

**DESCRIBING THE RELATIONSHIP BETWEEN THE COGNITIVE AND
LINGUISTIC COMPLEXITY OF A MATHEMATICAL LITERACY
EXAMINATION AND TYPES OF STUDENT ERRORS**

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If you want to go quickly, go alone. If you want to go far, go together.

African Proverb

To have reached this point, at the cost of that quicker route, I am deeply indebted to my family: Charles and Gail Vale, and Monica Coetzee.

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DECLARATION

I, Pamela Vale, have read and understood the University's policy on plagiarism. This is my own work and, where I have drawn on the work of others, I have referenced appropriately. This work has not been submitted to fulfil the requirements of a degree at any other university.

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ABSTRACT

Much prior research has shown that if students have a poor command of the language in which they are taught and assessed, they experience a complex and deep learning disadvantage (Barton & Neville-Barton, 2003). Abedi (2006) mentions, in particular, that unnecessary linguistic complexity can threaten the validity of examination items and thus compromises the fairness of the assessment for English language learners. In Clarkson's (1991, p. 31) research it was found that for the English language learners in the study "comprehension errors [made] up a high proportion of the errors made when...students attempt[ed] to solve mathematical word problems".

In an attempt to explore whether this was the case for a group of National Certificate (Vocational) [NC(V)] students at an FET college, the research conducted in this study focused on describing the cognitive and linguistic complexity of Level 4 Mathematical Literacy examination items as well as the types of responses from a sample of students. A mixed-methods case study design was selected. Student errors were classified as either due to mathematical literacy-related sources, or language-related sources and the question was asked as to how the cognitive and linguistic complexity of items might be related to the types of errors made.

Statistically significant correlations were found between the linguistic complexity of items and language-related errors, and between the cognitive complexity of items and all types of errors. It was also possible to identify which language features, in particular, were statistically significantly correlated with linguistic complexity, namely: prepositional phrases; words of 7 letters or more and complex/compound sentences.

As was expected, the majority of errors were categorised as mathematical literacy-related. However, as many as 19.22% of all errors made were identified as language-related. While the scope of the study prevents any generalisations from being made, the results indicate a need for a larger-scale study of this nature to determine if the complex and deep learning disadvantage mentioned by Barton and Neville-Barton (2003) does exist with regard to the assessment of Mathematical Literacy for NC(V) students who are English language learners (Barton & Neville-Barton, 2003).

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CHAPTER 1

INTRODUCTION

Where a lack of validity and fairness occurs in a final examination, the consequences for students are significant due to the high-stakes nature of this component. All assessment items, therefore, need to be carefully constructed with regard to the language demands, if students' abilities and subject knowledge are to be fairly assessed. Umalusi's (2010a) research, however, suggests that this might not be the case. This study examines the validity and fairness of Mathematical Literacy assessment in a vocational context.

1.1 CONTEXT OF THE RESEARCH

The Further Education and Training [FET] college landscape has undergone dramatic changes in the last decade with college mergers and name changes, as well as alterations in their qualification and programme structures. In making these changes, the South African government claims to be seeking to meet "government delivery imperatives in skills development and employment creation" (Wedekind, 2008, p. 10). The purpose of this shift in identity is to facilitate access to education for all adults, thereby enabling them to achieve a qualification at Level 4 of the National Qualifications Framework [NQF] (FET Round Table, 2010). A further aim is to provide a second chance to young adults who have either not successfully completed 12 years of schooling or who have completed their schooling without qualifying for entry to a university. In addition, youth who are not in education, employment or training [NEET] have the opportunity, through these colleges, to participate in study programmes that will enhance their employability (FET Round Table, 2010). Part of this process of change has been to introduce the National Certificate (Vocational) [NC(V)], which has been developed as a "sister qualification" (Umalusi, 2010b, p. 10) to the National Certificate Senior [NCS].

Technical and vocational education has a history of more than 100 years in South Africa (Wedekind, 2008), but there is no historical precedent for the newly implemented NC(V) in this education system (Umalusi, 2010c) The rationale for introducing the new curriculum, as outlined by Umalusi (2010b), is to provide an alternative Level 4 qualification to students that will equip them with both the theoretical background and practical experience required to master a trade or technical skill.

The curriculum currently offers students a selection of eighteen fields of study, all of which include compulsory fundamental subjects comprising one language, Life Orientation and either Mathematics or Mathematical Literacy. For NC(V) students to be certified at an NQF Level 4, a certain level of numeracy and literacy, commensurate with other Level 4 qualifications, must be demonstrated (Umalusi, 2007). It is for this purpose that the fundamentals have been included in the curricula of all fields of study.

1.2 RATIONALE FOR THE RESEARCH

National Census 2011 statistics revealed that only 9,6% of the population have English as their home language (Lehohla, 2012). However, the official language of learning and teaching is English in the majority of educational institutions (Probyn, 2004). These demographics imply that most students will have little contact with English speakers and a number of national surveys also reveal that they have little exposure to written language outside of the classroom (Probyn, 2004). This is particularly the case for FET colleges as, according to the FET Round Table report (2010), many of these students are from rural areas where their schooling has taken place in a socio-economically deprived environment.

The English proficiency of many of the students “frequently does not meet the demands of learning through the medium of English” (Probyn, 2004, p. 50). These students’ performance in other subjects is compromised by the fact that, where English is the language of learning and teaching, their language proficiency may not be sufficient to adequately comprehend lessons, texts and assessments.

According to the Select Committee on Education and Recreation (2006, p. 2), the preamble to the FET Colleges Bill tasks colleges with “redress[ing] past discrimination and ensur[ing] representivity and equal access [as well as] provid[ing] optimal opportunities for learning”. In order for the learning environment to be optimised, academic support must be made available to students who have been disadvantaged by a poor quality basic education. In FET colleges, the support for these students has been found to be weakly conceptualised and not optimally implemented, if it exists at all (FET Round Table, 2010). This lack of support may have an impact on students’ ability to cope with instruction and assessments, and may therefore compromise their ability to improve their linguistic competence.

Umalusi (2010a) monitors the quality of examinations and assessment for the NC(V). Their evaluation of the November 2009 NC(V) examinations revealed that poor editing forced external moderators to “grapple with poor language usage, incorrect spelling and typing errors” (Umalusi, 2010a, p. 17). In addition, it found that some editors were unaware that the changes they made with regard to language and structure could change the meaning of the items (Umalusi, 2010a). As only a sample of examination papers is externally moderated, the implication is that many more poorly constructed examination papers may find their way into the examination room. It is possible that a student with limited language proficiency in the language of the examination will be unable to accurately respond to such poorly constructed examination items and their results, consequently, may not accurately reflect their knowledge of that particular subject.

Clarkson’s (1991) research with students in Papua New Guinea, who were assessed in a language other than their home language, supports this view. His finding was that “comprehension errors make up a high proportion of the errors made when...students attempt[ed] to solve mathematical word problems” (Clarkson, 1991, p. 31). Barton and Neville-Barton (2003) explain that research has revealed complex and deep learning disadvantage in schools where the students’ language proficiency in the medium of instruction is extremely poor. Their own research revealed that students at a university level, who are learning in their second language “suffer a greater disadvantage in mathematics than is expected from the literature” (Barton & Neville-Barton, 2003, p. 9).

As Abedi (2006) points out, where there is unnecessary linguistic complexity in a test item, the validity of the item can be questioned, as the construct measured may no longer be that which is targeted. This complexity may be a source of measurement error (Abedi, 2006), which adds a construct to the assessment and in so doing introduces “a source of construct-irrelevant variance because it is not conceptually related to the content being measured” (Abedi, 2006, p. 782). Abedi’s (2006) research revealed an increasing performance gap between English home language and English second language students as the language demand of the assessment was increased. What needs to be clear to any assessor of Mathematical Literacy is what exactly constitutes unnecessary linguistic complexity, and what complex language structures are features of the subject itself, and cannot be avoided.

The Assessment Guidelines for Mathematical Literacy (Department of Education [DoE], 2007a) emphasize the principles of validity, as well as fairness and transparency in assessments. Methods referred to as unfair are, among others: “bias based on ethnicity, race, gender, age, disability or social class...and comparison of students’ work with other students, based on learning styles and language” (DoE, 2007a, p. 3; DoE, 2007b, p. 3).

The concept of bias and unfairness can be extended to include instances where assessments require a high level of language proficiency in order for students to comprehend items and formulate responses. As reflected in both Barton and Neville-Barton’s (2003) and Abedi’s (2006) research, students whose home language is not the language of learning and teaching and those who have not already experienced this language as the medium of instruction are disadvantaged. This impacts on the validity of assessments, as the linguistic competence of the student becomes what is assessed as opposed to the outcomes of the subject being examined.

Currently, the NC(V) examinations are evaluated for their linguistic complexity in a relatively superficial manner, assessing only whether the language is pitched at an appropriate level and that no bias is evident (Umalusi, 2010a). It is specified that “language used in all papers should be correct [and] there should be no grammar or typing errors” (Umalusi, 2010a, p. 22). It is mentioned that the editing process be rigorous, yet no guidance is offered as to specifically what elevates linguistic complexity to an inappropriate level, and what would constitute unnecessary linguistic complexity for a particular subject. For it to be possible to ensure that English language learners are not disadvantaged in any examination, this must be explored and understood.

1.3 RESEARCH GOALS

The aim of this study is to provide an in-depth description of the linguistic features and cognitive complexity of a Mathematical Literacy examination, and to explore how these may relate to the types of errors English language learners have made in this examination.

The research questions are:

- How can Mathematical Literacy examination items be described with regard to their linguistic complexity?

- How can Mathematical Literacy examination items be described with regard to their cognitive complexity?
- What types of errors are students making?
- How do linguistic complexity and cognitive complexity relate to the types of errors made?

1.4 ORGANISATION OF CONTENT

This research report is presented in five chapters. Chapter 1 serves to introduce the study, providing the context, rationale and research goals.

In Chapter 2 the literature relevant to and informing this study will be reviewed. Both seminal and current work is included in this review. Literacies are defined and discussed in the first section, providing the background to the discussion of what constitutes Mathematical Literacy and what can be considered to constitute English literacy.

The development of mathematical thought is described, with particular reference to the work of Jean Piaget, Lev Vygotsky and David Tall. Here it is explained how the early development of children, as it occurs within a particular home language environment, may influence their further development as they move into a schooling system that relies on a language of instruction which is not their home language.

Mathematical problem solving and mathematical language are described, as well as second language acquisition, which leads to a discussion regarding the nature of the relationship between English proficiency and mathematical proficiency. The assessment of Mathematical Literacy is also described, with particular reference to psychometric design, and the assessment of English language learners. A discussion of error analysis in mathematics concludes this chapter.

Chapter 3 describes the methodology of the study as well as the methods employed in the collection and analysis of data. The ontological, epistemological and methodological stance of the research is outlined, followed by a discussion of mixed methods research and the case study method, with particular reference to this study.

The selected site and participants will be described, as well as the pilot study informing the final decisions made with regard to data analysis. A summary of the lessons learned from the pilot study concludes this section.

The finer details of the data analysis will be presented. This is divided in accordance with the four research questions, i.e. first a description of the analysis of linguistic complexity of the examination; second a description of the analysis of cognitive complexity; third a description of the error analysis of student scripts and lastly a description of the analysis process which will allow the relationships between these three to be described. All tools for analysis are provided. The chapter concludes with a summary of the operational definitions of each construct analysed in the study after a discussion of reliability and validity and the ethical considerations in conducting this study

Chapter 4 includes the presentation and analysis of the data. This too is divided according to the four research questions. Quantitative and qualitative data is presented in order to provide the in-depth descriptions required to achieve the goal of the study.

In Chapter 5 the analysis of the data is further discussed in order to draw conclusions regarding the research questions asked. The shortcomings and limitations of the study are described, as well as the contributions made by the study and what avenues for further research exist. Included in this chapter is a discussion of the practical lessons that can be learned from the results of this study, by lecturers and assessors.

1.5 SUMMARY

In this chapter, the context of the study, its rationale and the specific research goals and questions have been outlined. An outline has also been provided in order to guide the reader as to the specific content of each chapter.

CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

In order to research linguistic difficulty as it applies to the assessment of mathematical literacy, it is necessary to review several fields of research. In this chapter an explanation of these different fields is provided as well as how they can be connected.

This study is concerned with students who are primarily developing and using literacies in their second language, therefore, it is essential to consider the variety of ways in which literacy itself is conceptualised. The chapter will begin with an overview of a range of theories that define what constitutes literacy in its broadest sense (Scribner, 1984; Gee, 1986; 1989; 1999; Street, 2005). Thereafter definitions of numeracy (Steen, 1990; Baker, Street & Tomlin, 2003; Barwell, 2004; de Lange, 2003) and mathematical literacy (Steen, 2001; Hoyles, Wolf, Molyneux-Hodgson & Kent, 2002; de Lange, 2003; Vithal & Bishop, 2006; Ojose, 2011) are provided and discussed.

It is also important to understand how mathematical thinking develops. The seminal theories of Jean Piaget (1964; 1972; Piaget & Chomsky, 1980) and Lev Vygotsky (1978; 1981; 1998) are outlined, as well as David Tall's work (2006; 2008a; 2008b; Pegg & Tall, 2005; Gray & Tall, 1994), which derives predominantly from the work of Piaget. It is explained how each of these theories overlap with the development of language proficiency, and what this implies for English language learners who are being taught and assessed through the medium of English.

After the discussion of how mathematical thinking develops, this thinking is described with particular reference to the process of problem solving (Polya, 1957; Schoenfeld, 1992; Lucangeli, Tressoldi & Cendron, 1998; Langley & Rogers, 2005; Programme for International Student Assessment [PISA] Problem Solving Expert Group, 2010). Included in this section is a brief discussion of the role of working memory and long term memory in problem solving. Thereafter theories of second language acquisition are described (Cummins, 1980; 1984; Cuevas, 1984; Cummins & Swain, 1986; Collier, 1987; Crain & Shankweiler, 1988; Perlovsky 2010; Saville-Troike, 2010) with reference to both mathematical thinking

and problem solving (Sweller, 1988; Lucangeli, Tressoldi & Cendron, 1998; Tuovinen & Sweller, 1999; van Merriënboer & Sweller, 2005; Barbu, 2010). This section concludes with a discussion regarding the cognitive consequences of bilingualism and whether or not this enhances problem solving ability (Dawe, 1983; Pavlenko, 2005; Dominguez, 2008; Planas & Setati, 2009; Bialystok & Craik, 2010; Kempert, Saalbach & Hardy, 2011).

Following the presentation of these language theories is a description focusing on mathematical language in particular (Cuevas, 1984; Halliday, 1989; Barton & Neville-Barton, 2003; Duval, 2006; Hammill, 2010; Bergqvist, Dyrvold & Österholm, 2012). Mathematical language can be distinguished from everyday language in a number of ways, particularly with respect to the specific vocabulary used and the inclusion of a large symbolic and visual component (MacKinley, 1986; Diezmann, Lowrie & Kozak, 2007; Lowrie & Diezmann, 2007; Diezmann & Lowrie, 2008; Kress, 2000).

The final field focussed on in this review is that of educational assessment. In the design of assessments there are various psychometric considerations to be taken into account, particularly with regard to the linguistic aspects of the assessment (Abedi, 2002; 2006; Herman & Abedi, 2004; Abedi & Gándara, 2006; Khisty, 2006; Halliday, 2010; Hammill, 2010). When assessing students in a multicultural and multilingual context, linguistic design becomes especially important, which necessitates a theoretical discussion of what constitutes linguistic complexity in assessment. A brief discussion of the use of taxonomies in assessment design is also included (DoE, 2007a; Mullis, Martin, Smith, Garden, Gregory, Gonzalez et al., 2003a; Berger, Bowie & Nyaumwe, 2010; Wu, 2010).

Theories regarding how to analyse and categorise student errors are also presented (Clements, 1980; PISA Governing Board, 2010). The knowledge of how to determine why a student is making a particular error is an essential tool in the teaching of mathematics. This categorisation allows for the planning of effective interventions. A student making errors due to limited language proficiency requires a different approach to remediation than a student comprehending the language yet not the mathematical concept itself, or how to use it.

All of the fields and theories presented in this chapter must be considered when attempting to understand why students perform in a particular manner in mathematical literacy examinations.

2.2 LITERACIES

“Literacy has never been more necessary for development; it is key to communication and learning of all kinds and a fundamental condition of access to today’s knowledge societies. With socio-economic disparities increasing and global crises over food, water and energy, literacy is a survival tool in a fiercely competitive world. Literacy leads to empowerment, and the right to education includes the right to literacy – an essential requirement for lifelong learning and a vital means of human development and of achieving the Millennium Development Goals (MDGs)”

Richmond, Robinson & Sachs-Israel, 2008, p. 9

2.2.1 General conceptions of literacy

In seeking to understand the notion of mathematical literacy, it is necessary to understand the concept of literacy itself. A common basic understanding of the term literacy would be the ability to read and write, but this definition is incomplete. How to understand literacy is an area of highly contested international debate, important in that it “informs education, human rights and development discourse” (Street, 2005a, p. 2).

2.2.1.1 Defining literacy

Krashen (1982) differentiates between the acquisition and learning of a language. Learning refers to conscious knowledge of the rules of a language and an ability to use them, whereas acquisition occurs through the processes of “implicit learning, informal learning and natural learning” (Krashen, 1982, p. 10). These are the processes by which we achieve competence in our home language, particularly in the oral mode (Gee, 1989). Gee (1989) explains that home language competence in this mode is achieved as a result of exposure to “socio-culturally determined” (Gee, 1989, p. 22) communication within the family. He defines this as the primary discourse which can be considered unique to that particular group of individuals or culture. The term ‘discourse’ is used by Gee (1989, p. 21) to describe:

...a socially accepted association among ways of using language, of thinking, and of acting that can be used to identify oneself as a member of a socially meaningful group or social network.

A discourse is “integrally connected [to] the identity or sense of self of the people who practice them” (Gee, 1986, p. 720).

As the child's social experiences begin to extend beyond the home and family it becomes necessary to use language in order to communicate with individuals who do not share the same primary discourse. Gee (1989, p. 23) describes this as a "secondary use of language". Schools expose children to the world views dominant in society (Gee, 1986) and competence in this secondary discourse is necessary for success in these societies. Gee's (1989, p. 23) definition of literacy, therefore, is the "control of secondary uses of language".

This is not, however, an uncontested definition. There is significant debate regarding the precise definition of how literacy is acquired and developed. Some academics emphasise the social aspect, with others favouring a skill-based definition. Jeanne Chall (Indrisano & Chall, 2006) is one influential researcher who favours the skills-based definition. Chall (Indrisano & Chall, 2006, p. 35) views reading as a "complex of abilities and skills that change with development" and writes that explicit instruction is required to teach these abilities and skills. She writes that children do need to read to learn words, but "they need to learn words directly [too], apart from the context (Chall, 1987, p. 15). Gee (1989) argues, however, that after initial enculturation in the home, achieving language competence and literacy occurs "through a mixture of acquisition and learning" (Gee, 1989, p. 20).

Many view the relationship between a skills-based and social view of literacy as dichotomous. Others propose that both views are legitimate and can complement each other. The ambiguity and debate regarding definitions of literacy holds significant consequences as each different understanding has implications for programme design, and particularly, what constitutes a "minimal functional competency" (Scribner, 1984, p. 23).

Functional literacy is a broad concept describing the "level of proficiency necessary for effective performance in a range of settings and customary activities" (Scribner, 1984, p. 23). Scribner's (1984) definition includes a strong socio-cultural and temporal aspect. What could be considered minimal competence at another time in history or in another culture will differ from what is now considered minimal competence in our 21st century globalised society.

The United Nations Educational, Scientific and Cultural Organisation [UNESCO] (2005) has attempted to consolidate the views of academics from a range of disciplines. They acknowledge four ways in which literacy can be understood (UNESCO, 2005, p. 148):

“literacy as an autonomous set of skills; literacy as applied, practised and situated; literacy as a learning process and literacy as text”.

Literacy as an autonomous set of skills

According to this view, literacy consists of the skills of reading and writing, independent of context and individual background. Those ascribing to this view are concerned with understanding how these skills may be learned.

Literacy as applied, practised and situated

From this perspective, literacy cannot be applied in a way that is “neutral and independent of social context” (UNESCO, 2005, p. 151). Social and cultural contexts vary and the practice of literacy varies accordingly. This viewpoint is aligned with Gee’s (1986; 1989) and Street’s (2003; 2005a) perspectives that “literacy is a social practice, not simply a technical and neutral skill...it is always embedded in socially constructed epistemological principles” (Street, 2005a, p. 13). UNESCO refers here to the concepts of literacy events and literacy practices, concepts proposed by scholars in the field of New Literacy Studies (Street, 2005a) as being key to this definition. Heath (1982, p. 50) defines the concept of a literacy event as “any occasion in which a piece of writing is integral to the nature of the participants’ interactions and their interpretative processes”. Literacy practices are defined by Street (2003, p. 79) as “broader cultural conception[s] of particular ways of thinking about and doing reading and writing in cultural contexts”.

Literacy as a learning process

Literacy can be viewed as “a process, rather than a product because individuals become literate through the learning process” (UNESCO, 2005, p. 151). Each student will bring a different socio-cultural reality to the classroom, which drives and directs learning. This learning broadens students’ realities and allows them to approach the realities from which they come, equipped with new skills and perspectives. Therefore, the learning process itself serves to challenge these realities and social processes (UNESCO, 2005) which introduces the notion of critical literacy, in which reading and writing serve as tools to probe, question and transform the social world (UNESCO, 2005).

Literacy as text

This understanding of literacy defines it by its subject matter: “the texts that are produced and consumed by literate individuals” (UNESCO, 2005, p. 152). The examination of these texts as discourse places the notion of literacy in a wider, socio-political realm where texts “construct, legitimate and reproduce existing power structures” (UNESCO, 2005, p. 152).

2.2.1.2 A tentative consensus: Literacy as human capital

Literacy is an important measure of human capital and contributes to social and economic growth (Organisation for Economic Co-operation and Development [OECD], 2009a; 2012). It is important for countries to measure the literacy levels of their populations both in order to assess the growth potential of their economies, as well as to “be able to benchmark their measures of human capital internationally for competitiveness and productivity” (OECD, 2012, p. 5). In the information-based economy of the 21st century it is especially important for countries to gauge the literacy levels of their populations to determine their potential for participation in the global economy.

A challenge to international comparisons, however, is the fact that literacy is understood differently in different countries, and literacy levels cannot be compared where the different measures have been used (OECD, 2009a). Large-scale international surveys have been designed and implemented, allowing countries to assess the skills of their population and to ascertain their level of preparation for the “challenges of the modern knowledge-based society” (Programme for the International Assessment of Adult Competencies [PIAAC] Literacy Expert Group, 2009, p. 3) relative to other nations. In order to achieve this, member countries need to all agree as to the definition of literacy put forward by these bodies. These definitions are therefore extremely influential and represent a form of global consensus as to what literacy is.

The Programme for International Reading Literacy Study [PIRLS] assessment, administered by the International Association for the Evaluation of Educational Achievement [IEA], is focussed on assessing the reading literacy of students in their fourth year of schooling (Mullis, Martin, Kennedy, Trong & Sainsbury, 2009). Reading literacy is defined for their purposes as “the ability to understand and use those written language forms required by society and/or valued by the individual” (Mullis et al., 2009, p. 11). A literate young reader is considered able to “construct meaning from a variety of texts [and] read to learn, to

participate in communities of readers in school and everyday life, and for enjoyment” (Mullis et al., 2009, p. 11).

The PISA assessment is an internationally standardised assessment administered to 15-year-olds (OECD, 1999). In PISA 2000, the domains of reading, mathematical and scientific literacy were assessed, not only in terms of whether the student had mastered the school curriculum, but also whether they possessed the requisite knowledge and skills for their adult life (OECD, 1999). Reading literacy is defined for PISA as “[u]nderstanding, using and reflecting on written texts in order to achieve one’s goals, to develop one’s knowledge and potential, and to participate in society” (OECD, 1999, p. 12; 2003, p. 15; 2006, p. 46). The PISA 2009 definition is updated to include “engaging with written texts” (OECD, 2009a, p. 23).

For the International Adult Literacy Survey [IALS], literacy is defined along a continuum according to the extent to which an adult is able to demonstrate competence in “us[ing] information to function in society and the economy” (OECD, 2000, p. x). Their specific definition of literacy is: “the ability to understand and employ printed information in daily activities, at home, at work and in the community – to achieve one’s goals, and to develop one’s knowledge and potential” (OECD, 2000, p. x).

The PIAAC Literary Expert Group (2009) revisited earlier conceptions of literacy in light of IALS results from the 1990s, as well as by considering the broadened range of competencies required in the 21st century’s information age. They adapted the IALS definition to read: “Literacy is understanding, evaluating, using and engaging with written texts to participate in society, to achieve one’s goals, and to develop one’s knowledge and potential” (PIAAC Literary Expert Group, 2009, p. 8).

The UNESCO definition encompasses all of the facets of literacy included in these definitions and represents the tentative consensus that appears to have been reached regarding how to operationally define literacy. The following was proposed as an internationally relevant operational definition of literacy:

Literacy is the ability to identify, understand, interpret, create, communicate and compute, using printed and written materials associated with varying contexts. Literacy involves a continuum of learning in enabling individuals to achieve his or

her goals, develop his or her knowledge and potential, and participate fully in community and wider society.

(UNESCO Expert Meeting, 2005, p. 21)

There is a thread of agreement evident in these definitions. The operational definitions of literacy adopted for the practical needs of international studies represent a traditional view of literacy, centred particularly on reading literacy. But, despite the appearance of consensus evident in these operational definitions, in reality, the literacy debate rages on. Gee (1999, p. 358) argues that it is possible for a child to show competence in basic reading tests, yet be unable to “use language (oral or written) to learn, to master content, to work in the new economy, or to think critically about social and political affairs”. This is increasingly the case in our global society, therefore for our 21st century purposes, a broadened view that considers more than linguistic competence in interpreting written texts is required.

2.2.2 Numeracy

Steen (1990, p. 211) writes that “[n]umeracy is to mathematics what literacy is to language” and as such comprises those mathematical skills required for participation in society, as the definitions for literacy similarly emphasise the ability to use linguistic skills to participate fully in society. A common view in the first half of the twentieth century, according to Steen (1990), was that literacy, and by extension numeracy, was regarded to be achieved at the end of the fourth year of formal schooling. The twenty-first century information age in which we now find ourselves, demands a far higher level of literacy and numeracy for effective participation in society.

De Lange (2003) writes that innumeracy is a significant societal problem that is often overlooked. One facet of innumeracy is the “inability to evaluate statements regarding problems and situations that invite mental processing and estimating” (de Lange, 2003, p. 75). Competence in evaluating such statements and problems is essential for a fully functional and independent adult in the Information Age. Statistics, graphs, percentages, rates and other forms of numerical information fill many, if not the majority, of the texts individuals are confronted with in their personal, academic and working lives.

Similar to the New Literacy Studies' differentiation between literacy events and practices (Street, 2005b), numeracy events and practices are defined by Baker, Street and Tomlin (2003). Numeracy events are: "occasions in which numeracy activity is integral to the nature of the participants' interactions and their interpretive processes" (Baker et al., 2003, p. 12). Numeracy practices are "broader cultural conceptions that give meaning to the event, including the models that participants bring to it" (Baker et al., 2003, p. 12). Literacy practices are predominantly focussed on the written mode, embedded in the oral mode and often make use of the visual mode (Baker & Street, 2004). Numeracy practices are similarly "enacted in a number of different modes such as speech, writing and visual representation...and may involve a mix of such modes, similar to other acts of communication" (Baker & Street, 2004). Numeracy can be considered a subset of literacy, being literacy practices involving so-called numerate texts and the social process of making meaning of these (Barwell, 2004).

Similar to the international drive to assess literacy, there is a need for numeracy levels of populations to be ascertained and compared in order to allow governments and other stakeholders to gauge the level of preparedness of their citizens and economies for the demands of the information age. Just as literacy is challenging to define across cultures and countries so is numeracy, yet a form of consensus is required for these studies to be relevant to all participating nations.

The OECD definitions (1999; 2003; 2006; 2009; 2012; PIAAC Numeracy Expert Group, 2009; PISA Governing Board, 2010) are particularly influential due to the number of such international studies they have commissioned. The construct is defined for the purposes of the PIAAC assessment as: "the ability to access, use, interpret, and communicate mathematical information and ideas, in order to engage in and manage the mathematical demands of a range of situations in adult life" (PIAAC Numeracy Expert Group, 2009, p. 22). Their definition of numerate behaviour is that it involves "managing a situation or solving a problem in a real context, by responding to mathematical content or ideas represented in multiple ways" (PIAAC Numeracy Expert Group, 2009, p. 22).

Numeracy is distinguishable from the mathematics that is learned in formal schooling. Numerate individuals require more than just the ability to work with abstract mathematical concepts, but should "understand the meaning of numbers [and] see the benefits of thinking

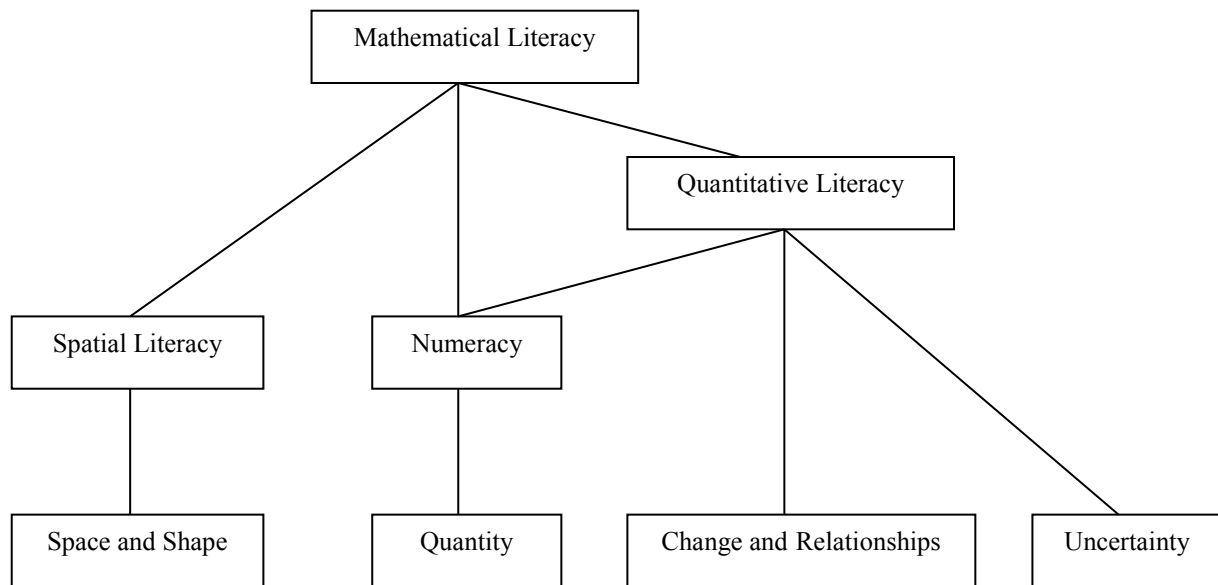
quantitatively about commonplace issues” (Steen, 2001, p. 15). School mathematics does not always address all aspects of numeracy. Numeracy, according to Barwell (2004), involves the use of diagrams, numbers and calculations in social practice, whereas mathematics is more focussed on abstraction. While diagrams, numbers and calculations are themselves abstractions, their use as located in social practice adds the dimension that distinguishes numeracy from pure mathematics.

2.2.3 Mathematical literacy

For the purposes of PISA 2012, the PISA Governing Board (2010, p. 5) have defined mathematical literacy as: “an individual’s capacity to recognise, do and use mathematics, including to reason mathematically in a variety of contexts, and to identify the role that mathematics plays in the world by describing, modelling, explaining and predicting phenomena”. It is described as a continuum, with those more mathematically literate being capable of making more informed decisions (PISA Governing Board, 2010).

Mathematical literacy, as it is explained by de Lange (2003, p. 81), is regarded as the “overarching literacy comprising all others”. Previously, numeracy has been referred to as the “minimum competency” (Hoyles, Wolf, Molyneux-Hodgson & Kent, 2002, p. 3) required of adults in the workplace. This has been replaced by mathematical literacy due to the changing nature of work at all levels of employment, particular as a result of the demand for competence in information technologies (Hoyles et al., 2002). This organisation of literacies is illustrated in Figure 2.1.

Figure 2.1 de Lange's (2003) organisation of mathematical literacies



from de Lange (2003, p. 81)

De Lange (2003) and Ojose (2011) provide definitions for each tier of this conceptualisation:

- Spatial literacy: “our understanding of the three dimensional world in which we live” (de Lange, 2003, p. 80)
 - Space and shape refers to how we need to “understand the properties of objects and their relevant positions... [and] learn to navigate through space and through constructions and shapes” (Ojose, 2011, p. 95).
- Numeracy: “the ability to handle numbers and data” (2003, p. 81)
 - Quantity refers to the “understanding of relative size, recognition of numeral patterns, and the ability to use numbers to represent quantifiable attributes of real-world objects” (Ojose, 2011, p. 94).
- Quantitative literacy subsumes a cluster of categories (de Lange, 2003):
 - Numeracy, as defined in 2.2.2
 - Uncertainty: This includes the topics of data, chance, statistics and probability (Ojose, 2011).
 - Change and relationships: The ability to translate different representations of relationships between phenomena and how these phenomena change, for example graphic or algebraic representations (Ojose, 2011).

One can also distinguish between general and specific mathematical literacy. General mathematical literacy would be that which is expected of all individuals, regardless of their life roles (de Lange, 2003). Specific mathematical literacy is that which is required by an individual in his or her specific position in the community, for example, the unique set of competencies required for one's career (de Lange, 2003). The particular terminology used by de Lange (2003) to make this distinction is basic mathematical literacy versus advanced mathematical literacy. These terms, however, cloud the fact that, depending on the career (e.g. manual labour), the mathematical literacy required in the workplace may be more basic than what De Lange (2003) would call basic mathematical literacy.

Hoyles et al. (2002) identified common key mathematical skills required by the majority of employees in the workplace. These are (Hoyles, et al., 2002, p. 24-25):

- Multi-stage calculations including percentages
- Ability to understand relationships (including indirect/multi-step) between variables
- Ability to read, interpret and transform data from charts and spreadsheets
- Ability to create formulae
- Confidence in identifying, appreciating and using concepts of risk and probability
- Ability to use approximations, estimates and formal probabilities to model likely events

The educational ideal, according to Vithal and Bishop (2006), is to create a more accessible and equitable mathematics. To this end, the South African Department of Education introduced Mathematical Literacy as a compulsory subject for students in the FET phase who do not choose Mathematics as a subject (Mbekwa, 2006).

Owen and Sweller (1985), in their research regarding problem-solving, note that rules and principles of mathematics are taught in the mathematics classroom, yet it is often the case that the principles of solving problems are not a focus. It therefore becomes possible for a student to solve many numerically-based real-life problems and "remain almost oblivious to the [problem] structure" (Owen & Sweller, 1985, p. 273). These students may find difficulty transferring the use of these rules and principles to problems that are framed differently.

Mathematical ability therefore cannot be considered synonymous with mathematical literacy which involves mathematics in the broadest sense. It differs significantly from the formal,

abstract and symbolic competency associated with the mathematics of the classroom, and refers rather to the intuitive, contextual and concrete application of numerical concepts (de Lange, 2003). This distinction is such that a student declared competent in the mathematics classroom may in fact be incapable of literate thought with regard to the application of this mathematics in the real world. Mathematics education is not only important for those students who wish to pursue a career requiring mathematics for entry into a course or for successful engagement with that subject matter, it is also essential for the entire school population to “enact their citizenry in a rapidly advancing scientific and technological world once they leave school” (Vithal & Bishop, 2006).

2.2.4 Mathematical Literacy as a subject

Mathematical Literacy was introduced as a subject in South Africa in order to consolidate and apply the mathematical content of the preceding General Education and Training Phase. The emphasis is on the use of these mathematical skills in applied contexts. This application increases in difficulty and complexity as the student progresses through the FET phase to demand higher levels of understanding and analysis (Venkatakrisnan & Graven, 2006). A shift is envisioned from the abstract mathematics they have learned in the GET Phase, to the concrete application of these mathematical skills (Venkatakrisnan & Graven, 2006).

The *Subject Guidelines: Mathematical Literacy Level 2* (Department of Higher Education and Training [DHET], 2012a, p. 1), define mathematical literacy as being: “An attribute of individuals...that involves managing situations and solving problems in everyday life, work, societal and lifelong learning contexts by engaging with mathematical concepts...presented in a wide range of different ways”. The Department of Basic Education [DBE] (2011, p. 10) lists the competencies that comprise mathematical literacy as: “the ability to reason, make decisions, solve problems, manage resources, interpret information, schedule events and use and apply technology”.

De Lange’s (2003) subdivision of mathematical literacies is reflected in the topics and subject outcomes of the NC(V) Mathematical Literacy course (DoE, 2007a, p. 4) as summarised in Table 2.1.

Table 2.1 Subdivisions of mathematical literacy

Topic	De Lange's (2003) Mathematical Literacy
TOPIC 1: Numbers	Quantity
TOPIC 2: Patterns, Relationships and Representations	Change and Relationships
TOPIC 3: Finance	Quantity; Change and Relationships
TOPIC 4: Space, Shape and Orientation	Spatial literacy
TOPIC 5: Information communicated through numbers, graphs and tables	Change and Relationships; Uncertainty

This structuring relates to the capacity of students to apply mathematical skills to the “myriad issues involving quantitative, spatial, probabilistic or relational reasoning” (OECD, 2009b, p. 19) in the workplace and in their personal lives as is a requirement for success in the 21st century information and knowledge society (OECD, 2009).

2.3 THE DEVELOPMENT OF MATHEMATICAL THINKING

In the majority of recent theories on the development of mathematical thinking, it is from the classic, seminal work of Jean Piaget (1896-1980) and Lev Vygotsky (1896-1934) that theorists take their departure. Those theorists taking a constructivist view frequently make reference to Piaget, with those favouring a social constructivist or sociocultural view deriving their theories from those of Vygotsky (Cobb, 1994).

It is Piaget who identified the “fundamental role of logic-like operations in human mental activity” (Bruner, 1997, p. 123) and Vygotsky who recognised that “individual human intellectual power depend[s] upon our capacity to appropriate human culture and history as tools of mind” (Bruner, 1997, p. 123). Both theorists understood development to be a “progression from a limited to a broader and more inclusive mastery over the environment and self” (Engeström, 1996, p. 129).

In addition to reviewing the theories of Piaget and Vygotsky, the work of David Tall, an influential theorist still actively researching today, will be reviewed in this chapter. Tall describes the development of formal mathematical thinking according to innate mental structures and the experiences that facilitate their development.

2.3.1 The constructivism of Jean Piaget

Bruner (1997) describes the genius of Jean Piaget as lying in his “recognition of the fundamental role of logic-like operations in human activity” (Bruner, 1997, p. 2). These inner logical processes mediate our knowledge of the world, and our knowledge is therefore a construction based on that mediation (Bruner, 1997).

Piaget (1964) begins by explaining the concept of an operation. In order to ‘know’ an object, he argues, we act on it. “To know is to modify, to transform the object, and to understand the process of this transformation, and as a consequence to understand the way in which the object is constructed” (Piaget, 1964, p. 20). He lists some examples of operations as: joining objects, measuring, counting, separating and adding (Piaget, 1964). These operations do not occur in isolation, but are joined together to form operational structures and are also reversible, as in addition and subtraction or joining and separating (Piaget, 1964). It is the “central problem of development ... to understand the formation, elaboration, organisation, and functioning of these structures” (Piaget, 1964, p. 20).

Piaget describes cognitive development as occurring in four intellectual stages (Piaget, 1964; 1972; Piaget & Chomsky, 1980):

- Sensorimotor
- Preoperational
- Concrete operations
- Formal operations

These stages describe development between birth and the ages of 12 to 15, during which intellectual structures slowly develop (Piaget, 1972). The stages have been shown to be regular in their sequencing between individuals, with only the speed of the development varying between individuals and sociocultural environments (Piaget, 1972).

In the sensorimotor stage, which occurs during the first 18 months, the practical knowledge required to spatially locate and manipulate objects is developed. The pre-operational stage is initiated as the language ability of the child begins and, therefore, symbolic functions and representational thought start to become evident (Piaget, 1964; 1972). Between the ages of 7/8 and 11/12, the first operations become evident in the concrete operational stage. The child achieves the logic of structures such as natural numbers, ordering, classifying, measuring and causality among others (Piaget, 1972). The development of formal operations occurs between the ages of approximately 11/12 and 14/15. This signals the achievement of complete logic, evident in “the capacity to reason in terms of verbally stated hypotheses... [and] to deduce the consequences that the hypotheses necessarily imply” (Piaget, 1972, p. 158).

Piaget (1964) explains that the progression from one stage to the next is due to four factors: physical maturation; experience of the physical environment; social transmission through, e.g., language and schooling and self-regulation. Linguistic expressions, such as “some of my flowers are yellow” (Piaget, 1964, p. 23) which indicate a part-whole understanding, will not be understood by a child who has not yet begun the concrete operational stage, but once this logic is achieved, linguistic expressions that they have been frequently exposed to become comprehensible (Piaget, 1964). The reverse also applies. Piaget explains the concrete operational stage as beginning with verbal ability (Piaget, 1964; 1972). It is clear, therefore, that language has a crucial role to play in movement from one stage to the next.

For children who experience the initial years of this development in a home language other than that which is used in school, the possibility exists that there may be delays or difficulty in subsequent mathematical success in expressing this logic in the second language. Von Glasersfeld (1982, p. 11), in his interpretation of Piaget’s work, notes that “logical-mathematical thought operates with elements which, in every particular operational context, are taken as pre-established”. It is possible that a student would be capable of that level of logic but would be unable to comprehend associated questions or express their competence in a language other than that in which this development initially occurred.

2.3.2 Lev Vygotsky’s theory of concept formation

Berger (2005) argues that Piaget’s theory and those theories of mathematical concept formation and development that are derived from his work are flawed in their lack of

attention to the “crucial role of language (or signs) and the role of social regulation” (Berger, 2005, p. 154). Vygotsky, in contrast, views the mental functioning of the individual as being derived from social and cultural processes (Wertsch & Tulvist, 1992) rather than from “interiorised actions” (Berger, 2005, p. 154) as is Piaget’s focus.

Vygotsky (1998) writes of the intellectual development of the adolescent in the “transitional age” (Vygotsky, 1998, p. 42) where the individual begins to think in terms of concepts and relies less on concrete thought. He writes that during this age, new content learned creates new types of behaviour (Vygotsky, 1998). Together with this shift to thinking in concepts, the adolescent is also initiated into the world of social ideology (Vygotsky, 1998). Vygotsky (1998) argues that although children are also exposed to sociocultural environments, their as yet incomplete mastery of intellectual skills means that they do not yet fully participate in this environment.

Children are still in the process of growing into their intellect and that of those around them (Vygotsky, 1978). Those that seem able to perform actions beyond their years are reliant on imitation and do not yet fully comprehend what they are doing, or why (Vygotsky, 1978). Vygotsky (1978) compares this to a child using a word they have learnt to say, but do not yet understand, in conversation with an adult. For adolescents, however, higher forms of thinking open up as the “inevitable result of the formation of concepts within the sphere of a particular societal ideology” (Vygotsky, 1998, p. 44).

The formation of concepts and subsequently, thinking in concepts, leads “to discovery of the deep connections that lie at the base of reality, to recognising patterns that control reality [and] to ordering the perceived world with the help of...logical relations” (Vygotsky, 1998, p. 48). Speech becomes a powerful means with which to analyse, classify, order and generalise, and the word becomes the carrier of concepts (Vygotsky, 1998). Vygotsky (1981, p. 163) claims that “[s]ocial relations among people genetically underlie all higher functions and their relations”.

In Vygotsky’s later work, a major theme was concept formation or conceptual development, with a focus on the achievement of mature academic conceptual thinking (van der Veer, 1994). The child using a word he or she does not fully understand in communication is making use of a pseudoconcept (Berger, 2006). As a result of this communication, this child

will find that the meaning of the word or concept evolves from this use. Therefore, the meaning of the concept “undergoes substantial development for the child as [s/he] uses the word or sign in communication with more socialised others” (Berger, 2005, p. 155)

In mathematics, Berger (2005) argues, the student is required to construct the concept such that its meaning agrees with how it is used by the mathematics community. This construction happens as the student communicates with learned others, for example, in interaction with a lecturer or by the use of a textbook. These mathematical concepts are thus socially integrated (Berger, 2005).

Vygotsky differentiates between three stages in the formation of a concept. Berger (2005, 2006) summarises these as follows:

1: Syncretic heap stage

A child groups unrelated objects or ideas in a seemingly random manner.

2: Complex stage

Objects and ideas become grouped in the child’s mind according to associations or common attributes that the child has recognised. This grouping is not due to logic, but rather based on experience.

3: Potential concept stage

A student learns to group objects according to their particular abstract characteristics, therefore using abstract thinking concurrently to complex thinking. This resultant group is referred to as a potential concept.

As Berger (2006, p. 17) explains, “[a]bstractions are inherent in the construction of any mathematical concept”, therefore Vygotsky’s notion of a potential concept is particularly relevant in considering the development of mathematical thinking. Students move from the use of pseudoconcepts, where the correct use of the concept is coincidental, to the use of a fully constructed genuine concept.

Vygotsky’s focus on social and cultural processes as shaping cognitive development implies that speech and understanding are inseparable and are “manifested identically in both social use of language as a means of communication and in its individual use as a means of thinking” (Vygotsky, 1998, p. 50). With the first stages of any child’s development occurring with primary caregivers in the home, the language and culture of the home becomes

influential in their development. If formal education is then conducted in a second language, according to Vygotsky's theory this change in language must influence development.

2.3.3 David Tall and the three worlds of maths

David Tall's theory of the development of mathematical thinking is based on Piaget's (1964; 1972) notion that cognitive growth proceeds through "actions on existing objects that become interiorised into processes and then encapsulated as mental objects" (Pegg & Tall, 2005, p. 189). Gray and Tall (1994) introduced the idea of a procept as an "amalgam of process and concept" (Gray & Tall, 1994, p. 116). Symbols act as pivots, which can switch from a process to a concept: adding two numbers would be a process (e.g. $3+4$), whereas the concept would be that the sum of $3+4$ is 7 (Pegg & Tall, 2005). Gray and Tall (1994) write that the development of procepts passes through three phases: process-procedure-procept.

Children who solve addition problems by 'counting all' items from first to last are using a process, e.g. 3 items + 2 items = (count 1,2,3,4,5) 5 items (Gray & Tall, 1994). Those who 'count on' understand the concept of 'three' and are said to be making use of a procedure, e.g. 3 items + 2 items = (count 3,4,5) 5 items; those who have developed proceptual understanding are able to use their proceptual knowledge that $3+2=5$ to bypass the process of counting altogether (Gray & Tall, 1994).

Gray and Tall (1994) explain that this proceptual fact is not the same as a rote learned fact. Proceptual understanding is characterised by a "rich inner structure which may be decomposed and recomposed to produce derived facts" (Gray & Tall, 1994, p. 118). This implies that flexibility has been achieved and the child is able to recognise equivalence, e.g. $2+3=5$; $5-3=2$; $5-2=3$, and is able to develop new proceptual facts, such as subtraction as related to addition.

David Tall (2006) expands this theory to describe long term development towards powerful mathematical thinking. He writes that development from new-born to adult requires "powerful ideas to be compressed into thinkable concepts that apply in new situations" (Tall, 2006, p. 1).

The development of mathematical thinking is described as resting on what Tall (2008a) terms set-befores and met-befores. Set-befores are innate mental structures which develop with brain maturation. Examples would include the ability to perceive colours, to perceive objects and to perceive movement, all of which develop as the visual structures of the brain mature (Tall, 2008a). Three set-befores, in particular, are essential for shaping the long-term development of mathematical thinking (Tall, 2008a, p. 6): “recognition of patterns; repetition of sequences of actions and the language to describe and refine how we think”.

Met-befores are experiences that facilitate personal growth in cognitive development. They allow neural connections and networks to become more refined by initiating the reformulation of “old information in new ways, changing how we think as we grow more mature” (Tall, 2008a, p. 2).

Tall (2008a, p. 1) further elaborates on his developmental model to outline what he terms the “three worlds of mathematics”: the conceptual-embodied world; the proceptual-symbolic world and the axiomatic-formal world. Conceptual embodiment arises from the set-before recognition, in which knowledge structures are built and categorised according to what we think about and perceive. This is facilitated by our practical interaction with the physical world.

Growing out of the embodied world is the proceptual-symbolic world in which physical actions and experiences become symbolised as “processes to do and concepts to think about” (Tall, 2008a, p. 7). Reliant on the cognitive development in these two worlds is the formal-axiomatic world where formal concepts are based on linguistic definitions that will have been formed during prior developmental stages (Tall, 2008a). This is the most sophisticated level of mathematical thinking.

The earliest stages of this development occur in the context of the home, in which this emergent mathematical thinking is mediated by the home language. Objects perceived and manipulated in the physical world are named, and processes described, in interactions using language. Language also allows for the compressed reference to these complex, structured entities. The ultimate progression is towards the complex, structured cognitive abilities of the axiomatic-formal world and this will rest on cognition acquired in an environment characterised by a specific language.

2.3.4 Conclusion

All of the developmental theories discussed imply a link between the development of mathematical thinking and language. Where there is discontinuity between the language used in the home, where initial development occurs, and the language used at school, the development of mathematical thinking must be influenced to some extent.

2.4 MATHEMATICAL THINKING AND PROBLEM SOLVING

Schoenfeld (1992, p. 28) describes mathematics as a search for patterns and argues that “‘doing’ mathematics...involves observation of patterns, testing of conjectures, and estimation of results” and it does not simply consist of calculations. It does not consist of knowledge, but rather the ability to apply the “process aspects of mathematics” (Schoenfeld, 1992, p. 28), in other words, to problem solve. In this section problem solving theories will be discussed, with reference to mathematical literacy and the solving of problems both in the real world, and in the mathematical literacy classroom.

2.4.1 Generic problem solving theories

Polya is considered by many to be the “father of the modern focus on problem solving in mathematics education” (Passmore, 2007, p.1). In his pioneering work, *How to Solve It* (Polya, 1957), he outlines a process that has informed conceptions of problem solving (OECD, 2003) and mathematical literacy (PISA Governing Board, 2010) still used today. His four-stage process of problem solving consists of (Polya, 1957):

1. Understanding the problem
In this first step, an understanding of the unknowns, data and conditions of the problem needs to be established.
2. Devising a plan
A connection needs to be found between the unknowns and the given data to arrive at a plan for finding the solution.
3. Carrying out the plan
4. Looking back

The solution obtained by carrying out the plan must be examined for its correctness.

The authors of the framework for the PISA 2012 (PISA Problem Solving Expert Group, 2010) assessment of problem solving have derived their four problem-solving processes from Polya's (1957) work. These processes do not necessarily form a linear information processing system but represent parallel processes (PISA Problem Solving Expert Group, 2010). They are:

1. Exploring and understanding
The problem situation is observed and interacted with in the search for information, limitations and obstacles in order to build a mental representation.
2. Representing and formulating
A coherent mental representation is formed by mentally organising information and integrating it with prior knowledge. This includes constructing tabular, graphical, symbolic or verbal representations; formulating hypotheses and critically evaluating information.
3. Planning and executing
Planning involves goal setting and devising a strategy to achieve the goal state and then carrying out this plan.
4. Monitoring and reflecting
As progress is made towards the goal state, intermediate and final results are checked with remedial action taken where necessary. On reflection, assumptions are critically evaluated and alternative solutions considered.

PISA Problem Solving Expert Group (2010, p. 20)

The PISA 2012 (PISA Problem Solving Expert Group, 2010, p. 10) definition of problem solving competence reads:

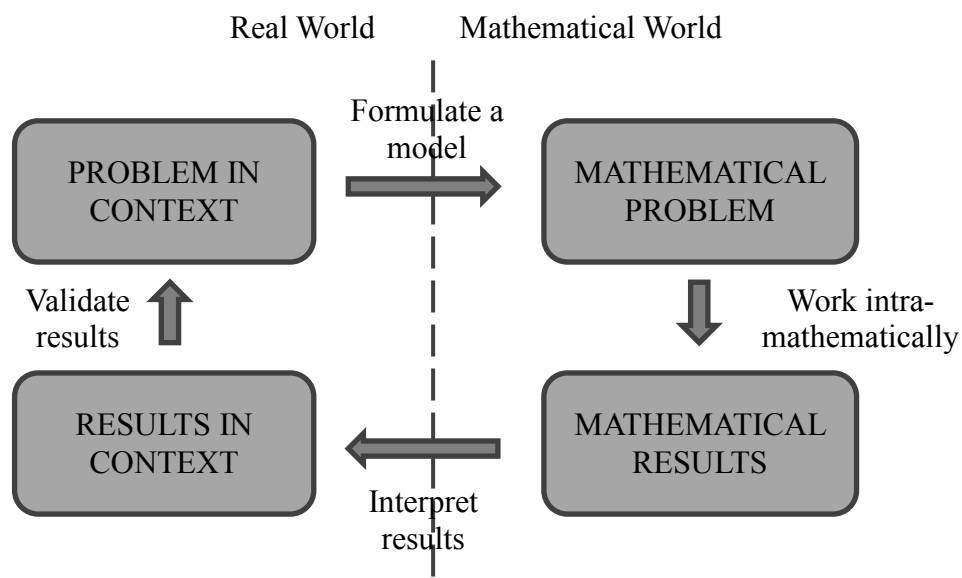
Problem solving competency is an individual's capacity to engage in cognitive processing to understand and resolve problem situations where a method of solution is not immediately obvious. It includes the willingness to engage with such situations in order to achieve one's potential as a constructive and reflective citizen.

2.4.2 Mathematical literacy in practice

The PISA Governing Board (2010) describes the specific process of solving a mathematical problem embedded in a context. This is referred to as mathematical literacy in practice. Greiff

(2012) writes that problem-solving research has followed two separate strands, with either a focus on generic cognitive ability, or “specific domain-bound ability of little generalizability” (Greiff, 2012, p. 50) with little conversation between these two traditions. The PISA 2012 (PISA Problem Solving Expert Group, 2010, p. 10) model brings these two strands together. It combines Polya’s (1957) four processes and the PISA problem solving processes (PISA Problem Solving Expert Group, 2010) to represent a mathematically domain-specific problem solving cycle. This model is illustrated in the figure below.

Figure 2.2 Mathematical literacy in practice



from PISA Governing Board (2010, p. 6)

The process of solving such a problem begins where a problem is posed in a given real world context, for example, the area of a floor to be tiled needs to be calculated by a contractor. The problem needs to be translated into a mathematical one by formulating a model. In this case, the model would require a procedure to calculate the area as well as knowledge of the relevant measurements. Once the model has been derived and the required information gathered, intra-mathematical work is done to arrive at a numerical solution. This numerical result must be interpreted in order to make the decisions for which the calculation was performed. A contractor may need the information from this example for a variety of decisions, such as to calculate the cost to tile this floor, in which case this information will become necessary for the next problem solving cycle.

2.4.3 Moving between the real world and the mathematical world

According to the PISA model of mathematical literacy in practice (PISA Governing Board, 2010), the problem solving cycle shifts between the real world and the mathematical world. In this section the movement between these two worlds will be discussed in greater detail.

Langley and Rogers (2005, p. 1) write that “problem solving abstracts from physical details”. The physical setting provides a form of external memory, therefore aiding problem solving, but the mental activity required to make sense of these details occurs at a more abstract level (Langley & Rogers, 2005). As soon as the real world problem is posed, the input is classified and filtered (Gardner, Rush, Konitzer & Teegarden, 2011). A mathematical model is formulated based on the classified and filtered information. The problem thus becomes represented in the mathematical world. Individuals set up a mental representation of the problem situation that can be manipulated to predict outcomes (Gardner et al., 2011).

This process is depicted in the PISA 2012 model (PISA Governing Board, 2010) where it refers to the formulation of a model, with which the information regarding the problem in context is transformed into a mathematical problem. These models result from “an internal understanding of the external world” (Gardner et al., 2011).

According to Lucangeli, Tressoldi and Cendron (1998), the construction of a mental model, or problem representation, seems specific to mathematical problem solving. With regard to mathematical word problems, they describe this construction as depending on four abilities. The first two involve reading for meaning: “the ability to *transform* each sentence in the text into a mental representation; and the ability to *integrate* the different pieces of information into a single coherent representation of the problem” (Lucangeli et al., 1998, p. 258). In addition, “the ability to *plan* the steps necessary to arrive at the solution and the ability to *execute* the plan” (Lucangeli et al., 1998, p. 258) are required.

Research conducted by Lucangeli et al. (1998) and Schoenfeld (1992) revealed another ability to be essential to mathematical problem solving: the metacognitive ability to self-evaluate. This corresponds with the fourth of Polya’s (1957) stages of problem solving: looking back; and the fourth process according to the PISA Problem Solving Expert Group (2010): monitoring and reflecting. Solutions need to be evaluated according to the adequacy

of the methods used and checked for accuracy. Where shortcomings are discovered, these need to be remedied before a result can be declared.

It is at this moment that the mathematical world again interacts with the real world. The verified mathematical result needs to be interpreted according to the original real-world problem context. This completes the cycle, as the abstract thought processes necessary to correctly model the problem and progress towards its solution are brought back to the physical context to implement the solution. The interplay between the world, the conceptual model and the mathematical model in which each is used to modify the other in the solution process continues in a cyclical fashion, as the PISA Governing Board (2010) model of mathematical literacy in practice depicts.

2.4.4 Working memory and problem solving

Despite the sophistication of the human brain, it is only able to process a certain limited number of pieces of information when using working memory (Tall, 2008b). Cognitive Load Theory (Sweller, 1988) explains the interface between mathematical literacy and English language proficiency in assessment. According to this theory, “performance on complex cognitive tasks depends on whether the amount of information presented to the [student] equals or exceeds the availability of working memory” (Barbu, 2010, p. 4). What this theory implies is that the devotion of cognitive resources to comprehending text would reduce the cognitive resources available for mathematical problem solving (Barbu, 2010; Lucangeli, Tressoldi & Cendron, 1998).

While working memory has a limited capacity, long-term memory is effectively limitless (Tuovinen & Sweller, 1999). Only two to four elements can be manipulated in working memory when the information that is presented is novel, as opposed to the seven elements possible when information is more familiar (Van Merriënboer & Sweller, 2005). This expands, however, when long-term memory is activated during the task. Long-term memory is a store of knowledge as organised cognitive schemata. A schema “categorises elements of information according to [the way in which] they will be used” (Sweller, van Merriënboer & Paas, 1998, p. 257). The entire schema then forms only one element to be held in working memory, allowing for more elements to be accommodated and manipulated (Van Merriënboer & Sweller, 2005). If practiced, these schemata can become automated thus

allowing for even more efficient use of long-term memory during tasks (Van Merriënboer & Sweller, 2005). This is not possible for working memory, where all processing is conscious (Sweller et al., 1998). Therefore the activation of schemata, particularly those that are automatised, will increase the availability of learned information during problem solving.

Sweller and colleagues (Sweller et al., 1998) use reading as an example. After sufficient practice, a reader is able to automatically process individual letters and words and, as a result, is able to attend to the meaning of a text. Less experienced readers, such as an English language learner, who has not had sufficient practice, will need to devote working memory to the processing of letters and words and will then be unable to hold additional information in their working memory to comprehend meaning.

2.5 ENGLISH AS A SECOND LANGUAGE

In South Africa, many students are studying Mathematical Literacy in a language other than that in which they experienced their early development. The process of second language acquisition, therefore, needs to be considered when teaching and assessing these students and when interpreting their results, particularly when comparing them to students who are assessed and taught in their home language.

2.5.1 Second language acquisition and mathematical thinking

For students whose second language is English, part of their learning will have been in their home language. Transferring the mathematical skills they have developed in their home language into contexts presented in a second language therefore becomes more complicated for these students. The movement from informal spoken language to formal, written mathematical language needs to occur at three levels: “from spoken to written language, from main language to English, and from informal to formal language” (Setati, 2002, p. 10). For English home language students, only two movements are required, i.e. spoken to written language and informal to formal language. Working in a second language, therefore, is more complicated.

2.5.1.1 Second language acquisition

The acquisition of a second language occurs cognitively along a similar path to that of the development of mathematical thinking. The information processing framework, outlined by

Saville-Troike (2010), explains that lower order component skills need to be in place for higher order skills to be learned. This is essentially the same as the learning process for other domains of knowledge.

The acquisition of these components demands controlled processing, which in turn demands considerable attention and mental effort (Saville-Troike, 2010). As the lower order component skills become automatised, mental space is made available, allowing the student to process new information and engage in higher order cognitive tasks (Saville-Troike, 2010). These higher order skills would include, therefore, the ability to study in a second language. Before the pre-requisite language skills have been automatised, the learning of other domains of knowledge in this second language requires mental space that is limited or not available at all to these students.

Cummins (1980; 1984) has contributed to the field of second language acquisition in the distinction he proposed between two types of language proficiency. His theory is still influential, more than three decades later. The distinction he makes is between basic interpersonal communicative skills [BICS] and cognitive academic language proficiency [CALP]. Cummins (1980, p. 177) defines CALP as “those aspects of language proficiency which are closely related to the development of literacy skills in [home language and second language]”, while BICS is defined as sociolinguistic competence, which may be independent of the level of CALP. BICS and CALP are not dichotomous, but are conceptualised as varying along a continuum with regard to the relative cognitive demand, and the degree of context-embeddedness of “communicative activities” (Cummins, 1984, p. 12).

2.5.1.2 Age of acquisition

Whereas home language acquisition begins at birth, the acquisition of a second language begins for many when they enter school, if not later. Much has been written about the optimal age for the acquisition of a second language. In research conducted with younger children, it has been found that it generally takes approximately 2 years to acquire BICS, and 5-7 years to acquire CALP (Collier, 1987).

Cummins (1980) writes that for older students whose CALP is well developed in their home language, the acquisition of CALP in a second language will be more rapid than for younger children. This phenomenon is the result of a common underlying proficiency which supports

the acquisition of this more cognitively demanding language proficiency (Collier, 1987; Cummins, 1980). It is only possible for the CALP in the home language to transfer properly to the second language if there is literature available to students in their home language. As explained by Jiang (2011), a student's reading performance in their second language is related to their reading ability in their home language. In South Africa, for students with an African language as their home language, this may be problematic as there are significantly fewer books available in these languages than for English and Afrikaans students. Having books in the home is a significant predictor of reading achievement for South African students (Howie, 2010).

Jiang (2011) points out that language proficiency in the second language also plays a role in the acquisition of reading proficiency in this language. Kern (2000, p. 118) explains that "proficient readers in their [home] language [are] often unable to apply their well-developed reading skills when reading in a second language". Therefore, if well-developed reading skills are achieved in an African home language, despite the limited literature available, this will not necessarily transfer to proficiency in reading in English. Cummins (1979) explains this in his Linguistic Threshold Hypothesis. This hypothesis states that a certain level of language proficiency needs to be achieved in the second language, as well as the reading proficiency in the home language, in order for students to transfer their home language reading skills to their second language.

The PIRLS 2006 study (Howie, Venter, van Staden, Zimmerman, Long, du Toit, Scherman, Archer, 2008) revealed that the percentage of African home language students achieving Grade 4 reading competence was significantly lower than English home language students, Afrikaans home language students as well as the Low International Benchmark. For these young students, this reveals a lack of CALP in their home language, which makes it more challenging for them to transfer skills to a second language. There are two things necessary for CALP to transfer from a home language to a second language:

- Students must have achieved CALP in their home language (Jiang, 2011)
- Students must have reached a threshold of language proficiency in their second language (Cummins, 1979).

2.5.1.3 Assessing students in their second language

The assessment of Mathematical Literacy, specifically the final examinations, is text-based. What is cognitively undemanding to a native English speaker will be more demanding for an English language learner (Cummins & Swain, 1986). This is particularly true for students who are in earlier stages of second language acquisition, as would be the case for students whose earlier schooling was in their home language. The processing of speech becomes highly automatic from the earlier stages of language acquisition, however, reading requires the reshaping of these processes to “interface with a new information source” (Crain & Shankweiler, 1988, p. 167). The processing of texts requires strategies of comprehension and production that are distinct from those required for everyday oral interactions (Cummins & Swain, 1986). These skills may be lacking in students who otherwise seem proficient in English based on their ability to orally communicate and interact effectively in English. The acquisition of these skills may require specific and sustained instruction (Cummins & Swain, 1986).

2.5.2 The consequences of bilingualism

It is a commonly held belief that English language learners learning mathematics in English generally underperform in assessments, when compared to students whose home language is English (Dawe, 1983). As Dawe (1983) points out, this picture does not describe the potential of these students but rather their ability to perform in the school settings. Much research, however, points to the fact that bilingualism holds both negative and positive consequences with regard to cognition (see Bialystok & Craik, 2010; Dawe 1983; Dominguez, 2008; Pavlenko, 2005; Planas & Setati, 2009; Kempert, Saalbach & Hardy, 2011). It is in students of minority language groups who are forced to learn in a second language that a deficit in general cognitive ability has been found (Dawe, 1983). Those of majority-language groups who are learning in a second language display positive associations with their bilingualism (Dawe, 1983).

Dawe (1983) writes that where language is a barrier to learning for a bilingual minority-language student, this disadvantage increases as the student progresses through their schooling. More recent research is accumulating evidence, however, that the lifelong effect of bilingualism is positive with regard to executive-control processes (Bialystok & Craik, 2010). Pavlenko (2005, p. 5) points out that although “speakers’ construction of the world may be

influenced by the structural patterns of their languages, as well as their discourses,...it may be changed through participation in alternative discourses, such as schooling, or through additional learning”.

Bilingualism can be defined in a number of ways. It can be understood as the use of two languages, both with “native-like control” (Planas & Setati, 2009, p. 38), or as speakers of one language who are capable simply of meaningful sentence construction in a second language (Planas & Setati, 2009). Planas and Setati (2009) write that it is best to regard bilingualism as existing on a continuum.

There is a pattern of negative and positive effects of bilingualism (Bialystok & Craik, 2010). Monolingual students show better performance in tasks that assess language proficiency, vocabulary knowledge and lexical access (Bialystok & Craik, 2010). Bilingual students, however, display “marked superiority over monolingual controls in flexibility of thought and ...a more diversified structure of intelligence” (Dawe, 1983, p. 328). Cognitive flexibility could be of value in the learning of mathematics (Dawe, 1983) and has indeed been shown to be associated with mathematical competence (Dominguez, 2008).

There are, however, specific conditions predicting whether a student experiences any advantages or disadvantages associated with their bilingualism. Barton and Neville-Barton (2003) explain that research on bilingualism and mathematics does show a relationship between language and mathematics, but that this effect is a complex one. The cognitive benefits are only achieved when based on adequately developed home language skills (Dawe, 1983). It is the level of abstraction, or CALP, achieved in the home language that is associated with mastery of mathematical conceptual operations (Dawe, 1983). Dominguez (2008) attributes mathematical underachievement by bilingual students to semilingualism, or limited bilingualism. He also indicates that the register of mathematics is challenging for all students and that “many difficulties that bilingual students encounter as they try to solve word problems are related to deficient knowledge of mathematical register” (Dominguez, 2008, p. 82).

In the South African context, there is a social imbalance in the contexts in which the two languages of a bilingual individual are used (Planas & Setati, 2009). This is due to the social circumstances in which they find themselves and the influence other people have on their

choice of language (Planas & Setati, 2009). Dominguez (2008) views bilinguals as possessing a cognitive resource, in that bilinguals are able to make a language choice, or ‘language switch’, in response to their needs to communicate mathematical ideas, and thus have different ways of saying things to express important mathematical ideas.

In assessments that require reading and writing in a second language, however, bilinguals who are advantaged by the ability to switch languages to explain mathematical ideas may experience a disadvantage if they have not simultaneously acquired an ability to express these ideas in the second language. Language switching leads to a cognitive cost in mathematical problem solving where “additional calculation processes [are] required when transferring knowledge from the language of instruction to the language of retrieval” (Kempert, Saalbach & Hardy, 2011, p. 2). Where the student possesses only a low command of the language of instruction, they do not possess sufficient mental representation, which results in poor school performance (Kempert, Saalbach & Hardy, 2011). In addition, when switching languages, cognitive resources are taken up, leaving fewer available for the actual task (Kempert, Saalbach & Hardy, 2011).

2.6 MATHEMATICAL LANGUAGE

Mathematics includes a language component (Bergqvist, Dyrvold & Österholm, 2012) as there are words, symbols, phrases and grammatical structures without which it cannot exist. According to Halliday (1989), it is not possible for basic and everyday language to adequately describe scientific or mathematical concepts. Barton and Neville-Barton (2003, p. 4) report that language has been identified as a “vehicle for mathematics learning” and is an important area for investigation.

Duval (2006, p. 108) writes that it is in mathematics that we find “the largest range of semiotic representation systems”. Included are common forms such as those used in natural language as well as forms such as symbolic notations and graphs which are specific to mathematics (Duval, 2006).

2.6.1 The nature of mathematical texts

A mathematical register refers to the specialised meanings of words from a reinterpretation of vocabulary and phrases from the natural language (Cuevas, 1984). The mathematical register constitutes the “styles of meaning and modes of argument...rather than the words and natural language structures” (K’Odhiambo & Gunga, 2010, p. 80). This mathematical speech and writing requires that students become proficient in both ordinary and mathematical English (Setati, 2002), therefore requiring a certain level of linguistic competence in the language of instruction. Patkin (2011, p. 2) describes mathematics as possessing “unique linguistic forms” and making frequent use of key terms, such as those signifying the four operations of addition, subtraction, multiplication and division (Patkin, 2011). In fact, key terms are involved in signifying any mathematical concept, as any important concept will result in the preponderance of terms.

The language of mathematics is “informationally dense and structurally complex” (Hammill, 2010, p. 1). Hammill (2010) lists several characteristics of mathematical texts: complex ideas are expressed in dense noun phrases; relationships are described by verbs; special terminology often conflicts with common use of the words and logical connectives are used extensively. These complex sentences have the effect of “obscur[ing] the presence of people, distanc[ing] the reader from the author, and portray[ing] the student as passive and mathematics as impersonal” (Hammill, 2010, p. 1). In this mathematical discourse, rhetorical information is not explicitly stated, but is implied (Flick & Anderson, 1980). This leads to students with English as a second language struggling to comprehend the whole meaning of a paragraph, despite comprehending each of the individual sentences (Flick & Anderson, 1980).

It is difficult to separate the process of reading from the process of problem solving (Bergqvist & Österholm, 2010). Lewis (1989) found that the majority of errors on solving mathematical word problems occurred due to the misrepresentation of the problem structure as communicated by the text and not errors in computation.

2.6.2 The symbolic component of mathematical texts

Mathematical texts are not only characterised by particular technical terms and grammatical structures, but are frequently multimodal (Hammill, 2010). Texts almost always contain symbolic notation and graphics in addition to the text (Hammill, 2010).

Symbols serve as a type of shorthand and are a means of condensing concepts into a manageable form that can be manipulated (K’Odhiambo & Gunga, 2010). The reading of symbols can be done as with English, from left to right. These symbols have their own syntax, but share the two-dimensional characteristics of diagrams (Hammill, 2010). They act as objects and can be worked with as such.

The reading of symbols requires some significant backtracking (Hammill, 2010). Hammill (2010) writes that novice readers neglect to pay attention to parentheses and select rather to focus on the individual operations. They “respond strongly to the visual structure of symbolic mathematics, independent of the semantic content” (Hammill, 2010, p. 4).

2.6.3 The visual component of mathematical texts

In mathematics, visual images are essential as a representation of abstract phenomena (Hammill, 2010). What an image adds to the meaning of a text can frequently not be achieved by written language alone, these modes, therefore, are distinct in terms of what they are able to achieve (Kress, 2000). Kress (2000, p. 339) writes: “Image is founded on the logic of display in space; writing (and speech even more so) is founded on the logic of succession in time. Image is spatial and nonsequential; writing and speech are temporal and sequential”.

It is complex and difficult to acquire meaning from visual displays, particularly when the student is offered no guidance in this regard (Hammill, 2010). The more challenging the content of the text, the more likely the student is to focus on the accompanying visual displays, and this can help to compensate for any difficulties a student may find in comprehending text (Chen, 2011). Lowrie and Diezmann (2007) explain that in order to interpret graphics successfully, students need to attend to and comprehend the mathematical content and context as well as the graphics

Graphs represent only one type of graphic used in communicating and analysing information (Lowrie & Diezmann, 2007). MacKinley (1986), in his work with software engineering and computer graphics, identified a basic set of graphical languages classified according to how information is encoded. These are summarised, with examples, in the table below:

Table 2.2 MacKinley’s (1986) graphical languages

Language	Information encoded by:	Example
Single-position languages	The position of a mark set on one axis	Horizontal axis, vertical axis
Apposed-position languages	A mark set that is positioned between two axes	Line chart, bar chart, plot chart
Retinal-list languages	One of the six retinal properties of the marks in a mark set independent of position	Colour, shape, size, saturation, texture and orientation
Map languages	Fixed positions with graphical techniques specific to maps	Road map, topographic map
Connection languages	A connected set of node objects with a set of link objects	Tree diagram
Miscellaneous languages	A variety of additional graphical techniques	Pie chart, Venn diagram

Summarised from MacKinley (1986, pp. 127- 130)

“Graphical language development is based on the assertion that graphical devices communicate information equivalent to sentences” (Nowell, Schuman & Hix, 2002, p. 2). The concept of these languages continues to be influential in current research regarding numeracy and mathematical proficiency and the role of information graphics in mathematical texts (Diezmann, Lowrie & Kozak, 2007; Diezmann & Lowrie, 2008; Lowrie & Diezmann, 2007). The knowledge students possess about how to decode graphics influences their mathematical proficiency and “impacts on whether they will be high or low performers” (Diezmann, Lowrie & Kozak, 2007, p. 2). High performers are able to perceive more information in a graphic representation than low performers and are therefore able to use these representations to derive information and make inferences (Diezmann, Lowrie & Kozak, 2007). Low performers who acquire more graphical knowledge are better able to experience success in numeracy tasks (Diezmann, Lowrie & Kozak, 2007).

2.6.4 The relationship between English proficiency and mathematical proficiency

Barton and Neville-Barton (2003) estimate that academic performance varies due to English language ability by up to 10%. Language as a “vehicle for mathematics learning” (Barton & Neville-Barton 2003, p. 4) is an important area for investigation and research as a deep and complex disadvantage is faced by students who have a poor command of the language of learning and instruction (Anthony & Setati, 2007; Barton & Neville-Barton, 2003).

Other than developing an understanding of the mathematical register, in order to acquire mathematical knowledge, students need to participate in the negotiation of meaning in the classroom which also requires competent use of English (Anthony and Setati, 2007). Cuevas (1984, p. 138) writes that the “mastery of mathematical concepts presupposes some facility with the language used to express, characterise, and apply those concepts”.

In South Africa, this is particularly important as “[t]he majority of [students]...learn mathematics in a language that they are not fluent in” (Setati & Barwell, 2008, p. 2). Research has revealed that the majority of students unsuccessful in Grade 12 are not studying in their home language (Setati & Barwell, 2008). There are additional factors which contribute to this picture, but it is undeniable that English proficiency is related to mathematical proficiency.

Sarah Howie’s (2005) secondary analysis of the South African results of the Third International Mathematics and Science Study supports this. Her key findings include that: “pupils could not communicate their answers in the language of the test” (Howie, 2005, p. 178) and that “pupils who [spoke] the language of the test more frequently at home...attain[ed] higher scores on the mathematics test” (Howie, 2005, p. 178). Howie’s (2005) analysis indicated a variety of statistically significant factors affecting the mathematics results of the student participants: achievement on an English language proficiency test; socio-economic status; perception of mathematics; exposure to English; the teachers’ view of their professional status; the mathematics teachers’ beliefs about mathematics; school location; extent of English use in the classroom; teachers’ time spent working and teachers’ preparedness for lessons. The data, however, specifically supports her conclusion that the “language component represented in a number of the variables” (Howie, 2005, p. 184) has a strong effect on achievement in mathematics.

Language frequently introduces ambiguity into the mathematics classroom. Barwell (2005) writes of two models that describe the use of language in the mathematics classroom: the formal model and the discursive model.

In the formal model, ambiguity is described as related to the dimension of formality (Barwell, 2005). Everyday language and formal mathematical language are frequently used interchangeably in the classroom, and the conflicting definitions which exist between the mathematics register and everyday language cause ambiguity and complicate meaning making (Barwell, 2005) This is particularly the case for students that have the language of learning as their second language (Barwell, 2005). K'Odhiambo and Gunga (2010) describe the method of mathematics as resting on argumentation and computation, therefore language is key and “[t]he first step in resolving any mathematical issue is to translate it into everyday language” (K'Odhiambo & Gunga, 2010, p. 86). Informal, or everyday use of language, is part of mathematical practice, thus this ambiguity needs to be navigated rather than eliminated from the mathematics classroom (Barwell, 2005).

The discursive model describes mathematics as constructed through discursive practices: “the use of spoken, written or symbolic interaction, including the use of gestures and other non-linguistic aspects of interaction” (Barwell, 2005, p. 120). An example of such an activity would be the defining of mathematical terminology, which can be done in a number of different ways. “Meaning arises through interaction” (Barwell, 2005, p. 120) and is related therefore to the broader social languages invoked by the utterances in the classroom (Barwell, 2005).

Mathematics “equips students with a concise and powerful means of communication” (K'Odhiambo & Gunga, 2010, p. 81). But, as Lewis (1989) points out, there is a need for explicit attention to be paid to developing the comprehension of the semantic structures of mathematical texts and there must be explicit instruction in representation skills in order to achieve this.

2.7 ASSESSMENT

The range of different forms of presentation, and information to be interpreted in the NC(V) subject are listed as: “collected and organised data obtained from numbers, tables and graphs” (DHET, 2012a, p. 5), all of which are framed by a given context. The way in which these contexts and concepts are presented is by “written inscription and language [which is] used to create, record and justify [mathematical] knowledge” (Anthony & Setati, 2007, p. 218). The language proficiency required for reading and writing in English, however, is not within the scope of the Mathematical Literacy curriculum, nor should it therefore interfere with the assessment of this curriculum.

2.7.1 The assessment of Mathematical Literacy in NC(V)

The PISA Governing Board (2010, p. 7) defines the key component of mathematical literacy as “an individual’s ability to apply mathematics in a variety of situations, and the definition acknowledges the importance of individuals being able to address a breadth and range of contexts”. In the real world, for which this subject aims to prepare students, contexts are not simply a creation of language, but in the classroom authentic sources are often abandoned for the convenience of a word sum or a contrived and simplified textual description. The language proficiency of the student can therefore become a stumbling block, and those who are less proficient in English are unable to access the context to even begin to solve the problem posed. There are certain issues that at times require language for their access, for example citizenship issues frequently require the use of text (e.g. in newspaper articles), but these issues themselves exist in the real-world and are not the constructions of texts.

When assessing these students, Cummins and Swain (1986, p. 141) point out that “educators’ implicit assumptions with regard to the nature of language proficiency are by no means innocuous”. Academic difficulties can be created by educators who are ignorant of the fact that ESL students require much longer than English home language students to attain grade and/or age appropriate levels of academic language proficiency levels (Cummins & Swain, 1986). Halliday (2010) also emphasises that assessment is a social practice, involving power relations, and therefore the principles of justice need to be deliberately applied by considering students’ language proficiency when interpreting results.

Furthermore, mathematical literacy examinations do not merely require the processing of text, but also the interpretation of multi-modal information presented in symbolic notation, diagrams, graphs and tables (Hammill, 2010). Kress (2000, p. 337) points out that “it is now no longer possible to understand language and its uses without understanding the effect of all modes of communication that are present in any text”. In Mathematical Literacy examinations, these multimodal sources of information are contextualised in language. The interpretation of multimodal mathematical texts is highly complex. When deliberate processing needs to occur in order to decode the text that contextualises these multimodal information sources, as with English language learners decoding English texts, working memory may become saturated. Such students would be unable to optimally engage in problem solving.

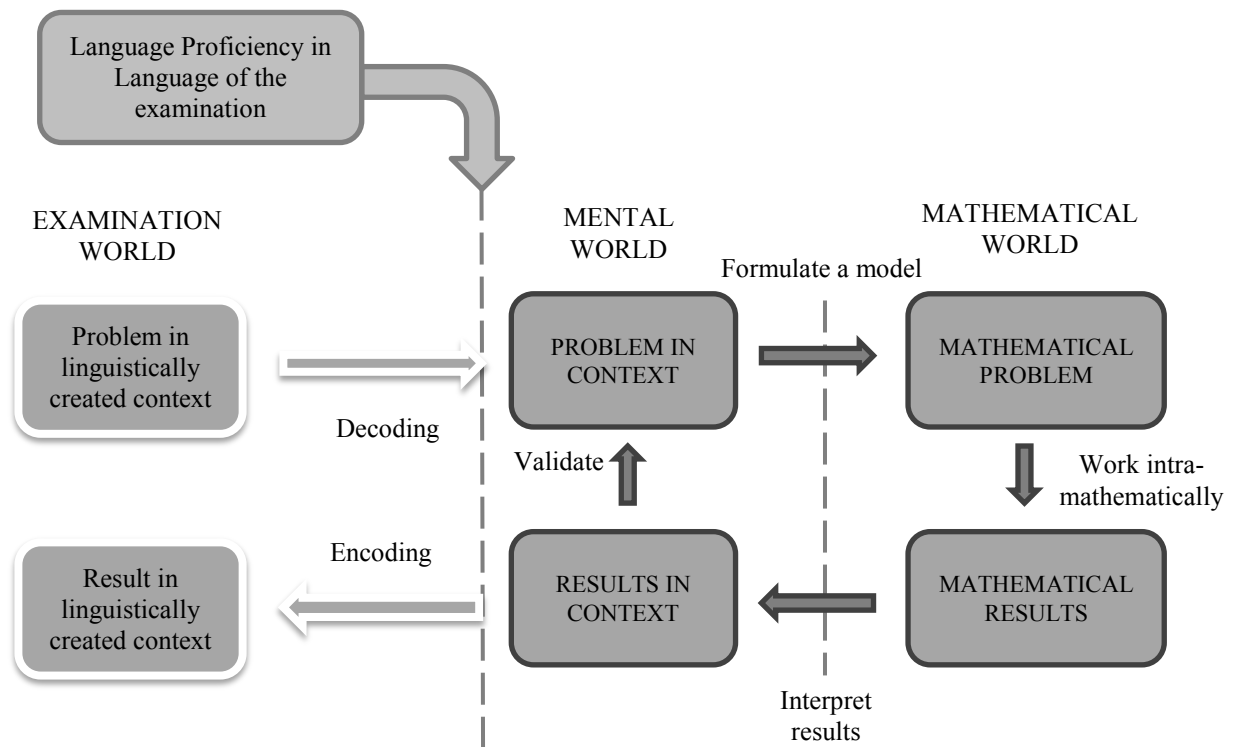
According to the DBE (2011, p. 10), it is “essential that the contexts [students] are exposed to [must be] authentic and relevant, and relate to daily life, the workplace and the wider social, political and global environments”. It is further mentioned that students “must be able to work with actual real-life problems and resources, rather than with problems developed around constructed, semi-real and/or fictitious scenarios” (DBE, 2011, p. 10). In real life we are never faced with simply a piece of paper, with no context and no interaction possible. Problem solving should not have as a pre-requisite an excellent command of language, but in final, high-stakes examinations, it is precisely this limited command of the language of assessment that may prevent students from solving the given problems accurately.

Figure 2.3 gives a graphic description of how language proficiency can impact on problem solving in an examination setting. In examinations, problems are linguistically created, and sources of information are contextualised through the use of language, for example, a scenario will be written in order to contextualise the information given in a table. The item requires decoding by reading the text as well as viewing the table.

The level of a students’ language proficiency will either allow them to understand the context and resume the mathematical literacy problem solving process with accurate information, or it will limit their understanding causing them to enter the problem solving process with inaccurate information. By the same token, once the solution has been calculated, the raw number obtained requires encoding back into the context through writing or presenting information in tables, graphs or symbolic notation. An additional characteristic of the

problem solving process in an examination that is worth noting, is the fact the problem is abstracted from the real world, and is located rather in the mental world in which it needs to be solved.

Figure 2.3 Problem solving in examinations



Adaptation of PISA Governing Board (2010, p. 6)

2.7.2 Psychometric design considerations

In general, it has been shown that English language learners do not perform as well as English home language students in language-based assessments (Abedi, 2002; 2006). In the NC(V), the majority of the students' final mark is based on their performance in a summative, high stakes, written examination. For students with low English proficiency, test item responses reveal reduced validity and low reliability (Abedi, 2002) and may be misleading (Abedi, 2006). There are, therefore, considerable psychometric issues challenging the fair assessment of English language learners.

2.7.2.1 Testing in a multicultural, multilingual context

There has been a global increase in linguistic and cultural diversity in classrooms (Khisty, 2006; Herman & Abedi, 2004), challenging educators to develop strategies of teaching and assessing that effectively address the needs of these students. In testing the subject matter competency of students for whom English is a second language, language ability may confound their results where they are assessed in English (Abedi, 2002; Abedi, Courtney & Leon, 2003a; Herman & Abedi, 2004). Abedi and Gándara (2006, p. 38) point out that English language learners' reading proficiency, in particular, "plays a major role in their assessment outcomes since without proficiency in reading, students will have difficulty understanding test questions", although proficiency in all domains of language is also essential for assessment to be valid (Abedi & Gándara, 2006). Abedi, Courtney and Leon (2003a) list three effects that language background and proficiency have on test results:

- students' language backgrounds affect their performance in content-based areas such as math and science
- the linguistic complexity of test items may threaten the validity and reliability of achievement tests, particularly for [English second language] students
- as the level of language demand decreases so does the performance gap.

(Abedi, Courtney & Leon, 2003a, p. 9)

2.7.2.2 Validity and reliability

Curriculum documents outlining the outcomes for the NC(V) Mathematical Literacy course explicitly bring to the attention of lecturers and assessors the principles of validity and reliability of assessment (DHET, 2012b). Both are technical requirements for any measure of ability (Wolfaardt & Roodt, 2005). Reliability is defined as how consistently a test produces a particular result (Wolfaardt & Roodt, 2005), in other words, whether the same result can be replicated if the context remains constant (DHET, 2012b). There are four basic types of reliability, outlined by Wolfaardt and Roodt (2005) that are applicable to written educational assessments:

- Test-retest reliability

This indicates whether two sets of scores obtained from an initial and repeated administration of a test are significantly correlated according to a coefficient of stability (Wolfaardt & Roodt, 2005)

- Alternate form reliability

When two equivalent forms of a test are administered, the set of scores for both should be significantly correlated. This reliability coefficient is known as the coefficient of equivalence (Wolfaardt & Roodt, 2005).

- Split-half reliability

When a test is split into two halves, the sets of scores obtained by administration of each half are required to be significantly correlated according to the calculated coefficient of internal consistency (Wolfaardt & Roodt, 2005).

- Kuder-Richardson reliability

This measure of reliability is relevant for tests where responses are dichotomous, in other words either correct or incorrect (Wolfaardt & Roodt, 2005). It is also known as inter-item consistency as it measures whether the item's scores correlate with one another (Wolfaardt & Roodt, 2005).

The DHET (2012b) specifically mentions validity as a requirement of assessments and explains that it refers to whether an assessment actually measures the outcome that it claims to be testing, and how well this is done. This points to construct validity and criterion-related validity, in particular. The construct validity of a measure refers to “the extent to which it measures the theoretical construct or trait it is supposed to” (Wolfaardt & Roodt, 2005, p. 35). Criterion-related validity describes the degree to which a predictor, the test score, and the criterion, the actual ability, are correlated (Wolfaardt & Roodt, 2005).

2.7.2.3 Fairness and bias in assessment

The latest assessment guidelines for Mathematical Literacy (DHET, 2012b) emphasize the principles of fairness and transparency in assessments. Methods referred to as unfair are, among others: “bias based on ethnicity, race, gender, age, disability or social class...and comparison of students' work with other students, based on learning styles and language” (DHET, 2012b, p. 3).

The concept of bias and unfairness can be extended to include instances where assessments require a high level of language proficiency in order for students to comprehend items and formulate responses. It is possible that students whose home language is the language of learning and teaching and those who have already experienced this language as the medium

of instruction may be favoured. This impacts on the validity of assessments, as the language proficiency of the student becomes what is assessed as opposed to the outcomes of the subject being examined.

2.7.2.4 Test accommodation

It is validity and fairness, in particular, which are lacking in the assessment of English language learners' mathematical achievement through the medium of English. Schoenfeld (2002) writes that where tests are of a high-stakes nature, as in examinations, this issue is of paramount importance. Alternative assessments or tests incorporating appropriate accommodation in order for test results to accurately reflect whether a student has mastered the content and thought processes required by the subject (Schoenfeld, 2002).

In an effort to increase the validity of assessments for English language learners, tests are frequently adjusted to accommodate these students. Strategies of accommodation do raise the concern that in adjusting the tests, construct validity could be compromised (Abedi, Courtney & Leon, 2003b; Abedi & Gándara, 2006). Abedi and Gándara (2006, p. 40) report that "many of the commonly used accommodations for English language learners...are neither effective in helping English language learners...with their language barriers nor are the results of these accommodated assessments valid". This does not mean that it is necessary for this to be the case, where design principles are skilfully applied it is possible for this to be avoided (Abedi & Gándara, 2006).

Test accommodations can be grouped into two types: test modifications and procedure modifications. Test modifications include:

- translation into students' home language
- items written in both English and the home language
- modification of the linguistic complexity of the test
- incorporation of the home language and/or English glossaries into the test.

(Abedi, Courtney & Leon, 2003b, p. 11)

Procedure modifications may include:

- allowing English language learners' to have extended time to take the test
- multiple testing sessions, small group administration or individual administration
- administration by a familiar test administrator

- availability of published dictionaries or bilingual glossaries
- simplified directions
- repeated instructions
- translating the directions
- reading the directions aloud

(Abedi, Courtney & Leon, 2003b, p. 11)

Another procedure modification would be that of language compensation. This is applied in South Africa where students who do not take English or Afrikaans as a home language receive a compensation of 1.05 for their non-language subjects (Umalusi, 2006; Mabizela, 2011; Rakometsi, 2011). MacFarlane (2011) explains that this involves multiplying a student's mark by 1.05, or adding 5% of their mark to their total score. In other words a student scoring 10% will receive an additional 5% of their 10%, which is 0.5 marks.

2.7.3 Linguistic design considerations

Unnecessary linguistic complexity of test items has been shown to “introduce a source of error in measurement and is considered as a construct-irrelevant factor in the assessment” (Abedi & Gándara, 2006, p. 39). As mentioned previously, these language factors could compromise the validity of the results obtained for English language learners being assessed through the medium of English. Abedi and Gándara (2006) report that research has revealed two significant conclusions: when linguistic complexity is reduced, the performance gap between English language learners and English home language students is reduced and in the process of reducing this complexity, the construct validity is not necessarily compromised.

According to Abedi (2006), any test involving language is, for all students, partly an assessment of their language skills. However, if a performance gap exists between English language learners and English home language students, and has been shown to be reduced on linguistically simplified tests (Abedi & Gándara, 2006), there is inequality in the extent to which scores are confounded and linguistic complexity must be attended to during test design.

There are several studies and theories that explain exactly what contributes to the linguistic complexity of tests (e.g. Halliday, 1989; Shaftel et al., 2006; Barbu, 2010; Bergqvist, Dyrvold & Österholm, 2012). These factors are described in 2.7.3.1, 2.7.3.2 and 2.7.3.3.

2.7.3.1 The word level

Bergqvist, Dyrvold and Österholm (2012) focus specifically on vocabulary and the complexity that word choice can add to a text. They outline five aspects of word difficulty: “word length; word form (e.g. nominalised verbs); word type (e.g. pronouns); common/uncommon words (word familiarity) and word meaning (e.g. a word’s potential for ambiguity). In categorising a particular word as being common or uncommon, it is necessary to do so in relation to a specific population or discourse (Bergqvist et al., 2012). Students’ oral, everyday language will differ from the technical vocabulary particular to mathematics. It is expected of students of mathematics, however, to navigate this language. Therefore, vocabulary uncommon to everyday language as well as uncommon with respect to the mathematical language it is expected of students to learn can be considered to contribute to an unnecessary linguistic complexity in a mathematical problem or test.

After analysing the difficulty of the individual words in a text, the difficulty of the text as a whole should be categorised. Some ways in which this can be indexed would include focussing on: “counting the number of difficult words [or] the proportion of difficult words” (Bergqvist et al., 2012, p. 6).

2.7.3.2 Grammatical complexity

Specific vocabulary is required if students are to understand concepts adequately. Halliday (1989) argues, however, that it is not sufficient to focus only on vocabulary, as the difficulty of mathematical texts can be more readily attributed to grammar. Halliday (1989) outlines seven aspects of mathematical texts:

1. Interlocking definitions

The definition of circle, radius, circumference and diameter interlock in that they are used to define each other, e.g. the concept of a radius cannot be defined without reference to a circle

2. Technical taxonomies

These definitions are also interlocked but are complex in the way in which they are ordered. There are different types of taxonomies, for example, superordination and composition. Referring to a square as being a type of quadrilateral is an example of a superordination taxonomy, whereas referring to a 90° angle as being a part of a square would be an example of a composition taxonomy.

3. Special expressions

This refers to special phrases rather than technical vocabulary that are particular to mathematics. These are special modes of expression that have evolved in the mathematical discourse. For example, ‘The graph will tell you how the value of your house appreciates with time.’

4. Lexical density

This is a measure of the number of lexical items per clause. Mathematical texts are often lexically dense, thus containing a larger amount of technical vocabulary per clause.

5. Syntactic ambiguity

The structure of mathematical texts can introduce ambiguity. Even simple sentences can be ambiguous.

6. Grammatical metaphor

This refers to “the substitution of one grammatical class, or one grammatical structure by another” (Halliday, 1989, p. 86). Halliday (1989, p. 86) provides the example of the phrases, “his departure” and “he departed”. The words can be considered equivalent, yet the change from the first to the second involves a change from a noun phrase to a verb clause while retaining its meaning.

7. Semantic discontinuity

Halliday (1989, p. 86) explains that writers sometimes “make semantic leaps, across which the reader is expected to follow them in order to reach a required conclusion”.

2.7.3.3 A linguistic complexity checklist

Shaftel et al. (2006) investigated which language features impacted student performance the most in a general mathematics assessment administered to Grade 4, 7 and 10 students. The assessment was administered in its original form. Items were coded according to their linguistic complexity, taking into account (Shaftel et al., 2006, p. 111): “total number of words, sentences, and clauses in each item; syntactic features such as complex verbs, passive voice, and pronoun use and vocabulary in terms of both mathematics vocabulary and ambiguous words”.

The results were analysed according to which linguistic features had a statistically significant relationship with the mean performance of the participants as they progressed from one grade’s test to the next. The statistically significant results are summarised in the table below:

Table 2.3 The relationship between linguistic features and participant performance

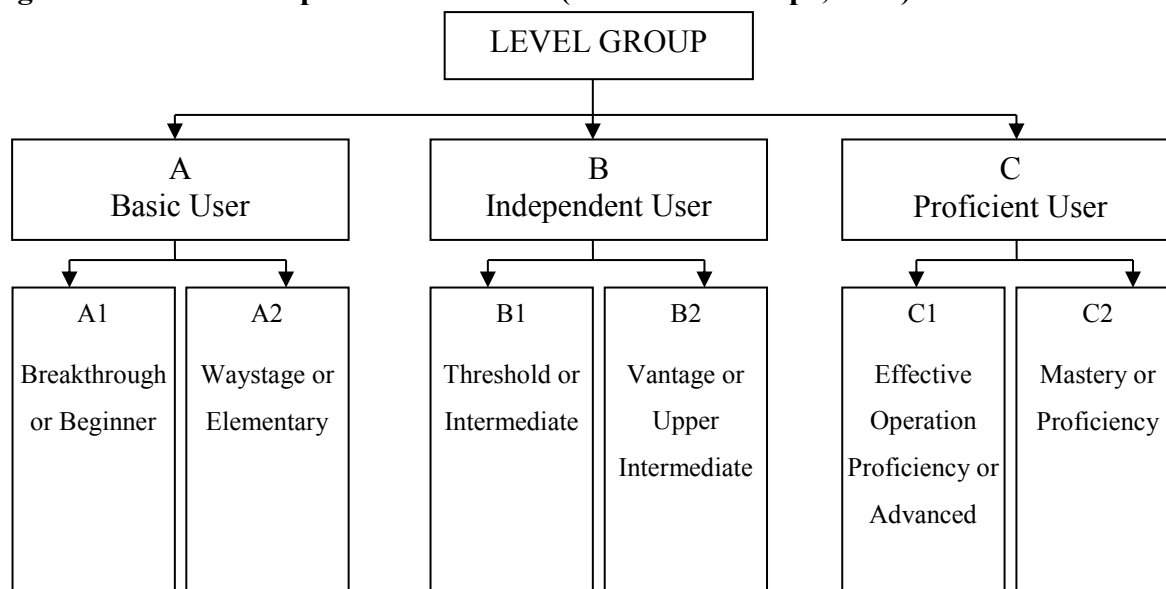
Linguistic features	Statistically significant correlations with item means		
	Grade 4	Grade 7	Grade 10
Prepositions	X (negative)		
Ambiguous words	X (negative)		
Complex verbs	X (negative)		X (positive)
Pronouns	X (negative)		
Math vocabulary	X (negative)	X (negative)	X (negative)
Comparatives		X (negative)	

These results reveal six aspects of linguistic complexity that “showed unique and statistical effects on item difficulty” (Shaftel et al., 2006, p. 117): prepositions, ambiguous words, complex verbs, pronouns, mathematical vocabulary and comparatives.

The European Association for Quality Language Services [EAQUALS]’s Core Inventory for General English [CIGE] (North, Ortega & Sheehan, 2010) is the result of a research project designed to develop a practical guide for teachers and assessors of English as a second language, to assist them in applying the Common European Framework of Reference [CEFR] for languages (Council of Europe, 2001) in their practice.

The CEFR “has been translated into 40 languages and is now *the* international standard for language teaching and learning” (North et al., 2010, p.4, emphasis in original). It was developed during the period 1992-1996 and endured a further four years of piloting before its current, published form. Therefore, although not specifically designed for the South African context, there is value in considering this carefully compiled and validated framework as a tool which could aid us in describing the linguistic complexity of a particular text. Levels of language proficiency are described as illustrated in Figure 2.4.

Figure 2.4 Level descriptors of the CEFR (Council of Europe, 2001)



from Lindhout, Teunissen & Lindhout (2012, p. 37)

Lindhout et al. (2012) extended the level descriptions to assign an approximate (Dutch) school grade level to each of the language proficiency levels A1 to C2.

Table 2.4 Dutch grade levels associated with CEFR level descriptors

Level	Description	Grade equivalent	Description
A1	Breakthrough/Beginner	7-8	Primary School
A2	Elementary/Waystage	9-10	Vocational
B1	Intermediate/Threshold	9-12	High School
B2	Upper Intermediate/Vantage	11,12	Technical College
C1	Advanced/Effective operation	13-18	University
C2	Mastery	19-23	Academic thesis

Adapted from Lindhout et al. (2012, p. 44)

The particular value which the CIGE (North, et al., 2010) adds is its detailed assignment of various grammatical features to corresponding language proficiency demands. This allows the categorisation of particular sentences according to the level of English proficiency of a reader. An example is the assignment of different types of modal verbs to categories A1 to C1: modals can/can't and can/could are categorised as comprehensible to a basic user (A);

modals must/have to and ought to are assigned to the independent user (B); and modals in the past tense, as with should have/might have require a proficient user (C) of the language (North et al., 2010). Similar break-downs have been constructed for tenses, questions, pronouns, prepositions, adjectives, adverbs, and more. All have been included in APPENDIX A. Detailed descriptions of each of the CEFR levels are provided as APPENDIX B.

2.7.3.4 Readability

In the assessment of mathematical literacy, it is relevant to consider the overall readability of the lead-in text, that which describes the context, as well as the readability of the items themselves. The degree of reading difficulty that can be ascribed to this text would need to be appropriate to the language ability of all students, including English language learners, for the construct validity of the assessment to be adequate.

Early research into learning and teaching was premised on the belief that subject matter presented to students needed to progress from simple matters to more complex ones (von Glasersfeld, 1971). Von Glasersfeld (1971) focused specifically on reading and extending knowledge of what would contribute to the difficulty of a text and therefore how to objectively assess it.

He lists three imperatives for this early research: developing a reliable measure of the difficulty students have in comprehending methods; developing a more sophisticated linguistic understanding of what affects comprehension and analysing data more deeply than had been the case until then (von Glasersfeld, 1971). Research had focussed on vocabulary, sentence length, human interest, prepositional phrases and clauses and von Glasersfeld (1971) noted the need to interrogate what might constitute linguistic complexity, and thereby increase the validity of measures of readability.

Oakland and Lane (2004) highlight the fact that in order for reading not to influence construct validity for students with lower reading ability, tests would need to have lower reading difficulty. Readability formulae are often utilised in order to match texts to the level of reader competence (Oakland & Lane, 2004; Rezaee & Norouzi, 2011). There are mixed views regarding the reliability and validity of these formulae based predominantly on the continued lack of comprehension despite simplifications made to texts according to the factors utilised by the formulae to calculate difficulty (Rezaee & Norouzi, 2011). Conversely, however,

studies have been conducted that demonstrate high negative correlations between comprehension and the level of readability as calculated by these formulae (Rezaee & Norouzi, 2011). Despite these conflicting views regarding the validity of readability formulae, their use is widespread among textbook writers and publishers (Oakland & Lane, 2004). They are considered valuable in calculating the difficulty of surface-level features of a text and are attractive in the ease with which they can be applied, despite their lack of credibility in assessing structure-level features of texts (Oakland & Lane, 2004).

Due to the widespread use of these formulae, regardless of the debate around their validity, they are useful to consider when needing an approximation of the readability of assessments. The following formulae will be briefly described, all of which have been validated in various studies (e.g. Thomas, Hartley & Kincaid, 1975; Homan, Hewitt & Linder, 1994; Allan, McGhee & van Krieken, 2005; Badgett, 2010; Crossley, Allen & McNamara, 2011):

- Flesch Reading Ease Index
- Flesch-Kincaid Grade Level Formula
- Automated Readability Index [ARI]
- Gunning-FOG Formula
- Homan-Hewitt Formula

The Flesch Reading Ease Formula

This formula makes use of the average sentence length of the text as well as the average number of syllables per word in a 100 word sample of a text. Scores range between 0 and 100, with ranges of scores assigned descriptions of the degree of ease (Allan et al., 2005; Crossley et al., 2011).

The formula is as follows (Allan et al., 2005, p. 5):

$$\text{Score} = 206.835 - (1.015 \times \text{average sentence length}) - (84.6 \times \text{average syllables per word})$$

The ranges are described in Table 2.5 (Allan et al., 2005, p. 5).

Table 2.5 Flesch Reading Ease ranges

Reading Ease Score	Description
0 to 30	Very Difficult
30 to 50	Difficult
50 to 60	Fairly Difficult
60 to 70	Standard
70 to 80	Fairly Easy
80 to 90	Easy
90 to 100	Very Easy

Lindhout et al. (2012) compared Flesch Reading Ease Scale and CEFR levels relative to Dutch school grades. The following table shows how these correspond:

Table 2.6 Comparison between CEFR levels and Flesch Reading Ease scores

Flesch Reading Ease	CEFR Level	Dutch grade equivalent	Description
70-100	A1	7-8	Primary School
50-70	A2	9-10	Vocational
35-50	B1	9-12	High School
20-35	B2	11,12	Technical College
10-20	C1	13-18	University
<10	C2	19-23	Academic thesis

Adapted from Lindhout et al. (2012, p. 44)

Flesch-Kincaid Grade Level Formula

This formula is a recalculation of the Flesch Reading Ease Formula. The same variables are utilised, but rearranged in order to give a result according to the United States school grade level a reader would need to have achieved in order to comprehend a text (Allan et al., 2005; Crossley et al., 2011). This formula is as follows (Allan et al., 2005, p. 5):

$$\text{Grade Level} = (0.39 \times \text{average sentence length}) + (11.8 \times \text{average syllables per word}) - 15.59$$

The resulting grade level cannot be equated with a South African grade level, but provides a useful estimate of the number of years of formal schooling a student would need to have in order to adequately comprehend the text.

Automated Readability Index

Three factors are used in calculating the ARI, which provides an estimated grade level: number of sentences; number of letters per word and number of words per sentence (Thomas et al., 1975). The resulting formula is as follows (Thomas et al., 1975, p. 150):

$$\text{Grade Level} = 0.50(\text{words per sentence}) + 4.71(\text{letters per word}) - 21.43$$

Gunning-Fog Formula

This formula calculates the number of years of formal schooling required for comprehension of a text on first reading. It is thus more useful when applied in a variety of contexts as it does not make use of grade levels specific to America as an indicator. It makes use of two variables: average sentence length and average number of difficult words, where difficult words are defined as those consisting of more than two syllables (Allan et al., 2005). The formula reads (Alan et al., 2005, p. 5):

$$\text{Grade Level} = 0.4 \times (\text{average sentence length} + \text{average number of difficult words})$$

Homan-Hewitt Formula

Whilst the majority of readability formulae require extended texts for a reliable and valid result to be calculated, Homan and Hewitt developed and validated a readability formula appropriate for use with single-sentence items (Badgett, 2010; Homan et al., 1994). Their formula consisted of three variables: number of difficult words; average word length; and sentence complexity (Badgett, 2010). The resulting formula reads (Homan et al., 1994, p. 351):

$$\text{Readability} = 1.76 + (0.15 \times \text{number difficult words}) + (0.69 \times \text{sentence complexity}) - (0.51 \times \text{word length})$$

This formula is the only one of its kind that is appropriate for the analysis of single-sentence items (Homan et al., 1994; Badgett, 2010).

All of the above theories give slightly different weightings to different components of linguistic difficulty, and reveal slightly different opinions regarding what constitutes linguistic difficulty. Used together, however, they can be effectively applied in the South African context to provide an approximation of the reading level a text requires of students.

2.7.4 The use of taxonomies in assessment design

The NC(V) Mathematical Literacy curriculum requires the examiner to adhere to strict guidelines regarding the spread of the cognitive domains assessed by items. These are represented in the taxonomy provided in the table below (DoE, 2007b, p. 20):

Table 2.7 NC(V) Mathematical Literacy examination design guidelines

TOPICS	Knowing 30%	Applying routine procedures in familiar contexts 30%	Applying multi-step procedures in a variety of contexts 20%	Reasoning and reflecting 20%
Numbers (20%)	<p style="text-align: center;">PAPER 1</p> <p style="text-align: center;">(150 marks)</p> <p style="text-align: center;">Paper 1 is intended to be a knowing and routine applications paper</p>		<p style="text-align: center;">PAPER 2</p> <p style="text-align: center;">(150 marks)</p> <p style="text-align: center;">Paper 2 is intended to be an applications and reasoning and reflecting paper</p>	
Patterns and Relationships (20%)				
Finances (20%)				
Space, Shape and Orientation (20%)				
Information communicated through numbers, graphs and tables (20%)				

This table also outlines the proportion of the examination that should be assigned to each of the topics. For Mathematical Literacy, these are: Numbers; Patterns and Relationships; Finances; Space, Shape and Orientation; and Information communicated through Numbers, Graphs and Tables. Each topic is allocated an even proportion of each examination.

Paper 1 is designed to test only knowledge and the application of routine procedures in the ideal mark ratio of 90:60. Paper 2 does not include knowledge items, but incorporates application of routine procedures, application of multi-step procedures and reasoning and reflecting in the ideal mark ratio of 30:60:60.

This taxonomy is based on the Trends in Mathematics and Science Study [TIMSS] (Mullis et al., 2003) 2003 mathematical cognitive domains. The four cognitive domains listed are: “knowing facts and procedures; using concepts; solving routine problems; and reasoning” (Mullis et al., 2003, p. 25). It is tempting to equate these four cognitive domains with the difficulty of the items, but within each domain easy as well as difficult items are possible (Wu, 2010; Berger, Bowie & Nyaumwe, 2010).

2.7.5 Summary

The design of an assessment that is fair, valid and reliable, and that simultaneously adheres strictly to a particular curriculum, is a detailed and complex undertaking. In the assessment of mathematical literacy, it is also essential to recognise the hidden dimension of linguistic complexity and the impact that this can have on the validity of the assessment.

2.8 ERROR ANALYSIS

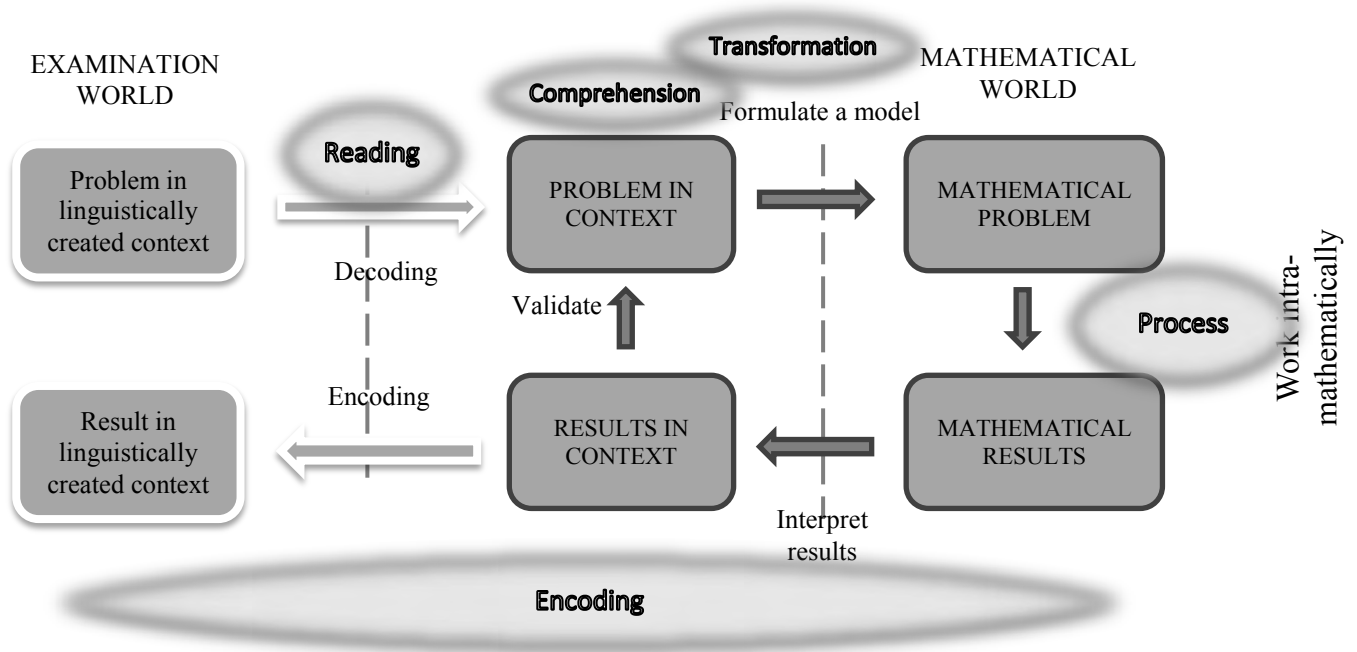
The analysis of errors has a long history in mathematics education. There are a great variety of ways in which students may respond incorrectly, but there are several models which have been developed to assist in broadly identifying where the student has gone wrong. Clements (1980) points out that it is difficult to distinctly categorise the source of any one particular error, as they closely interact and overlap. It is, however, possible to apply a theoretical hierarchy of errors to broadly classify what might have led to the student’s incorrect response.

Data from researchers who have constructed various analysis hierarchies confirm that a frequent source of errors in written mathematical tasks is “reading and reading comprehension” (Clements, 1980, p. 13). For ESL students, the linguistic item variables together with their own personal competence in the language will impact on their ability to respond correctly to any item. This leads to several recognisable errors.

Newman (1977) has proposed the following error categories into which student errors on mathematical tasks can be grouped: reading; comprehension; transformation; process; encoding. Reading errors arise when a student is not able to understand the actual words or phrases used in the problem. Comprehension errors are closely related, being those that are due to the student not understanding what it is that the item required. This could be related to the language proficiency of the student, but could also be an indication that the student has not yet fully comprehended the mathematics concept involved. A transformation error occurs when a student is unable to decide what needs to be done in order to solve the problem. Processing errors involve the inability to carry out the method that has been identified as appropriate during transformation, and an encoding error results from the inability to communicate the solution, whether in written or spoken form.

Each error occurs at a specific stage in the problem solving process. If Newman’s (1977) error categories are mapped over Figure 2.3, the model describing problem solving in examinations, it becomes clearer where these errors can be placed in this process.

Figure 2.5 Sources of error in Mathematical Literacy examinations



adaptation of PISA Governing Board (2010, p. 6)

On careful inspection of student responses to examination items, one can detect which factor/s contributed to the error as well as determine at what stage of the problem solving process this error occurred.

2.9 SUMMARY

This review of relevant literature has considered the ideas of seminal theorists as well as current thinkers in the fields of literacy, cognitive development, mathematical thinking and problem solving, second language acquisition and psychometric assessment. It is necessary to consider all of these fields of research in order to begin to comprehend the multitude of facets of assessing English language learners in mathematical literacy.

CHAPTER 3

METHODOLOGY

3.1 INTRODUCTION

Noor (2008) writes that the choice of research approach and research method depends on the nature of the research problem to be explored. In this study, the assessment and performance of a specific group of individuals, with a similar experience of learning in English where this is not their home language, is the focus. The research is descriptive in nature. Both quantitative and qualitative content analysis will be performed in order to develop an in-depth description of the case study.

In this chapter, the specific research questions will be introduced, as well as the selected research approach. The ontological position of the study is cognitive constructivism, with an epistemological view aligned with rationalism and a realist methodology. These positions will be expanded on and explained to demonstrate how they form a coherent whole. The method chosen is a case study, with a quantitative and qualitative analysis of the data.

The selected site and participants are described in detail as well as the pilot study which was conducted prior to the research itself. Firstly the existence of the phenomenon needed to be established, and the analysis technique tested, before the actual study could take place.

Finally, the data collection process as well as the method of analysis is described, and the questions of validity, reliability and ethics are addressed.

3.2 RESEARCH QUESTIONS

Mouton and Marais (1996, p. 103) explain the process of descriptive research: “Once the data have been generated, the researcher attempts to discover relationships or patterns by means of close scrutiny of the data”. This progression is reflected in the four research questions, which move from 3 questions requiring the gathering of data which contributes to the development of an in-depth description of the examination and the students’ scripts, to the final question in which patterns and relationships are sought between variables found in the descriptive data.

The Research Questions are as follows:

Research Question 1: How can Mathematical Literacy examination items be described with regard to their linguistic complexity?

Research Question 2: How can Mathematical Literacy examination items be described with regard to their cognitive complexity?

Research Question 3: What type of errors are student making when responding to Mathematical Literacy examination items?

Research Question 4: How do the linguistic complexity and cognitive complexity of Mathematical Literacy test items relate to the types of errors students are making?

3.3 RESEARCH APPROACH

In this study, the students whose performance is evaluated are viewed from a constructivist perspective. It is assumed that they arrive in the classroom with “multiple, apprehendable, and sometimes conflicting social realities” (Guba & Lincoln, 1994, p. 111), which have influenced their construction of knowledge.

3.3.1 Constructivism

Constructivists view knowledge as socially constructed and acknowledge the existence of multiple realities (Golafshani, 2003). Mouton (1998, p. 46) explains that, “complex mental structures are neither innate nor passively derived from experience, but are actively constructed in the mind”. As Phillips (1995) explains, a constructivist researcher does not assume that individuals are born with knowledge already present, neither do they believe that they are born with an idea of how to acquire knowledge. Rather, they agree that there is cognitive potential present at birth, but knowledge and the methods used to acquire knowledge, are constructed through interactions between human beings in our “multiple and diverse realities” (Golafshani, 2003, p. 603) within our social realities. That is, “our simple ideas may be mere reflections of nature, but complex ideas are [constructed] by the human mind” (Phillips, 1995, p. 8).

The current study is informed by this view of the individual student. The students whose examination scripts will be analysed will have constructed this knowledge, in part, in a language other than the language of the assessment. For these assessments to be fair and valid, the individual context in which students may have constructed the knowledge should not reflect in their responses. It is the existence of this knowledge and their proficiency in mathematics, and not the process and context of individual construction, which should be assessed.

Psychological researchers differentiate between two types of constructivism, differing slightly according to the degree to which they are realist or relativist. These are: cognitive constructivism and socioconstructivism.

3.3.1.1 Cognitive constructivism

Cognitive constructivism is slightly more realist in its approach than socioconstructivism and is concerned with how mental structures develop as an individual negotiates his or her social world (Schuh & Barab, 2008). Piaget is considered the major theorist to have contributed to this branch of constructivism. Various names have been used to describe it: radical constructivism; cognitive constructivism and psychological constructivism (Schuh & Barab, 2008).

Cognitive constructivists do not explicitly deny that there is an objective, real world in which we exist, thus positioning them slightly closer to the realist than the relativist schools of thought (Schuh & Barab, 2008). What categorises them as constructivists, however, is their acknowledgment that the individual mind is influenced by social context (Schuh & Barab, 2008). Therefore, depending on the context, each individual's mental constructions may differ.

Duit (1996, p. 42) summarises three key principles of cognitive constructivism:

1. Knowledge is not passively received but is built up by the cognising subject.
2. The function of cognition is adaptive and enables the [students] to construct viable explanations of experiences.
3. The process of constructing meaning [is] always...embedded within a social setting of which the individual is part.

This is a very particular view of knowledge construction and reflects the views of theorists Jean Piaget (1964; 1972; Piaget & Chomsky, 1980) and David Tall (2006; 2008a; 2008b; Pegg & Tall, 2005; Gray & Tall, 1994), as described in section 2.3 of Chapter Two.

3.3.1.2 Socioconstructivism

Socioconstructivism, also known as sociocultural constructivism, social constructivism or sociohistoricism (Schuh & Barab, 2008) can be distinguished from cognitive constructivism. Theorists working within this paradigm, most notably Vygotsky, contend that “knowledge creation is a shared rather than an individual experience and evolves through social negotiation” (Schuh & Barab, 2008, p. 74).

This remains similar to cognitive constructivism. What differentiates this position from that of the cognitive constructivists, however, is that, although they similarly propose that the individual is not the same as the environment, they perceive thoughts as only existing in a socially constructed world (Schuh & Barab, 2008). Learning is therefore “not an individual endeavour, [but] relies on the interactions in which the [student] participates” (Schuh & Barab, 2008, p. 78).

3.3.1.3 Positioning the current study

The table below summarises the differences, according to Schuh and Barab (2008), between cognitive constructivism and socioconstructivism in their placement on a continuum of realist to relativist approaches. The basic ontological assumption of constructivism is relativist (Guba & Lincoln, 1989); therefore both types of constructivism would lie closer to the relativist end of this continuum.

Table 3.1 Psychological perspectives: Epistemology, ontology, unit of analysis and dualism perspective

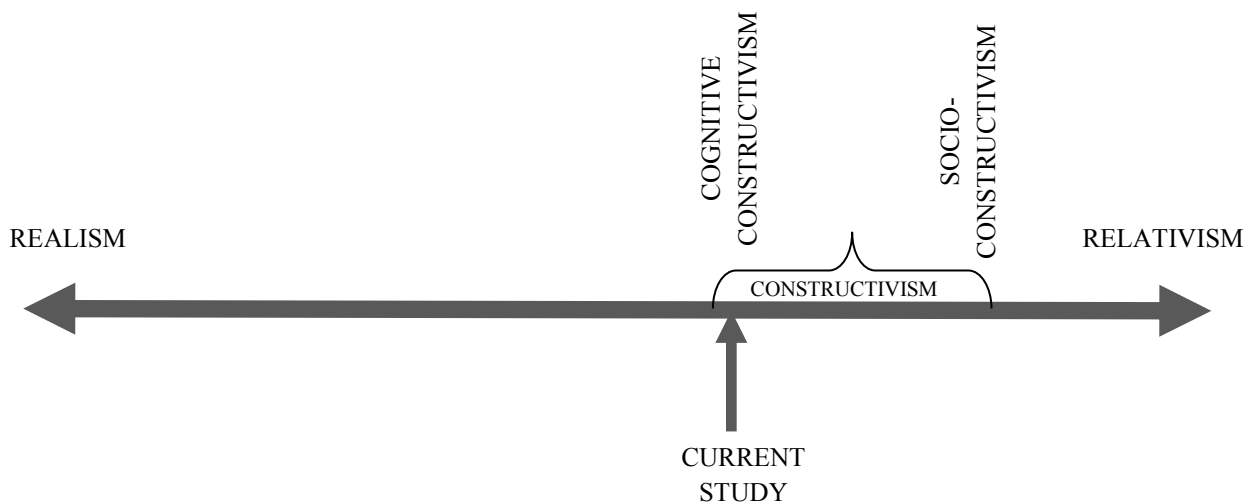
	Epistemology	Unit of Analysis	Dualism perspective
Cognitive constructivism	Rationalism Realism	Reorganisation of mental structures of an individual making sense of the world	Mind/environment
Socioconstructivism	Relativism	Relation (and processes) between the individual and society	Individual/environment

adapted from Schuh and Barab (2008, p. 76)

In order to acknowledge the myriad influences on the performance of any students in written examinations, these two types of constructivism must coexist to some extent in this study.

This study aims to focus on the cognition of the individual student in its exploration of individual responses in the examination, and will focus on how the examination is structured with regard to the complexities and cognitive demands which will be faced by students. These findings are interpreted and discussed in light of the environment in which they are learning, i.e. with English as a second language. It is the mental structures, and their expression, with which this study is concerned, and the focus will not explicitly be on how the individual exists in and relates to their environment. This study is therefore situated within cognitive constructivism, although still residing in constructivism as a whole.

Figure 3.1 Continuum of research approaches



3.3.2 Rationalism and realism in constructivist research

In Table 3.1, rationalism and realism are listed as associated with cognitive constructivist epistemology. For many, these contradict the ontological position of constructivism. This is referred to by Jupille, Caporaso and Checkel (2003, p. 11) as the “rationalist-constructivist divide”. They argue, however, that it is possible to integrate these views and maintain the ontological and epistemological cohesion of a study. This is more so for cognitive constructivism than socio-constructivism, thus it is possible for this study in particular to consider incorporating rationalist and realist views as suggested by Schuh and Barab’s (2008) summary in Table 3.1.

3.3.2.1 Rationalism as constructivist epistemology

Rationalism rests on an assumption of individualism, thus viewing individuals to be the most basic unit for social analysis (Jupille et al., 2003). This implies that “[b]oth individual and collective actions and outcomes are explicable in terms of unit-level (individual) properties” (Jupille et al., 2003, p. 12).

The individual student will act in a particular manner related to their unique process of constructing the knowledge being assessed. This will lead to the group outcome of, for example, a particular pass rate for a given class. This study contends that a pass rate cannot be fully understood unless each individual’s performance is closely examined for what might have influenced their performance. Rationalism is therefore not opposed to constructivism (Jupille et al., 2003); it simply places a greater emphasis on the individual as he or she exists in an acknowledged unique social environment, rather than the social whole. While commonalities did exist between individual student responses to examination items in this study, each response was in part the result of how the individual student had constructed the knowledge assessed.

According to Schuh and Barab (2008), this view would be more complementary to cognitive constructivism rather than socioconstructivism, as the focus is primarily on the individual mind as it exists in its environment. Jupille et al. (2003) argue that, when synthesised, rationalism and constructivism can more effectively facilitate our ability to explain the world in depth than would be possible if each approach is applied in isolation.

3.3.2.2 Realism as constructivist methodology

It cannot be denied that the present study has the aim of pointing towards a possible causal relationship between English language proficiency linguistic demands of the examination items and the errors made by students. Although there is no formal causal hypothesis, and the purpose is descriptive, in order for the study to meaningfully stimulate further research based on its methodology, an acknowledgment of the realist school of thought is required. This would appear to contradict the stated constructivist view, but Maxwell (2004) explains that these two approaches can be complementary and therefore coexist as ontology and methodology in a study.

Realism, as applied in the social sciences, emphasises “the context dependence of causal explanation” (Maxwell, 2004, p. 6), thus incorporating the constructivist focus on the social context and its influence on how individuals construct knowledge. Maxwell (2004, p. 6) summarises the realist position as: “mechanism + context = outcome”. Therefore any causal relationship is contingent on the social context in which the relationship is observed. Social scientists taking a realist stance “see the meanings, beliefs, values, and intentions held by participants in a study as essential parts of the causal mechanisms operating in that setting” (Maxwell, 2004, p. 7) – a distinctly constructivist view. In this study, while the students’ individual work was sampled, their immediate learning context was identical and any relationship observed needed to be considered in light of this social context.

3.3.4 Methods

3.3.4.1 Mixed methods research

Guba and Lincoln (1989) write that constructivism is particularly silent with regard to what methods, particularly whether quantitative or qualitative, should be used. Both methods are often appropriate (Guba & Lincoln, 1989). Golafshani (2003, p. 604) explains that “constructivism values the multiple realities that people possess... [t]herefore, to acquire valid and reliable data about multiple and diverse realities, multiple methods of searching or gathering data are in order”. Gerring (2004) supports this point of view, writing that where a case study is the chosen method, the selected number of cases can be as small as one, but may also include a large number of cases, and as a consequence both qualitative and quantitative evaluations may be used.

A strength of the realist view, as defined in Section 3.3.2.2, is that it provides a perspective that accepts both quantitative and qualitative research as being productive approaches (Maxwell, 2004). Maxwell (2004, p. 9) writes that: “realism supports the argument that qualitative research can be scientific in the full sense of the term, providing explicitly developed, stable explanations for the phenomenon studied”.

Another argument for the selection of a mixed method design is that it permits the application of both inductive and deductive reasoning in approaching the data analysis (Engel & Schutt, 2012). In quantitative research reasoning tends to be more deductive in nature. Concepts are developed “on the basis of theory” (Engel & Schutt, 2012, p. 89) and subsequently decisions are made as to “what should be observed to indicate that concept” (Engel & Schutt, 2012, p. 89). Inductive reasoning is a characteristic of qualitative research and “concepts emerge from the process of thinking about what has been observed” (Engel & Schutt, 2012, p. 89).

In this study, deductive reasoning is applied in the process of arriving at operational definitions of the constructs to be described in order to decide on a method of quantitative measurement. These operational definitions are informed by the writings of both seminal and contemporary theorists, as well as current research findings in the relevant fields. Based on these definitions, specific tools for measurement were selected. A summary of these definitions and tools can be found in Table 3.10.

As Stobart (2008, p. 13) points out, “the appeal of examinations has always been their fairness and their promise of meritocratic selection”. Examination results frequently hold significant consequences for students, and are assumed to be fair, despite the often unacknowledged “major theoretical issues of validity and reliability” (Stobart, 2008, p. 14). Assessment can never be a neutral event (Stobart, 2008) and embedded in the practice is the positivist philosophy that there is one truth that can be both defined and discovered (Terre Blanche & Durrheim, 2006). This study seeks to thoroughly interrogate the fairness and validity of mathematical literacy assessment in the NC(V), as indicated by student errors. It is, therefore, essential to include an element of quantitative analysis in an attempt to critique the inherently quantitative nature of educational assessment and to foreground this approach to data analysis.

Davis (1998) explains, however, that any investigation of assessment must take into account that this practice is “located in a social and political context” (Davis, 1998, p. 10). The students participating in the research will have actively constructed their cognitive schemas related to second language learning and mathematical literacy, and this will have been shaped by a multitude of contextual and innate factors. Thus, any attempt to acknowledge the unique perspectives offered by the participants must include a qualitative component.

Qualitative and quantitative data will be gathered simultaneously. These two forms of data will be gathered from the same document sources, thus while gathering quantitative data, qualitative observations of the examination items and student responses will be made. These will be noted and, although the qualitative data gathering will be planned before the research is started, adjustments may need to be made in light of what is discovered in this process.

The inclusion of this data will help to clarify areas that the quantitative data from this document-based analysis may fail to adequately describe. It is not sufficient to focus solely on quantitative data to explain a phenomenon embedded in a social context and holding social consequences.

This study makes use of the mixed method design, but with quantitative representation and analysis of the data being the dominant method, with qualitative analysis featuring as a non-dominant, supportive, method. Creswell, Shope, Plano Clark and Green (2006, p. 3) call this a “nested design, in which qualitative [data] plays a supporting role within a larger [study]”.

3.3.4.2 Case study

The case study method is an “empirical inquiry that investigates a contemporary phenomenon within its real life context using multiple sources of evidence” (Noor, 2008, p. 1602). As the aim of this study is to comprehensively describe the phenomenon of assessment of mathematical literacy, and will utilise both the examination paper and student scripts this is a suitable research method to employ.

This study will take the form of an instrumental case study. Stake (1998) describes this type of case study as placing a specific example of the phenomenon to be described under close scrutiny in order to facilitate the “understanding of a particular situation” (Baxter, 2008, p. 550). The overarching aim in this study is to explore and describe the assessment of

mathematical literacy in the NC(V) as a phenomenon. A very small case will be the focus in order to develop an in-depth description, which will provide indications as to whether this phenomenon warrants investigation on a larger scale.

This strategy has been chosen because the close examination of “the individual case can have wider implications...that would not have come to light through the use of a [larger] research strategy” (Denscombe, 2007, p. 36). This research will serve the specific context of the case study by providing insight into the assessment of NC(V) English language learners at this site and, if necessary, exposing aspects of assessment that the college itself is in a position to alter. As Gerring (2004) explains, although a case study relies on the investigation of a particular case, it can “illuminate features of a broader set of units” (Gerring, 2004, p. 343). As such, any practical implications of the findings for this specific college are most likely able to be useful to other colleges in similar situations.

The primary criticisms of the case study method are that there is a lack of scientific rigour and consequently problems with reliability (Noor, 2004). Case study research is also criticised for a perceived lack of representativeness, and therefore causal relationships that become evident for the single unit studied cannot be assumed to be true for any units that have not been included in the study (Gerring, 2004).

The case study method has, however, been selected for its advantages that are particularly valuable to the current research. The first is the depth of insight that a case study allows and the “round” (Gerring, 2004, p. 346) picture that is formed of the phenomenon. Gerring (2004) also argues that where cases are carefully chosen, the researcher can propose a possible causal relationship with a broader reach, thus stimulating further research which may stretch beyond the specific case examined. Thus, although a particular case study “does not make any causal inferences, [this] does not denigrate the possibility of causal analysis” (Gerring, 2004, p. 346) in later studies.

3.3.5 Summary

The following table provides a summary of the research approach of this study:

Table 3.2 Summary of research approach

	Ontology	Epistemology	Methodology	Method
Definition	What is the nature of being and reality? What is real in the world? What exists? (Schuh & Barab, 2008, p. 70)	What are knowledge and the nature of knowledge? How do we come to know what exists (Schuh & Barab, 2008, p. 70)	How can the inquirer (would-be knower) go about finding out whatever he or she believes to be known? (Schuh & Barab, 2008, p. 70)	What is the type of method the researcher will follow to gather data?
	Cognitive constructivism	Rationalism	Realism	Mixed methods Instrumental Case Study
How?	The view of the student, as well as of mathematical knowledge itself, is explained in terms of how each individual constructs and develops this thinking.	The research will focus on the individual responses of students as separate entities and use this individual analysis to summarise what is happening for the group as a whole.	The outcome is student errors is understood in terms of both the mental process of problem solving and taking into account the student's context in attempting to explain it. "mechanism + context = outcome" (Maxwell, 2004, p. 6).	Quantitative and qualitative analysis will be conducted on data gathered from close analysis of a small aspect (student errors) of a larger phenomenon (assessment of English language learners).

3.4 SITE SELECTION AND PARTICIPANTS

The selected site for this case study is an urban FET college in the Eastern Cape. Demographic statistics generated for this college, as a part of a study commissioned by the Department of Higher Education and Training (2011), show that 74.4% of the students are African; 10.6% Coloured; 0.3% Indian and 1.4% White. While race statistics do not directly translate into home language statistics, these numbers do imply that the majority of students

at this college do not have English as their mother tongue. English, however, is the medium of instruction and assessment at this institution.

The specific unit of analysis for this case study is the September 2011 Level 4 Trial Mathematical Literacy examination and a specific Level 4 NC(V) class's examination scripts for this examination.

This examination is internally set, that is, by lecturers from the selected site, and the results are weighted at 75% of a students' Internal Continuous Assessment [ICASS] mark. The ICASS result is a composite mark reflecting the student's performance during the academic year. Two examinations are written: the first paper is aimed at assessing basic knowledge and routine application of procedures and the second assesses the students' ability to reason and reflect on the procedures they apply (DoE, 2007b). The Level 4 examination has been selected as the fairness and validity of this examination affects students the most. If they do not pass Mathematical Literacy in this Level, they will not receive their qualification and will be required to redo the subject the following year instead of being able to move on from their studies.

The examination will be analysed in terms of: the cognitive complexity of the items; as well as the linguistic complexity; and the level of language proficiency required to comprehend the items. Thereafter, the student answer scripts will be accessed and an error analysis of their responses will be conducted. This will be done with the permission of the Head of Department.

3.4.1 The examination paper

The *National Certificates (Vocational) Assessment Guidelines: Mathematical Literacy NQF Level 4* (DoE, 2007b) outlines the specifications for the structuring of the two Mathematical Literacy examination papers. These specifications prescribe that an equal proportion of the examination be assigned to each topic. They also describe what proportion of each examination should be assigned to each cognitive domain.

Table 3.3 below shows the ideal mark allocation for each examination, as well as the actual mark allocation of the September 2011 Level 4 Mathematical Literacy examination selected

for the case study. This analysis was an important step in the selection of the question paper. If the papers deviate from the prescribed specifications, they cannot be considered valid or fit for use in this study.

If the combined mark allocations over both papers are considered, the weighting of both topics and cognitive domains does represent the DoE's (2007b) prescribed structure, with the exception that Cognitive Domain 4 is underrepresented.

Table 3.3 Analysis of adherence to prescribed specifications

	Topic				
IDEAL MARK ALLOCATION	1	2	3	4	5
60	55				
60		52			
60			63		
60				51	
60					79

	Cognitive Domain			
IDEAL MARK ALLOCATION	1	2	3	4
90	83			
90		116		
60			77	
60				24

3.4.2 The student sample

This case study focussed specifically on a class of 2011 NC(V) Level 4 students, with isiXhosa as their home language, of which there were 45 students. This particular language group was chosen as the college demographics reveal that isiXhosa is the home language of the majority of the students. Selecting students with other home languages could introduce a confounding variable and thus no other language groups were included.

Sampling within the case will be purposive, such that the selected students' scripts will provide a good example of the phenomena to be described (Durrheim, 2006; Stake, 1998). This will be achieved by an initial examination of the students' scripts. Those selected will be the fifteen which provide the most errors that include working out, and not those where answers have simply been left blank.

The table below provides a comparison between the whole group of 45 isiXhosa speaking students and the selected sample of 15 students.

Table 3.4 Comparison of all isiXhosa speaking students with selected sample

	Whole group	Sample
isiXhosa Home Language	45	15
Male	5	0
Female	40	15
Average age	22	22
% students passing Level 4	80	73
% students passing Mathematical Literacy	73	60

It is worth noting that, while NC(V) levels 2-4 correspond in NQF level to NCS Grades 10-12, the average age of the Level 4 students in this sample is as high as 22 as the only age restriction in the NC(V) is that a Level 2 student must be at least 16 years of age. Anyone meeting that criterion is able to enrol.

The following table shows the difference between the average percentages achieved by the full Level 4 class and the average percentages achieved by the students in the sample.

Table 3.5 Average percentages of all isiXhosa speaking students and the selected sample

	Mathematical Literacy Trial Examination Paper 1	Mathematical Literacy Trial Examination Paper 2	Mathematical Literacy ICASS result	Mathematical Literacy final result	English Additional Language ICASS result
GROUP AVERAGES	47%	33%	34%	37%	40%
SAMPLE AVERAGES	41%	25%	27%	31%	39%

This strategy of purposive selection, as well as the selection of students from one particular college and one particular NC(V) programme, limits the generalisability of the results. For practical reasons, however, the scope of this research allows the examination of a small sample and not one proportionally representative of the entire population. Therefore the number of examples of analysable units has been maximised.

3.5 PILOT STUDY

Guba and Lincoln (1989) describe constructivist evaluation as consisting of two phases: the discovery phase, and the assimilation phase. The discovery phase involves the researcher's effort to generally describe the phenomenon in order to provide the context on which the study will be constructed. The discoveries of the first phase are then incorporated into subsequent steps in the research process in the assimilation phase.

This broad approach has been followed in this study by the use of pilot studies aimed at refining the researcher's understanding of the context, as well as improving the means of data analysis, before embarking on the actual research process.

The pilot study took the form of a two phase exploration. In the first phase, the researcher sought to gain a broad view of what errors English second language students were making in a mathematical literacy test. This was done through a rudimentary error analysis simply grouping student errors into those marks lost due to mathematical errors and those lost due to misunderstandings of the text. Seven students were selected to participate. The students assisted the researcher in assigning errors to language and mathematical sources, as well as discussing their experiences of writing that particular test.

The second phase sought to refine the data analysis process, specifically how to categorise student errors and how to design a method of analysis that could meaningfully compare the data in order to answer the research question.

3.5.1 Pilot study: First phase

The stimulus for the research question of this study was a significant drop in the class average of an NC(V) Level 3 Mathematical Literacy class when a test containing a particularly large amount of reading was administered. It was the assumption of the researcher that the students' language proficiency could have influenced the class's results, and the insight of a group of students in this class was sought as a first step in piloting this study to explore the students' perceptions of what the causes may have been.

3.5.1.1 Collection of data

In an effort to understand this phenomenon, seven students of varying abilities were selected to assist the researcher in exploring the possible reasons for this decrease. These students were in NCV Level 3 and, therefore, were not part of the main study. One question in particular had been problematic, with no student achieving more than 25% for it. This multi-item question is included as APPENDIX C. When interviewed, all students reported that they had struggled to comprehend the text in this question. Other contributions to the discussion included that:

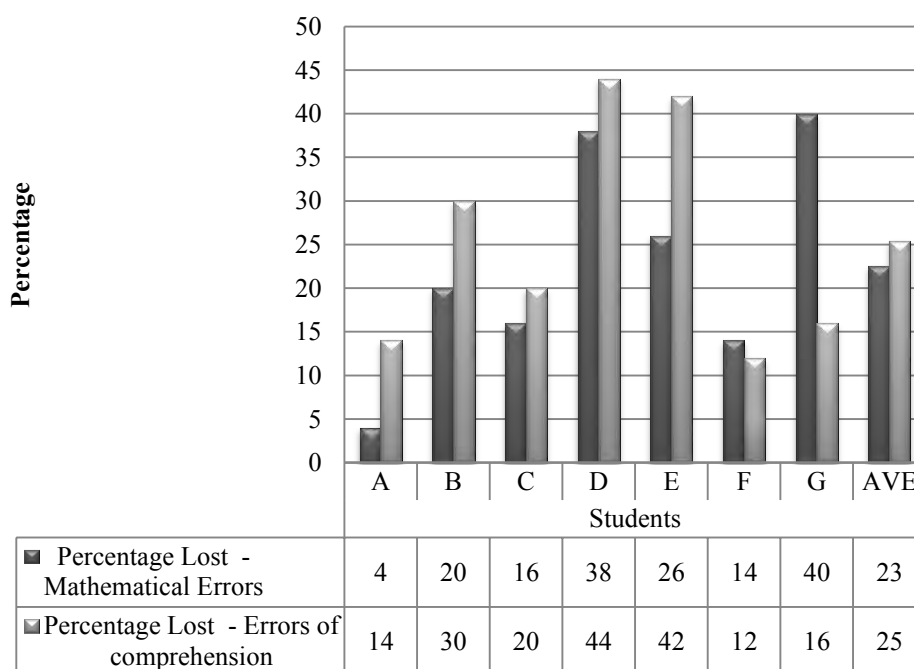
- Some vocabulary was not understood (this vocabulary was not part of the mathematical terminology that students are required to know).
- A diagram would have made the text more comprehensible (in questions other than the one given as APPENDIX C).
- The lead-in text to each question was considered confusing.
- Grammatical errors had been noticed by the students, although they were unable to identify them specifically.

Notably, a student who had achieved above 80% for the test commented that he had also struggled significantly. He mentioned that it had taken him an unusually long time and many attempts to answer certain questions because he had not easily understood what was required of him. This was evident in his answer book, where those attempts had been scratched out and the item re-attempted multiple times.

3.5.1.2 Results

Students assisted in categorising their errors as broadly attributable to reading comprehension and limited language proficiency versus those attributable to mathematical proficiency. The following results were obtained:

Figure 3.2 Error analysis of focus group students' test responses



For most students, more percentage points were lost due to comprehension than to mathematical errors.

3.5.1.3 Lessons from the first phase

It was clear from the discussion held with the students that they were aware that the linguistic aspects of the texts written for mathematical literacy test items were causing them to make errors. On discussion with the students, a rewording and explanation of the meaning of the text, not the mathematical content, resulted in their ability to solve the problem. Only slight mathematical inaccuracies were evident in their subsequent attempts, and not errors due to a misunderstanding of the problem.

It is possible that a 'practice effect' could better explain their ability to solve the item after explanation was offered. The students, however, were convinced that it was due to their newly acquired comprehension. They indicated that it was the item itself that was to blame, and not their reading comprehension. This implied that a close analysis of the examination paper would be required to determine the accuracy of the language used with regard to grammatical structures and spelling. It was also decided that the linguistic features of all items would need to be analysed.

It was noticed that it was reading comprehension, in particular, that seemed to be the underlying source of the language-related errors. This was due to the type of item selected for analysis, as this was the only language skill required to interpret the item. It was decided, therefore, that the error analysis would need to incorporate a distinction between different types of language-related errors, and that the items used for analysis in the study would need to represent more than just items requiring reading proficiency.

The comment by the strongest student in the group that he needed several attempts before managing to answer the item correctly was deemed significant. There was value in viewing his script from a qualitative perspective, as his numerous deleted attempts were evidence of his frustration in trying to comprehend the item. This suggests that a qualitative analysis of scripts could add depth to the analysis of the data collected in the study.

3.5.2 Pilot study: Second phase

The research questions of the present study were refined as a result of the first phase pilot study. In light of those discoveries, and following a review of the literature, a means of defining linguistic and cognitive demands, as well as how to ascertain the source of student errors was designed. This too required piloting.

3.5.2.1 Sample selection

The September 2011 Level 4 Paper 1 Mathematical Literacy trial examination, selected for use in the study itself, was used in this phase of the piloting process. From this examination paper, 15 items were selected. These were selected on the basis of their reading requirements, varying from a single sentence to a paragraph. They also represented a range of difficulty levels with regard to their mathematical content.

The English second language students' scripts were examined as to whether the student had attempted the items or not. Those who had left out more than 30% of the examination were eliminated to allow for the largest possible amount of analysable errors. From the remaining scripts, fifteen were selected for analysis. These students all achieved between 30 – 39% (elementary achievement) in the examination, but represented a wider range of language proficiency, with English First Additional Language results of 31% (elementary) to 51% (adequate).

3.5.2.2 Analysis of data

After correct responses had been eliminated, there were 131 errors available for analysis. Each error was assigned to one of the categories listed below:

- Decoding: Reading
- Decoding: Graphics, tables and symbolic notation (viewing)
- Encoding: Writing
- Encoding: Presenting data in graphs, tables and symbolic notation
- Mathematical calculation errors
- Other

These categories were developed inductively, based on close qualitative analysis of the errors made by these seven students. From the qualitative descriptions of the errors, a broad distinction was apparent between the decoding and encoding skills required in order to work between the ‘examination world’ and the ‘mathematical world’ (see Figure 2.5).

Errors attributed to reading were those where a lack of comprehension of the text contained in the problem was evident in the way in which data was interpreted and used in the calculation. Viewing errors were those where a lack of comprehension of symbolic notation, tables or graphs had led to the error.

Where students struggled to translate their solutions into written form, the error was attributed to limited ability to encode information in writing. Similarly, where students demonstrated comprehension of the item and an ability to perform calculations but were unable to summarise this information accurately in symbolic form or in a table or graph, the error was attributed to a lack of ability to present data accurately, as well as an encoding error.

In addition to decoding and encoding errors were those in which mathematical ability resulted in an incorrect response.

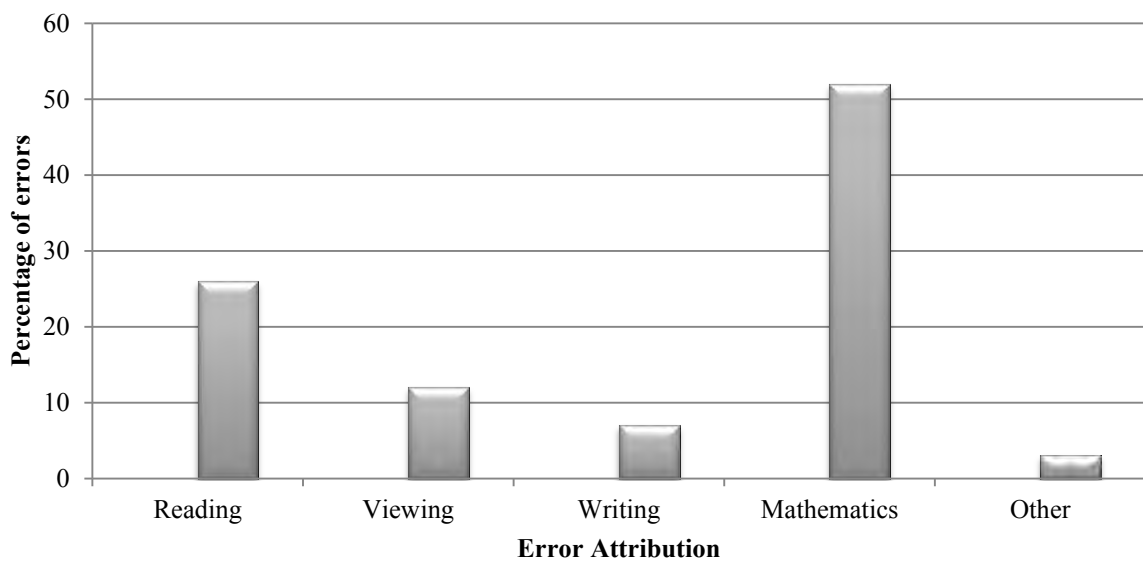
3.5.2.3 Results

The number of items available for analysis was 225, of which 131 were errors that were analysed according to what the underlying source was most likely to be. The results of the error analysis revealed mathematical ability to be the dominant source of error, accounting

for 52% of the total errors. Decoding errors were also prominent, 38% of the errors were attributed to decoding, of which 26% were reading comprehension errors and 12% were viewing errors owing to a lack of comprehension of symbolic notations, tables or graphs. Encoding errors only fell into the category of writing, due to the nature of the items, and this accounted for 7% of the errors.

A graphic display of these results is included below:

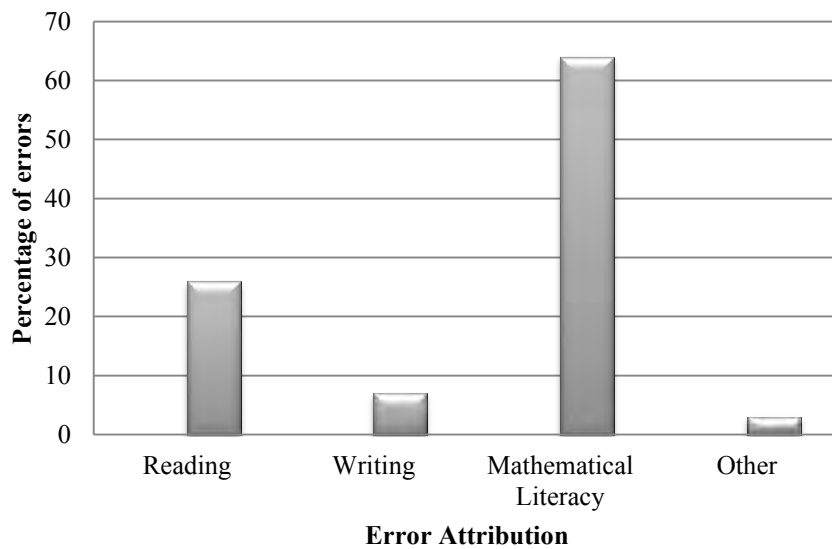
Figure 3.3 Bar graph showing percentage of errors per error attribution category



3.5.2.4 Lessons from the second phase

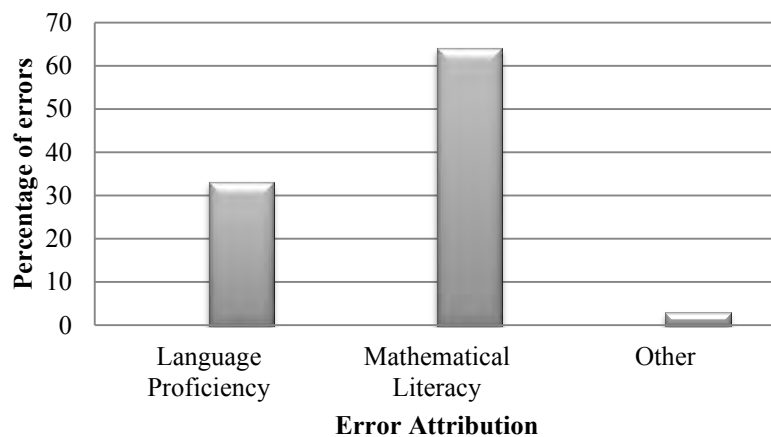
Definitions of mathematical literacy all contain an element of decoding of information presented in tables, graphs, numbers and symbolic notation. Therefore, if the aim of the error analysis is to separate errors based on language proficiency versus errors made due to mathematical literacy, it was decided that viewing and presenting could be incorporated into the mathematics category to create the category mathematical literacy. When this change was made to the pilot data, the rearranged result became that depicted below:

Figure 3.4 Percentages of mathematical literacy, reading and writing errors



As both reading and writing errors can be attributed to language proficiency, it was deemed possible to further simplify the picture if reading and writing was to be collapsed into one category: language proficiency. This would allow a comparison between the percentage of the mathematical literacy errors made and the percentage of errors due to limited language proficiency. This is shown in the graph below.

Figure 3.5: Mathematical literacy versus language proficiency in the attribution of errors



This progression in the analysis of the data generated from the error analysis was deemed to effectively show the relative contributions of language proficiency and mathematical literacy to the number of errors made. It also clearly revealed how the different components of these categories contributed, i.e. percentage of reading versus writing errors as components of language-related errors.

The second phase thus allowed a refinement of how results would be analysed.

3.5.3 Summary of lessons from the pilot studies

The following lessons were learnt from the first phase of the pilot study:

- The subjective experience of the students confirms that language characteristics of the item could be the source of certain errors.
- Students did not only lose marks due to language factors, for 6 out of the 7 students, more marks were lost due to language proficiency than mathematical proficiency.
- It is necessary to ascertain whether language-related errors are due to more than just reading.
- Items for the study must include those which require other skills, such as writing and viewing.
- The lead-in text could provide a source of confusion for students resulting in mistakes in subsequent items related to this text. These may not be related to the linguistic characteristics of the item.
- Attention should be paid in the research to the visuals that accompany lead-in texts. These could lead to errors or aid the student in their comprehension.
- Vocabulary demands, other than the mathematical technical terms included in the Mathematical Literacy curriculum, are also a possible source of error.
- Errors, both grammatical and spelling, need to be included in the analysis of the items.
- There is value in examining the students' answer scripts from a qualitative perspective in an attempt to identify the level of effort required in responding to the items. This would indicate to what extent the student was involved in the task, as well as serve to identify possibly automatised responses. This includes those responses of students who have answered the item correctly.

In the second phase items with varying language demands were used. Student errors were also analysed according to more than one category of language skills. Reading, viewing and presenting were included as sources of error. Diagrams and other visual sources of

information were considered as possible causes of student errors. The following was learnt from the second phase of the pilot study:

- Errors due to ‘viewing’ and ‘presenting’, as defined for this study, should contribute to the category of mathematical proficiency in terms of attribution of errors to either language of mathematical sources.
- The incorporation of ‘viewing’ and ‘presenting’ into the mathematical category transforms it into representing mathematical literacy as opposed to pure mathematical proficiency.

These findings were assimilated into the final design of the study.

3.6 DATA COLLECTION

Data collection was sequential (Denscombe, 2007), and carried out in the following two phases:

- a) The September 2011 Level 4 NC(V) Mathematical Literacy examination papers (Paper 1 and Paper 2) were obtained. These were analysed according to their linguistic complexity and cognitive complexity.
- b) Students’ scripts from the September 2011 examination were accessed. A preliminary analysis was performed on all student scripts, noting which students had answered the most items incorrectly, where working out had been shown. These were considered to be analysable errors. It was decided that 30% of the sample scripts would be selected for further analysis, thus the 15 scripts with the highest number of analysable errors were identified. A detailed error analysis was performed on these scripts, noting what the apparent source of their errors might be.

3.7 DATA ANALYSIS

Data analysis was carried out in three parts. The examination papers were described according to their linguistic and cognitive complexity. An error analysis was conducted in which students’ responses to all items in the examination were categorised according to the

most probable source of error. Finally, a quantitative analysis was conducted on the data already gathered to ascertain whether any relationship can be found between the linguistic complexity and cognitive complexity of the examination items, and the number and types of errors made per item.

While the most prominent form of analysis was quantitative, qualitative observations were made in each of these three phases. The reporting of the results of these two approaches to analysis was integrated as they contribute to the deep description of each component.

3.7.1 Student results

The decision to include a description of the students' Mathematical Literacy results is for the depth it adds to the description of the examination. The results for the full group of 45 students are reported. The pass rate as well as the spread of results according to the level descriptors provided in the *National Certificates (Vocational) Assessment Guidelines: Mathematical Literacy NQF Level 4* (DoE, 2007b) is provided. These descriptors are listed in the table below:

Table 3.6 Rating scale for interpretation of Mathematical Literacy results

CODE	RATING	MARKS
7	Outstanding	80 - 100
6	Meritorious	70 - 79
5	Substantial	60 - 69
4	Adequate	50 - 59
3	Moderate	40 - 49
2	Elementary	30 - 39
1	Not achieved	0 - 29

(DoE, 2007b, p. 7)

Also included in the presentation of data is a description of the students' language proficiency, as evident in the full class's final ICASS results for Level 4 English First Additional Language. It is not possible to pinpoint a student's precise language proficiency as a specific percentage, the seven point scale presented above is used to provide a rudimentary description of the student's language proficiency.

3.7.2 Description of examination items according to linguistic complexity

Examination items are defined, for the purposes of this study, as each item for which a student can acquire points. The linguistic complexity of examination items is described in three ways. A Linguistic Complexity Index is calculated per item according to the frequency of use of specific language features. This index is analysed quantitatively. Readability is also calculated, but the numeric values obtained are used to guide a more qualitative description of the reading proficiency required of students. Readability is calculated per examination scenario. Lastly, the overall language proficiency required of students is discussed with the guidance of the CEFR (Council of Europe, 2001) classifications of levels of English language proficiency according to the presence the language features listed in the CIGE (North et al., 2010). This is assessed per examination scenario. Both the readability data and the data regarding student language proficiency levels are analysed qualitatively, whilst a numerically based summary is included.

3.7.2.1 Calculating a Linguistic Complexity Index

A tool has been developed specifically for this calculation. The frequency of use of specific language features has been tallied according to the following three levels: word level; sentence level; and paragraph level. The individual language features tallied, according to those listed as linguistically complex by Shaftel et al. (2006), are:

Word level:	Words of 7 letters or more
	Pronouns
	Slang
	Homophones
	Homonyms
Sentence level:	Prepositional phrases
	Infinitives
	Complex verbs (3 words or longer)
	Complex or compound sentences
	Conditional constructions
	Comparative constructions
Paragraph level:	Number of culture- or experience-specific references

Each item was assessed and coded for the frequency of these language features by two independent raters, both experienced lecturers of NC(V) English First Additional Language. This followed a similar process as that employed by Shaftel et al. (2006): coding was initially done independently, but the method was adjusted to allow “coding questions and issues raised by the first rater [to redefine] some of the criteria for the second rater, with subsequent review and recoding of confusing criteria” (Shaftel et al., 2006, p. 111).

The raters made two additions to this checklist. At the word level, it was agreed that abbreviations and acronyms should be included, as these were frequently used in the Mathematical Literacy examination. The category of grammatical and spelling errors was also added, as this had been identified by the students in the first phase of the pilot study as a barrier to their comprehension of the text. These errors introduce ambiguity, which is identified by Shaftel et al. (2006) as being problematic for English language learners. One requirement is to count “the number of references to American holidays [and] the number of references to specific cultural events or situations” (Shaftel et al., 2006, p. 126). This was adjusted to become relevant to a broader context: number of culture- and/or experience-specific references.

The analysis of the linguistic complexity was guided by the Linguistic Complexity Checklist designed by Shaftel et al. (2006). This tool was selected as it had been specifically designed to assess mathematical test items. The full checklist, as used in the study, is included as APPENDIX D.

Table 3.7 Items included in the Linguistic Complexity Checklist (Shaftel et al., 2006)

LINGUISTIC COMPLEXITY CHECKLIST	
(A) BASIC	Number of sentences
	Number of words
(B) WORD LEVEL	Number of different words with 7 letters or more
	Number of pronouns
	Examples of slang, homophones and homonyms

	Number of abbreviations
(C) SENTENCE LEVEL	Number of prepositional phrases
	Number of infinitives
	Number of complex verbs
	Number of complex or compound sentences
	Number of conditional constructions
	Number of comparative constructions
(D) PARAGRAPH LEVEL	Number of cultural- and/or experience-specific references
	Number of grammatical and/or spelling errors

A Linguistic Complexity Index [LCI] was calculated based on these frequencies to allow the direct comparison of items regardless of their length. The number of instances of use of each language feature listed in the checklist was added, and the result divided by the number of sentences. The formula is given below:

$$\text{LCI} = (\text{Number of words} + \text{Sum B} + \text{Sum C} + \text{Sum D}) \div \text{Number of sentences}$$

The division of the sum by the number of sentences means that items of differing length can be compared. This scale provides measurement at the interval level. Each item and lead-in text will be assigned a Linguistic Complexity Index value. These index values will be analysed quantitatively.

Pearson product-moment correlations will be calculated between the language features contributing to the Linguistic Complexity Index and the index itself. This will serve to augment the data regarding the frequency of use of these features by assessing whether these features are in fact statistically significantly correlated to the index itself. It is not necessarily the case that those contributing the most to the linguistic index have a statistically significant correlation with the index, nor that they have any correlation at all.

3.7.2.2 Describing readability

Several measures exist which allow texts to be categorised according to the level of reading proficiency required to decode them. This adds a valuable dimension to the description of the linguistic complexity of items.

A number of online utility tools were explored, the most comprehensive of which was http://www.online-utility.org/english/readability_test_and_improve.jsp. This website allowed the copying and pasting of text into their analyser and readability measures were automatically computed. These included: the Gunning-Fog Formula; the Flesch Kincaid Grade Level Formula; the Automated Readability Index [ARI]; and the Flesch Reading Ease Index.

It was not possible to identify a tool which facilitated the use of the Homan-Hewitt Formula. As mentioned in Chapter 2 (Section 2.7.3.4), this formula would have been the most appropriate as it was developed for use with single sentence items. The complexity of the formula, as well as the lack of operational definitions for ‘difficult words’ and ‘sentence complexity’, which are included in the formula, prevented its use in the present study.

The particular readability formulae selected for use were the Gunning-Fog formula, the Flesch-Kincaid formula and the Flesch Reading Ease score. The advantage of selecting two readability measures is that an average can be calculated rather than relying on a single measure. The Gunning-Fog formula and the Flesch-Kincaid formula differ in their construction, thus the use of an average allows the inclusion of more variables than a single formula would provide. The Flesch Reading Ease Index generates a score which is described qualitatively and is also therefore not bound to a context-specific grade level description.

A weakness of these formulae is that they are not necessarily valid for single sentence items, such as the majority of the individual items making up the Mathematical Literacy examination. For this reason they have been applied to each scenario in the examination together with all of the associated items. A scenario is defined as the lead-in text (providing context details and explanations) with its associated items. This reduces the number of analysable units from the total number of examination items, to a total number of scenarios at 17. Nevertheless, the values calculated provide important information with regard to the linguistic complexity of the examination as a whole. These readability formulae provide

quantitative data at the ordinal level, but are used to guide a qualitative description of the reading proficiency required of students.

3.7.2.3 Describing the level of language proficiency required of students

The examination paper will be analysed qualitatively with the guidance of the CIGE (North et al., 2010). This framework provides an outline of different language features of texts, e.g. the use of simple present tense; and different functions of texts, like conveying precise information. These are matched with descriptions of the levels of language proficiency, as defined on the CEFR (Council of Europe, 2001), required of students to comprehend such texts. This is provided as APPENDIX B.

The CEFR levels of language proficiency are listed below (Lindhout, Teunissen & Lindhout, 2012), but have been discussed in more detail in 2.7.3.3 of Chapter 2:

- A1 – Breakthrough/Beginner
- A2 – Waystage/Elementary
- B1 – Threshold/Intermediate
- B2 – Vantage/Upper Intermediate
- C1 – Effective Operational Proficiency/Advanced
- C2 – Mastery/Proficiency

Each level listed in this tool is accompanied by a qualitative description of the competencies each student is assumed to have at each language proficiency level. This adds qualitative depth to the discussion of the resulting data.

The specific functions of each text, as well as the presence of certain language features, according to the list provided in APPENDIX A, have been noted. Each language feature and text function is associated with an approximate CEFR language proficiency level, for which detailed qualitative descriptions are provided by North et al. (2010). These descriptions appear as APPENDIX B. The predominant focus of these descriptions is on listening and speaking proficiency, but they can be used to deduce what reading and writing proficiencies are required.

It has only been possible to approximate the level group, as the CIGE itself provides a range of CEFR language proficiency groups for certain functions and features, and the texts each contain more than one grammatical feature, and possibly more than one function.

While a quantitative summary is provided of the frequency of use of each feature and function, the main form of analysis is qualitative in nature. Analysis will be performed on each scenario with its associated items as they are presented in the examination with the associated items, as opposed to analysing individual items. This decision was made due to the fact that the CIGE was developed for longer reading texts and not for single sentence items.

3.7.3 Description of examination items according to cognitive complexity

Cognitive complexity is described in two parts. A Cognitive Complexity Index [CCI] has been calculated per examination item based on the cognitive domain assessed, the relative difficulty of the item and the number of marks allocated to the item.

3.7.3.1 Calculating a Cognitive Complexity Index

The cognitive domain assessed by each item has been defined according to the taxonomy outlined in the *Trends in Mathematics and Science Study Assessment Frameworks and Specifications* (Mullis, Martin, Smith, Garden, Gregory, Gonzalez, Chrostowski & O'Connor, 2003). This taxonomy categorises items according to whether they require knowing facts and procedures; use of concepts; solving of routine problems or reasoning skills.

These are cognitive domains, rather than strictly an assessment of difficulty, therefore each of these four domains has been further analysed as to whether they are of low, moderate or high complexity as suggested by Berger, Bowie and Nyaumwe (2010). Although each domain can be assessed through use of easy or difficult items, there is no doubt that items requiring reasoning skills, even at the 'easy' level, are more demanding than those 'easy' items simply requiring knowledge of facts and procedures.

For this reason, cognitive complexity level has been assigned to each item according to a rubric designed for this research. It takes into account both cognitive domain and difficulty. These levels range from 1 to 6 and can be considered one measurement of the cognitive complexity of an item.

Table 3.8 Rubric for determining the cognitive complexity of an item

TIMSS Cognitive Domain (Mullis et al., 2003)	Low difficulty	Medium difficulty	High difficulty
Knowing facts and procedures	<i>1</i>	<i>2</i>	<i>3</i>
Using Concepts	<i>2</i>	<i>3</i>	<i>4</i>
Solving routine problems	<i>3</i>	<i>4</i>	<i>5</i>
Reasoning	<i>4</i>	<i>5</i>	<i>6</i>

This data is measured at the ordinal level. There is only one value to which each case can be assigned (Engel & Schutt, 2012) and these values offer categories which allow “greater than and less than distinctions” (Engel & Schutt, 2012, p. 91). An item with a cognitive complexity of 1 can be considered less complex than an item with a cognitive complexity of 2.

It is necessary to also consider the number of marks allocated to an item as an indicator of cognitive complexity. An item that requires reasoning at a high difficulty level would be placed at a cognitive complexity level of 6. This type of item will differ in the level of cognitive demand according to the length of response required. Therefore a Cognitive Complexity Index has been calculated which includes consideration of the cognitive complexity level as well as the mark allocation. For example, if an item is positioned on the rubric such that its Cognitive Complexity Level [CCL] is 4, the Cognitive Complexity Index value will be 4 (4 x 1) if it is allocated only one mark, but will have a Cognitive Complexity Index of 8 (4 x 2) should it be allocated 2 marks. Therefore, in order to account for the mark allocation, the ordinal scale is converted to an interval scale by multiplying the rubric value of the item with the number of marks allocated to that item.

The formula, therefore, for calculating the Cognitive Complexity Index is as follows:

$$CCI = CCL \times \text{Mark allocation}$$

This measure not only permits an ordering of the items according to their relative cognitive complexity, but “the gaps between the numbers are meaningful; a one-unit difference is the same at any point in the scale” (Engel & Schutt, 2012, p. 91). This index, more so than the ordinal rubric values, separates scores by more than single values, thus allowing a clearer and more accurate distinction between complexities of items.

The index calculated, being at an interval scale, is a more appropriate choice for comparison with the Linguistic Complexity Index. This aspect of cognitive complexity has been analysed quantitatively.

3.7.3.2 The contribution of graphic components to cognitive complexity

As outlined in Chapter 2 (section 2.6.3), visual displays add complexity to an examination item due to the need to attend to and comprehend the mathematical content and context as well as the graphics (Lowrie & Diezmann, 2007). In describing the cognitive complexity characteristics of the examination, a qualitative description of the graphical languages (MacKinley, 1986) used must be included.

These graphic information sources are not all of the same form. For this reason, MacKinley’s (1986) differentiation between categories of graphical languages will be used. The relevance of MacKinley’s (1986) model is evident in the frequent use of his work in current research (see Diezmann, Lowrie & Kozak, 2007; Diezmann & Lowrie, 2008; Lowrie & Diezmann, 2007).

This data is nominal. The number of graphic sources of information in the examination is 15.

Table 3.9 Graphical languages

Language	Information encoded by:	Example
Single-position languages	The position of a mark set on one axis	Horizontal axis, vertical axis

Language	Information encoded by:	Example
Apposed-position languages	A mark set that is positioned between two axes	Line chart, bar chart, plot chart
Retinal-list languages	One of the six retinal properties of the marks in a mark set independent of position	Colour, shape, size, saturation, texture and orientation
Map languages	Fixed positions with graphical techniques specific to maps	Road map, topographic map
Connection languages	A connected set of node objects with a set of link objects	Tree diagram
Miscellaneous languages	A variety of additional graphical techniques	Pie chart, Venn diagram

summarised from MacKinley (1986, pp. 127- 130)

3.7.4 Error analysis of students' scripts

The students' examination scripts were initially analysed according to which items had been answered incorrectly, in order to ascertain which items had more incorrect responses than others. Thereafter, these scripts were evaluated according to the types of errors the students appeared to be making.

Insight into the source of an error can be derived from the process the student has followed in attempting to respond to the item. Where the student has shown 'working out', the errors can be attributed to either a lack of mathematical ability or due to the student's inability to fully comprehend the item. This lack of comprehension has been carefully considered to determine whether it was due to a lack of mathematical literacy itself, or due to the linguistic complexity of the item and, therefore, due to the language proficiency of the student.

Newman's (1977) classification of errors has been used, as well as the insights gained from the pilot study, to generate the categories according to which the errors have been described.

Errors were categorised as due to one or more of the following sources:

- Language proficiency

- Reading comprehension
- Writing of solutions as textual descriptions
- Mathematical literacy
 - Viewing – comprehension and/or use of graphic sources of information
 - Mathematical calculation errors
- Carelessness
- Indeterminate cause

A raw count of these error types was made per examination item in order to later statistically analyse whether item complexity is correlated with the number and type of errors. This raw error count included both responses from students which resulted in a score of zero for the item, as well as those responses scoring less than full marks but more than zero. This was decided for the reason that both types of responses included at least one error.

A second approach to the quantitative analysis of the error count was to calculate the total number of marks lost per error type, per item for the sample of students. This data was used to calculate a percentage value, for each error type, of the total marks lost. These percentages have been summarised per topic as well as for the entire examination.

The error analysis described here was not only analysed quantitatively. A qualitative description of how students responded to examination items is also provided and this will include examples of actual student errors. This was guided by the categories into which errors will have been grouped and includes various other observations made during the analysis of the scripts.

3.7.5 Statistical analysis of correlations between types of errors and item complexity

Research Question 4 required exploration as to whether any possible relationships exist between types of errors and cognitive and/or linguistic complexity, and the types of complexity themselves.

Pearson product-moment correlations were calculated between the following variables:

Cognitive Complexity Index and Linguistic Complexity Index

Cognitive Complexity Index and Linguistic Complexity Index and:

- Number of errors per item
- Number of mathematical calculation errors
- Number of mathematical literacy-related errors
- Number of reading errors
- Number of language-related errors

Reading and mathematical calculation errors were selected due to the fact that many more examination items required these competencies rather than writing competence and the interpretation and use of graphic sources of information.

3.7.6 Summary

In summary, the following operational definitions and measurement tools have been employed in this study:

Table 3.10 Operational definitions and tools for quantitative analysis

Construct	Operational Definition	Tool	Level of measurement
Cognitive complexity of examination items	The cognitive complexity of an examination item is a function of: the cognitive domain assessed; the level of difficulty of the item; and the number of marks allocated to the item.	<u>Rubric of:</u> TIMSS 4 Cognitive Domains (Mullis et al., 2003) and three levels of difficulty: low, medium and high. Ordinal rubric value multiplied by the mark allocation of the item.	6 point ordinal scale Interval
Graphical Languages	The type of graphical language used as a stimulus in the examination item as defined by MacKinley (1986).	MacKinley's (1986) six graphical languages	Nominal
Linguistic complexity of examination items	The three factors contributing to the linguistic complexity of an examination		

Construct	Operational Definition	Tool	Level of measurement
	item are: the type of grammatical features; the type of graphical languages used; and readability		
Grammatical Features	Grammatical features as listed in the Linguistic Complexity Checklist (ShafteI et al., 2006) contribute to the linguistic complexity of a text at the word level, sentence level and whole text level	Index value calculated from dividing the sum of the counted values from applying a Linguistic Complexity Checklist (adapted from ShafteI et al., 2006) by the number of sentences in the text or item.	Interval level
Readability	The relative ease with which a text can be read and the number of years of formal schooling required for a student to be able to read a text with ease.	Gunning-Fog Formula (indicating the number of years of formal schooling required); Flesch-Kincaid Formula (indicating approximate US grade level) Flesch Reading Ease score (provides a value that can be interpreted as a description of the relative ease with which a text can be read)	Ordinal Ordinal Nominal
Language proficiency level required of students in an examination	A function of the particular linguistic features of a text; the specific function of the text; and the ease with which a text can be read.	Common European Framework of Reference for Languages: learning, teaching, assessment [CEFR] (Council of Europe, 2001) European Association for Quality Language Services	Nominal

Construct	Operational Definition	Tool	Level of measurement
		[EAQUALS]'s Core Inventory for General English [CIGE] (North et al., 2010) Flesch Reading Ease	Nominal
Sources of student errors in mathematical literacy examinations	Sources of student error in Mathematical Literacy examinations are due to: Decoding errors; Encoding errors; and Mathematical errors. These occur at different stages in the problem solving process	Clements' (1980) classification of errors can combined with the PISA Governing Board (2010) mathematical literacy problem solving process	Nominal

3.8 ETHICAL CONSIDERATIONS

Permission to access the examination scripts of the 2011 Level 4 class was obtained from the Head of Department. The identity of the students whose scripts were examined was unknown to the researcher. The only stage at which the student's name was used was to eliminate those students whose mother tongue was not isiXhosa as this needed to be done according to a class list and the demographic forms completed by each student at the beginning of the year. Once this was completed, each student's script which had been selected was allocated a code for identification, and no further reference to their names was required.

The identity of the college represented in this case study will also be protected at all times. The context in which the college exists has been described, but in a manner which does not allow any conclusion to be made regarding its name. Students from one specific division have been selected for inclusion in the study. It is not necessary to include mention of which division this is, thus it is eliminated from any description of the context.

The name of the lecturer whose students have been selected, as well as the names of the assessors who set the examination papers which are described in this study are known to the researcher. Their permission, however, was sought and granted, but their identity carefully protected.

An ethical shortcoming, however, was the lack of consultation with the students whose scripts were used in the description of this case. It was not possible to address the whole class and acquire permission as they did not attend college subsequent to writing the trial examination. Official exam protocol also prohibited the addressing of a group of students prior to their writing of a national examination. This was the only stage at which the entire class was again present. Nevertheless, it is a relevant ethical criticism that can be made.

Beneficence, one of the basic tenets of ethical research (Wassenaar, 2006), was one particular consideration when designing this study. It was desired that the outcomes of the research be packaged such that any insights gained might be shared with relevant academic staff members at the site of the case study. In Chapter 5 (section 5.3), practical suggestions are made as to how the findings might be applicable to NC(V) Mathematical Literacy lecturers

and assessors, as well as NC(V) English First Additional Language lecturers. These will be shared with staff at this site in a workshop after the conclusion of the study.

3.9 RELIABILITY AND VALIDITY

The combination of methods that a mixed method approach allows aids in the triangulation of methods and data. According to Freebody (2004, p. 84), this “offset[s]...localisation and apparent subjectivity” and strengthens any conclusions the researcher may arrive at after careful analysis of the data. Methodological triangulation has been achieved by combining quantitative methods and data with qualitative methods and data, as well as by including the consideration of more than one facet of each construct to be described in answering the research questions. Specifically: linguistic complexity is described according to a complexity index, readability and required language proficiency; cognitive complexity is described according to cognitive domains assessed and difficulty of items, as well as by a calculated index, and the presence of graphic information sources; and errors are described according to their apparent source, a raw error count, and the percentage of marks lost per error type.

Data triangulation, the use of a variety of information sources (Denscombe, 2007), has been included in the research design. The examination paper has been evaluated both quantitatively and qualitatively according to specific criteria; the students’ responses to the same examination items have been quantitatively analysed in terms of the types of errors they have made and the language proficiency of the students has been qualitatively discussed. Together these types of data have contributed towards painting a richer picture of the phenomenon as it pertains to this specific case.

The validity of the documentary sources must be considered and can be evaluated in terms of several basic criteria, among them authenticity (Denscombe, 2007). The examination and answer scripts are authentic documents from the September 2011 trial examination period. The request for access to the documents and permission to use them in the study was made after the examination paper was produced and after the students had written the examination. It was not possible, therefore, for the research process to have in any way affected the authenticity of these documents. The examination is a typical example of a summative assessment, set by an appointed examiner and subjected to a peer moderation process.

Prior to acceptance of the examination as fit-for-use, it was carefully analysed as to whether the DoE (DoE, 2000a) assessment guidelines had been adhered to. Although each paper alone revealed discrepancies, the papers when considered together adhered to the guidelines and were considered, therefore, to be fit-for-use.

In order to answer the research questions, it was necessary to consult numerous fields of study and attempt to combine their theories to design the methods employed. This brings to the fore questions regarding the validity of the design, operational definitions and tools used. In an attempt to address this, all quantitative analysis tools were either recognised tools from seminal researchers, or adaptations of such tools, therefore contributing to the validity of the quantitative analysis as whole. Constructs were operationally defined only after in depth consideration of both the perspectives of prominent theorists as well as current research in the relevant fields had been made.

The presence of various language features in items, as well as the classification of the cognitive complexity of items was done by the researcher with the assistance of a subject expert in English First Additional Language and a subject expert in Mathematical Literacy. This increases the validity of the calculated complexity indices. Error analysis was done by the researcher alone, but the involvement of a critical friend in the process compensates for the decrease in validity that this approach would otherwise have caused.

It can be argued that the quantitative measurement of linguistic and cognitive complexity is itself lacking in validity. For the purposes of this research, however, the search is not for absolute precision of measurement, but for the descriptive and comparative value that such quantification allows.

3.10 SUMMARY

This chapter opened with the presentation of the four research questions with which this study is concerned, and outlined the approach which has been taken in attempting to answer them. The constructivist ontological position taken for this research was described, with the distinction made between socioconstructivism and cognitive constructivism. Rationalist epistemology and realist methodology were then explained as they apply to the study. A mixed methods approach was selected for this case study.

The case study focuses on the assessment of Mathematical Literacy for a group of NC(V) Level 4 isiXhosa home language students at an FET college. The assessment selected was the September 2011 Trial examination. The site, examination and student sample were described.

The pilot study occurred in two phases, both of which were presented in the chapter. The lessons learned from the pilot were used to inform the final research design and methods of analysis.

Five broad areas for data analysis were described. These were the analysis of: student results; linguistic complexity of the examination; cognitive complexity of the examination; error types from student scripts and the relationships between the complexity indices and types of errors. Motivation was provided for the measurement tools selected, and where these were adaptations of an existing tool, a detailed description was given of the adaptation process and the final product thereof. The section describing data analysis was concluded with a summary of the operational definitions of all constructs to be measured.

The ethical considerations which were taken into account in preparation for and execution of the study were explained, with particular mention made of the aim to provide benefit to the college at which the research was conducted. The chapter was concluded with a discussion of reliability and validity.

CHAPTER 4

PRESENTATION AND ANALYSIS OF DATA

4.1 INTRODUCTION

The data presented in this chapter will address each of the four research questions. In addition, the students' results from the trial examination are presented and summarised as they will become important in later discussions of the data in Chapter 5.

Unless otherwise specified, results are given for Paper 1 and Paper 2 in combination and referred to as 'the examination'. Where a distinction is made between the two papers, they will be referred to as Paper 1, and Paper 2. Information has been gathered on an item by item basis, while some was gathered with reference to each of the 17 scenarios presented in the examination. Where scenarios were used this is indicated. An item approach was used in all other instances.

The examination papers that are described in this chapter are included as APPENDIX E and APPENDIX F. A summary of both the linguistic complexity data and the cognitive complexity data can be found in APPENDIX I.

4.2 STUDENT RESULTS

The first table, given below, summarises the full class's results for Trial Paper 1 and 2: first results are discussed separately; thereafter the class average trial examination results are discussed, and finally, their final ICASS results for Mathematical Literacy are discussed. The full list of results for the entire class is included as APPENDIX G, and those of the sample are summarised in APPENDIX H.

Table 4.1 Percentage of students scoring in each score range for mathematical literacy trial examination

CODE	RANGE	DESCRIPTION	EXAM PAPER 1	EXAM PAPER 2	TRIAL AVERAGE	FINAL MATH ICASS
1	0 – 29%	Not achieved	9%	42%	20%	33%
2	30 – 39%	Elementary	21%	33%	29%	36%
3	40 – 49%	Moderate	23%	13%	33%	22%
4	50 – 59%	Adequate	28%	7%	9%	7%
5	60 – 69%	Substantial	16%	4%	9%	2%
6	70 – 79%	Meritorious	2%	0%	0%	0%
7	80 – 100%	Outstanding	0%	0%	0%	0%
TOTAL			100%	100%	100%	100%
AVE %			47%	33%	40%	34%
PASS RATE			91%	58%	80%	67%

While 91% of students passed Paper 1, only 58% achieved the pass mark of 30% for Paper 2. Similarly, the average result of 47% for Paper 1 dropped to 33% for Paper 2, with the average result for the final ICASS mark a mere 1% above this at 34%. For the average trial results, only 51% of students were able to achieve beyond the elementary level. For the final ICASS results, only 31% were able to achieve beyond the elementary level.

The table that follows summarises the spread of marks for the final English First Additional Language ICASS result.

Table 4.2 Percentage of students scoring in each score range for English First Additional Language year mark

CODE	RANGE	DESCRIPTION	ENG ICASS
1	0 – 29%	Not achieved	13%
2	30 – 39%	Elementary	36%
3	40 – 49%	Moderate	40%
4	50 – 59%	Adequate	9%
5	60 – 69%	Substantial	2%
6	70 – 79%	Meritorious	0%
7	80 – 100%	Outstanding	0%
TOTAL			100%
AVE %			40%
PASS RATE			51%

The pass mark for this subject is 40% and the average percentage achieved by this group is exactly this mark. Only 51% of the students have achieved beyond this elementary level.

For both the Mathematical Literacy and English First Additional Language ICASS results, no students achieved meritorious or outstanding results, and only 2% were able to achieve beyond 60% (substantial). The picture that emerges from this data is that students are not achieving mastery in either of these subjects.

4.3 RESEARCH QUESTION 1: LINGUISTIC COMPLEXITY OF EXAMINATION ITEMS

The first research question calls for a description of the linguistic complexity of the Mathematical Literacy examination items. Linguistic features were evaluated on an item by item basis according to the Linguistic Complexity Checklist as outlined by Shaftel et al. (2006). Readability was calculated per examination scenario using the Gunning Fog and Flesch Kincaid readability measures, as well as by calculating the Flesch Reading Ease score.

Also evaluated was the required English language proficiency level, as explained by the CEFR language proficiency levels. Both readability and CEFR language proficiency are discussed qualitatively, while the results of the Linguistic Complexity Checklist are utilised for quantitative analysis.

It should be noted that Question 1.1 of Paper 1 was excluded from analysis as it consisted of calculations only, and contained no text.

4.3.1 The Linguistic Complexity Index

The description of linguistic complexity was guided by the Linguistic Complexity Checklist (Shaftel et al., 2006). Items were evaluated according to the categories offered in this checklist, as outlined in section 3.7.2.1. Numbers of words per sentence as well as language features were examined. Language features were divided into those at a word level, sentence level and paragraph level. A Linguistic Complexity Index value was assigned to each item as well as each lead-in text.

4.3.1.1 Number of words per sentence

Number of words per sentence was averaged per each of the 17 scenarios. The mean value was 14.42 words per sentence for the entire examination. The data varied from 8.8 words to 20.1 words.

4.3.1.2 Word level features

At the word level, the following features were counted for each item and lead-in text: words with 7 letters or more; slang, homophones and homonyms; pronouns; and abbreviations.

Number of words with 7 letters or more

Words with 7 letters or more were mostly those associated with Mathematical Literacy subject content.

Number of uses of slang, homophones or homonyms

Uses of slang, homophones or homonyms were infrequent but a particularly relevant set of homonyms was found in the Soccer World Cup scenario in Paper 1.

Types of matches were described as: the Opening Match; Group Matches; Round of 16; Quarter-Finals; Semi-Finals; 3rd/4th Place and the Final. The words third, fourth, opening and final all imply an ordinal relationship. Third and fourth refers to the ultimate placing of the competing teams in the World Cup, while opening and final refer to the ordering of the matches themselves. The words quarter and semi are fractions, round refers to a shape, and because of how '3rd/4th Place' is written, students unfamiliar with the context may read it as three quarters.

Number of pronouns

The type of pronoun used most frequently was the personal pronoun (e.g. he, she). Interrogative pronouns (e.g. who and what), relative pronouns, demonstrative pronouns and indefinite pronouns also featured, with no examples of reflective pronouns (e.g. himself, herself etc.).

Number of abbreviations

Several abbreviations were used, most directly related to content specifically taught in the Mathematical Literacy course, e.g. Value Added Tax [VAT]. Many did not include an

expansion where they were first used. These included: VAT; Greenwich Mean Time [GMT]; *Fédération Internationale de Football Association* [FIFA]; South Africa [SA]; Digital Video Disc [DVD]; United States of America [USA]; Number [No.] ; Departure [Dep.]; Arrival [Arr.]; Port Elizabeth [P.E.]; Unemployment Insurance Fund [UIF]; and Consumer Price Index [CPI].

4.3.1.3 Sentence level features

In assessing sentence level features, the following were taken into consideration: complex verbs; infinitives; complex/compound sentences; prepositional phrases; and conditional or comparative constructions.

Number of complex verbs and infinitives

Only verbs consisting of three or more words were counted. There were 38 such verbs found in the examination, for example: "...will you pay..." (Paper 1, Question 1.4) and "has already completed" (Paper 1, Question 3). Infinitives numbered 51.

Number of complex/compound sentences

67 sentences contained either two independent clauses, for example: "Vusi left later and she was travelling at an average speed of 120 km/h" (Paper 2, Question 2); or one independent clause with one or more dependent clauses, for example: "During the 2006/2007 tax year, Luka (36 years old) worked for a hotel chain" (Paper 2, Question 3).

Number of prepositional phrases

Prepositional phrases far outnumbered any other sentence level language feature at 345. The function of prepositions is to show relationships between nouns and pronouns. In this examination most prepositional phrases were essential in describing relationships essential to the solution of the mathematical problem, e.g. "Approximately how many kilometres will you travel *from* the gate *to* Manyane Rest Camp" (Paper 1, Question 5.4). This example requires the student to measure the distance between the two points on the map before making a conversion. Without the use of the prepositions 'from' and 'to' it would not be possible to describe between which two points on the map the distance is to be measured.

The average over the whole examination was 1.7 prepositional phrases per sentence. Many sentences contained significantly more. Question 1.3.2 in Paper 2 is such an example: "*On*

which day *of* the week and *at* what time does Siphwe have to leave her home *in* order to be *at* the departure point *on* time”.

Number of conditional or comparative constructions

There were relatively few examples of the use of conditional and comparative constructions in the examination at 44. It was noted, however, that each example of their use was essential in describing exactly what piece of information given in the scenario was to be used in the calculation. One such example, from Paper 2, reads: “How much would Sally have paid for the property over 25 years if she [had] decided to rent?” (Paper 2, Question 1.2.1). The use of the conditional “if she decided to rent” provides essential guidance as to what section of the given table held the information required to solve the problem.

4.3.1.4 Paragraph level features

At a paragraph level, the number of cultural and contextual or experiential references was counted, as well as the number of grammatical or spelling errors.

Number of cultural and contextual or experiential references

Many of the scenarios included references to situations with which students may not have been familiar. One such scenario is found in Question 9, of Paper 1. This scenario made use of the example of seamstresses manufacturing uniforms. This item had a high number of errors, for which the underlying source may have been the misunderstanding of the scenario, thereby creating confusion regarding how to answer the items.

Number of grammatical or spelling errors

There were many examples of both grammatical and spelling errors in the examination, which would add complexity for an English language learner due to the ambiguity it introduces.

One particular example was Paper 1, Question 2. Number 2.7 is shown below:

2.7	Calculate the price for the following match ticket combinations:	
2.7.1	4 of category 3 tickets for the opening match and 4 of category 2 tickets for Quarter Finals.	(3)
2.7.2	1 of category 3 tickets for the Quarter-Finals, 2 of category 1 tickets for Semi-Finals and 2 of wheel chair tickets for Finals.	(4)

This item should have read:

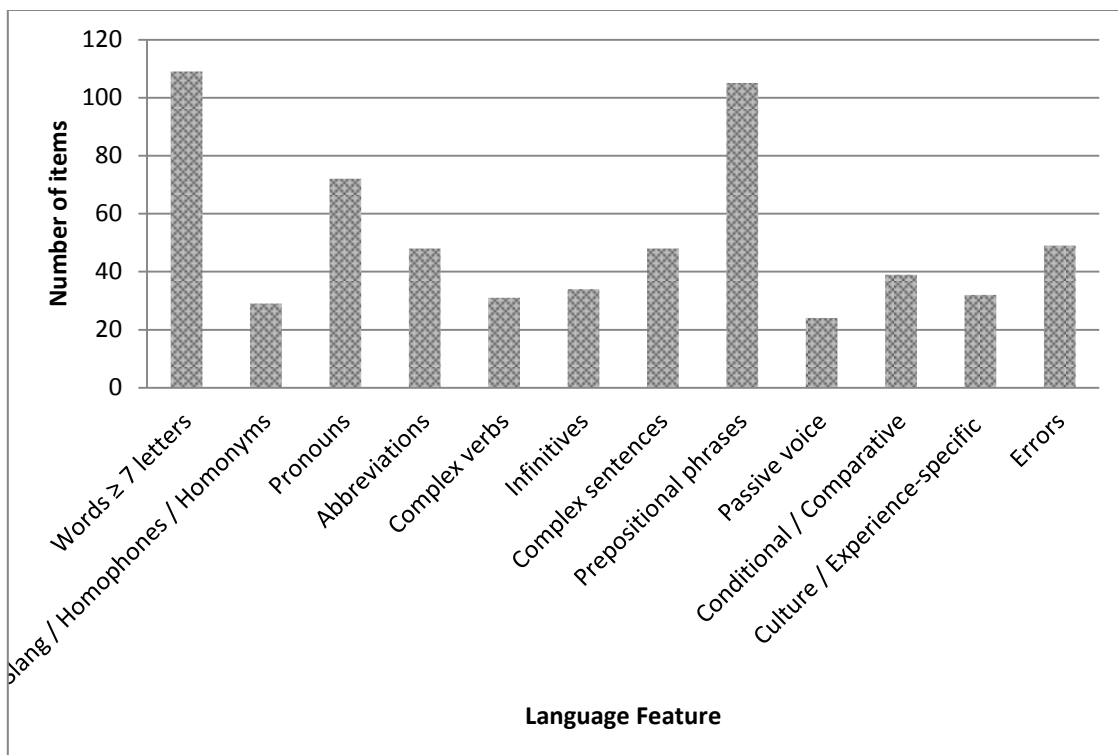
- 2.7 Calculate the price of the following match ticket combinations:
- 2.7.1 4 category 3 tickets for the *Opening Match* and 4 category 2 tickets for a *Quarter-Final*.
- 2.7.2 1 category 3 ticket for a *Quarter-Final*, 2 category 1 tickets for a *Semi-Final* and 2 wheel-chair tickets for *the Final*.

The repeated incorrect use of the preposition ‘of’, as well as the incorrect usage of the plural form of the word ‘ticket’ may have caused any language-related errors made in responding to this item.

4.3.1.5 Frequency of use of language features

The figure below shows the comparison of the frequency of use of each of the language features discussed above.

Figure 4.1 Frequency of use of language features in the Mathematical Literacy examination



It can be clearly seen that words with 7 letters or more and prepositional phrases appear the most, with pronouns third with substantially less use.

When the data regarding the frequency of use of the language features are summarised per topic, it emerges that they are similar to those for the entire examination paper. The following table summarises which 5 language features, per topic, appear the most.

Table 4.3 Language features with highest frequencies per topic

	FIRST	SECOND	THIRD	FOURTH	FIFTH
TOPIC 1	Words ≥ 7 letters	Prepositional phrases	Pronouns	Abbreviations	Complex verbs
TOPIC 2	Words ≥ 7 letters	Prepositional phrases	Errors	Pronouns	Abbreviations
TOPIC 3	Words ≥ 7 letters	Prepositional phrases	Pronouns	Abbreviations	Conditional / Comparative constructions
TOPIC 4	Words ≥ 7 letters	Prepositional phrases	Abbreviations	Errors	Slang / Homophones / Homonyms
TOPIC 5	Prepositional phrases	Words ≥ 7 letters	Errors	Pronouns	Culture / Experience-specific references

Words with seven letters or more and prepositional phrases appeared as the most frequent features for every topic as they did for the examination as a whole. Only the use of conditional/comparative constructions; slang/homophones/homonyms and use of culture/experience specific references do not appear as one of the language features in the top five for the whole examination.

For each language feature, the number of items for which it had the highest frequency in relation to the others was counted. The table below shows the percentage of items for which each language feature had the highest frequency. For example, prepositional phrases had a frequency higher than all other features in 28% of the items.

Table 4.4 Percentage of items for which each language feature has the highest frequency

Language feature	Words \geq 7 letters	Slang / homophones / homonyms	Pronouns	Abbreviations	Complex verbs	Infinitives	Complex / Compound sentences	Prepositional phrases	Passive voice	Conditional/ Comparative	Culture- / Experience-specific	Errors	TOTAL
% of items	47%	3%	6%	3%	0%	1%	1%	28%	1%	1%	1%	9%	100%

The results differ slightly from those presented in Figure 4.1. Words with 7 letters or more and prepositional phrases remained the features with the largest frequency, but grammatical or spelling errors contribute the most in 9% of the examples, with pronouns appearing as the top contributor in only 6% of the examples.

4.3.1.6 Descriptive summary of data associated with the Linguistic Complexity Index

The descriptive statistics summarising and comparing the number of examples of the language feature per examination item are provided in the table below. For example the maximum number of prepositional phrases used in any examination item was 17 such phrases in one item. The Linguistic Complexity Index values are also summarised in this table. While these statistics reduce and simplify a large amount of rich data ($n = 107$), and do not allow for any inferences to be made, they do provide a useful summary. In particular, the range of values obtained from the data and the mean, signifying the average number of features per sentence, are useful indicators as to the average linguistic complexity of the examination and frequency of use of features per sentence for the examination as a whole.

Two significant outliers were eliminated from this analysis according to their particularly high linguistic index. These items were 2.7.1 and 2.7.2 of Paper 1, with linguistic indices of 52 and 58 respectively. This high linguistic index was the result of a disproportionately high number of grammatical errors in each of these single sentence items. A third outlier that was excluded from analysis was Question 1 of Paper 1. This question had a particularly low linguistic index, at only 2, as it served to assess only the students' ability to calculate answers to various 'number sentences' and did not contain any contextualising text. As such, it contained only one phrase and two words of seven letters or more ("calculate" and "calculations"). 107 cases therefore remained for analysis.

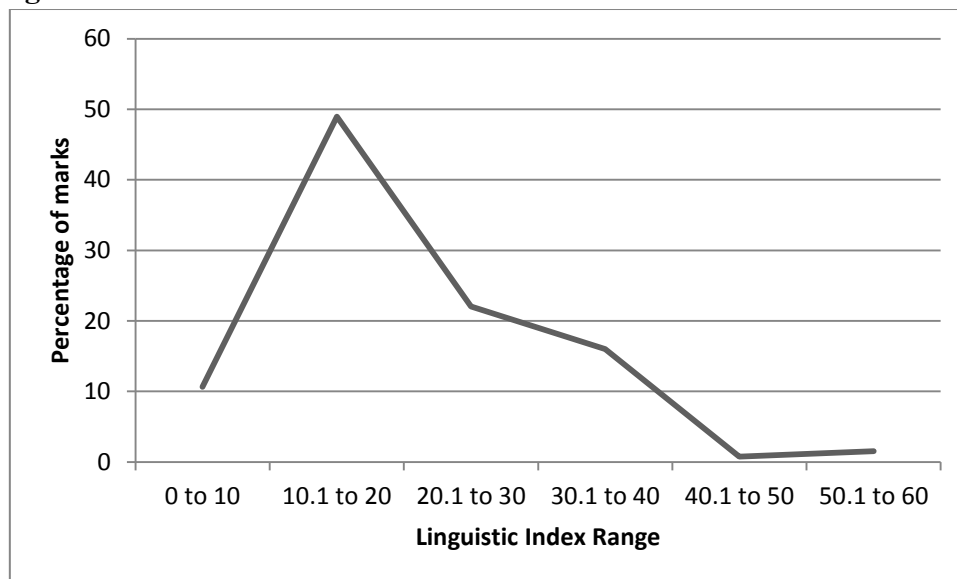
Table 4.5 Descriptive statistics for Linguistic Complexity Index and top language feature contributors

n = 107	Mean	Standard Deviation	Minimum	Maximum	Median
LINGINDEX	19.31	8.60	7.50	41.00	18.00
Prepositional phrases	3.01	3.01	0.00	17.00	2.00
Words ≥ 7 letters	4.53	4.62	0.00	26.00	3.00
Pronouns	1.19	1.43	0.00	9.00	1.00
Errors	1.05	1.90	0.00	8.00	0.00
Abbreviations	1.03	1.44	0.00	6.00	0.00

Spread of mark allocation per Linguistic Complexity Index range

The Linguistic Complexity Index values can be divided into six levels: 0 – 10; 10.1 – 20; 20.1 – 30; 30.1 – 40; 40.1 – 50; 50.1 – 60. The graph below shows the spread of mark allocation across these levels:

Figure 4.2 Graph showing percentage of marks allocated per Linguistic Complexity Index range



The range represented by the largest percentage of marks is the second level of complexity, with indices 10.1 to 20. It is followed by a sharp decrease in percentage of marks as the linguistic complexity level increases.

4.3.1.7 Percentages of marks allocated to each linguistic complexity range per topic

Linguistic complexity has been summarised in the four tables below as they appear per topic:

Table 4.6 Linguistic Complexity Index ranges per topic

	TOPIC 1	TOPIC 2	TOPIC 3	TOPIC 4	TOPIC 5	TOTAL
0 to 10	14%	6%	6%	12%	15%	53
10.1 to 20	50%	39%	63%	59%	35%	245
20.1 to 30	14%	22%	31%	12%	31%	110
30.1 to 40	21%	33%	0%	18%	8%	80
40.1 to 50	0%	0%	0%	0%	4%	4
50.1 to 60	0%	0%	0%	0%	8%	8
TOTAL	100%	100%	100%	100%	100%	500

For all topics, the level of linguistic complexity with the highest weighting was in the index range of 10.1 to 20. Topic 5 was the only topic to have a Linguistic Complexity Index spread across all 6 index ranges. Topic 3 had the smallest range of Linguistic Complexity Index values, spreading over only the first three ranges.

4.3.1.8 Calculation of Pearson product-moment correlations

Language features were ordered according to their frequency of use as well as the percentage of examples for which they were the top contributor to the Linguistic Complexity Index. The top 5 language features were selected for further analysis. These were:

1. Words with 7 letters or more (109/47%)
2. Prepositional phrases (105/28%)
3. Pronouns (72/6%)
4. Errors (49/9%)
5. Abbreviations (48/3%)

Due to the relatively high frequency of use (48) in comparison to the next highest, conditional and comparative constructions (39), complex/compound sentences were added to the list of features for further analysis. Its frequency of use was also exactly the same as that of abbreviations, selected as the 5th feature for further analysis. This was despite the fact that it was the top contributor to the Linguistic Complexity Index in only 1% of the examples.

Pearson product-moment correlations were calculated using the number of uses of each language feature per examination item as the independent variable and the Linguistic Complexity Index as the dependent variable.

Statistical significance was not revealed for any features when the full range was used. The range was therefore systematically narrowed, first from the bottom and then from the top. Statistical significance was only deemed fit for reporting if no less than $\frac{2}{3}$ of the sample remained. Therefore as soon as n decreased below 72, Pearson product-moment results were no longer calculated.

Statistical significance was only found for prepositional phrases, words with 7 letters or more and complex/compound sentences where $n \geq 72$.

The summary of the Pearson product-moment results are provided in the table below:

Table 4.7 Pearson product-moment correlations between Linguistic Complexity Index and selected language features

	n	r(x;y)	Strength	p	Statistical Significance (confidence level)	
Independent Variables	Prepositional phrases	107	-0.13	Relatively weak	0.173	none
	<5 phrases	90	0.23	Moderate	0.026	At 5% level
	<4 phrases	81	0.30	Moderately strong	0.006	At 1% level
	Words with ≥ 7 letters	107	-0.05	Weak	0.602	none
	<6 words	84	0.23	Moderate	0.034	none
	<5 words	74	0.36	Moderately strong	0.002	At 1% level
	Pronouns	107	-0.15	Relatively weak	0.133	none
	Errors	107	-0.18	Relatively weak	0.063	none
	Abbreviations	107	-0.08	Weak	0.390	none
	<2 abbreviations	76	0.05	Weak positive for the rest of the range	0.837	none
	Complex/Compound sentences	107	0.10	Relatively weak	0.287	none
	<3 sentences	102	0.33	Moderately strong	0.001	At 1% level
	<2 sentences	95	0.34	Moderately strong	0.001	At 1% level

The following table summarises which language features were statistically significantly correlated to the Linguistic Complexity Index:

Table 4.8 Summary of statistically significant correlations

	Statistical significance	Interpretation
Prepositional phrases	√ (pos)	1% confidence level, moderately strong
Words ≥ 7 letters	√ (pos)	1% confidence level, moderately strong
Pronouns	X	-
Errors	X	-
Abbreviations	X	
Complex / Compound sentences	√ (pos)	1% confidence level, moderately strong

4.3.2 Readability

Readability was calculated per scenario in order to provide a large enough piece of text for the measure to be valid. An exact value is provided by the measure, as reported in the table below, but the values are interpreted qualitatively according to the descriptions provided for each range in the interpretation tables.

The Gunning Fog and Flesch Kincaid readability measures both calculate an approximate United States [US] grade level of reading skill, but in some cases were found to have quite different results. For this reason an average was calculated for interpretation.

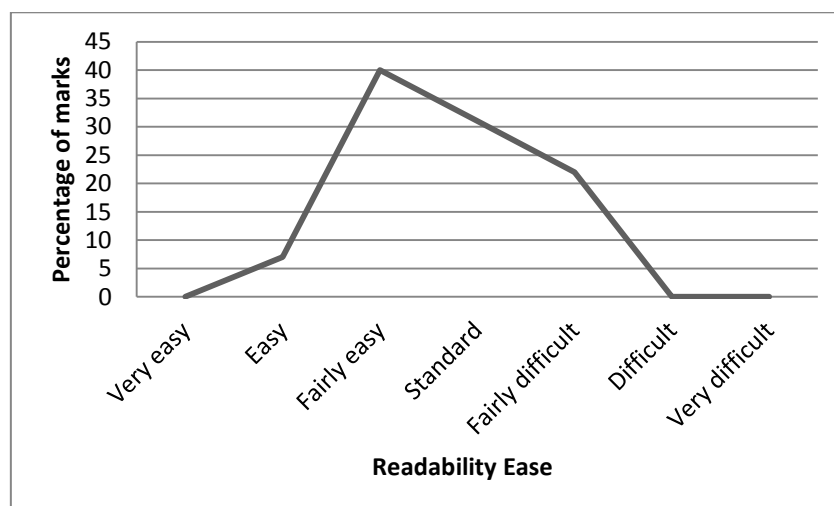
Table 4.9 Readability results per scenario presented in the examination

NUMBER	POINTS	APPROXIMATE US GRADE LEVEL			FLESCH READING EASE	READABILITY LEVEL
		GUNNING FOG	FLESCH KINCAID	AVE		
1	34	7.08	5.15	6.12	77.88	Fairly easy
2	16	12.66	10.74	11.70	52.42	Fairly difficult
3	11	8.11	8.51	8.31	59.51	Fairly difficult
4	17	8.56	7.29	7.93	68.62	Standard
5	17	11.18	9.09	10.14	65.79	Standard
6	9	7.59	4.95	6.27	80.20	Easy
7	12	8.66	9.90	9.28	57.18	Fairly difficult
8	18	7.31	6.13	6.72	74.73	Fairly easy
9	11	7.57	5.00	6.29	83.64	Easy
10	5	10.97	8.68	9.83	60.07	Standard
11	23	6.93	5.22	6.08	73.36	Fairly easy
12	13	7.34	5.69	6.52	73.35	Fairly easy
13	9	8.53	7.15	7.84	66.62	Standard
14	26	10.60	8.75	9.68	60.42	Standard
15	26	10.52	8.61	9.57	59.01	Fairly difficult
16	20	10.40	8.64	9.52	62.31	Standard
17	33	6.95	5.89	6.42	75.70	Fairly easy
AVE		8.88	7.38	8.13	67.69	Standard

Readability varied widely, from 5.00 as the Flesch Kincaid value for scenario 9 (in Paper 1), and 12.66 as the Gunning Fog value for scenario 2 (in Paper 1). The Flesch Kincaid Reading Ease score varied between Easy and Fairly Difficult, although averaging out as Standard.

The percentage of points allocated to each range of Reading Ease descriptions is provided in Figure 4.3.

Figure 4.3 Percentage of marks allocated to each Reading Ease range



4.3.3 Required level of language proficiency

The 17 scenarios found in the examination were again used for evaluation. The required level of language proficiency per scenario was first assessed individually, and thereafter an average calculated for the entire examination.

The CIGE was used in this analysis. These were translated into the CEFR language proficiency levels required to comprehend each type of part of speech or language function. Where a range of proficiency levels was associated with a particular part of speech or language function, the average was used in the analysis of the data. This necessitated the insertion of language proficiency categories between those provided in the CEFR tool, i.e. A1+; A2+; B1+ and B2+.

Not all of the language features provided in the CIGE were found in the examination; however certain language features appeared in all scenarios. Those appearing in every scenario included:

- Parts of speech
 - personal pronouns (subject)
 - common prepositions
 - prepositional phrases (time and movement)
 - prepositional phrases (place and time)

- definite and indefinite articles
- common adjectives
- Language function
 - numbers
 - giving precise information.

Those appearing in all scenarios were categorised at levels A1 and A1+, except for the function of giving precise information, categorised at B2.

Apart from ‘giving precise information’, those contributing to one of the highest three levels of language proficiency (B2; B2+ and C1) included: logical markers (6); speculating and hypothesising (4); past perfect tense (4); articles with abstract nouns (3); expressing certainty, probability and doubt (2); and expressing opinions (2). Appearing in only one scenario, but also located in one of the three highest levels were: generalising and qualifying; defending a point of view and future perfect tense. The table below summarises the number of scenarios in which each part of speech of language function appears:

Table 4.10 Number of scenarios in which each part of speech or language function appears

CEFR LEVEL	A1	A1+	A2	A2+	B1	B1+	B2	B2+	C1
Personal pronoun - subject	17								
Common prepositions	17								
Prepositional phrases (time & movement)		17							
Prepositional phrases (place & time)		17							
Definite, indefinite	17								
Common adjectives	17								
Function - Numbers	17								
Function - Giving precise information							17		
Imperatives		14							
Question forms		14							
To be	13								
Common phrasal verbs			11						
to + infinitive (express purpose)			11						

CEFR LEVEL	A1	A1+	A2	A2+	B1	B1+	B2	B2+	C1
Zero and 1 st conditional					11				
Comparative, superlative		11							
Prices	11								
Describing past experiences and storytelling				11					
Connecting words, and, but, because	11								
Wh-questions in the past				10					
Simple Present		8							
Future Time (will & going to)						8			
Possessive pronouns		7							
Ending -ed + -ing			7						
Simple adverbs of place, manner and time			7						
Simple Past	6								
Linking devices: logical markers									6
Should			5						
Speculating and hypothesising								5	
Present Continuous		5							
Telling time	4								
Past Perfect								4	
Might, may, will, probably					3				
Articles with abstract nouns							3		
Describing things			3						
Gerunds		2							
Can/could		2							
Very basic (very, really)		2							
Arrangements			2						
Expressing certainty, probability, doubt							2		
Expressing opinions									2
Have got	1								
Might, may			1						
Possibly, probably, perhaps			1						

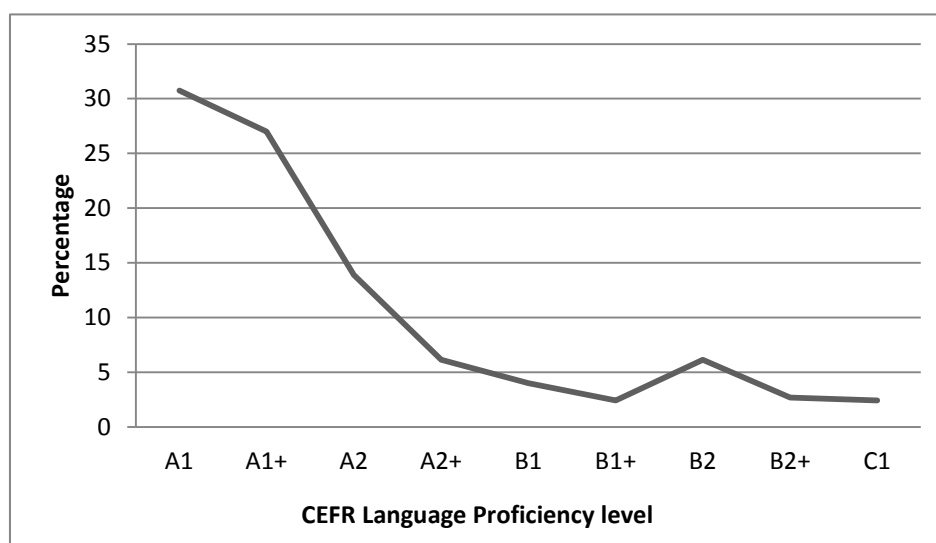
CEFR LEVEL	A1	A1+	A2	A2+	B1	B1+	B2	B2+	C1
Must/mustn't			1						
Have to			1						
Basic (e.g. any, some, a lot of)		1							
Directions	1								
Describing habits and routines		1							
Obligation and necessity			1						
Describing places				1					
Generalising and qualifying							1		
Expressing agreement or disagreement						1			
Defending a point of view									1
Past Continuous			1						
Future Perfect								1	
Linkers: sequential – past time					1				

The table below provides the sum of the frequency of each language proficiency level as per Table 4.9. Figure 4.2 provides the graphic representation of these values:

Table 4.11 Sum of language proficiency levels appearing in the examination

CEFR LEVEL	A1	A1+	A2	A2+	B1	B1+	B2	B2+	C1	SUM
Total Instances	115	101	52	22	15	9	23	10	9	373
Percentage of total	31%	27%	14%	6%	4%	2%	6%	3%	2%	100%

Figure 4.4 Percentage of CEFR levels for examination paper



By a large margin, level A1 and A1+ appear the most, with a sharp decrease to level A2, continuing downward with a slight peak at B2. Both A1 and A1+ are described as the “breakthrough or beginner” (Lindhout, Teunissen & Lindhout, 2012, p. 37) stage.

A student considered competent at the beginner level is assumed able to:

Interact in a simple way, ask and answer simple questions about themselves, where they live, people they know, and things they have, initiate and respond to simple statements in areas of immediate need or on very familiar topics.

(North et al., 2010, p. 23)

It is essential, however, that competence at the level of B2 (Upper Intermediate) has been achieved by students, as the function of the text in every scenario is to give precise information. This is the source of the peak at B2 in Figure 4.4. A full description of a student at this level of English language proficiency is included in APPENDIX B. In order to function at this level, a more sophisticated use of the language is required.

4.4 RESEARCH QUESTION 2: COGNITIVE COMPLEXITY OF EXAMINATION ITEMS

Cognitive complexity is described in this section according to the Cognitive Complexity Level of each item according to the rubric outlined in section 3.7.3.1. Thereafter, the Cognitive Complexity Index value is calculated per item, and a summary provided of the

percentage of marks allocated per linguistic complexity level per topic. A summary is also provided of the graphical languages used in the examination (MacKinley, 1986).

Question 1.1 of Paper 1 was again excluded from analysis as it was not assigned a Linguistic Complexity Index.

4.4.1 Item analysis according to Cognitive Complexity Level

An item analysis was performed to allocate a Cognitive Complexity Level to each. These levels ranged from 1 to 6, according to the rubric found in Table 3.10.

This value formed the starting point in calculating the Cognitive Complexity Index for each item. This was done for each item, as opposed to each piece of text, as with the Linguistic Complexity Index. Therefore, the lead-in text will have a Linguistic Complexity Index value, but will not have a matching cognitive complexity value.

An example of an item categorised at a Cognitive Complexity Level of 1 is the following:

1.2 Write the following ratio in its simplest form: 12 : 108	(2)
--	-----

This item involves ‘knowing facts and procedures’, specifically the simplification of ratios, and was deemed easy by both of the raters.

The following item also involves ‘knowing facts and procedures’ but was categorised as being of medium difficulty, therefore having a Cognitive Complexity Level of 2:

1.5 A computer game costs R499 excluding VAT. What will you pay for the game after VAT of 14% has been added?	(3)
---	-----

The example below is at a Cognitive Complexity Level of 2, but is placed on the rubric as involving the use of concepts (compound increase), although at an easy level for this domain, owing to the fact that students were provided the formula, and were not required to transform any of the values before using them in the required calculation.

1.9 R6 000 is invested at 5% compound interest per year. What will the investment be worth after FOUR years?
 Use the formula: $A = P(1 + i)^n$
 Where A = Final Amount
 P = Principal/Starting amount
 i = Interest rate (5% = 0,05)
 n = Time period in years (n = 4)

(3)

Question 1.10 was classified as being at a Cognitive Complexity Level of 3, as it involved ‘using concepts’ (time zones), at an agreed medium level of difficulty, as it required conversion across two time zones as opposed to one.

1.10 Use the information below to determine what the time will be in New York if it is 4:00 pm in South Africa.

NEW YORK GMT -4
 SOUTH AFRICA GMT +2

(3)

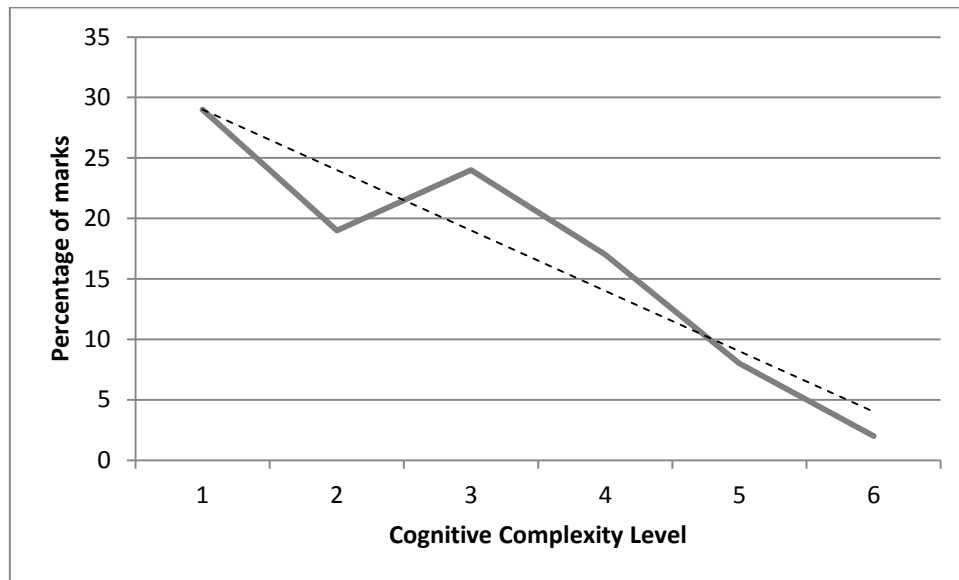
The following table summarises the number of marks allocated to each cognitive complexity category, according to the position of the item on the rubric. This summary is for the whole examination; therefore, the total is 300 marks.

Table 4.12 Number of marks per cognitive complexity category

COGNITIVE COMPLEXITY		COGNITIVE DEMAND		
		LOW	MED	HIGH
COGNITIVE DOMAIN	1	59	21	3
	2	34	59	23
	3	17	30	30
	4	2	8	14

The following graph summarises the percentage of items placed at each Cognitive Complexity Level:

Figure 4.5 Graph showing percentage of marks allocated to each Cognitive Complexity Level



4.4.2 Item analysis according to Cognitive Complexity Index values

The following two examples were categorised at a Cognitive Complexity Level of 4. They involved the use of concepts, and were deemed by the raters to both be of high difficulty.

4.3 Sketch a line graph that shows Eddy's income during May on the same system of axis with Eddy's expenditure. Clearly label the graph. Use the attached graph paper on ANNEXURE B to draw the graph. (6)

6.2.6 Determine the total number of cases reported in the two provinces during the period 2001/2002. (2)

The first example required the students to produce a graph, while the second required a two-step calculation based on information read from a graph. They differ in their mark allocation, as well as in the amount of time which would have been involved in responding to the items.

The first item would require more time to complete and would demand more working memory than the second. Their Cognitive Complexity Index value therefore differs. The first

item is indexed at 24 (4 multiplied by six), while the second is indexed at 8 (4 multiplied by 2).

After performing this operation on the Cognitive Complexity Level of each item, the following descriptive statistics summarise the resulting values:

Table 4.13 Descriptive statistics of Cognitive Complexity Index

Cognitive Complexity Index	
Mean	8.12
Standard Deviation	7.71
Minimum	1.00
Maximum	40.00
Median	6.00

One particularly high outlier (Paper 1, Question 3) was excluded from any further analysis. This large index resulted from the relatively high number of marks assigned to this question (11 marks). When the number of marks was multiplied by the Cognitive Complexity Level of 6, the Cognitive Complexity Index reached 66, which was more than double the second highest Cognitive Complexity Index of 28 (Paper 2, Question 5.2).

4.4.3 Percentages of marks allocated to each Cognitive Complexity Level per topic

The spread of Cognitive Complexity Levels per topic is given in the table below:

Table 4.14 Cognitive Complexity Level per topic

	TOPIC 1	TOPIC 2	TOPIC 3	TOPIC 4	TOPIC 5	TOTAL
1	29%	39%	5%	24%	40%	136
2	21%	17%	16%	24%	20%	97
3	29%	17%	21%	35%	23%	124
4	7%	11%	37%	12%	17%	84
5	7%	17%	16%	6%	0%	45
6	7%	0%	5%	0%	0%	12
TOTAL	100%	100%	100%	100%	100%	500

The level for which each topic is weighted the heaviest is indicated in bold. They vary per topic, with Topics 2 and 5 weighted heavily at Level 1. Topic 1 carries equivalent weighting for levels 1 and 3; Topic 3 is weighted most heavily at Level 4; and Topic 4 weighted most heavily at Level 3.

Only Topics 1 and 3 range across all 6 of the Cognitive Complexity Levels. Topic 2 was most heavily weighted at Level 1. Topic 5 had the heaviest of the weightings at Level 1.

4.4.4 Graphical languages

Three types of graphical languages were used in this examination: apposed-position languages; retinal list languages; and map languages. Apposed-position languages appeared in the form of tables, line graphs and a bar chart in a total of 11 scenarios; a map, a plan of a netball court and a diagram of a picture frame were found in the 3 scenarios using map languages; and retinal-list languages appeared in only 1 scenario, in the form of a diagram showing the shape of two containers.

4.5 RESEARCH QUESTION 3: TYPES OF ERRORS MADE BY STUDENTS

This section provides examples of different types of errors from student scripts, as well as a summary of the types of errors made by the sample of students. Summaries of the error counts per type and item and the percentage of marks lost per type and item are included as APPENDICES J and K.

4.5.1 Examples of error types

Examples of the following types of errors are provided in this section: graphic-related errors; mathematical calculation errors; reading errors; writing errors; errors due to carelessness; and those for which no type could be assigned.

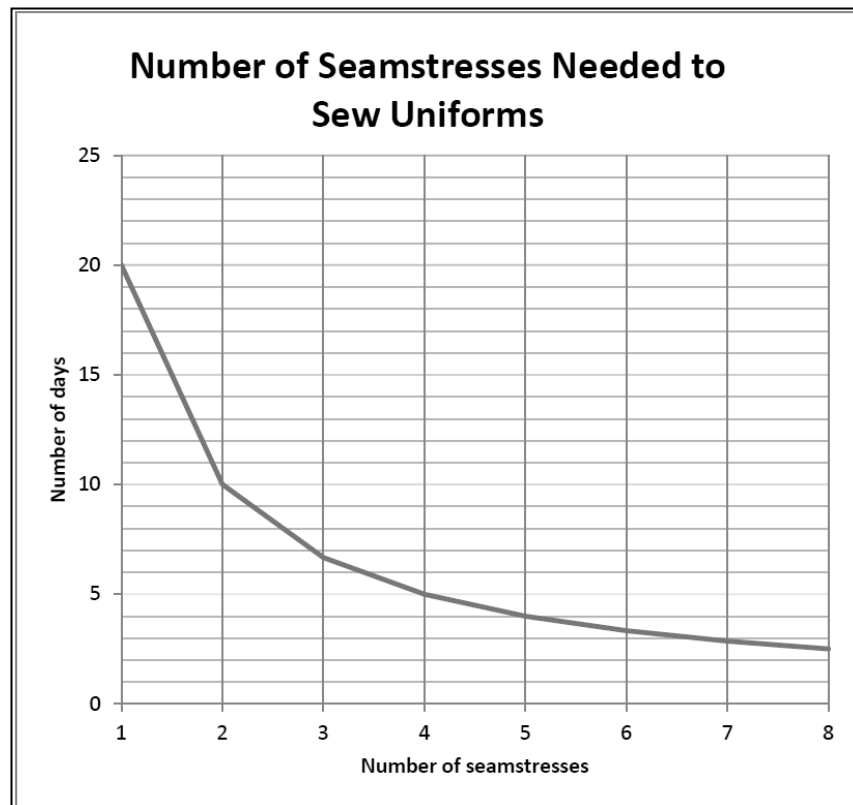
4.5.1.1 Graphic-related errors

The presentation and analysis of the data gathered regarding graphic-related errors will be grouped according to the types of graphical languages (MacKinley, 1986) used in the examination.

Apposed-position languages

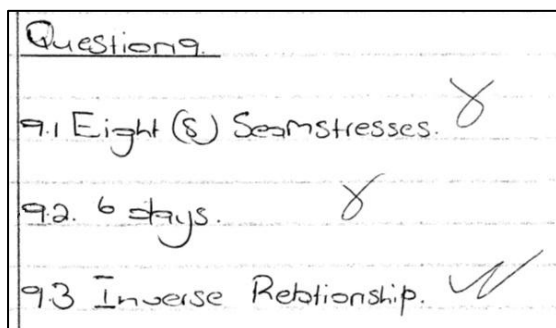
The scenario about seamstresses (Paper 1, Question 9) produced many errors (12 errors for 9.1 and 10 errors for 9.2) as a result of students misinterpreting the graph.

Figure 4.6 Line graph from Paper 1, Question 9



One response from a student script is given below:

Figure 4.7 Excerpt from student script – graphic-related error (graph)



The first item required students to read from the graph how many seamstresses were required to complete the job in 20 days. This student interpreted the two end points as being

equivalent, relating the 20 days at the start point to the 8 seamstresses at the end. It was not understood how x-axis and y-axis values are related and how to read co-ordinates on the line. Similarly, the second item asked students to estimate the number of days 6 seamstresses would need to complete the job. This students' inability to read co-ordinates from the graph and to understand the relationship of the x-axis values to the y-axis values is again evident in the response of 6 days.

Understanding that days are related to number of seamstresses is evident in their qualification of the numbers with the correct word, but the concept of co-ordinates and the information they provide is not understood.

The correct answer is given for Question 9.3, which asked what type of relationship is represented by the graph. This information could have been studied and produced automatically without any understanding of inverse relationships and how they function, which seems to be the case for this student, as it did for many of the others.

Question 7.4, from Paper 1, provided a different type of graphic stimulus, a table with information about the prices of various size boxes of [Compact Discs] CDs.

Figure 4.8 Table from Paper 1, Question 7.4

7.4 Eddy decides to buy 100 Compact Discs (CDs) to sell at the flea market. He receives the following quotation (prices in the table below):

	Description	Price
Option 1	1 CD	R2,49
Option 2	A box of 10 CDs	R19,99
Option 3	A box of 25 CDs	R52,99

Many students did not understand how to link the text horizontally across the table to the different prices. They failed to understand that the prices in the right-hand column corresponded to the product described as a single unit. That is, that a single box of 10 CDs cost R19.99, rather than that each CD in the box of 10 cost R19.99. The excerpt that follows shows an example of this type of error from a student script:

Figure 4.9 Excerpt from student script – graphic-related error (table)

7.4.1	$R2,49 \times 50 = R124,5$	✓✓
7.4.2	$R19,99 \times 50 = R999,5$	✓
7.4.3	$R52,99 \times 50 = R2649,5$	✓
7.4.4	OPTION 1.	✓

Students were asked to calculate, for each option, how much it would cost to purchase 50 CDs. This student understood from the table that for Option 2 one CD cost R19.99 and therefore 50 would cost R999.50, and similarly, that Option 3 indicated the price of one CD was R52.99, therefore the cost of 50 would be R2649.50.

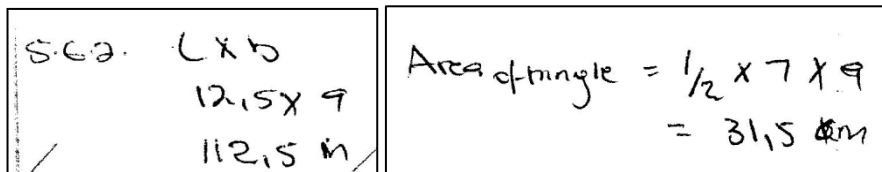
Map languages

Map languages were used in three scenarios. Similar errors were made by all students. Students were able to perform well in items related to a graphic, but that did not require any values to be read from the graphic.

For example, in Question 8.4, the volume of a box was to be calculated, but the values required for the calculation were provided with the items and required no reference to the accompanying visual. Every student from the sample answered this item correctly. However, for Question 8.2, which required a calculation for which the relevant value was to be found on the diagram, only 1 student achieved full marks.

An example from Paper 1, Question 5.6 is shown below. In this item, students were required to calculate the area of a rectangle and a triangle represented on a map. The relevant distances were to be found on the map. Every student in the sample achieved zero for this item. The student whose answers are shown below knew how to use the formulae given, but could not use the map to find the correct distances. The calculation itself is correct, but the values are not.

Figure 4.10 Excerpt from student script – graphic-related error (map language)



Retinal-list languages

Only one scenario included the use of retinal-list languages. No item, however, required direct reference to the images and therefore no graphic-related errors were found for this item. Errors included only those related to mathematical calculation and reading.

4.5.1.2 Math calculation errors

Some errors revealed that the student did not know how to carry out the actual mathematical calculation correctly, although showing that they understood what was required to respond to the item.

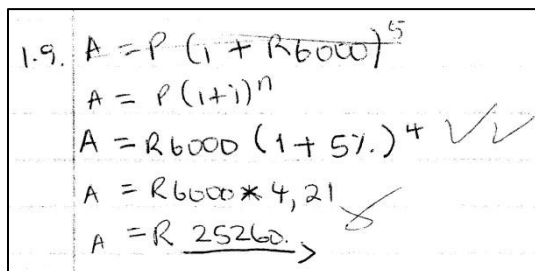
One item in which many such calculation errors occurred was Question 1.6 in Paper 1:

1.9 R6 000 is invested at 5% compound interest per year. What will the investment be worth after FOUR years?
 Use the formula: $A = P(1 + i)^n$
 Where A = Final Amount
 P = Principal/Starting amount
 i = Interest rate (5% = 0,05)
 n = Time period in years (n = 4)

(3)

Many responses showed the following error, or similar:

Figure 4.11 Excerpt from student script (calculation error)



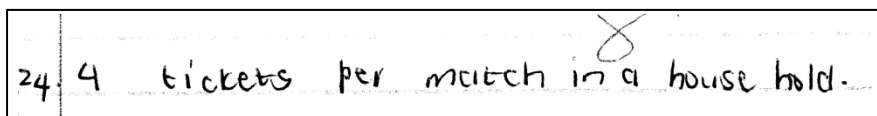
This student was able to substitute the given values in the correct place in the formula, but was unable to accurately use these values and operations in the calculation.

4.5.1.3 Reading and writing errors

There were a large number of reading errors in the student responses to Questions 2.4 and 2.5 in Paper 1.

In Question 2.4 students were asked to determine the maximum number of tickets a South African household could buy for the entire World Cup. The correct answer was 28 according to the table provided, which stated that each household could apply for a maximum of four tickets per match for a maximum of 7 matches.

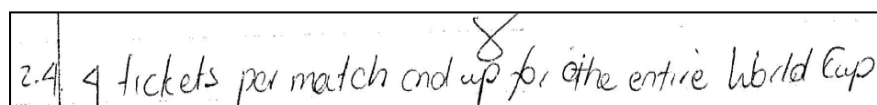
Figure 4.12 Excerpt from student script (reading error)



24. 4 tickets per match in a house hold.

This student did not read (or possibly understand) the second half of the relevant sentence stating that the maximum number of matches was 7.

Figure 4.13 Excerpt from student script (reading error)

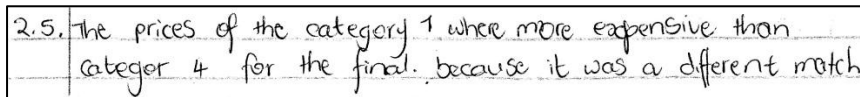


24. 4 tickets per match end up for the entire World Cup

The student's response above reveals a similar source of error as the first, although the answer is worded slightly differently.

Question 2.5 asked students what the difference was between the price of a Category 1 and Category 4 ticket for the Final. The student's response given below, reveals that it was understood that Category 1 tickets were more expensive, but not that the Final is one particular match. They have also not specifically noticed the word 'difference', which, in the context of a Mathematical Literacy examination, indicates that subtraction is required.

Figure 4.14 Excerpt from student script (reading error)



2.5. The prices of the category 1 were more expensive than
category 4 for the final. because it was a different match

There were several different types of reading errors that emerged in analysing the students' responses:

Not reading the entire text – usually ignoring the last section of a lead-in text or item

Not noticing a particular word

Not comprehending a particular word

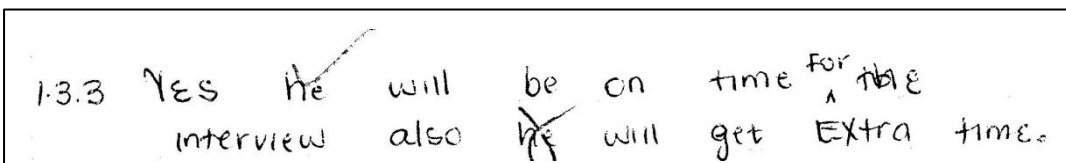
Not comprehending a particular phrase

Understanding the lead-in text, but not what the item required

Some errors pointed out the inability of students to express themselves in writing, although in their responses it was evident that this was not due to a lack of comprehension of the lead-in text or item.

Question 1.3.3 in Paper 2 asked students whether someone would be on time for an interview, given her time of departure and travelling time. The correct response involved saying that she would be in time for the interview. A second relevant piece of information provided by many students was that she would arrive with 5 minutes to spare. One student replied:

Figure 4.15 Excerpt from student script (writing error)



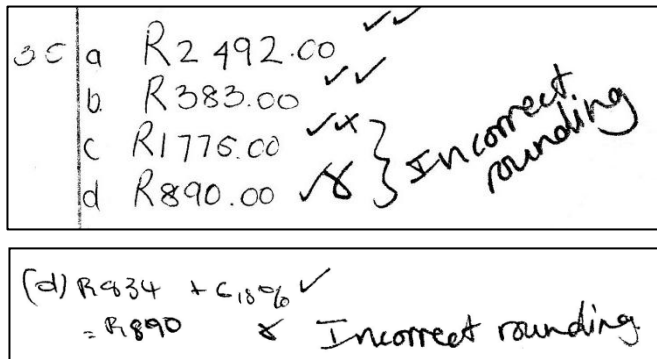
1.3.3 YES He will be on time ^{for} the
interview also ~~he~~ will get EXTRA time.

The marker did not award the student full marks as the person was not going to be *given* extra time, but would be *arriving early*. This was despite the fact that the student did seem to understand the information provided, but was unable to translate this understanding accurately into writing.

4.5.1.4 Carelessness

Errors were attributed to carelessness if the student had elsewhere demonstrated competence in the particular skill. The following two students, who rounded off incorrectly, had shown previously that they were able to round off accurately, but failed to do so in these particular answers.

Figure 4.16 Excerpt from student scripts (carelessness)



Most of the errors attributed to carelessness involved incorrect rounding off. Small errors at the end of longer calculations performed accurately until the last line, where students had elsewhere shown that they could perform the operation in the item, were also classified as due to carelessness.

4.5.1.5 Item characteristics

Some errors needed to be attributed to item characteristics where items introduced unnecessary ambiguity through their wording. In these cases, students answered correctly according to one possible meaning of the item, but their response was not marked as correct because they had not understood it as the examiner had intended.

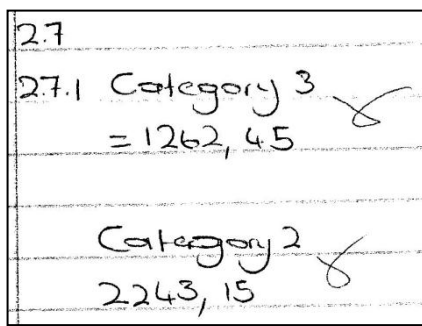
Question 6 in Paper 1 was particularly problematic in this regard. The graph provided information about number of persons driving under the influence for given time periods, in particular: 2001/2002; 2002/2003; 2003/2004; and 2004/2005. Confusion was evident in some students' responses when the item asked about 'year(s)' (Question 6.2.2) or 'year' (Question 6.2.5), as opposed to the other items which referred to periods. Several students, who were able to accurately respond to items where periods were referred to, were not able to answer 6.2.2 or 6.2.5, attempting to give a single year as an answer as opposed to a period.

These errors were categorised as due to a flaw in the item, rather than being the fault of the student.

4.5.1.6 Indeterminate

In some cases, as with the one below, it was not possible to determine how the student had arrived at their answer and, therefore, what the source of the error might be. The example given below is of one student's response to Question 2.7.1 of Paper 1. There was no apparent method that could be identified that would lead to the answers provided. Without being able to view this student's calculations, the error could not be assigned to any category.

Figure 4.17 Student error of indeterminate cause



4.5.2 Automatic and extended responses

It was noticed during the error analysis that several students revealed a larger degree of task involvement than others when responding to items. This was evident either in the way they showed more extended working out than others, as with the student's response in Figure 4.16, or in how they accompanied their responses with textual explanations, as evident in the student's response shown in Figure 4.17.

Figure 4.18 Extended working out shown in student's response

$$\begin{aligned}
 & 8.1 (2 \times 1) + 2 \times 6 \\
 & = (2 \times 90) + (2 \times 60) \\
 & 180 + 120 \\
 & 300 \text{ cm} \quad \checkmark \checkmark \checkmark
 \end{aligned}$$

Figure 4.19 Examples of student responses including textual explanations

$$\begin{aligned}
 & \frac{R3590}{100} \times 12.5\% = R448.75 \text{ discount} \\
 & \therefore R3590 - R448.75 \quad \checkmark \checkmark \checkmark \\
 & = R3141.25, \text{ amount to be payed.}
 \end{aligned}$$

Vusi travelled : $600 - 100 = 500 \text{ km}^{\text{h}}$ travelled, travelled 250 km at 8:30
 Thabo travelled : $780 - 120 = 660 \text{ km}^{\text{h}}$ distance travelled \times
 " the time of day he reached 450 km was at 10:30

Where students' work showed repeated attempts at any particular item, this was also considered evidence of task involvement. In order to reconsider their answers, these students needed to apply conscious effort. Examples are shown below:

Figure 4.20 Examples of student responses where items were reattempted

$R94780,100$ $R2270,70$ $R383,13$ $R1470,408$	$R2492,04$ $R333,13$ $R1635,00$ $R890,712$	$R94780,125$ $R24904$ $R383,13$ $R1666,392$	$G.1.1. R274,83$ $= 71,45$	$3 \checkmark$
--	---	--	-------------------------------	----------------

Another feature of student responses was the apparent automaticity of responses. This was particularly evident for items requiring addition or subtraction of percentages and those requiring the calculation of volume. Even weaker students, achieving between 30% and 39%, were able to complete certain items with very brief and accurate calculation. This was taken to suggest a degree of automaticity of these procedures for this sample. Several examples are shown below, all from students who achieved between 30% and 39% for the examination:

Figure 4.21 Student responses to items requiring addition or subtraction of percentages

$$7.3.2 \ 150\ 000 - 30\% \\ = 105\ 000$$

$$1.4 \ R\ 3590 - 12,5\% \\ = R\ 3141,25$$

$$1.5 \ R\ 499 + 14\% \\ = R\ 568,86$$

Figure 4.22 Student responses to items requiring calculation of volume

$$8.4. \ 91\text{cm} \times 61 \times 5 = 27755.$$

$$8.4 \ 91 \times 61 \times 5 \\ = 27755$$

One would expect this type of response to take very little time, and it is possible that the student had identified only key words in the item and used these as a cue to carry out procedures. The student may have understood the item and the reason for the particular calculation to be made, but no evidence of this is provided in these responses. This is supported by the observation that, while the items requiring the additional or subtraction of percentages were answered correctly, weaker students were attempting to apply this method in numerous other examples where the word percentage appeared.

Other item types which revealed a large number of apparently automatized responses included all those in which formulae had been provided. Every student who responded to these items showed exactly the same method, followed in exactly the same manner. These responses, however, were mostly inaccurate. Students appeared to substitute values in haste, without considering whether they were using the appropriate value. In addition, they did not apply the rules of ordering operations when performing the actual calculation.

4.5.3 Summary of data according to number of marks lost per error type per topic

The following section presents tables and graphs summarising the percentage of marks lost per error category for each topic. Graphic-related and mathematical calculation errors are grouped as mathematical literacy errors, and reading and writing errors are grouped as language errors. This is in accordance with the findings from the pilot study (section 3.5.2.4), which suggested that it be relevant to group graphic and mathematical calculation errors to form the category of mathematical literacy-related errors

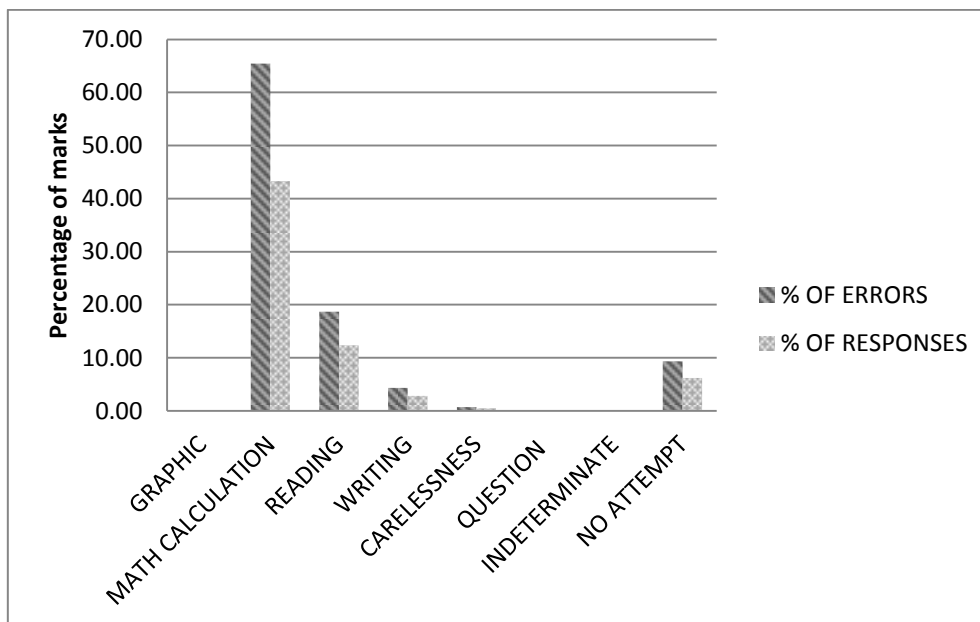
Percentages are given that reflect the marks lost if only errors are considered, as well as marks lost per error category if all responses, including correct responses, are considered.

4.5.3.1 Errors made for items based on Topic 1 – Numbers and Operations

Table 4.15 Percentage of marks lost per error category: Topic 1

Topic 1	% of errors	% of all responses		
Graphic-related	0.00	0.00	MATH LITERACY	
Math calculation	65.47	43.33	65.47	43.33
Reading	18.71	12.38	LANGUAGE	
Writing	4.32	2.86	23.03	15.24
Carelessness	0.72	0.48		
Question	0.00	0.00		
Indeterminate	0.00	0.00		
No attempt	9.35	6.19		
Total errors	100.00			
Partial marks		7.14		
Zero marks		59.05		
Full marks		33.81		
Total		100.00		

Figure 4.23 Graph showing percentage of marks lost per error category: Topic 1



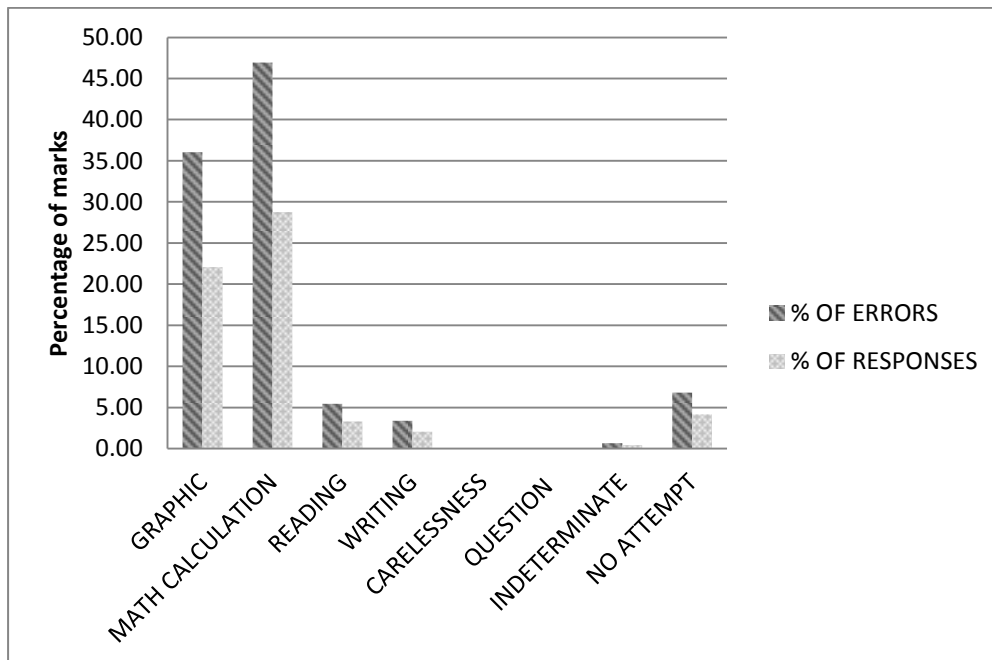
For Topic 1, the majority of all errors made, by a large margin, were due to math calculation errors. Reading and writing errors were also made, as well as a very small number of errors due to carelessness. No graphic items were included which explains the lack of graphic-related errors.

4.5.3.2 Errors made for items based on Topic 2 – Patterns and Relationships

Table 4.16 Percentage of marks lost per error category: Topic 2

Topic 2	% of errors	% of all responses	% of errors	% of all responses
Graphic-related	36.05	22.08	MATH LITERACY	
Math calculation	46.94	28.75	82.99	50.83
Reading	5.44	3.33	LANGUAGE	
Writing	3.40	2.08	8.84	5.41
Carelessness	0.00	0.00		
Question	0.00	0.00		
Indeterminate	0.68	0.42		
No attempt	6.80	4.17		
Total errors	100.00			
Partial marks		21.25		
Zero marks		40.00		
Full marks		38.75		
Total		100.00		

Figure 4.24 Graph showing percentage of marks lost per error category: Topic 2



For Topic 2, minimal reading and writing errors were made in comparison with the mathematical literacy-related errors attributable to graphics and math calculations. Topic 2 largely involves the interpretation of line graphs or tables of values showing relationships or

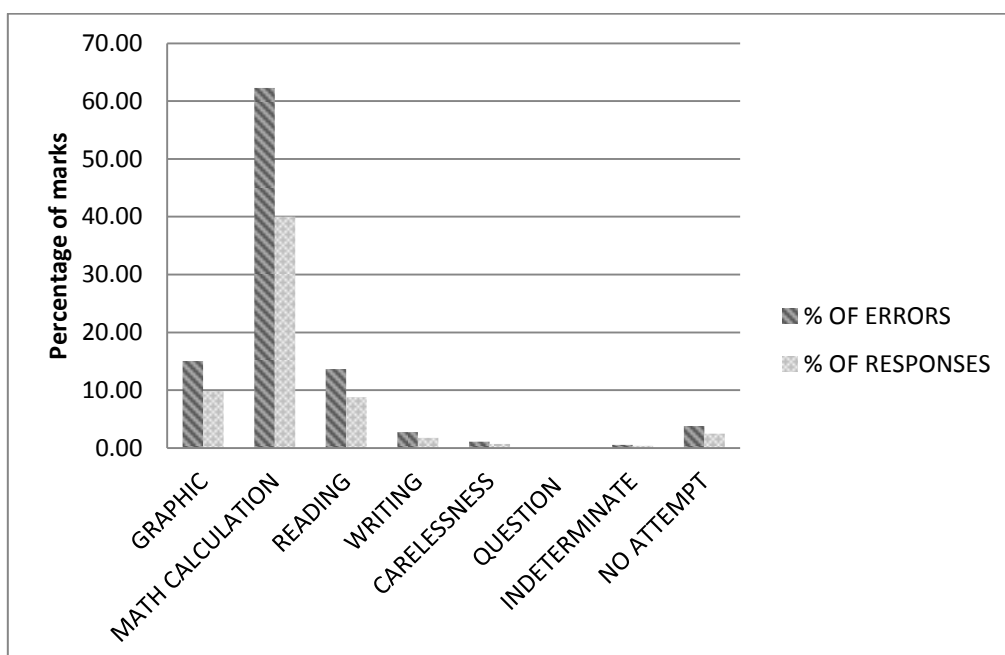
patterns, therefore minimising the possibility of reading or writing errors. Several students did not attempt certain items, but errors due to carelessness or question characteristics, for those who responded, were not made.

4.5.3.3 Errors made for items based on Topic 3 – Finance

Table 4.17 Percentage of marks lost per error category: Topic 3

Topic 3	% of errors	% of all responses		
Graphic-related	15.03	9.83	MATH LITERACY	
Math calculation	62.30	40.00	77.6	49.83
Reading	13.66	8.77	LANGUAGE	
Writing	2.73	1.75	16.39	10.52
Carelessness	1.09	0.70		
Question	0.00	0.00		
Indeterminate	0.55	0.35		
No attempt	3.83	2.46		
Total errors	100.00			
Partial marks		14.74		
Zero marks		49.47		
Full marks		35.79		
Total		100.00		

Figure 4.25 Graph showing percentage of marks lost per error category: Topic 3



Items based on Topic 3 required a substantial amount of reading in comparison with those based on Topics 1 and 2, and also required answers motivating certain opinions. This could explain the increase of reading and writing errors in this topic. Mathematical calculation, however, remains the reason for the majority of errors.

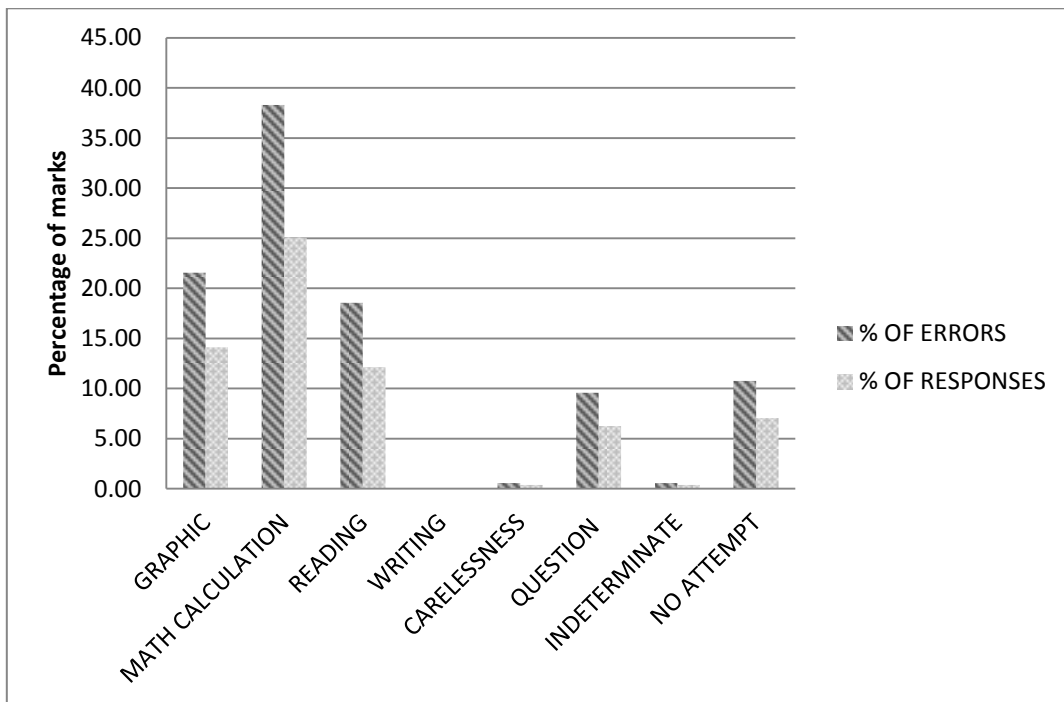
Errors due to carelessness did appear, and some students did not attempt some of the items. Items were posed in such a way that errors due to question characteristics were not made.

4.5.3.4 Errors made for items based on Topic 4 – Space, Shape and Orientation

Table 4.18 Percentage of marks lost per error category: Topic 4

Topic 4	% of errors	% of all responses	% of errors	% of all responses
Graphic-related	21.56	14.12	MATH LITERACY	
Math calculation	38.32	25.10	59.88	39.22
Reading	18.56	12.16	LANGUAGE	
Writing	0.00	0.00	18.56	12.16
Carelessness	0.60	0.39		
Question	9.58	6.27		
Indeterminate	0.60	0.39		
No attempt	10.78	7.06		
Total errors	100.00			
Partial marks		6.67		
Zero marks		58.82		
Full marks		34.51		
Total		100.00		

Figure 4.26 Graph showing percentage of marks lost per error category: Topic 4



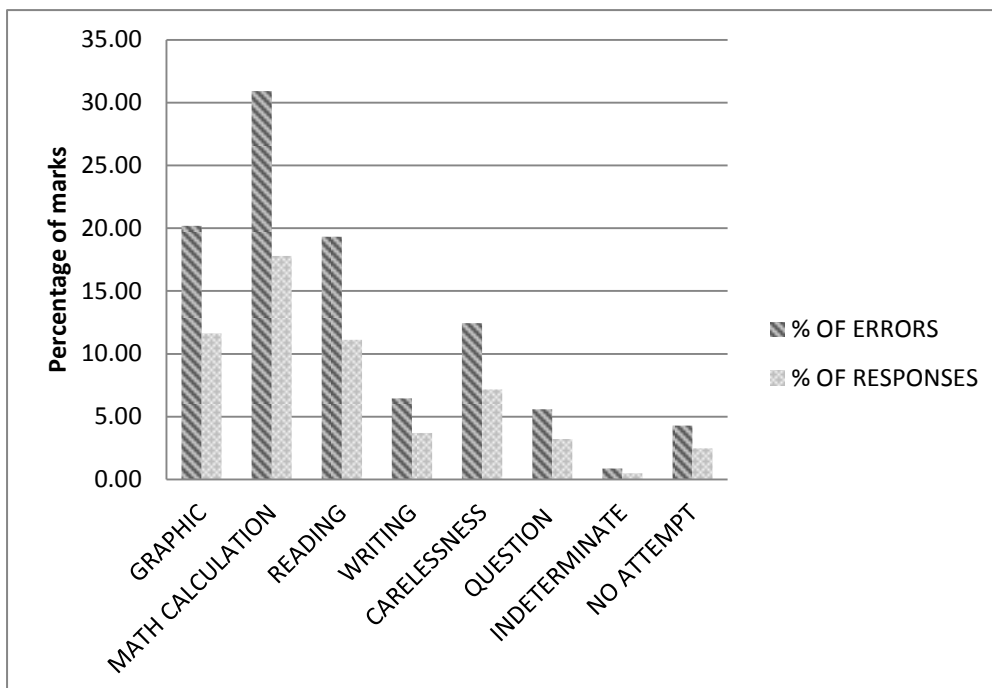
Items included in relation to Topic 4 led to errors mostly attributable to misinterpretation of graphics and errors due to mathematical calculation errors. Items based on this topic are heavily reliant on graphics, thus making the high percentage of reading errors surprising. No marks were lost due to writing errors, but carelessness did result in marks being lost.

4.5.3.5 Errors made for items based on Topic 5 – Information communicated through numbers, graphs and tables

Table 4.19 Percentage of marks lost per error category: Topic 5

Topic 5	% of errors	% of all responses	% of errors	% of all responses
Graphic-related	20.17	11.61	MATH LITERACY	
Math calculation	30.90	17.78	51.07	29.39
Reading	19.31	11.11	LANGUAGE	
Writing	6.44	3.70	25.75	14.81
Carelessness	12.45	7.16		
Question	5.58	3.21		
Indeterminate	0.86	0.49		
No attempt	4.29	2.47		
Total errors	100.00			
Partial marks		6.42		
Zero marks		51.11		
Full marks		42.47		
Total		100.00		

Figure 4.27 Graph showing percentage of marks lost per error category: Topic 5



One particular item, Question 6.2 of Paper 1, caused students to make errors as a result of ambiguity. The same students were able to complete unambiguous items in the same section. This resulted in Topic 5 being the only topic to feature errors attributable to question characteristics.

Large pieces of lead-in text were included to explain the graphs and tables used in Topic 5's items. This increased the opportunities for reading errors to be made. Writing was also required to answer certain items requiring motivation for preceding numerical answers, as well as features of the graphs, therefore similarly increasing the number of opportunities for writing errors to be made.

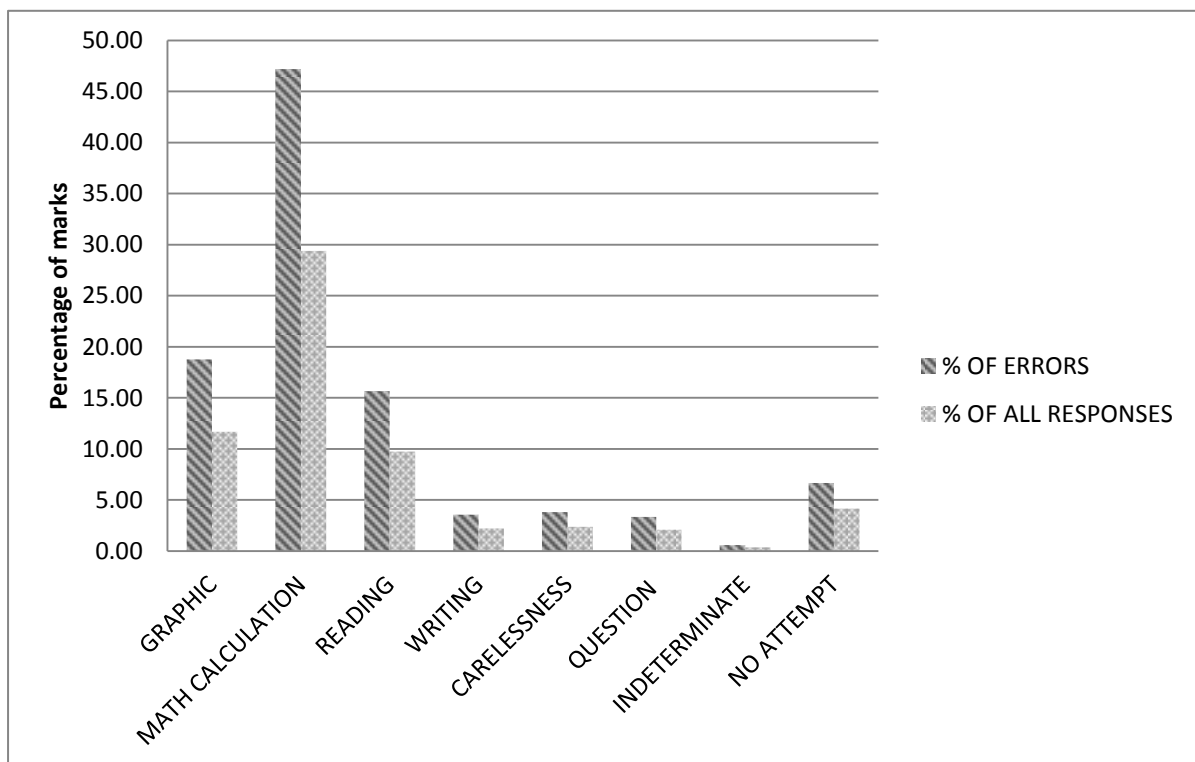
4.5.4 Summary of data according to number of marks lost per error type for whole examination

This section presents tables and bar graphs summarising the precise percentages of marks lost, for the sample of 15 students, for all examination items, per error category. Graphic-related and mathematical calculation errors are grouped as mathematical literacy errors, and reading and writing errors are grouped as language errors. Percentages are given that reflect the marks lost if only errors are considered, as well as marks lost per error category if all responses, including correct responses, are considered.

Table 4.20 Percentage of marks lost per error category: All topics

All topics	% of errors	% of all responses	% of errors	% of all responses
Graphic-related	18.75	11.68	MATH LITERACY	
Math calculation	47.18	29.39	65.93	41.07
Reading	15.65	9.75	LANGUAGE	
Writing	3.57	2.22	19.22	11.97
Carelessness	3.80	2.37		
Question	3.34	2.08		
Indeterminate	0.58	0.36		
No attempt	6.67	4.16		
Total errors	100.00			
Partial marks		10.82		
Zero marks		51.47		
Full marks		37.71		
Total		100.00		

Figure 4.28 Graph showing percentage of marks lost per error category



The majority of errors made were due to the mathematical-literacy related categories of graphic and calculation errors, as would be expected in a valid mathematical literacy examination. Reading, a language-related error, however, is only 3.1% (of all errors) lower than graphic-related errors. When considered together, reading and writing contribute 19.22% to the total errors made in the examination as opposed to the 65.93% contribution of mathematical-literacy related errors.

4.5.5 Complexity indices and error types analysed per topic

The two complexity indices can be compared according to their averages per topic and the percentage of marks lost per major error category, i.e. mathematical literacy-related errors and language-related errors. The following table summarises these values:

Table 4.21 Summary of average cognitive and linguistic complexity indices per topic

	TOPIC 1	TOPIC 2	TOPIC 3	TOPIC 4	TOPIC 5	WHOLE EXAM
Cognitive Complexity Index	8.00	8.28	13.00	8.71	5.29	8.12
Linguistic Complexity Index	18.51	22.31	17.19	19.11	23.76	20.63
Percentage of math literacy errors	65.47	82.99	77.60	59.88	51.07	65.93
Percentage of language errors	23.03	8.84	16.39	18.56	25.75	19.22

In interpreting this table it should be noted that the values for the Cognitive Complexity Index cannot be compared directly to the values for the Linguistic Complexity Index. These indices can only be internally interpreted according to their relative size.

Topic 5 represents both the highest linguistic complexity average and the lowest cognitive complexity average. As would be expected, it also represents the lowest percentage of math literacy errors, and the highest percentage of language-related errors. This pattern is, however, not consistent. In Topic 2, for example, the Linguistic Complexity Index is the

second highest, at 22.31, while the percentage of language-related error is the lowest by a large margin at 8.84.

From these results it would appear that when indices and errors are explored per topic, the nature of the items found in each topic, and not simply the relative cognitive and linguistic complexities, may be influencing the number of errors.

4.6 RESEARCH QUESTION 4: RELATING COGNITIVE COMPLEXITY, LINGUISTIC COMPLEXITY AND TYPES OF ERRORS

Research Question 4 asks how cognitive complexity and linguistic complexity are related, as well as how they each relate to the types of errors made by students. This section summarises the data gathered to address this question.

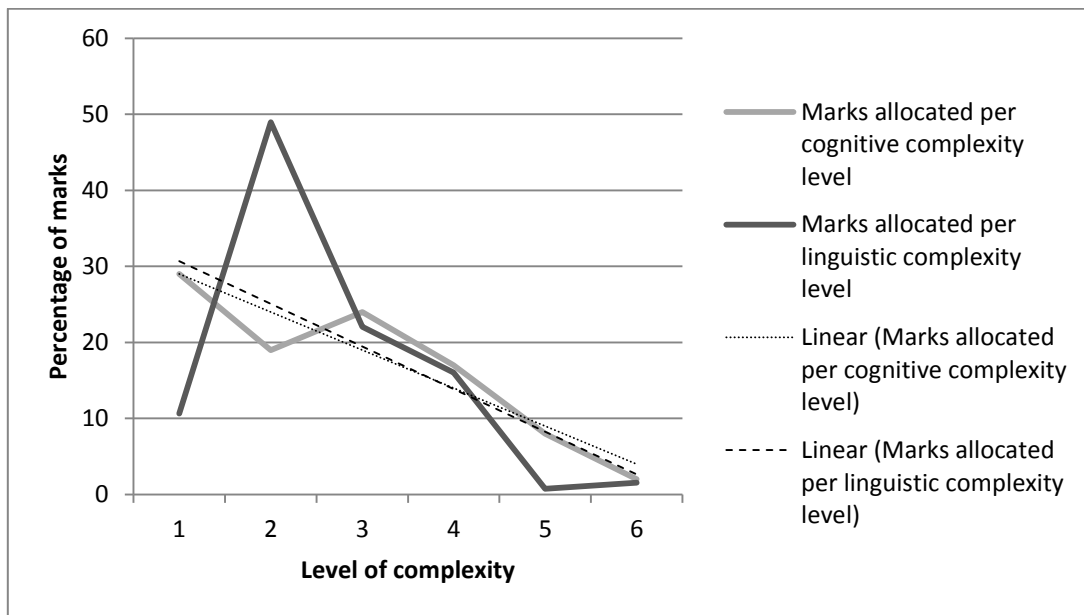
Pearson product-moment correlations were calculated to determine whether these variables are statistically significantly correlated and to what extent they are if this correlation exists.

4.6.1 Relating linguistic complexity and cognitive complexity

The linguistic complexity levels 1 to 6 can be graphically compared to the Linguistic Complexity Index if the index is divided into ranges corresponding to the same 6 levels as used in Figure 4.2. These levels and ranges are: 1 = 0 – 10; 2 = 10.1 – 20; 3 = 20.1 – 30; 4 = 30.1 – 40; 5 = 40.1 – 50; 6 = 50.1 – 60.

The graph below compares the percentage of marks allocated to each of the linguistic complexity and cognitive complexity levels:

Figure 4.29 Comparison between marks allocated to linguistic and cognitive complexity levels



In assessing whether linguistic complexity and cognitive complexity were statistically significantly correlated, outliers with respect to the Linguistic Complexity Index and the Cognitive Complexity Index were eliminated.

These indices were found not to be statistically significantly related ($r = 0.14$; $p = 0.179$).

4.6.2 Relating linguistic complexity and cognitive complexity with number of errors

When calculating the Pearson product-moment correlations for both the Linguistic Complexity Index and the Cognitive Complexity Index with the number of errors made, a statistically significant correlation was only found for the Linguistic Complexity Index when it was restricted to being <27 , reducing n to 73 from 103.

For the Cognitive Complexity Index a statistically significant, moderately strong correlation existed at the 1% level when the full range of values was used. This decreased to a statistically significant, moderate correlation at the 5% level when the Linguistic Complexity Index range was restricted to being <27 .

A summary is provided in Table 4.22:

Table 4.22 Pearson product-moment correlations for number of errors

Correlation with number of errors per item	n	r(x;y)	Strength	p	Statistical Significance (confidence level)
Cognitive complexity	103	0.35	Moderately strong	0.000	At the 1% level
<i>Linguistic index <27</i>	73	0.27	Moderate	0.019	At the 5% level
Linguistic index	103	-0.00	Weak	0.963	none
<i>Linguistic index <27</i>	73	0.25	Moderate	0.033	At the 5% level

4.6.3 Relating linguistic and cognitive complexity to types of errors

A Pearson product-moment correlation was calculated between the Linguistic Complexity Index and the Cognitive Complexity Index per item per type of error.

4.6.3.1 Mathematical literacy-related errors

Mathematical calculation errors

Mathematical calculation errors accounted for the majority of the mathematical literacy-related errors. For this reason it was decided to explore whether a statistically significant correlation may exist between it and the linguistic or cognitive complexity indices.

A statistically significant correlation was found between mathematical calculation errors and the Cognitive Complexity Index where number of mathematical literacy-related errors was restricted to being <14. No statistically significant correlation was found for the Linguistic Complexity Index.

Table 4.23 Pearson product-moment correlations for number of mathematical calculation errors

Correlation with number of mathematical calculation errors	n	r(x;y)	Strength	p	Statistical Significance (confidence level)
Cognitive complexity	100	0.06	Weak	0.547	none
<i>Math lit errors <14</i>	90	0.25	Moderate	0.017	At the 5% level
Linguistic index	98	-0.10	Weak	0.319	none
<i>Math lit errors <14</i>	87	-0.12	Weak	0.280	none

Mathematical literacy-related errors

For mathematical literacy-related errors as a whole, only the Cognitive Complexity Index was statistically significantly related. This, however, only existed if the range of the number of mathematical literacy-related errors was also restricted to being <14. This was a correlation of moderate strength at the 1% confidence level.

Table 4.24 Pearson product-moment correlations for number of mathematical literacy-related errors

Correlation with number of mathematical literacy errors	n	r(x;y)	Strength	p	Statistical Significance (confidence level)
Cognitive complexity	100	0.13	Relatively weak	0.183	none
<i>Math lit errors <14</i>	89	0.28	Moderate	0.008	At the 1% level
Linguistic index	97	0.08	Weak	0.421	none
<i>Math lit errors <14</i>	86	-0.05	Weak	0.664	none

4.6.3.2 Language-related errors

Reading errors

Reading errors accounted for the majority of language-related errors, thus it was decided to work out the Pearson product-moment correlations between reading errors and both complexity indices.

Both the Cognitive Complexity Index and the Linguistic Complexity Index were statistically significantly related to the number of reading errors, differing only in their relative strength and confidence level, as summarised in the table below. It was not necessary to restrict the range of any of the variables to find statistical significance for correlation between these variables.

Table 4.25 Pearson product-moment correlations for number of reading errors

Correlation with number of reading errors	n	r(x;y)	Strength	p	Statistical Significance (confidence level)
Cognitive complexity	100	0.23	Moderate	0.021	At the 5% level
Linguistic index	97	0.35	Moderately strong	0.001	At the 1% level

Language-related errors

When all language-related errors were used in the calculation of the correlations, both the Cognitive Complexity Index and the Linguistic Complexity Index were found to be statistically significantly related to the number of errors, both at the 1% level.

Table 4.26 below summarises these results:

Table 4.26 Pearson product-moment correlations for number of language-related errors

Correlation with number of language-related errors	n	r(x;y)	Strength	p	Statistical Significance (confidence level)
Cognitive complexity	100	0.28	Moderate	0.004	At the 1% level
Linguistic index	98	0.30	Moderately strong	0.002	At the 1% level

4.7 SUMMARY

Student results were discussed before the data specifically addressing the research questions were presented. This was due to their relevance for discussion when interpreting the data. The pass rate for Paper 1 was far higher than that of Paper 2, with the trial results averaging out at a pass rate of 80%. The average percentage, however, was only 40%, marginally above the elementary level of achievement.

The students' English results were also presented. The average percentage for the students' final ICASS result was 40%, identical to the pass mark. Only 51% of the class achieved a result above this mark.

Data describing the linguistic complexity of the examination, according to language features, readability and required language proficiency were presented, followed by a description of the cognitive complexity of the examination.

The data gathered in an attempt to describe the linguistic complexity of the examination items and the examination as a whole revealed that the following language features were particularly prominent:

1. Words with 7 letters or more (109/47%)
2. Prepositional phrases (105/28%)

3. Pronouns (72/6%)
4. Errors (49/9%)
5. Abbreviations (48/3%)

Complex/compound sentences were added to the further statistical analysis due to its relatively large contribution to the linguistic indices. Pearson product-moment correlations were calculated between each of the top six language features and the linguistic indices. When the range was restricted by eliminating the lower values of the language features, the following language features were shown to be statistically significantly related to the Linguistic Complexity Index: prepositional phrases; words with seven letters or more; and complex/compound sentences

Although readability values varied widely, ranging from an equivalent US Grade Level of 5.00 to 12.66, the readability could be described as of a standard level when averaged over the entire examination.

When the level of language proficiency required to comprehend the examination items was assessed, it emerged that CEFR levels A1 and A1+ were most prominent. These are both categorised as the “Breakthrough or Beginner” (Lindhout, Teunissen & Lindhout, 2012, p. 37) level of proficiency.

Cognitive complexity was described according to 6 levels allocated based on the cognitive domain assessed as well as the level of difficulty of the item. The spread of marks allocated per level was weighted heavily at levels 1, 2 and 3, with 4 to 6 represented by less than 40% of the marks. Thereafter, an index was calculated by multiplying the cognitive complexity level by the number of marks allocated to the item. This index ranged in value from 1 to 40, with a mean value of 8.21.

Examples of student errors were provided and summarised by grouping them into error categories. A comparison between percentages of marks lost per error category was provided for both the examination as a whole as well as for each topic. Finally, graphic-related errors and calculation errors were combined as mathematical literacy-related errors, and reading and writing errors were grouped to become language-related errors. The percentage of errors

attributable to each of these error types for the whole examination was 65.93% due to mathematical literacy errors and 19.22% due to language-related errors.

It also emerged that student responses could be analysed in terms of the apparent level of task involvement of the student for a particular item. Some responses revealed that the student had applied much effort in arriving at an answer, whether correct or incorrect, certain responses seemed to point to an automatic response. Relevant examples from student scripts were qualitatively discussed in this regard.

Finally, statistical relationships were sought between certain variables by calculating Pearson product-moment correlations. The Linguistic Complexity Index was found to not correlate statistically significantly with the Cognitive Complexity Index of the examination items. Statistically significant correlations included:

- cognitive complexity to number of errors; mathematical calculation errors; mathematical literacy-related errors; reading errors; and language-related errors
- linguistic complexity to number of errors; reading errors; and language-related errors

The data presented and analysed in this chapter describe the case being explored in this research. The examination is described in terms of its linguistic and cognitive complexity, the student scripts are described in terms of the responses and errors students have made, and the data are described with regard to how features of the examination relate to features of the errors students have made. In Chapter 5 this data will be further discussed in light of conclusions that can be made based on their evidence.

CHAPTER 5

DISCUSSION AND CONCLUSION

5.1 INTRODUCTION

Due to the descriptive nature of this case study, the answer to each research question lies in the detailed data presented and analysed in Chapter 4. This chapter will attempt to explain why the data says what it does, with reference to prior research and theories as presented in Chapter 2.

Research Question 1 is concluded after a revisiting of the findings with regard to linguistic complexity. Thereafter, the findings based on the Cognitive Complexity Levels and Indices, as well as the qualitative observations in this regard, are discussed, and a conclusion offered for Research Question 2. Error types are then discussed in order to arrive at a conclusion for Research Question 3. This conclusion includes a visual summary of the findings based on the adapted PISA Governing Board (2010) problem solving model, as developed throughout the thesis. Lastly, the statistical relationships between linguistic complexity, cognitive complexity and student error types are discussed in order to conclude Research Question 4.

The findings of this study can be used to arrive at several practical suggestions for Mathematical Literacy lecturers, Mathematical Literacy assessors, as well as English First Additional Language lecturers. These will be provided after the research questions have been addressed.

Finally, the limitations and shortcomings of the study will be presented, as well as what the study is able to contribute, including suggestions for several further avenues of research.

5.2 DISCUSSION OF FINDINGS

The findings per Research Question will be discussed and summarised in this section, and a conclusion offered for each.

5.2.1 Description of linguistic complexity

The following data were gathered in order to describe the overall linguistic complexity of the examination analysed for this study: language features as they contributed to a Linguistic Complexity Index; the required level of language proficiency and readability. The data is discussed in the sections below, and a conclusion drawn as to how the examination can be described with regard to its linguistic complexity.

5.2.1.1 Linguistic Complexity Index

After calculating the Linguistic Complexity Index for each examination item and lead-in text, it was noticed that a number of language features occurred with a much larger frequency than others. These language features were (in decreasing order of frequency): words of seven letters or more; prepositional phrases; pronouns; grammatical or spelling errors; abbreviations and complex/compound sentences. All of these features were listed by Shaftel et al. (2006) as features that contribute complexity to examination items.

Words with seven letters or more were the most frequently encountered language feature. They are considered to be a major source of linguistic complexity according to Bergqvist, Dyrvold and Österholm (2012). In this particular examination, the majority of these words were those which appeared in the mathematical literacy curriculum and as such were part of the vocabulary with which students should have been familiar. Mathematical vocabulary, specifically, was explored by Shaftel et al. (2006) and found to statistically significantly negatively correlate with the mean results of Grade 4, 7 and 10 students (see Table 2.3). This was the only language feature, of those explored by Shaftel et al. (2006), which correlated across all three grades that they had assessed.

Halliday (1989) argues that the difficulty of mathematical texts lies can, to a larger extent, be attributed to grammar rather than specific vocabulary. There are specific phrases, similar to the special vocabulary, which are characteristic of mathematical texts. Such an example from the examination paper was the sentence “R6000 is invested at 5% compound interest per year” (Paper 1, Question 1.9). This is a typical phrase which students of Mathematical Literacy would encounter in an examination paper. Halliday (1989), points out that such phrases are linguistically complex, thus English language learners may struggle to accurately comprehend it in an examination.

Certain language features were identified as being particularly important in indicating relationships between variables and communicating key information relevant to solving the problem posed. Of the language features contributing the most to the Linguistic Complexity Index for this examination, these were: words of 7 letters or more; prepositional phrases; and conditional/comparative constructions.

At the word level, many of the words of 7 letters or more were specific mathematical vocabulary. This vocabulary was part of what students should have been familiar with and the majority of these words could not have been replaced by any shorter term. Some examples of such terms used in this examination were: instalment, equation, formula, discount, perimeter, and expenses.

Prepositional phrases are necessary when describing how nouns relate to one another. The following list provides examples, from the examination, of texts where a prepositional phrase was essential in communicating information required to solve the problem:

- “...per month” (Paper 1, Question 4)
- “...in cell E4” (Paper 1, Question 5)
- “...on the graph” (Paper 1, Question 9)
- “...on Friday at 10:00” (Paper 2, Question 1.3)
- “...in kg” (Paper 2, Question 4)

Comparative constructions were also essential, where used, in communicating key information. For example, Question 6, in Paper 1, relied on such constructions to convey what information needed to be read from the graph, and what calculation had to be done. This item reads: “Which province had the greater increase in the number of cases reported over the period 2001 - 2005?” The increase would need to be calculated for both provinces, but the word “greater” indicates specifically what information should be provided as the answer.

Prepositions held a statistically significant negative correlation with the item means of Grade 4 students, in Shaftel et al.’s (2006) research, although not for the Grade 7 or 10 students they assessed. Comparative constructions only held such a correlation for the Grade 7 students assessed. In this study, it was prepositional phrases; words with 7 letters or more; and

complex/compound sentences for which a statistically significant correlation was found in relation to the value of the Linguistic Complexity Index.

Many grammatical and spelling errors were found in this examination, and these contributed the most to the Linguistic Complexity Index in 9% of the examination items (see Table 4.4). This is a worrying aspect of this examination, particularly due to the fact that the majority of students writing this examination do not have English as their home language. Cummins and Swain (1986) write that, while the reading of text in an examination may be relatively cognitively undemanding to a native English speaker, this is more demanding for an English language learner.

Where many errors are present in a text, it is to be expected that the text will become more demanding for a native speaker of English to accurately comprehend the text, and, therefore that much more demanding for an English language learner. It is possible that this feature of the examination may have been the source of the reading errors rather than the magnitude of the entire Linguistic Complexity Index.

A relevant example would be Question 2.7 of Paper 1. This example is discussed in section 4.3.1.4. Many of the errors are related to the erroneous use of prepositions in the items. For 2.7.1 and 2.7.2, it is the number of errors and not prepositional phrases that contribute the most to the Linguistic Complexity Index if these two items.

There were sentences that contained a particularly high number of certain language features. Statistical significance was only found when some of these higher frequencies were excluded and the range restricted to what would possibly be more representative of examinations that have been better written. For example, no statistically significant correlation was found with prepositional phrases and the Linguistic Complexity Index until the examples included were restricted to those less than 5 phrases ($n = 90$), and the confidence level improved from 95% to 99% when the range was further restricted to those items containing less than 4 prepositional phrases.

Despite Halliday's (1989) assertion that grammatical features of mathematical texts contribute more to the linguistic difficulty level, it would appear from Shaftel et al.'s (2006) research, as well as this study, that technical vocabulary does play a significant role in

contributing to the linguistic complexity. It is also clear that certain complex features cannot be eliminated from a mathematical text, if it is to retain its meaning.

5.2.1.2 Readability

Each examination scenario, with its associated items, stood as a separate entity in the examination, therefore each was examined independently for readability. Readability varied widely, at its lowest revealing a comparable US Grade level of 5.00, and at its highest a US Grade level of 12.66. The Flesch Reading Ease score (Allan et al., 2005) ranged between 52.42 (Fairly difficult) and 83.64 (Easy).

In order to enter NC(V) Level 2 a student must have passed Grade 9. These students may not have studied English as a home language but rather as an additional language, which would imply that their reading proficiency could not necessarily be classified as equivalent to Grade 9. The average readability level for the examination is at a US Grade level of 8, but 7 of the 17 scenarios have an average Grade level above 9. These scenarios represent 41% of the total marks for this examination.

5.2.1.3 Required language proficiency

The language features of the examination were also assessed according to the level of language proficiency. The two proficiency levels appearing the most in the examination were, by a large margin, levels A1 and A1+. These levels are described by the Council of Europe (2001) as the Breakthrough or Beginner stage. This indicates, according to Table 2.5, that students would not require more than a Grade 7 or 8 level of reading proficiency. According to the Dutch schooling system, this is at a primary school level (Lindhout et al., 2012).

The grade levels associated by Lindhout et al. (2012) with the CEFR proficiency levels (Table 2.6) specifically indicate the expected proficiency of English First Additional Language students at each grade (Lindhout et al., 2012). This comparison is therefore more relevant to the case described in this study, where all students in the sample do not have English as their home language. If the examination requires Beginner level proficiency (Council of Europe, 2001), the associated grade level is 7-8 (Lindhout et al. , 2012), and the prerequisite for entry into NC(V) is Grade 9, the required reading proficiency levels should have already been achieved by NC(V) Level 2 students.

The level which should have been attained by NC(V) Level 4 students, according to the relevant descriptors of the Dutch schooling system (Table 2.6), would be B2 Upper Intermediate (Technical College, Grades 11 and 12) (Lindhout et al., 2012) due to their three years of studying English First Additional Language. It would be expected, therefore, that the level of proficiency required in this examination was less than the level already achieved by these students. The only level which could be considered higher than what they had already achieved would be C1, which only appeared in 2% of the examination, according to Table 4.11.

Although Figure 4.4 shows that A1 to A1+ is the most represented language proficiency level in the examination, it is necessary to take note of the peak at B2. As mentioned in section 4.3.3, this reflects the presence of the language function ‘giving precise information’ in every scenario in the examination. For this reason, it is level B2 that needs to have been achieved by students in order to adequately comprehend the texts in the examination. Included in the description of the skills of such a student (APPENDIX B), is that the student should be able to provide: “explanations, arguments and comments,...[give] reasons in support of or against a particular point of view,...[evaluate] alternative proposals and [make] and [respond] to hypotheses” (North et al., 2010, p. 23). These are far more sophisticated than the simple interactions listed as characteristic of A1 beginner proficiency.

It could be reasonably expected that students in NC(V) Level 4 would be capable of comprehending the majority of the language used in the examination as their proficiency should be at or above B2. On analysing their English First Additional Language ICASS results, it is possible that the pass rate of 51% may indicate otherwise. More than half of the students did not achieve above the Elementary level, according to the level descriptors (see Table 4.2), possibly placing them at an A2 level of proficiency according to the CEFR (Council of Europe, 2001).

5.2.1.4 Concluding Question 1

While each aspect of linguistic complexity, i.e. the Linguistic Complexity Index; readability; and required language proficiency, revealed examples at the higher end of each of their continua, the heaviest weighting for each lay at a relatively simple level. For the Linguistic Complexity Index, this was in the range of 10.1 to 20; for readability this was at the level

‘relatively easy’; and the required level of language proficiency the heaviest weighting was at the Beginner level of A1 and A1+.

For these reasons, the examination can be considered, on average, to be appropriately pitched with regard to the language requirements. The total percentage of marks lost due to language-related errors, as summarised Table 4.20 however, suggests that it was not. The examples located higher on the relevant continua, rather than the average, must therefore be assessed as to whether they were related to the number of errors rather than the average. If only the average is considered, it would seem that it is rather the level of competence of the students that is the probable factor related to the number of these errors.

5.2.2 Description of cognitive complexity

The following data were gathered in order to describe the overall cognitive complexity of the examination analysed for this study: a Cognitive Complexity Level and Index; graphical languages used in the examination; and the level of cognitive involvement evident in students’ responses. The data is discussed in the sections below, and a conclusion drawn as to how the examination can be described with regard to its cognitive complexity.

5.2.2.1 Cognitive Complexity Levels and Indices

The way in which the taxonomy of the Mathematical cognitive domains is described in the *National Certificates (Vocational) Assessment Guidelines: Mathematical Literacy NQF Level 4* (DoE, 2007b), seems to reflect, in the lack of inclusion of difficulty levels, that cognitive domains have been equated with difficulty. This is evident in the number of marks allocated to each cognitive domain and their relative difficulty. If Table 4.12 is examined, items assessing ‘knowing facts and procedures’ are mostly of a low difficulty; those assessing the ‘use of concepts’ are mostly of medium difficulty; those requiring the ‘solving of routine problems are equally weighted at medium and high difficulty; and those requiring ‘reasoning’ are mostly at a high level of difficulty.

It is possible for easy and difficult items to be designed for each cognitive domain, although the temptation is to assume that difficulty levels automatically vary according to the cognitive domain being assessed (Wu, 2010; Berger, Bowie & Nyaumwe, 2010). This examination did not contain an appropriate spread of difficulty per domain, and therefore contains an

imbalance in this regard. This is despite the fact that the prescribed weighting according to the *National Certificates (Vocational) Assessment Guidelines Mathematical Literacy NQF Level 4* (DoE, 2007b) was adequately adhered to.

A topic by topic analysis revealed another imbalance. If Table 4.14 is examined, it is clear that topics differed with regard to the level of cognitive complexity at which they were weighted most heavily. Topics 1, 2 and 5 were weighted most heavily at the least demanding level (Level 1), while Topic 3 was weighted most heavily in as high a level as Level 4. For a student most competent and confident in Topic 3, for example, this represents a source of bias, as these items were pitched at a more demanding level. Because the cognitive domain assessed was closely related to the difficulty for this examination, it is reasonable to conclude that the spread of marks allocated to each cognitive domain, when examined per topic, was not appropriately balanced when compared to the guidelines provided in the Assessment Guidelines (DoE, 2007b).

A Cognitive Complexity Index was also calculated, which further differentiated between items according to the length of response required. This provided indices varying from 1 to 66. The value of 66 was considered an outlier and therefore eliminated from further analysis as it was more than 8 standard deviations above the mean.

5.2.2.2 Graphic contributions to cognitive complexity

Graphical sources of information were qualitatively described according to MacKinley's (1986) six graphical languages, outlined in section 2.6.3. These graphics provided information essential to the solution of the associated items, but, as Hammill (2010) points out, it is complex and difficult to interpret these graphics and relate them to the accompanying text and items. Kress (2000) explains that part of the complexity of this task lies in the fact that "image is spatial and nonsequential, [while] writing... [is] temporal and sequential" (Kress, 2000, p. 339). One can assume, therefore, that it would require the involvement of more working memory to solve these problems, thus increasing the cognitive complexity.

5.2.2.3 Evidence of cognitive involvement of students

An observation was made when examining the students' scripts that certain types of items appeared to have been answered with a measure of automaticity (see section 4.5.2), while

others required more controlled processing and mental effort. This was evident in both weaker and stronger students' responses. In particular, these were items requiring the calculation of volume of rectangular prisms, and those involving the addition and subtraction of percentages.

An automatic response would be one in which minimal mental effort and attention had been applied. Saville-Troike (2010) explains that in second language acquisition, as lower order skills become automatised the student is able to use the resulting mental space to engage in higher order cognitive tasks, such as problem solving.

In the case of this examination, the students whose scripts were analysed appeared to apply automatised methods indiscriminately. Adding a percentage to a value was one such method. As well as applying this to appropriate questions, students were carrying out the same calculation for items with a different problem structure, but the same key words. It is possible that students had merely identified these key words which gave them the cue to use the associated method. This could have been done without comprehension of the actual problem. The automatised skill may have allowed these students to avoid engaging in actual problem solving, a higher order cognitive skill.

Apparently automatised responses were evident even in those items predicted to be more cognitively challenging by their Cognitive Complexity Index. For this sample of students, therefore, the index value of these items is misleading, as many were able to answer accurately without engaging in any more sophisticated problem solving than reproducing a learned method.

Several students showed more extensive working out for some examples, showing more steps than were required, and including textual explanations alongside this working out. It was not always the stronger students who were doing this. Some students' responses were given with much extra detail included, yet they achieved zero for their answer. Another type of answer that was considered as evidence of a student experiencing the item as more cognitively challenging, were those where a student had attempted an item more than once.

These students displayed more cognitive involvement in their responses than others with more abbreviated working out. These answers were, however, not consistently evident for

any particular item, therefore it was not possible to conclude that the cognitive complexity values assigned to those particular items were too low for the group as a whole.

5.2.2.4 Concluding Question 2

Cognitive complexity was defined for this study as a function of: the cognitive domain assessed; the level of difficulty of the item and the number of marks allocated to the item (Table 3.10). This examination can be described as inappropriately balanced with regard to a topic by topic analysis (5.2.2.2) and an analysis per cognitive domain (5.2.2.1). The high number of graphical sources of information increased the cognitive complexity of the examination as a whole as compared to the Cognitive Complexity Levels and indices assigned to each example. In addition, certain items seem to have been experienced as less cognitively challenging by students providing seemingly automatised responses and more cognitively challenging by students who felt the need to provide more extensive responses or those who needed to attempt the item more than once before being satisfied with their answer.

Very few items resulted in accurate automatised responses. For this reason, the overall description of the cognitive complexity of the examination would be that the examination was more complex than both the breakdown of Cognitive Complexity Levels and the Cognitive Complexity Index, which included consideration of the number of marks allocated to each item. The high number of graphic sources of information, as well as the numerous examples of more extensive cognitive involvement of students supports this description.

5.2.3 Description of student errors

In this section, error types are discussed as they were categorised in section 4.5. A final summary is provided according to where in the problem solving process these error types occur, and a graphic summary provided of what the overall division between mathematical literacy-related errors and language-related errors looks like.

5.2.3.1 Discussion of error types

Errors were broadly classified as due to: mathematical calculation inaccuracies; an inability to comprehend or interpret graphics; an inability to decode or comprehend text or an inability to accurately encode mathematical answers in writing. Additional categories were:

carelessness; question characteristics; errors whose source could not be determined and items not attempted. Graphic-related and mathematical calculation inaccuracies were considered to be mathematical literacy-related, and the categories of reading and writing were grouped as language-related errors.

Apposed-position languages (MacKinley, 1986) were the most frequently used graphical languages in the examination, and all such instances resulted in numerous graphic-related errors (see section 4.4.4). Most of these errors were as a result of students' inability to use the axes to read values off the tables and graphs provided. The interpretation of this type of graphic is an aspect of Topics 2 and 5, yet they were used as a source of information in many items related to other topics.

Another type of graphical language used was that of map languages (MacKinley, 1986). As summarised in section 4.4.4, these were only used in items related to Topic 4. Again, it was the items requiring information to be found on the diagram and interpreted for which the most errors were counted. For items related to the graphic but not requiring the reading off of information, many more students achieved full marks.

One item, Question 4 of Paper 2, included the use of retinal-list languages (MacKinley, 1986), but this graphic only provided extra contextual information and its use was not required in any item (see section 4.4.4).

Mathematical literacy-related factors were the source of marks lost in 65.93% of the errors made. Of this percentage, 18.75% of the marks lost were due to graphic-related errors. This represents 28% of marks lost due to mathematical literacy-related sources. The number of graphic-related errors was high for every item involving the use of a graphical language.

In a similar manner, every item which involved the reading of relatively longer pieces of text resulted in numerous reading errors. The percentage of marks lost due to reading errors varied widely between topics. For Topic 1 items, reading was the source of 18.71% of the marks lost. For Topic 2 this was 5.44%; Topic 3, 13.66%; Topic 4, 18.56%; and for Topic 5, 19.31%. This was, most likely, a reflection of the amount of text used in the items for each topic. In this examination, Topic 1 items relied on word sums to describe what calculation would be required. In contrast, Topic 2 relied largely on graphic sources of information, and

items were briefer than in the other 4 topics. Although the Mathematical Literacy Subject Guidelines (DoE, 2007a) list Topic 4's subject outcomes as involving calculations using visual sources of information, in this examination these items involved the reading of large pieces of text describing context. As a result, the percentage of marks lost to language-related errors, for Topic 4, was the second highest, at 18.56%.

Writing was only indicated as the source of error where the student demonstrated an understanding of the source information and the item, but had lost marks due to an inability to convey this answer accurately in writing. These represented a small number of the marks lost due to language errors at 23% of the marks lost to language errors in the whole examination, and 3.57% of all marks lost in the examination. This is most likely a result of the low number of such items, but, similar to graphic-related and reading errors, for every example requiring written responses, a number of such errors were found.

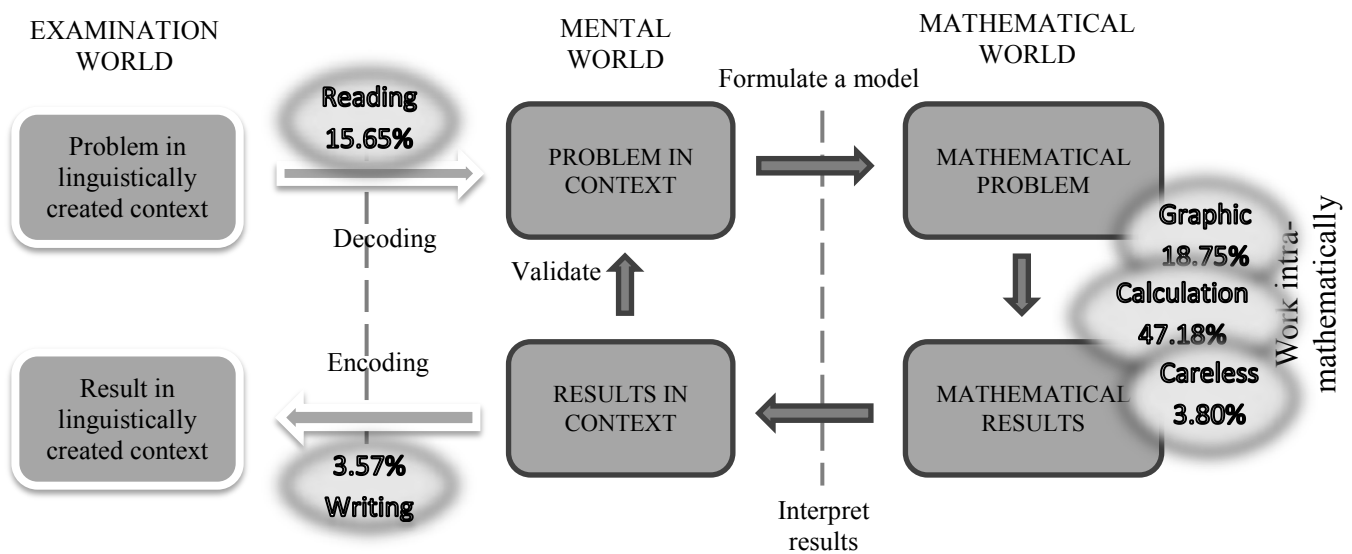
As pointed out in section 2.6.4, Barton and Neville-Barton (2003) have estimated the variation in academic performance due to English language ability as being up to 10%. In this particular case, this was potentially as high as 19.22%, according to the percentage of marks lost due to language-related errors, over the whole sample.

In section 4.5.2 it was noted that apparently automatised answers were found for certain types of items. Similarly, some students provided more extended responses than most of the other students. Both types of responses – those apparently automatised and those showing more extensive cognitive involvement – resulted in both answers scoring full marks and those achieving zero. Automatised, as well as increased cognitive involvement, was therefore not an indicator of a student's competence in that type of item.

5.2.3.2 Concluding Question 3

Figure 2.5 provides a visual summary of where in the problem solving process each type of error would occur. If the percentages of marks lost in this examination are summarised by inserting them into this figure, the following picture results:

Figure 5.1 Percentages of error types as summarised according to the problem-solving process



Adaptation of PISA Governing Board (2010, p.6)

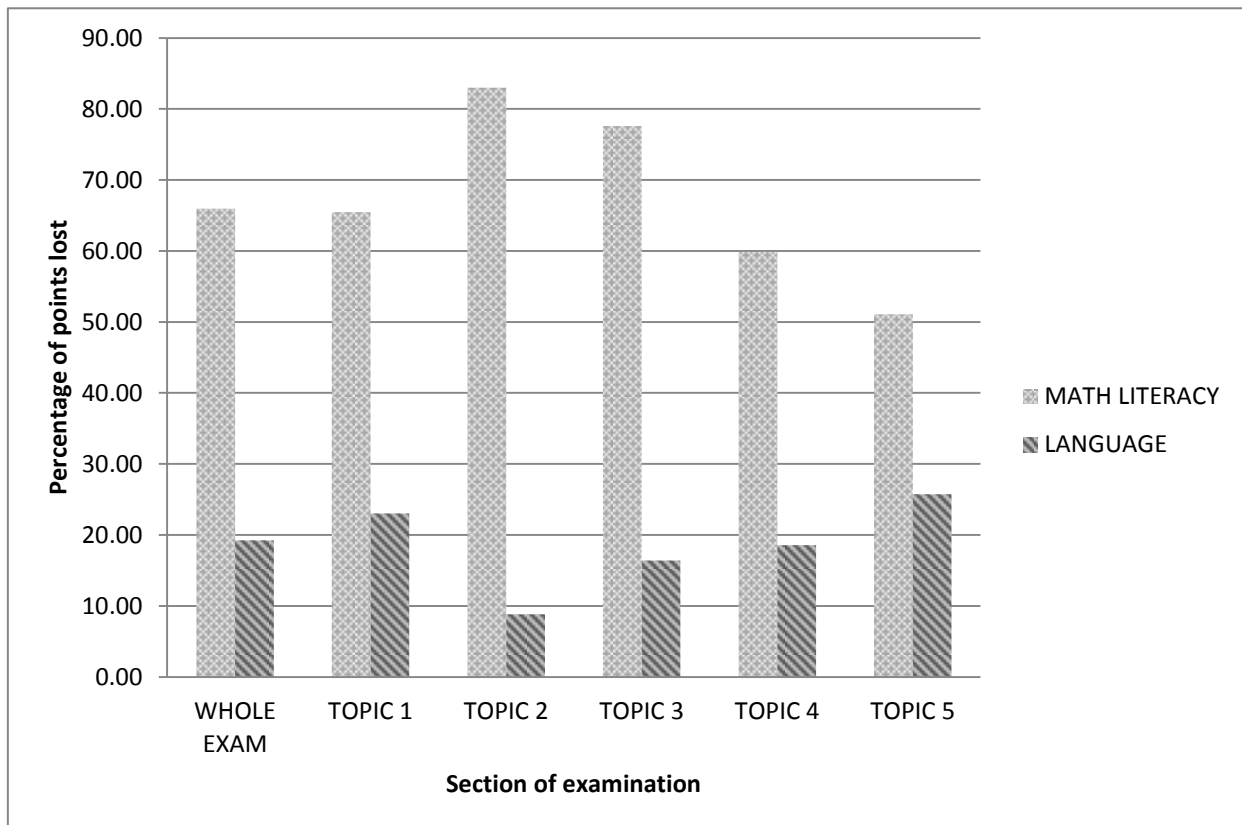
It is clear from this summary of error types as they are related to the problem solving process that the majority of the marks lost were during intra-mathematical work. Such errors were made after the students had correctly decoded the text, and accurately understood the problem in the context provided, sufficient to formulate a mathematical problem (where required). They did not, however, carry out the calculation accurately; made small, careless errors or did not correctly use the graphic information required.

Another major source of error, for these students, was in translating information between the mental world, in which they needed to work with the problem, and the symbolic, textual and graphic world of the examination. The majority of marks lost in this instance were due to inaccurate reading, or decoding, of the text. As a result, the problem was not understood correctly in the context of the scenario being presented in the text.

In Mathematical Literacy, an understanding of the context in which the problem exists is essential if the student is to correctly work intra-mathematically from their mental world. In examinations this is conveyed primarily through the use of text. The other source of language-related error, as positioned in the problem-solving process, lay in students incorrectly encoding their mathematical results from the mental world into a written answer for use in the examination world.

A summary is provided below of the percentage of errors made due to the language-related factors of reading and writing together, and the mathematical-literacy related factors of graphics and calculations together. This is shown per topic as well as for the whole examination.

Figure 5.2 Graph showing comparison between percentages of points lost for mathematical literacy versus language errors for all errors



5.2.3.3 Is mathematical literacy evident in student responses?

In examining student responses to the examination items, it was possible to observe the degree of mathematical literacy the students were displaying. It is not necessarily the fact that the student achieving the highest mark in the sample is the most mathematically literate.

Certain student responses included seemingly automatised answers, as discussed above. It is possible that in such cases students had identified key words and merely carried out a procedure rote-learned as appropriate for that specific type of item. It would be possible to extract the relevant values from the word problem on recognising the cue for the procedure, without engaging with the actual problem at all.

Further evidence that this may have been the approach for some students, was that when confronted with later problems including the same cue words, but requiring a different method of solving, the same automatic response was, in these cases, erroneously applied. According to the PISA 2012 definition a mathematically literate individual should be able to “recognise, do and use mathematics” (PISA Governing Board, 2010, p. 5). When these students were able to apply the automatised procedure correctly they could have been considered able to recognise relevant information, do the procedure, and in so doing use mathematics to arrive at a solution to an everyday type of problem.

There is another section, however, to this definition. In order to be considered mathematically literate, an individual also needs to be able to “identify the role that mathematics plays in the world by describing, modelling, explaining and predicting phenomena” (PISA Governing Board, 2010, p. 5). This aspect of mathematical literacy is not evident for students who indiscriminately apply the same automatised procedure to further examination items where the underlying problem structure is different. This is rather evidence that this student is not capable of mathematically literate thought, although being capable of applying a particular mathematical procedure.

It is, in fact, questionable that mathematically literate thought was required in this examination to achieve a passing mark. The evidence for this lies in the observation that, for items in which few errors were made, the method applied by each student was similar. This included the students not achieving full marks for the item. It is possible that this, too, was a method studied to be the appropriate response to the type of item, and a full understanding of the problem context and structure would not have been required in order to at least receive some of the marks available for the item.

The reverse was also evident in items where many errors were made. The same procedure was attempted by many of the students, which may signify that the lecturer had taught students to recognise cues and answer in a particular way. Students were not able to apply literate thought to work out that a slightly different approach was required to answer these items correctly.

As defined for the NC(V), mathematical literacy is “[a]n attribute of individuals...that involves managing situations and solving problems in everyday life, work, societal and

lifelong learning contexts by engaging with mathematical concepts...presented in a wide range of different ways” (DHET, 2012a, p. 1). Mathematical literacy is specifically described as a practical attribute relevant to everyday life. The real world in which these students would be expected to apply their mathematical knowledge is not equivalent to the mental world in which examination items are solved, nor is it equivalent to the examination world. Therefore, a student declared competent in Mathematical Literacy through an examination, is also not necessarily mathematically literate in a real-world sense.

The real answer to this, however, would require observation of the lectures to assess the lecturer’s approach to each topic, as well as think-aloud problem solving interviews with students. The observations of these students’ responses to this examination paper, however, do indicate that it is a possibility that mathematical literacy is not required to pass a Mathematical Literacy examination.

5.2.4 Description of statistical relationships

In order to answer Research Question 4, Pearson product-moment correlations were calculated between several variables and a graphic comparison made between the percentages of marks allocated to each linguistic and cognitive complexity level.

5.2.4.1 Discussion of results

The marks allocated per Cognitive Complexity Level and per linguistic complexity level were plotted on the same set of axes, and a trend line drawn. At level 2, linguistic complexity sharply increased, and cognitive complexity sharply decreased, but the trend lines revealed that the average percentages of marks allocated to both complexity indices were close to identical. This was despite the lack of a statistically significant correlation between the two complexities ($r = 0.14$; $p = 0.179$).

A statistically significant correlation did exist between both cognitive complexity and linguistic complexity and the total number of errors per item. It was, however, only when the Linguistic Complexity Index was restricted to only those values smaller than 27 that this correlation existed for both variables.

As was expected, the mathematical literacy-related Cognitive Complexity Level was the only complexity statistically significantly correlated with mathematical calculation errors and mathematical literacy-related errors as a whole. Both cognitive and linguistic complexity levels were, however, statistically significantly correlated with both reading errors and language errors as a whole. Linguistic complexity had the stronger correlation, with a higher confidence level for reading errors, and an equivalent confidence level for language-related errors. This indicates that the more cognitively challenging items were also more linguistically demanding, and this cognitive demand was also related to linguistic-type errors.

5.2.4.2 Concluding Question 4

These relationships reveal that the linguistic descriptions and features are important to consider as a source of error. Features contributing mathematical literacy-related complexity would be expected to hold statistically significant correlations with errors, but despite there being no statistically significant correlation between the two complexities, both are correlated with certain errors. Language-related errors contribute as much as 19.22% of the marks lost for the entire examination. Although this is relatively small in comparison with the 65.93% contribution of mathematical literacy-related errors, it remains a percentage large enough to indicate a cause for concern.

In concluding the findings regarding Question 1, it was mentioned that according to the low average linguistic complexity levels, it would seem that the low language proficiency of the students was to blame for the percentage of marks lost due to language-related errors. The statistically significant relationship between the Linguistic Complexity Index and the number of language-related errors indicates that this is most likely an inaccurate assumption.

Language errors were lower for linguistically simple items and higher for those more complex. The items at the higher end of the Linguistic Complexity Index, readability and required language proficiency continua therefore are a possible source of error. Despite the apparently acceptable linguistic design of the examination with regard to the averages, it is necessary to limit the items which would be classified as more linguistically complex. Data gathered by Howie (2005) indicated not only a correlation between mathematical achievement and the language component of an examination, but her data allowed the conclusion to be made that the language component affected achievement in mathematics. This causal conclusion cannot be made based on this data, but the statistically significant

correlations found do suggest the possibility that this might be the case for this sample in this examination.

5.3 PRACTICAL LESSONS FOR LECTURERS AND ASSESSORS

Several practical lessons can be derived from the descriptions and discussions in this case study. These can be applicable to Mathematical Literacy lecturers, Mathematical Literacy assessors, as well as English First Additional Language lecturers. The lessons would be particularly pertinent to lecturers working in the same college from which the student sample was drawn and whose internal examination was used, however, these suggestions are expected to be valuable guidelines to other lecturers working in both the NC(V) context, as well as those working with NCS students.

5.3.1 Lessons for Mathematical Literacy lecturers

Students for whom English is a second language need to be proficient in both ordinary and mathematical English (Setati, 2002). This is a complex task that requires more than just vocabulary lessons. Mathematics possesses “unique linguistic forms” (Patkin, 2011, p. 2), with characteristic “styles of meaning and modes of argument” (K’Odhiambo & Gunga, 2010, p. 80).

It is essential therefore, that sentences and paragraphs as a whole are carefully deconstructed for students during class time, such that they can begin to understand how mathematical English is constructed. This is essential if a student is to recognise problem structures through word sums, accurately solve them, and relate the answer to the context in which the problem is posed.

According to Bergqvist and Österholm (2010), it is difficult to separate the process of reading from the process of problem solving, and misrepresentation of the problem structure is frequently the result of a misunderstanding of the mathematical text (Lewis, 1989). The responsibility for assisting students in navigating this language lies with the Mathematical Literacy lecturer.

It is necessary to focus on vocabulary, as was clear from the description of linguistic complexity and its statistically significant correlates. Lecturers should deliberately draw students' attention to new vocabulary where it appears, and provide them with a definition. They should not assume that students will be capable of deriving the meaning of the new word from the context of the lesson.

It is not appropriate to teach students to compensate for a lack of comprehension by identifying key words as cues for deriving a mathematical procedure to answer examination items. In this way, such students are not being allowed an opportunity to engage properly with problems in such a way that they can begin to acquire mathematical literacy, they are simply becoming test-wise. This is not a skill which can be applied in any world other than the world of the examination.

It needs to be remembered that it is the students' acquisition of mathematical literacy with which the subject Mathematical Literacy is concerned, and a pure focus on teaching students to pass a Mathematical Literacy examination does not further this cause.

5.3.2 Lessons for Mathematical Literacy assessors

Unnecessary linguistic complexity will compromise the construct validity of a Mathematical Literacy examination as it is possible that this will be a source of measurement error (Abedi & Gándara, 2006). The data in this study showed which language features contributed to the linguistic complexity of the examination.

Several language features were identified as being essential in conveying specific mathematical relationships and concepts. In this examination, these included: prepositional phrases; conditional/comparative constructions; and words of 7 letters or more. It is therefore essential for certain complex language features to be used, but the number of such features per sentence should not be excessive. When many facts need to be given in a lead-in text, examiners should consider whether sentences can be broken up such that fewer ideas need to be processed in each sentence. It may also be helpful to present facts in a bulleted or numbered list. This would facilitate a reduction in the number of linguistically complex features per sentence and assist students in understanding the information conveyed in, for example, the prepositional phrases.

Some of the language features which contributed to linguistic complexity in this examination could be reduced without altering the meaning of the sentences. Abbreviations, for example, should be given alongside their extended form where they are first used. For example, write the full form of Unemployment Insurance Fund where the abbreviation UIF is first used. The use of passive voice is another example of a feature that added unnecessary complexity to the examination. Sentences should be written in the active voice where this is feasible and appropriate.

Cultural and contextual or experiential references can be avoided if scenarios are carefully considered with regard to the type of student who will be familiar with the context provided. The scenario provided in Question 9 of Paper 1, which described seamstresses completing a uniform order, is such an example. It is reasonable to assume that many of the students were not familiar with the word seamstress.

Errors are unacceptable and entirely avoidable. They compromise the quality of the examination and introduce ambiguity that English language learners may not be able to overcome. Examiners should double-check spelling and grammar.

It is not only language features and sentence structure that should be attended to. Readability is also a factor that should be kept in consideration when writing lead-in texts and examination items.

Examinations must be proofread by someone who is competent in editing texts. Editors should, however, ideally also be familiar with mathematical language in particular so that meaning is not lost during the editing process.

5.3.3 Lessons for English First Additional Language lecturers

Lessons can also be derived for English First Additional Language lecturers. Complexity was added to the examination through the use of specific language features, all of which should be explicitly focussed on the English classroom. It is the responsibility of the Mathematical Literacy lecturer to assist students in decoding mathematical texts, with their characteristic modes of argument and special phrases and vocabulary, but the meaning of the language features themselves needs to be taught by the English lecturer.

The particular language features which need to be focussed on are (in order of their frequency of use in the examination):

- Prepositional phrases
- Pronouns
- Complex/compound sentences
- Abbreviations
- Infinitives
- Complex verbs
- Slang, homophones and homonyms
- Passive voice

According to the CIGE tables, the text function of providing specific information, is located at proficiency level B2 (Upper Intermediate). This function was featured in all 17 scenarios used in the examination. In the English class, therefore, it is essential to focus on teaching students how to recognise relevant, precise pieces of information provided in texts. This can be done through the use of comprehension exercises using information texts, listening comprehensions or in summarising information texts, amongst others.

Reading was the greatest source of language-related errors, pointing to the need for students to improve their reading skills. Many of the reading errors involved students not noticing a key word or not reading a text attentively to the end, as well as not comprehending certain parts of the text. This indicates the need for students to practice close reading, which needs to be encouraged and facilitated in the English classroom.

These suggestions, based in this case on a Mathematical Literacy examination, will hold benefits for students in all of their academic subjects. NCV English First Additional Language lecturers need to be reminded that one of their primary purposes is to support the students' learning in their other subjects. This is very clear in the curriculum documentation (DHET, 2012c), but their explicit attention must be drawn to this. It is the English lecturer that is responsible for the academic literacy development of the students, and consequently their ability to decode information-laden texts such as those appearing as lead-in contextualising texts in Mathematical Literacy examinations. Their supportive role in the students' broad learning development in their NCV programme cannot be diluted.

5.4 SHORTCOMINGS AND LIMITATIONS OF THE CURRENT STUDY

Due to the small size of the student sample, and the fact that this sample had been taken from one specific division of one specific FET college, the results of this study cannot be generalised beyond the specific case that was explored. The examination paper which was analysed was not a national examination and thus conclusions that have been made cannot be generalised to the national examinations written by NC(V) students.

It was unfortunate that the students in the sample had not achieved a wider range of results for Mathematical Literacy, or for English First Additional Language. This was done in order to maximise the number of errors available for analysis. However, it was not possible to explore whether the pattern of errors made were any different for students achieving at higher levels in these subjects. A definite bias would be expected in the description of this case study, due to the poor English marks. The percentage of language-related errors was probably inflated.

The examination selected for use in this study was revealed, in the analysis, to have contained many grammatical and spelling errors. The paper had been moderated and approved for use, therefore it would be expected that these errors had been corrected during this process. The failure of this process would have led to inflated linguistic complexity indices. As this case study aimed to describe the linguistic complexity of the examination, it was possible to include, in the description, the fact that these errors had added to the complexity of the examination. However, although the number of these errors was not shown to be statistically significantly correlated with the Linguistic Complexity Index, their presence might have been the underlying cause of many of the reading errors made by students. This confounding variable was not controlled for in this study.

Many of the students' responses to examination items did not provide evidence of actual engagement with the problem. This included responses which achieved full marks, many of which appeared to have been answered with automatic, rote-learned methods. Without further investigation, it is not possible to conclusively make this statement. Should this, however, have been the case the examination of these scripts in particular would not have been valid, as these students' would have reflected their test-wiseness and not their mathematical literacy. In the same way, where their seemingly automatic responses resulted in full marks

for an item, this would not necessarily have implied that they had actually read and comprehended the text at all.

The use of student scripts from one particular class meant that one particular lecturer's students participated in the study. The reason for this approach was to avoid selecting students from the researcher's own class in order to eliminate the possibility of results being contaminated in this way. The disadvantage was that this lecturer's individual approach to the subject would have been reflected in the students' responses. In particular, items asking for the calculation of the volume of a rectangular prism and those asking for the addition or subtraction of percentages were answered with apparent automaticity and a high degree of accuracy. This could be a reflection of exam training of these students rather than the facilitation of actual problem solving by the lecturer.

Because these correct responses possibly reflected rote learning, rather than mathematical literacy, the error count per item would have been contaminated. Such responses could have been categorised as a form of error in that students had not engaged with the item and as such could not have been credited as showing either mathematical literacy or language competence in these responses. Without assessing the lecturer's particular approach and competency, areas in which students showed weakness could reflect how the lecturer had presented the material.

A particular shortcoming in the error analysis conducted was that the analysis was purely document-based. This only allowed a judgment-based categorisation of errors, which could have been overcome by including data from the students themselves in an attempt to gain insight into their actual approach to various items. The examination paper analysis was also purely document based. Involving the assessor in the research would have enhanced the validity of the conclusions made regarding the design of the examination. In order to increase validity two raters were involved in both the linguistic and cognitive complexity analyses, but the validity of their categorisation is affected by this isolation from the actors in the case study. As such, the resulting complexity indices should be interpreted with care.

These shortcomings, although presented subsequent to the conclusions made based on the data, need to be kept in mind when interpreting the data. This is particularly the case when

considering any possibility of generalising the results, even to other classes within the same division and at the same college.

5.5 CONTRIBUTIONS OF THE STUDY AND AVENUES FOR FURTHER RESEARCH

Despite the narrow scope of this study, it does make important contributions. Of these contributions, some are applicable to cases beyond that described in this study. One specific way in which this study can contribute to this field of research is in highlighting several interesting further avenues of research.

5.5.1 Contributions of the study

The consistent statistical significance in the correlations found between the Cognitive Complexity Index and many of the variables explored in this study suggests that the composition of this index could be relevant to other studies. Similarly, the approach to the calculation of linguistic complexity and its description could be used in evaluating subsequent examinations. Despite the shortcomings of these indices, they do provide value for any similar small-scale descriptive exploration as to whether an examination has been suitably designed.

The inclusion of an error analysis of student responses in this case study added a valuable aspect to the description of the complexity of the examination paper itself. The framework used permitted the categorisation of all of the error types observed in the students' scripts, and therefore suggests that this framework, too, may be of value to similar descriptive studies.

It was possible to derive several practical guidelines from the analysis of the data, which is a particular contribution that this study has been able to offer to Mathematical Literacy and English First Additional Language lecturers as well as Mathematical Literacy assessors at this college. These guidelines, outlined in section 5.3, have been based on specific observations made while gathering data for this case study, and as such are most suited to staff at this college. Their generic nature, however, suggests that they could be of value for such lecturers and assessors in other FET colleges.

5.5.2 Avenues for further research

Despite the shortcomings of this study, the in-depth, case-specific description does provide results which hint at several interesting avenues for further research.

5.5.2.1 Suggestions for further statistical analyses

Statistically significant correlations were found between certain language features and the Linguistic Complexity Index of items, as well as between the two complexity indices and the number of errors per error type. As explained in the section above, these correlations are specific to this case and cannot be generalised. The fact that correlations were found, however, points out the need for statistical research on a larger scale, with a sample representative of the national college population, in order to establish whether these same correlations exist. There are practical implications, on a national level, should any such statistical relationships be found. The lessons learned from such analyses could provide essential guidance regarding how best to construct examination papers for English second language students. Should the statistical analysis, however, reveal that papers are well designed, this would provide an important source of evidence defending the validity of the results obtained by the students.

Pearson product-moment correlations are not the only statistical tests of value in such a large-scale study. The increase in sample size and improvement in its representativeness would mean that more powerful tests could be used. One example would be a multiple regression analysis to ascertain the relative contribution of cognitive complexity and linguistic complexity to results, as well as the relative contributions of various language features.

The sole use of English language learners is a particular limitation to this study, which suggests an avenue for further research. If one were to increase the scale and include English home language students it would be possible to use Differential Item Functioning and construct Item Characteristic Curves to compare whether items function differently for English home language students and English language learners. This research is extremely important in South Africa. Where both types of students are writing examination in English, it is essential to explore whether any language-based bias exists in examination items. Again, should the results prove that there is no such bias, it makes the assessment and results of students more valid and therefore more defensible.

5.5.2.2 Suggestions for further descriptive research

The observation that student responses are possibly revealing a lack of mathematical literacy even in their correct responses, raises the question as to whether the subject, as it is taught and assessed at the moment, is in fact resulting in the ability of students to “[manage] situations and [solve] problems in everyday life, work, societal and lifelong learning contexts by engaging with mathematical concepts...presented in a wide range of different ways” Department of Basic Education (2011, p. 10) as the subject aims to do. Research is urgently required in this area, as these results could imply that: significant restructuring of the subject is needed; intervention as to how it is taught is required; and investigation is required into the validity of how it is being assessed. Should these results show that the subject, as it currently exists, is achieving this aim, the inclusion of Mathematical Literacy in the NC(V) becomes strongly defensible.

An avenue of research which would be interesting to pursue, and which addresses a particular shortcoming of this study, is to include the students themselves in any error analyses. The approach of a think-aloud interview would reveal, with more validity, the source of a student’s error. This would allow a more confident conclusion to be made as to the relative number of mathematical literacy-related and language-related errors. Insight could also be gained as to whether students reveal actual mathematical literacy in their correct responses or not.

5.6 CONCLUSION

Due to the descriptive goal of this study, the detailed description of the data, with the accompanying analyses, presented in Chapter 4, provided the full answers to the research questions. It was possible, however, in their discussion, to arrive at a more concise conclusion for each, as presented in this chapter.

It was found that the linguistic complexity of the examination was acceptable for an NC(V) Level 4 Mathematical Literacy examination, despite the fact that more complex language features were at times present in items. The language proficiency of the students themselves seemed the reason for the number of language-related errors they were making.

The cognitive complexity of the examination was revealed to be unbalanced with regard to the spread of cognitive complexity levels per topic assessed. Graphical languages added complexity to the items in which they appeared, and certain types of questions appeared to require more focused cognitive effort than others.

With regard to student errors, the most frequently made errors were due to mathematical literacy related sources, but language-related errors were also present, mostly due to reading errors.

When Pearson product-moment correlations were calculated between the complexity indices and types of errors, cognitive complexity was found to be statistically significantly correlated with all error types. Linguistic complexity was statistically significantly correlated with reading errors and language-related errors as a whole.

Several unanticipated findings were made in the course of this research. This included the observation that certain types of questions led to apparently more automatised responses from all students in the sample. These seemed to reflect a lack of engagement with the problem as questions with similar key words, but different problem structures, were answered in the same way by the majority of students. This led to a discussion as to whether mathematical literacy was actually evident in these correct responses, which suggests that dialogue is needed as to whether the subject is achieving its aim in the NC(V).

Due to the limited scope of the study as well as its shortcomings and its site-specific focus, it is not possible to generalise from the findings, but many avenues of further research can be suggested. This focused site-specific case study design, however, has provided findings from which many practical lessons can be drawn for lecturers and assessors in this college.

It is hoped that this study will indeed be a catalyst for the further research indicated as essential in this report, and that the practical lessons that have been derived from the study will in fact be implemented.

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APPENDICES

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APPENDIX A

European Association for Quality Language Services [EAQUALS]'s Core Inventory for
General English [CIGE]
(North, Ortega & Sheehan, 2010, p. 19)

FUNCTIONS					
Functions / Notions	CEFR LEVEL A1	CEFR LEVEL A2	CEFR LEVEL B1	CEFR LEVEL B2	CEFR LEVEL C1
Numbers					
Prices					
Telling time					
Directions					
Describing habits and routines					
Describing things					
Arrangements					
Obligation and necessity					
Describing places					
Describing past experiences and storytelling					
Giving precise information					
Expressing abstract ideas					
Expressing certainty, probability, doubt					
Generalising and qualifying					
Synthesising and evaluating					
Speculating and hypothesising					
Expressing opinions					
Expressing agreement or disagreement					
Defending a point of view					

TENSES					
	A1	A2	B1	B2	C1
Present					
Simple Present					
Present Continuous					
Present Perfect					
Present Perfect/Past Simple					
Present Perfect Continuous					
Past					
Simple Past				Narrative	Narrative
Past Continuous				Narrative	Narrative
Used to				Narrative	Narrative
Would expressing habit in the past					Narrative
Past Perfect				Narrative	Narrative
Past Perfect Continuous					Narrative

Future					
Future Time (going to)					
Future Time (present continuous)					
Future Time (will & going to)					
Future Continuous					
Future Perfect					
Future Perfect Continuous					

PARTS OF SPEECH					
	A1	A2	B1	B2	C1
Simple Verb Forms					
To be					
Have got					
Imperatives					
Phrasal Verbs					
Common phrasal verbs					
Extended phrasal verbs					splitting
Other Verb Forms					
Reported speech (range of tenses)					
Relative clauses					
Questions					
Question forms					
Wh-questions in the past					
Complex question tags					
Gerund & Infinitive					
I'd like					
Gerunds					
to + infinitive (express purpose)					
Verb + to + infinitive					
Conditionals					
Zero and 1 st conditional					
2 nd and 3 rd conditional					
Mixed conditionals					
Wish/if only & regrets					
Modals: Can					
Can/can't					
Can/could					
Modals: Possibility					
Might, may					
Possibly, probably, perhaps					
Might, may, will, probably					
Must/can't (deduction)					
Modals: Obligation & Necessity					
Must/mustn't					
Have to					
Must/have to					

PARTS OF SPEECH					
	A1	A2	B1	B2	C1
Should					
Ought to					
Need to/Needn't					
Modals: Past					
Should have/might have/etc.					
Can't have, needn't have					
Nouns					
Countable and uncountable	v. common				
There is/there are					
Pronouns					
Personal - subject					
Possessives					
Possessive adjectives					
Use of 's, s'					
Possessive pronouns					
Prepositions and prepositional phrases					
Common prepositions					
Prepositional phrases (time & movement)					
Prepositional phrases (place & time)					
Articles					
Definite, indefinite					
Zero articles with uncountable nouns					
Definite article with superlatives					
With countable and uncountable nouns					
With abstract nouns					
Determiners					
Basic (e.g. any, some, a lot of)					
Wider range (e.g. all, none, not (any), enough, (a) few)					
Broad range (e.g. all the, most, both)					
Adjectives					
Common					
Demonstrative					
Ending -ed + -ing					
Collocation of adjective					
Comparative, superlative					
Comparisons with fewer and less					
Adverbs					
Adverbs of frequency					
Simple adverbs of place, manner and time					
Adverbial phrases of time, place and frequency, incl. word order					
(Adjectives and) adverbs					
Adverbial phrases of degree, extent,					

PARTS OF SPEECH					
	A1	A2	B1	B2	C1
probability					
Comparative and superlative of adverbs					
Attitudinal adverbs					
Inversion (negative adverbials) Hardly...?					
Intensifiers					
Very basic (very, really)					
Basic (quite so, a bit)					
Broader range of intensifiers such as too, enough					
Wide range such as extremely, much too					
Collocation of intensifiers					
Passives					
Simple passives					
All passive forms					

APPENDIX B

Common European Framework of Reference [CEFR] for Languages - Level Descriptions (from North, Ortega & Sheehan, 2010, p. 23)

Level	Description
<p>C2</p> <p>MASTERY</p>	<p>Convey finer shades of meaning precisely by using, with reasonable accuracy, a wide range of modification devices; has a good command of idiomatic expressions and colloquialisms with awareness of connotative level of meaning; backtrack and restructure around a difficulty so smoothly the interlocutor is hardly aware of it.</p>
<p>C1</p> <p>ADVANCED / EFFECTIVE OPERATIONAL EFFICIENCY</p>	<p>Can express him/herself fluently and spontaneously, almost effortlessly. Has a good command of a broad lexical repertoire allowing gaps to be readily overcome with circumlocutions. There is little obvious searching for expressions or avoidance strategies; only a conceptually difficult subject can hinder a natural, smooth flow of language.</p> <p>Select a suitable phrase from a fluent repertoire of discourse functions to preface his remarks in order to get the floor, or to gain time and keep it whilst thinking; produce clear, smoothly-flowing, well-structured speech, showing controlled use of organisational patterns, connectors and cohesive devices</p>
<p>B2+</p> <p>STRONG UPPER INTERMEDIATE</p>	<p>Give feedback on and follow up statements and inferences by other speakers and so help the development of the discussion; relate own contribution skilfully to those of other speakers. Use a variety of linking words efficiently to mark clearly the relationships between ideas; develop an argument systematically with appropriate highlighting of significant points, and relevant supporting detail.</p>
<p>B2</p> <p>UPPER INTERMEDIATE</p>	<p>Account for and sustain his opinions in discussion by providing relevant explanations, arguments and comments; explain a viewpoint on a topical issue giving the advantages and disadvantages of various options; develop an argument giving reasons in support of or against a particular</p>

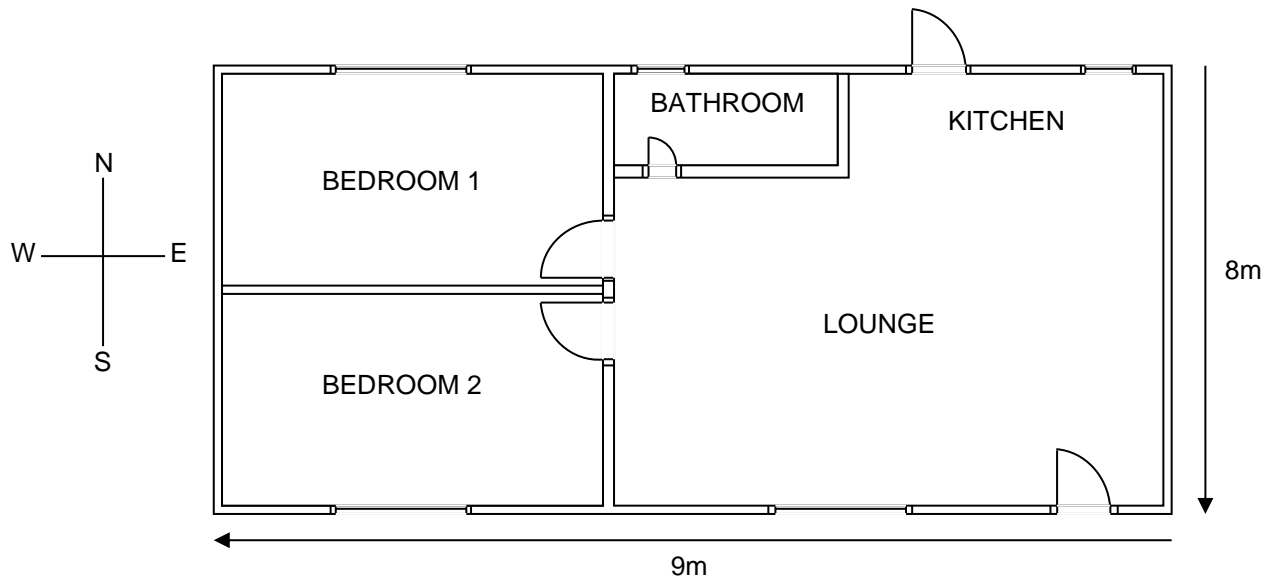
	<p>point of view; take an active part in informal discussion in familiar contexts, commenting, putting point of view clearly, evaluating alternative proposals and making and responding to hypotheses.</p> <p>Understand in detail what is said to him/her in the standard spoken language even in a noisy environment; initiate discourse, take his turn when appropriate and end conversation when he/she needs to, though he/she may not always do this elegantly; interact with a degree of fluency and spontaneity that makes regular interaction with native speakers quite possible without imposing strain on either party.</p> <p>Correct mistakes if they have led to misunderstandings; make a note of “favourite mistakes” and consciously monitor speech for it/them; generally correct slips and errors if he/she becomes conscious of them.</p>
<p>B1+</p> <p>STRONG INTERMEDIATE</p>	<p>Provide concrete information required in an interview/consultation (e.g. describe symptoms to a doctor) but does so with limited precision; explain why something is a problem; summarise and give his or her opinion about a short story, article, talk, discussion interview, or documentary and answer further questions of detail; carry out a prepared interview, checking and confirming information, though he/she may occasionally has to ask for repetition if the other person’s response is rapid or extended; describe how to do something, giving detailed instructions; exchange accumulated factual information on familiar routine and non-routine matters within his field with some confidence.</p>
<p>B1</p> <p>INTERMEDIATE</p>	<p>Generally follow the main points of extended discussion around him/her, provided speech is clearly articulated in standard dialect; express the main point he/she wants to make comprehensibly; keep going comprehensibly, even though pausing for grammatical and lexical planning and repair is very evident, especially in longer stretches of free production.</p> <p>Cope with less routine situations on public transport; deal with most situations likely to arise when making travel arrangements through an</p>

	agent or when actually travelling; enter unprepared into conversations on familiar topics.
A2+ STRONG ELEMENTARY	<p>Understand enough to manage simple, routine exchanges without undue effort; make him/herself understood and exchange ideas and information on familiar topics in predictable everyday situations, provided the other person helps if necessary; deal with everyday situations with predictable content, though he/she will generally have to compromise the message and search for words</p> <p>Express how he/she feels in simple terms; give an extended description of everyday aspects of his environment e.g. people, places, a job or study experience; describe past activities and personal experiences; describe habits and routines; describe plans and arrangements; explain what he/she likes or dislikes about something.</p>
A2 ELEMENTARY	<p>Use simple everyday polite forms of greeting and address; greet people, ask how they are and react to news; handle very short social exchanges; ask and answer questions about what they do at work and in free time; make and respond to invitations; discuss what to do, where to go and make arrangements to meet; make and accept offers.</p> <p>Make simple transactions in shops, post offices or banks; get simple information about travel; use public transport: buses, trains, and taxis, ask for basic information, ask and give directions, and buy tickets; ask for and provide everyday goods and services.</p>
A1 BEGINNER	Interact in a simple way, ask and answer simple questions about themselves, where they live, people they know, and things they have, initiate and respond to simple statements in areas of immediate need or on very familiar topic.

Mathematical Literacy Level 3 Pilot Study Test Question

QUESTION 1

Jabulani saved enough money to have his first house built in the town of Caledon. The plan below was drawn up by KDZ Architects and Simelani Contractors will be building the house.



	Width	Height
Walls		3m
Doors	82 cm	210 cm
Big Windows	240 cm	150 cm
Small Windows	120 cm	80 cm

Formulae: Area = L x B Volume = L x B x H

1.1 Calculate the total floor area of his house. 3

1.2 The foundation of the house is rectangular concrete (this is a mixture of sand, cement and stone). If the concrete floor is 0,3m thick, what is the volume of concrete that must be mixed? 2

1.3 The contractors need to order bricks for the house. It is always wise to order 5% more than needed because of breakages during offloading.
If 20 000 bricks are needed, how many should be ordered? 3

1.4 Jabulani wants to paint the outside walls. He must use the plan to calculate the area of the outside walls. All answers must be given in metres. (First convert the measurements from cm to m and then calculate the area of the windows and doors)

1.4.1 Total area of the outside doors? 3

1.4.2 Total area of the small windows? 3

1.4.3 Total area of the big windows? 3

1.4.4 What is the total area of walls that must be painted? Round off your final answer to the nearest m.
(Hint: Area of the 4 outside walls minus total area of windows and doors) 8

1.5 Use your answer from question 1.4.4 to calculate how many litres of paint he must buy to paint the outside walls with two coats of white paint.
Take note: He will need 1 litre of paint for every 4 m² of wall. 3

1.6 Jabulani wants to tile the whole house with the same tiles.



1.6.1 How many boxes must he buy to cover the whole house?
(The area of the house was calculated in Question 1.1)

2

1.6.2 What is the total cost of the tiles?

2

Linguistic Complexity Checklist
 Adapted from Shaftel et al. (2006, p. 126)

NUMBER OF SENTENCES: _____

A: BASIC

1. _____ Number of words in item

B: WORD LEVEL CHARACTERISTICS

1. _____ Number of different words with 7 letters or more
2. _____ Number of relative pronouns
3. _____ Number of examples of slang, homophones and homonyms
4. _____ Number of abbreviations

C: SENTENCE LEVEL CHARACTERISTICS

1. _____ Number of prepositional phrases
2. _____ Number of infinitives
3. _____ Number of complex verbs
4. _____ Number of complex / compound sentences
5. _____ Number of conditional constructions
6. _____ Number of comparative constructions

D: PARAGRAPH LEVEL CHARACTERISTICS

1. _____ Number of cultural- and/or experience-specific references
2. _____ Number of grammatical and/or spelling errors

Linguistic Complexity Index:

LCI = (Sum A + Sum B + Sum C + Sum D) ÷ Number of sentences

= _____

Mathematical Literacy Level 4 Trial Examination Paper 1, September 2011

QUESTION 1

- 1.1 Calculate the following:
- 1.1.1 $4^2 + \frac{3}{12}(80 \div 4)$ (Show ALL calculations) (3)
- 1.1.2 7,5% of R350 (2)
- 1.1.3 R2 000,00 - R200,20 x 3 (2)
- 1.1.4 $\frac{5}{6}$ of 540 females (1)
- 1.2 Write the following ratio in its simplest form: 12 : 108 (2)
- 1.3 Convert 9,585 ℓ to cm³ if 1 ℓ = 1 000 cm³ (2)
- 1.4 What will you pay for a TV that costs R3 590 if you receive a 12,5% discount for paying cash? (3)
- 1.5 A computer game costs R499 excluding VAT. What will you pay for the game after VAT of 14% has been added? (3)
- 1.6 Convert 4 hours to seconds. (3)
- 1.7 Four movie tickets cost R110. What is the price of 1 ticket? (2)
- 1.8 Calculate the volume (V) of a cone with a radius of 20 mm and a height of 30 mm. (3)
- Use the formula: $V = \frac{1}{3}\pi r^2 h$, where $\pi = 3,14$
- 1.9 R6 000 is invested at 5% compound interest per year. What will the investment be worth after FOUR years?
Use the formula: $A = P(1 + i)^n$
Where A = Final Amount
P = Principal/Starting amount
i = Interest rate (5% = 0,05)
n = Time period in years (n = 4) (3)
- 1.10 Use the information below to determine what the time will be in New York if it is 4:00 pm in South Africa. (3)
- NEW YORK GMT -4
SOUTH AFRICA GMT +2
- 1.11 Amanda runs a catering business. She has found that the probability of guests drinking tea is $\frac{4}{6}$. If 240 guests attend a morning tea, how many can she expect to drink tea. (2)

[34]

QUESTION 2

The table below shows the prices of tickets for soccer matches for the FIFA Soccer World Cup hosted by South Africa during June/July 2010. The prices of these tickets are given in South African Rand.

Study the table and answer the questions that follow:

MATCHES	CATEGORY 1	CATEGORY 2	CATEGORY 3	CATEGORY 4	WHEEL CHAIR
Opening Match (No 1)	3 150	2 100	1 400	490	490
Group Matches (No 2 to 48)	1 120	840	560	140	140
Round of 16 (No 49 to 56)	1 400	1 050	700	350	350
Quarter-Finals (No 57 to 60)	2 100	1400	1 050	525	525
Semi-Finals (No 61 & 62)	4 200	2800	1 750	700	700
3 rd / 4 th Place (No 63)	2 100	1 400	1 050	525	525
The Final (No 64)	6 300	4 200	1 800	1 050	1 050

- 1. Category 4 tickets are reserved exclusively for residents of the Republic of SA.**
- 2. Each household may apply for a maximum of 4 tickets per match and up for a maximum of 7 matches.**
- 3. All tickets ordered for a given match must be for the same ticket category, but you can order different categories for different matches.**

- 2.1 What was the price of a category 1 ticket for the final match? (1)
- 2.2 What was the price of the cheapest ticket for a World Cup Match? (1)
- 2.3 How many matches were played in the entire World Cup? (1)
- 2.4 Determine the maximum number of tickets a South African household could buy for the entire World Cup. (2)
- 2.5 What was the difference in price between a Category 1 and Category 4 ticket for the Final match? (2)
- 2.6 How many Euros did a person from Germany pay for a Category 1 ticket for the Quarter-Finals if the exchange rate was R9,21 for every Euro. (2)
- 2.7 Calculate the price for the following match ticket combinations:
- 2.7.1 4 of category 3 tickets for the opening match and 4 of category 2 tickets for Quarter Finals. (3)
- 2.7.2 1 of category 3 tickets for the Quarter-Finals, 2 of category 1 tickets for Semi-Finals and 2 of wheel chair tickets for Finals. (4)

[16]

QUESTION 3

- 3.1 Eddy buys a small franchise that sells DVD recorders at a local flea market on Saturdays and Sundays. He has created a budget for his expected income and expenses for the month of May. At the end of May he decides to complete a variance report to compare his budgeted amounts to his actual income and expenses.

He has already completed some parts of the report on ANNEXURE A. You have to help him to complete the report. Use the information below to complete the variance report on ANNEXTURE A.

Given below are all the actual expenses for May 2011:

Cell phone	R620
Stationary	R450
Transport	R2 150
New Stock	R47 000
Salaries	R16 800
Rent (Flea market + Warehouse)	R1 600
Savings	R20 000
Franchise cost	R1 300

Your report should show the following:

- ALL expenses
- Total expenses
- Total variance
- Surplus/deficit

[11]

QUESTION 4

Eddy decides to calculate his basic costs to sell his product at the flea market.

It costs Eddy R1 300 per month for this franchise. It also costs him R800 rental for the month of May for the space which he uses at the flea market. The cost of one DVD recorder is R1 500 and he sells them for R2 200 each.

- 4.1 The tables given below show the expenses and income for May 2011. Study the tables and calculate the missing values. DO NOT copy the tables, only write down the question number, your workings and your answer.

- 4.1.1 Calculate the value of (a)

Formula to calculate expenses per month:

$$\text{Expenditure} = \text{R2 100} + (\text{R1 500} \times \text{Number of DVD Recorders}) \quad (3)$$

No. of DVD Recorders	0	1	3	5	7	10
Expenditure (R)	2 100	3 600	6 600	(a)	12 600	17 100

- 4.1.2 Calculate the value of (b) and (c)

Formula to calculate income per month:

$$\text{Income} = \text{R2 200} \times \text{Number of DVD recorders sold} \quad (4)$$

No. of DVD Recorders sold	0	1	3	5	7	(c)
Income (R)	0	2 200	(b)	11 000	15 400	22 000

- 4.2 Calculate the profit if only 7 DVD recorders were sold per month.

Use formula: Profit = Income - Expenses (2)

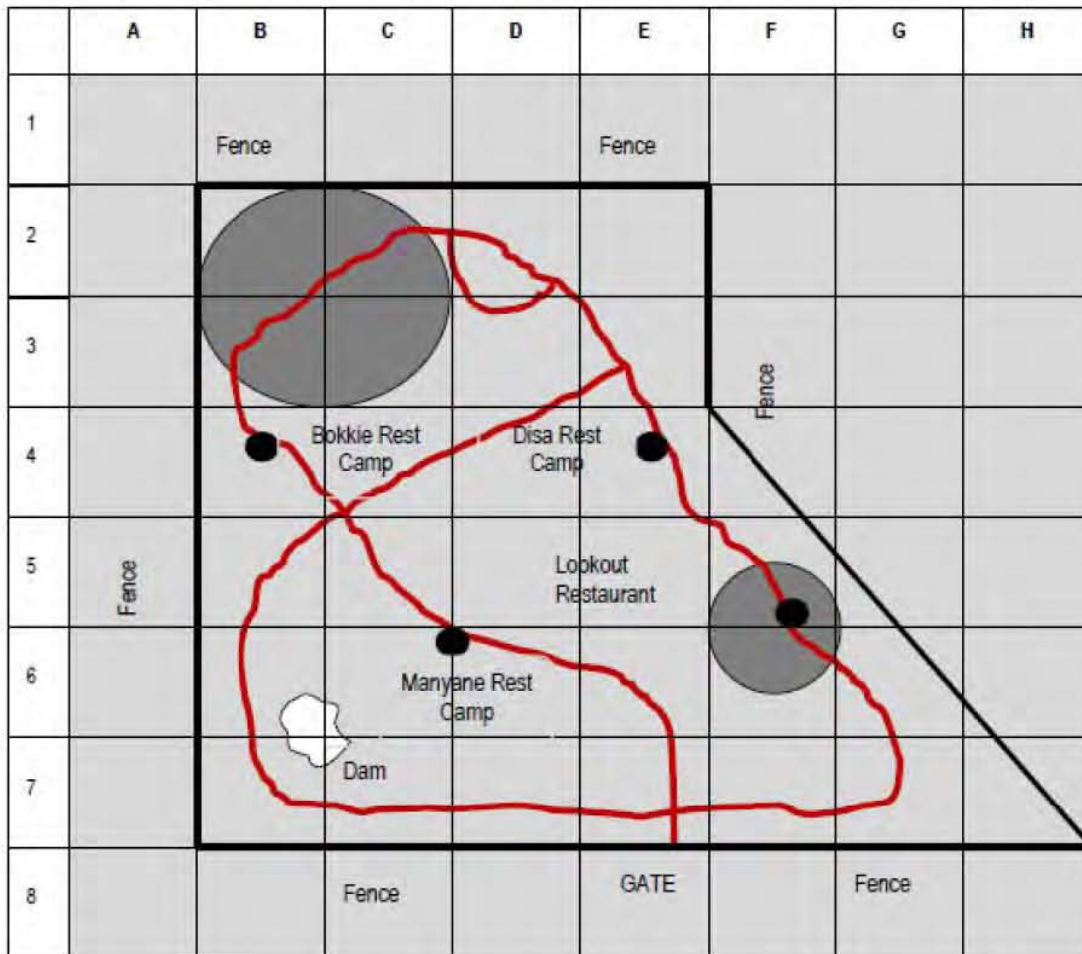
- 4.3 Sketch a line graph that shows Eddy's income during May on the same system of axis with Eddy's expenditure. Clearly label the graph. Use the attached graph paper on ANNEXURE B to draw the graph. (6)

- 4.4 How many DVD recorders should Eddy sell to break-even? (2)



[17]

QUESTION 5

Study the chart below and answer the questions that follow:



Key

-  Cellphone reception
-  Rest Camp



- 5.1 What is the name of the rest camp located in cell E4? (1)
- 5.2 What is the name of the rest camp that is located south-east of the Bokkie Rest Camp? (1)
- 5.3 How many cellphone reception areas are there in the park? (1)
- 5.4 Approximately how many kilometers will you travel from the gate to Manyane Rest Camp? (2)

- 5.5 Bokkie rest camp is 53 km from the gate. Calculate the time it will take you from the gate to Bokkie rest camp if you travel at an average speed of 15 km per hour. Write your answer in hours and minutes. (Round your answer off to the nearest minute)

Formula: $\text{time} = \frac{\text{distance}}{\text{speed}}$ (4)

- 5.6 Calculate the following:

- 5.6.1 The area of the triangular part of the park

Formula: $\text{Area of Triangle} = \frac{1}{2} \times \text{base} \times \text{height}$ (3)

- 5.6.2 The area of the rectangular part of the park

Formula: $\text{Area of Rectangle} = \text{length} \times \text{breadth}$ (3)

- 5.6.3 The total area of the park. (2)

[17]

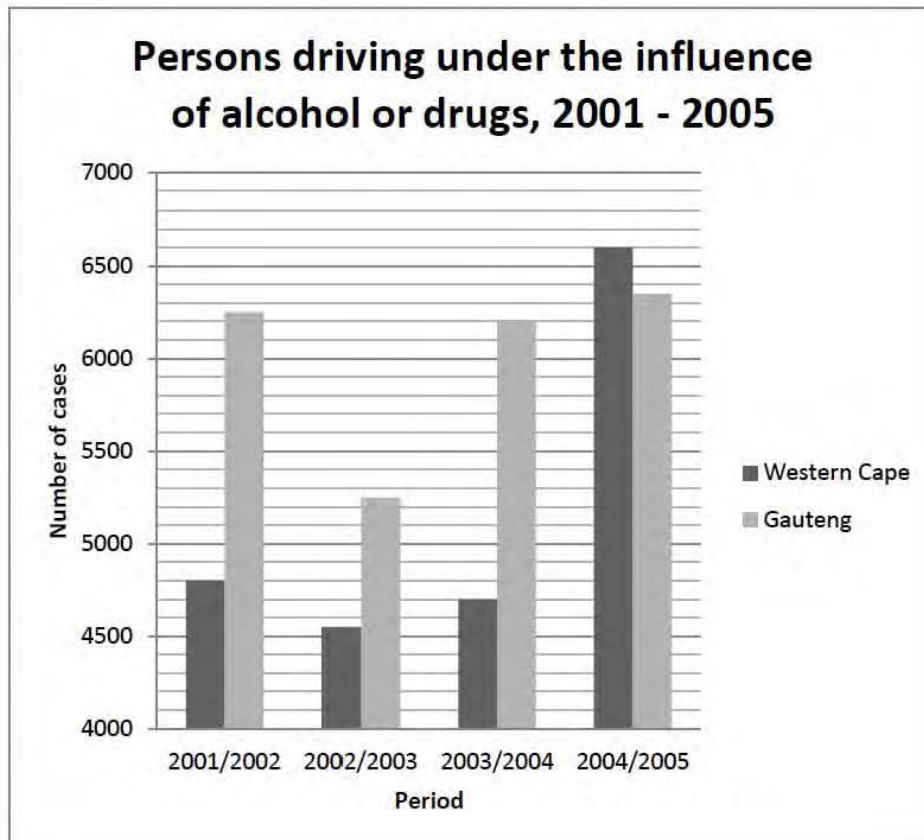
QUESTION 6

- 6.1 The prices of different types of 200 grams bath soap in a store are listed in the table below. Study the information in the table and answer the questions that follow.

The prices of different types of 200 grams bath soap									
R6,99	R7,49	R6,49	R8,80	R5,99	R7,99	R8,49	R7,80	R8,99	R7,50

- 6.1.1 Determine the range of the soap prices in the store. (2)
- 6.1.2 Determine the median value of this data set. (3)
- 6.1.3 What is the mean price of the soaps in the store? (2)
- 6.1.4 Determine the first and the third quartiles of this data set. (2)

- 6.2 The graph given below shows the number of persons reported as driving under the influence of alcohol or drugs in two provinces in South Africa. Study the information on the graph and answer the questions that follow:



- 6.2.1 How many cases were reported in 2002/2003 in Gauteng? (2)
- 6.2.2 In which year(s) were the lowest number of cases reported in both Gauteng and Western Cape.? (2)
- 6.2.3 Which province had a greater increase in the number of cases reported over the period 2001 – 2005? (2)
- 6.2.4 How many more cases were reported in Gauteng than in the Western Cape during the period 2003/2004? (2)
- 6.2.5 In which year did the number of reported cases in the Western Cape exceed that of Gauteng? (2)
- 6.2.6 Determine the total number of cases reported in the two provinces during the period 2001/2002. (2)

[21]

QUESTION 7

- 7.1 Your company needs to import a machine from the USA. How much will the machine cost your company in Rands if the machine cost \$4 700 and the exchange rate is R6,98 per \$1? (3)
- 7.2 Eddy paid R969 for his new cell phone. Value-added tax (VAT) at 14% is included in the price. Determine the price of the cell phone before VAT was added. (3)
- 7.3 Eddy buys a car for R150 000.
- 7.3.1 Eddy paid a 20% deposit on the car. How much (in Rand) did he pay as the deposit? (2)
- 7.3.2 Eddy expects the car to depreciate in value by 30% after 2 years. What will the value of the car be after 2 years. (3)
- 7.4 Eddy decides to buy 100 Compact Discs (CDs) to sell at the flea market. He receives the following quotation (prices in the table below):

	Description	Price
Option 1	1 CD	R2,49
Option 2	A box of 10 CDs	R19,99
Option 3	A box of 25 CDs	R52,99

Use the above table to calculate the following:

- 7.4.1 Price for 50 CDs on Option 1 (2)
- 7.4.2 Price per 50 CDs on Option 2 (2)
- 7.4.3 Price per 50 CDs on Option 3 (2)
- 7.4.4 If he decides to purchase 50 CDs which option is the cheapest option. (1)

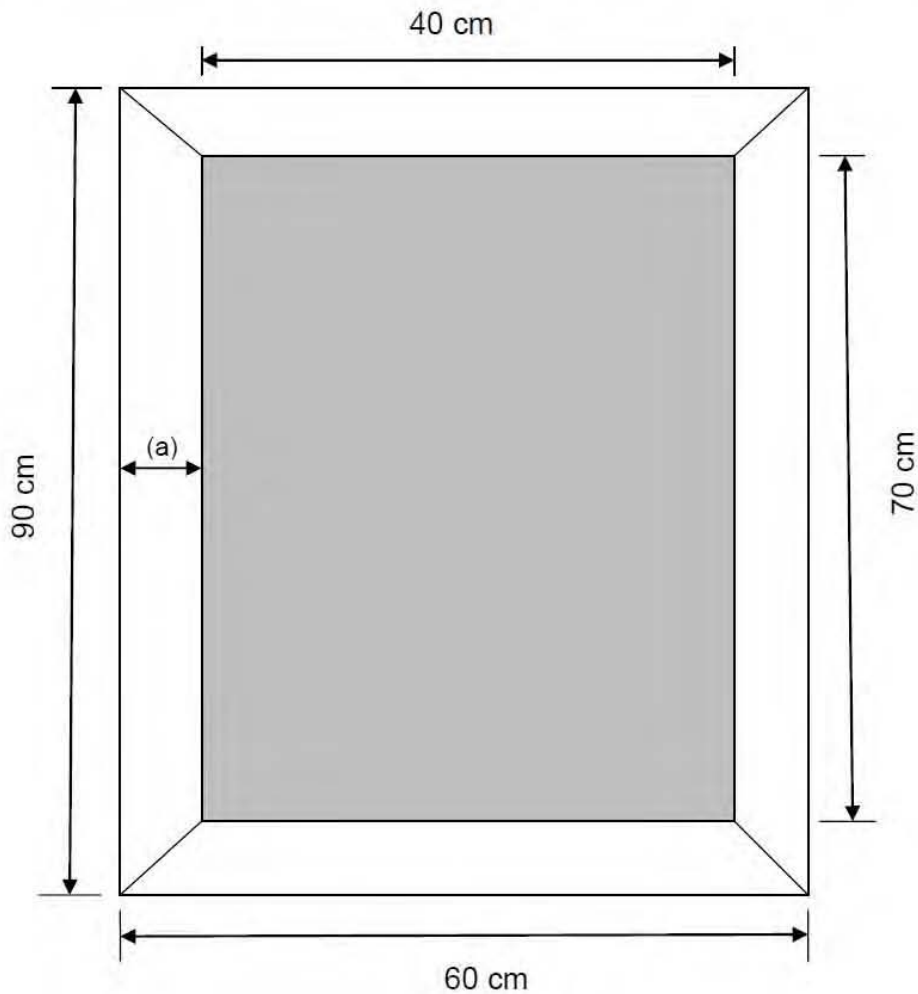
[18]

QUESTION 8

The diagram below is a framed mirror. The mirror has the following parts:

A mirror with a length of 70 cm and breadth of 40 cm.

A wooden frame with an outside length of 90 cm and breadth of 60 cm.



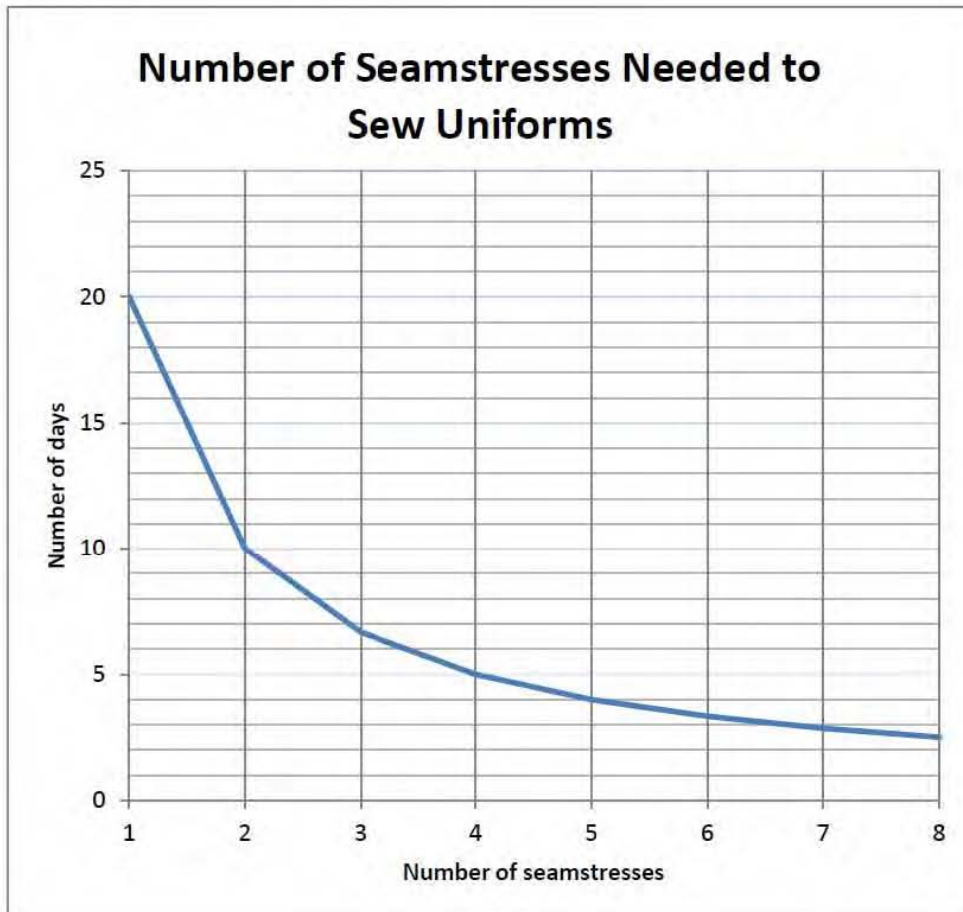
- 8.1 Determine the outside perimeter of the frame.
Formula: $\text{Perimeter} = (2 \times l) + (2 \times b)$ (3)
- 8.2 Calculate the width of the frame - indicated with (a) on the diagram. (3)
- 8.3 Convert the 90 cm length to meters. (2)
- 8.4 The mirror is sold in a box that is 91 cm long, 61 cm wide and it has a depth of 5 cm. Calculate the volume of the box. (3)
Formulae: $\text{Volume} = l \times b \times h$

[11]

QUESTION 9

Amanda runs a small factory which manufactures uniforms. She receives an order for 100 uniforms for a local school. She needs to calculate how long it will take to make the uniforms. She uses a graph to illustrate the number of days it will take a number of seamstresses to sew the uniforms.

Study the graph and answer the questions that follow:



- 9.1 How many seamstresses would she need if she had 20 days to complete the order? (1)
- 9.2 Estimate how many days it would take 6 seamstresses to complete the order? (2)
- 9.3 Is the relationship between the variables shown on the graph a direct relationship or an inverse relationship? (2)

[5]

TOTAL: 150

ANNEXURE A

NAME:

QUESTION 3.1**COMPLETE AND SUBMIT WITH YOUR ANSWER BOOK.**

BUDGET FOR MAY 2011				
	Item No:	Budget (Planned)	Actual (Realised)	Variance
INCOME		R	R	R
Sales	10001	94 600	95 000	400
TOTAL INCOME	10002	94 600	95 000	400
EXPENSES		R	R	R
Rent (Flea market & Warehouse)	10003	1 600	1 600	00
Transport	10004	2 000		
Franchise cost	10005	1 300		
New Stock	10006	50 000		
Cellphone	10007	500		
Savings	10008	20 000		
Salaries	10009	18 000		
Stationery	10010	300		
TOTAL EXPENSES	10011	93 700		
SURPLUS/DEFICIT	10012	900		

[11]

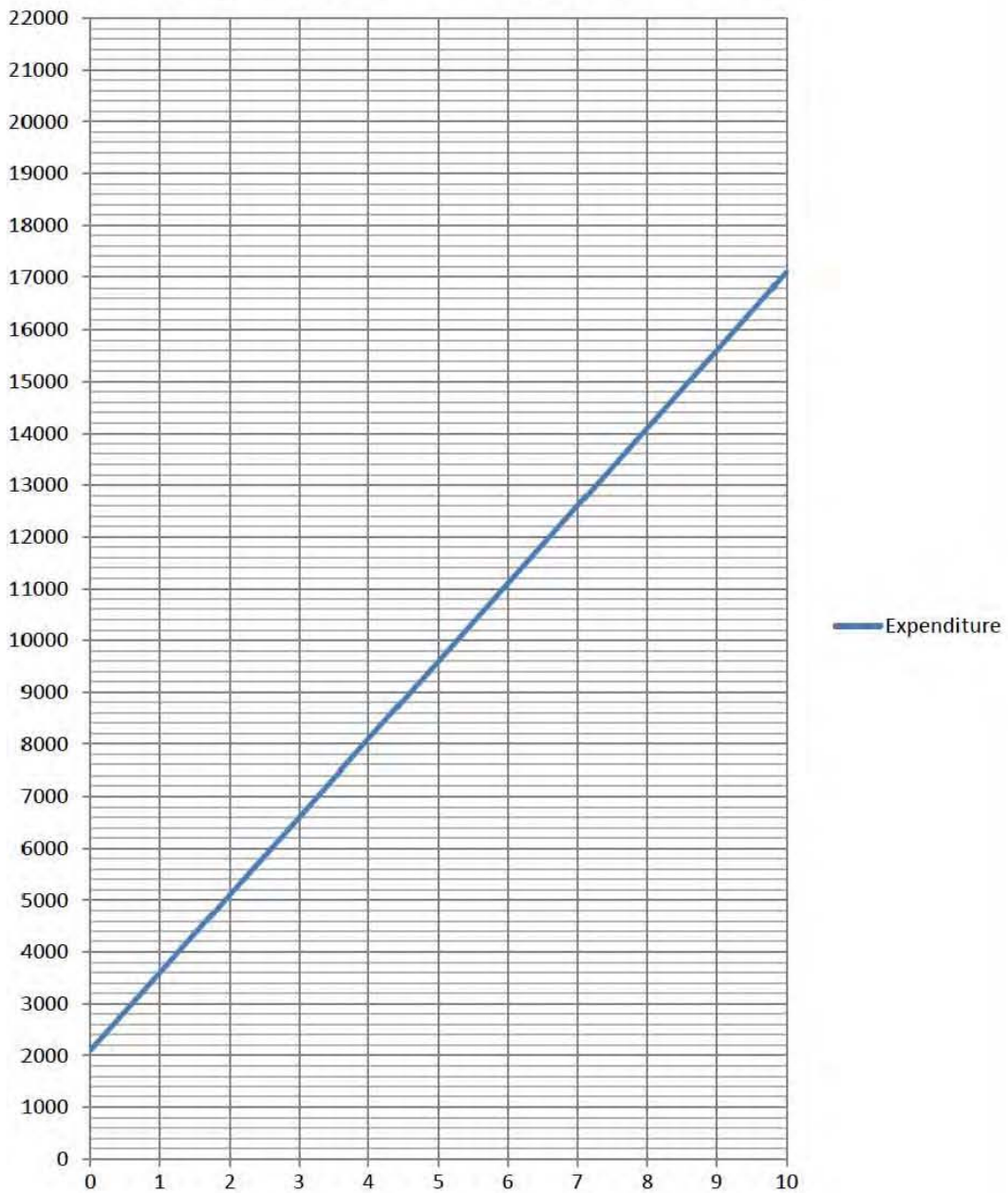
ANNEXURE B

NAME:

QUESTION 4.3

COMPLETE AND SUBMIT WITH YOUR ANSWER BOOK.

Eddy's income and expenditure graph



Mathematical Literacy Level 4 Trial Examination Paper 2, September 2011

QUESTION 1

1.1 Tiny is selling pies and samoosas at the college cafeteria. The table given below shows the sales of pies and samoosas. Study the table and answer the questions that follow.

ITEM	FEBRUARY	MARCH	APRIL	MAY	JUNE
PIES	700	656	500	525	400
SAMOOSAS	367	450	750	800	500

1.1.1 Which item showed the greater sales for this period?
Show ALL calculations. (3)

1.1.2 Sketch a line graph that shows the sales of pies and samoosas for this period. Clearly label the graph.
Use the graph paper on ANNEXURE A and submit it with the ANSWER BOOK. (9)

1.1.3 Calculate the mean sales for pies. (3)

1.1.4 What is the range for pies? (3)

1.1.5 Determine the median for samoosas. (3)

1.1.6 Give ONE possible reason for the sharp decrease in sales from May to June. (2)

1.3 Sally is renting an office for her catering company. The table below shows the cost of renting or buying a property in town in 2011. Study the table and answer the questions that follow.

Type of Property	Rent per month	Value of property	IF YOU BUY	
			Monthly Installment	Repayment Period
Office in town	R 2500	R 450 000	R 4784	25 years

1.2.1 How much would Sally have paid for the property over 25 years if she decided to rent? (3)

1.2.2 How much would Sally have paid for the property over 25 years if she decided to buy? (3)

1.2.3 The value of the property is R 450 000. It is estimated that the value of the property will increase by 3.5% per year. What will the value of the property be after 3 years?

Formula: $A = P (1 + i)^n$ where:
 A = Final Amount (Compound Increase/ Decrease)
 P = Principal Amount/Loan Amount
 i = Interest Rate (convert to decimal form)
 n = Number of years (5)

1.2.4 Do you think it is better to rent or to buy? Motivate your choice. (2)

1.3 Sipiwe is invited for an interview that is to be held at the Head office in Durban on Friday at 10:00. She will travel from Cape Town. The time table for Greyhound coaches is supplied below. Study the table.

GREYHOUND COACHES			
CAPE TOWN – BLOEMFONTEIN – DURBAN			
Service Number		Gdbc1130	
		DEPARTURE TIMES	
	DEPARTURE POINTS		Daily
Cape Town	Greyhound Office, 1 Adderley Street	Dep	11:00
Beaufort West	Engen One Stop – Cnr Swartberg and Rice Street	Dep	17:30
Bloemfontein	Tourist Centre, Park Road (Next to Municipal Swimming Pool)	Dep	23:45
Welkom	Shell garage, Corner of Tempest and Koppie Alleen Streets	Dep	01:35
Ladysmith	One Stop Tugela – Engen Service Station	Dep	06:10
Pietermaritzburg	Greyhound African Link, Cnr Burger and Commercial Streets	Dep	08:15
Durban	Motor Coach terminal, New Durban Station	Arr	09:15

It takes Sipiwe approximately 25 minutes to get to the departure point in Cape Town. It takes approximately 10 minutes to walk from New Durban Station to the Head Office.

1.3.1 On which day of the week and at what time does Sipiwe have to leave her home in order to be at the departure point on time? Allow 10 minutes for anything that might happen. (3)

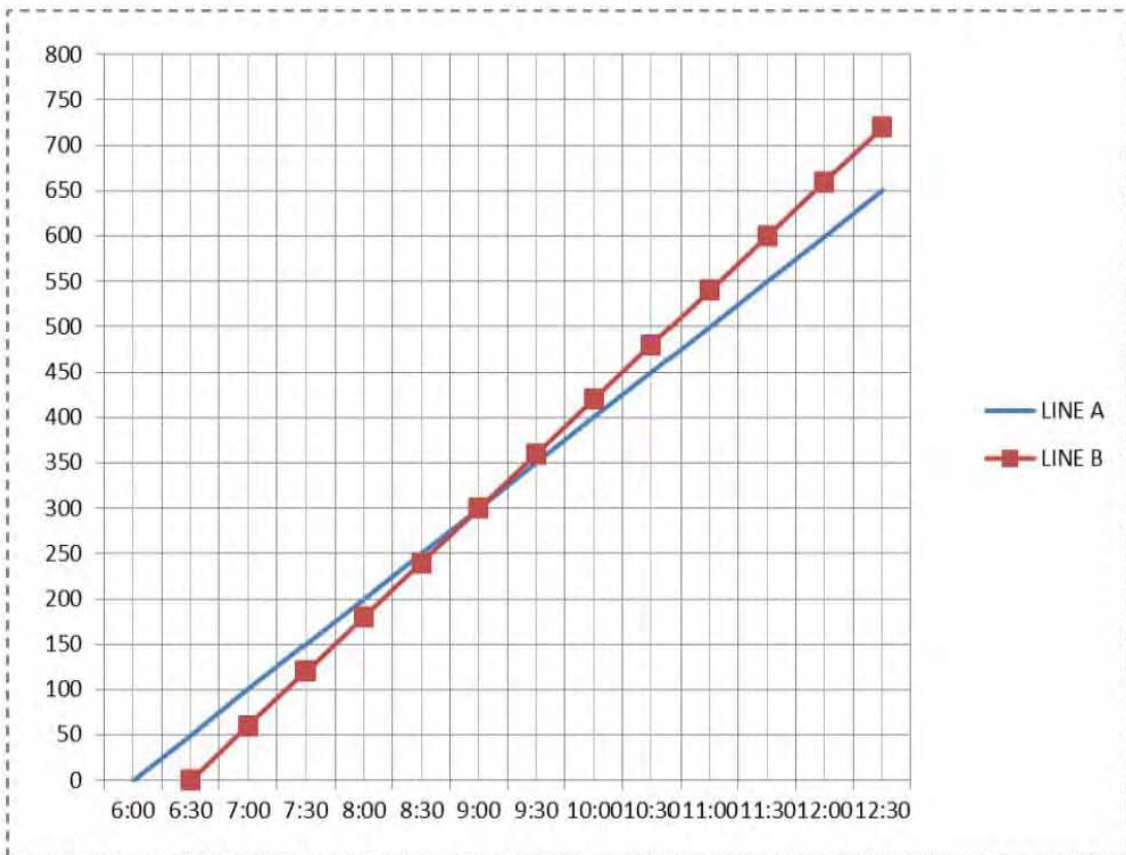
1.3.2 Determine the time Sipiwe will spend on the bus from Cape Town to the Durban station. (3)

1.3.2 Based on the above information, will Sipiwe be in time for the interview? Motivate your answer with the necessary calculations. (3)

[45]

QUESTION 2

Vusi and Thato were driving from the same place in Bloemfontein to P.E. Thato left at 6:00 and he was travelling at an average speed of 100 km/h (kilometers per hour). Vusi left later and she was travelling at an average speed of 120 km/h. The graph below shows the distances travelled by Vusi and Thato. Study the graph carefully and answer the questions that follow.



- 2.1 At what time did Vusi leave Bloemfontein? (2)
- 2.2 Consider the given information and the graph and write down the correct labels for the following:
- (a) Title for the graph (2)
 - (b) y-axis (2)
 - (c) x-axis (2)
- 2.3 Which line (line A or line B) on the graph represents Thato's journey? Explain your answer. (3)
- 2.4 Estimate from the graph at what time Vusi started overtaking Thato and the distance travelled at that time. (4)

2.5 Given below are the equations to calculate the distance travelled by Vusi and Thato.

d represents the distance in km and t represents the time of the day.

Vusi's travelling: $d = 100t - 600$

Thato's travelling: $d = 120t - 780$

Use the above equations to determine the following accurately:

2.5.1 The distance that was travelled by Vusi at 8:30 (Note: 8:30 = 8,5) (2)

2.5.2 The time of the day at which Thato reached 450km (5)

2.6 Use the following formula to calculate the time of the day at which Vusi will overtake Thato

Formula: $100t - 600 = 120t - 780$ (4)

[26]

QUESTION 3

During the 2006/2007 tax year, Luka (36 years old) worked for a hotel chain. Lukas's **gross annual** salary was **R127 650**.

Rates applicable to individuals		2006/2007	
Taxable Income	Rates of tax		
Rand			
0 – 122 000		+ 18% of each R1	
122 001 – 195 000	R20 410	+ 25% of amount over	R122 000
195 001 – 270 000	R40 210	+ 30% of amount over	R195 000
270 001 – 380 000	R62 710	+ 35% of amount over	R270 000
380 001 – 490 000	R101 210	+ 38% of amount over	R380 000
490 001 and more	R143 010	+ 40% of amount over	R490 000
Rebates:			
Primary rebate R8 280			
Additional rebate for persons 65 years or older R5 040			
Exemptions:			
Annual exemption on interest earned for individuals younger than 65 years R19 000			
Annual exemption on interest earned for individuals older than 65 years R27 500			

- UIF = 1% of the gross income
- Pension Fund = 7,5% of the gross income

- 3.1 Calculate his monthly contribution towards UIF (3)
- 3.2 Calculate his monthly contribution towards his pension fund. (3)
- 3.3 Use the above table and calculate Luka's monthly tax contribution (8)
- 3.4 What was his monthly net income? (4)
- 3.5 According to market researchers, people in Lukas's income bracket typically spend their money as shown in the table below.

Refer to the next table which lists the changes in CPI for each of the expenditure groups. Calculate how much he had to budget for items (a) to (d) for 2007. (8)

Expenditure Group	Typical monthly spend by Luka in January 2006	Percentage change in CPI for expenditure group	Anticipated monthly spend by Luka in January 2007
Food	R 2 280,00	9.3%	a
Clothing, footwear & accessories	R430,00	-10.9%	b
Housing & electricity	R1 626,00	9.2%	c
Transport	R834,00	6.8%	d
Medical & dental	R280,00	5.6%	R296,00
Insurance and funds	R456,00	-	R480,00
Personal care	R259,00	5.0%	R272,00
Communication	R259,00	0.2%	R260,00

[26]

QUESTION 4

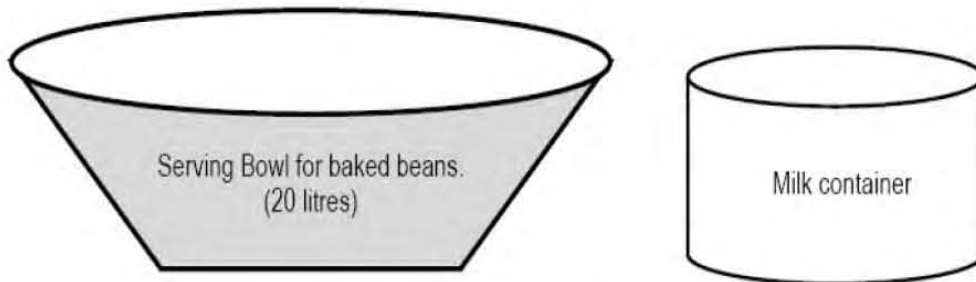
Vuyo's catering company was awarded a tender to cater for breakfast to 900 students at the college hostels. After thorough research Vuyo found that nearly all the students would like to have cornflakes, tea, bread and apricot jam for breakfast. 90% of these students would also like to have baked beans.



The table below shows the data Vuyo collected on the quantities of apricot jam and baked beans.

Item	Quantity	Price
Apricot jam	30 kg	R195,00
Baked beans	30 kg	R 150,00

Vuyo also bought serving bowls for the baked beans and large containers for milk.



Consider the above information and answer the questions that follow.

- 4.1 Determine the height of the milk container with a capacity of 30 litres when full and a radius of 20 cm. (1 litre = 1 000 ml = 1 000 cm³)

Formula: $V = \pi r^2 h$

$V = \text{volume}, r = \text{radius}, h = \text{height and } \pi = 3,14$ (5)

4.2 Vuyo estimates that each student will eat an average of 75 grams of baked beans.

4.2.1 Determine the quantity of baked beans in kg that the students will eat in one breakfast. Only 90% of students will eat baked beans. (5)

4.2.2 Calculate the number of 30 kg containers of baked beans that will be required to serve all the students. (3)

4.2.3 How much will it cost to serve baked beans for all the students per breakfast? (2)

4.3 A 25 kg apricot jam container has the following dimensions:

Length : 25 cm Breadth : 20 cm Height : 50 cm

Show by calculation that the total volume of the container corresponds with the weight of 25 kg. (1 000 cm³ = 1 kg)

Formula: Volume = length × breadth × height (5)

[20]

QUESTION 5

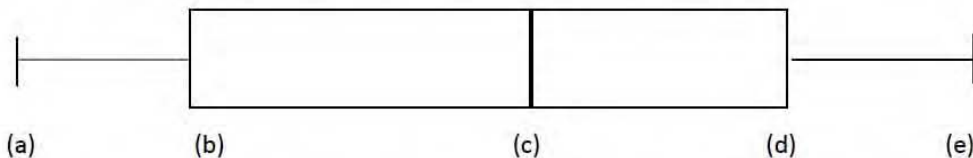
The picture below shows the dimensions of a netball court of a college. The court is divided into equal thirds.



FORMULAS	CIRCLE (use 3.14 for π)	RECTANGLE
AREA	πr^2	L x B
PERIMETER	$2\pi r$	2 (L + B)

- 5.1 If the students warm up before a match by running the perimeter of the court 5 times, how far would they have to run? (5)
- 5.2 The college decided to paint the goal shooting and centre areas (those are the two semi-circles and circle in the middle) a darker colour. Calculate the total area to be painted. (7)
- 5.3 The paint has a coverage of 4 m² per litre. How many litres of paint will be needed? (2)
- 5.4 It is cheaper to buy paint in 5 litre tins. How many will he have to buy? (3)
- 5.6 The following represents the last 10 scores the college netball team obtained during the season.

19 12 18 25 9 14 28 16 23 12



- 5.6.1 Give the correct term for points (a) to (e) on the box-and-whisker plot. (8)
- 5.6.2 Use the scores given above to calculate the value of (a) to (e). (8)

[33]

ANNEXURE A (Question 1.1.2)

(9 marks)

STUDENT :

A large rectangular area containing horizontal dotted lines for writing, with a solid vertical line on the left side and a solid horizontal line at the bottom.

APPENDIX G

Academic results for full Level 4 class

	MLIT FINAL RESULT	MLIT TOTAL ICASS	TRIAL P1	TRIAL P2	TRIAL AVERAGE RESULT	ENG TOTAL ICASS
	29	24	39	23	31	28
	24	15	40	20	30	46
A15	29	21	36	21	29	40
A19	36	27	47	30	39	36
A20	27	21	36	13	25	33
B4	36	29	49	17	33	44
	25	20	31	17	24	31
B10	24	24	31	21	26	46
B12	14	14	23	13	18	37
B20	22	16	32	17	25	36
B21	37	25	37	31	34	36
	18	19	17	18	18	13
B24	27	25	34	25	30	42
	29	22	23	29	26	26
	28	27	42	26	34	28
	35	35	37	35	36	37
	41	35	*	40	40	55
A6	33	31	51	18	35	51
A12	40	37	47	36	42	37
	41	32	50	31	41	45
A18	39	33	48	38	43	33
A21	39	32	53	35	44	36
	30	30	27	16	22	28
	36	30	42	29	36	45
A27	31	38	42	29	36	45
	33	34	56	31	44	45
	44	31	59	37	48	42
	33	31	48	23	36	49
	42	34	57	35	46	51
B17	37	30	43	38	41	39
	31	32	*	31	31	42
	*	44	51	42	47	47
	41	42	61	43	52	39
	51	47	59	38	49	38
	47	43	57	35	46	33
	48	47	63	62	63	45
	42	40	57	33	45	27
	46	43	53	42	48	46

	MLIT FINAL RESULT	MLIT TOTAL ICASS	TRIAL P1	TRIAL P2	TRIAL AVERAGE RESULT	ENG TOTAL ICASS
	40	41	49	34	42	35
	*	47	59	42	51	39
	48	45	67	48	58	41
	57	53	64	56	60	45
	63	59	66	61	64	47
	56	57	69	58	64	61
	65	60	65	52	59	59
AVE	37	34	47	33	40	40

APPENDIX H

Academic results for student sample

CODE	MLIT FINAL RESULT	MLIT TOTAL ICASS	TRIAL P1	TRIAL P2	TRIAL AVERAGE RESULT	ENG TOTAL ICASS
A6	33	31	51	18	37	51
A12	40	37	47	36	34	37
A15	29	21	36	21	29	40
A18	39	33	48	38	39	33
A19	36	27	47	30	30	36
A20	27	21	36	13	27	33
A21	39	32	53	35	35	36
A27	31	38	42	29	32	45
A24	36	29	49	17	31	44
B10	24	24	31	21	24	46
B12	14	14	23	13	27	37
B17	37	30	43	38	31	39
B20	22	16	32	17	29	36
B21	37	25	37	31	33	36
B24	27	25	34	25	30	42
AVE	31	27	41	25	31	39

APPENDIX I

Summary of complexity indicators per item

		PTS	TOPIC	LINGUISTIC COMPLEXITY INDEX	COGNITIVE DEMAND (DIFFICULTY)	COGNITIVE DOMAIN	COGNITIVE COMPLEXITY LEVEL	COGNITIVE COMPLEXITY INDEX	GUNNING FOG	FLESCH KINCAID	FLESCH READING EASE
QUESTION 1	1.2	2	1	12.00	1	1	1	2	7.08	5.15	77.88
	1.3	2	1	11.00	1	1	1	2			
	1.4	3	1	36.00	2	1	2	6			
	1.5	3	1	14.75	2	1	2	6			
	1.6	3	1	11.00	1	1	1	3			
	1.7	2	1	8.50	1	1	1	2			
	1.8	3	4	12.25	2	1	2	6			
	1.9	3	3	10.38	2	1	2	6			
	1.10	3	1	38.00	2	2	3	9			
	1.11	2	1	11.89	1	2	2	4			
QUESTION 2	2	LEAD-IN TEXT		13.78					12.70	10.74	52.42
	2.1	1	5	20.00	1	1	1	1			
	2.2	1	5	22.00	1	1	1	1			
	2.3	1	5	16.00	1	1	1	1			
	2.4	2	5	26.00	2	2	3	6			
	2.5	2	5	31.00	1	2	2	4			
	2.6	2	5	46.00	2	2	3	6			
	2.7.1	3	5	52.00	2	2	3	9			
	2.7.2	4	5	58.00	2	2	3	12			
QUESTION 3	3	11	3	13.13	3	4	6	66	8.11	8.51	59.51
QUESTION 4	4	LEAD-IN TEXT		16.75					8.56	7.29	68.62
	4.1	LEAD-IN TEXT		11.67							
	4.1.1	3	2	11.00	2	2	3	9			
	4.1.2	4	2	10.75	1	2	2	8			
	4.2	2	2	11.25	2	3	4	8			
	4.3	6	2	13.22	3	2	4	24			
	4.4	2	2	18.00	1	3	3	6			
QUESTION 5	5	LEAD-IN TEXT		24.00					11.20	9.09	65.79
	5.1	1	4	19.00	1	1	1	1			
	5.2	1	4	31.00	1	1	1	1			
	5.3	1	4	20.00	1	1	1	1			
	5.4	2	4	29.00	3	3	5	10			

		PTS	TOPIC	LINGUISTIC COMPLEXITY INDEX	COGNITIVE DEMAND (DIFFICULTY)	COGNITIVE DOMAIN	COGNITIVE COMPLEXITY LEVEL	COGNITIVE COMPLEXITY INDEX	GUNNING FOG	FLESCH KINCAID	FLESCH READING EASE
	5.5	4	2	10.28	1	3	3	12			
	5.6.1	3	4	12.25	2	2	3	9			
	5.6.2	3	4	12.50	2	2	3	9			
	5.6.3	2	4	14.00	1	2	2	4			
QUESTION 6	6.1	LEAD-IN TEXT		20.25					7.59	4.95	80.20
	6.1.1	2	5	18.00	1	1	1	2			
	6.1.2	3	5	16.00	1	1	1	3			
	6.1.3	2	5	16.00	1	1	1	2			
	6.1.4	2	5	19.00	3	2	4	8			
	6.2	LEAD-IN TEXT		23.25					8.66	9.90	57.18
	6.2.1	2	5	18.00	1	1	1	2			
	6.2.2	2	5	29.00	1	1	1	2			
	6.2.3	2	5	31.00	2	2	3	6			
	6.2.4	2	5	30.00	2	2	3	6			
	6.2.5	2	5	33.00	1	2	2	4			
6.2.6	2	5	27.00	3	2	4	8				
QUESTION 7	7.1	3	3	22.25	1	2	2	6	7.31	6.13	74.73
	7.2	3	3	10.56	2	2	3	9			
	7.3.1	2	3	10.33	1	2	2	4			
	7.3.2	3	3	12.00	2	3	4	12			
	7.4	LEAD-IN TEXT		15.22							
	7.4.1	2	3	28.00	1	1	1	2			
	7.4.2	2	3	28.00	2	2	3	6			
	7.4.3	2	3	28.00	2	2	3	6			
	7.4.4	1	3	28.00	1	3	3	3			
QUESTION 8	8	LEAD-IN TEXT		22.25					7.57	5.00	83.64
	8.1	3	4	8.75	1	2	2	6			
	8.2	3	4	26.00	2	2	3	9			
	8.3	2	4	13.00	1	1	1	2			
	8.4	3	4	11.44	1	2	2	6			
QUESTION 9	9	LEAD-IN TEXT		12.28					11.00	8.68	60.07
	9.1	1	2	29.00	1	1	1	1			
	9.2	2	2	25.00	1	1	1	2			
	9.3	2	2	27.00	1	1	1	2			
QUESTION 1	1.1	LEAD-IN TEXT		8.67					6.93	5.22	73.36

		PTS	TOPIC	LINGUISTIC COMPLEXITY INDEX	COGNITIVE DEMAND (DIFFICULTY)	COGNITIVE DOMAIN	COGNITIVE COMPLEXITY LEVEL	COGNITIVE COMPLEXITY INDEX	GUNNING FOG	FLESCH KINCAID	FLESCH READING EASE
	1.1.1	3	5	7.50	2	1	2	6	7.34	5.69	73.35
	1.1.2	9	5	9.00	2	2	3	27			
	1.1.3	3	5	10.00	2	1	2	6			
	1.1.4	3	5	11.00	1	1	1	3			
	1.1.5	3	5	10.00	2	1	2	6			
	1.1.6	2	5	22.00	1	4	4	8			
	1.2	LEAD-IN TEXT		12.11							
	1.2.1	3	3	30.00	2	3	4	12			
	1.2.2	3	3	30.00	2	3	4	12			
	1.2.3	5	3	12.31	2	3	4	20			
	1.2.4	3	3	9.50	1	4	4	12			
	1.3	LEAD-IN TEXT		10.25							
	1.3.1	3	5	20.25	3	2	4	12			
	1.3.2	3	1	23.00	2	2	3	9			
1.3.3	3	1	13.25	3	4	6	18				
QUESTION 2	2	LEAD-IN TEXT		10.80					10.60	8.75	60.42
	2.1	2	2	12.00	1	1	1	2			
	2.2.a	2	2	32.00	1	1	1	2			
	b	2	2	31.00	1	1	1	2			
	c	2	2	31.00	1	1	1	2			
	2.3	3	2	9.00	1	2	2	6			
	2.4	4	2	30.00	1	2	2	8			
	2.5	LEAD-IN TEXT		19.50							
	2.5.1	2	2	35.00	3	3	5	10			
	2.5.2	5	2	33.00	3	3	5	25			
2.6	4	2	33.00	3	3	5	20				
QUESTION 3	3	LEAD-IN TEXT		41.00					10.50	8.61	59.01
	3.1	3	3	16.00	3	2	4	12			
	3.2	3	3	20.00	3	2	4	12			
	3.3	8	3	18.00	3	3	5	40			
	3.4	4	3	11.00	3	3	5	20			
	3.5	8	3	21.56	2	4	5	40			
QUESTION 4	4	LEAD-IN TEXT		10.89					10.40	8.64	62.31
	4.1	5	4	35.00	1	3	3	15			
	4.2	LEAD-IN TEXT		25.00							

		PTS	TOPIC	LINGUISTIC COMPLEXITY INDEX	COGNITIVE DEMAND (DIFFICULTY)	COGNITIVE DOMAIN	COGNITIVE COMPLEXITY LEVEL	COGNITIVE COMPLEXITY INDEX	GUNNING FOG	FLESCH KINCAID	FLESCH READING EASE
	4.2.1	5	1	14.00	3	3	5	25			
	4.2.2	3	1	32.00	2	2	3	9			
	4.2.3	2	1	24.00	2	3	4	8			
	4.3	5	4	18.00	1	3	3	15			
QUESTION 5	5	LEAD-IN TEXT		16.00					6.95	5.89	75.70
	5.1	5	4	38.00	2	3	4	20			
	5.2	7	4	15.50	2	3	4	28			
	5.3	2	4	9.25	2	2	3	6			
	5.4	3	1	9.75	2	2	3	9			
	5.5	LEAD-IN TEXT		25.00							
	5.5.1a	1	5	20.00	1	1	1	1			
	b	2	5	20.00	1	1	1	2			
	c	2	5	20.00	1	1	1	2			
	d	2	5	20.00	1	1	1	2			
	e	1	5	20.00	1	1	1	1			
	5.5.2a	1	5	22.00	1	2	2	2			
	b	2	5	22.00	3	2	4	8			
	c	2	5	22.00	2	2	3	6			
	d	2	5	22.00	3	2	4	8			
e	1	5	22.00	1	2	2	2				

APPENDIX J

Error count per item and error type (n = 15)

ITEM	TOPIC	PTS	GRAPHIC	MATH CALCULATION	READING	WRITING	CARELESSNESS	QUESTION	INDETERMINATE	NO ATTEMPT	TOTAL ERRORS	TOTAL SCORING SOME MARKS	TOTAL SCORING ZEROS	TOTAL SCORING FULL MARKS	TOTAL
1.2	1	2		7						4	11	0	11	4	15
1.3	1	2		10							10	0	10	5	15
1.4	1	3		4	1		1				6	2	4	9	15
1.5	1	3		3	2						5	1	4	10	15
1.6	1	3		11						3	14	2	12	1	15
1.7	1	2			1						1	1	0	14	15
1.8	4	3		9			1			1	11	0	11	4	15
1.9	3	3		14	1						15	5	10	0	15
1.10	1	3		12							12	0	12	3	15
1.11	1	2		9							9	1	8	6	15
2.1	5	1								1	1	0	1	14	15
2.2	5	1		1							1	0	1	14	15
2.3	5	1	10						1		11	0	11	4	15
2.4	5	2	1	1	8		1			1	12	1	11	3	15
2.5	5	2			3	4					7	0	7	8	15
2.6	5	2	1	10					1		12	0	12	3	15
2.7.1	5	3		1	8						9	5	4	6	15
2.7.2	5	4		1	6						7	3	4	8	15
3	3	11	6	8					1		15	14	1	0	15
4.1.1	2	3		4							4	3	1	11	15
4.1.2	2	4		2							2	2	0	13	15
4.2	2	2	7	2	1						10	0	10	5	15
4.3	2	6	11		2					1	14	13	1	1	15
4.4	2	2	1	8	1				1	1	12	0	12	3	15
5.1	4	1									0	0	0	15	15
5.2	4	1									0	0	0	15	15
5.3	4	1	2								2	0	2	13	15
5.4	4	2	6	1							7	0	7	8	15
5.5	2	4		8	2					3	13	7	6	2	15
5.6.1	4	3	2	3	1			8		1	15	0	15	0	15
5.6.2	4	3	1	2	2			8		2	15	0	15	0	15
5.6.3	4	2		3	3					4	10	0	10	5	15
6.1.1	5	2		6						1	7	0	7	8	15
6.1.2	5	3		9							9	0	9	6	15
6.1.3	5	2		10							10	0	10	5	15
6.1.4	5	2		10						3	13	3	10	2	15
6.2.1	5	2	9	2				1			12	0	12	3	15
6.2.2	5	2	1					5			6	1	5	9	15
6.2.3	5	2			1			2			3	0	3	12	15
6.2.4	5	2	4	3	8						15	0	15	0	15
6.2.5	5	2	2		1			5			8	1	7	7	15
6.2.6	5	2	8		5						13	1	12	2	15
7.1	3	3		9						1	10	0	10	5	15
7.2	3	3		12	1						13	0	13	2	15
7.3.1	3	2		10							10	0	10	5	15

ITEM	TOPIC	PTS	GRAPHIC	MATH CALCULATION	READING	WRITING	CARELESSNESS	QUESTION	INDETERMINATE	NO ATTEMPT	TOTAL ERRORS
7.3.2	3	3		7			1			1	9
7.4.1	3	2	1	1							2
7.4.2	3	2	11				1				12
7.4.3	3	2	9								9
7.4.4	3	1				1					1
8.1	4	3	1	6						1	8
8.2	4	3	4	7				1	2		14
8.3	4	2		11					1		12
8.4	4	3									0
9.1	2	1	10						2		12
9.2	2	2	10								10
9.3	2	2		3							4
1.1.1	5	3				6					6
1.1.2	5	9	11								11
1.1.3	5	3		5							5
1.1.4	5	3		4					1		5
1.1.5	5	3		6							6
1.1.6	5	2		1		5					6
1.2.1	3	3			7						7
1.2.2	3	3	1		7						8
1.2.3	3	5		9							9
1.2.4	3	3									0
1.3.1	5	3		2	5				1		8
1.3.2	1	3		9		1					10
1.3.3	1	3			2	5					7
2.1	2	2	1								1
2.2.a-c	2	2	10		1	3					14
2.3	2	3				2					2
2.4	2	4	2	6							8
2.5.1	2	2		11	1						12
2.5.2	2	5		12	1				1		14
2.6	2	4		13					2		15
3.1	3	3		7	3				1		12
3.2	3	3		8	3				1		12
3.3	3	8		10	1				1		12
3.4	3	4		10	2				2		14
3.5	3	8		9		4					13
4.1	4	5		3	11				1		15
4.2.1	1	5		7	5				2		14
4.2.2	1	3		8	7						15
4.2.3	1	2		4	3				3		12
4.3	4	5		3	8				2		13
5.1	4	5	9	3	2				1		15
5.2	4	7	10	4	1						15
5.3	4	2	1	9	3				2		15
5.4	1	3		7	5				1		13
5.5.1a	5	1					14		1		15
5.5.2a	5	1					14		1		15
TOTAL			163	410	136	31	33	29	5	58	869

TOTAL SCORING SOME MARKS	TOTAL SCORING ZEROS	TOTAL SCORING FULL MARKS	TOTAL
2	7	6	15
1	1	13	15
0	12	3	15
0	9	6	15
0	1	14	15
1	7	7	15
1	13	1	15
1	11	3	15
0	0	15	15
0	12	3	15
10	0	5	15
4	0	11	15
4	2	9	15
6	5	4	15
0	5	10	15
0	6	9	15
0	6	9	15
4	3	8	15
1	7	7	15
0	9	6	15
0	0	15	15
1	7	7	15
0	10	5	15
4	3	8	15
0	1	14	15
10	4	1	15
2	0	13	15
0	8	7	15
0	12	3	15
0	14	1	15
0	15	0	15
3	9	3	15
2	10	3	15
0	12	3	15
0	14	1	15
10	3	2	15
0	15	0	15
1	13	1	15
2	13	0	15
1	11	3	15
10	3	2	15
0	15	0	15
1	14	0	15
3	12	0	15
0	13	2	15
0	15	0	15
0	15	0	15
0	15	0	15
151	718	526	1395

APPENDIX K

Marks lost per error type per item

ITEM	TOPIC	PTS	POINTS LOST: GRAPHIC	POINTS LOST: CALCULATION	POINTS LOST: READING	POINTS LOST: WRITING	TOTAL POINTS LOST	POINTS AVAILABLE = 15 X PTS
1.2	1	2	0	14	0	0	14	30
1.3	1	2	0	20	0	0	20	30
1.4	1	3	0	9	0	0	9	45
1.5	1	3	0	6	6	0	12	45
1.6	1	3	0	27	0	0	27	45
1.7	1	2	0	0	0	0	0	30
1.8	4	3	0	27	0	0	27	45
1.9	3	3	0	27	3	0	30	45
1.10	1	3	0	36	0	0	36	45
1.11	1	2	0	16	0	0	16	30
2.1	5	1	0	0	0	0	0	15
2.2	5	1	0	1	0	0	1	15
2.3	5	1	10	0	0	0	10	15
2.4	5	2	2	0	16	0	18	30
2.5	5	2	0	0	6	8	14	30
2.6	5	2	2	20	0	0	22	30
2.7.1	5	3	0	0	12	0	12	45
2.7.2	5	4	0	0	16	0	16	60
4.1.1	2	3	0	3	0	0	3	45
4.1.2	2	4	0	0	0	0	0	60
4.2	2	2	14	4	2	0	18	30
4.3	2	6	0	0	0	0	0	90
4.4	2	2	2	16	2	0	20	30
5.1	4	1	0	0	0	0	0	15
5.2	4	1	0	0	0	0	0	15
5.3	4	1	2	0	0	0	0	15
5.4	4	2	12	2	0	0	14	30
5.5	2	4	0	12	0	0	12	60
5.6.1	4	3	6	9	3	0	18	45
5.6.2	4	3	3	6	6	0	15	45
5.6.3	4	2	0	6	6	0	12	30
6.1.1	5	2	0	0	12	0	12	30
6.1.2	5	3	0	27	0	0	27	45
6.1.3	5	2	0	20	0	0	20	30
6.1.4	5	2	0	14	0	0	14	30
6.2.1	5	2	18	4	0	0	22	30
6.2.2	5	2	0	0	0	0	0	30
6.2.3	5	2	0	0	2	0	2	30
6.2.4	5	2	8	6	16	0	30	30
6.2.5	5	2	2	0	2	0	4	30
6.2.6	5	2	14	0	10	0	24	30
7.1	3	3	0	27	0	0	27	45
7.2	3	3	0	36	3	0	39	45
7.3.1	3	2	0	20	0	0	20	30
7.3.2	3	3	0	15	0	0	15	45

ITEM	TOPIC	PTS	POINTS LOST: GRAPHIC	POINTS LOST: CALCULATION	POINTS LOST: READING	POINTS LOST: WRITING	TOTAL POINTS LOST	POINTS AVAILABLE = 15 X PTS
7.4.1	3	2	2	0	0	0	2	30
7.4.2	3	2	22	0	0	0	22	30
7.4.3	3	2	18	0	0	0	18	30
7.4.4	3	1	0	0	0	1	1	15
8.1	4	3	0	18	0	0	18	45
8.2	4	3	9	21	0	0	30	45
8.3	4	2	0	18	0	0	18	30
8.4	4	3	0	0	0	0	0	45
9.1	2	1	10	0	0	0	10	15
9.2	2	2	0	0	0	0	0	30
9.3	2	2	0	0	0	0	0	30
1.1.1	5	3	0	0	0	6	0	45
1.1.2	5	9	45	0	0	0	45	135
1.1.3	5	3	0	15	0	0	15	45
1.1.4	5	3	0	12	0	0	12	45
1.1.5	5	3	0	18	0	0	18	45
1.1.6	5	2	0	2	0	10	12	30
1.2.1	3	3	0	0	21	0	21	45
1.2.2	3	3	3	0	18	0	21	45
1.2.3	3	5	0	45	0	0	45	75
1.2.4	3	3	0	0	0	0	0	45
1.3.1	5	3	0	6	12	0	18	45
1.3.2	1	3	0	27	0	3	30	45
1.3.3	1	3	0	0	6	3	9	45
2.1	2	2	2	0	0	0	2	30
2.2.a-c	2	2	0	0	2	6	8	30
2.3	2	3	0	0	0	0	0	45
2.4	2	4	8	24	0	0	32	60
2.5.1	2	2	0	22	2	0	24	30
2.5.2	2	5	0	60	5	0	65	75
2.6	2	4	0	52	0	0	52	60
3.1	3	3	0	21	0	0	21	45
3.2	3	3	0	24	3	0	27	45
3.3	3	8	0	80	8	0	88	135
3.4	3	4	0	40	8	0	48	60
3.5	3	8	0	0	0	0	0	135
4.1	4	5	0	15	55	0	70	75
4.2.1	1	5	0	35	20	0	55	75
4.2.2	1	3	0	24	15	0	39	45
4.2.3	1	2	0	8	4	0	12	30
4.3	4	5	0	0	0	0	0	75
5.1	4	5	45	15	10	0	70	75
5.2	4	7	70	21	7	0	98	105
5.3	4	2	0	0	0	0	0	30
5.4	1	3	0	21	15	0	36	45
5.5.1a	5	1	0	0	0	0	0	15
5.5.2a	5	1	0	0	0	0	0	15
TOTAL			329	1074	334	37	1774	3990

GLOSSARY OF ABBREVIATIONS AND ACRONYMS

FET – Further Education and Training

NQF – National Qualifications Framework

NEET – Not in education, employment or training

NC(V) – National Certificates (Vocational)

NCS – National Certificate Senior

DoE – Department of Education

PISA – Programme for International Student Assessment

UNESCO – United Nations Educational, Scientific and Cultural Organisation

OECD – Organisation for Economic Co-operation and Development

PIAAC – Programme for the International Assessment of Adult Competencies

PIRLS – Programme for International Reading Literacy Study

IEA – International Association for the Evaluation of Educational Achievement

IALS – International Adult Literacy Survey

DHET – Department of Higher Education and Training

DBE – Department of Basic Education

BICS – Basic Interpersonal Communication Skills

CALP – Cognitive Academic Language Proficiency

EAQUALS – European Association for Quality Language Services

CIGE – Core Inventory for General English

CEFR – Common European Framework of Reference

ARI – Automated Readability Index

TIMSS – Trends in Mathematics and Science Survey

ICASS – Internal Continuous Assessment

LCI – Language Complexity Index

CCI – Cognitive Complexity Index

CCL – Cognitive Complexity Level

VAT – Value Added Tax

GMT – Greenwich Mean Time

FIFA - Fédération Internationale de Football Association

SA – South Africa

DVD – Digital Video Disc

USA – United States of America

No. - Number

Dep. - Departure

Arr. – Arrival

P.E. – Port Elizabeth

UIF – Unemployment Insurance Fund

CPI – Consumer Price Index

US – United States

CD – Compact Discs