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THE DEVELOPMENT OF AN IMPROVED CODED-PULSE,  
VERTICAL-INCIDENCE IONOSONDE

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degree of Master of Science at

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by

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## LITERATURE CITED

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ABSTRACT

This thesis describes the theoretical development of a new ionospheric sounding system.

The different types of ionosonde, their prime objectives, and their relative merits and demerits are discussed.

The various types of code and their correlation functions are described.

The essential requirements of the new system are listed, and suitable codes are found for it. Computer calculations and mathematical derivations demonstrate the (theoretical) suitability of these codes under all conditions.

Essentials of the mode of operation of the system and details of its design are specified, and computer simulations are used to examine relevant aspects of its operation.

Finally, since the construction of the system is not complete and results cannot therefore be presented, the present state of construction of the system is described.

HISTORICAL NOTE

The origin of the "Bal" family can be traced back to 1946, when the National Institute for Telecommunications Research was developing the Wadley ionosonde. The delightful mis-translation of the terms "ionosphere sounder" into the Afrikaans "ysterbal klanker" (literally "iron ball sounder") caused much amusement, the new ionosonde was promptly nicknamed "Ysterbal", and the lineage was established.

Rhodes University continued the succession in the fifties and sixties with Ysterbals II, III and IV, and when a miniaturised model was born in the late sixties, it was appropriately named "Minibal".

This thesis describes the creation of "Microbal", the latest addition to the illustrious house of "Bal".

CHAPTER 1

INTRODUCTION

1.1 Presentation of contents.

In the interests of a coherent presentation of this thesis, it was decided that neither a purely logical, nor purely chronological arrangement of the material would be advisable, but rather that a suitable compromise between the two approaches should be reached. Thus the arrangement of the six chapters is intended to be a dominantly logical one, while the chronological development is emphasized within the chapters themselves. These remarks apply particularly to Chapters 3 and 4 which represent the main body of the work.

It is hoped that in retrospect the natural evolution of the project (from the modification of an old system, to the development of a new one) will be clearly seen.

1.2 Purpose of an ionosonde.

The prime object of a ground-based, vertical-incidence ionosonde (or ionosphere sounder) is to obtain a record of the variation with carrier frequency of

- (i) the virtual (or apparent) height of reflection of radio signals returned from the ionosphere, and
- (ii) the intensity of these reflected signals (or "echoes").

This essentially 3-dimensional record - called an ionogram - may be used for the estimation of a number of important properties of the upper atmosphere (Phillips, 1974).

Other uses of ionosondes, such as the study of fluctuations in the ionosphere or the state of polarization of the reflected signals are not relevant to this thesis and will not be discussed here; nor will the scaling and interpretation of ionograms.

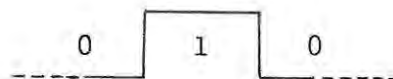
### 1.3 (Single-pulse) ionosondes.

#### 1.3.1 Some definitions and important concepts.

A number of terms, methods and basic concepts arise in connection with pulse ionosondes. Most can be found in the literature, but their use in this context will nevertheless be briefly explained here.

The simplest type of pulse ionosonde transmits a single, large, amplitude-modulated, radio-frequency pulse upwards towards the ionosphere, where it is dispersed by one or more of the ionospheric layers. Some of the energy in the pulse is returned to the ionosonde, where it is amplified and envelope detected.

If the pulse is one of a pulse train (or code or sequence) it may be called a bit, and it has a length (in time) called the bit length. The sinusoidal, radio-frequency (R.F.) pulse will usually be represented schematically by its rectified envelope, thus:



"One" and "zero" (or "negative one") may be used in the usual way to denote the presence or absence of pulses in a pulse train.

The time between transmission (at any set carrier frequency) of the outgoing pulse and arrival of the signal (or echo) pulse is called the delay or delay time, and will be denoted by  $\tau$ . It is mainly this information that the physicist seeks. The relation between the propagation distance,  $d$ , and the delay is then very nearly

$$d = c\tau$$

where  $c$  is the velocity of light in vacuo.

It should be noted here that due to the proximity of transmitter and receiver antennas in the usual vertical-incidence system, transmission and reception cannot take place at the same time, since the usually powerful transmitter would simply swamp the sensitive receiver. The maximum transmission time is thus usually constrained to be less than the minimum expected delay time.

The virtual height (v.h) is, of course, half the propagation distance.

$$\text{v.h.} = \frac{ct}{2}$$

The ionosphere is usually sounded to a v.h. of 1 000 km since echoes usually occur within this range (Delobbeau, 1971, p.48). A single scan of the ionosphere thus takes about 6,7 ms.

In the usual vertical-incidence system the frequency of the R.F. carrier is not kept constant for successive scans, but is swept from about 1 to 15 MHz. The range of frequencies used, and the fact that they are swept, are the essential features which distinguish an ionosonde from a conventional range radar.

A simple data-capture system consists of an oscilloscope whose beam is triggered by the transmitted pulse and z-modulated by the signal. A moving photographic film records the change of v.h. (and intensity) with frequency. Height and frequency markers may be added if desired. The resulting ionogram may be printed in the usual way or somehow projected onto a screen. (See Plate 1.1 below).

Although the interpretation of ionograms is not of interest in this text, two points should be noted:

- (i) the lowest echo usually occurs at a v.h. of approximately 100 km (delay time 0,67 ms.), and
- (ii) multiple echoes (due, for instance, to back-and-forth reflection between an ionospheric layer and the earth) are common.

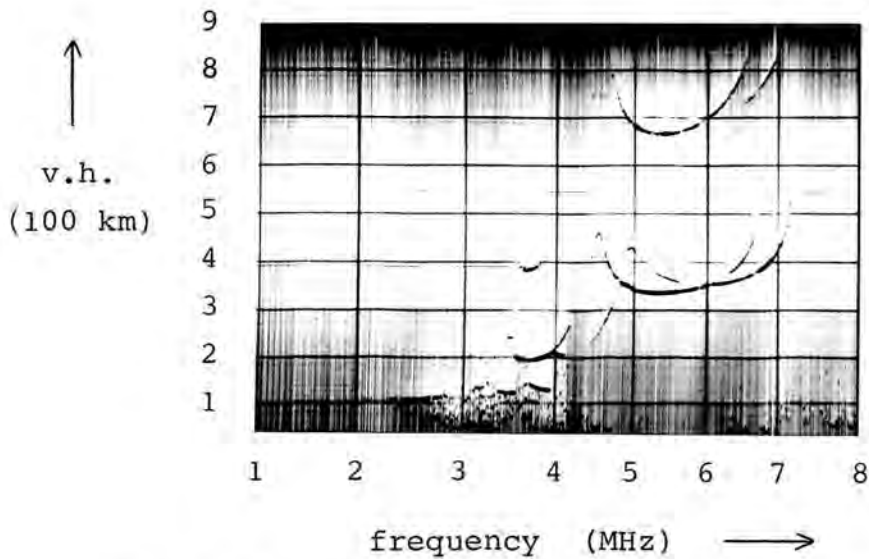


Plate 1.1 A typical ionogram ("Minibal").

1.3.2 Limitations.

There are two serious limitations which detract from the usefulness of a single-pulse ionosonde.

- (i) If the ionosonde is to be mobile, there is usually a limit to the lengths of its transmitter and receiver antennas. If the system is expected to be sensitive to signals with carrier frequencies as low as 1 MHz, and if simple half-wave dipoles are to be used, then these antennas should be about 150 m long. This specification is difficult to meet, especially on board an aircraft where fixed antennas are limited in length to about 20 m. If sufficiently long antennas are not practicable, other methods must be found to increase sensitivity, particularly at the lower end of the frequency range.
- (ii) The peak output power of a single-pulse ionosonde is limited by practical considerations such as increased expense, difficulty, and interference to others (Phillips, 1974). This means that other methods of improving the signal-to-noise ratio (S.N.R.) of the ionosonde (without loss of v.h. resolution) must be used. One such method is described below.

#### 1.4 Coded-pulse ionosondes.

##### 1.4.1 A brief introduction.

Instead of transmitting a single pulse per scan, a coded-pulse ionosonde transmits a suitably encoded amplitude-, phase- or frequency-modulated pulse train (or code) in the usual way, and feeds all incoming signals into an autocorrelator or matched filter. This unit performs an autocorrelation process on the demodulated signals, and its output (the autocorrelation function) yields the required delay-time information, which is recorded in the usual way. This process, also known as digital pulse compression (and sometimes as matched filtering), has been used in radar, ordinary ionospheric sounding, and incoherent-scatter applications (Skolnik, 1970, pp. 20-18 and 20-19; Coll and Storey, 1964 and 1965; Gray and Farley, 1973; Ioannidis and Farley, 1972).

Thus the S.N.R. of the system is improved, less peak power need be expended, and the v.h. resolution is not diminished since the autocorrelator responds only for the duration of a single pulse. Details of correlation processes will be discussed in Chapter 2.

##### 1.4.2 Limitations.

As was mentioned in section 1.3.1, the close proximity of transmitter and receiver in the usual vertical-incidence ionosonde imposes a limit on the time during which the transmitter may be operational. This means that for a fixed transmission time, the number of bits in the transmitted code can only be increased by decreasing the length of each bit. While increasing the number of bits entails an increase in the S.N.R. of the system, a decrease in bit length necessitates a broadening of receiver bandwidth and a subsequent decrease in overall S.N.R. This maximum transmission time constraint is the only serious limitation of the coded-pulse ionosonde.

### 1.5 Initial aims and objectives.

The initial aim of the project was to modify an already existent single-pulse ionosonde, so as to make it more efficient.

The ionosonde in question (nick-named "Minibal") transmits a single, high-power, R.F. pulse 50 times per second. Since it may be operated either on an aircraft or a ship, the lengths of its antennas are severely limited, which results in a loss of efficiency at low frequencies.

Thus the initial objectives of the project were, if possible,

- (i) to improve the S.N.R. of the system (especially at the low-frequency end) without decreasing height resolution,
- (ii) to expend less peak power in doing so, and
- (iii) to modify the system as little as possible in implementing these objectives.

It was proposed that the single-pulse system should be changed to a coded-pulse one, as described in section 1.4. It was envisaged that a 13-bit Barker code would be used, since it is the longest of the well-known codes having a suitable autocorrelation function (see Chapter 2). The S.N.R. of the system would thus be improved and less peak power need be expended, but it must be noted that in order to keep modifications to a minimum, the system was to remain essentially amplitude-modulated (A.M.), since phase- or frequency-modulation would require drastic changes (to the receiver in particular).

The practical implementation of the initial aims involved designing and building suitable circuitry to

- (i) generate a code for the transmitter, and
- (ii) correlate all incoming signals with the transmitted code after transmission had ceased.

This system, hereafter called the Barker system, will be more fully discussed in Chapter 3.

1.6 Subsequent suggestions.

Later, however, a somewhat novel mode of operation, similar to that of the Barry Research "Chirpsounder" (Phillips, 1974, part IIIC, p.789), was suggested by a colleague (Poole, 1975).

It was proposed that the ionosonde should transmit "semi-continuously" (that is, on average half of the time) and cyclically according to some suitable binary code, incoming signals being received during the transmitter off-times. A correlator would process any returning signals (obviously some would be lost) and so determine the delay between transmission and reception of the code. Inherent in this system are

- (i) semi-continuous (i.e. 50%) transmission, which would greatly increase transmitted signal energy, and
- (ii) amplitude modulation (or on-off-keying, O.O.K.), since the transmitter must be completely shut down to allow reception of incoming signals.

The most serious difficulty regarding the implementation of this system would be finding a code which would

- (i) allow approximately equal transmission and reception time,
- (ii) have an unambiguous autocorrelation function (see Chapter 2), and
- (iii) give a fairly constant return of signal energy for all possible delays.

This system, hereafter called the "Poole" system, will be more fully discussed in Chapter 4.

CHAPTER 2

CORRELATION PROCEDURES

2.1 Introduction.

It is clear from the preceding chapter that any discussion of the Barker or Poole systems requires a detailed explanation of the terms, methods and basic concepts which arise in connection with correlation procedures. Although most of these can also be found in the literature, their use in this context will be briefly explained in the next section. Section 2.3 discusses a special type of correlation.

2.2 Cyclic and non-cyclic digital autocorrelation.

There are two distinct types of correlation, digital and analogue, each of which may be subdivided into auto- and cross-correlation.

Since we will be concerned with sequences of discrete "rectangular" pulses which (if transmitted by an ionosonde) will, on return from the ionosphere, be correlated with themselves, only digital autocorrelation need be defined here. The cross-correlation function differs from the autocorrelation function only in that elements of two different sequences are multiplied and the products summed (see the definitions below).

Furthermore, digital autocorrelation may be subdivided into two common categories: cyclic and non-cyclic.

The non-cyclic digital autocorrelation function of a sequence or code  $a_i, i = 1, 2, \dots, N$  is defined as the set of ordered pairs  $(k, A(k))$  where

$$A(k) = \begin{cases} \sum_{i=1}^{N-k} a_i a_{i+k} & 0 \leq k \leq N-1 \\ 0 & k \geq N \end{cases}$$

with  $k$  an integer variable  
 and  $A(k) = A(-k)$   
 (Coll and Storey, 1964, p.1156).

Similarly one cycle of the cyclic function may be defined by

$$A(k) = \sum_{i=1}^k a_i a_{N-k+i} + \sum_{i=1}^{N-k} a_{i+k} a_i \quad 0 \leq k \leq N-1$$

with  $k$  an integer variable  
 and  $A(k) = A(-k)$  once more.  
 (c.f. Green, 1975, p.33; MacWilliams and Sloane, 1976, p.1718).

The autocorrelation function is sometimes "normalised" by dividing each  $A(k)$  by the number  $N$  of bits in the code. The elements  $a_i$  might be binary (we make an important distinction here between the "ordinary" binary elements 1 and 0, and the "bipolar" ones 1 and -1), ternary (for instance 1, 0 and -1) or multilevel (for instance 0,  $\pm 1$ ,  $\pm 2 \dots$ ). The ordinary and bipolar binary types will be used interchangeably throughout this text.

Practically, both types of binary autocorrelation function defined above may be easily calculated. Consider, for example, the bipolar sequence -1 1 1 1. If the code is written below itself with no "offset" ( $k = 0$ ) and a term-by-term multiplication is carried out and the products summed, the result ( $A(0)$ ) is 4. If the process is repeated with a single "offset" ( $k = 1$ ), the result is  $A(1) = 1$  if the overlapping bit is ignored (the non-cyclic case) and  $A(1) = 0$  if the overlapping bit is "cycled" back to the beginning (cyclic case).

code	-1 1 1 1	-1 1 1 1
offset code	-1 1 1 1	-1 1 1 1
	-1 1 1	-1 1 1 1
products	-1 1 1	-1 1 1 1
$A(1)$	1	0
	(non-cyclic)	(cyclic)

Instead of cycling the last bit back to the beginning in the

cyclic case, Skolnik (1970, p.16-27) achieves the same result by writing the code out twice in the first line, so that no overlap occurs.

Calculation of the  $A(k)$  for all  $N = 4$  possible values of  $k$  for both the cyclic and non-cyclic cases may be summarized as follows:

non-cyclic					cyclic						
k	-1	1	1	1	A(k)	k	-1	1	1	1	A(k)
0	-1	1	1	1	4	0	-1	1	1	1	4
1		-1	1	1	1	1	1	-1	1	1	0
2			-1	1	0	2	1	1	-1	1	0
3				-1	-1	3	1	1	1	-1	0

Table 2.1

This method is, of course, not the only convenient way of obtaining autocorrelation functions (Hovanessian, 1973, p.7-40).

A number of points should be noted in connection with the above definitions:

- (i) Since the autocorrelation functions are symmetric about  $k = 0$ , only half of them need be shown.
- (ii) The cyclic and non-cyclic bipolar autocorrelation functions are, of course, different.
- (iii) The functions may be conveniently (if somewhat loosely) represented by graphs (Coll and Storey, 1964, pp.1156-1157; Gray and Farley, 1973, p.125 and Ioannidis et al, 1972, p.763). (See Figure 2.1 below).
- (iv) It is to be expected that  $A(0) > A(k)$ ,  $0 < k \leq N - 1$ . Thus a peak or main lobe is expected in the autocorrelation function for  $k = 0$  (zero offset or "match") while lesser peaks or sidelobes may occur elsewhere. Important properties of a code or sequence are the height of the main lobe of the autocorrelation function, which is indicative - if the code is used

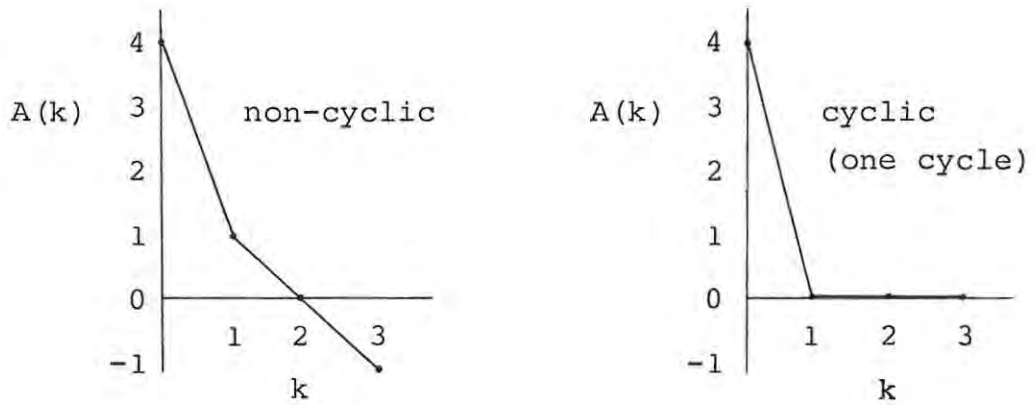


Figure 2.1

in (say) a radar application - of the energy of the signal, and the main-to-side-lobe ratio (M.S.R.), which is a measure of the "ambiguity" of the autocorrelation function.

Those functions which are described by

$$A(0) = N$$

$$A(k) \leq 1 \text{ for } 0 < k \leq N - 1$$

will be called optimum autocorrelation functions (Cook and Bernfeld, 1967, p.245) and those that have the additional property that all the  $A(k)$  are equal for  $0 < k \leq N-1$ , will be called perfect autocorrelation functions.

- (v) If the autocorrelation function of the ordinary binary sequence 0 1 1 1 (which has the same "shape" as the bipolar one, -1 1 1 1) is calculated in the same way, the results are somewhat different.

non-cyclic			cyclic		
k	0 1 1 1	A(k)	k	0 1 1 1	A(k)
0	0 1 1 1	3	0	0 1 1 1	3
1	0 1 1	2	1	1 0 1 1	2
2	0 1	1	1	1 1 0 1	2
3	0	0	3	1 1 1 0	2

Table 2.2

The non-cyclic one bears no resemblance to its bipolar counterpart (Table 2.1), but the cyclic one does. The cyclic one can, in fact, be translated into bipolar form, for instance by multiplying each of the  $A(k)$  by a constant (4) and subtracting another constant (8). A similar property will be derived in section 4.4.2 for a particular class of sequences.

In a radar application, however, this procedure is of little significance since it represents the amplification of a voltage signal and the removal of a d.c. level. If noise is present, the signal-to-noise ratio (S.N.R.) will not be improved. It must be accepted that in a physical application, bipolar binary signals are simply more useful since they have twice the excursion (and hence the energy) of ordinary ones. For this and other reasons given below (section 3.2.5), only bipolar codes will be used in the implementation of the above definitions of correlation.

### 2.3 "Extended" correlation.

Application of the formal definitions of autocorrelation to a coded-pulse ionosonde requires further careful consideration.

In the definitions, the relative positions (i.e. the offsets,  $k$ ) of the two codes being "correlated" are known and overlapping bits may be discarded or "recycled" at will. But in an ionospheric application the offsets are, of course, unknown since the position (in time) of the returned signal is unknown. The correlation process, in fact, occurs most of the time in total or partial absence of signal since the scan is about 10 times longer than the code. Thus the correlator is required to operate not only on the reflected signals alone, but on parts of the scan outside the incoming code as well, and this of course affects the results of the correlation process. This type of procedure should not be classed as an autocorrelation at all. It is rather a cross-

correlation between the basic code and an extended one consisting of the same basic code flanked appropriately by 0's or -1's.

In the idealised diagram below one scan is shown fixed in time while the "correlating" code moves with time along it, performing the necessary correlation operation as it does so. The code is once more -1 1 1 1.

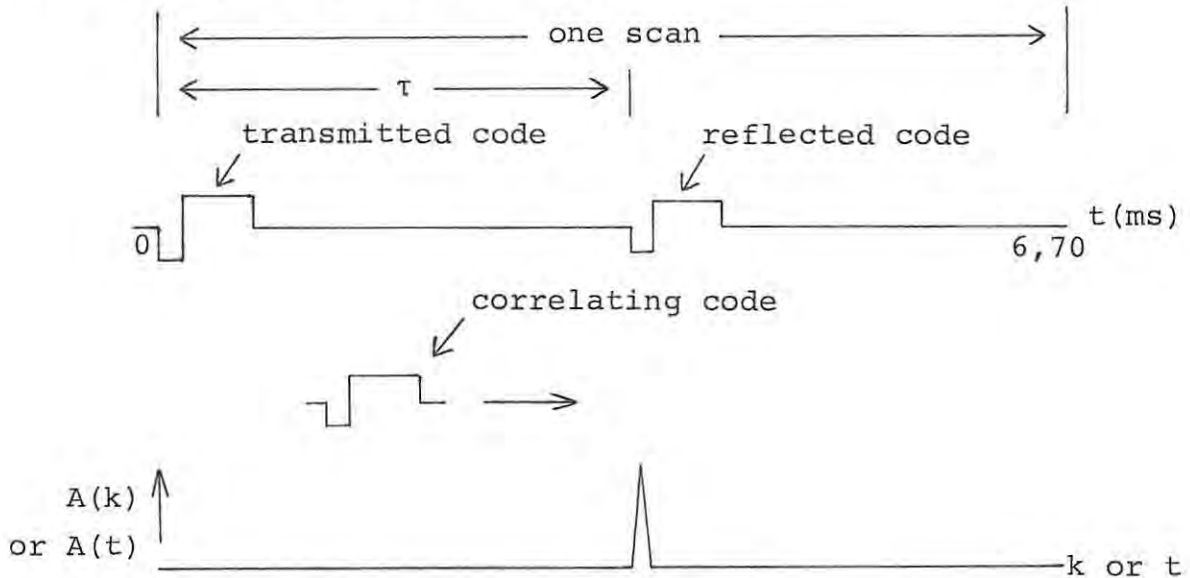


Figure 2.2

This consideration does not, however, affect phase- or frequency-modulated systems since these are in effect ternary, not binary ones. They may be designed to yield three signal levels after demodulation, viz. two values of phase or frequency (which may be denoted by 1 and -1) and a total absence of signal (denoted by 0). The corresponding amplitude-modulated or O.O.K. (on-off-keyed) system is strictly binary since there are only two possible signal levels - presence or absence of signal.

The correlation process required by phase- or frequency-modulated systems is thus equivalent to the non-cyclic bipolar case discussed above (Table 2.1), since multiplication of overlapping bits by 0 is equivalent (in terms of the definition of autocorrelation) to ignoring them. The following table describes the process in some detail.

k	0 0 0 0	-1 1 1 1	0 0 0 0	A(k)	
-4	-1 1 1 1			0	
-3		-1 1 1 1		-1	
-2			-1 1 1 1	0	
-1				-1 1 1 1	1
0				-1 1 1 1	4
1				-1 1 1 1	1
2				-1 1 1 1	0
3				-1 1 1 1	-1
4				-1 1 1 1	0

Table 2.3 Extended ternary correlation.

The process required by an ordinary binary (i.e. A.M. or O.O.K.) system, too, yields the same results as its non-cyclic counterpart in Table 2.2.

k	0 0 0 0	0 1 1 1	0 0 0 0	A(k)	
-4	0 1 1 1			0	
-3		0 1 1 1		0	
-2			0 1 1 1	1	
-1				0 1 1 1	2
0				0 1 1 1	3
1				0 1 1 1	2
2				0 1 1 1	1
3				0 1 1 1	0
4				0 1 1 1	0

Table 2.4 Extended ordinary binary correlation.

The function produced by a bipolar binary system, however, deviates from the previously-discussed types, since now the correlation process does not distinguish between a -1 indicating a complete absence of signal, and a -1 belonging to the code.

k	-1 -1 -1 -1	-1 1 1 1	-1 -1 -1 -1	A(k)
-4	-1 1 1 1			-2
-3	-1 1 1 1			-2
-2	-1 1 1 1			0
-1	-1 1 1 1			2
0	-1 1 1 1			4
1	-1 1 1 1			0
2	-1 1 1 1			-2
3	-1 1 1 1			-4
4	-1 1 1 1			-2

Table 2.5 Extended bipolar binary correlation.

Not only do the  $A(k)$  differ from the usual (non-cyclic) ones, but the function is not even symmetric as it was before. This type of correlation, then, represents a special case, and was called "extended bipolar (binary) correlation".

In order to avoid controversy and confusion about the type of correlation involved (i.e. auto- or cross-correlation), any correlation process which involves the correlation of a code with a delayed, superposed, filtered, extended or otherwise modified version of itself will in future simply be termed "correlation" and the resulting function the "correlation function". The elements of this correlation function will (for convenience) still be denoted by  $A(k)$ .

#### 2.4 Summary.

The definitions of cyclic and non-cyclic autocorrelation functions and various aspects of their application to binary codes were discussed. It was shown that while phase- and frequency-modulated systems are easy to implement in terms of these definitions, the correlation procedure required by an amplitude-modulated ionosonde depends on whether the system is ordinary binary or bipolar. The correlation required by a bipolar binary coded-pulse ionosonde is a special one and was called "extended bipolar correlation".

## CHAPTER 3

### THE "BARKER" SYSTEM

#### 3.1 Introduction.

Coded-pulse trains are at present being used in radar and ionospheric sounding applications, and have been widely discussed in the literature (Cook and Bernfeld, 1967, Chapter 8; Nathanson, 1969, Chapter 12; and Sparrius and Ribbens, 1969, in addition to those quoted in section 1.4.1. Phillips, 1974, part III gives a useful review of sounding techniques).

Most of the binary coded-pulse systems described in the literature, however, employ either phase or frequency modulation. As explained in section 2.3, the procedures necessary in a bipolar amplitude-modulated system, though similar in principle, differ in detail from those of (for instance) the phase-modulated system of Coll and Storey.

#### 3.2 Miscellaneous aspects.

##### 3.2.1 Properties of the code.

If a maximum transmission time (see section 1.4.2) of 600  $\mu$ s is chosen, which is equivalent to a v.h. of 90 km (the lowest ionospheric layer usually sounded - the E-region - lies at about 100 km), and a bit length of 50  $\mu$ s is assumed, the Barker system requires a code of length 12 bits.

A search was thus made for a code of length from about 10 to 17 bits with the desired correlation function.

##### 3.2.2 Correlation procedures.

Three Fortran IV computer programs were written, one to implement the "extended bipolar correlation" and labelled XBCO, the other two - called ACOR and CACO - to perform non-cyclic and cyclic autocorrelations respectively. ACOR and CACO

were used to check the autocorrelation properties of various codes against those given in the literature. All three programs are given in Appendix A.

The response of a number of codes to XBCO, ACOR and CACO will be discussed in section 3.3.

### 3.2.3 Multiple echoes.

There are 3 common causes of multiple vertical-incidence ionospheric reflections (see Plate 1.1):

- (i) partial reflection of the same transmitted signal by two or more ionospheric layers, which does not occur often,
- (ii) propagation via the ordinary (O) and extraordinary (E) modes, which is due to the earth's magnetic field, and therefore cannot be avoided unless the polarization of the reflected signals is taken into account (Ratcliffe, 1960, p.399),
- (iii) back-and-forth reflection of a signal between the earth and an ionospheric layer, which is not uncommon.

Since echoes of the third type are separated in time by at least the length of the transmitted code, they pose no problem to the system, but those of the second type in particular, deserve further attention.

Since the v.h's of reflection of the O and E echoes can, and in fact often do, approach and "cross over" each other with change in carrier frequency, a coded-pulse system must be able to resolve such echoes. In other words, it must be designed to cater for super-position of coded-pulse trains. Once again our system must be slightly modified.

Since the correlation function is linear (that is, the correlation of a code with the algebraic sum of two codes is the algebraic sum of the relevant correlations taken separately), the problem is easily overcome, because the codes can simply be summed and the correlation carried out on this sum.

However, since the algebraic sum of binary codes is not itself binary, the super-posed signal trains appearing in the receiver must not be converted into binary signals, as this would result in a needless loss of information. Thus the incoming signals must be sampled at more than one level. The physical implementation of this process is not difficult with an analogue-to-digital converter (A.D.C.), and will be discussed more fully below.

As an example of this phenomenon, consider the following simplified case of two bipolar binary signals returning from the ionosphere:

$$\begin{array}{rcccccccc}
 \text{E echo} & \dots & -1 & \boxed{-1 \ 1 \ 1 \ 1} & -1 & -1 & -1 & \dots \\
 \text{O echo} & \dots & -1 & -1 & -1 & \boxed{-1 \ 1 \ 1 \ 1} & -1 & \dots \\
 \hline
 \text{received signal} & \dots & -2 & -2 & 0 & 0 & 2 & 0 & 0 & -2 & \dots \\
 \text{correlating code} & & & \boxed{-1 \ 1 \ 1 \ 1} & \rightarrow & & & & & & 
 \end{array}$$

Some of the A(k) for this correlation are

$$\dots \quad 0 \quad 4 \quad 2 \quad 2 \quad -4 \quad -4 \quad -6 \quad -4 \quad \dots$$

These are clearly the result of a term-by-term algebraic sum of the original A(k) (Table 2.5) and the same A(k) offset by two bits. Thus:

$$\begin{array}{rcccccccc}
 \text{A(k)} & \dots & 2 & \boxed{4 \ 0 \ -2 \ -4} & -2 & -2 & -2 & \dots \\
 \text{offset A(k)} & \dots & -2 & 0 & 2 & \boxed{4 \ 0 \ -2 \ -4} & -2 & \dots \\
 \hline
 \text{sum} & \dots & 0 & 4 & 2 & 2 & -4 & -4 & -6 & -4 & \dots \\
 & & & \uparrow & & \uparrow & & & & & \\
 & & & & & & & & & & \text{E and O mainlobes}
 \end{array}$$

The importance of this process is clearly demonstrated in this example. The super-position has diminished the main-lobe of the O signal. In fact, it is clear that had the two signals been separated by 3 bits, any trace of the O echo would have been obliterated.

This procedure - called "super-posed extended bipolar correlation" - is thus an important test of the usefulness of a

code. Although the resultant  $A(k)$  can easily be calculated from the original ones, a computer program (called SXBC) was written to perform this operation for all possible superpositions, from first principles.

This program is also given in Appendix A.

#### 3.2.4 Signal averaging.

In a further effort to increase S.N.R., especially at the low-frequency end, it was decided to incorporate into the system some sort of summing or "averaging" device. Consecutive scans at a set frequency would be fed into the device, which would superimpose them appropriately and finally divide by the number of scans summed. Small signals in the presence of random noise would thus accrue, while the noise would settle to some d.c. level which could then be discriminated against by the correlator (Lynn, 1973, p.207).

Since it had already been proposed that the incoming signals should be converted to digital, the averaging device would, in this case, simply consist of a "memory" into which the signals could be added. When a predetermined number of scans had been superimposed and the necessary divisions done, the resultant information would be sent to the correlator. The entire process would then be repeated at the next frequency. The averaging device will be discussed in more detail later.

Furthermore, since the entire averaging and correlation process is highly amenable to simulation on any digital computer which has a random-number generator, programs were written to test all aspects of the system. One such program is discussed in Chapter 5.

#### 3.2.5 Signal levels and d.c. removal.

Although the transmitted code is O.O.K. (on-off-keyed) and can therefore be thought of as ordinary binary, it was decided (mainly for the reasons given below) that the proceeding correlation should be of the bipolar binary type.

To achieve this end, the incoming demodulated signals would be passed through a filter which would remove the d.c. (zero-frequency) component in the signal, thereby effectively changing the zero-level of the signal train, and changing the ordinary binary code into a bipolar one of the same total excursion. (Lynn, 1973, p.144). Naturally, the code with which the signal would be correlated (the "correlating" code) would also be bipolar.

This d.c. removal has not been written into the programs given in Appendix A, since it is convenient to work with integer arithmetic and bipolar codes. This means that all the  $A(k)$  of the correlations below must, in fact, be halved because an ordinary binary code like 1 0 0 1 (when it has its d.c. removed) becomes  $\frac{1}{2}$   $-\frac{1}{2}$   $-\frac{1}{2}$   $\frac{1}{2}$ , which will yield  $A(k)$  that are half those which result from the corresponding bipolar code 1 -1 -1 1, when correlated against 1 -1 -1 1.

The reasons for wanting the correlation to be bipolar are:

- (i) the correlation procedure is defined in terms of multiplication of bipolar sequences, and is easy to implement electronically. This is the least important reason.
- (ii) The summing or averaging of ordinary binary signals in the presence of noise would lead to the accumulation of enormous numbers in the averaging device, whereas conversion to bipolar would result in numbers whose average is approximately zero. This would also mean that the  $A(k)$  resulting from the correlation would have an average of zero, rather than some large value.
- (iii) A bipolar correlating code, which would ideally comprise as many 1's as -1's, would discriminate against any non-random (for instance, continuous-wave (C.W.)) signals, should they accumulate in the averaging device, giving (ideally) a correlation function which is zero everywhere.

Though the bipolar system described above is obviously not the only viable one, it was decided to use it, and that a

code would be sought to suit the system, rather than the converse.

3.3 Some of the codes.

3.3.1 The 13-bit Barker code, B13.

The properties of the Barker codes have been widely discussed in the literature and details will therefore not be discussed here. All that need be noted is that these codes have optimum non-cyclic autocorrelation functions, and that the longest one has 13 bits. This code, hereafter abbreviated B13, is

1 1 1 1 1 -1 -1 1 1 -1 1 -1 1

ACOR verifies that its non-cyclic autocorrelation function is optimum, and is

13 0 1 0 1 0 1 0 1 0 1 0 1

(Cook and Bernfeld, 1967, p.245; Nathanson, 1969, p.467).

It is of interest to note that its cyclic autocorrelation function as given by CACO, is perfect, viz:

13 1 1 1 1 1 1 1 1 1 1 1 1

Furthermore, the result of XBCO is highly satisfactory:

-3 -5 -3 -5 -3 -3 -3 -5 -3 -3 -1 -1 13  
-1 1 -1 1 -1 -1 -1 1 -1 -1 -3 -3 -5

This is, of course, by far the most significant result. The M.S.R. is 13, the same as that given by ACOR and CACO. (It should be noted here that for the purpose of calculating the M.S.R. of a correlation function whose sidelobes are all less than one, the maximum sidelobe is taken to be one. Clearly zero and negative integers in the denominator of such a ratio will cause unnecessary complications, and digital electronic circuitry can easily be designed to discriminate against such numbers).

The preponderance of large, negative A(k) resulting from XBCO is a result of the predominance of +1's in the 13-bit Barker

code, and is in itself of no consequence. However, a superposition of such pulse trains will cause the -5's, in particular, to greatly diminish the mainlobe of any correlation function, should they happen to coincide with it. SXBC illustrates this point rather well, especially when the superposed codes are separated by 5, 9, 11 and 13 bits.

separation	A(k)												
0	26	=2	2	=2	2	=2	=2	=2	2	=2	=2	=6	=6
1	12	12	0	0	0	0	=2	=2	0	0	=2	=4	=6
2	12	=2	14	=2	2	=2	0	=2	0	=2	0	=4	=4
3	10	=2	0	12	0	0	=2	0	0	=2	=2	=2	=4
4	10	=4	0	=2	14	=2	0	=2	2	=2	=2	=4	=2
5	8	=4	=2	=2	0	12	=2	0	0	0	=2	=4	=4
6	10	=6	=2	=4	0	=2	12	=2	2	=2	0	=4	=4
7	10	=4	=4	=4	=2	=2	=2	12	0	0	=2	=2	=4
8	10	=4	=2	=6	=2	=4	=2	=2	14	=2	0	=4	=2
9	8	=4	=2	=4	=4	=4	=2	0	12	=2	=2	=4	=2
10	10	=6	=2	=4	=2	=6	=4	=4	0	=2	12	=4	=2
11	8	=4	=4	=4	=2	=4	=6	=4	=2	=2	=2	10	=4
12	10	=6	=2	=6	=2	=4	=4	=6	=2	=4	=2	=4	10
13	8	=4	=4	=4	=4	=4	=4	=4	=4	=4	=4	=4	=4

The worst-case M.S.R. here is 5, for example at separation 4.

The 13-bit Barker code is therefore suitable for the system under discussion, but does not perform particularly well under super-posed conditions.

### 3.3.2 Coll and Storey's code, CS17.

Coll and Storey (1964, p.1156) successfully used a 17-bit binary sequence with M.S.R. 8,5 in an ionospheric sounding experiment. The code they used is

1 1 1 1 =1 =1 1 1 =1 1 1 =1 1 =1 1 =1 =1

Since the origin of this code was at the time not clear, it was simply named CS17 after the authors, and put to the usual tests.

ACOR verified that the non-cyclic autocorrelation function is (op.cit., Figure 1):

17 =2 =1 2 =1 0 =1 2 1 0 =1 2 1 =2 =1 =2 =1

The cyclic autocorrelation function has an even better M.S.R. The A(k) are

$$17 \quad -3 \quad -3 \quad 1 \quad -3 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad -3 \quad 1 \quad -3 \quad -3$$

However, XBCO yields an M.S.R. of only  $\frac{17}{3}$ . The A(k) are

$$\begin{matrix} -5 & -7 & -5 & -7 & -3 & -3 & -5 & -3 & -3 & -1 & -3 & -3 & -5 & -1 & -3 & -3 & 17 \\ -1 & 1 & 3 & 1 & 1 & 1 & 3 & 1 & 1 & -1 & 1 & 1 & -1 & -1 & -3 & -3 & -3 \end{matrix}$$

This somewhat inferior M.S.R. and the resulting super-posed correlation function mean that CS17, although suitable for Coll and Storey's phase-modulated system, proves inadequate for our application.

### 3.3.3 The 15-bit M-sequence, M15.

The derivation and properties of the M-sequences (linear feedback shift-register sequences or pseudo-random noise sequences) will be discussed fully in Chapter 4, where they are more relevant to the discussion. They have length  $2^n - 1$  bits (where n is a positive integer), so that a 15-bit code was chosen to be tested in the usual way. This code, as originally derived from the alpha-sequence 1 0 0 1 and starting condition 1 1 0 1 (see section 4.3.5) is, in bipolar form

$$1 \quad 1 \quad -1 \quad 1 \quad -1 \quad 1 \quad 1 \quad -1 \quad -1 \quad 1 \quad -1 \quad -1 \quad -1 \quad 1 \quad 1$$

Its cyclic autocorrelation (CACO) function is perfect. The A(k) are

$$15 \quad -1 \quad -1 \quad -1 \quad -1 \quad -1 \quad -1 \quad -1 \quad -1 \quad -1 \quad -1 \quad -1 \quad -1 \quad -1 \quad -1$$

The non-cyclic one is not even optimum (Skolnik, 1970, p.20-20).

However, since this code did not yield satisfactory results under the extended correlation, XBCO, all the possible orientations of the code, formed by cyclically shifting bits from (say) the beginning to the end of the code, were tested in an effort to optimize the correlation function. Only one such orientation was found to have an optimum XBCO. This code is

$$-1 \quad -1 \quad -1 \quad 1 \quad 1 \quad 1 \quad 1 \quad -1 \quad 1 \quad -1 \quad 1 \quad 1 \quad -1 \quad -1 \quad 1$$

Now since the XBCO operation is reversible in k (or time), reversal of the order of the bits will leave the M.S.R. un-

changed. The reversed code, hereafter named M15, is

1 -1 -1 1 1 -1 1 -1 1 1 1 1 -1 -1 -1

Its XBCO is entirely satisfactory.

=3 =3 -3 -1 =3 =3 -1 -1 1 =1 -3 -1 -1 =1 15  
 1 1 1 -1 1 1 -1 -1 =3 =1 1 -1 -1 -1 =1

That this particular orientation of the code is optimum is a remarkable and very fortunate coincidence, since the code now ends in three -1's (or zeros), which means that this 15-bit code has an effective length of only 12 bits and an M.S.R. of 15!

The super-posed "extended" correlation function is, not surprisingly, quite satisfactory, with a worst-case M.S.R. of 6.

separation

A(k)

0	30	2	2	2	-2	2	2	-2	-2	=6	=2	2	-2	-2	=2
1	14	16	2	2	0	0	2	0	-2	=4	=4	0	0	-2	-2
2	14	0	16	2	0	2	0	0	0	=4	=2	-2	-2	0	-2
3	14	0	0	16	0	2	2	-2	0	=2	=2	0	-4	-2	0
4	12	0	0	0	14	2	2	0	-2	=2	0	0	-2	-4	-2
5	14	-2	0	0	-2	16	2	0	0	=4	0	2	-2	-2	=4
6	16	0	-2	0	-2	0	16	0	0	-2	-2	2	0	-2	-2
7	14	2	0	-2	-2	0	0	14	0	=2	0	0	0	0	=2
8	14	0	2	0	-4	0	0	-2	14	=2	0	2	-2	0	0
9	12	0	0	2	-2	-2	0	-2	-2	12	0	2	0	-2	0
10	12	-2	0	0	0	0	-2	-2	-2	=4	14	2	0	0	-2
11	14	-2	-2	0	-2	2	0	-4	-2	=4	-2	16	0	0	0
12	12	0	-2	-2	-2	0	2	-2	-4	=4	=2	0	14	0	0
13	12	-2	0	-2	-4	0	0	0	-2	=6	=2	0	-2	14	0
14	12	-2	-2	0	-4	-2	0	-2	0	-4	=4	0	-2	-2	14
15	14	-2	-2	-2	-2	-2	-2	-2	-2	-2	=2	-2	-2	-2	=2

M15 is therefore highly suitable for use in this system.

### 3.3.4 Turyn's codes, T15A and T15B

Turyn ( 1968, p.213) lists a number of binary codes of length more than 13 bits, and having small non-cyclic autocorrelation sidelobes. One of these codes, it transpired, is CS17, and another was used by Gray and Farley (1973, p.124). Many of the shorter codes bear a close resemblance to the 13-bit Barker code

1 1 1 1 1 -1 -1 1 1 -1 1 -1 1

which in Turyn's somewhat devious notation would be written

5 2 2 1 1 1 1

Only 4 of the 20 codes tested gave optimum correlations under XBCO, two of length 15 bits, and one each of length 18 and 19 bits. The latter two were considered to be too long for use. The two 15-bit ones (named T15A and T15B) are

1 1 1 1 1 -1 -1 1 1 -1 1 =1 1 1 -1

and

1 1 1 1 1 1 -1 =1 1 1 -1 1 -1 1 1

Their XBCO functions are, respectively,

=7 =5 -3 -5 =3 =3 =3 =5 =3 =3 =5 =1 =3 =1 15  
1 =1 1 -1 1 1 =3 =1 1 1 -1 =1 =3 =5 =5

and

=5 =3 -5 -3 =5 =3 =3 =7 =5 =3 =3 =1 =1 1 15  
1 =1 1 -1 1 =1 =5 =1 1 =1 =1 =3 =3 =5 =7

and their SXBC functions are

separation

A(k)

0	30	2	=2	2	=2	2	2	=6	=2	2	2	=2	=2	=6	=10
1	14	16	0	0	0	0	2	=2	=4	0	2	0	=2	=4	=8
2	12	0	14	2	=2	2	0	=2	0	=2	0	0	0	=4	=6
3	14	=2	=2	16	0	0	2	=4	0	2	=2	=2	0	=2	=6
4	10	0	=4	0	14	2	0	=2	=2	2	2	=4	=2	=2	=4
5	12	=4	=2	=2	=2	16	2	=4	0	0	2	0	=4	=4	=4
6	12	=2	=6	0	=4	0	16	=2	=2	2	0	0	0	=6	=6
7	10	=2	=4	=4	=2	=2	0	12	0	0	2	=2	0	=2	=8
8	12	=4	=4	=2	=6	0	=2	=4	14	2	0	0	=2	=2	=4
9	12	=2	=6	=2	=4	=4	0	=6	=2	16	2	=2	0	=4	=4
10	12	=2	=4	=4	=4	=2	=4	=4	=4	0	16	0	=2	=2	=6
11	10	=2	=4	=2	=6	=2	=2	=8	=2	=2	0	14	0	=4	=4
12	12	=4	=4	=2	=4	=4	=2	=6	=6	0	=2	=2	14	=2	=6
13	10	=2	=6	=2	=4	=2	=4	=6	=4	=4	0	=4	=2	12	=4
14	8	=4	=4	=4	=4	=2	=2	=8	=4	=2	=4	=2	=4	=4	10
15	10	=6	=6	=2	=6	=2	=2	=6	=6	=2	=2	=6	=2	=6	=6

and

separation	A(k)															
0	30	2	=2	2	=2	2	=2	-10	=2	2	=2	=2	-6	=6	=10	
1	16	16	0	0	0	0	0	=6	=6	0	0	=2	-4	=6	=8	
2	14	2	14	2	=2	2	=2	-4	=2	=4	=2	0	-4	=4	=8	
3	14	0	0	16	0	0	0	=6	0	0	=6	=2	=2	=4	=6	
4	12	0	=2	2	14	2	=2	-4	=2	2	=2	=6	-4	=2	=6	
5	12	=2	=2	0	0	16	0	=6	0	0	0	=2	-8	=4	=4	
6	10	=2	=4	0	=2	2	14	-4	=2	2	=2	0	-4	=8	=6	
7	8	=4	=4	=2	=2	0	0	10	0	0	0	=2	=2	=4	=10	
8	12	=6	=6	=2	-4	0	=2	-4	14	2	=2	0	-4	=2	=6	
9	12	=2	=8	=4	-4	=2	=2	=6	0	16	0	=2	=2	=4	=4	
10	10	=2	=4	=6	=6	=2	=4	=6	=2	2	14	0	-4	=2	=6	
11	12	=4	=4	=2	=8	=4	=4	=8	=2	0	0	14	=2	=4	=4	
12	10	=2	=6	=2	-4	=6	=6	=8	=4	0	=2	0	12	=2	=6	
13	12	=4	=4	=4	-4	=2	=8	=10	=4	=2	=2	=2	=2	12	=4	
14	10	=2	=6	=2	=6	=2	=4	=12	=6	=2	=4	=2	-4	=2	10	
15	8	=4	=4	=4	-4	=4	=4	=8	=8	=4	=4	=4	-4	=4	=4	

The worst-case M.S.R.'s are both 5.

Both these codes, then, would be suitable for the system, especially T15A, which has an effective length of 14 bits.

### 3.4 Summary.

A number of codes were tested, particularly with reference to their special "extended" correlation (XBCO) functions, with a view to implementation in the Barker system. The most promising of these have been listed above; others include the 11-bit Barker code and the de Bruijn (or non-linear feedback shift-register) sequences, which will be discussed later. The results of the XBCO operation on B13, CS17, M15, T15A and T15B are summarized graphically in figures 3.1 to 3.5.

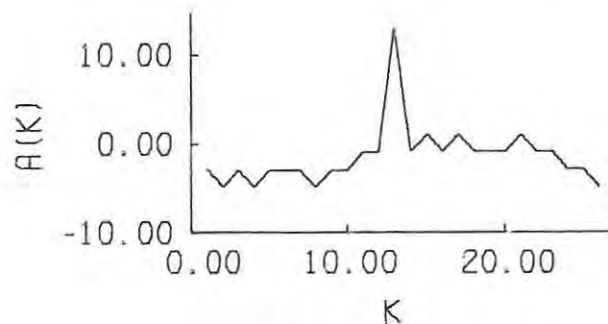


Figure 3.1 XBCO on B13

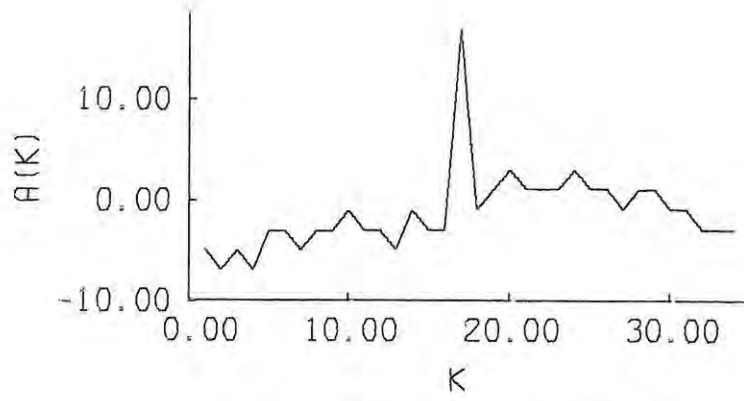


Figure 3.2 XBCO on CS17

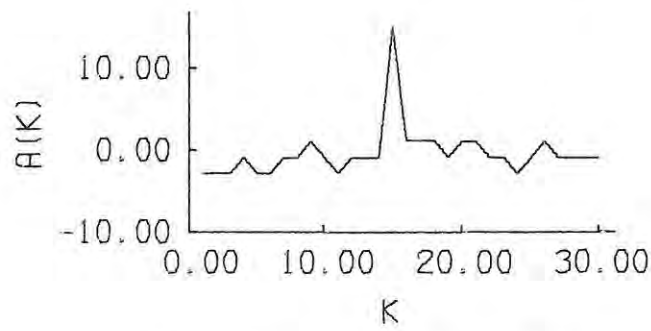


Figure 3.3 XBCO on M15

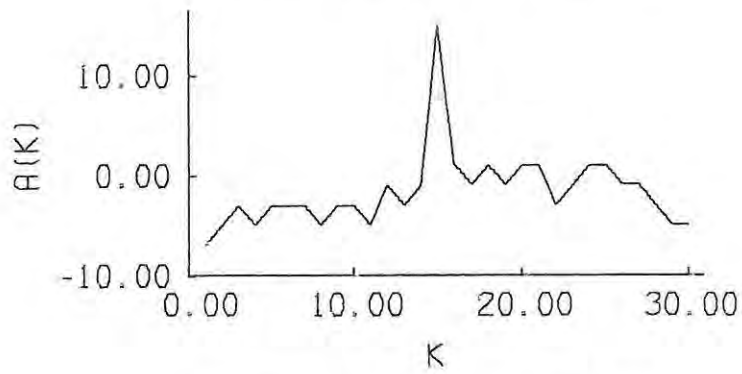


Figure 3.4 XBCO on T15A

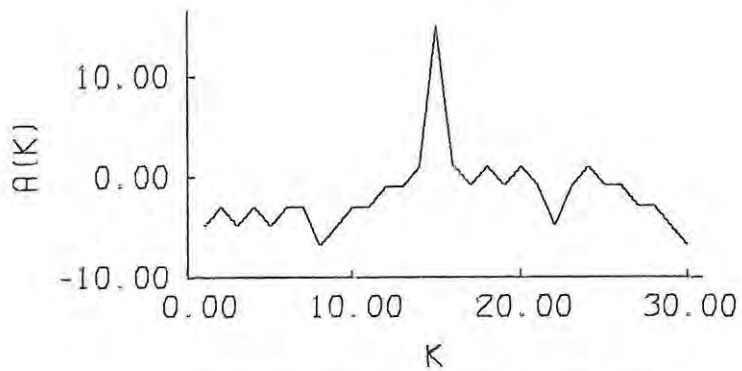


Figure 3.5 XBCO on T15B

Of the codes tested, the 15-bit M-sequence (with an M.S.R. of 15, an effective length of 12 bits and a minimum sidelobe of -3) emerged as the most suitable code and was therefore chosen to be used in the "Barker" system. It must be remembered, however, that (as mentioned in section 3.2.5) all the  $A(k)$  in the above correlations must be halved, and that the maximum  $A(k)$  is an absolute indication of the energy of the returned signal.

It has been shown, therefore, that an amplitude-modulated coded-pulse system, though more complicated and less efficient than its phase- and frequency-modulated counterparts, is nevertheless a viable proposition, and a suitable code has been found for such a system.

CHAPTER 4

THE "POOLE" SYSTEM

4.1 Introduction.

The basic aims of this system are the same as those of the Barker system, that is

- (i) to improve S.N.R. without loss of height resolution, and
- (ii) to expend less peak power in doing so.

An additional, implicit aim was to improve on the performance of the Barker system by increasing the amount of transmitted signal energy per scan.

The brief introduction given to the system in Chapter 1 will be enlarged upon here.

It was proposed that like the Barry Research Chirpsounder (mentioned by Phillips, 1974, p. 789), the ionosonde should transmit amplitude-modulated (O.O.K.) radio frequency pulses according to some binary code, and receive any reflected signals during the times when the transmitter was off. However, unlike the Chirpsounder, which obtains the delay-time information from the frequency of the returned signals, the necessary data would be contained in the amplitude of these signals, and would be derived from the digital correlation function of the code.

Since the ionosphere is normally sounded to a v.h. of 1000 km, the code should be transmitted in cycles each of length at least 6,7 ms, so as to be able to create an ionogram in minimum time if required. The incoming signal code could be fed into a memory or shift register with as many registers as bits in the code. The register could be clocked (shifted) at the bit rate of the signal. For each shift of the signal code the necessary term-by-term multiplication by the correlating code and summation of the products (i.e. correlation operation) could be carried out. The time between trans-

mission of the last bit in a cycle and the occurrence of the main lobe of the cyclic correlation function would yield the virtual height information. These ideas are illustrated in the graphs below (Figure 4.1). The simple case of only one of a number of cycles transmitted at a fixed frequency, and delayed by an integral number of bits, is considered. An arrow marks (for convenience) the beginning of the hypothetical code. Practical details of the correlation process will be given in Chapter 5.

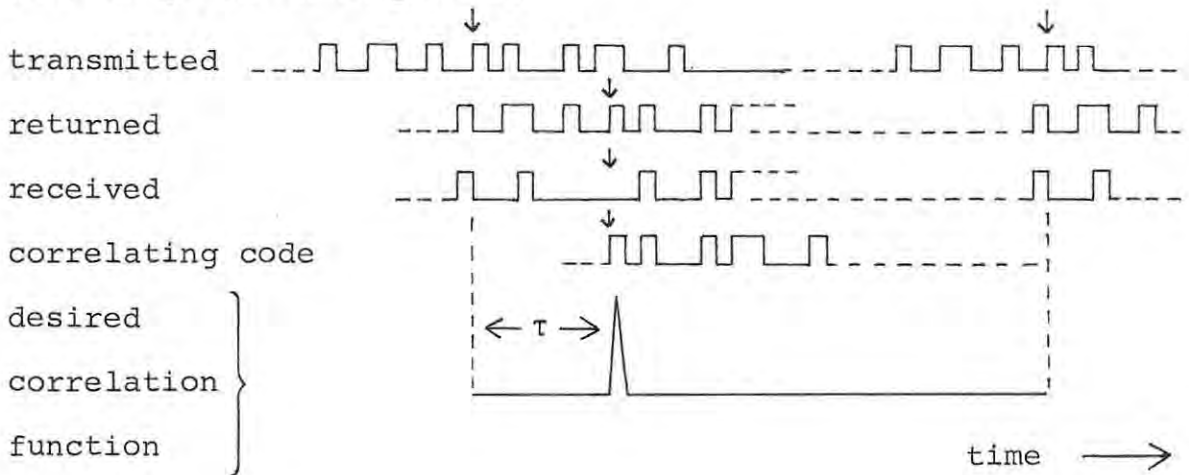


Figure 4.1

The modification of the returned code by the transmitted code is clearly illustrated in the figure. At the times when the transmitter is "on" (i.e. the transmitted code is 1) no reception is possible. The received code may be described by the simple formula

$$\text{received} = \text{returned} \times (1 - \text{transmitted})$$

if the codes are in ordinary binary form. This "masking" of the returned code by the transmitted one, is fundamental to the system.

It should be noted too that in Figure 4.1 the vertical axis of each of the first 4 graphs is calibrated in terms of (say) power, while that of the correlation function is, of course  $A(k)$ . Physically this digital  $A(k)$  may be easily converted into an analogue voltage signal. The offsets,  $k$ , are in this case offsets in time. The delay-time,  $\tau$ , may be

measured directly in real time or in terms of numbers of bits. This point, too, will be discussed below in Chapter 5.

The operation described graphically above is more simply carried out by considering the bits of the codes to be the binary elements 1 and 0 or 1 and -1. Since the correlation operation is more conveniently described by the bipolar elements, these will as usual be used in the computer calculations which follow. Thus reception will only take place when the transmitter code is -1.

#### 4.2 Miscellaneous aspects.

##### 4.2.1 Prerequisite properties of the code.

A number of prerequisite properties of the desired binary code are immediately intuitively obvious.

- (i) The code should consist of approximately as many 1's as -1's so as to allow both maximum transmission and reception time. It is to be expected that about half of the signal will be lost on reception.
- (ii) The received signal energy (area under a power vs. time graph) should be constant for all possible delays.
- (iii) The correlation function of the transmitted (or correlating) code and the received code should be optimum or perfect. That it should have a constant M.S.R. for all delays is implied by property (ii) above. The correlation must be a cyclic one.
- (iv) Since bits of length approximately  $50 \mu\text{s}$  give a suitable v.h. resolution, the code should contain about  $\frac{6,66 \text{ ms}}{50 \mu\text{s}} = 133$  bits. This is not a rigid requirement, however, and codes of various lengths were tested.

##### 4.2.2 Correlation procedures.

The cyclic correlation process in this case is in no way extraordinary. A Fortran IV program (called MASK) was

written to simulate the process, that is, the delay (by a predetermined, integral number of bits) of a code with respect to itself, the "masking" of the delayed code by the other, and the subsequent correlation to find the starting-point of the code and hence the delay-time.

Later MASK was extended to include a calculation of the M.S.R. for all possible non-zero delays, the output being in the form of a graph. These programs (MASK and PLOT) are given in Appendix B.

#### 4.2.3 Multiple echoes.

The ability of the ionosonde to resolve multiple echoes is even more important to this system than to the Barker system, since now the code covers the entire (say) 6,67 ms scan. Thus the incoming signals must, once again, be sampled at more than just two levels, that is, the sampling must be multilevel (see section 3.2.3).

#### 4.2.4 Signal averaging.

The averaging device of this system does not differ from that of the Barker system. Details will be discussed below under "Design specifications".

#### 4.2.5 D.c. removal.

The Poole system, like the Barker one, will also benefit from the d.c. removal described in section 3.2.5, and for the same reasons. Thus all the A(k) resulting from MASK and PLOT must again be halved, since d.c. removal halves the magnitude of the incoming ordinary binary signal if no noise is present.

### 4.3 Some of the codes.

#### 4.3.1 The "coin" code, C60.

A random binary code was obtained by a colleague by assigning a binary element to each of the faces of a coin, and tossing

it 60 times. The resulting 60-bit code (named C60) was the first to be tested under the Poole system and clearly illustrates the rigorous demands that the system makes on a code. The code is

```

-1  1 -1  1 -1 -1  1  1  1 -1 -1 -1  1 -1  1  1 -1 -1  1  1
 1 -1  1  1 -1  1  1 -1  1 -1 -1 -1 -1  1  1 -1 -1 -1 -1  1
-1  1  1  1 -1  1  1 -1  1 -1  1  1  1 -1 -1 -1  1 -1  1 -1
    
```

MASK produced (for an arbitrary delay) a correlation function with a small M.S.R., and PLOT confirmed that the M.S.R. was small and varied with delay (Figure 4.2).

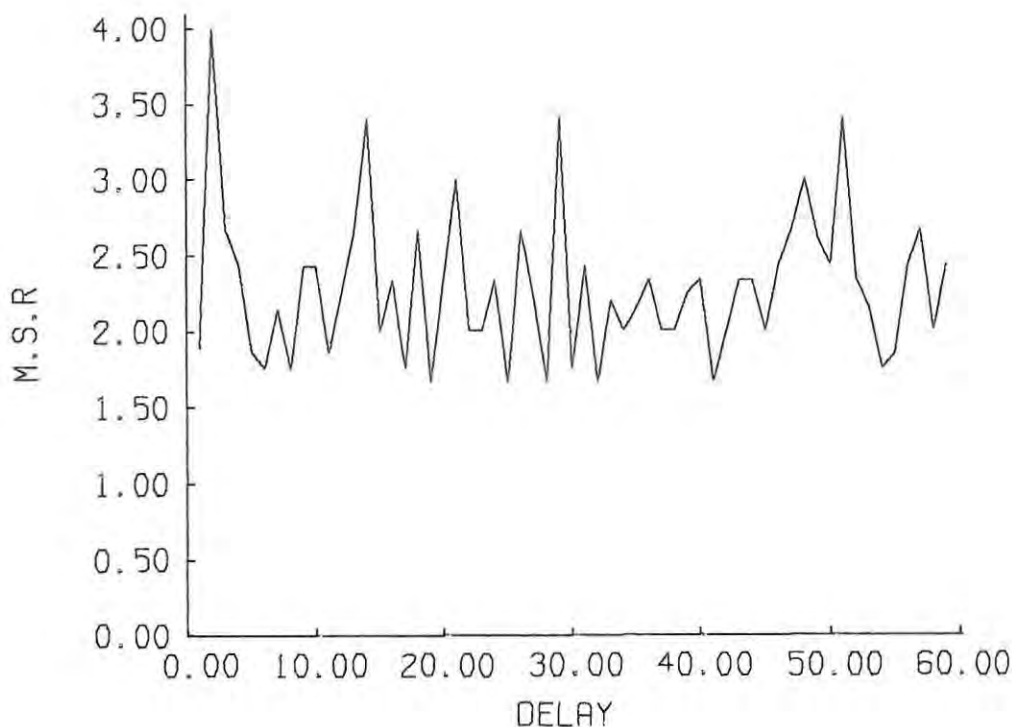


Figure 4.2 C60.

#### 4.3.2 The "progressive" code, P55.

The progressive code, P55, was so named because it consisted of progressive numbers of alternating 1's and -1's, resulting in a code of length 55 bits, 30 of them 1's, thus:

```

-1  1  1 -1 -1 -1  1  1  1  1 -1 -1 -1 -1 -1  1  1  1  1  1
 1 -1 -1 -1 -1 -1 -1 -1  1  1  1  1  1  1  1  1 -1 -1 -1 -1
-1 -1 -1 -1 -1  1  1  1  1  1  1  1  1  1  1
    
```

This code yielded M.S.R.'s which are close to unity for most delays, which means that it is totally unacceptable for this application. The results of PLOT are given in Figure 4.3.

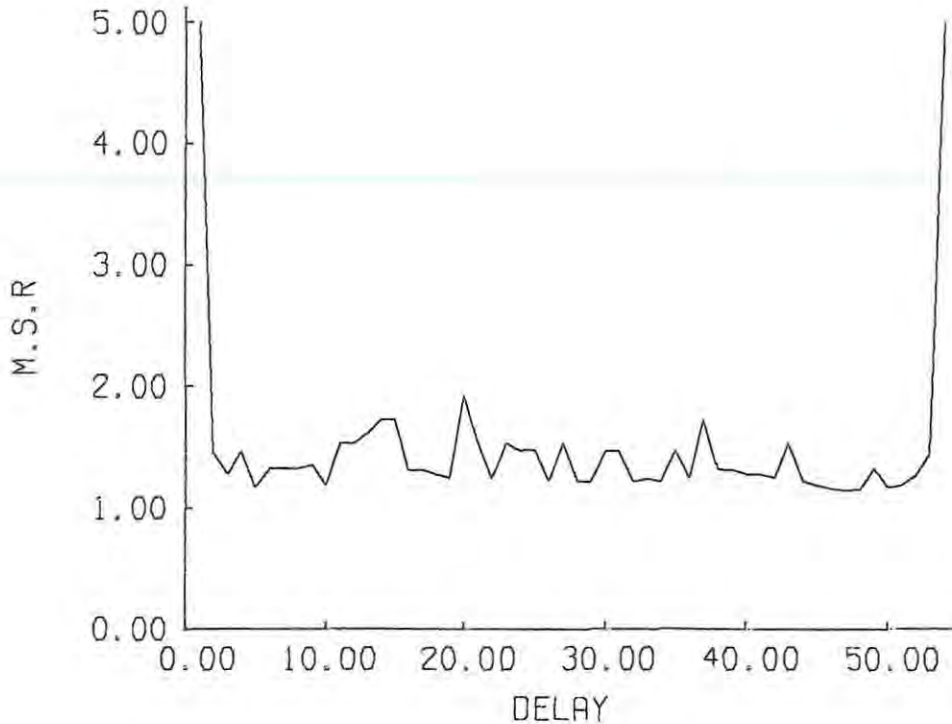


Figure 4.3 P55.

4.3.3 Combinations of codes. B 713.

It seemed reasonable to suppose that a cyclic code with the desired properties might be obtained by combining shorter, well-known codes (like the Barker codes) in various ways.

It is clear, however, that simply repeating a code a number of times is unsatisfactory, since (for example) a delay which is a multiple of the number of bits in the basic code, will cause the returned code to be completely masked by the transmitted code, resulting in no received code and thus an M.S.R. of zero.

The following idea seemed more promising: a 91-bit code was formed by combining seven 13-bit Barker codes according to the 7-bit code

1 1 -1 1 -1 -1 1

which is itself a Barker code. An ordinary 13-bit Barker code was written for each 1 and an "inverted" 13-bit Barker code

1 1 -1 -1 -1 1 1 -1 -1 1 -1 1 -1

was written for each -1. The resulting code was named B713.

Figure 4.4 shows the results of PLOT on B713. Even at extreme delays the M.S.R. is hardly satisfactory. No other combinations of well-known codes yielded better results.

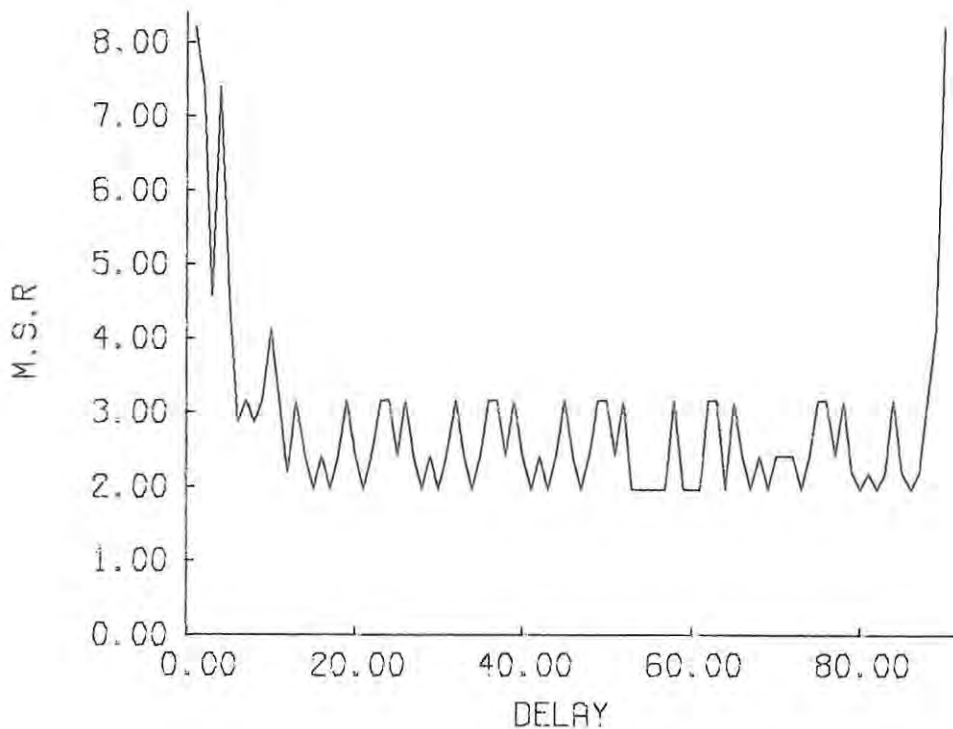


Figure 4.4 B713.

#### 4.3.4 The Wiener numbers.

The Wiener numbers are generated (Papoulis, 1962, p.258) by attaching first a 1 and then a -1 to the ends of "words" (groups of bits) generated in the same way from the initial words 1 and -1. The first 3 steps are:

1 1

1 1                      1-1                      -1 1                      -1-1

1 1 1   1 1-1   1-1 1   1-1-1   -1 1 1   -1 1-1   -1-1 1   -1-1-1

(Words brought down from the previous step are underlined.)

If at any step the words are strung together, the result is an  $n2^n$ -bit binary code (where  $n$  is the number of the step). The 64-bit code, named WN64, was chosen to be submitted to PLOT. The results (which speak for themselves) are given in Figure 4.5.

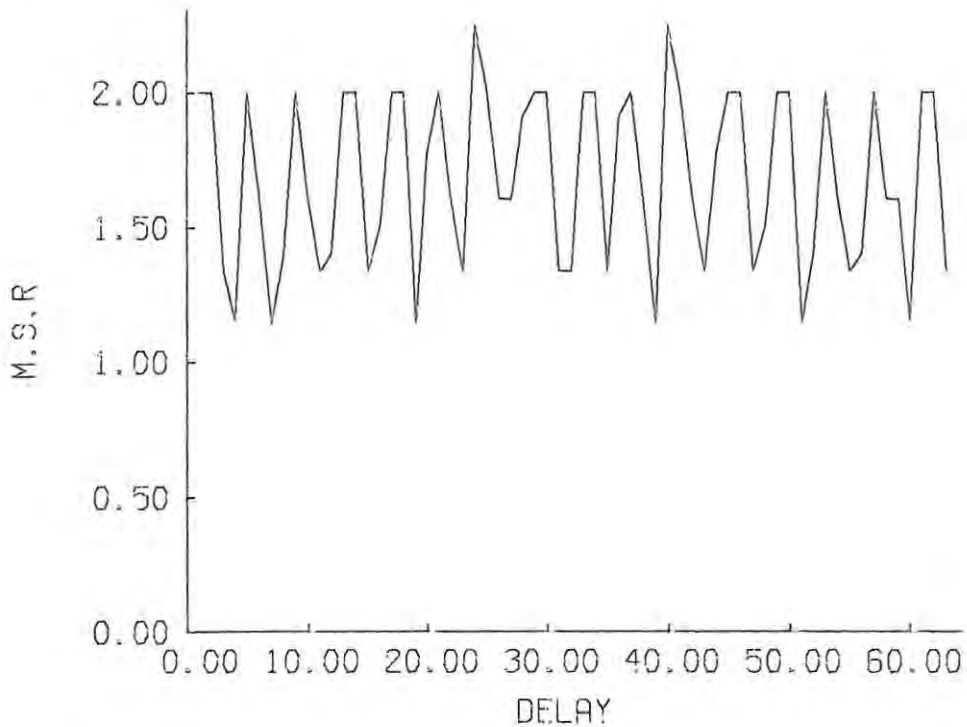


Figure 4.5 WN64.

#### 4.3.5 The M-sequences. M127.

The M-sequences (maximum-length sequences, linear feedback shift-register sequences or pseudo-random binary sequences) form a class of binary sequences or codes whose derivation and properties are the subject of at least one entire text (Golomb, 1967) and many shorter works (Cook and Bernfeld,

1967, pp.247-253; Green, 1975, Ch.2; Nathanson, 1969, sections 12.2 and 12.4; MacWilliams and Sloane, 1976 and Skolnik, 1970, pp.20-18 to 20-22). Only a brief, largely non-mathematical introduction will be given here to this rather extensive topic.

Each element,  $m_{i+1}$ , of the M-sequence is derived from the preceding  $n$  elements by the recurrence relation

$$m_{i+1} = \alpha_n m_i \oplus \alpha_{n-1} m_{i-1} \oplus \dots \oplus \alpha_1 m_{i-n+1}$$

where the symbol  $\oplus$  denotes addition modulo-2. Given the correct initial state (or starting sequence)  $m_i, i=1, \dots, n$  and "alpha-sequence"  $\alpha_i, i=1, \dots, n$ , this recurrence relation will generate a binary sequence which repeats itself after a total of  $2^n-1$  bits. Any other initial state or alpha-sequence will generate a sequence which repeats itself before  $2^n-1$  bits, hence the name maximum-length sequences.

The initial state is, in fact, arbitrary, the only impermissible combination of  $n$  binary elements being  $n$  zeros. The only alpha-sequences that will generate maximum-length sequences are those taken from the (binary) co-efficients of primitive, irreducible polynomials in the binary field (GF(2)). Tables containing such polynomials of order between 1 and 34 are given in Peterson (1961, Appendix C) and most of the references given above.

Consider, for instance, the case where  $n=3$ . The only polynomial which is both primitive (i.e. which divides into  $x^m+1$ , where  $m > 2^n-1$ ) and irreducible (unfactorable) is, according to Peterson (op.cit., pp.251 and 254),

$$x^3 + x + 1.$$

Although all irreducible polynomials have a non-zero co-efficient of  $x^0$ , this co-efficient is not included in the alpha-sequence, which in this case is 1 0 1. The initial state 1 1 0 will thus generate the 7-bit M-sequence

$$1\ 1\ 0\ 1\ 0\ 0\ 1$$

which is also, in fact, a different cyclic "phase" of the 7-bit Barker sequence (Cook and Bernfeld, 1967, p. 245;

Nathanson, 1969, p.467). It is clear that any non-zero binary word may be used as the initial state to generate this (cyclic) code since the code contains all 7 possible combinations of 3 binary elements excluding 3 zeros.

The recurrence relation given above makes the generation of M-sequences in shift-registers particularly easy, hence the name (linear) shift-register sequences. The  $n$  bits of the initial state (or starting sequence) are put into an  $n$ -bit shift-register, which is "tapped" at the output of each register. The taps are fed back through a series of switches and modulo-2 adders (or exclusive-OR gates) to the input of the shift-register. If the configuration of the switches is described by a valid  $n$ -bit alpha-sequence, then the shift-register, when clocked, will produce a  $(2^n - 1)$ -bit M-sequence.

Consider again the case when  $n=3$ , with initial condition 1 1 0 and alpha-sequence 1 0 1. The shift-register circuit may be represented as follows:

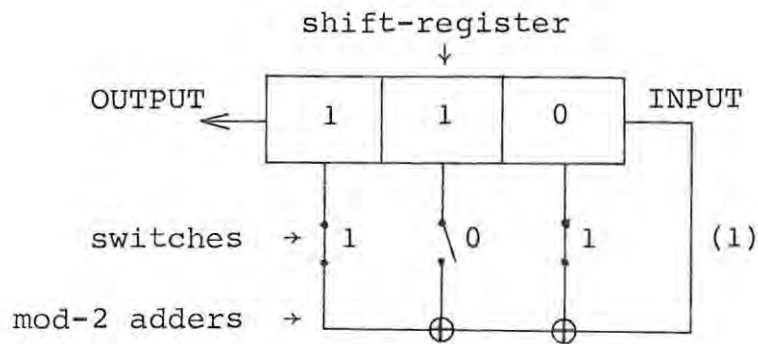


Figure 4.6

One author (Nathanson, 1969, p.464) interprets the constant term (coefficient of  $x^0$ ) as referring to the closing of the feedback loop to the input of the shift-register.

Since the expression on the right-hand side of the recurrence relation given above is linear in the  $m$ 's, these feedback shift-register sequences are called linear (Golomb, 1967, p.13). The non-linear feedback shift-register (or de Bruijn) sequences will be discussed in section 4.3.7.

The simplest alpha-sequences for n from 1 to 7 are (Nathanson, 1969, p.465).

n	$\alpha_1$	.....	$\alpha_7$				
1	1						
2	1	1					
3	1	0	1				
4	1	0	0	1			
5	1	0	0	1	0		
6	1	0	0	0	0	1	
7	1	0	0	0	0	0	1

The M-sequences have a number of interesting properties which will be discussed fully in section 4.4. The ones which prompted an investigation into their response to MASK and PLOT are the so-called "weighting" property and the autocorrelation property.

The weighting property states that the M-sequences consist of  $2^{n-1}$  ones and  $2^{n-1}-1$  zeros. This is a prerequisite property. The autocorrelation property states that their cyclic autocorrelation function is given by

$$A(k) = \begin{cases} 2^n - 1 & \text{for } k = 0 \pmod{2^n - 1} \\ -1 & \text{for } k \neq 0 \pmod{2^n - 1} \end{cases}$$

(Cook and Bernfeld, 1967, p.251). One cycle of this autocorrelation function may be represented graphically as follows:

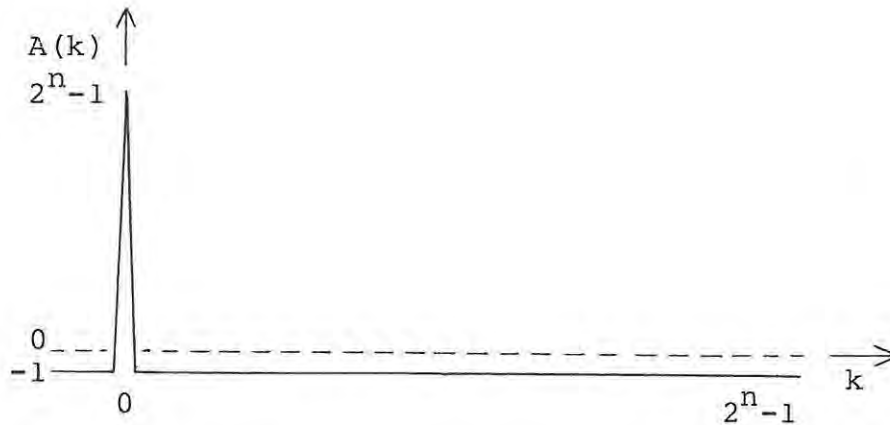


Figure 4.7

This property can be very easily verified for a 7-bit sequence (using bipolar notation and circles to denote the beginning of the code).

k	(1) 1 -1 1 -1 -1 1	A(k)
0	(1) 1 -1 1 -1 -1 1	7
1	1 (1) 1 -1 1 -1 -1	-1
2	-1 1 (1) 1 -1 1 -1	-1
3	-1 -1 1 (1) 1 -1 1	-1
4	1 -1 -1 1 (1) 1 -1	-1
5	-1 1 -1 -1 1 (1) 1	-1
6	1 -1 1 -1 -1 1 (1)	-1

CACO verified this property for all the M-sequences of length up to 127 bits. The non-cyclic autocorrelation function is not even optimum (Cook and Bernfeld, 1967, p.252; Nathanson, 1969, p.465; Skolnik, 1970, p.20-20).

The 127-bit sequence (called M127) generated from the alpha-sequence 1 0 0 0 0 0 1 is, in bipolar form

```

1 1 1 1 1 1 1 -1 -1 1 -1 1 -1 -1 1 1 -1 -1 1
1 1 -1 1 1 1 -1 1 -1 -1 1 -1 1 1 -1 -1 -1 1 -1
1 1 1 1 -1 1 1 -1 1 -1 1 1 -1 1 1 -1 -1 1 -1 -1
1 -1 -1 -1 1 1 1 -1 -1 -1 -1 1 -1 1 1 1 1 1 -1 -1
1 -1 1 -1 1 1 1 -1 -1 1 1 -1 1 -1 -1 -1 1 -1 1
1 1 1 -1 -1 -1 1 -1 1 -1 -1 -1 -1 1 1 -1 -1 -1 -1
1 -1 -1 -1 -1 -1 -1
    
```

Its response to MASK is most interesting. A programmed delay of 17 bits yielded the following correlation function:

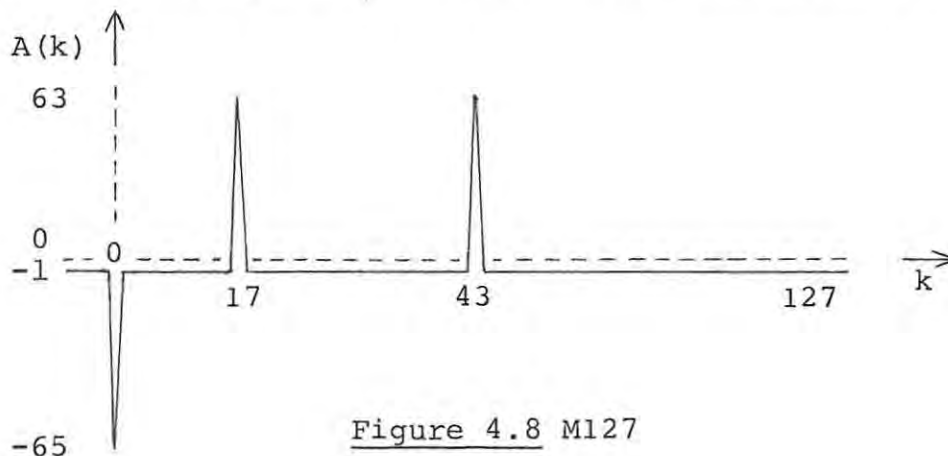


Figure 4.8 M127

Four points are worth noting:

- (i) there is a large negative "peak" at zero delay.
- (ii) There is a peak at a delay of 17 bits, and it is 63 bits high, as was hoped.
- (iii) There is, however, another peak (later named a "conjugate" peak) of the same height as the signal peak at a delay of 43 bits.
- (iv) Apart from these 3 peaks, the correlation function is perfectly flat.

Further simulations with different delays yielded similar results with the conjugate peak appearing seemingly randomly in the correlation function. Furthermore, MASK produced functions with the same basic shape for all other M-sequences.

Consider again the 7-bit sequence (in bipolar notation) and assume a delay of 3 bits. One cycle of the received code may be computed as follows:

$$\begin{array}{rcl}
 \text{transmitted} & \textcircled{1} & 1 \ -1 \ 1 \ -1 \ -1 \ 1 \\
 \text{returned} & -1 \ -1 \ 1 & \textcircled{1} \ 1 \ -1 \ 1 \\
 \hline
 \text{received} & -1 \ -1 \ 1 & \textcircled{-1} \ 1 \ -1 \ -1
 \end{array}$$

Once again the circles denote the beginnings of the respective codes. A correlation can now be carried out on the received code in order to locate its starting point.

k	-1 -1 1 $\textcircled{-1}$ 1 -1 -1	A(k)
0	$\textcircled{1}$ 1 -1 1 -1 -1 1	-5
1	1 $\textcircled{1}$ 1 -1 1 -1 -1	3
2	-1 1 $\textcircled{1}$ 1 -1 1 -1	-1
3	-1 -1 1 $\textcircled{1}$ 1 -1 1	3
4	1 -1 -1 1 $\textcircled{1}$ 1 -1	-1
5	-1 1 -1 -1 1 $\textcircled{1}$ 1	-1
6	1 -1 1 -1 -1 1 $\textcircled{1}$	-1

Once again  $A(0) = -(2^{n-1}+1)$  and  $A(k) = 2^{n-1}-1$  for the signal and conjugate peaks. In this case the conjugate peak appeared at  $k = 1$  for a delay of 3.

PLOT, which does not distinguish between a main-lobe and a sidelobe of the same height, implies that the correlation function is the same for all possible delays (Figure 4.9).

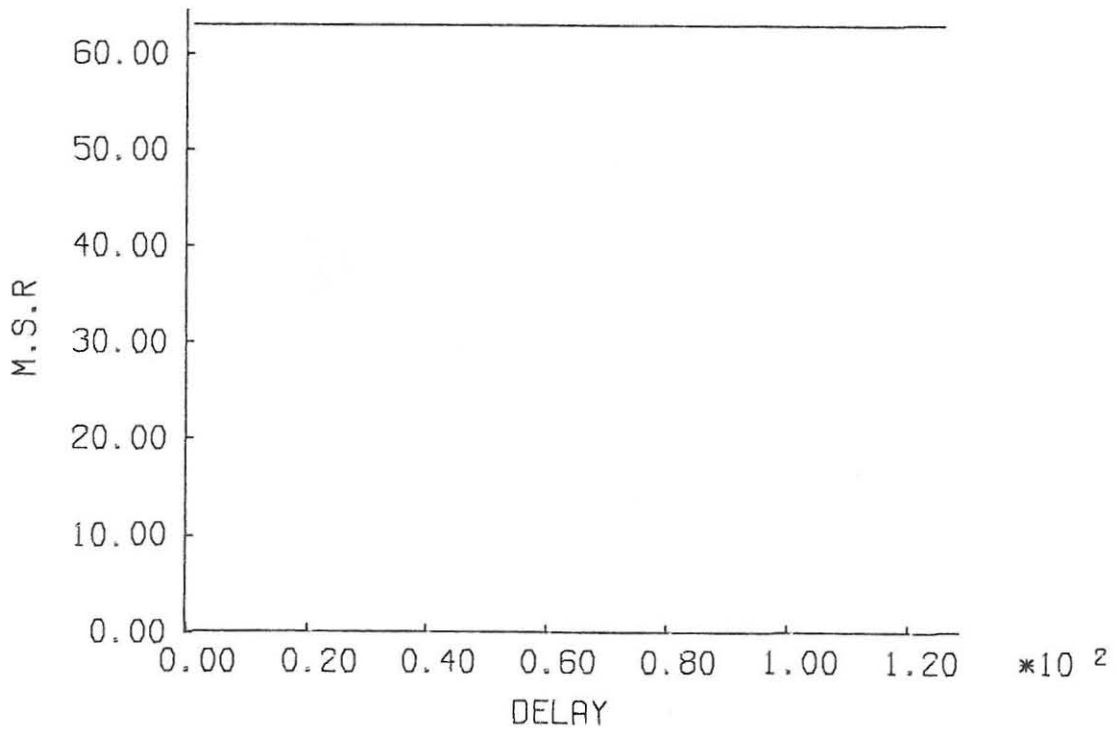


Figure 4.9 M127

The M-sequences, then, fulfil almost all the prerequisites of the desired code:

- (i) they have approximately as many 1's as -1's.
- (ii) The constant M.S.R. for all delays shows that the returned signal energy is constant for all delays.
- (iii) The correlation function (except for the negative and conjugate peaks) is perfect, and
- (iv) a code of length 127 bits exists, which is close to the suggested 133 bits (section 4.2.1).

The large negative peaks at zero delay pose no great threat to the system and can, in fact, be easily eliminated, but the conjugate peaks, although they would appear on an ionogram as random "noise" and would easily be discriminated against by the human eye, nevertheless detract from the obvious success of the M-sequences in this application.

The large negative peak at zero delay is a result of the masking of the received code by the transmitted code, and is present in the correlation functions of all the codes subjected to MASK. Since the received code can only be 1 when the transmitted code is 0, and is always 0 when the transmitted code is 1, the received code bears a resemblance to the "inverse" of the transmitted code (which of course has its beginning at zero delay) and therefore produces a large negative peak when correlated against the transmitted code at zero delay. This negative peak may be eliminated by converting the incoming ordinary binary signals to bipolar form (as was originally envisaged anyway) and then electronically gating this signal to zero when the transmitter is on, so that the following transformations of signal level result:

signal 1 → 1  
signal 0 → -1  
"transmitter" 0 → 0

The resulting ternary code, when correlated against the bipolar transmitted code will produce the same correlation function as before without the negative peak, since now the zeros due to the transmitter do not interfere with those of the signal. In other words, the additional influence of the transmitter on the received code has been effectively eliminated (remembering, of course, that half of the returned bits have been masked by the transmitter and therefore irretrievably lost).

A number of attempts were made to overcome the detrimental effect of the conjugate peaks in the correlation function of the masked M-sequences.

One made use of a "double correlation" technique whereby

(for example) 4 31-bit M-sequences would be transmitted instead of a single, long code and the returned codes correlated with another 4 31-bit codes, the first of which would be inverted (as before) according to the 4-bit Barker code -1 1 1 1 (which is optimum). The resulting correlation function would then be correlated with a 124-bit code consisting of the 4 bits -1 1 1 1, each of which would be followed by 30 zeros. This second correlation would then (hopefully) discriminate against any negative and conjugate peaks arising from the first correlation, and give the delay-time information unambiguously. It became clear, however, that since the delay is approximately constant at a fixed carrier frequency, the first correlation would produce negative, signal and conjugate peaks in the same relative positions for all four "sub-cycles" of the code. The second correlation would then simply reproduce the same ambiguous correlation function as before. No neat solution was found to this problem.

This attempt to solve the conjugate peak problem has not been described in any great detail because it transpired that the problem was soon circumvented in a very simple way.

#### 4.3.6 The "W"-sequences. W127.

A W-sequence is derived by inverting an M-sequence, that is, by multiplying each of its (bipolar) elements by -1. Although a number of authors (Green, 1975, p.33 and MacWilliams and Sloane, 1976, p.1718 for example) have, in order to establish the autocorrelation property, transformed the 1's and 0's of M-sequences into -1's and 1's (respectively), thereby unconsciously creating W-sequences, no direct reference to these new sequences has been found in the literature.

The W-sequence derived from the 127-bit M-sequence above is

```

1 1 1 1 1 1 -1 -1 -1 -1 -1 -1 -1 1 -1 1 -1 1 -1 1
1 -1 -1 1 1 -1 -1 -1 1 -1 -1 -1 1 -1 1 1 -1 1 -1 -1
1 1 1 -1 -1 1 -1 -1 -1 -1 1 -1 -1 1 -1 1 -1 -1 1 -1
-1 1 1 -1 1 1 -1 1 1 1 -1 -1 -1 1 1 1 1 -1 1 -1
-1 -1 -1 -1 1 1 -1 1 -1 1 -1 -1 -1 1 1 -1 -1 1 -1 1
1 1 -1 1 1 -1 -1 -1 -1 1 1 1 -1 1 -1 1 1 1 -1 1
-1 1 1 1 1 1 -1

```

Its response to MASK bears a remarkable resemblance to that of the corresponding M-sequence (for the same delay, 17 bits).

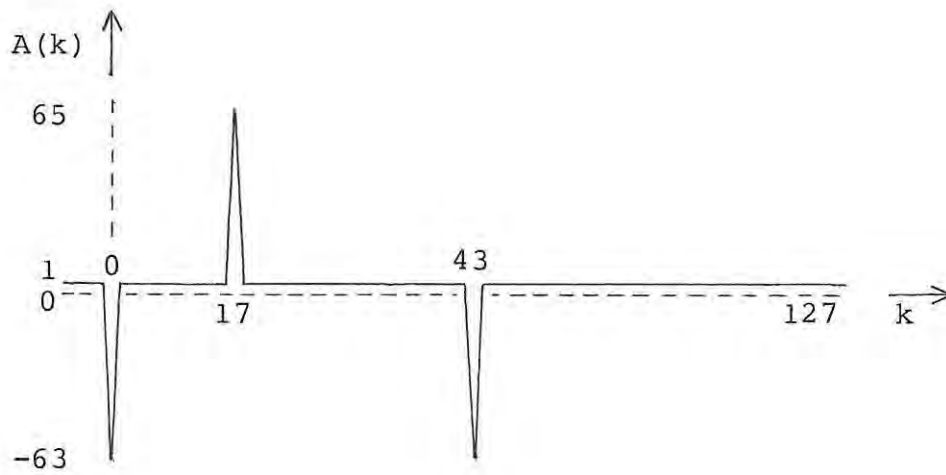


Figure 4.10 W127.

All the peaks appear at the same  $k$  as before, but for the W-sequences the conjugate peak is negative and the values of the  $A(k)$  are slightly changed. Further simulations, other W-sequences and different delays yielded similar results. PLOT confirmed these findings (Figure 4.11).

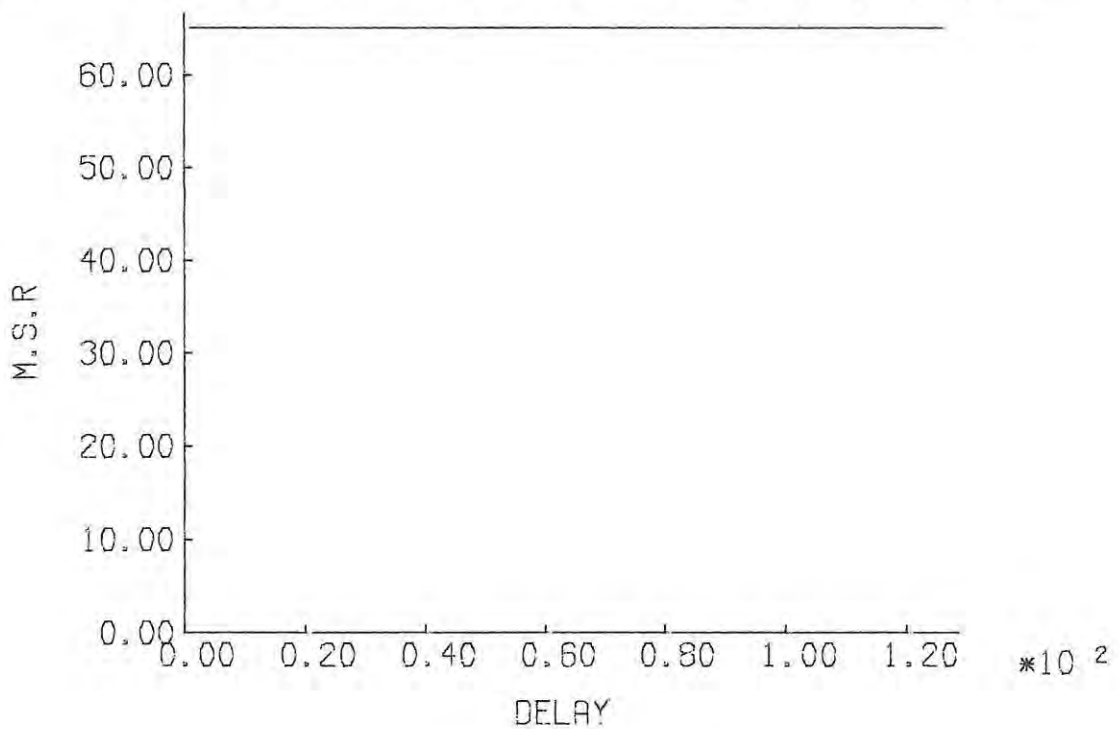


Figure 4.11 W127.

Although the inversion of the conjugate peak enables the position (in time) of the signal to be uniquely (unambiguously) determined, the existence of this apparently random negative peak in the correlation function does nevertheless pose a threat to the system. If multiple echoes occur such that one signal peak occurs at the same delay as the conjugate peak due to another, the signal peak will be obliterated. Furthermore, an unfortunate symmetry was found to exist between the signal and conjugate peak; if a delay of  $d_1$  bits produced a conjugate peak at  $d_2$  bits, then a delay of  $d_2$  bits caused a conjugate peak at  $d_1$  bits. This means that if a signal due to one echo did occur at the conjugate peak due to another, both signals would be obliterated. Although this is intrinsically a serious problem, the probability of such a situation arising is, of course, small.

The W-sequences, then, are highly suitable for implementation in the Poole system.

A further investigation of the properties of the M-and W-sequences is undertaken in section 4.4.

#### 4.3.7 The de Bruijn sequences.

The  $2^n$ -bit de Bruijn (or non-linear) feedback shift-register) sequences can be generated in a shift-register in much the same way as the M-sequences, except that the function describing the feedback configuration includes the "NAND" (or not-AND) operation. The AND and NAND operations are defined by the following truth-table.

A	B	A.B	$\overline{A.B}$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0


where the symbol  $\overline{\quad}$  stands for negation. If necessary, ordinary multiplication will henceforth be denoted by  $\times$ , so

as not to confuse it with AND.

The recurrence relation from which the de Bruijn sequences are derived is

$$m_{i+1} = \alpha_n m_i \oplus \alpha_{n-1} m_{i-1} \oplus \dots \oplus \alpha_1 m_{i-n+1} \oplus \overline{m_i \cdot m_{i-1} \cdot \dots \cdot m_{i-n+2}},$$

the alpha-sequence being that of the corresponding M-sequence. This relation is non-linear in the m's, hence the name non-linear feedback shift-register sequences.

The new recurrence relation requires the inclusion of an (n-1)-input NAND gate (represented by the symbol ) in the shift-register generator circuit. Consider again an n=3 generator, which may be represented as follows:

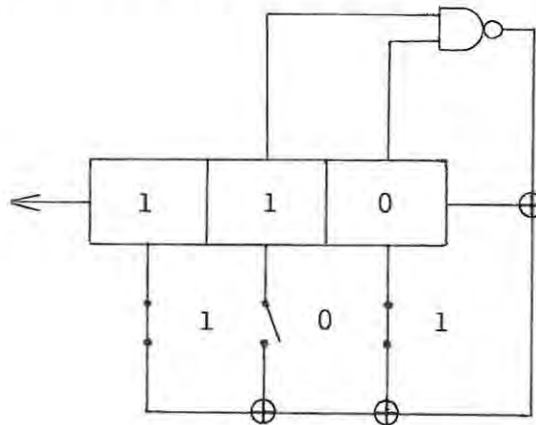


Figure 4.12

This configuration will generate the 8-bit de Bruijn sequence

1 1 0 0 0 1 0 1.

Note the inclusion of the run of n zeros in this sequence which causes it to be one bit longer than its linear counterpart. It is the NAND operation in the recurrence relation which allows the sequence to first "get into" and then "get out of" this run of n zeros. The cyclic de Bruijn sequences contain all  $2^n$  possible combinations of n of the binary elements 1 and 0.

Another of the properties of the de Bruijn sequences which is comparable to that of the M-sequences is the weighting property, which states that the weighting of 1's and 0's (or -1's) in the de Bruijn sequences is exactly equal. It is here that the similarity between the two types of shift-register sequence ends.

Their cyclic autocorrelation functions, for instance, are very different, especially for the longer sequences. CACO gives, for DB128, the 128-bit sequence

```

1 -1 -1 1 1 -1 1 1 -1 1 1 1 -1 -1 -1 1 1 1 1 -1
1 -1 -1 -1 -1 -1 1 1 -1 1 -1 1 -1 -1 -1 1 1 -1 -1 1
-1 1 1 1 -1 1 1 -1 -1 -1 -1 1 1 1 -1 1 -1 1 1 1
1 -1 -1 1 1 1 1 1 -1 1 1 1 1 1 1 -1 -1 -1 -1 -1
-1 -1 -1 1 -1 1 -1 1 -1 1 1 -1 -1 1 1 -1 -1 -1 1 -1
-1 -1 1 -1 1 1 -1 1 -1 -1 1 1 1 -1 -1 1 -1 -1 -1 -1
1 -1 -1 1 -1 1 -1 -1
    
```

generated by the alpha-sequence 1 0 0 0 0 0 1, an autocorrelation function with main lobe of height 128, maximum sidelobe 12 and minimum sidelobe -16.

With such a cyclic autocorrelation function, it seems unlikely that the de Bruijn sequences will respond satisfactorily to the masking effect of the system, and MASK confirms this supposition. PLOT summarizes this short-coming of the non-linear feedback shift-register sequences. (Figure 4.13).

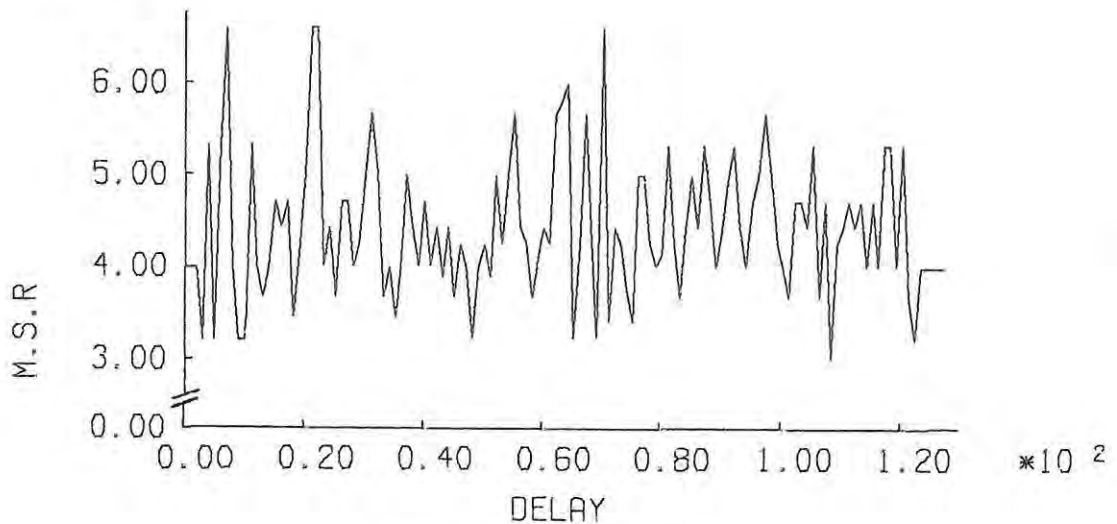


Figure 4.13 DB128

#### 4.4 Properties of the M- and W-sequences.

##### 4.4.1 Introduction.

Although the computer simulations above imply that the W-sequences are eminently suitable for the system, (and though it seems unlikely that a better code can be found), it nevertheless seemed worthwhile to undertake a more detailed study of the M- and W-sequences with a view to proving that the correlation functions given above occur for all possible delays and for all sequences.

##### 4.4.2 Properties of the M-sequences.

The M-sequences have a number of interesting and relevant properties which are well-documented in the literature (MacWilliams and Sloane, 1976, p.1718; Nathanson, 1969, p.458). In the interests of completeness those that have already been discussed will be mentioned again briefly (for the ordinary binary sequences).

- (i) The weighting (or balance) property states that the number of 1's in an M-sequence is given by  $2^{n-1} = \frac{N+1}{2}$  and the number of 0's by  $2^{n-1}-1 = \frac{N-1}{2}$ , where  $N = 2^n - 1$ .
- (ii) The "run" property states that the number of runs of  $m$  consecutive non-zero elements in an M-sequence is  $2^{n-m}$ , where  $1 \leq m \leq n$ . It should be noted here that (for example) the three bits 1 1 1 represent one run of 3 bits, two runs of 2 bits and three runs of 1 bit.
- (iii) The "window" property states that if a window of width  $n$  bits is slid along a (cyclic)  $N$ -bit M-sequence, each of the  $2^n - 1$  combinations of  $n$  elements (excluding  $n$  zeros) will be seen exactly once.
- (iv) The "shift-and-add" property states that if an M-sequence and a cyclic shift of itself are added modulo-2, the resulting sequence is yet another cyclic shift of the original sequence. Since this is an

important property (as will be seen below) a computer program (called CONJ and listed in Appendix B) was written for the purpose of further investigating this property. The 127-bit M-sequence given above has been written so that it begins with its only run of 7 ones. Consecutive cyclic shifts were thus added modulo-2 to this "zero-phase" sequence and the program then searched through the resulting sequence for the run of 7 ones, printing out the relevant data in columns (Table 4.1).

1	7	36	106	71	79	106	36
2	14	37	46	72	85	107	69
3	63	38	58	73	78	108	10
4	28	39	100	74	92	109	35
5	54	40	51	75	41	110	26
6	126	41	75	76	116	111	96
7	1	42	114	77	33	112	16
8	56	43	17	78	73	113	115
9	90	44	94	79	71	114	42
10	108	45	68	80	102	115	113
11	87	46	37	81	118	116	76
12	125	47	22	82	23	117	98
13	55	48	119	83	50	118	81
14	2	49	122	84	101	119	48
15	31	50	83	85	72	120	121
16	112	51	40	86	34	121	120
17	43	52	93	87	11	122	49
18	53	53	18	88	61	123	24
19	29	54	5	89	20	124	60
20	89	55	13	90	9	125	12
21	57	56	8	91	70	126	6
22	47	57	21	92	74		
23	82	58	38	93	52		
24	123	59	104	94	44		
25	105	60	124	95	65		
26	110	61	88	96	111		
27	66	62	30	97	32		
28	4	63	3	98	117		
29	19	64	67	99	103		
30	62	65	95	100	39		
31	15	66	27	101	84		
32	97	67	64	102	80		
33	77	68	45	103	99		
34	86	69	107	104	59		
35	109	70	91	105	25		

Table 4.1.

In other words, when the original ("zero phase") M-sequence is added to a version of itself shifted cyclically by the number of bits given in the first column of each pair, the result is the same sequence shifted by the corresponding number of bits given in the second column. For example, when the sequence is shifted 4 bits and added to itself, the result is the same code shifted 28 bits.

It should be noted, too, that if the code is shifted by 28 bits and added to itself, the result is a code shifted by 4 bits. This interesting symmetry between pairs of these numbers is due to the fact that, for any binary elements a, b and c, if

$$a \oplus b = c$$

$$\text{then } a \oplus c = b.$$

- (v) The autocorrelation property says that one cycle of the cyclic autocorrelation function of an N-bit M-sequence is given by

$$A(k) = \begin{cases} 2^n - 1 & \text{for } k = 0 \\ -1 & \text{for } 0 < k \leq N-1. \end{cases}$$

It can be shown (Green, 1975, p.33; MacWilliams and Sloane, 1976, p.1718) that property (v) follows from properties (i) and (iv). The proof given below, however, differs slightly from those in the literature, and requires a number of definitions which will be utilized throughout the rest of this section.

- (i) An ordinary N-bit M-sequence ( $N = 2^n - 1$ ) is given the symbol  $M_0$ , thus

$$M_0 \equiv m_1 m_2 \dots m_N.$$

This sequence shifted cyclically by d bits beomes

$$M_d \equiv m_{N-d+1} \dots m_N m_1 m_2 \dots m_{N-d}.$$

The shift-and-add property can then be written:

$$\text{for some } d \text{ and } e: M_0 \oplus M_d = M_e \quad (1 \leq d, e \leq N-1, d \neq e).$$

The sequence consisting of all 1's is given the symbol I, thus

$\Gamma \equiv 1\ 1\ 1\ \dots\dots\dots 1$  (N elements).

The sequence consisting of zeros only is denoted by  $\phi$ , thus

$\phi \equiv 0\ 0\ 0\ \dots\dots\dots 0$  (N elements).

(ii) The symbol  $\sum M_a$  (where  $M_a$  is an arbitrary M-sequence) means the decimal algebraic sum of the non-zero elements of the sequence, thus

$$\sum M_0 = 2^{n-1} = \frac{N+1}{2} \text{ (weighting property)}$$

(iii) The "exclusive-NOR" (or "inverted" addition modulo-2) operation is given the symbol  $\bar{\oplus}$  and defined by the following truth-table:

A	B	A $\bar{\oplus}$ B = $\overline{A \oplus B}$ = $\bar{A} \oplus B$ = $A \oplus \bar{B}$
0	0	1
0	1	0
1	0	0
1	1	1

(iv) Finally, comparison of the above truth-table with one defining the multiplication of bipolar binary elements, i.e.

A	B	A $\times$ B
-1	-1	1
-1	1	-1
1	-1	-1
1	1	1

shows that the two operations are equivalent if each of the ordinary binary elements in the former truth-table is multiplied by 2 and has 1 subtracted. It is clear, then, that if the exclusive-NOR operation is carried out N times and the results summed, the sum will be the same as the corresponding one for the bipolar elements if it is multiplied by 2 and has N subtracted. The definition of the cyclic auto-correlation function of an M-sequence can then be expressed in terms of both the ordinary and bipolar notations as follows

$$\begin{aligned}
 A(k) &= \sum (M_0 \times M_k) && \text{(bipolar)} \\
 &= 2\sum (M_0 \bar{\oplus} M_k) - N && \text{(ordinary)} \\
 &&& \text{for N elements, } 0 \leq k \leq N-1.
 \end{aligned}$$

Consider, for example, a 3-bit M-sequence:

multiplication				exclusive - NOR						
k	-1	1	1	A(k)	k	0	1	1	A(k)	2A(k)-3
0	-1	1	1	3	0	0	1	1	3	3
1	1	-1	1	-1	1	1	0	1	1	-1
2	1	1	-1	-1	2	1	1	0	1	-1

(A similar equivalence has already been noted in section 2.2.) It is convenient, henceforth, to consider ordinary binary M- and W-sequences only.

With the aid of these definitions it can easily be shown that property (v) follows from properties (i) and (iv).

Property (v). (M-sequences)

$$A(k) = \begin{cases} 2^n - 1 = N & \text{for } k = 0 \\ -1 & \text{for } 0 < k \leq N-1. \end{cases}$$

Proof.

(i) For  $k = 0$

$$\begin{aligned} A(0) &= 2 \sum (M_0 \oplus \bar{M}_0) - N \\ &= 2(\sum I) - N && \text{by definition of } \oplus \\ &= 2N - N \\ &= N \end{aligned}$$

(ii) For  $0 < k, 1 \leq N-1$  and  $M_0 \oplus M_k = M_1$

$$\begin{aligned} A(k) &= 2 \sum (M_0 \oplus \bar{M}_k) - N \\ &= 2(\sum \bar{M}_1) - N && \text{by property (iv)} \\ &= 2\left(\frac{N-1}{2}\right) - N && \text{by property (i)} \\ &= -1 \end{aligned}$$

#### 4.4.3 Corresponding properties of the W-sequences.

The properties of the W-sequences corresponding to properties (i), (ii) and (iii) for the M-sequences can easily be derived considering that a W-sequence is simply an M-sequence with each element "inverted". Property (iv), the shift-and-add property, however, provides an interesting deviation from the pattern.

Unlike the corresponding property for the M-sequences,

$$M_0 \oplus M_d = M_e,$$

the shift-and-add property for the W-sequences states

$$W_0 \oplus W_d = M_e.$$

This property can be illustrated using the 7-bit M- and W-sequences.

	M		W
	1 1 0 1 0 0 1		0 0 1 0 1 1 0
⊕	1 1 1 0 1 0 0	⊕	0 0 0 1 0 1 1
	0 0 1 1 1 0 1		0 0 1 1 1 0 1

i.e.  $M_0 \oplus M_1 = M_3$

$W_0 \oplus W_1 = M_3$

This important property follows from the simple fact that for any two binary elements a and b:

$$\bar{a} \oplus \bar{b} = a \oplus b.$$

The obvious result of this property is that property (v) - the autocorrelation property - of the W-sequences is exactly the same as that of the M-sequences. This can be proved by a process similar to that given above for the M-sequences.

#### 4.4.4 Further properties of the M- and W-sequences.

With the aid of those listed above, the author derived a new property of the M- and W-sequences, and from it ten theorems that prove that the correlation functions resulting from the masking of one sequence by another - such as happens in the Poole system - are indeed what the computer simulations showed them to be.

The new property follows directly from the shift-and-add property above. It has been appropriately called the shift-and-AND property and numbered (vi).

#### Property (vi), (shift-and-AND).

If  $M_0 \neq \phi$  and if  $M_0 \oplus M_d = M_e$ , then

- (a)  $M_0 \cdot M_d \neq M_0 \cdot M_e$
- and (b)  $\bar{M}_0 \cdot M_d = \bar{M}_0 \cdot M_e$ .

Proof.

$$\begin{aligned}
 \text{(a) } M_0 \cdot M_e &= M_0 \cdot (M_0 \oplus M_d) \\
 &= M_0 \cdot M_0 \oplus M_0 \cdot M_d \\
 &= M_0 \oplus M_0 \cdot M_d \\
 &= M_0 \cdot (I \oplus M_d) \\
 &= M_0 \cdot \bar{M}_d
 \end{aligned}$$

$$\begin{aligned}
 \text{(b) } \bar{M}_0 \cdot M_e &= \bar{M}_0 \cdot (M_0 \oplus M_d) \\
 &= \bar{M}_0 \cdot M_0 \oplus \bar{M}_0 \cdot M_d \\
 &= \phi \oplus \bar{M}_0 \cdot M_d \\
 &= \bar{M}_0 \cdot M_d
 \end{aligned}$$

(It should be noted here that any commutative, associative and distributive laws involving the AND, exclusive-OR and exclusive-NOR operations that are assumed in this section, are easily proved, for instance with the aid of truth-tables).

Similar reasoning shows that for the W-sequences, if  $W_0 \oplus W_d = M_e$ , then  $M_0 \oplus W_d = W_e$ , from which

$$\text{(a) } W_0 \cdot W_d = W_0 \cdot W_e$$

$$\text{and (b) } \bar{W}_0 \cdot W_d \neq \bar{W}_0 \cdot W_e.$$

Since the masking of a delayed M-sequence  $M_d$  by  $M_0$  is described by  $\bar{M}_0 \cdot M_d$ , it is easy to see that it is this important property that leads (as will be shown below) to ambiguity in the correlation function of such a masked sequence, while no such ambiguity appears in the corresponding function of the masked W-sequence  $\bar{W}_0 \cdot W_d$ .

$M_0 \cdot M_d$  and  $\bar{M}_0 \cdot M_d$  can, of course, be physically interpreted as the information lost, and the information returned (respectively) when  $M_d$  is masked by  $M_0$ . It is easy to see that

$$M_0 \cdot M_d \oplus \bar{M}_0 \cdot M_d = M_d.$$

The following notation will be used to simplify the proofs of some of the subsequent theorems:  $m_a$  will be used to denote an arbitrary element of the delayed M-sequence  $M_a$ , while the number of cases where corresponding elements of two arbitrary shifts  $M_a$  and  $M_b$  of an M-sequence  $M_0$  are (say) 1 and 0 respec-

tively, will be denoted by  $(m_a, m_b = 1, 0)$ . For example

$$\begin{aligned} \text{for } M_0 &= 1 \ 1 \ 0 \ 1 \ 0 \ 0 \ 1 \\ \text{and } M_2 &= 0 \ 1 \ 1 \ 1 \ 0 \ 1 \ 0, \\ (m_0, m_2 = 0, 1) &= 2 \\ \text{and } (m_0, m_2 = 0, 0) &= 1 \end{aligned}$$

Finally  $A(k)_d$  will be used to denote  $A(k)$  for the correlation of the delayed, masked sequence  $\bar{M}_0 \cdot M_d$  with  $M_0$ .

The following theorem is useful for demonstrating both M- and W-sequence properties:

Theorem 4.1.

$$\begin{aligned} \sum \bar{M}_a \cdot \bar{M}_b &= \frac{N-3}{4} \text{ and} \\ \sum M_a \cdot M_b &= \sum \bar{M}_a \cdot \bar{M}_b = \sum \bar{M}_a \cdot M_b = \frac{N+1}{4} \text{ for all } a \text{ and } b, a \neq b, N \geq 3. \end{aligned}$$

Proof.

$$\sum M_a = \sum M_b = \frac{N+1}{2} \quad (\text{weighting property})$$

$$\text{Let } x = (m_a, m_b = 1, 1).$$

$$\text{Then } \frac{N+1}{2} - x = (m_a, m_b = 1, 0)$$

$$\text{and } \frac{N+1}{2} - x = (m_a, m_b = 0, 1).$$

$$\text{Let } y = (m_a, m_b = 0, 0).$$

Then, since there are  $N$  elements in each code,

$$\begin{aligned} x + (\frac{N+1}{2} - x) + (\frac{N+1}{2} - x) + y &= N \\ \therefore y &= N - (N+1) + x \\ &= x - 1. \end{aligned}$$

$x$  may be evaluated as follows:

$$\sum (M_a \oplus M_b) = \sum M_c = \frac{N+1}{2} \text{ (shift-and-add property, } a \neq b).$$

Thus since  $1 \oplus 0 = 0 \oplus 1 = 1$ , while  $0 \oplus 0 = 1 \oplus 1 = 0$ ,

$$(m_a, m_b = 1, 0) + (m_a, m_b = 0, 1) = \frac{N+1}{2}$$

$$\therefore 2(\frac{N+1}{2} - x) = \frac{N+1}{2}$$

$$\therefore x = \frac{N+1}{4} (= 2^{n-2})$$

$$\text{from which } y = \frac{N-3}{4} (= 2^{n-2}-1)$$

$$\text{Thus } (m_a, m_b = 1, 1) = (m_a, m_b = 1, 0) = (m_a, m_b = 0, 1) = \frac{N+1}{4}$$

and  $(m_a, m_b = 0, 0) = \frac{N-3}{4}$ , and the result follows from the definition of the AND operation.

Corollary.

This result and a method similar to the one used to derive it, yield the following results:

$$\begin{aligned} \sum \bar{M}_a \cdot \bar{M}_b \cdot \bar{M}_c &= \frac{N-7}{8} \text{ and} \\ \sum M_a \cdot M_b \cdot M_c &= \sum M_a \cdot M_b \cdot \bar{M}_c = \sum M_a \cdot \bar{M}_b \cdot M_c = \sum M_a \cdot \bar{M}_b \cdot \bar{M}_c = \sum \bar{M}_a \cdot M_b \cdot M_c = \\ \sum \bar{M}_a \cdot M_b \cdot \bar{M}_c &= \sum \bar{M}_a \cdot \bar{M}_b \cdot M_c = \frac{N+1}{8} \text{ for all } a, b \text{ and } c, a \neq b \neq c, \\ M_a \oplus M_b &\neq M_c, N \geq 7. \end{aligned}$$

Summary of theorem 4.1.

	m <sub>a</sub>	m <sub>b</sub>			m <sub>a</sub>	m <sub>b</sub>	m <sub>c</sub>			
(i)	0	0	}	$\frac{N-3}{4}$	(i)	0	0	}	$\frac{N-7}{8}$	
(ii)	0	1			(ii)	0	0			1
(iii)	1	0			(iii)	0	1			0
(iv)	1	1			(iv)	0	1			1
					(v)	1	0	}	$\frac{N+1}{8}$	
			(vi)	1	0	1				
			(vii)	1	1	0				
			(viii)	1	1	1				

The following 5 theorems apply to the correlation function of the masked M-sequence:

Theorem 4.2 (M-sequence)

$$A(d)_d = \frac{N-1}{2} (= 2^{n-1}-1) \text{ for all } d, 0 < d \leq N-1.$$

Proof.

$$\begin{aligned} A(d)_d &= 2\{\sum (\bar{M}_0 \cdot M_d \oplus M_d)\} - N && \text{from section 4.4.2} \\ &= 2\{\sum (\bar{M}_0 \cdot M_d \oplus \bar{M}_d)\} - N \\ &= 2\{\sum \bar{M}_0 \cdot M_d + \sum \bar{M}_d\} - N && \text{since } (\bar{M}_0 \cdot M_d) \cdot \bar{M}_d = \phi, \\ \text{that is } (\bar{m}_0 \cdot m_d, \bar{m}_d = 1, 1) &= 0. && \text{"+" here means decimal addition.} \\ &= 2\{\frac{N+1}{4} + \frac{N-1}{2}\} - N && \text{from theorem 4.1 and prop. (i).} \\ &= \frac{N+1}{2} + N - 1 - N \\ &= \frac{N-1}{2}. \end{aligned}$$

Theorem 4.3 (M-sequence)

If  $M_0 \oplus M_d = M_e$ ,  $A(k)_d = A(k)_e$  for all  $k, 0 < k \leq N-1$ .

Proof.

$$\begin{aligned} A(k)_d &= 2 \sum (\bar{M}_0 \cdot M_d \oplus \bar{M}_k) - N \\ &= 2 \sum (\bar{M}_0 \cdot M_e \oplus \bar{M}_k) - N \quad \text{from the shift-and-AND property.} \\ &= A(k)_e. \end{aligned}$$

Theorem 4.4 (M-sequence)

If  $M_0 \oplus M_d = M_e$ ,  $A(d)_d = A(e)_d$ ,  $0 < d, e \leq N - 1$ .

Proof.

$$\begin{aligned} A(d)_d &= A(e)_e \quad (= \frac{N-1}{2}) && \text{from theorem 4.2} \\ \text{But } A(e)_e &= A(e)_d && \text{from theorem 4.3} \\ \therefore A(d)_d &= A(e)_d \end{aligned}$$

This means that the masked correlation function is "double-peaked", i.e. ambiguous.

Theorem 4.5 (M-sequence)

$A(0)_d = -\frac{N+3}{2}$  ( $= -2^{n-1}-1$ ) for all  $d$ ,  $0 < d \leq N - 1$ .

Proof.

$$\begin{aligned} A(0)_d &= 2 \{ \sum (\bar{M}_0 \cdot M_d \oplus \bar{M}_0) \} - N \\ &= 2 \{ \sum (\bar{M}_0 \cdot M_d \oplus \bar{M}_0) \} - N \\ &= 2 \{ \sum \bar{M}_0 \cdot (M_d \oplus I) \} - N \\ &= 2 \{ \sum \bar{M}_0 \cdot \bar{M}_d \} - N \\ &= 2 \{ \frac{N-3}{4} \} - N \quad \text{from theorem 4.1} \\ &= \frac{N-3}{2} - N \\ &= -\frac{N+3}{2}. \end{aligned}$$

Theorem 4.6 (M-sequence)

$A(k)_d = -1$  for all  $k$ ,  $0 < k \leq N-1$ ,  $k \neq d, e$  where  $M_0 \oplus M_d = M_e$ .

Proof.

$$\begin{aligned} A(k)_d &= 2 \{ \sum (\bar{M}_0 \cdot M_d \oplus \bar{M}_k) \} - N \\ &= 2 \{ \sum (\bar{M}_0 \cdot M_d \oplus \bar{M}_k) \} - N \end{aligned}$$

Now, from the definition of  $\oplus$ , elements of  $(\bar{M}_0 \cdot M_d \oplus \bar{M}_k)$  will be non-zero only when  $\bar{M}_0 \cdot M_d$  and  $\bar{M}_k$  are dissimilar. Thus

$$A(k)_d = 2 \{ ((\bar{m}_0 \cdot m_d), \bar{m}_k = 1, 0) + (\bar{m}_0 \cdot m_d, \bar{m}_k = 0, 1) \} - N.$$

From theorem 4.1 (or its summary, numbers (i), (iv), (v) and (vii)) this is

$$\begin{aligned}
 &= 2\left\{3\frac{N+1}{8} + \frac{N-7}{8}\right\} - N \\
 &= 2\left\{\frac{N-1}{2}\right\} - N \\
 &= N - 1 - N \\
 &= -1.
 \end{aligned}$$

Analogous theorems can be proved for the W-sequences:

Theorem 4.7 (W-sequence)

$$A(d)_d = \frac{N+3}{2} \quad \text{for all } d, \quad 0 < d \leq N-1.$$

Proof.

$$\begin{aligned}
 A(d)_d &= 2\left\{\sum (\bar{w}_0 \cdot w_d \ominus \bar{w}_d)\right\} - N \\
 &= 2\left\{\sum (\bar{w}_0 \cdot w_d \oplus \bar{w}_d)\right\} - N \\
 &= 2\left\{\sum \bar{w}_0 \cdot w_d + \sum \bar{w}_d\right\} - N && \text{see theorem 4.2} \\
 &= 2\left\{\sum M_0 \cdot \bar{M}_d + \sum M_d\right\} - N \\
 &= 2\left\{\frac{N+1}{4} + \frac{N+1}{2}\right\} - N && \text{from theorem 4.1 and}
 \end{aligned}$$

property (i), the weighting property

$$\begin{aligned}
 &= \frac{N+1}{2} + N + 1 - N \\
 &= \frac{N+3}{2}
 \end{aligned}$$

The W-sequences do not have the property analogous to theorem 4.3 since  $\bar{w}_0 \cdot w_d \neq \bar{w}_0 \cdot w_e$ . Thus we expect

$$A(k)_d \neq A(k)_e \quad \text{for W-sequences.}$$

Hence  $A(d)_d \neq A(e)_d$  for W-sequences, that is, their masked correlation functions are not perfectly ambiguous, as are those of the M-sequences.

The following theorems do, however, hold for the W-sequences:

Theorem 4.8 (W-sequences)

$$\text{If } M_0 \oplus M_d = M_e, \quad A(e)_d = -\frac{N-1}{2}, \quad 0 < d, e \leq N-1.$$

Proof.

$$\begin{aligned}
 A(e)_d &= 2\left\{\sum (\bar{w}_0 \cdot w_d \ominus \bar{w}_e)\right\} - N \\
 &= 2\left\{\sum (\bar{w}_0 \cdot w_d \oplus \bar{w}_e)\right\} - N
 \end{aligned}$$

$$\begin{aligned}
 &= 2\{\sum (M_0 \cdot \bar{M}_d \oplus M_0 \oplus M_d)\} - N \\
 &= 2\{\sum (M_0 \cdot (\bar{M}_d \oplus I) \oplus M_d)\} - N \\
 &= 2\{\sum (M_0 \cdot M_d \oplus M_d)\} - N \\
 &= 2\{\sum \bar{M}_0 \cdot M_d\} - N \\
 &= 2\left\{ \frac{N+1}{4} \right\} - N \quad \text{from theorem 4.1} \\
 &= -\frac{N-1}{2}
 \end{aligned}$$

This confirms that  $A(d)_d \neq A(e)_d$ .

Theorem 4.9 (W-sequence)

$$A(0)_d = -\frac{N-1}{2} \text{ for all } d, \quad 0 < d \leq N-1.$$

Proof.

$$\begin{aligned}
 A(0)_d &= 2\{\sum (\bar{W}_0 \cdot W_d \oplus W_0)\} - N \\
 &= 2\{\sum (\bar{W}_0 \cdot W_d \oplus \bar{W}_0)\} - N \\
 &= 2\{\sum (\bar{W}_0 \cdot (W_d \oplus I))\} - N \\
 &= 2\{\sum \bar{W}_0 \cdot \bar{W}_d\} - N \\
 &= 2\{\sum M_0 \cdot M_d\} - N \\
 &= 2\left\{ \frac{N+1}{4} \right\} - N \quad \text{from theorem 4.1} \\
 &= -\frac{N-1}{2}
 \end{aligned}$$

Theorem 4.10 (W-sequence)

$$A(k)_d = +1 \text{ for all } k, \quad 0 < k \leq N-1, \quad k \neq d, e \text{ where } M_0 \oplus M_d = M_e.$$

Proof.

Following the method of theorem 4.6:

$$A(k)_d = 2\{((\bar{W}_0 \cdot W_d), \bar{W}_k = 1, 0) + ((\bar{W}_0 \cdot W_d), \bar{W}_k = 0, 1)\} - N.$$

From theorem 4.1 (or its summary, numbers (ii), (iv), (v) and (viii)), this

$$\begin{aligned}
 &= 2\left\{4 \frac{N+1}{8}\right\} - N \\
 &= N + 1 - N \\
 &= +1
 \end{aligned}$$

These theorems confirm (for all M- and W-sequences and for all possible delays and offsets) the results obtained in sections 4.3.5 and 4.3.6. The existence of the conjugate peaks in the masked correlation functions can be attributed to the shift-and-AND property, while their positions are

determined by the shift-and-add property (as implied by Table 4.1).

Since the de Bruijn sequences possess neither the shift-and-add nor shift-and-AND properties, they do not have the same features as their linear counterpart.

#### 4.5 Summary.

A number of codes were tested, particularly with reference to their masked correlation functions, with a view to implementation in the Poole system.

Of the codes tested, the 127-bit W-sequence ("inverted" M-sequence) emerged as the most suitable and was therefore chosen to be used in the system.

It has been shown, therefore, that a system employing semi-continuous, cyclic, amplitude-modulated transmission, with reception taking place during the transmitter off-times and the delay-time information being obtained from the cyclic correlation function of the transmitted and received codes, is viable (in theory), and a suitable code has been found for such a system.

CHAPTER 5

DESIGN SPECIFICATIONS AND COMPUTER SIMULATIONS

5.1 Introduction.

Having found a code suitable for implementation in the Poole system, it was decided that since the requirements of this system differed so much from those of Minibal (especially as regards transmitter output power), an entirely new ionosonde incorporating both the Barker and Poole systems (as alternatives) would be built.

To this end, consideration was given to the design specifications and basic mode of operation of the new ionosonde (later named "Microbal"), and thereafter some final computer simulations of the entire system were done, in order to check these aspects of the design.

5.2 Design specifications.

5.2.1 Codes and bit length.

It has already been stated that the codes to be used are the 15-bit M-sequence (1 0 0 1 1 0 1 0 1 1 1 1 0 0 0 in ordinary binary notation) and the 127-bit W-sequence derived from the alpha-sequence 1 0 0 0 0 0 1, the latter being chosen assuming a bit length of 50  $\mu$ s. Both codes would be implemented in the same way, the shorter one being considered to be a 127-bit code consisting of the 15-bit code followed by 112 zeros.

Since the one code is approximately ten times longer than the other, and since about half the returning signal is lost due to masking by the transmitter, it is to be expected that the S.N.R. of the Poole system will be approximately 5 times better than that of the Barker system, all other considerations being the same.

#### 5.2.2 Scan time and range.

A 127-bit code with a bit length of 50  $\mu$ s results in a basic scan time of 6,35 ms (and hence a range of 952,5 km ), and a theoretical v.h. resolution of 7,5 km. These values were considered to be entirely satisfactory.

#### 5.2.3 Frequency sweep and increment.

It was proposed that the range of frequencies swept by the carrier should be selectable on the front panel of the instrument, but that a lower bound of 0,5 MHz, and an upper one of 30 MHz (in case oblique sounding was to be attempted), should be set. The carrier frequency would not be increased continuously, as in Minibal, but in discrete steps of 20, 50 or 100 kHz.

#### 5.2.4 Number of averages and "break" frequency.

In order to simplify electronic circuitry and facilitate front-panel control, it was decided to vary the number of scans summed by the averaging device in powers of 2, with a minimum of 1 and a maximum of 128. This would mean a maximum increase in S.N.R. of a factor of  $\sqrt{128}$  (Lynn, 1973, pp.207-8).

Furthermore, another front-panel control would allow the frequency sweep to be "broken" into two parts, so that the numbers of averages carried out in these ranges could be varied independently. The "break" frequency could be set anywhere in the frequency range.

#### 5.2.5 Receiver bandwidth and sampling rate.

The accepted optimum bandwidth B required for the transmission of rectangular pulses through a multi-stage I.F. amplifier is the reciprocal of the pulse (or bit) length (Schwartz, 1970, p.421) and the minimum sampling rate (the Nyquist rate) is then 2B (op.cit., p.119).

The masking effect of the Poole system, however, necessitates

modification of these values. Since the delay-time between transmission and reception is, of course, not necessarily an integral multiple of the bit length, it is clear that portions of bits will be lost on reception, due to the masking effect of the transmitter. However, it is a feature of the M-sequences that exactly the same amount of energy will be returned for all possible delays, which means that if a certain portion of a bit is lost at one time, exactly the same fraction of a bit will be returned at some other time. While the larger portions of bits will, to some extent, be passed by a receiver tuned to pass whole bits, the smaller portions will be greatly attenuated.

The worst-case masking occurs when the delay-time is a half-integral multiple of the bit length, when all bits affected by the masking will appear as half-bits. (A calculation using a 127-bit M-sequence showed that of the total 32 bits returned, 20 were half-bits and 22 were whole. This means that approximately  $\frac{1}{3}$  of the total non-zero M- and W-sequence signal will be affected.)

It was considered necessary, then, to double the receiver bandwidth in order to optimize reception of these worst-case half-bits. Portions smaller than half a bit will still be attenuated or lost, but half-bits and larger portions will be received.

The bandwidth (for the Poole system) was thus

$$B = 2\left(\frac{1}{50\mu\text{s}}\right) = 40 \text{ kHz.}$$

The optimum sampling rate is then 80 kHz (4 samples per bit or a sampling interval of 12,5  $\mu\text{s}$ .)

The increased bandwidth will, of course, cause an undesirable decrease in S.N.R., but is unavoidable.

#### 5.2.6 A.G.C., d.c. removal and sampling levels.

In order to minimize large-scale saturation of the receiver input stages, for instance by commercial radio stations, it

was proposed that an automatic gain control (A.G.C.) be included in the receiver circuitry.

Also included should be the filtering out of the zero-frequency component of the demodulated signal so as to change the signal levels and discriminate against continuous-wave interference (see section 3.2.5). Essentially this d.c. removal would entail passing the incoming signal through a suitable series capacitor.

Adequate resolution of multiple echoes (super-position of signal codes) necessitates sampling the bipolar signals at a number of levels, perhaps by an A.D.C. The actual number of levels sampled was left to be dictated by electronic considerations.

#### 5.2.7 Transmitter output power.

This aspect of the design of the new ionosonde differs greatly from that of the old. Since the ordinary binary W-sequences have very nearly the same number of 1's and 0's, the Poole system transmitter has a duty-cycle of very nearly 50%. Once again the choice of actual transmitter output power was left to the dictates of electronic circuitry, with the proviso that it should be the maximum that could be easily obtained at this duty-cycle, preferably with semiconductor devices. This power, it transpired, would be about 150 W (average power). This seemed a reasonable figure. Minibal has a duty-cycle of about 0,5% and a peak power of approximately 6 kW, which is equivalent to an average power of approximately 30 W.

#### 5.2.8 Antenna sharing.

It seemed desirable that since the transmitter and receiver never operate at the same time, they should share a single antenna. This would greatly facilitate operation on a ship or aircraft.

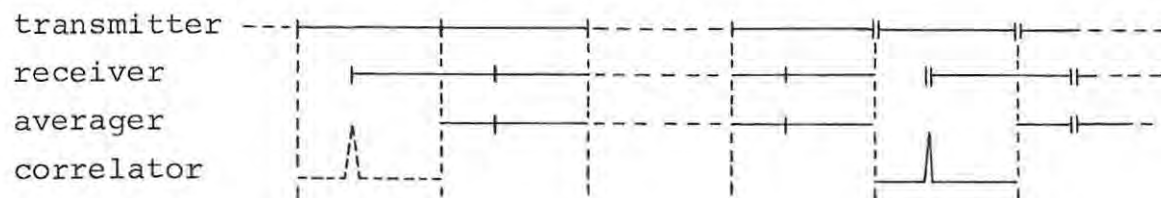
### 5.2.9 The correlation process.

The essentials of the correlation process were introduced in section 4.1, but two important problems arise immediately in connection with this, the basic mode of operation of the instrument.

- (i) Since the transmitter and receiver are tuned in unison, the frequency of (delayed) signals returning from the ionosphere might well fall outside the bandwidth of the receiver when, at the beginning of a cycle, it is tuned "up" to receive a higher frequency. Thus at any fixed frequency there may be a time after the end of the last scan, when signals are lost.
- (ii) Since incoming signals will be continuously fed into the averaging device, and since the correlation process (that is, the derivation of any one  $A(k)$ ) requires operation on all the bits in the averaging device at that time, it is preferable that the process of feeding incoming signals to the "averager" be halted for an entire cycle while the correlation is being done, otherwise an ever-increasing portion of the cycle being correlated will consist of one scan more than the rest. Halting the feeding in of incoming signals will, of course, result in an unfortunate loss of signal.

Both these losses can, however, be easily minimized by devoting the first scan at any frequency to the correlation of the signals obtained during the previous scan (at the previous frequency). This means a loss of one entire scan at each frequency, but has the advantage that only complete scans (with the delay-time information contained in them) are fed to the averaging device.

The transmitter, receiver, averager and correlator processes may be diagrammatically described as follows:



where  $\text{---}$  and  $\text{---}$  represent scans transmitted at different frequencies.

As mentioned in section 3.2.5, the incoming signals would be correlated against a bipolar form of the transmitted code. The delay-time information would be expressed in terms of bit number, the 508 bits (127 bits, each sampled 4 times) representing the range of v.h.'s from 0 to 952,5 km. Each 100 km of v.h. is thus equivalent to

$$\frac{508}{9,525} = 53\frac{1}{3}$$

quarter-bits. The output from the correlator would therefore comprise the correlation function with 100 km v.h. markers spaced  $53\frac{1}{3}$  quarter-bits apart (to the nearest), and converted to an analogue voltage and displayed (in z-modulated form as usual) on an oscilloscope. The 508-bit correlating code would comprise the appropriate 127-bit code, each bit of which would be followed by 3 zeros.

#### 5.2.10 Threshold device.

It was proposed that an electronic threshold device be incorporated into the display and data capture units, so that correlation peaks below a certain height could be ignored by the monitor and camera oscilloscopes. It would be a manual device, operated from the front panel and could be used to eliminate unwanted, low-level signals ("grass").

#### 5.2.11 Ionogram synthesis time.

The time required to synthesize a complete ionogram obviously depends on the frequency range swept, the frequency increment ( $\Delta f$ ) and the number of scans summed ( $i$ ) - it will be assumed that  $i$  is the same above and below the break frequency.

The total time ( $T$ ) needed to synthesize 1 MHz of an ionogram is

$$T = \frac{1000}{\Delta f (\text{kHz})} \times (i+1) \times 6,35 \times 10^{-3} \text{ s.}$$

The minimum and maximum times needed to obtain 1 MHz of an ionogram are thus

$$T_{\min} = \frac{1000}{100} \times 2 \times 6,35 \times 10^{-3}$$
$$= 0,127 \text{ s}$$

$$T_{\max} = \frac{1000}{20} \times 129 \times 6,35 \times 10^{-3}$$
$$\approx 40,1 \text{ s.}$$

5.2.12 Block diagram.

A block diagram showing the major component parts and indicating the mode of operation of the ionosonde, is given in Figure 5.1.

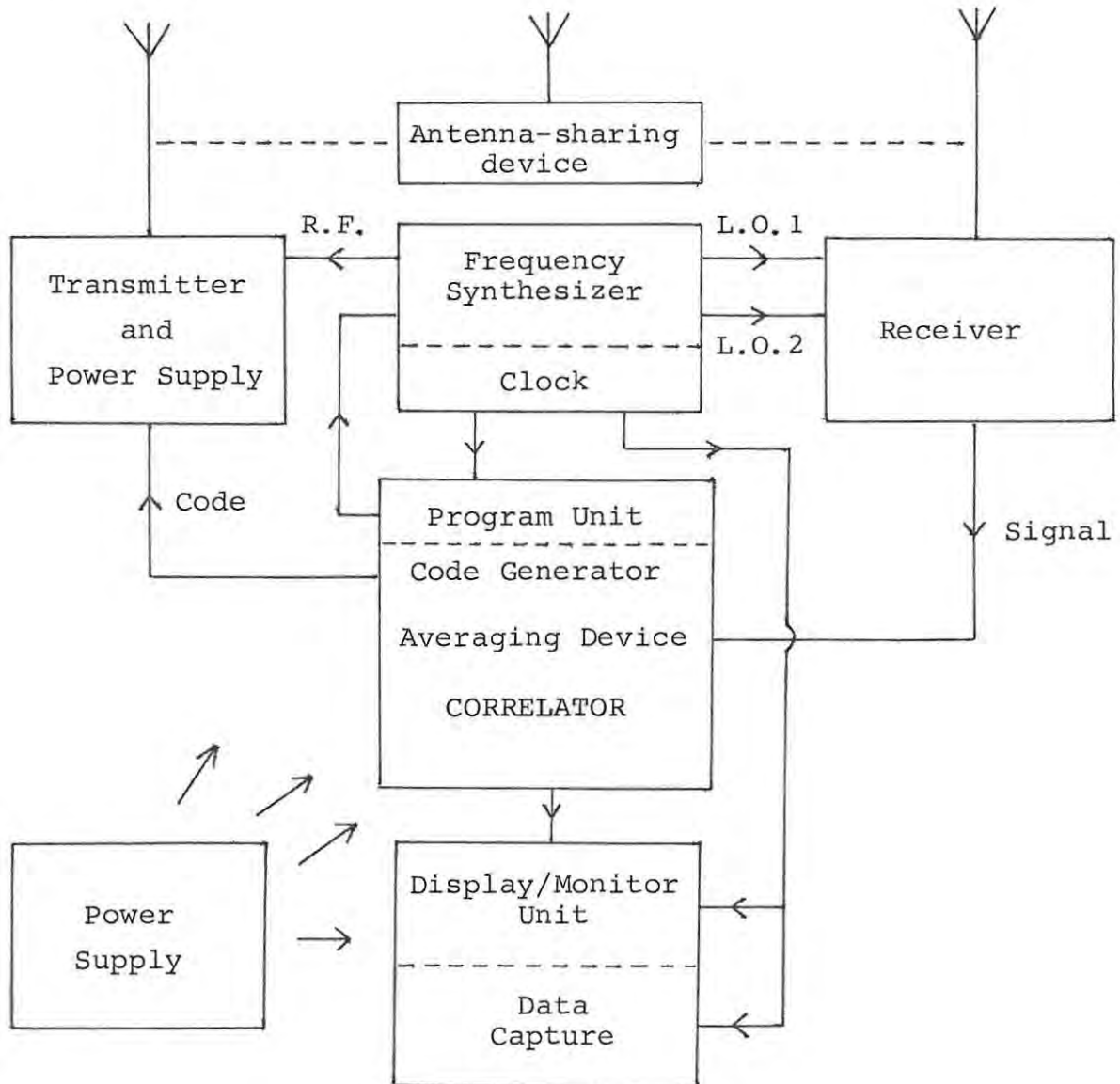


Figure 5.1.

### 5.3 Computer simulations.

#### 5.3.1 Introduction.

A Fortran IV computer program was written to simulate the entire operation of the ionosonde, that is, the delay and superposition of codes, addition of random noise, masking by the transmitter, filtering by the receiver, d.c. removal, averaging, and finally, correlation. A graph-plotting routine was used once more to display the correlation functions.

The program also allowed correlation with noise only, and the addition of a continuous-wave (C.W.) signal to the coded signal. The program (called SIMU), with suitable "comment" statements, and with examples of data subfiles, is given in Appendix C.

Three points concerning the program should be noted:

- (i) Simulation of the sampling process required expanding each bit of the transmitted and correlating codes into 4 bits, resulting in codes of length 508 bits. The delays put into and resulting from the program, therefore, are 4 times the delays submitted, for instance, to MASK above.
- (ii) the peak amplitude of the signal was assigned the integer variable ISIG, the (pseudo-) random bipolar noise added to the signal was called NL, and the C.W. signal, ICW.
- (iii) the filter in the program is a very simple digital low-pass filter which transforms a rectangular half-bit into a triangular one of the same height and twice the width. This is a rough but convenient approximation.

SIMU was applied first to the Barker system, and then to the Poole system. Detailed investigations (into superposition and resolution) were done on the Poole system only.

5.3.2 The Barker system.

Five simulations were done on the Barker system. The resulting correlation functions are given in figures 5.2 to 5.6, and the input data are tabulated below.

Delays	Signal (ISIG)	Noise (NL)	Averages (IAV)	C.W.Signal (ICW)	Figure
100 200 240 -	1	0	1	0	5.2
100 200 240 -	1	1	1	0	5.3
100 200 240 -	1	1	4	0	5.4
100 - -	1	0	1	5	5.5
- - - -	0	1	1	0	5.6

A number of points should be noted concerning these simulations:

- (i) the size of the mainlobes and shape of correlation function are approximately as predicted in Chapter 3, considering the  $A(k)$  have not been doubled here, and that filtering and d.c. removal will also change their values slightly (see Figures 5.2 and 5.5).
- (ii) the superposition of the signals delayed by 200 and 240 bits is quite satisfactory (Figure 5.2). The separation of these two signals is 10 whole bits - a worst-case separation (see section 3.3.3) - and the M.S.R. of the 200-bit delay signal reflects this fact.
- (iii) when bipolar noise of the same peak amplitude as the signal is added (Figure 5.3), the signals are still easily discernible. The largest sidelobe is about 4.
- (iv) the averaging process (see Figure 5.4) greatly reduces noisy sidelobes and enhances the signal, as expected. 4 averages reduces the largest sidelobe to about 2.
- (v) the d.c. removal and correlation with a bipolar (correlating) code discriminates very effectively against large-amplitude continuous-wave interference (Figure 5.5). Only one signal (at a delay of 100) was put into the simulation of Figure 5.5.
- (vi) Figure 5.6 shows that, as expected, correlation with random noise only, is poor.

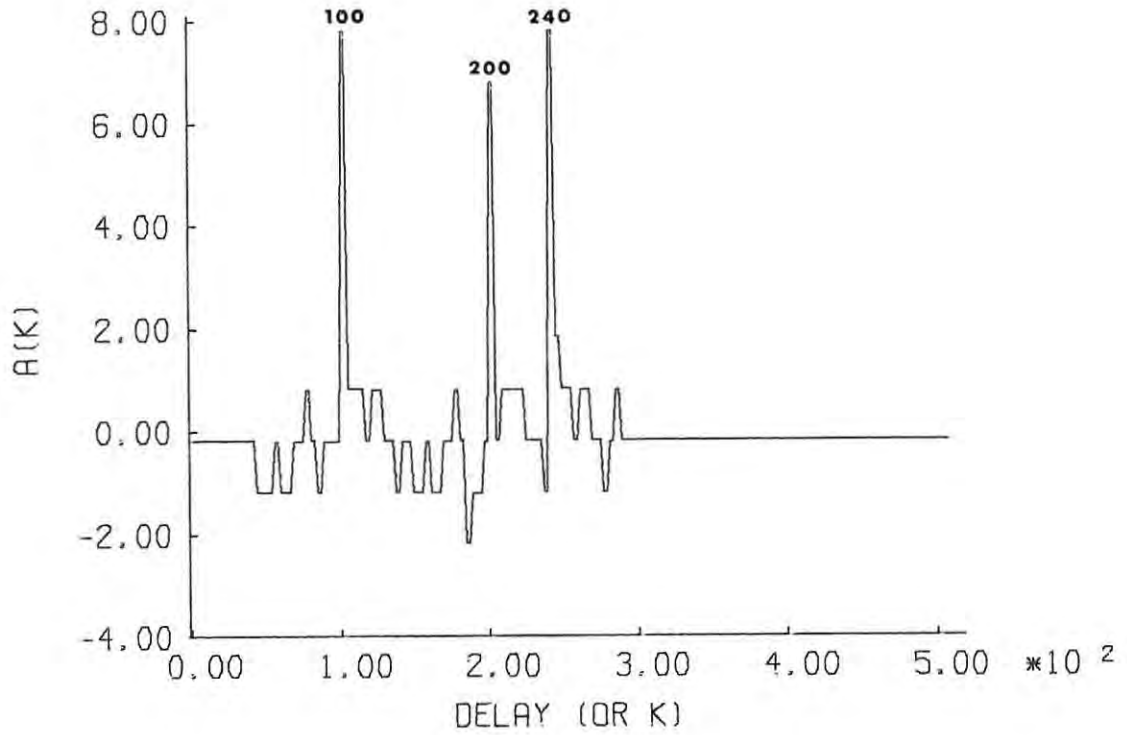


Figure 5.2 Signal 1, noise 0, averages 1, C.W. 0.

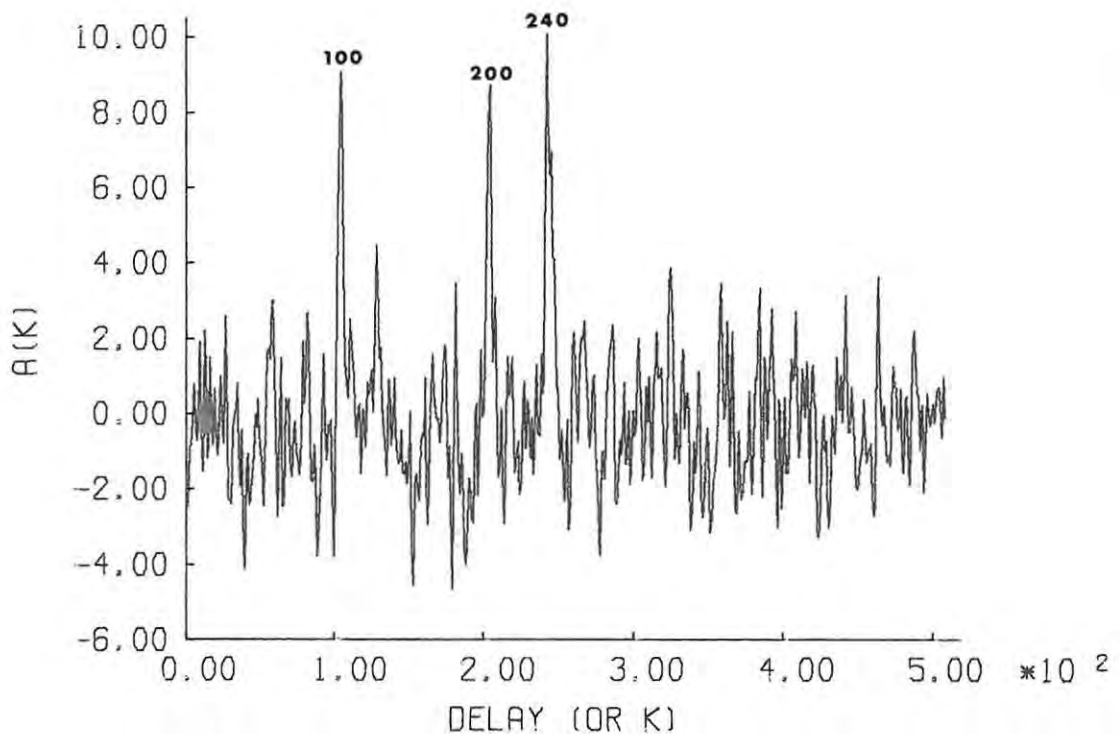


Figure 5.3 Signal 1, noise 1, averages 1, C.W. 0.

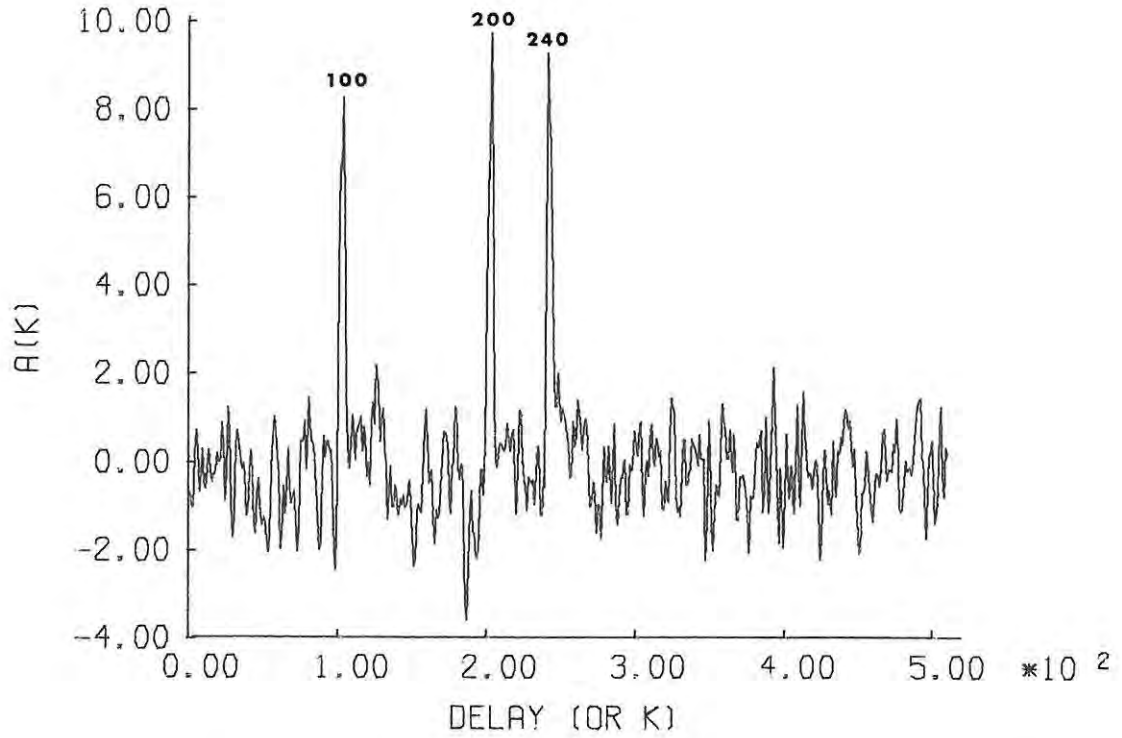


Figure 5.4 Signal 1, noise 1, averages 4, C.W. 0.

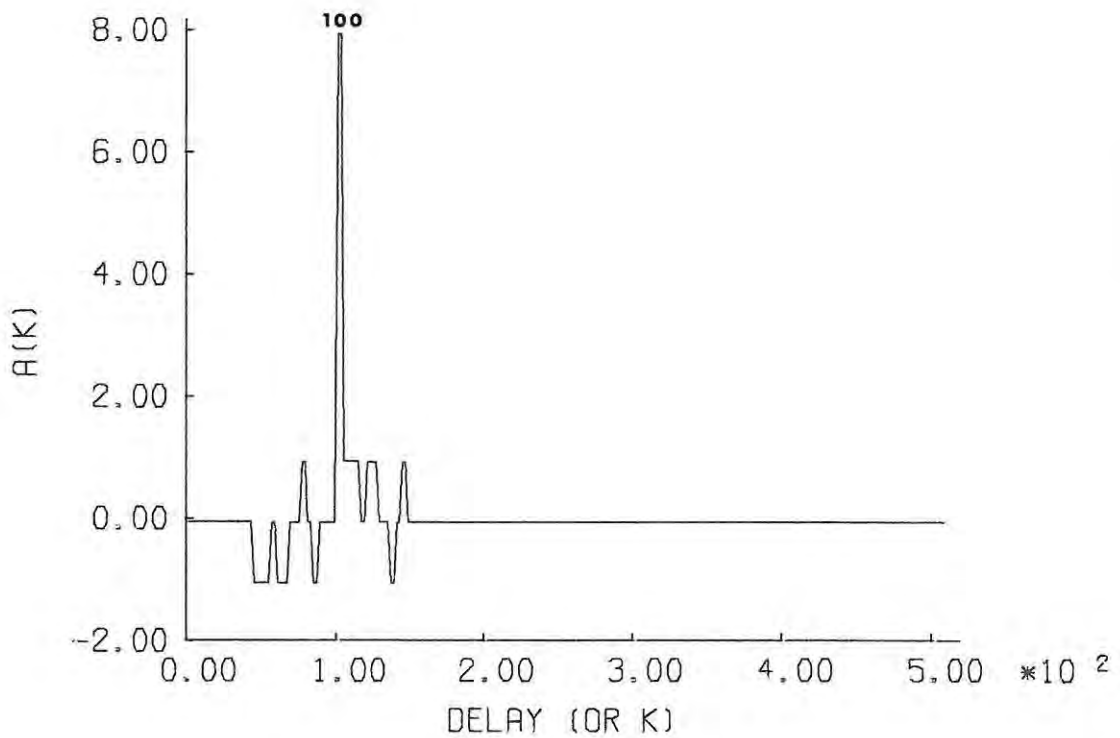


Figure 5.5 Signal 1, noise 0, averages 1, C.W. 5

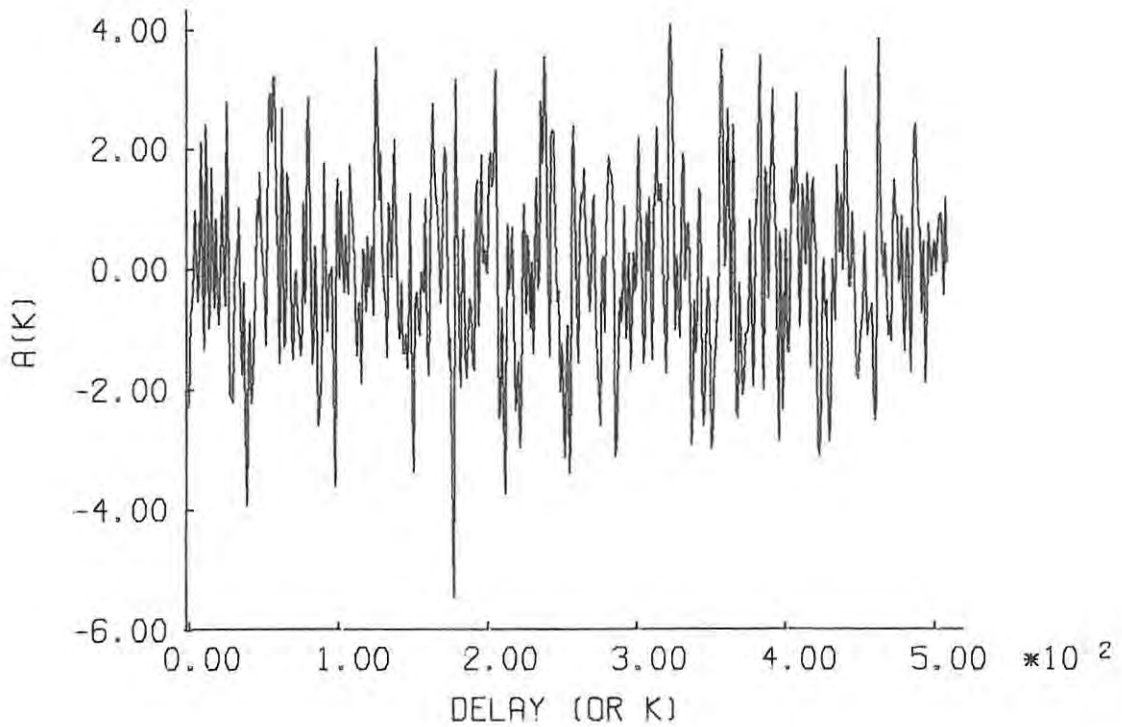


Figure 5.6 Signal 0, noise 1, averages 1, C.W. 0.

5.3.3 The Poole system.

Six simulations of this system were done (Figures 5.7 to 5.12). The input data were as follows:

Delays				Signal (ISIG)	Noise (NL)	Averages (IAV)	C.W.Signal (ICW)	Figure
68	69	280	285	1	0	1	0	5.7
68	70	280	286	1	2	1	0	5.8
68	71	280	287	1	2	4	0	5.9
68	72	280	288	1	10	128	0	5.10
68	172	280	-	1	0	1	5	5.11
-	-	-	-	0	1	1	0	5.12

The Poole system simulation shows analogous properties to those given above for the Barker system, with a number of additional features.

- (i) The M.S.R.'s are, of course, much better than those of the Barker system, and the correlation function

(in the absence of noise) is mostly flat. Negative conjugate peaks occur throughout, and have been labelled with the delay of their associated signal peaks and with a negative sign.

- (ii) "Non-integral" delays (in this case delays which are not multiples of 4) have two conjugate peaks, a larger one associated with the closest multiple of 4, and a smaller one associated with the next closest multiple (Figure 5.7, delays 68 and 69, for instance). As the signal peak moves away from one multiple of 4 towards another (see Figures 5.7 to 5.10, delays 69 to 72 respectively), the conjugate peaks increase and decrease in size, respectively.
- (iii) The Poole system is, understandably, less susceptible to noise than the Barker system (see Figure 5.8, where the noise level is 2). The averaging process is highly satisfactory (Figures 5.9 and 5.10).
- (iv) The obliteration of two signals occurring at "conjugate" delays (refer to section 4.3.6) is demonstrated in Figure 5.11 (whole-bit delays of 17 and 43. See table 4.1).

The following properties, although only demonstrated for the Poole system, hold for the Barker system as well.

- (i) Very closely-spaced signals (for instance Figure 5.7, delays 68 and 69) cause a signal peak of height greater than that of a single signal peak, as expected.
- (ii) The simulation system is only able to resolve signals separated by 5 quarter-bits (Figure 5.7, delays 280 and 285). This rather poor v.h. resolution is due to the computer program's rudimentary digital filter, discussed in section 5.3.1. The v.h. resolution of the physical system will depend ultimately on the transfer function of the multistage I.F. receiver amplifier. Superposed peaks due to signals separated by 1, 2, 3 and 4 bits (delay 68 and its adjacent peaks) and 5,6,7 and 8 bits (delay 280 and its adjacent peaks) are shown in Figures 5.7, 5.8, 5.9 and 5.10.

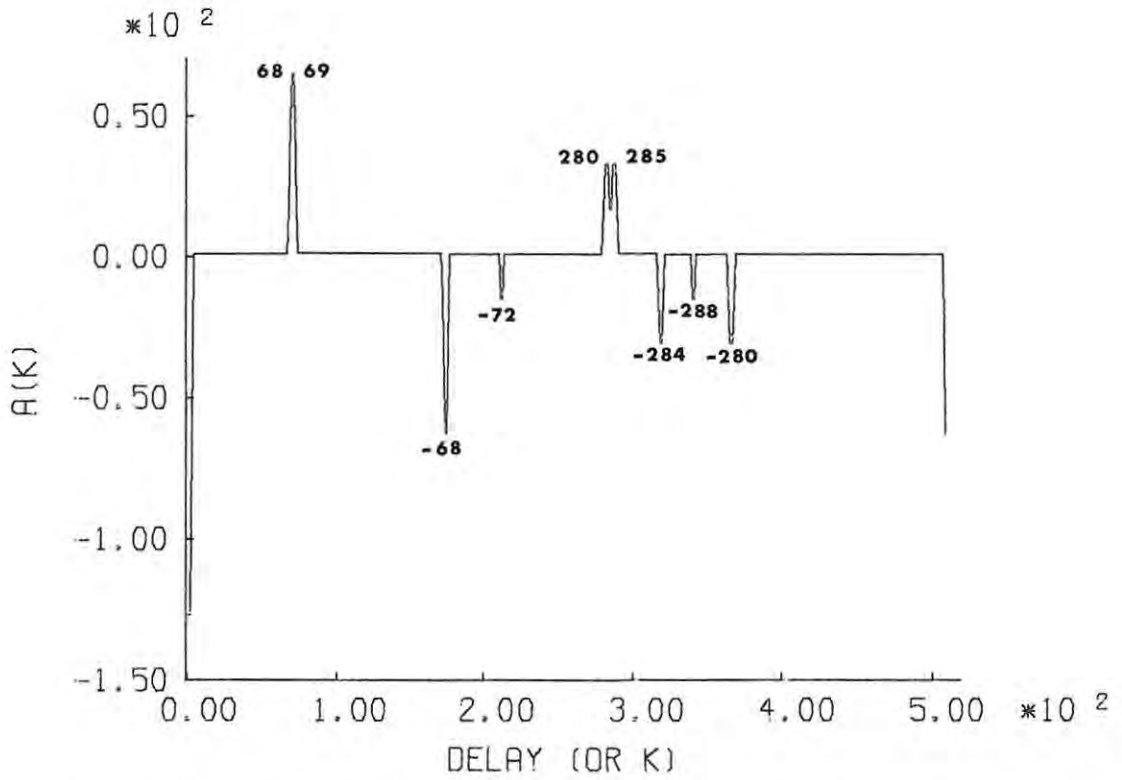


Figure 5.7 Signal 1, noise 0, averages 1, C.W. 0.

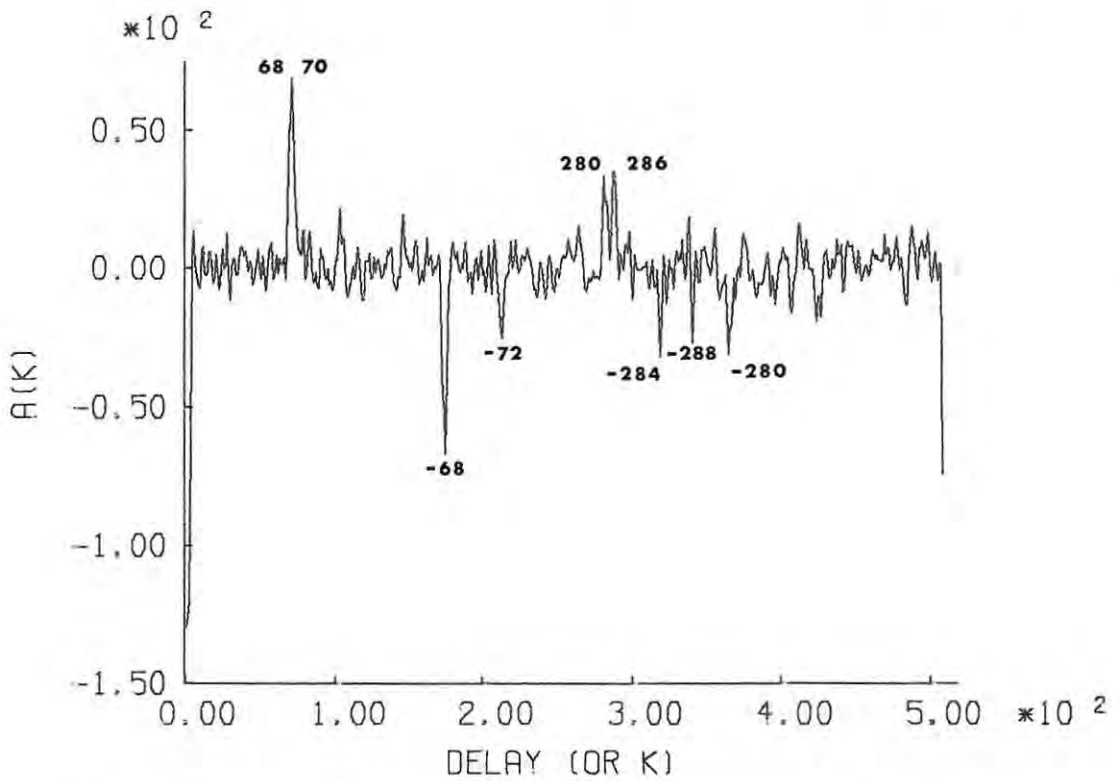


Figure 5.8 Signal 1, noise 2, averages 1, C.W. 0.

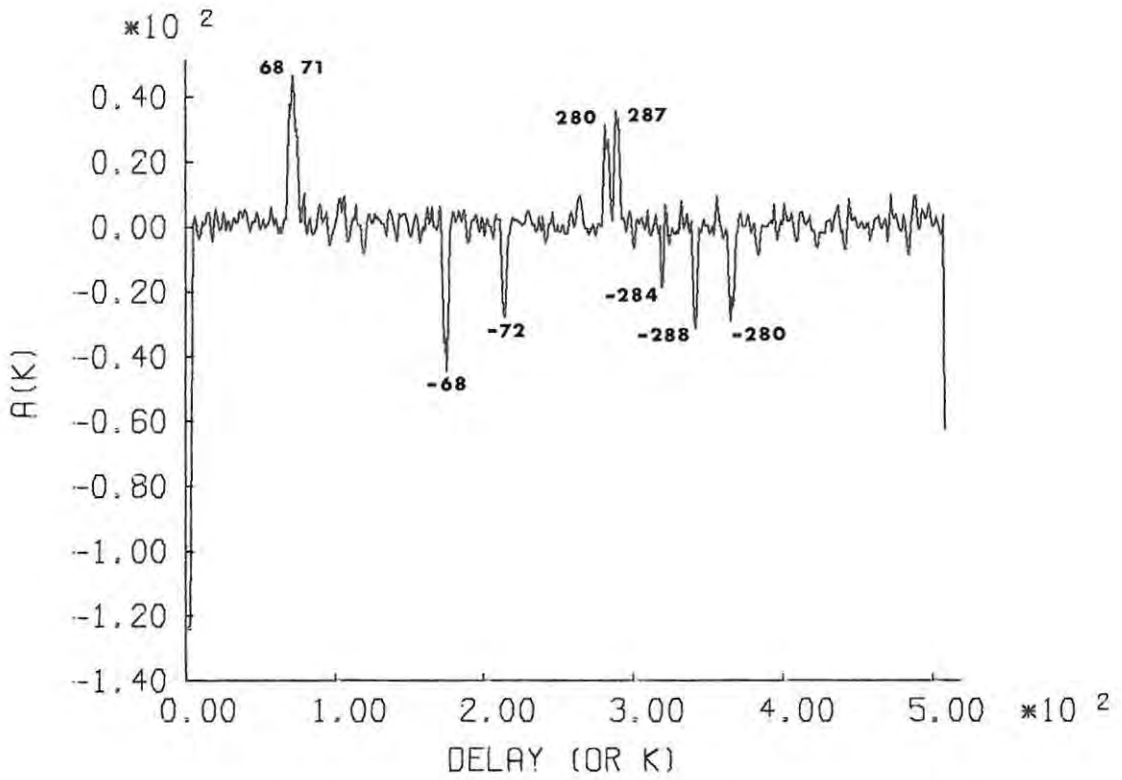


Figure 5.9 Signal 1, noise 2, averages 4, C.W.0.

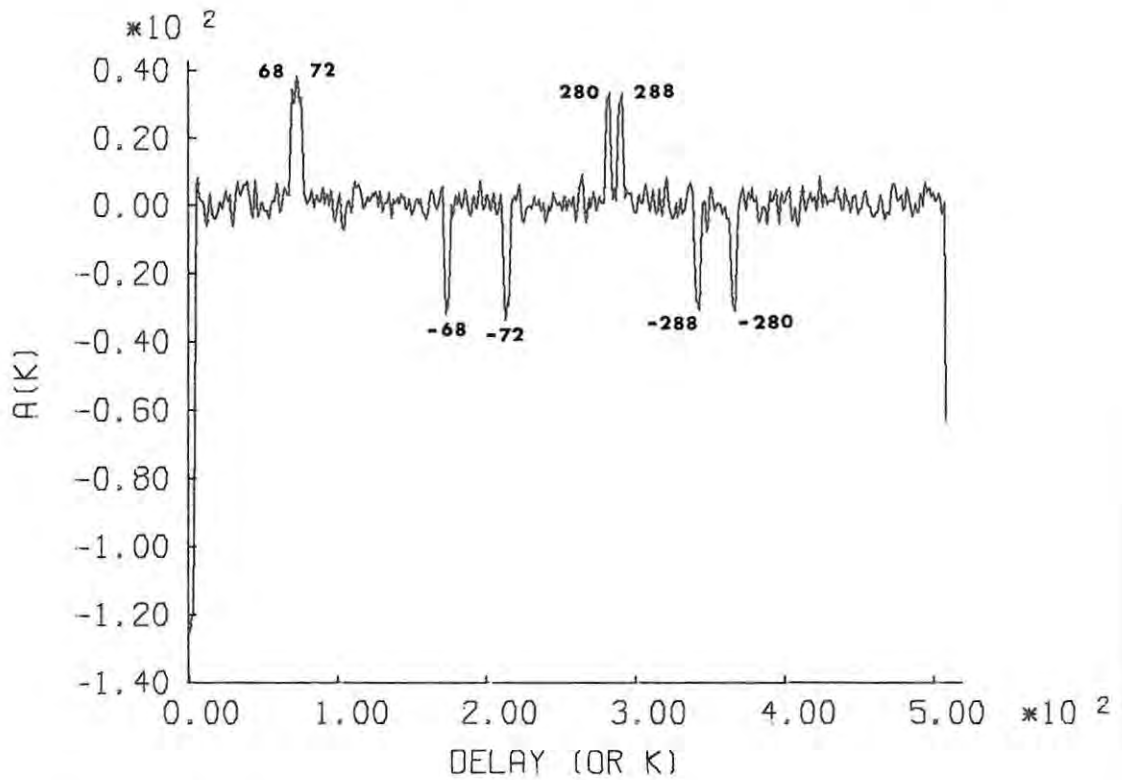


Figure 5.10 Signal 1, noise 10, averages 128, C.W.0.

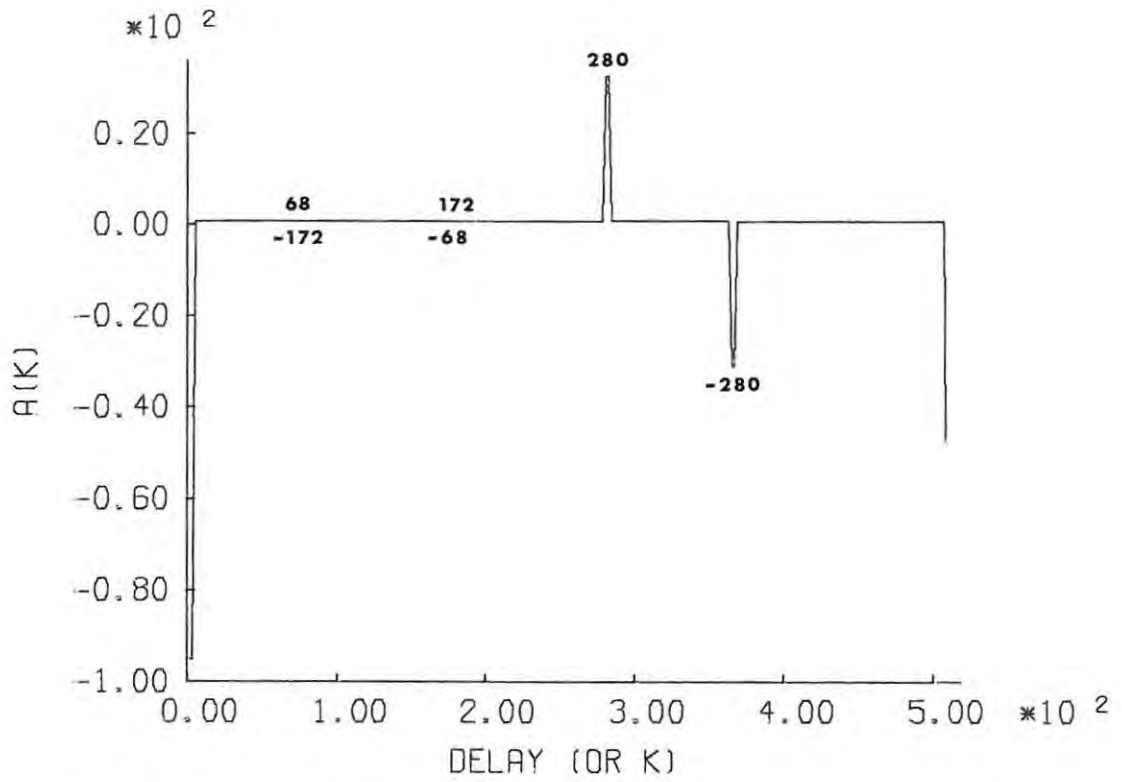


Figure 5.11 Signal 1, noise 0, averages 1, C.W.5.

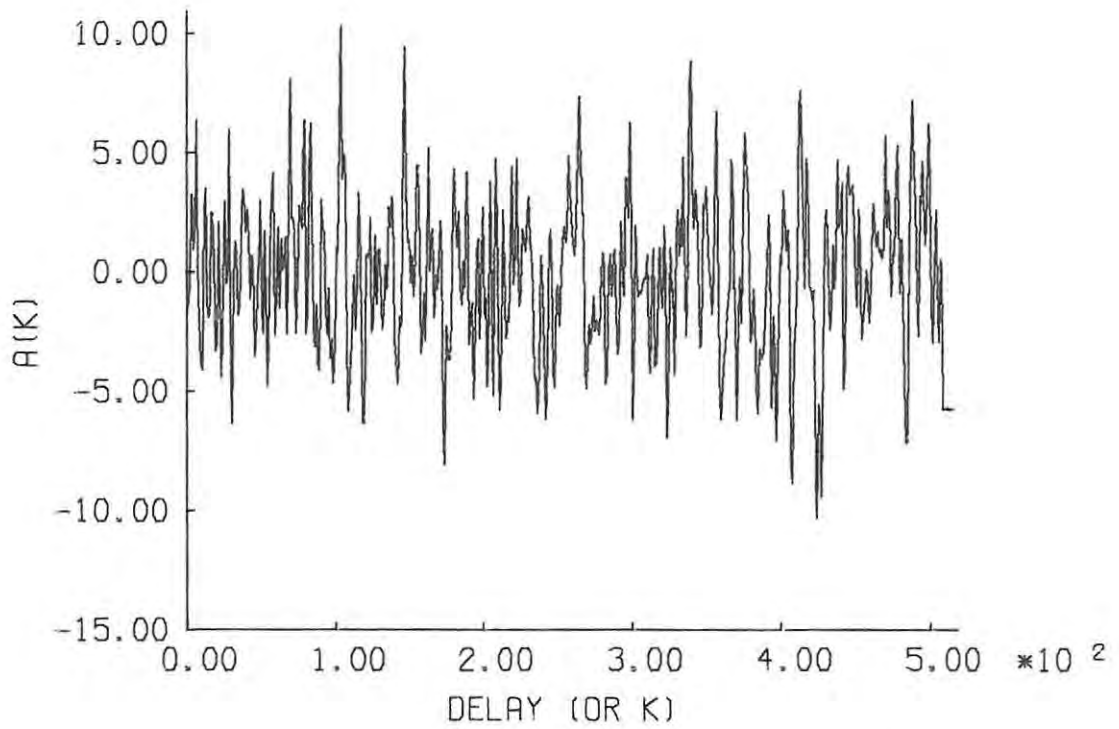


Figure 5.12 Signal 0, noise 1, averages 1, C.W.0.

#### 5.4 Afterthought.

A simple method of overcoming the detrimental effect of the conjugate peak in the correlation function of the W-sequences, was later suggested by the originator of the Poole system (Poole, 1977).

It was proposed that the time available for transmission at any fixed frequency should be divided into two equal intervals. During the first of these an M-sequence (say) would be transmitted, the correlation would be done, and the correlation function stored in a memory. During the second interval the same process would be carried out for the corresponding W-sequence. The two correlation functions (see sections 4.3.5 and 4.3.6 for examples) would then be added term-by-term (decimal) so that the conjugate peaks (the positive one of the M-sequence and the negative one of the W-sequence) cancel, while the signal peaks add.

This idea, although it meant devoting yet another scan at every frequency to the correlation process, was immediately incorporated into the system since it prevents any loss of signals that may occur at conjugate delays.

#### 5.5 Summary.

A number of parameters necessary for the design of the new ionosonde were specified and details concerning the mode of operation of the instrument were clarified. Computer simulation of the operation of the entire ionosonde confirmed what theory and intuition predicted.

CHAPTER 6

CONCLUSION

6.1 The present state of construction of the system.

6.1.1 Introduction.

The development of "Microbal", up to the stage of obtaining the basic design specifications and doing computer simulations, was completed at the end of 1975. The specifications were then handed to a colleague, Clive Way-Jones, who designed all the electronic circuits and supervised both electronic and mechanical aspects of the construction of the system.

At the time of writing, however, the system is unfortunately not sufficiently advanced to be able to produce any results, nor is there any prospect of it being ready for some months.

This chapter, therefore, describes the present state of construction of the system, how it should be tested when it is ready, and also suggests a further avenue of investigation which arises from the project.

Since it is the aim of this thesis to describe the overall theoretical development of the new ionosonde, the only details of the mechanical and electronic design which will be discussed are those which

- (i) may represent improvements or advances in their field,
- (ii) are relevant to the operation (not necessarily maintenance or repair) of the ionosonde, or
- (iii) describe the part played by the author in the construction of the system.

Some plates showing the existing system are given at the end of section 6.1.

### 6.1.2 Mechanical aspects.

The underlying philosophy behind the construction of the entire ionosonde was that it should be compact, robust and easy to service.

To this end, it was decided to house the system in four separate racks, each of which could be divided into modules which could easily be detached from front or back panels.

The transmitter and antenna-sharing device would be housed (with their own power supply) in one rack; the frequency synthesizer, clock, program unit, correlator and receiver in another; the other power supply and monitor oscilloscope in the third; and the camera and its oscilloscope in the fourth. Some space would also be left in the fourth for the possible inclusion of a digital data-capture system (probably a magnetic tape cassette unit) some time in the future. At present all but the monitor and data-capture systems are either complete or under construction (see Plate 6.1).

The transmitter (Plates 6.1 (top rack) and 6.2) has been built but must be tested under load conditions, and may require some modification. One interesting mechanical feature is its modular construction. It has two power supplies mounted on sliding extruded-aluminium heatsinks, while the final power stage is fixed to a detachable heatsink at the back of the rack (see Plate 6.2 (bottom)).

The "correlator" rack (Plates 6.1 (middle) and 6.3) is almost complete. It consists of 3 modules, each with its own front panel, and each detachable from the back panel, which may in turn be detached from the rack (Plate 6.5 (top)). This back panel is also made of heatsink material so as to provide heat dissipation for voltage regulators (Plate 6.3 (bottom)). Another feature of the correlator rack is the correlator module which is twice as wide as its neighbours and which can therefore house double-width printed-circuit boards.

All radio-frequency (R.F.) circuits in the transmitter and

correlator racks are enclosed in brass boxes and connected by shielded cables. R.F. signals are carried from one module to another or from one rack to another via connectors mounted in brass blocks which protrude through the back panels of their respective racks (Plates 6.2 and 6.3 (bottom)).

The power supply is complete and operational (Plates 6.1 (bottom rack) and 6.4). The individual power supplies (like those of the transmitter) are mounted on sliding extruded aluminium heatsinks, while the front and back panels may be detached from the rack but are connected to each other (Plate 6.5 (bottom)).

Most of the mechanical work involved in the "correlator" and power supply racks (including the layout and construction of the front and back panels) was done by the author.

#### 6.1.3 Electronic aspects.

The electronic circuitry, too, was designed to be compact, robust and wherever possible, modular. Certain aspects of the electronics are intimately related to the mechanical construction, and have been discussed above, but some other interesting features of the electronic circuitry will be described.

The final stage of the transmitter amplifier consists of a pair of power transistors in a push-pull arrangement, and can supply about 150 w. The output impedance is 50  $\Omega$ . The antenna-sharing device uses semiconductors to switch the antenna from transmitter to receiver according to the transmitter code, while shorting the unused input or output to ground. It also incorporates a lightning-protective component.

The frequency synthesizer and clock use as reference a 5 MHz, temperature-controlled quartz crystal oscillator which is accurate to about 0,1 Hz, and is adjustable on the front panel. The clock is constructed solely of CMOS (complementary metal-oxide semiconductor) components which draw very little power.

It may be reset and also speeded up or slowed down from the front panel. The display can be made to read days, hours and minutes, or minutes and seconds. The clock is also capable of distinguishing between leap years and "ordinary" years!

The frequency synthesizer consists essentially of a number of phase-lock loop circuits which are capable of synthesizing frequencies accurate to a few Hz in 100 MHz. These loops, with the aid of broad-band R.F. amplifiers, elliptic-function low-pass filters and double-balanced mixers, provide all the R.F. and local oscillator frequencies for the system, i.e. 0,50 to 29,99 MHz for the transmitter, and 60,50 to 89,99 and 58 MHz for the receiver. The variable frequency loop may be programmed automatically by the program unit, or manually by a set of thumbwheel switches on the correlator module front panel. The number of scans, "break" frequency and end frequency are also set by thumbwheel switches on this panel.

The synthesizer/clock module also incorporates an entirely independent frequency counter with a front-panel display. The entire module was built by the author and is all working perfectly except for the frequency synthesizer which, in spite of having been redesigned and rebuilt a number of times, still requires modification.

The correlator module comprises the program unit, "code generator", averaging device, and the correlator itself. The program unit consists entirely of CMOS components, and is required to switch the instrument on and off at the appropriate times and to drive the frequency synthesizer accordingly. The code is not actually generated, but is stored (in various forms) in a number of PROM's (programmable read-only memories). A front-panel switch allows either the "long" or the "short" code to be used. (Another allows the ionosonde to be used for reception only (R), normal transmission and reception (T/R) or alternate R and T/R for possible oblique-incidence sounding). The averaging device reads the appropriate

information from the memory, adds the latest (incoming) datum to it, and writes it back to memory. After the required number of scans have been summed, it divides each sum by this number before the correlator goes to work. The correlator is essentially a high-speed, highly-dedicated computer which is capable (in only 12,5  $\mu$ s) of reading 127 6-bit binary numbers from the memory, multiplying each by 1 or -1, writing them back into the memory, and summing the results of the multiplication.

The receiver is essentially a double-converted superheterodyne system with 60 and 2 MHz intermediate frequencies. It also incorporates bandwidth selection, the d.c. removal, gain control systems, and the analogue-to-digital converter, and houses a battery of Nickel-Cadmium cells which power the oscillator, clock and program unit in the event of a mains power failure. The receiver is not yet complete.

The power supply has the ability to run on either 24 V d.c. or else 115 or 230 V at 50, 60 or 400 Hz a.c. (an important prerequisite for operation on a ship or aircraft). One of the mains power supplies charges the Ni-Cad. cells, powers the oscillator, temperature-controlled oven, clock, and program unit, and is operational while the system is connected to the mains, irrespective of whether the rest of the circuitry is switched on or not. The power supply was built by the author.

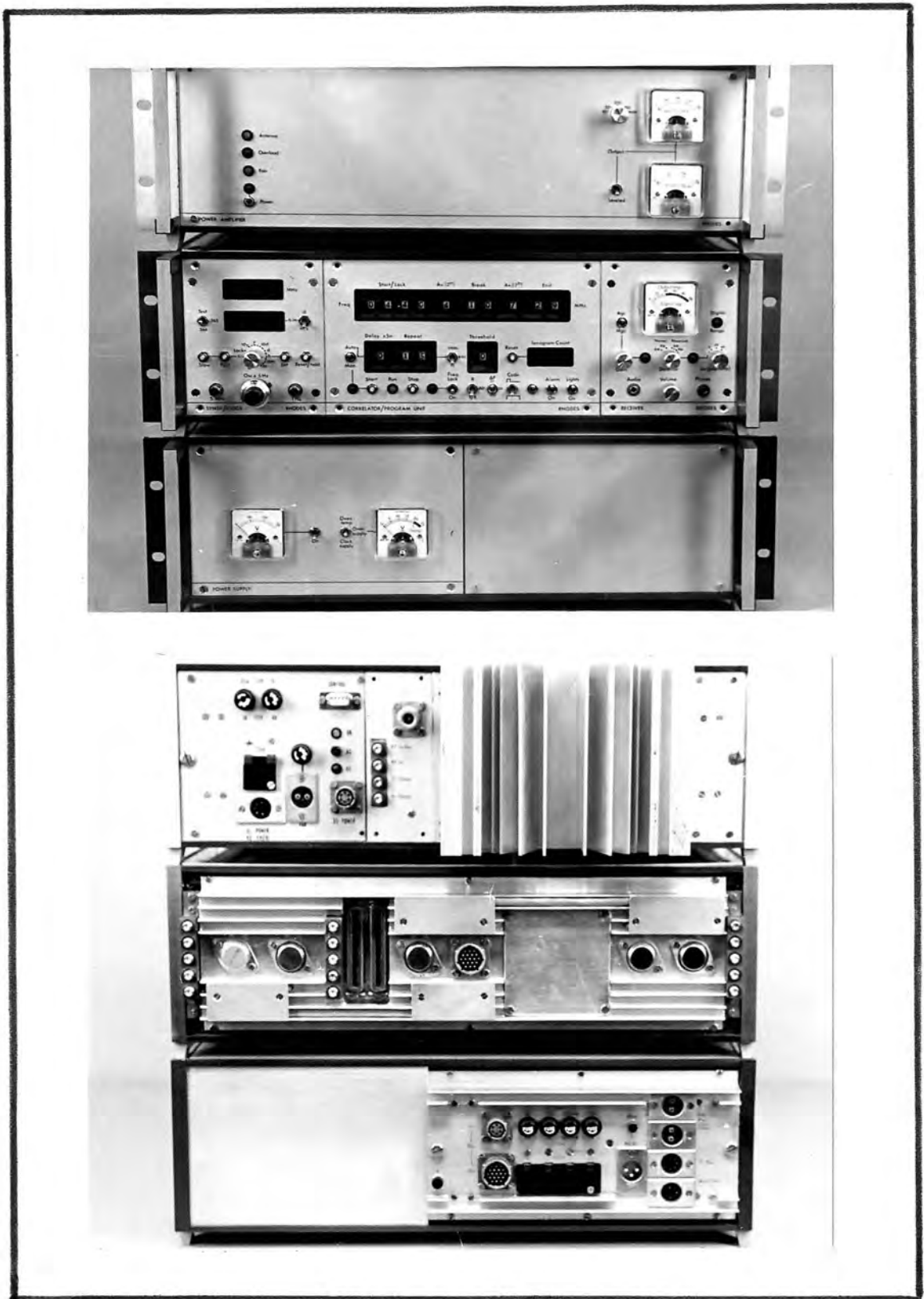


Plate 6.1 The system at present, front and back.

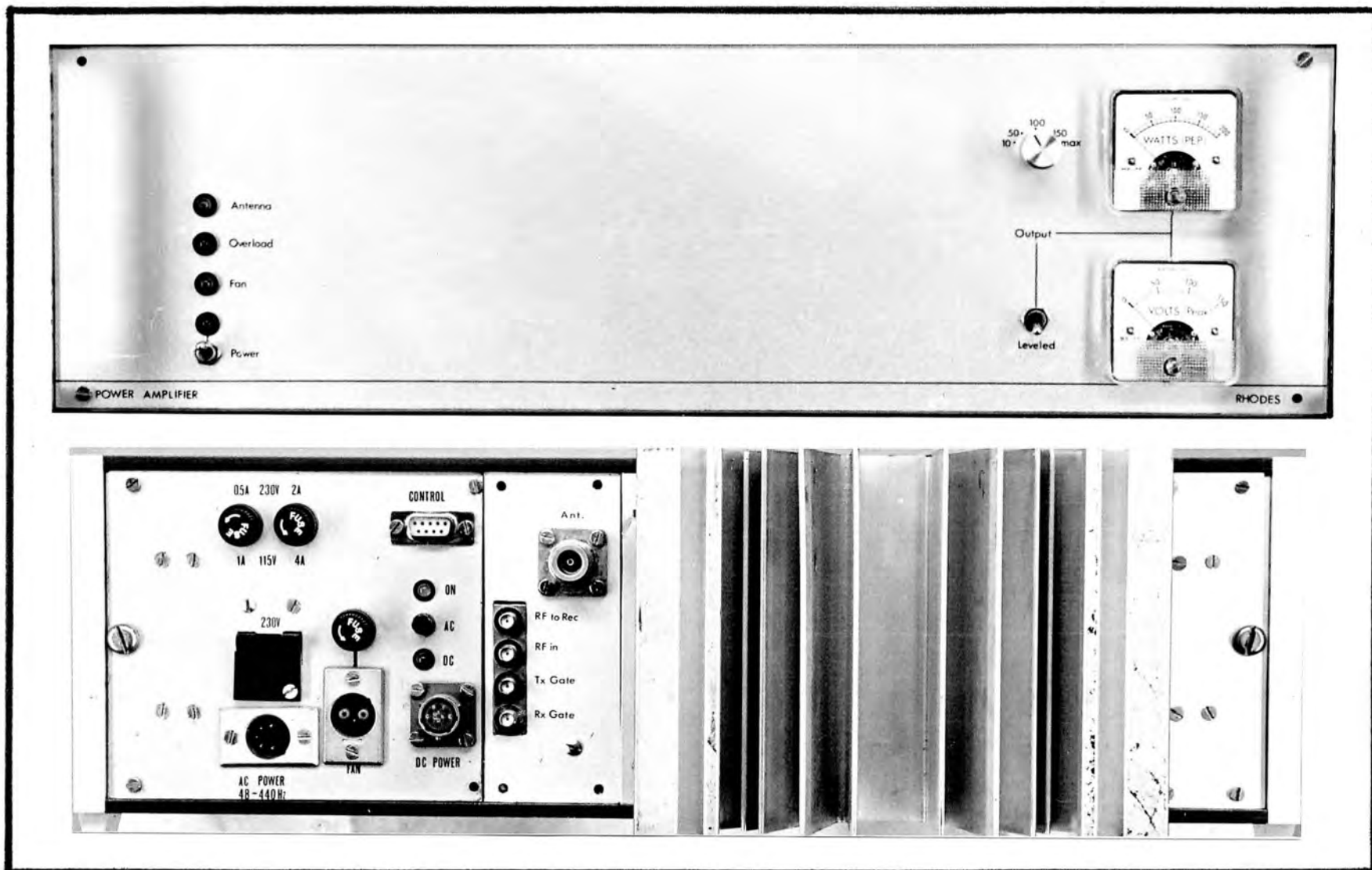


Plate 6.2 The transmitter front and back panels.

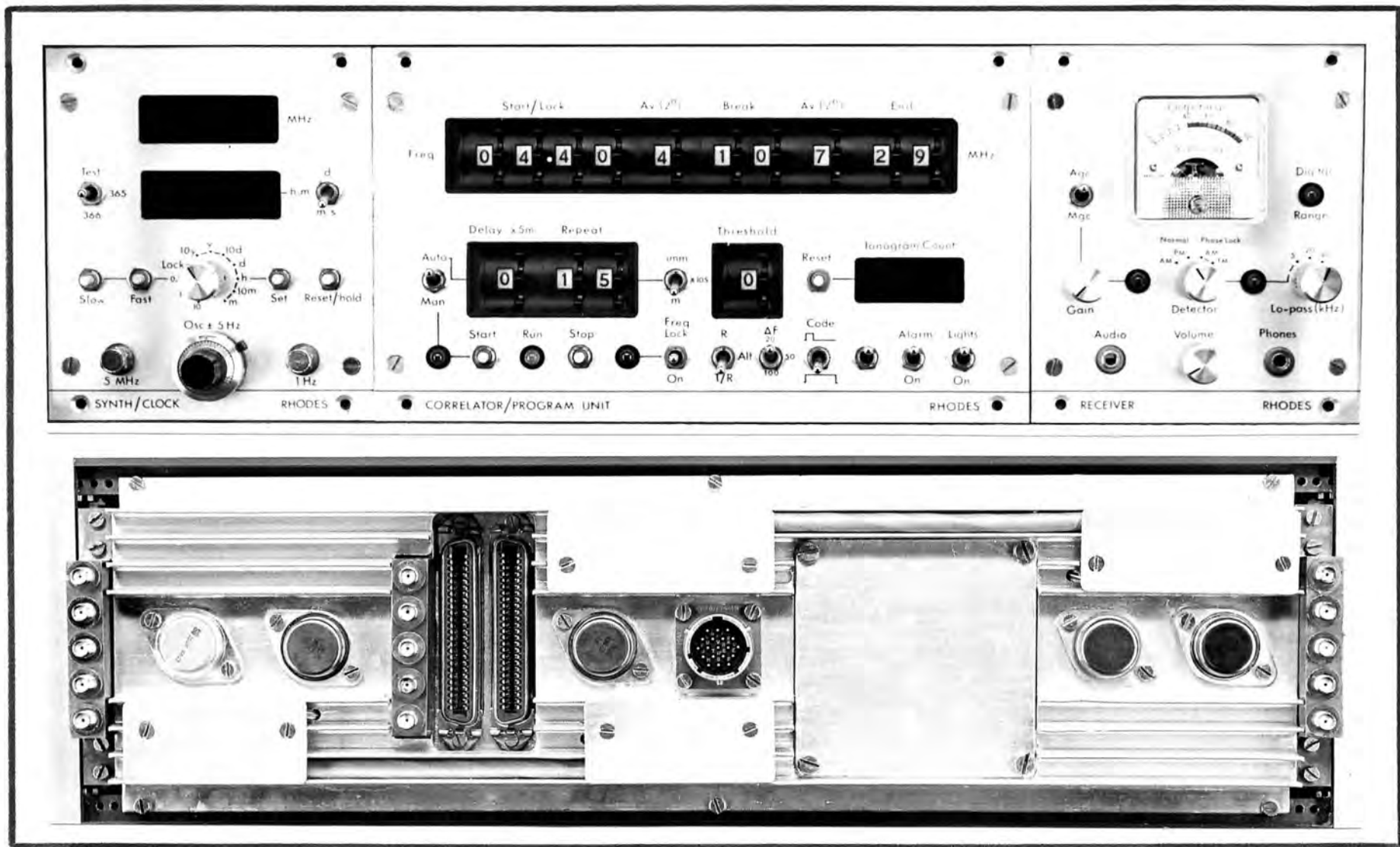


Plate 6.3 "Correlator" front and back panels.

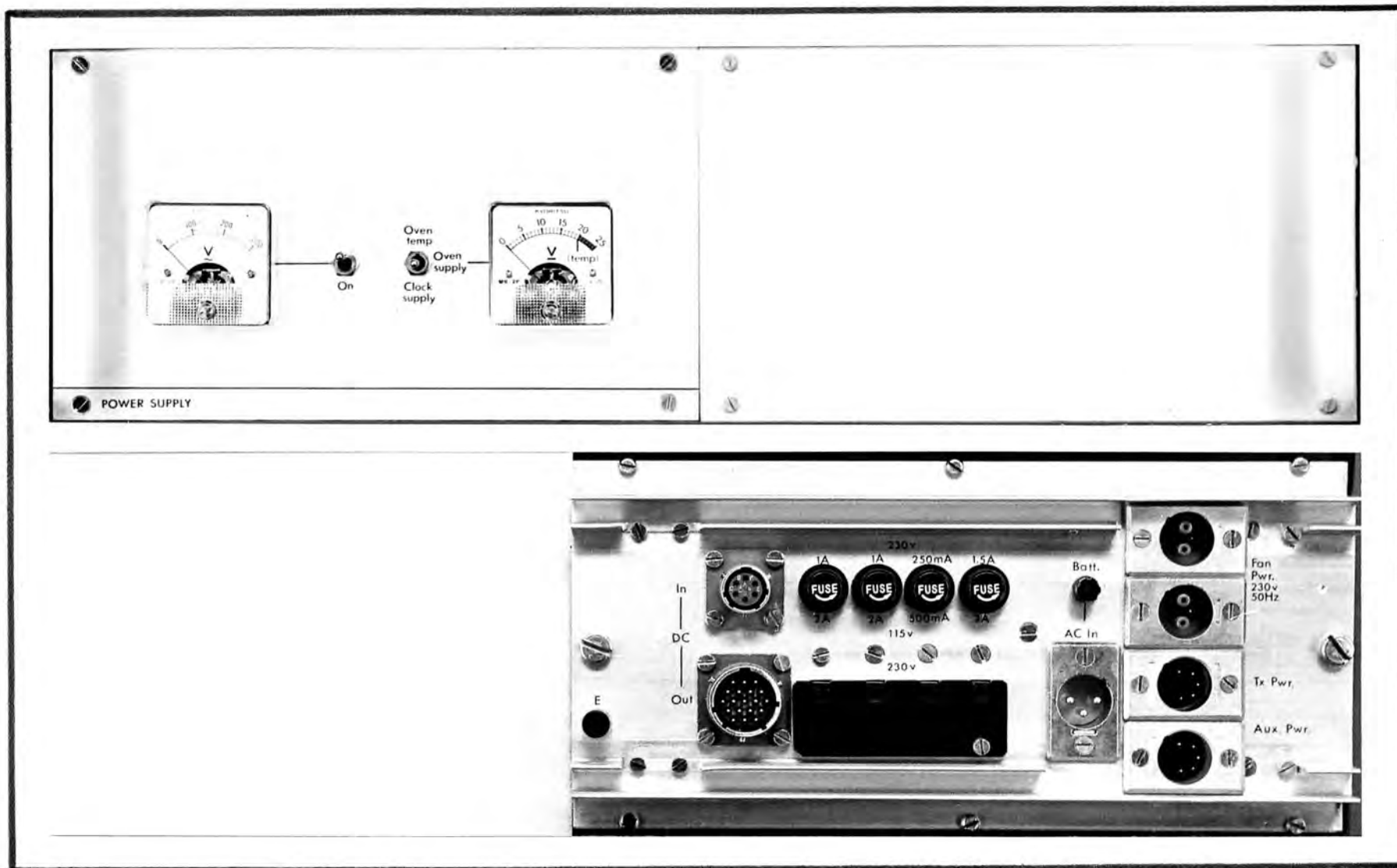


Plate 6.4 Power supply front and back panels.

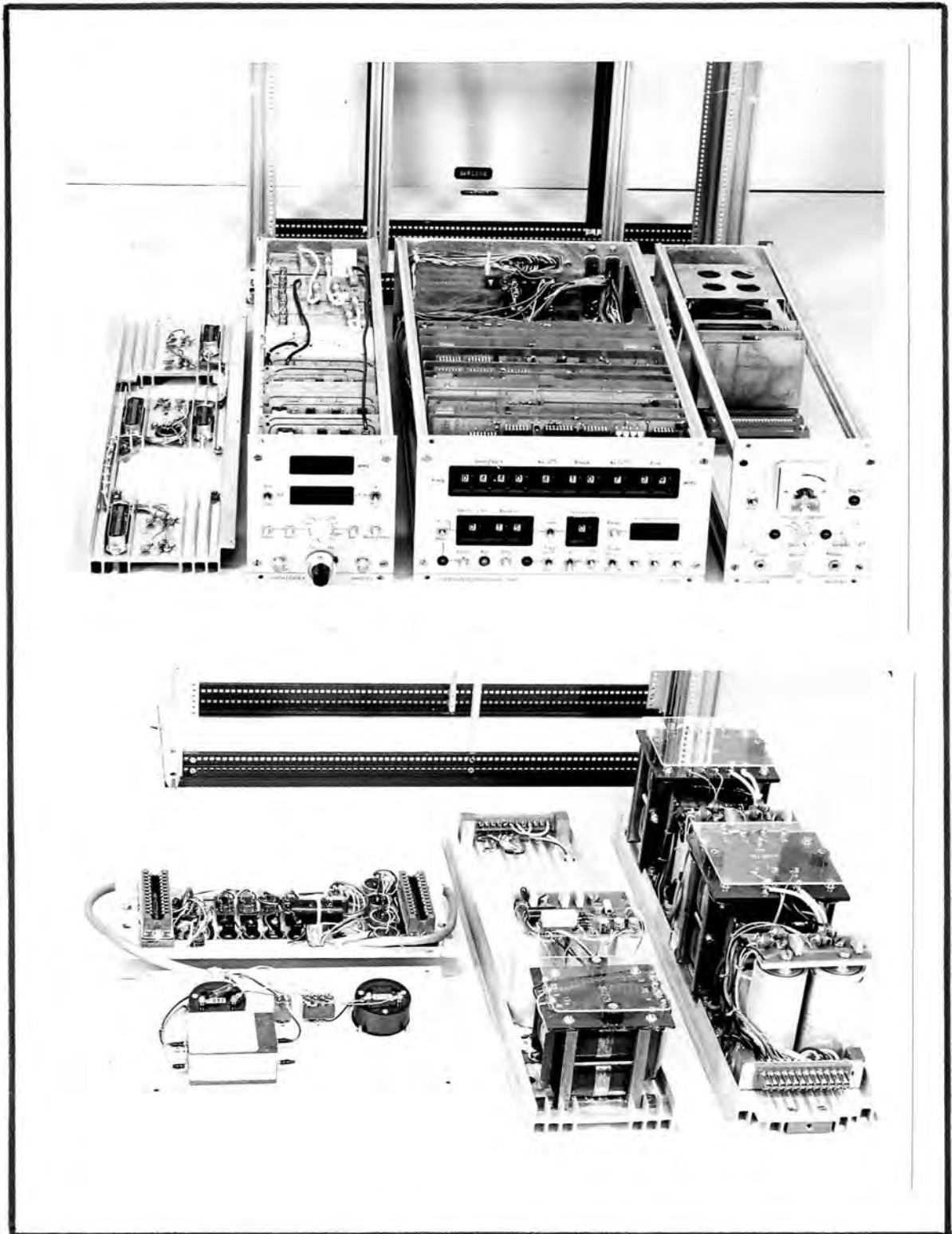


Plate 6.5 Modular construction: "correlator" rack (top),  
and power supply rack (bottom).

### 6.2 Testing the completed system.

It is suggested that in order to test the operation of the system in general, and the receiver bandwidth, averaging device and relative performances of the 15-bit and 127-bit codes in particular, at least 6 ionograms should be taken with the following codes, bandwidths (B), and numbers of averages (i):

	code	B (kHz)	i
1	M15	20	1
2	M15	20	128
3	W127	40	1
4	W127	40	128
5	W127	20	128
6	M127/W127	40	128

The d.c. removal, A.G.C., sampling rate and transmitter output power will be tested implicitly. Features which should be examined explicitly are the virtual height resolution and the effectiveness of the threshold device.

Clearly, a comparison with an appropriate Minibal or Chirp-sounder ionogram, would be instructive.

### 6.3 A suggestion for further investigation.

Although the linear feedback shift-register sequences adequately fulfil the requirements of the Poole system, it is possible that a code (or class of codes) may exist that is even better suited to such a system. One method of finding such a code is tentatively suggested here.

In essence, the search for a code suitable for implementation in the ionosonde involved setting up an equation (or writing a computer program) which described the operation of the system, putting various codes into this equation, and considering the suitability of the resulting correlation functions.

It is suggested, however, that a somewhat less empirical approach should be adopted towards the search for a suitable code; that a "system equation" should once again be written, but that appropriate restrictions should then be placed on the correlation function resulting from it, and that a code should be sought which would satisfy the equation within the limits of the restrictions placed on it.

For instance, for a hypothetical N-bit ordinary binary code C delayed by d bits, the system equation might resemble that given in Chapter 5 above, viz.

$$A(k)_d = 2\{\sum (\bar{C}_0 \cdot C_d \oplus C_k)\}^{-N},$$

where the notation has its usual meaning. The correlation function would then be restricted to having the desired properties, for example

$$\begin{aligned} A(k)_d &= \frac{N+1}{2} \quad \text{for } k = d \\ &= 0 \quad \text{for } k \neq d, \quad 0 \leq k \leq N-1. \end{aligned}$$

A solution would then be sought for this restricted equation, in the hope that it might yield a code (or recurrence relation which would generate such a code) which would completely satisfy the requirements of the system.

#### 6.4 Conclusion.

Although it is regrettable that final results were not able to be presented in this thesis, it should be noted that because of the developmental nature of the project (and the resulting sophistication and innovation of the associated electronics), more time is needed for the construction of the ionosonde than can be spared by the author.

Nevertheless it is felt that the foundations of the system have been firmly laid, and that all the prime objectives have been attained.

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## APPENDICES

- A (i) XBCO refer section 3.2.2
- (ii) ACOR refer section 3.2.2
- (iii) CACO refer section 3.2.2
- (iv) SXBC refer section 3.2.3
  
- B (i) MASK refer section 4.2.2
- (ii) PLOT refer section 4.2.2
- (iii) CONJ refer section 4.4.2
  
- C (i) SIMU refer section 5.3.1
- (ii) Data subfiles, M150 and W127

XBCO

```

C   PROGRAM CALCULATES (MINIBAL-
C   TYPE) CORRELATION FUNCTION
MASTER XBCO
DIMENSION IT(450),ISUM(300),IC(150)
READ (5,5000) M
READ (5,5001) (IC(J),J=1,M)
DO 1 J=1,M
  IT(J)=-1
  IT(J+M)=IC(J)
  IT(J+2*M)=-1
1  CONTINUE
  WRITE (6,6000) M
  DO 3 I=1,2*M
    ISUM(I)=0
    DO 2 J=1,M
      IA=IC(J)*IT(I+J)
      ISUM(I)=ISUM(I)+IA
    2  CONTINUE
  3  CONTINUE
  WRITE (6,6001) (ISUM(I),I=1,2*M)
  STOP OK
5000 FORMAT (I0)
5001 FORMAT (24I0)
6000 FORMAT (1H ,15H BITS IN CODE ;I3)
6001 FORMAT (1H ,36I3)
END
FINISH

```

## ACOR

```

C      PROGRAM CALCULATES (NON-CYCLIC)
C      AUTOCORRELATION FUNCTION
      MASTER ACOR
      DIMENSION IC(300),ISUM(150)
      READ (5,5000) M
      READ (5,5001) (IC(J),J=1,M)
      DO 2 I=1,M
      ISUM(I)=0
      DO 1 J=1,M-I+1
      IA=IC(J)*IC(I+J-1)
      ISUM(I)=ISUM(I)+IA
1      CONTINUE
2      CONTINUE
      WRITE (6,6000) M
      WRITE (6,6001) (ISUM(I),I=1,M)
      STOP OK
5000  FORMAT (I0)
5001  FORMAT (24I0)
6000  FORMAT (1H ,15H BITS IN CODE :I3)
6001  FORMAT (1H ,36I3)
      END
      FINISH

```

## CACO

```

C      PROGRAM CALCULATES (CYCLIC)
C      AUTOCORRELATION FUNCTION
      MASTER CACO
      DIMENSION IC(300),ISUM(150)
      READ (5,5000) M
      READ (5,5001) (IC(J),J=1,M)
      DO 1 J=1,M
      IC(J+M)=IC(J)
1      CONTINUE
      WRITE (6,6000) M
      DO 3 I=1,M
      ISUM(I)=0
      DO 2 J=1,M
      IA=IC(J)*IC(I+J-1)
      ISUM(I)=ISUM(I)+IA
2      CONTINUE
3      CONTINUE
      WRITE (6,6001) (ISUM(I),I=1,M)
      STOP OK
5000   FORMAT (I0)
5001   FORMAT (24I0)
6000   FORMAT (1H ,15H BITS IN CODE ;I3)
6001   FORMAT (1H ,36I3)
      END
      FINISH

```

SXBC

```

C      PROGRAM CALCULATES (SUPER=POSED)
C      CORRELATION FUNCTION.
      MASTER SXBC
      DIMENSION IX(120),IY(120),ISUM(60)
      READ (5,5000) M
      READ (5,5001) (IX(I),I=1,M)
      DO 1 I=1,M
      IX(I+M)=-1
1      CONTINUE
      DO 7 J=1,M+1
      DO 2 I=1,J
      IY(I)=-1
2      CONTINUE
      DO 3 I=1,2*M=J+1
      IY(I+J-1)=IX(I)
3      CONTINUE
      DO 4 I=1,2*M
      IY(I)=IX(I)+IY(I)
4      CONTINUE
      DO 6 I=1,M
      ISUM(I)=0
      DO 5 L=1,M
      IA=IX(L)*IY(I+L-1)
      ISUM(I)=ISUM(I)+IA
5      CONTINUE
6      CONTINUE
      N=J-1
      WRITE (6,6001) N,(ISUM(I),I=1,M)
7      CONTINUE
      STOP OK
5000  FORMAT (I0)
5001  FORMAT (24I0)
6001  FORMAT (1H ,I3,5X,36I3)
      END
      FINISH

```

MASK

```

C   PROGRAM SIMULATES POOLE-TYPE IONOSONDE OPERATION
      MASTER MASK
      DIMENSION ICODE(150),IREC(300),ISUM(150)
      READ (5,5000) M,N
      READ (5,5001) (ICODE(I),I=1,M)
      DO 1 I=1,N
        IREC(I)=ICODE(M-N+I)
        IF(ICODE(I),EQ,1) IREC(I)=1
        IREC(I+M)=IREC(I)
1     CONTINUE
      DO 2 I=N+1,M
        IREC(I)=ICODE(-N+I)
        IF(ICODE(I),EQ,1) IREC(I)=1
        IREC(I+M)=IREC(I)
2     CONTINUE
      DO 5 I=1,M
        ISUM(I)=0
        DO 3 J=1,M
          IPROD=IREC(I+J-1)*ICODE(J)
          ISUM(I)=ISUM(I)+IPROD
3     CONTINUE
5     CONTINUE
      WRITE (6,6000) M
      WRITE (6,6001) (ISUM(N),N=1,M)
      STOP OK
5000  FORMAT (2I0)
5001  FORMAT (24I0)
6000  FORMAT (1H ,15H BITS IN CODE :I3)
6001  FORMAT (1H ,24I3)
      END
      FINISH

```

## PLOT

```

C   PROGRAM SIMULATES POOLE-TYPE IONOSONDE OPERATION,
C   CALCULATES M,S,R. FOR ALL POSSIBLE DELAYS,AND
C   PLOTS A GRAPH OF M,S,R. VS. DELAY
MASTER PLOT
DIMENSION X(150),Y(150),H(4),ICODE(150),IREC(300),
*ISUM(150)
CALL AMGRAPH (6,0,0,0,4HARBC,Y,XS,YS,H,0)
9   READ (5,5000) M
    IF(M.EQ.1) GO TO 10
    READ (5,5001) (ICODE(I),I=1,M)
    DO 6 N=1,M=1
      ISIDE=-1
      IMAIN=0
      DO 1 I=1,N
        IREC(I)=ICODE(M-N+I)
        IF(ICODE(I).EQ.1) IREC(I)=-1
        IREC(I+M)=IREC(I)
1     CONTINUE
      DO 2 I=N+1,M
        IREC(I)=ICODE(-N+I)
        IF(ICODE(I).EQ.1) IREC(I)=-1
        IREC(I+M)=IREC(I)
2     CONTINUE
      DO 5 I=1,M
        ISUM(I)=0
        DO 3 J=1,M
          IPROD=IREC(I+J-1)*ICODE(J)
          ISUM(I)=ISUM(I)+IPROD
3     CONTINUE
        IF(ISUM(I).LE.ISIDE) GO TO 5
        IF(ISUM(I).LT.IMAIN) ISIDE=ISUM(I)
        IF(ISUM(I).GT.IMAIN) ISIDE=IMAIN
        IF(ISUM(I).GT.IMAIN) IMAIN=ISUM(I)
5     CONTINUE
        X(N)=FLOAT(N)
        IF(ISIDE.EQ.0) ISIDE=1
        SIDE=FLOAT(ISIDE)
        Y(N)=FLOAT(IMAIN)/ABS(SIDE)
6     CONTINUE
        M=M-1
        XS=10.0
        YS=8.0
        CALL AMAXISLABEL (5HDELAY,5HM,S,R,5,5)
        CALL AMGRAPH (6,1,M,-1,X,Y,XS,YS,15HM,S,R VS, DELAY,15)
        GO TO 9
10  CALL AMGRAPH (6,0,0,2,X,Y,XS,YS,H,0)
    STOP OK
5000 FORMAT (I0)
5001 FORMAT (24I0)
END
FINISH

```

## CONJ

```

MASTER CONJ
DIMENSION IC(127),IS(133)
READ (5,5000) M
READ (5,5001) (IC(I),I=1,M)
DO 1 J=1,M
  IC(J)=(IC(J)+1)/2
1 CONTINUE
DO 8 I=1,M=1
  DO 2 J=1,I
    IS(J)=IC(J)+IC(M+J=I)
    IF(IS(J).EQ.2) IS(J)=0
2 CONTINUE
DO 3 J=I+1,M
  IS(J)=IC(J)+IC(J=I)
  IF(IS(J).EQ.2) IS(J)=0
3 CONTINUE
DO 4 J=1,6
  IS(J+M)=IS(J)
4 CONTINUE
DO 6 K=1,M
  IX=0
  DO 5 J=1,7
    IX=IX+IS(J+K=1)
5 CONTINUE
  IF(IX.EQ.7) GO TO 7
6 CONTINUE
7 IY=K-1
  WRITE (6,6000) I,IY
8 CONTINUE
STOP OK
5000 FORMAT (I0)
5001 FORMAT (24I0)
6000 FORMAT (1H ,2I5)
END
FINISH

```

SIMU

```

C   PROGRAM SIMULATES ENTIRE IONOSONDE OPERATION,
    MASTER SIMU
    DIMENSION ID(5),ICORR(127),ICORR(508),IRET(508),
*      RET(508),REC(508),SUM(508),H(4),X(508),Y(508)
    CALL AMGRAPH (6,0,0,0,4HARBC,Y,XS,YS,H,0)
1   READ (5,5000) M,(ID(I),I=1,5),ISIG,NL,IAV,ICW
    IF(M.EQ.1) GO TO 24
    READ(5,5001) (ICORR(I),I=1,127)

C   CODE IS EXPANDED TO SIMULATE SAMPLING AT 4
C   TIMES THE BIT RATE.
    IS=4
    DO 3 I=1,127
    DO 2 J=1,IS
    ICORR(IS*I=J+1)=ICORR(I)
2   CONTINUE
3   CONTINUE

C   CODE OR C.W. SIGNAL (OR BOTH) SELECTED,
    IF(ICW.EQ.0) GO TO 5
    IF(ISIG.EQ.1) GO TO 5
    IF(ISIG.EQ.0) GO TO 9
    DO 4 I=1,508
    IRET(I)=ICW
4   CONTINUE

C   RETURNED CODE IS DELAYED BY ID(I),I=1,5 AND
C   SUPERPOSED.
5   DO 8 I=1,5
    IF(ID(I).EQ.0) GO TO 9
    DO 6 J=1,ID(I)
    IRET(J)=IRET(J)+(ICORR(508-ID(I)+J)+1)/2
6   CONTINUE
    DO 7 J=ID(I)+1,508
    IRET(J)=IRET(J)+(ICORR(J-ID(I))+1)/2
7   CONTINUE
8   CONTINUE

C   AVERAGING STARTS, IAV AVERAGES.

C   RETURNED CODE HAS PSUEDO-RANDOM NOISE ADDED
C   (NL TIMES THE SIGNAL SIZE).
9   R=0.123
    DO 15 J=1,IAV
    DO 10 I=1,508
    CALL FPMCRV(R)
    RET(I)=FLOAT(IRET(I))+NL*(2.0*R-1)
10  CONTINUE

C   RETURNED CODE IS MASKED BY CORRELATING CODE,
    DO 11 I=1,508
    IF(ICORR(I).EQ.1) RET(I)=0
11  CONTINUE

C   CODE IS "FILTERED" BY TAKING A RUNNING MEAN
C   OVER TWO BITS.

```

C (i) cont.

```
Z=(RET(508)+RET(1))/2
DO 12 I=1,507
RET(I)=(RET(I)+RET(I+1))/2
12 CONTINUE
RET(508)=Z

C DC LEVEL IS REMOVED FROM THE CODE.
DC=0
DO 13 I=1,508
DC=DC+RET(I)
13 CONTINUE
DC=DC/508
DO 14 I=1,508
RET(I)=RET(I)-DC
REC(I)=REC(I)+RET(I)
14 CONTINUE
15 CONTINUE
DO 16 I=1,508
REC(I)=REC(I)/IAV
IF(((I-1)/4)*4.NE.I-1) ICORR(I)=0
16 CONTINUE

C AVERAGING ENDS, CORRELATION BEGINS.
IF(M.EQ.127) GO TO 18
DO 17 I=61,508
ICORR(I)=0
17 CONTINUE
18 DO 22 J=1,508
SUM(J)=0
IF(J.EQ.1) GO TO 20
DO 19 I=1,J-1
SUM(J)=SUM(J)+REC(I)*ICORR(509-J+I)
19 CONTINUE
20 DO 21 I=J,508
SUM(J)=SUM(J)+REC(I)*ICORR(I-J+1)
21 CONTINUE
22 CONTINUE
DO 23 I=1,508
IRET(I)=0
IRET(I)=0
REC(I)=0
X(I)=I
Y(I)=SUM(I)
23 CONTINUE

C AUTOCORRELATION FUNCTION IS CONTAINED IN
C X(I) AND Y(I), SENT TO THE GRAPHPLOTTER.
XS=10.0
YS=8.0
CALL AMAXISLABEL (12HDELAY (OR K),4HA(K),12,4)
CALL AMGRAPH (6,1,508,-1,X,Y,XS,YS,20HCORRELATION
GO TO 1 FUNCTION,20)
24 CALL AMGRAPH (6,0,0,2,X,Y,XS,YS,H,0)
STOP OK
5000 FORMAT (10I0)
5001 FORMAT (24I0)
END
FINISH
```

W127

127 68 69 280 285 0 1 0 1 0  
1 1 1 1 1 1 =1 =1 -1 -1 =1 =1 =1 1 =1 1 =1 1 =1 1 1 =1 =1 1  
1 -1 -1 -1 1 -1 =1 =1 1 =1 1 1 =1 1 =1 =1 1 1 1 =1 =1 1 =1 =1  
=1 =1 1 =1 =1 1 =1 1 =1 =1 1 =1 =1 1 1 =1 1 1 =1 1 1 =1 =1  
=1 1 1 1 1 =1 1 =1 =1 -1 =1 =1 1 1 =1 1 =1 1 =1 =1 =1 1 1 =1  
=1 1 =1 1 1 1 =1 1 1 =1 =1 =1 =1 1 1 1 =1 1 =1 1 1 1 1 =1  
=1 1 1 1 1 1 =1

M150

150 100 200 240 0 0 1 0 1 0  
1 -1 -1 1 1 =1 1 =1 1 1 1 1 =1 =1 =1 =1 =1 =1 =1 =1 =1 =1 =1  
=1  
=1  
=1  
=1 =1 =1 =1 =1 =1 =1