AN INVESTIGATION INTO THE USE OF VISUALISATION PROCESSES AS A TEACHING STRATEGY TO ENHANCE NUMBER SENSE

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ABSTRACT

The literature agrees that in order to improve learning outcomes, instruction in classrooms should be improved first. Mathematics educationists therefore advocate that schools should make extensive and deep efforts to rethink their instructional programmes. Research furthermore suggests that increasingly, indicators on school performance and teaching reveal largely unacknowledged problematic teaching of mathematics in the great majority of South African schools. This research study is therefore a contribution towards rethinking the teaching strategies within mathematics classrooms. The study examined the use of visualisation processes in order to understand how these interact with the pedagogy of selected mathematics teachers when they teach number sense after participating in an intervention programme. This study argues that the effective use of visualisation processes enhanced the teaching of number sense.

The research study was framed as a case study that was grounded within the interpretive paradigm. The study was located in classrooms where the participating teachers promoted active learning after taking part in an intervention programme. A constructivist theoretical underpinning was therefore adopted. At the heart of the study was the Visualisation Intervention Programme (VIP), which involved seven Rhodes University Mathematics Education Project (RUMEP) teachers within the John Taolo Gaetsewe District of the Northern Cape Province. The content of the VIP was informed by initially working with five teachers of well-resourced schools within the Northern Cape who made interesting use of visualisation processes and manipulatives to teach number sense. With the assistance of these five teachers, the VIP was then implemented by seven selected RUMEP teachers to investigate the role of visualisation processes in the teaching of number sense understandings. The study employed a mixed method approach. Qualitative data was collected through observations and interviews, while quantitative data was collected with a series of pre- and post-tests.

The analysis of the findings of this research study revealed that the effective use of visualisation processes was instrumental in enhancing the teaching of number sense understandings. Furthermore, the use of visualisation processes by the selected teachers fostered independent thought and conceptual understanding of number sense topics on the part of their learners.

DEDICATION

This thesis is dedicated with honour and worship to Elohim: Abba Yahweh, Yeshua Messiah and Ruach Hakodesh.

I also dedicate this thesis to the loving memory of my late father, Lambert Jacobus Albertus Griqua, and my late mother, Francina Griqua. I am reminded of my father's instruction to do everything in the Name of Elohim. I remember with fondness the tender look in my mother's eyes when I told her that I was going to write a book one day…

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I honour and appreciate the five teachers who collaborated with me in crafting the Visualisation Intervention Programme (VIP).

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The special place that my three daughters and five grandchildren occupy in my life, served as encouragement to complete this study and set a benchmark for their future academic endeavours.

DECLARATION OF ORIGINALITY

I, Ronald Max Griqua (Student Number: 17G9354), declare that this thesis: An investigation into the use of visualisation processes as a teaching strategy to enhance number sense, is my own work and written in my own words. Where I have drawn upon the words or ideas of others, I have acknowledged the authors by using the reference practices as set out by Rhodes University Educational Guide to referencing.

29 November 2019

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Ronald Max Griqua (Signature) Date

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CHAPTER 1

INTRODUCTION AND CONTEXT OF THE STUDY

1.1 INTRODUCTION

This research study is part of the VIPROmaths project at Rhodes University which seeks to research the effective use of visualisation processes in the mathematics classroom in Southern Africa. The specific aim of my study is to investigate selected Grade 6 teachers' use of visualisation processes as a teaching strategy to enhance the number sense of their learners.

This chapter provides an introduction to the study and explains why research of this nature was undertaken. The chapter begins with providing the context within which the study was undertaken, coupled with background details. The research goals and questions are then provided. A brief summary of the research methodology employed is included, followed by a discussion about the significance of the study. Finally, an overview of the chapters is provided to explain the structure of the thesis.

1.2 CONTEXT AND BACKGROUND

This research study took place within my own work space. I am a staff member of RUMEP (Rhodes University Mathematics Education Project). RUMEP is an autonomous unit within the Faculty of Education at Rhodes University which is responsible for the delivery of the BEd (in-service) degree in Mathematics Education. The teachers who participate in RUMEP courses mostly reside in the rural areas of the Northern and Eastern Cape provinces of the Republic of South Africa.

RUMEP's in-service programme is largely informed by social constructivism as a theory of learning (Stoker, 2003). Its various cohorts of teachers are sensitised to value the errors and misconceptions of learners and use this as a vehicle to address possible learning difficulties by providing classroom support. In light of this, the report of the Department of Education about the poor numeracy skills of Grade 6 learners has aroused my interest to explore alternative teaching strategies which could enhance the number sense of learners.

According to the Department of Basic Education of the Republic of South Africa [DBE RSA] (2014), the Annual National Assessments (ANA) tests which were written in South Africa from 2012 to 2014,have flagged the poor numeracy skills of Grade 6 learners nationally. The average ANA percentage marks of Grade 6 learners in the Northern Cape Province, from 2012 to 2014, are sadly below the acceptable target of fifty percent. For the John Taolo Gaetsewe district, where I work with the RUMEP teachers, the average percentage marks of Grade 6 learners was thirty-three-and-a-half percent, which is a real cause for concern.

By virtue of my responsibilities at RUMEP, I work with many in-service teachers and their learners. As a result, I come across teachers who do use interesting visualisation processes in their teaching of mathematics. In this study, I wanted to interrogate the visual teaching of number sense understandings by working collaboratively with some of the teachers who employ visualisation processes as a teaching strategy and involving them in designing an intervention programme for teachers who do not use a visual approach in their teaching. This collaboration effort produced the Visualisation Intervention Programme (VIP) which was then implemented in seven classrooms of Grade 6 teachers who are part of the RUMEP cohort of 2018 to 2020.

Integral to the VIP were a set of four lessons that each participant taught, which focused on the nature of numbers, addition and subtraction, multiplication and division as well as number sense word problems. The VIP *inter alia* aimed at enabling learners to understand numbers and operations. Consequently, the ultimate outcome of the VIP was for learners to acquire the appropriate skills to apply their knowledge of numbers and operations within computational settings like number sense word problems.

In order to measure whether the implementation of the four VIP lessons of the seven RUMEP teachers who took part in the research led to discernable change in the performance of learners regarding number sense understandings, a pre- and post-test on number sense were administered to the learners of the participating teachers. The test was sourced from the Number Sense Item Bank designed by McIntosh et al. (1997). It consisted of fifty questions which were divided into four sections, namely whole numbers, fractions, decimals and percentages.

1.3 RESEARCH GOALS AND QUESTIONS

The overall goal of this study was to investigate the role which visualisation played when employed as a teaching strategy to enhance number sense as a result of participating in the Visualisation Intervention Programme (VIP).

The objective of the research was to investigate the role of visualisation processes as a teaching strategy to enhance number sense.

The following were sub-objectives of the research:

- To design and craft a Visualisation Intervention Programme (VIP) for implementation in the classroomsof the seven RUMEP teachers who took part in the research, by collaborating with teachers from well-resourced schools (Appendix A),
- To set a number sense test which could serve as both a pre- and post-test (Appendix B),
- To develop an analytical framework for considering visual teaching strategies for number sense (adapted from McIntosh et al., 1992) (Appendix C),
- To draft a working document for observing the lessons of the seven RUMEP teachers who took part in the research (Appendix D) and
- To plan and prepare a provisional semi-structured interview schedule for the video-stimulated recall interviews with the seven RUMEP teachers who took part in the research (Appendix E).

This study asked the following research question:

How can teachers use visualisation processes in a mathematics classroom in order to enhance their teaching of number sense as a result of participating in the VIP?

1.4 RESEARCH METHODOLOGY

This research study was designed as a case study and had two parts to it – one was qualitative and the other, quantitative in nature. The qualitative part involved working with seven selected RUMEP teachers in designing and implementing a set of number sense lessons using visualisation processes. The quantitative part involved the administering of pre- and post-tests to learners to determine changes in number sense performance.

The following research instruments were used:

- A number sense test (Appendix B)
- An observation schedule to critique the lessonsof the seven RUMEP teachers who took part in the research (Appendix D)
- A provisional semi-structured interview schedule for the video-stimulated recall interviews with the seven RUMEP teachers who took part in the research (Appendix E).

1.5 SIGNIFICANCE OF THE STUDY

The rationale for undertaking research on how to enhance the teaching of number sense, was found in the poor ANA results regarding the numeracy skills of Intermediate Phase learners in the John Taolo Gaetsewe district where I facilitate the Rhodes University Mathematics Education Project (RUMEP).

The study was furthermore inspired by my experiences with the RUMEP cohort and their learners with whom I was working. The fact that number sense forms the foundation for higher order mathematics heightened my interest in investigating the role of visualisation processes as a teaching strategy to enhance the number sense of Grade 6 learners.

RUMEP was established in 1993 as a teacher development institute. Its mission is to develop the mathematics teaching and learning skills of primary school teachers (Penlington, 2005). I have been working at RUMEP since 2016 with a particular focus on facilitating the project in the Northern Cape. The empirical field of this study was thus the RUMEP initiative in that province.

Kilpatrick (1988) cited mathematical instruction and the nature of mathematical knowledge as two important themes in mathematics education research. This research study was specifically interested in teacher practice as it could contribute to the broader narrative around the nature of the teaching and learning of number sense in mathematics (McIntosh et al., 1997). Teacher education and the learning processes of learners are important elements and factors to explore, if one wants to contribute to quality education in South Africa.

The role that South African teachers play is firmly rooted in the implementation of the Curriculum and Assessment Policy Statement (CAPS) document. For this reason, my research study is framed by the CAPS document. The CAPS document is the product of efforts to address the South African learners' poor levels of fluency and numeracy in mathematics. However, certain challenges exist regarding the implementation of CAPS, from the perspective of the visual teaching of number sense understandings. This study seeks to identify these challenges and offer an alternative within the context of the teaching and learning of number sense understandings.

In researching the use of visualisation processes to enhance the teaching of number sense, the directives for curriculum implementation within the CAPS document are therefore studied and described within the context of its advocacy for the teaching of number sense understandings. The study wishes to establish the effectiveness of the implementation of the CAPS document in relation to the development of number sense understandings.

This research study furthermore serves the purpose of contributing to the reflective practice process of RUMEP as the participating schools are all integral to its broader development agenda. The research study therefore investigated instructional techniques and strategies used by experienced teachers who were then involved in running the Visualisation Intervention Programme (VIP) for seven RUMEP teachers to implement some of these techniques. The level of learner performance by participating learners with regard to number sense as a result of the VIP was also determined through the number sense pre- and post-tests.

1.6 OVERVIEW OF THE CHAPTERS

This thesis is organised into five chapters. Chapter 1 provides an overview of the thesis. In this chapter, the context of the study coupled with the background to it, are first discussed. The research goals and questions are then set out, followed by the research methodology. The significance of the study is discussed to highlight its benefits for the teaching and learning of number sense understandings, given the poor numeracy skills of our South African learners that were recorded in the ANA results. The chapter concludes with an overview of the whole thesis structure.

Chapter 2 begins with a discussion of the literature pertaining to visualisation in mathematics education and a critique of visualisation teaching strategies. The Visualisation Intervention Programme (VIP) is then discussed as a teaching strategy. The number sense framework, the teaching of number sense, the relationship between number sense and the Curriculum and Assessment Policy Statement (CAPS) document as well as the relationship between number sense and visualisation are then discussed. Furthermore, the principles underlying constructivism and social constructivism are discussed. The relationship between social constructivism and the VIP then receives attention. Finally, a critique of social constructivism is offered.

Chapter 3 deals with the research methodology that was employed. The research orientation is first described, followed by the research question. The research methods are then communicated. Afterwards, the research design where the different phases are set out is discussed in detail. The data collection and analysis procedures are explained in detail. The ethical considerations and validity of the research study are finally presented to conclude this chapter.

Chapter 4 is primarily dedicated to the qualitative analysis of the implementation of the VIP in the classrooms of the seven selected RUMEP teachers who formed part of the study, as well as the quantitative analysis of the number sense test administered to the learners who formed part of the study. The focus group interview results are first reflected upon. Then an in-depth discussion follows on the views and responses of the seven participating teachers. The chapter concludes with an analysis of the pre- and post-test results.

Chapter 5 relates a comprehensive summary of the findings. The deliberations of the focus group interview, reflections on the observed lessons and a discussion of the pre- and post-test results are part of this summary. This is followed by a discussion on the significance of the research study. Recommendations based on the research findings are then made. The chapter concludes with suggestions for further research into the role of visualisation processes in the teaching of number sense, as well as some personal reflections.

1.7 CONCLUSION

This chapter provides the reader with an overview of my research study. I looked at the context and background of the study, the research goals, questions and methodology, its significance and an overview of all the chapters. The next chapter introduces the literature reviewed and key concepts underpinning my study about the use of visualisation processes to enhance the teaching of number sense.

CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

This study was undertaken against a backdrop of poor numeracy skills displayed by South African Grade 6 learners in the ANA results from 2012 to 2014 [DBE RSA] (2014).

The average percentage marks of Grade 6 learners in the Northern Cape Province for the ANA Mathematics tests in 2012, 2013 and 2014 are shown below in Table 2.1 (DBE RSA, 2014):

Table 2.1: The average percentage marks of Grade 6 learners in the Northern Cape in the 2012, 2013 and 2014 ANA Mathematics tests

Province	**2012**	**2013**	**2014**
Northern Cape	23.8%	35.6%	39.3%

Although Table 2.1 indicates an improvement in the learner performance over the three consecutive years, the average provincial scores remain below the acceptable achievement level of fifty percent or more (DBE RSA, 2014).

Table 2.2 shows that the average percentage marks for Grade 6 learners in the ANA Mathematics tests for districts in the Northern Cape, in the 2013 and 2014 ANA Mathematics tests, are the lowest for the John Taolo Gaetsewe District, where the experimental schools of this research study are located (DBE RSA, 2014):

Table 2.2: The average percentage marks for Grade 6 learners in the 2013 and 2014 ANA Mathematics tests for districts in the Northern Cape

Year	**John Taolo Gaetsewe**	**Frances Baard**	**Namakwa**	**Pixley ka Seme**	**ZF Mgcawu**
2013	31.2%	38.3%	42.3%	32.2%	36.1%
2014	35.8%	43.5%	42.9%	35.8%	37.4%

The ANA Mathematics tests were designed to ascertain the numeracy skill levels of South African learners. From Tables 2.1 and 2.2 it is clear that the numeracy skill levels of South African Grade 6 learners should be improved. Hence this investigation into the role of visualisation processes as a teaching strategy to enhance the number sense of learners.

Barber and Mourshed (2007) see quality instructional practices as the most important factor when it comes to the improvement of learner performance. In order for quality instruction to be delivered, teachers need to have acquired the appropriate skills and teaching repertoire towards this end. The quality of the schooling system is thus directly related to the quality of teachers who operate within it.

According to Barber and Mourshed (2007), the quality of an education system cannot exceed the quality of its teachers. They furthermore regard teacher quality as the most important variable for improving learner outcomes. An interesting finding of their research is that improving teacher quality can have a substantial impact on learner performance within a short time frame. It is therefore of vital importance that teachers should be held accountable for the quality of their instruction.

The research study done by Li, Ni, Li and Tsoi (2012) examined the influence of curriculum reform on teachers' perceived instructional practices. The research findings suggest a positive correlation between the professional development of teachers and their instructional practices. Some elements of the instructional practices influenced and shaped by the professional development of the teachers using the reformed-based curriculum, included self-learning activities for learners, instruction based on problem-solving, facilitating the connection of mathematics with real-life applications and introducing multiple assessment methods.

The research results of Li et al. (2012) indicated that teachers, who were using the conventional curriculum as opposed to the reformed-based curriculum employed in the research study, were still stuck in giving drills and exercises to their learners. Teaching within the confines of the conventional curriculum clearly constituted an instructional approach with a poor learning outcome.

Taylor (2008) concurs that in order to improve learning outcomes, instruction in classrooms should be improved first. This places great emphasis on the classroom practice of the teacher.

Taylor (2008:21) believes that "improving what teachers do in their classrooms is the key to improved learning". Taylor (2008) therefore regards the setting up of effective curriculum management systems and the improvement of instruction in classrooms as two instructional tasks upon which improved learning outcomes depend.

Interesting research was conducted by Yang (2005) to investigate whether mathematical diary writing could help learners to develop number sense. Learners were instructed to keep mathematical diaries and make public and private entries after each number sense class. The mathematical diary entries proved to help learners with making sense of fractions and also to "help them reflect upon their thinking and ideas about fractions" (Yang, 2005, p. 11).

The diary entry of one Grade 6 learner was clothed in rich descriptive language. The learner vividly described how he decided which of two fractions were the largest. He explained his decision-making process by drawing two identical circles to represent cakes of equal size, cutting it in the number of pieces as determined by the denominators of the two fractions and comparing the two fractions (Yang, 2005). Giving learners an opportunity to summarise their thinking in written form is one of many ways in which instructional practices could be improved for the enhancement of number sense (Reys, 1994).

Elmore and Fuhrman (2001) argue that the improvement of instructional practice is one of the factors that improve school performance. Other factors like past performance of a school are less important than a targeted focus on improving the nature of teaching. Not even the demographic characteristics of a school, i.e. the location and size of a school, the size of classes and the economic composition of the student population within the school are as important as the improvement of instructional practice. Elmore and Fuhrman (2001) further lament the fact that few of the American schools whose internal accountability systems they have studied, "appear to be making extensive or deep efforts to rethink their instructional programs" (Elmore & Fuhrman, 2001, p. 70). Researching mathematics outcomes within South African schools, McCarthy and Oliphant (2013, p. 3) emphatically state that "mounting indicators on school performance and teaching reveal largely unacknowledged poor teaching of mathematics in the great majority of schools". This study is therefore a contribution towards rethinking the teaching strategies within the Grade 6 mathematics classrooms.

Kroesbergen and van Dijk (2015) found that visual-spatial working memory as well as number sense of learners are related to mathematics performance. Kroesbergen and van Dijk (2015, p. 102) define working memory as "the ability to temporarily store and manipulate information necessary for complex cognitive tasks". Visual-spatial working memory therefore entails the ability to temporarily store and manipulate, inter alia, two- or three-dimensional figures. Kroesbergen and van Dijk (2015) tested whether learners with a specific weakness in visual-spatial working memory and/or number sense scored lower on mathematics fluency and problem-solving assessments than learners without such a weakness. The outcome of their test revealed that "children with a weakness in either visual-spatial working memory or number sense have lower mathematical ability than children without such a weakness and children with a weakness in both visual-spatial working memory and number sense have lower mathematical ability than children with a weakness in either of them" (Kroesbergen & van Dijk, 2015, p. 106).

This research is an indication of where teachers should focus in designing classroom strategies to improve learner performance as it has conclusively shown that number sense and visual-spatial working memory "are important predictors of mathematical ability in general, and of mathematical learning disabilities in specific" (Kroesbergen & van Dijk, 2015, p. 107).

My research study focuses on the visual teaching of number sense as a possible means of improving the performance of learners. In order to address the poor numeracy skills of Grade 6 learners, innovative teaching strategies should be implemented. In this chapter, I discuss the role of visualisation as an innovative strategy to teach number sense. At first, an overview of visualisation is given. The discussion is then funnelled down to visualisation in mathematics education and concludes with a critique of visual teaching strategies. The Visualisation Intervention Programme (VIP) is then discussed in order to explain the implementation of a visual teaching strategy intervention programme for the enhancement of number sense. The VIP is at the heart of this case study.

Furthermore, a number sense framework that forms the basis of my analytical framework is discussed. This framework was used to investigate the role that visualisation played as a teaching strategy in the teaching of selected teachers. The chapter concludes with a discussion and critique of social constructivism, coupled with a focus on the conceptual framework of teaching proficiency as advocated by Kilpatrick et al. (2001). The role of visualisation as a teaching strategy for number sense is the central theme throughout the chapter.

2.2 VISUALISATION

2.2.1 Introduction

Arcavi (2003, p. 215) stated that “vision is central to our biological and socio-cultural being”. The faculty of human vision is an asset for any individual as it serves to inform him or her about the world he or she lives in. The idea of seeing with more than human eyes was contemplated by Jackson (2002) in describing how the ability to visualise extends beyond blindness. In this regard, Rösken and Rolka (2006) citing Draaisma (2000) related how the blind Euler produced more than 355 papers as a mathematician making use of his visual imagination and outstanding memory.

According to Nemirovsky and Noble (1997), researchers who argue for the use of visualisation in mathematics refer to the work of past and contemporary mathematicians who have made use of visual representations in their mathematising. Many researchers like Zimmermann and Cunningham (1991), Davis and Anderson (1979) and Presmeg (1989) regard visualisation and visual reasoning as being important for learning mathematics. Nemirovsky and Noble (1997, p. 100) believes that visual thinking is “a resource that can open the way to different ways of thinking about mathematics than the linguistic and logico-propositional thinking of traditional proofs and the symbol manipulation of traditional algebra”.

The definition of Zazkis, Dubinsky and Dautermann (1996, p. 441) assists us by providing us with this overarching view of visualisation:

“Visualisation is an act in which an individual establishes a strong connection between an internal construct and something to which access is gained through the senses. Such a connection can be made in either of two directions. An act of visualization may consist of any mental construction of objects or processes that an individual associates with objects or events perceived by her or him as external. Alternatively, an act of visualization may consist of the construction, on some external medium such as paper, chalkboard, or computer screen, of objects or events which the individual identifies with object(s) or process(es) in her or his mind.”

Research within the field of the cognitive neurosciences which investigates mental imagery for spatial relationships is useful across a range of disciplines, including the domain of mathematics.

Thompson, Slotnick, Burrage and Kosslyn (2009, p. 1252) tested participants on spatial location and spatial transformation while monitoring brain activity and found that there is a "clear dissociation between two types of spatial imagery, and that a small, distinct set of brain areas is specific to each". This interesting neuroimaging experiment yielded results which proved a distinction in brain activity for the ability to locate real or virtual objects and the ability to manipulate or transform mental images. In turn, the researchers discovered that two different, broad types of spatial imagery exist. The two types of spatial imagery are constituted by the location of an object and the spatial relations and transformation of such an object.

The research done by Thompson et al. (2009) therefore suggests that visualising spatial location and mentally transforming spatial location depend on distinct neural networks, hence the activation of distinct brain areas when mental processing of location and transformation takes place. These findings have repercussions for designing instructional material for the visual teaching of mathematical concepts related to spatial location and transformation. The findings furthermore resonate with research done on identifying subcomponents of mental imagery, including those brain functions which underlie the processing of spatial relations (Thompson et al., 2009).

Against this background, I wish to focus on visualisation processes in the context of mathematics.

2.2.2 What is visualisation in mathematics?

To this day there is no seminal definition of what visualisation in mathematics entails. The literature points to different viewpoints which differ remarkably (van Garderen, 2006). On the one hand, the use of visualisation is seen as a powerful **process** for solving problems. On the other, visualisation refers to **visual imagery** which is seen to play an important role in clarifying the meaning of a problem, assisting in the choice between different problem-solving approaches and influencing the cognitive constructions of the one who employs such imagery (Owens & Clements, 1998). An important view of visualisation in mathematics education is the notion that it is a **mathematical process**, that is, it is used to reason in mathematics and to think logically (Presmeg, 1997).

According to van Garderen (2006, p. 496) there are also researchers who assert that after "examining the relationship between visualization and mathematical problem-solving performance, have found either a weak relationship or no relationship" in using visualisations for solving mathematical problems. An explanation for these inconsistent research findings is offered by van Garderen (2006) who believes that various definitions of visual imagery are used by different researchers. For example, Clements (1982, p. 36) recommends that mathematics educators embrace "simple, picture-in-the-mind notions of imagery". Presmeg (1986, p. 297), on the other hand, defines a visual image as "a mental scheme depicting visual or spatial information".

Fennema (1979) contends that spatial thinking is a prerequisite for solving all mathematical tasks. Research confirms the position that spatial ability positively correlates with measures of performance in mathematical tasks, especially in geometry and when solving complex problems (Battista, 1990). However, the strength of the relationship between spatial ability and visualisation has sparked wide debate (Lean & Clements, 1981; Krutetskii, 1976). According to van Garderen (2006), the manner in which spatial ability is defined and assessed may be a possible reason for the debate. However, in spite of the debate, van Garderen (2006, p. 496) asserts that "definitions of spatial factors, such as spatial visualization, appear to make use of visual imagery".

Zimmermann and Cunningham (1991) citing Davis and Anderson (1979), observe that the geometric and kinesthetic elements of mathematical instruction over the past century had to give way to the verbal, symbolic and analytical elements thereof. However, "the pendulum has recently begun to swing back to a more balanced view of mathematics which takes into account more fully the visual and intuitive dimensions" (Zimmermann & Cunningham, 1991, p. 2). Steen (1988, p. 616) believes that mathematics has developed into "the science of patterns" with the advent of computers and mathematicians will naturally try to visualise these patterns. Zimmermann and Cunningham (1991) assert that we first use mental images to understand and make sense of mathematical concepts and constructs before we solve mathematical problems.

For the purpose of this research project, Arcavi's (2003) definition of visualisation will underpin this work because it emphasizes both the process and the product of using visualisations in teaching. According to Arcavi (2003, p. 217), "visualization is the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds, on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understandings".

It is therefore important to narrow our focus to the teaching and learning of mathematics to fully appropriate Arcavi's (2003) view of the visualisation processes at work within the field of mathematics education.

2.2.3 Visualisation in mathematics education

Visualisation is recognised as a central issue in mathematics education. It helps us to understand how people make sense of mathematics. The call of Presmeg (2006, p. 233) to undertake extensive research in order to fully understand "how visualisation interacts with the didactics of mathematics" cannot be more relevant for my study. Given the challenges of our South African mathematics classrooms, visualisation as a tool for mathematics teaching and learning could be complementary to the analytical and verbal mode of lesson preparation and presentation that we are so accustomed to. Presmeg (2006) calls for research into an effective pedagogy which could enhance the use and power of visualisation in mathematics education. Hence, my study into the role that visualisation processes may play in enhancing the teaching of number sense <u>in particular</u>.

As indicated above, research has found a positive correlation between spatial ability and success in mathematics education. However, the wide use of visual images does not necessarily lead to effective problem-solving in mathematics (Lean & Clements, 1981). To investigate this phenomenon, research by Hegarty and Kozhevnikov (1999) clarified the relationship between visual imagery, spatial ability and mathematical problem-solving. When researching the mathematical performance of sixth-graders solving word problems, they found that there was a strong correlation between the mathematical proficiency of a student and the quality of the visual imagery such a student employs.

Hegarty and Kozhevnikov (1999, p. 684) stated that "visual imagery refers to the ability to form mental representations of the appearance of objects and to manipulate these representations in the mind". They drew a distinction between pictorial imagery and schematic imagery. Pictorial imagery is seen as "images that encode the visual appearance of objects or persons described" and schematic imagery as "images that encode the spatial relations described in a problem" (van Garderen, 2006, p. 497).

Schematic imagery is regarded as having a higher quality than pictorial imagery as it can be used for mathematical reasoning by learners. They create spatial images which can be manipulated in order to solve practical and theoretical problems in mathematics. It was found that gifted learners employed schematic imagery, while the average learners and those with learning disabilities employed pictorial imagery when solving mathematical word problems (van Garderen, 2006).

The research of Hegarty and Kozhevnikov (1999) has also confirmed schematic visual-spatial representations as being positively related to mathematical problem-solving, whereas pictorial representations are not necessarily related to success in mathematical problem-solving. Their research was instrumental in clarifying the reasons why previous research did not find any relationship between the use of visual-spatial representations and mathematical problem-solving. Hegarty and Kozhevnikov (1999) furthermore believe that in the light of the two distinct types of visual imagery, the categorization of learners being either visualisers or verbalisers is far too general. The visualisers should also be categorized as either schematic types or pictorial types.

Arcavi (2003) suggested a complementary role for visual thinking. He indicated that the visual solution of a mathematical problem may inform and support the symbolic solution attained by mathematical conventions. Visualisation provides an opportunity to more rigorously engage with the mathematical concept and tease out the mathematical meaning(s) inherent to the problem being solved. This complementary role of visualisation was echoed by Gorgorió et al. (1996) in their research into the interaction between geometry and visual processing. Their finding was that mental images play an important role in geometrical thought, which is constituted by the interaction between external representations and geometrical knowledge.

Gorgorió et al. (1996) outlined a process for visualisation as it unfolds in the use of a dynamic geometry package, called *Cabri-Géomètre*. They emphasized the complementary role of the visualisation process in the study of geometry. They regarded the visualisation process as unfolding progressively in three different steps, namely:

- Crude visualisation: Gorgorió et al. (1996, p. 2) asserted that "crude visualisation is seeing a diagram and being able to interpret its technical rules or constraints". During this step of the process, visualisation processes refer to the learner's understanding of how to employ the technical aspects of the geometry software package when manipulating a diagram. The learner is able "to interpret Cabri's technical rules to produce a drawing but this drawing does not embody the geometrical relationship present" (Gorgorió et al., 1996, p. 3).

- Visualisation as the reading of visual information: During this step, the learner is able to interpret the geometrical relationships present in the diagram under study.
- Visual processing: The final step involves the ability to mentally manipulate and transform visual representations and imagery which were created by exploring the diagram within the geometry software package.

This three-step visualisation process outlined above, illustrated "how the use of a dynamic geometry package such as *Cabri-Géomètre* both needs and contributes to developing visualisation in all these three senses" (Gorgorió et al., 1996, p. 1). The call to use technological tools is strengthened in the research of Gorgorió et al. (1996). The findings were made that *Cabri-Géomètre* mediates the problem-solving process and assists in developing mathematical visualisation. It was furthermore found that *Cabri-Géomètre* facilitates the visualisation process. It also assisted the learners in expanding their geometric knowledge.

The use of computer-based packages such as *Cabri-Géomètre*, *GeoGebra* and *Geometer Sketchpad* are well entrenched in many mathematics classroom the world over, including South Africa (Hölzl & Schäfer, 2013). Hölzl and Schäfer (2013, p. 45) echo the call to use technological tools as they advocate for "a more learner-centred, individualized style of teaching that incorporates conceptual understanding and an appropriate pedagogy suited to the use of interactive software such as *GeoGebra*".
Arcavi (2003, p. 236) observed that, "…many teachers may feel that analytic representations … seem to be more pedagogically appropriate and efficient". This implies that visualisation processes could be regarded to be not as efficient as the traditional algorithmic processes when doing pure mathematics. My research project takes the position that visual thinking should be employed as a support mechanism for the algorithmic or symbolic approach to solving mathematical problems. This position acknowledges the positive role which computer software packages can play in the teaching and learning of mathematics. In my experience, the synergies that education and computer technology create should be exploited more fully in the South African context.

Arcavi (2003) promoted the use of visualisations and highlighted the value of using visualisations in the mathematics classroom. He concurred that visual tools, like diagrams, are valuable for teaching as they support a discovery approach to problem-solving. He realized that proof of mathematical theorems is still largely done by conventional methods, and that the mathematics community as a whole still do not "…value visualisation as an integral part of doing mathematics" (Arcavi, 2003, p. 235). This reluctance to include visual reasoning as valid mathematical practice inspired me to research more deeply how the use of visualisation processes can indeed enhance the teaching of mathematics.

It is my assumption that mathematics teachers should be aware of the potential value of using visualisation processes in their teaching on the one hand, but also remain true to the rigours of accepted mathematical procedures on the other. Teachers have an important role to play in finding interesting ways of using visualisation in conjunction with algorithmic solutions to solve a mathematical task.

In my own practice I emphasize the importance of visualisation – particularly when teaching word problems. In my experience, when learners are presented with a word problem, many seem to be uncertain of how to approach the task. Many find it difficult to interpret the word problem situation in both numerical and symbolic terms. The connection that should be made between the narrative and the application of mathematics seems to get lost. It is often only when I start to facilitate the process by slowly reading through the word problem statement with my learners and scaffolding the content, that learners start to make the relevant connections. Integral to my scaffolding is guiding the learners to form a picture of the context. I find that this enables the learners to assign variables to the word problem to either create an algebraic equation or re-write the gist of the word problem.

Presmeg (2006) is very helpful in this regard. She suggested that it is beneficial to the learners to make connections between diagrams or imagery and symbolic representations of the word problem. It is important for teachers to facilitate the process of making sense of the algebraic context of the word problem and its associated visual imagery. Presmeg (2006) concurred with Stylianou (2001) that learnersof mathematics perceive visualisation as a useful tool and regularly attempt to use it in problem-solving. However, these learners may also encounter difficulties when applying this skill due to a lack of training in visual processing. Stylianou (2001) aptly suggests that mathematics teachers need to be able to help their visualising learners to exploit the strengths of visualisation strategically and appropriately.

2.2.4 Critique of visual teaching strategies

Arcavi (2003) holds that visualisation as a tool in learning and doing mathematics has become widely acknowledged. However, the literature cautions that the topic of visualisation and its use as an alternative to the formal, verbal, symbolic and algorithmic approach to mathematics has its challenges. Arcavi (2003) cited cultural, cognitive and sociological reasons for these difficulties which constitute a critique of visual teaching strategies for mathematics teaching and learning.

Arcavi's (2003) critique could be summarised as follows:

- Firstly, viewed from a cultural standpoint, the use of visualisations can easily be downplayed by the beliefs and values which exist within the mathematics community pertaining to what constitutes mathematics. The dilemma which exists within the mathematics community regarding whether visualisation should be accepted as a legitimate and acceptable mathematics methodology has the potential of negatively influencing classroom practice, curriculum design and teacher development. It brings with itself a negative attitude which devalues visualisation processes as an integral part of the teaching and learning of mathematics (Presmeg, 1997).
- Secondly, reasoning with mathematical concepts within a visual setting does not have the luxury of predefined, 'safe' routines as is the case with engaging mathematical content in an algorithmic manner with procedural safeguards. Furthermore, visualisation-rich conceptual images can place a high demand on cognition, as the required flexible and competent translation between the visual and analytic presentation may be difficult for learners of mathematics. This poses a threat to the learners' relational understanding of the mathematical concepts inherent to the image and the handling of multiple representations could be strenuous for the learner.
- Thirdly, in my experience, the general mathematics education community favours the analytical presentation in mathematics teaching and learning over visualisation, given its perceived sequential nature and pedagogical efficiency. Another sociological issue may be the cultural background of learners. Those coming from visually-rich social environments

may be visualisers who experience visualisation as complementary to the analytical, verbal mathematics methodology. The verbalising learners may experience certain deficits within the visual approach to the teaching and learning of mathematics.

Presmeg (1986) researched the preferences of high school learners to use visual methods when solving mathematical problems. The over-arching tenet of her research is that visualisation processes correlate positively with success in the teaching and learning of mathematics. However, she found certain limitations of the visual method. She discovered that visual teaching methods are more time-consuming than non-visual methods. Furthermore, many participating learners in her study who made use of visual techniques could not clearly communicate the mathematical concepts. Instead, they "stumbled over terminology" and would eventually draw diagrams to explain the concept (Presmeg, 1986, p. 45).

Some researchers have defined the role of visualisation in mathematical problem-solving as "ranging from being useful to being an impediment" (Rösken & Rolka, 2006, p. 457).In their study of how learners would use visualisation processes in solving problems regarding integrals, Rösken and Rolka (2006) found that the participants did not fully represent all aspects of the integral concept in their visual model. This flawed visual representation hindered their thinking in the follow-up task.

The researchers also found that visualisation processes should be used flexibly in order to be efficient and helpful. The participants used visualisation in other problems, but could not solve the problems correctly. Learners should possess the cognitive flexibility to use both visual and algorithmic methods (Rösken & Rolka, 2006; Arcavi, 2003). Furthermore, learners may choose to use algorithms and procedures instead of recognising the value of visualisation when the problem-solving context warrants it, just because they have a cognitive fixation on the conventional way of doing mathematics (Rösken & Rolka, 2006).

This literary critique of visual teaching strategies could serve as a directive for mathematics teachers who would want to rely mainly on visualisation processes in their classrooms. My research study advocates visualisation processes as complementary to the use of the verbal, analytical and algorithmic mathematics methodologies. Teachers and learners should employ visualisation processes without being enslaved by them (Arcavi, 2003; Tall, 1994). Makina (2010, p. 25) observes that "visualisation helps teachers to make instructional decisions about how to teach, the content and the nature of tasks. Furthermore, it aids teachers with the facilitation of lessons and with the ability to engage learners in realistic situations”.

2.3 THE VISUALISATION INTERVENTION PROGRAMME (VIP)

2.3.1 Introduction

As indicated above, the VIP (Appendix A) constitutes an intervention programme aimed at enhancing the teaching of number sense to Grade 6 learners. It is the result of collaboration between teachers who favour the visual approach to teaching. The VIP is employed as a research tool to investigate the extent to which participating teachers make effective use of visualisation processes in the teaching of number sense. As an intervention programme, the VIP provided a directive for the participating teachers to implement instructional techniques and strategies in a visual manner.

Presmeg (1986) defines teachers who prefer to teach visually as being of high teaching visuality. This teaching visuality refers to “the extent to which that teacher uses visual presentations when teaching mathematics” (Presmeg, 1986, p. 43). The VIP is therefore an attempt at increasing the teaching visuality of the participating teachers in this research study, in order to investigate whether and in what manner visual teaching could enhance the teaching of number sense understandings as depicted in Appendix C.

The implementation of the VIP facilitated classroom observations and interviews with the participating teachers as it provided a framework to extract the views of the participating teachers regarding the usefulness of visualisation when teaching number sense. This intervention program framed the research methodology as a case study. Key to the case is the incorporation of the views and perceptions of the participants (Tellis, 1997).

We now turn to an in-depth focus on the design of the VIP as a teaching strategy in order to link our discussion of visualisation processes in mathematics education to the teaching of number sense.

2.3.2 The VIP as a teaching strategy

The collaborating teachers took cognisance of the need to craft a VIP that could facilitate number sense understandings through visual teaching strategies as contemplated in Appendix C. The VIP was designed as an instructional programme which provided opportunities for visual processing as the teachers delivered their lessons. The focus was on creating a facilitative platform for visual, mathematical instruction. Boaler et al. (2016) suggest that neuroimaging has shown that our mathematical thinking is grounded in visual processing, even if we work on a number calculation with symbolic digits. The VIP sought to take advantage of this human predisposition for visual processing in solving mathematical tasks.

Furthermore, Boaler et al. (2016, p.2) believes that “good mathematics teachers typically use visuals, manipulatives and motion to enhance students’ understanding of mathematical concepts”. The VIP advocated visual teaching strategies relevant to the number sense understandings as concretised within the different problem contexts of the learner activities. These strategies included the use of whole number charts, fractional number charts, arrays of dots, the decimal fraction board, the fraction circle, the abacus, Gattegno chart, flard cards, the fraction wall, fraction strips, the decimal number line, the conversion chart, number line stickers, diagrammatical representations, spider diagrams, base ten blocks, the multiplication board and counters, as well as sticks and bands.

The VIP sought to move away from presenting mathematics as a numeric and symbolic subject only. Instead, the intention was to teach mathematics for the development of visual understandings (Boaler et al., 2016). As such, the lessons on number sense understandings were planned with accompanying visual aids. The lessons were developed around the nature of whole numbers and fractional parts, the relationship between whole numbers and fractional parts, the addition and subtraction of whole numbers and fractional parts, the relationship between addition and subtraction as inverse operations, the multiplication and division of whole numbers and fractional parts, the relationship between multiplication and division as inverse operations and also number sense understandings through word problems.

The learner activities were designed to facilitate a problem context for number sense understandings. These activities included oral and written questions. The oral questions served as an introduction to the lesson. The objective was to check learners’ prior knowledge and to do mental mathematics to prepare them for the lesson topic. The lessons were developed in four stages, namely introduction, mental mathematics, development of the lesson and consolidation of the lesson. The aim was to do justice to the constructivist perspective that instruction should allow learners opportunities to make connections between new information and their existing cognitive structures (Ausubel, 1968).

The VIP was introduced to the participating teachers with a view to impart the process of facilitating visual learning. In order for the participating teachers to properly implement the programme in this case study, they were taken through a series of workshops as well as one-on-one deliberations regarding the implementation of visual strategies for the teaching of four lessons related to number sense. To assist the participating teachers even further, the first lesson was used as a pilot, in order to refine their teaching proficiency regarding the use of visual and concrete manipulatives.

The participating teachers were made aware that the visual teacher regards visual aids as "a stimulating reference for abstract concepts" as these aids facilitate visual thinking (Alsina & Nelsen, 2006, p. 125). The challenge with preparing the teachers for the implementation of the VIP was that most of them were fixated on the conventional teaching style of approaching mathematics in a verbal, analytical and algorithmic manner. These teachers were generally non-visual as far as their mathematics classroom practices are concerned (Arcavi, 2003; Presmeg, 1986). Given the significance of the VIP in relation to the research objectives, it was important that the participating teachers bought into the vision of the planned and desired visual strategies.

As the VIP sought to enhance the teaching of number sense, a literary review of number sense is necessary.

2.4 NUMBER SENSE

2.4.1 Introduction

The Department of Basic Education in South Africa expects that learners who have entered the Intermediate Phase have a secure number sense and be fluent in dealing with operations. These skills should then be further developed to include knowledge of the meaning, relationship and relative size of different kinds of numbers. Coupled to that is knowledge of representing numbers in multiple ways, understanding the effect of operating with numbers and having the ability to estimate and check solutions (Stott, 2014;DBE RSA, 2011).

From my own experience of being a teacher and mathematics education professional in South Africa and from the literature, it is apparent that the expectation of the Department of Basic Education is not being met regarding the level of numeracy that it sets for Intermediate Phase learners. The ANA results are proof that the expected level of fluency and numeracy as set out in the Curriculum and Assessment Policy Statement (CAPS) document is not attained (DBE RSA, 2014). Furthermore, teachers and researchers still experience a classroom situation where "many learners are still reliant on concrete one-to-one methods of calculation such as finger counting or tally marks throughout the primary grades" (Stott, 2014, p. 3).

In order to discuss number sense more rigorously, we need to understand how it is defined in the literature. Yang et al. (2004), citing various reports and documents, agreed that number sense refers to a person's general understanding of numbers and operations and the ability to handle daily-life situations that include numbers. This ability is used to develop practical, flexible, and efficient strategies (including mental computation and estimation) to handle numerical problems.

McIntosh et al. (1992) also saw number sense as a person's general understanding of numbers and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgements and to develop useful strategies for handling numbers and operations. This suggests that numbers and quantitative methods are used as a means of communicating, processing and interpreting information.

This implies that numbers are important and central to mathematics. McIntosh et al. (1992, p. 3) asserted that "the acquisition of number sense is a gradual, evolutionary process…often evident at an early age as children think about numbers and try to make sense of them."
They furthermore concluded that the mathematical thinking of learners is influenced by the context in which learners encounter the mathematical problem - hence the importance of providing learners with rich situated activities to promote problem-solving and also to stimulate different components of number sense. Number sense may be triggered by the context in which mathematics evolves. To cite an example of how this contextual trigger may work, one can think of a scenario where a customer has to pay a certain amount for goods. If the amount is exorbitant, the customer may question the addition operation which determines the price of the grocery items (McIntosh et al., 1992).

2.4.2 The number sense framework

McIntosh et al. (1992) introduced a framework of how the **different components of number sense** relate to each other. See Table 2.3 below in tabular format:

Table 2.3: Overview of a framework for number sense (McIntosh et al., 1992)

MAJOR COMPONENTS	NUMBER SENSE UNDERSTANDINGS	NUMBER SENSE THEMES
KNOWLEDGE OF AND FACILITY WITH NUMBERS	Sense of orderliness of numbers	Place Value Relationship between number types Ordering numbers within and among number types
	Multiple representations for numbers	Graphical / Symbolic Equivalent numerical forms (including de-composition and re-composition) Comparison to benchmarks

	Sense of relative and absolute magnitude of numbers	Comparing to physical referent Comparing to mathematical referent
	System of benchmarks	Mathematical Personal
KNOWLEDGE OF AND FACILITY WITH OPERATIONS	Understanding the effect of operations	Operating on whole numbers Operating on fractions / decimals
	Understanding mathematical properties	Commutativity Associativity Distributivity Identities Inverses
	Understanding the relationship between operations	Addition / Multiplication Subtraction / Division Addition / Subtraction Multiplication / Division
APPLYING KNOWLEDGE OF AND FACILITY WITH NUMBERS AND OPERATIONS TO COMPUTATIONAL SETTINGS	Understanding the relationship between problem context and the necessary computation	Recognize data as exact or approximate Awareness that solutions may be exact or approximate
	Awareness that multiple strategies exist	Ability to create and / or invent strategies Ability to apply different strategies Ability to select an efficient strategy
	Inclination to utilize an efficient representation and / or method	Facility with various methods (mental, calculator, pencil and paper) Facility choosing efficient number(s)
	Inclination to review data and result for sensibility	Recognize reasonableness of data Recognize reasonableness of calculation

The framework in Table 2.3 above emphasizes that number sense is constituted by the learners' knowledge of numbers and operations. The learner then needs to apply that knowledge practically, efficiently and flexibly in different mathematical contexts. In order to respond successfully to any computational setting, the learner should become skillful in using numbers and operations. The framework suggests that number sense is "…a propensity for and an ability to use numbers and quantitative methods as a means of communicating, processing and interpreting information. It results in an expectation that numbers are useful and that mathematics has a certain regularity…" (McIntosh et al., 1992, p. 4).

The interconnections of the major components of number sense as per the framework above was presented by McIntosh et al. (1992) as figure 2.1 suggests below:

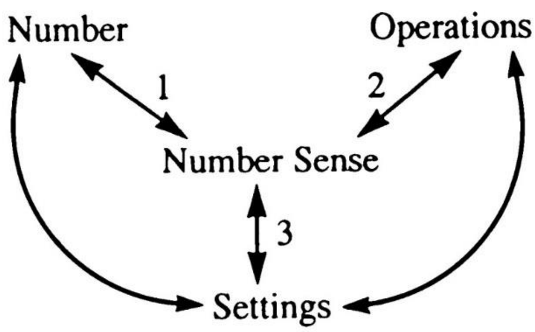


Figure 2.1: The interconnections of number sense (McIntosh et al., 1992)

The interconnections of number, operations and settings depicted in figure 2.1 represent the manner in which an individual could make sense of his or her own thought processes while engaging in a mathematical task related to the number sense understandings as shown in Table 2.3. An individual with good number sense is "thinking about and reflecting on the numbers, operations and results being produced" (McIntosh et al., 1992, p. 5). During the process of reflection, any (or a combination) of the major components of the number sense framework in Table 2.3could be involved as the individual contemplates the relationships between numbers, operations and the computational settings wherein which number sense is applied.

McIntosh et al. (1992) contended that there is a need for a comprehensive model of number sense in order to inform instruction, curriculum development and research. In later research, McIntosh et al. (1997, p. 63) identified six strands of number sense inherent to the framework described above in order to develop a number sense test. McIntosh et al. (1997, p. 8) regarded the number sense framework in Table 2.3 as well as the strands in Table 2.4 as being "useful in facilitating thinking and discussion about the multifaceted notions associated with number sense". The six strands were developed from the three major components of the framework above and can be listed as follows in Table 2.4 (McIntosh et al., 1997):

Table 2.4: Overview of how the six strands relate to the major components of number sense (McIntosh et al., 1997)

MAJOR COMPONENTS	**STRANDS**
KNOWLEDGE OF AND FACILITY WITH NUMBERS	• Understanding of the meaning and size of numbers • Understanding and use of equivalent forms and representations of numbers
KNOWLEDGE OF AND FACILITY WITH OPERATIONS	• Understanding the meaning and effect of operations • Understanding and use of equivalent expressions
APPLYING KNOWLEDGE OF AND FACILITY WITH NUMBERS AND OPERATIONS TO COMPUTATIONAL SETTINGS	• Computing and counting strategies • Measurement benchmarks

From these six strands, McIntosh et al. (1997) identified items to be included in a test for assessing the level of the number sense of learners. A review of available assessment instruments was conducted by McIntosh et al. (1997) in order to identify those items which would provide insight into the thinking of learners as related to the six strands cited above. The items were then constructed by McIntosh et al. (1997) to test the acceptable level of the conceptual knowledge of the primary school participants. Secondly, the questions for the number sense test were framed in non-routine environments to ensure that the participants use strategies which are based on understanding and not familiarity with the problem-type. Lastly, the questions were carefully considered to allow for the eliciting of components of number sense as identified by the six strands in Table 2.4. Afterwards, the items were compiled in a Number Sense Item Bank by McIntosh et al. (1997).

My research study made use of this "Number Sense Item Bank" (McIntosh et al., 1997, p. 9) in designing the number sense pre- and post-tests for my participating learners. The pre- and post-tests (Appendix B) are identical and were used as a quantitative research tool to investigate whether the VIP with its inherent visualisation processes did facilitate mathematics learning by enhancing the teaching of number sense.

2.4.3 The teaching of number sense

Most mathematics educators are in agreement that learners need to handle numbers in multiple ways rather than just through drilling exercises (Stott, 2014). It is important to engage learners in mathematical tasks and activities that require flexible thinking and sense-making, such as card and dice games. A typical card game will provide opportunities for the learners to, *inter alia*, develop their skills in applying the basic order of operations (Stott, 2014).

Anghileri (2000, p. 125) suggested teaching approaches which may be "most appropriate in developing children's number sense". She advocated that learners should be presented with opportunities within the classroom to talk about numbers in whatever real-life context they occur. In this manner, learners will be able to use numbers confidently. They will also identify mathematics as a vehicle for making sense of the world. Learners should therefore be assisted in transferring their informal knowledge of numbers, gained through experiences outside of school, to practical applications within the classroom (Anghileri, 2000).

Bobis (2008) suggested that a strong relationship exists between visualisation and the development of number sense. The spatial structuring ability of learners contributes to an emerging sense of numbers and number relationships. Bobis (2008, p. 5) believed that "we cannot 'make' children understand relationships between numbers in any direct way, but we can provide activities designed to enhance the construction of these relationships." These activities may include visual strategies, i.e. an array of dots to teach the decomposition of numbers.

Howden (1989) asserted that using concrete objects, learners can explore such numerical ideas as more than, less than, grouping, composition, decomposition, and different ways of modeling the same number. The use of concrete objects facilitates visualisation and this helps learners to discover new relationships and the properties of numbers (Howden, 1989). Calendar activities help to teach order, counting forward and counting back. Characteristics of even and odd numbers and multiples and factors of a given number can be discovered by arranging objects in appropriate arrays. It is not enough, however, simply to perform such activities. Recording the findings and talking about them are essential for recognising and understanding the relationships.

The *ten frame* manipulative which is an array of squares used to teach counting helps learners "to visualize numbers as doubles and neighbors and to look for groups of tens in adding numbers"

(Howden, 1989, p. 7). Howden (1989) explains how the ten frame was used by learners to organise a scoop of beans in groups of ten for counting purposes without the manipulative being presented by the teacher. The learners used previous knowledge of the ten frame and creatively put it to use. The ordering of the beans was followed up by an interpretation of the meaning of the number with its related place values. The learners used visual ways to count the scoop of beans as the symbolic paper-and-pencil method was inappropriate for this specific problem context.

My research study investigates the extent to which visual teaching strategies could enhance the teaching of number sense. It was therefore necessary to design an analytical framework to seek evidence of visual teaching strategies for number sense. The analytical framework for visual teaching strategies was informed by the need to connect classroom learning related to number sense with real-life contexts, through the use of concrete and virtual manipulatives, in order to resonate with the ideas expressed by Stott (2014), Anghileri (2000), Bobis (2008) and Howden (1989).

I adapted the cited number sense framework of McIntosh et al. (1992) for my analytical framework and used it to explore how visual teaching strategies could be utilized for each number sense theme. The adapted framework served as an analytical tool for the observed lessons of this research study. The connections between these number sense themes and the visual teaching strategies which may be employed are presented as Appendix C, a framework for considering visual teaching strategies for number sense (adapted from McIntosh et al., 1992).

As good number sense is ideal for handling computational settings successfully, teachers should go beyond teaching the standard curriculum and focusing merely on the mastery of algorithms. They should design and introduce instructional units which have the development of good number sense as their outcome (Yang et al., 2004). As indicated above, the VIP has been designed for this purpose and used as the empirical field for my study. The instructional units of the VIP deal with number sense understandings and the visual teaching thereof in order to facilitate the development of good number sense as advocated by Yang et al. (2004).

2.4.4 Number Sense and the Curriculum and Assessment Policy Statement (CAPS)

Post-Apartheid South Africa has seen major upheavals in the education sector through the implementation of a number of new and revised curricula. Curriculum 2005 was underpinned by outcomes-based education (OBE) perspectives (Jansen, 1998). Jansen (1998) predicted that OBE would have a negative effect on South African schools and that it would “undermine the already fragile learning environment in schools and classrooms of the new South Africa” (Jansen, 1998, p. 322). The major points of concern that Jansen (1998) had, were the assumption by the Ministry of Education that curriculum change would affect the South African economy positively, the notion that it would solve deeply entrenched pedagogical problems, the flawed assumptions about classroom practice and the instrumentalist view of knowledge. Furthermore, the limited participation of teachers in policy formulation, the administrative burden placed upon teachers coupled with the fact that the outcomes-based model trivialized school subject content were contributing reasons for the prediction that OBE would fail (Jansen, 1998).

This background sketch paints a picture of why the curriculum underwent constant change since 1994. The initial Curriculum 2005 paved the way for the National Curriculum Statement (NCS) in 2002. This was done due to the identified challenges of OBE. There was an attempt to streamline the outcomes-based curriculum and to make it more accessible to teachers. These amendments were effected in the NCS and the later revised NCS. In order to address the challenges which emanated from the revised NCS, the Curriculum and Assessment Policy Statement (CAPS) was introduced in 2012 (Gumede & Biyase, 2016).

This study is located within the CAPS document which articulates its mission to instil number sense (DBE RSA, 2011, p. 10). The Grade 6 mathematics curriculum is constituted by five content areas (DBE RSA, 2011). These content areas are:

- Numbers, operations and relationships
- Patterns, functions and algebra
- Space and shape (Geometry)
- Measurement
- Data handling

The weighting of the numbers, operations and relationship mathematics content area is fifty percent of the total curriculum. This is the content area of the curriculum which seeks to develop the number sense of learners. The curriculum progression for number sense is characterized by increases in the number range, the introduction of different kinds of numbers and change in calculation techniques. It is expected from Grade 6 learners that they develop more efficient techniques for calculations once place value and the properties of numbers and operations are understood adequately (Department of Basic Education of the Republic of South Africa [DBE RSA], 2011).

The DBE RSA (2011, p. 10) furthermore has the following Intermediate Phase specific content focus:

- The range of numbers developed by the end of the Intermediate Phase is extended to at least 9-digit whole numbers, decimal fractions to at least two decimal places, common fractions and fractions written in percentage form.
- In this phase, the learner is expected to move from counting reliably to calculating fluently in all four operations. The learner should be encouraged to memorise with understanding, multiply fluently, and sharpen mental calculation skills.
- Attention needs to be focused on understanding the concept of place value so that the learner develops a sense of large numbers and decimal fractions.
- The learner should recognize and describe properties of numbers and operations, including identity properties, factors, multiples, and commutative, associative and distributive properties.

Given the above specific content focus for Grade 6, the CAPS document does indeed resonate well with the number sense understandings as referred to in the number sense framework in Table 2.3 and the analytical framework for considering visual teaching strategies for number sense in Appendix C.

However, as Appendix A suggests, certain challenges regarding the <u>implementation</u> of the CAPS document do exist. These challenges can be summed up as follows:

- The nature of numbers (whole and fractional parts) are not being taught comparatively. Instead, they are being taught as separate topics in the curriculum as per the "Time Allocation per Topic: Grade 6" (DBE RSA, 2011, p. 212).

- It is difficult for teachers to facilitate an environment for learners to discover the connection between the whole numbers, fractions, decimals and percentages as these concepts are being taught in isolation (DBE RSA, 2011). Number sense understandings should be taught relationally to allow learners to connect these understandings to real life.
- Given the poor performance of our South African mathematics learners (Adler et al., 2016), the allocated weighting of fifty percent to inculcate number sense in Grade 6 is not enough.
- Extending of the range of whole numbers to nine digits is not as productive as teaching for conceptual understanding of numbers and operations by using smaller numbers.

The teaching guidelines within the CAPS document include the content areas and topics, concepts and skills to be developed, clarification notes and the allocated time per topic. Despite having constructivist underpinnings, the curriculum does not offer comprehensive guidance on how it should be implemented in the classroom as far as mathematics methodology is concerned. We will now turn to a discussion on how visualisation processes may relate to the teaching of number sense.

2.4.5 Number Sense and Visualisation

This research study seeks to investigate the role of visualisation processes as a strategy to enhance the teaching of number sense. As such, understanding the connection between number sense and visualisation is important. It is noteworthy that research suggests that learners benefit from exposure to different modes of representation (Moskal & Magone, 2000). These modes of representation are constituted by the manner in which information is communicated by the teacher and used by the learner. Such modes may be visual-spatial like concrete manipulatives and diagrams, verbal or syntactic like written and spoken words or formal notational such as mathematical symbols (Moskal & Magone, 2000).

Furthermore, Chrysostomou et al. (2012, p. 1) found that “spatial imagery, in contrast to the object imagery and verbal cognitive styles, is related to achievement in number sense”. Their research has found a strong relationship between the participants’ preference for using spatial imagery in solving number sense tasks and solving these tasks successfully. A distinction is being made between two types of imagery subsystems. These are termed ‘object’ and ‘spatial’ imagery subsystems. Object imagery refers to the visual processing of the properties of the object which is being visualised. Spatial imagery, in contrast, refers to visual processing of the relationships between the features of the object which is being visualised.

Research found that a learner's use of visual-spatial imagery positively correlates with the degree to which number sense tasks are understood and solved (Chrysostomou et al., 2012). Visual-spatial imagery facilitates productive visual processing of mathematical relationships. It was discovered that the successful solution of algebraic word problems is associated with the use of visual-spatial imagery. An emphasis is placed on the need to study the influence of cognitive styles on the learning of mathematics in order to inform our instructional practices for the development of number sense (Chrysostomou et al., 2012; van Garderen, 2006; Hegarty & Kozhevnikov, 1999).

Research confirms that when visual-spatial representations are included as a component of instruction and learners are urged to go beyond merely visualising the problem, the mathematics problem-solving skills of learners do seem to improve (van Garderen, 2006; Hegarty & Kozhevnikov, 1999). It therefore stands to reason that the teaching of number sense should accommodate strategies that could facilitate the use of visual-spatial imagery by learners. In turn, this could promote good number sense and ultimately improve their abilities to successfully solve mathematical problems.

The VIP (Appendix A) was therefore crafted with a view to facilitate a classroom environment which is conducive for the use of visualisation processes in the context of developing good number sense. Visualisation processes were employed as a strategy to teach number sense as well as an aid for learner engagement during the lessons.

2.5 THEORETICAL FRAMEWORK

2.5.1 Introduction

Any strategy which seeks to enhance the quality of teaching has the learning and development of learners as its focus. It is therefore important to understand how the learning of mathematics can best be facilitated. The theoretical aspect which is of integral importance to the learning and development of number sense in this study is social constructivism. This theory of how the learning of mathematics occurs provided a theoretical lens and conceptual framework through which the researcher could investigate the role of visualisation processes as a strategy to enhance the teaching of number sense.

Constructivism is an epistemological construct. It grew from an enquiry regarding different ways of knowing. It was generally accepted that knowledge was the product of transferred intellectual content from a knowledgeable individual to an uninformed one, until thought leaders transformed this view. The learner was no longer viewed as a mere absorber of knowledge. Instead, the learner was seen as having the mental capacity to engage in thinking, reasoning and understanding. This view recognised the learner as an active constructor of knowledge, instead of a passive receiver thereof (Piaget, 1972).

Piaget (1967) viewed constructivism as a theory of knowledge whereby human beings generate knowledge and meaning as a result of interactions between their experiences and their ideas. He held that an individual connects new learning to an existing mental scheme and then either assimilates the new idea into the existing scheme or accommodates the new idea by amending the existing scheme. His views represented a move away from the behaviourist position that the actions of teachers are paramount in ensuring that student learning and development takes place. Constructivism acknowledges that knowledge is not a product to be transferred by the teacher, but a process of discovery on the part of the learner.

From the work of Piaget and other leading cognitive theorists, a more refined theory of learning was developed. The learning environment is by definition social in nature. As an interplay of human agents informs learning, the theory of social constructivism entered the debate (Vygotsky, 1978).

2.5.2 Social constructivism

Vygotsky (1978) took the theoretical stance of constructivism a step further by emphasizing the social factor involved in the construction of knowledge – hence the idea of social constructivism. He regarded the role of communication, social interaction and instruction as paramount for student learning and development. Vygotsky branded the social interplay between the student and others as the "zone of proximal development" or ZPD. Vygotsky (1978, p. 76) defines the ZPD as "… the distance between the actual development level as determined by independent problem solving and the level of potential development through problem solving under adult guidance or more capable peers." To Vygotsky (1978), the experienced adult is responsible for providing scaffolding of the subject matter to support the understanding of the learner.

Jones et al. (2002) saw the work of Vygotsky as foundational to social constructivism in educational settings. Vygotsky's ideas were revolutionary in the sense that the focus was shifted from the individual to significant others within the learning environment. The social context within which learning takes place is extremely important. The degree to which such social contexts provide a climate conducive for learning and development determine the level of success which the learner will achieve. This social context is constituted by the role of those individuals whom the learners regard as significant to their learning (Jones et al., 2002, p. 5).

As significant others, teachers should provide scaffolding to learners for the development of number sense. Visual teaching strategies can be used as the scaffolding which will support the learner in his/her construction of number sense understandings as the framework of McIntosh (1992) in Table 2.3 suggests. Furthermore, teachers who prefer to teach visually should aim to make "connections between the subject matter and … the real world" in order to enhance the learner's development of number sense (Presmeg, 2006, p. 213). In this manner, the actual level of development in the Vygotskian ZPD, as far as number sense is concerned, will be improved.

In order for teachers to be proficient in their teaching, they should reflect on their own teaching and also encourage learners to engage in reflection. The errors and misconceptions of learners of mathematics should serve as an opportunity for the teacher to explore their conceptual understanding. It could also provide a platform for reflection on the teaching and learning of mathematics. The teacher who has empathy with the learners for their mathematical errors and misconceptions, is positioned to understand the cognitive functioning of learners from a constructivist perspective. Their errors and misconceptions should be viewed as rational and meaningful efforts to cope with mathematics and the teacher should be cognisant that it has derived from what they have been taught (Olivier, 1989).

Kilpatrick et al.'s (2001) work, which is based on social constructivism, provided a conceptual framework for teaching proficiency. They have extended their mathematical proficiency model to include a model for teaching proficiency as well. According to them, being proficient in teaching requires the following important and interrelated components which function in unison as different strands in a rope:

- Conceptual understanding of the core knowledge required in the practice of teaching: In this strand, the teachers' knowledge of mathematical content, their learners and pedagogy should be connected for intelligent use. This entails an elaborate, integrated knowledge of mathematics coupled with the knowledge of how the mathematical understanding of learners develop and the kind of pedagogy that accounts for the nature of mathematics and how learners should engage with it for optimal learning outcomes.
- Fluency in carrying out basic instructional routines: The characteristic feature of this strand is that the teacher should be able to tap into his or her acquired repertoire of instructional routines while interacting with the learners during the teaching of mathematics. These routines include classroom management, procedures for correcting and collecting homework, reacting to learners who demonstrate serious misconceptions, handling situations where the teacher may not understand a learner's answer and knowledge of how to deal with learners who do not have the critical skills needed for the lesson. Teachers should have several ways of approaching teaching problems in order to choose the most appropriate one when the need arises.
- Strategic competence in planning effective instruction and solving problems that arise during instruction: Teachers should take into account the scope of the mathematics curriculum, the level of learners' understanding, the best way in which learners learn and benchmarking the specific mathematics topic against best practices in teaching, when they plan and execute their instruction.
- Adaptive reasoning in justifying and explaining one's instructional practices and in reflecting on those practices so as to improve them: Teachers should have the ability to

analyse their instructional practices objectively. They should be reflective practitioners aimed at improving the manner in which they teach.

- Productive disposition toward mathematics, teaching, learning and the improvement of practice: It is important for teachers to see how mathematics, their understanding of how learners think and the way they teach, fit together to make sense. Furthermore, teachers should find sense in the analysis and scrutiny of their own teaching practices and that they are capable of learning from experience in order to improve their instructional practices.

This research study reflected on teaching proficiency within the context of social constructivism and as such, explored the extent to which the participating RUMEP teachers made use of visual teaching strategies for the construction of number sense understandings as a result of the VIP. This entailed a reflection on the above-mentioned interrelated components of teaching proficiency. The observation schedule presented as Appendix D was used for this reflection as the researcher looked for evidence of these teacher proficiencies in the lessons of the VIP which the participating teachers taught.

2.5.3 Social Constructivism and RUMEP practice

The participating teachers of this research study are part of the RUMEP cohort. Hence the importance to shed light on the nature of the classroom practice which RUMEP seeks to develop in its cohort. As indicated earlier, RUMEP functions as a teacher development institute which seeks to develop the mathematics teaching and learning skills of primary teachers (Penlington, 2005).

RUMEP subscribes to asocial constructivist perspective on learning. Knowledge is seen as a product of the interaction between the learner's experience and the current knowledge structures he or she possesses. As the learner actively participates in the construction of his or her own knowledge, it becomes the responsibility of the teacher to consider ways of supporting the learner in this quest. RUMEP therefore defines the role of teachers as agents who facilitate discussion, communication, reflection and negotiation as important elements in their classroom practice (Olivier, 1989).

The RUMEP classroom practice as defined above, informed the crafting of the VIP (Appendix A). It is therefore important to also focus on the connection between social constructivism and the VIP.

2.5.4 Social Constructivism and the VIP

The collaboration team who crafted the VIP (Appendix A) sought to define the role of the participating teachers in a manner which would facilitate a constructivist classroom. The teachers were advised on the type of visual aids to use as a tool to effect learner engagement. This would set up the environment for learners to discuss, communicate, reflect and negotiate the issues pertaining to the number sense tasks at hand. The teacher would facilitate and guide the participation of learners during the four lessons as per the VIP (Appendix A).

Implementation of the VIP would allow an opportunity to investigate whether visualisation processes contribute to the enhancement of the teaching of number sense. In this regard, the theoretical underpinning of social constructivism became imperative as it would define the manner in which learners would be supported in their endeavour to connect the visual imagery with their experiences. The lessons have therefore been sequenced to allow for the maximum participation of learners. As indicated earlier, the lesson sequencing was designed to check the previous knowledge of learners (their existing schemas) and to connect these with new schemas formed by the use of visual aids, which would set in motion visual processing by the learners (Ausubel, 1968).

The implementation of the VIP would ensure a constructivist classroom environment through the social interplay created between the stakeholders, the scaffolding provided to the learners and the reflections on teaching and learning. These factors are discussed below.

2.5.4.1 Social interplay during the VIP lessons

The VIP lessons were structured in a manner that would allow optimal social interaction between the participating teachers and the learners as well as the learners amongst themselves. As an introduction to the lesson, the teacher would use an oral questioning technique and flashcards to test embedded knowledge. The mental mathematics would also be done orally with visual aids to put the teacher in a position to guide the learners as the significant other. The exercises that were at the heart of the lesson, would be completed within a group work context where the teachers would provide support to the groups regarding the use of the relevant visual aids. The learners would also depend on more capable learners within the group to complete the challenges successfully. This social interplay that is created between the learners and the teachers would be instrumental in supporting the conceptual development of number sense understandings on the part of the learners.

2.5.4.2 Scaffolding during the VIP lessons

The participating teachers' use of visualisation processes to teach number sense understandings would provide a support base to their learners, which would manifest in a mathematically rich environment. Visual aids like the fraction wall, fraction strips, fraction circles, the number line, flashcards, the spider diagram and others were prescribed by the VIP to serve as the scaffold needed by learners to construct number sense understandings through self-discovery of the connections between the whole and fractional parts thereof.

2.5.4.3 Teacher and learner reflections during the VIP lessons

Reflection is an inherent part of learning. The VIP lesson activities would be completed within different learner groups and then presented to the whole class for their perusal and feedback. This would create an opportunity for learners to reflect on their own work. The errors and misconceptions of the learners would also enable the participating teachers to reflect on their teaching. The learners' actual level of development would then be improved due to the reflection exercise.

2.5.5 Critique of social constructivism

Researchers like Kirschner, Sweller, and Clark (2006) believe that constructivism "promotes a teaching style with unguided or minimally guided instructions for students" (Alanazi, 2016, p. 2). Alanazi (2016) is convinced that when minimal instruction is given during a lesson, the importance and structure of working memory during learning is being ignored to the detriment of learners who will become lost and frustrated. The teacher should provide good leadership in giving clear and concise instructions to the learners.

Clements (1997) asserts that the notion of learners actively constructing knowledge just because manipulatives are being used by the teacher, is flawed. Teachers should ensure that the manipulatives which are used serve the purpose of learner engagement. This calls for teacher proficiency regarding mathematical knowledge and the pedagogical content knowledge constituted by the skill, to use manipulatives in a didactically meaningful manner.

Teachers should guard against using manipulatives "to impose prescribed procedures for routine problem types" (Clements, 1997, p. 199). In such a case, the learners will only use manipulatives in a rote manner and not to construct knowledge (Clements & McMillen, 1996). Clements and McMillen (1996, p. 277) firmly believe that "reflecting on and discussing different models may indeed help students abstract the mathematical idea. Brief and trivial use, however, will not help; each manipulative should become a tool for thinking."

Constructivist classroom practice is not easy for teachers in general and new teachers in particular. Managing and controlling the class becomes more problematic when alternative teaching strategies which align to the social constructivist view of learning, as opposed to the transmission model, are employed (Orton & Wain, 1994). Furthermore, the mere use of cooperative groups does not make teaching constructivist. "The way students think and interact is more important than the size of the group in which they work" (Clements, 1997, p. 199). My experience as a teacher taught me that some learners will not take responsibility within a group. They simply wait for the outcome constructed by the minority. Teachers should therefore monitor the quality of engagement within the groups designed for cooperative learning.

Another point of critique centres around the time allocated for discussion of a topic. There is a real threat that a teacher might not be able to finish the prescribed curriculum by allocating too much time executing the lessons in the constructivist manner. Another factor contributing to the problem, is that some teachers are uncertain about when it is best to intervene in a discussion. Teachers are also concerned about the fact that the type of learner assessments may not be in line with the constructivist approach to learning (Orton & Wain, 1994).

2.6 CONCLUSION

I have discussed visualisation in this chapter as it forms the backbone of the study. In order to understand the role of visualisation processes as a strategy to enhance the teaching of number sense, a literary review of visualisation as product and process was necessary. The broad discussion was funnelled down to unpack the nuts and bolts of visualisation as it is manifested and experienced in mathematics education. In order to give a balanced account of the two mainstream positions regarding mathematics teaching methodology, namely visual and algorithmic, a critique of visual teaching strategies was offered.

The VIP was then discussed to inform the reader about how visualisation is connected to the teaching of number sense. The process of crafting the VIP as a collaborative effort was explained. Furthermore, an analysis was done on how the VIP served as a teaching strategy for this study, in order to investigate the role of visualisation processes regarding the teaching of number sense.

An in-depth discussion on number sense then followed as the study is concerned with how the teaching thereof could be enhanced. The cited framework for number sense, a literary review around the teaching of number sense, the relationship between number sense and the CAPS document, as well as the connection between number sense and visualisation were among the subtopics.

The theoretical framework of social constructivism provided the theoretical lens through which this research topic was examined. Social constructivism was discussed as a theory of learning upon which the VIP was crafted and implemented in order to investigate the role of visualisation processes in the teaching of number sense.

The literature pertaining to the use of visualisation processes in teaching and learning suggests that "using visual elements in teaching and learning yields positive results" (Stokes, 2001, p. 19). The emphasis of this chapter was on visualisation processes and how the use of dynamic or visual-spatial imagery relates to the teaching of number sense, which could have a positive learning outcome. In the course of these discussions, a number of issues were raised regarding the relationship between these visualisation processes and the teaching of number sense.

In the next chapter I examine the research methodology and focus on the qualitative and quantitative methods that were used for collecting and analysing the data.

CHAPTER 3

RESEARCH METHODOLOGY

3.1 RESEARCH ORIENTATION

The overall goal of this study was to investigate the role which visualisation played, when employed as a teaching strategy to enhance number sense as a result of participating in the Visualisation Intervention Programme (VIP).The study consisted of two data generating processes. The first one involved selected RUMEP teachers teaching number sense, and the second involved administering and analysing a set of pre- and post-tests written by learners. My empirical research with the RUMEP teachers was framed as a qualitative case study and was located in the interpretive paradigm. The analysis of the pre- and post-tests was quantitative.

Interpretive research describes and analyses educational activities (in this case teaching number sense that explicitly embraces a visualisation approach to teaching) from the perspective of the participants. The objective of this research study was to gain deeper insights into the classroom practice of selected teachers that adopted a visual approach to teaching and learning number sense, as a result of participating in the VIP.

Scotland (2012), citing Guba and Lincoln (1994), saw relativism as the ontological position of the interpretive paradigm. Reality is seen as a subjective experience and is dependent on the individual. Reality is therefore viewed as an individual construct. The researcher should thus base his/her understanding and interpretation of classroom practice on the participants' perspectives. As this study sought to investigate the use of visualisation as a teaching strategy to enhance number sense, the researcher enlisted the classrooms of seven RUMEP teachers. In these classrooms the subjective understanding of the participants pertaining to the use of visualisation in the teaching and learning of number sense was captured and shared.

3.2 RESEARCH QUESTION

This study asked the following research question:

How can teachers use visualisation processes in a mathematics classroom in order to enhance their teaching of number sense as a result of participating in the VIP?

3.3 RESEARCH METHODS

3.3.1 Introduction

This mixed method study was undertaken with a total of twelve Grade 6 teachers who teach mathematics. Firstly, I invited five teachers from well-resourced Northern Cape schools (see section 3.4.2.1) to collaborate with me in designing and crafting the VIP. Secondly, seven selected RUMEP teachers were then invited to participate in the VIP for a period of three terms. It is the latter group of seven teachers that constituted my case study for the empirical research. It is thus the VIP that formed the empirical field. In conjunction with the implementation of the VIP, I also administered a number sense pre- and post-test to learners of the seven RUMEP teachers, to ascertain whether discernable change in learner performance took place as a result of the implementation of the VIP.

Gerring (2004, p. 341) defined a case study as "… an in-depth study of a single unit (a relatively bounded phenomenon) where the scholar's aim is to elucidate features of a larger class of similar phenomena". In this research study, the case was the seven participating teachers teaching number sense themes to Grade 6 mathematics learners. The unit of analysis was the views and responses of the participating RUMEP teachers about the use of visualisation processes as a result of participating in the VIP. A further unit of analysis was the pre- and post-test results.

3.3.2 Sampling and Participants

Researchers typically select samples with a goal of identifying information-rich cases that will allow them to study a case in-depth when working within the interpretive paradigm (Mertens, 2005). The participants were five teachers from well-resourced schools and seven Grade 6 teachers from RUMEP schools. I believe that the latter provided me with rich and in-depth data which assisted me in answering my research question. Each of the participants had at least five years of teaching experience which was necessary to contribute meaningfully to my study.

Cohen et al. (2007:114-115) states that "in purposive sampling …researchers handpick the cases to be included in the sample on the basis of their judgement of their typicality or possession of the particular characteristics being sought." The seven RUMEP teachers (all within the John Taolo Gaetsewe District of the Northern Cape) were selected on the basis of their willingness to participate in this research project as well as on their level of interest and active participation during the workshops described in Phase Two of the research study.

The participating RUMEP teachers each received a set of RUMEP manipulatives which they used during the VIP workshops described in phase three and the lessons described in Phase Five of the research study. After the research was conducted, these teachers were allowed to keep the manipulatives as a gesture of gratitude.

3.4 RESEARCH DESIGN

3.4.1 Introduction

The research study followed six interlinked phases, according to the Implementation Schedule presented as Appendix F.

3.4.2 Phases of the Research Study

3.4.2.1 Phase One: Preparation with well-resourced schools

As part of my responsibilities at RUMEP, I visited twenty Northern Cape schools throughout 2017 to offer classroom support to the teachers who participated in the project. During these visits I came across well-resourced schools that used visualisation processes in their teaching of mathematics. For the purpose of this study well-resourced schools were those where, in the context of my school visits, teachers not only had access to teaching aids and resources, but also made regular use of them. I have seen that in the well-resourced schools, the mathematics teachers made interesting use of manipulatives and other teaching aids in order to create a rich mathematics learning environment. I recognized that I could not make the assumption that teachers from well-resourced schools were necessarily effective teachers. I thus wished to select those teachers that, in my observations, made interesting use of the resources that were available to them at their schools.

I re-visited some of these well-resourced schools in the Northern Cape and invited several teachers to collaborate with me in designing and drafting the VIP (Appendix A) for this research study. Five teachers from the well-resourced schools accepted my invitation and I then set up a series of meetings with them to design the VIP. Three face-to-face meetings were held with the collaborating teachers, totaling a duration of approximately four and a half hours. The first meeting lasted for one hour and forty minutes, the second meeting for one hour forty-four minutes and the third one for one hour.

The first meeting was used to orientate the collaborating teachers regarding their role in crafting the VIP. The teachers shared their understanding of visualisation in mathematics as making things real to learners and facilitating practical learning by using manipulatives and other visual aids during teaching. The meeting then discussed what they understood by learners having a sense of number, and how, from their experience, this number sense was being applied in computational settings. The meeting agreed that number sense understandings should be taught in relation to each other rather than in silos (compartments) as prescribed by CAPS.

During the follow-up meeting, CAPS was interrogated more rigorously with regard to its effectiveness in developing the number sense of learners. The third meeting was used to refine the first draft of the VIP. The collaborating teachers were then requested to send further input per e-mail and via the WhatsApp group that was created. The input of the collaborating teachers were processed and used to inform the final VIP, as attached in Appendix A. The collaboration exercise developed capacity-building on the part of the collaboration team as they got involved in the implementation of the VIP at schools which were not well-resourced.

3.4.2.2 Phase Two: Workshops with RUMEP schools

All RUMEP school teachers that I am involved with in the Northern Cape were informed about this research project and invited to a series of awareness workshops to discuss the VIP that was designed and drafted in Phase One. The workshops were integral to my ordinary RUMEP work schedule.

3.4.2.3 Phase Three: Selection of RUMEP teachers

After the workshops in phase two, I invitedseven RUMEP teachers to volunteer to participate in the research, based on their level of interest and active participation during the workshops described in phase two of the research study. I then ran a series of VIP workshops with these participants where we examined and explored what it means to incorporate a visual approach to teaching number sense.

The VIP designed in Phase One formed the basis for implementing the VIP ideas and lesson plans in the seven classrooms. The RUMEP teachers were advised to make use of the available visual aids issued to them by RUMEP. They were furthermore encouraged to create new visual aids based on their relevance to the number sense understandings under discussion.

3.4.2.4 Phase Four: Pre-test administration to RUMEP schools and one control group

I administered a pre-test on number sense to the learners of the seven selected RUMEP teachers in order to establish a benchmark for measuring any discernable change in learner performance as a result of the implementation of the VIP. The pre-test was designed by making use of the "Number Sense Item Bank" (McIntosh et al., 1997, p. 9). One Grade 6 mathematics class which none of the seven RUMEP teachers taught, also wrote the pre-test to serve as the control group of the research study.

3.4.2.5 Phase Five: Intervention within the seven RUMEP school classrooms

As Appendix F indicates, the implementation of the VIP took place in four cycles. Cycle 1coincided with Term 2, Cycle 2 and 3 with Term 3 and Cycle 4 with Term 4 of 2018. I video-recorded one lesson per teacher in every cycle. After each recorded lesson, I then ran a reflective workshop with each teacher. (See Appendix F.) This reflective workshop entailed individual stimulated recall interviews (Nguyen et al., 2013) and an analysis of the lesson using the lesson observation tool in Appendix D.

The first lesson in Cycle 1 constituted a pilot to allow refinement of the implementation of the VIP for the subsequent cycles, and to clear any misconceptions which may have arisen. At the end of the pilot lessons, I met with the seven participating teachers to conduct a focus group interview. The focus group interview reflected on the extent to which the implementation of the pilot lessons conformed to a visual approach to teaching number sense and how the participants could refine the implementation of the remaining VIP lessons.

The participating teachers then taught the first and second official lessons for the purpose of data collection in this research study. The third official lesson was taught in order to deepen the learners' conceptual understanding of number sense themes, but was not analysed. Video-stimulated recall interviews were conducted with the seven participating teachers immediately after they taught the pilot as well as the first and second official lessons of the VIP respectively.

3.4.2.6 Phase Six: Post-test administration to RUMEP schools and one control group

A post-test, which was identical to the pre-test, was written by the learners of the seven selected RUMEP teachers in order to compare the results to the pre-test, as a means of measuring any discernable change in learner performance as a result of the implementation of the VIP. The same control group mentioned in Phase Four also wrote this test.

3.5 DATA COLLECTION AND ANALYSIS

3.5.1 Techniques / Tools for Data Collection

3.5.1.1 Introduction

Data for the qualitative analysis was collected by observing the VIP lessons of the seven participants and interviewing them afterwards. Data for the quantitative analysis was collected from the pre- and post-tests that the learners of the participants wrote.

3.5.1.2 Observations

For each participating teacher, three lessons were observed and video-recorded. This thus made a total of twenty-one video recordings. Only fourteen of these video recordings were analysed byusing the lesson observation tool in Appendix D in conjunction with the analytical tool in

Appendix C. For details of the analysis see Section 3.5.2. The seven video recordings of the pilot lessons were only used during the focus group interview that was conducted with the participants, to refine the implementation of the follow-up VIP lessons.

3.5.1.3 Interviews

Data was collected through fourteen individual video-stimulated recall interviews with the participating RUMEP teachers during the reflective workshops after each lesson. Nguyen et al. (2013, p. 2) defines video-stimulated recall interviewing as a data collection method which "involves video-taping teachers during their normal teaching duties then playing them the video recordings of their own behaviour." These interviews served as reflection exercises where the teachers and I engaged with the recorded lessons as we were watching certain selected video moments to stimulate recall on the part of the participant. The engagements entailed the answering of questions on the use of visualisation processes after watching the video moments, as per the semi-structured interview schedules (Appendix E). After Cycle 1, a focus group interview with the seven participating teachers was conducted and video-recorded in order to collectively reflect on how the pilot lesson was taught and whether the use of visualisation processes could be refined for the remaining VIP lessons.

3.5.1.4 Tests

Pre- and post-tests on number sense (Appendix B) were administered to the learners of the seven participating RUMEP teachers to measure any discernable change in learner performance as a result of the implementation of the VIP. The same test served as the pre- and post-test. The test consisted of number sense items sourced from the Number Sense Item Bank designed by McIntosh et al. (1997), as depicted in Figure 3.1 below. Number sense items are <u>special questions</u> designed to test the conceptual understanding of learners. These questions are set in non-routine environments that are not familiar to learners and they elicit components of number sense (McIntosh et al., 1997).

The test was customized for South African learners and questions on four main number sense topics were set, namely whole numbers, fractions, decimals and percentages (Appendix B). The test counted 50 marks and was written for the duration of an hour.

Figure 3.1 indicates that the number sense framework of McIntosh (1992) in Table 2.3 forms the foundation from which McIntosh et al. (1997) derived the six strands of number sense in Table 2.4. The Number Sense Item Bank of McIntosh et al. (1997) was furthermore informed by the six strands of number sense. The pre- and post-test questions were sourced from the Number Sense Item Bank.

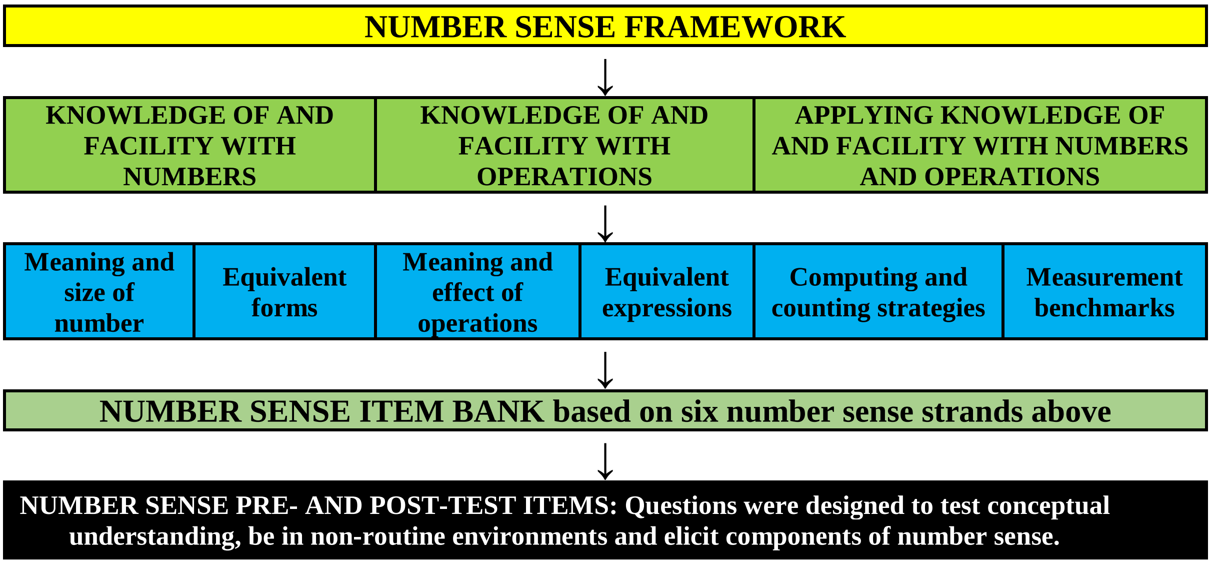


Figure 3.1: An illustration of how the pre- and post-test items were derived from the number sense framework of McIntosh (1992)

3.5.2 Data Analysis

Appendix C was used to check which number sense themes and visual teaching strategies were evident when the teachers taught the lessons prescribed by the VIP. The evidence was recorded as per Part A of Appendix D. Part B of Appendix D was used to analyse the extent to which the criteria of teaching proficiency (Kilpatrick et al., 2001) were evident in the observed lesson.

Merriam (2002) advises that for qualitative research, data analysis should be done simultaneously with data collection. She insists that simultaneous data collection and analysis allows the researcher to make adjustments along the way and to test emerging concepts and categories against subsequent data. Although I made use of the analytical tools (Appendices C and D) to analyse the video-recordings of the two official lessons, I remained open to concepts and categories that I did not anticipate emerging.

Throughout the data analysis, care was taken to organise excerpts from the transcribed video-stimulated recall interviews into groups of emerging concepts and categories. The key emerging concepts and categories centred on the views of the seven RUMEP teachers regarding the use of visualisation as a teaching strategy to enhance number sense. It informed the qualitative analysis of the transcribed video-stimulated recall interviews. The transcripts of the video-stimulated recall interviews were therefore carefully organised into categories that reflected the participants' views on visual teaching to bring about the acquisition of number sense.

I identified the following six categories:

- Insights of the participant
- Subjective meanings of the participant's responses
- Errors and misconceptions of the participant
- Consistencies in the participant's responses
- Inconsistencies in the participant's responses
- Common elements of the participant's responses

The following two subcategories were furthermore identified:

- Clarifications of the participant's responses (as a subcategory of insights)
- Perceptions of the participant (as a subcategory of subjective meanings)

The six categories and two subcategories were then entered into a qualitative data analysis computer software package called NVivo 12. The software was designed to assist qualitative researchers when deep levels of analysis on small or large volumes of text-based information are needed. The fourteen transcripts of the video-stimulated recall interviews concerning the two official lessons of the VIP were uploaded to NVivo 12 for qualitative analysis in terms of the six categories and two subcategories.

Every line of each transcript was scrutinized for inclusion into one of the categories or subcategories as a means of coding. The software produced copies of every category and subcategory with the relevantly coded references. These copies were printed and used to analyse the responses of the participating teachers to the semi-structured and video-stimulated recall interviews. Afterwards, the printed copies of the categories and subcategories were archived.

The pre- and post-test scripts were marked immediately after they were administered to the learners of the seven participating RUMEP teachers. A statistical analysis using descriptive statistics was used to analyse the test responses.

3.6 ETHICS AND VALIDITY

3.6.1 Ethical Considerations of the Research Study

This research study supported the following ethical principles:

3.6.1.1 Respect and dignity

I communicated the exact content of the research to the participants, not withholding any information pertaining to their participation or anything that could influence their choice to partake in the project. I made it explicitly clear to them that they had the right to withdraw their participation at any time. Their anonymity was guaranteed. The collected data was kept confidential and was only shared with my supervisor. Pseudonyms were used in the final write-up of the thesis.

3.6.1.2 Transparency and honesty

Written consent was requested and received from the Head of Department (HOD) of the Northern Cape Department of Education. I furthermore requested and received written consent from the principals of the Northern Cape schools where the participants are employed. The participants were informed that their participation was voluntary. The participants were also allowed to check and sign off on the transcribed interviews in order to ascertain whether they were a true reflection of the information provided to the researcher. The transcripts were archived after use.

3.6.1.3 Accountability and responsibility

It was my responsibility to reduce power relations as much as possible in order to facilitate an environment within which the participants were encouraged to give their honest opinion without fear of any repercussions. I assured them that their participation in this study would in no way have any compromised impact on their employment. As I am the RUMEP lecturer of the seven participants, it was my responsibility to also assure them that their participation in this research study would in no way impact negatively on their studies. These assurances were given as a letter written and signed by myself and my supervisor. The participants voluntarily consented to participate in the research study based on the given assurances. (See Appendices G and J.)

3.6.1.4 Integrity, academic professionalism and researcher positionality

I upheld the professional and academic standards and integrity expected by Rhodes University by adhering to the rules and standards as laid down by the institution. My findings were based on authentic data which I collected. My analysis was based on my empirical work and not on my own assumptions and opinions.

3.6.1.5 Confidentiality and anonymity of the participants

In order to guarantee anonymity and confidentiality of the participants, I used pseudonyms such as T1, T2 or T3 to represent their identities. The names of the schools were also kept anonymous in this study, by referring to the experimental schools as ES1, ES2, etc. and the control school as CS1. The video recordings were only used for analysis purposes with the participants. They were not used for any other purposes. If I wish to use them for academic purposes such as a conference presentation, I will seek specific consent from the teachers when the time comes.

3.6.1.6 Security of the data

I made it my responsibility to safely secure the data of this research on the hard drive of my computer and on an external hard drive for five years.

3.6.1.7 Informed Consent

The principle of informed consent was followed to ensure that the dignity of the participants in this research study was protected. Informed consent constitutes the right of any person to determine whether he/she wants to participate in a research project. Freedom of choice and self-determination are therefore at the heart of informed consent (Ruane, 2005). In upholding the principle of informed consent, I fully informed all stakeholders about all aspects of the research project. An informed consent letter was disseminated to the following stakeholders:

- The seven participating RUMEP teachers (Appendix G)
- The learners of the seven participants who wrote the pilot, pre- and post-test (Appendix H)
- The parents of the participating learners (Appendix I)
- The five teachers from the well-resourced schools who assisted in crafting and drafting the VIP (Appendix J)
- The principals of the seven RUMEP teachers (Appendix K)
- The Head of Department of the Northern Cape Department of Education (Appendix L)

Sieber (1992) advises that the following points of information be contained within a letter of consent:

- Identification of the researcher
- Explanation of the purpose of the study
- Request for participation, mentioning the right to withdraw at any time without impunity
- Explanation of research method
- Duration of research participation
- A description of how confidentiality will be maintained
- Mention of the subject's right of refusal without penalty
- Mention of the right to withdraw one's own data at the end of the session
- Explanation of any risk involved in the research project
- Description of any feedback and benefits to subjects
- Information on how to contact the person designated to answer questions about the subjects' right or injuries, and
- An indication that subjects may keep a copy of the consent

The informed consent letters of this research study conformed to the requirements for ethically responsible research as envisaged by Sieber (1992) above.

3.6.2 Validity of the Research Study

Creswell and Miller (2000, p. 124) affirmed that "there is general consensus …that qualitative inquirers need to demonstrate that their studies are credible." Qualitative researchers therefore employ one or more of a vast array of procedures to demonstrate validity of their studies. This includes member checking, triangulation, thick description, peer reviews and external audits.

3.6.2.1 Member Checking

Member checking was employed to ensure validity of this research study as participant observations and video-stimulated recall interviews were conducted. According to Creswell and Miller (2000), member checking is a process whereby data and interpretations are taken back to the participants in the study so that they can confirm the credibility of the information and narrative account.

3.6.2.2 Triangulation

Triangulation is a process whereby one searches for convergence among the participants, according to Denzin (1978). The validity procedure was a fitting one for this research study as seven teachers were observed and interviewed. The participants' views and responses converged in critical areas concerning the use of visualisation processes to enhance the teaching of number sense. The NVivo 12 software package discussed above in section 3.5.2, produced a cluster of coded references for every participating teacher under a certain category or subcategory. An analysis of these references enabled me to see how the participants agreed on aspects of the investigation into the role of visualisation processes as a strategy to enhance the teaching of number sense.

3.6.2.3 Thick description

Thick description was also employed to ensure validity of the study. Creswell and Miller (2000) believe that vivid detail is a good strategy to help readers understand that the account is credible. This procedure furthermore assisted in contextualising the participants in the study. The views and responses of four of the seven participants were analysed individually and in-depth about the use of visualisation processes as a result of participating in the VIP. A summary of the views and responses of the other three participants were then made. This ensured thick description of the views and responses of the participants to increase the credibility of the narrative in the research study.

3.6.2.4 Validity of the pre- and post-test

To ensure the validity of the pre- and post-test on number sense, a pilot test was written by five learners randomly selected from one of the RUMEP schools that was not included as a participant of this study. The pilot test was used to assist in flagging any ambiguities of interpretation which may have existed to be corrected before the tests were administered to the experimental and control group of learners.

The learners of the control group were orientated to check for ambiguities of interpretation and to verbally report such during the administration of the test. The five learners did not report any ambiguities of interpretation during the administration of the test. The five tests were immediately marked and then later archived after the experimental groups completed the tests.

3.7 CONCLUSION

This chapter has located the research study within the interpretive paradigm. Furthermore, the mixed method approach was described and justified more specifically. The research instruments used to collect the data were explained.

The research design was thoroughly discussed in its different phases. The manner in which data was collected for qualitative and quantitative analysis was clearly set out. Lastly, issues of ethical concern and validity were also addressed.

The next chapter describes the qualitative and quantitative analysis of the data collected.

CHAPTER 4

DATA ANALYSIS AND DISCUSSION

4.1 INTRODUCTION

In this chapter, the data collected by using the methodology discussed in Chapter 3 are analysed. This data analysis consists of collation, presentation and discussion of the results that were obtained from the focus group interview with the seven participants, the fourteen observations of the seven participants' VIP lessons and the fourteen video-stimulated recall interviews with the seven participants. The data are analysed by looking at the participants' experiences and views on engaging the VIP. This serves as the qualitative analysis of my case study which is an investigation into the role of visualisation processes in enhancing the teaching of number sense. The results of the number sense pre- and post-test which were administered to the learners of the participating teachers, are then analysed. The analysis of the pre-and post-test on number sense is quantitative.

The data analysis started after the implementation of Phase Four of the research design and continued throughout the other phases. I made use of Appendix D to observe <u>the first and second official lessons</u> per participant. Fourteen lesson observations and corresponding video-stimulated recall interviews were done as part of the data collection. In this chapter, the fourteen video-stimulated recall interviews are coded as Teacher 1, Interview 1 (T1, I1) and Teacher 1, Interview 2 (T1, I2). The same format of coding is repeated for the interviews with all seven participating teachers. Printed versions of the fourteen lesson observations and transcripts of the corresponding video-stimulated recall interviews are being archived. The electronic versions of the lesson observations and transcripts of the video-stimulated recall interviews are stored on an external hard drive.

The lesson observation tool (Appendix D) was designed to check whether the lesson is aligned with the analytical framework for making use of visual strategies (Appendix C). Furthermore, it was also designed to look for evidence of the teacher's proficiency in using visual strategies in the teaching of number sense within the context of Kilpatrick et al.'s (2001) five teaching proficiency components. The unit of analysis is the views and responses of the participating

RUMEP teachers about the use of visualisation processes as a result of participating in the VIP. The qualitative analysis was therefore done by extracting these views and responses from the seven participating RUMEP teachers and discussing it teacher by teacher. A further unit of analysis is the pre- and post-test results and the comparison thereof, which constitutes the quantitative analysis of the research study.

In order to analyse the views and responses of the seven teachers about the use of visualisation processes as a result of participating in the VIP, the narrative is structured across the teachers. The views and responses of four teachers are discussed individually and in depth. The views and responses of the remaining three teachers are then only summarised to complete the narrative. This summary was necessary to ensure that this thesis does not stretch beyond its word limitation and stays within the intended scope. The discussion focuses on the first and second official VIP lessons respectively. The completed lesson observations (Appendix D) are analysed first to ascertain how the teachers *responded* to the prescribed VIP lessons. The transcripts of the corresponding video-stimulated recall interviews are then analysed to ascertain the teachers' *reflection* upon the lessons.

Gall, Borg and Gall (1996, p. 767) define qualitative research as an "inquiry that is grounded in the assumption that individuals construct social reality in the form of meanings and interpretations, and that these constructions tend to be transitory and situational. The dominant methodology is to discover these meanings and interpretations by studying cases intensively in natural settings and subjecting the resulting data to analytic induction". In analysing the transcripts of the video-stimulated recall interviews, I therefore first looked for concepts which I could group into categories in order to "facilitate interpretation for meaning" (Hart, Smith, Swars & Smith, 2009, p. 29). The categories and subcategories constitute the conceptualization of the interview data as "codes are abstractions of the data, particularly patterns in the data, not mere summaries" (Glaser, 2010, p. 27).

The concepts which were found in the interview transcripts were grouped, and are described, discussed and interpreted in the following categories:

- Insights of the participant about the use of visualisation processes as a result of participating in the VIP.

- Clarifications of the participant's responses (as a subcategory of insights) about the use of visualisation processes as a result of participating in the VIP.
- Subjective meanings of the participant's responses about the use of visualisation processes as a result of participating in the VIP.
- Perceptions of the participant (as a subcategory of subjective meanings) about the use of visualisation processes as a result of participating in the VIP.
- Errors and misconceptions of the participant about the use of visualisation processes as a result of participating in the VIP.
- Consistencies in the participant's responses about the use of visualisation processes as a result of participating in the VIP.
- Inconsistencies in the participant's responses about the use of visualisation processes as a result of participating in the VIP.
- Common elements of the participant's responses about the use of visualisation processes as a result of participating in the VIP.

The qualitative analysis is first presented by discussing the outcome of the focus group interview, as well as the views and responses of the seven participating teachers about the use of visualisation processes as a result of participating in the VIP. Afterwards, the quantitative analysis is presented and discussed. A conclusion is then drawn to summarise the narrative and to give an account of the emerging concepts and categories of the qualitative and quantitative data.

4.2 THE FOCUS GROUP INTERVIEW

4.2.1 Introduction

The focus group consisted of the researcher, the five members of the collaboration team and the seven participating teachers. The meeting was arranged after the seven participants had completed teaching the pilot lesson of the VIP and the first semi-structured, video-stimulated recall interview was done. The video-stimulated recall interview served as a reflection on the lesson that the participant taught. The discussion of the focus group primarily focused on refining the implementation of the VIP and clearing any misconceptions that the seven participants may have had. In this regard, the researcher and the collaboration team, in conjunction with the seven participating teachers, critically reflected on the participating teachers' execution of the pilot lesson.

Certain video moments of the reflective workshops with the seven participants were replayed and then discussed in an open session to allow everyone an opportunity to comment and advise regarding the use of visual teaching strategies. The lesson observation tool (Appendix D) was supplied to the participants of the focus group. They were requested to complete the document as they watched the selective replay of the reflective workshops that were conducted with each of the seven participating teachers. In completing Appendix D, the participants used the analytical framework (Appendix C) as a guide to evaluate the extent to which the participating teachers made use of visual strategies for the teaching of number sense understandings. Afterwards, the completed lesson observations were discussed in the focus group.

The focus group discussion added value regarding the implementation of the VIP during the follow-up lessons. The focus group reflected on the quality of the visual teaching strategies which were employed, the time management of the participating teachers during the lesson and the general teaching proficiency which was displayed.

4.2.2 Quality of the Visual Teaching Strategies

The collaboration team criticized the manner in which one of the participating teachers made use of a fraction circle. The fraction circle was cut in equal parts marked as common fractions. It was therefore intended to be used for the teaching of common fractions. Instead, the teacher employed the visual aid for the teaching of decimal fractions without the parts being marked as decimals. In the end, the teacher was trapped in his explanation and reverted to the conventional way of teaching the division of decimals in an algorithmic manner.

Most teachers were applauded for preparing visuals before the lesson started, as this increased learner participation. The example of the teacher who made number cards for the learners to go and paste on a visual number line on the writing board in answer to the leading questions of the teacher, was regarded as a good standard for using visual aids. The collaboration team regarded the abundant use of visual aids by the participating teachers during the pilot lesson as being responsible for facilitating the conceptual understanding of learners, making them more responsive and stimulating mathematical thought as far as number sense understandings are concerned.

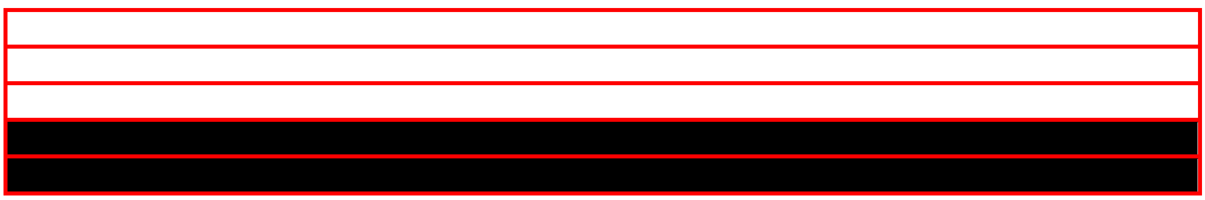
4.2.3 Time Management

The collaborating teachers from the well-resourced schools criticized the amount of time that most of the participating teachers spent on the introduction and mental mathematics part of the pilot lesson. The mental mathematics was treated as an activity for classwork instead of an opportunity to jog the minds of the learners and prepare them for the topic of the lesson. Mental mathematics should put time pressure on learners for it to have any benefit. Most of the teachers could not complete the pilot lesson due to the inefficient use of time by allocating the incorrect amount of time to the different elements of the lesson plan. The input from the collaborating teachers was very helpful in ensuring that better time management would be practised during the first and second official lessons of the VIP.

4.2.4 Teaching Proficiency

The collaborating teachers noticed that some of the teachers used wrong and inconsistent mathematical language whilst teaching decimal fractions during the pilot lesson. Instead of reading a decimal fraction as two comma six five (2,65), some teachers would pronounce the decimal as two comma sixty-five or two point sixty-five. The importance of the proper use of mathematical language was emphasized.

The reading of a common fraction as numerator over denominator (2 over 5) was also corrected. The meeting agreed that referring to fractions in this manner has no mathematical meaning. The desired naming convention should be numerator out of denominator (2 out of 5), which could be presented visually as the shaded parts of the diagrammatical representation below:



It was further noted that some of the teachers did not present their work on the writing board in a neat and orderly manner. The team pointed out that the writing board is a resource to be used for optimal benefit of the learners. Teachers also model behaviour to learners and as such, their work should be presented in an orderly manner and neatly. It was advised that teachers could divide the writing board in columns and start writing from the left upper corner. Completed work could then stay on the writing board as the lesson progresses to provide the learners with an opportunity for constant reflection. The participating teachers were advised to create a balance between the learners listening to the teacher and doing writing work in their books.

4.2.5 Results of the Focus Group Interview

The focus group interview generated a shared understanding about how to optimally use visualisation processes for the teaching of number sense understandings. The experience of the collaboration team regarding the use of visualisation processes in their classrooms, benefitted the seven teachers who participated in the VIP. The collaboration team offered several perspectives on how to contextualise the use of different visual aids.

All seven participants shared their feelings, opinions and perspectives in an attempt to gain better understanding of how to teach the remaining three VIP lessons. The lesson observation tool (Appendix D) provided the necessary prompts for individuals in the focus group to open up and engage with each other. This open discussion of the focus group provided the seven participants who taught the pilot lesson with an opportunity to seek clarification on the best way to implement the visual strategies relevant to the teaching of number sense understandings.

4.3 THE VIEWS AND RESPONSES OF THE PARTICIPATING TEACHERS

4.3.1 Teacher 1

4.3.1.1 First Official Lesson

(a) *Observation of Teacher 1*

The lesson topic was 'addition and subtraction' as per the VIP (Appendix A).

(i) The alignment of the participant's lesson with the analytical framework (Appendix C):

The lesson of Teacher 1 (T1) displayed the following number sense themes in her teaching of addition and subtraction:

- Operating on whole numbers: T1 used a physical balance scale to facilitate a mathematical environment for learners to explore the addition and subtraction of whole numbers. The physical balance scale provided a visual aid that made the addition and subtraction of whole numbers meaningful within the context of a mathematical equation where the left-hand side always has to equate the right-hand side. Learners were able to find the variable in the equation with ease by compensating for the difference, in order to balance the two sides.

  T1 also made use of the calibrated number line to teach the rounding of whole numbers for the sake of simplifying the addition thereof. The calibrated number line provided a visual aid and supported the operation of adding whole numbers more easily through rounding and compensating.

- Place Value: T1 used the calculator for playing the ZAP game. She verbally called out and wrote the number 12 508 (twelve thousand five hundred and eight) on the writing board, which the learners put in their calculators. She then asked them to remove the 5 as a digit by using one operation on their calculators. The 5 would then be 'zapped', as the name of the game suggests. The learners had to know place values and that they should use the subtraction operation in order to remove the specific digit and replace it with nought. After allowing the learners some time for processing, one of them gave the correct answer as 12 008on behalf of his group.

- Operating on fractions: T1 made use of the calibrated fractional number line which showed different start and end points in order to facilitate an understanding of the counting sequence of equal-sized fractions. One learner was invited to the writing board to demonstrate the addition and subtraction of fractions by making use of the calibrated fractional number line. She had no problem in finding the difference between $\frac{6}{7}$ and $\frac{2}{7}$ as $\frac{4}{7}$ by making use of the visual aid.

  T1 also used fraction circles to facilitate an environment for the learners to operate on fractions. She drew a fraction circle and decorated it like a pizza. She invited three learners to the writing board and then shared the pizza amongst them. T1 verbally discussed the act of sharing as she cut the different parts according to the problem scenario set forth in the VIP (Appendix A). She facilitated the conceptual understanding of fractions being 'parts of a whole' very efficiently. Through this diagrammatical representation, learners were better equipped to add and subtract the different parts.

- Operating on decimals: T1 did not use any visual strategy other than the decimal place value chart to add and subtract decimals. While decimals were added and subtracted algorithmically, the decimal place value chart was used. T1 could have used decimal fraction strips, decimal fraction circles or any other diagrammatical representation of decimal fractional parts within a whole unit to teach equivalent decimal fractions, in order for learners to understand the relationship between the equal parts.

(ii) The participant's proficiency in using visual strategies in the teaching of number sense:

An analysis was done to look for evidence of the teacher's proficiency in using visual strategies in the teaching of number sense within the context of the five teaching proficiency components of Kilpatrick et al. (2001). This analysis made use of a rating scale from 1 to 3, as outlined in Appendix D.

T1 was rated as follows:

TEACHING PROFICIENCY COMPONENT	ACTUAL RATING
Conceptual understanding of the core knowledge required in the practice of teaching	Abundant evidence (rating 3)was observed of the teacher's skill in connecting her own mathematical content knowledge with her understanding of how the mathematical knowledge of her learners develops and the most efficient way of teaching them in a visual manner. This connection of the said elements manifested in the visualisation processes that T1 used during the lesson to ensure optimal learning outcomes.
Fluency in carrying out basic instructional routines	Abundant evidence (rating 3) was observed of the teacher's skill in managing her classroom throughout the whole lesson. A mathematical and visually rich environment was facilitated by the teacher intentionally allocating diverse visual aids to the learners in the different groups, before the lesson started. T1 responded to learners of different abilities by making use of visualisation processes. She ensured learner participation in solving problems with visual aids. T1 furthermore demonstrated several ways of dealing visually with teaching problems during the lesson.
Strategic competence in planning effective instruction and solving problems that arise during instruction	Abundant evidence (rating 3) was observed of the teacher's skill in implementing visual strategies at the level of the learners' understanding. The manner in which T1 dealt with fractions, is reminiscent of best practice way of teaching the topic visually - by making use of fraction circles.
Adaptive reasoning in justifying and explaining one's instructional practices and in reflecting on those practices so as to improve them	Some evidence (rating 2) was observed of T1's skill in reflecting on her own lesson and in seeking ways to improve the visual teaching of it. She continually sought confirmation from the learners as to whether they understood the topic under discussion in order to inform herself about the need for teaching more visually.
Productive disposition toward mathematics, teaching, learning and the improvement of practice.	Some evidence (rating 2) was observed of T1's awareness of how mathematics, her understanding of how her learners think and the visual manner in which she teaches, fit together to make sense.

(b) <u>*Results and discussion of the video-stimulated recall interview with Teacher 1*</u>

T1's responses to the semi-structured and video-stimulated recall interview are categorized and discussed as follows:

- **Insights of the participant about the use of visualisation processes as a result of participating in the VIP:** The participant regarded the use of the number line as being important to enhance the number sense of her learners. She saw that the learners were able to skip count on the number line in order to calculate the difference between two whole numbers. She furthermore saw the use of counters as also being productive for the exercise of counting numbers correctly.

"Yes, it helped a lot…the counters. I can see all the learners are participating and if, they work in groups, if the other one counts and miss the other number, the others would say, take this. The counters are the group leader, they coach the learner to be able to count. The learners are able to see the numbers, counting the numbers." (T1, I1)

T1 regarded the balance scale as instrumental in assisting learners to seek equivalence between quantities. The number line was also used to assist with the setting of measurement benchmarks.

T1 cited various challenges with using visualisation processes. These challenges include blurry visuals and learners playing with calculators for fun and not for mathematical understanding, causing them to lose concentration. She advocated that visuals should be made of brightly coloured paper rather than ordinary white paper, in order to capture the attention of learners.

- **Clarifications of the participant's responses (as a subcategory of insights) about the use of visualisation processes as a result of participating in the VIP:** T1 clarified her insight about how the counters assisted the learners within the group work context. She regarded the counters as playing the role of actively guiding the group members. She furthermore explained the relationship between the counters and the number line as being complementary in enhancing the number sense of the learners.

  T1 explained the mathematical application of creating equivalence between two quantities through the analogy of the balance scale. As the scale only balances when the mass on both sides are equal, so the two mathematical quantities can only balance if the variable plus the known number on one side equate with the known number on the other side. This visualisation process guides the learner in solving the missing value of the variable. According to T1, the number line can be used to teach the meaning and size of numbers.

Learners are in a position to break up a whole number into two constituent parts and test the sum of the numbers through skip counting on a number line.

The participant related how the fraction circle was used to explain parts of a whole in a practical manner. The quarters and eighths are compared to deduce the equivalence of different fractions based on congruent parts of the circle. The clarification also addressed the use of fraction circles to demonstrate the sum of different fractions. When setting measurement benchmarks, learners could use the number line to plot the bigger number and the relatively smaller one, by virtue of their knowledge of the relative positions of quantities on the number line.

- **Subjective meanings of the participant's responses about the use of visualisation processes as a result of participating in the VIP:** T1 was convinced that the use of the balance scale is a means of relating numbers to the weight of objects. She has indicated that the language of mathematics should be mastered by learners as early as possible to facilitate better mathematical problem-solving skills. T1 saw the practical use of fraction circles as foundational to the learners' ability to write out the addition of fractions algorithmically. She regarded the sacrifice of investing time in the preparation of visual aids as worthwhile because of the empowering nature of the visuals as far as the learning of the learners is concerned.

- **Perceptions of the participant (as a subcategory of subjective meanings) about the use of visualisation processes as a result of participating in the VIP:** T1 perceived the use of the visual aids (number line, counters, fraction circles and balance scale) as being instrumental in the learners' acquisition of number sense. She emphasized the practical use of fraction circles resembling pizzas to be shared among a group of friends.

  "Yes, firstly, there are two pieces. So, the two pieces are being divided into different parts. The first one into four parts, and then the other one into eight parts. So I want them to know that even though that one is four parts, and that one is eight parts, they are the same pieces. The four parts, one quarter, is the same as two-eighths of the second pizza. And then, the one who gets a quarter of the first pizza, gets two-eighths of the second pizza." (T1, I1)

T1 acknowledged the positive role which the use of visualisation processes played in increasing the level of learner participation during the lesson.

- **Errors and misconceptions of the participant about the use of visualisation processes as a result of participating in the VIP:** No errors or misconceptions were found during the video-stimulated recall interview regarding the first official lesson.

- **Consistencies in the participant's responses about the use of visualisation processes as a result of participating in the VIP:** The participant consistently applauded the use of visual strategies in the teaching of number sense. She ascribed the use of visual aids to the learners' enjoyment of her mathematics lessons.
  "Yes. It works a lot and the learners enjoy my lesson and I won't be absent any day because we see that they participate a lot. They are able to talk to each other, they are free…they are free." (T1, I1)

- **Inconsistencies in the participant's responses about the use of visualisation processes as a result of participating in the VIP:** No inconsistencies were found in T1's responses during the video-stimulated recall interview.

- **Common elements of the participant's responses about the use of visualisation processes as a result of participating in the VIP:** T1 saw the time-consuming nature of preparing visual aids as a worthwhile exercise as it positively impacts on the learning of learners.

4.3.1.2 Second Official Lesson

(a) *Observation of Teacher 1*

The lesson topic was 'multiplication and division' as per the VIP (Appendix A).

(i) The alignment of the participant's lesson with the analytical framework (Appendix C):

The lesson of Teacher 1 (T1) displayed the following number sense themes in her teaching of multiplication and division:

- Operating on whole numbers: T1 used a spider diagram in combination with the array representation to facilitate a platform for multiplying and dividing whole numbers. The spider diagram provided a visual representation that displayed number patterns in both its input and output sides. Use of the array and grid representations complemented the use of the spider diagram, which assisted with multiplying and dividing whole numbers.

- Equivalent numerical forms (including decomposition and recomposition): T1 used a diagrammatical representation with smaller rectangles within a big rectangle, in order to facilitate an environment within which the number of small rectangles could be represented in different ways. The number of small rectangles of different colours were represented in multiple ways in relationship to the big rectangle.

- Operating on fractions: T1 also made use of the diagrammatical representation with smaller rectangles within a big rectangle to teach equivalent fractions, in order for learners to understand the relationship between the numerators and denominators of equivalent fractions.

(ii) The participant's proficiency in using visual strategies in the teaching of number sense:

An analysis was done to look for evidence of the teacher's proficiency in using visual strategies in the teaching of number sense within the context of the five teaching proficiency components of Kilpatrick et al. (2001). This analysis made use of a rating scale from 1 to 3, as outlined in Appendix D.

T1 was rated as follows:

TEACHING PROFICIENCY COMPONENT	ACTUAL RATING
Conceptual understanding of the core knowledge required in the practice of teaching	Abundant evidence (rating 3) was observed of the teacher's skill in connecting her own mathematical content knowledge with her understanding of how the mathematical knowledge of her learners develops and the most efficient way of teaching them in a visual manner. The visual aids, like the spider diagram and diagrammatical representation, were carefully chosen as they were the most efficient way of teaching multiplication and division.

Fluency in carrying out basic instructional routines	Abundant evidence (rating 3) was observed of the teacher's skill in managing her classroom throughout the whole lesson. A mathematical and visually rich environment was facilitated by the teacher intentionally allocating diverse visual aids to the learners in the different groups, before the lesson started. T1 responded to learners of different abilities by making use of visualisation processes. She ensured learner participation in solving problems with visual aids.
Strategic competence in planning effective instruction and solving problems that arise during instruction	Abundant evidence (rating 3) was observed of the teacher's skill in implementing visual strategies at the level of the learners' understanding. The spider diagram and diagrammatical representation were appropriate for assisting the learners in understanding how to multiply and divide whole numbers and also how to handle equivalent fractions.
Adaptive reasoning in justifying and explaining one's instructional practices and in reflecting on those practices so as to improve them	Some evidence (rating 2) was observed of T1's skill in reflecting on her own lesson and seeking ways of improving the visual teaching of it.
Productive disposition toward mathematics, teaching, learning and the improvement of practice.	Some evidence (rating 2) was observed of T1's awareness of how mathematics, her understanding of how her learners think and the visual manner in which she teaches, fit together to make sense.

(b) *Results and discussion of the video-stimulated recall interview with Teacher 1*

T1's responses to the semi-structured and video-stimulated recall interview are categorized and discussed as follows:

- **Insights of the participant about the use of visualisation processes as a result of participating in the VIP:** The participant regarded the use of the spider diagram in conjunction with a learner using her fingers as being helpful in enhancing the multiplication and division of whole numbers.

  "Okay, the learner is using this spider diagram. She can see the number and she can use the two sides together. I can see the learner is also using the fingers there with multiplying and dividing. There are two signs there. The learner can use the input and calculate the first sign…and get the answer for the first sign…and then use that number for the second sign and get the output. So, the learner can also have the sense of numbers when using the fingers and the spider diagram." (T1, I2)

T1 appreciated the diagrammatical representation which consisted of smaller rectangles shaded in different colours. According to her, the learners could form different fractions by using the differently shaded parts. T1 saw the diagrammatical representation as a visual aid to help learners identify halves, thirds and quarters.

- **Clarifications of the participant's responses (as a subcategory of insights) about the use of visualisation processes as a result of participating in the VIP:** T1 clarified her insight about how the dotted array could assist learners in understanding the commutative principle of multiplication. She furthermore clarified how the use of sticks could facilitate the decomposing and counting of big numbers.

- **Subjective meanings of the participant's responses about the use of visualisation processes as a result of participating in the VIP:** T1 was convinced that the use of the dotted array assisted the learners with sharing big numbers in equal parts. Sharing is achieved by grouping the dots as a means to divide whole numbers.

- **Perceptions of the participant (as a subcategory of subjective meanings) about the use of visualisation processes as a result of participating in the VIP:** T1 perceived the use of the correct mathematical language as an instrument to assist learners with writing the common papers as an external form of assessment. She made flashcards with the relevant mathematical terms to be used when multiplying and dividing whole numbers. T1 lamented the difficulty inherent to teachers' production of visual aids in general, but praised the benefits of teaching visually. She perceived the visual aids as a tool which assisted the learners with retaining mathematical concepts. T1 furthermore experienced how learners loved playing with the visual aids as part of the mathematical task at hand.

- **Errors and misconceptions of the participant about the use of visualisation processes as a result of participating in the VIP:** T1 confused mathematical signs with mathematical operations. In her explanation of how she has made use of visualisation processes, she would time and again refer to operational signs instead of one of the four mathematical operations (addition, subtraction, multiplication and division).

"But when they call it a dividend, they know that a dividend is the first number. It is the first number in the number sentence for the operational sign…division." (T1, I2)

- **Consistencies in the participant's responses about the use of visualisation processes as a result of participating in the VIP:** The participant applauded the use of visual strategies in teaching mathematics in general. She cited the example of using models in the practical teaching of 3D objects. According to her, the learners would be able to distinguish the faces, vertices and edges of the object more successfully if they use a model of the object.

  "Making skeletons about the 3-D…they are able to know the edges…when they see them…counting them. The vertices…they can count them…using those skeletons for the 3-D. They are next to them. The face…they can count them. The type of the 2-D shape out of the 3-D object…they can know this is a cube…but on the cube we are having the squares. How many squares…they can count them…and the visuals are so very much important." (T1, I2)

- **Inconsistencies in the participant's responses about the use of visualisation processes as a result of participating in the VIP:** In explaining how her learners used visuals to deduce equivalent forms of fractions, T1 resorted to the algorithmic method of multiplying the numerator and denominator with the same value. This constitutes an inconsistency, as the expectation was to extract her views on how visualisation processes could assist learners with forming equivalent fractions.

- **Common elements of the participant's responses about the use of visualisation processes as a result of participating in the VIP:** T1 saw the time-consuming nature of preparing visual aids as a good trade off compared to the quality of the learning experience for the learners.

4.3.1.3 General Findings

T1 was consistent in aligning both lessons with the analytical framework (Appendix C). Her choice of visual teaching strategies was appropriate for the number sense themes displayed during the lessons. The only challenge for T1 was finding a visual strategy for operating on decimals. T1 was rated favourably regarding her proficiency in using visual strategies in the teaching of number sense, as a result of her participating in the VIP.

The insights which T1 has demonstrated during the video-stimulated recall interviews, ranged from the value she could see in the use of visual aids, to a critique on the preparation of these visual aids. She confirmed her belief in the use of visual aids as a tool to enhance the teaching and learning of number sense. She, however, advocated that enough time should be allowed for preparing the visual aids.

The interviews with T1 also flagged some errors and misconceptions regarding her view and labelling of the four mathematical operations. T1 also confused the algorithmic method of deducing fractional forms with the visual way of finding equivalent fractions.

4.3.2 Teacher 2

4.3.2.1 First Official Lesson

(a) *Observation of Teacher 2*

The lesson topic was 'addition and subtraction' as per the VIP (Appendix A).

(i) The alignment of the participant's lesson with the analytical framework (Appendix C):

The lesson of Teacher 2 (T2) displayed the following number sense themes in his teaching of addition and subtraction:

- Operating on whole numbers: T2 used physical counters and the calibrated number line to facilitate a mathematical environment for learners to explore the addition and subtraction of whole numbers. The use of the physical counters complemented the use of the calibrated number line to assist the learners with the counting sequence and the addition of whole numbers.

- Operating on fractions: T2 made use of diagrammatical representation to teach the addition and subtraction of fractions. The nature of fractional parts was well taught through the use of diagrammatical representation.

- Operating on decimals: T2 made use of the decimal number line with a decimal start and end point. His choice was not a good visual strategy because the decimal number line should be calibrated to assist learners in exploring the counting sequence of decimal fractions which is a complex concept for Grade 6 learners.

(ii) <u>The participant's proficiency in using visual strategies in the teaching of number sense:</u>

An analysis was done to look for evidence of the teacher's proficiency in using visual strategies in the teaching of number sense within the context of the five teaching proficiency components of Kilpatrick et al. (2001). This analysis made use of a rating scale from 1 to 3, as outlined in Appendix D.

T2 was rated as follows:

TEACHING PROFICIENCY COMPONENT	ACTUAL RATING
Conceptual understanding of the core knowledge required in the practice of teaching	Some evidence (rating 2) was observed of the teacher's skill in connecting his own mathematical content knowledge with his understanding of how the mathematical knowledge of his learners develops and the most efficient way of teaching them in a visual manner.T2 created a visual environment for the learners to add and subtract whole numbers and fractions.
Fluency in carrying out basic instructional routines	Some evidence (rating 2) was observed of the teacher's skill in managing his classroom throughout the whole lesson. T2 responded to learners of different abilities by making use of visualisation processes. He ensured learner participation in allowing the use of physical counters within group work context.
Strategic competence in planning effective instruction and solving problems that arise during instruction	Some evidence (rating 2) was observed of the teacher's skill in implementing visual strategies at the level of the learners' understanding. T2 used diagrammatical representation as learners are generally competent in handling this visual aid to find equivalence among different fractional forms.
Adaptive reasoning in justifying and explaining one's instructional practices and in reflecting on those practices so as to improve them	Some evidence (rating 2) was observed of T2's skill in reflecting on his own lesson and seeking ways of improving the visual teaching of it. He continually sought confirmation from the learners as to whether they understood the topic under discussion.
Productive disposition toward mathematics, teaching, learning and the improvement of practice.	Some evidence (rating 2) was observed of T2's awareness of how mathematics, his understanding of how his learners think and the visual manner in which he teaches, fit together to make sense.

(b) Results and discussion of the video-stimulated recall interview with Teacher 2

T2's responses to the semi-structured and video-stimulated recall interview are categorized and discussed as follows:

- **Insights of the participant about the use of visualisation processes as a result of participating in the VIP:** The participant regarded the use of the number line as being important to assist his learners with counting forwards. He saw that most of the learners suffer from the misconception of adding the denominators of fractions and attempted to address it by making use of fraction strips when adding fractions. He furthermore saw the use of fraction circles as being productive for the exercise of assessing the size of fractional parts. T2 regarded the use of visualisation processes as beneficial to his own learning as well.

  "Even myself, as a teacher, with visual aids, I'm learning a lot because I've been preparing the lesson but what…while I was busy preparing for that lesson, I discovered that this learning aid, can be used for several activities. Then, by so doing, I'm a learner too." (T2, I1)

- **Clarifications of the participant's responses (as a subcategory of insights) about the use of visualisation processes as a result of participating in the VIP:** T2 clarified his insight about how the number line assisted the learners with associating counting forwards with addition. He furthermore saw value in the fraction strips as they emphasize the meaning of parts of a whole and equivalence between fractions.

- **Subjective meanings of the participant's responses about the use of visualisation processes as a result of participating in the VIP:** T2 was of the opinion that the fraction strips assisted the learners in comparing two fractions in their pursuit of understanding equivalence. He further saw visual aids as a tool which could facilitate self-learning due to the relationships between mathematical concepts that the learner forms while exploring the visuals.

  "The more they do it, that's when they can discover on their own. At first, it was halves, it was two equal parts, now they are measuring two parts of the four parts, is the same as one part of the two parts. So, hence we say on their own, they are hands on. Everything, they just pick it up. It's learning… on its own, from there, it just flows. These visuals, they help because things they just come out on its own." (T2, I1)

- **Perceptions of the participant (as a subcategory of subjective meanings) about the use of visualisation processes as a result of participating in the VIP:** T2 was convinced that teaching without making use of visualisation processes, tends to confuse the learners. When learners can see visuals of the concept under discussion, they acquire conceptual understanding much quicker. Allowing learners a hands-on approach with the visuals leads to self-discovery of mathematical facts.

- **Errors and misconceptions of the participant about the use of visualisation processes as a result of participating in the VIP:** No errors or misconceptions were found during the video-stimulated recall interview regarding the first official lesson.

- **Consistencies in the participant's responses about the use of visualisation processes as a result of participating in the VIP:** No noteworthy consistencies were found in T2's responses during the video-stimulated recall interview.

- **Inconsistencies in the participant's responses about the use of visualisation processes as a result of participating in the VIP:** Inconsistencies were found in T2's responses during the video-stimulated recall interview in his explanation of the value of the balance scale. He explained that the balance scale assisted the learners in understanding inverse operations. A more relevant visual aid to explain inverse operation, would be either a horizontal or vertical number line.

- **Common elements of the participant's responses about the use of visualisation processes as a result of participating in the VIP:** T2 saw the use of visual aids as a transformative exercise as it positively impacted on his enjoyment of teaching mathematics.

4.3.2.2 Second Official Lesson

(a) *Observation of Teacher 2*

The lesson topic was 'multiplication and division' as per the VIP (Appendix A).

(i) The alignment of the participant's lesson with the analytical framework (Appendix C):

The lesson of Teacher 2 (T2) displayed the following number sense themes in his teaching of multiplication and division:

- Equivalent numerical forms (including decomposition and recomposition): T2 used a diagrammatical representation with smaller rectangles within a big rectangle. This strategy facilitated a tool to represent the number of smaller rectangles in different ways. This way multiple representations for numbers could be made.

- Operating on fractions: T2 used the same diagrammatical representation with smaller rectangles within a big rectangle to represent equivalent fractions. The representation showed fractional parts within a whole unit, and as such, it assisted the learners to understand the relationship between the numerators and denominators of equivalent fractions.

- Operating on whole numbers: T2 used the spider diagram to assist with the multiplication and division of whole numbers. Due to the fact that the spider diagram is a visual representation which displays number patterns in both the input and output, the multiplication and division of whole numbers are grasped more effectively by the learners.

(ii) The participant's proficiency in using visual strategies in the teaching of number sense:

An analysis was done to look for evidence of the teacher's proficiency in using visual strategies in the teaching of number sense within the context of the five teaching proficiency components of Kilpatrick et al. (2001). This analysis made use of a rating scale from 1 to 3, as outlined in Appendix D.

T2 was rated as follows:

TEACHING PROFICIENCY COMPONENT	ACTUAL RATING
Conceptual understanding of the core knowledge required in the practice of teaching	Some evidence (rating 2) was observed of the teacher's skill in connecting his own mathematical content knowledge with his understanding of how the mathematical knowledge of his learners develops and the most efficient way of teaching them in a visual manner. T2 chose the spider diagram and diagrammatical representation (with the rectangles) as visual aids to teach multiplication and division.
Fluency in carrying out basic instructional routines	Some evidence (rating 2) was observed of the teacher's skill in managing his classroom throughout the whole lesson. T2 was ready with the relevant visual aids as the lesson unfolded. He responded to learners of different abilities by making use of relevant visualisation processes.
Strategic competence in planning effective instruction and solving problems that arise during instruction	Some evidence (rating 2) was observed of the teacher's skill in implementing visual strategies at the level of the learners' understanding. The spider diagram and diagrammatical representation were appropriate for assisting the learners in understanding how to multiply and divide whole numbers and also how to represent numbers in multiple ways.
Adaptive reasoning in justifying and explaining one's instructional practices and in reflecting on those practices so as to improve them	Some evidence (rating 2) was observed of T2's skill in reflecting on his own lesson and seeking ways to improve the visual teaching of it.
Productive disposition toward mathematics, teaching, learning and the improvement of practice.	Some evidence (rating 2) was observed of T2's awareness of how mathematics, his understanding of how the learners think and the visual manner in which he teaches, fit together to make sense.

(b) *<u>Results and discussion of the video-stimulated recall interview with Teacher 2</u>*

T2's responses to the semi-structured and video-stimulated recall interview are categorized and discussed as follows:

- **Insights of the participant about the use of visualisation processes as a result of participating in the VIP:** T2 saw that the diagrammatical representation assisted a learner in correcting a multiplication mistake. The learner discovered the result of multiplying two numbers as adding a certain number of groups of the second number. For instance, ten times ten means ten groups of ten.

  "With diagrams, we are talking about multiplication…where learners…initially you can see the learner there. She has made a mistake of twenty…instead of using times, she said ten times ten. The ten times ten is one hundred, but while she is on the board…because of visual aids…that is why she could see…it was only two groups of ten. The groups of tens…that is why she goes to multiplication. So, they can see how we use the diagrams in…according to the operations…ten times ten is not twenty…it is one hundred." (T2, I2)

- **Clarifications of the participant's responses (as a subcategory of insights) about the use of visualisation processes as a result of participating in the VIP:** The participant clarified the fact that the diagrammatical representation was valuable in helping the learner to distinguish between addition and multiplication. The visual aid assisted the learners in understanding the meaning of multiplication as being different from addition.
  "But ten plus ten…which gives us two groups of ten…will give us twenty. So they can see it simpler there, because initially…as I said…from the diagram. Using the multiplication…she said ten times ten is one hundred, but once she was there on the board using the visuals…now she discovered…what is the meaning of multiply by…what is the difference between multiply by and plus…because of those diagrams. Now you talk about two groups of ten. When you add them…it is actually addition, but it is not ten times ten." (T2, I2)

- **Subjective meanings of the participant's responses about the use of visualisation processes as a result of participating in the VIP:** T2 was convinced that the diagrams or visual aids were instrumental in helping learners understand the nature of fractions. According to him, learners discovered that the more an object was shared or cut into pieces, the smaller the pieces or fractions would be. The learners then understood that the fraction with the larger denominator is actually smaller in value.

- **Perceptions of the participant (as a subcategory of subjective meanings) about the use of visualisation processes as a result of participating in the VIP:** T2 perceived the use of visual aids as being helpful as they contribute to the learners' understanding of fractions. He believes that these visual aids provide concrete material from which learners can move to abstract and complex thinking. He experienced the visual aids as being instrumental for learners to visualise. These visualisation processes in turn, makes mathematics simpler for the learners. Mathematics then becomes fun to the learners.

- **Errors and misconceptions of the participant about the use of visualisation processes as a result of participating in the VIP:** No errors and misconceptions were found during the video-stimulated recall interview regarding the second official lesson.

- **Consistencies in the participant's responses about the use of visualisation processes as a result of participating in the VIP:** No noteworthy consistencies were found in T2's responses during the video-stimulated recall interview.

- **Inconsistencies in the participant's responses about the use of visualisation processes as a result of participating in the VIP:** No inconsistencies were found in T2's responses during the video-stimulated recall interview.

- **Common elements of the participant's responses about the use of visualisation processes as a result of participating in the VIP:** T2 regarded overcrowding and the lack of resources as the main problems as far as the use of visualisation processes in the teaching of number sense is concerned.

4.3.2.3 General Findings

T2 succeeded in aligning both lessons with the analytical framework (Appendix C). He chose good visual strategies in general, except for the visual strategy to operate on decimals. The participant was rated satisfactorily regarding his proficiency in using visual strategies in the teaching of number sense, as a result of him participating in the VIP.

The insights which T2 demonstrated during the video-stimulated recall interviews include the value of visualisation processes for the number sense understandings of the learners, the positive impact of the visual aids on his own learning as a teacher and the fact that visualisation processes added to his and the learners' enjoyment of mathematics. On the flip side, however, T2 lamented the lack of resources and overcrowding as factors which negatively impacted on the use of visualisation processes in the teaching of number sense.

Some inconsistencies were also found in T2's explanation that the balance scale facilitated an understanding of inverse operations. A number line is more strategic in explaining inverse operations, whereas the balance scale could be used to explain addition and subtraction within the context of an equation.

4.3.3 Teacher 3

4.3.3.1 First Official Lesson

(a) *Observation of Teacher 3*

The lesson topic was ‘addition and subtraction’ as per the VIP (Appendix A).

(i) The alignment of the participant’s lesson with the analytical framework (Appendix C):

The lesson of Teacher 3 (T3) displayed the following number sense themes in her teaching of addition and subtraction:

- Operating on whole numbers: T3 demonstrated the addition and subtraction of whole numbers by making use of the algorithmic (conventional) way of doing mathematics. She did not use any visual strategy. T3 could have used the calibrated number line and physical counters to assist learners with the counting sequence, rounding and compensating of numbers.

- Operating on fractions: T3 made use of the diagrammatical representation to teach the addition and subtraction of fractions. The nature of fractional parts was well taught through the use of the diagrammatical representation. The learners could demonstrate the addition and subtraction of fractional parts by making use of a fraction circle.

- Operating on decimals: T3 did not use any visual aids to teach decimal fractions. Instead, she used conventional algorithms to demonstrate the addition and subtraction of decimals. The participant could have used decimal fraction strips to demonstrate the relationship of decimal fractions to one whole.

(ii) The participant’s proficiency in using visual strategies in the teaching of number sense:

An analysis was done to look for evidence of the teacher's proficiency in using visual strategies in the teaching of number sense within the context of the five teaching proficiency components of Kilpatrick et al. (2001). This analysis made use of a rating scale from 1 to 3, as outlined in Appendix D.

T3 was rated as follows:

TEACHING PROFICIENCY COMPONENT	ACTUAL RATING
Conceptual understanding of the core knowledge required in the practice of teaching	Some evidence (rating 2) was observed of the teacher's skill in connecting her own mathematical content knowledge with her understanding of how the mathematical knowledge of her learners develops and the most efficient way of teaching them in a visual manner. Visual aids, like the fraction circle and diagrammatical representation, were used to operate on fractions. However, T3 did not use any visual strategy for operating on whole numbers and decimals.
Fluency in carrying out basic instructional routines	Some evidence (rating 2) was observed of the teacher's skill in managing her classroom throughout the whole lesson. T3 responded to learners of different abilities by making use of visualisation processes for operating on fractions. She ensured learner participation in solving problems with visual aids by allowing learners to demonstrate the addition and subtraction of fractional parts on the writing board, by making use of fraction circles.
Strategic competence in planning effective instruction and solving problems that arise during instruction	Some evidence (rating 2) was observed of the teacher's skill in implementing visual strategies at the level of the learners' understanding. The fraction circle and diagrammatical representation were appropriate for assisting the learners in understanding how to operate on fractions. These visual aids are also very simple to use.
Adaptive reasoning in justifying and explaining one's instructional practices and in reflecting on those practices so as to improve them	Some evidence (rating 2) was observed of T3's skill in reflecting on her own lesson and seeking ways of improving the visual teaching of it.
Productive disposition toward mathematics, teaching, learning and the improvement of practice.	Some evidence (rating 2) was observed of T3's awareness of how mathematics, her understanding of how her learners think and the visual manner in which she teaches, fit together to make sense.

(b) *Results and discussion of the video-stimulated recall interview with Teacher 3*

T3's responses to the semi-structured and video-stimulated recall interview are categorized and discussed as follows:

- **Insights of the participant about the use of visualisation processes as a result of participating in the VIP:** The participant regarded the use of fraction circles as being important to enhance the computing and counting strategies of her learners. She saw that the learners were able to divide the whole into parts. She furthermore observed that when they use fraction circles, even the slow learners could easily see the relationship between a fraction and a whole.

  "They can do the computing and counting strategies using the fraction circles. The fraction circles because they are the visual aids, the learners can see, and then, it will be easy for them to divide the whole into parts. So, it is easy for learners and even for the slow learners, they can easily see where that one quarter is coming from, from that fraction circle." (T3, I1)

  T3 regarded the decimal teaching visual aid(conversion chart) as instrumental in assisting learners to convert the decimal form of a fraction to a common fraction and percentage. The learners could then also compare tenths and hundredths to assess which one is bigger.

- **Clarifications of the participant's responses (as a subcategory of insights) about the use of visualisation processes as a result of participating in the VIP:** T3 clarified her insight about how the fraction circles assisted the learners in seeing the relationship between common fractions, decimal fractions and percentages as different representations with the same value. She furthermore explained how the conversion chart assisted the learners with understanding decimal fractions as part of a whole.

  "Yes, it helped them because on that visual aid of the decimals, on the column of the decimals, down there. When we add the decimals, they will get one whole. And when they add the fractions, they will get one whole. So it will be easier for them to subtract from one whole and then, they get the correct answer. And even if they add, two or more fractions, they will still get to one whole." (T3, I1)

- **Subjective meanings of the participant's responses about the use of visualisation processes as a result of participating in the VIP:** T3 was convinced that the use of the balance scale, fraction circles and decimal chart make mathematics interesting for the learners. Consequently, the learners start to like mathematics more. The participant saw the use of visual strategies for teaching addition and subtraction as beneficial to learners with different preferences for learning. She was of the opinion that learners now have an opportunity to choose from a pool of different strategies when solving problems related to addition and subtraction.

- **Perceptions of the participant (as a subcategory of subjective meanings) about the use of visualisation processes as a result of participating in the VIP:** T3 perceived the use of the visual aids as being instrumental in the learners' exploration of numbers. She emphasized the fact that the visual aids assisted the learners with understanding and manipulating equivalent expressions.

  "Yes, the use of this visual teaching strategies or this visual teaching aids help my learners to understand the equivalent expressions. Because it is easy for them to see it and how to use them to manipulate the expressions." (T3, I1)

- **Errors and misconceptions of the participant about the use of visualisation processes as a result of participating in the VIP:** No errors or misconceptions were found during the video-stimulated recall interview regarding the first official lesson.

- **Consistencies in the participant's responses about the use of visualisation processes as a result of participating in the VIP:** The participant did not demonstrate any consistency worth mentioning.

- **Inconsistencies in the participant's responses about the use of visualisation processes as a result of participating in the VIP:** No inconsistencies were found in T3's responses during the video-stimulated recall interview.

- **Common elements of the participant's responses about the use of visualisation processes as a result of participating in the VIP:** No common elements were found for the first official lesson in T3's responses during the video-stimulated recall interview.

4.3.3.2 Second Official Lesson

(a) *Observation of Teacher 3*

The lesson topic was 'multiplication and division' as per the VIP (Appendix A).

(i) The alignment of the participant's lesson with the analytical framework (Appendix C):

The lesson of Teacher 3 (T3) displayed the following number sense themes in her teaching of multiplication and division:

- Equivalent numerical forms (including decomposition and recomposition): T3 used a diagrammatical representation with smaller rectangles within a big rectangle, in order to facilitate an environment within which the number of small rectangles could be represented in different ways. The number of small rectangles of different colours were represented in multiple ways in relationship to the big rectangle.

- Operating on fractions and decimals: T3 also made use of the diagrammatical representation with smaller rectangles within a big rectangle to teach equivalent fractions in order for learners to understand the relationship between the numerators and denominators of equivalent fractions. The participant furthermore used the decimal number line, which assisted learners with the decimal counting sequence.

- Operating on whole numbers: T3 used the dotted array in combination with the hundred square. The dotted array and hundred square provided a platform which facilitated the multiplication and division of whole numbers.

(ii) The participant's proficiency in using visual strategies in the teaching of number sense:

An analysis was done to look for evidence of the teacher's proficiency in using visual strategies in the teaching of number sense within the context of the five teaching proficiency components of Kilpatrick et al. (2001). This analysis made use of a rating scale from 1 to 3, as outlined in Appendix D.

T3 was rated as follows:

TEACHING PROFICIENCY COMPONENT	ACTUAL RATING
Conceptual understanding of the core knowledge required in the practice of teaching	Abundant evidence (rating 3) was observed of the teacher's skill in connecting her own mathematical content knowledge with her understanding of how the mathematical knowledge of her learners develops and the most efficient way of teaching them in a visual manner. The visual aids, like the decimal number line as well as the diagrammatical representation, were carefully chosen as they were the most efficient manner in which multiplication and division could be taught.
Fluency in carrying out basic instructional routines	Abundant evidence (rating 3) was observed of the teacher's skill in managing her classroom throughout the whole lesson. A mathematical and visually rich environment was facilitated by the teacher intentionally allocating diverse visual aids to the learners in the different groups, before the lesson started. T3 responded to learners of different abilities by making use of visualisation processes. She ensured learner participation in solving problems with visual aids.
Strategic competence in planning effective instruction and solving problems that arise during instruction	Some evidence (rating 2) was observed of the teacher's skill in implementing visual strategies at the level of the learners' understanding. The decimal number line and diagrammatical representation were appropriate for assisting the learners in understanding how to multiply and divide whole numbers and also how to handle equivalent fractions.
Adaptive reasoning in justifying and explaining one's instructional practices and in reflecting on those practices so as to improve them	Some evidence (rating 2) was observed of T3's skill in reflecting on her own lesson and seeking ways of improving the visual teaching of it.
Productive disposition toward mathematics, teaching, learning and the improvement of practice.	Some evidence (rating 2) was observed of T3's awareness of how mathematics, her understanding of how her learners think and the visual manner in which she teaches, fit together to make sense.

(b) *<u>Results and discussion of the video-stimulated recall interview with Teacher 3</u>*

T3's responses to the semi-structured and video-stimulated recall interview are categorized and discussed as follows:

- **Insights of the participant about the use of visualisation processes as a result of participating in the VIP:** The participant regarded the use of the dotted array as being helpful in enhancing the multiplication and division of whole numbers.

“The dotted array supported the learners’ thinking skills around multiplication and division. The learners were able to identify that the dotted array was divided into small, equal groups. Which means if it is three groups of six, then they can multiply three by six and get eighteen or they can even make three groups of six…which is equal to eighteen.” (T3, I2)

T3 appreciated the diagrammatical representation which consisted of smaller rectangles shaded in different colours. According to her, the learners could form different fractions by using the differently shaded parts.

“The diagrammatical representation helped learners a lot, because learners learn a lot when they see especially when it was divided into coloured parts. So they could see that eight out of twenty is equal to two-fifths, whereas four out of twenty is one-fifth.” (T3, I2)

T3 observed that the learners were supported by the teacher using visual aids. This facilitated an environment conducive to the development of independent thought on the part of the learner. The participant’s remark sums up her insight in this regard:

“I can see that when I use the visual aid, they can see light. They can do things for themselves. They think for themselves. They do not always follow the method that we are using, they do it their own way.” (T3, I2)

- **Clarifications of the participant’s responses (as a subcategory of insights) about the use of visualisation processes as a result of participating in the VIP:** T3 clarified her insight about how the dotted array assisted learners with doing computations on fractions. She furthermore clarified how the fraction circles and fraction rectangles (diagrammatical representation) facilitated the division of numbers.

- **Subjective meanings of the participant’s responses about the use of visualisation processes as a result of participating in the VIP:** T3 was convinced that the use of the diagrammatical representation of fractional parts assisted the learners with developing and building multiplication facts. She furthermore stated that the visuals increased the learners’ level of thinking. The use of visual aids also contributed to the fact that the learners started to regard mathematics as an interesting subject.

- **Perceptions of the participant (as a subcategory of subjective meanings) about the use of visualisation processes as a result of participating in the VIP:** T3 perceived the use of the dotted array as an instrument which facilitated the understanding of the learners.

  "The dotted array increased the learners' understanding, because this learner is one of my average learners and I was surprised to see him breaking that thousand into those groups." (T3, I2)

- **Errors and misconceptions of the participant about the use of visualisation processes as a result of participating in the VIP:** No errors or misconceptions were found during the video-stimulated recall interview with T3.

- **Consistencies in the participant's responses about the use of visualisation processes as a result of participating in the VIP:** No consistencies worth mentioning were found during the video-stimulated recall interview with the participant.

- **Inconsistencies in the participant's responses about the use of visualisation processes as a result of participating in the VIP:** No inconsistencies were found during the video-stimulated recall interview with the participant.

- **Common elements of the participant's responses about the use of visualisation processes as a result of participating in the VIP:** T3 lamented the lack of resources and the time-consuming nature of preparing visual aids as factors which negatively impacted on the preparation of the visual aids. As she saw value in visually teaching number sense, she was prepared to sacrifice time and her own resources to optimally prepare for her mathematics lessons.

4.3.3.3 General Findings

T3 struggled to align her first official lesson with the analytical framework (Appendix C) when teaching how to operate on whole numbers and decimals. However, when she taught operations on fractions, she made use of a diagrammatical representation of fractional parts. This visual aid assisted learners with executing computations fluently as far as the addition and subtraction of fractions are concerned.

The second official lesson was consistently aligned with the analytical framework (Appendix C).Her choice of the diagrammatical representation with smaller rectangles within a big rectangle, the decimal number line, the dotted array and the hundred square were relevant for the number sense themes that were evident in the lesson on multiplication and division.T3 was rated favourably regarding her proficiency in using visual strategies in the teaching of number sense, as a result of her participating in the VIP.

The insights which T3 has demonstrated during the video-stimulated recall interview, were characterised by the value which she saw in teaching number sense understandings visually. T3 related how even the slow learners performed better than expected regarding the addition and subtraction of fractions when she explained operating on fractions using visualisation processes. The participant experienced how visual teaching succeeded in making mathematics more interesting for her learners. T3 furthermore claimed that the independent thought of her learners was stimulated by her visual teaching.

Like other participants, T3 levelled criticism at the preparation of the visual aids because of time constraints and challenges pertaining to the availability of resources. However, she considered the challenges worth overcoming as her faith in visual teaching was unwavering.

4.3.4 Teacher 4

4.3.4.1 First Official Lesson

(a) *Observation of Teacher 4*

The lesson topic was 'addition and subtraction' as per the VIP (Appendix A).

(i) The alignment of the participant's lesson with the analytical framework (Appendix C):

The lesson of Teacher 4 (T4) displayed the following number sense themes in his teaching of addition and subtraction:

- Operating on whole numbers: T4 balanced two bricks on a horizontal steel ruler to simulate a balance scale in order to facilitate a mathematical environment for learners to explore the addition and subtraction of whole numbers. He furthermore made use of a diagrammatical representation in order to show the learners how to 'remove' the subtrahend. The use of the simulated balance scale complemented the use of the diagrammatical representation to assist the learners with the addition and subtraction of whole numbers.

- Place value: T4 used flashcards showing numbers with one digit underlined. Learners were tasked with saying the value of the underlined digit. This visual strategy facilitated an environment for the exploration of place value.

- Operating on fractions: T4 made use of fraction circles that resemble pizzas to teach the addition and subtraction of fractions. The nature of fractional parts was well taught through the use of this diagrammatical representation. The conceptual understanding of 'parts of a whole' was consolidated by the use of these visual aids.

- Operating on decimals: T4 made use of the decimal place value chart while adding the decimal fractions. His choice was not the best visual strategy because decimal fraction strips or decimal fraction circles could have been used to demonstrate the relationship of decimal fractions to one whole.

- Comparison to benchmarks: T4 did not use any visual strategy other than the decimal place value chart to compare different decimal quantities. He could have used the conversion chart to change the decimal fraction to a common fraction in order to do a comparison between the different fractions.

(ii) The participant's proficiency in using visual strategies in the teaching of number sense:

An analysis was done to look for evidence of the teacher's proficiency in using visual strategies in the teaching of number sense within the context of the five teaching proficiency components of Kilpatrick et al. (2001). This analysis made use of a rating scale from 1 to 3, as outlined in Appendix D.

T4 was rated as follows:

TEACHING PROFICIENCY COMPONENT	ACTUAL RATING
Conceptual understanding of the core knowledge required in the practice of teaching	Some evidence (rating 2) was observed of the teacher's skill in connecting his own mathematical content knowledge with his understanding of how the mathematical knowledge of his learners develops and the most efficient way of teaching them in a visual manner. T4 created a visual environment for the learners to add and subtract whole numbers and fractions.
Fluency in carrying out basic instructional routines	Some evidence (rating 2) was observed of the teacher's skill in managing his classroom throughout the whole lesson. T4 responded to learners of different abilities by making use of visualisation processes during his lesson.
Strategic competence in planning effective instruction and solving problems that arise during instruction	Some evidence (rating 2) was observed of the teacher's skill in implementing visual strategies at the level of the learners' understanding. T4 used the diagrammatical representation and fraction circles as learners are generally assisted by these visual aids to develop a clear understanding of 'parts of a whole' and also to find equivalence among different fractional forms.
Adaptive reasoning in justifying and explaining one's instructional practices and in reflecting on those practices so as to improve them	Some evidence (rating 2) was observed of T4's skill in reflecting on his own lesson and seeking ways of improving the visual teaching of it. He continually sought confirmation from the learners as to whether they understood the topic under discussion as he experimented with different visual aids during the course of the lesson.

Productive disposition toward mathematics, teaching, learning and the improvement of practice.	Some evidence (rating 2) was observed of T4's awareness of how mathematics, his understanding of how his learners think and the visual manner in which he teaches, fit together to make sense.

(b) *Results and discussion of the video-stimulated recall interview with Teacher 4*

T4's responses to the semi-structured and video-stimulated recall interview are categorized and discussed as follows:

- **Insights of the participant about the use of visualisation processes as a result of participating in the VIP:** The participant regarded the use of the flash cards as being beneficial for teaching the concept of place value. These flashcards were furthermore used as a tool to exercise the ability of the learners to read out numbers. T4 was convinced that the place value chart was a good tool to enforce the meaning and size of numbers, especially when it comes to place value after the decimal comma.

- **Clarifications of the participant's responses (as a subcategory of insights) about the use of visualisation processes as a result of participating in the VIP:** T4 clarified his insight about how the simulated balance scale was used to foster the conceptual understanding of equivalence in mathematics. This enabled the learners to solve an algebraic equation with one unknown variable.

  "So I tried to use that balance scale to see whether the two bricks, will they be able to balance and that also teaches the learners of equivalence. If things are equivalent, it means that they are the same, meaning that the two bricks they have the same weight…The learners could have figured out that the missing number there is five, because when we add five to the twelve it will give us our seventeen. Seventeen is on one side and seventeen on the other side. Our scale will be able to balance." (T4, I1)

- **Subjective meanings of the participant's responses about the use of visualisation processes as a result of participating in the VIP:** T4 saw the use of the fraction circles as beneficial to the learners' understanding of parts of a whole. The learners physically took out a number of parts and could then ascertain the remaining parts of the same whole.

According to T4, this made the subtraction of fractional parts very practical. The learners could also practically experience the essence and meaning of equivalent fractions. Furthermore, the use of the decimal place value chart was applauded for being a "useful teaching tool for learners to visualise". (T4, I1)

According to T4, he found it difficult to contemplate a lot of teaching strategies for the topic of addition and subtraction.

"I think that when it comes to the constraining factors…one of the constraining factors would be…I found it difficult to come up with a lot of teaching strategies, especially for this topic, addition and subtraction. I did not have enough teaching strategies for learners to visualise, but I think in future, I will make a point to come up with different strategies and when it comes to addition and subtraction, what I have also realised is that I did not show the learners a variety of methods to come to the correct answers by using different visual teaching strategies.". (T4, I1)

- **Perceptions of the participant (as a subcategory of subjective meanings) about the use of visualisation processes as a result of participating in the VIP:** T4 was convinced that teaching without making use of visualisation processes makes it harder for the learners to develop understanding of the mathematical concepts under discussion. When learners can see visuals of the concept under discussion, they acquire conceptual understanding more easily.

  "So I tried to show them that by cutting it off and then I showed in that fraction circle, I showed them all the equal pieces in the fraction. How many pieces did we have there? But the learners could do that and got it correctly. So, it also assisted them and when they do these visualisation things, it makes it easier for them but if you just talk to them without showing the visuals, learners are not getting it easy…what the educator is actually trying to explain". (T4, I1)

- **Errors and misconceptions of the participant about the use of visualisation processes as a result of participating in the VIP:** No errors or misconceptions were found during the video-stimulated recall interview regarding the first official lesson.

- **Consistencies in the participant's responses about the use of visualisation processes as a result of participating in the VIP:** No noteworthy consistencies were found in T4's responses during the video-stimulated recall interview.

- **Inconsistencies in the participant's responses about the use of visualisation processes as a result of participating in the VIP:** Inconsistencies were found in T4's responses during the video-stimulated recall interview as far as his explanation of how the learners used a fraction circle, is concerned. The participant had difficulty in explaining the process of cutting off fractional parts to represent subtraction. His explanation did not match the actual algorithm he was trying to describe.

- **Common elements of the participant's responses about the use of visualisation processes as a result of participating in the VIP:** Like other participants, T4saw the accurate use of mathematical language as an important exercise as it assists learners when writing the common paper examinations in mathematics.

4.3.4.2 Second Official Lesson

(a) *<u>Observation of Teacher 4</u>*

The lesson topic was 'multiplication and division' as per the VIP (Appendix A).

(i) <u>The alignment of the participant's lesson with the analytical framework (Appendix C)</u>:

The lesson of Teacher 4 (T4) displayed the following number sense themes in his teaching of multiplication and division:

- Equivalent numerical forms (including decomposition and recomposition): T4 used a diagrammatical representation with smaller rectangles within a big rectangle. This strategy facilitated as a tool to represent the number of small rectangles in different ways. This way multiple representations for numbers could be made.

- Operating on fractions and decimals: T4used the decimal number line to assist the learners with the multiplication of decimals. The calibrated decimal number line was used to skip count in order to solve a multiplication operation as the sum of multiples or repeated addition.

- Comparisons to benchmarks: The decimal number line which T4 used, also assisted with comparing different decimal values in a benchmarking exercise. Learners understood that positive movement to the right increases quantity, and likewise, negative movement to the left decreases the quantity of the decimal fraction.

- Operating on whole numbers: T4 used the hundred square, the spider diagram, the diagrammatical representation and the lattice method for multiplication to assist with the multiplication and division of whole numbers. The diagrammatical representation provided a tool to display how the product of two numbers could be presented in diverse ways to show different sets of arrangements in packing the same amount of fruit. This facilitated an environment to ensure that the multiplication and division of whole numbers was more successfully grasped by the learners.

(ii) The participant's proficiency in using visual strategies in the teaching of number sense:

An analysis was done to look for evidence of the teacher's proficiency in using visual strategies in the teaching of number sense within the context of the five teaching proficiency components of Kilpatrick et al. (2001). This analysis made use of a rating scale from 1 to 3, as outlined in Appendix D.

T4 was rated as follows:

TEACHING PROFICIENCY COMPONENT	ACTUAL RATING
Conceptual understanding of the core knowledge required in the practice of teaching	Some evidence (rating 2) was observed of the teacher's skill in connecting his own mathematical content knowledge with his understanding of how the mathematical knowledge of his learners develops and the most efficient way of teaching them in a visual manner. T4 chose the spider diagram, the hundred square, the lattice method for multiplication and the diagrammatical representation (with the rectangles) as visual aids to teach multiplication and division.

Fluency in carrying out basic instructional routines	Some evidence (rating 2) was observed of the teacher's skill in managing his classroom throughout the whole lesson. T4provided the different learner groups with visual aids relevant to the concepts under discussion during every stage of the lesson. He therefore responded to learners of different abilities by making use of relevant visualisation processes.
Strategic competence in planning effective instruction and solving problems that arise during instruction	Some evidence (rating 2) was observed of T4's skill in implementing visual strategies at the level of the learners' understanding. The spider diagram and diagrammatical representation were appropriate for assisting the learners in understanding how to multiply and divide whole numbers and also how to represent numbers in multiple ways.
Adaptive reasoning in justifying and explaining one's instructional practices and in reflecting on those practices so as to improve them	Some evidence (rating 2) was observed of T4's skill in reflecting on his own lesson and seeking ways of improving the visual teaching of it.
Productive disposition toward mathematics, teaching, learning and the improvement of practice.	Some evidence (rating 2) was observed of T4's awareness of how mathematics, his understanding of how the learners think and the visual manner in which he teaches, fit together to make sense.

(b) *Results and discussion of the video-stimulated recall interview with Teacher 4*

T4's responses to the semi-structured and video-stimulated recall interview are categorized and discussed as follows:

- **Insights of the participant about the use of visualisation processes as a result of participating in the VIP:** T4 regarded the use of the multiplication chart in conjunction with the spider diagram as being helpful to the learners as they did not struggle to give the correct answers when prompted by the teacher. The participant saw that the number line assisted the learners in multiplying whole numbers with decimal numbers, where the whole number was the multiplicand and the decimal number acted as the multiplier. The learners regarded the decimal value (multiplier) as a constant difference on the number line and could thus add the multiples to solve the multiplication problem.

  "Yes, I think that…especially by using that number line. Learners can see very clearly that…especially where we had that thirteen multiplied by nought comma five…it is very clear that nought comma five will be…if we maybe use…can one say it is a constant…the constant difference…that of nought comma five." (T4, I2)

- **Clarifications of the participant's responses (as a subcategory of insights) about the use of visualisation processes as a result of participating in the VIP:** T4 clarified his insight about how the number line facilitated the multiplication of a whole number and a decimal number as an example of how the use of visual aids enhances the teaching of number sense.

  "So I show them on the number line that it will be thirteen times to get to the answer. So that they can see that…when you multiply now by fourteen…the very same thing…you can see that difference. Fourteen will be bigger than thirteen…the answer for fourteen multiplied by nought comma five. We will still have to take another step…one step forward." (T4, I2)

- **Subjective meanings of the participant's responses about the use of visualisation processes as a result of participating in the VIP:** T4 regarded visual teaching strategies as being responsible for learners' conceptual understanding of equivalent forms in fractions and the general enhancement of their number sense. T4 further advocated the correct use of mathematical language by the teacher in order to acquaint the learners with it. The participant felt that the learners would then be in a beneficial position to solve word problems successfully. T4 believes that if the learners are not familiar with the correct terminology in mathematics, it will act as a barrier to solving word problems.

- **Perceptions of the participant (as a subcategory of subjective meanings) about the use of visualisation processes as a result of participating in the VIP:** T4 was convinced that teaching by making use of visualisation processes assisted him and his learners tremendously because the learners could use their sensory processing abilities to acquire deeper understanding of the mathematical concepts under discussion.

  "Yeah. I think that, according to my experience, it assisted me and the learners…also for the benefit of the learners whereby they can see. If the learners can see and touch maybe, is where their understanding will…they will always remember things more…easily if they have seen it and maybe they have touched it. Not just by…maybe you just talk to them without showing them. But if you will show them, that thing will always be in their heads and it will be too difficult for them to get it out of their head." (T4, I2)

- **Errors and misconceptions of the participant about the use of visualisation processes as a result of participating in the VIP:** The error that was found during the video-stimulated recall interview regarding the second official lesson, involves a problem on long division. T4 confused the dividend with the divider when attempting to calculate the quotient.

  "And then…maybe in that case…I could have also shown them…maybe by using the long division…that three divided by one hundred and twenty-five…so that they can also see how we are dividing to get to the forty." (T4, I2)

- **Consistencies in the participant's responses about the use of visualisation processes as a result of participating in the VIP:** No noteworthy consistencies were found in T4's responses during the video-stimulated recall interview.

- **Inconsistencies in the participant's responses about the use of visualisation processes as a result of participating in the VIP:** An inconsistency was found in T4's responses during the video-stimulated recall interview in his explanation of how he used visual strategies to teach the learners the meaning and size of numbers. He explained the process in an algorithmic way instead of visual.

  "Okay, nought comma five…is that…? I think in that case…the only thing that I did not show them is that the nought comma six is bigger than the nought comma five. I could have shown them with a difference of nought comma one, so that they can see. Then we add nought comma one to nought comma five to get to nought comma six, so that they can see what is the difference in that case." (T4, I2)

- **Common elements of the participant's responses about the use of visualisation processes as a result of participating in the VIP:** T4 regarded the lack of resources as a constraining factor for visual teaching. He admitted that teachers can create their own resources, but that it is still a time-consuming exercise.

4.3.4.3 General Findings

T4 struggled to align his first official lesson with the analytical framework (Appendix C) when teaching how to operate on decimals. The decimal place value chart was not the best strategy for teaching the addition of decimal values. A series of calibrated decimal number lines are better suited for teaching how to operate on decimals as it can be displayed with different calibrations showing ever-decreasing distances between a start and end point. The participant did not use any visual strategy to show how to compare different decimal quantities. It would have been effective to convert decimal fractions to common fractions by making use of a conversion chart and then comparing the quantities.

The second official lesson was consistently aligned with the analytical framework (Appendix C). T4 chose the diagrammatical representation with smaller rectangles within a big rectangle, the decimal number line, the hundred square, the spider diagram and the lattice method as visual aids for the number sense themes that were evident in his lesson on multiplication and division. T4 was rated favourably regarding his proficiency in using visual strategies in the teaching of number sense, as a result of him participating in the VIP.

The insights which T4 demonstrated during the video-stimulated recall interview, were characterised by the value which he saw in teaching number sense understandings by making use of visualisation processes. T4 related how his learners performed better in mathematical tasks when they see and touch the visual aids. He attributed the use of the visual aids to the deeper conceptual understanding of his learners as far as number sense understandings are concerned.

T4 also criticized the lack of resources and the time-consuming nature of preparing the visual aids. However, he still considered the use of visualisation processes as imperative for the successful teaching of number sense understandings.

4.3.5 Summary of the views and responses of Teacher 5, Teacher 6 and Teacher 7

4.3.5.1 First Official Lessons

(a) Observations of Teacher 5, 6 and 7

The lesson topic was 'addition and subtraction' as per the VIP (Appendix A).

(i) The alignment of the participants' lessons with the analytical framework (Appendix C):

The lessons of Teacher 5 (T5), Teacher 6 (T6) and Teacher 7 (T7) displayed the following number sense themes in their teaching of addition and subtraction:

- Operating on whole numbers: T5 made use of a representation of the balance scale in order to facilitate a mathematical environment for learners to explore the addition and subtraction of whole numbers. This ensured that the addition and subtraction of whole numbers became meaningful within the context of an equation. T6 approached the number sense theme differently by testing whether his learners could read the numbers out loud from bright flash cards. This exercise tested the fundamentals of place values. T7 made use of the calibrated number line and a representation of counters to model the addition and subtraction of whole numbers.

- Operating on fractions: All three participants made use of fraction circles that resemble pizzas to be shared amongst friends in order to teach the addition and subtraction of fractions. The nature of fractional parts was well taught through the use of this diagrammatical representation. The conceptual understanding of 'parts of a whole' was consolidated by the use of these fraction circles and the demonstrated sharing amongst friends.

- Operating on decimals: T5 made use of the decimal place value chart while adding and subtracting the decimal fractions. Her choice was not the best visual strategy because decimal fraction strips or decimal fraction circles could have been used to demonstrate the relationship of decimal fractions to one whole. However, T6 and T7 did not employ any visual aid for adding and subtracting decimal fractions. They simply stuck to the conventional algorithmic method of adding and subtracting decimal fractions. All three teachers failed to align their teaching of how to add and subtract decimal fractions with the analytical framework in Appendix C. The best visual aids they could have used are the decimal fraction strips and circles that visually demonstrate the relationship of decimal fractions to one whole.

(ii) The participants' proficiency in using visual strategies in the teaching of number sense:

An analysis was done to look for evidence of the teachers' proficiency in using visual strategies in the teaching of number sense within the context of the five teaching proficiency components of Kilpatrick et al. (2001). This analysis made use of a rating scale from 1 to 3, as outlined in Appendix D.

T5, T6 and T7 were rated as follows:

TEACHING PROFICIENCY COMPONENT	ACTUAL RATING
Conceptual understanding of the core knowledge required in the practice of teaching	Some evidence (rating 2) was observed of the teachers' skill in connecting their own mathematical content knowledge with their understanding of how the mathematical knowledge of their learners develops and the most efficient way of teaching them in a visual manner. The three participants created a visual environment for the learners to add and subtract whole numbers and common fractions. As far as the teaching of how to add and subtract decimal fractions is concerned, the teachers did not employ efficient teaching strategies. Their interventions ranged from using the decimal place value chart to no visualisation processes for operating on decimals.
Fluency in carrying out basic instructional routines	Some evidence (rating 2) was observed of the teachers' skill in managing their classrooms throughout the whole lesson. They responded to learners of different abilities by making use of visualisation processes for operating on whole numbers and common fractions during their lessons.
Strategic competence in planning effective instruction and solving problems that arise during instruction	Some evidence (rating 2) was observed of the teachers' skill in implementing visual strategies at the level of their learners' understanding. All three participants used fraction circles as learners are generally assisted by these visual aids to develop a clear understanding of 'parts of a whole' and also to find equivalence among different common fractions.

Adaptive reasoning in justifying and explaining one's instructional practices and in reflecting on those practices so as to improve them	Some evidence (rating 2) was observed of the participants' skill in reflecting on their own lessons and seeking ways of improving the visual teaching thereof. They continually sought confirmation from the learners as to whether they understood the topic under discussion as they experimented with different visual aids during the course of the lessons.
Productive disposition toward mathematics, teaching, learning and the improvement of practice.	Some evidence (rating 2) was observed of the three teachers' awareness of how mathematics, their understanding of how their learners think and the visual manner in which they teach, fit together to make sense.

(b) *Results and discussion of the video-stimulated recall interviews with Teacher 5, 6 and 7*

The responses of T5, T6 and T7 to the semi-structured and video-stimulated recall interviews are categorized and discussed as follows:

- **Insights of the participants about the use of visualisation processes as a result of participating in the VIP:** T5 regarded the use of the decimal place value chart as being beneficial for teaching the concept of place value. She noticed that the learners easily confused place values before and after the decimal comma. The decimal place value chart therefore provided a visual tool to act as scaffolding that assisted the learners to differentiate between the place values of whole numbers and the place values of fractions.

  "I've used the decimal place value chart to help the learners to know the difference between the tens and the tenths and where do we write them on the place value chart." (T5, I1)

  T6 saw the use of fraction circles as a means to increase learner participation, as the visual aids stimulate the interest of learners. All three participants commended the use of fraction circles as they assisted the learners in understanding the sharing of one whole into fractional parts. Thus, the meaning and size of fractions were better understood by the learners.

  "Yeah, I think that here, the visual did help a lot, because… the aim here was to see….which portion is bigger than the others, so when representing it by means of a pie chart, it's easy for the learner to see as to which part is bigger than the others and which parts are equivalent. As you can see others got quarters and the other one got, I mean, a half, so looking at… the visual. The learner can understand which portion is bigger, is it a half or a quarter?" (T7, I1)

T7 applauded the use of the number line to teach addition and subtraction of whole numbers. The number line furthermore assisted with understanding the addition of inverse values which results in nought. The forward movement was cancelled out by the backward movement on the number line. T7 experienced that the number line also helped with benchmarks as learners could easily see which number is bigger or smaller than another one.

- **Clarifications of the participants' responses (as a subcategory of insights) about the use of visualisation processes as a result of participating in the VIP:** All three participants clarified their insight regarding the efficiency of visualisation processes for the teaching of number sense understandings. They concurred that the fraction circles contributed much to the learners' conceptual understanding of fractions. The scenario of sharing a pizza amongst friends provided a platform for identifying fractions, understanding the addition and subtraction of fractions and also converting fractional pieces to equivalent fractions.

"And when I say, bring a quarter, they know what they are holding is one quarter, so they bring it. They understand the concept that this is a quarter. When I was going on in the clip, I said bring the two quarters…the two halves together to make a whole. And then, two learners, they came you see, they do exactly that. One half and one half, it makes what? A whole…" (T6, I1)

T5 observed that the visual aid of fraction circles resembling a pizza assisted the learners to compare fractions and ascertain that two pieces out of four equal pieces are greater than two pieces out of eight equal pieces. This clarification suggests that visualisation processes assist with benchmarking exercises. It furthermore fosters an understanding of equivalent fractions.

T7 clarified how he used an array of circles to teach subtraction. Learners were required to draw the number of circles in an array which represented the minuend. They then cancelled out the number of circles represented by the subtrahend. The circles which were not cancelled in the original array, then represented the difference. He furthermore clarified his insight about the use of the number line. He emphatically stated that when a learner adds or subtracts using a number line, such a learner "will conceptually understand the meaning of addition and subtraction". (T7, I1)

- **Subjective meanings of the participants' responses about the use of visualisation processes as a result of participating in the VIP:** T5 believed that the learners who were exposed to the visual teaching of fractions would be in a position to apply this learning to other fractions in general as the use of fraction circles grounded their understanding of the nature of fractions.

  "They know, that we are having a whole. From there, they get a half and the other one gets a quarter and then the piece which is left is a quarter. They know very well, they have seen it happening. They were doing it themselves and then, I think they can be able to do so in other fractions." (T5, I1)

  T6 regarded his use of visual teaching strategies as a stimulant for learning. According to him, he experienced a greater degree of learner participation due to the use of visual aids. He also saw the visual aids as an instrument to facilitate mental mathematics. Use of the visual aids made fractions more practical to learners. They could cut out certain parts of the whole and name them correctly. T7 concurred with T6 in stating that learners learn better when they see something concrete while it is being discussed rather than just hearing about it.

- **Perceptions of the participants (as a subcategory of subjective meanings) about the use of visualisation processes as a result of participating in the VIP:** All three participants perceived of the visual aids as being helpful to the learners. T5 noticed that her learners knew the values of the different digits in a number. She attributed her learners' knowledge to the use of the place value chart. The learners of T5 and T6 were also able to arrange fractions in an ascending order due to learning fraction sizes through visualisation processes. The operations of addition and subtraction were implied when the learners of the three participants worked with fraction circles as they could see what happened when they 'took away' a quarter or a half of one whole fraction circle.
  "So they know that a quarter plus a quarter makes a half if you add them together. And then, if you take a half from four quarters then you are going to be left with two quarters. So that's what makes me very sure that they know using the operations that taking out… it is subtracting and then when you add the two, then, that is addition.**"** (T5, I1)

T6 perceived the use of visual aids as being responsible for stimulating his learners' understanding. He therefore lamented the fact that it is time-consuming to prepare the lessons with visual aids as time can become a scarce commodity in the busy life of a teacher.

- **Errors and misconceptions of the participants about the use of visualisation processes as a result of participating in the VIP:** No errors or misconceptions were found during the video-stimulated recall interview regarding the first official lessons of the three participants.

- **Consistencies in the participants' responses about the use of visualisation processes as a result of participating in the VIP:** T6 and T7 were consistent in applauding the benefits of visual teaching for the learning experience of the learners.

- **Inconsistencies in the participants' responses about the use of visualisation processes as a result of participating in the VIP:** No noteworthy inconsistencies were found in the three participants' responses during the video-stimulated recall interview.

- **Common elements of the participants' responses about the use of visualisation processes as a result of participating in the VIP:** The three participants were confident that they would continue using visualisation processes in their teaching of mathematics, as it positively impacted the slower learners. They also cited some challenges with visual teaching. The time-consuming nature of preparing the visuals and the risk of giving scissors to learners during group work were flagged as potential challenges to teaching visually in a sustainable manner.

4.3.5.2 Second Official Lessons

(a) *Observations of Teacher 5, 6 and 7*

The lesson topic was 'multiplication and division' as per the VIP (Appendix A).

(i) The alignment of the participants' lessons with the analytical framework (Appendix C):

The lessons of Teacher 5 (T5), Teacher 6 (T6) and Teacher 7 (T7) displayed the following number sense themes in their teaching of multiplication and division:

- Operating on whole numbers: The three participants used an array of dots, the spider diagram and a diagrammatical representation with smaller rectangles within a big rectangle. The array of dots was helpful in showing how the product of two numbers could be calculated in diverse ways, to represent different arrangements of packing the same amount of fruit. The spider diagram showed the patterns of numbers visually. The diagrammatical representation with smaller rectangles showed the different ways in which the rectangles could be arranged, thus facilitating multiple representations for whole numbers.

- Equivalent numerical forms (including decomposition and recomposition): All three participants used a diagrammatical representation with smaller rectangles within a big rectangle, in order to facilitate an environment within which the number of small rectangles could be represented in different ways. The number of small rectangles of different colours were represented in multiple ways in relation to the big rectangle.

- Operating on fractions and decimals: T5 used the fraction wall to represent equivalent fractions. This tool enables learners to understand the relationship between fractions and whole numbers. T6 used the decimal number line to assist with the multiplication of decimal fractions. If the multiplicand is a whole number and the multiplier a decimal fraction, then the decimal fraction will be counted in multiples on the number line. The multiplicand is the number of decimal multiples to be counted.

- Comparisons to benchmarks: The decimal number line which T6 used assisted with comparing different decimal values in a benchmarking exercise. Learners understood that positive movement to the right increases quantity, and likewise, negative movement to the left decreases the quantity of the decimal fraction.

(ii) The participants' proficiency in using visual strategies in the teaching of number sense:

An analysis was done to look for evidence of the teachers' proficiency in using visual strategies in the teaching of number sense within the context of the five teaching proficiency components of Kilpatrick et al. (2001). This analysis made use of a rating scale from 1 to 3, as outlined in Appendix D.

T5, T6 and T7 were rated as follows:

TEACHING PROFICIENCY COMPONENT	ACTUAL RATING
Conceptual understanding of the core knowledge required in the practice of teaching	Some evidence (rating 2) was observed of the teachers' skill in connecting their own mathematical content knowledge with their understanding of how the mathematical knowledge of their learners develops and the most efficient way of teaching them in a visual manner. The three participants chose the dotted array, spider diagram, the fraction wall, the decimal number line and the diagrammatical representation (with the rectangles) as visual aids to teach multiplication and division.
Fluency in carrying out basic instructional routines	Some evidence (rating 2) was observed of the teacher's skill in managing their classrooms throughout the whole lesson. They provided the different learner groups with visual aids relevant to the concepts under discussion during every stage of the lesson. They therefore responded to learners of different abilities by making use of visualisation processes relevant to the number sense themes inherent to the lesson.
Strategic competence in planning effective instruction and solving problems that arise during instruction	Some evidence (rating 2) was observed of the three participants' skill in implementing visual strategies at the level of the learners' understanding. The spider diagram and diagrammatical representation were appropriate for assisting the learners in understanding how to multiply and divide whole numbers and also how to represent numbers in multiple ways, whereas the fraction wall was optimally used to allow learners to explore different fractions. The decimal number line provided a visual tool for multiplying a whole number with a decimal fraction in the form of repeated addition, by making jumps equal to the fractional part.
Adaptive reasoning in justifying and explaining one's instructional practices and in reflecting on those practices so as to improve them	Some evidence (rating 2) was observed of the participants' skill in reflecting on their own lessons and seeking ways of improving the visual teaching of them.

Productive disposition toward mathematics, teaching, learning and the improvement of practice.	Some evidence (rating 2) was observed of the three participants' awareness of how mathematics, their understanding of how their learners think and the visual manner in which they teach, fit together to make sense.

(b) *Results and discussion of the video-stimulated recall interviews with Teacher 5, 6 and 7*

The responses of T5, T6 and T7 to the semi-structured and video-stimulated recall interviews are categorized and discussed as follows:

- **Insights of the participants about the use of visualisation processes as a result of participating in the VIP:** T5 regarded the use of the diagrammatical representation of smaller rectangles within a big rectangle as being beneficial for teaching the learners how to form a number sentence. The visual aid provided a mathematical context and clues to calculate the total number of rectangles in various ways. It furthermore provided a mathematically rich environment for learners to become creative with forming numbers sentences.

  "I have used the visual aid of the diagrammatical representation to help the learners learn by what they see. This visual aid helps the learners to formulate a number sentence on what they are looking at. They were given a diagrammatical representation of diagrams…so…where they have to form a number sentence which will tell the number of rectangles on the diagram and really, they did so." (T5, I2)

  T5 affirmed that using visual teaching strategies are useful for teaching towards the enhancement of number sense. She saw that learners could use a fraction wall and divide any whole number into equal fractional parts, because the fraction wall facilitated the principle of dividing any quantity into equal parts. T5 even experienced how her learners could use conversion tables and calendars to convert days into years, and the participant ascribed that to the inherent value of working with visuals. T6 related how his learners used the dotted array to solve different ways of packing fruit. The learners used the dots as boxes which contained a certain number of oranges. They could then experiment with packing the same amount of fruit in different ways. T6 applauded the manner in which his learners could visually solve the product of a whole number and a decimal fraction by using the number line. The learners were able to count the decimal value in its multiples, where the number of multiples represented the whole number. Done thus, the multiplication of the decimal fraction and the whole number was constituted by repeatedly adding the decimal fraction.

“Yeah, because in the number line, like I said, they started at nought on the number line. So nought, which is a whole number, and from there…because we are having nought comma five and nought comma six …so they started nought comma one …nought comma two …until nought comma nine and then they get a whole number which is one. So they know…that with the multiplication of the decimal number with the two digit whole number…what they are going to get. So they make sense out of that one.” (T6, I2)

T6 also commented on his insight regarding the wrong use of mathematical language. According to him, the wrong use of language within a mathematical context leads to misconceptions on the part of the learners. T7 pointed out that the visual aids, like a dotted array, facilitate an environment for learners to discover that the multiplication operation in effect means repeated addition.

- **Clarifications of the participants’ responses (as a subcategory of insights) about the use of visualisation processes as a result of participating in the VIP:** All three participants clarified their insight regarding the efficiency of visualisation processes for the teaching of number sense understandings. They concurred that the visual aids assisted the learners in solving the problems given to them in the different learner groups, whether it was dividing a big number, converting years into days, solving a problem of packing fruit or calculating with decimal fractions. T6 clarified in detail how the learners went about solving the packing problem:

“Unlike just multiplying, multiplying…using the numbers, but because of what they see there…they can make sense out of it. They can do what? They can add together. In the end, they get the number as five hundred for the first ten boxes, then another five hundred for the second ten boxes. Then, in the end…they do what? They add all the boxes together. Then they know that a farmer was using one thousand oranges and each box contains fifty oranges.” (T6, I2)

T7 explained the use of the dotted array for packing the fruit differently:

"As you had the question before, where a farmer had a certain number of fruits that are packed in twenty boxes. The farmer repacked the fruits into another setting or into new boxes. By the learners using the dotted array here…each dot there was representing a box, indicating as to how many fruits are going to be into that box. So after putting certain number of fruits into the box, the learner is going to add the total number of the fruits to see if whether it is still the same number as what the farmer had initially." (T7, I2)

Both T6 and T7 realised that the visual aid was helpful to the learners in their attempt to solve the word problem.

- **Subjective meanings of the participants' responses about the use of visualisation processes as a result of participating in the VIP:** T5 was convinced that a visual aid like the diagrammatical representation of rectangles strengthened the learners' ability to identify fractional forms in the diagram and also to do operations with fractions. The learners could visually calculate what portion of the diagram was represented by a fractional part of a fraction, a mathematical exercise that would have been complex if attempted algorithmically.

"I think this use of visual aid helps a lot because learners are able to understand based on what they see. For an example, these learners…they were looking at the diagrams and know two-fifths and then, that is why they were able to know where half of two-fifths is. And then secondly also, they were able to calculate the half of two-fifths, using other strategies." (T5, I2)

T6 and T7 also expressed their belief in the value of visualisation processes in teaching as being opposed to the conventional way of teaching.

"Definitely. As I said, it is easier for a learner to understand what he sees rather than what he hears. Because sometimes you can try to explain something by words without putting what you are explaining on the board and you will find that the learner does not understand. If you can explain and at the same time put a visual on the board, it is easy for the learner to understand." (T7, I2)

- **Perceptions of the participants (as a subcategory of subjective meanings) about the use of visualisation processes as a result of participating in the VIP:** T5 had the perception that even the challenged learners such as who find it difficult to learn or are hard of hearing do benefit from the visual teaching strategies due to the fact that the visual aids could be observed and as such communicate directly with the learners. T5 furthermore stated that the diagrams were helpful in assisting the learners in identifying fractions, knowing how to form a fraction and identifying the name of the particular fraction.

  T6 perceived that learners were encouraged to visualise when presented with the visual aids. This process led to greater learner participation during the lesson to the extent that even the apathetic learners started to participate.

  "Obviously yes, because the more they visualise…I have seen since I started this thing…the more they visualise, the more eager every one of them wanted to participate. And then, even those ones that will be hiding in their own shells…in their own corners there…they are now doing what? They are now coming out, because they can see this is very interesting." (T6, I2)

- **Errors and misconceptions of the participants about the use of visualisation processes as a result of participating in the VIP:** T7 struggled to use the diagrammatical representation as a visual tool to facilitate an environment for the learners to form fractions representing the parts shown in different colours.

  "Yeah. Here, even though the learners were really struggling…I also struggled to see as to…how can I use this diagram to convert…I mean…to make common fractions? I want to be honest with you, Sir. I could not relate that…I mean…I could not use this diagram to write common fractions or to convert it from common fractions to percentages. So that is why they also struggled here." (T7, I2)

- **Consistencies in the participants' responses about the use of visualisation processes as a result of participating in the VIP:** T5 and T7 were consistent in applauding the benefits of visual teaching for the learning experience of the learners. T7 saw improvement regarding the performance of his learners.

“The visual strategies did help. Since we have started this project of number sense and visualisation, I can see a lot of improvement on the performance of my learners. Initially we were just putting numbers on the board without putting some visuals, but if you can put the visuals on the board, the learners can see the difference between what you are saying.” (T7, I2)

T5 consistently declared that the visualisation processes can even help those learners who struggle to properly understand English as the medium of instruction.

“Even the learners who are unable to hear me, but cannot understand the language, but use their eyes to see based on what is in front of them and then try to make some sense out of what they see.”

- **Inconsistencies in the participants’ responses about the use of visualisation processes as a result of participating in the VIP:** No noteworthy inconsistencies were found in the three participants’ responses during the video-stimulated recall interview.

- **Common elements of the participants’ responses about the use of visualisation processes as a result of participating in the VIP:** The three participants repeatedly stated that the use of visualisation processes in their teaching of mathematics contributed positively to the conceptual understanding of learners. They also cited some challenges with visual teaching, like the time-consuming nature of preparing the visual aids.

4.3.5.3 General Findings

T5, T6 and T7 struggled to align the teaching of addition and subtraction of decimal fractions with the analytical framework (Appendix C) during their first official lesson. The decimal place value chart which T5 employed was not the best strategy for teaching the addition of decimal values. T6 and T7 reverted to the conventional way of teaching decimal fractions without seeking innovative visual methods. A series of calibrated decimal number lines could have been used for teaching how to operate on decimals as decimals can be displayed with different calibrations showing ever-decreasing distances between the same start and end point. Decimal number lines provide a context for exploring the magnitude of decimal fractions and the relationship between place values before and after the decimal comma.

The second official lessons of all three participants were consistently aligned with the analytical framework (Appendix C). The participants chose the diagrammatical representation with smaller rectangles within a big rectangle, the decimal number line, the spider diagram, an array of dots and the fraction wall as visual aids for the number sense themes that were evident in their lessons on multiplication and division. All three participants were rated favourably regarding their proficiency in using visual strategies in the teaching of number sense, as a result of them participating in the VIP.

The insights which the three participants demonstrated during the video-stimulated recall interview were characterised by the value which they saw in making use of visualisation processes when teaching number sense. The participants could vividly describe the learning process that was being facilitated by the use of the different visual aids. They deeply understood the exact manner in which their learners' conceptual understanding of number was developed through the use of visualisation processes.

The participants also criticized the time-consuming nature of preparing the visual aids. However, they still considered the use of visualisation processes as valuable for the successful teaching of number sense understandings and therefore promised to keep on implementing visual teaching strategies beyond the data collection process.

The views expressed by T5 and T6 were that challenged learners also benefit from visual teaching. This view is encouraging as it suggests that visualisation processes may facilitate an environment for optimal learning opportunities to ensure that struggling learners gain conceptual understanding. The use of visualisation processes in the teaching of number sense understandings therefore has the potential to turn around the sad reality of mathematics education in South Africa.

4.4 THE PRE- AND POST-TEST ANALYSIS AND RESULTS

4.4.1 Introduction

The analysis consisted of statistically analysing the pre- and post-test results of the Grade 6 learners of the seven participating teachers. The test results were analysed by using bar graphs as support in answering the research question. First, the pre- and post-test results of the seven experimental schools and the control school were compared to report on the change in learner performance.

The average change in learner performance at the experimental schools was then compared to that of the control school, to ascertain whether visualisation processes implemented within the experimental schools led to more improvement of the pre-test results than the change in learner performance within the control school where no intervention was done.

A comparative analysis of the experimental and control schools' performance regarding the different number sense topics was also done, to investigate those number sense areas where visualisation processes impacted more effectively. In the final analysis, each experimental school was separately analysed to investigate how learners of different capabilities performed after the implementation of the VIP.

4.4.2 Quantitative Findings

4.4.2.1 Comparison of the Number Sense Test Results

Table 4.1 and Figure4.1 show the percentage of change in the number sense pre- and post-test results of the experimental schools and the control school. All eight schools performed better in the post-test. Five of the seven experimental schools performed better than the control school concerning the incremental change.

Table 4.1: A comparison of the number sense test results in the seven experimental schools and one control school

Schools	Average Pre-Test %	Average Post-Test %	Average Change %
Experimental School 1	18	25	7
Experimental School 2	21	23	2
Experimental School 3	19	22	3
Experimental School 4	32	39	7
Experimental School 5	27	35	8
Experimental School 6	20	28	8
Experimental School 7	26	38	12
Control School 1	35	40	5

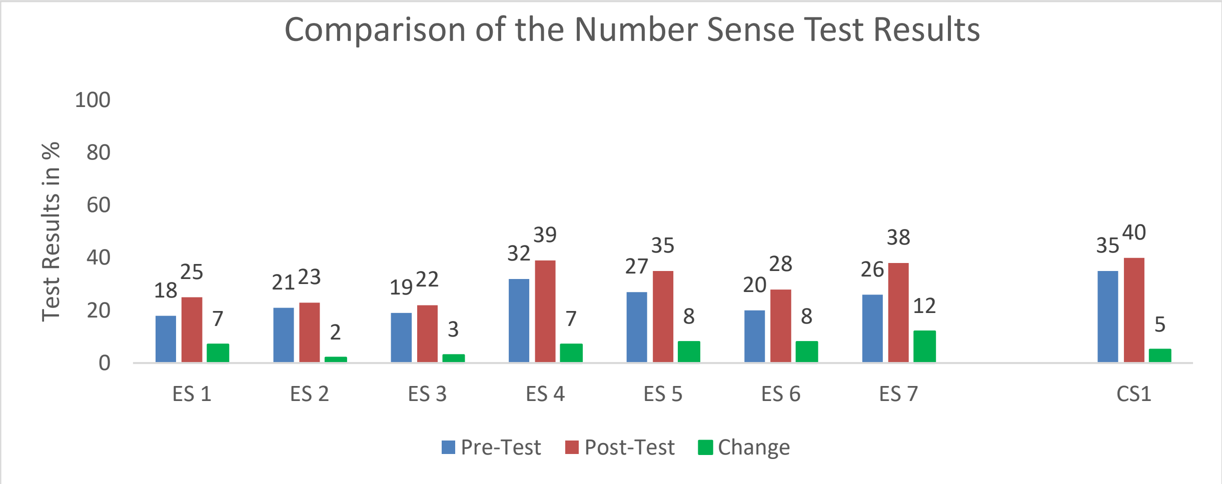


Figure 4.1: A comparison of all the average number sense pre- and post-test results

4.4.2.2 Learner Performance Change

In Table 4.2 and Figure4.2 the average pre- and post-test results of the experimental schools was calculated and compared to that of the control school. It is clear from the evidence that the control school generally performed better in both the pre- and post-test. However, most of the experimental schools (5 out of 7) demonstrated a better relative improvement than the control school.

Table 4.2: A comparison of the average number sense test results in the experimental schools and the control school

Schools	Pre-Test %	Post-Test %	Change %
Experimental Schools (Average)	23	30	7
Control School	35	40	5

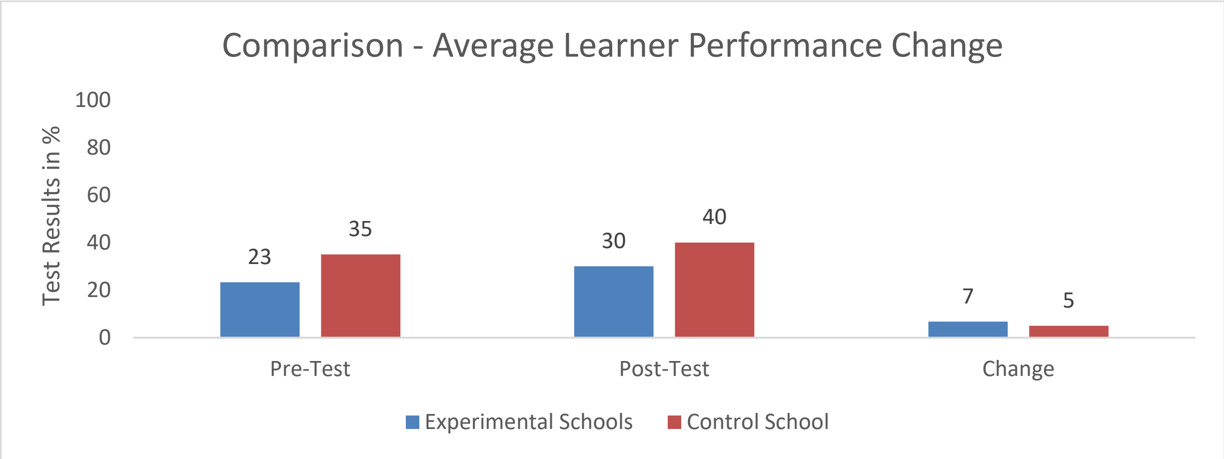


Figure 4.2: A comparison of the average number sense test results

4.4.2.3 Learner Performance in the Number Sense Topics

Table 4.3 and Figure4.3 show that the experimental schools improved their performance in all four number sense topics, although both the pre- and post-test results were generally poor. It highlights the fact that decimal fractions are most challenging for learners. The generally poor pre- and post-test results are also indicative of the real and continuous challenges that exist as far as the number sense of learners in South African public schools is concerned. This reality justifies my argument that teachers should teach number sense in a visual manner. The results below indicate that particularly the topics of ‘common fractions’ and ‘whole numbers’ have improved remarkably due to the use of visual aids.

Table 4.3: A comparison of the average learner performance in the number sense topics in the experimental schools

Number Sense Topics	Average Pre-Test %	Average Post-Test %
Whole Numbers	25	34
Fractions	22	30
Decimals	21	24
Percentages	26	32

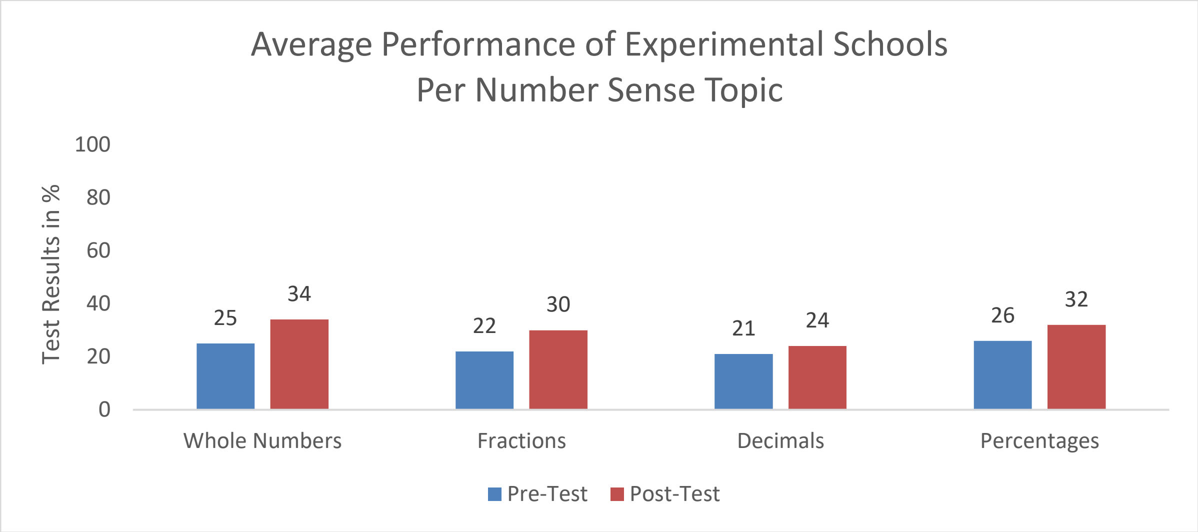


Figure 4.3: Average learner performance in all the number sense topics at the experimental schools

In Table 4.4 and Figure 4.4 the learner performance of the control school in the different number sense topics was analysed and compared to that of the experimental schools. The experimental schools had an eight percent increase in the topic of 'fractions', whereas the control school only improved its performance in the same topic by three percent. The experimental schools also showed more improvement in the topic of 'decimals'. The improvement in the topic of 'whole numbers' was the same for both the control and experimental schools. Although the control school performed better in the topic of 'percentages', the overall improved performance of the experimental schools was better than that of the control school.

Table 4.4: A comparison of learner performance in the number sense topics in the control school

Number Sense Topics	**Average Pre-Test %**	**Average Post-Test %**
Whole Numbers	47	56
Fractions	36	39
Decimals	24	26
Percentages	32	41

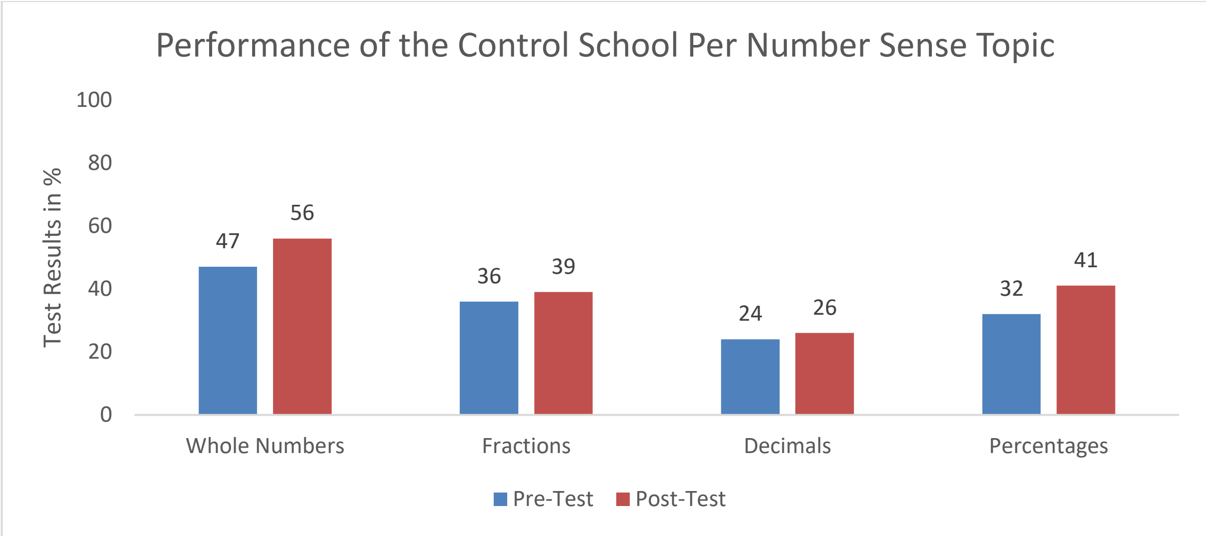


Figure 4.4: Learner performance in all the number sense topics at the control school

4.4.2.4 Individual Analysis of the Experimental Schools

An analysis of the seven experimental schools was done individually to ascertain the extent to which a visual approach to the teaching of number sense enhanced learner performance and to compare the pre- and post-test results of the experimental schools with each other.

Figure 4.5 demonstrates that Experimental School 1 had an improved performance of ten percent in the topic of 'percentages', which was the greatest improvement result of all number sense topics for the school. The results for all the other topics also improved. Performance in the topic of 'whole numbers' improved by eight percent. This may be due to the fact that T6 made use of flashcards for mental mathematics exercises during the implementation of the VIP. The exercises were designed to assist learners with thinking fast and calling out whole numbers with more than three digits. This is how T6 expressed himself in this regard during the video-stimulated recall interview:

"Because if I look at some cases, most of the learners cannot read the number that they have got more than three digits. So, the more they read, because they must make a sense out of what they see there. So, I do not read it for them. They read for me and in the same time, I do what, I ask the other learners whether the number is numerically correct or not." (T6, I1)

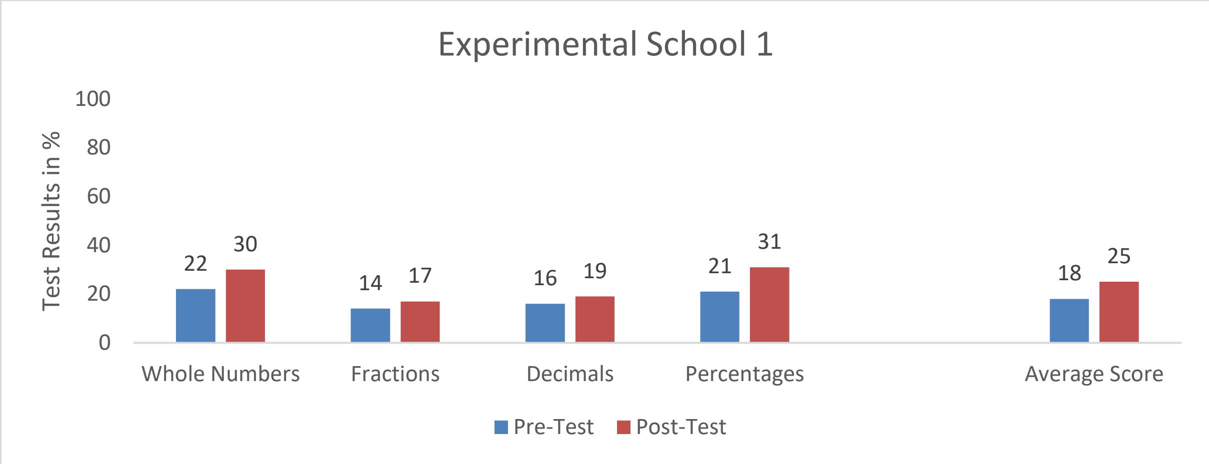


Figure 4.5: Learner performance in all the number sense topics at Experimental School 1

In Figure 4.6, the most improvement for Experimental School 2 was eight percent in the topic of 'common fractions'. This is an interesting finding as the conceptual understanding of fractions in the Intermediate Phase is a real concern. T5's use of visualisation processes in the teaching of 'common fractions' was responsible for the substantial improvement of eight percent for the topic of 'common fractions'. She used visual aids like the fraction circle, the fraction wall and diagrammatical representations to facilitate the teaching of 'common fractions' in both official lessons of the VIP. The improvement in 'whole numbers' was marginal and there was a decline in performance regarding the topic of 'percentages'. This decline may be ascribed to the fact that T5 did not take a visual approach in teaching the topic of 'percentages' to her learners when she implemented the VIP. She therefore did not explain in a visual way how percentages are related to common and decimal fractions.

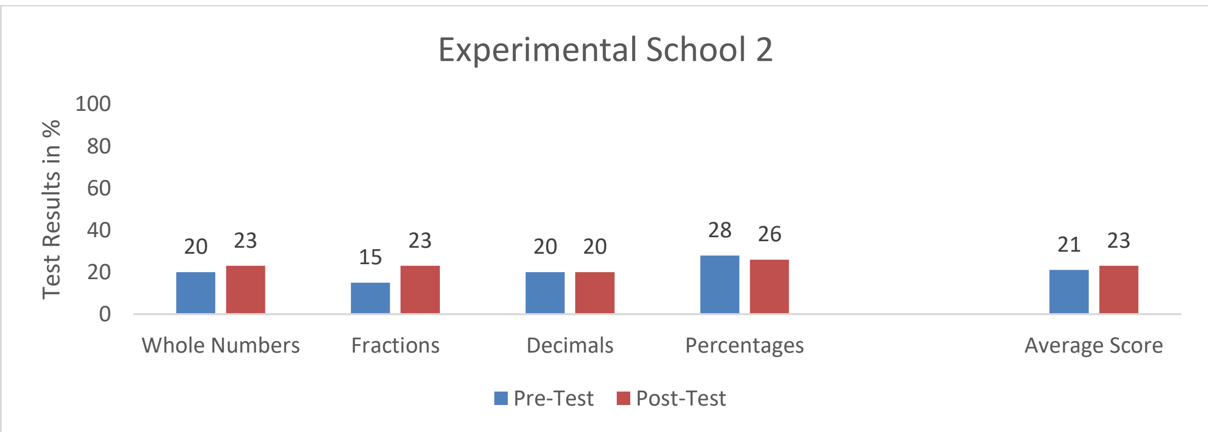


Figure 4.6: Learner performance in all the number sense topics at Experimental School 2

The Experimental School 3 results depicted in Figure 4.7, also indicate that an improvement of eight percent was attained for ‘common fractions’ while the performance in ‘decimal fractions’ declined with seven percent. This result points to the observation that T7 used a lot of visual aids to teach common fractions during the implementation of the VIP, while he did not use any visual aids for the teaching of decimal fractions. He did not consider the use of decimal fraction strips or decimal fraction circles to demonstrate decimals as part of a whole. This could have easily resulted in the learners having a rule-based approach to treating decimal fractions, instead of applying conceptual knowledge of decimal fractions in computational settings. The pre- and post-test required learners to know decimal fractions as part of a whole and to execute the computations within that context.

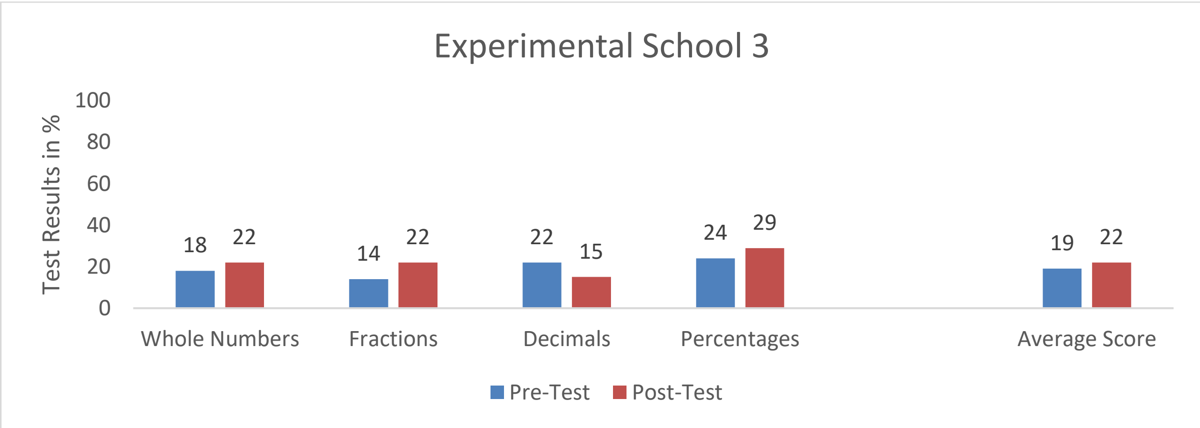


Figure 4.7: Learner performance in all the number sense topics at Experimental School 3

Experimental school 4 had a consistent improvement of seven percent across all number sense topics, with an improvement of eight percent for ‘common fractions’. This improvement in results is noteworthy given that T2 teaches an overcrowded class. The participant succeeded in facilitating a mathematically rich environment for the teaching of number sense understandings during the implementation of the VIP.

His learners used a combination of counters and the calibrated number line to add whole numbers. T2 used a diagrammatical representation of fractional parts within a whole unit, which contributed positively to the learners’ understanding of the nature of fractional parts and their relation to the whole. It furthermore enhanced the computational fluency of his learners in adding and subtracting fractions.

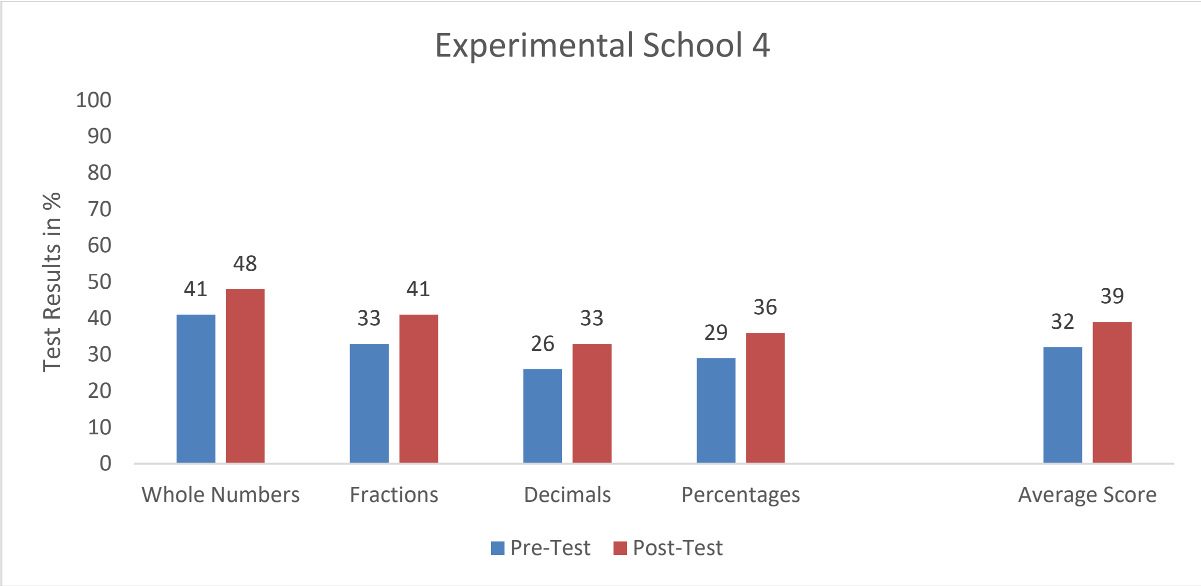


Figure 4.8: Learner performance in all the number sense topics at Experimental School 4

According to Figure 4.9, the improvement result of Experimental School 5 is seventeen percent for 'whole numbers' and nine percent for 'common fractions'. All number sense topics show an improvement. Abundant evidence was presented of T3's visual teaching practice and her ability to choose the most efficient and relevant combination of visual aids for teaching number sense understandings during the implementation of the VIP.

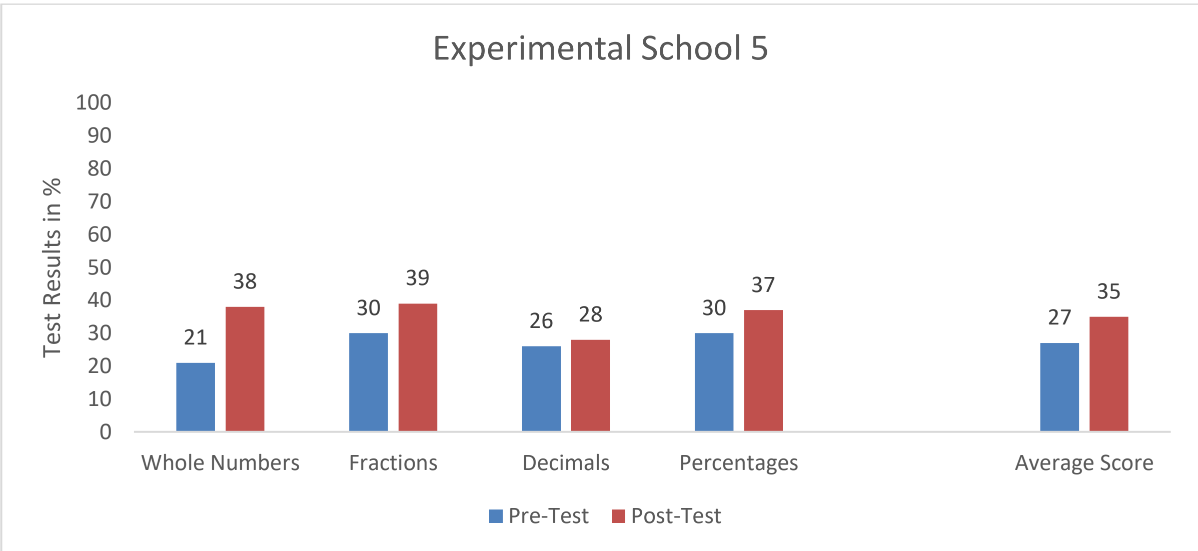


Figure 4.9: Learner performance in all the number sense topics at Experimental School 5

In Figure 4.10, Experimental School 6 shows consistent improvement in performance across all number sense topics. The topic of ‘common fractions’ showed the greatest improvement result of nine percent while the performance in ‘decimal fractions’ improved by eight percent. The improvement result in ‘decimal fractions’ is remarkable as it is the second highest improvement amongst all seven experimental schools, where one school had no improvement and another showed a decline of seven percent.

The reason for this outcome may be that T4 consistently applied visual teaching techniques in his teaching of number sense understandings during the implementation of the VIP. He taught the multiplication and benchmarking of decimals by using a decimal number line. The learners had to calculate the greater product between thirteen multiplied by nought comma five and fourteen multiplied by nought comma five by making use of a decimal number line. Learners innovatively calibrated the number line in equidistant parts of nought comma five. The learners recognised the operation of multiplication as repeated addition. They then skip counted thirteen times and realised that skip counting fourteen times would take them further down the decimal number line. Hence, their answer was that fourteen multiplied by nought comma five is greater than thirteen multiplied by nought comma five. I argue that the manner in which T4 made use of visualisation processes in the teaching of decimals, improved the conceptual understanding of his learners.

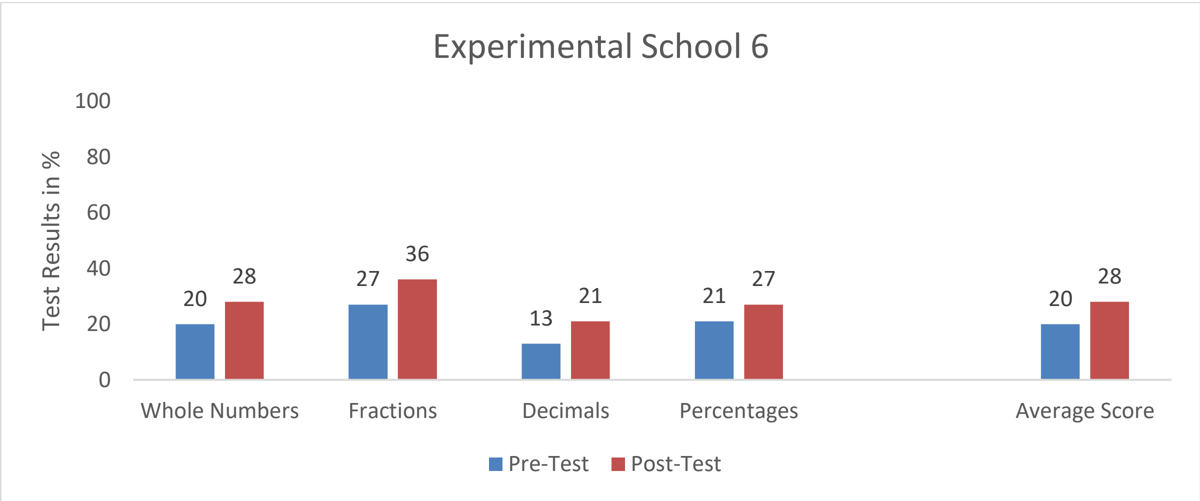


Figure 4.10: Learner performance in all the number sense topics at Experimental School 6

Figure 4.11 shows the results of Experimental School 7. This school showed the most improved results of all seven experimental schools. The teacher at Experimental School 7 is a visual teacher by nature. She implemented the VIP efficiently and confidently. During the observations, she regularly expressed her belief in the benefit of using visualisation processes in the teaching of number sense understandings.

T1 ensured that she provided every learner with a visually stimulating and mathematically rich environment in order to enhance the teaching of number sense understandings. Performance in the topic of 'whole numbers' improved by fifteen percent, 'common fractions' by twelve percent, 'decimal fractions' by ten percent and 'percentages' by eleven percent. The overall improvement of twelve percent is the highest of the other six experimental schools and the control school.

The consistent improvement results of Experimental School 7 is an indication that the VIP in that school had the desired effect. T1 used visual aids to represent fractional parts in such a manner that her learners could understand the relationship between common fractions, decimals and percentages. Teaching the different representations and equivalent forms of fractional parts assisted T1's learners in building conceptual understanding.

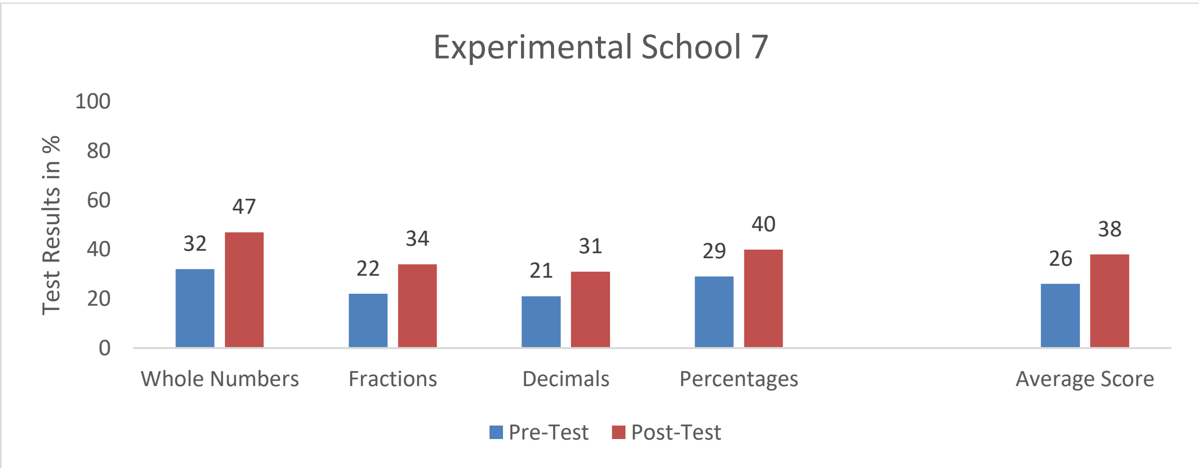


Figure 4.11: Learner performance in all the number sense topics at Experimental School 7

4.5 CONCLUSION

In this chapter, an analysis was made of the views and experiences of the seven teachers who participated in the VIP. This included an analysis of the focus group interview, the first and second official lessons of the VIP and the number sense pre- and post-test results. The focus group interview involved the seven participants who collaboratively reflected on their pilot lessons. The views and experiences of the participants were extracted by looking for emerging concepts in the interview transcripts. These concepts were then categorized for description, discussion and interpretation. Six categories and two subcategories were identified for the qualitative analysis. The interview transcripts provided a substantial amount of material and the interview responses verified and strengthened the analysis.

Furthermore, an analysis of the results of the pre- and post-test on number sense are also contained in this chapter. The results of the seven experimental schools were compared to those of one control school that share the same socio-economic context. The results indicate that the use of visualisation processes had a positive outcome for the teaching of number sense understandings as the experimental schools demonstrated greater improvement in the post-test results than the control school.

The next chapter uses the analysis of Chapter 4 to discuss the findings, consider some implications, draw conclusions from this study, formulate a number of recommendations and explore avenues for further research.

CHAPTER 5

CONCLUSION AND RECOMMENDATIONS

5.1 INTRODUCTION

My research study sought to answer the question about how selected teachers could usevisualisation processes in a mathematics classroom in order to enhance their teaching of number sense, as a result of participating in the Visualisation Intervention Programme (VIP) attached as Appendix A. The research study was undertaken with seven RUMEP teachers who participated in the VIP. The qualitative and quantitative sets of data discussed in this chapter were collected over a period of seven months.

In this chapter I firstly present and discuss a summary of the main findings. Then some implications of the study are considered. Conclusions are drawn from the research study regarding the use of visualisation processes in teaching number sense understandings. Further, recommendations are made about the use of visualisation processes as a strategy to enhance the teaching of number sense. Finally, avenues for further research are explored.

5.2 SUMMARY OF THE MAIN FINDINGS

The main findings are presented and discussed as per the deliberations of the focus group interview and reflections on the observed lessons. This constituted the qualitative data of my research study. The presentation and discussion of the pre- and post-test results constituted the quantitative data of the study.

5.2.1 Deliberations of the Focus Group Interview

The outcomes of the deliberations in the focus group interview showed that simply making use of visual aids during the teaching of number sense understandings may not be enough for promoting the conceptual understanding of learners regarding the number sense themes in Table 2.3. It is far more important to make use of visualisation processes that are relevant for the number sense theme under discussion, engage learners to explore the concept more fully and promote the learners' reflective thought regarding the number sense theme.

It further emerged that teachers who make use of visualisation processes that enhance their teaching of number sense, need to constantly collaborate with like-minded teachers in order to share ideas, offer support and clarify misconceptions regarding the use of visual aids during their lessons.

5.2.2 Reflections on the Observed Lessons

Implementation of the VIP assisted the participants to consistently align their lessons with the main constructs and elements of my analytical framework (Appendix C). The second official lesson was better aligned with these constructs than the first official lesson, because of the reflective workshop that was held between the two lessons. The participants ascertained which visual strategies might be most appropriate for the number sense themes inherent to their VIP lessons. Most of the chosen visual teaching strategies facilitated rich mathematical environments for promoting conceptual understanding of the number sense themes. All participants demonstrated some evidence of teaching proficiency in using visualisation processes according to the criteria of teaching proficiency (Kilpatrick et al., 2001), that were evident in their observed lessons.

All participants saw value in using visualisation processes in the teaching of number sense understandings. There was general consensus that the use of visualisation processes in the teaching of number sense understandings also promoted the participants' own development as teachers. The participants testified that as a result of the use of visual aids which introduced more fun-filled activities into the mathematics classrooms, most of their learners seemed to enjoy mathematics more, thus making the subject more interesting to them. Four participants observed how even the slow and challenged learners seemed to understand the number sense themes better and participated more freely in group discussions. They believed that the use of visualisation processes facilitated an environment for learning opportunities.

An interesting observation of one of the participating teachers, was that the use of visualisation processes in the teaching of number sense understandings stimulated independent thought on the part of the learners. The learners of the participant devised their own methods of solving problems, based on the principles they had mastered through the visual teaching that was offered to them. The participants were convinced that their learners performed better when they are able to see and touch the visual aids. The participating teachers expressed their belief in the benefits of visual teaching for learner development, but acknowledged that it was time-consuming to prepare the visuals for use during their VIP lessons.

It is noteworthy that some participants reverted to conventional teaching in cases where they could not use visualisation processes for the teaching of number sense understandings. One such case was the teaching of decimal fractions. The participants found it difficult to use their creativity to use the calibrated decimal number line, the conversion chart for benchmarking, decimal fraction strips or decimal fraction circles to visually teach decimal fractions.

One participant erroneously used a balance scale to teach inverse operations, whereas the calibrated number line could have been used more effectively in demonstrating counteractive movement for inverse operations. The participants generally had reservations about the sustainability of visual teaching beyond the implementation of the VIP, due to scarcity of resources.

5.2.3 Pre- and Post-Test Results

The experimental schools compared favourably to the control school regarding the average percentage of change in the test results, although the average pre- and post-test results of the experimental schools were lower than that of the control school. When comparing the average change in performance of the experimental schools with that of the control school, the result was most favourable for the experimental schools in the topic of 'common fractions'. This resonates with the finding that the participating teachers put a lot of effort into the visual teaching of 'common fractions' during the implementation of the VIP.

The experimental schools were analysed individually and the results that were obtained paints an interesting picture concerning the use of visualisation processes in the teaching of number sense. One experimental school showed a decline of two percent for the topic of 'percentages', while an improvement of eight percent was achieved for 'common fractions'. The participating teacher at this experimental school did not use any visualisation process for teaching 'percentages' and she also did not demonstrate the relationship between 'percentages' and 'common fractions'. At a second experimental school, a decline of seven percent was recorded for 'decimal fractions', while an improvement of eight percent was achieved for the topic of 'common fractions'. Once again, the analysis showed that the teacher did not use any visualisation process for teaching 'decimals'.

The participating teacher whose improvement result was the highest of all schools in the topic of'common fractions', demonstrated abundant evidence of proficiency in visual teaching. She innovatively looked for ways in which she could use different visuals in a complementary fashion to enhance the teaching of the number sense theme under discussion and thus promote the conceptual understanding of her learners.

One of the participants of my research study experienced less than desirable classroom circumstances, like overcrowding. However, the average percentage change in his learners showed consistent improvement across all number sense topics of the pre- and post-tests. The teacher also approached the teaching of his VIP lessons by taking great care to ensure a visually stimulating mathematical environment for the learners in his class. The results point favourably to the value of using visualisation processes in the teaching of number sense themes.

The learners of one of the experimental schools creatively skip counted on the calibrated decimal number line to turn the multiplication operation into one of repeated addition, and draw a conclusion as to the larger product of the two presented by the participating teacher. This interesting use of the visual aid corroborates the perception that the use of visualisation processes enhance the teaching of number sense understandings. The average change result for the topic of 'decimal fractions' was eight percent for this experimental school and this serves as further confirmation of the benefit inherent to visual teaching.

The best average performance improvement of all eight schools (the control school included), was recorded at one of the experimental schools where visual teaching has been adopted by the participating teacher. This teacher decorated her classroom with schematic wallpaper visuals that are rich in mathematical relationships. The learners are confronted with these visuals every day. During the VIP lessons, the teacher issued relevant visual aids to the different learner groups for the completion of the activities of the particular lesson.

The impressive improvement that was recorded for all number sense topics, vindicates the teacher's use of visualisation processes for the teaching of number sense understandings. Performance in all four number sense topics improved significantly. For the topic of 'whole numbers' an improvement of fifteen percent was recorded. For 'common fractions' it was twelve percent, for 'decimal fractions' the positive change was ten percent and 'percentages' improved by eleven percent. The overall average improvement of twelve percent was the highest of both the other six experimental schools and the control school.

5.3 SIGNIFICANCE OF THE STUDY

According to the Department of Basic Education of the Republic of South Africa [DBE RSA] (2014), the Annual National Assessments (ANA) Mathematics test performance of learners in the Intermediate Phase was a cause for concern. The ANA Mathematics tests evaluated the numeracy skills of South African learners. The DBE RSA further noted that in terms of the presidential targets of at least sixty percent of learners achieving acceptable levels of performance (fifty percent or more in test results), the Grade 6 Mathematics national results from 2012 to 2014 showed that learners were still below the target.

It is interesting to note that the Curriculum and Assessment Policy Statement (CAPS) was introduced in 2013 for the Intermediate Phase. However, the poor ANA results in 2013and 2014 indicated that the expected level of fluency and numeracy was not yet achieved (DBE RSA, 2014). This was perhaps due to some challenges with the implementation of the CAPS document in Grade 6. These challenges need to be examined as they were important for my research study. The most important challenges can be summed up as follows:

- Although the CAPS document resonates well with number sense understandings and the analytical framework (Appendix C), there is no serious drive in the curriculum to advocate the use of visualisation processes. The use of visual aids are simply recommended for some number sense themes in the clarification notes of the curriculum, but no guidelines are provided.
- Only fifty percent of the curriculum is currently allocated for the teaching of number sense topics. Good number sense is a prerequisite for mastering higher order mathematics during later phases. More time should therefore be allocated in the primary mathematics curriculum

for the teaching of number sense understandings. It is encouraging that the Mathematics Teaching and Learning Framework of the Department of Education states that issues in the curriculum such as "scope, depth, sequencing and time allocation need to be reviewed" in order to teach mathematics for conceptual understanding (DBE RSA, 2018, p. 10).

- The CAPS document compartmentalises number sense topics. This has led to number sense being taught in silos of the different number sense topics, which makes the effective and coherent development of number sense understanding difficult for learners. Whole numbers and fractional parts should be taught alongside each other using visualisation strategies to expedite the conceptual understanding of the learners.
- Although the CAPS document has constructivist underpinnings, no comprehensive guidance is given to teachers in terms of their classroom practice. The methods and practice of teaching number sense understandings should be infused with the use of visualisation processes, to ensure that learners make meaning of the number sense themes as per Table 2.3.

Herein lies the significance of the research study. The use of visual strategies which could enhance the teaching of number sense was investigated in this research study, in order to draft the VIP which could identify and address the compounded learning deficits concerning number sense themes in the John Taolo Gaetsewe District. This research study focused on Grade 6, in the Intermediate Phase in primary schools. The mathematics learning backlog that was apparent in the Intermediate Phase at the primary level is carried forward into the Senior Phase at high schools, as the 2013 ANA results showed that "achievement levels in Mathematics declined across the grades with progressively steeper declines from Grade 6 to Grade 9" (DBE RSA, 2014, p. 15).

There was therefore a need to explore the pedagogics of Intermediate Phase teachers with regard to number sense, in order to fully grasp the nature of the crisis in mathematics teaching and learning, and to contribute to the debate of how best to improve the mathematics performance in South African schools. My research study showed that the use of visualisation processes for the teaching of number sense understandings, led to the promotion of independent thought within the learner groups. The learners were able to construct their own knowledge by using their own calculating procedures and using the visual aids provided. Kamii, Lewis and Livingston (1993) emphasize that learners should be encouraged to invent and use their own calculating procedures.

These learners would then be able to solve more problems correctly, build their own meanings and understand mathematical concepts more competently than children who were taught the conventional, rule-based algorithm. This finding in my research study is a reflection on the extent to which visual teaching enhances the acquisition of number sense.

The development of the participating teachers was enhanced as a result of them participating in the VIP. In their reflections, the participants expressed their views on how the implementation of the VIP improved the quality of their own classroom practice. They interrogated the extent to which they applied the principles of 'social constructivism' as the theory of learning that they subscribe to. Participation in the VIP informed their teaching practice regarding the value they came to attach to the use of visualisation processes when teaching for their learners' improved conceptual understanding.

5.4 RECOMMENDATIONS

My research study suggests valuable information for teachers to consider when they use visualisation processes in the teaching of number sense. It is important to create a mathematically rich, stimulating and visual environment for the learner groups and guide them towards conceptual understanding of the number sense theme under discussion. Then allow them sufficient time for self-exploration in completing the lesson activities by making use of the visual aids. The learners soon appreciate the visual process of solving the problem, instead of copying the standard algorithms which are rule-based.

The focus should always be on facilitating an environment for independent and creative thought on the part of the learner. If standard algorithms become the norm for teaching number sense understandings, the thinking ability of learners will be compromised and stifled. Learners should reason with their peers and communicate their ideas about number sense understandings within a group work context, by using visualisation processes facilitated by the teacher. If this process is not followed in a non-threatening, visual classroom setting, the development of number sense can be constrained.

Teachers should always create situations that will enable them to understand the manner in which the learners think about the number sense concept and how they engage the visual aid to arrive at the solution of the problem. It is therefore advisable that learners are requested to visually present their group work findings to the whole class. This will give the class an opportunity to interrogate the solution and the learner group will be presented with an opportunity to justify their mathematical thinking in a visual manner.

The mathematics teacher should take care that the visual strategies employed for teaching number sense understandings are always executed in the most efficient, accurate and flexible manner. The proficiency to teach visually depends on the choice of the visual aid within a certain mathematical setting. The visualisation process employed must be represented by dynamic, schematic imagery which allows the learners to form mathematical relationships between whole numbers and fractional parts.

5.5 SUGGESTIONS FOR FURTHER RESEARCH

Given the challenges posed by the CAPS document and the need to steer Intermediate Phase mathematics in South Africa away from the mainstream teaching of standard algorithms, I suggest that an in-depth longitudinal study about the use of visualisation processes to enhance the teaching of number sense be undertaken with Grade 4, 5 and 6 teachers and learners.

My research study was located within a nodal zone, a rural area with mixed socio-economic conditions, ranging from villages with minimal resources to conditions of abject poverty. It would be interesting to see how the research results would differ if this research study was done within more affluent, urban areas. Research about the use of visualisation processes in the teaching of number sense for the Intermediate Phase could also be undertaken with pre-service teachers from three or more universities located within different provinces, to ascertain the level of professional development regarding the teaching of number sense understandings in the Intermediate Phase.

I further suggest that research be undertaken on the role of visualisation processes in the remedial teaching of Intermediate Phase learners who are challenged regarding number sense understandings. Lastly, research into an alternative curriculum rooted in visualisation processes is encouraged, due to the prevalent teaching and learning conditions in South Africa, where, since the birth of our democracy in 1994 curricula have not succeeded in substantially improving the fluency and numeracy skills of Intermediate Phase learners.

5.6 PERSONAL REFLECTIONS

Undertaking this research study was indeed an enriching experience of continuous learning for me. The theoretical lens of social constructivism was appropriate as the VIP was at the heart of the research study. As a General Education and Training Facilitator for RUMEP the study challenged me to do introspection regarding my own teaching style and the manner in which I apply social constructivism in dealing with my cohort.

Analysing the views and responses of the participating teachers was a worthwhile exercise, although I wished for a deeper level of understanding on the part of some of the participants whose responses were a bit superficial. It could be argued that perhaps the quality of the views and responses of the participants reflected, to some extent, the state of their classroom practice.

The lesson observations and video-stimulated recall interviews needed commitment to time schedules. I am pleased to say that the participants never disappointed me in this regard. All observations and interviews transpired at the scheduled time. The pre- and post-test results were very low and disappointing. However, the results are in concert with the low educational standards that prevail within rural communities. Rural schools are characterised by learner absenteeism, substance abuse and poor classroom management. Learners progress from previous grades with compounded learning deficiencies that the receiving teachers have to struggle with.

As I reflect on the shortcomings of the CAPS document, it is frustrating to know that curriculum change is an extremely slow, national process. However, it remains important to encourage teachers to think outside of the box and teach number sense understandings comparatively and visually for the sake of developing the conceptual understanding of learners.

5.7 CONCLUSION

My mixed method research study was located within the interpretive paradigm. It was underpinned by social constructivism as the theoretical lens and the conceptual framework of teaching proficiency, in order to investigate the use of visualisation processes for the teaching of number sense. The VIP formed the empirical field of the study. Data was collected through the use of a number sense test, observation schedule and semi-structured interview schedule.

The research study showed that the conventional, algorithmic approach to the teaching of number sense understandings was not very useful and did not inspire logical, mathematical thinking. Instead, taking a visual approach to the teaching of number sense understandings developed the conceptual understanding of learners, stimulated independent thought, made mathematics more exciting and increased learner participation.

The research study furthermore advocated that enhanced visual teaching of number sense understandings places conceptual understanding first, then only gives attention to procedural fluency. It furthermore demonstrated how the use of visualisation processes may be used as a vehicle towards facilitating the learners' development of conceptual understanding. Procedural fluency is needed in mathematics. However, in the context of my study, if conceptual understanding is achieved, the learners will understand why the procedure works and they will then be able to deduce the rules for themselves.

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APPENDICES

APPENDIX A

VISUALISATION INTERVENTION PROGRAMME

This Visualisation Intervention Programme (VIP) is the collaborative effort of teachers who use visualisation processes and manipulatives to teach NUMBER SENSE in well-resourced schools. Interaction with the teachers of these well-resourced schools occurred as an open process, allowing the teachers to freely share their ideas and experience regarding the visual teaching of number sense. These meetings have been video recorded.

The Collaboration Team concluded that the following principles should inform the VIP:

- Teaching aids like the fraction wall, flashcards, the spider diagram, the fraction circle, food items like a cake or smartie sweets, the geoboard, the abacus, the graduated ruler, number charts, the number line, sketches (drawings), counters (stones and beans) and mathematical games could be used to teach number sense understandings visually.
- Visualisation makes the learner confident to share his / her ideas with others.
- In order to gain conceptual understanding, learners should have an image of the concept under discussion.
- The nature of numbers (whole and fractional parts) should be taught comparatively as many ways of representing the same reality and not as separate topics with no connection to each other.
- An environment should be facilitated in such a manner that learners discover the connection between the whole, fractions, decimals and percentages for themselves.
- The grade six mathematics curriculum should devote much more time to the building of number sense and not only fifty percent as the CAPS document dictates. A strong sense of numbers and operations as well as their use in computational settings, will ultimately facilitate better conceptual understanding of other topics in mathematics.
- There is no need to extend the number range to 9-digit whole numbers as the CAPS document prescribes for grade six. It is more important to teach for the conceptual understanding of numbers and operations by using smaller numbers as a tool towards that end.
- The CAPS document has compartmentalised number sense understandings. It would have been more educational to explore all aspects of number sense during a lesson. Such an

endeavour would create relational understanding whereby a learner would connect number sense understandings to real life.

- The VIP should endeavour to identify and address learning deficits regarding number sense in Grade 6. Four lessons will be taught in seven RUMEP schools during the second, third and fourth term of 2018. These four lessons will be rolled out as follows:

DATE	**TERM**	**CYCLE**	**LESSON TYPE**	**LESSON TOPIC**
May 2018	Two	One	Pilot Lesson	The Nature of Numbers
July 2018	Three	Two	First Official Lesson	Addition and Subtraction
August 2018	Three	Three	Second Official Lesson	Multiplication and Division
October 2018	Four	Four	Third Official Lesson	NS Word Problems

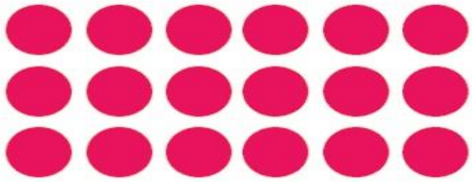
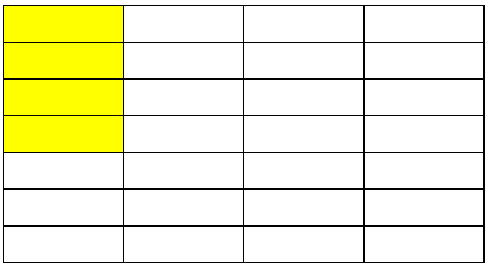
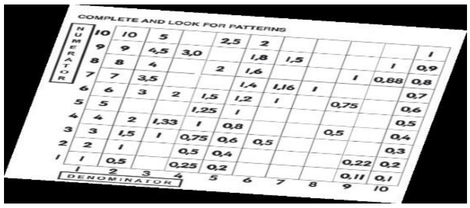
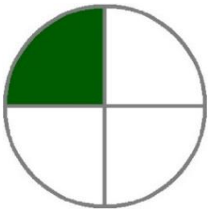
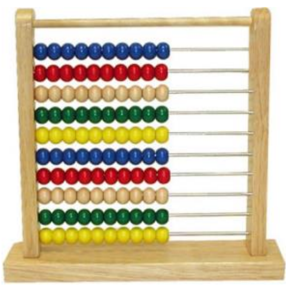
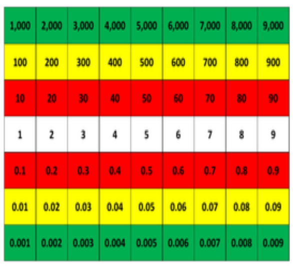
- In order to measure discernible change in learner performance regarding number sense understandings as a result of the implementation of the VIP, a number sense pre-test should be administered in one control school and seven RUMEP experimental schools. The post-test should be administered in these eight schools after the implementation of the VIP. The pre- and post-test are identical. The test is sourced from the Number Sense Item Bank of McIntosh et al. (1997) and will be piloted with five learners to check for any ambiguities before the administration of the pre-test to the eight schools.

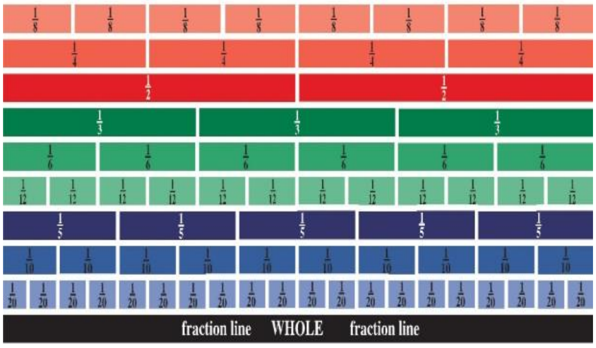
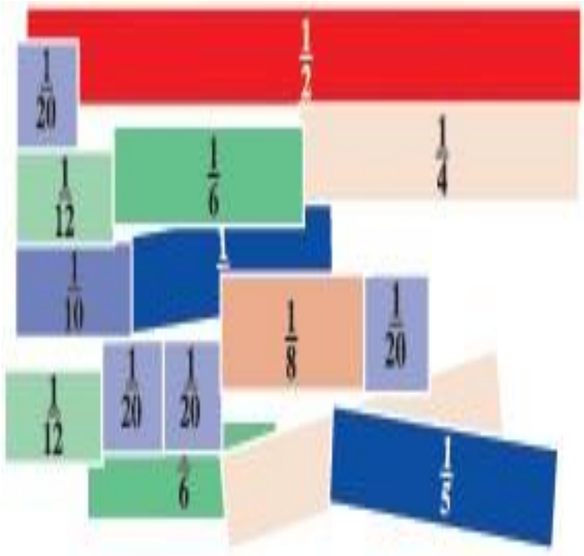
NUMBER SENSE LESSON PLANS FOR GRADE SIX

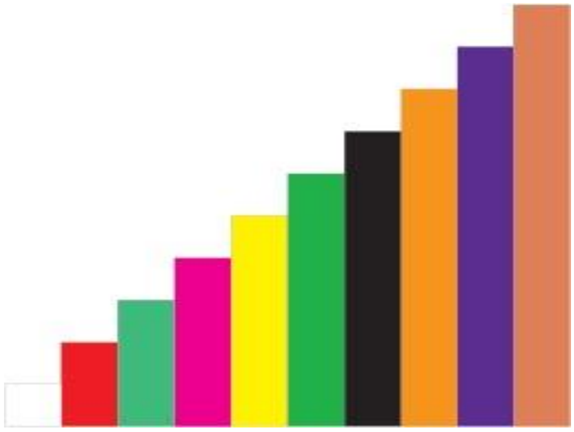
The Nature of Numbers

DATE	DURATION	TOPIC
_____ May 2018	One Hour	The Nature of Numbers

TEACHING SEQUENCE	LEARNER ACTIVITIES	VISUAL TEACHING STRATEGIES
Introduction:		
Check learners' prior knowledge on whole numbers and fractional parts by using a questioning technique. (5 minutes)	The following oral questions are put to the class: • Give the counting number before 500. • If you count in quarters, what comes after $2\frac{3}{4}$? • What is halve of 0.340? • What is double 25%?	 ***Number Charts***  ***Fractional Number Charts***

Mental Mathematics:		
Exercise the minds of the learners to prepare them for the lesson. (5 minutes)	The following oral questions are put to the class: • Look at the array. Form a number sentence to express the quantity of dots. • What fractional part has been shaded in this diagram? • Give the decimal fraction equal to a HALVE. • Look at the partly shaded circle. What percentage is NOT shaded?	 ***Array of dots***  ***Fraction diagram***  ***Decimal fraction board***  ***Fraction circle***
Development:		
The lesson develops in stages in order to address the nature of whole numbers and fractional parts. (40 minutes)	<u>**Activity 1**</u> The digits are: 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. Put one digit in each of the two boxes so that the answer will be as big as possible.	  ***Abacus Gattegno chart***

	(Do not repeat the digits.) **7 ☐☐ - 312 = ?** Explain why you chose those digits.	  ***Flard cards*** ***Sticks and bands***
	<u>**Activity 2**</u> You are given the following fractions: $\mathbf{\frac{1}{3}; \frac{1}{4}; \frac{1}{5}}$ **and** $\mathbf{\frac{1}{6}}$. Order the fractions from the smallest to the largest. Explain your answer by referring to the fraction strips.	 ***Fraction wall***  ***Fraction strips***

<table>
<tr><td></td><td></td><td>

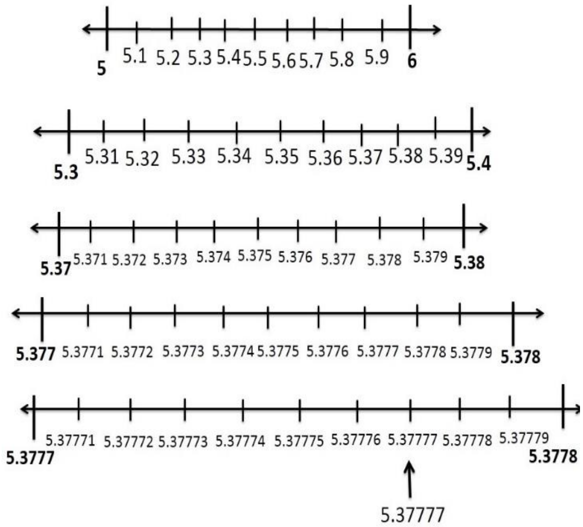
Cuisenaire rods</td></tr>
<tr><td></td><td>

<u>**Activity 3**</u>

Make use of the decimal number line and write down any three decimals between:

2,65 and 2,66

Explain your solution.

</td><td>

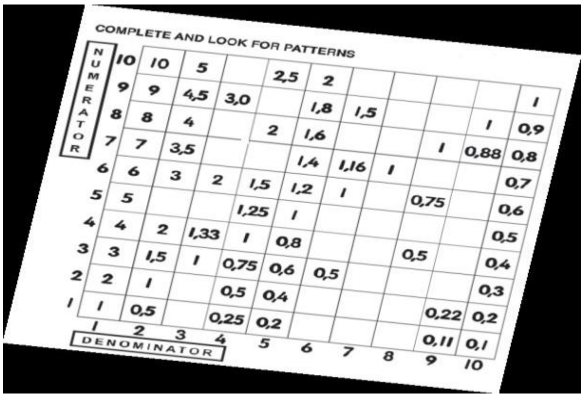
Decimal number line</td></tr>
<tr><td></td><td>

<u>**Activity 4**</u>

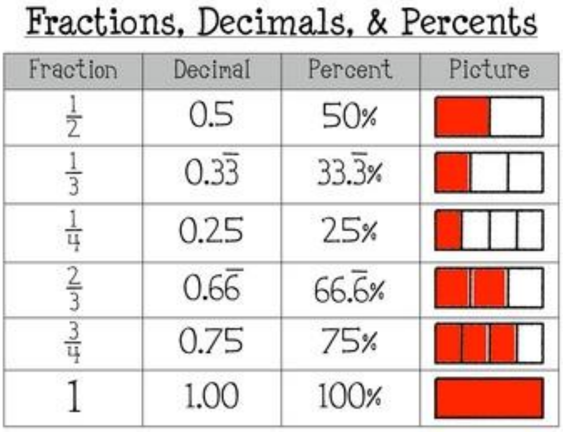
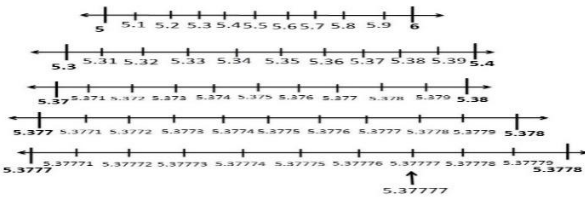
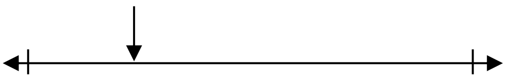
You are given the following numbers:

33% ; 0,25 ; $\frac{1}{5}$ and 0,81

Order the numbers from the smallest to the largest.

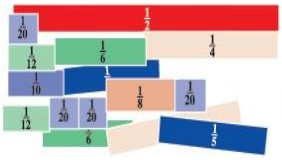
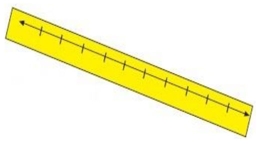
</td><td>

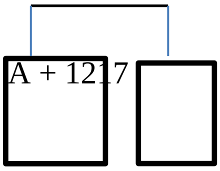
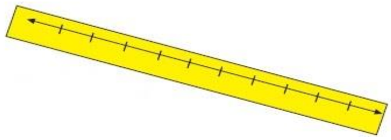
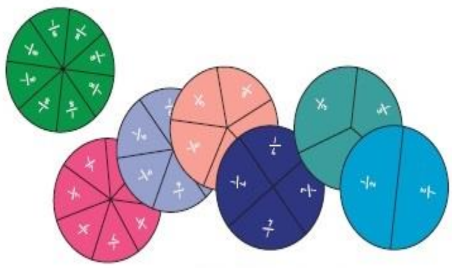
Decimal fraction board</td></tr>
</table>

	Explain your solution by making use of the decimal fraction board provided.	
Consolidation:		
The relationship between whole numbers and fractional parts are emphasized to consolidate the learners' understanding of the nature of numbers. (10 minutes)	**Activity 5** The following expression: 25% of 1230 is the same as … Circle your answer and explain your solution: A. 1230×25 B. $1230 \times \frac{1}{4}$ C. $25 \div \frac{1}{4}$ D. $1230 \div \frac{1}{4}$	 Fractions, Decimals, & Percents Fraction | Decimal | Percent | Picture $\frac{1}{2}$ | 0.5 | 50% | $\frac{1}{3}$ | $0.3\overline{3}$ | $33.\overline{3}$% | $\frac{1}{4}$ | 0.25 | 25% | $\frac{2}{3}$ | $0.6\overline{6}$ | $66.\overline{6}$% | $\frac{3}{4}$ | 0.75 | 75% | 1 | 1.00 | 100% | ***Conversion chart***
	Activity 6 Estimate the DECIMAL shown by the arrow on these number lines: (a) (b)	 ***Decimal number line***  

Addition and Subtraction

DATE	DURATION	TOPIC
_____ July 2018	One Hour	Addition and Subtraction

TEACHING SEQUENCE	LEARNER ACTIVITIES	VISUAL TEACHING STRATEGIES
Introduction:		
Check learners' prior knowledge on addition and subtraction by using a questioning technique. (5 minutes)	The following oral questions are put to the class: Use the "counting on" strategy and find the missing number: • $123 + \square = 135$ • $1{,}234 + \square = 1{,}239$ • $\frac{2}{7} + \square = \frac{6}{7}$ • $1\frac{1}{5} + \square = 2$	  ***Number charts*** ***Fractional charts***   ***Fraction strips*** ***Number line sticker***
Mental Mathematics:		
Exercise the minds of the learners to prepare them for the lesson. (5 minutes)	The following oral questions are put to the class: Use your calculator and ZAP (replace with nought) the	

	underlined number by making use of ONE operation: • **3 856** • **12 508** • **0,234** • **89,764**	 ***Calculator***
Development:		
The lesson develops in stages in order to address the addition and subtraction of whole numbers and fractional parts. (40 minutes)	**Activity 1** Look at the balance scale:  What is the value of A if the scale is balanced? Explain different ways in which you can solve this problem.	 ***Number line sticker***  ***Counters***
	Activity 2 Peter, Joan and Thapelo share a pizza. Peter gets $\frac{1}{2}$ of the pizza. Joan gets $\frac{1}{4}$ of it. What fractional part did Thapelo get?	 ***Fraction circles***

<table>
<tr><td></td><td>(Explain your calculations.)</td><td></td></tr>
<tr><td></td><td><u>Activity 3</u>

Ben, Thabo and Marco share a pizza. Ben gets 0,35 of the pizza. Marco gets 0,32 of it.

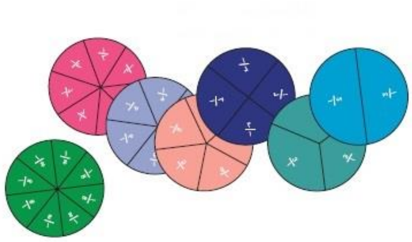
(a) What decimal fraction did Thabo get?

(b) Which boy got the smallest portion?

Explain your answer.</td><td>

Fractions, Decimals, & Percents
<table>
<tr><th>Fraction</th><th>Decimal</th><th>Percent</th><th>Picture</th></tr>
<tr><td>$\frac{1}{2}$</td><td>0.5</td><td>50%</td><td></td></tr>
<tr><td>$\frac{1}{3}$</td><td>$0.3\overline{3}$</td><td>$33.\overline{3}$%</td><td></td></tr>
<tr><td>$\frac{1}{4}$</td><td>0.25</td><td>25%</td><td></td></tr>
<tr><td>$\frac{2}{3}$</td><td>$0.6\overline{6}$</td><td>$66.\overline{6}$%</td><td></td></tr>
<tr><td>$\frac{3}{4}$</td><td>0.75</td><td>75%</td><td></td></tr>
<tr><td>1</td><td>1.00</td><td>100%</td><td></td></tr>
</table>

Conversion chart


Fraction circles</td></tr>
<tr><td>Consolidation:</td><td></td><td></td></tr>
<tr><td>The relationship between addition and subtraction as inverse operations is emphasized.

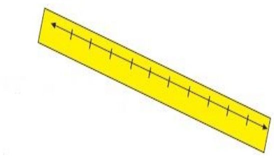
(10 minutes)</td><td><u>Activity 4</u>

Add the following numbers by grouping and rounding to the nearest ten. Then compensate for the rounding, for example:

328 + 234= (330 + 230) – 2 + 4

(a) 548 + 431

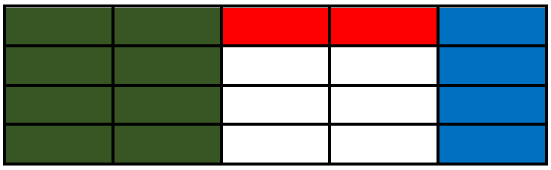
(b) 987 + 1 068

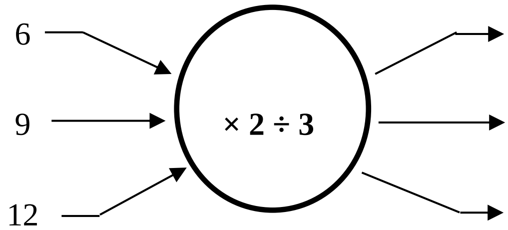
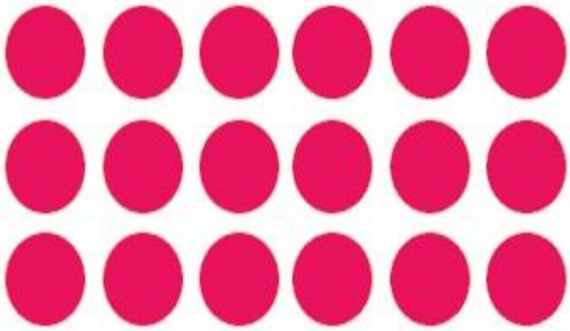
(c) 5 431 + 66</td><td> 
Number line stickerCounters


Sticks and bands</td></tr>
</table>

Multiplication and Division

DATE	DURATION	TOPIC
_____ August 2018	One Hour	Multiplication and Division

TEACHING SEQUENCE	LEARNER ACTIVITIES	VISUAL TEACHING STRATEGIES
Introduction:		
Check learners' prior knowledge on multiplication and division by using a questioning technique. (5 minutes)	The following oral questions are put to the class: Look at the diagram, representing rectangles: • Give a number sentence for the number of rectangles in the whole diagram, using the multiplication operation. • What fractional part is green? • Find $\frac{1}{2}$ of $\frac{2}{5}$ from the diagram. • What percentage	 ***Diagrammatical representation***

	is red and blue?	
Mental Mathematics:		
Exercise the minds of the learners to prepare them for the lesson. (5 minutes)	The following oral questions are put to the class: Complete the spider diagram:	 ***Spider diagram***
Development:		
The lesson develops in stages in order to address the multiplication and division of whole numbers and fractional parts. (40 minutes)	**<u>Activity 1</u>** A farmer has stored all his oranges in 50 boxes, with 20 oranges in a box. He now needs to repack them all in 20 new boxes. • How many oranges will there be in each new box? • Explain different ways in which you can solve this problem.	 ***Array of dots***

<table>
<tr>
<td></td>
<td><u>Activity 2</u>

Without calculating, select the larger amount:

(123 × 4) OR (123 + 124 + 125 + 126)

Explain why you have made your specific choice.</td>
<td>
Array of dots


Base ten blocks</td>
</tr>
<tr>
<td></td>
<td><u>Activity 3</u>

Without calculating, select the larger amount:

(120 ÷ 4) OR (120 − 30 − 29− 28)

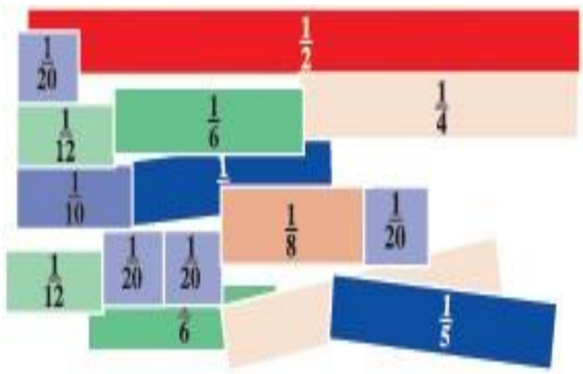
Explain why you have made your specific choice</td>
<td>
Array of dots


Base ten blocks</td>
</tr>
<tr>
<td></td>
<td><u>Activity 4</u>

You are given the following:

$\frac{1}{3} \times 120 = 40$

How much larger than 40 is:

$\frac{1}{3} \times 123$?</td>
<td>

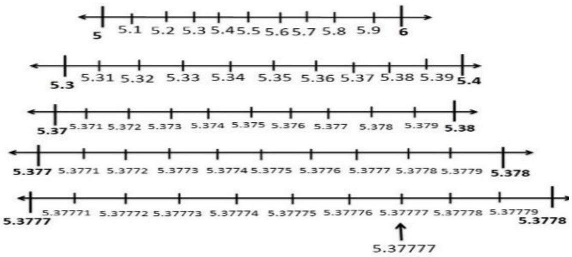
Fraction strips</td>
</tr>
</table>

<table>
<tr><td></td><td>Explain your answer.</td><td></td></tr>
<tr><td></td><td><u>Activity 5</u>

Without calculating the exact answer, circle the largest product and explain your choice:

A. 0,5 × 14

B. 13 × 0,5

C. 14 × 0,6</td><td>

Decimal number line</td></tr>
<tr><td>Consolidation:</td><td></td><td></td></tr>
<tr><td>The relationship between multiplication and division as inverse operations is emphasized.

(10 minutes)</td><td><u>Activity 6</u>

Mpho is in Grade 6. He says that he is 36 500 days old. Is that possible?

A. Yes

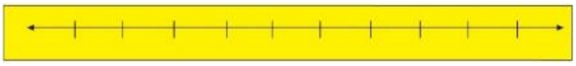
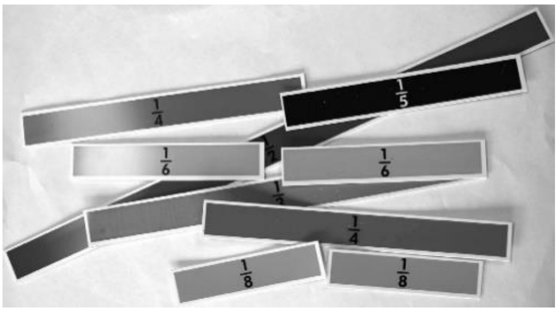
B. No

C. Maybe

Tell why you chose your answer and show your working.</td><td>LOOK FOR PATTERNS IN MULTIPLICATION TABLE
<table>
<tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr>
<tr><td>2</td><td>4</td><td>6</td><td>8</td><td>10</td><td>12</td><td>14</td><td>16</td><td>18</td><td>20</td></tr>
<tr><td>3</td><td>6</td><td>9</td><td>12</td><td>15</td><td>18</td><td>21</td><td>24</td><td>27</td><td>30</td></tr>
<tr><td>4</td><td>8</td><td>12</td><td>16</td><td>20</td><td>24</td><td>28</td><td>32</td><td>36</td><td>40</td></tr>
<tr><td>5</td><td>10</td><td>15</td><td>20</td><td>25</td><td>30</td><td>35</td><td>40</td><td>45</td><td>50</td></tr>
<tr><td>6</td><td>12</td><td>18</td><td>24</td><td>30</td><td>36</td><td>42</td><td>48</td><td>54</td><td>60</td></tr>
<tr><td>7</td><td>14</td><td>21</td><td>28</td><td>35</td><td>42</td><td>49</td><td>56</td><td>63</td><td>70</td></tr>
<tr><td>8</td><td>16</td><td>24</td><td>32</td><td>40</td><td>48</td><td>56</td><td>64</td><td>72</td><td>80</td></tr>
<tr><td>9</td><td>18</td><td>27</td><td>36</td><td>45</td><td>54</td><td>63</td><td>72</td><td>81</td><td>90</td></tr>
<tr><td>10</td><td>20</td><td>30</td><td>40</td><td>50</td><td>60</td><td>70</td><td>80</td><td>90</td><td>100</td></tr>
</table>
Multiplication board</td></tr>
</table>

Number Sense Word Problems

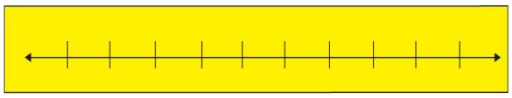
DATE	DURATION	TOPIC
____ October 2018	One Hour	Number Sense Word Problems

TEACHING SEQUENCE	LEARNER ACTIVITIES	VISUAL TEACHING STRATEGIES
Introduction:		
Check learners' prior knowledge on number sense word problems by using a questioning technique. (5 minutes)	The following oral questions are put to the class: • A book costs R10,25. How much will two books cost? • A farmer plants sunflower on one-third of his three hectares of land. How many hectares were used? • A piece of rope is 4 metres long. It is cut in 8 equal lengths. How long is each piece?	 ***Empty number line***  ***Fraction strips***

<table>
<tr><td>Mental Mathematics:</td><td></td><td></td></tr>
<tr><td>Exercise the minds of the learners to prepare them for the lesson.

(5 minutes)</td><td>The following oral questions are put to the class:

• Give two numbers with a sum of nine and a difference of one.
• Add together: $6\frac{1}{2}$, 5 and $3\frac{1}{2}$
• A train journey takes $7\frac{1}{2}$ hours. If the journey starts at 08:30, what time does it end?</td><td>
Counters


Empty number line</td></tr>
<tr><td>Development:</td><td></td><td></td></tr>
<tr><td>The lesson develops in stages in order to address number sense understandings through word problems.

(40 minutes)</td><td><u>Activity 1</u>

Ben, Mary and Tshepo only have R200 to spend on their entertainment on Saturday. They want to go to the stadium to watch soccer. It will cost them R30 per ticket. Afterwards, they intend to go to a movie which cost R25 each.</td><td>LOOK FOR PATTERNS IN MULTIPLICATION TABLE
<table>
<tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr>
<tr><td>2</td><td>4</td><td>6</td><td>8</td><td>10</td><td>12</td><td>14</td><td>16</td><td>18</td><td>20</td></tr>
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<tr><td>4</td><td>8</td><td>12</td><td>16</td><td>20</td><td>24</td><td>28</td><td>32</td><td>36</td><td>40</td></tr>
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<tr><td>6</td><td>12</td><td>18</td><td>24</td><td>30</td><td>36</td><td>42</td><td>48</td><td>54</td><td>60</td></tr>
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<tr><td>8</td><td>16</td><td>24</td><td>32</td><td>40</td><td>48</td><td>56</td><td>64</td><td>72</td><td>80</td></tr>
<tr><td>9</td><td>18</td><td>27</td><td>36</td><td>45</td><td>54</td><td>63</td><td>72</td><td>81</td><td>90</td></tr>
<tr><td>10</td><td>20</td><td>30</td><td>40</td><td>50</td><td>60</td><td>70</td><td>80</td><td>90</td><td>100</td></tr>
</table>
Multiplication board</td></tr>
</table>

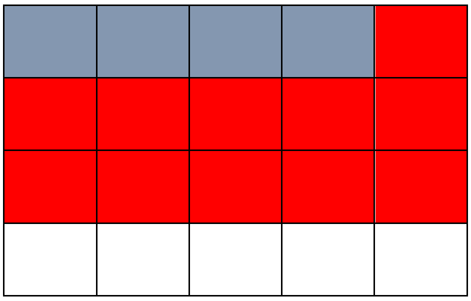
How much money will they have left over to spend on food?

Activity 2

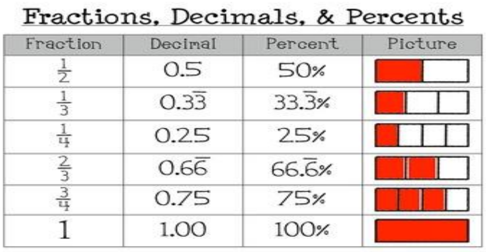
Graham and his father are milking cows. Graham has milked 4 cows and his father has milked 11 cows. Together they have milked $\frac{3}{4}$ of the cows that should be milked for the day.

(a) How many cows must be milked for the day?

(b) Express the cows that should still be milked as a decimal fraction and a percentage.



Diagrammatical representation



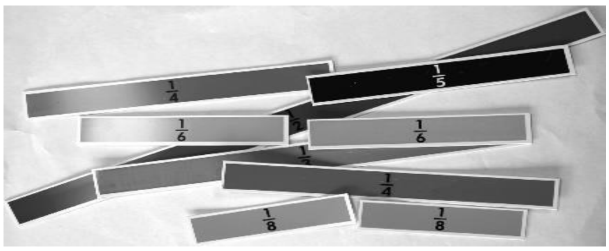
Fractions, Decimals, & Percents

Fraction	Decimal	Percent	Picture
$\frac{1}{2}$	0.5	50%	
$\frac{1}{3}$	$0.3\overline{3}$	$33.\overline{3}$%	
$\frac{1}{4}$	0.25	25%	
$\frac{2}{3}$	$0.6\overline{6}$	$66.\overline{6}$%	
$\frac{3}{4}$	0.75	75%	
1	1.00	100%	

Conversion chart

Activity 3

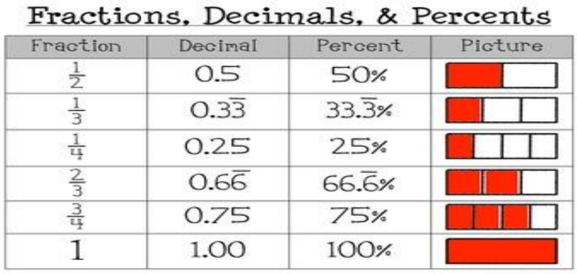
(a) Robert cuts a piece of wood in 50 pieces of equal size. He puts 12 pieces in a fire to burn. What percentage



Fraction strips

<table>
<tr>
<td></td>
<td>of the wood is left? Explain your answer.

(b) Write the 12 pieces of wood as a fraction.

(c) Write the part of wood that is left, as a decimal.</td>
<td>

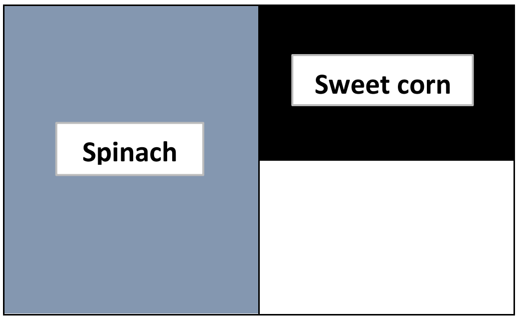
Fractions, Decimals, & Percents

Fraction	Decimal	Percent	Picture
$\frac{1}{2}$	0.5	50%	
$\frac{1}{3}$	$0.3\overline{3}$	$33.\overline{3}$%	
$\frac{1}{4}$	0.25	25%	
$\frac{2}{3}$	$0.6\overline{6}$	$66.\overline{6}$%	
$\frac{3}{4}$	0.75	75%	
1	1.00	100%	

Conversion chart</td>
</tr>
<tr>
<td>Consolidation:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The number sense understandings are consolidated to include whole numbers and fractional parts in the same computational setting.

(10 minutes)</td>
<td><u>Activity 4</u>

(a) Heidi plants a rectangular garden. In half of it she plants spinach. In half of the remaining garden, she plants sweet corn. What <u>percentage</u> of the garden has nothing planted in it? Use a diagram to explain your thinking.

(b) Write a number sentence to show the part of the garden with nothing planted in it.</td>
<td>

Diagrammatical representation</td>
</tr>
</table>

CONCLUSION

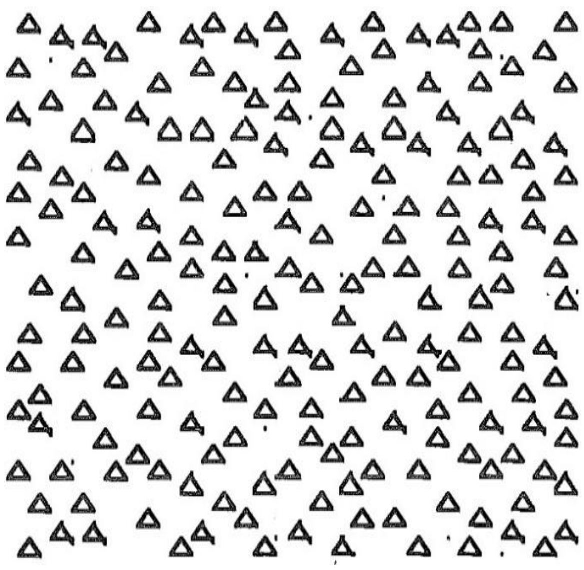
The four number sense lesson plans above constitute a visualisation intervention programme (VIP) which will employ visual teaching techniques to teach number sense understandings. These visual teaching strategies are offered as a guideline and could be replaced by visual strategies equal in nature and quality. The lesson plan activities show a line of progression to accommodate the slower learners, in case the teacher cannot finish the lesson within the allocated one hour period.

It is the hope of the Collaboration Team that the VIP will prove to enhance the teaching of number sense in the Intermediate Phase. Furthermore, the hope is that learner performance could reflect discernible positive change due to the efficient use of the visual teaching strategies.

Positive research results could transform the manner in which teachers approach this mammoth task of laying a sound mathematical foundation for Intermediate Phase learners to build on as they progress to higher order mathematics in the Senior and FET Phase.

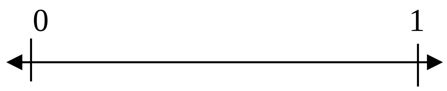
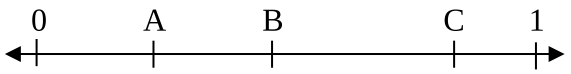
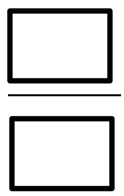
APPENDIX B

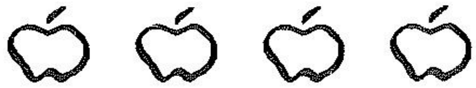
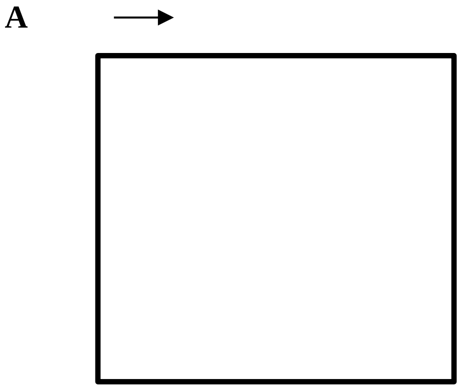
NUMBER SENSE TEST

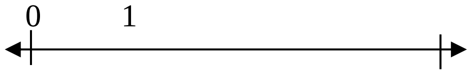
SECTION A: WHOLE NUMBERS	**ANSWER**
1. About how many days have you lived? Circle the nearest answer.	A. 420 B. 42000 C. 4200 D. 42
2. The South African Police Service is counting the number of cars on a highway in Johannesburg. The counting meter now reads: **1 0 2 9 9** What will it read after one more car passes by?	☐☐☐☐☐
3. I have R218,60 in my bank account. How many R10 notes will the bank be willing to give me if I withdraw the money?	___________________
4. About how many trianglesdo you see in this picture? Circle the nearest answer. 	A. 20 B. 50 C. 100 D. 200 E. 500

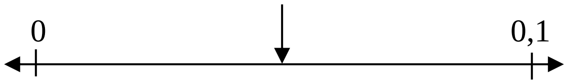
5. The digits are: 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. Put one digit in each of the two boxes so that the answer will be as big as possible. (Do not repeat the digits.)	**4 □ □ - 231 = ?**
6. Here are five digits: 3, 7, 1, 8, 5 Arrange all these digits to make the smallest number possible.	________________
7. Here are 5 digits: 4, 1, 0, 7, 3 Arrange them to make the number closest to 30 000.	________________
8. There are ten learners in a line at a classroom door. Mpho is seventh in line. How many learners are ahead of her?	________________
9. If a grandfather is more than 69 years old and less than 79 years old, write down three different ages he could be.	__________ ; __________ ; __________
10. When counting, what is the number that comes before 700?	________________
11. A school has 509 learners. If 97 learners are away on a trip, <u>about how many</u> are still at school? Circle your answer.	A. 300 B. 400 C. 500 D. 600
12. Cathy has R20. She has R4 less than Brenda. How much does Brenda have?	________________

SECTION B: FRACTIONS	**ANSWER**
13. <u>About</u> how much of this box is shaded? Give your answer as a fraction. 	________________

14. Place the numbers $\frac{1}{5}$ and $\frac{7}{10}$ in their correct positions on this number line:	
15. It is true that $\frac{7}{8}$ is a fraction between $\frac{1}{2}$ and 1. Write another fraction between $\frac{1}{2}$ and 1.	________________
 16. Which point on the number line above best represents $\frac{7}{9}$?	________________
17. Circle the fraction which represents the largest amount.	A. $\frac{5}{6}$ B. $\frac{7}{9}$ C. $\frac{8}{15}$ D. $\frac{2}{3}$
18. Put two of the following numbers **5 ; 7 ; 11** in the boxes to make a fraction as close as possible to $\frac{1}{2}$.	
19. Thabo cuts a pizza into four equal pieces. He eats two of the pieces. What <u>fraction</u> of the whole pizza is left?	________________
20. Shade $\frac{4}{5}$ of this rectangle.	
21. How many quarters make one rand?	________________

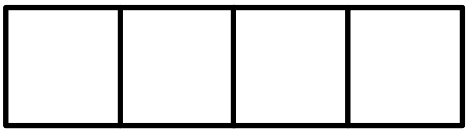
22. Boniswa took half of the apples from a bag. Here are her apples:  Peter took all the others from the bag. How many apples were there in the bag to start with?	__________________
23. A watermelon is cut into quarters. Then each quarter is cut in half. How many pieces of watermelon are there now? Circle your answer.	A. 2 B. 4 C. 6 D. 8
24. Thapelo buys a cold drink and gets R5 change. He wants his change in quarters. How many quarters will he get? Circle your answer.	A. 5 B. 4 C. 20 D. 25
25. You are going to walk once around a square-shaped field. You start at the corner marked **A** and move in the direction shown by the arrow. Mark with an **X** where you will be after $\frac{1}{3}$ of your walk.	

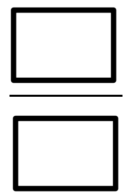
SECTION C: DECIMALS	ANSWER
26. For two years Thabo has been putting only 20 cent coins in his piggy bank. Yesterday he opened it and counted his money. He had R20,60. How many 20 cent coins were in his piggy bank?	__________________
27. Place the numbers 0,2 and 0,7 in their correct positions on this number line:	

28. Without calculating the exact answer, circle the best estimate for: $35 \times 0{,}97$	A. More than 35 B. Less than 35 C. Impossible to tell without working it out.
29. Estimate the decimal shown by the arrow on this number line: 	___________________
30. Estimate the decimal shown by the arrow on this number line: 	___________________
31. Which is greater? Circle your answer.	A. $23 \div 0{,}7$ B. $23 \times 0{,}7$ C. $23 + 0{,}7$ D. Impossible to tell without calculating.
32. Ten bottles of water cost R70,90 at one store. I can get 5 bottles for R35,20 at a second store. At which store is the water cheaper? Circle your answer and then fill in the blank.	A. First store B. Second store C. Tell how you decided: __________________________ __________________________
33. Portia ran 100 metres in 9,53 seconds. Lerato took 3 tenths of a second longer. How long did it take Lerato to run 100 metres? Circle your answer.	A. 12,53 seconds B. 9,83 seconds C. 9,56 seconds D. 9,50 seconds

34. How many different decimals are there between 1,43 and 1,44? Circle your answer and then fill in the blank.	A. None. Why? ____________________ B. One. What is it? ____________________ C. A few. Give two. _______________ and ______________ D. Lots. Give two. _______________ and _______________
35. Circle the decimal which best represents the shaded part of the box. 	A. 0,8 B. 0,01 C. 0,2 D. 0,02
36. The following expression: $0,5 \times 670$ is the same as … Circle your answer.	A. $670 \div 2$ B. $670 \div 5$ C. 670×5 D. 5×6700
37. <u>Without calculating</u>, decide which one of these answers is reasonable and circle it. Circle your answer.	A. $45 \times 1,05 = 39,65$ B. $4,5 \times 6,5 = 292,5$ C. $87 \times 1,076 = 93,61$ D. $589 \times 0,95 = 595,45$

SECTION D: PERCENTAGES	ANSWER
38. Lucia had R460,00 and spent 90 percent of the money on buying clothes. Without calculating an exact answer, circle the best estimate for how much she spent. Circle your answer.	A. Slightly less than R460.00 B. Much less than R460,00 C. Slightly more than R460,00 D. Impossible to tell without calculating
39. A tank holds 1000 fish. If I increase the number by 50%, how many fish will there be now in the tank? Circle your answer.	A. 500 B. 1050 C. 1500 D. 2000
40. David had R350. He spent 100% of it. How much money did he have left? Circle your answer.	A. R35 000 B. R450 C. R0 D. R250
41. Put these numbers in <u>ascending order</u>. Start with the smallest number on the top row and end with the largest number on the bottom row: **61%** ; **0,567** ; $\frac{3}{5}$; **0,36** ; **30%**	_______________ _______________ _______________ _______________ _______________
42. Jackson increased his Mathematics test mark from 40 to 50. What percentage increase is this? Circle your answer.	A. 10% B. 25% C. 50% D. 90%

43. Last month a pack of pencils cost R10. This month there is 10% off the cost of a pack of pencils. What is the cost of a pack of pencils this month? Circle your answer.	A. R8 B. R9 C. R7 D. R5
44. Shade in 75% of this rectangular shape:	
45. The following expression: 25% of 100 is the same as … Circle your answer.	A. 100×25 B. $100 \times \frac{1}{4}$ C. $25 \div \frac{1}{4}$ D. $100 \div \frac{1}{4}$
46. What answer goes in the box to make this sentence true? 93% + □ = 1 Circle your answer.	A. 0,7 B. 7% C. 0,7% D. 7
47. Which one is bigger? 15% or 0,17 Circle your answer.	A. 0,17 B. 15%
48. Cyril cuts a loaf of bread in 10 slices. He eats 4 slices. What percentage of the loaf of bread is left? Circle your answer.	A. 40% B. 10% C. 60% D. 100%

49. Put two of the following numbers **12 ; 8 ; 7** in the boxes to make a fraction as close as possible to 70%.	
50. Select the best answer for the following: 98% of 120 Circle your answer.	A. 121 B. 118 C. 87 D. 98

APPENDIX C

An analytical framework for considering visual teaching strategies for number sense (adapted from McIntosh et al., 1992).

NUMBER SENSE UNDERSTANDINGS	NUMBER SENSE THEMES	VISUAL TEACHING STRATEGIES
Sense of orderliness of numbers	Place Value	• **The abacus can be used to demonstrate place values and regrouping of a number (SAIDE, 2007, p. 36).** • **The use of flard cards whereby the different digits of a number are placed behind each other is helpful in teaching place values, because when the cards are exposed the values of the different digits become apparent (SAIDE, 2007, p. 37).** • **The ZAP calculator game facilitates an understanding of place values. It is being played as follows: One player calls out a number for the other players to enter onto their calculator displays (e.g. 4 789). The player then says 'ZAP the 8', which means that the other players must replace the 8 with the digit 0, using one operation (i.e. to change it into 4 709). The player who is the quickest to decide on how to ZAP the given digit could call out the next number (SAIDE, 2007, p. 38).** • **The Gattegno chart can be used to facilitate an understanding of place value (Anghileri, 2000, p. 12).**
	Relationship between number types	• **The number line can be populated with a starting and endpoint. The learner is required to insert a number or decimal value in between at a point indicated by the teacher (McIntosh et al., 1992, p. 6).**
	Ordering numbers within and among number types	• **The number track, calibrated and empty number lines are used for representations which focus on the ordering of numbers and**

		the counting sequence (Anghileri, 2000, p. 10).
Multiple representations for numbers	Graphical / Symbolic	• The hundred square serves as a symbolic aid to represent numbers in different ways, like patterns with a common difference as well as multiples (Anghileri, 2000, p. 12).
	Equivalent numerical forms (including de-composition and re-composition)	• Dotted and array cards can be used as concrete materials to facilitate multiple representations for numbers (Howden, 1989, p. 7).
	Comparison to benchmarks	• Benchmarking can be encouraged by making use of dot cards and array cards (Muir, 2012, p. 26).
Sense of relative and absolute magnitude of numbers	Comparing to physical referent	• Subitising, which is the instant recognition of the number of items in a group, can be facilitated by making use of dot cards and array cards (Muir, 2012, p. 26).
	Comparing to mathematical referent	• Ten frames can be used to assist in visualising numbers as doubles, neighbours and to look for groups of tens in adding numbers (Howden, 1989, p. 7).
System of benchmarks	Mathematical	• An estimation challenge, with a real-sized picture of the item being estimated printed on a laminated card, can be used to test how learners would apply numerical benchmarks (Muir, 2012).
	Personal	• An estimation challenge, with a real-sized picture of the item being estimated printed on a laminated card, can be used to test how a learner may use his / her personal experience of magnitude to solve a mathematical problem (Muir, 2012).
Understanding the effect of operations	Operating on whole numbers	• Card games can be used to exercise the four operations with whole numbers (Stott, 2014, p. 4). • A "pictorial representation of a number line can be an effective method for modeling the addition of integers" (Cope, 2015, p. 12).

		• **Array and grid representations assist with the multiplication of whole numbers (Barmby et al., 2010).**
	Operating on fractions / decimals	• **Diagrammatical representations of fractional parts within a whole unit can be used to teach equivalent fractions in order for learners to understand the relationship between the numerators and denominators of equivalent fractions (Askew et al., 1997, p. 30).** • **The tangram, a Chinese geometrical puzzle consisting of a square cut into seven pieces which can be arranged to make various other shapes, can be used to teach the relationship between fractional parts (Yang et al., 2008, p. 803).**
Understanding mathematical properties	Commutativity	• **Dotted and array cards can be used as concrete materials to enhance the understanding of mathematical properties (Howden, 1989, p. 7).** • **Grid representations highlight the commutativity of numbers (Barmby et al., 2010).**
	Associativity	• **Dotted and array cards can be used as concrete materials to enhance the understanding of mathematical properties (Howden, 1989, p. 7).**
	Distributivity	• **Dotted and array cards can be used as concrete materials to enhance the understanding of mathematical properties (Howden, 1989, p. 7).** • **The array representation assists teachers in showing the distributive properties of multiplication (Barmby et al., 2010, p. 48).**
	Identities	• **A pictorial representation of a number line can be an effective method for teaching the identity property for addition and subtraction (Cope, 2015).**

		• **The array representation can be used to teach the number "one" as a multiplicative identity (Barmby et al., 2010).**
	Inverses	• **A pictorial representation of a number line can be an effective method for teaching inverse values (Cope, 2015).**
Understanding the relationship between operations	Addition / Multiplication	• **Use of the array representation helps to "show that multiplication can be the same as repeated addition" (Barmby et al., 2010, p. 47).**
	Subtraction / Division	• **Use of the array representation helps to show that division can be the same as repeated subtraction (Barmby et al., 2010).**
	Addition / Subtraction	• **A pictorial representation of a number line can be an effective method for teaching addition and subtraction as inverse operations (Cope, 2015).**
	Multiplication / Division	• **Dotted and array cards can be used as concrete materials to enhance the understanding of multiplication and division as inverse operations (Howden, 1989).**
Understanding the relationship between problem context and the necessary computation	Recognize data as exact or approximate	• **An estimation challenge, with a real-sized picture of the item being estimated printed on a laminated card, can be used for allowing learners to explore magnitude as exact or approximate (Muir, 2012).**
	Awareness that solutions may be exact or approximate	• **An estimation challenge, with a real-sized picture of the item being estimated printed on a laminated card, can be used to create an awareness of the type of solutions needed within the context of the mathematical problem posed (Muir, 2012).**
Awareness that multiple strategies exist	Ability to create and / or invent strategies	• **An estimation challenge, with a real-sized picture of the item being estimated printed on a laminated card, can be used to test the learner's ability to create or invent solution strategies within the context of the mathematical problem posed (Muir, 2012).**

	Ability to apply different strategies	• **An estimation challenge, with a real-sized picture of the item being estimated printed on a laminated card, can be used to test the learner's ability to apply different strategies within the context of the mathematical problem posed (Muir, 2012).**
	Ability to select an efficient strategy	• **An estimation challenge, with a real-sized picture of the item being estimated printed on a laminated card, can be used to test the learner's ability to select an efficient strategy within the context of the mathematical problem posed (Muir, 2012).**
Inclination to utilize an efficient representation and / or method	Facility with various methods (mental, calculator, pencil and paper)	• **Allowing learners to experiment with unconventional visual algorithms like the lattice method of multiplication and the scaffolding algorithm for division will stimulate their natural ability to use various methods and representations in mathematical problem-solving (Groth, 2012, p. 159).**
	Facility choosing efficient number(s)	• **An array representation can be used to multiply two awkward numbers. The numbers are rounded up and part of the array is subtracted after the calculation to match the original numbers (Barmby et al., 2010).**
Inclination to review data and result for sensibility	Recognize reasonableness of data	• **An estimation challenge, with a real-sized picture of the item being estimated printed on a laminated card, can be used to test the learner's ability to recognize the reasonableness of data within the context of the mathematical problem posed (Muir, 2012).**
	Recognize reasonableness of calculation	• **An estimation challenge, with a real-sized picture of the item being estimated printed on a laminated card, can be used to test the learner's ability to recognise the reasonableness of calculation within the context of the mathematical problem posed (Muir, 2012).**

APPENDIX D

Sample of a working document for using the analytical framework (Appendix C) and the conceptual framework for teaching proficiency when observing the lesson

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VISUALISATION INTERVENTION PROGRAMME

LESSON OBSERVATION TOOL

School Name: ______________________________ Date: ______________________________

Name of Teacher:___________________________ Name of Researcher: _________________

Grade:____________________________________ Lesson Topic: ____________________

PART A: ALIGNMENT WITH THE ANALYTICAL FRAMEWORK FOR MAKING USE OF VISUAL STRATEGIES	**NUMBER SENSE THEME EVIDENT IN LESSON**	**VISUAL STRATEGY USED PER THEME**	**IS THE VISUAL STRATEGY APPROPRIATE FOR THE THEME?**	**COMMENT ON ALIGNMENT WITH THE ANALYTICAL FRAMEWORK**
	Place Value	Gattegno Chart	Yes, the chart is ordered to consolidate an understanding of place value.	This visual teaching strategy to address learner misconceptions about place value is advocated by the analytical framework for considering visual strategies for number sense.
	The commutativity mathematical property	A pictorial representation of the number line	No, a pictorial representation of the number line is more appropriate for teaching the identity property for addition and subtraction as well as inverses.	Dotted and array cards can be used more productively. Grid representations also highlight the commutativity of numbers.

PART B: PROFICIENCY IN USING VISUAL STRATEGIES IN THE TEACHING OF NUMBER SENSE	**TEACHING PROFICIENCY COMPONENTS**	**1**	**2**	**3**
	Conceptual understanding of the core knowledge required in the practice of teaching	×		
	Fluency in carrying out basic instructional routines		×	
	Strategic competence in planning effective instruction and solving problems that arise during instruction	×		
	Adaptive reasoning in justifying and explaining one's instructional practices and in reflecting on those practices so as to improve them		×	
	Productive disposition toward mathematics, teaching, learning and the improvement of practice.	×		

1	**No Evidence**
2	**Some Evidence**
3	**Abundant Evidence**

The evidence which the researcher is looking for in the lesson is constituted by the skill of the teacher in using visual strategies within the context of the five specified teaching proficiency components.

APPENDIX E

RHODES UNIVERSITY
Where leaders learn

SEMI-STRUCTURED INTERVIEW SCHEDULE (PROVISIONAL)

The following are the type of questions I have asked during the video-stimulated recall interviews. The precise nature of the questions depended on what unfolded in the video recordings.

1. How did you use visual teaching strategies to enhance the number sense of your learners? Please provide examples and identify on the video recording where this happened.

2. Did you find visual teaching strategies useful for teaching towards the enhancement of number sense? Please provide examples and identify on the video recording where this happened.

3. How did you use visual teaching strategies to enable learners to understand the meaning and size of numbers? Please provide examples and identify on the video recording where this happened.

4. Do you think that using visual teaching strategies equipped your learners with the necessary skills to understand and use equivalent forms and representations of numbers? Please elaborate and provide examples.

5. In your view and experience, did the use visual teaching strategies add to the learners' conceptual understanding of the meaning and effect of operations? Please elaborate and provide examples.

6. In your view, did the use of visual teaching strategies increase learner understanding and use of equivalent expressions? Please elaborate and provide examples.

7. Do you think that the use of visual teaching strategies assisted learners with implementing a variety of computing and counting strategies? Please elaborate and provide examples.

8. According to your experience, how did the use of visual teaching strategies help learners to set measurement benchmarks correctly? Please explain your answer.

9. Are you going to use visual strategies again in your teaching? Please explain your answer.

10. According to your experience, what were some of the constraining factors when using visual strategies in your teaching? Please elaborate.

APPENDIX F

The Implementation Schedule for the research study during 2018:

Phase	Date	Activity	Cycle	Term
PHASE 1	01 – 30 April	PREPARATION: WELL-RESOURCED SCHOOLS		TERM 2– 2018
PHASE 2	02 – 05 May	WORKSHOPS WITH RUMEP SCHOOLS		
PHASE 3	02 – 05 May	SELECTION OF RUMEP TEACHERS		
PHASE 4	07 – 10 May	PRE-TEST: RUMEP SCHOOL LEARNERS AND CONTROL GROUP		
PHASE 5	14 - 17 May	PILOT LESSON VIDEO RECORDINGS: 7 RUMEP TEACHERS	CYCLE 1	TERM 2& 3 - 2018
	21 - 24 May	REFLECTIVE WORKSHOP: INTERVIEW AND ANALYSIS (PILOT LESSON)		
	02 June 2018	FOCUS GROUP INTERVIEW WITH WELL-RESOURCED AND RUMEP TEACHERS		
	23 - 26 July	1ST OFFICIAL LESSON VIDEO RECORDINGS: 7 RUMEP TEACHERS	CYCLE 2	
	30 July – 03 Aug	REFLECTIVE WORKSHOP: INTERVIEW AND ANALYSIS (1STOFFICIAL LESSON)		
	27 – 31 Aug	2NDOFFICIAL LESSON VIDEO RECORDINGS: 7 RUMEP TEACHERS	CYCLE 3	TERM 3 & 4 - 2018
	03 - 07 Sept	REFLECTIVE WORKSHOP: INTERVIEW AND ANALYSIS (2NDOFFICIAL LESSON)		
	15 – 18 Oct	3RDOFFICIAL LESSON TAUGHT BY 7 RUMEP TEACHERS FOR POST-TEST	CYCLE 4	
PHASE 6	05 – 09 Nov	POST-TEST: RUMEP SCHOOL LEARNERS AND CONTROL GROUP		

APPENDIX G

Letter of consent to participating RUMEP teachers

RHODES UNIVERSITY
Where leaders learn

INVITATION TO PARTICIPANTS

Participant's Name
Address

Dear (participant's name)

RE: INVITATION TO PARTICIPATE IN A RESEARCH STUDY

You are invited to participate in a Rhodes University Mathematics Education Masters research study. The aim of this research is to investigate the use of visualisation as a teaching strategy to enhance number sense. Your participation in this study will provide an in-depth understanding of ways in which the use of visual teaching strategies can enhance number sense.

The research will be undertaken through an intervention programme that will consist of seven mathematics teachers (hopefully including you). You will be asked to participate in an intervention programme that consists of a series of workshops, implementing a learning programme and participating in four interviews. Further, your teaching with the use of visual strategies, will be video recorded on four occasions. The collection of this data will span over three terms. Your participation in the research is anonymous and your identity will not be revealed outside of this research project.

If you agree to participate, we will explain in more detail what would be expected of you, and provide you with the information you need to understand the research at our first meeting. These guidelines would include potential risks, benefits, and your rights as a participant. I attach a letter of ethical approval from the Ethics Committee of the Faculty of Education. Furthermore, please note that your participation in this study will in no way have any compromised impact on your employment and RUMEP studies.

Participation in this research is voluntary and a positive response to this letter of invitation does not oblige you to take part in this research. To participate, you will be asked to sign a consent form to confirm that you understand and agree to the conditions, prior to any (interview or observation) commencement. Please note that you have the right to withdraw at any given time during the study.

Thank you for your time and I hope that you will respond favourably to our request.

Yours sincerely,

Ronald Max Griqua (**17G9354)**

Supervisor: Prof. Marc Schäfer

APPENDIX H

Letter of consent to participating learners

RHODES UNIVERSITY
Where leaders learn

PERMISSION LETTER TO LEARNERS

Student address
Date

REQUEST FOR PERMISSION TO ADMINISTER A PRE- AND POST-TEST ON NUMBER SENSE

To whom it may concern

My name is **RONALD MAX GRIQUA**, and I am a Masters (degree) student at Rhodes University (RU) in Grahamstown, South Africa. The research I wish to conduct for my Master's full thesis requires me to administer a pre- and post-test on number sense to Grade 6 learners in order to ascertain whether discernable change in learner performance took place as a result of the implementation of an intervention programme.This research will be conducted under the supervision of Prof. Marc Schäfer.

This letter serves to seek formal consent from you as a learner of (name of school) to allow me to enter your classroom in order to administer a pre- and post-test. The study is on how teachers use visual teaching strategies in order to enhance number sense in mathematics. I intend to administer the pre-test from 22^{nd} March to 27^{th} March 2018 and the post-test from 18^{th} September to 21^{st} September 2018. I will not video-record any child in the class during the tests.

I further assure you that the intervention will not disturb any official school activities as it is planned to be executed as an integral part of normal school time. In the event that you do not wish to be part of the class during the administration of the tests, I will not include you in the intervention. This same assurance was given to the HOD and principal of the school.

If you have any further queries, please do not hesitate to contact me on the cell number XXX XXX XXXX and email address rmgriqua@gmail.com.

Thank you for your time and consideration in this matter.

Yours sincerely,

Ronald Max Griqua

XXX XXX XXXX

Rhodes University

I……………………………………..,a learner of XXX School, hereby give Mr Ronald Max Griqua permission to administer a pre- and post-test to me for the purpose of his research. It is understood that Mr Ronald Max Griqua will not video-record me for any purpose.

Signed: ……………………………………..

Date: …………………………….....……….

APPENDIX I

Letter of consent to the parents of the participating learners

RHODES UNIVERSITY
Where leaders learn

PERMISSION LETTER TO PARENT(S)

Student address
Date

REQUEST FOR PERMISSION TO CONDUCT RESEARCH ON THE MATHEMATICS TEACHERS OF YOUR CHILD

To whom it may concern

My name is **RONALD MAX GRIQUA**, and I am a Masters (degree) student at Rhodes University (RU) in Grahamstown, South Africa. The research I wish to conduct for my Master's full thesis requires me to observe and video-record mathematics teaching at a primary school and interview the teacher of a class. This research will be conducted under the supervision of Prof. Marc Schäfer.

This letter serves to seek formal consent from you as a parent of (name of learner) to allow me to enter the classroom of your child in order to video record his/her teacher. The study is on how teachers use visual teaching strategies in order to enhance number sense in mathematics. I intend to observe and video-record the teacher over a period of three terms from May to October 2018. I will not video-record any learners in the class, but they will be present when the teacher teaches. Your child will also write a pre- and post-test on number sense. I further assure you that the intervention will not disturb any official school activities as it is planned to be executed as an integral part of normal school time.

In the event that you do not wish your child to be part of the class during the time of my research on the teacher, I will not include your child in the intervention. Your child will then also not be requested to write the pre- and post-test on number sense. This same assurance was given to the HOD and principal of the school.

This research will be beneficial to the mathematics teacher of your child in that it will enhance the teacher's teaching and in turn benefit your child's learning.

If you have any further queries, please do not hesitate to contact me on the cell number XXX XXX XXXX and email address rmgriqua@gmail.com.

Thank you for your time and consideration in this matter.

Yours sincerely,

Ronald Max Griqua

XXX XXX XXXX

Rhodes University

==

I…………………………………….. , parent or legal guardian of ……………………………… hereby give Mr Ronald Max Griqua permission to video-record my child's teacher for the purpose of his research. It is understood that Mr Ronald Max Griqua will not video-record my child for any purpose. I furthermore give him permission to administer a pre- and post-test to my child as part of the research.

Signed: ……………………………………..

Date: …………………………...……….

APPENDIX J

Informed Consent Form

RHODES UNIVERSITY
Where leaders learn

INFORMED CONSENT FORM

Research Project Title:	An investigation into the use of visualisation as a teaching strategy to enhance number sense.
Principal Investigator(s):	RONALD MAX GRIQUA

Participation Information
• I understand the purpose of the research study and my involvement in it. • I understand the risks and benefits of participating in this research study. • I understand that I may withdraw from the research study at any stage without any penalty. • I understand that participation in this research study is done on a voluntary basis. • I understand that while information gained during the study may be published, I will remain anonymous and no reference will be made to me by name or student number. • I understand that XXX (other data collection requirements particular to this research, e.g. test results, personal information, video recording) may be used. • I understand and agree that the interviews will be recorded electronically. • I understand that I will be given the opportunity to read and comment on the transcribed interview notes. • I confirm that I am not participating in this study for financial gain.

Information Explanation
The above information was explained to me by:
The above information was explained to me in English and I am in command of this language:

Voluntary Consent	
I, ____________________ hereby voluntarily consent to participate in the above-mentioned research.	
Signature:	Date: / /
Investigator Declaration	

<table>
<tr><td colspan="2">I, Ronald Max Griqua, declare that I have explained all the participant information to the participant and have truthfully answered all questions asked by the participant.</td></tr>
<tr><td>Signature: </td><td>Date: / /</td></tr>
</table>

APPENDIX K

Letter of consent to the participating institution

RHODES UNIVERSITY
Where leaders learn

PERMISSION LETTER TO THE HEAD OF INSTITUTION

Student address

Date

REQUEST FOR PERMISSION TO CONDUCT RESEARCH AT XXX PRIMARY SCHOOL

To whom it may concern

My name is RONALD MAX GRIQUA, and I am a Masters (degree) student at Rhodes University (RU) in Grahamstown, South Africa. The research I wish to conduct for my Masters' full thesis requires me to observe and video-record mathematics teaching at a primary school and interview the teacher of the class, hence voice-record the interview twice. This research will be conducted under the supervision of Prof. Marc Schäfer.

This letter serves to seek formal consent to approach the teacher, Mr. XXX, the learners and the parents of the learners in his/her class as participants for this research. For this reason I request your permission to visit your school from the 22nd of March to the 7th of September 2018 to conduct my research as outlined in my research proposal.

I attach a copy of my research proposal which includes copies of the consent and assent forms to be used in the research process. I attach a letter of ethical approval from the Ethics Committee of the Faculty of Education. As part of this, I undertake to ensure that the name of the school and all participants will be replaced with pseudonyms and that all the materials I collect as part of the research will be accessible only to myself and my supervisor.

Upon completion of the study, I undertake to provide you and the participant teacher with access to the research findings. If you require any further information, please do not hesitate to contact me on the cell number XXX XXX XXXX and email address rmgriqua@gmail.com.

Thank you for your time and consideration in this matter.

Yours sincerely,

RONALD MAX GRIQUA

XXX XXX XXXX

Rhodes University

APPENDIX L

Letter of consent to the gatekeeper (Northern Cape Department of Education)

RHODES UNIVERSITY
Where leaders learn

CONSENT LETTER – HEAD OF DEPARTMENT

Student address

Date

RE: APPLICATION FOR A RESEARCH SITE

I am **RONALD MAX GRIQUA**, a part time student at Rhodes University in Grahamstown, South Africa. I am studying towards a Masters' degree in education in the field of Mathematics Education and my research interest lies in the use of visual teaching strategies in the mathematics classrooms. The title of my study is "an investigation into the use of visualisation as a teaching strategy to enhance number sense".

I intend to involve the following seven schools for my study: (names of schools). It is therefore my humble request to you as the HOD of the Northern Cape Department of Education for these seven schools, to afford me the opportunity of conducting my research in their research environments. This study will take at least 3 terms (from April until November 2018). I assure you that this study will not cause any disturbance to the normal school programme, as it will be integrated with the normal schedules of the school. The study will take a form of an intervention programme that will involve seven Grade 6 primary school teachers from (name of schools). Reflective stimulus group interviews and observations will be used as data collection tools in this study. I will video-record teachers' use of visual teaching strategies in the classrooms. I will also voice-record the group interviews as a means of collecting data for my research.

The aim of the study is to analyse how the use of visual teaching strategies can enhance number sense as well as gain insight into teachers' perceptions and experiences of using these visual teaching strategies after the implementation of the intervention. The participation of the seven teachers in this study will be beneficial to them (including me) in the sense that the use of visual teaching strategies will provide us with many opportunities for enhancing number sense.I also intend to send letters to the principals of the concerned schools should I be allowed by your office to go ahead with this research.

Should you have any queries regarding my request, please feel free to contact me at these contacts: Cell: XXX XXX XXXX / E-mail address: rmgriqua@gmail.com

I attach a letter of ethical approval from the Ethics Committee of the Faculty of Education.

Yours sincerely,

Ronald Max Griqua

XXX XXX XXXX

Supervisor: Prof. Marc Schäfer