Mathematics teachers' experiences of designing and implementing a Circle Geometry Teaching Programme using the van Hiele Phases of Instruction as a conceptual framework: A Namibian case study.

A thesis submitted in partial fulfillment of the requirements for the degree

of

Master of Education

(Mathematics Education)

of

Rhodes University

by

Beata Lididimikeni Dongwi

30 November 2012

DECLARATION OF ORIGINALITY

I **Beata Lididimikeni Dongwi(Student Number: 609D6388)** declare that this thesis ***Mathematics teachers' experiences of designing and implementing a Circle Geometry Teaching Programme using the van Hiele Phases of Instruction as a conceptual framework: A Namibian case study*** is my own work written in my own words. Where I have drawn on the words or ideas of others, these have been acknowledged using the reference practices according to the Rhodes University Education Department Guide to Referencing.

Beata L. Dongwi 30 November 2012

(Signature) (Date)

ABSTRACT

The aim of this case study was to examine, analyze and report on the findings of the experiences of selected mathematics teachers when they used the van Hiele phases of instruction in designing and implementing a Grade 11 circle geometry teaching programme.

The sample consisted of three selected mathematics teachers from the school where the researcher teaches. This school is located in the Oshikoto Education Region in Namibia. The school serves a multicultural group of 759 learners from a middle-class economic background. The site and participants were selected conveniently as the researcher had unrestricted access to both the facilities and the participants.

This research takes the form of a case study and is underpinned by the interpretive paradigm. Data for this research was collected using a variety of techniques such as interviews, classroom observation and document analysis. This facilitated easy triangulation of the data.

The findings of this research make four claims with regard to the experiences of the mathematics teachers with designing and implementing the circle geometry teaching programme using the five van Hiele phases of instruction as a conceptual framework. The findings revealed that firstly, all three participating mathematics teachers used and implemented **all** the five van Hiele phases of instruction in their lessons I observed. Secondly, the teachers navigated quite freely from one phase of instruction to the next, but also returned to the earlier phases for clarification and reinforcement in their teaching. Thirdly, the teachers saw the phases of instruction as a good pedagogical tool or template for planning and presenting lessons. Fourthly, the majority of the learners followed the instructions and seemed to obtain the answers faster than expected. The lesson presentations were lively and both teachers and learners communicated at length to discover angle properties of circles while developing and nurturing the technical language of geometry.

ACKNOWLEDGEMENTS

I would like to acknowledge my supervisor, Professor Marc Schafer for his supervision, inspiration and support throughout this study.

I would also like to acknowledge Mr. Robert Kraft for stepping in when I needed his input the most and for being so helpful.

Ms. Judy Cornwell for her golden touch in bringing my writing skills and referencing up to an acceptable standard.

I am very grateful to the school principal, the three mathematics teachers and learners for their participation and support.

I am also very grateful to Matron Naibes and Ms. Willemien Wannberg for their patience and kindness in dealing with the Rhodes University students.

I would like to thank my beloved mother Lavinia Ndaladipalekwa Dongwi, my two sisters Karina and Lavinia and my cousin Talohole for their help and support throughout my research journey.

Finally, I would like to thank the Almighty God for placing all these people in my path and for His guidance and abundant blessings.

DEDICATION

This thesis is dedicated to the late Dr. Humphrey Uyoyo Atebe for introducing me to the van Hiele theory. He opened doors for me that I never even knew existed. Look at me now; my research is unique in the Education Sector of the Republic of Namibia. I will be forever grateful; may your soul rest in eternal peace.

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ACRONYMS

DNEA	Directorate of National Examinations and Assessment
MoE	Ministry of Education
NSSC	Namibia Secondary School Certificate
SSP	Secondary School Phase
UPP	Upper Primary Phase
VHL3	Van Hiele Level 3

CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

In this chapter, I introduce the research project. The chapter begins by providing the background of the study and its context. I follow with an explanation of the rationale for the study. I then present the research question and the significance of the study as well as highlighting some limitations to the study. The chapter concludes with a brief overview of the study.

1.2 BACKGROUND TO AND CONTEXT OF THE STUDY

In my experience as a mathematics teacher, I have observed how many high school students struggle to recognize, make accurate constructions, accurately describe properties of plane geometry figures, and construct appropriate proofs in geometry. I noted with frustration how students often reached a dead end when it comes to solving geometrical problems. Some even believe that geometry is the most difficult aspect of mathematics which in turn creates a feeling of resentment towards geometry as a mathematical activity.

In my experience, mathematics teachers in Namibia highlight numerous factors that they believe contribute towards their students' poor performance and failure in geometry. These constraining factors include an inappropriate curriculum, weak textbooks, lack of teaching/learning aids and unmotivated pupils. Van Hiele's (1986) solution to overcoming these problems is for teachers to take responsibility for their own teaching and to make appropriate choices.

The difficulties that pupils experience with geometric conceptualization arise from various factors, but their inability to reason at a higher level of geometric thinking does not lie solely within their own learning ability or motivation. The teacher's instructions and choice of exercises also play an integral role in the pupils' learning. The renowned Dutch mathematics educator and researcher Van Hiele (1986) suggested that there are difficult moments that face every high school teacher in teaching geometry. He too encountered many moments where pupils failed to understand his teaching. According to van Hiele (1986, p. 45), "learning mathematics meant learning to think, and to be able to think precisely you should have attained the highest possible level". How then can mathematics teachers ensure that their pupils attain the highest possible level if the teachers themselves are not certain about what levels the pupils have achieved? While investigating this teaching challenge, I was particularly inspired by the van Hiele (1958, 1959, 1986, 1999) theory that sought to provide answers to the teaching and learning of geometry in high schools. "The van Hiele theory is a learning model that describes the geometric thinking students go through as they move from a holistic perception of geometric shapes to a refined understanding of geometric proof" (van Hiele, as cited in Genz, 2006, p. 4).

Many researchers (including Burger & Shaughnessy, 1986; Senk, 1983, 1989; Fuys, Geddes & Tishchler, 1984; Clement & Battista, 1992; Gutiérrez & Jaime, 1998; Serow, 2008; Atebe & Schafer, 2010) have based their van Hiele research specifically on the five levels of geometric thinking. My research however, will focus on the five van Hiele *phases of instruction* (van Hiele, 1986). My research is inspired by Serow's (2008), work that focused on the phases of instruction in the teaching of plane geometry. My research study was inspired by the fact that I have not encountered much research on deductive geometry that particularly engages the phases of instruction as a conceptual framework.

The van Hiele theory (1986) suggests that students progress through a hierarchy of five different levels of thinking as they learn geometry. Burger and Shaughnessy (1986, p. 31) named the five levels of thinking in geometry (from the lowest to highest level) visualization, analysis, abstraction, deduction and rigor. Burger and Shaughnessy (1986) explained that although high school geometry is taught in most high schools at a deductive level, most students are only capable of reasoning informally about geometric concepts when they enter high school. However, van Hiele (1959, as cited in Fuys, et al., 1984, p. 250) believes that the levels of geometric thought are inherent in the effective conceptualization of geometry; they are generally independent of the method of teaching used. He stresses however, that it is possible that certain methods of teaching do not permit the attainment of these higher levels, so that the methods of thought used at these higher levels remain inaccessible to the student. Furthermore, "each level has its own linguistic symbol and its own system of relations connecting these signs (van Hiele, as cited in Fuys, et al., 1984, p. 250) which need to be embraced by the learner in order to function effectively at each level.

Van Hiele (1986) thus recommends a set of phases of instruction that teachers should follow in order to facilitate the movement between the van Hiele levels of geometric thinking. Teachers are advised to guide their students into geometric conceptualization by employing the five phases of instruction in their practices. This would encourage pupils to proceed from lower to higher levels of thinking. A description of each phase of the five van Hiele phases of instruction is summarized in Table 1.1 below.

Table 1.1: Descriptions of the van Hiele Phases of Instruction

Phase	Description of phase focus
1. Information	Students become familiar with the working domain through **discussion and exploration**. Discussions take place between teacher and students that stress the content to be used. Through this discussion, the teacher discovers how students interpret the language and provides information to bring students to determined action and insight.
2. Directed Orientation	Students identify the focus of the topic through a series of **teacher-guided tasks**. At this stage, students are given the opportunity to exchange views. Through this exchange, there is a gradual implicit introduction of more formal language. The role of instruction during this phase is for students to be actively engaged in exploring objects to encounter the primary connections of the network of relations that is to be created.
3. Explicitation	Students become conscious of the new ideas and **express these in accepted mathematical language.** Care is taken to develop the technical language with understanding through the exchange of ideas.
4. Free Orientation	Students complete activities in which they are required to **find their own way** in the network of relations. The students are now familiar with the domain and ready to explore it more formally. Through their problem solving, the students' mathematical language develops further as they begin to identify cues to assist them.
5. Integration	Students **build an overview** of the material investigated. Summaries concern the new understandings of the concepts involved and incorporate the language of the new level. While the purpose of instruction is now clear to the students, there is less and less assistance from the teacher.

Adapted from Serow (2008, p. 446).

Groth (2005) summarized this as "the goal of most high school geometry courses is to have students reasoning at the deductive level by the end of the year" (p. 28). How do students develop such reasoning? Van Hiele (1999) believed that students' development into reasoning at the deductive level is "more dependent on instruction than on age or biological maturation and that certain types of instructional experiences can foster, or impede, development" (p. 311). Serow (2008, p. 446) thus asserted that the five phases of instruction "provides a structure on which to base a program of instruction". When developing the five phases of instruction, van Hiele (1959, as cited in Fuys, et al., p. 250),said that "the aim of the art of teaching is precisely to face questions of knowing how these phases are passed through, and how help can effectively be given to students."

The aim of my research study is to gain insight into the van Hiele phases of instruction by working with selected mathematics teachers and using the five phases of instruction to design, implement and reflect on a Grade 11 teaching programme that focuses specifically on circle geometry.

1.3 RATIONALE FOR AND PURPOSE OF THE STUDY

Geometry teaching and learning in Namibia and abroad has had its fair share of problems, and research provides abundant evidence to show that high school learners have difficulty with geometrical conceptualization. Studies including those by Burger and Shaughnessy (1986), De Villiers and Njisane's (1987), Senk (1989), Mateya (2008), Nikoloudakis (2009), Atebe and Schafer (2011) collectively, revealed that high school students who participated in their respective studies had a limited and notably inadequate knowledge of basic geometric terminology.

It is therefore the purpose of this study to look for ways to contribute to the improvement of geometric conceptualization among secondary school learners in Namibia. Hence, the goal of my research study is to design, implement and reflect on a Grade 11 'circle geometry teaching programme' based on the van Hiele phases of instruction.

1.4 RESEARCH QUESTION

The fundamental research question for this study is: What are the experiences of selected mathematics teachers when using the van Hiele phases of instruction in designing and implementing a Grade 11 circle geometry teaching programme?

1.5 SIGNIFICANCE OF THE STUDY

The van Hiele theory is not well researched in Namibia and where research has been done, it has focused on the van Hiele levels of geometric thinking. My study is thus unique as its primary focus is on the van Hiele phases of instruction. I not only diverted my focus away from the broadly researched van Hiele levels among high students, I involved teachers in their teaching practice and analyzed their experience in the design and implementation of a circle geometry teaching programme for Grade 11 mathematics classrooms in Namibia.

This study is significant because it opens doors for further research on the van Hiele phases of instruction in Namibia. The teaching programme designed in this study does not only apply to circle geometry alone. It can also be applied to the entire geometry curriculum, other topics in mathematics or even in other school subjects, to lead the learners to higher levels of thinking and conceptualization of the school curriculum at large.

1.6 LIMITATIONS

The following points reveal possible limitations of this study:

- The study is limited to only one school in the Oshikoto region in Namibia.
- Lack of sufficient van Hiele research in the country made it difficult for me to build on local research experiences and findings. This can however also be seen as a strength, since this study adds to the body of knowledge of geometry teaching and learning in Namibia.
- Over-commitment of the research participants hindered the researcher from collecting as much data as initially desired.

1.7 OVERVIEW OF THE STUDY

1.7.1 Chapter two

This chapter deals with the literature reviewed for this study. It starts with a description of geometry as it is generally viewed in the literature and discusses the problems with geometry globally and in the Namibian context.

The van Hiele theory is discussed with respect to the van Hiele levels of thinking and particularly to the five phases of instruction (information, directed orientation, explicitation, free orientation and integration). The implications for the phases of instruction for teaching are then discussed. It should be noted that the term "explicitation" is particular to the van Hiele theory, where it means to make the knowledge the learners gain explicit in the appropriate mathematical language.

1.7.2 Chapter three

This chapter presents and discusses the research methodology used in this study. It describes the research site, the selection of the participants, the orientation framing the study and the research methods used for this qualitative case study. The chapter also describes the design process of the circle geometry teaching programme. A final draft of the teaching programme that was implemented by the three mathematics teachers is also presented in this chapter. Finally, the chapter discusses the ethical considerations, validity, limitations, challenges and disappointments encountered during the research journey.

1.7.3 Chapter four

This chapter deals with the data analysis and findings of this study. The chapter provides an analysis of the experiences of the participants in implementing the teaching programme and gives details of how the three mathematics teachers experienced the design of the circle geometry teaching programme.

1.7.4 Chapter five

This chapter concludes the study. It presents a summary of the research findings, significance of the study, recommendations, limitations, avenues for further research and some personal reflections.

CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

The way in which mathematics teachers approach geometric instruction determines, to a large extent, the mathematical thinking strategies and dispositions that our learners attain and develop. Literature provides sufficient evidence to show that teaching and learning geometry is not an easy task (van Hiele, 1986). This chapter is aimed at reviewing the literature relevant to geometry and the Van Hiele theory. I structure this literature review in alignment with the goal of my research project which is to investigate selected mathematics teachers' experiences with a new geometry teaching programme that is aimed at developing learners' geometry skills to the highest level of thinking. My reading concentrated mainly on the van Hiele *phases of instruction* as this is my primary focus as opposed to most of the other research which focused on the van Hiele *levels of thinking* in geometry (De Villiers & Njisane, 1987; Senk, 1989; Guttiérrez & Jaime, 1991, 1998; Gal & Lew, 2008; Atebe & Schafer, 2008). Therefore I will follow in the footprints of researchers like Siyepu (2005), Serow (2008) and Abdullah & Zakaria (2011) who shared my research orientation.

The review starts with a general explanation of geometry. This serves as a basis to discuss how geometry is perceived both in Namibia and abroad. I then give a brief overview of the geometry curriculum, teaching and learning in Namibia. I finally present and explain the van Hiele theory, the levels of thinking and particularly the phases of instruction as well as the theory's implications for teaching.

2.2 GEOMETRY

School geometry is defined by Clement and Battista (2004, p. 420) as "the study of those spatial objects, relationships, and transformations that have been formalized (or mathematized) and the axiomatic mathematical systems that have been constructed to represent them". The Faculty of Mathematics at the University of Waterloo defines geometry as the study of shapes and configurations that attempts to understand and classify space in various mathematical contexts (Waterloo Mathematics, 2009). The *Oxford Study Mathematics Dictionary* on the other hand defines school geometry as the "area of mathematics relating to a study of space and the relationships between points, lines, curves and surfaces" (Tapson, 1999, p. 52).

These definitions refer to geometry as a study of a discipline or body of knowledge. Geometry however can also be defined in terms of a process. For example, the Namibian Mathematics Syllabus for Grades 5-7 defines geometry as "the mathematical understanding of space and shapes" (Namibia. Ministry of Education [MoE], 2010a, p. 2). Importantly, geometry is an essential part of the mathematics curriculum that focuses on the development and application of spatial concepts through which children learn to present and make sense of the world (Thompson, as cited in Alex & Mammen, 2010, p. 203). Spatial concepts all relate to specific relationships between shapes and their contexts. There are numerous definitions pertaining to the nature and understanding of geometry from ancient to modern times. Jones (2002) maintains that a useful contemporary definition of geometry is accredited to the highly respected British mathematician, Sir Christopher Zeeman who says that "geometry comprises those branches of mathematics that exploit visual intuition (the most dominant of our senses) to global insight" (p. 124). Schafer (2004) suggests that geometric conceptualization is intertwined with the broader meaning of the space concept and proposes that space can mean many things to many people (p. 10).

My view is that geometry is a crucial aspect of mathematics that is necessary for our understanding of the spatial world around us. If taught in an interesting way it could inspire curiosity in mathematics even in those learners who have no interest in the subject. Now that I have looked at the definition(s) of geometry and its nature, I would like to review the aims and objectives of teaching geometry in schools. The Royal Society/JMC (as cited in Jones, 2002) suggested the following aims of geometry:

- to develop spatial awareness, geometrical intuition and the ability to visualize;
- to provide a breadth of geometrical experiences in 2 and 3 dimensions;
- to develop knowledge and understanding of and the ability to use geometrical properties and theorems;
- to encourage the development and use of conjectures, deductive reasoning and proof;

- to develop skills of applying geometry through modeling and problem solving in real world contexts;
- to develop useful ICT skills in specifically geometrical contexts;
- to engender a positive attitude to mathematics, and
- to develop an awareness of the historical and cultural heritage of geometry in society, and of the contemporary application of geometry. (p. 124)

The preceding definitions and the aims of teaching geometry are similar to the official aims of teaching/learning geometry statements in the Namibian context. The general objectives laid down in the secondary school geometry curriculum (Namibia. MoE, 2010b) suggests that by the end of the geometry topic, learners will:

- know and use geometrical terms and the vocabulary of simple plane figures and simple solids;
- measure lines and angles and construct simple geometrical figures using straight edges, compasses, protractors and set squares;
- recognize properties of simple plane figures directly related to their symmetries;
- calculate unknown angles using the geometrical properties of intersecting and parallel lines and of simple plane figure, and
- determine the locus (path) of a point under certain conditions. (pp. 6-7)

These objectives are fundamental to the teaching of geometry. “The study of geometry contributes to helping students develop the skills of visualization, critical thinking, intuition, perspective, problem-solving, conjecturing, deductive reasoning, logical argument and proof” (Jones, 2002, p. 125). These objectives however are not explicit about how geometry should be taught. They focus more on what learners should be able to do. Geometry curriculum designers and implementers also need however, to focus on the teaching process of mathematics and geometry.

2.2.1 PROBLEMS WITH GEOMETRY TEACHING AND LEARNING

"There is no king road to geometry"

That was the answer that Euclid gave to the king of Egypt when he asked him to explain his Elements in an easier way. (Heath, as cited in Dimakos, Nikoloudakis, Ferentinos & Choustoulakis, 2007, p. 90). This illustrates that the difficulties in understanding geometry is not an unusual phenomenon, it has existed since the ancient times. Nikoloudakis (2009) observes that similar research on the understanding of geometric concepts by learners has shown that learners in general find defining and recognizing geometric shapes and the use of deductive thinking in geometry problematic (p. 17). Burger and Shaughnessy (1986) echo this sentiment in their study where a number of secondary school learners interviewed in their clinical study had incomplete notions of basic shapes and their properties. "This observation might explain some of the frustrations students and teachers have with secondary school geometry courses. Students are not sufficiently grounded in basic geometry concepts and relation to 'reinvent' Euclidean geometry. Memorization may be their only recourse (p. 46)

Many other researchers have also reported on learners' geometry related difficulties in their various fields of study. Weber (2003, as cited in Nikoloudakis, 2009) found that learners find it very difficult to successfully write simple geometry proofs. Senk (1989) on the other hand stated that many secondary school learners in the United States were not prepared for geometry classes. Fuys, et al., (1988) found that there was too much emphasis placed on formal symbolism and identification in the elementary school geometry curriculum, while relational understanding was underestimated. Research done in Southern Africa revealed similar problems when it came to assessing why learners struggle with formal geometry.

De Villiers and Njisane's (1987, as cited in de Villiers, 1996, p. 12) study revealed that about 45% of learners in Grade 12 (Std 10) in KwaZulu Natal had only mastered Level 2 or lower, whereas the examination assumed mastery at Level 3 and beyond. De Villiers (1996) further attributes the failure of geometry in many secondary schools to the role of language that creates communication gaps between the teacher and the learners. Atebe and Schafer (2011) in their study showed that participating secondary school learners "had a limited and arguably inadequate knowledge of basic geometric terminology…" (p. 63). This is also attributed to the traditional curriculum which is typically presented at a higher level than those of the learners (Van Hiele, 1986).

As in South Africa (de Villiers, 2010), Namibia has a geometry curriculum that is heavily loaded in secondary school with formal geometry. I often feel frustrated when teaching the lower secondary learners at the lack of geometry knowledge and experience many of them bring from the primary school. In my view, learners in the primary schools do not spend enough time dealing with geometric ideas in a conceptual manner – their geometric understanding is often shallow and lacks conceptual understanding.

Research revealed that difficulties with geometric conceptualization are often a result of various factors. Learners' apparent inability to reason at a higher level of geometric thinking does not only lie within their own learning patterns or motivation. The teacher's instructions and choice of exercises also play an integral and important role in the learners' learning. Burger and Shaughnessy (1986) explained that high school geometry as it is taught in most high schools is taught at a deductive level while most learners are only capable of reasoning informally about geometric concepts upon entrance into geometry. De Villiers (2010) echoes this sentiment and states that within the South African context the main reason for the failure of the traditional geometry curriculum is because its expectations are set at a higher level than that of the learners' ability. For example, the curriculum might require the learners to reason at van Hiele level three of geometric understanding while the learners are only able to reason up to the second van Hiele level.

In my experience as a mathematics teacher, I have observed that many of my high school learners struggle to recognize shapes, make accurate constructions, accurately describe properties of plane geometry figures, and construct appropriate proofs in geometry. This leads to a negative view of geometry as a mathematical activity that might affect all learners, even those who are mathematically oriented.

In conversation with a Computer Studies specialist (a Peace Corps volunteer) who helped out with one Grade 8 mathematics class during his time at my school I asked him how he felt about geometry in general, and this was his response: "I hate geometry! I am an Algebra guru." Do not get me wrong, this teacher has a sound knowledge of formal geometry but only 'hates' geometry “because it is too complicated or rather people like to complicate it instead of just calling a spade a spade” (Personal communication, Q. Lee, July 12, 2012). The question I ask is geometry really that complicated or could there be other factors that contribute towards ‘geometric’ confusion among individuals?

Another factor that could contribute to the problems in geometry conceptualization in secondary schools in Namibia and beyond could be the prescribed textbooks. In my experience, many textbooks deal with geometry in a very technical and procedural manner at the expense of developing deep conceptual understanding, thus inhibiting learners from reaching higher van Hiele levels of geometric thinking. Van Hiele, (1986) states that the possibility exists that some methods of teaching do not permit learners to attain higher levels of geometry thinking.

This study looks specifically at possible ways of teaching geometry so that it facilitates the attainment of higher van Hiele levels. The intervention that forms the core of this study is intended to inspire and encourage teachers to design their own curriculum in such a way that it enables learners to develop meaningful geometric thinking.

Despite many curriculum reforms over the decades the geometry curriculum remains inaccessible to many learners. The renowned Dutch mathematics educator and researcher Van Hiele (1986) discovered that there are difficult moments that face every high school teacher in teaching geometry. He too encountered many such moments when learners failed to understand his teaching.

Therefore, like teaching any other topic in mathematics, teachers should develop geometric concepts in such a way that learners are able to work from the less abstract to the more complex. This involves a carefully planned sequence of teaching events that develops geometric conceptualisation. The van Hiele theory provides a model to do just that. The rationale and aspiration that underpin this literature review are not only anchored in my personal experience as a mathematics teacher, but have been inspired by the van Hiele theory (van Hiele, 1958, 1959, 1986, 1999; de Villiers, 1987, 2010; Atebe & Schafer, 2008, 2010, 2011) that sought to provide answers to the teaching and learning problems of geometry in secondary schools in Namibia and beyond.

If learning geometry means learning to think and being able to attain the ‘highest possible level’ of conceptualization (van Hiele, 1986) then all mathematics teachers should be well versed in the nature of good geometry teaching. Let us now venture into geometry teaching and learning in Namibia.

2.2.2 GEOMETRY TEACHING AND LEARNING IN NAMIBIA

As mentioned earlier, geometry is a fundamental branch of the Namibian Mathematics Curriculum. Upper Primary Phase (UPP) learners (age 12-14 years) should enter the Secondary School Phase (SSP) with sufficient knowledge of basic geometry. They should at least be able to visualize and recognize basic properties of geometrical shapes and measure lines and angles of geometric figures (Namibia. MoE, 2010a). The UPP (Grades 5-7) is the phase where learners begin to explore properties of basic geometric shapes in their immediate surroundings. The grades 11-12 geometry curriculum is built on the Upper Primary curriculum. I start this section by clarifying the ‘Namibian levels’ of teaching/learning mathematics in the Grade 11-12 Namibian curriculum.

The Grade 11-12 mathematics curriculum is divided into two levels: the Ordinary Level and the Higher Level. According to the Namibian Senior Secondary Certificate (NSSC) Mathematics syllabus the Mathematics Ordinary and Higher Level is designed as a two-year course leading to a national examination by the end of the second year (Grade 12).

The Mathematics Ordinary Level is divided into two syllabuses: the Core Syllabus and the Extended Syllabus. Candidates who follow the Core Syllabus write the national examination papers 1 and 3 and they are eligible for the award of passing grades C to G only. Candidates who follow the Extended Syllabus write the national examination papers 2 and 4 and they are eligible for the award of passing grades A+ to E only.

The Mathematics Higher Level syllabus on the other hand enables students to study more advanced topics than the Ordinary Level curriculum. The Higher Level candidates are eligible for awards of passing grades 1 to 4 only, grade 1 being the highest passing grade. The illustration below (Figure 1) helps for an easy understanding of the current mathematics curriculum in Namibia.

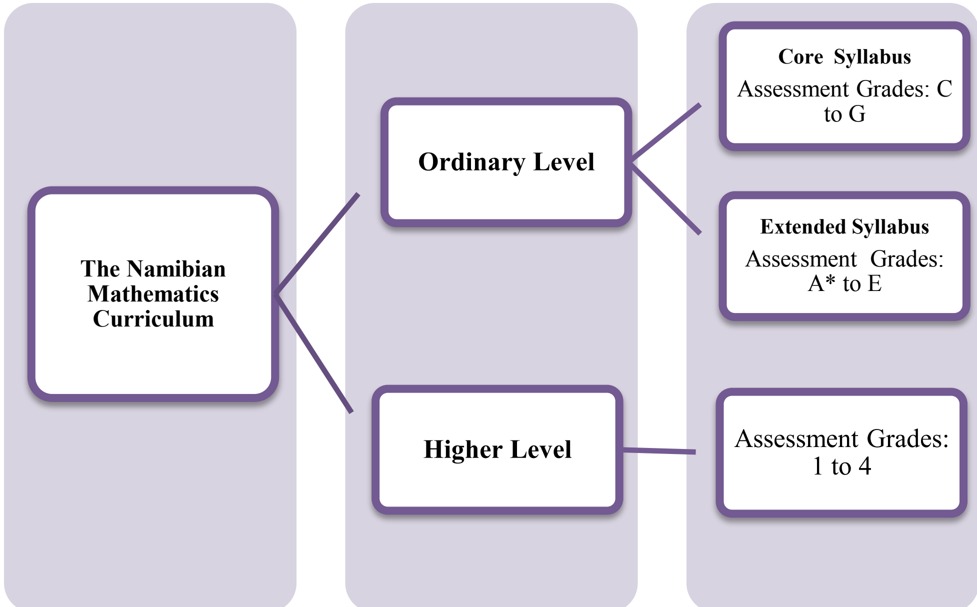

Figure 2.1: The levels in the Namibian Mathematics Syllabus, implemented in January 2010.

2.2.3 CIRCLE GEOMETRY

Circle geometry is geometry that deals with the relationships of circles. It is a rich branch of geometry that offers a wide range of concepts, properties and axioms to the geometry curriculum. The circle geometry curriculum includes concepts such as: circle, centre, circumference, radius, diameter, chord, semi-circle, sector, segment, arc, secant, tangent, concyclic points, cyclic quadrilateral, circumcircles and concentric circles. Many of the geometric concepts developed in circle geometry are applied to real-life problems. These concepts can for example be used to plan a wide range of activities that would be of particular interest to learners. Engaging in geometric activities, learners should be able to deduce definitions, properties of circle geometry and conjecture circle geometry theorems such as such 'an angle at the *centre* of a circle is twice the angle on the *circumference*'.

My interest in circle geometry is inspired by the diversity of concepts inherent in that part of the curriculum. This diversity allows me to plan innovative and exciting learning activities as well as to present the content in a more lively and interesting way. It also allows the teachers to do so much in a single lesson presentation, having only to use one or two illustrations involving circles. The continued poor performance in geometry at secondary school level (Examiner's Report, 2010) is of concern – thus my interest in researching this matter further.

I believe that meaningful teaching and learning of geometry is predominantly about doing (hands-on). Therefore, circle geometry offers a wide range of opportunities for teachers to plan learning activities with more insight. Hands-on activities enhance learners' concentration and participation in the lesson.

Table 2.1 below shows a section of the circle geometry mathematics curriculum extracted from the grades 11-12 mathematics syllabi:

SPECIFIC OBJECTIVES **Learners should be able to:**		
TOPIC: Geometry	**Everybody**	**Extended/Higher Level only**
Geometrical terms and relationships	- Use and interpret vocabulary of circles	
Symmetry	Recognize and use the following symmetry properties of circles: - Equal chords are equidistant from the centre	Recognize and use the following symmetry properties of circles: - *The perpendicular bisector of a chord passes through the centre of a circle* - *Tangents from an external point are equal in length*
Angle properties	Calculate unknown angles using the following geometrical properties (reasons may be required but no formal proofs): - Angle in a semi-circle - Angle between tangent and radius	Calculate unknown angles using the following geometrical properties (reasons may be required but no formal proofs): - *Angle at the centre of a circle is twice the angle at the circumference* - *Angles in the same segments are equal* - *Angles in opposite segments are supplementary*
Locus	Use the following loci and the method of intersecting loci for sets of points in two dimensions: - Which are at a given distance from a given point Which are equidistant from tow given points	

Table 2.1: The Circle Geometry Curriculum. Adapted from the Grades 11-12 mathematics syllabi (Namibia. MoE, 2010b, 2010c).

The sections of the geometry curriculum written in italics in Table 1 above are taught at the Extended and Higher Level only. However, that part of the syllabus that is written in non-italics is for everybody in Grades 11-12, irrespective of the learners' choice of a level of study.

Many Namibian learners perform poorly in geometry. According to the Examiner's Report (2010), many secondary school learners in Namibia performed below average in the 2009 national examination. The report revealed that grade 12 learners who sat for the November 2009 national examinations had trouble expressing themselves accurately when attempting geometry questions. The report further explained that learners lack knowledge of theorems and proof writing in geometry (Examiner's Report, 2010).

The Examiner's Report consists of an analysis carried out by 'Chief Examination Markers' committee of the Directorate of National Examination and Assessment (DNEA). After the marking of national examination (Grades 10 & 12), the DNEA committee analyzed learners' responses to examination questions per subject. The committee then compiled a report known as the 'Examiner's Report' consisting of all subjects offered in Namibian schools. These reports are sent to schools during the first quarter of the following academic year with the aim of advising the teachers about problems in various subjects. Such reports are crucial to teachers as they discover the learners' weakness in various topics across the subject curricula. They also help researchers in the education field to acquire primary information about the learners' performance such as the strengths and weaknesses of the learners. They also assist in identifying opportunities for both teachers and learners to help address problems and difficulties.

According to the Examiner's Report of 2010, Grade 12 Ordinary Level students answered geometry questions poorly in comparison to their Higher Level counterparts. Many of the Ordinary Level students struggled with angles, while many of the Higher Level students could not clearly prove geometry questions. Clearly both Ordinary and Higher level students experienced difficulties of some kind in their respective levels of study. Could these difficulties be partly attributed to what happens in junior primary grades? I am tempted to agree with Teppo (1991) who claims that "systematic instruction in the middle grades is necessary to prevent students from entering high school at low levels of geometric concept development" (p. 217).

This research aims at designing and implementing a circle geometry teaching programme using the van Hiele phases of instruction as a conceptual framework. Before I discuss the van Hiele phases of instruction, I will briefly explain the van Hiele level framework.

2.3 THE VAN HIELE THEORY

The van Hiele theory is a "learning model that describes the geometric thinking students should go through as they move from a holistic perception of geometric shapes to a refined understanding of geometric proof" (Genz, 2006, p. 4). This theoretical model was developed by the renowned educators, Pierre Marie van Hiele and his wife Dina van Hiele-Geldof about six decades ago. The van Hieles proposed and developed this learning model as a result of the frustration that they experienced with their learners' poor conceptualization of geometric reasoning. The model proposes five levels of thinking that learners sequence through in order for them to master geometric concepts.

The van Hiele theory identifies a sequence of five hierarchical levels of geometric thinking. These thinking levels are recognition, analysis, ordering, deduction and rigor. According to the van Hiele theory, "students move sequentially from one level of thinking to the next [level] as their capability increased"(van Hieles, as cited in Gutierrez, Jaime & Fortuny, 1991, p. 237).

2.3.1 THE VAN HIELE LEVELS

The van Hiele theory described the five levels of thinking in geometry using a 0-4 scale. I aligned my numbering scale in this study with current van Hiele research practices and numbered them on a 1-5 scale. Learners who are performing at a level below van Hiele level 1 are thus considered to be at the original van Hiele level 0. I briefly describe each of the five levels as cited in the following texts: van Hiele (1999), Clements and Battista (2004), de Villiers (2010) and Atebe (2011).

Level 1 - Recognition/Visualization level

At this level, learners recognize geometric shapes as a whole. They are able to usually recognize and name geometric figures such as triangles, rectangles and parallelograms without explicit identity of these shapes. For example, a square is a square because it looks like one. If a shape is drawn out of its standard position then it is no longer the same shape. Understanding of properties of a shape is not necessary at this level.

Level 2 - Analysis level

At the analysis level, learners recognize and can characterize shapes by analyzing their properties. However, they only focus on the necessary properties of shapes (e.g., a square has four right angles and four sides the same length) (Burger & Shaughnessy, 1986; Groth, 2005). At this stage, learners are able to apply the appropriate technical language, though they do not understand the interrelationships between these properties and between different shapes. Therefore, learners tend not to appreciate proof writing in geometry as they might or even often think that geometrical theorems can be established by rote memorization.

Level 3 - Ordering/Abstract level

At this level, learners can logically order the properties of figures and form abstract definitions for shapes. Hence, the relationships between the various properties of the figure are now understood. It is at this level that learners start to make sense of the formal definitions of shapes (Khoh, 1992, p. 33). However, they still lack understanding of the roles of axioms, theorems and proofs.

Level 4 - Deduction level

In level 4, learners go beyond just identifying characteristics of shapes; they observe the complete structure of geometry. They begin to develop longer sequences of statements and start to understand the significance of the deduction, role of axioms, theorems and proof. They are able to create proof based on their own argument.

Level 5 – Rigor

Learners start to reason formally about mathematical systems in the last van Hiele level. They manipulate geometric statements such as axioms, definitions, and theorems. Non-Euclidean geometry can be studied at this level.

It is clear that the levels of thinking comprise a hierarchical nature. They are logically structured to suggest that learners move from lower to higher levels of thinking in geometry. The current level is a prerequisite for the previous level. For example, “the recognition of a figure at Level 1 feature is an essential prerequisite for Level 2. The consideration of properties at Level 2 will eventually lead to Level 3 understanding where students see relationships between them, i.e., how one or two properties lead to a third” (Pegg, 1992, p. 21). The fourth level leads to conceptual understanding of geometrical proof and development and of theorems and postulates.

The van Hiele theory has been a study of much research in mathematics education. Some researchers assigned van Hiele levels to students (Mayberry, 1983) some looked at assessment using levels of geometric thought (Burger & Shaughnessy, 1986; Gutiérrez & Jaime, 1998). Other extensive research was conducted looking at students’ achievement in writing geometry proofs (Senk, 1989; Dink & Jones, 2008). Other interesting research that interests me is Serow’s (2008) research that looked at the ‘phases’ approach of the van Hiele theory and Siyepu’s (2005) circle geometry research. Some research was done for comparison purposes. For example, Atebe and Schafer’s (2008) research compared geometric thinking of Nigerian and South African learners. Genz’s (2006) research on the other hand aimed at tracing the differences between the van Hiele aligned curriculum and the non-standard based curriculum.

Groth (2005) paraphrases that, “the goal of most high school geometry courses is to have students reasoning at the deductive level by the end of the year” (p. 28).However, at most secondary school geometry is only taught up to level 3 (Cassim, 2006, p. 26). This is confirmed by van Hiele (1986) who warns that lower secondary learners are only able to reach the third level of the van Hiele model, which is the ordering level. I believe this might be the reason why many studies on the van Hiele theory only did their research to the ordering level (Ding & Jones, 2006, 2007; Gutierrez & Jaime, 1998).

2.3.2 PROPERTIES OF THE VAN HIELE LEVELS OF THINKING

In order to understand the van Hiele theory of geometric thought, I discuss important features/properties associated with the levels of thinking in geometry. I align my discussion to that of Pegg’s (1992, pp. 21-22) as follows:

1. Learners in the same mathematics classroom may be on different van Hiele levels for different concepts. “However, once one concept has been raised to a higher level, it will take less time for other concepts to reach that level” (Pegg, 1992, p. 21).

2. It is inherent in the nature of the levels that in understanding geometry, learners must go through the levels of thinking in a sequential order. Learners cannot attain a higher level without first passing through the lower levels of thinking. Mayberry (1983) mediates that if the levels of thinking form a hierarchy, then learners performing at level n are expected to perform at level n and lower but not at level $n + 1$. Pegg (1992) warns however that "pupils can simulate higher levels by learning rules or definitions by rote or by applying routine algorithms that they do not understand" (p. 21).
3. To move learners from one level of thinking to the next level, except perhaps for brilliant students, requires direct instruction hence; progress is more dependant on instruction than on age or biological maturation (van Hiele-Geldof, as cited in Fuys, et al., 1984; van Hiele, 1986, 1999). Learners are expected to relate new information to what is already known by linking the known to the unknown (Etchberger & Shawn, 1992, as cited in Siyepu, 2005, p. 10).
4. To attain a higher level of thinking, learners need to confront a personal "crisis of thinking". They may not be forced to think at a higher level as this may hinder their potential to master reasoning at a higher level.
5. "Level reduction" occurs when structures are at a higher level than that of the learners and are re-interpreted at a lower level. This occurs for example when using basic circle geometry concepts to prove theorems at a higher level. For example, learners should be able to recognize and conjecture properties of cyclic quadrilaterals in order for them to prove that a certain (given) quadrilateral is cyclic.
6. Each level of thinking has its own language and it is up to the teacher (instruction) to ensure that learners' progression through the levels is directed in the correct language. Thus, a language structure at each level is a critical factor in the movement through the levels of thinking. Hence, if two people reason at two different levels they will not understand each other. This occurs mostly between students in the same classroom, the learner and the teacher, learner and the textbook or learning domain, and as well as between the teacher and the mathematics syllabus.
7. Like linguistic symbols are a feature of the levels, each level has its own organizational relationships. Teachers should be aware that what may appear to be accurate at one level may not appear to be correct at a higher level.

8. Concepts implicitly understood at one level become explicitly understood at the next level.
9. The learning process is discontinuous. Thus, if students reach one level of thinking, they take time to reach the required maturity of that level before proceeding to the next level. Forcing learners to learn at a higher level or teaching them content higher than that of their level maturity will not succeed until they are geometrically mature enough to proceed to a higher level.
10. Rote learning or application of routine algorithms without understanding leads to no level attainment.

Difficulties in teaching geometry persisted in the van Hieles' years of teaching despite their change of geometric instruction over the years. Van Hiele (1986, p. 39) chronicled; "in the years that followed, I changed my explanation many times, but the difficulties remained. It always seemed though I were speaking a different language". They then developed a framework of teaching phases that helped teachers to move their learners from one level to the next.

To confirm the properties of the levels of thinking, van Hiele-Geldof (1958, as cited in Fuys, et. al., 1984) stressed that learners cannot progress through the levels of thinking without proper instruction. Hence, it is important that the teachers' instruction is pegged at the appropriate van Hiele level to enable learners to attain the highest possible level in their learning environments. Let us now examine the phases of instruction which, "in the process of apprenticeship, lead to a higher level of thought" (Van Hiele as cited in Fuys, et. al., 1984).

2.3.3 THE VAN HIELE'S PHASES OF INSTRUCTION

Van Hiele (1986) recommends a set of instructional phases that teachers should follow in order to facilitate the students' movement between the van Hiele levels of geometric thinking. The phases of instruction are: information, guided orientation, explicitation, free orientation and integration. Teachers are advised to guide their learners' geometric conceptualization by employing these five phases of instruction in their practices (van Hiele, as cited in Fuys, et. al., 1984, Mistretta, 2000; Clements & Battista, 2004; Groth, 2005; Ding & Jones, 2007, Serow, 2008; Abdullah & Zakaria, 2011). This would encourage learners to proceed from lower to higher levels of thinking. A description of each phase of instruction is summarized below.

Phase 1: Information/Inquiry

During the information phase, the teacher provides inquiry-based activities in which learners carry out ‘experiments’ and make inductive reasoning and conjectures with regard to the objects learnt. Furthermore, the teacher introduces the vocabulary and concepts necessary for completing a task and for the learners to become familiar with the working domain through discussion and exploration. Discussion takes place between the teacher and the learners to foreground the objects to be used. It is through this discussion that the teacher discovers how learners interpret the language and then provides the necessary information to bring them to purposeful action and perception. Hence, Crowley (1987) reasons that the teachers engage with activities at Phase 1 so that they “learn what prior knowledge the students have about the topic *while* [italics added] the students learn what direction further study will take” (p. 5).

In this phase, the teacher can for example introduce the content to be learned say, circle geometry. She/he then engages the learners in a discussion about fundamental concepts inherent in circular shapes. This can be done by sketching circles on the chalkboard and asking the learners to draw and label all the parts of the circle they can remember. By doing this, the teacher familiarizes the learners with the content to be learned by engaging them in a content-based discussion in order to test their pre-knowledge of the subject content. She/he introduces the correct terminology i.e. two radii instead of two radiuses.

Phase 2: Guided Orientation

In the second phase, the teacher guides learners to uncover connections and to identify the focus of the subject matter. At this stage, learners are given the liberty to exchange ideas during classroom discussion. They engage with the concepts in order to begin to develop an understanding of them and the connections between them. This enables the learners to learn by exploring the subject matter during the teacher-guided discussions.

At this stage, the teacher is more involved in the learning processes. She/he directs the learners where she/he wants them to go. For example, the teacher sketches a circle and only labels the closely related concepts, say a diameter, radius, chord and secant. She/he then guides the learners to differentiate between these concepts. More unknown concepts are introduced while the teacher guides the learners to use the proper circle geometry terminology. However, the learners’ own language is still acceptable at this stage though the teacher's role is to correct their language into a more accepted technical language. Hence, "much of the material will be short tasks designed to elicit specific responses" (Crowley, 1987, p. 5)

Phase 3: Explicitation

In the course of the third phase, explicitation takes place. Learners learn to verbalize their understandings of the concepts and connections. They become more conscious of the new ideas and express these in accepted mathematical language. The concepts now need to be made explicit using accepted language. The teacher takes care that learners advance the use of technical language with understanding through the exchange of ideas. Hence, learners' new knowledge is formed through experience and integrating this with past knowledge.

The teacher further extends the learning arena for example by putting 'theory into practice'. This is where conceptual terminologies are applied to real geometrical problems. For example*, what is the diameter of the circle if the radius is 3.5cm*? This problem will trigger the learners' thoughts to the definition property of a diameter (twice the radius). The teacher allows the learners to work independently or in pairs. The leaners are then required to discuss their answers with the rest of the class while giving reasons for each answer by making use of the accepted geometric language. For example the diameter of the circle is 7cm because the diameter is twice the radius which measures 3,5cm. Hence, the teacher's role will also be to ensure that the accepted technical language is maintained at this phase.

Phase 4: Free Orientation

In the free orientation phase, learners are challenged to solve problems. They can now complete complex tasks (related to the concepts at hand) that require a number of steps and can be solved in many different ways. Learners are required to find their own way and locate themselves in the network of relations to complete such tasks. They are now familiar with the learning domain and are ready to explore it. Through problem solving, learners' language develops further as they start to identify hints to assist them.

The use of investigations is still largely unknown at this stage, but the learners are encouraged to complete activities in which they are required to find their own solution strategies. Through problem solving, learners' mathematical language develops, as the teacher provides capacity for logical thought.

The teacher's role in this phase, according to Clements and Battista (2004, p. 431) is "to select appropriate materials and geometric problems" that require certain levels of thinking to solve geometric problems successfully. For example;

A circle with centre O has $\angle$AOB subtended at the centre of the circle. A and B intersect on the major sector at M, and $\angle$AOB is 2a°. Explain why $\angle$AMB is a°.

This type of (circle) geometric problem enhances the leaners' technical language, permits high thinking performance and encourages them to elaborate and reflect on the problem and their solutions to the problem (Clements & Battista, 2004). A characteristic of this particular phase is that the teacher withdraws him/herself from the learners' learning process. She/he becomes an observer of the learning process and less of a facilitator.

Phase 5: Integration

In the integration phase, learners build an overview of the content studied. The teacher helps them to reflect upon the observations they have made and they begin to understand the overall structure of the concepts and where those structures fit in the scheme of formal mathematics. In the same vein, learners summarize the new understanding of the concepts involved and incorporate the appropriate language of the new level. It is important that these summaries only include what the learners already know – no new material should be introduced at this stage. The purpose of instruction is now clear to the learners hence, there is less and less assistance from the teacher. Learners could for example be asked to summarize the content learned over a short period of time, say, a particular theme (e.g. circle geometry) and would do this with diligence.

By the completion of this phase (integration), learners have reached a new level of thinking. This new level of thinking replaces the previous level of thinking and learners are once again ready to repeat the five phases of learning at the next level of the van Hiele model of thinking. The cycle keeps on repeating until learners attain the highest possible level of geometric thinking for the content under study.

Each instructional learning phase builds upon and adds to the thinking of the previous van Hiele level (Genz, 2006). Serow (2008, p. 446) illustrates that the five phases of instruction "provides a structure on which to base a program of instruction". When developing the five phases of instruction, van Hiele (as cited in Fuys, et al), suggests that educators should be clear about the aim of the art of teaching geometry at each phase. That is "precisely to face questions of knowing how these phases are passed through, and how help can effectively be given to students" (p. 250).

A UK study of geometry teaching (Royal Society, as cited in Ding & Jones, 2006) concludes that "the most significant contributions improvements in geometry teaching will be generated by the development of good models of pedagogy, supported by carefully designed activities and resources (p. 45)." Hence, instructions planned to nurture development from one level to the next level of thinking should include a sequence of activities, "beginning with an exploratory phase, gradually building concepts and related language, and culminating in summary activities that help students integrate what they have learned into what they already know" (van Hiele, 1999, p. 311).

Table 2.2 below summarizes the descriptions of the van Hiele Phases of Instruction

Phases	Description of phase focus; some illustrative examples
6. **Information**	Teacher familiarizes the learners with content to be learned by engaging them in a content-based discussion. Teacher learns and tests their pre-knowledge of the subject content and how they interpret the language. The learner gets acquainted with the working domain; uses materials presented to him/her; examines examples and non-examples.
7. **Directed Orientation**	The teacher guides learners to uncover the connection and to identify the focus of the subject matter. Learners engage with the concepts in order to begin to develop understandings of them and the connections between them. Learners' own language is still acceptable at this stage though the teacher's role is to correct their language into a more accepted technical language.
8. **Explicitation**	Learners learn to verbalize their understandings of the concepts and connections. They become more conscious of the new ideas and express these in accepted mathematical languages. For examples, expresses ideas about the properties of a figure. The teacher's main role is to ensure that accepted technical language is maintained at this phase.
9. **Free Orientation**	Learners can complete complex tasks that require a number of steps and can be solved in many ways. Learners are able to complete activities in which they are required to find their own way in the network of relations. Through problem solving, learners' mathematical language develops. The teacher's role is to select appropriate materials and geometric problems that require certain of level of thinking to solve geometric problems successfully.
10. **Integration**	Learners begin to understand the overall structure of the concepts and where those structures fit in the scheme of formal mathematic. They begin to summarize the new understanding of the concepts involved and incorporate language of the new level. A new level of thinking is attained.

Summary idea adapted from Serow (2008) and Fuys, et al., (1988)

2.3.4 IMPLICATIONS OF THE PHASES OF INSTRUCTION FOR TEACHING

The implementation of the five van Hiele phases has, in my view, significant potential in the teaching of mathematics. There has unfortunately been relatively little research in this area (Clements & Battista, 1992, p. 434). The phases of instruction are important because they enable the teacher to lead the students from one level of thinking to the next level. Freudental (1973, as cited in Patsiomitou & Emvalotis, 2010) confirms that "good geometry instruction can mean… leading learners to understand why some organizations, some concepts, some definition is better that another" (p. 23). In the van Hieles' framework, this means leading the learners through the phases of instruction to attain the highest possible level of thinking in geometry. So what are the implications for a teacher who uses the five phases of instruction?

In the first phase of instruction (information), the teacher should inform the learners about the subject matter. The teacher should place the learning materials at the learners' disposal (Clements & Battista, 1992); and engage them in a discussion to talk and learn about their pre-knowledge.

At phase two (Directed Orientation), the teacher's role is to direct the learners' learning activity by providing well planned tasks at the right van Hiele level. The teacher should ensure that the aim of teaching/learning is well known to the learners (Clements & Battista, 1992).

At the third phase of instruction (Explicitation), the teacher should accept the learners' own language during content discussions. However, the teacher should ensure that "relevant mathematical terminology" is explicated to the students (Clements & Battista, 1992, p. 431).

The fourth phase is the Free Orientation phase. The teacher should select appropriate learning materials and geometrical problems that the learners should complete under his/her observation (to ensure that the correct geometrical terminologies are applied to accurate components). It is therefore the teacher's role to "introduce terms, concepts and relevant problem-solving needed *to lead the students to higher level of thinking* [italics added]" (Clements & Battista, 1992, p. 431).

In the last phase of instruction (Integration), the teacher should encourage the learners to summarize and reflect on the geometric knowledge of the topic learned. At this phase, the teacher should ensure that the learners use the correct technical language in order to lead them to the next level of thinking in geometry. The teacher should also ensure that learners are comfortable with the reasoning and subject content they have just mastered and are ready to take on the content of the next level of thinking.

These implications are vital to the intervention aspect of this study, which is to design and implement a grade 11 circle geometry teaching programme using the van Hiele phases of instruction as a conceptual framework. Therefore, to ensure successful implementation of this teaching programme, teachers should be aware of the implications of the van Hiele phases as discussed above.

2.4 CONCLUSION

In this chapter, I focused my discussion on the van Hiele theory and its implications for teaching. I started by giving a broad overview of geometry both in Namibia and abroad. I then framed my geometry discussion to more specific aspects such as problems in teaching and learning geometry in secondary schools, the current Namibian geometry curriculum, including the circle geometry section of the geometric curriculum. I then engaged with the levels of thinking and the van Hiele phases of instructions.

I compare the overall van Hiele theory to a staircase. The steps that one takes to go up the stairs are the levels of thinking while, the height of each stair (the motion of lifting a foot up to the next stair) represents the phases of instruction. As one needs to lift one foot at a time to take the next step on the stairs, it also takes one phase of instruction at a time to reach the fifth phase of geometric thought. The literature review provided the framework underpinning this study. In the next chapter, I discuss the research methodology I employed in my research study.

CHAPTER 3

RESEARCH METHODOLOGY

3.1 INTRODUCTION

What is the point of researching if you do not share your discoveries; or embarking on an adventure if you do not tell of your experiences? In my opinion, a tale-free journey is not worth taking in the first place. Every journey has its own encounters depending on where, when, how, and why it happened. That is why there is always a purpose for every journey taken irrespective of its outcome. The main purpose of this chapter is to describe a very important aspect of my research journey.

This chapter outlines the methodology used to conduct the study and the rationale for the choices made with regard to the design of the instruments. The methodology journey is expressed in terms of orientation, design and process. The chapter begins with a brief discussion of the research goal, sample, orientation, methodology and design. I report on the events of the teaching programme design which includes the data that emerged during the introductory workshop as well as the one-on-one discussions with each of the teachers.

I then discuss the tools used for data collection. The chapter also describes the sampling methods I used, the analysis procedures and ethical considerations which are crucial features of any research project. Finally, the chapter concludes by highlighting some of the limitations of the research.

3.2 RESEARCH GOAL AND QUESTION

The main goal of my research was to design, implement and reflect on a ‘circle geometry teaching programme’ based on the van Hiele phases of instruction.

The fundamental research question is: What are the experiences of selected mathematics teachers when using the van Hiele phases of instruction in designing and implementing a Grade 11 circle geometry teaching programme?

3.3 SAMPLE: RESEARCH SITE AND PARTICIPANTS

I conducted this research project in the school where I teach. It is an urban school in Oshikoto Region in Namibia. It has a multicultural in student profile and the students predominately come a from middle-class economic background. I selected the school primarily for convenience reasons as I had easy and regular access to it.

There are currently five grade 11 and five Grade 12 classes at the school. One Grade 11 class and one Grade 12 class do Mathematics Higher Level and the rest do Mathematics Ordinary Level. These two levels differ in that the Higher Level covers a major part of circle geometry while the Ordinary Level covers only the basics of circle geometry. Consequently, I selected the Higher Level Grade 11 learners and their three mathematics teachers for the study because their curriculum includes sufficient circle geometry.

So the sample is located in a Secondary urban school in Namibia, and consists of 3 Grade 11 mathematics teachers and the 18 learners selected from the Grade 11 class doing Mathematics Higher Level.

3.4 ORIENTATION

3.4.1 The interpretive paradigm

The study is oriented within an interpretive paradigm. According to Cohen, Manion and Morrison (2007, p. 21), the central endeavour in the context of the interpretive paradigm is "to understand the subjective world of human experience. To retain the integrity of the phenomenon being investigated, efforts are made to get inside the person's head and understand from within". By employing the interpretive paradigm, I located myself accordingly to engage deeply with the data and analyze my participants' experiences with designing and implementing a circle geometry teaching programme using the van Hiele phases of instruction as a conceptual framework. I interacted with the Grade 11-12 mathematics teachers over a period of three months. I observed the actual implementation of the teaching programme and engaged the participating teachers in lesson reflections on the whole implementation process.

As the van Hiele phases of instruction are embedded within a social constructivist perspective (van Hiele, 1986), my designed teaching intervention is underpinned by the social constructivist pedagogy, and thus is aligned with the objectives of the Namibian learner-centered curriculum. Von Glasersfeld (as cited in Ernest, 1990) asserted that:

> …knowledge is not passively received but actively built by the cognizing subject; the function of cognition is adaptive and serves the organization of the experiential world, not the discovery of ontological reality. (p. 3)

Jackson (1995) suggests that the nature of the interpretive paradigm "stresses the importance of interpretation *individuals* put on their actions and reactions to others" (p. 9). It is by this virtue that the interpretive paradigm is most suitable for this study. This research project aims to investigate the teachers' 'experiences'. The best possible way to do that is to provide rich descriptions of the phenomenon and, if possible, to develop some explanations for it (Ellis, 1992).

Research design associated with the interpretive approach relies mainly on a case study which involves a single or a few cases that are described in detail. The research design also emphasizes verbal description rather than numeral analyzes of data – i.e. qualitative research (Jackson, 1995, p. 10).

3.4.2 Qualitative research

Jackson (1995) observes that:

> Qualitative research emphasizes verbal description and explanations of human behavior. Rather than concerning itself primarily with a representative sample, qualitative research emphasizes careful and detailed descriptions of social practice in an attempt to understand how participants explain their own world. (p. 17)

Furthermore, Lincoln and Norman (as cited in Anderson, 2000) describe qualitative research as follows:

> Qualitative research is multi-method in focus, involving an interpretive, naturalistic approach to its subject matter. This means that qualitative researchers study things in their natural setting, attempting to make sense of, or interpret phenomena in terms of the meanings people bring to them. Qualitative research involves the studied use and collection of a variety of empirical materials – case study, historical personal experience, introspective, life story, interview, observation, historical, interactional, and visual texts – that describe routine and problematic moments and meanings in individuals' lives. (pp. 119-120)

My decision to work with qualitative data is linked to the type of my inquiry which is to try to understand how mathematics teachers experienced the designing and implementing of a van Hiele teaching programme. Anderson (2000) asserts that:

> The fundamental assumption of a qualitative research paradigm is that a profound understanding of the world can be gained through conversations [interviews] and observation in natural settings rather than through experimental manipulation under artificial conditions. (p. 119)

I observed the actual implementation of the teaching programme and analyzed documents such as the circle geometry curriculum for grade 11-12 as well as the Examiner's Reports for national examinations. I interviewed the participating teachers when it was time to seek that 'understanding' which is the core basis of this study.

Henning, van Rensburg and Smit (2004) state that a qualitative inquiry as "a research form, approach or strategy allows for a different view of the theme that is studied and in which the participants have a more open-ended way of giving their views and demonstrating their actions" (p. 5). The choice of adopting a qualitative strategy is influenced by the nature of the question asked. My research question has to do with a specific focus – teachers' experiences. That is why I am comfortable that the use of a qualitative approach is appropriate for this study. However, Anderson (2000) cautions that the researcher should "learn that being in the right place at the right time is all important. Moreover, the researcher's perspective also influences what might be found" (p. 119).

3.5 RESEARCH METHODOLOGY

3.5.1 Case Study

The interest of this study is in “the process rather than outcomes, in context rather than a specific variable, in discovery rather than a confirmation” (Henning, et al., 2004, p. 41). Hence, a case study method was used to generate rich data for the research investigation. Anderson (2000) defines a case study as “an investigation defined by an interest in a specific phenomenon within its real-life context”. It is a qualitative form of inquiry that relies on multiple sources of information (p. 121).

I engaged three Grade 11-12 mathematics teachers in the study. My focus was to elicit stories from them with regard to their experiences in designing and implementing a circle geometry teaching programme. Thus, the unit of analysis for this case study is their experience of this process. In a case study, “the interaction of the unit of study with its context, is a significant part of the investigation” (Babbie & Mouton, 2001, p. 281). It is also important in case studies “for events and situations to be allowed to speak for themselves, rather than to be largely interpreted, evaluated or judged by the researcher” (Cohen, et al., 2007, p. 254). I encouraged the participants to be reflective practitioners and share their experiences about the programme implementation with each other and me during the focus group interviews.

3.6 RESEARCH DESIGN

3.6.1 Designing the teaching programme

In this section, I present a synopsis of how the final draft of the teaching programme evolved. The programme design is however not the main focus of the analysis. It is the implementation of the teaching programme that forms the main focus of the analysis.

The design of the final draft of the teaching programme resulted from my initial consultations with my supervisor followed by discussions with and suggestions from the three participating teachers. This is a brief report of this process:

I started by developing the outline of an overall programme design in consultation with my supervisor who contributed ideas on how it should be structured. I then prepared handouts explaining in simple and brief terms the van Hiele theory (levels of thinking and phases of instruction) and gave these to the three teachers to read before our workshop. I also collected a variety of activities on circle geometry from old question papers, a mathematics web-site and the Namibian mathematics textbooks. We drew on these to populate the lessons.

I then asked the teachers to read the van Hiele handouts in order for them to acquaint themselves with the theory. I planned for the workshop presentation to be as brief as a possible because the teachers had indicated to me that they did not have much time to spare due to other commitments in their after school hours. I also gave the teachers a handout on the circle geometry curriculum that I had extracted from the Grade 11-12 mathematics syllabus. I asked the teachers to review the circle geometry curriculum while paying attention to the structure of circle geometry topics and sub-topics.

While the teachers familiarized themselves with the reading materials in their own time, I did further work on the programme design. I also extended my reading on the van Hiele phases to look for different types of interventions to see if I could select some that resonated with the intervention I had in mind. I drew on ideas about teaching interventions from Serow's (2008) research which used the van Hiele phases of instruction (referred to as phases of teaching and learning in her research) to align my selection of learning activities with the structure and sequence of her teaching intervention. The aim of my teaching programme is very similar to that of Serow's. I also drew from Courtney-Clarke's (2008) teaching intervention which had a reasonable structure of teaching instructions and learning activities.

I then held my first formal workshop with 2 of the teachers (the third teacher was not available at the time) which took about 40 minutes. When I explained the van Hiele theory to the teachers during the introductory workshop, I started with the levels of thinking as the phases of instruction are embedded within these levels. I explained the features of each level up to the third level of thinking (the abstract level). "At level 3, students can follow a short proof based on properties learned from concrete experiences, but they may not be able to derive such proofs themselves" (Senk, 1989, p. 310). As the teaching programme had to be designed for secondary school learners, it was only necessary to design it with criteria up to the third level of thinking. In fact, "the goal of most high school geometry course is to have students reasoning at the deductive (abstract) [italics added] level by the end of the year" (Groth, 2005, p. 28).

Using the phases of instruction I explained the transition from one level of thinking to the next and from this background information I explained the importance of their role in teaching using the van Hiele theory/model. Van Hiele (1986) recommends a set of instructional phases that teachers should follow in order to facilitate the students' movement between the van Hiele levels of geometric thinking. I explained the features of the van Hiele phases as discussed by the van Hieles (as cited in Fuys, et. al., 1984); Clements and Battista; 1992, Groth, 2005; Ding and Jones, 2007; Serow, 2008; Abdullah and Zakaria, 2011 and Mistretta, 2000.

During the workshop, I also reviewed the circle geometry curriculum with the teachers as it appears in the Broad Curriculum. The purpose of this review was to allow the teachers to discuss and decide upon the criteria of the teaching programme – given the stipulated basic competencies in the Grade 11-12 mathematics syllabus. If the circle geometry teaching programme is aimed at leading the learners to the abstract level of thinking in geometry, it was necessary to align its criteria to these basic competencies which targeted a similar goal. From there we started selecting suitable activities and discussed their appropriate sequencing. I took note of all the preferred activities and made copies that I used to compile a worksheet-handout that teachers gave to the learners during the lessons. This process of sequencing and activity selection continued beyond this formal workshop into the one-on-one sessions that followed with each of the teachers.

I met with each of the teachers on a number of occasions in one-on-one sessions to discuss improvements to the first draft of the teaching programme and to clarify ideas that were not clear to them. The teachers commented on the drafts of the teaching programme as they unfolded. In response to these comments, I altered the programme as often as necessary.

However, the teachers expressed surprise at the additional design work that I expected them to participate in. Teacher A said during our first one-on-one meeting that "I thought all I had to do was just to teach." Adding the workshop and programme design to the teachers' agendas was a burden that they agreed to carry. I explained the necessity of conducting the workshop in order to learn about the phases of instruction that they were required to use when implementing the teaching programme.

When I felt that the teaching programme was ready for implementation, I gave each teacher a copy of the most recent draft in order for them to prepare for lesson presentations (See Appendix E for the draft of the teaching programme). When the teachers felt they were ready to implement the programme, we worked out time slots for when they could carry out the actual implementation.

The reader should however note that the teachers could if they chose to, rely entirely on the teaching programme, as they were not expected to write their own lesson plans for the presentations. However, the teachers could choose to select their own 'normal' activities used for assessment purposes. They could do so from the list of activities provided, or opt to use their own. At this stage, teachers seemed to have a clear picture of how to align the activities to the van Hiele phases at each level of thinking. Teacher A reasoned during the introductory workshop. "This is a level two activity because it requires the learners to analyze the properties of the angles in this circle."

Furthermore, the teaching programme was designed and implemented in a limited period of time; I therefore had to be fairly directive in the pre-teaching training I did with them. The teachers started implementing the teaching programme and I then distanced myself from their actual practices and merely observed. These lessons were videotaped. I interviewed them after every lesson to find out how they felt about their practices. Data from classroom observations and interviews is analyzed in the next part of this chapter.

Table 3.1: The Circle Geometry Teaching programme

Guidelines for the teacher

[1]The intervention is sequenced using the van Hiele phases of instruction: information, directed orientation, explicitation, free orientation, and integration.

Activity and Phase	Teacher's Instruction/Activity Description
A. Circle geometry concepts **Information and Directed Orientation Phases**	1.1.1 Pupils work through simple constructions in their note books, using a mathematical set. Instructions include: a) Construct a circle with centre O. b) Draw a diameter and name it AB at the point of contact with the

Activities 1: angles in a semi-circle, Tangent of the circle, and tangents from an external point **Explicitation and Free Orientation Phases** →	circumference of the circle. c) Draw two chords, from A and B and let them intersect at point C on the circumference. d) Measure $\angle$ACB. Draw another pair of chords from A and B and let them intersect at C'. Measure $\angle$ACB. What did you notice? What conclusions can you draw from these angles?
Integration Phase	At this stage, the pupils should notice that an angle in a semi-circle equals 90 º. The teacher should ensure that the pupils master the correct technical language at this stage such as: semi-circle (the teacher should not sketch any figure at this stage; rather inspect pupils' constructions and ensure they all follow instructions accurately).
Directed Orientation Phase **Activities 2:** Angles between a radius and a tangent of the circle **Explicitation and Free Orientation Phases** **Activities 3:** Tangent of the circle, and tangents from an external point **Integration**	2.1.1 Construct another circle with centre O. a) Draw a radius and name it OT. Draw a tangent to the circle at T and name it ATB. b) Measure $\angle$OTB and $\angle$OTA. What did you notice? c) Draw another tangent from point A on the opposite side of the circle. Draw a radius to this tangent and mark the point of intersection P hence, the tangent APQ. d) Measure $\angle$OPA and $\angle$OPQ. What did you notice? e) Draw chord PT and measure the lengths of sides AP and AT. What did you notice? What conclusions can you draw from these angles? At this stage, pupils have mastered the concepts that angles formed between the radius and the circle equal 90º. Hence, the tangent and the radius of the circle are perpendicular. The teacher helps them to conclude that tangents from the same point are equal in length.
Directed Orientation and Explicitation Phases **Activities 4:** Angles subtended by the same arc are	3.1.1 The teacher is aware that the pupils' level of thinking has improved if not increased hence; instruction should also be a high level. At this stage, only the correct geometric language should be accepted. The teacher should assist the pupils where necessary for example, by sketching figures. Pupils are instructed to:

equal; angles in the same segment are equal

a) In figure 1, measure $\angle$ACB, $\angle$ADB and $\angle$AEB. Now measure angle AOB. What did you find?

Teacher: In figure 1, $\angle$AOB is an angle at the centre of the circle subtended by arc AB. Angels ACB, ADB and AEB are at the circumference of the circle subtended by arc AB (show arc AB to the pupils).

Free Orientation

b) In figure 2, reflex angle AOB is at the centre of the circle subtended by the major arc AB (show the major arc AB). Angle ACB is an angle at the circumference of the circle, subtended by the same major arc AB. Measure reflex angle AOB and $\angle$ACB. What did you notice?

c) Compare with the first circle. (You said earlier that the angle in a semi-circle equals 90º. Is this angle also on the circumference? Is so, what is the size of the angle at the centre of the circle?

Integration Phase

Let pupils refer to Figure one and introduce the concept of angles subtended by the same arc. Draw chord AB and talk about angles in the same segment.

Activity: Find the size of the angles *c, d, e* and give a reason for your answer.

At this stage, pupils have mastered the analysis level (van Hiele Level 2). They are now prepared and ready to take on thinking of the third level of thinking. This means that the teachers instruction will also be at Level 3 (higher than before), and so do the activities.

Information and Directed Orientation

4.1.1 The teacher directs the lesson toward the concepts of cyclic quadrilateral. She/he presents pupils with ready drawn figures as shown below. [The diagrams below are not to scale. *O* is the centre of the circle].

The teacher starts by drawing a circle on the chalkboard with two opposite angles ABC and $\angle$ADC subtended by the same chord (diameter AB). The idea here is to conceptualize concepts of cyclic quadrilateral by realizing that angles in opposite segments are supplementary.

Pupils are instructed to:

a) In each diagram, fill in the sizes of the angles at *O* and at *B* and *D*.

b) Calculate the sum of $\angle$B + $\angle$D. What did you find?

Explicitation and Free Orientation Phases

c) What is the sum of $\angle$BAD and $\angle$BCD? Give a reason for your answer.

5.1.1 Draw a cyclic quadrilateral ABCD.

	a) Measure all the angles of the cyclic quadrilateral. b) Find the sum of the two pairs of opposite angles. c) What can you conclude about angles in cyclic quadrilaterals?
Integration	In the following activities, inquire the pupils to give reasons for their answers using complete sentences in the correct geometric language. Pupils should be operating at VHL3 in order for them successfully complete this activity.

[1]The teaching sequence is adapted from Serow (2008, pp. 448-449). Activities adapted from Courtney-Clarke and Coulson (2008, pp. 203-213).

3.7 DATA COLLECTION METHODS

Anderson's (2000) study suggests that irrespective of the way you chose to collect data (e.g. document analysis, observation, and interviews) "it is imperative that the researcher try to understand phenomena and interpret the social reality from two perspectives, etic and emic" (pp. 123-125). I discuss the two perspectives according to Anderson (2000, p. 125) as follows:

Etic perspective: Understanding how the world looks from an etic perspective requires that the researcher constantly looks at phenomena and asks what the event or interaction mean to the individual. For example, I looked at the van Hiele phases of instruction, scrutinized the teaching programme and tried to understand how participants viewed their teaching in terms of the van Hiele phases of instruction.

Emic perspective: From the emic perspective the researcher acknowledges conceptual and theoretical understanding of the participants' social reality. The researcher considers the participants' perspective with the goal of trying to define, unravel, reveal or explicate their world. This perspective also requires a qualitative researcher to take cognizance of his or her own emic perspective.

Data collection methods included the following techniques: document analysis, observations and interviewing. I discuss my engagement with each method below:

3.7.1 Document analysis

The first stage of data collection process involved analyzing documents pertinent to this research study. I started the data collection by reviewing both the grade 11 mathematics syllabus and the examiner's reports of the grade 12 national examinations for the 2010 academic year.

Together with the participants' assistance, we reviewed the National Mathematics Syllabus for Grades 11-12 (Namibia. MoE, 2010c) does not specifically outline 'circle geometry' as its own domain within the geometry discipline. Thus, we focused only on the circle geometry portion of the curriculum. (See discussion of circle geometry curriculum in the previous chapter).

3.7.2 Observation

Classroom observations making use of video recordings

I undertook classroom observations during the second stage of data collection process. Maree (2010) defines observation as "the systematic process of recording the behavioural patterns of participants, objects and occurrences without necessarily questioning or communicating with them (pp. 83-84). In addition to our senses, intuition is used to gain a deep insight and understanding of the phenomenon being observed.

Anderson (2000) expounds that "observational data bring to the analysis and interpretation of a setting a type of information which cannot be garnered any other way" (p. 128). Depending on the nature of the research project, the research process can select from three methods of data observation within the research setting. Anderson (2000) distinguishes these as a complete observer, complete participant and participant observer.

For the purpose of this research, I became a complete observer as I entered the research setting and remained physically detached from the activities and social interactions (Anderson, 2000, p. 128). I observed two lessons from each of the three mathematics teachers within a period of three months. The purpose of classroom observation was to videotape the actual programme implementation. The videos were then used in the focus group interviews to reflect on the lessons in terms of the research questions asked.

Besides the videos, I used an additional and indispensable data source known as field notes. Patton (as cited in Anderson, 2000) warns that "field notes should contain a written comment of everything the researcher finds worthwhile; do not leave it to recall" (p. 128). Maree (2010) cautions about field notes and observation making the following points:

- Your field notes should be as accurate as possible. Record what you see, hear and experience as if you are seeing it for the first time.
- Always write up field notes as you make your observation. You may record your observation notes with a tape recorder, but make sure that you are clear about the context and participants.
- When recording events or behavior in a social setting make sure that you record both verbal and non-verbal behavior.
- Reflection on your observation should be done as soon as possible after the event and should include your hunches (it appears; it seems to be). (p. 86)

I wrote my field notes in a book where I kept records of all research related events and occurrences. Sometimes I recorded notes as the event unfolded. In cases where I forgot to bring the book along I would always record my immediate thoughts on a piece of paper. I kept records of the transition between the van Hiele levels during classroom observations. In the video recordings, I carefully noted how the teacher passed his/her pupils through the van Hiele phases of instruction. I wrote these down whenever I got a chance or else spoke softly into the camcorder so that I could transcribe it afterwards, when viewing the videos. For me the classroom observation process was the most exciting part of the data collection process.

3.7.3 Interviews

Maree (2010) describes an interview as a "two-way conversation in which the interviewer asks the participant questions to collect data and to learn about the ideas, beliefs, views, opinions and behaviours of the participant" (p. 87). The main aim of qualitative interviews is to obtain "rich descriptive data" that will help the researcher "to understand the participant's construction of knowledge and social reality" through his/her lens (Maree, 2010). I used two types of interviews for this study: focus group and open-ended interviews as discussed below.

Focus group interviews

Focus group interviews (Babbie & Mouton, 2001) were conducted with the teachers to explore their experiences with the introductory workshop, and designing and implementing the circle geometry teaching programme. It was during this stage of the research study that the data provided insight into how the teaching programme worked for the teachers. It was my role as a researcher to encourage and inspire the participants to share information as generously as possible.

Maree (2010) assumes that the focus group interview will be "productive in widening the range of responses, activating forgotten details of experience and releasing inhibitions that may otherwise discourage participants from disclosing information" (p. 90). Moreover, Anderson (2000) characterizes focus group interviews as follow:

> A focus group is a group process in which:
>
> - There is a group discussion of a predetermined issue or topics;
> - Group members share certain common characteristics;
> - The group is led by a facilitator or moderator;
> - The responses are usually recorded by an assistant moderator.

Experiences with the van Hiele phases of instruction were the predominant topic of the focus group interviews held with the research participants. The participants are all mathematics teachers in both the junior and senior secondary phase. I facilitated this group and asked a Computer Literacy teacher to video and audio record the responses. I chose to videotape the focus group discussions in order to capture any gestures that the participants might make. This was appropriate given my research goal which was to try to understand the teachers' 'experiences' with the circle geometry teaching programme.

Open-ended interviews

In addition to focus group interviews, I conducted an open-ended interview with each participant after the lesson presentations. This was done for reflection purposes. Maree (2010) maintains that an open-ended interview often takes the form of a conversation with the intention that the researcher explores with the participant his or her views, ideas, beliefs and attitudes about certain events or phenomena. I started these interviews by asking the teachers to talk about their practices in general and then to share with me their experiences with the circle geometry teaching programme in particular.

3.7.4 Triangulation

Cohen, et al. (2007) define triangulation as “the use of two or more methods of data collection in a study of some aspect of human behaviour” (p. 141). Maxwell (as cited in Maree, 2007) argues that triangulation “reduces the risk of chance associations and systematic bias, and relies on information collected from a diverse range of individuals, teams and settings, using a variety of methods” (p. 39). Different research techniques were used to collect data for this study and methodological triangulation was applied. I used different methods on the same object of study (Cohen, et al., 2007, p. 142). “By analogy, triangular techniques in the social science attempts to map out or explain more fully, the richness and complexity of human behaviour by studying it from more than one standpoint…” (Cohen, et al., 2007, p. 141).

3.8 SUMMARY OF THE DESIGN AND TOOLS

Phase	Techniques	Purpose	Analysis
1	Introductory workshop and discussions	Familiarize teachers with the van Hiele theory, particularly the phases of instruction.	Teachers’ experiences and views with the introduction of the van Hiele theory
2	Document analysis	To allow teachers to review the Broad Curriculum and to discuss and decide upon the teaching programme criteria.	Teachers’ experiences with documentation (the current geometry curriculum and the comments on pupils’ performance in geometry)
3	Classroom observation (video recording)	To capture actual implementation of the teaching programme by individual teachers	Video clips of the lessons
4	Focus group interviews Semi-structured interviews	Watch video recordings together. Discuss and share video recordings, personal experiences with the whole process	Personal and professional opinions and experiences (fears, interests, and way forward) about the teaching programme using the van Hiele phases as a conceptual framework.

Table 3.2: Summary of data collection tools

3.9 ANALYSIS

According to Cohen, et al., (2007), qualitative data analysis is about "making sense of data in terms of the participants' definitions of the situation, noting patterns, themes, categories and regularities" (p. 461). In my research project, data was analyzed as it was obtained. The analysis process included categorization of pre-conceived and emerging themes from phase one to four.

I did not have a structured observation schedule *per se* but I did have pre-conceived themes to alert me. These were the van Hiele phases of instruction: information, directed orientation, explicitation, free orientation and integration. The circle geometry teaching programme was designed using these phases. The presence of the phases of instruction in the teachers' practices was of most interest to me. I let the lessons unfold and waited for the phases of instruction to emerge. Although I expected the teachers to follow the teaching sequence, I could not rule out their possible diversion from the lesson objectives. That is why I took notes of any changes (diversions) from the structured circle geometry teaching programme. Data from focus group and open-ended interviews was transcribed and analyzed according to categories that aligned with the five van Hiele phases. I then synthesized the findings of each theme (phase).

3.10 ETHICAL CONSIDERATIONS

According to Cohen, et al., (2007), when conducting educational research, the researcher needs to address issues of confidentiality, anonymity, informed consent, gaining access to and acceptance in the research setting. In this study, I negotiated access to the school with the Headmaster by obtaining written permission from him.

I consulted with the teachers who participated in the study to explain my research plans and the benefits thereof. In addition, the consent of both pupils and their parents or guardians was obtained in a written form as well. I explained in the permission letters the nature of my research and assured them I would adhere to the principles of confidentiality and anonymity. Pupils whose parent(s) or guardian(s) for whatever reason refused to permit their children to be part of the research project, were not included in the research-planned lessons.

3.11 VALIDITY

Cohen, et al., (2007) define validity as a demonstration that a particular instrument measures what it signifies to measure (p. 133). Winter (as cited in Cohen, et al., 2007) adds that in qualitative research, "validity might be addressed through the honesty, depth, richness and scope of the data achieved, the participants approached, the extent of triangulation and the disinterestedness or objectivity of the researcher" (p. 133).

I triangulated the research by using three methods of data collection – documents analysis, observation and interviews. I tried to keep as distant as possible during the classroom observations in order to avoid manipulating the participants' reactions and responses. Validity was also ensured by adapting and using the work of other researchers. For example I adapted and used Serow's (2008) and Courtney-Clarke and Coulson's (2008) work to inform my teaching programme.

Worth noting is that conversely, while seeking to be an outsider and not interfere during any of the lessons, I was a very active participant and co-designer with the teachers of the teaching programme. This also needs to be considered when viewing the findings of the study.

3.12 LIMITATIONS AND CHALLENGES

Like all research, qualitative research has its limitations and challenges. I found it very difficult to congregate all focus group participants in the same place at the same time although they lived in the same geographical area (Cohen, et al., 2007). In fact, the school's calendar of activities made it almost impossible for me to conduct my research project. The teachers at this school are always busy with sports and other extramural activities from Mondays to Thursday as well as on Saturdays. This study taught me that that every second counts and every encounter is precious and crucial.

Because of the small sample, no generalizations beyond my case can be made of the research findings.

I encountered several disappointments along this research journey. These included lateness for lesson presentations and focus group discussions, absenteeism, time constraints, delays, school calendar mismatches, and the like. I constantly needed to be flexible to adapt my original plan to suit the routines of my participants.

3.13 CONCLUSION

For me, this was a very interesting yet emotional chapter to write. Although I had to adapt my original planned journey, I enjoyed telling this story.

The chapter presented a detailed account of the research paradigm, design and methodology according to which research is conducted. This case study research is oriented within the interpretive paradigm hence the use of the qualitative approach. The case study focused on three mathematics teachers. I employed three qualitative methods of data collection i.e. document analysis, classroom observation and interviewing.

The chapter ends with a discussion on triangulation, ethical issues and limitations and the challenges I encountered. The next chapter analyzes and describes the findings of the collected data.

CHAPTER FOUR

DATA ANALYSIS AND DISCUSSIONS

4.1 INTRODUCTION

In this chapter, I discuss the findings and analysis of my research project. The nature of my research is underpinned by the van Hiele theory of geometric thinking and reasoning. Therefore, the data collected for this research is analyzed in alignment with the van Hiele phases of instruction as a conceptual framework.

Qualitative data analysis is a process of making sense of collected data "in terms of the participants' definitions of the situation, noting patterns, themes, categories and regularities" (Cohen et al., 2007, p. 461). Similarly, McMillan and Schumacher (2001) defined data analysis as "describing data in meaningful terms" (p. 213).

I analyzed the data on the implementation of the teaching programme. I concluded with the research findings based on the purpose of my research which was to investigate the mathematics teachers' experiences with the designing and implementing a circle geometry teaching programme.

4.2 ANALYSIS OF IMPLEMENTING THE TEACHING PROGRAMME

In the next section I analyze the data from the three mathematics teachers in terms of the five phases of instruction. I discuss the actual implementation of the circle geometry teaching programme by the three mathematics teachers named: Teacher A, Teacher B and Teacher C. I present and analyze the classroom observations as they happened during the actual implementation of the teaching programme.

4.2.1 A reminder of the five phases of instruction

According to the van Hiele (1986) the movement from one level of geometric thinking to the next level incorporates five phases of instruction: information, directed orientation, explicitation, free orientation and integration. It is useful to provide a short reminder about the key characteristics of the van Hiele phases of instruction as discussed in the literature review chapter. The five phases form the basis and framework of the narrative for this chapter.

The first phase of instruction is the *information phase*. During this phase, the teacher places the learning materials at the learners' disposal (Clements & Battista, 1992) and engages them in a discussion talking and learning about their pre-knowledge. Hence, learners are introduced to the subject matter to be studied at this stage.

The second phase of instruction is the *directed orientation phase*. During this phase, the teacher guides the learners towards the content to be studied while at the same time instilling the correct usage of geometric language. However, learners' own language may also be accepted, but often under correction. It is the teacher's role to direct the learners' learning activities by providing them with well-planned tasks, at the right van Hiele level. The teacher should also ensure that the aim of teaching and learning is well known to the learners at this phase (Clements & Battista, 1992).

The third phase is the *explicitation phase* which is also known as the explanation phase. During this phase, the teacher requires learners to explicate their responses while attending to their technical geometric language usage. Clements and Battista (1992) assert that during the explicitation phase, the teacher should accept the learners' own language during content discussions. However, the teacher should ensure that "relevant mathematical terminology" is explained to the learners (p. 431). The learners are made aware of the proper and appropriate geometric terminologies.

The fourth phase is the *free orientation* phase. During this phase, the teacher prepares the learners for advanced reasoning in geometry by selecting appropriate geometrical problems that learners should complete under the teacher's observation. The teacher provides limited or no assistance to the learners at this phase. Nevertheless, it remains the teacher's role to "introduce terms, concepts and relevant problem-solving needed *to lead the learners to a higher level of thinking in geometry* [italics added]" (Clements & Battista, 1992, p. 431).

The last phase is the *integration phase*. The teacher's role during this phase is to encourage the learners to draw conclusions from the geometric knowledge learnt at each van Hiele level of geometric thinking. Language is an important instructional aspect of the van Hiele theory hence the teacher needs to make explicit the language required for that particular level (Teppo, 1991, p. 213).

In the next section I analyze the data from the three participants in terms of the five phase of instruction.

4.3 INFORMATION PHASE

4.3.1 Teacher A

Teacher A introduced the lesson by telling the learners that they were going to learn about circle geometry. She then wrote the heading ‘Circle Geometry’ on the chalkboard and underlined it. Although Teacher A might have believed that at Grade 11, learners are expected to have an understanding of circles, she wanted to make sure that her assumptions were accurate. The teacher therefore conversed at length with the learners throughout the early introduction of the lesson. This is crucial to van Hiele phase one of instruction as it states that it is by communicating with the learners that we learn about their pre-knowledge of the subject matter.

I observed how the teacher encouraged the learners when she asked them to label as many parts of the circle as they possibly could. In this way, the teacher learned exactly what the learners knew and did not know, and how she would go about teaching them. Teacher A asserted in the interview that “*learners do not really know everything they claim to know so, as a teacher I must make sure that I do not get a wrong impression of their existing knowledge*.” In the van Hiele phases of instruction, the information phase is necessary for the introduction part of the lesson. Therefore, it is important that the teacher establishes the learners’ pre-knowledge as accurately and as early as possible.

The teacher further asserted that she knew that the learners might label many parts of the circle to impress her or the researcher in their class without really thinking about their own learning. “*Learning without thinking is not proper learning*” exclaimed Teacher A. I can therefore conclude at this juncture that the events of the classroom observations during the information phase aligned well with what the teacher said in the interviews. The teacher openly practiced what she said she would. I observed her lesson closely and could see how she organized her instructions according to phase one’s features.

4.3.2 Teacher B

Like Teacher A, Teacher B also introduced the lesson by letting the learners know that they were going to learn about circle geometry. The teacher informed the learners that they were expected to have in their possession a fully equipped mathematical set in order for them to cope during the lesson. The teacher further informed the learners that all drawings were to be done in pencil only and that they should sharpen their pencils to avoid drawings lines that were not clear and thus might be inaccurate.

The teacher assumed that informing the learners about the necessity of using a pencil in geometrical constructions is an important aspect of the van Hiele phases of instruction. "*You told me about the phases of instruction, so I think that the sooner the learners learn about the stationery required in circle geometry the better. In fact, this is the information phase and I was just trying to inform them*." I concur with Teacher B. The information phase is aimed exactly at telling the learners what is expected of them. This includes the technicalities of geometric constructions.

I asked Teacher B to enlighten me more on his experiences with the information phase. Teacher B stated that it was I who introduced him to the van Hiele phases of instruction and that the first thing that a teacher should do is to inform the learners about the subject matter to be studied i.e. circle geometry. Teacher B explained that in addition, as required in the information phase, he needed to find out what the learners knew about circle geometry. "*That is why I began the lesson by asking the learners to draw and label all the circle geometry concepts that they could remember*".

Teacher B added that the information phase of the van Hiele phases of instruction is very crucial as it is during this phase that learners are made aware of what is that they had to learn. The teacher therefore seized the opportunity to discover the learners' existing knowledge. "*I know that most of them could label parts of the circle accurately but they were unable to apply them at this stage. That was the reason why I only asked them to label and not say much about the concepts as I still wanted to find out what they knew about circle geometry*."

4.3.3 Teacher C

Similarly, Teacher C also introduced her lesson by informing the learners about the content to be studied. She then asked a few questions to test the learners' pre-knowledge of circles in general before leading them to basic competencies of the lesson. The teacher told the class that they were going to learn about circle geometry concepts and angle properties of circles. "*Today we are going to look at geometry. We are going to talk about angles in circles but before we proceed, we need to know what a circle actually is*."

The teacher asked the learners to define a circle. She listened to their different definitions and used these to summarize a definition of a circle that she then wrote on the chalkboard. The teacher also asked learners to name and define circle geometry concepts such as radius, diameter, circumference, chord, arc etc. before she drew and labeled a circle with these concepts. She encouraged the learners to participate actively in the classroom activities by giving each learner an equal chance to engage.

I asked the teacher to describe her actions during the information phase in relation to what happened in the classroom. The teacher alleged that she was aware that learners in Grade 11 would be familiar with circle geometry concepts and a few theorems which is why she started by asking them to name and define circle geometry concepts as a way of introducing the lesson. "*I knew that they learned geometry and circles in previous grades, I also know that they applied those concepts during problem-solving*." It therefore remained the teacher's role during the information phase to determine how much learners knew about circle geometry to enable her to locate an appropriate area of instruction for the remaining van Hiele phases of instruction. I noted that the information phase helped the teacher to introduce her lesson while at the same time assessing the learners' pre-knowledge as well as gaining insight for the way forward.

4.3.4 Synthesis of Teacher A, B and C

It is evident that all three teachers were interested determining their learners' pre-knowledge of circle geometry. All teachers started their lessons by asking the learners to describe what they know about circles and specific parts of the circle. Inquiring about the learners' background knowledge of the subject matter is embedded within van Hiele's phases of instruction framework. Therefore, this engagement from the teachers ascertaining the learners' prior knowledge and informing the learners about the direction the lesson will take fits in exactly with the van Hiele requirements (Crowley 1987, p. 5).

Although all three teachers used the designed teaching programme to plan and teach their lessons, I noticed several differences among their practice. This is because the three teachers used different teaching methodologies to present their lessons. Teacher A asked the learners to draw a circle and they filled in circle geometry concepts together. Teacher B asked learners to draw a circle and fill in all the circle geometry concepts that they could remember. Learners did so without the teacher's help. Teacher C on the other hand asked learners to talk about concepts first and then draw the circle later. Even though the three teachers' approaches were different, they all reached a similar objective of obtaining information about the learners' existing knowledge of circle geometry.

4.4 DIRECTED ORIENTATION PHASE

4.4.1 Teacher A

During the directed orientation phase, Teacher A instructed the learners to differentiate between similar related concepts of the circle that were dealt with during the information phase i.e. chord and secant, sector and segment. The teacher stated that learners need to be thorough with circle geometry concepts as they are a prerequisite for unpacking the angle properties of circles.

Thereafter, Teacher A asked learners to draw a few circles that she planned to use during the lesson presentation for instructional purposes at a later stage. The teacher then instructed the learners to consider the first circle and draw two radii: *OA* and *OB*. As she wanted all learners to have similar drawings, she instructed them how she wanted the radii to be drawn in the circles. "*Let the angle subtended by the radii at the centre of the circle equal 120º*." The teacher explained to the children that the $\angle 120^\circ$ is called the angle at the centre. That is the angle subtended at the centre of the circle by two radii and an arc of the circle. "*Now I want you to draw two chords; one from points A and the other from point B and let them intersect at a certain point C on the circumference of the circle*." The teacher then asked the learners to measure the size of the angle formed at point *C*; that is $\angle ACB$ at the circumference of the circle. "*Now I want you to draw for me another pair of chords from each A and B and let them intersect* *at a certain point D on the circumference.*" The teacher asked the learners to measure the size of $\angle ADB$ and asked the learners to explain what they found.

The learners engaged in a productive discussion with the teacher about their discoveries. "*They discovered the two angle properties of circle geometry during that activity*." The teacher directed the learners towards the discovery of 'an angle at the centre is twice the angle at the circumference theorem' and 'angles in the same segment are equal'. The teacher was excited about the progress of the lesson and provided a class activity immediately after the learners learnt about the two angle properties. "*The reason for the activity was to determine whether the learners could apply what they were taught and learnt*."

Activity

In figure 4.1, points L, M and N lie at the circumference of the circle centre O. $\angle MON$ *at the centre of the circle equals 98º.*

Instruction*: Calculate the size of* $\angle MLN$*. Show your method and give a reason for your answer.*

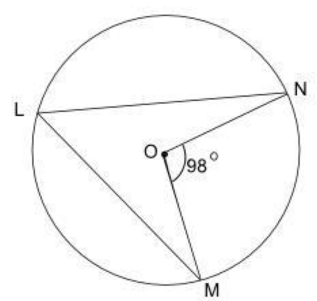

Figure 4.1

The learners did not waste any time computing the solution to the above activity. As the teacher expected, learners divided 98º by 2 to get 49º which is the size of $\angle MLN$. The teacher was confident that learners would not to get a wrong answer as she considered the activity to be easy and less challenging. "*They discovered the theorem on their own and I did not expect them to fail at all*."

I asked the teacher to discuss her views on the directed orientation phase and this is what she said: "*The directed orientation phase is easy as it comes right after you learn about the learners'* *background knowledge. So, what I did was to give them an exercise that required knowledge of circle geometry concepts and basic angle properties to complete the activity successfully. As you might have observed, that was exactly what happened*."

Learners gave the solution to the activity without hesitating because they were well directed by the teacher at the beginning. Consistent naming of angles also helped to ease the teacher's instructions and made it easy for everybody to understand each other when giving solutions as they all had the same drawings in their books. Their reasoning aligned well with the teacher's expectations as she said earlier that she did not expect them to fail.

4.4.2 Teacher B

The teacher moved the learners from the information phase to the direct orientation phase using a variety of instructions. Before Teacher B gave the learners an activity, he first assessed their work on the basic concepts of circle geometry. The teacher also assessed the learners' drawings, labels and spelling of circle geometry terminologies. This was because the teacher wanted to see if learners labeled the correct name to its corresponding concept in the circle. Unexpectedly, some learners mixed up the concepts and the teacher assisted them to rectify their mistakes. Teacher B preferred that learners recognize their own mistakes and work by themselves to correct them. That is why he kept on probing them until they realized what was wrong with their labeling. The teacher also wanted the learners to think for themselves and to avoid relying too much on the teacher to get correct answers. "*I prefer they fight hard to understand the concepts rather than relying too much on me*."

Teacher B instructed the learners to draw a circle "*in a way that it has a set of points that are 5 cm away from the fixed point*". The whole class looked puzzled and the teacher immediately realized that there was something in his instruction that bothered them. Teacher B immediately acknowledged that he did not define a circumference of the circle as a set of points that is a particular fixed distance from the centre. Therefore, learners felt like he was talking about something they had not yet learnt. When the teacher realized that learners did not understand his instruction, he posed the question in a different way in a language which was more familiar to them.

Teacher B later asked the learners to draw a diameter and name it AB at the points where it intersected the circumference of the circle. Instruction continued until an angle in a semi-circle was drawn. The teacher used the learners' constructions to explain the theorem of angles in semi-circles, allowing them to discover angle properties on their own.

Furthermore, Teacher B instructed the learners to draw two more circles and used them to explain the properties of angles in the semi-circle, tangents from the same external point, angle at the centre and angle in the circumference, as well as angles in cyclic quadrilaterals. This took the teacher some time to explain before he provided a class activity for assessment purposes. Here is an example of one of the questions that Teacher B asked his learners to complete.

Activity: *Angles between the tangent and radius of the circle; tangents from the same external points; angle at the centre-angle at the circumference; angles in the same segment.*

The teacher instructed the learners to draw a circle centre *O* and two radii *OA* and *OB* that are 140º apart at the centre of the circle. When the learners were done with this construction, Teacher B drew the same figure on the chalkboard and informed the learners that they would discover the angle properties together. The teacher instructed the learners to draw two tangents to the circle at point *A* and then at point *B*. He told them to let the two tangents intersect at an external point *T*. Teacher B stated in the interview that the purpose of drawing the radii the same angle apart for every learner was so that the whole class produced the same figures eventually and would therefore furnish the same answers when asked. "*This helps for better understanding and also to clear out any misconceptions that learners might have brought with them to the classroom*." The teacher instructed the learners to use their protractors and measure the angles between the tangents and the radii of the circle say $\angle OAT$ and $\angle OBT$. The learners were also instructed to use their rulers and measure the lengths of the two tangents, *AT* and *BT* and explain their findings.

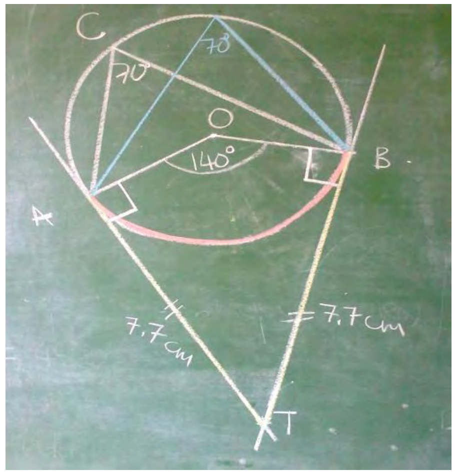

Figure 4.2: Snapshot of Teacher B's lesson presentation

Teacher B also used the circle in figure 4.2 to explain the 'angle at the centre-angle at the circumference' theorem. The teacher asked the learners to draw two chords, from points *A* and *B* that intersect at a certain point *C* at the circumference of the circle. Teacher B then asked the learners to use their protractors and measure the size of $\angle ACB$; an angle at the circumference of the circle. He further instructed them to draw another pair of chords from A and B and let them intersect at a certain point D at the circumference of the circle. "*Measure the size of angle ADB using your protractor and tell me what you found. What conclusion can you draw about angles AOB, ACB and ADB*?"

The learners answered that the two angles at the circumference are a half each of the angle at the centre. Teacher B congratulated the learners for their discovery and summarized four theories learned from Figure 4.2 above. These are: *1. An angle between a radius and a tangent of the circle is always 90º. 2. Tangents drawn from the same external points have the same length. 3. An angle at the centre of the circle is twice the angle at the circumference subtended by the same arc (i.e. arc AB). 4. Angles in the same segment subtended by the same arc are equal.*

When I asked him what he noticed about the directed orientation phase, he expressed that his lesson was a success as the learners performed as expected. The teacher revealed that it is thanks to the van Hiele teaching programme that his learners discovered the theorems which were central for the activity provided. "*They discovered all the angle properties on their own. All that I did was only to instruct and direct them"*. Directing the learners is the core purpose of the second phase of instruction. The teacher was amazed by his own practice as he confessed that he had never experienced such liveliness in his learners before. "*I did not even tell them anything; I only helped them with their constructions and they managed to do everything else on their own*." At this stage, learners were prepared to work more independently and the teacher planned for more challenging activities as the learners' performance improves.

4.4.3 Teacher C

Under the directed orientation phase, Teacher C provided structured activities that allowed learners to become more acquainted with circle geometry. She instructed them to construct a circle with centre *O*, radii OA and OB with a reflex angle of 200º at the centre. "*Draw two chords drawn from each point A and B and let them intersect at a certain point C at the circumference of the circle*." After the learners were done with the constructions, the teacher instructed them to measure the size of the angle that the two chords formed at the circumference; that is $\angle ACB$. She also asked them to draw another pair of chords form *A* and *B* that intersect at the major arc *AB*, that is, at the opposite side of the drawn $\angle ACB$. See figure 4.3 below for a completed shape.

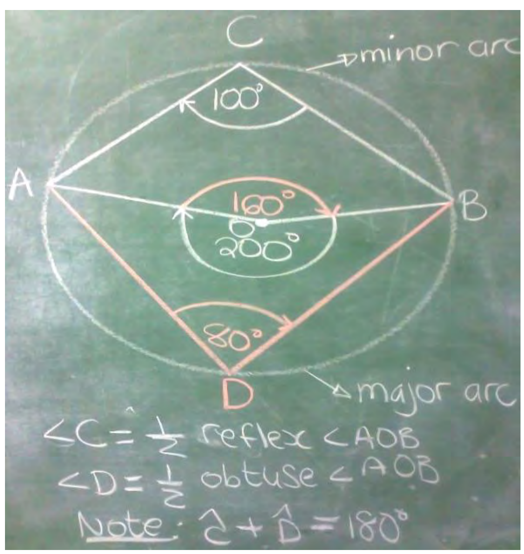

Figure 4.3: Snapshot of Teacher C's lesson presentation

Teacher C stated that the purpose of this activity was to elicit specific responses while at the same time assessing the learners' usage of circle geometry terminologies. The teacher also wanted to explain angles at the centre-angle at the circumference property. She claimed during the interview that learners are used to constructing the acute and obtuse angles subtended by an angle at the centre ≤180º. She therefore used the reflex angle at the centre to ensure that learners were provided with sufficient knowledge of all the angle properties of circles. "*If the learners are well informed and directed early in their learning, then they will experience only minor problems with the application of knowledge learned. But if they knew too little, then they will always struggle with assessment activities and their performance will remain questionable*."

The teacher ensured that the learners followed her instructions during lesson presentations by insisting that they listened while she explained the content and by observing their work while they were busy. The teacher walked between the rows to see that the learners did as instructed. She also corrected the learners own language that she found inappropriate for certain reasoning and replaced it with what she called the accepted language of geometry.

Teacher C's instructions during the directed orientation phase was very similar to that of Teacher B as she led her learners to discover angles between tangents and radii of circles. She also directed her learners to discover the lengths of tangents drawn from the same external point theorems as it was the case with Teacher B. It was interesting to note that as the lesson proceeded, Teacher C gave fewer and fewer instructions to the learners. For example, towards the end of the lesson Teacher refrained from reminding the learners to mark off the centre when she asked them to construct a circle.

The teacher acknowledged during the reflective interview that she was not sure about the van Hiele phases and levels when she taught the first lesson. However, when I informed her about the van Hiele theory in the interview she excitedly realized that she indeed passed her learners through van Hiele level one and two. She also realized that her instructions were framed within the phases of instruction framework. *"At first I have not noticed much because I was not really sure if I understood this van Hiele theory but I think I started from the beginning like you are telling them what to do and then you are letting them to apply what you have told them onto the board and show their working".*

Teacher C confessed in the interview that she felt the intensity of the lesson when she taught using the van Hiele phases of instruction and her confidence in teaching grew as the lesson progressed. The teacher added that the structure of the teaching programme framed by the van

Hiele phases of instruction helped her with the organization of both the lesson presentation and the activities planned to enhance learning. "*I was not very confident in the beginning but when I realized that learners understood and followed my instruction; I was sure that I was actually doing the right thing*."

4.4.4 Synthesis of Teacher A, B and C

All teachers instructed learners to draw circles and discover angle properties of circle geometry on their own before writing and explaining the theorems to them. I also noticed that all three teachers asked their learners to use the same labeling in their constructions i.e. radii *OA* and *OB*. This was because the teachers used the same teaching programme that we designed during our introductory workshop as part of the research process. Where assessment activities were provided, they all aligned with the lesson objectives with the purpose of eliciting geometric reasoning from the learners' side.

However, the three teachers also differed in the way they approached the planned content in the teaching programme. As I stated earlier, I did not expect the teachers to follow the teaching programme to the book as it is flexible in its design. Teacher B and C used similar approaches as they asked the learners to draw several angles to discover the theorem of angles in the semi-circle on their own. Teacher A on the other hand preferred to draw everything on the chalkboard and explain to the learners. She would then provide an activity after every explanation to assess if learning really took place.

4.5 EXPLICITATION PHASE

4.5.1 Teacher A

During the explicitation phase, Teacher A instructed the learners to refer to the circle drawn earlier in order for them to answer her questions. She started by defining basic concepts of circle geometry i.e. centre, radius, diameter, circumference, and chord etc. She then asked the learners to differentiate between closely related circle geometry concepts such as a chord and a secant and a sector and a segment. The teacher told the learners that she was aware of their familiarity with circle geometry concepts but she was going to ask them to explain certain concepts anyway.

When asked to define the circumference of the circle, Teacher A realized that learners still relied on their own language. They defined circumference as a line that runs around the circle. The teacher was not satisfied with the response and asked them to copy the definition she wrote on the chalkboard. She defined the circumference of the circle as a set of points that is equidistant (equal distance) from the fixed point (centre of the circle).

It was clear that at this phase, learners were still going through a transformation between their own language and the accepted geometric language. They answered most of the tasks in a non-geometric language and the teacher corrected them without discouraging the use of their own language. "*If I discourage them maybe they will stop participating. My aim was to have them answer the questions however way they could and I will give them new definitions of concepts alongside their own*". The teacher also required that learners use full sentences when answering questions especially those that needed reasoning of any kind.

One learner differentiated between a sector and a segment that "*a sector is an area subtended by an arc and two radiuses* [*sic*] *while a segment is a region subtended by an arc and a chord of the circle*". The teacher immediately noted the incorrect usage of the plural form of radius. She corrected the learner and cautioned the class about the correct usage of the geometric language once again.

I reminded Teacher A during the first interview that I noticed how persistent she was about the application of geometric language. The teacher noted that she had to ask the leaners for definitions because she did not believe that they had a clear understanding of them. "*I wanted to see whether they had a clear understanding of what they were talking about. You saw how they used simple language in the beginning but improved as I kept on correcting them*".

During the second lesson that I observed of Teacher A, I could not help but notice how differently she approached the content under study in comparison to her first lesson. She reminded the learners that they were going to apply knowledge of known basic concepts to seek solutions to circle geometry problems. Hence, she provided activities that allowed learners to become more acquainted with the materials being taught (Mistretta, 2000, p. 367). Here is an example of one of the questions in the learning activity:

Activity

In figure4.4 below, points S, U and T lie on the circumference of the circle centre; O. RS and RT are tangents to the circle at S and T respectively. $\angle SRT$ equals 40º.

***Instructions**:*

1) *Write down all the circle geometry concepts that you can see i.e. centre O.*
2) *Write down all the circle geometry theorems that we learned.*
3) *Calculate the size of $\angle SUT$. Show all your working and give a reason for your answer.*

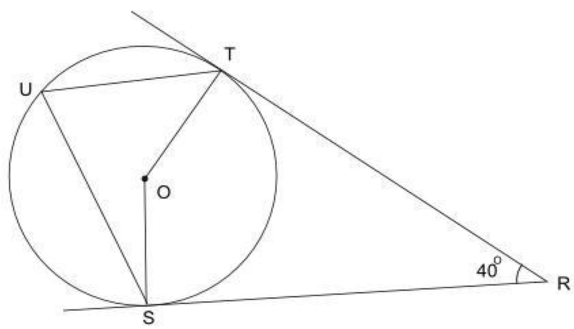

Figure 4.4

There were several steps that learners needed to remember in order to find a solution to this problem. It was also necessary that they consulted the notes provided in the information phase to enable them find the size of the required angle with ease. These notes included information on an angle between a radius and tangent theorem, opposite angles of cyclic quadrilaterals and the angle at the centre twice angle at the circumference property which in return provided the solution to the learning activity.

Teacher A said that the purpose of this activity was not only to calculate the size $\angle SUT$. She also wanted the learners to revisit circle geometry concepts and angle properties as described above in order for them to have a clear understanding of the application of the basic concepts when calculating. It is important that leaners acquire knowledge that is conceptual in nature. It is this knowledge that often acts as a springboard when learners carry out procedural tasks.

I asked Teacher A what she would like to share with me regarding the explicitation phase and this is what she said: "Y*ou told me that the explicitation phase is like an explanation. So, I asked the learners to explain their solutions as you would have observed in the lesson. Most of them were familiar with the concepts and knew very well how to apply them to solve geometric problems*". I noticed that not only the learners explained their solutions to geometric problems, the teacher also did her part by ensuring that they applied the correct geometric language at all times. After all, it is the purpose of the explicitation phase that learners become aware of their language and the language of circle geometry while the teacher ensures that terminologies are accurately explicated.

4.5.2 Teacher B

Teacher B was not only careful about how learners explained terminologies but also how they spelt conceptual words. While observing their work, Teacher B came across a certain learner who misspelt the word tangent (spelt as tagent). The teacher asked the learner to read the word out loud but the learner had difficulty reading the word. "*When I asked the learner to read the word out loud I knew that she would not do so as she did not spell the word accurately. I wanted her to realize that she had made a mistake and that was why I did not correct her right away.*"

In the second lesson that I observed, Teacher B's instructions were more advanced in comparison to how she instructed the learners during the first lesson. He used scaffolding to instruct learners to construct a circle and discover angle properties of cyclic quadrilaterals. Teacher B encouraged learners to use descriptive words in their explanations as a way of nurturing their geometric language. Even though the teacher accepted the learners' own language at this phase, he ensured that he instilled the usage of correct terminologies and definition of concepts. He instructed them to complete a quick activity that required appropriate knowledge of circle geometry concepts. Here is an example of the activity that Teacher B gave to the learners to complete during the lesson:

Activity 1

In figure 4.5 below, points A, B, C and D lie at the circumference of the circle with the centre indicated. Angles A and C equals x and y respectively.

Instruction: *Use the given diagram to find the sizes of each of the following angles and give reasons for your answer.*

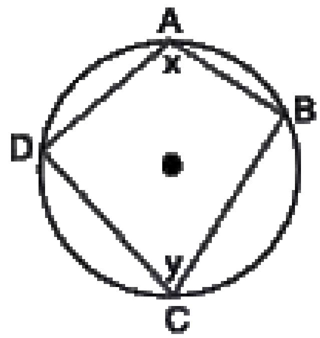

Figure 4.5

(a) *If $\angle y = 70^o$, find the size of $\angle x$.*
(b) *What is the sum of $\angle ABC$ and $\angle ADC$?*

Teacher B asked the learners for the sum of $\angle ABC$ and $\angle ADC$ as he wanted them to apply the knowledge of opposite angles in cyclic quadrilateral that they just learnt. "*I wanted them to draw a conclusion with regard to the sum of the two opposite angles*". At this stage, learners are expected to have mastered reasoning based on the analysis of the properties of a figure (van Hiele Level 2). They are now prepared and ready to tackle more complicated tasks as the teacher's instructions become more complex.

I asked the teacher about his experience with the explicitation phase and this was what he said: "*I think that the explicitation phase is the one that is mostly used by the teachers to thoroughly explain the content to the learners. For example, if you ask learners to give reasons for their answers i.e. in the exercise that I gave them on cyclic quadrilateral, you will sometimes find them reasoning that the sum of two opposite angles is 180º because the quadrilateral is inside a circle. Sometimes they are right but just not using the correct language at that particular moment*". It therefore remains the teacher's role to ensure that the acceptable geometric language is instilled throughout the explicitation phase.

Language is an important aspect of learning in the van Hiele theory as it gives structure to teachers' instructions during the lesson. Therefore, "language is useful because by the mention of the word parts of a structure can be called up" (van Hiele 1986, p. 86). The use of correct language at each phase enhances understanding of concepts at that particular phase while the teacher acts as a driving force behind that understanding. This is why the phases of instruction are important. It remains the teacher's responsibility to ensure that learners are well equipped with the appropriate geometric knowledge and language of each of the van Hiele phases of instruction and necessary level of thinking in geometry. Fortunately, Teacher B understood the van Hiele theory well from the beginning and had few difficulties aligning his teaching to the van Hiele phases of instruction. His lesson went well and he was glad that despite minor problems, the learners managed to reason well when he asked them to motivate their answers.

4.5.3 Teacher C

At the beginning of the explicitation phase, Teacher C asked the learners to define a diameter of a circle. The learners answered that a diameter is any line that passes through the centre of the circle and has its endpoints at the circumference of the circle. The teacher drew a circle, marked off the centre and drew a number of lines on the circle that passed through the centre of the circle (Fig. 4.6). Teacher C asked the learners to identify which lines are not diameters of the circle. This exercise confused some learners as they did not know how to make their choices.

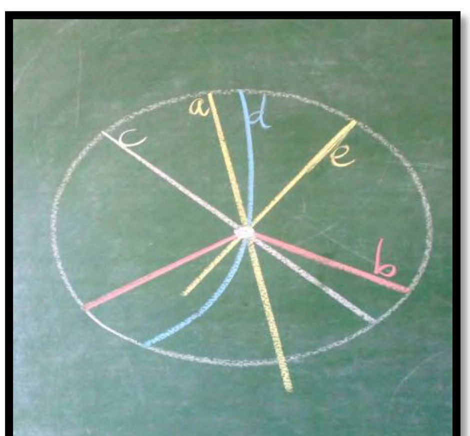

Figure 4.6: Snapshot of Teacher A's lesson

Teacher C stated that she provided the exercise deliberately to help clear out that confusion. "*That was a counter example of the diameter. I knew that many learners did not have a strong* *understanding of basic concepts of circles that is why some of them chose the curved line to be one of the diameters of that circle*." The teacher warned the learners about the following non-examples of diameters of circles:

- *It does not mean that if a line goes through the centre of the circle is always a diameter of that circle. It must satisfy all features of the diameter first.*
- *A line passing through the centre of the circle must be a straight line (not curved) in order for it to be considered a diameter of that circle.*
- *A straight line that passes through the centre of the circle and does not intersect one side or both sides of the circumference is not a diameter of that circle.*

The teacher explicitly defined a diameter of the circle and cleared any misconceptions that learners might have brought to class that day. It was a good exercise that enhanced knowledge not only for that particular lesson but for further applications in problem solving in geometry as well.

Teacher C then instructed the learners to draw a circle centre O and diameter AB. "*I want you to draw two chords from both point A and B and let them intersect at a certain point C at the circumference of the circle*". The purpose of this instruction was for the learners to draw an angle in a semi-circle with the teacher's direct assistance. Without observing the work at that point, Teacher C asked the learners to measure an angle that is formed by the two chords at the circumference of the circle; that is $\angle ACB$. The teacher required the learners to write down their answers and continue to follow instructions. She asked them again to draw another pair of chords that intersect at points D, E and F at the circumference. "*I want you to measure all the angles and tell me what you found*."

Some learners answered that all angles equaled 90º while some learners protested that that was not always the case. Teacher C immediately demanded to see everybody's work when she realized that some learners had not adhered to instructions. Some of them made the chords intersect in the segment instead of the circumference. After the learners rectified their mistakes, they all drew conclusions from their findings using the teacher's scaffold. They concluded that angles formed by two chords drawn from either end of the diameter always intersect at a right angle.

One learner concluded that if two chords are drawn from either end of the diameter and intersect at the circumference, then such chords will always be perpendicular to each other. Teacher C wrote the circle geometry theorem of angles formed in the semi-circles in addition to the learners' findings. "*Have you noticed that these learners only need a little push and then their thinking is already in order*?" It was impressive to see the satisfaction on Teacher C's face given that she was initially nervous about the whole programme implementation idea. "*I cannot believe I was so nervous about something so interesting like this teaching programme*."

From the teacher's instructions, the learners learned that the pairs of chords that they were to draw were not supposed to intersect anywhere in the circle but only at the circumference of the circle. As the lesson progressed, the teacher directed them less and less when they discovered other angle properties of circles such as angles between tangents and radii of circles and opposite angles in cyclic quadrilaterals. The teacher later asked the learners to complete a task about the property they just learned. Here is an example of the activity that Teacher A gave to the learners completed during the lesson:

Activity 1

In figure 4.7 below, points A, B and C lie on the circumference of the circle.

Instruction*: Use the circle below to answer the following questions.*

a) *Identify and define the diameter of the circle.*
b) *If $\angle BAC$ is 26^o, calculate the size of $\angle ACB$ and give a reason for your answer.*

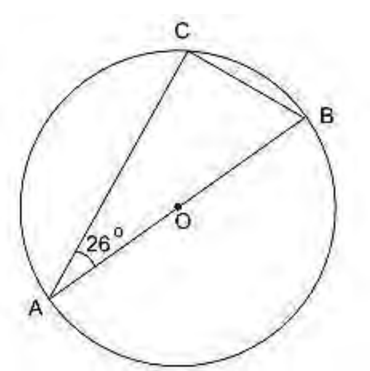

Figure 4.7

The learners had to find the size of the required angle and motivate their answers. As expected, the learners subtracted 26º from 90º reasoning that $\angle$B is in the semi-circle and $\angle$C equals 90º.Some learners subtracted 55º + 90º from 180º with a reason that figure *ABC* is right angled triangle whose sum of the interior angles equals 180º.

The teacher stated in the interview that the learners were thinking at an advanced level and she was going to change her instructions and expectations in order to increase their knowledge capacity of circle geometry. She further stated that she was going to set more complex activities as the learners' progress improved. The teacher also emphasized the correct usage of technical language. She did this by constantly correcting the learners' own language whenever necessary.

I observed that Teacher C constantly asked for clarity from the learners as they responded to her questions. For example, when the learners gave a solution, the teacher asked for an explanation and the lesson learned from the problem solved. The teacher sometimes used this opportunity to test if the learners were sure of their solutions. She would pretend that she was not sure of the solution herself. Thus, the learners were taught to think carefully about their responses and trust their own solutions if they believed that they were correct.

When asked her opinion of the explicitation phase Teacher C immediately asked if this was not the phase where the explanation of concepts occurs in more detail. "*This is where the explanation of concepts occurs isn't it? So, I think I tackled this one very well. I gave them definitions of circle geometry terminologies and kept on asking the learners if they understood my explanation. I also gave them time to explain their solutions which went well because they knew that if they gave invalid answers then they will have to explain them*".

Teacher C's lesson progressed from less structured circle geometry related tasks to more structured tasks. Learners participated well in the lesson activities and explained their answers where necessary. At this stage learners received substantial help from the teacher especially with the correct geometric language.

4.5.4 Synthesis of Teacher A, B and C

The teachers provided learners with activities that aligned well to the phases of instruction. All teachers gave a variety of tasks that required explanation of concepts after instructing their learners to discover various properties of circle geometry. All teachers prompted the usage of correct geometric language without discouraging the learners from participating.

However, each teacher used a different teaching approach for the assessment activities. Teacher A asked the learners to define basic circle geometry concepts every time they wanted to use them in their answers. She instilled correct usage of geometrical terminologies as she constantly assessed their progress. Teacher B looked at spelling of concepts. He said that it was crucial for learners to spell geometrical terminologies accurately to enable them to pronounce them properly. He also encouraged his learners to acknowledge their own mistakes and learn from them as their geometric reasoning improved. Teacher C on the other hand allowed ample time after posing a question – enabling the learners to consolidate their responses as they knew that they were under scrutiny from the teacher. Therefore, I can conclude in this instance that teacher C practised what she preached.

4.6 FREE ORIENTATION PHASE

4.6.1 Teacher A

Teacher A was aware from the beginning that during the free orientation phase, learners are encouraged to work independently. Thus, she provided structured planned activities for the learners to complete by themselves in the classroom while she assessed their progress. These activities were arranged from easy to difficult. The teacher stated that this type of arrangement helped the learners to organize their own thinking and reasoning when they answered the questions. Teacher A further noted that she was not supposed to help the learners much during the free orientation phase. She thus clearly stated her criteria before the learners attempted the activities, and then let the learners get on with the task. This enabled learners to work independently, yet it provided them with a supportive framework.

The questions included in these activities required advanced knowledge of both circle geometry concepts and angle properties of circles. Teacher A said that this part of the lesson was aimed at empowering the learners to think independently and to solve challenging problems without her assistance. Here are some of the questions that the teacher included in the activities of the free orientation phase:

Activity 1

In figure 4.8 below, points A, B, C and D lie on the circumference of the circle with centre O.

Chord AB and CD intersect at X.

$\angle BAD = 92^o$ *and* $\angle ABC = 57^o$.

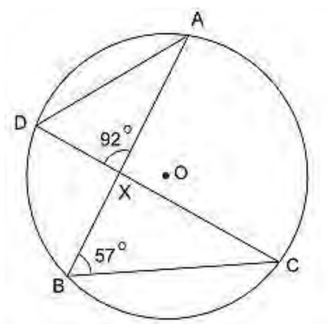

Figure 4.8

Instruction: *Calculate (giving reasons) the sizes of the following angles:*

(a) *ADC*
(b) *BAD*
(c) *AXC*
(d) *DCB*

In the question above, Teacher A provided key information to the learners that they could use to find solutions to the question. In order to find the size of $\angle ADC$ learners needed knowledge of the angles in the same segment theorem. The teacher helped the learners with this concept as they focused mainly on the sum of the interior angles of triangle *ADX* instead of the angles in the same segment property.

Angle *BAD* was well responded to as many learners used the sum of the interior angles of a triangle property. The learners then used the answer that they got in (a) to find the size of $\angle BAD$ which was 31º. Everybody noticed the vertically opposite angles property of $\angle BXC$ and reasoned why it equalled 92º. The learners then applied the teacher's explanation of the first question (a) to respond to question (d) which was an angle in the same segment with $\angle BAD$

subtended by chord *DB*. The second activity of the free orientation phase was different, complex and required algebraic skills in addition to circle geometry theorems.

Activity 2

Solve for x

Opposite angles of a cyclic quadrilateral are 21x – 2 and 38x + 5. Solve for x and hence, find the size of each angle.

I asked Teacher A how she decided on which activities to select for the free orientation phase. The teacher stated that she planned the activities in order of their structure. "*I provided the less structured questions first and left the tougher ones for when I realize that learners would be able to cope without my help*." Teacher A acknowledged that the van Hiele phases teaching programme helped her to plan appropriate activities for each phase. The teaching programme included succinct instructions on how to align one's teaching with the five phases of instruction. The activities planned for the programme are not necessarily level bound and the teacher could plan her own activities freely provided that they aligned to the phases of instruction framework. This is one of the reasons why Teacher A's activities are different from those in the teaching programme presented in chapter three of this research project.

I asked Teacher A to share with me what she learned from the free orientation phase. The teacher felt that it was a good move that we designed the teaching programme first. She found it to be a great help in making her teaching of circle geometry a success. "*I feel that this teaching programme can do some good to every mathematics teacher especially the teachers who are teaching for the first time. It really helped me to plan my lessons and activities that I taught without experience problems*." Although she did not quite answer the question, it is evident that the teacher had a positive experience with the circle geometry teaching programme during the free orientation phase. Her activities and instructions aligned very well with the phases of instruction and her learners participated well in the teaching activities. However, it is interesting to note that Teacher A sometimes revisited earlier phases, mostly the information and the explicitation phases, in order to reinforce the activities for the learners if she saw that they did not understand them.

The teacher distanced herself more and more from the learners' discussions as they thought of the solution to the problem. The learners had to figure out their own way of solving problems during the free orientation phase. The learners performed well and found the sizes of all the required angles as well as the appropriate geometric reasoning as requested by the teacher. Teacher A was pleased with her learners' progress as she revealed her excitement during the interview. "*It makes me happy to teach something and get it back when I ask for it. Those learners made me proud today*."

4.6.2 Teacher B

During the free orientation phase, Teacher B provided the learners with pre-prepared worksheets for them to complete in class. He handed out the worksheets and observed as the learners completed them. Here is an example of some of the questions from the worksheet:

Activity 1

In the figure 4.9 below, A, B, C and D are points at the circumference of the circle centre O.

Calculate the size of the following angles and give a reason for your answer in each case:

(a) XAD
(b) CXD
(c) *CBD*

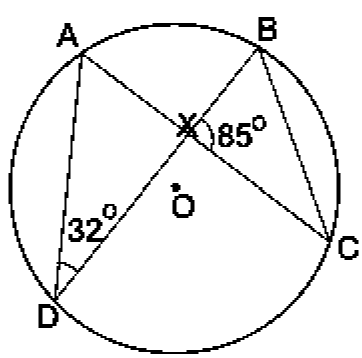

Figure 4.9

Activity 1 was aimed at testing the learners' knowledge of basic circle geometry theorems that the learners had learned earlier. The teacher assessed their progress and provided more activities such as:

Activity 2

In circle centre O, diameter AB, radii OC and OD as well as chords AC, CB and AD are all drawn.

a) *If* $\angle AOC = 80^{\circ}$*, find with reasons the sizes of angle:*

i) *ADC*
ii) *ABC*
iii) *ACO*

b) *Prove that* $\angle ACB = \angle CAD$ *and give reasons for your answer.*

At this stage of the lesson, the learners were prepared to think abstractly with less assistance from the teacher. The teacher said that he had revised the questions that he included in the worksheet before he considered them. Nevertheless, the learners complained that this activity was very confusing. The teacher explained that was the whole point of the activity, "*though it was not necessarily meant to confuse them. The purpose of this activity was to make them aware that there are different geometrical problems out there and they must always expect the unexpected*."

Teacher B was aware that the free orientation phase points towards the end of one van Hiele level of thinking. Hence, learners should be prepared to orientate themselves via structured geometrical activities to enable them to cope at a higher level of thinking in geometry. That is why during the free orientation phase the teacher only accepted reasoning that was in the correct geometric language and penalized language that was too informal and vague.

Other tasks that were given during the free orientation phase related to the one given above though they required different circle geometry theorems such as opposite angles in cyclic quadrilaterals in order to complete them successfully. It is necessary that tasks are presented to the learners in numerous ways so that they stimulate a variety of solution strategies. Although learners at this stage worked independently, they could still get help from the teacher when necessary.

4.6.3 Teacher C

Like the other two teachers, Teacher C's aim during the free orientation phase was to pass her learners through to higher levels of thinking in geometry. Teacher C wanted the learners to discover angle properties of circles on their own. She thus prepared various activities for the free orientation phase accordingly, and in each case she required that learners worked independently as individuals and sometimes in pairs or as a group. "*As long as they did not have to ask any explanation from my side, I was satisfied*." Teacher C structured the activities for this phase as follows: firstly, she asked the learners to construct their own circle and solve the accompanying problems. Secondly, she provided a drawn figure and asked the learners to use it to answer the questions. Lastly, Teacher C provided some abstract activities (word problem) where learners were required to draw a sketch before attempting to find the solution. Here are the three activities that were presented by Teacher C:

Activity 1

During this activity, Teacher C instructed her learners as follows:

1) *Construct a circle centre O and diameter MP. Draw two chords from each end of the diameter and let them intersect at point N at the circumference.*
2) *Without measurements, state the size of $\angle$MNP and give a reason for your answer.*
3) *Draw another pair of chords from M and P and let them intersect at a certain point Q on the circumference.*
4) *State the sum of $\angle$N and $\angle$Q and tell me what you found.*

The teacher wanted the learners to discover the theorem of opposite angles of cyclic quadrilaterals with as little assistance from her as possible. She encouraged the learners to recall and apply knowledge previously learned in circle geometry saying that this was just a continuation of what they already covered. The learners progressed well and worked independently and Teacher C's instructions and activities continued during the free orientation phase.

Activity 2

WXYZ is a cyclic quadrilateral with chord XW extending into a secant XWV (Fig. 4.10).

∠XYZ equals 108º.

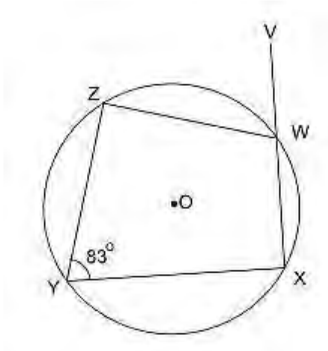

Figure 4.10

a) *Calculate the size of ∠VWZ. Show all your working and give a reason for your answer.*

The aim of this activity was to test the learners' ability to interpret diagrams in order to extract the required solutions. In other words, Teacher C required more autonomous thinking from every learner during this activity. Learners applied their knowledge of circle geometry concepts and angle properties of circles in order to complete this activity.

When the teacher realized that the learners' performance in the two previous activities was up to standard, she provided the third activity that required more than just concrete thinking and/or reasoning. Here is an example of the third activity:

Activity 3

In circle O, diameter $\overline{AB}$, radius $\overline{OC}$, and chord $\overline{BC}$ are all drawn. If ∠AOC = 50°, find ∠OCB. Show all your working.

During the third activity, the teacher encouraged the learners to read the information provided carefully and give reasons for all the answers that they gave. As I stated earlier, learners could draw a sketch of the information first and then compute their answers from the sketch. I observed how little Teacher C assisted the learners during this activity. "*They needed to prove to me that they were capable of taking on thinking at higher levels that is why I assisted less and less*." The teacher wanted the learners to figure out that ∠*AOC* is indeed the angle at the centre and ∠*OCB* is an isosceles angle of triangle *OCB*. The learners were therefore expected to apply various circle geometry theorems to find the required solution.

When I asked Teacher C about her views on the free orientation phase she stated that during this phase, learners were expected to revisit all the three phases in order to perfect their reasoning of the free orientation phase. For example, by revisiting the information and directed orientation phases, learners would be able to incorporate basic knowledge with the new knowledge to enable them to target their thinking and reasoning of circle geometry to greater heights.

Teacher C further reported that the order in which she provided the learning activities enhanced good order thinking and reasoning from the learners' side. Firstly, the learners had to apply basic conceptual knowledge. Secondly, they had to specifically apply the basic knowledge of circle geometry for the second activity. Finally, the learners were expected to apply a combination of the requirements for the two activities to find solutions for the third activity. The teacher said she gave the learners a chance to discover more circle geometry theorems. "*I also gave them a chance to learn about their own thinking and to discover the theorem of opposite angels in cyclic quadrilaterals on their own*".

4.6.4 Synthesis of Teacher A, B and C

All three teachers provided learning activities that were arranged in an ascending order of difficulty – the easy questions came first. This arrangement worked well as learners did not hesitate to answer the first question and shout out their responses as soon as they were done. However, the last questions were not so easy, and the teachers encouraged the learners to think back to what they learnt previously. It was therefore the teachers' roles during the lesson to assess the learners' progress while they were still busy completing the tasks.

All teachers' second activities were word problems that required learners to make a sketch in order for them to answer the questions. Teachers provided little assistance to the learners in this phase as they encouraged the learners to work on their own.

Despite the teachers' common approaches to the content during the free orientation phase, there were differences in the presentation of the lessons and activities. Teacher B's approach for example was somewhat different from the other two teachers who asked the learners to construct their own circles to discover angle properties of cyclic quadrilaterals. Teacher B preferred to use pre-drawn figures to explain the angle properties of circle geometry as he reasoned that it saved time to use readymade figures.

Teacher C did not agree with Teacher B's comment during the focus group interview. She stated that our learners in Namibia (as reported in the Examiner's reports of various years) lack basic conceptual knowledge across curricula with geometry and graphs of functions topping the list. Teacher C therefore argued that the introduction of programs such as the van Hiele phases be taken seriously and be introduced to all mathematics teachers countrywide if possible.

4.7 INTEGRATION PHASE

4.7.1 Teacher A

Teacher A instructed the learners to draw their own conclusions of every circle geometry theorem that was engaged with in the lessons. They had to articulate the theorems themselves. For example, after the learners were introduced to the angles in semi-circle properties, they were provided with a learning activity that required them to give reasons for their answers. Learners were then encouraged to draw their own conclusions on the specific theorem before they proceeded to the next theorem.

I asked Teacher A what she thought of the integration phase of instruction. *"Integration is like conclusion right? I told my learners to conclude by summarizing all that we have learned about circle geometry. You see, this was easy as long as you do not ask them to give reasons for their answers. I noticed that more than half of the class understood the concepts and theorems very well. The remaining percentage only need to work on their language and all would be fine with them".*

4.7.2 Teacher B

Teacher B believed that diversity in his well planned activities helped to lead the learners to a higher level of thinking. The teacher claimed that he used a variety of activities that stimulated the learners' thinking capacity hence they did not have any difficulties summarizing the content that they just learned. "*They did well with different tasks and I was very comfortable that learning took place in that class. As you can see they came a long way; from mixing up basic circle geometry concepts to confusing theorems*," proclaimed the teacher.

To conclude his teaching, Teacher B instructed the learners to summarize the content learned over the two lessons that they had together. The teacher asked the learners to compare their summaries with each other to ensure that they grasped everything that they were taught.

4.7.3 Teacher C

Teacher believed that her learners gained the necessary knowledge of circle geometry to enable them draw appropriate conclusions about what was learned. Teacher C commended the employment of the phases of instruction for this teaching project as a major contribution to the lesson outcomes. To the question of how the teacher led the learners throughout the phases of instruction, Teacher C acknowledged that the circle geometry teaching programme contributed to the success of the whole teaching process. "*That teaching programme was amazing. I relied on it countlessly as I sometimes only had to check what was next and read it out to the learners*."

Teacher C's class worked particularly hard as they assimilated the correct use of geometric language. I observed how the teacher asked the learners to conclude every aspect along the way until the end of the teaching programme.

4.7.4 Synthesis of Teacher A, B and C

In a nutshell, I would say that the three teachers' lessons were very similar not only in structure but also in presentation. All teacher asked learners to draw conclusions on the conceptual theorems learned. This could be because they all used the designed teaching programme to plan their lessons and activities. The learners summarized the circle geometry content from the basic concepts to the more complex angle properties of circle geometry. The role of the teachers at this phase was to ensure that the learners summarized correctly, and to see whether learners used appropriate geometric language when reasoning.

Teacher C had a unique preference on how she wanted her learners to conclude the content. She asked the learners to draw and label a circle with all the concepts and circle geometry theorems that they learned. She later instructed the learners to write down all the angle properties learned in order of presentation. She then encouraged her learners to always refer to that summary in case they were not sure of what property to use when asked to complete a task.

4.8 FINDINGS

My analysis supports four central claims:

Claim 1: All three participating teachers used and implemented **all** the five van Hiele *phases of instruction* in their lessons that I observed.

The three teachers' lesson presentations used the common programme on circle geometry developed jointly with them. They either followed it exactly or aligned it according to their own instructions and activities.

All three teachers explicitly went through all five van Hiele phases of instruction during their lessons and also instructed the learners according to the main features of the van Hiele phases of instruction. For example, during the *information phase*, the teacher is expected to inform the learners about the content to be taught and learns about their existing knowledge. Teacher A informed the learners that they were going to learn about circle geometry and wrote 'Circle Geometry' on the chalkboard. She then asked them about the circle geometry concepts and asked them to construct a circle and draw and label as many circle geometry concepts as they could possibly remember, all to test their prior knowledge.

During the *directed orientation phase*, (which is to direct the learners' learning activities by providing well planned tasks that build on the established prior knowledge during the information phase), Teacher B, for example continued after establishing their pre-knowledge on some circle geometry concepts. He then instructed the learners to "*construct a circle centre O,* ***diameter*** *AB. Draw two* ***chords*** *from each A and B and let them intersect at a certain point C on the* ***circumference*** *of the circle.*" The teacher's aim at this point was see how the learners applied the concepts (**in bold**) learnt in Phase 1 to construct the required circle.

During the *explicitation phase*, (in which the learners and the teacher engage in a discussion about the content whereby the teacher is expected to accept the learners' own language) Teacher C for example engaged the learners in a discussion about the diameter of the circle. The learners defined the diameter as any line that passed through the centre of the circle with its endpoints on the circumference of the circle. To test their understanding and further use of geometric language, the teacher gave the learners an activity about the examples and non-examples of a diameter of the circle. She did this by drawing a circle with various lines passing through the centre of the circle and asked the learners to identify the diameter of that circle (See figure 4.6 in part two of this analysis).

During the *free orientation phase*, (in which the teacher is expected to select appropriate geometrical problems that learners should complete under his/her observation) Teacher A for example provided structured activities for the learners to complete independently while assessing their progress. These activities required the learners to have mastered an advanced knowledge of circle geometry concepts and angle properties to solve them as accurately as possible. Teacher B and C also provided similar activities and observed their learners progress during the lessons (See the activities under the free orientation phase of this chapter).

During the *integration phase* (which is to encourage the learners to summarize and reflect on the circle geometry knowledge learned) Teacher A for example asked her learners to explain the angle properties of a circle such as: angles in the same segment, angles between tangents and radii of the circle and angle properties of cyclic quadrilaterals. The learners did this orally. Teacher B asked his learners to write a summary of the definitions of the parts of the circle and on the angle properties learnt. Teacher C on the other hand drew a circle on the chalkboard and asked the learners one by one to draw the circle geometry concepts that they learned. The teacher also asked the learners to define each concept as they differentiated them from each other.

The above synopsis of the actual implementation of the van Hiele teaching programme shows how all three teachers, collectively, used the five phases of instruction to teach their lessons. During the interview of her first lesson, I asked Teacher C if she had been aware of using the van Hiele phases during her lesson.

Interviewer*: Did you at all notice if you used the van Hiele phases during your lesson?*

Teacher *C: Err… Not really that I noticed much coz I don't really… hmm… but I think I have because I started from the beginning. Like you're telling them what the topic is all about and what to do and then you're letting them to apply what you've told them onto the chalkboard to show their working.*

Interviewer*: Are those now the first two phases?*

Teacher C*: Yeah*

Interviewer*: You gave them the information and then you told them what to do?*

Teacher C*: Hmm… and then I let them do it or identify or label – put the labels on the parts of the circle that we learned. I also asked them to give reasons for their answers when I told them to complete the exercises on the chalkboard.*

The teacher acknowledged that she was not sure if she understood the van Hiele phases/levels. She could not name them in order from the information to the integration phase but understood them in her own way. The above interview transcript showed that Teacher C used the phases of instruction in her first lesson.

Teacher A also acknowledged that she did not really understood "*this van ... theory*" but she used the teaching programme to teach her lessons thoroughly. Teacher B on the other hand kept on reminding me that it was I who introduced him to the van Hiele phases of instruction and he only did at each phase what I instructed him to do. For example, when I asked him about his instructions during the information phase of his first lesson, he said that "*you told me about the phases of instruction. So, I think that the sooner the learners learn about the stationery required in circle geometry the better.*" Teacher B further explained that he was just trying to inform the learners [probably at the appropriate phase of instruction]. Teacher B also commented on how he felt about his practice during the explicitation phase. "*I think the explicitation phase is the one that is mostly used by the teachers to thoroughly explain the content to the learners*."

I was interested in whether these participating teachers really understood the phases of instruction as the van Hieles themselves presented them in their theory, or had they adopted their own parallel framework to that of the van Hieles. When I asked the teachers if they used the phases of instruction to teach, they said that they did, but did not particularly mention the phases per se i.e. in the information phase, I did this and that. They rather sequenced what they did first, second, third, etc. When I analyzed this sequence, I found out that it was very similar to the five van Hiele phases of instruction (Fig. 4.11).

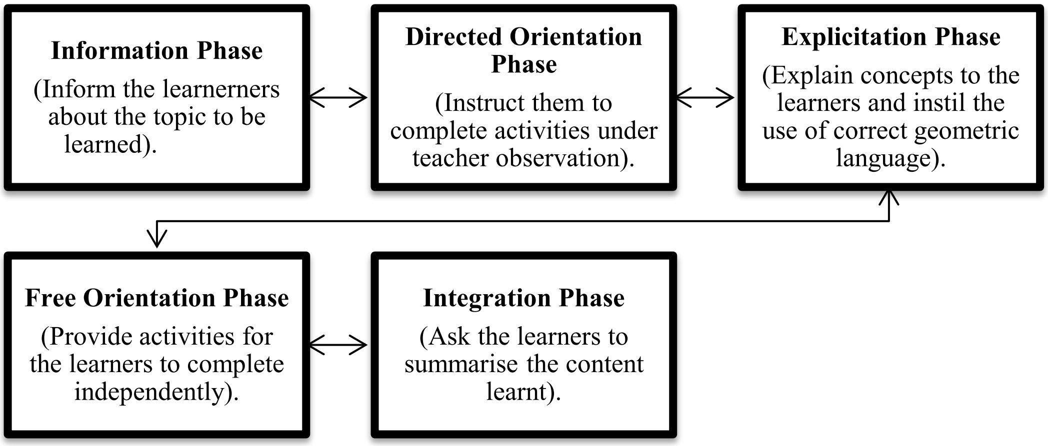

Figure 4.11: Appropriate understanding of the teachers' understanding of the van Hiele phases of instruction.

The content inside the brackets shows how the teachers viewed the use of the phases of instruction in teaching. It is clear that the teachers had their own understanding of the van Hiele phases of instruction and used this understanding to align their own instructions to each van phase.

Claim 2: The teachers navigated quite freely from one phase of instruction to the next, but also returned to the earlier phases.

During the classroom observations, I noticed that the teachers moved backwards and forwards between the phases of instruction and yet could still stay on track with the programme. For example, when operating in the free orientation phase, the teachers could refer back to earlier phases of information and directed orientation phase, and sometimes even to the explicitation phase to clarify missing/unclear concepts. They would then go back to the free orientation phase and continue with what they were doing.

A typical example is from Teacher B's first lesson when he asked the learners to construct a circle which should be a set of points that were 5cm from a fixed point. When the teacher noticed how puzzled his learners looked, he decided to define the circumference of the circle (explicitation phase) as a set of points which are a fixed distance (pointed at a radius) from a fixed point (centre of the circle). The teacher explained the concept while providing sufficient information to supplement his explanation. He then proceeded with his initial instruction when he noticed that the learners were coping.

The assessment activities that the teachers gave to the learners during the classroom observations showed how the teachers required their learners to navigate between the phases of instruction to find solutions to geometric problems. Teacher A for example asked the learners to write down all the circle geometry concepts that they could see in figure 4.4. The first two instructions that the teacher gave to the learners where not necessarily aimed at the purpose of the activity – the teacher saw fit to take the learners back to the information phase (to recall concepts and angle properties of circles) before she allowed them to operate at the explicitation phase again (See Fig. 4.4). After the teacher was satisfied with the learners' responses, she gave them the remaining instruction which was to *"calculate the size of* $\angle$*SUT. Show all your working and give a reason for your answer*."

Since the activity in figure 4.4 required knowledge of angle properties, which in turn needed knowledge of circle geometry concepts, Teacher A deemed it necessary to navigate between the phases of instruction to give clear instructions to the learners. Hence, the teacher's role during the explicitation phase is "to bring the objects of study *i.e. Circle geometry concepts and theorems* [italics added] to an explicit level of awareness by leading students' discussion of them in their own language" (Clements & Battista, 1992, p. 431). The three teachers encouraged the learners to actively engage in assessment activities both as individuals and as a group. They did not accept the learners' use of informal language as they replaced it with the correct geometric language.

Furthermore, in the introductory workshop, Teacher A advised that I should inform Teacher C on the flexibility of the van Hiele phases of instruction. "*Don't forget to explain to Teacher C that when you are teaching, you always start with the information phase then the directed orientation phase and so forth. But when you are explaining something, the order of the phases does not matter. You can always go back to the information or any other phase to make the learners understand*." Teacher C in return navigated well between the explicitation and information phases during the task on examples and non-examples of a diameter of the circle (Figure 4.6).

Claim 3: The phases of instruction are seen by the teachers as a good pedagogical tool or template for planning and presenting lessons.

The circle geometry teaching programme was an effective tool for sequencing activities that involved circle geometry. The programme enabled the teachers to align the activities to the five phases of instruction. The learning activities that teachers used for explanation on the chalkboard for example were approached in a manner which is supported by the van Hiele phases of instruction. For example, Teacher B drew a circle on the circle when he introduced circle geometry to his class. When the teacher labelled the parts of the circle in alphabetical order, he labelled the centre of the circle as (a), the circumference (b), the radius (c) etc. as he needed these to inform parts that followed. For example: the diameter passes through the *centre* and is twice the *radius*, the chord has its endpoints on the *circumference*, and so forth. This is how the phases of instruction also support each other. The information informs all the other phases of instruction and vice versa.

Towards the end of the introductory workshop, Teacher A expressed that this was a good teaching programme which might work with other sections as well. She suggested that we could also use the teaching programme to plan lessons on Graphs and Functions in which our learners also experience difficulties. "*These phases of instruction can also be used in Graphs and Functions and also Transformations chapters where most of our learners also have difficulties.*" Teacher C commented that she could use the phases of instruction across the curriculum. "*I think I can also use these phases in Life Science*."

Teacher B also appreciated the engagement of the teaching programme as he said that it was efficient for his practice. The teaching programme is designed with well prepared teacher instructions and assessment activities. Teacher B was excited about this design as he said that there was no need for him to take the design along to his lesson presentation. He said that he read the paper several times and was ready to teach. "*That paper is very clear so, I don't think I need to take it along to class. I read it many times and everything is clear enough for me to teach without it.*"

I noticed during the classroom observations that the teachers were very confident about their own practices. I also noticed when I reviewed the videotapes that as the teachers' lessons progressed;

they were not sidetracked by my visits in their classrooms. Various comments during the reflective interviews also showed how effective this teaching programme had been for them. For example, when I asked Teacher A how she felt about the teaching programme during the free orientation phase, she stated, "*I feel that this teaching programme can do some good to every mathematics teacher especially the teachers who are teaching for the first time. It really helped me to plan my lessons and activities even though I kept calling you to guide me*."

Even Teacher C who was very nervous at the beginning of her first lesson, revealed how effective the teaching programme had been to her pedagogical practices. "*Have you noticed that these learners only need a little push and then their thinking is already in order? ... I cannot believe I was so nervous about something so interesting like this teaching programme.*"

Claim 4: The majority of the learners followed instructions and seem to obtain the answers faster than expected.

Commonly, good practice enhances competent outcomes. All the teachers claimed that they did their part, and the learners performed very well during the assessment activities – often faster than they expected.

In addition to the overall learner involvement, the learners remained on-task for the entirety of the learning time and throughout each activity. They conversed at length with each other and the teachers as they progressed from informal to formal language usage and reasoning. During the free orientation phase, "the students engage in more open-ended activities that can be approached by several different types of solutions (Teppo, 1991, p. 212). Here is one example of the learners' activity that required them to use the correct geometric language and reasoning:

In circle O, diameter $\overline{AB}$ *, radius* $\overline{OC}$ *, and chord* $\overline{BC}$ *are all drawn. If* $\angle AOC = 50°$*, find* $\angle$ *OCB. Show all your working.*

The learners were expected to find their own method of solving the problem as no further information is given (this is a typical *free orientation phase* activity). The learners were placed in a situation where they were required to use various geometry concepts and angle properties of circles in order to compute a solution to this problem. They applied different methods and solved the problem. Their reasoning about their solutions was also very sound. (See Appendix G for the learners' solutions to this question).

4.9 CONCLUSION

In this chapter, I analyzed the data collected for this study. I wrote vignettes of actual lesson presentations by the three participating teachers. The findings of this reaserch were expressed in terms of asserted claims. These claims were used to reveal the three teachers' experiences with the designing and implementing a circle geometry teaching programme that used the van Hiele phases of instructio as a conceptual framework.

The next chapter concludes the end of this research journey.

CHAPTER 5

CONCLUSION AND RECOMMENDATIONS

5.1 INTRODUCTION

This chapter provides a summary of the entire research process. It includes a synopsis of the research findings as discussed in the previous chapter, and highlights the significance of the study. The chapter also presents recommendations and articulates limitations encountered during the research process. I finally share my personal reflections of the whole research journey.

5.2 SUMMARY OF FINDINGS

The findings of this research revealed that:

- All three participating mathematics teachers used and implemented **all** the five van Hiele phases of instruction in their lessons that I observed.

During the classroom observations, I noticed that the teachers matched their instructions and assessment activities to the five van Hiele phases when they implemented the teaching programme.

- The teachers navigated quite freely from one phase of instruction to the next, but also returned to the earlier phases for clarification and reinforcement.

The teachers could move back and forth between the phases of instruction and yet could still stay on track with the teaching programme. They could for example, when operating in the free orientation phase, refer back to the earlier information and directed orientation phases, to clarify concepts, then go back to the free orientation phase and continue with what they were doing.

- The teachers saw the phases of instruction as a good pedagogical tool or template for planning and presenting lessons.

The teachers appreciated the application of the van Hiele phases of instruction to teach circle geometry. They saw the teaching programme as a good lesson planner with reliable assessment activities. However, teachers also provided their own assessment activities to supplement the ones embedded within the teaching programme.

- The majority of the learners followed instructions and seem to obtain the answers faster than expected.

Good learners' performances usually come as a result of good teaching. Most of the learners performed beyond basic expectations and they were actively engaged throughout the lesson presentations.

5.3 SIGNIFICANCE

Like Mateya's (2008) study on the van Hiele levels in two schools in Namibia, this study is also unique in the Namibian mathematics education research arena, as it focused on the van Hiele phases. I built on Mateya's (2008) work in discussing possible applications of the van Hiele theory in Namibia when I conducted my study in a secondary school in Oshikoto Education Region.

The purpose of my study was to explore the experiences of the mathematics teachers with designing and implementing a circle geometry teaching programme, using the van Hiele phases of instruction as a conceptual framework.

The findings of this research are intended to inspire teaching programmes of a similar kind in other schools in the region and finally the whole country. I also plan to use the findings of this research to persuade the educational tertiary institutions in Namibia to design their geometry mathematics curricula for the pre-service teachers in alignment with the van Hiele phases of instructions.

5.4 RECOMMENDATIONS AND AVENUES FOR FURTHER RESEARCH

When I embarked on this research journey, I had an abundance of international literature on the van Hiele theory. I tried to find research conducted on the van Hiele phases of instruction in Namibia but could only locate some conducted in the broader Southern Africa instead. Therefore, based on the findings of this study, I would like to make the following recommendations:

- More research in this field should be conducted in Namibia. Research using the van Hiele theory, particularly the phases of instruction, would have benefits for learning about interesting and innovative teaching practices in Namibia.
- Building on the findings of the circle geometry teaching programme designed and implemented in this study, further research is needed to design and implement a similar teaching programme in other subjects across the curriculum.
- The teaching programme should not be seen as a substitute for thorough individual lesson planning, preparation and presentation. The teachers should always be encouraged to go the extra mile to refine their own instructions and use a variety of their own learning and assessment activities to engage their learners critically.

5.5 LIMITATIONS

This study used a very small sample and its findings cannot be generalized due to the following reasons:

- The research was conducted in a single urban secondary school with a multicultural student and teacher profile, predominantly from a middle-class economic background. The results can therefore not be generalized as there could be different findings from a school in a rural area, and/or a school in any other town in Namibia.
- Time constraints and over-commitment of the teachers made it very challenging for me to secure consistent participation. Also we could not finish teaching all of the intended content of the teaching programme, due to its length and the limited time allocation I had with the teachers.
- Unfortunately, one of the teachers did not attend the official introductory workshop. The teacher could have contributed so much more in our group situation. Instead I had to rely on our one-on-one sessions and informal conversations with him for his input and information about his experience.

5.6 REFLECTIONS

5.6.1 Critical reflections of the research findings in terms of the existing literature

The current research literature on the van Hiele theory focused more on the van Hiele levels of thinking in geometry in comparison to the van Hiele phases of instruction. This research focused on the van phases of instruction hence, can easily be situated in the existing literature on the van Hiele theory. Therefore, the findings of this research contribute to the current research literature as discussed in the preceding research recommendations.

This research might not be viewed as one of a kind by the entire research world but it is surely one of a kind in the Republic of Namibia. The uniqueness of this research landed the Namibian mathematics research arena a spot in the global research of the van Hiele theory. This means that Namibia is now one of the fewer countries in the world with a research that focused on the van Hiele phases of instruction. Furthermore, the findings of this research would as well enrich the current research literature in that it would be beneficial to the novice researchers in mathematics education; particularly geometry related studies.

The fourth finding of this research project revealed that 'the majority of the learners followed instructions and seem to obtain the answers faster than expected'. This comes as a result of well designed teaching instructions that ensured effective content conceptualization. This highlights yet another unique contribution that the 'Circle Geometry Teaching Programme' designed in this research has on the current research literature.

Lastly, I would like to highlight to the reader that the findings of this research do not contradict the current research literature. On the contrary, the results contribute to the existing literature by emphasizing the necessity of the van Hiele phases in designing, planning and implementing a Circle Geometry Teaching Programme using the van Hiele phases of instruction as a conceptual framework.

5.6.2 Personal reflections

When I embarked on this research journey, I always knew that it would be fascinating and inspiring but I never thought that it would be as challenging as it turned out to be. This was a stony path to travel for two years. It took me a while to identify and refine my research topic. My supervisor always reminded me of the importance of reading and locating my research in the broader literature.

Doing this research enabled me to develop discernibly professionally, academically and as a member of my society. This fills me with a sense of gratitude for the opportunity to have conducted research at this level. I am grateful to Rhodes University and its dedicated and professional staff for the support they have given me for the duration of my study.

The three mathematics teachers who participated in my study did a wonderful job. They were very helpful during our one-on-one sessions and implemented the circle geometry teaching programme within a limited timeframe. It is no wonder the school in which they teach has been awarded a school of excellence trophy for the eighth consecutive year.

5.7 CONCLUSION

This final chapter concludes the whole research process. It provided a summary of the research findings and highlighted the significance of the study. It also offered some recommendations and some avenues for further research. I discussed the limitations and then reflected on the impact that this study had on my current personal, academic and professional life.

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APPENDICES

APPENDIX A – LETTER TO THE PRINCIPAL

Dear Sir.

RE: Research site and participants permission requisition

I am a registered part-time student at Rhodes University, Grahamstown (student number 609D6388). I have been studying for a Master's degree in Mathematics Education since January2011. I would be most grateful if you would allow me to use the Grade 11 classes of Etosha SS as research sites for the research report, which I am required to write.

The aim of my research is to discuss and document the experiences of mathematics teachers when using van Hiele phases of instruction in designing and implementing a 'Grade 11 circle geometry teaching programme'. Should you agree to allow me to use Grade 11 classes as research sites, I will arrange with the teachers involved to teach circle geometry in the afternoon. Data for analysis will be collected from focus group discussions with mathematics teachers (workshops and interviews), observations and field notes. The teachers and parents will be asked for permission to audio-tape record interviews, and video-tape record their children respectively.

The school, learners and parents will be assured of anonymity in the final research report and will be welcome to proofread drafts of the report to ensure that details are accurately recorded and reported.

Should you have any concerns or questions about this request, please do not hesitate to contact me personally or at 0812104831.

Yours sincerely

……………….

B. L. Dongwi (Ms.)

Consent Form

Beata Dongwi is hereby given permission to use the Grade 11 Classes of Etosha Secondary School as research sites for the research report she is required to write for the completion of her Master's degree in Mathematics Education. I understand that data for analysis will be collected from focus group interviews, observations and field notes, and that information from these may be used in the final report. I have been assured that my school, my learners and parents will have anonymity in that report.

Principal's signature: .... Date: ....

SGB Chairperson's signature: .... Date: ....

APPENDIX B – LETTER TO THE MATHEMATICS TEACHERS

March 5, 2012

Dear Mrs. (*Name removed*)

RE: Research participants permission requisition

I am a registered part-time student at Rhodes University, Grahamstown (student number 609D6388). I have been studying for a Master's degree in Mathematics Education since January2011. I would be most grateful if you would agree to become my research participant for the research report that I am required to write.

The aim of my research is to discuss and document the experiences of mathematics teachers when using van Hiele phases of instruction in designing and implementing a 'Grade 11 circle geometry teaching programme'. Should you agree to become my research participant, I would arrange with you to teach circle geometry to Grade 11 classes during the afternoon sessions in days yet to be arranged. You will use the van Hiele Phases of instruction to design a teaching curriculum. Data for analysis will be collected from focus group discussions with mathematics teachers (workshops and interviews), observations and field notes. I would also like to ask for your permission to audio-tape record interviews, and video-tape actual lesson presentations.

I would like to assure you of anonymity and confidentiality in the final research report and you would be most welcome to proofread drafts of the report to ensure that details are accurately recorded and reported.

Should you have any concerns or questions about this request, please do not hesitate to contact me personally or at 0812104831.

Yours sincerely

……………………

B. L. Dongwi (Ms.)

Consent Form

I agree to become a research participant in Beata Dongwi's research project by doing the following:

a) Agreeing to attend research related workshops in the afternoon.
b) Agreeing to teach a Grade 11 class that is identified as a research site during afternoon sessions (preferably from 15H30 to 16H30 on Mondays/Wednesdays).
c) Agreeing to be tape and video recorded during focus group interviews (workshops and discussions) and actual lesson presentations.

I understand that data for analysis will be collected from focus group interviews, observations and field notes, and that information from these may be used in the final report. I have been assured of confidentiality and anonymity in that report.

Teacher's signature: .... Date: ....

APPENDIX C – LETTER TO PARENTS/GUARDIANS

Inquiries: Ms. B. Dongwi

February 29, 2012

Dear Parents

I am a registered part-time student for a Master's degree in Mathematics Education with the Education Department at Rhodes University, Grahamstown, South Africa. To qualify for my Master's degree, I am required to conduct a research project based on a research proposal that is already approved by the University's high degree committee last. I have chosen to focus on circle geometry in the grade 11 curriculum. In other words, my research is aimed at investigating mathematics teachers' experiences with the geometry teaching programme using van Hiele's phases of instruction as a conceptual framework.

The 'van Hiele's phases of instruction' is a teaching tool that is designed to be used by mathematics teachers to teach geometry in grade 11. Because geometry is viewed and accepted by many as difficult to learn and fully understand, I decided to vent research in circle geometry, design and implement a teaching tool that might just be an answer that our Namibian teacher and students have been waiting for. This means that your child will not only gain experience as a participant in research of this magnitude but also expertise in circle geometry as a mathematics domain.

Please complete the attached consent form if you are willing to assist me with this research:

a) By allowing me to observe your child, make field notes and keep samples of videos records of him/her participating in my research project.
b) By allowing him/her to be video-recorded while working during lesson presentations and to use these videos as evidence in the research write up.
c) By allowing him/her to please come to school in the afternoon, preferably from 15H30 to 16H30 to attend lessons. The dates are yet to be decided upon in due course.

Please be rest assured that video tapes and field notes whereby your child is a participant will be confidentially stored and will not be viewed to anybody without your consent.

I hope to get a positive response from you.

Yours in Education

....

Beata Dongwi

(A mathematics teacher: *Name of the school removed*).

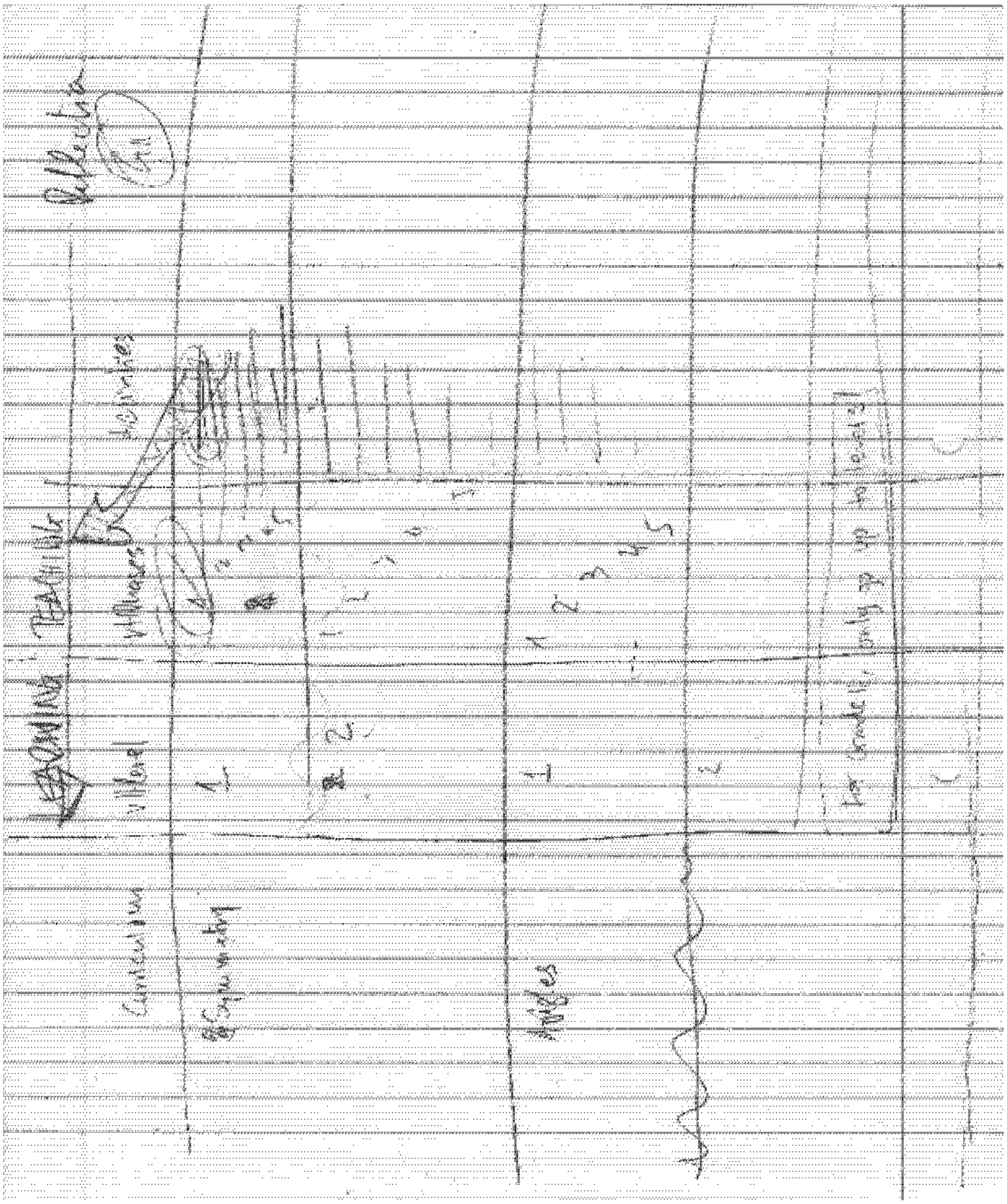
Consent Form

I hereby agree to assist Beata Dongwi in her research. I understand that she will be:

- Observing my child, making field notes, keeping samples or photocopies of his/her work and recording videos to use in the research report.
- Information collected will be kept confidential and permission will be granted whenever videos are to be viewed for purposes other than current study.

Signed: .... Date: ....

APPENDIX D – FIRST PROGRAMME STRUCTURE FROM MY SUPERVISOR

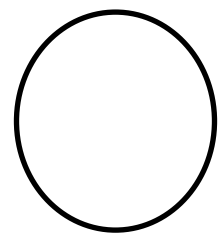


APPENDIX E – DRAFT OF THE TEACHING PROGRAMME

The Circle Geometry Teaching intervention

Guideline for the teacher

The intervention is sequenced using the van Hiele phases of instruction: information, direct orientation, explicitation, free orientation, and integration.

Activity and Phase	Teacher's Instruction/Activity Description
B. Geometrical Concepts **Information and Directed Orientation Phases** **Activities 1:** angles in a semi-circle, Tangent of the circle, and tangents from an external point	6.1.1 Students work through simple constructions in their note books, using a mathematical set. Constructions include: e) Construct a circle with centre O. f) Draw a diameter and name it AB at the point of contact with the circumference of the circle. g) Draw two chords, from A and B and let them intersect at point C on the circumference. h) Measure angle ACB. Draw another pair of chords from A and B and let them intersect at C'. Measure angle AC'B. 
Explicitation and Free Orientation Phases	What did you notice? What conclusions can you draw from these angles? At this stage, the learners should notice that an angle in a semi-circle equals 90 º. The teacher should ensure that the learners master the correct technical language at this stage such as: semi-circle (the teacher should not sketch any figure at this stage; rather inspect learners' constructions and ensure they all follow instructions accurately).
Directed Orientation Phase **Activities 2:** Angles between a radius and a	7.1.1 Construct another circle with centre O. f) Draw a radius and name it OT. Draw a tangent to the circle at T and name it ATB.

tangent of the circle

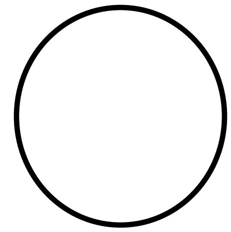
Explicitation and Free Orientation Phases

Activities 3: Tangent of the circle, and tangents from an external point

Free Orientation Phase

g) Measure angles OTB and OTA. What did you notice?

h) Draw another tangent from point A on the opposite side of the circle. Draw a radius to this tangent and mark the point of intersection P hence, the tangent APQ.



i) Measure angles OPA and OPQ. What did you notice?

j) Draw chord PT and measure the lengths of sides AP and AT. What did you notice? What conclusions can you draw from these angles?

At this stage, learners have mastered the concepts that angles formed between the radius and the circle equal 90º. Hence, the tangent and the radius of the circle are perpendicular. The teacher helps them to conclude that tangents from the same point are equal in length.

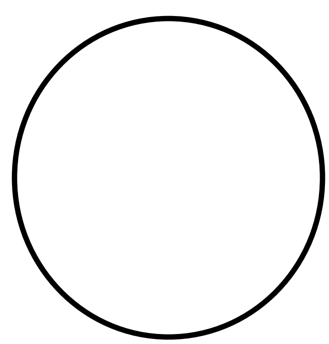
8.1.1 The teacher is aware that the learners' level of thinking has improved if not increased hence; instruction should also be a high level. At this stage, only the correct geometric language should be accepted. The teacher should assist the learners where necessary for example, by sketching figures. Learners are instructed to:

Direct Orientation and Explicitation Phases

Activities 4: Angles subtended by the same arc are equal; angles in the same segment are equal

d) In figure 1, measure angles ACB, ADB and AEB. Now measure angle AOB. What did you find?

Teacher: In figure 1, angle AOB is an angle at the centre of the circle subtended by arc AB. Angels ACB, ADB and AEB are at the circumference of the circle subtended by arc AB (show arc AB to the learners).



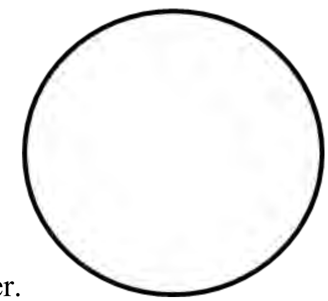
e) In figure 2, reflex angle AOB is at the centre of the circle subtended by the major arc AB (show the major arc AB). Angle ACB is an angle at the circumference of the circle, subtended by the same major arc AB. Measure reflex angle AOB and angle ACB. What did you notice?

f) Compare with the first circle. (You said earlier that the angle in a semi-circle equals 90º. Is this angle also on the circumference? Is so, what is the size of the angle at the centre of the circle?

Let learners refer to Figure one and introduce the concept of angles subtended by the same arc. Draw chord AB and talk about angles in the same segment.

Integration Phase

Activity: Find the size of the angles *c, d, e* and give a reason for your answer.



At this stage, learners have mastered the analysis level (van Hiele Level 2). They are now prepared and ready to take on thinking of the third level of thinking. This means that the teachers instruction will also be at Level 3 (higher than before), and so do the activities.

9.1.1 The teacher directs the lesson toward the concepts of cyclic quadrilateral. She/he presents learners with ready drawn figures as shown below. [The diagrams below are not to scale. *O* is the centre of the circle].

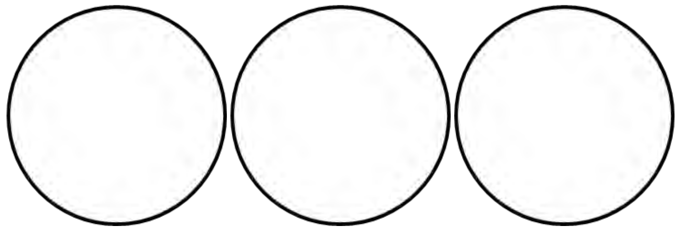
The teacher starts by drawing a circle on the chalkboard with two opposite angles ABC and ADC subtended by the same chord (diameter AB). The idea here is to conceptualize concepts of cyclic quadrilateral by realizing that angles in opposite segments are supplementary.

Learners are instructed to:

d) In each diagram, fill in the sizes of the angles at *O* and at *B* and *D*.

e) Calculate the sum of angle B + angle D. What did you find?

f) What is the sum of angle BAD and angle BCD? Give a reason for your answer.



10.1.1 Draw a cyclic quadrilateral ABCD.

d) Measure all the angles of the cyclic quadrilateral.

e) Find the sum of the two pairs of opposite angles.

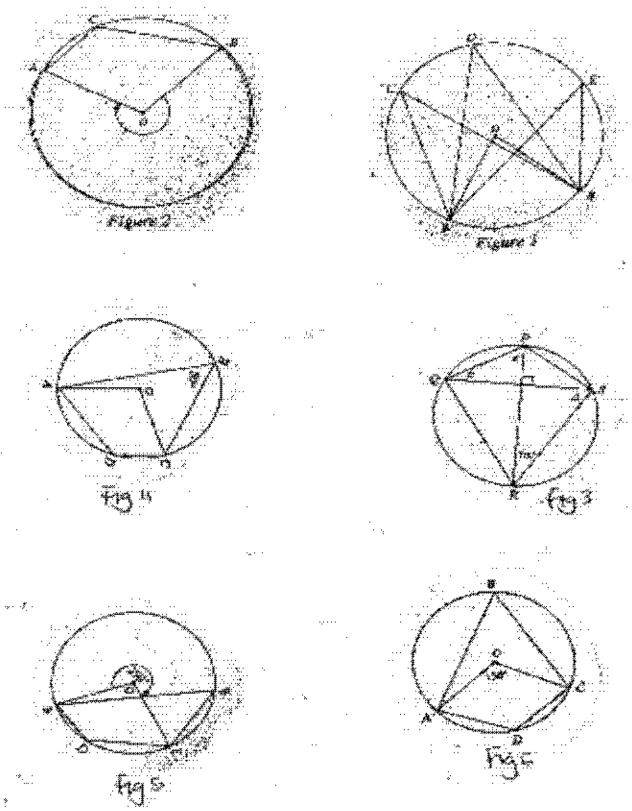
f) What can you conclude about angles in cyclic quadrilaterals?

In the following activities, inquire the learners to give reasons for their answers using complete sentences in the correct geometric language. Learners should be operating at VHL3 in order for them successfully complete this activity.

The teaching sequence is adapted from Serow (2008, pp. 448-449). Activities adapted from Courtney-Clarke and Coulson (2008, pp. 203-213).

APPENDIX F – WORKSHEET CONTAINING CIRCLE GEOMETRY ACTIVITIES

Worksheet 1

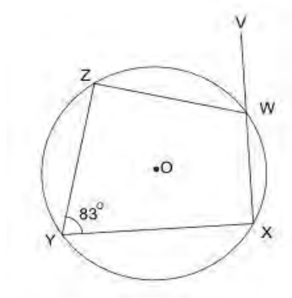


Source: Mathematics for life Grade 10

The following activities are downloaded from the mathematics website.

Retrieved October 25, 2012 from

http://www.kutasoftware.com/FreeWorksheets/GeoWorksheets/11-Inscribed%20Angles.pdf

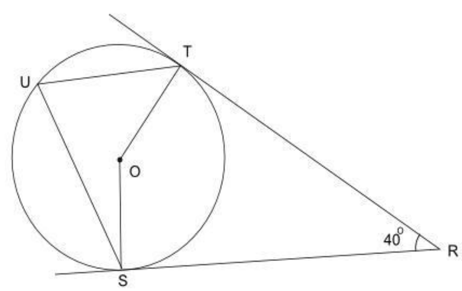


WXYZ is a cyclic quadrilateral drawn inside a circle center O and V is on the line XW extended.

∠XYZ = 83°

What is the size of ∠VWZ?

Geometry (Plane) (High School Geometry, Hard)

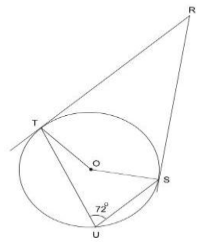


RS and RT are tangents to the circle center O.

∠SRT = 40°

What is the size of ∠SUT?

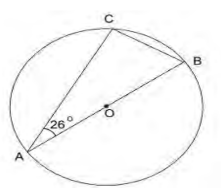
Geometry (Plane) (High School Geometry, Hard)



RS and RT are tangents to the circle center O.
∠SUT = 72°

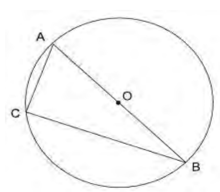
What is the size of ∠SRT?

Geometry (Plane) (High School Geometry, Easy)



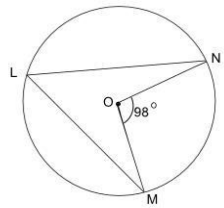
AB is a diameter of a circle, center O. C is a point on the circumference of the circle, such that ∠CAB = 26°.
What is the size of ∠CBA?

Geometry (Plane) (High School Geometry, Easy)



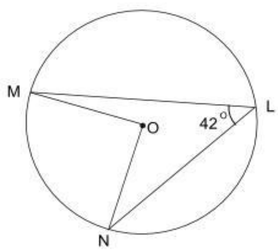
AB is a diameter of a circle, center O.
C is a point on the circumference of the circle, such that ∠CAB = 2 × ∠CBA
What is the size of ∠CBA?

Geometry (Plane) (High School Geometry, Easy)



L, M and N are points on the circumference of a circle, center O.
∠MON = 98°.
What is the size of ∠MLN?

Geometry (Plane) (High School Geometry, Easy)



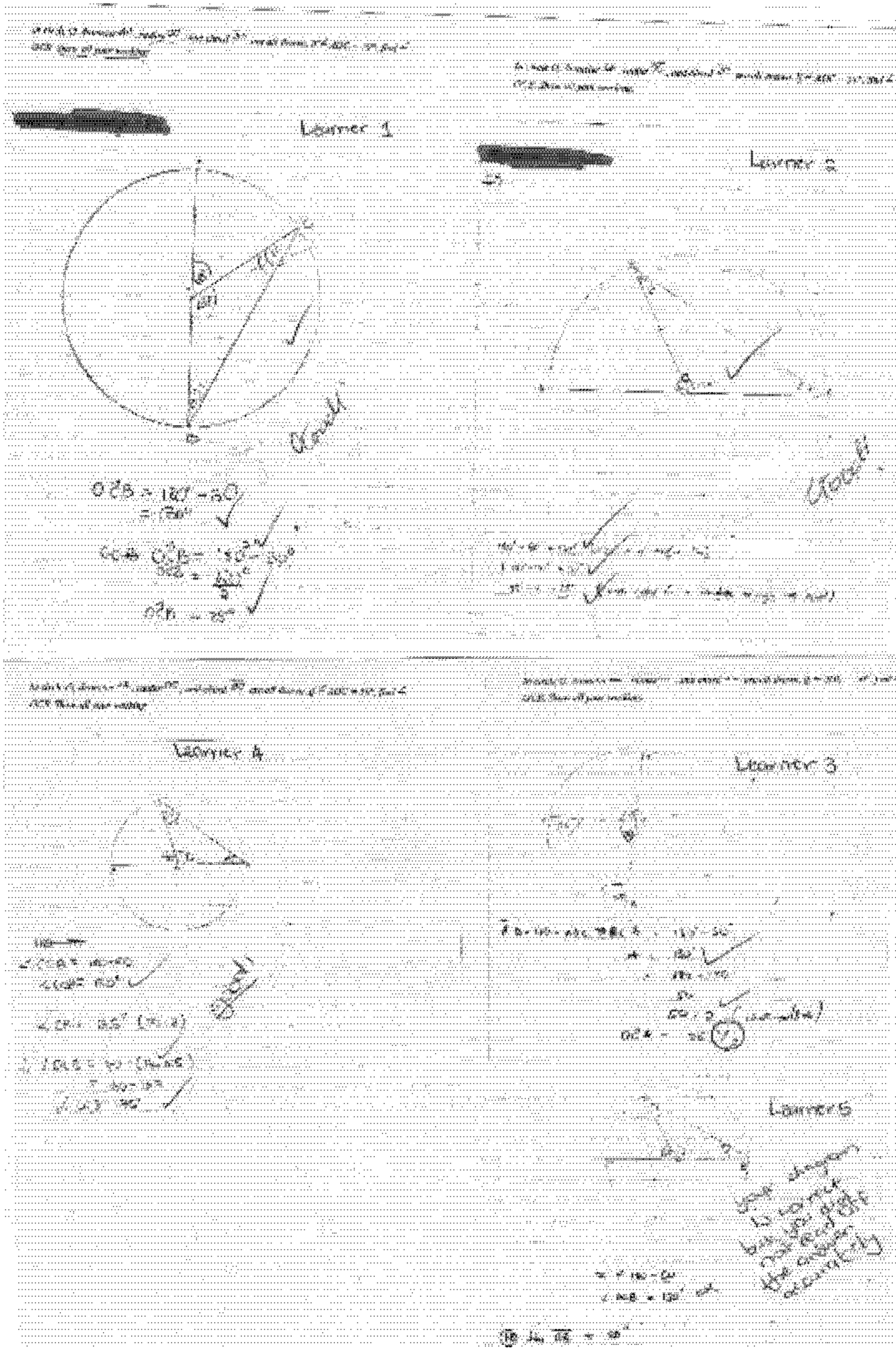
L, M and N are points on the circumference of a circle, center O.

∠MLN = 42°.

What is the size of ∠MON?

In circle O, diameter $\overline{AB}$, radius $\overline{OC}$, and chord $\overline{BC}$ are all drawn. If $\angle$AOC = 50°, find $\angle$ OCB. Show all your working.

APPENDIX G – SELECTED LEARNERS' RESPONSES TO AN ASSESSMENT ACTIVITY DONE IN CLASS



APPENDIX H – REFLECTIVE INTERVIEW QUESTIONS

Reflective interview questions – in terms of the five phases

1. Under the information phase, what can you say?
2. Under directed orientation, what worked and what did not work?
3. What can you tell me about the explicitation phase?
4. What about the directed orientation phase?
5. How did you lead the pupils through to the integration phase?
6. How did you find teaching using the teaching programme? (Would you recommend it to other mathematics teachers? Why/why not?
7. After I introduced you to the van Hiele phases of instruction have you ever tried using the phases of instruction to plan your own lessons/activities?
8. Was the whole research process worth it? What do you think?