AN INVESTIGATION INTO THE NATURE OF MATHEMATICAL CONNECTIONS SELECTED GRADE 7 TEACHERS MAKE WHEN TEACHING FRACTIONS: A NAMIBIAN CASE STUDY

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DECLARATION OF ORIGINALITY

I, undersigned, hereby declare that the work contained in this thesis is my own original work and has not previously in it's entirely or in part been submitted at any university for a degree.

L. M. Amupolo   03 December 2014

ABSTRACT

The general understanding of mathematics as a subject and its implications is, in reality alarmingly low. Evidence of this is evident in learners' performance and their reaction towards the subject. Fractions as a domain of Mathematics are no exception. The majority of the learners do not learn Fractions comfortably. The causes of this may be varied. However, it is believed that one way of ensuring meaningful teaching and learning is to make use of appropriate connections.

The significance and the important role of the teacher in making mathematical connections in learning for understanding are well documented in the literature. This study focuses on the nature of mathematical connections selected Grade 7 teachers make when teaching Fractions, as well as their perceptions of the importance of making such connections.

This qualitative case study was conducted in three schools in the Oshana region. The purpose was to investigate how mathematics teachers make connections in fractions. Underpinned by an interpretive paradigm, the study made use of observations and interviews to generate data. The framework borrowed from Businkas' (2008) study was used in analysing and coding the nature of connections used in the lessons observed. An individual conversation on the nature and perceptions of the connections made in the observed lessons was undertaken with each teacher followed by a focus group discussion that aimed at analysing deeper perceptions on connections.

The main findings of the study revealed that teachers made use of all the different types of connections as per Businkas's framework. The frequency of occurrence showed that Instruction-Oriented Connection and Multiple Representation connections topped the list of connections used. Teachers pointed out that connections to prior knowledge and making multiple representations were most significant, as they related to learners' existing knowledge and pointed to different ways of solving a problem. The teachers were, however, not familiar with the other connections identified as this was their first experience of interrogating connections. They, however, agreed on the importance of making those connections.

The teachers agreed that meaningful connections indeed helped with their conceptual understanding of Mathematics. They believed that connections can increase learners' interest in school and help reduce negative views of fractions, in particular, and mathematics in general.

However, they felt that the limited number of resources, poor teaching approaches and the inability of creating fraction sense may hinder them from making appropriate connections.

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DEDICATION

I dedicate this thesis to my Mom, Dad, my brothers and sisters.

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ABBREVIATIONS

MOE – Ministry of Education

MR – Multiple Representations

PWR– Part Whole Relationships

IM– Implications

P– Procedural

IOC– Instruction Oriented Connections

PCK– Pedagogical Content Knowledge

LCE–Learner Centered Education

LCM– Lowest Common Multiple

LCD– Lowest Common Denominator

CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

Mathematics is a universal subject and compulsory in all schools in Namibia. It is, however, a subject that not every learner feels comfortable with. In my experience as a teacher, many learners have developed a phobia for it. This is particularly true for fractions. It is, therefore, a big challenge for all mathematics teachers to ensure that the teaching and learning of Mathematics is meaningful and relevant.

Making connections is believed to be one of the many teaching approaches that enable teachers to link Mathematics teaching and learning effectively. In my view teachers are one of the pillars in the process of learning – learners look to them for their knowledge, skills, interesting solutions of problems and guidance. They look to them to show them the reality of Mathematics. This study specifically focuses on how teachers make connections when teaching fractions.

1.2 BACKGROUND TO, AND CONTEXT OF, THE STUDY

Mathematics is both an abstract and a concrete subject with applications in everyday life. It is an integral part of almost every subject and it can be argued that without Mathematics it would be very difficult to make sense of the world. It is thus important that learners are able to use Mathematics sensibly and that teachers facilitate access to it by making meaningful connections. This research project is focused on the nature of mathematical connections that teachers make when teaching fractions in Grade 7. The importance of mathematical connections in teaching and learning for understanding will be discussed in more details in the next chapter.

Mhlolo, Venkat and Schafer (2012) defined connections as a relationship, a process or an association between ideas in Mathematics that can be used to link topics in Mathematics. Haines and Hoffman (2010) defined connections as a way of combining all subjects instead of focusing on them as totally separate, different subjects. Furthermore, Haylock and Thangata (2007) defined making connections as the process in learning whereby the pupil constructs understanding of mathematical ideas through an awareness of the relationship formed between the physical example, language, pictures and mathematical symbols. Connections are concerned with what Mathematics is: "it is the heart of the definition of Mathematics" (Mwakapenda, 2008, p. 190).

Making mathematical connections according to the given definitions is thus about creating relations, links, patterns or networks in Mathematics. The act of doing this cannot be achieved in one day but is a process - a progression practice continually unifying domains in Mathematics. These links can be either between Mathematics and Mathematics, Mathematics and other subjects or Mathematics and the outside world (McGregor, 2007). It is thus important that students should be made aware of possible mathematical connections (Mhlolo, 2012, p.49) when they engage in mathematical activities.

Many researchers emphasize the importance of making use of connections in a mathematics class. Students' understanding becomes deeper and more lasting through relationships. Sawyer (2008) alluded that mathematical connections play a major role in numeracy development. Of course, a learner cannot be proficient in Mathematics if he /she lack skills in numbers. If the link is created between the mathematical symbol and its language, the ability to work with numbers will improve. Being able to make connections is therefore important for independent learning. Haines and Hoffman observed that "when students begin to make their own connections they become one step closer to becoming an independent learner" (2010. p. 1). Thus, Mhlolo, et al. (2012) emphasized that it is through making strong mathematical connections that learners become numerate and independent thinkers.

Haylock and Thangata (2007) suggested that Piaget's concepts of accommodating and assimilating new experiences into old ones are only achievable if teachers create opportunities for learners to make connections. To strengthen this point, Anthony and Walshaw (2009) reinforced the argument that the idea behind mathematical connections is the ability for the students to connect what they know in different ways. Haines and Hoffman (2010) noted that connections make it possible for learners to acquire a better perception of the content of the subject. Making mathematical connections thus helps learners to make good judgments of new concepts and skills rather than relying on procedures.

There are varieties of techniques that teachers can make use of in making meaningful connections in their teaching. Sawyer (2008) advised that teachers should recognize and use connections between mathematical concepts and be able to understand how these concepts are interconnected. Mhlolo's, et al. (2012) study on the similar issue concluded that different representation of the same concept is one of the techniques that can help learners make connections and understand better. This simply means that a mathematical idea or concept can be presented in different modes, for example symbols, graphs, pictures or concrete materials and by so doing, a teacher is interconnecting different ideas and representations.

Recognizing learners' prior knowledge can also help teachers make connections. Haines and Hoffman (2010) suggested that teachers should either refer back to other topics or subjects previously covered, or help learners to relate their own experiences to the one being shared in the class. This will help learners to connect what they have learned to current events. Further to this, Haylock and Thangata (2007) also proposed that teachers should be aware of the need to provide rich maths experiences to enhance connections building as the foundation for developing learners' understanding. This means that teachers should create opportunities for learners to build on their experiences in order for them to make connections in the future.

Anthony and Walshaw (2009) emphasized that teachers effectively support students by creating connections between different ways of solving problems and how, and why, they yield the same results. This aligns with Sawyer's (2008) argument when he suggested that teachers should give learners chances to justify and explain their solutions, as well as allowing learners to explore connections between mathematical ideas and real life experiences and to use them to solve problems. By so doing, learners are developing their conceptual understanding of mathematics.

Learners need to see how subjects are interconnected; and this can be made possible if teachers interconnect subjects and different topics where the relations are observed. Haines and Hoffman (2010) pointed out that a teacher can create connections by integrating different subject areas in the lesson instead of teaching a subject in isolation. Another method that teachers can use to create connections, according to Anthony and Walshaw (2009), is through constructive class discussion where learners share their solutions and solutions strategies. This inspires other learners to also think critically and make different connections.

1.3 RATIONALE FOR, AND PURPOSE OF THE STUDY

The rationale behind this project is twofold. It is rooted in my own experience of how I was taught Mathematics, how I reacted to it, how I teach Mathematics, and how my learners react to it. As far as I can remember, I only came to like Maths when I went to secondary school. Mathematics in my primary school was a set of numbers with lots of rules and formulas to be memorised. I could not understand why I had to learn that difficult subject; I could not see where on earth I would use it, nor could I understand why we had to be beaten when we failed a test.

My primary experience with learning Mathematics was not enjoyable and is generally a bad memory. My high school Australian subject teacher opened my mind to the reality that I did not have to memorize the formulas or memorise the rules but that I should understand them. In the process, learning Mathematics became enjoyable. I started practicing it with ease, even though there was a gap that I needed to fill before I could learn a new topic. This gap was to make use of what I had learned in previous grades in some of the calculations. Revisiting what I learnt previously was quite a challenge to me because I did not understand. I did not know what I was doing. I could not connect.

When I think of my past, I think of how my own learners feel about Mathematics. I am sure that the majority see it the way I did. This is evident from their reactions towards the subject, the little time they spend doing it and the type of questions they ask in the class.

Just like me, some of my learners believe that Mathematics is very difficult. I, however, do not want them to continue living my own experience. My teaching on most occasions aims at developing their conceptual understanding by creating links as opposed to relying on mere procedures. When I was introduced to connections, the only knowledge I was aware of was connecting Mathematics to real life situations. Little did I know that connections are far more comprehensive and involve many more aspects of my subject. It occurred to me that, perhaps one of the reasons why my learners find Mathematics difficult is because I am not making all the appropriate connections in my teaching.

My introduction to connections triggered me to critically analyse two main documents that teachers closely work with. The first one is the National Mathematics Subject Policy Guide

(2009) and the Mathematics Syllabuses for the primary, upper primary, junior secondary and senior secondary syllabuses. The main purpose was to ascertain what these documents say about making connections. I was surprised to see that the need and the importance of making connections were made clear in all the documents. The making of connections should thus be part of every teaching process as learners are expected to see and use Mathematics in their everyday lives.

The National Mathematics Subject Policy Guide reminds teachers that “mathematical skills, knowledge, concepts and process, enable the learner to investigate, model, and interpret numerical and spatial relationships and patterns that exists in the world” (Namibia, Ministry of Education [MOE] , 2009, p. 2). The mathematics syllabus guide for 1-4 (2005) was issued to teachers with instructions to make learners aware of the usefulness Mathematics has in life. The mathematics syllabus for grade 5-7 (2010) *inter alia* aims at promoting learners’ positive attitudes towards Mathematics and their ability to apply Mathematics in everyday life. The mathematics syllabus grade 8-10 required that what learners learn should provide a broad foundation “so that all learners can experience mathematics as relevant and worthwhile” (Namibia, Ministry of Education [MOE], 2010, p. 1). The Mathematics syllabus grade 11-12 (2010) further sees Mathematics as “living product” (p. 2). Learning Mathematics is more than an accumulation of skills and knowledge. Learners should, therefore, apply Mathematics in everyday situations and develop an understanding of the role it plays in daily life.

The rationales given in the above referenced documents have significant implications for teachers. The documents are clear on what roles teachers should play as far as making connections is concerned. This implies that learners’ learning will be compromised if the teaching does not make appropriate connections. Connections are among the key experiences to be gained by learners (Mwakapenda, 2008).

Looking at my own experience and what is stipulated in the policy and the syllabuses, the question as to what teachers are doing in the classrooms to achieve these goals interested me and motivated me to conduct this research. Specifically, I am curious about the nature of connections teachers make in their teaching in order to make it possible for learners to investigate, model, interpret and experience Mathematics as relevant and worthwhile; make it valuable and sensible, with learners able to apply and appreciate the uses of Mathematics - a requirement as specified in the syllabus and the subject policy.

1.4 THEORETICAL AND CONCEPTUAL CONSIDERATION

The research done by Businskas (2008) about connections, in particular on the forms or nature of connections, inspired my thinking on how teachers make connections in their teaching of Mathematics. Businskas (2008) proposed a framework for identifying mathematical connections in practice as follows:

- *Different representation connections*
- *Part-whole relationship connections*
- *Implication connections*
- *Procedural connections*
- *Instruction-oriented connections*

This framework is at the heart of my research project and informs my theoretical engagement and my research design

1.5 RESEARCH QUESTIONS

This study aims to answer two main questions:

1. What are the natures of connections that selected teachers make when teaching Fractions?
2. What are the perceptions of selected teachers on the importance of making connections?

1.6 METHODOLOGY

This study is located in the interpretive paradigm (Cohen, Manion and Morrison, 2011). It deals with practices and experiences of teachers (Mackenzie and Knipe, 2006). I used a qualitative approach to analyse data from lesson observations and interviews. The research is inductive and based on the phenomena of real world actions, i.e. classroom practices (Blanche, Durkheim and Painter, 1999).

A case study approach was used to generate data for the research study. Cohen, et al., (2011) suggested that "case studies strive to portray what it is like to be in a particular situation in order to catch the close up reality and thick descriptions of participants' lived experiences and feelings about a situation" (2011, p. 290). I worked with three grade 7 mathematics teachers and my unit of analysis was the nature of connections they make in their teaching and also their perceptions of the importance of making connections.

The research was conducted in three selected combined schools in the Oshana region of Namibia. A Grade 7 group was selected because it is the most fundamental grade in the upper primary phase in which learners build their mathematical abilities.

1.7 SIGNIFICANCE OF THE STUDY

It is important to use connections when teaching fractions as they will assist learners to visualize and use fractions in a real-world context, as well as apply them to other domains in Mathematics. Fractions can be difficult for learners to learn without experiences that allow them to build their own understanding and connections. Fractions instruction should, thus, include the integration of content, multiple representations as well as real-world examples, which are all forms of connections. It is desirable to represent fractions in a way that stimulates curiosity and encourages exploration which can enhance students' learning and their attitudes towards Mathematics.

This study will assist teachers to appreciate the value of using different connections in teaching Mathematics and fractions. It will also assist teachers to interpret the subject policy and the syllabus in order to make appropriate connections in the teaching of Mathematics. This will, I trust, help in changing their practices to make Mathematics more relevant and worthwhile to the learners.

1.8 LIMITATIONS

Making connections is very important and, according to the subject policy and the syllabus, every teacher should make use of them for the reasons alluded to in the documents. However, according to Sawyer (2008) some teachers do not intentionally make connections for learners to learn and some teachers do not even recognize that they are making connections.

As this is a small-scale study, the connections identified in the observed lessons do not provide a comprehensive picture of the entire participants' teaching practice. They are mere snapshots of the connections made at the specific time of videotaping the lessons.

The sample size of this study is very small and the results, therefore, pertain only to the participants of this study.

1.9 OVERVIEW OF THE STUDY

The thesis consists of five chapters:

Chapter 1 presents an overview of the context of the study, the rationale behind the study, the research goals and an overview of the theoretical framework and the research methodology.

In **Chapter 2** I reviewed the different literature related to the study. The theoretical framework is also explained in more detail, as well as other vital information that has to do with this study as far as connection-making is concerned.

Chapter 3 is a presentation of the methodology used. The paradigm that informed this study is justified. Different data gathering tools, the research site, participants, and the issue of ethics, validation and limitations of the study are also explained.

Chapter 4 is a narrative description of the results gathered. Participants' own words are extensively used in this chapter along with my comments and interpretations.

Finally, **Chapter 5** concludes the thesis and provides a summary of the findings, some recommendations for further research and implications for teachers. It ends with a personal refection of the whole research journey.

CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

Mathematics is a universal subject and one of the most emphasized subjects in schools. In many countries, including Namibia it is a compulsory subject for every learner. Mathematics is fundamental to the development of many other subject disciplines and fields. However, learners' achievement in this subject is generally very low and researchers argue that the quality of teaching mathematics can be improved through meaningful connection makings.

Making connections in Mathematics is a well-researched area in mathematics education, however relatively little research in this regard has been done with Fractions, specifically on teacher's practices on teaching fractions. My research, therefore, focuses on investigating the nature of connections teachers make in teaching fractions.

It is claimed that good teaching of Mathematics enables learners to see the significance of Mathematics they are learning and to see the connections between school mathematics and the everyday life. It enables learners to make good judgments about the value of Mathematics and about how different mathematical domains are connected and intertwined. It is difficult for learners to make connections by themselves; teachers should teach lessons that will assist learners to make sense of mathematics through making connections (Mhlolo, Venkat & Schafer (2012).

This chapter is an analysis of different researchers' work on connections in teaching, specifically when teaching fractions. The chapter began with a brief examination of what connections is all about as a foundation for understanding my research project. I looked at the nature of connections, connections in teaching in general and making connections specifically in teaching Fractions. Proficiency teaching and content knowledge of teachers in terms of making connection are also discussed. The chapter also explained what the Namibian curriculum says about connection making and concludes with a discussion on the theoretical framework that underpins this study.

2.2 WHAT ARE CONNECTIONS?

To reach consensus about what connections are all about, the following broad definitions of mathematical connections will be used in this project:

- Mhlolo, Venkat & Schafer (2012) defined mathematical connections as a relationship between ideas that one can use to link topics in Mathematics. It can also be a process of making links between mathematical ideas, or associations a person might make between two or more mathematical concepts.
- Haines & Hoffman (2010) defined connections as a way of combining all subjects instead of focusing on them as separate entities.
- Businskas (2008) suggested that mathematical connections relate to causal or logical interdependence between mathematical ideas.
- Haylock & Thangata (2007) defined making connections as the process in learning whereby the pupil constructs understanding of mathematical ideas through a growing awareness of relationships between concrete experiences, language, pictures and mathematical symbols.
- "Connecting mathematical ideas means linking new ideas to related ones and solving challenging mathematical tasks by seeking familiar concepts and procedures that may help in new situations"(Leikin and Levav-Waynberg, 2007, p. 350).

In short, mathematical connections can be defined as a linking feature that makes explicit relationships and processes that connect mathematical ideas for more meaningful understanding.

2.3 NATURE OF CONNECTIONS IN TEACHING PRACTICE

My research project was inspired by Businskas (2008) study on connections, specifically with regard to the characteristics of different connections that a teacher can make. My interest is specifically focused on how teachers make connections in their teaching of fractions. She proposed a framework to identify mathematical connections in teaching practices. The framework consists of seven connection categories:

- Alternate representations
- Equivalent representations
- Common features
- Inclusion
- Generalizations
- Implications
- Procedure

Although Businskas (2008) identified seven categories, I am going to use only five of them with ideas borrowed from the study by Mhlolo (2012) and Mhlolo, Venkat & Schafer (2012). Their five categories consolidated Businskas (2008) framework and include the following:

- Multiple Representations Connections (MR)
- Part-Whole Relationships Connections (PWR)
- Implications Connections (IM)
- Procedural Connections (P)
- Instructional-Oriented Connections (IOC)

To obtain a deeper understanding of the above categories I discussed each of them below:

2.3.1 Different representation connections

Different representation suggests that teachers should be able to communicate, link and connect the same idea from "different modes" (Businskas, 2008), in different forms or examples by using words, graphs, symbols, tables and diagrams to express similar mathematical ideas. E.g. a half (word) is the same as 0, 5 (symbol) and $\frac{1}{2}$ (symbol). These are the different ways of writing the same number.

According to the National Council of Teachers of Mathematics, NCTM (1989):

> Different representations of problems serve as different lenses through which students interpret the problems and the solutions. If students are to become mathematically powerful, they must be flexible enough to approach situations in a variety of ways and recognize the relationships among different points of view (p. 84).

It is argued that learners' learning and reasoning abilities are enhanced if they are encouraged to use different representations. This notion is asserted by Blanton & Kaput, (2005) who said that multiple representations can be used as tools to support and extend mathematical reasoning and sense-making. Through the use of multiple representations, the understanding and development of mathematical ideas is strengthened. Cleaves (2008) further noted that multiple representations help develop independent solvers. They encourage learners to better grasp the implications and connections of mathematical thoughts and ideas.

Cautions on using multiple representations

As noted above, multiple representations are necessary for conceptual understanding in Mathematics. However, some researchers cautioned that using multiple representations can only be beneficial to the learners if they are being used in a suitable and appropriate way. Using multiple representations alone, without the guidance of the teacher, does not necessarily help learners to develop their mathematical understanding. De Jong & Van der Meij, (2003) observed that learners need to understand, relate, and translate different representations. It is thus important for teachers to facilitate the representations accordingly in order for them to be meaningful and support learners in their conceptual development.

Some representations serve as a basis for the development of other representations. Teachers, or learners, thus need to understand the fundamental representation first if they want to use it as a starting point to develop others and to avoid forming stumbling blocks to learning new concepts.

2.3.2 Part-whole relationship connections

This type of connection applies in the context of the teacher's presentation when teaching from the concrete to the abstract, i.e. from specifics to general instances. These types of connections are very crucial as they help learners to see the connection in the content from the general through to the particular, or vice versa. Mathematics is a subject where the generalization of patterns is central; however, the generalization that teachers make should not be made without using practical experiences and specific examples.

To form a strong connected network between the concrete and the abstract Liebeck, as cited in Mhlolo, (2012, p.4) suggested that children's understanding of Mathematics takes place in a sequential form that involves building up connections between concrete experiences, then adding in the appropriate language and perhaps pictures, and finally the mathematical symbols. With that sequence learners should be able to clearly see how a general statement or a result was arrived at. For example, when teaching fractions, a teacher should give many examples before coming to a conclusion that division by zero is undefined.

The absence of this type of a connection creates the gap in the structure and sequence of what should be learned first and why. The link developed from specific to general will therefore help learners to clearly see the development of Mathematics. Mathematics, I believe, as an abstract subject forced some learners to memorize algorithms and formulas. However, if the teaching is done in such a way that an association is made before a generalization, learners will not attempt for example to add two denominators when they are adding fractions.

2.3.3 Implication connections

This connection highlights the dependence of one concept on another (Businkas, 2008). Making this connection provide learners with abilities to justify, prove, and therefore develop logical reasoning. By using this type of connection learners will learn how to look for evidence, thereby making arguments on what is true or false in various situations. The acquisition of this type of connection will prevent concepts from being learned in isolation. It will also promote connections between other topics that have been neglected. It is, therefore, of great advantage that learners make connections between Mathematics related contexts that they are learning and between what they have learned. Learners need to use the insight gained in one context to investigate and speculate about another concept.

Mathematics is an integrated subject. However, some teachers or learners see it as a subject with totally unrelated topics. Connecting different concepts in Mathematics helps learners to broaden their knowledge of Mathematics. If leaners are able to connect between different facts, methods, ideas and examples this will form a network of rich information that will lead to a deep understanding of the subject and not a subject with many rules to be memorized.

When learning fractions, learners first need to know some fundamental procedures that will help them compute more advanced problems. For example, learners need to know how to multiply fractions before they learn how to divide fractions. This type of connection is, therefore, important as it gives the link between different concepts and procedures, when they should be used and why.

Implications in fractions are essential because fraction concepts depend much on each other. Some fractions concepts use concepts from other domains in Mathematics. Making these types of connections will therefore help learners not to only understand the concept they are learning, but to understand other concepts as well.

2.3.4 Procedural connections

Procedural connections involve algorithms that use prescribed steps that lead to a specific outcome, which is often the calculation of something (Mhlolo, 2012). Learners should be able to make connections between mathematical procedures in order to use them appropriately. For example, when doing fractions a teacher who makes strong procedural connections should be able to make the learners understand that using Highest Common Factors is a procedure for simplifying fractions.

Learners need to develop procedural fluency, which was defined by Kilpatrick, et al. (2001) as the skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Developing this proficiency requires a teacher to balance and emphasize the connection between conceptual understanding and computational proficiency.

Assessment in Mathematics is a powerful tool to assess how well learners can connect different procedures in Mathematics. Learners should not only use different procedures, but they should also know the connection between them and why they yield the same answer.

Being conscious about this does not only help them to understand mathematics but to also help them make sense of it and become proficient individuals.

2.3.5 Instruction-oriented connections

This connection includes the prior knowledge and skills that should be known in order to understand a new concept. Prior knowledge plays a major role in this form of connection where learners need to link new concepts to old ones. It is a powerful starting point for learning. "The learner brings to the school a wealth of knowledge and social experience gained from the family, the community and interaction with the environment. This knowledge and experience has potential that can be utilized and drawn upon in teaching and learning" (Thekwane, 2001, p. 2).

Learners need to see the sequences of topics and how they build on each other. They should be able to use previous experiences to acquire new knowledge. According to Namibia's National Curriculum for Basic Education (2010), it is stated that:

> Teaching which does not build on experience and learning will limit the learners' thinking, and the learners will not see the connection between the world outside school and what is taught and learnt in school. Teaching should always begin with helping the learners realize what they might already know about something, or what ideas or questions they might have about it even if they do not know, and by relating to the environment within and around the school (p. 29- 30).

Donovan and Bransford (2007) acknowledged that "new understandings are constructed on a foundation of existing understandings and experiences" (p. 4). Recognizing prior knowledge is one of the characteristics of effective teachers. Anthony and Walshaw (2009) viewed that effective teachers are those who "put their students' current knowledge and interests at the centre of their instructional decision making" (p. 9). Further to this Stylianides & Stylianides (2007) observed that teaching for understanding involves building upon children's' learning and experiences.

Children use what they know to form their new understandings.   In the book Towards Education for All (1993) it is stated that "teaching begins with the interests of the learners, their level of maturity, and their previous experiences" (p. 60). Mubita (1998) clearly argued for recognition to prior knowledge and maintained that "the teacher should respect and acknowledge informal skills, which learners have gained outside the school situation, because they form the foundation for further learning" (p. 12).

Learners come to school with different ideas, knowledge, skills and misconceptions that need to be built upon, shaped or adapted. Donovan and Bransford (2005) asserted that "while prior learning is a powerful support for further learning, it can also lead to the development of conceptions that can act as barriers to learning" (p. 5). In contrast Anthony and Walshaw (2009) challenged that, "prior knowledge should not be a stumbling block to understanding. To make sense of a new concept or skill, students need to be able to connect it to their existing mathematical understandings, in a variety of ways". (p. 10).

Pointing to the critical role played by teachers, Weinberg (2001) noted that it is difficult to find out whether the learners recognized the connection made in the class. It is, therefore, important that learners are made explicitly aware of these different possible mathematical connections through instruction that emphasizes the interrelatedness of mathematical ideas. Sarama & Clements (2009) noted that Mathematics requires a knowledgeable teacher to create a supportive environment and form challenges, suggestions, tasks, and language. That way, students' understanding becomes deeper and more lasting, and learners come to view mathematics as a coherent whole, as connected with other subjects and as connected to their own interests and experiences.

2.4 MAKING CONNECTION IN TEACHING PRACTICE

Every lesson taught should inspire learners to value and make sense of what they are learning. Making connections should, therefore, be a central reason why children learn Mathematics in schools and why fractions are essential. Mathematics teaching should thus be effective in a sense that it should make sense to the learners.

Mathematics' goals align and they are common in many countries across the globe. With regard to connections, for example, the goals and objectives for School Mathematics according to the National Council of Teachers of Mathematics (National Council of Teachers of Mathematics, as cited in Sawyer, 2008 p. 429) stated that mathematics teaching should allow learners to:

- Recognize and use connections among mathematical ideas;
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole;
- Recognize and apply mathematics in contexts outside of mathematics.

Sawyer (2008, p. 434) pointed out a number of strategies that can be used by teachers to help students learn to make mathematical connections. These are:

- Assist students to become competent in using a range of mathematical procedures.
- Require students to select the knowledge and procedures that will assist with the solution of mathematical problems. Don't always tell students what procedures to use.
- Expect students to explain and justify methods selected for working out problems.
- Encourage students to draw on ideas from other disciplines or from their own experience when solving problems or recording their thinking.
- Assist students to re-frame ideas or information from other disciplines or their own experience so that they are expressed using the specialized language of mathematics. Model how to do this when evaluating students' suggestions and chosen methods.
- Respond positively when students themselves identify connections between diverse bodies of disciplinary knowledge, or between mathematical knowledge and real life.

In teaching practice, the above ideas require proper lesson planning and a deeper knowledge of the subject and the learners.

2.5 SIGNIFICANCE OF TEACHING FRACTIONS IN SCHOOLS

A fraction is a practical concept. It is one of the pillars in mathematics. What learners learn in fractions should help them to be proficient in all other mathematical activities. Although many of us are not always explicitly aware that we are using fractions, it is a living concept; hence it should be conveyed as such so that we can make sense of it.

Proficiency in fractions is essential for learners to be successful in mathematical problem-solving activities. It serves as a backbone for other mathematical domains. Many algebra concepts involve fractions. Algebraic fractions, in particular, use procedures like common fractions and other complex fractional manipulations. It is thus important that learners feel confident and comfortable with computing with fractions.

2.6 MAKING CONNECTIONS IN TEACHING FRACTIONS

Fractions are integral to any primary, secondary and tertiary school mathematics curriculum. However, McLeod and Newmarch (2006, p. 12) observed that "all too often learners think of 'fractions' as being a discrete (and often difficult) topic that has no real connection with any other area of Maths". Bezuk & Cramer (1989) saw the need to properly develop fractions concepts and its relations either within itself or with other domains in mathematics. It is thus important that teachers integrate fractions in different grades or in different subjects.

Meaningful connections making in mathematics makes learning easy and understanding. Achieving this is not an easy task for teachers. Thekwane (2001, p. 10) suggested the following "generic probing questions" that teachers can use in the class with the aim of helping learners to make mathematical connections:

- How does this relate to...?
- What ideas that we have learned before were useful in solving this problem?
- Have we ever solved a problem like this before?
- What use of mathematics did you find in the newspaper yesterday morning?
- Can you give me an example?

In my view, these and other questions could apply very well in the context of teaching fractions. Learners need to make sense of different connections made in the class for them to view fractions positively and mathematics as a significant subject.

Different researchers suggested different ways that teachers can make meaningful connections. These are as follows:

2.6.1 Connections to whole numbers

It is commonly agreed that learners find it very difficult to learn fractions and teachers find it difficult to teach them. Connecting fractions to whole numbers can be a fundamental starting point to introduce fractions and ease the way of teaching and learning. Kilpatrick, Swafford and Findell, (2001) emphasized the following:

> Rational numbers are numbers. That fact is so fundamental that it is easily overlooked. A rational number is like a single entity just as the number 5 is a single entity. Each rational number holds a unique place (or is a unique length) on the number line. As a result, the entire set of rational numbers can be ordered by size, just as the whole numbers can (p. 235).

Just like in algebra, grasping fractions content is quite a challenge to most learners. Whole numbers can be used as a stepping stone in the understanding of fractions, for example using the number line.

2.6.2 Bridging fractions with everyday use

Learners can learn fractions better if fractions are contextualized. Kilpatrick, et al., (2001) noted that “students’ informal notions of partitioning, sharing, and measuring provide a starting point for developing the concept of rational number” (p. 232). Referring to learners, Kilpatrick, et al., (2001) further noted that “their experience in sharing equal amounts can provide an entrance into the study of rational numbers” (p. 232). This was also supported by McLeod and Newmarch (2006) when they advised teachers to “use the knowledge and experience that all adult learners already have of fractions, particularly halves and quarters, and also the fractions that are part of everyday language” (p. 12). This means that teachers should expose learners to lots of problems involving sharing or dividing for learners to better understand what fractions are and what is involved when using them.

2.6.3 The use of manipulative

Learning fractions with concrete materials can help learners work with fractions better because the nature and abstractness of fractions makes it difficult for learners to compute with fractions. It also helps them to make connections between the numeric form of a fraction and its natural counterpart. "Fraction concepts can be explained by teachers and students using a combination of external representations such as written symbols, spoken language, concrete materials, pictures, and real world examples" (Lesh, et al., 1983 cited in (Wong and Evans, 2007, p. 825).

Learners might have a clear picture of what a half is but when a half is presented with other fractions they often get confused. Kilpatrick et al., (2001) reported that:

> Physical representations serve as tools for mathematical communication, thought, and calculation, allowing personal mathematical ideas to be externalized, shared, and preserved. They help clarify ideas in ways that support reasoning and build understanding. These representations also support the development of efficient algorithms for the basic operations. Mathematics requires representations. In fact, because of the abstract nature of Mathematics, people have access to mathematical ideas only through the representations of those ideas (p. 94).

2.6.4 Using multiple representations

As mentioned earlier, learners should be able to link mathematical ideas by means of multiple representations. This also applies to fractions. McLeod and Newman (2006) emphasized that:

> Learners need to be familiar with multiple representations of fractions, and should always be given more than one representation. These can include: area diagrams using a range of different shapes, number lines, words, symbols, some decimal equivalents and percentages, fractions as a result of division. Pictorial representations of a particular fraction may be of different sizes and different shapes. For example, don't always use shaded sections of circles, and interesting discussions can be had from drawing half of a small square and a quarter of a larger square and asking which is the larger fraction (p. 10).

2.6.5 Integration of fractions in other domains of Maths

Fractions are an integral part of many topics in mathematics. As outlined above, they serve as stepping stones, or rather as a foundation for the understanding of other mathematical concepts. McLeod and Newmarch (2006) recommended that "work on fractions needs to be integrated into other maths topics; number, shape, data handling, and particularly every sort of measure of weight, length, capacity, time, and simple probability" (p. 12).

Learners should be able to show that "a fraction represents a number with many names" (Wong and Evans2007, p. 826). "Instead of feeling that 'decimals' or 'percentages' are completely different topics, learners need plenty of experience, even at an early stage, of seeing that these are simply other ways of representing fractions" (McLeod and Newmarch, 2006, p. 12).

2.7 TEACHING FOR MATHEMATICAL PROFICIENCY

Learners need to be proficient in Mathematics. This achievement will help them to make meaningful connections and hence be at ease when doing Mathematics. Kilpatrick et al. (2001) described mathematical proficiency as "the integrated attainment of conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition" (p. 313). Similarly, Schoenfeld & Kilpatrick (2008) defined proficient teachers as teachers that have multiple ways of conceptualizing the current grade-level content, can represent it in a variety of ways, understand the key aspects of each topic, and see connections to other topics at a similar level.

Kilpatrick et al. (2001) proposed five interwoven strands for teaching for mathematical proficiency. These strands are intertwined and interdependent in the development of mathematical proficiency. The authors believe that having these proficiencies allows students to connect concepts and use their understandings in future problem solving. Developing these interrelated strands promotes retention and fluency, which is more powerful than memorization.
The strands are described in terms of what the teacher would do, as explained below Kilpatrick et al. (2001, p. 380):

- ***conceptual understanding:*** of the core knowledge required in the practice of teaching;
- ***procedural fluency*** in carrying out basic instructional routines;
- ***strategic competence*** in planning effective instruction and solving problems;
- ***adaptive reasoning*** in justifying and explaining one's instructional practices;
- ***Productive disposition*** toward mathematics, teaching, learning, and to improving practice.

Kilpatrick's model of teaching for proficiency emphasises the notion that teaching and learning of Mathematics is the result of interaction between the teacher, learner and the content (mathematics). They refer to it as the *instructional triangle* (p. 313). The achievement from these interactions can vary depending on how teachers, students, and content interact in contexts to produce teaching and learning.

A teacher teaching for mathematical proficiency uses multiple ways of conceptualising the content, presents it differently and continuously makes connections. I concur with Kilpatrick's et al. (2001) view that it is only teachers with a solid conceptual understanding of mathematical content who will be able to engage their learners in fruitful and meaningful conversations about multiple ways of solving mathematical problems, instead of simply demonstrating procedures to learners and then giving them repetitive opportunities to practice the procedures (Kilpatrick et al., 2001).

2.8 PEDAGOGIC CONTENT KNOWLEDGE (PCK) OF TEACHERS AND CONNECTIONS MAKING

Subject teachers possess a specialised form of knowledge known as PCK. This type of knowledge was proposed by Shulman (1986) and is composed of knowledge of the subject, knowledge of teaching and knowledge of the curriculum.

Rich PCK is a necessary requirement for a teacher to teach with connections. For example, prior knowledge of the learners assists the teacher in lesson planning. Comprehensive knowledge of the curriculum helps the teacher to align the lesson with the curriculum. Further, knowledge of different teaching approaches helps the teacher to vary the lessons and make them more meaningful. All this enables the teacher to make rich and powerful connections.

Knowledge of the subject is a key component of a teacher's PCK. Leong (2013) pointed out that a teacher's interest in teaching depends on the knowledge of the subject. Further to this Carpenter, Femmena, Peterson & Carey (1988) found that a deep knowledge of the subject creates a web of domains. It is of fundamental importance that, in order to be able to make connections between different mathematical domains, a teacher is proficient in the mathematical content being taught.

Kilpatrick, et al., (2001), indicated that teachers who possess strong mathematical knowledge of the subject are more likely to increase their students' ability to reason, conjecture, and problem-solve, as well as identify their misconceptions. Hill, Rowan & Ball (2005) further found out that the teacher's knowledge of the subject influence students' achievements. Teachers are unable to connect if they posses little knowledge of the subject and students are unable to achieve if they cannot connect. Ball (2008) therefore concluded that teacher's competency in the subject influences connections ability.

2.9 CONNECTION MAKING IN THE NAMIBIAN CONTEXT

2.9.1 Curriculum and policies

The teaching of mathematics, like any other subject in Namibia is guided by documents such as the syllabus, the subject policy, National Broad Curriculum on Basic Education and the policy on learner-centred education. All these documents give teachers opportunities to see mathematics as a subject that has links within itself and other subjects. Although the documents are not clear on strategies that teachers can use in the teaching of mathematics, I argue that it gives sufficient perspectives to teachers on how to plan for their lessons to have links to other disciplines.

Mathematics is one of the main key learning areas in the education curriculum of Namibia, from pre-primary to tertiary institutions. Mathematics as a subject is one of the fundamental cornerstones of basic education in Namibia. The need to make necessary connections in mathematics is clearly stipulated in educational policies and documents

I took the following points from different educational policies, which highlighted the need for teachers to make connections:

- The new National Curriculum for Basic Education [MOE] (2010 insisted that learners "use basic number and mathematical concepts and operations, and numerical notation, and *apply mathematics in everyday life*" (p, 9).

- The National Mathematics Subject Policy Guide reminds teachers that "mathematical skills, knowledge, concepts and process, enable the learner to *investigate, model, and interpret numerical and spatial relationships and patterns that exists in the world*" (Namibia, Ministry of Education [MOE] , 2009, p. 2).

- It is clearly stipulated in Namibia's National Curriculum for Basic Education, 2010 that Mathematics is an *indispensable tool for everyday life*. It is also indispensable for the *development of science, technology and commerce*.

- According to the Namibian Mathematics Policy (Ministry of Education, 2005, p. 2), the purpose of learning mathematics in schools is to *develop their mathematical knowledge and oral, written and practical skills* in a way which encourages confidence and provides satisfaction and enjoyment, to *apply mathematics in every situation* and develop an *understanding of the part which mathematics plays in the world around them,* to *recognize when and how a situation may be represented mathematically*, to identify and interpret relevant factors and, where necessary, to *select an appropriate mathematical method to solve the problem*.

Mathematics is a compulsory subject in Namibia. I went through the syllabus for grades 1-4; 5-7; 8-10 and 11-12. In each syllabus I analysed the aims and goals given. The following are the main common goals and expectations of teaching mathematics. Teaching and learning mathematics requires teachers and learners to:

- Recognize and apply mathematics in context and in their daily lives;
- Understand how mathematics ideas are interconnected and build on one another;
- Recognize and use connections among mathematical ideas.

The points mentioned above make clear references to the importance of making connections as discussed in this literature review. The points also give reference to the role of the teacher in the mathematics classroom. It is therefore clear that mathematics is about making connections. If every teacher is teaching to achieve these goals, it is quite obvious that they are making connections.

2.9.2 Connections making in a learner-centred paradigm

Namibia went through an education transformation after it gained independence in March 1990. The underlying transformation was underpinned by a learner-centred approach.

Mubita (1998) on supporting a learner-centred approach said that:

> A learner-centered curriculum is a holistic curriculum because it is integrated. That implies it is holistic in the sense of its connectedness. Integrated and holistic refers to the notion of connectedness in the sense that the learner will be able to make connections of what he or she learns. In this case home and school curriculum should be interrelated in order to make sense for the learners (p. 8).

In terms of mathematics, a learner-centred approach gives opportunities to all the learners to learn. It is inclusive and gives ideas that every learner is unique. The challenge lies in the hands of the learner to get in touch with what they already know and by so doing they are connecting to their prior knowledge. The role of the teacher is therefore to help learners make a link between their home experiences and what they are instructed at school.

The National Curriculum for Basic Education (2010) on learner-centred education supports knowledge development. It emphasized building new knowledge on what learners already know as a profound ingredient for learners’ learning and understanding. Further to this, Thekwane (2001) concluded that “learners are conceptualized as active mathematical thinkers who try to construct meaning and make sense for themselves of what they are doing on the basis of their personal experience” (p. 3).

Through this construction of knowledge, learners are forming webs of connections. This point gives teachers the role to facilitate and to distance themselves from lecturing, which is viewed to be teacher-centred. Below is part of a table summary that I took from Thekwane (2001) that illustrates the relationship between connections in a leaner-centred versus a teacher-centred education paradigm.

Table 2.2.9.1. The relationship between mathematics in a teacher-centred and learner-centred paradigm.

	Teacher-centered	**Learner-centered**
Mathematical connection	• Learning isolated topics • Developing skills out of context. • Rote learning	• Connecting mathematics to other subjects and to the real world. • connecting topics within mathematics • Applying Mathematics.

Much research has been done on learner-centred education. Donovan and Bransford, (2007) concluded that paying attention to students' individual abilities and backgrounds, as well as their cultural values is fundamental for learning in a learner-centred classroom. Leaners should be given challenging tasks that will foster their thinking and teachers should facilitate them to make connections in the completion of those tasks.

In the book Towards Education for All (1993) the learner-centred approach posited that the starting point of learning is the learners' existing knowledge, skills, interests and understanding. These are derived from previous experiences in and out of school and the empowerment of learners to learn to investigate and to make sense of a widening world.

The change to learner-centred education challenges teachers to make use of connections and offer learners opportunities to find logic in mathematics. The LCE method is consistent with making connections. From the above discussion it is clear that a learner-centred approach to teaching aligns well with a pedagogy that emphasizes the use of making connections.

Several of Businskas (2008) forms of connections discussed earlier in this chapter fit in well with this policy of learner-centred education. These are:

- In an LCE approach teachers should be able to show their leaners that mathematics is made up of related concepts by connecting similar concepts as well as exploring different mathematical ideas. This fits well with Businkas's implication connections.
- Motivating learners to answer a mathematical problem in different ways is one of the objectives in a learner-centred approach. This fits well with Businkas' procedural connections.
- LCE suggests the use of physical materials to enhance learning. Physical materials can mean physical manipulatives, which are modes of multiple representations, a form of mathematical connection suggested by Businkas (2008).
- Building on learners experiences is the starting point of any lesson in a learner-centred classroom. This is similar to Businskas's instruction oriented form of connections.

2.10 THEORETICAL FRAMEWORK: SOCIAL CONSTRUCTIVISM

The theoretical framework underpinning my study is constructivism. Constructivism is defined in numerous ways and its definition is drawn from the developmental works of Vygotsky (1978), Piaget (1977), and the theoretical work of Bruner (1966). Below are some of the definitions of constructivism:

- According to Vygotsky (1986), social constructivism is a theory of learning and an approach to education that lays emphasis on how people create meanings of the world through a series of individual constructs within a social environment.

- Maxwell, (1992, p. 1) defined constructivism as a "specific learning theory in which students learn based upon experimentation, observation, and speculation about their own experiences".

- Jaworski (1997) postulated that constructivism is not about teaching but about knowledge and learning. In her view about constructivism, she categorized her views into two principles namely:

    - Knowledge is actively constructed by the learner, not passively received from the environment.
    - Coming to know is a process of adaptation based on, and constantly modified by, a learner's experience of the past.

In sum, constructivism is a learning theory. It promotes a possible approach to teaching and learning based on the notion that learning is the outcome of mental construction; a practice of assimilating mental images to accommodate new experiences. The framework of constructivism highlights that knowledge is not received from external factors only but also by assimilating, accommodating and reflecting on experiences and by connecting new information to what is already known.

LCE is grounded in social constructivism, because social constructivism takes into account the social settings of the learning environment as a cooperative atmosphere between the teachers and learners. Glasgow (1996, p. 34) indicated that LCE involves an approach where "students learn to decide what they need to find success within the class and educational format". Learner-centred education places the student at the centre of education. It begins with understanding the educational context from which a student comes.

The view of the learner and learning described in the curriculum aligns well with the broad considerations of constructivism, particularly with social constructivism. Perry, Geoghegan, Owen & Howe (1995), agreed that, according to the social constructivist view of learning mathematics, students construct their own mathematical knowledge rather than receiving it in finished form from the teacher or a textbook. This means that students create their own internal representations of their interactions with the world and build their own networks of representations (p. 453) and connections.

Social constructivism goes further and explores the implications of learning as a dialectic interaction between ourselves and our community through which we internalize our experiences and actively construct our knowledge and understanding. Skemp (1987) noted that math is a way of using our minds that greatly increases the power of our thinking. This means that math has great impact on how we construct our knowledge.

Constructivism emphasizes making connections in teaching mathematics; it promotes making connections. A mathematical connection is associated with making links in order to make sense of what is happening in the world. This view of mathematics enables learners to relate their existing knowledge of mathematics into the newly established knowledge, which is in line with the constructivist perspective.

Learners have informal knowledge of mathematics. Learners make connections by forming relations between the constructed or built knowledge and the knowledge that is taught. Construction of knowledge is done in the mind; therefore connections should be made between the mind and the concrete example at hand.

Learning takes time. Knowledge is needed to learn. It is not easy to assimilate new knowledge without connecting it to existing knowledge formed from previous experiences. Therefore there is a need of making connections based on previous knowledge.

In the above discussion, I reflected on the possible nature of connections that can be made by teachers in their teaching of mathematics. I further argued the significance behind teaching with connections in mathematics and fractions. I thought of teachers' practices and decided to look inside the classrooms to code the nature of connections that they make in their teaching of fractions. This would answer my first research question.

Teachers' own understanding of what Mathematics is and their own view of how Mathematics should be taught, as well as what they know about their learners may influence the type or nature of connections they make. It was therefore important to question their own perspectives about the importance of making connections. This would answer the second research question.

2.11 CONCLUSION

In this chapter I analysed and reviewed literature on the importance of making connections in the teaching of mathematics in general, and of fractions in particular. I began with broad definitions of connections followed by an overview of the nature of connections as pointed out by Businskas (2008). I discussed the idea of connections in teaching, followed by making connections in fractions – the focus of my study. Kilpatrick's framework on teaching for mathematical proficiency as well as PCK of teachers was also discussed in relation to making connections. I also described education in the Namibian context and how an LCE approach is fundamental to connection makings.

I discuss the research methodology and research design in the next chapter.

CHAPTER 3

METHODOLOGY

3.1 INTRODUCTION

The nature of connections and its impact on mathematics teaching and learning, as described in the previous chapter directly influenced the development of my research methodology and affected the choices I made around the research design of this research project. Making connections, as argued by many researchers, influences learners' understanding of Mathematics. For this reason it was important to observe and listen to teachers' own actions and expressions on how they make and understand connections.

This project deals with human beings - hence I have been very sensitive to the needs of my participants when observing them. I was very conscious of the ethical considerations and took great care in gathering information in a sensitive manner. I discuss this in more detail later in this chapter.

What follows is an outline of my research process. This chapter begins by describing the goals and focus of the research project. In section 3.3 I discuss the research orientation and provide a rationale and justification for the study. The research methodology section (section 3.4) describes the actual research approach adopted by the study. Section 3.5 outlines the selection of the research participants and the site. I describe the summary of the design tool of the research process in section 3.6. The data analysis process is described in 3.7, followed by a discussion on the ethical and validity aspects. The chapter finishes by explaining the limitations and challenges encountered in the study.

3.2. RESEARCH GOALS, QUESTIONS AND SUB-QUESTIONS

Mathematics is an abstract subject. Its abstractness often makes it difficult for the learners to grasp and value the content in all the domains comfortably. As discussed in the previous chapter, one way of making mathematics more explicitly significant, worthwhile and relevant to the learners is through making meaningful connections. Thus, this study aims to investigate the nature of mathematical connections teachers make in their teaching of fractions. This is guided by using the following research questions.

1. What is the nature of connections that selected teachers make when teaching fractions?
2. What are the perceptions of selected teachers on the importance of making connections?

3.3 RESEARCH ORIENTATION

The study seeks to deeply understand connections. In order to achieve this I used numerous data collection tools to better understand the phenomenon under investigation, therefore this study is qualitative. Leedy and Ormrod (2005) described qualitative research as a type of research "used to answer questions about the complex nature of phenomena, often with the purpose of describing and understanding the phenomena from the participants' point of view" (p. 94).

Within the qualitative approach, I used an interpretive paradigm for the research. The interpretive approach targets an understanding of human experiences (Cohen, Manion and Morrison, 2011). It provides an understanding of the multidimensional domain of existing knowledge through the opinion of those who live or experience it (Flick, 2006). In order to obtain more insight into the problem under investigation, I made an attempt at understanding the thinking of the persons being researched in order to understand better how those individuals interpret their own experiences (Cohen, et al., 2007). After observing my participants I used a stimulated-video recall interview methodology to gain deeper insights into the perceptions that teachers have about using connections. Interpretive research also deals with "subjective" (Cohen et al., 2011) data. I thus focused on the perceptions that individual teachers either expressed orally or demonstrated through their teaching actions.

The interpretive paradigm suited my research study well as I was interested in practices and experiences of teachers in the real world (Mackenzie and Knipe, 2006). The data I obtained from the interviews is subjective since it reflects an individual's own experiences. This research is also inductive and relies on real world actions, i.e. classroom practices (Blanche, Durkheim and Painter, 2006).

3.4 RESEARCH METHODOLOGY

This study was conducted in the form of a case study. Walliman (2011) suggested that a case study design is suitable for a qualitative study such as mine. Wisker (2001) described a case study as a research approach grounded in empirical research which concentrates on the specific issue, in a specific context. It could make use of multiple methods of data collection techniques. A case study provides a distinctive instance of real people in real situations, allowing readers to understand ideas with depth and rich insight. (Cohen, et al., 2007). A characteristic feature of a case study is that it "give rich data on specific situations and new ideas can emerge" (Blanche, et al., 1999, p. 461).

A case study approach was appropriate for my study because I focused on real circumstances, focusing on specific phenomena, which are connections. Creswell (1998) indicted that in a qualitative and interpretive approach, the researcher is directly involved in the process of data collection and analysis. In this study, the phenomenon of investigating the nature of mathematical connections teachers make was the main focus. Hence my case is a single case of three mathematics teachers, and my unit of analysis is the teachers' practice with regard to making mathematical connections and their perceptions about using these connections.

3.4.1 Research design

The research process was designed around five phases:

Phase 1: During this phase I piloted the instrument (observation schedule) before administering it. I observed one teacher in order to validate the observation instrument using the connection framework.

Phase 2: In this phase fractions lessons from participating teachers were observed and video-recorded. It is during this phase that I recorded the nature of connections teachers made. I initially planned to observe each teacher twice but one teacher was transferred before all the lessons could be observed. This reduced my observed lessons from six to five.

Phase 3: In this phase, I analysed the video recordings alone and classified the connections that the teachers made according to my observation schedule and according to the framework I used. This assisted in my preparation of the interview questions that I would ask during the joint video analysis with the teacher in phase 4. Phase 3 and Phase 4 enabled me to answer the first research question.

Phase 4: In this phase I had a one-on-one discussion/interview with each teacher in the form of a joint stimulus recall video interview to analyse their specific lessons. I probed their views on the different connections they made. I also focused on each teacher's perceptions on the role of connections in their teaching of fractions. This discussion helped me to answer the first and second research questions. The conversation also helped me to enrich and validate my own analysis of phase 3. It further enriched my own understanding of the teachers' perceptions of connections they used in their lessons.

Phase 5: I conducted a focus group interview with 2 teachers. I did this to further consolidate, enrich and validate (Cohen, et al., 2011) the lesson analysis in phases 3 and 4. This enabled me to further answer the second research question.

3.4.2 Data collecting techniques

According to Van As and Van Schalkwyk (2008) a very effective approach in qualitative studies is to make use of multiple sources in collecting data in order to obtain rich evidence. Using numerous sources can make the data much more consistent and trustworthy, as it reveals ideas from different sources (Cohen, et al., 2007). Three methods of data gathering tools were employed during the study: observation, a stimulated video-recall interview and a focus group discussion. I describe each in more detail below.

a) Observation

An old saying "action is better that words", applies well to research specifically when observing. According to Walliman (2011) observations are used to reveal peoples' understanding of the process better by observing their actions rather than merely listening to their spoken explanations. Observation data is well known for its empirical real-life evidence. (Wisker, 2001, p. 178). Observers have the advantage of capturing what teachers say and do at the same time.

In order to find out the nature of connections that the participating teachers use, I observed two teachers twice and one teacher once. Both lesson observations took place during normal lessons and they were all video-taped in order to give me an “opportunity to replay data uncontaminated by assumptions at the time of recording” (Silverman, 2010, p. 212). The observation schedule was designed according to Businskas’ framework that I used during the observation (refer to Appendix F).

b) Interview

Two different types of interviews were used in the study and they were:

i) One-on-one interview

A one- on- one discussion in the form of a stimulated video recall interview**,** (Lyle, 2003) with the individual teachers was done after I analysed the lessons on my own. This kind of interview assisted me in obtaining deeper insight into the nature of connections the teachers made as well as their perspectives and purposes of using such connections. The interviews were semi-structured for the purpose of “further questioning and clarity” (Thomas, 2011, p. 163). The whole interview was audio-recorded and transcribed. Appendix J is an example of the stimulated video recall interview I had with the teachers.

ii) Focus group interview

Focus group discussions provided more in-depth data for the study, specifically on teachers’ perceptions about connection making. Leedy and Ormrod (2005) noted that people feel more comfortable talking in a group than alone.

I conducted a focus group interview with the two remaining teachers that I observed at the end of the data gathering process. The purpose of this interview was to pull together what individual teachers said about connections. These also enriched and validated the lessons observed and analysed, gaining further insights into teachers’ perspectives about connections.

I conducted all forms of the interviews discussed above by using semi- structured questions, which permitted me to probe for further clarification when the need arose. A semi-structured interview uses a mixture of structured and unstructured questions, where a number of questions may be predetermined.

As an interviewer I was free to deviate from structured questions and explore further issues raised by participants. I used a semi-structured approach because it allowed flexibility during the interview process, where I was able to inquire and also allow participants to express their ideas and experiences freely (Wisker, 2001) (refer to Appendix K for the list of main interview questions that were used).

3.5 SAMPLING

The selection of participants in this study was purposive because of the nature of the subject studied. According to Cohen, et al., (2007) purposive sampling is used to access 'knowledgeable people' for the purpose of generating rich data from the experts. Such an approach consists in sampling those who have in-depth knowledge or experience about the particular issues under investigation.

The research was conducted in three selected combined schools in the Oshana region of Namibia. I purposefully (Silverman, 2010, p. 141.) selected three Grade 7 mathematics teachers, one from each school. All the schools are within a radius of not more than seven km from my school making it easier for me to access them. I chose Grade 7 because it is the last Grade in the upper primary phase in which learners builds their mathematical knowledge, which prepares them for a more advanced phase, Junior Secondary. It is thus important that Grade7 learners embody connections in their learning.

3.6 SUMMARY OF THE DESIGN AND TOOLS

The table below provides a summary of the research design and tools used.

Table 3.1: summary of the research design and tools

Phase	Method/Tools	Purpose	Data
Phase 1	Piloting	To assess the viability and validity of the research tools and make changes where necessary.	General data on were to make changes to the instrument.
Phase 2	Classroom observation	To obtain evidence on the nature of connections teachers make.	Qualitative
Phase 3	Video analysis	To classify the nature of connections made and to prepare interview questions needed for the stimulus recall interview.	Qualitative data
Phase 4	One-on-one interview using a stimulated call video interview	To obtain insight on teachers' perspectives about connections and the nature of connections they make.	Qualitative data
Phase 5	Focus group interview	To consolidate, enrich and validate lessons observed and analysed in phase 3 and 4.	Qualitative data

3.7 ANALYSIS

I used an observation sheet (refer to Appendix F) to code the nature of connections based on Businskas' framework. Under each connection I described possible indicators of the connections that I could observe from the teacher. The analysis of the observed data was done as follows. I went through the transcriptions of the observed lessons, identified the nature of connections teachers made, colour coded them accordingly and categorised them according to the framework shown in the table 3.2 below. In Appendix G, H and I give a clear picture of how this was done.

Table 3.2: The indicators of the nature of connections

Nature of connections	Coding	Observable indicators or descriptions
Multiple Representations	MR	▪ Uses representations of different modes of the same concept. ▪ Allow learners to use different ways to answer the question. ▪ Uses equivalent representations ▪ Uses diagrams, models, notations, images, analogies, stories, games, etc.

		▪ Uses manipulative and videos ▪ Uses different descriptions and differences in different methods.
Part-whole relationships	PWR	▪ Makes connections between the general and the specific through particular examples. ▪ Uses practical examples to move to the abstract; a gradual teaching process from specific to abstract. ▪ Make use of particular examples, ideas, concepts and techniques. ▪ Generalizes concepts or actions. ▪ There is a hierarchy in the teaching-learning process.
Implications Connections	IM	▪ Shows how one concept depends on another. ▪ Able to justify relationships between the concepts ▪ Uses logical reasoning and draws conclusions from premises. ▪ Asks learners to justify their answers. ▪ Uses clear distinctions for sentences involving and, or, not, if-then some, all, etc. ▪ Provides deductive reasoning on why procedures make sense. ▪ Has a problem solving skills and tests assumption.
Procedural Connections	P	▪ Uses written algorithms to develop the concept. ▪ Shows fluency in computational skills. ▪ Does not dictate the learners to what procedure to use. ▪ Realizes that there are many ways to solve a problem.
Instruction-oriented Connections	IOC	▪ Shows a hierarchical nature of concepts to be learned. ▪ Shows that there is an extension of what learners know (linking prior-knowledge to new concepts). ▪ Knows learners 'backgrounds and aware of their thinking and ideas. ▪ Connects teaching to learners sense making ways. ▪ Recognizes and aware of learners prior knowledge. ▪ Uses the content to cater learners' different abilities. ▪ Borrows ideas from other domain or disciplines.

3.8 ETHICAL CONSIDERATION

It is important that appropriate ethical principles are used in any research (Eisenhaver, Orb & Wynaden, 2011). Gaining ethical approval for the research study is important for any research project (Silverman, 2010, p.30). Research ethics are adhered to in order to protect the welfare of research participants (Blanche, et al., 2006, p.61).

Ethical considerations were considered in several ways in this study. Before I began with the research, I sent a letter asking for permission to conduct my research to the office of the circuit inspector (refer to Appendix A), the principals of the schools (refer to Appendix B) and the selected teachers (refer to Appendix C). I met with each teacher and discussed the purpose of the research. It is at that juncture that I made participants aware of what is expected so that they could make an informed and voluntary decision on whether or not to participate. I ensured them of their rights to withdraw at any stage from the study. I also assured them of their anonymity and confidentiality (Cohen, et al., 2011). I changed other identifying information on the documents, such as places and names on the transcripts and audio records. I kept information for the purposes of the study only. Letters of consent were given (refer to Appendix D and E).

3.9 VALIDITY

Using several instruments in the research project is essential for validity as it provides trustworthy, rich and in-depth data. A primary focus in the qualitative research approach is for researchers to capture authentically the lived experiences of people, and in order for the data to be valid they should accurately depict the participants' experiences and behaviours, as well as the meaning they give to events (Maxwell, 1992).

I ensured reliability, validity and authenticity through a process of triangulating the data. Triangulation involves finding evidence from different sources of information, different investigators or different methods of data collection (Van As & Van Schalkwyk, 2008). In order to validate and triangulate data I collected, I used three main sources of data collection: observation, stimulated video-recall interview and a focus group interview as discussed above.

3.10 LIMITATIONS AND CHALLENGES

Making connections is very important and every teacher should make use of them. However, some teachers do not intentionally make connections for learners to value why they are learning mathematics, and some teachers do not even recognize that they are making connections. During the one-on-one discussions it was thus important to reach consensus about what constitutes different types of connections. This was time-consuming and impacted on the length of the research process.

Another limitation of the research was the small sample size. This meant that I could not claim to represent how all mathematics teachers in the Oshana Region perceived and experienced connection making. The small sample size prohibited me from making any generalization of the results. In addition, in some cases the participants may have said what they thought the researcher would like to hear and, therefore, may not have given genuine responses to all the questions.

3.11 CONCLUSION

In conclusion, this chapter focused on the research design and the methodology of the study. I have described the whole research process, which includes the orientation, data collection techniques, the sampling strategy, the commitment to research ethics, a discussion on the validity and limitations, as well as describing the data analysis. The data that informed this study are presented in the next chapter.

CHAPTER 4

FINDINGS AND DISCUSSION

4.1 INTRODUCTION

The data for this chapter was derived from the observations, stimulated video-recall interviews and the focus group discussion. This chapter is organised according to the lessons observed. Each lesson is divided into three minute intervals. The individual connections as per framework in chapter 1 are identified in bold and then discussed. To remind the reader the following abbreviations are used:

IOC -- Instruction oriented connection,
P -- Procedural connection
MR -- Multiple representation connection
PWR --Part-whole relations connection
IM -- Implications connection

4.2 LESSON OBSERVATIONS

4.2.1 Teacher: Jacobs Lesson 1 Grade 7B

Ordering of fractions, multiplying a fraction with another fraction and multiplying a fraction with a whole number.

This lesson took place in grade 7B and was about arranging fractions in ascending order as well as multiplication of fractions with another fraction and with whole numbers. The teacher gave one example on ordering fractions, which composed of $\frac{2}{5}; \frac{6}{11}; 0.21; 0.79;\ 2\frac{7}{9}$. This was a continuation of what was previously done when they looked at arranging common fractions only.

Another aspect that was covered in the lesson was multiplication of fractions. The teacher gave some examples on how to multiply a fraction with another fraction and how to multiply a fraction with a whole number. This was taught mostly procedurally. The teacher did not use any other method, diagram or any other learning resource.

0-3 min

The teacher started off with an **IOC** as a connection by asking learners if they can still remember what they did yesterday *" we looked at how to arrange fractions from the biggest to the smallest can you still remember that?* The teacher supplemented what they knew already when he said "we *looked at common fractions only, now, we have a mixture of forms of fractions"*. This is an IOC because it reflects a hierarchy of concepts learned and links the new concept to prior knowledge of the leaners.

The teacher made a **P** connection when he said that "*we can either convert all the fractions to percentages or to decimals"*. Here the teacher provides two different procedural options. Another **P** connection was made when the teacher told learners what to do when converting common fractions to decimal fractions when he said "*when you want to convert to decimal fraction, you don't need to write all the numbers after the decimal point, 3 or 4 numbers after the comma are enough"*. This is also a way of telling learners what to do, which is procedural without any explanation of why.

An **MR** connection and an **IM** connection were identified when the teacher explained that "*these numbers are the same,* $\frac{2}{5}$ *is the same as* $0{,}4$ *it is just because they are written in different forms; as a decimal or as a common fraction but they are of the same value".* This is both an MR and IM because it shows another way of writing a number in an equivalent form. It also falls under IM because it emphasizes that one is a common fraction and the other is a decimal fraction – but both are forms of fractions. After all the numbers where converted to decimal fractions, the teacher made a **PWR** connection when he said "*this one having zeros means they are all small"*. This is a general statement the teacher made showing how numbers relate to each other.

3-6 min

After the ordering of fractions was done, the teacher continued with the lesson and he made an **IOC** when he said "*we are going to continue with something different but still include different* *forms of fractions*." After he wrote $\frac{2}{5} \times \frac{3}{4}$ he made another **IOC** when he asked learners if they were familiar with this and could calculate it. The two statements above are both IOC since there is an extension to what learners already know (different forms of fractions) and recognizes their prior-knowledge (if they are familiar with it).

6-9 min

The answer to the calculation was $\frac{3}{10}$. The teacher made another **IOC** when he said "*but again remember what we always say, all the answers in fractions should be*"? This shows that the teacher emphasized the learners' prior- knowledge. The teacher made a **P** connection when he wrote down what to do when multiplying fractions "*if you want to multiply fractions, a numerator should multiply another numerator and a denominator should multiply another denominator"*. This shows that he is teaching the prescribed steps. The teacher, who teaches steps of what to do to work out the sum, is making procedural connections.

9-12 min

The teacher gave a problem and called on one learner to work it out on the chalkboard. The teacher made an **IM** connection when he asked the learner to justify how he worked out the answer. "*How did you work out that one? How did you get 24? Can you explain to us? Where did you get 133"?*

12-24min

The teacher gave a class work activity to the learners. Therefore no connections were identified.

24- 27 min

After the teacher marked all the learners' class work he continued with multiplication of fractions with whole numbers. He wrote $\frac{1}{2} \times 5$ on the chalkboard and asked one learner how it should be worked out. The learner replied that it should be $\frac{1}{2} \times \frac{5}{100}$. By recognizing this misconception, the teacher made an **IOC**, **IM, MR and PWR** connection when he said "*now we changed the value of the number now. This side remains* $\frac{1}{2}$ *but this one is not 5 anymore, if we* *divide it* $\frac{5}{100}$ *it will give us 0,05.* (**IM/IOC**). *We changed the value of the number and we don't have to change the value of the number. We were not suppose to make it over 100 but"? "Because if we make it over one, we did not change the value of that number because if we divide 5 with 1 it will still give us 5.* $\frac{5}{1}$ *is the same as 5. This is a fraction and this one is a whole number but they are the same* ***(MR).*** *We did not change anything it remains 5. Now all the whole numbers if we want to make them fractions we should make then over 1*" **(PWR).**

In the above teacher's explanation, the teacher used both IOC and IM connections because he distinguished between true and false and connected his teaching to learners' sense making ways when he explained why 5 should be written over 1 and not over 100. An equivalent representation, which is a form of MR connection was made since the same information was made in more than one form; a whole number and a common fraction ($\frac{5}{1}$ is the same as 5). A generalization and reasoning was also made from the explanation when the teacher said "*all the whole numbers if we want to make them fractions we should make then over 1".*

27-30 min

After the teacher explained why $\frac{1}{2} \times 5$ will be $\frac{1}{2} \times \frac{5}{1}$, he asked one learner how it should be worked out and the learner responded that it should be made a reciprocal. The teacher made an **IOC** when he asked "*where did you learn that word reciprocal? But where did you use it"?* This shows that the teacher is aware of the learners' background and their prior knowledge. He explained where the reciprocal is used and in his explanation he made an **IM** connection when he said "*dividing two fractions is the same as multiplying with its reciprocal or its inverse*". This is IM since it shows how one concept depends on another (division of fractions depends on the knowledge of reciprocal).

At the end of the lesson the teacher made a **P** connection when he wrote down how fractions with whole numbers should be calculated or worked when he wrote "*for you to multiply a fraction with a whole number, make a whole number a fraction first by making it over one, then you multiply the numerators and again the denominators".*

Table 4.1 below provides the frequency of the connections made in this lesson.

Types of connection	**Frequency**
Multiple Representation (MR)	2
Part-Whole Relationships (PWR)	2
Implications connections (IM)	4
Procedural connections (P)	5
Instruction-Oriented Connection (IOC)	7

4.2.2 Teacher: Jacobs Lesson 2 grade 7B

Multiplying fractions of quantity

This lesson was conducted in grade 7B, the same class was observed. The lesson was about multiplication of fractions with quantities. The teacher started the lesson by explaining what quantities are. The teacher gave some examples and explained to learners what is expected from them.

0-3

At the beginning of the lesson, the teacher wrote $\frac{1}{4}$ of 20 and asked learners to calculate it. Before the learners shared their ideas the teacher made an **MR** connection together with an **IOC** by using sweets to represent the quantities that are to be worked out in the problem. He said "*let me say you have 20 sweets and then I said give me* $\frac{1}{4}$ *of the sweets that you have, how many sweets are you going to give me?"* The sweets used are related to the social context of the learners, which is an example of an MR connection. This statement also made an IOC since it focused on helping learners to understand what quantities are, as the teacher explained at the beginning, and hence helped them to make sense of the problem, mostly on how it can be solved.

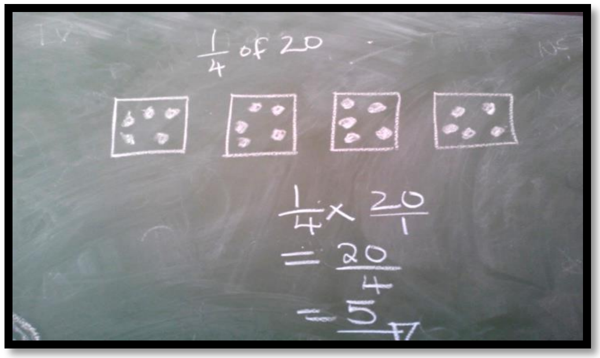

Figure 4.1 An MR of fraction as quantity

From the answer that the teacher received from the learner, he made an **IM** connection when he asked the learner to explain how she got 5. The teacher asked "*how did you get that answer? Why did you divide with 4, why not with 3 or 5?"*. This type of a question seeks reasoning and an explanation from the learner - this can be classified as an IM connection.

The teacher's question about why 4 was used to divide into 20 was not answered, and it was during that time that the teacher made an **IOC** when he referred them back to what they did with the question "*what did we say about a quarter? What is a quarter of something?"* This shows that the teacher was aware of the learners' prior knowledge and referred to what they were doing in fractions so that the learners could give a logical reason.

The teacher used dots to represent sweets and used the diagram to represent $\frac{1}{4}$ of 20, as show in diagram 4.1 above. That was a type of **MR** and **PWR** connection because obtaining the answer was represented in different modes, which is an MR connection. The connection was made through a practical example, from general to specific. This kind of connection is also a PWR connection.

3-6 min

The teacher's explanation using the diagram was not the only method he used. He used another method of calculating $\frac{1}{4}$ of 20 procedurally. When he was solving the problem, he made a **PWR** connection when he generalized a statement that "*every whole number is written over 1"*. The steps involved in solving $\frac{1}{4}$ of 20 were procedural, which is a **P** connection. The emphasis was on what to do and not on focusing on why. After the teacher worked out the problem with the second method, he gave another problem to be solved. The problem was 3 of $\frac{2}{7}$. Here the teacher made an **MR** connection when he used an image of bread to represent 3. He said "*imagine you have 3 breads and out of those three breads you gave away* $\frac{2}{7}$ *how will you know how many you gave away?"*

6-9 min

No connection was made during this time because learners were busy calculating.

9-12 min

The teacher explained how two learners worked out 3 of $\frac{2}{7}$. Her explanation was procedural – hence a P connection.

Table 4.2 below provides the frequency of the connections made in this lesson

Types of connection	Frequency
Multiple Representation (MR)	3
Part-Whole Relationships (PWR)	2
Implications connections (IM)	1
Procedural connections (P)	2
Instruction-Oriented Connection (IOC)	2

4.2.3 Teacher: Brianna Lesson 1 grade 7B Fractions as a part of a whole.

This lesson was an introduction to fractions as a part of the whole. The teachers revisited content that was covered in previous grades. The teacher presented the lesson well and the learners' interaction with the teacher was very good.

0-3 min

The teacher started with an **IOC** when she asked the learners to divide the circles in two, three, four, and five equal parts. This is an IOC because she knew their backgrounds (they learned fractions in previous grades). When she was doing this she made an **MR** when she said "*imagine you are three of you and you are so hungry and you only have one apple that you have to divide among the three of you right. So make sure you divide it into three equal parts because the moment you give someone a smaller piece that one will be angry with you so makes sure that u divides it into 3 equal parts"*. The teacher placed Mathematics in a social context that is familiar to the learners.

3-6 min

Learners were busy dividing their circle. Most of them were struggling to divide their circles into thirds, and no connections were made during this time.

6-9 min

The learners continued to divide the circles into different parts. An **IOC** was made again when the learners were busy dividing the fourth circle when the teacher said "*imagine you are 4 of you and you need to divide this apple among the four of you and it should be the same parts…. 4 equal parts otherwise someone will want to fight"*. After the learners divided all the circles, the teacher also drew them on the chalkboard and divided one circle into two equal parts. It is here that the teacher made another **IOC** since she was aware of what they did in previous grades. She asked for their prior-knowledge when she asked "*how do we call something when we divided it into two equal parts? In fractions, how do we call that? You learnt these fractions already, is it not so"?*

12-15 min

The teacher continued with a circle which was divided into three unequal parts. She asked learners again for the name of something divided into 3 equal parts. Learners did not know this and the teacher gave them clues until they got it right saying it's a third. The teacher shaded one part of the circle and asked learners if it is a third, all the learners agreed. This is where the teacher made an **MR** and **IOC** connection by saying: "*let's think this as if it is an apple (pointing to the circle). You are 3 of you and u need to share this apple. Remember if you share this apple and give someone this part (pointing the smaller part), eeee that person will be very angry. Now, is this a third? Learners said no! No its not! So you need to divide it into 3 **equal** parts for you to call it a third"*.

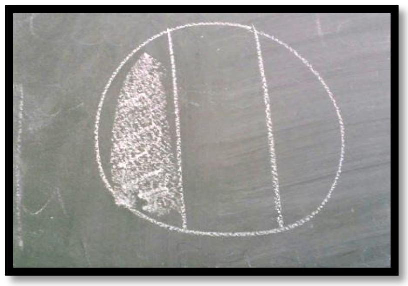

Figure 4.2: Is this a third?

The teacher placed mathematics in a social context by linking a circle to an apple. The diagram was used in the explanation which is an example of an **MR** connection.

The teacher again connected her teaching to learners' understanding by showing them why it is not a third and when to call a piece a third, as shown in figure 4.2 above. This is an **IOC.**

An **IM** connection was made when the teacher deduced an if-then connection from the above instance and generalized it when she said "*always remember, **if** you just divide something into three parts which are not equal **then** we are not talking about a third we are talking about something different*".

Another **MR** connection was made here when the teacher said "*now can I have 6 people…….who can come and divides this people into thirds?* This is a concrete example used in the class to represent a third and concrete representation is part of MR.

15-18 min

The teacher continued to test learners' knowledge about different fractions. *"How do we call something when it is divided into four equal parts? How do we call something when it is divided* *into five equal parts? How do we call a circle that I did not divide at all*"? When the teacher asks all these questions she is making an **IOC** because she is testing the learners' prior-knowledge.

18-21 min

The teacher drew a circle and divided it into two equal parts then shaded one part (**MR** connection**).** "*If I shade this one and they ask you to identify the fraction which is shaded. First of all I divided this one into how many parts? (Writing 2 below the division line). How many parts are shaded? (Writing 1 in top of a division line) how do we call the number on top? …… and how do we call the bottom one"*?

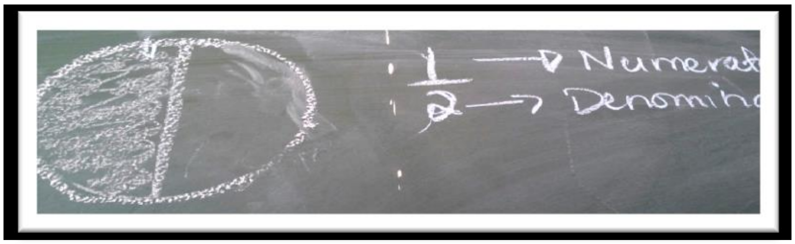

Figure 4.3: An illustration of a Numerator and a Denominator

The teacher made an **MR** connection because she used a diagram to illustrate a half and also to show where the numerator and a denominator are placed using the diagram, as indicated in figure 4.3 above. The teacher used both the pictorial and a symbolic representation of a half. The teacher also made an **IOC** because she asked learners what the top and the bottom numbers are called.

21-24 min

The teacher made some connections again when she said "*who can tell us, what does this numerator tells us and what does this denominator tells us? We have an example here……where did we get this one from?* (Showing 2 on the chalkboard) *we got it from here* (showing the two parts on the circle). *Is it not so? This* (showing 1) *is this one (showing the part shaded) so what does the numerator and the denominator tells us"?* The teacher made a **PWR** connection here because she used a practical example (a diagram) for reasoning. She also made an **IOC** where she tried to connect her teaching to a more sensible way when she showed what the numerator and the denominator represent.

24-27 min

During this time, the teacher made some connections when she said "*now in whole numbers we learnt that we can count in ones like 1, 2, 3, 4, 5, 6 or count in twos like 2, 4, 6, 8, 10…. In fractions for example we can count in halves. This one is one half. Is it not so? Yes. The second one will be what? 2 half; 3 half, 4 half, 5 half, 6 half"*. This is an **MR** connection because the teacher connected this to other domains of mathematics. In this case she used whole numbers and connected them to fractions. The teacher also made an **IOC** when she referred back to what they had learned previously by using the knowledge of counting in whole numbers and extending this to what she wanted to teach the learners. This introduced the concept of improper fractions and mixed numbers.

The teacher wrote $\frac{1}{2}; \frac{2}{2}; \frac{3}{2}; \frac{4}{2}; \frac{5}{2}; \frac{6}{2}$ on the chalkboard and continued "*now, we know that this is this one (pointing a half and a circle on the chalkboard previously drawn). This one will be then that (pointing $\frac{2}{2}$ and a circle that is divided into equal parts with all equal parts shaded), which means it a …a whole. 2 divided by 2 is what? Its 1. A 1 that we say it's a whole. How many parts are shaded? 2, so meaning that it will be a whole. We have shaded the whole thing"*. This is an **MR** connection whereby the teacher used diagrams to show the learners the connection between the pictorial and symbolic representation of $\frac{1}{2}$ and $\frac{2}{2}$.

The teacher continued "*I am sure that some of you were taught how to change improper fractions to mixed numbers. Now how do we use diagrams to show this mixed number (pointing $\frac{3}{2}$ in the list)? We will then say is 1 whole (drawing a circle divided it into two equal parts and shade them) and here is a half" (drawing a circle and shaded one part). "Let me ask you. 6 divided by 2 what is the answer? 3 so it will be 1 2 3 (she drew 3 circles divided them into halves) that's what it means. Let's count. How many halves? 1 half, 2 half. 3 half 4 half, 5 half 6 half. Meaning we have 3"*. This is also similar to the above instance where the teacher used diagrams to show how the three in $\frac{6}{2}$ was calculated, as indicated in figure 4.4 below. Therefore she had made an **MR** connection.

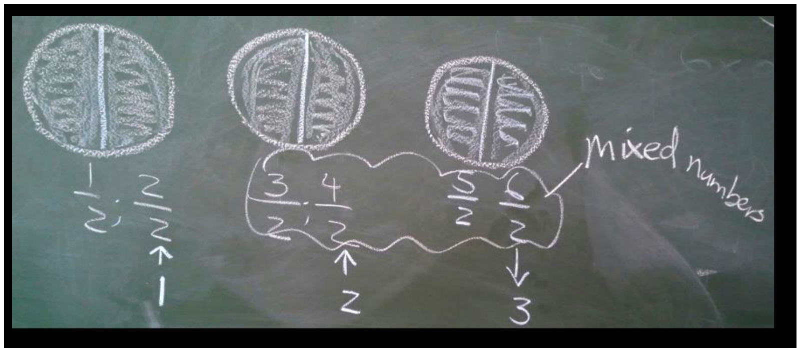

Figure 4.4: Pictorial representation of mixed numbers

27-30 min

The lesson ended with the teacher summarizing what was learnt in the lesson. Here she made an **IM** connection when she said "***if*** *we divided a whole into equal parts,* ***then*** *is when we talk of fractions; a part of a whole"*. This is a generalization of an if-then connection, which was drawn from examples presented in a hierarchical manner.

Lastly, the teacher again made an **MR** connection in her statement "*these are the things we do in real life situations. Sometimes you bought a chocolate and it is having 18 bars and you are two of you; you need to divide that one into 2 equal parts"*. Here the teacher placed mathematics in a social context where an analogy of MR is used to show learners how they use fractions in real life.

Table 4.3 below provides the frequency of the connections made in this lesson

Types of connection	**Frequency**
Multiple Representation (MR)	8
Part-Whole Relationships (PWR)	2
Implications connections (IM)	2
Procedural connections (P)	0
Instruction-Oriented Connection (IOC)	9

4.2.4 Teacher: Brianna Lesson 2 Grade 7B Word problems

This lesson was about word problem solving in fractions. In general, solving word problems requires the basic knowledge of the four main operations and how they work. This means that learners are expected to have knowledge of how to add, subtract, multiply and divide.

Operating with fractions to solve problems is not simple for learners in junior secondary phase. Learners need a very good understanding of problem solving strategies and that is why, in most cases, learners are given simple steps to follow in solving word problems.

0-3 min

The teacher started this lesson by making an **IOC** when she asked for learners' ideas on how to solve word problems. Some of the questions she asked learners were "*what is the first thing that we have to do when we are solving word problems … after we read a sentence we need to do what … and then the next thing"?…* and many others; that aim to seek what learners know about word problems.

The teacher emphasized what learners said about the steps involved in solving word problems. One of them was when she told learners to make sense of their final answer. By doing so she made another **IOC** which can also be an **MR** connection when she said "*you cannot just say for example there are 11 people and in a car only 5 people will fit, if they ask you for example how many cars do this people need, are you going to say they need 2,5 cars? Is there a car which is a half? … so you are going to say they need 3 cars because the other person will need a car, that why we said you look back and see if the answer you got really makes sense"*. This was an IOC because she connected this to the learners' understanding. It was also an MR connection because the example was borrowed from a social context so that learners could visualise how this type of question related to a real-life situation.

3-6 min

The teacher gave the first word problem to be solved. The problem referred to a real-life situation. After one learner solved the problem, the teacher made an **IM** connection where she asked the learner to justify the answer when she asked "*now, how did you get 400g? Can you tell* *us how you worked it out? ... Why with a 2? Why did you divide it with 2?*" All these types of questions require learners to provide reasons and justifications for their answers.

6-9 min

Solving the first word problem continued. Regarding the question why they divided 800 by 2 one learner replied that it was because it would not leave a remainder. It is during this time that the teacher explained the answers as follows: "*didn't we say a half is something divided into two equal parts? Can you remember what we did last time?* **(IOC)** *(The teacher drew a circle and divided it into 2 equal parts) This is a half because it is divided into two equal parts. Now if this is your cake which is 800g and if it is half of it that means you divided it in halves. Now, how much is each half*? **(MR** connection)**)**.

The teacher continued, "$It\ is\ like\ \frac{1}{2}\times\frac{800}{1}=\frac{800}{2}=400g\ (\boldsymbol{P})\ or\ \frac{800}{2}=400g?\ that\ means\ \frac{1}{2}\times\frac{800}{1}\ is\ the\ same\ as\ \frac{800}{2}; they\ will\ all\ give\ you\ 400g.$ *So multiplying with a half is the same as dividing by 2"* **(PWR/IM/ MR/ IOC**). There are a number of connections evident in this piece. First of all the teacher referred to what was previously done and that meant she knew their prior-knowledge. A drawn circle was used to explain what the question entails and this was an MR, since it was a pictorial representation of a half. Another MR connection was made where the teacher worked out the answer by using $\frac{1}{2}\times\frac{800}{1}$ and using $\frac{800}{2}$ which are equivalent representations under multiple representations. The teacher worked this out procedurally using two ways. This meant that there is a connection between the two methods used, yielding the same answer.

The teacher generalized that $\frac{1}{2}\times\frac{800}{1}$ was the same as $\frac{800}{2}$; they will all give you 400g, so multiplying with a half was the same as dividing by 2. This was done following an example and a justification from the example. When a general statement is made from a specific instance, it is a PWR connection. The same statement can be an IM connection as well because the teacher showed the link between the two methods and why they all work. The explanation was logical and made sense. The explanation was also made in such a way that it made sense to the learners and catered for different learning abilities. This was supported when the teacher used the diagram, and two methods of working out the problem, which can be classified as an IOC.

The teacher continued with the lesson and again made an **IOC** when she said "*there was something we did when we were doing some fraction calculations, there was something like this; just to refresh your mind* $1-\frac{3}{4}$*; we said we are going to change a whole into a fraction* $\frac{4}{4}$*".* This showed that the teacher was aware of the learners' prior-knowledge and background. The teacher again made an equivalent representation when she said *"we said that* $\frac{4}{4}$ *is the same as a whole (1).* $\frac{4}{4}$ *gives you 1 meaning that it is a whole".* This is an **MR** connection whereby $\frac{4}{4}$ and 1 are the same but written in different formats: as a fraction and as a whole number.

The teacher solved this problem by herself and by using connections as well. When the teacher said "*if I subtract this is going to be* $\frac{4}{4}-\frac{3}{4}=\frac{1}{4}$ *. Can you remember when we were adding fractions? We said if the denominators are the same like here; we only add the numerators and keep the denominator the same*", she made an **IOC** when she asked the learners if they can remember what they did previously. She also made a **P** connection when she set out the procedure involved in solving out the problem.

9-12 min

The teacher asked learners to work out the second word problem by themselves. The problem read: "*Maria has learned* $\frac{2}{3}$ *of a poem. How much she still has to learn?"* No connections were made because the learners where busy working out the problem

12-15 min

The teacher asked one learner to calculate the problem on the chalkboard. She made an **IM** connection when she asked the learner to explain to the whole class how she worked it out.

15-18 min

Together with the class, the teacher moved on to the next word problem. The problem was quite tricky and most learners struggled with it. The teacher gave learners a clue by making an **IOC** connection when she said "*remember that we said if you are adding fractions with different denominators you need to find the LCM first*". This indicated that the teacher knew the learners, background and was familiar with what they already knew.

18-21 min

There was no connection made during this time because learners were busy working out the problem.

21-24 min

The teacher decided to solve the problem with the class because none of the learners were any closer to the answer. She made an **MR** connection when she drew a circle to represent a pizza. The circle was a pictorial representation of a pizza which is an MR connection. The teacher continued explaining how the problem would be solved and again repeated an **IOC,** similar to the one she made previously when she reminded them of the LCM that they should consider when adding fractions of different denominators. After they worked out the part of the pizza that was given away, the teacher made another **MR** connection as she said "*we need to know that the pizza was a whole, a whole (1) that represents* $\frac{12}{12}$". This is an equivalent representation of one thing, a whole and $\frac{12}{12}$.

Table 4.4 below provides the frequency of the connections made in this lesson

Types of connection	Frequency
Multiple Representation (MR)	5
Part-Whole Relationships (PWR)	1
Implications connections (IM)	2
Procedural connections (P)	1
Instruction-Oriented Connection (IOC)	7

4.2.5 Teacher: Bella Lesson 1 Grade 7C Addition of fractions

Addition of fractions is one of the competencies that should be achieved in upper primary phase. The lesson was about adding fractions and simple fractions were used. From my experience as a mathematics teacher, most of the learners find it very difficult to add fractions. This can be due to a poor conceptual understanding of what is involved in adding fractions.

This lesson took place in grade 7 C. The teachers' presentation was very satisfying and aimed to make learners understand why the LCM is used in adding fractions specifically where denominators are different. The teacher used the diagram to show how fractions are added and also calculated it using the method.

0-3 min

The teacher started off by making an **IOC** when she asked learners to work out $\frac{3}{5}+\frac{2}{3}$ on their own. This was done with the aim of testing how well they can remember, from previous grades, how fractions can be added. She was assessing their prior knowledge.

3-6 min

Together with the learners, the teacher made some connections when solving the problem. One of the connections made was an **IM** connection where she talked of the LCM as the first step to be performed on adding fractions with different denominators. This shows how adding fractions depends on the concept of LCM. In her emphasis, the teacher made a **PWR** connection when she generalized that "*you cannot add fractions with different denominators*". This statement is a clear generalization of what to do when adding fractions.

After the class found the LCM to add the two fractions, the teacher's teaching was procedural because there was no explanation made as to why it should be done that way. Therefore, since the teacher focused on the steps, this is identified as a **P** connection. After the problem was solved, an **MR** connection was made when the teacher said: "*That is not the only method that we can use, that was only a calculation we can also do that in practical meaning with diagrams"*. This is an MR connection since it shows the teacher's ability of recognizing that there are multiple ways to answer the question.

6-9 min

In the second method, the teacher made an **IOC** when she asked learners the meaning of a denominator and a numerator that she presented in a diagram. This shows the teacher's awareness on learners' prior knowledge and therefore used that knowledge to continue with what is to be learned. An **MR** connection of both $\frac{3}{5}$ and $\frac{2}{3}$,as shown in the diagram below was made in symbolic and pictorial format.

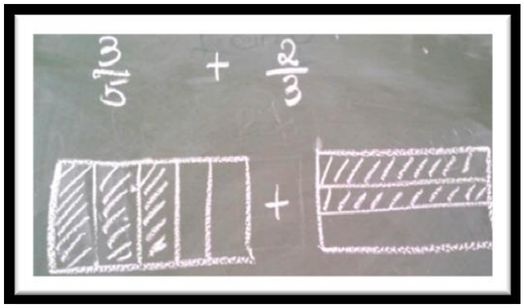

Figure 4.5: Symbolic and pictorial representation of fractions

The teacher made a **PWR/ IM** connection when she said that "*now we want to add those pieces together by filling the unshaded parts in this shape with the shaded parts from this diagram. But we cannot just take one piece from* $\frac{2}{3}$ *and fit them here, it won't fit because as you can see* $\frac{1}{3}$ *here is bigger than* $\frac{1}{5}$ *because the shapes are not divided in the same parts. This one is divided in 5 and this one in 3. So to add these pieces together you have to divide the shapes in the same pieces and that is how the LCM comes in*".

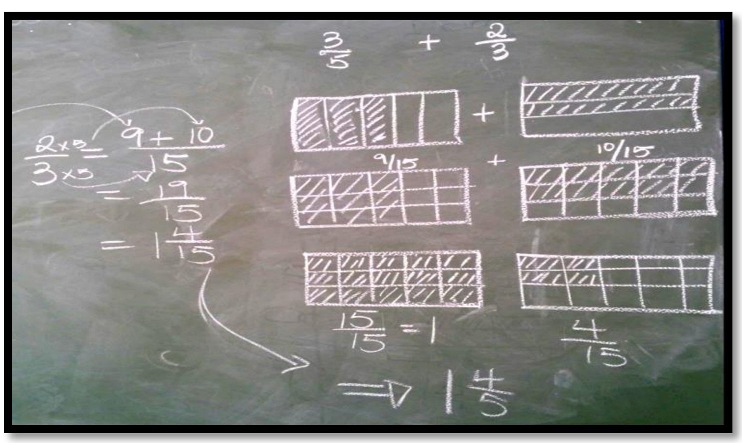

Figure 4.6: Addition of fraction with diagrams

The teacher in her statement tried to connect the general to the specific using the diagram and to show how the concept of using the LCM/LCD came about in adding fractions, as indicated in figure 4.6 above, which is part of PWR connections. The teacher also showed the link of why the LCD is used in adding fractions by drawing conclusions from the example itself, which is part of IM connections. The teacher's explanation was done in a logical way, from simple to complex and following a good sequence until she got the answer, which was the same as the one done with mere calculations.

9-12 min

The teacher made a **P** connection when she told learners to use any method in the examination or in the test as long as it bears the correct answer. The teacher who does not dictate to learners what method they should use shows the P connection.

Table 4.5 below provides the frequency of the connections made in this lesson

Types of connection	**Frequency**
Multiple Representation (MR)	2
Part-Whole Relationships (PWR)	2
Implications connections (IM)	2
Procedural connections (P)	2
Instruction-Oriented Connection (IOC)	2

The above observed lessons that were analysed generated a comprehensive sense of the nature of connections made by each teacher. All the connections that were made in all the lessons are summarized in Table 4.6 below.

Table 4.6 Summary of all connections used

Types of connection	**Frequency**
Multiple Representation (MR)	20
Part-Whole Relationships (PWR)	9
Implications connections (IM)	11
Procedural connections (P)	10
Instruction-Oriented Connection (IOC)	27

A synthesis of the table will be provided in section 4.4.

4.3 INTERVIEWS

4.3.1 Stimulated recall video analysis

As indicated in the methodology chapter, after the lesson observations I analysed the videos with the teachers to obtain a deeper insight into what was really happening in the class. I looked at the connections they made in the lesson and the discussion below indicates the reasons why such connections were made. In most cases, teachers were not aware of all the connections they made. This is because of poor knowledge of the different types of connections. After a satisfying explanation on the different types of connections, they gave their ideas on why that specific connection was important.

4.3.1.1 Teacher: Jacobs Lesson 1 grade 7B Ordering of fractions, multiplying a fraction with another fraction and multiplying a fraction with a whole number.

The lesson was specifically about arranging decimal fractions and multiplication of fractions. The teacher made all different types of connections as suggested by Businkas although some were used more than others.

The common connection used was IOC and the teacher believed that it is always good to test learners' knowledge, experiences and their understanding because it directs the teacher what to focus on in the lesson. He said that his class is made up of learners from different schools. Although he knew that they had already learnt fractions, according to the syllabus, this did not guarantee that they had all done it, or that they all remembered what they had learnt. The teacher clarified this by stating that *"it is not good just to teach what you need to teach. It is good to know what they know so that we can identify specifically what they acquired and what skills and knowledge they gained*". Asking questions on previous work and testing them on what they can remember helps the teacher to identify their gaps and misconceptions.

The teacher made some procedural connections as well where he wrote what learners need to do when they are multiplying fractions. To the questions why he wrote this the teacher stressed that "*procedures are needed for the learners to be able to work out a mathematic problem quickly especially in the examination. You can teach this (multiplication) with diagrams or using a number line but this can be time consuming in the examination.*"

This teacher did not use other methods in the lesson to either order fractions or multiply fractions. He was, however, aware of the importance of using different procedures or methods in teaching. Mr. Jacobs noted that teaching different procedures gives learners a choice of different methods to use in the test or examination and this caters for their differences in abilities.

The implication connections were made in the lesson on the ordering of fractions. The teacher believed that it is necessary for the learners to connect between decimal fractions and common fractions so that they would understand how they depend on each other. Mr. Jacobs noted that *"common and decimal fractions are two different mathematical notations but their results are the same".* He observed that most learners could not see that 0.25 and $\frac{1}{4}$ is the same. It was therefore crucial that they made this connection for future learning and better understanding of fractions.

In most cases, the teacher asked learners to explain their answers. By so doing he is making instruction-oriented connections. To the question why he asked those questions, the teacher answered that it was because he wanted to know what they were thinking and if they can use the procedures provided to them to solve the problem. The teacher further noted that some of the learners do not really concentrate in the class and some might have different ideas of solving the problem. Asking them to explain their steps would prove if they understood and whether they were concentrating.

Mr. Jacob further argued that *"some learners cannot express themselves clearly in mathematics, especially mathematics symbols. U will find that a learner cannot read this for example* $\sqrt{64}$ *or some can read 0.25 as zero point twenty five instead of zero point two five".* I believe that a teacher cannot discover this if he does not ask for learners' explanations.

The use of MR connections in his teaching were only done between the symbols. The teacher said that it was done with a purpose of showing learners different ways of writing a number that means the same thing: "I *don't want my learners to treat 0, 25 and* $\frac{1}{4}$ *differently, I want them to see them as one number that why I represent them in different forms".* He extended this to the idea that fractions, decimals and percentages need to be taught well because they are important in different contexts. He gave an example of percentages that are appropriate when comparing learners' achievement rather than using decimals.

In the lesson, the teacher also used some **PWR** connections when he generalized as follows: "*all the whole numbers, if we want to make then fractions, we should make them over 1*". The teacher believed that it is necessary to generalize as this applies to all the whole numbers and not just the ones used in the class. The teacher, therefore, wanted his learners to use this in all cases whenever they were working with whole numbers and where they needed to convert them to common fractions.

4.3.1.2 Teacher Jacobs Lesson 2 grade 7B

Multiplying fractions of quantity

The teacher used different types of connections in this lesson. He used MR connection when he used different methods, such as using the diagram and working out answers procedurally. He said that this was done so that he could cater for different learners and for the learners to see the practical example of how the answer can be worked out, instead of only focusing on the procedures without any understanding.

The connection to prior knowledge was also made in the lesson where the teacher asked the learners to explain how she got the answer in solving $\frac{1}{4}$ of 20. The teacher asked why a 4 was used to divide 20 and not a 3, or a 5. Mr. Jacobs explained that this connection was made because he wanted to see if the learners could still recall what a quarter really means and if she could use it properly in solving the problem.

The learner failed to answer this question. From this reaction the teacher pointed out that *"see now, this learner worked out the problem correct but does not know what she was doing because she cannot defend her answer"*. The teacher, therefore, sees it as crucial to ask questions related to what was learned or what learners did in previous grades to further develop their understanding of fractions.

The question was also asked to see if the learner has any misconception of the method she used to solve the problem. The teacher believed that "*some answers came by coincident, but sometimes the reality is the method used might be totally wrong and will never work for other similar problems"*.

A general statement which is a PWR connection was made in the lesson as well when the teacher said "*every whole number is written over 1*". According to the teacher, he did this because he noticed that some learners confuse this with 100: "*They have learnt that percentages are out of 100, you will find learners using 100 instead of 1 even if percentages are not stated at all, and therefore this might help them differentiate the two and when to use 100 and when to use 1*".

The procedural connection on how to work out $\frac{20}{1} \times \frac{1}{4}$ was mainly done for the learners to know what to do instead of why, according to the teacher. This would also help them in tests or in the examination where they need to work out the problem procedurally.

Although the teacher used several connections here and there, his perceptions as to why he used such connections were quite similar. I therefore excluded some of the reasons to avoid repetitions.

4.3.1.3 Teacher Brianna Lesson 1 grade 7B

Fractions as a part of a whole.

The teacher made IOC connections with the purpose of testing the learners to see if they could remember what they did in previous grades. She experienced that fractions is a big challenge to her learners and therefore had to revisit their prior-knowledge to find out what they know, what they don't know and where they get confused. She believed that she could not continue with the lesson if learners had not caught up previous work quite well.

The MR connections used in the lesson were done for the learners to make a connection between the symbol and a diagram. "*I mostly use diagrams to represent a fraction that I want to talk about so that my learners will know and see what I mean. It is useless to talk of a quarter when learners do not know how it looks like….. I think my learners learn better from that so diagrams really help*".

The teacher used diagrams and the concept of whole numbers to present the concept of mixed numbers. The diagrams represented whole and half numbers that were taught at the beginning of the lesson. The teacher said that *"learners learn mixed numbers without a thorough knowledge of why they are composed of a whole number and a fraction and how they came about. So you see I used the whole numbers first to count in halves and then transferred that to fractions where mixed numbers were also included*".

The teacher did not use any concrete material or a manipulative in the lesson. She, however, used the mental images of apples and chocolate which are forms of a multiple representation. Ms. Brianna said that this was done to encourage the learners to make sure that they divided their circles in equal parts. *"They will try to make sure that all the parts are equally divided",* she said.

An if-then connection under **IM** connections was made in the lesson where the teacher emphasized that "*if you just divided something into three parts which are not equal, then we are not talking about a third we are talking about something different".* According to the teacher, she made this connection to show that a fraction is part of an equally divided whole. This statement therefore helped the learners to make sure that the whole is divided into three equal parts if they are to talk of a third and to avoid further misconceptions.

Another connection made in the lesson was the **PWR** connection whereby the teacher generalized from specific examples (using the diagrams) and moved to the abstract (using the symbols) following a sequence of practical examples which were presented from simple to complex. The teacher did this for the learners to see how fractions came about.

She gave an example of how the symbol of a mixed number was derived from common fractions and whole numbers. She clearly stated that *"it is not good to just talk about a half when children do not know what you mean. You cannot say this is a mixed number when the children don't know why u know it should be done carefully in a sequence otherwise learners won't understand"*.

4.3.1.4 Teacher: Brianna Lesson 2 grade 7B

Word problems

The teacher mostly used **IOC** in this lesson. Firstly, this was done to test their prior knowledge and to see if they can use what they learned (the four basic operations) to solve word problems. The teacher said that "*word problem solving only uses addition, subtraction, multiplication, and division and they learnt how to add, subtract, divide and multiplying fractions.....it's better for me to see how well they can use that knowledge to solve problems instead of me telling them what to do"*.

Multiple representation connections which were used by the teacher aimed at helping learners to make sense of the problem so that they could solve it better. The teacher used a circle to represent a pizza. The teacher noticed that most of the learners were struggling to solve the problem and therefore using some drawings would help in their thinking and help them to be more focused on what they needed to do. The teacher pointed out that *"diagrams helps learners to understand the problem better as it gives a better connection between what is in their minds and what they are reading"*.

The teacher used implication connections whereby she talked of LCM in adding fractions. She experienced that some learners believed that they can only use LCM when they are simplifying fractions not presented in word problems. She observed that *"if you gave a leaner* $\frac{2}{3}+\frac{4}{5}$ *for example, that leaner will think of the LCM, but if you gave that in a word problem, that learner will struggle a lot"*. That means they isolate procedures learned in one concept and do not use them to solve another concept. It was therefore important for them to be shown that the procedure for adding fractions applies to any problem that involves adding fractions whether presented in a word problem or not.

The relationship was formed when the teacher said that “$\frac{1}{2} \times \frac{800}{1}$ *is the same as* $\frac{800}{2}$.″ This connection made by the teacher served the purpose of helping learners to see the connection between different methods yielding the same answer. She observed that *“most of the learners do not know that a half is something divided into two parts. Of course they know it like if you give them an apple and ask them to divide it in halves they will do it. But when it comes to asking them why* $\frac{1}{2} \times \frac{800}{1}$ *is the same as* $\frac{800}{2}$*, that learner won’t tell you. You saw that the leaner said he divided by 2 because it won’t leave a remainder, so I think that connection is really important”*. The general statement of saying either method will give the correct answer will help learners to see the connection between the methods, and a good explanation will help them know why and how that can be possible.

4.3.1.5 Teacher Bella Lesson 1 Grade 7C

Addition of fractions

The teacher used **MR** connections when she used two different methods of adding fractions. This was done for the learners to see the main reason for using the LCD when adding fractions with different denominators. She said that “*we were only taught to use the LCD when adding fractions with different denominators. Until I went to the teacher’s college I didn’t know why. So presenting this using the diagrams can really help learners at least to know why, and they can hardly forget that because they saw it*”.

The teacher also observed that the reason why learners add denominators together is because they do not understand why denominators cannot be added either. She said that *“most of the learners do not know that a denominator represent a whole and so you cannot add them together; I think with diagrams this will help them not to add denominators anymore”*. Ms. Bella concluded that diagrams help learners to easily understand the method.

Using the LCD is also an implication connection because adding fractions depends on the knowledge of LCD. To the question as to why LCD is crucial in fractions and why she had to remind learners of the LCD, the teacher said that *"learners need to know that what they learned previously or in previous grades does not end there; it can be used in future, in other topics in Mathematics or in other grades. Reminding them of that will help them not just to study for the sake of passing but to be aware that they are learning something for life*".

The teacher used the procedure for adding fractions, which is a form of procedural connection. According to the teacher, this connection is very important and was done to enable the learners to work with ease in the examination or in tests. She believed that, even if learners can use either method, knowing the procedures helps them simplify the problem quicker than using the diagrams, which are quite time consuming. She further noted that *"knowing the procedures is kind of a backbone to the knowledge of other concepts like learners cannot find the LCM if they do not understand it. That means if they want to know how to add fractions with different denominators, then they must know how to find the LCD first*". The knowledge of the procedures will help in developing knowledge of other concepts.

The Part-whole relationship connection in the lesson was demonstrated when the teacher generalized that "you cannot add fractions with different denominators". The teacher made this statement because she believed that learners would always remember it whenever they were adding fractions, whether they were doing it to solve a word problem or just simplifying fractions. A generalization helps to refine the procedures. Ms. Bella said that *"if learners know that they cannot add fractions with different denominators, there is no way they will try that mistake unless they are ignoring it so they will be forced to know what to do".*

The teacher also made use of an IOC connection when she first asked one leaner to work out the problem. This was done with the purpose of testing the learners to see if they can still remember previous work done in previous grades. She experienced that some of the learners did not know that topics in Mathematics build on each other "*they believe that* *what they learned in grade 1 is just for grade 1 and what they learnt in grade 6 ends in grade 6 so asking them will maybe help them think that is not true".*

Another IOC connection was made when the teacher asked the learners the meaning of a numerator and the denominator. Ms. Bella said that she made that connection because learners cannot add fractions if they do not understand the meaning of a denominator and a numerator. She believed *that "knowing what the denominator is will prevent learners to add them together".*

4.3.2 Focus group interview

A focus group interview was conducted with two teachers in order to get their perceptions about making connections in teaching fractions and the importance of making such mathematical connections. This type of interview answered the second research question of my study. Their responses are presented in themes that emerged.

a) Learners' view of fractions and connections

This theme provides insight into how learners react to fractions. The teachers shared mixed opinions according to their experiences when teaching fractions. One teacher agreed that most of the learners view fractions as a difficult topic because they could not link them to real life. The teacher claimed: *"I think there is a lack of connection between what they learn at school and what happens in real life, so they take it as a topic they learn in maths without linking it so they actually do fractions without knowing that they are doing fractions".*

On the other hand, Mr. Jacob suggested that his learners reacted quite well to fractions because of the link that was created between fractions, decimals and percentages. He affirmed that his learners know that even if the problem was given in a common fraction and were told to give the answer in decimals, it meant the same thing; for example 0.5 is just a half or 50% and that meant one and the same thing to them.

b) Connections and learners' interest in school

The teachers emphasized the importance of making connections as it helps learners to be more interested in school and value mathematics specifically. Mr. Jacobs believed that *"connections should be a daily routine, it should be used frequently in order to influence learners' beliefs of the values of Mathematics in daily lives and to other subjects, we can even ask questions that will force and encourage our learners to connect Mathematics to the real world"*. The teachers further believed that when teachers are using real examples from learners experiences and backgrounds, from situation learners know and hear about, they will recognize that what they are learning at school puts value on what they really do and that they can use the knowledge to solve real life problems.

According to the teacher, learners need to apply the mathematics they are learning separately from the problems used in the textbooks. Mr. Jacobs said that *"learners should not see that the problems in textbooks are there for the purpose of the subject but they need to see how those problems are connected to real life, how they relate to them and to know that they are there for them to see how they can use the subject in reality*". They believed that teaching should therefore aim at promoting mathematical connections and take into account learners experiences and interests. Ms. Brianna said that *"teaching without connection to real life is a waste of time because there is no way learners will appreciate why they are in school and maybe one of the reasons why learners drop out is because they think school is of no use to them, because they don't learn anything related to the reality of life".*

According to the teachers learners find it difficult to make connections by themselves. They say that if teachers are not there to show them the connection between the school content and what is happening in the world, the learners will find it difficult to find out the relationship between the subject and the role it plays in reality. They will eventually consider mathematics as a subject they are learning in vain.

c) Teaching approaches and strategies for connections

From her experience as a learner, Ms. Brianna observed how a poor sequence of teaching fractions can result in poor understanding of fractions. *She said that "you will only be told that* *this is a numerator and this is a denominator without knowing why".* Mr. Jacobs added that *"we were not taught how fractions are used in real life".* This results in poor connections between mathematics and real life.

In order to combat this to create a room for connections, the participants thought that using a proper teaching strategy might help in making some connections. The old teaching method was teacher-centred. The participants were of the opinion that bringing reality into the classroom could help in making connections. Using concrete materials, drawings, different methods of solving a problem, quizzes, dramas, games and asking for their prior knowledge are some of the approaches teachers can use to make connections.

Ms. Brianna suggested that allowing learners to talk about fractions in the classroom can indeed help them understand mathematics better. A constructive talk on where fractions are used in real life can help learners see the connection between school and home. Mr. Jacob added that it could be an interesting experience if the lesson about fractions was taken outside the classroom so that learners could see fractions in nature.

d) Developing fractions sense to make connections

The teachers said that it is quite obvious that most of us do fractions without recognizing it. It is therefore very important that teachers develop fraction sense in the learners by asking them questions that force them to make connections. Ms. Brianna thought that it is very necessary to make learners aware of how their action might involve fractions and how they are involved by asking them questions that force them to think of how fractions are part of their daily activities. Mr. Jacob suggested that it could be helpful if learners are forced to think and recognize fractions. As he put it: *"you can also ask them in the class like what did you do at home that involves fraction? From their answers, there might be other learners who did the same thing but did not recognize that there are fractions there, so from that conversation they will learn of different activities that involve fractions and why and when they go home they will have that knowledge. You can also like ask them to record things they did at home that involved fractions maybe for a week and at the end they will then be able to see how fractions is part of daily activities and how important they are*". Further to this, Brianna added that "*you can also ask them to look around and see if they can spot out things that are either written in fractions or* *involving fractions. You might find one saying like I went to the shop and I found one packet written 20% that can also be fraction* $\frac{20}{100}$.

e) Resources and connection making

Participants believed that the lack of resources might be a stumbling block for teachers to make connections. Some of the topics in fractions might be difficult to connect and therefore limit connections making. Most of the teachers depend on textbooks and most of these do not provide ways of making connections. Ms. Brianna observed how learners' questions of where the concepts they are learning are used in reality became a habit. She noted that, *"if learners found that the Math you are teaching them is difficult, they will ask you, "Ms., where are we going to use this in real life"? So sometimes I don't know also and if there is nothing in the textbook I can't answer that question right away".*

Most of the schools are not connected to the internet. Mr. Jacobs emphasized the advantage of the internet as a helpful tool in finding ideas on making connections. He said that *"you can get very good ideas from internet. You can even get a lesson plan that you can successfully use. Sometimes if you don't know the real application of a certain topic, you can get it from internet, now if there if no internet, you cannot answer some of your learners' questions"*. This makes it difficult for the teachers to make meaningful connections.

4.4 SYNTHESIS OF THE RESEARCH FINDINGS

4.4.1 Observation

Different teachers used different connections in their teaching, as reflected in the tables. All the different forms of connections, as per Businkas' Framework, were recorded. I, however, observed that some connections were not clearly made. Looking at the frequencies of occurrence on the nature of connections, which were under observation, the study displayed that the Instruction-Oriented Connections (IOC) was used most frequently. This may, however, differ from teacher to teacher and from lesson to lesson.

It was observed, almost in every lesson that teachers asked for the learners' ideas first before they gave their contributions. They asked the learners if they could still remember this or that from previous lessons or from previous grades. I have also observed how the nature of the topic may influence the ability of the teacher to make connections and the nature of the connections to be made. This is an indication that some of the topics can be connected much easier than others and the nature of connections may depend on the topic itself.

Multiple representations in the lesson were mainly made between different symbols or different forms of writing the same number. For example $\frac{12}{12}$ is the same as 1, as a common fraction or as a whole number displaying the same number. The diagrams were not commonly used even in circumstances where they were supposed to be used for the conceptual development of the learners.

The procedural connections were mostly done on the algorithms and little focus was made between different procedures that can be used to work out the problem. This is a signal that the participating teachers were aiming at developing procedural fluency of the learners. The results also indicated how the teachers tried to connect different concepts by making implication connections. It was observed however, that connections to other subjects were not done. This can potentially influence learners to treat subjects differently because of the lack of this type of connection.

The least-used connection was part-whole relationships that have to do with generalizations. Teachers were clear on what to do and not to do in fractions. Some of the strong generalizations recorded were:

- All the whole numbers are written as a fraction over 1.
- If the parts are not divided into three equal parts, then you are not talking of a third.
- You cannot add fractions with different denominators.

Apart from the main purpose of the project, it was interesting to discover that mathematics teachers' ways of interacting with the learners are quite similar. I observed that they question, get the answer and repeat the answer, ask the whole class to repeat the answer and say it again.

1. Ask the question
2. Get the feedback from the learners

3. Repeat the feedback
4. Ask the whole class to repeat it
5. Write it down or repeat it again.

This form of interaction keeps order in the class and I observed how learners' attention was captured.

4.4.2 Stimulated video recall interview

The participating teachers agreed that they used IOC most often in their lessons. This is because they saw it as important to remind the learners what they learnt previously and how that knowledge can be used to learn a new topic. This also helps to connect different domains of mathematics together and give some insights to the learners that they should not treat topics in mathematics separate but as interrelated groups of topics building on each other in such a way that the knowledge of the first concepts helps in the understanding of the next.

Multiple representation connections were also used often by the teachers. Different forms of representing a fraction as well as to make a connection between a symbolic and a pictorial representation of a fraction were mostly used under this category. Teachers who used pictures in their lessons agreed that learners at that stage learn better by visualizing, which is why using the diagrams can be an effective way of explaining the concept and thus facilitating learners' understanding.

Although they were not aware of other forms of connections besides the multiple representations and the connections to prior knowledge, their responses recognized the value of generalizing as it makes learners aware of what to do and satisfies the need to know the procedures for easy computation in tests or examinations.

4.4.3 Focus group interview

The frequency of IOC in the lesson observed aligns well with what the participating teachers said in the stimulated video recall interview and in the focus group interview. The teachers believed that a connection to prior knowledge is very important in developing learners' understanding since it puts mathematics in context.

The teachers agreed that connections are indeed important as they act as a bridge between school and home experiences. They felt, however that they knew little about other forms of connections and suggested the need for workshops where the issue of connections could be explicitly discussed.

They agreed that some topics are really difficult to connect and lack of resources could even make the situation worse. Teachers' own backgrounds - how they were taught or how they were trained - influence their abilities to make connections. Textbooks give little or nothing at all about connections and therefore the internet can facilitate this. Not all the schools however are connected to the internet, making it even harder for the teachers to make connections and for the learners to value Mathematics.

Apart from the lack of resources, talking about fractions in the classroom and taking some lessons outside might help in making connections. Teachers should not be limited to what they have at hand. Creating fractions sense by asking learners to record their daily activities where fractions are involved can indeed help them value mathematics and fractions.

4.5 CONCLUSION

The chapter discussed the findings of the research project. It highlighted the nature of connections made by selected teachers and their perceptions about making connections in teaching fractions. The results indicated that IOC is the connection used most frequently because linking to prior knowledge is a very powerful tool in knowledge acquisition.

CHAPTER 5

CONCLUSION

5.1 INTRODUCTION

This chapter presents a summary of the research findings for the study and its limitations. It also suggests some recommendations that could be implemented in the teaching of fractions. These recommendations can potentially contribute to the effective teaching and learning of fractions, especially in making sense of fractions and making meaningful connections. The chapter also suggests possible areas for further study. It ends with some critical reflections.

5.2 SUMMARY OF THE FINDINGS

The study focused on the nature of mathematical connections selected Grade 7 mathematics teachers make in their teaching of fractions. It also looked at the teachers' perceptions on the importance of making connections. This study was driven by my own experience of being a learner and a teacher. It was inspired by the session we had about mathematical connections and was motivated by a research study conducted by Businkas and Mhlolo. All this prompted me to look into the classrooms regarding the nature of connections teachers make and to discover their opinions on the importance of making such connections.

This study was a case study and a qualitative method was used to generate data from observations and interviews. Convenient sampling was used and ethical issues were taken into consideration throughout the study.

Initially, the teachers had little knowledge of the different connections that could be made. It was quite a learning opportunity for them when I explained the different types of connections that can be made. The connection that they were very familiar with was the connection to prior knowledge and the connection to real life situations. They admitted that the other types of connections as per Businkas' framework, such as procedural and implication connections, were done unconsciously. They said that they had no idea that they were making these connections.

It was found that the selected teachers did make connections in their teaching of fractions. All the different types of connection as identified by Businkas were recorded during lesson observations. The IOC was the dominant connection that was used most often. Teachers believed that it is by using and recognizing learners' prior knowledge that learners become interested and engage in the subject. It is then that learners are able to make a connection between what they learnt and what they needed to learn. The teachers further believed that connecting to the learners' prior knowledge would assist the learners in making sense of the subject. It is then that their existing ideas are connected to what they have learnt at school.

This finding relates well to constructivism - the theory that underpins this study. It is well documented that constructivism is a powerful learning theory that takes into account learners' existing knowledge to build new concepts. It is not surprising; therefore to see that the connection that was most frequently used (IOC) has to do with existing knowledge of the learners. Looking at this, a logical conclusion can be made that the participating teachers aligned well with constructivist practices.

The study further found that Multiple Representations connections were also often used. Teachers believed that representing a concept in different forms helps with the conceptual development of fractions as well as enabling greater understanding and learning of fractions. The use of diagrams was often employed and teachers believed that this facilitated learning. The use of diagrams helped learners to see **why** things are done rather than merely seeing **how** they are done.

Teachers believed that making connections is indeed important. This is because it helps the learners to value mathematics in particular and education in general. It was found from the teacher perceptions that if learners are made aware of the real uses of mathematics through meaningful connections, they will value its importance instead of seeing it as a difficult subject. They will appreciate its significance.

The study findings therefore provided some insightful answers to my research questions. As noted earlier, teachers are trying their level best to make connections. Although they are doing some of the connections unconsciously, lack of knowledge, lack of teaching and learning resources, and lack of connections orientation hinders them to do this optimally. This should motivate teacher educators, teacher training institutions, authors of textbooks, and workshop facilitators to design appropriate solutions so that teaching with meaningful connections can be enhanced.

5.3 RECOMMENDATIONS

The following recommendations resulted from the research findings:

It is important that meaningful workshops are held where teachers can share their experiences with making connections in the teaching of fractions.

It is important that teachers make learners aware of the different connections they are making. Informing learners of the different connections made in the lesson will eventually help them to make sense of the mathematics they are learning.

This study was conducted in town schools, using only three teachers. It is recommended that similar research is conducted in different geographical settings. It is further recommended that a bigger sample be used for similar research.

5.4 LIMITATIONS OF THE STUDY

The study was carried out in only three schools with only three teachers. The focus was only on teachers and the unit of analysis was specifically on their teaching practices. Only the voices and actions of these teachers were considered for the analysis. As a result, the research findings cannot be generalized. It would have been helpful if the voices of learners could be heard as well. This would have widened the perceptions of the importance of using connections.

Teachers were not aware of other different types of connections until they were revealed in the joint video analysis. It would have thus been helpful to conduct a workshop before the lesson observation process.

5.5 AREAS FOR FUTURE RESEARCH

The research articulated some of the possible connections that can be made in teaching practice. As mentioned above, similar research should be conducted with a bigger sample. Also, different locations would possibly yield different and insightful results.

I already indicated that the insights of learners would add to a new research design.

Further research could be done in the form of an intervention programme, whereby different types of connections would be introduced to the teachers and the teachers would then practice and present these.

The participants have a good knowledge about some of the common connections. Their level of putting these connections in practice is, however, not very substantial. There is thus potential for further research on the structure of the curriculum and how teacher training institutions should facilitate the development of these types of connections.

5.6 REFLECTION

The journey I went through to make this research project materialize magnified my view of mathematics education. The study enabled me to develop new insights about knowledge acquisition, teaching and learning. When I was working through the different literature and texts, I learned that there are many complex and intertwined factors involved in teaching mathematics. I learnt that a solid orientation on what comprises mathematics education needs to be employed for effective teaching and learning.

If I were to conduct this research again, I would be more conscious of the development of my observation schedule that I can use comfortably. I would also pilot the video recall interviews so that I would be more aware of possible follow-up questions. I discovered that sometimes I follow up with wrong questions that had nothing to do with answering my research questions.

Overall, I found my research journey very enriching. It has made me more aware of connections and it has enhanced my own practice in ensuring that I make appropriate mathematical connections for my learners.

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APPENDICES **APPENDIX A**

RHODES UNIVERSITY
Grahamstown • 6140 • South Africa

January 2014

The Inspector of Education

Ompundja Circuit

Oshakati

Request for permission to conduct research in schools

Dear Madam

My name is Loide Amupolo and I am a postgraduate student at Rhodes University in Grahamstown in South Africa. I wish to conduct a research study for my Masters' dissertation that involves investigating the nature of mathematical connections that Grade 7 teachers make in their teaching of Fractions. The need for this study is coupled with the mathematics aims and objectives stipulated in the subject policy and the syllabus informing teachers of the need to make connections so as to make Mathematics relevant and worthwhile to the learners.

I am hereby seeking your consent to approach 3 schools in the region to secure 3 participants for this study. This project will be conducted under the supervision of Professor Marc Schafer from Rhodes University and it will take a period of one year to complete.

I wish to observe each of the three teachers twice during the course of the 1st term in the specific topic mentioned above. I also wish to interview them after the videotaping to obtain their views of the mathematical connections that they made in their teaching. The participants and the schools will be protected by anonymity, confidentiality and minimal disturbance.

I will be grateful if you give me consent by signing the attached form. If you require any further information, please do not hesitate to contact me at 081 200 9524 or 065-231844 or loidemwetu@gmail.com.

Thank you for your consideration on this matter. I will be grateful if you authorize me to do the project by signing the attached form.

Yours Sincerely

Loide Amupol

APPENDIX B

RHODES UNIVERSITY
Grahamstown • 6140 • South Africa

January 2014

The School principal

…………………….School

Request for permission to conduct research at your schools

Dear Headmaster

My name is Loide Amupolo and I am a postgraduate student at Rhodes University in Grahamstown in South Africa. I wish to conduct a research study for my Masters' dissertation that involves investigating the nature of mathematical connections that Grade 7 teachers make in their teaching of Fractions. The need for this study is coupled with the mathematics aims and objectives stipulated in the subject policy and the syllabus informing teachers of the need to make connections so as to make Mathematics relevant and worthwhile to the learners.

I am hereby seeking your consent to approach your school to secure one participants; a grade 7 mathematics teacher for this study. This project will be conducted under the supervision of Professor Marc Schafer from Rhodes University and it will take a period of one year to complete.

I wish to observe each of the three teachers twice. I also wish to interview them after the videotaping to obtain their views of the mathematical connections that they made in their teaching. The participants and the schools will be protected by anonymity, confidentiality.

I will be grateful if you give me consent by signing the attached form. If you require any further information, please do not hesitate to contact me at 081 200 9524 or 065-231844 or loidemwetu@gmail.com.

Thank you for your consideration on this matter. I will be grateful if you allow me to do a research at your school by signing the attached form.

Yours Sincerely

Loide Amupolo

APPENDIX C

RHODES UNIVERSITY
Grahamstown • 6140 • South Africa

Dear colleague

My name is Loide Amupolo and I am a postgraduate student at Rhodes University in South Africa. I would like to invite you to take part in the research study. I wish to conduct a research for my dissertation that involves investigating the **nature of mathematical connections that teachers make in their teaching of Fractions**. Connections are also referred to as links; relations; applications; uses of mathematics in real life that can be used to make Mathematics relevant and worthwhile to the learners I would like to learn from you on how you make connections consciously or unconsciously in your teaching of Mathematics specifically in Fractions. This study may reveal valued matters that may change our perceptions of teaching and learning of Mathematics.

I am hereby seeking your consent as a mathematics teacher to be one of the participants for this study. The study will comprise of two methods of which you will be observed twice when you are teaching fractions; and interviewed once through a stimulus-recall video interview. Both the observation and the interview will be video/audio recorded subjected to your permission. The observation will be made during normal school lessons; and the interview will be done after school. I would also like you to join me for a focus group interview that will be done after observation.

Please note that this study is not an evaluation of your teaching but merely looking at different mathematical connections you make. Feel free to participate as the school and you as a participant will be protected by strict anonymity and confidentiality because the name of the school or you as a participant will NOT be revealed in the research report.

If you would like more information about the study, do not hesitate to contact me at 081 200 9524 or 065-231844 or loidemwetu@gmail.com

Thank you for your consideration on this matter and thank you for taking part in the study. I will be grateful if you accept to participate in this project by signing the attached form.

Yours sincerely

____

Loide Amupolo

APPENDIX D

Consent form (for the inspector and the principals)

I …………………………..……in my capacity as a……………………………… hereby giving consent to Ms. Loide Amupolo in her capacity as a Masters student at Rhodes University to conduct a research.

Signed: ………………………………………………………………..

Date: ……………………………………………………………….

APPENDIX E

Consent form

I ……………………….……in my capacity as a teacher at ……………………………… hereby accepting to participate in the research project that will be conducted by Ms. Loide Amupolo in her capacity as a Masters student at Rhodes University.

Signed: ………………………………………………………………..

Date: ……………………………………………………………….

APPENDIX F

Lesson Observation Sheet

Lesson: ….. **Teacher's name:** ………………………….…… **Grade:** …..

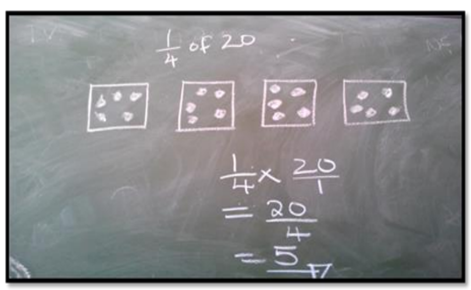
Topic: ……………………………………………………………………..

Time	Code	Description of a connection	Justification
03:00			
06:00			
09:00			
12:00			
15:00			
18:00			
21:00			
24:00			
27:00			
30:00			
33:00			

Lesson Observation **APPENDIX G**

Lesson 2 Teacher's name: **Jacob** Grade: **7B**

Topic: Multiplying fractions of quantity.

Time	Code	Description of a connection	Justification
03:00	**MR/IOC**	*let me say you have 20 sweets and then I said give me $\frac{1}{4}$ of the sweets that you have, how many sweets are you going to give me*	sweets used are related to the social context of the learners help them to make sense of the problem
	IM	*how did you get that answer? Why did you divide with 4, why not with 3 or 5?*	seeks reasoning and an explanation from the learner
	IOC	*what did we say about a quarter? What is a quarter of something*	aware of the learners' prior knowledge
	MR		different mode (diagram was used)
	PWR		specific from a particular
06:00	**PWR**	*every whole number is written over 1*	Generalized
	P	this is a calculation for the diagram here	Using the steps
	MR	*imagine you have 3 breads and out of those three breads you gave away $\frac{2}{7}$ how will you know how many you gave away*	an image representation or an analogy of a multiple representation.
09:00			
12:00	**P**	Explanations in a procedural way.	Emphasized on the steps.

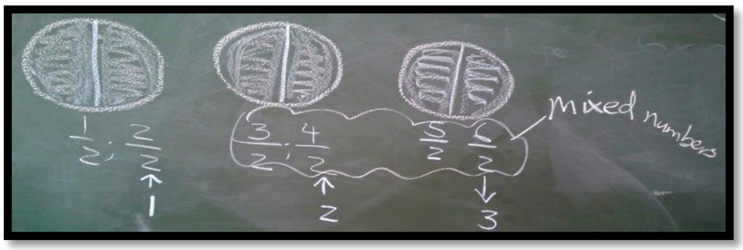
Lesson Observation **APPENDIX H**

Lesson 1 Teacher's name: **Brianna** Grade: **7B**

Topic: Fractions as a part of a whole.

Time	Code	Description of a connection	Justification
03:00	**IOC** **MR**	Draw 5 circles and divide them in halves, thirds , quarters, fifth "*Imagine you are three of you and you are so hungry and you only have one apple that you have to divide among the three of you right. So make sure you divide it into three equal parts because the moment you give someone a smaller piece that one will be angry with you so makes sure that u divides it into 3 equal parts.*	knows their backgrounds (they learned fractions in previous grades) placed Mathematics in a social context that is familiar to the learners
06:00			
09:00	**IOC** **IOC**	*imagine you are 4 of you and you need to divide this apple among the four of you and it should be the same parts…. 4 equal parts otherwise someone will want to fight* *how do we call something when we divided it into two equal parts? In fractions, how do we call that? You learnt these fractions already, is it not so"*	placed Mathematics in a social context that is familiar to the learners aware of the learners prior knowledge.
12:00	**MR/ IOC**	*let's think this as if it is an apple (pointing to the circle). You are 3 of you and u need to share this apple. Remember if you share this apple and give someone this part (pointing the smaller part), eeee that person will be very angry. Now, is this a third? Learners said no! No its not! So you need to divide it into 3* ***equal*** *parts for you to call it a third*	linking a circle to an apple connects her teaching to learners' ways of sense making by showing them why it is not a

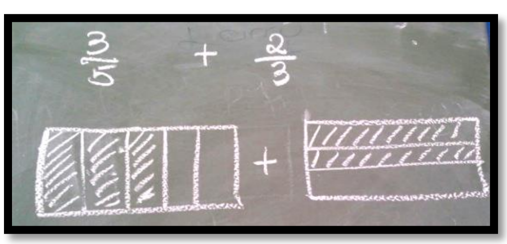
	IM MR	 ***if*** *you just divide something into three parts which are not equal* ***then*** *we are not talking about a third we are talking about something different* *now can I have 6 people…….who can come and divide this people group into thirds*	third and when to call a piece a third an if-then connection concrete examples used to represent a third
15:00	**IOC**	*"How do we call something when it is divided into four equal parts? How do we call something when it is divided into five equal parts? How do we call a circle that I did not divide at all*	Testing learners prior knowledge
18:00	**MR** **IOC**	 *If I shade this one and they ask you to identify the fraction which is shaded. First of all I divided this one into how many parts? (Writing 2 below the division line). How many parts are shaded? (writing 1 in top of a division line) how do we call the number on top? …… and how do we call the bottom one.*	a diagram to illustrate a half and also to show where the numerator and a denominator comes in using the diagram asking for the learners prior knowledge
21:00	**PWR** **IOC**	"*who can tell us, what does this numerator tells us and what does this denominator tells us? We have an example here……where did we get this one from?* (Showing 2 on the chalkboard) *we got it from here* (showing the two parts on the circle). *Is it not so? This* (showing 1) *is this one (showing the part shaded) so what does the numerator and the denominator tells us*	she used a practical example (a diagram) for reasoning connect her teaching to a more sensible way
24:00	**MR**	*now in whole numbers we learnt that we can count in ones like 1, 2, 3, 4, 5, 6 or count in twos like 2, 4, 6, 8, 10…. In fractions for example we can count in halves. This one is one half. Is it not so? Yes. The second one will be what? 2 half; 3 half, 4 half, 5 half, 6 half*	connects this to other domains of Mathematics. In this case she used whole numbers and connects them

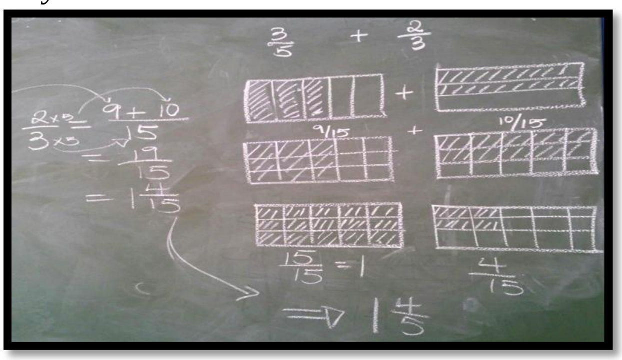
	MR	*now, we know that this is this one (pointing a half and a circle on the chalkboard previously drawn). This one will be then that (pointing $\frac{2}{2}$ and a circle that is divided into equal parts with all equal parts shaded), which means it a …a whole. 2 divided by 2 is what? Its 1. A 1 that we say it's a whole. How many parts are shaded? 2, so meaning that it will be a whole.* 	to fractions the teacher used diagrams to show to the learners the connection between the pictorial representation and symbolic representation of $\frac{1}{2}$ and $\frac{2}{2}$ and others
	MR	*I am sure that some of you were taught how to change improper fractions to mixed numbers. Now how do we use diagrams to show this mixed number (pointing $\frac{3}{2}$ in the list)? We will then say is 1 whole (drawing a circle divided it into two equal parts and shade them) and here is a half" (drawing a circle and shaded one part). "Let me ask you. 6 divided by 2 what is the answer? 3 so it will be 1 2 3 (she drew 3 circles divided them into halves) that's what it means. Let's count. How many halves? 1 half, 2 half. 3 half 4 half, 5 half 6 half. Meaning we have 3*	the teacher used diagrams to show where three in $\frac{6}{2}$ came from
27:00	IM	***if** we divided a whole into equal parts, **then** is when we talk of fractions; a part of a whole*	generalization of an if-then connection
	MR	*Sometimes you bought a chocolate and it is having 18 bars and you are two of you; you need to divide that one into 2 equal parts*	placed Maths in a social context where an analogy of MR is used to show learners how they use fractions in real life.

Lesson Observation **APPENDIX I**

Lesson 1 Teacher's name: **Bella** Grade: **7C**

Topic: Fractions as a part of a whole.

Time	Code	Description of a connection	Justification
03:00	**IOC**	Who can work out $\frac{3}{5}+\frac{2}{3}$	Getting learners' ideas to work out the problem.
06:00	**IM** **PWR** **P** **MR**	*we need to find the lowest common multiple of the denominators first* *you cannot add fractions with different denominators* *First of all you find the LCM, add the numerators and keep the denominator unchanged.* *that is not the only method that we can use, that was only a calculation we can also do that in practical meaning with diagrams*	adding fractions depends on the concept of LCM a generalization that applies in all cases when adding fractions teacher focused on the teaching of the prescribed steps teacher's ability of recognizing that there are multiple ways to answer the question-
09:00	**IOC** **MR** **PWR/ IM**	*What is the meaning of a denominator and a numerator?*  *Now we want to add those pieces together by filling the unshaded parts in this shape with the shaded parts from this diagram. But we cannot*	Aware of the learner's prior knowledge. Different representations in symbolic and a pictorial forms connect the general to

		just take one piece from $\frac{2}{3}$ and fit them here, it won't fit because as you can see $\frac{1}{3}$ here is bigger than $\frac{1}{5}$ because the shapes are not divided in the same parts. This one is divided in 5 and this one in 3. So to add these pieces together you have to divide the shapes in the same pieces and that is how the LCM comes in	the specific using the diagram and to show how the concept of using the LCM came about in adding fractions showed the link between what implies the LCM in adding fractions by drawing conclusions from the example
12:00	**P**	*So you can*  *either use this one or this one. You can use the calculation or the diagram. They will give you the same answer.*	does not dictate learners to which method they can use

APPENDIX J

VIDEO ANALYSIS RECALL INTERVIEW:

MR. Jacob; Lesson 1:

Ordering of fractions, multiplying a fraction with another fraction, and multiplying fractions with whole numbers.

Thank you very much sir for sparring your time so that we can look at this lesson together. The main aim is just to get more insight on what was really happening in your class, specifically on the nature of connection. We can start now;

R: first of all as a Math teacher, why should learners learn Mathematics?

J: if you just look around in this classroom, Math is the building plan. Learners need to learn maths so that they will make sense of the word. How will you imagine a classroom without corners?

R: are you saying that maths is all around us?

J: sure. Aaaaa….mathematics is the subject that comprises of many things which are also part of other subject. Learning maths in school will help learner make sense of other subjects because maths is part of every subject.

R: What do you mean by that? What do you mean by saying math is part of every subject?

J: you know maths is composed of numbers. Anything that use numbers is doing maths. I'm not sure in all the subjects but like in natural science, they talk of the distance between the earth and the sun, in social studies they have directions, the angle between north and east is 90 degrees. Like our learners are doing design and technology, they use to construct many things and they need to measure.

R: let's focus specifically on your lesson, why should learners learn fractions?

J: fractions are part and parcel of everyday life. I don't think any person could skip a day without doing fractions. On a daily basis, we share, divide, measure and so on. So learners need to learn fractions so tat they will make sense of their lives. Fractions make them to learn practical things.

R: can you give me some practical examples of decimal fractions.

J: a simple example is money. N$ 12.45 is a decimal and learners need to know why that's why they have to learn fractions.

R: in the lesson you asked learner to convert fractions in descending order. U gave a mixture of fractions, what was the reason behind?

J: a fraction can be a common fraction, decimal, mixed, proper, improper etcetera. I want learners to see that there are many ways of writing fractions and a single number can be written in different forms. For example 5 and $\frac{5}{1}$ is one number but one is a fraction and the other is a whole number. I gave them a mixture of number so that they can know different forms of writing numbers.

R: what type of connection do you think you made there?

J: mmmmmm. I think aaaa is this one Multiple representation because it have two forms of writing a number.

R: Here you asked learners to convert all the fractions to decimal fractions, why?

M: It is quite difficult for the learners to tell which common fraction is big or small if they are not either written with the same denominator or with the same numerator. The reason for converting all the fractions to decimal fractions is to make it easier for them to convert and also to be able for them to see that there are different ways of writing one number. You know that $\frac{2}{5} = 0{,}4 = 40\%$. this means if I gave one learner $\frac{2}{5}$ of 10 sweets and the other 0, 4 of 10 sweets and the other 40% of 10 sweets, they all received the same number of sweets. Learners need to know that there are many ways of representing one number. Common and decimal fractions are two different mathematical notations but their results are the same. Fractions, decimals and percentages need to be taught well because they are important in different context. For example percentages are better to be used when you want to see the top learner.

R: when you are arranging the numbers you advised them to use original numbers and not the other way of writing it. Why did you do that if they are the same thing?

J: yes they are the same but they have to answer the question. They were given this numbers (pointing the first row) and not those ones (pointing the second row), that's why they have to use what was given to them. Although a decimal fraction is just another way of writing a certain number they need that procedure to be easier for them to arrange with ease.

R: where do ordering fractions used in real life? Why should you teach this?

J: In sport for instance, decimals are used to determine the winner. For example, if one runner recorded maybe 10: 12 another 10: 14 and other 9: 58, all this numbers are decimals, but who is the first, second and third? Of course you have to arrange those decimals in order to find out.

R: I observed that you asked learners if they have learned this and that in the previous grade or if they can remember. Why is that the case?

J: Mathematics is a subject that builds on each other. We can not really say we are learning a new thing but we are just building on what learners have learned. One reason is to asses their prior-knowledge so that we can build on what they know already. It is not good just to teach what you need to teach. It is good to know what they know so that we can identify specifically what they acquired and what skills and knowledge they gained.

R: what type of a connection do you think you made there?

J: I think that was IOC here. Is it? Yaaah, because it used prior knowledge.

R: That right here again, after you worked out the example, you wrote down the procedure for multiplying fractions, what was the main purpose behind that?

J: procedures are needed for the learners to be able to work out a mathematic problem quickly especially in the examination. You can teach this with diagrams or using a number line but this can be time consuming in the examination.

R: let us pause a bit. Now you said that diagrams can also be used to teach multiplication of fractions apart from just relying on procedures right?

M: yes. Yaah, there are many ways to teach the same idea in maths.

R: but y didn't you use it in your lesson?

J: Of course we would like to use them but sometimes you need a lot to cover for the term, they can be time consuming really and in the exam they do not usually ask learners to use diagrams or to represent their answers in different form but only to work out, so teaching how to do it is better for all.

R: is there any advantage of teaching in different ways like as you said, using diagrams and number lines?

J: yaah, yes a lot of them. Number one is that learners have different learning methods. Pictures can help them understand the concept better.

R: according to this, which connection you think you made here?

J: I think that could beeeeee implications? No, procedural. Nee?

R: yes, a learner in this example said 5 should be written as a fraction out of 100 and after a very good correction that you made here what connections you want your learners to see between 5 and $\frac{5}{1}$?

J: 5 and $\frac{5}{1}$ is just one number but one is a whole and the other is a common fraction but not $\frac{5}{100}$ because it will give you a different number totally.

R: A learner here mentioned a reciprocal and asked where they took if from. Why did you ask that question?

J: mhhhh… sometimes learners have a lot in mind but they don't know where a certain thing is applicable. Some learners search for information before they come in class and yet that information might not be clear to them, some learners have learned things in previous grades but that concept didn't develop well. I asked that question so that I can correct it and to update my self with learners' prior knowledge so that I can build on them when we reach that topic of reciprocals.

R: do you think you made any connection here?

J: ou, nooo,or let me see? Nooo

R: I think you did. By asking that questions you are referring to prior knowledge, which could be IOC.

J: Really I didn't know. So if I asked something related to what learners did in previous grades is IOC?

APPENDIX K

FOCUS GROUP INTERVIEW QUESTIONS

1. How do learners view or react to fractions? What do you think they have such a view?
2. How do you handle this view or reaction when teaching fractions?
3. What are the values of making fractions meaningful to the learners?
4. Some researchers suggested that one way of teaching effectively is through making meaningful connections. What can you say about this idea? (What are your opinions on the importance or advantages of making connections?)
5. What are the teaching strategies or methods that you use so that you can make connections (to link fractions to real life situations) give examples… especially in fractions?
6. What type of connections do you make when teaching fractions?
7. What resources does u use in order to teach with connections?
8. Learners are not empty vessels, what do you do to help them make sense of fraction?
9. It's believed that teachers make connections consciously or unconsciously, do you think learners are aware of the connections we are making? What should teachers do for the learners to recognise the connection?
10. One researcher concluded that "a connection is a unifying entity (bring together lots of things)" what can you say about that? Can you link this to fractions teaching?