A SURVEY OF THE DEVELOPMENT OF PITCH PERCEPTION THEORIES, THEIR APPLICATION TO BELL SOUNDS AND AN INVESTIGATION OF PERCEIVED DIFFERENCES BETWEEN RINGING AND CHIMING BELLS

THESIS

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Abstract

A brief overview of the workings of the human auditory system is followed by a review of literature concerning both the theories and experimental investigations of human pitch perception. The application of these theories to the inharmonic complex tones produced by bells is discussed, and further experiments using bell sounds are reviewed. A methodology for psychoacoustic experiments with specific reference to those investigating pitch perception of inharmonic complex tones is presented. This methodology is then implemented in an experimental investigation of pitch perception of ringing and chiming bell sounds. A pitch matching experiment using ringing and chiming sounds from four bells aimed to determine perceived pitch differences between ringing and chiming bells. This experiment was inconclusive because insufficient data was collected. Known experimental results, such as the inability of non-musicians to match the pitches of sounds with different timbres were confirmed. Spectral analyses of the stimuli were performed. The presentation of stimuli at a low level of sensation is questioned, as this might have prevented pseudo high frequency noise resulting from stronger upper partials in the ringing sound from being audible, and hence the pitch differences between ringing and chiming bells would not be observed.

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Contents

List of Figures

List of Tables

Introduction

Psychology of music has been defined by McAdams (1987) as a field that 'studies all forms of musical behaviour with the accepted methods of experimental psychology'. Some of the topics of concern in music psychology are the perception of musical qualities of sound, cognitive processes involved in storing and organising music, emotional and aesthetic responses to music and the definition of learned and acquired musical aptitudes. Because the topics of concern to the music psychologist are so broad, knowledge and expertise needs to be drawn from many diverse fields such as acoustics, psychology, brain sciences and music theory (McAdams, 1987).

The branch of music psychology with which this thesis is concerned is that of psychoacoustics. Psychoacoustics is a sub branch of a modern branch of experimental psychology known as psychophysics. The purpose of psychophysical research is to relate the physical stimuli received by the sense organs to the perceived sensation (Hedden, 1980). Psychoacoustic experimentation is aimed specifically at defining the relationships between sound stimuli and the resulting perceptual experiences.

An excellent history of the development of thought and studies of human perception from the time of the ancient Greek philosophers to today was written by Hamlyn (1961). Psychoacoustics grew directly out of the work of physicists and physiologists of the late 19th century such as Helmholtz and Fecher (McAdams, 1987). Helmholtz not only contributed enormously to knowledge of the auditory system, see Helmholtz (1877), but also made an extensive study of optical perception (Warren and Warren, 1968).

Electronic measuring, recording and synthesising devices invented in the twentieth century have made it possible for the stimuli to be accurately and systematically controlled and for perceptual experiences to be quantified accurately. This has lead to an enormous growth in the field in the latter part of the twentieth century (McAdams, 1987).

One of the topics of concern in the field of psychoacoustics is pitch perception, particularly the pitch perception of inharmonic complex tones. It has been found that the perceived pitch of a complex tone does not always correspond to one of the frequencies present in the sound, but rather that it is constructed from the spectral information of the sound at a higher level of mental processing (Goldstein, 1973). What is more is that the perceived pitch of particularly an inharmonic complex tone can be influenced by the musical context in which it is placed (Roederer, 1995).

Bell sounds, being inharmonic complex tones, have attracted much attention from researchers investigating pitch perception. However, much of the research into pitch perception has concentrated on spectrally simple sounds, much less research has gone into investigating spectrally dense sounds such as bells (Eggen and Houtsma, 1986). There is still room for much more experimentation on spectrally dense inharmonic complex tones.

The present study begins with a brief description of the functioning of the human auditory system in Chapter 1. Chapter 2 contains a literature survey of experiments and theories concerning pitch perception. Pitch perception of bell sounds, and experiments in this regard are discussed in Chapter 3. Chapter 4 presents the methodological aspects of psychoacoustic research, with specific reference to investigating pitch perception of inharmonic complex tones. Chapters 5 and 6 contain the results from a psychoacoustic experiment aimed at investigating the perceived pitch differences between ringing and chiming bells sounds.

Chapter 1 - The Human Auditory System

1.1 Structure and Function of the Ear

A schematic diagram of the human ear is given in **Figure 1.** The ear is divided into the outer, middle and inner ear because different functions are performed by each portion. This diagram is not to scale.

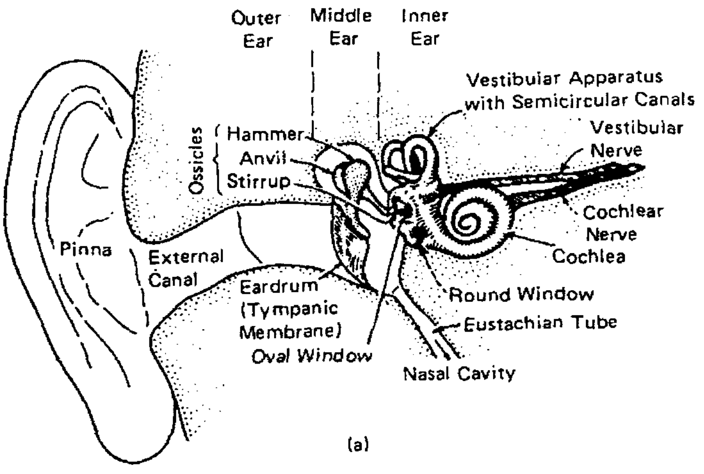

Figure 1 A schematic representation of the human auditory system. (Roederer, 1995, Figure 2.6)

1.1.1 Outer Ear

The sound is captured by the pinna of the outer ear and funnelled into the external canal. The dimensions of this canal cause resonance at about 3800 Hz, strengthening frequencies in that region the most (Campbell and Greated. 1987). Directional hearing is aided both by the shape of the pinna and by the spatial separation of the two ears. The pinna helps to determine the height of the sound source relative to the listener, as has been shown by experiment (Campbell and Greated 1987). The position of the sound on a horizontal plane is determined from the phase differences between the sounds arriving at the individual ears (Roederer, 1995).

1.1.2 Middle Ear

The middle ear is a boney cavity separated from the outer ear by the ear drum or tympanic membrane and from the inner ear by the round and oval windows. The tympanic membrane vibrates in sympathy with the sound transferring the motion to the three bones of the middle ear: the hammer, anvil and stirrup (or malleus, incus and stapes). These in turn translate the vibration into pressure oscillations in the fluid in the cochlear of the inner ear by setting the oval window into motion. A schematic diagram of the bone chain is given in **Figure 2**.

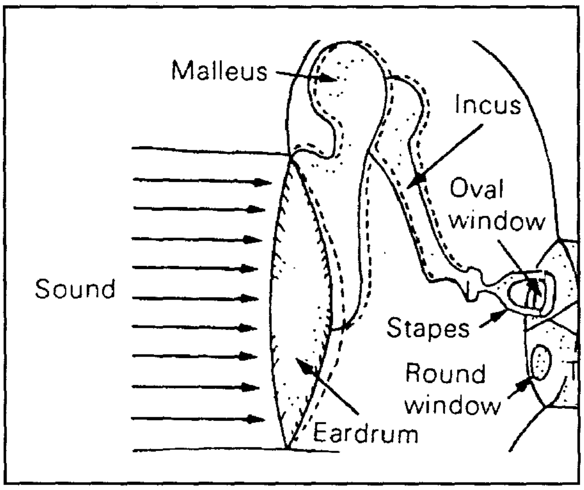

Figure 2 The bone chain of the middle ear (Coren *et al.*, 1994, p. 191).

The middle ear serves as a protection mechanism for the inner ear from excessively loud sounds. In the presence of dangerously loud sounds a muscle in the middle ear pulls the stirrup away from the oval window to reduce the amount of energy transmitted to the inner cochlear. This is called the acoustic reflex (Coren *et al.*, 1994).

1.1.3 Inner Ear

The semicircular canals are responsible for our sense of balance. The cochlear is a long thin fluid filled tube which is rolled up giving it a snail like appearance. **Figure 3** shows a simplified version of the cochlear uncoiled. The cochlear of an adult is about 34 mm long (Roederer, 1995).

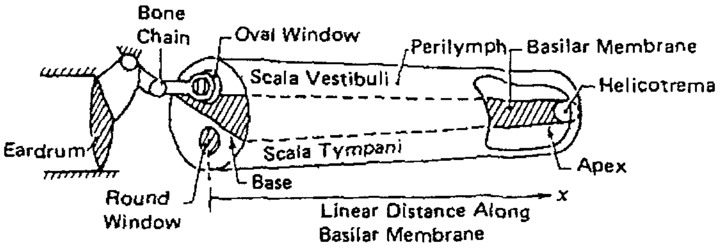

Figure 3 A simplified schematic representation of the uncoiled cochlear. (Roederer, 1995, Figure 2.6)

The cochlear is divided lengthwise by the basilar membrane into the vestibular canal and the tympanic canal. These two canals are fluid filled and connected by an opening called the helicotrema. The round window deforms to serve as a pressure release mechanism for the cochlear (Coren *et al.*, 1994). The basilar membrane, because of its changing dimensions, resonates in different places for sounds of different frequencies. The intensity of the sound determines the width of the resonance region on the basilar membrane. On the basilar membrane there are about 16000 “hair cells” which when caused to sway back and forth by the motion of the membrane send electrical impulses to their adjoining nerve cells (Roederer, 1995). The sound is finally translated into neural impulses which are sent to the brain via the auditory nerve for further analysis[1]. When two tones are very close in frequency and heard simultaneously there

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[1] For a more in depth explanation of the workings of the inner ear, the pathway of the auditory information to the brain and the processing of information in the auditory cortex see Coren *et al.* (1994, pp. 190-200) and Bekesy (1960, parts 2 and 3).

is a substantial overlap in their resonance regions on the basilar membrane and they cause almost exactly the same group of hair cells to fire. When this happens the tones are said to be within one critical bandwidth (Campbell and Greated, 1987). These closely grouped sounds are processed by the auditory system as if they were indistinguishable (Benade, 1976). If two tones which are initially at the same frequency are gradually separated, at first beats are heard which gradually become more rapid and later give way to a sensation of roughness before becoming perceptible as two distinct tones. The critical band is at the changeover point between the sensation of roughness and a sensation of two distinct tones (Hodges, 1980). The critical bandwidth as a function of frequency is shown in **Figure 4**.

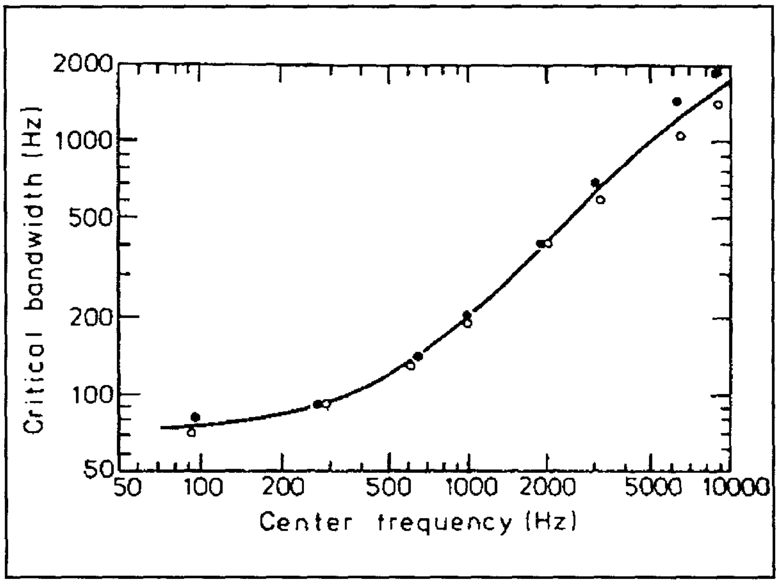

Figure 4 The critical bandwidth over a range of frequencies (Rasch *and Plomp*, 1982, Fig. 3. p. 6).

Theories of pitch perception attempt to explain the functioning of the cochlear and the nature of the interactions between the brain and cochlear. Some of these theories are discussed in the next chapter.

1.1.4 Auditory Nerve and Auditory Cortex

The auditory nerve carries electrical impulses in the form of potential spikes. Each spike corresponds to a maximum compression of the cochlear fluid. No one hair cell responds to all the maxima, but rather the additive effect of the firing of a collection of cells is representative of the repetition rate of the sound stimulus. This is referred to as the *volley principle* and was first proposed by Wever in 1949 (Eggen, 1986). The volley principle is illustrated in **Figure 5** (Campbell and Greated, 1987). Numbers (a) to (e) represent possible hypothetical firings from five different hair cells as a result of the sinusoidal sound wave at (g). The sum of these firings, (f) represents the repetition rate of the sound.

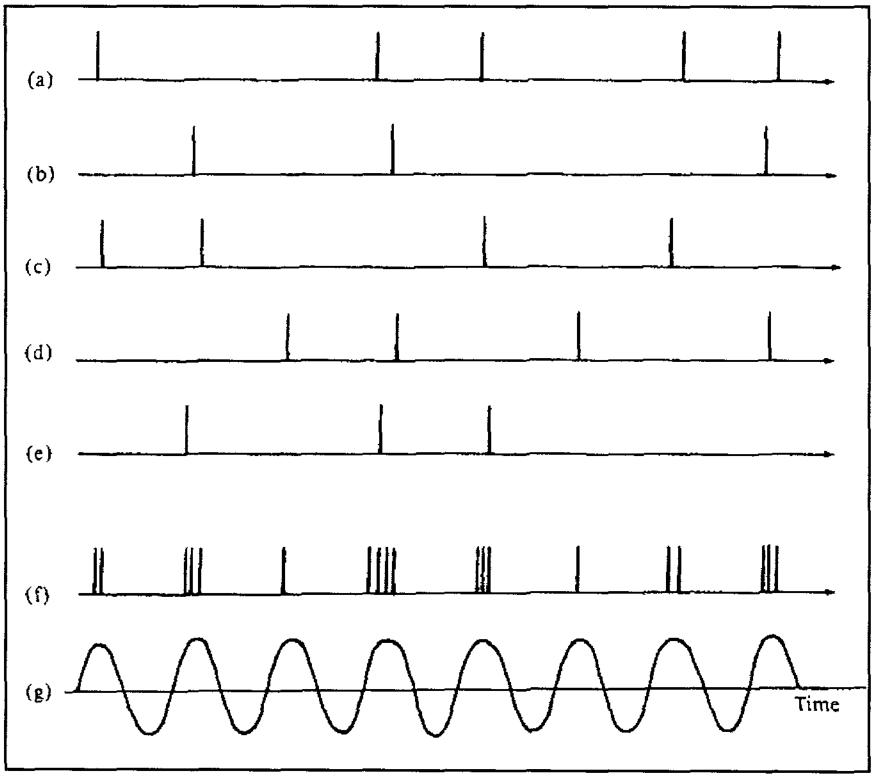

Figure 5 Impulses from 5 different nerve cells, (a) to (e), excited by a pure tone, (g). The sum of (a) to (e) is (f) (Campbell and Greated, 1987).

Figure 6 shows the intricate pathway taken by the auditory information on its way to the ultimate destination in the brain, the auditory cortex.

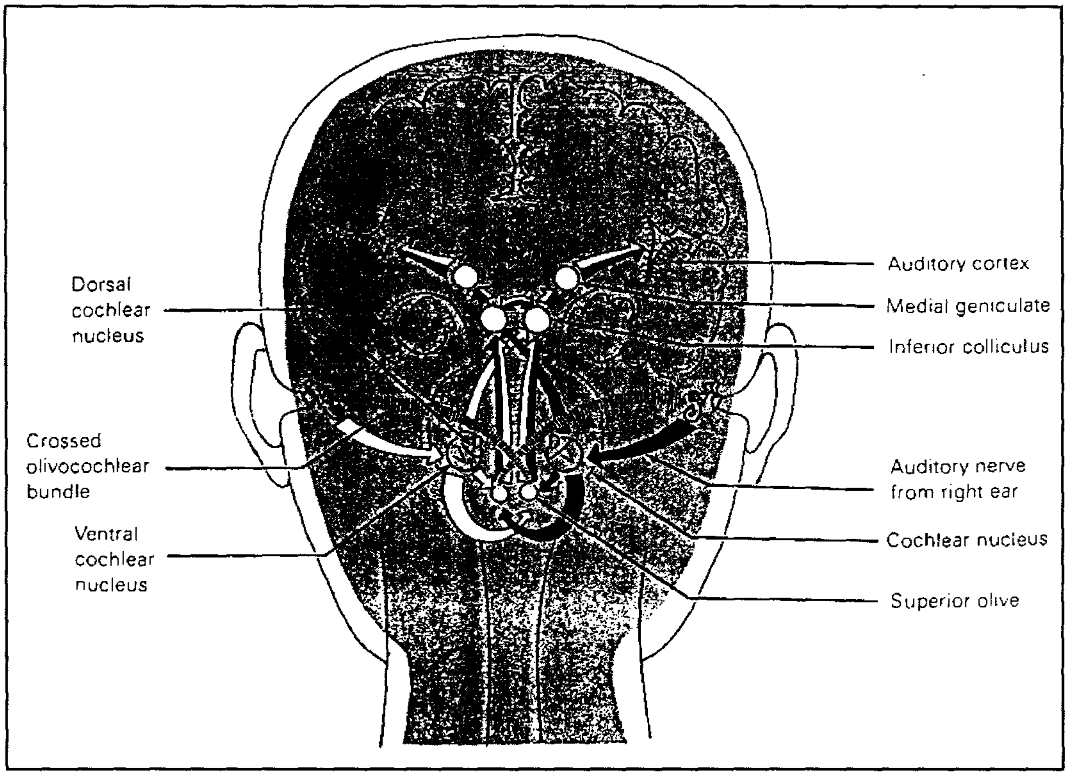

Figure 6 The pathway of the auditory information to the auditory cortex (Coren *et al.*, 1994, p. 204).

The responses of the auditory cortices of animals have been studied extensively. Experiments on cats revealed several different responses from neurons in the auditory cortex to a sinusoid as illustrated in **Figure** 7.

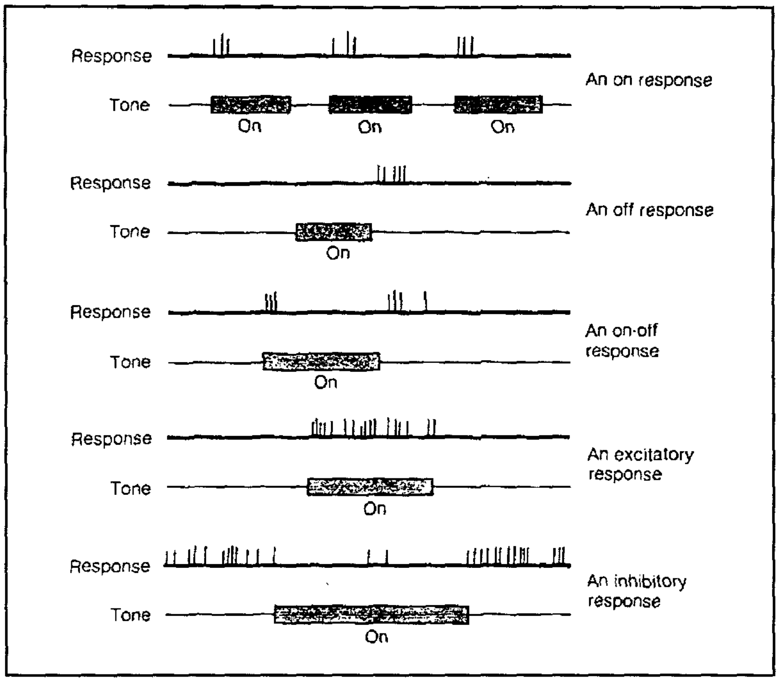

Figure 7 Neural responses detected from the auditory cortex of a cat (Coren *et al.*, 1994, p. 208).

Ultimately, in order to test any hypotheses about the functioning of the human auditory system it is necessary to gather information on the actual auditory perceptions and experiences of people. This is the role of the psychoacoutic experimenter. The next section discusses some of the basic facts about auditory perception deduced through such experimentation.

1.2 Limits of Perception

All physical systems are constrained by their dimensions and other physical properties. The human auditory system is no exception. The limit imposed on a sense organ due to physical factors and the brain is called the physiological limit. Experiments have been performed in order to determine these limits of auditory perception for the average person. Thresholds of hearing with respect to loudness, range of pitch and pitch discrimination have been determined (Hodges, 1980).

The range of frequencies audible to human beings is from about 20 Hz to 20 kHz. The upper limit decreases with age, with the result that most adults can only hear up to about 15 kHz. Most musical sounds range from about 100 Hz to 4 kHz, so this is not a serious loss (Hodges, 1980).

The intensity of the sound determines to a large extent how loud it is. But, because the human ear is more suited to amplifying certain frequencies, the loudness of sounds with the same intensities changes depending on their frequencies. **Figure 8** equal loudness contours for sounds

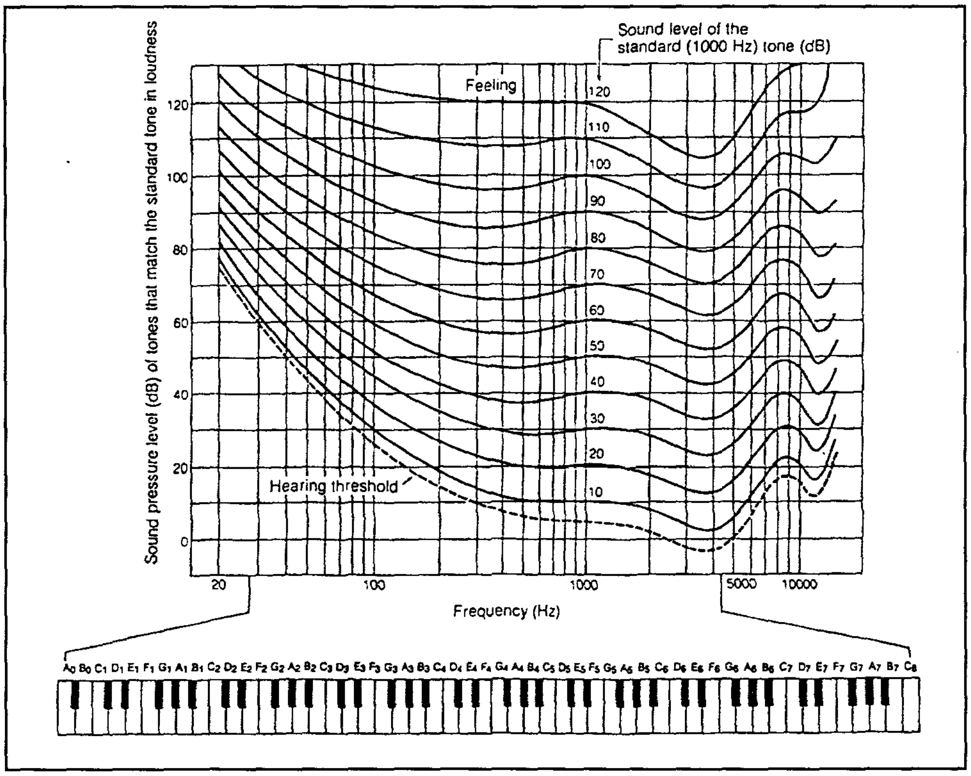

Figure 8 Contours of equal loudness over the range of frequencies from 20 Hz to 20 kHz (Coren et al, 1994, p. 234, Fig. 7-11).

from about 20 to 20000 Hz.

From this diagram it is evident that the ear has a peak of sensitivity between 3 and 4 kHz[2] and is most sensitive to sounds in the range of frequencies from about 200 Hz to 5 kHz, a range within which most musical instruments produce their sound.

Pitch discrimination is an ability that varies enormously among individuals. There are three ways in which this has been tested (Hedden, 1980):

1) Two tones are presented successively.
2) One tone is gradually or rapidly modulated.
3) Two tones are presented simultaneously.

The method of testing makes a big difference to the accuracy with which people can detect the frequency differences. When the tones are presented together they have to lie further apart than one critical bandwidth in order to be discerned as two tones. With the other two methods this is not necessary and hence smaller frequency differences are detectable. The modulation of a single tone enables the most accurate discrimination, and the more sudden the change the better the chance of discrimination (Hedden, 1980).

Other factors that influence pitch discrimination are the complexity of the tone, the loudness, the pitch of the reference signal and the subject's ability to move relative to the sound source. When the reference signal is between 500 Hz and 1 000 Hz pitch discrimination is at its best. If sounds are too soft or the subject is unable to move in the sound field, the process is inhibited.
Plomp (1964) discovered that the frequency difference needed to distinguish the pitch of two tone stimuli was smaller than the critical bandwidth. Discrimination has been found to be better for complex tones than for sinusoids if the reference tone is below 4 kHz (Hedden, 1980). Above this the complexity of the tone makes no difference (Hedden, 1980). A discrimination of 3 Hz for a sinusoidal stimulus at a reference frequency of 440 Hz is considered average for the successive method of presentation (Seashore, 1967).

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[2] The peak of sensitivity at 3 kHz corresponds well to the resonance region of the external canal discussed in section 1.1.1.

A minimum duration of about 250 ms is required before it is possible to experience a clear, unchangeable pitch for a sound (Gulick, 1971). Sounds of a shorter duration than this can cause the sensation of pitch, but it is changeable and often inaccurately perceived. If sounds are too short they are just experienced as clicks (Gulick, 1971). The duration required for some sensation of pitch is frequency dependent as shown in **Figure 9**.

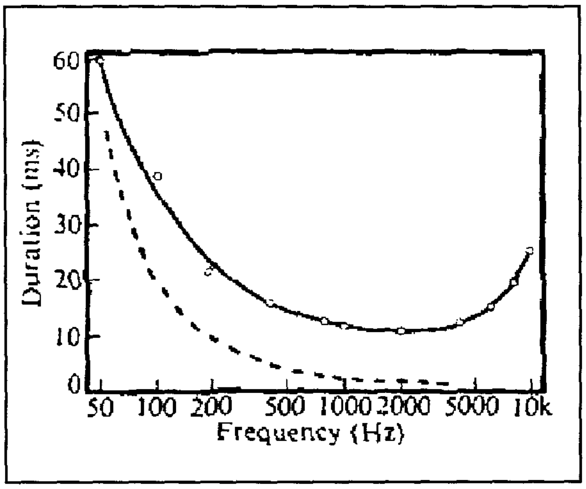

Figure 9 Duration of sound required for pitch perception as a function of frequency (Rossing, 1989, Fig 7.4, p.113).

The next chapter will deal more specifically with the topic of pitch perception.

Chapter 2 - Pitch Perception

2.1 Introduction

Pitch, timbre and loudness are subjective qualities that we use in order to describe our perception of a physical sound stimulus. Although they are strongly influenced by the frequency, spectrum and intensity of the sound respectively, they are not, as was once thought, independent of the other physical properties of the sound or of the context in which they are heard. The perceived pitch of a sound has even been known to differ by as much as half a semitone depending on which ear the listener used (Campbell and Greated, 1987).

The perceived pitch of a pure tone (when only one frequency is present) is easily related to the physical stimulus as it is almost entirely dependent on the frequency of the tone, although the intensity of the sound does have a small effect. When dealing with a *complex tone*, i.e. a sound that is composed of more than one frequency, the perceived pitch becomes more difficult to predict.

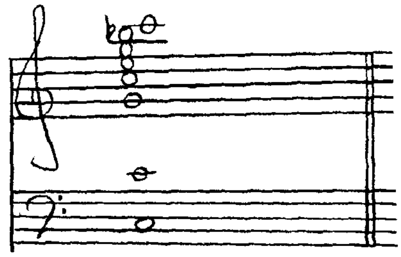

Figure 10 Harmonic series with the fundamental at C_3.

When the other constituent frequencies of a sound are related to the lowest partial, f, in the sequence f, 2f, 3f, 4f, 5f etc they are called harmonics. The perceived pitch of sounds with harmonically related constituent frequencies, which represents most musical sounds, corresponds to f, the fundamental frequency. This series of frequencies can be represented in musical notation as in **Figure 10.** Here the fundamental frequency is at the pitch of the C below middle

C.

During the 19th century it was discovered that the perceived pitch of an harmonic series with the fundamental missing (i.e. 2f, 3f, 4f, 5f etc) was still that of the fundamental frequency. This effect is utilised in the lower register of the bassoon and in speakers of small dimensions. These objects are too small to produce any substantial power at the low frequencies that we believe we are hearing. This phenomenon has a parallel in visual perception. We are able to recognise the object represented by an arrangement of straight lines as being a house, window, table etc. even though there is a substantial amount of information missing (Benade, 1976).

When the partials are not harmonic as is the case with most percussive instruments (for example bells) the perceived pitch is more difficult to predict. The perceived pitch is most often not at any frequency present in the sound spectrum (Beerends *and Houtsma*, 1986; Goldstein, 1973) and occasionally the musical context can cause the same people to perceive different pitches for the same sound (Roederer, 1995).

When the perceived pitch is not directly as a result of a frequency component of the sound it is referred to as the *virtual pitch*, *subjective pitch*, *missing fundamental* or *residue pitch*. The perception of the virtual pitch is often referred to as *fundamental tracking*. The process of perceiving a single pitch for the complex sound is often referred to as *fusion* whereas if individual partials of a complex sound are heard separately from the sound as a whole it is referred to as *fission* or *analytic perception*.

Four interesting characteristics of the virtual pitch according to Roederer (1995) and Noorden (1982) are:

1) Cochlea fluid oscillations at the frequency of the virtual pitch are negligibly small. This is what distinguishes this effect from combination tones (difference and summation tones) for which there is a considerable fluid oscillation at the relevant frequency.
2) A virtual pitch can not be made to beat with any other sound.
3) Virtual pitch is heard even if the necessary frequencies for its formation are presented dichotically (one into each ear).
4) Virtual pitch is most readily perceived for pitches corresponding frequencies below 1500 Hz. Above this the fundamental needs to be present.

These facts all indicate a higher order analysis of the incoming sound (Roederer, 1995). Several theories of pitch perception have developed in order to explain these phenomena.

2.2 Some Theories of Pitch Perception

2.2.1 Historical Developments

The first person to attempt a scientific explanation of pitch perception was Ohm in 1843. He proposed that in order for a certain pitch to be perceived the sound stimulus must have sufficient energy at the frequency corresponding to that pitch (Eggen, 1986). This is known as *Ohm's Acoustical Law* (Moore, 1977).

The partials of most musical sounds are harmonically related and usually most of the energy is at the fundamental frequency. For such sounds Ohm's law holds as the perceived pitch is indeed that of the fundamental frequency.

Seebeck, a psychologist and a contemporary of Ohm, discovered the phenomenon of the "missing fundamental" while experimenting with sirens. The energy at the fundamental was almost insignificant and yet there was a strong sensation of pitch at the fundamental frequency for these sounds (Eggen, 1986). This contradicted Ohm's law.

Helmholtz (1877) enthusiastically defended Ohms law against the claims of Seebeck. He attempted to explain the missing fundamental in terms of the difference tones (distortion in the ear) created by the adjacent harmonic partials. This meant that the cochlear was receiving energy at the repetition rate of the fundamental, but as a consequence of the difference in frequency between the other harmonics. The theory of Helmholtz assumed that the transverse fibres in the basilar membrane (hair cells) would resonate at frequencies determined by their length, mass and tension. This would enable the ear to act like a frequency analyser. It was later discovered that the fibres could not resonate independently, but that the basilar membrane vibrated as a whole (Eggen, 1986). Helmholtz was the first to propose a mechanism of pitch perception dependent on the locality of the motion of the cochlear (Dowling , 1986).

During the first half of the twentieth century two types of theories of pitch perception developed alongside one another, the pattern recognition theories (*place theory*) and the temporal theories (*time theory*) (Moore, 1977)..

Schouten in 1940 proposed his *residue theory* of pitch perception which used concepts from both the place and time theories in an attempt to explain the phenomenon of the missing fundamental (Eggen, 1986; Moore, 1977).

Although most researchers seemed inclined to reject one or the other of these theories, experiments involving "jnd's" (just noticeable differences) of pitch seemed to indicate that both temporal and pattern recognition mechanisms could be operating at once (Pierce, 1990).

All the theories of pitch perception developed up to this point relied on the interaction of the sound components on the cochlear, and relied only on the output of the cochlear for the determination of the pitch of complex tones (Goldstein, 1973). These theories were therefore not able to explain the results from experiments performed by Goldstein and Houtsma concerning the pitch of two tone complexes (Eggen, 1986; Roederer, 1995; Moore, 1977). Goldstein and Houtsma discovered in 1972 that even when the two tones were presented *dichotically*[3] a residue pitch was perceived (Goldstein, 1973). This indicated that higher order neural processes were involved in assigning pitch to a complex tone and lead to the development of models incorporating a central processor such as the *optimum processor* and *virtual pitch* theories (Eggen, 1986; Moore, 1977).

All five of the above mentioned theories have had a significant impact on research into pitch perception of complex tones and will therefore be discussed in more detail in the next section. No attempt has been made in this chapter to discuss all theories of pitch perception proposed to date, as this is far beyond the scope of this thesis. Only the most influential theories have been discussed.

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[3] One into each ear

2.2.2 Recent Theories of Human Pitch Perception

2.2.2.1 Place Theory

The place theory is referred to as such because the vibrations (which occur in time) are converted into a spatial pattern of vibrational maxima along the basilar membrane. This in turn sets the hair cells into motion which trigger a spatial pattern of neural activity (Eggen, 1986; Frances, 1988). The place on the basilar membrane that experiences a maximum displacement is determined by the frequency. In other words the ear is performing a frequency analysis on the incoming stimulus.

This theory was rejected by some researchers for a variety of reasons according to Frances (1988):

1) The hair cells are not able to vibrate completely independently as they are physically joined to one another.
2) There are not enough hair cells to account for the extremely acute pitch discriminating abilities of the musical ear.
3) Louder sounds set larger areas of the basilar membrane into motion, which according to the place theory should decrease the ability to discriminate pitch. No such effect is recorded.

2.2.2.2 Time or Periodicity Theory

According to this theory the incoming vibrations cause neural signals that vary periodically. This time distribution of neural activity is decoded by the central nervous system. In this theory the ear is not seen as performing a spectral analysis (as in the place theory) but instead a time analysis. Wever proposed his volley principle in 1949 which attempted to explain how the neural firings from the cochlear could represent the frequency of the sound wave[4] (Eggen, 1986).

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[4] See section 1.1.4 for an explanation of the volley principle.

2.2.2.3 Residue Theory

Schouten (1938) proved that the perception of the missing fundamental could not be attributed to distortion as was hypothesised by Helmholtz in an attempt to explain Seebeck's siren experiments. He developed the Residue Theory which borrows concepts from both the Place and Time Theories. Schouten (1940 a) proposed that the lower frequencies in the sound are sufficiently distinct in frequency to excite separate, clearly defined regions of the cochlear. This part of the sound is perceived according to the Place Theory. The higher frequency partials are closer together and therefore do not cause the vibration of clearly distinguishable portions of the cochlear. These frequencies were termed the *residue*. Schouten (1940 b) proposed that the residue lead to a time distribution of spikes in the auditory nerve which were analysed by the brain according to the Time Theory (Eggen, 1986). When all the constituent frequencies of the sound are added together, the amplitude envelope of the resulting wave will have a certain repetition rate. According to the residue theory, this repetition rate is what determines the perceived pitch of the sound (Lehr and Ayres, unpublished). For harmonic sounds the overall repetition rate of the envelope does correspond to the fundamental frequency.

2.2.2.4 Optimum Processor Theory

The Optimum Processor Theory was first introduced by Goldstein (1973) in an attempt to explain the results from his famous experiments involving the dichotic presentation of two tone complexes[5]. Interaction of sound components on the basilar membrane was rejected in favour of a central processor which determined the perceived pitch on the basis of the spectral information sent to it by the ears (Moore, 1977).

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[5] Refer to section 2.3.3 for information about the experiments performed by Goldstein.

Goldstein proposed four stages in the pitch identification of complex tones (Eggen, 1986; Goldstein, 1973):

1.) Each ear acts as a spectrum analyser, resolving the incoming stimulus into its frequency components.
2.) The frequency components alone are conveyed to the optimum central processor along independent channels. The phase and amplitude data are ignored.
3.) The incoming frequencies each have their own uncertainties due to the finite resolution capabilities of the ear. The processor compares its input to an harmonic template in order to approximate (optimise) the incoming frequencies to an harmonic series.
4.) The pitch is perceived as the fundamental of the harmonic series that best corresponds to the actual sound.

The optimum processor theory is well illustrated by the flow chart in **Figure 11** taken from Goldstein (1973).

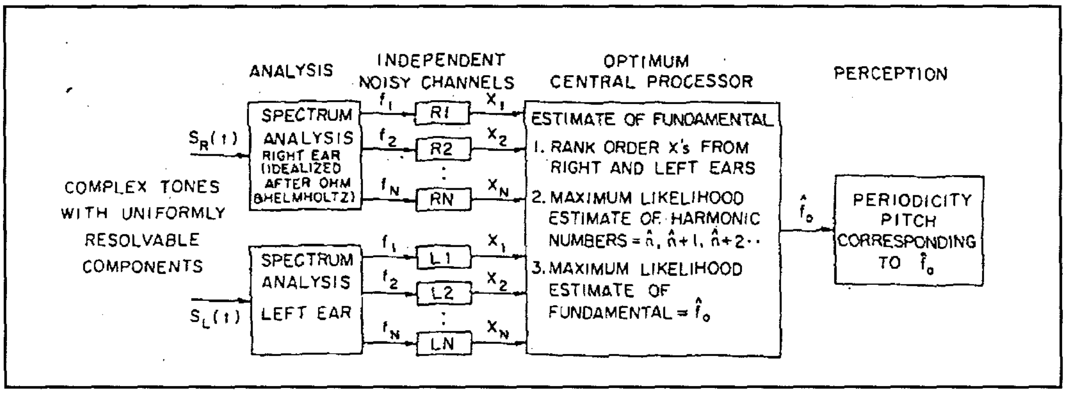

Figure 11 Illustration of the flow of auditory information to and from the optimum processor, according to the optimum processor theory (Goldstein, 1973, Fig. 1.).

The first 6 to 8 harmonics of a complex sound have clearly separated resonance regions, but after this they begin to fall within a critical bandwidth of each other. The central processor can therefore only use information from the first few harmonics for the comparison with the harmonic template (Hodges, 1980).

The optimum processor theory was later revised (Gerson and Goldstein, 1978) to account for the perceived pitch of two tone complexes composed of odd harmonics. According to the original theory the processor assumes that neighbouring components of the sound are neighbouring harmonics. In the revised version of the theory this is not a necessity, but rather the most probable course of events.

2.2.2.5 Virtual Pitch Theory

Terhardt (1979) developed the Virtual Pitch Theory in an attempt to explain the human ability to perceive both an overall pitch for a complex tone (synthesis or virtual pitch perception) and "hear out" individual partials (analysis or spectral pitch perception) (Terhardt, 1979). He adapted and elaborated on a theory proposed by Walliser (Moore, 1977) in which two steps are proposed:

1) The pitch difference between neighbouring partials is approximated.
2) The subharmonic of the lowest partial present which is closest to this pitch difference is chosen as the pitch.

Terhardt proposed that the subharmonic was not necessarily that of the lowest partial but that of a dominant partial (one that is resolvable from the rest of the sound). When partials become too close together to be resolved, no residue pitch will be heard (Moore, 1977).

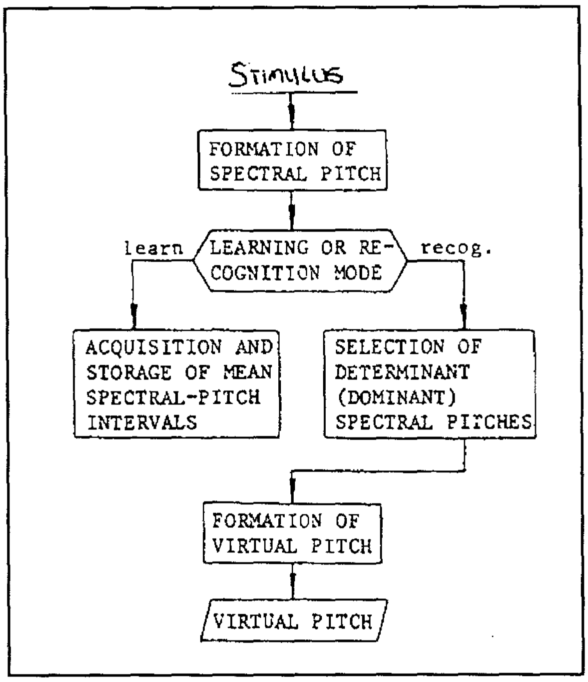

Figure 12 A flow chart representation of the virtual pitch theory. (Terhardt, 1979, Fig. 1. p. 157).

The workings of the virtual pitch theory are well summarised in **Figure 12.**None of the mathematical formulation of the theory devised by Terhardt (1979) will be given here, but the essential features of each stage will be discussed. The input parameters in the first box in **Figure 12** are the frequencies and magnitudes of the partials of the sound stimulus. The formation of spectral pitch is considered in two parts (Terhardt, 1979):

1) determining whether the stimulus conditions support spectral pitch perception
2) establishing the relationship between the input parameters and the spectral pitch

Spectral pitch can only be perceived for a particular partial if (Terhardt, 1979):

1) the partial is above the threshold of hearing or,
2) the masking of that partial by the other partials does not prevent its perception.

Once it has been established that the spectral component can be perceived as a separate spectral pitch, the relationship of this spectral pitch to the spectral component is calculated. The spectral pitch is dependent mainly on the frequency of the partial, but *pitch shifts* can occur due to changes in the sound pressure level and masking by the other partials or noisy component of the sound (Terhardt *et al.*, 1982(a)).

Acquisition and storage of the mean spectral pitch intervals takes place when the person is first exposed to sounds composed of several components. The first sounds that are most commonly heard are those of the human voice, which are largely harmonic complex tones. This is how the virtual pitch theory proposes to explain why we come to expect harmonic relationships between partials.

In the recognition mode the information stored in the learning mode is applied to the dominant spectral pitches of the sound leading to the identification of several subharmonics. This process could lead to competition between several coexisting subharmonics for the status as the virtual pitch of the sound (Terhardt, 1979). Two of the factors determining the choice of virtual pitch are (Terhardt, 1979):
1) The pitch interval in which the greatest number of subharmonics coexist.
2) The pitch value which corresponds to the smallest subharmonic number.

Both the Virtual Pitch Theory and the Optimum Processor Theory can account for most of the experimental data concerning pitch perception of complex tones (Eggen, 1986). In the next section some of the results from experiments investigating the nature of human pitch perception are discussed.

2.3 Experiments Probing Pitch Perception

2.3.1 Pitch Perception of Two-tone Complexes and the Effect of a Musical Context

Many of the experiments aimed at explaining human pitch perception have made use of two-tone complexes. Much less research has been directed at spectrally dense complex tones (Eggen and Houtsma, 1986). Some two-tone experiments have investigated the effects of a musical context on perceived pitches of complex tones. It has been found that the pitch of simultaneous complex tones in a musical context is more easily perceived than that of isolated sounds (Beerends and Houtsma, 1986).

Composers have utilised for many years the fact that ascending pitch sequences give rise to the impression of a crescendo and descending pitches to that of a decrescendo. This is one example of where the musical context influences our perception of loudness[6] (Krumhansl, 1992). The influence of a musical context on our perception of pitch is more difficult to demonstrate and comprehend.

Organ builders have known at least since the end of the 16th century that the musical context of a melody can cause people to hear low notes that are not really there (Roederer, 1995). They began to build organs with stops that sounded a 5th above the written note. When this is played in isolation a clear interval of a 5th is heard, but when in a suitable melodic context the effect is that of strengthening the octave below the written note. This makes sense because the octave below the written note in this case is the fundamental of an harmonic series in which the written note and the 5th above are the second and third harmonics.

A clear example of where the context of a sound can influence the perceived pitch is in the two tone pitch matching experiments performed by Smoorenburg in 1970 (Roederer, 1995). He presented subjects with two tone stimuli of short duration. They were asked to match the pitch of a test tone to that of the stimuli. His two types of stimuli were:

1) Two sine waves which were adjacent harmonics eg 800 Hz and 1000 Hz
2) Two sine waves that were not adjacent harmonics eg 900 Hz and 1100 Hz. These are the 9th and 11th harmonics of a series starting on 100 Hz.

These were presented both in and out of musical contexts. He confirmed that subjects perceived the residue pitch with much greater ease when a musical context was used. The pitch of the two tone complexes consisting of harmonically related tones was consistently that of the missing fundamental. In the above example this is 200 Hz. Smoorenburg also found that the missing fundamental became more difficult to hear as higher order pairs of adjacent harmonics were used (Roederer, 1995).

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[6] The diagram in **Figure 4**, p.6 could partially explain this effect.

The inharmonically related tones could however be caused to take on one of two pitches depending on their context. To simplify the explanation it is best to consider two tones, also 200 Hz apart, but which are not adjacent harmonics as in the example above. Smoorenburg discovered that the perceived pitch was not that of the true repetition rate (200 Hz) but could either be about 180 Hz or 220 Hz depending on the context. These pitches seemed to correspond to two pseudo repetition rates (Roederer, 1995). This is best illustrated as in **Figures 13** and **14. Figure 13** shows the waveform resulting from the addition of the adjacent harmonics and **Figure 14** shows the result of adding the non-adjacent harmonics. In both cases the frequency difference is 200 Hz.

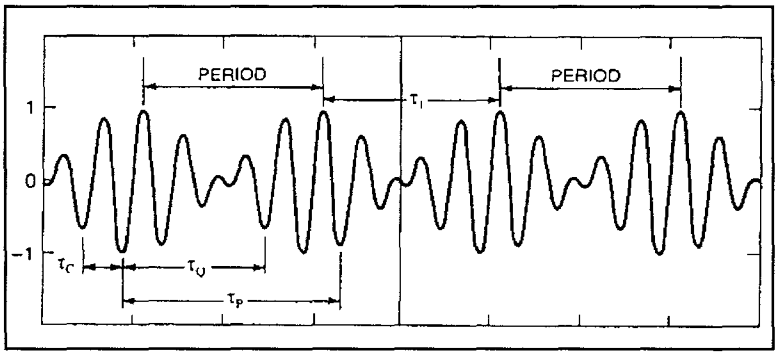

Figure 13 The waveform resulting from the addition of adjacent harmonics with 200 Hz difference. (Roederer, 1995, Figure 2.20)

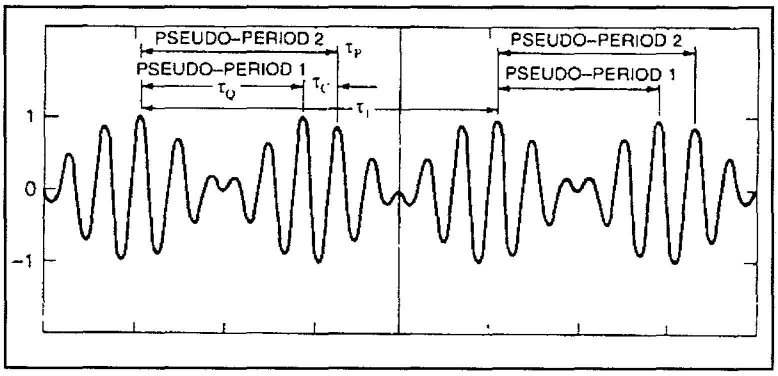

Figure 14 The waveform resulting from the addition of non-adjacent harmonics with 200 Hz difference. (Roederer, 1995, Figure 2.20)

Comparing the two diagrams one can see that the repetition rate of the envelope in both cases is the same (200 Hz). The wave in **Figure 13**, however only exactly repeats itself at a frequency of 100 Hz. The surprising result was that the perceived pitch of the waveform in **Figure 13** corresponded to neither the frequency of the envelope nor the true repetition rate, but rather to the pseudo repetition rates indicated on the diagram.

In 1972 Goldstein and Houtsma investigated the pitch perception of two-tone complexes. The two-tone stimuli were placed in a context such that their missing fundamentals would form a melody. They proved that given a melodic context, all that was necessary for the missing fundamental to be perceived was two adjacent harmonics (Campbell and Greated1987; Roederer, 1995). Their next experiment was considered a landmark in the field of pitch perception. The individual components of the two-tone stimuli were presented dichotically. The astounding discovery was that the residue pitch was still readily perceived, given the suitable melodic context. This showed that the virtual pitch was not a result of mechanical processes in the ear, but that the central nervous system was involved (Roederer, 1995; Campbell and Greated, 1987).

Most of the tones used up to this point to produce stimuli for studying complex tones were made up of successive harmonics (Houtsma, 1979), in other words if the two sounds making up a two-tone complex are denoted by t_1 and t_2 then t_2 will be related to t_1 related according to equations **(1)** and **(2)**.

$$if \quad t_1 = nf_o \tag{1}$$

$$then \quad t_2 = (n+1)f_o \tag{2}$$

Houtsma (1979) performed a series of experiments in which the two tones were non-adjacent harmonics. Now t_2 would be related to t_1 by equation **(3)**, where m is a whole number different from one.

$$t_2 = (n+m)f_o \tag{3}$$

Houtsma (1979) then compared the results from his experiments to the pitches predicted by three theories of pitch perception; the optimum processor theory, the virtual pitch theory and the pattern transformation theory. Subjects were required to identify melodic intervals formed between two two-tone complexes formed from non-adjacent harmonics. These results were also compared to the results obtained by Goldstein and Houstma in 1972 where successive harmonics were used.

The results from these experiments were (Houtsma, 1979):

1) Subject's performance became worse with increasing *m* and *n*.
2) Performance became worse more rapidly with increasing *n* for the higher values of f_0 (800 and 1000 Hz) than for lower values (200, 400 and 600 Hz).
3) Subjects performed better when *n*, *m+n* or both had the values 1,2,4 or 8.

Comparing these results to pitches predicted by the three theories of pitch perception showed that (Houtsma, 1979):

1) The optimum processor theory was able to correctly predict the pitch in most cases.
2) The virtual pitch theory, could with some small modifications, predict the results correctly.
3) No modification could be found for the pattern transformation theory such that it would agree with the experimental results.

The theories of pitch perception evolved in order to explain the sensation of pitch experienced when a complex tone was heard in isolation from others, and did not consider the case where simultaneous streams of pitches are heard, as is the case in polyphonic music (Beerends and Houtsma, 1986). Relatively few experimenters have investigated the pitch of simultaneously sounding complex tones (Beerends and Houtsma, 1986). Beerends and Houtsma (1986) experimented both with single two-tone complexes and with pairs of these two-tone complexes presented dichotically. They found that most subjects processed the pitches of the two complexes separately (Beerends and Houtsma, 1986).

2.3.2 Pitch Perception of Harmonic and Inharmonic Complex Tones With Many Partials

Schouten, Ritsma and Cardazo (1962) used three-tone complexes to investigate the phenomenon of *pitch shift*. Pitch shift occurs when the partials of a sound are modified slightly from their harmonic norm. The resulting residue pitch of the complex is shifted approximately linearly to the shift of the average of all the frequencies (Schouten *and Ritsma.*, 1962). Their experiments lead to several discoveries and conclusions (Schouten *and Ritsma.*, 1962):

1) Pitch shift could be demonstrated for complex tones consisting of only three components.
2) Possible ambiguity of the residue pitch was experimentally confirmed.
3) A pitch extracting mechanism, separate to the pitch analyser (basilar membrane) must exist.
4) The pitch extractor does not use only the information provided by the envelope of the signal, but is effected by the fine structure of the waveform.

Experiments have shown that there is a region of frequencies (500 Hz to 2 kHz) which is dominant in determining the perceived pitch of complex tones (i.e. dominant in effecting the pitch shift). Partials that fall within this region of frequencies have the greatest effect on the perceived pitch of the tone (Ritsma, 1967). This effect is even noticeable with the sounds from piano strings. The 5th partial of the low C_2 played on a piano is in the dominance region and is sharper than a true harmonic due to the finite mass of the string. The pitch experienced for this note is as a consequence sharper than that of a sinusoid at the fundamental frequency of the sound
(Campbell and Greated, 1987).

Moore and Glasberg (1984) investigated the relative dominance of individual partials in determining the pitch of complex tones. For their first series of experiments complex tones consisting of many harmonics with equal amplitudes were used. The subjects adjusted a perfectly harmonic *comparison tone* until they were satisfied that its pitch matched that of a *test tone*, in which one of the harmonics had been mistuned. The test tone and comparison tone were presented alternately. In a second set of experiments the effect of the relative amplitude of an harmonic on its dominance in determining the pitch of the complex was investigated.

Some of the conclusions drawn from these experiments were (Moore and Glasberg, 1984):

1) When the frequency of one partial of a complex tone is shifted from its harmonic value the residue pitch of the whole sound is affected. The shift of the residue is linearly related to the shift of the partial until the shift becomes greater than about 3%. When shifted more than this, the partial begins to have less of an effect on the partial as a whole and eventually becomes eventually audible as a separate tone.
2) When complex tones have harmonics that are equal in amplitude, the dominant harmonics are usually within the first six. Fundamental frequencies of 100, 200 and 400 Hz were used. Some of these results contradicted the concept of the dominant region introduced by Ritsma (1967).
3) The dominance of a single partial decreased when its intensity was decreased below that of its adjacent harmonics. The opposite effect was observed for some subjects when the intensity of one partial was increased above that of its surrounding harmonics.

Faulkner (1985) investigated whether subjects matched the residue pitches of two complex tones or whether they matched the pitches of components within the two complex tones. Subjects performed two types of pitch matching tasks:

1) The test and trial tones had similar residue pitches, but did not have any coincident partials.
2) The test and trial tones had nearly coincident partials and similar residue pitches.

From his observations Faulkner concluded that subjects only matched the residue pitches when there were no frequency components within the two sounds that could be matched.

Moore (1987) took the first conclusion listed above from Moore and Glasberg (1984) as his starting point for a number of interesting experiments concerning the pitch perception of very slightly inharmonic complex tones.

He investigated the threshold of mistuning required for a partial to be heard separately from the complex tone. The duration of the shortest stimulus was chosen as 50 ms and the longest was 1610 ms.

Moore drew two important conclusions from this experiment:

1) Even when the mistuned partials were being perceived as separate pure tones, they were still influencing the residue pitch of the complex tone as a whole.
2) He also discovered that the sensitivity of the human pitch perception mechanism to mistuned partials was decreased for sounds of shorter duration. The longer the sound, the more likely the subject was to perceive the mistuned partial separately. He concluded from this that the duration of an inharmonic complex tone could influence its perceived pitch.

Moore (1987) also performed some pitch matching experiments in order to test the claim by Faulkner that subjects compare the pitches of corresponding harmonics when performing pitch matching tasks involving two complex tones. The subjects were required to match the pitches of harmonic complex tones to those of inharmonic tones. The mistuned partial of the inharmonic tone was only present in half of the harmonic comparison tones. This experiment, in contradiction to Faulkner's conclusions, established that subjects match the residue pitches of tones rather than the pitches of individual components.

Houtsma and Smurzynski (1990) returned to the question of the effect of unresolvable high order harmonics (those that fall within a critical bandwidth) on the pitch of tone complexes. They found that subjects were still capable of identifying melodic intervals between complexes consisting entirely of harmonics between the 20th and 30th harmonics (which are definitely no longer resolvable). The also found that the *jnd* (just noticeable difference) between sounds became larger as the harmonic number of the lowest partial was increased. After the lowest harmonic was above the 12th harmonic, the jnd no longer increased as rapidly.

2.3.3 Perceptual Interactions Between the Auditory Dimensions of Pitch, Timbre and Loudness

The information reaching our ears in the form of sound is distributed among a number of perceptual dimensions simultaneously, such as pitch, timbre, loudness and duration. The purpose of these studies was to investigate how these dimensions influence one another and how the listener eventually integrates the information into a coherent sound object (Krumhansl and Iverson, 1992).

Auditory dimensions are said to interact if a variation in one leads to a perceived variation in the other. The subject is told to concentrate on the *relevant dimension* while the *irrelevant dimension* can undergo one of three treatments (Melara and Marks, 1990) :

1) The irrelevant dimension is held constant. These are called *baseline tasks* and the relevant dimension is said to experience *orthogonal variation.*
2) The variation of the irrelevant dimension is uncorrelated to that of the relevant dimension. These are called *filtering tasks*.
3) *Correlated tasks* require that the variation of the irrelevant dimension is correlated to that of the relevant dimension.

Dimensions exhibit *Garner interference* if filtering tasks are performed poorly compared to baseline tasks (Melara and Marks, 1990). When performance is better for correlated tasks than for baseline tasks the dimensions are said to experience *redundancy gain* (Melara *and Marks*, 1990).

Experiments performed in the 1980's showed that loudness did interact with other musical attributes such as pitch (Krumhansl, 1992). Subjects experienced substantial Garner interference when pitch and loudness were varied (Melara, 1990). The effect of loudness on the perceived pitch has been quantified experimentally. When the frequencies were low (at about the bottom G of the bass clef) a drop in pitch was experienced as the amplitude increased. The opposite was observed for very high notes whereas notes in the midrange (on and just above the treble clef stave) were not susceptible to intensity related pitch changes (Campbell and Greated, 1987).This effect is often noticed in wind bands. The instrumentalists (particularly flautists) hear their instruments from a closer range and due to the loudness experience difficulty in judging how well they are tuned (Campbell and Greated, 1987).

Melara and Marks (1990) found from their experiments that timbre and pitch interacted giving rise to Garner Interference. They concluded, together with their previous results concerning the interactions between pitch and loudness, that the pitch, loudness and timbre of a sound are processed jointly by the human perceptual system. This means that even if one of these dimensions is manipulated independently, the perception and memory of the other dimensions is influenced.

The experiments performed by Krumhansl and Iverson (1992) investigated the interactions between pitch and timbre both for single tones and for tones presented in a sequence. They confirmed the findings of Melara and Marks (1990) that subjects could not concentrate on the pitch of a tone without being influenced by the timbre and *vice versa.*

2.3.4 Influence of Musical Training on the Perception of Pitch and Timbre

Beal performed experiments in 1985 in which musicians and nonmusicians were asked to judge whether chords played on a guitar, piano or harpsichord contained the same notes. Not surprisingly the task was more difficult when the two chords were played on different instruments. Musicians were far better at performing these tasks. In other words the interactions between the dimensions of pitch and timbre caused less confusion amongst the musicians (Krumhansl, 1992).

Pitt (1994) conducted several experiments in an attempt to quantify the differences between nonmusicians' and musicians' perception of timbre and pitch. Subjects were presented with two consecutive tones and had to identify whether the tones differed in pitch, timbre or both, or whether they stayed the same. He discovered that nonmusicians had great difficulty in determining whether the pitch had changed when the timbre was changed. Musician's pitching abilities were, however, not affected by changes in timbre.

Another effect addressed in the article (Pitt, 1994) was that musicians and nonmusicians (although the nonmusicians were more susceptible to this) perceived a complex tone with the same fundamental as another to be higher in pitch if it had higher harmonics. Pitt (1994) refers to experiments performed by Hirsh in 1992 in which subjects compared the pitches of complex tones composed of six harmonics which varied in harmonic number, but which all shared the same fundamental.

Experiments investigating the differences between musicians' and nonmusicians' abilities to recognise and remember melodies were performed at the University of Notre Dame (Radvansky, 1995). They found that nonmusicians relied far more heavily on timbre for recognition and memory of pitch sequences.

2.3.5 Cyclic Pitch Illusions and the Bi-dimensional Nature of Pitch

The similarity of two notes an octave apart has lead some researchers to postulate two different attributes (or dimensions) associated with the sense of pitch; *tone height* and *tone chroma* (Campbell and Greated, 1987). The *tone chroma* is a synonymous concept to the *pitch class* in musical set theory. These two attributes of pitch are often illustrated using the pitch spiral as shown in **Figure 15**.

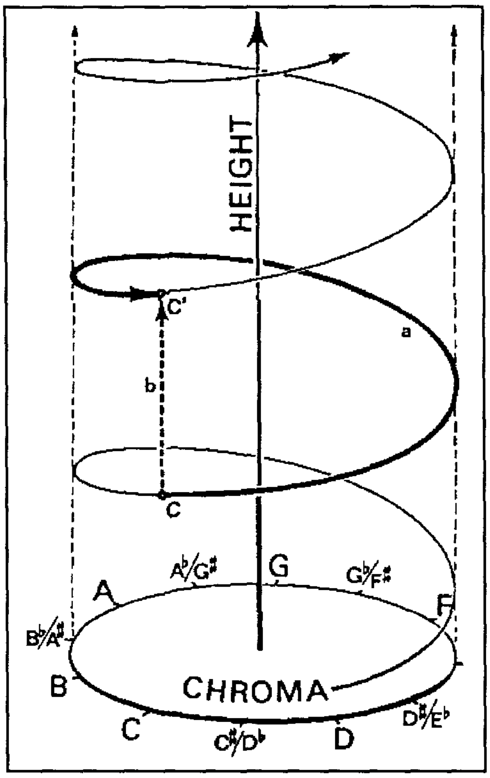

Figure 15 The helix or spiral of pitch (Shepard, 1982, Fig. 1. p.353)

An illusion similar to that of the eternally ascending staircase (shown in **Figure 16**) is also found to exist with certain sequences of complex tones. This illusion can cause a listener to experience a continually ascending series of pitches, when in reality the same few tones are simply being repeated. The effect was first described by Shepard in 1962 (Nakajima *et al.*, 1991).

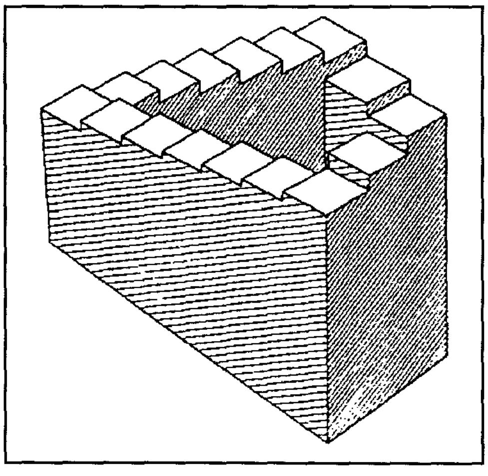

Figure 16 The illusion of an eternally ascending staircase (Shepard, 1964, Fig.2.)

Shepard (1964) used complex tones made up purely of octaves and arranged them in an ascending chromatic sequence. Each tone spanned ten octaves and was sounded for 0.12 seconds with a 0.84 period of silence in between. When these twelve tones were presented repeatedly in the ascending order, listeners heard an eternally ascending sequence of pitches, and *vice versa* for the twelve notes presented in descending order (Shepard, 1964). Listeners seemed to track the relative motion of partials within the sound, rather than the fundamental frequency (Burns, 1981).

Shepard (1964) also found that if an upward shift of less than a tritone was used between successive tones, the pitch shift was perceived as upwards, but if the interval exceeded a tritone, the shift was perceived in the downward direction (Shepard, 1964). This is astounding because the pitch is perceived as going down when the fundamental has gone up in frequency (Terhardt, 1979). This is a clear example of where the perceived pitch is strongly dependent on the context in which it is heard (Terhardt *et al.*, 1982(b)).

Pollack (1978) investigated the roles of the signal envelope, lowest component frequency, number of tonal components, frequency density and spacing of adjacent complexes in contributing to the Shepard illusion. Pollack concluded from his observations that the signal envelope has no significant effect, but that the minimum spacing between adjacent sounding complexes needed for the illusion to work was effected by the number of components, and the frequency density of the components.

Shepard's demonstration of pitch circularity was at first generally accepted as proof of octave equivalence (Nakajima *et al.,* 1991). Octave equivalence was later challenged by researchers such as Burns (1981), Nakajima *et al.* (1988) and Nakajima *et al.* (1991). Burns showed that this circularity of pitch could occur even if the complexes were made up of non-octave components. Nakajima *et al.* (1988) showed that there was a learning process involved in the perception of the illusion, because some subjects only experienced the illusion after a few attempts. Nakajima *et al.* (1990) confirmed that the Shepard illusion was not dependent on octave equivalence.

Chapter 3 - Pitch Perception of Bell Sounds

3.1 Introduction

A great variety of instruments have been classified as bells. It is therefore necessary at the outset to clarify that the type of bell being considered is the large church bell known to Western European society. The sounds from this type of bell are examples of inharmonic complex tones (Jones, 1930; Rossing, 1989). This gives an illusive nature to the pitch of a ringing bell, for example, heard on its own rather than in combination with other ringing bells (Rayleigh, 1890).

The prominent partials of an ideally tuned bell with the second lowest partial at middle C could be represented on a musical stave as shown in **Figure 17**.

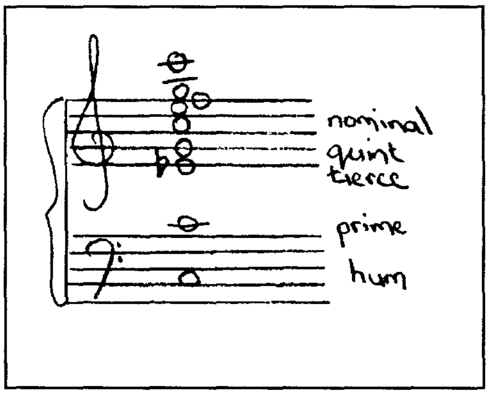

Figure 17 Prominent partials of an ideal bell written in musical notation with the prime at middle C.

Comparing **Figure 10** to **Figure 17** we can see that the bell has a minor third partial whereas the harmonic series contains a major third. The bell is the only instrument that has this minor third partial which is what gives the sound its unique quality (Lehr and Ayres, unpublished). The ratios of these main partials relative to the prime, their names and their note names for the prime at middle C are given in **Table 1.**

Table 1 Frequency ratios of the partials of a well tuned bell. Modified from Perrin et al. (1995, Table 1) and Fletcher and Rossing (1991, Table 21.1 p. 583).

Partial Name	Frequency Ratio to the Prime	Note Name
Hum	0.5	C
Prime / Fundamental	1	C1
Tierce / Minor Third	1.2	E♭1
Quint / Fifth	1.5	G1
Nominal / Octave	2	C2
Major Tenth	2.5	E2
Eleventh	2.6	F2
Twelfth	3	G2
Double Octave	4	C3-

The perceived pitch of a perfectly tuned bell would coincide with that of the second lowest partial, the prime. For this reason the prime is sometimes rather confusingly referred to as the fundamental.

When the bell partials are not well tuned, the perceived pitch of the sound may not correspond to any of the partial frequencies (Walker, 1984). This subjective pitch of the bell is called the *strike note*. The timbre of the *strike note* has a metallic quality, whereas the lower partials, if heard out from the rest of the sound have a more pure timbre (Lehr and Ayres, unpublished).

Bells with a great variety of internal tunings can have the same strike note. **Table 2** shows the internal tuning of five bells which all have a strike note at C_4.

Table 2 The internal tuning of five bells which all have a strike note at C_4 (Lehr and Ayres, unpublished).

Partial	**Bells**				
Hum	C3	C3	B2	C♯3	C3
Prime	B3	C♯4	C4	C4	C4
Tierce	E♭4	E♭4	E♭4	E♭4	E♭4
Quint	F♯	G♯4	G4	G4	G4
Nominal	C5	C5	C5	C5	C5

Perception of the strike note is more difficult if the bell is imperfectly tuned, especially if the prime is out of tune (Lehr and Ayres, unpublished). The strike note is more easily identified if the bell is sounded in succession with other bells, because the ear tends to follow the scale to which the bells are tuned (Lehr and Ayres, unpublished).

A number of experiments have been performed in an attempt to establish which physical characteristics of the bell sound have the greatest effect on the pitch of the strike note. These experiments are discussed in section 3.2.

3.2 Experimental Investigations of the Strike Note

Jacob van Eyck, in 1636, was the first person to mention the strike note (Lehr, personal communication). Lord Rayleigh was the first to document systematic investigations of the strike note (Rayleigh, 1890). Using Helmholtz resonators and calibrated tuning forks he determined some of the partial frequencies of three church bells. Much to his surprise, the perceived pitch of the bells did not correspond to the pitch of the lowest partial.

"A question , to which we shall recur in connexion with church bells, here suggests itself. Which of the various coexisting tones characterizes the pitch of the bell as a whole? ... My first attempts upon church bells were made in September 1879, upon the second bell (reckoned from the highest) of the Terling peal; and I was much puzzled to reconcile the pitch of the various tones , determined by the resonators, with the effective pitch of the bell, when heard from a distance in conjunction with the other bells of the peal" (Rayleigh, 1890).

The partial and effective pitches (strike notes) of two of the bells studied by Lord Rayleigh are shown in **Table 3.**

Table 3 The partial frequencies and strike notes of two of the church bells studied by Lord Rayleigh.

Strike Note	Partial Frequencies
B1	D1, A♯1, D2, G♯2+, **B2**
D2	E1, C2, F2+, B♭2, **D3**, F2

From information like this for seven bells he concluded that the perceived pitch must be an octave below the 5th partial of the sound (the nominal) (Rayleigh, 1902). This hypothesis is now referred to as the *octave rule* (Eggen and Houtsma, 1986).

Jones (1930) attempted to explain the octave rule in terms of the onset times of the partials. He proposed that the pitch of the strike note was determined, except for its octave, by the nominal (5th partial) of the sound because this partial was the first prominent partial to be heard after the strike. The prime and tierce (2nd and 3rd partials) which were more sluggish in obtaining their maxima were according to him the determining factor in the octave of the pitch.

Meyer and Klaes discovered in 1933 that the strike note could not be heard when only the first five partials were present, but could be heard when the 7th partial was added. They formulated their *difference tone rule.* They concluded that the strike note was formed as a result of a difference tone between the 5th and 7th partials of the bell (Eggen and Houtsma, 1986).

Arts (1938) did some acoustic measurements on several church bells in an attempt to choose between the octave rule and the difference tone hypothesis. He favoured the octave hypothesis on the basis of his measurements. Arts (1938) documented an interesting observation of two bells, which when rung separately appeared to have strike notes that were a quarter tone out of tune with each other, but when rung simultaneously, seemed to be perfectly in tune with one another. He attributed this to the fact that their nominals were in tune, further justifying his support for the octave hypothesis. Arts (1939) also investigated the secondary strike note. He attributed its existence to a partial in the bell sounds which was an octave above the secondary strike notes.

In 1940 Schouten developed his residue theory[7]. This theory explains the strike note as a residue pitch created by the nominal, twelfth and double octave which are in the frequency ratio 2,3,4. It also predicts the existence of the secondary strike note, about a perfect 4th higher than the strike note, due to other partials forming a portion of an harmonic series (Eggen and Houtsma, 1986). Very large bells (800 kg or more) exhibit this secondary strike note (Fletcher and Rossing, 1991). An explanation of why smaller bells do not seem to have a secondary strike note could be the fact that the higher partials are too high to facilitate the perception of the residue pitch i.e. higher than about 1500 Hz (Roederer, 1995). This could also be used to explain why very small bells often do not have a strike note at all (Fletcher and Rossing, 1991).

Bagot (1986) described some experiments that he performed using recorded bell sounds and various filters. The bell that he used had a strike note of about 358 Hz. The results he obtained can be summarised as follows:

1) When a low pass filter was applied at 500 Hz, the strike note was not audible. All that was heard was a mixture of frequencies with the hum being the loudest.
2) When a high pass filter was applied at 500 Hz, the strike note was clearly audible.
3) When a band reject filter was applied at the frequency of the nominal, the strike note also vanished.

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[7] See chapter 2 page 19 for an explanation of the Residue Theory.

This proved that the strike note had no physical existence in the sound of the bell but was constructed in the human auditory system from the information provided by some of the higher partials, the nominal being one of the most important (Bagot, 1986).

Eggen and Houtsma attempted to pinpoint which bell partials had the greatest influence on the pitch of the strike note. They performed a series of pitch matching experiments in which the partials of a recorded bell sound were systematically varied, first one at a time and then in pairs, and the variation in the pitch of the strike note was monitored.

A summary of their results is illustrated in **Figures 18** and **19**. **Figure 18** shows the effects of shifting one partial at a time and **Figure 19** shows the case for shifts of clusters of partials. The relative change in the pitch of the strike note is given along the $\Delta P/P$ axes and the relative shift imposed on the frequencies of the partials is shown on the $\Delta F/F$ axes. Long and short bell sounds were used in this experiment, but little difference was observed for the different durations.

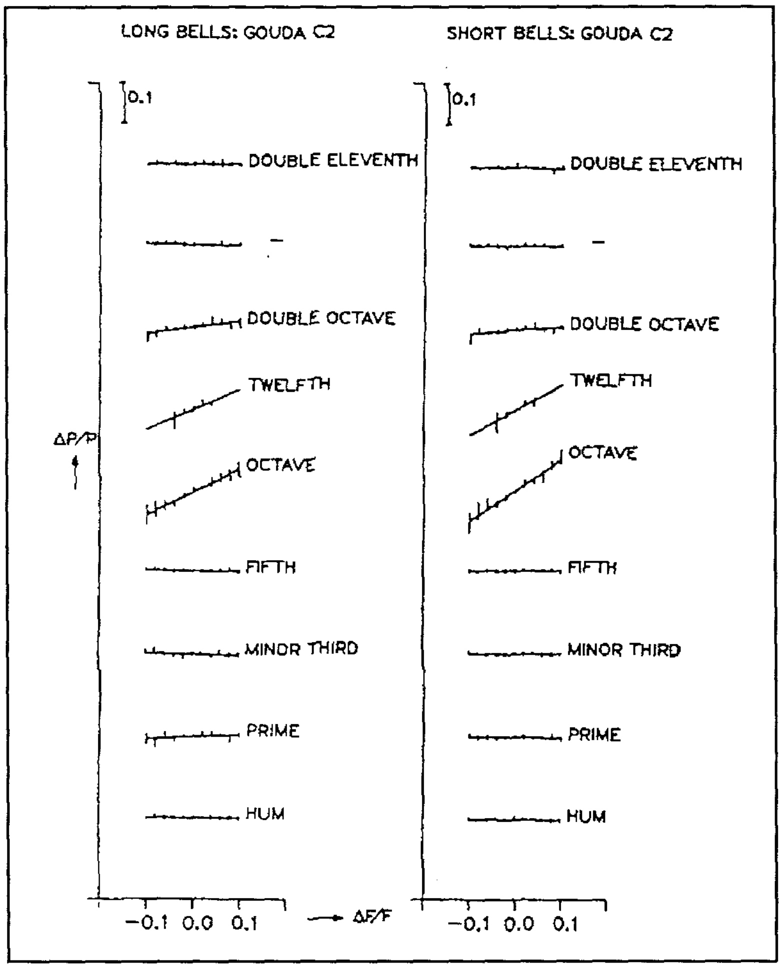

Figure 18 The relative shift of the strike note (Δ P/P) when one partial frequency is shifted (Δ F/F) at a time. (Eggen and Houtsma, 1986, Figure 1)

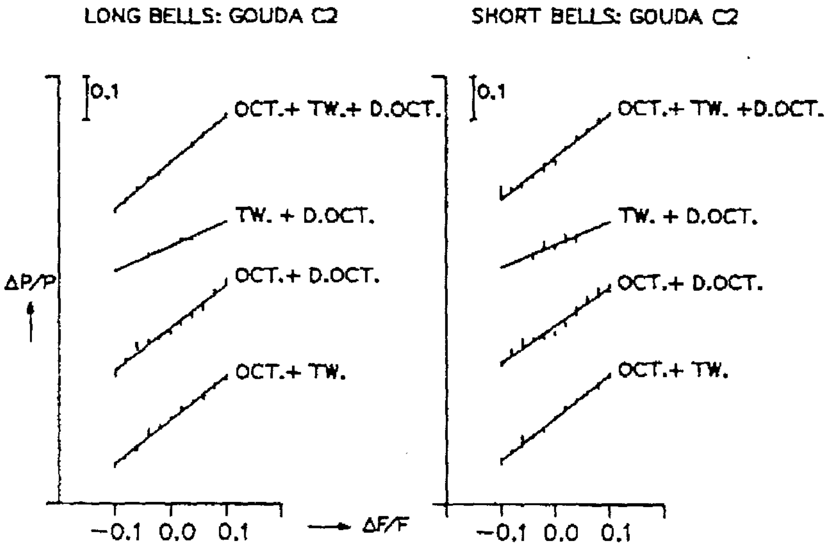

Figure 19 The relative shift of the strike note (ΔP/P) for shifts of groups of partial frequencies (Eggen and Houtsma, 1986, Figure 2)

3.3 Octave Error Judgments of the Primary and Secondary Strike Notes

Some listeners hear the strike or secondary strike notes an octave above the pitch predicted by the residue theory. The *mistaken octave theory* has been proposed to explain these octave uncertainties (Lehr and Ayres, unpublished). The theory claims that the complexity of the sound during the strike leads the ear to search for some component of the sound which is easily identified because of its stronger intensity. The nominal has the strongest intensity at the strike, and is therefore a likely candidate. There are however so many overtones with this partial that, that an uncertainty is introduced into the perceived octave of the sound (Lehr and Ayres, unpublished).

Schouten showed in a demonstration in 1960 (Lehr and Ayres, unpublished) that high frequency noise interferes with the perception of a residue pitch, but that a pure tone can still be heard above high frequency noise. When a bell is struck very hard, for example when it is *rung full circle*[8], the energy imparted to it is sufficient to excite the higher partials. These higher partials deviate substantially from the harmonic series and are therefore similar to high frequency noise. When the bell is struck lightly for example when *chimed or clocked* the energy is not sufficient to support the upper partials and the high frequency noise is reduced. It is possible that the high frequency noise present in the sound of a ringing bell could prevent the perception of the strike note as the pitch of the sound and encourage the hearing out of the nominal instead, leading to a perceived pitch an octave above the strike note (Lehr and Ayres, unpublished). The possibility of the method of striking influencing the perceived pitch of a bell has yet to be investigated (Lehr and Ayres, unpublished).

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[8] For definitions of ringing and chiming see chapter 5 pp 56 and 58.

Chapter Four - Methodology for the Determination of the Perceived Pitch of Inharmonic Complex Tones

There are six essential stages to consider when setting up any experiment that attempts to relate a physical sound to the resulting subjective experience (i.e. a psychoacoustic experiment) according to Freed and Martin (1986) :

1) Subject selection, evaluation and training
2) Stimulus selection and manipulation
3) Perceptual measurements
4) Statistical analysis of perceptual measurements
5) Acoustic measurements
6) Relating the acoustic and perceptual measurements

The methodology for each of these steps will be discussed in this chapter with specific reference to pitch matching experiments and particularly those involving inharmonic complex tones such as bell sounds. The methodology presented in this chapter was used for the research presented in chapters 5 and 6 of this thesis.

4.1 Subject Selection, Evaluation and Training

Subjects chosen for listening experiments must be able to hear sufficiently well to perform the tasks at hand to a large degree of accuracy and consistency. When dealing with pitch identification tasks it is necessary for the subject to posses both a good threshold of hearing over a wide range of frequencies and good pitch discriminating abilities.

Several tests of musical aptitude exist which can be used to test pitch discriminating abilities, for example the Seashore and Bentley Tests. According to Seashore (1967) the average pitch discrimination ability at 440 Hz is 3 Hz. A hearing threshold of 15 dB or better in the range from 250 to 8 000 Hz is considered sufficient (Bech, 1992).

The above abilities are physiologically determined and can hence not improve with training or experience. However the training, experience, age and knowledge of the participants can influence their responses (Bech, 1992).

A subject who is experienced in the procedures of psychoacoustic experimentation will be more likely to give consistent responses than an inexperienced subject. In an experiment involving the identification of loudspeaker sounds it was found that most subjects reached an asymptotic level of performance after four training experiments (Bech, 1992). Other experimenters seem to be content with two trial experiments (Radvansy, 1995).

The knowledge and degree of musical training of subjects can often have an undesired effect on their responses. Musical training and a knowledge of the spectrum of the bell sound could lead to an increased tendency towards analytic perception[9] (Helmholtz, 1877). Both Pitt (1994) and Radvansky (1995) defined musicians as those who had experienced at least ten years of formal musical training. They disagree however on their definitions of the nonmusician. Pitt (1994) defined a nonmusician for the purpose of his experiments as someone with less than two years of formal musical training. Radvansky (1995) considered less than five years to be adequate. Subjects with perfect pitch may, for example, tend to replicate their responses due to their memory of the comparison tone as opposed to providing a consistent response to the test tone. These possibilities will be explored in chapter 5.

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[9] Analytic perception is when the partials of a complex sound are heard individually instead of as a perceptual whole. This is explained in Chapter 2.

4.2 Stimulus Selection and Manipulation

Until the 20th century the experimenter was restricted to the use of naturally occurring sounds because sound synthesis and editing techniques as well as recording equipment had not yet been invented. This restriction meant that it was not possible to exercise precise control over the stimuli, and the results were of necessity less informative.

Now that technology has provided a means for stricter control over the selected stimuli they can be constructed in such a way that they test only the quantities required by the experimenter. Simplicity of the stimuli is necessary in order to minimise the number of variables that need to be taken into account when examining the results. On the other hand, stimuli that are oversimplified loose their resemblance to frequently encountered sounds and hence the results from such an experiment risk losing their applicability to the real life situation.

The experimenter also needs to consider the physiological limits of audition when choosing the stimuli. It has been determined that in order for a pitch to be perceived comfortably, the aural stimulus must last for at least 250 ms (Gulick, 1971). In a pitch matching experiment performed by Eggen (1986) synthesised bell sounds of 1 536 ms (long bell stimuli) and 100 ms (short bell stimuli) duration were used. The correspondence between the responses measured for the short and long stimuli were good and hence Eggen concluded that all the information needed to determine the pitch of a bell sound was contained within the first 100 ms of sound.

Another factor to consider when dealing with the residue pitch of inharmonic complex tones is that the louder the stimulus the more likely the subject will be to experience analytic perception. As this is undesirable the stimulus should be presented at as low a decibel level as possible. For example Eggen (1986) first determined the subject's threshold of hearing and then set the stimuli to 20 dB above this threshold.

4.3 Perceptual measurements

Perceptual measurements in psychoacoustics are performed in order to identify the subjective response or sensation resulting from a particular sound stimulus. Four methods commonly employed are *magnitude estimation, magnitude production, ratio estimation* or *ratio production* (Hedden, 1980).

When using the magnitude estimation technique the experimenter assigns numbers to stimuli, usually at the extremes of some range of sensation. The subject is presented with this numerical and perceptual range first and then asked to assign numbers to numerous stimuli within the range. Magnitude production differs from magnitude estimation in that the subject is required to turn a dial which manipulates the stimulus until they are satisfied that the stimulus matches some point on a numerical scale (Hedden, 1980).

A ratio estimation task requires the subject to provide a number that best describes the relationship between two stimuli. Ratio production differs again only in that the subject is required to produce a given relationship by manipulating the stimuli (Hedden, 1980).

The pitch discrimination tests used in most tests of musical aptitude such as the Seashore test (Seashore, 1967) and the Bentley test (Bentley, 1966) employ a form of the ratio estimation technique. In these tests, two tones are presented sequentially and the subject is required to decide how the pitch of the second tone relates to that of the first i.e. is it higher, lower or the same as the first.

Pitch matching tasks are commonly used in order to determine the perceived pitch of complex tones. Pitch matching is essentially the simplest ratio production task where the subject must adjust one of the two stimuli until it is identical in pitch to the other. The adjustable stimulus is usually a simple sine wave while the pitch of the complicated stimulus remains static (Nakajima *et al.*, 1988).

Several important considerations relating to pitch matching tasks are discussed by Benade (1976) :

1) If analytic perception occurs for the complex stimulus, it is likely that the subject will match the sine wave to one of the partials of the sound rather than to the residue pitch.
2) Pitch matches made between alternately presented sounds do not always agree with those for which the same two sounds were presented simultaneously[10].
3) Musicians use the absence of beating to determine whether two sounds are at the same pitch. This leads to two problems :
    3.1) This works only if the two sounds both have harmonic partials or one of the sounds is a sine wave. If there are non-harmonic partials, beating is likely to occur between some of the higher constituent frequencies. This will make pitch matching more difficult and will probably yield a different result to the case where a sine wave is used (Benade, 1976).
    3.2) The residue pitch of an inharmonic complex tone has no physical existence and therefore does not beat with any other sound (Roederer, 1995).

Pitch matching tasks are commonly used for determining the pitch of bell sounds and other sorts of complex tones (Smoorenburf, 1970; Goldstein, 1973; Eggen and Houtsma, 1986; Gerson and Goldstein, 1978; Rayleigh, 1890, Nakajima *et al.*, 1988).

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[10] This stands to reason considering the effect of the critical bandwidth. See Chapter 1.

4.4 Statistical Analysis of Perceptual Measurements

Statistical analyses of the results from listening experiments are necessary in order to determine if any trends exist in the data and to iron out any chance or random fluctuations in the responses of the subjects. Both the responses of different subjects to the same stimulus and the repeated response of the same subject to a particular stimulus need to be checked for consistency. The three statistical tests of significance used to analyse data in chapter 5 were the chi square test, the binomial distribution and analysis of variance.

4.4.1 Chi square test

The chi square test is useful for determining whether or not the results from an experiment are significantly different from the expected results (Sprent, 1981). When using the chi-squared test the *null hypotheses* is that there is no sigificant discrepancy between the observed frequency and the expected frequency of an occurrence (Phelan and Reynolds, 1996). The null hypotheses, as its name implies, represents the negation of what the experimenter is hoping to prove (Behr, 1988).

The chi-squared test is well suited to experimental situations where there are, for example, a specified number of possible responses to a task that was repeated a specified number of times. It can also be applied to determine whether there is a significant difference between the responses to two different stimuli. In this case the null hypotheses is that there is no significant difference between the responses to the two different stimuli (Hayslet, 1974).

The frequency of these responses is compared to the expected frequency using Equation **(4)** (Hayslett, 1974).

$$\chi^2 = \sum_{i=1 j=1}^{nm} \frac{(O_{ij}-E_{ij})^2}{E_{ij}} \qquad \textbf{(4)}$$

Here O_{ij} are the observed frequencies, E_{ij} are the expected frequencies, n is the total number of responses and m is the total number of tasks or attempts.

The data is presented as a table to facilitate the calculations (Behr, 1988). **Table 4** is an example of 3 possible responses to two different tasks.

Table 4 Tabulated quantities for calculating chi-squared. Modified from (Behr, 1988).

	Response 1	**Response 2**	**Response 3**
Task 1	$O_{11} - E_{11}$	$O_{21} - E_{21}$	$O_{31} - E_{31}$
Task 2	$O_{12} - E_{12}$	$O_{22} - E_{22}$	$O_{32} - E_{32}$

The number of degrees of freedom for the system can be calculated from equation **(5)**.

$$\nu = (m-1)(n-1) \tag{5}$$

Using equation **(5)** in the above example we get

$$(3-1)(2-1) = 2$$

Chi-squared distribution values for different degrees of freedom are readily available in tables published in statistics textbooks (Behr, 1988; Phelan and Reynolds, 1996; Sprent, 1981; Hayslett, 1974). . By comparing the calculated chi-squared value to the tabulated values one can determine the probability of an observed deviation from an expected result, i.e. the null hypotheses.

4.4.2 Cumulative Binomial distribution

The cumulative binomial distribution gives the probability of getting at least r sucesses in a total of n trials. For example, the probability of achieving four at least four heads in six tosses of a coin. This probability (P) is expressed in equation **(7)** (Stoker, 1979).

$$P(x \leq r) = \sum_{x=0}^{r} \binom{n}{r} \pi^{x} (1-\pi)^{n-x} \qquad \textbf{(7)}$$

Here π is the probability that the event will happen in any one trial and x assumes the integer values from 0 to r.

4.4.3 Analysis of variance

Variance is a measure of dispersion, i.e. a measure of to what degree a number of observations within one category differ from one another. The variance (S^2)of a group of observations is calculated using equation **(8)**.

$$S^2 = \frac{\sum (x - \bar{x})^2}{n-1} \qquad \textbf{(8)}$$

Here x is the mean of the observations $x_1, x_2, x_3, \ldots, x_n$ and , n is the number of observations. Σ denotes the sum over n.

Analysis of variance separates the variations present in data into independent components. For example if there are three categories of observations, the variance within each category and the variance between categories can be calculated independently.

Ratios of these component variances are used to test hypotheses based on the data. The significance of the differences between sample variances is tested by the variance ratio test, or the F test. F distribution tables have been widely published for use, for example by Stoker (1979).

After obtaining a significant result for the overall F test, individual means can be compared using the t test. The t test is used when dealing with small numbers of observations. The t distribution is dependent on a single parameter known as the number of degrees of freedom. The degrees of freedom are the number of independent pieces of information available. Degrees of freedom are usually denoted by df or DF. For example in the case of n observations, one degree of freedom is used for the calculation of the mean and there are *(n-1)* degrees of freedom remaining, hence the *(n-1)* divisor in the calculation of S^2, equation **(8)**.

The t distribution is defined by equation **(9)**.

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} \tag{9}$$

In equation **(9)** μ is the mean of the theoretical infinite populatin of x values of which the observations $x_1, x_2 \ldots x_n$ are a subset, and the remaining symbols are a previously defined. As in the case of the F tables, t tables are also widely published for use in statistical analyses.

4.5 Acoustic Measurements and Analyses

Acoustic measurements and analyses are performed in order to quantify the physical nature of the stimuli and to enable different stimuli to be compared to one another. This is a vital step if the causes of perceived differences between stimuli are going to be explained or understood. The information contained in chapter 2 implies that when studying pitch perception of complex tones the quantities of importance are the frequencies, relative magnitudes and the time evolution of the partials.

Complex tones are usually analysed using Fourier analysis techniques, unless of course the tones were synthesised specifically for the experiment, in which case their spectra and time evolutions were predetermined.

The *Fourier transform* is a mathematical operation which transforms information about the sound from the time domain (the waveform) into the frequency domain (the frequency spectrum). In order to be applicable to physically measured sound, the Fourier transform must be used in the discrete form (Bracewell, 1978). This means that the value of the waveform need not be known at all times, but only at discrete intervals of time. The process of observing the amplitude of the waveform at regular intervals of time is referred to as *sampling*. The rate of sampling is expressed in number of samples per second and hence in the units of Hertz (Hz). The time interval between each sample (t) is the reciprocal of the sample rate.

The sampling rate determines the maximum frequency within the sound which can be captured by the sampling process and later reproduced on sound reproduction equipment. The sampling rate must be twice that of the maximum frequency (Bracewell, 1978). Compact disks use a sampling rate of 44.1 kHz and can hence only capture frequencies up to about 22 kHz. This is ample considering that most adults can only hear up to 15 kHz (Hodges, 1980).

The discrete Fourier transform is usually performed on 2^n samples, where n is an integer, because this simplifies the computing algorithms substantially. The number of significant output data points that can be calculated is equal to the number of input samples. The spacing of the data points in the frequency domain is related to the time interval over which the samples were taken, by equation **(10)** (Bracewell, 1978).

$$\Delta f = \frac{1}{T} \tag{10}$$

Here Δf is the distance in Hertz between two output points and T is the total length of time over which the sampling took place. T can be expressed in term of the number of samples and the sample spacing, as in equation **(11)**.

$$T = (2^n)(t) \tag{11}$$

4.6 Relating the Acoustic and Perceptual Measurements

The ultimate purpose of any psychophysical experiment is to increase or supplement our understanding of the mechanisms of human perception. Without attempting to relate the acoustic and perceptual results the process would be incomplete and we would not have gained any new information.

Many experimenters have worked on the problem of pitch perception of inharmonic complex tones including bell like sounds. The important results from such experiments have been discussed in Chapters 2 and 3. This thesis does not attempt to propose any new theories concerning the pitch perception of complex tones, but rather uses existing knowledge as a basis for discussing and accounting for the experimental results obtained in Chapter 5.

Chapter Five - Investigating Pitch Perception of Ringing and Chiming Bells

5.1 Introduction

Western church bells can be sounded in a variety of ways. The bells of a *carillon* remain stationary while their clappers are moved by motors controlled electronically from a keyboard.

The word *ring* is defined by Strucket (1985) as follows : "to sound a bell by full circle swinging by rope and wheel". Bells mounted for *change ringing* swing through almost a full circle (*ringing full circle*). The bell ringers stand in a chamber below the bells, and each controls one bell using ropes attached to large circular wooden mountings (wheels). This is illustrated in **Figure 20** below.

Because the bells start and come to rest in the upright position, most of the force required to ring the bell is provided by the weight of the bell. The ringer effectively upsets the balance of the bell by tugging on the rope (hand or force stroke). The bell swings through almost 360° and slows as it approaches the uppermost position. The ringer then pulls on the rope again in order to assist the reversal of the swinging direction (back stroke). **Figure 21** is a schematic diagram showing the back and fore strokes.

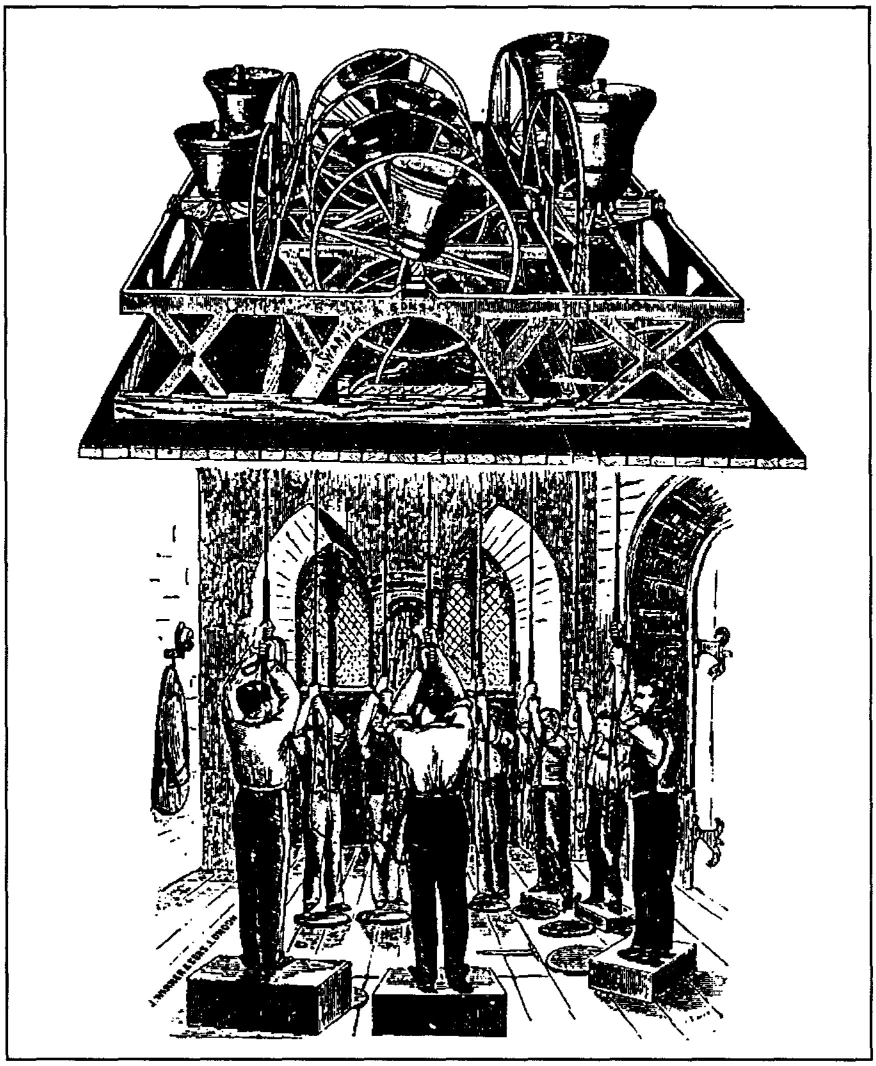

Figure 20 Bells positioned for change ringing (Morris, 1974, p. 28, Fig. 23.)

The clapper strikes the bell as it slows down approaching the upper position. The bell is therefore struck twice before returning to its starting position.

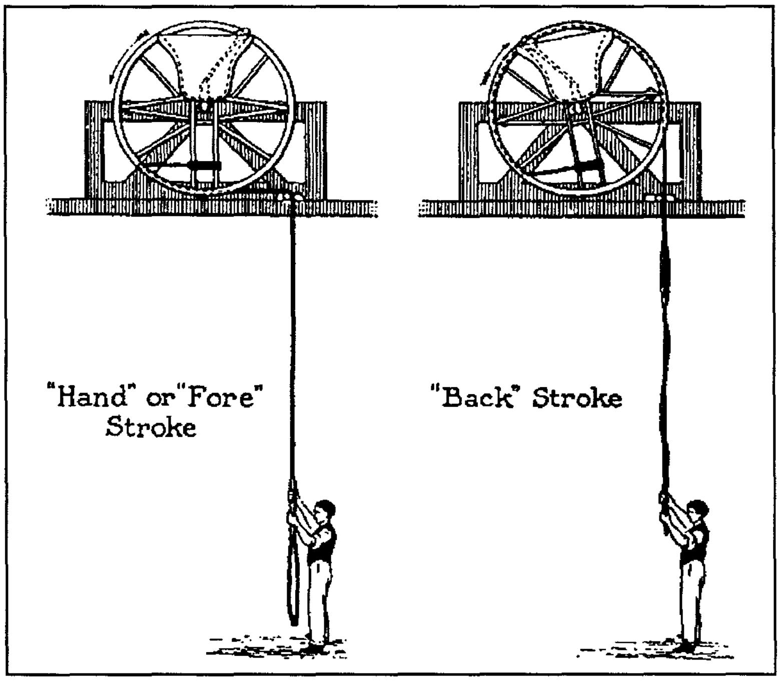

Figure 21 Schematic diagram showing the bell and bell ringer during the back and force strokes (Morris, 1977, p. 17, Fig. 13.).

The word *chime* is defined by Struckett (1985) as: "to cause a clapper to strike on one side only by swinging the bell through a small arc". In order for this to happen the bell has to begin moving from a stationary state in the downward position.

A ringing bell is much louder than one that is being chimed because the clapper strikes harder when the bell swings more vigorously. This causes the upper partials to be more audible. Bells number 2, 5, 8 and 10 from the peal of 10 in the tower of the Cathedral of St Michael and St George in Grahamstown were recorded while ringing and then while chiming. It was necessary to use more than one bell in the pitch matching experiment, to avoid biassed responses due to the subjects memory of the previous stimulus. If too many bells were used in the experiment, the length of the test would become unreasonable. Therefore four bells covering the range of the peal were chosen.

The strike notes of the four bells are given in musical notation in **Figure 22**.

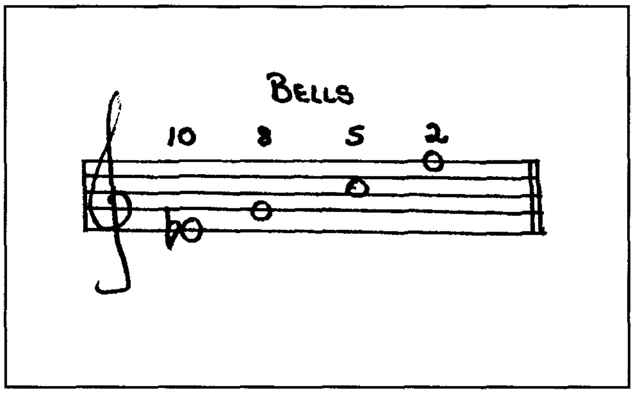

Figure 22 Strike notes of bells number 2, 5, 8 and 10.

5.2 Apparatus and Production of Stimuli

Bell stimuli to be used in pitch matching tasks were produced from recordings of the four bells in the Cathedral in Grahamstown using a Tascam, DA-P1 DAT recorder and AKG C 414 EB microphones. The microphones were placed at a distance of about 5 m from the bells in the chamber above the bells. A schematic diagram of the recording apparatus in the bell tower is given in **Figure 23** below.

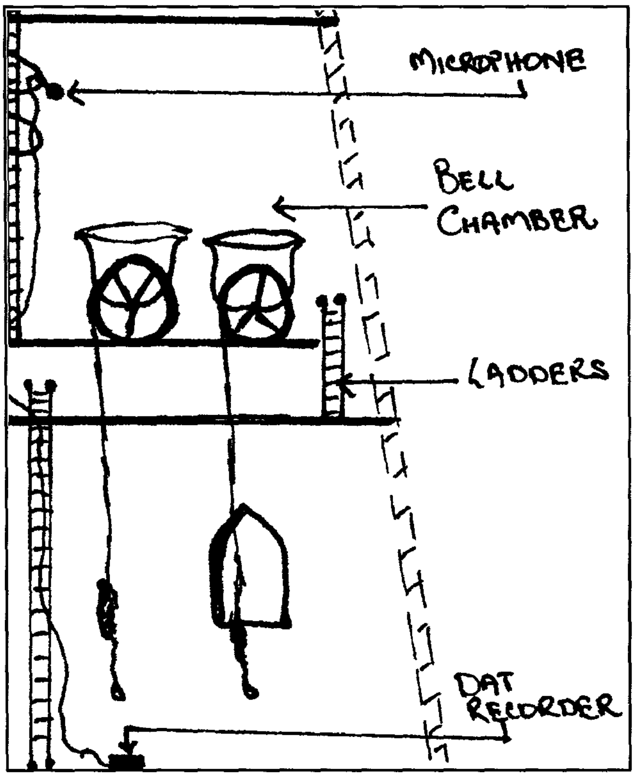

Figure 23 Schematic diagram of the recording set up used to record the four bells in the cathedral in Grahamstown.

The recordings were transferred from the DAT tape and converted into sound files using a DAT drive, and software called DATman and SoundFiler. The sound files were edited using CoolEdit and CreativeWave. One well captured ring and chime from each bell was isolated and reproduced to produce eight different test tones. The test tones consisted of bell stimuli which were approximately 0.7 seconds long and alternated with 1.5 seconds of silence. The stimulus duration was chosen to be 0.7 seconds because this was in between the durations of the long (1536 ms) and short (350 ms) duration bell stimuli used by Eggen (1986).

Tones used for trial pitch matching attempts were produced using CoolEdit. A sine wave and square wave were used for this purpose and presented in a similar manner to the bell stimuli.

A pitch discrimination test was devised using similar principles to the tests in the Bentley and Seashore test batteries (Bentley, 1966; Seashore, 1967). One of the reasons for not using a recognised test battery was that many of the subjects had administered these tests to their pupils and were therefore familiar with the sequence of responses. A more important reason was that the recognised musical aptitude tests only presented each interval once. This could lead to the guesses of the subjects going unnoticed. A test was therefore devised in which each interval was presented three times, decreasing the risk of false conclusions of pitch discrimination due to guessing. CoolEdit was used to generate sine waves with frequencies of 440, 442, 438, 445. 435. 450, 430, 460 and 420 Hz. The 440 Hz sine wave (of 1 s duration) was followed after a silence of 0.5 s by one of the other sine waves, or itself. Thirty such combinations[11] were constructed in which each of the frequencies different from 440 Hz was used three times. A trial test consisting of four pitch discrimination tasks was devised in the same way.

The pitch discrimination and pitch matching tasks were recorded to a compact disk. The order of the tracks is shown in **Table 5**.

Table 5 Order of the CD tracks

Track Number	**Recording**
1	Trial pitch discriminating tasks
2	Pitch discriminating tasks
3	Trial pitch matching task 1
4	Trial pitch matching task 2
5	Announcement of pitch matching tasks 1 to 24
6 - 29	Pitch matching tasks 1 - 24

---

[11] The order of these sine wave combinations is given in Appendix 3.

The order of the bell sounds used for pitch matching tasks 1 to 24 is shown in **Table 6**. Here the letter refers to the method of excitation (R - ringing, C - chiming) and the number (2, 5, 8 or 10) to the bell in question.

Table 6 Order of sounds used for pitch matching tasks 1 - 24.

Task Number	Sound	Task Number	Sound
1	R2	13	R10
2	C8	14	C5
3	C5	15	C8
4	R10	16	R2
5	C2	17	R8
6	R5	18	C10
7	C10	19	R5
8	R8	20	C2
9	C2	21	R10
10	R5	22	C5
11	C10	23	C8
12	R8	24	R2

This random order of the presentation of the bell stimuli was in an attempt to avoid biassed responses from the subjects on the basis of their memory of the previous stimulus.

The layout of the apparatus is illustrated in **Figure 24.** For the sake of the comfort of the subject and for ease of administration, headphones were not used.

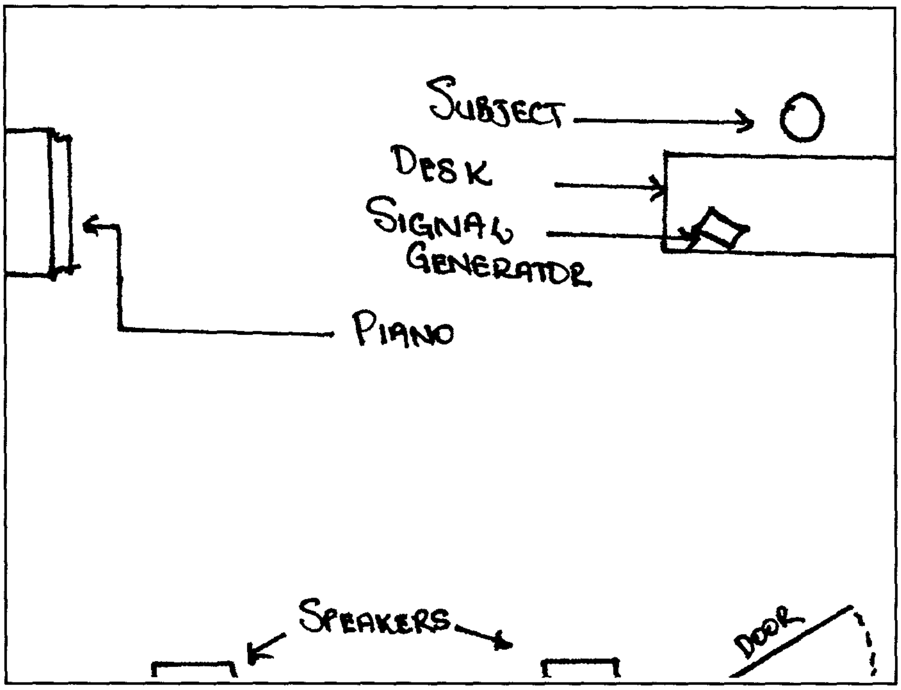

Figure 24 Apparatus and Layout for the pitch discrimination test and pitch matching experiment.

The subjects could modify the volume of the comparison tone to suit themselves. The sound level of the test tone (stimulus) was set at a low level to minimise analytic perception.

5.3 Procedure

The subjects were first required to fill in a questionnaire pertaining to their musical backgrounds, medical backgrounds (with respect to any permanent hearing loss), experience with listening tests and knowledge of bells[12].

The subjects' pitch discriminating abilities were tested using the test constructed as described in section 5.2 above. They were asked to define how the second tone compared to the first. If it was higher than the first tone, an *h* was to be written in the space provided, if the second tone was lower, an *l* and if the two tones were the same an *s* was required. First the four trial tasks were attempted to ensure that they understood what was required, and then the pitch discriminating test was performed.

Subjects were then given an opportunity to become familiar with the knobs on the signal generator and allowed to practice tuning it to notes played on the piano. Usually, middle C, one very high note and one very low note were used. Once they were a little more familiar with the device, the two trial pitch matching tasks were presented. The purpose of these preliminary matches was not just to further familiarise the subjects with the equipment, but to draw their attention to the necessity of matching both the pitch chroma and pitch height correctly.

The twenty-four pitch matching tasks were then attempted. Each track on the CD with a bell stimulus lasted about 50 s. The subjects were however given as long as they pleased to make the match (i.e. the track was repeated if necessary). Between each matching attempt the reading on the signal generator was taken and the subject was required to turn the volume and pitch knobs right down. The digital display of the signal generator was hidden from the view of the subject to avoid number matching in stead of pitch matching.

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[12] This questionnaire is available in Appendix 2.

Subjects were advised to first listen to the bell sound before beginning the matching process, and to turn down the volume on occasion, if necessary to listen to the bell sound on its own. Some subjects preferred to have the CD paused once they had listened to the test tone a few times, and then they tuned the generator to their memory of the sound. They checked their setting for fine tuning afterwards with the bell sound playing. This approach was only possible with musically trained subjects. Because the pitches of adjacent test tones were substantially different it was not necessary to introduce a random offset into the frequency dial as done by Eggen (1986).

The whole procedure lasted between twenty minutes and an hour, depending on how long the subject needed to make the matches. The pitch matching tasks were performed on an individual basis. Where possible the pitch discriminating test was administered to groups of between two and five people.

5.4 Results

5.4.1 Subject Evaluation

The subjects were mostly volunteers drawn from the staff and students of schools in Grahamstown and of the Rhodes University music and physics departments.

Subjects with between five and ten years of practical music training were classed as musicians, those with more than ten years were classed as experienced musicians, and those with less than five years were classed as non-musicians. The number of subjects in each group is tabulated according to age in **Table 7.**

Table 7 Age and degree of musical training of the subjects.

Musicianship	**Age 10 - 20**	**Age 20 - 30**	**Age 30 - 50**	**Age 50 and above**	**Total**
Non-musicians	1	2	2	2	7
Musicians	3	1	2	2	8
Experienced Musicians		3	4	1	8
Total	4	6	8	5	**23**

Two of the subjects had perfect pitch. None of the subjects had taken part in any psychoacoustic pitch matching experiments before and none of the subjects had any knowledge of the acoustics of bells.

5.4.2 Pitch Discrimination

All the subjects were capable of hearing to an accuracy of 5 Hz at a starting frequency of 440 Hz. Comparisons between the pitch discriminating abilities of persons falling into the categories non-musicians, musicians and experienced musicians were made. The results of these comparisons are given in **Table 8** and the details of the statistical analyses are presented in Appendix 4.1.

Table 8 Comparisons of pitch discrimination for non-musicians, musicians and experienced musicians.

Degree of Musical Training	**Number of Subjects Tested**	**Mean number of correct pitch discriminations out of 30 trials**
Non-musicians	7	22.1 a *
Musicians	8	26.5 b
Experienced musicians	8	26.4 b
General mean		25.2
Standard error of a single observation		3.125

* Means followed by the same letter are not significantly different

In the overall analysis of variance the variance ratio (F) for 2 and 20 degrees of freedom was significant at the 5% probability level. Using the t test it was shown that the mean obtained for non-musicians was significantly lower ($P < 0.05$) than the means obtained for both categories of musically trained subjects. There was no significant difference in pitch discrimination ability between the musicians and experienced musicians.

5.4.3 Pitch Matching Experiment

Incidences of spectral pitch perception needed to be separated from those of virtual pitch perception. Any match more than one quarter of a semitone different from the strike note or one of the notes an octave up or down from the strike note was not considered a valid match of the strike note. This is reasonable because all the subjects proved in the test of pitch discrimination that they were capable of hearing to a greater precision than this.

A number of comparisons were made using the data from the pitch matching experiment. These are presented in the next four sections.

5.4.3.1 Comparison of non-musicians, musicians and experienced musicians abilities to match the residue pitch of the four bells

The number of successful matches with the strike note or secondary strike note made by the non-musicians, musicians and experienced musicians was compared. The results of these comparisons are presented in **Table 9**.

Table 9 Number of successful matches of the strike or secondary strike note made by non-musicians, musicians and experienced musicians.

Musical Training	**Number of subjects tested**	**Mean number of correct pitch matchings out of 24 trials**
Non-musicians	7	2.1 a *
Musicians	8	14.9 b
Experienced musicians	8	11.8 b

* Means followed by the same letter do not differ significantly

The differences between the non-musicians and the two categories of musicians was so great that no statistical analysis was needed to highlight them. As expected the variance ratio for the overall analysis of variance was highly significant ($P < 0.01$) and a comparison of means using the two sided t test for 20 degrees of freedom showed no significant difference between musicians and experienced musicians. The details of the statistical analyses performed on the pitch matching data is given in Appendix 4.2.

15.4.3.2 Comparison of the incidence of virtual pitch perception for the four bells

Individual subjects often seemed to have difficulty with perceiving the residue pitch of a particular bell. In order to establish whether the virtual pitch of any one bell was more illusive than the others a comparison was done. For this comparison of the ease with which the virtual pitch of the four bells was perceived all the successful matches of virtual pitch were combined. The observed frequencies of correct identification were compared with the expected frequencies using the chi square (χ^2) test. The null hypothesis for this comparison was that there was no significant difference between the ease with which the virtual pitches of these four bells was perceived. The results are displayed in **Table 10**.

Table 10 Observed and expected correctly identified virtual pitch perceptions for the four bells.

Bell Number	Number of virtual pitch identifications	
	Observed	**Expected**
2	63	57
5	54	57
8	44	57
10	67	57

Calculated χ^2 3df = 5.509 NS
Tabulated χ^2 3df (P = 0.05) = 7.815

The non-significant result obtained for this chi square test indicates that there is no justification for rejecting the null hypothesis. There is therefore no significant difference between the ease with which the virtual pitches of the four bells was perceived by the subjects.

This is interesting because the inner harmonies of bell 2 are fairly different to those of the other bells (see chapter 6 for spectral analyses of the bell sounds). Bell 2 was cast by

Whitechapel Bell Foundry in 1996, whereas the others were cast by Warners in 1878.

7.4.3.3 Comparison of the Incidences of virtual pitch perception for ringing versus chiming bell sounds

The ability of the test group of subjects to correctly identify virtual pitch for ringing and chiming bell sounds was investigated. For this chi square test the null hypothesis was that correct virtual pitch identification could be done equally well for ringing and chiming bells. The alternative hypothesis was that the incidence of virtual pitch identification differs for ringing and chiming bell sounds. The results are presented in **Table 11.**

Table 11 Observed and expected frequencies of correct virtual pitch identification for ringing and chiming bells.

	Frequency of correct identification of virtual pitch	
	Observed	**Expected**
Ringing bells	118	114
Chiming bells	110	114

Calculated χ^2 1df = 0.28 NS
For probability (P) = 0.05
Tabulated χ^2 = 3.841

The calculated value of 0.28 is less than the tabulated value of 3.841 required for statistical significance at the 5% level of probability. The non-significant result for the χ^2 test indicates that the null hypothesis cannot be rejected.

There is therefore no evidence to support the alternative hypothesis that identification of the virtual pitch of a bell sound could be more difficult depending on the manner in which the bell was sounded.

5.4.3.4 Octave error judgments of the strike note

An octave error judgments occurred when the subjects matched the pitch of the signal generator to an octave above or below the strike note. The number and types of octave error judgments were compared for :

1.) Ringing or chiming bell sounds

2.) Non-musicians, musicians and experienced musicians

The null hypothesis was that octave error judgments were equally likely for both the ringing and chiming sounds and that octave errors are independent of musical training. **Table 12** shows a comparison of the octave error judgments made for the two types of bell sounds (ringing and chiming).

Table 12 Comparison of the octave error judgments made for the ringing and chiming sounds of the four bells.

Octave identification	Frequency of octave identification		Total
	Ringing bells	**Chiming bells**	
Octave up error	5	1	6
Octave down error	27	14	41
Correct octave identification	86	95	181
Totals	118	110	228

The probabilities of achieving the frequency of octave up errors shown in **Table 12**, if octave up and down errors have equal probability ($P = 0.5$), were calculated using the cumulative binomial distribution.

The calculated probabilities of achieving the octave up errors shown in **Table 12** are so small ($P < 0.001$) that the null hypothesis that octave up and octave down errors are equally likely must be rejected. An alternative hypothesis, that octave down errors occur with a greater frequency must therefore be adopted.

Table 13 shows the results from the comparison investigating the effects of musical training on octave error judgments in the experiment.

Table 13 Frequency of octave error judgments and correct matches of the strike note made by musically trained and untrained subjects.

	Extent of Musical Training			**Total**
	Non-musicians	**Musicians**	**Experienced musicians**	
	Frequency of errors			
Octave up error	3	0	3	6
Octave down error	2	24	15	41
Correct octave identification	10	95	76	181
Matches of the strike note	15	119	94	228
Total number of matching attempts	168	192	192	552
	Percentages			
Correct identification of the strike note	8.9	62.0	49.0	41.3

The sharp contrast between the percentage of correct matches of the strike note for the musically trained and untrained subjects highlights once again the poor performance of the non-musicians in identifying the strike notes of the bells.

An interesting trend is the considerably higher frequency of octave down errors by comparison with octave up errors. Once again the calculated probabilities of obtaining these frequencies leads to a clear rejection of the null hypothesis that for trained musicians octave up and octave down errors are equally likely. The probabilities calculated for these observations, using the cumulative binomial distribution, are extremely low ($P < 0.001$).

5.5 Discussion and Conclusions

As the ability to discriminate small differences in pitch is not improved with musical training (Seashore, 1964), there was no expected difference between the amateur and experienced musicians. The inability of the non musicians to discriminate pitches as well was not as a result of less training, but rather, this inability was probably one of the causes of their not having acquired more musical training.

Although the non-musicians did not perform as well in the test of pitch discrimination, their almost inability to perform the pitch matching tasks was not expected. A possible reason for the poor performance of the non-musicians could be the timbral differences between the test and comparison tones. Many of the non-musicians explicitly complained about the differences in tone quality between the test and comparison tones. This complaint was not expressed by any of the musicians. This stands to reason, as musicians, and particularly the experienced musicians would have had plenty of practice with tuning their instruments to other instruments in either orchestral or chamber ensembles. This possible explanation for the poor performance of the non-musicians, despite their adequate pitch discriminating ability is supported by experimental evidence discussed by Pitt (1994) and Radvansky (1995)[13]. Another complaint received from the non-musicians concerned the difference between the piecewise presentation of the bells sounds as opposed to the continuous nature of the comparison tone.

Many of the non-musicians were incapable of distinguishing between a consonant match, and a match of a unison. Often they would sing or whistle the strike note and then tune the generator to a major third, or perfect fifth from what they were singing. These "consonant" matches were often made with the notes of the piano and with the trial pitch matching sounds as well (both of which were harmonic). One non-musician even insisted that a unison had been obtained when the signal generator was a tritone away from the pitch of one of the harmonic trial test tones!

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[13] Refer to section 2.3.4, page 32-33 for a description of these experiments.

The above discussion suggests that only people with musical training should be used as subjects in experiments of this nature.

One of the most obvious differences between the sounds of ringing and chiming bells is the intensity of the sound. The ringing sounds were therefore not heard at as low a sensation level as the chiming sounds. This fortunately did not result in an increased incidence of analytic perception for the ringing sounds, as was initially feared.

An unexpected result, was the greater frequency of downward octave error judgments. It is possible that a flaw in the experimental procedure, rather than a perceptual tendency could have lead to this trend. Three possible explanations are:

1.) The frequency dial of the signal generator was always returned to the 0 Hz position between each task. If the subjects are likely to settle for the first appropriate match encountered as the dial is turned this could account for the more frequent downward octave errors. If the tasks were all started with the dial at a high frequency, the result may have been the opposite.
2.) Many subjects seemed to find it easier to match the bell pitch with lower frequencies from the signal generator. Musicians often made the match at a low frequency, and then adjusted to the correct octave once the tone chroma had been successfully matched.
3.) The younger subjects in particular seemed to associate low intensity with low pitch. The downward octave error judgments could be linked to this associative tendency.

Although octave error judgments seem at a glance to be more numerous for ringing than for chiming bell sounds, there were not enough data points to establish any statistically significant differences. Many more observations would be needed in order to establish this.

The tenor bell had a very prominent secondary strike note. Matches of the secondary strike note were almost as frequent as for the primary strike note.

The subjects' memory of the previous match often influenced their responses to the subsequent bell stimulus. Some subjects were unable to escape from one particular frequency for up to four consecutive matches. The frequency of one match seemed to draw their attention to partials in the next sound which were not too different in frequency. The two subjects with perfect pitch claimed to recognise the stimuli at the second repetition.

The generally quite high incidence of analytic perception could have several causes. It could be attributed to the fact that the test and comparison tones could be heard simultaneously. This meant that many of the musicians used beats to guide them in the tuning process, which lead to many analytic matches which were typically between a minor 2nd and minor 3rd away from the strike note. It became clear that when musicians tuned to their memory of the sound (tuned the generator while the bell was not sounding) they found the strike note more easily. Shortening the test tones[14] and presenting them in some sort of musical context may have facilitated perception of the strike note.

Three changes that would be implemented in the event of a continuation of this research would be:

1.) Including some sort of musical context to facilitate subjective pitch perception.
2.) Using only trained musicians as subjects
3.) Ensuring that the test and comparison tones can not be heard simultaneously.

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[14] Moore (1987) found that analytic perception of inharmonic partials became more difficult for sounds of shorter duration. See page 29 of this thesis.

Chapter Six - Spectral Analyses of the Ringing and Chiming Sounds of Four Bells

6.1 Introduction

Since this chapter presents an analysis of the sounds of four bells, it seems appropriate to briefly mention the physical characteristics of the bell that give rise to their unique spectra.

The relationships between the partial frequencies of an ideally tuned bell have been given in the introduction to chapter 3. These partials each arise from a different *mode* of vibration (vibrational shape) of the bell, just as the overtones present in the sound of a piano string, for example, each have their origin in one of the vibrational shapes of the string. The vibrational shapes of stretched strings are often illustrated as in **Figure 25.**

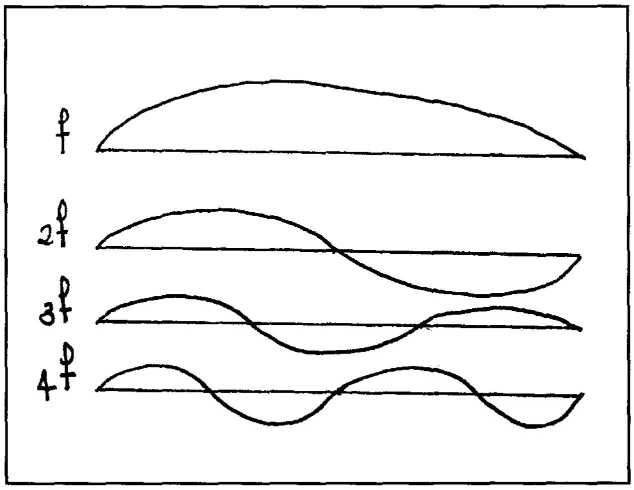


Figure 25 Shapes of the vibrational modes of a stretched string.

The portions of the string that remain stationary are called the *nodes* and the portions that move the most are called the *antinodes*. If the string is plucked at the node of a particular mode, this mode will have less energy.

When caused to vibrate, bells also deform in well defined ways. The shapes of the vibrational modes of a bell are necessarily more complex than those of a stretched string due to the more sophisticated geometry of a bell. Because the bell is a three dimensional object the *nodes* are no longer at points as in the case of the stretched string, but are now along lines or curves. There are two types of nodal lines. The latitudinal nodes form circles around the circumference of the bell at various heights. The longitudinal nodes begin at the top of the bell and go vertically downward on either side. The nodes of the vibrational modes responsible for the first five partials of a bell are illustrated in **Figure 26.**

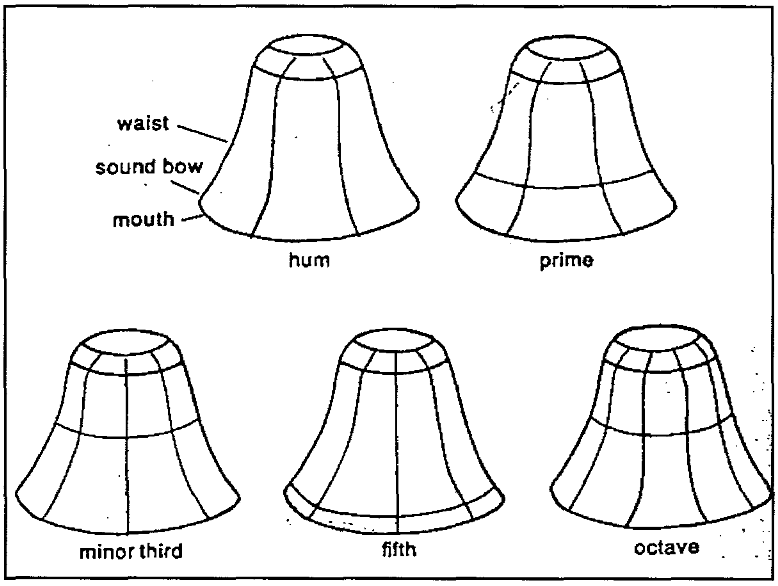


Figure 26 Nodes of the vibrational modes responsible for the first five partials of a bell (Rossing, 1984, p. 442)

Each partial frequency is therefore as a result of a very specific deformation of the bell shape. These deformations are best illustrated by viewing the bell from below (**Figure 27**) and by viewing a vertical slice through the centre of the bell (**Figure 28**).

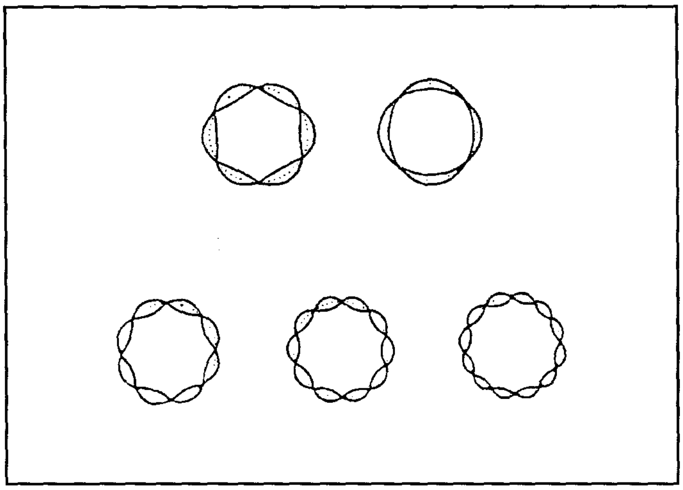


Figure 27 A view from the bottom of the bell showing shapes of deformation and the longitudinal nodes (Fletcher and Rossing, Fig. 21.3 p. 579).

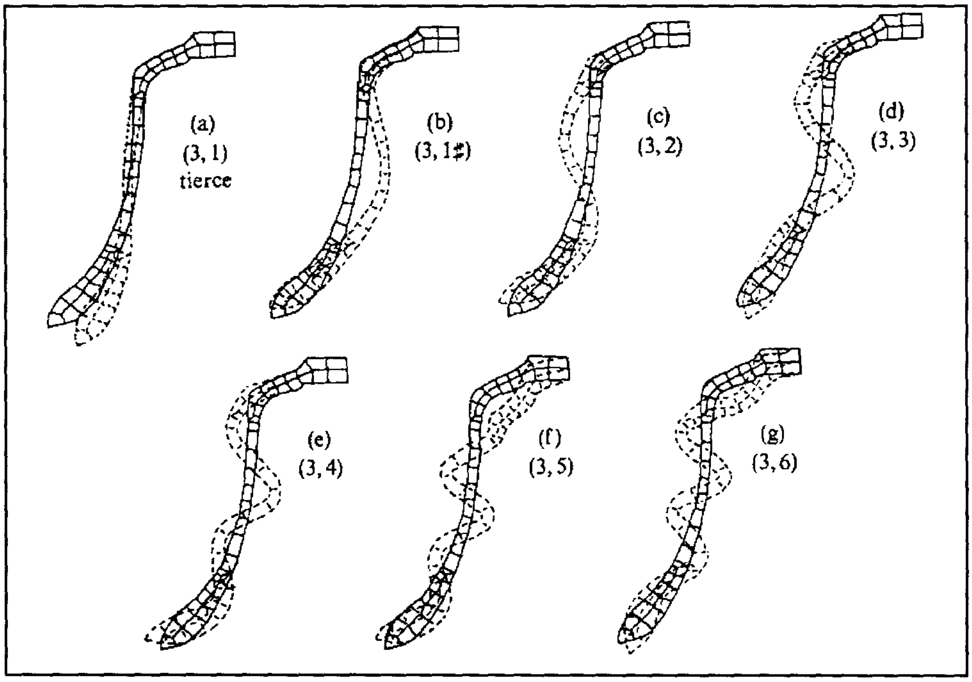
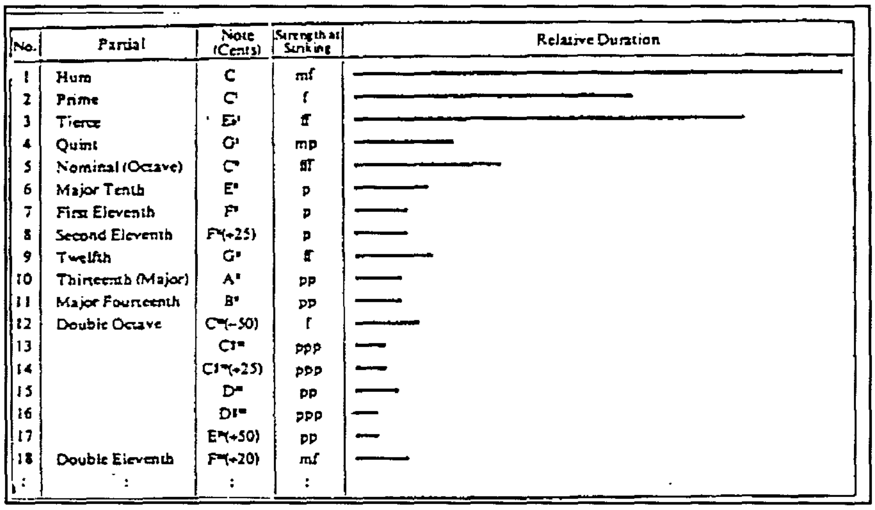


Figure 28 Deformation of the shape of a bell viewed on a vertical slice through the centre. This shows the latitudinal nodes (Fletcher and Rossing, Fig. 21.5 p. 581).

The vibrational modes have been divided into ring driven and shell driven modes. The ring driven modes have nodal circles at the *waist* (about half way up the bell) whereas the shell driven modes have their nodal circles at the *soundbow* (the bottom of the bell). The relative pitches of partials within one of these groups has been found to remain more or less the same during the tuning process (Lehr, 1986).

Partials of bell sounds are not all equally loud at the strike and do not all decay at the same rate. The nominal, tierce and prime respectively are the loudest partials soon after the strike, but die away more rapidly than the hum which is weak at the outset, but because of its slow decay rapidly becomes the strongest component (Slaymaker and Meeker, 1954). The quint is a very weak partial from the time of the strike (mainly because the clapper is positionsed close to the soundbow, which is on the nodal circle of the mode shape of the quint) and decays very rapidly.

The relative strengths at striking and relative durations of the partials are illustrated in **Figure 29**.



No.	Partial	Note (Cents)	Strength at Striking	Relative Duration
1	Hum	C	mf	
2	Prime	C'	f	
3	Tierce	E♭'	ff	
4	Quint	G'	mp	
5	Nominal (Octave)	C''	fff	
6	Major Tenth	E''	p	
7	First Eleventh	F''	p	
8	Second Eleventh	F''(+25)	p	
9	Twelfth	G''	ff	
10	Thirteenth (Major)	A''	pp	
11	Major Fourteenth	B''	pp	
12	Double Octave	C'''(-50)	f	
13		C♯'''	ppp	
14		C♯'''(+25)	ppp	
15		D'''	pp	
16		D♯'''	ppp	
17		E'''(+50)	pp	
18	Double Eleventh	F'''(+20)	mf	
:	:	:	:	

Figure 29 Relative strengths and durations of the partials of a bell (Lehr and Ayres, unpublished, p. 2-19).

A bell with a longer decay time is often considered to be of better quality than one for which the partials decay more quickly. Typical durations of the five lowest partials of a well cast bell with a strike note of middle C, weighing about two tons are give in **Table 14**.

Table 14 Typical durations of the five most important partials of a bell (Lehr and Ayres, unpublished).

Partial Name	**Duration**
Hum	100
Prime	55
Tierce	75
Quint	20
Nominal	30

The resonance of the bell as a whole takes a finite amount of time to reach a maximum. This is because the disturbance takes a finite amount of time to propagate from the point of impact with the clapper, through the metal to the other parts of the bell. The time taken to reach the maximum also differs for the different modes of vibration.

6.2 Procedure

6.2.1 Selection and Preparation of Waveforms for Analysis

The waveforms of the recorded sounds from bells number 2, 5, 8, and 10 were analysed in order to establish the partial frequencies and observe the relative intensities and decay times of the partials. Waveforms exhibiting *clipping* or in which the bell was struck twice by the clapper were rejected for the purposes of this analysis.

Two acceptable waveform files were chosen for each bell, one of the ringing and one of the chiming sound. These were divided into smaller files containing 8192 points each and converted into ASCII format using CoolEdit. The sample rate chosen for the recordings was 44.1 kHz (CD sampling rate). Each ASCII file therefore represented a portion of the waveform that was about 0.185 s long.

6.2.2 Fourier Analysis of the Waveforms

The magnitudes of the samples and the sample spacing were used as input for a Fortran program[15] which performed the Fourier Transforms. The program produced an ASCII file containing 8192 magnitudes with their corresponding frequencies. The results from the transforms were too large for the conventional spreadsheets to import or plot. A linux based plotting program called *wip* was therefore initially used to plot these results. The unimportant parts of the frequency range were then discarded in order that a spreadsheet called *QuattroPro* could be used for further analyses and comparisons.

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[15] This Fortran program was written by Prof G. Poole of the Rhodes Physics Department for use in his research into upper atmosphere physics.

6.3 Results

For each of the four bells the following is provided:

1.) ringing and chiming spectra for t = 0 to 0.2 s after clapper impact
2.) A table giving the frequencies, frequency ratios to the prime, note names and intensities of the partials for both the ringing and chiming bell sounds (from 0 to 0.2 s after the strike).

Frequencies were rounded off to the nearest 1 Hz and had an uncertainty of ± 2.5 Hz. Intensities were rounded off to one decimal place. Only comparisons of relative intensity were made. No comparison of absolute intensity was possible as the microphone attenuation and recording level were changed for each bell recording. No calibration tones of known intensity were recorded at the time and hence the absolute intensities are unknown.

At first only partials with intensities of five or above for the ringing sound were tabulated. The hum and quint partials of bells 10 and 8 were softer than this, therefore the spectra were scrutinised a second time in search of these partials. The hum partials, although very soft, were found. The quints were however indistinguishable from the noise, i.e. they were less than about 0.1.

The equally tempered scale with A4 = 440 Hz was used for comparison with the partials. Diatonic notes of the minor key in which the strike note is the tonic were used where possible. Plus (+) and minus (-) signs were allocated to partials that were 1/4 of a semitone or more sharp or flat of the equally tempered note. The convention of using numbers to denote the octave of the note and taking middle C as C4 was used. The strike notes of the four bells are, beginning with the lowest (bell 10), E♭4, G4, C5 and F5.

Long term decay of the partials was not calculated because background noises such as cars, hooters, people, pigeons and the sound of the howling gale outside were substantially louder than the bell sound once it had decayed for longer than about three seconds.

6.3.1 Bell number 10 (Tenor)

Spectra of the ringing and chiming sounds of the tenor bell in the first 0.2 seconds after clapper impact are given in **Figures 30** and **31.**

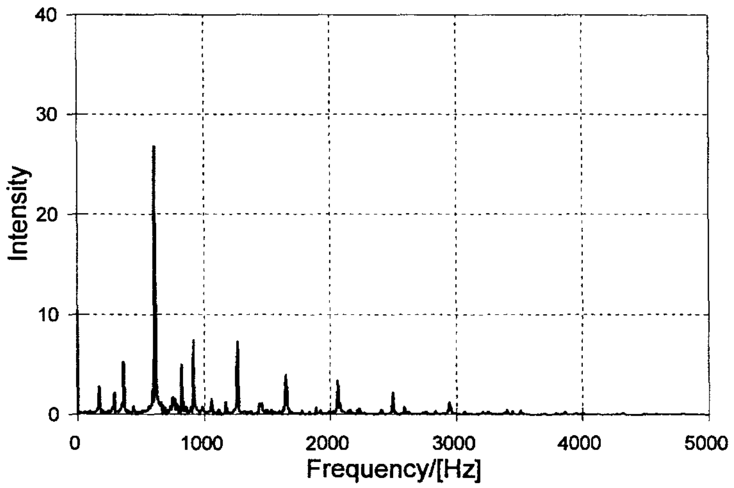

Figure 30 Spectrum of the chiming sound of the tenor bell from 0 to 0.2 s after clapper impact.

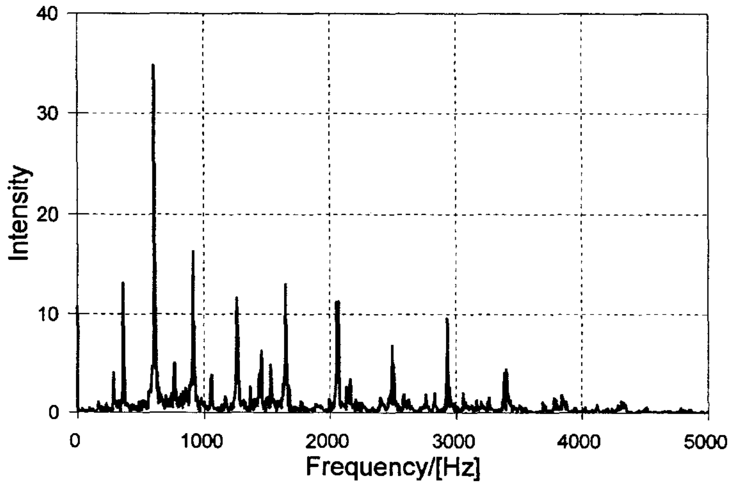

Figure 31 Spectrum of the ringing sound of the tenor bell from 0 to 0.2 s after clapper impact.

Table 15 gives the frequencies of the partials for the tenor bell, the notes to which these frequencies correspond, as well as their initial amplitudes for the ringing and chiming spectra.

Table 15 Partial frequencies, frequency ratios, corresponding note names and initial intensities of the partials for the ringing and chiming sounds of the tenor bell.

Partial number	**Partial Name**	**Frequency (*f*)**	**Ratio of *f* to prime**	**Note Name**	**Ringing intensity (Ri)**	**chiming intensity (Ci)**	**% (Ci/Ri)**
1	Hum	173	0.60	F3	1.3	0.5	26.0
2	Prime	**286**	**1.00**	D♭4 +	4.1	2.2	53.7
3	Tierce	362	1.27	G♭4 -	13.2	8.3	62.9
4	Nominal	610	2.13	E♭5 -	34.9	26.8	76.8
5	Major tenth	769	2.69	G5 -	6.1	1.7	27.9
6	Twelfth	918	3.21	B♭5 -	16.4	7.4	45.1
7	Double Oct	1269	4.44	E♭6 +	11.7	7.3	62.4
8		1458	5.10	G♭6 -	6.3	1.2	19.0
9		1534	5.36	G 6-	4.9	0.5	10.2
10		1653	5.78	A♭6	13.1	5.0	38.2
11		2063	7.21	C 7 -	11.3	3.4	30.1
12	Triple Oct	2495	8.72	E♭7	6.9	2.6	37.7
13		2507	8.77	E♭7 +	5.0	0.3	6.0
14		2933	10.26	G♭7 -	9.6	0.6	6.3
15		3387	11.84	A♭7 +	4.1	0.3	7.3
16		3403	11.90	A 7 -	4.4	0.4	9.1

6.3.2 Bell number 8

Figures 32 and **33** show the chiming and ringing spectra of bell number 8 for the waveform between 0 and 0.2 s after clapper impact.

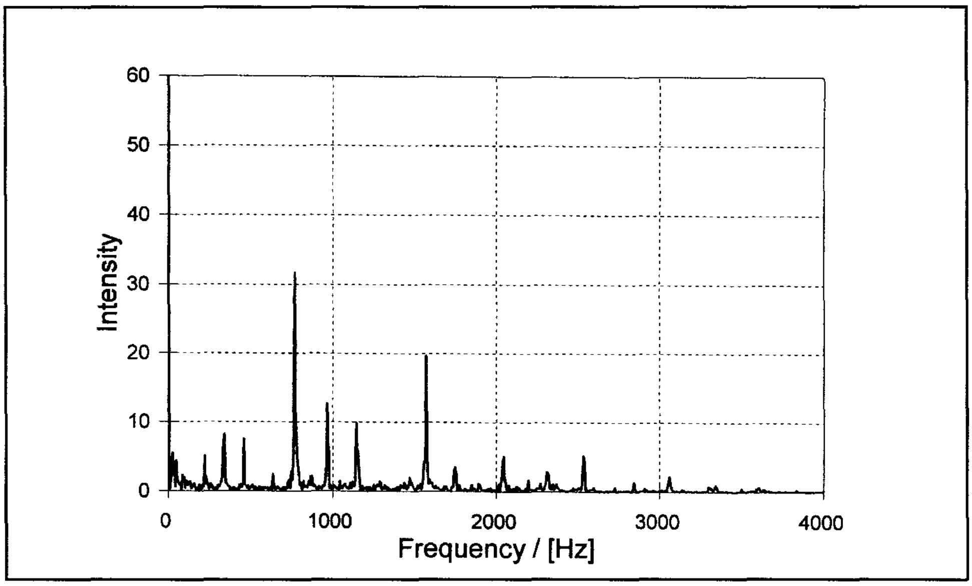

Figure 32 Spectrum of the chiming sound of bell number 8 from 0 to 0.2 s after clapper impact.

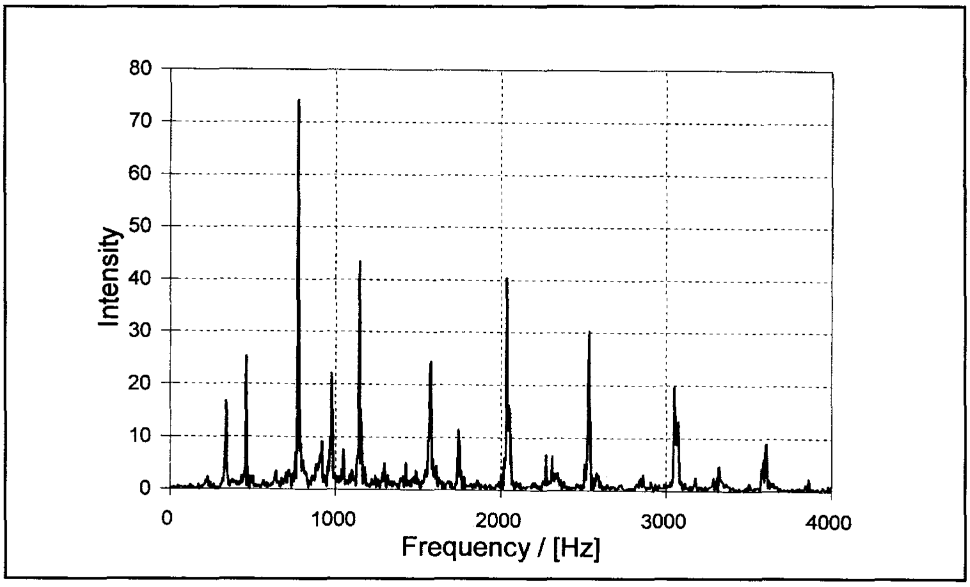

Figure 33 Spectrum of the ringing sound of bell number 8 form 0 to 0.2 s after clapper impact.

Table 16 gives the frequencies of the partials for bell number 8, the notes to which these frequencies correspond, as well as their initial amplitudes for the ringing and chiming spectra.

Table 16 Partial frequencies, frequency ratios, corresponding note names and initial intensities of the partials for the ringing and chiming sounds of bell number 8.

Partial number	**Partial Name**	**Frequency (*f*)**	**Ratio of *f* to prime**	**Note Name**	**Ringing intensity (Ri)**	**chiming intensity (Ci)**	**% (Ci/Ri)**
1	hum	226	0.66	A3 +	2.6	0.7	37.1
2	prime	**340**	**1.00**	F4 -	18.6	8.7	46.8
3	tierce	495	1.45	B4	25.5	7.6	29.8
4	nominal	772	2.27	G5	74.8	31.2	41.7
5	eleventh	1047	3.08	C6	7.7	1.5	19.5
6	twelfth	1145	3.37	D6 -	43.8	9.6	21.9
7	twelfth	1155	3.39	D6 -	12.4	6.3	50.8
8	double oct.	1577	4.64	G6 +	24.3	19.4	79.8
9		1744	5.13	A6 -	11.8	3.2	27.1
10		2036	5.99	B6 +	40.2	4.2	10.4
11		2047	6.02	C7 -	16.4	5.4	32.9
12		2273	6.69	C♯7 +	6.1	1.3	21.3
13		2311	6.80	D7 -	6.2	2.1	33.9
14		2533	7.45	E7 -	30.2	5.3	17.5
15	triple oct	3051	8.97	G7 -	19.1	2.1	11.0
16	triple oct	3073	9.04	G7 -	13.8	2.1	15.2
17		3581	10.53	A7 +	4.3	0.2	4.7
18		3591	10.56	A7 +	6.1	0.5	8.2
19		3602	10.59	A7 +	9.6	0.7	7.3

6.3.3 Bell number 5

The chiming and ringing spectra of bell number 5 are shown in **Figures 34** and **35.**

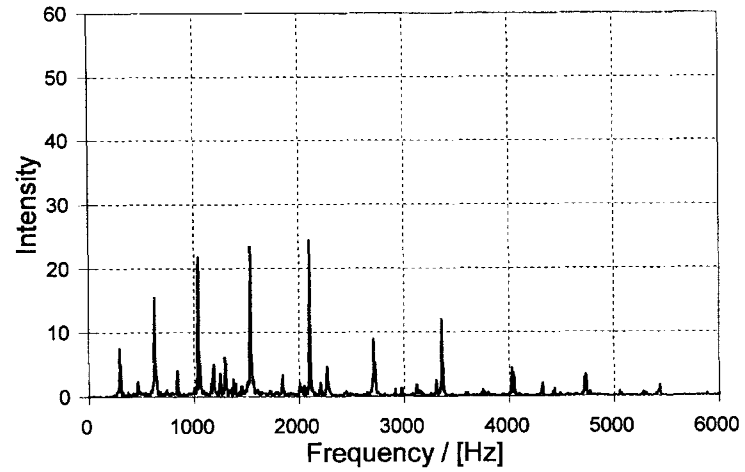

Figure 34 Spectrum of the chiming sound of bell number 5 from 0 to 0.2 s after clapper impact.

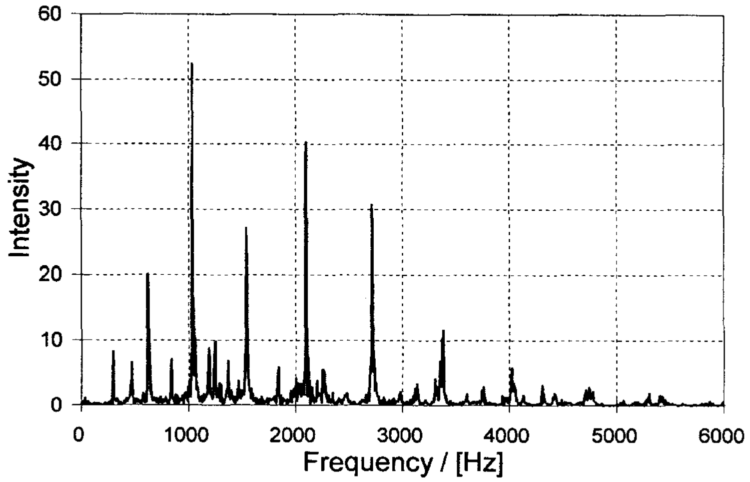

Figure 35 Spectrum of the ringing sound of bell number 5 from 0 to 0.2 s after clapper impact.

Table 17 gives the frequencies of the partials for bell number 5, the notes to which these frequencies correspond, as well as their initial amplitudes for the ringing and chiming spectra.

Table 17 Partial frequencies, frequency ratios, corresponding note names and initial intensities of the partials for the ringing and chiming sounds of bell number 5.

Partial number	**Partial Name**	**Frequency (*f*)**	**Ratio of *f* to prime**	**Note Name**	**Ringing intensity (Ri)**	**chiming intensity (Ci)**	**% (Ci/Ri)**
1	hum	297	0.63	D4 +	8.4	7.5	89.3
2	prime	**470**	**1.00**	B♭4 +	6.7	2.1	31.3
3	tierce	621	1.32	E♭5	20.8	15.2	73.1
4	quint	843	1.79	A♭5 +	7.6	4.3	56.6
5	nominal	1037	2.21	C 6 -	52.4	21.7	41.4
6		1058	2.25	C6 +	10.9	8.4	77.1
7	major tenth	1188	2.53	D6 +	8.7	4.2	48.3
8	1st eleventh	1247	2.65	E♭6	9.8	3.1	31.6
9	2nd eleventh	1371	2.91	F6 -	6.3	2.1	33.3
10	twelfth	1539	3.27	G6 -	27.9	23.5	84.2
11	thirteenth	1841	3.92	A6 +	5.4	3.2	59.3
12	double oct	2101	4.47	C7 +	40.7	24.6	60.4
13		2257	4.80	C♯7 +	5.4	4.1	75.9
14		2268	4.83	C♯7 +	5.2	4.2	80.8
15	double eleventh	2716	5.78	F7 -	30.9	8.6	27.8
16		3381	7.19	A♭7 +	11.5	0.4	3.5
17		4024	8.56	B7 +	5.4	4.1	75.9

6.3.4 Bell number 2

Figures 36 and **37** show the frequency spectra of the chiming and ringing sounds of bell number 2.

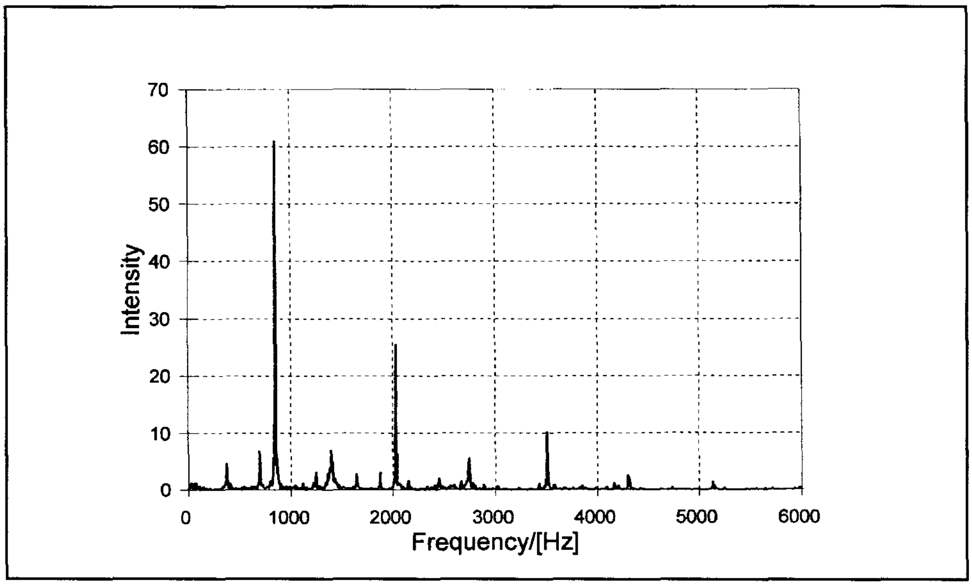

Figure 36 Spectrum of the chiming sound of bell number 2 from 0 to 0.2 s after clapper impact.

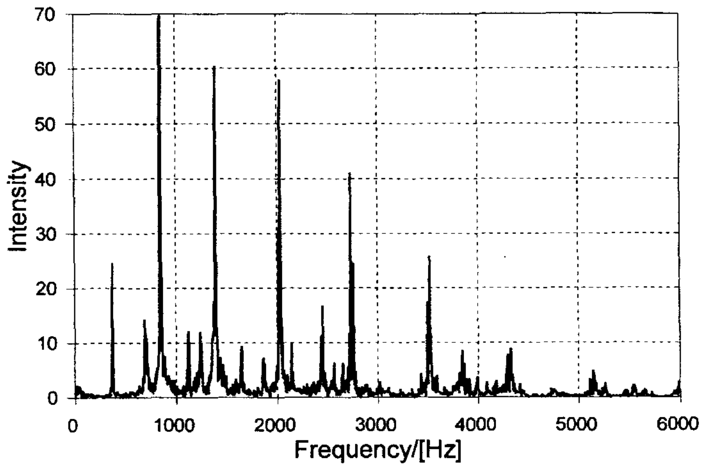

Figure 37 Spectrum of the ringing sound of bell number 2 from 0 to 0.2 s after clapper impact.

Table 18 gives the frequencies of the partials for bell number 2, the notes to which these frequencies correspond, as well as their initial amplitudes for the ringing and chiming spectra.

Table 18 Partial frequencies, frequency ratios, corresponding note names and initial intensities of the partials for the ringing and chiming sounds of bell number 2.

Partial number	**Partial Name**	**Frequency (*f*)**	**Ratio of *f* to prime**	**Note Name**	**Ringing intensity (Ri)**	**chiming intensity (Ci)**	**% (Ci/Ri)**
1	hum	373	0.54	F♯4 +	24.5	4.1	16.7
2	prime	**696**	**1.00**	F4	15.8	6.1	38.6
3	tierce	853	1.23	A♭4 +	69.9	61.9	88.6
4	quint	1123	1.61	D♭5 +	12.8	1.5	11.7
5	quint	1242	1.78	E♭5	12.7	3.2	25.2
6	nominal	1394	2.00	F5	60.8	7.3	12.0
7		1653	2.38	A♭5	9.9	2.2	22.2

8		1868	2.68	B♭5	7.6	3.8	50.0
9	twelfth	2026	2.91	C6 -	31.9	25.9	81.2
10	twelfth	2107	3.03	C6 +	4.3	0.6	14.0
11	thirteenth	2150	3.09	D♭6 -	10.3	1.6	15.5
12	thirteenth	2160	3.10	D♭6 -	4.3	0.3	7.0
13	fourteenth	2441	3.51	E♭6 -	11.4	0.8	7.0
14	fourteenth	2457	3.53	E♭6 -	16.4	2.9	17.7
15		2722	3.91	F6 -	11.6	1.1	9.5
16		2738	3.93	F6 -	41.4	5.4	13.0
17	Double octave	2749	3.95	F6 -	21.8	5.2	23.9
18	Double octave	2760	3.97	F6 -	24.8	2.7	10.9
19		3500	5.03	A6 -	17.2	10.4	60.5
20		3521	5.06	A6 -	25.9	0.1	0.4
21	Double eleventh	3846	5.53	B6 -	8.2	0.7	8.5
22		4296	6.17	C7 +	7.4	0.4	5.4
23		4326	6.22	C7 +	8.3	0.6	7.2

6.4 Discussion and Conclusions

The methods used to find the spectra of the bell sounds seemed rather like "re-inventing the wheel" considering that there are devices that exist for the sole purpose of performing Fourier transforms on sound. In all, five different computers had to be used just to convert the DAT recordings into something that could be used as input for the program that performed the transforms, after which a further two computers had to be used in order to view the results. This number of data conversions and transfers slowed the process considerably and could have been avoided with the correct equipment. These sorts of difficulties unfortunately often do not become apparent until the procedure is well under way.

6.4.1 Comparison of the Spectra of the Ringing and Chiming Bell Sounds

The most obvious difference between the ringing and chiming sounds of the bells is the overall intensity of the sound. The ringing bell sounds are louder than the chiming sounds. The upper partials of the chiming sound (those higher than the double octave) are generally below 5 and often below 1 in intensity. The use of calibration tones to establish the intensity corresponding to the threshold of hearing would have been useful in establishing which sounds were still audible.

6.4.2 Comparative Discussion of Spectral and Pitch Matching Results.

Considering the inharmonicity of the partials of all these bells, it is not surprising that the subjects experienced analytic perception with such ease and frequency. The low level of presentation, which was necessary in order to minimise analytic perception may have caused the higher harmonics to drop below the threshold of hearing. This would have lead to an absence of pseudo high frequency noise even in the ringing sounds, and could account for the absence of the possible octave error judgments predicted by Lehr and Ayres (unpublished).

In order to rectify this problem of the absence of high frequency noise, the level of sensation would have to be increased, which in turn would increase the incidence of analytic perception. Some sort of musical context would have to be used in performing experiments using louder bell stimuli if a reasonable level of subjective pitch perception is to be maintained.

The strike notes and prime and tierce frequencies of the four bells are shown in musical notation in **Figure 38**.

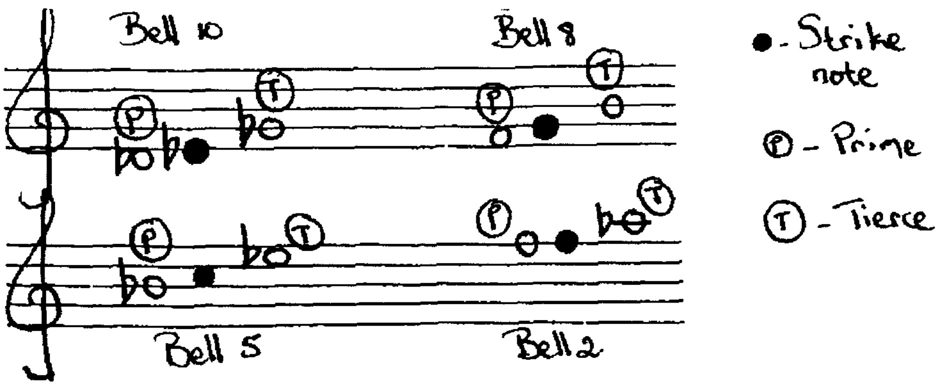

Figure 38 Strike notes and prime and tierce partials of the four bells written in musical notation.

The intervals between the strike note and the prime for bells 5, 8 and 10 is about a major 2nd. The tierce partials are mostly between a minor and major third away from the strike note. This could account for the tendency of the subjects to match the generator to intervals of between a minor 2nd and minor 3rd away from the strike notes.

The tenor bell had a particularly strong *metallic fourth* or *secondary strike note*. None of the other bells seemed to exhibit this characteristic. Inspection of **Table 15** reveals that partials number 7, 10, 11 and 12 are fairly well tuned to the notes E♭6, A♭6, C7 and E♭7, which are in the frequency ratios 3:4:5:6 when compared with the perceived metallic fourth at A♭4. Most of these frequencies are below 2000 Hz, whereas the corresponding partials of the other bells are mostly above 2000 Hz. Perception of the subjective pitch is more difficult if the frequencies contributing to that pitch are above about 2000 Hz (Roederer, 1995). Therefore it is to be expected that only the tenor bell could sustain the perception of the secondary strike note.

Conclusions

The literature reviews contained in Chapters 1, 2 and 3 showed that the field of psychoacoustics has gone a long way towards explaining the mechanisms of human pitch perception, not only in a qualitative sense, but also quantitatively in the form of modern theories such as the optimum processor and virtual pitch theories. Most of the experimental work however has concentrated on pitch perception on spectrally simple sounds, usually with only two or three spectral components. Although the atomistic approach of using simple stimuli in experiments is a necessary precursor to experiments using more complicated stimuli, the results from such simple experiments can not always be directly applied to more complicated situations. Sounds encountered in real life are very complex in nature, and so in order to develop a comprehensive understanding of human hearing, more experiments need to be performed using more complicated and true to life stimuli.

Experiments investigating pitch perception of complicated stimuli such as bell sounds have been limited in the past by the experimenters' inability to modify the stimuli in a systematic and well defined manner. Technology developed in the 20th century has liberated experimenters from the use of simple or ill-defined sound stimuli and given them the ability to exercise precise control over the experimental parameters.

The purpose of psychophysics is to relate physical stimuli to the resulting perceptual experience in an attempt to gain understanding of the mechanisms of perception. The methodology laid out in Chapter 4 provides a framework within which to achieve these goals.

Results from the pitch matching experiment in Chapter 5 confirmed several known experimental outcomes such as the fact that non-musicians are particularly bad at matching the pitches of sounds with different tone qualities, and that inharmonic complex tones presented without some sort of musical context lend themselves to analytic perception. Another factor contributing to analytic perception seemed to be the subjects' memory of previous pitch matches.

Due to the high incidences of analytic perception, not enough data were available to draw any statistically viable conclusions about the perceived pitch differences between ringing and chiming bell sounds.

If perceived differences in pitch between ringing and chiming bells could result from pseudo high frequency noise caused by the louder upper partials of the ringing sound, then presenting the bell stimuli at a low level of sensation (which is done to minimise analytic perception) could prevent the effect from being observed if the upper partials become softer than the threshold of hearing. An experiment would need to be devised which incorporated some form of musical context to facilitate subjective pitch perception, enabling the stimuli to be presented at a higher level of sensation.

Appendix 1 - Glossary of Terms

Analytic Perception - When the pitch of a single overtone of the sound is heard in favour of the overall pitch of the complex tone, also called *fission*.

Antinode - A position on a vibrating body where the displacement is a maximum.

Beating - When two tones are sufficiently close in frequency to cause a noticeable periodic fluctuation in the amplitude envelope.

Carillon - A collection of bells with a range of not less than two octaves, played using a keyboard.

Chiming - A bell is said to chime when the bell moves through a small arc causing the clapper to strike it.

Clapper - The mallet-like metallic object suspended from the centre of the top of the bell and able to move freely in order to strike the bell.

Clipping - Loss of information from the waveform due to the finite capability of the recording apparatus to respond to intensity. Occurs in other words when the sound is too loud for the equipment to capture it.

Combination Tone - A sensation of pitch formed when two pitch components are sufficiently loud to cause distortion in the inner ear.

Comparison Tone - The tone used in pitch matching experiments over which the subject has control in order to implement a match.

Complex Tone - A sound composed of more than one constituent frequency.

Difference Tone - A type of combination tone where the perceived pitch is at the difference in frequency between the component tones.

Dichotic - If two sounds are presented separately to the different ears of a subject they are dichotic.

Fission - When the pitches of individual components of the sound (overtones) are heard, see also *analytic perception.*

Fundamental - The lowest frequency of a series of harmonically related frequencies. Usually denoted by f_o.

Fusion - The perception of a single pitch for a tone made up of many frequencies.

Harmonic - A frequency component of a complex tone that is harmonically related to the fundamental, i.e. it is an integer multiple of the fundamental frequency.

Hum - The lowest partial in the sound spectrum of a bell. It has the longest decay time.

Node - The position on a vibrating body where the displacement is zero.

Nominal - The fifth lowest partial in the sound spectrum of a bell. Usually about an octave above the perceived pitch.

Overtone - An harmonic frequency component of a complex tone that is different from the fundamental.

Partial - A component of a complex tone that need not be harmonically related to any other frequencies within the sound.

Pitch Matching - An experimental procedure whereby subjects are asked to match the pitch of a *comparison tone* to that of a *test tone.*

Prime - The second lowest partial in the sound of a bell.

Pure Tone - A tone consisting of only one frequency, i.e. it has no overtones, partials or harmonics.

Quint - The fourth lowest partial in the sound of a bell. It is ideally tuned a fifth away from the *prime*.

Residue Pitch **-** The overall pitch perceived for a tone with many frequency components, also *subjective* or *virtual pitch.*

Ringing **-** When the bell sounded by making it move through an entire circle.

Sine Wave **-** An oscillation at only one frequency (*pure tone*).

Spectral Pitch **-** The pitch of a frequency component of a complex tone.

Stimulus **-** The sound under consideration in a psychoacoustic listening experiment.

Strike Note **-** The *virtual, subjective* or *residue pitch* of a bell sound.

Subject **-** A participant in a psychological experiment.

Subjective Pitch **-** The single pitch perceived for a tone consisting of many frequency components. Also called the residue or virtual pitch.

Test Tone **-** The tone to which the subject is expected to make the match in a pitch matching experiment.

Tierce **-** The third lowest partial of the bell spectrum. It is ideally a minor third away from the *prime*.

Virtual Pitch **-** The single pitch perceived for a tone with many constituent frequencies. Also called *residue* or *subjective pitch.*

Appendix 2 - Questionnaire, Answer Sheets and Instructions to Subjects

QUESTIONNAIRE

1.) General :

1.1) Surname and initials ____

1.2) Age ________

1.3) Have you ever suffered any permanent hearing loss due to medical reasons? Specify.

____

____

2.) Musical background

2.1) Have you ever taken practical music lessons? ________

2.2) Which instrument/s and for how many years

____

____

____

2.3) What level or grades did you achieve on the above instruments?

____

____

____

2.4) What is your highest qualification in music theory?

____

3.) Experience with listening tests

3.1) Have you ever experienced a test of musical aptitude? State which one if you can remember.

_______ ____________________

3.2) Have you ever taken part in a listening experiment before? If so, give a brief description of the tasks required.

____

____

____

4.) Knowledge of bells

4.1) Have you ever attended a course in bell ringing? __________

4.2) Do you know what the intervals are between the five lowest overtones of a bell?

____

PITCH DISCRIMINATION

Trial Attempts

1.) _____ 2.) _____ 3.) _____ 4.) _____

Pitch Discrimination

1.) _____

2.) _____

3.) _____

4.) _____

5.) _____

6.) _____

7.) _____

8.) _____

9.) _____

10.) _____

11.) _____

12.) _____

13.) _____

14.) _____

15.) _____

16.) _____

17.) _____

18.) _____

19.) _____

20.) _____

21.) _____

22.) _____

23.) _____

24.) _____

25.) _____

26.) _____

27.) _____

28.) _____

29.) _____

30.) _____

PITCH MATCHING

Trial Attempts

1.) _____ 2.) _____

Tasks 1 to 24

1.) _____ 9.) _____ 17.) _____

2.) _____ 10.) _____ 18.) _____

3.) _____ 11.) _____ 19.) _____

4.) _____ 12.) _____ 20.) _____

5.) _____ 13.) _____ 21.) _____

6.) _____ 14.) _____ 22.) _____

7.) _____ 15.) _____ 23.) _____

8.) _____ 16.) _____ 24.) _____

Appendix 3 - Order of presentation of the sine waves in the pitch discrimination test

The order of presentation of the second sine waves in each pair used in the test of pitch discrimination is given in **Table 3.1.** The first sine wave always had a frequency of 440 Hz.

Table 3.1 The order of the frequencies of the second sine waves in each of the pairs used in the test of pitch discrimination.

Number	Frequency	Number	Frequency
1	460	**16**	430
2	435	**17**	440
3	420	**18**	438
4	445	**19**	442
5	440	**20**	438
6	420	**21**	440
7	438	**22**	460
8	460	**23**	435
9	440	**24**	420
10	450	**25**	445
11	442	**26**	440
12	430	**27**	442
13	445	**28**	430
14	435	**29**	440
15	450	**30**	450

Appendix 4 - Statistical Analyses

Appendix 4.1 - Pitch discrimination test

Table 4.1 Analysis of variance

Source of variation	**Degrees of freedom**	**Sums of squares**	**Mean squares**	**Variance ratio (F)**
Between categories of musical training	2	98.074	49.035	4.94 *
Within categories of musical training	20	198.357	9.918	
Total	**22**	**296.435**		

The variance ratio (F) for 2 and 20 df

P = 0.05 F = 3.49

P = 0.01 F = 5.85

Table 4.2 Student t test comparisons. Two tail test for t with 20 df.

Comparison	**Calculated t value**	**Tabulated t value**
Non musicians versus musicians	2.673 *	
Non musicians versus experienced musicians	2.827 *	**P** **t** 0.05 2.068 0.01 2.845
Musicians versus experienced musicians	0.159 NS.	* Significant at P = 0.05 ** Significant at P = 0.01 NS Not significant

Appendix 4.2 - Pitch matching abilities of subjects with different degrees of musical training

Table 4.3 Analysis of variance

Source of variation	Degrees of freedom	Sums of squares	Mean squares	Variance ratio (F)
Between categories of musical training	2	646.593	323.296	8.79 *
Within categories of musical training	20	735.232	36.761	
Total	**22**	**1381.826**		

The variance ratio (F) for 2 and 20 df

P = 0.05 F = 3.49

P = 0.01 F = 5.85

Table 4.4 Student t test comparisons. Two tail test for t with 20 df.

Comparison	**Calculated t value**	**Tabulated t value**	
Non musicians versus musicians	4.057 **		
Non musicians versus experienced musicians	3.061 **	**P** 0.05 0.01	**t** 2.068 2.845
Musicians versus experienced musicians	1.031 NS	* ** NS	Significant at P = 0.05 Significant at P = 0.01 Not significant

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