A STUDY OF THE CONSUMPTION CAPITAL ASSET PRICING MODEL'S APPLICABILITY ACROSS FOUR COUNTRIES

A thesis submitted in partial fulfilment of the

requirements of the degree

MASTER OF COMMERCE (FINANCIAL MARKETS)

of

RHODES UNIVERSITY

by

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ABSTRACT

Historically, the Consumption Capital Asset Pricing Method (C-CAPM) has performed poorly in that estimated parameters are implausible, model restrictions are often rejected and inferences appear to be very sensitive to the choice of economic agents' preferences. In this study, we estimate and test the C-CAPM with Constant Relative Risk Aversion (CRRA) using time series data from Germany, South Africa, Britain and America during relatively short time periods with the latest available data sets. Hansen's GMM approach is applied to estimate the parameters arising from this model. In general, estimated parameters fall outside the bounds specified by Lund & Engsted (1996) and Cuthbertson & Nitzsche (2004), even though the models are not rejected by the *J*-test and are associated with relatively small minimum distances.

Keywords: Econometric and Statistical Methods and Methodology, Investment Decisions, Asset Pricing, CAPM

JEL Classification: C1, G11, G12

DECLARATION

Except for the references that have been accurately cited and discussed herein, the content of this thesis represents my own efforts. The entire thesis has neither been nor is concurrently being submitted to any other academic institution for the purpose of obtaining a degree.

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07/2013

ACKNOWLEDGEMENTS & DEDICATION

I am most thankful to Doctor E. Seedat (Department of Economics, Rhodes University) who has provided supervision for this study as well as expanded my knowledge and assisted me in many ways in the last year.

I dedicate this work to my father whose support has enabled me to complete this study.

The financial assistance of the National Research Foundation (NRF) towards this research is hereby acknowledged. Opinions expressed and conclusions arrived at, are those of the author and are not necessarily to be attributed to the NRF.

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LIST OF ABBREVIATIONS & NOTATION

max	Maximum (Optimization)
argmin	Argument θ
lim	Limit
CAPM	Capital Asset Pricing Model
C-CAPM	Consumption Capital Asset Pricing Model
CRRA	Constant Relative Risk Aversion
GMM	Generalized Method of Moments
RRA	Relative Risk Aversion
SDF	Stochastic Discount Factor
A	Hessian matrix of dimension $P \times P$
A^{-1}	Inverse of the Hessian matrix
r	Simple (net) rate of return
C	Agents' consumption
β	Discount factor parameter
U(C)	Period utility of agents' consumption
U'(C)	Marginal utility of aggregate consumption
P	Price of asset
M	SDF/Inter-temporal marginal rate of substitution /Asset pricing kernel
R	Gross return
D	Share Dividend (cash)
Cov	Covariance
N	Number of single asset units
γ	Relative Risk Aversion parameter
X	Level of habit
x	Vector of dimensions $P \times 1$
MA(p)	Moving average process of order p
S	Surplus consumption ratio
s	Natural Logarithm of S
l	Optimal rate of lag growth
H_i	Sequence of matrices
HJ	Hansen & Jagannathan (1997) distance measurement
φ	Regulates the persistence of the log surplus consumption ratio
$\bar{s}$	Steady state level of s
$\tau(s)$	Sensitivity function (determines innovation in consumption)
E	Expectation operator
ML	Maximum Likelikhood
P	Number of moment conditions
K	Number of unknown parameters
θ	Parameters
z	Vector of instruments
f(.,.)	Function
∇	Gradient of a function
Θ	Parameter space
$\Re$	P-dimensional Euclidean space
g()	Vector of orthogonality conditions
q	Number of instruments
Q	Objective function
W	Weighting matrix
V	Asymptotic covariance matrix
J	J-statistic for over-identifying restrictions
h()	Disturbance term
SD	Spectral density matrix
SD^{-1}	Consistent estimator of weighting matrix, W
n	n staged iterative process
ϵ	Arbitrarily small positive constant
Y	Set maximum number of iterations
DAX	DAX 30 Index level

LIST OF ABBREVIATIONS & NOTATION (CONTINUED)

S&P	Standard & Poor's 500 Composite Index level
JSE ALSI	Johannesburg Stock Exchange All Share Index
∂	Partial derivative
D	Partial derivative of a vector of orthogonality conditions divided by partial derivative of parameter vector
v()	Vector of random variables
α	Consumption growth rate
$>, \geq$	Greater than, greater than or equal to
$<, \leq$	Less than, less than or equal to
$\equiv$	Is the same as
$\int_x^y$	Integral with lower limit of integration of x and upper limit of y
$\sum$	Summation
+	Addition
$\|\|.\|\|$	Euclidean norm

Subscripts not related to notation appearing earlier in this list

T	Sample set size
0	Date on which sample set starts
t	A date between 0 and T ($0 < t < T$)

Superscripts not related to notation appearing earlier in this list

T	Transpose of a real vector
*	True values/optimal values
^	Estimated value

CHAPTER 1. INTRODUCTION

1.1 Brief Context

Considerable research has been done to determine the forces that influence the behaviour of asset returns (Dimson & Mussavian, 1990). Such research has culminated in the development of the Capital Asset Pricing Model (CAPM) by Sharpe (1964), Lintner (1965) and Black (1972). The CAPM offers useful insights about the relationship between expected stock returns and risk, as well as predictions concerning how risk is measured. The CAPM is still widely used by practitioners and academics despite its frequent poor empirical performance. This may be due to the CAPM being vitiated by numerous simplifying assumptions (Fama & French, 2004).

The CAPM has been extended by several authors to consider individual agents' dynamic optimization behaviour under uncertainty in particular. Lucas (1978) and Breeden (1979) introduced the Consumption based Capital Asset Pricing Model (C-CAPM) to represent the dynamic relationship between asset returns and the inter-temporal marginal rate of substitution. As resource allocation in static microeconomics is determined by relative prices, so the inter-temporal resource allocation in the dynamic model is determined by asset returns (Hamori, 1992).

Financial assets, like securities, are prominent in the construction of the C-CAPM in that their presence permits consideration of inter-temporal consumption smoothing. Agents hold these securities to transfer purchasing power from one period to another, therefore, if an agent has no assets, their consumption would be determined solely by current income. If an agent holds assets, then these assets can be sold to finance consumption when current income is low. An individual asset will be more desirable if the return is expected to be high when consumption is expected to be low (Cuthbertson & Nitzsche, 2004). Cuthbertson & Nitzsche (2004) state that the systematic risk of an asset is therefore determined by the covariance between the asset's return and consumption, which differs to the covariance of the CAPM. This covariance is determined by the return on the market portfolio.

Since Hall (1978) introduced the random walk model of consumption modelled using Euler equations[1], there has been considerable interest in testing the implication of individual agents' consumption choices influence on asset returns.

In order to estimate the C-CAPM, Hansen & Singleton (1982) used the Generalized Method of Moments (GMM) within the C-CAPM framework of Lucas (1978). Since then, substantial theoretical and empirical research utilizing the GMM approach in estimating the C-CAPM has arisen (Auer, 2011).

The C-CAPM has proven to be empirically disappointing. For example, Hansen & Singleton (1983) rejected the power utility representative agent formulation of Lucas (1978). Breeden et al. (1989) and Campbell (1996) state that poor empirical evidence has led to the conclusion that the C-CAPM is rarely an 'improvement' on the traditional CAPM. Despite frequent rejection of the C-CAPM, Lettau & Ludvigson (2001) emphasise that the reputation of the theoretical paradigm itself remains intact.

The C-CAPM is a very general theory of equilibrium asset returns and is based on views about agents' tastes. The theory holds for any individual consumer who has the choice of investing in securities and for any asset or portfolio of assets for any time horizon of returns, such as a month or year (Cuthbertson & Nitzsche, 2004). Returns should be explained only by the different risk factors and sensitivity of the returns to the risk factors, whereas the C-CAPM measures the risk of a security by the covariance of its return with per capita aggregate consumption (Elton, Gruber, Brown & Goetzman, 2007).

1.2 Aims of Study

The aim of this study is to determine whether consumption decisions are related to stock index returns for Germany, South Africa, United Kingdom (UK) and the United States of America (America) during different periods. Specifically, the Euler equation relation of Lucas' (1978) C-CAPM, using the Constant Relative Risk Aversion (CRRA), specification must hold. Lucas' (1978) CRRA C-CAPM will represent the relation between such consumption decisions and the relevant countries' stock index returns, thereby augmenting the existing literature on this topic. The chosen time periods differ to those used in the extant literature in estimating the C-CAPM. In particular, the sample will include the beginning of the period of economic turbulence for all four countries. Mehra (2003) states that after a

[1] Hall (1978) showed under rational expectations that consumption should be martingale (i.e. should be unpredictable).

major downturn the realised equity premium will be low, whereas the expected premium will be high. This is consistent with low consumption during a downturn.

The habit formation specification of the C-CAPM will not be estimated, because Auer (2011), Engsted & Moller (2011) and Engsted, Hyde & Moller (2005) find that the CRRA specification of the C-CAPM either performs at least as well as the habit formation or estimation of the habit formation does not yield additional explanatory power.

1.3 Method and Structure

In order to establish a relation between consumption and asset returns, we estimate the coefficient of relative risk aversion and the rate of time preference, which comprise the Stochastic Discount Factor (SDF) in Lucas' (1978) C-CAPM with the standard CRRA power utility function. The SDF contained in the Euler equation of the foregoing model will be constructed via the maximization of a representative agent's expected life time utility, following the CRRA specification using the GMM method following convention (Hamori, 1991; Lund & Engsted, 1996, Campbell, 2006). The estimated returns are compared to the actual returns and inferences are made regarding the accuracy and viability of the chosen model as follows. First, Hansen's (1982) J-test for over-identifying restrictions implied by the model is used when there are more orthogonality conditions than parameters to be estimated. Second, Hansen and Jagannathan's (1997) distance measurement test is applied as it provides a useful economic measure of model fit.

The study is structured as follows. Chapter 2 introduces consumption-based asset pricing models, the SDF and empirical applications thereof. Specific attention is given to Lucas' (1978) CRRA power utility model. In addition, C-CAPM estimation issues are considered. Chapter 3 introduces the data set and the main results of the tests including a discussion on the robustness of the empirical work. The final chapter contains concluding remarks and suggestions for further research.

CHAPTER 2. LITERATURE SURVEY

This chapter consists of a discussion of the theoretical foundations and empirical implementation of the SDF within the context of the C-CAPM. In Section 2.1, consumption and asset pricing is discussed, making special reference to the SDF's influence on stock prices. In Section 2.2 the classic power utility and the habit formation forms of the C-CAPM are examined in more detail, given that the habit formation form is a popular alternative to classic power utility C-CAPM. Section 2.3 comprises the discussion of how the SDF and C-CAPM are estimated. A summary of the relevant literature referred to in this chapter appears in Appendix 1.

2.1 Consumption and Asset Pricing

Theories of capital asset pricing rely on the principle of the expected present discounted value of future dividends and the hypothesis of efficient capital markets (Cashin & McDermott, 1998). Moreover, the behaviour of aggregate stock prices has been of perennial interest to investors, policy-makers and economists as they have struggled to understand the patterns observed in stock prices (Campbell, 2003). Campbell (2003) states that "in the last 20 years stock markets have continued to show some familiar patterns, including high average returns and volatile and pro-cyclical price movements". Jagannathan & Wang (1996) agree that investors demand a higher expected premium[2] for riskier investments.

However, the evaluation techniques in assessing the risk of cash flows, and therefore the demanded risk premium, is still only partially understood. For the traditional Sharpe (1964), Lintner (1965) and Black (1972) CAPM, asset returns depend on beta, assuming that agents consume all their wealth after one period. Merton (1973) showed that the traditional CAPM would not hold in general, in a dynamic environment and would therefore require multiple betas. Breeden (1979) confirmed that one beta would be sufficient inter-temporally, given that the correct beta is measured by the marginal contribution of an asset to consumption flow rather than wealth. Lucas (1978) and Breeden (1979) demonstrate that in equilibrium, in an inter-temporal economy, the 'consumption betas' are proportional to expected excess returns (Breeden, Gibbbons & Litzenberger, 1989).

[2]Kocherlakota (1996) states that equity premium with regards to the covariance between consumption growth and stock returns is greater than the covariance between consumption growth and bond returns, therefore, investors see stocks as a poorer hedge against consumption risk and so stocks must earn a higher average return.

Like the standard CAPM, the C-CAPM predicts a positive linear relationship between expected return and systematic (beta) risk. Investors in assets utilizing the CAPM are concerned with market beta risk, whereas in the C-CAPM the consumption beta risk is considered by investors. The consumption beta measures asset returns' sensitivity to the growth rate of aggregate consumption (Faff & Oliver, 1997). Mankiw & Shapiro (1986) suggest that the covariance between aggregate consumption growth and asset returns is a better measure of systematic risk.

Due to the nature of risk-averse consumers, such economic agents use (dis)investments in the asset to assist in the smoothing of consumption, therefore, creating a direct effect between consumption growth and asset returns (Engsted & Moller, 2010). In the asset pricing models of Lucas (1978) and Breeden (1979), a representative investor influences their consumption plans by trading shares within a competitive market. Hansen & Singleton (1983) state that an implication of such trading is that stock returns are serially correlated with both consumption expenditure and the degree of risk aversion of investors.

Moreover, the rejection of the linear present value formula for stock prices suggests that investors consider consumption risk when making portfolio decisions (Grossman & Shiller, 1981). Nieto & Rubio (2011) argue that exclusively, neither the volatility of consumption growth nor the volatility of market returns can forecast economic cycles. Instead, Nieto & Rubio (2011) find that the important factors in forecasting are the simultaneous effect of the volatilities together with the covariance between consumption growth and market return as well as the influence of preference parameters. The volatility of the consumption-based SDF jointly captures all of these effects (Nieto & Rubio, 2011). Given the absence of arbitrage opportunities the SDF exists, such that the equilibrium price of a traded asset can be represented as the conditional expectation of the future payoff, discounted by the SDF (Brennan, Wang & Xia, 2004).

Previous empirical finance literature has extensively utilized the SDF method for econometric evaluation within asset-pricing models. An asset-pricing model ascertains a particular SDF that is a function of the model parameters and observable variables (Jagannathan & Wang, 2002). The advantage of the SDF method, which is a combination of the SDF representation and the GMM, is that it can be used in the analysis of linear as well as non-linear asset pricing models, therefore, providing an appropriate means of estimating and testing the C-CAPM (Cochrane, 2001). Despite doubts about the estimation efficiency of the

SDF given its generality, Jagannathan & Wang (2002) concluded that the asymptotic precision of the SDF method is as efficient as the beta method (using Ordinary Least Squares) for estimating risk premia in linear factor pricing models. Lucas (1978) first introduced the general theoretical specification and Hansen & Singleton (1982) were the first to exploit the GMM in estimating consumption based asset pricing models which was a radical departure from the use of Ordinary Least Squares and Maximum Likelihood estimation in the case of the Sharpe (1964), Litner (1965) and Black (1972) CAPM.

Following the approach of Hansen & Singleton (1982), Mehra & Prescott (1985) analysed the S&P 500 Composite Index during 1889 – 1978 period within the American market and find that the real average return is 6.9 percent, whereas the American Government risk-free bonds yielded real average returns of 0.8 percent. The historical American average equity premium is around 6 percent and is of magnitude greater than can be rationalized in the context of the standard neoclassical paradigm of financial economics, and this regularity is termed "the equity premium puzzle". This puzzle stems from the finding that equity risk, as measured by the covariance between consumption growth and stock returns within the context of the C-CAPM, is too low in order to warrant the large risk premium on stocks (Smoluk & Neveu, 2002).

It is evident that the C-CAPM is completely incapable of replicating the high observed equity premium once reasonable parameter values are incorporated into the model (Campbell, 2003). Kocherlakota (1996) provides a possible solution for the higher average return on stocks whereby stock returns covary more with consumption growth than do bond returns, therefore, investors view stocks as a poorer hedge against consumption risk. Thus, higher average returns must be earned. Considerable research has attempted to solve this puzzle. Mehra & Prescott (1985) were unable to explain the equity premium using the inter-temporal C-CAPM within the American market. Campbell (1996) attempts to uncover the equity premium puzzle and tests alternative C-CAPM preference models in several countries. Campbell (1996) concludes that the equity premium puzzle is evident in almost all countries tested and, therefore, is an international phenomenon. For example, Hassan & van Biljon (2010) find that the South African equity premium exists, wherein the observed premium is too high to be explained by the standard C-CAPM and the results are comparable to evidence from advanced economies.

Smoluk & Neveu (2002) states that the C-CAPM thus far shows that investors in aggregate must be extremely averse to consumption risk (corresponding to a high Relative Risk Aversion (RRA)) to justify the demand for such a large equity premium. Kocherlakota (1996) states that if investors have coefficients of RRA larger than ten, contrary to Mehra & Prescott (1985), the equity premium puzzle vanishes. Only a handful of economists believe that investors can be this highly risk averse. Kandel & Stambaugh (1991) find that investors are more risk averse than commonly thought. Cochrane (2005b) states that studies of well-known puzzles should be declining, because there are preferences that can coherently describe the data using high risk aversion parameters.

Assuming that many identical, infinitely-lived consumers exist admits an examination of the decisions of a representative consumer and the implications for asset pricing (Danthine & Donaldson, 2005). For instance, Lucas (1978) specifies a basic pricing equation wherein a representative investor seeks to maximize expected lifetime utility, which is applicable to a broad class of inter-temporal asset pricing models. The representative agent acts to maximize the expected present value of the utility of consumption discounted over their lifetime (Danthine & Donaldson, 2005). The representative agent is able to freely trade some asset i and receive a gross simple rate of return, $1 + r_{i,t+1}$, on an asset from time t to time $t + 1$. If the agent consumes C_t at time t and has a time-separable utility (alternative utility preference exists in previous empirical literature and will be taken into account in Section 2.3) with a discount factor, β which is the rate of time preference and period utility function $U(C_t)$, then the first-order condition can be specified as (Campbell, 2003):

$$U'(C_t) = \beta E_t[(1 + r_{i,t+1})U'(C_{t+1})] \ldots\ldots [1]$$

where $U'(C)$ denotes the marginal utility of consumption. Greater marginal utility value implies that consumers would prefer to consume tomorrow by saving today, i.e. consumers place higher value on future consumption (Smoluk & Neveu, 2002). The first-order condition implies that the marginal utility cost of consuming one real unit less at time t is equal to the expected marginal utility benefit of investing the same unit in asset i at time t and then selling it at time t+1 and consuming the resultant return (Campbell, 2003). In order to obtain the Euler equation, Equation 1 can be rewritten as:

$$E_t\left[(1 + r_{i,t+1})\beta \frac{U'(C_{t+1})}{U'(C_t)}\right] = 1 \ldots\ldots [2]$$

where $M_{t+1} = \beta \frac{U'(C_{t+1})}{U'(C_t)}$ is the SDF which is also known as the inter-temporal marginal rate of substitution. In complete markets, the SDF is unique because agents can trade with one another in order to remove any idiosyncratic variation within their utility function (Campbell, 2003). Engsted & Moller (2010) redefine the asset pricing relationship where the gross return of investing in an asset at time *t* and selling it at *t+1* as $R_{t+1} = [P_{t+1} + D_{t+1}]/P_t$ as:

$$0 = E_t[M_{t+1}R_{t+1}] - 1 \text{.....[3]}$$

Engsted and Moller (2010) state that consumption-based models' risk adjustment occurs when multiplying the raw return by the SDF. Equation 3 incorporates the fundamental idea that risk-adjusted equilibrium returns are unpredictable and allows for the empirical estimation and testing using GMM. The GMM process allows asset returns and the SDF to be serially correlated, leptokurtic and conditionally heteroscedastic (Jagannathan, Skoulakis & Wang, 2002). The main feature of the estimation is the minimization of a quadratic form in the sample which has a resemblance to a population moment condition, which presents sufficient information to identify the unknown parameters (Hall, 2005). The GMM will be explained further in Section 2.3.

The equilibrium expected return is required to include the crucial correlation between consumption growth and returns. From Equation 2, expected returns are:

$$E_t[R_{t+1}] = 1 - \frac{Cov[R_{t+1}, M_{t+1}]}{E_t[M_{t+1}]} \text{.....[4]}$$

Equation 4 indicates that an asset with a lower covariance between asset return and the SDF, is expected to have a higher return (Auer, 2011).Such an asset is expected to have low returns when agents have high marginal utility. The asset is risky because it fails to deliver wealth when investors value wealth the most, therefore, investors demand a higher risk premium to hold it (Campbell, 2003).

The possible preference models used in C-CAPM estimation differ with respect to the form of the assumed utility function and the resultant SDF (Hyde, Cuthbertson & Nitsche, 2005). The main preference models are discussed below.

2.2 Consumption C-CAPM

The CAPM of Sharpe (1964), Lintner (1965) and Black (1972) is inconsistent with several empirical regularities of cross-sectional asset pricing data (Lettau & Ludvigson, 2001). Lettau

& Ludvigson (2001) provide a possible explanation into the failure to the CAPM, namely that its static specification fails to consider the effects of time-varying investment opportunities into the calculation of risk within an asset.

Lucas (1978) and Breedan (1979) devised the C-CAPM to represent the dynamic relationship between asset returns and the SDF. If the C-CAPM relationship holds, the expected stock returns are equal to zero, in which case consumption behaviour determines stock returns. In the C-CAPM an asset's systematic risk is associated with the state of the economy, namely consumption. In the C-CAPM, Hansen & Singleton (1983) assume identical agents, within a production-exchange economy and choose consumption and investment plans so as to maximize the expected value of the time-additive von Neumann-Morgenstern utility function. To derive the joint distributional restrictions, it is necessary to specify a distribution function and to parameterise preferences (Hansen & Singleton, 1983).

Despite the possible superiority of the C-CAPM versus the CAPM, poor empirical evidence has led to the conclusion that the C-CAPM is rarely an 'improvement' on the traditional CAPM. For example, Mankiw & Shapiro (1986) find that the traditional CAPM outperforms the C-CAPM using cross-sectional data during the 1959-1982 period for 464 American companies, within the context of the CRRA utility function. The reasons for these results, apart from defects in data sets, are a combination of errors in model specification (especially the use of utility functions in SDF's that are inconsistent with the data (Hyde, Cuthbertson & Nitzsche, 2005)). In addition, it is necessary to consider defects in model estimation, testing and inferences, too few countries being included and the consideration of relatively short time periods.

Despite the negative results, Cochrane (2005b, 267) states, "at some level, the consumption-based model must be right if economics is to have any hope of describing stock markets." Moreover, Auer (2013a) states that there is really no alternative to the C-CAPM, as other models are approximates or can expressed as a special cases of the C-CAPM. The volume of literature on the traditional C-CAPM is limited due to the poor performance of the C-CAPM empirically.

Breeden et al. (1989) tests the CRRA C-CAPM, using monthly returns during the 1926-1982 period for the American stock market, using betas based on both consumption and the portfolio having the maximum correlation with consumption. Breeden et al. (1989, 260) concludes that the market price of risk is significantly positive and that "while the CCAPM is

by no means a perfect description of the data, we found the fit better than we anticipated". Although Breeden et al (1989) finds the fit is better than anticipated, Hansen & Singleton (1982, 1983) rejected the time separable CRRA C-CAPM using America data. The traditional C-CAPM has rarely demonstrated positive results, but empirical evidence has been lacking as most of the traditional C-CAPM tests were performed on American data. Hamori (1992) and Lund & Engsted (1996) estimated the C-CAPM for Japan and European countries, respectively. Hamori (1992) finds that the C-CAPM was partially useful in explaining the Japanese stock returns, but like Chen & Ludvigson (2009) and Grossman, Melino & Shiller (1987) obtained large RRA parameter estimates. Hyde & Sherif (2005b) find that the CRRA C-CAPM has the ability to explain the equity premium without excessively large values of RRA.

Lund & Engsted (1996) estimate the C-CAPM with Danish (1922-1990), German (1885-1913 and 1952-1990), Swedish (1918-1990) and UK (1919-1987) stock returns and find a lack of empirical support in describing the time-varying discount rates in terms of the consumption-based models. Lund & Engsted (1996) obtained high RRA values with a small percentage of estimates being significantly different from zero. The RRA coefficient was negative sign which is inconsistent with economic theory. A negative RRA coefficient would imply a negative rate of time preference.

Lund & Engsted (1996), as in most other studies, use the CRRA utility function. Hassan & van Biljon (2010), using the CRRA utility function, find that the C-CAPM results for South Africa (1900-2005) are as poor as the results of studies of advanced economies over equally long sample periods. Kasa (1997) tested the C-CAPM for the US, Japan, UK, Germany and Canada for the 1972-1993 period. The C-CAPM results were no better than previous empirical research; the model did not perform unless allowance was introduced for the time variation in the price of risk.

The frequent lack of empirical support of the CRRA C-CAPM may be attributed to the fact that measured consumption is too "smooth" to rationalize the variability observed in asset returns (Chapman, 1997). Moreover, Ludvigson (2012) mentions that the CRRA C-CAPM has difficulty in explaining several pricing phenomena. For example, the inability to explain the high ratio of equity premium to the standard deviation of stock returns, together with the stable growth of aggregate consumption. In recognition of these limitations, subsequent empirical research involved alteration of the standard C-CAPM to consider new preference

orderings, restrictions of the dynamics of cash flow fundamentals, or new market structures (Ludvigson, 2012). Thereafter, very little empirical research was conducted on the traditional C-CAPM.

To address the flaws in the power utility C-CAPM, research extended preference orderings and market structures to include habit formation, time-separable consumption and world consumption. Abel (1990), Constantinides (1990) and Campbell & Cochrane (1999) introduced habit formation. This model consists of individuals developing habits for high or low consumption, such that risk aversion becomes time-varying and countercyclical (Engsted, Hyde & Moller, 2010). Abel (1990) presents an external habit model, "catching up with the Joneses", whereby habit depends on aggregated consumption which is unaffected by any one agent's decisions. Li & Zhong (2005) finds that the predictability of returns is explained in part by changing prices or risk associated with consumption relative to habit. Auer (2011) concludes that for German stocks the habit formation model is similar to the standard C-CAPM in terms of an overall high level of RRA and both models cannot be rejected, but the habit formation estimates a plausible time discount factor. Hyde, Cuthbertson & Nitzsche (2005) find that incorporating habit formation only partially decreased the implied levels of RRA for French and German stocks. The habit formation models are thought to be the most promising in describing stock market behaviour, but Chen & Ludvigson (2009) state that further empirical research, particularly outside of the American asset market, is required.

The C-CAPM has been extended to include a world C-CAPM, a special case of the C-CAPM, wherein variations in expected market returns across countries can be explained by varying consumption risk exposure (Darrat, Li & Chul Park, 2011). The world C-CAPM includes partial and perfectly integrated world capital models (Campbell, 2003). Stulz (1981) states that asset prices from all countries are determined by a common SDF under the assumption of complete international market integration, where there is complete consumption risk sharing. Darrat et al. (2011) finds that the CRRA world C-CAPM explains about 11 percent of the variations in stock returns across countries, but the model required an implausibly large coefficient of RRA. Li & Zhong (2010) find that a large RRA parameter is needed to explain the equity premium for the CRRA world C-CAPM, therefore, reverting to the original problem found in the power utility C-CAPM. Auer (2013b) finds that the world Campbell & Cochrane (1999) habit formation C-CAPM has higher explanatory power compared to the world CAPM and the world CRRA C-CAPM, in explaining the cross section

of G7 countries' equity risk premia under complete market integration. Auer (2013b) concludes that the world Campbell & Cochrane (1999) habit formation C-CAPM explains more than ninety percent of the cross-sectional variation in risk premia.

In addition, the C-CAPM has been extended to include limited participation consumption, wherein an asset-holder's consumption is distinguished from non-asset-holder consumption (Bach & Moller, 2011). Bach & Moller (2011) find that in a C-CAPM study for the US bond market, asset-holder consumption outperforms non-asset-holder and aggregate consumption data. The flaw in this model is the difficulty in obtaining a value for asset-holder consumption. To be specific, Bach & Moller (2011) had to survey a representative sample of American households over a period of time which may be a true representation of asset-holder versus non-asset holder consumption.

Lund & Engsted (1996) state that the empirical failure of the simple and theoretically consistent C-CAPM 'poses a major challenge to the profession'. Lettau & Ludvigson (2001) stated that despite the empirical flaws in the C-CAPM, the reputation of the model remains intact. This is due to the measure of systematic risk and the asset's covariance with marginal utility of consumption, remaining unmatched by other asset pricing models. Due to the significance of the model, it is important to extend inquiry to include a greater number of countries and a longer time period, as this will provide an indication whether the results and models are robust (Lund & Engsted, 1996:498).

2.2.1 Power Utility C-CAPM

In contrast to the CAPM, inter-temporal general-equilibrium models clearly identify the underlying economic influences investors gain by accepting risk of the possible equity premium (Carmichael, 1998). Mankiw & Shapiro (1986) state that from a theoretical perspective, consumption betas should offer a better measure of systematic risk.

Lucas (1978) introduces the CRRA C-CAPM which includes the stochastic behaviour of equilibrium asset prices in a single good, pure exchange economy with identical consumers. Lucas (1978) describes a representative agent as having a standard utility function and chooses to maximize the expected present value of lifetime utility. In Lucas' (1978) economy, dividends are aggregate stock market payments, which are equivalent to aggregate output and this aggregate output is equivalent to aggregate consumption. Chen (2003) states that due to the assumptions of Lucas (1978), when employing the first-order Euler conditions

for the model, the exogenous consumption process and utility specification can be used to derive the equilibrium SDF.

Lucas (1978) states that a representative agent[3] wishes to choose consumption streams so as to maximize utility, which can be specified by:

$$\max\ E\{\sum_{t=0}^{\infty}\beta^t U(C_t)\}\text{.....[5]}$$

subject to the budget constraint:

$$C_t + N_{t+1}Pt < (D_t + P_t)N_t\text{.....[6]}$$

where C_t is consumption, N_t represents single asset units, P_t is the price per unit of the asset, D_t is the dividend and β is the discount factor $(0 < \beta < 1)$. The agents' degree of risk aversion is determined by the concavity of the utility function. Equation 5 is the representative agent's maximization of consumption stream and is dependent on the budget constraint of Equation 6. Any representative agent will attempt to maximize utility when applying Equation 3, which is parameterized with a state-independent and a time separable utility function. A utility function is state independent if the utility an individual receives from consumption is independent of the state of the world, therefore, implying the utility function remains the same regardless of whether the state of the world is good or bad (Smoluk & Neveu, 2002).

Lucas (1978) assumes preferences which imply a negative relationship between risk aversion and inter-temporal substitution[4] for which the appropriate utility function, U(.), is the Constant Relative Risk Aversion (CRRA) given by:

$$U(C) = \frac{C^{1-\gamma}}{1-\gamma}\text{.....[7]}$$

with $U'(C) = C^{-\gamma}$ and $\gamma > 0$ is the degree of RRA.

The feature that makes Equation 7 the "preference function of choice", in the growth literature and the Real Business Cycle literature is that it is scale-invariant. The advantage of this feature is that although the levels of aggregate economic variables have increased over

[3] The existence of representative economic agents can be justified in the usual way if preferences are homothetic, common to all consumers and consumers are similar in most respects, with the exception of differing wealth levels (Epstein & Zin, 1989).

[4] The measure of inter-temporal substitutability is obtained by solving the consumer's optimization problem under certainty.

time, the equilibrium return process is stationary. In addition, the CRRA preference function is one of two preference functions that allow for aggregation of investors and a representative agent formulation that is independent of the initial distribution of endowments (Mehra, 2003). The main flaw in the CRRA utility function is that the CRRA function links risk preferences to time preferences, therefore, the coefficient of RRA is the reciprocal of the elasticity of inter-temporal substitution (Mehra, 2003). Thus, with CRRA preferences, investors who prefer to smooth consumption across various states of nature also prefer to smooth consumption over time.

The combination of Equation 3 and Equation 7, give the first-order Euler equation of the Lucas CRRA model:

$$E_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} R_{it}\right] - 1 = 0 \text{…….[8]}$$

where $\frac{C_{t+1}}{C_t}$ denotes consumption growth (Smoluk & Neveu, 2002). An interpretation of Equation 7 is identical for all assets, where weights correspond to the marginal rate of substitution. Thus, during periods of low marginal utility of consumption (high levels of consumption), the weights will be low as returns delivered in these periods barely augments utility. From Equation 8 it is evident that is the SDF is approximated as:

$$M_{t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \text{…….[9]}$$

The main interest is to estimate values for β and γ (Lund & Engsted, 1996). β is the rate of time preference and a higher (lower) value of β implies that a consumer attaches greater (less) value to future consumption. An economically plausible β should be less than one, indicating that consumers discount future consumption. Economically plausible estimates for γ are less certain. Mehra & Prescott (1985) suggest that γ should be greater than zero but less than or equal to 10. The higher RRA implies extremely risk-averse consumers, who prefer low consumption growth and prefer less volatile consumption over time. A restriction imposed by the CRRA preference is that γ is the reciprocal of the elasticity of inter-temporal substitution. The coefficients cannot be below zero as this contradicts Equation 7 by implying non-convex preferences (increasing marginal utility) and that the inter-temporal equilibrium for asset prices does not exist (Smoluk & Neveu, 2002).

For the consumption-based CRRA C-CAPM unrealistically high RRA would imply the acceptance of exceptionally high excess returns on stocks. Given that the CRRA C-CAPM has often been associated with a very low covariance between stock returns and consumption growth. The low covariance is required to explain the historically high excess returns on stocks. Although accounting for a high RRA implies an economically unrealistic value for the risk-free rate (Engsted et al., 2010).

Mehra (2003) states that there is no *a priori* reason for an individual that is averse to variation of consumption in different states, at a particular point in time, being averse to consumption over time. On average, consumption grows over time and investors, in the Mehra & Prescott (1985) scenario, have little incentive to save therefore the demand for bonds is low and the risk-free rate is counterfactually high as a result. Mehra & Prescott (1985) utilized the CRRA C-CAPM which may introduce discrepancies given the omission of habit consumption. A utility function that is time-separable implies that habits are ignored, whereby utility today does not influence utility tomorrow (Smoluk & Neveu, 2002). Equation 3's utility preference function is modified to consider alternative preferences such as habit formation. The inclusion of habit formation has the advantage that a high γ does not imply that the investors will want to smooth consumption over time (Mehra, 2003).

2.2.2. Habit Formation C-CAPM

The habit preference model considers the persistence of previous consumption and its consequent effects on current utility (habit formation) (Hyde & Sherif, 2005a). Engsted & Moller (2010) state the basic idea behind habit formation models is that people become accustomed to living in a certain way and the utility of some consumption level at time t will be lower (higher) if the previous period's consumption was high (low) than if the previous period's consumption was low (high).

Sundaresan (1989) and Constantinides (1990) state that the habit formation preference is important and should be included in the C-CAPM framework as the level of today's consumption will influence tomorrow's marginal utility of consumption. The absolute level of consumption is no longer important, as the habit formation models consider consumption relative to some previous benchmark level (Cuthbertson & Nitszche, 2004).

Cuthbertson & Nitszche (2004) state that agents are concerned about decreases in consumption relative to previous levels, whereas the decrease in absolute consumption may appear minimal. Constantinides (1990) introduces this modification of preferences by

incorporating habit formation where the utility is not only affected by current consumption but also by past consumption. The time-non-separable internal habit persistence model incorporates the notion that utility is a decreasing function of past consumption, whereas marginal utility is an increasing function of past consumption. The RRA of investor consumption, no longer measures the degree of risk aversion since risk aversion, varies with consumption relative to habit. This allows the investor to be highly averse to consumption risk even when risk aversion is slight, therefore, a slight change in consumption can cause large changes in marginal utility (Mehra, 2003).

Three modelling issues arise that lead to the differentiation of the habit formation models. First, the choice of functional form, second, the effect of an agent's own decisions on future levels of habit, and finally the speed at which habit reacts to consumption of the individual or economic agents in aggregate (Campbell, 2003). Campbell (2003) mentions that Constantinides (1990), Campbell & Cochrane (1999) and Boldrin, Christiano & Fisher (2001) use a power functional form of the difference $C_t - X_t$, whereas Abel (1990) states that the utility function should be a power function of the form C_t/X_t. However, Abel's (1990) model precludes the variation of RRA over time.

Another modelling issue arises from agents' future levels of habit being determined internally or externally. The internal habit model, which is discussed by Sundaresan (1989) and Constantinides (1990), consists of an agent's own decisions on consumption determining the habit level. The external habit model considered by Abel (1990) and Campbell & Cochrane (1999) among others use aggregate consumption to determine habit and habit is unaffected by any individual agent's decisions.

The final modelling issue considered above relates to the reaction speed of habit, where habit depends on one lag of consumption or gradually reacts to consumption. For instance, Abel (1990) and Ferson & Constantinides (1991) let habit depend on one lag of consumption which means that only previous periods consumption is relied upon in order to determine the habit associated with consumption. Constantinides (1991), Heaton (1995), Campbell & Cochrane (1999) and Boldrin, Christiano & Fisher (2001), assume that habit gradually reacts to changes in consumption, which implies that the price of risk is time-varying and countercyclical. Allowing habit to gradually react to changes in consumption when consumption is above the habit in cyclical upswings, the price of risk is low. Thus, low expected returns and high asset prices ensue and when consumption is close to habit, the

price of risk is high which leads to high expected returns and low asset prices (Moller, 2009). This allows the utility framework to better represent observed consumption behaviour, by accounting for different preferences.

Campbell & Cochrane (1999) state that if habit moves slowly in response to consumption there are persistent movements in volatility and it is easier to forecast long horizon returns. Chen & Ludvigson (2009) find that the habit specification is better expressed as a nonlinear function of past and current consumption, rather than described as a linear function.

Several authors, including Campbell & Cochrane (1999), mention that the nonlinear habit specification function is crucial in allowing the model to account for the joint behaviour of aggregate consumption and asset returns[5]. Campbell & Cochrane (1999) suggest the addition of a slow-moving external habit to Lucas' (1978) CRRA C-CAPM, assuming an independently and identically distributed consumption growth process. Hyde & Sherif (2005b) find that β and γ estimates appearing in the Campbell & Cochrane C-CAPM are both economically plausible and highly significant. Therefore, Hyde & Sherif (2005b) recommend that the C-CAPM uses the Campbell & Cochrane (1999) habit formation specification.

Identical agents maximize Equation 5 adjusting for habit via the term $C_t - X_t$ which yields:

$$E\left(\sum_{t=0}^{\infty} \frac{\beta^t (C_t - X_t)^{1-\gamma} - 1}{1-\gamma}\right) \ldots\ldots [10]$$

where X_t is the level of habit, the evolution of the level of habit is determined by aggregate (per capita) consumption and is not affected by individual consumers' consumption choices (Campbell & Cochrane, 1999). Previous empirical literature utilizes the history of aggregate consumption in the calculation of habit. For convenience, Campbell & Cochrane (1999) define the *surplus consumption* ratio as $S_t \equiv (C_t - X_t)/C_t$, which captures the relationship between consumption and habit. This ratio increases (decreases) when consumption increases (decreases) and, therefore, S_t approaches one (zero) as consumption increases (decreases) relative to habit in a good (bad) state. Incorporating S_t in the SDF yields:

$$M_{t+1} = \beta\left(\frac{S_{t+1}}{S_t}\frac{C_{t+1}}{C_t}\right)^{-\gamma} \ldots\ldots [11]$$

[5] Alternative habit formation models, in the context of the C-CAPM, are studied in detail by Campbell (2003) but this chapter mainly focuses on the most popular Campbell & Cochrane (1999) model.

Substitution of Equation 11 into Equation 8 yields the pricing equation:

$$E\left[\beta\left(\frac{S_{t+1}}{S_t}\frac{C_{t+1}}{C_t}\right)^{-\gamma}R_{t+1}-1\right]=0\text{…….[12]}$$

Compared to the CRRA C-CAPM Equation 8, growth in the S_t ratio is an extra variable included in the SDF. RRA is measured by γ/S_t, indicating that RRA is now time-varying and countercyclical (Auer, 2011). Unlike the CRRA model, accepting a high RRA value will not imply an implausibly high risk-free rate. This model explains time-varying and countercyclical ex ante returns, which implies pro-cyclical stock prices, as a result of time-varying and countercyclical risk-aversion (Engsted & Moller, 2010).

Allowing habit to adapt non-linearly to the history of consumption, Campbell & Cochrane (1999) can keep marginal utility of consumption above habit therefore the habit will always remain finite[6] and postive. To ensure habit is below consumption at all times, $s_t \equiv \ln(S_t)$ is modelled as a first order autoregressive process:

$$s_{t+1}=(1-\varphi)\bar{s}+\varphi s_t+\tau(s_t)\epsilon_{c,t+1}\text{…….[13]}$$

where the parameter φ regulates the persistence of the log surplus consumption ratio, $\bar{s}$ is the steady state level of s_t and $\tau(s_t)$[7] is the sensitivity function that determines innovation in consumption growth, $\epsilon_{c,t+1}=C_{t+1}-C_t-g$. Li (2001) find the conditional covariance between returns on an asset and the consumption growth too close to a constant level when time varies. Moller (2009) states that this lack of time-variation in the volume of risk indicates that time-varying expected returns depend on the price of risk time-variation. The time-variation in the price of risk is a non-linear function of S_t.

Campbell & Cochrane (1999) focused mainly on explaining the stylized facts of the equity market and, therefore, set the real risk-free rate equal to a constant and flat yield curve (Bach & Moller, 2011). Calibrating their model with a constant risk-free rate, Campbell & Cochrane

[6] Chapman's (1998) model allowed consumption to fall below habit and resulted in undesirable consequences.

[7] The sensitivity function $\tau(s_t)$ is defined as:

$$\tau(s_t)=\begin{cases}\frac{1}{\bar{s}}\sqrt{1-2(s_t-\bar{s})}-1 & s_t\le s_{max}\\ 0 & else\end{cases},$$

where

$$\bar{s}=\sqrt{\frac{\sigma_c^2\gamma}{1-\varphi}},\ s_{max}=\bar{s}+\frac{1}{2}\left(1-\bar{s}^2\right)\text{ and }\bar{s}=\ln(\bar{S})$$

Specifying $\tau(s_t)$ in this way reveals an important feature of the model - that average risk aversion over time can be high but the risk-free rate remains low and stable (Auer, 2011).

(1999) chose parameter values to match certain moments of post-war American data and find that this model explains stock return predictability. The purpose of not setting a constant risk-free rate would be in order to observe features of the term structure of interest rates, as done by Wachter (2006). Instead of using Wachter's (2006) calibration method, Engsted et al. (2010) includes the use of an iterated GMM approach to estimate all the parameters of the model.

Moller (2009) finds that the habit persistence model, along both cross-sectional and time-series dimensions, is valid using the Campbell & Cochrane (1999) specification for the American stock market during the 1947-2005 period. Using Danish data for the 1922-2004 period, Engsted & Moller (2009) conclude that the Campbell & Cochrane (1999) model did not perform much better than the CRRA C-CAPM. In addition, using Belgian, Canadian, French, German, Italian, Swedish, British and American data, Engsted et al. (2010) find that the Campbell & Cochrane (1999) model is unable to consistently explain cross-country stock returns and results are not consistent in any of the countries. However, Engsted et al. (2010) find that the Campbell & Cochrane (1999) model in Belgium, Italy, Sweden, the UK and the American has some empirical support in various dimensions, such as parameters that are economically plausible and/or predictable stock returns which are statistically significant. Canada and France have mixed results with some economically plausible parameter estimates, but Germany has no empirical support for the Campbell & Cochrane (1999) model. In addition, Hyde et al. (2005) test the CRRA and Campbell & Cochrane (1999) C-CAPM for France (1971-2000) and Germany (1965-2000) and find that Campbell & Cochrane's (1999) model yields smaller pricing errors and decreases the implied RRA in comparison to the CRRA C-CAPM. Moreover, Hyde & Sherif (2005a) test the CRRA and Campbell & Cochrane (1999) C-CAPM for the UK during the 1965-2000 period find both models yield mixed results which implies that the Campbell & Cochrane (1999) model may offer few advantages over the CRRA C-CAPM.

2.3 Estimation Issues

Extensive empirical evidence analyses strategies for obtaining econometric estimates of structural parameters of rational expectations models (Tauchen, 1986). Hansen (1982) developed the GMM, which subsumes several standard econometric estimates. Since then, GMM has been widely applied to estimate economic and financial data sets (Hall, 2005).

Hansen & Singleton (1983) utilize a procedure for estimating the parameters of a nonlinear rational expectations model, where the only *a priori* information specified is a subset of the economic environment. Many standard estimators, including OLS and Instrumental Variables are considered to represent special cases of the GMM estimator.

The generality of the GMM approach has made it a popular tool in the asset pricing literature, because an efficient GMM has the advantage of consistency when heteroskedasticity is present (Baum, Schaffer & Stillman, 2003). To determine the popularity of the GMM versus other techniques such as ML, Hansen & West (2002) surveyed a limited range of articles published between 1990 and 2000. Hansen & West (2002) found that parametric linear models dominated the literature, followed by Hansen's (1982) GMM technique. The GMM has a growing influence in literature and has various applications, such as dynamic optimization models implying moment restrictions (Hansen & West, 2002).

The GMM model implies the existence of a family of orthogonality conditions that contain any theoretical economic restrictions that can be imposed or tested. Heuristically, identification requires at least as many orthogonality conditions as there are coordinates in the parameter vector, in order for estimation to occur. In the orthogonality conditions, there exists an unobservable disturbance which renders the estimation unresolved. The unobservable variables can be replaced by equivalent expressions consisting of the true parameter vectors and the observed variables (Hansen, 1982). Using this idea, the GMM estimates parameters of the model, in order to match the moment conditions of the model as closely to the observable sample moment conditions (Zhou, 1994). The moment conditions and the consequent estimated model parameters are based on the orthogonality conditions implied by the model[8] (Engsted & Moller, 2010). The GMM technique admits estimation of the unknown parameters by setting the estimating sample moment functions as close to zero. The nonlinearities are a direct result of the nonlinearity introduced in the agent's objective function and indirectly introduced through estimating the "optimal" weighting matrix (Tauchen, 1986). As early as, Hansen (1982) this problem could be addressed by setting a linear combination of sample cross products equal to zero. The resulting estimators will make

[8] In an informationally efficient market, it is assumed that there is less available information to an econometrician than the representative agents, therefore, it should not be possible to explain equity premia based on the information available to the econometrician. GMM allows an econometrician to estimate the model parameters and the moment conditions that arise in the efficient market (Jagannathan et al., 2002).

the sample moment conditions of the population orthogonality conditions as close to zero as possible according to some metric or distance measure.

The GMM is a method for estimating the parameters of an economic model under more realistic assumptions than previous estimation techniques[9] (Jagannathan et al., 2002). Despite the advantages of using the GMM technique, other methods such as ML may be more efficient. The ML method specifies the assumptions concerning the distribution of errors, therefore, all moments are incorporated whereas the GMM typically uses moment conditions generated by the SDF representation of the chosen preferences. If the distributional assumption is valid, then the ML method estimates the model parameters more efficiently than the GMM. If the distributional assumptions do not follow the known distribution, the GMM offers superior parameter estimates. Given that the probability density function of the errors encountered in the GMM technique deviates from the normal distribution for example, the GMM technique offers a means of estimating parameters which arise from non-normally distributed samples (Jagannathan et al., 2002). The ML can be computationally tedious as the likelihood function must be maximized subject to moment condition constraints for each t (Hall, 2005). Jagannathan & Wang (2002) tested the ML and GMM[10] method and concluded that the GMM is as efficient as the ML method, therefore, supporting the empirical application of the GMM.

The dynamic optimization problem of economic agents generally implies a group of stochastic Euler equations that in turn imply a set of population orthogonality conditions. The orthogonal population conditions depend in a nonlinear way on observable variables.

Following Hansen (1982), Hansen & Singleton (1982) construct samples of the orthogonality conditions close to zero, according to a certain metric. The GMM procedure uses the first-order condition (Equation 2) to estimate the unknown known parameters θ. In the Lucas' CRRA C-CAPM, GMM selects the parameter vector $\theta = (\beta, \gamma)$ (Lund & Engsted, 1996). The set of orthogonality conditions are generated and used to construct a criterion, whereby the minimiser is the estimate of θ^*[11] (Hansen & Singleton, 1982). The unpredictability of the

[9] The assumption relaxes the nature of the stochastic processes control over the temporal evolution of exogenous variables (Jagannathan et al., 2002).
[10] The GMM method when applied to the moment conditions generated by the SDF representation of the linear asset-pricing models (Jagannathan & Wang, 2002).
[11] The manner in which the criterion function is constructed guarantees that the parameter estimator is consistent, asympototically normal and has a consistently estimated asymptotic covariance matrix (Hansen & Singleton, 1982).

GMM at time t leads to the inclusion of a vector of instruments, z_t, which comprises variables known at the time the expectation is formed, time t (Tauchen, 1986). Hansen & Singleton (1982) begin with a general population framework that encompasses all cases of nonlinear function and then is extended to include model specifications.

The population moment conditions involves a function f(.,.) of the observable vector of random variables, v_t and the unknown (Kx1) parameter vector, θ_0. The (Kx1) parameter vector θ_0 is an element of the parameter space Θ, which is a subset of the p-dimensional Euclidean space $\Re^p$[12]. The population moment condition is:

$$E[f(v_t, \theta_0)] = 0 \ldots\ldots [14]$$

In order for the estimation to take place, the population moment conditions must provide enough information to determine θ_0 uniquely. θ_0 is only uniquely determined if the moment condition does not equal zero for all other values of θ, therefore, the population moment condition can be said to identify θ_0. The parameter space at which the population moment condition is equal to zero is denoted by θ_0 (Hall, 2005).

The GMM specification can be classified as under-identified, just-identified and over-identified. The under-identified case, where there are fewer moment conditions than parameters (P<K), will lead to finitely many solutions, whereas there are no solutions in the over-identified case when there are more moment conditions than parameters (P>K). In the just-identified case, the number of moment conditions, P, is equal to the number of unknown parameters, K, to be estimated thus there is a unique solution. It is possible to set the sample average of the moments equal to zero in the just-identified case, but this is not feasible in the over-identified or under-identified case (Imbens, 2002). The GMM splits the original moment conditions into the just-identifying restrictions and the over-identifying restrictions. The just-identifying restrictions contain information actually used in the estimations, whereas the over-identifying restrictions represent the remainder.

The GMM estimations yield two fundamental statistics that are associated with these restrictions. The estimator θ^* is a function of the information contained in the identifying

[12]Restrictions are placed on v_t and $f(.,.)$ whereby v_t follows a strictly stationary process therefore implying that all expectations of functions of v_t are independent of time and $f(.,.)$ contains regular conditions (Hall, 2005).

restrictions, whereas the estimated sample moment $g(\theta^*)$ is a function of the information contained in the over-identifying restrictions (Hall, 2001).

Given information at time t, the moment condition for the data and the parameters:

$$g(\theta) = E[f(\theta, z_t, R_{t+1}, \propto_{t+1})] = 0 \ldots\ldots [15]$$

where $g(\theta)$ is an (Mx1) vector of orthogonality conditions. In Lucas' (1978) CRRA C-CAPM the vector of random variables is $v_t = (z_t, R_{t+1}, \propto_{t+1})$, where $\propto_{t+1} = C_{t+1}/C_t$, consumption growth rate. Lund & Engsted (1996) state that the sample counterpart vector that corresponds to the vector of population orthogonality conditions (Equation 15) is[13]:

$$g_T(\theta) = \frac{1}{T}\sum_{t=1}^{T} f(\theta_0, z_t, R_{t+1}, \propto_{t+1}) \ldots\ldots [16]$$

If θ^* are the true preference parameters of the representative agent in Lucas' (1978) model, then the first order conditions imply that:

$$E\{f(\beta^*, \gamma^*, z_t, R_{t+1}, \propto_{t+1})\} = 0 \ldots\ldots [17]$$

Therefore, $g(\theta^*)$ tends towards zero with probability one, as the number of observations grows larger (Kocherlakota, 1990). In the just-identified case generally, θ^* can be estimated by replacing expectation with a sample average that yields a unique solution by solving Equation 18 (Imbens, 2002). Hansen (1982) defined the GMM estimator as the value of θ that generalizes the optimization problem to the minimization of the quadratic form, thereby minimizing the distance:

$$Q_T(\theta) = g_T(\theta)' W_T g_T(\theta) \ldots\ldots [18]$$

where W is a positive, definite $M \times M$ symmetric matrix and $\hat{\theta} = (\hat{\beta}, \hat{\gamma})$ is an asymptotically efficient estimator given the orthogonality conditions. Hansen (1982) shows that if:

$$W = W^* = \{Ef(f(\theta, z_t, R_{t+1}, \propto_{t+1}) f(f(\theta, z_t, R_{t+1}, \propto_{t+1})'\}^{-1} \ldots\ldots [19]$$

Then by definition the GMM estimator is given by:

$$\hat{\theta} = argmin_{\theta \epsilon \Theta} Q_T(\theta) \ldots\ldots [20]$$

[13] To apply the GMM method, $f(\theta, z_t, R_{t+1}, \propto_{t+1})$, must be stationary for all parameter values (Lund & Engsted, 1996).

where $argmin$ is the value of $argument - \theta$ which minimizes the function $Q_T(\theta)$ (Hall, 2005). As W^* is not observable to the econometrician, the estimator is generally not feasible (Imbens, 2002). In the just-identified case, the choice of W is insignificant because $\hat{\theta}$ will be equal to the value that sets the average moments exactly equal to zero[14] (Imbens, 2002). In the over-identified case, the choice of weighting matrix is significant. The statistically optimal choice for W is given by Hansen (1982) as the inverse of the asymptotic covariance matrix of the moments or the Spectral Density Matrix denoted by SD, $W^* = {SD_T}^{-1}$, given by $T^{-\frac{1}{2}} g_T({\theta_T}^*)$. Hansen (1982) shows that under general conditions if $\hat{\theta}$ is a consistent estimator of θ^* then SD^{-1} is a consistent estimator of W^*. It is optimal in that it yields θ with the smallest asymptotic variance.

Utilizing the foregoing result, Bach & Moller (2011) estimated the GMM that is based on two weighting matrices - the identity matrix and the statistically optimal weighting matrix. Bach & Moller (2011) state that the purpose of using an identity matrix is economically appealing as it allows the GMM to pay equal attention to all moment conditions. Therefore, enabling a comparison of the magnitude of the estimated pricing errors across different consumption measures applied. A statistically optimal weighting matrix does not allow for such a comparison as it places a higher weight on the linear combination of moments with the lowest variance, which may change with different consumption measures. The statistically optimal weighting matrix does provide efficient estimates which the identity matrix does not (Bach & Moller, 2011).

Since a consistent estimator of θ^* is needed to compute the optimal weighting matrix, an iterated GMM procedure is often applied (Lund & Engsted, 1996). Hansen (1982) provides a suitable solution to obtain an initial but consistent estimate of θ^*, although the estimate is generally inefficient. The estimate can be calculated by minimizing $J(\theta)$ using an arbitrary positive definite $P \times P$ weighting matrix, like the identity matrix of dimension P (Imbens, 2002). The first estimator can be understood as the second element in a series of estimates converging to the iterated GMM estimator. The consequent estimator $\widehat{\theta(1)}$ is used to calculate the efficient GMM estimator θ^* from the optimal weighting matrix $W^* = SD^{-1}$.

This process can be iterated by obtaining the residuals from the two-step GMM and using the results to calculate a new $\widehat{SD}$. The new $\widehat{SD}$ can be used to calculate the three-step feasibly

[14] At least in large samples (Imbens, 2002).

efficient GMM estimator (Baum et al., 2003). The process can be iterated further to the point where the estimator converges to $\hat{\theta}^*$ in the n staged GMM (Lund & Engsted, 1996). Kocherlakota (1990) provides an algorithm for the n staged GMM. The minimization routine always starts the search for the true parameters through the construction of the J-statistic for each data set using the Y-step estimator described in the following algorithm:

Let $W(0) = I$ (identity matrix)

(1) $[\widehat{\theta(n)}] = \min_\theta g_T(\theta)' W[n-1]\, g_T(\theta)$;

(2) Set $W_T(n) = W_T[\theta(n)]$. If max $|W_T(n) - W_T(n-1)| < \epsilon$ or $n \geq Y$ go to (3), otherwise go to (1);

(3) $\left(\hat{\theta}^*\right) = (\theta(n))$.

where ϵ is an arbitrarily small positive real number with a typical value for ϵ being 10^{-6} (Hall, 2005). Using the above steps, Ferson & Foerster (1994) examine both the two-staged and iterated GMM estimators as authors generally employ one of the two[15]. In simple models (few equations and instruments) the two-step GMM is reliable, the coefficient estimates are approximately unbiased and the goodness-of-fit statistics conform well to the asymptotic distribution. The two-step and iterated GMM procedure is employed in more complex models, but with a small sample. Ferson & Foerster (1994) find that the CRRA C-CAPM is rejected too often when applying the two-step procedure, whereas the iterated GMM test statistics conform more closely to the asymptotic distribution. The coefficients estimated using the iterated GMM can still be unreliable in small samples. Therefore, Ferson & Foerster (1994) concluded that the iterated GMM outperforms the two-step GMM.

Hall (2005) states the two-step is sufficient to obtain the efficiency bound, but after simulating data it was found that the iterated procedure can offer considerable gains. Gains include improvements in the quality of asymptotic theory, assuming that the large sample properties of the GMM estimates are true in small samples, but this can also lead to "over rejecting" the model. Ferson & Foerster (1984) and Kocherlakota (1990) find that the small sample properties of the two-step estimator performs worse that the iterated GMM estimators. In practice, Y is often set to equal 2 but Kocherlakota (1990) set Y to equal 70 to give the GMM technique the best chance of reaching an optimal estimation through iteration.

[15] Both the two-staged and iterated GMM estimators have the same asymptotic properties (Ferson & Foerster, 1984).

In calibrating the model with Y, Kocherlakota (1990) tested seven different GMM estimators differentiated by returns and instruments using an artificial economy calibrated to conform to American asset pricing and consumption data, where there exists a single agent with a high RRA. Kocherlakota (1990) found that for several commonly used GMM estimators the J-test tends to 'over reject' the over-identifying restrictions in the model.[16]

Given that the CRRA model is associated with a matrix of rank two,[17] implies that z_t requires at least two elements. Although the selection of z_t can affect the parameter estimates, as the latter can vary with the choice of z_t (Hall, 1993). In addition, there is little guidance from the previous literature with regards to choosing lag lengths in the formation of the instruments (Tauchen, 1986). The most common set of instruments using financial market data consists of current and lagged values of the consumption growth rate, asset rate of returns and unity (Tauchen, 1986). Lund & Engsted (1996) tested several different sets of instrumental variables, whereby the first set contains a constant and lagged consumption growth rates and returns, whereas the second set extends set one to include two lags. The parameters appear less plausible from the inclusion of additional lags, which may be the result of a loss of power when instruments lagged by two periods are included. As an extension, Cashin & McDermott (1998) test the CRRA C-CAPM for Pakistan, Jordan and Turkey using several sets of instruments consisting of the rate of consumption growth and returns, lagged up to five periods. The plausibility of the parameter estimates are invariant to the choice of lag length. However, for Pakistan the optimal lag length is determined to be one whereas Jordon and Turkey had an optimal lag length of four.

Ferson & Constantinides (1991) state that it is preferable to use unrelated variables as instruments, because measurement errors may lead to biased parameter estimates and the over-identifying restrictions may be spuriously rejected. Lund & Engsted (1996) consider the unrelated variables argument and use an instrument set containing the two-lagged values of

[16] Hall (2005) states that there is evidence, motivated by the development of an alternative method of attaining efficient bounds, that the asymptotic approximation of the two-step and iterated GMM may be poor in some circumstances. Hansen, Heaton & Yaron (1996) tested the two-step and iterative GMM estimators and also introduce an alternative estimation method termed the continuous-updating estimator. Unlike the iterative process, which takes the weighting matrix as given in each step of the GMM estimator, the continuous-updating estimator takes the covariance matrix as a continuously changing as $\theta$is changed in the minimization step. The first order conditions for the minimization problem have altered to include an extra term in comparison to the fixed weighting matrices. According to Hansen et al. (1996), the extra term does not distort the estimators limiting distribution. The added advantage of this estimator is that it does not change depending on how the moment conditions are scaled. Hansen et al. (1996) conclude through a Monte Carlo procedure that the continuous-updating estimator might have a lower bias relative to other GMM estimators.

[17] See Equation 3.9, page 56 of Hall (2005).

the dividend price ratio. Lund & Engsted (1996) conclude that the estimates of RRA vary considerably with different definitions of instrumental sets. Tauchen (1986) finds large divergences in the estimation of parameters, which can be encountered in practice when different lag lengths are used in the formation of instrument sets. As the number of instruments increase, the estimators' variance decreases around increasingly biased parameter values. Tauchen (1986) concludes that estimators can perform reasonably well, but appear to be sensitive to the number and choice of instruments within a set, therefore, the GMM performs better when the number of instruments is limited.

When the number of orthogonality conditions exceeds the number of parameters to be estimated, tests of the restrictions implied by the model are available[18] (Hansen, 1982). Mis-specification may lead to inconsistent results, therefore, rendering all subsequent inferences misleading. Thus, it is imperative to test whether the model is correctly specified (Hall, 2005). From Equation 14, v_t satisfies the population moment condition, therefore, it is necessary to test whether the sample is consistent with the hypothesis that this condition holds in the population. Since K>P, the over-identifying restrictions are available to test the model specification. Hansen (1982) suggests the following specification test for over-identifying restrictions in order to determine whether the whole model is mis-specified:

$$J_T = T\left\{ g_T\left(\hat{\theta}_T^*\right) \hat{W}_T g_T\left(\hat{\theta}_T^*\right)\right\} \text{.....[21]}$$

This test is associated with the minimum chi-squared estimator with a null hypothesis of:

$$H_0 : E\{f(v_t, \theta_0)\} = 0 \text{.....[22]}$$

In Equation 21 correct model specification implies that the J-statistic is chi-squared distributed with degrees of freedom equal to P-K. Hansen & Singleton (1982) assumes that in a small sample the J-statistic has an approximate chi-squared distribution with P-K degrees of freedom. They found that the model is rejected at the 5 percent level of significance thus the difference between the estimated and observed value is statistically significant.

Rejection of the model can occur if the sample size is small as is identified by Hall (2005). Hall (2005) identifies two main approaches in the analysis of mis-specified nonlinear

[18] As already noted, the asymptotic model parameter estimation sets K linear combinations of P sample moments equal to zero. Therefore there are P-K linearly independent combinations of the orthogonality conditions, when the model is true, that are not set to zero but should be close to zero (Hansen, 1982).

dynamic models. The approaches are based on the non-local and local form of mis-specifiation. In the non-local form, there is no value of θ for Equation 16 to hold and the 'size' of the mis-specification is the same for all t for any T. Thus, if the model is incorrect then the situation will not change regardless of a sample size increase. The local mis-specification is found when the model is mis-specified for finite T, but mis-specification decreases as T increases, therefore, the model is correct in the limit. The sequence of processes that generate the data become closer to satisfying Equation 22 in the limit as T increases. When the C-CAPM is over-identified, Hall (2005) states that it is not possible to set every moment equal to zero. The size of the sample is dependent on information available on per capita consumption, the consumer price index and returns on the relevant share index.

The estimation of the C-CAPM requires information on per capita consumption, the consumer price index and returns on the relevant share index. The factors selected for the various C-CAPM models require *a priori* information as in Chen et al. (1986). Various inter-related consumption data issues arise in the estimation of the C-CAPM, according to Campbell (2003). These include, time-averaging, measurement errors, seasonal adjustment and whether durable goods should be included. The reported consumption data is subject to measurement problems not found in stock indices (Campbell, 2003).

Estimation and testing is complicated by the C-CAPM's predictions which are related to the instantaneous flow of consumption and the point-in-time asset values, whereas the available data is a time-average of the consumption flow (Campbell, 2003). Campbell (2003) finds that the contemporaneous correlation between real consumption growth and stock returns is sensitive to the timing convention. The estimate of RRA may be substantially biased, because of problems with time-aggregation (Grossman, Melino & Shiller, 1987). The C-CAPM estimates the asset price with respect to changes in the aggregate consumption rate between two intervals of time. The C-CAPM is constructed by assuming that consumption decisions only occur once during the two intervals (Lund & Engsted, 1996). The measured aggregate consumption is closer to the average of the period of time than the spot consumption, thus a "summation bias" arises. Smoluk & Neveu (2002) agree that employing aggregate consumption data might cause specification errors, since a significant proportion of consumers live from paycheck-to-paycheck and are, therefore, constrained in inter-temporally allocating consumption. In addition, Breeden et al. (1989) examine the measurement problem of reporting an average consumption rate rather than a point-in-time consumption rate. Breeden et al. (1989) find the summation bias lowers the variance of

measured consumption growth and creates a positive autocorrelation, even when the true consumption rate has no autocorrelation.

To attempt to mitigate the "summartion bias", Campbell (2003) finds that the "beginning-of-quarter"[19] timing convention produces higher contemporaneous correlation between stock returns and consumption growth. Moreover, Campbell (2003) observes the timing conventions have less importance when the data is measured for longer periods.

Separate to the issue of timing is the measurement of consumption. The flow of consumption is defined as per capita real consumption expenditure on nondurables and services less expenditure on education services, medical care services, housing services, personal business services and footwear. Parker (2001) mentions that this approach is valid, because utility arises from service flows from durable goods rather than from expenditures on durable goods. Breeden et al. (1989) measure consumption expenditure with nondurable and services following Hall (1978), in an attempt to minimize the measurement problems arising from the use expenditure instead of current consumption of on goods and services.

Availability of consistent consumption data on nondurables and services is limited over a longer period of time and is only available in a few selected countries. For example, Germany does not have available consumption data on nondurables and services, therefore, Hyde et al. (2005) measure German consumption expenditure as total consumers' expenditure. Hyde et al. (2005) state that consumption expenditure, generally, that omits durable goods is a better measure for time-separable preferences than total consumption. However, it is anticipated that the use of total consumption would not affect the results of the C-CAPM to a considerable extent.

Another issue relating to the measurement of consumption expenditure in the C-CAPM is that of seasonal adjustment. Seasonal adjustment, which smoothens expenditure, is only desirable if the transformed expenditure is a better representation of actual consumption (Campbell, 2003). In examining the stylized facts of seasonal fluctuations in American aggregate consumption purchases, Miron (1986) finds that the non-seasonally adjusted data obtain results for the C-CAPM, which are superior to those using seasonally adjusted data in

[19]"Beginning-of-quarter" is the timing convention wherein a given quarter's consumption data is measured at the beginning of the quarter, than consumption growth in the following quarter is divided by this quarter's consumption (Campbell, 2003).

the same model. However, English, Miron & Wilcox (1989) find that the seasonally adjusted data performs at least as well as the non-seasonally adjusted data using the C-CAPM[20].

Measurement error in the consumption data is another issue inhibiting the accurate estimation of the C-CAPM (Campbell, 2003). In re-examining Hansen & Singleton's (1983) study of the C-CAPM, Wheatley (1988) finds that by simulation, the measurement errors in consumption created a bias in the original test statistics. After correction for this bias, the C-CAPM was not rejected. Wheatley (1988) concludes that the serial dependence of a vector of asset returns and consumption growth can be biased towards rejection, if consumption is measured with error. Measurement errors are reduced as the sample set for consumption expenditure is increased. (Breedan et al., 1989).

CHAPTER 3. EMPIRICAL ANALYSIS

In order to determine whether the C-CAPM is valid for Germany, South Africa, UK and America, this chapter will consist of the estimation of the C-CAPM for these countries based on the empirical approaches discussed in Chapter 2. In Section 3.1, the data is described and the rates of change in the first four moments of the critical input data series are reported and analysed. Section 3.2 contains a description of the CRRA C-CAPM method within the context of GMM, which is to be estimated for the four countries in question. Finally, Section 3.3 consists of an analysis of the results.

3.1 Data

The C-CAPM will be estimated using the quarterly stock return and consumption data described in Table 3.1. Appendix 2 contains certain details regarding these data sets. Estimation periods will be split for Germany and the American data in order to determine whether the estimated coefficients are more plausible and associated diagnostic tests are more valid in some periods than others. Period breaks are determined according to significant changes in political regime or economic events.

[20] English et al. (1989) found that there was a bug in the software program (TSP) used by Miron (1986) to estimate the model. English et al. (1989) finds opposite results from Miron's (1986) original paper.

Table 3.1: Stock Return and Consumption Data Description

Country	Full Start and End Period	Total number of Observations	Period Break	Number of Observations in Break	Base Period for Consumer Price Index (CPI)
Germany	1970:2-2012:4	170	1970:2-1990:3	81	2010
			1990:4-2012:4	89	
South Africa	1998:3-2010:2	47			2012
UK	1988:3-2011:4	94			2005
U.S.A	1964:1-2011:2	189	1964:1-1973:4	40	1984
			1974:1-2011:2	149	

Source: Own Calculation

The period break for Germany was chosen as 1990:3 when the reunification of East and West Germany occurred (October 3, 1990). Thus, by 1990:4 the consumption decisions of East Germany are taken into consideration with West Germany (Fuchs-Schündeln, 2008). During the reunification period, East German household income levels (and therefore consumption) increased from around thirty-five percent in 1990:3 to an average value of eighty percent in 1994 (Fuchs-Schündeln, 2008). The period break for the American data occurs after the Organization for Petroleum Exporting Countries (OPEC) producers raised oil prices to a significant extent in 1973. By 1974:1 the sharpest decrease in global share prices since the Great Depression and World War Two occurred. The American economy, like most developed economies was then affected by a severe recession along with South Africa and the United Kingdom[21] (Davis, 2003).

Since the CRRA C-CAPM will be estimated for individual countries, each stock market is treated as a separate entity, which assumes that there is no integrated world capital market and the national economies are entirely closed[22] (Campbell, 2003). This assumption follows Campbell (2003) who treats each national economy and, therefore, stock market as separate entities with their own asset pricing model. It is assumed that consumers make economic decisions that occur simultaneously with the sampling interval of the data, therefore, consumption decisions during quarter t take place at the beginning of quarter t. This also follows Campbell (2003) who finds that the "beginning-of-quarter" timing convention produces higher contemporaneous correlation between stock returns and consumption growth

[21] These countries' estimated did not include the above mentioned time period.
[22] Li (2010) extends the C-CAPM to include complete market integration assumptions and Darrat et al. (2011) tested the extensions of both a partial and perfectly integrated world capital markets.

and is, therefore, less likely to lead to specification errors. Following Smoluk & Neveu (2002), we assume that consumers do not inter-temporally store items purchased.

In the measurement of consumption data for the four countries in question, the availability of data is limited for a long period of time. Therefore, consumption is measured as seasonally adjusted private final consumption expenditure. Hyde et al. (2005) acknowledge that such a measure is appropriate for time non-separable preferences, whereas time separable preferences are suited to a proxy of consumption data which omits consumption on durable goods[23]. Hyde et al. (2005) state that the utilization of total consumption expenditure will not have an observable impact on the results of the CRRA C-CAPM's estimation.

A possible estimation issue noted previously was the use of employing aggregate consumption data and the consequent conclusions producing specification errors. Smoluk & Neveu (2002) tested the C-CAPM for the American data by separating consumers according to income and indirectly testing if liquidity constraints[24] affect the C-CAPM. Smoluk & Neveu (2002) did not find any discernible pattern over income groups, therefore, this analysis will employ aggregate consumption data.

Following Hamori (1992), Chapman (1997) and Auer (2011), nominal consumption is converted to real units using the relevant seasonally adjusted CPI described in Table 3.1. Real per capita consumption is constructed using the relevant countries' interpolated population data. This measure is constructed as follows. Given that the population data is reported on a yearly basis, linear interpolation is applied to obtain a quarterly population estimate. The consumption, price and population data appearing in Table 3.1 is obtained from Thomson DataStream.

The C-CAPM will be estimated using the stock index returns of the aforementioned countries as proxies. The German stock market index used is the DAX 30, the British stock market index is the FTSE 100, the American stock market index is the S&P 500 Composite and the FTSE JSE All Share index is used for South Africa. All quantities in Lucas' (1978) model are assumed to be real as agents are supposed to interact within a pure exchange economy with no money or wealth-preserving tools for perishable goods. Thus, the C-CAPM requires real

[23] Campbell (2003) utilizes the consumption proxy which omits durable consumption but unlike South Africa, UK and the American data, Germany does not have available data on consumption expenditure of non-durables and consumption.
[24] Comparison of unconstrained (high-income) consumers to constrained (low-income) consumers (Smoluk & Neveu, 2002).

returns. As with nominal consumption, the nominal returns are converted to real units using the CPI with the base years supplied by the respective national statistically agencies and specified in Table 3.1.

The first four moments of the data sets appearing in Table 3.2, which are the main inputs into the CRRA C-CAPM (which will be estimated in Section 3.2), are reported for the following reason. The GMM method, unlike ML for instance, does not require the input parameters to follow a normal distribution which can be ascertained from the first four moments. A variable follows the normal distribution when skewness and excess kurtosis are equal to zero[25].

Rates of change in German, British and American consumption data do not follow the normal distribution. The rate of change in South African consumption data does not follow the normal distribution, but is fairly close to it. On the other hand, the rate of change in German and American stock returns do not follow the normal distribution. While the rate of change in South African stock return data does not follow the normal distribution it is fairly close to it. The rate of change in British stock return data deviates considerably from the normal distribution. For the pre-reunification period the rate of change in German consumption data is slightly more volatile and declines at a more substantial rate than during the post-reunification period. The rate of change in German consumption data is less normal during the post-reunification period. Stock returns are more volatile and positive post-reunification

[25] Excel's equation which was used in order to estimate the first four moments, has a built-in correction that will give a kurtosis of zero for a normal distribution.

Table 3.2: Descriptive Statistics

Country	Period	Rate of Change in Real Consumption Expenditure per Capita				Real Stock Index Returns			
		Mean (%)	Standard Deviation (%)	Skewness	Excess Kurtosis	Mean (%)	Standard Deviation (%)	Skewness	Excess Kurtosis
Germany	1970:2 - 2012:4	-0.49	1.20	-0.39	2.91	1.00	11.13	-0.48	1.57
Germany	1970:2 - 1990:3	-0.74	1.23	0.04	1.70	-0.12	9.76	-0.64	1.96
Germany	1990:4 - 2012:4	-0.27	1.14	-0.84	6.67	2.03	12.23	-0.50	1.33
South Africa	1998:3 - 2010:2	0.79	1.31	-0.34	-0.38	2.22	12.49	-0.65	0.11
UK	1988:3 - 2011:4	0.50	0.97	-0.77	1.45	1.01	10.62	2.70	24.72
USA	1964:1 - 2011:2	0.45	0.85	-0.64	2.52	0.80	8.10	-0.46	0.84
USA	1964:1 - 1973:4	0.78	0.87	0.19	-0.89	-0.11	6.80	-0.43	0.96
USA	1974:1 - 2011:2	0.36	0.83	-0.98	3.36	1.04	8.47	-0.50	0.80

Source: Own Calculation

and follow the normal distribution more closely. The rate of change in American consumption data post-oil price shock deviates from the normal distribution to a greater extent, is slightly less volatile and grows at a slower rate than before the oil shock. After the oil price shock American stock returns are more volatile and grow rather than decline (as in the pre-oil price shock period).

One possible reason for the lack of studies confirming the validity of the CRRA C-CAPM is that the standard deviation of consumption may be too small to warrant a large equity premium. It can be observed from Table 3.2 that the standard deviation in the rate of change in German, British and American consumption data are considerably less than the standard deviation of in the rate of change of corresponding stock return data, which would imply a demand for a large equity premium. The standard deviation in the rate of change in South African consumption data and the respective standard deviation of the rate of change in South African stock index is of similar magnitude to potentially produce a smaller equity premium.

3.2 Method

In order to estimate the CRRA C-CAPM, the GMM approach discussed in Section 2.3 will be used and is suitable for non-normally distributed data sets. Adapting the Euler Equation 8 to include a set of instrumental variables denoted by the $q \times 1$ dimensional vector, z_t yields:

$$E_t\left[(\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} R_{t+1} - 1)z_t\right] = 0 \text{.....[23]}$$

where the instruments must be uncorrelated with the Euler Equation 8 at time t. Equation 23 implies the population moment condition as stated in Smoluk & Neveu (2002):

$$g(v_t, \theta_0) = (\beta_0\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma_0} R_{t+1} - 1) \otimes z_t \text{.....[24]}$$

where $\theta_0 = (\beta_0, \gamma_0)^T$ and $v_t = (C_{t+1}, C_t, R_{t+1}^T, z_t^T)^T$ and $\otimes$ is the Kronecker product. GMM selects the parameter vector $\widehat{\theta_T} = (\beta, \gamma)^T$ in order for the sample average of moment conditions of Equation 18 (denoted by $g(\theta_T)$) to minimize Equation 23. If the number of orthogonality conditions is greater than the number of parameters to be estimated, then a test for over-identifying restrictions is employed (Smoluk & Neveu, 2002).

The non-linearity of the model results in more moment conditions than parameters to be estimated, therefore, the weighting matrix, W, determines the relative importance of the specified moment conditions. Ogaki (1993) provides a general discussion of the alternative

weighting matrices. Hansen (1982) and Cumby, Huizinga & Obstfeld (1983) write the asymptotic covariance matrix of θ as:

$$V_T = (G_T{}'W_T G_T)^{-1} G_T{}'W_T SD_T W_T G_T (G_T{}'W_T G_T)^{-1} \ldots\ldots [25]$$

where $G_T = \sum_{t=1}^{T} \frac{E[g(\theta)]}{T}$ is a vector of sample moments of $g(\theta)$ and $SD_T = \frac{1}{T}\sum_{t=1}^{T} g(\theta)g(\theta)'$. Consistent estimation of the asymptotic covariance matrix is essential for hypothesis testing (Newey & West 1978). Hansen (1982), shows under general conditions if $\hat{\theta}$ is a consistent estimator of θ^* then SD^{-1} is a consistent estimator of W^*. Optimal θ (in the sense that V_T is as small as possible) yields the least asymptotic variance. Hansen's (1982) efficient weighting matrix gives a consistent estimator of θ^*:

$$SD = \frac{1}{T}\sum_{t=1}^{T} h_t^2 z_t z_t{}^T + \frac{1}{T}\sum_{j=1}^{q}\sum_{t=j+1}^{T} h_t h_{t-j}(z_t z_{t-j}^T + z_{t-j} z_t^T) \ldots\ldots [26]$$

where $h(x_{t+1}, \hat{\theta})$ is the disturbance term and $h_t = h(x_{t+1}, \hat{\theta}) = \hat{\beta} R_{t+1}^{j} \alpha_{t+1}^{\hat{\gamma}-1} - 1$and $\widehat{\theta}$ is a consistent estimator of θ^*. The choice of a weighting matrix depends on the estimation requirements. Hansen's (1982) SD matrix has a theoretical value for lag lengths of zero and no overlapping returns. Time aggregation may cause the disturbance term ($h(x_{t+1}, \hat{\theta})$) in Equation 26 to follow a MA(1) process (Ferson & Constantinides, 1991), therefore, instrumental variables should only include variables known at time $t-1$. If the lag length is greater than zero, then the weighting matrix may not be positive definite in finite samples.

To prevent results obtained not using a positive definite weighting matrix, the Newey & West (1987) weighting scheme can be implemented. By weighing the autocovariances, a heteroskedasticity, autocovariance consistent covariance estimator is produced which guarantees a positive definite matrix. Newey & West (1987) computes the foregoing estimator using the long-run covariance with a vector moving average moment process and uses the sample autocovariances to estimate SD. Lund & Engsted (1996) did not employ the Newey & West (1987) weighting scheme, as Hansen's (1982) Equation 26 always turned out to be positive definite in the sample set. In order for Equation 26 to be a consistent estimator of an optimal weighting matrix, the Newey & West (1987) weighting matrix requires the lag length to be larger than the value implied in the MA process (Ogaki, 1993).

An optimal weighting matrix is required to estimate an optimal θ, but a consistent estimator of θ_0 is needed to compute the optimal weighting matrix. To solve the aforementioned

dependency, a two-step procedure is often applied. Previous empirical evidence (Section 2.3) reveals that the identity matrix is conventionally set as the initial weighting matrix. The estimator of θ_0 is then used to compute W^*. In the second step, W^* is used to calculate the efficient GMM estimator θ. Following Engsted et al. (2010) and Engsted & Moller (2010) the spectral density matrix in this analysis will be estimated using the Newey & West (1987) approach with a lag truncation of $l = T^{1/3}$ which Ogaki (1993) considers to be the optimal lag growth.

The iterated version of the GMM is employed in Section 3.2 as it has been shown that the estimate produces better small sample properties (Ferson & Foerster, 1994, Kocherlakota, 1990). Lund & Engsted (1996) note that at each stage of the iterative process, a non-linear function is minimized which itself requires an iterative procedure. This iterative procedure will be estimated using the quasi-Newton approach, which is discussed by Press et al. (2007). The quasi-Newton algorithm to be used in this study is the Broyden-Fletcher-Goldfarb-Shanno approach. The main aim of this quasi-Newton approach, like its counterparts, is to accumulate information from successive line minimizations, in order for *P* such line minimizations to produce an exact minimum of a quadratic form in *P* dimensions. The quasi-Newton method iteratively constructs a sequence of matrices H_i which provides a good approximation of the inverse Hessian matrix A^{-1}, with $\lim_{i\to\infty} H_i = A^{-1}$. By using Newton's method, a minimum is found by searching for a zero of the gradient of the function with:

$$\nabla f(x) = \nabla f(x_i) + A.(x - x_i) \ldots\ldots.[27]$$

where ∇ denotes the gradient of the function, *x* is an $P \times 1$ dimensional vector and *A* is of dimension $P \times P$. Setting $\nabla f(x) = 0$ the next iteration, as stated in Nocedal & Wright (2006), can be determined with:

$$x - x_i = -A^{-1}.\nabla f(x_i) \ldots\ldots.[28]$$

To get the exact minimum, the left-hand side of Equation 28 represents the requisite finite step, whereas the expression appearing on the right-hand side of the same equation arises when the $H \approx A^{-1}$ is as accurate as possible. The quasi-Newton approach differs from the Newton approach, because the former uses an approximation of the Hessian matrix of *f*, while the latter uses the actual Hessian matrix of *f*. Such an approach is superior in certain

circumstances if descent directions of *f* at x_i are considered. Descent directions are those along which *f* decreases. For Equation 28, which is also stated in Nocedal & Wright (2006) to represent a valid descent direction:

$$\nabla f(x_i).(x - x_i) = -(x - x_i).A.(x - x_i) < 0 \text{ .....[29]}$$

Equation 29 holds if *A* is positive definite although, in general, this is not assured far from a minimum. The quasi-Newton approach consists of starting with a positive definite, symmetric approximation to *A* and then constructing the approximate H_i's so that H_i remains positive definite and symmetric. Near the minimum, Equation 28 approaches the true Hessian with quadratic convergence of Newton's method (Press et al., 2007).

Wooldridge (2001) states that instruments must be carefully chosen for valid GMM estimators to be obtained. Thus, in this study, the instrumental variables chosen relate to the data set and the fewest possible instruments are chosen in recognition of Tauchen's (1986) findings (discussed earlier). The set of instrumental variables used in this study, which consist of a constant (unity) and lagged values of return and consumption growth rates, is the same as the set used by Lund & Engsted (1996) and similar to those used by Hansen & Singleton (1982), Cashin & McDermott (1998), Boldrin, Christiano & Fisher (2001), Hyde & Sherif (2005a) and Engsted & Moller (2009).

To test the significance of the model, the standard errors and p-values of the parameter estimates are reported. The non-linear Euler Equation 24, is over-identified and, therefore, the J-test needs to be applied in order to test the validity of the model as a whole. The model validity needs to be tested, because it is not possible to equate every moment to zero. The J-test can be used in order to determine whether the given model's moment conditions are significantly different from zero (Engsted et al., 2010). The computed J-test statistic can be compared to the critical value which has a chi-square distribution. The degrees of freedom equal the sum of the number of instruments in the set minus the number of parameters to be estimated (Hamori, 1992). The asymptotic covariance matrix of the GMM estimator, as stated in Lund & Engsted (1996), is given by:

$$cov(\theta) = \frac{1}{T}\left[\frac{\partial g(\hat{\theta})}{\partial \theta}^T SD^{-1}\frac{\partial g(\hat{\theta})}{\partial \theta^T}\right]^{-1} = \frac{1}{T}(D^T SD^{-1}D)^{-1} \text{.....[30]}$$

The model is rejected if the pricing errors are large. To be specific, the model's estimated returns and observed returns are significantly different from each other. Besides the use of formal tests of the model an alternative to the J-test, is the Hansen & Jagannathan (1997) mis-specification measure (HJ). Hansen & Jagannathan (1997) devise the following measure of mis-specification of the CRRA C-CAPM (as well as other asset pricing models):

$$min_M \|M_{t+1} - M\| \ldots\ldots [31]$$

where $\|.\|$ represents the Euclidean norm. The maximum pricing errors also represent the least squares distance between the SDF M_{t+1} of a given model and the set of true SDFs that correctly assign prices to the vector of assets (Hansen & Jagannathan, 1997).

The HJ model provides a useful economic measure of model fit and is given by the HJ distance (Engsted & Moller, 2010):

$$HJ = [E(M_{t+1}R_{t+1} - 1)^T E(R_{t+1}R_{t+1}^T)^{-1} E(M_{t+1}R_{t+1} - 1)]^{1/2} \ldots\ldots [32]$$

HJ can be rationalized as the maximum percentage pricing error, therefore, if HJ=0.25 the asset prices implied by the model deviate from observed prices by approximately 25 percent (Hyde et al., 2005). The chi-squared statistic is a quadratic form in the pricing error vector $E(M_{t+1}R_{t+1} - 1)$. The distance matrix is the quadratic form, $E(R_{t+1}R_{t+1}^T)^{-1}$, which is invariant to the choice of proxy and is independent of the matrix computed in the large sample chi-square test (J-test). The J-test is computed using the null hypothesis wherein the pricing error vector is zero (Hansen & Jagannathan, 1997). The HJ measures the maximum percentage pricing errors associated with a given model and is comparable across models irrespective of preferences (Engsted & Moller, 2010).

Hyde et al. (2005) computed the HJ distance for both annual and quarterly French and German share data. The HJ distance decreases as the data reporting period decreases. For example, during the 1965-2002 period for the German CRRA C-CAPM assuming different values of the RRA, the minimum distance is between 52.53 percent and 64.52 percent. However, the quarterly data produces a minimum distance between 6.93 percent and 7 percent. In this study given the use of quarterly data, it is expected that the minimum distance will be less than obtained by Hyde et al. (2005) results which were derived from annual data.

Other studies such as Engsted et al. (2010) computed the CRRA C-CAPM with quarterly Belgian, Canadian, French, German, Italian, Swedish, British and American data and

obtained lower values for the HJ measure than Hyde et al.'s (2005) results, which were obtained from annual data. Engsted et al. (2010) find that the distance varies from 1.44 percent for Italian data to 20.42 percent for the British data. Given the HJ test calculates the distance between the actual share price and the estimated share price, a large variance in results across countries, vitiates the estimation technique.

3.3 Results

The CRRA C-CAPM parameter estimates obtained via the iterative GMM procedure appear in Table 3.3 (below). This model was estimated with an initial starting subjective discount rate (β) of 0.99, which appears to be the largest value tested in C-CAPM models (Hyde et al., 2005). The coefficient of relative risk aversion (γ) is set equal to 10, which is considered the maximum plausible level by Mehra & Prescott (1985). *A priori*, the following must hold:

$$\text{Subjective discount rate: } 0 < \beta < 1$$

$$\text{Measure of relative risk aversion: } \gamma > 0$$

Lund & Engsted (1996) state that realistic values of β should lie within the 0.95 to 0.98 range. From Table 3.3, the German β for all periods falls within the *a priori* bounds and adhere to Lund & Engsted's (1996) realistic parameter range. Prior to re-unification, German consumers preferred current consumption to future consumption, which is indicated by a higher subjective discount rate in comparison to the post re-unification period. The values of β being greater than one for South Africa and Britain might be construed as representing a preference for current consumption over future consumption, if the estimate is correct. The value of β exceeding one for the American data during the entire period and the pre-oil shock period can be interpreted similarly. However, during the post-oil shock period the β falls within the *a priori* bound (but only slightly).

The standard deviation for the subjective discount factor is low, between 0.009 and 0.03 indicating that the CRRA is estimated with more certainty, except for the American period 1963:4-1973:4 and the UK subjective discount factor, which have a slightly higher standard deviation of 0.24 percent and 0.26 percent respectively.

All of the countries' subjective discount rates are not significant, except Germany for the whole period and the post re-unification period, where the subjective discount rate is significant at 10 percent and 5 percent level, respectively. Since the German β values for all

periods, except prior re-unification, are significant we conjecture that in comparison to other countries' β values (which we acknowledge are not statistically significant), German consumers appear to be more patient than consumers in the other sampled countries. This conjecture is based on Wang et al. (2011) who find that Germans (out of 45 countries sampled in a choice experiment) are the most patient when given the choice of receiving a sure amount now versus receiving a greater amount in the future. This behaviour can be extrapolated to German consumption behaviour.

Inspection of Table 3.3 reveals that in every case $\gamma > 0$ as required. Cuthbertson & Nitzsche (2004) state that γ should be in the range of five to eighteen based on experimental data. For all time periods, German $\gamma < 5$ implying that the German consumers are relatively risk averse, which is consistent with Wang et al. (2011) in that German consumers are relatively patient. However, such a result falls outside of Cutherbertson & Nitzsche's (2004) plausible γ bounds. It appears that German consumers were more risk averse prior to re-unification than after re-unification, but this may be due to the fact that East German consumers' behaviour were introduced into the data set after re-unification. All German γ coefficients are not statistically significant at all levels.

South African investors appear to have a relative risk preference given that γ is slightly less than five, although the coefficient is also not statistically significant at all levels. If the coefficient was significant the results may be attributed to a high proportion of South Africa's population being impoverished compared with countries in the sample[26] (although, as we shall observe later the American data γ values are lower).

British consumers appear to be relatively risk averse although the coefficient is not statistically significant. If this coefficient is valid then it may be argued it is consistent with Wang et al. (2011) finding that British consumers are more patient than the average consumer based on a cross country comparison (note that German consumers were considered to be much more patient than British consumers in the study by Wang et al. (2011)).

American consumers are relatively less risk averse, with a statistically significant γ only at the ten percent level. American consumers appear to become more risk averse after the oil shock, which is intuitively plausible but γ is only statistically significant at the ten percent level post-oil shock. Lund & Engsted (1996) note that a common characteristic in most

[26] May (2010) discusses South African poverty trends during the relevant period.

consumption-based asset pricing model studies is the inability to obtain precise estimates of γ.

The CRRA C-CAPM was re-estimated for different values of the β in order to check whether initial impatient levels affect parameter estimates compared with the initial estimation. The change in results was negligible across all countries. Hyde et al. (2005) find that when estimating the CRRA C-CAPM for France and Germany for different $\beta = \{0.95, 0.97, 0.99\}$ there is no qualitative difference between estimations.

The foregoing results can be considered from the perspective of Lund & Engsted's (1996) assertion that estimates of β and γ are negatively correlated, therefore, unrealistic estimates of β imply unrealistic estimates of γ. For example, in the South African case $\beta = 1.0074$ is associated with $\gamma = 4.2617$ which both lie outside of the bounds suggested by Lund & Engsted (1996) and Cuthbertson & Nitzsche (2004), respectively.

All estimations are done with the Grocer 1.55 add in to Scilab 5.4.1. In Appendix 3 an example of the code for the estimation and testing of the German CRRA C-CAPM is provided. In the disc appended to this study the Excel spreadsheets of the consumption and asset return data for each country is supplied as well as the salient results of the estimation. Note that 1000 iterations are applied for all countries. The Newey & West (1987) weighting matrix is used and the number of lags differ per country. The number of lags are chosen following Ogaki (1993), whereby the cubed root of the number of observations is used. Specifically, six lags will be used for the German and American data sets whereas three and five lags are used for the South African and British data sets, respectively.

Table 3.3: C-CAPM Results

β	s(β)	*t*-stat	*p*-value	γ	s(γ)	*t*-stat	*p*-value	*J*-test	*p*-value	HJ
Germany 1970:2-2012:4										
0.9651	0.0192	-1.8212	(0.0686)***	4.7909	3.6782	1.3025	0.1927	0.0489	(0.0686)***	0.0378
Germany 1970:2-1990:3										
0.9798	0.0172	-1.1782	0.2387	2.3999	2.1483	1.1172	0.2639	0.6448	0.2639	0.0102
Germany 1990:4-2012:4										
0.9682	0.0131	-2.4259	(0.0153)**	3.9263	3.4211	1.1477	0.2511	1.5013	(0.0153)**	0.0647
South Africa 1998:3-2010:2										
1.0074	0.0307	0.2410	0.8096	4.2617	2.9235	1.4577	0.1449	0.3961	0.8096	0.1158
UK 1988:3-2011:4										
1.0236	0.2620	0.9012	0.3675	5.4396	3.9234	1.3864	0.1656	0.933	0.3675	0.0661
USA 1963:4-2011:2										
1.0040	0.0096	0.4149	0.6782	2.5584	1.3912	1.8390	(0.0659)***	1.0756	0.6782	0.0471
USA 1963:4-1973:4										
1.0146	0.2480	0.5902	0.5551	1.6104	2.9282	0.5499	0.5824	0.0017	0.5551	0.0702
USA 1974:1-2011:2										
0.9998	0.0100	-0.0184	0.9853	2.8105	1.6331	1.721	(0.0853)***	0.4919	0.9853	0.0425

Source: Own Calculations. Note: Table 3.3 reports the CRRA power utility model of Lucas (1978) parameter estimates using the iterated GMM approach described in Section 2.3 with the standard errors as well as the test statistic reported on both parameters. Hansen's (1982) *J*-test of over-identifying restrictions, computed as in Equation 22, is reported with the asymptotic *p*-value. HJ is the minimum distance test of Hansen & Jagannathan (1997), as computed in Equation 32. The *p*-values for the respective parameters and *J*-test analysed for statistical significance and are reported in parenthesis with the levels of significance denoted by: * (significant at the 1 percent level), ** (significant at the 5 percent level), *** (significant at the 10 percent level).

Recall from Section 3.2 that the *J*-test is applied in order to test the validity of the CRRA C-CAPM. The *J*-test and *p*-values for the *J*-test are reported in Table 3.3. Specifically, the purpose of the *J-test* is to ensure that the components of the θ vector in Equation 30 are as small as possible in order for the model's estimated returns to be as close as possible to the observed returns. The null and alternative hypotheses, stated by Lund & Engsted (1996), are:

$$H_0 : g(\theta) = 0$$
$$H_A : g(\theta) \neq 0$$

The CRRA C-CAPM is not accepted at the one, five and ten percent significance levels if the J-test statistic is greater than the chi-squared statistic (H_0 is not accepted). Engsted et al. (2010) state that if the parameter estimates seem economically implausible, but the *J*-test of over-identifying restrictions does not statistically reject the model at conventional significance levels, this may be the result of the low power of the test.

The model is rejected at the five percent level for Germany during the post re-unification period and at the ten percent level for Germany during the whole period, therefore, valid inferences cannot be made. The German (prior re-unification), South African, British and American (all periods) CRRA C-CAPM are not rejected at the one, five and ten percent levels of significance indicating that valid inferences are possible.

Cochrane (2005b) and Hansen & Jagannathan (1997) state that if the CRRA C-CAPM is not rejected by the *J*-test, this does not necessarily imply low pricing errors. Engsted & Moller (2010) consider HJ pricing errors to be low at levels of less than ten percent. The minimum distance for Germany, the UK and the American is low by these standards whereas the South African pricing error is greater than ten percent. For Germany, the minimum distance increases after re-unification by 5.45 percentage points, which implies that the CRRA C-CAPM better fits the pre re-unification period data. The minimum distance for America is slightly lower after the oil shock than during the preceding era (2.77 percentage points).

This models' weak results prompt further inquiry into the consumption-stock return nexus as the results herein may be due to model mis-specification among other deficiencies described in section 2.3.

CHAPTER 4. CONCLUSION

Despite the plausibility of a relationship between asset returns and consumption expenditure, which has led to the development of several consumption-based asset pricing models (like the CRRA C-CAPM and its variants) cross-country empirical analysis of such models yield inconclusive results. However, the CRRA C-CAPM has generally lacked consistency with observed stock price formation patterns. Despite this, it remains useful as a measure of systematic risk (Lettau & Ludvigson, 2001). Evidence of this usefulness arises from this model still being estimated into the second decade of the twenty-first century by, for instance, Engsted et al. (2010), Hassan & Van Biljon (2010) and Auer (2011). An important consideration in undertaking this research agenda is that the estimation of the model must occur for the widest possible range of countries across long time periods. With respect to time periods, it is also important to estimate the CRRA C-CAPM during relatively short time periods by dividing data sets and using the most recent available data.

In recognition of the aforementioned ideas, the contribution of this study to the extant literature, concerning the CRRA C-CAPM, is that this model is estimated for several countries, during relatively short periods of time, with more recent data sets than were used in previous studies of this model where, the data sets related to the same countries.

Thus the CRRA C-CAPM was estimated for Germany, Britain, South Africa and the American data in order to obtain the coefficient of the Relative Risk Aversion (γ) and the subjective discount rate (β). The estimation occurred using the GMM approach. Specifically, the Newey & West (1987) weighting matrix was used as the optimal weighting matrix with the lag truncation set to the values suggested by Ogaki (1993). When the estimated returns were compared to the actual returns, the accuracy of the CRRA C-CAPM model could be assessed.

It was found that the German β for all periods falls within the realistic parameter range, but is only statistically significant for the whole period and the period prior re-unification. The estimation of β has indicated that German consumers appear to be more patient than consumers sampled in other countries, based on the Wang et al. (2011) study. The remaining countries' β values fall within Lund & Engsted's (1996) realistic bounds and *a priori* bounds and are not statistically significant. The only exception is for America after the post-oil shock (β falls within the *a priori* suggested by Hamori (1990) among others, although β still

remains statistically insignificant). South Africa's β fell outside of Lund & Engsted's (1996) bounds and was also not statistically significant at all levels.

For the American γ coefficient, in the case of the whole period and post-oil shock period, it is statistically significant, but falls out of bounds specified by Cutherbertson & Nitzsche (2004). The British γ fell within the aforementioned bounds but was not statistically significant. The estimation of γ indicated that German consumers were more risk averse prior to re-unification, but this may be attributed to the inclusion of East German consumers into the data set after re-unification. South Africa's γ coefficient was not within Cutherbertson & Nitzsche (2004) bounds and was not statistically significant at all levels.

The *J*-test and Hansen & Jagannathan (1997) minimum distance measures were applied to all time periods to test the validity of the model and how far the estimated results deviate from the actual results. Each CRRA C-CAPM was not rejected at the one, five and ten percent levels of significance, indicating that valid inferences can be made for Germany (prior re-unification), South Africa, the UK and America (all periods). However, the β values in most of these cases are implausible thus indicating a low power of the test (Engsted et al., 2010).

In the aforementioned countries, the *J*-test was not rejected, but this does not necessarily imply minimal pricing errors which are evident in the CRRA C-CAPM test for South Africa, which has a pricing error of 11.58 percentage points. The remaining countries' pricing errors are minimal in accordance with Engsted & Moller (2010). Therefore, the use of HJ reveals that the CRRA C-CAPM may yield accurate results, but a combination of all inferences made imply otherwise. In order to make valid inferences, further tests should be conducted, as the *J*-test is a general specification test of the CRRA C-CAPM model.

Like Lund & Engsted (1996), Cashin & McDermott (1998), Chen (2003) and Hassan & Van Biljon (2010) we find that the CRRA C-CAPM does not accurately represent the hypothesised relation between stock returns and consumption growth.

There are several suggestions for further research. The CRRA C-CAPM is the basis of the literature on the equity premium puzzle, therefore, another area of investigation is to determine if this study would aid in the explanation of the aforementioned countries' equity premiums.

As mentioned earlier, subsequent studies of the CRRA C-CAPM should include to account for certain forms of misspecification, which cannot be detected by application of the *J*-test.

For example, to test for structural instability or non-constancy of the parameter vector over time (Lund & Engsted, 1996).

Lund & Engsted (1996) test for structural instability using the test developed by Ghysels & Hall (1990). This test is based on the sample moment conditions from one sub-sample being evaluated at the parameter estimates in the second sub-sample. This test requires prior knowledge of the structural breaks, but this is not always possible. Therefore, Andrews & Ploberger (1994) extends the initial stability tests to allow for unknown break points. Another potential structural stability test is the Hall & Sen (1999), approach which is utilized by Hyde & Sherif (2005b). This test decomposes the structural stability null hypothesis into orthogonal conditions consisting both identifying and over-identifying restrictions and determines when instability is solely in the parameters or when the instability affects other aspects of the model. A possible suggestion for further research is to test for a structural break at the assumed break dates in this study (re-unification and the oil price shock in the German and American data respectively). Future research can also include tests for instability in the unknown periods using the aforementioned tests.

The assumption of CRRA preferences may not capture actual stock price movements, therefore, the introduction and estimation of alternative preferences is needed. For example, the habit-persistence preferences take the past consumption decisions into consideration in the SDF. Specifically, the Campbell & Cochrane (1999) C-CAPM as this habit preference is the most intuitively appealing in that the risk aversion moves counter-cyclically and return predictability is based on the surplus consumption ratio (Engsted et al., 2010). Hyde & Sherif (2005b) testing the Campbell & Cochrane (1999) model for British data, finds the model describes observed data and further recommend the specification in testing the C-CAPM.

To account for greater international financial integration, stock prices from all countries may be determined in the C-CAPM by a common stochastic factor (Li & Zhong, 2005). Such a study would require the assumption of complete international market integration and complete consumption sharing (although tests of incomplete market integration and consumption sharing may also be useful).

Finally, the C-CAPM can be extended to include a distinction between an asset-holder's consumption and a non-asset-holder's consumption. Further research can account for the limited participation consumption by conducting surveys of a large representative sample within the aforementioned countries during the longest possible period of time.

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APPENDICES

Appendix 1: Literature Table

Author	Year	Theoretical = 0, Empirical =1	Countries	Period	Does CCAPM relationship hold	Range of risk aversion	Range of rate of time preference	Model Specification	Method	Weighted matrix	Data Used	Instruments
ABEL	1990	0	NA	NA	NA	NA	NA	Abel habit preference model	NA	NA	NA	NA
AUER	2011	1	GERMANY	1988-2001	Parametric analysis (GMM) shows that both the CRRA and CC model are not rejected but require high risk-aversion to be consistent with the data.	40.18 (CRRA), 85.09 (CC).	1.0027 (CRRA), 0.7391 (CC).	Power utility model, cc habit preference model	GMM & Nonparametric approach of Hansen & Jagannathan (1991)	Identity matrix	Following Engsted et al (2010) German consumption is measured as seasonally adjusted private total consumption. real per capita consumption constructed using interpolated annual population data.(from IMF international Financial statistics). Investment funds chosen: German institute BVI offers a database of German and foreign investment funds mainly sold to German investors. - fund returns are calculated based on data extracted from Thomson Financial datastream.	NA
AUER	2013	1	Germany	1981-2009	SDF that varies with money aggregates can have reduced pricing errors versus models with constant parameters. However large portion of cross-sectional variation remainds unexplained.	-37.83 to -31.42	0.98	simple C-CAPM using monetary conditioning information	GMM	Identity matrix where the spectral density matrix is the Newy & West (1987) estimator with Bartlett kernel	Consumption is measured as seasonally adjusted private total consumption expenditures. Analyse cross-section of German stock porfolios therefore use stocks listed on the Frankfurt Stock Exchange that are traded in Euro and the ISIN beings with DE, to construct portfolios	NA
AUER	2013	1	Canada, France, Germany, Italy, Japan, UK and USA	1970 - 2010	CC has superior explanatory power versus CRRA and CAPM.	NA	NA	CC, CRRA, CAPM	GRACH & GMM approach combined	Newey & West (1987) with four lags	Returns calculated from MSCI stock market indices. Private total consumption in each country is used.	lagged values of world consumption growth rate, USA term spread, USA consumption-wealth ratio

Author	Year	Theoretical = 0, Empirical =1	Countries	Period	Does CCAPM relationship hold	Range of risk aversion	Range of rate of time preference	Model Specification	Method	Weighted matrix	Data Used	Instruments
BACH & MOLLER	2011	1	USA	1982-2004	Model accuracy in pricing bonds improves substantially when using the consumption of households that do invest in comparison to using non-assetholder consumptionand aggregate consumption. CC model with assetholder consumption has the ability to explain the large equity premium with a plausible level of risk aversion.	AH: 3.478(I Matrix) & 11.53 (S Matrix), Agg: 8.874(I Matrix)& 19.627 (S Matrix)	AH: 0.905 (S Matrix) & 0.922 (I Matrix), Agg: 0.8 (S Matrix) & 0.820 (I Matrix)	CC with limited market participation	GMM	Identity & statisticlly optimal weighting matrix (Inverse of aysmptotic covariance matrix)	quarterly aggregate per capita consumption of non-durables and services, asset and non-assetholder consumption data of Government bonds with maturities of 1,2,5,7 and 10 years.	NA
BAUM, SCHAFFER & STILLMAN	2003	0	NA	NA	NA	NA	NA	NA	GMM	Optimal weighting matrix is one which minimizes the asymptotic variance of the estimator	NA	NA
BLACK	1972	0	NA	NA	NA	NA	NA	NA	CAPM	NA	NA	NA
BLACK	1972	0	NA	NA	NA	NA	NA	CAPM	NA	NA	NA	NA
BOLDRIN, CHRISTIANO & FISHER	2001	1	USA	USA Retuns: 1892-1987, Consumption data: 1964-1988	NA	NA	NA	Real Business Cycle modificated -1. Habit preference & 2. aggregate allocation of capital and labour are determined before current-period realization.	GMM	NA	Returns is the S&P 500 composite. Consumption sector is measured as index of hours work in service sector and index of hours worked in the nondurable manufacturing sector.	Lagged consumption growth and lagged returns.
BRAV, CONSTANTINIDES & GECZY	2002	1	USA	1982-1996	Evidence supports the hypothesis of incomplete consumption insurance and evidence that the representative consumer model can account for the equity premium when limited participation is taken into account is sensitive to the experimental design.	Tested for values between 0 & 20	NA	C-CAPM with incomplete consumption insurance assumption and limited participation	GMM	NA	Household-level quarterly consumption data is the Consumer Expenditure Survey (produced by Bureau of Labour Statistics). Quarterly consumption of nondurables and services. Treasury bill return (nominal monthly risk free rate of interest), value weighted nominal, monthly market return (capital and dividends) is an arithmetic return which are calculated from a pooled sample of stocks listed on NYSE and American Stock Exchange.	NA
BREEDEN	1979	0	NA	NA	NA	NA	NA	CRRA	NA	NA	NA	NA

Author	Year	Theoretical = 0, Empirical =1	Countries	Period	Does CCAPM relationship hold	Range of risk aversion	Range of rate of time preference	Model Specification	Method	Weighted matrix	Data Used	Instruments
BREEDEN, GIBBONS & LITZENBERGER	1989	1	US	1929-1982	Performance of traditional CAPM and C-CAPM are about the same.	NA	NA	CRRA & CAPM	ML & GLS	NA	Monthly returns on individual securities collecterd from Center for Research in Security Prices (CRSP). Twelve portfolios of these numbers are formed by grouping firms using first two digits of SIC numbers. From 1929 to 1939 use nominal personal income less transfer payments as a proxy of aggregate consumption. For the rest of the period consumption data is based on expenditures on nondurables and services, following Hall (1978).	NA
BRENNAN, WANG & XIA	2004	1	USA	1952-2000	NA	NA	NA	Intertemporal Capital Asset Pricing Model (ICAPM) & Fama-French Three-Factor Model	GMM	Inverse of variance-covariance matrix of pricing errors	Monthly observations of the yields on eight synthetic constant maturity zero-coupon bonds (USA Treasury bonds with maturities of 3 and 6 months, and 1, 2, 3, 4, 5 and 10 year bonds)	NA
CAMPBELL & COCHRANE	2000	0	NA	NA	Generate artificial time series and find the CAPM, using the wealth portfolio return correlated with consumption, is better than the standard consumption-based model. The CC can explain why the CAPM is better asset pricing model than the standard consumption-based models.	NA	NA	NA	NA	NA	NA	NA
CAMPBELL & COCHRANE	1999	0	NA	NA	NA	NA	NA	CC	NA	NA	NA	NA
CASHIN & MCDERMOTT	1998	1	Jordan, Turkey, Pakistan	1986-1992 (Jordan), 1988-1993 (Turkey) & 1986-1991 (Pakistan).	Equity prices movements are not consistent with C-CAPM	0.27-0.65 (Jordan), -9.92 to -1.35 (Turkey), 2.77 - 5.87 (Pakistan)	0.93-0.94 (Jordan), 0.76-0.82 (Turkey), 0.91-0.92 (Pakistan)	CRRA	GMM	Optimal weighing matrix (asymptotic covariance matrix)	Quarterly data on aggregate private consumption. Respective countries local stock exchange index.	Unity constant, lagged consumption growth (up to 5 lag lengths) and lagged stock returns (up to 5 lag lengths).
CHAPMAN	1997	1	USA	1954-1991	Not typically rejected by overall measures of model fit but produces statistically and economically large pricing errors on individual assets. Accuracy improves with increase in additional future values of consumption growth (motivates the appeal to time nonseperable preferences).	NA	NA	CRRA	GMM	Inverse of covariance matrix of models distubances	Quarterly personal consumption expenditures on nondurable goods. Three month Treasury bills, portfolio of high grade corporate bonds, portfolio of long-term government bonds and monthly reutns on ten size portfolios of NYSE.	Divident yield on the S&P 500 Composite Index, spread of Baa rated versus Aaa rate corporate conds and logarithmic twelfth difference in the Federal Reserve Board's index of total industrial production.

Author	Year	Theoretical = 0, Empirical =1	Countries	Period	Does CCAPM relationship hold	Range of risk aversion	Range of rate of time preference	Model Specification	Method	Weighted matrix	Data Used	Instruments
CHEN	2003	1	TAIWAN	1991-2000	CAPM outperforms the C-CAPM, C-CAPM fails to explain Taiwan stock market although consumption beta should offer better measure of systematic risk theoretically	0.90-1.20	0.97-0.99	CRRA & CAPM	OLS	NA	Market composite stock price index is the monthly Taiwan capitalization-weighted stock index (TWSE) of the Taiwan Stock Exchange	NA
CHEN & LUDVIGSON	2009	1	USA	1952-2001	The empirical results indicate that the estimated habit function is better described as internal rather than external and the time-preference paramater is sensible. The habit functions generate positive SDF and perform well in explaining the cross sectional stock return data. The internal habit SDF proxy can explain the cross-sectional equity returns better than the Fama-French model, scaled consumption CAPM, external habit SDF proxy, the classic CAPM and CRRA model.	0.76	0.985	CAPM, CRRA, Scaled C-CAPM, Fama-French, internal and external habit formation	GMM	Second moment matrix of returns	Consumption is measured as expenditures on non-durables and services (excluding shoes and clothing). Asset returns from NYSE, AMEX and NASDAQ.	Constant, linear terms, squared terms and pair-wie cross products of each variable
CONSTANTINIDES	1990	0	NA	NA	NA	NA	NA	Generate model with habit persistence	NA	NA	NA	NA
DARRAT, LI & PARK	2011	1	17 MSCI Country Indices	1970 - 2007	World C-CAPM has very little power in explaining variations of expected returns across MSCI country indices as it requires an economically implausible large coefficient of RRA. The heterogeneous world CCAPM and world surplus CCAPM outperform the world C-CAPM in determining excess returns acrsoss countries and the the return on stock is higher if its return is more negatively correlated with consumption dispersion (partly explain the equity premium puzzle)	NA	NA	classic world C-CAPM, heterogenous C-CAPM, world CC C-CAPM and Abel's world habit C-CAPM	GMM	Identity and covariance matrix (inverse of the long run matrix)	Indices for Morgan Stanley Capital International are value-weighted and adjusted for dividend reinvestment. Household (private) total consumption as data on non-durable and service consumption was unavailable for most countries. Hh total consumption	NA
DAVIS	2003	0	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
DIMSON & MUSSAVIAN	1999	0	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA

Author	Year	Theoretical = 0, Empirical =1	Countries	Period	Does CCAPM relationship hold	Range of risk aversion	Range of rate of time preference	Model Specification	Method	Weighted matrix	Data Used	Instruments
DIMSON & MUSSAVIAN	1999	0	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
ENGLISH, MIRON & WILCOX	1989	0	NA	NA	Results of Miron (1986) were incorrect due to a problem in the softward program (TSP). Therefore seasonally adjusted consumption is the preferred method.	NA	NA	NA	NA	NA	NA	NA
ENGSTED & MOLLER	2009	1	Denmark	1922-2006 (annual data), 1977-2006 (quarterly data)	CC Model does not seem to perform marketly better than the CRRA model (especially over longer time period measured annually). For the shorter time period there is some evidence of the RRA being counter-cyclical time-variant but the CC does not produce lower pricing errors or more plausible parameter estimates than the CRRA model. Neither model is statistically rejected by Hansen's J test and the pricing errors are the same magnitude for both models.	8.4589- 9.5197 (CRRA, annual data), 13.34-21.69 (CRRA, quarterly data), 7.49 - 8.42 (CC, annual data), 15.24-24.09 (CC, quarterly data).	0.97-0.99 (CRRA, annual data), 0.98-0.99 (CRRA, quarterly data), 0.89-0.92 (CC, annual data), 0.94-0.97 (CC, quarterly data).	CCRA, CC surplus consumption habit formation model	GMM	Inverse covariance matrix of the sample orthogonality conditions	Quarterly data measures consumption as per capita, seasonally adjusted, expenditures on non-durables and services. Adopt Campbell (2003) beginning of period timing assumption. Utilize dividend-adjusted stock market return from Morgan Stanley Capital International. Additionally utilize long-term (10 years) and short-term (3 month) government bond returns.	Lags of stock returns, bond returns, consumption growth and lagged log price-dividend ratio.
ENGSTED, HYDE & MOLLER	2010	1	Belgium, Canada, France, Germany, Italy, Sweden, UK & US.	Starts between 1949 & 1953 depending on country and end 2004.	CC model does not give a perfect description of the data in any of the countries (to be expected due to highly stylized nature of model). But for majority of countries (Belgium, Italy, Sweden, UK and US) the CC model gets empirical support in a variety of different dimensions (economically plausible estimates of preference parametsrs, time varying counter cyclical risk aversion, statistically significant return predictability for both stocks and bonds based on surplus cosnumption ratio) Canada and France - mixed results. Germany - no empirical support for CC model. Therefore large cross country differences in the CC's model ability in explaining financial market returns.	Identity matrix: 7.45-90.17 (CRRA), 25.14-75.40 (CC). HJ Weighting matrix: 11.39-119.54 (CRRA), 18.41-58.26 (CC)	Identity matrix: 1.23-2.38 (CRRA), 0.36-0.91 (CC). HJ Weighting matrix: 1.22-2.36 (CRRA), 0.6793-1.02 (CC).	CRRA & CC	GMM	Identity matrix & HJ weighting matrix	Consumption is measured as private total consumption from IMF international financial statistics. Adopt Campbell (2003) that consumption during year t takes place at beginning of year t. Returns on stocks, stocks and long-term (10 year) bonds collected.	constant (can only use constant if consumption follows a random walk), log price-divedend ratio
EPSTEIN & ZIN	1989	0	NA	NA	NA	NA	NA	Epstein-Zin	NA	NA	NA	NA
EPSTEIN & ZIN	1989	1	USA	1959-1986	Performance of utility model and tests of the expected utility hypothesis are sensitive to choich of both consumption measure and instrumental varialbes.	NA	-0.0074 to 0.0016	NA	GMM	NA	Four different measures of consumption -1. expenditure on nondurable goods, 2. expenditure on nondurable goods excluding shoes and clothing, 3. expenditure on nondurable goods and services, 4. expenditure on nondurable goods and services excluding shoes, clothing and medical expenditure. Nominal return is measured with the value-weighted indexs of shares traded on NYSE.	Constant, lagged consumption growth, lagged market return.

Author	Year	Theoretical = 0, Empirical =1	Countries	Period	Does CCAPM relationship hold	Range of risk aversion	Range of rate of time preference	Model Specification	Method	Weighted matrix	Data Used	Instruments
ENGLISH, MIRON & WILCOX	1989	0	NA	NA	Results of Miron (1986) were incorrect due to a problem in the softward program (TSP). Therefore seasonally adjusted consumption is the preferred method.	NA	NA	NA	NA	NA	NA	NA
ENGSTED & MOLLER	2009	1	Denmark	1922-2006 (annual data), 1977-2006 (quarterly data)	CC Model does not seem to perform marketly better than the CRRA model (especially over longer time period measured annually). For the shorter time period there is some evidence of the RRA being counter-cyclical time-variant but the CC does not produce lower pricing errors or more plausible parameter estimates than the CRRA model. Neither model is statistically rejected by Hansen's J test and the pricing errors are the same magnitude for both models.	8.4589- 9.5197 (CRRA, annual data), 13.34-21.69 (CRRA, quarterly data), 7.49 - 8.42 (CC, annual data), 15.24-24.09 (CC, quarterly data).	0.97-0.99 (CRRA, annual data), 0.98-0.99 (CRRA, quarterly data), 0.89-0.92 (CC, annual data), 0.94-0.97 (CC, quarterly data).	CCRA, CC surplus consumption habit formation model	GMM	Inverse covariance matrix of the sample orthogonality conditions	Quarterly data measures consumption as per capita, seasonally adjusted, expenditures on non-durables and services. Adopt Campbell (2003) beginning of period timing assumption. Utilize dividend-adjusted stock market return from Morgan Stanley Capital International. Additionally utilize long-term (10 years) and short-term (3 month) government bond returns.	Lags of stock returns, bond returns, consumption growth and lagged log price-dividend ratio.
ENGSTED, HYDE & MOLLER	2010	1	Belgium, Canada, France, Germany, Italy, Sweden, UK & US.	Starts between 1949 & 1953 depending on country and end 2004.	CC model does not give a perfect description of the data in any of the countries (to be expected due to highly stylized nature of model). But for majority of countries (Belgium, Italy, Sweden, UK and US) the CC model gets empirical support in a variety of different dimensions (economically plausible estimates of preference parametsrs, time varying counter cyclical risk aversion, statistically significant return predictability for both stocks and bonds based on surplus cosnumption ratio) Canada and France - mixed results. Germany - no empirical support for CC model. Therefore large cross country differences in the CC's model ability in explaining financial market returns.	Identity matrix: 7.45-90.17 (CRRA), 25.14-75.40 (CC). HJ Weighting matrix: 11.39-119.54 (CRRA), 18.41-58.26 (CC)	Identity matrix: 1.23-2.38 (CRRA), 0.36-0.91 (CC). HJ Weighting matrix: 1.22-2.36 (CRRA), 0.6793-1.02 (CC).	CRRA & CC	GMM	Identity matrix & HJ weighting matrix	Consumption is measured as private total consumption from IMF international financial statistics. Adopt Campbell (2003) that consumption during year t takes place at beginning of year t. Returns on stocks, stocks and long-term (10 year) bonds collected.	constant (can only use constant if consumption follows a random walk), log price-divedend ratio
EPSTEIN & ZIN	1989	0	NA	NA	NA	NA	NA	Epstein-Zin	NA	NA	NA	NA

Author	Year	Theoretical = 0, Empirical =1	Countries	Period	Does CCAPM relationship hold	Range of risk aversion	Range of rate of time preference	Model Specification	Method	Weighted matrix	Data Used	Instruments
GROSSMAN, MELINO & SHILLER	1987	1	USA	Differing sets of data: 1890-1981, 1890-1980, 1953-1983, 1953-1980, 1947-1981, 1947-1980	Quantitative difference in average yields can only be rationalized by high RRA but taking account of measuring consumption as unit averaged, substantially decreases the RRA required to rationalize the data. The orthogonality condition is rejected by the data.	2.12-154.47	0.77-1.27	Continuous-time intertemporal C-CAPM	MLE	NA	Real per capita, seasonall adjusted consumption on non-durables and services. Real return on corporate stocks, real return on short debt and real return on long-term bonds.	NA
HAMORI	1991	1	Japan	1980-1988	Usefulness of CCAPM in Japanese asset market which differs from empirical results in previous papers. Model accepted at conventional significance level and the estimated parameter values are reasonable.	-0.23-0.27	0.984-0.997	CRRA	GMM	NA	Real consumption and Tokyo stocks, long-term government bonds, long-term coporate bonds and short-term real interest rates.	NA
HANSEN	1982	0	NA	NA	NA	NA	NA	NA	GMM	NA	NA	NA
HANSEN, HEATON & YARON	1996	0	NA	NA	NA	NA	NA	NA	GMM	Test different weighting matices. Firstly testing where the weighting matrix is is estimated using an initial (consistent estimator) of the parameter vector, secondly where the weighting matrix is iterated and converges and lastly where the weighting matrix is changed for every hypothetical parameter value.	NA	NA
HANSEN & JAGANNATHAN	1997	1	USA	1959-1990	The estimated specification errors are a little smaller but still exceed errors of twenty-five percent of norm, for CAPM style models (SDF is a linear combination of a constant plus a scale factor times a meausre of the market returns).	Set between 0-15	Set to 0.95 or 1.00	CRRA, Abel's habit formation, linear factor model	GMM	Second moment matrix of returns,distance matrix in quadratic form (proportional to inverse of asymptotic covariance matrix)	Returns calculated from NYSE stocks. Consumption of nondurables.	NA

Author	Year	Theoretical = 0, Empirical =1	Countries	Period	Does CCAPM relationship hold	Range of risk aversion	Range of rate of time preference	Model Specification	Method	Weighted matrix	Data Used	Instruments
HANSEN & SINGLETON	1982	0	NA	NA	NA	NA	NA	CRRA	GMM	NA	Two measures of consumption expenditure namely non-durable consumption and non-durable plus services consumption. Stocks listed on NYSE used as return proxy.	Lagged return and lagged consumption growth
HANSEN & SINGLETON	1983	1	USA	1959-1978	Evidence against restrictions.	-2.721 to 0.359	0.9965 to 1.0007	CRRA	ML	Covariance matrix	Two different measures of consumption: 1. nondurables plus services, 2. nondurables. Return series of aggregation across common stok were considered (an average return on all stocks listed on NYSE and returns on individua members of Dow Jones Industrial).	NA
HANSEN & WEST	2002	0	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
HASSAN & VAN BILJON	2010	1	South Afirca	1900-2000	Observed equity premium is too high to be explained by standard C-CAPM. South Africa produces parameter maginitudes comparable to evidence from advanced economies.	-593 to 233	NA	CRRA	NA	NA	Equity index as proxy for aggregate sstock market (consolidate JSE All Share and JSE Actuaries index to form one combined equity index for South Africa). Per capita consumption of non-durables and services.	NA
HYDE & SHERIF	2005	1	UK	1965-2000	Evidence supporting both habit formation and traditional power utility C-CAPM but results are mixed. Epstein-Zin specification rejected.	OLS: 25.51 (CRRA), GMM: 0.03-2.14 (CRRA)	OLS: 1.10(CRRA), GMM: 1.0009 (CRRA)	CRRA, Epstein-Zin, Abel, CC	GMM & OLS	NA	Two measures of consumption expenditure namely non-durable consumption and non-durable plus services consumption. Market return is the FTSE All Share index.	Constant, lagged consumption growth, lagged returns, lagged bond yield
HYDE, CUTHBERTSON & NITZSCHE	2005	1	FRANCE & GERMANY	France: 1971-2000, Germany: 1965 - 2000	The CCAPM is consistent with asset returns but only if high risk aversion is assumed. The external habit formation decreases implied risk aversion and the minimum distance test (Hansen & Jagannathan) produces the smallest pricing errors. The vertical distance test shows that all specifications generate mean-standard deviation pairs which are not statistically different from the bound.	Test for large range of values	Choose 0.97 but test sensitivity of results by alternative values (0.95, 0.97. 0.99).	CRRA, CC, Abel, Epstein-Zin	Did not estimate parameter but tests validity of different combinations of parameters (test vertical distance and HJ distance)	NA	Tested both annual and quarterly data. Availability of consistent consumption data over a long period is limited therefore consumption is measured as total consumers' expenditure for Germany and proxied by retail sales for France but it is anticipated that using total consumption will not impact results substantially. Two different asset returns are used, firstly, gross real return on stocks and secondly real gross return on short-term (3 month) money market instruments.	NA

Author	Year	Theoretical = 0, Empirical =1	Countries	Period	Does CCAPM relationship hold	Range of risk aversion	Range of rate of time preference	Model Specification	Method	Weighted matrix	Data Used	Instruments
IMBENS	2002	0	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
JAGANNATHAN & WANG	1996	1	USA	1963-1990	NA	NA	NA	CAPM & Conditional CAPM	GMM	Hansen & Jagannathan (1994) weighting matrix	Assume that return on value-weighted portfolio of all stocks listed on NYSE and AMEX are a reasonable proxy for the return on market portfolio. Returns are calculated on stock of nonfinancial firms listed in NYSE and AMEX.	NA
JAGANNATHAN, SKOULAKIS & WANG	2002	0	NA	NA	NA	NA	NA	NA	GMM	NA	NA	NA
KASA	1997	1	Canada, German, Japan, UK & USA	1972-1993	Production-based CAPM performs better than the C-CAPM.	NA	NA	C-CAPM & production CAPM	GMM	Optimal variance-covariance orthogonailty conditions	Quarterly end-of-period observations of Morgan Stanely Capital International Indices. Consumption on nondurables and services (Japan had to be interpolated). Real investment measured as gross private fixed investment.	Lagged price-dividend ratio (US only), lagged population-weighted average of national consumption growth rates, lagged capital growth rates.
KOCKERLAKOTA	1996	0	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
KOCKERLAKOTA	1990	0	NA	NA	NA	NA	NA	NA	GMM	NA	NA	NA
LI	2010	1	Australia, Austria, Belgium, Cananda, Denmark, France, Germany, Hong Kong, Italy, Japan, Netherlands, Norway, Spain, Sweden, Switzerland, UK and USA.	1970-2007	The classic world C-CAPM requires large and economically implausible coefficient of RRA. The heterogeneous, CC and Abel's habit world C-CAPM estimate the coefficient of RRA at lower and economically reasonable estimates. However, the overall fit of the model for the heterogeneous, CC and Abel's habit world C-CAPM are worse than for the classic world C-CAPM.	63.06 (classic world C-CAPM), 3.56-30.08 (heterogenous world C-CAPM), (1.97) world CC C-CAPM & (18.63) Abel's world habit C-CAPM	NA	classic world C-CAPM, heterogenous world C-CAPM, world CC C-CAPM and Abel's world habit C-CAPM	GMM	Newey & West (1987) heteroskedasticity and autocorrelation consistent matrix with two lag length	Morgan Stanley Capital International stock market indices for the respective countries. Household or private consumption is obtained for each country.	Lagged world consumption growth, lagged USA consumption-wealth ratio, lagged USA term Spread.

Author	Year	Theoretical = 0, Empirical =1	Countries	Period	Does CCAPM relationship hold	Range of risk aversion	Range of rate of time preference	Model Specification	Method	Weighted matrix	Data Used	Instruments
LI & ZHONG	2010	1	Australia, Austria, Cananda, France, Germany, Italy, Japan, Norway, Spain, Sweden, Switzerland, UK and USA.	1970-2000	The predictability of expected returns from most devleoped countries equity markets are partly explained by time-varying consumption relative to habit associated with a common world SDF. The world CC outperforms unconditional world C-CAPM and CAPM and the three-factor international model incorporating returns on a world market portfolio, the exchange rate and the real interest rate as risk factors.	NA	NA	CC testing local and world risk	GMM	NA	Quarterly data of 17 national stock price indices from Morgan Stanely Capital International which represent value-weighted portfolios of large firms traded in the national equity markets. Consumption is meaused as non-durables and services for the USA but for the remaining countires, consumption data is measured as household or private total consumption within a country	Associated with disturbance term
LI & ZHONG	2002	1							GMM			
LINTNER	1965	0	NA	NA	NA	NA	NA	CAPM	NA	NA	NA	NA
LUCAS	1978	0	NA	NA	NA	NA	NA	CRRA	NA	NA	NA	NA
LUND & ENGSTED	1996	1	Denmark, Germany, Sweden & UK	1922-1990 (Denmark), 1885-1913 & 1952-1990 (Germany), 1918-1990 (Sweden), 1919-1987 (UK).	The statistical tests are unable to rejct the C-CAPM but the estimates of the RRA are mostly implausible and imprecise.	-1.71 to 4.68 (Denmark), -16.63 to 10.36 (Germany), 1.23 to 0.16 (Sweden), -26.12 to 10.81 (UK)	0.86 to 0.99 (Denmark), 0.74 to 1.11 (Germany), 0.93 to 0.94 (Sweden), 0.47 to 0.98 (UK)	CRRA	GMM & VAR	I & Asymptotic covariance matrix	Stock indices used from countries Stock exchanges and private consumption	Constant unity, lagged values of return and consumption growth (1 and 2 lags, respectively), two-lagged values of dividend price ratio.
MANKIW & SHAPIRO	1986	1	USA	1959-1982	No support for C-CAPM when compared to traditional CAPM but C-CAPM is preferable on theoretical grounds.	NA	NA	CAPM & C-CAPM	GLS, OLS, WLS (weighted least squares), IV (instrumental variable procedure)	Variance-covariance matrix is diagonal with elemnts proportional to the variance of asset returns divided by the variance of market returns	Cross-section of stocks include all companies listed on NYSE continuously during sample period. Market return is the return on the S&P composite. Consumption measured as real consumer expenditure on nondurables and services.	NA
MEHRA	2003	0	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
MEHRA & PRESCOTT	1985	0	USA	NA	NA	NA	NA	CRRA	NA	NA	NA	NA
MERTON	1973	0	NA	NA	NA	NA	NA	Intertemporal CAPM	NA	NA	NA	NA

Author	Year	Theoretical = 0, Empirical =1	Countries	Period	Does CCAPM relationship hold	Range of risk aversion	Range of rate of time preference	Model Specification	Method	Weighted matrix	Data Used	Instruments
MIRON	1986	1	USA	1948-1982	Previous reject may be as a result of the treatment of seasonal fluctuations. Find that when seasonal fluctuations in consumption purchases are included in analysis there is no evidence in aggregate data against the model. Find that the parameters are plausible and model is not rejected in overidentifying restrictions.	NA	NA	Intertemporal Consumption choice model (equilibrium pricing function of the kind derived by Lucas (1978)).	Least Squares	NA	Adjust data to seasonally unadjusted data on real consumption purchases for different consumption categories.	constant, three seasonal dummies, time, time-squared and lagged values of the included variables.
MOLLER	2009	1	USA	1947-2005	The CC model has difficulties in explaining cross-sectional variation in expected stock returns but has the ability to explain time-variation in expected stock return.	3.472-5.837	0.79-0.90	CC	GMM	Identity and covariance matrix	Consumption is measured as expenditures on non-durables and services. Beginning of time convention of Campbell (2003) and use of real per capita consumption. Returns are calculated on the value weighted CRSP index including NYSE, AMEX and NASDAQ frims.	NA
NEWEY & WEST	1987	0	NA	NA	NA	NA	NA	NA	GMM	Simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix	NA	NA
NIETO & RUBIO	2011	1	USA	1985-2006	Recursive and Yogo specification predict economic cycles at horizons of 1, 2 and 4 quarters while long-run preferences specifications are better at 8 and 12 quarters. The paper finds that the volatiity of popular consumption based SDFs is a reasonable measure of exante economic uncertainty.	24 (Recursive), 25 (Yogo), 1.6 (Recursive long), 2.50 (Yogo long)	0.99 (Recursive), 0.988 (Yogo), 0.991 (Recursive long), 0.98 (Yogo long)	Recursive preferences with non-durables consumption, recursive preferences non separable preferences with non-durable and durable consumption and Yogo, long-run version of the two previous cases.	OLS	Identity matrix	Quarterly seasonally adjusted aggregate real per capital consumption expenditure of non-durable and services. Stock mkt data measured as quarterly and monthly real returns on a value-weighted stock market portfolio and excess returns of 10 sized-sorted equally weighted portfolio returns. High-frequency data from S&P 500 Index and monthly data for the volatility index. BBB corporate bond yields, treasury yields.	NA
SAUER & MURPHY	1992	1	Germany	1968-1988	results provide evidence that CAPM explains risk-return relationship in Germany significantly better than the CCAPM	NA	NA	CAPM, CCAPM	OLS & GLS	NA	Total returns (both dividend and capital gains) on 249 German stocks actively traded on Frankfurt stock exchange (stocks not continuously traded on the exchange are excluded from initial sample in order to avoid parameters with limited number of observations. Quarterly returns on consumption.	NA
SHARPE	1964	0	NA	NA	NA	NA	NA	CAPM	NA	NA	NA	NA

Author	Year	Theoretical = 0, Empirical =1	Countries	Period	Does CCAPM relationship hold	Range of risk aversion	Range of rate of time preference	Model Specification	Method	Weighted matrix	Data Used	Instruments
SMOLUK & NEVEU	2002	1	USA	1984-1996	Using consumption expenditure data spilt into quintiles based on consumer income produces erratic behaviour across income groups.	-0.1343 to 0.70	0.97-0.99	CRRA with consumption delineated by consumer income	GMM	NA	The consumption data is obtained through a survey which is composed in two separate parts that are later integrated (diary and interview survey). The diary survey captures small, frequently purchased items and the the interview survey captures large and regularly recurring purchased items. Approximately 5000 consumer units are sampled only looking at nondurable goods and services. Complete income reports are ranked in ascending order. Total stock returns are based on S&P 500 index.	lagged real 3-month Treasurey bill rates, lagged real total return on S&P 500, lagged real consumption growth and lagged log dividend to price ratio.
STOCK & WRIGHT	2000	1	USA	1959-1990	The habit formation/durability model is largely rejected. The S-sets point to higher degrees of risk aversion than suggested by conventional GMM analysis (weak-identification asymptotic theory introduced).	Different instrumental sets: -3.33 to 6.436 (CRRA)	Different instrumental sets: 0.822 to 0.999 (CRRA)	CRRA, habit formation/durability & Epstein-Zin	GMM	Nonrobust covariance matrix	First data set contains annual USA data on stock returns, bond returns and cosnumption. The stock returns are based on Cowles Commission index and are followed by the annual average price of the S&P composite index. The consumption is measured as real consumption of nondurables and services. The second data set contains month data on asset returns sorted by sector and consumption expenditure on nondurables.	Lagged Stock returns, lagged bond returns, lagged consumption growth, long bond rate minus short interest rate (spread), lagged divideend yield.
STULZ	1981	0	NA	NA	NA	NA	NA	Intertemporal model of international asset pricing (admits differences in consumption opportuntity sets across countries.	NA	NA	NA	NA
SUNDARESAN	1989											
TAUCHEN	1986	0	NA	NA	NA	NA	NA	NA	GMM	NA	NA	NA
WHEATLEY	1988	1	USA	1959-1981	Simulations were used to show that tests of model's restrictions can be biased towards rejection when consumption is measured with a small amount of error. RRA is implausibly high.	-137.50 to 113.28	NA	CRRA	MLE	NA	Two measures of consumption were used, nondurables and nondurables and services. Stock return series were taken from CRSP of the value-weighted index of the NYSE stocks.	NA

Author	Year	Theoretical = 0, Empirical =1	Countries	Period	Does CCAPM relationship hold	Range of risk aversion	Range of rate of time preference	Model Specification	Method	Weighted matrix	Data Used	Instruments
ZHOU	1994	1	USA	1941-1986	Evidence that GMM offers good finite sample properties.	NA	NA	NA	GMM	Test alternative GMM tests based on arbitrary weighing matices.	Monthly stock returns from CRSP data base.	Constant, lagged return on the equal-weighted index in excess of the Treasury bill rate, lagged monthly returns of differenct bewteen 90-day bill and 30-day bill, dummy variable for January, lagged difference of yield on Moody's BAA rated bonds and Moody's AAA-rated bonds, lagged dividend yield spread.

Abbreviations in Literature Table: Generalized Method of Moment (GMM), Constant Relative Risk Aversion (CRRA), Cochrane & Campbell (1999), Stochastic Discount Factor (SDF), Asset-holders consumption (AH), Aggregate consumption (Agg), Identity (I), Spectral Density (S), Maximum Likelihood estimation (ML), Generalized Least Squares estimation (GLS), Ordinary Least Squares Estimation (OLS), Instrumental variable procedure (IV), Weighted Least Squarest estimations (WLS), New York Stock Exchange (NYSE), American Stock Exchange (AMEX), Morgan Stanley Capital International (MSCI), Johannesburg Stock Exchange (JSE), Hansen & Jagannathan (1994) (HJ), Vector Autoregression (VAR), Centre for Research in Security Prices (CRSP).

Appendix 2: Data Description

All data is collected from Thomson Datastream. Personal consumption expenditure classifies each commodity by the following conventions: Durable goods are commodities that can be stored or inventoried and have an average life span of at least three years, nondurable goods are the rest of the commodities that can be inventoried, and services are commodities that occur and are consumed at the place and time of purchase therefore they cannot be stored.

Specifically, for Germany, final consumption expenditure consists of expenditure incurred by residents on goods and services used for the direct satisfaction of members of the community which includes private non-profit organisations. Results are reported as seasonally adjusted, current prices.

For the UK, the resident population is defined as all those usually resident in an area including armed forces stationed in England and Wales, students in halls of residence and long term prisoners. CPI is the Harmonized Indices of Consumer Prices (HICPs) and are the harmonized inflation figures required by the Treaty of Amsterdam under Article 121 which was an amendment of the Treat of the European Union. The Treaty required the adjustment of the base year to 2005 in order to provide a common reference point for all HICPs of the Member States of the EU. The HICP is a price index and is not seasonally adjusted.

Finally, for America, personal consumption expenditure comprises goods and services purchased by persons residing in the America. Most of the personal consumption expenditure consists of purchases of new goods and services by individuals from business. Included in the measurements of consumption are the purchase of goods and services by non-profit institutions, purchases abroad of goods and services by American residents travelling or working in foreign countries and certain services provided by the government such as tuition payments for higher education and charges for medical care. Results are reported as seasonally adjusted, current prices.

Appendix 3: SCILAB Code Example

SCILAB code (GROCER add-in) for the estimation of the German C-CAPM with reference to Germany2.xls. Parameters and associated diagnostics for the first four iterations appear below.

```
function [m, e]=gmmCcapmM(b, gmmopt, Y, X, Z)

// PURPOSE: Moments condition for power utility CCAPM example.
//---------------------------------------------------------------------
// INPUTS:
// . b = k-vector of model parameters
// . gmmopt = gmm options tlist
// . X = (nobs x k) matrix of exogenous variables
// . Y = (nobs x neq) matrix of endogenous variables
// . Z = (nobs x nz) vector of instruments
//---------------------------------------------------------------------
// OUTPUTS
// . e = (nobs x neq) matrix of conditions of interest
// . m = north-vector of orthog. cond. from stacking Z'h
//---------------------------------------------------------------------
// DESCRIPTION:
//   X = [c r], Y = [1 1], Z = [1 lag(c) lag(r)]
//   c:    nobs-vector of consumption growth
//   r:    (neq x 2) matrix of returns, [re rf] (1.1 = 10//)
//   imrs: nobs-vector of IMRSs
//   e:    (nobs x neq) IMRS*r=1
//
//   m_(t) = beta*[c_(t)/c_(t-1)]^-gamma
//---------------------------------------------------------------------
// written by:
// Mike Cliff,  Purdue Finance  mcliff@mgmt.purdue.edu
// E. Michaux (2007) for the scilab translation

[T, neq] = size(Y);
k = size(Z,2);
north = neq*k;
R = X(:,2:size(X,2));
cg = X(:,1);

imrs = b(1)*cg.^(-b(2));
//imrs = b(1) + b(2)*[R(:,2)-1];
//e = (imrs*ones(1,neq)).*R - Y;

e = repmat(imrs,1,neq).*R - Y;
```

```
m = matrix(Z'*e/T,north,1);

endfunction

//C-CAPM parameter and J-test calculation using GMM procedure of Scilab Grocer

[fd,SST,Sheetnames,Sheetpos] = xls_open('/Users/kayleighspurway/Desktop/scilab
examples/germany2.xls'); //Open data into Scilab
[Value,TextInd] = xls_read(1,Sheetpos(1)); //read excel spreadsheet

funcprot(0);

gmmdata = Value(1:171,:);
nz = 1 //Number of lages used as instruments
T = size(gmmdata,1)-nz; //sample size
neq = size(gmmdata,2)-1; //number of equations

cg = gmmdata(1+nz:T+nz,1); //C_(t+1)/C_(t)
R = gmmdata(1+nz:T+nz,2); //(R_(t+1)-R_(t))/R_(t)

y = ones(T,1); //vector of ones

b=[.99,10]; //starting values for beta and gamma respectively (parameter)

X = [cg R];
Z = ones(T,1);
for i = 1:nz
 Z = [Z gmmdata(1+nz-i:T+nz-i,2:3)];
end //instruments - constant, lagged consumption and return


disp(' We give GMM instructions through the gmmopt tlist:')
disp(' We refernce the moment conditions from Scilab grocer built in consumption capital
asset pricing model for nonlinear estimation')
disp(' and specify a Newey-West weighting matrix with optimal lags lags')
disp('Starting value for beta and gamma are 0.99 and 10 respectively, taken from Mehra and
Prescott (1983)')


gmmopt = tlist(['gmm';'momt';'gmmit';'W0';'W';'S';'lags';'namep';'prt';'null']);
gmmopt('momt')='gmmCcapmM';          // moment conditions
gmmopt('gmmit') = 1000;          // Number of GMM iterations
gmmopt('W0') = 'I';          // Initial weighting matrix (identity)
gmmopt('W')='S';              // Subsequent wtg matrix optimal
gmmopt('S')='NW';              // Optimal Newey West Matrix
gmmopt('prt')=1;              //1 if user wants to print optimization steps
gmmopt('lags')=6;              // Lags in weighting matrix
gmmopt('namep') = ['beta','gamma'];    // variable names
b=[.99;10];              // Starting values
gmmopt('null') = [1;0];          // Null hypothesis
```

```
gout=gmm('y',gmmopt,'exo="X"','ivar="Z"','parm0=b'); //output (tlist that displays results)


//Hansen-Jagannathan distance calculation
//HJ=[E(M_((t+1) * R_(t+1) - 1)'*inv(E(R_(t+1)*R_(t+1)')) *E(M_(t+1) * R_(t+1) - 1)]^1/2
//SDF = M_(t+1) = beta*[c_(t+1)/c_(t)]^-gamma
//R_(t+1) is the return at period t+1
//M_((t+1) * R_(t+1) - 1 is the Euler Equation ...
//R_(t+1)*R_(t+1)' is the weighting matrix of the vector of returns times by the transposed
vector of returns

[fd,SST,Sheetnames,Sheetpos] = xls_open('/Users/kayleighspurway/Desktop/scilab
examples/germany2.xls'); //Open data into Scilab
[Value,TextInd] = xls_read(1,Sheetpos(1)); //read excel spreadsheet

funcprot(0);

gmmdata = Value(1:171,:);
nz = 1 //Number of lages used as instruments
T = size(gmmdata,1)-nz; //sample size
neq = size(gmmdata,2)-1; //number of equations

cg = gmmdata(1+nz:T+nz,1); //C_(t+1)/C_(t)
R = gmmdata(1+nz:T+nz,2); //(R_(t+1)-R_(t))/R_(t)

y = ones(T,1); //vector of ones

b=[.99,10]; //starting values for beta and gamma respectively (parameter)

X = [cg R];
Z = ones(T,1);
for i = 1:nz
 Z = [Z gmmdata(1+nz-i:T+nz-i,2:3)];
end //instruments - constant, lagged consumption and return



SDF=b(1)*cg.^(b(2)); //SDF = beta*[c_(t+1)/c_(t)]^-gamma

n = SDF.*R;
g = n - y; //Euler equation

W = [R*R']; //weighting matrix

HJ1 = mean(g')*inv(mean(W))*mean(g) //Expectation operator denoted by mean

HJ = sqrt(HJ1) //square root of the Hansen Jagannathan distance (least squares estimation)
```

```
Starting GMM iteration: 1
  Weights attached to moments

      Moment 1   Moment 2   Moment 3
beta   0.6559081  0.6620077  -0.3179157
gamma  0.5414365  0.5412204  -0.0826569

                  *
                *    *

  optimization step...


***** enters -qn code- (without bound cstr)
dimension=         2, epsq=  0.1490116119384766E-07, verbosity level: imp=         3
max number of iterations allowed: iter=     10000
max number of calls to costf allowed: nap=     10000
-------------------------------------------------
 iter num    1, nb calls=   1, f= 0.6882E-02
 linear search: initial derivative=-0.2640
             step length= 0.1000E-01, df=-0.6910E-03, derivative= 0.2502
             step length= 0.5134E-02, df=-0.6778E-02, derivative=-0.6482E-10
 iter num    2, nb calls=   3, f= 0.1044E-03
 linear search: initial derivative=-0.3971E-04
             step length=  1.000    , df=-0.5974E-07, derivative=-0.3970E-04
             step length=  10.00    , df=-0.5966E-06, derivative=-0.3961E-04
             step length=  100.0    , df=-0.5892E-05, derivative=-0.3861E-04
             step length=  1000.    , df=-0.5095E-04, derivative=-0.2760E-04
 iter num    3, nb calls=   7, f= 0.5345E-04
 linear search: initial derivative=-0.2903E-04
             step length= 0.9036    , df=-0.4894E-04, derivative= 0.3042E-05
 iter num    4, nb calls=   8, f= 0.4508E-05
 linear search: initial derivative=-0.3970E-04
             step length=  1.000    , df=-0.4273E-05, derivative=-0.2266E-05
 iter num    5, nb calls=   9, f= 0.2352E-06
 linear search: initial derivative=-0.2266E-05
             step length=  1.000    , df=-0.2119E-06, derivative=-0.1090E-06
 iter num    6, nb calls=  10, f= 0.2329E-07
 linear search: initial derivative=-0.1042E-06
             step length=  1.000    , df=-0.5814E-09, derivative=-0.1956E-09
 iter num    7, nb calls=  11, f= 0.2271E-07
 linear search: initial derivative=-0.3881E-08
             step length=  1.000    , df=-0.2708E-13, derivative=-0.2285E-10
 iter num    8, nb calls=  12, f= 0.2271E-07
 linear search: initial derivative=-0.1944E-09
             step length=  1.000    , df=-0.9782E-18, derivative= 0.2981E-13
 iter num    9, nb calls=  13, f= 0.2271E-07
 linear search: initial derivative=-0.2534E-13
             step length=  1.000    , df=-0.2614E-21, derivative=-0.1552E-13
```

```
 iter num   10, nb calls=  14, f= 0.2271E-07
 linear search: initial derivative=-0.1552E-13
              step length=  1.000    , df= 0.6022E-21, derivative=-0.2753E-13
              step length= 0.1000    , df=-0.3110E-21, derivative= 0.9937E-15
 iter num   10, nb calls=  16, f= 0.2271E-07
***** leaves -qn code-, gradient norm=  0.8432245998224710E-13
Norm of projected gradient lower than   0.8432246D-13.

  ...optimization ended

  Value of the parameters at the end of iteration  1
beta   0.9640787
gamma  4.9988319

                    *
                  *   *


   Starting GMM iteration: 2
  Weights attached to moments

       Moment 1   Moment 2    Moment 3
beta   0.5069930  0.8176678   -0.3246607
gamma  1.1262614  -0.2961239  0.1698625

                    *
                  *   *

  optimization step...


***** enters -qn code- (without bound cstr)
dimension=         2, epsq=  0.1490116119384766E-07, verbosity level: imp=         3
max number of iterations allowed: iter=     10000
max number of calls to costf allowed: nap=     10000
-----------------------------------------------
 iter num    1, nb calls=   1, f= 0.2986E-03
 linear search: initial derivative=-0.3572E-02
              step length= 0.1000E-01, df= 0.8514    , derivative=  17.02
              step length= 0.1000E-02, df= 0.8483E-02, derivative=  1.698
              step length= 0.1000E-03, df= 0.8161E-04, derivative= 0.1666
              step length= 0.1000E-04, df= 0.4942E-06, derivative= 0.1345E-01
              step length= 0.2099E-05, df=-0.3753E-07, derivative= 0.3907E-09
 iter num    2, nb calls=   6, f= 0.2986E-03
 linear search: initial derivative=-0.1764E-03
              step length= 0.8589E-03, df=-0.7498E-07, derivative=-0.1760E-03
              step length= 0.8589E-02, df=-0.7430E-06, derivative=-0.1728E-03
              step length= 0.8589E-01, df=-0.6748E-05, derivative=-0.1408E-03
              step length= 0.4253    , df=-0.1855E-04, derivative= 0.6197E-06
 iter num    3, nb calls=  10, f= 0.2801E-03
 linear search: initial derivative=-0.2796E-05
```

```
            step length=  1.000    , df=-0.1014E-08, derivative= 0.1350E-07
 iter num    4, nb calls=  11, f= 0.2801E-03
 linear search: initial derivative=-0.1082E-07
            step length=  1.000    , df=-0.6385E-13, derivative= 0.3083E-09
 iter num    5, nb calls=  12, f= 0.2801E-03
 linear search: initial derivative=-0.3074E-09
            step length=  1.000    , df=-0.6196E-16, derivative= 0.9233E-10
 iter num    6, nb calls=  13, f= 0.2801E-03
 linear search: initial derivative=-0.9234E-10
            step length=  1.000    , df= 0.2049E-16, derivative=-0.3986E-09
            step length= 0.1000    , df=-0.6343E-17, derivative=-0.1076E-09
            step length= 0.2000    , df=-0.3795E-18, derivative= 0.6814E-09
 iter num    7, nb calls=  16, f= 0.2801E-03
 linear search: initial derivative=-0.6813E-09
            step length= 0.7749E-01, df= 0.2385E-17, derivative= 0.1797E-09
 iter num    8, nb calls=  17, f= 0.2801E-03
 linear search: initial derivative=-0.1079E-09
            step length=  1.000    , df= 0.6180E-17, derivative=-0.3301E-09
            step length= 0.1000    , df= 0.1057E-16, derivative=-0.5254E-09
 iter num    8, nb calls=  19, f= 0.2801E-03
***** leaves -qn code-, gradient norm=  0.2763647807754043E-09
Norm of projected gradient lower than   0.2763648D-09.

  ...optimization ended

  Value of the parameters at the end of iteration  2
beta   0.9650970
gamma  4.7888354

                    *
                  *    *

   Starting GMM iteration: 3
  Weights attached to moments

       Moment 1    Moment 2     Moment 3
beta   0.4859057  0.8679344   -0.3538401
gamma  1.11509    -0.2889513  0.1738613

                    *
                  *    *

  optimization step...


***** enters -qn code- (without bound cstr)
dimension=         2, epsq=  0.1490116119384766E-07, verbosity level: imp=         3
max number of iterations allowed: iter=      10000
max number of calls to costf allowed: nap=      10000
-----------------------------------------------
```

```
 iter num    1, nb calls=   1, f= 0.2875E-03
 linear search: initial derivative=-0.3293E-03
              step length= 0.1000E-01, df= 0.8690    , derivative=  17.38
              step length= 0.1000E-02, df= 0.8687E-02, derivative=  1.738
              step length= 0.1000E-03, df= 0.8657E-04, derivative= 0.1735
              step length= 0.1000E-04, df= 0.8361E-06, derivative= 0.1705E-01
              step length= 0.1000E-05, df= 0.5398E-08, derivative= 0.1409E-02
              step length= 0.1894E-06, df=-0.3119E-09, derivative=-0.2532E-10
 iter num    2, nb calls=   7, f= 0.2875E-03
 linear search: initial derivative=-0.1828E-05
              step length= 0.6145E-02, df=-0.5722E-09, derivative=-0.1526E-05
              step length= 0.3716E-01, df=-0.1887E-08, derivative=-0.4389E-09
 iter num    3, nb calls=   9, f= 0.2875E-03
 linear search: initial derivative=-0.5342E-09
              step length=  1.000    , df=-0.1396E-15, derivative=-0.1408E-09
 iter num    4, nb calls=  10, f= 0.2875E-03
 linear search: initial derivative=-0.1410E-09
              step length=  1.000    , df= 0.2542E-16, derivative= 0.3662E-09
              step length= 0.2830    , df= 0.1155E-16, derivative=-0.4745E-09
              step length= 0.2830E-01, df= 0.1138E-17, derivative=-0.4915E-09
              step length= 0.2830E-02, df= 0.3144E-17, derivative=-0.1229E-09
 iter num    4, nb calls=  14, f= 0.2875E-03
***** leaves -qn code-, gradient norm=  0.2088383395118356E-09
Norm of projected gradient lower than   0.2088383D-09.

  ...optimization ended

  Value of the parameters at the end of iteration  3
beta   0.9650854
gamma  4.7908994

                    *
                  *   *

   Starting GMM iteration: 4
  Weights attached to moments

       Moment 1   Moment 2    Moment 3
beta   0.4861079  0.8674294   -0.3535374
gamma  1.1151906  -0.2890125  0.1738219

                    *
                  *   *

  optimization step...


***** enters -qn code- (without bound cstr)
dimension=         2, epsq=  0.1490116119384766E-07, verbosity level: imp=         3
max number of iterations allowed: iter=      10000
```

```
max number of calls to costf allowed: nap=     10000
------------------------------------------------
 iter num    1, nb calls=   1, f= 0.2875E-03
 linear search: initial derivative=-0.3585E-05
              step length= 0.1000E-01, df= 0.8689    , derivative=  17.38
              step length= 0.1000E-02, df= 0.8689E-02, derivative=  1.738
              step length= 0.1000E-03, df= 0.8688E-04, derivative= 0.1738
              step length= 0.1000E-04, df= 0.8685E-06, derivative= 0.1737E-01
              step length= 0.1000E-05, df= 0.8653E-08, derivative= 0.1734E-02
              step length= 0.1000E-06, df= 0.8330E-10, derivative= 0.1702E-03
              step length= 0.1000E-07, df= 0.5104E-12, derivative= 0.1379E-04
              step length= 0.2063E-08, df=-0.3699E-13, derivative=-0.4659E-10
 iter num    2, nb calls=   9, f= 0.2875E-03
 linear search: initial derivative=-0.1712E-07
              step length= 0.9045E-04, df=-0.6657E-13, derivative=-0.1341E-07
              step length= 0.3733E-03, df=-0.1681E-12, derivative=-0.1502E-08
 iter num    3, nb calls=  11, f= 0.2875E-03
 linear search: initial derivative=-0.1512E-08
              step length=  1.000    , df=-0.1341E-14, derivative= 0.1102E-08
 iter num    4, nb calls=  12, f= 0.2875E-03
 linear search: initial derivative=-0.1103E-08
              step length=  1.000    , df= 0.2406E-15, derivative= 0.1203E-08
              step length= 0.3191    , df= 0.2152E-16, derivative=-0.2483E-10
              step length= 0.9011E-01, df=-0.6668E-17, derivative= 0.1017E-09
              step length= 0.3867E-01, df=-0.9433E-17, derivative= 0.2709E-09
 iter num    5, nb calls=  16, f= 0.2875E-03
 linear search: initial derivative=-0.2709E-09
              step length=  1.000    , df=-0.2439E-17, derivative= 0.6674E-09
 iter num    6, nb calls=  17, f= 0.2875E-03
 linear search: initial derivative=-0.6674E-09
              step length=  1.000    , df=-0.5367E-17, derivative= 0.1155E-09
 iter num    7, nb calls=  18, f= 0.2875E-03
 linear search: initial derivative=-0.1166E-09
              step length=  1.000    , df= 0.7860E-17, derivative=-0.5224E-09
 iter num    7, nb calls=  19, f= 0.2875E-03
***** leaves -qn code-, gradient norm=  0.3857918809684193E-09
Norm of projected gradient lower than   0.3857919D-09.

  ...optimization ended

  Value of the parameters at the end of iteration  4
beta   0.9650855
gamma  4.7908799
....

GMM estimation results
'dependent' variables: y
'exogenous' variables: X_1,X_2
instruments: Z_1,Z_2,Z_3
number of observations: 170
```

2 parameters, 3 moment conditions
1 equations model, 3 instruments
initial Weighting Matrix: I
weighting Matrix: Optimal
spectral Density Matrix: Newey-West (6 lags)

Parameters estimates

Parameters	Coeff	Std Err	Null	t-stat	p-val
beta	0.9650855	0.0191705	1	-1.8212631	0.0685669
gamma	4.7908799	3.678198	0	1.3025073	0.1927430

Moment conditions

	Moment	Std Err	t-stat	p-val
Moment 1	0.0000191	0.0000862	0.2210671	0.8250402
Moment 2	-0.0001878	0.0008496	-0.2210650	0.8250419
Moment 3	-0.0004346	0.0019659	-0.2210653	0.8250416

J-stat = 0.0489 Prob[Chi-sq.(1) > J] = 0.0686

HJ =

0.0378017