

**STUDENTS' VAN HIELE LEVELS OF GEOMETRIC THOUGHT  
AND CONCEPTION IN PLANE GEOMETRY: A COLLECTIVE  
CASE STUDY OF NIGERIA AND SOUTH AFRICA**

**A thesis submitted in fulfilment of the requirements for the degree of**

**DOCTOR OF PHILOSOPHY**

**of**

**RHODES UNIVERSITY**

**by**

**HUMPHREY UYOUYO ATEBE**

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## APPENDIX 1

### LETTERS OF INTRODUCTION AND CONSENT FORMS

#### APPENDIX 1.A

##### Supervisor's letter of introduction

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**RHODES UNIVERSITY**

*Where leaders learn*

Education Department, Grahamstown.

Tel: +27 (046) 603 8383/4

13 June 2006

Dear Sir

Mr Humphrey Atebe is a PhD student in the Education Department doing research in the field of spatial conceptualization in geometry education. He is particularly interested in applying the van Hiele model of geometric thinking in explaining possible reasons why learners find geometry difficult. Mr Atebe's research consists of a collective case study of Nigerian and South African Grade 10 – 12 learners. The study promises to yield very interesting and significant data. His proposal was recently passed by the Education Department Higher Degrees committee with its full support.

I am approaching you as Mr Atebe's supervisor and Head of the RU Education Department to invite your school to be the main participant in the South African aspect of the study. He is seeking the participation of 72 Grade 10 – 12 learners and their Mathematics teachers from your school to complete a number of tests. Mr Atebe will ensure that the participation of the learners will incur minimal interference with any school activities. He further ensures that the names of your learners, teachers and school will not be revealed in the thesis.

I encourage your participation in this study as it will contribute not only to the development of the learners and their teachers in their understanding of geometry, but also to the wider mathematics education community.

I thank you in advance for your co-operation and collaboration.

Yours sincerely,

Marc Schäfer (Dr)  
Head of Department  
Supervisor

## APPENDIX 1.B

### Researcher's letter of introduction to school Principals

---

Education Department  
Rhodes University  
P.O. Box 94  
Grahamstown 6140  
South Africa.

Date-----

The School Principal  
-----  
-----  
-----

Dear Sir

#### **PERMISSION TO DO RESEARCH IN YOUR SCHOOL.**

I am a PhD. Student at Rhodes University, South Africa, and I am currently undertaking my research study in the field of Mathematics Education. My study involves schools in Nigeria and South Africa. I am seeking your permission and support to conduct some of my research in your school.

My research study concerns an exploration into students' thinking levels about geometric shapes. Participants in this study would be a sample of grade 10, 11 and 12 mathematics learners and their teachers.

The students' role in this study will involve writing four sets of tests in geometry which have been carefully developed taking into account both the Nigerian and South African curricular requirements for the learners at the three grade levels. Kindly find details about the contents and form of these tests in the attached captioned 'Further details about the study'.

The teachers' role will be to assist in class organization during each test-taking exercise. Furthermore, because this study also seeks to investigate how geometry is taught in schools, one or two geometry lessons will be video-recorded in each of the three grades. The students' end-of-the-year scores in mathematics will be obtained (from the mathematics teachers) and compared with the students' tests' scores obtained in this study so as to establish a relationship.

The identity of the participants (students and teachers) and that of the school will be kept confidential. In other words, neither the names of the participants nor that of your school will be mentioned either in the final report of this study or any publication for which the information collected about your school may be used.

Please find attached a letter of introduction from my university and also a brief overview of my study.

I would be delighted to answer any question you may have concerning this study or the practical details of conducting the tests within the school setting. I will very much like to arrange a meeting with you to discuss this matter further. My phone number is:  
-----

I would very much appreciate your prompt and favourable response to this letter.

Yours truly,

Humphrey U. Atebe (Student No: \_\_\_\_\_).

### **Further Details about the Study**

My study is in the field of mathematics education. My research topic is titled *Students' van Hiele levels of geometric thought and conception in plane geometry: A collective case study of Nigeria and South Africa*. The study focuses on the geometric thinking levels of the Nigerian and South African mathematics learners based on both the Nigerian and South African curricula as mandated by the Nigerian National Policy on Education (1998) and the South African Curriculum Statement Grades 10 – 12 (2003).

The study utilizes a theory of levels of geometric thinking developed by two Dutch mathematics educators, Pierre van Hiele and his wife, Dina van Hiele-Geldof. The van Hiele Theory unravels and enables explorations into why many students encounter difficulties in their geometry courses.

In the more developed nations, like the U.K and the USA, research evidence has linked students' mathematical competencies to their geometric thinking levels. It has also been asserted that currently, the best description of students' thinking about geometric concepts is the van Hiele Theory.

Despite the fact that this theory has been applied to many curricula to improve geometry classroom instruction in many developed nations, little about this theory has been investigated in Nigeria and South Africa.

Only in the more recent times have a few research studies in South Africa utilized the van Hiele Theory. Van der Sandt and Nieuwoudt (2003), for example, made use of the van Hiele Theory to investigate grade 7 teachers' and prospective teachers' content knowledge of geometry in the North-West Province in South Africa. Their study yielded interesting results. Yet, the same cannot be said of the Eastern Cape.

My study seeks to investigate students' van Hiele levels of geometric thinking in Nigeria and South Africa. It also seeks to explore the structure of geometry instruction in both countries.

The tools for this investigation include four sets of well-designed tests in geometry to be written by 144 mathematics learners (72 from Nigeria and 72 from South Africa) drawn from grade 10, 11 and 12. One or two geometry classroom lessons will be video-taped in each of the three grade levels so as to explicate the nature and pattern

of geometry instruction in these grade levels. The tests that the students will write are as follows.

1. Terminology in Plane Geometry Test (TPGT): This is a multiple-choice test containing sixty questions. This test examines students' understanding of some basic terminology/concepts in geometry.
2. Geometric Items Sorting Test (GIST): This test will require the students to identify, describe and state relationships among the properties of some simple geometric shapes. These shapes will be given to them in the form of cardboard cut-outs. Each student will be given worksheets to record his/her activities.  
**N. B: Not everybody in the study will be involved in this test. Those to be involved will be determined by their performance on the TPGT.**
3. Conjecturing in Plane Geometry Test (CPGT): Worksheets will be given to the students for this activity. On the worksheets, they will be required to follow a set of step-by-step instructions that would enable them to discover the properties of and formulate conjectures about some geometric shapes.
4. The van Hiele Geometry Test (VHGT): This test consists of two parts. Part A is a 20-item multiple-choice objective test. Part B is an essay test and it contains three questions.

Each of these tests will last for about an hour, but no two or more tests will be written on the same day. I intend to administer the tests during the normal school hours, but with minimal interruptions on the normal school activities.

### **Benefits to your school**

Your school may benefit in the following ways by being involved in this study.

- The students will be exposed to a wide range of terminology and concepts in geometry that they may not as yet been privileged to learn.
- All the questions in the test items for this study are developed in such a way that they are consistent with the geometry curriculum meant for these students. By taking part in this study, your students will be privileged to identify some aspects of their geometry curriculum that warrant more of their attention.
- A very important benefit to your school for taking part in this study is that the mathematics teachers in your school may gain new insights into students' thinking about geometric shapes and may as a result plan their classroom instructions in ways that would further improve the students' understanding in geometry.

As stated earlier, please feel free to ask any question on any aspects of this study for which you need further clarifications.

Thank you.

Humphrey U. Atebe.

## APPENDIX 1.C

### Consent Form for the learners

---

**DEPARTMENT OF EDUCATION  
RHODES UNIVERSITY  
South Africa.**

#### Research Subject's Assent Form

**Title of the Study:** Students' van Hiele levels of geometric thought and conception in plane geometry: A collective case study of Nigeria and South Africa.

**Investigator:** Atebe, Humphrey Uyouyo.

**You are being asked to participate in a research study.**

#### **Purpose**

The purpose of this research study is to determine how much understanding you have had of the geometry that you are expected to learn in your current grade.

Your understanding of geometry meant for your grade would be measured using a theory of levels of geometric thinking developed by a husband-and-wife team of Dutch mathematics educators, Pierre van Hiele and his wife, Dina van Hiele-geldof. By taking part in this study, you will be assigned one of five levels of thinking about geometric concepts based on the van Hiele Theory.

Further, this study also intends to investigate how your assigned van Hiele level relates to your achievement in general mathematics (algebra and geometry) and geometry only. For this reason, your end-of-the-year examination scores in mathematics will be compared with your van Hiele test scores in order to establish a relationship.

#### **Your Role in this Study**

If you agree to take part in this study, you will be required to provide answers to **four sets** of questions about geometric concepts.

1. Terminology in Plane geometry Test (TPGT): This is a multiple-choice test containing sixty questions. You only need to choose the correct answer to each question from among four given options. This test examines your understanding of some basic terminology/concepts in geometry. The test requires neither a calculator nor does it involve numerical calculations.
2. Geometric Items Sorting Test (GIST): This test will require you to identify, describe and state relationships among the properties of some simple geometric shapes. These shapes will be given to you in the form of cardboard cut-outs. You will be given worksheets to record your activities. Again, this test does not require a calculator.

**N. B: Not everybody in the study will be involved in this test. Those to be involved will be determined by their performance in the test on terminology.**

3. Conjecturing in Plane Geometry Test (CPGT): Worksheets will be given to you for this activity. On the worksheets, you will be required to follow a set of step-by-step instructions that would enable you to discover the properties of and formulate conjectures about some geometric shapes. You will need construction tools (e.g. mathematical set) for this activity.
4. The van Hiele Geometry Test (VHGT): This test consists of two parts. Part A is a 20-item multiple-choice objective test. You will only need to choose the correct answer from among five given options and shade your answer in the computer answer sheet that you will be given. Part B contains three questions. Question 1 requires you to apply a theorem to solve a problem; question 2 requires you to write statements to complete a simple proof, and in question 3, you will need to write a simple proof that you have studied.

### **Costs to You**

You do not need to pay anything in order to take part in this study.

### **Payment to You**

No money will be paid to you for being in this study.

### **Risks/Discomforts**

There are no risks involved in this study. There are also no discomforts other than the ones that you are used to in writing a test or an examination in your school.

### **Benefits for Participating**

- One direct benefit for being in this study is that you will be exposed to a wide range of terminology and concepts in geometry that you may not as yet been privileged to learn.
- More importantly, all the questions in the tests for this study are drawn with due consultations with your curriculum. By taking part in this study, you are privileged to know those aspects of your curriculum that demand greater attention than you probably have experienced.
- A less direct, but very important benefit for being in this study is that your teachers may better be informed about your weaknesses and may plan their classroom instructions in ways that would better improve your understanding of geometry.

### **Your Rights**

- Although you will be required to write your name in each of the answer sheets given to you in this study (this is done to enable me match your van Hiele test scores with your end-of-the-year examination score in mathematics), your performance in the tests will not be revealed to anybody except me.
- In particular, no mention of your name would be made or associated with any of your scores in the tests in the final report of this study.
- You do not have to be in this study if you do not want to be.
- You can change your mind and withdraw from this study at any stage without any problem.

**Questions**

Should you have any questions about this study, you can ask your mathematics teacher or your parents, or talk to me about them.

If you sign below, it means that you have read the information on this sheet, and you would like to be in this study.

**Your Name:** .....

**Signature:** ..... **Date:** .....

## APPENDIX 1.D

### Consent Form for the parents of 'minor' subjects

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**DEPARTMENT OF EDUCATION  
RHODES UNIVERSITY  
South Africa.**

#### **Research Permission Form for Parents of Minor Subjects**

**Title of the Study:** Students' van Hiele levels of geometric thought and conception in plane geometry: A collective case study of Nigeria and South Africa.

**Investigator:** Atebe, Humphrey Uyoyoyo (Student No :.\_\_\_\_\_)

I am currently a PhD. student at Rhodes University and wish to undertake my research in the field of Mathematics Education. I am seeking your permission to allow your child to participate in this study.

#### **Purpose**

In Nigeria and South Africa, mathematics occupies a central position in the school curricula because of its useful links to many other fields of human endeavours.

In the more developed nations of the world, like the USA, research evidence has linked students' mathematical competencies to their geometric thinking levels. Currently, the best description of students' thinking about geometric shapes (or concepts) is offered by a theory of levels of geometric thinking developed by two Dutch mathematics teachers, Pierre van Hiele and his wife, Dina van Hiele-Geldof.

Although this theory has been applied to many curricula to improve geometry classroom instructional practices in many developed nations, little about this theory has been investigated in Nigeria and South Africa.

By allowing your child to participate in this study, you would be assisting the education community of mathematics educators to establish how applicable the van Hiele Theory is in an African context.

Your child is not alone in this study. In all, 144 students (72 in Nigeria and 72 in South Africa) will take part in this study.

#### **Procedures**

Should you agree to allow your child to be in this study, his/her part will involve writing four sets of tests in geometry. Two of these tests would be objective, in which case he/she will only need to choose the correct answer from a given number of options. One other test will require your child to sort geometric shapes in the form of cardboard cut-outs into various classes of figures. The fourth test will require your child to formulate conjectures about the properties of some geometric shapes.

In the test involving the sorting of geometric shapes, your child might be interviewed and his responses audio-recorded. Further, in trying to know about how your child is taught in geometry class, one or two lessons in which your child is being taught by his/her teacher will be video-recorded. Your child's end-of-the-year score in mathematics will be obtained from his/her school records and compared with his/her van Hiele geometry test score so as to establish a relationship.

Each test will last for about an hour, but no two or more tests will be written on the same day. All the tests will be taken during the normal school hours.

### **Risks**

There are no risks involved as a result of your child's participation in this study.

### **Benefits**

Your child may benefit in the following ways as a result of being in this study:

- He/she will be exposed to a wide range of terminology and concepts in geometry that he/she may not as yet been privileged to learn.
- All the questions in the tests for this study are developed in such a way that they are consistent with your child's school curriculum. By taking part in this study, your child will be privileged to identify some aspects of his/her geometry curriculum that warrant more of his/her attention.
- A less direct, but a very important benefit to your child for being in this study is that your child's teachers may better be informed about your child's capacity and may plan their classroom instructions in ways that would further improve your child's understanding of geometry.

### **Confidentiality**

In order to keep your child's personal information confidential, I will undertake the following procedures.

- The data collected about your child will be used only for the purpose of this study, and will be kept private to other people not associated with this study.
- Although your child will be required to write his/her name in each of the test answer sheets (this is done so as to be able to match his/her van Hiele test score with her end-of-the-year examination score in mathematics), his/her name and any other fact that might point to his/her identity will not appear when the results of this study are presented or published.
- Your child's records, including audio-taped and video-recorded data, from this study will be kept in locked files and nobody will be allowed access to these files.

### **Costs to You**

You do not have to pay anything for your child to be in this study.

### **Participants' Rights**

- Your child's participation in this study is voluntary.
- You reserve the right to change your mind and withdraw your child from the study at anytime, and there won't be any penalty for this.

### **Questions about the Study**

Should you have any questions about this study, please contact me on my cell phone number ----- (or send an e-mail to: -----).

**If you sign below, it means that you have read (or have had read to you) the information given in this permission form, and you would like your child to be in this study.**

**Your child's Name:** .....

**Parent's Signature:** .....; **Date:** .....

**Researcher's signature:** .....; **Date:** .....

Thank you.

## APPENDIX 2

### TEACHER'S QUESTIONNAIRE

---

**DEPARTMENT OF EDUCATION  
RHODES UNIVERSITY  
SOUTH AFRICA**

**Teacher's Personal/Class information Form**  
(To be completed by the teacher)

**Please go through this form carefully and supply the required information**

#### Personal Information

1. School-----
2. Teacher's Name -----
3. Gender: Male----- Female----- Tick one.
4. Teaching Qualifications (Please write out the degrees in full, e.g. "Bachelor of Education in Mathematics" for BEd (Maths)):
  - a) -----
  - b) -----
  - c) -----
  - d) -----
  - e) -----
5. How long (in years) have you been teaching mathematics? -----
6. How much (in years) of this time have you spent teaching Secondary School mathematics? -----
7. Have you had any in-service training in the last FIVE years? ----- Please list them, stating the year you had them.
  - a) -----
  - b) -----
  - c) -----
  - d) -----

#### Classroom-related Information

8. Grade level: -----
9. Today's Topic-----
10. Period of Day-----
11. Number of times class meet per week-----
12. Normal length of period (in minutes) -----
13. Number of students in your class-----
14. School Average number of students per class-----
15. National Average number of students per class-----
16. In the geometry aspects of the mathematics that you teach in this class, are the students expected to be able to write proofs? ----- If your answer is No, please explain why, and if your answer is yes, please go to item No. 17 (next page).

**Space for your explanation.**

17. Here are some standard geometry theorems. Please circle all letters that apply to the students in your class. Circle all appropriate options.

N if the students are **not** expected to know this theorem.

K if the students are expected to **know** the statement of this theorem

A if the students are expected to be able to **apply** this theorem in proofs.

P if the students are expected to be able to **prove** this theorem.

D if the students **must/ought** to have **done** this theorem in some earlier grades.

- |  |           |
|--|-----------|
| (a) The base angles of an isosceles triangle are congruent.  | N K A P D |
| (b) The sum of the angles of a triangle is $180^\circ$ .   | N K A P D |
| (c) The Pythagorean Theorem.   | N K A P D |
| (d) The diagonals of a rectangle are congruent.  | N K A P D |
| (e) If two parallel lines are cut by a transversal, alternate interior angles are congruent.   | N K A P D |
| (f) Two triangles are congruent if three sides of one are congruent respectively to three sides of the other (SSS congruence).                     | N K A P D |
| (g) Two triangles are similar if two angles of one triangle are congruent to two angles of the other (AA similarity).                              | N K A P D |
| (h) A line parallel to one side of a triangle divides the other two sides proportionally   | N K A P D |
| (i) If two triangles are equiangular, their corresponding sides are proportional.  | N K A P D |
| (j) If the corresponding sides of two triangles are proportional, then the two triangles are equiangular (converse of theorem i).                  | N K A P D |
| (k) The line from the centre of a circle to the midpoint of a chord is perpendicular to the chord.   | N K A P D |
| (l) The line from the centre of a circle perpendicular to a chord bisects the chord.   | N K A P D |
| (m) The angle which an arc of a circle subtends at the centre is twice the angle it subtends at any point on the remaining circumference.          | N K A P D |
| (n) Angles in the same segment of a circle are equal.  | N K A P D |
| (o) The angle in a semicircle is a right angle.  | N K A P D |
| (p) The opposite angles of a cyclic quadrilateral are supplementary.   | N K A P D |
| (q) A tangent to a circle is perpendicular to the radius at the point of contact.  | N K A P D |
| (r) The tangents to a circle from the same external point are equal in length.   | N K A P D |
| (s) The angle between a tangent to a circle and a chord drawn from the point of contact is equal to the angle in the alternate (opposite) segment. | N K A P D |

**APPENDIX 3**

**STUDENTS' QUESTIONNAIRE 1**

**APPENDIX 3.A**

**Terminology in Plane Geometry Test (TPGT)**

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**TERMINOLOGY IN PLANE GEOMETRY TEST**

**(TPGT)**

**Date:** .....

**Time Allowed: 50 minutes**

**INSTRUCTIONS:**

1. Do not start until you are told to do so.
2. While you are waiting, please fill the appropriate information in the spaces below.

**Name (Surname first):** .....

**Name of school:** .....

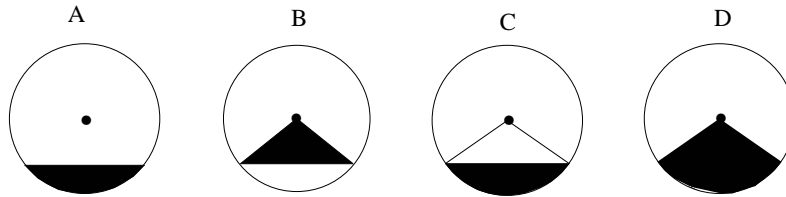
**Grade:** .....

**Age (in years):** ..... **Sex:** .....

3. This is an objective test. There are **sixty questions** in all. You are expected to **answer all** the questions. **Do not mark your answers on this test booklet.**
4. Each question is followed by **four** options lettered **A to D**. Choose the correct option for each question and **shade in pencil** on your answer sheet the answer space which bears **the same letter** as the option you have chosen. **Give only one answer to each question.**
5. Use an **HB** pencil throughout.
6. The **diagrams** in this test are **not necessarily** drawn to scale.

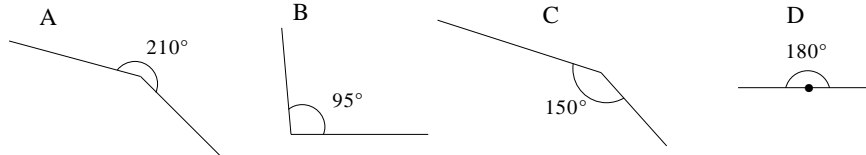
- What is the straight line that joins any two points on the circumference of a circle called?
  - an arc
  - a diameter
  - a radius
  - a chord

- Which of the shaded regions represent a sector of a circle?



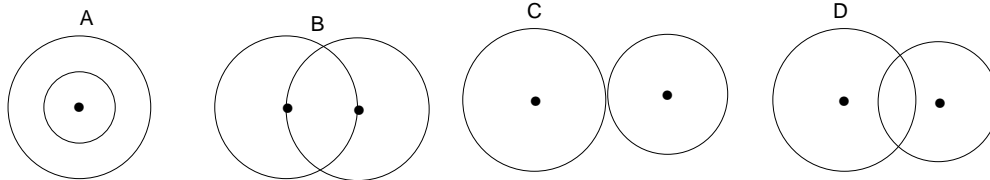
- Which of the following best describes the radius of a circle?
  - The straight line drawn from the centre to any point on the circumference.
  - The straight line that passes through the centre of a circle.
  - The straight line that joins any two points on the circumference of a circle.
  - The straight line that touches a circle at one and only one point.

- Which of the angles below is a reflex angle?



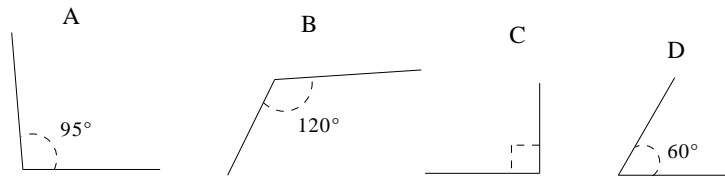
- What is the name of the chord that passes through the centre of a circle?
  - A tangent
  - A radius
  - An arc
  - A diameter

- Which of the following represents concentric circles? The centres are indicated by a dot (•).



- What is the name of the straight line that touches a circle at one and only one point, no matter how long it is produced at both ends?
  - A tangent
  - A chord
  - A diameter
  - A perimeter

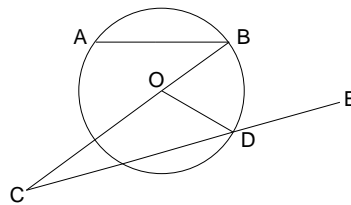
8. Which of the following is an acute angle?



9. Which of the following statements is **true** of a scalene triangle?

- A. It is necessarily an acute-angled triangle
- B. It cannot be a right-angled triangle
- C. No two of its sides have equal measure
- D. It must be an obtuse-angled triangle

10. In the diagram, O is the centre of the circle. Which line segment represents a chord?

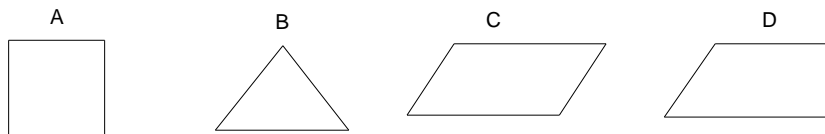


- A. OD
- B. AB
- C. CE
- D. CD

11. Which of these statements is **true** of an isosceles triangle?

- A. It cannot be an obtuse-angled triangle
- B. Two of its angles must necessarily add up to  $90^\circ$
- C. It cannot be a right-angled triangle
- D. Two of its angles must necessarily have equal measure

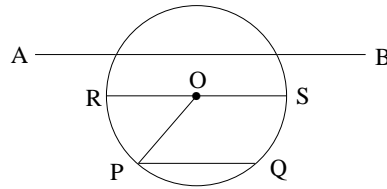
12. One of the following shapes **does not** belong to the same class of shapes as the others. Which one is it?



13. Which is **true**? If two straight lines intersect, then vertically opposite angles are

- A. right angles
- B. equal
- C. complementary
- D. supplementary

14. In the diagram, O is the centre of the circle. Which line segment represents a radius?

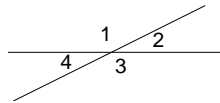


- A. AB
- B. ROS
- C. PQ
- D. OS

15. Which of the following best describes a cyclic quadrilateral?

- A. It is a rectangle in which a circle is inscribed
- B. It is a circle in which a rectangle is inscribed
- C. It is a quadrilateral in which a circle is inscribed
- D. It is a circle in which a quadrilateral is inscribed

16. Use the diagram below to determine a pair of vertically opposite angles.



- A. 1 and 3
- B. 2 and 3
- C. 3 and 4
- D. 1 and 2

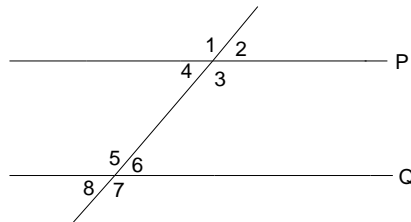
17. The straight line that divides a given geometrical shape into two identical halves is called

- A. the diagonal
- B. the line of symmetry
- C. the altitude
- D. the angle bisector

18. Which of the following pairs of lines are perpendicular?

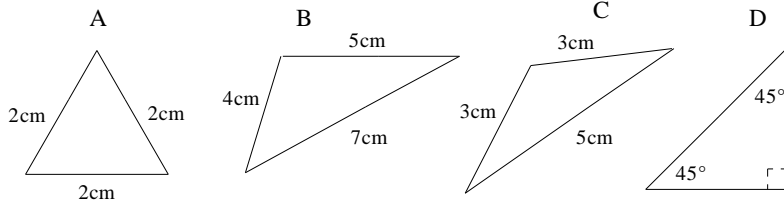


19. In this figure, lines P and Q are parallel. Two angles whose measures must add up to  $180^\circ$  are



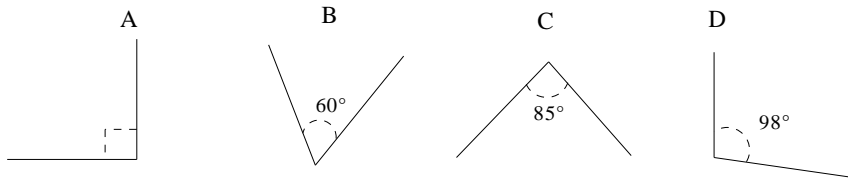
- A. 2 and 4
- B. 4 and 5
- C. 5 and 7
- D. 2 and 8

20. Which of the following triangles is an equilateral triangle?



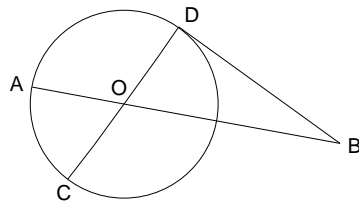
21. A reflex angle is an angle that is
- greater than  $90^\circ$ , but less than  $180^\circ$
  - $180^\circ$
  - greater than  $180^\circ$ , but less than  $360^\circ$
  - $360^\circ$

22. Which of the following represents a right angle?



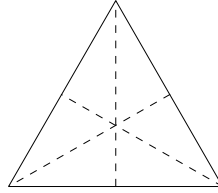
23. If two or more circles are concentric, then they have
- the same centre
  - the same radius
  - different centres
  - different radii

24. In the diagram, O is the centre of the circle. Which of the following is a diameter?



- AOB
  - DB
  - COD
  - OB
25. An acute angle is one that is
- less than  $90^\circ$
  - $90^\circ$
  - greater than  $90^\circ$ , but less than  $180^\circ$
  - $180^\circ$

26. How many sides and diagonals are there in the accompanying diagram?

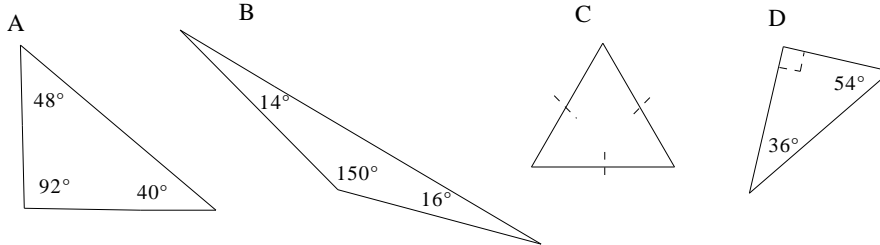


- A. 3 sides, 3 diagonals
- B. 0 sides, 3 diagonals
- C. 3 sides, 0 diagonals
- D. 0 sides, 0 diagonals

27. A right angle is one that is

- A. less than  $90^\circ$
- B.  $90^\circ$
- C. greater than  $90^\circ$ , but less than  $180^\circ$
- D.  $180^\circ$

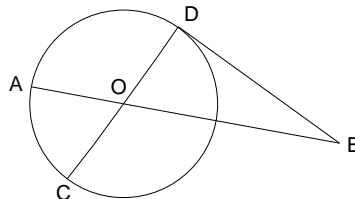
28. Which one of the following is a right-angled triangle?



29. An arc of a circle is

- A. the same as the circumference
- B. the region between two radii
- C. a part of the circumference
- D. a semicircle

30. In the diagram, O is the centre of the circle. Which of the following is a tangent?

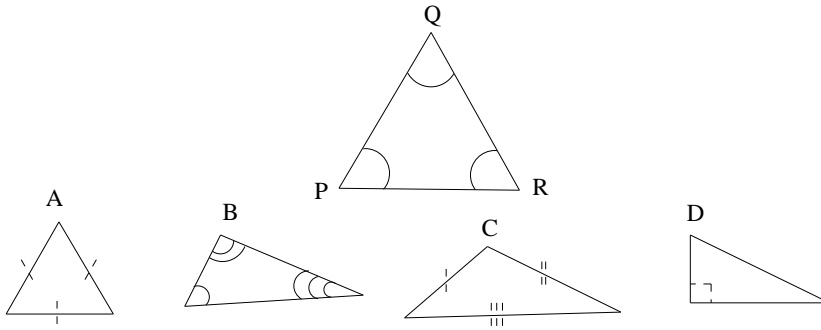


- A. AOB
- B. DB
- C. COD
- D. OB

31. Which is **correct**? If two lines are perpendicular to each other, then the two lines

- A. are also parallel to each other
- B. intersect at right angles
- C. intersect at their midpoints
- D. have the same measure

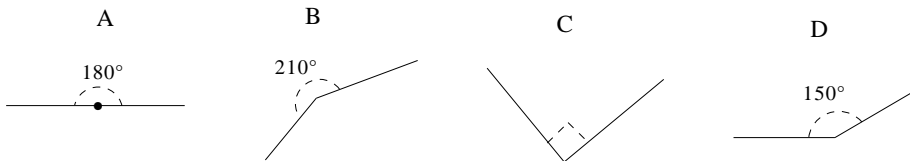
32. Which of the following triangles is similar to  $\triangle PQR$ ?



33. Which is correct? A polygon having four sides is best described as

- A. a rectangle
- B. a parallelogram
- C. a quadrilateral
- D. a trapezium

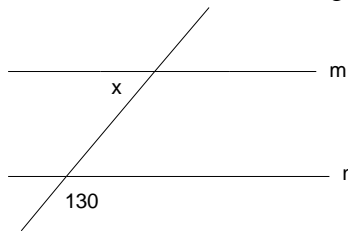
34. Identify an obtuse angle in the following diagrams.



35. Two angles are said to be complementary if

- A. their sum is  $180^\circ$
- B. they are vertically opposite angles
- C. their sum is  $90^\circ$
- D. they are corresponding angles

36. Lines  $m$  and  $n$  are parallel. What is the measure of the angle marked  $x$ ?

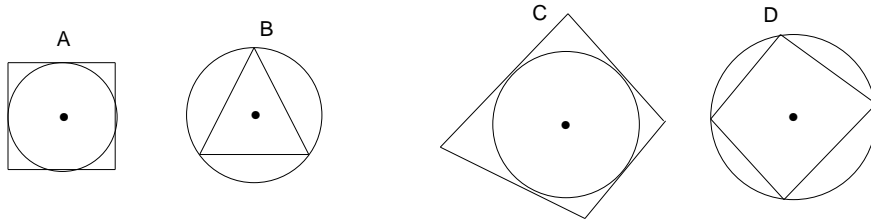


- A.  $30^\circ$
- B.  $40^\circ$
- C.  $50^\circ$
- D.  $130^\circ$

37. Which is **correct**? If two lines are parallel, then

- A. they have equal length
- B. they are also perpendicular
- C. they have a common starting point
- D. they have a constant distance between them

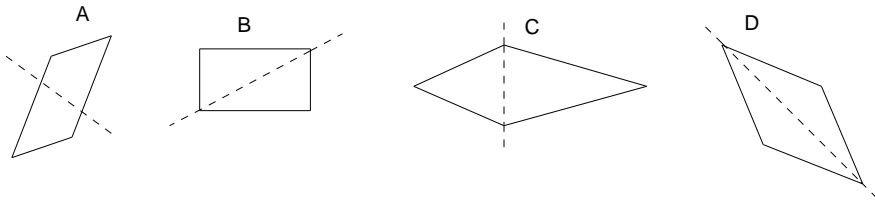
38. Which of the following diagrams represents a cyclic quadrilateral?



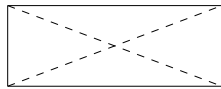
39. Two angles are said to be supplementary, if

- A. their sum is  $180^\circ$
- B. they are corresponding angles
- C. their sum is  $90^\circ$
- D. they are vertically opposite angles

40. If the broken lines (---) represent lines of symmetry, in which of the following has the line of symmetry been drawn correctly?

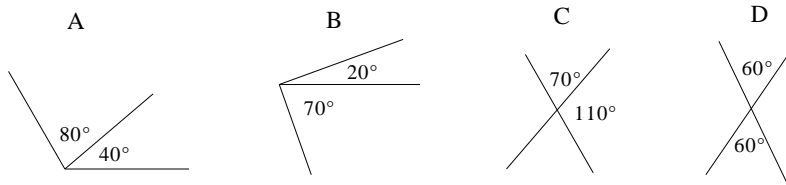


41. Consider the diagram below. How many sides and diagonals does the shape have?



- A. 4 sides, 3 diagonals
- B. 6 sides, 0 diagonals
- C. 4 sides, 2 diagonals
- D. 6 sides, 2 diagonals

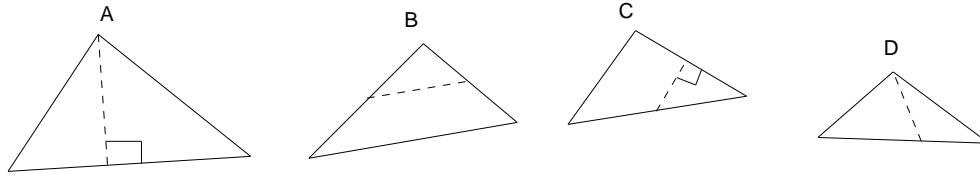
42. Which of the following diagrams illustrates complementary angles?



43. A line that cuts a set of lines is called a transversal. Which is **true** of corresponding angles? If two or more parallel lines are cut by a transversal, then corresponding angles

- A. are equal
- B. add up to  $90^\circ$
- C. are the same as adjacent angles
- D. add up to  $180^\circ$

44. Identify the triangle in which the altitude has been drawn correctly among the following.



45. Which of these statements is **true** of a right-angled triangle?

- A. One of its angles could still be an obtuse angle.
- B. It has two angles which are complementary
- C. It cannot be an isosceles triangle
- D. It has two angles which are supplementary

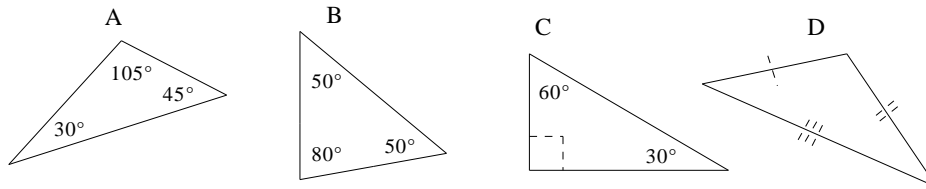
46. A line that cuts a set of lines is called a transversal. Which is **true**? If two or more parallel lines are cut by a transversal, then

- A. co-interior angles are congruent
- B. adjacent angles are complementary
- C. co-interior angles are supplementary
- D. adjacent angles are congruent

47. Which of these statements is **not true** of an equilateral triangle?

- A. All its angles must have the same measure.
- B. It has an angle whose measure is  $90^\circ$ .
- C. It is necessarily an acute-angled triangle.
- D. All its sides must have the same measure.

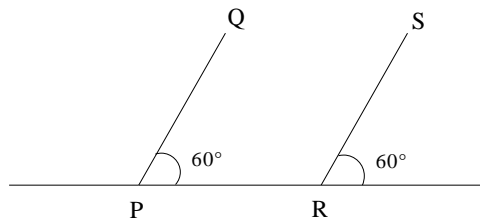
48. Identify an isosceles triangle in the following diagrams.



49. A line that cuts a set of lines is called a transversal. Which is **true** of alternate angles? If two or more parallel lines are cut by a transversal, then alternate angles are

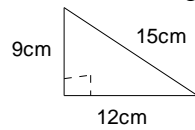
- A. congruent
- B. complementary
- C. supplementary
- D. reflex angles

50. Consider the diagram below. Which statement is **true**?



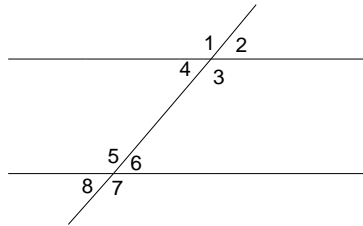
- A. PQ and RS are parallel
- B. PQ and RS are perpendicular
- C. PQ and RS must have the same measure
- D. PQ and RS will intersect each other at an angle of  $60^\circ$

51. What is the area of the triangle shown below?



- A. 36 cm sq.
- B. 54 cm sq.
- C. 108 cm sq.
- D. 1620 cm sq.

52. Identify one pair of alternate angles in the diagram shown below.

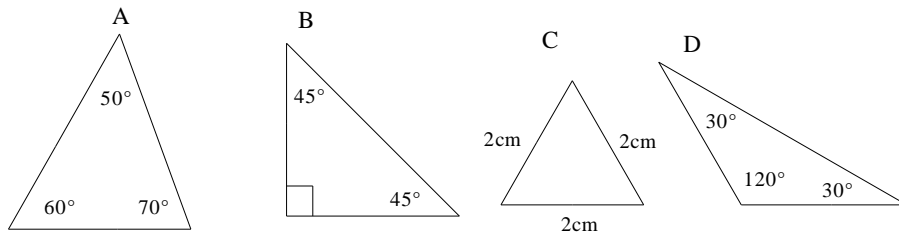


- A. 1 and 3
- B. 3 and 5
- C. 2 and 4
- D. 1 and 5

53. Which is **correct**? The perpendicular line drawn from an angle of a triangle to the side facing the angle is called

- A. a line of symmetry
- B. the diagonal
- C. an altitude
- D. the angle bisector

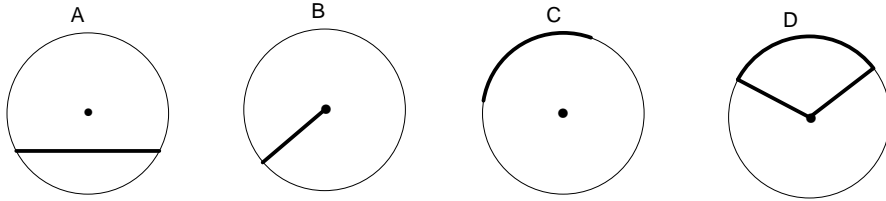
54. Which one of the following is a scalene triangle?



55. The measure of an obtuse angle is

- A. greater than  $180^\circ$ , but less than  $360^\circ$
- B.  $360^\circ$
- C. greater than  $90^\circ$ , but less than  $180^\circ$
- D.  $180^\circ$

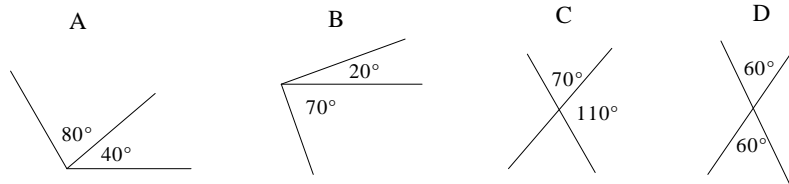
56. In one of the following diagrams, an arc has been represented by a thick line. Which one is it?



57. Which of the following is **true** of two similar triangles?

- A. They are necessarily congruent.
- B. Their corresponding sides are necessarily congruent.
- C. Their corresponding angles are necessarily congruent.
- D. They have the same area.

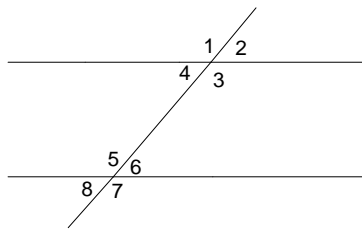
58. Which of the following diagrams illustrates supplementary angles?



59. The region of a circle enclosed by two radii (which do not form a diameter) and an arc is called

- A. length of arc
- B. a sector
- C. a segment
- D. perimeter of sector


60. Identify one pair of corresponding angles in the diagram shown below.



- A. 1 and 3
- B. 3 and 5
- C. 2 and 4
- D. 1 and 5

# APPENDIX 3.B



## Multiple-choice answer sheet for the TPGT



Rhodes University

**IMPORTANT:**

- Use Black or Blue pen only or Pencil
- Darken the circle completely
- Do not make any stray marks on this form

CORRECT MARK  INCORRECT MARKS 

STUDENT NUMBER	SEAT	TASK	
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8 1 1 B 1 1 1 1	GH <input type="radio"/> B <input type="radio"/> 1 <input type="radio"/> 1	1 1	2 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E
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X	X		24 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E
Y	Y		25 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E
Z	Z		26 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E

DATE		
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41 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	46 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	51 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	56 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E
42 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	47 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	52 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	57 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E
43 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	48 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	53 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	58 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E
44 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	49 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	54 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	59 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E
45 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	50 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	55 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	60 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E
61 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	66 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	71 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	76 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E
62 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	67 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	72 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	77 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E
63 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	68 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	73 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	78 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E
64 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	69 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	74 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	79 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E
65 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	70 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	75 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	80 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E
81 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	86 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	91 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	96 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E
82 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	87 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	92 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	97 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E
83 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	88 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	93 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	98 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E
84 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	89 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	94 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	99 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E
85 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	90 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	95 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	100 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E
101 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	106 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	111 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	116 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E
102 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	107 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	112 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	117 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E
103 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	108 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	113 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	118 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E
104 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	109 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	114 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	119 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E
105 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	110 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	115 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E	120 <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E

## APPENDIX 3.C

### Individual learner's performance on the TPGT

#### APPENDIX 3.C.1

#### NS grade 10 learners' performance and the TPGT

**Grade 10 Learners' Results on the TPGT  
NS Subsample**

#	AGE	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	T	%						
1	15	A	D	A	A	A	A	D	B	B	C	D	B	B	D	D	A	B	D	B	B	A	C	A	A	C	A	C	B	D	C	B	B	A	C	D	C	C	D	D	A	D	C	B	A	A	B	C	B	B	A	A	B	B	C	A	C	D	D	B	B	18	30						
2	16	1	Absent																																				0	0																													
3	14	1	B	D	A	A	C	A	C	B	B	A	C	B	C	B	B	A	A	D	D	A	A	A	B	C	A	A	C	D	C	D	A	A	B	A	C	D	A	B	A	B	A	A	B	B	B	D	A	D	A	B	B	C	C	A	B	A	B	C	D	A	B	B	C	C	C	19	32
4	15	1	B	D	C	A	C	B	B	A	C	B	C	B	B	D	D	A	B	C	B	A	A	A	B	C	D	A	B	D	D	B	D	D	A	A	D	A	D	A	A	B	B	A	B	D	A	B	B	D	D	C	B	B	D	D	C	D	D	D	C	B	29	48					
5	16	1	B	D	D	D	A	B	D	C	B	B	D	C	D	A	C	B	A	D	A	A	A	B	C	A	A	B	D	B	C	A	B	D	B	C	C	A	D	B	D	C	A	B	A	D	C	B	D	A	C	B	D	C	D	C	D	A	C	B	D	30	50						
6	14	1	Absent																														0	0																																			
7	14	2	C	D	A	D	B	C	C	B	B	C	B	B	B	D	A	C	A	B	D	A	B	A	A	C	A	A	C	C	A	D	A	A	B	D	B	B	A	B	B	A	C	C	C	D	B	C	A	D	A	C	B	C	D	B	D	C	B	B	C	17	28						
8	13	2	Absent																														0	0																																			
9	13	2	D	B	A	D	C	B	A	D	C	B	D	B	C	C	A	C	C	A	A	*	A	A	B	C	A	A	B	A	A	D	A	A	B	B	A	D	A	A	B	A	C	D	A	A	B	C	D	C	D	A	A	A	B	D	D	B	D	C	C	19	32						
10	13	2	C	D	C	B	C	A	A	A	C	C	A	B	B	C	C	A	D	A	A	C	C	A	A	C	C	B	A	D	A	A	C	A	A	C	A	D	A	D	A	A	B	D	A	B	A	D	A	B	D	D	A	D	B	A	B	D	19	32									
11	13	1	C	D	A	C	D	B	A	D	C	D	D	B	C	C	A	D	A	A	C	C	A	C	A	C	A	A	B	D	B	B	D	D	B	B	D	D	A	B	B	D	A	A	B	A	B	A	A	B	B	D	C	A	D	C	D	C	A	D	C	D	16	27					
12	14	1	D	B	B	B	B	A	D	C	C	A	A	B	C	B	C	D	D	B	D	D	D	B	B	C	B	A	D	A	B	D	A	B	A	B	A	A	B	A	B	B	D	A	B	B	D	D	A	B	B	D	B	A	C	D	A	B	A	C	D	A	B	20	33				
13	16	2	C	A	A	B	B	B	C	C	C	A	A	C	B	D	B	B	B	D	D	C	A	B	B	D	C	B	A	C	D	C	B	A	C	D	D	A	B	B	D	D	A	B	B	D	D	B	B	D	D	C	D	A	A	B	A	C	D	C	10	17							
14	15	2	B	A	A	D	A	B	B	C	A	A	B	D	A	B	C	C	C	A	A	B	C	C	A	A	B	A	A	C	A	A	B	A	A	C	A	A	A	C	A	A	B	A	C	D	C	C	A	A	A	A	A	A	D	A	A	A	D	A	C	13	22						
15	15	2	D	A	A	D	A	C	C	D	A	A	B	B	A	C	D	C	C	B	A	A	C	D	A	A	B	A	A	C	A	C	A	A	B	B	D	D	B	B	D	D	C	D	A	A	B	A	C	D	C	B	C	D	B	16	27												
16	15	2	B	C	B	D	B	C	B	D	C	B	D	B	C	B	D	B	C	D	A	B	D	B	B	C	A	A	C	C	B	D	A	C	B	D	A	A	C	B	D	B	C	B	A	D	C	D	B	A	C	B	A	B	C	B	17	28											
17	15	1	B	D	C	C	B	B	C	B	A	B	C	A	B	D	B	A	A	B	A	A	B	A	A	B	B	A	B	A	D	A	B	A	D	C	A	B	A	D	C	D	B	C	A	C	C	C	B	A	B	B	A	B	C	B	26	43											
18	14	2	A	D	B	C	B	A	C	D	C	C	D	B	C	B	D	D	B	B	C	A	A	A	A	A	A	A	A	D	C	B	D	A	C	B	A	A	A	B	D	D	A	B	B	D	D	C	D	A	C	A	B	C	B	A	A	B	C	C	C	B	21	35					
19	16	1	D	D	A	C	D	A	C	D	D	C	A	A	C	D	C	A	A	A	D	A	B	A	A	C	B	A	B	A	A	C	A	B	A	A	C	A	B	B	D	D	D	A	A	A	D	C	D	D	A	B	B	B	B	C	A	B	A	A	A	C	D	C	A	D	D	35	58
20	14	1	B	D	C	A	D	B	B	D	C	B	D	B	D	D	B	A	B	A	A	A	C	A	A	C	A	A	B	D	B	D	D	A	C	D	B	D	A	A	A	D	C	D	D	A	B	B	B	B	C	A	B	B	B	C	A	B	A	A	A	C	14	23					
21	15	2	B	A	B	D	C	C	C	A	C	C	D	B	C	B	C	D	A	B	B	B	C	B	A	C	B	C	B	A	C	A	A	A	C	C	A	A	A	A	B	B	D	D	A	A	C	B	D	D	A	A	C	B	D	20	33												
22	13	1	B	D	C	A	D	B	B	D	C	A	D	A	C	D	B	B	C	D	D	A	A	A	B	C	A	A	B	A	B	B	C	A	B	A	C	C	A	B	A	C	B	A	A	B	A	B	A	A	B	A	D	A	B	D	A	15	25										
23	17	1	B	D	C	B	D	B	A	C	B	A	A	D	A	D	C	D	B	D	D	D	A	D	C	C	D	A	D	A	B	C	C	D	C	C	D	C	D	C	C	D	C	C	C	B	B	B	A	A	B	C	D	A	B	D	A	15	25										
24	16	1	C	D	A	C	D	B	A	D	B	A	B	D	B	C	D	C	C	A	D	A	C	A	B	A	A	B	B	D	C	C	A	C	B	D	C	B	D	B	D	C	B	D	B	D	C	C	A	C	A	A	C	B	D	B	A	26	43										

Green = correct answer  
Red = wrong answer

## APPENDIX 3.C 2

### NS grade 11 learners' performance and the TPGT

**Grade 11 Learners' Results on the TPGT**  
NS Subsample

#	AGE	G	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	T	%
1	16	1	D	C	C	A	D	A	A	D	C	B	D	B	B	D	D	A	B	D	B	A	C	A	A	C	A	A	B	D	B	B	B	A	C	D	C	C	D	D	A	D	C	B	A	A	B	C	B	B	A	A	B	B	C	A	C	D	B	C	B	D	54	90
2	14	2	D	C	C	A	D	A	A	D	C	B	D	B	B	B	C	A	B	D	B	A	C	A	A	C	A	A	B	D	B	B	B	A	C	D	C	C	D	D	A	D	C	B	A	A	B	C	B	B	A	A	D	B	C	A	C	C	C	B	D	48	80	
3	15	1	D	D	C	A	D	A	A	D	C	B	D	B	B	B	D	A	B	D	B	A	C	A	A	C	A	A	B	A	B	B	C	A	B	A	C	C	A	B	A	C	C	D	B	A	D	B	C	D	C	A	C	D	B	B	A	C	C	B	B	D	28	47
4	17	1	D	D	C	A	D	B	C	A	C	B	D	B	B	D	D	B	D	A	A	A	A	A	A	C	A	A	B	A	C	A	C	A	A	D	B	C	D	D	D	A	D	C	D	A	C	A	B	B	A	A	B	B	C	A	C	C	C	B	B	D	39	65
5	16	1	B	D	C	C	A	D	A	D	D	C	B	D	B	B	C	D	D	B	A	B	D	A	C	A	A	C	A	A	B	A	C	A	C	D	D	D	A	C	D	D	B	B	C	C	C	A	B	B	D	A	C	A	A	A	D	C	B	26	43			
6	17	1	B	D	C	C	A	D	B	A	D	C	B	D	D	A	B	C	B	D	C	B	A	A	A	A	C	A	A	B	D	D	B	C	A	C	B	B	B	C	C	A	B	B	C	A	D	A	A	D	C	C	B	C	B	D	D	C	17	28				
7	17	2	A	D	B	A	B	A	B	B	A	B	D	D	A	B	C	A	D	B	D	D	A	D	B	C	C	C	B	A	B	A	A	B	C	A	A	C	A	C	A	C	D	A	B	C	D	C	B	C	A	B	A	D	C	C	B	C	B	D	D	C	19	32
8	16	2	A	A	D	C	A	B	A	D	C	A	D	B	C	A	D	C	D	B	A	A	A	B	C	D	D	C	A	C	A	C	A	C	C	A	C	C	A	B	B	D	A	A	D	A	A	B	C	A	A	A	A	A	A	B	D	13	22					
9	19	1	A	A	B	D	A	C	B	B	C	A	B	B	A	C	B	A	B	C	D	D	A	A	A	B	A	A	B	A	A	A	A	D	D	A	C	A	C	C	C	B	B	A	A	C	A	A	A	C	A	A	A	D	D	D	A	D	B	D	D	A	16	27
10	19	1	A	B	D	C	B	A	A	C	A	B	B	B	C	C	C	A	A	D	C	D	A	A	A	C	A	A	D	A	B	C	A	A	C	C	B	B	A	B	B	D	C	B	C	C	B	A	C	C	A	B	A	D	A	B	A	D	C	C	C	D	21	35
11	15	2	C	A	A	B	A	B	A	C	C	B	B	B	C	C	C	A	A	D	C	D	B	B	A	B	C	A	A	B	C	B	A	B	C	C	C	C	C	D	D	D	C	C	D	A	A	B	A	D	A	A	D	B	D	D	A	17	28					
12	15	2	B	D	B	B	A	B	A	C	C	B	B	B	C	C	B	A	A	D	C	D	B	B	A	B	C	A	A	B	C	B	A	B	C	C	C	C	D	C	C	A	C	B	A	C	B	D	C	B	C	20	33											
13	16	2	C	D	B	C	B	B	A	C	C	B	B	B	C	C	B	A	C	A	D	C	B	B	A	B	C	A	A	B	C	B	A	B	C	C	C	C	D	C	C	A	C	B	A	C	B	D	C	B	D	19	32											
14	16	1	C	D	A	D	C	B	A	D	D	C	B	B	C	B	C	A	B	D	C	A	A	A	C	A	A	A	A	C	A	A	A	C	D	D	B	B	A	B	B	B	A	B	B	D	A	B	B	A	D	A	D	B	D	20	33							
15	16	2	A	D	D	D	C	C	A	D	D	A	B	B	D	B	A	A	A	D	B	A	A	A	A	C	A	A	A	D	B	B	A	A	D	B	B	A	B	B	B	D	A	B	B	A	D	A	B	B	A	D	D	D	C	D	C	17	28					
16	16	2	B	D	A	B	D	D	D	A	C	B	A	A	D	C	B	B	A	B	D	A	C	A	A	D	A	A	A	B	B	B	A	B	A	B	B	A	D	B	B	D	A	D	C	D	A	A	B	B	A	D	D	D	C	D	C	29	48					
17	16	2	B	D	B	B	D	A	B	C	D	B	D	B	B	B	D	B	A	B	D	A	C	A	A	D	A	A	A	B	B	B	A	B	A	B	B	A	D	C	D	A	A	B	B	A	D	D	D	C	D	C	15	25										
18	15	1	B	D	A	C	B	D	D	A	C	B	A	A	D	C	B	A	B	A	B	A	C	D	B	C	B	A	A	B	B	A	A	B	A	B	A	D	B	B	B	D	A	D	B	B	D	B	C	C	A	C	D	B	A	D	15	25						
19	18	1	C	D	A	A	B	A	B	C	C	A	D	B	A	B	C	C	A	B	D	A	C	D	B	A	C	A	A	B	C	B	A	B	C	C	C	C	A	C	A	B	B	C	C	D	D	A	C	B	D	A	C	D	B	A	D	18	30					
20	16	2	C	D	A	C	B	B	C	C	B	B	B	A	C	D	B	D	B	A	D	A	A	A	A	B	A	B	B	D	A	A	B	B	D	A	A	B	B	D	D	A	C	B	B	D	C	B	A	D	C	C	D	C	A	27	45							
21	14	2	D	D	C	A	B	B	C	D	A	D	D	B	A	C	C	A	B	B	D	A	C	A	B	C	D	B	B	D	D	B	A	B	C	D	B	D	A	C	B	D	C	B	A	C	B	D	C	B	A	B	C	C	D	B	C	22	37					
22	17	2	C	D	B	A	B	B	C	C	C	C	D	C	C	C	B	A	A	C	A	A	C	B	C	D	A	A	B	D	B	A	A	C	A	A	A	B	A	D	D	D	D	A	D	B	A	B	A	A	A	D	C	A	A	25	42							
23	19	2	B	C	A	A	D	B	C	D	C	B	A	D	C	C	A	A	D	D	D	A	A	A	A	A	A	A	B	B	A	A	A	A	D	D	A	A	A	D	D	C	B	D	A	D	C	B	D	A	D	D	D	C	A	A	23	38						
24	17	1	A	C	D	A	D	A	D	B	A	A	D	B	A	D	A	A	A	A	A	A	C	A	C	A	A	C	A	D	A	A	C	A	A	A	A	C	A	A	B	D	B	A	C	C	C	A	A	D	D	B	D	C	D	23	38							

Green = correct answer  
Red = wrong answer

APPENDIX 3.C.3

NS grade 12 learners' performance and the TPGT

Grade 12 Learners' Results on the TPGT  
NS Susample

#	AGE	G	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	T	%		
1	16	2	B	D	C	B	D	D	B	B	D	D	D	B	B	D	D	A	B	D	B	A	C	A	A	C	A	A	B	C	D	C	B	B	A	C	D	C	C	D	D	A	D	C	B	A	A	B	C	B	B	A	A	B	B	C	A	C	D	D	B	B	B	10	17	
2	15	1	D	D	C	A	D	A	A	D	C	B	D	B	B	D	D	A	C	D	C	A	C	A	A	C	A	C	A	B	D	B	B	B	A	C	D	C	C	D	D	A	A	C	C	A	A	B	C	C	B	B	A	A	C	B	A	D	D	C	B	D	B	D	22	37
3	17	1	B	D	C	A	C	B	B	D	C	B	B	B	A	B	C	D	A	B	B	A	D	A	B	C	A	C	B	A	A	A	A	A	A	A	B	C	B	A	A	D	C	C	A	A	B	C	B	C	D	C	D	B	A	A	D	C	C	D	B	B	B	24	40	
4	17	1	B	C	C	A	C	C	C	D	C	B	D	B	A	B	C	C	A	B	B	A	D	A	B	C	A	A	B	D	A	A	A	A	A	A	B	C	A	A	A	B	C	C	A	A	B	A	B	C	D	D	D	D	D	A	C	C	A	C	B	B	32	53		
5	19	1	B	D	C	A	D	A	B	D	C	B	D	B	A	C	D	A	A	D	B	A	C	A	A	C	A	A	B	D	C	D	A	A	A	D	D	D	D	B	A	B	C	C	D	B	B	C	B	D	D	B	B	B	A	A	C	C	A	C	B	D	41	68		
6	17	1	B	D	C	C	D	A	C	D	C	B	D	B	A	C	D	A	A	D	B	A	C	A	A	C	A	A	B	D	C	D	A	A	A	D	D	D	D	B	A	B	C	C	D	B	B	C	B	D	D	B	B	B	A	A	C	C	A	C	B	D	44	73		
7	17	1	D	D	C	A	D	A	A	D	C	B	D	D	A	D	D	A	A	D	B	A	C	A	A	C	A	A	B	D	C	D	A	A	A	D	D	D	D	B	A	B	C	C	D	B	B	C	B	D	D	B	B	B	A	A	C	C	A	C	B	D	35	58		
8	17	1	D	D	C	A	D	B	A	D	C	B	D	B	B	D	C	A	A	D	B	B	A	C	A	A	A	A	A	B	D	B	A	A	C	B	B	D	B	A	A	B	C	C	D	B	B	D	A	D	A	C	D	B	D	A	C	C	C	D	B	B	25	42		
9	16	1	D	D	B	C	A	D	A	A	D	C	B	D	B	B	D	A	B	A	B	B	A	D	A	B	C	A	A	B	D	D	B	A	A	A	A	B	C	B	A	A	B	C	C	A	C	B	D	A	D	A	A	C	A	A	A	A	C	B	A	D	D	25	42	
10	17	1	A	D	C	A	C	B	A	D	C	B	C	B	A	B	B	B	A	B	A	A	D	A	B	C	A	A	A	A	A	A	A	B	C	A	C	C	A	A	A	B	C	B	A	A	C	D	A	D	A	A	A	D	A	A	A	C	B	A	D	D	24	40		
11	18	1	D	A	C	A	D	B	C	A	D	B	D	B	B	D	A	B	A	B	B	A	A	A	A	A	C	A	A	A	A	A	A	B	C	B	A	B	B	B	D	B	C	D	B	A	B	C	D	B	A	B	B	A	B	C	A	B	C	B	D	D	24	40		
12	18	1	B	D	C	A	D	B	C	C	A	D	B	B	A	B	C	A	A	D	B	D	C	A	A	C	A	A	A	C	A	A	A	A	A	C	A	D	A	A	B	D	D	C	D	B	A	C	C	D	B	A	C	A	C	D	B	22	37							
13	17	2	B	D	B	A	B	A	D	C	C	B	B	B	A	B	C	A	A	D	B	D	A	A	A	C	A	A	A	C	A	A	A	A	C	A	A	A	A	B	D	C	C	D	C	A	B	D	C	D	A	B	C	D	B	C	D	21	35							
14	17	1	C	B	A	D	D	B	B	A	B	C	D	A	D	B	C	A	A	D	D	B	C	C	A	D	A	B	B	D	A	A	B	B	D	D	B	C	A	A	B	D	C	C	A	A	A	B	A	A	A	A	D	D	C	D	D	C	B	C	B	B	36	60		
15	17	1	D	D	C	A	D	A	A	A	B	B	D	A	B	D	D	D	A	D	D	C	A	A	A	C	A	A	A	B	D	C	D	C	A	A	D	A	A	D	B	B	C	C	A	A	D	D	C	B	A	B	C	D	B	A	B	22	37							
16	16	2	B	D	C	C	C	B	B	D	C	A	D	B	D	D	C	A	A	D	D	C	C	C	B	A	C	C	A	B	D	C	D	A	A	D	B	C	A	A	D	B	C	D	B	A	B	B	D	A	D	C	D	C	D	D	B	19	32							
17	18	2	B	D	B	D	C	A	D	A	B	D	D	A	B	D	C	A	A	D	A	D	A	A	A	C	A	B	A	B	D	C	B	C	B	D	C	C	A	A	D	D	D	B	D	D	B	C	B	A	A	A	D	D	C	D	D	27	45							
18	18	1	B	A	C	A	D	A	B	C	C	A	D	A	A	C	C	A	A	C	C	A	A	A	C	A	A	A	A	C	A	A	C	B	D	C	B	C	A	A	D	B	B	D	D	B	D	B	D	D	B	D	C	B	C	B	19	32								
19	18	1	D	D	C	B	A	B	B	C	C	A	D	A	B	D	B	B	D	D	D	A	D	D	C	D	A	C	A	A	C	A	A	A	A	D	A	A	A	B	D	C	D	C	A	B	A	C	A	D	A	D	B	B	A	C	24	40								
20	16	2	B	A	A	A	C	A	C	D	C	D	B	B	B	D	D	A	C	D	D	A	C	A	A	C	A	A	A	C	A	A	A	B	D	C	B	B	D	D	B	C	C	A	A	B	C	C	A	A	B	A	C	D	D	A	17	28								
21	16	2	D	B	C	A	D	A	A	D	C	B	D	B	B	D	D	A	D	D	D	D	A	D	A	C	A	A	A	A	C	A	A	A	D	C	B	B	C	C	B	D	C	A	B	B	C	C	D	D	A	A	C	D	D	A	13	22								
22	17	2	C	A	B	A	C	B	A	C	B	D	B	A	C	B	D	B	A	C	B	A	A	D	B	C	B	A	B	D	C	B	B	B	C	C	B	B	C	C	B	A	A	B	D	D	D	C	A	B	B	21	35													
23	16	2	C	A	B	A	C	B	A	C	B	D	B	A	C	B	D	B	A	C	B	A	A	D	B	C	B	A	B	D	C	B	B	B	C	C	B	B	C	C	B	A	A	B	D	D	C	D	A	A	D	A	D	B	D	A	21	35								
24	16	2	B	D	C	A	B	C	B	C	B	C	A	C	C	A	C	A	C	A	A	A	C	A	C	A	A	A	A	A	A	A	A	D	B	B	D	B	C	C	A	A	D	C	D	A	A	D	A	D	B	D	A	21	35											

Green = correct answer  
Red = wrong answer

APPENDIX 3.C.4

SAS grade 10 learners' performance and the TPGT

Grade 10 Learners' Results on the TPGT  
SAS Subsample

#	AGE	G	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	T	%				
1	16	1	C	D	A	C	D	B	D	D	A	A	C	B	C	D	D	A	D	A	B	B	A	C	A	A	C	A	A	B	D	D	C	B	B	A	C	D	C	C	D	D	A	D	C	B	A	A	B	C	B	B	A	A	B	B	C	A	C	C	C	C	B	D	30	50		
2	16	1	D	D	B	A	B	B	B	D	A	A	D	D	B	B	D	A	D	C	B	B	A	C	C	A	A	C	A	C	B	D	D	C	A	B	A	B	D	D	A	C	C	C	D	B	C	C	A	C	A	A	B	B	C	C	A	D	D	A	A	D	36	60				
3	14	1	B	A	C	A	D	B	B	D	C	C	D	B	B	D	D	A	B	D	B	B	A	C	C	A	A	C	A	A	B	D	D	B	D	A	A	B	D	D	A	C	C	D	A	A	B	C	B	D	A	B	A	B	B	A	A	C	C	A	A	A	A	D	32	53		
4	15	2	C	D	A	A	D	B	C	D	D	B	D	B	B	C	B	C	C	A	A	D	A	A	C	C	A	A	C	A	A	B	D	B	A	A	A	A	B	D	D	A	C	D	C	D	D	C	C	A	A	A	A	B	B	D	A	C	C	A	A	A	A	D	34	57		
5	15	1	C	D	A	A	D	A	B	C	C	B	D	B	B	A	B	C	A	B	C	B	B	A	D	A	B	A	C	A	A	A	B	D	D	A	C	C	D	D	D	A	C	C	D	A	D	D	C	C	D	D	A	A	D	B	D	C	C	A	A	A	A	D	31	52		
6	15	2	C	D	A	A	D	A	B	D	C	B	D	B	B	B	C	A	B	B	B	B	A	D	A	B	A	C	A	A	A	B	D	B	A	A	A	B	D	D	A	C	C	C	D	D	C	C	A	A	B	A	A	D	B	C	C	A	A	A	A	D	27	45				
7	15	2	C	B	A	A	D	A	D	D	C	A	A	B	C	B	B	C	A	C	D	C	A	B	D	A	B	A	C	A	A	A	B	D	D	A	C	C	D	A	D	D	C	C	D	A	D	D	C	C	D	A	D	A	A	B	C	A	A	C	B	D	A	B	30	50		
8	16	2	B	D	C	A	B	B	C	D	C	A	D	B	B	D	D	D	B	A	B	B	A	C	C	A	B	C	A	A	A	B	D	D	B	C	C	B	D	C	A	D	A	B	A	A	A	B	B	A	A	A	A	D	B	C	C	A	A	D	A	C	27	45				
9	17	1	B	D	C	D	C	A	C	D	C	A	D	B	B	D	D	D	B	A	B	B	A	D	B	A	B	B	A	C	C	A	B	C	A	A	A	C	C	C	D	D	B	B	D	C	C	B	C	A	A	D	B	B	A	A	A	D	B	C	C	A	A	D	A	C	29	48
10	15	1	B	D	B	A	B	A	C	D	D	A	C	D	C	D	D	B	A	C	A	B	A	C	C	A	B	C	A	A	A	B	D	D	B	C	C	A	D	C	A	C	D	C	A	A	A	A	D	B	C	C	A	A	D	A	C	26	43									
11	17	1	C	D	A	C	B	A	C	D	D	A	D	D	C	D	D	D	A	D	C	C	C	A	A	B	C	A	B	B	D	D	B	A	C	A	D	A	D	C	C	B	B	B	C	C	A	A	D	D	B	C	A	A	B	D	B	C	A	25	42							
12	16	1	B	D	A	C	B	A	C	D	B	B	D	D	B	D	D	A	B	D	B	B	A	A	B	A	A	C	C	A	A	A	A	A	A	B	D	D	A	C	C	D	D	C	C	C	B	B	B	A	B	D	C	A	A	D	D	C	25	42								
13	21	1	C	B	D	A	D	B	D	D	B	D	C	D	D	D	C	A	A	D	C	B	C	A	A	B	B	C	C	A	B	D	D	A	C	C	C	B	B	C	C	D	A	A	A	B	D	C	D	C	A	D	D	C	19	32												
14	15	1	C	D	D	A	D	A	C	D	D	A	A	A	C	C	B	C	A	C	C	B	C	A	A	B	B	C	C	A	B	B	C	A	B	B	A	C	A	B	D	D	D	D	D	C	C	B	D	A	A	A	B	C	D	28	47											
15	15	1	B	A	B	A	A	A	C	A	A	B	B	B	B	B	D	C	A	A	B	D	C	B	B	D	C	B	C	A	B	D	D	A	A	A	C	A	B	B	B	A	C	A	B	B	D	C	A	A	D	D	C	C	A	A	D	A	C	28	47							
16	16	2	C	D	A	D	D	A	B	D	A	A	B	D	B	B	C	A	A	D	B	B	C	A	A	C	C	B	B	B	A	C	A	B	B	B	A	C	A	B	D	D	D	D	D	C	C	B	D	A	A	B	C	D	22	37												
17	15	2	C	A	B	A	C	A	C	D	A	A	D	B	B	B	D	D	A	D	B	B	C	A	A	C	C	A	A	B	D	D	A	A	A	C	A	B	B	B	D	D	D	C	C	B	D	A	A	D	B	A	A	D	D	C	C	A	A	D	A	24	40					
18	21	1	C	D	D	A	B	B	C	D	D	D	C	C	D	B	C	A	A	C	C	A	A	A	A	B	B	C	C	A	A	A	A	C	A	A	B	A	C	A	E	A	D	C	C	D	C	B	A	D	D	A	D	28	47													
19	16	2	C	D	A	A	B	B	C	B	B	D	C	B	B	B	D	D	A	B	B	B	D	A	A	B	B	C	C	A	A	A	A	A	A	B	D	B	C	C	C	A	A	B	B	A	A	A	A	B	B	A	A	C	C	C	A	A	D	A	24	40						
20	16	2	C	B	B	A	B	B	B	D	C	D	A	B	B	B	D	D	A	B	B	B	D	A	A	B	B	C	C	A	A	A	A	A	A	B	D	B	C	A	B	C	A	D	C	C	B	D	C	C	A	B	D	26	43													
21	16	1	C	D	A	C	D	B	A	D	A	A	D	B	B	B	D	D	A	B	B	B	D	A	A	B	B	C	C	A	A	A	A	A	A	B	D	B	C	A	B	C	A	D	C	A	A	A	A	B	A	A	A	A	B	B	A	A	C	C	C	A	A	D	A	24	40	
22	16	1	C	D	B	A	B	A	C	D	B	A	B	A	C	B	A	A	B	B	D	B	C	A	A	B	B	C	C	A	A	A	A	A	A	B	D	B	C	A	B	C	A	D	C	A	A	A	A	A	B	D	C	20	33													
23	15	2	C	B	C	A	D	A	B	D	D	A	D	D	B	B	C	A	A	C	A	B	C	A	A	B	B	C	C	A	A	A	A	A	A	B	C	B	D	A	A	B	C	D	C	A	A	A	A	B	C	D	23	38														
24	15	2	C	D	A	A	D	B	C	D	C	A	B	B	D	B	C	A	C	B	B	D	C	C	B	B	D	C	C	A	A	A	A	B	B	D	A	D	C	A	A	B	A	A	A	A	B	C	B	A	C	D	23	38														

Green = correct answer  
Red = wrong answer

APPENDIX 3.C.5

SAS grade 11 learners' performance and the TPGT

Grade 11 Learners' Results on the TPGT  
SAS Subsample

#	AGE	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	T	%					
1	16	C	D	A	C	D	B	D	C	C	B	A	D	D	B	D	D	A	C	B	A	A	D	C	D	B	A	C	D	B	C	B	A	C	D	C	C	D	D	A	D	C	B	A	A	B	C	B	B	A	A	B	B	C	A	C	D	B	C	A	C	20	33					
2	20	D	D	C	A	D	A	A	D	A	A	B	D	C	B	D	A	C	D	D	C	C	A	C	C	A	A	B	D	D	A	B	A	D	D	A	C	C	D	B	C	B	A	D	A	A	A	B	D	D	D	C	A	D	B	D	A	C	B	B	C	D	34	57				
3	15	C	D	C	A	B	D	C	D	D	A	D	B	B	B	D	C	B	A	B	D	C	A	C	C	A	A	B	D	D	C	D	D	B	B	A	D	D	B	B	A	C	C	D	A	D	B	C	B	B	A	A	D	B	C	C	B	D	C	C	27	45						
4	16	C	D	A	B	D	B	D	D	C	B	D	D	B	B	C	C	B	A	B	D	C	A	C	C	A	A	B	D	D	D	A	B	C	D	A	C	C	D	A	D	A	C	A	A	B	B	B	C	A	B	A	C	D	C	A	D	D	B	B	31	52						
5	15	B	D	C	A	A	D	C	D	B	A	D	B	B	B	B	C	B	D	C	A	C	A	B	C	A	C	A	B	D	D	A	B	C	D	A	C	C	D	A	B	B	B	D	C	A	A	B	D	A	C	A	D	A	A	C	B	D	C	A	29	48						
6	16	C	D	B	A	D	B	C	D	C	A	B	B	C	B	C	A	B	D	B	A	A	B	D	B	A	C	A	B	C	C	A	B	A	C	C	C	B	A	C	C	C	B	D	C	A	A	D	A	A	C	A	B	B	A	C	A	D	A	A	C	B	D	C	A	27	45	
7	17	C	A	B	A	B	A	C	D	C	B	D	D	C	B	C	A	D	B	B	A	A	D	B	B	A	D	A	C	D	A	D	A	D	B	B	B	A	C	C	B	A	C	C	C	B	A	A	B	D	C	A	A	D	C	D	A	B	B	A	C	A	B	B	A	C	27	45
8	16	C	D	C	A	D	A	C	D	B	A	A	B	B	B	D	D	D	B	B	A	D	D	B	B	A	D	D	B	B	A	A	A	B	D	A	C	C	B	A	C	C	A	A	D	C	D	A	A	D	C	D	A	B	B	A	C	A	B	D	C	A	C	C	A	D	18	30
9	19	B	C	A	D	C	A	A	B	B	A	D	D	D	B	D	D	A	D	D	B	B	A	C	C	B	B	A	C	C	B	A	C	A	C	A	C	C	C	B	A	C	C	C	B	A	A	D	C	A	A	B	19	32														
10	19	B	B	B	B	B	A	C	D	C	A	D	D	D	B	D	A	A	B	B	D	C	A	A	A	C	A	B	D	C	D	A	A	B	D	A	C	C	D	A	A	C	C	D	A	B	A	A	B	D	C	A	A	B	D	A	B	D	B	A	A	21	35					
11	18	D	D	B	C	B	A	B	B	D	A	B	A	B	B	C	A	A	B	C	B	C	A	A	A	C	C	A	A	B	D	A	A	B	D	C	D	A	A	C	D	B	C	C	B	B	A	A	B	D	C	D	A	A	B	D	B	D	B	A	21	35						
12	18	C	D	A	A	D	A	D	A	A	B	B	C	B	A	D	A	A	B	D	B	A	A	B	D	B	D	D	D	D	C	C	A	C	A	C	C	A	B	D	C	A	A	B	D	C	A	B	D	C	A	B	B	A	D	22	37											
13	18	C	D	A	A	D	A	C	D	A	A	B	D	A	A	D	B	D	D	D	C	D	D	B	C	A	C	B	D	A	B	A	A	A	C	D	C	D	A	D	B	B	C	C	A	A	A	D	D	B	C	D	D	B	A	25	42											
14	17	B	D	C	A	D	B	B	D	A	B	C	D	B	B	C	C	A	B	C	A	C	A	C	C	A	A	B	D	A	D	A	A	A	D	D	B	C	D	A	D	D	D	B	B	A	C	C	D	C	A	B	B	C	C	A	A	B	A	21	35							
15	18	C	D	A	A	C	B	B	D	B	A	B	A	D	B	D	A	A	B	D	B	A	A	B	A	C	A	B	D	D	C	D	C	B	A	B	B	B	A	D	C	C	A	D	A	A	B	B	C	C	A	A	A	D	C	16	27											
16	19	C	D	A	C	B	B	D	B	B	A	B	A	D	C	D	A	A	B	A	C	C	A	B	B	C	C	A	B	A	C	D	C	B	A	B	A	A	B	A	C	C	C	D	A	A	B	B	C	C	A	A	A	D	C	22	37											
17	19	B	D	A	A	C	B	B	D	B	A	B	A	D	A	D	A	A	B	D	B	A	A	B	A	C	A	B	D	D	C	D	C	B	A	B	B	B	A	C	C	C	A	D	A	B	C	A	C	D	B	A	A	A	C	A	C	A	C	D	B	26	43					
18	18	C	D	A	C	D	B	D	D	B	B	D	D	C	B	D	D	D	C	B	D	A	A	B	C	A	A	B	C	A	A	A	A	C	B	A	A	D	D	C	A	A	A	C	D	C	C	A	C	D	D	D	B	B	21	35												
19	17	B	D	D	A	B	B	D	A	C	B	C	B	A	C	C	A	A	B	D	B	A	A	B	D	B	C	A	A	C	A	A	A	A	C	B	A	A	A	C	C	C	A	C	A	C	A	A	B	D	D	C	B	D	D	23	38											
20	17	C	D	A	C	D	A	D	C	B	A	D	D	B	D	A	A	D	C	B	D	A	A	B	C	D	C	A	B	A	A	A	D	D	C	A	A	A	C	D	D	D	A	B	B	A	B	D	C	A	B	B	D	36	60													
21	17	B	D	C	A	D	A	A	D	C	B	D	B	B	B	D	A	A	B	D	B	A	A	B	D	B	C	A	A	C	A	A	A	B	D	A	A	A	B	D	A	C	C	A	D	C	D	C	B	A	A	A	B	D	B	27	45											
22	17	C	D	A	C	D	B	D	B	A	D	D	B	B	D	A	D	B	B	A	A	B	D	B	A	A	A	B	D	B	A	A	B	D	A	A	A	B	D	A	C	C	A	D	C	D	C	B	A	A	A	B	D	B	22	37												
23	18	B	B	D	A	B	D	D	B	D	C	D	D	B	B	C	B	A	B	D	C	A	A	A	A	D	C	A	A	A	D	D	B	C	C	B	B	A	B	A	D	A	D	B	A	C	C	C	A	C	22	37																
24	16	C	A	A	A	D	A	D	D	C	A	B	D	A	C	A	A	D	B	B	D	A	A	B	A	A	A	B	A	A	A	A	C	A	A	A	A	C	A	C	C	D	B	D	C	C	C	A	D	D	B	C	C	22	37													

Green = correct answer  
Red = wrong answer

### APPENDIX 3.C.6

### SAS grade 12 learners' performance and the TPGT

Grade 12 Learners' Results on the TPGT  
SAS Subsample

#	AGE	G	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	T	%									
1	17	2	D	A	C	A	D	C	A	D	D	B	D	D	B	D	D	A	A	D	B	A	C	A	A	C	A	A	B	D	D	C	B	B	A	C	D	C	D	D	D	A	A	C	C	A	D	B	C	B	B	A	A	B	D	D	A	C	C	A	C	B	D	38	63								
2	17	2	D	A	C	A	D	B	A	D	B	B	D	B	B	D	D	A	B	C	D	A	C	A	A	C	A	A	B	D	D	B	B	A	C	D	B	C	D	D	A	B	C	B	B	A	A	B	B	C	D	C	A	B	C	B	D	44	73														
3	19	2	D	C	C	A	D	C	B	D	D	B	D	B	B	D	D	A	B	D	B	A	C	A	A	C	A	A	B	D	D	C	B	B	A	C	D	B	C	D	D	A	B	C	B	B	A	A	B	B	C	D	C	C	A	C	C	D	43	72													
4	17	2	D	C	C	A	D	B	A	D	B	B	D	B	B	D	B	A	A	D	B	A	C	A	A	C	A	A	B	D	D	C	B	B	A	C	D	A	C	A	D	A	D	C	B	B	A	A	B	B	C	A	C	D	A	C	C	D	52	87													
5	18	1	D	A	C	A	D	A	A	D	C	B	D	B	B	D	D	A	B	D	C	A	C	A	A	C	A	A	B	D	D	C	B	B	A	C	D	A	C	D	D	A	D	C	B	A	A	B	C	B	B	A	A	B	B	C	A	C	52	87													
6	17	1	D	D	B	A	D	A	A	D	C	B	D	B	B	D	D	A	C	D	B	A	A	A	A	C	A	A	B	D	D	C	B	B	A	D	D	C	C	D	D	A	D	C	C	A	D	C	C	B	B	D	A	B	B	D	A	C	36	60													
7	18	1	D	A	C	B	D	A	A	D	C	B	D	B	B	D	D	A	C	D	B	A	C	A	A	C	A	A	B	D	D	C	B	B	A	D	D	C	C	D	D	A	D	C	B	A	A	B	C	B	B	C	A	B	B	C	D	A	D	A	C	B	D	47	78								
8	18	1	B	D	C	A	D	B	A	D	C	B	D	B	B	D	D	A	B	B	B	A	C	A	B	C	A	A	B	D	D	C	B	D	A	B	B	B	D	D	D	C	D	C	B	A	A	C	B	B	D	A	A	B	B	A	A	C	36	60													
9	18	2	D	D	C	C	D	B	A	C	D	B	D	D	B	D	D	A	D	D	B	A	C	A	B	C	A	A	B	D	D	B	B	A	D	D	B	B	A	A	D	B	C	B	D	A	A	B	A	A	C	C	A	D	D	C	B	36	60														
10	19	2	C	B	A	A	D	B	B	D	C	D	D	B	B	C	C	A	B	A	B	B	C	A	A	C	A	A	B	D	D	B	B	A	D	D	C	C	A	D	D	C	C	A	D	A	B	B	B	D	D	B	B	B	D	D	B	C	D	C	A	D	22	37									
11	19	2	D	A	B	A	A	B	D	C	D	A	D	B	B	A	D	A	B	A	A	A	B	A	D	C	C	B	A	D	D	B	B	A	D	A	B	A	A	C	C	A	C	B	D	A	A	A	D	D	C	C	A	D	D	C	C	A	D	C	C	B	27	45									
12	20	2	B	B	D	A	D	B	C	D	A	A	D	B	B	C	C	A	B	A	B	B	D	A	A	C	A	A	B	D	D	B	B	A	D	D	B	D	C	D	D	C	C	A	B	A	B	A	B	D	A	A	A	C	B	D	B	D	B	C	C	A	B	C	A	B	40	67					
13	22	1	B	B	C	A	D	B	A	D	A	B	D	B	B	D	B	A	D	D	C	D	D	A	C	A	B	A	B	D	C	B	A	D	D	A	C	D	D	A	B	C	D	D	A	B	C	B	B	C	C	A	B	B	A	C	D	A	C	B	D	28	47										
14	18	1	D	B	C	A	D	B	A	D	C	B	D	B	B	D	D	A	B	D	B	A	C	A	A	C	A	A	B	D	D	B	B	A	D	D	B	B	A	C	D	C	C	D	D	A	B	C	B	D	A	A	A	B	B	A	A	D	A	C	B	D	31	52									
15	18	1	C	B	A	A	D	C	A	D	A	B	D	B	D	D	D	A	A	D	B	C	C	A	B	C	C	A	B	D	D	C	B	B	A	A	B	B	B	A	A	B	A	C	D	D	B	D	C	D	D	B	A	C	D	D	B	A	D	A	C	B	D	28	47								
16	18	1	C	B	A	A	D	C	A	D	A	B	D	B	D	D	D	A	A	D	B	C	C	A	B	C	C	A	B	D	D	C	B	B	A	A	B	B	B	A	A	B	A	C	D	D	B	D	C	D	D	B	A	A	B	B	D	D	D	A	C	B	D	24	40								
17	18	2	B	A	C	A	A	B	A	D	B	B	D	B	B	D	D	A	B	D	B	A	D	A	C	C	A	A	B	D	B	B	A	A	B	A	A	A	B	B	A	C	D	D	A	D	C	D	D	B	A	C	B	B	D	A	B	B	A	C	D	28	47										
18	19	2	B	D	D	A	A	B	A	D	A	B	D	B	B	C	D	A	B	D	C	C	D	A	B	A	A	A	B	A	A	C	A	A	B	A	A	A	A	D	B	A	D	A	D	A	D	C	D	A	A	D	C	D	A	A	D	C	B	C	A	B	A	A	D	C	B	24	40				
19	20	2	B	C	D	C	C	B	B	C	D	B	D	B	B	C	C	A	D	B	B	A	A	C	A	A	B	A	B	D	D	B	B	A	A	B	A	A	A	C	D	B	D	D	D	A	A	C	C	A	A	A	B	B	C	C	A	A	D	A	C	A	A	D	D	C	22	37					
20	18	2	B	A	B	A	C	A	B	D	B	A	B	B	B	D	D	A	C	A	D	B	C	A	C	A	A	A	B	B	A	A	A	C	B	B	A	A	A	C	D	D	D	D	A	A	C	C	A	D	C	D	A	B	C	D	B	A	B	25	42												
21	20	1	D	C	C	A	D	A	A	D	B	B	D	B	B	D	D	A	A	D	B	A	C	A	A	C	A	A	B	D	B	B	A	A	C	D	A	A	C	C	D	D	D	D	A	A	C	C	B	A	D	A	A	D	C	A	A	D	A	C	A	A	D	B	A	25	42						
22	19	1	B	D	A	A	A	B	C	D	C	A	D	B	B	B	C	A	B	C	D	A	A	A	B	A	A	A	B	A	C	A	A	B	A	A	A	B	A	C	A	A	B	D	A	A	A	B	D	A	A	A	D	A	A	A	D	B	A	25	42												
23	19	1	B	D	D	A	C	A	A	D	C	A	D	A	B	D	D	A	D	D	A	A	C	A	B	C	A	A	B	D	A	A	A	B	A	A	A	A	C	D	B	C	D	D	B	C	C	C	B	A	D	D	A	D	B	A	A	D	B	B	A	A	B	C	B	28	47						
24	21	1	C	A	C	A	D	A	A	D	B	C	A	A	B	B	D	D	B	B	B	A	C	A	A	C	A	C	B	C	B	A	A	A	C	C	B	A	A	B	C	B	C	D	B	A	B	C	D	B	A	B	A	B	A	A	A	A	D	C	C	A	A	D	B	B	A	A	D	B	B	28	47

Green = correct answer  
Red = wrong answer

## APPENDIX 3.D

### Item analysis of the TPGT

#### APPENDIX 3.D.1

#### Item analysis of the TPGT: All NS subsamples

Item Number	Percentage with Choice				% Correct	Item Number	Percentage with Choice				% Correct		
	A	B	C	D			Blank	A	B	C		D	Blank
1	13	42	19	<u>26</u>	0	26	31	49	<u>19</u>	11	19	1	19
2	17	7	9	<u>67</u>	0	67	32	<u>75</u>	16	6	3	0	75
3	<u>27</u>	22	43	7	0	27	33	41	20	<u>23</u>	16	0	23
4	<u>46</u>	13	20	20	0	46	34	19	29	10	<u>42</u>	0	42
5	13	22	20	<u>45</u>	0	45	35	24	22	<u>16</u>	38	0	16
6	<u>39</u>	46	9	6	0	39	36	22	16	<u>35</u>	27	0	35
7	<u>33</u>	29	27	10	0	33	37	33	32	9	<u>26</u>	0	26
8	11	11	26	<u>51</u>	0	51	38	27	29	19	<u>24</u>	0	24
9	13	11	<u>65</u>	10	0	65	39	<u>33</u>	36	7	23	0	33
10	32	<u>51</u>	7	9	1	51	40	9	26	17	<u>48</u>	0	48
11	10	<u>26</u>	9	<u>55</u>	0	55	41	13	1	<u>80</u>	4	1	80
12	19	<u>64</u>	3	14	0	64	42	19	<u>23</u>	27	27	3	23
13	30	<u>27</u>	32	10	0	27	43	<u>45</u>	11	29	11	3	45
14	1	38	27	<u>33</u>	0	33	44	<u>80</u>	7	10	3	0	80
15	16	14	35	<u>32</u>	3	32	45	16	<u>45</u>	17	22	0	45
16	<u>49</u>	24	14	11	0	49	46	10	51	<u>22</u>	17	0	22
17	30	<u>22</u>	14	33	0	22	47	23	<u>22</u>	43	11	0	22
18	22	<u>36</u>	10	<u>32</u>	0	32	48	22	<u>23</u>	11	42	1	23
19	14	<u>27</u>	17	<u>38</u>	3	27	49	<u>38</u>	14	22	26	0	38
20	<u>65</u>	6	11	14	3	65	50	<u>52</u>	11	16	20	0	52
21	30	13	<u>48</u>	9	0	48	51	42	<u>23</u>	20	13	1	23
22	<u>80</u>	7	6	7	0	80	52	20	<u>38</u>	19	22	1	38
23	<u>54</u>	35	9	1	1	54	53	23	22	<u>30</u>	23	1	30
24	24	9	<u>64</u>	3	0	64	54	<u>24</u>	11	13	48	3	24
25	<u>57</u>	20	10	13	0	57	55	38	16	<u>33</u>	11	1	33
26	77	7	<u>14</u>	1	0	14	56	6	10	<u>41</u>	42	1	41
27	10	<u>71</u>	10	9	0	71	57	23	26	<u>22</u>	26	3	22
28	39	1	27	<u>32</u>	0	32	58	17	28	<u>32</u>	22	1	32
29	19	36	<u>26</u>	19	0	26	59	14	<u>45</u>	20	19	1	45
30	27	<u>29</u>	29	14	0	29	60	13	20	27	<u>38</u>	1	38

NB: Percentage correct are made bold and underlined  
 N = 69 (three learners were absent)

APPENDIX 3.D.2

Item analysis of the TPGT: NS grade 10 learners

NS Grade 10 Learners' Achievement on the TPGT

Item Analysis

Item Number	Percentage with Choice					% Correct	Item Number	Percentage with Choice					% Correct
	A	B	C	D	Blank			A	B	C	D	Blank	
1	9	48	24	<b><u>19</u></b>	0	19	31	52	<b><u>0</u></b>	5	38	5	0
2	19	9	5	<b><u>67</u></b>	0	67	32	<b><u>71</u></b>	19	9	0	0	71
3	<b><u>48</u></b>	19	28	5	0	48	33	38	43	<b><u>9</u></b>	9	0	9
4	<b><u>24</u></b>	19	24	33	0	24	34	14	48	5	<b><u>33</u></b>	0	33
5	14	24	19	<b><u>43</u></b>	0	43	35	24	29	<b><u>19</u></b>	28	0	19
6	<b><u>33</u></b>	52	14	0	0	33	36	33	19	<b><u>19</u></b>	28	0	19
7	<b><u>24</u></b>	28	38	9	0	24	37	33	33	9	<b><u>24</u></b>	0	24
8	9	14	28	<b><u>48</u></b>	0	48	38	24	38	24	<b><u>14</u></b>	0	14
9	14	24	<b><u>57</u></b>	5	0	57	39	<b><u>28</u></b>	43	14	14	0	28
10	48	<b><u>28</u></b>	19	5	0	28	40	5	24	9	<b><u>62</u></b>	0	62
11	19	28	24	<b><u>28</u></b>	0	28	41	14	0	<b><u>81</u></b>	0	5	81
12	14	<b><u>62</u></b>	5	19	0	62	42	19	<b><u>9</u></b>	28	38	5	9
13	14	<b><u>24</u></b>	52	9	0	24	43	<b><u>38</u></b>	14	33	9	5	38
14	0	38	28	<b><u>33</u></b>	0	33	44	<b><u>81</u></b>	5	9	5	0	81
15	24	19	24	<b><u>33</u></b>	0	33	45	33	<b><u>38</u></b>	14	14	0	38
16	<b><u>29</u></b>	24	19	28	0	29	46	5	71	<b><u>19</u></b>	5	0	19
17	24	<b><u>33</u></b>	33	9	0	33	47	38	<b><u>14</u></b>	38	9	0	14
18	33	33	9	<b><u>24</u></b>	0	24	48	14	<b><u>24</u></b>	19	43	0	24
19	28	<b><u>14</u></b>	9	43	5	14	49	<b><u>28</u></b>	24	14	33	0	28
20	<b><u>52</u></b>	9	14	19	5	52	50	<b><u>52</u></b>	14	24	9	0	52
21	48	14	<b><u>28</u></b>	9	0	28	51	48	<b><u>24</u></b>	19	9	0	24
22	<b><u>62</u></b>	14	14	9	0	62	52	24	<b><u>38</u></b>	24	14	0	38
23	<b><u>33</u></b>	48	19	0	0	33	53	9	24	<b><u>38</u></b>	28	0	38
24	33	9	<b><u>57</u></b>	0	0	57	54	<b><u>9</u></b>	14	9	62	5	9
25	<b><u>43</u></b>	33	9	14	0	43	55	49	19	<b><u>24</u></b>	9	0	24
26	90	5	<b><u>5</u></b>	0	0	5	56	5	14	<b><u>19</u></b>	62	0	19
27	19	<b><u>52</u></b>	14	14	0	52	57	19	33	<b><u>28</u></b>	19	0	28
28	43	0	24	<b><u>33</u></b>	0	33	58	28	28	<b><u>19</u></b>	24	0	19
29	24	33	<b><u>33</u></b>	9	0	33	59	9	<b><u>33</u></b>	24	33	0	33
30	9	<b><u>24</u></b>	33	33	0	24	60	19	19	38	<b><u>24</u></b>	0	24

NB: The percentage correct are made bold and underlined  
 N = 21 (three learners were absent)

APPENDIX 3.D.3

Item analysis of the TPGT: NS grade 11 learners

NS Grade 11 Learners' Achievement on the TPGT

Item Analysis

Item Number	Percentage with Choice				Blank	% Correct	Item Number	Percentage with Choice				Blank	% Correct
	A	B	C	D				A	B	C	D		
1	25	29	25	<u>21</u>	0	21	31	54	<u>25</u>	4	16	0	25
2	12	4	16	<u>67</u>	0	67	32	<u>63</u>	25	4	8	0	63
3	<u>29</u>	25	29	16	0	29	33	42	8	<u>33</u>	16	0	33
4	<u>46</u>	12	25	16	0	46	34	16	25	16	<u>42</u>	0	42
5	21	33	8	<u>38</u>	0	38	35	29	16	<u>16</u>	38	0	16
6	<u>38</u>	46	8	8	0	38	36	16	21	<u>42</u>	21	0	42
7	<u>46</u>	16	21	16	0	46	37	42	21	4	<u>33</u>	0	33
8	12	12	33	<u>42</u>	0	42	38	4	29	29	<u>38</u>	0	38
9	21	4	<u>63</u>	12	0	63	39	<u>29</u>	33	4	33	0	29
10	25	<u>58</u>	0	12	4	58	40	12	16	25	<u>46</u>	0	46
11	12	29	0	<u>58</u>	0	58	41	25	4	<u>67</u>	4	0	67
12	8	<u>71</u>	0	21	0	71	42	12	<u>42</u>	29	12	4	42
13	33	<u>16</u>	38	12	0	16	43	<u>42</u>	8	33	16	0	42
14	4	46	29	<u>21</u>	0	21	44	<u>83</u>	0	12	4	0	83
15	8	12	33	<u>38</u>	8	38	45	8	<u>50</u>	16	25	0	50
16	<u>58</u>	25	16	0	0	58	46	12	42	<u>21</u>	25	0	21
17	38	<u>33</u>	4	25	0	33	47	16	<u>29</u>	46	8	0	29
18	16	<u>42</u>	12	<u>29</u>	0	29	48	25	<u>25</u>	12	38	0	25
19	12	<u>25</u>	12	46	4	25	49	<u>42</u>	16	33	8	0	42
20	<u>63</u>	0	12	21	4	63	50	<u>50</u>	12	4	33	0	50
21	25	16	<u>50</u>	8	0	50	51	38	<u>29</u>	21	8	4	29
22	<u>83</u>	4	4	8	0	83	52	16	<u>29</u>	21	29	4	29
23	<u>63</u>	29	4	0	4	63	53	29	25	<u>38</u>	4	4	38
24	12	16	<u>63</u>	8	0	63	54	<u>29</u>	16	21	29	4	29
25	<u>58</u>	16	8	16	0	58	55	33	12	<u>42</u>	8	4	42
26	79	8	<u>12</u>	0	0	12	56	8	8	<u>38</u>	42	4	38
27	8	<u>67</u>	16	8	0	67	57	16	25	<u>25</u>	25	8	25
28	46	4	29	<u>21</u>	0	21	58	8	21	<u>42</u>	25	4	42
29	12	54	<u>12</u>	21	0	12	59	16	<u>50</u>	12	16	4	50
30	29	<u>29</u>	42	0	0	29	60	21	0	29	<u>46</u>	4	46

NB: The percentage correct are made bold and underlined  
N = 24

APPENDIX 3.D.4

Item analysis of the TPGT: NS grade 12 learners

NS Grade 12 Learners' Achievement on the TPGT

Item Analysis

Item Number	Percentage with Choice				Blank	% Correct	Item Number	Percentage with Choice				Blank	% Correct
	A	B	C	D				A	B	C	D		
1	4	50	8	<b><u>38</u></b>	0	38	31	42	<b><u>29</u></b>	25	4	0	29
2	21	8	4	<b><u>67</u></b>	0	67	32	<b><u>92</u></b>	4	4	0	0	92
3	<b><u>8</u></b>	21	71	0	0	8	33	42	12	<b><u>25</u></b>	21	0	25
4	<b><u>67</u></b>	8	12	12	0	67	34	25	16	8	<b><u>50</u></b>	0	50
5	4	8	33	<b><u>54</u></b>	0	54	35	21	21	<b><u>12</u></b>	46	0	12
6	<b><u>46</u></b>	42	4	8	0	46	36	16	8	<b><u>42</u></b>	33	0	42
7	<b><u>29</u></b>	42	25	4	0	29	37	25	42	12	<b><u>21</u></b>	0	21
8	12	8	16	<b><u>63</u></b>	0	63	38	54	21	4	<b><u>21</u></b>	0	21
9	4	8	<b><u>75</u></b>	12	0	75	39	<b><u>42</u></b>	33	4	21	0	42
10	25	<b><u>63</u></b>	4	8	0	63	40	8	38	16	<b><u>38</u></b>	0	38
11	0	21	4	<b><u>75</u></b>	0	75	41	0	0	<b><u>92</u></b>	8	0	92
12	33	<b><u>58</u></b>	4	4	0	58	42	25	<b><u>16</u></b>	25	33	0	16
13	42	<b><u>42</u></b>	8	8	0	42	43	<b><u>54</u></b>	12	21	8	4	54
14	0	29	25	<b><u>46</u></b>	0	46	44	<b><u>75</u></b>	16	8	0	0	75
15	16	12	46	<b><u>25</u></b>	0	25	45	8	<b><u>46</u></b>	21	25	0	46
16	<b><u>58</u></b>	25	8	8	0	58	46	12	42	<b><u>25</u></b>	21	0	25
17	29	<b><u>0</u></b>	8	63	0	0	47	16	<b><u>21</u></b>	46	16	0	21
18	16	33	8	<b><u>42</u></b>	0	42	48	25	<b><u>21</u></b>	4	46	4	21
19	4	<b><u>42</u></b>	29	25	0	42	49	<b><u>42</u></b>	4	16	38	0	42
20	<b><u>79</u></b>	8	8	4	0	79	50	<b><u>54</u></b>	8	21	16	0	54
21	21	8	<b><u>63</u></b>	8	0	63	51	42	<b><u>16</u></b>	21	21	0	16
22	<b><u>92</u></b>	4	0	4	0	92	52	21	<b><u>46</u></b>	12	21	0	46
23	<b><u>63</u></b>	29	4	4	0	63	53	29	16	<b><u>16</u></b>	38	0	16
24	29	0	<b><u>71</u></b>	0	0	71	54	<b><u>33</u></b>	4	8	54	0	33
25	<b><u>67</u></b>	12	12	8	0	67	55	33	16	<b><u>33</u></b>	16	0	33
26	63	8	<b><u>25</u></b>	4	0	25	56	4	8	<b><u>63</u></b>	25	0	63
27	4	<b><u>92</u></b>	0	4	0	92	57	33	21	<b><u>12</u></b>	33	0	12
28	29	0	29	<b><u>42</u></b>	0	42	58	16	33	<b><u>33</u></b>	16	0	33
29	21	21	<b><u>33</u></b>	25	0	33	59	16	<b><u>50</u></b>	25	8	0	50
30	42	<b><u>33</u></b>	12	12	0	33	60	0	42	16	<b><u>42</u></b>	0	42

NB: The percentage correct are made bold and underlined  
N = 24

APPENDIX 3.D.5

Item analysis of the TPGT: SAS subsample

Item Analysis of the TPGT: All SAS subsamples

Percentage with Choice						% Correct	Percentage with Choice						% Correct
Item Number	A	B	C	D	Blank		Item Number	A	B	C	D	Blank	
1	0	33	47	<u>19</u>	0	19	31	<u>31</u>	10	11	0	31	
2	18	14	7	<u>60</u>	1	60	32	<u>89</u>	8	1	0	89	
3	<u>36</u>	18	33	<u>13</u>	0	36	33	22	<u>28</u>	22	0	28	
4	<u>76</u>	4	15	4	0	76	34	6	14	18	<u>63</u>	63	
5	6	21	11	<u>61</u>	1	61	35	33	17	<u>18</u>	32	18	
6	<u>43</u>	46	6	6	0	43	36	10	7	<u>58</u>	25	58	
7	<u>31</u>	19	28	21	1	31	37	28	18	14	<u>40</u>	40	
8	7	6	8	<u>79</u>	0	79	38	13	22	22	<u>43</u>	43	
9	24	19	<u>38</u>	19	0	38	39	<u>36</u>	29	8	25	36	
10	44	<u>43</u>	6	6	1	43	40	8	26	25	<u>40</u>	40	
11	8	19	17	<u>54</u>	1	54	41	8	4	<u>86</u>	1	86	
12	6	<u>58</u>	0	<u>36</u>	0	58	42	15	<u>24</u>	35	26	24	
13	8	<u>60</u>	17	15	0	60	43	<u>43</u>	13	24	19	43	
14	6	53	11	<u>31</u>	0	31	44	<u>81</u>	7	1	10	81	
15	4	7	35	<u>54</u>	0	54	45	17	<u>26</u>	26	31	26	
16	<u>76</u>	8	7	8	0	76	46	26	15	<u>51</u>	7	51	
17	25	<u>39</u>	11	25	0	39	47	13	<u>42</u>	24	21	42	
18	14	31	15	<u>39</u>	1	39	48	28	<u>28</u>	14	31	28	
19	4	<u>51</u>	13	32	0	51	49	<u>35</u>	19	31	15	35	
20	<u>53</u>	13	24	11	0	53	50	<u>69</u>	10	10	11	69	
21	13	6	<u>67</u>	15	0	67	51	57	<u>25</u>	8	10	25	
22	<u>96</u>	0	0	4	0	96	52	11	<u>51</u>	7	31	51	
23	<u>32</u>	39	22	6	1	32	53	17	22	<u>22</u>	38	22	
24	22	6	<u>65</u>	7	0	65	54	<u>29</u>	4	28	36	29	
25	<u>74</u>	11	14	1	0	74	55	22	8	<u>63</u>	4	63	
26	71	10	<u>15</u>	4	0	15	56	19	17	<u>28</u>	32	28	
27	1	<u>89</u>	4	4	1	89	57	18	19	<u>14</u>	46	14	
28	18	4	7	<u>71</u>	0	71	58	17	18	<u>44</u>	17	44	
29	13	32	<u>33</u>	22	0	33	59	29	<u>22</u>	32	14	22	
30	29	<u>46</u>	14	11	0	46	60	15	29	4	<u>49</u>	49	

NB: Percentage correct are made bold and underlined  
N = 72

APPENDIX 3.D.6

Item analysis of the TPGT: SAS grade 10 learners

SAS Grade 10 Learners' Achievement on the TPGT

Item Analysis

Item Number	Percentage with Choice					% Correct	Item Number	Percentage with Choice					% Correct
	A	B	C	D	Blank			A	B	C	D	Blank	
1	0	25	71	<u>4</u>	0	4	31	63	<u>16</u>	12	8	0	16
2	12	16	0	<u>71</u>	0	71	32	<u>83</u>	12	4	0	0	83
3	<u>46</u>	25	16	12	0	46	33	21	42	<u>12</u>	25	0	12
4	<u>79</u>	0	12	8	0	79	34	0	8	<u>16</u>	<u>75</u>	0	75
5	0	33	8	<u>54</u>	4	54	35	33	12	<u>16</u>	38	0	16
6	<u>46</u>	46	4	4	0	46	36	12	16	<u>50</u>	21	0	50
7	<u>8</u>	29	38	21	4	8	37	29	29	12	<u>29</u>	0	29
8	8	4	8	<u>79</u>	0	79	38	8	25	42	<u>25</u>	0	25
9	29	8	<u>46</u>	16	0	46	39	<u>38</u>	21	4	33	4	38
10	58	<u>25</u>	4	12	0	25	40	4	16	21	<u>58</u>	0	58
11	8	21	21	<u>50</u>	0	50	41	12	4	<u>83</u>	0	0	83
12	8	<u>58</u>	0	33	0	58	42	12	<u>21</u>	38	29	0	21
13	8	<u>58</u>	25	8	0	58	43	<u>46</u>	8	25	16	4	46
14	0	71	0	<u>29</u>	0	29	44	<u>79</u>	4	0	16	0	79
15	8	4	42	<u>46</u>	0	50	45	29	<u>16</u>	8	46	0	16
16	<u>79</u>	8	8	4	0	79	46	25	12	<u>58</u>	4	0	58
17	12	<u>33</u>	21	33	0	33	47	21	<u>25</u>	25	29	0	25
18	21	33	29	<u>16</u>	0	16	48	42	<u>12</u>	12	33	0	12
19	8	<u>46</u>	12	33	0	46	49	<u>21</u>	29	33	16	0	21
20	<u>42</u>	21	33	4	0	42	50	<u>67</u>	8	8	16	0	67
21	8	4	<u>71</u>	16	0	71	51	38	<u>21</u>	16	25	0	21
22	<u>100</u>	0	0	0	0	100	52	12	<u>42</u>	4	42	0	42
23	<u>38</u>	46	8	8	0	38	53	25	25	<u>16</u>	33	0	16
24	29	8	<u>58</u>	4	0	58	54	<u>38</u>	4	29	29	0	38
25	<u>83</u>	8	8	0	0	83	55	29	4	<u>63</u>	4	0	63
26	54	21	<u>21</u>	4	0	21	56	21	16	<u>29</u>	25	8	29
27	0	<u>96</u>	0	0	4	96	57	4	8	<u>21</u>	63	4	21
28	25	4	0	<u>71</u>	0	71	58	29	12	<u>29</u>	21	8	29
29	16	21	<u>38</u>	25	0	38	59	33	<u>12</u>	29	21	4	12
30	42	<u>38</u>	8	12	0	38	60	21	33	0	<u>42</u>	4	42

NB: The percentage correct are made bold and underlined  
N = 24

APPENDIX 3.D.7

Item analysis of the TPGT: SAS grade 11 learners

SAS Grade 11 Learners' Achievement on the TPGT

Item Analysis

Item		Percentage with Choice					% Correct	Item		Percentage with Choice					% Correct
Number	A	B	C	D	Blank		Number	A	B	C	D	Blank			
1	0	33	58	<b><u>8</u></b>	0	8	31	58	<b><u>21</u></b>	16	4	0	21		
2	8	4	4	<b><u>79</u></b>	4	79	32	<b><u>87</u></b>	8	0	0	4	87		
3	<b><u>50</u></b>	16	25	8	0	50	33	25	25	<b><u>25</u></b>	25	0	25		
4	<b><u>63</u></b>	8	25	4	0	63	34	12	8	21	<b><u>58</u></b>	0	58		
5	4	25	12	<b><u>58</u></b>	0	58	35	33	25	<b><u>8</u></b>	33	0	8		
6	<b><u>46</u></b>	42	0	12	0	46	36	8	4	<b><u>63</u></b>	25	0	63		
7	<b><u>12</u></b>	12	38	38	0	12	37	21	16	21	<b><u>42</u></b>	0	42		
8	12	12	4	<b><u>71</u></b>	0	71	38	25	38	21	<b><u>16</u></b>	0	16		
9	16	25	<b><u>38</u></b>	21	0	38	39	<b><u>21</u></b>	38	12	29	0	21		
10	54	<b><u>29</u></b>	8	4	4	29	40	8	38	25	<b><u>29</u></b>	0	29		
11	12	29	8	<b><u>50</u></b>	0	50	41	8	4	<b><u>83</u></b>	4	0	83		
12	8	<b><u>38</u></b>	0	54	0	38	42	21	<b><u>21</u></b>	42	16	0	21		
13	12	<b><u>38</u></b>	25	25	0	38	43	<b><u>25</u></b>	8	42	25	0	25		
14	8	75	16	<b><u>0</u></b>	0	0	44	<b><u>79</u></b>	8	4	4	4	79		
15	4	4	38	<b><u>54</u></b>	0	54	45	21	<b><u>16</u></b>	38	25	0	16		
16	<b><u>58</u></b>	12	12	16	0	58	46	25	33	<b><u>29</u></b>	12	0	29		
17	42	<b><u>33</u></b>	4	21	0	33	47	8	<b><u>38</u></b>	33	16	4	38		
18	12	38	8	<b><u>38</u></b>	4	38	48	38	<b><u>16</u></b>	16	29	0	16		
19	0	<b><u>50</u></b>	12	38	0	50	49	<b><u>33</u></b>	12	38	16	0	33		
20	<b><u>50</u></b>	12	16	21	0	50	50	<b><u>67</u></b>	12	8	12	0	67		
21	21	4	<b><u>63</u></b>	12	0	63	51	<b><u>87</u></b>	<b><u>4</u></b>	4	4	0	4		
22	<b><u>87</u></b>	0	0	12	0	87	52	8	<b><u>58</u></b>	4	29	0	58		
23	<b><u>12</u></b>	42	38	4	4	12	53	12	29	<b><u>16</u></b>	38	4	16		
24	21	8	<b><u>54</u></b>	16	0	54	54	<b><u>16</u></b>	8	16	50	8	16		
25	<b><u>63</u></b>	16	16	4	0	63	55	16	8	<b><u>63</u></b>	4	8	63		
26	75	4	<b><u>12</u></b>	8	0	12	56	4	29	<b><u>33</u></b>	29	4	33		
27	0	<b><u>79</u></b>	8	12	0	79	57	12	29	<b><u>12</u></b>	42	4	12		
28	16	8	12	<b><u>63</u></b>	0	63	58	16	25	<b><u>38</u></b>	16	4	38		
29	8	38	<b><u>33</u></b>	21	0	33	59	38	<b><u>16</u></b>	25	16	4	16		
30	29	<b><u>21</u></b>	29	21	0	21	60	21	29	8	<b><u>38</u></b>	4	38		

NB: The percentage correct are made bold and underlined  
N = 24

APPENDIX 3.D.8

Item analysis of the TPGT: SAS grade 12 learners

SAS Grade 12 Learners' Achievement on the TPGT

Item Analysis

Item		Percentage with Choice					% Correct	Item		Percentage with Choice					% Correct
Number	A	B	C	D	Blank		Number	A	B	C	D	Blank			
1	0	42	12	<b><u>46</u></b>	0	46	31	25	<b><u>54</u></b>	0	21	0	54		
2	33	21	16	<b><u>29</u></b>	0	29	32	<b><u>96</u></b>	4	0	0	0	96		
3	<b><u>12</u></b>	12	58	16	0	12	33	21	16	<b><u>46</u></b>	16	0	46		
4	<b><u>87</u></b>	4	8	0	0	87	34	4	25	16	<b><u>54</u></b>	0	54		
5	12	4	12	<b><u>71</u></b>	0	71	35	33	12	<b><u>29</u></b>	25	0	29		
6	<b><u>38</u></b>	50	12	0	0	38	36	8	0	<b><u>63</u></b>	29	0	63		
7	<b><u>71</u></b>	16	8	4	0	71	37	33	8	8	<b><u>50</u></b>	0	50		
8	0	0	12	<b><u>87</u></b>	0	87	38	4	4	4	<b><u>87</u></b>	0	87		
9	25	25	<b><u>29</u></b>	21	0	29	39	<b><u>50</u></b>	29	8	12	0	50		
10	21	<b><u>75</u></b>	4	0	0	75	40	12	25	29	<b><u>33</u></b>	0	33		
11	4	8	21	<b><u>63</u></b>	4	63	41	4	4	<b><u>91</u></b>	0	0	91		
12	0	<b><u>79</u></b>	0	21	0	79	42	12	<b><u>29</u></b>	25	33	0	29		
13	4	<b><u>83</u></b>	0	12	0	83	43	<b><u>58</u></b>	21	4	16	0	58		
14	8	12	16	<b><u>63</u></b>	0	63	44	<b><u>83</u></b>	8	0	8	0	83		
15	0	12	25	<b><u>63</u></b>	0	63	45	0	<b><u>46</u></b>	33	21	0	46		
16	<b><u>91</u></b>	4	0	4	0	91	46	29	0	<b><u>67</u></b>	4	0	67		
17	21	<b><u>50</u></b>	8	21	0	50	47	8	<b><u>63</u></b>	12	16	0	63		
18	8	21	8	<b><u>63</u></b>	0	63	48	4	<b><u>54</u></b>	12	29	0	54		
19	4	<b><u>58</u></b>	12	25	0	58	49	<b><u>50</u></b>	16	21	12	0	50		
20	<b><u>67</u></b>	4	21	8	0	67	50	<b><u>75</u></b>	8	12	4	0	75		
21	8	8	<b><u>67</u></b>	16	0	67	51	46	<b><u>50</u></b>	4	0	0	50		
22	<b><u>100</u></b>	0	0	0	0	100	52	12	<b><u>54</u></b>	12	21	0	54		
23	<b><u>46</u></b>	29	21	4	0	46	53	12	12	<b><u>33</u></b>	42	0	33		
24	16	0	<b><u>83</u></b>	0	0	83	54	<b><u>33</u></b>	0	38	29	0	33		
25	<b><u>75</u></b>	8	16	0	0	75	55	21	12	<b><u>63</u></b>	4	0	63		
26	83	4	<b><u>12</u></b>	0	0	12	56	33	4	<b><u>21</u></b>	42	0	21		
27	4	<b><u>91</u></b>	4	0	0	91	57	38	21	<b><u>8</u></b>	33	0	8		
28	12	0	8	<b><u>79</u></b>	0	79	58	4	16	<b><u>67</u></b>	12	0	67		
29	12	38	<b><u>29</u></b>	21	0	29	59	16	<b><u>38</u></b>	42	4	0	38		
30	16	<b><u>79</u></b>	4	0	0	79	60	4	25	4	<b><u>67</u></b>	0	67		

NB: The percentage correct are made bold and underlined  
N = 24

**APPENDIX 4**  
**STUDENTS' QUESTIONNAIRE 2**

**APPENDIX 4.A**  
**Geometric Items Sorting Test (GIST)**

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**GEOMETRIC ITEMS SORTING TEST**  
**(GIST)**

**Geometric shapes Item Sorting Activities: The Manipulatives.**

**Date: .....**

**Time Allowed: 90 minutes**

**INSTRUCTIONS:**

1. Do not start until you are told to do so.
2. While you are waiting, fill in the appropriate information in the spaces below.  
**Name (surname first):** .....  
**Name of school:** .....  
**Grade:** .....  
**Age (in years):** ..... **Sex:** .....
3. This activity involves the identification, description and classification of some geometrical shapes in the form of cardboard cut-outs.
4. You are allowed to make use of straightedges (rulers), protractors, dividers, or any other mathematical instruments in the mathematical set that you are supplied.
5. Answer the questions in the spaces provided for each question.

**Question 1**

Placed before you are a set of numbered geometrical shapes. You are required to identify each of the shapes by writing the name of each shape and giving a reason to explain how you know it is that shape. An example is given.

<b>Shape No.</b>	<b>Name of shape</b>	<b>Reason</b>
Example 0.	Regular Pentagon	It is a polygon with five equal sides.
1.		
2.		
3.		
4.		
5.		
6.		
7.		
8.		
9		

<b>Shape No.</b>	<b>Name of shape</b>	<b>Reason</b>
10.		
11.		
12.		
13.		
14.		
15.		
16.		
17.		
18.		
19.		
20.		
21.		
22.		

<b>Shape No.</b>	<b>Name of shape</b>	<b>Reason</b>
23.		
24.		
25.		
26.		
27.		
28.		
29.		
30.		

**Question 2**

i). Sort the shapes on your desk into two broad groups. List the shape numbers only.

<b>Group A shapes</b>	<b>Group B shapes</b>

ii). What would you tell someone to look for in order to pick out, from among these shapes, a shape that belongs to:

a). Group A shapes;

b). Group B shapes?

**Answer.**

a). For Group A shapes: .....  
.....  
.....

b). For Group B shapes: .....  
.....  
.....

iii). (a). What is the general/common name for all the shapes in Group A?

**Answer:** .....

(b). What is the general/common name for all the shapes in Group B?

**Answer:** .....

**Question 3**

i). Now, make a further sorting of the shapes in (your) Group A into as many smaller groups as you can, putting together shapes that are alike in some way, by completing the following table.

**NOTE: A shape may belong to two or more groups.**

<b>Group</b>	<b>Shape Nos.</b>	<b>Common name for these shapes</b>	<b>How are these shapes alike?</b>
I.			
II.			
III.			
IV.			
V.			
VI.			
VII.			

ii). Now, make a further sorting of the shapes in (your) Group B into as many smaller groups as you can, putting together shapes that are alike in some way, by completing the following table.

**NOTE: A shape may belong to two or more groups.**

<b>Group</b>	<b>Shape Nos.</b>	<b>Common name for these shapes</b>	<b>How are these shapes alike?</b>
I.			
II.			
III.			
IV.			
V.			
VI.			
VII.			
VIII.			

**Question 4.**

**i).** What would you tell someone to look for in order to pick out all the **parallelograms** from among these shapes?

**Answer:** .....  
.....  
.....  
.....

**ii).** What would you tell someone to look for in order to pick out all the **rectangles** from among these shapes?

**Answer:** .....  
.....  
.....  
.....

**iii).** What would you tell someone to look for in order to pick out all the **rhombuses** from among these shapes?

**Answer:** .....  
.....  
.....  
.....

**iv).** What would you tell someone to look for in order to pick out all the **squares** from among these shapes?

**Answer:** .....  
.....  
.....  
.....

**v).** What would you tell someone to look for in order to pick out all the **trapeziums** from among these shapes?

**Answer:** .....  
.....  
.....  
.....

**vi).** What would you tell someone to look for in order to pick out all the **isosceles triangles** from among these shapes?

**Answer:** .....  
.....  
.....  
.....

**Question 5.**

**i). (a).** Is shape No. 23 a rectangle?

**Answer:** .....

**b).** How do you know?

**Answer:** .....

.....

.....

**ii). (a).** Is shape No. 17 a parallelogram?

**Answer:** .....

**b).** How do you know?

**Answer:** .....

.....

.....

**iii). (a).** Is shape No. 6 a rhombus?

**Answer:** .....

**b).** How do you know?

**Answer:** .....

.....

.....

**iv). (a).** Is shape No. 1 a parallelogram?

**Answer:** .....

**b).** How do you know?

**Answer:** .....

.....

.....

**v). (a).** Is shape No. 30 a scalene triangle?

**Answer:** .....

**b).** How do you know?

**Answer:** .....

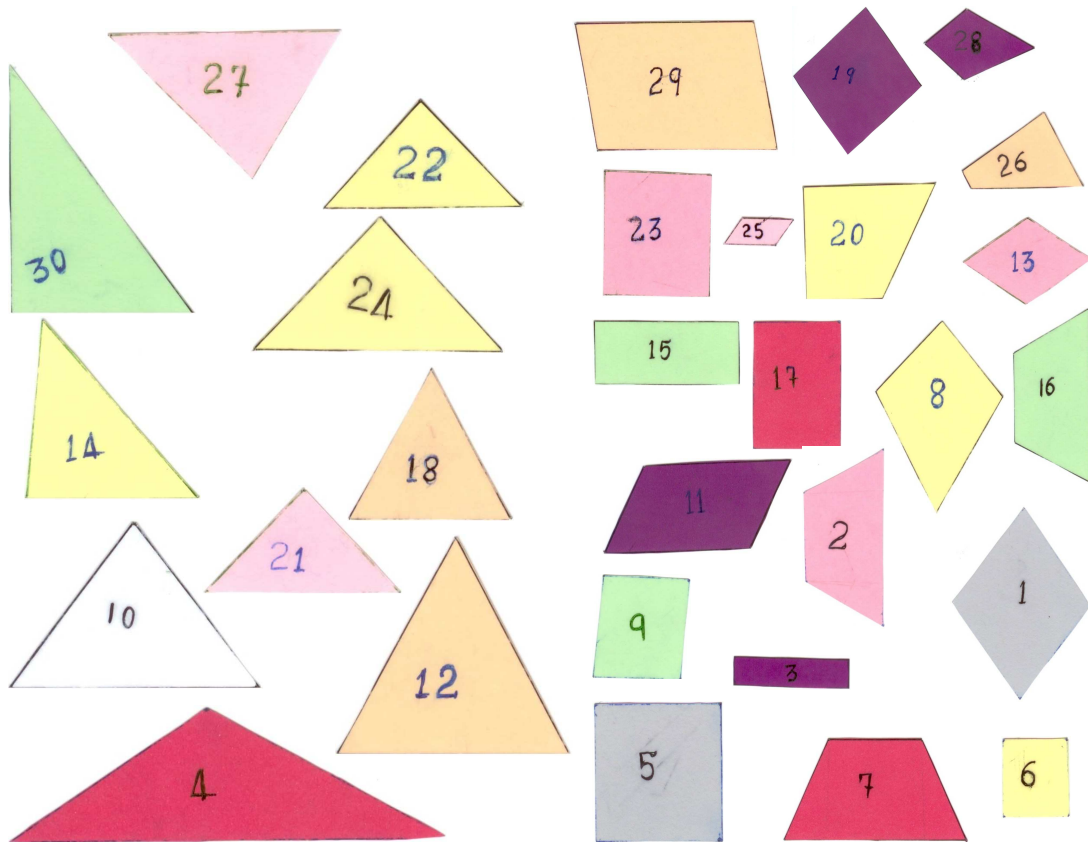
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## APPENDIX 4.B

### Concept cards of triangles and quadrilaterals used for the GIST

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## Appendix 4.C

### Marking Scheme for the GIST

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#### MARKING GUIDE FOR THE GIST

##### Question 1.

- 0 – Student does not name the shape at all, or names the shape incorrectly.
- 1 – Student names the shape correctly, but gives wrong/inadequate reason (e.g. the student names a square correctly, but states that “all four sides are equal” as the only reason).
- 2 – Student names the shape correctly and gives reasons that are both correct and adequate (e.g. the student names a square correctly and gives reasons as: “It is a parallelogram; All 4 sides are equal; It has 4 right angles” Or gives a correct definition of a square as the reason).

**Marks Obtainable: 60.**

##### Question 2.

###### (i).

- 0 – Student makes partial/incomplete groupings of shapes
- 1 – Student makes correct and complete groupings of shapes.

###### (ii).

- 0 – Student associates wrong number of sides or wrong name with shapes of the group concerned.
- 1 – Student associates correct number of sides or correct name with shapes of the group concerned.

###### (iii).

- 0 – Student gives incorrect/inappropriate general/common name for the group shapes.
- 1 – Student gives correct general/common name for the group shapes.

**Marks Obtainable: 3.**

**Question 3.** Award marks for any **five subgroups** each of the **two major groups** of shapes as follows:

- 0 – Student does not sort shapes into subgroups at all; OR **leaves out 2 or more** members (or includes non-members) of the subgroup, gives incorrect common name of shapes in the subgroup and/or does not tell (or does so, but incorrectly) how shapes in the subgroup are alike.
- 1 – Student sorts shapes into subgroups (without class inclusion), but **omits not more than 1** member of the subgroup, gives the correct common/general name of members of the subgroup, but does not tell (or does tell incorrectly) how shapes in the subgroup are alike.
- 2 – Student sorts shapes into subgroups **without omitting any member** of the subgroup (with class inclusion), gives the correct common/general name of shapes in the subgroup, tells correctly how the shapes in the subgroup are alike.

**Marks Obtainable: 20.**

##### Question 4. (i) – (vi).

- 0 – Students writes nothing. OR lists incorrect properties of the shape and/or gives wrong definition of the shape.
- 1 – Student lists only a few examples /properties of the shape; OR gives partially correct definition of the shape.
- 2 – Student lists unique properties of the shape; OR gives a clear and accurate definition of the shape.

**Marks Obtainable: 12.**

**Question 5. (i) – (v).**

- 0 – Student writes nothing. Or student gives incorrect answer.
- 1 – Student gives the correct answer to part (a), but gives incomplete reason (Part b); OR student gives wrong answer to Part (a), but justifies his/her answer by writing a sequence of statements that is invalid only because it is based on a faulty reasoning about the shape (Part b).
- 2 – Student gives the correct answer to Part (a), and lists (or defines) the defining **properties of the more inclusive shape** (Part b).

**Marks Obtainable: 10.**

**TOTAL MARKS OBTAINABLE: 105.**

**APPENDIX 5**  
**STUDENTS' QUESTIONNAIRE 3**

**APPENDIX 5.A**

**Conjecturing in Plane Geometry Test (CPGT)**

APPENDIX 5.A.1

Worksheet 1 of the CPGT for the grade 10 learners

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**CONJECTURING IN PLANE GEOMETRY TEST  
(CPGT) GRADE 10**

**Date:** .....

**Time Allowed: 90 minutes**

**INSTRUCTIONS:**

1. Do not start until you are told to do so.
2. While you are waiting, please fill the appropriate information in the spaces below.

**Name (Surname first):** .....

**Name of school:** .....

**Grade:** .....

**Age (in years):** ..... **Sex:** .....

3. This activity involves the use of geometrical construction (or drawings) to **identify, describe, define, and make conjectures** about the properties of some common geometrical shapes.
4. For each investigation, you are to **draw or construct** a geometric shape. You are required to **follow the instructions** detailed out in each of the **steps** under each investigation.
5. Your drawings or construction should be made in the **spaces provided** for each investigation.
6. Where an investigation requires you to **measure an angle or a line segment**, please do so and write down your answer in the **spaces provided** for that purpose.
7. Feel free to make use of any of the **instruments (rulers, compasses, protractors, dividers, setsquares etc.) in the mathematical set** that you are provided.
8. Where an investigation requires you to answer some questions, please supply your answers in the spaces provided for that purpose.
9. Feel free to ask me any question.

**Investigation 1:**

**Step 1:** Using a ruler, draw any triangle ABC, with each side of length greater than or equal to 5cm.

**Step 2:** Use a protractor to measure the angles of the triangle.

$\angle A = \dots\dots\dots$ ;  $\angle B = \dots\dots\dots$ ;  $\angle C = \dots\dots\dots$

**Step 3:** Find the sum of the angles of  $\Delta ABC$ .

$\angle A + \angle B + \angle C = \dots\dots\dots$

Compare your result with those of other students near you.

**Space for your diagram.**

**Question:**

What can you conclude about the sum of the angles of a triangle?

**Answer:** .....  
.....  
.....  
.....  
.....  
.....  
.....

**Investigation 2:**

**Step 1:** Draw  $\triangle ABC$ , such that  $|AB| = 6\text{cm}$ , and  $|BC| = 6\text{cm}$ .

**Step 2:** Draw another  $\triangle PQR$ , such that  $|PQ| = 5\text{cm}$  and  $|QR| = 5\text{cm}$ .

**Step 3:** Use a protractor to measure the following angles.

- (i)  $\angle A = \dots\dots\dots$ ; (ii)  $\angle C = \dots\dots\dots$ ; (iii)  $\angle P = \dots\dots\dots$ ; (iv)  $\angle R = \dots\dots\dots$

**Space for your diagram.**

**Questions:**

1) What are  $\triangle ABC$  and  $\triangle PQR$  called?

**Answer:** They are both called .....

2) In  $\triangle ABC$ , how does  $\angle A$  compare with  $\angle C$ ?

**Answer:** .....

.....

3) In  $\triangle PQR$ , how does  $\angle P$  compare with  $\angle R$ ?

**Answer:** .....

.....

4) From your answers to question Nos. 2 and 3 above, what can you conclude about the angles of a triangle in which two sides are equal?

**Answer:** .....

.....

.....

.....

.....

.....

**Investigation 3:**

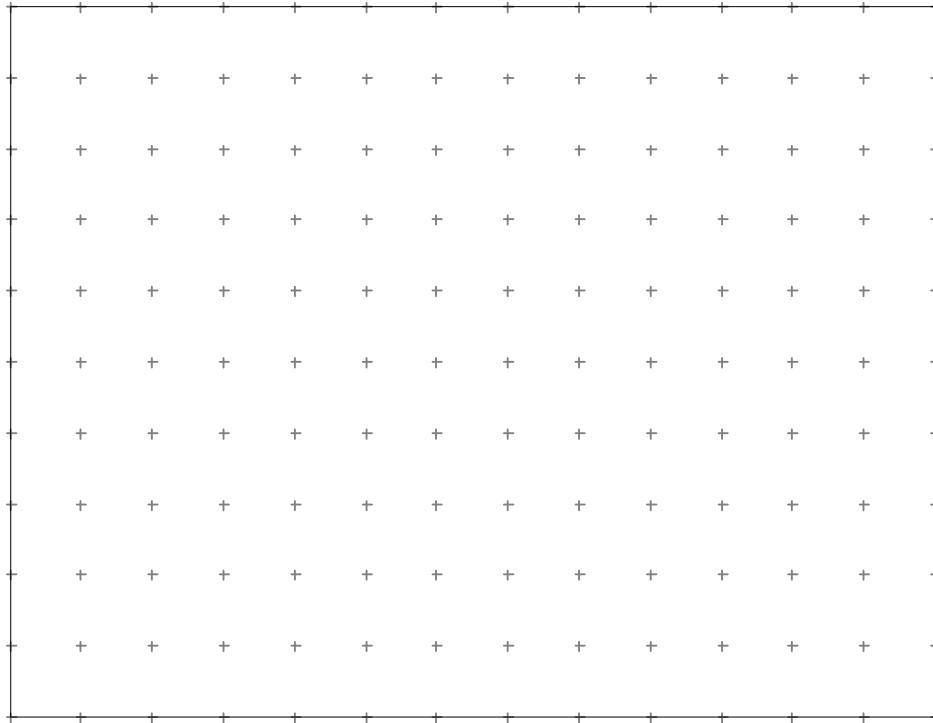
**Step 1:** Draw any  $\triangle ABC$ , such that all the sides are equal. Let the sides be at least 5cm long.

**Step 2:** Measure all the angles of  $\triangle ABC$ .

$\angle A = \dots\dots\dots$ ;  $\angle B = \dots\dots\dots$ ;  $\angle C = \dots\dots\dots$ ;

Compare your results with those of others near you.

**Space for your diagram.**



**Questions:**

1) What is  $\triangle ABC$  called?

**Answer:** .....

2) From your observation in **step 2 above**, what can you conclude about the angles of a triangle in which all the sides are equal?

**Answer:** .....

.....

.....

3) Suppose in a certain  $\triangle PQR$ , it is given that **all the angles** are equal, does this mean that **all the sides** are also equal?

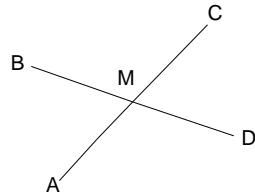
**Answer:** .....

**Investigation 4:**

**Step 1:** Use a ruler to draw any straight line AC (slanting upwards from left to right) which is greater than or equal to 6cm.

**Step2:** Locate the midpoint of the line AC and label it as M.

**Step 3:** Draw another straight line BD which is equal in length to line AC (slanting downward from left to right) and intersecting (or crossing) AC in such a way that M is also its midpoint. Your diagram from steps 1 – 3 should look like this:



**Step 4:** Use a ruler to join AB, BC, CD and AD.

**Step 5:** Measure the following.

i)  $AB = \dots\dots\dots$ ;  $DC = \dots\dots\dots$ ;  $BC = \dots\dots\dots$ ;  $AD = \dots\dots\dots$

ii)  $\angle ADC = \dots\dots\dots$ ;  $\angle DAB = \dots\dots\dots$ ;  $\angle ABC = \dots\dots\dots$ ;  $\angle BCD = \dots\dots\dots$

Compare your results with those of others near you.

**Space for your diagram.**

**Questions.**

1. What type of parallelogram is ABCD?

**Answer:** .....

2. How do you know for sure that ABCD is the parallelogram that you have named in question No. 1 above? List as many reasons as you possibly can.

**Answer:** i) .....

ii) .....

iii) .....

iv) .....

v) .....

3. From your diagram and those of others near you, what can you **conclude** about a parallelogram whose diagonals are equal?

**Answer:** .....

.....

4. Give a very short definition of the shape ABCD.

**Answer:** .....

.....

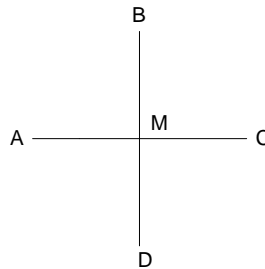
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**Investigation 5:**

**Step 1:** Draw any horizontal straight line AC, which is about 6cm long.

**Step 2:** Locate the midpoint of AC and label it as M.

**Step 3:** Draw another straight line BD having the same length as AC, and in such a way that M is its midpoint and AC is perpendicular to BD (i.e.  $\angle AMB = 90^\circ$ ). Your drawing should look like this:



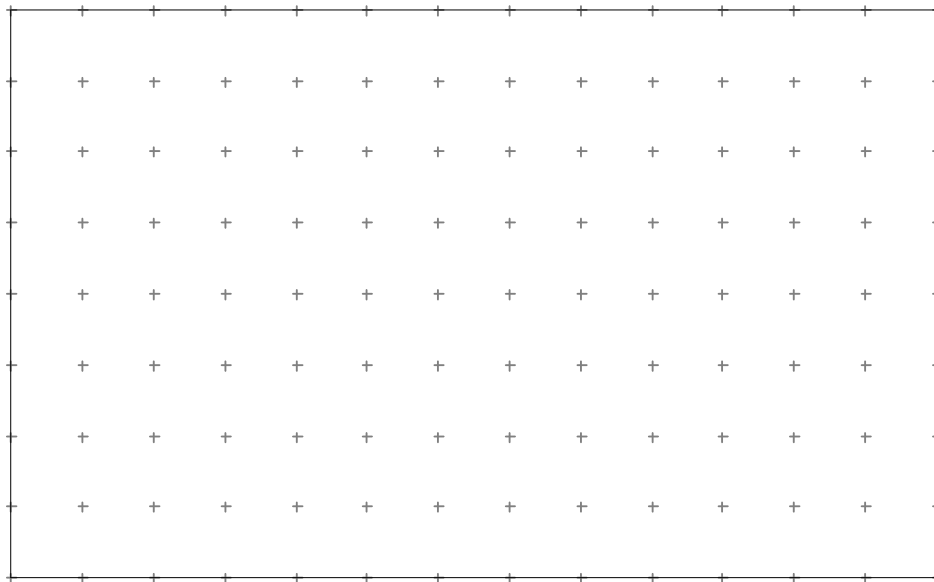
**Step 4:** Join AB, BC, CD and AD.

**Step 5:** Measure the following.

- i) AB = .....cm; BC = .....cm; AD = .....cm; DC = .....cm;
- ii)  $\angle ADC = \dots\dots\dots$ ;  $\angle DAB = \dots\dots\dots$ ;  $\angle ABC = \dots\dots\dots$ ;  $\angle BCD = \dots\dots\dots$

Compare your results with those of others near you.

**Space for your diagram.**



**Questions:**

1) What type of parallelogram is ABCD?

**Answer:** .....

2) How do you know for sure that ABCD is the parallelogram that you have named in question No. 1 above? List as many reasons as you possibly can.

**Answer:** i) .....

ii) .....

iii) .....

iv) .....

v) .....

3) What properties does this shape have that the shape you drew in investigation 4 does not have?

**Answer:** i) .....

ii).....

iii) .....

4) From your diagram and those of others near you, what can you **conclude** about a parallelogram which has equal diagonals that bisect each other at right angles?

**Answer:** .....

5) Give a very short definition of the figure that you have drawn.

**Answer:** .....

.....

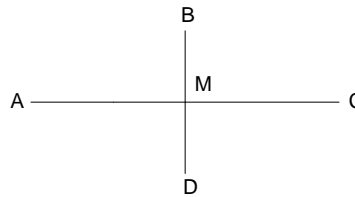
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### Investigation 6:

**Step 1:** Draw any horizontal straight line AC which is greater than or equal to 8cm long.

**Step 2:** Locate the midpoint of AC and label it as M.

**Step 3:** Draw another straight line BD, shorter (or longer) than AC in such a way that M is its midpoint and AC is perpendicular to BD. Your drawing should look like this:



**Step 4:** Join AB, BC, CD and AD.

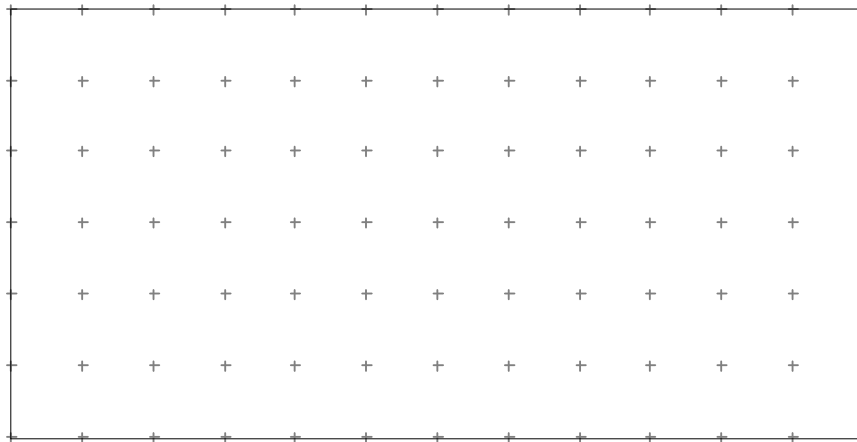
**Step 5:** Measure the following.

i) AB = .....; BC = .....; AD = .....; DC = .....

ii)  $\angle ADC = \dots\dots\dots$   $\angle ABC = \dots\dots\dots$   $\angle DAB = \dots\dots\dots$   $\angle DCB = \dots\dots\dots$

Compare your results with those of others near you.

**Space for your diagram.**



**Questions:**

1. What type of parallelogram is ABCD?

**Answer:** .....

2. How do you know for sure that ABCD is the parallelogram that you have named in question No. 1 above? List as many reasons as you possibly can.

**Answer:** i) .....

ii) .....

iii) .....

iv) .....

v) .....

3. List the properties that are common to the shape that you drew in investigation 5 and this new shape that you have just drawn.

**Answer:** i) .....

ii) .....

iii) .....

iv) .....

4. Which property (or properties) does the shape in investigation 5 have that this new shape does not have?

**Answer:** .....

.....

.....

.....

.....

5. Does the shape in investigation 5 belong to this class of new shape that you have just drawn? Give a reason.

**Answer:** .....

**Reason for your answer:** .....

.....

.....

.....

6. Give a very short definition of the shape that you have just drawn.

**Answer:** .....

.....

.....

APPENDIX 5.A.2

Worksheet 2 of the CPGT for the grade 11 learners

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**CONJECTURING IN PLANE GEOMETRY TEST  
(CPGT) GRADE 11**

**Date :** .....

**Time Allowed: 90 minutes**

**INSTRUCTIONS:**

1. Do not start until you are told to do so.
2. While you are waiting, please fill the appropriate information in the spaces below.

**Name (Surname first):** .....

**Name of school:** .....

**Grade:** .....

**Age (in years):** ..... **Sex:** .....

1. This activity involves the use of geometrical construction (or drawings) to investigate the conditions for **two triangles to be similar and formulate conjectures** about similar triangles. **The form that your conjectures should take has been exemplified in investigation 1 below.**
2. For each investigation, you are to **draw** or **construct** a triangle. You are required to **follow the instructions** detailed out in each of the **steps** under each investigation.
3. Your drawings or construction should be made in the **spaces provided** for each investigation.
4. Where an investigation requires you to **measure an angle or a line segment**, please do so and write down your answer in the **spaces provided** for that purpose.
5. Feel free to make use of any of the **instruments (rulers, compasses, protractors, dividers, setsquares etc.) in the mathematical set** that you are provided.
6. Where an investigation requires you to answer some questions, please supply your answers in the spaces provided for that purpose.
7. Feel free to ask me any question.

**Investigation 1:**

**Step 1.** Draw any horizontal straight line, XY of length greater than or equal to 10cm. Locate two points B and C on XY such that  $|BC| = 6\text{cm}$ .

**Step 2.** Using a ruler and a protractor, locate a point, A above BC, such that  $\angle ABC = 60^\circ$  and  $|AB| = 8\text{cm}$ . Join AB and AC so as to form  $\triangle ABC$ .

**Step 3.** Use your ruler and setsquares to draw a line segment DE which is parallel to BC, where D is any point on AB and E is any point on AC.

**NOTE: Observe that  $\triangle ABC$  and  $\triangle ADE$  are similar.**

**Step 4:** Measure AD and AB; AE and AC; DE and BC.

$|AD| = \dots, |AB| = \dots; |AE| = \dots, |AC| = \dots; |DE| = \dots |BC| = \dots$

**Step 5:** Calculate the following ratios (or proportions):

$$\frac{AD}{AB} = \dots; \frac{AE}{AC} = \dots; \frac{DE}{BC} = \dots;$$

Compare your results with the results of others near you and **state your observation as conjecture No. 1.**

**Conjecture No. 1:** If two triangles are similar, then their corresponding sides are .....

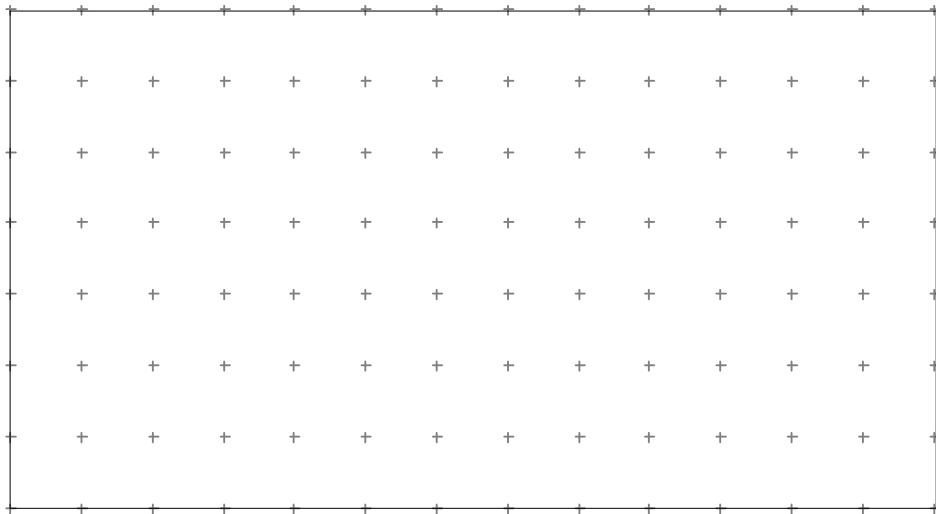
**Step 6:** Measure the following angles;

$\angle ADE = \dots, \angle ABC = \dots, \angle AED = \dots, \angle ACB = \dots,$

Compare your results with the results of others near you and **state your observation as conjecture No. 2.**

**Conjecture No. 2:** If two triangles are similar, then their corresponding angles are ...

**Space for your diagram.**



**Question.**

From your conjectures 1 and 2 above, what do you think are the **necessary and sufficient conditions** for two triangles to be similar?

**Answer:** .....

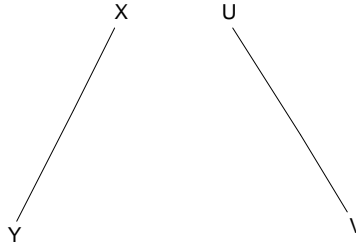
.....

.....

.....

**Investigation 2:**

**Step 1:** Draw any two non-parallel and non-horizontal straight lines XY and UV which are at least 5cm apart. Make each line to be about 6cm long. Your drawing for step 1 should look like this:



**Step 2:** Using a ruler and setsquares, construct three parallel lines, AP, BQ and CR, each to intersect XY and UV such that A, B and C are points on XY and P, Q and R are other points on UV.

**NOTE:** Lines XY and UV are called transversals.

**Step 3:** Measure the following:

$|AB| = \dots\dots\dots, |BC| = \dots\dots\dots; |PQ| = \dots\dots\dots, |QR| = \dots\dots\dots$

**Step 4:** Calculate the following ratios:

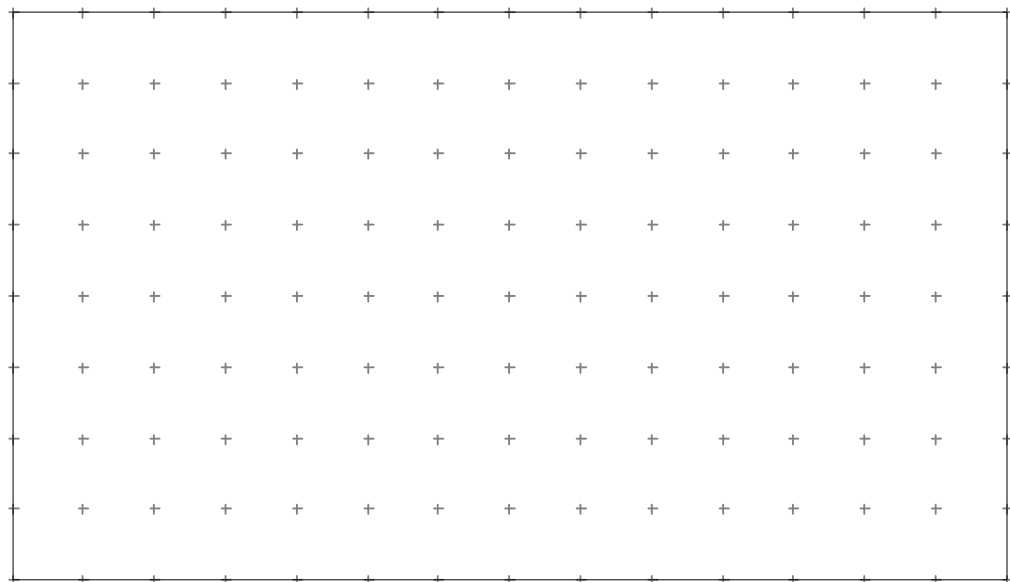
$\frac{AB}{BC} = \dots\dots\dots; \frac{PQ}{QR} = \dots\dots\dots;$

Compare your results with those of others near you.

State your observation as a conjecture.

**Conjecture:** If three parallel lines are cut by a pair of transversals, then the corresponding intercepts cut off on each are in the same .....

**Space for your diagram.**



**Question:**

Suppose XY and UV are produced so that they meet at a point Z, what shapes would be formed by ZAP, ZBQ and ZCR?

**Answer:** These shapes would all be .....

**Investigation 3:**

**Step 1:** Draw any  $\triangle ABC$ , with  $\angle A$  pointing to the top of the page.

**Step 2:** Using a ruler and setsquares, construct a line  $DE$  parallel to  $BC$ , where  $D$  is any point on  $AB$  and  $E$  is any other point on  $AC$ .

**Step 3:** Measure the following:

$|AD| = \dots\dots\dots, |DB| = \dots\dots\dots; |AE| = \dots\dots\dots, |EC| = \dots\dots\dots$

**Step 4:** calculate the following ratios:

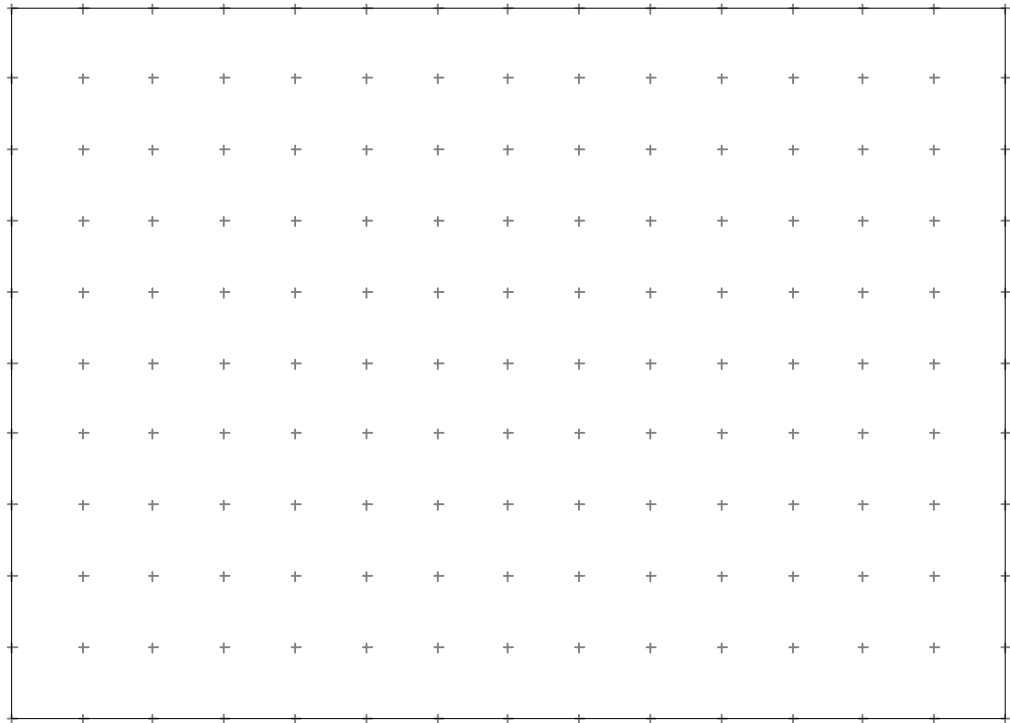
$\frac{AD}{DB} = \dots\dots\dots; \frac{AE}{EC} = \dots\dots\dots;$

Compare your results with those of others beside you.

**Question:** How does  $DE$  divide  $AB$  and  $AC$ ?

**Answer:** .....  
.....  
.....

**Space for your diagram.**



**Conjecture:** State your observations as a conjecture: .....  
.....  
.....  
.....  
.....

**Investigation 4:**

**Step 1:** Draw any  $\triangle ABC$ , with  $\angle A$  pointing to the top of the page.

**Step 2:** By suitable constructions, locate the midpoint, M of AB and the midpoint, N of AC. Join M to N.

**Step 3:** Measure  $\angle ABC$  and  $\angle AMN$ :

$\angle ABC = \dots\dots\dots$ ;  $\angle AMN = \dots\dots\dots$

Compare your results with those of others near you.

**Question (i):** From your measurement of these angles, what can you conclude about the lines MN and BC?

**Answer:**  $\dots\dots\dots$

**Question (ii):** From your answer to question (i) above, what can you **conclude** about the line that joins the midpoints of two sides of a triangle? Give your answer as a conjecture.

**Answer:**  $\dots\dots\dots$

**Step 4:** Measure lines MN and BC.

$|MN| = \dots\dots\dots$ ;  $|BC| = \dots\dots\dots$

**Question (iii):** How does the line MN compare with line BC?

**Answer:**  $|BC| = \dots\dots\dots \times |MN|$  (or  $|MN| = \dots\dots\dots$  of  $|BC|$ ).

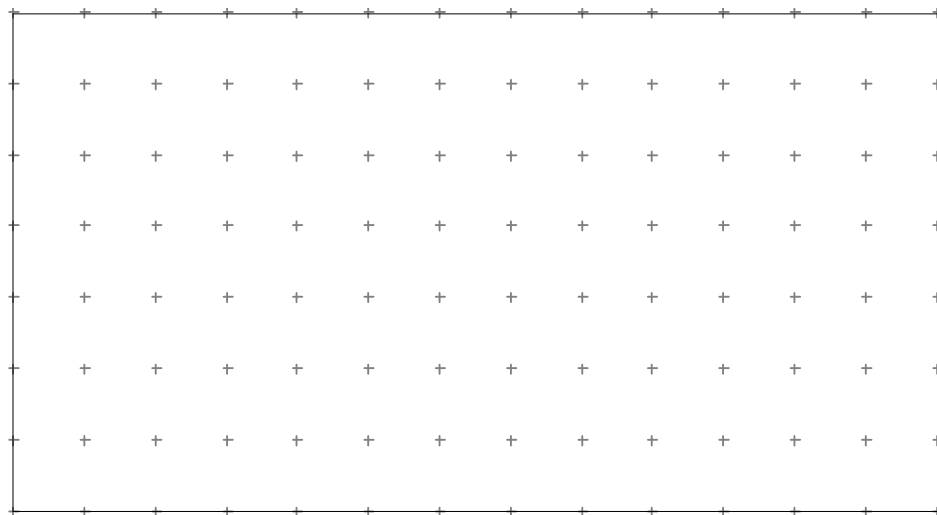
Compare your results with those of others near you and state your observation as a conjecture.

**Conjecture:**  $\dots\dots\dots$

**Question (iv):** From your answers to questions (ii) and (iii) above, how does the line that joins the midpoints of two sides of a triangle relate to the third side of the triangle? State your answer as a conjecture.

**Answer:**  $\dots\dots\dots$

**Space for your diagram.**



**Investigation 5:**

**Step 1:** Using a ruler and a protractor, draw any two non-congruent triangles, ABC and PQR such that  $\angle A = \angle P$ ;  $\angle B = \angle Q$ ; and  $\angle C = \angle R$ .

**NOTE: Recall that  $\Delta ABC$  and  $\Delta PQR$  are similar triangles.**

**Step 2:** Measure lines AB and PQ; AC and PR; BC and QR:

$|AB| = \dots\dots, |PQ| = \dots\dots; |AC| = \dots\dots, |PR| = \dots\dots; |BC| = \dots\dots, |QR| = \dots\dots$

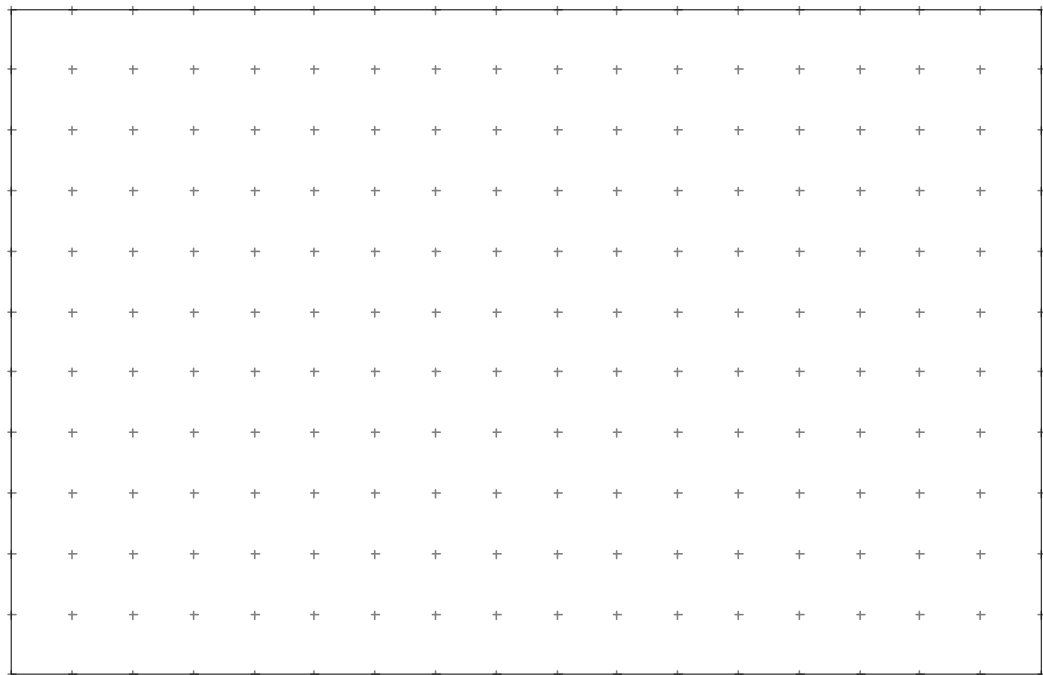
**Step 3:** Calculate the following ratios:

$$\frac{AB}{PQ} = \dots\dots; \frac{AC}{PR} = \dots\dots; \frac{BC}{QR} = \dots\dots$$

Compare your results with the results of others near you and state your findings as a conjecture.

**Conjecture:** .....  
.....  
.....  
.....  
.....

**Space for your diagram.**



## Investigation 6:

**Step 1:** Draw (or construct) any two non-congruent triangles, ABC and PQR such that the corresponding sides, AB and PQ, AC and PR, BC and QR, are in a constant ratio, that is the corresponding sides are proportional (e.g.  $|AB| = 1.5 \times |PQ|$ ).

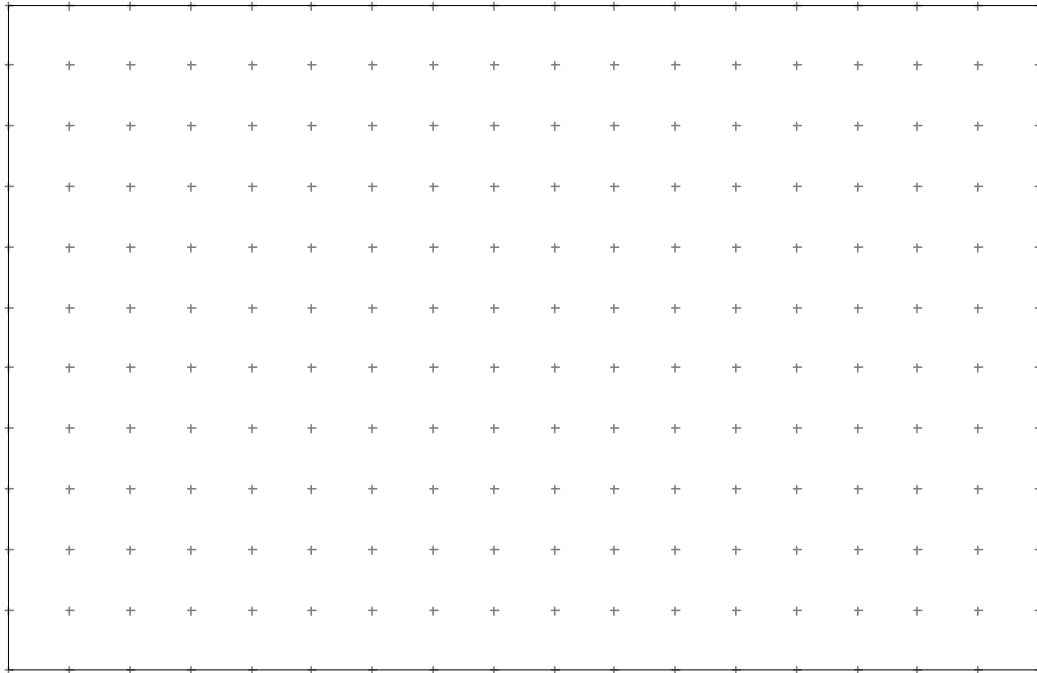
**Step 2:** Measure the following angles:

$\angle A = \dots\dots\dots$ ,  $\angle P = \dots\dots\dots$ ;  $\angle B = \dots\dots\dots$ ,  $\angle Q = \dots\dots\dots$ ;  $\angle C = \dots\dots\dots$ ,  $\angle R = \dots\dots\dots$ ;

Compare your results with those of others near you and state your observation as a conjecture.

**Conjecture:** .....  
.....  
.....  
.....  
.....

**Space for your diagram.**



APPENDIX 5.A.3

Worksheet 3 of the CPGT for the grade 12 learners

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**CONJECTURING IN PLANE GEOMETRY TEST  
(CPGT) GRADE 12**

**Date:** .....

**Time Allowed: 90 minutes**

**INSTRUCTIONS:**

1. Do not start until you are told to do so.
2. While you are waiting, please fill the appropriate information in the spaces below.

**Name (Surname first):** .....

**Name of school:** .....

**Grade:** .....

**Age (in years):** ..... **Sex:** .....

3. This activity involves the use of geometrical construction (or drawings) to investigate some of the **properties of circles and to formulate conjectures** about these properties. **The form that your conjectures should take has been exemplified in investigation 1 below.**
4. For each investigation, you are to **construct** or **draw** a circle. You are required to **follow the instructions** detailed out in each of the **steps** under each investigation.
5. Your drawings or construction should be made in the **spaces provided** for each investigation.
6. Where an investigation requires you to **measure an angle or a line segment**, please do so and write down your answer in the **spaces provided** for that purpose.
7. Feel free to make use of any of the **instruments (rulers, compasses, protractors, dividers, setsquares etc.) in the mathematical set** that you are provided.
8. Where an investigation requires you to answer some questions, please supply your answers in the spaces provided for that purpose.
9. Feel free to ask me any question.

**Investigation 1:**

**Step 1:** Using a pair of compasses with a radius greater than or equal to 3cm, construct a circle. Label the centre, O.

**Step 2:** Use a ruler to draw any two non-congruent chords AB and PQ which do not intersect. Make sure that neither chord is a diameter, and that the two chords are **NOT** on the same side of the circle.

**Step 3:** Use a ruler to measure the lengths of the chords and locate the midpoint of chords AB and PQ. Label the midpoint of AB as M and the midpoint of PQ as N.

**Step 4:** Use your ruler to join O to M and O to N.

**Step 5:** Using your protractor, measure angles AMO and PNO

$\angle AMO = \dots\dots\dots$ ,  $\angle PNO = \dots\dots\dots$ ,

Compare your results with those of others near you and write down your observation as a conjecture.

**Conjecture:** The line segment drawn from the centre of a circle to the midpoint of a chord is  $\dots\dots\dots$  to the chord.

**Space for your diagram.**

**Investigation 2:**

**Step 1:** Construct a circle, centre O.

**Step 2:** Construct any two non-parallel and non-congruent chords AB and PQ that are not diameters. Make sure that the two chords are **NOT** on the same side of the circle.

**Step 3:** Construct the perpendiculars from O to AB and from O to PQ. Label M, the point of intersection of the perpendicular from O to AB, and label N, the point of the intersection of the perpendicular from O to PQ.

**Step 4:** Measure AM and MB; measure also PN and NQ.

$|AM| = \dots\dots\dots$ ,  $|MB| = \dots\dots\dots$ ;  $|PN| = \dots\dots\dots$   $|NQ| = \dots\dots\dots$

Compare your results with those of others near you

**Question:** How does the perpendicular from the centre of a circle to a chord divide the chord? State your observation as a conjecture.

**Answer:** .....  
.....  
.....  
.....

**Space for your diagram.**

**Investigation 3:**

**Step 1:** Construct a circle, centre O and radius greater than or equal to 3cm.

**Step 2:** Construct two non-parallel congruent chords, AB and PQ which are not diameters, and are on different sides of the circle.

**Step 3:** Construct the perpendiculars from O to meet AB at M and to meet PQ at N.

**Step 4:** Measure OM and ON.

$|OM| = \dots\dots\dots$ ;  $|ON| = \dots\dots\dots$

Compare your results with those of others near you and state your observation as a conjecture.

**Conjecture:** .....  
.....  
.....  
.....

**Space for your diagram.**

**Investigation 4:**

**Step 1:** Construct a circle, centre O and radius greater than or equal to 3cm.

**Step 2:** Construct two non-parallel congruent chords, AB and PQ which are not diameters, and on different sides of the circle.

**Step 3:** Draw radii, OA, OB, OP and OQ.

**Step 4:** Use your protractor to measure angles AOB and POQ.

$\angle AOB = \dots\dots\dots$ ,  $\angle POQ = \dots\dots\dots$

Compare your result with those of others near you and state your observation as a conjecture.

**Conjecture:** .....  
.....  
.....

**Space for your diagram.**

**Investigation 5:**

**Step 1:** Construct a circle, centre O with a radius greater than or equal to 3cm.

**Step 2:** Locate two points A and B on the circumference of the circle such that arc AB is a minor arc.

**Step 3:** Join OA and OB.

**Step 4:** Locate another point, P on the major arc.

**Step 5:** Join P to A and P to B.

**Step 6:** Measure angles AOB and APB.

$\angle AOB = \dots\dots\dots$ ,  $\angle APB = \dots\dots\dots$

How does  $\angle AOB$  compare with  $\angle APB$ ? Compare your result with those of others near you and state your observation as a conjecture.

**Conjecture:** .....  
.....  
.....  
.....  
.....

**Space for your diagram.**

**Investigation 6:**

**Step 1:** Construct a circle, centre O with a radius greater than or equal to 3cm.

**Step 2:** Locate two points, A and B such that arc AB is a minor arc.

**Step 3:** Join O to A and O to B.

**Step 4:** Locate two other points, P and Q on the major arc.

**Step 5:** Join P to A and B. Join Q to A and B.

**Step 6:** Measure angles APB and AQB.

$\angle APB = \dots\dots\dots$ ,  $\angle AQB = \dots\dots\dots$

Compare your result with those of others near you and write down your observation as a conjecture.

**Conjecture:** .....  
.....  
.....

**Space for your diagram.**

**Investigation 7:**

**Step 1:** Construct a circle, centre O with a radius greater than 2cm.

**Step 2:** Construct a diameter with A and B as endpoints.

**Step 3:** Locate any point, P on the circumference of the circle.

**Step 4:** Join P to A and P to B.

**Step 5:** Locate another point, Q on the same semicircle as P. Join Q to A and Q to B.

**Step 6:** Measure angles APB and AQB:

$\angle APB = \dots\dots\dots$ ,  $\angle AQB = \dots\dots\dots$

State your observation as a conjecture.

**Conjecture:** .....  
.....  
.....  
.....

**Space for your diagram.**

### Investigation 8:

**Step 1:** Construct a circle, centre O with a radius greater than or equal to 3cm.

**Step 2:** Locate points, P, Q, R and S on the circle such that P and Q are on the same side of the circle, and R and S are both on the opposite side of the circle to P and Q.

**Step 3:** Draw straight lines to join P and Q, Q and R, R and S, and S and P.

**NOTE: Observe that PQRS is a cyclic quadrilateral.**

**Step 4:** Measure the four interior angles of the quadrilateral. That is, measure  $\angle PQR$ ,  $\angle QRS$ ,  $\angle RSP$  and  $\angle SPQ$ .

$\angle PQR = \dots\dots\dots$ ;  $\angle QRS = \dots\dots\dots$ ;  $\angle RSP = \dots\dots\dots$ ;  $\angle SPQ = \dots\dots\dots$ ;

**Step 5:** Add up pairs of opposite angles:

$\angle PQR + \angle RSP = \dots\dots\dots$ ;  $\angle QRS + \angle SPQ = \dots\dots\dots$

Compare your result with those of others near you and state your observation as a conjecture.

**Conjecture:** .....

.....

.....

.....

**Space for your diagram.**

**Investigation 9:**

**Step 1:** Construct a circle, centre O with a radius greater than or equal to 2cm.

**Step 2:** Draw a radius OT, where T is any point on the circumference of the circle.

**Step 3:** Draw any straight line, ATB such that ATB touches the circle externally at T.

**NOTE: Recall that ATB is a tangent to the circle at T.**

**Step 4:** Measure angles OTA and OTB:

$\angle OTA = \dots\dots\dots$ ;  $\angle OTB = \dots\dots\dots$

Compare your result with those of others near you and state your observation as a conjecture.

**Conjecture:** .....  
.....  
.....  
.....

**Space for your diagram.**

**Investigation 10:**

**Step 1:** Construct a circle, centre O with a radius 2cm.

**Step 2:** Locate any point P outside the circle such that line OP is about 6cm. Join O to P.

**Step 3:** Bisect line OP and label as M, the point of intersection of OP and the bisector.

**Step 4:** With M as centre and radius equals MO (or MP), draw two arcs to cut the circle at Q and R.

**Step 5:** Join P to Q, and P to R. Also, join O to Q and O to R.

**Step 6:** Measure lines PQ and PR.

$|PQ| = \dots\dots\dots$ ,  $|PR| = \dots\dots\dots$ ,

Compare your result with those of others near you and state your observation as a conjecture.

**Conjecture:** .....  
.....  
.....  
.....

**Space for your diagram.**

## APPENDIX 5.B

### Marking schemes for the CPGT

#### APPENDIX 5.B.1

#### Marking scheme for worksheet 1

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#### MARKING GUIDE FOR WORKSHEET 1 (GRADE 10)

##### QUESTION 1.

- 0 – Student does not draw any triangle at all, or does so, but does not obtain (or does obtain, but incorrectly) the sum of the angles of the triangle as  $180^\circ$ .
- 1 – Student draws a triangle and obtains the sum of the angles to be  $180^\circ$ , but fails to conjecture (or does so, but incorrectly) that the sum of the angles of a triangle is  $180^\circ$ .
- 2 – Student draws a triangle, finds the sum of the angles to be  $180^\circ$ , and conjectures correctly that the sum of the angles of a triangle is  $180^\circ$ .

**Marks Obtainable: 2.**

##### QUESTION 2.

- 0 – Student does not draw any triangles, or does draw the two triangles, but does not tell (or does tell, but incorrectly) what the triangles are called (i.e. isosceles  $\Delta$ s). Student does not tell (or does tell, but incorrectly) how  $\angle A$  compares with  $\angle C$  and  $\angle P$  compares with  $\angle R$ .
- 1 – Student draws the two triangles, and tells correctly what they are called (i.e. isosceles  $\Delta$ s), but does not tell (or does tell, but incorrectly) how  $\angle A$  compares with  $\angle C$  and  $\angle P$  compares with  $\angle R$ .
- 2 – Student draws the two triangles and tells correctly what they are called (i.e. isosceles  $\Delta$ s), and tells how  $\angle A$  compares with  $\angle C$  (i.e.  $\angle A = \angle C$ ), and  $\angle P$  compares with  $\angle R$  (i.e.  $\angle P = \angle R$ ), but does not conjecture (or does so, but incorrectly) that if in a triangle two of the sides are equal, then two of its angles are necessarily equal.
- 3 – Student draws the two triangles, tells correctly what they are called, tells correctly how  $\angle A$  compares with  $\angle C$  and  $\angle P$  compares with  $\angle R$ , and conjectures correctly that if two sides of a triangle are equal, then two of the angles of the triangle are also equal.

**Marks Obtainable: 3**

##### QUESTION 3.

- 0 – Student does not draw any triangle, or does draw the required triangle, but does not tell (or does tell, but incorrectly) the name of the triangle as **equilateral triangle**.
- 1 – Student draws the required triangle and tells, correctly, the name of the triangle as **equilateral triangle**, but does not conjecture (or does so, but incorrectly) that if in a triangle all the sides are equal, then all the angles are also equal.
- 2 – Student draws the required triangle, names it correctly as an **equilateral triangle**, and conjectures correctly that if in a triangle all the sides are equal, then all the angles are necessarily equal.

**Marks Obtainable: 2**

##### QUESTION 4.

- 1).
- 0 – Student does not tell (or does tell, but incorrectly) the name of parallelogram ABCD as a **rectangle**.
- 1 – Student tells correctly the name of parallelogram ABCD as a **rectangle**.
- 2).
- 0 – Student writes nothing, or writes/lists meaningless statements/properties.
- 1 – Student writes/lists at least THREE properties of (or gives a definition that uniquely describes) a **rectangle**.

**N.B.** Accept as correct statement such as “it has two opposite equal sides” instead of “it has two pairs of opposite equal sides”. Interviews reveal that learners meant the latter statement.

- 3).  
 0 – Student does not conjecture (or does so, but incorrectly) that if the **diagonals** of a parallelogram are equal, then the parallelogram is a rectangle.  
 1 – Student conjectures correctly that if the diagonals of a parallelogram are equal, then the **parallelogram is a rectangle**.
- 4).  
 0 – Student writes nothing, or writes only meaningless statements.  
 1 – Student gives a correct definition of a rectangle, e.g. “A rectangle is a parallelogram with at least one right angle”, or “A rectangle is a parallelogram having equal diagonals”.

**Marks Obtainable: 4**

**QUESTION 5.**

- 1).  
 0 – Student does not tell (or does so, but incorrectly) the name of parallelogram ABCD as a **square**.  
 1 – Student names, correctly, parallelogram ABCD as a square.
- 2).  
 0 – Student writes nothing, or writes/lists only meaningless statements/properties.  
 1 – Student lists at least **TWO specific properties** of (or gives a definition that uniquely describes) a **square**. Examples of acceptable properties: “It has four equal sides”, “It has four right angles”, etc.
- 3).  
 0 – Student writes nothing, or gives properties that do not **distinguish a square from a rectangle**.  
 1 – Student lists at least ONE correct property of a square that a rectangle does not have, e.g. “It has four equal sides”, “The diagonals bisect (or intersect) each other at right angles”, “The diagonals bisect the angles”, etc.
- 4).  
 0 – Student does not conjecture (or does so, but incorrectly) that a parallelogram which has **diagonals of equal length** that bisect each other at right angles is a **square**.  
 1 – Student conjectures correctly that a parallelogram which has **diagonals of equal length** that bisect each other at right angles is a **square**.
- 5).  
 0 – Student writes nothing, or writes only meaningless statements/definition.  
 1 – Student gives a correct definition of a square, e.g. “A **square is a rectangle** with at least one pair of adjacent sides equal”, OR “a **square is a rhombus** with an angle a right angle”, OR “a **square is a rectangle with four equal sides**”.

**Marks Obtainable: 5**

**QUESTION 6.**

- 1).  
 0 – Student does not tell (or does so, but incorrectly) the name of parallelogram ABCD as a **rhombus**.  
 1 – Student names, correctly, parallelogram ABCD as a **rhombus**.
- 2).  
 0 – Student writes nothing, or writes/lists only meaningless statements/properties.  
 1 – Student lists at least **TWO specific properties** of (or gives a definition that uniquely describes) a **rhombus**.
- 3).  
 0 – Student writes nothing, or lists properties that are common **NOT ONLY** to a square and a rhombus, e.g. (i) Both have four sides, (ii) Both have angles whose sum is  $360^\circ$  are unacceptable as correct responses.

- 1 – Student lists at least **TWO** specific properties common to a square and a rhombus, e.g. (i) Both have four equal sides, (ii) In both, the diagonals bisect the angles, (iii) The diagonals bisect each other at right angles, etc.
- 4).  
0 – Student writes nothing, or gives properties that do not **distinguish a square from a rhombus**.  
1 – Student lists at least ONE correct property of a square that a rhombus does not have, e.g. each angle of a square measures  $90^\circ$ ; the diagonals of a square have equal length, etc.
- 5).  
0 – Student writes nothing (or does write something, but answers incorrectly), or student answers correctly, but does not give any reason (or does so, incorrectly) to support his/her answer.  
1 – Student writes the correct answer, and gives at least ONE valid reason to support his/her answer.
- 6).  
0 – Student writes nothing, or writes only meaningless statements/definition.  
1 – Student gives a correct definition of a rhombus, e.g. a rhombus is a parallelogram with at least one pair of adjacent sides equal; or a rhombus is a parallelogram with four equal sides.

**Marks Obtainable: 6**

**TOTAL MARKS OBTAINABLE: 22**

## APPENDIX 5.B.2

### Marking scheme for worksheet 2

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#### MARKING GUIDE FOR WORKSHEET 2 (GRADE 11)

##### QUESTION 1.

- 0 – Student does not draw any triangles at all, or does so, but does not obtain (or does obtain, but incorrectly) the **ratios** of the given **corresponding sides** of **similar triangles ABC** and **ADE** to be equal (i.e.  $AD/AB = AE/AC = DE/BC$ ).
- 1 – Student draws the required triangles and obtains, correctly the **ratios** of the given **corresponding sides** to be **equal/constant**, but fails to conjecture (or does so, but incorrectly) that if two triangles are similar, then their corresponding sides: **are in a constant ratio; OR are proportional; OR have equal/the same ratio.**
- 2 – Student draws the required triangles, **obtains the ratios** of the given corresponding sides to be equal/constant, and **conjectures** correctly that if two triangles are similar, then their corresponding sides are proportional (or are in a constant ratio), but does not obtain (or does so, but incorrectly) **corresponding angles** of the two similar triangles to be **equal**, and/or student does not conjecture (or does so, but incorrectly) that **if two triangles are similar, then their corresponding angles are equal.**
- 3 – Student draws the two similar triangles and obtains the **ratios** of their **corresponding sides** to be **equal/constant**; student **conjectures** correctly that if two triangles are similar, then their **corresponding sides** are **proportional** (or are in a constant ratio); student obtains corresponding angles of the two similar triangles to be **equal**, and **conjectures** correctly that if two triangles are similar, then their corresponding angles are equal; **but student does not** state (or does so, but incorrectly) the **necessary and sufficient conditions (NASCO)** for two triangles to be **similar**.
- 4 – Student draws the two similar triangles, obtains the ratios of the corresponding sides to be the same, and obtains corresponding angles to be equal. Student conjectures correctly that if two triangles are similar, then their corresponding sides are proportional and their corresponding angles are equal, and expresses these as the NASCO for two triangles to be similar.

**Marks Obtainable: 4.**

##### QUESTION 2.

- 0 – Student writes nothing, or does write, but does not obtain (or does so, but incorrectly) the **ratios of the given intercepts (or sides) to be equal.**
- 1 – Student obtains the **ratios** of the given intercepts to be **equal**, but does not conjecture (or does so, but incorrectly) that if three parallel lines are cut by a pair of transversals, then the corresponding intercepts cut off on each one are in the same ratio.
- 2 – Student obtains the **ratios** of the given intercepts to be **equal, and conjectures** correctly that if three parallel lines are cut by a pair of transversals, then the corresponding intercepts cut off on each one are in the same ratio, but does not tell (or does so, but incorrectly) the shapes that would be formed by producing lines XY and UV until they meet.
- 3 – Student obtains the **ratios** of the given intercepts to be equal, and conjectures correctly that if three parallel lines are cut by a pair of transversals, then the corresponding intercepts cut off on each one are in the same ratio, and states correctly that the shapes that would be formed by producing lines XY and UV until they meet **are triangles.**

**Marks Obtainable: 3.**

##### QUESTION 3.

- 0 – Student writes nothing, or does write, but does not obtain (or does obtain, but incorrectly) the ratios of the given sides of  $\triangle ABC$  and  $\triangle ADE$  to be equal (i.e.  $AD/DB = AE/EC$ ).
- 1 – Student obtains the **ratios** of the given corresponding sides of  $\triangle ABC$  and  $\triangle ADE$  to be equal (i.e.  $AD/DB = AE/EC$ ), but does not tell (or does so, but incorrectly) how the line DE divides lines AB and AC.
- 2 – Student obtains the **ratios** of the given corresponding sides of  $\triangle ABC$  and  $\triangle ADE$  to be the same and tells correctly how the line DE divides lines AB and AC (i.e. **DE divides AB and AC proportionally**), but does not conjecture (or does so, but incorrectly) that **the line drawn parallel to one side of a triangle divides the other two sides proportionally.**

- 3 – Student obtains the **ratios** of the given corresponding sides of  $\triangle ABC$  and  $\triangle ADE$  to be equal, tells correctly how the line  $DE$  divides lines  $AB$  and  $AC$ , and **conjectures** correctly that **the line drawn parallel to one side of a triangle divides the other two sides proportionally**.

**Marks Obtainable: 3.**

**QUESTION 4.**

- i).  
 0 – Student writes nothing, or does write something, but fails to tell (or does tell, but incorrectly) that the lines  $MN$  and  $BC$  of  $\triangle ABC$  and  $\triangle AMN$  are **parallel**.  
 1 – Student states correctly that the lines  $MN$  and  $BC$  are **parallel**.

- ii).  
 0 – Student does not conjecture (or does so, but incorrectly) that the line that **joins** the **midpoints** of two sides of a triangle is **parallel** to the **third side**.  
 1 – Student conjectures correctly that the line that **joins** the **midpoints** of two sides of a triangle is **parallel** to the **third side**.

- iii).  
 0 – Student writes nothing, or does write something, but does not tell (or does tell, but incorrectly) **how line  $BC$  compares with line  $MN$**  (i.e.  $|BC| = 2 \times |MN|$  (or  $|MN| = \frac{1}{2}|BC|$ )).  
 1 – Student states correctly **how line  $BC$  compares with line  $MN$**  (i.e.  $|BC| = 2 \times |MN|$ , or  $|MN| = \frac{1}{2}|BC|$ ), but does not conjecture (or does so, but incorrectly) that the line that **joins** the **midpoints** of two sides of a triangle is **equal to half** of the length of the third side.  
 2 – Student states correctly that  $|BC| = 2 \times |MN|$ , (or  $|MN| = \frac{1}{2}|BC|$ ) and conjectures correctly that the line that **joins** the **midpoints** of two sides of a triangle is **equal to half** of the length of the third side.

- iv).  
 0 – Student does not conjecture (or does so, but incorrectly) that the line that **joins** the **midpoints** of two sides of a triangle is **parallel to the third side** and **equal to half** of it.  
 1 – Student conjectures correctly that the line that **joins** the **midpoints** of two sides of a triangle is **parallel to the third side** and **equal to half** of it.

**Marks Obtainable: 5.**

**QUESTION 5.**

- 0 – Student writes nothing, and/or fails to obtain (or does obtain, but incorrectly) the **ratios** of the given **corresponding sides** of similar  $\triangle s$   $ABC$  and  $PQR$  to be **equal**.  
 1 – Student obtains the **ratios** of the given **corresponding sides** of similar  $\triangle s$   $ABC$  and  $PQR$  to be **equal**, but does not conjecture (or does so, but incorrectly) that if the corresponding angles of two triangles are equal, then their corresponding sides are proportional (or are in a constant ratio) and the triangles are similar.  
 2 – Student obtains the **ratios** of the given **corresponding sides** of similar  $\triangle s$   $ABC$  and  $PQR$  to be **equal**, and **conjectures** correctly that if the **corresponding angles** of two triangles are equal, then their corresponding sides are **proportional** (or are in a constant ratio) and the **triangles are similar**.

**Marks Obtainable: 2.**

**QUESTION 6.**

- 0 – Student writes nothing, and/or fails to obtain (or does obtain, but incorrectly) the given **corresponding angles** of similar  $\triangle s$   $ABC$  and  $PQR$  to be **equal**.  
 1 – Student obtains the given **corresponding angles** of similar  $\triangle s$   $ABC$  and  $PQR$  to be **equal**, but does not conjecture (or does so, but incorrectly) that if the **corresponding sides** of two triangles are **proportional**, then their **corresponding angles** are **equal**, and the triangles are **similar**.  
 2 – Student obtains the given **corresponding angles** of similar  $\triangle s$   $ABC$  and  $PQR$  to be **equal**, and conjectures correctly that if the **corresponding sides** of two triangles are **proportional**, then their **corresponding angles** are **equal**, and the triangles are **similar**.

**Marks Obtainable: 2.**

**TOTAL MARKS OBTAINABLE: 19**

## APPENDIX 5.B.3

### Marking scheme for worksheet 3

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#### MARKING GUIDE FOR WORKSHEET 3 (GRADE 12)

##### QUESTION 1.

- 0 – Student writes nothing, and/or fails to obtain  $\angle AMO = 90^\circ$  and  $\angle PNO = 90^\circ$ .
- 1 – Student obtains  $\angle AMO = 90^\circ$  and  $\angle PNO = 90^\circ$ , but does not **conjecture** (or does so, but incorrectly) that the line segment drawn from the **centre** of a **circle** to the **midpoint** of a **chord is perpendicular** to the chord.
- 2 – Student obtains  $\angle AMO = 90^\circ$  and  $\angle PNO = 90^\circ$ , and **conjectures** correctly that the line segment drawn from the **centre** of a **circle** to the **midpoint** of a **chord is perpendicular** to the chord.

**Marks Obtainable: 2.**

##### QUESTION 2.

- 0 – Student writes nothing, and/or does not obtain (or does so, but incorrectly)  $|AM| = |MB|$  and  $|PN| = |NQ|$ .
- 1 – Student obtains correctly  $|AM| = |MB|$  and  $|PN| = |NQ|$ , but does not **conjecture** (or does so, but incorrectly) that the line drawn from the **centre** of a circle **perpendicular** to a chord **bisects the chord**.
- 2 – Student obtains correctly,  $|AM| = |MB|$  and  $|PN| = |NQ|$ , and **conjectures** correctly that the line drawn from the **centre** of a circle **perpendicular** to a chord **bisects the chord**.

**Marks Obtainable: 2.**

##### QUESTION 3.

- 0 – Student writes nothing, and/or fails to obtain  $|OM| = |ON|$ .
- 1 – Student obtains, correctly  $|OM| = |ON|$ , but does not **conjecture** (or does so, but incorrectly) that **equal chords** are **equidistant** from the **centre of a circle**.
- 2 – Student obtains, correctly  $|OM| = |ON|$ , and **conjectures** correctly that **equal chords** are **equidistant** from the **centre of a circle**.

**Marks Obtainable: 2.**

##### QUESTION 4.

- 0 – Student writes nothing, and/or fails to obtain  $\angle AOB = \angle POQ$ .
- 1 – Student obtains  $\angle AOB = \angle POQ$ , but **does not conjecture** (or does so, but incorrectly) that **equal chords subtend equal angle at the centre of a circle**.
- 2 – Student obtains  $\angle AOB = \angle POQ$ , and **conjectures**, correctly, that **equal chords subtend equal angle at the centre of a circle**.

**Marks Obtainable: 2.**

##### QUESTION 5.

- 0 – Student writes nothing, and/or fails to obtain  $\angle AOB = 2 \times \angle APB$ .
- 1 – Student obtains  $\angle AOB = 2 \times \angle APB$  (or  $\angle APB = \frac{1}{2} \times \angle AOB$ ), but **does not conjecture** (or does so, but incorrectly) that the **angle** which an **arc of a circle subtends** at the **centre** is **twice** that which it subtends at **any point** on the remaining **circumference**.
- 2 – Student obtains  $\angle AOB = 2 \times \angle APB$  (or  $\angle APB = \frac{1}{2} \times \angle AOB$ ), and **conjectures** correctly that the **angle** which an **arc of a circle subtends** at the **centre** is **twice** that which it subtends at **any point** on the remaining **circumference**.

**Marks Obtainable: 2.**

##### QUESTION 6.

- 0 – Student writes nothing, and/or fails to obtain  $\angle APB = \angle AQB$ .
- 1 – Student obtains  $\angle APB = \angle AQB$ , but **does not conjecture** (or does so, but incorrectly) that the angles subtended by **the same arc** of a circle **are equal**.
- 2 – Student obtains  $\angle APB = \angle AQB$ , and **conjectures** correctly that the angles in **the same segment** of a circle subtended by **the same arc are equal**.

**Marks Obtainable: 2.**

**QUESTION 7.**

- 0 – Student writes nothing, and/or fails to obtain  $\angle APB = \angle AQB = 90^\circ$ .
- 1 – Student obtains  $\angle APB = \angle AQB = 90^\circ$ , but **does not conjecture** (or does so, but incorrectly) that the angle subtended by the **diameter** of a circle is a **right angle**.
- 2 – Student obtains  $\angle APB = \angle AQB = 90^\circ$ , and **conjectures** correctly that the **angle subtended** by the **diameter of a circle** is a **right angle** (or the angle in a semicircle is a right angle).

**Marks Obtainable: 2.**

**QUESTION 8.**

- 0 – Student writes nothing, and/or fails to obtain  $\angle PQR + \angle RSP = 180^\circ$  and  $\angle QRS + \angle SPQ = 180^\circ$ .
- 1 – Student obtains  $\angle PQR + \angle RSP = 180^\circ$  and  $\angle QRS + \angle SPQ = 180^\circ$ , but **does not conjecture** (or does so, but incorrectly) that the **opposite angles** of a **cyclic quadrilateral** are **supplementary** (i.e. add up to  $180^\circ$ ).
- 2 – Student obtains  $\angle PQR + \angle RSP = 180^\circ$  and  $\angle QRS + \angle SPQ = 180^\circ$ , and **conjectures** correctly that the **opposite angles** of a **cyclic quadrilateral** are **supplementary**.

**Marks Obtainable: 2.**

**QUESTION 9.**

- 0 – Student writes nothing, and/or fails to obtain  $\angle OTA = \angle OTB = 90^\circ$ .
- 1 – Student obtains  $\angle OTA = \angle OTB = 90^\circ$ , but **does not conjecture** (or does so, but incorrectly) that a **tangent** to a circle is **perpendicular** to the **radius** at the **point of contact**.
- 2 – Student obtains  $\angle OTA = \angle OTB = 90^\circ$ , and **conjectures** correctly that a **tangent** to a circle is **perpendicular** to the **radius** at the **point of contact**.

**Marks Obtainable: 2.**

**QUESTION 10.**

- 0 – Student writes nothing, and/or fails to obtain  $|PQ| = |PR|$ .
- 1 – Student obtains  $|PQ| = |PR|$ , but **does not conjecture** (or does so, but incorrectly) that **tangents** to a circle **from the same external point** are **equal** in length.
- 2 – Student obtains  $|PQ| = |PR|$ , and **conjectures** correctly that **tangents** to a circle **from the same external point** are **equal** in length.

**Marks Obtainable: 2.**

**TOTAL MARKS OBTAINABLE: 20**

**APPENDIX 6**

**Students' questionnaire 4**

**APPENDIX 6.A**

**Van Hiele Geometry Test (VHGT)**

**APPENDIX 6.A.1**

The VHGT for grade 10 learners

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**VAN HIELE GEOMETRY TEST (VHGT)  
GRADE 10**

**Date:** .....

**INSTRUCTIONS:**

1. Do not start until you are told to do so.
2. While you are waiting, please fill the appropriate information in the spaces below.

**Name (Surname first):** .....

**Name of school:** .....

**Grade:** .....

**Age (in years):** ..... **Sex:** .....

3. This paper consists of **two parts**: Part A and Part B. You are expected to **answer all the questions from both parts. Part A will last for 30 minutes, while Part 2 will last for 20 minutes.**

**Instructions for Part A**

4. **Part A** is an objective test, consisting of **20 multiple-choice** questions. Provide your answers on the **computer answer sheet** that you are given. **Do not mark your answers on the test booklet.**
5. Each question is followed by **five** options lettered **A to E**. There is only one correct answer to each question. Choose the correct option for each question and **shade in pencil** on your computer answer sheet the answer space which bears **the same letter** as the option you have chosen. **Give only one answer to each question.**
6. Use an **HB** pencil for this part.

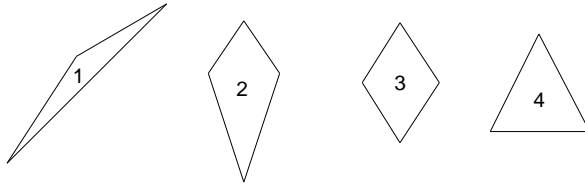
**Instructions for Part B**

7. Part B is an essay paper and it contains **three questions** in all. Answer all the questions in the spaces provided for each question in this booklet. Use either blue or black ink to write your answers.
8. Do not start Part B until you are told to do so.

**NOTE:** The **diagrams** in this test are **not necessarily** drawn to scale.

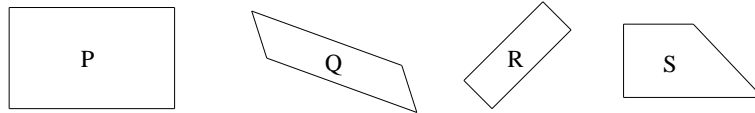
**PART A: 30 MINUTES**

1. Which of these are triangles?



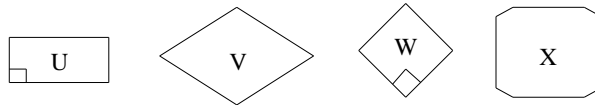
- A. All are triangles
- B. 4 only
- C. 1 and 2 only
- D. 3 only
- E. 1 and 4 only

2. Which of these are rectangles?



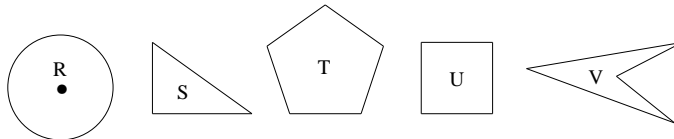
- A. P and R only
- B. Q and S only
- C. P only
- D. R only
- E. All are rectangles

3. Which of these are squares?



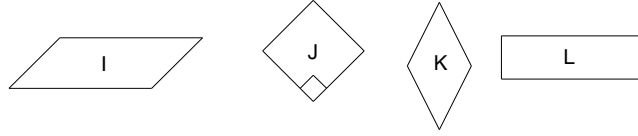
- A. X only
- B. U and W only
- C. W only
- D. V and X only
- E. U only

4. Which of these are quadrilaterals?



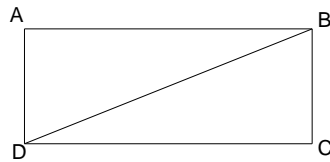
- A. None of these are quadrilaterals
- B. T only
- C. U and V only
- D. R only
- E. S and V only

5. Which of these are parallelograms?



- A. I only
- B. L only
- C. I and K only
- D. J and L only
- E. All are parallelograms

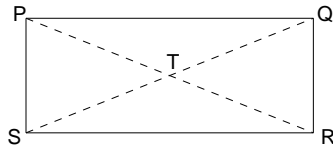
6. ABCD is a rectangle. One of its diagonals, BD is drawn.



Which is **true in every** rectangle?

- A. BD bisects  $\angle B$  and  $\angle D$ .
- B. BD divides ABCD into two congruent right-angled triangles.
- C. BD is a line of symmetry.
- D. BD divides ABCD into two congruent isosceles triangles.
- E. BD and DC have the same measure.

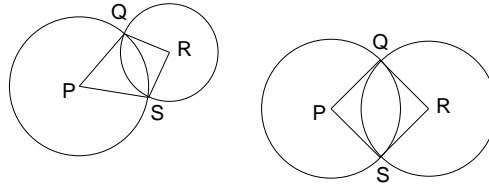
7. PQRS is a rectangle. The diagonals, PR and SQ intersect at T.



Which of the following is **not true** in **every** rectangle?

- A. PR and QS have the same measure.
- B. T is the midpoint of both PR and QS.
- C. PR and QS have different measures.
- D. There are four right angles.
- E. PS and QR are parallel.

8. P and R are the centres of two circles which intersect at Q and S to form a 4-sided figure PQRS. Two examples are given below.



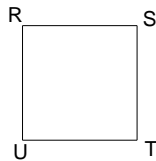
Which of (A) – (D) is **not** always **true**?

- A. PQRS will have at least two angles of equal measure.
  - B. PQRS will have two pairs of sides of equal length.
  - C. Angles P and R will have the same measure.
  - D. The lines PR and QS will be perpendicular.
  - E. All of (A) – (D) are true.
9. An equilateral triangle is a triangle with all the three sides equal in length. Two examples are given below.



Which of (A) – (D) is **true in every** equilateral triangle?

- A. Each angle is an acute angle.
  - B. The measure of each angle must be  $60^\circ$ .
  - C. Each angle bisector is a line of symmetry.
  - D. Each angle bisector must also bisect the opposite side perpendicularly.
  - E. All of (A) – (D) are true.
10. RSTU is a square. Which of these properties is **not true** in all squares?



- A. RS and SU have the same measure.
- B. The diagonals bisect the angles.
- C. RT and SU have the same measure.
- D. RT and SU are lines of symmetry.
- E. The diagonals intersect at right angles.

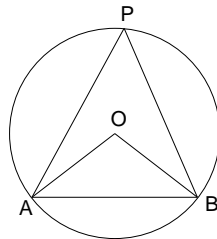
11. What do all rectangles have that some parallelograms **do not** have?

- A. Opposite sides are parallel.
- B. Diagonals are equal in length.
- C. Opposite sides are equal in length.
- D. Opposite angles have equal measure.
- E. None of (A) – (D).

12. Which is **true**?

- A. All properties of rectangles are properties of all parallelograms.
- B. All properties of squares are properties of all rectangles.
- C. All properties of squares are properties of all parallelograms.
- D. All properties of rectangles are properties of all squares.
- E. None of (A) – (D) is true.

13. In the diagram, O is the centre of the circle. AB is a chord and P is any point on the circumference. Which relationship is **true in every** circle?



- A.  $\triangle AOB$  is isosceles.
- B. AP and BP have equal measure.
- C. OA and OB have equal measure.
- D.  $\angle AOB = \angle APB$ .
- E. (A) and (C), above are true.

14. Consider these two statements.

**Statement R:** In  $\triangle ABC$ ,  $\angle A$  and  $\angle C$  are complementary.

**Statement S:**  $\triangle ABC$  is a right-angled triangle.

Which is **true**?

- A. If **R** is true, then **S** is false.
- B. If **R** is false, then **S** is true.
- C. If **R** is true, then **S** is true.
- D. **R** and **S** cannot both be false.
- E. **R** and **S** cannot both be true.

15. Here are two statements.

**Statement S:** Figure 1 is congruent to Figure 2.

**Statement T:** Figure 1 is similar to Figure 2.

Which is **true**?

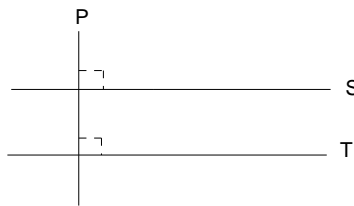
- A. If **S** is true, then **T** is false.
- B. If **S** is true, then **T** is true.
- C. If **S** is false, then **T** is false.
- D. **S** and **T** cannot both be true.
- E. None of (A) – (D) is true.

16. If a line P is perpendicular to a line Q, and line Q is perpendicular to line R, which of the following is **true**?
- A. P and R are perpendicular.
  - B. P and R have equal length.
  - C. Q and R have equal length.
  - D. P and R are parallel.
  - E. P and Q have equal length.

17. Examine these three statements.

- (i). Two lines perpendicular to the same line are parallel.
- (ii). A line that is perpendicular to one of two parallel lines is perpendicular to the other.
- (iii). If two lines are equidistant, then they are parallel.

In the figure below, it is given that lines S and P are perpendicular and lines T and P are perpendicular.



Which of the above statements could be the reason that line S is parallel to line T?

- A. (i) only
  - B. (ii) only
  - C. (iii) only
  - D. Either (ii) or (iii)
  - E. Either (i) or (ii)
18. Given a parallelogram, P from which of the following can we be sure that P is a square?
- A. P has four sides all of which are equal in length.
  - B. P has diagonals of equal length.
  - C. P has diagonals of equal length which bisect each other at right angles.
  - D. P has four right angles and diagonals of equal measure.
  - E. P has diagonals that intersect at right angles and also has four equal sides.

19. In Figure 1, a student was asked to find the sum of  $x$  and  $y$ . The student's answer was:  $x + y = 180^\circ$ . This student was correct!

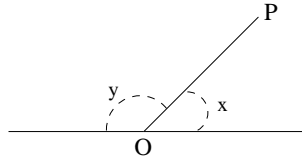


Figure 1

The teacher decided to increase the measure of angle  $x$  as shown in Figure 2 below.

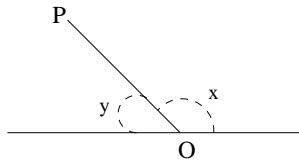


Figure 2

Which is **true** of the new value of  $x + y$ ?

- A.  $x + y$  is now greater than  $180^\circ$ .
  - B.  $x + y$  is still  $180^\circ$ .
  - C.  $x + y$  is now less than  $180^\circ$ .
  - D.  $x + y = 90^\circ$
  - E.  $x + y = 270^\circ$
20. Here are three properties of a figure.
- Property P:** It has diagonals of equal length.
- Property Q:** It is a square.
- Property R:** It is a rectangle.
- Which is **true**?
- A. P implies Q which implies R.
  - B. P implies R which implies Q.
  - C. Q implies R which implies P.
  - D. R implies Q which implies P.
  - E. R implies P which implies Q.

**DO NOT OPEN TO THE NEXT PAGE. SUBMIT YOUR COMPUTER ANSWER SHEET AND WAIT UNTIL YOU ARE TOLD TO START PART 2.**

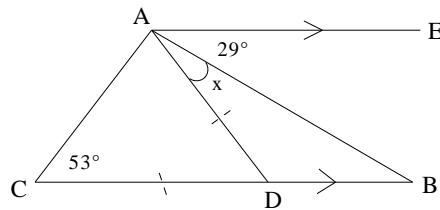
Name: .....

**PART 2: 20 MINUTES**

**NOTE:** Attempt all the questions. Even when you do not know the complete answer, provide answer for the parts that you know, because marks would be awarded even for partial answers.

**Question 1**

In the diagram,  $AE \parallel CB$  and  $|AD| = |CD|$ .  $\angle BAE = 29^\circ$  and  $\angle ACD = 53^\circ$ . Find the value of  $x$ . You are to show your workings, giving reasons for each step.



**Space for your answer**

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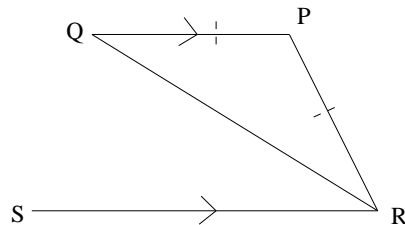
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**Question 2**

You only need to write statements and give reasons to complete this proof.

Given:  $\triangle PQR$  is an isosceles triangle.  $|PQ| = |PR|$  and  $PQ \parallel RS$ .

Prove:  $QR$  bisects  $\angle PRS$ .



Proof:

Steps	Statements	Reasons
1	$PQ \parallel RS$	Given
2	$\angle Q = \angle QRS$	..... .....
3	$ PQ  =  PR $	Given
4	$\therefore \angle Q = \angle PRQ$	..... .....
5	From (2) and (4), .....	By substitution.
6	Hence, .....	Definition of angle bisector.



APPENDIX 6.A.2

The VHGT for grade 11 learners

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**VAN HIELE GEOMETRY TEST (VHGT)  
GRADE 11**

**Date:** .....

**INSTRUCTIONS:**

1. Do not start until you are told to do so.
2. While you are waiting, please fill the appropriate information in the spaces below.

**Name (Surname first):** .....

**Name of school:** .....

**Grade:** .....

**Age (in years):** ..... **Sex :**.....

3. This paper consists of **two parts**: Part A and Part B. You are expected to **answer all** the questions from both parts. **Part A will last for 30 minutes, while Part B will last for 20 minutes.**

**Instructions for Part A**

4. **Part A** is an objective test, consisting of **20 multiple-choice** questions. Provide your answers on the **computer answer sheet** that you are given. **Do not mark your answers on the test booklet.**
5. Each question is followed by **five** options lettered **A to E**. There is only one correct answer to each question. Choose the correct option for each question and **shade in pencil** on your computer answer sheet the answer space which bears **the same letter** as the option you have chosen. **Give only one answer to each question.**
6. Use an **HB** pencil for this part.

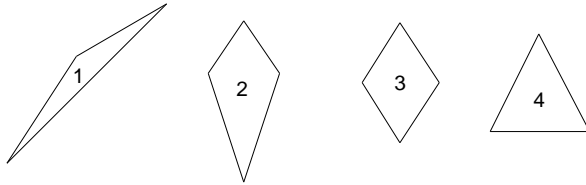
**Instructions for Part B**

7. Part B is an essay paper and it contains **three questions** in all. Answer all the questions in the spaces provided for each question in this booklet. Use either blue or black ink to write your answers.
8. Do not start Part B until you are told to do so.

**NOTE:** The **diagrams** in this test are **not necessarily** drawn to scale.

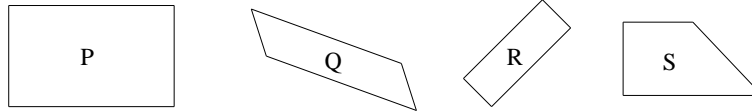
**PART A: 30 MINUTES**

1. Which of these are triangles?



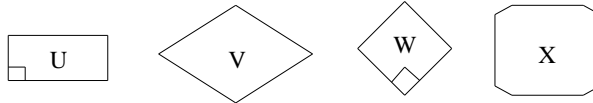
- A. All are triangles
- B. 4 only
- C. 1 and 2 only
- D. 3 only
- E. 1 and 4 only

2. Which of these are rectangles?



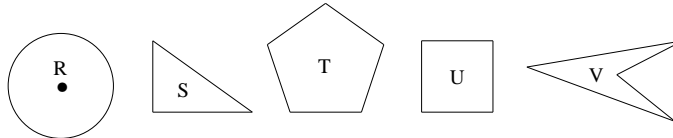
- A. P and R only
- B. Q and S only
- C. P only
- D. R only
- E. All are rectangles

3. Which of these are squares?



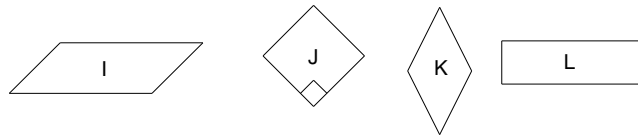
- A. X only
- B. U and W only
- C. W only
- D. V and X only
- E. U only

4. Which of these are quadrilaterals?

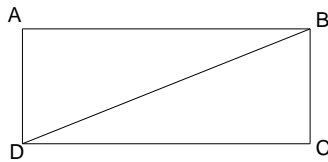


- A. None of these are quadrilaterals
- B. T only
- C. U and V only
- D. R only
- E. S and V only

5. Which of these are parallelograms?

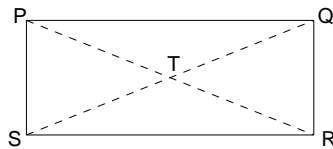


- A. I only  
B. L only  
C. I and K only  
D. J and L only  
E. All are parallelograms
6. ABCD is a rectangle. One of its diagonals, BD is drawn.



Which is **true in every** rectangle?

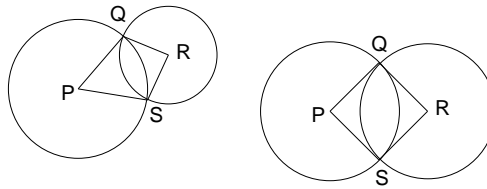
- A. BD bisects  $\angle B$  and  $\angle D$ .  
B. BD divides ABCD into two congruent right-angled triangles.  
C. BD is a line of symmetry.  
D. BD divides ABCD into two congruent isosceles triangles.  
E. BD and DC have the same measure.
7. PQRS is a rectangle. The diagonals, PR and SQ intersect at T.



Which of the following is **not true in every** rectangle?

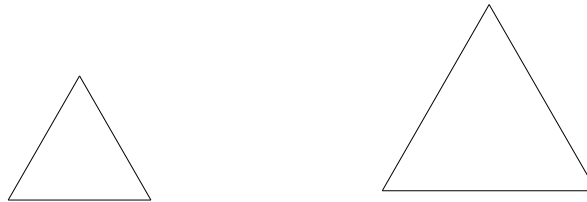
- A. PR and QS have the same measure.  
B. T is the midpoint of both PR and QS.  
C. PR and QS have different measures.  
D. There are four right angles.  
E. PS and QR are parallel.

8. P and R are the centres of two circles which intersect at Q and S to form a 4-sided figure PQRS. Two examples are given below.



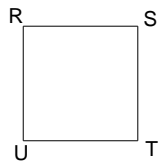
Which of (A) – (D) is **not** always true?

- A. PQRS will have at least two angles of equal measure.
  - B. PQRS will have two pairs of sides of equal length.
  - C. Angles P and R will have the same measure.
  - D. The lines PR and QS will be perpendicular.
  - E. All of (A) – (D) are true.
9. An equilateral triangle is a triangle with all the three sides equal in length. Two examples are given below.



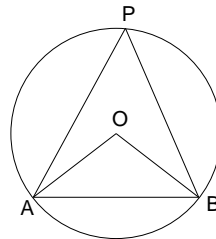
Which of (A) – (D) is **true in every** equilateral triangle?

- A. Each angle is an acute angle.
  - B. The measure of each angle must be  $60^\circ$ .
  - C. Each angle bisector is a line of symmetry.
  - D. Each angle bisector must also bisect the opposite side perpendicularly.
  - E. All of (A) – (D) are true.
10. RSTU is a square. Which of these properties is **not true** in all squares?



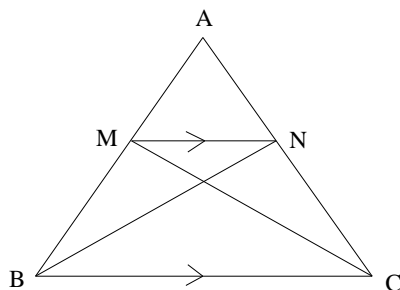
- A. RS and SU have the same measure.
- B. The diagonals bisect the angles.
- C. RT and SU have the same measure.
- D. RT and SU are lines of symmetry.
- E. The diagonals intersect at right angles.

11. What do all rectangles have that some parallelograms **do not** have?
- Opposite sides are parallel.
  - Diagonals are equal in length.
  - Opposite sides are equal in length.
  - Opposite angles have equal measure.
  - None of (A) – (D).
12. Which is **true**?
- All properties of rectangles are properties of all parallelograms.
  - All properties of squares are properties of all rectangles.
  - All properties of squares are properties of all parallelograms.
  - All properties of rectangles are properties of all squares.
  - None of (A) – (D) is true.
13. In the diagram, O is the centre of the circle. AB is a chord and P is any point on the circumference. Which relationship is **true in every** circle?



- $\triangle AOB$  is isosceles.
  - AP and BP have equal measure.
  - OA and OB have equal measure.
  - $\angle AOB = \angle APB$ .
  - (A) and (C), above are true.
14. Consider these two statements.
- Statement R:** In  $\triangle ABC$ ,  $\angle A$  and  $\angle C$  are complementary.
- Statement S:**  $\triangle ABC$  is a right-angled triangle.
- Which is **true**?
- If **R** is true, then **S** is false.
  - If **R** is false, then **S** is true.
  - If **R** is true, then **S** is true.
  - R** and **S** cannot both be false.
  - R** and **S** cannot both be true.
15. Here are two statements.
- Statement S:** Figure 1 is congruent to Figure 2.
- Statement T:** Figure 1 is similar to Figure 2.
- Which is true?
- If **S** is true, then **T** is false.
  - If **S** is true, then **T** is true.
  - If **S** is false, then **T** is false.
  - S** and **T** cannot both be true.
  - None of (A) – (D) is true.

16. In the diagram,  $MN \parallel BC$ .



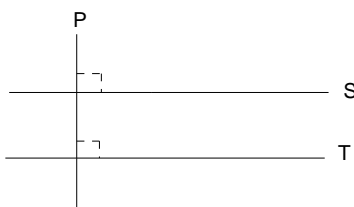
Which is **true in every** triangle?

- A.  $|MN| = \frac{1}{2}|BC|$
- B.  $\triangle AMN$  is congruent to  $\triangle ABC$
- C.  $\triangle ABC$  is isosceles
- D.  $|MN| = |NC|$
- E.  $\triangle AMN$  is similar to  $\triangle ABC$

17. Examine these three statements.

- (i). Two lines perpendicular to the same line are parallel.
- (ii). A line that is perpendicular to one of two parallel lines is perpendicular to the other.
- (iii). If two lines are equidistant, then they are parallel.

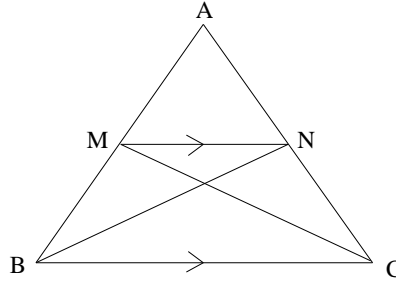
In the figure below, it is given that lines S and P are perpendicular and lines T and P are perpendicular.



Which of the above statements could be the reason that line S is parallel to line T?

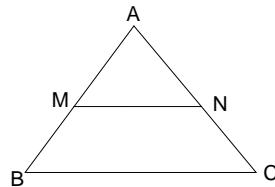
- A. (i) only
- B. (ii) only
- C. (iii) only
- D. Either (ii) or (iii)
- E. Either (i) or (ii)

18. In the diagram,  $\triangle ABC$  is such that  $MN \parallel BC$ . The lines  $BN$  and  $MC$  are drawn so as to form  $\triangle MNB$  and  $\triangle MNC$ .



Which of these may **not be true in every triangle**?

- A.  $|MN| = |NC|$
  - B.  $\frac{AM}{MB} = \frac{AN}{NC}$
  - C.  $\frac{AM}{AB} = \frac{AN}{AC}$
  - D. In  $\triangle AMN$  and  $\triangle ABC$ , corresponding angles are equal
  - E.  $\triangle MNB$  and  $\triangle MNC$  have equal area.
19. In the diagram,  $M$  is the midpoint of  $AB$  and  $N$  is the midpoint of  $AC$  in  $\triangle ABC$ .



Which is **not correct** in every triangle?

- A.  $|MN| = \frac{1}{2}|BC|$
  - B.  $\angle AMN = \angle ABC$ .
  - C.  $|AM| \times |AC| = |AN| \times |MB|$
  - D. Lines  $MN$  and  $BC$  are parallel
  - E. None of (A) – (D) is correct.
20. Here are three properties of a figure.
- Property P:** It has diagonals of equal length.
- Property Q:** It is a square.
- Property R:** It is a rectangle.
- Which is **true**?
- A. P implies Q which implies R.
  - B. P implies R which implies Q.
  - C. Q implies R which implies P.
  - D. R implies Q which implies P.
  - E. R implies P which implies Q.

**DO NOT OPEN TO THE NEXT PAGE. SUBMIT YOUR COMPUTER ANSWER SHEET AND WAIT UNTIL YOU ARE TOLD TO START PART 2.**

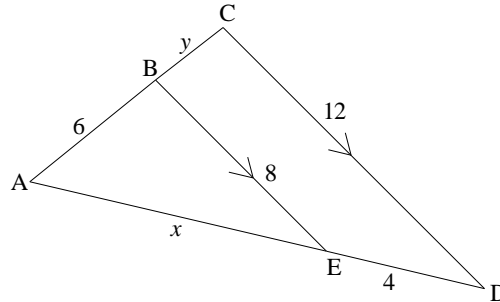
Name: .....

**PART 2: 20 MINUTES**

**NOTE:** Attempt all the **three** questions. Even when you do not know the complete answer, provide answer for the parts that you know, because marks would be awarded even for partial answers.

**Question 1**

Find the values of  $x$  and  $y$  in the diagram. Show your workings in the space provided.



**Space for your answer**

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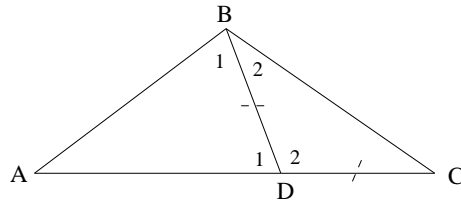
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### Question 2

You only need to write statements and give reasons to complete this proof.

Given: In the diagram, D is the midpoint of line AC.  $|BD| = |DC|$   
 Prove:  $\angle ABC = 90^\circ$ .

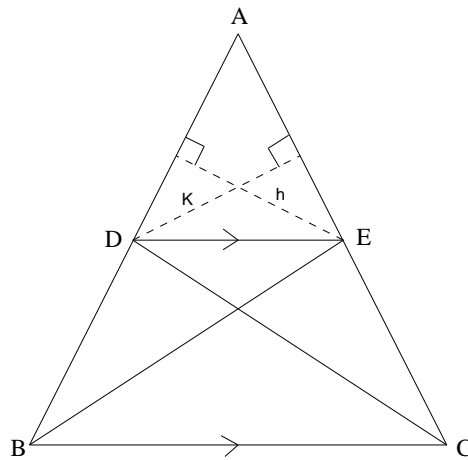


Proof:

Steps	Statements	Reasons
1	$ BD  =  DC $	Given
2	$\therefore \angle B_2 = \angle C$	..... .....
3	Also, $ AD  =  BD $	D is the midpoint of AC and $ BD  =  DC $
4	$\therefore$ .....	Base angles of isosceles $\triangle ABD$ .
5	So, $\angle A + \angle B_1 + \angle B_2 + \angle C = 180^\circ$	..... .....
6	i.e. $2\angle B_1 + 2\angle B_2 = 180^\circ$	.....
7	Hence, $\angle ABC = 90^\circ$	$\angle ABC = \angle B_1 + \angle B_2$

**Question 3**

Write this proof in the space provided. Give a reason for each step in your proof.



Given: In  $\triangle ABC$ ,  $DE \parallel BC$ . From angle E,  $h$  is the height of  $\triangle ADE$ , while from angle D, the height of  $\triangle ADE$  is  $k$ .

Prove:  $\frac{AD}{DB} = \frac{AE}{EC}$ .

**Space for your answer**

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APPENDIX 6.A.3

The VHGT for grade 12 learners

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**VAN HIELE GEOMETRY TEST (VHGT)  
GRADE 12**

**Date:** .....

**INSTRUCTIONS:**

1. Do not start until you are told to do so.
2. While you are waiting, please fill the appropriate information in the spaces below.

**Name (Surname first):** .....

**Name of school:** .....

**Grade:** .....

**Age (in years):** ..... **Sex:** .....

3. This paper consists of **two parts**: Part A and Part B. You are expected to **answer all** the questions from both parts. **Part A will last for 30 minutes, while Part B will last for 20 minutes.**

**Instructions for Part A**

4. **Part A** is an objective test, consisting of **20 multiple-choice** questions. Provide your answers on the **computer answer sheet** that you are given. **Do not mark your answers on the test booklet.**
5. Each question is followed by **five** options lettered **A to E**. There is only one correct answer to each question. Choose the correct option for each question and **shade in pencil** on your computer answer sheet the answer space which bears **the same letter** as the option you have chosen. **Give only one answer to each question.**
6. Use an **HB** pencil for this part.

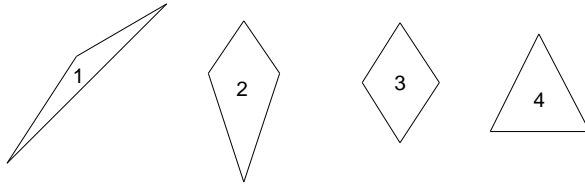
**Instructions for Part B**

7. Part B is an essay paper and it contains **three questions** in all. Answer all the questions in the spaces provided for each question in this booklet. Use either blue or black ink to write your answers.
8. Do not start Part B until you are told to do so.

**NOTE:** The **diagrams** in this test are **not necessarily** drawn to scale.

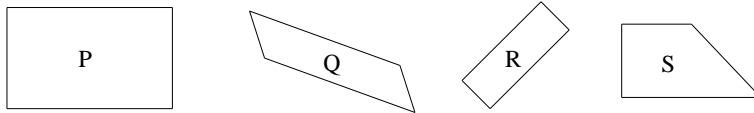
**PART A: 30 MINUTES**

1. Which of these are triangles?



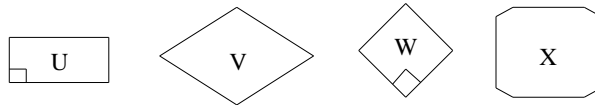
- A. All are triangles
- B. 4 only
- C. 1 and 2 only
- D. 3 only
- E. 1 and 4 only

2. Which of these are rectangles?



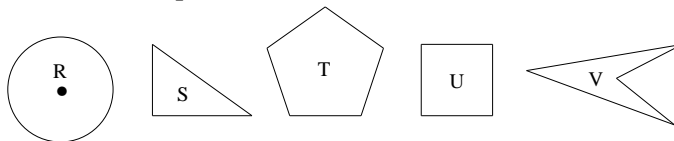
- A. P and R only
- B. Q and S only
- C. P only
- D. R only
- E. All are rectangles

3. Which of these are squares?



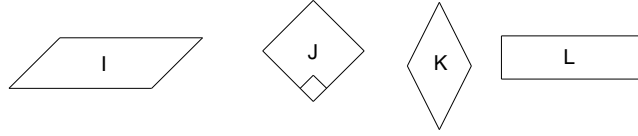
- A. X only
- B. U and W only
- C. W only
- D. V and X only
- E. U only

4. Which of these are quadrilaterals?



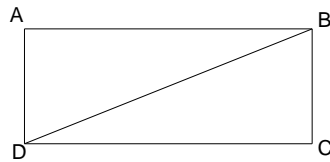
- A. None of these are quadrilaterals
- B. T only
- C. U and V only
- D. R only
- E. S and V only

5. Which of these are parallelograms?



- A. I only
- B. L only
- C. I and K only
- D. J and L only
- E. All are parallelograms

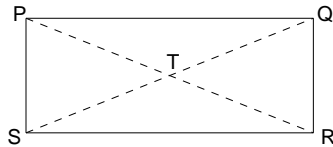
6. ABCD is a rectangle. One of its diagonals, BD is drawn.



Which is **true in every** rectangle?

- A. BD bisects  $\angle B$  and  $\angle D$ .
- B. BD divides ABCD into two congruent right-angled triangles.
- C. BD is a line of symmetry.
- D. BD divides ABCD into two congruent isosceles triangles.
- E. BD and DC have the same measure.

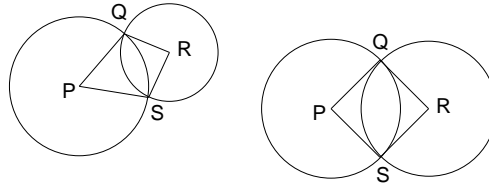
7. PQRS is a rectangle. The diagonals, PR and SQ intersect at T.



Which of the following is **not true** in **every** rectangle?

- A. PR and QS have the same measure.
- B. T is the midpoint of both PR and QS.
- C. PR and QS have different measures.
- D. There are four right angles.
- E. PS and QR are parallel.

8. P and R are the centres of two circles which intersect at Q and S to form a 4-sided figure PQRS. Two examples are given below.



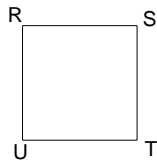
Which of (A) – (D) is **not** always **true**?

- A. PQRS will have at least two angles of equal measure.
  - B. PQRS will have two pairs of sides of equal length.
  - C. Angles P and R will have the same measure.
  - D. The lines PR and QS will be perpendicular.
  - E. All of (A) – (D) are true.
9. An equilateral triangle is a triangle with all the three sides equal in length. Two examples are given below.



Which of (A) – (D) is **true in every** equilateral triangle?

- A. Each angle is an acute angle.
  - B. The measure of each angle must be  $60^\circ$ .
  - C. Each angle bisector is a line of symmetry.
  - D. Each angle bisector must also bisect the opposite side perpendicularly.
  - E. All of (A) – (D) are true.
10. RSTU is a square. Which of these properties is **not true** in all squares?



- A. RS and SU have the same measure.
- B. The diagonals bisect the angles.
- C. RT and SU have the same measure.
- D. RT and SU are lines of symmetry.
- E. The diagonals intersect at right angles.

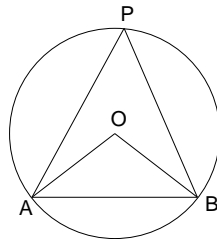
11. What do all rectangles have that some parallelograms **do not** have?

- A. Opposite sides are parallel.
- B. Diagonals are equal in length.
- C. Opposite sides are equal in length.
- D. Opposite angles have equal measure.
- E. None of (A) – (D).

12. Which is **true**?

- A. All properties of rectangles are properties of all parallelograms.
- B. All properties of squares are properties of all rectangles.
- C. All properties of squares are properties of all parallelograms.
- D. All properties of rectangles are properties of all squares.
- E. None of (A) – (D) is true.

13. In the diagram, O is the centre of the circle. AB is a chord and P is any point on the circumference. Which relationship is **true in every** circle?



- A.  $\triangle AOB$  is isosceles.
- B. AP and BP have equal measure.
- C. OA and OB have equal measure.
- D.  $\angle AOB = \angle APB$ .
- E. (A) and (C), above are true.

14. Consider these two statements.

**Statement R:** In  $\triangle ABC$ ,  $\angle A$  and  $\angle C$  are complementary.

**Statement S:**  $\triangle ABC$  is a right-angled triangle.

Which is **true**?

- A. If **R** is true, then **S** is false.
- B. If **R** is false, then **S** is true.
- C. If **R** is true, then **S** is true.
- D. **R** and **S** cannot both be false.
- E. **R** and **S** cannot both be true.

15. Here are two statements.

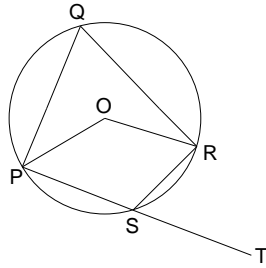
**Statement S:** Figure 1 is congruent to Figure 2.

**Statement T:** Figure 1 is similar to Figure 2.

Which is **true**?

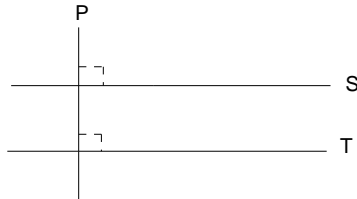
- A. If **S** is true, then **T** is false.
- B. If **S** is true, then **T** is true.
- C. If **S** is false, then **T** is false.
- D. **S** and **T** cannot both be true.
- E. None of (A) – (D) is true.

16. PQRS is a cyclic quadrilateral. O is the centre of the circle. Line PS is produced to a point T, outside the circle.



From this diagram, one can prove that  $\angle Q = \angle RST$ . What would you conclude from this proof?

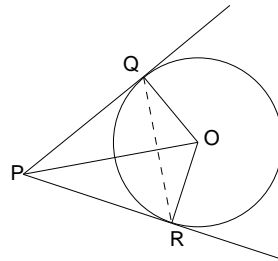
- A. Given any cyclic quadrilateral PQRS with PS produced to T, then  $\angle Q = \angle RST$ .
  - B. Only in this cyclic quadrilateral can we be sure that  $\angle Q = \angle RST$ .
  - C. Given any quadrilateral, PQRS with PS produced to T, then  $\angle Q = \angle RST$ .
  - D. Only when the quadrilateral, PQRS looks like a kite can we be sure that  $\angle Q = \angle RST$ .
  - E. Only in some, but not all cyclic quadrilaterals PQRS, can we prove that  $\angle Q = \angle RST$ .
17. Examine these three statements.
- (i). Two lines perpendicular to the same line are parallel.
  - (ii). A line that is perpendicular to one of two parallel lines is perpendicular to the other.
  - (iii). If two lines are equidistant, then they are parallel.
- In the figure below, it is given that lines S and P are perpendicular and lines T and P are perpendicular.



Which of the above statements could be the reason that line S is parallel to line T?

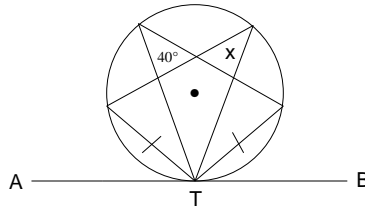
- A. (i) only
- B. (ii) only
- C. (iii) only
- D. Either (ii) or (iii)
- E. Either (i) or (ii)

18. In the diagram, lines PQ and PR are tangents to the circle at Q and R respectively. O is the centre of the circle.



Which of these is **not true**?

- A. PQOR is a kite.
  - B. PQ and PR have the same measure.
  - C. PO bisects angles P and O.
  - D. QR intersects PO at right angles.
  - E. QR is perpendicular to PQ.
19. In the diagram, ATB is a tangent. Three students, **F**, **G** and **H** were asked to find the value of **x**, giving a reason to support their answers.



Here are their answers alongside their reasons.

Student F:  $x = 40^\circ$  (*angles subtended by equal chords*)

Student G:  $x = 40^\circ$  (*angles in the same segment of a circle*)

Student H:  $x = 40^\circ$  (*angle between a tangent and a chord equals angle in the alt. seg.*)

Which of these students gives the **correct reason** for their answers?

- A. F only
  - B. G only
  - C. H only
  - D. F and H only
  - E. All of F, G and H.
20. Here are three properties of a figure.
- Property P:** It has diagonals of equal length.
- Property Q:** It is a square.
- Property R:** It is a rectangle.

Which is **true**?

- A. P implies Q which implies R.
- B. P implies R which implies Q.
- C. Q implies R which implies P.
- D. R implies Q which implies P.
- E. R implies P which implies Q.

**DO NOT OPEN TO THE NEXT PAGE. SUBMIT YOUR COMPUTER ANSWER SHEET AND WAIT UNTIL YOU ARE TOLD TO START PART B.**



## Question 2

You only need to write statements and give reasons to complete this proof. Refer to the diagram below.

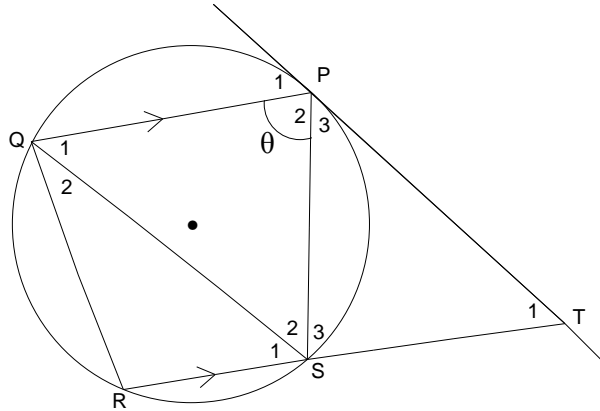
Given:

(i) PQRS is a cyclic quadrilateral.  $PQ \parallel RS$ .

(ii) PT is a tangent at P.

(iii)  $\angle P_2 = \angle S_2 = \angle P_1 = \theta$ .

Prove: PQRT is a parallelogram.




Proof:

Steps	Statements	Reasons
1	$P_2 = P_1 = \theta$	Given
2	$\therefore T_1 = P_1 = \theta$	..... .....
3	$S_3 = P_2 = \theta$	.....
4	$S_3 = \angle PQR = \theta$	Exterior angle of a cyclic quadrilateral = the interior opposite angle.
5	$\angle QPT = 180^\circ - \theta$	..... .....
6	Now, $\angle R = 180^\circ - \theta$	Opposite angles of a cyclic quadrilateral.
7	From (5) and (6): .....	By substitution
8	And from (2) and (4): $T_1 = \angle PQR$	..... .....
9	Hence, from (7) and (8), PQRT is a parallelogram.	Pairs of opposite angles are equal.



## APPENDIX 6.B

### Multiple-choice answer sheet for the VHGT



**IMPORTANT:**

- Use Black or Blue pen only or Pencil
- Darken the circle completely
- Do not make any stray marks on this form

Rhodes University

**CORRECT MARK**    **INCORRECT MARKS**

●
○
○
○

	STUDENT NUMBER	TASK				
■	6 0 0 A 0 0 0 0	0 0	1	A B C D E	26	A B C D E
■	8 1 1 B 1 1 1 1	1 1	2	A B C D E	27	A B C D E
■	2 2 C 2 2 2 2	2 2	3	A B C D E	28	A B C D E
■	3 3 D 3 3 3 3	3 3	4	A B C D E	29	A B C D E
■	4 4 E 4 4 4 4	4 4	5	A B C D E	30	A B C D E
■	5 5 F 5 5 5 5	5 5	6	A B C D E	31	A B C D E
■	6 6 G 6 6 6 6	6 6	7	A B C D E	32	A B C D E
■	7 7 H 7 7 7 7	7 7	8	A B C D E	33	A B C D E
■	8 8 I 8 8 8 8	8 8	9	A B C D E	34	A B C D E
■	9 9 J 9 9 9 9	9 9	10	A B C D E	35	A B C D E
■	K		11	A B C D E	36	A B C D E
■	L		12	A B C D E	37	A B C D E
■	M		13	A B C D E	38	A B C D E
■	N		14	A B C D E	39	A B C D E
■	O		15	A B C D E	40	A B C D E
■	P		16	A B C D E	41	A B C D E
■	Q		17	A B C D E	42	A B C D E
■	R		18	A B C D E	43	A B C D E
■	S		19	A B C D E	44	A B C D E
■	T		20	A B C D E	45	A B C D E
■	U		21	A B C D E	46	A B C D E
■	V		22	A B C D E	47	A B C D E
■	W		23	A B C D E	48	A B C D E
■	X		24	A B C D E	49	A B C D E
■	Y		25	A B C D E	50	A B C D E
■	Z					

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## APPENDIX 6.C

### Individual learner's performance on the VHGT

#### APPENDIX 6.C.1

#### NS grade 10 learners' performance on the VHGT

##### Grade 10 Learners' Results on the VHGT NS Subsample

#	G	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	VH Score	TOT	% Score	
		E	A	C	C	E	B	C	C	E	A	B	D	C	C	B	D	A	C	B	C				
1	2	B	C	D	B	A	B	B	D	E	A	E	E	E	E	B	A	D	D	E	C	0	2	5	25
2	1	B	B	D	B	A	A	D	B	C	A	B	C	B	C	B	A	C	B	A	C	0	4	5	25
3	1	B	C	E	A	A	D	B	B	B	E	C	E	E	B	A	E	A	B	D	A	0	0	1	5
4	1	B	D	A	C	A	C	C	B	C	A	D	E	B	B	E	B	C	A	A	A	0	0	3	15
5	1	B	C	E	B	A	B	B	D	B	A	D	B	B	A	C	A	C	A	A	D	0	0	2	10
6	1	B	C	E	A	A	E	B	A	E	B	A	E	B	E	E	C	A	A	A	C	0	0	3	15
7	2	E	A	C	C	C	E	D	E	B	B	C	A	C	B	D	B	C	D	B	A	0	1	6	30
8	2	E	A	D	A	A	C	C	E	B	E	B	E	C	C	D	E	A	A	B	D	0	4	8	40
9	2	B	C	E	B	E	B	B	C	E	A	C	B	B	E	A	A	B	B	A	A	0	2	5	25
10	2	B	C	D	E	A	B	C	C	C	A	D	C	B	B	A	D	B	A	D	C	0	2	6	30
11	1	E	A	E	A	A	C	C	E	B	E	C	E	C	D	E	C	E	C	D	B	0	0	5	25
12	1	D	C	B	A	D	A	E	C	D	E	A	C	A	A	D	A	D	A	A	D	0	0	1	5
13	2	E	A	E	B	A	C	C	B	B	A	C	A	D	B	B	D	E	A	B	E	0	0	7	35
14	2	B	C	E	A	A	A	D	D	A	A	A	C	A	A	B	A	B	C	B	D	0	0	4	20
15	2	B	C	E	A	A	B	D	D	A	A	A	E	B	A	B	A	B	C	C	D	0	0	4	20
16	2	B	C	C	B	A	B	A	C	A	B	C	A	C	D	A	E	A	C	C	A	0	0	6	30
17	1	E	C	C	B	B	D	A	A	A	B	A	C	E	D	A	E	B	B	E	A	0	0	2	10
18	1	E	D	C	C	A	C	A	E	A	B	E	A	D	C	A	D	A	D	A	A	0	1	6	30
19	1	E	A	C	B	C	C	C	A	A	E	B	E	B	B	A	E	E	A	D	A	0	1	5	25
20	1	E	A	A	C	A	B	C	E	E	A	E	C	B	E	E	C	B	C	B	E	0	3	9	45
21	2	B	C	E	B	A	A	A	C	B	D	D	C	E	B	C	E	B	A	E	A	0	0	1	5
22	1	A	A	D	B	A	B	C	C	E	B	C	E	E	C	C	A	B	A	E	A	0	2	6	30
23	1	E	A	E	A	E	C	C	A	E	E	D	A	E	D	C	D	A	A	D	E	0	1	7	35
24	1	E	A	A	C	A	B	C	E	E	A	E	C	B	E	E	C	B	C	B	E	0	3	9	45

Green = correct answer  
Red = wrong answer

APPENDIX 6.C.2

NS grade 11 learners' performance on the VHGT

Grade 11 Learners' Results on the VHGT  
NS Subsample

#	G	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	VH Score	TOT	% Score	
		E	A	C	C	E	B	C	C	E	A	B	D	C	C	B	E	A	A	C	C				
1	1	E	A	C	C	E	B	C	B	E	A	E	E	E	C	E	A	A	A	C	A	8	11	13	65
2	2	E	E	B	C	E	B	D	E	A	B	E	E	E	B	B	A	A	A	C	A	8	9	8	40
3	1	E	A	B	C	E	C	A	E	A	E	E	E	E	A	D	E	D	A	D	A	0	1	6	30
4	1	E	D	C	A	A	A	C	C	D	B	E	D	E	B	D	C	E	B	C	E	0	0	6	30
5	1	E	A	C	C	A	B	C	C	A	A	D	E	E	D	E	A	C	B	C	A	0	3	9	45
6	1	E	A	C	C	A	B	E	C	E	E	E	E	E	E	C	E	A	C	C	A	8	11	10	50
7	2	B	C	A	B	B	D	C	C	E	B	B	E	C	C	B	E	E	E	E	E	0	6	8	40
8	2	E	C	B	B	A	E	C	E	D	D	E	E	D	B	C	B	D	A	A	C	0	0	4	20
9	1	E	D	E	B	B	B	D	B	E	D	C	E	D	E	D	A	C	E	C	A	0	0	4	20
10	1	Absent															0	0	0	0					
11	2	E	A	B	B	C	B	C	C	E	E	E	D	D	A	B	B	D	A	E	A	0	2	9	45
12	2	B	A	B	C	E	B	D	A	B	D	E	B	E	B	B	B	B	E	C	C	0	1	7	35
13	2	E	A	A	C	B	C	D	E	E	C	E	E	E	C	B	C	E	A	C	E	0	1	8	40
14	1	Absent															0	0	0	0					
15	2	E	A	E	C	A	E	B	C	E	E	E	A	C	B	E	C	D	E	C	A	0	1	7	35
16	2	E	E	C	C	E	B	C	C	A	B	D	E	A	D	D	A	E	D	B	A	0	3	7	35
17	2	E	A	E	C	A	B	C	C	B	E	D	E	E	A	E	A	D	E	C	A	0	3	7	35
18	2	A	E	B	A	E	B	C	B	E	B	E	B	E	B	B	E	B	E	C	A	0	2	7	35
19	1	E	A	B	A	A	D	C	C	E	B	B	C	E	B	B	E	B	D	A	B	0	2	8	40
20	2	B	C	A	A	A	B	A	C	B	C	A	E	E	C	A	C	C	E	A	B	0	0	3	15
21	2	A	A	D	E	E	B	E	B	D	C	D	A	E	C	A	E	B	D	B	C	0	0	6	30
22	2	B	C	E	A	A	D	B	C	E	B	E	A	D	C	A	C	B	C	A	B	0	0	3	15
23	2	B	C	E	A	A	B	B	B	B	A	D	A	E	B	A	B	A	A	D	A	0	0	4	20
24	1	Absent															0	0	0	0					

Green = correct answer  
Red = wrong answer

APPENDIX 6.C.3

NS grade 12 learners' performance on the VHGT

Grade 12 Learners' Results on the VHGT  
NS Subsample

#	G	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	VH Score	TOT	% Score	
		E	A	C	C	E	B	C	C	E	A	B	D	C	C	B	A	A	E	A	C				
1	2	E	A	C	A	A	E	E	D	E	A	C	A	E	E	B	D	A	B	E	A	0	1	7	35
2	1	E	A	C	C	C	B	C	C	E	A	B	E	E	D	B	A	E	C	B	A	0	3	11	55
3	1	E	A	B	A	A	B	C	C	B	D	E	E	A	B	A	A	C	C	B	A	0	2	7	35
4	1	E	A	E	C	E	B	C	B	B	A	E	A	B	E	B	D	B	E	D	A	0	3	9	45
5	1	B	C	A	B	A	B	A	B	A	B	D	D	E	D	B	D	C	D	A	B	0	0	4	20
6	1	E	C	C	C	A	B	C	E	B	A	D	E	A	A	A	A	C	C	E	A	0	3	7	35
7	1	E	A	C	C	E	B	D	C	E	D	B	A	E	E	B	E	C	E	B	A	0	3	11	55
8	1	E	A	C	C	E	B	C	D	E	B	D	E	B	A	B	D	A	E	B	D	0	3	11	55
9	1	E	D	A	A	A	B	C	B	B	A	E	A	E	B	A	C	D	C	E	D	0	2	4	20
10	1	E	A	B	C	D	B	C	E	E	E	E	A	B	A	B	A	E	C	B	D	0	3	8	40
11	1	E	A	C	A	E	B	C	B	B	A	D	D	B	B	E	B	A	C	C	B	0	3	9	45
12	1	E	A	A	A	A	B	B	B	B	A	D	D	B	B	B	A	B	B	C	C	0	0	8	40
13	2	E	E	A	A	E	B	C	C	E	B	E	E	E	B	E	E	E	A	C	B	0	2	6	30
14	1	E	A	C	A	A	B	D	A	E	C	E	E	D	B	A	E	D	D	D	A	0	1	5	25
15	1	E	A	B	C	E	B	C	E	B	D	E	E	A	B	D	A	D	C	D	B	0	1	7	35
16	2	E	E	B	A	E	B	C	E	B	E	A	E	D	B	B	E	E	A	D	C	0	0	6	30
17	2	A	A	D	C	E	A	C	E	B	B	A	B	D	C	B	A	D	E	D	C	8	9	9	45
18	1	E	A	C	B	A	B	A	C	A	E	B	E	E	E	B	A	E	E	A	C	8	9	11	55
19	1	E	A	C	A	A	B	C	C	E	D	E	E	E	A	B	A	E	A	E	E	0	3	9	45
20	2	A	A	E	B	E	D	C	C	A	D	B	E	D	A	C	C	A	B	B	A	0	0	6	30
21	2	E	A	B	C	C	B	C	E	E	E	E	A	B	E	B	A	E	C	B	A	0	3	8	40
22	2	Absent																				0	0	0	0
23	2	E	C	E	B	A	B	C	D	C	A	D	B	D	B	A	D	C	E	A	B	0	2	6	30
24	2	E	A	B	C	D	E	B	D	B	B	D	E	B	B	A	C	A	D	B	E	0	1	4	20

Green = correct answer  
Red = wrong answer

APPENDIX 6.C.4

SAS grade 10 learners' performance on the VHGT

Grade 10 Learners' Results on the VHGT																				VH Score	TOT	% Score			
SAS Subsample																									
#	G	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20				
		E	A	C	C	E	B	C	C	E	A	B	D	C	C	B	D	A	C	B	C				
1	1	E	A	C	B	D	B	C	E	A	A	B	E	C	D	B	D	E	D	B	C	8	15	12	60
2	1	E	A	C	B	A	B	C	C	E	A	E	B	E	D	C	D	E	E	B	E	0	3	10	50
3	1	B	A	C	C	A	B	C	C	A	E	B	E	E	A	B	D	E	C	B	B	8	11	11	55
4	2	E	A	C	A	A	B	D	D	D	E	E	E	E	B	C	D	B	E	B	C	8	9	7	35
5	1	E	A	C	C	E	B	E	A	A	B	E	D	E	C	B	D	A	A	B	E	8	13	12	60
6	2	E	A	C	C	C	C	C	C	A	A	E	E	E	A	B	D	A	A	B	D	8	11	11	55
7	2	E	A	B	A	C	B	C	B	B	B	C	A	E	A	B	D	A	A	B	B	8	8	8	40
8	2	E	A	B	A	C	B	E	A	B	B	E	A	E	B	B	D	D	D	B	B	0	0	6	30
9	1	E	A	C	C	E	A	C	E	E	D	E	E	E	C	E	A	A	E	B	E	0	1	10	50
10	1	E	A	C	E	A	B	C	A	E	D	B	E	B	C	E	D	D	A	B	D	0	3	10	50
11	1	B	D	C	B	A	B	D	D	A	A	C	A	B	E	B	D	A	B	B	A	8	8	7	35
12	1	B	C	E	B	A	B	C	C	C	A	E	A	D	A	B	A	A	A	B	C	8	10	8	40
13	1	E	A	C	B	A	B	A	B	B	B	B	A	C	B	A	D	C	A	B	A	0	1	8	40
14	1	E	C	B	C	E	B	D	D	B	B	E	A	E	C	B	A	B	D	B	D	0	1	7	35
15	1	E	A	A	A	A	A	C	C	B	D	E	A	B	B	D	D	B	A	B	B	0	0	6	30
16	2	E	D	E	A	A	B	C	D	A	B	D	E	A	C	E	D	B	E	B	A	0	0	6	30
17	2	E	E	B	C	E	D	D	B	C	B	E	C	B	B	C	A	B	A	B	E	0	1	4	20
18	1	B	C	C	A	E	D	C	A	A	A	A	E	C	B	D	A	A	E	B	A	0	0	7	35
19	2	E	A	B	B	E	B	D	E	B	A	E	A	B	B	E	A	B	A	B	A	0	1	6	30
20	2	E	A	C	C	E	D	C	B	E	D	E	A	E	C	E	B	B	E	B	C	0	1	10	50
21	1	E	A	E	B	A	E	D	B	B	C	D	A	E	A	E	A	A	C	B	C	8	8	6	30
22	1	B	C	C	B	A	C	B	A	C	A	A	A	B	D	E	B	B	A	A	C	0	0	3	15
23	2	E	A	C	B	A	A	B	C	E	D	E	E	B	B	D	A	A	E	B	A	0	1	7	35
24	2	C	A	C	B	A	D	B	B	A	C	A	A	D	D	E	D	B	A	B	E	0	0	4	20

Green = correct answer  
 Red = wrong answer

APPENDIX 6.C.5

SAS grade 11 learners' performance on the VHGT

Grade 11 Learners' Results on the VHGT																					VH Score	TOT	% Score					
#	G	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19				20				
		E	A	C	C	E	B	C	C	E	A	B	D	C	C	B	E	A	A	C	C							
1	1	B	C	D	A	C	0	E	B	D	B	C	0	A	E	C	D	B	0	B	D	E	C	A	0	0	3	15
2	1	E	A	C	A	A	1	B	E	C	B	A	2	A	E	C	B	D	0	C	A	B	B	D	0	3	8	40
3	2	B	A	C	B	E	1	D	C	B	C	A	0	E	E	E	E	C	0	C	E	D	C	A	0	1	6	30
4	2	E	D	C	B	C	0	E	C	D	B	D	0	C	C	D	A	C	0	E	B	A	D	B	0	0	5	25
5	2	E	A	C	C	A	1	D	C	C	C	E	0	B	E	E	D	B	0	D	D	B	C	E	0	1	9	45
6	2	E	D	A	C	A	0	E	C	D	A	B	0	C	A	B	E	A	0	E	E	D	D	B	0	0	4	20
7	2	E	A	E	A	A	0	B	C	D	B	B	0	C	A	E	A	A	0	E	C	E	A	A	0	0	5	25
8	2	B	A	C	A	C	0	B	D	C	B	C	0	D	E	B	A	D	0	B	C	B	B	A	0	0	4	20
9	1	B	C	C	E	D	0	B	D	E	E	A	2	E	E	E	C	D	0	D	A	C	E	E	0	2	6	30
10	1	Absent					0						0						0	0	0	0						
11	2	B	A	C	A	C	0	E	B	C	C	A	0	B	D	E	A	D	0	E	D	D	B	C	0	0	8	40
12	2	E	E	C	A	A	0	E	C	E	B	A	0	E	A	E	C	A	0	E	C	E	A	B	0	0	6	30
13	2	E	E	A	C	E	1	A	C	E	A	A	0	E	A	E	C	B	0	C	C	A	E	E	0	1	8	40
14	2	E	A	C	C	A	1	B	C	D	E	B	2	B	A	E	D	A	0	B	C	A	A	D	0	3	9	45
15	2	E	A	B	B	E	1	B	C	D	E	A	2	C	B	D	A	C	0	E	C	A	C	E	8	11	10	50
16	2	E	D	C	B	C	0	B	B	D	A	A	0	A	E	B	A	A	0	A	A	A	B	D	0	0	6	30
17	2	C	A	B	B	E	0	A	C	C	B	A	2	B	D	E	C	C	4	A	A	A	B	C	8	14	11	55
18	2	E	A	C	A	A	1	A	C	C	A	A	2	E	C	A	A	B	0	E	C	A	A	B	0	3	9	45
19	2	B	D	C	A	E	0	D	C	E	E	B	0	E	E	E	B	E	0	E	E	E	E	A	0	0	5	25
20	2	B	C	A	D	E	0	A	D	C	B	A	0	E	D	E	E	A	0	E	E	A	C	C	8	8	8	40
21	2	B	B	C	C	E	1	B	C	D	B	A	2	B	B	E	A	B	0	E	C	B	C	A	0	3	10	50
22	1	E	C	C	C	A	1	C	C	E	B	D	0	C	E	C	E	D	0	E	E	D	A	B	0	1	6	30
23	2	E	A	D	E	C	0	B	C	E	B	A	2	D	C	D	B	A	0	E	C	A	C	A	8	10	8	40
24	2	E	C	E	B	A	0	D	A	C	B	A	0	D	C	B	*	B	0	B	C	D	D	D	0	0	4	20

Green = correct answer  
Red = wrong answer

APPENDIX 6.C.6

SAS grade 12 learners' performance on the VHGT

Grade 12 Learners' Results on the VHGT  
SAS Subsample

#	G	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	VH Score	TOT	% Score		
		E	A	C	C	E	B	C	C	E	A	B	D	C	C	B	A	A	E	A	C					
1	2	E	A	C	C	E	B	C	C	D	E	E	B	B	C	B	0	A	B	B	A	C	8	11	11	55
2	2	E	D	C	C	E	B	A	C	E	A	E	A	E	C	B	0	A	A	E	A	B	8	11	14	70
3	2	E	E	B	A	E	B	C	D	B	A	E	B	E	B	E	0	C	A	E	B	A	0	2	7	35
4	2	E	A	C	A	A	B	C	E	B	A	E	A	E	B	E	0	A	A	E	A	A	8	11	10	50
5	1	B	A	C	C	A	B	C	E	E	A	B	E	E	C	E	0	A	C	E	A	B	8	11	12	60
6	1	E	A	C	C	E	B	C	C	E	A	E	D	C	E	B	4	A	E	E	A	D	8	15	16	80
7	1	E	D	C	A	C	B	D	E	A	A	D	D	C	A	B	4	E	A	E	A		8	12	10	50
8	1	E	A	C	C	A	B	C	C	E	A	C	E	E	C	D	0	B	A	B	A	E	0	3	12	60
9	2	E	A	C	B	E	C	C	C	B	A	E	C	E	E	B	0	A	C	E	D	A	0	3	10	50
10	2	E	D	C	A	B	E	E	D	A	C	A	B	D	E	C	0	B	A	C	B	E	0	0	3	15
11	2	B	C	C	D	A	D	D	C	E	A	B	C	E	B	D	0	A	A	D	B	D	0	2	7	35
12	2	E	A	D	A	C	B	D	C	D	A	B	D	C	C	B	4	C	D	D	D	E	0	6	10	50
13	1	E	A	A	B	B	A	C	C	B	A	C	D	C	A	B	4	B	A	D	E	C	0	6	10	50
14	1	E	A	D	C	E	A	D	C	B	A	D	A	E	D	B	0	E	E	E	D	A	0	1	8	40
15	1	C	A	B	C	E	A	C	D	A	A	B	E	A	C	B	4	E	D	B	A	D	0	5	9	45
16	1	E	A	B	A	B	D	C	E	B	B	E	A	E	D	B	0	B	E	D	D	C	0	0	5	25
17	2	E	A	C	C	E	B	D	D	B	A	C	A	A	B	B	0	A	B	C	D	A	0	1	9	45
18	2	E	B	E	A	C	E	C	C	B	A	D	A	D	E	B	0	A	C	C	E	C	0	2	7	35
19	2	E	A	B	A	C	B	E	C	A	B	A	D	E	A	C	0	B	A	E	B	D	0	0	7	35
20	2	E	E	B	D	D	B	C	C	B	D	B	B	A	C	A	0	B	E	D	C	B	0	2	6	30
21	1	E	A	C	C	A	B	C	E	B	B	E	E	A	C	E	0	B	E	E	A	C	8	9	10	50
22	1	E	A	C	A	A	B	C	A	A	A	E	A	E	C	A	0	C	A	D	D	B	0	3	8	40
23	1	E	A	C	B	E	B	C	E	A	A	D	C	D	A	C	0	E	A	C	E	C	0	3	9	45
24	1	B	A	D	B	A	C	D	C	A	B	B	A	B	C	A	0	C	B	E	B	A	0	0	5	25

Green = correct answer  
Red = wrong answer

## APPENDIX 6.D

### Item analysis of the VHGT

#### APPENDIX 6.D.1

#### Item analysis of the VHGT: NS subsample (% with choice)

Levels	Choice	Item No.	1	2	3	4	5
1	A		7	<u>54</u>	15	34	56
	B		26	1	20	26	6
	C		0	29	<u>28</u>	<u>37</u>	7
	D		1	7	10	0	4
	E		<u>65</u>	7	26	3	<u>26</u>
	Blank			0	0	0	0
2		Item No.	6	7	8	9	10
	A		9	11	9	19	<u>32</u>
	B		<u>60</u>	15	20	31	26
	C		13	<u>53</u>	<u>34</u>	6	6
	D		9	15	11	6	13
	E		9	6	25	<u>38</u>	22
Blank			0	0	0	0	0
3		Item No.	11	12	13	14	15
	A		11	22	7	19	24
	B		<u>13</u>	9	25	35	<u>38</u>
	C		13	13	<u>9</u>	<u>16</u>	10
	D		25	<u>7</u>	16	11	11
	E		37	49	41	18	16
Blank			0	0	1	0	0
4		Item No.	16	17	18	19	20
	A		*	<u>22</u>	*	*	46
	B		*	24	*	*	13
	C		*	12	*	*	<u>16</u>
	D		*	16	*	*	11
	E		*	20	*	*	13
Blank			*	0	*	*	0

NB: Percentage correct for each item is underlined.

NB: Item Nos. 16, 18 & 19 differ for grades 10, 11 & 12 learners, hence they are excluded here.

N = 68

APPENDIX 6.D.2

Item analysis of the VHGT: NS grade 10 learners (% with choice)

NS Grade 10 Learner' Achievement on the VHGT (% with choice)							
Item Analysis							
Levels	Choice	Item No.	1	2	3	4	5
1	A		4	<u>38</u>	12	33	75
	B		50	4	4	42	4
	C		0	50	<u>21</u>	<u>21</u>	8
	D		4	8	21	0	4
	E		<u>42</u>	0	42	4	<u>8</u>
	Blank		0	0	0	0	0
2		Item No.	6	7	8	9	10
	A		16	16	16	25	<u>46</u>
	B		<u>38</u>	21	16	29	25
	C		29	<u>42</u>	<u>25</u>	12	0
	D		8	16	16	4	4
	E		8	4	25	<u>29</u>	25
Blank		0	0	0	0	0	
3		Item No.	11	12	13	14	15
	A		21	21	8	16	29
	B		<u>12</u>	8	42	29	<u>21</u>
	C		29	33	<u>16</u>	<u>16</u>	16
	D		21	<u>0</u>	8	16	12
	E		16	38	25	21	21
Blank		0	0	0	0	0	
4		Item No.	16	17	18	19	20
	A		33	<u>25</u>	46	29	42
	B		8	38	16	<u>25</u>	4
	C		16	16	<u>25</u>	<u>8</u>	<u>16</u>
	D		<u>16</u>	8	12	21	21
	E		25	12	0	16	16
Blank		0	0	0	0	0	

The correct answer to each question is underlined.  
N = 24

APPENDIX 6.D.3

Item analysis of the VHGT: NS grade 11 learners (% with choice)

NS Grade 11 Learner' Achievement on the VHGT (% with choice)							
Item Analysis							
Levels	Choice	Item No.	1	2	3	4	5
1	A		9	<u>52</u>	14	28	48
	B		24	0	33	19	14
	C		0	24	<u>24</u>	<u>48</u>	5
	D		0	9	5	0	0
	E		<u>66</u>	14	24	5	<u>33</u>
	Blank			0	0	0	0
2		Item No.	6	7	8	9	10
	A		5	9	5	19	<u>14</u>
	B		<u>62</u>	14	24	19	33
	C		9	<u>48</u>	<u>52</u>	0	14
	D		14	19	0	14	14
	E		9	9	19	<u>48</u>	24
	Blank		0	0	0	0	0
3		Item No.	11	12	13	14	15
	A		5	19	5	14	19
	B		<u>9</u>	9	0	38	<u>33</u>
	C		5	5	<u>9</u>	<u>28</u>	9
	D		24	<u>9</u>	19	9	19
	E		57	57	66	9	19
	Blank		0	0	0	0	0
4		Item No.	16	17	18	19	20
	A		28	<u>19</u>	<u>33</u>	19	57
	B		19	24	9	9	14
	C		24	14	9	<u>52</u>	<u>14</u>
	D		0	24	14	9	0
	E		<u>28</u>	19	33	9	14
	Blank		0	0	0	0	0

The correct answer to each question is underlined.  
N = 21 (three learners were absent)

APPENDIX 6.D.4

Item analysis of the VHGT: NS grade 12 learners (% with choice)

NS Grade 12 Learner' Achievement on the VHGT (% with choice)							
Item Analysis							
Levels	Choice	Item No.	1	2	3	4	5
1	A		8	<u>74</u>	17	39	43
	B		4	0	26	17	0
	C		0	13	<u>39</u>	<u>43</u>	8
	D		0	4	4	0	8
	E		<u>87</u>	8	13	0	<u>39</u>
	Blank			0	0	0	0
2		Item No.	6	7	8	9	10
	A		4	8	4	13	<u>35</u>
	B		<u>83</u>	8	22	43	32
	C		0	<u>70</u>	<u>26</u>	4	4
	D		4	8	17	0	22
	E		8	4	30	<u>39</u>	17
Blank			0	0	0	0	0
3		Item No.	11	12	13	14	15
	A		8	26	8	26	22
	B		<u>17</u>	8	30	39	<u>61</u>
	C		4	0	<u>0</u>	<u>4</u>	4
	D		30	<u>13</u>	22	8	4
	E		39	52	35	22	8
Blank			0	0	4	0	0
4		Item No.	16	17	18	19	20
	A		<u>43</u>	<u>22</u>	13	<u>13</u>	39
	B		4	8	13	35	22
	C		13	22	35	13	<u>17</u>
	D		22	17	13	22	13
	E		17	30	<u>26</u>	17	8
Blank			0	0	0	0	0

The correct answer to each question is underlined.  
N = 23 (one learner was absent)

## APPENDIX 6.D.5

### Item analysis of the VHGT: SAS subsample (% with choice)

Levels	Choice	Item No.	1	2	3	4	5
1	A		0	<u>63</u>	7	32	41
	B		23	3	17	28	4
	C		4	14	<u>61</u>	<u>30</u>	20
	D		0	13	7	4	4
	E		<u>73</u>	7	8	4	<u>31</u>
	Blank		0	0	0	1	0
2		Item No.	6	7	8	9	10
	A		14	4	8	27	<u>55</u>
	B		<u>54</u>	8	10	41	21
	C		7	<u>59</u>	<u>38</u>	8	7
	D		14	21	23	4	11
	E		11	7	21	<u>20</u>	6
Blank		0	0	0	0	0	
3		Item No.	11	12	13	14	15
	A		11	35	8	24	15
	B		<u>21</u>	10	18	21	<u>38</u>
	C		14	11	<u>14</u>	<u>28</u>	14
	D		13	<u>13</u>	11	13	14
	E		41	31	48	13	18
Blank		0	0	0	1	0	
4		Item No.	16	17	18	19	20
	A		*	<u>34</u>	*	*	27
	B		*	18	*	*	18
	C		*	20	*	*	<u>21</u>
	D		*	10	*	*	15
	E		*	18	*	*	17
Blank		*	0	*	*	1	

NB: Percentage correct for each item is underlined.

NB: Item Nos. 16, 18 & 19 differ for grades 10, 11 & 12 learners, hence they are excluded here.

N = 71

APPENDIX 6.D.6

Item analysis of the VHGT: SAS grade 10 learners (% with choice)

SAS Grade 10 Learner' Achievement on the VHGT (% with choice)							
Item Analysis							
Levels	Choice	Item No.	1	2	3	4	5
1	A		0	<u>71</u>	4	25	54
	B		21	0	21	42	0
	C		4	16	<u>63</u>	<u>29</u>	12
	D		0	8	0	0	4
	E		<u>75</u>	4	12	4	<u>29</u>
	Blank			0	0	0	0
2		Item No.	6	7	8	9	10
	A		12	4	21	33	<u>33</u>
	B		<u>58</u>	12	25	29	29
	C		8	<u>50</u>	<u>25</u>	12	8
	D		16	25	16	4	21
	E		4	8	12	<u>21</u>	8
	Blank		0	0	0	0	0
3		Item No.	11	12	13	14	15
	A		12	50	4	21	4
	B		<u>16</u>	4	29	33	<u>38</u>
	C		8	4	<u>12</u>	<u>25</u>	12
	D		8	<u>4</u>	8	16	12
	E		54	38	46	4	33
	Blank		0	0	0	0	0
4		Item No.	16	17	18	19	20
	A		33	<u>38</u>	46	4	25
	B		8	38	4	<u>96</u>	16
	C		0	4	<u>8</u>	<u>0</u>	<u>25</u>
	D		<u>58</u>	8	12	0	12
	E		0	12	29	0	21
	Blank		0	0	0	0	0

The correct answer to each question is underlined.  
N = 24

APPENDIX 6.D.7

Item analysis of the VHGT: SAS grade 11 learners (% with choice)

SAS Grade 11 Learner' Achievement on the VHGT (% with choice)							
Item Analysis							
Levels	Choice	Item No.	1	2	3	4	5
1	A		0	<u>48</u>	13	35	39
	B		35	4	8	26	0
	C		4	22	<u>61</u>	<u>26</u>	26
	D		0	17	8	4	4
	E		<u>61</u>	8	8	8	<u>30</u>
	Blank		0	0	0	0	0
2		Item No.	6	7	8	9	10
	A		17	4	0	17	<u>61</u>
	B		<u>39</u>	13	4	52	17
	C		4	<u>65</u>	<u>35</u>	13	8
	D		17	13	35	0	8
	E		22	4	26	<u>17</u>	4
Blank		0	0	0	0	0	
3		Item No.	11	12	13	14	15
	A		13	22	4	35	30
	B		<u>22</u>	8	17	13	<u>26</u>
	C		22	17	<u>13</u>	<u>17</u>	17
	D		13	<u>13</u>	13	13	22
	E		30	39	52	17	4
Blank		0	0	0	4	0	
4		Item No.	16	17	18	19	20
	A		8	<u>17</u>	<u>39</u>	22	30
	B		17	4	17	22	22
	C		13	43	4	<u>30</u>	<u>13</u>
	D		8	13	22	13	17
	E		<u>52</u>	22	17	13	17
Blank		0	0	0	0	0	

The correct answer to each question is underlined.  
 N = 23 (one learner was absent)

APPENDIX 6.D.8

Item analysis of the VHGT: SAS grade 12 learners (% with choice)

SAS Grade 12 Learner' Achievement on the VHGT (% with choice)							
Item Analysis							
Levels	Choice	Item No.	1	2	3	4	5
1	A		0	<u>71</u>	4	38	29
	B		12	4	21	16	12
	C		4	4	<u>58</u>	<u>33</u>	21
	D		0	12	12	8	4
	E		<u>83</u>	8	4	0	<u>33</u>
	Blank		0	0	0	4	0
2		Item No.	6	7	8	9	10
	A		12	4	4	29	<u>71</u>
	B		<u>63</u>	0	0	42	16
	C		8	<u>63</u>	<u>54</u>	0	4
	D		8	25	16	8	4
	E		8	8	25	<u>21</u>	4
Blank		0	0	0	0	0	
3		Item No.	11	12	13	14	15
	A		8	33	16	16	12
	B		<u>25</u>	16	8	16	<u>50</u>
	C		12	12	<u>16</u>	<u>42</u>	12
	D		16	<u>21</u>	12	8	8
	E		38	16	46	16	16
Blank		0	0	0	0	0	
4		Item No.	16	17	18	19	20
	A		<u>38</u>	<u>46</u>	0	<u>38</u>	25
	B		29	12	12	21	16
	C		16	12	16	4	<u>25</u>
	D		0	8	25	25	16
	E		16	21	<u>46</u>	12	12
Blank		0	0	0	0	4	

The correct answer to each question is underlined.  
N = 24

## APPENDIX 7

### Learners' School Examination in Mathematics (SEM) scores

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#### APPENDIX 7.A

#### NS and SAS learners' SEM scores

Cand. No.	Grade level SEM score (%)					
	NS			SAS		
	10	11	12	10	11	12
1	54	77	65	58	41	45
2	60	58	70	63	34	45
3	56	72	68	30	26	55
4	70	52	45	38	12	55
5	50	78	50	31	25	55
6	64	57	65	63	21	55
7	58	58	70	48	11	45
8	56	68	67	40	16	45
9	60	75	58	63	8	35
10	65	70	75	57	7	45
11	62	77	68	41	9	35
12	66	65	75	46	8	10
13	58	70	68	36	13	55
14	70	68	70	55	8	35
15	68	75	58	37	9	35
16	58	60	60	39	9	35
17	55	54	70	50	5	22
18	60	55	78	22	8	10
19	58	60	80	24	16	10
20	70	64	76	19	6	10
21	50	70	54	30	7	35
22	60	48	58	11	8	10
23	72	58	66	13	10	22
24	74	60	74	20	13	22

## APPENDIX 8

### Transcripts of the videotaped lessons

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#### APPENDIX 8.A

#### Mr Adeleke's lesson (NS grade 10 teacher)

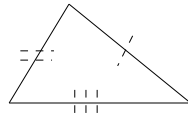
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The students were in their class, as is customary in most public schools in Nigeria, chatting away until Mr Adeleke, the teacher, enters the class. At the signal of the student monitor, the students rose and greeted in unison "Good morning sir". Mr Adeleke, in response greets the students "good morning, now sit down". The lesson is now officially underway.

Elapsed  
Time

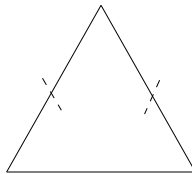
Last week, we discussed about types of triangles. I told you that we have, 1) scalene triangle, ok? "Yes" [replied the students]. And then I tried to draw the diagram for you, that for scalene, we have these things, that none of the sides are what? Equal, "equal" [repeated the students almost simultaneously with the teacher. Mr Adeleke draws the following sketch on the chalkboard as he explains].

1). Scalene triangle.



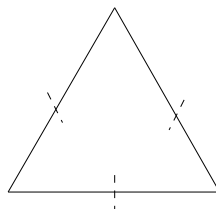
I also talked about isosceles, isosceles triangle [he writes on the chalkboard as he speaks], ok? "Um" [answered the students]. Isosceles triangle where the two opposite sides are what? "Equal" [the teacher says this together with the students, but with his voice leading].

2). Isosceles triangle.



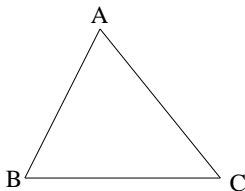
And also, we talked about equi, equilateral triangle where all the sides are equal.

3). Equilateral triangle



5 minutes

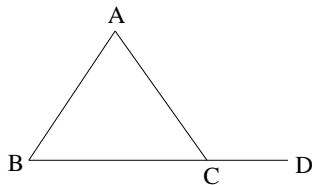
Today, we are going to look at how we can prove that the sum of a triangle, ok, is equal to one eighty degrees [he writes this on the chalkboard with the students verbalising after him]. The sum of a triangle is equal to  $180^\circ$ . We are going to prove today that the sum of a triangle, A, B, C, that is, angle A plus angle B plus angle C is equal to one eighty ... **“degrees”**. [He writes this on the chalkboard as follows].



$$\hat{A} + \hat{B} + \hat{C} = 180^\circ$$

How do we prove this? First and foremost, we need to draw our diagram, ok? **“Yes”**. This is the diagram here, alright? [He draws it on the chalkboard as shown hereunder].

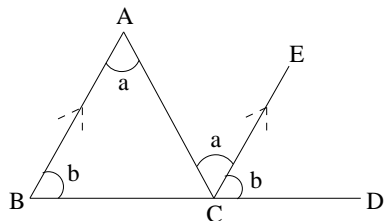
1) Diagram



We could do this line, call this A, B, produce line BC to meet at em, D, ok? **“Yes”**. Then, the first thing now after drawing the diagram is required, required to prove. Require to prove [he writes it on the chalkboard].

Required to Prove: We want to prove that  $\hat{A} + \hat{B} + \hat{C} = 180^\circ$ . Then number three, we want to construct. Construction, construction, construction.

3) Construction: What are we constructing? Assuming this is parallel [puts arrowheads on line AB in the diagram above], draw AB, ok, parallel to EC. What I am trying to say is that, draw this, AB parallel to what? **“EC”** [draws EC parallel to AB as shown in the diagram below].



20 minutes

Label this small 'a', that is, the angle, label this 'b'. Label this small 'a', label this small 'b', ok? [Mr Adeleke labels the diagram as shown above].

Now, let's start the proof now. In that proof, we are saying that angle, angle A, sorry [rubs off A], angle BAC is equal to angle ACE. That is alternate angles, ok? Alternate what? **"Angles"** [chorused the students].

Mr Adeleke states the proof of the angle sum of a triangle on the chalkboard all by himself, seeking only corroborative questions from the students. The chalkboard surface was so rough and untidy that the complete proof which he wrote on the chalkboard was rendered illegible as captured by the camera.

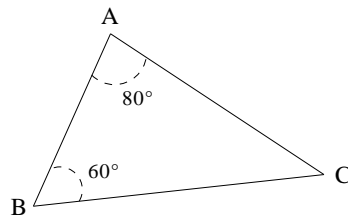
We are saying that this angle here [touches small letter 'a' at angle A], this angle here is equal to this angle here [touches small letter 'a' at angle C in the diagram above]. Alternate angles, what makes them alternate angles? Because AB is parallel, AB is parallel to what? **"EC"** [replied the students, but with his voice leading]. Also, angle ABC is equal to angle ECD, is equal to angle ECD. Corresponding what? **"Angles"**. The same thing [referring to the reason]. AB parallel to what, AB parallel ... **"CE"** [says the students, but with his voice leading]. But look at this now, angle BAC plus angle ABC, again plus angle, am coming. Angle A, angle A, am talking about this angle now, angle ACB, this angle here, this angle here, angle ACB [uses his fingers to trace the angle]. Angle ACB plus angle ACE, plus angle ACE plus angle, now angle ECD, here [shows it to the students in the diagram on the chalkboard]. Angle ABC, that is, this angle, plus this angle, plus this angle [touches the angles at C in the diagram] is equal to what? One eighty ... **"degrees"** [the students completed it as expected]. Who can tell me the reason? Because this is what? **"Straight line"**. It is a straight eh ... **"line"**. That is a straight angle, straight angle, ok?

But we have also seen that em, we have also seen that, we have also seen that, this is our ACB. Now we also know that angle ACB plus angle A, angle ACB, ACB plus angle ACE, angle ACE, what is angle ACE? Is what? **"a"** [chorused the students, following the cue given by the teacher who places his finger on small letter 'a' at angle C], plus 'a' plus angle ACD, angle ACD. What is angle ACD? Angle A ...,

ACD, “b”. ACD is what? “b” [replied the students]. Is what? “b”. Is A ..., not ACD, it is supposed to be angle ECD, ok? [He makes the necessary correction on the chalkboard]. Angle EC what? “D”. Is what, is what? “b”. Correct, it is em, “b” [says the students following Mr Adeleke’s leading voice], which equal to one eighty ..., “degrees” [he says this together with the students, with his voice leading]. But, we can see that angle ACB is our C here, is our what? “C”. Is that not so? This is C [touches angle ACB in the diagram on the chalkboard]. So, this will give us, C + a + b = 180 what? “Degrees” [chorused the students]. Ok? “Yes”. Therefore, angle A, you can rearrange, plus angle B plus angle C is equal to 180 what? “Degrees” [answered the students].

You can see that we have come back to what we already stated that was required to prove, that angle A plus angle B plus angle C is equal to 180 ... “degrees”. That is why we say that the sum of a triangle is 180 ... “degrees”.

How can we use this now in solving triangle? I will do only one for you. Suppose I have ABC and here is 80° while here is 60° [draws and labels triangle ABC on the chalkboard as follows].

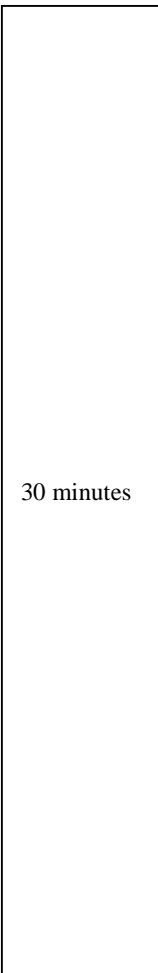


I may say find angle C. Find angle what? “C” [replied the students]. You will say that angle A plus angle B plus angle C equals What? 180 ... “degrees”. [Mr Adeleke writes this on the chalkboard as follows].

$$\hat{A} + \hat{B} + \hat{C} = 180^\circ \text{ (sum of triangle).}$$

What is your reason? You will say sum of triangle, and of course, that is what we have here [referring to the theorem just proven]. Sum of what? “Triangle”. Ok? “Yes”. You will say sum of a triangle is one eighty ... “degrees”. Therefore, our A is what? 80°. Our B is what? 60° plus what? C, equals 180° [he writes this on the chalkboard].

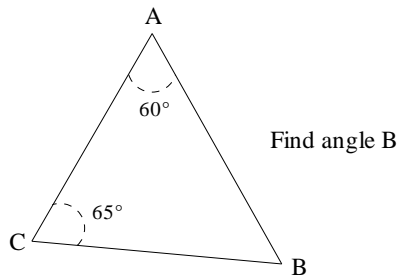
$$80^\circ + 60^\circ + \hat{C} = 180^\circ$$



$$\begin{aligned} \text{That is, } 140^\circ + \hat{C} &= 180^\circ \\ \therefore \hat{C} &= 180^\circ - 140^\circ \\ \hat{C} &= 40^\circ \end{aligned}$$

That is one forty plus C, ok? **“Yes Sir”**. One forty plus C equals one eighty degrees. What is 80 plus 60 class? **“140”**. One forty, good, plus C equals one eighty degrees. Therefore, it means that C equals one eighty minus what? **“140”**. And one eighty minus one forty is what? **“40”**. Forty degrees, so you can now see that C is em ... **“40”** [*chorused the students*]. Forty degrees.

Therefore, I want you to do this. Quickly do this and see whether you understand what I have done so far. [*The students now took out their notebooks for class work while Mr Adeleke writes the problem on the chalkboard*]. This is 65° and this 60°. Just do this and ..., sorry, find angle B, find angle B, find angle B.



[*Mr Adeleke circulates around the room, not actually inspecting what the students were doing, but just allowing them time to complete the task*].

Has anybody worked it out? [*A number of students raised up their hands, indicating that they had solved the problem*]. Can you [*to a student*] come and demonstrate how you do it? What is your name, what is your name? Ade ... **“no, Ajose”** [*says the female student*]. Ajose, let Ajose come and see ..., help us to work it out and find out, let's see how she got it [*Ajose went to write the solution on the chalkboard as follows*].

$$\begin{aligned} A + B + C &= 180^\circ \\ 60 + B + 65 &= 180^\circ \\ 60 + 65 + B &= 180^\circ \end{aligned}$$

35 minutes

Ask the class, ask the class [Mr Adeleke tells Ajose to ask the class as she struggles with the sum of 60 and 65]. 60 plus 65 class, what is it? **“125”** [chorused the students]. **“One twenty-five minus one eighty?”** [Ajose asks the class]. No, one eighty plus [instead of minus] one twenty-five class? [Mr Adeleke interjects]. What is it? **“55”**, fifty-five, so angle B is fifty-five what? **“Degrees”**.

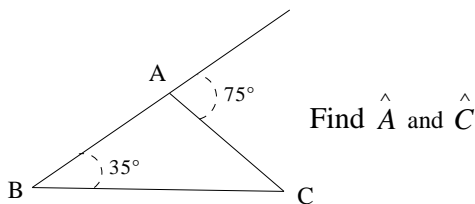
$$125 + B = 180^\circ$$

$$B = 180^\circ - 125$$

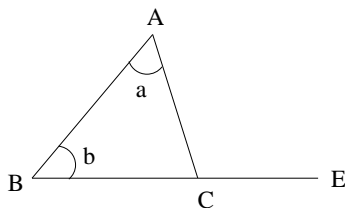
$$\therefore B = 55^\circ.$$

Can you put your hands together for her? [The class applauded Ajose as she returns to her seat].

Alright. So, you've seen that it is possible for us to use this rule [instead of explicitly using the word 'theorem'] to solve em, angles in em, a triangle, ok? Now, I want you to go home with this. Go home with this [he writes a problem on the chalkboard as shown hereunder] and em, I want you to go home with this and find angle A and em, angle C. Find angle A and angle C.

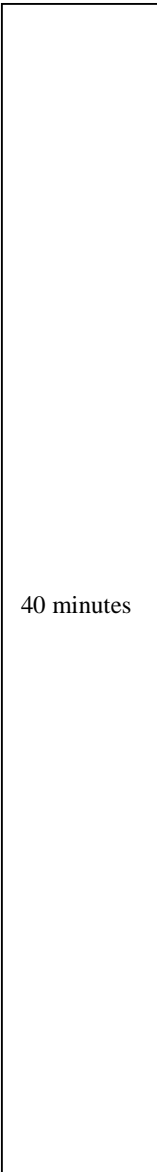


So that by the time we meet next week, we shall be talking about the interior angles of a triangle [draws a diagram on the chalkboard as shown below].



The two interior opposite angles, 'a', 'b' is equal to the exterior angle Z, or let's say C. We are going to prove that this angle 'a' plus 'b' is equal to angle ACE. That is what we will be doing next week. Thank you. **“Thank you Sir”** [Replied the students and the lesson is now over. Mr Adeleke ends the lesson 5 minutes before the official time].

As with most public schools in Nigeria, the teacher leaves the class at the end of his lesson while the students sit back and wait for another teacher for the next lesson.



## APPENDIX 8.B

### Mr Balogun's lesson (NS grade 11 teacher)

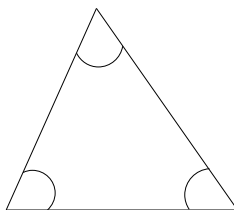
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*Mr Balogun, the teacher, had just entered the class. He moves straight to the chalkboard and writes the date, the subject and the topic for the day (kinds and properties of triangles) on it before turning to greet the students. "Good day everybody", he greets. "Good morning Sir", replied the students, and the lesson is now officially underway.*

Elapsed  
Time

We want to talk about the kinds and properties of triangles, types and properties of triangles. Now, before we go on, let's talk about the definition of triangle. A triangle is three-sided figure with three lines joined together with three angles. I will take it again. A triangle is three lines joined together to form three angles [*Mr Balogun circulates around the classroom as he states the definition, while the students were just listening but not writing*].

An example of ..., we have em, a line like this [*draws a horizontal straight line*], another one like this [*draws another line joining the first*], from here you join it to this place [*he completes the triangle as shown hereunder*].



This is what now? [*Mr Balogun uses arcs to mark off the angles of the triangle as shown above*]. "Angles" [*replied the students*]. Three angles, three-sided, are you with me? "Yes Sir". Now, apart from that one, let's talk about the types, types of triangles. If we want to consider a triangle, how do we do it? We normally know that a triangle, you talk about a triangle we all know that the sum is 180 degrees [*he writes it on the chalkboard as follows*].

Sum of a triangle is 180°.

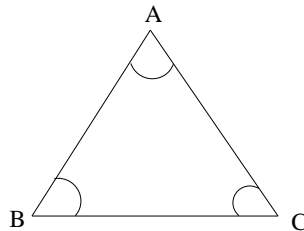
5 minutes

Am I correct? **“Yes Sir”**. Good. Now, before that, let’s talk about kinds and properties of a triangle [*he moves to consult his note of lesson which he keeps on a desk in the front of the class. He does this consultation quite often*]. The first one we will talk about, let’s talk about the types of triangles that we have. We have equalateral [*his own spelling and pronunciation*] triangle, we have isosceles triangle and we have scalene. Are you with me? **“Yes Sir”**. Now, equalateral triangle, we have kinds now, we have em... one, we have em ..., two we have em... three we have em... [*He lists the following as he speaks*].

Kinds of triangles

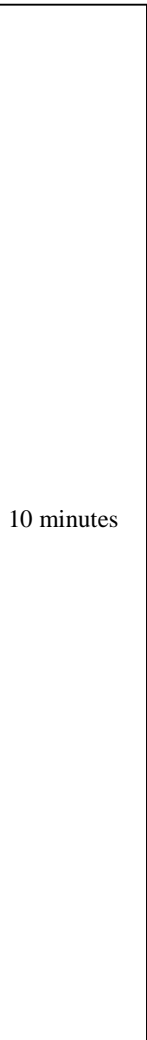
- Scalene triangle
- Isosceles triangle
- Equalateral triangle

Now we talk about, these are the three types of triangles. The first one we have em ... scalene triangle. In a scalene triangle, no sides are equal, no sides, we have em, in a triangle we have em, let’s say we have em ... A, B and C [*he labels the triangle above*]. Are you with me? **“Yes Sir”**. Let’s draw something new so that em ..., call this one A, B and C [*he draws and labels another triangle on the chalkboard as shown below*].



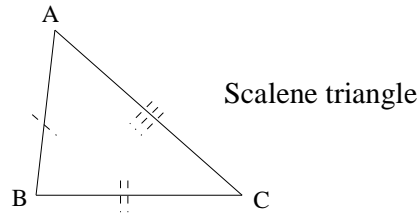
Angle A, am I correct? **“Yes”**. The size of this side, this side is known as ‘a’ [*points to the side opposite angle A, i.e. side BC in the diagram above*], side ‘a’. Opposite of the angle here [*touches angle B*] is known as ‘b’, small, capital, capital letter small letter ‘b’ [*he stutters*]. The angle here [*he marks an arc at angle C*] is known as C. Are you with me? **“Yes Sir”**. The opposite there [*he touches line AB*] is known as ‘c’. So, this side is known as ‘c’ [*places his fingers on line AB, the line opposite angle C*]. Are you with me? **“Yes Sir”**.

Now, after I said that, I said that em, a scalene triangle is a, a, a [*he stutters*] type of triangle that has all the sides are not equal. No side there are equal. For example, let



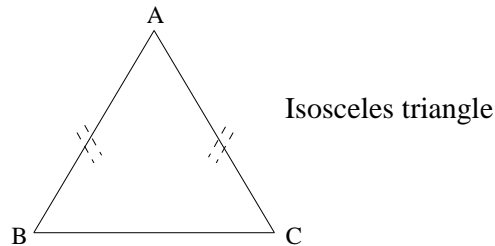
10 minutes

me just give you ... This one [*he touches the scalene triangle drawn on the chalkboard*].



We say this one is not equal to this one, is not equal to this one [*Mr Balogun marks the sides of the scalene triangle as shown above to indicate that they (the sides) are not equal to one another*]. Are you with me? **“Yes Sir”**. So, this one is known as scalene, no sides are equal, is it clear? **“Yes Sir”**. Is it clear? **“Ay”**. Is clear? **“Yes”**. So let’s give this one A, call this one A, call this one B and C [*labels the angles of the triangle drawn above*].

Now, isosceles triangle: Isosceles triangle is a triangle with two sides that are equal. Two sides, two of the sides are equal. Example of it, we have em, ..., and em, ..., in an isosceles triangle, two angles, when I say isosceles, I said that two sides are equal, we have em, ... [*he draws an isosceles triangle on the chalkboard as shown hereunder*].

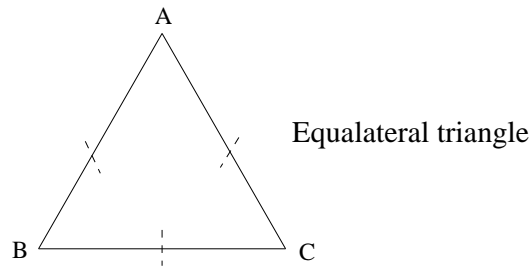


Call this one A, B, C [*Mr Balogun labels the angles of the triangle*]. This one is equal to this [*marks off the two equal sides on the isosceles triangle as indicated in the diagram above*]. Are you with me? **“Yes”**. Yes, so this is called isosceles, ..., **“triangle”** [*the students completed the statement*], triangle. Is it ok? **“Yes”**.

Now, equilateral triangle: That is a triangle, are you writing? **“Yes”**. **“Come again Sir”** [*some of the students requested Mr Balogun to repeat what he has just dictated*]. Which one? [*Meanwhile, the students are busy copying all that Mr Balogun has*

written on the chalkboard]. Are you there? [He asks to know whether the students have finished copying from the chalkboard]. **“Yes Sir”** [replied the students].

Now, the last one, there is equilateral [his own spelling and pronunciation] triangle. Equilateral triangle has, is a triangle with all the sides equal to each other. All, from the word ‘equal’, equal [Mr Balogun uses his hand to underscore ‘equal’ on his ‘equilateral’ that he has written on the chalkboard], equilateral [he emphasizes]. If we have em, AB is 5 cm [he explains using a diagram on the chalkboard], BC will be 5 cm and BC will be what? **“5 cm”** [chorused the students]. That is all sides are what? **“Equal”**. That is how we come across with the word equilateral. Should I repeat it? **“Yes Sir”**. Now, let’s write that one down, and em, when we have em ... [wipes the chalkboard and draws an equilateral triangle].



An equilateral triangle is that all the sides are equal, ok? **“Yes”**. All the sides and all the angles are equal. If the angle here [touches angle B on the diagram above] is  $60^\circ$ , A will be 60, C will be 60. So, all the angles, we have [writes on the chalkboard], we have A, B and em, C, are all equal. So, each is 60, 60, 60, 60 which will make up 180. That is the sum of a triangle. Am I correct? **“Yes”**.

Now, that is the types, the kinds of triangles we talk about. Now, let’s go, let’s go to the properties. Now, is there anything that you don’t understand on the kinds of triangles? Is there any question, before we move on? Any question before we move on? [No response from the students who were all busy writing from the chalkboard]. Yes, questions. Maybe you don’t understand anything, let me know now, now. Which one, which one? [He asks a student who now says she does not understand a portion of the lesson]. Which one? **“That one”** [she points to scalene triangle on the chalkboard].

15 minutes

Scalene, I said that in a scalene, we have, no sides are equal and no angles are equal as well. We have, since the sides, ..., this is the type of scalene [*touches a scalene triangle drawn on the chalkboard*], ok? The angle here is not equal to this, is not equal to this one. Are you with me? **“Yes”**. And look at the, the, the sides. This is the shortest side [*touching side AC of his scalene triangle, even though that side was actually the longest, see diagram above*], followed by this [*referring to side AB*], this is the longest [*touching side BC*], ok? No side is equal to each other. Are you with me? **“Yes”**. As there is no side equal to each other, then angles are not going to be equal. This one maybe, maybe higher than this, this one maybe higher than this [*Mr Balogun touches the three sides of a triangle on the chalkboard in turn as he explains*]. Is that not so? **“Yes”**. Is it clear? **“Yes Sir”**. It is clear? **“Yes”**. Yes, yes, yes, any other one? Ask me questions.

Now, let us move down to the properties, the properties, properties [*he moves to consult his lesson note and goes to write on the chalkboard the following subheading*].

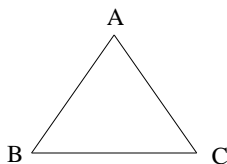
### Properties of triangles

Now, let's talk of, we have equilateral here [*touches it*], am I right? **“Yes Sir”**. Now let's talk about the properties of equilateral here. One, all the sides are equal [*he lists them on the chalkboard*]. Am I right? We are talking of equilateral now, properties of ... [*He was interrupted by a student who had a question*]. Yes, **“I don't understand it”** [*says the female student*]. Properties of triangles now, we said that we have, we are taking it one after the other. We said that, this is equilateral triangle, all the sides are equal. Let me explain [*he rubs off a section of the chalkboard and writes*], in an equilateral triangle, all the sides are equal [*a student seeks further clarification, so Mr Balogun repeats what he has written on the chalkboard*]. In an equilateral triangle all the sides are equal, comma, then all, ..., I said it before, that is if we have, let this angle A equal to angle B is equal to angle C [*he writes this as:  $\angle A = \angle B = \angle C$* ]. Am I correct? **“Yes”**. That is we have 60, 60, 60. Is that clear? **“Yes”**.

In an isosceles triangle, two of the sides are equal. I hope that one is clear? **“Yes”**. Two of the sides are what? **“Equal”** [*chorused the students*].

25 minutes

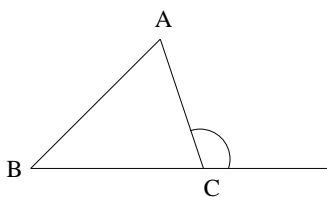
That is to say, in the example I gave you initially, I said that em, isosceles triangle has two equal sides [draws an isosceles triangle on the chalkboard]. Am I correct? “Yes”.



A, B and C [he labels the triangle as shown in the diagram above]. Two of the sides are equal. Are you there? “Yes”. Then, the first one is, we talked about the scalene [Mr Balogun writes ‘scalene triangle’ on the chalkboard stating that none of the sides are equal to each other and then moves on to discuss ‘sum of the angles of a triangle’].

The other one now, you know this [points to the definition of a scalene triangle that he has written on the chalkboard]. This is, is clear to everybody. “Yes Sir”. And this one [he has written that ‘the sum of a triangle is 180°’] is also clear to everybody? “Yes Sir”.

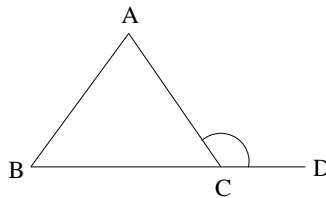
Now, the last one we are going to talk about is this, in a triangle, where you have em, we have em, something like this [draws the following diagram on the chalkboard].



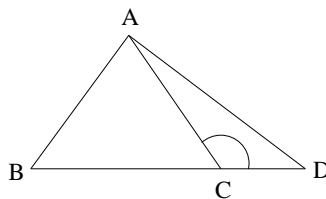
Are you with me? “Yes Sir”. Now, we have this angle A, B and em, ..., there is an angle here [uses an arc to indicate exterior angle C of  $\Delta ABC$  as shown in the diagram above]. What do we call this one? [He is referring to exterior angle C]. This one [exterior angle C] is going to be, ..., the addition of this and this [touches angles A and B in the diagram above] will give us this [touches exterior angle C]. Is it clear? “Um” [few voices from the students]. Is clear? “Um”. Let’s say you have em, let’s say you have em, you know that em, the sum of a triangle is 180°. Am I correct? “Yes Sir”. A equals to, let’s say A is equal to ..., “60°” [suggested the students]. B equals

80°, and we don't know C. You know we don't know C here [*touches interior angle C of  $\Delta ABC$  drawn above*]. Are you with me? **“Yes Sir”**. What you will do is that em we have em, ...,  $A + B = 140^\circ$ , ok? Equals one forty, one forty degrees. Am I right? **“Yes”**. So that em,  $180^\circ = A + B + C$ . Am I right? **“Yes”**. We know this one [*circles  $A + B$* ]. We have  $180^\circ - 140^\circ = 40^\circ$ .

So, this one [*exterior angle C*] is going to be what, is going to be what? **“40”** [*answered the students*]. No, it is going to be  $140^\circ$ , because we said that the sum of this and this [*touches interior angles A and B*] will give us this [*touches exterior angle C*]. [*Mr Balogun draws another triangle ABC with side BC produced to a point D and uses it to explain further that the exterior angle of a triangle is equal to the sum of the interior opposite angles*].



We have A, C and D now; are you following? **“Yes”**. The sum of this one, B and A is going to be em, the angle marked here [*touches angle ACD in the diagram above*]. So, let's see, we have em, let's join this line [*joins points A and D to form triangle ABD as shown below*].



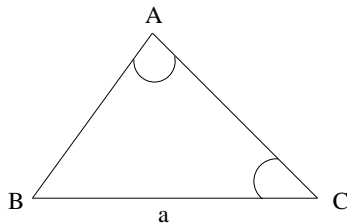
You see that, the sum of this, this A, A and B will give us this one [*exterior angle C*], am I correct? **“Yes”**. So, this one here [*touches interior angle C of  $\Delta ABC$* ], will be  $40^\circ$  to give us the sum of  $180^\circ$ . We have  $A = 60$ ,  $B = 80$  and C here will give us, ..., **“40”** [*says the students*]. So this one [*exterior angle C of  $\Delta ABC$* ] is 140, ok? **“Um”**. Then, this one [*touches angle CAD*] and this one [*touches angle D*] will have to be 140, I mean 180, 180, we have em,  $180 = 140 + A + D$ , am I correct? **“Yes”**. So, ..., **“Sir, which one is 180?”** [*Interrupted by a student*]. The sum, if, ..., the angle sum is

30 minutes

what? 180. Then what do we have? We have 180 minus 140 is ... **“40”**. Are you with me? **“Yes”**.

This type of angle [*exterior angle C of  $\Delta ABC$* ] is called an obtuse angle, ok? **“Yes”**. We have different types of angles: we have obtuse, we have em, an em, we have obtuse, we have acute, am I correct? **“Yes”**. And we have em, right-angled triangle. I hope that one is clear? **“Yes Sir”**.

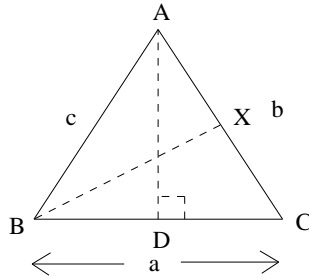
Now, now, the next one we have to talk about is ... [*Consults his lesson note*]. We will talk about the area of a triangle. The area of a triangle. Now the area of a triangle [*Mr Balogun writes it on the chalkboard as a subheading*]. Now, we can't talk about the area of a triangle without talking of the real thing of triangle. In triangle we have notations. How do we get it? We have em, we have em, a triangle ... [*draws a triangle on the chalkboard*].



Call this A, B and call this C [*he labels the triangle as shown above*]. Am I correct? **“Yes Sir”**. The angle here [*marks angle A with an arc*] is known as A, the one, the line opposite it is known as side 'a'. This is angle B, the one opposite it is known as side 'b', small letter 'b', the one opposite angle C is known as side 'c', ok? **“Yes”**. Are you with me? **“Yes Sir”**. Now, if that is the case, we said that em, the sum [*instead of the area*] of a triangle is half base times height. Now let us look at it. How do you get half base times height?

Now, assuming you have something like this, we have em, a triangle ... [*draws the following triangle on the chalkboard*].





Are you following? **“Um”**. Now, let me, let me just tell you something. In a triangle, the perpendicular line, when you divide this, this dot, dot, dot is called the perpendicular line [*uses his fingers to trace line AD*]. From A, you draw a perpendicular line from A to join the line BC to form D. Are you with me? **“Yes”**.

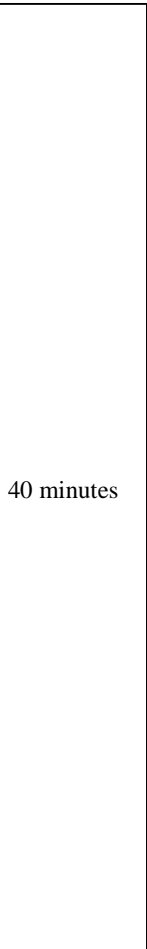
So, in this case now, we have different triangles here. We have ABC and we have ABD, the triangle there, we have ADC, ok? **“Yes Sir”**. So, definitely we have different triangles here. Are you with me? **“Yes”**.

Now, even if you want, say, from the perpendicular from A to cut, sorry, from B to cut line AC meeting at where? X. So, we have ABX, we have BXC. Are you following? **“Yes Sir”**. Good. Now, don't forget one thing: this line, one, nine ... ninety degrees, ok? [*He indicates a right angle at D*]. The angle on a straight line, perpendicular line is what? **“Equals 90°”** [*says the students*]. Ok ?

Now, let us not continue with that one. Let's do something today. Let us consider a particular thing. We said that the sum of a triangle is half base times height, ok? **“Um”**. [*He writes it on the chalkboard as follows*].

$$\text{Sum of a triangle} = \frac{1}{2} \times \text{base} \times \text{height}.$$

And look something: in some cases they may write perpendicular height. Some may write half, may write perpendicular, some may write altitude, altitude side or altitude line. All these things still narrow down to the same thing [*points at half base times height*]. Are you with me? **“Yes Sir”**. Are you with me? **“Yes”**. Now, look at this, now let us see, sum of a triangle is half times base times height [*reads from the chalkboard*]. Look up everybody. In a triangle, we have em, half, the base of this particular, this particular angle, I mean this particular triangle [*referring to  $\triangle ABC$  in the diagram above*], this is the base, abi? [*'abi' is a Pidgin word for 'is that not so?'*].



40 minutes

“Yes Sir”. BC, the, the height, the height [Mr Balogun looks at the students expecting an answer], here is the base, the height is A, are you with me? AD [runs his fingers from A to D and writes the following on the chalkboard].

$$\begin{aligned} \text{Sum of a triangle} &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2} \times BC \times AD \end{aligned}$$

But in some cases, you may say that  $\sin B$ , they may say that, that  $\sin B$  is equal to opposite, we have the opposite here as what? Here [he runs his fingers along line AD], the opposite here, AD over AB [he writes this on the chalkboard as:  $\sin B = \frac{AD}{AB}$ ], am I correct? “Yes”. We say that  $\sin B$  here, the Sine here,  $\sin B$  here is the same thing as opposite over hypotenuse. Am I correct? “Yes”. Good.

Now, at this point, I said that make this one the subject of the formula, we have AD equals to AB  $\sin B$  [he is referring to the formula,  $\sin B = \frac{AD}{AB}$  stated above]. Are you with me? “Um”. Your AB here, AB, what, what is this one, AB  $\sin B$ , we have em, half times, your BC here, BC is ‘a’. Am I correct? “Yes”. [All this while, Mr Balogun is at the chalkboard and his explanations are from  $\Delta ABC$  in which two altitudes are drawn as shown in the diagram above]. Am I correct? “Yes”. ‘a’ times your AB, AB is what? AB is what? “c” [chorused the students, since Mr Balogun runs his fingers along that side of the triangle]. Are you following? “Yes”. ‘c’ Sine what?  $\sin B$ . So, we have half times ac  $\sin B$ . [At this point, Mr Balogun has written his formula for the ‘sum of a triangle’ completely on the chalkboard as follows].

$$\begin{aligned} \text{Sum of a triangle} &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2} \times BC \times AD \\ &= \frac{1}{2} \times a \times c \times \sin B \\ &= \frac{1}{2} \times ac \sin B \end{aligned}$$

Is it clear to everybody? “Yes”. Is it clear to everybody? “Yes”. Is it clear? “Yes”. It is clear? “Yes”.

Now, let me explain this point to you again. I said that em, the side, I mean the sum of a triangle is half times base times height. Are you with me? **“Yes Sir”**. The side, the height here is AD, the perpendicular line [*he is using the diagram on the chalkboard*]. Are you following? **“Yes Sir”**. Now, what we have again is BC [*touches line BC on  $\Delta ABC$* ], ok? I said that in triangle ABC, triangle ABC we have your BC, ok? **“Um”**. Half of BC times AD. Where did you get, where did you get your AD? AD is the perpendicular line, ok? Now, the perpendicular line should be equal, AD will be equal to  $AB \sin B$ . We have em,  $\sin B$  is the same thing as opposite over hypotenuse, A, AD over AB. Are you with me? **“Yes”**. Here is your AB and using change of subject of formula, we have em, are you with me?  $AD = AB \times \sin B$ . So, where is the half AB? AB is the same thing as side ‘c’. Am I correct? **“Yes Sir”**. That is how we get our ‘c’ [*using the diagram on the chalkboard to explain*].

So, let’s go back to where, where we started from. We talked about em, definition, the definition of triangle, which I said that a triangle has three angles and em, three sides. Am I correct? **“Yes”**. Then from there we moved down to the types of triangles that we have. We talked about scalene, scalene, isosceles triangle and em, equilateral triangle. Then from there I moved forward to say that em, we have three kinds of triangles and the properties. We talked about the sides, how many sides has isosceles? I said to you that it has two sides equal, am I right? **“Yes”**. In scalene, I said that, I said that em, no sides are equal, no angles are equal [*the students verbalise this with Mr Balogun*].

So also, again we talked about equilateral, I said that ‘equal’ means when all the sides are ... **“equal”** [*interjects the students*] listen, are the same. Then I said that the sides are equal. Are you with me? **“Yes”**.

Then from there we talked about the em, the properties of triangles and I said that in equilateral, all the angles are equal. Am I right? **“Yes”**. Then at the same time, we talked about the area<sup>1</sup> of a triangle which states that half base times height where we have the perpendicular line that crosses the triangle in the middle, is known as the

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<sup>1</sup> Although Mr Balogun now correctly uses the word ‘area’, he still does not realise that he has been using ‘sum of a triangle’ instead of ‘area of a triangle’ for that section of his lesson that dealt with the formula for the area of a triangle. Consequently, he does not correct his error before leaving the class.

45 minutes

perpendicular line. So, that is it, and we talked about the sum of a triangle, how we now arrived at  $\frac{1}{2} \times ac \sin B$ . Is that clear? **“Yes Sir”**. Any question? Any question? **“Sir, Sir”** [one of the male students wants more clarification about the formula for Mr Balogun’s ‘sum of triangle’ stated on the chalkboard]. I said that in this triangle, AB and D, ok, ABD makes this, you know we have, we have this one [touches line AB of  $\triangle ABC$ ], angle B, ok, when we talk of angle, angle B now, we said that angle B is the same thing as opposite over hypotenuse, you get that one? **“Um”**.



$\sin B$ ,  $\sin B$  now is the same thing as AD all over AB [he writes this as  $\sin B = \frac{AD}{AB}$ ], which is this, ok? Now, make this one [touches AD in the formula stated above] the subject of the formula, we don’t know this one. AB, I mean AD now is the same thing as this one and this one, it is just like cross multiplying, we have em, ... [he writes it on the chalkboard as follows].

$$\frac{\sin B}{1} = \frac{AD}{AB}$$

So,  $AD = AB \sin B$ . Are you following? **“Yes”**. [Mr Balogun gives some further explanations, not audibly captured by the camera, using the diagram on the chalkboard]. So, altogether now, we have  $\frac{1}{2} \times ac \sin B$ . Is that ok? **“Yes Sir”**. Any other question before I go? If you have any question talk before I go. Girls ask questions, boys have been asking questions, girls try. Ask a question. Ask a question. [No student asks any further question]. Good day. **“Thank you Sir”**.



With this exchange of greetings, the lesson has officially come to an end and Mr Balogun left the class. Mr Balogun’s lesson period was extended by 5 minutes beyond the 45 minutes official time.

## APPENDIX 8.C

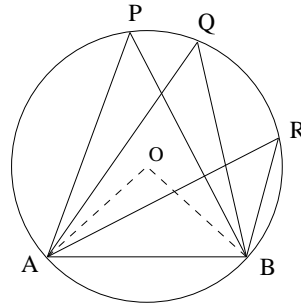
### Mr Lawal's lesson (NS grade 12 teacher)

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By the time I got to the class, Mr Lawal, the teacher was already there. So, the camera missed the usual exchange of greetings between the teacher and the students.

Elapsed  
Time

Em, class, last week we talked about angle subtended on a circle. So, if we have a circle like this [Mr Lawal draws a circle on the chalkboard] and you have a chord AB, if this one is extended here, we have this, and if this one is extended here we have this [he connects two points, P and Q, and later, a third, point R on the major arc to points A and B of the chord AB as shown hereunder].

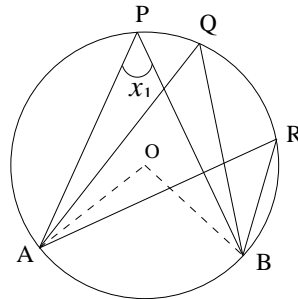


If there is a centre zero here, ok? **“Yes”**. You can draw a line AO and a line OB [he connects points A and B to O, the centre of the circle], so that if this is P, Q, we can have another line here, R [he labels points P, Q and R on the circle as shown in the diagram above], so that this angle, APB and AQB and ARB is subtending this circle at the major arc here [places his hand on the major arc], ok? **“Ay”** [answered the students]. At the major what? **“Arc”** [chorused the students]. Arc, here, ok? By this em, minor ... this is a minor arc, is that not so? **“Yes Sir”**. And this is a major arc, ok? [Mr Lawal uses his hands to trace the minor and the major arcs using line AB as the reference chord for the students to see]. **“Um”**. So, this also is a segment. This can be a segment. This can be what? **“Segment”** [chorused the students]. Are you getting me now? **“Yes”**. This is the arc, ok? But this is the major segment. They are angles subtended under the major segment by this minor arc [Mr Lawal illustrates all these using the diagram on the chalkboard].

5 minutes

What we want to do today, what we want to do today is to prove that angles in the same segment of a circle are equal. So, we have this, draw this circle [*he draws another circle on the chalkboard as shown hereunder*]. The first procedure is the diagram. The first procedure when you are proving is what? **“Diagram”** [*chorused the students*]. You draw the diagram.

1) Diagram



We have our, our arc here, ok? **“Yes”**. We call this one AB, we call this one P, Q, R. That is in the, in the arc. At that point, the angles there, they are on the circumf ... circumference of the what? **“Circle”**, circle [*Mr Lawal repeats after the students*]. P, Q, R they are on the circumference of the circle.

Then, number two, you say required to prove. Required to what? **“To prove”** [*echoed the students*]. Required to prove, what do you want to prove? We want to prove that angle AP, AP, AP, let’s call it  $x_1$ . APB is equal to angle AQB, is equal to angle ARB [*he writes these on the chalkboard as follows*].

Required to prove:  $\hat{A}P\hat{B} = \hat{A}Q\hat{B} = \hat{A}R\hat{B}$ .

That is what we want to prove, ok? **“Yes”**.

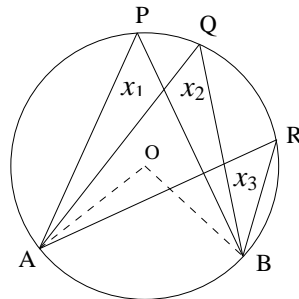
Number three, construction, construction.

Construction: Join A and B to the centre of the circle.

You can see the way I am listing it step by step, is that not so? **“Yes”**. Number three is construction, ok? Join line [*instead of points*] A and B to the centre of the circle. The centre of what? **“Of the circle”**. The centre of the circle is here, so you join this A and em, B to O. Do you see? Ok? [*All this time, the students were copying whatever Mr Lawal writes on the chalkboard*].

15 minutes

Then, the next thing now is proof, proof, proof. By, ... by, ... let  $x_1$ ,  $x_2$  and  $x$  what? **“Three”**. Call this one  $x_2$  and this one  $x$  what? **“Three”**. Ok? **“Yes”**. [*His diagram now looks like this*].



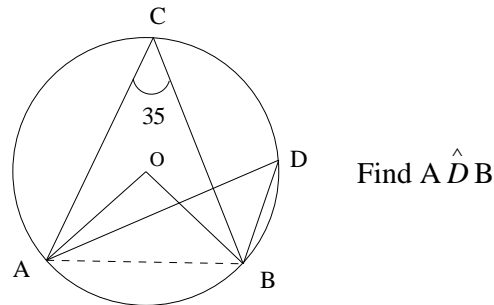
So, the next thing, therefore the next thing to do here is that angle, angle APB is equal to half, is equal to half of angle AOB, is equal to half of angle what? **“AOB”** [*chorused the students*]. What will be our reason, class? [*Mr Lawal did not wait for the students to respond before stating the reason by himself*] The reason is that angle at centre, angle at centre is twice angle at what? Circumference. Angle at centre is equal to two times angle at circumference, ok?

25 minutes

Also, angle AQB, ok? Is equal to half angle AOB. Same, give the same reason. The same what? **“Reason”** [*chorused the students*]. That angle at centre is twice angle at what? **“Circumference”**. Also, angle ARB is half of AOB. Is what? The same reason. So, students at this junction, let me show the last one. We can continue to produce a line like this [*draws lines from points A and B to form more angles on the major arc*].

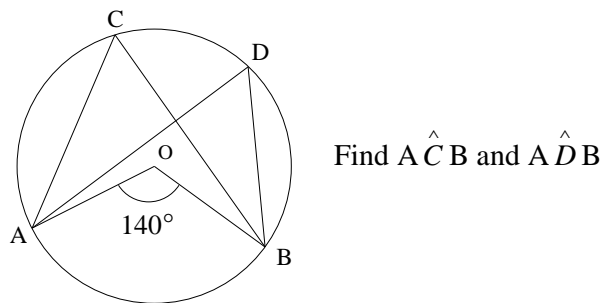
Therefore, we have seen that, from our proof we have seen that this is  $x_1$ , abi? [*‘Abi’ , as used here by Mr Lawal is a confirmation seeking Pidgin word for ‘is that not so’?*]. **“Yes”**, equals  $x_2$  abi? **“yes”**, equals  $x_3$ , is that not so? **“Yes”**. Therefore, angle AQB equals angle A ..., sorry, angle APB equals angle AQB equals angle ARB [*he writes this as:  $\hat{A}P B = \hat{A}Q B = \hat{A}R B$* ]. So, you have seen the proof here now. So, we’ve come from the general to the single, to a single proof. We wanted to prove that this angle is equal to this one and this one [*touches angles P, Q and R in turn*]. So, how can we apply it? How can we apply this em ... proof here now?

Let's say we have this em ..., we have this circle, ok? [Draws a circle on the chalkboard and poses a problem for the students as shown hereunder]



Now find angle, angle ADB, .find angle what? **“Angle ADB”**. Alright, you see that I have given you, they are angles in the same, in the same segment abi? **“Um”**. Because, the two of them have a common chord, abi? [He connects A and B with a dashed line as in the diagram]. **“Yes”**. Ok? **“Um”**. So, they are angles in the same segment. Therefore, there is no need for you to begin to add and subtract. Just say angle ADB equals to angle ACB, ok? That is,  $\hat{A}DB = \hat{A}CB$ . If it is of em ..., a theory paper, you will say angles in the same what? **“Segment”**. [Mr Lawal solved this problem by himself asking only corroborative questions from the students. He writes  $\angle ADB = \angle ACB$  ( $\angle$ 's in same segments). Ok? **“Yes”**].

Now, another way to apply this is this [poses another problem on the chalkboard as shown below]



Now, I have to find angle C and D. Find angle ACB and em ..., angle AD what? **“B”**. Do you see? **“Yes”**. From what is on the board, we've already said that angle at centre is twice angle at what? **“Circumference”**. So, that means, two of angle A ... ACB is equal to angle AO what? **“AOB”**. Is that not? **“Yes Sir”**. Is that not? **“Yes Sir”**. [Mr Lawal solves the problem on the chalkboard as he speaks].

$$2\hat{A}CB = \hat{A}OB$$

$$= 140^\circ$$

$$\hat{A}CB = \frac{140}{2}$$

$$\hat{A}CB = 70^\circ$$

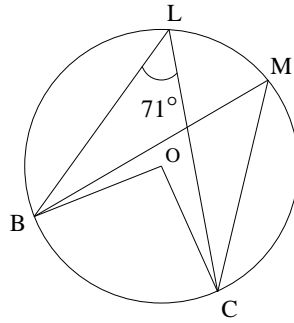
But angle AOB is what? “140”. Therefore, angle ACB is equal to one forty over what? “2”. And therefore, angle ACB is equal to seventy degrees.

Do you get the point? “Yes Sir”. Alright, now we can find ADB. Of course, angle ACB is equal to angle ADB which is what? Seventy degrees. Of course, here too, if it is theory, you would have said that angle at centre is equal to angle at what? “Circumference”. You must advance the reason for it, if it is theory so that the examiner will know that you know what you are doing. But if it is objective [*he means multiple-choice*], you may not need to do so. You just need to state that therefore this is equal to this.

I don't know whether you have em, any question to ask. Class do you have any ..., yes [*to a student who raises her hand that she has a question to ask*]. “**What about this one, angle ADB, is it not supposed to be angle ACB first before angle ADB?**” [*This student is referring to the first rider where the teacher wrote  $\angle ADB = \angle ACB$ . Her question here is that why not  $\angle ACB = \angle ADB$ ?*] Em ,..., it doesn't mean, it doesn't mean because, that is a very good question, the question she has asked is this: I said  $\hat{A}DB = \hat{A}CB$ , she is saying that why not  $\hat{A}CB$  before  $\hat{A}DB$ ? It doesn't even mean because we are saying that they are equal, is that not so? “**Yes**” [*answered the students*]. It is just like saying two equals two, you are getting me now? “**Um**”. You can bring this to this side, but you are still in order. Let's say  $\hat{A}CB$ , ok, equals angle  $\hat{A}DB$  [*he rewrites this as:  $\hat{A}CB = \hat{A}DB$* ].

So with that class can you quickly do this? Let me see whether you really understand what I have been saying since, do this for me [*Mr Lawal poses another rider on the chalkboard for the students to do as shown hereunder*].

30 minutes



Find BMC and BOC

35 minutes

[The students copied the problem in their notebooks and began to solve it individually, as is usually the case in most public schools in Nigeria. Mr Lawal just circulates around the room not necessarily attending to any students].

If you have done it, let me know. Please can you come and demonstrate how you arrived at your answer [says to a student who said he has finished the class work. The student comes to the front and begins to solve the problem on the chalkboard].

$$\hat{BLC} = \hat{BMC} \quad (\text{angles in the same segment})$$

$$\text{Since } \hat{BLC} = 71^\circ$$

$$\therefore \hat{BMC} = 71^\circ$$

I hope that is correct class, is that not so? **“Yes Sir”**.

**“Then, for  $\hat{BOC}$ ”** [the student continues],

$$\hat{BOC} = 2 \hat{BLC}$$

$$\hat{BOC} = 2 \times 71$$

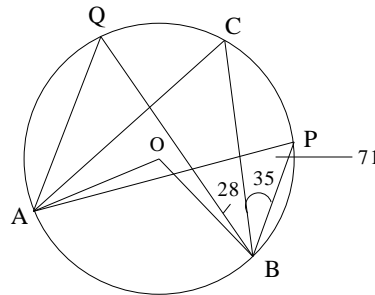
$$\therefore \hat{BOC} = 142^\circ$$

Class is it correct? **“Yes Sir”**. [The class applauded the student as he returns to his seat].

40 minutes

You can see that this em, theorem, circle theorem, circle geometry theorem we have just proved, and you have to apply it. The theory [instead of the theorem] holds provided they have a common chord at the minor what? **“Segment”**, segment, yes. If they don't have the same common chord, the law does not hold. Alright? **“Yes”**.

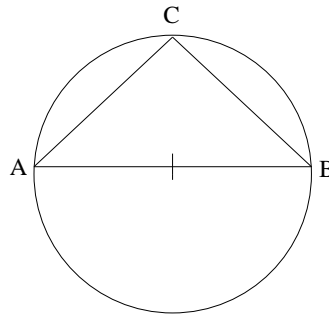
But then, while you will have this as em, assignment, have this as an assignment and try and see if you can em, you can get it [Mr Lawal writes a problem on the chalkboard as homework for the students].



Find: i)  $\angle BAC$   
ii)  $\angle AOB$

This one is  $28^\circ$ , ok? Find angle BAC. Find angle BAC, ok? Find angle what? **“BAC”**. Here is em...  $71^\circ$ , ok? **“Um”**. Angle BAC and AQB. Find the two angles, find the two. Find these two, ok? **“Yes Sir”**. So that when we meet we shall be talking about angle in a semicircle, angle in a semicircle is equal to  $90^\circ$ . In fact, we would be looking at something like this [draws a diagram to illustrate].

45 minutes



This is an angle in a semicircle, is that not so? **“Yes”**. We shall be saying that angle ACB is equal to  $90^\circ$ . We will be proving that. Thank you very much. **“Thank you Sir”**. [With this exchange of greetings, the lesson has come to an end and the teacher leaves the class].

## APPENDIX 8.D

### Mr John's lesson (SAS grade 10 teacher)

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*A few minutes before his class, Mr John, the teacher, was busy arranging the class for the next lesson. In a moment, the students started filing into the class individually and in groups of two, chatting away.*

Elapsed  
Time

Four people around a table [*Mr John instructs the students*]. In groups of four, yeah. Sit down there [*pointing to a student that was standing*]. Go there [*to another student*].

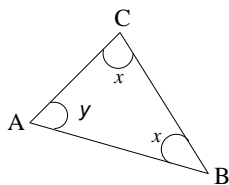
Some of you were in a group you are doing as a research project. So, this is the last part of the research where he [*referring to me*] is going to photograph video a whole group in real action. A group of students, engaging with the details and how they mark exams [*Mr John tries to prepare students' mind for the camera*].

So, me, I am going to the same topic we did last year, that is about the triangle. And I've got some worksheets for you to complete. So these are the worksheets [*Mr John circulates around the room giving each student a worksheet that he had prepared for the lesson*]. You will see the four triangles, each one of you is going to select one and do it in your notebook and then you are going to complete it on this drawing sheet [*raises sheets of drawing papers for the students to see*]. So, this sheet [*referring to the worksheet which contains some instructions*] will also serve the same work as your worksheet. You can read through it very quickly and then I will explain it on the board what I expect you to do. Take one, take this, take, take, take this [*he circulates around the room distributing the worksheets and the drawing sheets to the students*]. So, two people can read one [*he reckons that the worksheets will not go round*].

5 minutes

You are short of compass class, so two of you need to use one [*meant the school is short of compass for the students*]. Please, take all the pencils to me at the end of this lesson [*he distributes pencils among the students*]. We have 17 [*he counts the number of compasses as he distributes them*]. Take the compass. Take one compass. Take

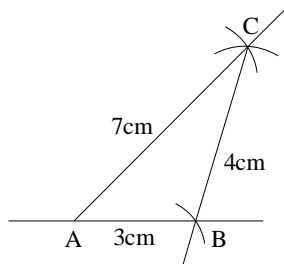




So, a triangle has three sides and three angles. Also, in your junior class you must have learned that the sum of the angles of a triangle is equal to  $180^\circ$  [*students singing along with him*].  $\hat{A} + \hat{B} + \hat{C} = 180^\circ$  [*Mr John reads them aloud as he writes: angle A plus angle B plus angle C is equal to one hundred and eighty degrees*]. You are going to verify this at the end of this your lesson. You are going to measure angles of all these triangles [*referring to the four triangles on the worksheet*], and then you make sure that the sum is equal to  $180^\circ$ . You are going to investigate four types of triangles. One is called em ..., what is its name? **“scalene”** [*the students chorused*]. Scalene triangle, the other one is called **“isosceles”** [*the students chorused*], you have them in your book [*referring to the worksheets and students’ textbooks*], isosceles triangle. You are going to represent the worksheet on the board here [*Mr John lists the four types of triangles on the chalkboard*]. Scalene, and then you have isosceles triangle. Then you have number three as ... **“equilateral triangle”** [*students chorused*], equilateral triangle [*Mr John repeated*]. And then finally you have... **“Right-angled triangle”** [*the students chorused*], right-angled triangle [*Mr John repeats and writes the four types of triangle on the chalkboard, misspelling isosceles as “isosles”*]. You can abbreviate by  $\perp \triangle$  [*for right-angled triangle*].

The sides are too small [*referring to the dimensions of the given scalene triangle on the worksheet: 2cm, 4cm and 5cm*]. You will find it difficult to measure. So, I’m going to change the sides to 3, 4, and 7 so that it will be easy to measure them. Unfortunately, I don’t have good ruler but I will make use of my tools here [*he had come to the class with some construction tools for the chalkboard: a long flat wooden ruler, a pair of compasses and a protractor*]. Right, my meter rule, we’ve been struggling for one year to get a meter rule [*he mutters to himself*]. So, to make use of one of the sides, let’s say this is AB 3cm [*uses the wooden ruler to draw a straight line on the chalkboard and marks off two points A and B on it*].

Now, making use of your compass – you can use a protractor also to cut the triangle, but I want you to use compass [*fixes a piece of chalk into his chalkboard compass*]. You know how to draw the triangle using a compass? **“No”** [*many of the students answered*]. Some of you can do it [*he insists*]. Right, here we’ve taken AB as equal to 3cm [*moves to and indicates it on the chalkboard*]. Now, on your ruler, you take the other two sides. I said how much? [*Referring to the dimensions of the triangle*], 3, 4 and ... and 7. So, 4cm here on the meter rule [*measures 4cm from a protractor having a meter rule attached to it*], 4cm on one side, let’s use this side BC is equal to 4cm [*Mr John draws an arc on the chalkboard with B as centre, radius 4cm*]. Please, make a long arc, we don’t know whether it is going to cut. And take the other as 7cm. I’ve told you the problem I am getting [*his difficulty with the chalkboard compass*], so the 4cm, the measurements are wrong [*mutters to himself, and the students laughed*]. I’m not measuring, am not having a proper ruler, that’s the problem [*he mutters to himself again*]. But, anyway, you’ve got proper rulers and protractors so, you draw these as the other two sides [*draws the triangle of sides 3, 4 and 7cm on the board*]. For the convenience of measuring using the protractor, we need to extend these lines a bit [*referring to the lines that form the sides of the triangle*].



You will see it when you measure it [*referring to the need to extend the lines forming the sides of the triangle*]. What is the problem? [*Attending to a student*]. So this point [*point where the 4cm and 7cm arcs meet*] is called C. That is 4 here and this is 7cm [*labelling the sides of the triangle*]. After this, you need to measure each of these angles; angle A, angle B and angle C and then you will write it down. Also, you read through your worksheet and answer various questions on the other page [*the one page worksheet was a photocopy of two pages from a maths textbook – Study and Master Maths*]. And then the question 8 says that as a class, discuss the definitions of triangles. You said a triangle is sides bounded by, a plane figure bounded by three straight lines. Think about other ways of explaining what is a triangle. Alright? Have you read it? Any doubt? Any doubt? [*He repeats as no students responded*].

25 minutes

So what you are going to do is that in your notebook, each one of you, you must trace one triangle, whether you do it wrongly or not, alright?. Then you start drawing it and start answering all aspects of the questions mark here on the page 1. Then it is a question of working in a group answering all the questions and putting all those figures which you have drawn in your notebook in drawing all these things. Then you discuss it again. But if you don't complete it today, maybe you will discuss the, the rest questions tomorrow. But try to do up to the fourth one, ok? **“Yes”** [*chorused the students*]. I will go round and see the work you are doing. You want compass? Here.

*Mr John circulates around the room answering questions, giving instructions and correcting errors as the students all began to work in their various groups. You have to draw it by measurement [directing a student who was drawing just any triangle].*

Have you selected one each? So, I take this you take that [*asking each member of a group to choose and draw only one of the four triangles each*]. Do that first. Is that clear? **“Yes”**. Yes, and then draw one in your notebook. Very fast. I will give you 5 minutes.

You are to measure. Some of you are copying the figures. You have to measure and draw it [*giving guidance to a group of students*]. *The students, though sat in their groups, each member of a group was doing individual work at first – drawing a different triangle each in their notebook, while Mr John circulates around the room giving guidance as needed. The students chatted to one another as they work in their respective groups.*

If you're stuck somewhere please call me [*Mr John announces to the students who are now busy with their work*].

You can use your worksheet to answer all those questions. You don't have to draw it again. Use this worksheet to answer the questions. Investigation means what? In investigation as a group you come across many difficulties. Alright? Yeah. And I am not interested in your single answer. Don't think that you are doing something wrong, no, as long as you can read and take measurements, you are investigating. [*Mr John*

33 minutes

*gives further explanation of what investigation is, but this was not audibly captured by the camera as his voice was subdued by the ensuing noise from the students*]. All these difficulties will come in your group and you are going to discuss it. That is the purpose of investigation. Mathematics and also life is not em, straightforward business: You come across your problem, then you think about it, then you solve it.

38 minutes

Did any group complete all the four triangles? Did any group? [*No response, the students engrossed in their work*]. If you complete it you must show me. And then you start drawing it on the drawing sheet as a group. I am going to mark it as a group. It is a group assessment. And you don't get the same marks for copying in a group.

You are just copying that figure [*he says to a student*]. In a group, you can discuss that because the sides are small, it was difficult to measure .... If you have drawn one of the triangles by now you can put it together on the drawing sheet. So, start one by one. But at least you are doing the right things [*addressing a group of students*]. You are going to answer this part, the person who has drawn the triangle must answer it. Then you discuss with the other people and explain to them, 'this is what I did', 'this is what I did'. Then you are going to see a lot of discussions coming up.

43 minutes

Also, because it is group, different ideas will generate and you come to an agreement, then if you can't get it, you call me and I will explain to you what is the problem. [*The students in their groups are busy doing and discussing the activities on the worksheets, while Mr John circulates around the room answering questions and giving guidance*]. I think you should start drawing on the drawing sheet now.

50 minutes

*It is now 50 minutes and the school bell signalling 'change of lesson' has rung and the students started filing out of the class as Mr John told them that they will complete the work the next day.*

## APPENDIX 8.E

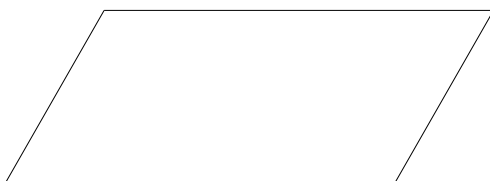
### Mr Shlaja's lesson (SAS grade 11 teacher)

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*It is time for another lesson and the students started walking into the class where Mr Shlaja, the teacher, has just finished arranging the seats in preparation for the lesson. Before long, all the students have settled down and the lesson is about to begin.*

Elapsed  
Time

Today's lesson is an introductory one [*says Mr Shlaja, the teacher*]. It is about the properties of parallelograms, neh? [*Neh could be interpreted to mean, 'ok'? in Xhosa language. Mr Shlaja uses it quite often*]. **“Yes”** [*replied the students*]. Let me start here [*he circulates around the room distributing worksheets on parallelogram, which he has prepared for the lesson, to each of the students*].

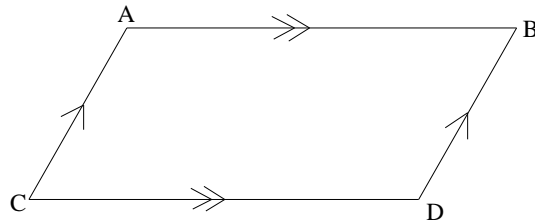


You are going to work in groups, neh? **“Ay”**. Groups of two. You will write on this parallelogram on the worksheet as A, B, C, D, neh. A, B, C, D [*Mr Shlaja wants the students to label the angles of the parallelogram above with those letters*]. What you are going to do is to measure the sides, neh? The sides of the parallelogram. You are going to work together neh. Hey, hey, shhhh [*tries to control the noise from the student that was beginning to subdue his own voice*]. You have to measure from here to here [*he touches points A and B of side AB of the parallelogram as he explains to a group of students what they are expected to do. Although the students are to work in pairs, every member of each pair had his/her own worksheet. The students were each busy with taking measurement and recording their values*].

5 minutes

Right, what I will like you to do neh, you write KB neh, don't write AB out there. AB and CD [*the students pronouncing along with him*]. Now, I want you to measure AB, neh? Measure AB and you write it down. When you finish measuring AB, you measure CD, neh [*uses the parallelogram he has drawn and labelled on the*

chalkboard to show the students what dimension to measure]. Then after finishing that, measure AC and BD [students' voices louder than Mr Shlaja's voice in mentioning 'AC and BD']. Because this is a parallelogram, but yours is not a parallelogram, unless you put this [he puts arrowheads on the parallelogram on the chalkboard which now looks like the one below], ok? **“Yes”** [replied the students].



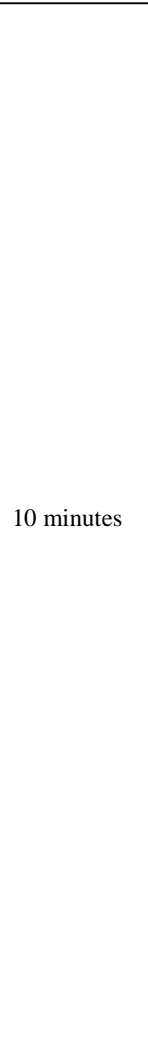
Two on AB and two on CD, one on AC and one on BD [referring to the arrowheads]. So, you are going to measure AB and CD.

If, if you finish, neh, do a construction CB, CB, neh, construction 1, CB, yeah and AD. Construction again on, construction on AD, AD, join AD [he shows students points to join on the chalkboard].

Have you finished with the construction? **“Yes”**. When you finish with the construction, neh, you measure the angles. Angles A, B, C, D, neh [he circulates around the room and distributes protractors to the students]. Angle A, B ... Measure angles A, B, C, and D, neh. [The students were each busy, trying to measure the angles].

There is an announcement, sorry, from the Principal [two students had just come into the class with a notebook containing some information for the students. Mr Shlaja read the information to the class as follows]. “Makana house meeting which has been rescheduled for next week has been rescheduled and brought forward to Saturday, this Saturday, neh. This means that we have to have our houses tomorrow, that is Wednesday at the latest so that the teachers can then work ...”

Alright, now, some of you have said ... shhhhh [hushes the students to keep the noise down]. Some of us do not know how to measure angles, neh? **“Yes”** [answered the students rather enthusiastically]. You don't measure like this. It means you put it there at this point, at zero, neh? Hush, stop the noise [he says this in Xhosa language].

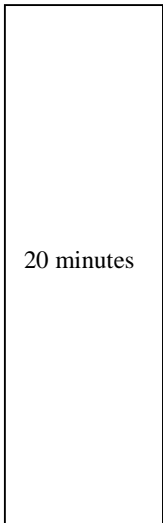


10 minutes

At this point, you put it at zero [*uses a chalkboard protractor to demonstrate how to measure angles to the students*]. Right? **“Yes”**. Measure from here, we have 10, 20 and so on, neh? And then on the other side, outside here, we have 180, 170. We are not going to use 180. You are going to use 0, 10 and so on neh? Siyavana? [*a Xhosa word meaning ‘do you understand?’*] **“Yes”** [*answered the students*]. 0, 10, 20, 30, 40, 50, 60, 70, 80, then, if it is 80, mine is 70, neh? Mine is 70, from here to here is 70, siyavana? **“Um”** [*answered the students meaning ‘yes’*]. On all the angles, A, B, C, D ... no talking [*tries to control the rising noise from the students*].

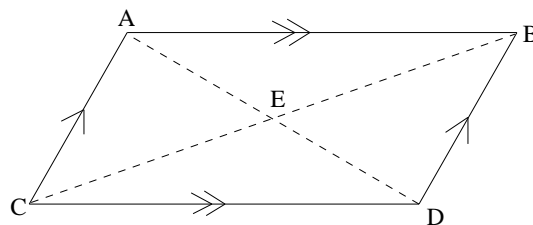
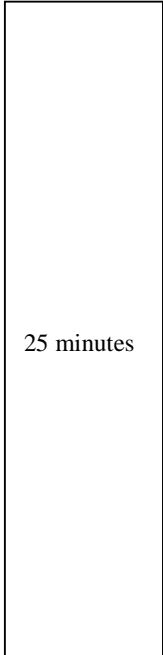


Start from zero [*directing a group of student as he circulates around the room*]. Stop talking, wena [*wena is a Xhosa word for ‘you’*]. When you finish, you should talk so that I give you the other work that you are to do. [*Mr Shlaja circulates around the room giving guidance to the students on the measurement of the angles of the parallelogram using a protractor*]. The angles, your angles are measured in degrees neh? **“Yes”**. So, you write  $\hat{A}$ , and your answers in degrees.



*Mr Shlaja moves from one seat to another, answering questions from the students, correcting their errors and giving them guidance on the measurement and recording of the angles of the parallelogram.*

Hey, hey, look at the board, look at the board. These things you people are doing, you can't get it right, it is wrong [*picks a chalkboard ruler and goes to the chalkboard to explain once again what is expected that the students should do*]. I don't want you to measure ... Here is my construction [*draws the diagonals of the parallelogram on the chalkboard*]. Here is my construction neh? Here is my other construction [*he joins A to D and B to C as shown below*].



Here [*touches angle A*], we have two angles, right? **“Um”** [*responded the students*]. B, BAD, I don't want BAD, I want A. And A starts from here to here [*uses an arc from*

line AB to line AC to mark off angle A as he explains], understand? **“Yes”**. And angle B starts from here and ends there [draws an arc from AB to BD in order to mark off angle B]. If you start here and end there, it is not B [draws an arc from BC to BD to clarify]. It is DBC, siyavana? **“Yes”** So, when you measure A [he meant B] as a whole, you measure, you measure from here to there [he indicates angle ABD with his hands], and when you measure, sorry B [his earlier mistake of mentioning A instead of B]. And for A, you measure from there to there [uses his fingers to trace the required angles on the parallelogram drawn on the chalkboard]. So, do it quickly, thank you.

Listen, listen, when you measure, I said put the zero there [he positions the protractor at angle ABD], this one occupies this 70, right? **“Um”**. If you take this end [indicating angle CDB], it is more than 70. So, which one do we use here? [The students pointed to the side of the protractor that is greater than 70]. The smaller side? **“The bigger one”** [replied the students]. The bigger one. [The students returned to their work, discussing in groups of two].

Now listen, listen again. If you have finished measuring the angles, I will like you to name this point as E [touches the intersection of the diagonals of the parallelogram as shown above]. This point, name this point as E. When you finish naming this point as E, I want you to measure EB and EC, neh? Then AE and ED siyavana? [using his fingers to show each of these dimensions on the parallelogram]. Siyavana? **“Yes”**. Yeah, I want us to start. [The students began to work while Mr Shlaja goes round to inspect their work, correcting their errors].

Right, now stop. Please stop. You were to name the angles, measure the angles neh? These angles are angle A, angle B, angle C and angle D, neh? [He writes these letters, A, B, C, and D on the chalkboard]. I don't want noise. Angle A equals ...? **“120”**. Hundred and twenty neh? **“Yes”**. Angle B ... **“60”**. Sixty degrees? **“Yes”**. And angle C ...? **“60”** and angle D ...? **“120”**. One twenty degrees, right? **“Yes”** [Mr Shlaja then writes these on the chalkboard as follows].

$$\hat{A} = 120^\circ$$

30 minutes

$$\hat{B} = 60^\circ$$

$$\hat{C} = 60^\circ$$

$$\hat{D} = 120^\circ$$

Now, go to, use the first property of, you know the first? What can you say about the opposite angles of a parallelogram? **“They are equal”** [*said the students*]. They are equal neh? **“Yes”**. Good, that is one of the properties of a parallelogram. Opposite angles are equal. Now, opposite angles neh? **“Yes”**. Opposite angles equal [*he writes this against the listed angles values on the chalkboard as ‘opposite  $\angle$ s = ’*].

Let’s move to the sides. You’ve measured the sides neh? **“Ay”**. What are the sides? Side one ... **“AB”**. AB neh? **“Yes”**. AB and ... **“CD”**. And ... **“AC”**. AC and ... **“BD”**. BD. Measurements: **“8.8”**, **“8.9”**. Eight point eight? **“8.9”**, **“8.9”** [*the students read out the values they had for side AB of the parallelogram*]. Ok, 8.9, that is for line AB, neh? **“Yes”**. CD ... **“9”** [*many voices, some mentioning 6*]. And AC ... **“6”**, **“6”** [*many voices*], and BD ... **“6”**. Six again? **“Yes”**. Now what can you say about the sides, the opposite sides? Remember this is opposite side. **“Equal”** [*replied the students*]. Equal neh? **“Yes”**. Opposite sides equal [*he writes this on the chalkboard*].

Now, we have measured CB neh? **“Um”**. We have measured CB. **“Um”**, and we have measured AD and we have measured CB, neh? [*Mr Shlaja was referring to the diagonals AD and BC of the parallelogram*]. **“Um”**. I don’t want CB, I want EB and EC. **“Six comma five centimetre”** [*meaning 6.5*]. Six comma five, EB and EC? **“EB is 6.5”**, **“6.4”** [*the students stating their various values for the length of EB*]. EB 6.5 and EC ...? **“Six comma five”**. Six point five? **“Yes”**. Em, AE ...? **“4 cm”**. And ED equal to ...? **“4 cm”**. [*Mr Shlaja clearly writes all these values on the chalkboard as the students offer them*].

Diagonals, the diagonals, what can you say about the diagonals? Diagonals AD and BC, what can you say about the diagonals? **“All angles are on a straight line”** [*offers one of the students*]. The value of EC is ... **“6.5”** and EB is ... **“6.5”**. Also, the value of AE is ... **“4”** and ED ... **“4”**. **“Bisect”** [*says one of the students*]. Opposite sides of a parallelogram bisect each other neh? **“Yes”** [*chorused the students*]. Siyavana?

40 minutes

“Um”. Now, this is another property of a parallelogram: the diagonals bisect each other.

We’ve measured that this one [*indicating EC*] is 6.5 neh? And this one [*indicating EB*] is 6.5 neh? “Yes”. And this [*AE*] is 4 and this one [*ED*] is 4. This one [*touches line BC*] cuts this one [*touches line AD*] into two equal parts and this one [*AD*] cuts this one [*BC*] into two equal parts, right? “Um”.

These are not the only properties of parallelograms, ok? “Yes”. The diagonals cut each pair of opposite angles into two, two angles [*shows this on the chalkboard*]. What you measured today was the whole angle [*places his fingers at angle CDB*]. But, there are two angles at D. [*Mr Shlaja uses his fingers to trace angles ADB and ADC*]. Now, as your homework, I want you to measure the size of the two angles at each angle of the parallelogram, siyavana? “Ay”. I want them written down and you will show them to me tomorrow, ok? “Yes”. [*The ‘change of lesson’ bell has rung and the lesson is now over*].

## APPENDIX 8.F

### Mr Andile's lesson (SAS grade 12 teacher)

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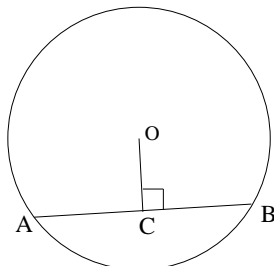
Afternoon class [Mr Andile, the grade 12 teacher greets his students who were already settling down for the day's lesson]. **“Good afternoon Sir”** [replied the students in unison].

Em ... Today I want to ... [interrupted as a student walked into the class], I have prepared ... [pauses and directs the student to a sit]. I have prepared a lesson. It is a revision lesson on circle geometry [he announces the field of study]. Em, I'm saying this is a revision lesson because I know that you've done, you've gone through this lesson last year. But because this lesson is still part of your, your syllabus for grade 12 this year that is why we are going to go through this lesson today, ok?

Em, there is a theorem in circle geometry – a theorem which states that if you are given a circle and there is a line that is, em, drawn from the centre of the circle to the midpoint of a chord, then that line is perpendicular to the chord [he states this theorem offhand, circulating around the room while the students just listen to him].

Now, today we will focus only for those learners who have been registered in maths, ok? Ok. This theorem is a converse of a theorem that states that if you're given a circle and there is a line drawn from the centre of that circle to the midpoint of a chord, then that line is perpendicular to the chord [he writes statements of the converse theorem being referred to on the chalkboard and illustrates it with a diagram as shown hereunder].

1). A line from the centre of a circle perpendicular to a chord bisects the chord.



2).

5 minutes

8 minutes

This one is a converse, and this converse, em, says a line from the centre of the circle perpendicular to the chord, em, bisects the chord [*Mr Andile reads from the chalkboard*].



Ok. When you are to prove a theorem, there are certain steps that you have to follow, ok? Step number 1, you are to understand, the line from the centre of a circle perpendicular to a chord bisects the chord, ok? It means we are through with step number 1 [*he itemizes the steps on the chalkboard*].

The second step, you are to draw the diagram, ok? I've already drawn the diagram for you. You are to draw the diagram, ok? [*Referring to the diagram above*].

Then the third step, you are to write the given information. What are the given information? [*No response from the students*]. Look at the diagram and write down all the given information, ok? What are we given here? [*Mr Andile points to the diagram*]. Um? [*Seeking clarification from a student who mumbled something inaudibly*]. Just look at the diagram and tell me, what are we given? [*A couple of students offered answers not clearly captured by the camera, but also not satisfactory to Mr Andile*]. We are given a circle with a centre O, ok? This is the given information [*Mr Andile offers to answer his question by himself and writes it on the chalkboard*].

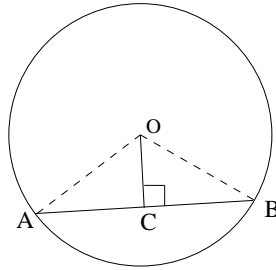
13 minutes

3). Given: Circle with centre O,  $OC \perp AB$ .

What else? Ok. Ok. [*Responding to suggestions from the students*]. Ok. We are given two lines, we are given a line drawn from the centre which is perpendicular to AB. So, we are given the line OC which is perpendicular to AB, ok? [*He then includes this information in the step 3 above*]. But in the following given information, you must underline the keywords, ok? [*He now underlines "perpendicular" and "bisects" in the statement of the theorem written on the chalkboard*]. Now, it is given that OC is perpendicular to AB. We have to prove that OC bisects line AB. To bisect, it means that OC divides line AB into two equal parts, ok? You are required to prove that OC, no, not OC. AC is equal to BC [*he writes this on the chalkboard symbolically as follows*].

4). R.T.P:  $AC = BC$ . That is the required to prove, ok? Then, construction necessary, you write the construction. You construct a ... by joining OA, and when you construct, you must always use a dotted line, ok? **“Yes”** [*chorused the students*].

5). Constr. [*He adds dashed lines to the above diagram which now looks like this*]



OC in triangle BOC, BOC, ok. For the proof of this theorem, you have to understand certain concepts that you have learned in grade 11 or 10.

The first one is the concept of congruence, congruence, ok. But you have to use that concept and prove the congruency between these two triangles [*referring to  $\triangle AOC$  and  $\triangle BOC$  in the diagram*]. And congruency, by definition, means if two triangles are congruent to one another it means that they are equal in all respect. They are equal in area, like their corresponding sides are equal, ok? **“Yes”**. [*Few voices from the students*]. Now, to join OA and BO, then the proof, ok? In the proof, I am saying we are to compare  $\triangle AOC$  and  $\triangle BOC$  [*he writes these on the chalkboard as follows*]:

6). Proof: In  $\triangle AOC$  and  $\triangle BOC$ ,

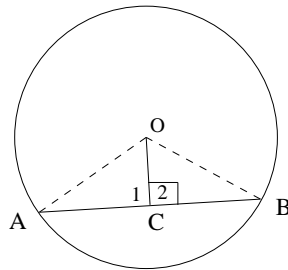
You are to compare the sides, the angles. Remember that there are four cases of congruence. Do you remember them? **“Ay”** [*the students responded indicating “yes”*]. There are four cases of congruencies: side, side, side (SSS), ok? The second one, angle, angle, side (AAS), the third side, angle, side (SAS), and the fourth one is right angle, hypotenuse and side (RHS), [*the students repeating after him in each case*]. Ok. I am glad that you still remember those cases of congruence.

Now, let's look at these two triangles. Ok, which side in this triangle AOC which is equal to another side in triangle BOC? And therefore give a reason why. [*No response from the students*].

AO = BO isn't it? Is it by construction? Eh? Is it by construction? [No response from the students still]. The radii of the same circle are equal. AO is equal to BO, radii, ok? This means em, em the, the radius anywhere is equal to any other radius of the same circle. Now, which other side of this triangle [i.e.  $\triangle AOC$ ] is equal to the side, to this one [touching  $\triangle BOC$  drawn on the chalkboard]. Just compare sides or angles if you want. EAC is equal to ...? What is the reason for that? Is it given? [Some of the students nodding, others saying "yes"]. Eh, is it given? If you say it is given, then go back to the given information again, if it is not there that is it is not given, ok? Look at OC. You see that OC belongs to both triangles. That means OC is common, ok? "Yes" [chorused the students]. OC is equal to OC which is a common side  
 $OC = OC$  (common side), ok?

What else? What else? Then which side that can still compare with this triangle? [I.e.  $\triangle BOC$ ]. " **$C_1$  and  $C_2$  are equal, because they are  $90^\circ$** " [a female student answered]. Ok, that is this angle, angle ACO, these two angles each is equal to  $90^\circ$  [pointing and touching the angles at point C in the diagram]. These two angles, ok, if we want to make things easy for you, we call them  $C_1$  and  $C_2$  [Mr Andile labels them as shown below].

20 minutes



Ok,  $C_1$ , angle  $C_1$  is equal to angle  $C_2$  [he writes it as follows]:

$$\hat{C}_1 = \hat{C}_2.$$

What is the reason? What is the reason for that? Each equal  $90^\circ$ , we have got three, we got two sides and an angle equal to  $90^\circ$ . Therefore we can conclude that  $\triangle AOC$  is congruent to  $\triangle BOC$  [he writes this in symbol on the chalkboard].

$$\therefore \triangle AOC \equiv \triangle BOC$$

What is the reason for that congruence? “**side, side, side**” [*chorused the student*]. Side, “**angle, side**” [*completed the student*]. Side, angle, side? “**Yes**” [*Replied the students*]. Side, angle, side; side, angle, side; side, angle, side. And the angle is included angle. In this case the angle must be an included angle. Right angle, this is hypotenuse and side. Right angle, hypotenuse, hypotenuse means right angle and side [*Mr Andile now writes the condition for the congruency on the chalkboard*].

$$\triangle AOC \equiv \triangle BOC \quad (\text{R.H.S}).$$

Ok, what is the implication of that congruence? What conclusion can you draw from that congruence? Ok? Because I have told you already that if triangles are congruent, it means they are equal in all respect. Their corresponding sides will be equal. Then from this congruence, we can conclude that because triangle AOC is congruent to triangle BOC we can say that AC is equal to BC [*which he writes as*]:

$$AC = BC \quad (\text{corr. sides of congruent triangles}).$$

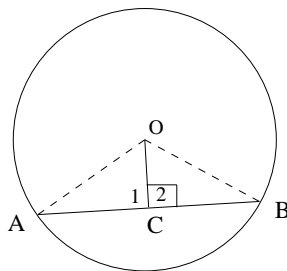
Reason, corresponding sides of congruent triangles. Or you can say congruent, ok. Corresponding sides of congruent triangles, ok?

*By this time, Mr Andile has written the complete proof of the theorem on the chalkboard as follows.*

27 minutes

1). A line from the centre of the circle perpendicular to a chord bisects the chord.

2).



3). Given: Circle with centre O,  $OC \perp AB$ .

4). R.T.P:  $AC = BC$ .

5). Constr: Join OA and BO.

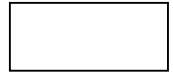
6). Proof: In  $\triangle AOC$  and  $\triangle BOC$ ,  $AO = BO$  (radii)

$$OC = OC \quad (\text{common side})$$

$$\hat{C}_1 = \hat{C}_2 \quad (\text{each} = 90^\circ)$$

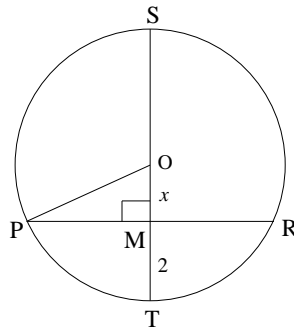
$$\therefore \triangle AOC \equiv \triangle BOC \quad (\text{R.H.S})$$

$$\therefore AC = BC \quad (\text{corr. sides of congruent } \triangle s)$$



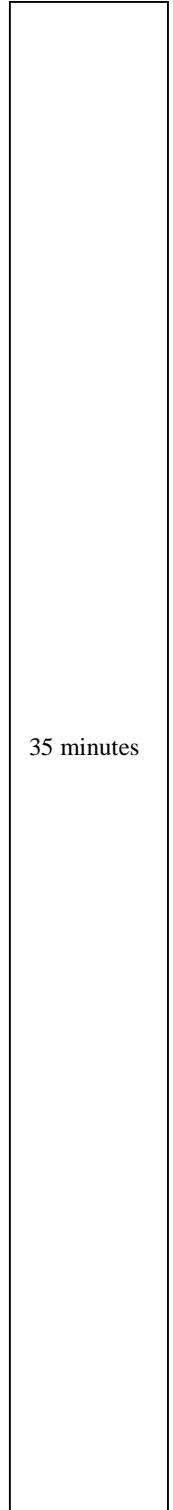
Hence, if you then take the syllabus, after proving the theorems, you have to apply the theorems in solving the geometric riders, ok? You will be given geometric riders where you are to apply all the proven theorems, ok, to solve, em the geometric riders. I've got a nice geometric rider here for you. Take one [*Mr Andile circulates around the room, distributing worksheets containing geometric riders to the students*]. Take one.

All are the Matrics of 2003, ok? In the diagram alongside, PR is a chord of the circle with centre O, ok? [*He reads from the worksheet*], as you can see, and diameter ST is perpendicular to PR at M. Diameter ST is perpendicular to PR at M and then we are given the, the size of the sides of PR = 8 cm, MT = 2 cm, OM =  $x$  cm [*the diagram referred to is as shown below*].

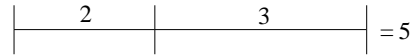


35 minutes

Now, we are to write OP in terms of  $x$  and a number. Write OP, look at OP here, OP is the hypotenuse of this triangle [*referring to  $\triangle POM$  in the diagram*] on OPM and it is also a radius of the circle. It is the hypotenuse and it is also a radius of the circle. And this is a right-angled triangle. So we now prove, we are required to write OP in terms of  $x$  and a number, eh. And it is given that, em, OM, OM this distance is  $x$ , and MT is 2. This one is  $x$ , this one is 2 [*touching those segments in the diagram*] and we know that the radii of the same circle are ... **“equal”** [*completed the students*], then we have to express OP in terms of  $x$  and a number. What is the size of OP if we are to express OP in terms of  $x$  and a number, eh? OP, I want you to look at this diagram, OP and OT, OP and OT [*touches lines OP and OT in the diagram reproduced on the chalkboard as shown above*]. Putting an emphasis on these two lines, OP and OT, ok,



you know part of OT is  $x$  and the other part of OT is 2. OP, the total size of OP will be ... “ $2x$ ” [answered the students]. Eh? “ $2x$ ” [insisted the students]. The total size of OT, the total size of OT, one part is  $x$ , and the other part is 2. What will be the total part of OT? “ $2x$ ” [the students maintained]. Eh? [Draws a line with two segments as shown below in order to prompt the students for the correct answer].



The total is 5, ok? This part is 3, let's make it 2 [referring to the figure above]. What will be this part? “ $3$ ” [answered the students]. It will be 3, for the total is 5. Now I've given you a portion of OT which is equal to  $x$  [places his fingers on that part of OT, i.e. OM] and another portion of OT which is equal to 2. What will be the total size of OT? [The students mumbled a wrong answer]. How did you get 5 here? [Points to the line segment above] How did you get 5? [Noisy indiscernible response from the students]. Eh? Add. “**Yes**” [agreed the students]. You do the same thing. What will be the size of OT? “ $x + 2$ ” [just a couple of the students answered].  $x$  plus 2 [re-echoed Mr Andile], good. It is  $x$  plus 2. Now, that statement, OP is equal to OT, these are the radii of the same circle, eh? Radii of the same circle, and if OT is equal to  $x$  plus 2, it goes without saying that OP must also be equal to  $x$  plus ... “ $2$ ” [completed the students], one mark only [referring to the memo for that exam question]. Express OP in terms of  $x$  and a number. Express OP in terms of  $x$  and a number.

The second question, what is the length of PM? [Referring to the circular diagram above]. PM, PM we are given line PR, ok? Or you take this line out of the circle, ok [draws a straight line PR].



This is PR, ok and from the theorem, from this theorem, this is a direct application of this theorem [referring to the theorem just proved]. We got the line drawn from the centre of the circle perpendicular to the chord. This line must bisect the chord, ok. According to this theorem, a line drawn from the centre of a circle perpendicular to the chord must bisect the chord. And if that is the case, and we know that PR, PR is given is equal to 8 cm, ok, and OM bisects PR, bisects means you divide the line into two equal halves. Then what is the size of PM? “ $4$ ” [answered the students]. PM is 4, ok? “**Yes**” [replied the students].

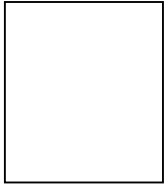
Then, the other question, calculate the length of the radius of the circle. What is the length of the radius? What is the length of the radius? [No response from the students]. Now, we are to calculate this line, the radius, which is the hypotenuse, ok? “Ay” [answered the students]. In  $\triangle POM$ , ok, I want you to look at this triangle [uses his fingers to trace  $\triangle POM$  in the diagram]. In  $\triangle POM$ , ok, PO, PO squared is equal to PM squared plus OM squared, according to theorem, Pythagoras [Mr Andile writes this algebraically on the chalkboard as follows]:

$PO^2 = PM^2 + OM^2$  (Pyth.), eh? In right-angled triangle, the square on the hypotenuse is equal to the sum, the sum of the squares on the other two sides of the triangle, ok? Then from this statement just stated, with side OP which is the same as PO is equal to  $x + 2$ , eh. PM, PM we say PM is equal to ... “4” [completed the students] you square that and OM is equal to  $x$ , you square that, ok? Now you use a little bit of algebra. You know already method of squaring a binomial, eh? [Here, Mr Andile meant expanding a binomial,  $(x - 2)^2$ ]. Squaring the first term, “ $x^2$ ”, you get  $x$  squared. The first term multiplied by the second term times 2, “plus  $4x$ ” [chorused the students]. You square the second term, “plus 4” [added the students]. Plus 4 [re-echoed the teacher] and which is equal to ... “16” [says the students]. 16 plus  $x^2$  and if you collect the like terms, or transpose the terms, your  $4x$  will be equal to, or if you transpose this, your  $x^2$  will cancel this one ok? “Um” [the students answered]. And then if we transpose the one from the left to the right, we have 16 minus 4 which is equal to 12. We then solve for  $x$ . Therefore, if you divide both sides by 4, your  $x$  will be ... “3” [answered the students].

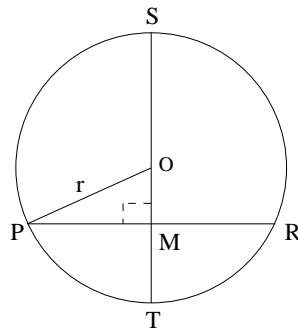
[Mr Andile solves the problem by himself, the students merely agreeing with his statements. By this time he has written the complete solution of the problem that required the students to calculate OP, the radius of the circle, in the circular diagram above, as follows]:

$OP = OT$ (radii) $OP = x - 2$ $PM = 4$ cm In $\triangle POM$ , $PO^2 = PM^2 + OM^2$ (Pyth.) $x^2 + 4x + 4 = 16 + x^2$	$\therefore 4x = 12$ $x = 3$ $\therefore \text{Radius} = 3 + 2$ $= 5$
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Ok. What was the question? The question that says, calculate the length of the radius of the circle. Calculate the length if the radius of the circle is OP, ok. Therefore, the radius which is OP is equal to  $x$  which is 3 plus 2 which is equal to 5. So, the radius of the circle is equal to ... “5” [chorused the students] centimetres.



That is, that was a direct application of the theorem that we’ve just proved, ok? You will get a similar one to this one. This is the one I want you to do, ok? [The teacher reads from the second worksheet for the student]. Correct, correctly complete the wording of the following theorem: A line, as far as you understand the wording of the theorem, it is very important for you to understand the wording. A line through the centre of a circle, perpendicular to a chord ..., ok. Complete that statement “bisects ... .” Bisects the chord [Mr Andile repeats after the students] for 1 mark. Bisects the chord. Then in the diagram below, PR is a chord of the circle with centre O [he was referring to this diagram shown hereunder].



38 minutes

Diameter ST is perpendicular to PR at M. PR equal to 8 cm and MT equal to 2 cm. What is the difference between this rider [the solved one] and this one [the current one diagrammed above]? Let’s compare the two riders. They are very, very similar, ok. What is the difference between the two? Can you figure out the difference? Eh, can you see the difference? [No response from the students, some in pairs discussing the problem, others were just looking].

You will do this one as class work. If you don’t finish it today, you will finish it as homework, ok? The only difference between these two riders is that in this one [the solved one], it is given that OM is equal to  $x$ . In this one [the current one], it is not given. You are to give this distance [referring to OM in the diagram above] a

variable. Give this distance a variable, call it  $x$ , call it  $m$ , call it  $y$ , ok, in order to solve for the variable. That is the only difference, ok. Do it now. Do it now.



*[The students began to solve the problem while Mr Andile circulates around the room, answering questions from and giving guidance to the students].*

$r$  is the radius *[answering a student]*.  $r$  stands for the radius. Use the bottom part of that paper to solve that rider, ok? I want to see the solution.

40 minutes

Another thing we have to understand is that there is a difference when you solve a rider and when you prove a theorem, ok? When you prove a theorem, you are to go through all these steps, ok... Sometimes it is unnecessary to redraw in rider, unless you want to make a construction.

Give OM a variable, ok. Let that part be equal to any variable, but you are to avoid using the variables that are already in the sketch, ok? Yes, you can call it  $y$  *[responding to a student]*. And then you follow the same approach, you will get the solution. 8 cm ok, and OT is perpendicular to PR, which means OT bisects PR. I want to see clarification. OT bisects PR, OM, bisects ... *[inaudible utterances as he attends to a group of students]*.

45 minutes

You will finish that as homework, ok? Thank you. *[The school bell has rung, indicating the end of the lesson period, and the students filed out of the classroom].*