The use of the van Hiele theory in investigating teaching strategies used by Grade 10 geometry teachers in Namibia

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BY

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DECLARATION OF ORIGINALITY

I ***Augustinus Muyeghu*** (**Student number: 07M6236**) declare that this thesis on ***the use of the van Hiele theory in investigating teaching strategies used by Grade 10 geometry teachers in Namibia*** is my own work written in my own words. Where I have drawn on the words or ideas of others, these have been acknowledged using the reference practices according to the Rhodes University Education Department Guide to Referencing.

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ABSTRACT

This study reports on the extent to which selected mathematics teachers facilitate the teaching and learning of geometry at the van Hiele levels 1 and 2 at a Grade 10 level in selected schools in Namibia. It also addresses and explores the teaching strategies teachers employ in their classrooms.

Kilpatrick et al.'s model for proficient teaching and the van Hiele model of geometric thinking were used to explore the type of teaching strategies employed by selected mathematics teachers. These two models served as guidelines from which interview and classroom observation protocols were developed. Given the continuing debate across the world about the learning and teaching of geometry, my thesis aims to contribute to a wider understanding of the teaching of geometry with regard to the van Hiele levels 1 and 2.

There are no similar studies on the teaching of geometry in Namibia. My study concentrates on selected Grade 10 mathematics teachers and how they teach geometry using the van Hiele theory and the five Kilpatrick components of proficient teaching. As my research looks at teaching practice it was important to deconstruct teaching proficiency with a view to understanding what makes good teachers effective.

The results from this study indicated that the selected Grade 10 mathematics teachers have a good conceptual understanding of geometry as all of them involved in this study were able to facilitate the learning and teaching of geometry that is consistent with the van Hiele levels 1 and 2.

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ACRONYMS

BED	Bachelor of Education degree
BETD	Basic Education Teachers Diploma
cf	Cross-reference
DEAL	Diploma in Education African Languages
I	Interview
LCE	Learner-centred education
MBEC	Ministry of Basic Education and Culture
MED	Master of Education
MoE	Ministry of Education
NCTM	National Council of Teachers of Mathematics
SIMS	Second International Mathematics Study
TC	Teacher-centred
VTO	video-taped observation

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CHAPTER ONE

INTRODUCTION

1.1 INTRODUCTION

Namibia is a developing country which needs a strong education system in order to compete in the global world of mathematics. For this to occur Namibia needs quality teachers who can constantly improve the teaching strategies in mathematics classroom situations. Teachers should act as facilitators of learning and their teaching should be consistent with what has been shown to be effective. In the context of this study this means that geometry teachers should plan and teach in a way which is consistent with the framework described by van Hiele. This study focuses particularly on the teaching and learning at levels 1 and 2 of the van Hiele theory.

This chapter gives a brief overview of what prompted this study, what it hopes to achieve and provides the context of the research. In addition, it discusses the research goal and the research site and presents a short outline of the structure of the thesis

1.2 CONTEXT OF THE STUDY

The National Curriculum echoes Plato's belief that geometry is an important subject which deserves significant attention in the teaching of mathematics (Burger & Shaughnessy, 1986). In the Grade 10 classroom, learners discover many important patterns and concepts of geometry. They construct triangles and other plane figures, they study and build tessellations. These are some of the activities that require children to build shapes which focus their attention on the attributes or characteristics of the shape they are constructing. It is important that children engage in these activities to enhance their geometric development (Burger & Shaughnessy, 1986). This study makes specific use of the van Hiele (1986) model of geometric thinking to analyse how teachers teach geometry. It also utilises the Kilpatrick et al.'s (2001) model to analyse teachers' practice.

1.3 RESEARCH GOAL

The goal of the study is to explore the use of van Hieles theory in investigating geometry teaching strategies employed by Grade 10 mathematics teachers in Namibia. To achieve this goal the study endeavoured to answer the following research question:

- To what extent do the selected Grade 10 mathematics teachers facilitate geometry learning and teaching consistent with levels 1 and 2 of the van Hiele theory of geometric thinking?

1.4 VAN HIELE MODEL

Van Hiele (1986) established an enabling framework to interrogate the development of cognition in geometry. He established a five level structure which has been the subject of much research around the world.

1.4.1 Van Hiele levels

Many studies confirm the usefulness of the five levels of geometric development described by van Hiele in the 1950s (Burger & Shaughnessy, 1986). The five levels, from one to five, are:

- Recognition/Visualisation
- Analysis
- Informal deduction
- Deduction
- Rigor

To achieve my research goal I specifically focused on the van Hiele levels 1 and 2.

1.4.2 The van Hiele phases of teaching

Indirectly, this study also made use of the van Hiele phases which specifically look at teacher instruction. The phases are:

- Inquiry/information
- Directed/guided orientation
- Explicitation/explanation

- Free orientation
- Integration

The main goal of instruction during these phases is to ensure learners are actively engaged in exploring the acquired concepts of geometry (Clements & Battista, 1992). They describe the role of the teacher as a facilitator in helping the learners summarise their key ideas while learners build and develop a summary of all they have learned about the concepts of geometry.

In short, these five phases of teaching should occur within each level as learners move from one level to the next. By employing these phases of teaching, teachers are able to identify the type of activities appropriate for learners at the visual level and at the analysis level as proposed by the van Hiele theory.

1.5 GENERAL BACKGROUND TO THE RESEARCH AND THE RESEARCH SITE

It could be argued that if learners are unable to provide reasons for studying geometry, their teachers have failed to teach geometric concepts properly. I have been employed as an Education Officer advising on mathematics for eight years. It is my perception that many teachers are not fulfilling all the curriculum requirements for the teaching of geometry. Many are not facilitating a learning environment conducive to the development of the appropriate skills their students need to develop an understanding of geometry. It is for these reasons that I have an interest in exploring more rigorously the way geometry is taught and in particular how Grade 10 teachers of mathematics facilitate the learning of geometry.

My research is conducted within the interpretive orientation. According to Cohen, Manion and Morrison (2000:180) the interpretive orientation "affords the researcher an opportunity to understand and interpret the world in terms of its actors". This orientation is suitable for my study as it unfolds or opens up research based on information-rich data. The primary research method is a case study which aligns well with the interpretive orientation. According to Cohen et al. (2000:180), a case study is "a careful study of some social unit which attempts to determine what factors led to its success or failure". It gives an opportunity for one aspect of a problem to be studied in some depth within the limited time of the study. This case study comprises a sample of three mathematics teachers in their mathematics classrooms.

The research design is divided into a number of phases. In phase one, I analyse what geometry the sample teachers are teaching and what their teaching plans are. I do this using van Hiele's lenses. In particular, I determine what the teachers do in the geometry class to develop level 1 and level 2 skills described by van Hiele.

In phase two, I determine how teachers teach geometry. I analyse their teaching strategies and describe their methodology in developing essential geometry skills. I make use of classroom observations and interviews to generate data. Interviews are the main data-collection method of this research study. In addition, I use observations to complement the interview data. These techniques will triangulate the data and add validity to the study (Polit, Beck & Hungler, 2001:312).

I make use of Kilpatrick, Swafford, & Swindell's (2001) framework of proficient teaching to develop the observation schedule which enables a more detailed analysis of how teachers teach geometry. The framework of proficient teaching of geometry designed by Kilpatrick et al. (2001:380) consists of the following five interwoven strands:

- Conceptual understanding
- Procedural fluency
- Strategic competence
- Adaptive reasoning
- Productive disposition.

Good teaching practice should promote the development of these strands of proficient teaching of geometry. Moreover, since these strands are "interwoven and interdependent" (Kilpatrick et al., 2001:380) good teaching practices cannot focus on just one or two of these strands.

Purposeful sampling was used to select three mathematics teachers from schools in the Rundu Circuit in the Kavango Region of Namibia. In purposeful sampling, researchers handpick the cases to be included in the sample on the basis of their judgement of their typicality (Cohen et al., 2000:103). In wishing to work with participants who would generate rich data, I selected teachers who are regarded as proficient and who have a good reputation in the teaching community.

Ethical measures were taken into account and the anonymity of research participants was protected.

My position as an Education Officer for mathematics in the Kavango region was a significant limitation of and the greatest threat to the validity of my study. It was anticipated that teachers might withhold information or be scared to tell the truth. I made a firm undertaking to the participants that the results would be specific to this research only and anonymous in all aspects. The teachers were assured that their participation would in no way compromise their professional standing or their position with the Ministry of Education.

A further limitation was the small number of teachers used in the study. The findings therefore cannot represent the views and practice of all mathematics teachers in the region. I will, therefore, not overly generalise and only make recommendations within the context of the study.

1.6 OVERVIEW OF THE THESIS

This thesis is divided into five chapters and is structured as follows:

1.6.1 Chapter one

This chapter provides an introduction, the context of the study, general background of the research and the research site, the research goal and an overview of the thesis.

1.6.2 Chapter two

Chapter two presents a review of selected literature on geometry teaching and learning consistent with the van Hiele levels 1 and 2. It covers the importance of geometry in the context of teaching strategies. The concepts and learning approaches associated with teaching and learning are discussed.

1.6.3 Chapter three

This chapter is a discussion of the methodology used in the study and explains the research orientation. The data collection methods and techniques, data analysis and the ethical and validity issues are also discussed in this chapter.

1.6.4 Chapter four

This chapter is devoted to the data presentation, analysis and discussion. The responses of the participants in the study are presented and arranged into data themes. Chapter four also discusses the findings emerging from the data. This discussion takes into account the literature presented in Chapter two.

1.6.5 Chapter five

Chapter five concludes the study. It presents a summary, draws conclusions from the findings and makes recommendations and suggestions for future research.

CHAPTER TWO

THEORETICAL PERSPECTIVE

LITERATURE REVIEW

2.1 INTRODUCTION

This study explores the extent to which selected Grade 10 mathematics teachers in Namibia facilitate geometry learning and teaching that is consistent with levels 1 and 2 of the van Hiele theory of geometric thinking. This research study specifically explores the teaching geometry strategies used by Grade 10 mathematics teachers to develop van Hiele levels 1 and 2 at a Grade 10 level.

Chapter two critically analyses and reviews the literature which has shaped and informed the research. Brink (2000:76) states "the literature review is a process involving finding, reading, understanding and forming conclusions about published research theory on a particular topic". According to Burns and Grove (as cited in Kasokonya, 2007:7), relevant literature refers to those sources which are pertinent or highly important in providing the in-depth knowledge needed for studying a selected research topic. A literature review provides the foundation and framework for any study. It also presents an introduction to the van Hiele theory.

The van Hiele theory is so far un-researched in Namibia. In this country teacher practice and teaching strategies consistent with the van Hiele theory of geometry have yet to be examined in a rigorous way. An understanding and application of the van Hiele teaching phases would influence teacher practice and teaching strategies in teaching geometry (Andrew, 2001; Ernest, 1991; Gates, 2001; Kilpatrick et al., 2001). Teacher practice and teaching strategies are instruments which can activate learners pre-knowledge in the development of geometric understanding. Thus if geometry teachers in the Kavango Region had knowledge of the van Hiele theory, it could possibly improve the learners' performance in this region of Namibia.

My research is carried out within a context of the educational reform process in Namibia. This chapter covers an in-depth discussion and critical analysis of the van Hiele theory and describes the five van Hiele teaching phases. An overview of international and local research studies follows. Chapter two begins by giving a brief overview of the importance and development of geometric conceptualisation in some countries worldwide. This is followed by an analysis of student performance and achievement which highlights the current poor performance and problem issues which may cause misconceptions in geometry. Under the section concerning the Namibian context I review the reform policies of Namibia on general education, learner-centred education, and the Grade 8-10 curriculum of mathematics with a particular focus on the Grade 10 syllabus.

The literature review then moves to a discussion of the five components of proficient teaching of geometry according to Kilpatrick et al.'s model. I use Kilpatrick's et al. (2001:381-384) framework of proficiency in teaching geometry to develop an observation schedule which enabled me to investigate how teachers teach geometry. Chapter two concludes with a synopsis of the whole chapter.

2.2 GEOMETRIC CONCEPTUALISATION

This section analyses the importance of geometry and the development of geometric conceptualisation.

2.2.1 The importance of geometry

The importance of geometry is reflected in how well geometry is taught in many countries worldwide, including Namibia. In the past, most teachers in primary and secondary school spent very little time on geometry (Van de Walle, 2001). This is possible because teachers felt inadequate to teach specific geometry topics or regarded geometry as unimportant. In addition, the traditional norm-referenced approach did not give a lot of weight to geometric thinking. These arguments have been advanced by the National Council of Teachers of Mathematics [NCTM] (2000) and Van de Walle (2001). Thanks to the increased emphasis on geometry which demands its inclusion in the national curriculum, geometry is now being taught in most areas of the schools, including those schools in the Kavango Region.

Helping learners become proficient in geometry is an essential goal for all schools. This will be discussed further in section 2.6 under the Kilpatrick et al.'s model. The geometry content component is very important because it develops mathematical problem-solving skills. In the geometry component of the curriculum learners are expected to:

- Use visualisation and spatial reasoning to analyse characteristics and properties of geometric shapes;
- Identify and justify geometric relations, formally and informally;
- Apply transformations and symmetry to analyse problem solving situations;
- Apply coordinate geometry to analyse problem solving situations (Namibia. Ministry of Education [MoE], 2007:1-2).

Mastering problem-solving skills helps to develop spatial reasoning (Van de Walle, 2001; Schäfer, 2004). Similarly, the understanding and development of spatial and visual skills further develop problem-solving abilities (Van de Walle, 2001). Schäfer (2004) emphasises that geometry is used on a daily basis by many people. For example, scientists and professionals such as architects, artists and land developers - to mention just a few - use geometry regularly. Schäfer (2004:10) also suggests that geometric conceptualisation is intertwined with the broader meaning of the space concept and proposes that space can mean many things to many people. The important role of spatial thinking should be recognised by teachers and incorporated in geometry (Clements, 2003; Clements & Battista, 1992). It can be argued that one of the major reasons for studying geometry is to solve problems around us.

Van de Walle (2001:23) describes the importance of geometry as follows:

- Can provide a more complete appreciation of the world
- Can be found in the structure of the solar system, in geological formations, in rocks and crystals, in plants and flowers, even in animals
- Plays major role in art, architecture, cars, machines, and virtually everything that humans create have elements of geometric form
- Can develop problem-solving skills through geometric explorations
- Plays a key role in the study of other disciplines of mathematics. For example, fraction concepts are related to geometric part-to-whole construct.
- Helps to build a fence, design a house and plan a garden

Van de Walle (2001) also suggests that if geometry increases learners' fondness for mathematics, the effort is worthwhile. I would suggest that in order to increase learners' interest and appreciation of mathematics and geometry in particular, teachers should encourage learners to think imaginatively, to make conjectures, to generalise, to abstract, to ask and answer questions in their own words. This view has been supported by Higgins (2005) who states that when learners become proficient in geometry their understanding of the importance of geometry is shaped. Geometry is one of the living subjects in schools, and Higgins describes the importance of geometry as

- Providing experiences that help students develop understanding of shapes and their properties
- Enabling learners to solve relevant problems and to apply geometric properties to real-world situations (p. 16).

In conclusion, geometry is indeed a basic skill which should be taught to learners of all ability groups in all schools. This will contribute to them to becoming effective citizens of their country.

Having discussed the importance of geometry, it is important to provide an overview of the development of geometric conceptualisation.

2.2.2 Development of geometric conceptualisation

The Namibian Curriculum Panel Committee designs the National Broad Curriculum for education. In the National Broad Curriculum, geometry is regarded as one of the disciplines of mathematics (Namibia. MoE, 2007b:2). As a core discipline within mathematics it is vital the subject is taught effectively and I would argue that geometry should be taught according to the five strands of teaching proficiency as outlined in the model of Kilpatrick et al. (2001:369), which is discussed later in this chapter. The Kilpatrick model emphasises how teachers teach and learners learn geometry to develop geometric understanding. This model demonstrates that through hands-on activities learners understand geometry content much better. This has been verified by many research studies carried out worldwide such as Crowley (1987) and Senk (1985; 1989).

Much research (Clements, 2003; Clements & Battista, 1992; Clements, Swaminathan, Hannibal & Sarama, 1999) has been undertaken in the development of geometric conceptualisation. In the geometry classes, learners should be engaged in geometric activities to enhance their geometric understanding (Crowley, 1987; Swafford, Jones & Thornton, 1997; Senk, 1983, 1985; Hoffer, 1983). According to Clements and Battista (1992:422), "children's conception of space is constructed through the progressive organisation of the child's motor skills". The representation of space is developed from prior active manipulation of the environment which influences the child. This view is supported by research by Clements and Battista (1992:422). The effective development of geometric conceptualisation requires learners to construct their own meaning and to have ownership of their meanings (Von Glasersfeld, 1995). The construction of learners' knowledge is based on subjective experiences.

As learners construct their own knowledge their geometric understanding is also developing. The aspects of thinking, understanding, reasoning and applying geometry knowledge should be emphasised by teachers. As geometry requires the knowledge and skills of learners to construct shapes around them, it is expected that teachers will facilitate reflective thinking amongst learners. According to Stein et al. (1994:26) "in constructivist classrooms learners are participating actively while teachers facilitate the process of teaching and learning".

Research on geometric reasoning by Fuys, Geddes and Tischler (1988) suggests that proficient teaching in geometry is developed through appropriate activities, which enables learners to move to the next level of reasoning. My research seeks to shed light on why learners in Grade 10 in Namibia struggle to perform well in geometry.

2.3 THE PERFORMANCE AND ACHIEVEMENT OF STUDENTS IN GEOMETRY

2.3.1 Poor performance in geometry

According to Clements and Battista (1992:421), "primary and secondary school learners in many countries, especially in the US are failing to learn basic geometric concepts and become proficient in geometric problems solving". In my view, the reason for this is that learners are unprepared for the study of more sophisticated geometric concepts, especially in comparison with learners from some other nations such as Japan and Taiwan (Stigler, 2006). It has been shown that learners from Japan and Taiwan in grade 5 perform much better on geometry tests than similar age US learners (Stigler, 2006).

Similarly, Japanese learners in grade 5 scored much higher than US learners on tests of visualisation and paper-folding (Clements & Battista, 1992:421). Data from the Second International Mathematics Study (SIMS) also show that, in geometry, some grade 8 and 10 learners in United States are scoring below the pass rate (Schoenfeld, 1985). In addition, Usiskin (1987) reports that less than 90% of 13 year old learners could find the unknown angle in a triangle. Usiskin (1987) further states that of learners who enrolled in geometry, only 63% of them are able to correctly identify triangles that are presented along with properties. Therefore learners' performance in identifying common geometric shapes is a matter of concern in many countries. This is supported by Clements and Battista (1992:421) who showed that learners' performance in dealing with properties of shapes, visualisation of shapes and applications was poor. For instance, only 10% of grade 7 learners could find the area of a square and less than 90% of them could identify a triangle.

It is of concern that the number of secondary school learners who enrol in geometry is relative small compared to other subjects in schools. According to Higgins (2005), many people today believe geometry has failed learners because it is difficult. In my opinion, the reason is that it is viewed as difficult because it is imposed on learners in a non-creative and uncritical fashion. Learners and teachers view geometry as a system of rules and algorithms to be memorised and applied. It is not surprising therefore that few learners view geometry as a creative subject. However if they are able to find this creativity, some learners get the opportunity to appreciate the historical problems associated with the development of geometry as a central discipline of mathematics. Creativity in geometric conceptualisation also encourages learners to think through problems and apply knowledge and logical reasoning. It develops skills which are necessary in the development of mathematical proof (Senk, 1989).

In my experience, teachers pre-empt possible opportunities for learners to actively participate in classroom activities. The learners nod their heads to show if they understand the lesson. In reality this nod often does not necessarily mean that the learners understand. This might lead to poor performance of learners in geometry. The poor performance of learners through didactic teaching is the reason why Namibia introduced a learner-centred education (LCE) approach in schools, and this has been shown to increase the performance of the learners in geometry (Namibia. MoE, 2003:16). In a LCE approach teachers act as facilitators who guide and support but do not prescribe too much in the learning process.

Generally learners do not learn geometry by passively sitting and listening to a teacher talking but learn most when they interact with geometric activities. This has been shown in the work of Atebe and Schäfer (2008) who believe that learners learn best by actually practising geometry. In my view it is true that learners who are encouraged to experiment will understand and apply geometric concepts correctly.

According to Mayberry (1983), most learning of geometric concepts has been through rote learning. This would also appear to be the case in Namibia. Due to meaningless rote learning learners frequently do not properly understand the properties of shapes, relationships and the implications thereof. Mayberry (1983) blames the school geometry curriculum as a primary cause of poor performance because of how topics are developed and how they are delivered to learners. According to Clements and Battista (1992:422), "the main focus of the school geometry curriculum is for the learners to recognise and name geometric shapes".

In this context, learners are required to develop skills in the use of tools in geometric concepts by using a compass and protractor. Usiskin (1987) argues that because there is not an adequate geometry curriculum at primary school level, learners entering secondary school are not performing well in geometry. Usiskin (1987:29) found in his research that the secondary school geometry curriculum was studied by only a few learners. He found that only one third of these learners could perform well in geometry.

In Namibia geometry is offered right through from primary school to secondary school and is a compulsory part of the mathematics curriculum. It is, therefore, important to research the teaching and learning of geometry. In Namibia, my research will focus on the teaching of geometry in schools in the Kavango region specifically.

2.3.2 Problem issues and misconceptions

Misconceptions of geometry are widespread throughout primary and secondary school. A research study conducted by Senk (1989) showed that only 30% of high school geometry learners enrolled in geometry are able to perform a geometric proof. It is suggested that misconceptions of geometry explain the failure of the other 70% of learners (Senk, 1989). Misconceptions are considered to be one of the greatest contributing factors to the high failure rate of learners in geometry (Rosser et al., 1988). The following are examples of common misconceptions:

- An angle must have one horizontal ray
- A right angle is an angle that points to the right
- To be a side of a figure a segment must be vertical
- A segment is not a diagonal if it is vertical or horizontal
- A square is not a square if its base is not horizontal
- The only way a figure can be a triangle is if it is equilateral
- The height of a triangle of parallelogram is a side adjacent to the base
- The angle sum of a quadrilateral is the same as its area
- The Pythagorean theorem can be used to calculate the area of a triangle
- If a shape has four sides, then it is a square
- The area of a quadrilateral can be obtained by transforming it into a rectangle with the same perimeter

Rosser et al. (1988) suggest that teachers should make use of a sequential set of exercises to assist in the mastery of conceptualisation of geometry operations in order to reduce the misconceptions of concepts. Namibian teachers could make similar use of these activities to facilitate their teaching and the learning processes. The sequence is:

- Reproducing a geometric pattern, constructed from blocks
- Reproducing a similar pattern, which was covered after an initial 6 second observation period
- Reproducing the result after rotation of a similar pattern, which was covered after an initial 6 second observation period and then rotated

- Reproducing a perspective view of a similar pattern, with the original pattern remaining available

Misconceptions naturally contribute to learners' poor performance and achievement. Research on learners' poor performance and achievement conducted by Usiskin (1982) in mathematics reveals:

- 64% of the learners could not identify a triangle within a parallelogram
- Only 16% of them could find the area of a rectangle

Studies have shown that misconceptions are a major factor in the poor performance of learners in geometry; my research aims to shed light on this within the Namibian education system. The findings will help suggest possible ways of overcoming those misconceptions. In the research study of Usiskin (1982) findings indicated that many learners who studied high school geometry did not have the expected levels of competence in geometry. For example:

- 84% of them could not calculate the area of a rectangle
- 66% of them could not find the area of a right-angled triangle
- 48% of students at the time of entering secondary school could not state the area of a square when its sides were given

According to Clements and Battista (1992:421), "less than 25% of learners in grade 11 could correctly identify which figures had lines of symmetry". A similar result was found by Siyepu (2005) in South Africa.

It is clear that misconceptions of concepts are playing a major role in the poor performances of geometry. Therefore my research study, in the context of Namibia, looks at how misconceptions play a part in the poor performance in geometry.

2.4 NAMIBIAN CONTEXT

2.4.1 The general education reform and learner-centred education

This research study is carried out in Namibia. I have reviewed documents such as the Constitution, the Education Act, the National Broad Curriculum and the Mathematics syllabus 8-10 to examine the education system in Namibia with regard to geometry. The Education Act, No 16 of 2001 sets out the principles and the intended learning outcomes for basic education (Namibia. Ministry of Basic Education and Culture [MBEC], 2001). Furthermore, the National Broad Curriculum outlines the following guidelines (Namibia. MoE, 2007b:1-2):

- Recognise that learning involves developing values and attitudes as well as knowledge and skills;
- Promote self-awareness and an understanding of the attitudes, values and beliefs of others in a multi-lingual and multi-cultural society;
- Encourage respect for human rights and freedom of speech;
- Provide insight and understanding of crucial global issues in a rapidly changing world which affect quality of life, i.e. the AIDS pandemic, global warming, environmental degradation, distribution of wealth, expanding and increasing conflicts, the technological explosion and increased connectivity;
- Recognise that as information in its various forms becomes more accessible, learners need to develop higher cognitive skills of analysis, interpretation and evaluation to use information effectively;
- Seek to challenge and to motivate learners to reach their full potential and to contribute positively to the environment, economy and society.

This curriculum aims to develop positive attitudes towards mathematics and enable learners to understand and master the basic geometric concepts (Namibia. MBEC, 1996:5). The curriculum provides a framework for subject syllabi and gives guidance for the planning of appropriate teaching materials.

The goal of the education curriculum and the syllabi is to empower learners to help develop Namibia into a knowledge-based society (Namibia. MoE, 2006:8). Vision 2030 requires a knowledge-based society which creates, transforms and uses knowledge for innovation, so as to improve the quality of life (Namibia. Government of the Republic of Namibia [GRN], 2004:7).

The Ministry of Education has advocated a LCE approach to replace the teacher-centred approach. The teacher-centred approach considers the learner to be simply a recipient of what the teacher has to provide, and this attitude promoted rote learning in schools. With the inception of LCE, the learners are placed at the centre of the teaching and learning process, while the teacher's role is to facilitate the process of teaching and learning. The LCE approach implies the active involvement of all learners (Chaka, 1997). Chaka maintains that LCE also provides learners with hands-on activities and experiences which stimulate them to think and interrogate the questions which face them. An LCE approach ought to promote unity and cooperation between teachers and learners. The teacher should guide, support and provide the necessary assistance to learners in order for them to be actively involved and acquire the required information to progress. The mathematics curriculum gives guidance on how to provide appropriate activities for learners, to help them master the basic requirements of geometry.

2.4.2 Mathematics curriculum (general and Grade 10)

The study of mathematics at junior secondary level contributes to the learner's ability to think logically, work systematically and accurately, and solve real-world problems. These are good reasons why mathematics should be a compulsory subject, and why it should be taught in such a way as to cater for a wide range of learner abilities. In Namibia, the Mathematics syllabus for geometry for Grades 8-10 (Namibia. MoE, 2007a:2) aims to develop:

- an understanding of spatial concepts and relationships;
- the acquisition of mathematical knowledge and skills in a way which encourages confidence and provides satisfaction and enjoyment;
- the ability to apply mathematics, in the context of everyday situations and of other subjects that learners may be studying; and
- an understanding of geometry principles.

As geometry is a crucial discipline in the field of mathematics, the National Mathematics syllabus for Grades 8-10 (Namibia. MoE, 2007a) outlines the basic competences to be achieved. These are:

- Grade 8 learners should be able to use terminology pertaining to lines, angles and triangles; construct and measure lines, angles and triangles; find line and rotational symmetry of plane figures; apply the properties of angles on lines and of angles in triangles.
- Grade 9 learners should be able to apply the Theorem of Pythagoras; construct perpendicular and parallel lines and bisectors; interpret and draw reflections and rotations; apply properties of angles in quadrilaterals.
- Grade10 learners should be able to construct and describe enlargements, scale drawings and nets; apply the properties of similar triangles, regular and irregular polygons, and angles in circles.

At entry into the Secondary Phase, all learners are expected to be confident and competent in handling these basic competencies. The Ministry of Education expects learners to have:

- an understanding of the geometric system,
- the ability to solve geometry problems in a variety of contexts, and
- an understanding of basic two- and three-dimensional shapes and their properties.

All learners are required to achieve the basic competencies outlined in the Mathematics Teaching Syllabus and it is expected that teachers will facilitate the learning process in geometry (Namibia. MoE, 2006). With the increasing emphasis on science in today's world, learners should be motivated to have numerical skills. These numerical skills involve and create logical models of understanding and can be developed through geometric concepts (Dindyal, 2007).

Geometric understanding enables learners to investigate, model and interpret spatial relationships and patterns which exist in the world. It is thus suggested that learners should be given opportunities to work with concrete instruments, drawings and symbolic notation in order to progress faster through the levels of understanding and skill as defined in the van Hiele framework.

2.5 VAN HIELE THEORY

In the 1950s van Hiele developed a theory which outlined a series of thought levels describing stages of sophistication in understanding geometry (Teppo, 1991; Hoffer, 1983, Mayberry, 1983; Usiskin, 1992; Senk, 1989). Van Hiele (1986) described five levels, but Clements (2003:152) has condensed these into the following four levels:

- visual
- descriptive/analytic
- abstract/relational
- proof

Clements (2003) proposed a re-conceptualisation of the visual level to better describe the way in which learners at this level perceive shapes. Clements and Battista

(1992:429) have argued for a level prior to level 1 (a level 0 when the 1-5 notation is used). This is the pre-recognition level, where learners cannot reliably distinguish between different shapes: e.g. they may be able to distinguish between a square and a circle, but not between a square and a rectangle.

In the original research the five van Hiele levels were numbered 0-4, but in recent years they have been renumbered as levels 1-5. This later convention was adopted by Swafford et al. (1997), and is the one used in this study.

2.5.1 Van Hiele levels

Van Hiele (1986) established an enabling framework to interrogate the development of cognition in geometry. He established a five-level framework which has been the subject of much research around the world. The van Hiele theory does not offer a deterministic view of a fixed progression, but rather an empirical description of relatively stable stages which can provide educators with guidance on structuring learners' experiences appropriately.

Many studies (Atebe and Schäfer, 2008; Teppo, 1991; Malloy, 1999; Hoffer, 1983, Mayberry, 1983; Usiskin, 1992; Senk, 1989; Crowley, 1987; Mason, 1998; Fuys et al., 1988; Usiskin & Senk, 1990; Usiskin, 1982; Senk, 1983; De Villiers & Njisane, 1987; Wilson, 1990; Pegg, 1995, Burger & Shaughnessy, 1986) confirm the five levels of geometric development originally described by van Hiele in the 1950s, but questions have been raised about certain aspects of the theory. I will discuss these later.

The five levels are as follows:

Level 1: Recognition/Visualisation
According to Fuys, Geddes and Tischler (1988), a learner at this level identifies, names, compares and operates on geometrical shapes such as triangles, angles and parallel lines according to their appearances. At level 1 a learner can recognise shapes by their appearance but cannot identify specific properties of shapes. For example, a rectangle may be recognised as a door because it looks like a door (Mason, 1998;

Fuys et al., 1988; Usiskin & Senk, 1990). Although learners at Level 1 may be able to recognise individual characteristics, they do not use them for complex recognition.

Some researchers (Clements et al., 1999) believe that learners at this level often use a combination of verbal declarative and visual knowledge to differentiate between shapes. Murray (1997) argues that at this level geometric understanding is based on a statement of belief, and not on logical conclusions. Mason (2003:5) states that learners at this level identify only a subset of the visual characteristics of a shape, resulting in an inability to distinguish between shapes. For example, they may not distinguish between a rhombus and a parallelogram (Mason, 2003).

Level 2: Analysis

At this level a learner analyses the attributes of shapes and the relationships between the attributes of shapes (Fuys et al., 1988). For example, a learner might think of a rectangle as a shape with four sides, and label all shapes with four sides, rectangles. But s/he might refuse to accept a square as a rectangle because it is square (Mason, 2003; Fuys et al, 1988; Usiskin & Senk, 1990). According to Mason (2003:81), learners discover properties and rules through observation and investigation. In other words, learners analyse shapes in terms of their components and the relationships between those components and discover the properties of shapes empirically. The properties are then used to solve problems.

Murray (1996; 1997) notes, that at Level 2, the terminology and symbols are meaningful to learners and they can formulate their own definitions. Definitions are accepted as binding for logical arguments and discussions. For example, this shape must be a square because it has four equal sides and four right angles.

Level 3: Informal deduction

At this level a learner discovers and formulates generalisations about previously learned properties and rules (Mason, 1998). Many studies reveal that learners develop informal arguments based on what they have learned. They also recognise the relationships among shapes (Mason, ibid). Murray (1997) suggests that the network of related concepts developed at the analysis level becomes complete and stable at this level. Precise definitions are understood and accepted. Learners understand the relations within and between shapes, for example, that the opposite sides of a parallelogram are parallel, and therefore the opposite angles are equal; or that a square has all the properties of a rectangle, therefore must also be a rectangle.

Level 4: Deduction

Hoffer (1983) argues that at this level learners can construct proofs, understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions. Hoffer further states that at this level learners prove theorems deductively and establish interrelationships amongst networks of theorems. Learners also develop sequences of statements that logically justify conclusions, constructing simple, original proofs (Hoffer, 1983). Learners understand the structure of the geometric system. High school geometry is usually taught up to this level.

Level 5: Rigor

Hoffer (1983) argues that at this level a learner is able to establish theorems in different systems of postulates and compares or analyses deductive systems. Hoffer further states that learners understand the formal aspects of deduction, such as establishing and comparing geometrical systems. Learners can also understand the use of geometric proof and appreciate the science of geometry in general.

Van Hiele emphasises that in facilitating movement from one level to another, teachers are not merely transmitters of knowledge but also, importantly, facilitators of learning (Ernest, 1991).

2.5.2 Van Hiele phases of teaching

According to Hoffer (1983:208), van Hiele also identified five phases of learning which teachers must understand in order for learners to advance to the next level in geometric thinking. Thus the van Hiele model includes both levels of geometric thinking and phases of learning. Progress from one level to the next depends not on biological development but on the teaching and learning process, especially on the teacher's providing guidance about expectations. So, progression of learners from one learning level to the next involves a corresponding five phases of teaching. Each successive phase of teaching involves a higher level of thinking. When teaching is planned in keeping with the phases, appropriate activities should be designed (Fuys et al., 1988 & Presmeg, 1991). As learners proceed from level to level sequentially (and no level can be omitted), the activities given to them to perform are crucial (Hoffer, 1983).

The sequence of five phases of teaching as suggested by van Hiele is as follows:

Phase 1: Inquiry/information

In this phase the teacher engages learners in two-way conversations about the topic (Hoffer, 1983:208). The teacher also establishes vocabulary and sets rules for learning. According to Hoffer (1983) learners are given activities to be discussed in order to develop geometric thinking. Through discussion, the teacher identifies gifted as well as slow learners in the classroom. In order for the teacher to spend sufficient time with slow learners, challenging activities are provided for gifted learners. As learners are kept busy working on various activities, they gather information which stimulates them to acquire content-based knowledge (Fuys et al., 1988; Clements & Battista, 1992). The teacher has also to provide feedback to learners on what they have written down (Clements & Battista, 1992). Through discussion the teacher learns how learners interpret the concepts of geometry, and thus can provide them with information to sharpen their geometric understanding (Clements & Battista, 1992). If learners are encouraged to sort out shapes, for example, sorting quadrilaterals and squares; they quickly learn the names and characteristics of the shapes. Through engaging learners in activities the teacher ensures their conceptual understanding is also developed (Kilpatrick, et al., 2001).

In this phase the teacher not only gives activities to learners but also requires them to reflect on what they have discovered. Through reflection the learner further understands and assimilates concepts of geometry.

Phase 2: Directed/guided orientation

Here the teacher directs the path of exploration in such a way as to ensure that a learner becomes familiar with specific key ideas related to the topic (Hoffer, 1983:208). In this phase, the teacher ensures that learners explore specific concepts of geometry. Furthermore, learners explore sets of carefully sequenced activities such as folding, measuring and constructing (Pegg, 1995; Mason, 1998; 2003). The teacher provides appropriate information on how learners should complete activities assigned to them, activities aimed at developing geometric understanding (Clements & Battista, 1992:431). The main goal of instruction during this phase is to ensure that learners are actively engaged in exploring the concepts of geometry that they have acquired (Clements & Battista, ibid).

It is evident that when teachers reflect on and articulate what has been learned with their learners, the learners subsequently demonstrate levels of reasoning well beyond their earlier performance on the properties of shapes (Kilpatrick et al., 2001:285). According to Clements and Battista (1992:431), the teacher's role is to guide learners through appropriate explorations that are carefully structured and sequenced. In such tasks learners manipulate objects so as to encounter specific geometric concepts and procedures. Teachers have to select materials and activities in which the targeted concepts and procedures are central and which further develop geometric conceptualisation, for example the activities of folding, measuring and finding symmetry.

Phase 3: Explicitation/explanation

During this phase, the teacher provides geometric activities to learners and asks them to explain the meaning of geometric concepts in their own words: for example, learners express ideas about the properties of shapes (Fuys et al., 1988 & Presmeg, 1991). Clements and Battista (1992) suggest that the role of the teacher is to explain the geometric concepts to learners, to give them an explicit level of awareness. This teaching is carried out by leading the discussion of the learners in an appropriate way. Once learners have demonstrated a degree of geometric understanding and have discussed it in their own words, the teacher can introduce other relevant geometric terminologies.

In phase 3 the learners work much more independently, refining their understanding and use of vocabulary (Hoffer, 1983:208). Mason (2003) has demonstrated that in this phase learners describe in their own words what they just have learned about the topic. Learners also express views and ask questions about what they have learned. They become aware of geometric concepts and explain them using appropriate geometric language. Thus, in this phase, learners become explicitly aware of their geometric conceptualisation and describe the geometric conceptualisations in their own words.

Phase 4: Free orientation

In this phase the learners now encounter challenging activities and gain experience in finding their own way of resolving tasks (Hoffer, 1983:208). They can complete complex tasks to develop their understanding of geometry. In this phase learners solve problems whose solutions require the synthesis and utilisation of concepts and relations previously elaborated (Clements & Battista, 1992). Mason (1998) adds that in this phase learners apply the methods they have learned to solve problems in open-ended tasks. The role of the teacher is to:

- select an appropriate teaching concept which is in line with geometric activities and give clear instructions to learners;
- encourage learners to elaborate on the geometric problems;
- introduce the geometrical terms and concepts relevant to the lesson of the day (Clements & Battista, 1992:431).

During this phase, the teacher provides the required geometrical terms and gives more complex tasks to the learners to develop their understanding. The teacher also requires learners to identify the properties of shapes hitherto unknown to them, for example, a kite.

Phase 5: Integration

In this phase learners review and reflect on their learning and understanding (Hoffer, 1983:208). Clements and Battista (1992) describe the role of the teacher as a facilitator in helping the learners summarise their key ideas. Learners go over the main points of what they have learned and reflect on them. In this phase learners build and develop a summary of all they have learned about the concepts of geometry. They integrate their geometric knowledge into a coherent network which can easily be described and applied (Clements & Battista, 1992). This network is described in the language and conceptualisations of mathematics (Clements & Battista, ibid). Furthermore, the teacher gives the learners an opportunity to reflect on and evaluate what has been learned about concepts of geometry. According to Clements and Battista (1992:443), the role of the teacher in this phase is "to encourage learners to reflect on and consolidate their geometric knowledge". Mason (1998) states that learners who summarise and integrate what they have learned can develop geometric understanding. Clements and Battista (1992) maintain that at the completion of this phase a new level of thinking is attained.

The five phases of teaching should occur within each level as learners move from one level to the next. Clements and Battista (1992) state that "as learners work their way through the levels in succession, the teacher should include geometric language, tools, symbols and relationships appropriate to the topic or objective to be taught". A number of researchers have indicated the types of activities appropriate for learners at the level 1 (recognition or visualisation) and level 2 (analysis) stages of the van Hiele theory. For example, Murray (1996) suggests that learners at the visual level should be given tasks in which situations are authentic, and that the new topic should form a natural part of this situation. She proposes that given tasks:

- should not be more complicated than the actual concepts involved;
- should allow freedom of interpretation; and
- should also be realistic.

Murray (1996) also suggests that teachers can assist learners in moving through the visual level by encouraging discussion and argument, by introducing more formal definitions, and by providing exact terminology as required. Holmes (1995) advises that learners at the visual level should be given activities which require them to manipulate and identify geometric shapes, sort and arrange shapes, draw and construct shapes, describe shapes in their own words and solve problems involving shapes. Furthermore, Holmes (1995) argues that activities for learners at the analysis level should be similar to those used at the recognition level, but that at the analysis level the focus should be on the properties of the shapes and involve the classifying of shapes according to properties. This requires learners to derive generalisations inductively, that is, by studying examples and employing the use of appropriate vocabulary.

Teachers who implement the suggestions proposed by Murray (1996) and Holmes (1995) about which activities should be given to learners at different phases would promote and increase the learners' understanding of geometric concepts. This would enable learners to understand why

- Geometry should be studied.
- There is a need to learn formulae to calculate, for example the formula to calculate the area of square.
- Learners apply the rule to substitute numbers to find the answer (Higgins, 2005).

In my view if learners are unable to provide reasons for studying geometry, it could imply that their teachers have failed to teach geometric concepts properly.

2.5.3 Strengths and weaknesses of the van Hiele theory

According to Malloy (2002:143) one of the strengths of using the van Hiele framework is "that the progression of learners from one level to the next depends more on teaching than on the age of the learner". Primary school teachers are advised to employ the five van Hiele teaching phases in teaching geometry to enhance the development of geometric conceptualisation (Fuys et al., 1988). These five phases appear within each level as learners move one level to the next (Clements & Battista, 1992). The strengths of using the van Hiele theory are that the informal activities at levels 1 and 2 provide appropriate conceptual knowledge for the formal activities at the next level.

However, research shows that there are weaknesses in the theory. For instance, Dindyal (2007) claims that one of the limitations of the van Hiele theory is that it appears to lack generality and thus each strategy of the teaching phases may need to be revised for different content knowledge. Senk (1989) claims that van Hiele did not acknowledge the existence of the level 0 (pre-recognition). But van Hiele (1986) asserts that all learners enter geometry at ground level (level 0) and have the ability to identify common geometric features. Other researchers such as Gutierrez, Jaime and Fortuny (1991) suggest that learners may develop two consecutive van Hiele levels of reasoning at the same time. They found that, depending on the complexity of the problem, learners used several levels of reasoning. However, they claim that this is not to be interpreted as a rejection of the hierarchical structure of the van Hiele theory, but rather that the theory should be adapted to the complexity of the human reasoning processes.

In research done by De Villiers (1996), it was found that in general the curriculum was presented at a higher level than the conceptual understanding of the learners. For example, at the end of grade 5 only 10-15% of the learners were at level 2. It was suggested that the main reason for this was that insufficient attention is given to geometry in the primary school. Consequently no amount of effort and good teaching methods at the secondary school will be successful unless there is a major revision of the primary school geometry curriculum presented along van Hiele lines.

2.5.4 International research studies

Many studies (Clements, 2003; Clements, Swaminathan et al., 1999; Crowley, 1987; Swafford et al., 1997; Senk, 1983, 1989; Hoffer, 1983) focus on the development of geometric conceptualisation. This review of international research concentrates on more recent studies; and the following selection has been included because of their relevance to my own work.

Malloy (2007) examines the ways in which a reform of geometry instruction shapes learners' development during the upper primary and junior secondary phases. Ding and Jones (2006) focused on geometry teaching at the lower secondary school level in Shanghai. Their study suggests that teachers use classroom strategies which attempt to reinforce visual and deductive approaches in order to develop learners' thinking in the transition to deductive geometry education.

Stigler (2006) suggests that in order to teach geometric concepts well, knowledge, skill and judgement are required. The education system in Namibia needs to take note of this advice as improving the teaching of geometry should involve sustainable improvements to instructional routine.

Research has shown that an important step in learners' development of geometric thinking is to move away from intuitive, visual-holistic reasoning about geometric shapes to a more analytic conception of the relationships between the parts of shapes.

This view is shared by Battista (1998) and Clements and Battista (1992). Conceptualising and reasoning about the properties of shapes is a major step in this development (Battista, 1998). Diezmann (1996) focuses on creative thinking within the context of geometry. This creativity depends on spatial sense, which can be developed by geometric experiences. Spatial sense is defined as "an intuitive feel for one's surroundings and the objects in them" (NCTM, 1989:49). According to Clements and Battista (1992:432) it is spatial reasoning rather than geometry which is deemed essential in creative thought. This view is supported by the positive correlation between spatial ability and mathematical achievement (Schäfer, 2004). Thus, teaching with more emphasis on geometric proficiency is required to facilitate creative thinking (Kilpatrick et al., 2001).

Rosser et al. (1988) focus on the order of acquisition of related geometry competencies in young children. They found that the operations associated with the lower level tasks are required for solving the higher level tasks. They concluded that the fixed order of task mastery, and progression in the development of geometry ability, are expected to occur in geometry teaching. Clements et al. (1999) also discuss the criteria pre-school children use to distinguish between shapes, with reference to the van Hiele levels. The research indicates that an extra level should be added before the recognition level.

Cannizzaro and Menghini (2001) aim to enhance teachers' didactical awareness in managing learners' transition from the perception of an elementary shape to its definition and from definitions to inclusive definitions. They use the van Hiele theory to encourage teachers to reflect on their own practice. Konyalioglu, Konyalioglu, Ipek, and Isik (2003) investigate the role of a visualisation approach on learners' conceptual understanding. Konyalioglu et al. (2003) suggest geometric shapes are a tool for the visualisation of the abstract concept in mathematics. They argue that by using a visualisation approach many geometric concepts can become concrete and clear for learners to understand. Their study focuses on visualisation, conceptual knowledge and procedural knowledge. Baykul (1999) defines procedural knowledge as the symbols, rules and knowledge used in solving mathematical problems and conceptual knowledge as the relationships between mathematical concepts.

An alternative framework for studying geometric thinking combines the van Hiele theory with other theories. For example, Olive (1991) analyses his study by employing the van Hiele levels of thinking and Skemp's (1987) model of mathematical understanding. Lawson and Chinnapan (2000) explore the relationship between problem solving performance and the organisation of learners' knowledge. Their findings are based on the extent to which high and low achieving groups of learners undertake geometric activities.

Many research studies examine the cognitive aspects of learning geometry. For example, Chinnapan (1998) examines the nature of prior mathematical knowledge which facilitates the construction of useful problem presentations in geometry. Dindyal (2007) focuses on the need for an inclusive framework for learners' thinking in school geometry. He was interested in how learners think in geometry given its importance in the school mathematics curriculum. As a result his study raises issues about geometric thinking and the need to conceptualise geometric thinking within a broad framework. Although his study did not focus on levels of geometric thinking he suggests that the progression of a learner from one level to the next depends on the quality of experience that learner is exposed to (Dindyal, 2007).

According to Crowley (1987:231) "a learner's experience in geometry at the primary school level appears to be critical to success in later schooling". Many researchers show that those learners who are only at the van Hiele level 1 or level 2 when presented with senior secondary school geometry have a very poor chance of success, an argument supported by Crowley (1987). Learners who already begin senior secondary school geometry at the van Hiele level 3 have at least a 50% chance at succeeding (Senk, 1989). Research by Senk (1989) supports the notion that a proof-oriented geometry course requires thinking at least at level 3 in the van Hiele hierarchy. Thus it is important to find out whether, within the Namibian context, grade 10 mathematics teachers facilitate learning at the van Hiele levels 1 and 2.

De Villiers and Njisane (1987) have indicated that the use of a hierarchical classification might not be necessary for formal deductive thinking. They also suggested that the van Hiele theory needs refinement with regard to the levels at which deduction can occur, and propose that simpler intuitive deductive reasoning might be possible at levels lower than the ordering level. De Villiers and Njisane (1987) note that there is some confusion within the writings about the van Hiele theory as to how class inclusion is understood. For example there are different types of isosceles triangles, namely right-angled triangles, acute angled triangles and obtuse triangles. But all have the properties which define them within the class “isosceles triangle”.

Teppo (1991) claims that van Hiele suggests children may learn geometry along the lines of a structure for reasoning that they developed in the 1950s. However she writes that in 1989 the NCTM recognised the importance of the van Hiele model of teaching sequential learning and an activity-based approach.

Burger and Shaughnessy (1986) characterised the van Hiele levels of development in geometry and assigned various activities to the different levels. These activities include drawing shapes, sorting shapes, determining a mystery shape, establishing properties of parallelograms and comparing components of a mathematical system. Fuys et al. (1988) also examine and use the van Hiele theory. They provide suggestions on how teachers can select activities to encourage the development of geometric understanding. Other researchers (Murray, 1996; Holmes, 1995; Presmeg, 1991) identify the types of activities teachers can give to learners to perform in the classroom and at home.

Mayberry (1983) suggests that learners’ levels of thinking might be different across the basic competencies depending on how they have been learned. She designed tasks for the first four levels of the van Hiele framework using seven common geometric concepts, namely squares, right triangles, isosceles triangles, circles, parallel lines, similarity and congruence.

In China, Ding and Jones (2006) used the van Hiele theory to analyse the teaching of geometrical proof at grade 8 in Shanghai. The analysis indicates that though the first three of the van Hiele teaching phases were found in the Chinese lessons, the instructional intention of the guided orientation was not exactly the same as that identified by van Hiele.

2.5.5 Nigeria, South Africa and Namibia

This section summarises and discusses relevant research carried out in Nigeria, South Africa and Namibia. To date there is little or no research based in Namibia about how teachers teach geometry employing the van Hiele theory. The situation is different in other southern African countries such as Nigeria and South Africa in particular. Numerous studies (Schäfer & Atebe, 2008; Schäfer, 2004; Siyepu, 2005; Sanni, 2007; Atebe & Schäfer, 2008) have been undertaken in South Africa, one of which included a Nigerian context. Concept geometry, especially relating to the role of teachers in the teaching of geometry, is one discipline of mathematics which is well-researched in South Africa.

In South Africa, Sanni (2007) carried out research on how geometry is taught in schools as an investigative rather than instructive process. Sanni (2007) presented a sequence of activities to be used in the teaching of geometry and surface areas of solid shapes in a Grade 8 classroom. The sequence of activities is consistent with teaching geometric concepts as an investigative rather than instructive process.

Atebe and Schäfer (2008) compared the van Hiele geometric thinking of Nigerian and South African mathematics learners. They administered the van Hiele geometry tests to Grade 10, 11 and 12 learners in Nigeria and South Africa. The size of the sample was initially 144, but a few learners were absent (one from the South African and four from the Nigerian subsamples respectively). Atebe and Schäfer (2008) used the van Hiele levels of geometric thinking in the analysis of their study. In Nigeria 68 learners completed the test. Thirty six (36) or 50% were found to be working at the pre-recognition level, 15 were at level 1, 16 at level 2, 1 at level 3 and none at level 4. In South Africa 71 learners took part. Twenty nine (29) were at the pre-recognition level, 16 at level 1, 17 at level 2, 2 at level 3 and 4 at level 4. In the Nigerian subsample three learners could not be assigned to any van Hiele level. Results of the study clearly show that many learners in higher grades do not have the prerequisite skill to follow the content of the higher grade geometry curriculum.

Furthermore, I looked at the study of Siyepu (2005) who focussed on the use of the van Hiele theory to explore problems encountered in circle geometry using a Grade

11 case study. Siyepu (2005:12) suggests that "the teachers need to be aware that what may appear to be correct at one level may not be correct at a higher level, or vice versa". Siyepu further states that teachers should not expect learners to master the recognition and analysis aspects of geometry until level 3, for example, that a square is seen to belong to the set of rectangle (Murray, 1997). The van Hiele theory emphasises that learners who are at a lower level of thinking cannot be expected to understand instructions presented to them at a higher level of thinking (Teppo, 1991). Schäfer (2004) who focused on the worldview and the conceptualisation of space in geometry asserts that space should not be conceptualised in terms of objective features because space can mean many things to many people.

Other researchers in South Africa, such as Murray (1996; 1997), Presmeg (1991) and Holmes (1995), focused on what the teachers teach in geometry classes to develop the van Hiele level 1 and level 2 skills.

2.6 KILPATRICK ET AL.'S MODEL OF TEACHING PROFICIENCY

According to Kilpatrick et al. (2001:369) proficiency in teaching is "related to effectiveness that consistently helps learners learn worthwhile mathematical content". They further state that proficiency entails the ability to work effectively with a wide variety of learners in different environments and across a range of geometry content. It is evident that proficient teaching of geometry is a complex activity. Mathematical proficiency involves interwoven strands that are interrelated, as does geometry. Within the Kilpatrick model these components have been described as:

- Conceptual understanding of the core knowledge required in the practice;
- Procedural fluency in carrying out basic instructional routines;
- Strategic competence in planning effective instruction and solving problems that arise during instructions;
- Adaptive reasoning in justifying and explaining one's instructional practices and in reflecting on those practices so as to improve them;
- Productive disposition toward mathematics, teaching, learning and the improvement of practice (Kilpatrick et al., 2001:380).

With an understanding of these five components, teachers would view the teaching and learning of geometry as the product of interactions between the teacher, learners and geometry. This would affect what and how teachers teach (Malloy, 2007).

According to Kilpatrick et al. (2001:333) “what is learned depends on what is taught”. The way teachers choose content, decide on how to present it, and determine how much time to allocate to it can affect learning (Ding & Jones, 2006). Therefore teachers must do careful preparation when teaching geometry because proficiency in teaching geometry is acquired over time. This view has been supported by Kilpatrick et al. (2001:314). Teachers of geometry should have the goal of providing learners with knowledge and understanding in order to allow them to function independently within geometry.

2.6.1 Conceptual understanding

Baykul (1999) defines conceptual knowledge as mathematical concepts and their relationship to each other. In addition, Kilpatrick et al. (2001) state that conceptual understanding consists of those relationships constructed internally and connected to already existing ideas. It also involves the understanding of mathematical ideas and procedures. Learners use conceptual understanding of mathematics when they

- Identify properties of shapes
- Define geometric concepts
- Compare shapes by their appearance or properties (Olive, 1991:42).

When learners acquire conceptual understanding skills they can solve new and unfamiliar problems and eventually new knowledge or ideas are generated. The conceptual understanding in the practice of teaching requires knowledge which is connected to the real world. Teachers need to make connections between their knowledge of geometry, the learners and pedagogy. According to Kilpatrick et al. (2001:381) “knowledge that makes a difference in teaching practice develops the students’ geometric understanding and learning practices”. In my view, geometric knowledge and learners’ knowledge must be connected to classroom practice if teachers are to help learners develop proficiency in geometry. In this context, I suggest that regular in-service workshops focusing on proficient teaching of geometry be conducted for geometry teachers in schools. This will help teachers develop their own conceptual understanding of geometric concepts (Mayberry, 1983).

2.6.2 Procedural fluency

Baykul (1999) defines procedural knowledge as symbols, rules and knowledge used in solving mathematical problems. Kilpatrick et al. (2001) define procedural fluency as the skill in carrying out procedures flexibly, accurately, efficiently and appropriately. This includes the step by step routines needed to perform arithmetic operations. It is also refers to being fluent in procedures from other disciplines of mathematics. For example, learners should be able to measure the size of an angle using a protractor. Accuracy and efficiency with procedures are developed through understanding. If learners remember the procedures or steps that are followed to solve problems they should make fewer common computational errors (Battista, 1998).

Researchers (Ding & Jones, 2006; Stigler, 2006; Diezmann, 1996; Swafford, et al., 1997) have shown that expert teachers have a large repertoire of instructional routines at their disposal. These findings have been confirmed by Kilpatrick et al. (2001:382). Here teachers can readily draw upon instructional routines as they interact with learners in teaching geometry. For example, teachers need to know how to respond to learners who demonstrate a lack of understanding of concepts. It is important that teachers are prepared, and have the ability to deal with learners who lack the critical prerequisite skills of learning geometric concepts. Teachers should prepare lesson plans giving consideration to the learners' level of understanding. The pace of lessons should reflect the learners' procedural fluency and learning abilities.

The importance of language is another factor in learners' procedural fluency. Van Doreen (2002:46) reminds us that language is coequal with mathematics in its capacity to enable our minds. In this context language is a system that boosts any understanding of human nature.

2.6.3 Strategic competence

According to Kilpatrick et al. (2001:383) strategic competence is "the ability to plan effective instruction". It is also the ability to formulate, represent and solve mathematical problems which arise during instruction. Most of the problems learners encounter in real-life are process problems (Teppo, 1991). Solutions for these problems involve the integration of conceptual understanding and procedural knowledge. To this end learners need to have a broad range of strategies upon which to draw. Geometry instruction must include the teaching of many strategies to empower all learners to become successful problem solvers (Cannizzaro & Menghini, 2001; Gal, 2007).

Kilpatrick et al. (2001) suggest that teachers need to find out what learners know, know how to respond to learners' questions and assist learners accordingly. Teachers should know how to deal intelligently with problems occurring in the classroom. They must also take into account what their learners understand and how they learn geometric concepts (Channapan, 1998). The activities and teaching practices which teachers employ in teaching geometry should be transparent and effective. Kilpatrick et al. (2001:383) suggest teachers should learn to recognise teaching strategies involved in solving problems so that they can address the problems in reasonable and intelligent ways.

2.6.4 Adaptive reasoning

According to Kilpatrick et al. (2001:383) "teachers can learn from their teaching by analysing the difficulties students have encountered in learning a particular topic; what the students have learned; how the students responded to particular representations, questions and activities and the like". In this component of proficient teaching of geometry, teachers become reflective practitioners. For example teachers can pose a particular geometry problem to their classes and discuss the geometric thinking they observe (Sanni, 2007). Reflection is essential to improve teacher practice (Lawson & Channapan, 2000) and teachers also need to reflect on their own professional development.

2.6.5 Productive disposition

This component of teaching proficiency focuses on one's own knowledge, practice and learning. Here teachers develop a productive disposition toward geometry and believing that geometry makes sense, they can figure it out (Kilpatrick et al., 2001:384). Teachers recognise that geometry makes sense when their understanding of the children's thinking and their teaching practices fit together (Dindyal, 2007). Teachers are facilitators of the teaching and learning processes. They have to learn to listen to their learners so that they analyse their teaching practice effectively.

These five components of proficiency in teaching geometry are not independent of each other. They are integrally related and all need to be included in the teaching process. When learners have successfully been taught these five components of proficiency in geometry they will:

- Understand the concepts of geometry and become proficient with the skills of mathematics
- Communicate and reason mathematically
- Become problem solvers by using appropriate tools and strategies (Kanyalioglu, 2003).

2.7 CONCLUSION

The teaching of geometry concepts, employing the use of the van Hiele theory, has been widely researched worldwide. In this chapter I discussed the van Hiele theory in association with the use of the Kilpatrick model in the teaching of geometry. Given the continuing debate across the world about the learning and teaching of geometry, my thesis aims to contribute to a wider understanding of the teaching of geometry with regard to the van Hiele levels. In my research I focus on teaching at levels 1 and 2. The use of the van Hiele model of teaching phases is of vital importance in my study as I investigate the teaching strategies employed by mathematics teachers in selected Namibian classrooms.

It is unfortunate that there are no studies about the teaching of geometry in Namibia. My fellow MED colleague, Mateya Muhongo, in the Kavango region of Namibia, is working on a study focussing on the learning of geometry but this is not yet complete.

As I am not aware of similar studies on the teaching of geometry in the Kavango Region I have chosen to research this area in order to provide some understanding of how geometry teachers teach at levels 1 and 2 of the van Hiele framework. My study concentrates on selected Grade 10 mathematics teachers and how they teach geometry using the van Hiele theory and the five components of proficiency in geometry teaching as discussed above. As my research looks at teaching practice it is important to deconstruct teaching proficiency with a view to understanding what makes good teachers effective.

My own research study is substantiated by previous investigations into the five teaching phases of the van Hiele theory and the five components of proficient teaching in geometry. I have adopted the Swafford et al. (1997) numbering notation for the five levels of the van Hiele framework. At the van Hiele level 1, teachers engage learners in different geometry activities. These activities involve the identification of shapes, naming of shapes, comparison of shapes and operating with geometric shapes according to their appearance. At the van Hiele level 2, teachers allow learners to discover properties of shapes empirically.

My field of investigation concentrates on the extent to which selected Grade 10 mathematics teachers facilitate geometry learning and use teaching activities and approaches that are consistent with levels 1 and 2 of the van Hiele theory of geometric thinking.

CHAPTER THREE

RESEARCH METHODOLOGY

3.1 INTRODUCTION

The literature review in the previous chapter outlined the theoretical foundation and framework that informed this study. This chapter describes the research design and explains the procedure used for conducting the study.

The data collection techniques, the selection of samples, the transcription and analysis of data, ethical considerations, validity and reliability of the study and triangulation are all discussed. The three data collection techniques employed in this study were interviews, classroom observation and document analysis.

The chapter also discusses my qualitative research approach within an interpretive orientation.

3.2 THE ROLE OF THE RESEARCHER IN DATA COLLECTION

Prior to beginning data collection, permission for conducting the research was obtained from the Regional Director of Education (see Appendix A). Once the permission was granted, arrangements were made for selected Grade 10 mathematics teachers to be interviewed and observed. Data collection also included analysis of the teachers' written records and lesson plans. The information obtained through the interview process was recorded and transcribed. Written notes were made from observations and an examination of the documents. In addition to the formal interview framework it was interesting to have the subjective views of the teachers, talking freely about their own experiences. Where appropriate, information from these subjective views was incorporated in the analysis of data.

The skills of the researcher in data collection are crucial. Whites (2000) maintains that validity in qualitative methods hinges to a great extent on the skills, competence and the rigour of the person doing the field work.

It is the responsibility of the researcher to create good relationships with the participants so that they feel free to communicate their experiences. It is also essential to treat all participants with appropriate respect. Mutual respect builds relationships of trust. In my study I adopted a professional manner and avoided leading questions, excessive guidance and any other behaviour which might cause distortion.

Three selected Grade 10 mathematics teachers were interviewed individually and their lessons observed. Interview and classroom observation schedules were made available to the teachers. During the interviews an audio-recorder was used and the lessons I observed were video taped. As a precaution, written notes were made in case the audio or video recorder failed to work.

3.3 INTERPRETIVE ORIENTATION AND METHODOLOGY

This research is conducted within an interpretive orientation. According to Cohen et al. (2000:180) the interpretive orientation "affords the researcher an opportunity to understand and interpret the world in terms of its actors". This orientation is suitable for this study, as it opens the research for information-rich data. I adopted a case study methodology as it aligns consistently with an interpretive orientation. According to Cohen et al. (2000:180) a case study is "a careful study of some social unit that attempts to determine what factors led to its success or failure". It gives an opportunity for one aspect of a problem to be studied in some depth within a limited time frame. The aim of a case study is to help understand, in the broadest possible terms, not the products of scientific inquiry but the process itself (Whites, 2000:20). This case study uses a sample of three selected mathematics teachers.

It can be argued that all case studies are inductive in that they report on the particular and specific, and then relate that to the general picture. Case studies can be used either to generate theory and ideas about a topic, or to test out a theory in a real-life situation.

A characteristic feature of case studies is that they employ a variety of different techniques including interviews, observation and document analysis.

3.4 DATA COLLECTION TECHNIQUES

This study employs interviews, observation and document analysis to address the stated research goal. According to McMillan and Schumacher (2001:180) data collection techniques are methods used to gather quantitative or qualitative information and the purpose is to try to answer the research question. For Cohen et al. (2000:44) methods are "that range of data collection techniques used in educational research to gather data which are to be used as a basis for inference and interpretation, for explanation and prediction.

3.4.1 Interviews

According to Cohen et al. (2000:267) an interview is "an interchange of views between two or more people on a topic of mutual interest". It sees the centrality of human interaction for knowledge production and emphasises the social context of research data. In addition, McMillan and Schumacher (2001:41) state that "interviews enable participants to discuss their interpretations of the world in which they live and to express how they regard situations from their own point of view". It could be argued that an interview is a two-way conversation initiated by the interviewer for the specific purpose of obtaining research-relevant information. This information focuses on content specified by the research aims of systematic description, prediction or explanation. It involves the gathering of data through direct verbal interaction between individuals.

The real benefit of an interview is that the researcher is face-to-face with the respondents, so that any misunderstandings can be cleared up immediately (Whites, 2000:30). Respondents can question what they do not understand and the researcher can re-word or re-order the questions, if something unexpected happens. Thus the interview schedule is semi-structured. The developed interview protocols were based on the van Hiele levels 1 and 2 (see appendix D). A full outline of these interview protocols is shown in appendix D.

During the interviews I explored how teachers engage learners in geometric thinking. Furthermore, the interview process was used to explore the teaching strategies employed by the selected mathematics teachers in geometry learning and teaching against the background of the van Hiele levels 1 and 2.

3.4.2 Observation

The second data collection technique employed was classroom observation. Whites (2000:30) describes observation as "an accurate record of what people do and say in real-life situations". Classroom observation data affords the researcher an opportunity to gather live data from live situations (Cohen et al., 2000:305). In this study a strategy of non-participant observation was employed. Cohen et al. (2000) describes non- participant observation as a "technique where the researcher does not become completely involved in the situation which is being researched". This was the case in the current study. I endeavoured to remain detached and simply recorded what happened.

I developed the observation protocols based on the van Hiele model and the Kilpatrick model, as shown in Table 3.1 and Table 3.2 below. The full detailed observation protocols of these tables are outlined in Appendix E and F respectively. These protocols were used to analyse teaching methodology and in particular how teachers teach geometry. In each classroom observation I observed the way the selected mathematics teachers interacted with learners in their geometry classes, the use of the chalkboard and the way the given activities were tackled. These activities included naming and identifying of shapes - which is in line with the van Hiele level 1. Sometimes learners were required to identify the types of the shapes such as isosceles or scalene triangles. Other observations were made on the classroom management and time management skills of the selected teachers.

Table 3.1: The van Hiele observation schedule

Activities	Level	Weak	Moderate	Strong	Comments
The teacher displays a variety of different readymade geometric shapes to the class.	1				
The teacher asks learners to list examples of shapes in the outside world.	1				
The teacher uses informal language to describe shapes.	1				
The teacher encourages learners to recognise and identify figures and shapes.	1				
The teacher introduces a typical topic on properties of shapes.	2				
The teacher uses formal language to describe shapes.	2				
The teacher asks learners to list properties of shapes.	2				

I constructed this schedule by taking into account the findings of researchers on the knowledge and skills learners and teachers need to master and facilitate at the van Hiele levels 1 and 2 respectively. This schedule was developed based on these activities. For example I looked at the role of the teacher at the visual level as well as at the analysis level. The activities at the visual level (L1) and at the analysis level (L2) are shown in this schedule. These activities were evaluated against the teacher's practice to determine the teaching strategies the selected teachers used to facilitate the development of geometric thinking at the van Hiele levels 1 and 2. I assessed the teaching skills of the teacher on a three-point scale. They are weak, moderate and strong. In the table I left an additional column for any comment that I wished to make.

Table 3.2: The van Kilpatrick observation schedule

CONCEPTUAL UNDERSTANDING	Weak	Moderate	Strong	Comment
The teacher is proficient in subject knowledge.				
The teacher uses prior knowledge of the students				

INSTRUCTIONAL ROUTINES	**Weak**	**Moderate**	**Strong**	**Comment**
Teacher is sensitive to process, not only product.				
The teacher starts the lesson well.				
The teacher checks homework.				
The teachers demonstrate acceptable English language proficiency.				
STRATEGIC COMPETENCE	**Weak**	**Moderate**	**Strong**	**Comment**
The teacher has planned the lesson.				
The teacher encourages participation.				
The teacher elicits responses from specific learners.				
The teacher gives individual and group activities or tasks.				
The teacher monitors group/individual/pair work.				
ADAPTIVE REASONING	**Weak**	**Moderate**	**Strong**	**Comment**
The teacher figures out how to adapt material so that it is appropriate for a given group of students.				
The teacher evaluates what the students have learned and incorporates that in his/her teaching.				
PRODUCTIVE DISPOSITION	**Weak**	**Moderate**	**Strong**	**Comment**
The teacher values learners' contributions.				
The teacher is passionate about mathematics.				
The teacher shows interest in the students.				

I also constructed the Kilpatrick observation schedule by taking into account the five components of proficient teaching (see Table 3.2). These components include the various activities that teachers assess or teach to develop geometric thinking of the learners in their classrooms.

For example, in Table 3.2, I observed the way teachers facilitate conceptual understanding (use of prior knowledge of the students), instructional routine (teacher is sensitive to process, not only product), strategic competence (teacher has planned the lesson), adaptive reasoning (teacher figures out how to adapt material so that it is appropriate for a given group of students) and productive disposition (teacher shows an interest in the students). The focus was to observe the way the geometry content component is taught and learned by the learners to develop geometric understanding.

These activities were rated or weighed against the teacher's practice to determine the way the selected teachers facilitated the development of geometric thinking in their teaching of geometry. I once again adopted a three-point scale, similar to the one in the previous observation schedule.

3.4.3 Document analysis

According to Lincoln and Guba (1985:276) document analysis and records are "useful sources of information, readily available, stable and easily accessible". This method of collecting information also serves the purpose of triangulation. The following documents were analysed: The Mathematics syllabus, the National Assessment Policy, Vision 2030, the National Broad Curriculum, lesson plans, schemes of work, lesson preparations, tasks or activities, the subject policy, the grade 8-10 mathematics curriculum and the grade 10 mathematics syllabi.

All three teachers were observed with regard to how they used the syllabi in terms of their curriculum preparation. The emphasis was on how they interpreted the syllabus in relation to their scheme of work, lesson plan and homework. The compliance with the rules and regulations was analysed. It was evident that all teachers taught the subject as laid down in the syllabi. For example the scheme of work was developed from the teaching syllabus and the lesson plans were developed from the scheme of work. I further analysed the homework given to learners with activities at the end of the lesson presentations and this homework had to be written in the learners' workbooks. The given activities showed that teachers assess learning mainly through information recall, and this assessment is in line with the national assessment policy.

I also developed a research journal. It is believed that the research journal is a route or ongoing written account of the research process. This journal included comments about activities, needs, responses, plans, ideas and questions from the selected mathematics teachers. The research journal contains notes of all data related to my research study.

Finally I located and recorded citations to books and other documents which contained useful information and views on the area of study.

3.5 PHASES OF DATA COLLECTION AND ANALYSIS STRATEGIES

Cohen et al. (2000:267) claim that data analysis is not a separate phase which can be marked out at some singular time during the inquiry. Information gathered from the data analysis is used to complement or confirm interview and observation data. In this study I was guided by the five research phases to collect data recommended by McMillan and Schumacher (2001:405-407). These phases demonstrate the interactive processes of sampling and selecting, data recording, analysis and display and tentative interpretations during the data collection period. The five phases are:

3.5.1 Phase 1: Planning

When planning a qualitative study the focus is on the research goal and the research question. The research begins with a description of the type of setting; the selection of the participants and seeking permission to carry out the research in the chosen settings (see appendices A, B and C).

3.5.2 Phase 2: Beginning data collection

This phase relates to the first days in the field when it is necessary to establish rapport, trust and reciprocal relationships with the individuals to be interviewed and observed.

In this phase I secured the participation of selected teachers. I also held initial interviews and refined my interview and observation schedule (see appendix D).

3.5.3 Phase 3: Basic data collection

In this phase I completed my interviews and observations and began the transcription process (see appendices G and H).

3.5.4 Phase 4: Closing data collection

At this stage attention was given to possible interpretations and the verification of emerging findings.

3.5.5 Phase 5: Completion

In this phase I finalised the analysis and sought a holistic sense of the relationship of the parts to the whole. I concluded the thesis by outlining the findings that emerged from the data which were compared with the literature study; followed by the potential value of the study; limitations of the study and recommendations for further research.

3.6 SAMPLING

In this study purposeful sampling was employed to select the three grade 10 mathematics teachers from Rundu town in the Kavango Region of northern Namibia. In purposeful sampling "researchers handpick the cases to be included in the sample on the basis of their judgement of their typicality" (Cohen et al., 2000:103). For Whites (2000:65) purposeful sampling "allows for variation and enables particular choices to be made relative to a particular research situation".

In order to generate rich data, the selection criteria for the teacher participants was based on their teaching proficiency. This was determined by their reputation of being perceived as good teachers by their peers, and by the examination results of their classes in the last three years.

3.7 DATA ANALYSIS

McMillan and Schumacher (2001:213) refer to data analysis “as describing the data in meaningful terms”. Data analysis requires researchers to be comfortable with developing categories and making comparisons. This includes being open to the possibility of seeing contradictory or alternative explanations. The data analysis generally begins with editing and coding of data (Zikmund, 2003:72). My data emerged from interviews, observations and documents. My analysis was done through themes that were extracted as discussed in the chapter four. My analysis was constantly framed by my research goals which was to investigate the teaching strategies teachers use to facilitate the geometry learning and teaching that is consistent with the van Hiele levels 1 and 2 at a grade 10 level.

The van Hiele model as well as the Kilpatrick model was used to develop the interview and observation protocols. The actual data was analysed based on the developed models.

3.8 VALIDITY

According to Whites (2000:20) validity “means the research design properly addresses the questions and objectives the researcher is trying to answer and achieve”. Threats to validity were minimised by choosing an appropriate time scale, selecting appropriate instrumentation for gathering the data and using an appropriate sample.

A most practical way of achieving greater validity is to minimise the amount of bias. Harper (1991) defines bias “as allowing a particular influence to have more importance than it really warrants”. This relates to the attitudes, opinions, and expectations of both the respondents and the researcher. Other issues are the misperceptions and misunderstandings of both the respondents and the researcher of what is being said or asked which may also cause bias. The participants were allowed to seek clarity on the questions they could not understand. Where possible questions were reworded and re-directed in order to overcome any bias.

3.9 TRIANGULATION

"The idea of combining different methods to cross validates data and findings, is termed triangulation. Triangulation has been described as cross-validation between data sources, data collection techniques, time periods and theoretical schemes" (McMillan & Schumacher, 2001:478). In order to validate data, information and results from different sources, situations and methods were compared to determine common patterns. According to Johnson (1994), a case study is "an enquiry which uses multiple sources of evidence".

Using a number of methods allows triangulation of the research and this makes it more robust and valid. According to Polit et al. (2001:312) triangulation is "another technique to enhance credibility". In this research, interviews are the main method of data-collection, complemented by observations and document analysis. Interviews and observation methods were used to check information (Whites, 2000:67).

3.10 ETHICAL CONSIDERATIONS

Ethics is the study of moral principles and values which govern the way an individual or group conducts its activities (Churchill, 1995:26). In the current study ethical measures are taken into account. The research data is kept confidential and not used for any other purpose. The anonymity of research participants is protected. The essence of anonymity is that information provided by participants should in no way reveal their identity (Cohen et al., 2000:62).

Other ethical issues were addressed by following the recommendations of McNiff (1996:35):

- Negotiating access (written letter to principal, teachers and parents);
- Promising confidentiality;

- Ensuring the right of withdrawal from the research at any time;
- Keeping good faith and right to information.

I employed a code of good practice as outlined by Churchill (1995) when carrying out this study. The three selected mathematics teachers were involved in the research only with their full consent and knowledge (see appendix C). Participants were given enough information about the research to make an informed decision as to whether to take part or not. During the research participants retained the right to withdraw. I refrained from coercing any participant to take part in the research. Information about the nature of the research was truthfully explained and not withheld. Participants were never asked to say or do anything which might destroy their self-confidence or self-determination. The participant's right to privacy was respected and all participants were treated with consideration and respect.

The permission to conduct the research study in the selected schools of the Kavango region in Namibia was obtained from the Regional Director of Education, the school principals, and the teachers involved in this study. Obtaining the participants' informed consent to participate in the study was of vital importance. The participants were assured that their participation in no way compromised their professional standing or their position with the Ministry of Education. Participants also had the right to know the results of the research before publication.

3.11 SUMMARY

This chapter described how the research was designed and conducted within an interpretive orientation. The use of a qualitative approach to research was highlighted and the methods used to obtain and analyse data have been described. The validity of the research is addressed and the measures used to ensure validity clarified.

Data for the study was collected from a sample of three grade 10 mathematics teachers who were teaching in schools of the Kavango region in Namibia. The selection of the sample was described. Data collection was categorised into five research phases, which demonstrated the interactive processes of sampling and selecting, data recording, analysis and display, and tentative interpretations during the data collection period.

The van Hiele model and the Kilpatrick model were applied to analyse the data from interviews, classroom observations and documents analysis. Use of different techniques triangulated the data and added validity and credibility to this study.

Themes were extracted from the data and these themes or categories were used to clarify and present the data. This is discussed in detail in the next chapter.

CHAPTER FOUR

DATA PRESENTATION, ANALYSIS AND DISCUSSION

4.1 INTRODUCTION

Data obtained from this case study are presented and analysed in order to explore to what extent selected Grade 10 mathematics teachers facilitate geometry learning and teaching consistent with levels 1 and 2 of the van Hiele theory of geometric thinking. This chapter also highlights the teaching strategies used by the selected Grade 10 mathematics teachers to develop the van Hiele levels 1 and 2 at a Grade 10 level.

Data is presented in such a way as to provide as authentic a picture of the responses of the participants as possible. I felt it important to include many of the original comments. This adds a richness and authenticity to the data presentation. Thus, where possible the actual words of the respondents have been used as supporting evidence.

Where verbatim comments are reproduced, either from interviews or class observations, reference is given which corresponds to the reference point of the interview (e.g. 4 I , 17 I) or video transcript (e.g. 1 V, 56 V). Transcripts are given as appendices to this study. The full transcripts of interviews and classroom observations are included as Appendix G and H respectively.

The themes discussed in this chapter emerged from the analyses of both the recorded and transcribed interviews and video-taped classroom observations. Before analysing the individual interviews and classroom observations, a short profile of each teacher is provided.

4.2 PROFILES OF THE RESPONDENTS

The size of the sample of mathematics teachers was initially four, but one female teacher from School A did not wish to be interviewed and video-taped. For the ethical reasons outlined in chapter three, she was not asked to explain her refusal to take part and her decision was accepted.

4.2.1 Teacher B

Teacher B is female and has been a mathematics teacher since January 2007. She holds a Bachelor of Education degree (BED) from the University of Namibia. Mathematics was one of her major subjects. She teaches mathematics at school B which has 1089 learners and 43 teachers. Last year she taught from Grades 8 to12 but in 2008 she only taught one class of Grade 10 learners and the remainder of her classes were Grade 11 and 12 learners.

4.2.2 Teacher C

Teacher C is male and has been a mathematics teacher for 14 years. He holds a Basic Education Teacher's Diploma (BETD) from the Rundu College of Education. He teaches mathematics at junior secondary level (Grades 8-10) at school C which has 665 learners and 22 teachers. His specialist subjects are mathematics and natural science although he has also taught history.

4.2.3 Teacher D

Teacher D is female and has been a mathematics teacher for 12 years. She holds a Basic Education Teacher's Diploma (BETD) from the Rundu College of Education and a Diploma in Education African Languages (DEAL) from the University of Namibia. In the BETD programme, mathematics was one of her major subjects, the other being natural science. As a teacher of mathematics Teacher D currently teaches the Grade 8, 9 and 10 learners at school D which has 508 learners and 22 teachers.

4.3 RESPONSES OF TEACHERS

The interview protocols were developed in conjunction with the van Hiele model in order to tease out teaching practice around the van Hiele levels 1 and 2. Although the selected teachers are currently teaching mathematics to Grade 8, 9 and 10 learners, the focus of this investigation was on Grade 10. The following categories or themes emerged from the interviews. They also represent some of the characteristic teaching elements of the van Hiele levels 1 and 2. They form the framework of the analysis narrative:

- enjoyment in teaching geometry lessons
- introduction of a typical geometry lesson
- the best way of describing shapes
- recognition or visualisation of shapes
- comparing or sorting of shapes
- the difference between a square and a rectangle
- analysis of parallelograms in terms of their properties
- analysis of rectangles in terms of their properties
- provision of hands-on activities
- an example of geometrical problems, and
- any other relevant factors about geometry

4.3.1 Responses regarding enjoyment in teaching geometry lessons

The first question asked of teachers was to state whether they enjoy teaching geometry, as this is fundamental in facilitating the learning and teaching of any subject. If teachers do not enjoy teaching, learning is compromised. When teachers were interviewed about how much they enjoy teaching geometry, all teachers indicated that they really enjoy teaching geometry lessons. For example, Teacher B said that she enjoys teaching mathematics in general, but geometry in particular (16I). Teacher C said he especially appreciates geometry because it shows the different shapes in nature and Teacher D responded that she likes seeing learners touching shapes (427I).

My conclusion is that these three teachers enjoy teaching geometry and thus facilitate the learning of geometry. This is consistent with teaching at the van Hiele levels 1 and 2 because when learners are engaged and enjoy their learning they are better able to develop their geometrical thinking. Enjoyment is enhanced and understanding improved when learners visualise and touch shapes. Innovative teachers encourage, stimulate and enable learners to like learning.

Furthermore, the evidence shows that learners were exposed to shapes in the classroom; they could see and touch which helps to encourage geometric thought. This is an indication of facilitating geometry learning and teaching consistent with the van Hiele levels 1 and 2.

4.3.2 Responses regarding the introduction of a typical geometry lesson

The second question asked of teachers referred to their introduction of a typical geometry lesson. Responses showed some link with teaching that is consistent with van Hiele levels of thinking. For example, Teacher B reminded learners that their forefathers were dealing with geometrical shapes when constructing traditional houses. Those houses feature a square, a triangle and a rectangle - especially the roofs of the huts. The teacher also displayed these shapes in the classroom in order for the learners to see or visualise them.

The following comments about the introduction of lessons were made by the three teachers.

Teacher B responded as follows:

> "My learners have to be taught algebra before geometry. I ask about their previous knowledge. They can have an example of a square such as a photo frame and an example of a triangle - like in Kavango our traditional houses are built with the shape of triangle". (20I)

Teacher C said:

> "I introduce the lesson for geometry by asking learners to visualise shapes. This is to make learners see the object itself in the real-life". (219I)

Teacher D stated that:

> "I have to show them what the shape looks like. For example, a rhombus, and then I explain all the properties of the rhombus". (436I; 438I)

On the evidence of responses made by the teachers it was clear that they introduced geometry lessons by assessing the pre-knowledge of the learners, and showing them examples of shapes such as a triangle. The traditional houses served as an example in one of the teacher's lesson presentations. This demonstrates implementation of the van Hiele theory and teaching consistent with the van Hiele levels 1 and 2 at a Grade 10 level as recognition of these shapes is relevant to the knowledge and experiences of the learners.

4.3.3 Responses regarding the best way of describing shapes

The third question posed to teachers related to the best way to describe shapes such as squares or rectangles. They responded that learners should see and touch shapes in the classroom, such as small cupboards, notice boards, tables and desks. Furthermore they stated that learners should be encouraged to discover the properties of these shapes by themselves, for example to identify the relationship of the sides and the angles.

Teachers responded individually as follows.

Teacher B said:

> "The best way when they learn it as they see it at home. Discover a square is having all the sides equal and all the four angles they add up to 360 degree". (44I; 46I)

Teacher C responded:

> "The best way is bring those shapes into the classroom and learners have to discover about the properties of that particular shape based on angles and sides". (284I)

Teacher D stated:

> "In my opinion the best way of describing is to show the learners the shapes. For example, rectangle, show them the opposite sides which are all equal and the four angles. A square is in the form of a classroom, notice board, a table or teacher's table and learners' table as well". (514I)

All three teachers agreed that the best way of describing shapes is to bring those shapes into the classroom and give learners the opportunity to discover the properties of that particular shape from experience. For example, in finding out about a square, learners identify that all four sides and angles are equal and the size of each angle is always 90 degrees. Similarly they learn the size of each angle in a rectangle is equal to 90 degrees.

In summary, all the teachers interviewed said that shapes should be brought into the classroom in order for the learners to see and discover for themselves the properties of that specific shape. In the process learners are able to identify and provide the name of that shape. These teachers provide the learners with opportunities to discover and share information with others. The learners explain the geometric shape relationship and experiment with various situations, e.g. measuring the three angles in a variety of triangles or working out areas through comparing an unknown figure (e.g. a tile) to a known figure [e.g. a rectangle]. Further they analyse outcomes of their experiments. Prompted by teacher's questions, they discuss and explain their analyses. This shows the link between the teaching strategies employed and the description of shapes within the parameters of the van Hiele model.

4.3.4 Responses regarding recognition or visualisation of shapes

The fourth question asked of teachers concerned the teaching of geometry lessons when learners are encouraged to recognise or visualise figures or shapes. Teacher B, for example, responded that learners should be encouraged to remember the Z-shape form, F-shape and U-shape form to identify the alternate angles, corresponding angles and co-interior angles within parallel lines respectively. This is consistent with the research of Pegg (1995).

Teacher B said:

> "Let's talk about angles formed within parallel lines, alternate angles, corresponding angles and co-interior angles. I draw a parallel line and they see clearly that the alternate angle is kind of like a Z-shape form". (59I)

Teacher C responded:

> "I give them shapes to identify and they have to give the names of the shapes". (228I)

Teacher D stated that:

> "The learners must see a certain shape - for example see a square, a rectangle, triangle, etc - that encourages learners to remember them very easily". (449I)

The word or phrase commonly used by the respondents "see, naming of shapes" indicates that teachers recognise that learners have to recognise and visualise the shape for them to learn. The visualisation is very important and is one of the teaching strategies teachers used to facilitate geometry learning and teaching that is consistent with the van Hiele levels 1 and 2 at Grade 10 level.

4.3.5 Responses regarding the comparing or sorting of shapes

The fifth question asked was how teachers facilitate comparison and sorting of figures or shapes. They were asked to describe an example of such a lesson. All teachers stated that they show different shapes and then ask learners to compare them. Teacher C required learners to discover shapes through measurement. Teacher D explained the relationship between shapes such as a rhombus and a square. She displayed the shapes in order for learners to visualise them.

Teacher B said:

> "This is what learners have to see that a square and a rhombus are more or less the same. A square when pushed forms a rhombus. Then a parallelogram and a rectangle are also more or less equal. When rectangle is pushed it forms a parallelogram. The both parallel sides and angles are equal". (88I)

Teacher C stated that:

> "My approach is for learners to discover for themselves (self discovering) for example, let them measure for themselves. In that way they get more confidence in the lesson". (249I)

Teacher D said:

> "What I do is explain to the learners how the rhombus and the square are related by using their length, the length of their sides and I show shapes to help the learners understand. With a rectangle and a parallelogram I show them similarities between the two shapes. It will make some learners to have doubts if taught without seeing them". (482I)

Teachers displayed shapes for the learners to differentiate between, for example, the rhombus from the square or a rectangle from a parallelogram. In this way learners were taught comparison and sorting of shapes. Sometimes teachers grouped similar shapes together for the learners to sort. These activities are compatible with teaching at van Hiele levels 1 and 2 at Grade 10 level.

4.3.6 Responses regarding the difference between a square and a rectangle

The sixth question enquired how teachers teach their learners to distinguish the difference between a square and a rectangle. Teacher B teaches these shapes by outlining the properties of the square and rectangle. Teacher C displays the two shapes in the classroom for learners to view and requires learners to measure them and then determine whether the shape is a square or rectangle. Teacher C uses the door, the classroom and window as examples of a rectangle whilst tiles are examples of squares. Teacher D encouraged learners to distinguish between these two shapes by applying the properties of a square and a rectangle. She also mentioned a door as an example of a rectangle.

Teachers referred to real life examples to assist learners in differentiating a square from a rectangle. For example a square was equated to tiles whilst a rectangle was equated to a door, the shape of the classroom or a window. This suggests teachers address the teaching of differentiation of shapes in ways which fit in with the van Hiele model.

4.3.7 Responses regarding the analysis of parallelograms in terms of their properties

This question aimed to investigate how teachers motivate their learners to work with parallelograms. Teacher B stated that learners get involved when you ask them questions to identify parallel lines and diagonals. Teacher B displays these shapes in the classroom. Through viewing shapes the interest of the learners is stimulated. Teacher C motivated learners by giving them activities to identify the properties of a parallelogram. Teacher D displays many parallelograms in the classroom but first discusses the properties of a rectangle.

This shows the teachers facilitate the geometry learning and teaching consistent with the van Hiele levels 1 and 2 at Grade 10 level.

4.3.8 Responses regarding the analysis of rectangles in terms of their properties

The eighth question raised the issue of how teachers encourage learners to analyse rectangles in terms of their properties. Only one of the teachers (Teacher B) stated that she encourages learners to identify the lengths, breadths and diagonals of the rectangles. Teacher B said that:

> "Learners have to analyse the lengths and widths or breadths of a rectangle by measuring them. If the sides of the two lengths of a rectangle are equal than from there a deduction can be made regarding the properties. This shape has angles that are equal with diagonals which bisect at 90 degrees". (119I)

Asking learners to analyse a rectangle in terms of its properties is a skill learners should master to develop geometric thought consistent with the van Hiele level 2.

4.3.9 Responses regarding provision of hands-on activities

The ninth question focused on the hands-on activities teachers give learners to establish the properties of shapes. Teacher B puts learners in groups and asks them to cut out papers. Teacher C provides various activities based on the properties of shapes, for example, learners might draw or identify the required or given shapes. Teacher D also groups learners and asks them to draw shapes such as rectangles or parallelograms. Teacher B said:

> "I send learners to cut shapes. This may be done individually or as a group project. I ask them to cut a triangle in a form of isosceles or equilateral one". (124I; 126I)

Teacher C responded:

> ” I can instruct them to draw the line of symmetry and then count how many lines of symmetry on different shapes”. (358I)

Teacher D stated:

> “I will ask them to draw parallelogram. I explain that a parallelogram is similar to a rectangle but “bent a bit” and that helps the learners to make sense of it.” (580I)

The teachers in the study provided activities to learners. For example, Teacher B set an activity whereby learners had to cut papers in the form of an isosceles or an equilateral triangle. Teacher C and Teacher D asked learners to draw the lines of symmetry and a parallelogram respectively. These activities demonstrate thinking and practice in line with the van Hiele level 1.

4.3.10 Responses regarding an example of geometrical problems

The tenth question asked whether teachers could provide an example of how they teach learners to use properties to solve particular geometrical problems. Teacher B stated that she usually draws a shape with unknown angles - such as a parallelogram - on a chalkboard. Learners are required to use the U-shape form (Pegg, 1995) to find the value of the unknown sides or angles. Teacher C engages learners in activities which have unknown angles while Teacher D informs learners to remember the formula for calculating the area of a square.

Teacher B said:

> “I ask my learners to draw a parallelogram with one unknown angle. So the moment they know these other angles then they can use the properties of a parallelogram to find the solution”. (133I)

Teacher C said:

> “For example let’s take a quadrilateral shape. The learners can find the sum of the interior angles of the quadrilateral shape”. (375I)

Teachers often ask learners to solve routine problems by operating on the shapes rather than referring to the properties in general. An overemphasis of this can be problematic if teachers are to facilitate geometry learning and teaching that is consistent with the van Hiele levels 1 and 2. Providing activities to learners which require them to focus on the properties of shapes is one of the important teaching strategies teachers should use to facilitate the development of geometric thinking consistent with the van Hiele levels 1 and 2.

4.3.11 Responses regarding anything else about the teaching of geometry

The last question invited teachers to say anything else they would like to about how they teach geometry. Teacher D did not comment further. Teacher B said she teaches geometry according to the syllabus, not just following the textbook because not the entire syllabus is covered in the prescribed textbooks (154I). Teacher C used many ways to teach geometry; for example he often takes the class outside the classroom into the environment and requires learners to discover shapes by themselves. Learners are sent into the community to identify shapes such as triangles and pentagons on the stop signs and on road signs (398I).

In summary the interview responses demonstrate that the selected teachers are generally teaching in a way which will facilitate geometry learning consistent with the van Hiele levels 1 and 2. Shapes are displayed in front of the class for learners to view and touch. Teachers also reported that various activities are provided to learners, for example, to cut papers in a form of a shape of a rectangle or triangle - consistent with the van Hiele level 1. Many of the teaching strategies used facilitate geometry learning and develop the geometric thinking of learners. These teaching strategies are compatible with the van Hiele levels 1 and 2.

4.4 THE THREE CLASSROOM OBSERVATIONS

Following the interviews, classroom observations were carried out in order to explore how the participating teachers facilitated geometry teaching and learning consistent with the van Hiele levels 1 and 2. Each teacher was observed once in a Grade 10 class. The classroom observation protocol that was designed, incorporated elements of the van Hiele levels 1 and 2 (see Appendix E), and the Kilpatrick et al.'s model of proficient teaching (see Appendix F). I was specifically interested in observing the structure of lessons, introductions, presentations, evaluations, assessments and classroom management of lessons with particular reference to the criteria that frame the van Hiele levels 1 and 2.

THE OVERALL LESSONS

In the lessons observed, Teacher B was teaching about angle properties of triangles; Teacher C and Teacher D were teaching learners about lines of symmetry. It is interesting to note that the two lessons on finding the lines of symmetry of plane figures are not part of the Grade 10 learning contents but part of the Grade 8 curriculum. As lines of symmetry are found only in the grade 8 syllabuses yet were being taught to a Grade 10 class, suggests that there was a gap in the understanding of the learners so the teachers taught this topic as revision.

The themes discussed in this section emerged from the classroom observations. They also represent some of the characteristic teaching elements of the van Hiele levels 1 and 2. I report on the observations according to the following emergent themes:

- Observations regarding displaying of shapes
- Observations regarding identifying and naming of shapes
- Observations regarding comparing and sorting of shapes
- Observations regarding use of language to describe shapes
- Observations regarding asking learners to construct shapes
- Observations regarding explaining the difference between shapes
- Observations regarding asking learners to solve routine problems
- Observations regarding providing activities to learners
- Observations regarding introducing a typical topic on properties of shapes
- Observations regarding teaching learners how to establish the properties of a shape

4.4.1 Observations regarding displaying of shapes

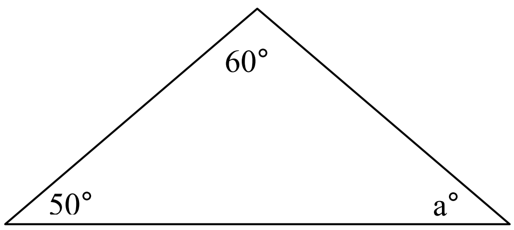
In each classroom observation teachers had teaching aids or instruments such as readymade geometric shapes. These shapes were squares, rectangles and triangles. It was also noted that teachers provided examples of the properties of shapes

The use of displaying shapes in classes was the strategy most frequently employed by the teachers. It was used to stimulate and motivate learners, through a visualisation approach; learners were able to visualise shapes to develop geometric thinking. The teachers helped each learner to develop a visual diagram of what was happening in the problems. The different shapes in nature such as squares, rectangles and triangles were made available and displayed to facilitate the teaching and learning of geometry. This strategy helped to create a classroom environment conducive to learning. The following examples illustrate how each participating teacher used this strategy.

Teacher B:

> She drew a triangle on the chalkboard, and asked: "What is the sum of in the interior angles in a triangle?" Learners responded "180 degrees, which is $a + b + c = 180°$". The teacher wrote the problem on the chalkboard. Find the value of a. You boy, what is the answer (3V)? The boy in the class says a° + 50° + 60° = 180°. She wrote the solution on the chalkboard as follows:
> a° + 50° + 60° = 180°; a° + 110° = 180°;
> a = 180° – 110°;
> a = 70°. So the value of 'a' is 70°," said the teacher. (12V)

Figure A



Teacher C unfolded the paper then drew the attention of the learners to the ready-made drawings displayed on the poster pasted on the chalkboard. He asked the learners whether they saw or visualised a rectangle drawn on the poster. The teacher also asked the learners to give the number of lines of symmetry seen after the paper is unfolded. (51V)

Teacher D was teaching lines of symmetry and shape identification. She reinforced the fact that a line of symmetry is a line which divides shapes into equal parts and a learner demonstrated that the letter A has one line of symmetry. In the same class a learner correctly identified lines of symmetry in a paper square pasted onto the chalkboard.

In summary, the teachers displayed a variety of different ready-made geometric shapes to the class in line with the van Hiele level 1. For example learners copied a shape or line on paper, which assesses the visual level. One of the teachers drew a typical shape of a triangle on the chalk board and this is also in line with the van Hiele level 1. Teacher B also asked learners to solve routine problems by operating on the shapes using the area and perimeter formula rather than referring to the properties in general. This strategy is consistent with the van Hiele level 1, as learners remember perimeter and area formulae from the previous grades; then through measuring the angles of a triangle with a protractor they can discover and learn for themselves. Learners were encouraged to recognise and identify geometric shapes which enabled them to solve the given problems through the use of formulae.

To complement the van Hiele theory, the Kilpatrick model was employed to observe when Teacher B performed calculations efficiently. For example, she asked learners to find the value of *a* in Figure A. This teacher was sensitive to process, not only product as she worked out the solution on the chalkboard. Further she monitored the previous homework properly. Both activities were assessed according to an instructional routine which is defined in the Kilpatrick framework. Another instructional routine was when Teacher C engaged learners in doing activities such as asking learners to give the number of the lines of symmetry seen after the paper is unfolded.

4.4.2 Observations regarding identifying and naming of shapes

The observations were aimed at checking whether the teacher described shapes, whether they asked the learners to identify and bring shapes from home, whether they asked learners to cut out shapes and whether teachers encouraged learners to recognise and identify shapes and figures. The observations also looked at whether teachers described shapes or a class of shapes in terms of their properties.
In all classroom observations it was noted that teachers teach learners to identify and test relationships between parts of shapes, e.g. congruence of sides. Teachers gave learning activities which stimulated learners to reason inductively to discover a relationship through experimenting. For example, a learner defined that a square has four congruent sides and four right angles. The strategy of identifying and naming of shapes was commonly used by all three teachers. It was also common for teachers to

pose questions to learners requiring them to identify and provide the names of the displayed shapes.

For example, Teacher D pasted a square piece of paper on the chalkboard, asked learners to name the shape and then asked them to identify how many types of triangles could be made in the square (121V). This strategy of asking learners to identify shapes develops geometric thinking and indicates that teachers are facilitating geometry learning and teaching in an appropriate way. This is consistent with van Hiele level 1 as, from experimenting with various squares and rectangles, the learners develop their own explanations as to why a square is classified as a rectangle.

Each observation showed that teachers ensured that learners explored specific concepts. For example, Teacher B pasted a square piece paper onto the chalkboard and asked them to name the shapes. By doing this, the instructional routine that develops geometric thought was demonstrated.

4.4.3 Observations regarding the comparing and sorting of shapes

The teachers were observed finding out if or how they taught learners to compare and sort shapes by appearance, and how they taught comparison of the shapes according to the relationships between their parts.

Comparison and sorting were not commonly taught by the teachers, possibly because it did not feature in the Grade 10 learning content for that period. Evidence from the classroom observations showed that teachers did not use this type of strategy. For example, they did not ask learners to sort or compare shapes on the basis of appearance as a whole, or compare shapes which are similar. Thus in this respect the teachers did not implement teaching in line with either the van Hiele theory or the Kilpatrick model.

4.4.4 Observations regarding the use of language to describe shapes

Observations were made to determine whether teachers used informal as well as formal language to describe shapes, and whether teachers encouraged learners to describe in their own words the properties of a typical shape.

The use of both informal and formal language to describe shapes was frequently employed by the teachers. The strategy of asking learners to describe in their own words helped teachers gauge how well learners understood geometric concepts.

The following are examples of such questioning and answering techniques:

Teacher C

> The teacher invited learners to define or give their own understanding of a line of symmetry. One learner answered "the line that divides the shapes". (39V)

Teacher D:

> "Who can remember what rotational symmetry is? One boy answered "the shape is rotated when it fits exactly on the original shape". (145V)

This implies that teachers encourage learners to describe concepts in their own words using informal language (van Hiele level 1). This type of questioning strategy assists teachers to understand and guide the learners accordingly and develops their geometric thinking. As stated in chapter two, Von Glasersfeld (1995) claimed that the effective development of geometric conceptualisation requires learners to construct their own meaning and to have ownership of their meanings. This construction of learners' meaning is found in the language. Therefore, the importance of the language cannot be overstated. Van Doreen (2002:46) reminds us that language is coequal with mathematics in its capacity to enable our minds. In this context language serves as an effective tool for facilitating mathematics and a system that boosts or develops any understanding of human nature. The way teachers motivate learners to make use of their own language to describe shapes encourages learners to communicate. It also assists teachers to gauge the background of their learners in terms of their knowledge and understanding of geometric concepts. In the observation teachers were language proficient. This was evident when Teacher C invited learners to define or give their own understanding of a line of symmetry. With this view in mind teachers evaluated learners according to the conceptual understanding that develops geometric thinking as defined in the Kilpatrick framework.

4.4.5 Observations regarding asking learners to construct shapes

Classroom observations showed how teachers asked learners to construct shapes according to their appearance or properties. The teachers demonstrated shapes by drawing and copying a typical shape on the chalkboard. This was seen to be effective in capturing the interest of the learners and motivated them in the lesson.

All teachers used a similar technique to get learners to construct shapes, namely asking them to draw the shape or asking them to draw lines of symmetry on the chalkboard. For example, Teacher B asked learners to draw the shape of an equilateral triangle and then asked them to give its characteristics (7V). Teacher D asked learners to draw lines of symmetry on a square (125V). As learners are engaged in the construction of the shapes their geometric understanding also develops. Thus this teaching strategy can help learners to develop the geometric thought consistent with the van Hiele theory

This activity demonstrated the van Hiele theory at level 1. The fact that teachers chose materials and activities that target the key concepts and procedures under consideration meant that the instructional routine was evaluated. In addition teachers were also creative and innovative when they asked learners to name shapes such as the equilateral triangle. This implies that teachers assessed learners according to the strategic competence as outlined by the Kilpatrick et al.'s model model.

4.4.6 Observations regarding explaining the difference between shapes

Observation showed how teachers explained to learners how to recognise the difference between shapes. For example, Teacher B drew an isosceles triangle and an equilateral triangle on the chalkboard. She then asked learners to name the characteristics of an isosceles and an equilateral triangle. Learners responded that an isosceles triangle is a triangle that has two equal sides, its two base angles are equal and line of symmetry divides the base in half. An equilateral triangle has three equal sides and each angle equals 60°”. (6V)

Before the teacher B introduced a topic on properties of shapes she asked learners to list the properties of shapes such as an isosceles and an equilateral triangle. This teaching strategy encourages learners to describe a typical shape in terms of their properties in their own words. It was observed that the way the difference between shapes was explained is also consistent with the van Hiele level 2. The observation revealed that learners listed the properties of the given shapes but failed to explain why all rectangles are parallelograms.

The Kilpatrick strands also featured in the observation when teachers evaluated what the learners learned and incorporated that in their teaching. For example Teacher B asked learners to name the characteristics of an isosceles triangle. This teacher assessed learners according to an adaptive reasoning which is a strand of proficient teaching. Further in the observation, teachers also engaged in analysing how the learners responded to particular questions and activities (adaptive reasoning), teachers also valued the learners’ contributions (productive disposition). All these teachers demonstrated teaching according to the Kilpatrick model.

4.4.7 Observations regarding asking learners to solve routine problems

Classroom observations showed teachers asking learners to solve routine problems by operating on the shapes, rather than only referring to the properties in general. For example, learners used trial and error methods to find the sum of a triangle of interior angles. It was also evident that teachers asked learners to solve a certain problem by using known properties of shapes For example Teacher D asked learners to measure the length and breadth of a rectangle to determine the area of the shape.

A further example of this technique is as follows:
Teacher B reminded the learners to remember to use the formula of the sum of a triangle of interior angles equals 180°. This assesses learners according to the van Hiele levels 1 and 2. At this level learners identify shapes as a whole but they may not recognise, for example, that a scalene triangle has three different angles which add up to 180°. They may also exclude the necessary attributes of a triangle with curved sides. A learner works out the how to find the area of a new shape by dividing it up into shapes of known area.

The fact that Teacher B was inspired and showed interest in the learners, showed a productive disposition as defined in the Kilpatrick model.

4.4.8 Observations regarding providing activities to learners

Teachers were expected to provide activities to learners which required them to focus on the properties of shapes and to use vocabulary appropriately. Observations showed that all the teachers did this. Also observations showed that teachers asked learners to list examples of shapes in the outside world. Examples of the kinds of activities given to learners during and after the lesson presentations are as follows:

Teacher B:

> "Teachers asked: what is the value of 'c' in the following expression: c + d + 65° = 180°, if the triangle is isosceles? What is the value of c?" asked the teacher". (18V; 19V)

Teacher C:

> The teacher provided class activities to the learners asking them to draw the lines of symmetry, identify the number of lines of symmetry and give names of the shapes. (83V)

Teacher D:

> "The teacher provided exercises and moved round to check what the learners were doing and gave guidance immediately. "Help one another," she said. She moved from desk to desk to see how the learners' progressed. (166V)

The teachers gave the learners activities that required them to focus on the properties of the shapes. They further provided activities to learners which required them to use appropriate vocabulary for parts and relationships. For example in explaining that opposite sides and diagonals bisect each other, learners need to recall that a square has four opposite sides and the diagonals bisect each other. Both activities were consistent with the van Hiele level 2 which requires learners to think critically. Hence the teachers used appropriate strategies to facilitate geometry learning and teaching to develop the geometric thought.

To ask learners to list examples of shapes in the outside world implies that the activity is in line with the van Hiele theory at level 2. For example, Teacher D provided exercises and moved round to check what the learners were doing and gave guidance immediately. This also implies that the activity given is consistent with the van Hiele theory at level 2. This showed strategic competence in the Kilpatrick model.

4.4.9 Observations regarding introducing a typical topic on properties of shapes

Many strategies were used by teachers to introduce topics on properties of shapes. For example:

Teacher B drew a triangle on the chalkboard, and asked learners as follows: "What is the sum of the interior angles of a triangle? How is the exterior angle of a triangle related to two opposite interior angles of the same triangle (1V; 3V)"? Observations showed that it was a good to start lessons in that way because it motivated and stimulated learners to open up classroom discussions.

Teacher C asked learners to provide the definition of the lines of symmetry. "Can any one of you tell us about line of symmetry?" This was done before introducing a typical topic on the properties of shapes (36V).

Teacher D was prepared to teach lines of symmetry but she started the presentation with a question based on what the learners already knew. The question read as follows: “How many sides does the pentagon shape have? Or what is the sum?” One learner responded “540” (99V).

These teachers’ presentations commenced with a good, stimulating questioning strategy which encouraged learners to use informal and formal language to describe geometric concept in their own words. This questioning and answering strategy can positively mobilise the interest of learners. For example teachers presented the topic on the properties of shapes after the discussion on the lines of symmetry. Another observation made was when teachers explained that a rectangle looks like a square, a parallelogram is a “slanty” rectangle and angles are arms of a clock, etc. Knowing that a parallelogram has opposite sides parallel, a learner will note that this is also the case with rectangles and squares. All of these techniques are consistent with the van Hiele theory and operate at van Hiele levels 1 and 2.

In addition, I also observed that teachers had planned their lessons and they communicated openly and showed genuine interest in knowing the names of the learners (Teacher B). This is in line with strategic competence as defined in the Kilpatrick framework.

4.4.10 Observations regarding teaching learners how to establish the properties of a shape

In all classroom observations teachers showed learners how to establish empirically the properties of a typical shape. It was evident that this strategy was commonly used. For example, Teacher B drew an isosceles triangle and asked learners to provide the characteristics of isosceles triangles (5V) while Teacher C asked learners to give definitions of scalene, acute and obtuse. When the learners could not come up with a definite explanation about scalene, the teacher led them by saying that a scalene has all three sides which are not equal and all the angles are also not equal (93V). This type of activity assessed the van Hiele theory at level 1. In addition, the Kilpatrick et al.’s model was also reinforced when Teacher B discussed and clarified materials or shapes - an explanation about scalene serves as an example. This is consistent with instructional routine in the Kilpatrick et al.'s model.

The teachers provided activities to learners which required them to focus on the properties of the shapes. Teaching learners how to empirically establish the properties of a typical shape is regarded as level 2 geometric thought within the van Hiele theory. For example, after shading congruent angles in a triangular shape, the learner notes the sum of angles in each triangle is 180° and then tries to find out whether this is the case with all triangles. In other observations learners described a square as having four sides, four right angles, all sides equal and opposite sides parallel. However, learners could not explain how, in a parallelogram, the idea that opposite angles are equal follows from a knowledge that opposite sides are parallel. Given certain properties of the shape, the learner can identify what shape it is.

4.5 DOCUMENT ANALYSIS

Following classroom observations, a document analysis was carried out. This included documents such as the Mathematics syllabus, the National Broad Curriculum the national assessment policy, lesson plans or lesson preparations, schemes of work, and tasks or activities. The lesson plans related to the lessons observed as part of this study were analysed to discover the utilisation of the syllabus versus the scheme of work.

Observation of the ways teachers employed the learning contents of the syllabi
All three teachers were observed with regard to how they employed the teaching syllabi in their curriculum preparations. The emphasis was on how teachers interpret the syllabus in relation to their scheme of work, lesson plans and homework. It became evident that all the selected teachers complied with the rules and regulations laid down in the syllabus. For example, the scheme of work was developed from the teaching syllabus while the lesson plans were developed from the scheme of work. And at the end of each lesson presentation, homework was given to learners with activities which are in line with the national assessment policy and this homework was written in the learners' homework books. For example Teacher B asked her

learners to work out an activity that expected of learners to use properties of shapes to solve particular geometrical problems. Teacher C and Teacher D asked their learners to identify and name the lines of symmetry of plane figures.

When I analysed the Grade 8-10 mathematics curriculum and the Grade 10 mathematics syllabus, I found that these documents are used, reviewed and revised annually. However the review is not complete as certain topics in geometry are not included in these syllabi.

Generally, most of the geometry learning content in the Grades 8-10 curriculum is not consistently taught in all grades, such as the Theorem of Pythagoras, which is only found in Grade 9 learning content. This theorem is not included in the Mathematics syllabus for Grades 8 and 10, yet it is often assessed in the national examination of Grade 10. This omission actually hinders learners' progress to perform well in the national examination.

4.6 DISCUSSION OF FINDINGS

In the preceding sections of this chapter, data was presented which describes how the three selected Grade 10 mathematics teachers responded to the interview questions and how they presented geometry lessons during the classroom observations. In addition a brief document analysis was conducted. The main purpose of this study was to explore how mathematics teachers facilitated the teaching and learning of geometry consistent with the van Hiele levels 1 and 2. In this section, the emerging issues from the evidence or data in the findings will be discussed further in relation to or in the light of the literature reviewed in chapter two. I discuss the data against a framework of the following themes:

- The learner-centred education (LCE) approach
- Visualisation
- Content knowledge
- Motivation
- Participation and communication
- Assessment

4.6.1 The learner-centred education (LCE) approach

The learner-centred education (LCE) approach came out strongly in the findings as an area in which mathematics teachers still need support. As shown in the discussion to the reform policy (chapter two), the LCE approach was adopted with the purpose of improving the quality of learning in the classroom through a more consistent approach (Namibia. MoE, 2003). This approach was based on the needs of the Namibian society and beyond. The aim is to have a knowledge-based society and productive citizens which creates, transforms and uses knowledge for innovation to improve the quality of life.

A LCE, therefore, is adopted in the Namibian education system because of the many benefits it has for learners. Some of these benefits include equipping learners with relevant knowledge and skills which will help them to be self-dependent or self-reliant in the public sector. Ultimately this could lead to the promotion of confidence and the determination to succeed (Vision, 2030:15). This approach further promotes unity and cooperation between teachers and learners (Lawson & Channapan, 2000).

The responses expressed by the teachers about the importance of geometry coincided with those of Van de Walle (2001), Schäfer (2004), Higgins (2005) and others (chapter two). In developing country such as Namibia, LCE outlines the way to improve and facilitate quality teaching and learning and it offers a practical solution to many problems (see chapter two). Van de Walle, Schäfer, Higgins and others further claimed that geometry is very important because it develops mathematical problem-solving skills. It is for this reason Namibia introduced a LCE approach in schools to support the teaching and learning of geometry, which is believed to be increasing the performance of the learners in geometry (MoE, 2007:16). In these observations learners were engaged in analysing shapes in terms of their components and the relationships between those components and discover the properties of shapes empirically. Further learners explored sets of carefully sequenced activities such as folding, measuring and constructing (Teacher C). With this view in mind I support that a LCE approach implies the active involvement of all learners with the aim to have a deeper understanding of geometric concepts (Chaka, 1997).

These teachers also provided learners with hands-on activities and experiences which stimulate them to think and interrogate the questions which face them (Sanni, 2007). This approach demands teaching strategies that can lead to a deeper conceptual understanding in a learner about what is being taught and learnt. These are the strategies which focus on the holistic development of a learner. In the three schools involved in the study the teachers found it difficult to implement a LCE approach because they were hampered by the lack of teaching and learning materials such as mathematical instruments which could be employed to construct shapes.

By knowing how to implement the van Hiele theory and the Kilpatrick model, teachers can facilitate an understanding of geometric concepts and relationships in their learners and develop their mathematical knowledge and skills in a way, which encourages confidence and provides satisfaction and enjoyment (see chapter two). This geometric understanding will enable learners to investigate relationships and patterns which exist in the world as outlined in the Mathematics Syllabi 8-10. For that reason the main purpose of the syllabi is to equip learners with knowledge and skills that will enable them to solve geometrical problems in a variety of concepts (MoE, 2006). It therefore follows that learners should be given every opportunity to help them progress faster through the levels of understanding and skills as defined in the van Hiele framework, complemented by the Kilpatrick model.

Based on the very positive responses by the three teachers in this study about the importance of geometry, it would seem that they concurred with other researchers who emphasised the need for geometry in the curriculum. However, as the results show, these positive responses are not generalisable due to the small sample that was involved in the study.

The lessons observed in the three schools did include practical work which involved drawings of shapes. Learners acquired these skills when their teachers engaged them in the activities involving the identifying and naming of shapes, and calculating the value of unknown angles. I therefore support the ideology that in a LCE environment the role of the teacher is to facilitate the teaching and learning process (Ernest, 1991). In light of the data analysed, participating teachers seemed to be trying to work within a LCE approach.

4.6.2 Visualisation

It was evident that teachers aimed to create visualisation of shapes as part of their teaching. In both interviews and observations they used words or phrases like "touch or see/visualise (29I, 219I); show (125V, 520I), draw (1V, 5V); what is it (19V, 20V); identify, give and name (3V)". All these instructions stress the visualisation approach. Also drawing or showing shapes on the chalkboard gave learners an opportunity to understand geometric concepts through visual means. In the observations, Teacher B drew a triangle on the chalkboard, and asked learners to calculate the sum of the interior angles of a triangle. Observation showed that it was a good start to the lesson because it motivated and stimulated learners to open up classroom discussions.

Teacher C asked learners to provide the definition of the lines of symmetry. This was done before introducing a topic on properties of shapes (36V). Teacher D pasted a square piece of paper on the chalkboard, asked learners to name the shape and the asked them to identify how many types of triangles could be made in the square (121V). Also observations showed that teachers asked learners to list examples of shapes in the outside world.

As discussed in the literature review, Konyalioglu et al. (2003) suggest that geometric shapes are tools for visualisation of the abstract concept in mathematics (see chapter two). They argue that by using a visualisation approach many geometric concepts can become concrete and clear for learners to understand. According to Fuys et al. (1988) a learner at the visual level identifies, names, compares and operates on geometrical shapes such as triangles, angles and parallel lines according to their appearances.

In the interviews, teachers also reported that various activities are provided to learners, for example, to cut papers in a form of a shape of a rectangle or triangle. Shapes are displayed in front of the class for learners to view and touch. This compares with Clements and Battista (1992:421) who found that a visualisation and paper-folding approach increases learners' performance in dealing with properties of shapes, visualisation of shapes and applications. Through viewing shapes the interest of the learners is stimulated (Teacher C). Therefore, mastering the visualisation approach develops conceptual understanding (Kilpatrick et al., 2001; Baykul, 1999) which helps learners to develop their geometric thinking at the van Hiele levels 1 and 2.

Outcomes of the current study show that the teachers were applying the visualisation techniques to teach the learners and were working to develop their conceptual understanding as outlined in the Kilpatrick model. At level 1 a learner can recognise shapes by their appearance but cannot identify specific properties of shapes. For example, a rectangle may be recognised as a door because it looks like a door (see chapter two). This implies the van Hiele level 1 is practiced by the selected teachers to develop geometric thought of learners.

4.6.3 Content knowledge

The notion of content knowledge features prominently in the findings. It cuts across all themes emerging from the interviews, video-taped classroom observations and document analysis.

Geometry cannot be taught or learned in a classroom only. Good teacher practice implies that one introduces topics by referring to pre-knowledge (33I; 35I). For example Teacher B reminded learners that their forefathers were dealing with geometrical shapes when constructing traditional houses. Those houses feature a square, a triangle and a rectangle - especially the roofs of the huts. The teacher also displayed these shapes in the classroom in order for the learners to see or visualise them. In this case the subject knowledge is compared to the real-life situation which broadens the conceptual understanding. This means that this teacher used many teaching and learning strategies that could help facilitate effective learning and teaching to happen.

A research study by Olive (1991) discussed in chapter two, found that learners use conceptual understanding when they identify properties of shapes, define geometry concepts and compare shapes by their appearance or properties. For example in this study the respondents emphasised that algebra should be mastered first by the learners before geometry is taught (20I). Thus MoE (2006) expected that all learners are required to achieve the basic competencies outlined in the Mathematics Teaching

Syllabus 8-10 and it is expected that teachers will facilitate the learning process in geometry. I observed the teachers stimulating learners to partake in discussions, group work and demonstrations. I also observed learners having an understanding of geometric shape and their properties. When selected teachers employed group work and the question-and-answer methods as well as demonstration in the classroom, the evidence revealed that strategies were often used in a reflective approach. Learners were asked to express their views or illustrate activity in their own way. Of the lessons observed Teacher B and Teacher C did not use a demonstration method that requires active involvement of the learners. The use of a demonstration method involves the integration of conceptual understanding and procedural knowledge, which develop geometric thinking (Teppo, 1991).

Other examples of promoting geometry content knowledge occurred when teachers engaged learners in activities such as "measuring the size of shapes" (230I; 467I), "self discovering", "presenting the local examples of a rectangle such as a door and the teacher's table (317I; 530I) and squares such as tiles" (337I). These relate to common shapes which are encountered by learners in real-life situations.

Observations however showed that learners could not calculate the value of an unknown angle in a triangle. This means that these learners might lack some geometry content knowledge and basic operation skills. The danger is that learners will view geometry as a system of rules and algorithms to be memorised and applied. I therefore suggest that learners should engage actively with geometric activities for them to learn most effectively. The work of Atebe and Schäfer (2008) shows that learners learn best by actually practising geometry (see chapter two).

Overall, this study revealed that the participating teachers were competent in geometry content. They also valued the learners' experience through acknowledgement and comments made. I also observed that the teachers were sensitive to process (procedural fluency), not only product. For example, teachers performed calculations efficiently. During the lessons, I observed how teachers checked and monitored the class work. However, there was great variation in their approach to classroom instruction.

The teaching observed was in line with the van Hiele theory and the Kilpatrick et al.'s model. Geometry lessons were generally well planned and organised, the resources provided by teachers contributed positively to accomplishing the lessons' purposes, and the geometry content was significant and worthwhile. However, while teachers appeared to take into account learners' prior knowledge in planning and delivering instruction, there was great variety in the instructional strategies used by teachers to meet the diverse learning needs of the learners

4.6.4 Motivation

Motivation as a key element in the facilitation of teaching and learning of geometry emerges as a strong theme in these findings. Mathematics teachers in this study indicated that learners should see and touch the geometric shapes. They responded that these shapes should be displayed in front of the class to motivate them. The interviews further revealed that learners should be shown all the opposite angles which are equal (117I), and the length and width or breadths (119I) by way of comparing the sizes.

In the observation Teacher B reminded the learners to remember what they were told, that $a° + 50° + 60°$ add up to 180° (12V). Teacher D, who demonstrated the number of line of symmetry a rectangle has (137V), used two rectangles which crossed with each other at a right angle (90°); one was placed vertically while the other one was placed horizontally. Further in the observation a rectangle was also used to illustrate the number of order of symmetry (152V). Further, the respondents involved learners by inviting them to the chalkboard to demonstrate shapes (44V). Through these activities, learners are stimulated and motivated to partake in the discussions that in turn promote geometric thinking.

Numerical and geometric skills are necessary for the modern world. The teachers tried hard to motivate learners to make use of these skills. For example, Teacher B asked learners to draw the shape of an equilateral triangle and then asked them to give its characteristics (7V). Teacher D asked learners to draw lines of symmetry on a square (125V). This teaching strategy can help learners to develop the geometric thought.

In the lessons observed, teachers engaged in analysing how the learners responded to particular questions and activities. For example Teacher C invited learners to demonstrate paper-folding in front of the class. This view was supported by Holmes (1995), who suggests that learners at the visual level should be given activities which require them to manipulate and identify geometric shapes, sort and arrange shapes, draw and construct shapes, describe shapes in their own words and solve problems involving shapes to develop their geometric thinking. Added to this is the fact that these activities for learners at the analysis level should be similar to those used at the recognition level, though at the analysis level the focus is on the properties of the shapes and involves the classifying of shapes according to properties.

This was demonstrated in the current study and evidence revealed that the teachers showed an interest in the learners by valuing their contributions. This study however found that lessons were not always paced appropriately. Teachers did however use group or individual activities and these were carefully monitored.

4.6.5 Participation and communication

Teachers used words such as "drawing and experimenting" which suggests they aimed at learners actively participating in lessons. For example teachers were observed to draw parallel lines and learners could see clearly the alternate angle (61I). The aim was for the learners to discover and experiment with shapes for themselves (126I). Sometimes learners were given measurements to draw a square" (467I).

When the teacher came prepared for the lessons, with appropriate teaching aids, teachers and learners worked well together and the teachers encouraged participation by eliciting responses from specific learners. Teacher C involved or engaged learners in self-discovery when he displayed the different shapes in nature like squares, rectangles and triangles. In this case teachers engaged learners in analysing the shapes to discover their properties.

Regarding the discovering of the properties of shapes, as mentioned by teachers, a similar concern was noted by Mason (2003:81) who suggested that learners discover properties and rules through observation and investigation. In other words, learners analyse shapes in terms of their components, and the relationships between those components, and discover the properties of shapes empirically. For example, after shading congruent angles in a triangular shape the learners noted the sum of angles in each triangle is 180° and then tried to find out whether this is the case with all triangles. From the interviews with teachers I also observed the way learners identified shapes that belongs to square and rectangle in the classroom.

Examination of the classroom culture in the geometry classes observed in this study showed that generally learners were encouraged to actively participate in the lessons and there was respect for learners' ideas, questions and contributions.

Another example of encouraging participation was when a teacher asked learners to draw the shape of equilateral triangle and required them to verbally give its characteristics (7V). In this example, the teacher invited learners to illustrate the lines of symmetry by folding the paper until the sides corresponded (64V). This was carried out in group work and learners were asked not to work out the activities individually (165V).

4.6.6 Assessment

Assessment, one of the key skills required by mathematics teachers, features prominently in these findings, with evidence from interviews and classroom observations. The mathematics teachers employed different ways of assessing learners in their teaching strategies. For example teachers set learners to cut papers (124I), and the assessment was done in a group or individually. Teacher B preferred group work because it saves time and encourages learners. Learners communicated within their groups instead of teachers talking to that specific person directly (128I). Further, one of the respondents indicated that a test can be provided to learners to see their performance, which will show whether or not they have improved. Various activities were given as class work (35V) and homework (168V) which assessed the learners' skills and development in line with the van Hiele theory.

Other class activities provided by teachers required the learners to draw lines of symmetry, identify the number of line of symmetry and give names of the shapes

(83V). Other activities asked learners to calculate an unknown figure through substitution of numbers in a given problem, such as when calculating the perimeter of square (627I). These teachers engaged learners in doing activities. As indicated in chapter two by Clements and Battista (1992), these teachers also directed learners' activity by guiding them in appropriate explorations that were carefully structured, sequencing tasks in which learners manipulated objects so as to encounter specific concepts and procedures of geometry, for example the paper-folding illustrated by Teacher C.

In support of Hoffer (1983) as indicated in chapter two, these learners were given activities, which were then discussed in order to develop geometric thinking. Engaging learners in activities monitored by teachers, the conceptual understanding of learners will be developed (Kilpatrick et al., 2001).

From the above I can conclude that these provided activities will enhance learners' understanding of geometry. It is extremely important that learners are supported by their teachers to develop geometric thinking that ensures effective learning.

4.7 CONCLUSION

This chapter gives an account of the research data and its analysis. In trying to understand the data, the respondents' responses to the interview questions were compared with class observations to determine how the teachers actually presented geometry in their schools. Many factors were considered, such as the teachers' teaching and the type of learning and teaching materials available in their schools. Thereafter, the mathematics syllabus document was analysed regarding the implementation of geometry in schools. In order to present a more informed picture of the data, literature was consulted to verify the findings, and used to demonstrate the most acceptable way of implementing the mathematics curriculum in the schools.

Generally data emerging from this study resonates strongly with the literature research. The intent of this research was to arrange responses from interviews and observations into themes or categories. This process was not entirely smooth as there was repetition and at times contradictions and some respondents spoke at length when asked to explain an issue.

Because of questions arising from the differences in performances in the school mathematics final examinations, learners' geometry expectations have received renewed focus (the Namibian Mathematics syllabus Grades 8-10, 2007). Specific questions have arisen regarding whether all learners are sufficiently prepared to handle geometric content by Grades 8, 9 or 10, and whether teachers have different expectations for learners based on the learners' prior geometry knowledge performance. In the context of this research, there is evidence that the teachers made use of the prior knowledge of learners to enhance learning.

This study concentrated on the teaching strategies used by teachers and whether they were consistent with the van Hiele levels 1 and 2 at a Grade 10 level. The findings obtained from the classroom observations and interviews indicate that much of the teaching and learning of Grade 10 geometry teachers in the selected schools is structured in such a way to support the development of geometric thinking as described in the van Hiele theory, particularly at levels 1 and 2.

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

5.1 INTRODUCTION

This chapter focuses on the conclusions and recommendations with regard to the study undertaken. Limitations of the study and implications for future research are also addressed.

In order to make effective use of the van Hiele theory in developing geometric thinking, mathematics teachers should ideally have the necessary knowledge and skills and be trained in the theory and application of the van Hiele model. The aim of this study was to explore to what extent currently practising mathematics teachers facilitate the teaching and learning of geometry at the van Hiele levels 1 and 2 at Grade 10 level.

The findings suggest that the selected teachers in the study generally used teaching strategies which supported learning at the van Hiele levels 1 and 2, even though they had not received any direct training or instruction about the van Hiele theory. However there was also evidence indicating that gaps exist with regard to the facilitation of the teaching and learning of geometry in the schools studied, and this seems to be the case in many schools in Namibia.

5.2 CONCLUSIONS FROM DATA ANALYSIS

This study has highlighted the unique personal experience and teaching strategies of the selected participating mathematics teachers. Data gathered through interviews and observation were compared and extended through the personal reflections of the teachers. From the analysis of data certain key themes emerged:

Learner participation

Generally teachers aimed to involve learners in activities. Activities such as asking learners to construct or draw shapes according to their appearance or properties were in keeping with geometric thinking at van Hiele levels 1 and 2. This teaching approach encouraged participation of the learners as advocated by learner centred education (LCE) and also assisted in developing thinking and learning consistent with the van Hiele model. Learners also took part in discussions and expressed geometrical concepts in their own words. Other activities incorporating learner participation included cutting out shapes. These activities demonstrate application at the van Hiele level 1.

Use of visualisation

The teachers made use of "visualisation". They asked learners to identify and bring shapes from home. Through displaying these shapes in the classroom and using them to support teaching, teachers enabled learners to visualise and thus develop their geometric thinking. This activity is consistent with the van Hiele theory at level 1.

Content knowledge

The focus on content knowledge came out strongly in the findings, for example when teachers asked learners to list examples of shapes in the outside world. The learners linked traditional houses to shapes such as squares, rectangles and triangles as in the roof of a hut. The content knowledge assessed was based on the van Hiele levels 1 and 2.

Motivation

Teachers motivated learners to analyse typical shapes in terms of their properties. They also encouraged learners to describe a typical shape in terms of their properties, using their own words. In addition, they communicated clearly and effectively with learners through questioning, instruction, explanation and feedback.

Properties of shapes

Teachers asked learners to solve routine problems which required them to focus on the properties of the shapes. For example learners were asked to name the characteristics of an isosceles and an equilateral triangle. These activities given to the learners were consistent with the van Hiele levels 1 and 2 respectively.

In many of the key issues relating to teaching and learning at the van Hiele levels, the outcomes of this study correlate with the views put forward by other researchers in same field, as outlined in the Literature Review (chapter two).

With regard to teaching methodology, it was found that the teaching strategies employed by the selected mathematics teachers to facilitate the teaching and learning of geometry were reasonably consistent with the five strands of Kilpatrick et al.'s model in Kilpatrick et al. (2001:380), i.e.:

- conceptual understanding
- instructional routines
- strategic competence
- adaptive reasoning
- productive disposition

5.2.1 Conclusions with regard to conceptual understanding

It emerged from this study that conceptual understanding plays a major role in the development of geometric thinking. It involves the understanding of mathematical ideas and procedures as indicated by Olive (1991:42), who claims that learners use conceptual understanding of mathematics when they identify properties of shapes, define geometric concepts and compare shapes by their appearance or properties.

Drawing on the evidence from the classroom observations in this study it was clear that teachers understood and could explain the objectives of their lessons. Further in the observations, it was evident that teachers used the prior knowledge of learners, as when teachers called for ideas from the learners.

5.2.2 Conclusions with regard to instructional routines

This teaching proficiency is of vital importance and was demonstrated by the teachers who were generally sensitive to process and not only product. The teachers started the lessons well and they promoted interaction between themselves and the learners in teaching mathematics.

In all aspects of the facilitation, teachers discussed and clarified materials. They also assessed how learners interpret knowledge by engaging learners in practical activities.

The teachers chose materials and tasks which targeted the key concepts and procedures under consideration. The teaching was stimulating, enthusiastic and consistently challenging.

5.2.3 Conclusions with regard to strategic competence

The teachers came prepared for the lesson (with teaching aids, etc), which made it evident that they had planned their lessons. The teachers elicited responses from specific learners, communicated openly and showed genuine interest in learners (knowing learners names, interest, and backgrounds). They gave and monitored individual and group activities. The teachers gave clear instructions and guidance for activities which were followed by frequent, supportive and appropriate feedback.

5.2.4 Conclusions with regard to adaptive reasoning

Teachers adapted material to make it appropriate for a given group of learners and were flexible in adapting their teaching approaches according to learners' participation. Additionally, the teachers continuously engaged in analysing the difficulties their learners encountered in learning a particular concept and adapted their teaching accordingly. The teachers analysed how the learners responded to particular representations, questions, and activities, evaluated what the learners learned and incorporated it in their teaching.

5.2.5 Conclusions with regard to productive disposition

Teachers valued learners' contributions and they were positive and optimistic. The teachers were approachable and friendly which led them to be sensitive and fair. They were passionate about and patient in their teaching of mathematics. The teachers were also confident, enthusiastic and humble but they were firm and strict. The teachers were motivated and showed an interest in their learners.

5.3 POTENTIAL VALUE OF THE STUDY

As previously stated, the main thrust of this study was to explore to what extent grade 10 mathematics teachers facilitate the teaching and learning of geometry at the van Hiele levels 1 and 2. The potential value of the study lies in the identification of key issues which can inform geometry teachers how to use the van Hiele model to underpin their teaching and thus develop the geometric thinking, understanding and achievement of the learners

Evidence from this study illustrates how the participating teachers taught in ways consistent with the van Hiele levels and in keeping with the Kilpatrick et al.'s model. Key points are summarised as follows:

5.3.1 The visual level

Evidence to show activities at this level included observations of the following:

- The teachers displayed a variety of different ready made geometric shapes to the class
- The teachers asked learners to list examples of shapes in the outside world
- The learners were required to cut out shapes
- The learners were encouraged to recognise and identify figures and shapes

5.3.2 The analysis level

At this level the following activities were given:

The teachers

- introduced properties of shapes,
- used formal language to describe shapes,
- asked learners to list properties of shapes,

- asked learners to construct shapes according to their properties.

In this way teachers were able to promote an environment which stimulates communication and explanation, and provided a framework for the learning of geometric thinking.

5.3.3 Lesson planning

Research shows that in order to be successful with geometric thinking, learners should be able to visualise and touch shapes. This needs to be included in good lesson plans. This was shown in the lesson plans examined, and additionally there was evidence of planning according to the Kilpatrick model. For example, teachers evaluated the previous learning of the learners and incorporated that in their teaching plans. Furthermore activities were in line with the national assessment policy documents.

It was evident that all the selected teachers complied with the rules and regulations laid down in the syllabus. Their schemes of work were developed from the teaching syllabus while the lesson plans were developed from the scheme of work.

The overall conclusion is that the Grade 10 mathematics teachers in this study generally taught in such a way as to facilitate learning at the van Hiele levels 1 and 2. The teachers used appropriate strategies to facilitate geometry learning and their teaching aimed to develop the geometric thinking of the learners.

5.4 LIMITATIONS OF THE STUDY

As this study was a small scale case study, the research findings are not necessarily generalisable in a broader context. The availability of mathematics teachers willing to participate in this study was limited. Since only a small number of Kavango mathematics teachers were involved, the same results may not apply to other parts of Namibia, or even to other parts of the Kavango region.

There is also a possible limitation in terms of how fully participants responded in interviews and how they performed in the classroom. Given that the researcher works as an advisor in the Kavango region, the teachers participating in this research were all known to him. It is possible this influenced the responses given. The respondents may have felt the need to maintain a certain degree of restraint for the researcher.

To date, no literature is available on the application of the van Hiele theory in a Namibian context. Thus, there was no data to use for comparison within the Namibian situation.

The findings and recommendations must be viewed in the context of these limitations but nevertheless this study identified teaching strategies used in geometry classrooms in some schools of the Kavango region and demonstrated that these strategies were, by and large, consistent with the van Hiele levels and Kilpatrick et al.'s model.

Based on the very positive responses by the three teachers in this study about the importance of geometry, it would seem that they concurred with other researchers who emphasise the importance of geometry in the mathematics curriculum.

5.5 RECOMMENDATIONS

5.5.1 Recommendations for teaching practice

In order to strengthen the teaching within the context of the van Hiele levels 1 and 2, the following teaching strategies should be encouraged:

- Providing or displaying examples of ready made geometric shapes or of the properties of shapes (van Hiele level 1)
- Using both informal and formal language to describe shapes (van Hiele levels 1 and 2)
- Describing, recognising, naming and identifying shapes or a class of shapes in terms of their properties (van Hiele levels 1 and 2)
- Comparing and sorting shapes by their appearance or relationships between their parts (van Hiele level 2)

- Asking learners to construct shapes according to their appearance or properties (van Hiele levels 1 and 2)
- Explaining to learners how to recognise the difference between shapes (van Hiele level 1)
- Asking learners how to solve a certain problem by using known properties of shapes (van Hiele level 1)
- Asking learners to solve routine problems by operating on the shapes rather than referring to the properties in general (van Hiele level 1)
- Introducing a typical topic on properties of shapes (van Hiele level 2)
- Providing activities to learners which require them to focus on the properties of shapes or to use vocabulary appropriately (van Hiele level 2)
- Teaching learners how to identify and test relationships between parts of shapes, e.g. congruence of sides (van Hiele level 2)

5.5.2 Recommendations for teacher training

- All university and college lecturers involved in the training of mathematics teachers should be motivated and encouraged to facilitate the teaching and learning of geometry consistent with the van Hiele theory.
- Mathematics teachers should be equipped with the necessary knowledge of the van Hiele theory and the skills of the Kilpatrick model in their training. This would enhance the geometry content knowledge and teaching strategies employed in the geometry teaching and learning in the schools in general and in the classrooms in particular.

5.5.3 Recommendations for future research

Since no other research has been conducted on the van Hiele theory in the Namibian context, it is hoped that this investigation will act as a catalyst for further research. This study presents several research opportunities and the following recommendations are made:

- A bigger and more heterogeneous sample of mathematics teachers from school teachers to university lecturers is recommended for further research

- Similar studies should be conducted with mathematics teachers in other regions of Namibia
- Similar studies should be conducted across other grades in Namibia

5.6 PERSONAL REFLECTION

I have gained a unique experience through this research study at this university. I also found the exercise of conducting research very stimulating and adventurous. This included developing the ability to be open-minded and respectful of other views; to display optimism and enthusiasm, confidence and decisiveness; to understand setbacks that derail an important initiative; and be flexible and willing to try a different approach if the first effort runs into roadblocks.

5.7 FINAL WORD

In conclusion, there is a dearth of data on the van Hiele theory in the teaching and learning of geometry in Namibia. This is regrettable and needs to be addressed if the geometric thinking of learners is to be developed and Namibian schools gain a competitive edge in the teaching of mathematics.

Although extensive data on the application of the van Hiele theory is lacking, it is my opinion that the van Hiele theory should reflect a view of instructional design in the teaching profession. In this regard the van Hiele theory, complemented by the Kilpatrick model, should be taught to teachers and implemented in a wide range of authentic settings, with ample opportunities for reflection and feedback. In this way teachers in the field of mathematics could contribute towards the existing body of knowledge of these theories and thus assist in developing the geometric thinking of the learners and meeting the learners' learning needs.

I hope that the findings and recommendations for future research made in this study will inform many mathematics teachers in Namibia.

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Appendices

Appendix A: Formal letter to the Regional Director

Muyeghu A
P O Box 1037
Rundu
Namibia
16 April 2008

Ministry of Education

Kavango Education Region

Private Bag 2134

Rundu

Dear Sir

RE: Request to conduct a research study in schools of your region

I am an M Ed (Mathematics Education) part time student at Rhodes University (Student number: 07M6236) and currently conducting a research study on the topic: To explores to what extent selected Grade 10 mathematics teachers facilitate geometry learning and teaching that is consistent with levels 1 and 2 of the van Hiele theory of geometric thinking. I would be most grateful if you would allow me to use your schools of the Kavango region in Rundu as the research sites for the research report which I am required to write. If you agree to allow me to use your facilities the following schools have been selected to participate in the study sample, namely, Elia Neromba SS, Rundu SS, Dr AM Mbamba JS and Sarusungu CS. This exercise will kick off as from 19-23 May 2008. This letter serves to seek your permission to use the selected samples for my research exercise. Information collected through this exercise will be treated confidentially. Should you have any concerns or questions about this request, you can contact me at 081 257 7964.

Thank you for your assistance.

Yours faithfully,
Augustinus Muyeghu

Appendix B: Formal letter to the school

Muyeghu A
P O Box 1037
Rundu
Namibia
16 April 2008

The School Principal
………………………………………
………………………………………
Rundu

Dear Sir/Madam

RE: Request to conduct a research study in school

I am an M Ed (Mathematics Education) part time student at Rhodes University (Student number: 07M6236) and currently conducting a research study on the topic: To explores to what extent selected Grade 10 Mathematics teachers facilitate geometry learning and teaching that is consistent with levels 1 and 2 of the van Hiele theory of geometric thinking. This exercise will commence as from 19-23 May 2008. The assistance that I would like to ask of you includes arranging for me to:

- Interview Grade 10 Mathematics teacher and
- Observe him/her within the given period

Time schedule:

- From 06h30 in the morning to 17h00 in afternoon

I shall be grateful if you could co-operate with me in this very important matters. Information collected through this exercise will be treated confidentially. The school, teacher will be assured of anonymity to ensure that details are accurately recorded and reported. Should you have any concerns or questions about this request, you can contact me at 081 257 7964.

Thank you for your assistance.

Yours faithfully,
Augustinus Muyeghu (Mr)

Appendix C: Consent Form

Augustinus Muyeghu an M Ed (Mathematics Education) student is hereby given permission to use the following schools having been selected to participate in his research study namely, Elia Neromba SS, Rundu SS, Dr AM Mbamba JS and Sarusungu Combined School as the research sites for the research report he is required to write for the completion of his Master's degree. I understand that data for analysis will be collected from observations, document analysis and interviews with the selected Grade 10 mathematics teachers. This information from these participants may be used in the final report. I have been assured that my schools and my teachers will have anonymity in that report. The school principals concerned should avail venue on the date of interviews and observations as stated in his letter (19-23 May 2008). Finally you are requested to sign here as witnesses.

……………………………………………… ……………………..
The Kavango Regional Director of Education Date

……………………………………………… …………………….
The Principal: Elia Neromba Secondary S Date
(School A)

……………………………………………… ……………………
The Principal: Rundu Secondary School Date
(School B)

……………………………………………… ……………………
The Principal: Dr Alpo Mauno Mbamba JS Date
(School C)

……………………………………………… ……………………
The Principal: Sarusungu Combined School Date
(School D)

Appendix D: Interview schedule

Goal/Aim: Teaching of van Hiele levels 1 and 2 by Grade 10 mathematics teachers

Research site: ……………………………………………………………………………

Interview date:………………………. Interview number:…………………..

Interview starts:……………………… Interview ends:………………………

Interviewer:……………………………… Interviewee:………………………….

Introduction: The researcher introduces himself and provides the purpose of the interview. **Informed consent question**: Do you consent freely to participate in this tape-recorded interview?

1. How long you have been a mathematics teacher? Probes …
2. Do you enjoy teaching geometry lessons? Please elaborate
3. When you teach a typical geometry lesson, how do you introduce your lesson? Probes…
4. How do you teach a geometry lesson where your learners are encouraged to recognize or visualize figures or shapes? Please describe an example of such a lesson.
5. How do you teach comparing or sorting of figures or shapes? Please describe an example of such a lesson.
6. What, in your opinion, is the best way to describe shapes such as a square or a rectangle?
7. How do you teach your learners to distinguish the difference between a square and a rectangle? Provide examples.
8. How do you motivate your learners to analyse parallelograms in terms of their properties? Please provide an example.
9. How do you encourage your learners to analyse rectangles in terms of their properties? Provide examples
10. What hands-on activities do you do with your learners to establish the properties of shapes? Provide examples.
11. Can you please provide an example of how you teach learners to use properties to solve particular geometrical problems?
12. Is there anything else you would like to tell me about how you teach geometry?
13. Thank you very much for talking to me.

Appendix E: Observation schedule based on the van Hiele theory

Subject:……………………………………
Date:…………………………………
Lesson starts:……………………………… Lesson ends:…………………………
Teacher observed:…………………………
Observer:…………………………….
Grade 10:…………
Topic/Theme:…………………………………………………..
Objective/s:………………………………………………………………………

Observation schedule based on the van Hiele levels 1 and 2

Activities	Level	Weak	Moderate	Strong	Comments
The teacher displays a variety of different readymade geometric shapes to the class.	1				
The teacher asks learners to list examples of shapes in the outside world.	1				
The teacher describes shapes (example…).	1				
The teacher uses informal language to describe shapes.	1				
The teacher asks learners to identify and bring shapes from home.	1				
The teacher asks learners to cut out shapes.	1				
The teacher encourages learners to recognise and identify figures and shapes.	1				
The teacher teaches learners how to compare and /or sort shapes by their appearance.	1				
The teacher explains to learners how to recognise the difference between shapes.	1				
The teacher constructs, draws or copies a typical shape on the chalk board.	1				
The teacher asks learners to construct shapes according to their appearance.	1				

The teacher asks learners to solve routine problems by operating on the shapes rather than referring to the properties in general.	1				
Activities	**Level**	**Weak**	**Moderate**	**Strong**	**Comments**
The teacher introduces a typical topic on properties of shapes.	2				
The teacher uses formal language to describe shapes.	2				
The teacher asks learners to list properties of shapes.	2				
The teacher encourages learners to describe a typical shape in terms of their properties in their own words.	2				
The teachers compare two shapes according to their relationships between their parts.	2				
The teacher asks learners to construct shapes according to their properties.	2				
The teacher describes a class of shapes in terms of their properties.	2				
The teacher teaches learners how to compare and/or sort shapes by their properties.	2				
The teacher provides examples of the properties of shapes.	2				
The teacher teaches learners how to solve a certain problem by using known properties of shapes.	2				
The teacher provides activities to learners which require them to focus on the properties of the shapes.	2				

The teacher teaches learners how to empirically (i.e. by investigation) establish the properties of a typical shape.	2				
The teacher motivates and encourages learners how to analyse a typical shape in terms of their properties.	2				

Activities	**Level**	**Weak**	**Moderate**	**Strong**	**Comments**
The teacher teaches learners how to identify and test relationships between parts of shapes, e.g. congruence of sides.	2				
The teacher provides activities to learners which require them to use level 2 vocabulary appropriately.	2				

Appendix F: Observation schedule based on Kilpatrick model

School ……………………………. **Date…………………………………….**

Length of lesson…………………. **Teacher observed………………………**

Observer………………………….. **Grade 10…………**

Aims/objectives:………………………………………………………………………

……..………………………………………………………………………………….

Observation schedule based on Kilpatrick model of proficient teaching

CONCEPTUAL UNDERSTANDING	**Weak**	**Moderate**	**Strong**	**Comment**
The teacher is proficient in subject knowledge.				
The teacher values learners' experience.				
The teacher shares ideas and calls for ideas from the students.				
The teachers are gender sensitive/fair.				
The teacher is language proficient.				
The teacher displays knowledge about student development and learning.				
The teacher encourages cooperative learning.				
The teacher is resourceful and vibrant.				
The teacher uses prior knowledge of the students				
The teacher understands and can explain the objectives of the lesson.				

INSTRUCTIONAL ROUTINES	**Weak**	**Moderate**	**Strong**	**Comment**
Teacher is sensitive to process, not only product.				
Teacher listens to the students' contribution.				
Teacher performs calculations efficiently.				
Teacher promotes interaction between students in teaching mathematics.				

The teacher starts the lesson well.				
The teacher checks homework.				
The teacher knows how to respond to a student who gives an answer the teacher does not understand.				
The teacher knows how to respond to a student who demonstrates a serious misconception.				
The teacher has several ways of approaching mathematical problems. S/he is flexible in solving mathematical problems.				
The teacher dresses and behaves as a positive role model.				
The teacher monitors the previous homework properly.				
The teacher engages the learners in two-way conversation about the topic.				
The teacher gathers information by working with examples and of concepts.				
The teacher discusses and clarifies materials.				
The teacher learns how learners interpret the language.				
The teacher engages learners in doing activities.				
The teacher directs the path of exploring in such a way as to ensure that the student becomes familiar with specific key ideas related to the topic.				
The teacher ensures that students explore specific concepts.				
The teacher directs learners' activity by guiding them in appropriate explorations that are carefully structured, sequenced tasks in which learners manipulate objects so as to encounter specific concepts and procedures of geometry.				

The teacher chooses materials and tasks which target the key concepts and procedures under consideration.				
The teaching is stimulating, enthusiastic and consistently challenging.				
The teachers demonstrate acceptable English language proficiency.				

STRATEGIC COMPETENCE	**Weak**	**Moderate**	**Strong**	**Comment**
The teacher has planned the lesson.				
The teacher comes prepared for the lesson (teaching aids etc).				
The teacher is proficient in teaching methods/style.				
The teacher displays good time management.				
The teacher teaching decisions swiftly and efficiently.				
The teacher is creative and innovative.				
The teacher shows initiative.				
The teacher encourages participation.				
The teacher elicits responses from specific learners.				
The teacher communicates openly and shows genuine interest in learners (knows learners names, interest, backgrounds).				
The lesson is coherent (it flows smoothly).				
The lesson flows logically.				
The lesson is paced to allow the amount of time for each lesson component.				
Short well planned cryptic notes on a neat black board summarise the lesson.				
The subject content and learning tasks are presented in a clear and stimulating manner.				

The learning material demonstrates a logical progression with learners' prior experience and knowledge.				
The teacher communicates clearly and effectively with learners through questioning, instruction, explaining and feedback.				
The teacher integrates skills (reading, writing, spelling and speaking).				
The teacher gives individual and group activities or tasks				
The teacher uses a variety of activities or tasks (games, presentations, written work, role plays, practical work, etc).				
The teacher gives clear instructions and guidance for activities.				
The teacher monitors group/individual/pair work.				
The teacher gives frequent, supportive and appropriate feedback.				

ADAPTIVE REASONING	**Weak**	**Moderate**	**Strong**	**Comment**
The teacher figures out how to adapt material so that it is appropriate for a given group of students.				
The teacher is flexible in adapting his/her teaching approach according to students' participation.				
The teacher continuously engages in analysing the difficulties his/her students have encountered in learning a particular concept and adapts his/her teaching accordingly.				
The teacher evaluates what the students have learned and incorporates that in his/her teaching.				

The teacher engages in analysing how the students respond to particular representations, questions, and activities.				

PRODUCTIVE DISPOSITION	**Weak**	**Moderate**	**Strong**	**Comment**
The teacher values learners' contributions.				
The teacher is positive and optimistic.				
The teacher is approachable and friendly.				
The teacher is sensitive and fair.				
The teacher is passionate about mathematics.				
The teacher is patient.				
The teacher is confident and enthusiastic.				
The teacher is humble.				
The teacher is firm and strict.				
The teacher shows interest in the students.				
The teacher is motivated.				