Re- imagining the use of the abacus as a visualization tool to develop number sense in Grade 3 learners

By

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ABSTRACT

The poor performance of learners in mathematics has long been a matter of concern in Namibia. After more than three decades after independence, the country's education system is still seeking ways of addressing the high rate of poor performance in mathematics. Research previously conducted pointed out the lack of number sense in learners, as one of the contributing factors to the poor performance in mathematics. This research study is a contribution towards using the abacus as a visualization tool to develop number sense in the Foundation Phase. The study was premised on the thought – supported by literature – that effective use of concrete equipment and manipulatives, of which the abacus is part of, does affect the teaching and learning of number sense. The study further examines the use of the abacus as a visualization tool to develop number sense by the four selected Grade 3 teachers. The study argues that the effective use of the abacus develops number sense in learners.

This study was framed as a multiple case study that was grounded within an interpretive paradigm and informed by the constructivist learning theory. The qualitative data of this study were collected using questionnaires, observation, reflective journals, and semi-structured interviews. The data were then analyzed using thematic analysis and an analytical tool developed from relevant literature. A survey was conducted using 50 Foundation Phase teachers in the //Kharas region, while the intervention programme consisted of four purposively selected teachers from the four primary schools in the Kalahari circuit in the Keetmanshoop district.

The findings of this study revealed that the majority of teachers were not aware of abacus use in teaching mathematics before the intervention programme. It was also revealed that the few teachers that use the abacus as a visualization tool to develop number sense, employ it as a counting tool for explaining certain concepts and as well as for teaching simple arithmetic. In this research study, the selected teachers use the abacus to link the abstract mathematics content to a concrete way of doing mathematics. In the absence of the abacus in classrooms, various manipulatives are used to develop learners' number sense. Mathematical games, verbalizing mathematics concepts, and drawing pictures to visualize abstract concepts among others are used by the selected teachers to enhance the development of number sense in the Foundation Phase.

The use of the abacus by the selected teachers effectively fostered the visualization process and the conceptual understanding of number sense in learners. Through the abacus, teachers led their learners into visualizing number sense concepts such as subitizing, computing, performing mental mathematics and physical representation of numbers in different ways. The abacus was used by teachers to enhance listening skills, improve concentration and strengthen the memory of learners.

On the other hand, the study also revealed that despite the various benefits of the abacus, it is time consuming. The lack of abacus use in previous grades has a huge impact on the use of it and the development of learners' number sense in the grades being studied in this research study.

Teachers are recommended to make their own abacuses, encourage learners to make their own abacuses from the readily available materials, and to allow the learners to realize that the mathematics they are doing in classrooms is around them. The use of re-imagining, re-envisioning, re-conceptualizing and re-examining of so-called 'old teaching tools' such as the abacus, needs to be encouraged through in-service and pre-service teacher training programmes.

The study concludes that the use of traditional algorithm methods do not promote conceptual understanding and visual strategies for Foundation Phase learners and should be discouraged. It is hoped that this study will contribute towards improving the practices of mathematics teachers primarily in the Keetmanshoop district, //Kharas Region and in the rest our Namibian schools.

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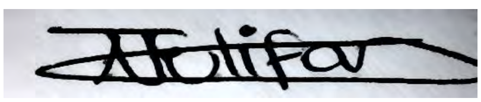
DEDICATION

I would like to dedicate this thesis to my late parents: My mother **Ndeshitila Sarastine Kamushietango** and my father **Angula Elifas Iyambo Tashiya** for educating me in their absence. Dad, Mom you made sure that even in your absence I was able to go to the best schools, you prepared my future in your absence.

I hope when you look down on earth you are proud of what I have achieved.

DECLARATION OF ORIGINALITY

I, Elifas Taimi Ndinelago student number 19E9935 declare that this thesis entitled “Re- imagining the use of the abacus as a visualization tool to develop number sense in Grade 3 learners is my work, and a product of my research. The contents of this thesis represent my unaided work, and that the thesis has not previously been submitted for academic examination towards any qualification. Where I have drawn on ideas of people from other publications or other sources, I have fully acknowledged these in accordance with Rhodes University, Education Department reference guide. Furthermore, it represents my own opinions and not necessarily those of the Rhodes University.



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Elifas Taimi Ndinelago (Signature) Date

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LIST OF ACRONYMS

COVID	Corona Virus Disease
DV	Dave’s videos (1, 2, 3, 4, 5)
ICT	Information and Communication Technologies
IDS	Inclination to review Data and results for Sensibility
IJARM	International Journal of Academic Research in Management
IUE	Inclination to Utilize an Efficient Representation
LV	Lebby’s videos (1, 2, 3, 4, 5)
MoEAC	Ministry of Education Arts and Culture
MRN	Multiple representations of numbers
NCTM	National Council of Teachers of Mathematics
QV	Quasi’s videos (1,2,3,4,5)
SAN	Sense of Absolute Magnitudes of Numbers
SON	Sense of Orderliness of Numbers
SSI	Semi-structured Interviews
TQ	Teacher Questionnaire
TV	Tyler’s videos (1, 2, 3, 4, 5)
UEO	Understanding the Effect of Operation
UK	United Kingdom
UMP	Understanding Mathematical Properties
URO	Understanding the Relationships between Operations

CHAPTER 1

CONTEXT OF THE STUDY

1.1 INTRODUCTION

Mathematics education for young children is crucial and needs to be prioritized. Mathematics has been and will remain a key component of early childhood education around the world. The Egyptians expressed numbers using separate symbols for 1, 10, 100, 1000, etc. Since ancient times, people from different civilizations have used physical objects to help them solve mathematical problems. In the early 1900s, Italian educator Maria Montessori continued with the idea that manipulatives are important to education. She designed several materials to help elementary students learn the basic ideas of mathematics. Since the 1900s, manipulatives have come to be considered essential in teaching mathematics at the elementary school level. The National Council of Teachers of Mathematics (NCTM) has recommended the use of manipulatives in teaching mathematical concepts at all grade levels. The ancient civilization of Southwest Asia used counting boards as teaching manipulatives and the Romans created the first abacus based on the counting board. For many centuries the abacus was used as a calculating tool for performing arithmetic processes in many parts of the world until the modern numerical system was adopted by many countries worldwide.

The original modern inhabitants of Namibia were the San and the Khoe people known as the Bushmen. With no formal schooling the Bushmen identified their herds of cattle and sheep through facial and pattern recognition. They developed their own ways of knowing that none of their animals was missing when they took stock of them. They further developed a geometrical thinking for tracking their animals by getting to know the shape of their dung to learn which animal had passed by and how long ago. The young Bushmen were taught by the elders, the mathematical understanding and construction skills to build houses. The Bushmen used a structure similar to Stonehenge as a calendar to count days by following the shadow of the setting sun. Stones were placed to track the movement of the sun which cast shadows on the rock. In all these ways, one can argue that the element of number sense was embedded even though these terms were not used. The Bushmen were also were very visual beings in their approaches and in the way they conducted their day-to-day activities.

This research study is part of the ViproMath project at Rhodes University which seeks to research the effective use of visualization processes in mathematics. This study aimed to investigate and explore how the Grade 3 teachers in the four primary schools in the Keetmanshoop district use the abacus as a visualization tool when teaching mathematics at Foundation Phase level to develop number sense. The study will point out how Grade 3 teachers teach, using the abacus, their perceptions, and experiences on the use of the abacus.

This chapter provides the introduction of this study on why the research was taken. The chapter further outlines the research goal and the research questions that guided this study. Further, the chapter gives a brief summary of the methodology employed, the theoretical framework, and the significance of the study. The chapter ends with the outline structure of the whole thesis.

1.2 BACKGROUND TO THE STUDY

The Namibian context: The Mathematics Curriculum

The National Curriculum for Basic Education builds on the experience and achievements of the first curricula and syllabi that were introduced in the 1990s. The Namibian curriculum recently went under review, intending to give direction to Basic Education toward the realization of Namibia Vision 2030 as the curriculum for the future (Namibia. Ministry of Education Arts and Culture [MoEAC], 2016). The ministry emphasized that:

> The curriculum needed to be revised so that it provides a coherent and concise framework to ensure consistency in the delivery of the curriculum in schools and classrooms throughout the country. It describes the goal, aims and rationale of the curriculum, the principles of teaching, learning and assessment, the language policy, and curriculum management at school level (MoEAC, 2016, p. 1).

In the newly revised curriculum, mathematics became a compulsory subject for every learner from Grade 1 up to Grade 12, regardless of whether they are competent in mathematics or not.

> Mathematics is an indispensable tool for the development of science, technology, and commerce. Mathematical skills, knowledge, concepts, and processes enable the learner to investigate, model, and interpret numerical and spatial relationships and patterns that exist in the world (MoEAC, 2016, p. 15).

In support of this, the Ministry of Education argued that the whole nation needs to be literate in the area of mathematics if national development is to be realized (MoEAC, 2016).

The National Curriculum for Basic Education calls teachers to make use of different teaching materials including Information and Communication Technologies (ICTs) so that they can enhance learning and make teaching fun (Namibia. Ministry of Education Arts and Culture [MoEAC], 2016). The Ministry of Education, through the Mathematics Subject Policy Guide of 2016, emphasized that although a calculator is an ICT component, it is recommended that learners at the primary level (Grades 0-7) should not be allowed to use calculators. This is due to the notion that the Primary School grades are at the foundation where learners need to learn all the basic arithmetical concepts in mathematics such as order and effects of operations , relationships and magnitudes of numbers, and that this lays foundation for all further learning (MoEAC, 2016).

Despite the directive in the revised curriculum for teachers to make use of learning materials when teaching, teachers at the Foundation Phase find it difficult to come up with materials that help learners to develop number sense (Miranda & Adler, 2016). As an upper primary mathematics teacher (Grade 4 – 7) for more than five years, I have observed and experienced how learners struggle with basic arithmetic. I have also noted that learners that come to Grade 4 struggle with basic mathematics concepts such as order of operations, word problems, and relationships between numbers and patterns. Neergard (2013) believes that learners who lack strong number sense have trouble developing the foundation needed for even simple arithmetic.

At the moment, in Namibia there is no recorded and accessible study in the public domain that was done on the use of the abacus to develop number sense yet, but there are related studies done on how to develop number sense in mathematics, see for example Courtney-Clarke (2012), Naukushu (2016) and Peters (2016). In my experience as a mathematics teacher, the abacus is not being used to its full potential in Junior Primary classrooms.

According to the Ministry of Education Arts and Culture (2015), numeracy is the core function of primary education and is thus very important in the Foundation Phase. The newly revised curriculum aims to develop numeracy and mathematical thinking and positive attitudes toward mathematics, and acquire basic number concepts and numerical notation. It also stipulates that learners should be able to understand and master mathematical concepts and apply mathematics in everyday life.

Table 1.1: Intended learning objectives and basic competencies for Grade 3 number sense (Namibia. Ministry of Education Arts and Culture [MEAC], 2015)

Learning objectives	Competencies
• count concrete objects up to 20 and backward from 20 - 0 • count mechanically up to 100, in 2s up to 30, in 5s and 10s up to 100	• Count concrete objects up to 100 and in 2s to 100, 3s to 60, 4s to 80, 5s, and 10s to 150 • Re-arrange a collection of objects to make them easier to count, e.g. groups of 2, 5, 10
Learners will subitize, estimate and apply re-arranging skills to handle spatial relationship and patterns	• Estimate the number of objects up to 30 by appropriate grouping, then check by counting • Recognize that the same number in different arrangements of objects can represent the same number
Learners will recognize digit position and place value	• Recognize, in three-digit numbers up to 1000, that the digit positions in numbers show the quantity. • Identify the place value of hundreds, tens, and units in three-digit numbers
Learners will recognize, read and write numbers	Read, pronounce and identify numbers up to 1000 on a number chart and number line and explain the reason for the position of the number
Learners will understand basic mathematical concepts to master subtraction, addition, multiplication, division and numerical notation	• Subtract any two- or three-digit number from a three-digit number between 100 and 500, subtract a single-digit number from a three-digit number mentally. • Add two numbers up to 100 mentally and add two or more different numbers with a sum between 0 to 500 • Multiply any two-digit number up to 50 by any number between 1 and 10 through repeated addition or doubling, • Recall and apply simple division facts related to multiplication facts
Learners will solve story problems about everyday contexts using addition, subtraction, grouping, or sharing and using any logical strategies	• Record appropriate ways to find solutions by using concrete objects, discussing/sharing ideas, finding patterns, acting out a play, and combining drawings, diagrams, and numbers. • Use different strategies to solve problems and write story problems using number sentences in the number range 1 - 100 by applying the four operations.

When examining this extract from the syllabus one notices that most of the learning objectives in Grade 3 focus on numeracy and developing number sense. The basic competencies cover number sense themes such as operations, the relationship between operations, knowledge, and facility of numbers, and computational strategies. These are the number sense themes examined in my study.

This study employed an intervention programme that enabled teachers to practise on how an abacus can be utilized as a visualization tool to teach number sense in their classes. The intervention at the heart of this study was aimed at training teachers to teach, using the abacus and develop their learner's number sense in the process. The study used the analytical tool adopted from McIntosh, Reys, and Reys' (1992a) Number Sense Framework. The teachers taught their lessons using the abacus and incorporating the number sense themes in their teaching. Through the teaching, the learners were able to visualize the content being presented. The use of the abacus integrated the constructivism and the visualization process and it enhanced the conceptual understanding of number sense in learners.

1.3 RESEARCH GOAL AND QUESTIONS

1.3.1 Research Goal

This study aims to investigate how selected teachers utilize the abacus as a visualization tool to develop number sense in Grade 3 classes as a result of participating in the intervention.

1.3.2 Research Questions

1. How did teachers use the abacus when teaching Grade 3 mathematics classes prior to participating in the intervention programme?
2. How can the abacus be utilized as a visualization tool to develop number sense in a Grade 3 class when teaching mathematics?
3. What are the teachers' experiences and perceptions of using an abacus as a visualization tool in developing learners' number sense after participating in the intervention programme?

1.4 RESEARCH METHODOLOGY

This research project is oriented in the interpretive paradigm and employed qualitative data collection and interpretation. Cohen, Manion and Morrison (2018) explained that interpretive approaches in research have the intention of understanding the world of human experience that is socially constructed (Willis, 2007). This study used two sets of participants. Firstly, 50 Foundation

Phase teachers in the //Kharas region took part in the survey questionnaire on the current use of the abacus in classrooms. Secondly, four participants were selected to take part in the intervention programme. For ethical consideration in this study, the four participants are identified by their pseudonyms as Mr Lebby, Ms Dave, Mr Quasi, and Ms Tyler. The study used a multiple case study method. The cases involved the four Grade 3 mathematics teachers that were purposively selected.

The study unfolded in five phases which were: pilot and administration; securing of a research site and selection of participants; abacus workshop, implementation of the intervention programme, and interviewing teachers. During the intervention, the four teachers taught the five lessons which were planned at the intervention workshop, and data were collected through lesson observation and interviews. Teachers were also asked to keep reflection journals, where they kept records of their journey from the beginning to the end of the intervention programme.

Constructivism forms the theoretical underpinning that supported this study. The constructivist theory posits that knowledge is constructed by the knower, based on mental activity (Connors, 2007). Constructivists emphasize the importance of the active involvement of learners in constructing their knowledge and building new ideas or concepts based on current knowledge and past experience (Vey, 2005).

1.5. RATIONALE OF THE STUDY

It is hoped that his study will not only add to the literature but will also help curriculum developers and advisory services for the Foundation Phase to consider integrating the abacus into the curriculum to develop number sense and improve mathematics performance at the Foundation Phase level. The study will encourage the Namibian Foundation Phase teachers to use the abacus and other manipulatives as visuals in enhancing the development of number sense. Lastly, the findings of this study may help create awareness of the use of manipulatives and concrete materials in understanding and developing the learners' number sense.

1.6 STRUCTURE OF THE STUDY

1.6.1. Chapter 1

This thesis is organized into five chapters. Chapter 1 provides an overview of the thesis. In this chapter, the context of the study coupled with its background, are first discussed. The research goals and questions are then set out, followed by the research methodology. The significance of the study is discussed to highlight its benefits for using the abacus for teaching and learning of number sense understandings. The chapter ends with an overview of the whole research study.

1.6.2. Chapter 2: Literature review

This chapter discusses the main concepts used in this study in relation to the three main research questions that guided the study. Number sense, the abacus, the visualization process, and the constructivism theory are then discussed.

1.6.3 Chapter 3: Methodology

This chapter describes the method used to collect data in this study. Special reference to the research orientation and research method is made i.e. the interpretive paradigm, case study method, the qualitative approach, and how data was collected and generated. The research site, selection of the participants, and a short description of the data analysis are presented. The chapter closes by discussing the ethical considerations, validity and limitations of the study.

1.6.4 Chapter 4: Data analysis and presentation

The data analysis chapter presents the data collected from the four research instruments, namely the questionnaires, the observation, reflective journals, and the interviews. The data collected are analyzed in detail to generate the findings for the three research questions of this study. The data are further linked to the literature in Chapter 2.

1.6.5 Chapter 5: Conclusion and recommendations

This is the final chapter of this study. It presents a summary of the findings, the significance of the study, and its limitations. It outlines some suggestions for further research, personal reflections and the chapter ends with a conclusion.

1.7. CONCLUSION

This chapter has endeavoured to provide an overview of this research study. The chapter is aimed at understanding the background and context of the Namibian education system and gives an overview of the research goals and research questions that guided the study. The methodological approach used to collect data for this study and the importance of the study are briefly discussed. It ends with an outline of how the thesis as awhole look like.

In the following chapter, I look at a range of literature related to my research question in order to develop a better understanding of how the re-imagining of the abacus can be used a visualization tool to develop number sense in the Foundation Phase.

CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

Number sense is a foundational pillar of early childhood mathematics learning. Success of learners in mathematics in the later years of their school and academic life is strongly rooted in the way in which their number sense abilities were developed in their formative years. This study was undertaken against the backdrop of the lack of number sense understanding in the Foundation Phase in Namibian schools. The literature chapter presents the groundwork of this study.

While I argue for the use of manipulatives in mathematics classrooms for number sense development I am equally aware of the challenges associated with the use of the manipulatives. To carry out this study on how number sense can be developed in the Foundation Phase using the abacus I provide literature in this chapter on aspects that are key to my study. Firstly, I discuss what number sense is and the characteristics associated with it. Secondly, I discuss literature on the abacus as a teaching manipulative tool that can be used to develop number sense. Thirdly, visualization as a process that can be used in the development of number sense is discussed in detail. Lastly, I also discuss the literature on the theoretical framework that is underpinning this study – which is the constructivist theory of teaching. The chapter concludes with a summary.

2.2. NUMBER SENSE

Research done by Maghfirah and Mahmudi (2018) shows that number sense has become an important topic in mathematics education in the last few decades. Number sense, for the most part, refers to an individual general comprehension of numbers and flexibility in using the operations for making mathematical judgments. In general, number sense is not seen as an issue that is directly taught as a subheading within a chapter or a specific topic. Instead, it is assumed as the result of mathematical experience, where learners employ their senses in understanding circumstances involving numbers without exerting a standard algorithm (Maghfirah & Mahmudi, 2018a).

2.2.1. What is number sense?

Many authors have noted difficulties defining number sense or disparities in the definition and description found in literature. Andrew and Sayers (2015) referred to number sense as a poorly defined construct. Berch (2005) described number sense as a single construct which is defined and interpreted in different way.

Number sense is defined by the National Council of Teachers of Mathematics [NCTM] (1989) as an intuitive feel for numbers and a common-sense approach to using them. Number sense involves an understanding of how different types of numbers (such as fractions and decimals) are related to each other, and how they can best be used to describe a particular situation (NCTM, 1989). McIntosh et al. (1992b) also saw number sense as a person's general understanding of numbers and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgments and to develop useful strategies for handling numbers and operations. McIntosh et al. (1992, p. 3) stress that "the acquisition of number sense is a gradual, evolutionary process…often evident at an early age as children think about numbers and try to make sense of them".

The NCTM found that number sense is one of the foundational ways in which learners understand numbers and there are characteristics that a learner has to portray for one to say they have or lack number sense.

2.2.2. Characteristics of Number Sense

The NCTM described learners with numbers sense understanding as those who understand number meaning, develop multiple relationships among numbers, know the relative size of numbers, and comprehend how arithmetic operation affects results (Howden, 1989). For learners to have an understanding of number sense, mathematics teachers have an important role to play in the classroom environment they create, teaching strategies they use, and in activities they select.

Number sense can be developed in various ways in learners, however, the development of number sense depends on the environment where they are encouraged (Hope & Small, 1994). Learners develop their number sense when they work with concrete materials and familiar ideas. Howden (1989, p. 9) states that "number sense in learners develops gradually, and varies as a result of exploring numbers, visualizing them in a variety of contexts, and relating them in ways that are not limited by traditional algorithms". Number sense includes the children's skills related to counting, recognizing number patterns, comparing numbers, and estimating. As children work with numbers, they enhance these skills and deepen their understandings and thoughts about numbers. Furthermore, they represent and count numbers in different ways and develop perceptions about operations. Spontaneously, as they use operations and different solution strategies for operations, they continue to deepen their number sense (Berch, 2005).

Using real-life experiences, physical materials and technology to construct meaning for whole numbers leads to the development of number sense in the learners. Real-world situations should be incorporated into the learners' experiences so that they can become aware of the existence of other numbers, such as negative numbers. Using the models related to whole numbers helps learners represent the equivalent form of the same number. In the process of using concrete materials when learning mathematics, learners learn to generate equivalent forms of many different kinds of numbers. Learners learn to understand that 367= three hundreds, six tens and seven ones is equal to three hundreds, five tens and seventeen ones and in these ways their confidence feeling about place value is increased and they also get a clear understanding of multi-digit computation.

Interestingly, Burns (2007) observed that number sense develops gradually by using different teaching strategies in mathematics. Modeling different methods for calculating is one method of developing number sense in learners. A teacher demonstrating different approaches to solving a problem exposes the learners to think of different possibilities they have not yet considered. Allowing learners to think that there is only one way to solve a problem tends to put their focus on that method, instead of thinking about what makes sense of a number. Emphasizing mental calculation when doing computation has the benefit of encouraging learners to build on their knowledge about numbers and their numerical relationships. Mental computation will not bound learners to memorization procedures but will enable them to think more flexibly and efficiently – which will, in turn, lead them to think of alternative ways to solve problems

Furthermore, Burns (2007) added that estimation should be made part of computing in the process of developing number sense because learners use estimation every day of their lives, for example, estimating the distance between the school and their houses; how many hours they have to drive between different towns; the amount of money they have spent on groceries, etc. Thus estimation relies not on mental mathematics but on reasoning. Mathematics is about reasoning and most importantly, it should make sense.

McIntosh et al. (1992a) developed a number sense framework with different key components and number sense themes to describe the characteristics of number sense. These number sense components: knowledge of and facility with numbers; knowledge and facility with operations; and applying knowledge of and facility of mixed numbers and operations to computational strategies were later used to design a tool that I have used to analyse data in Chapter 4 of this study. The framework by McIntosh et al. (1992) is discussed below.

Knowledge of and facility with numbers

Various methods have been advocated to help children develop a conceptual understanding of numbers, including the use of manipulatives and/or models, the use of number lines, a study of various number bases, and units of instruction targeted at 'place-value'. When a learner shows a knowledge and facility of numbers she/he shows the following characteristics: a sense of orderliness of number; multiple representations for numbers; a sense of the relative and absolute magnitude of numbers; and a system of benchmarks

a) ***Sense of orderliness of numbers* –** An understanding of the number system helps the learner to mentally organize, compare, and order numbers encountered in a mathematical environment. For example, a learner that is learning to count beyond twenty comes to appreciate the patterns identified both orally and in written form inherent in the number system. The system of place value (including its application to whole and decimal numbers) and understanding rational numbers (including how they are represented) is an important component in the orderliness of numbers. Malofeeva, Day, Saco, Young and Ciancio (2004) belives that number sense involves number recognition, its vocabulary and meaning. It entails being able to both identify a particular number symbol from a collection of number symbols and name a number when shown that symbol, typically up to twenty (Clarke & Shinn, 2004; Gurganus, 2004; Yang, Li, & Lin, 2008). Significantly, children who experience difficulty with number recognition generally tend to experience mathematical problems in higher grades (Lembke & Foegen, 2009) and particularly with subitizing, a key process of early arithmetic (Koontz, 1996; Stock, Desoete, & Roeyers, 2010).

b) **Multiple representations for numbers** – Numbers appear in different contexts and may be expressed in a variety of symbolic and graphical representations. Number sense includes the recognition that numbers take many forms and can be thought about and manipulated in many ways to benefit a particular purpose. Decomposition/recomposition involves expressing a number in an equivalent form as a result of recognizing how this new equivalent form facilitates operating on the recomposed numbers.

Andrews and Sayers (2015) stress that learners understand that numbers can be represented in different representations. Such representations can include number lines, the different partitions of a number, the use of fingers and various manipulatives. Buxton (1997) alluded that number representation can be presented in three modes, namely language, spatial and symbolic. These modes can be used as modes of representation into the logical and useful technique for teaching number sense, for example if the decimal 0.16 is chosen, learners represent their understanding of 0.16 by writing it as a fraction: $\frac{16}{100}$ and writing it in words as sixteen hundredths. The inability of learners to link the different representations is a sign of a weak understanding of multiple representations of numbers.

c) ***Sense of relative and absolute magnitude of numbers*** **–** learners with this types of number sense can recognize the relative value of number quantity in relation to each other and can sense the general size of a given number. This means that learners in Grade 3 will have a different notion of the numeral '1000′ if the teacher asks questions that allow them to think of a thousand in a personal context, for example asking learners how long does it take them to walk 1000 steps or whether they have lived for 1000 days? In addition to McIntosh et al.'s (1992) sense of relative and absolute magnitude of numbers, Griffin (2004); Ivrendi (2011); Jordan, Kaplan, Nabors-Olah, and Locuniak (2006) discussed number sense as including anawareness of magnitude and comparison between different magnitudes and suggest deploying language such as 'bigger than' and 'smaller than'. In particular, children who are magnitude aware have moved beyond counting as a memorized list and a mechanical routine without attaching any sense of numerical magnitude to the words. Moreover, being magnitude aware supports the development of other mathematical skills, particulary subitizing; (Nan, Knösche, & Luo, 2006; Stock et al., 2010).

d) ***System of benchmark* –** The numerical benchmark provides learners with the necessary mental reference for thinking about numbers. Benchmarks can be used to judge the size of an answer or roundness of a number so that they can be easy to process mentally. This skill is comprised of the ability to determine and use reference points that can vary according to situations. Mathematics is about thinking and making sense of the world through numbers, measurements, and symbols. In classroom mathematics, teachers should use numerical measurements from everyday life experiences, where learners can find the answers from various sources. This may lead to learners developing the spirit of inquiry (Pee; Tin; Berinderjeet, 2014).

Knowledge and facility with operations – In school mathematics this number sense theme aims to help learners understand operations, including how numbers are formed. In primary school, for learners to understand mathematic addition, subtraction, division, and multiplication they are provided with specific skills to perform these operations by paper/pencil procedures. Knowledge and facility with operation components in mathematics include: understanding the effect of the operations, the relationship between operations, the mathematics properties, and computational strategies.

a) ***Understanding the effect of operation*** – to fully understand operations means understanding the effect of various operations on various numbers including whole numbers and rational numbers. Visuals are used to help the learners understand the concept of the operation e.g. using an abacus to model multiplication as a repeated addition gives learners concrete ideas to think about multiplication and work it out. Using different visuals such as number lines and arrays may help the learners to see multiplication in a variety of ways. A high level of thinking and enhancement of number sense is created when learners reflect on the interaction between the operations and numbers.

In some cases, teachers pressurize learners to remember rules in multiplication and division computation sums However, learners may forget or become more confused as they have no understanding of why certain rules work. It is therefore the responsibility of the teachers to strongly encourage the learners to use their understanding of operations (Pee et al., 2014)

b) ***Understanding mathematics properties*** **–** commutative properties of mathematics, associativity, and distributivity have been included in the school curriculum. When learners use mathematics properties it is evident that they have developed this number sense characteristic. Learners with good number sense use the connections and are comfortable applying these mathematical properties in different situations.

Sayers, Andrews, and Björklund (2016) say that different mathematical properties can be used to help learners to intuitively recognize and use other equivalent expressions in different ways. Leaners should be allowed to discuss how they deduce the pair of equivalent expressions and the thinking they use.

c) ***Understanding relationships between operations*** **–** Learners can solve problems when they can see the connections between operations. For instance, if the learners are asked about the number of wheels on a tricycle the learners may apply repeated addition or may add by grouping or applying multiplication. All answers will show the different ways of thinking about the problem. To understand the relationships between operations, learners need to first understand each operation. The inverse relations between operations is another valuable connection that helps learners to think about problems.

This number component, according to Şengül (2013), is related to the ability to recognize how the result will change when operations or numbers are changed in a calculation. Reflecting on the relations between the operation and numbers promotes higher-order thinking and further enhances number sense. In the same vein, Van Luit and Schopman (2000) see number sense as an awareness of the relationship between numbers and quantity. They add that learners do not understand only by the one-to-one correspondence between the number's name and the quantity it represents, but also that the last number in a count represents a total number of objects. Number sense incorporates quantity discrimination whereby learners recognize that eight represents a quantity that is bigger than six but smaller than ten. Quantity discrimination according to Kroesbergen, Van Luit, Van Lieshout, Van Loosbroek and Van De Rijt (2009) is a predictor of a child's later mathematics achievement.

Applying knowledge of and facility with numbers and operations to computational settings – When solving a real-world problem which expects learners to reason with numbers involves decisions: on what type of answer is appropriate; what computational tool is efficient; and then selecting strategies and applying them. This component expects learners to understand the relationship between the problem context and the necessary computation.

Individual problem solving without resorting to written calculation and estimation, to investigate the appropriateness of the results, emphasizes the ability to do mental calculations. Learners who lack the conceptual knowledge of whole numbers may find mathematics difficult when it comes to calculation. Although sometimes learners can compute and perform basic arithmetic operations they also need to make realistic assumptions and decision that require reasoning to find answers (Pee et al., 2014)**.**

Inclination to utilize an efficient representation – Learners who pose this characteristic of number sense are aware that some strategies are more efficient at times than others. On the other hand, the learners with little number sense often use the more difficult method when they are calculating. The learners can examine the answers in light of the problem at hand when the solution is provided, to determine if the answer makes sense.

2.2.3. The importance of number Sense

The importance of number sense cannot be overemphasized. A young child with more advanced number sense may be able to turn simple addition facts into a more complex idea by breaking down or decomposing numbers (Heath, 2012). Gersten, Jordan and Flojo (2005) also added that number sense development in young children has been linked to future mathematics achievement in the same way that phonological awareness has been linked to reading achievement. Additionally, researchers (Heath, 2012; Politylo, 2015) found that the most valid means of predicting mathematics difficulties in young children involves some of the basic principles of number sense. Number sense helps learners understand what number means and also improves mental mathematics in learners. A learner with a well-developed number sense can look at mathematics in the outside world and make comparisons. A good sense of number builds learner confidence, which in turn impacts their views on mathematics and enables them to apply mathematics and cope better with more advanced mathematics. This kind of access and success can open up career opportunities and it is also useful in other fields for learners who specialise in mathematics in higher grades (Howden, 1989).

Lack of number sense affects the performance of learners in mathematics. This is supported by Akkaya (2015) who asserted that lack of number sense in learners leads to serious problems in learning other mathematical subjects. A strong foundation of a high level of mathematics can be achieved when a learner's number sense is well developed. Research conducted by Tsao and Lin (2011) suggests that high achieving learners use number sense more, when compared to low achieving learners. When solving a problem like 18+9 a learner with a strong number sense can solve it as in 20+7 while the learner that lacks number sense will struggle to get the answer. Learners lack number sense because they don't use numbers flexibly; they are accustomed to using memorization techniques from a young age instead of manipulating and interacting with the numbers flexibly. The learners with a lack of number sense memorize procedures and they are unable to reason why their solutions are reasonable. Learners with a lack of number sense are likely to have trouble developing the foundation needed for even simple arithmetic, let alone more complex mathematics.

2.3. THE ABACUS

The abacus is an ancient calculating device used primarily in Asian culture for performing arithmetical processes. It is specifically designed for visualizing while performing calculations. Young (2015) defines an abacus as a mathematical device or tool used by people for performing addition, subtraction, multiplication, and division at the Foundation Phase level. According to Skoumpourdi (2016), the abacus is a cultural instrument that follows the theoretical development of mathematics and which is very commonly found in early childhood classes because it focuses on the polynomial representation of natural numbers.

Bruekler (2017) regards the abacus as an ideal tool that demonstrates the basic arithmetic operations in various number systems. He further states that the abacus is the first and the most well-known calculating tool that is still widely used in the European and Asian countries and has survived through centuries due to its simplicity, adaptability, and creativity. Three main forms of the abacus are still used today in some education systems. These are the Chinese abacus, the Japanese abacus, and the Russian abacus (Figure 1) (Lutjens, 2015).

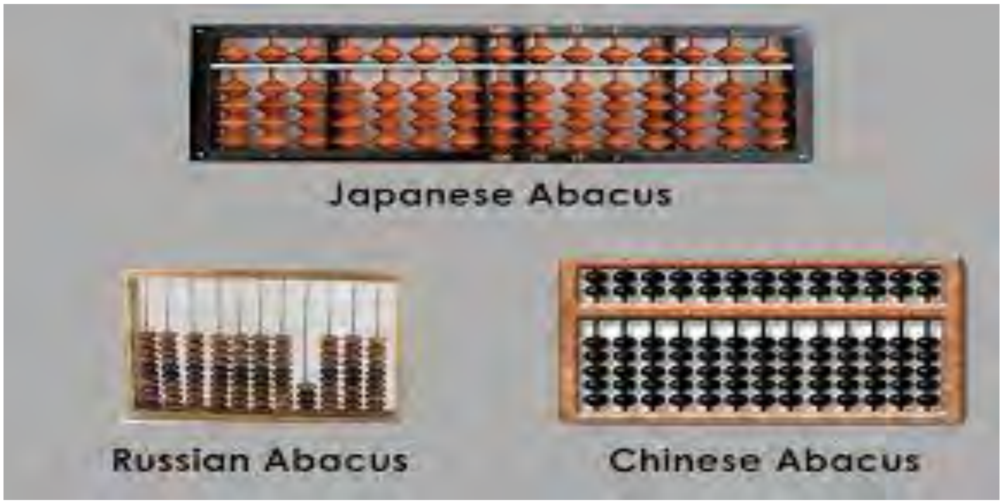

Figure 2.1: Japanese, Russian, and Chinese Abacuses (Lutjens, 2015)

At present, in many classes in other parts of the world, a modernized abacus has been introduced. The modern abacus consists of a combination of the abacus and the counting board. It consists of 10 horizontal rows with 10 beads on each row grouped in fives in contrasting colors (Figure 2.2).

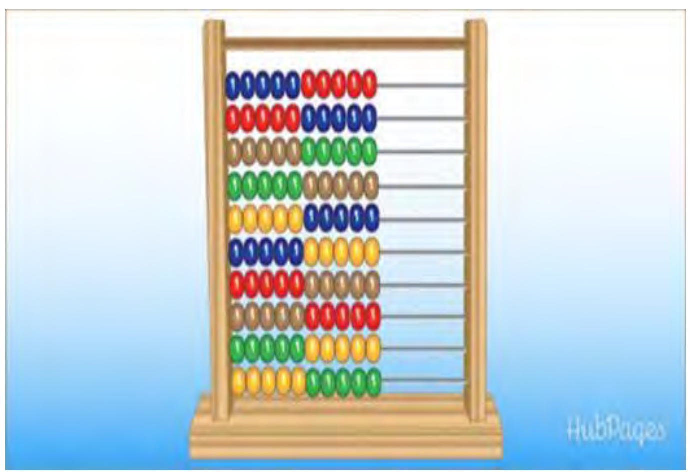

Figure 2.2: The modern Abacus (Skoumpourdi, 2016)

2.3.1. An abacus as a teaching Manipulative

Manipulatives

Since the early 1900s, manipulatives have come to be considered essential in teaching mathematics at the elementary school level. The advocacy for the use of manipulatives stem from the learning theories of Piaget, Bruner, and Montessori: that learners develop and build knowledge as they move from concrete experience to abstract thinking (McNeil & Jarvin, 2007). Piaget (1980) hypothesized that children are not mentally mature enough to grasp abstract mathematical concepts if their teachers only present the concept in writing (using words, numbers, and symbols). According to Piaget, children need several experiences with concrete materials and drawings to learn abstract concepts. According to the Piagetian theory, learners need to physically manipulate objects and then they should be encouraged to reflect upon the meaning of the results of their physical actions (Baroody, 1989). In mathematics, teaching for understanding includes the presentation of the curriculum through multiple representations. These multiple representations that can be used in mathematics education include physical (concrete), pictorial (static visual), and virtual (dynamic electronic) representations. The mathematics educators can then use manipulatives either as physical or visual manipulatives. Marshall and Swan (2008) defined a mathematics manipulative material as an object that can be handled by an individual in a sensory manner during which conscious and unconscious mathematical thinking will be fostered. According to Sarama and Clements (2016) there are two types of manipulative: (a) physical and (b) virtual manipulatives.

(a) Physical Manipulatives

Physical manipulatives are the type of manipulative that can be touched by learners with their hands. Physical manipulatives can also be looked at as concrete objects which learners use to explore mathematical concepts through their visual and tactile senses (McNeil & Jarvin, 2007). Physical manipulatives are designed to be moved by hands as a means of developing motor skills or understanding abstraction, especially in mathematics (Cope, 2015). Physical manipulatives range from low cost simple everyday items such as buttons, dominoes, bottle tops, counters and stones, to more complex and discipline-specific items such as calculators, two colour counters, decimal tiles, tangrams, *Geoboards* and abacuses (Bellonio, 2001).

(b) *Virtual manipulatives*

Virtual manipulatives are not concrete in a physical sense but can be displayed on a computer screen. Johnson, Campet, Gaber and Zuidema (2012) described the virtual manipulative as those that allow teachers to combine the pictorial, verbal, and symbolic representation of problems more easily while still allowing learners to move objects, in a similar way to physical manipulatives. Akkan (2012) discovered that only a few teachers can confidently make use of the virtual manipulative. Virtual manipulatives are associated with some weaknesses: for instance they are less flexible; they require less input from teachers; sometimes they make it more difficult for the learners to learn concepts perhaps because they encourage memorization.

In today's mathematics classrooms the use of visual aids is accepted. They are believed to encourage learners to think in visuals when solving problems. Johnson et. al., (2012) further added that virtual manipulatives expose learner's understanding and problem solving strategies in several ways. Educational manipulatives in classrooms play an important role in the discovery and expression of mathematical relationships and encourage mathematical argumentation. According to Cain-Castron (1996) and Heuser (1999), the use of manipulatives helps to improve the working environment in a mathematics classroom: when students work with manipulatives and then are given a chance to reflect on their experience, not only is mathematical learning enhanced, but mathematics anxiety is greatly reduced.

In the effort to make mathematics less abstract, the use of manipulatives helps to bring it to the concrete level (National Council of Teachers of Mathematics [NCTM], 2000). The NCTM advocates for the use of manipulatives as they help learners represent numbers and develop number sense and also bring meaning to learners' use of written symbols (NCTM, 2000). Furthermore, Dunlap and Brennan (1979) saw manipulatives as tools that can help learners understand and develop mental images of mathematics concepts and provide learners with a concrete basis from which abstract thinking is developed.

The abacus as a physical manipulative

An abacus is a physical object, where learners manipulate the beads with their hands on the physical abacus while they visualize it in their brains. The abacus, unlike other manipulatives, not only gives learners a simple and fast way to represent numbers, but an efficient way to perform any math operation with any sized number. The abacus teaches learners to add and subtract using number complements. Complements are the most efficient way to add and subtract and this is also how computers perform these mathematics operations (Vasuki, 2013). During the process of using the abacus, the learners develop number representation skills and this enables them to see concepts concretely and bridge the gap between concrete and mathematics symbolism (Metallo, 1988). According to Tang (1993), the use of an abacus during the mathematics lesson helps learners to see the relationship between numbers and abstract concepts. The manipulations of beads on the abacus help learners to understand the concept of place value.

The abacus has been used to teach arithmetic for a long time in many countries. One question that may arise is: why emphasize the use of an old instrument in this modern time when more technological advancements have been made? Amoah, Bour, Boampong, and Arkoh (2019) assert that even if technological advancements make it possible for human beings to perform complex calculations and to explore new dimensions in the field of knowledge, young minds require concrete objects to manipulate freely so that rules for processing are formed solely by their physical experiences. With the help of the abacus, simple arithmetical functions are easy to learn –especially multiplication – which may be difficult for some learners to understand (Amoah et al., 2019). Children are born with a rudimentary sense of number that can develop (Marmasse, Bletsas, & Marti, 2000). Hence, as their schema for comparing merges with that of counting, new brain structures are developed that help them to develop number sense (Chikiwa, 2017). Thus, I see the use of the abacus as a visualization tool in the Foundation Phase to develop number sense as important.

2.4. VISUALIZATION

2.4.1. Introduction

Using visualization when teaching mathematics in the early stages provides teachers with an interesting and meaningful approach when teaching different mathematical concepts (Budaloo, 2016). Budaloo (2016) asserts that learners also have opportunities to discover new ways of understanding these concepts. It is also argued that visualization is a powerful tool with which learners can construct mental and physical representations that correctly mirror mathematical relationships and concepts (Kadunz & Straber, 2004)**.** Teachers in classrooms use visualization as a powerful tool when teaching mathematics and make the instructional decisions both on how content can be taught and the nature of the different tasks that are being used (Kadunz & Straber, 2004). In many areas of the curriculum, for example mathematics, science and technology, visualization is an important feature .

As the world moves toward the information age, the ability to reason visually in education is increasingly becoming important. Budaloo (2016) notes that reasoning visually in mathematics contributes to the recognition of the role of visualization in mathematical thinking and solving world problems.

2.4.2. Definition of Visualization

Literature states that visualization can be mental or physical. Zazkis, Dubinsky and Dautermann (1996) define visualization as an act in which individuals establish a strong connection between an internal construct and something to which access is gained through the senses. An act of visualization may consist of any mental construction of objects or processes that an individual associates with objects or events, perceived by an external source (Zazkis et al., 1996). Similar to Zazkis et al.'s (1996) definition are those of Bishop (1989), Hershkowitz (1989) and Zimmermann and Cunningham (1991) who define visualization as a process, ability, and product of the reflection, usage, interpretation, and creation of pictures, images, and diagrams in our minds, on paper, or by portraying with technological tools.

Zazkis et al.'s (1996) explanation of visualization emphasizes that the capacity is not merely an act that is added to the repertoire of knowing, but conveys a mutually determining phenomenon of acting and knowing. This definition implies that visualization involves knowing that something occurs as a result of performing actions on a target object, concept, or process.

Presmeg (2006) saw visualization through visual imagery as an effective way to communicate both abstract and concrete ideas. According to Presmeg (1986, p. 9) “visualization is an aid to understanding and thus one can speak of visualizing a concept or a problem”. Therefore, to visualize a problem means that one needs to understand a problem in terms of the visual (mental) image. It is then a process that involves visual imagery with or without diagrams as part of the solution (Presmeg, 2006).

From a mathematical perspective, different authors have defined visualization in many ways. Arcavi (2003) describes the process of visualization as:

> Visualisation is the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds, on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understanding (p. 217).

Arcavi's definition does not make any claim that privileges internal constructs in either mentally or explicitly constructing a relevant object, which seems to be the suggestion of Zazkis et al, (1996) in their definition above. Arcavi's definition stresses that in mathematics learning, visualization can be a powerful tool to discover mathematical problems and to give meaning to mathematical concepts and the relationships between them. According to Rösken and Rolka (2006), visualization reduces complexity when dealing with a multitude of information. From the different definitions of visualization, one can deduce that it helps us to understand by not only seeing what is within our sight (physical) but also seeing and interpreting what we mentally see in our minds, pages, or when using technological tools and software. The visualization process also helps people to communicate and expand their understanding of information and ideas and improve people's abstract thinking abilities (Özkan, Arikan, & Özkan, 2018).

2.4.3. Types of visualization

There are a number of visualization forms that can be used in mathematics lessons at both primary and high schools. Depending on the "degree of correspondence between the mathematical situation and the concrete way of representation, there can be several different types of visualization (Guzman, 2011). Guzman (2011) further identified different types of visualization as described below.

Isomorphic visualization- In mathematics two structures are isomorphic if the form of the two structures is the same, but the elements are differently named. This kind of visualization is used when there is a strong relation between visualization and the math problem. The concrete visualization gives direction to solve an abstract problem. Most mathematical visualization is part of this category. Mathematicians mainly accept this type of visualization. Visualization is dependent on traditions and various kinds of conventions. It is true that "an image is worth a thousand words", but this dictum presupposes that we know a specific sign system and we can decode and understand it.

Homeomorphic visualization - In mathematics two structures are homomorphic if the elements of the two structures are the same, but differ in form. In this visualization, the relationships of different kinds of abstract objects is visualized, which helps to understand guesses and proofs. Here you cannot find a direct relationship between the visual signs and mathematical content, therefore the visualization process is quite subjective and sometimes it is not easy to communicate.

Analogical visualization-This type of visualization is the most complex. Here we mentally substitute the object we are working with, with another that relates between them the two in an analogous way that is perhaps easier to handle.

Diagrammatic visualization – This type of visualization of our mental objects and their mutual relationships is represented by diagrams. In some cases diagrams become generalized, but in many cases, they are of a very personal, individual nature and cannot be shared easily with others. The tree diagram we use in probability, the Venn-diagram in set theory or the Hasse diagram in number theory can be found in this category.

2.4.4. The role of visualization in mathematics

Visualization is commonly believed to play an important role in mathematics teaching. Ben-Chaim, Lapan, and Houang (1989) drew attention to the role of visualization in the development of inductive, deductive, and proportional reasoning. According to Ben-Chaim et al. (1989), visualization is a central component of many processes and is essential to fulfilling the transformation of thinking from the concrete to the abstract.

Research has highlighted the importance of visual representation both for learners and teachers in the teaching and learning of mathematics. The role of visualization is an important part of teachers' knowledge of mathematics and the teachers play an important role in the explanation of the mathematical ideas (Leinhardt & Smith, 1985). The visual representation can highlight specific aspects of a mathematical concept and therefore support the explanation of mathematical concepts (Ainsworth, 1999; Kaput, 1991). Visual representation enables learners to make connections between their own experience and mathematical concepts and therefore gain insight into these abstract mathematical ideas.

According to Bishop (1989), visualization helps teachers to make an instructional decision about how to teach mathematical content and the nature of the task that is being taught. Furthermore, visualization helps the mathematics teachers with the facilitation of lessons and with the ability to engage learners in a realistic situation – for example, an understanding of the precise limitation of children's imagery could indicate to teachers on how to teach using visual imagery.

Primarily visualization leads to successful learning and understanding of mathematics. Mathematical thinking, according to Duval (2000), requires the coordination of at least two registers: multi-functional (open to multiple representations in verbal and geometric register) and monofunctional (open to one interpretation in symbolic and graphic representation).

Using visual representation leads to learners having a good mathematical understanding. Mathematical understanding is achieved as learners learn to discriminate between and coordinate semiotic systems of representation. The semiotic systems of representation, or registers, refer to four classifications of systems of representation: generation, inspection, transformation, and use (Kosslyn, 1994). Some learners' difficulties in the construction of concepts are linked to the restriction of representations in their learning experiences.

It is generally agreed that visualization develops the power of multidimensional thinking in individuals (Wheatly, 1991). Wheatly (1991) further states that multidimensional thinking helps the learners to develop the capacity for collective discussion and exchange of ideas by looking at the concepts from a different perspective. If we are to include visualization in the mathematics learning process, we are building a learner's ability to perceive, set up, troubleshoot, and face challenges in their own lives in the future.

From my experience as a teacher, I have noticed that teachers bring visuals into lessons only when they notice that the learners are struggling to grasp a concept. Some believe visual mathematics is for struggling or young learners and that learners should only work with visuals as an introduction to more advanced or abstract mathematics. In addition, teachers also look at the use of visuals as time-consuming and this results in them teaching without the use of visuals and concrete materials.

In an effort to make use of and improve learners' capacity to visualize, the importance of visualization should be emphasized in schools. Visualization is very important in the process of education because it improves creative thinking, making it wider, more open, and more versatile. Mathematics is very suitable for introducing the different kinds of visual aids which can improve creative thinking. Learners need to be taught different approaches to mathematics and in this way, gain access to a deep and new understanding of different mathematical concepts. Typically, a good mathematics teacher uses visuals, concrete manipulatives, and motion to enhance learners' understandings of different mathematical concepts (Furner, 2017). In the Foundation Phase, one of the manipulatives that can be used by the teachers to develop number sense and conceptual understanding in learners is the abacus.

2.4.5. The abacus and visualization

Visualization is a very important feature in "teaching for understanding" in mathematics because it aids the teacher by engaging learners in realistic situations (Makina, 2010). Learners may be enabled to represent different equations given to them on the abacus. Visualization not only enables faster calculations, but also takes the abacus user into a mental plane which is a unique move from the physical to the mental. Using the abacus makes photographic memory much stronger and it makes objects clear in the mental plane, where objects are clearer and the distraction is minimized.

According to Kim (2015), studies reveal that training with an abacus can promote mental visualization and the moving of beads on the abacus helps to create concrete images of numbers with which learners can perform mental calculations. Stigler (1984) explained that skilled abacus users visualize a mental abacus and perform mental arithmetic by manipulating beads. In addition to visualizing the abacus when doing calculations, Stigler (1984) argues that using a physical abacus leads to learners developing a mental abacus. Mental abacus users are trained to visualize an abacus and to move imaginary beads on this abacus to perform arithmetic calculations. Learners move these imaginary beads using their hands and thus move their hands in the air as they perform calculations, suggesting that motor representation somehow interfaces with the number representation created by the mental abacus.

Stigler, Chalip, and Miller (1986) reported that in Japan, where the abacus training is part of the math curriculum, the abacus training helped children to develop number sense, mental arithmetic, and mathematical application such as problem-solving as well as rapid and accurate calculation. Although technology is successfully being used these days as a tool to improve education at all levels, its improper usage is curbing the imagination of the student community, leading to a decrease in their thinking capacity and ability to focus and concentrate. As attention is a vital cognitive feature of any learning process, students these days are not coping well with this process. Although computers have taken over recently, the abacus is still widely used in Japanese and Chinese elementary schools where it is part of the curriculum (Geeta & Gavas, 2014).

Some children need to be taught how to use the ancient tool abacus in this technological world. For example, blind children are taught to use the abacus simply because they cannot perform calculations on paper. The abacus has proved to be an ideal tool for those learners as it is reliable and allows them to calculate easily and quickly. For the beginner the use of the abacus is easy to understand and it also helps in developing the minds of the children (Manju & Kaur, 2014). Visualization can be used as a process to develop number sense when teaching mathematics in the Foundation Phase using different manipulatives like the abacus.

2.4.6. The use of the abacus as a visualization tool to develop number sense

Research has indicated that number sense develops gradually and over time, resulting from an exploration of numbers, visualizing numbers in a variety of contexts, and relating to numbers in different ways (Burns, 2007). Maghfirah and Mahmudi (2018b) are of the opinion that number sense develops as a result of mathematical experience where learners can employ their senses in understanding circumstances involving numbers without exerting standard algorithm. Similary, Sao and Rung (2007) emphaized that teachers play an important role in developing number sense in the type of classroom environment they create, in the teaching practice they employ and the activity they select.

Bobis (2008) observed that many children lacked a good "sense of number" despite having been exposed to formal mathematics schooling. She noted that this contributed to the recognition by researchers of the importance of studying young children's visual representations of numbers, in their attempts to explore earliest understandings of number concepts. It was further noted that activities that focused on the visual identification of groups of numbers, such as subitizing rather than counting one by one, helped learners develop an understanding of part-whole relationships, especially in the decomposition of ten.

Manipulative tools such as counting boards and abacuses enhance young children's ability to imagine and visualize a quantity. Children must be able to subitize, that is, to recognize the number of a quantity immediately without counting (Bobis, 2008). Subitizing has long been recognized as an important skill for developing number sense (Clements, 1999). However, people cannot recognize and visualize quantities of more than six without some type of grouping. Visualization can be emphasized through the use of a counting board and an abacus with a grouping of five structures. To understand that our number system is based on tens, according to the Japanese Council of Mathematics Education (Cotter, 2000), children must experience the pattern of trading: ten ones for one ten, ten tens for one hundred, etc., something that can only be done with the use of the abacus. The abacus shows children how to visualize problems. It does this by using a simple concept such as arithmetic which is the basis of math and science. Learners that use the abacus perform rapid mental arithmetic by manipulating a mental representation of an abacus which is used as visual imagery (Swinson, 2018).

Swinson (2018) identified the important benefits of an abacus in developing number sense as:

- **Higher sense of spatial reasoning** – An abacus is a multi-sensory tool. Because it can behave in many different, yet controlled ways, it shows children how to equate physical objects with abstract thinking. This builds both their imagination and spatial reasoning.
- **Critical thinking skills** – Because the abacus is taught through system knowledge, it teaches children how to critically and analytically think about a problem.
- **Highly developed long-term memory** – When children learn the abacus, they learn how to solve a problem. This technique is applied to problems outside of arithmetic.
- **Independence** – When a children learn how to solve a problem, their sense of confidence grows. They want to try the next problem and the next one. They want to show you what they have done all on their own.

An abacus is a physical object, where learners manipulate the beads with their hands on the horizontal rods while they visualize the calculations in their brains. An abacus is a manipulative that starts with very simple number representations but quickly advances from other mathematics manipulatives because it grows with the child's mathematics learning. Unlike other manipulatives, the abacus not only gives learners a simple and fast way to represent numbers but an efficient way to perform any mathematics operation with numbers of any size. The abacus teaches learners to add and subtract using number complements. Complements are the most efficient way to add and subtract; this is also how computers perform these mathematics operations (Freeman & Club, 2014). Using manipulatives like abacuses increases students' understanding of place value, (Phillips, 1989). It can also help students to reduce errors and increase their scores on tests that require them to solve problems (Carroll & Porter, 1997).

Teaching for conceptual understanding is directly related to the effective use of manipulatives and visual representation (Eisenhart et al., 1993). Thus, using an abacus as a visualization tool to develop number sense gives opportunities to the learners to gain new knowledge and this may also help develop the conceptual understanding of basic mathematics at the Foundation Phase level (Grade 3) through the relationship between the existing knowledge and new information that they are learning. The use of manipulatives such as abacuses aid in learners having a better understanding of the mathematical concepts and assist them on how to learn with more than simply the teacher modeling a procedure on the chalkboard.

Number sense is a topic of great interest in the school mathematics curriculum. The National Council of Teachers of Mathematics (2000) note that the development of number sense is important in mathematics education. The NCTM (2000, p.32) in their "principles and standard for school mathematics", noted that number sense is one of the foundational ideas in mathematics in that students: understand numbers, ways of representing numbers, relationshins among numbers, understand meaning of operations and how they are related to one another and compute fluently. Every learner is born with the ability to succeed in mathematics but the degree of success is directly related to the strength of their success. Learners will only develop strong number sense to the extent that their mathematics teachers encourage them to understand mathematics, as opposed to the memorization of rules and mechanical application of algorithms. The above-discussed number sense themes, how they are developed, their importance in a mathematic classroom, and the main components of number sense (*knowledge of and facility with numbers, knowledge of facility with operation and applying knowledge and facility with numbers and operation to computational setting*) are adapted and adopted as observable indicators which form part of my analytical tool in Appendix H. The next section will discuss the theoretical framework that underpins this study.

2.5. THEORETICAL FRAMEWORK

2.5.1 Constructivism

This research is theoretically informed by the constructivist theory. Constructivism is a learning theory that finds its roots with theorists such as Jean Piaget and Lev Vygotsky. The theory is based on the idea that knowledge is constructed by the knower based on mental activity (Connors, 2007). Constructivists emphasize the importance of the active involvement of learners in constructing their knowledge and building new ideas or concepts based on current knowledge and past experience (Vey, 2005). Similarly, Brooks and Brooks (1993) note that constructivism theory emphasises learners' construction of the meaning of what they learn by building from existing knowledge. The theory views learners as active members of their learning process (Jaramillo, 1996). In constructivism, a learner is not viewed as a passive receiver of knowledge, an "empty vessel" into which the facilitator must "pour" knowledge.

Wheatly (1991) asserts that conceptual knowledge cannot be transferred ready-made and intact from one person (the facilitator) to another person (the learner). Rather, the learner is viewed as an active participant who constructs his/her own knowledge. The learner comes to the learning situation with his own existing knowledge; new ideas are understood and interpreted in the light of the learner's existing knowledge, built up out of his previous experience. Learning from this perspective entails that the learner must re-organize and re-structure his present knowledge structures, and this can only be done by the learner himself. Learning is a social process. Learners learn from each other (and the facilitator) through discussion, communication, and sharing of ideas, by actively comparing different ideas, reflecting on their own thinking, and trying to understand other people's thinking by negotiating a shared meaning

A paraphrase of constructivism without doing any harm to its meaning would be: Constructivism is a paradigm or world view postulate that learning is an active, constructive process with the learners being information constructors and actively constructing or creating their own subjective representation of objective reality (Connors, 2007). Involving learners actively in their learning help them to link new information to their prior knowledge. In simple terms, constructivism states that learning is an active process of constructing knowledge rather than acquiring it.

Connors's (2007) definition of constructivism is rooted in Jean Piaget's notion of constructivism. Piaget's theory of constructivism proposes that humans cannot be given information in which they understand immediately. Instead, learners must construct their own knowledge. The learners then build their knowledge through experience. The experiences they build help them create schemas which are mental models of the world. Schemas are changed, enlarged, and made more sophisticated through the process of assimilation and accommodation. These two processes are identified as key in the interaction of constructing new knowledge from the learner's prior knowledge. During the process of constructing new knowledge, learners assimilate new information and incorporate it into the existing framework without changing it. Piaget suggests that the accommodation process reframes one's mental representation of the external world to fit new experiences. Piaget rejected the idea that learning is the passive assimilation of given knowledge. He instead proposed that learning processes have different stages of adaptation to reality during which learners construct knowledge by creating and testing their own theories.

One can argue that constructivism is an approach to learning where learners will not understand knowledge if they are merely taught facts as preexisting entities. In elaborating on constructivism, Arends (1998) stated that constructivists believe in the personal construction of meaning by the learners through experience and meaning is influenced by the interaction of prior knowledge.

2.5.2. Radical constructivism

The idea of radical constructivism was developed by von Glasersfeld (1974). Radical constructivism believes that knowledge is not passively received but rather actively constructed by the individual. Radical constructivism does not ignore the existence of an external world but denies that one knowledge can objectively reflect it; it replaces the truth with viability but accepts abstract mathematics. Radical constructivism is an idea that all learning must be constructed and there is no utility or meaning in instruction that is teacher or textbook-driven. It is often referred to with reference to mathematics but it can be difficult to understand.

Radical constructivism is an existing theory of how best to teach mathematics. In radical constructivism, when learners are learning basic operations standard algorithms are not needed; what is important is how learners understand the concepts underlying these operations. Teachers using radical constructivism design conceptual investigations, questions and activities that allow learners to explore the mathematical ideas that are relevant to their everyday lives. Learners can design their own problem solving and deal with questions involving arithmetics from a very young age.

2.5.3. Constructivism and teacher practice

Constructivism is a view of learning based on the belief that knowledge is not a thing that can be simply given by the teacher. Rather, learners through an active engagement process of development, construct knowledge. Learners are the builders and creators of meaning and knowledge. Amineh (2015) identifies four principles of constructivism. The first is that learning depends on what we already know. Secondly, new ideas occur as we adapt and change our old ideas. Thirdly, learning involves inventing ideas rather than mechanically accumulating facts. Lastly, meaningful learning occurs through rethinking old ideas and coming to new conclusions about new ideas which conflict with our old ideas. A productive, constructivist classroom then consists of learner-centered, active instruction. Teachers should therefore use the abacus to teach the meanings of operations, related properties of natural numbers and rules of the decimal system.

In a constructivist classroom, the teacher provides learners with experiences that allow them to hypothesize, predict, manipulate objects, pose questions, research, investigate, imagine, and invent. The teacher's role is to facilitate a process of learning in which the learners are encouraged to be responsible and autonomous. Crawford and Witte (1999) state that putting mathematics instruction and learning in context when teaching helps learners see that mathematics is part of their world. It also enables the learners to construct meaning that makes sense to them, which in turn, helps make sense from new situations and problems.

Brooks and Brooks (1993) identify the four tenets of constructivism for constructivist teachers:

- Discover the learner’s point of view.
- Develop lessons that will challenge learners ideas
- Recognize that learners want to know the relevance of the concept in the curriculum.

- Evaluate learners’ learning in the context of the daily classroom and not as a separate event.

According to Connors (2007), the theory advocates that the learning environment should stimulate learners’ interest to enable them to develop and construct their own meaning. Brooks and Brooks (1993) state that constructivist views of education advocate that teachers should use raw data and primary resources, along with manipulatives and interactive and physical materials. A constructivist approach to teaching should encourage and accept learners’ autonomy and initiative. Vygotsky (1962, p. 3) asserts that “Learners learn through interacting with peers and teachers as well as manipulative tools and their contextual setting”. When teachers use manipulative tools to teach, learners are enabled to explore and discover concepts that can lead to understanding and construction of knowledge. Harasim (2018) adds that the main principle of constructivist theory is that learning is an active process in which learners use sensory input to construct meaning. Using the abacus as a visualization tool in developing number sense transforms the learners from passive recipients of information to active participants in the learning process.

Piaget argues that children do not have the mental maturity to grasp abstract mathematical concepts presented in words or symbols alone, and they need many experiences with concrete materials and drawings for learning to occur (Piaget, 1980). Concrete materials are objects that are designed to represent abstract mathematical ideas explicitly and concretely, e.g. base ten blocks, counters, number lines and abacuses (Moyer, 2001).

2.5.4. Constructivism and teaching manipulatives

An important characteristic of a constructivist environment is the common use of teaching and learning manipulatives in the classroom. The use of manipulatives is constructivist because learners are actively engaged in discovery during the learning process. According to Piaget’s cognitive constructivist theory, children construct knowledge through concrete experiences in their world, which implies that manipulatives have a positive effect on children’s learning of mathematics, in particular the abstract concepts. Piaget also believed that because of a young child’s mental immaturity, the use of concrete materials in learning would supplement the words and symbols they are yet to develop.

D’angelo and Iliev (2012) argued that teachers who implement manipulatives in their mathematics lessons create classrooms that are constructivist-based. Golafshani (2013) also recommends that we need to "recognize that teachers' teaching styles will become more constructivist through instructional practices with manipulatives" (p. 157). The use of manipulatives is also supported by Brooks and Brooks (1999) who suggest that in "constructivist theory curricular activities rely [more] heavily on primary sources of data and manipulative materials… than on textbooks and workbooks" (p. 17). It is further claimed that manipulatives help learners to construct, reinforce and connect several representations of mathematical ideas that are meaningful, in order for learning to occur (Clements, 1999).

2.5.5. Constructivism, Visualization, and Abacus

An important characteristic of a constructivist environment is the common use of teaching and learning manipulatives in the classroom. The abacus is one of the visual manipulatives that is used in the Foundation Phase in schools. Hands-on experience of using an abacus is necessary to understand an arithmetic system. Using the abacus as an educational tool helps learners to see not only the ones and tens displayed, but they are also able to recognize their relative position. The use of the abacus allows visualization of the exchange of a group of ten units of the next series when it comes to the algorithm for addition and subtraction.

The abacus allows learners to represent all kinds of numbers on the physical tool and also do mental calculations with any kinds of numbers in their minds. Thus, using visualization processes in re-imagining an abacus as a visualization tool gives opportunities to learners to learn new knowledge of computing numbers. This may develop their number sense at the foundational stage. Conceptual understanding develops "through the creation of the relationship between existing knowledge and new information that is just entering the system" (Hiebert & Lefevre, 1986, p. 27).

According to Muenke and Cohen (2000), using an abacus as a visualization tool has various benefits to learners such as: **improvement of memory**: learners achieve this by imagining the beads moving in a specific order, hence increasing their photographic memory; **increase in auditory skills:** learners respond quickly to verbal instructions that have to do with solving arithmetic calculations; **increase in concentration:** learners visualize the abacus and the transformations of the beads at every stage while ignoring environmental noise and other distractions; **improvement of mathematical perception and ability:** when learners visualize the virtual/imaginary abacus in their minds, they give incredibly fast and accurate responses; **increase in self-esteem and self-confidence**: with the increase in mathematical perception and ability, learners are no longer afraid of numbers and mathematics in general; and an **increase in concurrency and simultaneous task ability**: learners simultaneously listen, imagine, think and give the answer to complex and concurrent arithmetic calculations.

2.6. CONCLUSION

I have discussed number sense in depth in this chapter as it forms the backbone of the study. I have reviewed literature around the teaching of number sense, the concept of number sense, and the connections between number sense, the abacus and visualization. The theoretical framework of constructivism provided the theoretical lens through which this research topic was examined. Constructivism was discussed as a theory of learning based on the belief that knowledge is not a thing that can be simply given by the teacher. Rather, learners through an active engagement process of development, construct knowledge; learners are the builders and creators of meaning and knowledge.

The next chapter discusses the research methodology that was used to gather data for this study on how an abacus can be used as a visualization tool by the Grade 3 teachers to develop number sense.

CHAPTER 3

METHODOLOGY

3.1 INTRODUCTION

This chapter discusses the research design and methodological processes that were followed in my research study. The study aimed to investigate how selected teachers utilized the abacus as a visualization tool to develop number sense in Grade 3 classes, as a result of participating in the intervention. My study was thus guided by the following three research questions:

1. How did teachers use the abacus when teaching Grade 3 mathematics classes prior to participating in the intervention programme?
2. How can the abacus be utilized as a visualization tool to develop number sense in a Grade 3 class when teaching mathematics?
3. What are the teachers' experiences and perceptions of using an abacus as a visualization tool in developing learners' number sense after participating in the intervention programme?

In this chapter, I briefly discuss the research paradigm that guided my research process. I further discuss in detail the data collection tools I used in my study, their validity and reliability as well as how data were analyzed. In data collection, the focus is on the site, the sampling of participants, and research instruments. The research instruments employed to find out how teachers used the abacus to develop number sense in this study were questionnaires, reflective journals, observation, and semi-structured interviews. I also discuss data analysis, where I focus on data collected from a survey, classroom observations, and reflective journals. Finally, I concluded the chapter with ethical considerations.

3.2. RESEARCH ORIENTATION

This research is oriented in the interpretive paradigm. A paradigm is defined by Lincoln, Lynham, and Guba (2011) as a philosophical assumption or set of beliefs that guide the actions and define the worldview of a researcher. This means that a paradigm creates the intellectual beliefs and principles that show how a researcher sees, interprets, and acts within that world (Romani, Primecz, & Topçu, 2011). Kuhn (1962) regarded a paradigm as a "way of looking or researching phenomena, a worldview or a view of what is counted as accepted or correct scientific knowledge" (p. 23). According to him, the term 'paradigm' refers to a research culture with a set of beliefs, values, and assumptions that a community of researchers has in common regarding the nature and conduct of research. In addition, Cohen et al. (2018) assert that the paradigm doesn't necessarily drive the research as research is driven by the purpose of the research. Paradigms can clarify and organize the thinking about the research. In the same vein, Lather (1986) explains a research paradigm as a conceptual lens through which the researcher examines the methodological aspects of their research projects to determine the research method that will be used and how data will be analyzed. This is very important in every decision in the research process, including the choice of method to be used in the study.

According to Candy (1998), paradigms can be grouped into three main categories, namely positivist, interpretivist or critical paradigms. Rehman and Alharthi (2018) assert that positivist paradigm assumes that reality exists independently of human. According to them, reality is not mediated by our senses and it is governed by immutable laws. The positivist strive to understand the social world like the natural world and they view the world as 'being out there' and available for study in a more or less static form. On the other hand, Mertens (2008) notes that critical researchers emphasize the importance of the interactive relationship between the researcher and the participant and the impact of social and historical facts that influences them. The critical paradigm assumes that the reality exists but it has been shaped by cultural, political, ethnic, gender, religious factors which interact with each other to create social system.

My study was informed by the interpretive paradigm which I now discuss in the next section.

Interpretive paradigm

The interpretive paradigm is concerned with understanding the world as it is from the subjective experiences of individuals (Guba & Lincoln, 1989). In the classrooms, the subjective understanding of participants regarding the use of the abacus as a visualization tool to develop number sense, was captured and shared. The interpretive paradigm uses meaning (versus measurement) oriented methodologies, such as interviewing or participant observation, that rely on a subjective relationship between the researcher and subjects. The interpretive paradigm is underpinned by observation and interpretation, whereby observation is used to collect information about events, while interpretation refers to making meaning of information. Meaning can be made by drawing inferences or by judging the match between information and an abstract pattern (Aikenhead, 1997). In this study, observation was used to collect information on how teachers teach using the abacus to develop learners number sense. Interpretive researchers set out to understand their interpretation of the world around them. They use approaches such as understanding and hermeneutics to see the social world through the eyes of the participants rather than as an outsider, according to Cohen et al. (2018).

In presenting the tenets of the interpretive paradigm, Grix (2004) writes that according to interpretivism the world is constructed through the interactions of individuals. The natural and social worlds are not distinct and researchers are part of that social reality, not detached from the subject they are studying.

Hammersley (2013) argued that the interpretivist paradigm is originally rooted in the fact that the method used to understand knowledge related to human and social sciences cannot be the same as in physical science because humans interpret their world and then act, based on such interpretation while the world does not. The interpretive paradigm helped me to engage with my participants and interpret data related to teaching practice that involves abacus use in classrooms. Hammersley (2013) further emphasized that since multiple interpretations are developed among human relationships, interpretivist researchers try to understand diverse ways of seeing and experiencing the world through different contexts and cultures, avoiding bias when studying events and people with their own interpretations. The interpretive paradigm in this study helped me to collect data that relates to the participants' perceptions and experiences on the use of the abacus as a visualization tool to develop number sense.

Creswell (2007) added that interpretivist researchers gain a deeper understanding of the phenomenon and its complexity in its unique context instead of trying to generalize the source of understanding for the whole population. This study did not only describe the objects, humans and events, but deeply engaged with the social context and was conducted in a natural setting.

My study aims to understand human behaviour and action concerning teaching practice using the abacus as a visualization tool to develop number sense. In this study, an empathetic understanding of how teachers felt and interpreted experiences regarding the use of the abacus was done through interpretivism. To understand how the teachers teach mathematics using the abacus, I needed to be in the classrooms where they actively teach. The interpretive paradigm allowed me to view the world through the perceptions and experiences of the four participants in this study.

3.3 RESEARCH METHODS

Research methods are techniques that researchers use in research (Walliman, 2011). He further added that the research methods represent the tools of trade and provide a researcher with ways of collecting, sorting and analyzing data to arrive to conclusions. Using the right research method enables the researcher to convince readers that the conclusions made have some validity and that new knowledge discovered is soundly based.

3.3.1. Case study

This research study employed a qualitative case study method. Scholars have defined case studies differently. Yin (2012) explained a case study as a research strategy that helps one to understand phenomena in real-life situations and the strategy is common in a range of fields such as psychology and sociology. Case studies differ from other research methods in that theystudy real-life situations where the investigated phenomena depend on the context. On the same note, Gerring (2004) described a case study as: "an in-depth study of a single unit (relatively constrained phenomena) where the researcher aims to clarify features of a larger class of similar phenomena" (p. 342). As per Gerring's definition of a case study, one can understand it as an intensive study of a single unit to generalize a larger set of the unit.

A case study is an appropriate methodology to use when the researcher wishes to understand in some depth the participant's views and experiences of the problem under investigation. The third research question of this study focused on collecting data from the participants' views and experiences of how they taught, using the abacus in their mathematics classrooms. Yin (2003) added that the case study is considered when: (a) the focus of the study is to answer how and why questions; (b) you cannot manipulate the behaviours of those involved in the study; and (c) you want to cover contextual conditions because you believe they are relevant to the phenomenon under study. It follows therefore, that for the first research question of this study focusing on how the teachers used the abacus in their classrooms, their responses were not manipulated.

Through case study methods, a researcher can go beyond the quantitative statistical results to understand the behavioural conditions through the actor's perspective. In this study too, participants's experiences and perceptions were collected to understand their behavioural conditions. In qualitative data, a case study helps explain both the process and outcome of a phenomenon through complete observation, reconstruction and analysis of the cases under investigation (Tellis, 1997). In a case study, one or more cases can be investigated. When one case is examined it is referred to as a singular case, while several cases arereferred to as a multiple case study. This study used a multiple case study approach.

Multiple case studies

Yin (2003) explains that when a researcher chooses to do a multiple case study she/he can analyze the data within each situation and also across different situations, unlike when a single case study is chosen. Yin (2003) further argues that multiple case studies can be used to either predict contrasting results for expected reasons or else predict similar results in the study. A multiple case study enables the researcher to explore differences within and between cases. The goal is to replicate findings across cases. Because comparisons will be drawn, the cases must be chosen carefully so that the researcher can predict similar results across cases, or predict contrasting results based on a theory (Yin, 2003). A multiple case study includes extensive and comprehensive data collection of multiple cases through exhaustive and careful procedures such as interviews, documents, and artifacts (Creswell, 2014).

As suggested by Yin, in this multiple case study I explored the differences between the four selected teachers. Multiple case studies can be implemented real-life events that show numerous sources of evidence through replication, rather than through sampling (Yin, 1984). In this multiple case study my research questions underwent a theoretical evolution of my research question. The multiple case studies allowed me to analyze the data within each situation and across different situations. Furthermore, the multiple case study let me understand the similarities and differences between the cases.

Gerring and McDermott (2007) explained the unit of analysis as the actual object or entity being studied and that it (the unit) must be at the same level as the object of the proposition. In this study, the multiple case study included four cases (four Grade 3 mathematics teachers from the four primary schools in the Kalahari circuit of the //Kharas educational region in Namibia) that generated similar results, with the intent to understand how Grade 3 teachers use the abacus to develop their learners' number sense. The four Grade 3 teachers therefore used a physical manipulative in the form of an abacus as a visualization tool to develop number sense. Their experiences and perceptions of the use of the abacus in developing number sense were examined. The unit of analysis of this study was the teachers' practice on the use of an abacus as a visualization tool in developing number sense in a Grade 3 class.

3.3.2 Qualitative Approach

Research approaches are plans and the procedures for research that span the steps from broad assumptions to detailed methods of data collection, analysis, and interpretation. Research approaches according to Bhawna and Gobid (2015) can be classified as quantitative, mixed method and qualitative research.

Aliaga and Gunderson (2000) argues that quantitative research is explaining phenomena by collecting numerical data that are analyzed using mathematically based methods. The methodology of a quantitative research maintains the assumption of an empiricist paradigm (Creswell, 2003). Quantitative Research is the systematic empirical investigation of observable phenomena via statistical, mathematical or computational techniques. Creswell (2003) explained a mixed methods research design as a procedure for collecting, analyzing, and "mixing" both quantitative and qualitative research and methods in a single study to understand a research problem. A mixed methods approach is one in which the researcher tends to base knowledge claims on pragmatic grounds. It employs strategies of inquiry that involve collecting data either simultaneously or sequentially to best understand research problems.

A qualitative approach was used in this study as this research approach is predominantly concerned with the understanding of human experience (Jackson, Drummond, & Camara, 2007). In a qualitative approach, the researcher is interested in understanding how participants make meaning of the situation and the experience they have in the world (Merriam, 2015). A qualitative research approach views human thought and behaviour in a social context and covers a wide range of phenomena to understand and appreciate them thoroughly. This approach facilitates an investigation of a phenomenon within its natural setting using a variety of data. Additionally, Patton (2002) stresses that a qualitative case study is an approach that ensures the issue being researched is not explored through one lens but rather a variety of lenses, which allows for multiple facets of a phenomenon to be revealed and understood.

Farr (2008) believes that qualitative research can provide complex textual descriptions of how people experience a given research issue. It provides information about the "human" side of an issue, that is, the often contradictory behaviours, beliefs, opinions, emotions, and relationships of individuals. Qualitative methods are also effective in identifying intangible factors, such as social norms, socioeconomic status, gender roles, ethnicity and religion, whose role in the research issue may not be readily apparent.

Qualitative research approaches are designed to help researchers understand people and the social and cultural contexts within which they live (Palmer & Bolderston, 2006). The qualitative research approach was used in this study to understand the meaning that the participants ascribed to their perceptions and experiences on the use of the abacus to develop number sense. Cohen et al. (2018) add that multiple sources of evidence can provide convergent and concurrent validity in a case, and the researcher is required to have an ability to handle and synthesize many kinds of data simultaneously.

This study used different data collection instruments as lenses to explore how the Grade 3 teachers utilize the abacus as a visualization tool to develop number sense in mathematics. The study is thus qualitative because of the kinds of data collection and analysis techniques that were used. According to Cohen et al. (2018), qualitative research provides in-depth, intricate, and detailed understanding of meanings and actions; non-observable as well as observable phenomena; attitudes, intentions and behaviours; which are all well served by naturalistic inquiry. They further add that the qualitative research approach gives voice to participants and it probes issues that lie beneath the surface of presenting behaviours and actions. The in-depth interviews, observations, and reflective journal data in this study were collected in the participants' natural settings, attempting to make sense and meaning of their teaching. The use of a qualitative approach helped me gain an in-depth understanding of how the Grade 3 selected teachers use an abacus as a visualization tool to develop the learners' number sense.

3.3.3 Sample and Participants

The quality of research falls and stands by the appropriateness of its methodology and the suitability of the sampling strategy that has been adopted (Cohen et al., 2018). The teachers were the participants in this study and they were carefully selected purposively. Hameed (2016) explains purposive sampling as a strategy in which particular settings, persons or events are selected deliberately to provide important information. Following this, Bertram and Christiansen (2015) say that in purposive sampling the researcher makes specific choices about which people or groups to include in the sample. In purposive sampling, the researcher decides what needs to be known and sets out to find who can and are willing to provide the information, by the knowledge and experience they have.

Selection of the surveyed participants

Participants for this research study were selected at two levels. Purposive sampling was used in this study by selecting the fifty Foundation Phase teachers and the four Grade 3 teachers. Schatzman and Strauss (1973) stress that after several observation visits to the sites, the researcher will know who to sample for the study. They further add that the researcher selects participants according to the aim of the research. The first participants of this study were chosen with the aim of investigating how Foundation Phase teachers are currently using the abacus to develop number sense in their classes. The fifty participants were selected according to the following criteria*: (a) Foundation Phase teachers from the //Kharas educational region; (b) the Foundation Phase teachers (pre-grade to Grade 3) that are currently teaching at this phase at the time of the research; and (c) fully qualified mathematics teachers for the Foundation Phase.* The fifty foundation teacher's survey questionnaire was aimed to answer the first research question of this study. The study used open ended questions in the questionnaire. The participants were not restricted to a set of pre-determined answers. Participants in this study were allowed to fully articulate their thoughts, opinions and experiences on the use of the abacus as a visualization tool to develop number sense in the Foundation Phase. The surveyed participants were all from the //Kharas region, where the sample of the 12 participants was selected from them to take part in the intervention programme.

Selection of the intervention programme participants

Secondly, twelve participants from the four primary schools in Keetmanshoop took part in the intervention workshop. The twelve participants were purposively selected for the intervention workshop on the basis of their willingness to attend the workshop and that they were geographically conveniently located to the workshop venue. This was to minimize travelling costs and time required to get to the venue, since all the workshops and intervention activities were done in the afternoons. Hameed (2016) emphasizes that it is doubtful whether it is possible for the researcher to collect data from all the cases. The researcher should therefore reduce the number of cases. From the twelve participants, I further selected four Grade 3 mathematics teachers from the four primary schools in the Kalahari circuit to be part of the intervention programme. For this study, the four teachers were selected based on their involvement and willingness to participate in the Phase 1 Workshop.

My research questions played a key role in deciding the type of participants I was to work with in this study. Tongco (2007) argues in purposive sampling, the question the researcher is interested in answering is of utmost importance for decisions about the methodology and the selection of the participants. Sargeant (2012) notes that the researcher needs to be clear on the research question(s), and select participants who can best and most broadly inform the question(s). He further argues that the participants should be recruited purposefully until saturation or a thorough understanding of the phenomenon is achieved. The four participating teachers were thus selected purposively according to the following criteria:

a) they had to be qualified Grade 3 mathematics teachers;
b) they had to be teaching Grade 3 at the time of this research;
c) they were from the four primary schools in the Kalahari circuit;
d) they showed willingness to use the abacus as a visualization tool to develop number sense in their Grade 3 classes during the Phase 1 Workshop; and
e) they were willing to take part in this study.

The five criteria above were used because the study primarily aims to find out how the Grade 3 teachers utilize the abacus to develop number sense in their learners. The criteria above provided me with the data I needed to answer the second research question of this study.

3.3.4. Profiling and coding of participants

For easy understanding of the data analysis section in the next chapter, the teachers were coded for easy identification purposes and to keep them anonymous throughout the study. The first set of fifty Foundation Phase teachers and were coded as Tr 1; Tr 2, Tr 3...Tr 50. Tr in this study represents 'teacher' and there were 50 survey participants, so the coding ranges from Tr1 to Tr 50. Allen and Wiles (2016) argue that renaming participants by allocating pseudonyms to confer anonymity to both participants and the content of the research, is common practice.

For the reflective journals, the lesson observations and the interviews, the following pseudonyms were used to keep the anonymity of the four participants: Mr Lebby, Ms Dave, Mr Quasi, and Ms Tyler. The teachers each presented five lessons that were recorded, and the recorded lessons were coded as shown in Table 3.1. For example the first lesson for Ms Dave is coded as DV1. The recorded lessons were presented in the afternoons so that they did not affect the normal teaching time of the learners. After the lesson presentations, a semi-structured interview was conducted between the researcher and each participant. The interview was aimed at collecting open ended data, explore participants' thoughts, experiences and perceptions on the use of the abacus in the Foundation Phase. The interview data in this study were coded as shown in Table 3.1; for example the semi-structured interview for Mr Lebby is coded as LSSI.

Table 3.1: Coding and profiling of teachers

Participants	Qualifications	Years of teaching experience	Video 1	Video 2	Video 3	Video 4	Video 5	Interview
Mr Lebby	Bachelor of Education with Honours	6 years	LV1	LV2	LV3	LV4	LV5	LSSI
Ms Dave	Bachelor of Education with Honours	6 years	DV1	DV2	DV3	DV4	DV5	DSSI
Mr Quasi	Diploma in Education	10 years	QV1	QV2	QV3	QV4	QV5	QSSI
Ms Tyler	Diploma in Education	8 years	TV1	TV2	TV3	TV4	TV5	TSSI

Lesson description

The four participants each presented five lessons and these lessons were cooperatively planned during the intervention workshop. Lessons were cooperatively planned with the aim of encouraging one another and focusing on effective differentiation, misconceptions and consider alternative ways of teaching using the abacus. Each of the lessons was 40 minutes long. The lessons, which were all presented to Grade 3 learners are described and summarized in Table 3.2 below:

Table 3.2: Lesson descriptions

Lesson	Learning objectives
1. Place Value	Learners should be able to recognize three-digit numbers up to 1000, so that the digit position in the number shows the quantity e.g. given the digits '1', '8' and '4',order the numbers from the biggest to the smallest three-digit number: 841 and 148.
2. Addition	Learners should be able to break down and build up two and three-digit numbers in different ways. They should be able to add numbers in tens up to 500; add two numbers up to 100 mentally (without writing); add two or more numbers with a sum of between 0 and 500, by breaking down and building up the numbers using place value e.g. 342 + 185=(300+100) +(40+80)+ (2+5).

3. Subtraction	Learners should be able to subtract two or three-digit numbers from a three-digit number between 100 and 500 using subtraction of tens, using place value, breaking down and building up. They should be able to subtract a single-digit number from a three-digit number mentally and explore the number pattern for example: **100-37=63; 200-37=163; 300-37=263**
4. Multiplication	Learners should be able to multiply any two-digit number by 2, 3, 4, 5 and 10 with a product of between 100 and 500 using decomposition. They should be able to multiply any two-digit number up to 50 by any number between 1 and 10, through repeated addition for example 12×4= 12+12+12+12.
5. Division	Learners should be able to apply and recall simple division facts related to the multiplication facts of 2, 3, 5 and 10, for example 20÷2=10; 2×10=20 (division as inverse operation of multiplication)

3.4. RESEARCH DESIGN

Research design is explained by Creswell (2009, p. 3) "as the plan and procedures for research span the decision from broad assumptions to detailed methods of data collection and analysis". Considering Creswell's definition, research design means that the plan provides the overall framework of how data are collected and the results provided can be judged as credible. This study employed a qualitative research design. Qualitative research design used in this study helped with collecting primarily data and derives meaning from participants' perspectives. The research design of this study unfolded in five phases. This was necessary to enhance validity and reliability, and for triangulation through the inclusion of a variety of data collection techniques and methods.

The five phases of this study are detailed below.

Phase 1: Survey- Pilot and administration

According to Denscombe (2014) piloting refers to the conduct of the preliminary research, prior to the main study. Piloting provides structured opportunity for informed reflection, modification of, the research design, the research instrument and timing.

Piloting- In phase 1 of this study, I designed questionnaires and piloted them first. The piloted questionnaires helped me with checking ambiguities in the questions, use of simple language and the time required to complete the questionnaires, among others. During the piloting process I identified some ambiguities in the questionnaire, such as some keywords and phrases which were unclear; there were double-barreled questions and some questions were biased. After the piloting process, I revised and refined the questionnaire by making use of precise language, proofreading, and spell checked the questions. To avoid ambiguity, and the double-barreled questions, I broke them down to ask one question. For example, before piloting the questionnaire, question 6 read: *Do you use the abacus as a secondary tool and support*? and after piloting it changed to *In your opinion is the abacus a useful teaching tool? Explain your answer*. Abbreviations, acronyms, and words that meant more than one thing were also removed after the piloting process. The questionnaire before piloting consisted of abbreviations like JP for Junior primary, AB for the abacus, NS number sense, which were later removed after piloting.

Distribution of the Questionnaire- The revised and refined questionnaires were then distributed to 50 foundation teachers in the //Kharas region. The 50 questionnaires were all returned to the researcher. Some questionnaires were partially completed and some questions were not elaborated on by the participants as required. The survey questionnaire served as one of my primary data collection tools. According to Cohen et al. (2018), typically surveys gather data at a particular point in time, intending to describe the nature of existing conditions, identifying standards against which existing conditions can be compared, or determining the relationships that exist between specific events.

Phase 2: Securing of a research site and selection of participants

In this phase, I requested permission from the Director of Education as well as from the school principals where the research was to be conducted (See Appendices B and C). Participants were selected as per the criteria mentioned in Section 3.4.3. Ethics considerations were adhered to as mentioned in Section 3.9 of this study.

Phase 3: Abacus workshop

The planning of the intervention workshop took place at the Teacher Resource Center in Keetmanshoop, which is at the central point of all the schools in the district and reachable by all participating teachers. The intervention workshop was conducted over two weeks, three days a week, in the afternoons. The workshop was attended by twelve Grade 3 teachers from the four primary schools (three Grade 3 teachers from each school) in the Kalahari circuit. The participants had to complete consent documents (See Appendix D) to take part in the intervention workshop. The twelve and four participants were selected as per the criteria mentioned earlier in Section 3.3.3 above.

During the workshop, the participating teachers and I designed abacuses that were used during the intervention programme. Participating teachers were part of the designing and producing of abacuses so that they could make their own abacuses at a later stage. The participating teachers all produced reflective journals, where they provided an account of their work in progress, reflections on the abacus workshop and their experiences and perceptions. The information obtained in these reflective journals formed part of my data collection and helped me to answer research questions 2 and 3.

In this phase, participating teachers learned how to do calculations using the abacus to help learners develop number sense in the Foundation Phase. In addition to this, different ways of developing the learners' number sense in mathematics using the abacus were discussed at the workshop. During the workshop the four participating teachers cooperatively planned five lessons on place value, addition, subtraction, division, and multiplication.

Piloting of lesson observation

Lesson observation was piloted with Grade 4 teachers. Two Grade 4 mathematics teachers were observed teaching the concept of place value to their learners. The camera was placed and the back of the class to test the video recording instruments and the researcher used the analytical tool in Appendix H for the lesson observation. After the lesson observation, it was discovered that there was a number sense theme in the analytical tool that did not apply to the five lessons taught and it was removed.

Phase 4: Implementation of the intervention programme

Each of the four selected teachers was observed teaching his/her five lessons planned in Phase 3. The observation focused on these teachers' use of the abacus to develop number sense during their teaching of mathematics. In this phase, all lessons presented by the teachers were video recorded and transcribed for data analysis purposes. The participating teachers were required to use their reflective journals to reflect on each lesson they presented using the abacus. The lesson observation and video recordings were analyzed using the analytical tool in Appendix H. The data collected in this phase was used to answer the second research question.

Phase 5: Interviewing teachers

In this phase, semi-structured Interviews (SSI) were done with the participants to understand their perceptions and experiences of using the abacus as a visualization tool. Each participant was interviewed once after teaching his/her five lessons, with the interviews lasting for 30 minutes each. The interviews were audio recorded and later transcribed. The data obtained in this phase, together with the teachers' reflective journals, answered the third research question of my study.

3.5 DATA COLLECTION TECHNIQUES

Case studies typically include multiple data collection techniques since the data are collected from multiple sources. The use of these multiple data collection techniques and sources strengthens the credibility of the outcomes in the data analysis. Yin (2014) asserts that data collection techniques include interviews, observations, questionnaires and relevant documents. This study is a qualitative case study and it employed questionnaires, reflective journals, observations and interviews as the data collection methods.

3.5.1. Questionnaires

One of the primary data collection instruments used in this study was the questionnaire. Invented by Garton Francis, a questionnaire is the backbone of any survey, the success of which lies in its design which aims at collecting primary data (Roopa & Rani, 2017). Roopa and Rani (2017) further argue that a questionnaire is simply a list of mimeographed or printed questions that are completed by or for a respondent to give his opinion. A questionnaire is defined by Abawi (2013) as a data collection instrument consisting of a series of questions and other prompts to gather information from respondents. According to Denscombe (2014), a research tool qualifies as a questionnaire when it meets the following criteria: firstly, it should be designed to collect information that can subsequently be used as data analysis; secondly, it should consist of set of questions; and lastly, it should gather information by asking people directly about the points concerned with research.

According to Roopa and Rani (2017) a questionnaire enables data to be collected in a standardized way so that the data are internally consistent and coherent for analysis. They further assert that questionnaires should always have a definite purpose that is related to the objectives of the research, and that from the outset it needs to be clear how the findings will be used. In this study, the questionnaire was necessary because it provided me with a lot of data on the current use of the abacus in the Foundation Phase in the //Kharas region.

This study is a qualitative study that employed a qualitative questionnaire. The questionnaires were necessary to find out whether the abacus is currently being used in the Foundation Phase and if it is being used, what it is used for. The questions in the questionnaire were chosen in a way that they gave answers on the current use of the abacus and the development of number sense in the Foundation Phase. The questionnaire (See Appendix G) consisted of open-ended questions which allowed the participants to narrate their understanding, experiences and opinions in their own ways. The open-ended questions used in the questionnaire gave the participants extra detail to help qualify and clarify their responses. The open-ended questions allowed the participants to convey their feedback and ideas in their own voices. To collect data from the questionnaires, 50 questionnaires were distributed to 50 Foundation Phase teachers in the //Kharas region. The questionnaires focused on finding out whether or not the Grade 3 teachers use the abacus in teaching mathematics and if they use the abacus, on what occasions they use it and how.

3.5.2. Classroom observations

Observation is used by researchers to "obtain real-time data about subject's behaviour, interactions, and experiences from the actual environment in which they naturally occur" (Picardi & Masick, 2013, p. 132). Mack, Woodsong, Macqueen, Guest, and Namey (2005) define observation as a qualitative method with roots in traditional ethnographic research, whose objective is to help researchers learn the perspectives held by populations under study. Cohen et al. (2007) reiterate that the distinctive feature of observation as a research process is that it offers an investigator an opportunity to gather live data from naturally occurring situations. The data obtained from observation are first-hand, due to the reason that the researcher goes to the site of the study, which in educational settings can be a school, a classroom or a staffroom, to collect these kinds of data. When it comes to the observation as a research instrument, the researcher looks directly at what is taking place in a situation rather than relying on a second hand account.

The observations were necessary for this study because it provided the researcher with information that participants could have been unwilling to discuss in the interviews. Mack et al. (2005) assert that participant observation always takes place in community settings, in locations believed to have some relevance to the research questions. In this study, the four participants were observed by the researcher teaching their planned lessons. The lesson observations were conducted in the afternoons after the intervention programme so that they did not affect the normal teaching time of the learners. During the observations, the researcher used the analytical tool in Appendix H which was designed with number sense observable indicators for teaching using the abacus. The analytical tool consisted of the number sense themes that the teachers developed during their teaching, using the abacus. During observation, the researcher gathered data by watching the behaviour and events or noting down physical characteristics in their natural settings. The observations were done to collect data to answer the second research question of this study. By judging and drawing inferences from the participants' lesson presentations, I could interpret whether participants' presentations were answering my research question. I video recorded all twenty lessons that the teachers presented, using the abacus as a visualization tool to develop number sense. The lessons were video recorded to collect data that answered the second research question of this study.

3.5.3 Reflective journals

The participating teachers all produced reflective journals, where they provided an account of their work in progress, reflections on the abacus workshop and their experiences and perceptions from the beginning of the intervention programme until the end. Russell and Kelly (2002) indicate that keeping self-reflective journals during the analysis process is a strategy that facilitates reflexivity, by using the researchers' journals to examine "personal assumptions and goals" and to clarify "individual belief systems and subjectivities" (p. 2). They further explain that a reflective journal includes the details of what the researchers or the participants did, thought, and felt during the research process, and the rationale behind those thoughts and percepts are then recorded.

The use of the reflective journals in this study allowed me to hear the voices of my participants through the chance given to them to express their perceptions and changes they experienced as a result of participating in the intervention programme. The reflective journals allowed my participants to talk about their presuppositions, experiences, actions and rationales during the course of the intervention programme. The data from the reflective journals were analyzed using thematic analysis. The data collected from these reflective journals formed part of my data collection and helped me with answering Research Questions 2 and 3, which seek to understand how teachers use the abacus, and their perceptions and experiences of the use of the abacus to enhance number sense in their learners.

3.5.4. Interviews

Interviews were one of my other data collection techniques. The qualitative research interview seeks to describe the meanings of central themes in the life world of the subjects. The main task in interviewing is to understand the meaning of what the interviewees say (Kvale, 1996). (Hochschild, 2009) notes that the interview can do what surveys cannot, which is to explore issues in depth, to see how and why people frame their ideas in the ways that they do, and how and why they make connections between ideas, values, events, opinions, behaviours, etc. Denscombe (2014) views research interviews as a method of data collection that uses people's answers to researchers' questions as to their source of data. The interview in general has something in common with the questionnaire as they both depend on the data that comes from what people tell the researcher.

The four participants were interviewed after presenting their five planned lessons. The interviews were based on the questions on the perceptions and experiences of using the abacus as a visualization tool to develop number sense in mathematics (See Appendix J). The use of the interview in this study allowed me to explore the lived experience of the participating teachers. The interviews aimed at understanding my research participants' terms and how they make meaning of the world in which they live, in experiences and cognitive processes. The interviews helped me to collect data and produce knowledge that could not directly be observed during the observation stage. The data collected from the semi-structured interviews answered the third research question which seeks to understand the participant's experiences and perceptions of the use of the abacus as a visualization tool to develop number sense.

3.6 DATA ANALYSIS

According to Taylor and Gibbs as cited in Cohen et al. (2018), qualitative data analysis concerns how we move from the data to understanding, explaining, and interpreting the phenomena in question. They further add that qualitative data analysis implies making sense of data in terms of participants' definitions of the situation, noting patterns, themes, categories, and regularities. Data captured in this qualitative research study were interpreted, analyzed, and presented systematically.

Thematic analysis

Research Questions 1 and 3 of this study were analyzed using thematic analysis. Braun and Clarke (2006) define thematic analysis as a "method for identifying, analyzing, and reporting patterns. They further argue that thematic analysis is an approach to analysing qualitative data to answer broad or narrow research questions about people's experiences, views and perceptions, and representations of a given phenomena. In this study thematic analysis was used to analyze participants' views and experiences about abacus use in mathematics to develop learners' number sense. The themes analyzed in this study were those developed by research questions in the questionnaires, interviews, reflective journals and emergent themes from the collected data. Boyatzis (1998) explains thematic analysis as a qualitative analysis that is used to analyze, classify and present themes (patterns) that relate to the data. Thematic analysis in this study allowed me to see and make sense of the shared meanings and experiences of the teachers of the use of the abacus to develop number sense.

The data analysis in this study was done in three stages as shown in Table 3.3 below:

Table 3.3: Data analysis table

Stage	Data source	Analysis	Purpose of the Instrument
Stage 1	Survey questionnaire	Analyzed questionnaires to find out about the use of the abacus in schools. Analyzed teachers other ways of teaching number sense. Establish the emerging themes.	Provide data for Research Question 1
Stage 2	Observation	The transcribed video was taken during teaching.	Provide data for Research Question 2

		Analyzed the lessons taught as per the analytical tool in A**ppendix H**.	
Stage 3	Interview and reflective journals	Analyzed the interview and reflective journal using thematic analysis. Establish emerging themes.	Provide data for Research Question 3

Stage 1

The first stage analyzed data from the survey questionnaire. The survey focused on how the Foundation Phase teachers teach number sense, whether or not they use the abacus in teaching mathematics, if they do use it when they use it, and how. Data from the questionnaires were analyzed according to the themes as they arose from different questions in the questionnaire. I used the following organizational themes shown below to classify the qualitative data drawn from the survey questionnaires: **the current use of the abacus; developing number sense and the use of other manipulatives; benefits and the usefulness of using the abacus; sense-making using the abacus.** The themes were generated from the questionnaire questions and some emerged from the questionnaire data.

Stage 2

The second stage analyzed qualitative data from the classroom observations. During qualitative analysis, words, text, or images are combined into categories of information that are presented to show the variety of ideas that were gathered during the data collection process (Creswell & Plano Clark, 2007). The analytical tool in **Appendix H** was used to analyze data in this stage to observe teachers using the abacus to develop number sense themes in their Grade 3 classes. The analytical tool has observable indicators of number sense themes. The themes and the emergent themes were categorized with the codes using color codes. For example, when a teacher presented her/his lesson when teaching the number sense theme 'multiple representations of numbers', I then indicated the theme code MRP from the analytical tool beside it.

Stage 3

Stage 3 analyzed data from the teacher's reflective journals and interviews. This stage of analysis involved identifying emergent themes from the interviews and reflective journals, using thematic analysis. Braun and Clarke (2006) define thematic analysis as a "method for identifying, analyzing, and reporting patterns (themes) within data" (p. 6.). The reflective journals were analyzed using the themes developed from literature such as: understanding the meanings and sizes of numbers; Understanding the meanings and effects of operations; understanding and use of equivalent expressions; and flexible computing and counting strategies for mental computation and measurement benchmarks. The themes that emerged from the journal were analyzed as aspects that emanated from teachers' responses in their journals.

The data collected from the interview were analyzed by using the inductive approach in looking at the teachers' experiences and perceptions of using the abacus after the intervention programme. The themes used to analyze the interviews were derived from the research questions in Appendix I. The data that emerged from the interview data were also analyzed to form part of the data that answered the third research question. Neuman (2003) states that the inductive approach aims to generate meanings from the data set collected to identify patterns and relationships among data.

3.7 VALIDITY AND RELIABILITY

According to Ary, Jacobs, Razavieh and Sorensen (2006) as cited in Cohen et al. (2018), validity is explained as the extent to which the interpretation of data is warranted by the theories and evidence used. They further emphasize that researchers must indicate the grounds and the evidence that they use to connect their data with the claims made from, or conclusions drawn from, the data. "For research to be reliable it must demonstrate that if it were to be carried out on a similar group of respondents in a similar context (however defined), then similar results would be found" (Cohen et al., 2018, p. 235).

Reliability reflects consistency and replicability over time. Furthermore, reliability is seen as the degree to which a test is free from measurement errors, since the more measurement errors occur the less reliable the test (Fraenkel &Wallen, 2003). To address the issue of validity and reliability, I used a variety of data generation methods to triangulate the data, namely: questionnaires, reflective journals, observations, and interviews. Cohen et al. (2018) define triangulation as the use of two or more methods of data collection in the study of some aspect of human behaviour. In the same vein, Creswell (2012, p. 259) defines "triangulation as the process of substantiating evidence from different individuals, types of data, or methods of data collection in descriptions and themes in qualitative research". The first phase of this study employed a survey questionnaire. The second phase constituted a qualitative approach where reflective journals, observations, and interviews were the main data collection techniques. Cohen et al. (2018) recommend that qualitative validity should be addressed through the honesty, depth, richness, and scope of the data achieved. Furthermore, data were interpreted based on underpinning theory and analytical frameworks, following the case study and are therefore not my view and opinions.

Moreover, to ensure the validity of my research project, I piloted the questionnaire, which helped me with checking ambiguities and language in the questions, and the time required. The interview was piloted to test the interview questions and make sure they answer the third research question. The lesson observations were piloted using Grade 4 mathematics teachers, and the video recording ensures effective and productive filming.

The latter was done by using the member-checking technique. I returned the results to the participants to check for accuracy and resonance with their experiences, and in this way, explored the credibility of the results. This ensured that the information collected from this study was not biased or factually flawed.

3.8 ETHICS

Ethics has been defined as "a matter of principled sensitivity to the rights of others" (Cavan, 1977, p.810). Educational researchers must take into account the effects of the research on participants; they have a responsibility to participants to act in such a way as to preserve their dignity as human beings. This research study adhered to research ethics by ensuring the following ethical principles**:**

As a researcher first I sought verbal permission from the 50 participants through their school principals. Secondly, I sought permission for access to the four schools where my research was conducted. I wrote a letter to the //Kharas regional director to seek permission for carrying out the research. Furthermore, four letters asking for permission were sent to the four school principals in the Kalahari circuit of the //Kharas educational region where the research observation took place. I also sought consent from the four participating teachers in my research project. Cohen et al.,

(2018) consider that the research participants should take part voluntarily, free from any coercion or under influence, and their rights, dignity, and (when possible) autonomy should be respected and appropriately protected. In this study before the intervention programme, I explained to the participants their rights, that the research was voluntary and they were allowed to withdraw at any time during the research study if they wish to.

The UK Economic and Social Research Council, as cited by Cohen et al. (2018), determine that individual research participant and group preferences regarding anonymity should be respected and participant requirements concerning the confidential nature of information and personal data should be respected. They further stress that the essence of anonymity is that information provided by participants should in no way reveal their identity. To guarantee the anonymity and confidentiality of my research participants I did not use my participants' real names when presenting data. Pseudonyms such as Tr1, Tr 2...Tr 50 for the survey study, and Dave, Lebby, Quasi, and Tyler were used for observations and interviews. The names of the schools were also kept anonymous in this study as School A, B, C and D. Parents and guardians of the Grade 3 learners were also asked for consent and the anonymity and confidentiality of their children was assured. Learners signed assent letters and agreed to be present during the observation process. I explained to the parents and learners that the research was voluntary and the learners could leave at any stage of the research if they no longer felt like being part of the study.

In this study I demonstrated honesty and transparency by explaining the purpose of the research to the participants. The participants were all informed about the intention of the study. Further transparency in this study was exercised by making sure that information related to the study was dessiminated to all the participants. Brooks, Te Rielle and Maguire (2014) remark: "power relations are imminent in all research settings" (p. 106) and researchers may hold different social and power positions from participants. As I am a Master's student teacher, I ensured that power relations were reduced as much as possible by encouraging the participants to give their honest opinions without fear of any repercussion. I assured the participants that their participation in this study would in no way have any compromised impact on their participation in my class. As I am an Upper Primary teacher and they are all in the Lower Primary, I explained to the learners the purpose of my presence in their classrooms and that they should not get worried about it.

This research study was guided by honesty and integrity and through this research study, I upheld the professional academic standards and integrity demanded by Rhodes University by adhering to the laid down rules. The findings were based on authentic data I collected. The collected data were presented in the way they were collected without manipulating or influencing them. Data analysis for this study was based on my empirical work and not on my assumptions.

Denscombe (2014) explains research risk as the probability of harm (physical, psychological, social, legal, or economic) occurring as a result of participation in a research study. He further adds that both the probability and the magnitude of possible harm may vary from minimal to significant. In this study, there were minimal social risks and they were considered when it came to ethical principles. During the lesson presentation, some of the learner's faces popped up in the video, and I therefore blurred the faces to hide the children's identities. Some of the learners were upset by the video and mechanical equipment that were present in their classroom, so I explained the purpose of my presence to the learners that they should not get worried about it. Teachers had a sense of uneasiness when presenting the observed lessons due to my presence. I therefore explained to the teachers clearly what the purpose of the observation was, that the data collected would only be used for this specific study and that only my supervisor and I would have access to the collected data.

This research study has benefits associated with it**.** The findings of this study can be used by mathematics teachers in primary schools for their teaching by learning how to integrate the abacus more strategically in their lesson presentations. This would ultimately contribute positively to learner engagement and interest in mathematics.The teachers that formed part of the study came together and shared knowledge they gained in the intervention programme. The participants shared the challenges they experienced in the intervention and how they overcame these challenges. The participants also shared other ways of developing number sense at Foundation Phase level.

3.9. CONCLUSION

This chapter outlines the research design. I have elaborated further on the methodology, namely the case study approach, the methods of data collection employed and detailed the research process used in this study. Also, as the procedure was qualitative in nature, the methods of data collection used were consistent with the tenets of qualitative research. Finally, issues pertaining to the reliability and validity of this research, as well as ethical considerations that guided me in the process were discussed in relation to the research site, the nature of participation and data collection methods.

In the next chapter a detailed description of the findings and data analysis is presented.

CHAPTER 4

DATA ANALYSIS AND PRESENTATION

4.1. INTRODUCTION

This chapter presents, analyzes, and discusses the results from the data collected on the investigation of how the abacus can be utilized as a visualization tool to develop number sense in a Grade 3 class. In addition, the study also looked at the teachers' experiences and perceptions on the use of the abacus in the Foundation Phase to develop number sense. The data were collected through questionnaires, reflective journals, lesson observations, and semi-structured interviews. Data from the survey questionnaire was used to answer Research Question 1, Research Question 2 was answered using data collected from the reflective journals and lesson observation, and lastly, data collected from the interviews and reflective journals on the experiences and perceptions of the teachers were used to answer Research Question 3. This chapter attempts to carefully analyze the use of the abacus to teach mathematics to develop number sense in learners in the four primary schools in the Kalahari circuit of the //Kharas educational region.

The findings of this study thus provide necessary information and evidence to answer my three main research questions which are:

1. How did teachers use the abacus when teaching Grade 3 mathematics classes prior to participating in the intervention programme?
2. How can the abacus be utilized as a visualization tool to develop number sense in a Grade 3 class when teaching mathematics?
3. What are the teachers' experiences and perceptions of using an abacus as a visualization tool in developing learners' number sense after participating in the intervention programme?

4.2. RESULTS FROM SURVEY QUESTIONNAIRE

The survey questionnaire was designed to find an answer to the first research question which is: ***How did teachers use the abacus when teaching Grade 3 mathematics classes prior to participating in the intervention programme?*** The data were analyzed using the thematic analysis method. The themes used in this study emanated from the data and some themes were developed from the questions presented in the questionnaire.

4.2.1 The current use of the abacus

The teacher questionnaire (TQ) was distributed to 50 Foundation Phase teachers (n = 50) in the //Kharas region. According to the findings of the TQ on the current use of the abacus, 66% of the respondents (see Figure 4.1) did not use the abacus in their classrooms while the remaining 34% indicated that they were currently using the abacus.

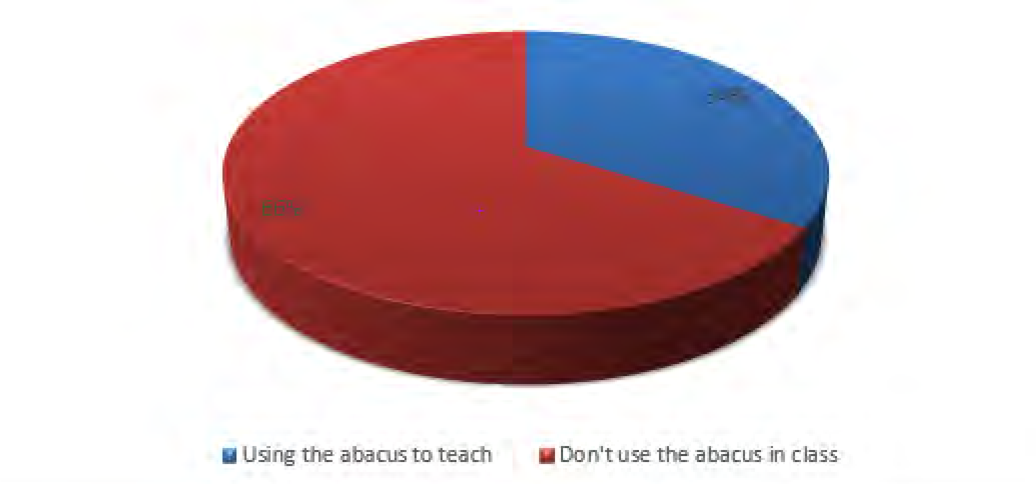

Figure 4.1: The use of the abacus before the intervention

As indicated in Figure 4.1, only a few of the surveyed teachers were using the abacus when teaching mathematics to the Foundation Phase classes. It was necessary to look at the teachers' current use of the abacus; this kind of data provided me with insight into what was currently going on in the classroom when using the abacus as a visualization tool in developing number sense. The teachers' responses helped me to plan the intervention that looked at how an abacus can be used to develop number sense in learners.

4.2.2 The use of the abacus

The surveyed teachers in this study indicated what they use the abacus for in the Foundation Phase: they indicated that they use it (the abacus) for various purposes and teaching different content as indicated in their responses in Table 4.1.

Table 4.1: Responses of the teachers on the use of the abacus

Teacher	Responses
Tr 1	Counting from one to 100. I use it sometimes when the learners are struggling to count.
Tr 5	I use it as a learning aid counting, grouping, and teaching colour
Tr 13	Sometimes to teach the concept of place value and explaining the position of digits to determine their value
Tr 18	To teach addition whereby learners add up, to teach subtraction whereby learners take away and division for grouping.
Tr 28	Place value, reading numbers, expanded notation, addition, and subtraction.
Tr 31	Linking of the abstract mathematics content to the concrete way of doing mathematics
Tr 34	We use it for counting manipulative and counting of concrete objects when we do mathematics

As shown in Table 4.1, Tr 1 indicated that she uses the abacus as a 100-bead counting tool only in cases where learners are struggling with counting. Tr 5 used the abacus as a learning aid for grouping of numbers and he also uses it for teaching colour. In Table 4.1, Tr 13 indicated that he uses an abacus to explain the concept of place value of the individual digits in a given number and the position of digits to determine their value. Some teachers explained that they use the ancient tool when explaining the position of each digit to determine its value in relation to the written symbols, and this idea can be extended for an undefined number of places to the left (for whole numbers) or right (for decimal fractions).

Tr 18 and Tr 28 indicated that they use the abacus to do calculations with all basic operations, which they perform very similarly to the standard pencil and paper algorithm, and also as a way to get learners to approach numbers in different ways. In the teaching of addition, learners add beads, and for subtraction the participants physically showed the learners that they needed to take away beads – which is an easier method to get the answer compared to the traditional algorithm using pencil and paper..

Tr 31's response on the use of the abacus was that he uses it for linking abstract mathematics content to concrete ways of doing mathematics. Table 4.1 shows that Tr 34 indicated that she uses the abacus as a concrete tool when learners have mathematics learning problems. She further added that developing a concrete understanding of mathematics concepts helps learners to perform math skills and this enables them to understand the concepts at the abstract level.

4.2.3 Developing of number sense and the use of manipulatives

Teachers in the //Kharas region used various types of manipulatives to teach number sense (See Figure 4.2). In this section of the study, I was interested in finding out other types of manipulatives apart from the abacus', that the teachers use in developing their learners' number sense in the Foundation Phase.

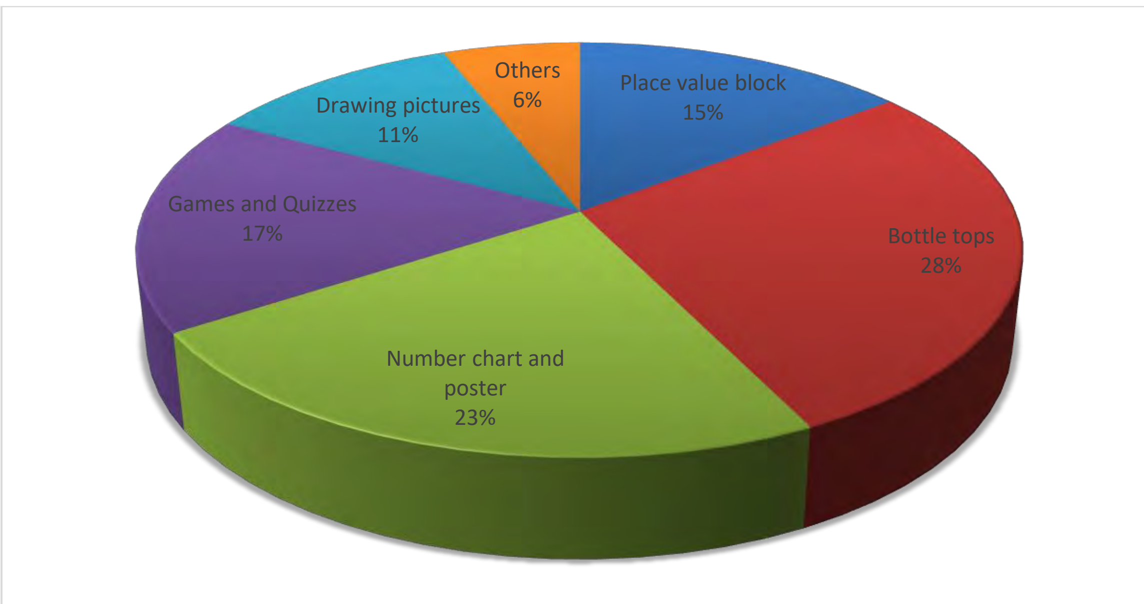

Figure 4.2: Types of manipulatives used to teach mathematics

The survey data indicated that 30% of the teachers (see Figure 4.2) used bottle tops as a means of counting and performing arithmetic when doing computation to develop number sense. A secure number sense is the ability to use numbers flexibly, to understand what numbers mean and their relationships to one another. It enables learners to be competent and confident with numbers and calculations.

According to the survey data in Figure 4.2, 24% of the teachers used number charts as teaching manipulatives in their Foundation Phase classes. Tr 13 indicated: "*We draw pictures that let the learners visualize the content they are learning and make sense of them and let the learners write number sentences and their own number stories".* This echoes the theorist Piaget that suggests that children do not have the mental maturity to grasp abstract mathematical concepts presented in words or symbols alone and need many experiences with concrete materials and drawings for learning to occur (Piaget, 1970).

As presented in Figure 4.2, 17% of the teachers used mathematical games and quizzes to develop number sense when teaching mathematics in the Foundation Phase. Tr 20 indicated that: *"learners get actively involved in the lessons when playing games. When teaching mathematics using games gives the learners a sense of suspense, joy, and fun."* Tr 38's response to what manipulatives he used, was: *"...playing games and math quizzes digital or manual and modeling of dough for digit formation."* Research literature has shown that using games in teaching mathematics leads to an improved attitude towards mathematics, enhanced motivation and support for the development of children's problem-solving abilities. It is also argued that the mathematical discussions that happen while playing mathematical games leads to the development of mathematical understanding (Skemp, 1993).

Other teachers (6%) use different concrete materials that learners can touch and thus connect real-world objects to abstract math content they are learning in the classroom.

4.2.4 Benefits and the usefulness of the abacus

Figure 4.3 below shows the benefits of using the abacus as indicated by the surveyed teachers. The benefits of using the abacus in this study were summarized into four emergent themes, namely improving the proficiency of mathematics; developing logical understanding and reasoning; improving learning of kinesthetic learners; and sharpening concentration and improving learners' memory.

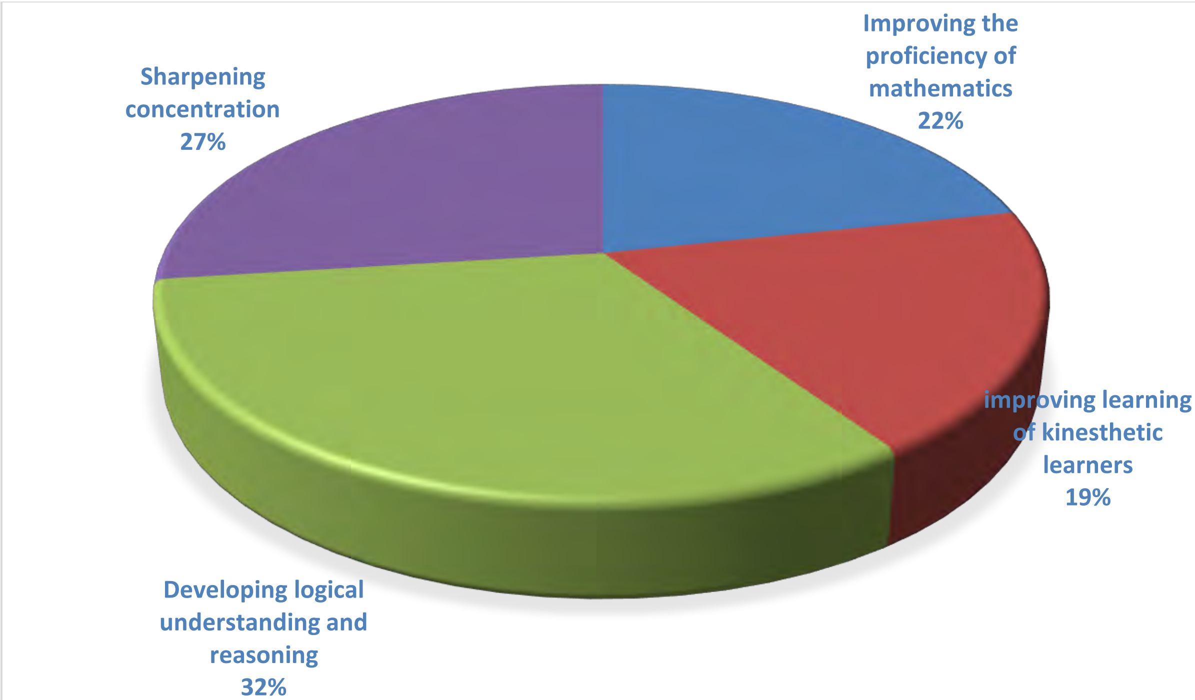

Figure 4.3: The benefits of using the abacus

As indicated in Figure 4.3, 22% of the teachers pointed out that improvement in mathematics proficiency is one of the benefits of using the abacus. Tr 1 said that using the abacus improves the learner's mathematical thinking and this is supported by Tr 30 who added that "*it improves visualization and leads to a better understanding of concepts*".

Figure 4.3. indicates that 32% of the surveyed participants mentioned that the use of the abacus leads the learners to develop logical reasoning in mathematics. Tr 6 said that the learners tend to apply logic based on the movement of the beads on the abacus in order to calculate. Tr 10 further responded that the abacus helps learners to develop their fine motor skills and eye-hand coordination. Tr 14 talked of the positive use of the abacus producing logical understanding for the learners by enabling them to visualize a mental abacus where they move beads in their minds when doing calculations. Tr 2 stated that the abacus helps the learners to develop mental math ability which they use in their everyday lives. Some of the teachers stressed that the use of the abacus generates and explores new mathematical ideas and learners learn mathematical relationships through mathematical activities and discussions. Piaget (1970) also emphasized the significance of manipulatives in a child's construction of logical-mathematical knowledge.

From the response in Figure 4.3, it can be seen that 19% of teachers in the //Kharas region indicated that the abacus helps learners who need to manipulate objects to learn. These kinesthetic learners need to move in order to learn and tactile learners need something to manipulate to fully understand the concept. Tr 7 said that "*When learners are touching and moving the beads to do simple arithmetic and counting it improves their mathematics performances and understanding"*. Similar to Tr 7, Tr 28 responded that: "*the problem-solving abilities improve in learners using the abacus and manipulating the abacus enables learners to understand other mathematical ideas and they connect these mathematical ideas across different areas of mathematics".*

Teachers also responded that the abacus sharpens concentration and improves the learner's memory when doing mathematics. In Figure 4.3, 27% of the teachers mentioned concentration and memory improvement as some of the abacus benefits. Tr 40 stated that the use of the abacus makes learners understand the logic behind the mathematical problems and that it sharpens concentration and improves their memory which then helps with basic calculations. Tr 26 added that: "*learning mathematics using the abacus enhances the concentration of the children as the abacus requires the children to be focused on the number and perform the calculation fast and precisely".*

4.2.5 Sense-making using the abacus

Teachers in this study were asked how they use the abacus to make sense of what the learners are learning. Some of the teacher's responses are given in Table 4.2 below.

Table 4.2: Teachers' ways of using the abacus for sense-making

Teacher	Responses
Tr 44	Do mathematical reasoning to connect ideas and gain a deeper conceptual understanding.
Tr 1	Involve learners in number talks on the conversation about mental mathematics. Learners are given problems to solve using the abacus physically and later solve the same problem mentally which will generate the same results. Through problem-solving, the learners communicate their thinking to the class whereby they make sense of the concept they are learning.
Tr 7	During the use of the abacus in class with my learners, we share ideas on the concepts that can be taught and learned using the abacus
Tr 17	I make sense of it by making the connection or relation to the daily situation, daily life experiences such as sharing sweets among siblings.
Tr 21	Connect the abstract mathematics in the classroom to the real world and make the connection to everyday life experience
Tr 37	Subitizing, recognizing several quantities without counting, and visualizing beads as numbers. I use the abacus to enhances visualization in learners where they visually put numbers in groups when counting and support the development of the part-whole relationship in the early number sense. When I teach place value I teach learners to learn patterns for trading for example ten for one ten, ten tens for one hundred, and one hundred tens for one thousand which can only be done with the abacus

As presented in Table 4.2, Tr 44 responded that when using the abacus, she let the learners see mathematics through explanation and reasoning when doing computation. She indicated that: "*learners do reason and through mathematical reasoning, the learners connect ideas and gain a deeper conceptual understanding of the mathematic they are learning*".

Tr 7's response (see Table 4.2) shows that he let learners share ideas to get different strategies when doing computation with the abacus and also makes the learners aware of the mathematical relationships between the operations. Tr 7 is supported by Tr 21 who reiterated that she connects abstract mathematics in the classroom to the real world to relate the concepts to everyday life experiences using the abacus.

Tr 37's response in Table 4.2 above shows that he used the abacus for sense-making by using the processes of subitizing and visualization. Tr 37 indicated that when teaching place value, she teaches patterns for trading for a simple understanding of place value by using the abacus.

4.2.6 Summary of the survey

From the surveyed teacher's responses, it is noted that most teachers who took part in the survey barely use the abacus in their classes to teach mathematics. In instances where teachers use the abacus they only use it for counting and then only in some cases when learners are struggling with a certain concept. Although surveyed teachers hardly use the abacus their responses indicated that they use other manipulatives such as bottle tops, place value blocks, number charts, games, quizzes, etc. to develop number sense in their learners. In spite of this, the teachers asserted that the use of the abacus benefits the learners in various ways and that it is a good manipulative to develop number sense in the Foundation Phase.

4.3. RESULTS FROM CLASSROOM OBSERVATIONS

This section presents the data collected from the lesson observations of the four Grade 3 teachers. During their lesson obsevations, I focused on the number sense themes from the literature I reviewed. The lesson observation data answered my second research question.

The teachers' lessons were observed and analyzed using the analytical tool shown in Appendix H.. Each teacher was observed five times, and the lessons were video recorded and coded as shown in Table 4.3. below. The codes that I used for the extracts from the teachers' lessons are as follows: a teacher's direct excerpt in lesson 1 line 17 is coded as **Le1L17.**

Table 4.3: Coding and profiling of teachers

Participants	Video 1	Video 2	Video 3	Video 4	Video 5	Interview
Mr Lebby	LV1	LV2	LV3	LV4	LV5	LSSI
Ms Dave	DV1	DV2	DV3	DV4	DV5	DSSI
Mr Quasi	QV1	QV2	QV3	QV4	QV5	QSSI
Ms Tyler	TV1	TV2	TV3	TV4	TV5	TSSI

4.3.1 Sense of orderliness of numbers (SON).

Table 4.4: Excerpts of teachers presenting the lessons that relates to SON

Lebby	Dave	Quasi	Tyler
Le1L7*: On the abacus, you need to understand that how many bottle tops are in one row. How many digits do we have in mathematics?* ***Learners response:*** *9 digits.* ***Le1L7:*** *The first number I have is 523 can someone represent it on the abacus? And another one to write the same number in expanded notation*	***Le1L3:*** *On your abacus, each row represents all Ten digits from 0 to 9. Now, each row represents a single digit of a base 10 number system. We can represent any number on the abacus for example 325815*	***Le1L17****: The number values change when you move from one place to the next. E.g. when you move a $1 coin from the unit place value to the tens place value its value now becomes a $10*	***Le1L29:*** *Every digit in a number has its value for example The seven in 270 represent 7tens or 70 conversely the 7 in 7980 represents 7 thousands or 7000.*
Le3L11: *When we subtract the value of the number decreases. If I take 9 bottle tops and take away 4 units what will be my answer?*	**Le1L2***: You can use the abacus to explore what happens when you compute with a certain operation.*	**Le1L23***: Representing number 105 on the spike abacus: what will you do with the units? How many units do we have? since we have 0 at the tens place value how many bottle tops are we adding there? And the hundreds?*	**Le1L32:***When we are adding 25+4 it means that we add 4 more to the value of 25 which becomes 29.*
Le1L24*: represent the number 523 on the abacus and at the same time decompose it according to its values.*	**Le4L2:** *Can someone tell us the following definitions: Multiplicand; multiplier and product.*	**Le3L10:** *When objects are taken away, they decrease in value, and when they have added the value increased.*	**Le3L 7:** *Let's count from 100 to 0 in 10s. what happens to the number of beads every time we remove a 10?*

From the lessons taught by the four teachers (see Table 4.4) the use of the abacus becomes visible in the theme SON. In the extracts from video data, LV1 (Mr Lebby) demonstrated the different place values on the abacus and he further explained the number system to his learners. He explained place value as a system for writing numerals where the position of each digit determines its value and each value is a multiple of a common base of ten in our decimal system. Mr Lebby demonstrated to his learners the difference between the number and the digit in words and presented it on the abacus. He used the abacus to regroup the given number, for example he used the number 523 and broke it down according to its value (500+20+3). Learners visualized the beads as numbers and manipulated them to get the needed answers. This aligns with Tang (1993) who asserted that the abacus helps learners to see the relationship between numbers and abstract concepts, and that the manipulation of the beads on the abacus helps learners understand the concept of place value.

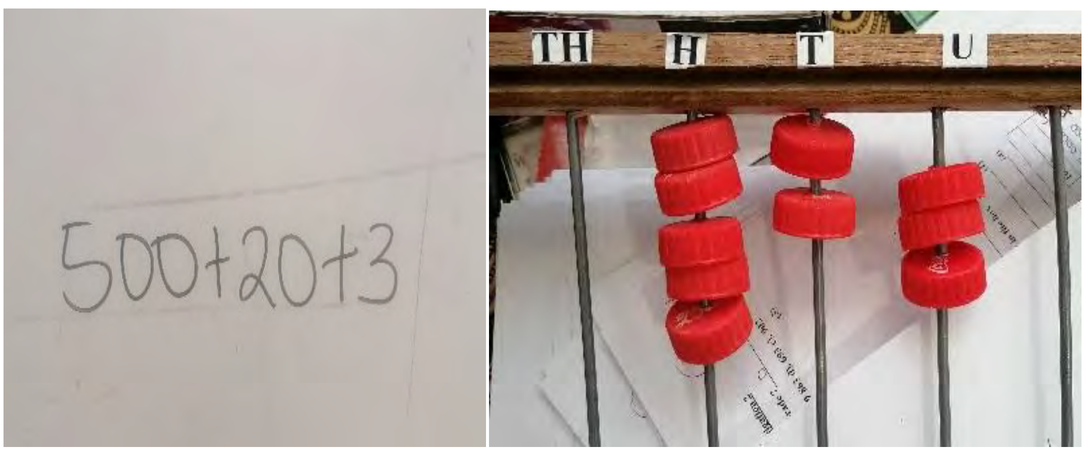

Figure 4.4: Extract from Mr Lebby's lesson on the decomposition of the number 523

In the same vein, Ms Dave explained the place value of a number as the value represented by a digit in a number, based on its position in the number. For easy understanding, Ms Dave then created an open abacus (Spike abacus) where learners put bottle tops to represent numbers.

Figure 4.5: Ms Dave demonstrating with the abacus the place value and the value of the number 325 815

In the extract from Mr Quasi's lesson in Table 4.4, meaningful questions were asked about place value. Mr Quasi explained to his learners that place value defines the value of a specific number in its position and it also allows the number to change its value according to where it is placed. Number value changes when you move a digit from one place value holder to the next – for example, if you move one dollar from the unit place value to the tens place value, its value changes and it becomes a ten dollar. In QV1, Mr Quasi used his abacus to explain place value as shown in Figure 4.6 below. He represented various abstract numbers physically on the abacus. He then showed learners what changes occur when you move a number from one place value holder to the next, using one dollar as an example and houses representing place value. He placed the one dollar in the unit place value holder and moved it to the tens and hundreds place value holders while asking questions that helped the learners understand the sense of orderliness of numbers.

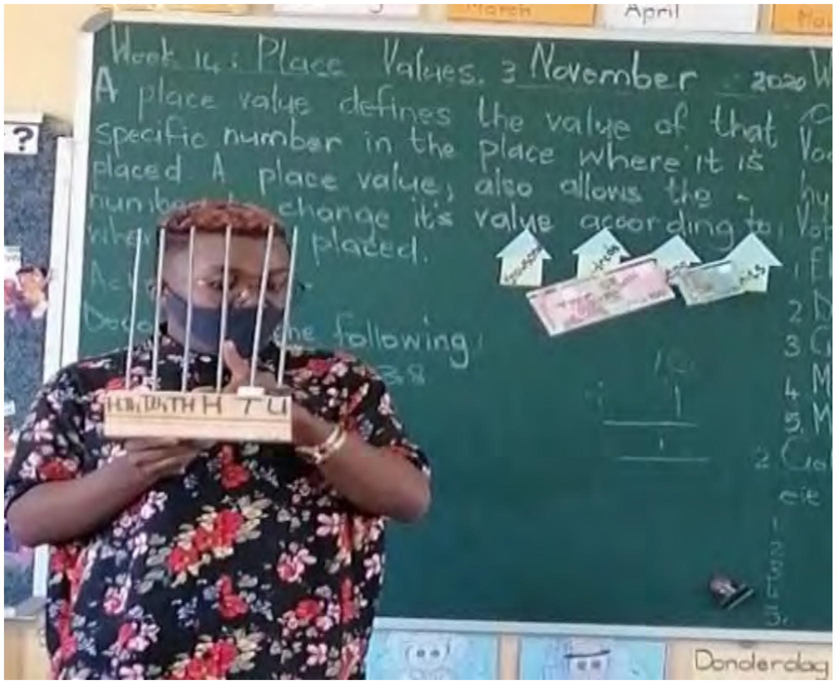

Figure 4. 6: Writing and representing numbers according to their place values

In Figure 4.7 Mr Quasi asked the learners how to write the number 105 (See Table 4.4.) in their place value holders. Mr Quasi illustrated the number 105 on the abacus (physical representation) and used the place value houses to place each number under its house or place value holder.

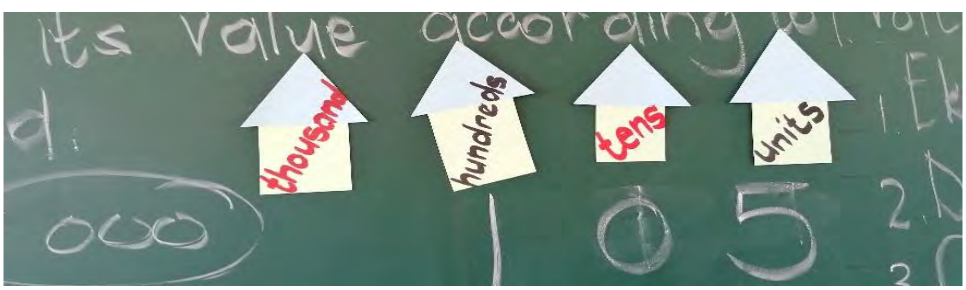

Figure 4.7: Mr Quasi presenting the number 105 in their place values

As seen in Table 4.4 excerpts from Ms Tyler's lesson, 1 saw her describing the place value of a number as the value of each digit in a number, for example, the 7 in 270 represents seven tens or seventy, conversely the 7 in 7980 represents seven thousands or 7000. She further explained that the abacus is read from left to right and it represents the places of the digits in increasing order (1; 10; 100; 1000; 10 000). The abacus represents in multi-digits and the places on the abacus are denoted by U, T, H, TH, etc. Ms Tyler explained this by physically showing the learners on the abacus after each verbal explanation she gave to her learners. This echoes with Cotter (2000) who emphasizes that children should experience the pattern of trading ten ones for one ten, one hundred for ten tens and so on.

Ms Tyler used her spike abacus to show how one can add 25+4, using place value and the abacus. She explained that two bottle tops in the place value holder of the tens plus five bottle tops in the unit place value will give you 25 and if you add four more bottle tops in the units place value holder, the answer will be 29. She used the traditional algorithm to add 25+4 and later represented the same number as a physical representation on the spike abacus. The learners realized that four more bottle tops make the total 29 using the abacus as seen in Figure 4.8 below. This computation taught the learners to compute with the abacus, sums given in the traditional algorithm way.

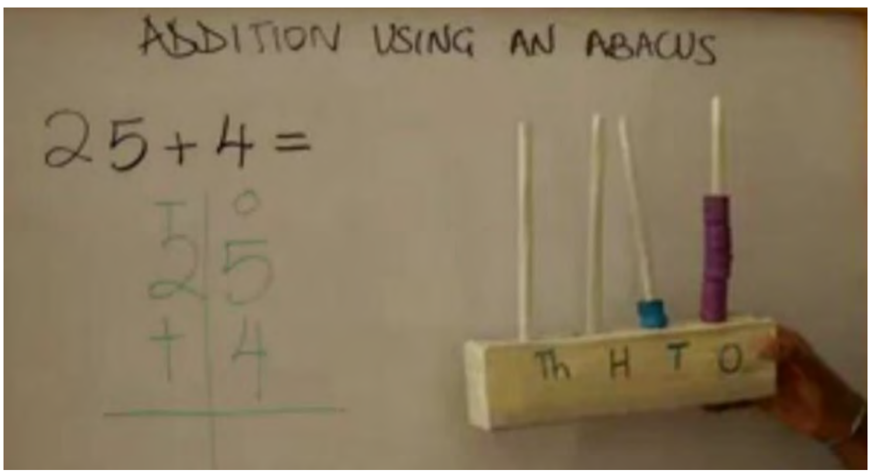

Figure 4.8: Addition of 25+4 on the chalkboard and the abacus

In his presentation as shown in extracts in Table 4.5, Mr Lebby, when explaining the concept of subtraction, emphasized that when subtracting, the value of the number decreases in the final answer. As a reflection on place value, he underscored with the learners that ten beads/ bottle tops in the top row equal to one bead in the row below, demonstrating it with the abacus. The learners then had to visualize the rows on the abacus and keep in mind which row represented a certain place value. This discussion sought the learner's knowledge of SON.

Similarly, Mr Lebby took nine bottle tops from the units row place value and slide back the five, while explaining and leaving behind on the left side of the abacus, four bottle tops (See Figure 4.9

below). Learners were able to take away the subtrahend and count out aloud the difference on the abacus while they were physically showing it to their peers.

Figure 4.9: Mr Lebby demonstrating steps of subtraction

In Table 4.4, the extract from Ms Dave shows her explanation that one can use the abacus to compute with any of the four basic operations and explore what happens to the numbers. In DV4 Ms Dave explained the different terminologies associated with multiplication (multiplicand, multiplier, and product) to ensure that one's representation of the product falls neatly in the correct place value rows when multiplying using the abacus.

Overall, the teachers used their abacuses in their lessons to explain the concepts of place value, grouping, and decomposing of numbers which are some of the main characteristics of the sense of orderliness of numbers. During the teaching of this theme, the teachers led their learners to develop an understanding of place value concepts and numeration in relation to counting. The observed teachers also helped learners to group numbers by visually representing them on the abacus and comfortably computing with the abacus.

4.3.2 Multiple representations of numbers (MRN)

Table 4.5 below shows the excerpts from the teacher's lessons on how the abacus was used to represent numbers in multiple ways. Representing numbers on the physical abacus helped the learners to connect a physical representation of numbers (bottle tops) to that same quantity as shown in the drawing and finally to the actual written symbols for that number (abstract). This aligns with Andrews and Sayers (2015) who mentioned that numbers can be represented in different ways such as using fingers and various manipulatives.

Table 4.5: Multiple representations of numbers

Lebby	Dave	Quasi	Tyler
***Le1L17**: In the number 407, how many units are there? And how many tens are there and the hundred?*	**Le3L14***: Let's decompose the number 314+105 on the chalkboard and one to represent it on the physical abacus for us. What about 24+35?*	***Le3L23**: Decomposition means breaking down the numbers according to their values. When we have 38 is the same as 30+8*	**Le1L4:** *The number 132 can be represented on the abacus physical and symbolic abacus*
The addition means we must put it together. 89+65 this is the mathematics problem we have, the purpose of this addition is to develop a sense of adding and you can develop a sense of adding by recognizing the value of a digit in a given number. Now quickly represent 89 plus 65 on this 100 bead abacus.	*What happens when one subtracts? Learners response: the number becomes smaller. Alright, today we are going to subtract 610 from 621.*	*312+543=855(800+50+5)* *Now can we represent this on the symbolic abacus?*	**Le2L11:***Let's add 32+4 using our board with the pen and paper method and then represent it on the symbolic abacus.*
Le3L3*: 10 bottle tops at the tens place values equal to 1 bottle top at the hundreds place value***. NB***: **10 Bottle tops up are equals to 1 bottle top below**.* *If we have 85-13, we can subtract the two numbers according to their values i.e. 80-10 =70 and 5-3= 2. What will be our final answer?*	**Le4L19:** *We are multiplying 10x13 using model multiplication. We're going to model multiply using the abacus to understand the meaning of division and multiplication through manipulating it.*	**Le3L7:** *Multiplication can be written as repeated addition: when you add sums together you will get the same answer as to when you are multiplying the same numbers*	**Le3L4***:Model multiplication can be written as repeated addition e.g. 8x 5 one can take five rows with 8 beads on each row to make up 40*

In Table 4.5. above, the teachers used the abacus to represent numbers in different ways. Figure 4.10 below shows Mr Lebby representing the number 407 on the symbolic abacus whilst asking the questions recorded in Table 4.5. Additionally, he asked the learners to represent different numbers on the symbolic abacus. The teacher in this case helped the learners to manipulate beads when doing computation on the physical abacus and then asked the learners to draw the symbolic abacus similar to the physical one that the teacher was previously using to compute and manipulate beads. They pictured the dots they were drawing as numbers and represented the given numbers in drawings without using the physical abacus. Representing numbers on the symbolic abacus helped learners to understand how to symbolically represent numbers on the abacus. Kim (2015) said that moving beads on the abacus promotes visualization and helps create images of numbers. She further added that the use of the physical abacus leads to a mental abacus where learners end up moving imaginary beads using their hands. Using imaginary beads suggests that motor representation somehow connects with the number representation created by the mental abacus.

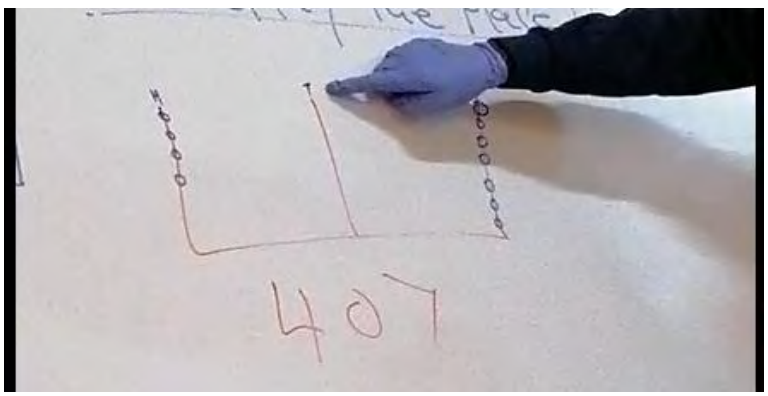

Figure 4.10: Mr Lebby representing number 407 on a symbolic abacus

Mr Lebby demonstrated adding 89+65 using the abacus (see excerpts in Table 4.5). He first asked the learners the place value of the two given numbers and represented them on the physical abacus. With an explanation, he decomposed the two numbers (80+60+9+5). In the process of decomposing numbers, learners came to realize that a larger number can be broken down into smaller numbers. Mr Lebby helped the learners construct the number 89 on the abacus. He first represented 89 on the abacus and explained again the concept that ten bottle tops in the row above is equal to one bottle top on the row below, before adding the number 65. In video data VL2 Mr Lebby explained how to add the number 65 to 89 with six bottle tops in the place value holder of the tens and five bottle tops in the unit place value holder as shown in Figure 4.11 below.

Figure 4.11: Mr Lebby representing number 89 on the abacus.

In the previous lesson Mr Lebby (as indicated in Table 4.5) explained the concept of numbers with different place values. In video data LV2 the learners confused the concept of place value when they were not using the physical abacus. Instead of saying ‘ten’ because the bottle top in question was in the ‘tens’ row, they referred to it as ‘one’, not recognizing the bottle top’s place value . With the abacus, Mr Lebby illustrated to the learners how to determine possible ways to make up numbers. With the example of making number thirteen with three different numbers using the bottle tops on the abacus, the learners presented it as follows: 6+4+3. Making up numbers by breaking them down is referred to as ‘complements’. Complements in the development of number sense are supported by Freeman and Club (2014) who emphasizes that complements are the most efficient way to add and subtract numbers. In the previous example, the learners realized that a single number can be formed up by smaller numbers and that numbers can be represented in many different ways.

To represent numbers in different ways, Ms Dave decomposed the number 314+ 105 on the abacus and the chalkboard. She gave an example of 24+35 where in this case she moved two tens and three tens and then moved four and five bottles tops into the units place value holder without regrouping or carrying over. Learners counted in sequence on the physical abacus from one to a hundred thousand and learned to use the bottom row to keep track of each counting circle. The addition of numbers of different place value is considered here as helping the learners to learn the fundamental property of numbers and place value. Ms Dave gave a sum of 10+5; 20+5 and 30+5 in which the learners noticed they were very similar patterns

In Figure 4.12, Mr Lebby was demonstrating the decomposition and recomposition of 85–13 using the abacus (eight tens plus five units minus 1 ten and 3 units).

Figure 4.12: Mr Lebby demonstrating the subtraction of 85–13= 72 (VL3)

He gave the example of recomposing 24–7 as 24–3–4=20–3=17. With small numbers, he let the learners draw symbolic abacuses in their books and subtract by cancelling out the subtrahends and getting the answers. With the given classwork of 304–285 learners used the strategy of subtracting using the abacus to find the answers. With the number 30, learners were struggling because of the ‘0’ in the units place value holder and Mr Lebby had to intervene and explain the concept of the place value holders using the abacus. Learners were given classwork with diagrams of the abacus with beads wherethey had to determine the numbers represented by each diagram and write down the illustrated numbers. The diagrammatic representation in each case helped learners understand that numbers can also be represented in diagrams.

In the excerpts in Table 4.5, **Le3L14,** Ms Dave asked the learners what happens when they subtract and the learners replied: “*the number becomes smaller*.” She further explained that subtracting using the abacus means that you remove the total number of beads/ bottle tops represented by the subtrahend. The teacher, with the aid of the abacus, allowed the learners to explore what happens when subtracting 610 from 621. In the DV3 the teacher demonstrated the equation 621–610 and the learners observed the grouping of this number using the abacus and the answer, which is 11.

As shown in Figure 4.13 below Mr Lebby is explaining the meaning of repeated addition of 8 × 6 using the chalkboard and the abacus for the learners to understand the concept of inverse operations.

Figure 4.13: Mr Lebby presenting the concept of inverse operations of multiplication and addition.

In the video data, LV4 (as shown in Figure 4.13 above) Mr Lebby illustrates the principle that a number can be illustrated in several ways. Mr Lebby explained the multiplication of 8×6 using the abacus by first asking the learners how many groups of 8 do I need? He wrote 8×6 as 8+8+8+8+8+8 = 48, and every time he made a group of 8 on the abacus, he drew a tally mark. In the end these tally marks had to correspond with the number of bottle tops, forming the answer. Representing numbers in different ways is supported by Bobis (1996) who referrs to number sense as an organized conceptual framework of number information that enables a person to understand numbers and their relationships and to solve with flexibility mathematical problems that are not bound to traditional algorithm.

Ms Dave used a verbal representation to explain 10×13 while using the chalkboard and later she represented it physically on the abacus. Ms Dave broke numbers down into meaningful and useful units that could easily be recomposed as shown below:

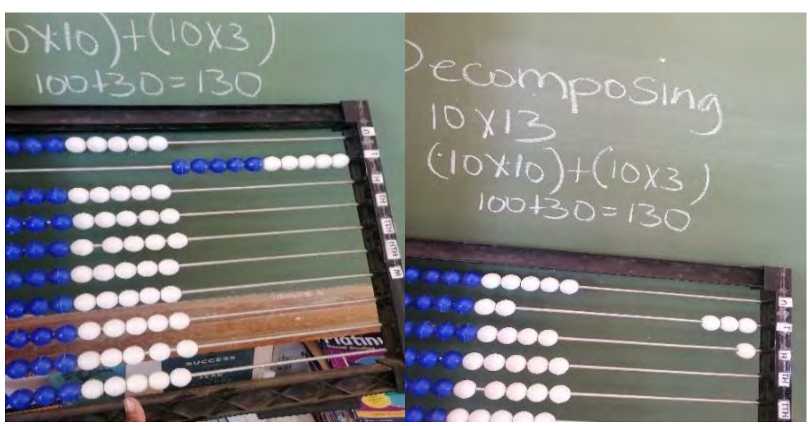

Figure 4.14: Decomposing of 10×13 on the physical abacus

Table 4.5 excerpts show Mr Quasi explaining what decomposition is and how one can write different numbers in an expanded notation as decomposition. Mr Quasi used different numbers and presented them on the abacus, breaking them down and writing the examples onto the chalkboards**.** He then placed the bottle tops on the abacus and learners read out aloud the presented number. In QV1 Mr Quasi, as shown in Figure 4.15 below, demonstrated how a specific number can be represented on the abacus.

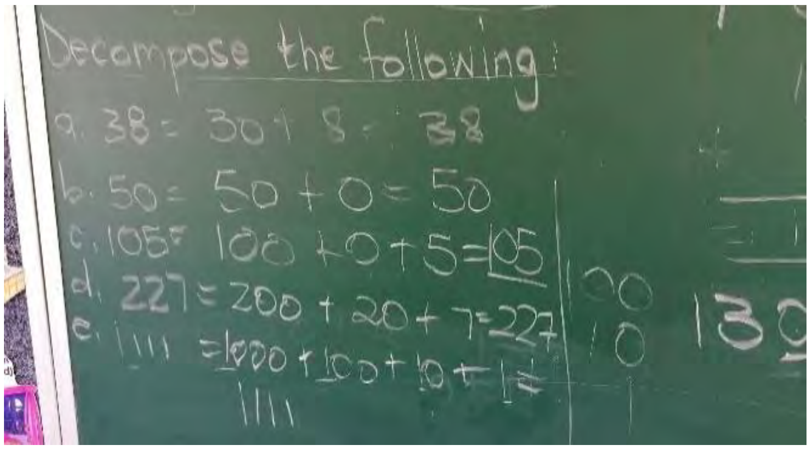

Figure 4.15: Decomposition of numbers by Mr Qoasi.

In the video data QV2, Mr Quasi demonstrated the concept of adding 312+543 as demonstrated in Figure 4.16. The teacher used bottle tops to represent the numbers in the given equation.

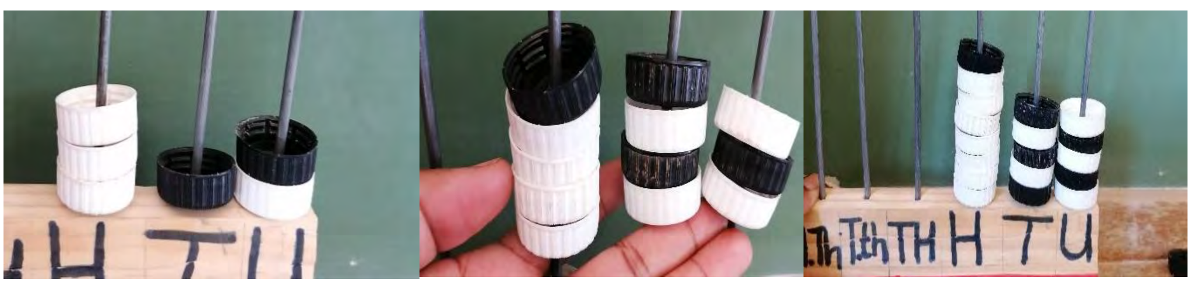

Figure 4.16: Mr Quasi demonstrating adding numbers: 312+543=855

In the video data, QV4 Mr Quasi defined multiplication as repeated addition (See Table 4.5). According to him, numbers that are added together will achieve the same result as when the same numbers are multiplied. Mr Quasi used the loose bottle tops as shown in Figure 4.17 below to explain on the board the concept of repeated addition and later presented it on the abacus, giving an example of 4×3 =12. The physical counters used by Mr Quasi helped learners with the counting sequence and compensating of numbers.

Figure 4. 17: Mr Quasi explaining 4×3 on the abacus and as repeated addition

The excerpts in Table 4.5 show Ms Tyler telling her learners that the number 132 can be represented on the abacus. She went further to demonstrate it physically as shown in Figure 4.18 below.

Figure 4.18: Ms Tyler demonstrating the addition of 100+30+2 on the abacus

The teacher, with the aid of the abacus, explained the concept of model multiplication. The concept of model multiplication used in this case helped the learners to understand how to model multiply any of the given numbers using the abacus. This then helped the learners to see numbers in all sorts of patterns. With the aid of the abacus, Ms Tyler demonstrated how to multiply 8×5 (See Figure 4.19) using model multiplication. She explained the *concept* of model multiplication with the example of 8×5 as shown in the excerpts in Table 4.5: *"Moving of 8 beads on the five neighboring* *wires which can also be used as repeated addition*." She split 8 into 5+3, multiplied 5×5 and then added the multiplication: 3×5. The blue beads below show 5×5 and the white beads show 3×5. The model multiplication 8×5=40 shows eight beads on the five neigbouring wires representing 8+8+8+8+8=40, which is repeated addition.

Figure 4.19: Model multiplication of 5×8 (repeated addition)

Table 4.5 shows extracts of Ms Tyler's lesson where she explained the addition of 32+4 on the board using the pen and paper method, and later drawing the symbolic representation of the same number. In this case, the symbolic abacus was used to demonstrate how one can add. Figure 4.20 below shows the video screenshot of the extract.

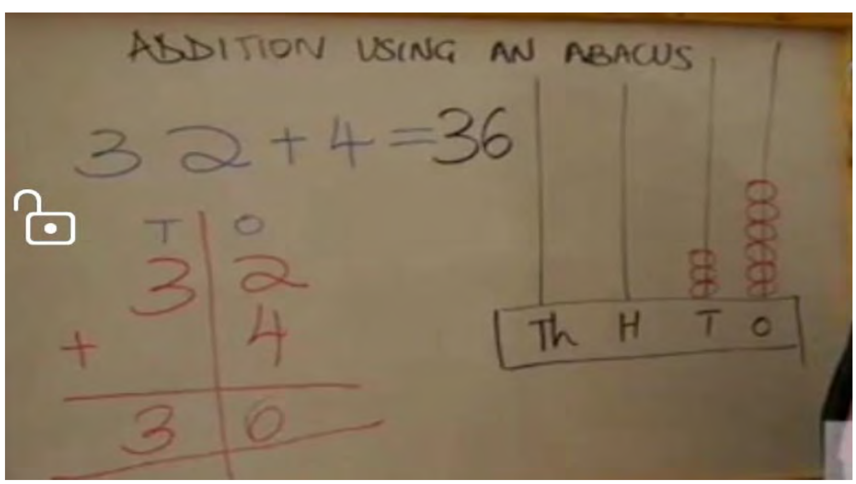

Figure 4.20: Ms Tyler's representation of 32+4 on a symbolic abacus

With the number sense theme MRN, the teachers used the abacus in their teaching differently. The teachers kept engaging the learners in the discussion, using the abacus to represent numbers in multiple ways such as: representing numbers both on the symbolic and physical abacus; writing numbers in expanded notation (decomposition of numbers); and verbal model representation. Teachers used the abacus to multiply, add, and subtract different numbers using the abacus. The participants further explained how multiplication can be presented as repeated addition with the concept of model multiplication, by visually demonstrating it in their teaching. Howden (1989) is of the opinion that learners develop number sense when they work with concrete materials and familiar ideas. He further added that learners need to explore numbers in a variety of contexts and relate them to ways that are not limited to the traditional algorithm.

4.3.3 Sense of relative and absolute magnitudes of numbers (SAN)

Table 4.6: Extracts from video transcripts of teachers that relate to the SAN

Lebby	Dave	Quasi	Tyler
How many beads or bottle tops are these? Why are you guys saying they are 50 bottle tops?	*Sometimes we take objects without counting them and we call it subitizing.*	*These are place value houses. We can put numbers under their place values to recognizes their sizes*	*Without counting the number of beads can we divide 20/5? How?* *Learners' responses: we take the whole two rows and divide them by separating the beads according to their colors.*

Excerpts from the lesson transcripts show that this number sense indicator was less evident in the lessons presented, and in some cases it was absent. In the excerpts in Table 4.6, Ms Dave explained the concept of subitizing. Ms Dave's learners visualized numbers, then they grouped numbers according to the multiplier e.g. 3×4. The learners grouped the beads into four groups on the abacus and drew tally symbols each time they made a group. Representing numbers in groups of four on the physical abacus and drawing tally marks helped the learners visualize numbers in different contexts. In the video data LV4, Mr Lebby let the learners organize the bottle tops according to their quantity and judge the groups without counting. This conceptual subitizing helped the learners and the teacher during the lesson on decomposing and composing of numbers. According to Clement (1999), subitizing is recognized as an important skill for developing number sense. In addition, he claimed that people can not recognize and visualize quantities immediately without counting and that skill of visualization can be highlighted through the use of the counting board and the abacus with its groupings of five structures.

Burns (2007) notes that number sense develops gradually by using different teaching strategies in mathematics and modeling different methods for calculating. When Mr Quasi used the concept of a repeated addition when delivering the multiplication lesson, he used model multiplication and not the concept of using tally marks when multiplying. With model multiplication, the learners managed to subitize the answers without counting the beads**.** Mr Quasi further grouped the bottle tops when dividing and the colorful beads on the abacus also helped the learners with subitizing the dividends and immediately detected the divisor.

Ms Tyler used the abacus for subitizing when she presented her fourth lesson which was on division. She took the number 20 and divided it by 5. The learners subitized the groups of five without counting, but by dividing the beads according to their colors. She further gave the second example of 20÷2 where learners were able to see the computation visually and count the groups of two formed when 20 is divided by 2. While such skills may be as a result of other practices or ways learners engage with content, I argue that the visual aspect of the abacus as used by the teacher enhanced these learners' subtizing abilities.

Figure 4.21: Ms Tyler subitizing 20÷5=4 and 20÷2=10

This number sense theme (SAN) was poorly presented in the observed lessons. Learners were not allowed to think of numbers in their contexts to help them better understand the size of numbers in varieties of contexts. The absolute magnitude of numbers was less evident in the lessons presented because the teachers failed to give the learners the ability to recognize the relative value of the size of numbers in relation to each other while physically using the abacus.

To adequately develop this number sense theme, the teachers should have used different numbers in personal contexts for the learners to better understand the size and magnitude of numbers. The abacus is one of the best tools to teach the meaning and size of numbers but its visuality in this theme was absent in most of the lesson presented.

4.3.4 Understanding mathematical properties (UMP)

Table 4.7 below shows data extracted from the video transcripts of the teachers using the abacus to explain the concept of **understanding mathematical properties.** The mathematical properties (commutativity, associativity, and distributivity) have been long used in school mathematics using the pen and paper method and are viewed as a statement of the obvious. Sayers et al. (2016) saw that mathematical properties can be used to help learners to intuitively recognize and use the equivalent expressions in different ways. Learners with good number sense are comfortable when applying mathematical properties.

Table 4.7. Excerpts from the teachers' lessons that relate to the understanding of mathematical properties

Lebby	Dave	Quasi	Tyler
Le2L31*: Can we all recall the 4+5 we added at the beginning of the lesson?4+5=5+4 and this we call it* ***commutative property****. The order in which you write the numbers doesn't matter.* *Another property of mathematics is the* ***associative property****, with this property you associate the numbers e.g. 3x4x2= (3x4) x2.* ***Le4L 9****: Distributive property: For example, if asked to mentally find the product of 54 x 3, many of us would decompose 54 into 50 and 4. We could then say that (50 x 3) + (4 x 3) = 150 + 12 = 162".*	**Le4L13:** *The commutative, distributive, and associative properties do not apply to subtraction, but subtraction has a property called the* ***subtractive property of zero****. The subtractive property states that if we subtract 0 from any number the answer will be the number itself.*	**Le3L10**: ***Commutative Property*** **2+3+4=3+4+2** *changing the order of addends does not change the sum.* **Associative property:** *the sum of three or more numbers remains the same regardless of numbers are grouped.* ***Le4L27: Distributivity property*** *can mean that you can distribute, divide or give a share of addition e.g.3x(2+3)= (3x2)+(3x3)*	***Le3L19: Commutative Property:*** *The number that we operate can be moved or swapped from its position without making any difference to the answer.* ***Le3L34:*** *subtraction and division are not associative*

Commutative property: The excerpts of Mr Lebby's lesson in Table 4.7 show that he clarified the commutative property of addition by referring to his first example and explained that 4+5 =9 and 5+4= 9 (the order in which you write the numbers doesn't affect the answer). He visually demonstrated this number property using the nine bottle tops on the unit row of the abacus on both occasions. Mr Lebby also explained to the learners that while commutative property works for addition, this doesn't apply for subtraction – he gave an example of 23+9=9+23 but 23-9 ≠ 9-23. When learners were busy doing this addition they were paying attention to what happening to the numbers as they were computing, looking at patterns and generalizing the patterns into properties.

Mr Quasi explained that commutative means that changing the order of addends does not change the sum. Excerpts **Le4L27** in Table 4.7 indicate Mr Quasi giving an example of 2+3+4= 3+4+2. "*When we add two or more whole numbers, their sum is the same regardless of the order of the addends"*. He used his abacus to physically demonstrate this to his learners by taking three groups of two, three, and four and swopped places on the spike abacus so that the group of three took the place of the group of two, the group of four took the place of the three and two took the place where the group of four was. The learners realized that in both cases the answer remains the same – 9.

Associative property: -In the extracts in Table 4.7 Mr Quasi explained the associative property by using the same numbers as the ones used when he explained the commutative property. He explained that with associative property, changing the grouping of addends does not change the sum. After verbal representation of the associative property Mr Quasi, with the aid of the abacus demonstrated that: (2 + 3) + 4 = 2 + (3 + 4) and when three or more numbers are added, the sum is the same, regardless of the grouping of the addends i.e. (4 + 2) + 3 = (4 + 3) + 2. He gave the learners numbers to represent as associative properties on the chalkboard, visually represent the same numbers on the abacus and explain it to their peers. The learners thus demonstrated to their peers the concept of the associative property by using the abacus and moving the same number of beads in both cases.

Distributive Property: Table 4.7 extracts show that Mr Lebby explained how to multiply 54×3. He further elucidated that multiplication can be distributed over addition and subtraction. In this case, he used contextual representation to explain the concept of the distributive property. Mr Lebby taught the learners that in the distributive property, factors can be spread out so that the numbers are easier to work with. He used his abacus to multiply 54×3 by multiplying 50 by 3 first and then multiplying 3×4. Mr Lebby then added the two answers (150+12) and arrived at 162 as the final answer. We often use this property when multiplying mentally. "*For example, if asked to mentally find the product of 54 x 3, many of us would decompose 54 into 50 and 4. We could then say that (50 x 3) + (4 x 3) = 150 + 12 = 162".* After giving the example he let the learners perform a similar sum mentally and went on to demonstrate it visually on the abacus to check the accuracy of their answers. Performing this property on the abacus enables learners to visually see the process of the distributive property.

Ms Tyler (as indicated in Table 4.7) stressed that subtraction and division are not associative. Using her abacus, Ms Dave explained that in the distributive property, one or both multiplication factors can be decomposed into two or more parts, giving an example of 6×8 is equivalent to (8×4)+(8×2).

Mr Quasi explained distributivity as "distribute" means to divide something or give a share or part of something, meaning that multiplying the sum of two or more addends by a number will give the same result as multiplying each addend individually by the number and then adding the products together. He gave an example of 3× (2+3) = (3×2)+(3×3) (see Table 4.7). He used the loose bottle tops to explain the distributive property because the learners were finding it difficult to grasp the concept of distributivity. This helped the learners to physically and visually understand the process being discussed and make sense of the concept.

Ms Dave used verbal representation with her learners to explain the three (Commutativity, distributivity, and associativity) properties of multiplication, with examples as shown in Figure 4.22 below.

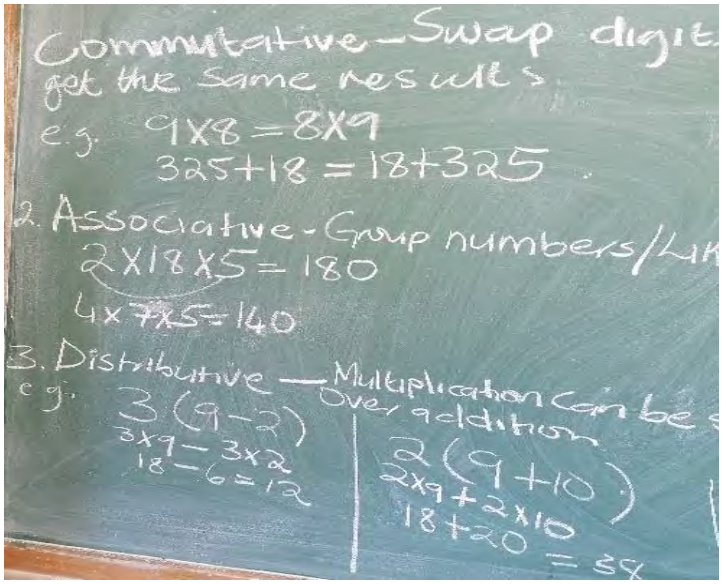

Figure 4.22: Ms Dave explaining the three properties of multiplication

The teachers presented the five lessons using the abacus to teach the three mathematical properties (commutative, associative and distributive). During their lesson presentations, the teachers engaged learners by giving them similar sums to demonstrate the properties of mathematics on the abacus. They explained the different properties using the abacus so the learners could make connections between the concrete and the abstract.

4.3.5 Understanding the relationships between operations (URO).

In the Foundation Phase, teachers need to instil in the learners a solid understanding of each operation. The learners also need to understand how the operation they are using relates to other operations. McIntosh et al. (1992b) noted that when learners have an understanding of the relationships between operations they are able to solve problems and develop their number sense. Table 4.8 below shows the excerpts from the video transcripts for the four teachers that relate to the number sense theme **understanding the relationships between operations.**

Table 4.8 Extracts from the video transcripts of data related to understanding the relationship between operations

Lebby	Dave	Quasi	Tyler
Le3L18: ***Addition/Multiplication:*** *When you add 3 two times is the same as multiplying 3 with two i.e. 3+3 is the same as 3x2. We can demonstrate this with the 6 beads on our unit row of the abacus only.*	**Le4L15:** *What is 4x4? On our abacus, if we multiply 4 by 4 we get 16 and we can also get 16 if we add 4+4+4+4.Use your abacus and perform 4x4. With the 16 beads as your answer demonstrate the repeated addition.*	**Le4L4***:3x4=12: three groups of four can make up 12*	
Le5L1: *When you divide does your answer become bigger or smaller? What happens when you subtract? Now* ***Division and subtraction*** *are inverse operations. What did we say the inverse operation is in the previous lesson? Now let's take* ***12/3:*** *12÷3=4 and as repeated subtraction is: 12- 3=****9-****3=****6****-3=****3****-3=****0***	**Le5L17***: Using our abacus we can show that division can be written as repeated subtraction: for example, 16÷4= 4 as repeated subtraction it can be written as 16-4=****12****-4=****8-****4=****4****-4=****0***	**Le3L1***: started the lesson with a song of 5 ducks that went swimming and every time they come back one is missing. This song clearly explained and shows that division can also be used as repeated subtraction*	
Le3L19*: Inverse operation means that one operation can undo another operation.* ***Addition and subtraction*** *are inverse operations i.e. 3+4=7 and 7-4=3*		***Le3L23:*** *subtraction can be the same as repeated addition for example of 30-15= 15 and 15+15=30. Using the abacus: 30-15*	
Le4L 24*: With multiplication and division being inverse of one another means that we can check our multiplication answer by division. For example, 108÷9=12 is the same as 12x9=108*		***Le4L: 15:multiplication and division*** *are inverse operations for example 25÷5= 5 and 5x5=25*	***Le5L14****: 20÷2=10. 10 is the number of times you can subtract the groups of two from 20 before you get zero.*

Addition and Multiplication – Multiplication can be written as repeated addition, and this is demonstrated in the extracts in Table 4.8 where Mr Lebby used his abacus as an aid to demonstrate 3+3=6 and multiply 3×2= 6. As an example, he used the abacus to physically show the addition and multiplication relationship. He also presented the equation of 8×3= 24 on the chalkboard and ask the learners to represent it on the abacus as repeated addition. Using 24 beads the learners put them into groups of eight and ended up with three groups of eight. The learners recognized the operation of multiplication as repeated addition. Mr Quasi used the abacus to demonstrate the concept that multiplication can be the same as repeated addition. For the equation 4×3 he used repeated addition of 3+3+3+3 with bottle tops (See Table 4.8) to make the explanation clear to the learners.

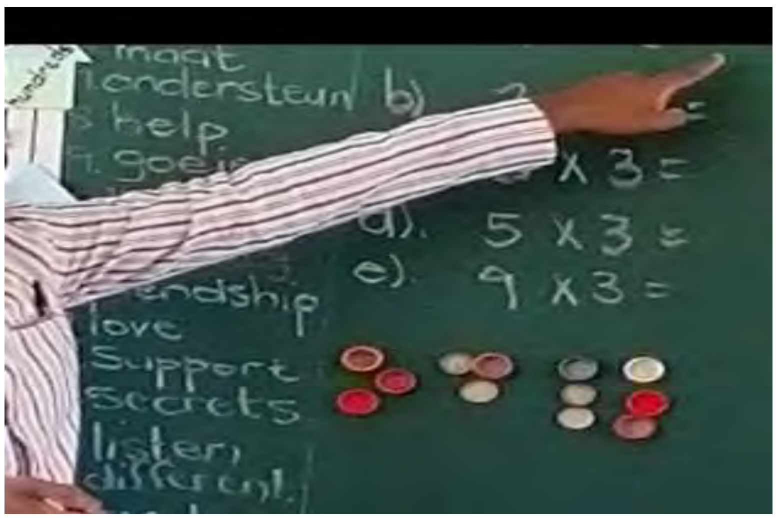

Figure 4.23: Mr Quasi illustrating the concept of repeated addition using loose bottle tops

With the aid of the same bottle tops on the abacus, Ms Dave explained multiplication as repeated addition by putting sixteen bottle tops on the abacus and performed repeated addition after multiplying 4×4.

Addition and Subtraction – As indicated in Table 4.8, Mr Lebby explained the term 'inverse operation' to the learners and gave the example where the learners added using their abacuses and performed subtraction as an inverse operation. With the abacus, Mr Lebby (as shown in Table 4.8) explained *"If you start with 3 and add 4, you get 7. Then, when you subtract 4, you undo the original addition and arrive back at 3: 3 + 4 = 7 → 7 – 4 = 3."*

Ms Dave explained that the inverse operation of subtraction is addition, meaning that subtraction undoes addition. She, too, used both counters and the abacus to illustrate to her learners the concept of addition and subtraction as inverse operations. Ms Dave used a symbolic representation of the abacus to explain further to her learners. This enhanced the understanding of the relationship between operations and improved her learners' number sense. She explained that an addition sentence can be rearranged using the abacus and wrote it on the chalkboard as shown in Figure 4.24 below:

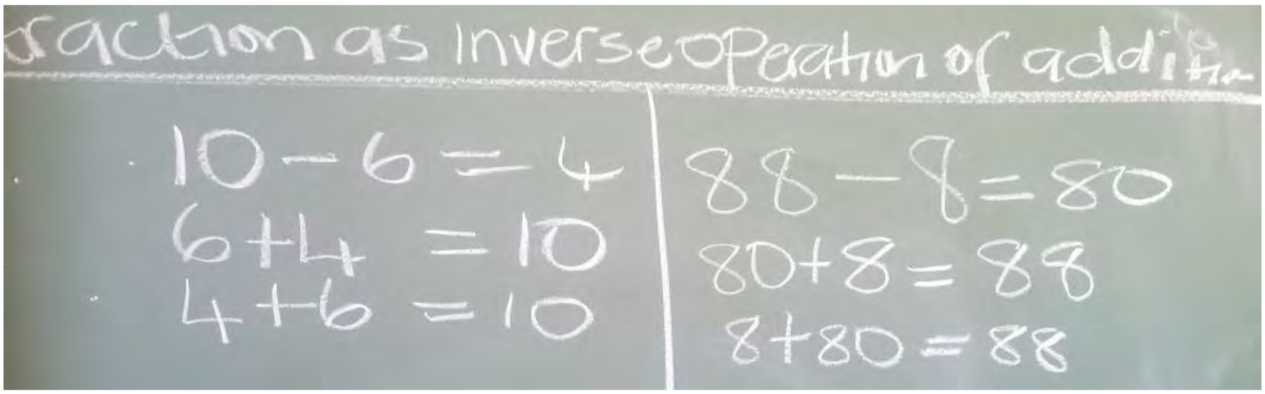

Figure 4.24: Ms Dave's explanation of subtraction as an inverse operation of addition

Subtraction and Division – With the abacus as a visualization tool to develop number sense, Mr Lebby (as indicated in excerpts in Table 4.8 above) presented the division problem 12÷3 and asked the learners to demonstrate repeated subtraction. He then explained to his learners that they should subtract three until the remainder is zero. In this case, they would have to subtract three groups, four times to get zero. With the help of the abacus the learners physically demonstrated repeated subtraction to get three groups of four as their final answer.

Division and Multiplication – Ms Tyler demonstrated with an explanation, that division is the same as repeated subtraction. The extracts in Table 4.8 from Ms Tyler's lesson show the problem 20÷2=10. Figure 4.25 below shows Ms Tyler using her abacus to explain that 20÷2=10 and 10x2=20. With the aid of the abacus, the learners saw the meaning of division as an inverse operation of multiplication.

Figure 4.25: Ms Tyler demonstrating division as inverse operation of multiplication using the problem 20÷2

Figure 4.26 shows the worksheet Ms Tyler gave the learners to practise the inverse operation of multiplication. In this worksheet, learners used their abacuses to reverse the multiplication by division and it helped them to use the other operation to check the accuracy of their answers.

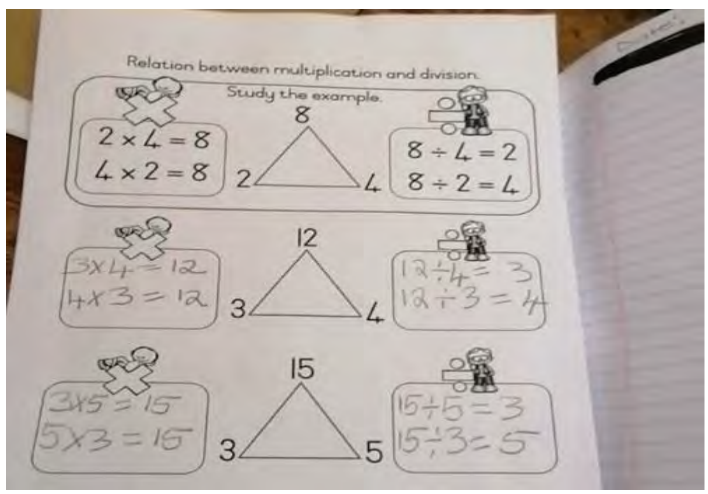

Figure 4.26: Division and multiplication as inverse operations

To enhance the learners' understanding of the effects of operations the teachers used the abacus to explain and demonstrate the meaning of inverse operations in mathematics. The learners came to understand that the inverse operation is when one operation reverses the effect of the other operation. Recognizing the inverse relationships between the operations equips learners with greater flexibility in working with numbers than if facts are learned in isolation. In the lesson, the learners applied basic concepts and operations on the abacus and understood the concept of inverse operations.

4.3.6 Inclination to review data and results for sensibility (IDS)

Table 4.9 Teachers' excerpts on IDS

Lebby	Dave	Tyler
Le2L34: *take the number 408+205: write it in expanded notation and after we get the answer, we should demonstrate how we arrive at it with the aid of the abacus*	**Le5L37:** *on Page 23 lets us do the word problems and we check the reasonableness of our answer with the abacus.*	**Le1L19:** *lets us regroup the following numbers according to their values.: 254 and 762 and grouping them we can now represent it on the abacus.*

As indicated above in Table 4.9 the excerpts from Mr Lebby's lesson, he asked his learners to add 408 and 205 together mentally and then demonstrate how they got the answers. By visually showing their procedures and explaining how they arrived at their different answers, the learners became more confident. According to Burns (2007), mental computation does not force learners to memorize but makes learners think more flexibly and efficiently to solve problems.

For the number sense theme IDS Ms Dave asked the learners to work on a word problem (see Table 4.9) and then use the abacus to check the answer. The exercises on page 23 were designed to assist learners in thinking fast and calling out whole numbers with more than three digits. Using the abacus helped the learners understand the abstract content and the visual representation on the abacus.

Ms Tyler presented the number sentence 245+ 762 (as shown in Table 4.9 above) to the learners who showed their thinking by decomposing the numbers as 200+700+40+60+7. After they had the answers they checked and verified their results using the abacus. Decomposing numbers into place value holders, in this case into tens and ones, is one characteristic of number sense. This feature is key in developing children's number sense and their understanding of place value. Ms Tyler asked her learners to check the reasonableness of their answers each time they performed a calculation. She then used the board to group the number sentence 245+762 using the place value indicators as indicated in Figure 4.27 below, before it was demonstrated on the abacus by the learners.

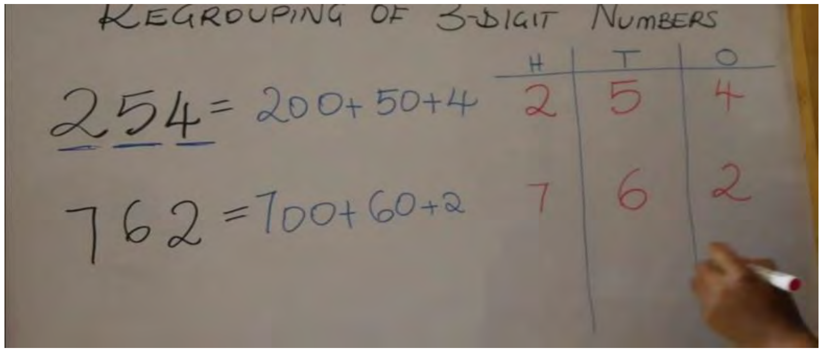

Figure 4.27: Ms Tyler regrouping 254+762

Ms Tyler explained about the misconception that multiplication makes the number bigger and division makes the number smaller, as when working with fractions, this is not the case. She used the abacus to model multiplication problems using distributive properties e.g. 321×3= (300×3) + (20×3) + (3×1)

Throughout these lessons, when the learners found answers they used the abacus to explain why they were are correct. This is where the learners really grasped the idea of using the abacus when using mental mathematics to add, to prove their answers and to explain why they were correct. In the presented lessons the four teachers used word problems which their learners solved using the abacus. According to Basham and Marino (2013), Bybee (2013) and DeJarnette (2012) word problems require the integration of mathematics to help in solving real-world problems. Word problems are calculations that require one to apply mathematical knowledge to solve them. In such cases, learners may be able to manipulate physical objects to find the solution to their answers.

4.3.7 Inclination to utilize efficient representations or methods (IUE)

Table 4.10: Teacher's excerpts on the IUE number sense theme

Lebby	Dave	Quasi	Tyler
Use mental mathematics to calculate: 456-89 and use the abacus to check if your estimated answer is correct.	*Multiplying with 10,100,1000 you move the invisible comma to the right according to the number of 0 present.*	*Estimation and guessing of the answer and then use the abacus to test its accuracy.*	*Multiplication has a misconception that makes the number bigger and division makes the number smaller, this doesn't apply when working with fractions.*

With this number sense, theme, learners examined answers in hand to determine if they were making sense. Şengül (2013) contests that some strategies are more efficient at certain times than others. The excerpts from Table 4.10 show Mr Lebby using mental mathematics to subtract 89 from 456, after which he asked the learners to use the abacus to check their answers. Mr Lebby's video data on addition shows the learners working on word problems, using traditional algorithms and then using the abacus to check the accuracy of their answers. Mr Lebby used verbal instructions to illustrate to his learners how to use the inverse operation to check their answers and to give reasons why they thought their answers were correct. Learners visually represented the teacher's verbal instruction by manipulating the beads on their own abacuses. Multiplicative reasoning involves considering three quantities simultaneously: the number of groups, the equal quantities in each group and the total quantity/ value.

Ms Dave used mental calculations of multiplying and dividing with ten, one hundred and one thousand (see Table 4.10), she led her learners to use the abacus to represent their answers and then to prove it by the pen and paper method. Mr Quasi in his lessons first asked his learners to represent given numbers by drawing them in the air. Visualizing and drawing numbers in the air helped the learners to know the shapes of the different numbers before writing them down. Mr Quasi's learners used pens to do a traditional algorithm and then worked out the sums on the abacus to test whether their answers were correct. Mr Quasi and his learners used visual algorithms and representations in solving the mathematical problems given.

Ms Tyler explained to the learners the misconception of division and allowed the learners to support their generalizations using the abacus. When learners were skip counting for a known fact e.g. 6×3, they could think of 5×3 then 15+3=18. Ms Tyler learners represented numbers on the abacus while focusing on subtraction and applying mental mathematics strategies such as skip counting. At the same time they used the inverse operation of addition to double-check the accuracy of their answers. To estimate calculations the learners used front end estimation (the digit with the largest value is used even if the estimate might be low.

4.4 SUMMARY OF THE LESSON OBSERVATIONS

In the classrooms, as was observed in the activities and worksheets given to the learners, the use of the abacus helped them to develop their number sense successfully. The abacus was utilized as a visualization tool by the teachers in the following ways: to explain the multiple representations of numbers; to help their learners understand mathematical properties; to enable greater understanding of the effects of operations and to develop the sense of orderliness of numbers. In the lesson presentations, learners were guided to visualize numbers in the form of beads and bottle tops on the abacuses. In the observed teachers' classes, learners were observed using the abacus and performing rapid and precise mental arithmetic. It can be inferred that they manipulated mental representations of the abacus based on visual imagery.

The teachers' consistent use of the abacus during the lessons seemed to solidify and consolidate their learners' understanding of mathematics. The abacus was utilized to the extent that the abstract nature of some concepts turned into a delight. During the observations, it was noted that the learners were paying attention and were actively involved in the lessons. These learners were noted as being able to reason logically on the computations they were doing. The abacus thus improved learner motivation. This was because the observed teachers had not previously used the abacus in their classes for teaching any mathematics concepts, but the use of the abacus remarkably improved interactions between them and their learners. Learners also asked questions during the lessons. Use of visual tools such as the abacus can thus only be ingored at the expense of improved teaching of mathematical concepts such as number sense in the Foundation Phase. It is therefore important for mathematics teachers of these formative years to be encouraged to re-think how best they can incorporate an old tool such as the abacus to become relevant in modern day classes.

4.5. RESULTS FROM REFLECTIVE JOURNALS AND INTERVIEWS

Data collected from the interviews are presented in this section to answer the third research question: *What are the teachers' experiences and perceptions of using an abacus as a visualization tool in developing learners' number sense after participating in the intervention programme?*

Mr Lebby indicated his awareness of the term 'visualization' in mathematics. He described visualization as a theory of learning where one says "*whatever you see you can remember but what you do you will always know"*. He added that visualization is very important in mathematics because it shows the learners how a specific answer was obtained. He affirmed that using the abacus contributed to visualization:

> *With the abacus, you can demonstrate how much is taken away or how much you are going to group into. This means that visualization plays a vital role when it comes to developing number sense since it stimulates thinking among the learners but also helps the teacher to deliver effective lessons… without visualization, many of the kids will not be able to comprehend the mathematics taught to them.* **(LL4)**

When asked about her understanding of visualization Ms Dave said that visualization is forming a mental image about a specific problem. For example,

> *... when talking about the area of the rectangle one can be able to explain it by drawing and visualizing the area by taking the abstract idea to concrete. In another way, I understand it as a way of creating images/diagrams to demonstrate specific abstract content that one needs her/his kids to understand (***DL1).**

Mr Quasi's understanding of visualization was the ability to observe and see to interpret. He marked that*: "I can take it back to Bloom taxonomy you have to comprehend, understand and put it in your own words and apply with the knowledge what you see".* Ms Tyler added she did not have any idea of what visualization was before the intervention programme.

The participants indicated a shared understanding of what visualization in mathematics is. They all described it as being able to form something mentally in their minds about a specific problem.

4.5.1 Understanding of number sense

To the question of number sense understanding, Mr Lebby said that he understands number sense as a well-organized conceptual framework of number information that enables the person to understand numbers, the relationship between the numbers as well as to solve problems that are not bound to the traditional algorithm: He feels that

> *…number sense goes beyond the four basic operations…it is the concept that helps learners that are dealing with mathematics to create some relationship between numbers before they do the traditional method of addition, multiplication, division, and subtraction*.

Ms Dave's understanding of number sense is "*the ability to understand numbers to use them in real life and outside of the relationship between number and operation"*. She pointed out that if one understands how a specific operation affects the number, then one can say that person has a good number sense.

> *In the lesson when I used the abacus, the learners were able to see clearly what happens when you multiply and what happens to the number when you divide them. The abacus in the lessons helped the learners to compute complex numbers without using the traditional algorithm and at the same time learners understand numbers and solve the problem given to them using the abacus.(* **QL2**).

Ms Dave regarded the use of the abacus as being important to enhance the number sense of her learners: "*I saw that my learners were able to demonstrate an abstract on the abacus to calculate and arrive at answers".*

Mr Quasi explained that he understood number sense as the ability of the learners to understand the magnitude of numbers, work with numbers, identify numbers and build number relationships.

> *When I was teaching using the abacus I saw that my learners were able to understand the numbers and their magnitude and how the operation sign affects them when they are operated on* (**QL2**).

Mr Tyler added that:

> *I can look at it as the way the learners visualize and freely or comfortably talk about numbers and use them. In my lesson, the learners visualized the abacus bottle tops and beads as numbers. They utilized the abacus and tried to understand the relationship between the numbers and the beads or bottle tops that they are manipulating* **(TL4).**

Participants looked at number sense as an intuitive understanding of numbers, their magnitude, their relationships and how they are affected by operations. They indicated that using the abacus in their teaching helped their learners develop a sense of computing complex numbers without using traditional algorithms. Teachers felt that the abacus led to their learners being able to compute abstract numbers due to the number sense they had developed through using the abacus. The teachers experienced how their learners used the abacus to understand numbers, their magnitude and how they are affected by operations.

4.5.2 The usefulness of the abacus in teaching

With regard to his teaching using the abacus, Mr Lebby attested that the abacus is very useful with the intention of developing number sense in the Foundation Phase. He explained that:

> *When it comes to understanding and reading numbers the person has to understand the place value. The abacus is one of the best tools to teach for understanding the concept of place value…any child who wants to know how to read numbers must recognize the value and the position of the individual digit in a given number and with the abacus, they can visually see the concepts…. the abacus has been useful in teaching when it comes to the demonstration of different position of individual digits* **(Ll2)**.

He added that when he used the abacus he experienced how his learners used it to do fast, accurate calculations and it also boosted the their confidence. In addition, Mr Lebby argued that the abacus promoted a stress-free way of teaching mathematics concepts because once if learners do not find the correct answer, they still have the chance to redo it until the answer is correct. During teaching the abacus enabled problem-solving skills, particularly in multiplication. Mr Lebby stated that the abacus assisted his learners to recognize and visualize that in order to make one ten they need ten units to make up a ten, etc… He affirmed that the abacus strengthens visualization and memory compared to the pen and paper method where a child may make mistakes during the process of transferring and carrying over the number when doing computation.

On the other hand, Ms Dave pointed out that:

> *My learners were able to do basic calculations and secondly the concentration on the moving of beads and bottle tops and getting correct answers improved. When I was teaching using the abacus learners were trying to focus on the abacus so that they can keep track of the task they were doing since they were involved in drawing tally especially when it comes to division and multiplication. I experienced how the learners were able to differentiate between place value and value of a digit. From my teaching observation, the abacus developed positive attitudes towards mathematics; apart from doing complex calculations the dependency on the abacus reduced because they ended up doing some sums mentally (moving beads with hands without physical abacus"(* ***DL14)***.

According to Mr Quasi, the abacus allowed his learners to strengthen their memory. He argued that learners find it very interesting and the colour scheme on the abacus improves concentration. He affirmed that:

> *… as I said earlier they find it very interesting and they concentrated on what was happening, which in the same vein improves the listening skills, visualization and strengthen the memory because it is easier for them after using it in the lesson for them to recall what they did in that lesson, for them to remember because they used the tool* **(QL13)**.

Ms Tyler conceded that:

> *As a visual tool in my teaching it didn't help me only in the basic calculation but it is also helped me to explain and demonstrates sums that learners struggle with when using the pen and paper method*(**TL7**).

She added that it was useful in the way that learners were able to visualize the given numbers and work on them using the abacus: "*The other thing I experienced through manipulating the abacus was that learners who often struggle with long division with big numbers were able to compute complex numbers using the abacus."* Ms Tyler pointed out that her learners later grasped the skills of calculating using the abacus and the pace of finding correct answers increased.

All the participants attested that the abacus is an ideal and useful tool to teach mathematics with the aim of developing number sense. They explained that they felt that the abacus is the best tool to teach concepts such as the place value and arithmetics. The participants found that the abacus helped their learners monitor themselves when computing, promoted stress free way of teachimg, strengthened visualization and generated positive attitudes towards mathematics.

4.5.3 Benefit of the abacus to the learners

Mr Lebby indicated that his learners who struggled to read numbers managed to read them from the visual object (abacus) and he was able to demonstrate to the learners the difference between the units, tens and hundreds. Moreover, the abacus helped his learners to know the relationships between numbers:

> *I was able to demonstrate that ten units are equivalent to one bottle top at the place value of the ten and also explain the concept of ten bottle tops on the row above on the abacus is equals to one bottle tops on the row below. It helped with the development of number sense during the calculation using the basic operations….*

Like I said earlier if you have 37+5 the learners will first display the number 37 on the abacus and add 5 to the 7 bottle tops on the unit row, unfortunately, there are only 10 bottle tops on each row the learners then realized they have to apply the concept of 10 units is equal to 1 ten to finish adding 5 to 7 and get the correct answer. In another word, learners learned that you can't have two digits in one place value"(**LL18).**

In his reflective journal, Mr Lebby noted:

The learners develop the strategy of decomposition and this decomposition of numbers made the calculation easier for the learners breaking numbers down according to their values… the use of the abacus encouraged learners to think about how certain numbers have "complements" or "partners." For instance, thirteen is made by partnering up ten plus three or partnering seven plus six. Decomposition gave the learners a better underlying sense of how math works.

Ms Dave cited various benefits of using the abacus to develop number sense. These benefits include **DL20:** *"...self-confidence; problem-solving ability; math proficiency and different* strategies".

Learners were able to demonstate how they got the answers, they were confindent about explaining it because they have the abacus and they can physically demonstrate how they arrived at certain answers. Learners often experience challenges with word problem solving ability. With the aid of the abacus my learners were able to get the abstract in words and perform the calculation on the abacus. Another benefits I observed in my teaching using this ancient tool is the logical reasoning in my learners and improvement in mathematics proficiency. Learners were reasoning logically when I ask them questions and they confidently demonstrate why they think their answers are correct. My learners develop the positive attitudes towards mathematics. They further develop the different strategies of solving problems apart from the pen and paper method ***(DL20).***

In her reflective journal Ms Dave wrote:

The abacuses we are currently constructing consist of place values on it, unlike the one present in our classrooms. With the place values holders visible on the abacus learners will be able to identify the values of given digits without experiencing any difficulties. The bottle tops used to make these abacuses will help learners realize that mathematics is around them and not all mathematics tools need to be bought some can be recycled.

On the question of how the use of the abacus is beneficial to the teaching of mathematics Mr Quasi responded that his *"…learners had an opportunity to use the abacus themselves. Apart from visualizing it, they had an opportunity to use it, they can move beads on the lines of the units, the tens, and the hundreds."* Mr Quasi explained that he observed how his learners were sliding the counters back and forth when doing computations. He also felt that his learners were positively interacting in the lesson because of the way the were manipulating the beads and bottle tops on the abacus when solving problems. Later, he narrated "… *since we are in the modern world the abacus calculation in school has faced out and his learners finds it very interesting using the abacus to find answers."*

The teachers maintained that the abacus is a very useful tool and it has many benefits. They pointed out that the abacus helped them to physically demonstrate the difference between place value and the value of a digit. Teachers perceived the abacus as being an excellent tool that could help them in their teaching of complements and relationships between numbers. They further added that the abacus has various benefits, including promoting self-confidence; improving problem-solving ability; developing math proficiency and encouraging different strategies of solving mathematics problems.

4.5.4 Challenges experienced during the use of the abacus

Learner-related challenges

Mr Lebby explained that his learners experienced challenges during his teaching using the abacus. He was of the view that:

> *Although some learners know how to read the numbers they can't relate the reading of the numbers to the place values, which means if you use the place value to display the number few learners will struggle to find what type of number it is since all the bottle tops used on the abacus are the same size and only two different colors" (**LL38**).*

He explained that with an example of number eleven,

> *… some of my learners counted it as 2 because they didn't recognize that one bottle top is at the place value of the tens and the other one is at the place value of the units. All they see is two bottle tops and they make it number two instead of looking at the place values of individual digits.*

Mr Lebby also indicated that he experienced a challenge with the concept of division using the abacus.

> *When dividing with a bigger number with a unit, for example, and it comes to the exchanging of the tens with the units it becomes a little bit problematic and my learners loose tracks through this… Let me say 21÷3 you only have two bottle tops at the place value of the tens and 1 bottle tops at the place value of the units, what happens is that the child is unlikely to group the bottle tops in groups of 3, they will take the three displayed bottle tops and say 21÷3 =1 because they didn't comprehend the fact that the two bottle*

tops are at the place values of the tens and not units. This brought much confusion to the learners **(LL40)**.

With the above-mentioned challenges, Mr Lebby advocated that:

> *The teachers should make realistic examples where the learners must be able to recall or establish themselves as far as number sense is concerned. The lesson should be made very interesting the learners must be able to ask questions of what is going on. The more the learners ask questions it will help them to think critically when it comes to the computation of mathematics.*
>
> *Once the learners develop their number sense the next thing to be done is to focus on the process and not the answer. As a teacher you don't need to be concerned about the correct answer, see if the process that the child is using is correct and by saying correct process you don't need to dictate what method must be used to find the correct answer but the process must be relevant toward getting the correct answer***.**
>
> *In this case with the scenario of 21 that I made use of the learner must be able to recognize that we are talking about two bottle tops in the position of the tens and one bottle tops in the position of the units, which means if the child has to replace 2 bottles tops which are in the position of the tens they must be replaced with the ten units appearing there two times. So that in that case a learner will be able to recognize that I need 21 bottle tops and from those 21 bottle tops I'm able to group them into a group of 3 (***LL49)**.

According to Ms Dave, her learners with special needs found it difficult to catch up or understand the content and this was connected to the introduction of the abacus in the previous grade.

> *I observed that previous grade 0-2 learners were not using the abacus, they only use it for counting as they call it the 100 bead abacus. It became difficult when introducing the concept of using the abacus because the learners find it difficult to find the values and place values of the number. The concept of one bead below is equaled to ten beads up also gave the learners hard time grasping it and understand it very well.*

Ms Dave believes that to combat the challenges the following should be done:

> *We as the junior primary teachers have to introduce the abacus from grade 0 at schools since the use of calculators is not allowed in the Namibian primary education and make it compulsory across the curriculum. Introducing the abacus curriculum in lower primary will improve learners' mathematics abilities and confusion they have in different concepts e.g. The place values and division. More practice is required especially when it comes to multiplication and division (* **DL27).**

Ms Tyler experienced that her learners had difficulties with the basic number concepts, complements, being slow at the beginning and lacking self-confidence. Some of the learners had problems in computing with big numbers. Only a few abacuses were available and some learners had to share when doing calculations.

Mr Quasi is convinced that learners should be motivated to participate in the use of the abacus and teachers should make sure that lessons are real and interesting. Relating the mathematics work to real-life experiences so that they are able to use it in their everyday lives is also one way of removing the challenges. The abacus should be made imaginable and visionable. The teachers should use visualization in the form of the abacus from the pre-primary, in order for learners to get used to it. Learners should be allowed to find materials to construct their own abacuses, so they become aware that the mathematics they are doing in class is all around them.

Teacher-related challenges

In his journal, **Mr Lebby** iterated that the lack of confidence at the beginning of the intervention programme was caused by a lack of understanding of what visualization and number sense is and how the abacus can be used in the two processes for learners to understand the mathematics concepts and make sense of them.

Ms Dave indicated that the concept of multiplication and division with two or more numbers was quite challenging to her. As a teacher she first had to rehearse for the lesson before presenting it to her learners:

> *When it came to multiplication and division of the abacus with two or more numbers even I have to rehearse several times before the lesson and learners also found it difficult. There were parts of the lesson where the learners lose track and have to redo the whole computation over again.*

Participant journals revealed that they first experienced problems when they started teaching with the abacus. Ms Tyler indicated in her journal that:

> *I first had to practice using the abacus when doing multiplication and division because the two operations are tricky to be performed on the abacus before I was observed by the researcher.*

She further added that addition and subtraction with small numbers are easy and thus do not need too much attention after one grasps the procedure. Ms Tyler also mentioned that she experienced challenges of not having enough abacuses for her learners. Some of her learners were simply watching how she presented the lesson using the abacus. She believes that if each of her learners had their own abacus then they would have all be involved in the lesson the same time.

Curriculum -related challenges

The teachers in the interviews pointed out the challenges they experienced when using the abacus that were curriculum-related. Mr Lebby stated that he experienced challenges associated with the transitional school sytem. He had this to say:

> *The first challenge is that learners are in a transitional school system whereby they are being taught by different teachers in Pre-primary, Grade 1, 2 and 3, so for my first experience is that more or less the abacus wasn't used at the beginning and the learners didn't comprehend the use of the abacus properly that's is why I find learners in grade 3 that cannot read the numbers and identify the values and place value of individual digits placed on the abacus. The abacus use should be made compulsory in the Foundation Phase to avoid having learners in Grade 3 that have problem with simple arithmetics.* **(LL35).**

Time constraints emerged from Mr Quasi's lesson presentations as one of the challenges of using the abacus. Mr Quasi explained that lessons are 40 minutes long and that this was not enough time to successfully deliver a lesson using the abacus. A double period would be preferable. He indicated that:

> *The use of the abacus is* ***time-consuming****, when it comes to introducing the concept then you have to explain it and monitors the learners when they are performing different sums using the ancient tools. He further stated that one needs to be* ***stationed*** *when explaining a certain task because learners are using concertation skills and if you move around the learners lose focus and if one is stationed at the front then some learners at the back will not be able to see* **(QL28)**.

Mr Quasi experienced challenges when his learners found it **difficult to calculate** big numbers because they were only in Grade 3 and some of them still had problems in counting big numbers. Their number sense had not yet developed.

> *In a class setting we have a different level of learners: average, slow and gifted when I introduced the abacus it was very challenging to my slow and average learners because of the content and concept that we use e.g. when it comes to multiplication and division concepts around these two is a lot to take in by these types of learners (QL30).*

With the challenges above Mr Quasi thought of the teachers to have enough abacuses for each learners so that the teacher can be stationed at one position while all the learners are working with their abacuses.

> *As I said before Grades 1 and 2 learners are too slow and busy with the readiness programme and teachers hardly have time to introduce the concept of using the abacus. Grade 3 grasp the concept but are challenged and once the learner goes challenged there is a lack of participation and lack of encouragement. In addition the mathematic periods in the foundation should be doubled at least twice a week to be able to use different manipulatives when teaching mathematics and develop our learners number sense.*

Ms Tyler advocated that basic number concepts should be taught first so that learners are able to count and know complements that make up numbers. The abacus should be used to teach basic number concepts for learners to visualize and not forget the concepts taught. Learners should be encouraged to use various methods in mathematics to develop their number sense. She cited that familiarity with the abacus should start as early as pre-primary so learners use it as a counting tool and teachers should encourage positive attitudes about the use of the abacus.

With the challenges experienced by the participants, they are of the opinion that a double period at least twice a week should be allocated to mathematics. The teachers also felt that the curriculum developers should intergrate the use of the abacus into the curriculum and that it is made compulsory in the Foundation Phase. In their interviews, the teachers indicated that extra support should be given to learners with learning difficulties in order to develop their number sense and be on a par with their peers.

4.5.5 Changes as a result of the intervention programme

According to Mr Lebby, as a result of the intervention programme and being taught with the abacus, more and more learners realized the difference between the value of the digits as well as the position of the digits that we refer to as place value. He said:

> *As you teach mathematics using the abacus we also recognize that you don't need to use the traditional algorithm for the four basic operations but you can also demonstrate by recognizing the place value of the digit and can carry out the four basic operations with the abacus. As the result of this intervention the pace at which learners are getting the answer improved and they get answers too fast and with confidence… The lesson becomes effective because I was able to demonstrate how we arrived at certain answers.*

On the other hand, the slow and average learners also improved their understanding of mathematics and number sense as a result of using the abacus.

Ms Dave recounted that as a result of using the abacus in her teaching, she observed the following changes in her learners:

> *The accuracy of getting the answers improved and the pace at which the learners are getting the answers changes. My learners are now able to do complex calculations using the abacus without any fear (self-confidence). My learners further, develop different strategies of arriving at answers using the abacus when they are doing different computation. Learners develop logical reasoning when explaining their answers. In addition to that, the learners also developed problem-solving abilities and solved word problems without and difficulties* **(DL40)**.

Mr Quasi narrated in the interview that:

> *My learners got motivated and look at mathematics positively. The use of the abacus reduced level of fear in mathematics. The learners show improvement in their thinking skills about numbers. They enjoy using the abacus and became persistent and determined to solve the problems. Cognitive abilities greater ability to understand what was taught, when they see a number they see the value of a given number.*

4.5.6 Emergent themes regarding other ways of developing number sense

During the interviews, the participants cited various methods/ways in which the Foundation Phase teachers can develop number sense in their learners. These methods include game playing, role-playing, using number lines, verbalizing math concepts and development of number concepts.

Mr Lebby suggests that apart from using the abacus, he recommends game playing as one-ways in which teachers can develop number sense.

Game playing: gives meaning to numbers e.g. playing cards can be used to teach addition and number sense.

> *You can tell the learners today we are going to play cards using certain rules, they can also play it by matching the same number. Some learners cannot recognize the difference between numbers because we teach these learners with the assumption that they already know the shape of the digit but in reality, there might be some that don't know, so you can ask them to match the numbers that look the same (* **LL56)**.

Mr Quasi also stated that: "*Learners are interested in technological work and software games can be used to develop their number sense".*

Role-playing:Mr Lebby pointed out that teachers can use role-playing to develop number sense. He suggested giving the learners mathematical stories that they have to dramatize using numbers:

"*you as a teacher can allow them to play small business by buying and getting change and by doing so the learners will develop their number sense"*.

Using the number line: This supports students' development of number sense by helping them create a mental representation of the order and magnitude of numbers. Besides this, explicitly sequencing instruction to transition from the concrete to visual to abstract representations of mathematics concepts, supports students' conceptual understanding.

> *In the Foundation Phase, teachers can use many other ways that enhance understanding of numbers and develop. Foundation mathematics is very important because it enables learners to develop number sense and if learners leave the lower grades without fully developing their number sense it might affect their mathematics in upper grades*.

According to Ms Dave

> *the use of a number line can make the learners understand the number system, and then the learners can also be able to understand the difference between addition and subtraction for instance if you draw a number line and start adding a grade 3 learner can see that numbers are increasing and when subtracting the numbers are decreasing.*

Verbalize the mathematics concepts: Ms Dave was of the opinion that teachers should let the learners explain the methods they use to arrive at certain answers and explain in their own words how they solved a problem. This can also develop number sense. She further argued that the use of manipulatives should be encouraged:

> *Teachers can use concrete manipulatives like place value block, stones, flashcards, counters, etc…the use of manipulative help learners make use of quantity e.g. 7x5 then the learners can take 7 manipulatives five times which also encourage the concept of inverse operation. As teachers, we need to visually show learners with physical manipulative why certain answers are the way they are* **(DL38)**.

Mr Quasi's response to different ways of developing number sense was that:

> *Teachers should also encourage mental mathematics in learners by allowing learners to count in skipping numbers, in doing this it is not only mental mathematics that is encouraged but the concept of decomposing numbers is also developed.*

Mr Quasi indicated that:

> *Lastly, I can say if we have to develop the number sense at the foundation level the development of number concepts should also be developed when it comes to teaching mathematics. Learners should be able to understand the relationship between numbers. Using the math concepts like the identity property, zero product property, commutative, associative and distributive property, if the learners understand this concept then their*

number sense will improve and this will benefit them in their future mathematics later in life.

Ms Tyler added that

> *learners should be helped to visualize numbers in a variety of contexts and that mathematics should be authentic and based on real-life experiences. Teachers should give learners multiple strategies, allowing them to choose a strategy that works best. She suggested that theachers should model and encourage mental computation. Allowing peer teaching to discussproblem solving can also help the learners and at the same time encourages the process of learner-centered education. Provides manipulatives for problem-solving rather than the pen and pencil method.*

4.6 COMMONALITIES AND DIFFERENCES IN THE FINDINGS ON THE USE OF THE ABACUS IN DEVELOPING NUMBER SENSE

In this section, I summarize the analysis of the data presented above, considering the themes of the similarities and differences from the perspectives of the four participants. The summary aligns with the three research questions of this study:

- How did teachers use the abacus when teaching Grade 3 mathematics classes prior to participating in the intervention programme?
- How can the abacus be utilized as a visualization tool to develop number sense in a Grade 3 class when teaching mathematics?
- What are the teachers' experiences and perceptions of using an abacus as a visualization tool in developing learners' number sense after participating in the intervention programme?

It is evident from the questionnaire data that most teachers do not use the abacus when teaching mathematics even though they have abacuses in their classrooms. Some teachers only used it as a counting tool. Although they did not use the abacus the teacher's acknowledged that using visuals when teaching is of utmost importance. The teachers highlighted other ways of developing number sense such as base ten blocks, concrete objects and drawing pictures to visualize content. This aligns with the NCTM (2000) who advocate for the use of concrete materials and manipulatives to develop number sense and bring meaning to learners' written symbols.

The classroom observations revealed that all four participants used the abacus to teach their five planned lessons. The teachers utilized the abacus in ways where they explained the content by demonstrating what was being taught. They taught their learners to visualize the beads and bottle tops as numbers and manipulated them to perform various computations. As advanced by Kim (2015), an abacus can promote mental visualization and moving and manipulating of beads on the abacus helps to create concrete images of numbers, with which learners can perform mental calculations. The data from the lesson observations shows that learners paid more attention to the lessons and they were actively involved. The engagement between the teachers and learners was interesting and one could see that they were enjoying the lesson.

The audio data from the interviews are in agreement with the data collected from the questionnaires and lesson observations. In the interviews, the participants said that the abacus helped to teach the place value concept and that they had observed changes in their teaching and in the learning of their learners. They echoed that using the abacus helped the learners to actively participate in the lessons and develop self-confidence, logical reasoning, and mathematical proficiency by visualizing and manipulating the ancient tool. This confirms with Swinson (2018), who reports that the abacus has the following benefits in mathematics: it encourages critical thinking, it helps with high spatial reasoning and creates independence in learners. In addition, the participants reported that learners visualized the abacus and read numbers out aloud; later they physically demonstrated the processes on the abacus. Some indicated that even though using the abacus is time consuming, it is the best way to make mathematics fun and interesting.

During the data analysis, similarities were grouped according to the major themes of number sense: *sense of orderliness of numbers; multiple representations of numbers; sense of the relative magnitude of numbers; understanding the effects of operations; understanding mathematical properties; the relationship between operations and the inclination to review data.*

Sense of orderliness of numbers – number sense implies the understanding of how the number system works with place value being an important component of it (McIntosh, Reys, & Reys, 1992b). Using the abacus, the teachers taught their lessons so that their learners were able to understand the concept of place value and used the abacus to physically demonstrate the concept. Mr Lebby attested: *"The abacus is one of the best tools to teach for an understanding of place value because it is visual in its nature."* Cotter (2000) emphasized that children must experience the pattern of trading: ten ones for one ten, ten tens for one hundred, etc. – something that can only be done with an abacus.

Multiple representations of numbers – In the lessons the teachers presented, the different representations of numbers was evident. Teachers used numbers in a variety of contexts to develop their learners develop number sense understanding. The teachers let the learners present numbers using the symbolic abacus, the physical abacus, physical counters and traditional algorithms to represent numbers. I observed that learners could see, and at the same time use numbers in different representations, which showed their understanding of number sense. Buxton (1997) mentioned that an inability of learners to use number sense in different representations is a sign of a weak number sense.

Understanding the effects of operations – In all the lessons, teachers gave their learners opportunities to use the abacus and demonstrate how different operations affect numbers. The teachers presented their lessons so that their learners could understand the effects of operations on various numbers and number sentences. In this study, the participants modeled multiplication using the abacus as repeated addition and this provided a concrete way of helping learners to think about multiplication and carry out their computations.

Pee et al. (2014) indicate that in some cases, teachers pressurize learners to remember rules for multiplication and division computations However, learners may forget or become more confused as they have no understanding of why certain rules work. It is then the responsibility of the teachers to strongly encourage learners to use their understanding and quantities and operations.

Understanding the relationships between operations – In the lessons presented, teachers taught and presented activities using the abacus that were aimed at helping their learners understand the relationships between operations. In order to help their learners to easily understand this theme, the teachers explained the concept of model multiplication (which is repeated addition) and performed other inverse operations using the abacus. Sengul (2018) says that if someone understands the relationships between operations, then they have high order thinking skills and it further enhances their number sense. The use of the abacus to show the relationships between operations is supported by Tang (1993) who explains that it the helps learners to see the relationship between numbers and abstract concepts.

Understanding mathematical properties – The teachers used their abacuses to demonstrate the three mathematical properties, then explained and demonstrated how certain properties do not apply to some of the operations. The participants taught this theme with the intention of illustrating and linking practical applications to the development of understanding mathematics. Number sense often manifests itself as learners intuitively apply arithmetical properties when inventing procedures for computing.

Inclination to review data – When the solution is produced, learners with number sense examine their answers in the light of the original problem. This number sense theme was delivered by learners defending their answers by visualizing and manipulating their abacuses.

Inclination to utilize efficient representations – The teachers helped their learners to get answers by using different strategies with the abacus; some that are more efficient at certain times than others. On the other hand, it was shown that learners with little number sense often use a more difficult method for are calculating. With the abacus, learners can examine their answers in light of the problem at hand when the solution is provided to determine if their answer makes sense.

4.7 CONCLUSION

In this chapter an in-depth analysis of this research study was presented. The data collected from the survey was analyzed using thematic analysis. In addition, the observation data that aimed at answering Research Question 2 was analyzed using the analytical tool in Appendix H. The views and experiences of the participants on the use of the abacus were extracted by looking at the emergent themes in the reflective journals and interview transcripts. The interview data answered Research Question 3. The chapter ends with this conclusion.

The next chapter uses the analysis of Chapter 4 to discuss the findings and analysis, consider some implications, draw conclusions from this study, formulate a number of recommendations and explore avenues for further research.

CHAPTER 5

CONCLUSION AND RECOMMENDATIONS

5.1 INTRODUCTION

The purpose of this study was to explore and understand how an abacus can be utilized as a visualization tool to develop number sense in the Foundation Phase. The research questions focused on teaching the uses of the abacus, with the aim of developing number sense. In this study I was interested in how this ancient tool can be re-imagined and re-envisioned to enhance the conceptual understanding of number sense in learners.

In this chapter, I present a summaryof the main findings drawn from the study. This is followed by a discussion on the significance of this study and the limitations thereof. Further, I present my recommendations emanating from my findings on the use of the abacus as a tool to enhance the teaching of number sense. Finally, I have presented my own reflections of this three year research process as a whole and the avenues I suggest for further research.

5.2. SUMMARY OF THE MAIN FINDINGS

I present my main findings according to the three research questions of this study. The survey questionnaire was designed to answer the first research question and to give an overview of the current use of the abacus in the Foundation Phase in the /Kharas educational region. During the intervention period, lesson observations were carried out to answer the second research question. A number of themes that were evident and those that emerged in relation to number sense, were discussed in the previous chapter. The participants were asked to keep a reflective journal to record their experiences and perceptions of the use of the abacus in developing number sense in Grade 3 classes, from the beginning of the intervention programme until the end. The teachers' reflective journals contributed to answering the second and third research questions. Finally, semi-structured interviews were conducted after the lesson observations to provide answers to the last research question of this study.

Research question 1

How did teachers use the abacus when teaching Grade 3 mathematics classes prior to participating in the intervention programme?

Few of teachers who participated in this study indicated that they used the abacus during their teaching. More than half of the Foundation Phase teachers who took part in this survey did not use the abacus at all. It was noted that more than half of the respondents were not aware that the abacus can be used to perform arithmetical work with different operations. The few participants who indicated that they use the abacus, explained that they employ it as a counting tool, for explaining certain concepts and for teaching simple arithmetic.

Some teachers felt that the abacus is the ideal tool to demonstrate basic arithmetic with the basic operations and counting. The surveyed teachers also indicated that they used the abacus to link abstract mathematics content to the concrete mathematics.

The teachers identified various manipulatives that they use to develop their learners number sense.: playing mathematical games, verbalizing mathematics concepts and drawing pictures to visualize abstract concepts, among others.

The surveyed teachers identified several benefits of using the abacus as described in section 4.2.5 for the development of number sense in mathematics. The few teachers that used the abacus responded that they were using it for sense-making through logical reasoning and explanations, for learners to gain a deeper conceptual understanding of numbers.

The data from the survey indicated that not much use of the abacus was evident in the Foundation Phase classes in the //Kharas Region. This then prompted me to design an intervention programme that would help teachers re-think, re-envision, re-explore and re-examine the use of an abacus in the teaching of number sense. Despite the abacus being an old tool still in existence during these modern times where technological advancements have been made, learners in the Foundation Phase require concrete manipulatives to develop their number sense. The intervention was aimed at helping Foundation Phase teachers to re-think and re-imagine how best an old tool such as an abacus can be used to meet teaching and learning requirements of a modern classroom environment. As will be explained below, the re-imagining, re-conceptualizing and re-envisioning of the abacus use in the intervention led to the teachers being able to teach simple arithmetic while using this ancient tool to develop their learners' number sense. Amoah et al. (2019) emphasizes that young minds require concrete objects to manipulate without rules for processing data, and that data is added with rules for processing later being formed solely by one's physical experiences. They further add that the abacus helps learners in lower grades to learn concepts such as multiplication, that may be difficult to understand if such manipulatives are not used.

Research Question 2

How can the abacus be utilized as a visualization tool to develop number sense in a Grade 3 class when teaching mathematics?

During the classroom observation, I noted that participants consistently aligned their lessons with the main number sense themes as discussed in Chapter 4 using the abacus to teach the concepts pertaining to number sense in the Foundation Phase. The teachers used the abacus so that the learners were led to visualize the beads and bottle tops on the abacus as numbers. In this study, the observed teachers' use of the abacus in the teaching of number sense, brought about and strengthened the visuality element in computing. In the lessons, the observed teachers used the abacus to link abstract mathematics to concrete ways of computing by manipulating beads and subitizing numbers.

I observed the teachers introducing the symbolic abacus and this helped the learners to realize that the abacus can be represented symbolically by drawing it. The teachers' use of the abacus in its concrete and symbolic forms enhanced the physical representation of numbers in different ways. From the teachers' comments, the abacus helped their below average and challenged learners to understand, through the use of the abacus, number sense themes more efficiently and they participated more freely in class discussions. The inclusion of this old tool thus brought inclusivity into the classrooms and increased class participation during learning. Teachers successfully used the abacus to demonstrate how to compute using different mathematical properties and the relationships between operations. They also used the abacus to test mental mathematics and problem solving in their learners.

The abacus in all lessons observed was used concurrently with verbal explanations followed by physical representations of the concept by the teachers. Teachers presented different mathematical abstract concepts, while encouraging their learners to use the abacus to visualize the abstract content and solve problems using the abacus. In the observed lessons teachers used the abacus to compute mathematics that the learners use in their daily lives. This use of the abacus helped the learners to link abstract mathematics to concrete ways of computing and this resulted in the enhancement and understanding of number sense. In the Foundation Phase where the calculators are not allowed, these participant teachers used the abacus for teaching arithmetic and counting.

Overall, the abacus was used to visualize, compute, subitize and perform mental mathematics. The learners visualized abacus beads and bottle tops as numbers and performed different calculations while re-imagining the tool as a calculator. The abacus was used in such a way that it linked the abstract to concrete mathematics. The introduction of a symbolic abacus enhanced the physical representation of numbers and it brought inclusivity to the learners.

Research Question 3

What are the teachers' experiences and perceptions of using an abacus as a visualization tool in developing learners' number sense after participating in the intervention programme?

In this study, participating teachers spoke about the various experiences that they went through and how their perceptions of the abacus were influenced after participating in the intervention programme. With the re-imagining of the abacus as a visualization tool, the participants believed that the use of the abacus worked well for them. Teachers acknowledged that the abacus was indeed a functional visual tool that aided in developing number sense in learners. They further stated that through visualizing the abacus, the learners were able to compute using complex numbers without traditional algorithms. The teacher participants were of the opinion that the abacus helped learners to perform better when they were manipulating beads and bottle tops and visualizing them as numbers at the same time. Throughout the lesson presentation the participants considered that learners talked comfortably about numbers and used numbers in a variety of ways. The teachers also noticed that learners developed positive attitudes towards mathematics and that their dependency on the abacus reduced over time. The teachers also appreciated and conceded that the abacus, to a great extent, simplified the complexities that could have arisen in the teaching of number sense.

Participants are of the opinion that the abacus is an ideal tool to teach the number system, improve visualization listening skills and concentration, boost learner's confidence, and strengthen their memory. The participants also feel that the use of the abacus increases learners' engagement and enjoyment in lessons. Additionally, teachers claim that the abacus is effective in supporting links underpinning deep conceptual understanding of number sense.

The teachers acknowledged that the use of the abacus and its benefits of visual teaching promoted live and interactive classroom environments that helped learners to better understand number sense. However, they were all quick to say that incorporating the abacus was time-consuming in terms of preparing for the lessons and when conducting the lessons. They lamented the lack of time due to various factors related to the curriculum and school programmes.

The teachers pointed out that the abacus brought to their learners many advantages associated with learning by seeing and that the abacus needs to be a compulsory subject in the curriculum. They asserted that to make abacus learning a compulsory subject in the Foundation Phase, teachers need to have a high level of computation skills with the abacus.

Lastly, they maintained that even though the abacus has numerous advantages it has its disadvantages. The main challenges the participants had in common was the lack of abacus use in previous grades and insufficient time to teach using the abacus.

5.3 SIGNIFICANCE OF THE STUDY

The poor performance of Namibian learners in mathematics is a major concern. Many factors may be contributing to this. The Namibia mathematics subject policy for the Foundation Phase emphasizes the use of concrete materials. The subject policy indicates that the use of concrete materials increases learners' ability to reason independently and reflect critically on their thinking. However, in some instances, primary school mathematics teachers in some parts of Namibia find it challenging to use concrete materials as teaching and learning tools.

This study contributed to the use of the abacus in the Foundation Phase to develop number sense in mathematics. The re-imagining of using the abacus as a tool to develop number sense can help with the current poor performance of mathematics in Namibia. The findings of this study provided insightful information on how an abacus can be utilized as a visualization tool by teachers to develop number sense in the Foundation Phase in the //Kharas region, especially with the participating teachers in this study. Further, it enabled the researcher and teachers to re-imagine using the old abacus in modern classrooms (where calculators are not allowed) to develop number sense in learners. This understanding will lead to the improvement of lack of number sense in the Foundation Phase, particularly in the //Kharas educational region.

5.4. CHALLENGES AND LIMITATIONS

The challenges encountered in this study were in collecting the survey questionnaire from the 50 participants. Some participants argued that there was not enough time to complete the questionnaire. Others further claimed that they do not use the abacus in their classes and it was difficult to complete the questionnaire on the use of the abacus. In addition, some questionnaires I received were incomplete, partially completed and some answers were not elaborated on where needed.

The pandemic (Covid-19) delayed the data collection process. The schools were closed due to Covid in 2020 when it was time for data collection. Since this study is interpretive, the intervention workshop and lesson observation had to be put on hold until the schools re-opened and face-to-face teaching started again.

The other challenge experienced in this study was using the abacus to teach at the intervention workshop. Some teachers found it difficult to grasp the concept of teaching mathematics using this ancient tool. They did not understand that the abacus can be used to teach number sense. In the intervention workshop, teachers had to integrate the abacus into their lessons.

This study has limitations resulting from the quantity of data collected. Due to the nature of this study, five teachers were meant to take part and only four did. The study was confined to a particular geographical area and followed a qualitative approach in which data was collected from a small sample, with the result that its findings cannot be generalized. The study focused on the teachers' use of the abacus to develop number sense in the Foundation Phase. However, only some number sense themes were explored using the abacus in this study. The use of the abacus and other manipulatives with some of the number sense themes should be researched as well. Similar case studies on the use of the abacus and manipulatives should be carried out to make a significant contribution to the understanding of why Namibian learners have poor number sense and poor performance in mathematics is high. Future researchers may also include the upper grades and explore the use of the abacus in other topics in mathematics as well.

5.5. RECOMMENDATIONS

This research study suggests evidence that teachers of primary education in Namibia should consider when using visualization processes in developing number sense with their learners. It is of utmost importance for teachers to create a rich mathematical environment and guide the learners towards the understanding of number sense concepts during teaching. In doing this, a careful consideration of re-introducing the abacus in the Foundation Phase should be given. Teachers at this phase need to develop a habit of using different manipulatives and concrete materials to enhance the development of number sense.

Teachers need to be encouraged to have positive attitudes towards the use of the so-called 'old' teaching resources such as the abacus. This is important because in modern Foundation Phase classrooms calculators are not allowed to be used. Thus, an abacus may be used to perform the same calculator functions for younger minds while allowing other associated advantages to be enjoyed by the learners. Teachers should make lessons as interesting as possible for the learners to participate. This is possible if they are assisted to re-envision and re-conceive of the abacus as a potentially innovative and good manipulative to assist in doing computation in their lessons. Teachers, together with their learners, can construct abacuses for their classes. These type of abacuses can be made from readily available materials in their own environments such as bottle tops, beads, wood, and wire. Encouraging learners to make their abacuses can also help learners realize that the mathematics they are doing in the classroom is around them and at the same time have positive attitudes towards the subject and the tool.

The abacus should be used to encourage the process of visualization in the teaching and learning of mathematics, especially core concepts in the Foundation Phase such as number sense. Using the abacus as a visualization tool will help the learners to see and realize that mathematics can be taught and learned in different ways that are easier to follow than the traditional methods. Use traditional algorithms devoid of conceptual understanding should be discouraged. The use of tools like the abacus brings many advantages that teachers need to tap into: it allows learners to self explore in class activities and they are low cost manipulatives that can be made from cheaply and locally avaible materials. Teachers are thus encouraged to help their learners appreciate the visuality of solving problems using the abacus instead of only using the pen and paper computations which are rule based and highly procedural.

The participant teachers pointed out that manipulatives and abacus use are time-consuming. However, they should follow the Ministry of Education's recommendations on the use of concrete materials to improve mathematics performance. Time may be regarded as one of the most valuable resources in a mathematical classroom. Using the abacus may be time-consuming in the short term, but it is necessary for enhancing conceptual understanding. The use of manipulatives is compulsory in the Foundation Phase, according to ministerial recommendations for the development of number sense in learners. Teachers thus need to be trained to use such important teaching and learning resources such as the abacus. The use of and re-imagining, re-envisioning re-conceptualising and re-examining of so called ‘old teaching tools’ is important. The workshops, observations, team planning and engagements that I had with the four teachers attest to the urgent need for in-service teacher training and the establishment of collaboration that will foster improved teaching of mathematics.

5.6 SUGGESTIONS FOR FURTHER RESEARCH

As this study was carried out on only five components (addition, subtraction, division, multiplication, and place value) of number sense in Grade 3, it will be interesting if this research is expanded to other components in the Grade 3 syllabus.

I suggest an in-depth longitudinal study about the use of manipulatives to enhance the visualization process to develop number sense be conducted with pre-primary, Grade 1 and Grade 2 learners. As this interpretive study was located in Keetmanshoop, it would be more interesting to see how the results would differ if the study was done on farm schools in the //Kharas region.

Further research on the use of visuals and manipulatives to develop number sense could be conducted to train the Namibian Foundation Phase teachers to incorporate it in their teaching. Further studies should also be conducted to find ways in which learners' number sense can be developed fully at the Foundation Phase to prevent and improve the poor performance of mathematics in Namibian schools. Research on the use of the abacus as a visualization tool in mathematics in all primary grades could be undertaken in the future.

5.7. PERSONAL REFLECTIONS

Undertaking this research study journey was an experience for me in the research field. The theory of constructivism used as a lens of this interpretive study was appropriate.

As a mathematics primary educator, the concept of visualization in this study challenged me to do introspection on the use of manipulatives and concrete materials in my teaching. The importance of number sense emphasized in this study made me realize the importance of number sense in the Foundation Phase as the lack of number sense is the most cause of poor performance in mathematics.

The experience I gained in this study will encourage me to get involved in more research studies in the future. Overall, the writing of this research study has improved my grammar, academic writing and reading.

5.8. CONCLUSION

My research study was located within the interpretive paradigm. It was underpinned by constructivism as the theoretical lens to investigate and explore how an abacus can be utilized as a visualization tool to enhance number sense understanding in learners. The data of this study were collected through questionnaires, observations, reflective journals and semi-structured interviews. The findings of this research study revealed that the abacus can be used as a visualization tool in various ways to develop learner's number sense understanding in the Foundation Phase. The study further advocated that the use of the abacus in mathematics stimulates independent thoughts in learners, makes mathematics lessons more interesting and enhances the learner-centered approach in classrooms.

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APPENDICES

Appendix A: Ethical Clearance

RHODES UNIVERSITY
Where leaders learn

Human Ethics subcommittee
Rhodes University Ethical Standards Committee
PO Box 94, Grahamstown, 6140, South Africa
t: +27 (0) 46 603 8055
f: +27 (0) 46 603 8822
e: ethics-committee@ru.ac.za

www.ru.ac.za/research/research/ethics
NHREC Registration no. REC-241114-045

25 February 2020

Taimi Elifas

Review Reference: 2020-0775-3275

Email: g19e9935@campus.ru.ac.za

Dear Taimi Elifas

Re: Re- imagining the use of the Abacus as a visualization tool to develop number sense in Grade 3 learners.

Principal Investigator: Doctor Clemence Chikiwa

Collaborators: Ms Taimi Ndinelago Elifas, Professor Marc Schafer

This letter confirms that the above research proposal has been reviewed and **APPROVED** by the Rhodes University Ethical Standards Committee (RUESC) – Human Ethics (HE) sub-committee.

Approval has been granted for 1 year. An annual progress report will be required in order to renew approval for an additional period. You will receive an email notifying when the annual report is due.

Please ensure that the ethical standards committee is notified should any substantive change(s) be made, for whatever reason, during the research process. This includes changes in investigators. Please also ensure that a brief report is submitted to the ethics committee on completion of the research. The purpose of this report is to indicate whether the research was conducted successfully, if any aspects could not be completed, or if any problems arose that the ethical standards committee should be aware of. If a thesis or dissertation arising from this research is submitted to the library's electronic theses and dissertations (ETD) repository, please notify the committee of the date of submission and/or any reference or cataloging number allocated.

Sincerely

Prof Roman Tandlich

Chair: Human Ethics sub-committee, RUESC- HE

Appendix B: Approval letter from the Director of education

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||KHARAS REGIONAL COUNCIL
DIRECTORATE of EDUCATION, ARTS and CULTURE

Tel: (063) 227000
Fax: (063) 223800

Private Bag 2160
KEETMANSHOOP

Enquiries: |Awebahe J ||Hoeseb
Date: 24 February 2020
File Ref. No.: 15/1

Regional Director of Education, Arts and Culture: ||Kharas

Taimi Ndinelago Elifas
Rhodes University
Drosty Road
GRAHAMSTOWN
6139

jonenelago@gmail.com

Dear Taimi Ndinelago Elifas

SUBJECT: Approval to conduct research at public primary schools in ||Kharas Region

1. Receipt of your letter, dated 24 February 2020, Subject: **Request for permission to conduct a research**, bears reference.

2. Permission is hereby granted to you **Taimi Ndinelago Elifas**, to conduct research on the topic **"Re-imaging the use of the Abacus as a visualization tool to develop number sense in Grade 3 learners"**, amongst Grade 3 teachers at public primary schools in the ||Kharas Region resorting under the Directorate of Education, Arts and Culture in the ||Kharas Regional Council.

3. The aforementioned authorization is granted on condition that **only Grade 3 teachers from public primary schools located in the ||Kharas Region will** form part of the intended research sample and that the anonymity of those respondents from the selected public schools, thus the teachers, would be guaranteed.

4. Please present this letter to the Principals of the selected public primary schools in the ||Kharas Region when you visit a school and agree on suitable time-slots for research engagements with any individual/group of the Grade 3 teacher/s with the Principals.

5. Note must however be taken of the limitation that the engagement with the Grade 3 teachers is not supposed to lead to the disruption and or interruption of teaching and learning programmes and or processes, or discontinuation of teachers' teaching duties.

6. Lastly, the Regional Management Committee of the Directorate of Education, Arts and Culture in the ||Kharas Regional Council would like to request you to share the research findings with this regional directorate to be used for insightful planning purposes and for adding value to inputs geared towards policy-making and or policy review.

Yours faithfully

24/02/2020
....

|Awebahe Johannes ||Hoeseb
Regional Director of Education, Arts and Culture
||Kharas Regional Council

Appendix C: Approval letters from school principals

School A

School A

SCHOOL PRINCIPAL CONSENT FORM

CONSENT FORM

I the under signed, in my capacity as a School Principal, give consent to Ms. T.N Elifas a part time Master of Education (Mathematics Education) student at Rhodes University, Grahamstown South Africa, to conduct Research at-[redacted]-Primary School **on the use of the Abacus as visualization tool to develop number sense in the Grade 3 learners** as outlined in the attached request letter. Both parties understand that this consent can be revoked at any time without giving reasons.

Principal Name [redacted] ....Student Name: Ms. T.N. Elifas.

Signature [redacted] ....Signature..... TElifas

Witness [redacted]

Signature and D[redacted]

REGIONAL COUNCIL
OF EDUCATION ARTS AND CULTURE
[redacted]
2020 -02- 28
REPUBLIC OF NAMIBIA

School B

School B

SCHOOL PRINCIPAL CONSENT FORM

CONSENT FORM

I the under signed, in my capacity as a School Principal, give consent to Ms. T.N Elifas a part time Master of Education (Mathematics Education) student at Rhodes University, Grahamstown South Africa, to conduct Research at--[redacted]--Primary School **on the use of the Abacus as visualization tool to develop number sense in the Grade 3 learners** as outlined in the attached request letter. Both parties understand that this consent can be revoked at any time without giving reasons.

Principal Name [redacted] ......Student Name:.Elifas Taimi N

Signature........ ......Signature.....TNElifas.......................

Witness..

Signature and Date:............ 2020-02-27.................................

IIKHARAS REGIONAL COUNCIL
DIRECTORATE OF EDUCATION, ARTS AND CULTURE
THE PRINCIPAL
[redacted]
2020 -02- 27
PO Box 705, Keetmanshoop
Tel: 063 - 222843 Fax: 063 - 221039
REPUBLIC OF NAMIBIA

School C

School C

SCHOOL PRINCIPAL CONSENT FORM

CONSENT FORM

I the under signed, in my capacity as a School Principal, give consent to Ms. T.N Elifas a part time Master of Education (Mathematics Education) student at Rhodes University, Grahamstown South Africa, to conduct Research at-[redacted]------------------------Primary School **on the use of the Abacus as visualization tool to develop number sense in the Grade 3 learners** as outlined in the attached request letter. Both parties understand that this consent can be revoked at any time without giving reasons.

Principal Name:[redacted] .Student Name: Elifas Taimi

Signature: [redacted] .Signature: ....

Witness: [redacted] ....

Signature and Date: 24-02-2020 ....

School D

SCHOOL PRINCIPAL CONSENT FORM

CONSENT FORM

I the under signed, in my capacity as a School Principal, give consent to Ms. T.N Elifas a part time Master of Education (Mathematics Education) student at Rhodes University, Grahamstown South Africa, to conduct Research at--[redacted]-------------Primary School **on the use of the Abacus as visualization tool to develop number sense in the Grade 3 learners** as outlined in the attached request letter. Both parties understand that this consent can be revoked at any time without giving reasons.

Principal Name[redacted] Student Name:...Elifas Tcumi...

Signature...[redacted] Signature....T.N Elifas.........................

Witness...[redacted] ....

Signature and Date:..24-02-2020...

!!KHARAS REGIONAL COUNCIL
DIRECTORATE OF EDUCATION, ARTS AND CULTURE
[redacted]
2020 -02- 24
PO Box 1335, Keetmanshoop
Tel: 063 - 222519 Fax. 063 - 226053
Email: donboscops@iway.na
REPUBLIC OF NAMIBIA

Appendix D: Participant teachers consent forms

Ms Tyler

APPEDIX 4: PARTICIPANT INFORMED CONSENT
INFORMED CONSENT DECLARATION
(Participant)

Project Title: **Re- imagining the use of the Abacus as a visualization tool to develop number sense in Grade 3 learners**

Elifas Taimi Ndinelago from the Department of Education, Rhodes University has requested my permission to participate in the above-mentioned research project.

The nature and the purpose of the research project and of this informed consent declaration have been explained to me in a language that I understand.

I am aware that:

1. The purpose of the research project is to investigate how teachers use the abacus to develop number sense in the grade 3 learners.
2. The Rhodes University has given ethical clearance to this research project and I have seen the clearance certificate.
3. By participating in this research project I will be contributing to gaining knowledge and more insights on how to use the abacus to develop umber sense when teaching mathematics. I also expect this project to increase the number of teachers using abacus and other manipulatives to develop number sense when teaching mathematics in the //Kharas region.
4. I will participate in the project by participating in a training programme on how I can use the abacus when teaching mathematics to develop number sense. After the training I will be involved in planning and designing lessons on the use of abacus to develop number sense. I will teach five of these lessons that will be video recorded for observation purposes by the researcher.
5. My participation is entirely voluntary and should I at any stage wish to withdraw from participating further, I may do so without any negative consequences.
6. I will not be compensated for participating in the research.
7. There may be risks associated with my participation in the project. I am aware that
   a. The following risks are associated with my participation: I may work after normal working hours without compensation as the workshop will take place after school working hours.
   b. The following steps have been taken to prevent the risks: The lesson will be observed during the normal teaching time. ………
   c. There is 40% chance of the risk materialising
8. I understand that the study will involve taking video and voice recordings of my teaching and responses to interviews.
9. The researcher intends publishing the research results in the form of a thesis, research report, conference proceedings, journal articles and book chapters. However, confidentiality and anonymity of records will be maintained and that my name and identity will not be revealed to anyone who has not been involved in the conduct of the research.
10. I will receive feedback in the form of a 'summary of the key findings' regarding the results obtained during the study.
11. Any further questions that I might have concerning the research or my participation will be answered by Elifas Taimi N. Email: jonenelago@gmail.com cell: 0817204763.
12. By signing this informed consent declaration I am not waiving any legal claims, rights or remedies.
13. A copy of this informed consent declaration will be given to me, and the original will be kept on record.
14. Request to take pictures, video and voice recording for this study

I, ………………………………………………………………………… have read the above information / confirm that the above information has been explained to me in a language that I understand and I am aware of this document's contents. I have asked all questions that I wished to ask and these have been answered to my satisfaction. I fully understand what is expected of me during the research.

I have not been pressurised in any way and I voluntarily agree to participate in the above-mentioned project.

[redacted]

Participants signature **Witness** **Date** 17/07/2020

Rhodes University, Research Office, Ethics
Ethics Coordinator: ethics-commitee@ru.ac.za
t: +27 (0) 46 603 7727 f: +27 (0) 86 616 7707
Room 220, Main Admin Building, Drostdy Road, Grahamstown, 6139

Mr Lebby

APPEDIX 4: PARTICIPANT INFORMED CONSENT
INFORMED CONSENT DECLARATION
(Participant)

Lebby

Project Title: **Re- imagining the use of the Abacus as a visualization tool to develop number sense in Grade 3 learners**

Elifas Taimi Ndinelago from the Department of Education, Rhodes University has requested my permission to participate in the above-mentioned research project.

The nature and the purpose of the research project and of this informed consent declaration have been explained to me in a language that I understand.

I am aware that:

1. The purpose of the research project is to investigate how teachers use the abacus to develop number sense in the grade 3 learners.
2. The Rhodes University has given ethical clearance to this research project and I have seen the clearance certificate.
3. By participating in this research project I will be contributing to gaining knowledge and more insights on how to use the abacus to develop umber sense when teaching mathematics. I also expect this project to increase the number of teachers using abacus and other manipulatives to develop number sense when teaching mathematics in the //Kharas region.
4. I will participate in the project by participating in a training programme on how I can use the abacus when teaching mathematics to develop number sense. After the training I will be involved in planning and designing lessons on the use of abacus to develop number sense. I will teach five of these lessons that will be video recorded for observation purposes by the researcher.
5. My participation is entirely voluntary and should I at any stage wish to withdraw from participating further, I may do so without any negative consequences.
6. I will not be compensated for participating in the research.
7. There may be risks associated with my participation in the project. I am aware that
   a. The following risks are associated with my participation: I may work after normal working hours without compensation as the workshop will take place after school working hours.
   b. The following steps have been taken to prevent the risks: The lesson will be observed during the normal teaching time. ………
   c. There is 40% chance of the risk materialising
8. I understand that the study will involve taking video and voice recordings of my teaching and responses to interviews.
9. The researcher intends publishing the research results in the form of a thesis, research report, conference proceedings, journal articles and book chapters. However, confidentiality and anonymity of records will be maintained and that my name and identity will not be revealed to anyone who has not been involved in the conduct of the research.

10. I will receive feedback in the form of a 'summary of the key findings' regarding the results obtained during the study.
11. Any further questions that I might have concerning the research or my participation will be answered by Elifas Taimi N. Email: jonenelago@gmail.com cell: 0817204763.
12. By signing this informed consent declaration I am not waiving any legal claims, rights or remedies.
13. A copy of this informed consent declaration will be given to me, and the original will be kept on record.
14. [redacted]ing for this study

I, …..[redacted]……………… have read the above information / confirm that the above information has been explained to me in a language that I understand and I am aware of this document's contents. I have asked all questions that I wished to ask and these have been answered to my satisfaction. I fully understand what is expected of me during the research.

I have not been pressurised in any way and I voluntarily agree to participate in the above-mentioned project.

[redacted]

Participants signature **Witness** **Date**

Rhodes University, Research Office, Ethics
Ethics Coordinator: ethics-commitee@ru.ac.za
t: +27 (0) 46 603 7727 f: +27 (0) 86 616 7707
Room 220, Main Admin Building, Drostdy Road, Grahamstown, 6139

Ms Dave

APPEDIX 4: PARTICIPANT INFORMED CONSENT
INFORMED CONSENT DECLARATION
(Participant)

Dave

Project Title: **Re- imagining the use of the Abacus as a visualization tool to develop number sense in Grade 3 learners**

Elifas Taimi Ndinelago from the Department of Education, Rhodes University has requested my permission to participate in the above-mentioned research project.

The nature and the purpose of the research project and of this informed consent declaration have been explained to me in a language that I understand.

I am aware that:

1. The purpose of the research project is to investigate how teachers use the abacus to develop number sense in the grade 3 learners.
2. The Rhodes University has given ethical clearance to this research project and I have seen the clearance certificate.
3. By participating in this research project I will be contributing to gaining knowledge and more insights on how to use the abacus to develop umber sense when teaching mathematics. I also expect this project to increase the number of teachers using abacus and other manipulatives to develop number sense when teaching mathematics in the //Kharas region.
4. I will participate in the project by participating in a training programme on how I can use the abacus when teaching mathematics to develop number sense. After the training I will be involved in planning and designing lessons on the use of abacus to develop number sense. I will teach five of these lessons that will be video recorded for observation purposes by the researcher.
5. My participation is entirely voluntary and should I at any stage wish to withdraw from participating further. I may do so without any negative consequences.
6. I will not be compensated for participating in the research.
7. There may be risks associated with my participation in the project. I am aware that
   a. The following risks are associated with my participation: I may work after normal working hours without compensation as the workshop will take place after school working hours.
   b. The following steps have been taken to prevent the risks: The lesson will be observed during the normal teaching time. ………
   c. There is 40% chance of the risk materialising
8. I understand that the study will involve taking video and voice recordings of my teaching and responses to interviews.
9. The researcher intends publishing the research results in the form of a thesis, research report, conference proceedings, journal articles and book chapters. However, confidentiality and anonymity of records will be maintained and that my name and identity will not be revealed to anyone who has not been involved in the conduct of the research.
10. I will receive feedback in the form of a 'summary of the key findings' regarding the results obtained during the study.
11. Any further questions that I might have concerning the research or my participation will be answered by Elifas Taimi N. Email: jonenelago@gmail.com cell: 0817204763.
12. By signing this informed consent declaration I am not waiving any legal claims, rights or remedies.
13. A copy of this informed consent declaration will be given to me, and the original will be kept on record.
14. [redacted] is study

I, ……[redacted]…….. have read the above information / confirm that the above information has been explained to me in a language that I understand and I am aware of this document's contents. I have asked all questions that I wished to ask and these have been answered to my satisfaction. I fully understand what is expected of me during the research.

I have not been pressurised in any way and I voluntarily agree to participate in the above-mentioned project.

……[redacted]

Participants signature **Witness** **Date**

Rhodes University, Research Office, Ethics
Ethics Coordinator: ethics-commitee@ru.ac.za
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Room 220, Main Admin Building, Drostdy Road, Grahamstown, 6139

Mr Quasi

Quasi

APPEDIX 4: PARTICIPANT INFORMED CONSENT
INFORMED CONSENT DECLARATION
(Participant)

Project Title: **Re- imagining the use of the Abacus as a visualization tool to develop number sense in Grade 3 learners**
Elifas Taimi Ndinelago from the Department of Education, Rhodes University has requested my permission to participate in the above-mentioned research project.
The nature and the purpose of the research project and of this informed consent declaration have been explained to me in a language that I understand.

I am aware that:

1. The purpose of the research project is to investigate how teachers use the abacus to develop number sense in the grade 3 learners.
2. The Rhodes University has given ethical clearance to this research project and I have seen the clearance certificate.
3. By participating in this research project I will be contributing to gaining knowledge and more insights on how to use the abacus to develop umber sense when teaching mathematics. I also expect this project to increase the number of teachers using abacus and other manipulatives to develop number sense when teaching mathematics in the //Kharas region.
4. I will participate in the project by participating in a training programme on how I can use the abacus when teaching mathematics to develop number sense. After the training I will be involved in planning and designing lessons on the use of abacus to develop number sense. I will teach five of these lessons that will be video recorded for observation purposes by the researcher.
5. My participation is entirely voluntary and should I at any stage wish to withdraw from participating further, I may do so without any negative consequences.
6. I will not be compensated for participating in the research.
7. There may be risks associated with my participation in the project. I am aware that
   a. The following risks are associated with my participation: I may work after normal working hours without compensation as the workshop will take place after school working hours.
   b. The following steps have been taken to prevent the risks: The lesson will be observed during the normal teaching time. ....
   c. There is 40% chance of the risk materialising
8. I understand that the study will involve taking video and voice recordings of my teaching and responses to interviews.
9. The researcher intends publishing the research results in the form of a thesis, research report, conference proceedings, journal articles and book chapters. However, confidentiality and anonymity of records will be maintained and that my name and identity will not be revealed to anyone who has not been involved in the conduct of the research.
10. I will receive feedback in the form of a 'summary of the key findings' regarding the results obtained during the study.
11. Any further questions that I might have concerning the research or my participation will be answered by Elifas Taimi N. Email: jonenelago@gmail.com cell: 0817204763.
12. By signing this informed consent declaration I am not waiving any legal claims, rights or remedies.
13. A copy of this informed consent declaration will be given to me, and the original will be kept on record.
14. [redacted] udy

I, [redacted]... have read the above information / confirm that the above information has been explained to me in a language that I understand and I am aware of this document's contents. I have asked all questions that I wished to ask and these have been answered to my satisfaction. I fully understand what is expected of me during the research.

I have not been pressurised in any way and I voluntarily agree to participate in the above-mentioned project.

[redacted] 27/07/2020

Participants signature **Witness** **Date**

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Appendix E: Learner consent form

CHILD PARTICIPANT'S ASSENT FORM
INFORMED CONSENT DECLARATION (Child participant)

Project Title: Re- imagining the use of the Abacus as a visualization tool to develop number sense in Grade 3 learners

Researcher's name: Elifas Taimi Ndinelago

Name of participan[redacted]..........................

1. Has the researcher explained what s/he will be doing and wants you to do?
   YES ✓ NO
2. Has the researcher explained why s/he wants you to take part?
   YES ✓ NO
3. Do you understand what the research wants to do?
   YES ✓ NO
4. Do you know if anything good or bad can happen to you during the research?
   YES ✓ NO
5. Do you know that your name and what you say will be kept a secret from other people?
   YES ✓ NO
6. Did you ask the researcher any questions about the research?
   YES ✓ NO
7. Has the researcher answered all your questions?
   YES ✓ NO
8. Do you understand that you can refuse to participate if you do not want to take part and that nothing will happen to you if you refuse?
   YES ✓ NO
9. Do you understand that you may pull out of the study at any time if you no longer want to continue?
   YES ✓ NO
10. Do you know who to talk to if you are worried or have any other questions to ask?
    YES ✓ NO
11. Has anyone forced or put pressure on you to take part in this research?
    YES NO ✓
12. Are you willing to take part in the research?
    YES ✓ NO

[redacted] 0 June 2020

Signature of Child **Date**

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Appendix F: Parent Consent form

PARENT AND GUARDIAN'S INFORMED CONSENT
INFORMED CONSENT DECLARATION (Parent or Guardian)

Project Title: **Re- imagining the use of the Abacus as a visualization tool to develop number sense in Grade 3 learners**
Elifas Taimi Ndinelago from the Department Education, Rhodes University has requested my permission to allow my child to participate in the above-mentioned research project.
The nature and the purpose of the research project and of this informed consent declaration have been explained to me in a language that I understand.

I am aware that:

1. The purpose of the research project is to use the abacus as a visualization tool to develop number sense in the junior primary phase.
2. The Rhodes University has given ethical clearance to this research project and I have seen/ may request to see the clearance certificate. [**Certificate number**]
3. By participating in this research project my child/ward will be contributing towards teachers gaining knowledge and more insight about how to use the abacus to develop number sense at the foundation phase. As a result of this study teachers are expected to use the abacus when teaching mathematics to help learners develop their number sense.
4. My child/ward will participate in the project by being in a class where the teacher is teaching using the abacus to develop number sense while being observed by the researcher.
5. My child's participation is entirely voluntary and if my child/ward is older than seven (7) years, s/he must also agree to participate.
6. Should I or my child/ward at any stage wish to withdraw my child from participating further, we may do so without any negative consequences.
7. My child may be asked to withdraw from the research before it has finished if the researcher or any other appropriate person feels it is in my child's best interests, or if my child does not follow instructions.
8. My child will not be observed or interviewed during this project.
9. Neither my child nor I will be compensated for participating in the research.
10. The work of my child will not be reviewed as part of this project.
11. There may be risks associated with my child's participation in the project. I am aware that
    a. the following risks are associated with participation:
       i) My child's face may appear in the videos.
       ii) My child might not concentrate in class because of the researcher present.
    b. the following steps have been taken to prevent the risks:
       i) The video camera will be placed at the back of classroom to avoid capturing children's faces
       ii) The researcher will introduce herself to the learners and her reason of being in class with a video camera and explain to the learners to be free at all time.
    c. there is a 25% chance of the risk materializing
12. The researcher intends to publish the research results in the form of a thesis, research report, conference proceedings, journal articles and book chapters. However, confidentiality and anonymity of records will be maintained and that my child's name and identity will not be revealed to anyone who has not been involved in the conduct of the research.
13. I will receive feedback regarding the results obtained during the study, if I ask for it.
14. Any further questions that I might have concerning the research or my participation will be answered by Elifas Taimi N., jonenelago@gmail.com Cell. 0817204763l5.
15. By signing this informed consent declaration I am not waiving any legal claims, rights or remedies that I or my child/ward may have.
16. A copy of this [redacted] given to me, and the original will be kept on record.

I, [redacted] have read the above information / confirm that the above information has been explained to me in a language that I understand and I am aware of this document's contents. I have asked all questions that I wished to ask and these have been answered to my satisfaction. I fully understand what is expected of my child during the research.
I have not been pressurised in any way to let my [redacted] child take part. By signing below, I voluntarily agree that my child/ward [redacted] (**insert name of child**), who is ...9... years old, may participate in the above-mentioned research project.

[redacted]

Parent/Guardian's signature **Witness** **Date**

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Appendix G: Survey Questionnaires

The information that will be obtained from the questionnaires will treated as confidential and collected information will only be used for this research purpose.

1. Are you using the Abacus in your classroom? Explain your answer.

2. What do you use the Abacus for? Explain the instances when you use it.

3. How do you teach number sense? What other manipulatives do you use to teach number sense?

4. How do the learners respond when you use the Abacus to teach mathematics?

5. What are the benefits of using the Abacus in your teaching?

6. In your opinion is the Abacus a useful teaching tool? Explain your answer.

7. How do you use the Abacus to make sense of what the learners are learning and advanced mathematical thinking and reasoning?

Appendix H: Analytical tool……………………………….

Table 1: Analytical Tool: The abacus as a visualization tool for developing number sense (McIntosh et al., 1992)

Number sense understanding	**Code**	**Number sense theme**	**Visual teaching strategies** **The teacher:**
Sense of orderliness of numbers	SON	Place Value	**SON1:** uses an abacus to demonstrates place values **SON2:** uses an abacus to demonstrate regroup of a number **SON3:** uses the place value system to make the learners understand the number system
		Relationship between number types	**SON3:** uses an abacus to show a decimal place.
Multiple representations for numbers	MRN	Equivalent numerical Forms (including decomposition and recomposition)	**MRN1**: uses an abacus to facilitate multiple representations for numbers
		Graphic/ Symbolic	**MRN2**: uses an abacus to represent numbers in different ways (e.g. patterns with a common difference as well as multiples)
Sense of relative and absolute magnitude of numbers	SAN	Comparing to physical Referent	**SAN1:** Uses an Abacus for Subitizing
		Comparing to mathematical referent	**SAN2:** Uses an abacus to assist in visualizing numbers as doubles and to look for groups of tens in adding numbers
Understanding the effect of Operations	UEO	Operation on whole numbers	**UEO1**:Uses an abacus as a method for modeling the addition of integers
		Operating on fractions /decimals	**UEO2**:Uses an abacus to teach equivalent fractions in order for learners to understand the relationship between the numerators and denominators
Understanding mathematical properties	UMP	Associativity	**UMP1**: uses an abacus enhance the understanding of mathematical properties
		Commutativity	**UMP2:** uses an abacus to enhance the understanding of mathematical properties
		Distributivity	**UMP3:** uses an abacus to enhance the understanding of mathematical properties
Understanding the relationship between operations	URO	Addition / Multiplication	**URBO1:** Uses an abacus to show that multiplication can be the same as repeated addition
		Subtraction / Division	**URBO2:** Uses an abacus to show that division can be the same as repeated subtraction
		Addition / Subtraction	**URBO3**: Uses abacus to teach addition and subtraction as inverse operations.
		Multiplication / Division	**URBO4:** Uses an abacus to enhance the understanding of multiplication and division as inverse operations
Inclination to review data and result for sensibility	IDS	Recognize reasonableness of Calculation	Uses an abacus to recognize the reasonableness of calculation within the context of the mathematical problem
Inclination to utilize an efficient representation and / or method	IUE	Facility with various methods (mental, calculator, pencil and paper)	Uses an abacus to experiment with unconventional visual algorithms and representations in mathematical problem-solving

Appendix I: **Semi-Structured interview schedule for teachers**

Good afternoon Teacher: ……..…thank you for making time to let me interview you. I would like to ask you some questions about your experience and perception of using the Abacus as a visualization tool to develop number sense in a grade 3 class.

I hope to use this information to help write my Master's thesis and to inform all the teachers who will read my thesis about the use of an abacus as a visualization tool to develop number sense in a Grade 3 class. The interview will take about 30 minutes. I hope you will be available to respond to my few questions at this time.

(a) Body

1. What is your understanding of number sense?

---

2. During your lesson you have been teaching learners using an Abacus. How has the Abacus been a useful tool in your teaching?

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3. How did your learners benefited from your use of the abacus when you were teaching?

---

4. What challenges did you experience using an Abacus to teach? How can these challenges be overcome?

---

5. What changes if any, have occurred in your teaching as a result of this intervention?

---

6. What other ways do you think can be used by the junior primary teachers to develop number sense in their learners when teaching mathematics?

(b) Closing

We have come to an end of our interview. Thank you very much for your time. Is there anything else you wish to add or talk about?

Taimi N. Elifas

Appendix J: Lesson Plans

<u>LESSON PLAN 1: PLACE VALUES</u>

Teacher…………………………………………………Date………………...Grade: 3___

Subject: Mathematics…………………………………………………………Time: 40 Minutes

Theme and Topic: Place Values

Teaching Aids: Chalkboard and Abacus

Learning objectives: Learners will recognize digit position and place value

Basic Competencies: Learners should be able to recognize in three-digit number up to 1000 that the digit position in number shows the quantity e.g. given the digit 1,8,4 form the biggest and the smallest three-digit number: 841 and 148.

1. **Monitoring of Home work done:** Peer marking of the previous work
2. **Introduction:** Check the learner's prior knowledge of place value by asking them questions and introduce the topic.
3. **<u>Teacher's activities</u>**

Teacher introduces the abacus to the learners and explains to them what it is used for. Define what place values are to the learners with examples. With aid of the Abacus the teacher will explain how different place value can be represented on an abacus.

Explain how to identify place values of hundreds, tens and unit in a three digit numbers. The teacher will also demonstrate the basics decimals on the Abacus.

4. **<u>Learner's activities</u>**

Show the digit position of numbers on the abacus. Learners will represent the given digit with their place values on the physical abacus. With the use of the abacus the learners will identify place values of hundreds, tens and unit in a three-digit number e.g. 856 the value of 8 is 800, the value of 5 is 50 and the value of 6 is 6. The learners will round off given digit on the abacus to the nearest 10.

5. **Consolidation**: put different three-digit number on the board and ask the learners to represent their value and place value on the physical abacus.
6. **Opportunities to develop English reading and writing**: Read out loud the place values of different digits on the physical abacus and write them down in the exercise books.
7. **<u>Learning Support:</u> Slow learners**: Get help from the teacher and the gifted peers.

Gifted Learners: Help the slow peers and get extra exercise to complete when they finish earlier compared to the other.

Reflection:

LESSON PLAN 2: ADDITION

Teacher:………………………………………………………Date:………………..Grade: 3___

Subject: Mathematics………………………………………………...….Time: 40 Minutes

Theme and Topic: Addition

Teaching Aids: Chalkboard, Abacus, bottle tops and exercise books

Learning objectives: Learners will understand basic mathematical concepts to master addition and numerical notation.

Basic Competencies: Learners should be able to break down and build up two and three digit number in different ways. Add decade numbers up to 500. Add two numbers up to 100 mentally (not writing). Add two or more different numbers with a sum between 0 and 500 by breaking down and build up number using place values e.g. 342 + 185=(300+100) +(40+80)+ (2+5)

1. **Monitoring of Home work:** Do follow up of the previous work and mark the learners books.
2. **Introduction:** Ask learners to represent on the physical abacus values and place values of given numbers as they learned it in the previous lesson.
3. **Lesson presentation: Teacher's activities**

Take different decades from 0 to 500 and add them by forming up equation using the abacus. Take three digit numbers and break it down according to their values e.g. 589= 500+80+9.

Explain how to add numbers up to 100 mentally without writing them down.

Demonstrate to the learners that you can add the numbers regardless of how they are grouped using the abacus e.g. **(2+7)+5=2+(5+7)**

9+5+=2+12

14=14

Using the abacus the teacher will demonstrate to the learners that moving numbers in addition the answer will still remain the same for example: **25+20=20+25.**

4. **<u>Learner's activities</u>**

The learners will listen to the teacher's explanation on how to add decades, break down the number and group them using the abacus. The teacher will write down different sums on the chalkboards. Learners with the help of the teacher will represent the given sum by the teacher on the abacus.

They will add numbers mentally without writing them down (pen and paper algorithm). And explain how they calculated their answers mentally.

5. **Consolidation**: Recap on the lesson by asking learners to do simple addition given to them on the abacus.

Ask questions on how Associativity and Commutativity affects the arrangement of numbers.

6. **Opportunities to develop English reading and writing:** the learners will read out loud the numbers they are breaking down and grouping and do mental calculation. Write notes and copy down the given activity.

7. **<u>Learning Support</u>**

Slow learners: Get more time to finish working out their answers and with the help of the teacher and peers.

Gifted Learners: Help the slow peers with breaking down of numbers and re-grouping them.

Reflection:

LESSON PLAN 3: MULTIPLICATION

Teacher:………………………………………………………Date:………………..Grade: 3___

Subject: Mathematics………………………………………….……………..Time: 40 Minutes

Theme and Topic: Multiplication

Teaching Aids: Chalkboard and Abacus

Learning objectives: Learners will understand basic mathematical concepts to master multiplication and numerical notation.

Basic Competencies: Learners should be able to multiply any two digit number by 2,3,4,5, and 10 with a product between 100 and 500 using decomposition. Multiply any two digit number up to 50 by any number between 1 and 10 through repeated addition for example: $12\times4= 12+12+12+12$.

Learners should be able to recall and apply multiplication table

1. **Monitoring of Home work done:** Mark the previous work and do the correction
2. **Introduction:** Recap on the previous lesson to evaluate the understanding of the learners and explain to the learners that multiplication is **repeated addition** with examples and demonstrate it with the physical abacus.
3. **Lesson presentation: Teacher's activities**

The teacher will use the abacus to demonstrate to the learners that multiplication can be the same as repeated addition.
Explain with the use of the abacus the distributive properties of operation for example: $\mathbf{3\times (1\times4) = (3\times1) \times4}$. With the abacus as a teaching aid the teacher will also present the multiplication table using the abacus from 1 to 12.

4. **Learner's activities**

Present different multiplication as repeated addition on the abacus. Learners will show the distributive properties of multiplication.

By showing or moving the abacus beads the learners will recall the multiplication table between 1 and 12.

5. **Consolidation**: Give the activity on the board on multiplication as repeated addition and ask learner to represent it on the physical abacus.

   Leaners will recall the multiplication in their mind (Mentally)

6. **Opportunities to develop English reading and writing:** The learners will read out loud the numbers they are breaking down and grouping and do mental calculation. Write notes and given activity.

7. **<u>Learning Support</u>**

Slow learners: Get help from the teacher and the gifted peers.

Gifted Learners: Help the slow peers

Reflection:

____

____

LESSON PLAN 4: SUBTRACTION

Teacher:…………………………………………………Date:………………..Grade: 3_______

Subject: Mathematics…………………………………………………….....Time: 40 Minutes

Theme and Topic: Subtraction

Teaching Aids: Chalkboard and Abacus

Learning objectives: Learners will understand basic mathematical concepts to master Subtraction and numerical notation.

Basic Competencies: Learners should be able to subtract two or three digit number from a three digit number between 100 and 500 using subtraction of decades, using place values, break down and build up.

Subtract a single digit number from a three digit number mentally. Explore the number pattern for example: **100-37 =63**

200-37=163

300-37=263

1. **Monitoring of Home work done:** Do follow up of the previous work

2. **Introduction:** Check the learner's prior knowledge by asking questions related to the topic and introduce the lesson topic.

3. **Lesson presentation: Teacher's activities**

Firstly, explain to the learners how the pen and paper method of subtraction work (traditional algorithm). Using the abacus the teacher will subtract two or three digit number from a three digit number. Subtract decades between 100 and 500 using the abacus.

With the concept of the place values the teacher will demonstrate how to subtract, breakdown and build up numbers.

Explore number pattern for example: **100-25= 75**

200-25=175

300-25=275.

4. **Learner's activities**

Listen to the teacher's explanation and ask questions where they don't understand. The learners will use the abacus to subtract the two digit number from a three digit number. Using the concept of place values the learners will subtract two or three digit number from a three digit numbers.

5. **Consolidation**: Recap on the lesson by asking learners to do simple subtraction given to them on the abacus.

6. **Opportunities for developing English reading and writing:** the learners will read out loud the numbers they are breaking down and grouping and do mental calculation. Write notes and given activity.

7. **Learning Support**

Slow learners: Get help from the teacher and the gifted peers.

Gifted Learners: Explain to the peers that are finding it difficult to grasp the concept for the lesson in their vernacular.

Reflection:

LESSON PLAN 5: DIVISION

Teacher:………………………………………………………Date:………………..Grade: 3___

Subject: Mathematics……………………………………………………...Time 40 Minutes

Theme and Topic: Division

Teaching Aids: Chalkboard and Abacus

Learning objectives: Learners will understand basic mathematical concepts to master Division and numerical notation.

Basic Competencies: Apply and recall simple division facts related to multiplication facts of 2,3,5 and 10 for example: $20 \div 2 = 10$; $2 \times 10 = 20$ (Division as inverse operation of multiplication)

1. **Monitoring of Home work done:** Do follow up of the previous work
2. **Introduction:** Learners will reflect on previous lesson by recalling what they have learned on the previous lesson. Check the prior knowledge of the learners on previous inverse operation signs and introduce Division as an inverse operation of multiplication.
3. **Lesson presentation: Teacher's activities**

Explain to the learners that division is the inverse operation of multiplication with examples. With the physical abacus the teacher will present will present how division is the inverse operation of multiplication. Together with the learners the teacher will do the activity of division and repeat it as an inverse of multiplication. Demonstrate that division can be the as repeated subtraction and show it on the physical abacus.

4. **<u>Learner's activities</u>**

Divide given number on the abacus and demonstrate how it is the inverse operation of multiplication. Listen to the teacher's explanation and ask questions where they don't understand.

Do problem solving for division and multiplication on the abacus. The learners will get exercise based on the lesson they had and work it out on the abacus.

5. **Consolidation**: Do problem solving activities orally together with the learners and mentally.

6. **Opportunities for developing English reading and writing:** Write notes and given activity.

7. **<u>Learning Support</u>**

Slow learners: Get help from the teacher and the gifted peers.

Gifted Learners: Help the slow peers .

Reflection: