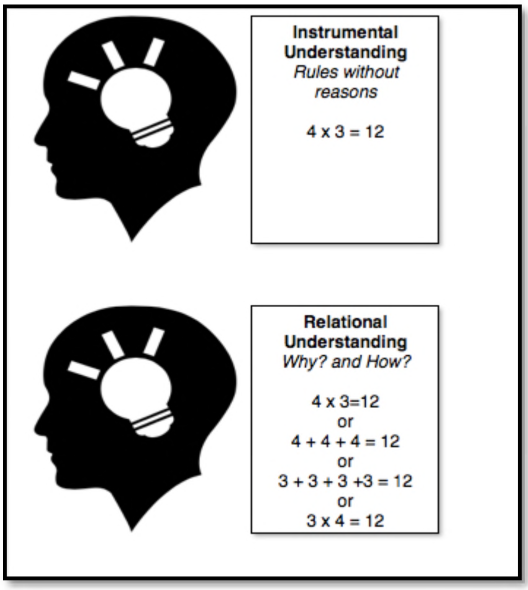


Figure 2.2 Delineating the difference between instrumental and relational understanding

Kilpatrick et al. (2001) suggested that along with procedural fluency and conceptual understanding, there are 3 other competencies required to be proficient in mathematics: adaptive reasoning, strategic competence and productive disposition. These 5 competencies or "strands" of proficiency are inter-related and work together like the strands of a rope woven together to increase its strength. This research is focused on how learners use adaptive reasoning to develop conceptual understanding.

2.3 MATHEMATICAL REASONING

Von Glasersfeld (1995) holds the view that reasoning is the process of learning. More clarity is necessary for the purpose of research in mathematics education. With an understanding of mathematical competency and delineation between procedural and conceptual understanding, one can look more directly at the process of reasoning in mathematics. Kilpatrick et al. (2001 p. 116) defines adaptive reasoning as a "capacity for logical thought, reflection, explanation, and justification."

A Mathematical Competency Resesarch Framework (MCRF) was developed by Lithner et al. (2010) and applied in Boesen et al. (2014) as a way to more clearly define the competencies described in Kilpatrick et al. (2001) for research purposes. The MCRF describes mathematical reasoning as "the explicit act of justifying choices and conclusions by mathematical arguments." (Lithner et al., 2010 p. 161). With this notion of reasoning as being able to justify decisions through mathematical arguments, this research can begin to develop a framework for what makes for sound reasoning. This can be done by focusing on what makes a sound mathematical argument. A sound mathematical argument can be evaluated by how it justifies an individual's interpretation/imagination, doing and using/concentration, and judgement/generalization processes used in a mathematical task. This definition is important because it lays out a means for evaluating mathematical reasoning. For the purposes of this research this definition is used to clearly state that by analyzing mathematical arguments, one can evaluate a learner's mathematical reasoning abilities.

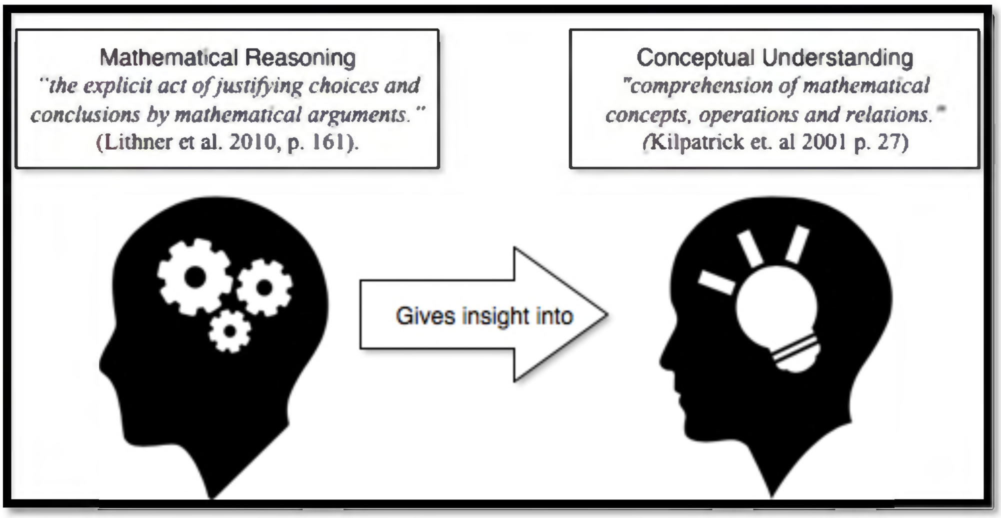


Figure 2.3 Creative mathematical reasoning gives insight into conceptual understanding

Further clarity is required however, because arguments and proofs also require further defining. Some would adhere to the idea that an argument is a strict logical sequence, or set of proofs. Others like Toulmin (1964) in philosophy, and Polya (1954) in mathematics, would state that the heuristics of plausible reasoning are just as valuable as strict logic. This suggests that like mathematical understanding, there are different ways of expressing mathematical reasoning.

2.4 IMITATIVE AND CREATIVE MATHEMATICAL REASONING IN MATHEMATICS

If Skemp (1976) and Kilpatrick et al. (2001) make the distinction between procedural fluency understanding, and conceptual understanding, it is also important to delineate between procedural reasoning and conceptual reasoning, because different outcomes require different processes for arriving at conclusions. A learner using reasoning that shows procedural understanding could justify their understanding by documenting a set algorithm learned in class to reach an answer. A learner required to justify their reasoning in a way that shows conceptual understanding requires a different set of observable indicators to determine if they have a sound understanding of the processes being employed, or is able to explain why they arrived at the answer they did (Yackel, 2001).

2.4.1 Imitative Reasoning for Procedural Fluency

The process of a learner being able to follow set procedures or algorithms to arrive at a solution is what Lithner (2008, 2010) refers to as *imitative reasoning*. In other words, one must be able to use imitative reasoning to gain instrumental/procedural understanding of a mathematical problem. Skills required in this process require a learner to a) memorize a set strategy or system, and b) be able to apply the strategy correctly given a new set of data. Lithner (2008, 2010) further distinguishes between two types of imitative reasoning. Firstly, memorized reasoning is merely the ability to memorize a proof. For example 4x3=12. A learner using memorized reasoning need only to write the answer 12 every time he sees the two factors 4 and 3 together in a number sentence. Secondly, algorithmic reasoning requires a learner to recall a sequence of rules, or an algorithm, to arrive at an answer. In our 4x3 example, a learner could show their ability by counting by 4s, because this is the rule they learned, 3 times.

2.4.2 Creative Mathematical Reasoning for a Conceptual Understanding

Lithner's (2008) notion on creative mathematical reasoning is based on Polya's (1954) notion of plausible reasoning which focuses not only on rigid mathematical proofs, but also on other ways of solving problems that are meaningful and sound. A learner who finds alternative ways to approach and implement strategies to solve a mathematical task, is using creative mathematical reasoning (Mann, 2006). Given our 4x3 example, a learner who can demonstrate an alternative approach to the task shows an indication of greater conceptual understanding. Observing that 4x3 is the same as 3x4 or that 4x3 is the same as 4x2 + 4 shows a greater conceptual or relational understanding of multiplication. While using imitative reasoning to achieve procedural fluency is important, observing the creative mathematical reasoning process reveals a depth of conceptual understanding (Sternberg, 2006).

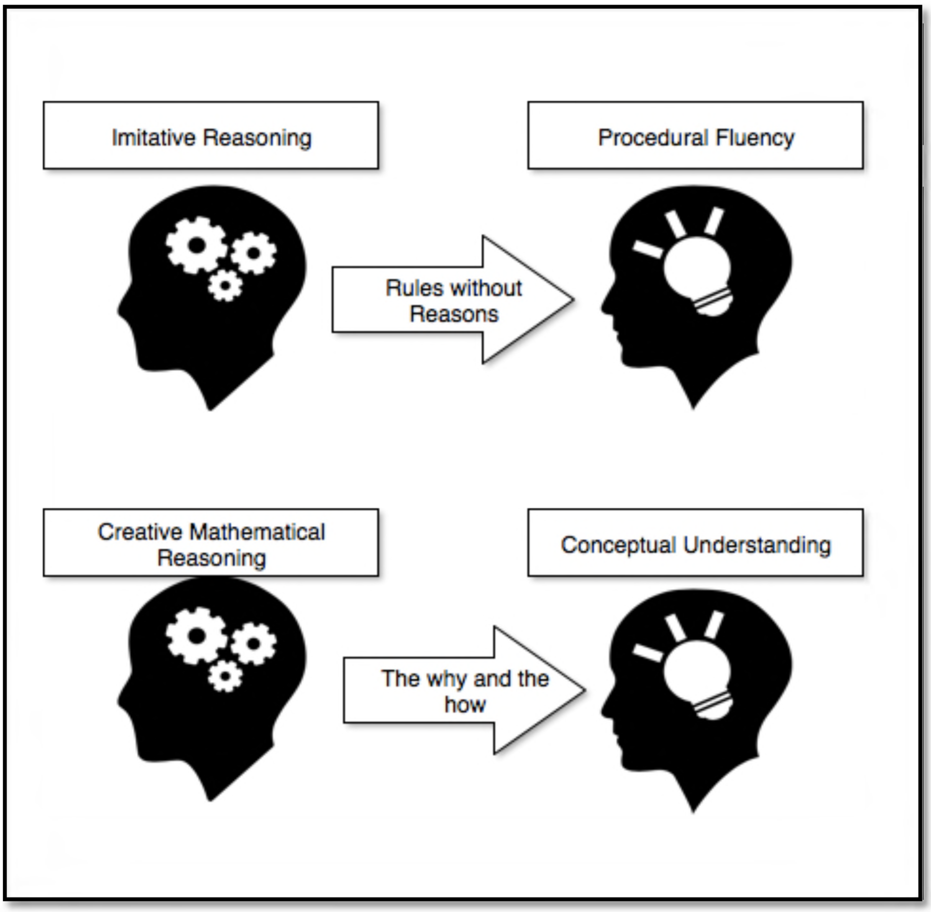


Figure 2.4 Delineating Imitative Reasoning and Creative Mathematical Reasoning

Defining creativity

Like the term reasoning, creativity is a buzzword that is not often well defined. It is often associated with the arts and people who think "outside the box." Simply stated, creativity is "the ability to make new things or think of new ideas." (Merriam-Webster Online). Within the context of mathematics education there are two different broad understandings of creativity (Yackel & Hanna, 2003). The classical view of creativity is that original, unique ideas come as a stroke of genius to a certain few (Mann, (2006). With this notion of creativity in mathematics, only a select few learners have a genius or gifted ability to do mathematics in unique ways, seemingly spontaneously.

In contrast to the genius view of creativity, Sternberg (2006) suggests a contemporary view of creativity that is accessible to all, not just a select few. The investment theory of creativity suggests that people who invest up-front time in metacognitive planning and conceptual understanding when approaching novel tasks, are more creative thinkers. In essence, creativity, or coming up with new ways of solving problems is an indicator of deep conceptual understanding and flexible knowledge about a subject (Yackel & Hanna, 2003). This notion is important to this research in that the depth of conceptual understanding of a learner can be evaluated by how creatively they solve problems (Balka, 1974; Silver 1997; Lithner 2000).

2.5 INDICATORS OF CREATIVE MATHEMATICAL REASONING

Balka (1974) as cited by Silver (1997) suggests that there are three key indicators of creativity in mathematical reasoning: *novelty, flexibility,* and *fluency.* Novelty is how unique or new the idea is to the learner. Fluency refers to the amount of different interpretations, strategies or solutions employed in *exploring and generating ideas* in solving a mathematical task. Flexibility is a learner's ability to *justify* their approaches or strategies and their solutions in multiple ways.

Campos (2010), in his reflection on Charles S. Pierce (1898), viewed his notion of mathematical reasoning similarly to Lithner (2010) and Silver (1997). The abilities of imagination, concentration and generalization according to Campos (2010) were critical to being able to solve mathematical problems. Imagination, similar to creativity, is the ability to manipulate mathematical strategies and ideas to formulate hypothetical solutions. Concentration refers to the ability to use deductive reasoning to concentrate on important information and applying strategies to solve it. Generalization is the abductive process of taking the solution to a problem and generalizing it as a rule or principle that can be applied to other situations (Pierce, 1992).

Lithner (2008) also suggests that a learner's reasoning must also be grounded in *intrinsic mathematical properties,* and that there must be plausible arguments made to justify strategies and solutions to the mathematical task. While learners may be able to creatively solve a problem, arrive at a solution, and generalize the concept to similar situations, it is important for the purposes of mathematics education that their solutions are grounded in mathematical properties. Figure 2.5 illustrates how creative mathematical reasoning gives insight into conceptual understanding.

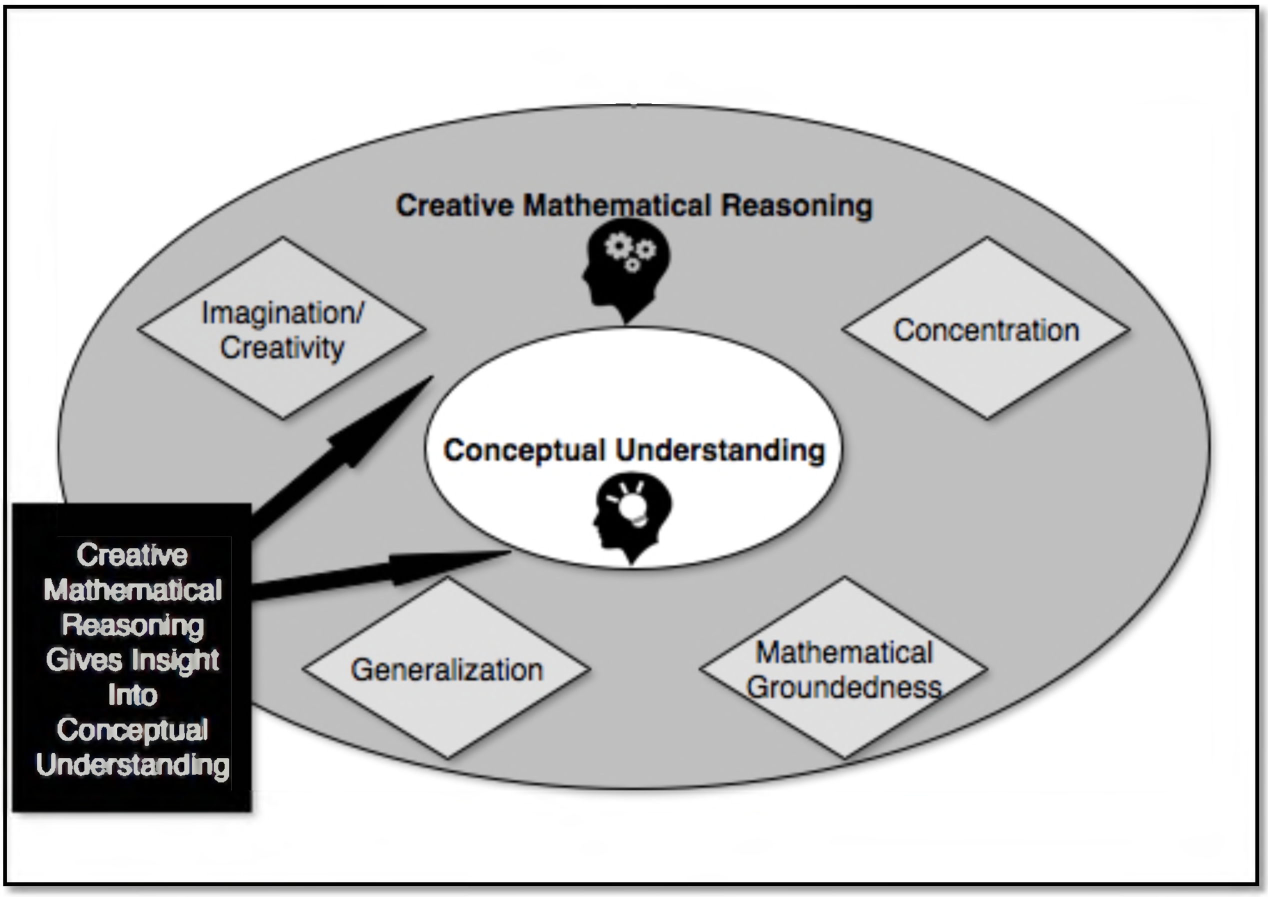


Figure 2.5 Observation of Creative Mathematical Reasoning gives insight into conceptual understanding (Adapted from the works of Lithner (2008) and Pierce (1992))

2.6 OBSERVING CREATIVE MATHEMATICAL REASONING

With mathematical reasoning defined as an ability to justify choices and conclusions by mathematical arguments (Lithner 2010; Boesan et al. 2014), mathematical arguments and argumentation becomes a tool that researchers can use to observe and evaluate mathematical reasoning. By observing how learners justify choices and conclusions in the form of a mathematical argument, researchers can gain insight into the conceptual understanding of learners (Brodie, 2010). To understand this more clearly we need to understand what a mathematical argument is and also how a researcher can discern between a weak and strong argument.

2.6.1 Plausible arguments as a tool for observing and evaluating creative mathematical reasoning

Polya (1954) in his work "Mathematics and Plausible Reasoning" emphasized the importance of creating opportunities for learners to guess, use insight, and discover new methods and solutions to novel problems. He suggested using heuristic or practical methods of problem solving where learners could discover mathematical truths for themselves. These heuristic methods are considered plausible in that learners develop plausible guesses to solutions given reasonable assumptions gathered from their exploration. This is opposed to more rigid proofs, as well as algorithms already established in the field (which were originally discovered through plausible reasoning). Plausible reasoning implies that while a learner's argument does not require strict proofs to be valid, there should be a way to evaluate the strength of their argument (Schoenfield, 1992).

2.6.2 Substantial arguments for the evaluation of plausible reasoning

Toulmin (1965) proposes a method for measuring the plausibility of reasoning as opposed to measuring strict logical reasoning. His method or tool is substantial argumentation. Essentially, if a person making an argument can justify the decisions, or systems used to solve a problem, then the person's argument is valid. If the justifications are true, then their argument is sound (valid and true). This method is helpful for viewing learners' reasoning, because in conversation we can analyze how learners can justify their thinking even from a young age by focusing less on formal logic and more on how strongly arguments are supported. (Yackel and Cobb, 1996). Even if a learner has not developed a strong logically strict thinking method, a researcher can still validate the learner's attempts, through interviews and observations of peer interaction, and their manipulation of mathematical tools. Toulmin's method of measurement of plausible reasoning through substantial argument is illustrated in Figure 2.6. If learners have data to justify their conclusion (in our case a solution to a mathematical task), they will make a stronger case if they can explain how the data supports or is evidence of a conclusion (warrant) (Prusak, Herskowitz, &Schwarz, 2012). If they can go further to illustrate how the data supports the conclusion (backing), then their conclusion has an even stronger case (see Figure 2.6).

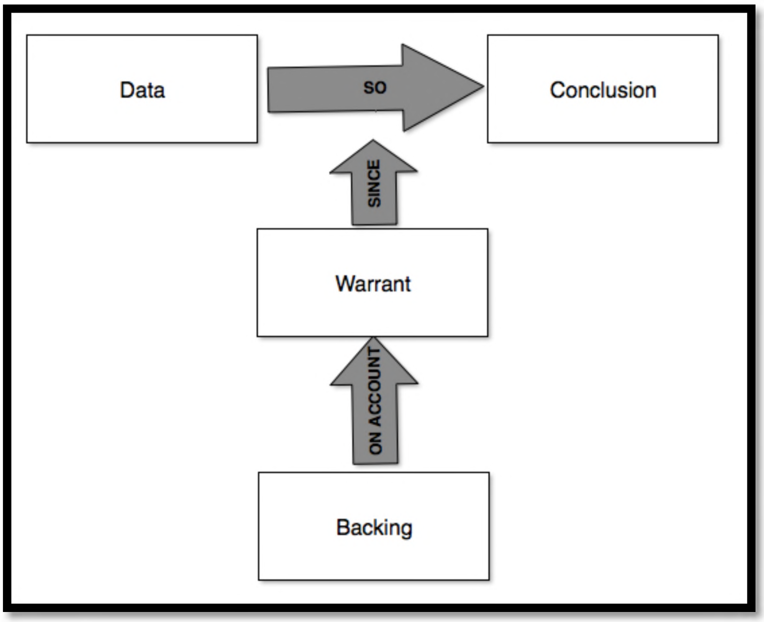


Figure 2.6 Toulmin's (1965) model for evaluating plausibility of arguments

Let us take an example from this current research. Learners are given the task of using a combination of angles to make a complementary angle. A simple mathematical argument would look as follows: = 90" (see Figure 2.7).

"Let's try 60˚ and 30˚. Look we did a right angle again". A learner just saying 60˚ and 30˚ makes a right angle does not make for a strong argument. However if the learner can show how they use a protractor to make two angles fit to make 90 degrees, they have a stronger case. A learner could go even further to do the maths to back their argument. In this case, they could also say "60 + 30
= 90" (see Figure 2.7).

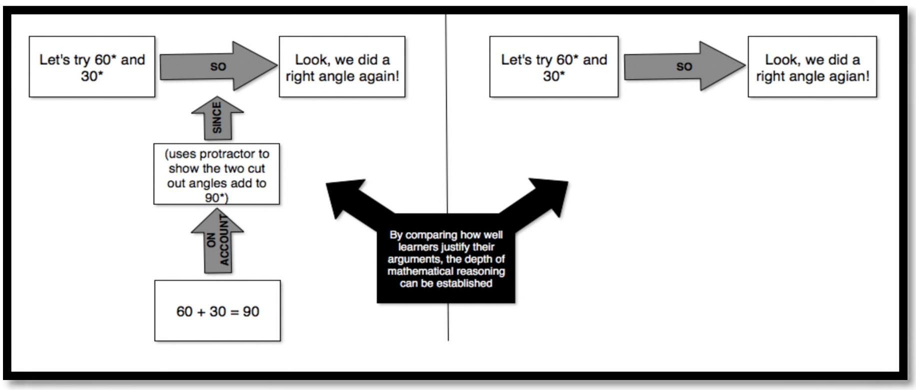


Figure 2.7 Two arguments are compared for their plausibility

2.7 OBSERVING ARGUMENT AS PRODUCT

One can view reasoning through argument in two ways. Firstly, an argument is the final product of one's claim. The defensible claim, or product, of an argument can indicate the plausibility of a learner's reasoning. By using ideas from Silver (1997), Lithner (2010) and Campos (2010), it is possible to develop a framework to evaluate the plausibility of a group's mathematical argument.

Creative Mathematical Reasoning	Observable Indicators
Imagination/Creativity: The flexibility of and uniqueness of strategies employed to solve the task.	**Flexibility:** # of correct strategies used **Fluency:** # of attempts at different strategies **Novelty:** Uniqueness, departure from the canonical. Is the reasoning sequence new or re-created for the reasoner?
Concentration: The ability to select the appropriate strategy and systematically employ the strategy to solve the task.	**Sequentiality:** Is there an order to their method? **Continuity:** Does the selected strategy respond to the objective and did it lead them to their conclusion?
Constructiveness: The ability to anchor their solution to mathematical properties, and apply these properties to other situations. This includes student ability to share this understanding so peers can understand their thinking.	**Mathematically Anchored:** Were relevant mathematical properties used to back their conclusion? **Generalization:** Are learners able to extend the conclusion or strategy to other circumstances? **According to sociomathematical norms:** Can the strategy hold up to criticism from peers or teachers?
Plausibility: Learners may interpret open-ended questions differently, but one can still measure if their interpretation of the premises if the solution is plausible mathematically, and is recognized by peers as so.	**Conceptual:** Given their interpretation of premises is the solution plausible mathematically? **According to sociomathematical norms:** Is the class able to follow and agree with their conclusion?

Figure 2.8 A working document on how this study intends to measure an argument as *product*

2.8 OBSERVING ARGUMENT AS PROCESS

There is however another way to evaluate the strength of a learner's argument, by looking at the process of argumentation. Argumentation process is the dialogue, interaction or debate that occurs between two opposing claims (Kuhn & Udell, 2003).

Observing the process of arguing has the potential to reveal different kinds of information in addition to evaluating the final argument (Nussbaum, 2008, 2011). It is the back and forth between learners that can reveal how learners think through mathematical tasks. In a small group setting, learners interact with each other, build familiarity with prior knowledge and make connections. In this way, peers' understanding and social experiences contribute to developing the depth of their understanding. (Tudge 1990, p. 159). By sharing thoughts in whatever language they are most comfortable with in a social group setting, learners are more readily able to express their thoughts, questions or concerns that bring insight into their conceptual understanding (Enyedy et al. 2008).

Through observing how learners work together in the *process* of developing an argument, a researcher can evaluate what learners spend their time discussing, how deeply they justify their thoughts to one another, and how well they interact to develop new arguments. (Krummheuer, 1995; Conner, 2014). With a common goal of solving a problem, a group of learners works together, as opposed to adversarial discussion to "win" an argument. This is called **collaborative argumentation** (Nussbaum, 2011). More specifically, Golanics and Nussbaum (2008) define collaborative argumentation as "a social process in which individuals work together to construct and critique arguments."

Learners participate in mathematical discussions in different ways, which can be an indicator of their creative mathematical reasoning. Krummheuer (2007) uses 4 terms to discriminate the types of contribution learners make in mathematical discussions from the work of Goffman (1981). This is important to mathematical reasoning, because part of creativity is the uniqueness or novelty of learners' ideas (Silver, 1997; Balka, 1974). If it is possible to determine how unique a learner's contribution is to an argument, it is an indicator of the creativity of the learner's reasoning (Levinson, 1988). The types of participation are indicated in Figure 2.9.

Types of Participation
Author: Expresses their own thoughts in their own words. (Original thoughts and expressions)
Relayer: Expresses someone else's thoughts in someone else's words. (Parroting)
Ghostee: Explains their own thoughts in someone else's own words. (Shares new understanding using another's words)
Spokesperson: Explains someone else's thoughts in their own words. (Paraphrasing, clarifying)

Figure 2.9 Types and nature of participation helps to identify the originality of ideas and words during collaborative discourse

By observing the novelty of learner contributions as well as the depth of their justifications and explanations, it is possible to begin to evaluate creative mathematical reasoning abilities (Sfard, 2001; Brodie, 2010). By looking at how learners justify their reasoning by using Toulmin's (1965) substantial argument tool (see Figure 2.8), it is possible to observe the depth of plausibility in learner interactions (Prusak et al. 2012).

2.9 SETTING THE STAGE FOR COLLABORATIVE ARGUMENTATION

Careful consideration must be made when determining the types of mathematical tasks learners must solve to be able to say a learner exercises appropriate reasoning skills in demonstrating conceptual understanding (Conner et al., 2014). Collaborative argumentation requires a researcher to address two key aspects when posing a mathematical task:

* The task will require the learner to exercise mathematical reasoning skills that reveal conceptual understanding
* The mathematical task must elicit dialogue and interaction between learners (Lampert, & Cobb, 2000).

Cohen (1994) suggests two key components that will allow mathematical reasoning and cooperative interaction to take place. Firstly, the task must be open-ended, and include more than just computational or algorithmic problems where there is one right answer (Stein, Engle, Smith & Hughes, 2008). The second required component is that the tasks must be non-structured, meaning there must not be a prescribed procedure or system inherent in the problem in order to solve it (Stylianides, & Stylianides, 2014). It must be a problem "with no one right answer and a learning task that will require all students to exchange resources; achievement gains will depend on the frequency of task-related interaction." (Cohen 1994, p. 8).

The present study selected Visual Technology for the Autonomous Learning of Mathematics (VITALmaths) clips as the format for introducing open-ended mathematical tasks to learners. VITALmaths are 1to 3 minute silent video clips that pose mathematical tasks to stimulate conversation and reasoning among small multi-age, multi-ability peer groups (Linneweber-Lammerskitten, Schafer & Samson, 2010). These visual mathematical tasks require only fundamental mathematical knowledge and intellectual demands, but allow learners to make observations, conclusions and considerations (Linneweber-Lammerskitten, Schafer & Samson, 2010). By using VITALmaths clips learners have visual prompts to understand the problems, as well as hands-on manipulatives to work out the problems as a group which helps those who struggle with maths or English language learners to have multi-media input (Mayer, 2005) . This provides a rich setting to observe how learners work together using creative mathematical reasoning to build their conceptual understanding. Figure 2.10 is a screenshot of one of the VITALmaths clips selected for this study.

Figure 2.10 A screenshot from one of the selected VITALmaths clips

Kramarski & Mevarech (2003) suggest that metacognitive prompts also help learners to express their reasoning. By creating worksheets that coincide with Polya's (1945) steps to solving problems, VITALmaths tasks can be broken into subsections with questions to prompt discussion. By requiring learners to think about their thinking, learners are also required to explain and justify their thinking (Schoenfield, 1992). The subsections are; understand the problem, devise a plan, carry out the plan, and present your argument. The types of metacognitive questions that help them to think about how they are solving the problem are classified into three types: comprehension questions that help them ensure they understand the problem, strategic questions that prompt how they will systematically solve the problem, and connection questions that help them to generalize their ideas, or help them make a constructive conclusion. (Kramarski & Mavarech, 2003, p. 286). Figure 2.11 is an example of the supplementary worksheets with meta-cognitive prompts participants used to solve the tasks.

Cohen (1993) suggests an additional way of increasing participation of group members in discussion. Assigning roles for each learner requires their participation. For the present research, three roles were identified based on suggestions by Cohen (1993) and pilot sessions with learners. These roles were not strictly adhered to, but reminded learners of how they should be interacting. The facilitator's role was to make sure every learner contributed to the discussion of each question even if it was only to say 'I agree'. The recorder's role was to answer the questions in full on the worksheet, and the presenter's role was to communicate with the teacher if they had completed a section, or if they needed clarification on a question. These roles reminded learners that they all needed to participate, answer the questions on the worksheet in full, complete each section and ask for assistance if they needed guidance.

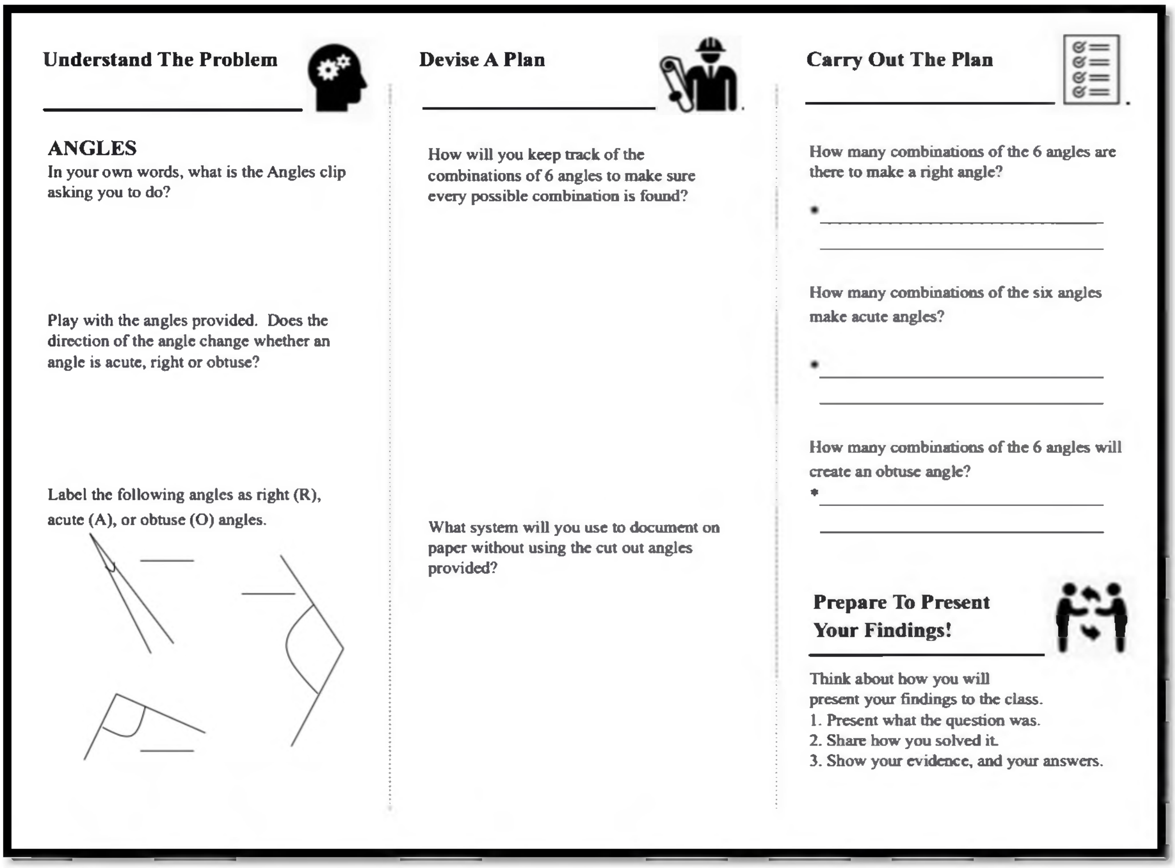


Figure 2.11 An example of the worksheet format used for each of the clips to guide learner interaction

2.10 THEORETICAL CONSIDERATIONS

2.10.1 Introduction

This section is a critical review of the learning theories and theoretical frameworks that are the foundation of research around creative mathematical reasoning, autonomous learning and collaborative argumentation discussed in Chapter two. The two main learning theories that are the foundation of this research are often referred to under one theory called Social

Constructivism. Social Constructivism is derived from two learning theories namely Constructivism and Social Learning theory. This critical review breaks down the two theories to better understand social constructivism.

2.10.2 Constructivism

Constructivism is the most basic theoretical foundation that frames the research and methodologies of this study. In essence, constructivism works from the notion that knowledge is constructed from our perceptions and experiences and prior knowledge (Simon 1995, Piaget 1970). Constructivism requires active participation from individuals and therefore an individual's knowledge or understanding is *subjective* to a variety of factors in the social environment and from prior knowledge. (Phillips 2011, Simon 1995). The challenge of constructivism is being able to interpret and individual's conceptual understanding and determine if it is compatible with objective reality (Prawat, 1992). This research poses open ended mathematical tasks to determine the depth of creative reasoning. By observing how groups construct knowledge and present their arguments we can interpret whether or not their conceptual understanding matches objective reality (Wadsworth, 1978).

Radical Constructivism

Radical constructivism is a subset of constructivism that focuses on the individual's abilities to synthesize knowledge (von Glasersfeld, 1995). It works on the notion that knowledge is malleable and continually evolving and adapting according an individual's prior knowledge, perspectives, and experiences (von Glasersfeld, 2001). This coincides with the Silver (1997) and Balka (1974) notion of creativity being a combination of flexibility, fluency and novelty of understanding and reasoning and the Campos (2010) and Pierce (1992) notion of mathematical reasoning being the imagination, concentration and generalization of mathematical concepts.

2.10.3 Social Learning Theory

Social learning theory focuses less on the individual's psychological aspects of learning, rather it focuses on the sociological aspects of learning. Bandura (1971) and Vygotsky (1978) recognized that a purely psychological approach to learning did not take into account the complexity of the social environment that impacts how people learn. Vygotsky (1978) looked at the types of social supports and structures that impacted a person's ability to learn (Moll, 1990). Bandura (1971) looked at learned behaviors and how they are reinforced positively or negatively in a social environment.

Symbolic interactionism

Symbolic interactionism is a social learning theory that focuses attention on the development of socio-mathematical norms (Yackel & Cobb, 1996). Socio-mathematical norms are the rules set up by educators and learners to develop mathematical understanding (Blumer, 1969). Symbolic interactionism goes further to say that by using definitive concepts, one can interpret a learner's understanding by observing learner interactions with other peers and or the teacher (Hanzel, 2011). This current research has used the theoretical framework of symbolic interactionism to create definitive terms to analyze and evaluate the process of learner interactions solving the VITALmaths tasks.

2.10.4 Social Constructivism

Social constructivism is a term used to describe the combination of the psychological perspective of constructivism, and the sociological perspective of Social Learning theories (Simon, 1995, Yackel & Cobb 1996). It acknowledges the importance of considering social and environmental impacts on how a learner constructs their own meaning and understanding. Social constructivism also recognizes that individual learners constructing their understanding impacts the social environment as participants construct knowledge together (Phillips, 1995).

2.11 CONCLUDING REMARKS

Creative mathematical reasoning is the mental process that the South African Curriculum and Assessment Policy (CAPS) refers to when they use the terms logical and critical thinking and problem solving. By using creative mathematical reasoning, we can better understand the relationships between mathematical concepts and the world around us. However, mathematical reasoning is difficult to observe without interaction. To prompt a small group of high school learners to talk about and solve mathematical tasks is no simple feat. By using 1 to 3 minute VITALmaths silent stop-motion video clips that pose a mathematical task that stimulates conversation and reasoning, researchers can set the stage for collaborative argumentation. By using supplementary worksheets that prompt metacognitive discussions and ensure that learners take close consideration of the task, a stage can be set to view how learners build conceptual understanding through creative mathematical reasoning. Figure 2.12

illustrates how all the different aspects of creative mathematical reasoning work together, and how VITALmaths clips and worksheets act as the stimulus to a creative mathematical reasoning response through collaborative argumentation.

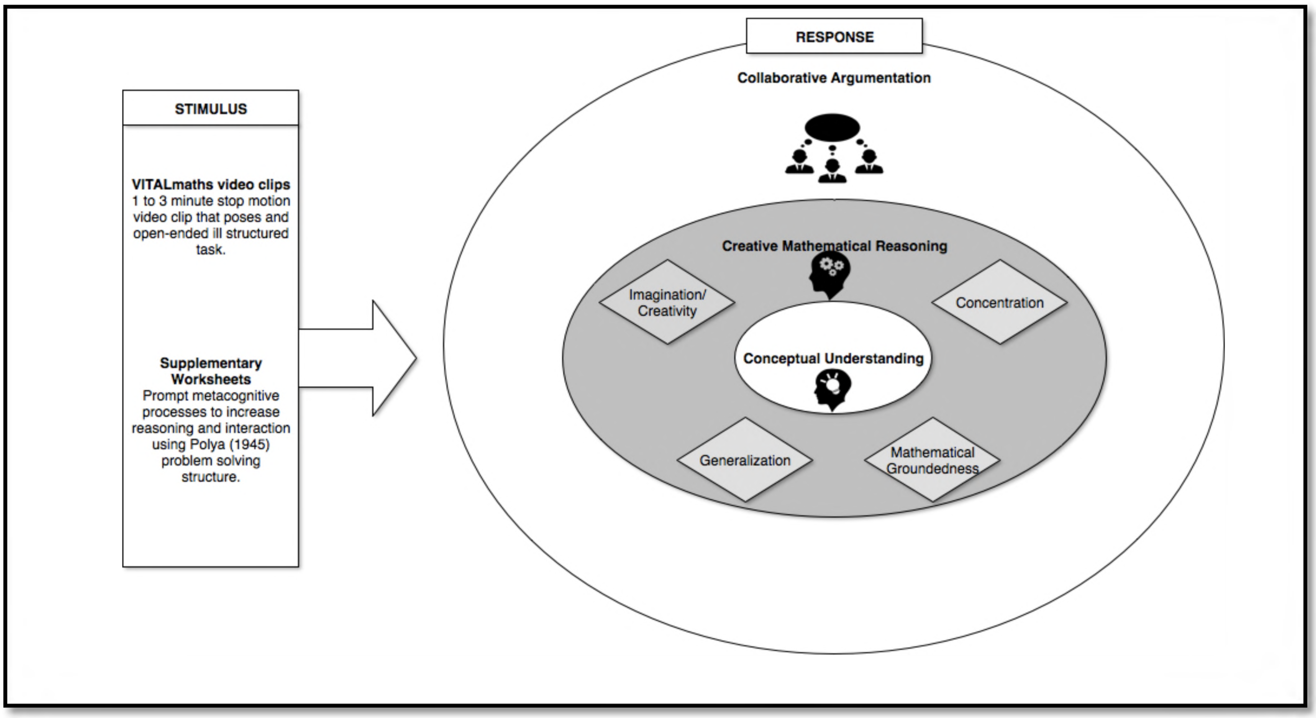


Figure 2.12 A visual showing how VITALmaths clips and worksheets are the stimulus for collaborative argumentation, which allows us to observe creative mathematical reasoning

This current study has a foundation in a Social Constructivist perspective which takes elements from radical constructivism and symbolic interactionism theories. Symbolic interactionism is used in this study as a theoretical framework for analysis and evaluation the arguments and interactions that took place as learners solved the VITALmaths tasks. The study starts from the theoretical foundation that the way learners argue and interact in a group gives insight into how learners construct their own mathematical reasoning and understanding. Chapter three will discuss the research design methodology.

CHAPTER THREE: RESEARCH DESIGN METHODOLOGY

3.1 RESEARCH ORIENTATION

A key element of social constructivist theory is that an individual's conceptual understanding of mathematical concepts is *subjective* (Ernest, 1991). An individual takes objective mathematical concepts, and re-creates the knowledge so that it fits with his or her personal understanding based on prior knowledge, perspectives and experiences (Yacke & Cobb, 1996). This new *subjective* understanding of *objective* mathematical concepts also takes into consideration the social milieu in which the reasoning takes place (Krummheuer, 2000; Yackel, 2001). Observing and analyzing creative mathematical reasoning through collaborative argumentation requires a significant amount of "context-specific" interpretation of social interactions and presentations, which makes a qualitative orientation most suitable for this study. According to Golafshani (2003), "qualitative research is a naturalistic approach that seeks to understand phenomena in a context-specific setting (p.600)".

3.2 RESEARCH GOALS

The specific goals of this study are to:

1. Develop VITALmaths clips, and through an interactive process create supplemental worksheets that support collaborative argumentation.
2. Develop and implement analysis tools that help gain insight into the creative mathematical reasoning of selected Grades 9 and 10 learners.

3.3 RESEARCH QUESTIONS

Given the above goals, this research aims to answer the following questions:

A. Do learners show creative mathematical reasoning abilities in interaction with peers (*process)?*

B. Do learners show creative mathematical reasoning abilities as they justify their claims (*product)?*

3.4 RESEARCH METHODS

This research is a case study of six high school learners, divided into two groups, attending non-school fee public schools in Grahamstown in the Eastern Cape of South Africa. Yin (2009) describes a case study as "an empirical inquiry that investigates a contemporary phenomenon within its real-life context." (p.13). Due to the complexity of individual and social processes involved in creative mathematical reasoning and collaborative argument, a case study was the most relevant method for research. Having two groups allowed for an effective analysis of trends in the process and product of creative mathematical reasoning and collaborative argumentation.

A case study was selected due to the exploratory nature and the contextual conditions of the research (Yin, 2009). The tools developed for analysis required an in-depth analysis of interaction between small groups. By using a case study method, I was able to identify, adapt and explore the tools for analysis, which could then be generalized to a larger analysis in future studies. As a researcher, I was already working with a small group of Grades 9 and 10 learners over a 2-year period. I had a privileged insight into the learning processes of this small group and so a case study seemed the best approach to take full advantage of the richness of contextual conditions. The two units of analysis for this case study were the process and product of creative mathematical reasoning.

For the purposes of this research, collaborative argumentation is seen as a social manifestation of creative mathematical reasoning which can be analyzed using definitive observable indicators. Kuhn & Udell (2003) point out that argument can be viewed as both *process* and *product*. Brodie (2010); Ball & Bass (2003) share the same view of mathematical reasoning. This research operates under this same notion that creative mathematical reasoning can be observed in how learners argue and interact to solve the task (*process)* and by how they present a final argument as to why their solution is correct (*product)*. This research has used different tools to analyze the process and product of creative mathematical reasoning (see research questions A and B in section 3.3).

Because qualitative data can often be interpreted in different ways, it was very important to ensure that the observable indicators for process and product of creative mathematical reasoning were clearly defined (Hanzel, 2011). In conjunction with the works of Lithner

(2008) and Campos (2010), an analysis framework from Krummheuer (2007) was adapted to analyze the interactive processes of learners solving VITALmaths tasks. Video and audio recordings were transcribed and translated. Each line of interaction was labeled according to Lithner (2008), Polya's (1945) reasoning structure and Toulmin's (1969) plausible argument framework. Each line of interaction was then analyzed a second time according to Krummheuer's (2000) participation roles.

To analyze the final solutions to the VITALmaths tasks, tools from Lithner (2008), Campos (2011) and Silver (1997) were adapted to create observable indicators that coincided with creative mathematical reasoning. Video and audio recordings of the groups presenting their findings, and written work presented, were analyzed according to the selected observable indicators of creative mathematical reasoning.

With definitive observable indicators identified, and tools for analysis prepared, it was possible to identify VITALmaths clips and create supplemental worksheets that would set the stage for collaborative argumentation. After several pilot tests, 6 VITALmaths clips were selected and supplemental worksheets produced.

A mixed methods approach of both qualitative and quantitative measures was adopted for two reasons as suggested by Johnson, Onwuegbuzie, & Turner (2007). Firstly, by using qualitative and quantitative measures, I was able to add a richness to the data. Specifically, in regards to process evaluation, the transcribed interactions lent itself to a quantitative analysis, which offered interesting insights into how participants spent their time interacting and solving the tasks. In the same regard, merely looking at the product of the groups' solutions to the task from a quantitative perspective did not allow for an interpretation of the strategies employed or the uniqueness of their approach, that the depth of a qualitative approach was able to provide (Johnson, et al. 2007). Secondly, by using a mixed method approach, I was able to provide a new vantage point from which to view creative mathematical reasoning (Creswell, & Plano, 2007). By merely looking at either qualitative or quantitative measures, I would not have been able to gain such an in-depth perspective from which to view how learners interacted while solving the tasks and presenting their findings.

3.5 RESEARCH DESIGN

3.5.1 Participant selection

Participants for this research were selected from an after-school programme that provides academic support in non-fee paying public schools in the underprivileged areas surrounding Grahamstown (Goerge, Torreano & Kellen, 2014). Participants were purposefully selected from the Inkululeko project due to the privileged insight I had in working with the learners in the programme for over a year. By having this privileged insight, I was able to interpret more clearly the social complexities within which the learners worked (Cohen, Manion, & Morrison, 2011). I was also able to get more honest and genuine responses and feedback from the learners due to having a close relationship with them. 3 boys and 3 girls in Grades 9 and 10 at 3 different schools, were selected. Learners were selected according to specific criteria. Firstly, it was decided to have a balance of gender. The 6 learners all showed a commitment to their learning based on attendance at club, and their interaction with the researcher. Motivation and willingness to engage in a challenge was also a factor as it was important to get as much interaction as possible, identified by the level of engagement at Inkululeko, in leadership and explorative opportunities. Learners' mathematics achievements ranged from low to high based on school marks, so there was a spectrum of abilities represented in each group. It was decided to have 3 members per group to ensure that all the learners would be able to contribute to the interactions. By having mixed ability, mixed gender, and mixed age groups, the focus was less on competitiveness, and more on positive interaction (Topping, Campbell, Douglass & Smith, 2010). Group 1 consisted of LS, LK, and LN. LS was a high achieving Grade 10 learner at the top of her class academically. LK was a moderate/high achieving Grade 9 boy, and LN was a low achieving Grade 10 girl. Group 2 consisted of LT, LP, and LD. LT was a moderately achieving Grade 10 boy. LP was a high achieving Grade 9 boy, and LD was a moderate/low achieving Grade 9 girl.

3.5.2 Research Design

Phase 1: Generation and selection of VITALmaths Clips and Worksheets to address Research Goal 1

Cowley (2014) in her research with a similar population of earners in the Eastern Cape found that spatial reasoning was a challenge for Grade 10 learners. This mathematical focus of spatial reasoning was inspired by her research, which looked at learner conceptual misunderstanding of spatial relationships. To ensure maximum participation of my learners, VITALmaths clips that only required a fundamental mathematical knowledge and that would stimulate conversations about reasoning, were selected and/or created (Linneweber-Lammerskitten, Schafer & Samson, 2010). Specific attention was also paid to ensuring directions were non-regimented and solutions were open-ended to prompt creativity in team problem solving (Cohen, 1994; Stein et al., 1996). The first VITALmaths clip created and piloted for students was Train Tracks which played with the concept of parallelism. A supplemental worksheet was also created. From this first clip, it was identified that more meta-cognitive prompts (prompts to think about one's mental processes) needed to be included in the supplementary worksheets, as suggested by Kramarski & Mevarech (2003), and the solution needed to be more open-ended (Stylianides & Stylianides, 2014). What evolved from the piloting process was a supplementary worksheet that was broken up into sections with specific meta-cognitive questions following Polya's (1957) reasoning structure. A simple presentation question was also created to ensure groups discussed their interpretation of what the problem was, an explanation of how the task was solved, and a presentation of evidence and their solution.

Through the piloting process of four sessions, the six participants were split into two groups of three to ensure maximum participation. Group members were selected to ensure a blend of gender and abilities. Methods for recording video and audio with the best sound quality were also determined. Six VITALmaths clips were developed and six supplemental worksheets were designed for each task to elicit conversation from learners about their thinking as they worked through the task, selected a strategy, implemented the strategy and presented the solutions (see Figures 4.1.0.1, 4.2.0.1, 4.3.0.1, 4.4.0.1, 4.5.0.1 and 4.6.0.1). A simple worksheet/framework was designed to guide learners in what needed to be displayed in the final presentation of solutions (Stein, 2008; Schoenfield, 1992). During the piloting of VITALmaths clips, learners were asked if they preferred to have the clips in English or isiXhosa, and interestingly, the participants preferred to solve the tasks in English. This was done to ensure learners could interact in the language they felt most comfortable (Enyedy et al., 2008).

Phase 2: Recording of learners solving of VITALmaths tasks and presenting solutions to address Research Goal 2

Group 1 and Group 2 were recorded solving six VITALmaths tasks over a three month period. Cohen (1994) suggests that to increase the amount of active participation in small groups, researchers should assign roles to participants as a reminder of what is expected. The three roles assigned and rotated from clip to clip were director, recorder and presenter (Cohen, 1994). The director's role was to ensure everyone contributed to interactions even if their contribution was a simple yes or no answer. The recorder documented the work on paper, and the presenter was responsible for telling the researcher when they were done with a section and for requesting help if clarification during the task was needed.

After the researcher introduced the task and defined roles, learners watched the 3-minute VITALmaths clip and used the supplemental worksheet to guide discussion around solving the task. These worksheets are found in Appendix 2. The worksheet was designed around Polya's (1945) reasoning structure and had 4 sections. The first section "Understand the Problem" focused on learners understanding the task and the core mathematical concepts necessary to solve the task. This allowed the researcher to see if there were any misunderstandings about what was required or about the mathematical concepts involved (Stein et al., 1996). The second section "Devise a Plan" prompted learners to discuss the strategies they were going to implement to solve the task. Questions like "How will you document your work?" and "What were your first two solutions?" The "Carry out the Plan" section focused on implementing the strategy and solving the task. The final section "Prepare to present your Findings" prompted participants to prepare to present their findings. (Kramarski, & Mevarech, 2003)

Once learners had completed the worksheet, they prepared a presentation on paper or on the whiteboard following a simple framework worksheet. The prompts for the presentation were "What was the problem?", "How did you solve it?", "Present your evidence" and "What was your solution?" This framework is found in Appendix 3.

The researcher played a minimal role in introducing the task and answering only specific questions pertaining to understanding the task. (Stein et al, 2008; Kramarski, & Mevarech, 2003). Given that English was a second language for these learners, it was important that they were able to seek clarification from the researcher. (Enyedy et al., 2008) Once learners had presented their findings, the researcher asked questions to clarify what the group was thinking if responses were vague. (Cohen, 1994).

3.6 DATA ANALYSIS INSTRUMENTS

Following the audio and video recording of the groups solving the VITALmaths tasks, I collected the written work, including informal notes or scribbles made, and took photos of any work done on the whiteboard. The audio recording was then transcribed and translated from isiXhosa to English with a colleague from the Inkululeko project who also had privileged insight into the meanings of comments made. Video recording helped to verify interactions and manipulation of objects to solve the task, and to clarify what participants were referring to in their interactions.

Each line of interaction was then coded according to two sets of observable indicators. The first set of coding was done according to Krummheuer's (2000), Levinson (1988), and Goffman (1981) participation roles. *Author* statements were original contributions to interactions. *Spokesperson* roles were statements that paraphrased another person's comments. *Relayer* comments were parroted comments from what others said or the reading of instructions. *Ghostee* comments were parroted statements that held new meaning by how they were said or what they referred to. The second set of coding was a combination of Lithner (2008), Polya's (1957) reasoning structure and Toulmin's (1964) plausible argument framework. Each interaction was classified according to the following reasoning structure: Comprehension, Strategy Choice, Strategy Implementation, Argument (which was broken down into claims, warrants and backing), Conclusion, Presentation and Side Conversation. Appendix 4 shows the breakdown of interactions according to the coding systems used.

From the analysis of interactions, it was possible to gather quantitative data around the amount of arguments made, the roles played by individual participants and as a group, the balance of interaction between group members, the amount of interactions spent within the reasoning structure, and the depth and length of arguments made solving the VITALmaths tasks. This data informed the evaluation of *process* of creative mathematical reasoning (Research Question A).

Group 1 and Group 2's written work and presentation of the final solution was analyzed to inform the *product* evaluation of creative mathematical reasoning. The evidence and presentation was analyzed according to the number of correct solutions compared to the number of possible solutions. By studying the evidence, it was also possible to analyze the strategies implemented by the learners in solving the task and thereby see if there was a sequence to their recording of evidence, and continuity of strategy from beginning to end of solving the task. Learner responses in the presentation also gave insight into the novelty of solutions, and depth of mathematical understanding and justification.

3.7 EVALUATION INSTRUMENTS

3.7.1 Process evaluation

The process evaluation of creative mathematical reasoning had 6 observable reasoning abilities that were considered. The evaluation was supported by the analysis of supplementary worksheets and transcribed and coded interactions from the video and audio recordings. A 5-point marking system was created to evaluate each reasoning ability. A score of 1 showed no evidence of the ability. A score of 2 showed weak evidence of the ability, and 3 showed moderate evidence. A score of 4 showed good evidence of the ability, while a 5 showed strong evidence of the ability (see Figure 3.1 as a sample).

BRICK LAYING PROCESS EVALUATION	GROUP 1	GROUP 2
Reasoning Abilities	**Observable Indicators**	**Observable Indicators**
Flexibility: # of Arguments Made	41 Arguments Made / 4 Correct Solutions 1 2 3 **4** 5	8 Arguments Made/ 5 Correct Solutions 1 **2** 3 4 5
Fluency: Sustained Interaction	7 lines of sustained interaction 1 **2** 3 4 5	5 Lines of sustained interaction 1 **2** 3 4 5
Initiative: Authored Participation	60.6% Author Statements 1 2 **3** 4 5	57.7% Author Statements 1 2 **3** 4 5
Concentration: Interactions across the reasoning structure.	30% of the time was spent understanding the task. Even distribution of strategy choice and strategy implementation. 1 2 **3** 4 5	48% of interactions were on understanding the task. Only 3 comments on strategy choice and 4 interactions around strategy implementation. **1** 2 3 4 5
Plausability: Depth of Mathematical Justifications	Limited **mathematical** justification. 5 warrants and 1 backing statements were made 1 **2** 3 4 5	Limited **mathematical** justification. 5 warrants were used, and 0 backing statement. **1** 2 3 4 5
Constructiveness: Balance of Contributions/Incorporating new ideas	LS contributed 42% and LK contributed 35%while LN contributed to 16% of interactions. 19.9% combined spokesman statements. 1 2 **3** 4 5	LT contributed to 51% of interactions wile LP contributed to 11%, and LD with 27%. 8% combined spokesman statements 1 **2** 3 4 5
	TOTAL: 17/30	**TOTAL: 11/30**

Marking Criteria:

1	2	3	4	5
No Evidence	Weak Evidence	Moderate Evidence	Good Evidence	Strong Evidence

Figure 3.1 Process evaluation of creative mathematical reasoning

Flexibility was evaluated by the number of arguments made during interaction (Silver, 1997; Balka 1974). Fluency was evaluated according to the amount of sustained interaction between participants around one specific argument (Kuhn & Udell, 2003). Initiative looked at the amount of authored participation in the group (Krummheuer, 2007). Concentration was evaluated by the balance of interactions across the reasoning structure. Plausibility looked at the depth of mathematical justifications (Prusak, et al., 2012; Toulmin, 1964). Constructiveness was evaluated by an analysis of the balance of participants' contributions, and the amount of spokesperson comments made in the group to support each other's ideas (Krummheuer, 2000; 2008). Based on the evaluation of the reasoning abilities, a score out of 30 was given. This evaluation helped to answer Research Question B.

3.7.2 Product Evaluation

The product evaluation of creative mathematical reasoning had the same 5-point marking criteria ranging from no evidence to strong evidence of the reasoning ability. The 6 reasoning abilities had similar names, but different observable indicators that specifically looked at the final solution of learner work. Group written and audio recorded presentations as well as written evidence helped inform the evaluation of the final product.

BRICK LAYING PRODUCT EVALUATION	GROUP 1	GROUP 2
Reasoning Abilities	**Observable Indicators**	**Observable Indicators**
Flexibility: # of Correct Solutions	4/Many Solutions 1 2 3 4 5	5/Many Solutions 1 2 3 4 5
Fluency: Strategies implemented.	Used blocks and photo prompts. 1 2 3 4 5	Referred to the video, limited discussion about strategy choice. 1 2 3 4 5
Novelty: Uniqueness of strategies.	Good use of pictures and blocks, but not enough consideration into each aspect. 1 2 3 4 5	Very rushed implementation of strategies. 1 2 3 4 5
Concentration: Sequentiality and Continuity of Strategy Implementation	Students looked at each individual picture prompt when considering solutions. Limited depth of considerations. 1 2 3 4 5	There was no evidence of sequentiality or continuity of strategy implementation. 1 2 3 4 5
Plausability: Mathematically Anchored Sociomathematical norms	Limited **mathematical** justification. Mathematical terms were used, but did not specify spatial relationships. 1 2 3 4 5	Limited **mathematical** justification. Explanations did not focus on mathematical concepts or terminology. 1 2 3 4 5
Constructiveness: Generalization to other concepts or experiences	Good evidence of using previously covered concepts. Limited specific usage of terminology. 1 2 3 4 5	No evidence of adopting strategies or prior-knowledge concepts from previous tasks. No terminology was used to build their argument. 1 2 3 4 5
	TOTAL: 17/30	**TOTAL: 9/30**

Marking Criteria:

1	2	3	4	5
No Evidence	Weak Evidence	Moderate Evidence	Good Evidence	Strong Evidence

Figure 3.2 Product evaluation of creative mathematical reasoning

Flexibility looked at the number of correct solutions compared to the number of possible solutions (Silver, 1997; Balka, 1974). Fluency looked at the number of strategies implemented to solve the task (Silver, 1997; Balka, 1974). Novelty was evaluated according to the uniqueness of the strategies implemented to solve the task (Krummheuer, 2000; 2007). Concentration was marked according to the sequence and continuity of strategies implemented, based primarily on reviewing the evidence shown to justify their solution (Lithner, 2008). Plausibility looked at the justifications in presentations and how mathematically anchored these justifications were (Lithner, 2008; Polya, 1954). Constructiveness was evaluated according to how well concepts from previous clips or prior mathematical knowledge were generalized into solving the task being evaluated (Campos, 2010; Pierce, 1992). Figure 3.2 shows a sample of a product evaluation of one of the VITALmaths tasks. The Group received a score out of 30 possible points similar to the process evaluation. This allowed for a comparison between Group 1 and Group 2 to address Research Question B.

3.7.3 Comprehensive Evaluation

Once all 6 VITALmaths Tasks were analyzed and evaluated, it was possible to find comprehensive averages for each observable indicator. It was important to look for trends across all 6 VITALmaths tasks and to find an average score as a means for comparison between the two groups. This allowed for an analysis of what reasoning abilities were evident in solving the tasks, and which abilities did not show strong evidence.

3.8 ETHICAL CONSIDERATIONS

Orb, Eisenhauer & Wynaden (2000) identified autonomy, beneficence, and justice as ethical principles, which must be considered when doing qualitative research. *Autonomy* refers to respecting the individual rights of research participants (Orb et al., 2000). This includes ensuring the voluntary participation and informed consent of the persons enrolled. In the case of working with minors, as in this research, parent permission is also important. All participants involved in this study were involved in the piloting of the 4 VITALmaths clips and worksheets, as it formed part of the Inkululeko programme. Those who expressed an interest were provided with consent forms, which were then signed, by participants and their parents. Learners were informed that this study was about how participants worked together to solve the tasks of signed informed consent. (see Appendix 1 for a sample of the form sent to parents and participants).

Beneficence in qualitative research addresses the "first do no harm" aspect of research (Orb et al., 2000). This specifically addresses maintaining anonymity and confidentiality of participants (BERA, 2011). To ensure anonymity in this study, learners received pseudonyms, and had their faces covered in any photos used in the research. Research was also conducted in a private, closed-door setting. Video and audio recordings were also kept confidential and were only viewed by the researcher and the translator during the course of the study (Orb, et al., 2000).

The principle of *justice* refers to avoiding exploitation and abuse of participants (Orb et al., 2000). This study did not interrogate psychosocial vulnerabilities of participants, but had the potential to impact on learner confidence. Working in groups and taking the focus off individuals helped to alleviate the anxiety or pressure to perform. The VITALmaths clips and supplemental worksheets had only fundamental mathematical concepts which made the tasks approachable (Linnewebber-Lammerskitten, Schafer & Samson, 2012). Given the open-ended nature of the tasks, groups were able to solve all the tasks to some extent. It is my belief that this research was beneficial to the participants because it provided an opportunity for participants to practice mathematical reasoning and collaborative argumentation skills while solving the tasks.

3.9 RELIABILITY AND VALIDITY OF RESEARCH METHODS

Golafshani (2003) indicates credibility, confirmability, consistency, and transferability as 4 essential criteria for reliability and validity in qualitative research. Hanzel (2011) states that to establish *credibility* in qualitative research, it is critical that observable indicators are clearly and precisely defined. This study is grounded in the definitive terminologies research methodologies of Lithner (2008), Polya (1945), Krummheuer (1995), Campos (2010), Toulmin (1954) and Silver (1997), which gives credibility to the observable indicators selected and observed.

Confirmability refers to the ability of a researcher to collect a sufficient variety and amount of data to support analysis. This study was able to ensure a triangulation of data by using video recordings, audio recordings, written work, and informal interviews after sessions to ensure data was conclusive and accurate (Golafshani, et al., 2003) The rigor of the data collection was also sufficient. Two groups were each recorded solving 6 VITALmaths tasks which added up to over 6 hours of video and audio recording of which 2161interactions were translated and transcribed.

The *consistency* of this research was evident through ensuring that the role of the researcher engaged in under 10% of group interactions. The researcher was restricted to only introducing the VITALmaths tasks, and answering questions from students that related to understanding the task. By having 6 VITALmaths tasks, and two groups as a means of comparison, this study was able to look for trends of consistency across all 6 tasks and between both groups in the case study. This ensured a consistency of results.

Transferability or *applicability* refers to an ability to replicate the study in different settings and circumstances (Golafshani, 2003). The VITALmaths tasks in this study were created in partnership with a university in Switzerland and the clips presented are or will be available in English, German and isiXhosa. The observable indicators used were quantifiable, and allowed for consistent interpretation (Hanzel, 2001).

3.10 CONCLUDING REMARKS

This research was a case study that used qualitative and quantitative measures. Its goals were to develop and utilize VITALmaths tasks, supplementary worksheets and an analysis framework that allowed for the observation and analysis of the process and product of creative mathematical reasoning. The research questions focus on creative mathematical reasoning abilities in the process of solving tasks as well as in the product of their final solutions. This study went through several phases to develop the tasks and video and audio recording of groups solving the tasks, as well as analysis and evaluation of groups solving the tasks. This case study ensured that the research was done ethically to protect the rights of participants. The study also put measures in place to ensure the validity and reliability of results. Chapter 4 describes the detailed analysis of results from the research.

CHAPTER FOUR: DATA ANALYSIS AND DISCUSSION OF RESULTS

PART A VERTICAL ANALYSIS

4.0 INTRODUCTION

Analysis Structure

With an understanding of the theoretical foundations and literature that support the methods for this research, this chapter aims to analyze the data, explain and discuss the outcomes and what they might say about learners' creative mathematical reasoning. This chapter analyzes the data in two sections. Section 1 is a detailed comparison of how two groups performed in each of the six VITALmaths clips. Section 2 evaluates how each group and the individuals within each group performed across the six VITALmaths clips. Analysis of the groups is consistent across this chapter. (See Figure 4.0.1)

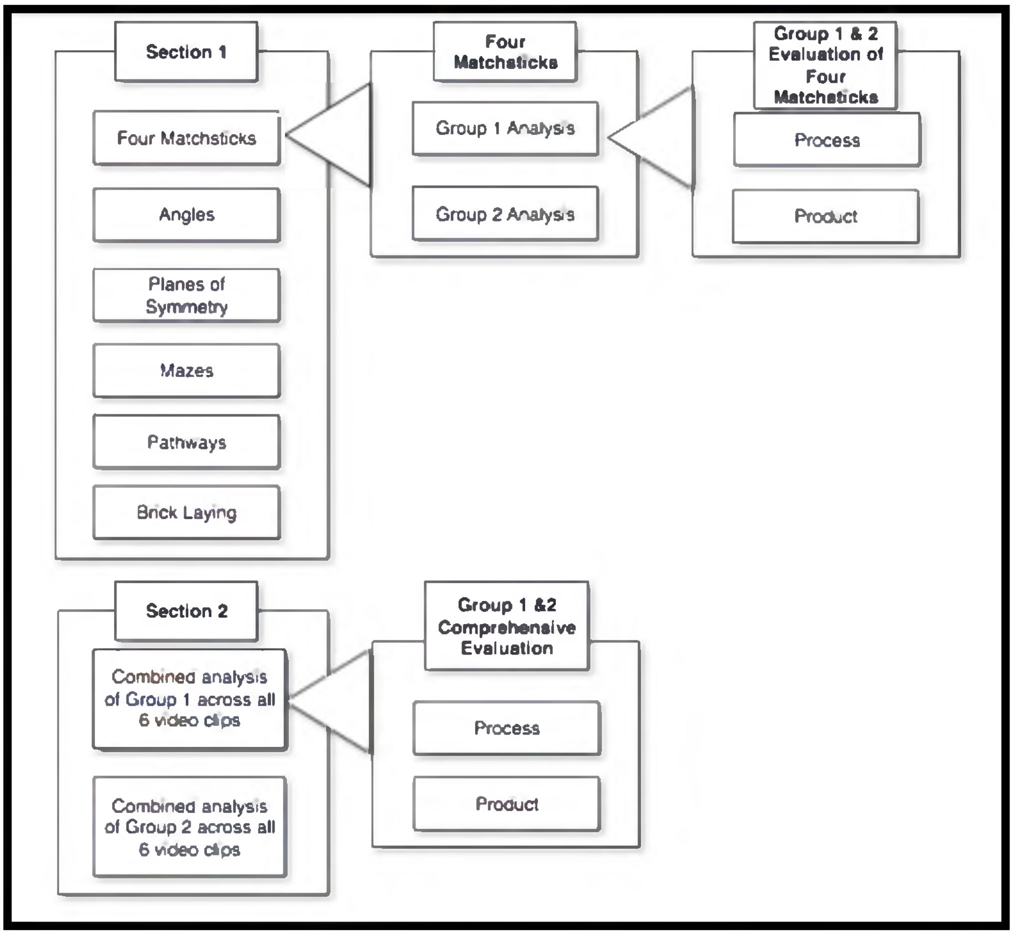


Figure 4.0.1 Map of data analysis for this research

Process analysis and evaluation procedures

With each group, for each video clip, analysis consisted of two components, *process* and *product*. The process of learners' arguments was evaluated using two tools; participation roles (Goffman, 1981; Levingson, 1987; Krummheuer, 2007), and plausible argument and reasoning structures (Toulmin, 1964; Polya, 1957; Krummheuer, 2007; Lithner, 2010). Data used includes video footage as well as transcripts of learners solving the problem, worksheets and scratch paper of their work, and their final presentation documents. Figure 4.0.2 illustrates the terms and observable indicators used to analyze the interaction of learners as they solved the VITALmaths tasks.

Process Analysis of Group Interaction and Argumentation	
Interaction Analysis	**Observable Indicators**
Flexibility	# of arguments made/final solutions
Fluency	Longest sustained interaction on a single mathematical concept
Initiative	Novelty of ideas in contribution to interactions
Concentration	Contributions to interaction **within the reasoning structure** Interactions around **developing strategies and implementing them**
Plausibility	Depth of **mathematical justifications** **Mathematical properties used**
Constructiveness	Balance of overall contributions to group interaction Percentage of interactions that incorporated other group members' ideas

Figure 4.0.2 System for analysis of the process of argumentation

By looking at the roles that learners play in their interactions, I was able to analyze the flexibility and novelty components of creativity (Krummheuer, 2007; Silver, 1997; Balka 1974). By observing how learners could transition between coming up with their own ideas (author), and being able to paraphrase or assimilate other learners' ideas, I was able to get an indication of the learners' ability to be flexible within their understanding (Krummheuer, 2007; Levinson, 1987; Goffman, 1981). By identifying the amount of original contributions to interactions as authors of new ideas, I was able to analyze the novelty of learner reasoning. The concentration component of creative mathematical reasoning was analyzed by observing the length and depth of back-and-forth interactions along one topic of discourse (Prusak et al. 2012, Krummheuer, 2007;, Toulmin, 1964)

Toulmin's (1964) tool for observing plausible arguments was valuable for analysis of the fluency, constructiveness, and concentration components of creative mathematical reasoning

(Prusak, 2012, Krummheuer, 2000). Fluency was analyzed by looking at the amount of arguments learners engaged in to solve the problem (Lithner, 2008). By looking at how learners justify individual arguments, it was possible to analyze how mathematically grounded learner reasoning was (Lithner, 2008; Boesen et al. 2014). By categorizing where learners spend their interactions within Polya's (1957) reasoning structure, one could identify how constructive those interactions were, and how systematic their interactions were in leading them towards an answer (Lithner, 2008, Boesen et al. 2014).

Product Analysis and Evaluation Procedures

To analyze the product of each groups' argument, this research used concepts from Lithner's (2010) Creative Mathematical Reasoning framework as well as Campos's (2010) concepts of mathematical reasoning and Balka's (1974) notion of creativity to evaluate the strength of the groups' argument via Silver (1997). This evaluation was based on learners' written work on worksheets and data used as supporting evidence, as well as a video recording of their presentation. Figure 4.0.3 illustrates the terms and observable indicators used to evaluate the product of Group 1 and Group 2, which would determine if and how creative mathematical reasoning was used.

Product Analysis of Argumentation	
Creative Mathematical Reasoning	**Observable Indicators**
Flexibility	# of correct solutions compared to # of solutions possible
Fluency	Strategies chosen and implemented
Novelty	Uniqueness, departure from the canonical. Are the strategy choices and their implementation new or re-created?
Concentration	**Sequentiality:** Is there an order to their strategy implementation based on their documented evidence? **Continuity:** Does the selected and implemented strategy respond to the objective and did it lead them to their conclusion?
Plausibility	**Mathematically Anchored:** Given their interpretation of premises, is the solution plausible mathematically? Were relevant mathematical properties used to justify their conclusion? **According to socio-mathematical norms:** Were they able to clearly articulate and justify their solution to the researcher?
Constructiveness	**Generalization:** Are students able to extend the solution or strategy to other circumstances? Were they able to utilize prior mathematical knowledge in solving the problem?

Figure 4.0.3 Analysis framework for evaluation of the product of learner argumentation

VITALmaths Clip Selection and Sequence

The six VITALmaths clips were selected and created based on 3 basic criteria. Firstly, the open-ended nature of the tasks: in all the clips, there were limited suggestions of strategies to use and there was limited specificity as to what a correct answer would entail (Cohen, 1994, Stein et al. 2008). Secondly, videos were selected on their accessibility to foundational geometry concepts such as angles, rotations, symmetry, horizontal and vertical (Cowley, 2014). The third criteria was based on the opportunity for learners to exercise spatial skills in a tangible way (Mayer, 2005). Given these three criteria, each video clip brought a unique aspect of learner creative mathematical reasoning to light. This chapter presents a vertical analysis of how Groups 1 and 2 performed in each of the 6 video clips. Due to the large amount of data collected and analyzed, the last three task analyses of Mazes, Pathways, and Bricklaying have been abridged.

4.1 VITALMATHS VIDEO CLIP FOUR MATCHES ANALYSIS AND EVALUATION

Introduction to the problem task

The VITALmaths video clip Four Matches requires learners to determine how many different shapes can be made using 4 matchsticks where the matchsticks are aligned vertically or horizontally and each matchstick meeting at least one other matchstick end to end. It has an open-ended nature in that a set number of shapes to find is not provided.

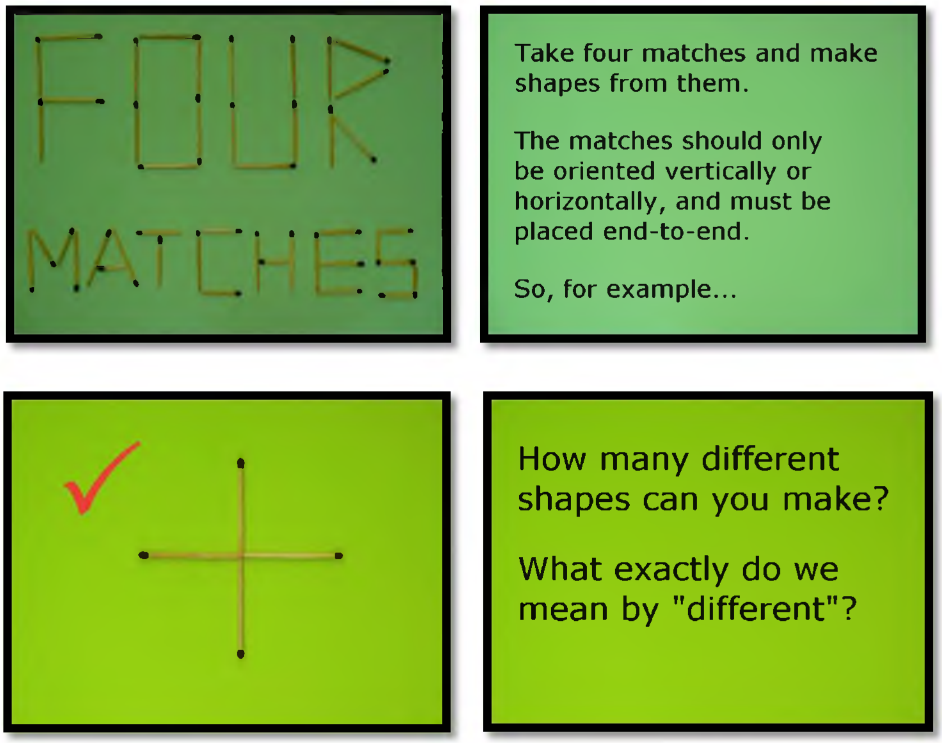


Figure 4.1.0.1 Screenshots of Four Matches VITALmaths video clip illustrate the problem task

The notion of "different" is not defined, which requires the learner to distinguish reflections as similar or different. Learners are required to use mathematical concepts of vertical, horizontal, similarity, rotations, and reflections. This task is beneficial for evaluating creative mathematical reasoning for several reasons. The number of matchstick shapes created indicates the flexibility of thinking (Silver, 1997), and their discrimination between which shapes are similar and which are different is an indicator of how mathematically grounded their arguments are (Lithner, 2010). This is a good generalization exercise, yielding observations about their mathematical understanding of reflections, rotations, similarity, and vertical and horizontal spatial relationships in a contextually rich task. Figure 4.1.0.1 shows screen shots that explain the critical information required to solve the Four Matches task.

Potential solutions to Four Matches problem task

Depending on a learner's notion of "different", there are two potential answers. If learners considered reflections to be the same, then the shapes with an asterisk can be omitted and there would be 14 solutions. If reflections are considered as "different" then there are 25 potential solutions. This solution was organized by how many combinations there were based on consecutive vertical matchsticks. Figure 4.1.0.2 displays one way of solving the Four Matches task.

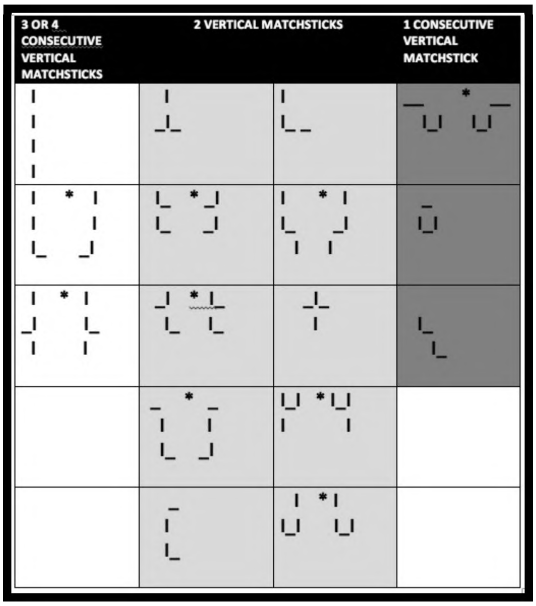


Figure 4.1.0.2 Four Matchsticks potential solutions

4.1.1 Group 1 Documentation of Four Matches Task

Group 1 was video and audio recorded over 30:45 minutes and 259 interactions as learners solved the Four Matches task. Interactions were transcribed and translated for analysis of their participation, argumentation and interaction within the reasoning structure. (See Figures 4.1.1.1 and 4.1.1.2). Figure 4.1.1.3 is a scanned image of the worksheet that acted as a scaffold for interactions. Group 1 elected to document their work on the whiteboard, which is illustrated in the adjacent image (see Figure 4.1.1.4) for analysis. Figure 4.1.1.5 was presented to the researcher as Group 1's final argument for the Four Matches Task.

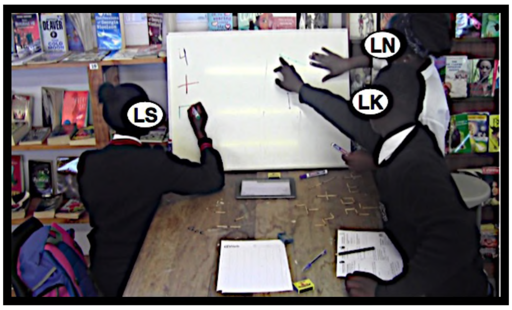


Figure 4.1.1.1 Screenshot of video footage of Group 1 solving the Four Matches task.

GROUP 1 ANALYSIS SUMMARY - MATCHES

Amount of Interaction			Argument Interaction	
Time of Interaction		30:43	Claims Made	66
Lines of Interaction		259	Warrants	21
Individual Contributions To Interaction			Backing	5
LS		95	Sustained Interactions Around One Argument	13
LK		75	**Reasoning Structure**	
LN		61	Comprehension	47
R		28	Argument	92
Participation			Strategy Choice	15
LS	Author	72	Strategy Implementation	40
	Relayer	11	Conclusion	11
	Spokesman	12	Presentation	33
	Ghost	0	Side Conversation	6
LK	Author	54		
	Relayer	10		
	Spokesman	9		
	Ghost	2		
LN	Author	43		
	Relayer	9		
	Spokesman	8		
	Ghost	0		
R	Author	11		
	Relayer	1		
	Spokesman	17		
	Ghost	0		

Figure 4.1.1.2 Displays the analysis of Group 1 interactions during the Four Matches problem task

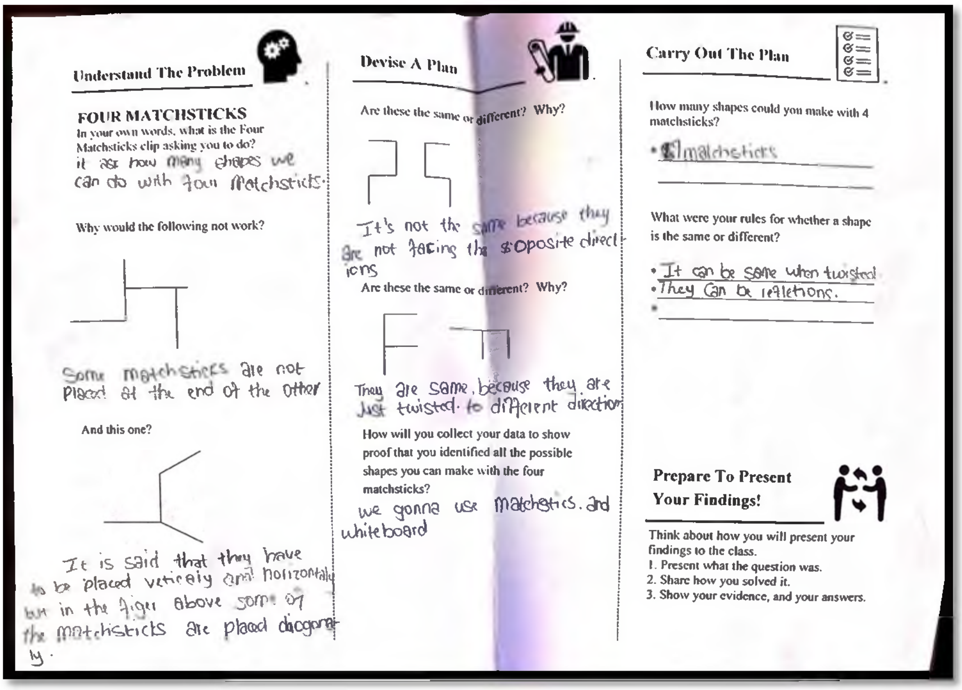


Figure 4.1.1.3 Documentation of Group 1 worksheet used to solve the Four Matches task

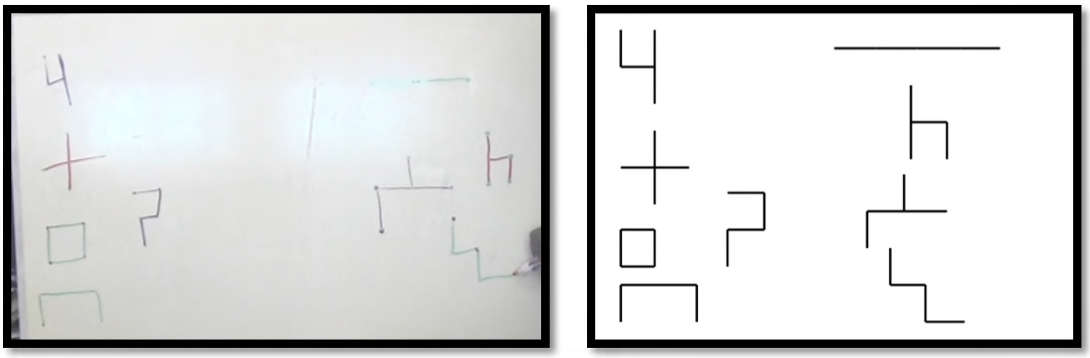
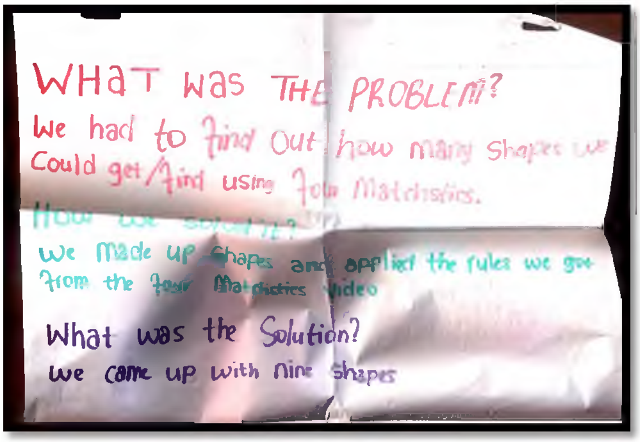


Figure 4.1.1.4 Documentation of Group 1 evidence presented as part of their solution to Four Matches task



Group 1 Four Matchsticks Argument:

What was the problem?
We had to find out how many shapes we could get/find using four matchsticks.

How did you solve it?
We made up shapes and applied rules we got from the four matchsticks video

What was the solution?
We came up with 9 shapes

4.1.1.5 Written documentation of Group 1 solution to the Four Matches task

4.1.2 Group 2 Documentation of Four Matches Task

The video and audio recording of Group 2 solving the Four Matches task was recorded over 26:34 minutes (see Figures 4.1.2.1 and 4.1.2.2). During this recording, 90 interactions were transcribed, translated and analyzed according to learner participation, argumentation, and interaction within the reasoning structure (see Figure 4.1.2.2). Figure 4.1.2.3 is a screen shot of the worksheet they filled out as they solved the problem. Figure 4.1.2.5 shows what learners produced to present their final argument. Figure 4.1.2.3 is documentation of student work as they solved the task while Figures 4.1.2.4 and 4.1.2.5 is the evidence of Group 2's solution to the task. While Group 2 eventually came up with 8 shapes, the researcher prompted them to keep looking when they had only found 4.

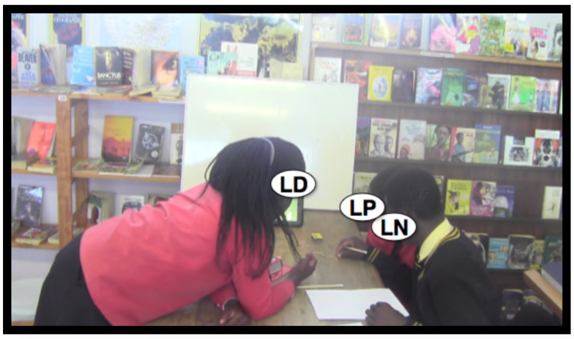


Figure 4.1.2.1 Screenshot of video footage of Group 2 solving the Four Matches task

GROUP 2 ANALYSIS SUMMARY - MATCHES

Amount of Interaction			Argument Interaction	
Time of Interaction		26:34	Claims Made	18
Lines of Interaction		90	Warrants	10
Individual Contributions To Interaction			Backing	0
LT		19	Sustained Interactions Around One Argument	4
LP		24	**Reasoning Structure**	
LN		39	Comprehension	47
R		8	Argument	28
Participation			Strategy Choice	8
LT	Author	12	Strategy Implementation	6
	Relayer	6	Conclusion	2
	Spokesman	1	Presentation	0
	Ghost	0	Side Conversation	0
LP	Author	21		
	Relayer	3		
	Spokesman	0		
	Ghost	0		
LN	Author	29		
	Relayer	5		
	Spokesman	5		
	Ghost	0		
R	Author	3		
	Relayer	0		
	Spokesman	5		
	Ghost	0		

Figure 4.1.2.2 Analysis of Group 2 interactions during the Four Matches task

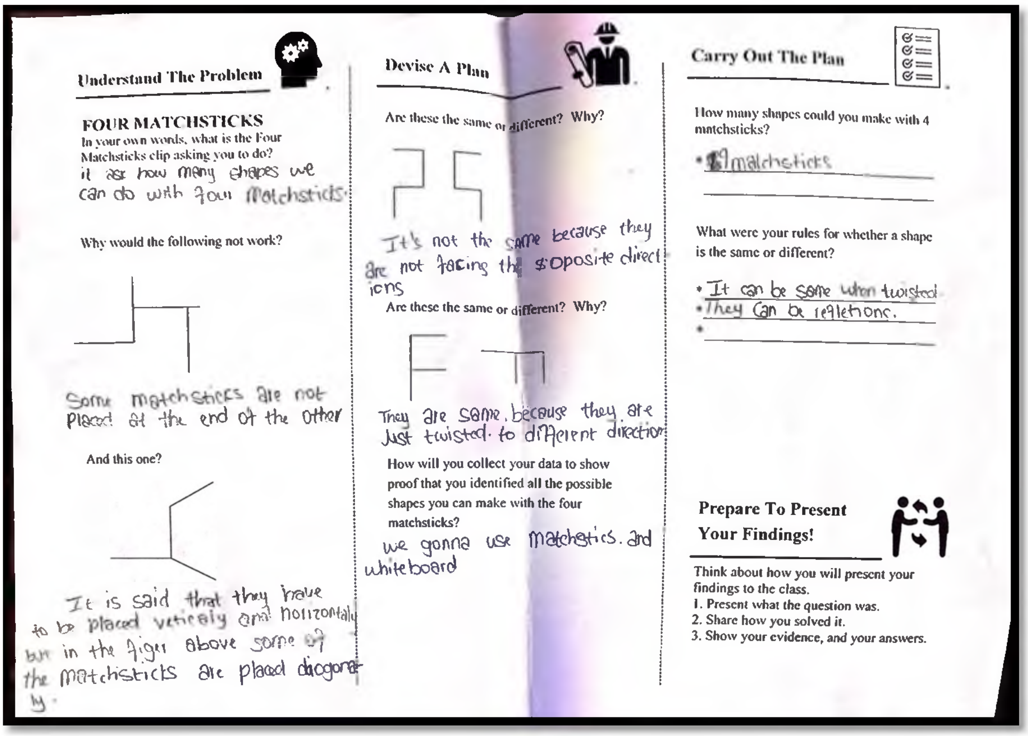


Figure 4.1.2.3 Documentation of Group 2 worksheet used to solve the Four Matches task

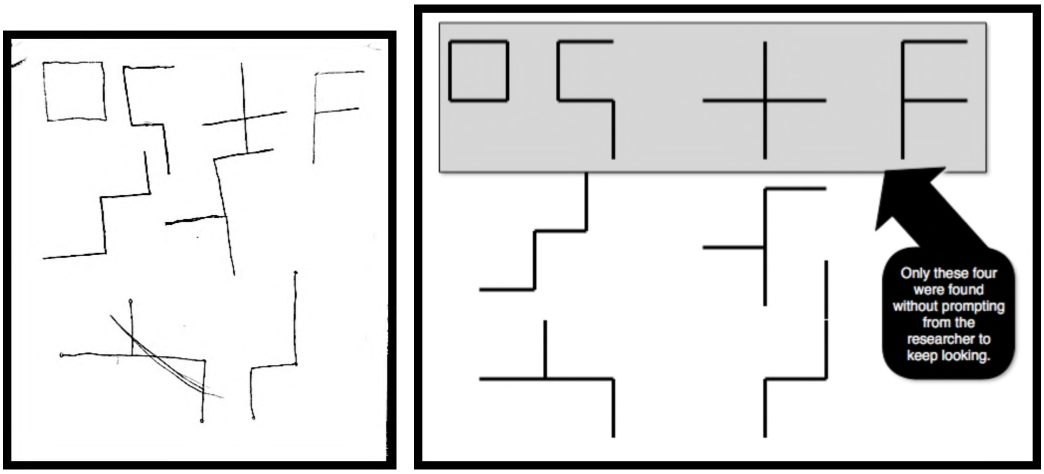
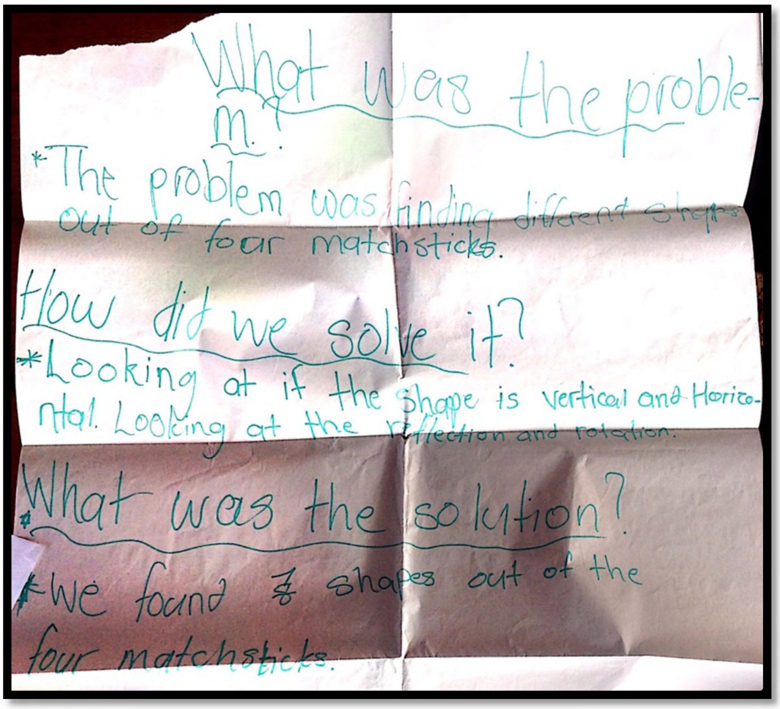


Figure 4.1.2.4 Documentation of Group 2 evidence presented as part of their solution to Four Matches task



Group 2 Four Matchsticks Argument:

What was the problem?
The problem was finding different shapes out of four matches.

How did you solve it?
Looking at if the shape is vertical and horizontal. Looking at the reflection and rotation.

What was your solution?
We found 8 shapes out of the four matchsticks.
(ONLY FOUR FOUND WITHOUT PROMPTING FROM THE RESEARCHER)

Figure 4.1.2.5 Written documentation of Group 2 solution to Four Matches task

4.1.3 Analysis of Group 1 and 2 Interactions Solving the Four Matches Task

Group 1 and Group 2 interacted in very different ways to each other during this task. Group 1 had 65% more interactions than Group 2 and 72% more claims as they solved the matches task (see Figure 4.1.3.1).

Description	Group 1	Group 2
Claims	66	18
Total interactions	259	90

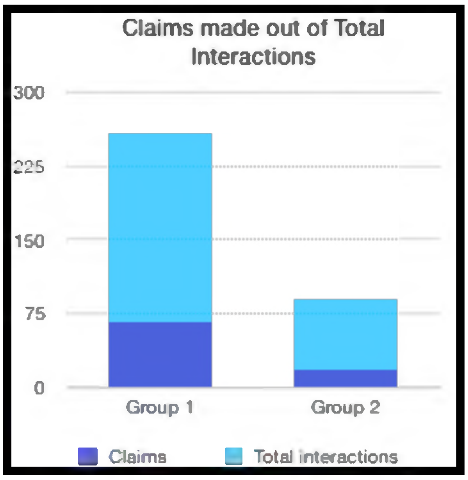


Figure 4.1.3.1 Compares the amount of interactions and the amount of claims made during interaction between Group 1 and Group 2 during the Four Matches task

Each group showed one learner who interacted the most, and all the learners initiated their own ideas in most of their interactions (see Figure 4.1.3.2). In Group 1, LS directed most of the interactions, while LK and LN share near equal contributions. They also had similar relayer and spokesman participation, whereas significant differences showed in the author participation. Group 2 displayed a different participation pattern. LD contributed to most of the interactions, and also contributed more relayer and spokesman participation roles than her peers.

Description	LS	LK	LN	R
Author	72	54	43	11
Relayer	11	10	9	1
Spokesman	12	9	8	17
Ghost	0	2	0	0
Total	**95**	**75**	**60**	**29**

Description	LT	LP	LD	R
Author	12	21	29	3
Relayer	6	3	5	0
Spokesman	1	0	5	5
Ghost	0	0	0	0
Total	**19**	**24**	**39**	**8**

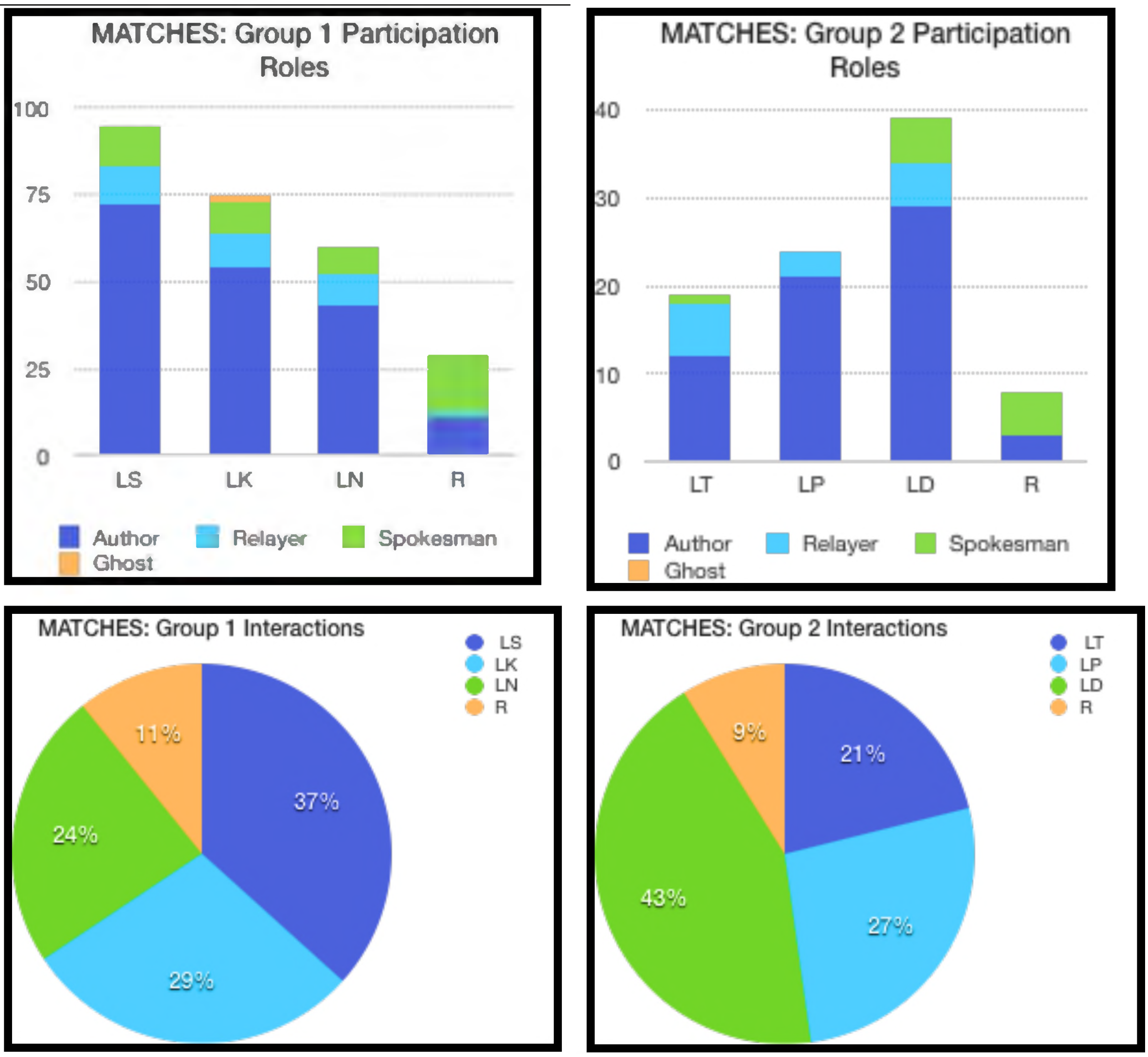


Figure 4.1.3.2 Compares individual participation to the group, comparing how original each learner's statements are in interaction with their peers

Group 1 and Group 2 spent their time completing the Four Matches task in different ways. While Group 1 spent 37% of their time solving the problem (argument), Group 2 spent 52% of their time understanding the task (comprehension) (see Figure 4.1.3.3). Very little time was spent selecting and implementing a strategy for systematically solving the problem, and even less time determining if they had reached an accurate conclusion. Group 1 spent more interactions implementing a strategy than selecting a strategy while Group 2 spent more time selecting the strategy than implementing (though by a difference of only 2 interactions).

Reasoning Structure	interactions
Comprehension	47
Strategy Choice	15
Strategy Implementation	42
Argument	92
Conclusion	11
Presentation	33
Side Conversation	6

Reasoning structure	interactions
Comprehension	47
Strategy Choice	8
Strategy Implementation	6
Argument	28
Conclusion	2
Presentation	0
Side Conversation	0

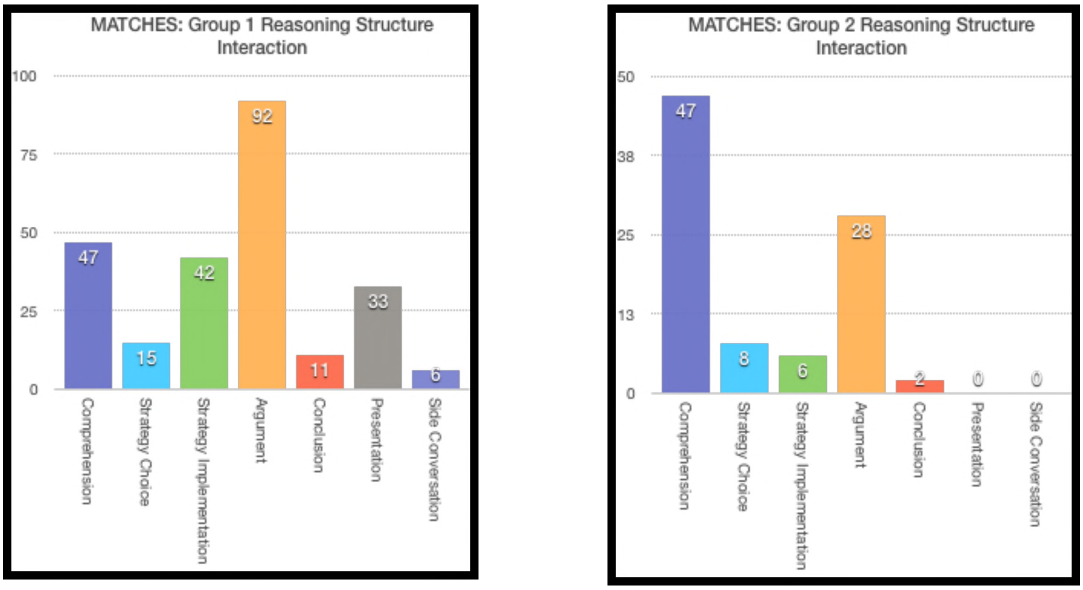
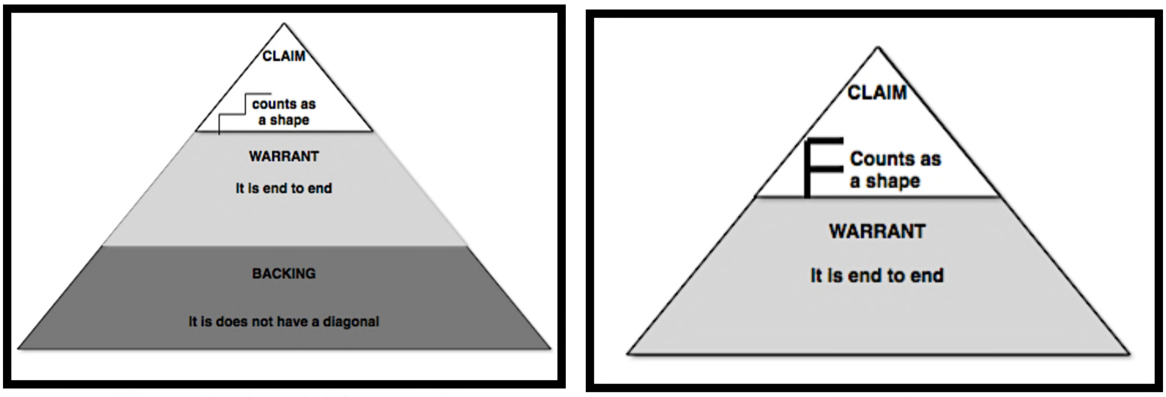


Figure 4.1.3.3 Illustrates where learners in Group 1 and 2 spent their time within the reasoning structure as they solved the Four Matches task

Group 1 supported their arguments with a warrant and a backing 5 times, whereas Group 2 only made warrant justifications (see Figure 4.1.3.4). Group 1 had 13 lines of sustained interactions around a challenge, while Group 2 sustained 4 interactions around a challenge within the task. Group 1 had deeper justifications to their individual claims by adding warrants and backing to their claims.



GROUP 1 **GROUP 2**

Figure 4.1.3.4 Compares the depth of the strongest argument made during interactions. When Group 1 said it does not have a diagonal, they meant each matchstick met at a straight angle or right angle

4.1.4 Evaluation of Process of Group 1 and Group 2 Solution to Four Matches Task

Both groups showed a lot of initiative solving the Four Matches task. However, Group 1 and Group 2 worked in very different ways. Group 1 shared a lot of interaction, while Group 2 did not. Figure 4.20 shows the process evaluation of Group 1 and Group 2 interactions during the Four Matches Task. Group 1 received a score of 19 out of 30. The group showed flexibility in the amount of interaction that took place around the task. They spent a moderate amount of time on conceptually understanding the mathematics of the task. The learners in group one showed good evidence of expressing initiative in solving the task. They received a “2” for concentration due to limited interactions around choosing strategies to solve and implement the task to its completion. Group 1 did show evidence of a depth in their justifications, but little of it was based on mathematical concepts. A lot of guess and check was used without developing a system for analyzing how many possible solutions there may be. While the interactions between learners in Group 1 were balanced, only a small portion of the interactions was responded to or assimilated in each other’s statements.

Group 2 received a 13 out of 30 score in the evaluation of their interactions while solving the problem. They had a limited amount of interaction in their argumentation as they solved the problem. Most interactions were short and did not show evidence of engagement around the mathematical concepts, and were often one-sided exclamations of what was a valid claim or not. All the members showed initiative in their interactions in proportion to the amount of interaction they had. The group spent very little time on actually selecting a strategy and following through with the strategy. Learners in Group 2 provided little evidence to suggest plausibility to their claims, and there was very little mathematical grounding to their claims. Group 2 received a “2” because LD dominated conversations and only a limited amount of interaction was spent assimilating other group members’ ideas in their interactions.

FOUR MATCHES PROCESS EVALUATION	GROUP 1	GROUP 2
Reasoning Abilities	**Observable Indicators**	**Observable Indicators**
Flexibility: # of Arguments Made	66 Arguments/8 Shapes Found 1 2 3 **4** 5	18 Arguments/4 Shapes Found 1 **2** 3 4 5
Fluency: Sustained Interaction	13 Lines of Sustained Interaction 1 2 **3** 4 5	4 Lines of Sustained Interaction **1** 2 3 4 5
Initiative: Authored Participation	74% Author Statements 1 2 3 **4** 5	76% Author Statements 1 2 3 **4** 5
Concentration: Interactions across reasoning structure	Limited interaction around strategy choice and implementation or conclusion. 1 **2** 3 4 5	Extensive interaction spent on comprehension, limited interaction around strategy choice and implementation. 1 **2** 3 4 5
Plausability: Depth of Mathematical Justifications	Some evidence of warrant and backing to claims, limited **mathematical** justification 1 2 **3** 4 5	Limited evidence of warrants to claims. Limited **mathematical** justification 1 **2** 3 4 5
Constructiveness: Balance of contributions/Incorporating new ideas	Even distribution of interaction between learners. Limited evidence of incorporating group member ideas. 1 2 **3** 4 5	LD contributed 43% of interactions. Limited evidence of incorporating group member ideas. 1 **2** 3 4 5
	TOTAL: 19/30	**TOTAL: 13/30**

Marking Criteria:

1	2	3	4	5
No Evidence	Weak Evidence	Moderate Evidence	Good Evidence	Strong Evidence

Figure 4.1.4 Framework of Group 1 and Group 2 Evaluation of Process in solving the Four Matches Task

4.1.5 Evaluation of Product of Group 1 and Group 2 Solution to Four Matches Task

Group 1 and Group 2 were creative in the ways they approached the task, but struggled to systematically find a comprehensive list of shapes. Figure 4.1.5 shows the product evaluation of Group 1 and Group 2 Solutions to the Four Matches Task. Group 1 identified 8/25 possible solutions. They used guess and check to solve the task, and did not develop a system to their method to ensure they found as many solutions as possible. Strategies for solving the task were limited. They did label different shapes with unique names, but did not use them to identify other shapes. If the group had tried finding all the letter shapes they could, this would have shown more novelty of strategy. There was no apparent sequentiality or continuity of strategy implementation. Mathematical justifications were limited, and they were unable to defend their method for ensuring they found all the possible solutions. The group showed limited constructiveness in their ability to generalize what they learned in previous tasks, although they attempted to use personal knowledge of the shapes to which they referred.

FOUR MATCHES PRODUCT EVALUATION	GROUP 1	GROUP 2
Reasoning Abilities	**Observable Indicators**	**Observable Indicators**
Flexibility: # of Correct Solutions	8/25 Solutions 1 | 2 | **3** | 4 | 5	4/25 Solutions 1 | **2** | 3 | 4 | 5
Fluency: Strategies implemented.	Guess and check strategy 1 | **2** | 3 | 4 | 5	Guess and check strategy 1 | **2** | 3 | 4 | 5
Novelty: Uniqueness of strategies.	Giving shapes names to refer. "F" "Dice" "Chair" 1 | **2** | 3 | 4 | 5	None evident. **1** | 2 | 3 | 4 | 5
Concentration: Sequentiality and Continuity of Strategy Implementation	There was no system evident. Attempts seemed sporadic and disconnected. **1** | 2 | 3 | 4 | 5	There was no system evident. Attempts seemed sporadic and disconnected **1** | 2 | 3 | 4 | 5
Plausability: Mathematically Anchored Sociomathematical norms	Some evidence, limited **mathematical** justification 1 | **2** | 3 | 4 | 5	Limited evidence. Limited **mathematical** justification. 1 | **2** | 3 | 4 | 5
Constructiveness: Generalization to other concepts or experiences	Did not use terminology learned in previous clips. Connections to shapes and personal knowledge of what they resembled. 1 | **2** | 3 | 4 | 5	Used rotation and reflections in their work as they solved the problem. Limited connections to experiences. 1 | **2** | 3 | 4 | 5
	TOTAL: 12/30	**TOTAL: 10/30**

Marking Criteria:

1	2	3	4	5
No Evidence	Weak Evidence	Moderate Evidence	Good Evidence	Strong Evidence

Figure 4.1.5 Framework of Group 1 and Group 2 Evaluation of Product of solutions to Four Matches Task

Group 2 identified 4 out of 25 possible solutions without prompting from the researcher to find more. This showed a limited amount of evidence of flexibility in their solution. Guess and check strategy was used to verify their solutions, but no system was implemented for ensuring they could find all the possible solutions. There was no evidence of novelty in the strategies employed; only random guess and check seemed evident. There was no evident system in their justifications. Attempts seemed sporadic and disconnected. When explaining how the problem was solved, Group 2 used the terms rotation and reflections, but did not explain how these pertained to their solutions. The terms were present but meaningless in their use of the terminology. Group 2 received a “2” for constructiveness because of their familiarity with the terms horizontal and vertical, rotations and reflections, but limited connections to previous experiences.

4.1.6 Summary of Four Matches analysis and evaluation

During the process of solving the Four Matchsticks task, both groups struggled with developing a strategy for finding a comprehensive solution to the task. There was no system to the guess and check methods and while some other solutions may have been found, they were not documented systematically. Group 1 was able to sustain conversations around specific challenges, and made more and stronger arguments as they solved the task.

In the evaluation of the final product of Group 1 and Group 2 solving the Four Matches task, both groups performed poorly. Group 2 in particular gave up after finding only 4 solutions out of a possible 25 solutions. Neither group used mathematical justifications to support their arguments, and explanations on how they solved the task were superficial and did not illustrate any system or mathematical method to their solving of the task.

4.2 VITALMATHS VIDEO CLIP ANGLES ANALYSIS AND EVALUATION

Introduction to the problem task

In the VITALmaths video clip Angles, learners explored the notion of acute right and obtuse angles. Learners were given pre-cut angles, the same as in the video clip, to be able to manipulate the different angles and come up with combinations. The clip was open-ended in that the clip does not specify how many combinations should be found. The mathematical concepts the clip addresses are the concepts of acute, right, and obtuse angles and their relationships to one another, as well as the notion that angles can be combined to make complementary angles. This clip was unique for evaluation of creative mathematical reasoning, in that flexibility of learner thinking could be observed by looking at how many combinations the students could find. It also allowed the opportunity to see how they justified their thinking, using cut out angles or addition to ensure they were less than 90°, 90° or greater than 90°. This task also provided a clear way of observing if learners developed constructive systems to identifying combinations. Figure 5.2.0.1 shows the screenshots of the angles in the VITALmaths clip critical to solving the task.

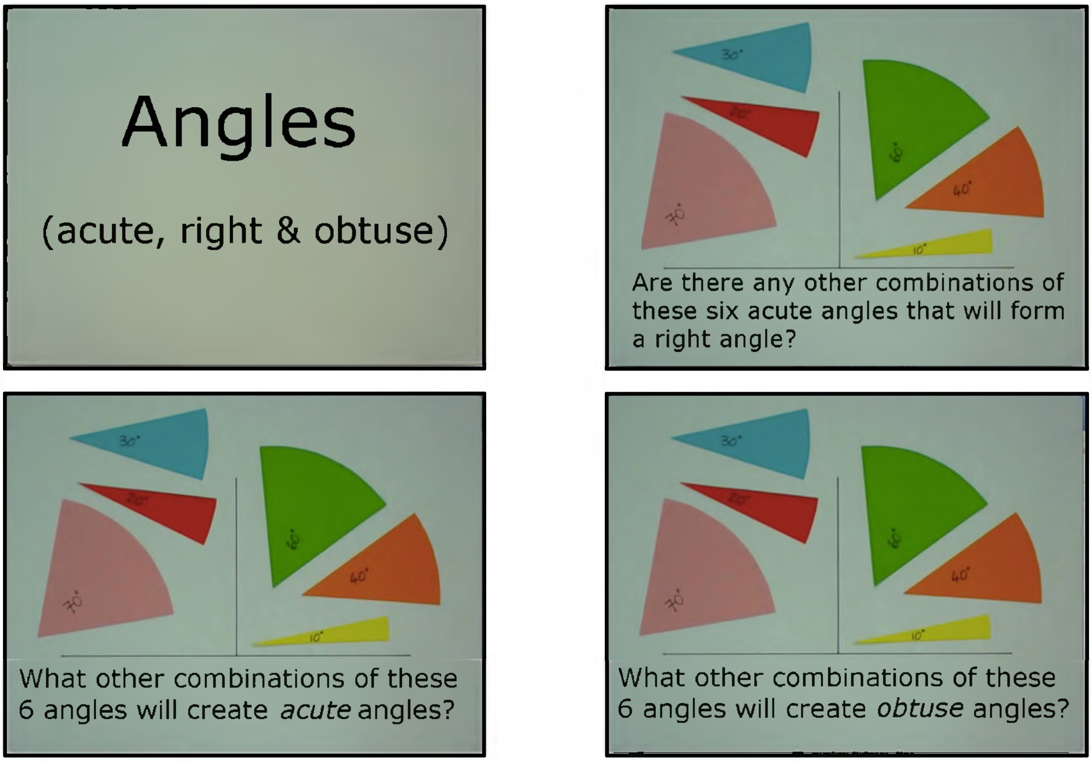


Figure 4.2.0.1 VITALmaths video clip Angles - profile in brief

Potential solutions

There are several ways to solve the Angles task. Figure 5.2.0.2 begins with finding all the possible angles with the largest angles first, and systematically finds the angles with progressively smaller angles. With acute and obtuse angles, the largest angles closest to 90° and 180° were found first and systematically moved towards finding the smallest possible angles.

Right Angles		Acute Angles
4 Combinations		**12 Combinations**
70° + 20° = 90°		70° + 10° = 80°
60° + 30° = 90°		60° + 20° = 80°
60° + 20° + 10° = 90°		60° + 10° = 80°
40° + 30° +20° = 90°		40° + 30° + 10° = 80°
		40° + 30° = 70°
		40° + 20° + 10° = 70°
		40° + 10° = 50°
		30° + 20° + 10° = 60°
		30° + 20° = 50°
		30° + 10° = 40°
		20° + 10° = 30°

Obtuse Angles		Straight Angles
25 Combinations		**2 Combinations**
70° + 60° + 40° = 170°		70° + 60° + 30° + 20° = 180°
70° + 60° + 30° + 10° = 170°		70° + 60° + 40° + 10° = 180°
70° + 60°+ 20° + 10° = 160°		
70° + 60° + 30° = 160°		
70° + 60° + 20° = 150°		
70° + 60° + 10° = 140°		
70° + 60° = 130°		
…….		
40° + 30° + 20° + 10° = 100°		

Figure 4.2.0.2 Angles Task solutions

4.2.1 Group 1 documentation of Angles task

The recording of Group 1 video and audio recording took 27:55 minutes and 218 interactions were transcribed and translated for analysis of process of argumentation, as seen in Figures 4.2.1.1 and 4.2.1.2. Figure 4.2.1.3 illustrates the nature of the worksheet they filled out as they solved the problem. Group 1 selected to document their work on the whiteboard (see Figure 5.2.1.4). Figure 4.2.1.5 shows what learners produced to present their final argument.

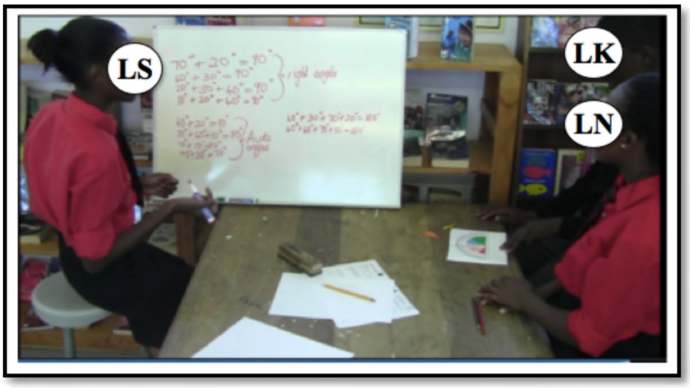


Figure 4.2.1.1 Interactions were transcribed and labeled according to learner role, and the focus of the interaction within the reasoning structure

GROUP 1 ANALYSIS - ANGLES

Amount of Interaction			Argument Interaction	
Time of Interaction		27:55	Claims Made	45
Lines of Interaction		210	Warrants	14
Individual Contributions To Interaction			Backing	9
LS		93	Sustained Interactions Around One Argument	14
LK		51	**Reasoning Structure**	
LN		40	Comprehension	63
R		26	Argument	68
Participation			Strategy Choice	31
LS	Author	69	Strategy Implementation	7
	Relayer	15	Conclusion	6
	Spokesman	7	Presentation	18
	Ghost	2	Side Conversation	0
LK	Author	35		
	Relayer	12		
	Spokesman	3		
	Ghost	1		
LN	Author	18		
	Relayer	10		
	Spokesman	7		
	Ghost	3		
R	Author	11		
	Relayer	0		
	Spokesman	15		
	Ghost	0		

Figure 4.2.1.2 Analysis of Group 1 interaction during Angles task

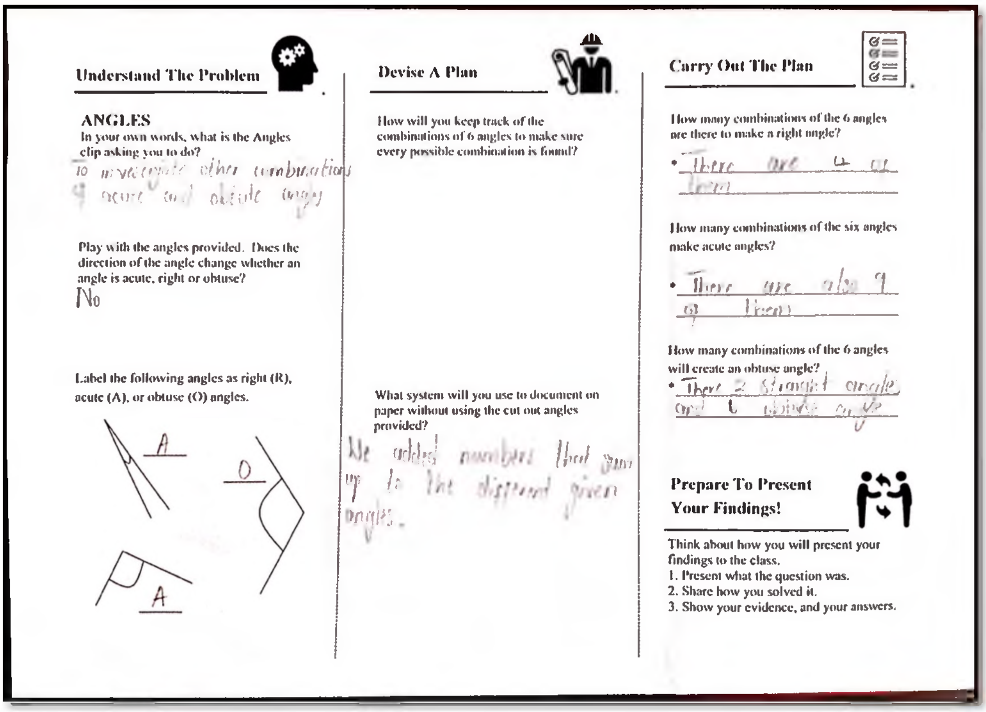
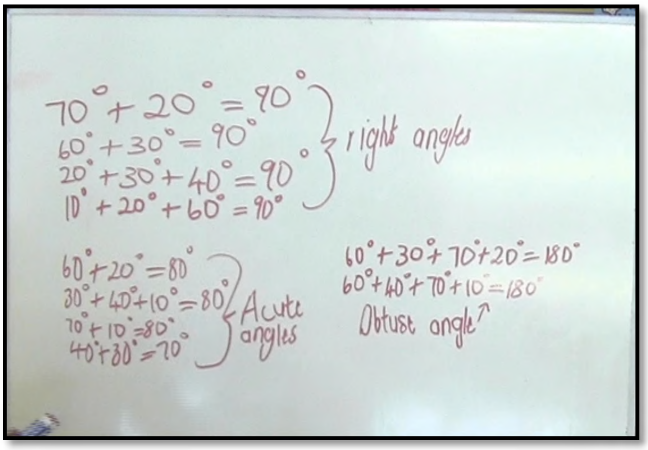
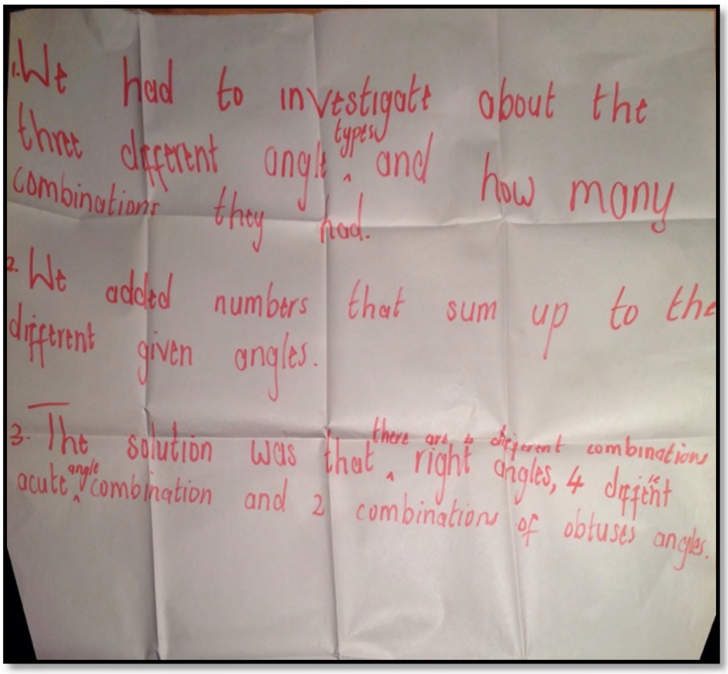


Figure 4.2.1.3 Group 1 responses to the worksheet provided to guide learner interaction of Angles task



Right Angles	Acute Angles	Obtuse Angles
70° + 20° = 90°	60° + 20° = 80°	60° + 30° + 70° + 20° = 180°
60° + 30° = 90°	30° + 40° + 10° = 80°	60° + 40° + 70° + 10° = 180°
20° + 30° + 40° = 90	70° +10° = 80°	
10° + 20° + 60° = 90°	40° + 30° = 70°	

Figure 5.2.1.4 Group 1 documented their work by writing solutions on the whiteboard for clarity; the learner's work is typed below the image



What was the problem?	We had to investigate about the three different angle types and how many combinations they had.
How did you solve it?	We added numbers that sum up to the different given angles.
How can you prove it?	(Evidence provided is found in figure 5.6)
What was the solution?	The solution was that there are 4 different combinations for right angles, 4 different acute angle combinations, and 2 combinations of obtuse angles.

Figure 4.2.1.5 Documentation of Group 1 presentation of their solutions to the problem

4.2.2 Group 2 documentation of Angles task

The recording of Group 2 solving the Angles Task took 29:30 minutes (see Figures 4.2.2.1 and 4.2.2.2). 256 lines of interaction were translated, transcribed and then coded by participation role of the individual learners and the purpose of the statement according to the reasoning structure. Group 2 selected to document their work on paper (See Figure 4.2.2.4A). Figure 4.2.2.3 is photo documentation of the worksheet Group 2 used to guide them through the Angles task. Figure 4.2.2.4B is the data presented as evidence for the solution to the task, and Figure 4.2.2.5 was the final argument presented to the researcher.

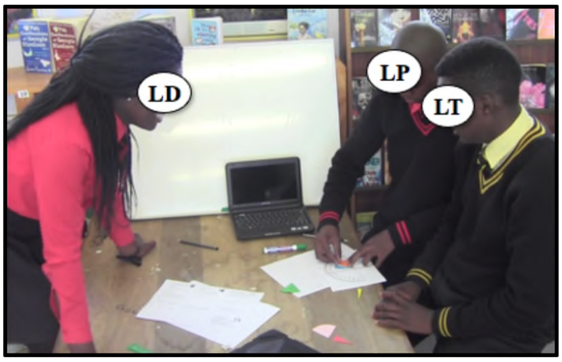


Figure 4.2.2.1 Video and Audio recording was taken of Group 2 over 29:30 minutes, and 256 interactions were transcribed and labeled according to learner role, and the focus of the interaction within the reasoning structure

GROUP 2 ANALYSIS SUMMARY - ANGLES

Amount of Interaction			**Argument Interaction**	
Time of Interaction		29:30	Claims Made	52
Lines of Interaction		256	Warrants	19
Individual Contributions To Interaction			Backing	3
LT			Sustained Interactions Around One Argument	17
LP			**Reasoning Structure**	
LN			Comprehension	48
R			Argument	74
Participation			Strategy Choice	32
LT	Author		Strategy Implementation	30
	Relayer		Conclusion	6
	Spokesman		Presentation	20
	Ghost		Side Conversation	2
LP	Author			
	Relayer			
	Spokesman			
	Ghost			
LN	Author			
	Relayer			
	Spokesman			
	Ghost			
R	Author			
	Relayer			
	Spokesman			
	Ghost			

Figure 4.2.2.2 Analysis of Group 2 interaction during Angles task

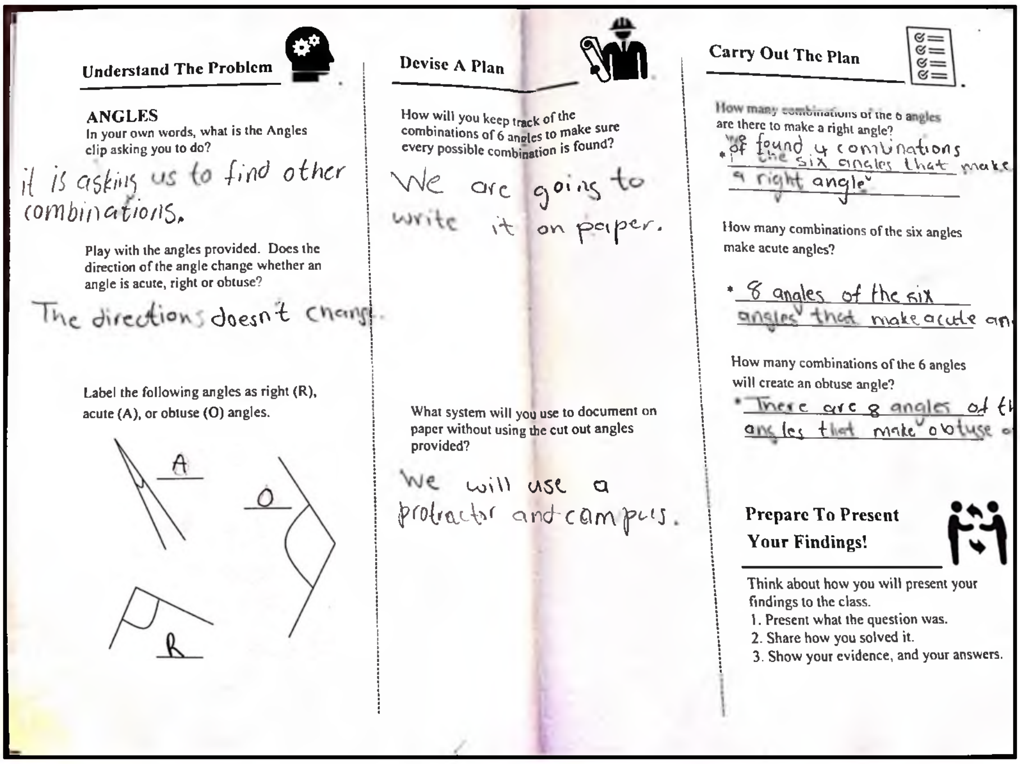


Figure 4.2.2.3 Group 2 responses to worksheet provided to guide learner interaction of Angles task

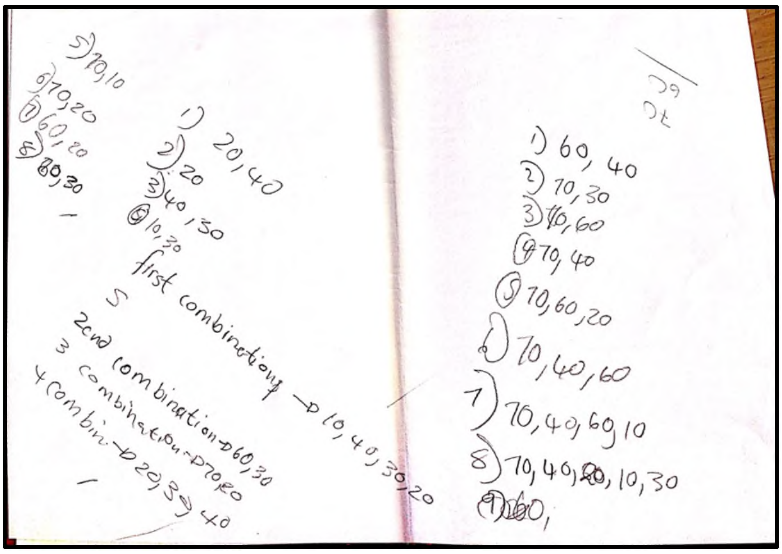
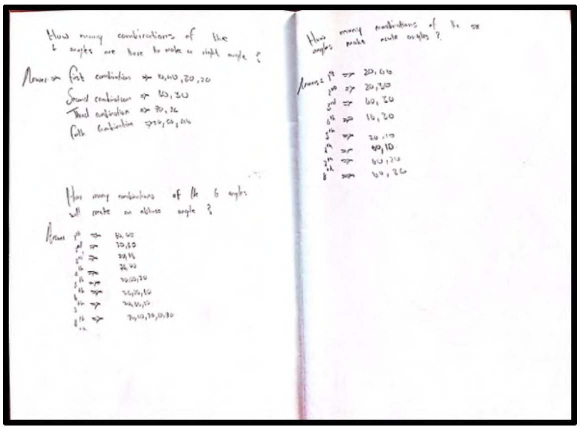
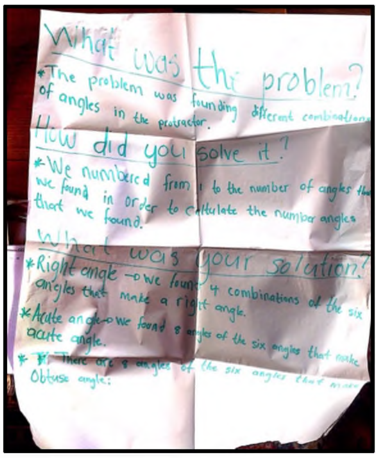


Figure 4.2.2.4A Group 2 informal notes taken during the Angles task



How many combinations of the 6 angles are there to make a right angle?	How many combinations of the six angles make acute angles?	How many combinations fo the 6 angles will create an obtuse angle?
Answer =>	Answer =	Answer
First Combination =>10, 40, 30, 20	1st => 20, 40	1st => 60, 40
Second Combination => 60, 30	2nd => 20, 30	2nd => 70, 30
Third Combination => 70, 20	3rd => 40, 30	3rd => 70, 60
Fourth Combination => 20, 30, 40	4th => 10, 30	4th => 70, 40
	5th => 70, 10	5th => 70, 60, 20
	6th => 60, 10	6th => 70, 40, 60
	7th => 60, 20	7th => 70, 40, 20, 10, 30
	8th => 60, 30	

Figure 4.2.2.4B Group 2 data presented to support their argument for Angles task



What was the problem?	The problem was finding different combinations in the protractor.
How did you solve it?	We numbered from it to the number of angles we found in order to calculate the number of angles that we found.
How can you prove it	(Evidence provided is found in Figure 4.2.2.4B)
What was the solution?	Right angle - We found 4 combinations of the six angles that make a right angle Acute angle – we found 8 angles of the six angles that make an acute angle. There are 8 angles for the six angles that make an obtuse angle.

Figure 4.2.2.5 Group 2 written presentation of their solution to the ANGLES Task

4.2.3 Analysis of Group 1 and Group 2 solving the Angles task

Argumentation can be analyzed as both process and product (Kuhn, & Udell, 2003). By looking at how learners engage and interact around an open-ended mathematical task, I analyzed the process of argumentation in which learners interacted around the task of finding different combinations of angles to make right, acute and obtuse angles. Figure 5.2.3.1 illustrates that out of 210 interactions, Group 1 made 45 arguments. Group 1 had 8 correct solutions out of 41 possible solutions. Group 2 had 52 arguments out of 256 interactions and identified 13 out of 41 possible solutions.

Description	Group 1	Group 2
Claims	45	52
Total Interactions	210	256

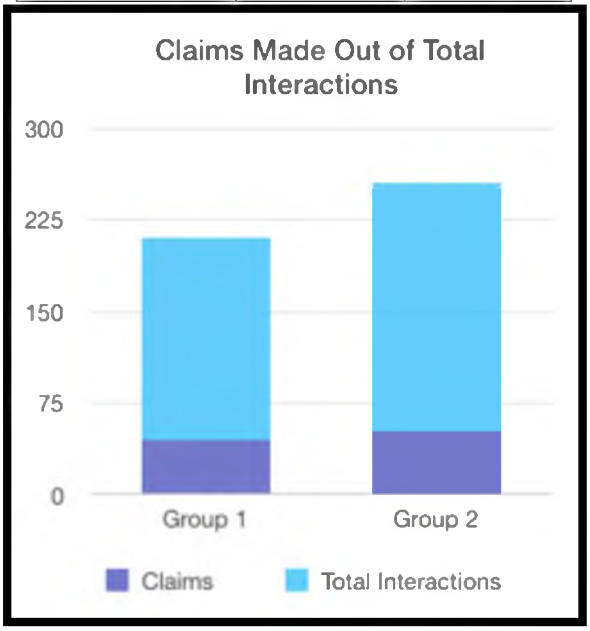


Figure 4.2.3.1 Comparison of the amount of interactions and the amount of claims Group 1 and Group 2 made during the Angles task

To evaluate the initiative of group members and the constructiveness of interactions between group members, each interaction was analyzed according to the type of contribution made (Krummheuer, 2007) (See Figure 5.2.3.2). In Group 1, LS made 44% of contributions, of which 71% of her statements were author statements and 7.5% spokesman statements. LK made 24% of contributions, of which 68% of his contributions were author statements and 5.8% spokesman statements. LN made 19% of contributions, of which 45% were author statements and 17.5% spokesman statements.

Group 2 interactions showed similarly that one person was more dominant in conversations. They showed more spokesman statements for each member. LT made the most contributions with 45% of interactions, of which 58% were author statements and 26.9% were spokesman statements. LD made 27% of interactions, of which 50% were author statements and 15.7% were spokesman statements. LP made 21% of interactions, of which 76.5% were author statements and 16.3% were spokesman statements.

Description	LS	LK	LN	R
Author	69	35	18	11
Relayer	15	12	10	0
Spokesman	7	3	7	15
Ghost	2	1	3	0
Total	**93**	**51**	**40**	**26**

Description	LD	LT	LP	R
Author	35	67	42	6
Relayer	21	13	4	0
Spokesman	11	31	9	12
Ghost	3	4	0	0
Total	**70**	**115**	**55**	**15**

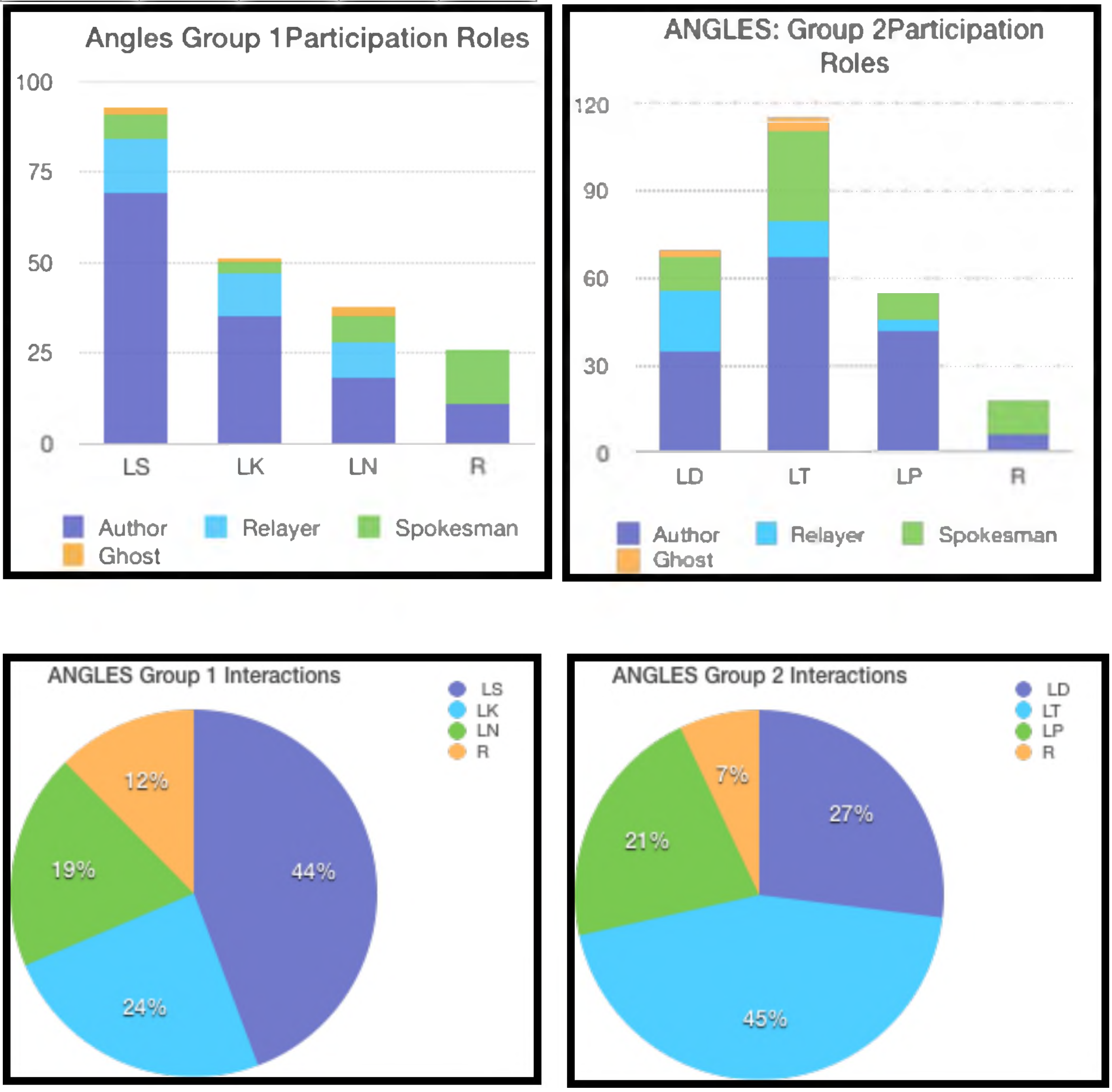
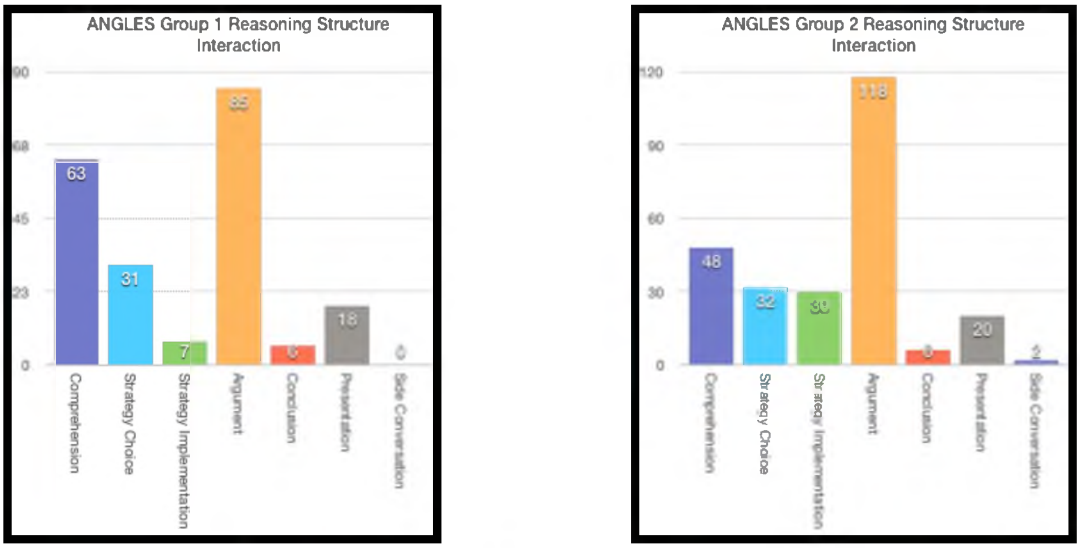


Figure 4.2.3.2 Analysis of each learner's contribution to the interactions as a whole

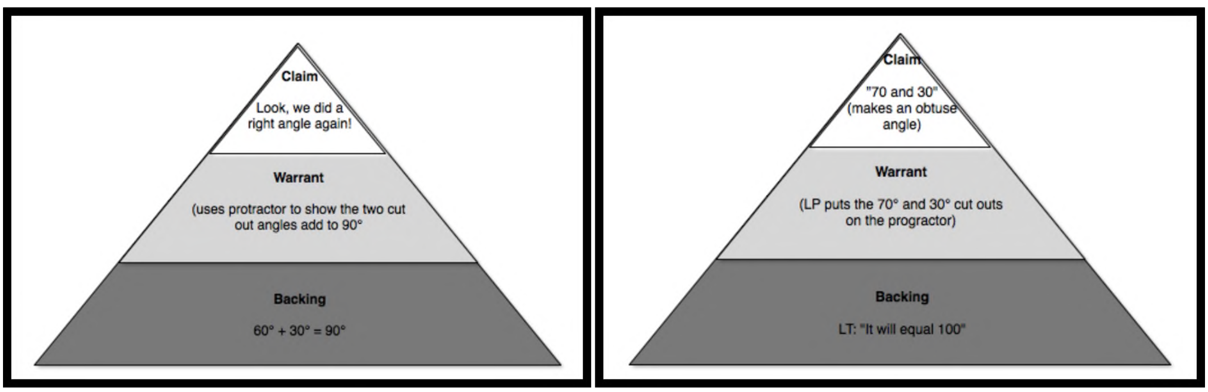
Figure 5.2.3.2 allows for an evaluation of the concentration of Group 1 and Group 2's interaction. By identifying how the groups spent their time, patterns emerge as to how the groups solved problems. Group 1 spent 31% of their interactions understanding the problem, compared to Group 2's 18.7%. When looking at how much interaction was spent on identifying strategies and implementing them, Group 1 spent 18% compared to Group 2's 24%. 40% of Group 1's interactions were spent on solving the task, compared to Group 2 who spent 46% of their interactions solving the task.



Reasoning Structure	Interactions
Comprehension	63
Strategy Choice	31
Strategy Implementation	7
Argument	85
Conclusion	6
Presentation	18
Side Conversation	0

Reasoning Structure	Interactions
Comprehension	48
Strategy Choice	32
Strategy Implementation	30
Argument	118
Conclusion	6
Presentation	20
Side Conversation	2

Figure 4.2.3.3 Comparison of the distribution of interactions within the reasoning structure



Group 1 Argument Depth **Group 2 Argument Depth**

Figure 4.2.3.4 Comparison of the depth of arguments Group 1 and Group 2 made during the Angles task

Figure 4.2.3.4 illustrates the depth of each group's argument and the foundation of the arguments made. Both Groups 1 and 2 used warrants and backing to support their arguments, and so the solutions the groups found were well grounded, but no arguments were made in regard to whether or not they found all the possible solutions.

Group 1 **Right Angles**	**Solutions** **Right Angles**		**Group 1** **Acute Angles**	**Solutions** **Acute Angles**
4 Combinations	**4 Combinations**		**4 Combinations**	**12 Combinations**
70° + 20° = 90°	70° + 20° = 90°		60° + 20° = 80°	70° + 10° = 80°
60° + 30° = 90°	60° + 30° = 90°		30° + 40° + 10° = 80°	60° + 20° = 80°
20° + 30° + 40° = 90	60° + 20° + 10° = 90°		70° +10° = 80°	60° + 10° = 80°
10° + 20° + 60° = 90°	40° + 30° +20° = 90°		40° + 30° = 70°	40° + 30° + 10° = 80°
				40° + 30° = 70°
				40° + 20° + 10° = 70°
				40° + 10° = 50°
				30° + 20° + 10° = 60°
				30° + 20° = 50°
				30° + 10° = 40°
				20° + 10° = 30°
Group 1 **Obtuse Angles**	**Solutions** **Obtuse Angles**		**Group 1** **Straight Angles**	**Solutions** **Straight Angles**
0 Combinations	**25 Combinations**		**2 Combinations**	**2 Combinations**
Created Straight	70° + 60° + 40° = 170°		60° + 30° + 70° + 20° = 180°	70° + 60° + 30° + 20° = 180°
Angles	70° + 60° + 30° + 10° = 170°		60° + 40° + 70° + 10° = 180°	70° + 60° + 40° + 10° = 180°
	70° + 60°+ 20° + 10° = 160°			
	70° + 60° + 30° = 160°			
	70° + 60° + 20° = 150°			
	70° + 60° + 10° = 140°			
	70° + 60° = 130°			
	…….			
	40° + 30° + 20° + 10° = 100°			

Figure 4.2.3.4.A Group 1 responses compared to possible solutions to each sub-question

Group 2 Right Angles	Solutions Right Angles		Group 2 Acute Angles	Solutions Acute Angles
3 Combinations	**4 Combinations**		**7 Combinations**	**12 Combinations**
INCORRECT	70° + 20° = 90°		INCORRECT	70° + 10° = 80°
10° + 40° + 30° + 20° = 100°	60° + 30° = 90°		60° + 30° = 90°	60° + 20° = 80°
	60° + 20° + 10° = 90°			60° + 10° = 80°
CORRECT	40° + 30° +20° = 90°		CORRECT	40° + 30° + 10° = 80°
60° + 30° = 90°			20° + 40° = 60°	40° + 30° = 70°
70° + 20°= 90°			20° + 30° = 60°	40° + 20° + 10° = 70°
20° + 30° + 40° = 90°			40° + 30° = 70°	40° + 10° = 50°
			10° + 30° = 40°	30° + 20° + 10° = 60°
			70° + 10° = 80°	30° + 20° = 50°
			60° + 10° = 70°	30° + 10° = 40°
			60° + 20° = 80°	20° + 10° = 30°
Group 2 Obtuse Angles	**Solutions Obtuse Angles**			
7 Combinations	**25 Combinations**			
60° + 40° = 100°	70° + 60° + 40° = 170°			
70° + 30° = 100°	70° + 60° + 30° + 10° = 170°			
70° + 60° = 130°	70° + 60°+ 20° + 10° = 160°			
70° + 40° = 110°	70° + 60° + 30° = 160°			
70° + 60° + 20° = 150°	70° + 60° + 20° = 150°			
70° + 40° + 60° = 170°	70° + 60° + 10° = 140°			
70° + 40° + 20° + 10° + 30° = 170°	70° + 60° = 130°			
	…….			
	40° + 30° + 20° + 10° = 100°			

Figure 4.2.3.4.B Group 2 solutions to the Angles task, compared to the possible combinations

Figures 4.2.3.4A and 4.2.3.4B illustrate how each group documented their work. This allows for identifying if and what strategies were used to solve the problem. Group 1 showed a strategy of looking for all the possible combinations from the largest acute angle down to the smallest angle that could be made. The strategy showed little continuity as the students only found 4 out of 12 solutions. While Group 1 showed accuracy, they did not show flexibility in finding all the possible solutions. Group 1 also showed conceptual misunderstandings in what an obtuse angle is, as they found possible straight angles instead. Group 2 had more correct solutions in total, but also had more incorrect attempts, which shows they struggled with verifying solutions. Group 2 used a less mathematically grounded approach by just using the cut out angles placed onto the given protractor. Had the students added up their solutions to verify their accuracy, they would have had fewer incorrect solutions. Group 2 showed only one strategy for ensuring they had a comprehensive list of solutions when looking for obtuse angles. Group 2 began with the smallest possible obtuse angle first, but only used the strategy for the first two solutions showing that there was little continuity in the strategy's implementation.

4.2.4 Evaluation of the process of Group 1 and Group 2 as they solved the Angles task

Group 1 and Group 2 both spent much time developing arguments as they solved the task, but showed little evidence that the arguments reached a comprehensive conclusion. Figure 4.2.4 shows the evaluation of Group 1 and Group 2 interactions while solving the Angles task. When looking at flexibility of interactions, there was a lot of engagement of both groups. There were significant sustained interactions in solving the task, showing that the groups were able to engage for extended periods on challenges they faced.

To evaluate the initiative of Group 1 and Group 2 in this task, the amount of authored statements was evaluated. Group 1 had 63% authored statements, while Group 2 had 58% authored statements. This indicates that the group was willing to contribute their own ideas to the interactions.

ANGLES PROCESS EVALUATION	GROUP 1	GROUP 2
Reasoning Abilities	**Observable Indicators**	**Observable Indicators**
Flexibility: # of Arguments Made	45 Arguments Made / 8 Correct Solutions 1 2 **3** 4 5	52 Arguments Made/ 17 Correct Solutions 1 2 **3** 4 5
Fluency: Sustained Interaction	14 lines of sustained interaction 1 2 3 **4** 5	17 Lines of sustained interaction 1 2 3 **4** 5
Initiative: Authored Participation	63% Author Statements 1 2 **3** 4 5	58% Author Statements 1 2 **3** 4 5
Concentration: Interactions across the reasoning structure.	A large percent of time was spent on comprehension of the task, and very little spent on strategy implementation. 1 **2** 3 4 5	An even distribution between strategy choice and strategy implementation. 1 2 3 **4** 5
Plausability: Depth of Mathematical Justifications	Limited **mathematical** justification. 14 warrants and 9 backing statements were made 1 2 **3** 4 5	Limited **mathematical** justification. 19 warrants were used, and 3 backing statements. 1 2 **3** 4 5
Constructiveness: Balance of Contributions/Incorporating new ideas	LS contributed to 45% of conversation and LN only contributed 19%. Limited evidence of incorporating peer ideas (spokesman) 1 **2** 3 4 5	LT contributed to 45% of interactions wile LP contributed to 21%. LT incorporated peer ideas well (27" spokesman statements). 1 2 **3** 4 5
	TOTAL: 17/30	**TOTAL: 20/30**

Marking Criteria:

1	2	3	4	5
No Evidence	Weak Evidence	Moderate Evidence	Good Evidence	Strong Evidence

Figure 4.2.4 Process evaluation of Angles Task

When evaluating the concentration of Group 1 and Group 2 interactions, Group 1 spent 31% of their time understanding the task, which took time from the actual solving of the task, while Group 2 showed a balance of interactions across the reasoning structure.

In evaluating the plausibility of Group 1 and Group 2 interactions, both groups used limited backing and grounding statements to mathematically justify their arguments. Group 1 used more grounding statements than Group 2; however, Group 2 made nearly the same amount of supporting statements in total.

When evaluating the constructiveness of interaction in Group 1 and Group 2, I looked at the balance of interaction and amount of spokesman statements that showed learners incorporating each other's ideas. Group 1 had one student who contributed 44% of interactions, while 15.2% of interactions incorporated other group members' ideas. Group 2 had 1 student who contributed to 44% of interactions, while 24.6% of interactions incorporated group members' ideas.

In all, Group 1 received a 17 and Group 2 received a 20 out of 30 marks in this evaluation of the process the groups underwent in developing a conclusive argument.

4.2.5 Evaluation of the product of Group 1 and Group 2 as they solved the Angles task

The Four Matches task proved to be a difficult task for both Group 1 and Group 2 in regards to finding as many angles as possible, although their attempts to solve the task were very creative. Figure 4.2.5 shows the product evaluation of Group 1 and Group 2 solutions to the angles task.

Group 1 and Group 2 showed very limited flexibility when looking for solutions, as provided in their conclusion. Group 1 had 8 out of 41 possible solutions, while Group 2 found 17. When evaluating the fluency in regards to the amount of strategies used to solve the task, Group 1 did use a strategy to verify if their solution was correct or not, by adding the angles to ensure they were within the range of acute, right or obtuse angles. Group 2 used only the protractor and shapes to verify their answers. Group 1 showed some uniqueness in verifying their answer by both using the protractor and adding the angles.

In evaluating the concentration of Group 1 and Group 2, there was limited evidence of sequentiality in their strategies and no evidence of continuity throughout the task for both groups. Group 1 used slightly more mathematical justification in their work, while Group 2 did showed none. Neither group showed evidence of using prior knowledge or generalizing across the sub-tasks when solving the task. In all, Group 1 earned 11 and Group 2 earned 10 out of 30 marks in the evaluation of the product of their arguments. Because Group 1 had a conceptual misunderstanding of what an obtuse angle was, they had fewer correct solutions than Group 2, but showed an only slightly better ability to justify their answers.

ANGLES PRODUCT EVALUATION	GROUP 1	GROUP 2
Reasoning Abilities	**Observable Indicators**	**Observable Indicators**
Flexibility: # of Correct Solutions	8/41 Solutions **1** 2 3 4 5	17/41 Solutions 1 **2** 3 4 5
Fluency: Strategies implemented.	Checking with both protractor and addition. 1 **2** 3 4 5	Relied on the protractor 1 **2** 3 4 5
Novelty: Uniqueness of strategies.	Different language was used to describe the symmetry of shapes and their symmetry. No strategies for ensuring they have all possible answers. 1 2 **3** 4 5	Used the protractor and cut outs only to verify answers. No system for ensuring completeness of conclusion. 1 **2** 3 4 5
Concentration: Sequentiality and Continuity of Strategy Implementation	There was some evidence of sequentiality with acute angle starting with largest angles they could find. 1 **2** 3 4 5	There was some evidence of starting with smallest possible obtuse angles, but no continuity of strategy. 1 **2** 3 4 5
Plausability: Mathematically Anchored Sociomathematical norms	Limited **mathematical** justification. Verfified answers to see if they added up correctly 1 **2** 3 4 5	Limited **mathematical** justification. **1** 2 3 4 5
Constructiveness: Generalization to other concepts or experiences	Limited discussions about prior-knowledge, there was confusion over terminology of obtuse angles, as they thought it needed to add up to 180° **1** 2 3 4 5	Limited discussions about prior knowledge, they did not refer to other ways to determine or verify their answers. **1** 2 3 4 5
	TOTAL: 11/30	**TOTAL: 10/30**

Marking Criteria:

1	2	3	4	5
No Evidence	Weak Evidence	Moderate Evidence	Good Evidence	Strong Evidence

Figure 4.2.5 Product evaluation of Angles task

4.2.6 Summary of Angles task analysis and evaluation

Both Group 1 and Group 2 showed evidence of interaction and depth of argument in the process of solving the task but this did not translate to evidence of a good final product. In both process and product evaluation, there was limited depth of mathematical systems or justifications in finding a comprehensive solution. Group 1 had mathematical misconceptions that showed a limited ability to generalize from prior knowledge. While there was much sustained interaction around solving the task, both groups implemented strategies and justifications that were superficial and unsystematic in their interactions and in their final product.

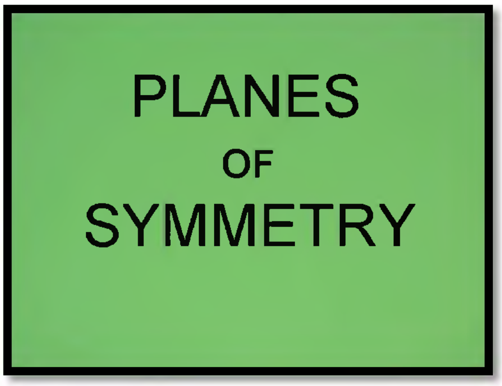
4.3 VITAL MATHS VIDEO CLIP PLANES OF SYMMETRY ANALYSIS AND EVALUATION

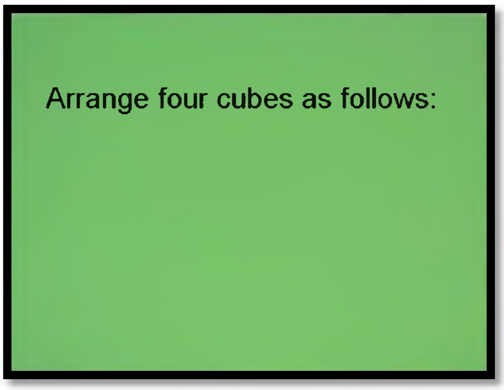
Introduction to problem task

The VITALmaths video clip Planes of Symmetry focuses on the mathematical concept of symmetry. Learners are required to manipulate one or two wooden blocks around a set structure of cubes to find new structures that have a vertical plane of symmetry. This task requires spatial reasoning skills of manipulation and rotation of structures, in order to find vertical planes of symmetry (see Figure 4.3.0.1).

The Planes of Symmetry task is open-ended because there are no suggestions on how to find the new structures. Learners were given two examples and asked to find 12 total structures with vertical lines of symmetry, but it was left to the learners to develop a system for solving and documenting the results.

This task was selected to evaluating creative mathematical reasoning by evaluating flexibility of thinking (the amount of correct solutions found). This task also allowed for the observation and evaluation of learner concentration skills because one could observe the systems (or lack thereof) that learners used to sequentially identify all possible structures.





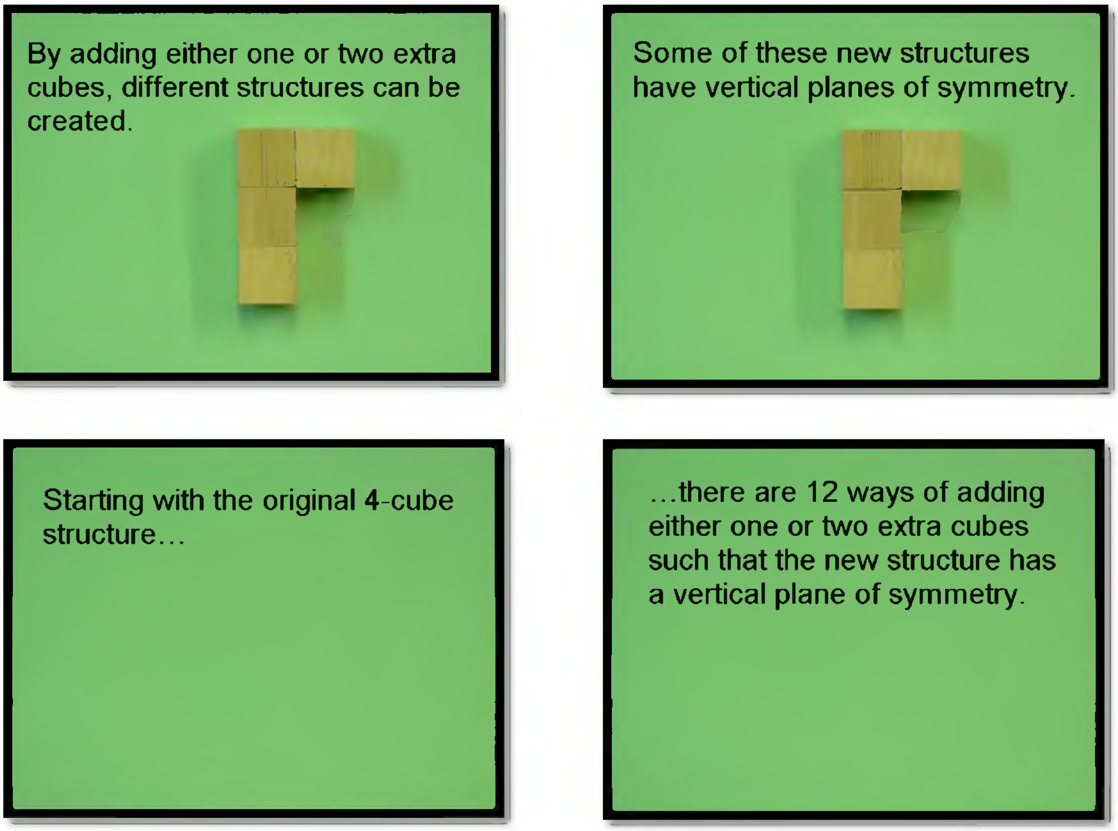


Figure 4.3.0.1 Screen Shots of Planes of Symmetry VITALmaths video clip illustrate the problem task

Potential solutions to Planes of Symmetry task

Figure 4.3.0.2 illustrates one way in which learners could have approached the task to find all 12 lines of symmetry. Each row from top to bottom represents the possible structures as the first block is added and moves counter-clockwise around the original structure.

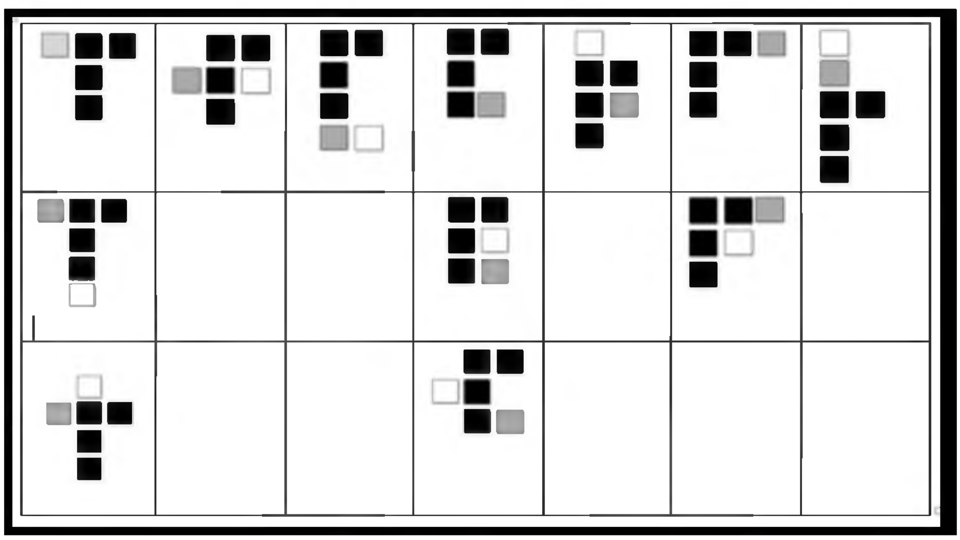


Figure 5.3.0.2 Planes of Symmetry solutions

This figure represents one possible way of systematically solving the task. The gray cube in each cell represents the first cube introduced to the original structure. The white square represents the second cube introduced to the original structure to form a symmetrical shape. From the left column to the right column, the first cube rotates counter-clockwise to show all possible solutions, with the gray cube in its place during rotation of the structure.

4.3.1 Group 1 documentation of Planes of Symmetry Task

While solving the Planes of Symmetry task, Group 1 was audio and video recorded during their 32:49 minutes of interaction (see Figure 4.3.1.1). 267 lines of interaction were transcribed and translated (see Figure 4.3.1.2). Figure 4.3.1.3 is photo documentation of learners' work on the provided worksheet. Group 1 selected to document their solutions on scratch paper (Figure 4.3.1.4) and wrote their arguments on the provided sheet (Figure 4.3.1.5).

Figure 4.3.1.1 Screenshot of video footage of Group 1 solving the Planes of Symmetry Task

GROUP 1 ANALYSIS SUMMARY - SYMMETRY

Amount of Interaction			Argument Interaction	
Time of Interaction		32:49	Claims Made	65
Lines of Interaction		267	Warrants	12
Individual Contributions To Interaction			Backing	6
LS		97	Sustained Interactions Around One Argument	11
LK		120	**Reasoning Structure**	
LN		35	Comprehension	49
R		15	Argument	83
Participation			Strategy Choice	46
LS	Author	60	Strategy Implementation	34
	Relayer	9	Conclusion	10
	Spokesman	27	Presentation	18
	Ghost	1	Side Conversation	27
LK	Author	104		
	Relayer	4		
	Spokesman	8		
	Ghost	4		
LN	Author	31		
	Relayer	1		
	Spokesman	3		
	Ghost	0		
R	Author	4		
	Relayer	0		
	Spokesman	11		
	Ghost	0		

Figure 4.3.1.2 Analysis of Group 1 interaction during the Planes of Symmetry Task

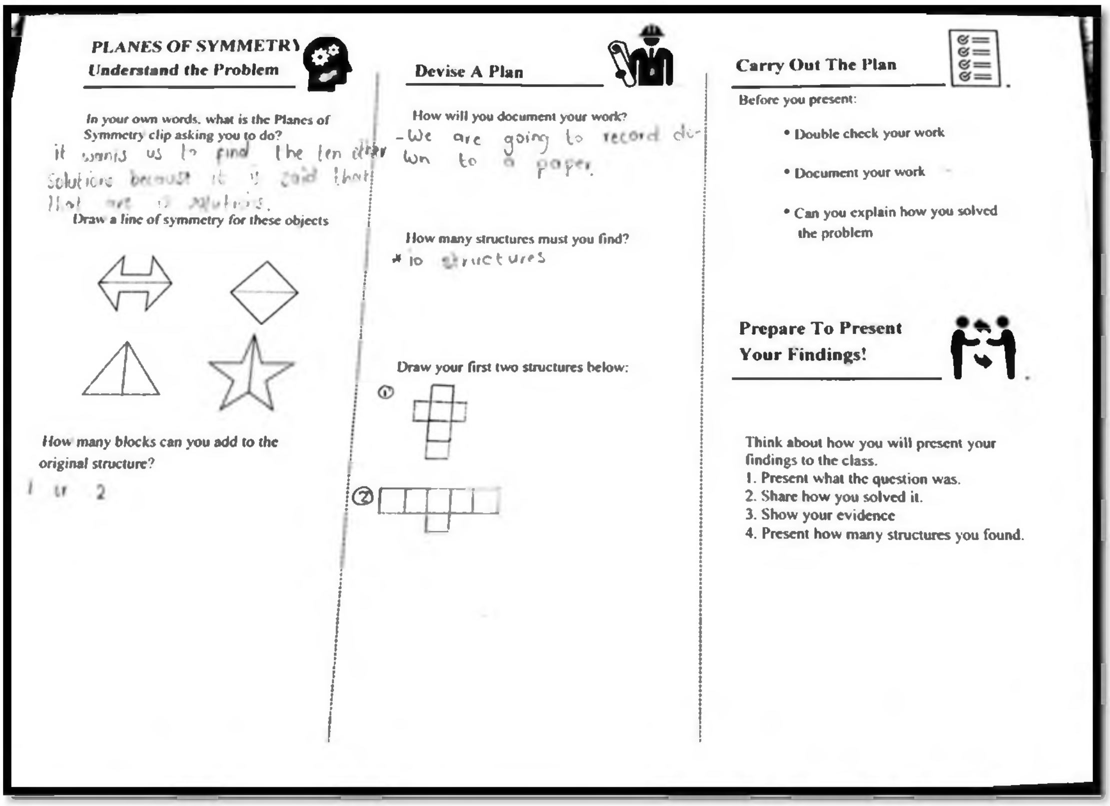


Figure 4.3.1.3 Documentation of Group 1 worksheet used to solve Planes of Symmetry task

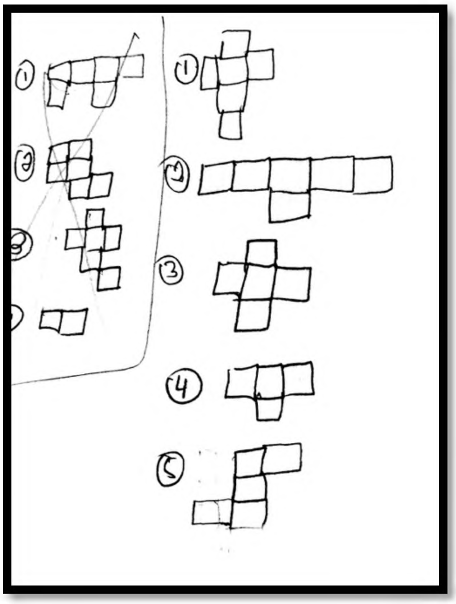
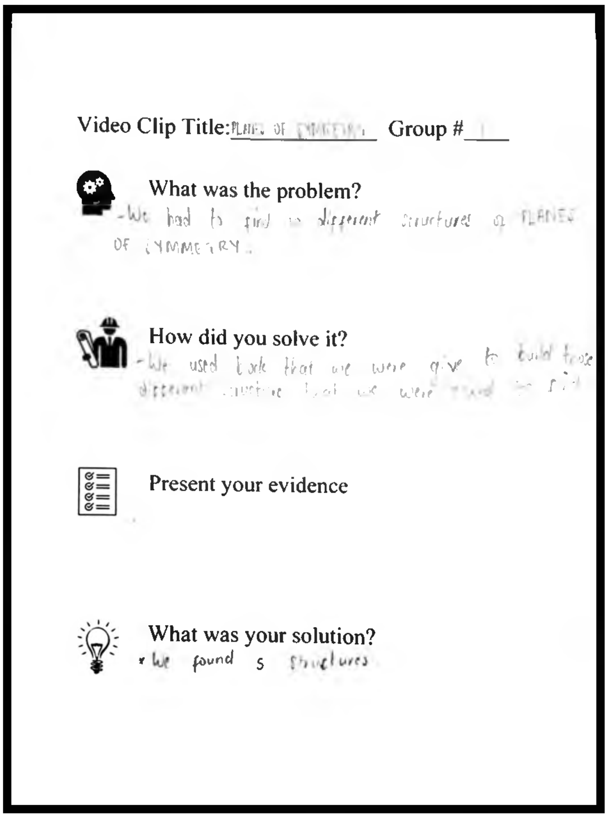


Figure 4.3.1.4 Documentation of Group 1 solutions to the Planes of Symmetry Task



Video Clip Title: PLANES OF SYMMETRY Group # 1

What was the problem?

-We had to find 10 different structures of PLANES OF SYMMETRY.

How did you solve it?

-We used block that we were give to build those

Present your evidence

What was your solution?

* We found 5 structures

Group 1 Planes of Symmetry Argument
What was the problem?
We had to find 10 different structures with planes of symmetry.
How did you solve it?
We used the blocks that we were given to build those different structures that we were asked to find.
What was your solution?
We found 5 structures

Figure 4.3.1.5 Documentation of Group 1 argument for their solution to Planes of Symmetry Task

4.3.2 Group 2 documentation of Planes of Symmetry Task

Group 2 was audio and video recorded during their 27:21 minutes of interaction solving the Planes of Symmetry task (see Figure 4.3.2.1). 161 lines of interaction were transcribed and translated (see Figure 4.3.2.2). Figure 4.3.2.3 is photo documentation of learners' work on the provided worksheet. Group 2 elected to document their solutions on scratch paper (Figure 4.3.2.4) and wrote their arguments on the provided sheet (Figure 4.3.2.5).

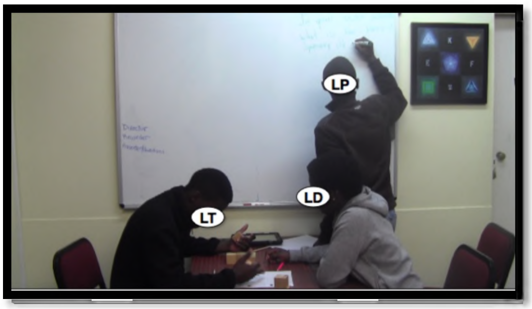


Figure 4.3.2.1 Screenshot of video footage of Group 2 solving the Planes of Symmetry task

GROUP 2 ANALYSIS SUMMARY - PLANES OF SYMMETRY

Amount of Interaction			Argument Interaction	
Time of Interaction		27:21	Claims Made	16
Lines of Interaction			Warrants	2
Individual Contributions To Interaction			Backing	0
LT		46	Sustained Interactions Around One Argument	6
LP		57	**Reasoning Structure**	
LN		42	Comprehension	42
R		16	Argument	18
Participation			Strategy Choice	10
LT	Author	25	Strategy Implementation	16
	Relayer	12	Conclusion	18
	Spokesman	8	Presentation	22
	Ghost	1	Side Conversation	5
LP	Author	46		
	Relayer	6		
	Spokesman	5		
	Ghost	0		
LN	Author	22		
	Relayer	12		
	Spokesman	8		
	Ghost	0		
R	Author	9		
	Relayer	0		
	Spokesman	7		
	Ghost	0		

Figure 4.3.2.2 Analysis of Group 2 interactions during Planes of Symmetry task

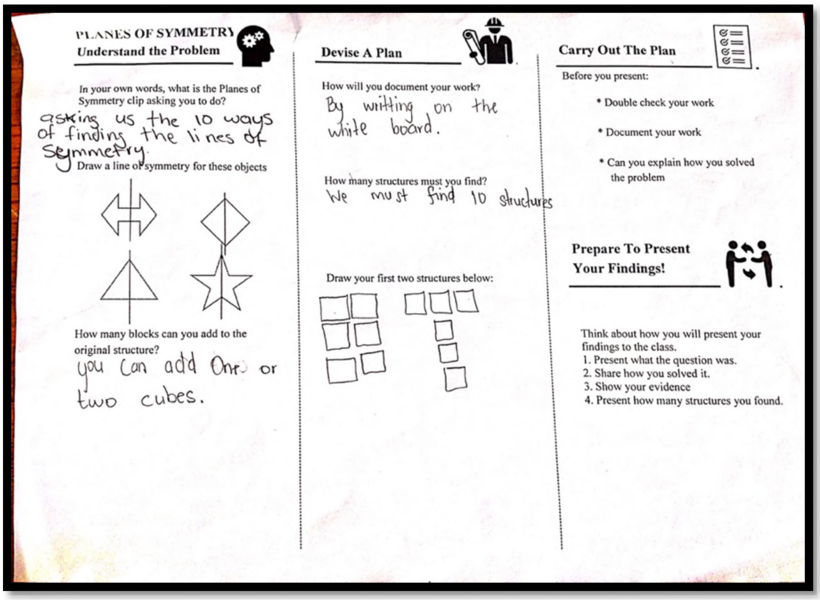


Figure 4.3.2.3 documentation of Group 2 worksheet used during Planes of Symmetry task

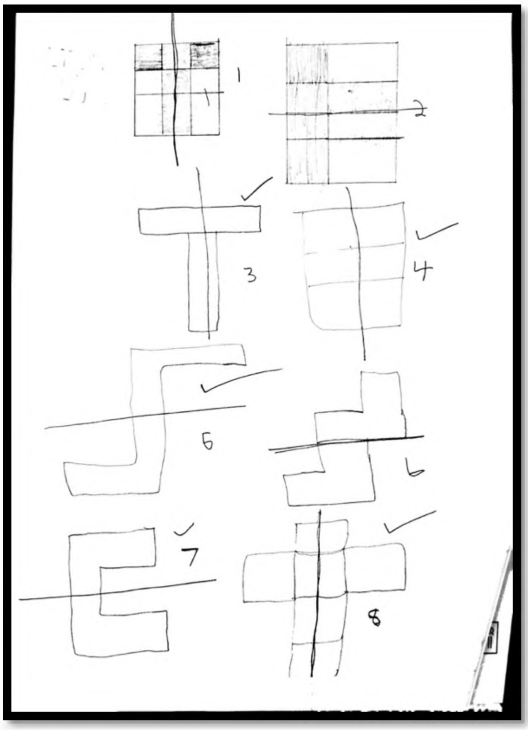
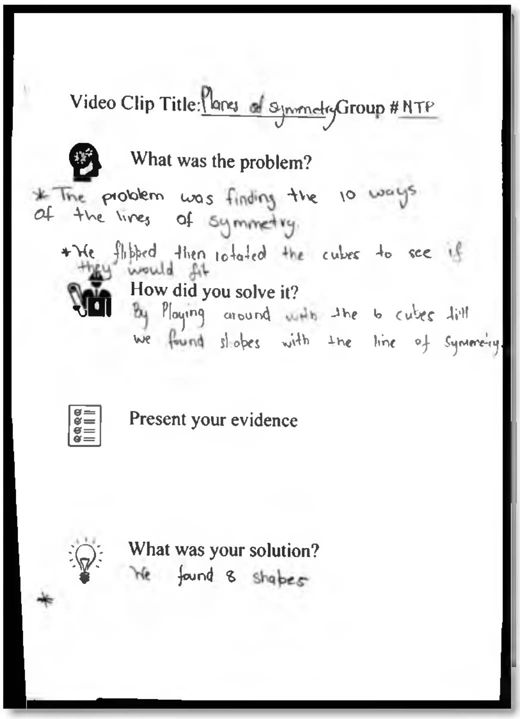


Figure 4.3.2.4 Documentation of Group 2 learner evidence of solution to the Planes of Symmetry task


Video Clip Title: Planes of Symmetry Group # NTP

What was the problem?

*The problem was finding the 10 ways of the lines of symmetry.

*We flipped then rotated the cubes to see if they would fit

How did you solve it?

By Playing around with the 6 cubes till we found shapes with the line of symmetry.

Present your evidence

What was your solution?

We found 8 shapes

*

Group 2 Planes of Symmetry Argument:

What was the problem?
The problem was finding the 10 ways of the lines of symmetry.

How did you solve it?
We flipped then rotated the cubes to see if they would fit. By playing around with the 6 cubes until we found shapes with the line of symmetry.

Present your evidence:
(See Figure 4.3.2.4)

What was your solution?
We found 8 shapes

Figure 4.3.2.5 Documentation of the argument Group 2 made while solving the Planes of Symmetry task

4.3.3 Analysis of Group 1 and 2 solving the Planes of Symmetry task

Group 1 and Group 2 interacted in very different ways from each other. Group 1 had far more interactions and made significantly more arguments than Group 2. In comparing how Group 1 interacted to Group 2, Group 1 had 60.3% more interaction than Group 2 (see Figure 4.3.3.1). Group 1 made claims in 24% of their interactions, while Group 2 made claims in 9.9% of their interactions (see Figure 4.3.3.1).

Description	Group 1	Group 2
Claims	65	16
Total interactions	267	161

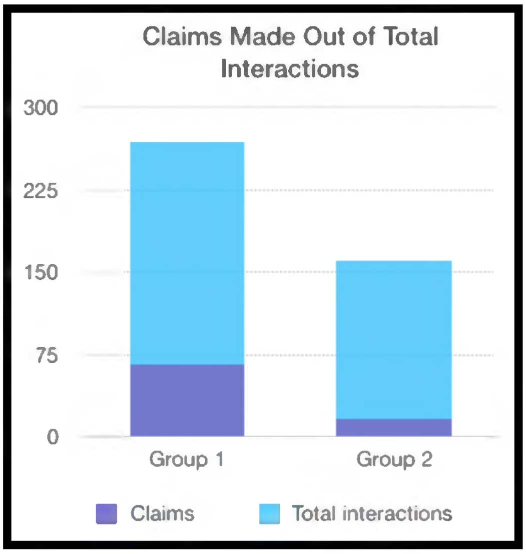
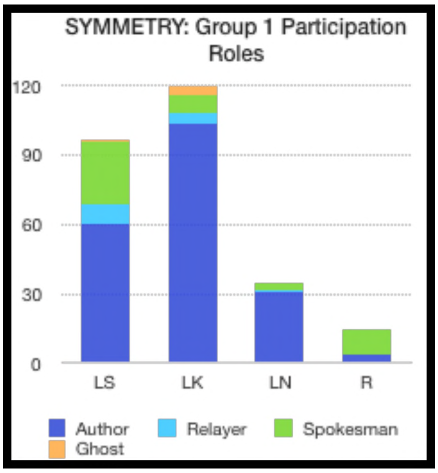


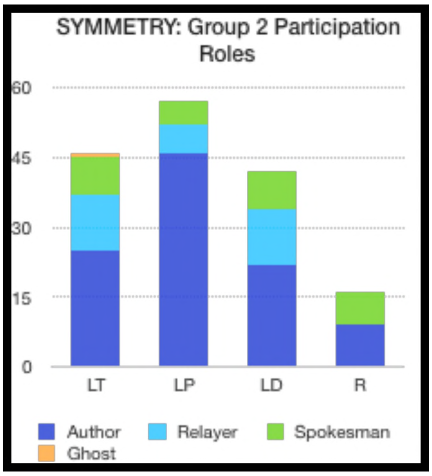
Figure 4.3.3.1 Comparison of Group 1 and Group 2 total interactions, as well as the amount of arguments made during the Planes of Symmetry task

There was a dominant speaker in both Group 1 and Group 2, although Group 2 showed a greater distribution of interaction (see Figure 4.3.3.2). 74% of Group 1 interactions were author statements (original statements that did not incorporate other members' ideas). Group 2, with 63%, also had a large percentage of author statements (see Figure 4.3.3.2). Group 1 did very little relaying or repeating of information (5.2%) compared to Group 2's interactions (18.6%). Both groups made very few ghost comments.

Description	*LS*	*LK*	*LN*	*R*
Author	60	104	31	4
Relayer	9	4	1	0
Spokesman	27	8	3	11
Ghost	1	4	0	0

Description	*LT*	*LP*	*LD*	*R*
Author	25	46	22	9
Relayer	12	6	12	0
Spokesman	8	5	8	7
Ghost	1	0	0	0





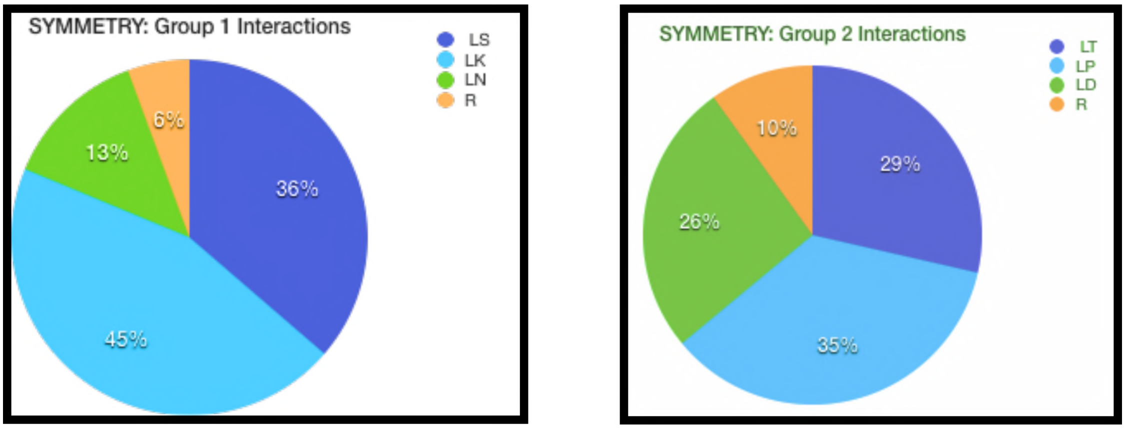
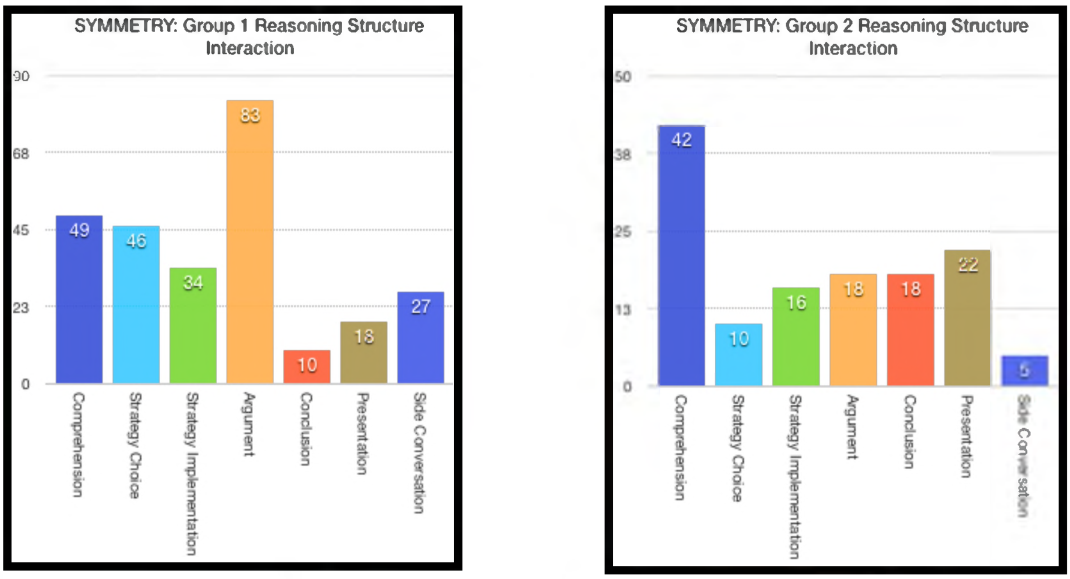


Figure 4.3.3.2 Comparison of the originality of individual participant statements from Group 1 and Group 2 as they solved the Planes of Symmetry task

Figure 4.3.3.3 shows the allocation of time spent within the reasoning structure. While both Groups 1 and 2 used the same number of interactions, proportionately to their total interactions, comprehending the task, with Group 1 making 49 interactions and Group 2 making 42, each group was very different. Group 2 spent a significant amount of time understanding the problem (26%) compared to Group 1 (18%). Group 1 spent 31% of their time making arguments and justifying their thinking, compared to Group 2 that spent 11% of their time making arguments (see Figure 4.3.3.3).

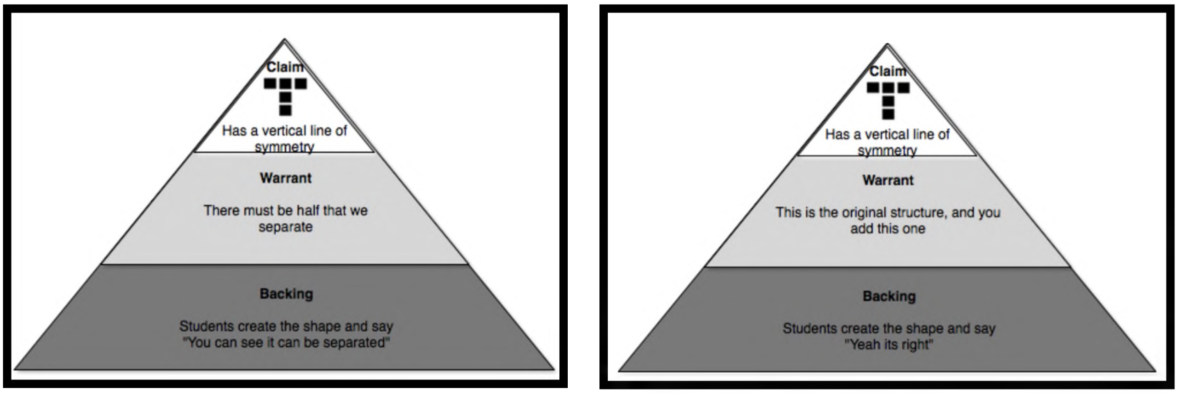


Reasoning Structure	Interactions
Comprehension	49
Strategy Choice	46
Strategy Implementation	34
Argument	83
Conclusion	10
Presentation	18
Side Conversation	27

Reasoning Structure	Interactions
Comprehension	42
Strategy Choice	10
Strategy Implementation	16
Argument	18
Conclusion	18
Presentation	22
Side Conversation	5

Figure 4.3.3.3 Comparison of the amount of interactions spent within different areas of the reasoning structure

Both Group 1 and Group 2 used warrants and backing in their arguments. Group 1 used a more precise mathematical language to articulate their arguments as seen in Figure 5.33.4. Group 1 used backing statements 6 times and warrant statements 12 times compared to Group 2 who only used one backing and one warrant in their interactions (see Figures 5.3.1.2 and 5.3.2.2).



Group 1 Argument Depth **Group 2 Argument Depth**

Figure 4.3.3.4 Comparison of depth of arguments made during the Planes of Symmetry task

4.3.4 Evaluation of the process of Group 1 and Group 2 as they solved the Planes of Symmetry task

To analyze the process by which Group 1 and Group 2 solved the Planes of Symmetry task, six reasoning abilities were evaluated: flexibility, fluency, initiative, concentration, plausibility, and constructiveness (Figure 4.3.4). To evaluate the flexibility of the groups as they interacted, the amount of arguments made was evaluated. For this task, Group 1 had 65 interactions focused on argumentation, which showed much interaction was spent solving the task compared to Group 2, who made 16 arguments. This was a significant difference between the two groups. Interestingly, Group 2, while they argued much less, had more correct solutions than Group 1. Fluency of interaction focused on the ability of the groups to sustain interaction on one item. Group 1 had 11 lines of interaction while Group 2 had only 4 lines of sustained interaction.

Initiative abilities in the groups were evaluated by identifying how many statements were original contributions to the interaction. Group 1 had 74% of their statements as author statements. Group 2 had 64% of their statements identified as author statements. The concentration abilities of the groups were evaluated by observing the balance of interactions across the reasoning structure. Group 1 showed moderate evidence of balance across the reasoning structure while Group 2 showed weak evidence of balanced interaction. A large proportion of their time was spent in understanding the task, compared to Group 1 who had a significant amount of interaction choosing and implementing strategies to solve the task.

To evaluate the plausibility of the groups' reasoning as they solved the Planes of Symmetry task, the mathematical justification as well as the amount of supporting statements they made in support of their arguments were analyzed. Both Groups 1 and 2 used limited amounts of mathematical justifications, and very few arguments were made about how comprehensive their final solution was. Group 1 did use 12 warrants to support their arguments and 6 comments to further back their arguments. Group 2 had only 1 warrant and 1 backing statement in their arguments.

PLANES OF SYMMETRY PROCESS EVALUATION	GROUP 1	GROUP 2
Reasoning Abilities	**Observable Indicators**	**Observable Indicators**
Flexibility: # of Arguments Made	65 Arguments Made / 2 Correct Solutions 1 2 3 **4** 5	16 Arguments Made/ 4 Correct Solutions 1 **2** 3 4 5
Fluency: Sustained Interaction	11 lines of sustained interaction 1 2 **3** 4 5	4 Lines of sustained interaction 1 **2** 3 4 5
Initiative: Authored Participation	74% Author Statements 1 2 3 **4** 5	63% Author Statements 1 2 **3** 4 5
Concentration: Interactions across the reasoning structure.	A balance of interaction across the reasoning structure. 1 2 **3** 4 5	A large percentage of the time was spent comprehending the problem 1 **2** 3 4 5
Plausability: Depth of Mathematical Justifications	Limited **mathematical** justification. 12 warrants and 6 backing statements were made 1 2 **3** 4 5	Limited **mathematical** justification. Only 2 warrants were used, and no backing statements. 1 **2** 3 4 5
Constructiveness: Balance of Contributions/Incorporating new ideas	LK contributed to 45% of conversation and LN only contributed 13%. Only LS incorporated other learner ideas consistently. 1 **2** 3 4 5	Very balanced contributions. Significant amount of incorporation of ideas 1 2 3 **4** 5
	TOTAL: 19/30	**TOTAL: 15/30**

Marking Criteria:

1	2	3	4	5
No Evidence	Weak Evidence	Moderate Evidence	Good Evidence	Strong Evidence

Figure 4.3.4 Process evaluation of Planes of Symmetry task

The constructiveness of the interactions in solving the Planes of Symmetry task was evaluated by looking at the balance of contributions amongst team members as well as the groups' ability to incorporate each other's ideas into the plan to solve the task. Group 1 showed an imbalance of interaction between LS (45%) and LN (13%). LS did however show some ability to incorporate other members' ideas into her own thinking based on the amount of spokesman comments that rephrased what others had said. Group 2 had an even balance of interaction and a significant amount of incorporating others' ideas by parroting what others had said or rephrasing other group members' comments.

4.3.5 Evaluation of the product of Group 1 and Group 2 solutions to the Planes of Symmetry task

To evaluate the final solution to the Planes of Symmetry task, this research used flexibility, fluency, novelty, concentration, plausibility and constructiveness as terms to look at essential components of the conclusive argument that indicate creative mathematical reasoning (see Figure 5.3.5). Flexibility was evaluated by analyzing how many correct solutions were found out of the total possible solutions (Silver, 1997; Lithner, 2010). Group 1 found 2 out of 12 solutions compared to Group 2, which found 4 out of 12 solutions. Fluency of reasoning was observed by identifying how many strategies were implemented. Both groups used random guess and check strategies. No strategies were implemented to ensure they had a conclusive number of solutions, or in verification of their solutions.

PLANES OF SYMMETRY PRODUCT EVALUATION	GROUP 1	GROUP 2
Reasoning Abilities	**Observable Indicators**	**Observable Indicators**
Flexibility: # of Correct Solutions	2/12 Solutions **1** 2 3 4 5	4/12 Solutions 1 **2** 3 4 5
Fluency: Strategies implemented.	Random Guess and Check Strategy 1 **2** 3 4 5	Random Guess and Check Strategy 1 **2** 3 4 5
Novelty: Uniqueness of strategies.	Different language was used to describe the symmetry of shapes and their symmetry. 1 **2** 3 4 5	None evident. 1 **2** 3 4 5
Concentration: Sequentiality and Continuity of Strategy Implementation	There was no system evident. Attempts seemed sporadic and disconnected. **1** 2 3 4 5	There was no system evident. Attempts seemed sporadic and disconnected **1** 2 3 4 5
Plausability: Mathematically Anchored Sociomathematical norms	Limited **mathematical** justification **1** 2 3 4 5	Limited **mathematical** justification. Learners identified the line of symmetry in their evidence. 1 **2** 3 4 5
Constructiveness: Generalization to other concepts or experiences	None evident. **1** 2 3 4 5	No evidence **1** 2 3 4 5
	TOTAL: 8/30	**TOTAL: 10/30**

Marking Criteria:

1	2	3	4	5
No Evidence	Weak Evidence	Moderate Evidence	Good Evidence	Strong Evidence

Figure 4.3.5 Evaluation of product of Planes of Symmetry task

Novelty was evaluated by analyzing the uniqueness of strategies employed (Silver, 1997, Lithner, 2010). Group 1 used a unique way of labeling the shapes they found but did not help in the solving of the task, and Group 2 did not show any uniqueness in their strategies. To evaluate the ability of the groups to use concentration, an analysis of the groups' abilities to implement strategies with sequentiality and continuity was used (Lithner, 2010). This aspect of reasoning proved to be the most challenging for both groups, as there was no evidence of sequence or continuity amongst their implementation of strategies. Plausibility was also a challenge when evaluating their ability to mathematically justify their claims (Toulmin, 1964). Group 1 did not mark any line of symmetry on their evidence, while Group 2 did. Neither group presented any mathematical justifications in their written argument. To evaluate constructiveness of the groups' abilities, an analysis of generalization abilities was undertaken. There did not appear to be any evidence of generalization of concepts or abilities from prior knowledge in the strategies implemented to provide a conclusion.

4.3.6 Summary of Planes of Symmetry analysis and evaluation

Both Group 1 and Group 2 performed better in their process of solving the solution than in actually arriving at a well-justified final argument. Group 1 particularly had a large amount of sustained interaction around solving the task, but both groups failed to develop a systematic strategy that was consistently implemented throughout the task. The ability to generalize the geometric concepts of symmetry and similarity, and reflections and rotations, proved a challenge in this task.

4.4 VITALMATHS VIDEO CLIP MAZES ANALYSIS AND EVALUATION

Introduction to the problem task

The Mazes task poses the challenge of identifying how many possible pathways a marble can move through a 3 x 3 array. The marble can move through or around the 9 squares in the Maze, but can only move down and to the right starting from the top left corner and traveling to the bottom right corner (see Figure 4.4.0.1).

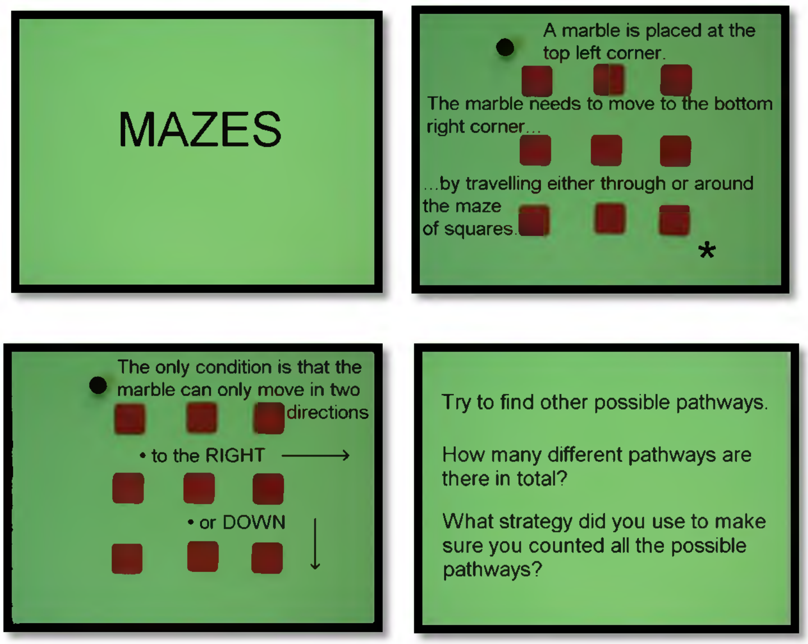


Figure 4.4.0.1 Screen Shots of The Maze VITALmaths video clip that illustrate the task

The mathematical concepts necessary to complete the task are very simple, but require spatial reasoning skills and systematic thinking to exhaust all the possible pathways through the Maze. This task requires creative mathematical reasoning because of its open-ended nature. No specific number of pathways is given and no strategies are provided; this requires flexibility of thinking to find as many solutions as possible, and fluency of thinking to develop and implement effective strategies, to identify a comprehensive conclusion.

Potential Solution to the Mazes Task

One way to find a systematic solution is to move down 3 and across 3 and move across the Maze until one moves right 3 times and down 3 times as shown in figure 4.4.0.2. This task requires learners to develop some type of system or algorithm for ensuring they have found all the possible solutions. Working from bottom to top, left to right is one possible system. Many other possible systems could be used, that would lead to the conclusion that there are 20 possible solutions. This solution shows both the visual of the Maze pathways as well as the symbolic use of letters indicating directions D for down and R for right, as there are only two allowable directions to be used in the task.

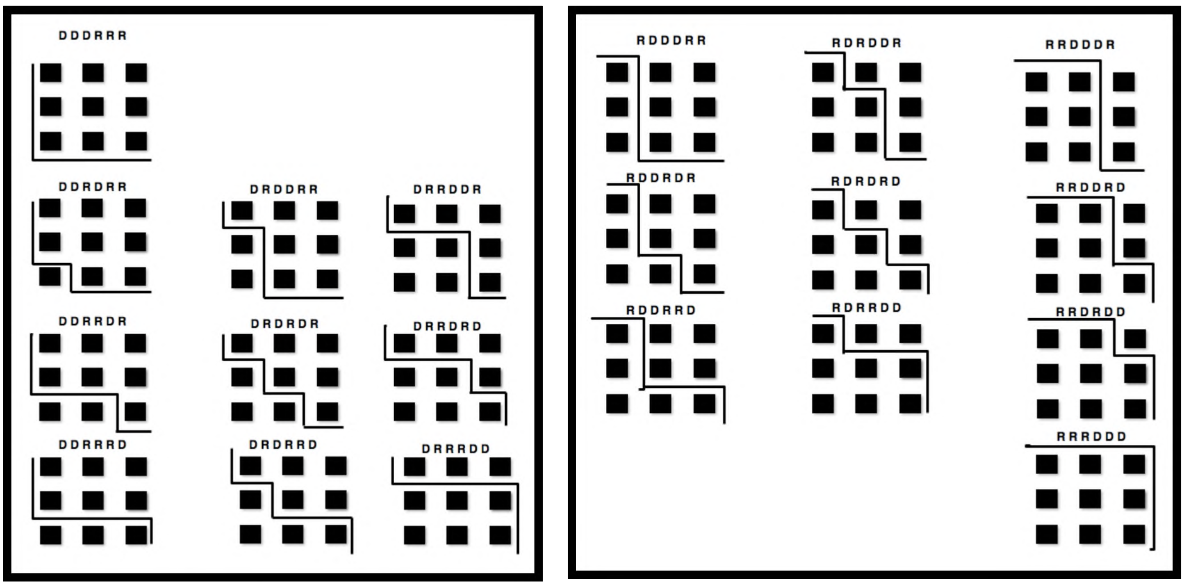
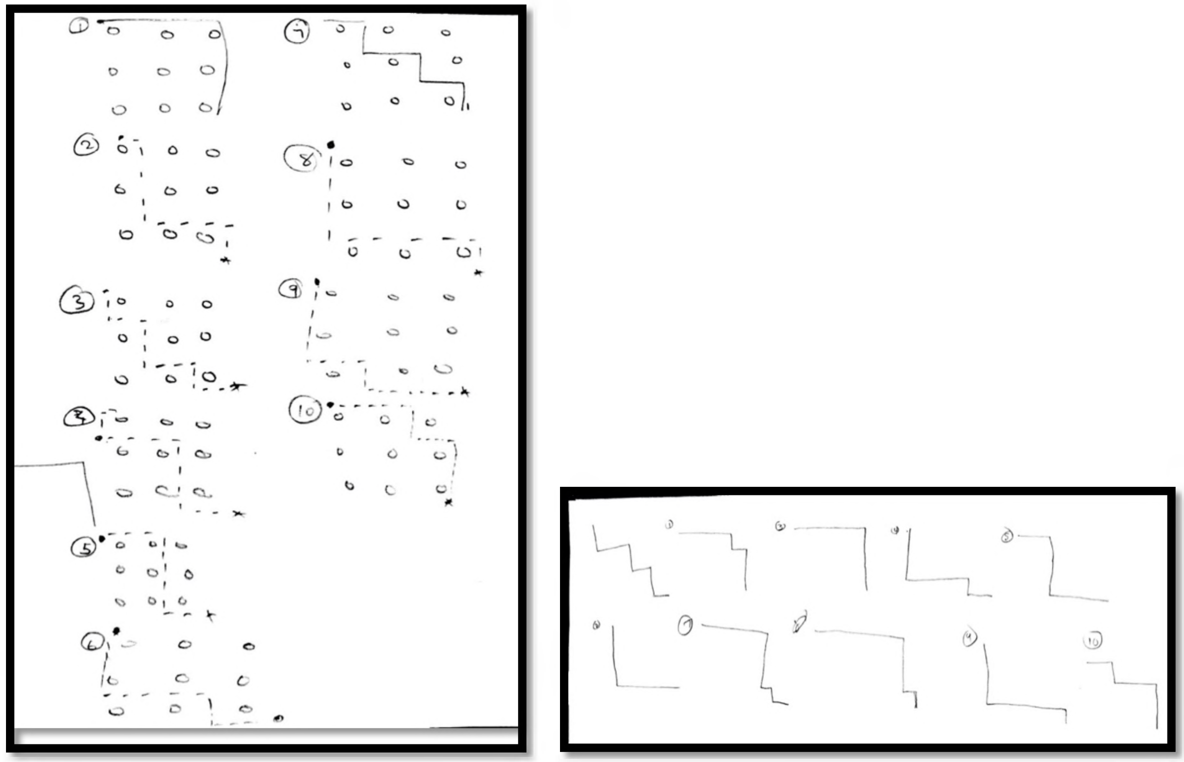


Figure 4.4.0.2 Potential solutions to The Maze task

4.4.1 Final Product Documentation of Group 1 and Group 2 Solutions to the Maze Task

Group 1 and Group 2 both found 10 different pathways as a solution to the task. By looking at the order in which each group found the pathways, it is evident that no strategy was implemented to ensure a comprehensive list of pathways. The responses to how the problem was solved focused on materials that were used to solve the task, not on a strategy that they implemented to solve the task. Group 1 evidence was easier to follow as they provided the dots around which the pathways moved, compared to Group 2 that only showed the pathways itself, with no reference to the Maze through which the pathways traveled (see Figure 4.4.1).



Group 1 Mazes Argument	Group 2 Mazes Argument
What was the problem? The problem was finding the possible ways that the marble could travel using down and rightwards routing **How did you solve it?** We used blocks and a toy car to find the ways that the car could travel. **What was your solution?** We found 10 ways	**What was the problem?** The problem was finding different pathways along which the marble could move. **How did you solve it?** By playing around with the movements of the car. **What was your solution?** We found 10.

Figure 4.4.1 Documentation of Group 1 and Group 2 solutions to The Mazes task

4.4.2 Evaluation of process as Group 1 and Group 2 solved the Mazes task

To evaluate the creative mathematical reasoning process that Group 1 and Group 2 used to solve The Maze task, the uses of flexibility, fluency, initiative, concentration, plausibility and constructiveness were analyzed (see Figure 4.4.2). Fluency was analyzed by looking at the number of arguments that were made to solve the task in comparison to the amount of correct solutions that were found. Group 1 made 40 different arguments and in that process found 10 correct solutions. Group 2 made 26 arguments and found 10 correct solutions. To evaluate fluency, the length of sustained interaction was evaluated. Group 1 had 6 lines of sustained interaction while Group 2 had 7 lines of sustained interaction. To evaluate the initiative abilities of the groups, the amount of authored statements contributing to interaction was analyzed. 76% of Group 1 interactions were author statements compared to Group 2, which had 58% of author statements.

To measure concentration abilities, or the ability to deductively and systematically reach a conclusion, analysis of the interactions across the reasoning structure was used (Polya, 1957; Lithner, 2008). Group 1 and Group 2 spent a similar amount of time across the reasoning structure, although limited time was spent discussing the conclusiveness of their argument. Plausibility was evaluated by analyzing the depth of mathematical justifications, looking specifically at the amount of warrant and backing statements made as well as the mathematical foundation of the arguments (Toulmin, 1964; Krummheuer, 2007). Both groups used limited amounts of mathematical justifications. Group 1 made 8 warrant statements and 2 backing statements in interactions, while Group 2 made 12 warrant statements and 0 backing statements.

MAZES PROCESS EVALUATION	GROUP 1	GROUP 2
Reasoning Abilities	**Observable Indicators**	**Observable Indicators**
Flexibility: # of Arguments Made	40 Arguments Made / 10 Correct Solutions 1 2 **3** 4 5	26 Arguments Made/ 10 Correct Solutions 1 **2** 3 4 5
Fluency: Sustained Interaction	6 lines of sustained interaction 1 2 **3** 4 5	7 Lines of sustained interaction 1 2 **3** 4 5
Initiative: Authored Participation	76% Author Statements 1 2 3 **4** 5	58% Author Statements 1 2 **3** 4 5
Concentration: Interactions across the reasoning structure.	An even distribution across the reasoning structure, although limited time was spent discussing conclusiveness of their argument. 1 2 **3** 4 5	An even distribution across the reasoning structure, although limited time was spent discussing conclusiveness of their argument. 1 2 **3** 4 5
Plausability: Depth of Mathematical Justifications	Limited **mathematical** justification. 8 warrants and 2 backing statements were made 1 **2** 3 4 5	Limited **mathematical** justification. 12 warrants were used, and 0 backing statements. 1 **2** 3 4 5
Constructiveness: Balance of Contributions/Incorporating new ideas	LS contributed and LK contributed an equally (29% each) while LN contributed to 17% of interactions. 15.7% combined spokesman statements. 1 2 **3** 4 5	LT contributed to 49% of interactions wile LP contributed to 30%, and LD with 15%. 23% combined spokesman statements 1 2 **3** 4 5
	TOTAL: 18/30	**TOTAL: 16/30**

Marking Criteria:

1	2	3	4	5
No Evidence	Weak Evidence	Moderate Evidence	Good Evidence	Strong Evidence

Figure 4.4.2 Evaluation of process of The Mazes task

To evaluate the constructiveness of the interactions, analysis of the balance of interactions between learners and how often group members were able to adapt other group members' ideas into their planning, was conducted. Group 1 members had an uneven distribution of interactions. LS and LK both contributed to 29% of the interactions while LN only contributed to 17% of interactions. They had an accumulative 15.7% of interactions that showed learners were paraphrasing the ideas of the other members. Group 2 interactions showed 49% of interactions came from LT, while 30% were attributed to LP, and LD had 15%. Group 2 showed that 23% of interactions were spokesman interactions, where ideas were paraphrased from the ideas of the other members.

4.4.3 Evaluation of product of Group 1 and Group 2 solutions to the Maze task

An evaluation of the conclusive argument Group 1 and Group 2 presented for The Mazes task was based on an analysis of the flexibility, fluency, novelty, concentration, plausibility and constructiveness of their final presentation and evidence (see Figure 4.4.3). Flexibility was evaluated on the number of correct solutions out of the number of possible solutions (Silver, 1997; Lithner 2008). Both groups found only 10 out of 20 possible solutions to the task. Fluency ability was evaluated by analyzing the number of strategies implemented during the solving of The Maze task. Group 1 and Group 2 used random guess and check methods to find pathways. There did not seem to be a logical system employed to find the variety of possible ways in which the problem could have been approached.

MAZES PRODUCT EVALUATION	GROUP 1		GROUP 2	
Reasoning Abilities	**Observable Indicators**		**Observable Indicators**	
Flexibility: # of Correct Solutions	10/20 Solutions	1 2 **3** 4 5	10/20 Solutions	1 2 **3** 4 5
Fluency: Strategies implemented.	Random Guess and Check	**1** 2 3 4 5	Random guess and check	**1** 2 3 4 5
Novelty: Uniqueness of strategies.	Random guess and check, no strategy to find conclusiveness.	**1** 2 3 4 5	Random guess and check, no strategy to find conclusiveness.	**1** 2 3 4 5
Concentration: Sequentiality and Continuity of Strategy Implementation	There was no evidence of sequentiality or continuity of strategy implementation.	**1** 2 3 4 5	There was no evidence of sequentiality or continuity of strategy implementation.	**1** 2 3 4 5
Plausability: Mathematically Anchored Sociomathematical norms	Limited **mathematical** justification. Drew dots and pathways to show as evidence	1 2 **3** 4 5	Limited **mathematical** justification. Not dots to indicate the pathway taken in evidence, making solutions unclear.	1 **2** 3 4 5
Constructiveness: Generalization to other concepts or experiences	No evidence of adopting strategies or prior-knowledge concepts from previous tasks.	**1** 2 3 4 5	No evidence of adopting strategies or prior-knowledge concepts from previous tasks.	**1** 2 3 4 5
	TOTAL: 10/30		**TOTAL: 9/30**	

Marking Criteria:

1	2	3	4	5
No Evidence	Weak Evidence	Moderate Evidence	Good Evidence	Strong Evidence

Figure 4.4.3 Product evaluation of the Mazes task

Novelty was evaluated by analyzing the uniqueness of strategies used; however, no strategies were used to solve the task or ensure that they had a conclusive argument, thus novelty was evaluated as limited. Concentration of group reasoning abilities was analyzed by observing the sequentiality and continuity of learner strategy implementation (Lithner, 2008; Campos, 2010). For this task both Group 1 and Group 2 showed no evidence of this ability. To evaluate plausibility, depth of mathematical justification and ability to present ideas were analyzed. Both groups showed limited mathematical justifications in their presentation and the pictures that were drawn to show the pathways. Group 1 did draw dots to indicate the pathways used, while Group 2 only drew lines that did not show the pathways clearly. To evaluate constructiveness of the presented arguments, evidence of generalization to other concepts or of prior knowledge was analyzed (Campos, 2010). There was no evidence of referring to prior knowledge to solve this task.

4.4.4 Summary of the Mazes task analysis and evaluation

The Mazes task was beneficial for the analysis of Creative Mathematical Reasoning in that the task required learners to identify and implement specific strategies for finding all the possible pathways through the Maze. By observing how learners used the manipulatives and transcribed interactions, and evaluating the final product of their argument, it was evident that both groups struggled to identify and systematically implement a strategy for solving the task. Superficial strategies for documenting the task were employed, but both groups only found half of the possible pathways through the Maze. Group 1 did show evidence of more group interaction, and argumentation in solving the task, but this did not effect a better outcome in the final argument.

4.5 VITALMATHS CLIP PATHWAYS ANALYSIS AND EVALUATION

Introduction to the problem task

The Pathways VITALmaths video clip poses the challenge of identifying how many different end-points can be reached in 4 vertical or horizontal moves on a 4 x 4 grid. The clip further questions whether there were any positions that could not be reached. The mathematical concepts necessary are simply horizontal and vertical lines (see Figure 4.5.0.1). This task was selected for the research for several reasons. Firstly, it is a similar task to the Mazes VITALmaths clip and so allows for observing if learners could generalize between clips. This task is also a tool for evaluating creative mathematical reasoning because of its open-ended nature. There is no given number of points required to find, requiring learners to be exhaustive in their search to find a solution. No strategies for finding the solution were provided or suggested.

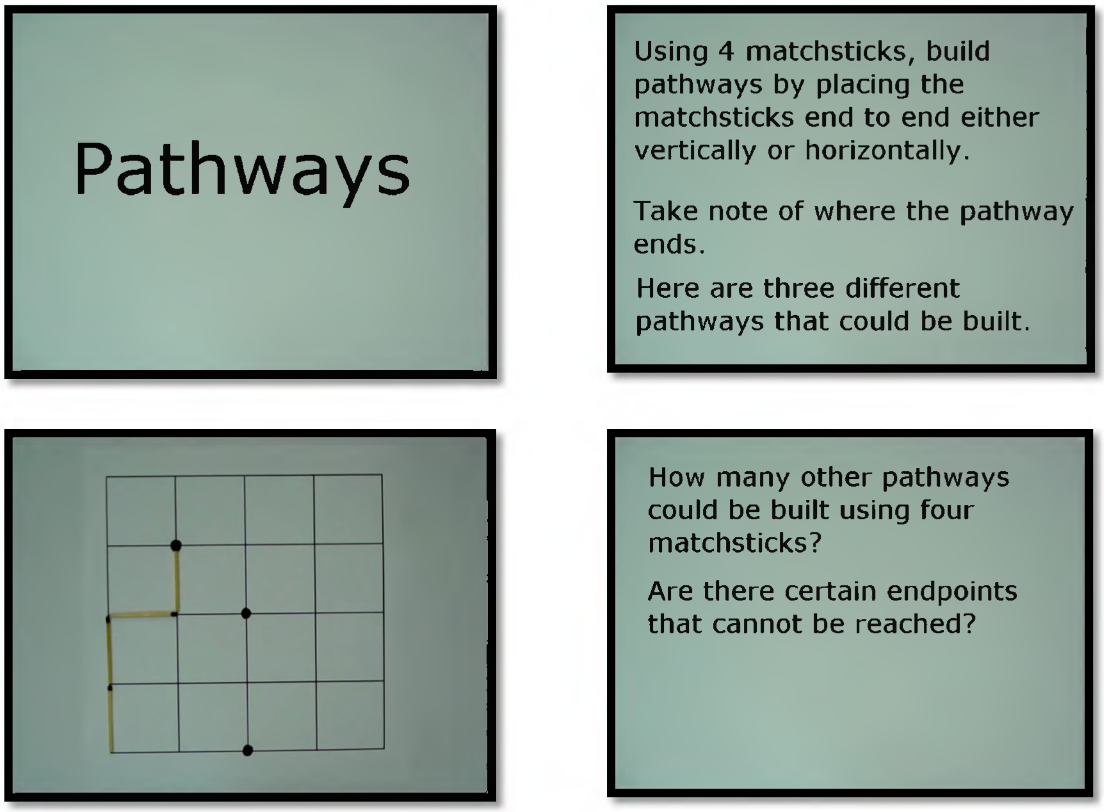


Figure 4.5.0.1 Screen shots of Pathways VITALmaths video clip that illustrates the task

Potential Solution to the Pathways task

There are many ways to solve the Pathways task. Figure 4.5.0.2A illustrates one way this task could be solved. The grid shows how the pathways were found from left to right, and bottom to top. There are 9 possible end-points with the understanding that one must start from the bottom left corner and not have to walk back along the same path. It is interesting to note that the starting point was not stipulated in writing in the video clip, which led one of the groups to operate on the notion that they could start from the bottom left corner but walk back along the same path and go a different direction; this allowed them to reach spaces that required an odd number of moves. This was an unintended misunderstanding but according to their understanding of the rules, they were able to be flexible in their reasoning (see Figure 4.5.0.2B).

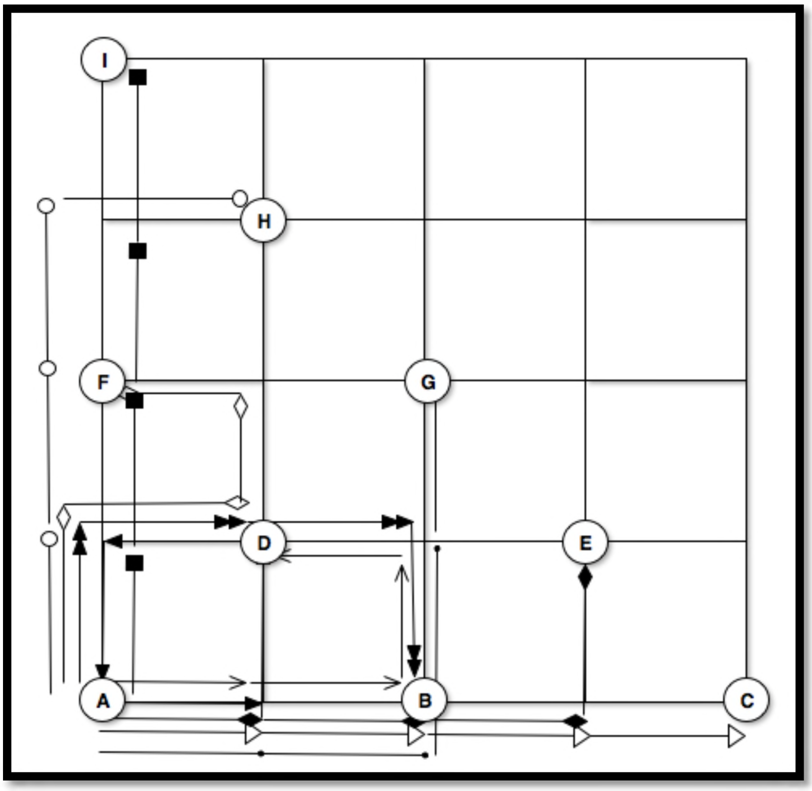


Figure 4.5.0.2.A Possible solution for Pathways task

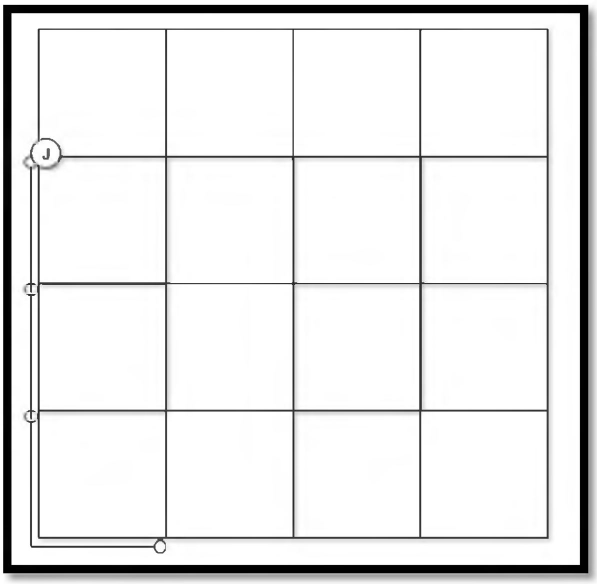
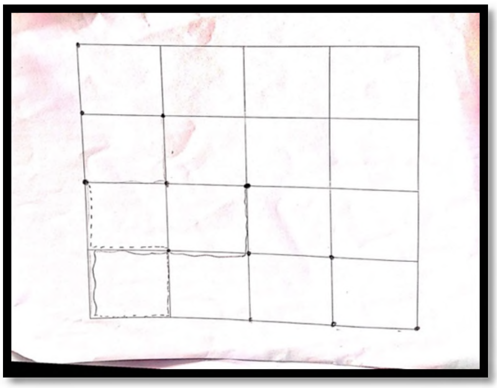


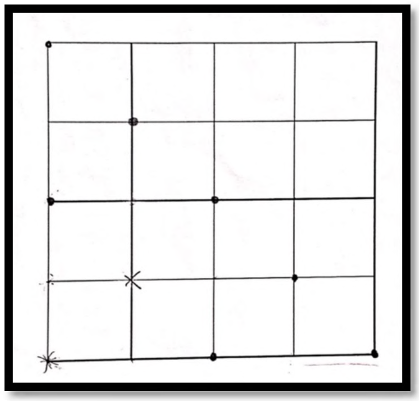
Figure 4.5.0.2.B Group 1 interpreted the task differently because the guidelines did not specifically state the location of the starting point

4.5.1 Documentation of Group 1 and Group 2 solutions to the Pathways task

Group 1 and Group 2 had two very different solutions to the Pathways task. Group 1 claimed to find 12 different end-points. Upon further investigation and conversation after the presentation, it transpired that learners had re-interpreted the rules, having started from the same corner but moving the matchstick in one direction and then returning in another direction (see Figure 4.5.1). This allowed the group to reach points that Group 2 had thought, by interpreting the rules differently, were not allowed. Though the starting point had been shown in the examples, it was not specified in the written directions. Neither group had very specific responses to the questions on how they solved the task. Group 2 had understood the problem to be to specifically find the end-points, not necessarily the pathways (see Figure 4.5.1).



Group 1 Pathways Argument
What was the problem?
The problem was find the pathways using four matchsticks
How did we solve it?
We used matchsticks and built pathways that we could find.
What was the solution?
We found 12



Group 2 Pathways Argument
What was the problem?
The problem was finding different end-points.
How did you solve it?
By using matchsticks
What was the solution?
We found 7 end-points

Figure 4.5.1 Group 1 and Group 2 solutions to the Pathways task

4.5.2 Evaluation of process as Group 1 and Group 2 solved the Pathways task

Group 1 and Group 2 were evaluated on how they interacted during the solving of the Pathways task. Flexibility, fluency, initiative, concentration, plausibility, and constructiveness were criteria for evaluation based on transcribed audio recordings, video recordings, learner work, and their final presentations of their conclusive argument (see Figure 4.5.2). To measure flexibility, an analysis of the amount of arguments made during the interaction was compared with the amount of correct solutions found. Group 1 made 23 arguments and found 12 correct solutions, while Group 2 made 20 arguments, and found 7 correct solutions.

Fluency was evaluated by analyzing the sustained interactions around one specific topic in interactions. Group 1 had 6 lines of sustained interaction while Group 2 had 18 lines of sustained interactions. To evaluate initiative, the amount of authored statements was analyzed. Group 1 had 66% of their arguments as author statements while Group 2 had 63% author statements.

Concentration looked at how balanced interactions were across the reasoning structure. 26% of Group 1's interactions was spent understanding the task compared to Group 2, which spent 34% of interactions comprehending the task. To evaluate plausibility, the depth of mathematical justifications was analyzed. Group 1 had 7 warrants and 3 backing statements made to justify their thinking, while Group 2 used 6 warrants and 1 backing statement.

PATHWAYS PROCESS EVALUATION	GROUP 1	GROUP 2
Reasoning Abilities	**Observable Indicators**	**Observable Indicators**
Flexibility: # of Arguments Made	23 Arguments Made / 12 Correct Solutions 1 | 2 | [3] | 4 | 5	20 Arguments Made/ 7 Correct Solutions 1 | 2 | [3] | 4 | 5
Fluency: Sustained Interaction	6 lines of sustained interaction 1 | 2 | [3] | 4 | 5	18 Lines of sustained interaction 1 | 2 | 3 | [4] | 5
Initiative: Authored Participation	66% Author Statements 1 | 2 | [3] | 4 | 5	63% Author Statements 1 | 2 | [3] | 4 | 5
Concentration: Interactions across the reasoning structure.	26% of the time was spent understanding the task. 13% of interactions were side conversation. 1 | [2] | 3 | 4 | 5	34% of interactions were on understanding the task. . 1 | [2] | 3 | 4 | 5
Plausability: Depth of Mathematical Justifications	Limited **mathematical** justification. 7 warrants and 3 backing statements were made 1 | [2] | 3 | 4 | 5	Limited **mathematical** justification. 6 warrants were used, and 1 backing statement. 1 | [2] | 3 | 4 | 5
Constructiveness: Balance of Contributions/Incorporating new ideas	LS contributed 35% and LK contributed 36% while LN contributed to 22% of interactions. 23% combined spokesman statements. 1 | 2 | [3] | 4 | 5	LT contributed to 39% of interactions wile LP contributed to 25%, and LD with 20%. 25% combined spokesman statements 1 | 2 | [3] | 4 | 5
	TOTAL: 16/30	**TOTAL: 17/30**

Marking Criteria:

1	2	3	4	5
No Evidence	Weak Evidence	Moderate Evidence	Good Evidence	Strong Evidence

Figure 4.5.2 Evaluation of Process of Pathways task

Constructiveness of group interactions looked at the balance of contribution amongst group members as well as the amount of statements made that showed group members were actively listening and incorporating peer ideas (Krummheuer, 2007). Group 1 had a relatively even contribution amongst peers. 35%, 26%, and 22% between LS, LK, and LN. 23% of interactions were spokesman statements. Group 2 had 39%, 25%, and 20% of interactions, distributed between LT, LP, and LD. They contributed 25% of interactions as spokesman statements.

4.5.3 Evaluation of the Final Product of Group 1 and Group 2 Solution to the Pathways task

The final solution to the Pathways task was evaluated by looking at 6 specific criteria: flexibility, fluency, novelty, concentration, plausibility, and constructiveness (see Figure 4.5.3). The flexibility criterion looked at the number of correct solutions compared to the number of possible solutions. Group 1 interpreted the task differently than Group 2 and found 12 out of 12 possible solutions while Group 2 found 7 out of 9 possible solutions, given their interpretation of the task. To evaluate fluency, the amount of strategies implemented to solve the task were analyzed. Both groups only used random guess and check methods, and did not have a verifying strategy in place to ensure they had found all the possible solutions.

PATHWAYS PRODUCT EVALUATION	GROUP 1	GROUP 2
Reasoning Abilities	**Observable Indicators**	**Observable Indicators**
Flexibility: # of Correct Solutions	12/12 Solutions 1 | 2 | 3 | 4 | **5**	7/9 Solutions 1 | 2 | 3 | **4** | 5
Fluency: Strategies implemented.	Random Guess and Check. **1** | 2 | 3 | 4 | 5	Random guess and check **1** | 2 | 3 | 4 | 5
Novelty: Uniqueness of strategies.	Random guess and check. Became flexible with the directions in finding end points that were impossible to reach given the visual demonstration. 1 | 2 | 3 | **4** | 5	Random guess and check, no apparent uniqueness. **1** | 2 | 3 | 4 | 5
Concentration: Sequentiality and Continuity of Strategy Implementation	There was no evidence of sequentiality or continuity of strategy implementation. **1** | 2 | 3 | 4 | 5	There was no evidence of sequentiality or continuity of strategy implementation. **1** | 2 | 3 | 4 | 5
Plausability: Mathematically Anchored Sociomathematical norms	Limited **mathematical** justification. Attempt to draw pathways to the end points, but not comprehensive. Explanations did not explain how exactly they solved the task 1 | 2 | **3** | 4 | 5	Limited **mathematical** justification. Explanations did not explain how exactly they solved the task 1 | **2** | 3 | 4 | 5
Constructiveness: Generalization to other concepts or experiences	No evidence of adopting strategies or prior-knowledge concepts from previous tasks. **1** | 2 | 3 | 4 | 5	No evidence of adopting strategies or prior-knowledge concepts from previous tasks. **1** | 2 | 3 | 4 | 5
	TOTAL: 15/30	**TOTAL: 10/30**

Marking Criteria:

1	2	3	4	5
No Evidence	Weak Evidence	Moderate Evidence	Good Evidence	Strong Evidence

Figure 4.5.3 Evaluation of Product of Pathways task

To evaluate novelty, an analysis of the uniqueness of strategies was made (Silver, 1997). In this case, Group 1 was creative in being flexible with the directions given; otherwise, both groups only used guess and check strategies. To evaluate concentration, the sequence and continuity of strategy implementation was analyzed. Both groups showed no use of sequence or continuation in their solving of the Pathways task.

Plausibility of the Pathways task analyzed the depth of mathematical justifications in their solutions to the task (Toulmin, 1964, Prusak et al., 2003). Group 1 attempted to draw the pathways to show evidence of how they arrived at the end-points. Neither group had specific explanations as to how they solved the task. To evaluate constructiveness of the groups' solutions to the task, evidence of adopting strategies from previous tasks was analyzed. Neither group showed evidence of this.

4.5.4 Summary of the Pathways Analysis and Evaluation

The Pathways task was similar in nature to the Mazes task, which gave some insight into the groups' abilities to generalize skills learned from previous tasks. Groups 1 and 2 spent very little time interacting during the task, which may be because of their perceived familiarity with the solving of the task. Both groups were similarly ad hoc in their implementation of strategies. Group 1 however was very creative in their interpretation of the rules given for the task, which showed a flexibility of thinking. Even after being questioned by the researcher as to their methods, the learners were able to justify their interpretation. Overall, the evaluation of the learners' ability to solve the task corresponds with that of other tasks, showing that implementation of strategies in solving the task was a challenge.

4.6 VITALMATHS CLIP BRICKLAYING ANALYSIS AND EVALUATION

Introduction to the problem task

The Bricklaying VITALmaths video clip poses the task of providing guidelines that an engineer should use while building a strong brick wall (see Figure 4.6.0.1). The mathematical concepts necessary for the task include an understanding of parallelism, perpendicularity, vertical and horizontal, symmetry, and the notion of *level* amongst a variety of other concepts in varying depths of understanding. The task is very open-ended, and allows an evaluator to see just how much depth of mathematical understanding learners may have. A variety of concepts must be applied to learners' arguments to be mathematically sound. These mathematical concepts were introduced in previous VITALmaths clips and so is an indicator of the ability of students to generalize concepts with which they have had previous interaction. This is also a good generalization exercise for observing if learners can apply their mathematical understanding to a contextually rich challenge. The number of mathematically grounded rules problem solvers come up with indicates the flexibility of thinking, and an indication of how mathematically grounded their arguments are.

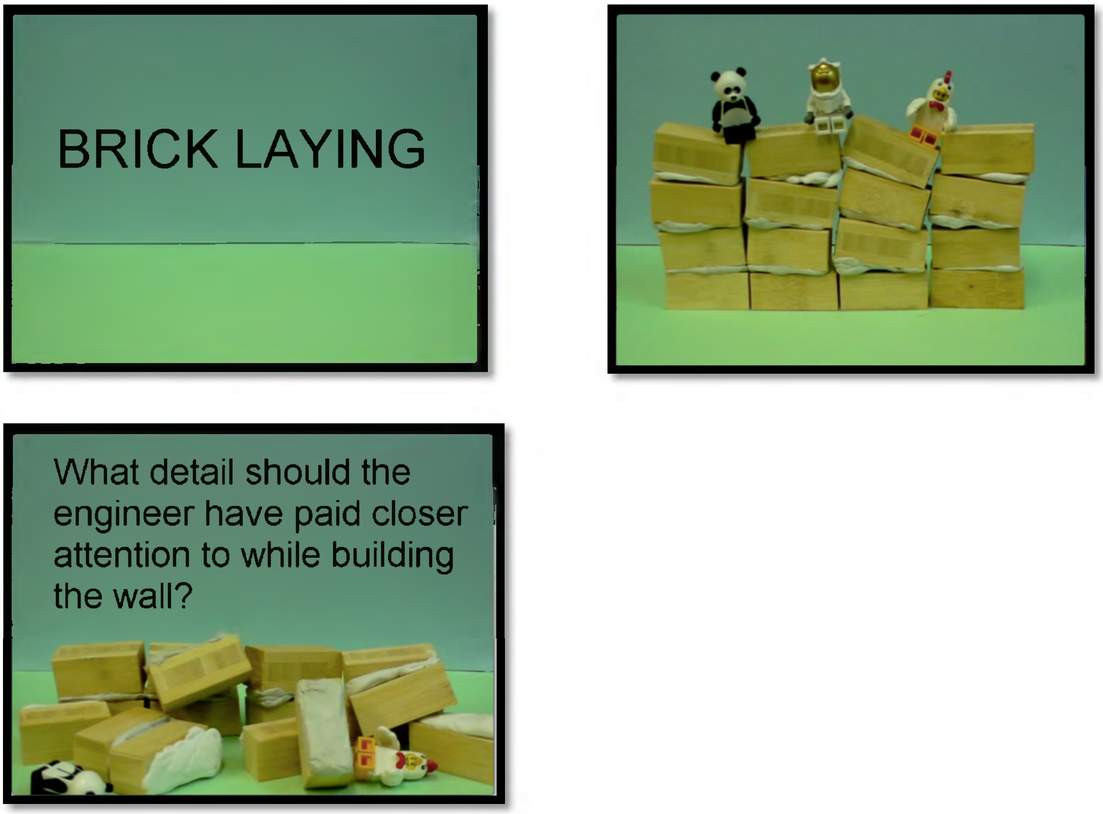


Figure 4.6.0.1 Screen shots of the Bricklaying VITALmaths video clip that illustrates the task

Potential solution to the Bricklaying task

There are many ways to solve the Bricklaying task. Figure 4.6.0.2 illustrates some potential, mathematically grounded rules that could apply to the task. The more complex terms that could be used are *plumb*, *square* and *level*, but they can be explained in different terms and to varying degrees of specificity. To be plumb means the vertical lines of the wall are parallel with the force of gravity. To be square means that all of the corners meet at right angles. To be level means that horizontal lines of the wall are perpendicular to the force of gravity. When looking at the bricks themselves, in the case of this wall, they must all be symmetrical, or the combination of bricks must be the same length, breadth and height in each row and/or column.

Concerning the mortar, it must be spread evenly to ensure the wall remains plumb, square and level. There are other non-mathematical factors that were raised by students and that are worthy of note. For instance, the engineer must not consume too much alcohol, which could impact on his or her attention to detail. The engineer must also communicate effectively with the other bricklayers. He or she must also consistently check and recheck his calculations throughout the building of the wall.

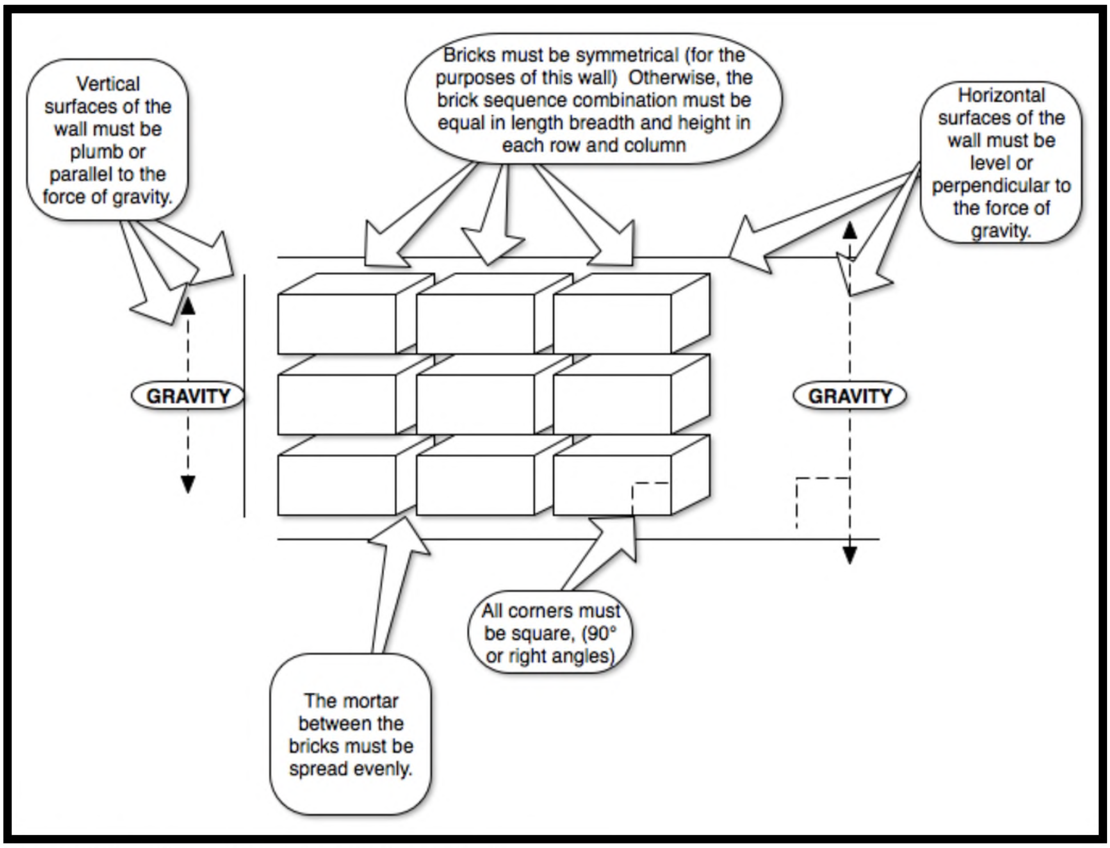


Figure 4.6.0.2 Possible solution for Bricklaying task

4.6.1 Group 1 and Group 2 solutions to the Bricklaying task

Figure 4.6.1 shows the solutions that Group 1 and Group 2 came up with to solve the Bricklaying task. Neither group drew pictures to help illustrate their rules. The worksheet provided showed some images that prompted specific issues a wall could have. Group 1 attempted to use mathematical terms in their rules, but were not specific in the spatial relationships, for instance "His structures must be straight or leveled" does not refer specifically to what must be straight or leveled. Group 2 was far vaguer in their explanations and referred less to mathematical concepts.

Group 1 Bricklaying Argument

What was the problem?

We had to help the bricklayer build a proper wall.

How did you solve it?

We used our maths skill to spot the problem and used blocks to solve it and we also used mathematical terms.

What was your solution?

1. His structures must be straight or leveled
2. When building a structure, he must think of horizontal and vertical lines to make sure his model is stable.
3. Use the same types of blocks to make a stable structure.
4. Place your bricks in a correct horizontal sequence.

Group 2 Bricklaying Argument

What was the problem?

To find a more stable way of creating a wall.

How did you solve it?

By looking at the structure that the character created.

Present your evidence:

The wall from the video.

What was your solution?

We gave the engineer tips on building a stable wall.

1. You will need a strong foundation so your wall does not fall.
2. Make sure the cement is strong and dries quickly.
3. Measure all your cement when putting it on your bricks.
4. Pay attention to the shape of the wall.
5. Make sure it is not leaning.

Figure 4.6.1 Group 1 interpreted the task differently because the guidelines did not specifically state the location of the starting point.

4.6.2 Evaluation of process as Group 1 and Group 2 solved the Bricklaying task

The process Group 1 and Group 2 went through in solving the Bricklaying task, flexibility, fluency, initiative, concentration, plausibility and constructiveness were evaluated through an analysis of video and audio recordings as well as written work produced during the task (see Figure 4.6.2). Flexibility was evaluated by analyzing the number of arguments that were made in comparison to how many correct solutions were found. Group 1 had 41 arguments and 4 correct solutions, compared to Group 2 which had 8 arguments and 5 correct solutions. Group 1 was able to have many interactions to solve the task while Group 2 had very few interactions. Fluency of the groups solving the Bricklaying task was evaluated from an analysis of the sustained interactions or the depth of interactions around one topic. Both Groups 1 and 2 showed very limited sustained interactions. Initiative evaluation was based on the amount of original statements contributed by peers. In this task, both Groups 1 and 2 had around 60% of their interactions as author statements. Group 1 had 60.6% authored interactions, while Group 2 had 57.7% of their interactions as author statements.

Concentration evaluation considered the balance of interaction across the reasoning structure. Group 1 showed 30% of interactions were focused on understanding the task, which is significant. There was an even amount of interaction between choosing strategies and implementing them. Group 2 spent 48% of interactions on understanding the task, which is a large proportion of time. There were very few interactions on selecting strategies and implementing them.

To determine the plausibility of the arguments made during interactions, the number of warrant and backing statements were analyzed. Group 1 had 5 warrant statements and 1 backing statement while Group 2 had 5 warrant statements and 1 backing statement.

BRICK LAYING PROCESS EVALUATION	GROUP 1	GROUP 2
Reasoning Abilities	**Observable Indicators**	**Observable Indicators**
Flexibility: # of Arguments Made	41 Arguments Made / 4 Correct Solutions 1 2 3 4 5	8 Arguments Made/ 5 Correct Solutions 1 2 3 4 5
Fluency: Sustained Interaction	7 lines of sustained interaction 1 2 3 4 5	5 Lines of sustained interaction 1 2 3 4 5
Initiative: Authored Participation	60.6% Author Statements 1 2 3 4 5	57.7% Author Statements 1 2 3 4 5
Concentration: Interactions across the reasoning structure.	30% of the time was spent understanding the task. Even distribution of strategy choice and strategy implementation. 1 2 3 4 5	48% of interactions were on understanding the task. Only 3 comments on strategy choice and 4 interactions around strategy implementation. 1 2 3 4 5
Plausability: Depth of Mathematical Justifications	Limited **mathematical** justification. 5 warrants and 1 backing statements were made 1 2 3 4 5	Limited **mathematical** justification. 5 warrants were used, and 0 backing statement. 1 2 3 4 5
Constructiveness: Balance of Contributions/Incorporating new ideas	LS contributed 42% and LK contributed 35%while LN contributed to 16% of interactions. 19.9% combined spokesman statements. 1 2 3 4 5	LT contributed to 51% of interactions wile LP contributed to 11%, and LD with 27%. 8% combined spokesman statements 1 2 3 4 5
	TOTAL: 17/30	**TOTAL: 11/30**

Marking Criteria:

1	2	3	4	5
No Evidence	Weak Evidence	Moderate Evidence	Good Evidence	Strong Evidence

Figure 4.6.2 Evaluation of the process of Group 1 and Group 2 interactions during the Bricklaying task

4.6.3 Evaluation of the product Group 1 and Group 2 solution to the Bricklaying task

The product evaluation of Group 1 and Group 2 solutions to the Bricklaying task was based on an analysis of written documentation and oral presentations given by the groups upon conclusion of the task (see Figure 4.6.3). Flexibility, fluency, novelty, concentration, plausibility and constructiveness of the solutions were considered.

To measure the flexibility of the groups' solution, the amount of correct solutions was analyzed compared to the possible solutions (Lithner, 2008). The task was very broad, and so multiple solutions could have been made. Group 1 had 4 solutions while Group 2 had 5 solutions. Group 1 had 4 of their solutions focused on mathematical concepts, while Group 2 had none that attempted to use descriptive mathematical terminology. To measure fluency, evidence of the strategies implemented was analyzed. Group 1 referred to the pictures they used to support their conclusions as well as the blocks provided on the table they used to build a "proper" wall. Group 2 had limited evidence of using any strategies to solve the task. It seemed to be a random selection of ideas.

The novelty of the conclusions Group 1 and Group 2 made in solving the Bricklaying task looked at the uniqueness of the strategies employed in solving the task (Lithner, 2008; Silver, 1997). Group 1 did use a variety of strategies to solve the task but the solutions did not show uniqueness, as the solutions seemed incomplete. Group 2 showed a very rushed implementation of strategy choice and implementation. Concentration looked at the systematic nature of the solutions. Group 1 did show some focused thought but did not follow their ideas through to completion as well as they might have. Group 2 showed no evidence of sequential thought and solutions seemed a collection of spontaneous thoughts.

BRICK LAYING PRODUCT EVALUATION	**GROUP 1**		**GROUP 2**	
Reasoning Abilities	**Observable Indicators**		**Observable Indicators**	
Flexibility: # of Correct Solutions	4/Many Solutions	1 2 **3** 4 5	5/Many Solutions	1 2 **3** 4 5
Fluency: Strategies implemented.	Used blocks and photo prompts.	1 2 **3** 4 5	Referred to the video, limited discussion about strategy choice.	**1** 2 3 4 5
Novelty: Uniqueness of strategies.	Good use of pictures and blocks, but not enough consideration into each aspect.	1 2 3 **4** 5	Very rushed implementation of strategies.	**1** 2 3 4 5
Concentration: Sequentiality and Continuity of Strategy Implementation	Students looked at each individual picture prompt when considering solutions. Limited depth of considerations.	1 2 **3** 4 5	There was no evidence of sequentiality or continuity of strategy implementation.	**1** 2 3 4 5
Plausability: Mathematically Anchored Sociomathematical norms	Limited **mathematical** justification. Mathematical terms were used, but did not specify spatial relationships.	1 2 **3** 4 5	Limited **mathematical** justification. Explanations did not focus on mathematical concepts or terminology.	1 **2** 3 4 5
Constructiveness: Generalization to other concepts or experiences	Good evidence of using previously covered concepts. Limited specific usage of terminology.	1 2 **3** 4 5	No evidence of adopting strategies or prior-knowledge concepts from previous tasks. No terminology was used to build their argument.	**1** 2 3 4 5
	TOTAL: 17/30		**TOTAL: 9/30**	

Marking Criteria:

1	2	3	4	5
No Evidence	Weak Evidence	Moderate Evidence	Good Evidence	Strong Evidence

Figure 4.6.3 Evaluation of the product of Group 1 and Group 2 solution to the Bricklaying task

Plausibility of the groups' final arguments considered how mathematically anchored solutions were and how well they were presented. Group 1 showed a good attempt at using mathematical terminology, but the terms were not used precisely enough to explain the spatial relationships being described. Group 1 had good ideas but none was mathematical in nature and no terminology was used despite prompts to do so. Constructiveness of the task was evaluated upon analysis of whether or not there was evidence that the groups could refer to previous experiences or concepts learned. Group 1 did show evidence based on their attempt to use mathematical terminology from previous clips in this research. Group 2 however did not.

4.6.4 Summary of Bricklaying analysis and evaluation

The Brick Laying task was an outlier in format and task expectations, compared to the other 5 clips. The open-ended nature of the task lent itself to different interpretations and conceptual understanding. It was the final task given to students because it became an indicator of learners' abilities to generalize their understandings from the previous clips that had looked at similarity of shapes, perpendicular and parallel relationships, and knowledge and use of the terms vertical and horizontal. Group 1 showed an attempt to understand the terminology used, but were not specific in the spatial relationships. Group 2 showed very little interaction in solving the task, and did not use the picture prompts to help them solve the task, as Group 1 was able to do.

4.7 VERTICAL ANALYSIS CONCLUDING REMARKS

The analysis and evaluation of each task provided insight into how learners interacted and presented a final argument. The Four Matches, Angles and Planes of Symmetry tasks provided more structured supports using modeled terminology and gave insight into how the learners interacted with presented concepts, and their mathematical misconceptions. The Maze and Pathways tasks were similar in nature, and required few mathematical concepts, but gave insight into the systematic implementation of strategies, and their flexibility of interpretation of the rules in the task. The Bricklaying task gave insight into the creativity of reasoning and mathematical justification of their reasoning. The task required learners to generalize their conceptual knowledge from previous tasks to provide mathematically grounded solutions. Both Groups 1 and 2 showed that they were consistently challenged in the implementation of strategies to solve the tasks and in generalizing conceptual knowledge from prior knowledge, as is evident from the comprehensive horizontal Analysis of groups across all the tasks found in Chapter 5.

CHAPTER FIVE: DATA ANALYSIS AND DISCUSSION OF RESULTS

PART B HORIZONTAL ANALYSIS

5.0 INTRODUCTION

Chapter 5 gives a horizontal analysis, which consolidates the evaluations from Chapter 4 Part A, by using the data from all 6 tasks Four Matches, Angles, Planes of Symmetry, Maze, Pathways and Bricklaying. With the consolidated data, I look for common trends or inconsistencies that occurred across the 6 tasks. After an analysis of the comprehensive data, this chapter will then discuss the overall evaluation across the 6 VITALmaths Clips.

Analysis tools explained

Interpretive data was collected from audio and video recordings as well as written work and presentations of Group 1 solving 6 VITALmaths tasks. Video and audio recordings were transcribed and analyzed according to specific criteria. Each line was analyzed according to the role the learner comment played in the interaction, according to Krummheuer's (1995; 2007) tools, to determine the novelty of individual statements. Data was sorted according to how it fits into Polya's (1954) and Lithner's (2008) reasoning structures. The analysis of data gave a picture of the amount of interaction that occurred and how it compared to the claims made, the participation roles group members played in the clips, the total participation of each group member, a comparison of interactions spent within the reasoning structure, and a measurement of the depth of argument justifications across all 6 tasks. An analysis of this data allowed for an evaluation of Creative Mathematical Reasoning.

Evaluation tools explained

Creative Mathematical Reasoning was evaluated in two distinct ways, firstly by how learners interacted in the *process* of solving the task, and secondly by looking at the final *product* of learner presentations. Observable indicators of process evaluation included flexibility, fluency, initiative, concentration, plausibility and constructiveness (see Figure 4.2). Product evaluation used similar terms with different observable indicators to evaluate creative mathematical reasoning. Terms used for product evaluation were flexibility, fluency, novelty, concentration, plausibility, and constructiveness (see Figure 4.3).

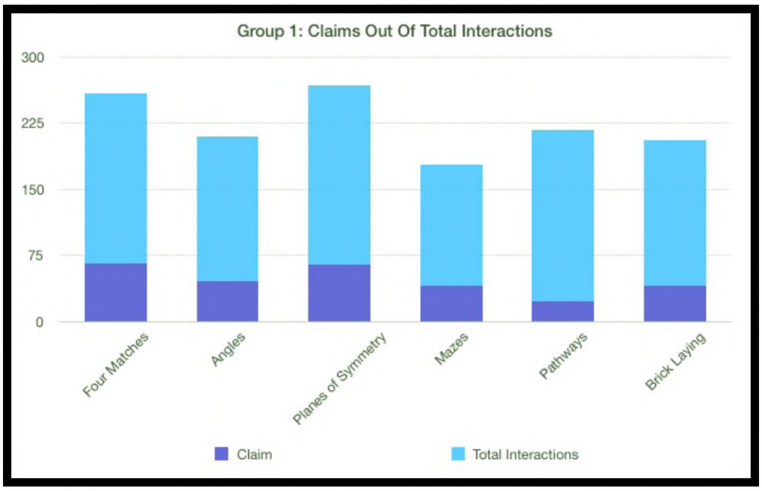
5.1 GROUPS 1 AND 2 COMPREHENSIVE ANALYSIS ACROSS THE SIX VITALMATHS TASKS

The comprehensive analysis of Group 1 and Group 2's creative mathematical reasoning was based on interpretation of transcribed records of video and audio recording as well as written work and presentations. This section explains the quantitative analysis that was extracted from the transcribed interactions.

5.1.1 Groups 1 and 2 claims out of total interactions

Group 1 analysis of claims out of total interactions

Group 1 had an average of 222.83 interactions in solving the mathematical tasks. Figure 5.1 illustrates the number of interactions made during the tasks compared to the number of claims made. The number of claims made per clip followed a general trend of proportionality to the amount of interactions made during the clip which averaged 46.66 claims per VITALmaths problem task. 20.9% of interactions were claims made by group members. The Pathways task was the only task that did not follow the trend of proportionate interactions to claims; there were fewer claims made during this task.

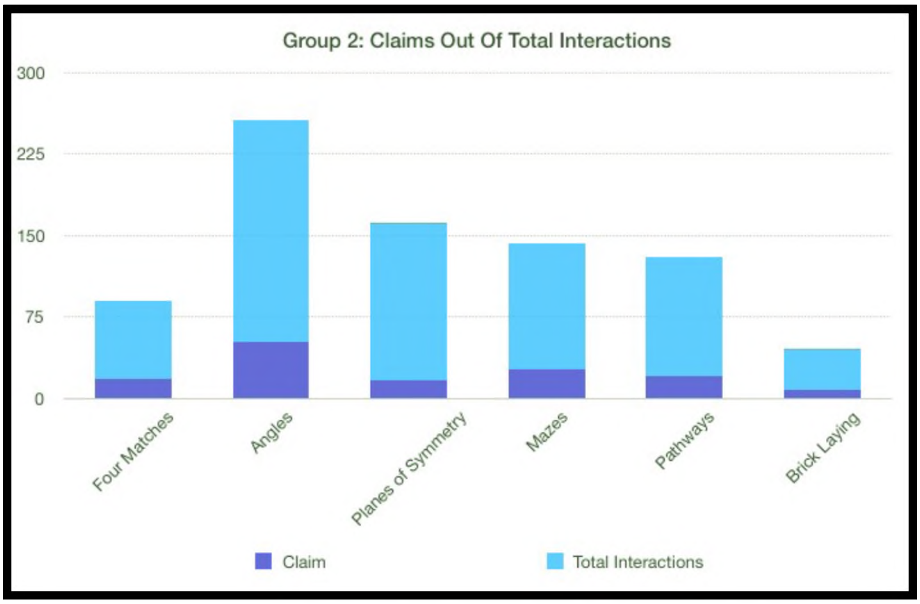


CLIP	CLAIMS	TOTAL INTERACTIONS
Four Matches	66	259
Angles	45	210
Planes of Symmetry	65	267
Mazes	40	178
Pathways	23	217
Bricklaying	41	206
TOTAL	280	1337
AVERAGE	46.66	222.83

Figure 5.1 Group 1 claims out of total interactions across all 6 VITALmaths tasks

Group 2 analysis of claims out of total interactions

Across all 6 VITALmaths tasks, Group 2 had an average of 137.33 interactions solving the tasks. There was no real pattern as to how much interaction occurred from task to task. The range was from 45 interactions (Bricklaying task) to 256 interactions (Angles task) (see Figure 5.2). The average amount of claims made per task was 23.33. The range of claims made per clip was from 8 to 52 claims. Across all 6 clips, the rate of claims per interaction was 16.98%.



CLIP	CLAIMS	TOTAL INTERACTIONS
Four Matches	18	90
Angles	52	256
Planes of Symmetry	16	161
Maze	26	142
Pathways	20	130
Bricklaying	8	45
TOTAL	140	824
AVERAGE	23.33	137.33

Figure 5.2 Group 2 claims out of total interactions across all 6 VITALmaths tasks

Comparative analysis of Group 1 and Group 2 claims and overall interactions

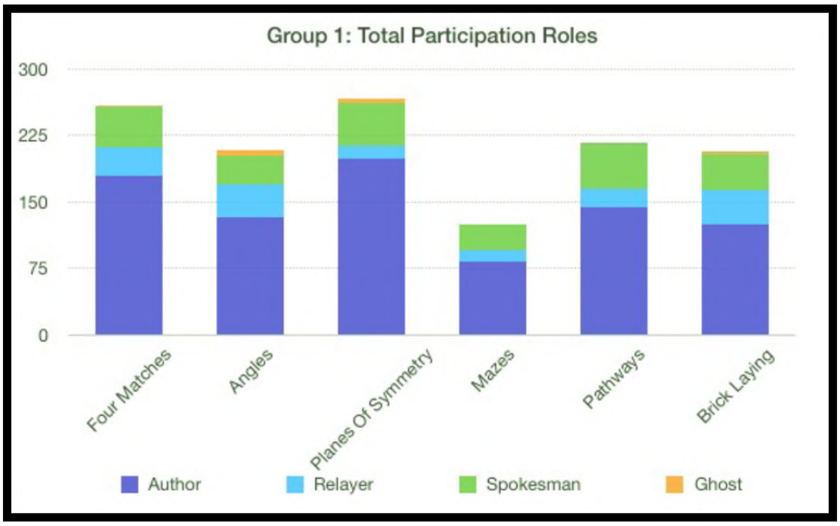
Group 1 had 513 more interactions across the 6 VITALmaths tasks than Group 2, which had 38.4% fewer interactions than Group 1. Group 1 also had exactly twice the number of claims across the 6 VITALmaths tasks. The range between the most interactions per clip versus the least interactions was also close to 2 times greater between Group 1 and Group 2. Though there were many differences in how much Group 1 and Group 2 made claims and interactions, the average rate of claims to interactions was very similar. Group 1's rate of claims to interactions was 20.9% compared to Group 2's of 16.98%.

5.1.2 Groups 1 and 2 participation roles and interactions

Group 1 analysis of participation roles and interactions

Given the participation roles of Author, Relayer, Spokesman and Ghostee (Krummheuer, 2005), each line of interaction was analyzed according to the originality of statements.

An average of 67% of interactions were author statements (original thoughts), 12.11% were relayer statements (parroted thoughts), 19.3% of interactions were spokesman statements (paraphrased thoughts), and 1.3% were ghostee statements (parroted statements that had new meaning). Figure 5.3 illustrates a general trend across the 6 clips where well over half of interactions were author statements, followed by spokesman statements of around 20%.

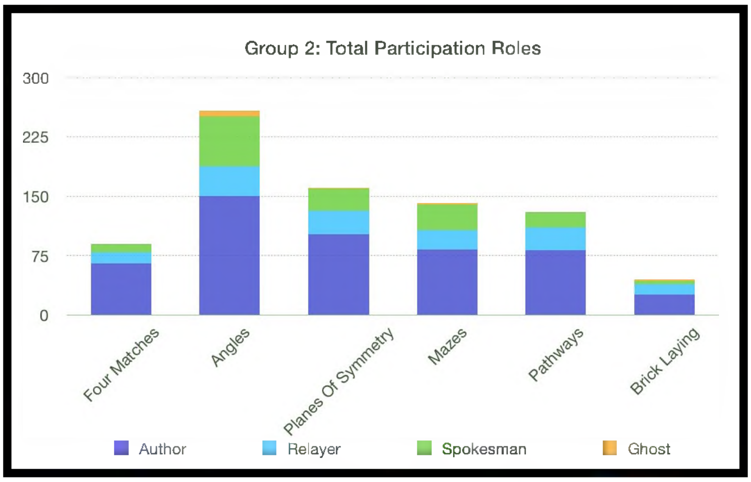


CLIP	AUTHOR	RELAYER	SPOKESMAN	GHOST
Four Matches	180	31	46	2
Angles	133	37	32	6
Planes of Symmetry	199	14	49	5
Maze	83	13	28	1
Pathways	144	21	52	0
Bricklaying	124	39	40	3
TOTAL	863	155	247	17
AVERAGE	143.33	25.83	41.16	2.83

Figure 5.3 Group 1 participation role interactions across all 6 VITALmaths tasks

Group 2 analysis of participation roles and interactions

Group 2 participants interacted irregularly across the 6 clips (see Figure 5.4). On average, 61% of interactions were author statements (original contributions). Author statements across all the clips were in the majority. On average, spokesman statements (paraphrased contributions) at 19.1% were used slightly more often than relayer statements (parroted contributions) of 17.9%. The Angles task showed a significant amount of spokesman statements compared to relayer statements (see Figure 5.4). The Bricklaying task showed an opposite trend with significantly more relayer comments (28%) than spokesman comments (8%).



Problem Task	AUTHOR	RELAYER	SPOKESMAN	GHOST
Four Matches	65	14	11	0
Angles	150	38	63	7
Planes of Symmetry	102	30	28	1
Maze	83	24	33	2
Pathways	82	29	19	0
Bricklaying	26	13	4	2
TOTAL	508	148	158	12
AVERAGE	84.66	24.66	26.33	2

Figure 5.4 Group 2 participation role interactions across all 6 VITALmaths tasks

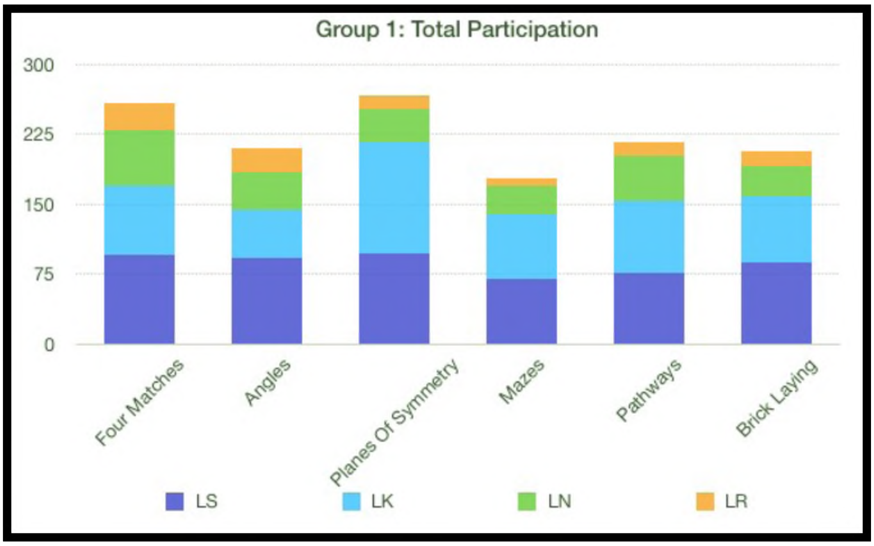
Comparative analysis of Group 1 and Group 2 participation roles

Groups 1 and 2 both made significantly more author statements across all the clips at 67% and 61% respectively. On average, about 20% of interactions were spokesman statements. Group 1 had 19.3% spokesman statements compared to Group 2's of 19.1%. Group 2 had a slightly higher percentage of relayer statements. On the whole, the both groups interacted in a similar way with close to 3/5 of interactions being author statements and 1/5 of interactions being spokesman statements.

5.1.3 Groups 1 and 2 balance of learner interactions

Group 1 analysis of balance of interactions

Across all 6 clips, participant LN played a relatively small role in the interactions that occurred, at 18% on average, while LS and LK participated on average about the same amount at 38% and 35% respectively. The researcher (LR) maintained an 8% interaction role which shows there was little engagement or support from the researcher beyond giving or clarifying instructions. Participant LK showed significantly more engagement during the Planes of Symmetry task and the Four Matches task showed a more even contribution between all three members of the group (see Figure 5.5). This may be due to the accessibility of the mathematical task where all group members were sufficiently confident to make contributions.

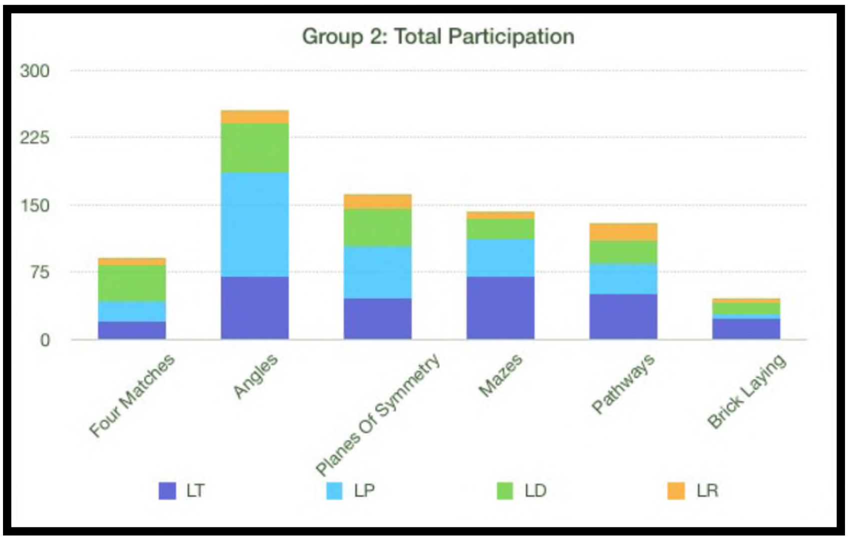


Problem Task	LS	LK	LN	LR
Four Matches	95	75	60	29
Angles	93	51	40	26
Planes of Symmetry	97	120	35	15
Maze	69	70	31	8
Pathways	76	78	48	15
Bricklaying	87	72	32	15
TOTAL	517	466	246	108
AVERAGE	86.16	77.66	41	18
% OF INTERACTIONS	38%	35%	18%	8%

Figure 5.5 Group 1 balance of interactions between participants across all 6 VITALmaths clips

Group 2 analysis of balance of interactions

Participants LT and LP both contributed nearly equal amounts on average across all 6 VITALmaths clips at 34% and 34% respectively. The majority of interactions from clip to clip varied between the participants (see Figure 5.6). LD, while her participation was less than her peers, still contributed to 24% of interactions on average, and contributed more than her peers during the Four Matches task. The researcher LR participated in only 9% of interactions which illustrates the limited engagement of the researcher in interactions.



Problem Task	LT	LP	LD	LR
Four Matches	19	24	39	8
Angles	70	115	55	15
Planes of Symmetry	46	57	42	16
Maze	69	43	22	8
Pathways	51	33	26	20
Bricklaying	23	5	12	5
TOTAL	278	277	196	72
AVERAGE	46.33	46.16	32.66	12
% OF INTERACTION	34%	34%	24%	9%

Figure 5.6 Group 2 balance of interactions between participants across all 6 VITALmaths clips

Comparative analysis of Group 1 and Group 2 balance of interactions

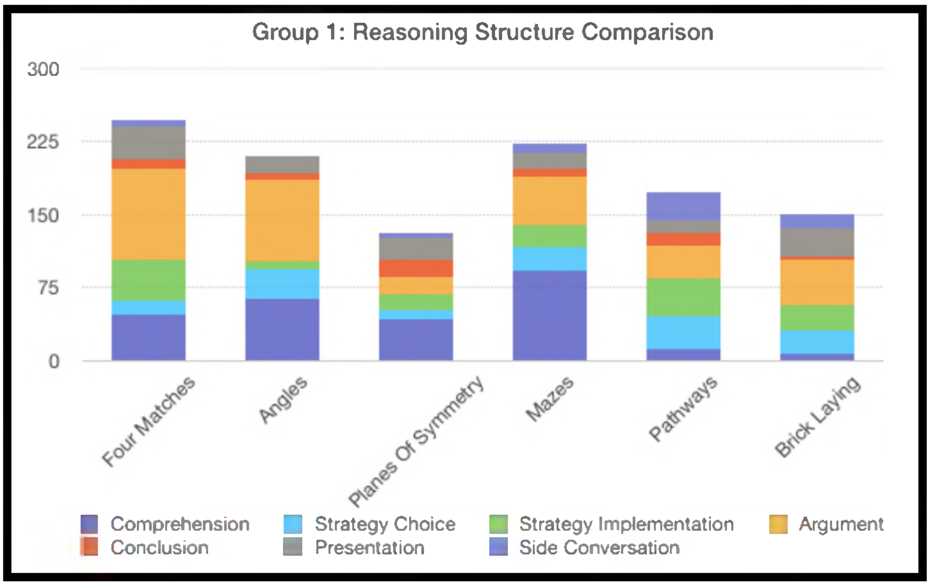
Group 1 and Group 2 engaged differently to each other with regard to the balance of interactions between participants. Group 1 had two more dominant participants, while Group

2 had a nearly balanced contribution across the 3 participants. In both groups, the researcher contributed the same amount of around 8% of interactions.

5.1.4 Group 1 and 2 reasoning structure interactions

Group 1 analysis of reasoning structure

On average, 54% of Group 1 interactions was spent understanding the task (26%) and in argument (28%), while 25% were in strategy choice (12%) and strategy implementation (13%) (see Figure 5.7).



Problem Task	Comp.	Strategy Choice	Strategy Implement	Argument	Conclude	Present	Side Convo
Four Matches	47	15	42	92	11	33	6
Angles	63	31	7	85	6	18	0
Planes of Symmetry	42	10	16	18	18	22	5
Maze	35	24	22	50	8	17	10
Pathways	56	34	39	33	14	12	29
Bricklaying	62	25	25	47	4	29	14
TOTAL	305	139	151	325	61	131	64
AVERAGE	50.8	23.16	25.16	54.16	10.16	21.8	10.66
% OF INTERACTION	26%	12%	13%	28%	5%	11%	5%

Figure 5.7 Group 1 analysis of reasoning structure across the 6 VITALmaths tasks

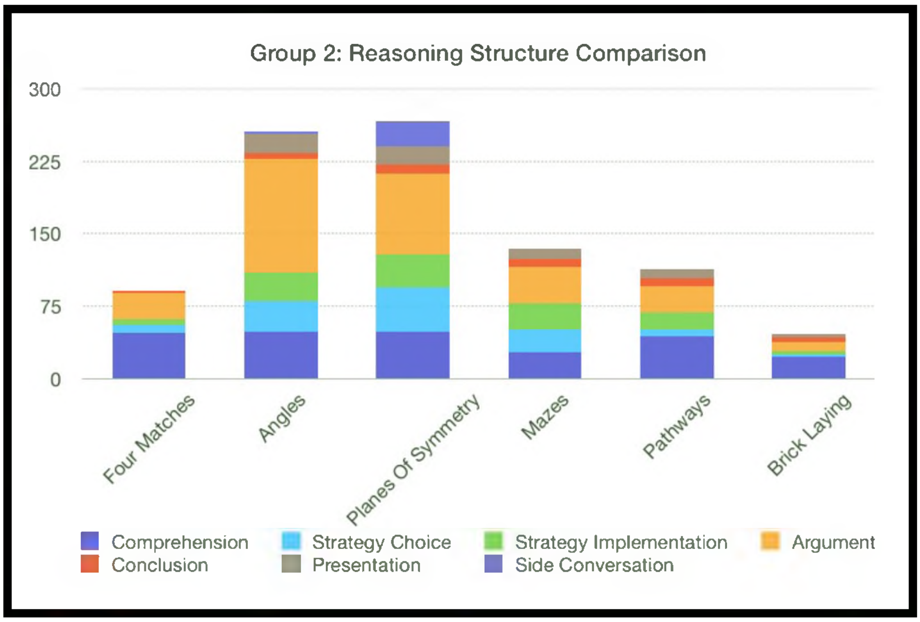
There was some variance between clips of how many interactions were strategy choice compared to strategy implementation. Only 5% of interactions were on ensuring Group 1 had a conclusive solution. The Planes of Symmetry task as well as the Pathways task showed a more even distribution of interactions between strategy choice and implementation, as well as argument and conclusion interactions. This may be due to the nature of the task that required more focus strategy.

Group 2 analysis of reasoning structure

Group 2 spent on average 32% of their interactions in argument, and 25% of interactions understanding the task (see Figure 5.8). An equal percentage of interactions were in selecting a strategy (13%) and implementing the strategy (13%). Only 4% of interactions was focused on ensuring a correct solution to the task. There was a consistent trend of allocation of interactions within the reasoning structure across all 6 VITALmaths clips, with the exception of the Bricklaying task.

Comparative analysis of Group 1 and Group 2 interactions within the reasoning structure

Group 1 and 2 shared some similarities in their interactions within the reasoning structure. Both Group 1 and Group 2 spent 25% of their time understanding the task and 5% of their time coming up with a conclusion to the task. Both groups also spent an equal amount of time (around 26%) on interactions selecting strategies and implementing them. Group 2 spent a slightly greater percentage in argument (see Figures 5.7 and 5.8).



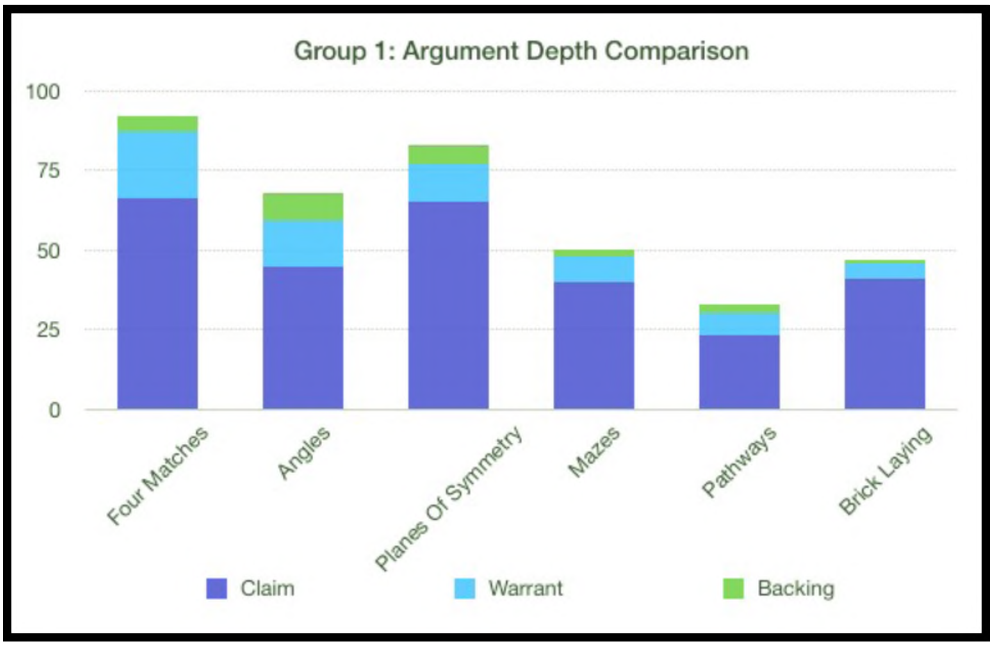
Problem Task	Comp.	Strategy Choice	Strategy Implement	Argument	Conclude	Present	Side Convo
Four Matches	47	8	6	28	2	0	0
Angles	48	32	30	118	6	20	2
Planes of Symmetry	49	46	34	83	10	18	27
Maze	27	24	27	38	8	11	0
Pathways	44	7	18	27	8	9	0
Bricklaying	22	3	4	9	4	4	0
TOTAL	237	120	119	303	38	62	29
AVERAGE	39.5	20	19.83	50.50	6.33	10.33	9.67
% OF INTERACTION	25%	13%	13%	32%	4%	7%	6%

Figure 5.8 Group 2 analysis of reasoning structure across the 6 VITALmaths tasks

5.1.5 Group 1 and Group 2 analysis of depth of argument

Group 1 analysis of depth of argument

Across all 6 VITALmaths clips, Group 1 showed a relationship between number of claims and warrants and backing (see Figure 5.9). The average ratio of claims to warrants to backing was 46.66:11.17:4.33. Group 1 made on average 46.6 claims per task. Roughly simplified for the purposes of comparison, this represents a ratio of about 4:1:.25. This means that for comparison purposes, for every 4 claims made, 1 backing statement was made. Furthermore 1 in 12 claims had a backing statement.

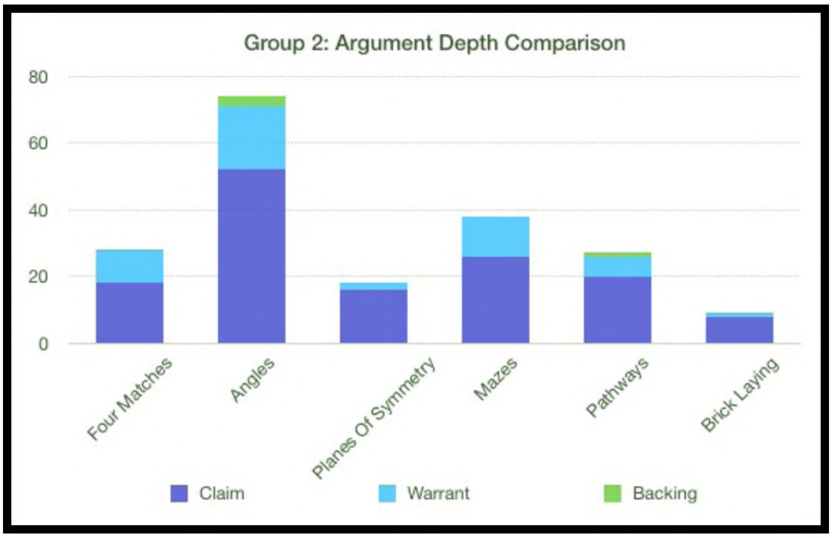


CLIP	CLAIMS	WARRANTS	BACKING
Four Matches	66	21	5
Angles	45	14	9
Planes of Symmetry	65	12	6
Maze	40	8	2
Pathways	23	7	3
Bricklaying	41	5	1
TOTAL	280	67	26
AVERAGE	46.67	11.17	4.33

Figure 5.9 Depth of arguments made during Group 1 interactions

Group 2 analysis of depth of argument

Group 2 did not show consistency across the 6 VITALmaths tasks. The participants only made backing statements during two tasks, Angles and Pathways (see Figure 5.10). The average ratio of claims made compared to warrant and backing statements was 23.33:8.33:.66, or in simplified form, 3:1:.08. For every 3 claims 1 warrant statement was made and 1 in 35 claims had a backing statement.



CLIP	CLAIMS	WARRANTS	BACKING
Four Matches	18	10	0
Angles	52	19	3
Planes of Symmetry	16	2	0
Maze	26	12	0
Pathways	20	6	1
Bricklaying	8	1	0
TOTAL	140	50	4
AVERAGE	23.33	8.33	.66

Figure 5.10 Depth of arguments made during Group 2 interactions

Comparative analysis of Group 1 and Group 2 depth of argument

On average, Group 1 made twice the number of claims per task. This influenced the ratio of claims to warrants to backing. The claims to warrants ratio was similar for both groups; for Group 1 it was 4:1 and for Group 2 it was 3:1. The real contrast between Group 1 and Group 2 was the backing statements to claims ratio. Group 2 made far fewer backing statements on average in comparison to Group 1.

5.1.6 Conclusion to comprehensive analysis

The data analyzed in 5.1.1 informed the evaluation of process of reasoning abilities discussed in section 5.2 below. Figures in section 5.1.1 Claims out of Total Interactions gave insight into the flexibility of reasoning. The figures in section 5.1.2 Participation Roles informed the evaluation of Initiative and Constructiveness of reasoning. Section 5.1.3 analysis on the Balance of Interactions also informed the constructiveness of interactions. Section 5.1.4 on the interactions across the reasoning structure gave insight into the Concentration abilities of the group, and the Argument Depth Comparison in section 5.1.5 was used for analysis of Plausibility of group reasoning abilities.

5.2 COMPREHENSIVE PROCESS EVALUATION ACROSS 6 VITALMATHS TASKS

Group 1 comprehensive process evaluation

Across all 6 VITALmaths tasks, Group 1 scored an average of 17.67 out of 30 marks in the process evaluation (see Figure 5.11). The areas of greatest strength were in flexibility and initiative. Group 1 was very interactive and made many claims indicating a flexibility of knowledge. With an average mark of 3.5 in Initiative, Group 1 participants made many original authored statements in their interaction. Areas in need of improvement are concentration and plausibility reasoning abilities. In general, strategy choice and strategy implementation interactions were not a large proportion of interactions, which gave Group 1 lower concentration marks. Group 1 did use warrants and backing, but the ratio of claims to warrants was high (4:1). Across the 6 VITALmaths task, there was a 4-point variance between the highest score and lowest score. The Pathways task had the lowest score of 16, while Four Matches had a score of 19. Group 1 showed a fair amount of consistency across the 6 tasks.

GROUP 1 COMPREHENSIVE PROCESS EVALUATION	Four Matches	Angles	Planes of Symmetry	Maze	Pathways	Brick-laying	Total	Average
Reasoning Abilities								
Flexibility: # of Arguments Made	**4**	**3**	**4**	**3**	**3**	**4**	**21**	**3.5**
Fluency: Sustained Interaction	**3**	**4**	**3**	**3**	**3**	**2**	**18**	**3**
Initiative: Authored Participation	**4**	**3**	**4**	**4**	**3**	**3**	**21**	**3.5**
Concentration: Interactions across the reasoning structure	**2**	**2**	**3**	**3**	**2**	**3**	**15**	**2.5**
Plausibility: Depth of Mathematical Justifications	**3**	**3**	**3**	**2**	**2**	**2**	**15**	**2.5**
Constructiveness: Balance of Contributions/Incorporating new ideas	**3**	**2**	**2**	**3**	**3**	**3**	**16**	**2.67**
TOTAL:	**19/30**	**17/30**	**19/30**	**18/30**	**16/30**	**17/30**	**106/ 180**	**17.67/ 30**

Marking Criteria:

1	**2**	**3**	**4**	**5**
No Evidence	Weak Evidence	Moderate Evidence	Good Evidence	Strong Evidence

Figure 5.11 Comprehensive process evaluation of Group 1

Group 2 comprehensive process evaluation

Group 2 was given an average mark of 15.33 across all 6 VITALmaths tasks (see Figure 5.12). On average, the area of greatest strength was in initiative. Participants made a proportionately significant amount of original authored statements which showed initiative in developing new ideas. This was consistent across all 6 tasks. There was a real variance in marks specifically in concentration abilities and constructiveness. In the Angles task, Group 2 scored a 4 for concentration based on interactions around strategy choice and implementation, and scored a 1 in concentration during the Bricklaying task, with little interaction around solving the task. In the Planes of Symmetry task, Group 2 showed constructiveness by balancing interactions between peers and responding to each other's ideas using spokesman statements, compared to in the Four Matches task and Bricklaying task, which showed one person playing a more dominant role and interactions being focused on their own ideas. There was a 9-point variance between the lowest score, which was 11 in the Bricklaying task, and the highest score, which was 20 in the Angles task. This seems to show a significant inconsistency in problem solving interactions.

GROUP 2 COMPREHENSIVE PROCESS EVALUATION	Four Matches	Angles	Planes of Symmetry	Maze	Pathways	Brick-laying	Total	Average
Reasoning Abilities								
Flexibility: # of Arguments Made	**2**	**3**	**2**	**2**	**3**	**2**	**14**	**2.33**
Fluency: Sustained Interaction	**1**	**4**	**2**	**3**	**4**	**2**	**16**	**2.67**
Initiative: Authored Participation	**4**	**3**	**3**	**3**	**3**	**3**	**19**	**3.17**
Concentration: Interactions across the reasoning structure	**2**	**4**	**2**	**3**	**2**	**1**	**14**	**2.33**
Plausibility: Depth of Mathematical Justifications	**2**	**3**	**2**	**2**	**2**	**1**	**12**	**2**
Constructiveness: Balance of Contributions/Incorporating new ideas	**2**	**3**	**4**	**3**	**3**	**2**	**17**	**2.83**
TOTAL:	**13/30**	**20/30**	**15/30**	**16/30**	**17/30**	**11/30**	**92/ 180**	**15.33/ 30**

Marking Criteria:

1	**2**	**3**	**4**	**5**
No Evidence	Weak Evidence	Moderate Evidence	Good Evidence	Strong Evidence

Figure 5.12 Comprehensive process evaluation of Group 2

Comparison between Group 1 and Group 2 Process evaluations

Group 1 showed a higher average mark of 17.67 compared to Group 2, which was 15.33. Group 2 did have the highest mark of 20 in solving the Angles task, but was inconsistent in other tasks. Both groups showed a strength and consistency in initiative. A large proportion of interactions were author statements. Plausibility of arguments proved to be the area of greatest need for improvement; the ratio of claims to warrants to backing was high, as participants did not provide warrants or backing to support their claims.

5.3 COMPREHENSIVE PRODUCT EVALUATION ACROSS 6 VITALMATHS TASKS

Group 1 comprehensive product evaluation

Group 1 scored an average of 12.17 across all 6 VITALmaths clips (see Figure 5.13). The strongest areas of reasoning were in flexibility and novelty. In the Pathways task, Group 1 found all the correct pathways to solve the task. The Angles and Planes of Symmetry tasks proved a challenge in finding all the possible solutions. The reasoning abilities of greatest challenge were constructiveness and concentration across all 6 VITALmaths tasks.

GROUP 1 COMPREHENSIVE PRODUCT EVALUATION	Four Matches	Angles	Planes of Symmetry	Maze	Pathways	Brick-laying	Total	Average
Reasoning Abilities								
Flexibility: # of Correct Solutions	**3**	**1**	**1**	**3**	**5**	**3**	**16**	**2.67**
Fluency: Strategies implemented	**2**	**2**	**2**	**1**	**1**	**3**	**11**	**1.83**
Novelty: Uniqueness of strategies	**2**	**3**	**2**	**1**	**4**	**4**	**16**	**2.67**
Concentration: Sequentiality and Continuity of Strategy Implementation	**1**	**2**	**1**	**1**	**1**	**3**	**7**	**1.17**
Plausibility: Mathematically Anchored Sociomathematical norms	**2**	**2**	**1**	**3**	**3**	**3**	**14**	**2.33**
Constructiveness: Generalization to other concepts or experiences	**2**	**1**	**1**	**1**	**1**	**3**	**9**	**1.5**
TOTAL:	**12/30**	**11/30**	**8/30**	**10/30**	**15/30**	**19/30**	**73/ 180**	**12.17/ 30**

Marking Criteria:

1	2	3	4	5
No Evidence	Weak Evidence	Moderate Evidence	Good Evidence	Strong Evidence

Figure 5.13 Comprehensive product evaluation of Group 1

The ability to transfer knowledge from task to task was difficult to observe in Group 1, hence a low score in constructiveness. Strategy choice and implementation were difficult for Group 1 as the strategies implemented seemed ad hoc and inconsistent. The scores across the task were inconsistent. The lowest score was an 8 and the highest score a 19 which showed an 11-point variance.

Group 2 comprehensive product evaluation

Group 2 had an average of 9.67 marks across all 6 VITALmaths tasks (see Figure 5.14). The area of greatest strength was in flexibility of reasoning, as they found correct solutions to the tasks. The area of greatest room for improvement was in concentration and constructiveness reasoning abilities. Implementing strategies consistently was limited in the presentations of evidence. Generalizing skills across the tasks and from prior knowledge was limited. On the whole, Group 2 marks across the 6 VITALmaths tasks were consistent with a variance of 1.

GROUP 2 COMPREHENSIVE PRODUCT EVALUATION	Four Matches	Angles	Planes of Symmetry	Maze	Pathways	Brick-laying	Total	Average
Reasoning Abilities								
Flexibility: # of Correct Solutions	**2**	**2**	**2**	**3**	**4**	**3**	**16**	**2.67**
Fluency: Strategies implemented	**2**	**2**	**2**	**1**	**1**	**1**	**9**	**1.5**
Novelty: Uniqueness of strategies	**1**	**2**	**2**	**1**	**1**	**1**	**8**	**1.33**
Concentration: Sequentiality and Continuity of Strategy Implementation	**1**	**2**	**1**	**1**	**1**	**1**	**7**	**1.17**
Plausibility: Mathematically Anchored Sociomathematical norms	**2**	**1**	**2**	**2**	**2**	**2**	**11**	**1.83**
Constructiveness: Generalization to other concepts or experiences	**2**	**1**	**1**	**1**	**1**	**1**	**7**	**1.17**
TOTAL:	**10/30**	**10/30**	**10/30**	**9/30**	**10/30**	**9/30**	**58/ 180**	**9.67/ 30**

Marking Criteria:

1	2	3	4	5
No Evidence	Weak Evidence	Moderate Evidence	Good Evidence	Strong Evidence

Figure 5.14 Comprehensive product evaluation of Group 2

Comparison between Group 1 and Group 2 Product evaluations

Group 2 showed similar areas of strengths and concern. On average, both groups scored 2.67 in flexibility of reasoning. Both groups also struggled with concentration and constructive reasoning abilities. Being able to select a strategy and see it through to a comprehensive conclusion was a challenge for both groups. The evidence presented did not show a clear system of solving the tasks. It was also a challenge for both groups to assimilate concepts from other tasks and generalize them to a new task. Group 2 was consistent in their scores across the 6 tasks, while Group 1 performed better in the Pathways and Bricklaying tasks, which increased their average marks.

5.4 CONCLUDING REMARKS

In doing a horizontal analysis of each group across 6 VITALmaths tasks, I was able to identify trends across the 6 clips and compare the analysis of each group. Group 1 had 38.4% more interactions than Group 2 but had a similar ratio of average of claims to interactions. The participation roles were very similar between the two groups showing a majority of interactions being author statements (between 61% and 67%). Group 1 participation of group members was imbalanced compared to Group 2. Group 1 had two participants who participated considerably more than the third participant in the group across the 6 VITALmaths clips. In Group 2 all three group members participated more equally. Both Groups 1 and 2 had similar interactions across the reasoning structure although Group 1 had significantly more argument statements when solving the tasks. While Group 1 made more arguments, Group 2 had a greater ratio of supporting statements to claims. Group 1 provided more backing statements on average than Group 2.

Based on this data analysis of interactions, Group 1 had marks that are more consistent compared to Group 2 in process evaluation. Group 2 had more consistent marks across the 6 VITALmaths tasks in product evaluation. Both groups showed strengths in flexibility of reasoning, but struggled with constructiveness and concentration reasoning abilities. Chapter 6 will discuss what can be gathered as potentially important findings from this analysis and what might be beneficial to further research.

CHAPTER SIX: CONCLUSION

6.1 INTRODUCTION

The focus of this research has been on the analysis of creative mathematical reasoning (Lithner, 2008). To analyze creative mathematical reasoning, this case study prompted collaborative argumentation through the use of VITALmaths clips and supplemental worksheets. Through qualitative and quantitative methods, the study analyzed the process of creative mathematical reasoning by evaluating the observable reasoning abilities of flexibility, fluency, initiative, concentration, plausibility and constructiveness (see Figure 4.0.2). The study also analyzed the product of creative mathematical reasoning by evaluating the observable reasoning abilities of flexibility, fluency, novelty, concentration, plausibility and constructiveness of their solutions to the tasks (see Figure 4.0.3).

This chapter summarizes the findings from the research in the analysis of creative mathematical reasoning. I will discuss the significance of this case study and what it may tell us about how learners solve mathematical tasks, from the unique perspective of this case study. Assumptions and limitations to the study and suggestions for further research to the study will also be considered.

6.2 SUMMARY OF FINDINGS

This case study analyzed the process and product of creative mathematical reasoning as two separate units of analysis. Two groups solved six VITALmaths tasks and were evaluated according to specific observable indicators. This evaluation of creative mathematical reasoning was done for both groups for each individual VITALmaths task (vertical analysis) and comprehensively across all six VITALmaths tasks (horizontal analysis). This section is a summary of findings of the process and product analysis of the groups' creative mathematical reasoning abilities.

6.2.1 Summary of process findings

How did learners interact with each other (Fluency, Initiative and Constructiveness)

The amount of interactions and sustained interactions around one concept (fluency) between participants while solving the VITALmaths tasks varied from task to task. This seemed to be more dependent on the challenge of the task, and group dynamics, so it was difficult to identify common threads or trends in fluency.

Both initiative and constructive reasoning abilities were observed and analyzed according to the participation roles played in the interactions (Krummheuer 1995). "Author" statements showed more initiative when participants contributed new ideas to the discussion. "Spokesperson" statements made by participants showed constructiveness as they were able to paraphrase other participants' ideas and integrate the new idea into their thinking. 60%-70% of interactions were original or authored statements which was an indication of the initiative of participants to contribute new ideas. 19% of participant interactions incorporated other group member ideas in their contributions. Observing interactions between participants was rather haphazard and disconnected. One participant would "pop" up with an idea, and then another participant would "pop" up with another unrelated idea. There was often a disconnectedness between ideas and contributions. Participants would share their own ideas around the same topic, but did not engage other group members' contributions.

How did learners interact with the task within the reasoning structure (Concentration)

The reasoning ability of concentration in this research focused on how participants interacted across a reasoning structure adapted from Polya (1945), and Lithner (2008). Interactions were coded according to the following categories: comprehension of the task, strategy choice, strategy selection, strategy implementation, argument, conclusion, presentation, and side conversations.

Both groups shared similar trends in the way they engaged across the reasoning structure. On average, 25% of interactions were focused on understanding the task, while 13% of interactions were spent selecting strategies and 13% of interaction was spent discussing implementation of strategies. The strategies selected and implemented were mostly superficial with a focus on how work would be documented. About 30% of interactions were spent solving the tasks. Only 4% of interactions were spent ensuring correctness and exhaustiveness of solutions. Very little time was spent on ensuring their tasks were conclusive. More up front time discussing strategy choice and implementation would have been beneficial for ensuring more correctness and completeness of solutions.

How did learners justify their reasoning (flexibility and plausibility)

While the amount of time spent solving the tasks varied from task to task, the proportion of interactions spent in argument was consistently between 17% and 20% for both groups. This was an indication of the flexibility of reasoning. As group 1 spent more time in interactions than Group 2, they also spent more time in collaborative argument.

Plausibility of reasoning in this research looked at the depth of justification in reasoning (Toulmin 1964). Group 1 had a 4:1 claims to warrants ratio, while Group 2 had a 3:1 ratio. Group 1 however had significantly more backing statements to further justify their thinking. As mentioned previously regarding constructiveness, the arguments made in interaction were haphazard, and disjointed. This may have impacted the ratios of claims to warrants. Having spent more focused effort in establishing and implementing strategies to solve the task (concentration) may have allowed for more plausible reasoning interactions.

6.2.2 Summary of product findings

Correctness and exhaustiveness of solutions (flexibility)

All of the VITALmaths tasks had an open-ended nature. The tasks required participants to find as many solutions as they could without specifying how many solutions they could find. This allowed the researcher to identify how exhaustive their solutions were. This gave insight into their flexibility of reasoning.

The correctness and exhaustiveness of solutions of group solutions to tasks varied from task to task. On average both groups performed about the same across all six tasks. Both groups received an average mark of 2 (weak evidence) in flexibility which measured the number of correct solutions. The limited correctness and exhaustiveness to solutions possibly relates to the limited evidence of strategy implementation (concentration). Based on group presentations, and documentation of evidence, there was limited consideration of how many possible solutions there were, or verifying the correctness of the solutions they had.

Novelty, fluency and concentration of strategies implemented

The novelty of solutions to the VITALmaths tasks considered the uniqueness of strategies employed to solve the tasks (Silver, 1997, Lithner, 2008). Both groups did show a uniqueness or novelty in the way they approached the VITALmaths tasks and presented their evidence. Most strategies, however, were superficial in nature only focusing on how the evidence was presented. Only few strategies were focused on ensuring correctness and exhaustiveness of solutions. This impacted on the fluency of reasoning, as only few strategies were implemented in each task.

Concentration of reasoning in this study analyzed the sequentiality and consistency of strategy implementation (Polya, 1957, Lithner, 2008). This proved to be a challenge for both groups. It was difficult to follow a sequence or consistency of reasoning from the beginning to end of the evidence presented. Similar to the interactions of participants in solving the tasks, solutions were presented in an ad hoc, disconnected way.

Strength and depth of arguments to justify solutions (plausibility and constructiveness)

This study proved to be difficult in finding evidence of generalizability of tasks, so marks were generally low in the area of constructiveness of learning. There were not many opportunities prompting learners to make connections between tasks and with prior knowledge, and there was little evidence from the presentations that participants were making these types of generalizations on their own.

Learners did attempt to justify their reasoning mathematically. Without sequentiality and inconsistency of the implementation of strategies, it was difficult for participants to provide a depth of plausible justification.

6.2.3 Answering the research questions

A. Do learners show creative mathematical reasoning abilities in interaction with peers (process)?

The participants of this case study did show evidence of creative mathematical reasoning in the interactive *process* of working together to solve the VITALmaths tasks. All the participants were very active and engaged in their attempts to solve the tasks as a group. What became evident over the vertical and horizontal analysis of groups was that the key components that allow for learners reasoning to be fully expressed lies not necessarily in the amount of initiative and novelty of ideas, but more in the concentration of strategic implementation that leads to a correct and comprehensive conclusion. Also, the ability to constructively integrate participants' new ideas into a deeper and stronger conceptual foundation of understanding is important for turning creative mathematical ideas into mathematically plausible arguments.

B. Do learners show creative mathematical reasoning abilities as they justify their claims (product)?

The learners of this case study did show strong evidence of creative mathematical reasoning in the *product* analysis of group solutions to the VITALmaths. Groups were able to present their solutions clearly and precisely, however the strategies employed to solve the task were somewhat superficial. The depth and concentration of strategic implementation of their novel ideas was key to developing correct and exhaustive mathematically plausible solutions. Without, sequential, and consistent strategy implementation, the evidence of flexibility, constructiveness and plausibility of solutions was negatively impacted (Johanning, 2006).

6.3 SIGNIFICANCE OF THE STUDY

This case study is significant in the richness of data that was collected to gain a new vantage point from which to view how learners work together to solve open-ended mathematical tasks. From this vantage point I was able to identify the creativity evident in participants' mathematical reasoning.

The reasoning abilities indicated in this research impact one another. For the participants in this study, there were two key reasoning abilities that impacted the depth of mathematical understanding. These key abilities that impacted the strength of participants' creative mathematical reasoning most are constructiveness and concentration.

Within the context of the Inkululeko project, which has an aim to build autonomous learners who can develop conceptual mathematical understanding on their own or with peers who have limited support, this research give insights into how to best support the creative reasoning of its learners.

Firstly, support can be given by modeling and practicing mathematical discussions that integrate prior knowledge, and other's ideas into a new understanding. By allowing scaffolded opportunities for mathematical discussions that integrate or generalize new ideas may help to develop constructiveness of reasoning.

Secondly, structuring activities with an emphasis on strategy selection and sequential and consistent strategy implementation may improve learner's ability to justify their reasoning in mathematically plausible ways. It may also improve the ability of learners to find more conclusive solutions and show a depth of mathematical understanding.

While this case study is specific to an after-school club focused on autonomous learning of underprivileged youth in the Eastern Cape of South Africa, the methodology may be an effective analysis tool for educators in different contexts to observe how their learners work together to solve mathematical tasks, and pin-point the reasoning abilities that need to be exercised and developed.

By having VITALmaths tasks developed for this study, educators can focus on developing creative mathematical reasoning in their classrooms. The frameworks for process and product evaluation of creative mathematical reasoning provides a vantage point for educators to observe what is happening while learners' try to solve open-ended mathematical tasks.

6.4 LIMITATIONS

There are several limitations that must be considered to ensure a transparency of results. These limitations are:

Language barriers: Participants expressed preference in having VITALmaths clips and worksheets presented in English. Students speak isiXhosa and often spoke isiXhosa while solving the tasks. During the piloting of this research, participants expressed a preference to having the VITALmaths clips presented in English. The researcher and his assistant from the Inkululeko project, who is fluent in isiXhosa, went to great lengths in the translation, transcription and interpretation of the audio and video recordings

Use of mathematical tools: Limited incorporation of physical manipulation of objects while solving the tasks. The tasks presented to students required physical movements and manipulation of objects such as matches, cut out angles, and marbles to solve the tasks. While these actions were considered and referenced in the interaction analysis, it was not a focus of this study.

Evaluating the constructiveness in the product evaluation: It was difficult in this analysis to find evidence of constructiveness or generalization from other task. This was a limitation of the tool used.

***Small sample size*:** Due to the complexity of analysis tools and the context of the study, only 6 participants were included in this study. This limited the opportunity to generalize findings on a larger scale.

6.5 SUGGESTIONS FOR FURTHER RESEARCH

This study was a very broad analysis of the process and product of creative mathematical reasoning. Through the analysis tools, specific areas of reasoning abilities stood out as critical abilities. Concentration of strategic implementation was one such ability. Studies focused on the depth of strategies implemented, and the types of strategies implemented in the solving of open-ended mathematical tasks would provide more insight on what types of strategies are most important and an evaluation of depth of these strategies could further inform creative mathematical reasoning abilities.

Further research could be done to analyze the constructiveness of learner arguments. While this study was able to identify the lack of cohesive integration of ideas, more research could be done to further analyze the "popcorn" phenomenon of participants contributing new ideas but not integrating other's ideas into their thinking.

This study could also be applied to different contexts as a comparative analysis. Extending the sample size and contexts of research to other countries, or educational settings would give insight into how learners from different contexts engaged with one another to solve mathematical tasks. Replicating the study with older and younger participants would allow for an analysis of whether or not creative mathematical reasoning abilities are developmental.

6.6. PERSONAL REFLECTIONS

I thoroughly enjoyed the process and developing the VITALmaths clips, supplemental worksheets and analysis tools used in this research because I was able to engage with this research from beginning to end.

I had been away from University for 10 years before returning to do my Masters in Mathematics education, so the re-learning process really made the work challenging. Translating, transcribing and coding 2161 lines of interaction proved to be the most time consuming, and cumbersome process. In translating and transcribing learner interactions, I really developed a deep appreciation for the way learners worked on the tasks. Often when learners are speaking in isiXhosa during Inkululeko, I wondered if they were on task, and if they understood what I was asking. When I translated, and transcribed the interactions between participants, I was so impressed with their understanding of the tasks and their incredible insights and efforts expressed while solving the tasks.

There were several very humorous side conversations. Participants spoke about why there was crime in some parts of Grahamstown and not others, and what they liked and disliked about being at the Inkululeko project. The greatest moment was when a group was near completion of a task. A participant commented to the group about how “Xhosa people are smart” He stated this out of pride for the completion of the task.

Trying to complete this study between running two education development projects proved challenging. However, the way this research has informed the way I view mathematics, education, and research has proved to be worth the time and effort.

6.7 CONCLUDING REMARKS

I continue to firmly believe in Paulo Freire’s (1970) words that “Knowledge emerges only through invention re-invention, the restless, impatient, continuing, hopeful inquiry human beings pursue in the world, with the world, and with each other.” From this study, I have found that for learners to catch the spark that is autonomous life-long learning, it is important to guide them in this process of invention and re-invention of ideas.

To do this in a mathematical context requires opportunities to exercise creative mathematical reasoning abilities. More specifically within the context of Inkululeko, an after-school project based in the township of Grahamstown, learners can develop their creative mathematical reasoning abilities by better integrating peer ideas in the selecting of strategies to solve mathematical tasks. These learners can also exercise their ability to concentrate their strategic implementation more systematically and consistently.

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APPENDICES

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Parental Permission for Children Participation in Research

Title:

Observing and evaluating creative mathematical reasoning through selected VITALmaths video clips and collaborative argumentation.

Introduction

The purpose of this form is to provide you (as the parent of a prospective research study participant) information that may affect your decision as to whether or not to let your child participate in this research study. The person performing the research will describe the study to you and answer all your questions. Read the information below and ask any questions you might have before deciding whether or not to give your permission for your child to take part. If you decide to let your child be involved in this study, this form will be used to record your permission.

Purpose of the Study

If you agree, your child will be asked to participate in a research study about how they solve math problems in a group of 3 learners. The purpose of this study is observe how learners work together to solve math problems and find out how they work well together and what they struggle with when solving the problems.

What is my child going to be asked to do?

If you allow your child to participate in this study, they will be asked to watch a video clip that explains a math problem. The students will be video and audio recorded as they work together to solve the problem, and then they will present their solution to me. I will analyze how they solved the problem and how good their solution is. This study will take one hour per video recording. Students will solve about 5 video problems. There will be 6 learners in this study all together (2 groups of 3). Your child will be audio and video recorded.

What are the risks involved in this study?

There are no foreseeable risks to participating in this study.

What are the possible benefits of this study?

On the 27th of ~~May~~ April, I hope to take learners to complete the study and provide them with breakfast and take them to eat at a restaurant for late lunch. Besides having a nice meal learners will have had opportunities to work with their friends on fun math problems and be given feedback on how they worked as a group. These learners will also be able to be leaders and help direct doing the problems at Inkululeko the after school project in which they attend.

Does my child have to participate?

No, your child's participation in this study is voluntary. Your child may decline to participate or to withdraw from participation at any time. Withdrawal or refusing to participate will not affect their relationship with Inkululeko Rhodes University in anyway. You can agree to allow your child to be in the study now and change your mind later without any penalty.

What if my child does not want to participate?

In addition to your permission, your child must agree to participate in the study. If you child does not want to participate they will not be included in the study and there will be no penalty. If your child initially agrees to be in the study they can change their mind later without any penalty.

Will there be any compensation?

Neither you nor your child will receive any type of payment participating in this study.

How will your child's privacy and confidentiality be protected if s/he participates in this research study?

Your child's privacy and the confidentiality of his/her data will be protected. Learners names and images will not be used in the published study. They will be identified as learner 1, learner 2 etc. Only myself and the translator will be analyzing the data.

If it becomes necessary for the Institutional Review Board to review the study records, information that can be linked to your child will be protected to the extent permitted by law. Your child's research records will not be released without your consent unless required by law or a court order. The data resulting from your child's participation may be made available to other researchers in the future for research purposes not detailed within this consent form. In these cases, the data will contain no identifying information that could associate it with your child, or with your child's participation in any study.

If you choose to participate in this study, your child will be audio and video recorded. Any audio and video recordings will be stored securely and only the research team will have access to the recordings. Recordings will be kept for 2 years and then erased.

Whom to contact with questions about the study?

Prior, during or after your participation you can contact the researcher Matthew Kellen at **078-646-5856** or send an email to matt.e.kellen@gmail.com for any questions or if you feel that you have been harmed.

Signature

You are making a decision about allowing your child to participate in this study. Your signature below indicates that you have read the information provided above and have decided to allow them to participate in the study. If you later decide that you wish to withdraw your permission for your child to participate in the study you may discontinue his or her participation at any time. You will be given a copy of this document.

Printed Name of Child

Signature of Parent(s) or Legal Guardian — Date: 20/04/15

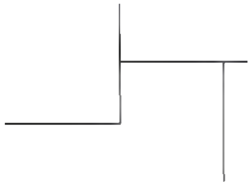
Signature of Researcher — Date: 20/04/15

Understand The

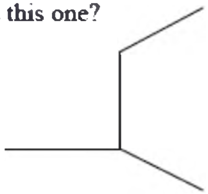
FOUR MATCHSTICKS

In your own words, what is the Four Matchsticks clip asking you to do?

Why would the following not work?

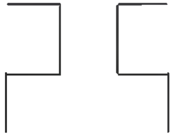


And this one?

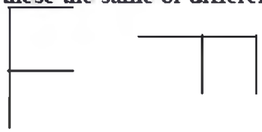


Devise A Plan

Are these the same or different? Why?



Are these the same or different? Why?



How will you collect your data to show proof that you identified all the possible shapes you can make with the four matchsticks?

Carry Out The Plan

How many shapes could you make with 4 matchsticks?

* ______________________

____

What were your rules for whether a shape is the same or different?

* ______________________
* ______________________
* ______________________

Prepare To Present Your Findings!

Think about how you will present your findings to the class.

1. Present what the question was.
2. Share how you solved it.
3. Show your evidence, and your answers.

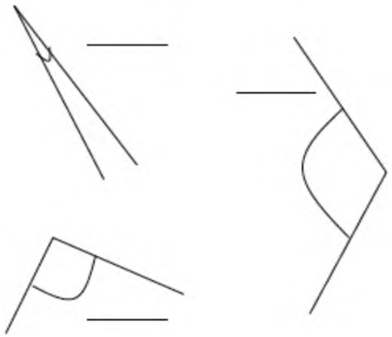
Understand The Problem

ANGLES

In your own words, what is the Angles clip asking you to do?

Play with the angles provided. Does the direction of the angle change whether an angle is acute, right or obtuse?

Label the following angles as right (R), acute (A), or obtuse (O) angles.



Devise A Plan

How will you keep track of the combinations of 6 angles to make sure every possible combination is found?

What system will you use to document on paper without using the cut out angles provided?

Carry Out The Plan

How many combinations of the 6 angles are there to make a right angle?

* ______________________

____

How many combinations of the six angles make acute angles?

* ______________________

____

How many combinations of the 6 angles will create an obtuse angle?

* ______________________

____

Prepare To Present Your Findings!

Think about how you will present your findings to the class.

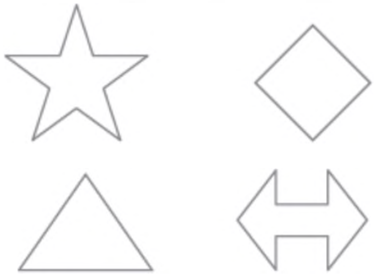
1. Present what the question was.
2. Share how you solved it.
3. Show your evidence, and your answers.

What is the problem

PLANES OF SYMMETRY

In your own words, what is the Planes of Symmetry clip asking you to do?

Draw a line of symmetry for these objects



How many blocks can you add to the original structure?

Devise A Plan

How will you document your work?

How many structures must you find?

Draw your first two structures below:

Carry Out The Plan

Before you present:

* Double check your work
* Document your work
* Can you explain how you solved the problem

Prepare To Present Your Findings!

Think about how you will present your findings to the class.

1. Present what the question was.
2. Share how you solved it.
3. Show your evidence
4. Present how many structures you found.

Understand The Problem

MAZES

In your own words, what is the Mazes clip asking you to do?

Where must your starting point be?

Where must your end point be?

Devise A Plan

How will you document your work?

Draw your first two pathways below:

Carry Out The Plan

How many pathways could you take?

Prepare To Present Your Findings!

Think about how you will present your findings to the class.

1. Present what the question was.
2. Share how you solved it.
3. Show your evidence
4. Present how many structures you found.

Understanding the Problem

PATHWAYS

In your own words, what is the Pathways clip asking you to do?

How many matchsticks can you use at one time?

Where must your starting point be?

Devise A Plan

How will you document your work?

Draw your first two pathways below:

Carry Out The Plan

How many points could you reach?

Prepare To Present Your Findings!

Think about how you will present your findings to the class.

1. Present what the question was.
2. Share how you solved it.
3. Show your evidence
4. Present how many structures you found.

Understand The Problem

BRICK LAYING

In your own words, what is the Brick Laying clip asking you to do?

What things did you notice about the wall that was built?

Do these pictures help?

Devise A Plan

How will you organize your thoughts in a professional way so the engineer does not make the same mistakes?

How will you show us? Do you need pictures or tools? (drawings, blocks, a protractor)

What important math terms will you need to use? (parallel, perpendicular)

Carry Out The Plan

Write what you will share with the engineer here. (You may prepare additional materials on a separate paper, take pictures, or use other tools as well)

- ______________________________
  ____
- ______________________________
  ____
- ______________________________
  ____
- ______________________________
  ____

Prepare To Present Your Findings!

Think about how you will present your findings to the class. Pretend you are speaking to a group of engineers learning to build.

Video Clip Title:_______________ Group #_____

What was the problem?

How did you solve it?

Present your evidence

What was your solution?