# INSTABILITY IN THE MAGNETOTAIL

by

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Note re title.

The analysis has been applied to a neutral point in a magnetic field, and in particular to a field whose lines are orthogonal to, and do not vary in the direction of one of three Cartesian axes. Thus effectively the neutral point becomes a neutral line. This was done at the request of my first supervisor, who suggested that a suitable model would be the field obtaining between two equal and parallel line currents. Although most authorities consider that the magnetotail takes the form of a neutral sheet, some authorities lean to the theory of a neutral line, e.g. Schindler [15]. My first supervisor suggested that I apply the results of my analysis to the magnetotail, using the parallel line current model to represent the neutral line of the latter theory, this being at roughly the same distance from the earth as the near end of the neutral sheet of the former theory. It was necessary to extract, from the literature, likely values of variables in this region (Chapters I and II), and to demonstrate how the parallel line current model might be considered applicable (Chapters III and IV). All this work was approved by my first supervisor before I started on the instability analysis (Chapters V, VI).

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# CONTENTS.

* In this paper, the word "constant" means "constant in space", and "steady" means "constant in time".

# I N T R O D U C T I O N

The magnetic induction field due to the Earth only would, if undisturbed by any outside agency, resemble macroscopically the field due to a magnetic dipole. However the field is disturbed by the interplanetary magnetic field, of which the most important component is that of the Sun.

If the Sun's magnetic field were effectively steady, it would also be a dipole field, and approximately constant in the region within about twenty earth-radii from the earth. Also, if we treat the Sun as a dipole, its dipole axis is roughly normal to the ecliptic plane. The Earth, treated as a dipole, has an axis which is inclined to the normal to the ecliptic plane at an angle which varies daily from a few degrees to nearly a third of a right angle. However, in this paper, it is proposed to treat both dipole axes as contra-parallel and effectively normal to the ecliptic plane, so that a general idea of the combined field can be obtained. Then the effect of a steady field due to the Sun, on the Earth's field would be the formation of a "neutral ring" surrounding the Earth; that is, a closed "neutral line", this being a line of points at each of which the net magnetic induction is zero. As the point of observation passes through this line, the field changes direction.

However the Sun's magnetic field is not steady, in fact it is caused to be time-dependent by the flow of plasma from the Sun, commonly known as the "solar wind". This, in a way explained by Alfven. [ 1 ], Cowling [ 2 ], Walen [ 3 ], Spitzer [ 4 ], Dessler [ 6 ] and others, causes the field lines of the interplanetary field to move outwards from the Sun. An important effect of this moving field is the formation of the phenomenon known as the "magnetopause", where the solar wind impinges on the Earth's field and gives rise to a surface current.

The explanation of this feature has been given by many writers, in particular by Piddington [ 5 ]. Inside it, the relevant part of the combined field near the earth is distorted in such a way that field lines do not cross the magnetopause. However, at large distances from the Earth on the far side from the Sun, the effectiveness of the magnetopause as a magnetic shield wears off with distance, and a proportion of the inter-planetary field passes through the magnetopause.

The behaviour of the neutral ring in the direction opposite to that of the sun has been the subject of considerable research. The region is referred to as the "magnetotail". Some writers have assumed that the neutral line, under the conditions described above, has become a surface, known as the "neutral sheet". Among others, the phenomenon has been investigated by the following :-

Dungey [ 7 ] considered a simplified model of a magnetic neutral point, and found that it was unstable to small perturbations in the case of steady and constant pressure and of infinite conductivity. However, after a qualitative discussion, it was pointed out that, under the conditions of growing instability, the assumed approximations became invalid, and also that the growth rate is likely to be reduced by finite conductivity.

The same writer [9] also suggested, after receiving information from one of the earliest space probes of the confirmed existence of the interplanetary magnetic field, that neutral points around the earth must be expected. A qualitative model was given in outline, and it was indicated that primary auroral particles would be accelerated at these neutral points. The quantitative model was in general agreement with the observed current system. Near the neutral points the flow was expected to be controlled by the strong current density existing there. Although this article appears to have been written without cognisance of the magnetopause or of the solar wind, it laid the foundation for the idea, subsequently confirmed experimentally, that there is a connection between the earths magnetic field lines relevant to the circum-polar auroral region, and those in the magnetotail (whose existence was not considered at the time of writing of the article referred to in this paragraph.)

Piddington [8 ] considered the plasma drift in the neighbourhood of a neutral sheet, and the relevant dispersion time.

Speiser [10,11] discussed the orbits of charged particles entering a neutral sheet and being accelerated.

Tendys [12] , making use of these orbits, analysed the current due to a wave perturbation, considering particularly the case where the system is at marginal stability : this is the case when the waves radiate out of the sides of the neutral sheet at the same rate as that of the energy introduced by the particles. This led to the existence of a negative conductivity; that is, the current is contra-parallel to the electric intensity, and as a result the growth rate for the waves is large.

Petschek [13] treated the phenomenon of annihilation of magnetic field lines, with particular reference to the conditions in solar flares.

Yeh and Axford [14], following the work of Petschek, considered the "X" type of configuration near a straight neutral line. The fluid velocity and magnetic induction vector both have components parallel to this line which were taken as constant, and the hydromagnetic flow was regarded as steady and incompressible. The magnetic induction field lines were then found to be carried from two sides toward, and from the other two sides away, from the neutral line. Near the planes forming the "X", shock surfaces may form in which case viscosity plays an important part. The effect of finite conductivity was also studied. It was found that the fluid must flow from the obtuse-angle wedges of the "X" to the acute-angle wedges, and that the reconnection process is such that oppositely directed magnetic induction field lines move towards the neutral line in the obtuse wedges, get reconnected at the line, and move away in the acute wedges. Further, it was found that the solution for the flow and for the magnetic induction fields is essentially unaffected by finite conductivity and by finite viscosity near the neutral line, and that the same applied to the rate of reconnection of the magnetic induction field lines.

Schindler [15] suggested that the magnitude of the magnetic induction field increased in a quasi linear manner as the point of measurement receded from the neutral line, until at a certain distance it adopted an approximately constant value. The existence of the magnetopause caused the neutral ring round the Earth to split, into a northern and a southern arc, on the side of the Earth nearest to the Sun. Schindler demonstrated that a further split, into lines at different distances from the Earth, could take place on the side of the Earth away from the Sun, and that, near the neutral lines in this region, the magnetic induction field is essentially unstable. In this, partly qualitative, analysis, it was assumed that the fluid velocity could be treated as negligible.

During the period devoted by the writer to the preparation and writing of this paper with its attendant calculations, there have been a number of space probes by artificial satellites through the magnetopause, and a considerable amount of information has been received concerning the magnetic field, plasma flow, etc., at a variety of positions within the magnetopause, and in particular in the region of, and near to, the magnetail. Indeed a report of what was probably the first probe of this type was published in 1965. From all this information it has been inferred that the neutral line in the direction of the Earth away from the Sun is indeed a neutral sheet, also that the composition of the magnetotail is one of considerable complexity. It could be said that the existence of the sheet was suspected from the space probe information received in the second half of the last decade, and that the suspicion became a certainty, as a result of further space probe information, during the first part of the present decade, that is, while this paper was being prepared. The Earth's magnetic field lines which emanate from a broad ring several degrees wide, and with mean diameter approximately fifteen degrees round the South pole, and eventually return to a similarly defined area round the North pole, constitute a region known as the Polar cusp, and it is these lines which form the magnetotail, being generally contra-parallel at the neutral sheet where merging takes place. Due to this, and to other phenomena within the magnetopause, there is considerable deviation from any simple "neutral line" model and the relevant behaviour requires methods beyond those of simple mathematical analysis for its explanation. Therefore the findings of this paper are of academic interest only.

A postcript (chapter VII) has been added which gives a more detailed description of the interior of the magnetopause, together with references to recent research articles on the subject,and an attempt to collate the practical information and theoretical predictions.

Meanwhile, it is intended to allow certain perturbations to take place in an "X" type field near a neutral line, and see if they lead to stable or unstable behaviour. The perturbations will be to the first and second space differentials of the scalar pressure, scalar conductivity, velocity and magnetic induction vectors that appear in the induction-diffusion equation and in the equations of motion and of conductivity.

# CHAPTER I

## VARIABLES IN THE MAGNETOTAIL

### §1.1 Basic Units.

Under laboratory conditions, the most suitable units for the study of hydromagnetic behaviour are those of the metre-kilogramme-second system. One advantage of this system lies in the fact that Maxwell's equations, as expressed in the relevant units, are freed from the numerical constant $4\pi$ and from the constant speed of electromagnetic waves in vacuo, c. Another advantage is that the equations bring into prominence the two interdependant properties of the medium concerned, these being the permittivity and the permeability, both of which have values, in the metre-kilogramme-second system, very different from unity.

Up to the time that the Ninth General Conference of Weights and Measures, meeting in 1948, finally ratified the metre-kilogramme-second system for general use, electro-magnetic theory and practise had been bedevilled by the existence of several sets of units: centimetre-gramme-second-electrostatic, centimetre-gramme-second-electromagnetic, Gaussian, international, etc. In addition there had been an understandable, if unfortunate, tendency to label the units of the various electromagnetic quantities with the names of the famous men of science responsible for important and relevant discoveries; in the exciting new field of electromagnetic study this may have appeared to be the most suitable way of erecting a durable monument to these pioneers. Thus, quantities were referred to by so many coulombs, ampères, volts, ohms, farads, watts, gauss, gilberts, maxwells, henries, oersteds, etc. The 1948 General Conference cleared up the first nuisance, providing science with a single , simple and self-consistent set of units, but made only a half-hearted attempt to tidy up the second one. Many units, such as the volt and watt, had by that time become well-established in the connected industries, and so had to be retained, and what little simplification of unit names that took place was partly offset by the adoption of a new one, the weber. This plurality of unit names has not been found necessary in the older science of mechanics, in which, with a few exceptions such as the joule and the newton (or their centimetre-gramme-second system equivalents, the erg and the dyne), all relevant quantities are normally expressed in terms of the applicable units of length, mass and time: an attempt to use the term "galileo" for the unit of acceleration does not appear to have achieved general acceptance.

All electromagnetic units can also be expressed in terms of length, mass and time and of one other phenomenon - it does not matter particularly which this is: it could, for instance, be current or magnetic flux. However since all electromagnetic phenomena owe their existence to electric charge, either at rest or in motion, (16) it is proposed to use the metre-kilogramme-second unit of charge, the coulomb, as the fourth unit, and to refer to the combined system as the CKMS system, this meaning the coulomb-kilogramme-metre-second system. It is now necessary to analyse the various electromagnetic quantities so that they may be correctly expressed in CKMS units. Small letters are used for each of the four relevant units except for coulomb, for which a capital C is required to avoid confusion with the symbol for the speed of "light". Where symbols are used in this paper, opportunity is taken here to introduce them. Electrostatic and electrodynamic units present no difficulty:-

| Symbol. | Quantity. | Unit. | |
|---|---|---|---|
| - | charge: | $C^1$. | |
| - | current: | $C^1 s^{-1}$. | |
| J | Current density: | $C^1 m^{-2} s^{-1}$, | since this quantity is a surface density. |
| E | Electric intensity: | $C^{-1} k^1 m^1 s^{-2}$, | since electric intensity is force per charge. |
| $\epsilon_0$ | Permittivity: | $C^2 k^{-1} m^{-3} s^2$, | obtained from the formula for the force between two point charges. |
| - | charge density: | $C^1 m^{-3}$ | |
| - | Displacement: | $C^1 m^{-2}$ | ,obtained from Maxwell's relevant constitutive equation, or from his equation connecting $\tilde{E}$, $\epsilon_0$ and charge density. |

| Symbol. | Quantity. | Unit. |
|---|---|---|
| $\sigma$ | Conductivity: | $C^{2}k^{-1}m^{-3}s^{1}$, obtained from Maxwell's relevant constitutive equation. |

Other electrostatic and electrodynamic units, e.g. capacitance, may be similarly expressed, however they are not needed in this paper.

For magnetic units it is necessary to start with the permeability, making use of what many writers regard as the definition of this quantity, viz. that the product of the permeability, the permittivity and the square of the speed of electromagnetic waves in a medium is dimensionless and is unity. This provides:

| Symbol. | Quantity. | Unit. |
|---|---|---|
| $\mu$ | Permeability: | $C^{-2}k^{1}m^{1}$. |

The only other magnetic unit required in this paper is:

| | | |
|---|---|---|
| $\underset{\sim}{B}$ | Magnetic Induction: | $C^{-1}k^{1}s^{-1}$. |

This identity is obtained from the fact that the square of the magnitude of the magnetic induction is $2\mu$ times the resulting energy density.

For record purposes, a few mechanical units are now tabulated, with their symbols and CKMS units:

| Symbol. | Quantity. | Unit. |
|---|---|---|
| L | length: | $m^{1}$. |
| – | Mass: | $k^{1}$. |
| T | Time: | $s^{1}$. |
| V | Velocity: | $m^{1}s^{-1}$. |
| D | Density: | $k^{1}m^{-3}$. |
| P | Pressure: | $k^{1}m^{-1}s^{-2}$. |
| $m^{-1}\underset{\sim}{\nabla}$ | Gradient: | $m^{-1}$. |

Normally, the operator $\underset{\sim}{\nabla}$ has dimension $(\text{length})^{-1}$ and so adds a factor $m^{-1}$ to the units of the quantity on which it operates. However it is proposed to use the symbol $\underset{\sim}{\nabla}$ to mean a differential with respect to a small change in the number of metre units defining the position of a variable point of measurement. Therefore it is necessary to replace the symbol for the gradient operator, normally written $\underset{\sim}{\nabla}$, by $m^{-1}\underset{\sim}{\nabla}$. Similarly $\underset{\sim}{\nabla}x$, becomes $m^{-1}\underset{\sim}{\nabla}x$, $\underset{\sim}{\nabla}^{2}$

becomes $m^{-2}V^2$ etc. When, later in the paper, quantities are replaced by dimension-free variables, capital letters (such as B) are replaced by small letters (such as b). Finally, there is one question that merits consideration, and that is, whether or not these units are of suitable size to measure phenomena in the magnetotail. They were chosen fortuitously to measure terrestrial phenomena on the laboratory scale: it might seem that, in particular, the unit of length is small for use in describing phenomena in the magnetotail. However this smallness is unimportant in view of the fact that length itself only appears in the governing equations in the form of the differential operator.

Meanwhile the units of mass, length and time are at least consistent, as may be shown by the following rough argument:

In the physical world the smallest known existing mass is that of the electron, which is (17) $\sim 10^{-30}k^1$. The largest known mass is that of the universe, believed to be $\sim 10^{54}k^1$, calculated from the believed density, $\sim 10^{-27}k^1m^{-3}$ (18) and volume (20). Therefore the kilogramme occupies a position $5/14^{ths}$, in the scale of order of sizes, from the smallest to the largest known mass. The smallest length, that exists in its own right, may be taken to be (19) the classical radius of the electron, $\sim 10^{-15}m^1$. The largest, which is what is believed to be the radius of the universe, is (20) $\sim 10^{27}m^1$. The metre therefore also occupies a position $5/14^{ths}$ on the order of scale size, from the smallest to the largest known length.

A similar result can be obtained for the second, if the smallest known time that can be measured with any certainty be taken to be the period of a molecular vibration. This, for e.g. the nitrogen molecule is (21) $\sim 10^{-10}s^1$. Similar, and also considerably shorter periods have been calculated for the half-lives, or "lifetimes" of unstable elementary particles. Powell, et al. (22) have listed a lifetime of $\sim 10^{-10}s^1$ for the decay of a $\Sigma^+$ hyperon into a neutron and a pion, $\sim 10^{-13}s^1$ for that of a $\Sigma^0$ hyperon into a lambda particle and

$\gamma$ radiation, and $\sim 10^{-16}s^{1}$ for pions into positron-electron pairs and/or $\gamma$ radiation.

Later in the same work the possibility of life-times as short as $\sim 10^{-22}s^{1}$ are mentioned. However these periods are subject to the uncertainty connected with the laws of probability: thus a particular pion may decay after any period between $10^{-\infty}\ s^{1}$ and $10^{\infty}\ s^{1}$. Therefore it is proposed to discount these decay life-times in the present rough analysis of the position of the second on the time scale, and use, for the shortest accurately known period one of the order of $10^{-10}s^{1}$. The largest known time is what is believed to be the life-expectancy of the universe, which is (23) $10^{18}s^{1}$ (see also Hawking,(24),(25)). Therefore the second is again 5/14ths on the order scale from the smallest to the largest known time.

However, judged by this criterion, the coulomb turns out to be a comparatively small unit of charge. The smallest known charge is that of the electron (17): $10^{-19}C^{1}$. One may make an estimate of the largest known charge by considering a universe, mass $10^{54}k^{1}$, as above, consisting entirely of hydrogen. This would require, taking a proton and electron mass together as (17)$\sim 10^{-27}\ k^{1}$, $\sim 10^{81}$ protons, so that the total positive charge would be $\sim 10^{81} \times 10^{-19}$, i.e. $\sim 10^{62}\ C^{1}$. In view of the possibility of large numbers of free or potentially free (in radiation) positrons in intergalactic space the order may be even higher. A unit of charge, 5/14ths in order along the range from $10^{-19}$ to $10^{\geqslant 62.2}\ C^{1}$, is $10^{N}\ C^{1}$, with $N \geqslant 10$.

§1.2: Permeability.

The permeability of a non-ferromagnetic medium differs from that of free space, that is, of a vacuum, by an amount of (26) proportionate order $\sim 10^{-6}$. Therefore it is sufficient, in this approximate analysis, to use the value of the permeability of a vacuum, for which the symbol $\mu$ will be used. This is normally expressed as (27) as $\sim 10^{-6}$ Henry per metre. However the latter unit is based on the metre-kilogramme second system, therefore one may write:-

$$\mu \sim 10^{-6} C^{-2} k^{1} m^{1}.$$

§1.3 Magnetic Induction and its Space Differentials.

In this and in the next four paragraphs it is proposed to obtain, by interpolation between values of space variables, known not exactly but at least correctly to an order of magnitude, likely values of these variables in the magnetotail region: such values being reliable at least to within the limit of an order of magnitude. The results will not, of course, bring into cognisance such local variations as may exist in the magnetotail region as a result of unstable or of overstable behaviour: in fact is is the purpose of this paper to enquire into the possible nature of such instabilities and overstabilities; the values obtained in this chapter will serve as starting, or boundary, values.

Values for the variables are given in the literature in certain regions in space which, by nature of the particular radiation received from them, or by their accessibility, have lent themselves to more detailed study than have general regions elsewhere. Such regions will be indicated by a suffix in the following manner: if A, say, is a space variable, then:

$A_S$ means: as measured in the Sun's corona, near the Sun,

$A_E$ " " " on the earth's surface,

$A_G$ " " " in the nearest HII region in the Galaxy.

We shall also have occasion to use suffixes for two other regions:-

$A_Q$ , meaning: as measured at the nearest stellar distance where the mean Galactic density tends to a constant value, and

$A_R$ , meaning: "as measured in a gas-cloud in the Galaxy". Finally, for the resulting value, as expected in the magnetotail region, the suffix M will be used:-

$A_M$ means: as expected in the magnetotail region, due to interpolation from known distant values, and disregarding possible large local fluctuations; in short: "as due to distant effects."

Capital letters will be used for all space variables in order that, later, the corresponding small letter may be used for dimensionless quantities: this does not apply to conductivity or to permeability, which will in due course be absorbed in derived variables.

For the magnitude B of the magnetic induction vector $\underset{\sim}{B}$, the first known value to consider is $B_E$, which is known (28) accurately. In fact, taking a mean value: $B_E \sim 10^{-5} C^{-1} k^{1} s^{-1}$.

In the reference quoted, the value, $\sim 10^{-1}$, given is for the magnetic intensity; it is necessary to multiply the figure by the permeability (using a Maxwell constitutive equation), see §1.2, and to adjust from centimetre-gramme-second electromagnetic units to CKMS units. Figures given in the reference are for various points at the Earths surface, however they are all of the same order. Thus: given magnetic intensity ( $\sim 10^{-1}$ ) x conversion factor to CKMS ( $\sim 10^{2}$), x permeability ($\sim 10^{-6}$) gives $B_E$.

As the distance from the Earth's centre increases, the magnetic intensity, and with it, the magnetic induction both together fall off in magnitude approximately as the inverse cube of the distance, from the earth's centre, of the point of measurement (with direction constant), in accordance with the known (29) behaviour of a dipole field.

The order of distance, from the Earth's centre, of the nearest point of the magnetotail is $\sim 10^{1}$ Earth radii as will be shown in §3.1. Therefore, we may expect that, due to the Earth:

$$B_M \sim 10^{-3} B_E, \sim 10^{-8} C^{-1} k^{1} s^{-1}.$$

Meanwhile, the second known value is $B_S$, which is (30):

$$B_S \sim 10^{-2} C^{-1} k^{1} s^{-1}.$$

The same remarks, regarding adjustment of the figure ($\sim 10^{2}$) given for magnetic intensity in the reference, apply as in the case of $B_E$, the units used are gauss, i.e. ampere $cm^{-1}$, so that $10^{2}$ gauss = $10^{4}$ ampere $m^{-1}$, which is a CKMS unit.

The Sun's magnetic field is also approximately a dipole field, and so although the field in the corona is largely due to local disturbances, we may expect that any magnetic field due to the Sun will fall off in magnitude approximately with the universe cube of the distance, at constant direction, from the centre of the Sun to the point of measurement. Under this assumption it will shortly be shown that $B_M$, due to the Sun, is of the same order as that due to the Earth. Therefore, if we were to take some point in the corona, near to the point at which $B_S$ is measured and collinear with it and the Earth, as the point from which distance is measured, we would obtain a very much smaller value for $B_M$, due to the Sun, the distance proportion being very much greater. The value of $B_M$ required is, of course, that due to Earth, Sun, and all other causes, therefore it will suffice to use the Sun's centre as reference point. The fact that the corona field is carried away radially from the Sun's centre by the solar wind lends weight to the advisability of this procedure. It also leads to the possibility that the field reduces approximately as the inverse square of the Sun's distance. This will lead to a larger likely figure for $B_M$, below, but will not affect the finding in the remainder of the paragraph, viz. that $|dB/dL|_M$ and $|d^2B/dL^2|_M$ are affected more strongly by the Earth than by the Sun.

The order of distance (31) of the corona from the Sun's centre is $\sim 10^9 m^1$, and that of the magnetotail is $\sim 10^{11} m^1$, and so we may expect that $B_M$, due to the Sun, is $\sim 10^{-2} \times (10^2)^{-3}$, i.e. $\sim 10^{-8} C^{-1} k^1 s^{-1}$. Therefore in the magnetotail region, the magnitude of the magnetic induction vector is approximately equally affected by the Sun and by the Earth, and it is safe to say that :

$$B_M \sim 10^{-8} C^{-1} k^1 s^{-1} .$$

For the purpose of calculating the first and second space differentials the magnetic induction vector, we need a third measurement. This is (30):

$$B_G \sim 10^{-9} C^{-1} k^1 s^{-1} ,$$

the figure ($10^{-5}$ Gauss) in the reference being adjusted in the same way as for $B_S$ above. Distances, L, (31) of points of measurement, are :-

(i) From the Sun's centre: $L_S \sim 10^9 m^1, L_M \sim 10^{11} m^1, L_G \sim 10^{17} m^1$. In the last figure it is assumed that the nearest HII region is distant approximately 3 parsecs. At this stage it is convenient to make use of a prime, meaning logarithm to base 10 of number of units in the value of the unprimed symbol. Thus:

$$A' \equiv \log_{10} \text{(no. of units in A)}.$$

This avoids cumbersome expressions. Fig. 1 shows a sketch of $B'$ plotted against $L'$, the letters referring to the region of measurement in this and in later figures. Inspection of Fig. 1 shows that : (App. B. p(ix))

$$|dB' / dL'|_S \sim 10^1 , \quad |dB'/dL'|_M \sim 10^0 , \quad |dB'/dL'|_G \sim 10^{-1} .$$

Then, since $dB'/dL' = (L/B)(dB/dL)$, we see that :

$$|dB/dL|_S \sim 10^1 (B/L)_S , \sim 10^{-10} C^{-1} k^{1} m^{-1} s^{-1} ,$$

$$|dB/dL|_M \sim 10^0 (B/L)_M , \sim 10^{-19} C^{-1} k^{1} m^{-1} s^{-1} ,$$

$$|dB/dL|_G \sim 10^{-1} (B/L)_G , \sim 10^{-27} C^{-1} k^{1} m^{-1} s^{-1} .$$

Fig. 2 shows a sketch of $|dB/dL|'$ plotted against $L'$. (p.(ix))

Then, since $d|dB/dL|'/dL' = (L/|dB/dL|)(d^2B/dL^2)$, and, from the sketch:

$$|d|dB/dL|'/dL'|_M \sim 10^0 ,$$

We see that: $|d^2B/dL^2|_M \sim 10^0(|dB/dL|/L)_M$,

i.e. $|d^2B/dL^2|_M \sim 10^{-30} C^{-1} k^{1} m^{-2} s^{-1}$.

(ii) From the Earth's centre: $L_E \sim 10^7 m^1$, $L_{M*} \sim 10^8 m^1$, $L_G \sim 10^{17} m^1$: the asterisk is to avoid confusion between $L_{M*}$ & $L_M$ in (i) above, not necessary in the case of G. Fig. 3 shows a sketch of $B'$ plotted against $L'$, and shows (p.(x)) that: $|dB'/dL'|_E \sim 10^1$, $|dB'/dL'|_{M*} \sim 10^0$, $|dB'/dL'|_G \sim 10^{-1}$. In fact, dipole theory (see § 3.1) indicates that, on the ecliptic, $|dB'/dL'| = 3$ as long as the field is due to the dipole only: this supports the first two figures above. Therefore, as in (i)

$$|dB/dL|_E \sim 10^1 (B/L)_E , \sim 10^{-11} C^{-1} k^{1} m^{-1} s^{-1} ,$$

$$|dB/dL|_{M*} \sim 10^0 (B/L)_{M*} , \sim 10^{-16} C^{-1} k^{1} m^{-1} s^{-1} ,$$

$$|dB/dL|_G \sim 10^{-1} (B/L)_G , \sim 10^{-27} C^{-1} k^{1} m^{-1} s^{-1} , \text{ as before.}$$

Fig. 4 shows a sketch of $|dB/dL|'$ plotted against $L'$, and shows that: (p.(x))

$$|d|dB/dL|'/dL'|_{M*} \sim 10^0 ,$$

whence, as in (i) $|d^2B/dL^2|_{M*} \sim 10^0 (|dB/dL|/L)_{M*}$,

i.e. $|d^2B/dL^2|_{M*} \sim 10^{-24} C^{-1} k^{1} m^{-2} s^{-1}$.

Taking the larger order values, i.e. those of sub para. (ii), and dropping the asterisk since it is no longer needed, we note that rough maximum values can be taken as:

$$B_M \sim 10^{-8} C^{-1} k^{1} s^{1} ,$$

$m^{-1}|\nabla B|_M$, $m^{-1}|\nabla \times B|_M$ both $\sim |dB/dL|_M$, i.e. $\sim 10^{-16} C^{1} k^{1} m^{-1} s^{1}$ ,
and $m^{-2}|\nabla^2 B|_M \sim |d^2B/dL^2|_M$ , i.e. $\sim 10^{-24} C^{1} k^{1} m^{-2} s^{-1}$ .

§ 1.4. Conductivity.

For the conductivity of inter-planetary and inter-stellar space, figures are only available in the corona and in galactic regions. Dungey (30) gives, for $\sigma/c^2$ :- Corona, $10^{-5}$; HII, $10^{-11}$ ; the units used are based on the volt, amp etc., and on the centimetre-gramme-second system. Referring back to §1.1. for the dimension of $\sigma$, and taking $c^2$ as $\sim 10^{21} cm^2 s^{-2}$ we get $\sigma_S \sim 10^{16} C^2 gm^{-1} cm^{-3} s^1$, $\sigma_G \sim 10^{10} C^2 gm^{-1} cm^{-3} s^1$. Adjusting to CKMS units:- $\sigma_S \sim 10^{25} C^2 k^{-1} m^{-3} s^1$, $\sigma_G \sim 10^{19} C^2 k^{-1} m^{-3} s^1$.

It will be noted that $\sigma_S/\sigma_G \sim 10^6$, and from §1.3., that $B_S/B_G \sim 10^7$ . Therefore it will be assumed that $\sigma_M/\sigma_G$ is of the same order as $B_M/B_G$ , (thus avoiding the effect of possible high fluctuation values of $B_S$ ), i.e. $\sigma_M/\sigma_G \sim 10^1$, whence:

$$\sigma_M \sim 10^{20} C^2 k^{-1} m^{-3} s^1 .$$

§ 1.5. Density.

For the density of extra terrestrial space, figures are again only available in the corona and in galactic regions. Adjusting his figures from $cm^{-3}$ to $m^{-3}$, Dungey (30) gives, for n, the neutron number density:-

$$n_S \sim 10^{13} m^{-3}, \quad n_G \sim 10^6 m^{-3} .$$

Multiplying these figures by the mass of a neutron which is (17) $10^{-27} k^1$ we obtain: $D_S \sim 10^{-14} k^1 m^{-3}$ , $D_G \sim 10^{-21} k^1 m^{-3}$ .

Further figures have been given by Sciama (32): the net density of the galaxy, which can be written as $D_Q$, $\sim 10^{-21} k^1 m^{-3}$ , and although the figure includes the stars, their presence does not affect the figure **by** as much as an order. Therefore the density in the nearest HII region is of the same order as that throughout the galaxy away from stars and planetary systems. Further it is obvious that $D_S$ is decreasing rapidly as L increases. Therefore it is possible to construct a $D' - L'$ curve noting that the slope, $dD'/dL'$, i.e. $(L/D)(dD/dL)$ is infinite in the corona region and nil at the nearest HII region; also it is effectively nil at the "Q" region.

Fig. 5 shows the sketch, drawn with these considerations, from which (p.(xi)) we see that :

$$D_M \sim 10^{-20} k^1 m^{-3}.$$

§1·6. Pressure, and its space differential.

Little appears to be known about inter-stellar hydrostatic pressure. Dungey (33) indicates that P/D is of the same order as $\frac{1}{2}V^2$, whence we may write $P_M \sim (DV^2)_M$, $\sim(10^{-20}(10^4)^2 \simeq)$ $10^{-12} k^1 m^{-1} s^{-2}$, for $V_M$ see §1.7. Later, (34) in the same monograph, an ionization pressure of $10^{-10}$ dyne$^1$ cm$^{-2}$ is given for the outer atmosphere: if we reduce this by an order, to get a likely value for $P_M$, and adjust to CKMS units, we again obtain the above result: $P_M \sim 10^{-12} k^1 m^{-1} s^{-2}$.

To get an idea of the way in which P varies with distance, a further measure of P and of D in space is required, so that a possible equation of state, at least at constant temperature, (a likely condition) may be derived. G.B. Field gives, in his lecture "The Physics of the Interstellar and Intergalactic Medium", (35) : $P_R \sim 10^{-13} k^1 m^{-1} s^{-2}$, in a gas cloud in which the neutron density is (36) $n_R \sim 10^6 m^{-3}$ at a temperature of 100°K, i.e. where (§1.5.) $D_R \sim 10^{-21} k^1 m^{-3}$. So that $P_M/D_M$ and $P_R/D_R$ are both $\sim 10^8 m^2 s^{-2}$, and we may assume that a rough form of Boyle's Law is applicable to the pressure and density of extra-terrestrial space, in the form $P \sim 10^{8} D\, k^{1} m^{-1} s^{-2}$.

Then $|dP/dL|_M = 10^{8} |dD/dL|_M = 10^{8} (D/L)_M |dD^{\prime}/dL^{\prime}|_M$, so that by inspection of Fig. 5, we see that $|dP/dL|_M \sim 10^{8} (10^{-20} 10^{-11}) 10^{0}\ k^{1} m^{-2} s^{-2}$,

i.e. $m^{-1}|\underset{\sim}{\nabla} P|_M \sim |dP/dL|_M, \sim 10^{-23} k^{1} m^{-2} s^{-2}$.

§1.7 Velocity and its space differentials.

Dungey (30) gives very nearly the same value for $V_S$, the particle velocity, in the corona, $\sim 10^{5} m^{1} s^{-1}$, as for $V_G$, the particle velocity in the HII region, $\sim 10^{4} m^{1} s^{-1}$. Field (37) gives $V_Q \sim 10^{4} m^{1} s^{-1}$, due to turbulence and mass motion in interstellar space. It seems likely that V is increased by one order in the corona, due to the high temperatures obtaining in that region, and that it is possible to regard the order of V elsewhere as constant, giving $V_M \sim 10^{4} m^{1} s^{-1}$,

and: $m^{-1}|\underset{\sim}{\nabla}\underset{\sim}{V}|_M$ and $m^{-1}|\underset{\sim}{\nabla} \times \underset{\sim}{V}|_M$ both $\sim |dV/dL|_M, \sim 10^{-\infty}$ i.e. negligible, and similarly $m^{-2}|\underset{\sim}{\nabla}(\underset{\sim}{\nabla}\cdot\underset{\sim}{V})|_M$ and $m^{-2}|\nabla^{2}\underset{\sim}{V}|_M$ both $\sim|d^{2}V/dL^{2}|_M, \sim 10^{-\infty}$, i.e. negligible, and since the kinematic viscosity $\nu$ of extra-terrestrial space is likely to be a very small quantity in CKMS units the term (38): $m^{-2}\nu\left(\nabla^{2}\underset{\sim}{V} + \frac{1}{3}\underset{\sim}{\nabla}(\underset{\sim}{\nabla}\cdot\underset{\sim}{V})\right)$, in the equation of motion may be ignored in the analysis of hydromagnetic phenomena in the magnetotail region.

1.8. Derived variables.

As will be seen later (Ch. II), the quantities $\mu$ and $\sigma$ only appear, in the governing equations of hydromagnetics, in the magnotail region, in combination with each other or with the density D. Therefore it is convenient to introduce "derived variables" $\Lambda, \Gamma$ by:

$$\Lambda \equiv (\mu D)^{-1}, \quad \text{and} \quad \Gamma \equiv (\mu\sigma)^{-1}.$$

Then the likely order of size of these derived variables, due to distant effects is (see §§1.2, 1.4 and 1.5):-

$\Lambda_M \sim 10^{26} C^{2} k^{-2} m^{2}$,

and $\Gamma_M \sim 10^{-14}\ m^{2} s^{-1}$.

## 1.9 Summary of Variables

It is convenient now to summarise the likely value of relevant variables in the magnetotail region, due to distant, rather than local, influences; the suffix M will be discontinued at this stage, being no longer needed:-

From §1.3 :- $B \sim 10^{-8}\ C^{-1} k^{1} s^{-1}$,

" " :- $m^{-1}|\nabla B|$ & $m^{-1}|\nabla \times B| \sim 10^{-16}\ C^{-1} k^{1} m^{-1} s^{-1}$,

" " :- $m^{-2}|\nabla^2 B| \sim 10^{-24} C^{-1} k^{1} m^{-2} s^{-1}$,

From §1.5 :- $D \sim 10^{-20} k^{1} m^{-3}$,

" §1.6 :- $P \sim 10^{-12} k^{1} m^{-1} s^{-2}$,

" " :- $m^{-1}|\nabla P| \sim 10^{-23} k^{1} m^{-2} s^{-2}$,

" §1.7 :- $|V| \sim 10^{4} m^{1} s^{-1}$,

" " :- $m^{-1}|\nabla V|$ & $m^{-1}|\nabla \times V| \sim 10^{-\infty} s^{-1}$,

" §1.8 :- $\Lambda \sim 10^{26} C^{2} k^{-2} m^{2}$, &

" " :- $\Gamma \sim 10^{-14} m^{2} s^{-1}$.

A remark may be made at this stage that all the above calculations are very rough: this particularly applies to $\sigma_M$ and hence to $\Gamma_M$. However if we were to write, generally, that $10^{19} < \sigma_M < 10^{25} C^{2} k^{-1} m^{-3} s^{1}$, this would mean that $10^{-13} > \Gamma_M > 10^{-19} m^{2} s^{-1}$, i.e.that the order of $m^{-2}\Gamma|\nabla^2 B|_M$ lies between $10^{-37}$ and $10^{-43}$, that is, between $10^{-25}$ and $10^{-31}$ of the term involving $|V \times B|_M$.

Also, $V_a$ the Alfvén speed, $\Lambda^{\frac{1}{2}} B$ (39), with the above values of $\Lambda$, B in the magnetotail region is seen to be such that $10^{4} < V_a < 10\ m^{1} s^{-1}$.

## 1.10 The Unit of Charge, effect of change.

As will be seen in Chapter II, the leading term governing the rate of change of the magnetic induction is $m^{-1}\nabla \times (V \times B)$. In the expansion of this term, we may ignore : $m^{-1} B\cdot\nabla V$ ( §1.9), $m^{-1}(\nabla\cdot B)\ V$ (Maxwell's equations), and $m^{-1}(\nabla\cdot V) B$ (§1.9). This leaves only $m^{-1} V\cdot\nabla B$ which, as can be seen from §1.9 has the order $10^{-12} C^{-1} k^{1} s^{-2}$. Meanwhile the other term,

$m^{-2}\Gamma\nabla^2 \underset{\sim}{B}$ has the order $10^{-38} C^{-1} k^1 s^{-2}$, and so is of the order $10^{-26}$ when compared with the first term, irrespective of what units are used.

The leading term governing the acceleration is $m^{-1}\Lambda( (\underset{\sim}{\nabla} \times \underset{\sim}{B}) \times \underset{\sim}{B})$. This, as can be seen from §1.9 has the order $10^2 m^1 s^{-2}$. The other terms, $m^{-1} \underset{\sim}{V} \cdot \underset{\sim}{\nabla} \underset{\sim}{V}$ and $m^{-1} D^{-1} \underset{\sim}{\nabla} P$ have respectively the orders $10^{-\infty}$ and $10^{-3}$ $m^1 s^{-2}$, and so the largest of these is of order $10^{-5}$ compared to the term involving $\Lambda$, again irrespective of what units are used.

Thus, the equation of motion, as it stands, is in a form suitable for perturbation analysis, a pertubation of unit order being of order $10^{-2}$ compared to the leading term, due to distant effects, in the equation.

It will be convenient to adjust the magnetic induction equation so that, for this equation also, the leading term is of order $10^{2\pm1}$ of its units. This may be effected by using a new unit of charge, given by the symbol f, where $1f^1 = 10^{13} C^1$. The result is that the summary of §1.9 is replaced by :

$B \sim 10^5 f^{-1} k^1 s^{-1}$,

$m^{-1}|\underset{\sim}{\nabla} \underset{\sim}{B}|$ and $m^{-1}|\underset{\sim}{\nabla} \times \underset{\sim}{B}| \sim 10^{-3} f^{-1} k^1 m^{-1} s^{-1}$,

$m^{-2}|\nabla^2 \underset{\sim}{B}| \sim 10^{-11} f^{-1} k^1 m^{-2} s^{-1}$,

$D \sim 10^{-20} k^1 m^{-3}$,

$P \sim 10^{-12} k^1 m^{-1} s^{-2}$,

$m^{-1}|\underset{\sim}{\nabla} P| \sim 10^{-23} k^1 m^{-2} s^{-2}$,

$|\underset{\sim}{V}| \sim 10^4 m^1 s^{-1}$,

$m^{-1}|\underset{\sim}{\nabla}\underset{\sim}{V}|$ and $m^{-1}|\underset{\sim}{\nabla} \times \underset{\sim}{V}| \sim 10^{-\infty} s^{-1}$,

$\Lambda \sim 10^0 f^2 k^{-2} m^2$

$\Gamma \sim 10^{-14} m^2 s^{-1}$.

This unit of charge is a large one, even allowing for the influence of the discussion at the end of Chapter I. However its use has the advantage that the order of size of the two governing equations of hydromagnetics in the magnetotail region is nearly the same. In addition it may be noted that $10^{13} C^1$ is the total charge on (17) $\sim 10^{32}$ protons or electrons: taking a proton mass as (17) $\sim 10^{-27} k^1$, this is the charge on $\sim 10^5 k^1$. Therefore $1f^1$ is the total proton charge on, say, $10^3 m^3$ of a neutron free element such as liquid hydrogen (40), which is not a large volume even by terrestrial standards.

# CHAPTER II

## DIMENSION-FREE EQUATIONS.

### 2.1. The Induction-Diffusion Equation.

The connection between $\underset{\sim}{V}$ and $\underset{\sim}{B}$ in a magnetofluid is given by the equation (41):-

$$\partial\underset{\sim}{B}/\partial T = m^{-1}[\underset{\sim}{\nabla} \times (\underset{\sim}{V} \times \underset{\sim}{B})] + \Gamma m^{-2}\nabla^2\underset{\sim}{B}, \quad f^{-1}k^{1}s^{-2};$$

the variables and operator $m^{-1}\underset{\sim}{\nabla}$ being defined in §1.1.

To render this equation dimension free, we introduce the dimension-less variables:-

$$\underset{\sim}{b} = \underset{\sim}{B}/B_o,$$
$$t = T/T_o,$$
$$\underset{\sim}{v} = \underset{\sim}{V}/V_o,$$

and $\gamma = \Gamma/\Gamma_o$, where:

$$B_o = 1 \; f^{-1}k^{1}s^{-1},$$
$$T_o = 1 \; s^{1},$$
$$L_o = 1 \; m^{1},$$
$$V_o = 1 \; m^{1}s^{-1},$$
$$\Gamma_o = 1 \; m^{2}s^{-1},$$

and multiply the L.H.S. term by $T_o/B_o$, the first term in the RHS by $L_o^2\,(V_o B_o)^{-1}$, and the second term on the R.H.S. by $L_o^4\,(\Gamma_o B_o)^{-1}$. The dimensions of these multipliers are, respectively: $1\; f^{1}k^{-1}s^{2}$, $1\; f^{1}k^{-1}m^{1}s^{2}$, and $1\; f^{1}k^{-1}m^{2}s^{2}$ so that the result is the dimension-free equation :-

$$\partial\underset{\sim}{b}/\partial t = [\underset{\sim}{\nabla}\times(\underset{\sim}{v} \times \underset{\sim}{b})] + \gamma\nabla^2\underset{\sim}{b}.$$

From §1.10, it will be seen that, in the magnetotail region, and due to distance influences only, the first term on the R.H.S. is of order $10^{1}$, the second term is of order $10^{-25}$, of which a factor $10^{-14}$ is due to $\gamma$ .

### §2.2. The Equation of Motion

The connection between P, $\underset{\sim}{V}$ and $\underset{\sim}{B}$ in a magnetofluid is given by the equation of motion (42):-

$$\partial \underset{\sim}{V}/\partial T = -m^{-1}(\underset{\sim}{V}\cdot\underset{\sim}{\nabla})\underset{\sim}{V} + \Lambda m^{-1}\{(\underset{\sim}{\nabla}\times\underset{\sim}{B})\times\underset{\sim}{B}\} - D^{-1}m^{-1}\underset{\sim}{\nabla}P \quad m^1 s^{-2}.$$

In this equation, the external body force, and effects due to temperature and kinematic viscosity have been ignored, see §1.7 and §1.6, which provides the inference that conditions are nearly isothermal. Using the dimensionless variables of para §2.1, where applicable, and also in addition:-

$\lambda = \Lambda/\Lambda_o$,

$\rho = D/D_o$,

$p = P/P_o$, where:

$\Lambda_o = 1\ f^2 k^{-2} m^2$,

$D_o = 1\ k^1 m^{-3}$,

$P_o = 1\ k^1 m^{-1} s^{-2}$,

and multiplying the L.H.S. term by $T_o/V_o$ and the first term on the R.H.S. by $L_o^2\ V_o^{-2}$, the second by $L_o^2\ (\Lambda_o B_o^2)^{-1}$ and the third by $L_o^2\ D_o/P_o$, these multipliers having the respective dimensions $m^{-1}s^2$, $s^2$, $s^2$, $s^2$, we obtain the dimension-free equation:

$$\partial \underset{\sim}{v}/\partial t = -(\underset{\sim}{v}\cdot\underset{\sim}{\nabla})\underset{\sim}{v} + \lambda\{(\underset{\sim}{\nabla}\times\underset{\sim}{b})\times\underset{\sim}{b}\} - \rho^{-1}\underset{\sim}{\nabla}p.$$

From §1.10 it will be seen that, in the magnetotail region, and due to distant effects only, the first term in the R.H.S. is of negligible order, the second is of order $10^2$, and the third of order $10^{-3}$.

## 2.3 The equation of continuity

The connection between D and $\underset{\sim}{V}$ in any fluid is given by the equation of continuity (43):-

$$\partial D/\partial T = -m^{-1}\underset{\sim}{\nabla}\cdot(D\underset{\sim}{V})$$

This equation can be put into dimension-free form by methods similar to those used in §2.1., §2.2. :-

$$\partial \rho/\partial t = -\underset{\sim}{\nabla}\cdot(\rho\underset{\sim}{v}).$$

The order of the term on the R.H.S. will be discussed in the next paragraph.

§2.4. The equation of state

The equation of state in a plasma has been discussed by many writers. Linhart (44) provides, for the case of isotropic pressure, constant stored magnetic energy density and zero fluid velocity, an equation of the form $\ln(P/D) = 2\alpha^{-1}\ln(D)$ + constant, where $\alpha$ is the number of degrees of freedom. This is based on the behaviour of a monomolecular gas and considers terrestrial densities, so that $\alpha$ is finite. However it may be argued that in extra terrestrial space, where a large variety of types of particle may exist, $\alpha$ is a large number and we may write tentatively (this incidentally agrees with the deduction of §1.6) :-

$$P/D = \text{constant}.$$

$$\text{i.e.}\quad m^{-1}D\nabla P = m^{-1}P\nabla D.$$

Putting this equation into dimension-free form in the manner of the preceding paragraphs of this chapter, we obtain:

$$\rho\nabla p = p\nabla\rho.$$

Then $\nabla\rho = \rho p^{-1}\nabla p$, which, from §1.10, indicates that, due to distant influences, $\nabla\rho \sim 10^{-20} \times 10^{12} \times 10^{-23}$, i.e. $\nabla\rho \sim 10^{-31}$.

Thus the term $-\nabla(\rho v)$ in §2.3, which can be replaced by $v\cdot\nabla\rho$ since $\nabla\cdot v$ is negligible (§1.10), is of order $10^{4}\times 10^{-31}$, i.e. of order $10^{-27}$.

Both terms, in above $p,\rho$ equation, due to distant influences are of order $10^{-43}$.

§2.5. Permissible Approximations in the Magnetotail

If we write the equations of §§2.1-4 as:

$$\partial b/\partial t = \nabla \times (v \times b),$$

$$\partial v/\partial t = -(v\cdot\nabla)v + \left((\nabla \times b)\times b\right),$$

$$\partial\rho/\partial t = -\nabla\cdot(\rho v) = 0,$$

and

$$\rho\nabla p = p\nabla\rho = 0,$$

then, by ignoring the terms $\gamma\nabla^2 b$ and $\rho^{-1}\nabla p$, and by setting the last two equations as equal to zero identically, we are assuming that:

(i) local variations of $\gamma, \underset{\sim}{\nabla}\rho, \underset{\sim}{\nabla}p$, and of $\nabla^2 \underset{\sim}{b}$ are of the same order in the magnetotail region as those due to distant influences,

(ii) $\lambda \sim 10^o$, constant and steady, since $\rho \sim$ constant and steady.

(iii) $\rho(\underset{\sim}{\nabla}\cdot\underset{\sim}{v})$ is not of higher order than $\rho(\sim 10^{-20})$, even if $\underset{\sim}{v}$ varies locally and $\underset{\sim}{\nabla}\cdot\underset{\sim}{v}$ is finite, and

(iv) terms of order $10^{-3}$ and smaller may be ignored, although, due to local variation of $\underset{\sim}{v}$, the term $(\underset{\sim}{v}\cdot\underset{\sim}{\nabla})\underset{\sim}{v}$ can be retained.

With this approximation, the only equations that need be analysed are the first two, since they do not depend on any variation in p or $\rho$.

Of course, at a neutral point, the first equation becomes identically zero and if it is assumed that the velocity also disappears at a neutral point, the second equation becomes zero, also, there.

Therefore in the analysis to be carried out in later paragraphs, attention will be directed to the behaviour of $\partial(\underset{\sim}{\nabla}\underset{\sim}{b})/\partial t$ and of $\partial(\underset{\sim}{\nabla}\underset{\sim}{v})/\partial t$, at the neutral point, as a result of perturbations in elements of $\underset{\sim}{\nabla}\underset{\sim}{b}, \underset{\sim}{\nabla}v$, and, subsequently, of possible perturbations in $\nabla^2 \underset{\sim}{b}, \underset{\sim}{\nabla}p$ and $\gamma$.

# CHAPTER III.

## THE NEUTRAL LINE.

### §3.1. A dipole in a uniform field.

In this and in the next paragraph, an idea will be obtained of the form taken by the magnetic induction field near the neutral ring as defined in the introduction, para (i), and ignoring the effect of the solar wind.

A Cartesian frame $OX_1Y_1Z_1\hat{\underset{\sim}{x}}\hat{\underset{\sim}{y}}\hat{\underset{\sim}{z}}$ is defined as follows: O is at the Earth's centre, $OX_1\hat{\underset{\sim}{x}}$ points directly away from the Sun's centre, $OY_1\hat{\underset{\sim}{y}}$ is along the Earths axis of rotation and North: in this analysis the axis of rotation is taken to be perpendicular to the plane of the ecliptic; allowance for inclination complicates the mathematics, but produces a similar final result. Since the frame is righthanded, $OZ_1\hat{\underset{\sim}{z}}$ is contra-parallel to the earth's velocity in revolution.

It is convenient to use plane polar coordinates $(R,\phi)$ for the plane $OX_1Z_1$ with $\phi = 0$ co-parellel $\hat{\underset{\sim}{x}}$, $\phi = \frac{\pi}{2}$ co-parallel $-\hat{\underset{\sim}{z}}$, and to assume for simplicity that the magnetic axis of the Earth is parallel to its axis of rotation. Then, since the Earth's "North Magnetic Pole" is in fact a "South" pole, for regions well away from the Earth's surface we may write down the magnetic induction vector $\underset{\sim}{B}_\oplus$, due to the Earth only, as that due to a magnetic dipole of strength $\underset{\sim}{\beta}$, say, $= -\beta\hat{\underset{\sim}{y}}$, at O. Then (45), at a point P, where $\vec{OP} = \underset{\sim}{r}_1$,

$$\underset{\sim}{B}_\oplus = -\frac{\mu}{4\pi}\, m^{-1}\underset{\sim}{\nabla}\{(\underset{\sim}{\beta}\cdot\underset{\sim}{r}_1)/r_1^{\,3}\}.$$

The Sun may also, to a close approximation, be treated as a magnetic dipole as far as its effects are felt well away from the Sun's surface: in this case the sense of the Sun's dipole is (46) directly opposed to that of the earth. Further, since the representative dipole of the Sun is of the order of $10^9$ that of the earth, one may expect that the neutral ring is at a distance from the Earth small compared to the distance from the Earth to the Sun; in fact it is $\sim 10^{-3}$ of the Sun's distance. Therefore, the magnetic induction vector, $\underset{\sim}{B}_\odot$, due to the Sun only, may be treated as a constant in the region near the Earth and including the neutral ring:

$$\underset{\sim}{B}_{\odot} = -\frac{\mu\beta\zeta}{4\pi}\,\hat{\underset{\sim}{y}},\ \zeta \text{ constant}.$$

The net field is therefore $\underset{\sim}{B}$, with:

$$\underset{\sim}{B} = \underset{\sim}{B}_{\oplus} + \underset{\sim}{B}_{\odot} = -\frac{3\mu\, RY_1}{4\pi(R^2+Y_1^2)^{5/2}}\,\hat{\underset{\sim}{R}} + \frac{\mu\beta}{4\pi}\left\{\frac{(R^2-2Y_1^2)}{(R^2+Y_1^2)^{5/2}} - \zeta\right\}\hat{\underset{\sim}{y}}.$$

The neutral ring will be where both terms in the R.H.S. of the above equation disappear, i.e. it will be where $Y_1 = 0$ and $R = \zeta^{-\frac{1}{3}} = A$, say: these are the equations giving the neutral ring.

As a matter of interest, if $Y_1 = 0$, $\underset{\sim}{B}_{\oplus} = \frac{\mu\beta}{4\pi R^3}\hat{\underset{\sim}{y}}$, so that on the Earth's surface ($R \sim 10^7 m^1$), from §1.3, $\frac{\mu\beta}{4\pi} \sim 10^{16}\, C^{-1} k^1 m^3 s^{-1}$, whereas (§1.3) $\frac{\mu\beta\zeta}{4\pi} \sim 10^{-8} C^{-1} k^1 s^{-1}$, which shows that $R$, for the neutral ring, $\sim 10^8 m^1$ as has already been claimed.

Now, for convenience, we choose the point D with Cartesian coordinates (A,0,0) as a new origin of a Cartesian frame $DXYZ\hat{\underset{\sim}{x}}\hat{\underset{\sim}{y}}\hat{\underset{\sim}{z}}$, and express the field $\underset{\sim}{B}$ in these new coordinates using the shift transforms:-

$$X = X_1 - A,\quad Y = Y_1\quad Z = Z_1 .$$

We obtain:

$$\underset{\sim}{B} = \frac{\mu\beta}{4\pi}\left[-\frac{3Y\{(X+A)\,\hat{\underset{\sim}{x}} + Z\hat{\underset{\sim}{z}}\}}{\{(X+A)^2 + Y^2 + Z^2\}}\right] + \frac{\mu\beta}{4\pi}\left[\frac{\{(X+A)^2 - 2Y^2 + Z^2\}}{\{(X+A)^2 + Y^2 + Z^2\}^{5/2}} - \zeta\right]\hat{\underset{\sim}{y}}.$$

§3.2. The Neutral Ring.

When X/A, Y/A and Z/A are small, so that only up to third order terms are kept:-

$$\underset{\sim}{B} = -\frac{\mu\zeta\beta}{4\pi}\left\{\frac{3Y}{A}\left(1 - \frac{4X}{A} + \frac{10X^2}{A^2} - \frac{5Y^2}{2A^2} - \frac{5Z^2}{2A^2}\right)\hat{\underset{\sim}{x}}\right.$$
$$+\left(\frac{3X}{A} - \frac{6X^2}{A^2} + \frac{7Y^2}{A^2} + \frac{4Z^2}{A^2} - \frac{10X^3}{A^3} - \frac{35XY^2}{A^3} - \frac{5XZ^2}{A^3}\right)\hat{\underset{\sim}{y}}$$
$$\left. + \frac{3YZ}{A^2}\left(1 - \frac{5X}{A}\right)\hat{\underset{\sim}{z}}\right\}.$$

Ignoring third order terms:-

$$\underset{\sim}{B} = -\frac{\mu\zeta\beta}{4\pi}\left\{\frac{3Y}{A}\left(1 - \frac{4X}{A}\right)\hat{\underset{\sim}{x}} + \left(\frac{3X}{A} - \frac{6X^2}{A^2} + \frac{7Y^2}{A^2} + \frac{4Z^2}{A^2}\right)\hat{\underset{\sim}{y}} + \frac{3YZ}{A^2}\hat{\underset{\sim}{z}}\right\}.$$

Ignoring second order terms:-

$$\underset{\sim}{B} = -\frac{3\mu\zeta\beta}{4\pi A}\left(Y\hat{\underset{\sim}{x}} - X\hat{\underset{\sim}{y}}\right)$$

Thus, very near D, the $\underset{\sim}{B}$ lines are parallel to a radius vector $\underset{\sim}{S}$, $= X\hat{\underset{\sim}{x}} + Y\hat{\underset{\sim}{y}} + Z\hat{\underset{\sim}{z}}$, if $\underset{\sim}{S}\times\underset{\sim}{B} = \underset{\sim}{0}$, i.e. if $Y^2 - X^2 = 0$: a pair of lines through D at right angles to each other and bisecting the angles between the OX and OY axes, with $\underset{\sim}{B}$ pointing towards D in the first and third quadrant, away from D in the other two quadrants. The particular pair of $\underset{\sim}{B}$ lines, in a plane of lines, that pass through the neutral point will be referred to hereon as "D-lines".

To find the form for the $\underset{\sim}{B}$ lines, allowing for second order terms, on the Z = 0 plane, we find their equation by requiring that $\delta\underset{\sim}{S} \times \underset{\sim}{B} = \underset{\sim}{0}$, where $\delta\underset{\sim}{S} \equiv \delta X\hat{\underset{\sim}{x}} + \delta Y\hat{\underset{\sim}{y}}$. This leads to the differential equation:-

$$\frac{dY}{dX} = \frac{3XA - 6X^2 + 7Y^2}{3Y(A - 4X)}$$

This gives the slope of the $\underset{\sim}{B}$ lines. By the expedient of putting $X = \pm\varepsilon$, $Y = \pm\varepsilon$ (four trials) with $\varepsilon$ small, we may investigate the slope, near D, of the D-lines. These are both convex in the $OX\hat{\underset{\sim}{x}}$ direction, forming a pair symmetric about DY, both tangential at D to the corresponding straight line of the first order case.

The third order terms will have an additional distortional effect,.but it is not necessary to investigate these.

Fig. 6 shows a sketch of $\underset{\sim}{B}$ lines due to the Earth and Sun together near the Earth, and going out to the neutral ring and a small way beyond, with the simplifications stipulated in §3.1. The plane of the figure is any plane through $OY_1$, but for later convenience it is to be regarded as the $OX_1Y_1$ plane. It will be noted that some of the lines avoid the Earth, and that the remainder pass through the Earth. These will be referred to hereon as $\underset{\sim}{B}_\odot$ and $\underset{\sim}{B}_\oplus$ lines respectively; when distorted they retain their identity. The neutral ring passes through the plane of the diagram at the points A,**D**. At these points, particular pairs, one each, of $\underset{\sim}{B}_\odot$ and $\underset{\sim}{B}_\oplus$ lines, becoming D-lines, meet, with their magnitudes approaching zero. (p.(xi))

§3.3. <u>Effect of Solar Wind</u>.

According to the accepted theory, as already referred to in the introduction, para (ii), the magnetopause, hereon referred to as MP, is of roughly paraboloidal shape with its axis on the line $\odot\oplus$ , i.e. from the Sun's to the Earth's centre, apex pointing towards $\odot$ , and with $\oplus$ approximately at the focus.

Outside MP, the $\underset{\sim}{B}$ lines near the Earth are $\underset{\sim}{B}_{\odot}$ . Inside it, at least near the Earth, they are $\underset{\sim}{B}_{\oplus}$ . As the $\underset{\sim}{B}_{\odot}$ lines are carried away from the Sun, MP forms a nearly but not quite rigid barrier: clearly at a distance very far from $\oplus$, in the direction opposite to that of $\odot$, its effect wears off in the manner discussed in the introduction. The $\underset{\sim}{B}_{\odot}$ lines, on the $OX_1Y_1$ plane, might therefore appear as in Fig. 7.

Meanwhile the $\underset{\sim}{B}_{\oplus}$ lines of Fig. 6 will have become distorted to fit inside the paraboloid. Those near D, on the side of $\oplus$ away from $\odot$, are prevented from becoming parallel by the existence of $\underset{\sim}{B}_{\odot}$ lines that have filtered through (i.e. a proportion of lines pass through) a distant part of MP, these latter are marked I in Fig. 7. (p. (xiii))

In the absence of current density inside MP, $\underset{\sim}{B}$ is the negative gradient of a harmonic function, single valued since the region is simply connected. Therefore it would be possible, if difficult, to solve this potential problem, probably requiring numerical methods, if sufficient boundary conditions could be found. These latter, however, are not known for certain, particularly as affecting the extent to which MP ceases to be a rigid barrier at large distances. Also, the practical problem is complicated by the inclination and rotation of the Earth's and Sun's magnetic axes, to the normal to the ecliptic and about the rotation axis, itself inclined, respectively. However there is a generally accepted form of the resulting $\underset{\sim}{B}$ field, inside and outside MP, near $\oplus$, in the literature, see Fig. 8. (p. (xii))

It will be noted from the figure that the neutral point D is now rather further from $\oplus$ than it was in fig. 6, also that the point A has become two points, marked A & C in the figure.

If one may imagine this distortion taking place in steps, an intermediate stage would appear as in Fig. 9, in which it will be noticed that at D the neutral ring is almost unaffected, but that a less pronounced separation of A and C has taken place. The neutral ring would now appear as in the three-dimensional sketch of Fig. 10. In the final configuration, that is, the configuration of Fig. 8, the neutral ring will appear as in Fig. 11. Thus, it is not split near D. In fact, looking down on the ecliptic plane DXZ, the part of the neutral ring near D will approximate in shape to the arc of a circle, with a centre of curvature K somewhere between D and ⊕. (p.(xiii)).

Before the distortion, the $\underset{\sim}{B}$ lines due to ⊙ and ⊕ lay on $\phi$ = constant planes, see §3.1. After distortion, one may expect that they will be close to planes through a line perpendicular to the ecliptic $OX_1Z_1$ plane, through K. Therefore, to a close approximation, near D, since KD ≫ length of small departure from D, it is correct to state that the $\underset{\sim}{B}$ lines lie on planes parallel to DXY. This has the important consequence that $\underset{\sim}{B}\cdot\hat{\underset{\sim}{z}} = 0$ at D and at all points close to D, and we need only consider the DXY plane itself.

## §3.4. The Magnetic Induction Field near the Neutral Line.

However it is no longer certain that the D-lines, as defined in §3.1. are of the form analysed in that paragraph. Inspection of Fig. 8 show that they are more likely to be approximately symmetrical about both DX and DY, have a point of inflexion at D, and no longer necessarily meet at an angle of $\pi/2$: in fact it is possible that they meet at an angle of $< \pi/2$ - it will be shown later that this requires the existence of a current density parallel to OZ. The inflexion is such that the D-lines in $X > 0$ are concave in the $O\hat{\underset{\sim}{X}}$ direction, and those in $X < 0$ are concave in the $-O\hat{\underset{\sim}{X}}$ direction. In the next paragraph a model will be constructed that gives a field of this type.

## §3.5. A Model for the above Field

The first order field of §3.1. can be put into dimensionless form, in the manner of Chapter II, using $L_0 x = X$, etc.:-

$$\underset{\sim}{b}_1 = -g(y\hat{\underset{\sim}{x}} + x\hat{\underset{\sim}{y}}),$$

where g is a dimensionless constant > 0. If we now add a field:

$$\underset{\sim}{b}_2 = -h(y\hat{\underset{\sim}{x}} - x\hat{\underset{\sim}{y}}),$$

where h is a dimensionless constant, > 0, then $\underset{\sim}{b}_2$ is the dimensionless magnetic induction field due to a constant current density co-parallel $\hat{\underset{\sim}{z}}$. The combined field is :

$$\underset{\sim}{b}_3 = -\{(g+h)y\hat{\underset{\sim}{x}} + (g-h)x\hat{\underset{\sim}{y}}\},$$

And if $\underset{\sim}{s} \equiv x\hat{\underset{\sim}{x}} + y\hat{\underset{\sim}{y}}$ and $\underset{\sim}{s} \times \underset{\sim}{b}_3 = \underset{\sim}{0}$, then $(g+h)y^2 - (g-h)x^2 = 0$: this is a pair of straight lines, and are therefore D-lines for the $\underset{\sim}{b}_3$ field, that intersect at an angle 2 arc tan$\{(g-h)/(g+h)\}^{\frac{1}{2}}$, i.e. $< \pi/2$.

Now we add a field:-

$$\underset{\sim}{b}_4 = -k(1 - e^{-\ell s^2})(y\hat{\underset{\sim}{x}} - x\hat{\underset{\sim}{y}}),$$

where $k, \ell$ are dimensionless constants, $> 0$,

then $\underset{\sim}{b}_4$ is the dimensionless magnetic induction field due to a current density which is co-parallel $\hat{\underset{\sim}{z}}$, zero at D, but increases with s up to a distance where it reaches a maximum, after which, with further increase of s, it tends to a constant value. This field, when added to $\underset{\sim}{b}_3$, will therefore further deflect the D-lines towards the Ox axis for higher values of s, thus it will provide the type of field considered at the end of §3.4.

The combined field is then:

$$\underset{\sim}{b} = -\{(g + h + k - u)y\underset{\sim}{x} + (g - h - k + u)x\hat{\underset{\sim}{y}}\},$$

where $u = ke^{-\ell s^2}$.

As s becomes fairly large, $u \to 0$, and the slope of the $\underset{\sim}{b}$ lines will remain positive in the first and third quadrants, and negative in the second and fourth quadrants, as long as $g - h - k$ remains positive, i.e. as long as g,h,k are chosen such that $g > h+k$, in which case $(g + h + k - u) > (g - h - k + u) > 0$, all $s > 0$.

Now if we put $h = 0$, this means that we assume that there is no current density at D; also near D, where terms in x and y higher than first order may be ignored, $\underset{\sim}{b}_4$ is zero as well as $\underset{\sim}{b}_2$ and the above $\underset{\sim}{b}$ field coincides with the $\underset{\sim}{b}_1$ field. However, current density away from D, increasing with distance from D, will give rise to afield whose D-lines intersect at right angles at D, but bend towards each other as in §3.4., on each side of DY.

A field of this type, that may serve as a model, is that due to two co-parallel line currents, each carrying current which has an electron-drift velocity parallel to $\hat{z}$, through the points $(\pm a,0,0)$.

This field will be analysed in the next chapter. In the subsequent chapters, investigations will be made as to the stability of the field to perturbations. In order to do this, it is necessary to consider starting conditions and also likely types of perturbation, as follows:-

(i) Starting conditions: The effect of a perturbation will be most marked if just before the perturbation occurs, the medium is at rest. Calling this instant of time "time 0", we may therefore say that, at time 0, $v = 0$, and all the elements of $m^{-1}\nabla v$ are zero. Also, at and near the neutral point, $b$ is either $0$ or very small, since its components each have a factor varying with a position component. Therefore we may put $b = 0$ generally, the effect of doing so being to ignore terms which contain, as a factor, a component of $b$ at time 0. Then, with both $b$ and $v$ zero, $\partial v/\partial t = 0$ (§2.5), and so $v$ remains zero.

Also, since $b$ has no component parallel to Dz, then this component, $b_3$, $= 0$ and $\nabla b_3 = 0$, at time 0.

From the remarks at the end of §3.3 we may also regard the system, at time 0, as being independent of z. Therefore, at time 0, $\partial b/\partial z = 0$.

(ii) It is unlikely that a perturbation taking the form of a disturbance velocity field will occur with the $b$ field remaining locally undisturbed. This would mean, if it happened, that the perturbation in $v$ would be due, at the neutral point, to a perturbation in $v$ at some comparatively distant point. This effect can only be propagated from the distant point through the medium at the speed of sound in the medium, which is $(P/D)^{\frac{1}{2}} \sim 10^4 m^1 s^{-1}$, a speed which is of the same order as the particle drift speed V at points in extra-terrestrial space where it is finite. This is a slow speed by extra-terrestrial standards. This leads us to conclude that perturbations will not, at and near the neutral point, take the form of sudden non-zero values in any of the components of $v$ or of any of the elements of $\nabla v$. However, the effect of an impulsive small $w$ ( $= \nabla \times v$)

near D, || Oz, i.e. of $\partial v_2/\partial x$, $\partial v_1/\partial y$ will be tried.

Again, it can be argued that a sudden non-zero value of a component of $\underset{\sim}{b}$, at D could be caused by a change in current density at a distant point, the effect reaching D rapidly at the speed of "light", $\sim 10^8 m^1 s^{-1}$. However the result of this would be that D would cease to be a neutral point, in fact the neutral point itself would shift position. In this paper, attention is directed particularly to instability at a neutral point; therefore perturbations in components of $\underset{\sim}{b}$ will not be considered.

But a perturbation can be accepted as possible in any of the elements of $\underset{\sim}{\nabla}\underset{\sim}{b}$ as long as we exclude $\partial b_3/\partial x$ and $\partial b_3/\partial y$ (since $b_3 = 0$ in Oxy) and also exclude $\partial b_1/\partial z$ and $\partial b_2/\partial z$ (system should remain independent of z.) It is felt that perturbations in $\underset{\sim}{J} \propto \underset{\sim}{\nabla} \times \underset{\sim}{B}$ are more likely. Perturbations will therefore only be considered in $\partial b_1/\partial y$ and $\partial b_2/\partial x$, and not in the three terms of $\underset{\sim}{\nabla}\cdot\underset{\sim}{b}$.

# CHAPTER IV.

## THE CO-PARALLEL LINE CURRENT MODEL.

### §4.1 The Magnetic Induction Field of Co-parallel Line Currents.

If line currents, each of current strength I, and each with the electron-drift, constituting the current, flowing in the direction $\hat{\mathbf{z}}$, are on the lines $X = \pm A, Z = 0$, in a frame DXYZ as in §3.1., then the resulting magnetic induction field $\mathbf{B}$ at a general point P is given by:-

$$\mathbf{B} = \frac{\mu I}{2\pi}\left(\frac{\hat{\mathbf{t}}_1}{R_1} + \frac{\hat{\mathbf{t}}_2}{R_2}\right)$$

where $\mu$ is the permeability of the medium, $R_1$ is the length of the perpendicular $PA_1$ to the line on $X = +A$ from $P(X,Y)$ and $\hat{\mathbf{t}}_1$ is a unit vector defined by:-

$$\hat{\mathbf{t}}_1 = \hat{\mathbf{z}} \times \widehat{\mathbf{A_1P}} \;;$$

$R_2, \hat{\mathbf{t}}_2$ are similarly defined for the line on $X = -A$.

Then, since $R_1^2 = (X - A)^2 + Y^2$,

and $\widehat{\mathbf{A_1P}} = \{(X - A)\hat{\mathbf{x}} + Y\hat{\mathbf{y}}\}/R_1.$

then $\hat{\mathbf{t}}_1 = \{-Y\hat{\mathbf{x}} + (X - A)\hat{\mathbf{y}}\}/R_1,$

and, similarly, $\hat{\mathbf{t}}_2 = \{-Y\hat{\mathbf{x}} + (X + A)\hat{\mathbf{y}}\}/R_2,$

and we may write the above expression for $\mathbf{B}$ in dimensionless form:-

$$\mathbf{b} = -\frac{1}{2}g\left\{\frac{y\hat{\mathbf{x}} + (a - x)\hat{\mathbf{y}}}{(1 - \frac{x}{a})^2 + (\frac{y}{a})^2} + \frac{y\hat{\mathbf{x}} - (a + x)\hat{\mathbf{y}}}{(1 + \frac{x}{a})^2 + (\frac{y}{a})^2}\right\},$$

where $aL_o = A$ and $\frac{1}{2}g$ is a dimensionless constant replacing $\mu I/(2\pi AB_o a)$, $L_o$ and $B_o$ were given in §2.1., also $g$ can be the same number as in §3.5. by choice of the current I. Then

$$\mathbf{b} = -g\left\{\frac{(1 + \frac{x^2}{a^2} + \frac{y^2}{a^2})y\hat{\mathbf{x}} + (1 - \frac{x^2}{a^2} - \frac{y^2}{a^2})x\hat{\mathbf{y}}}{(1 + \frac{x^2}{a^2} + \frac{y^2}{a^2})^2 - \frac{4x^2}{a^2}}\right\}.$$

It is convenient to write the denominator, to facilitate expansion later, as

$$1 - \frac{2(x^2 - y^2)}{a^2} + \left\{\frac{x^2 + y^2}{a^2}\right\}^2$$

## §4.2. The Field near the Neutral Line

When x,y are small, that is, for the region near D, it is permissible to use the binomial theorem. Expanding to the fifth order in $\frac{x}{a},\frac{y}{a}$, we obtain:-

$$b = -g\left\{(1+\frac{x^2}{a^2}+\frac{y^2}{a^2})\,y\hat{\underset{\sim}{x}} + (1-\frac{x^2}{a^2}-\frac{y^2}{a^2})x\hat{\underset{\sim}{y}}\right\}\left\{1+\frac{2(x^2-y^2)}{a^2} + \frac{(x^2-3y^2)(3x^2-y^2)}{a^4}\right\}$$

$$\text{i.e. } b = -g\left\{(1+\frac{3x^2-y^2}{a^2} + \frac{5x^4-10x^2y^2+y^4}{a^4})\,y\hat{\underset{\sim}{x}} + (1+\frac{x^2-3y^2}{a^2}+\frac{x^4-10x^2y^2+5y^4}{a^4})x\hat{\underset{\sim}{y}}\right\}.$$

We may compare this field with that of §3.5 , with h = 0 and with $e^{-\ell s^2}$ expanded for the first three terms only:-

$$b = -g\left[\left\{1+\frac{k\ell}{g}(x^2+y^2) - \frac{k\ell^2}{2g}(x^2+y^2)^2\right\}y\hat{\underset{\sim}{x}} + \left\{1-\frac{k\ell}{g}(x^2+y^2)+\frac{k\ell^2}{2g}(x^2+y^2)^2\right\}x\hat{\underset{\sim}{y}}\right].$$

There is a dissimilarity, which is to be expected, since the fields are due to different current systems. Meanwhile, so that $|\underset{\sim}{\nabla}\,\underset{\sim}{b}| \sim 10^{-3}$ (§1.10) at D, we require $|g| \sim 10^{-3}$. But as will be seen in Chapter V, we shall be investigating fieldsthat start from impulsive values. Therefore $|g|$ will be given the value $10^{0}$, i.e. $10^{3}$ times its value to match that due to external influences.

## §4.3. Stability of Fields near a Neutral line.

If a dimensionless magnetic induction field $\underset{\sim}{b}$ has a neutral point at the origin D, then this means simply that $\underset{\sim}{b}(\underset{\sim}{r},t)\to\underset{\sim}{0}$ as $\underset{\sim}{r}\to\underset{\sim}{0}$, where $\underset{\sim}{r}$ is the dimensionless position vector of the point of measurement of $\underset{\sim}{b}$.

Referring to the Cartesian frame Dxyz, and assuming that the components of $\underset{\sim}{b}$ orthogonal to $\hat{\underset{\sim}{z}}$ are independent of z, we may write:-

$$\underset{\sim}{b} = F_1(x,y,t)\hat{\underset{\sim}{x}} + F_2(x,y,t)\hat{\underset{\sim}{y}} + F_3(x,y,z,t)\hat{\underset{\sim}{z}}.$$

Then the requirement that $\underset{\sim}{\nabla}\cdot\underset{\sim}{b} = 0$ gives a differential equation for $F_3$, as a function of z. In fact, for given constant values of x,y: $dF_3/dz$ = known function of x,y = constant. Integrating, and allowing x,y to vary, we have :-

$$F_3 = zF_4(x,y,t) + F_5(x,y,t).$$

As we have seen, $F_4$ depends on $F_1$ and $F_2$, and we may assume that $F_5$ is also determinable, possibly from conditions imposed on $\nabla\times \underset{\sim}{b}$ away from D, also $F_5$ must tend to zero as $\underset{\sim}{r} \to \underset{\sim}{0}$.

So, once the behaviour of $F_1, F_2$ near D and on the plane $z = 0$ has been analysed, it is a simple matter to investigate the field near the origin, with z finite but small.

Therefore for the rest of this paragraph we will consider the field on the $z = 0$ plane:

$$\underset{\sim}{b} = F_1(x,y,t)\hat{\underset{\sim}{x}} + F_2(x,y,t)\hat{\underset{\sim}{y}}.$$

The function $F_1$ is assumed to be such that it can be expanded as a power series in x,y,t :-

$$F_1 = \sum_{\alpha=-\infty}^{\infty} \sum_{\beta=-\infty}^{\infty} \sum_{\gamma=-\infty}^{\infty} Q_{\alpha\beta\gamma}\, x^{\alpha} y^{\beta} t^{\gamma}.$$

We require that $F_1 = 0$ when $x = 0$ and $y = 0$, all t, and that $F_1$ be finite when $t = 0$. This means altering the range of summation :-

$$F_1 = \sum_{\alpha=0}^{\infty} \sum_{\beta=0}^{\infty} \sum_{\gamma=0}^{\infty} Q_{\alpha\beta\gamma}\, x^{\alpha} y^{\beta} t^{\gamma}, (Q_{00\gamma} = 0).$$

The question whether $F_1\hat{\underset{\sim}{x}}$, the x - component of $\underset{\sim}{b}$, is stable or unstable is then decided by the behaviour of the above expansion for $F_1$ as t increases from 0, having determined the $Q_{\alpha\beta\gamma}$ from boundary conditions on $F_1$ for given values of x,y,t. The result may be stable (that is, steady or decreasing oscillations, or exponential decrease), or unstable (that is, increasing oscillations, i.e. overstability, or exponential increase), etc.

When x,y are small such that only first order terms in x,y need be included - this assumes that $\sum_{\gamma} Q_{\alpha\beta\gamma} t^{\gamma}$ is finite for all $\alpha, \beta$ - then we have the situation near the neutral line, where the $\underset{\sim}{b}$ field has a x-component $F_1$ which may be written, with obvious change of coefficients:-

$$F_1 = \Big(\sum_{\gamma=0}^{\infty} P_{\gamma} t^{\gamma}\Big)x + \Big(\sum_{\gamma=0}^{\infty} Q_{\gamma} t^{\gamma}\Big)y,$$

or, more generally:-

$$F_1 = p(t)x + g(t)y.$$

The above analysis applies also to $F_2$, so that, near the neutral line on $z = 0$, we may write :-

$$\underset{\sim}{b} = (px + qy)\hat{\underset{\sim}{x}} + (rx + sy)\hat{\underset{\sim}{y}},$$

where p,q,r and s are functions of t. As they vary with t, each one may pass from a positive to a negative value or vice-versa, and it is of interest to investigate the form taken up by the $\underset{\sim}{b}$ field on $z = 0$ as p,q,r, and s vary with time. If the field is unstable, the following analysis will give an indication of the way in which the instability grows.

If the field is to have real D-lines, then writing as before:

$$\underset{\sim}{s} = x\hat{\underset{\sim}{x}} + y\hat{\underset{\sim}{y}},$$

and requiring that : $\underset{\sim}{s} \times \underset{\sim}{b} = \underset{\sim}{0}$,

we find that a real value for $(y/x)$, on a $\underset{\sim}{b}$ line that is tangential to the position vector at the point $(x,y)$, and so points to or away from D, can only be obtained if :

$$(p - s)^2 \geqslant -4qr.$$

If this condition is satisfied, two real values for $(y/x)$ can be found: these values are dependant only on p,q,r and s, and so the lines through D at these slopes coincide with $\underset{\sim}{b}$ all the way along the lines; thus in this case the D lines are a straight pair.

If the condition is not satisfied, then $(y/x)$ is imaginary even as x and $y \to 0$ together, and so there are no real D-lines.

In the case where D-lines exist, it is of interest to investigate whether $\underset{\sim}{b}$ points towards or away from D on both **D**-lines, or towards **D** on one line of the pair and away from D on the other. Let $\underset{\sim}{s}_1$ , $\underset{\sim}{s}_2$ be the position vectors of any two points, one on each of the pair of D-lines, and let $\underset{\sim}{b}_1$ , $\underset{\sim}{b}_2$ be the corresponding values of the field at these points. Then the first of the above conditions will apply if :

$$(\underset{\sim}{b}_1 \cdot \underset{\sim}{s}_1)(\underset{\sim}{b}_2 \cdot \underset{\sim}{s}_2) > 0.$$

But $x_1$, $y_1$, the components of $\underset{\sim}{s}_1$, satisfy the connection:

$$y_1/x_1 = \left[-(p - s) + \{(p - s)^2 + 4qr\}^{\frac{1}{2}}\right]/2q,$$

and similarly $x_2, y_2$ satisfy a corresponding connection with a negative radical. Substituting for $\underset{\sim}{b}_1$, $\underset{\sim}{b}_2$, $y_1$, $y_2$ and dividing the result by $(x_1x_2/q)^2$, we obtain, after some cumbersome algebra, the condition:

$$(ps - qr)\left\{(p - s)^2 + (q + r)^2\right\} > 0;$$

That is, the condition is : $ps > qr$.

To investigate the nature of the $\underset{\sim}{b}$ field elsewhere than on the D-lines, if they exist, it is necessary to obtain the equation of the $\underset{\sim}{b}$ lines. This may be resolved by the following device: we require that, for an element $\delta\underset{\sim}{s}$ of a $\underset{\sim}{b}$ line:

$$\delta\underset{\sim}{s} \times \underset{\sim}{b} = \underset{\sim}{0},$$

which leads to : $$\frac{dy}{dx} = \frac{rx + sy}{px + qy}.$$

The solution to this differential equation is obtained, in the usual way, i.e. by the substitution $y = xu(x)$. The result is the unexpectedly complicated equation:

$$\left\{y + \left(\frac{p-s}{2q} + w\right)x\right\}^{\frac{p+s}{2q} - w} = k^2\left\{y + \left(\frac{p-s}{2q} - w\right)x\right\}^{\frac{p+s}{2q} + w}$$

where $w = +\left\{\left(\frac{p+s}{2q}\right)^2 + \frac{r}{q}\right\}^{\frac{1}{2}}$

Here, $k^2$ is a parameter: the D-lines are given by the limiting values $0, \infty$ of $k^2$. It will be noted that the $\underset{\sim}{b}$ lines will all pass through D, if, for all k, it is possible for both x and y to approach zero. Clearly this can only happen if the two exponents are of the same sign, then if they are both positive, and w is positive by definition, we require $\frac{p+s}{2q} > w$.

If they are both negative we still require $\left|\frac{p+s}{2q}\right| > w$.

In either case the inequality leads to the condition:-

$$ps > qr.$$

Thus, there are three types of field, as follows:-

(i) $(p-s)^2 < -4qr$ : No D-lines, irrespective of whether $ps \gtrless qr$. In fact by substracting $(p+s)^2$ from each side of the inequality, one can see that $ps > qr$. Since $\underset{\sim}{b}$ lines can never point to or away from D, they follow paths that circumnavigate D in the form of an oblique spiral. Therefore this type of field will be referred to as a "spiral-type". Fig. 12 shows the appearance of the field for the case : (p. (xiv))

$$p = 3, q = 4, r = -2, s = -1.$$

The resemblance to an A-type spiral nebula is remarkable, and has led to attempts to correlate the existence of spiral arms to the presence of a nebular magnetic field of this type.

(ii) $(p-s)^2 > -4qr$ ; $ps > qr$ : All $\underset{\sim}{b}$ lines are D-lines and one pair is straight ; $\underset{\sim}{b}$ lines only point to or from D if the point of measurement is on one of this pair of straight lines, otherwise not. Fig. 13 shows the appearance ofthis type of field, which will be referred to as a "bar-type", for the case : (p.(xiv))

$$p = 4, q = 3, r = -2, s = -1.$$

There is again a remarkable resemblance to a nebula, in this case to a bar-nebula.

(iii) $(p-s)^2 > -4qr$ ; $ps < qr$. Only one pair of D-lines exist, which are straight. The $\underset{\sim}{b}$ lines point towards D on one line of the pair, and away from D on the other line. Elsewhere the $\underset{\sim}{b}$ lines resemble hyperbolae, with the D-lines as asymptotes. This type of field will be referred to as an "X-Type". Fig. 14 gives an example for the case: (p.(xv))

$$p = 4, q = 3, r = -2, s = -2.$$

There do not seem to be any nebulae which have this appearance. This is the type of field analysed by Dungey (6), as mentioned in the introduction, also by Yeh and Axford (14) and Schindler (15).

There are also three limiting types, or "boundary cases". It may be noted that there is no boundary case between a "spiral-type" and an "X-type" field, since this would require:

$$(p - s)^2 = -4qr, \; ps < qr, \text{ that is } (p - s)^2 < - 4ps, \text{ or equivalently:}$$

$$(p+s)^2 < 0,$$

which is impossible for real p,s. Therefore the three boundary cases are:

(iv) $(p-s)^2 = -4qr$, $ps > qr$. This is the boundary case between a "spiral-type" and a "bar-type" field. The two D-lines of the "bar-type" coincide. Fig. 15 gives an example for the case: (p. (xv))

$$p = 4,\ q = 3,\ r = -3,\ s = -2$$

(v) $(p-s)^2 > -4qr$, $ps = qr$. This is the boundary case between a "bar-type" and an "X-type". It will be noted that the condition:

$$ps = qr,$$

means that: $\mathbf{b} = \{(px + py)(p\hat{\mathbf{x}} + r\hat{\mathbf{y}})\}/p$ ,

that is; the $\mathbf{b}$ lines are constant in direction; the particular set of $\mathbf{b}$ lines that pass through D form one of the D-lines $(rx - py = 0)$, and $\mathbf{b}$ changes sense if the point at which it is measured follows a $\mathbf{b}$ line and crosses the other D line $(px + qy = 0)$. This field has an oblique resemblance to a line source or a line-sink field, so that no diagram is needed.

(vi) $(p-s)^2 = -4qr$, $ps = qr$. This is the common boundary case where the conditions for all three type of field meet. The two equations connecting p,q,r and s show that $s = -p$, and that $r = -p^2/q$, so that :

$$b = (px + qy)(q\hat{\mathbf{x}} - p\hat{\mathbf{y}})/q.$$

The two D-lines of Type (v) have coalesced; $\mathbf{b}$ is parallel to the combined D line $(px + qy = 0)$ everywhere, points in different directions on each side of the D line and is zero on it. The field resembles that due to a sheet-vortex, apart from the fact that b increases with distance from the D-line. Fig. 16 gives an example of this type of field for the case:- (p.(xvi))

$$p = 2,\ q = 1,\ r = -4,\ s = -2.$$

This is the type of field analysed by Piddington (7), Jukes (9), Stix (8), Speiser (11), (12) and by Tendys (13).

If we divide each term of the expression for $\mathbf{b}$ by q, and write $\xi = p/q$, $\eta = r/q$, and $\zeta = s/q$, the expression becomes :-

$$\mathbf{b}/q = (\xi x + y)\hat{\mathbf{x}} + (\eta x + \zeta y)\hat{\mathbf{y}},$$

giving a field which is similar to any $\mathbf{b}(p,q,r,s)$ field other than in sense and

and scale. The possible types of field may then be defined in a frame $\Omega\xi\eta\zeta$ (Cartesian), which contains the surfaces:-

$$\Sigma_1 \equiv (\xi - \zeta)^2 + 4\eta = 0$$

and $\Sigma_2 \equiv \xi\zeta - \eta = 0$

Rotating axes, about $\Omega\eta$, through an angle $\pi/4$, and reflecting in $\Omega\xi\zeta$, by the transforms:-

$$\xi = 2^{-\frac{1}{2}}(\xi' - \zeta')$$

$$\eta = -\eta'$$

$$\zeta = 2^{-\frac{1}{2}}(\xi' + \zeta')$$

the surfaces become, in the frame $\Omega\xi'\eta'\zeta'$:

$$\Sigma_1 \equiv \zeta'^2 - 2\eta' = 0$$

$$\Sigma_2 \equiv -\xi'^2 + \zeta'^2 - 2\eta' = 0$$

Figs. 17, 18 give an impression of these surfaces. (p.(xvii)) They clearly meet where $\xi' = 0$. The tangent plane of $\Sigma_1$ at a point $\xi_1', \eta_1', \zeta_1'$ has the equation:

$$\zeta'\zeta_1' - (\eta' + \eta_1') = 0,$$

that is, since $\eta_1' = \frac{1}{2}\zeta_1'^2$ :-

$$\zeta'\zeta_1' - \eta' - \tfrac{1}{2}\zeta_1'^2 = 0$$

The tangent plane to $\Sigma_2$ at a point $\xi_2', \eta_2', \zeta_2'$ has the equation

$$-\xi'\xi_2' + \zeta'\zeta_2' - (\eta' + \eta_2') = 0,$$

that is, since $\eta_2' = \frac{1}{2}(\zeta_2'^2 - \xi_2'^2)$ :-

$$-\xi'\xi_2' + \zeta'\zeta_2' - \eta' - \tfrac{1}{2}(\zeta_2'^2 - \xi_2'^2) = 0.$$

The points where, in each case, the tangent plane touches the relevant surface, are identical on the line where the planes meet. This, as we have seen, lies on $\xi' = 0$.

Thus the tangent planes to $\Sigma_1, \Sigma_2$, where they meet are obtained from the above expressions by writing:-

$$\xi_1' = \xi_2' = 0,\ \eta_1' = \eta_2' = \tfrac{1}{2}\zeta_1'^2,\ \zeta_1' = \zeta_2',$$

providing two identical equations, both of which are :-

$$\zeta'\zeta_1' - \eta' - \tfrac{1}{2}\zeta_1'^2 = 0$$

Therefore the tangent planes to the two surfaces, where they meet, coincide: this means that the two surfaces touch each other. This happens on the line where either surface meets the $\xi' = 0$ plane, i.e. on the line given by the equations:-

$$\xi' = 0$$

$$\zeta'^2 = 2\eta'$$

This line is a parabola on the $\xi' = 0$ plane, through the origin $\Omega$ and with the $\Omega\eta'$ axis as axis of symmetry. The important point that is demonstrated by this analysis is that the whole infinite region, spanned by the frames $\Omega\xi'\eta'\zeta'$ and, equivalently, $\Omega\xi\eta\zeta$, is divided into three sub-regions only by the surfaces. This explains why there are only three types of $\underset{\sim}{b}$ field, and only two boundary cases, in addition to the expected one common-boundary case.

Finally it is evident that an "X-type" field, referred to axes Oxy which bisect the angles between the D-lines will by symmetry be such that when $x = 0$, $\underset{\sim}{b} \,||\, \underset{\sim}{\hat{x}}$, so $s = 0$ and when $y = 0$, $b \,||\, \underset{\sim}{\hat{y}}$, so $p = 0$. Then $qr > 0$. The field $\underset{\sim}{b}_1$, where $q = r = -g$, is of this type.

# CHAPTER V

## FIRST DIFFERENTIAL EQUATIONS

### §5.1 Infinite conductivity and constant pressure

The assumption of infinite conductivity implies that $\gamma = 0$ (§§ 1.8, 2.1), so that the equation in §2.1 reduces to:

$$\partial b_i/\partial t = -v_i b_{ik} + v_{ik} b_k - v_{kk} b_i , \quad i,k = 1(1)3.$$

This is the same equation as that in §2.5, in suffix form, making use of Maxwell's equation $b_{kk} = 0$, summation convention, and with the convention that the second suffix refers to the gradient component, i.e.:

$$v_{ik} \equiv \text{ ik element of } \nabla v.$$

This equation holds even for cases where $b_{ikk}$ may have large local values. The assumption of constant pressure means that a term containing $p_i$, the $i^{th}$ component of $\nabla p$, may be ignored: this was done in §2.5 in which $\nabla p$ was taken to be negligible. The equation in §2.2, using suffixes again and the commutator operator [ ], reduces to:

$$\partial v_i/\partial t = -v_k v_{ik} + b_k b_{[ik]} .$$

In this equation $\lambda$ has been replaced by unity (§§1.10, 2.3). The affect of varying density, due to the equation of continuity, will be investigated in §5.2.

As explained in §2.5, no further equations are needed for stability investigation. Following Dungey (6), and so that permutations in elements of $\nabla b$ can be used ( §3.5, end), we take the gradient of the above equations and, in the result, set $b = 0$ and $v = 0$ (§3.5), obtaining, with $j = 1(1)3$:

$$\partial b_{ij}/\partial t = -v_{kj} b_{ik} + v_{ik} b_{kj} - v_{kk} b_{ij},$$

$$\partial v_{ij}/\partial t = -v_{kj} v_{ik} + b_{kj} b_{[ik]} .$$

These time-dependent differential equations may be transformed into recurrence equations by the following device: for any time-variable A, $\partial A/\partial t$ is closely represented by $(A^{n+1} - A^n)/\Delta t$, where $\Delta t$ is a constant small time period and $A^n$ is the value of A at time $t = n\Delta t$.

We set $\Delta t = 1\sec \div T_0 = 1$, which represents what seems intuitively to be a sufficiently short period of time to give an accurate investigation of extra-terrestrial phenomena. The equations become:

$$b_{ij}^{n+1} = b_{ij}^{n} - v_{kj}^{n} b_{ik}^{n} + v_{ik}^{n} b_{kj}^{n} - v_{kk}^{n} b_{ij}^{n} ,$$
$$v_{ij}^{n+1} = v_{ij}^{n} - v_{kj}^{n} v_{ik}^{n} + b_{kj}^{n} b_{[ik]}^{n} .$$

In these equations, when $n = 0$, we consider the values of the variables at an instant when conditions are due to external influences only, but certain of them are given a "perturbation value", that is the original value plus a small disturbance value. The perturbation values, as explained at the end of §3.5., are limited to $b_{\iota\kappa}^{o}$, where $\iota,\kappa = 1(1)2$. The perturbance may only take place, impulsively, at time 0 ($n = 0$), since the recurrence equations then determine the values of all subsequent $b_{ij}^{n}$ ($n \neq 0$). Remembering, from the argument in §3.5, that $b_{ij}^{o}$, $v_{ij}^{o}$ are zero if either i or j, but not both, are equal to 3, the equations for the case $n = 0$ appear when expanded as follows:-

$$b_{11}^{1} = b_{11}^{o} (1 - v_{kk}^{o}) - v_{21}^{o} b_{12}^{o} + v_{12}^{o} b_{21}^{o} ,$$
$$b_{12}^{1} = b_{12}^{o} (1 - v_{kk}^{o}) - v_{12}^{o} b_{11}^{o} - v_{21}^{o} b_{22}^{o} + v_{11}^{o} b_{12}^{o} + v_{12}^{o} b_{22}^{o} ,$$
$$b_{13}^{1} = 0,$$
$$b_{21}^{1} = b_{21}^{o} (1 - v_{kk}^{o}) - v_{11}^{o} b_{21}^{o} - v_{21}^{o} b_{22}^{o} + v_{21}^{o} b_{11}^{o} + v_{22}^{o} b_{21}^{o} ,$$
$$b_{22}^{1} = b_{22}^{o} (1 - v_{kk}^{o}) - v_{12}^{o} b_{21}^{o} + v_{21}^{o} b_{12}^{o} ,$$
$$b_{23}^{1} , b_{31}^{1} , b_{32}^{1} \text{ all } = 0,$$
$$b_{33}^{1} = b_{33}^{o} (1 - v_{kk}^{o}),$$
$$v_{11}^{1} = v_{11}^{o} (1 - v_{11}^{o}) - v_{21}^{o} v_{12}^{o} + b_{21}^{o} b_{[12]}^{o} ,$$
$$v_{12}^{1} = v_{12}^{o} (1 - v_{\kappa\kappa}^{o}) + b_{22}^{o} b_{[12]}^{o} ,$$
$$v_{13}^{1} = 0,$$
$$v_{21}^{1} = v_{21}^{o} (1 - v_{\kappa\kappa}^{o}) + b_{11}^{o} b_{[21]}^{o} ,$$
$$v_{22}^{1} = v_{22}^{o} (1 - v_{22}^{o}) + b_{12}^{o} b_{[21]} ,$$
$$v_{23}^{1} , v_{31}^{1} , v_{32}^{1} \text{ all } = 0,$$
$$v_{33}^{1} = v_{33}^{o} (1 - v_{33}^{o}).$$

From these equations we elicit the information that, even if the other dyad elements are non-zero when $n = 0$, if $b^0_{13}$ , $b^0_{23}$ , $b^0_{31}$ , $b^0_{32}$ , $v^0_{13}$ , $v^0_{23}$ , $v^0_{31}$ , and $v^0_{32}$ are zero then so are $b^1_{13}$ , $b^1_{23}$ , $b^1_{31}$ , $b^1_{32}$ , $v^1_{13}$ , $v^1_{23}$ , $v^1_{31}$ , and $v^1_{32}$ . The same thing will happen over the next period, i.e. $b^2_{13}$ , etc., are zero. The inference is that, in this case $b^n_{13}$ , $b^n_{23}$ , $b^n_{31}$ , $b^n_{32}$ , $v^n_{13}$ , $v^n_{23}$ , $v^n_{31}$ and $v^n_{32}$ are zero for all n.

Further, if $v^0_{33}$ , an element of $\underset{\sim}{\nabla}\underset{\sim}{v}$ at time 0 is zero (§3.5), the last equation shows that $v^n_{33}$ is zero for all n. The equation in $b^n_{33}$ , using Maxwell, can then be written, for all n : $b^{n+1}_{33} = (b^n_{11} + b^n_{22})(v^n_{\kappa\kappa} - 1)$.

Hence $b^{n+1}_{33}$ depends on values of $b^n_{\iota\kappa}$, $v^n_{\iota\kappa}$, deducible from the remaining equations. If an instability is discovered in the latter, this will automatically affect $b^n_{33}$ also, and so it is not necessary to investigate its behaviour.

With these considerations in mind, omitting the vanishing equations in $b^n_{13}$ , etc. and that in $b^n_{33}$ , and also noting that $b^n_{[21]} = -b^n_{[12]}$ and making some adjustmenst for tidiness, the equations may now be expanded for a general value of n :-

$$
\begin{aligned}
b^{n+1}_{11} &= b^n_{11}(1 - v^n_{\kappa\kappa}) + v^n_{12} b^n_{21} - v^n_{21} b^n_{12} , \\
b^{n+1}_{12} &= b^n_{12}(1 - v^n_{\kappa\kappa}) + v^n_{11} b^n_{12} - v^n_{12} b^n_{11} + v^n_{12} b^n_{22} - v^n_{22} b^n_{12} , \\
b^{n+1}_{21} &= b^n_{21}(1 - v^n_{\kappa\kappa}) - v^n_{11} b^n_{21} + v^n_{21} b^n_{11} - v^n_{21} b^n_{22} + v^n_{22} b^n_{21} , \\
b^{n+1}_{22} &= b^n_{22}(1 - v^n_{\kappa\kappa}) - v^n_{12} b^n_{21} + v^n_{21} b^n_{12} , \\
v^{n+1}_{11} &= v^n_{11}(1 - v^n_{11}) - v^n_{12} v^n_{21} + b^n_{21} b^n_{[12]}, \\
v^{n+1}_{12} &= v^n_{12}(1 - v^n_{\kappa\kappa}) + b^n_{22} b^n_{[12]}, \\
v^{n+1}_{21} &= v^n_{21}(1 - v^n_{\kappa\kappa}) - b^n_{11} b^n_{[12]}, \\
v^{n+1}_{22} &= v^n_{22}(1 - v^n_{22}) - v^n_{12} v^n_{21} - b^n_{12} b^n_{[12]}.
\end{aligned}
$$

It will be noted that the last four equations all include the factor $b^n_{[12]}$. Therefore to get full value from the investigation it is advisable to give different perturbance values to $b^0_{12}$ , $b^0_{21}$ , using the inference that a current density parallel to Dz and so proportional to $b^{o}_{[12]}$, occurs impulsively, due to some cause or other at time $t = 0$.

It will also be noted that, if we rotate axes of reference through an angle $\pi$ about the Dy axis, and use the same symbols, the effect will be to change the sign of a component $C_1$ of a vector $C$, and of the operator $\partial/\partial x$; thus $C_{12}$, $C_{21}$, $C_{13}$, $C_{31}$ change sign, whereas $C_{11}$, $C_{22}$ maintain sign. Therefore the same type of instability will result from the perturbations $b^{o}_{12} = \ell$, $b^{o}_{21} = m$, say ($\ell$,m constants) as from $b^{o}_{12} = -\ell$, $b^{o}_{21} = -m$.

Further, the equations are symmetrical about the plane $x = y$. Therefore the same type of instability will result from the perturbations $b^{o}_{12} = \ell$, $b^{o}_{21} = m$, as from $b^{o}_{12} = m$, $b^{o}_{21} = \ell$. There three points will be borne in mind when applying perturbation values to $b^{o}_{ik}$.

The model to be chosen for investigation is the co-parallel line current model of Chapter IV, with $\underset{\sim}{b}^{o}$ near D given by the second equation in §4.2. From the argument in §4.2 (end) we may put $g = -1$ since the sign of g does not affect stability investigation. Then $b^{o}$ approaches a value, $\sim 10^{5}$, due to external influences (§1.10), at distances of value $\sim 10^{5}\,m^{1}$ from D, and this distance has a value $\sim 10^{-3}$ of the distance of D from the Earth (§1.3ii). This distance can be looked on as being "near D" as long as the imaginary line currents, causing the model $\underset{\sim}{b}$ field near D, are distant from D also at lengths of the order $10^{8}\,m^{1}$.

From the equation, we see that $b^{o}_{12} = b^{o}_{21} = 1$. Let the disturbance values be of the order $10^{-1}$, so that the perturbed value is $10^{o} \pm 10^{-1}$. Then a table for perturbation values of $\underset{\sim\sim}{\nabla b}$, covering all possibilities, is as follows:-

| | | | |
|---|---|---|---|
| $b^{o}_{12}$ | 1.1 | 1.2 | 0.9 |
| $b^{o}_{21}$ | 1 | 1.1 | 1.1 |

and, in each case: $b^{o}_{11} = 0$, $b^{o}_{22} = 0$ (§4.2), $v^{o}_{ik} = 0$ (§3.5)

The result will be given in §5.4.

Alternatively we may try disturbances in $v_{12}^{o}$ , $v_{21}^{o}$ (§3.5) with $b_{12}^{o}$ , $b_{21}^{o}$ both equal to 1 and all other $b_{ik}^{o}$ , $v_{ik}^{o}$ = 0. Being careful to avoid values such that $|v_{12}^{o}| = |v_{21}^{o}|$ , since both the sum and difference of these quantities appear as factors in the equations and an instability effect might be cloaked by the equality, and bearing in mind the three above points about possible values for $b_{ik}^{o}$,- applicable also to $v_{ik}^{o}$ ,-all accepted possibilities are included in the following table:

| | | | |
|---|---|---|---|
| $v_{12}^{o}$ | 0.1 | 0.2 | -0.2 |
| $v_{21}^{o}$ | 0 | 0.1 | 0.1 |

and, in each case: $b_{ik}^{o} = 1 - \delta_{ik}$ (§4.2), $v_{11}^{o} = 0$, $v_{22}^{o} = 0$(§3.5).

The result will be given in §5.4.

Dungey (6) investigated the cases $b_{12}^{o} \neq 0$, $b_{21}^{o} = 0$, and also $v_{12}^{o} \neq 0$, $v_{21}^{o} = 0$, without numerical details and also without noticing the effects, mentioned above, of notational ($\pi$) symmetry about Dy and symmetry about $x = y$.

## §5.2 Constant conductivity and variable pressure

If the conductivity is constant, and finite, this implies that $\gamma$ = constant and finite (§§1.8, 2.1) and so we may write $10^{-14}$ for $\gamma$ (§1.10).

The equation in §2.1., adjusted as in §5.1 above, leads to the set of recurrence equations for $b_{ik}^{n}$ :-

$$b_{11}^{n+1} = b_{11}^{n}\ (1 - v_{kk}^{n}) + v_{12}^{n}\, b_{21}^{n} - v_{21}^{n}\, b_{12}^{n} + 10^{-14}\, b_{11kk}^{n},$$

$$b_{12}^{n+1} = b_{12}^{n}\ (1 - v_{kk}^{n}) + v_{11}^{n}\, b_{12}^{n} - v_{12}^{n}\, b_{11}^{n} + v_{12}^{n}\, b_{22}^{n} - v_{22}^{n}\, b_{12}^{n} + 10^{-14}\, b_{12kk}^{n},$$

$$b_{21}^{n+1} = b_{21}^{n}\ (1 - v_{kk}^{n}) - v_{11}^{n}\, b_{21}^{n} + v_{21}^{n}\, b_{11}^{n} - v_{21}^{n}\, b_{22}^{n} + v_{22}^{n}\, b_{21}^{n} + 10^{-14}\, b_{21kk},$$

$$b_{22}^{n+1} = b_{22}^{n}\ (1 - v_{kk}^{n}) - v_{12}^{n}\, b_{21}^{n} + v_{21}^{n}\, b_{12}^{n} + 10^{-14}\, b_{22kk}^{n} ,$$

in which $b_{ijk\ell}^{n}$ , $\ell = 1(1)3$, have been set at zero, by virtue of arguments similar to those affecting $b_{ij}^{n}$ in §3.5, for k = 3, and/or $\ell$ = 3.

It may be noted that, from the equations in §5.1 for $b_{ij}^{n+1}$ we obtain, on adding a term $\gamma b_{ijkk}^{n}$ (§2.1) to the original equation and differentiating :-

$$b^{n+1}_{ij\ell\ell} = b^{n}_{ij\ell\ell}(1 - v^{n}_{kk}) - 2v^{n}_{kk\ell}b^{n}_{ij\ell} - v^{n}_{kk\ell\ell}b^{n}_{ij} - v^{n}_{kj\ell\ell}b^{n}_{ik} - 2v^{n}_{kj\ell}\, b^{n}_{ik\ell}$$
$$- v^{n}_{kj}\, b_{ik\ell\ell} + v^{n}_{ik\ell\ell}\, b^{n}_{kj} + 2v^{n}_{ik\ell}\, b^{n}_{kj\ell} + v^{n}_{ik}\, b^{n}_{kj\ell\ell} + \gamma b^{n}_{ijkk\ell\ell}\,.$$

This shows that, if the $b^{o}_{ijkk}$ be given, then in order to determine the $b^{1}_{ijkk}$, $b^{2}_{ijkk}$, ......, we must at each stage be given the values of a number of other quantities; e.g. for $b^{1}_{ijkk}$ we need to know the $v^{o}_{kk\ell}$, $b^{o}_{ij\ell}$, ....., $b^{o}_{ijkk\ell\ell}$ and to find all these out from recurrence equations, obtainable by differentiation of the original gradient equations, we must be given values of differentials, of higher and higher order as the process continues, of $b^{o}_{ijk}$ etc. leading eventually to an infinite number of starting conditions.

As this process appears to have no future, and in order to solve the recurrence equations for $b^{n}_{ik}$, we must clearly assume values for the $b^{n}_{ikkk}$. The problem now arises:- will a result of interest be found if we take the values to be of impulsive type, i.e. if we take the $b^{o}_{ikkk}$ to be finite and large and the $b^{n}_{ikkk}$ to be zero for $n \neq 0$, or, should we ascribe finite values to all $b^{n}_{ikkk}$? The answer to this question may be obtained from the following reasoning: if, by the first method, stability results, then a small oscillation will accrue from a given set of disturbance values for $b^{n}_{ikkk}$, not large - these are perturbation values since the unperturbed value of $b^{o}_{ikkk}$ is zero (§4.2). A further small oscillation will accrue from a set of disturbance values for $b^{1}_{ikkk}$, not large, combining with the first oscillation into a stable behaviour. Similarly for all $b^{n}_{ikkk}$. This reasoning is helped by the fact that the equations for $b^{o}_{ik}$ are linear in $b^{o}_{ik}$ and in $b^{o}_{ikkk}$. Unfortunately the corresponding set of equations, to be given later, for $v^{n}_{ij}$ are not linear in $b^{n}_{ij}$ or, for that matter, in $v^{n}_{ij}$ either, so that the reasoning breaks down at this point. However, if, as a result of non-linearity, a continuous applied disturbance in the $b^{n}_{ikkk}$, all n, were to change what was stable behaviour, due to impulsive $b^{o}_{ikkk}$, into unstable behaviour it would be necessary to apply an infinite number of starting conditions, i.e. ascribe sets of values to $b^n_{ik kk}$ , all n, $0 < n < \infty$. No attempt will be made to do this in this paper and it is proposed to investigate only the effect of an impulsive disturbance, as being more likely to give a stable result than a continuous one. Also, as has already been stated (§3.5), a disturbance is more likely to occur in the "shear" terms $b^o_{12kk}$ and $b^o_{21kk}$ than in the other two. Therefore arbitrary disturbance values will be given to $\gamma b^o_{12kk}$ and to $\gamma b^o_{21kk}$ and $b^o_{11kk}$ , $b^o_{22kk}$ will be set at zero. Since we have decided on an impulsive disturbance, $b^o_{12kk}$ and $b^o_{21kk}$ can be large, say $\sim 10^{13}$ and with $\gamma \sim 10^{-14}$ the terms will be $\sim 10^{-1}$ .

If the pressure varies, then $\lambda$ varies through the density (§§1.8, 2.2, 2.4).

The equation in §2.2, adjusted as in §5.1, leads to the set of recurrence equations for $v^n_{ij}$ :-

$$v^{n+1}_{11} = v^n_{11}\ (1 - v^n_{11}) - v^n_{12}\, v^n_{21} + \lambda^n b^n_{21}\, b^n_{[12]} - \tau^n_1\, p^n_1 - \tau^n p^n_{11}\ ,$$

$$v^{n+1}_{22} = v^n_{22}\ (1 - v^n_{kk}) + \lambda^n\ b^n_{22}\, b^n_{[12]} - \tau^n_2 p^n_1 - \tau^n p^n_{12}\ ,$$

$$v^{n+1}_{21} = v^n_{21}\ (1 - v^n_{kk}) - \lambda^n\ b^n_{11}\, b^n_{[12]} - \tau^n_1 p^n_2 - \tau^n p^n_{12}\ ,$$

$$v^{n+1}_{22} = v^n_{22}\ (1 - v^n_{22}) - v^n_{12}\, v^n_{21} - \lambda^n b^n_{12}\, b^n_{[12]} - \tau^n_2\, p^n_2 - \tau^n\ p^n_{22}\ ,$$

where $\tau = \rho^{-1}$, $\lambda$ has been retained and terms due to the term $\tau \underset{\sim}{\nabla} p$ are included, the term in $\lambda_j b_k b_{[ik]}$ , of course, falls away since $b_k = 0$,§ 5.1. Remembering, from §1.6, that $\tau^n \simeq 10^8 (p^n)^{-1}$ , so that $\tau^n_i \simeq -10^8 (p^n)^{-2}\, p^n_i$ , and also from §1.8. that $\lambda^n = \tau^n/\mu$ , then,with $\mu \sim 10^{20} f^{-2} k^1 m^1$, $\lambda^n \simeq 10^{-12} (p^n)^{-1}$ (see §1.2), the equations can be written

$$v^{n+1}_{11} = v^n_{11}\ (1 - v^n_{11}) - v^n_{12}\, v^n_{21} + 10^8 (p^n)^{-2} \{ 10^{-20}\, p^n b^n_{21}\, b^n_{[12]} + (p^n_1)^2 - p^n p^n_{11} \},$$

$$v^{n+1}_{12} = v^n_{12}\ (1 - v^n_{kk}) + 10^8 (p^n)^{-2} \{ 10^{-20}\, p^n b^n_{22}\, b^n_{[12]} + p^n_1\, p^n_2 - p^n\ p^n_{12} \},$$

$$v^{n+1}_{21} = v^n_{21}\ (1 - v^n_{kk}) + 10^8 (p^n)^{-2} \{ -10^{-20}\, p^n b^n_{11}\, b^n_{[12]} + p^n_1\, p^n_2 - p^n\ p^n_{12} \},$$

$$v^{n+1}_{22} = v^n_{22}\ (1 - v^n_{22}) - v^n_{12}\, v^n_{21} + 10^8 (p^n)^{-2} \{ 10^{-20}\, p^n b^n_{12}\, b_{[12]} + (p^n_2)^2 - p^n p^n_{22} \}.$$

Meanwhile, the equation in §2.3 can be written, using (§1.6), $\rho \sim p$:-

$$\frac{\partial p}{\partial t} = p_k v_k - p v_{kk},$$

which leads to the recurrence equation, as in §5.1:-

$$p^{n+1} = p^n - p_k^n v_k^n - p^n v_{kk}^n ,$$

with first differential:

$$p_i^{n+1} = p_i^n - p_{ik}^n v_k^n - p_k^n v_{ki}^n - p_i^n v_{kk}^n - p^n v_{kki}^n ,$$

and second differential :-

$$p_{ij}^{n+1} = p_{ij}^n - p_{ijk}^n v_k^n - p_{\langle ik}^n v_{kj\rangle}^n - p_k^n v_{kij}^n - p_{ij}^n v_{kk}^n - p_{\langle i}^n v_{kkj\rangle}^n - p^n v_{kkij}^n ,$$

where $\langle\,\rangle$ is the anti-commutator operator on unlike indices. Actually (§3.5), $v_k = 0$. It is clear that, to solve the recurrence equation for $v_{\iota\kappa}^n$, the $p_{\iota\kappa}^{n+1}$ equation is of no help. Thus, supposing that we ascribe values to $v_\kappa^o$, $v_{\iota\kappa}^o$, $p^o$, $p_\kappa^o$ and to $p_{\iota\kappa}^o$. Then we may determine $v_{\iota\kappa}^1$ and $p^1$ but to determine $v_{\iota\kappa}^2$, and $p^2$, we require to know the values for $p_\kappa^1$ and for $p_{\iota\kappa}^1$, and for these to be found from the last two equations we must be given some of the values for $v_{\iota\kappa\omega}^o$, $p_{\iota\kappa\omega}^o$ and* for $v_{\iota\kappa\omega\nu}^o$. A similar vicious circle has arisen as in the case of the magnetic induction equation, due to the non-linearity of the equation of continuity.

A line of reasoning, which leads to rescue from this impasse, is to assume that the variation in pressure, with position, is a constant both in quantity and in direction, and also in time. Thus, due to some external effect, at and near D the pressure gradient becomes a non-zero but a space- and time-constant vector. Then $p_\kappa^n$ = constant, all $\kappa$ and n, but $p_{\iota\kappa}^n = 0$, all $\iota,\kappa$, and n. Let $p_1^n = 10^{-16}\alpha$, $p_2^n = 10^{-16}\beta$, and let $(p^n)^{-1} = 10^{12} q^n$, where $\alpha$ and $\beta$ are constants. Now the equation §2.3, assuming pq is constant, can be written

$$\partial q/\partial t = -q_k v_k + q v_{kk},$$

or, in recurrence form: $q^{n+1} = q^n - q_k^n v_k^n + q^n v_{kk}^n = q^n(1 + v_{kk})$, since $v_k = 0$ (§3.5).

It is now possible to combine the equations of this paragraph and to consider likely starting conditions. Omitting terms in $b_{11kk}^n$ and in $b_{22kk}^n$, and also,-using Kronecker's delta-writing $\theta\delta_{no}$ for $10^{-14} b_{12kk}^n$ and $\phi\delta_{no}$

* $\omega,\nu = 1(1)2$

for $10^{-14} b^n_{21kk}$ , the combined equations become:-

$$b^{n+1}_{11} = b^n_{11}(1 - v^n_{\kappa\kappa}) + v^n_{12} b^n_{21} - v^n_{21} b^n_{12} ,$$

$$b^{n+1}_{12} = b^n_{12}(1 - v^n_{\kappa\kappa}) + v^n_{11} b^n_{12} - v^n_{12} b^n_{11} + v^n_{12} b_{22} - v^n_{22} b^n_{12} + \theta\delta_{no} ,$$

$$b^{n+1}_{21} = b^n_{21}(1 - v^n_{\kappa\kappa}) - v^n_{11} b^n_{21} + v^n_{21} b^n_{11} - v^n_{21} b^n_{22} + v^n_{22} b^n_{21} + \phi\delta_{no} ,$$

$$b^{n+1}_{22} = b^n_{22}(1 - v^n_{\kappa\kappa}) - v^n_{12} b^n_{21} + v^n_{21} b^n_{12} ,$$

$$v^{n+1}_{11} = v^n_{11}(1 - v^n_{11}) - v^n_{12} v^n_{21} + q^n(b^n_{21} b^n_{[12]} + q^n\alpha^2)$$

$$v^{n+1}_{12} = v^n_{12}(1 - v^n_{\kappa\kappa}) + q^n(b^n_{22} b^n_{[12]} + q^n\alpha\beta),$$

$$v^{n+1}_{21} = v^n_{21}(1 - v^n_{\kappa\kappa}) + q^n(-b^n_{11} b^n_{[12]} + q^n\alpha\beta),$$

$$v^{n+1}_{22} = v^n_{22}(1 - v^n_{22}) - v^n_{12} v^n_{21} + q^n(-b^n_{12} b^n_{[12]} + q^n\beta^2),$$

$$q^{n+1} = q^n(1 + v^n_{\kappa\kappa}).$$

It has already been noted that suitable disturbance values for $\theta, \phi$ are $\sim 10^{-1}$ Values for the $q^n$ come from the last equation and depend on the derived previous set of values of the $v^n_{\iota\kappa}$ and of $q^o$ which can be taken as 1, the undisturbed value (§ 1.10). Perturbation values for $b^o_{\iota\kappa}$ , $v^o_{\iota\kappa}$ can be the same as those in § 5.1 but, to these must be added zero disturbance values, i.e. $b^o_{\iota\kappa} = 1$, $v^o_{\iota\kappa} = 0$, all $\iota, \kappa$, so that the effect of a disturbance in <u>only</u> $b^o_{\iota\kappa kk}$ and $p^o_\iota$ $(= p^n_\iota)$ can also be investigated. It is advisable to ascribe different disturbance values for $p^n_\iota$ , i.e. for $\alpha$ , $\beta$ : this is because, e.g. the $b^{n+2}_{12}$ equation contains the term $(v^{n+1}_{11} - v^{n+1}_{22})b^n_{12}$ , and that itself contains the term $(q^n)^2 (\alpha^2 - \beta^2)$. Also, the term $b^1_{[12]}$ occurs in all $v^2_{\iota\kappa}$ equations, and $b^1_{[12]}$ includes the term $\theta - \phi$ , so that it is advisable to set $\theta \neq \phi$. From § 1.10 we see that the unperturbed value of $p^n_\iota$ is $\sim 10^{-23}$ , so that of $\alpha$ and $\beta$ is $10^{-7}$. So that acceleration in the medium does not become at once unmanagably large, the effect of, say $\alpha \sim 10^{\psi}$ must be checked. This gives $|\nabla p| \sim 10^{\psi-16}$ , whence (§1.10), $\rho^{-1}|\nabla p|$ , the acceleration magnitude, $\sim 10^{\psi+4}$ . Therefore, since unperturbed $|v| \sim 10^4$ (§1.10), we must have $\psi < 0$. However, $\alpha$ , $\beta$ appear as squares or products in the v equations, giving terms of order $< 10^{-2}$ if $\psi < -1$.

The effect of such small terms would take long to show up, and so the best course is to choose $\psi$ such that $-1 < \psi < 0$. Tentatively, we put $\psi = -\frac{1}{2}$, and choose values for $\alpha$ , $\beta$ near $10^{-\frac{1}{2}}$.

By including zero values for $\theta, \phi$ but non-zero ones for $\alpha, \beta$ we may allow for the case where the conductivity is regarded as infinite but a constant perturbed pressure gradient exists.

The above arguments are covered by perturbation and disturbance values:

$$\left\{\begin{array}{l} b^o_{12} = \\ b^o_{21} = \end{array}\left|\begin{array}{c}1\\1\end{array}\right|\begin{array}{c}1.1\\1\end{array}\left|\begin{array}{c}1.2\\1.1\end{array}\right|\begin{array}{c}0.9\\1.1\end{array}\right\|\Bigg\}, \left\{\begin{array}{l} b^o_{11} = 0 , \; b^o_{22} = 0, \\ v^o_{ik} = 0 , \; q^o = 1 \end{array}\right\}, \text{ OR}$$

$$\left\{\begin{array}{l} v^o_{12} = \\ v^o_{21} = \end{array}\left|\begin{array}{c}0.1\\0\end{array}\right|\begin{array}{c}0.2\\0.1\end{array}\left|\begin{array}{c}-0.2\\0.1\end{array}\right\|\Bigg\}, \left\{\begin{array}{l} b^o_{ik} = 1 - \delta_{ik} , \\ v^o_{ii} = 0 , \; q^o = 0 \end{array}\right\}, \text{ and, for all 7 cases :-}$$

$$\left\{\begin{array}{l} \theta = \\ \phi = \end{array}\left|\begin{array}{c}0\\0\end{array}\right|\begin{array}{c}0.1\\0\end{array}\left|\begin{array}{c}0.2\\0.1\end{array}\right|\begin{array}{c}-0.2^{*}\\0.1\end{array}\right\|\Bigg\}, \left\{\begin{array}{l} \alpha = \\ \beta = \end{array}\left|\begin{array}{c}0\\0\end{array}\right|\begin{array}{c}0.3\\0\end{array}\left|\begin{array}{c}0.3\\0.4\end{array}\right\|\right\}$$

and the same sets with either $\theta$ replaced by $\phi$ and vice versa
and/or $\alpha$ " " $\beta$ " " " } if non-zero.

The last remarks are necessary because, in spite of symmetry about $x = y$, a different result might accrue from e.g. $\left\{\begin{array}{l} b^o_{12} = 1.1 \\ b^o_{21} = 1 \end{array}\right\} \& \left\{\begin{array}{l} \theta = 0.1 \\ \phi = 0 \end{array}\right\}$ as from $\left\{\begin{array}{l} b^o_{12} = 1.1 \\ b^o_{21} = 1 \end{array}\right\} \& \left\{\begin{array}{l} \theta = 0 \\ \phi = 0.1. \end{array}\right\}$. The trial where $\theta, \phi, \alpha, \beta = 0$ will give the result for §5.1, as affected by unsteady density.

Thus, for each of the 8 cases involving $b^o_{ik}$ , $v^o_{ik}$ , there are 35 trials to be carried out, giving a total of 245 trials.

The result will be given in §5.4.

At the end of Chapter VI it is proposed to consider the time scale of changes in any variations in the magnetic field and other properties which may

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* Could be $-0.1$, but this choice allows easier programming.)

be found to arise as a result of disturbances, and the type of field that may result in stable cases.

It must be remembered that the assumption that $\underset{\sim}{b} = \underset{\sim}{0}$ , $\underset{\sim}{v} = \underset{\sim}{0}$ near **D** , on which the equations in this chapter are based, no longer applies, if, as a result of perturbations and disturbances, these vectors grow rapidly in the region surrounding D. Thus, if any one of the values of, say, $b^n_{\iota\kappa}$ reaches a number, say N, this means that, at a distance of $10^4$ metres from D $|\underset{\sim}{b}| \sim N \times 10^4$ The likely value of $|\underset{\sim}{b}|$ , due to external influences, is $10^5$ (§1.10). It seems hardly likely that a stable state will result after an impulsive disturbance has caused $|\underset{\sim}{b}|$ , near D , to reach its general value in extra-terrestrial space. Therefore it is proposed to apply a maximum value for $b^n_{\iota\kappa}$ of 10 , in the investigations : if a $b^n_{\iota\kappa}$ reaches this figure, it will be assumed that conditions are unstable. By similar arguments the limit for the $v^n_{\iota\kappa}$ will also be set at 10 .

## §5.3. Variable Conductivity and Pressure

In the above two paragraphs, the conductivity, $\sigma$ , and hence $\gamma$ , were taken to be space constant. However, when finding a likely value for $\sigma$ in the magnetotail region (§1.4), $\sigma$ was shown to be a space-variable. In fact, combining the figures of §1.4 with those for L in §1.31, we have, in CKMS units :

$$L_S \sim 10^9 m^1 , \qquad L_M \sim 10^{11} m^1, \qquad L_G \sim 10^{17} m^1$$

$$\sigma_S \sim 10^{25} C^2 k^{-1} m^{-3} s^1 , \qquad \sigma_M \sim 10^{20} C^2 k^{-1} m^{-3} s^1 , \sigma_G \sim 10^{19} C^2 k^{-1} m^{-3} s^1 .$$

A plot, Fig 19, of $\sigma'$ against $L'$ shows that $|d\sigma'/dL'|_M \sim 10^0$, and thence that $|d\sigma/dL|_M = (\sigma/L)_M |d\sigma'/dL'|_M = 10^9 \times 10^0 = 10^9 C^2 k^{-1} m^{-4} s^1$ . (p(xviii))

Thus, if we work on these figures, we find that $|d\Gamma/dL|_M$ (§1.8), i.e. $|\mu^{-1}\sigma^{-2} d\sigma/dL|_M , \sim 10^6 \times 10^{-40} \times 10^9 , \sim 10^{-25} m^1 s^{-1}$, and therefore that (§1.10):-

$$m^{-1} |\underset{\sim}{\nabla}\Gamma| \sim 10^{-25} m^1 s^{-1} .$$

A first derivative of the equation in §2.1, adjusted as in §5.1, would

take the form : $\partial b_{ij}/\partial t = -v_{kj} b_{ik} + v_{ik} b_{ij} - v_{kk} b_{ij} + \gamma_j b_{ikk} + \gamma\, b_{ijkk}$,
if the space variance of γ is allowed for, the fourth term in the RHS being the one not previously considered. Due to external influences, as seen above and from §1.10, it has the order $10^{-25} \times 10^{-11} = 10^{-36}$.

The order of the last term can be obtained using Fig 4, from which we may write, - with L now as in §3.1 **ii** referring to distance from the Earth,- in CKMS units:-

$$|d(|dB/dL|')/dL'|_E \sim 10^{1}, \quad |d(|dB/dL|')/dL'|_M \sim 10^{0+}, \quad |d(|dB/dL|')/dL'|_G \sim 10^{0},$$
$$L_E \sim 10^{7}, \quad L_M \sim 10^{8}, \quad L_G \sim 10^{17}.$$

The first line means : $|d^2B/dL^2|_E \sim \{(|dB/dL|/L)\times 10^{1}\}_E \sim 10^{-17}$,
$|d^2B/dL^2|_M \sim 10^{-24}, \quad |d^2B/dL^2|_G \sim 10^{-44}$.

A plot of $|d^2B/dL^2|'$ against $L'$ is given in Fig 20 from which it can be seen (p.(xviii)) that :

$$|d(|d^2B/dL^2|')/dL'|_M \sim 10^{1},$$

that is: $|d^3B/dL^3|_M \sim \{|d^2B/dL^2|/L\}_M \times 10^{1}, \sim 10^{-24} \times 10^{-8} \times 10^{1}, \sim 10^{-31} C^{-1} k^{1} m^{-3} s^{-1}$ .
So that the order of $m^{-3}|\nabla(\nabla B)|$ is $10^{-18} f^{-1} k^{1} m^{-3} s^{-1}$ .
Therefore the order of the last term, in the above equation for $\partial b_{ij}/\partial t$, due to external influences is $10^{-32}$ .

However there is a possibility that, because of instability leading to unusual conditions near the neutral point, the terms containing $\gamma_j$ and γ may become of similar order to the others. This possibility was allowed for (in §5.2 by allotting high impulse values to the $b_{ijkk}$ at time 0) as regards γ ; for $\gamma_j$ an alternative approach is available.

Hirose [47] carried out an experiement with plasma in which measurements were taken of σ, V and of E, the magnitude of the electric intensity. Fig. 21, p.(xix) shows his readings, adjusted to CKMS units. It will be remarked that, under these laboratory conditions, the order of σ is very much lower, and that of V is higher than that of the corresponding values to be expected in the magnetotail region due to external influences. Hirose and his co-workers attempted to justify the readings in terms of a connection between σ and E , where N is a number based on the kinetic theory of plasma: one part of the curve (slope $\sim -1$) indicates a connection of the form $\sigma \propto E^{-1}$, and another (slope $\sim -\frac{1}{2}$) indicates $\sigma \propto E^{-\frac{1}{2}}$. The authors were able to obtain such connections from the theory, with an explanation of why the same connection does not apply right through the range of the experiment. The kinetic theory of plasma is a subject with wide ramifications, and many different formulae can be extracted from it by making certain assumptions. The experimental results lead to a suspicion that in view of the scatter of the readings, the $\sigma/E$ connections obtained by the authors are not necessarily correct. At the top of the row of points, it is claimed, using four points only, that $\sigma$ is constant at 60 CKMS units, and at the bottom it is claimed that $\sigma$ is constant at 1.1 CKMS, although the graph would indicate a value closer to 0.S CKMS. Incidentally the graph is in fact on the logarithmic scale, and so indicates the variation of $\sigma'$ with $E'$, where the units of $\sigma$ are $C^2 k^{-1} m^{-3} s^1$, and those of E are $C^{-1} k^1 m^1 s^{-2}$.

However it may be possible to use these readings by regarding them as a connection between $\sigma$ and V . This procedure is considered advisable because although in Hirose's paper the connection between $\sigma$ and E is regarded as of greater importance than that between $\sigma$ and V , yet in extra-terrestrial space it appears that $\sigma$ varies with E in a manner which cannot be correlated to the manner in which $\sigma$ and E behave together under the conditions of Hirose's experiment. To see this, we may plot, as in Fig 22, (p.(xix)) the two end points of Hirose's experiment (to the nearest integer in $\sigma$ ,E , and with suffixes H,K,) together with three points applicable to extra-terrestrial space, as follows:-

| | | |
|---|---|---|
| HII | $\sigma'_G = 19$ , | $E'_G = -14$, |
| Magnetotail | $\sigma'_M = 20$ , | $E'_M = -13$, |
| Corona | $\sigma'_S = 25$ , | $E'_S = -7$, |
| Hirose | $\sigma'_H = 2$ , | $E'_H = 1$, |
| " | $\sigma'_K = 0$ , | $E'_K = 4$ . |

In the above table, first three rows, the figures for $\sigma'$ were given at the start of this paragraph. Those for $E'$ are due to Dungey, who gives values for

$E_S$ and $E_G$ in two references. The first one [48] indicates that $E_S \sim 10^{-7}$ CKMS, and that $E_G \sim 10^{-15}$ CKMS. The second one [49] indicates that $E_S \sim 10^{-7}$ CKMS, as before, whereas $E_G \sim 10^{-13}$ CKMS. A geometric mean of the two measurements, viz. $10^{-14}$CKMS, for $E_G$ has here been adopted. Meanwhile $E_M$ has been calculated on the assumption that the order of E/B in space is approximately constant (see §1.3).

A value, $10^{-2}C^{-1}k^{1}m^{1}s^{-2}$ for E in the ionosphere is given by Piddington [50]: it is felt that this value has little bearing on the problem in hand, since, in the ionosphere where regions of finite charge density exist, one may expect high local values for E.

Therefore, in Fig. 23, (p.(xx)) a plot of $V'$ is given against $\Gamma'$, where, from §§1.2, 1.8, $\Gamma' = -\mu'-\sigma' = 6-\sigma'$: the graph includes the end points of Hirose's readings, approximated as above, and elsewhere values for $V'$ are from §1.7. This graph gives the impression that $\Gamma'$ may possibly increase with $V'$ as terrestrial (laboratory, i.e. as in Hiroses experiment) conditions are approached from extra-terrestrial (magnetotail) space, although $\Gamma'$may possibly decrease as $V'$ increases near the corona. In view of the fact that conditions in the corona can hardly be expected to be regarded as typical of extra-terrestrial space, it seems more likely that the first condition is applicable, so that :

$$(d\Gamma'/dV')_M \sim 10^{1}, \text{ i.e. } (d\Gamma/dV)_M \sim 10^{1}(\Gamma/V)_M, \sim 10^{-17}\ m^{1}.$$

This is a very small number : it is unaffected by units of charge and will give a connection $\gamma_j \sim 10^{-17}(\sqrt{v_k v_k})_j$ . But a disturbance in the magnetotail region may conceivably cause $V'$ to increase to over 5 , and then it is possible that Hirose's values for $\Gamma'$are applicable.

Fig. 24 shows a detailed plot, without approximations, of $\Gamma'$and $V'$ , (p.(xx)) taken from Hirose's experiment. It will be seen that there is considerable scatter, but that, over a sensible area of the graph, very nearly : $(d\Gamma'/dV') \sim 1$. That is : $\Gamma = V \times$ constant. We may put the constant as $10^{\frac{1}{4}}$ , to give $\Gamma' = 5\frac{3}{4}$

when $V = 5\frac{1}{2}$, as on the graph. The region concerned is, however, unlikely to be resembled in density, electric intensity, etc., in the magnetail region; therefore, in order to investigate the effect of perturbations leading to varying conductivity, it is proposed to consider a small reduction of an order in $\Gamma/V$, and write $\Gamma/V \simeq 1\ m^{1}$, whence $\gamma \simeq v$. Since we wish to make use of the vector components of $V$ it is necessary to square this equation:

$$\gamma^2 \simeq v_k v_k .$$

Taking the gradient and dividing by $2\gamma \simeq 2v$ :-

$$\gamma_i \simeq (v_k v_{ki})/v .$$

This equation can be written using the unit vector $\hat{\underset{\sim}{v}}$, components $\hat{v}_i$ :-

$$\gamma_i \simeq \hat{v}_k v_{ki} .$$

When we investigate instability, the effect of $\hat{\underset{\sim}{v}}$ can be allowed for as follows: although $\underset{\sim}{v} \to \underset{\sim}{0}$ as the neutral point is approached, near the neutral point $\underset{\sim}{v}$ is finite though small and $\hat{\underset{\sim}{v}}$ is defined. We assume, from the usual symmetry conditions about the Dxy plane, that $\hat{v}_3 = 0$ everywhere, and choose a particular point on this plane, or near it, where $\hat{\underset{\sim}{v}}$ is co-parallel $O y \underset{\sim}{j}$ . It can be argued that, as $\underset{\sim}{v}$ varies under impulse, this point may vary. However this does not matter: the previous investigations (§5.1,5.2) having referred to the whole area around D: if instability arises at this point, then it can be reasonably assumed that the whole system, near D, is unstable. Therefore we write $\hat{\underset{\sim}{v}} = \underset{\sim}{j}$, that is : $\hat{v}_k = \delta_{2k}$, using Kronecker's delta. The equation for $\gamma$ becomes :

$$\gamma_i = \delta_{2k} v_{ki} = v_{2i} .$$

With $\gamma$ varying in space, a first differential of the equation in §2.1 is:

$$\partial b_{ij}/\partial t = -v_{kj} b_{ik} + v_{ik} b_{kj} - v_{kk} b_{ij} + \gamma_j b_{ikk} + \gamma b_{ijkk} ,$$

where, as in §5.2 , $\underset{\sim}{v}$ and $\underset{\sim}{b}$ are treated as zero. Substituting for $\gamma_j$ :--

$$\delta b_{ij}/\delta t = -v_{kj} b_{ik} + v_{ik} b_{kj} - v_{kk} b_{ij} + v_{2j} b_{ikk} + \gamma b_{ijkk} .$$

The corresponding recurrence equation, as in §5.2 is :

$$b_{ij}^{n+1} = b_{ij}^{n} - v_{kj}^{n} b_{ik}^{n} + v_{ik}^{n} b_{kj}^{n} - v_{kk}^{n} b_{ij}^{n} + v_{2j}^{n} b_{ikk}^{n} + \gamma^{n} b_{ijkk}^{n} .$$

We now make the same assumption as in §5.2, viz. that $b_{ij}^{o}$ , $v_{ij}^{o}$ are zero if either $i$ or $j$, but not both is equal to 3. Then if $j = 3$, we have the three equations for $n = 0$ :-

$$b_{13}^{1} = \gamma^{o} b_{13kk}^{o} , \quad b_{23}^{1} = \gamma^{o} b_{23kk}^{o} , \quad b_{33}^{1} = b_{33}^{o} (1 - v_{kk}^{o}) + \gamma^{o} b_{33kk}^{o} .$$

The three equations for $v_{i3}^{1}$ are as in §5.2 . This means that :

$b_{13}^{2} = \gamma^{1} b_{13kk}^{1}$ , $b_{23}^{2} = \gamma^{1} b_{23kk}^{1}$ , $b_{33}^{2} = b_{33}^{1} (1 - v_{kk}^{1}) + \gamma^{1} b_{33kk}^{1}$ , and so on, and that $v_{13}^{n} = v_{23}^{n} = 0$, $v_{33}^{n} = fn(v_{33}^{o})$ as before. The terms $b_{i3}^{n}$ , $b_{i3kk}^{n}$ , and $v_{i3}^{n}$ do not appear in any of the other equations apart from $v_{33}^{n}$ as a member of $v_{kk}^{n}$ in , e.g. the equation for $b_{ij}^{n+1}$ . Thus, these equations form an independant group and, if they are to be solved, then the $\gamma^{n} b_{i3kk}^{n}$ must be prescribed for all n. In other words, except for the term $b_{33}^{n} (1 - v_{kk}^{n})$ in the equation for $b_{33}^{n+1}$ , all $b_{i3}^{n}$ must be prescribed, and there is no scope for investigation. These equations will therefore be omitted.

If $i = 3$, $j \neq 3$ we have the two equations for $n = 0$ :-

$$b_{31}^{1} = v_{21}^{o} b_{3kk}^{o} + \gamma^{o} b_{31kk}^{o} , \quad b_{32}^{1} = v_{22}^{o} b_{3kk}^{o} + \gamma^{o} b_{32kk}^{o} ,$$

and $v_{31}^{1} = v_{31}^{2} = 0$, as before, leading to the general equations :

$$b_{3i}^{n+1} = v_{2i}^{n} b_{3kk}^{n} + \gamma^{n} b_{3ikk} , \quad v_{3i}^{n} = 0 , \quad \text{all } n.$$



The terms $b_{3kk}^{n}$ , $\gamma^{n} b_{3ikk}^{n}$ , $v_{3i}^{n}$ do not appear in any of the other equations, and the values of $v_{2i}^{n}$ depend on the other equations. Therefore the values of the $b_{3i}^{n}$ depend on the other equations and on prescribed values for $b_{3kk}^{n}$ and $\gamma^{n} b_{3ikk}^{n}$ ; thus, the other equations form an independant group, and, once they are solved, these equations in $b_{3i}^{n}$ depend on the result and on prescribed perturbations. It seems likely that, if the other equations are unstable, those in $b_{3i}^{n}$ are also unstable, due to the effect of the $v_{2i}^{n}$ . Therefore, these equations also will be omitted.

We are left with :

$$b_{\iota\kappa}^{n+1} = b_{\iota\kappa}^{n} (1 - v_{kk}^{n}) - v_{\omega\kappa}^{n} b_{\iota\omega}^{n} + v_{\iota\omega}^{n} b_{\omega\kappa}^{n} + v_{2\kappa}^{n} b_{\iota kk} + \gamma^{n} b_{\iota\kappa kk} .$$

We now make the same assumption, as in §5.2 , and for the same reasons, that the non-shear terms of $b^n_{\iota\kappa kk}$ , i.e. the terms with $\kappa = \iota$ , are zero. The four equations, then, in detail, are, (putting $v^0_{33} = 0$ as before ):-

$$b^{n+1}_{11} = b^n_{11}(1 - v^n_{\kappa\kappa}) + v^n_{12}b^n_{21} - v^n_{21}b^n_{12} + v^n_{21}b^n_{1kk} ,$$

$$b^{n+1}_{12} = b^n_{12}(1 - v^n_{\kappa\kappa}) + v^n_{11}b^n_{12} - v^n_{12}b^n_{11} + v^n_{12}b^n_{22} - v^n_{22}b^n_{12} + v^n_{22}b^n_{1kk} + \gamma^n b^n_{12kk} ,$$

$$b^{n+1}_{21} = b^n_{21}(1 - v^n_{\kappa\kappa}) - v^n_{11}b^n_{21} + v^n_{21}b^n_{11} - v^n_{21}b^n_{22} + v^n_{22}b^n_{21} + v^n_{21}b^n_{2kk} + \gamma^n b^n_{21kk} ,$$

$$b^{n+1}_{22} = b^n_{22}(1 - v^n_{\kappa\kappa}) - v^n_{12}b^n_{21} + v^n_{21}b^n_{12} + v^n_{22}b^n_{2kk} .$$

The equations for $v^{n+1}_{\iota\kappa}$ are as in §5.2. No solution is possible unless, as explained in §5.2, the $b^n_{1kk}$ , $b^n_{2kk}$ , $\gamma^n b^n_{12kk}$ and $\gamma^n b^n_{21kk}$ are prescribed for all n. It should be noted here that no attempt to replace $\gamma^n$ by $v^n$ has been made, since the connection referred to at the beginning of this paragraph is an approximate, and not exact, equality. The problem now arises, what disturbance values should be prescribed for these four variables? - remembering that, due to external influences, they are negligibly small, - that is, they can be treated as zero, so that the disturbance values may be referred to as perturbation values.

If we allot these values only for when $n = 0$, the problem becomes no different from that of §5.2. On the other hand, if values are allotted for all n, not only do we run into the difficulty of infinite data, already referred to in §5.2, but also we have no way of investigating the effect of space - dependant conductivity. The best plan would appear to be to allot "impulse" disturbance values to the $\gamma^n b_{12kk}$ and $\gamma^n b^n_{21kk}$ , in the same way as, and for the same reasons as discussed in §5.2 ( in fact, we may use $\theta\delta_{n0}$ , $\phi\delta_{n0}$ as before), but for the $b^n_{\iota kk}$ , which are the components of the dimensionless form of $m^{-2}\nabla^2\underset{\sim}{B}$ , we may argue as follows :-

From Maxwell's equations, $m^{-2}\nabla^2\underset{\sim}{B} = -m^{-2}\underset{\sim}{\nabla} \times (\underset{\sim}{\nabla}\times\underset{\sim}{B}) = - m^{-1}\mu(\underset{\sim}{\nabla}\times\underset{\sim}{J})$. Also from $m^{-1}(\underset{\sim}{\nabla}\times\underset{\sim}{B}) = \mu\underset{\sim}{J}$ , we note that, when $b^n_{12}$ , $b^n_{21}$ are non-zero and unequal, therefore $J_3$ is non-zero, hence $\underset{\sim}{J}$ exists as a finite vector. However the governing equations provide us information as to whether or not $\underset{\sim}{J}$ is irrotational.

We may therefore imagine that, due to some extraneous disturbance, $\underset{\sim}{J}$ acquires a solenoidal part that does not change rapidly with time. Then, approximately, we may write : $m^{-1}\underset{\sim}{\nabla} \times \underset{\sim}{J} \simeq \underset{\sim}{C}$, constant vector, whence the $b^n_{ikk}$ are the components of a dimensionless constant vector. Thus, one may write : $b^n_{1kk} = \chi$, $b^n_{2kk} = \psi$, both constant, all n.

The four equations therefore become :-

$$b^{n+1}_{11} = b^n_{11}(1 - v^n_{\kappa\kappa}) + v^n_{12}\, b^n_{21} - v^n_{21}(b^n_{12} - \chi),$$

$$b^{n+1}_{12} = b^n_{12}(1 - v^n_{\kappa\kappa}) + v^n_{11}\, b^n_{12} - v^n_{12} b^n_{11} + v^n_{12} b^n_{22} - v^n_{22}(b^n_{12} - \chi) + \theta\delta_{n0},$$

$$b^{n+1}_{21} = b^n_{21}(1 - v^n_{\kappa\kappa}) - v^n_{11}\, b^n_{21} + v^n_{21} b^n_{11} - v^n_{21}(b^n_{22} - \psi) + v^n_{22}\, b^n_{21} + \phi\delta_{n0},$$

$$b^{n+1}_{22} = b^n_{22}(1 - v^n_{\kappa\kappa}) - v^n_{12}\, b^n_{21} + v^n_{21}\, b^n_{12} + v^n_{22}\,\psi$$

$v^{n+1}_{\iota\kappa}$, $q^n$ as in §5.2

Since, in three of the above four equations for $b^{n+1}_{\iota\kappa}$, the $b^n_{\iota kk}$ ($\psi$ or $\chi$) perform the function of subtractions from the values of three of the $b^n_{\iota\kappa}$, an indication of the order of size of the prescribed constant $b^n_{\iota kk}$ is needed. Being a higher differential, it is likely to be a little less than the $b^n_{\iota\kappa}$ for all n, that is, in an unstable case, less than the $b^o_{\iota\kappa}$. Looking at the prescribed perturbation values for the $b^o_{\iota\kappa}$ in §5.2, we note that they are of the order of unity. Therefore a likely value for the $b^n_{\iota kk}$, large enough to affect the issue, is of the order $10^0$. Thus, we may prescribe :

| | | | | | |
|---|---|---|---|---|---|
| $\chi$ | = | 1 | 0 | 1 | These values will be tried with each of the trials |
| $\psi$ | = | 0 | 1 | 1 | of §5.2, total 735 trials; result in §5.4. |

## §5.4 Stability Criteria

As will be seen from page (xxxiii) of Appendix B "Computational Details" there appear to be no stable cases for the trails made in §§5.1 - 3. The program was adjusted to include the incidences of infinite conductivity and constant pressure (§5.1), constant finite conductivity and variable pressure (§5.2) and of variable conductivity and pressure (§5.3). This was done as follows:-

For §5.1 it was necessary to set $\theta, \phi, \alpha, \beta, \chi$ and $\psi$ all at zero. This was done by instructions 47, and by 90, 91, 108, 109, 62 and 63 respectively, the instructions 90 ... 109 being part of the repeat process; see pages (xxii), (xxiii). In the program, these variables are given the symbols, Y,Z,U,V,W,X respectively.

For §5.2, non-zero values were given to $\theta,\phi,\alpha,\beta$ by instructions 48 ... 51, by instructions 93 ... 105, and by instructions 111 ... 118 in the repeat process, see pages (xxii), (xxiii).

For §5.3, non-zero values were given to $\chi,\psi$ by instructions 48 ... 51, and by instructions 121, 123 in the repeat process, see pages (xxii), (xxiii).

The instructions 48 ... 51 enable the program to be tried again, if required, with all disturbance and perturbation values altered by an order.

To check stability, the program was adjusted in such a way that, if none of the variables has reached a value of 10, that is, of order $10^2$ of the starting values of the variables, then the starting and final values of the variables are printed out after 10 applications of the difference equations : since $q^n$ is proportional to the inverse of the pressure, the above test must be applied to $(q^n)^{-1}$, see instruction 188 on page (xxiv). Although this would not necessarily indicate stability, it is possible to try such starting values over a longer period, thus checking for stability : the only alterations necessary to effect this would be to instructions 45 and 46, page (xxii). In fact, 8 cases were found, with instructions 45 and 46 both set at 10. {see xxxiii 45 set at 30} For these 8 doubtful cases, instruction 45 was changed to 30 and 46 to 100, the whole program was run again and showed that, in all cases, at least one variable had reached order $10^3$ times its starting value after < 30 applications of the difference equations, indicating instability. The result of this test is the one shown on page (xxxiii') of Appendix B.

The program, Appendix B pages (xxii) - (xxv), has been made as elastic as possible to enable different time periods between jumps, etc. to be used. In particular, the instructions in the following list may be adjusted as indicated :-

Instruction no :-

42 : This makes $\Delta t$ as 1 sec., and can be altered if different time periods between jumps are preferred. However it must be borne in mind that $\Delta t$ does in effect represent a vanishingly small time element, and therefore should not be made too large otherwise the results will not present a true picture of what is going on.

43 : This makes $\lambda = 1$ i.e. $\mu\rho = 1$ in fkms units ; alterable.

44 : This is the proportion between $v$ and $\gamma$ ( = $\mu^{-1}\sigma^{-1}$), set at 1 as decided in §5.3. This may be altered if desired.

45 : Maximum period for stability check, alterable as discussed above.

46 : Criterion for instability, if set at 10 this means that one variable reaches order $10^2$ of its starting disturbance or perturbation value in the number of applications of the differential equations decided by instruction 45. This may be altered as discussed above.

47 - 50 : These are for setting starting disturbances and perturbations. They may be altered, as discussed above, so that by a small change in the program a different set of starting disturbances and perturbations may be tried.

59 : This instruction is not in fact used, but a similar one exists in the second program (ref. §6). However it is left in, to allow for the possibility of inserting a stop instruction to prevent the machine from wasting time over attenuating cases, etc., and to avoid disturbing the number order of instructions.

60 : This zero instruction is changed to unity by instruction 67, that is, after the cases of §5.1 have been calculated : the effect is to bring variation from unity in the $q^{n+1}$ equation in §5.2. Instruction 67 can be altered from unity if desired.

61 : This unity instruction is changed to zero after one time space (i.e. second) by instruction 176 : (alterable, if, e.g., $\theta$ is to be regarded as a constant and not as an impulsive perturbation).

67 : See above (60).

68 : Avoids wasting machine time on steady cases of no interest.

176 : See above (61).

To investigate the type of $\underset{\sim}{B}$ and of $\underset{\sim}{v}$ fields resulting from the slower unstable cases, the program was run with instruction 45 on page xxii altered to 5(secs) instead of 10. This resulted in a large number of results in the print out, of cases where no variable had reached a value of 10 or more after five applications of the difference equations. It would take up too much space to provide all these results in this paper. Instead, the results were inspected to see if any of the results indicated a $\underset{\sim}{B}$ or $\underset{\sim}{v}$ field which had changed its type while displaying an unstable nature. Again there were a large number of these, all of which indicated the same phenomenon, viz. that the B field starts as an X-type and becomes a spiral type, whereas the $\underset{\sim}{v}$ field starts as a "Couette" or plain shear type and becomes an X-type. One was chosen for more detailed inspection, for which starting values and constants were as follows :-

$b_{11}^{o} = 0$, $b_{12}^{o} = 1$, $b_{21}^{o} = 1$, $b_{22}^{o} = 0$ (i.e. $\underset{\sim}{b}$ field unperturbed)

$v_{11}^{o} = 0$, $v_{12}^{o} = 0.1$, $v_{21}^{o} = 0$, $v_{22}^{o} = 0$ (i.e. $\underset{\sim}{v}$ field perturbed only in $v_{12}^{o}$ )

$\alpha = 0$, $\beta = 0$ (no perturbation in pressure second gradient)

$\chi = 1$, $\psi = 1$ ( i.e. $b_{1kk}^{n} = 1$, $b_{2kk}^{n} = 0$)

$\theta = 0$, $\phi = 0$ (i.e. no perturbation in the second gradient of $\nabla^2 \underset{\sim}{b}$)

Successive values, to three significant figures, after application of the difference equations were :

| $n$ | $b_{11}^{n}$ | $b_{12}^{n}$ | $b_{21}^{n}$ | $b_{22}^{n}$ | $v_{11}^{n}$ | $v_{12}^{n}$ | $v_{21}^{n}$ | $v_{22}^{n}$ | $q^{n}$ |
|---|---|---|---|---|---|---|---|---|---|
| 1. | 0.10 | 1.20 | 1.10 | -0.10 | 0.00 | 0.10 | 0.00 | 0.00 | 1.00 |
| 2. | 0.21 | 1.18 | 1.10 | -0.21 | 0.11 | 0.09 | -0.01 | -0.12 | 1.00 |
| 3. | 0.31 | 1.31 | 0.85 | -0.32 | 0.19 | 0.07 | -0.03 | -0.23 | 0.99 |
| 4. | 0.40 | 1.63 | 0.52 | -0.43 | 0.54 | -0.06 | -0.17 | -0.86 | 0.94 |
| 5. | 0.60 | 3.62 | -0.18 | -0.81 | 0.78 | -0.55 | -0.64 | -3.32 | 0.64 |

It will be seen that the $\underset{\sim}{b}$ field changes from an X-type to a spiral type on the fifth application of the difference equations, whereas the $\underset{\sim}{v}$ field changes from a "Couette" type to an X-type on the second application, remaining an X-type thereafter.

The $\underset{\sim}{J}$ field that is caused by all these unstable cases will be parallel to 0z, since in the system of this paragraph, $\underset{\sim}{J} = \mu^{-1}(\underset{\sim}{\nabla} \times \underset{\sim}{B})$. Writing $J_o = 1f^1m^{-2}s^{-1}$, and $\underset{\sim}{j} = \underset{\sim}{J}/J_o$, we find that $\underset{\sim}{j} = (B_o/J_o)\mu^{-1}m^{-1}(\underset{\sim}{\nabla} \times \underset{\sim}{b})$. Then, since $B_o/J_o = 1\ f^{-2}k^1m^{-2}$, $\mu = 10^{20}f^{-2}k^1m^1$, we have $j_3 = 10^{-20}b_{[12]}$. In the particular case chosen for examination, $j_3^0 = 0$ and $j_3^5 = 10^{-20} \times 3.8$, giving a current density $\underset{\sim}{J}$ of $10^{-20} \times 3.8f^1m^{-2}s^{-1}$, i.e. $10^{-7} \times 3.8C^1m^{-2}s^{-1}$ $(A^1m^{-2})$.

# CHAPTER VI

## SECOND DIFFERENTIAL EQUATIONS.

### §6.1. Infinite Conductivity and Constant Pressure.

If we take the gradient twice of the first equation in §5.1, we obtain, with $\ell = 1(1)3$ :-

$$\partial b_{ijk}/\partial t = -v_{\ell<j}\, b_{ik>\ell} - v_{\ell jk}\, b_{i\ell} + v_{i\ell}\, b_{\ell jk} + v_{i\ell<j}\, b_{\ell k>} - v_{\ell\ell}\, b_{ijk} - v_{\ell\ell<j}\, b_{ik>}.$$

In this equation the operator $< >$ refers to the two outside members of the contained symbols. Also, as in §5.1, we have ignored terms in $\gamma$ (§§1.8,2.1), due to infinite conductivity, and have assumed that $\underset{\sim}{v}$ and $\underset{\sim}{b}$ are effectively zero near D.

Similarly, the gradient taken twice of the second equation in §5.1 provides, with $\underset{\sim}{v} = \underset{\sim}{b} = \underset{\sim}{0}$ as above, $\lambda = 1$ and $\underset{\sim}{\nabla} p = \underset{\sim}{0}$ as in §5.1 :-

$$\partial v_{ijk}/\partial t = - v_{\ell<k}\, v_{ij>\ell} - v_{i\ell}\, v_{\ell jk} + b_{\ell<k}\, b_{[i\ell]j>} + b_{[i\ell]}\, b_{\ell jk}.$$

As in §5.1, with t set at unity as before, these two equations can be transformed into the recurrence equations :-

$$b^{n+1}_{ijk} = b^{n}_{ijk} - v^{n}_{\ell<j}\, b^{n}_{ik>\ell} - v^{n}_{\ell jk}\, b^{n}_{i\ell} + v^{n}_{i\ell}\, b^{n}_{\ell jk} + v^{n}_{i\ell<j}\, b^{n}_{\ell k>} - v^{n}_{\ell\ell}\, b^{n}_{ijk} - v^{n}_{\ell\ell<j}\, b^{n}_{ik>},$$

$$v^{n+1}_{ijk} = v^{n}_{ijk} - v^{n}_{\ell<k}\, v^{n}_{ij>\ell} - v^{n}_{i\ell}\, v^{n}_{\ell jk} + b^{n}_{\ell<k}\, b_{[i\ell]j>} + b^{n}_{[i\ell]}\, b^{n}_{\ell jk}.$$

To get an idea as to how this system behaves under perturbation, it is proposed in this chapter to use again the co-parallel line current model of Chapter IV (§4.2) as a model for the unperturbed $\underset{\sim}{b}$ field near D when $t = 0$, i.e. when $n = 0$; this is the same model as was used in Chapter V. In this system, the only non-zero $b^{0}_{ij}$ are $b^{0}_{12}$ and $b^{0}_{21}$ , both equal to (-g) which may be equated to unity as in §5.1. For the unperturbed $\underset{\sim}{v}$ field when $n = 0$ it is proposed again, as in Chapter V and based on the discussion in §3.5, to assume that all elements of $\underset{\sim}{\nabla}\underset{\sim}{v}$ and of $\underset{\sim}{\nabla}\underset{\sim}{\nabla}\underset{\sim}{v}$ are zero when $n = 0$.

No perturbations will be prescribed for the $b^{0}_{ij}$ or for the $v^{0}_{ij}$ , since the effects of such perturbations were analysed in Chapter V. It can be easily checked, from the equations for $b^1_{ij}$ , $v^1_{ij}$ in § 5.1 that, if $b^o_{12} = b^o_{21} = 1$, all other $b^o_{ij}$ and all $v^o_{ij} = 0$, then $b^n_{12} = b^n_{21} = 1$, all n, and all other $b^n_{ij}$ and all $v^n_{ij} = 0$, all n : the system is **steady**. Thus, by not perturbing the $b^o_{ij}$ , $v^o_{ij}$ , the constant or zero values of the $b^n_{ij}$ , $v^n_{ij}$ , all n, may be substituted in the above equations for $b^{n+1}_{ijk}$ and $v^n_{ijk}$ .

Perturbations will be prescribed for the $v^o_{ijk}$ and not for the $b^o_{ijk}$ . The reason for this action is as follows : there are 18 independent elements of $b^o_{ijk}$ , since $b^o_{ijk} = b^o_{ikj}$ . Of these, 15 are either one of the terms in $\nabla^2 \underset{\sim}{b}$ or one of the terms in $\underset{\sim}{\nabla}(\underset{\sim}{\nabla}\cdot b)$. In dimension form, $m^{-2}\nabla^2\underset{\sim}{B} = -\mu m^{-1}(\underset{\sim}{\nabla}\times\underset{\sim}{J})$ by Maxwell's relevant time-independent equation. If we assume that the perturbation is connected with an irrotational current density, as seems likely, then $\nabla^2 \underset{\sim}{b} = \underset{\sim}{0}$, and $\underset{\sim}{\nabla}(\underset{\sim}{\nabla}\cdot\underset{\sim}{b}) = \underset{\sim}{0}$ in any case. Thus, although it is possible for the individual terms in the expansion of both these expressions to be zèro, it seems that these terms themselves, and the three $b^o_{ijk}$ in which i,j and k are all different, are less likely to be subject to high individual perturbation values. These restrictions do not apply to the 18 elements of $v^o_{ijk}$ .

Since we can write all $v^n_{ij}$ and $b^n_{[ij]}$ as zero, the 36 equations for $b^{n+1}_{ijk}$ , $v^{n+1}_{ijk}$ become :-

$$b^{n+1}_{ijk} = b^n_{ijk} - v^n_{\ell jk}\, b^n_{i\ell} + v^n_{i\ell<j}\, b^n_{\ell k>} - v^n_{\ell\ell<j}\, b^n_{ik>} ,$$

$$v^{n+1}_{ijk} = v^n_{ijk} + b^n_{\ell<k}\, b^n_{[i\ell]j>} .$$

It will be noticed that, since the $b^n_{ij}$ are independent of n, these equations are linear, and that therefore they may be solved analytically, once the starting conditions have been prescribed. However, before doing so, it is important to observe from the detailed form of the equations, as presented below, that the set of equations can be divided into four separate groups, each of which is entirely independant of the other three. These are :

(i) The 8 equations for $b^n_{113}$ , $b^n_{223}$ , $b^n_{311}$ , $b^n_{322}$ , $b^n_{333}$ , $v^n_{123}$ , $v^n_{213}$ and $v^n_{312}$ . In fact, the $b^n_{333}$ equation is independent of the others -

$b^n_{333}$ is steady - but it is included here for tidiness sake. These equations are symmetrical in suffixes 1,2.

(ii) The 10 equations for $b^n_{112}$, $b^n_{211}$, $b^n_{222}$, $b^n_{233}$, $b^n_{323}$, $v^n_{111}$, $v^n_{122}$, $v^n_{133}$, $v^n_{212}$ and $v^n_{313}$. It will be seen that $v^n_{122}$ and $v^n_{133}$ are steady, but they appear in the other equations in this group.

(iii) The 8 equations for $b^n_{123}$, $b^n_{213}$, $b^n_{312}$, $v^n_{113}$, $v^n_{223}$, $v^n_{311}$, $v^n_{322}$ and $v^n_{333}$. These equations are symmetrical in suffixes 1,2, and $v^n_{333}$ is steady.

(iv) The 10 equations for $b^n_{111}$, $b^n_{122}$, $b^n_{133}$, $b^n_{212}$, $b^n_{313}$, $v^n_{112}$, $v^n_{211}$, $v^n_{222}$, $v^n_{233}$ and $v^n_{323}$. It will be seen that $v^n_{211}$ and $v^n_{233}$ are steady, but appear in the other equations in this group. However, this group is similar to the equations of group (ii), and in fact could be obtained from those of group (ii) by interchange of suffixes 1,2. Because of this, expected, feature, this group will not be considered further.

The 36 equations, with $b^n_{ij}$ all zero except $b^o_{12} = b^o_{21} = 1$ are (groups indicated) :-

(iv) $b^{n+1}_{111} = b^n_{111} - v^n_{211} + 2v^n_{112}$ , (ii) $v^{n+1}_{111} = v^n_{111} + 2b^n_{112} - 2b^n_{211}$ ,

(ii) $b^{n+1}_{112} = b^n_{112} - 2v^n_{212} + v^n_{122} - v^n_{313}$ , (iv) $v^{n+1}_{112} = v^n_{112} + b^n_{122} - b^n_{212}$ ,

(i) $b^{n+1}_{113} = b^n_{113} - v^n_{213} + v^n_{123}$ , (iii) $v^{n+1}_{113} = v^n_{113} + b^n_{123} - b^n_{213}$ ,

(iv) $b^{n+1}_{122} = b^n_{122} - 3v^n_{222} - 2v^n_{323}$ , (ii) $v^{n+1}_{122} = v^n_{122}$ ,

(iii) $b^{n+1}_{123} = b^n_{123} - 2v^n_{223} - v^n_{333}$ , (i) $v^{n+1}_{123} = v^n_{123}$ ,

(iv) $b^{n+1}_{133} = b^n_{133} - v^n_{233}$ , (ii) $v^{n+1}_{133} = v^n_{133}$ ,

(ii) $b^{n+1}_{211} = b^n_{211} - 3v^n_{311} - 2v^n_{313}$ , (iv) $v^{n+1}_{211} = v^n_{211}$ ,

(iv) $b^{n+1}_{212} = b^n_{212} - 2v^n_{112} + v^n_{211} - v^n_{323}$ , (ii) $v^{n+1}_{212} = v^n_{212} + b^n_{211} - b^n_{112}$ ,

(iii) $b^{n+1}_{213} = b^n_{213} - 2v^n_{113} - v^n_{333}$ , (i) $v^{n+1}_{213} = v^n_{213}$ ,

(ii) $b^{n+1}_{222} = b^n_{222} - v^n_{122} + 2v^n_{212}$ , (iv) $v^{n+1}_{222} = v^n_{222} + 2b^n_{212} - 2b^n_{122}$ ,

(i) $b^{n+1}_{223} = b^n_{223} - v^n_{123} + v^n_{213}$ , (ii) $v^{n+1}_{223} = v^n_{223} + b^n_{213} - b^n_{123}$ ,

(ii) $b^{n+1}_{233} = b^n_{233} - v^n_{133}$ , (iv) $v^{n+1}_{233} = v^n_{233}$ ,

(i) $b^{n+1}_{311} = b^n_{311} + 2v^n_{312}$ , (iii) $v^{n+1}_{311} = v^n_{311} + 2b^n_{312} - 2b^n_{213}$ ,

(iii) $b_{312}^{n+1} = b_{312}^{n} + v_{311}^{n} + v_{322}^{n}$ ,

(iv) $b_{313}^{n+1} = b_{313}^{n} + v_{323}^{n}$ ,

(i) $b_{322}^{n+1} = b_{322}^{n} + 2v_{312}^{n}$ ,

(ii) $b_{323}^{n+1} = b_{323}^{n} + v_{313}^{n}$ ,

(i) $b_{333}^{n+1} = b_{333}^{n}$ ,

(i) $v_{312}^{n+1} = v_{312}^{n} + b_{311}^{n} - b_{113}^{n} + b_{322}^{n} - b_{223}^{n}$,

(ii) $v_{313}^{n+1} = v_{313}^{n} + b_{323}^{n} - b_{233}^{n}$ ,

(iii) $v_{322}^{n+1} = v_{322}^{n} + 2b_{312}^{n} - 2b_{123}^{n}$ ,

(iv) $v_{323}^{n+1} = v_{323}^{n} + b_{313}^{n} - b_{133}^{n}$ ,

(iii) $v_{333}^{n+1} = v_{333}^{n}$ .

When considering the equations (i) and (iii) we note that they all effect terms of the form $b_{\iota\kappa 3}^{n}$ , $b_{3\iota\kappa}^{n}$ , $b_{333}^{n}$ , $v_{\iota\kappa 3}^{n}$ , $v_{3\iota\kappa}^{n}$ and $v_{333}^{n}$ . Now if we assume that the disturbances, when $n = 0$, to the vector fields, are such that $b_{\iota\kappa}^{o}$ , $v_{\iota\kappa}^{o}$ do not vary near D appreciably with z, and that the 0z component of $\underset{\sim}{b}^{o}, \underset{\sim}{v}^{o}$ is sufficiently close to the zero constant, that, for each, the double gradient vanishes, then all of the above terms, including the constant ones, are zero when $n = 0$ except $b_{333}^{o}$ and $v_{333}^{o}$ . However, $b_{333}^{o}$ is steady and does not affect the other equations, and so may be conveniently set at zero. Then these equations reduce to :

for (i): $b_{113}^{1}$ , $b_{223}^{1}$ , $b_{311}^{1}$ , $b_{322}^{1}$, $b_{333}^{1}$, $v_{123}^{1}$ , $v_{213}^{1}$ , $v_{312}^{1}$ are all zero, and

for (iii): $b_{123}^{1} = b_{213}^{1} = -v_{333}^{o}$ , $v_{333}^{1} = v_{333}^{o}$ ; $b_{312}^{1}$ , $v_{113}^{1}$ , $v_{223}^{1}$ , $v_{311}^{1}$ , $v_{322}^{1}$ are all zero.

Thus, in the case of (i), the same starting conditions apply for $n = 1$ as for $n = 0$, hence by induction the terms are zero for all n.

In the case of (iii), if $v_{333}^{o}$ is finite, then $b_{123}^{n}$ is unstable, since $b_{123}^{2} = b_{123}^{1} - v_{333}^{1} = -2v_{333}^{o}$ , etc., and similarly for $b_{213}^{n}$ . If we are interested in a stable solution, therefore, it is necessary to prescribe a zero value for $v_{333}^{o}$ . Then all terms in equations (i) and (iii) are zero for all n.

This leaves only equations (ii). In these, $b_{222}^{n}$ only appears in the equation for $b_{222}^{n+1}$ , and not elsewhere, so that once the other equations have been solved, and $v_{212}^{n}$ is known for all n, then $b_{222}^{n}$ may be deduced for all n : whether it is stable or not will depend on the other equations, and so this equation may be omitted from our investigation. Further, unless $v_{133}^{n}$ (steady) is zero, then $b^n_{233}$ is unstable, and so $v^o_{133}$ will be set at zero, and then $b^n_{233}$ is steady. Then, if $c_n \equiv b^n_{323} + v^n_{313}$, we see that $c_{n+1} - 2c_n + \text{constant} = 0$. This first order recurrence equation with independent term may be transformed into the second order recurrence equation without independent term :-

$c_{n+2} - 3c_{n+1} + 2c_n = 0$, with solution, to suit starting conditions $c_o, c_1$ as given : $c_n = 2c_o - c_1 + (c_1 - c_o)2^n$. This is unstable unless $c_1 = c_o$, in which case $c_n = c_o$, steady.

So for possible stability we require that $b^n_{323} + v^n_{313}$ is steady for all n. The equation for $b^{n+1}_{323}$ then shows, not only that $b^n_{323}$ is steady for all n, but also that $v^n_{313}$ is steady at zero for all n ; the equation for $v^{n+1}_{313}$ then shows that $b^n_{323} = b^n_{233}$, i.e. $b^n_{233}$ is also steady for all n. Thus, omitting equations for steady $b^n_{ijk}$, $v^n_{ijk}$, we are left with the following set :

$$b^{n+1}_{112} = b^n_{112} - 2w^n_{212} ,$$
$$b^{n+1}_{211} = b^n_{211} - 3v^n_{111} ,$$
$$v^{n+1}_{111} = v^n_{111} + 2b^n_{112} - 2b^n_{211} ,$$
$$w^{n+1}_{212} = w^n_{212} - b^n_{112} + b^n_{211} ,$$

where $w^n_{212} = v^n_{212} - \frac{1}{2}v^n_{122}$, introduced to eliminate the steady $v^n_{122}$. Group (ii) of equations, in fact, contain these four, the $v^n_{122}$ and $v^n_{133}$ equations, and a separate independent sub-group. The above four equations may be solved in the usual way by trial: $b^n_{112} = Px^n$, $b^n_{211} = Qx^n$, etc. This approach leads to the characteristic determinant equation for x :

$$\begin{vmatrix} (x-1) & 0 & 0 & 2 \\ 0 & (x-1) & 3 & 0 \\ -2 & 2 & (x-1) & 0 \\ 1 & -1 & 0 & (x-1) \end{vmatrix} = 0$$

Of this, the four roots are $x = 1,1,1 \pm 2\sqrt{2}$ . Thus, the equations are degenerate, so that the solution for $b^n_{112}$ is : -

$$b^n_{112} = P_1(1)^n + nP_2(1)^n + P_3(1 + \sqrt{8})^n + P_4(1 - \sqrt{8})^n ,$$

with a similar type of solution for $b^n_{211}$ , etc.

It is at once clear that, unless the $P_2$, $P_3$ and $P_4$ are zero, $b^n_{112}$ is unstable : if they are all zero, then $b^n_{112}$ is not only stable but steady. The same applies, of course, to $b^n_{211}$ ($Q_2$, $Q_3$ and $Q_4$ zero, otherwise unstable) and similarly for $v^n_{111}$, $w^n_{212}$ : in fact, if all four variables are steady, the equations, taken in order, show that $w^n_{212} = 0$ , $v^n_{111} = 0$ , $b^n_{211} = b^n_{112}$ . Thus, $v^n_{122} = 2v^n_{212}$ , both steady from the original $v^n_{212}$ equation. And so for (steady) stability we must prescribe :

$b^o_{211} = b^o_{112}$ (arbitrary), $v^o_{111} = 0$, $v^o_{122} = 2v^o_{212}$ (arbitrary),

and then $b^n_{222}$ is also steady and $b^o_{222}$ may be arbitrary.

If arbitrary values are given to the relevant $b^o_{\iota\kappa\omega}$ , $v^o_{\iota\kappa\omega}$ which do not agree with all the above conditions, the system is unstable. And, having been able to solve the equations without resource to mechanical means, opportunity will be taken here to obtain the time scale. This is taken to be the period that elapses from time $t = 0$ ($n = 0$) up to the moment when at least one of the variables has increased by two orders of ten from its starting value. Therefore, if we make the starting values of the order $10^{-1}$, we calculate the time taken for one variable to reach a value, order $10^1$ .

The starting values determine the $P_1$, ..., $P_4$, etc. It is only necessary to obtain these for one variable, say $b^n_{112}$ . Direct manipulation of the equations provides $b^1_{112}$, $b^2_{112}$ and $b^3_{112}$ as functions of $b^o_{112}$, $b^o_{211}$ , etc. Then the resulting four simultaneous equations for $P_1$, ..., $P_4$ are solved. For the other variables, this work need not be repeated, in fact once $b^n_{112}$ is known, the first equation gives $w^n_{212}$, then the last equation gives $b^n_{211}$ and the second equation gives $v^n_{111}$ .

Prescribing impulsive disturbances : zero for $b^o_{112}$ and $b^o_{211}$ as previously indicated, and $v^o_{111} = 0.1$ , $v^o_{212} = -0.1$, $v^o_{122} = v^n_{122} = 0.2$, so that $w^o_{212} = -0.2$, the detailed solutions are :-

$$b^n_{112} = (64)^{-1} \{ 14.4n + (0.7)\sqrt{8}(1+\sqrt{8})^n - (0.7)\sqrt{8}(1-\sqrt{8})^n \} ,$$
$$b^n_{211} = (64)^{-1} \{ 14.4n - (2.1)\sqrt{8}(1+\sqrt{8})^n + (2.1)\sqrt{8}(1-\sqrt{8})^n \} ,$$
$$v^n_{111} = (8)^{-1} \{ -0.6 + (0.7)(1+\sqrt{8})^n + (0.7)(1-\sqrt{8})^n \} ,$$
$$v^n_{212} = (16)^{-1} \{ -0.2 - (0.7)(1+\sqrt{8})^n - (0.7)(1-\sqrt{8})^n \} .$$

In these expressions, the terms whose magnitude increases most rapidly with $n$ are the middle terms of $b^n_{211}$ and of $v^n_{111}$, and both these variables are greater then ten when $n \geqslant 4$.

Thus the time-scale for instability in this case is $4 \times \Delta t = 4$ sec. Incidentally, the solution for $b^n_{222}$ is :

$$b^n_{222} = (8)^{-1} \{ -1.8n - (0.7)\sqrt{8}(1+\sqrt{8})^n + (0.7)\sqrt{8}(1-\sqrt{8})^n \} .$$

Similar analysis will apply, of course, to the group of equations marked (iv). Since, under these disturbances, the system is shown to be unstable there seems to be little point in investigating equations (i) and (iii). However, a short discussion of their stability under the conditions of this paragraph will be given at the end of the next paragraph.

## §6.2 Constant conductivity and variable pressure

The first equation in §6.1, with the same assumptions regarding $\underset{\sim}{b}$, $\underset{\sim}{v}$, but with the last term of the equation in §2.1 retained, becomes, treating $\gamma$ as a constant :-

$$\partial b_{ijk}/\partial t = -v_{\ell<j}b_{ik>\ell} - v_{\ell jk}b_{i\ell} + v_{i\ell}\, b_{\ell jk} + v_{i\ell<j}b_{\ell k>} - v_{\ell\ell}\, b_{ijk} - v_{\ell\ell<j}\, b_{ik>} + \gamma b_{ijk\ell\ell} ,$$

which in recurrence form, is :

$$b^{n+1}_{ijk} = b^n_{ijk} - v^n_{\ell<j}b^n_{ik>\ell} - v^n_{\ell jk}b^n_{i\ell} + v^n_{i\ell}\, b^n_{\ell jk} + v^n_{i\ell<j}b^n_{\ell k>} - v^n_{\ell\ell}\, b^n_{ijk} - v^n_{\ell\ell<j}b^n_{ik>} + \gamma b^n_{ijk\ell\ell} .$$

Similarly, if the pressure varies, and, with it, the density (§2.4) so that the last term of the equation in §2.2 is retained, then, with $\underset{\sim}{b} = \underset{\sim}{v} \quad \underset{\sim}{0}$ near D, the second equation in §6.1 becomes :-

$$\partial v_{ijk}/\partial t = -v_{\ell<k}v_{ij>\ell} - v_{i\ell}v_{\ell jk} + \lambda_{<j}b_{[i\ell]}b_{\ell k>} + \lambda b_{\ell<k}b_{[i\ell]j>} + \lambda b_{[i\ell]}b_{\ell jk}$$
$$- \tau_{jk}p_i - \tau_{<j}p_{ik>} - \tau p_{ijk} ,$$

which, in recurrence form is :-

$$v^{n+1}_{ijk} = v^n_{ijk} - v^n_{\ell<k}\, v^n_{ij>\ell} - v^n_{i\ell}\, v^n_{\ell jk} + \lambda^n_{<j} b^n_{[i\ell]}\, b^n_{\ell k>} + \lambda^n b^n_{\ell<k}\, b^n_{[i\ell]j>} + \lambda^n b^n_{[i\ell]}\, b^n_{\ell jk}$$
$$- \tau^n_{jk}p^n_i - \tau^n_{<j}p^n_{ik>} - \tau^n p^n_{ijk} .$$

The effect of perturbations in the $b^o_{ij}$, $v^o_{ij}$ and in the $\gamma\, b^o_{ijkk}$ were investigated in Chapter V. Therefore, in this paragraph, these variables will be set at their unperturbed values, zero or unity (§6.1). And here an important point arises : it will be noted, from the recurrence equations given in detail in §5.2, that, even if we leave $b^o_{12}$ and $b^o_{21}$ unperturbed at 1 and $v^o_{ik}$ undisturbed at zero, and also make $\theta = \phi = 0$ , the equations are unstable. In particular, to the second order in $\alpha^2$, $\beta^2$, and for finite n, $v^n_{12} = v^n_{21} = n\alpha\beta$, $b^n_{12} = 1 - \beta^2 n!$ , $b^n_{21} = 1 - \alpha^2 n!$, and $q^n = 1 - \frac{1}{2}n(n-1)(\alpha^2 - \beta^2)$.

The effect of a non-zero $p^n_i$ is, therefore, to render the equations unstable if $b^o_{ij}$ , $v^o_{ij}$ and $b^o_{ijkk}$ are undisturbed. But if, in addition, $p^n_i = 0$, i.e. $\alpha = \beta = 0$, then the $b^n_{ij}$ , $v^n_{ij}$ and $q^n$ remain steady. Therefore, in order to investigate the field, due to impulsive or other disturbances in the second differentials, since $v^n_{ij}$ , $b^n_{ij}$ , etc. appear in the equations it is necessary to insist that $p^n_i$ is zero, otherwise the equations will be unstable due to the $v^n_{ij}$ and the $b^n_{ij}$ .

By making the $p^n_i$ zero, and leaving the $b^o_{ij}$ , $v^o_{ij}$ and $b^o_{ijkk}$ unperturbed, then $b^n_{12} = b^n_{21} = 1$, all n, and all other $b^n_{ij}$ , all $b^n_{[ij]}$ , and all $v^n_{ij}$ are zero, all n.

Also, as in §5.2, we have $\tau^n \simeq 10^8 (p^n)^{-1}$ , $\tau^n_i \simeq -10^8 (p^n)^{-2}\, p$ , and now $\tau^n_{ij} \simeq 10^8 (p^n)^{-3} (2p^n_i\, p^n_j - p^n p_{ij})$, $\lambda \simeq 10^{-12} (p^n)^{-1}$ , and now $\lambda^n_i \simeq -10^{-12} (p^n)^{-2}\, p^n_i$ . And so, with $\tau^n_i$ also zero, due to its dependence in $p^n_i$ , both terms containing $p^n_{ij}$ disappear, which means that any disturbance values prescribed for the $p^n_{ij}$ are ineffective, only those prescribed for the $p^n_{ijk}$ will affect the equations. As in §5.2 we let $(p^n)^{-1} = 10^{12} q^n$ , and now the $q^n$ equation becomes :

$q^{n+1} = q^n(1 + v_{KK}) = q^n$, so that $q^n = q^o$, steady, and may be equated to 1 as in §5.2. Then $\lambda^n = 1$, steady, $\lambda_i^n$ disappears from the equations because of the factor $b^n_{[ij]}$, and it is convenient to write $r^n_{ijk}$ for $\tau^n p^n_{ijk}$, $= 10^{20} q^n p^n_{ijk} = 10^{20} p^n_{ijk}$. The equations for $b^{n+1}_{ijk}$, $v^{n+1}_{ijk}$, therefore, reduce to (with $s^n_{ijk} \equiv \gamma b^n_{ijk\ell\ell}$):-

$$b^{n+1}_{ijk} = b^n_{ijk} - v^n_{\ell jk} b^n_{i\ell} + v^n_{i\ell<j} b^n_{\ell k>} - v^n_{\ell\ell<j} b^n_{ik>} + s^n_{ijk} ,$$

$$v^{n+1}_{ijk} = v^n_{ijk} + b^n_{\ell<k} b^n_{[i\ell]j>} - r^n_{ijk} .$$

Now, any one term $s^n_{ijk}$, that is, with prescribed numbers allotted to i,j,k only appears once, in the equation for the corresponding $b^{n+1}_{ijk}$. The same feature applies to the $r^n_{ijk}$. Therefore the division of the equations into groups, as in §6.1, is still applicable. A further point is that the $s^n_{ijk}$ and the $r^n_{ijk}$ must be prescribed for all n, for the same reason as discussed in §5.2 for the $b^n_{ijkk}$ case - the undisturbed values in each case being zero. The problem now arises, what sort of disturbance values to give. If we treat them both as impulsive, that is, make the $s^o_{ijk}$ and the $r^o_{ijk}$ finite but keep the $s^n_{ijk}$ and the $r^n_{ijk}$ at zero when $n \neq 0$, then the above equations for $b^{n+1}_{ijk}$, $v^{n+1}_{ijk}$ merely revert to those of §6.1 for $n > 0$, and we have shown that, apart from somewhat artificial starting conditions, these equations are unstable.

The only possible alternative plan is to treat the $s^n_{ijk}$ and the $r^n_{ijk}$ as steady for all n, otherwise the number of possible solutions becomes infinite. Thus the superscript n may be removed from the s and r tensors, which become constants of the equations. It is worth remarking at this stage that:

$$r_{ijk} = r_{jik} = r_{jki} \text{ , whereas } s_{ijk} = s_{ikj} .$$

The group (ii) of equations now become, in detail, and including that for $v^n_{122}$, no longer steady, but still excluding that for $b^n_{222}$ and those for $b^n_{ijk}$, $v^n_{ijk}$ where any suffix is 3 for the reasons discussed in §6.1 :-

$$b^{n+1}_{112} = b^n_{112} - 2v^n_{212} + v^n_{122} + s_{112} \; ,$$
$$b^{n+1}_{211} = b^n_{211} - 3v^n_{111} + s_{211} \, ,$$
$$v^{n+1}_{111} = v^n_{111} + 2b^n_{112} - 2b^n_{211} - r_{111} \, ,$$
$$v^{n+1}_{122} = v^n_{122} - r_{122} \, ,$$
$$v^{n+1}_{212} = v^n_{212} - b^n_{112} + b^n_{211} - r_{122} \, .$$

Clearly, unless $r_{122} = 0$, $v^n_{122}$ is unstable and therefore so are the other variables, interdependant. And so we must set $r_{122} = 0$, and then $v^n_{122}$ is steady as before. Then :

$$v^{n+1}_{111} + 2v^{n+1}_{212} = v^n_{111} + 2v^n_{212} - r_{111} \; ,$$

and so $v^n_{111} + 2v^n_{212}$ is unstable unless $r_{111}$ is also zero. If we now write:

$$v^n_{111} = x^n_{111} + \tfrac{1}{3}s_{211}, \; v^n_{212} = y^n_{212} + \tfrac{1}{2}v^n_{122} + \tfrac{1}{2}s_{112}, \; r_{111} = 0,$$ we obtain :

$$b^{n+1}_{112} = b^n_{112} - 2y^n_{212} \; ,$$
$$b^{n+1}_{211} = b^n_{211} - 3x^n_{111} \; ,$$
$$x^{n+1}_{111} = x^n_{111} + 2b^n_{112} - 2b^n_{211} \; ,$$
$$y^{n+1}_{212} = y^n_{212} - b^n_{112} + b^n_{211} \; ;$$

these are a similar set of equations to those obtained in §6.1, shown to be unstable except for the adjusted starting conditions (causing steadiness):-

$$b^0_{211} = b^0_{112} \text{ (arbitrary)}, \quad s_{211} = 3v^0_{111} \text{ (arbitrary)}, \quad s_{112} = 2v^0_{212} - v^0_{122} \text{ (both arbitrary)}$$

$r_{111} = r_{122} = 0$. The last condition, affecting the pressure, need cause no surprise, because a differential pressure field, if unbalanced by applied or magnetic forces, is, from the Navier Stokes equation, bound to have an effect on the medium acceleration. Also, clearly, for $b^n_{222}$ stable, we need $s_{222} = 0$, and then $b^n_{222}$ is steady and arbitrary.

Referring now to the remaining equations in group (ii), that is, those in which one of the subscripts of $b^n_{ijk}$, $v^n_{ijk}$ is equal to 3, and allowing disturbances in these variables at time 0, we may make the following deductions from the detailed equations in §6.1, each of which now, of course, contains a term $+s_{ijk}$ (for the $b^{n+1}_{ijk}$ equations) or $-r_{ijk}$ (for the

$v_{ijk}^{n+1}$ equations) ;-

For stability ; from the $v_{133}^{n+1}$ equation, $r_{133} = 0$. Then $v_{133}^{n}$ is steady. From the $b_{233}^{n+1}$ equation, $v_{133}^{o} = s_{233}$ , then $b_{233}^{n}$ is steady. Adding the $b_{323}^{n+1}$ and the $v_{313}^{n+1}$ equations, $b_{233}^{o} = s_{323}$ , and we also need :- $b_{323}^{o} + v_{313}^{o} = 0$. Then $b_{323}^{n} = - v_{313}^{n}$ . Substracting them, $b_{323}^{n} - v_{313}^{n} = 2b_{233}^{o}$ , and so $b_{323}^{n} = -v_{313}^{n} = b_{233}^{o}$ , steady. Thus, the only stable solution is one in which all variables are steady. Of course, a non-zero $v_{313}^{o}$ has an effect on the group (ii) equations previously considered: the one for $b_{112}^{n+1}$ now has an added term $-v_{313}^{o}$ and that for $b_{211}^{n+1}$ , $-2v_{313}^{o}$. This does not affect the analysis, but does affect the stability conditions, which are, in full : for group (ii):- $b_{112}^{o} = b_{212}^{o}$ , $v_{313}^{o} = -b_{233}^{o} = -b_{323}^{o}$ , $s_{111} = 0$, $s_{112} = 2v_{212}^{o} - v_{122}^{o} - b_{233}^{o}$ , $s_{211} = 3v_{111}^{o} - 2b_{233}^{o}$ , $s_{233} = v_{133}^{o}$ , $s_{323} = b_{233}^{o}$ , $r_{111} = r_{122} = 0$ : if these conditions are met, all the relevant variables are steady; one of $b_{112}^{o}$ , $b_{212}^{o}$ , one of $b_{233}^{o}$ , $b_{323}^{o}$ , and $b_{222}^{o}$ , and all relevant $v_{ijk}^{o}$ except $v_{313}^{o}$ are arbitrary.

Similar conditions apply to group (iv) with interchange of suffixes 1,2.

Referring now to the equations of group (iii), after some cumbersome and intricate but not difficult work with seven interconnected equations, the following conclusion is reached : stability is only possible if the following equalities hold : -

$r_{113} = -r_{223} = b_{123}^{o} - b_{213}^{o} + v_{113}^{o} - v_{223}^{o} + \frac{1}{2}s_{123} - \frac{1}{2}s_{213}$ ; $r_{333} = 0$ ;

$s_{123} + s_{213} = 2(v_{113}^{o} + v_{223}^{o} + v_{333}^{o})$, and $s_{312} = b_{123}^{o} + b_{213}^{o} - 2b_{312}^{o} - v_{311}^{o} - v_{322}^{o}$, otherwise all the relevant $b_{ijk}^{o}$ , $v_{ijk}^{o}$ and one of the pair $s_{123}$ , $s_{213}$ are arbitrary. If these conditions hold, then, writing

$2K_1 = v_{113}^{o} - v_{223}^{o} + \frac{1}{2}s_{123} - \frac{1}{2}s_{213}$ , $2K_2 = b_{123}^{o} + b_{213}^{o} - 2b_{312}^{o}$ , the solution is :-

$$(b_{123}^{n} - b_{123}^{o}) = -(b_{213}^{n} - b_{213}^{o}) = -(v_{113}^{n} - v_{113}^{o}) = (v_{223}^{n} - v_{223}^{o})$$
$$= K_1 \{1 - (-1)^{n}\} ;$$

$(b^n_{312} - b^o_{312}) = K_2\{1 - (-1)^n\}$ ; $(v^n_{311} - v^o_{311}) = -(K_1 + K_2)\{1 - (-1)^n\}$,
$(v^n_{322} - v^o_{322}) = (K_1 - K_2)\{1 - (-1)^n\}$ and $(v^n_{333} - v^o_{333}) = 0$

Unless these conditions are exactly satisfied, the solution of the equations include terms containing the factor $3^n$ . This shows that the instability leads to an increase in the magnitude of the relevant variable by order $10^2$ in 4 secs.

The group (i) of equations may be dealt with quickly. Omitting details, it is found necessary for stability that : -
$r_{123} = 0$ ; $s_{333} = 0$; $s_{113} + s_{223} = 0$; $v^o_{123} = v^o_{213}$ ; and $s_{311} = s_{322} = b^o_{113} + b^o_{223} - b^o_{311} - b^o_{322} - 2v^o_{312}$ . This means that all the relevant $b^o_{ijk}$, and all the relevant $v^o_{ijk}$ , except one of the pair $v^o_{123}$ , $v^o_{213}$ , are arbitrary ; also one of the pair $s_{113}$, $s_{223}$ is arbitrary. Under these conditions, $b^n_{113}$ , $b^n_{223}$ , $b^n_{333}$ , $v^n_{123}$ and $v^n_{213}$ are steady, and, if $2K_3 = b^o_{113} + b^o_{223} - b^o_{311} - b^o_{322}$ , then $b^n_{311}$ , $b^n_{322}$ and $v^n_{312}$ oscillate as follows :-

$$(b^n_{311} - b^o_{311}) = (b^n_{322} - b^o_{322}) = -(v^n_{312} - v^o_{312}) = K_3\{1 - (-1)^n\} .$$

If these conditions are not satisfied, instability occurs owing to the possible presence of terms proportionate to $3^n$ , so that the instability time is again 4 secs.

A summary of the findings of this paragraph follows :-
$b^o_{ijk}$ may all be arbitrary except for the connections :-
$b^o_{112} = b^o_{211}$ ; $b^o_{122} = b^o_{212}$ , $b^o_{133} = b^o_{313}$ , $b^o_{233} = b^o_{323}$ .
$v^o_{ijk}$ may all be arbitrary except for the connections :-
$v^o_{123} = v^o_{213}$ , $v^o_{313} = -b^o_{233}$ , $v^o_{323} = -b^o_{313}$
None of the $s_{ijk}, \equiv \gamma b^o_{ijk\ell\ell}$, are arbitrary, except for one member of the sums :-

$$s_{113} + s_{223} = 0 \; ; \; s_{123} + s_{213} = 2(v^o_{113} + v^o_{223} + v^o_{323}).$$

Otherwise $s_{111} = s_{222} = s_{333} = 0,$

$$\left\{\begin{array}{l} s_{112} = 2v^{\circ}_{212} - v^{\circ}_{122} - b^{\circ}_{233} \\ s_{212} = 2v^{\circ}_{112} - v^{\circ}_{211} - b^{\circ}_{133} \end{array}\right\} ; \left\{\begin{array}{l} s_{122} = 3v^{\circ}_{222} - 2b^{\circ}_{133} \\ s_{211} = 3v^{\circ}_{111} - 2b^{\circ}_{233} \end{array}\right\} ; \left\{\begin{array}{l} s_{133} = v^{\circ}_{233} \\ s_{233} = v^{\circ}_{133} \end{array}\right\} ; \left\{\begin{array}{l} s_{313} = b^{\circ}_{133} \\ s_{323} = b^{\circ}_{233} \end{array}\right\} ;$$

$$s_{311} = s_{322} = b^{\circ}_{113} + b^{\circ}_{223} - b^{\circ}_{311} - b^{\circ}_{322} - 2v^{\circ}_{312} ;$$

$$s_{312} = b^{\circ}_{123} + b^{\circ}_{213} - 2b^{\circ}_{312} - v^{\circ}_{311} - v^{\circ}_{322} .$$

None of the $r_{ijk}$ , $\backsim p_{ijk}$ , are arbitrary :-

$$r_{111} = r_{112} = r_{122} = r_{123} = r_{133} = r_{222} = r_{233} = r_{333} = 0 ;$$

$$r_{113} = -r_{223} = b^{\circ}_{123} - b^{\circ}_{213} + v^{\circ}_{113} - v^{\circ}_{223} + \tfrac{1}{2}s_{123} - \tfrac{1}{2}s_{213} .$$

It is now of interest to note that, reverting to the case of infinite conductivity and constant pressure, the application of zero values to all $s_{ijk}$, $r_{ijk}$ in the above conditions gives again the conditions of §6.1, with the additional conditions for second differentials, either of $Oz$ components of $\underset{\sim}{v}$ , $\underset{\sim}{b}$ , or containing at least one operator $\partial/\partial z$ : stability is possible if, in addition to the conditions of §6.1 :

$$b^{\circ}_{133} = b^{\circ}_{233} = b^{\circ}_{313} = b^{\circ}_{323} = 0, \text{ remaining } b^{\circ}_{ijk} \text{ arbitrary} ; \quad v^{\circ}_{133} = v^{\circ}_{233} = v^{\circ}_{313} = v^{\circ}_{323} = 0;$$

$$v^{\circ}_{113} = -\tfrac{1}{2}b^{\circ}_{123} + \tfrac{1}{2}b^{\circ}_{213} - \tfrac{1}{2}v^{\circ}_{333} ; \; v^{\circ}_{223} = \tfrac{1}{2}b^{\circ}_{123} - \tfrac{1}{2}b^{\circ}_{213} - \tfrac{1}{2}v^{\circ}_{333} ; \; v^{\circ}_{123} = v^{\circ}_{213} ;$$

$$v^{\circ}_{311} + v^{\circ}_{322} = b^{\circ}_{123} + b^{\circ}_{213} - 2b^{\circ}_{312} ; \; v^{\circ}_{312} = \tfrac{1}{2}(b^{\circ}_{113} + b^{\circ}_{223} - b^{\circ}_{311} - b^{\circ}_{322}) .$$

Otherwise $v^{\circ}_{ijk}$ are arbitrary (this means only $v^{\circ}_{333}$ , and one each of the pairs $v^{\circ}_{123}$ , $v^{\circ}_{213}$ and $v^{\circ}_{311}$ , $v^{\circ}_{322}$ ).

The above analysis considers steady values for $\gamma b^{n}_{ijk\ell\ell}$ , $p^{n}_{ijk}$ . If they are to vary with n, then, as already explained, they must be prescribed. Clearly a stable solution could always be obtained by suitable and arbitrary choice of these numbers as required : however, this would be artificial and unlikely to resemble true physical behaviour. But since there are an infinity of random choices for these numbers, it is not feasible to enquire into their effect on stability.

Generally, under the conditions of this paragraph, if the system is unstable in any way the instability period is approximately 4 secs.

The variables in group (ii), (iv), containing a suffix 3, if unstable, have values approximately proportional to $2^n$, leading to an instability time of 7 secs. However, if unstable, one of them ($v^n_{313}$ or $v^n_{323}$) causes a faster instability in the remaining variables of the group.

§6.3 Variable Conductivity and Pressure.

If the conductivity varies, it is advisable to follow the reasoning and analysis of §5.3, taking up at the point where it was shown that we may set $\gamma_i = v_{2i}$, and may follow the behaviour of the variables at a point near D where $\underset{\sim}{v}$(small) is co-parallel $\underset{\sim}{j}$. Then :

$$\gamma_{ij} = v_{2ij} ,$$

and the first equation in §6.2 will have the extra terms $v_{2jk}b_{i\ell\ell} + v_{2<j}\, b_{ik>\ell\ell}$. However, the effects of non-zero $v^o_{ij}$, under conditions of variable conductivity and pressure, were investigated in §5.3 : therefore, as explained in §6.2, it is advisable to set the $b^n_{ij\ell\ell}$, $p^n_i$ and $v^o_{ij}$ at zero, and the $b^o_{ij}$ at unperturbed values, in order to investigate the second differential effect. Thus, the terms $v_{2<j}\, b_{ik>\ell\ell}$ may be omitted. With $q^n = 1$ as before, the reduced occurrence equations of §6.2 become :-

$$b^{n+1}_{ijk} = b^n_{ijk} - v^n_{\ell jk}\, b^n_{i\ell} + v^n_{i\ell<j}\, b^n_{\ell k>} - v^n_{\ell\ell<j}\, b^n_{ik>} + v^n_{2jk}\, b^n_{i\ell\ell} + \gamma^n\, b^n_{ijk\ell\ell},$$

$$v^{n+1}_{ijk} = v^n_{ijk} + b^n_{\ell<k}\, b^n_{i\ell>j} - r^n_{ijk} .$$

The presence of the term $v^n_{2jk}$ in the first equation has the effect that the equations may no longer be separated into independent groups : in fact they are all interconnected. Also, the equations are now non-linear and thus are not amenable to analysis. Therefore it is advisable to make assumptions and arguments similar to those in Chapter V, leading to the reduction of the problem to one in two dimensions only. Since, using the model of Chapter IV, $b^n_{\iota\kappa}$ is steady and in fact equal to the alternator $\epsilon_{\iota\kappa}$, $\equiv 1 - \delta_{\iota\kappa}$, the above equations become :

$$b^{n+1}_{\iota\kappa\omega} = b^n_{\iota\kappa\omega} - v^n_{\nu\kappa\omega}\epsilon_{\iota\nu} + v^n_{\iota\nu<\kappa}\epsilon_{\nu\omega>} - v^n_{\nu\nu<\kappa}\epsilon_{\iota\omega>} + v^n_{2\kappa\omega} b^n_{\iota\nu\nu} + \gamma^n b^n_{\iota\kappa\omega\nu\nu},$$

$$v^{n+1}_{\iota\kappa\omega} = v^n_{\iota\kappa\omega} + \epsilon_{\nu<\omega} b^n_{\iota\nu>\kappa} - r^n_{\iota\kappa\omega}, \text{ where } \omega,\nu = 1(1)2.$$

These equations expand into the twelve equations :-

$$b^{n+1}_{111} = b^n_{111} + v^n_{211}(b^n_{1\nu\nu} - 1) + 2v^n_{112} + \gamma^n b^n_{111\nu\nu}, \quad v^{n+1}_{111} = v^n_{111} + 2b^n_{112} - 2b^n_{211} - r^n_{111},$$

$$b^{n+1}_{112} = b^n_{112} + v^n_{212}(b^n_{1\nu\nu} - 2) + v^n_{122} + \gamma^n b^n_{112\nu\nu}, \quad v^{n+1}_{112} = v^n_{112} + b^n_{122} - b^n_{212} - r^n_{112},$$

$$b^{n+1}_{122} = b^n_{122} + v^n_{222}(b^n_{1\nu\nu} - 3) + \gamma^n b^n_{122\nu\nu}, \quad v^{n+1}_{122} = v^n_{122} - r^n_{122},$$

$$b^{n+1}_{211} = b^n_{211} + v^n_{211} b^n_{2\nu\nu} - 3v^n_{111} + \gamma^n b^n_{211\nu\nu}, \quad v^{n+1}_{211} = v^n_{211} - r^n_{112},$$

$$b^{n+1}_{212} = b^n_{212} + v^n_{212} b^n_{2\nu\nu} - 2v^n_{112} + v^n_{211} + \gamma^n b^n_{212\nu\nu}, \quad v^{n+1}_{212} = v^n_{212} + b^n_{211} - b^n_{112} - r^n_{122},$$

$$b^{n+1}_{222} = b^n_{222} + v^n_{222} b^n_{2\nu\nu} - v^n_{122} + 2v^n_{212} + \gamma^n b^n_{222\nu\nu}, \quad v^{n+1}_{222} = v^n_{222} + 2b^n_{212} - 2b^n_{122} - r^n_{222}.$$

In these equations, we have 6 $\gamma^n b^n_{\iota\kappa\omega\nu\nu}$ and the 4 $r^n_{\iota\kappa\omega}$, to which, as explained in §5.2, arbitrary values must be given for all n. If, as indicated in §5.2 $b^n_{11\nu\nu}$ and $b^n_{22\nu\nu}$ can be set at zero for the whole region near D, then near D we may assume that $b^n_{11\omega\nu\nu}$ (and, equivalently, $b^n_{1\omega 1\nu\nu}$) and $b^n_{22\omega\nu\nu}$ (and equivalently $b^n_{2\omega 2\nu\nu}$) are also zero. This leaves only $b^n_{122\nu\nu}$ and $b^n_{211\nu\nu}$, which are, in fact, under these conditions and in this system, the only surviving terms in the two components of $\nabla^2(\nabla^2 \underset{\sim}{b}^n)$. If we ignore the space variation of $\gamma$, the equation in §2.1 can be operated on to provide :

$$\gamma\nabla^2(\nabla^2 \underset{\sim}{b}) = \partial(\nabla^2 \underset{\sim}{b})/\partial t + \{\underset{\sim}{\nabla} \times \nabla^2(\underset{\sim}{v} \times \underset{\sim}{b})\} .$$

In §6.1 it was pointed out that $\nabla^2 \underset{\sim}{b}$ is zero in the likely case of an impulsive or continuous disturbed irrotational current density. However the curl term in the above expression has, for $i^{th}$ component, using the Levi-Civita third order alternator tensor :

$$\epsilon_{ijk} v_{j\ell\ell} b_k + 2\epsilon_{ijk} v_{j\ell} b_{k\ell} + \epsilon_{ijk} v_j b_{k\ell\ell} ;$$

the centre term of this does not disappear when $\underset{\sim}{v}, \underset{\sim}{b}$ are set at zero, but only drops out by virtue of the fact that, as explained earlier in this paragraph, $v_{j\ell} = 0$. Therefore, in order to investigate the effect of varying conductivity, we must accept the possibility that the disturbed current density has rotation which changes with time, so that the term $\partial(\nabla^2 \underset{\sim}{b})/\partial t$ is finite. Therefore the effect of the current density cannot be impulsive, which indicates the advisability of prescribing steady values for $\gamma^n b^n_{122\nu\nu}$ and for $\gamma^n b^n_{211\nu\nu}$. Then the effect of a current density of increasing rotation over a period of a finite number of seconds, as well as of varying conductivity, may be checked.

Considering now the $r^n_{\iota\kappa\omega}$, it is clear that, for possible stability, we must have $r^n_{112} = r^n_{122} = 0$. The remaining terms, $r^n_{111}$, $r^n_{222}$, must therefore be given finite disturbance values so that the effect of varying pressure can be checked simultaneously.

Therefore, we may put $\gamma^n b^n_{122\nu\nu} = \mu$, $\gamma^n b^n_{211\nu\nu} = \zeta$, $v^n_{122} = \xi$, $v^n_{211} = \eta$, $r^n_{111} = \delta$, $r^n_{222} = \varepsilon$, and the remaining $\gamma^n b^n_{\iota\kappa\omega\nu\nu}$, $r^n_{\iota\kappa\omega}$ as zero, all these being steady values. The difference equations become :

$$b^{n+1}_{111} = b^n_{111} + \eta(b^n_{1\nu\nu} - 1) + 2v^n_{112}, \qquad b^{n+1}_{222} = b^n_{222} + v^n_{222} b^n_{2\nu\nu} + 2v^n_{212} - \xi,$$
$$b^{n+1}_{112} = b^n_{112} + v^n_{212}(b^n_{1\nu\nu} - 2) + \xi, \qquad v^{n+1}_{111} = v^n_{111} + 2b^n_{112} - 2b^n_{211} - \delta,$$
$$b^{n+1}_{122} = b^n_{122} + v^n_{222}(b^n_{1\nu\nu} - 3) + \mu, \qquad v^{n+1}_{112} = v^n_{112} + b^n_{122} - b^n_{212},$$
$$b^{n+1}_{211} = b^n_{211} + \eta b^n_{2\nu\nu} - 3v^n_{111} + \zeta, \qquad v^{n+1}_{212} = v^n_{212} + b^n_{211} - b^n_{112},$$
$$b^{n+1}_{212} = b^n_{212} + v^n_{212} b^n_{2\nu\nu} - 2v^n_{112} + \eta, \qquad v^{n+1}_{222} = v^n_{222} + 2b^n_{212} - 2b^n_{122} - \varepsilon.$$

Consideration must now be given to the choice of starting disturbances. There are 16 quantities to which trial disturbance values must be allotted, these being the 6 $b^\circ_{\iota\kappa\omega}$, 4 $v^\circ_{\iota\kappa\omega}$ in the last set of 10 equations, and the 6 constants $\mu, \zeta, \ldots, \varepsilon$. Correctly each should be given a positive value and a negative value (of unequal magnitude, so that cancellation effects, cloaking possible instabilities, may be avoided), and a zero value (so that the effect of only a few of them disturbed may also be investigated. Thus, each quantity must have 3 trial values, for each of which all possible combinations, of the 3 trial values for each of the others, must be investigated. The process will therefore require $3^{16} \approx 2 \times 10^7$ trials.

With the computer time available, judging each trial to require approximately 10 seconds, this large number of trials is clearly impracticable. A suggested way out of this difficulty, which will be followed, is to prescribe the disturbances only to the 6 constants, $\mu,\zeta \ldots \varepsilon$ , This will require $3^6 \approx 700$ trials, which is feasible. It may also be remarked that, with this arrangement and the 6 $b^{o}_{\iota\kappa\omega}$ and 4 remaining $v^{o}_{\iota\kappa\omega}$ set at 0, all of the $b^{1}_{\iota\kappa\omega}$ and 2 of the $v^{1}_{\iota\kappa\omega}$ will be non-zero, and all of the $b^{2}_{\iota\kappa\omega}$ and all of the $v^{2}_{\iota\kappa\omega}$ will be non-zero, meanwhile the disturbed members of the set $\mu,\zeta, \ldots\varepsilon$ remain steady. Thus, the effect of disturbances to the $b^{o}_{\iota\kappa\omega}$, $v^{o}_{\iota\kappa\omega}$ is investigated, to some extent, with a delay of 1 - 2 seconds.

Previously (§6.1) suggested disturbance values for the $v^{o}_{\iota\kappa\omega}$, that is, for the constants $\xi$, $\eta$ were given as of order $10^{-1}$ .

Likely values of $\delta,\varepsilon$, due to external distant causes, that is, of the dimension-free form of (P/D, $\sim 10^{8}$, constant)$\times\, |\nabla\nabla\nabla\, D| \times 10^{20}$ (§6.2) can be obtained from Fig. 5 in the same way as those for $|\nabla\nabla\, B|$ were obtained from Figs. 1,2 and 20 in succession. Details will not be presented here, the result is a likely value of $\sim 10^{-15}$ . As will be seen from the equations for $v^{n}_{111}$, $v^{n}_{222}$ , the effect of steady disturbance values for $\delta,\varepsilon$ is cumulative. Also, if $\delta,\varepsilon$ are given impulse values, i.e. non-zero only for n = 0, the effect is the same as it would be if the disturbance value for $v^{o}_{111}$, $v^{o}_{222}$ were suitably altered, and then the equations checked for stability with $\delta,\varepsilon$ both zero for all n : this investigation is included in any case. Therefore $\delta,\varepsilon$ must be given steady values, and these will be of the order of $10^{-1}$ also, so that their effect can be compared at the same order as $\xi,\eta$ (which would be zero if due only to external, distant causes).

Likely values of $\mu$, $\zeta$ , due to external, distant, causes, may similarly be obtained from Fig. 20, taking the process one step further. Again, details will be omitted; the result turns out to be $\sim 10^{-54}$. The same remarks regarding impulse values and useful order of steady values apply, to these quantities, as to $\delta,\varepsilon$ above. Therefore $\mu,\zeta$ will be set at order $10^{-1}$ .

Perturbations to be tried have been chosen by a random process :

| $\xi$ | $\eta$ | $\mu$ | $\zeta$ | $\delta$ | $\varepsilon$ |
|---|---|---|---|---|---|
| 0.09 | 0.11 | -0.08 | 0.10 | -0.07 | -0.10 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| -0.11 | -0.09 | +0.07 | -0.12 | +0.08 | + 0.12 |

The results will be given in §6.4 .

§6.4 Stability Criteria.

As will be seen from page (xxxiv) of Appendix B "Computational Details", there are no stable cases for the trials made in §6.3. In fact, the system seems to be generally unstable, with a mean instability time of 6½ seconds needed for a variable to reach a value of $\sim 10^3$ of that of the order of the disturbance variables, see (xxxiv").

The same maximum period for test, and stability criterion were used as for § 5(i) - (iii). The parameters $\xi$(i.e. $v^\circ_{122}$ , written as UU), $\eta$(i.e. $v^\circ_{211}$ , written as VV), $\mu$(i.e.$\gamma^\circ b^\circ_{122\nu\nu}$, written as WW), $\zeta$(i.e. $\gamma^\circ b^\circ_{211\nu\nu}$, written as XX), $\delta$(i.e. $10^{20} p^\circ_{111}$, written as YY), and $\varepsilon$(i.e. $10^{20} p^\circ_{222}$ , written as ZZ), are set in the repeat process, instructions 58 - 97, using constants set by instructions 34 - 38. The figures on page (xxxiv) are, in fact, those for a stability criterion of $10^2$ . With criterion $10^1$ , the mean time was 5 , all unstable. As in §5 the criterion was increased to make certain.

The remarks made in §5.4, regarding instructions 42 - 46, apply equally and respectively in this program to instructions 33 - 37. These refer to the values of $\Delta t$, $\mu\rho$, $v \div \gamma$ ($=v\mu\sigma$), maximum period and criterion respectively.

The values after 5 seconds of the variables have been calculated for the first set of disturbances to be put through the machine. These were : all zero except $\varepsilon = -0{\cdot}1$. The results were :-

$$b^5_{111} = b^5_{212} = 3, \quad b^5_{122} = -10.32, \quad v^5_{112} = -5.4, \quad v^5_{222} = 11.3 .$$

## §6.5 Time Scales and Resulting Fields in Stable Cases.

In this paragraph, a summary of findings concerning both first-differential disturbances (§5) and second differential disturbance (§6) will be discussed.

(i) First differential disturbances : it seems clear that the type of $\underset{\sim}{B}$ field and of disturbance, considered as a model, is unstable in all cases tried, with a time scale of approximately 5·7 seconds.

(ii) Second differential disturbances : the inference from the calculations, and from the mechanical solution in the case of variable conductivity and pressure, is that the field is unstable : the only exceptions being for the physically unlikely condition of exact disturbance values as discussed in §6.2. Otherwise, for the type of $\underset{\sim}{B}$ field and of disturbance considered, the time scale is approximately 4 seconds for infinite or constant conductivity, but increases to 5·1 seconds for the case of variable conductivity considered.

(iii) Effect on current density : we can get an idea of the effect of unstable breakdown on the current density from the case discussed at the end of §5.4. We may infer from this that $j_3$ does in fact take part in the unstable growth of variables.

(iv) Effect on particle density : the figures mentioned in §5.4 indicated that density is particularly sensible to instability. Generally, the density tended either to increase or decrease in an exponential manner, leading to either an "explosion type" field or to a field with near-vacuum conditions. In such fields, the original equations become inapplicable. The unstable velocity field tends either to direct matter towards, or to draw it away from the neutral line.

(v) General comment : the model used, and types of disturbances tried (after testing for likelihood in order) indicates instability in nearly all cases. Therefore we would expect that, if the conditions were nearly correct and the model nearly applicable, there would be violent magnetic disturbances at frequent intervals, or continuously, in the space on the side of the Earth far from the Sun at a distance where the neutral line might be expected to exist : these disturbances would, apart from anything else, cause considerable background noise and the latter would by now have been not only noticed but also traced to its source. However, this does not happen, and a few remarks are not out of place at this stage to indicate the reason. It is suggested that the use of a line-current model is not the cause, because although the line current model does differ from the magnetic field pattern that we may expect near the neutral line but at finite distances from it (§§3.5, 4.1), it must be emphasised that close to the neutral line they are both "X" type fields, and this is the type of field that has been investigated. A suggestion, to explain why unstable disturbances do not occur is put forward as follows :-

Let the symbol $\Delta$ refer to that part of the neutral line centred about the point on the neutral line which is opposite to the Earth from the Sun. $\Delta$ may be regarded as a portion of the neutral line of about 0.1 of the Earths diameter. Further, to assist the imagination, we may regard $\Delta$ as a sausage-shaped domain of finite thickness, say about 0.01 of the Earths radius, whose centre line is the neutral line. The region of space in which the investigations have been made can then be regarded as being "in $\Delta$".

Firstly, we have assumed that, in $\Delta$, the magnetic field if undisturbed, is steady. In fact it is not steady, because the Earth itself is not axi-symmetric, and therefore its magnetic field, at a fixed point away from the Earth, varies as the Earth rotates. This variation in the magnetic field could be looked upon as the disturbances that cause the instability. Further, the solar wind itself, and, with it, the configuration of the magnetopause, varies slightly with time. Finally, as the Sun, with the Earth in revolution, moves on its orbital path through the galaxy, the Earth will pass through changing regions of the interstellar magnetic field. Therefore, as a result of all these changes, the magnetic field lines inside the magnetopause are constantly on the move. This means that $\Delta$ is not a fixed region, relative to the Earth and Sun, but continually on the move. Thus, although the disturbances caused by changes in the magnetic field, as indicated above, may cause instability, long before the instability can start building up the region, $\Delta$ , where it might happen, has moved elsewhere, and the process must start afresh.

Secondly, it was assumed in §3.5 that the velocity field, undisturbed, is zero in $\Delta$ . This is not the case, a fact that has already been indicated in §1.10. As the Earth revolves around the Sun, and as the Sun revolves about the galactic centre, the solar system drifts through interstellar matter. The Sun's gravitation will, of course, affect the motion of the interstellar matter, but not enough to bring it to relative rest with respect to the Earth. This is particularly true for the very small particles of interstellar matter, a fact which can be demonstrated by example more quickly than by mathematical analysis. Thus, large planets of size comparable to the Earth (Venus and Mars) move in orbits of period of the same order as that of the earth, and if one of them happened to have the same mean disturbance from the Sun as the Earth, then it would move at nearly the same speed. But meteors enter the earth's atmosphere at high relative velocity, and the particles of the solar wind pass the Earth at even higher relative speeds. Therefore we may expect that, in $\Delta$ , there is a drift of matter, in a variable direction, at high speed. The effect of this would be to carry away and dissipate any strong local variation in density long before it had time to build up.

Thirdly, the inclination of the Earth's axis of rotation has an effect. In a frame in which the Earth's and Sun's centres appear at rest, the Earth's axis traces a cone once a year. Thus its magnetic field is continually moving, and $\Delta$ would appear to rise up through, and then sink down through, the plane of the ecliptic. The effect of this motion of $\Delta$ has already been discussed.

# CHAPTER VII

## POSTSCRIPT

### 7.1 The Magnetosphere

In this paragraph, an attempt will be made to give a broad description of the magnetosphere, this being the domain enclosing the Earth itself and the magnetotail. Attention will be mainly restricted to the region on the tailward side of the Earth where there is a possibility of neutral lines forming. Generally the processes going on in the magnetosphere are extremely complex, depending as they do on a number of variables, each of which affects the others. Therefore the behaviour of the magnetosphere is beyond simple mathematical analysis; however, in certain relatively small restricted domains some analysis can be carried out after making justifiable simplifications and approximations. But, apart from such cases, the behaviour of the magnetosphere must be discovered by experiment, that is, by the use of readings from space probes and satellites. Up till 1957 human technology had not reached the stage at which the launching of space probes and satellites was possible, and the behaviour of the magnetosphere was very much a matter of conjecture; the only relevant phenomenon experienced on the Earth's surface being the aurora, already correctly guessed to be connected with magnetospheric processes. Subsequently, a number of space probes and satellites has been sent through the magnetosphere and a considerable amount of information has been collected. As a result of this information, a detailed picture of the magnetosphere is beginning to emerge, and various authorities have published the information with their deductions, together with, in some cases, a diagram of the magnetosphere. The diagrams agree with each other in general and so it may be said that we are now beginning to obtain an accurate picture of the magnetosphere and of its internal processes.

The correlation of space-probe and satellite information into likely processes in various parts of the magnetosphere is very much a matter of practical physics, in which mathematics plays a minor role. Therefore, in prefacing this paragraph, the writer, not being a physicist, is on extremely unfamiliar ground. However it is hoped that the description to be given is reasonably intelligible. An important point should be made at this stage: some values of properties, such as magnetic induction, in the magnetotail have in fact been measured and recorded. However these values appertain to a hydromagnetic domain that has already been disturbed by a series of catastrophic events: therefore these values should not be used as undisturbed values in the analysis of §§5 , 6, since the analysis attempts to explain why the catastrophic events themselves occur in the first place.

This paragraph was prepared early in 1977, and is based mainly on the relevant literature published in 1975 and in 1976: no attempt has been made to include the findings of articles published this year (1977). Nor will more than slight reference be made to literature published before 1974, because some of the 1975 and 1976 articles themselves contain a resumé of previous discoveries and research. However, reference will be made to certain older articles of up to 30 years ago which either provide important basic features or are in the nature of 'classics'.

To start with, a sketch of the magnetosphere, with its various domains, is given in Fig.25(p.xxa). This shows the section of the magnetosphere by a plane which includes the Sun and the Earth and is perpendicular to the ecliptic plane. A section of this type does in fact portray most of the salient features of the magnetosphere. However, no attempt has been made to allow for the varying inclination, to the ecliptic normal, of the Earth's magnetic axis: this rotates daily about the Earth's axis of rotation, which itself rotates annually, as seen in the frame of the diagram, about the ecliptic normal. Indeed it may be said that this feature precludes the attainment of any sort of steady state: however, as has been discovered, conditions in the magnetotail appear to be already unsteady for reasons not connected with this rotation. A discussion of the possible effect, of the varying inclination, on the configuration of the magnetosphere, with a diagram, and reference to readings from the space probe IMP-1, is given by Ness (51). Meanwhile the sketch on p.xxa has been made to include, as far as possible, features provided in articles by Dessler (6), McCormac and Evans (52), Akasofu and Lanzerotti (53), Frank (54), Frank and Ackerson (55), and Dungey (56). A legend ascribes the generally accepted names to the various domains.

The interplanetary magnetic field is the field due to the Sun and to other extra-terrestrial bodies: it will be referred to in this paragraph as the IMF. The extent to which the IMF penetrates into the magnetosphere, and the region where this happens, has been considered by various authorities. For instance, Lanzerotti (57) considers the behaviour of energetic solar particles in the magnetosphere. Measurements indicate trapping of such particles on closed magnetospheric field lines, and also provide the inference that the particle access point is probably at a distance of 1000 $R_e$ from the Earth in a tailward direction: here $R_e$ is the Earth's radius. This suggests that IMF lines beyond the earth are bent towards the Earth and merge with Earth lines somewhere in the Magnetotail at $< 1000\ R_e$ . The Earth lines affected are those emanating from high latitudes: this is according to Paulikas (58) who asserts that merging takes place at a distance of about 150 $R_e$, and that the aurora is largely due to these Earth lines. Heikkila (59) finds that merging also takes place near the nose: this phenomenon is also reported by Kennel (60). Kennel states that the reconnected lines are then dragged by the solar wind over the poles and into the magnetotail, where merging again takes place: this phenomenon was reported earlier by McCormac and Evans (61). In these articles it is assumed that the solar wind carries the IMF with it; i.e. 'frozen-in' conditions apply; however Alfvén (62), the discoverer of the 'frozen-in' theory, himself casts doubts on its rigid application under all circumstances.

The region, tailward from the Earth, where merging takes place is the neutral sheet. In it and on each side (Northward and Southward) there is plasma. The cause of the existence of plasma in this region, the 'plasma sheet', is discussed by Frank (63): he suggests that the polar wind, which is the upward flow of plasma from the polar ionosphere, modifies the plasma in the far magnetosphere, and in fact that the polar cusp supplies plasma to the magnetotail. This view is supported by McCormac and Evans (64). The behaviour of the plasma in the plasma sheet is of interest, since it is clearly affected by phenomena due to instability in the neutral sheet. It has been discovered that this plasma is in rapid motion for most of the time, mainly in either an Eastward or a Tailward direction. It is proposed to revert to this subject shortly.

Reference has already been made in the introduction (pp.v-viii) to articles in which a qualitative analysis is applied to the phenomenon of merging of magnetic lines, together with forecasts of likely results. Another article is of interest, and may be mentioned here: this is the paper by Parker (65): he takes a different approach and uses a model in which consideration is given to the effect of pushing together two tube-shaped domains: each tube is straight, they are not parallel to each other, and each contains a magnetic fluid and magnetic field lines parallel to the tube axis. The mathematical treatment indicates that fluid is squeezed out and moves away, nearly parallel to the bisector of the acute angle between the tubes, from the point where the tubes cross. Although the analysis referred to sunspots, the result is of general application, and goes some way in explaining the behaviour of plasma in the magnetotail. Dessler (66) suggests that merging takes place at 10 - 30 $R_e$ in the solar wind wake: the plasma density there is insufficient for a current sheet, and so conditions are vacuum-like, and frozen-in field conditions apply. Frank (67), using readings from the space-probe IMP-5, notes that merging takes place at the day-side of the polar cusp and in the neutral sheet in the tail. McCormac and Evans (68) suggest a neutral line at 15 - 35 $R_e$, where field line reconnection is triggered by substorms. Antonova and Shabanskiy (69) consider a model of the magnetosphere with the Earth's magnetic field replaced by a dipole, inclined to the ecliptic normal, and discuss the effect of this feature on reconnection of field lines. The problem of reconnection of field lines is analysed mathematically by Hill (70). He asserts that the plasma is collisionless, so that MHD conditions do not apply; also that plasma pressure is anisotropic: when the difference between the maximum and minimum principal values of the pressure is equal to $B^2/\mu_0$ (with the usual meaning of symbols) then merging ceases. Analysis is also applied to the case where the opposing magnetic fields have arbitrary magnitude and direction. It is found that two pairs of oblique MHD shock waves are formed, which deflect the moving plasma and form accelerated plasma flows away from the neutral line. If conditions are steady the shock waves are stationary. Calculations show that the merging speed depends on the upstream Alfvén speed, the densities above and below the reversal plane, and the angle between this plane and the upstream magnetic field. The plasma flux is perpendicularly towards the neutral sheet, bringing with it the magnetic field lines, and at the sheet protons and elections are ejected with mainly opposite accelerations along the sheet. The analysis of the article is based on the kinetic theory of plasma, and, to some extent, bears out the findings of Parker's earlier article. However, the result is questioned by Cowley (71). The possibility of merging of IMF and of geomagnetic field lines at the Earth end of the plasma sheet is discussed by Franck (73); he points out that the locality is at a distance from the Earth which varies as the plasma sheet thins and thickens:when the sheet is thinnest the distance is at a minimum of 10 $R_e$. This article uses readings from VELA satellites. Vasilunyas (73) reviews the knowledge of merging to-date, comparing it with the theoretical predictions made in Parker's earlier article and also by Pekcheck (13) and Axford (14), and subjects the problem to a mathematical treatment. As a result of this, he predicts the formation of an X-type MHD shock wave at the neutral point. Another review is that of Frank and Ackerson (74): they find that merging takes place in the dayside magnetopause, open field lines are formed and are swept into the magnetotail where they reconnect and form closed lines. This allows the IMF to enter the magnetosphere and causes plasma acceleration to the magnetotail. The reconnection is the cause of substorms.

The exact nature of substorms await further exploration for its definition. Hones (75) notes that they are the source of release of energy into the auroral zones. A general qualitative description has been given by Akasofu and Lanzerotti (76); also Kropotkin (77) constructs a simplified model subject to mathematical analysis and shows how a substorm can result from instability.

In this paragraph interest is mainly directed to the effect of substorms on the plasma and on field lines in the magnetotail rather than on the precise nature of substorms. It seems clear that there are expansive phases of substorms, followed by phases usually referred to as substorm recovery; during the expansive phase the plasma in the plasma sheet moves tailward and the sheet gets thin, whereas during substorm recovery the plasma moves Earthward and the sheet thickens. This phenomenon has been noted by the following authorities:-

(a) Frank (78), who finds, from readings of election energy densities made by the space-probe OGO-3, an indication that, at distances greater than 18 $R_e$, the local magnetic field is unstable and distorted.

(b) Hones (79), who finds that the thickening of the plasma sheet, at 18 $R_e$ during substorm recovery, is due to merging of IMF lines with magnetospheric field lines, the thickening being caused by stretched field lines contracting rapidly Earthward.

(c) Rostoker (80),

(d) Hones et al. (81) and (82), from readings from VELA satellites and from Explorer 34. They find that during recovery, the neutral line moves tailward - as reported by McCormac and Evans (52); also that the Earthward plasma flow lasts for a few minutes. Beyond the neutral line there is a thin plasma sheet during expansion, with plasma moving tailward.

(e) Frank (83) who uses readings from the space probes IMP-6 and IMP-7 and also VELA readings. The thinning of the plasma sheet lasts for about one hour.

(f) Kropotkin (84),

(g) Chang and Lanzerotti (85), who consider the generation of magneto-acoustic waves.

(h) Bowling (86), who uses readings from the space-probe Explorer 34. He finds periods of high geomagnetic activity, occurring irregularly and lasting a few hours.

(i) Russell(87),

(j) Roeloff et al., (88) and (89). They record measurements made, by the space probe IMP-7 (Explorer 47) at 35 $R_e$, of speeds of $\geqslant$ 50 KeV protons and of $\geqslant$ 30 KeV electrons. Quiet time speeds, in random directions, in the magnetotail and in the plasma sheet are about $7 \times 10^4$ $ms^{-1}$, but, during substorm expansion, the speeds reach $1.25 \times 10^6$ $ms^{-1}$ Tailward, and, during recovery, Earthward speeds reach $9.5 \times 10^5$ $ms^{-1}$.

(k) Frank and Ackerson (90), who report proton speeds of up to $5.5 \times 10^6$ $ms^{-1}$.

(l) Maezawa (91), who notes that the total time taken by expansion and recovery is about one hour.

(m) Hones et al. (92), who use readings made by the space probe IMP-6. The component of the magnetic field normal to the ecliptic is Southward during the expansion phase, and Northward during recovery. During the substorm, and for about nine hours afterwards, plasma speed is between 3 and $10 \times 10^4 ms^{-1}$ : these readings agree with the quiet time speeds read by Roeloff et al., see (j) above.

(n) Toichi and Miyazaki (93), who find flapping motions of the magnetotail during substorm expansion.

There does not seem to be entire agreement among the various authorities regarding the existence of a neutral line or a neutral sheet or both. The existence of a neutral sheet is claimed in a number of papers already quoted: (51),(67),(58),(59),(73),(87)&(92). Other authorities who favour the possibility are:-

(o) Russell and Brody (94), who suggest that the neutral sheet is curved.

(p) Heikkala (95),

(q) Alekseyev et al. (96),

(r) Bowling and Russell (97), who use readings from Explorer 34, and find that the neutral sheet is hinged at a distance of less than 10 $R_e$ .

(s) Fan et al. (98), who use readings from the space-probe IMP-7.

However a few authorities, particularly Dessler (99) and McCormak and Evans (100), consider that there must be a neutral line; in fact Dessler argues that it should be at about 30 $R_e$, with a neutral sheet of finite thickness further out.

From the consideration of these articles, it is to be inferred that, if a neutral line does form, possibly with an evanescent nature, it is likely to be at a varying distance from the Earth, and not of the fixed type considered in §§ 5,6. For an analysis of the stability conditions at such a neutral line it would be necessary to be in possession of knowledge of its speed, so that co-moving coordinates could be used, with the resulting effect of these on the components of the medium velocity and of the magnetic field.

7. (ii) Applicability of Findings

It has been found that the equations in Chs.5,6 treated as recurrence equations are unstable to certain disturbances. The criterion of instability was that the equations are regarded as unstable if any one variable reaches, within 10 time periods, a size of order 100 times the order of the initial disturbances. However the criterion of instability was satisfied, in every case, after only a few time periods. It is possible that the use of a time period of 1 second, which turns out to be large for the investigation of these particular instabilities, has led to a false result, viz. that what are in fact stable differential equations appear, on transform to recurrence equations, as unstable. Unfortunately a calculating machine cannot cope with a differential equation : if time is the independent variable, the best that can be done is to shorten the time period as much as possible consistent with excessive use of machine time : if the resulting recurrence equation is stable, then the inference, from consideration of trajectories, is that the original differential equation is also stable. Alternatively, if the differential equation is unstable it will have trajectories that curve away rapidly from the zero axis. The corresponding recurrence equation will have a series of points that move away less rapidly : the shorter the steps between the points, the more closely will they follow the trajectories of the differential equation, and the greater will be the apparent instability. Thus we may expect that, if the differential equation is in fact unstable, the corresponding recurrence equation is likely to show more and more marked instability as the time period is decreased.

This paragraph has been written : firstly, to indicate how the use of an insufficiently short time period may lead to a false result; secondly, to analyse a method whereby a time period may be chosen with confidence that there is no risk of a false result accruing; and thirdly, to apply the result to the findings of Chs 5,6. Since the equations under review are all first order, the analysis has been restricted to a first order equation; also a simple first order equation, whose solution when in differential form is known, has been chosen as an example. The analysis has been carried out from first principles, since the writer has only a slight knowledge of

Numerical Analysis.

Thus, we consider the equation, of a type which arises in Mathematical Biology:

$$\dot{x} = 2x(5-x),$$

the dot meaning d/dt. We note that, when $x = 5$, $\dot{x} = 0$, so that the trajectories on the t,x plane are asymptotic to the line $x = 5$; they never cross it. They are also clearly asymptotic to the line $x = 0$. Also, $\ddot{x} = 0$ when $x = 2.5$, so that the trajectories have points of inflection, with slope 12.5, on the line $x = 2.5$. Thus, between $x = 0$ and $x = 5$, the trajectories closely resemble the curves, $x = 2.5\{1 + \tanh 5(t-c)\}$, where c is a parameter : the shape of these curves is well-known . Confining our interest to the domain $x > 0$, we note that, when $x \gg 5$, the slope is steep and negative; so, for the domain $x > 5$, the curves resemble the positive parts of the rectangular hyperbolae $2(x-5)(t-c) = 1$.

The corresponding recurrence equations are :

(i) With $\Delta t = 1 : x_{n+1} = 11x_n - 2x_n^2$,

(ii) With $\Delta t = 0.1 : x_{n+1} = 2x_n - 0.2x_n^2$,

(iii) With $\Delta t = 0.01 : x_{n+1} = 1.1x_n - 0.02x_n^2$.

These recurrence equations do not admit of analytical solutions. However, as can be easily checked, (i) is highly unstable, whereas (ii) and (iii) generally indicate behaviour similar to the solution of the differential equation, which is:

$$x = \frac{5x_o}{x_o + (5-x_o)e^{-10t}}, \text{ where } x_o = x(t=0).$$

However, even (ii) and (iii) indicate instability if $x_o$ is large enough.

An insight into the behaviour of the recurrence equation can be obtained by setting $\Delta t$ at $\frac{1}{m}$ , and setting $x_n$ at 5+e. The false impression given by (i), and also by (ii), (iii) for large $x_o$, lies partly in the fact that the iterative solution, for successive $x_n$, makes the value of $x_n$ cross over the lines $x = 5$, $x = 0$, asymptotic to the solution of the differential equation : this is to be avoided.

The result of the substitution is :

$$x = (\frac{10}{m} + 1)\ (5 + e) - \frac{2}{m}(5 + e)^2 = (5 + e)(1 - \frac{2e}{m}).$$

It will be found that, if $e \gtrless 0$, so that $x_n \gtrless 5$, then $x_{n+1} \gtrless 5$ if $m > 2(5 + e)$. This is the condition for the recurrence equation iterative solution to resemble the differential equation solution, at least insofar as the solving points approach the line $x = 5$ asymptotically. Clearly, the maximum value of $\frac{1}{m}$ in this case depends on $e$, i.e. on $x_0 - 5$ if $n = 0$, so that we have found a criterion that is dependant on a starting perturbation. However this is no great disadvantage, because if we are investigating stability we may make the initial disturbance small, that is, in this case we may set $x_0$ at 5 (the equilibrium value) $\pm$ (small perturbation), so that $|e| \ll 5$. So, for this equation, a suitable value for the corresponding recurrence equation, for $\Delta t$ would be $\frac{1}{m}$, with $m > 10+$, i.e. $\frac{1}{11}$. Then the recurrence equation is $x_{n+1} = \frac{21}{11} x_n - \frac{2}{11} x_n^2$.

The above analysis only applies to the particular equation chosen as an example. It is now proposed to explore a form of reasoning which may lead to a more general result.

We consider the equation : $\dot{x} \equiv \frac{dx}{dt} = p(x)$,
where $p$ is a given function. For the corresponding recurrence equation, we start at an agreed base point $P_n(x_n, t_n)$ which may act as a constant of integration for the above equation, i.e. $x = x_n$ when $t = t_n$. The recurrence equation then calculates $P_{n+1}(x_{n+1}, t_{n+1})$ from the relations :-

$$x_{n+1} - x_n \equiv \Delta x_n = \Delta t_n \times (\text{slope at } P_n) = \Delta t_n\ p(x_n),$$

and $t_{n+1} - t_n \equiv \Delta t_n$ a chosen constant step in the value of $t$.

Thus $P_{n+1}$ lies on the tangent to the differential equation's solving trajectory, at $P_n$, and on the line $t = t_n + \Delta t_n$. The distance, $P_n P_{n+1}$ along the tangent is $\Delta t_n \sec \psi_n$, where $\tan \psi_n$ is the slope at $P_n$. Therefore, writing $p_n$ for $p(x_n)$, we have :

$$P_n P_{n+1} = \Delta t_n (1 + p_n^2)^{\frac{1}{2}}.$$

Over this short step, the solving trajectory will closely follow its circle of curvature at $P_n$ , and we want to make sure that, if $K_n$ is the centre of curvature at $P_n$ , then $K_nP_{n+1}$ does not differ from $K_nP_n$ by more than a small proportion. Let this proportion be $q$ , of order, say , $10^{-2}$.

Then $K_nP_{n+1} - K_nP_n \leq qK_nP_n$ .

Now, since the radius $K_nP_n$ is at right angles to the tangent, $(K_nP_{n+1})^2 = (K_nP_n)^2 + (P_nP_{n+1})^2$ . Let $\rho_n = K_nP_n$ , so that $\rho_n$ is the radius of curvature of the trajectory at $P_n$ . Using the above expression for $P_nP_{n+1}$ we have :

$$+\{\rho_n^2 + \Delta t_n^2(1+p_n^2)\}^{\frac{1}{2}} - \rho_n \leq q\rho_n$$

Meanwhile, dropping the suffix, noting that $\ddot{x} = p'\dot{x}$ with $p' \equiv \frac{dp}{dx}$ , and using the formula for $\rho$ , we have :
$$\rho = \left|\frac{(1+p^2)^{\frac{3}{2}}}{pp'}\right|.$$

Since $\Delta t_n$ is independent of $n$, we may replace it by $\Delta t$, and ignoring $q^2\rho^2$ we find that :
$$\Delta t^2 \leq \frac{2q\rho^2}{1+p^2},$$

i.e.
$$\Delta t \leq +\sqrt{2q}\,\left|(1+p^2)/pp'\right|.$$

Investigation must therefore be made to find the minimum value of $(1+p^2)/pp'$ as a function of $x$. By application of simple calculus methods, this turns out to be tantamount to solving the equation :

$$pp''(1+p^2) + p'^2(1-p^2) = 0.$$

Now, if $p$ is a linear function of $x$, the differential equation can be solved easily, and recurrence methods are not needed. The same applies to the case where $p$ is a quadratic function, as in our earlier example, $p(x) = 2x(5-x)$. However the equations in Chs. 5,6, while not admitting analytic solution, are of quadratic type; that is, they contain mostly terms in one variable or a product of two variables* (with a trivial exception : the equations for $v_{ij}^{n+1}$ in §5.2 contain terms in $q^n\, b_{ij}^n\, b_{[k\ell]}^n$ ; however these are more likely to increase instability than decrease it). Also, the coefficients on the R.H.S. of the equations are nearly all unity. Therefore, to obtain a rough idea of the necessary criterion, we will, for the sake of comparison, try our above analysis with $p(x) = x(1-x)$. Then $p' = 1-2x$, $p'' = -2$, and the above equation containing $p''$ becomes :

$$2x^6 - 6x^5 + 7x^4 - 4x^3 - 5x^2 + 6x - 1 = 0,$$

*and with unit coefficient for each term, see pp. 36,58 for which a close solution is found, using Newton's method, to be $x = 0.8$ . With this value of $x$, the minimum value of $|(1+p^2)/pp'|$ is approximately 10.7.

Therefore we require $\Delta t \leq \sqrt{2q} \times 10.7$, so that if we set $q$ at 0.01, as previously suggested, the criterion is : $\Delta t \leq 1.52$.

Thus the inference is that by choosing $\Delta t$ as 1 in the program, a sufficiently small $\Delta t$ has, in fact, been used. But, before leaving the matter, it would be pertinent to apply a comparison to the one set of equations, viz. those in §6.1, for which an analytical solution was possible.

Writing $x$ for $b_{112}$, $y$ for $b_{211}$, $z$ for $v_{111}$, and $w$ for $w_{212}$, the four equations in differential form are : $\dot{x} = -2w$, $\dot{y} = -3z$, $\dot{z} = 2(x-y)$, $\dot{w} = -(x-y)$. Using the same starting conditions as in §6.1, viz $x_0 = y_0 = 0$, $z_0 = 0.1$, $w_0 = -0.2$, the solution is :-

$$x = +\frac{7\sqrt{2}}{160} \sinh\sqrt{8}t + \frac{9t}{40} ,$$

$$y = -\frac{21\sqrt{2}}{160} \sinh\sqrt{8}t + \frac{9t}{40} ,$$

$$z = \frac{7}{40} \cosh\sqrt{8}t - \frac{3}{40} ,$$

$$w = -\frac{7}{80} \cosh\sqrt{8}t - \frac{9}{80} .$$

Of these variables, the ones with the greatest growth rates are $y$ and $z$, (as in the recurrence case), and each reaches a value whose magnitude is 10 when $t = 1.675$ : thus the differential equations are more unstable than would be indicated by the corresponding recurrence equations with $\Delta t = 1$ (which were found to have an "instability time" of 4). This is to be expected, as remarked earlier in this paragraph, and shows that the choice of unity for $\Delta t$ has not given a misleading idea of the behaviour of the differential equations.

If, in this instance, the time step $\Delta t$ had been chosen at 0.1, we might expect that the corresponding recurrence equations would show an "instability time" of between 1.675 and 4 . In fact, this is so: if the resulting solutions are inspected, and the term with the highest growth rate is selected, then this term turns out to include a factor $(1 + 0.1\sqrt{8})^n$, which means that the corresponding variable reaches a value $\geq 100$ when $n = 19$. Thus the "instability time" is $19 \times 0.1 = 1.9$.

Finally to make quite certain that the recurrence equations of Chs. 5,6, as calculated by computer, have in fact given a true picture, the program was run again with $\Delta t$ altered from 1 to 0.1. The criterion, 10, (i.e. 100 × order of disturbances) was left unaltered, but it was necessary to alter the maximum period, which counted time steps, from 10 to 100. The programs were, in fact, written with cognisance of the possibility of the necessity of making alterations such as this. The results are shown on pages (xxxiii') and (xxxiv'). The mean instability times are : First Differential Equations : 2.9 sec,

Second Differential Equations: 2.4 sec.

Originally, with $\Delta t$ set at 1 and maximum period at 30, the mean instability times were, from pages (xxxiii) and (xxxiv) :-

First Differential equations : 5.7 secs

Second Differential Equations: 5.1 secs

Thus reduction of $\Delta t$ from 1 to 0.1 has resulted in reducing the instability time, in the case of First Differential Equations, from 5.7 to 2.9 sec., (with no set amenable to analysis for comparison) and in the case of Second Differential Equations from 5.1 to 2.4 secs (with one set, amenable to analysis, indicating a limiting value of 1.675 sec. as $\Delta t \to 0$).

In conclusion it may be said that the problem of checking whether or not a time-differential equation is stable, by the method of turning it into a recurrence equation - using an adjustable time-step - has been discussed by many experts in the field, e.g. by G.E. Forsythe, M.A. Malcolm and C.B. Moler, in "Initial Value Problems in Ordinary Differential Equations", (Prentice Hall, 1977 (see, in particular, 6.5, "Stiff Equations".) It is the general opinion that, if the recurrence equation is stable, then it is almost certainly true that the corresponding differential equation is also stable. However one cannot be at all sure that the connection applies for instability; that is, if the recurrence equation is unstable it is not certain that the corresponding differential equation is also unstable : indeed there seems to be no accepted formal method of analysis applicable to this problem, particularly when the equations are simultaneous with many independent variables, as in Chs. 5,6. It is submitted therefore that the method outlined in this sub-paragraph is the most suitable one that can be found under the circumstances.

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APPENDIX A

FIGURES REFERRED TO IN THE TEXT

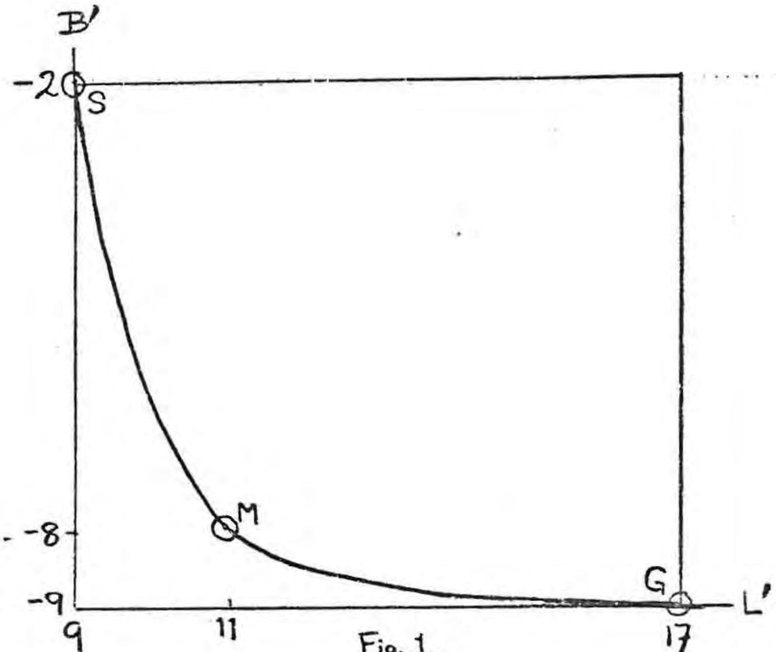


Fig. 1.

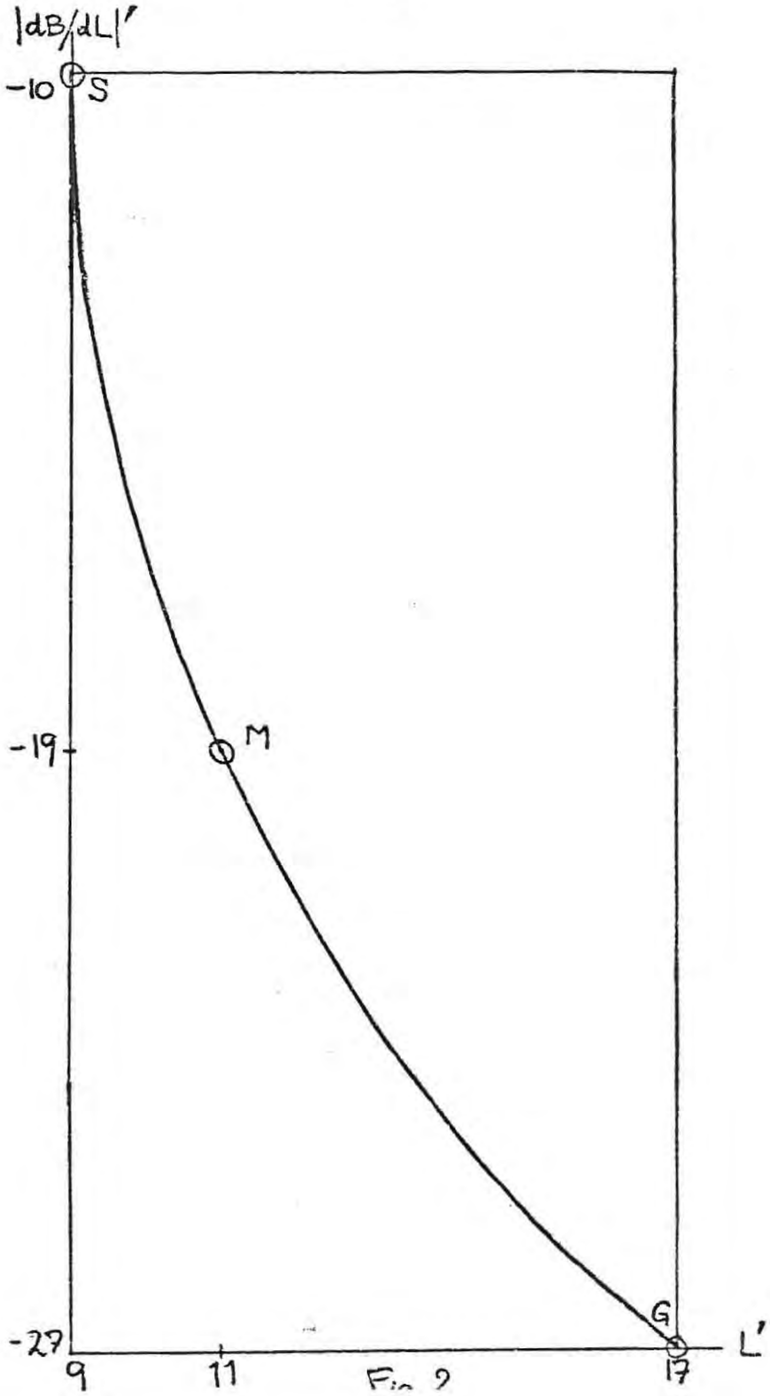


Fig. 2

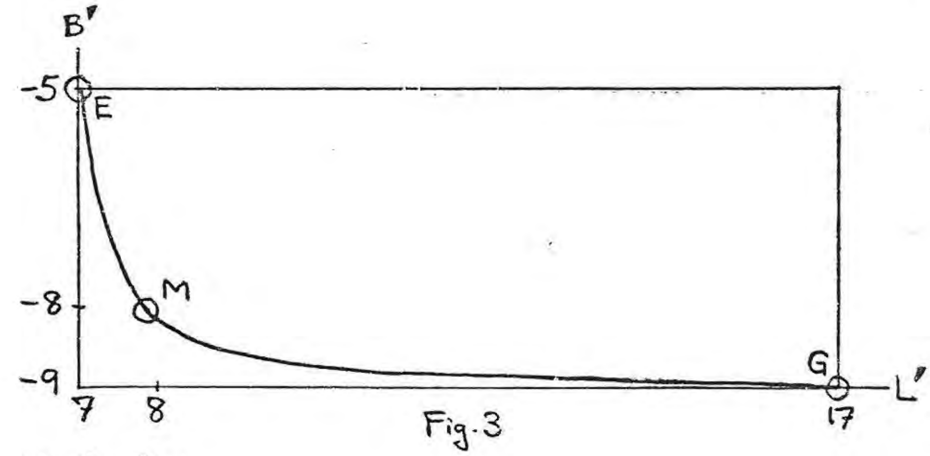


Fig. 3

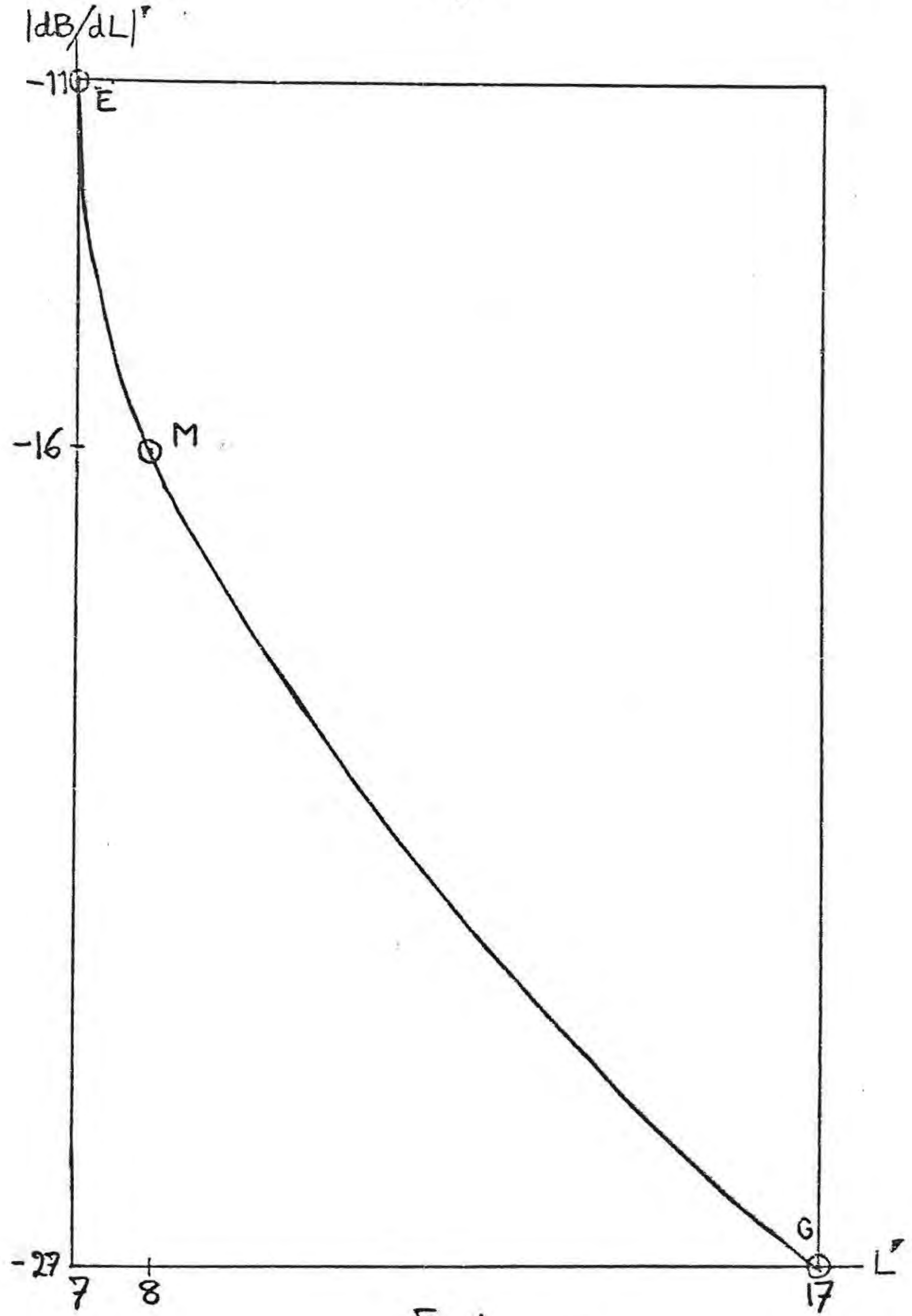


Fig. 4.

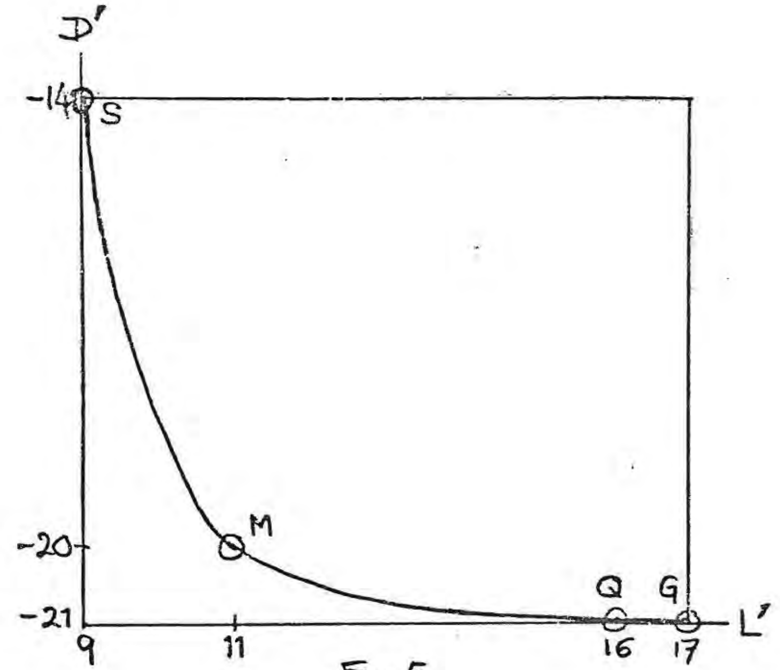


Fig. 5

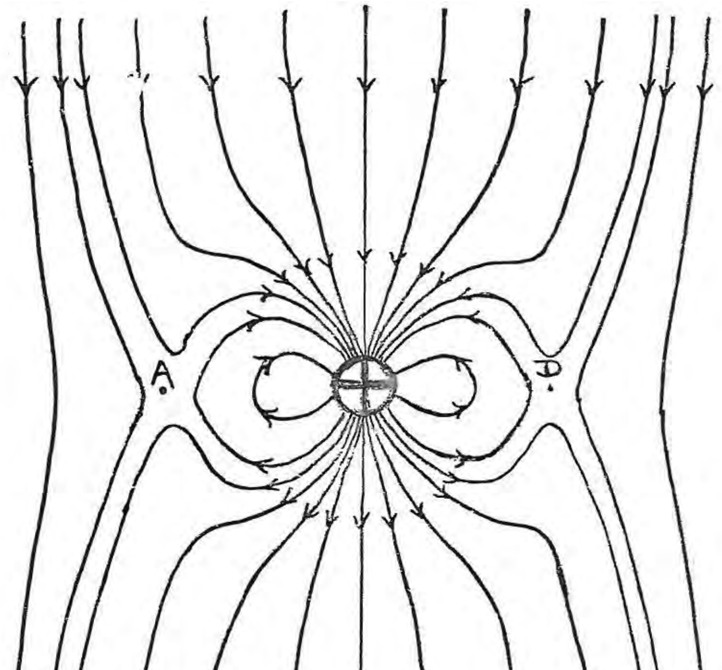


Fig. 6

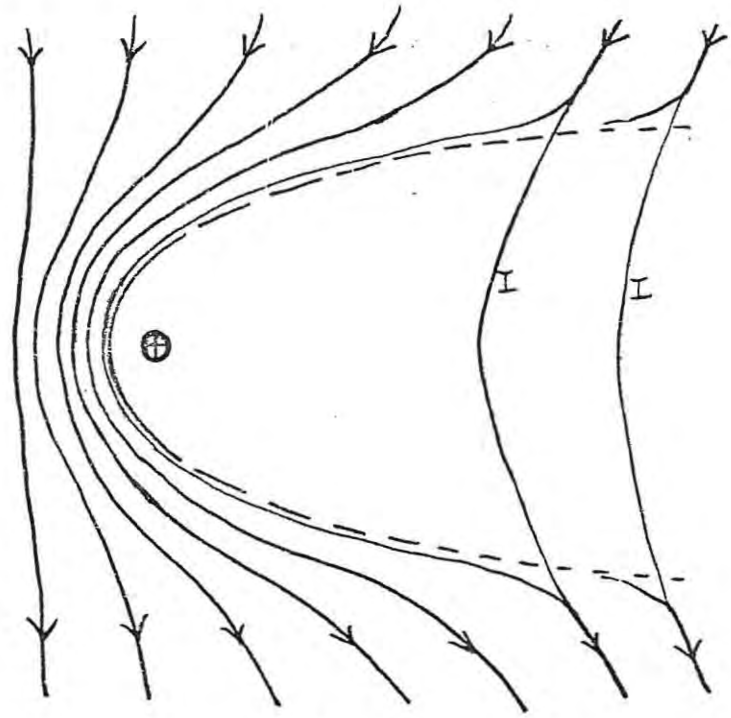

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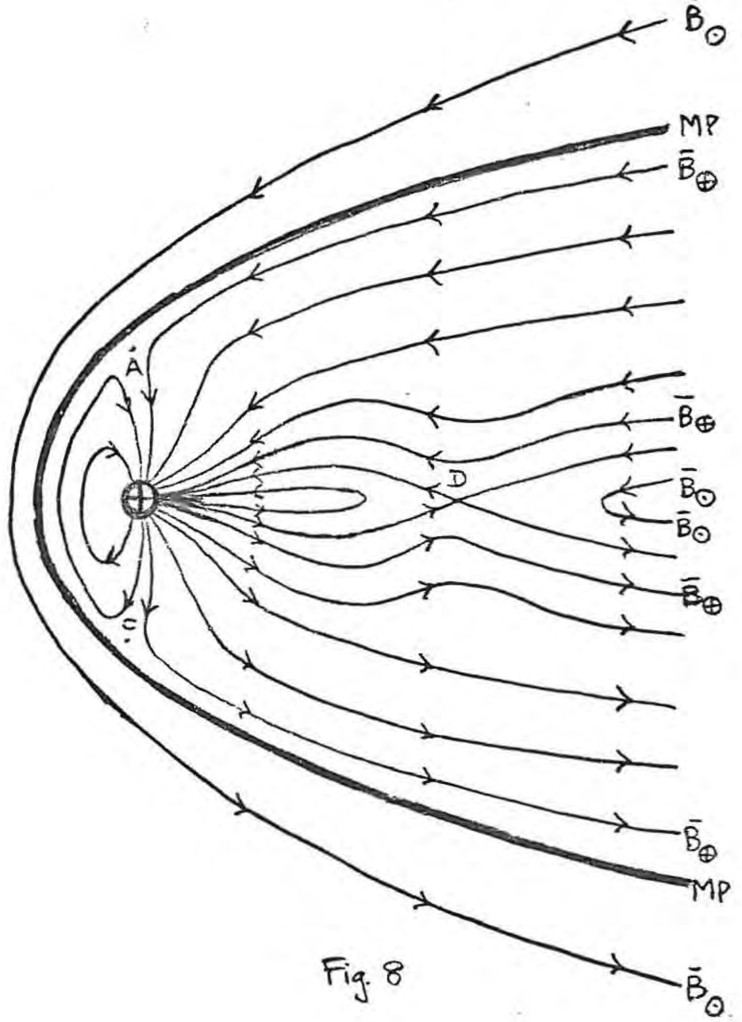
Fig. 7



$\bar{B}_\odot$
MP
$\bar{B}_\oplus$
A
D
$\bar{B}_\oplus$
$\bar{B}_\odot$
$\bar{B}_\odot$
$\bar{B}_\oplus$
C
$\bar{B}_\oplus$
MP
$\bar{B}_\odot$


Fig. 8

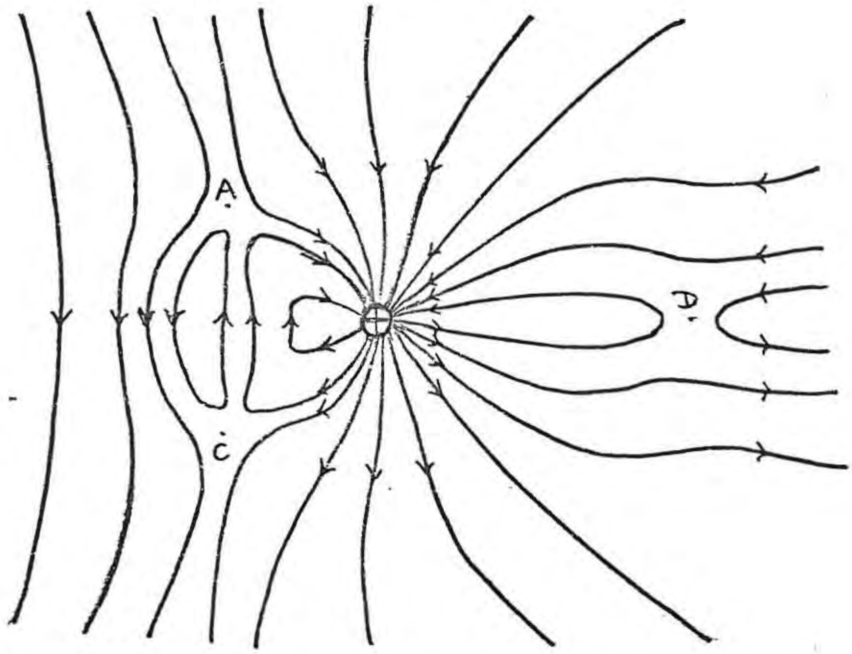



Fig 9

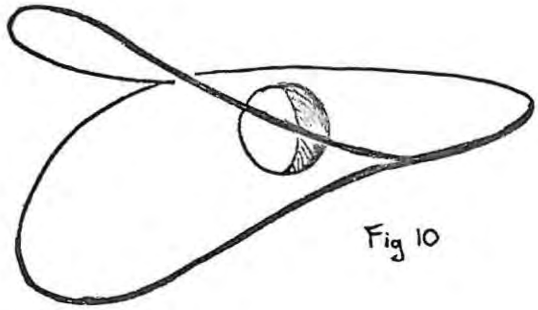


Fig 10

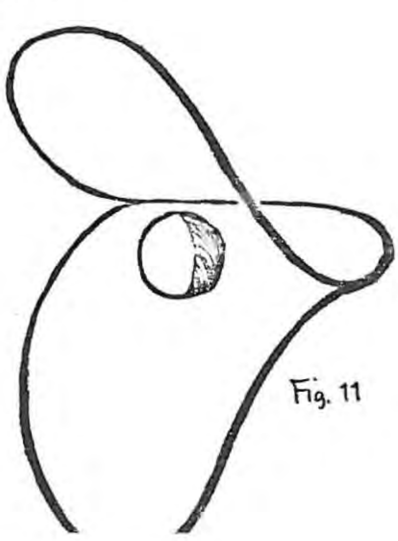


Fig. 11

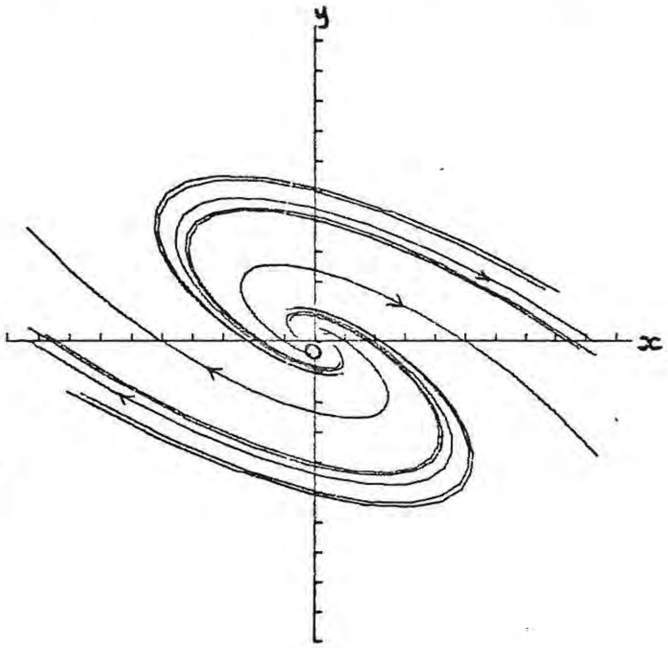



Fig. 12

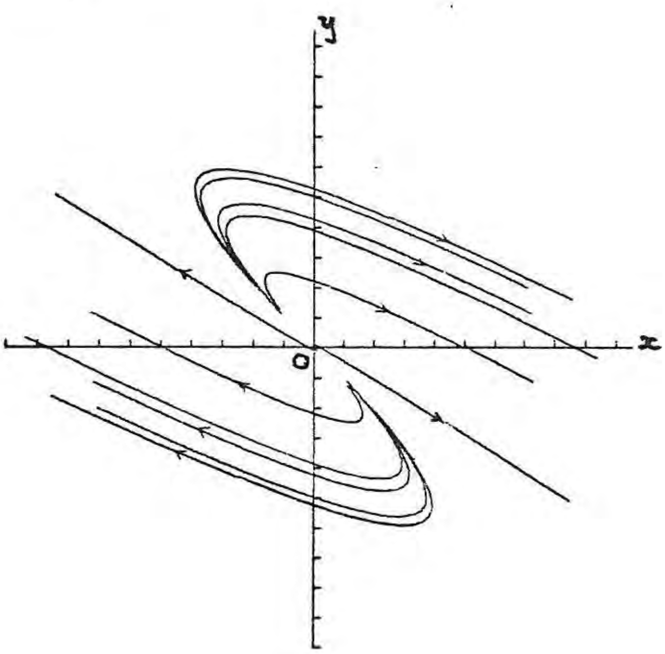



Fig. 13

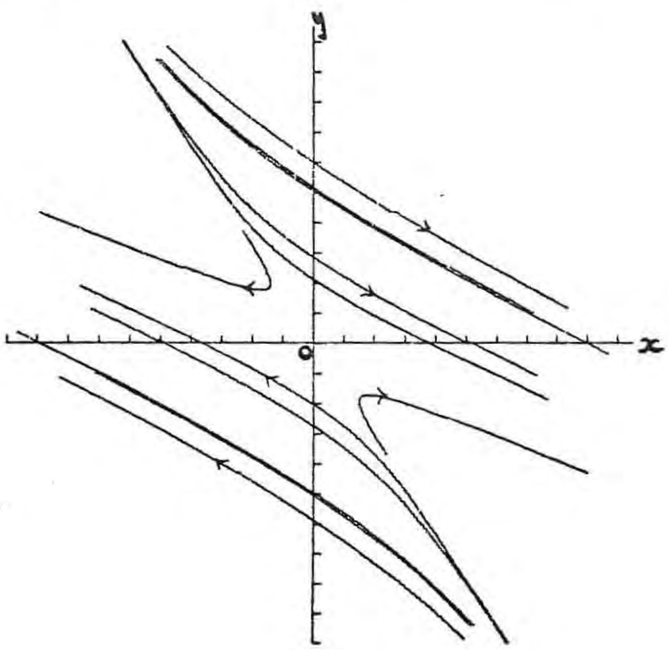



Fig. 14

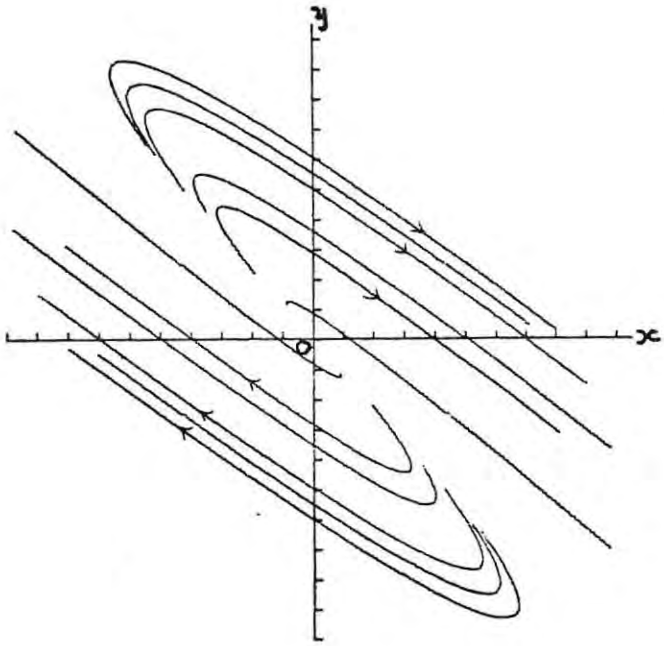



Fig. 15

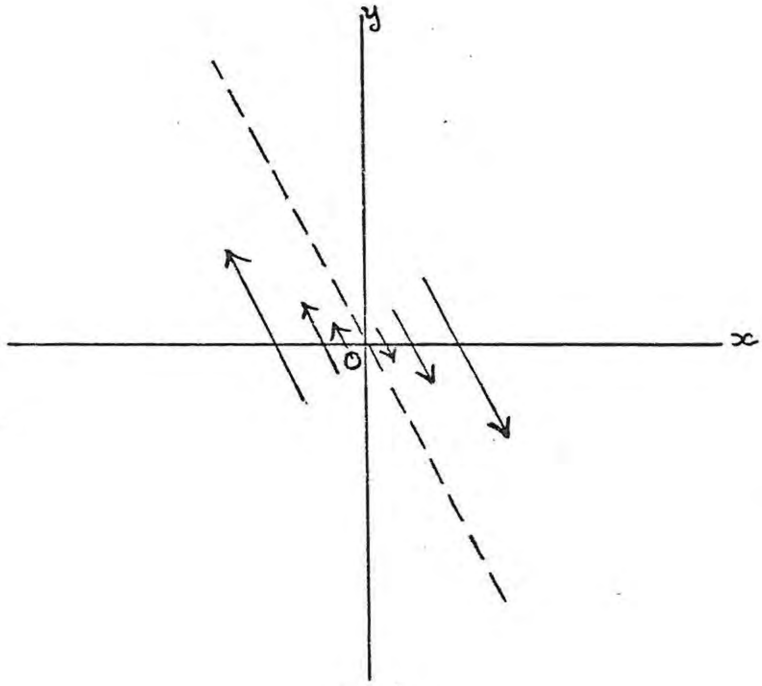



Fig. 16

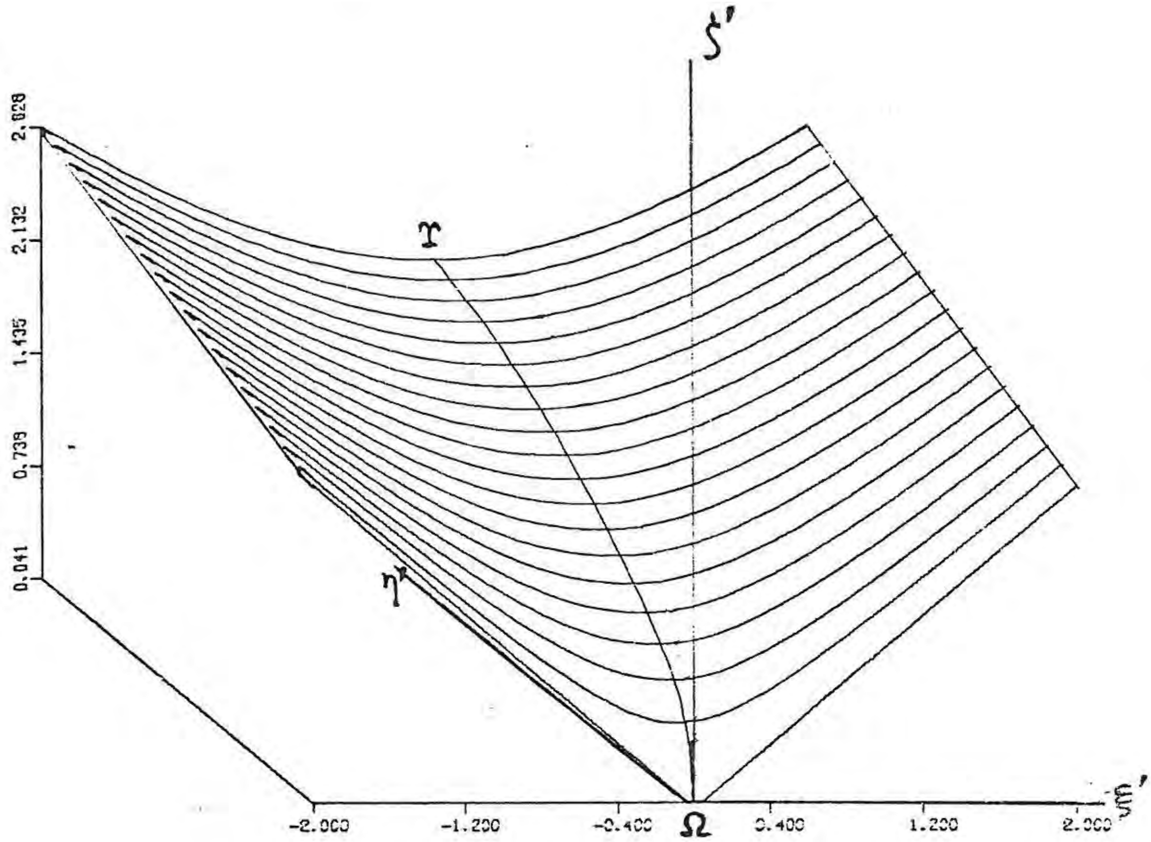



Fig 18

Surfaces touch on line ΩΥ

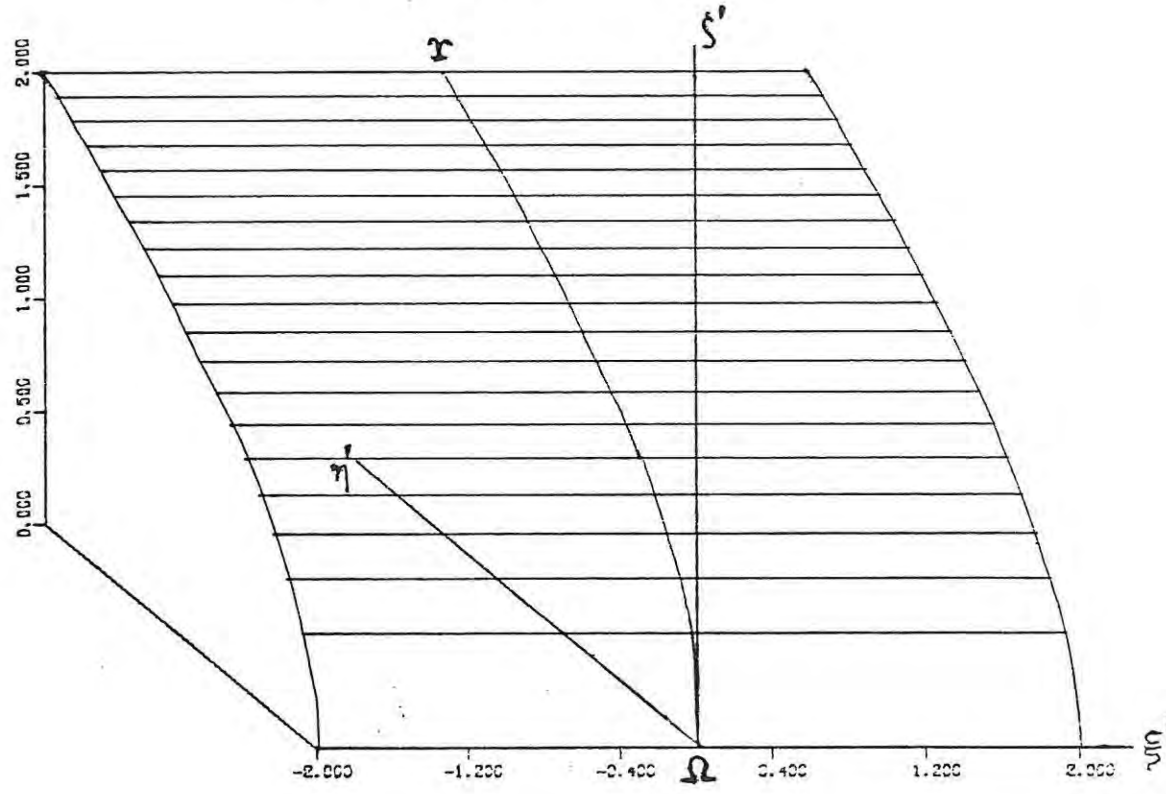



Fig 17.

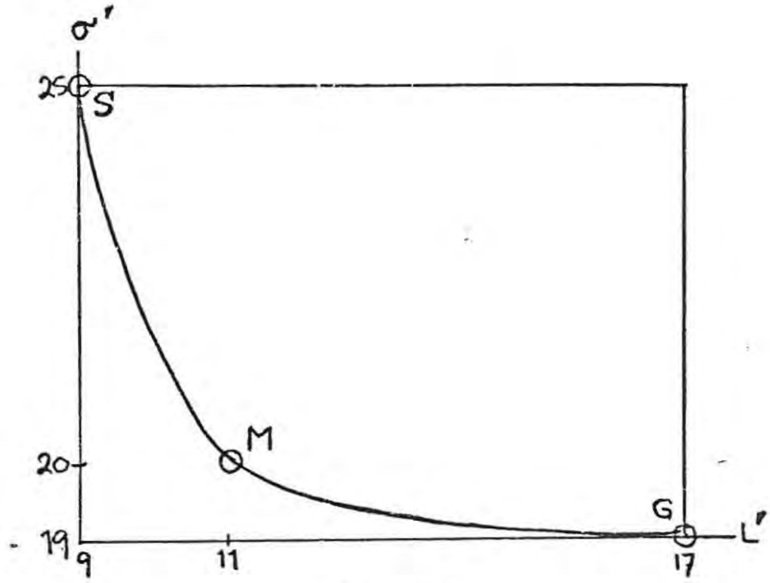



Fig. 19

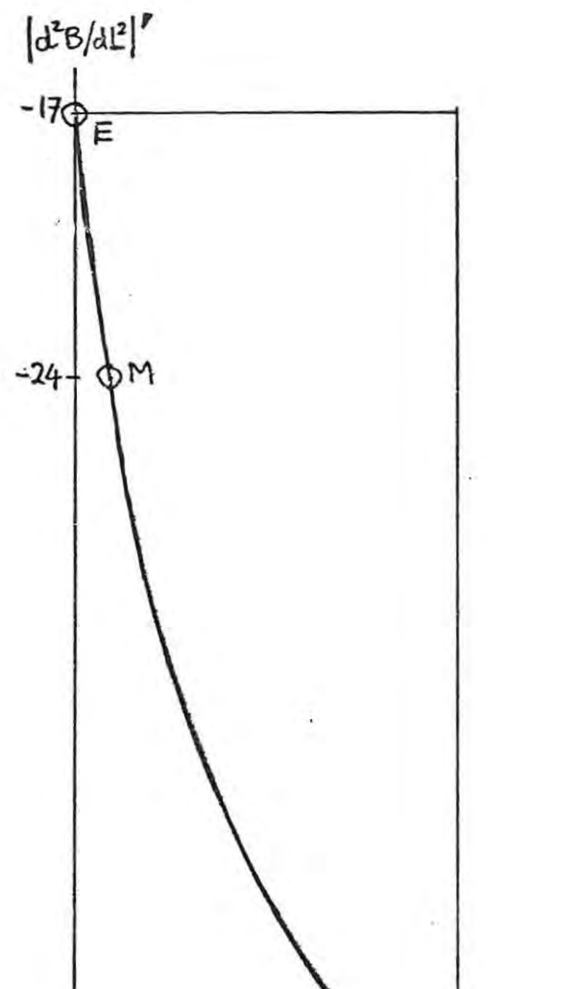
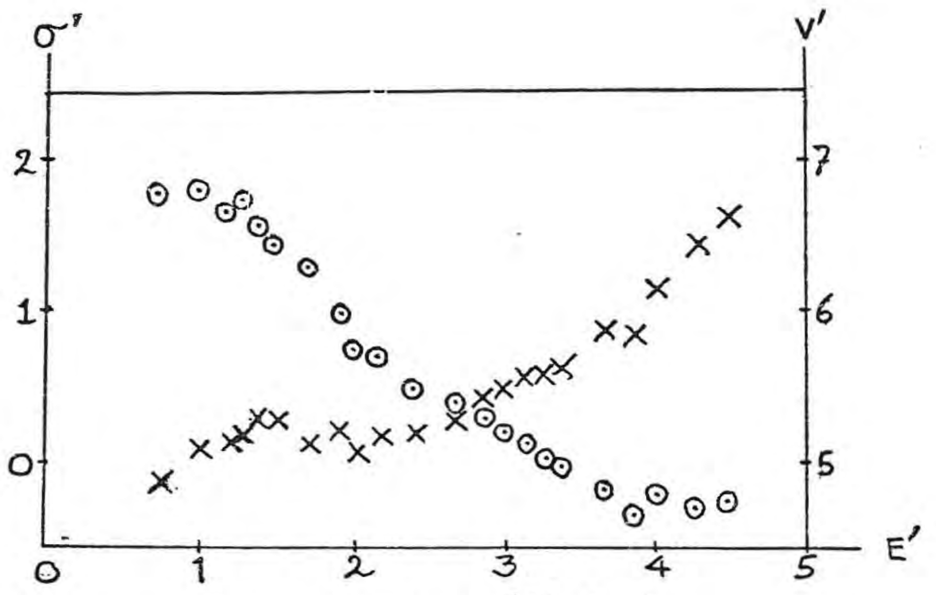



Fig. 20




E is in volt $m^{-1} = C^{-1} k^{1} m^{1} s^{-2}$

⊙ → σ is in $C^{2} k^{-1} m^{-3} s^{1}$

X → V is in $m^{1} s^{-1}$ Fig. 21

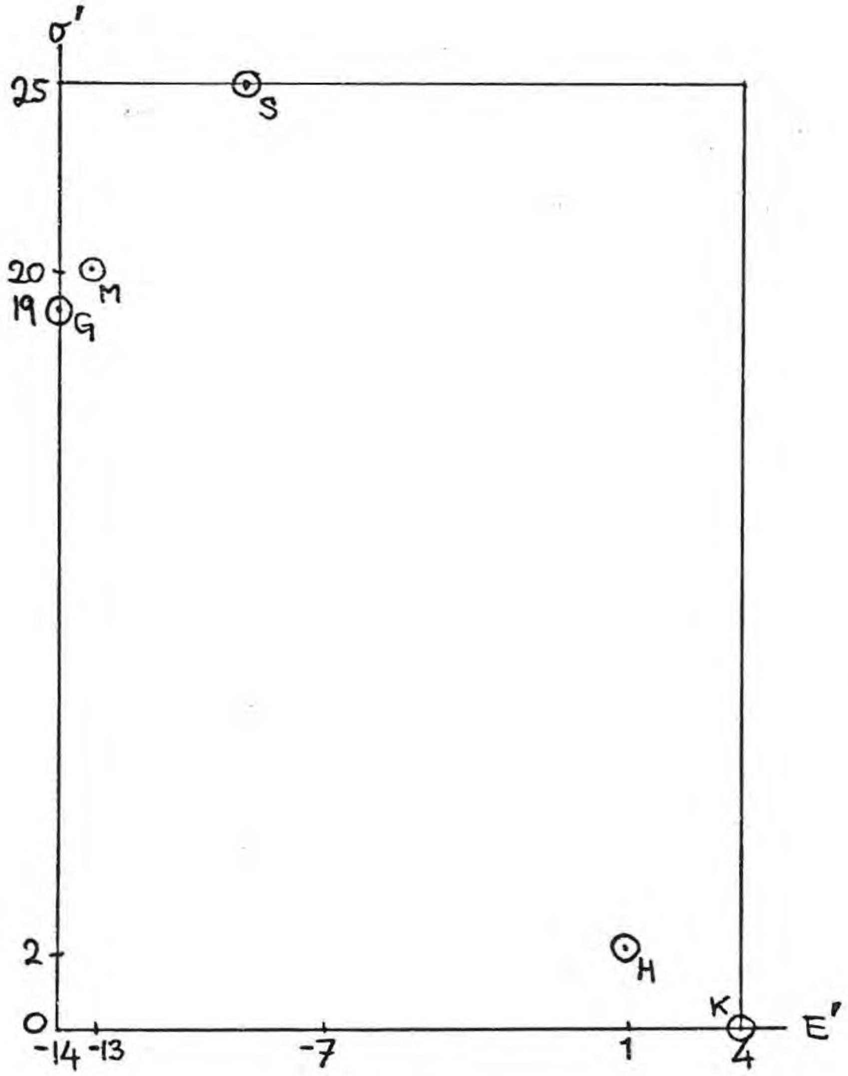
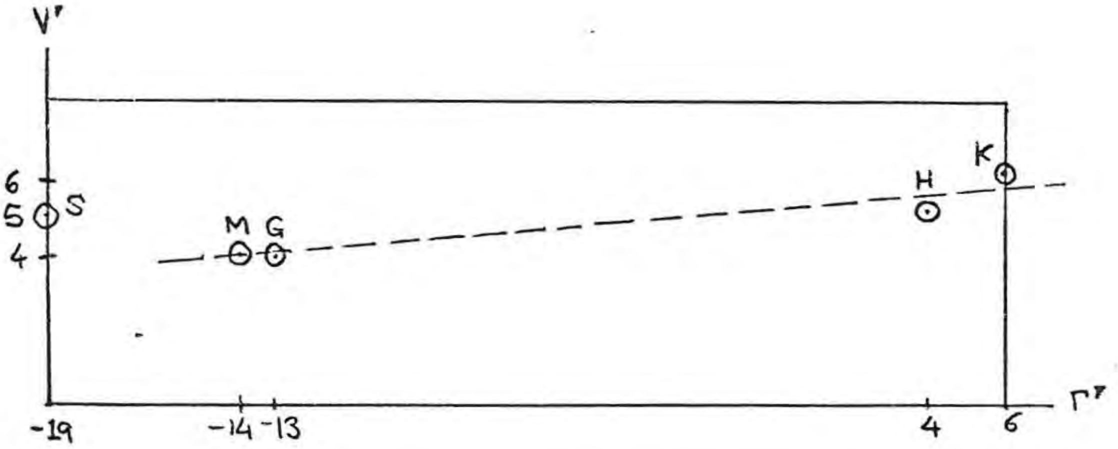



Fig. 22




$(d\Gamma'/dV')_M$ is read without cognisance of points S, G : ie from dotted line

Fig. 23

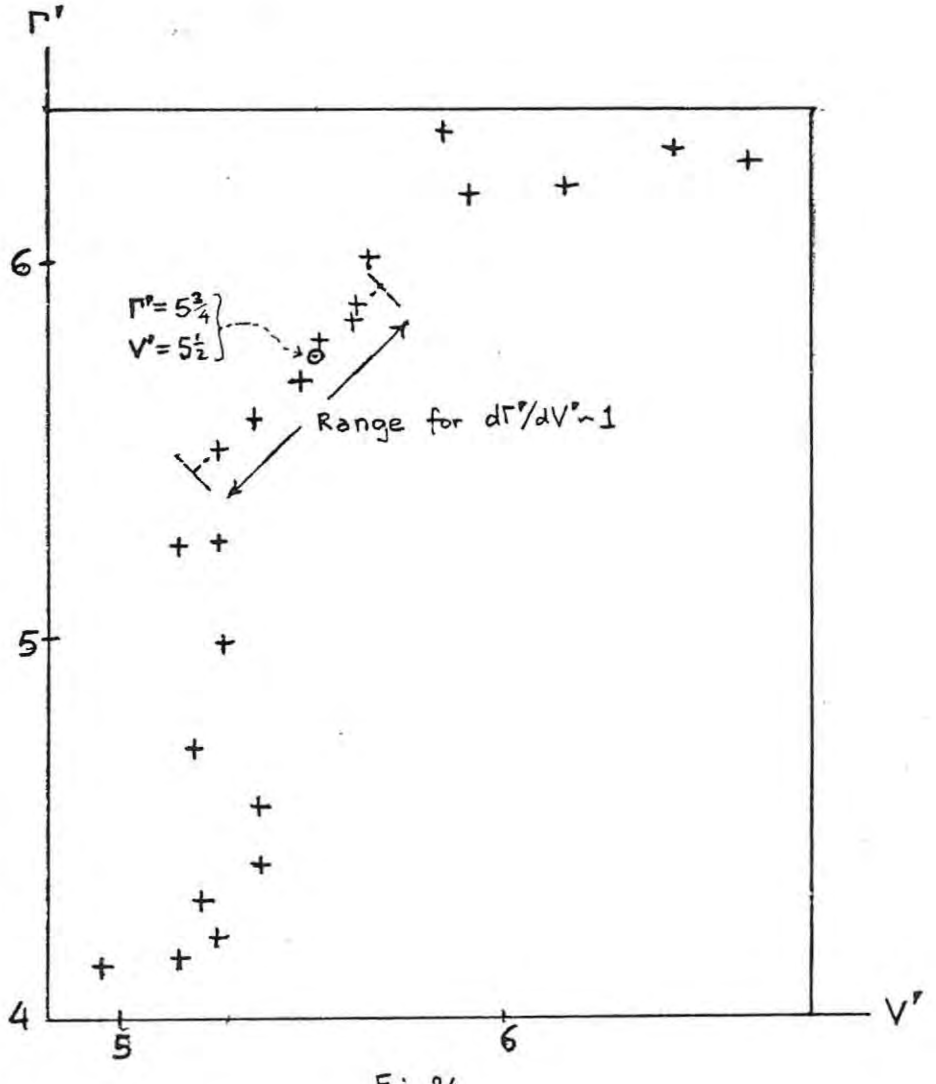
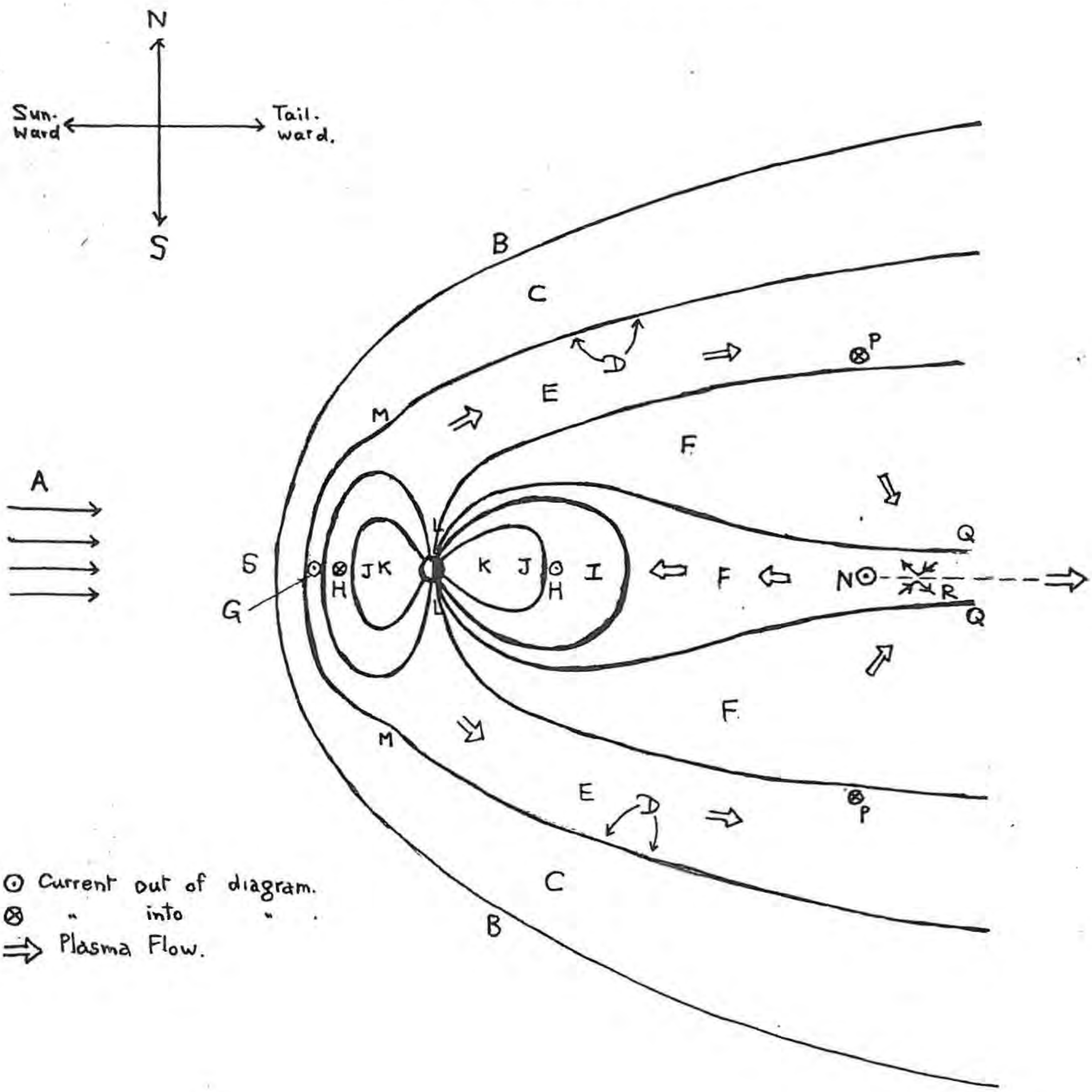



Fig. 24

# MAGNETOSPHERE.




## LEGEND

| | | | |
|---|---|---|---|
| A | Solar Wind | J | Plasma-Sphere |
| B | Bow Shock | K | Trapped Particles |
| C | Magnetosheath | L | Polar Cusp |
| D | Magnetopause | M | Cleft |
| E | Mantle | N | Neutral Sheet Current |
| F | Magnetotail | P | Tail Current, closed with N |
| G | Magnetopause Current | Q | Plasma Sheet |
| H | Ring Current | R | Neutral Sheet, with field line [merging |
| I | Plasma Pause | S | Nose |

APPENDIX B

## COMPUTATIONAL DETAILS

Index | Pages



---o0o---

# FORTRAN PROGRAM, paras 5.1 - 5.3

```
JOB MAJO,AMPT,TERRY                                    RUN BY GEORGE 2/MK9F ON
MINIMOP AMPT                                              06/04/76  AT 15.57
MAXTIME 200
VOLUME 1000
FORTRANCOMP FORT
EXECUTE FORT
****

FORTRAN COMPILATION BY #XFAT MK 5A    DATE  06/04/76  TIME  15/57/32

                 LIST
                 LIBRARY (AMPTROUTINES)
                 PROGRAM (FORT)
                 COMPACT
                 COMPRESS INTEGER AND LOGICAL
                 INPUT 5 = CR0
                 OUTPUT 6 = LP0
                 TRACE 1
                 END

                 TRACE 0
                MASTER INSTABILITY 1

                LOGICAL END
                REAL LAMBDA,JJ,II,I1,J1,I2,J2,NN(2000)
                DIMENSION A1(100),B1(100),C1(100),D1(100),E1(100),F1(100),
               *          G1(100),H1(100),I1(100),J1(100)
                DIMENSION A2(100),B2(100),C2(100),D2(100),E2(100),F2(100),
               *          G2(100),H2(100),I2(100),J2(100)
                DIMENSION U1(100),V1(100),W1(100),X1(100),Y1(100),Z1(100),

                END=.FALSE.

          C READ END OF LOOP VALUES

                READ (5,5000) IDMAX,ICMAX,IBMAX,IAMAX
                X=0.0
          C START LOOPS
                W=0.0
                I=1
                J=1
                DO 300 IDD=1,IDMAX
                ID=IDD-1
                DO 300 ICC=1,ICMAX
                IC=ICC-1
                DO 300 IBB=1,IBMAX
                IB=IBB-1
                DO 300 IAA=1,IAMAX
                IA=IAA-1

          C RESET INITIAL PARAMETERS

                DT=1.0
                LAMBDA=1.0
                F=1.0
                MAXPERIOD=10
                CRITERION=10.0
                A=0.0
                B=1.0
                C=0.1
                D=0.01
                E=0.001
                AA=A
                BB=B
                CC=B
                DD=A
                EE=A
                HH=A
                II=B
                JJ=0.0
                DENSITY=0.0
                THETA=1.0
```

```

C SET SPECIFIC FARAMETERS

      IF (ICC.GT.1) DENSITY=1.0
      IF (IA+IB+IC.EQ.0) GO TO 300
      GO TO (20,21,22,23,24,25,26) , IAA
20    FF=A
      GG=A
      GO TO 29
21    BB=B+C
      GO TO 29
22    BB=B+2.0*C
      CC=B+C
      GO TO 29
23    BB=B-C  & CC = B+C
      GO TO 29
24    BB=B
      CC=B
      FF=C
      GO TO 29
25    FF=2.0*C
      GG=C
      GO TO 29
26    FF=-2.0*C
      GG = C
29    GO TO (30,31,32,33,34,35,36) , IBB
30    Y=A
      Z=A
      GO TO 39
31    Y=C
      GO TO 39
32    Y=2.0*C
      Z=C
      GO TO 39
33    Y=-2.0*C
      GO TO 39
34    Y=A
      GO TO 39
35    Y=C
      Z=2.0*C
      GO TO 39
36    Z=-2.0*C
39    CONTINUE
      GO TO (40,41,42,43,44) , ICC
40    U=A
      V=A
      GO TO 49
41    U=3.0*C
      GO TO 49
42    V=4.0*C
      GO TO 49
43    U=A
      V=3.0*C
      GO TO 49
44    U=4.0*C
49    CONTINUE
      GO TO (59,50,51,52) , IDD
50    W=B
      GO TO 59
51    X=B
      GO TO 59
52    W=A
59    CONTINUE

C ADVANCE DIFFERENCE EQUATIONS

C FIRST STORE INITIAL VALUES

      A1(J)=AA
      B1(J)=BB
      C1(J)=CC
      D1(J)=DD
      E1(J)=EE
      F1(J)=FF
      G1(J)=GG
      H1(J)=HH
      I1(J)=II
      J1(J)=JJ
      U1(J)=U
      V1(J)=V
      W1(J)=W
```

```
      X1(J)=X
      Y1(J)=Y
      Z1(J)=Z

      DO 100 N=1,MAXPERIOD

C CALCULATE INCREMENTS

      DELA=-AA*(EE+HH)+FF*CC-GG*(BB-W)
      DELB=-2.0*BB*HH-FF*(AA-DD)+HH*W+THETA*Y
      DELC=-2.0*CC*EE+GG*(AA-DD)+GG*X+THETA*Z
      DELD=-DD*(EE+HH)-FF*CC+GG*BB+HH*X
      DELE=-EE*EE-FF*GG+II*(LAMBDA*CC*(BB-CC)+II*U*U)
      DELF=-FF*(EE+HH)+II*(LAMBDA*DD*(BB-CC)+II*U*V)
      DELG=-GG*(EE+HH)+II*(-LAMBDA*AA*(BB-CC)+II*U*V)
      DELH=-HH*HH-GG*FF+II*(-LAMBDA*BB*(BB-CC)+II*V*V)
      DELI=II*DENSITY*(EE+HH)
      DELJ=0.0

C AUGMENT VARIABLES

      AA=AA+DT*DELA
      BB=BB+DT*DELB
      CC=CC+DT*DELC
      DD=DD+DT*DELD
      EE=EE+DT*DELE
      FF=FF+DT*DELF
      GG=GG+DT*DELG
      HH=HH+DT*DELH
      II=II+DT*DELI
      JJ=JJ+DT*DELJ
      THETA=0.0

C TEST FOR INSTABILITY

      IF (ABS(AA).GT.CRITERION) GO TO 200
      IF (ABS(BB).GT.CRITERION) GO TO 200
      IF (ABS(CC).GT.CRITERION) GO TO 200
      IF (ABS(DD).GT.CRITERION) GO TO 200
      IF (ABS(EE).GT.CRITERION) GO TO 200
      IF (ABS(FF).GT.CRITERION) GO TO 200
      IF (ABS(GG).GT.CRITERION) GO TO 200
      IF (ABS(HH).GT.CRITERION) GO TO 200
      IF (ABS(1.0/II).GT.CRITERION) GO TO 200
      IF (ABS(JJ).GT.CRITERION) GO TO 200
100   CONTINUE
      N=MAXPERIOD+1

C SYSTEM IS STABLE
C STORE FINAL VALUES

      A2(J)=AA
      B2(J)=BB
      C2(J)=CC
      D2(J)=DD
      E2(J)=EE
      F2(J)=FF
      G2(J)=GG
      H2(J)=HH
      I2(J)=II
      J2(J)=JJ

C INCREASE STABLE COUNTER

      J=J+1

C TEST BUFFER AREA FULL
      IF (J.GT.100) GO TO 400

C STORE PERIOD USED

200   NN(I)=N
      I=I+1
300   CONTINUE
      END=.TRUE.

C PRINTOUT

400   CONTINUE
      WRITE (6,6000)
      WRITE (6,6001) (NN(ICT),ICT=1,I-1)
```

```
                 IF (J-1.LE.0) GO TO 600
                 WRITE (6,6002)
                 DO 500 JC=1,J-1
                 WRITE (6,6003) A1(JC),B1(JC),C1(JC),D1(JC),E1(JC),F1(JC
                *              G1(JC),H1(JC),I1(JC),J1(JC)
                 WRITE (6,6004) A2(JC),B2(JC),C2(JC),D2(JC),E2(JC),F2(JC
                *              G2(JC),H2(JC),I2(JC),J2(JC)
                 WRITE (6,6005) DENSITY,CRITERION,U1(JC),V1(JC),W1(JC),
                *               X1(JC),Y1(JC),Z1(JC)
         500     CONTINUE
                 J=1
         600     IF (END) STOP
                 GO TO 200

         5000    FORMAT (4I0)

         6000    FORMAT (1H1,40X,39HMAGNETOTAIL INSTABILITIES. PROGRAM 1
                *        ,/,1H0,50X,12HTEST PERIODS                    /)
         6001    FORMAT (1H ,24F5.1)
         6002    FORMAT (1H1,36HSTART AND END VALUES - STABLE CASES
         6003    FORMAT (1H0,10F12.5)
         6004    FORMAT (1H ,10F12.5)
         6005    FORMAT (1H ,10HPARAMETERS,9F12.5)
                 END

END OF SEGMENT, LENGTH   907, NAME  INSTABILITY1

                 FINISH

END OF COMPILATION -  NO ERRORS

S/C SUBFILE:            20 BUCKETS USED


CONSOLIDATED BY XPCK 12F    DATE    06/04/76   TIME 15/59/09

PROGRAM FORT
COMPACT DATA (15AM)
COMPACT PROGRAM (DBM)
CORE            13056

         SEG     INSTABILITY1
         SEG     ABS
```

# FORTRAN PROGRAM Para 6.3

ST , NORMELIST : LFOO ON 13/06/75 AT 11.25

```
          CARD LIST.  RUN ON 13/06/75
         MASTER INSTABILITY 2

         LOGICAL END,STARTED
         REAL LAMBDA,JJ,II,I1,J1,I2,J2,NN(2000)
         DIMENSION A1(101),B1(101),C1(101),D1(101),E1(101),F1(101),
        *          G1(101),H1(101),I1(101),J1(101)
         DIMENSION U1(101),V1(101),W1(101),X1(101),Y1(101),Z1(101)


C READ END OF LOOP VALUES

         READ (5,5000) IFMAX,IEMAX,IDMAX,ICMAX,IBMAX,IAMAX

C START LOOPS
         ZZ=0.0
         I=1
         J=1
         DO 300 IFF=1,IFMAX
         IF=IFF-1
         DO 300 IEE=1,IEMAX
         IE=IEE-1
         DO 300 IDD=1,IDMAX
         ID=IDD-1
         DO 300 ICC=1,ICMAX
         IC=ICC-1
         DO 300 IBB=1,IBMAX
         IB=IBB-1
         DO 300 IAA=1,IAMAX
         IA=IAA-1

C RESET INITIAL PARAMETERS

         DT=1.0
         LAMBDA=1.0
         F=1.0
         MAXPERIOD=10
         CRITERION=10.0
         A=0.0
         B=0.1
         C=0.01
         D=0.001
         E=0.0001
         AA=A
         BB=A
         CC=A
         DD=A
         EE=A
         FF=A
         GG=A
         HH=A
         II=A
         JJ=A

C SET SPECIFIC PARAMETERS

         IF (IAA+IBB+ICC+IDD+IEE+IFF.EQ.6) GO TO 300
         GO TO (20,21,22) , IAA
   20    ZZ=A
         GO TO 29
   21    ZZ=-B
         GO TO 29
   22    ZZ=B+2.0*C
   29    CONTINUE
         GO TO (30,31,32) , IBB
   30    YY=A
         GO TO 39
   31    YY=-B+3.0*C
         GO TO 39
   32    YY=B-2.0*C
   39    CONTINUE
         GO TO (40,41,42) , ICC
   40    XX=A
         GO TO 49
   41    XX=B
         GO TO 49
   42    XX=-B-2.0*C
   49    CONTINUE
         GO TO (50,51,52) , IDD
   50    WW=A
```

```
      GO TO 59
 51   WW=-B+2.0*C
      GO TO 59
 52   WW=B-3.0*C
 59   CONTINUE
      GO TO (60,61,62) , IEE
 60   VV=A
      GO TO 69
 61   VV=B+C
      GO TO 69
 62   VV=-B+C
 69   CONTINUE
      GO TO (70,71,72) , IFF
 70   UU=A
      GO TO 79
 71   UU=B-C
      GO TO 79
 72   UU=-B-C
 79   CONTINUE

C ADVANCE DIFFERENCE EQUATIONS

      DO 100 J=1,MAXPERIOD

C FIRST STORE INITIAL VALUES

      A1(J)=AA
      B1(J)=BB
      C1(J)=CC
      D1(J)=DD
      E1(J)=EE
      F1(J)=FF
      G1(J)=GG
      H1(J)=HH
      I1(J)=II
      J1(J)=JJ
      U1(J)=UU
      V1(J)=VV
      W1(J)=WW
      X1(J)=XX
      Y1(J)=YY
      Z1(J)=ZZ

C CALCULATE INCREMENTS

      DELA=F*VV*(AA+CC)-VV+2.0*HH
      DELB=F*II*(AA+CC)-2.0*II+UU
      DELC=F*JJ*(AA+CC)-3.0*JJ+WW
      DELD=F*VV*(DD+FF)-3.0*GG+XX
      DELE=F*II*(DD+FF)-2.0*HH+VV
      DELF=F*JJ*(DD+FF)+2.0*II-UU
      DELG=2.0*LAMBDA*(BB-DD)-YY
      DELH=LAMBDA*(CC-EE)
      DELI=LAMBDA*(DD-BB)
      DELJ=2.0*LAMBDA*(EE-CC)-ZZ

C AUGMENT VARIABLES

      AA=AA+DT*DELA
      BB=BB+DT*DELB
      CC=CC+DT*DELC
      DD=DD+DT*DELD
      EE=EE+DT*DELE
      FF=FF+DT*DELF
      GG=GG+DT*DELG
      HH=HH+DT*DELH
      II=II+DT*DELI
      JJ=JJ+DT*DELJ
      THETA=0.0

C TEST FOR INSTABILITY

      IF (ABS(AA).GT.CRITERION) GO TO 200
      IF (ABS(BB).GT.CRITERION) GO TO 200
      IF (ABS(CC).GT.CRITERION) GO TO 200
      IF (ABS(DD).GT.CRITERION) GO TO 200
      IF (ABS(EE).GT.CRITERION) GO TO 200
      IF (ABS(FF).GT.CRITERION) GO TO 200
      IF (ABS(GG).GT.CRITERION) GO TO 200
      IF (ABS(HH).GT.CRITERION) GO TO 200
      IF (ABS(II).GT.CRITERION) GO TO 200
```

```
      IF (ABS(JJ).GT.CRITERION) GO TO 200
100   CONTINUE
      J=MAXPERIOD+1

C SYSTEM IS STABLE
C STORE FINAL VALUES

      A1(J)=AA
      B1(J)=BB
      C1(J)=CC
      D1(J)=DD
      E1(J)=EE
      F1(J)=FF
      G1(J)=GG
      H1(J)=HH
      I1(J)=II
      J1(J)=JJ
      U1(J)=UU
      V1(J)=VV
      W1(J)=WW
      X1(J)=XX
      Y1(J)=YY
      Z1(J)=ZZ

      DO 500 JC=1,J,MAXPERIOD
      WRITE (6,6003) A1(JC),B1(JC),C1(JC),D1(JC),E1(JC),F1(JC),G1(JC),
     *                H1(JC),I1(JC),J1(JC)
500   CONTINUE
      WRITE (6,6006) IA,IB,IC,ID,IE
      WRITE (6,6005) DENSITY,CRITERION,U1(J),V1(J),W1(J),
     *               X1(J),Y1(J),Z1(J)
200   NN(I)=J
      I=I+1
300   CONTINUE

C PRINTOUT

400   CONTINUE
      WRITE (6,6000)
      WRITE (6,6001) (NN(ICT),ICT=1,I-1)
       STOP
      GO TO 200

5000  FORMAT (10I0)

6000  FORMAT (1H1,40X,39HMAGNETOTAIL INSTABILITIES. PROGRAM 2
     *       ,/,1H0,50X,12HTEST PERIODS                    /)
6001  FORMAT (1H ,24F5.1)
6002  FORMAT (1H1,36HSTART AND END VALUES - STABLE CASES          /)
6003  FORMAT (1H ,10F12.5)
6004  FORMAT (1H ,10F12.5)
6005  FORMAT (1H ,10HPARAMETERS,8F12.5/)
6006  FORMAT (1H ,'IA IB IC ID IE  ',5I2)
      END

      FINISH
```

FORTRAN PROGRAM FOR FIGS. 12 - 16, § 4.3

(RUN BY GEORGE 1S/1C ON 31/08/74)

```
DOC JOB PRINT          IXKEI     31/08/74

       JOB AMPDT-TES1,1-2,TERRY
       FORTRANCOMP FURT
       IN /0
       ENTER
       ****

DOC AMPD               IAMPD     31/08/74

FORTRAN COMPILATION BY #XFAE MK 4D      DATE  31/08/74

      LIST
      WORK(ICLF-DEFAULT)
      RUN

       LIBRARY (AHROUTINES)
       LIBRARY (SUBGROUPSRGP)
       PROGRAM (FURT)
       OUTPUT 4 = LP0
       OUTPUT 6 = /NONE
       INPUT 5 = CR0
       COMPRESS INTEGER AND LOGICAL
       TRACE 2
       END

       MASTER TERRY
       DIMENSION X(200),Y(200),H(4),F(3),G(3)
       COMMON A1,B1,C1,D1
       DATA H(1) /32H
       U=10.
       V=10.
       U=8.
       V=8.
       READ (5,100) XMIN,XMAX,YMIN,YMAX
       CALL ANGRAPH (6,0,8,0,4HAMPT,Y,U,V,H,0)
2      CONTINUE
       X(1)=XMIN
       X(2)=XMAX
       Y(1)=YMIN
       Y(2)=YMAX
       CALL ANGRAPH (6,3,2, 1,X,Y,U,V,H,0)
       READ (5,100) A1,B1,C1,D1,DS
       IF (A1.EQ.99999999.) GO TO 5
       WRITE (4,101) A1,B1,C1,D1,DS
       DO 10 JK=1,2
       NX=20
       NX=12
       DTH=360./(FLOAT(NX))*0.017453293
       TH=0.0
```

```
          R0=0.5*ABS(XMAX)
          NP=20
          DO 1 I=1,NX
          X0=R0*COS(TH)
          Y0=R0*SIN(TH)
          F(1)=0.
          F(2)=X0
          F(3)=Y0
          X(1)=F(2)
          Y(1)=F(3)
          M=1
          DO 3 K=2,NP
          CALL ADAMSBASH (F,G,3,DS,M)
          X(K)=F(2)
          Y(K)=F(3)
          IF (X(K).LT.XMIN) GO TO 4
          IF (X(K).GT.XMAX) GO TO 4
          IF (Y(K).LT.YMIN) GO TO 4
          IF (Y(K).GT.YMAX) GO TO 4
3         CONTINUE
          K=NP+1
4         CALL AMGRAPH (6,2,K-1,1,X,Y,U,V,H,O)
          TH=TH+DTH
1         CONTINUE
10        DS=-DS
          GO TO 2
5         CONTINUE
          CALL AMGRAPH (6,0,0,2,X,Y,U,V,H,O)
          STOP
100       FORMAT (5E0.0)
101       FORMAT (1H0,8F12.4)
          END

END OF SEGMENT, LENGTH   403, NAME  TERRY


          SUBROUTINE DYBYDY (Y,F,N)
          DIMENSION Y(3),F(3)
          COMMON A1,B1,C1,D1
          F(1)=1.0
          F(2)=A1*Y(2)+B1*Y(3)
          F(3)=C1*Y(2)+D1*Y(3)
          F(3)=-F(3)
          RETURN
          END

END OF SEGMENT, LENGTH    94, NAME  DYBYDY


          FINISH

END OF COMPILATION -  NO ERRORS

S/C SUBFILE:      15 BUCKETS USED

DOC AMPD              LAMPD     31/03/74
```

```
CONSOLIDATED BY XPCK 12D   DATE      31/08/74

PROGRAM FORT
COMPACT DATA (15AM)
COMPACT PROGRAM (DBM)
CORE          12544

              SEG      TERRY
              SEG      AMGRAPH
              SEG      FLOAT
              SEG      ABS
              SEG      COS
              SEG      SIN
              SEG      ADAMSBASH
              SEG      DYBYDY
              CUV      PLOTGUBBINS
              CUV      PLOTGUBBINS2
              CUV      PLOTGUBBINS3
              SEG      COPY8
              SEG      IABS
              SEG      HGPLOT
              SEG      HGPSYMBL
              SEG      HGPWHERE
              SEG      HGPNUMBER
              SEG      ALOG10
              SEG      INT
              SEG      AINT
              SEG      SIGN
              SEG      SQRT
              SEG      RKG
              CLV      HGPM3
              SEG      HGPNUM
              CLV      HGPOOL
              CLP      HGPI
              ENT      HGPTSYMB1
              SEG      HGPTSYMB
              CLV      HGPAREA
              CLP      HGPOSITION
              CLP      HGPCMINCH
              SEG      HGPOUT
              SEG      NINT
              SEG      FPROLOG
              SEG      WEP
              ENT      FEPILOG
              CLP      HGPAX
              SEG      GETAH
              SEG      CONVR
              SEG      NEWL
              SEG      ORC
              CUV      HGPBUFFER
              ENT      HGPSETUP
              ENT      HGPUPDAT
              ENT      FTRAP
              ENT      FRESET

LAST BUCKET USED OF PROGRAM DUMP(4095) IS      77

DOC AMPD           LAMPD     31/08/74
```

| | | | | | |
|---|---|---|---|---|---|
| (Fig 16) | 2.0000 | 1.0000 | 4.0000 | 2.0000 | 0.1000 |
| (Fig 12) | 3.0000 | 4.0000 | 2.0000 | 1.0000 | 0.1000 |
| Natural | 4.0000 | 3.0001 | 1.0000 | 2.0000 | 0.1000 |
| (Fig 15) | 4.0000 | 3.0000 | 3.0000 | 2.0000 | 0.1000 |
| (Fig 13) | 4.0000 | 3.0000 | 2.0000 | 1.0000 | 0.1000 |
| (Fig 14) | 4.0000 | 3.0000 | 2.0000 | 2.0000 | 0.1000 |
| | (p) | (q) | (-r) | (-s) | |

(xxxiii)

RESULT OF PROGRAM, para 5·1–5·3

$\Delta t = 1$, Maxperiod = 30, Criterion = 10

MAGNETOTAIL INSTABILITIES, PROGRAM 1

TEST PERIODS

| | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 5,0 | 5,0 | 5,0 | 8,0 | 6,0 | 7,0 | 6,0 | 5,0 | 4,0 | 5,0 | 6,0 | 6,0 | 7,0 | 6,0 | 4,0 | 4,0 | 5,0 | 6,0 | 6,0 | 6,0 | 5,0 | 7,0 | 6,0 | 4,0 |
| 5,0 | 5,0 | 5,0 | 6,0 | 5,0 | 5,0 | 5,0 | 6,0 | 5,0 | 6,0 | 6,0 | 5,0 | 5,0 | 5,0 | 6,0 | 5,0 | 6,0 | 5,0 | 4,0 | 4,0 | 8,0 | 5,0 | 5,0 | 5,0 |
| 6,0 | 5,0 | 5,0 | 6,0 | 6,0 | 6,0 | 7,0 | 5,0 | 4,0 | 4,0 | 7,0 | 5,0 | 5,0 | 6,0 | 5,0 | 4,0 | 4,0 | 7,0 | 5,0 | 5,0 | 5,0 | 6,0 | 6,0 | 6,0 |
| 5,0 | 6,0 | 5,0 | 6,0 | 7,0 | 5,0 | 5,0 | 5,0 | 8,0 | 9,0 | 9,0 | 7,0 | 5,0 | 5,0 | 5,0 | 8,0 | 6,0 | 8,0 | 5,0 | 4,0 | 4,0 | 6,0 | 5,0 | 5,0 |
| 5,0 | 7,0 | 6,0 | 5,0 | 5,0 | 6,0 | 6,0 | 7,0 | 7,0 | 5,0 | 5,0 | 5,0 | 7,0 | 6,0 | 6,0 | 7,0 | 5,0 | 5,0 | 5,0 | 7,0 | 6,0 | 6,0 | 5,0 | 7,0 |
| 7,0 | 4,0 | 5,0 | 5,0 | 6,0 | 6,0 | 7,0 | 6,0 | 5,0 | 5,0 | 5,0 | 7,0 | 6,0 | 6,0 | 6,0 | 5,0 | 5,0 | 5,0 | 7,0 | 6,0 | 5,0 | 5,0 | 7,0 | 7,0 |
| 7,0 | 6,0 | 6,0 | 6,0 | 5,0 | 5,0 | 6,0 | 6,0 | 6,0 | 7,0 | 5,0 | 5,0 | 5,0 | 7,0 | 6,0 | 7,0 | 7,0 | 5,0 | 5,0 | 5,0 | 7,0 | 6,0 | 6,0 | 5,0 |
| 6,0 | 6,0 | 4,0 | 5,0 | 5,0 | 5,0 | 5,0 | 6,0 | 6,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 6,0 | 6,0 | 4,0 | 5,0 | 5,0 | 5,0 | 6,0 | 5,0 | 4,0 | 6,0 |
| 6,0 | 6,0 | 6,0 | 7,0 | 5,0 | 5,0 | 6,0 | 7,0 | 8,0 | 6,0 | 6,0 | 5,0 | 4,0 | 8,0 | 6,0 | 6,0 | 5,0 | 6,0 | 4,0 | 4,0 | 7,0 | 6,0 | 6,0 | 5,0 |
| 6,0 | 7,0 | 6,0 | 5,0 | 5,0 | 5,0 | 7,0 | 7,0 | 5,0 | 5,0 | 5,0 | 7,0 | 6,0 | 6,0 | 7,0 | 5,0 | 5,0 | 5,0 | 7,0 | 6,0 | 6,0 | 5,0 | 4,0 | 4,0 |
| 7,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 8,0 | 6,0 | 8,0 | 6,0 | 5,0 | 5,0 | 5,0 | 6,0 | 8,0 | 7,0 | 6,0 | 5,0 | 4,0 | 5,0 | 6,0 | 7,0 | 6,0 |
| 5,0 | 7,0 | 8,0 | 4,0 | 5,0 | 5,0 | 5,0 | 6,0 | 5,0 | 5,0 | 5,0 | 6,0 | 5,0 | 6,0 | 6,0 | 5,0 | 5,0 | 5,0 | 6,0 | 5,0 | 6,0 | 5,0 | 4,0 | 4,0 |
| 7,0 | 5,0 | 5,0 | 5,0 | 6,0 | 5,0 | 5,0 | 6,0 | 6,0 | 7,0 | 7,0 | 5,0 | 5,0 | 4,0 | 9,0 | 5,0 | 6,0 | 6,0 | 5,0 | 4,0 | 4,0 | 10,0 | 5,0 | 6,0 |
| 6,0 | 6,0 | 6,0 | 6,0 | 5,0 | 6,0 | 5,0 | 6,0 | 7,0 | 5,0 | 5,0 | 5,0 | 7,0 | 7,0 | 9,0 | 7,0 | 5,0 | 5,0 | 5,0 | 7,0 | 7,0 | 8,0 | 5,0 | 4,0 |
| 4,0 | 6,0 | 5,0 | 5,0 | 5,0 | 7,0 | 6,0 | 5,0 | 5,0 | 7,0 | 6,0 | 7,0 | 7,0 | 5,0 | 5,0 | 6,0 | 7,0 | 6,0 | 6,0 | 7,0 | 5,0 | 5,0 | 6,0 | 7,0 |
| 6,0 | 6,0 | 5,0 | 8,0 | 8,0 | 5,0 | 5,0 | 5,0 | 7,0 | 6,0 | 7,0 | 6,0 | 5,0 | 6,0 | 5,0 | 7,0 | 6,0 | 6,0 | 6,0 | 5,0 | 6,0 | 5,0 | 7,0 | 6,0 |
| 5,0 | 5,0 | 7,0 | 7,0 | 8,0 | 5,0 | 6,0 | 6,0 | 5,0 | 5,0 | 6,0 | 6,0 | 7,0 | 11,0 | 5,0 | 5,0 | 5,0 | 9,0 | 6,0 | 9,0 | 11,0 | 5,0 | 5,0 | 5,0 |
| 9,0 | 6,0 | 9,0 | 5,0 | 6,0 | 6,0 | 4,0 | 5,0 | 5,0 | 5,0 | 6,0 | 6,0 | 6,0 | 5,0 | 5,0 | 5,0 | 6,0 | 5,0 | 6,0 | 6,0 | 5,0 | 5,0 | 5,0 | 5,0 |
| 6,0 | 5,0 | 5,0 | 7,0 | 6,0 | 6,0 | 6,0 | 7,0 | 5,0 | 5,0 | 7,0 | 8,0 | 7,0 | 6,0 | 6,0 | 5,0 | 5,0 | 7,0 | 7,0 | 8,0 | 5,0 | 6,0 | 5,0 | 4,0 |
| 7,0 | 6,0 | 8,0 | 5,0 | 7,0 | 7,0 | 6,0 | 5,0 | 6,0 | 5,0 | 7,0 | 8,0 | 5,0 | 5,0 | 6,0 | 7,0 | 6,0 | 6,0 | 8,0 | 5,0 | 5,0 | 6,0 | 7,0 | 6,0 |
| 6,0 | 5,0 | 4,0 | 4,0 | 7,0 | 5,0 | 6,0 | 5,0 | 5,0 | 5,0 | 5,0 | 8,0 | 5,0 | 6,0 | 6,0 | 5,0 | 5,0 | 5,0 | 6,0 | 5,0 | 6,0 | 6,0 | 5,0 | 4,0 |
| 5,0 | 6,0 | 5,0 | 6,0 | 5,0 | 7,0 | 8,0 | 4,0 | 5,0 | 4,0 | 5,0 | 6,0 | 5,0 | 5,0 | 5,0 | 6,0 | 5,0 | 5,0 | 6,0 | 5,0 | 5,0 | 5,0 | 6,0 | 5,0 |
| 5,0 | 5,0 | 4,0 | 4,0 | 7,0 | 5,0 | 6,0 | 6,0 | 6,0 | 5,0 | 5,0 | 6,0 | 6,0 | 6,0 | 9,0 | 5,0 | 5,0 | 4,0 | 9,0 | 5,0 | 8,0 | 7,0 | 5,0 | 4,0 |
| 4,0 | 10,0 | 5,0 | 7,0 | 6,0 | 6,0 | 6,0 | 6,0 | 5,0 | 6,0 | 5,0 | 5,0 | 7,0 | 5,0 | 5,0 | 5,0 | 7,0 | 5,0 | 6,0 | 7,0 | 5,0 | 5,0 | 5,0 | 7,0 |
| 5,0 | 6,0 | 5,0 | 4,0 | 4,0 | 6,0 | 5,0 | 6,0 | 6,0 | 7,0 | 6,0 | 6,0 | 5,0 | 6,0 | 5,0 | 7,0 | 7,0 | 5,0 | 5,0 | 5,0 | 7,0 | 5,0 | 7,0 | 7,0 |
| 5,0 | 5,0 | 5,0 | 7,0 | 5,0 | 7,0 | 5,0 | 6,0 | 6,0 | 4,0 | 5,0 | 4,0 | 5,0 | 6,0 | 8,0 | 7,0 | 5,0 | 5,0 | 5,0 | 6,0 | 6,0 | 7,0 | 7,0 | 5,0 |
| 5,0 | 4,0 | 5,0 | 7,0 | 5,0 | 5,0 | 7,0 | 6,0 | 6,0 | 6,0 | 6,0 | 6,0 | 5,0 | 5,0 | 6,0 | 5,0 | 5,0 | 11,0 | 5,0 | 5,0 | 5,0 | 10,0 | 5,0 | 6,0 |
| 10,0 | 5,0 | 5,0 | 5,0 | 10,0 | 5,0 | 6,0 | 5,0 | 6,0 | 6,0 | 4,0 | 5,0 | 4,0 | 5,0 | 6,0 | 6,0 | 6,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 6,0 | 6,0 |
| 5,0 | 5,0 | 5,0 | 5,0 | 6,0 | 5,0 | 5,0 | 7,0 | 6,0 | 8,0 | 6,0 | 8,0 | 5,0 | 5,0 | 6,0 | 7,0 | 5,0 | 6,0 | 6,0 | 5,0 | 5,0 | 7,0 | 7,0 | 6,0 |
| 6,0 | 6,0 | 5,0 | 5,0 | 7,0 | 7,0 | 6,0 | 6,0 | 6,0 | 8,0 | 6,0 | 5,0 | 5,0 | 5,0 | 6,0 | 7,0 | 6,0 | 5,0 | 5,0 | 6,0 | 5,0 | 7,0 | 7,0 | 5,0 |
| 5,0 | 5,0 | 6,0 | 5,0 | 7,0 | 5,0 | 5,0 | 4,0 | 7,0 | 5,0 | 7,0 | 5,0 | 5,0 | 5,0 | 5,0 | 8,0 | 5,0 | 5,0 | 6,0 | 5,0 | 4,0 | 5,0 | 6,0 | 6,0 |
| 6,0 | 6,0 | 4,0 | 4,0 | 5,0 | 6,0 | 6,0 | 6,0 | 5,0 | 7,0 | 6,0 | 4,0 | 5,0 | 4,0 | 5,0 | 6,0 | 5,0 | 5,0 | 5,0 | 6,0 | 5,0 | 5,0 | 6,0 | 5,0 |
| 5,0 | 5,0 | 6,0 | 5,0 | 5,0 | 5,0 | 4,0 | 4,0 | 8,0 | 5,0 | 5,0 | 6,0 | 6,0 | 5,0 | 5,0 | 6,0 | 6,0 | 6,0 | 7,0 | 5,0 | 4,0 | 4,0 | 7,0 | 5,0 |
| 6,0 | 7,0 | 5,0 | 4,0 | 4,0 | 7,0 | 5,0 | 6,0 | 7,0 | 6,0 | 6,0 | 6,0 | 5,0 | 6,0 | 5,0 | 5,0 | 7,0 | 5,0 | 5,0 | 5,0 | 8,0 | 5,0 | 6,0 | 7,0 |
| 5,0 | 5,0 | 5,0 | 8,0 | 5,0 | 6,0 | 5,0 | 4,0 | 4,0 | 6,0 | 5,0 | 5,0 | 5,0 | 6,0 | 6,0 | 6,0 | 5,0 | 5,0 | 5,0 | 6,0 | 7,0 | 5,0 | 5,0 | 5,0 |
| 6,0 | 5,0 | 7,0 | 7,0 | 5,0 | 5,0 | 5,0 | 6,0 | 5,0 | 7,0 | 5,0 | 6,0 | 6,0 | 4,0 | 5,0 | 4,0 | 5,0 | 5,0 | 7,0 | 7,0 | 5,0 | 5,0 | 5,0 | 5,0 |
| 5,0 | 7,0 | 7,0 | 4,0 | 5,0 | 4,0 | 5,0 | 7,0 | 5,0 | 5,0 | 6,0 | 7,0 | 6,0 | 6,0 | 6,0 | 6,0 | 5,0 | 5,0 | 6,0 | 5,0 | 5,0 | 7,0 | 5,0 | 5,0 |
| 5,0 | 7,0 | 5,0 | 5,0 | 7,0 | 5,0 | 5,0 | 5,0 | 7,0 | 5,0 | 5,0 | 5,0 | 6,0 | 6,0 | 4,0 | 5,0 | 4,0 | 4,0 | 5,0 | 6,0 | 6,0 | 5,0 | 5,0 | 5,0 |
| 5,0 | 5,0 | 6,0 | 6,0 | 4,0 | 5,0 | 5,0 | 5,0 | 6,0 | 5,0 | 4,0 | 6,0 | 6,0 | 7,0 | 8,0 | 7,0 | 5,0 | 5,0 | 6,0 | 7,0 | 5,0 | 7,0 | 6,0 | 5,0 |
| 5,0 | 6,0 | 7,0 | 6,0 | 6,0 | 6,0 | 5,0 | 4,0 | 6,0 | 7,0 | 6,0 | 6,0 | 6,0 | 8,0 | 7,0 | 5,0 | 5,0 | 5,0 | 5,0 | 7,0 | 5,0 | 5,0 | 5,0 | 6,0 |
| 5,0 | 7,0 | 7,0 | 5,0 | 5,0 | 5,0 | 6,0 | 5,0 | 7,0 | 5,0 | 4,0 | 4,0 | 8,0 | 5,0 | 6,0 | 5,0 | | | | | | | | |

(xxxiii')
RESULT OF PROGRAM, para 5.1-5.3.
Δt = 0.1, Maxperiod = 100, Criterion = 10

MAGNETOTAIL INSTABILITIES, PROGRAM 1
TEST PERIODS

| | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 22,0 | 20,0 | 23,0 | 39,0 | 31,0 | 35,0 | 36,0 | 21,0 | 20,0 | 25,0 | 41,0 | 35,0 | 41,0 | 36,0 | 21,0 | 20,0 | 23,0 | 40,0 | 35,0 | 40,0 | 30,0 | 23,0 | 22,0 | 22,0 |
| 29,0 | 26,0 | 29,0 | 36,0 | 22,0 | 21,0 | 22,0 | 33,0 | 29,0 | 32,0 | 36,0 | 22,0 | 21,0 | 22,0 | 33,0 | 29,0 | 32,0 | 30,0 | 20,0 | 19,0 | 25,0 | 31,0 | 33,0 | 37,0 |
| 29,0 | 21,0 | 20,0 | 28,0 | 29,0 | 30,0 | 32,0 | 27,0 | 20,0 | 19,0 | 30,0 | 28,0 | 29,0 | 30,0 | 27,0 | 20,0 | 19,0 | 29,0 | 28,0 | 28,0 | 30,0 | 34,0 | 22,0 | 21,0 |
| 26,0 | 35,0 | 41,0 | 44,0 | 30,0 | 21,0 | 20,0 | 27,0 | 31,0 | 32,0 | 34,0 | 30,0 | 21,0 | 20,0 | 27,0 | 30,0 | 32,0 | 34,0 | 26,0 | 20,0 | 19,0 | 34,0 | 26,0 | 27,0 |
| 28,0 | 34,0 | 28,0 | 25,0 | 24,0 | 30,0 | 28,0 | 39,0 | 36,0 | 26,0 | 24,0 | 24,0 | 31,0 | 29,0 | 38,0 | 35,0 | 26,0 | 24,0 | 24,0 | 31,0 | 28,0 | 38,0 | 29,0 | 32,0 |
| 29,0 | 23,0 | 27,0 | 25,0 | 39,0 | 32,0 | 29,0 | 26,0 | 23,0 | 29,0 | 27,0 | 39,0 | 32,0 | 28,0 | 26,0 | 23,0 | 29,0 | 27,0 | 39,0 | 42,0 | 25,0 | 23,0 | 26,0 | 35,0 |
| 31,0 | 39,0 | 29,0 | 26,0 | 24,0 | 22,0 | 28,0 | 26,0 | 28,0 | 30,0 | 25,0 | 23,0 | 22,0 | 29,0 | 27,0 | 29,0 | 30,0 | 25,0 | 23,0 | 22,0 | 29,0 | 27,0 | 29,0 | 26,0 |
| 30,0 | 27,0 | 21,0 | 26,0 | 24,0 | 26,0 | 27,0 | 27,0 | 24,0 | 22,0 | 27,0 | 26,0 | 27,0 | 27,0 | 26,0 | 24,0 | 22,0 | 27,0 | 25,0 | 27,0 | 34,0 | 24,0 | 22,0 | 23,0 |
| 32,0 | 30,0 | 32,0 | 34,0 | 22,0 | 21,0 | 31,0 | 39,0 | 40,0 | 31,0 | 32,0 | 22,0 | 21,0 | 33,0 | 35,0 | 35,0 | 30,0 | 32,0 | 22,0 | 20,0 | 32,0 | 35,0 | 35,0 | 30,0 |
| 42,0 | 24,0 | 22,0 | 28,0 | 42,0 | 38,0 | 35,0 | 36,0 | 23,0 | 21,0 | 29,0 | 44,0 | 49,0 | 32,0 | 35,0 | 23,0 | 21,0 | 29,0 | 44,0 | 49,0 | 32,0 | 29,0 | 21,0 | 20,0 |
| 39,0 | 32,0 | 31,0 | 28,0 | 24,0 | 23,0 | 24,0 | 40,0 | 29,0 | 42,0 | 41,0 | 24,0 | 22,0 | 24,0 | 51,0 | 30,0 | 64,0 | 41,0 | 24,0 | 22,0 | 24,0 | 50,0 | 30,0 | 63,0 |
| 32,0 | 26,0 | 24,0 | 25,0 | 30,0 | 26,0 | 32,0 | 38,0 | 25,0 | 25,0 | 25,0 | 35,0 | 28,0 | 36,0 | 38,0 | 25,0 | 25,0 | 23,0 | 35,0 | 28,0 | 36,0 | 34,0 | 23,0 | 22,0 |
| 26,0 | 36,0 | 36,0 | 41,0 | 31,0 | 23,0 | 22,0 | 32,0 | 32,0 | 37,0 | 34,0 | 30,0 | 23,0 | 21,0 | 34,0 | 31,0 | 35,0 | 33,0 | 30,0 | 23,0 | 21,0 | 34,0 | 31,0 | 34,0 |
| 32,0 | 35,0 | 25,0 | 23,0 | 28,0 | 37,0 | 75,0 | 40,0 | 32,0 | 24,0 | 22,0 | 30,0 | 33,0 | 40,0 | 35,0 | 32,0 | 25,0 | 22,0 | 30,0 | 33,0 | 40,0 | 35,0 | 29,0 | 22,0 |
| 21,0 | 51,0 | 29,0 | 32,0 | 31,0 | 41,0 | 29,0 | 27,0 | 27,0 | 36,0 | 26,0 | 41,0 | 41,0 | 28,0 | 26,0 | 27,0 | 39,0 | 29,0 | 39,0 | 41,0 | 26,0 | 26,0 | 27,0 | 39,0 |
| 29,0 | 39,0 | 37,0 | 32,0 | 29,0 | 25,0 | 31,0 | 26,0 | 40,0 | 41,0 | 30,0 | 27,0 | 26,0 | 34,0 | 27,0 | 42,0 | 41,0 | 30,0 | 27,0 | 25,0 | 34,0 | 27,0 | 42,0 | 42,0 |
| 27,0 | 25,0 | 29,0 | 41,0 | 32,0 | 37,0 | 32,0 | 29,0 | 26,0 | 25,0 | 31,0 | 27,0 | 32,0 | 34,0 | 28,0 | 25,0 | 24,0 | 33,0 | 28,0 | 34,0 | 34,0 | 27,0 | 25,0 | 24,0 |
| 33,0 | 28,0 | 34,0 | 28,0 | 32,0 | 29,0 | 22,0 | 28,0 | 25,0 | 28,0 | 30,0 | 29,0 | 27,0 | 25,0 | 30,0 | 26,0 | 31,0 | 30,0 | 29,0 | 27,0 | 23,0 | 29,0 | 26,0 | 30,0 |
| 46,0 | 26,0 | 24,0 | 25,0 | 40,0 | 31,0 | 45,0 | 33,0 | 25,0 | 25,0 | 42,0 | 39,0 | 45,0 | 31,0 | 32,0 | 24,0 | 22,0 | 41,0 | 36,0 | 48,0 | 30,0 | 32,0 | 24,0 | 22,0 |
| 41,0 | 36,0 | 49,0 | 30,0 | 38,0 | 26,0 | 24,0 | 33,0 | 44,0 | 35,0 | 33,0 | 34,0 | 25,0 | 25,0 | 39,0 | 42,0 | 41,0 | 31,0 | 34,0 | 25,0 | 23,0 | 38,0 | 41,0 | 41,0 |
| 31,0 | 31,0 | 23,0 | 22,0 | 41,0 | 34,0 | 47,0 | 29,0 | 24,0 | 25,0 | 24,0 | 40,0 | 23,0 | 27,0 | 41,0 | 24,0 | 22,0 | 24,0 | 50,0 | 24,0 | 28,0 | 40,0 | 24,0 | 22,0 |
| 24,0 | 49,0 | 24,0 | 27,0 | 32,0 | 26,0 | 24,0 | 25,0 | 30,0 | 22,0 | 25,0 | 38,0 | 25,0 | 25,0 | 23,0 | 35,0 | 23,0 | 26,0 | 38,0 | 25,0 | 23,0 | 23,0 | 34,0 | 23,0 |
| 26,0 | 34,0 | 23,0 | 22,0 | 26,0 | 36,0 | 25,0 | 30,0 | 31,0 | 25,0 | 22,0 | 32,0 | 32,0 | 30,0 | 58,0 | 30,0 | 25,0 | 21,0 | 34,0 | 31,0 | 32,0 | 52,0 | 30,0 | 25,0 |
| 21,0 | 33,0 | 31,0 | 32,0 | 52,0 | 35,0 | 25,0 | 23,0 | 28,0 | 37,0 | 27,0 | 33,0 | 32,0 | 24,0 | 22,0 | 30,0 | 33,0 | 28,0 | 40,0 | 32,0 | 23,0 | 22,0 | 30,0 | 33,0 |
| 28,0 | 40,0 | 29,0 | 22,0 | 21,0 | 50,0 | 29,0 | 39,0 | 41,0 | 35,0 | 32,0 | 29,0 | 24,0 | 29,0 | 22,0 | 30,0 | 35,0 | 31,0 | 28,0 | 25,0 | 30,0 | 25,0 | 31,0 | 35,0 |
| 31,0 | 27,0 | 24,0 | 30,0 | 23,0 | 31,0 | 29,0 | 41,0 | 33,0 | 25,0 | 27,0 | 21,0 | 27,0 | 32,0 | 34,0 | 30,0 | 24,0 | 28,0 | 22,0 | 29,0 | 31,0 | 33,0 | 29,0 | 24,0 |
| 28,0 | 22,0 | 28,0 | 41,0 | 29,0 | 26,0 | 26,0 | 33,0 | 24,0 | 37,0 | 32,0 | 29,0 | 26,0 | 25,0 | 31,0 | 25,0 | 25,0 | 34,0 | 28,0 | 25,0 | 24,0 | 33,0 | 25,0 | 26,0 |
| 34,0 | 27,0 | 25,0 | 24,0 | 33,0 | 23,0 | 26,0 | 28,0 | 32,0 | 29,0 | 22,0 | 28,0 | 22,0 | 24,0 | 30,0 | 29,0 | 27,0 | 23,0 | 30,0 | 22,0 | 24,0 | 30,0 | 29,0 | 27,0 |
| 23,0 | 29,0 | 22,0 | 24,0 | 45,0 | 26,0 | 24,0 | 25,0 | 41,0 | 24,0 | 27,0 | 40,0 | 26,0 | 24,0 | 31,0 | 42,0 | 26,0 | 40,0 | 37,0 | 25,0 | 23,0 | 32,0 | 44,0 | 26,0 |
| 38,0 | 37,0 | 25,0 | 23,0 | 31,0 | 44,0 | 26,0 | 38,0 | 42,0 | 28,0 | 25,0 | 27,0 | 41,0 | 24,0 | 39,0 | 44,0 | 26,0 | 24,0 | 29,0 | 42,0 | 25,0 | 40,0 | 43,0 | 26,0 |
| 24,0 | 29,0 | 42,0 | 25,0 | 40,0 | 34,0 | 24,0 | 23,0 | 37,0 | 43,0 | 26,0 | 36,0 | 22,0 | 20,0 | 23,0 | 38,0 | 24,0 | 26,0 | 36,0 | 21,0 | 20,0 | 23,0 | 40,0 | 24,0 |
| 26,0 | 35,0 | 21,0 | 20,0 | 23,0 | 40,0 | 24,0 | 26,0 | 30,0 | 23,0 | 22,0 | 22,0 | 29,0 | 22,0 | 24,0 | 36,0 | 22,0 | 21,0 | 22,0 | 33,0 | 23,0 | 25,0 | 36,0 | 22,0 |
| 21,0 | 22,0 | 33,0 | 23,0 | 25,0 | 30,0 | 20,0 | 19,0 | 25,0 | 31,0 | 26,0 | 28,0 | 29,0 | 21,0 | 20,0 | 28,0 | 29,0 | 31,0 | 35,0 | 27,0 | 20,0 | 19,0 | 30,0 | 28,0 |
| 34,0 | 40,0 | 27,0 | 20,0 | 19,0 | 29,0 | 28,0 | 34,0 | 40,0 | 34,0 | 22,0 | 21,0 | 26,0 | 35,0 | 27,0 | 29,0 | 30,0 | 21,0 | 20,0 | 27,0 | 30,0 | 29,0 | 32,0 | 30,0 |
| 21,0 | 20,0 | 27,0 | 30,0 | 29,0 | 32,0 | 26,0 | 20,0 | 19,0 | 34,0 | 26,0 | 38,0 | 43,0 | 28,0 | 31,0 | 27,0 | 25,0 | 27,0 | 22,0 | 26,0 | 29,0 | 29,0 | 26,0 | 23,0 |
| 27,0 | 22,0 | 27,0 | 29,0 | 29,0 | 26,0 | 23,0 | 27,0 | 22,0 | 26,0 | 26,0 | 42,0 | 35,0 | 22,0 | 25,0 | 21,0 | 24,0 | 27,0 | 34,0 | 29,0 | 22,0 | 26,0 | 22,0 | 25,0 |
| 27,0 | 33,0 | 28,0 | 22,0 | 26,0 | 22,0 | 25,0 | 32,0 | 27,0 | 24,0 | 24,0 | 29,0 | 23,0 | 28,0 | 29,0 | 26,0 | 24,0 | 22,0 | 28,0 | 22,0 | 24,0 | 30,0 | 25,0 | 25,0 |
| 22,0 | 29,0 | 23,0 | 24,0 | 30,0 | 25,0 | 23,0 | 22,0 | 29,0 | 23,0 | 24,0 | 26,0 | 30,0 | 27,0 | 21,0 | 26,0 | 21,0 | 22,0 | 27,0 | 27,0 | 24,0 | 22,0 | 27,0 | 22,0 |
| 23,0 | 27,0 | 26,0 | 24,0 | 22,0 | 27,0 | 22,0 | 23,0 | 34,0 | 24,0 | 22,0 | 23,0 | 33,0 | 24,0 | 25,0 | 41,0 | 25,0 | 22,0 | 27,0 | 44,0 | 26,0 | 38,0 | 40,0 | 23,0 |
| 21,0 | 27,0 | 42,0 | 27,0 | 38,0 | 39,0 | 23,0 | 21,0 | 27,0 | 42,0 | 27,0 | 38,0 | 43,0 | 25,0 | 23,0 | 25,0 | 33,0 | 24,0 | 34,0 | 41,0 | 24,0 | 22,0 | 26,0 | 38,0 |
| 25,0 | 40,0 | 41,0 | 24,0 | 22,0 | 26,0 | 38,0 | 25,0 | 40,0 | 34,0 | 22,0 | 20,0 | 30,0 | 41,0 | 29,0 | 39,0 | | | | | | | | |

(xxxiii'')
RESULT OF PROGRAM, para 5.1 - 5.3
$\Delta t = 1$, Maxperiod = 30, Criterion = 100

## MAGNETOTAIL INSTABILITIES, PROGRAM 1
### TEST PERIODS

| | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 6.0 | 5.0 | 6.0 | 8.0 | 7.0 | 8.0 | 7.0 | 5.0 | 5.0 | 6.0 | 7.0 | 7.0 | 7.0 | 6.0 | 5.0 | 5.0 | 6.0 | 7.0 | 7.0 | 7.0 | 6.0 | 8.0 | 7.0 | 5.0 |
| 6.0 | 6.0 | 6.0 | 7.0 | 6.0 | 6.0 | 5.0 | 7.0 | 6.0 | 6.0 | 6.0 | 6.0 | 6.0 | 5.0 | 6.0 | 6.0 | 6.0 | 6.0 | 5.0 | 5.0 | 9.0 | 6.0 | 6.0 | 6.0 |
| 7.0 | 5.0 | 5.0 | 7.0 | 7.0 | 7.0 | 8.0 | 6.0 | 5.0 | 5.0 | 8.0 | 6.0 | 6.0 | 6.0 | 6.0 | 5.0 | 5.0 | 8.0 | 6.0 | 6.0 | 6.0 | 7.0 | 7.0 | 6.0 |
| 6.0 | 7.0 | 6.0 | 6.0 | 8.0 | 6.0 | 5.0 | 6.0 | 9.0 | 10.0 | 10.0 | 8.0 | 6.0 | 5.0 | 6.0 | 9.0 | 9.0 | 9.0 | 6.0 | 5.0 | 5.0 | 7.0 | 6.0 | 6.0 |
| 6.0 | 8.0 | 7.0 | 6.0 | 6.0 | 7.0 | 6.0 | 8.0 | 8.0 | 6.0 | 6.0 | 6.0 | 8.0 | 7.0 | 7.0 | 8.0 | 6.0 | 6.0 | 6.0 | 8.0 | 7.0 | 7.0 | 6.0 | 8.0 |
| 8.0 | 5.0 | 6.0 | 6.0 | 7.0 | 7.0 | 8.0 | 7.0 | 6.0 | 6.0 | 6.0 | 8.0 | 7.0 | 7.0 | 7.0 | 5.0 | 6.0 | 6.0 | 8.0 | 7.0 | 6.0 | 5.0 | 8.0 | 8.0 |
| 8.0 | 6.0 | 7.0 | 6.0 | 6.0 | 6.0 | 7.0 | 6.0 | 7.0 | 8.0 | 6.0 | 6.0 | 6.0 | 8.0 | 7.0 | 8.0 | 8.0 | 6.0 | 6.0 | 6.0 | 8.0 | 7.0 | 8.0 | 6.0 |
| 7.0 | 7.0 | 5.0 | 6.0 | 5.0 | 6.0 | 6.0 | 7.0 | 7.0 | 5.0 | 6.0 | 6.0 | 6.0 | 6.0 | 7.0 | 6.0 | 5.0 | 6.0 | 6.0 | 6.0 | 7.0 | 5.0 | 5.0 | 7.0 |
| 7.0 | 7.0 | 7.0 | 8.0 | 6.0 | 6.0 | 7.0 | 8.0 | 9.0 | 7.0 | 7.0 | 5.0 | 5.0 | 8.0 | 7.0 | 7.0 | 6.0 | 7.0 | 5.0 | 5.0 | 6.0 | 7.0 | 7.0 | 6.0 |
| 7.0 | 8.0 | 7.0 | 6.0 | 6.0 | 6.0 | 8.0 | 8.0 | 6.0 | 6.0 | 6.0 | 8.0 | 7.0 | 7.0 | 8.0 | 6.0 | 6.0 | 6.0 | 8.0 | 7.0 | 7.0 | 6.0 | 5.0 | 5.0 |
| 8.0 | 6.0 | 6.0 | 6.0 | 6.0 | 6.0 | 6.0 | 9.0 | 7.0 | 9.0 | 7.0 | 6.0 | 5.0 | 6.0 | 7.0 | 8.0 | 8.0 | 7.0 | 5.0 | 5.0 | 6.0 | 7.0 | 8.0 | 7.0 |
| 6.0 | 8.0 | 9.0 | 5.0 | 6.0 | 6.0 | 6.0 | 7.0 | 6.0 | 6.0 | 6.0 | 7.0 | 6.0 | 7.0 | 7.0 | 6.0 | 6.0 | 5.0 | 6.0 | 6.0 | 7.0 | 6.0 | 5.0 | 5.0 |
| 8.0 | 6.0 | 6.0 | 6.0 | 7.0 | 6.0 | 6.0 | 7.0 | 7.0 | 8.0 | 8.0 | 6.0 | 5.0 | 5.0 | 10.0 | 6.0 | 7.0 | 7.0 | 6.0 | 5.0 | 5.0 | 11.0 | 6.0 | 6.0 |
| 6.0 | 7.0 | 7.0 | 7.0 | 6.0 | 7.0 | 6.0 | 7.0 | 8.0 | 6.0 | 6.0 | 6.0 | 8.0 | 8.0 | 10.0 | 8.0 | 6.0 | 6.0 | 6.0 | 8.0 | 8.0 | 9.0 | 6.0 | 5.0 |
| 5.0 | 7.0 | 6.0 | 6.0 | 6.0 | 8.0 | 7.0 | 7.0 | 6.0 | 7.0 | 6.0 | 8.0 | 8.0 | 6.0 | 6.0 | 7.0 | 8.0 | 7.0 | 7.0 | 9.0 | 6.0 | 6.0 | 6.0 | 9.0 |
| 7.0 | 7.0 | 6.0 | 8.0 | 8.0 | 5.0 | 6.0 | 6.0 | 8.0 | 7.0 | 8.0 | 7.0 | 6.0 | 6.0 | 6.0 | 8.0 | 7.0 | 7.0 | 7.0 | 6.0 | 6.0 | 6.0 | 8.0 | 7.0 |
| 6.0 | 6.0 | 8.0 | 8.0 | 9.0 | 6.0 | 7.0 | 7.0 | 6.0 | 6.0 | 7.0 | 6.0 | 7.0 | 11.0 | 6.0 | 6.0 | 6.0 | 11.0 | 7.0 | 10.0 | 13.0 | 6.0 | 6.0 | 6.0 |
| 11.0 | 7.0 | 11.0 | 6.0 | 7.0 | 7.0 | 5.0 | 6.0 | 5.0 | 6.0 | 6.0 | 7.0 | 7.0 | 5.0 | 6.0 | 6.0 | 6.0 | 6.0 | 7.0 | 7.0 | 5.0 | 6.0 | 6.0 | 6.0 |
| 7.0 | 6.0 | 5.0 | 8.0 | 7.0 | 7.0 | 7.0 | 8.0 | 6.0 | 6.0 | 8.0 | 8.0 | 7.0 | 7.0 | 7.0 | 6.0 | 5.0 | 8.0 | 8.0 | 8.0 | 6.0 | 7.0 | 6.0 | 5.0 |
| 8.0 | 7.0 | 9.0 | 6.0 | 7.0 | 8.0 | 8.0 | 6.0 | 7.0 | 6.0 | 8.0 | 8.0 | 6.0 | 6.0 | 7.0 | 8.0 | 7.0 | 7.0 | 8.0 | 6.0 | 6.0 | 6.0 | 8.0 | 7.0 |
| 7.0 | 6.0 | 5.0 | 5.0 | 8.0 | 6.0 | 7.0 | 6.0 | 6.0 | 6.0 | 6.0 | 8.0 | 6.0 | 6.0 | 7.0 | 6.0 | 5.0 | 6.0 | 7.0 | 6.0 | 7.0 | 7.0 | 5.0 | 5.0 |
| 6.0 | 7.0 | 6.0 | 7.0 | 6.0 | 8.0 | 9.0 | 5.0 | 6.0 | 5.0 | 5.0 | 7.0 | 6.0 | 6.0 | 6.0 | 7.0 | 5.0 | 6.0 | 7.0 | 6.0 | 6.0 | 5.0 | 6.0 | 5.0 |
| 6.0 | 6.0 | 5.0 | 5.0 | 8.0 | 6.0 | 7.0 | 7.0 | 7.0 | 6.0 | 6.0 | 7.0 | 7.0 | 7.0 | 9.0 | 6.0 | 5.0 | 5.0 | 10.0 | 6.0 | 9.0 | 8.0 | 6.0 | 5.0 |
| 5.0 | 11.0 | 6.0 | 3.0 | 8.0 | 7.0 | 7.0 | 7.0 | 6.0 | 7.0 | 6.0 | 6.0 | 8.0 | 6.0 | 6.0 | 6.0 | 8.0 | 6.0 | 7.0 | 8.0 | 6.0 | 6.0 | 6.0 | 8.0 |
| 6.0 | 7.0 | 6.0 | 5.0 | 5.0 | 7.0 | 6.0 | 6.0 | 6.0 | 7.0 | 7.0 | 7.0 | 6.0 | 7.0 | 6.0 | 8.0 | 8.0 | 6.0 | 6.0 | 6.0 | 7.0 | 6.0 | 8.0 | 8.0 |
| 6.0 | 6.0 | 6.0 | 7.0 | 6.0 | 8.0 | 6.0 | 7.0 | 7.0 | 5.0 | 6.0 | 5.0 | 6.0 | 6.0 | 8.0 | 8.0 | 6.0 | 6.0 | 5.0 | 6.0 | 6.0 | 8.0 | 8.0 | 5.0 |
| 6.0 | 5.0 | 6.0 | 7.0 | 6.0 | 6.0 | 7.0 | 9.0 | 7.0 | 7.0 | 7.0 | 7.0 | 6.0 | 6.0 | 7.0 | 6.0 | 6.0 | 12.0 | 6.0 | 6.0 | 6.0 | 11.0 | 6.0 | 6.0 |
| 11.0 | 6.0 | 6.0 | 6.0 | 11.0 | 6.0 | 6.0 | 6.0 | 7.0 | 7.0 | 5.0 | 6.0 | 4.0 | 5.0 | 6.0 | 7.0 | 7.0 | 5.0 | 6.0 | 5.0 | 6.0 | 6.0 | 7.0 | 7.0 |
| 5.0 | 6.0 | 5.0 | 6.0 | 7.0 | 6.0 | 5.0 | 8.0 | 7.0 | 8.0 | 9.0 | 8.0 | 6.0 | 6.0 | 7.0 | 8.0 | 6.0 | 7.0 | 7.0 | 6.0 | 6.0 | 8.0 | 8.0 | 6.0 |
| 7.0 | 7.0 | 6.0 | 5.0 | 7.0 | 8.0 | 6.0 | 7.0 | 7.0 | 8.0 | 8.0 | 6.0 | 6.0 | 5.0 | 7.0 | 8.0 | 6.0 | 6.0 | 6.0 | 7.0 | 6.0 | 7.0 | 8.0 | 6.0 |
| 6.0 | 6.0 | 7.0 | 6.0 | 7.0 | 6.0 | 5.0 | 5.0 | 8.0 | 7.0 | 8.0 | 6.0 | 6.0 | 5.0 | 6.0 | 8.0 | 6.0 | 6.0 | 7.0 | 5.0 | 5.0 | 6.0 | 7.0 | 6.0 |
| 7.0 | 6.0 | 5.0 | 5.0 | 6.0 | 7.0 | 6.0 | 7.0 | 6.0 | 8.0 | 7.0 | 5.0 | 6.0 | 5.0 | 5.0 | 7.0 | 6.0 | 6.0 | 5.0 | 6.0 | 6.0 | 6.0 | 6.0 | 6.0 |
| 6.0 | 5.0 | 6.0 | 5.0 | 6.0 | 6.0 | 5.0 | 5.0 | 9.0 | 6.0 | 6.0 | 7.0 | 7.0 | 5.0 | 5.0 | 7.0 | 7.0 | 6.0 | 8.0 | 6.0 | 5.0 | 5.0 | 8.0 | 6.0 |
| 7.0 | 8.0 | 6.0 | 5.0 | 5.0 | 8.0 | 6.0 | 7.0 | 8.0 | 7.0 | 7.0 | 6.0 | 6.0 | 6.0 | 6.0 | 6.0 | 8.0 | 6.0 | 5.0 | 6.0 | 9.0 | 6.0 | 6.0 | 8.0 |
| 6.0 | 5.0 | 6.0 | 9.0 | 6.0 | 6.0 | 6.0 | 5.0 | 5.0 | 7.0 | 6.0 | 6.0 | 6.0 | 7.0 | 7.0 | 7.0 | 6.0 | 6.0 | 6.0 | 7.0 | 8.0 | 6.0 | 6.0 | 6.0 |
| 7.0 | 6.0 | 8.0 | 8.0 | 6.0 | 6.0 | 6.0 | 7.0 | 6.0 | 8.0 | 6.0 | 7.0 | 7.0 | 5.0 | 6.0 | 5.0 | 6.0 | 6.0 | 8.0 | 8.0 | 5.0 | 6.0 | 5.0 | 6.0 |
| 6.0 | 8.0 | 8.0 | 5.0 | 6.0 | 5.0 | 6.0 | 8.0 | 6.0 | 5.0 | 7.0 | 8.0 | 7.0 | 7.0 | 7.0 | 6.0 | 6.0 | 6.0 | 7.0 | 6.0 | 6.0 | 8.0 | 6.0 | 6.0 |
| 6.0 | 8.0 | 6.0 | 6.0 | 8.0 | 6.0 | 6.0 | 6.0 | 8.0 | 6.0 | 6.0 | 6.0 | 7.0 | 7.0 | 5.0 | 6.0 | 5.0 | 5.0 | 6.0 | 7.0 | 7.0 | 5.0 | 6.0 | 5.0 |
| 6.0 | 6.0 | 7.0 | 6.0 | 5.0 | 6.0 | 5.0 | 5.0 | 7.0 | 5.0 | 5.0 | 7.0 | 7.0 | 8.0 | 9.0 | 8.0 | 6.0 | 6.0 | 6.0 | 8.0 | 6.0 | 7.0 | 7.0 | 6.0 |
| 5.0 | 7.0 | 8.0 | 7.0 | 7.0 | 7.0 | 5.0 | 5.0 | 7.0 | 8.0 | 7.0 | 7.0 | 6.0 | 8.0 | 8.0 | 6.0 | 6.0 | 5.0 | 6.0 | 8.0 | 6.0 | 6.0 | 6.0 | 7.0 |
| 6.0 | 8.0 | 8.0 | 6.0 | 6.0 | 6.0 | 7.0 | 6.0 | 8.0 | 6.0 | 5.0 | 5.0 | 8.0 | 6.0 | 7.0 | 6.0 | | | | | | | | |

XXXiV
Result of Program, paras 6:1-3
Δt = 1, Maxperiod = 10, Criterion = 10

MAGNETOTAIL INSTABILITIES, PROGRAM 2
TEST PERIODS Δt=1

| | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 5,0 | 5,0 | 6,0 | 5,0 | 5,0 | 6,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 6,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 6,0 |
| 5,0 | 5,0 | 5,0 | 5,0 | 6,0 | 5,0 | 5,0 | 6,0 | 5,0 | 5,0 | 6,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 6,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 |
| 5,0 | 5,0 | 5,0 | 5,0 | 6,0 | 6,0 | 6,0 | 5,0 | 6,0 | 6,0 | 5,0 | 6,0 | 6,0 | 5,0 | 5,0 | 5,0 | 5,0 | 6,0 | 6,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 |
| 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 6,0 | 6,0 | 5,0 | 5,0 | 4,0 | 7,0 | 5,0 | 4,0 | 6,0 | 5,0 | 4,0 | 5,0 | 5,0 | 4,0 | 5,0 | 5,0 | 4,0 | 6,0 | 5,0 |
| 4,0 | 5,0 | 5,0 | 4,0 | 5,0 | 5,0 | 4,0 | 5,0 | 5,0 | 4,0 | 6,0 | 5,0 | 4,0 | 6,0 | 5,0 | 4,0 | 6,0 | 5,0 | 4,0 | 5,0 | 5,0 | 4,0 | 5,0 | 5,0 |
| 4,0 | 6,0 | 5,0 | 4,0 | 5,0 | 5,0 | 4,0 | 5,0 | 5,0 | 4,0 | 5,0 | 5,0 | 4,0 | 6,0 | 6,0 | 5,0 | 6,0 | 6,0 | 5,0 | 6,0 | 5,0 | 5,0 | 5,0 | 5,0 |
| 5,0 | 5,0 | 6,0 | 5,0 | 6,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 6,0 | 5,0 | 6,0 | 6,0 | 6,0 | 5,0 | 6,0 | 6,0 | 5,0 | 6,0 |
| 6,0 | 5,0 | 5,0 | 5,0 | 5,0 | 6,0 | 6,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 6,0 | 6,0 | 5,0 | 7,0 | 5,0 | 5,0 | 6,0 |
| 5,0 | 5,0 | 6,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 7,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 6,0 | 5,0 | 5,0 | 5,0 |
| 6,0 | 5,0 | 5,0 | 6,0 | 5,0 | 5,0 | 6,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 6,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 |
| 6,0 | 5,0 | 6,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 8,0 | 5,0 | 5,0 | 7,0 | 5,0 | 5,0 | 6,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 |
| 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 6,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 6,0 | 5,0 | 5,0 | 6,0 | 5,0 | 5,0 | 6,0 | 5,0 | 5,0 | 5,0 | 5,0 |
| 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 6,0 | 6,0 | 5,0 | 5,0 | 5,0 | 5,0 | 6,0 | 6,0 | 5,0 | 6,0 | 6,0 | 5,0 | 6,0 | 6,0 | 5,0 | 5,0 |
| 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 4,0 | 5,0 | 5,0 | 4,0 | 5,0 | 5,0 | 4,0 | 7,0 | 5,0 | 4,0 | 7,0 | 5,0 |
| 4,0 | 6,0 | 5,0 | 4,0 | 5,0 | 5,0 | 4,0 | 5,0 | 5,0 | 4,0 | 5,0 | 5,0 | 4,0 | 5,0 | 5,0 | 4,0 | 5,0 | 5,0 | 4,0 | 5,0 | 5,0 | 4,0 | 6,0 | 5,0 |
| 4,0 | 6,0 | 5,0 | 4,0 | 6,0 | 5,0 | 4,0 | 5,0 | 5,0 | 4,0 | 5,0 | 5,0 | 4,0 | 5,0 | 5,0 | 4,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 6,0 |
| 5,0 | 6,0 | 6,0 | 5,0 | 6,0 | 6,0 | 5,0 | 6,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 6,0 | 6,0 | 5,0 | 5,0 |
| 5,0 | 5,0 | 6,0 | 6,0 | 5,0 | 6,0 | 6,0 | 5,0 | 6,0 | 6,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 6,0 |
| 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 7,0 | 5,0 | 5,0 | 7,0 | 5,0 | 5,0 | 6,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 |
| 5,0 | 5,0 | 5,0 | 6,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 6,0 | 5,0 | 5,0 | 6,0 | 5,0 | 5,0 | 6,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 |
| 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 6,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 7,0 |
| 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 6,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 6,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 |
| 5,0 | 5,0 | 5,0 | 5,0 | 6,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 6,0 | 5,0 | 5,0 | 5,0 | 6,0 | 6,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 |
| 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 6,0 | 6,0 | 5,0 | 5,0 | 5,0 | 5,0 | 6,0 | 6,0 | 5,0 | 5,0 | 4,0 | 5,0 | 5,0 | 4,0 | 6,0 | 5,0 | 4,0 | 5,0 | 5,0 |
| 4,0 | 5,0 | 5,0 | 4,0 | 5,0 | 5,0 | 4,0 | 5,0 | 5,0 | 4,0 | 7,0 | 5,0 | 4,0 | 5,0 | 5,0 | 4,0 | 6,0 | 5,0 | 4,0 | 5,0 | 5,0 | 4,0 | 6,0 | 5,0 |
| 4,0 | 5,0 | 5,0 | 4,0 | 5,0 | 5,0 | 4,0 | 5,0 | 5,0 | 4,0 | 5,0 | 5,0 | 4,0 | 6,0 | 5,0 | 4,0 | 5,0 | 5,0 | 4,0 | 6,0 | 5,0 | 5,0 | 5,0 | 6,0 |
| 5,0 | 6,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 6,0 | 5,0 | 6,0 | 5,0 | 5,0 | 5,0 | 6,0 | 5,0 | 6,0 | 5,0 |
| 5,0 | 5,0 | 6,0 | 6,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 6,0 | 6,0 | 5,0 | 6,0 | 6,0 | 5,0 | 6,0 |
| 6,0 | 5,0 | 5,0 | 5,0 | 5,0 | 6,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 7,0 | 5,0 | 5,0 | 6,0 |
| 5,0 | 5,0 | 6,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 6,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 |
| 6,0 | 5,0 | 5,0 | 6,0 | 5,0 | 5,0 | 6,0 | 5,0 | | | | | | | | | | | | | | | | |

XXXIV'
Result of Program, paras 6-1-3
Δt = 0·1, Maxperiod = 100, Criterion = 10

MAGNETOTAIL INSTABILITIES. PROGRAM 2

TEST PERIODS Δt=·1

| | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 23.0 | 32.0 | 28.0 | 23.0 | 29.0 | 28.0 | 23.0 | 28.0 | 27.0 | 23.0 | 28.0 | 32.0 | 23.0 | 33.0 | 24.0 | 22.0 | 25.0 | 26.0 | 23.0 | 27.0 | 24.0 | 22.0 | 25.0 | 31.0 |
| 23.0 | 33.0 | 24.0 | 21.0 | 36.0 | 24.0 | 21.0 | 29.0 | 24.0 | 21.0 | 28.0 | 24.0 | 21.0 | 27.0 | 24.0 | 21.0 | 34.0 | 23.0 | 21.0 | 25.0 | 24.0 | 21.0 | 27.0 | 23.0 |
| 21.0 | 24.0 | 24.0 | 21.0 | 32.0 | 34.0 | 27.0 | 30.0 | 29.0 | 26.0 | 29.0 | 28.0 | 26.0 | 29.0 | 27.0 | 26.0 | 28.0 | 34.0 | 27.0 | 31.0 | 25.0 | 24.0 | 25.0 | 27.0 |
| 25.0 | 27.0 | 25.0 | 24.0 | 25.0 | 33.0 | 27.0 | 31.0 | 23.0 | 20.0 | 41.0 | 23.0 | 20.0 | 28.0 | 23.0 | 20.0 | 27.0 | 23.0 | 20.0 | 27.0 | 23.0 | 20.0 | 32.0 | 22.0 |
| 20.0 | 24.0 | 23.0 | 20.0 | 26.0 | 22.0 | 20.0 | 24.0 | 23.0 | 20.0 | 31.0 | 21.0 | 19.0 | 25.0 | 21.0 | 19.0 | 25.0 | 21.0 | 19.0 | 25.0 | 21.0 | 19.0 | 25.0 | 21.0 |
| 19.0 | 25.0 | 21.0 | 19.0 | 23.0 | 21.0 | 19.0 | 24.0 | 21.0 | 19.0 | 23.0 | 21.0 | 19.0 | 25.0 | 27.0 | 22.0 | 34.0 | 26.0 | 22.0 | 28.0 | 26.0 | 22.0 | 28.0 | 25.0 |
| 22.0 | 27.0 | 27.0 | 22.0 | 34.0 | 24.0 | 21.0 | 24.0 | 25.0 | 22.0 | 26.0 | 23.0 | 21.0 | 24.0 | 27.0 | 22.0 | 32.0 | 33.0 | 31.0 | 29.0 | 29.0 | 28.0 | 30.0 | 29.0 |
| 27.0 | 29.0 | 28.0 | 27.0 | 28.0 | 34.0 | 30.0 | 30.0 | 25.0 | 25.0 | 25.0 | 27.0 | 26.0 | 28.0 | 25.0 | 24.0 | 25.0 | 34.0 | 30.0 | 30.0 | 42.0 | 23.0 | 31.0 | 20.0 |
| 23.0 | 29.0 | 28.0 | 23.0 | 29.0 | 27.0 | 23.0 | 28.0 | 34.0 | 23.0 | 32.0 | 25.0 | 23.0 | 25.0 | 27.0 | 23.0 | 27.0 | 25.0 | 23.0 | 25.0 | 32.0 | 23.0 | 32.0 | 31.0 |
| 35.0 | 28.0 | 29.0 | 29.0 | 29.0 | 29.0 | 28.0 | 29.0 | 28.0 | 28.0 | 29.0 | 31.0 | 35.0 | 28.0 | 25.0 | 25.0 | 26.0 | 27.0 | 27.0 | 28.0 | 25.0 | 25.0 | 25.0 | 31.0 |
| 33.0 | 28.0 | 27.0 | 23.0 | 28.0 | 25.0 | 23.0 | 25.0 | 38.0 | 23.0 | 32.0 | 36.0 | 23.0 | 32.0 | 29.0 | 23.0 | 30.0 | 27.0 | 23.0 | 28.0 | 24.0 | 22.0 | 24.0 | 23.0 |
| 21.0 | 23.0 | 26.0 | 23.0 | 27.0 | 24.0 | 21.0 | 28.0 | 23.0 | 21.0 | 25.0 | 24.0 | 21.0 | 36.0 | 24.0 | 21.0 | 37.0 | 24.0 | 21.0 | 29.0 | 24.0 | 21.0 | 27.0 | 23.0 |
| 21.0 | 24.0 | 22.0 | 20.0 | 23.0 | 24.0 | 21.0 | 27.0 | 28.0 | 26.0 | 29.0 | 25.0 | 24.0 | 26.0 | 35.0 | 27.0 | 30.0 | 35.0 | 27.0 | 30.0 | 29.0 | 27.0 | 30.0 | 29.0 |
| 26.0 | 28.0 | 24.0 | 23.0 | 25.0 | 23.0 | 22.0 | 23.0 | 27.0 | 25.0 | 27.0 | 23.0 | 20.0 | 27.0 | 23.0 | 20.0 | 25.0 | 23.0 | 20.0 | 39.0 | 23.0 | 20.0 | 36.0 | 23.0 |
| 20.0 | 29.0 | 23.0 | 20.0 | 27.0 | 22.0 | 20.0 | 24.0 | 21.0 | 20.0 | 22.0 | 23.0 | 20.0 | 26.0 | 21.0 | 19.0 | 25.0 | 21.0 | 19.0 | 24.0 | 21.0 | 19.0 | 25.0 | 21.0 |
| 19.0 | 25.0 | 21.0 | 19.0 | 25.0 | 21.0 | 19.0 | 25.0 | 21.0 | 19.0 | 23.0 | 20.0 | 19.0 | 22.0 | 21.0 | 19.0 | 24.0 | 26.0 | 22.0 | 28.0 | 24.0 | 22.0 | 25.0 | 27.0 |
| 22.0 | 34.0 | 27.0 | 22.0 | 35.0 | 26.0 | 22.0 | 29.0 | 26.0 | 22.0 | 27.0 | 23.0 | 21.0 | 24.0 | 22.0 | 21.0 | 23.0 | 25.0 | 22.0 | 26.0 | 29.0 | 27.0 | 29.0 | 26.0 |
| 25.0 | 26.0 | 33.0 | 30.0 | 29.0 | 33.0 | 30.0 | 29.0 | 30.0 | 28.0 | 30.0 | 28.0 | 27.0 | 28.0 | 25.0 | 24.0 | 25.0 | 23.0 | 23.0 | 24.0 | 27.0 | 26.0 | 28.0 | 28.0 |
| 23.0 | 29.0 | 25.0 | 23.0 | 26.0 | 43.0 | 23.0 | 31.0 | 37.0 | 23.0 | 31.0 | 29.0 | 23.0 | 30.0 | 28.0 | 23.0 | 28.0 | 24.0 | 22.0 | 25.0 | 23.0 | 22.0 | 23.0 | 26.0 |
| 23.0 | 27.0 | 29.0 | 28.0 | 29.0 | 26.0 | 26.0 | 26.0 | 31.0 | 35.0 | 28.0 | 31.0 | 35.0 | 28.0 | 30.0 | 30.0 | 29.0 | 28.0 | 28.0 | 28.0 | 25.0 | 25.0 | 25.0 | 24.0 |
| 23.0 | 24.0 | 28.0 | 27.0 | 28.0 | 26.0 | 23.0 | 27.0 | 31.0 | 23.0 | 33.0 | 24.0 | 22.0 | 25.0 | 24.0 | 22.0 | 24.0 | 26.0 | 23.0 | 27.0 | 23.0 | 21.0 | 23.0 | 36.0 |
| 23.0 | 32.0 | 28.0 | 23.0 | 28.0 | 28.0 | 23.0 | 29.0 | 24.0 | 21.0 | 27.0 | 24.0 | 21.0 | 32.0 | 23.0 | 21.0 | 25.0 | 23.0 | 21.0 | 24.0 | 24.0 | 21.0 | 26.0 | 22.0 |
| 20.0 | 23.0 | 24.0 | 21.0 | 37.0 | 24.0 | 21.0 | 28.0 | 24.0 | 21.0 | 29.0 | 27.0 | 25.0 | 28.0 | 33.0 | 27.0 | 31.0 | 25.0 | 24.0 | 25.0 | 24.0 | 23.0 | 25.0 | 26.0 |
| 25.0 | 27.0 | 23.0 | 22.0 | 23.0 | 35.0 | 27.0 | 30.0 | 28.0 | 26.0 | 29.0 | 29.0 | 26.0 | 29.0 | 23.0 | 20.0 | 26.0 | 23.0 | 20.0 | 31.0 | 22.0 | 20.0 | 24.0 | 22.0 |
| 20.0 | 24.0 | 23.0 | 20.0 | 25.0 | 21.0 | 20.0 | 22.0 | 23.0 | 20.0 | 36.0 | 23.0 | 20.0 | 27.0 | 23.0 | 20.0 | 26.0 | 21.0 | 19.0 | 24.0 | 21.0 | 19.0 | 25.0 | 21.0 |
| 19.0 | 23.0 | 21.0 | 19.0 | 23.0 | 21.0 | 19.0 | 24.0 | 20.0 | 19.0 | 22.0 | 21.0 | 19.0 | 25.0 | 21.0 | 19.0 | 25.0 | 21.0 | 19.0 | 25.0 | 25.0 | 22.0 | 27.0 | 27.0 |
| 22.0 | 32.0 | 23.0 | 21.0 | 24.0 | 23.0 | 21.0 | 24.0 | 24.0 | 22.0 | 26.0 | 22.0 | 21.0 | 23.0 | 27.0 | 22.0 | 35.0 | 26.0 | 22.0 | 28.0 | 26.0 | 22.0 | 28.0 | 28.0 |
| 26.0 | 28.0 | 34.0 | 29.0 | 30.0 | 25.0 | 24.0 | 25.0 | 25.0 | 24.0 | 25.0 | 27.0 | 26.0 | 27.0 | 23.0 | 23.0 | 23.0 | 33.0 | 30.0 | 29.0 | 28.0 | 27.0 | 29.0 | 29.0 |
| 28.0 | 30.0 | 27.0 | 23.0 | 28.0 | 32.0 | 23.0 | 32.0 | 25.0 | 23.0 | 25.0 | 24.0 | 22.0 | 25.0 | 26.0 | 23.0 | 27.0 | 23.0 | 22.0 | 23.0 | 37.0 | 23.0 | 31.0 | 28.0 |
| 23.0 | 29.0 | 29.0 | 23.0 | 29.0 | 28.0 | 28.0 | 28.0 | 31.0 | 33.0 | 28.0 | 25.0 | 25.0 | 25.0 | 25.0 | 25.0 | 25.0 | 27.0 | 27.0 | 28.0 | 23.0 | 23.0 | 24.0 | 31.0 |
| 35.0 | 28.0 | 29.0 | 28.0 | 29.0 | 30.0 | 29.0 | 30.0 | | | | | | | | | | | | | | | | |

(xxxiv'')

# RESULT OF PROGRAM, para. 6·3

Δt = 1   Maxperiod = 10   Criterion = 100

MAGNETOTAIL INSTABILITIES, PROGRAM 2

TEST PERIODS

| | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 6.0 | 8.0 | 7.0 | 6.0 | 8.0 | 7.0 | 6.0 | 7.0 | 7.0 | 6.0 | 7.0 | 8.0 | 6.0 | 7.0 | 6.0 | 6.0 | 7.0 | 6.0 | 6.0 | 7.0 | 6.0 | 6.0 | 7.0 | 7.0 |
| 6.0 | 7.0 | 6.0 | 6.0 | 8.0 | 6.0 | 6.0 | 7.0 | 6.0 | 6.0 | 7.0 | 6.0 | 6.0 | 7.0 | 6.0 | 6.0 | 8.0 | 6.0 | 6.0 | 6.0 | 6.0 | 6.0 | 7.0 | 6.0 |
| 6.0 | 6.0 | 6.0 | 6.0 | 8.0 | 7.0 | 7.0 | 7.0 | 7.0 | 7.0 | 7.0 | 7.0 | 7.0 | 7.0 | 7.0 | 6.0 | 7.0 | 7.0 | 7.0 | 7.0 | 6.0 | 6.0 | 7.0 | 7.0 |
| 6.0 | 7.0 | 6.0 | 6.0 | 7.0 | 7.0 | 7.0 | 7.0 | 6.0 | 5.0 | 8.0 | 6.0 | 5.0 | 7.0 | 6.0 | 5.0 | 7.0 | 6.0 | 5.0 | 7.0 | 6.0 | 5.0 | 8.0 | 6.0 |
| 5.0 | 6.0 | 6.0 | 5.0 | 6.0 | 6.0 | 5.0 | 6.0 | 6.0 | 5.0 | 8.0 | 6.0 | 5.0 | 7.0 | 6.0 | 5.0 | 6.0 | 6.0 | 5.0 | 6.0 | 6.0 | 5.0 | 6.0 | 6.0 |
| 5.0 | 7.0 | 6.0 | 5.0 | 6.0 | 6.0 | 5.0 | 6.0 | 6.0 | 5.0 | 6.0 | 6.0 | 5.0 | 7.0 | 7.0 | 6.0 | 8.0 | 7.0 | 6.0 | 7.0 | 7.0 | 6.0 | 7.0 | 6.0 |
| 6.0 | 7.0 | 7.0 | 6.0 | 7.0 | 6.0 | 6.0 | 6.0 | 6.0 | 6.0 | 7.0 | 6.0 | 6.0 | 6.0 | 7.0 | 6.0 | 7.0 | 7.0 | 8.0 | 7.0 | 7.0 | 7.0 | 7.0 | 7.0 |
| 7.0 | 7.0 | 7.0 | 7.0 | 7.0 | 7.0 | 7.0 | 7.0 | 7.0 | 6.0 | 7.0 | 7.0 | 6.0 | 7.0 | 6.0 | 6.0 | 7.0 | 7.0 | 7.0 | 7.0 | 10.0 | 6.0 | 7.0 | 7.0 |
| 6.0 | 7.0 | 7.0 | 6.0 | 7.0 | 7.0 | 6.0 | 7.0 | 8.0 | 6.0 | 7.0 | 6.0 | 6.0 | 7.0 | 7.0 | 6.0 | 7.0 | 6.0 | 6.0 | 7.0 | 8.0 | 6.0 | 7.0 | 8.0 |
| 7.0 | 6.0 | 7.0 | 7.0 | 6.0 | 7.0 | 7.0 | 6.0 | 7.0 | 7.0 | 6.0 | 7.0 | 7.0 | 6.0 | 7.0 | 6.0 | 6.0 | 7.0 | 7.0 | 6.0 | 7.0 | 6.0 | 7.0 | 7.0 |
| 7.0 | 6.0 | 7.0 | 6.0 | 7.0 | 6.0 | 6.0 | 7.0 | 9.0 | 6.0 | 7.0 | 8.0 | 6.0 | 8.0 | 7.0 | 6.0 | 8.0 | 7.0 | 6.0 | 7.0 | 6.0 | 6.0 | 6.0 | 6.0 |
| 6.0 | 6.0 | 6.0 | 6.0 | 7.0 | 6.0 | 6.0 | 7.0 | 6.0 | 6.0 | 6.0 | 6.0 | 6.0 | 8.0 | 6.0 | 6.0 | 8.0 | 6.0 | 6.0 | 8.0 | 6.0 | 6.0 | 7.0 | 6.0 |
| 6.0 | 6.0 | 6.0 | 5.0 | 6.0 | 6.0 | 6.0 | 7.0 | 7.0 | 6.0 | 7.0 | 6.0 | 6.0 | 7.0 | 7.0 | 7.0 | 7.0 | 7.0 | 7.0 | 7.0 | 8.0 | 7.0 | 7.0 | 7.0 |
| 7.0 | 7.0 | 6.0 | 6.0 | 7.0 | 6.0 | 6.0 | 6.0 | 7.0 | 6.0 | 7.0 | 6.0 | 5.0 | 7.0 | 6.0 | 5.0 | 6.0 | 6.0 | 5.0 | 8.0 | 6.0 | 5.0 | 8.0 | 6.0 |
| 5.0 | 7.0 | 6.0 | 5.0 | 7.0 | 6.0 | 5.0 | 6.0 | 6.0 | 5.0 | 6.0 | 6.0 | 5.0 | 6.0 | 6.0 | 5.0 | 6.0 | 6.0 | 5.0 | 6.0 | 6.0 | 5.0 | 7.0 | 6.0 |
| 5.0 | 7.0 | 6.0 | 5.0 | 7.0 | 6.0 | 5.0 | 6.0 | 6.0 | 5.0 | 6.0 | 5.0 | 5.0 | 6.0 | 6.0 | 5.0 | 6.0 | 6.0 | 6.0 | 7.0 | 6.0 | 6.0 | 6.0 | 7.0 |
| 6.0 | 7.0 | 7.0 | 6.0 | 8.0 | 7.0 | 6.0 | 8.0 | 6.0 | 6.0 | 7.0 | 6.0 | 6.0 | 6.0 | 6.0 | 6.0 | 6.0 | 6.0 | 6.0 | 7.0 | 7.0 | 7.0 | 7.0 | 7.0 |
| 6.0 | 7.0 | 7.0 | 8.0 | 7.0 | 7.0 | 8.0 | 7.0 | 7.0 | 7.0 | 7.0 | 7.0 | 7.0 | 7.0 | 6.0 | 6.0 | 7.0 | 6.0 | 6.0 | 6.0 | 7.0 | 6.0 | 6.0 | 7.0 |
| 6.0 | 7.0 | 6.0 | 6.0 | 7.0 | 8.0 | 6.0 | 7.0 | 9.0 | 6.0 | 7.0 | 7.0 | 6.0 | 7.0 | 7.0 | 6.0 | 7.0 | 6.0 | 6.0 | 6.0 | 6.0 | 6.0 | 6.0 | 6.0 |
| 6.0 | 7.0 | 8.0 | 7.0 | 6.0 | 7.0 | 6.0 | 6.0 | 7.0 | 7.0 | 6.0 | 8.0 | 7.0 | 6.0 | 7.0 | 7.0 | 6.0 | 7.0 | 7.0 | 6.0 | 7.0 | 6.0 | 7.0 | 6.0 |
| 6.0 | 7.0 | 7.0 | 7.0 | 6.0 | 7.0 | 6.0 | 7.0 | 7.0 | 6.0 | 7.0 | 6.0 | 6.0 | 7.0 | 6.0 | 6.0 | 6.0 | 6.0 | 6.0 | 7.0 | 6.0 | 6.0 | 6.0 | 8.0 |
| 6.0 | 8.0 | 7.0 | 6.0 | 7.0 | 7.0 | 6.0 | 8.0 | 6.0 | 6.0 | 7.0 | 6.0 | 6.0 | 8.0 | 6.0 | 6.0 | 6.0 | 6.0 | 6.0 | 6.0 | 6.0 | 6.0 | 6.0 | 6.0 |
| 5.0 | 6.0 | 6.0 | 6.0 | 8.0 | 6.0 | 6.0 | 7.0 | 6.0 | 6.0 | 7.0 | 7.0 | 6.0 | 7.0 | 7.0 | 7.0 | 7.0 | 6.0 | 6.0 | 7.0 | 6.0 | 6.0 | 7.0 | 7.0 |
| 6.0 | 7.0 | 6.0 | 6.0 | 6.0 | 7.0 | 7.0 | 7.0 | 7.0 | 7.0 | 7.0 | 7.0 | 7.0 | 7.0 | 6.0 | 5.0 | 7.0 | 6.0 | 5.0 | 8.0 | 6.0 | 5.0 | 6.0 | 6.0 |
| 5.0 | 6.0 | 6.0 | 5.0 | 6.0 | 6.0 | 5.0 | 6.0 | 6.0 | 5.0 | 8.0 | 6.0 | 5.0 | 7.0 | 6.0 | 5.0 | 7.0 | 6.0 | 5.0 | 6.0 | 6.0 | 5.0 | 7.0 | 6.0 |
| 5.0 | 6.0 | 6.0 | 5.0 | 6.0 | 6.0 | 5.0 | 6.0 | 5.0 | 5.0 | 6.0 | 6.0 | 5.0 | 7.0 | 6.0 | 5.0 | 6.0 | 6.0 | 5.0 | 6.0 | 6.0 | 6.0 | 7.0 | 7.0 |
| 6.0 | 7.0 | 6.0 | 6.0 | 6.0 | 6.0 | 6.0 | 6.0 | 6.0 | 6.0 | 7.0 | 6.0 | 6.0 | 6.0 | 7.0 | 6.0 | 8.0 | 7.0 | 6.0 | 7.0 | 7.0 | 6.0 | 7.0 | 7.0 |
| 7.0 | 7.0 | 7.0 | 7.0 | 7.0 | 6.0 | 6.0 | 7.0 | 6.0 | 6.0 | 7.0 | 7.0 | 6.0 | 7.0 | 6.0 | 6.0 | 6.0 | 7.0 | 8.0 | 7.0 | 7.0 | 7.0 | 7.0 | 7.0 |
| 7.0 | 7.0 | 7.0 | 6.0 | 7.0 | 8.0 | 6.0 | 7.0 | 6.0 | 6.0 | 7.0 | 6.0 | 6.0 | 6.0 | 6.0 | 6.0 | 7.0 | 6.0 | 6.0 | 6.0 | 9.0 | 6.0 | 7.0 | 7.0 |
| 6.0 | 7.0 | 7.0 | 6.0 | 7.0 | 7.0 | 7.0 | 6.0 | 7.0 | 7.0 | 6.0 | 7.0 | 6.0 | 7.0 | 7.0 | 6.0 | 7.0 | 7.0 | 7.0 | 6.0 | 6.0 | 6.0 | 6.0 | 8.0 |
| 7.0 | 6.0 | 7.0 | 7.0 | 6.0 | 7.0 | 7.0 | 6.0 | | | | | | | | | | | | | | | | |

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