Investigating how problem solving skills can be developed using a collaborative learning environment

by
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in partial fulfilment of the requirements
for the degree of

Master of Education
(Mathematics Education)

Rhodes University, Grahamstown
November 2013

DECLARATION

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DEDICATION

I dedicate this thesis to my parents Mr and Mrs Parekh, back in Leicester, UK for the endless sacrifices they made for my brothers and me. They ensured that we were educated and that we could attain our aspirations. My parents gave up many luxuries so that we could have all we needed to become the people we are today. Without their constant love and support, this would not have been possible.

ACKNOWLEDGEMENTS

I would like to extend grateful thanks to my supervisor Prof. Mellony Graven, for all her endless support and guidance especially in times of despair. I would also like to thank my family for always encouraging me to go on even when I felt like giving up. I could not have completed this mammoth task without all their help. I thank the learners who participated in this study both for their time and their willingness to share their thoughts with me in all aspects of the research process. I also thank my school for the permission granted to me to conduct this research.

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LIST OF ACRONYMS USED

ANA	Annual National Assessment
AMESA	Association for Mathematics Education of South Africa
CAPS	Curriculum Assessment Policy Statement
DBE	Department of Basic Education
DoE	Department of Education
ZPD	Zone of Proximal Development

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ABSTRACT

This thesis examines whether problem solving strategies develop and improve through working in a collaborative environment and, if so, how. The study explored the way peer-to-peer discussions which are focussed on finding solutions to mathematical problems might shape learners' attitudes and participation in mathematical problem solving. I use the Vygotskian (1978) socio-cultural perspective where the process of learning takes place within the Zone of Proximal Development (ZPD).

Polya's problem solving heuristics (Polya, 1973) and Kilpatrick's "Instructional Triangle" (Kilpatrick, Swafford & Findell, 2001) provided the analytical framework for the study. Seven grade 7 learners from a Ex-Model C school, volunteered to participate in the study. The data gathering process involved an initial problem solving assessment, a written questionnaire, observations and video recordings of the seven learners during a series of after school problem solving sessions and post intervention learner interviews.

The study showed that group discussion can have a positive impact on learners' problem solving in several respects:

My key findings point to:

- Mathematical communication does play a role in development of problem solving strategies.
- A more knowledgeable other, with regards to Vygotsky's (1978) ZPD and Kilpatrick et al's (2001) instructional triangle is a critical factor in the development of problem solving strategies.
- All five strands of Kilpatrick et al., (2001), strands for mathematical proficiency are required for correct solutions to be calculated.
- At times Polya's (1973) steps for problem solving move at a rapid pace and are difficult to notice. These steps develop at different speeds for different people.

CHAPTER 1: RATIONALE

RATIONALE FOR THE STUDY

Mathematical proficiency is at a crisis level in South African schools. Fleisch (2008), observes that 70% - 80% of school children from disadvantaged schools, finish primary school with very little basic knowledge and understanding of mathematics and have limited, if any, proficiency in even basic arithmetic. In contrast, children who attend relatively well resourced schools are more proficient and competent at mathematics and show a higher academic achievement. Responding to this crisis, teaching tends to focus on 'the basics' and 'higher level' problem solving is in danger of being ignored. The Department of Education report (2012) indicates that of all the grade 6 learners at schools across South Africa in 2000 only 2,1% were able to solve problems where information was extracted from tables, charts, etc. in order to solve multi-step problems. Of the abstract problem solving (which involves solutions that are algebraic or symbolic) the figure was only 1,3% in 2000 and 0,6% in 2007. This affirms my own experience over years of teaching that the vast majority of learners have weak mathematical problem solving skills. They are able to do other familiar and more routine and practised 'mathematics' with confidence; but are unable to master mathematical problem solving involving less familiar problems for which the solution or solution strategies are not immediately clear. This becomes particularly evident when they participate in the AMESA (Association for Mathematics Education of South Africa) maths challenges and other mathematics competitions.

From my experience learners are overly dependent on the teacher and this, combined with a fear of solving problems where the solution strategy is not immediately obvious, is problematic. Learners tend to give up if they are not sure of what should be done and thus the learners' habits, dispositions and attitudes towards maths problem solving are 'negative' or unproductive and can become obstacles to learning. I am aware that language may also be an added factor in learners' difficulty in interpreting a problem. Many of the learners at my school are second language English speakers while the language of instruction and the language of the mathematics problems they are given is in English. Problem solving usually requires more reading and understanding and involves several technical terms as well as instruction terms such as *add*, *simplify*, *find x, factorise* etc. Therefore interpreting what the problem is asking is more difficult for second language English learners. In the school where I teach English is a 2nd or even 3rd language for approximately 70% of the learners.

The focal point of my research will be to understand how learners problem solve and to explore the strategies they develop when provided with opportunities for peer-to-peer collaborative discussion. This research is driven by my wish to find ways to support the learners to improve their problem solving skills. In order to investigate learners' problem solving strategies and discussion I established a volunteer based after maths problem solving club.

My sample included seven grade 7 learners (which includes one volunteer who did not attend all the sessions) selected from the school where I work, which is a girls Ex-Model C primary school. I observed these learners in five after school sessions. While the focus of these five sessions was to understand how the learners' problem solve, I also wanted to observe how their problem solving strategies evolved and possibly improved through their interaction and engagement with the problems and each other. This progression over time is what I wanted to observe rather than only learners at one point in time, which is why my research spans five sessions.

From my teaching perspective I have noticed that some of the stronger performers of mathematics in tests and exams perform poorly in the AMESA Mathematics Challenges. This could be due to the fact that they are faced in these, with unfamiliar problems that require the generation of new strategies.

The following anecdotal vignette illustrates my motivation.

My story of two learners:

Since joining our school, two learners Sue and Sally (pseudonyms) have always performed very well in mathematics, obtaining either 1st or 2nd positions at the end of each academic year. Yet when they enter the AMESA Mathematics Challenges they perform poorly. Each year they were both encouraged to enter AMESA Mathematics Challenges. This encouragement was based on their teachers' expectations that they would do well. However, this was not the case. In fact they performed poorly, seldom getting enough problems correct to go on to the final round. My suspicions led me to consider that while they were strong on familiar types of problems, they were unable to interpret what should be done when faced with unfamiliar questions. While their calculation and basic number sense was strong, when faced with problems (such as the ones that appear in the AMESA Mathematics Challenges) they became frustrated and lost interest in participating. Their low performance in these challenges seemed to reinforce for them their sense that their participation was a waste of time. Since it was not part of the school curriculum the incentive to overcome the frustration and to find a way forward was not deemed necessary.

Each year our school participates in the AMESA Maths Challenge from grades 4-7. The AMESA Mathematics Challenges are written individually and learners are simply given their marks. If they get a sufficiently high mark they go through to write the second round. Since the Challenges are not part of the school curriculum there is little discussion by learners about the questions as well as little incentive to participate. My assumption of how learning occurs (discussed later) led me to wonder whether learners would benefit from discussing the problems with each other. Additionally the communication foregrounds the importance of problem. Thus CAPS (Curriculum Assessment Policy Statement) which forms part of the National Curriculum Statement (NCS) stresses the importance of developing problem solving skills. Its aims are mentioned below:

> *"The National Curriculum Statement Grades R-12 aims to produce learners that are able to*
>
> - *Identify and solve problems and make decisions using critical and creative thinking*
> - *Work effectively as individuals and with others as members of a team*

- *Organise and manage themselves and their activities responsibly and effectively*
- *Communicate effectively using visual, symbolic and/or language skills in various modes and the environment and the health of others*
- *Demonstrate and understanding of the world as a set of related systems by recognising problem solving contexts do not exist in isolation"*

(Department of Basic Education, 2011 p.5)

THE PURPOSE OF THE STUDY

The purpose of the study is:

1. To understand how learners problem solve (in the context of the Mathematics Challenge)
2. To understand why they struggle to problem solve.
3. To understand why they use the strategies they use.
4. To understand how attitudes towards and strategies used in problem solving might evolve given the opportunity for problem solving through discussion and interaction with peers.

The purpose of the study is therefore to investigate how learners' problem solve, the strategies they use and how such strategies might improve given the opportunity for peer-to-peer discussion of solution strategies to AMESA Mathematics Challenge problems. Social interactions including collaboration, co-operation and discussion provide stimuli for learners' internal meaning-making activity (Lerman, 2000). Lerman states that when the opportunity is given to interact with others, whether collaboratively or in peer-to-peer discussion, learners make more meaning of the activity and therefore provides opportunity for learning to improve.

According to the premise of the AMESA Mathematic Challenges the main aims of this challenge are to :

- *generate an interest in mathematics (to popularise mathematics)*
- *promote a broader perspective on the nature of mathematical activity, including that mathematical activity is more than calculating*
- *promote problem solving in mathematics education*

- *promote the perspective that the calculator is a useful and necessary tool in mathematical activity (the calculator cannot solve problems for learners)*
- *emphasise the importance of reading in mathematical activity*
- *provide a diagnostic tool to enable teachers to identify learners' misconceptions*
- *develop and disseminate materials that may contribute to meaningful mathematical activity in classrooms.*

  (AMESA, n.d.)

The problems that are presented in the AMESA Mathematics Challenges are thus informed by these aims. The aims also state that the organisers wish to 'popularise' mathematics and generate an interest in the topic. Locally Graven and Schafer (in press) argue that participation in such challenges provide numerous opportunities for developing a love of playfulness with mathematics. This is similar to Kilpatrick et al. (2001) who talk of encouraging a learner's disposition and inclination towards mathematics. As problem solving is the central focus of the AMESA Mathematics Challenges it seems reasonable that I choose it as the context, and the problems it presents as a basis for my research into problem solving. My research focuses on four key research questions:

RESEARCH QUESTIONS

1. What is the nature of the six grade 7 learners' problem solving strategies they applied to the mathematical problems included in the AMESA Mathematics Challenges?
2. How do learners' problem solving strategies develop (if at all) through working on AMESA Mathematics Challenge problems collaboratively?
3. What is the nature of the learners' attitudes towards the mathematical problems included in the AMESA Mathematics Challenges?
4. In what way, if at all, might peer-to-peer discussions, focused on finding solutions to mathematical problems, shape learners' attitudes and participation in mathematical problem solving?

In the next chapter I discuss the framework and literature consulted in order to address these critical questions.

CHAPTER 2: THEORETICAL PERSPECTIVE, CONCEPTUAL FRAMEWORK AND LITERATURE REVIEW

THEORETICAL PERSPECTIVE ON LEARNING

As indicated in chapter 1 and based on my teaching experiences my assumption about learning is that knowledge is constructed through interaction with others and meaningful knowledge is acquired when the individual is engaged in social activity. These assumptions underpin a socio-constructivist theory of learning (Vygotsky, 1978). A socio-constructivist perspective implies that social learning situations where interaction and communication is taking place enable and promote learning. In my study this 'interaction' and 'communication' will be enabled through the creation of a collaborative and co-operative learning environment in the form of an after school voluntary problem solving club.

Vygotsky (1978) states that learning is a 'social' process and thinking skills are formed through a 'shared' process through social interaction. Learners are able to perform tasks that they would not otherwise be able to do by having an adult or a 'more knowledgeable other' to engage with. This is explained quite clearly in Vygotsky's model of the Zone of Proximal Development (ZPD). This refers to the level of development that can be attained when children are engaged in social behaviour, interaction, collaboration and discussion. The ZPD is the distance between what the child can do by him/herself independently and what the child is capable of attaining with the guidance of an adult or in conjunction with other more capable peers. This assumption of the value of peer-to-peer learning and collaboration and interaction resonates with some of the key literature on collaborative learning and mathematical problem solving discussed in the next section.

Cohen and Ball (2001) state that the interactions among teachers and students in the context of education are important in cognitive development. In this model of learning, as in Kilpatrick et al.s' instructional triangle (Kilpatrick et al., 2001). Instruction is defined as the interaction amongst the three elements; the teacher/adult, the students and the mathematical content. Hiebert (cited in Nipper and Sztajn, 2008) suggests that the instruction is moulded by the knowledge of the teachers and students and interaction that takes place between them.

If peer-to-peer discussions support learning in solving mathematical problems then my interest lies in what conditions exist that account for this learning. Three conditions for co-operative working and learning are highlighted by Johnson, Skon and Johnson (1980):

1. Within the workings of a co-operative group, superior cognitive problem solving strategies are being developed – maybe as part of the interaction and discussion that takes place whilst trying out strategies and the discussion that follows if strategies do not work.
2. The exchange of information via discussions between the higher achievers and low/middle achievers benefits both parties; in particular the low/middle achievers perform better.
3. The incentive to work and find solutions is increased by support and encouragement from peers. This is particularly important, as often the low achievers tend to give up and not even try as their disposition is so negative towards mathematical problem solving.

These conditions for co-operative learning tie in with Vygotsky's (1978) ZPD whereby through discussion with a 'more knowledgeable other', in this case some-one who is a peer, the cognitive levels of problem solving can be fostered and improved. However, point 2 above, suggests that all parties learn. This learning by all is due to the way in which the social learning situation enables the learners to articulate and reflect on their strategy and thinking from a meta-level. Such reflection is powerful and many mathematics teachers say they have learnt more mathematics from teaching it than in all their years of study. The need to reason, articulate and explain a mathematical concept leads to a greater understanding of the underlying rationale of that concept. In this respect my study takes a participatory rather than an acquisitionist view to learning (Sfard, 1998). Sfard comments that a participatory view would involve the learner being actively involved, a participant in the process of gaining knowledge which is what occurs in my research where the learners are actively involved and participating through peer-to-peer discussions.

Kilpatrick et al. (2001), also working within a social constructivist perspective, talk about an 'instructional triangle'. In this they suggest that the view of teaching and learning of mathematics is a product of interactions between the teacher, the learner and mathematical content. This is shown in figure 1 below:

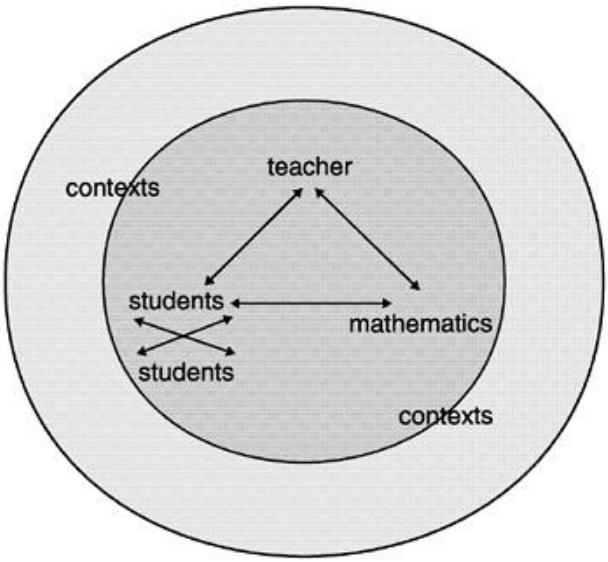


Figure 1. Kilpatrick's Instructional Triangle

Here interactions (discussions, collaborative co-operative working) exist between students, the teacher and the mathematical content. Thus in a collaborative learning environment, the students would have the opportunity available to them to discuss and engage with other students and also the teacher if necessary. Studies, for example Johnson et al., (1980) have shown that a collaborative environment leads to higher achievement of goals for individuals within the group for problem solving, especially because interaction and exchange of information is taking place and is a vital component of understanding.

The 'instructional triangle' as illustrated in figure 1 refers to the peer-to-peer (student to student) mathematical discussion made available through collaborative learning. This is but one part in the 'communication' that will take place; another would be what the learners actually produce in writing as a result of that peer-to-peer discussion. Both forms of communication form the unit of analysis of my study.

Studies done by Johnson et al., (1980) which look at the comparison of performance by learners who work in different situations / environments on problem solving (i.e. working individually or co-operatively) had very different results both in terms of success (in this case solving the question) and in developing a productive disposition towards mathematics, depending on in which situation they worked.

Johnson et al., describe a co-operative environment as one where all members of the group work together to achieve the same goal and a competitive environment would be one where the learners are working alone. They suggest that learners who work in a co-operative environment bring about higher achievement of goals (i.e. successfully solving the question)

for individuals than those learners who work in a competitive environment. This is because when working in a co-operative environment where peer-to-peer discussion is acceptable then successfully solving the problem makes them more inclined towards a positive "I can do this" attitude. On the other hand, learners within a competitive environment will need to be quite successful early on in order to feel the same way. Additionally if a learner is unable to solve the problem they may feel as though they have failed and if this happens often enough they develop a negative disposition as in the case of the two girls in my earlier vignette. The importance of some form of collaboration in a learning environment relates well with Vygotsky's ZPD (1978). This is defined as:

> *"the distance between the actual developmental level as determined by independent problem solving and the level of potential development through problem solving under adult guidance or in collaboration with more capable peers."* (p.33)

Conceptual Framework for Mathematical Proficiency

Kilpatrick et al. (2001), state that certain mathematical strands need to be achieved for mathematical proficiency to be attained. Mathematical proficiency is made up of five strands. These strands are interlinked and interwoven and do not work in isolation. The strands identified by Kilpatrick et al. (2001) which would enable mathematical proficiency in problem solving are as follows:

- Conceptual understanding – this is the understanding of the concepts such as the four basic operations, and relations.
- Procedural fluency – this refers to the skill, flexibility, and accuracy with which mathematical procedures are carried out.
- Strategic competence – this refers to the ability to formulate ideas, represent and solve mathematical problems
- Adaptive reasoning – the ability to reflect on outcomes, possess logical thought and explain and justify findings
- Productive disposition – refers to the inclination of the individual to see mathematics as sensible, useful and worthwhile, i.e. their attitude.

Kilpatrick et al. (2001) argue that if a learner is to do well and develop a proficiency in mathematics and therefore conceptual understanding, procedural fluency, strategic competence and adaptive reasoning then the learner already has a firm belief that

mathematics is worthwhile and therefore can be said to have a positive productive disposition. This strand of mathematical proficiency develops when the other four strands are developed. In essence the more conceptual understanding they have the more procedural fluency they possess and the more adaptive reasoning they have then the more they are able to make sense of mathematics and therefore have a better disposition towards mathematics. They go on to suggest that those learners who only see mathematics as a fixed entity with the tests in place to measure their ability in mathematics are unlikely to tackle challenging problems. These learners are more likely to be disheartened and therefore more likely to fail. Learners need to see mathematics as their chance to learn and should be encouraged to utilise their skills in all five strands in order to become what Owen and Sweller (1989) term 'experts' at problem solving. For Kilpatrick et al (2001), a mathematical productive disposition refers to,

> *"the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics. If students are to develop conceptual understanding, procedural fluency, strategic competence, and adaptive reasoning abilities, they must believe that mathematics is understandable, not arbitrary; that with diligent effort, it can be learned and used; and that they are capable of figuring it out."* (p.131)

Similarly Carr & Claxton (2002; 2004) refer to the '*notion of steady effort of work"*. They argue that this is important in developing resilience in the learning disposition of learners. Carr & Claxton define resilience as the

> *"inclination to take on (at least some) learning challenges where the outcome is uncertain, to persist with learning despite temporary confusion or frustration and to recover from setbacks or failures and rededicate oneself to the learning task."*(p.14)

My two learners in the anecdotal vignette, Sue and Sally, are both generally proficient in mathematical conceptual understanding and procedural fluency, but less proficient in terms of strategic competence and adaptive reasoning. Additionally, while their disposition towards school maths is productive in Kilpatrick et al.'s terms their disposition towards AMESA Mathematics Challenges is not. Since all five strands are considered necessary to enable mathematical proficiency in problem solving, the framework enables me to unpack which

strands might be under-developed and thus are likely to be leading to difficulty with problem solving.

According to Kilpatrick et al. (2001) problem solving would particularly require the strands of strategic competence and adaptive reasoning that is;

> "*the ability to formulate mathematical problems, represent them and solve them"*
>
> and
>
> "*the capacity to think logically about the relationships among concepts and situations"* respectively.

Here the learners will be required to try and formulate an idea or strategy that will be suitable to solve a problem; they will also need to be able to justify their solution as plausible and reasonable. The extent which learners are able to do this with ease will influence their disposition towards mathematical problem solving.

Part of my research interest relates to the attitudes towards mathematical problem solving, which Kilpatrick et al. (2001) clearly states is the 'worthiness' of maths or the 'productive disposition' of the learner. My interest is specifically in exploring the relationship between how the failure/success in developing solution strategies for mathematics problems such as those in the AMESA Mathematics Challenges leads to negative/positive attitudes which relate to Kilpatrick et al.'s productive dispositions (2001). In the case of repeated failure at problem solving I have noticed that learners tend to lose the inclination to even try and solve problems. If a learner does not see mathematical problem solving as a mathematical activity that has worth or if they find problem solving so difficult and taxing cognitively that they feel that it is beyond their grasp then he/she could lose interest in problem solving. This links to Vygotsky's notion of the ZPD where problems would need to be within the zone of the learner rather than way beyond.

LITERATURE REVIEW

CONCEPTUALISATION OF CO-OPERATIVE PROBLEM SOLVING

Davidson and Kroll (1991) define co-operative learning as any learning that takes place through the interactions within small groups of learners. This interaction is in the form of shared ideas to complete academic tasks. Co-operative learning in mathematics can be used to practice skills, discover new concepts, problem solving and discussion of concepts.

Davidson and Kroll (1991) found that co-operative learning was far more likely to be effective in promoting achievement and productivity than competitive and individualistic learning environments. Competitive individualistic learning environments are ones where only the individual benefits from goal achievement (correct solution) rather than the whole group as in the co-operative learning environment. They go on to argue that where differences in achievement do exist, they seem to be supportive of the small group co-operative environment as opposed to an individual working environment. So this suggests to me that learners working in small groups benefit from the interaction and communication that happens in a peer-to-peer collaborative environment. Cobo and Fortuny (2000) state that a co-operative learning environment is not enough and the *nature* of co-operation and interaction is just as important a factor as the environment itself within which learning takes place.

Thus a challenge in this respect is to develop learners ways of engaging and discussing that enable and support learning. Research conducted by Johnson, Maruyama, Johnson, Nelson & Skon (1981) indicated that a co-operative learning environment is more effective than a competitive environment especially in terms of goal achievement. Within a co-operative learning environment each member of the group (in my research each member of a pair) plays a part in its final outcome ie. The problem is solved correctly or it is not solved correctly. All members of this group/pair benefit especially the low achievers who benefit from the interaction and communication with a more able member.

I aim to examine what the features are that enable or hinder the improvement of problem solving skills, looking specifically at a collaborative, co-operative learning environment which will include a community of practice where a sense of belonging is developed.

I also aim to examine what the learners' disposition is with regard to mathematical problem solving and how this may or may not evolve through collaborative working. This will be undertaken within a grade 7 after school maths club that focuses on problem solving. My

rationale for this is as discussed above is that from my own experience of teaching mathematics, maths problem solving skills are weak and the learners have a very negative disposition towards problem solving.

The literature discussed above makes assumptions about the way learners' problem solve, i.e. through collaborative and co-operative teaching methods. I hope to investigate the strategies that are needed to become 'expert' problem solvers. The literature suggests that problem solving skills can be improved through collaborative and co-operative learning environments such as an after school maths club. I also investigate whether disposition and social relations are supportive in developing problem solving skills.

LOCAL PERSPECTIVE AND NATIONAL CONTEXT

Generally mathematics performance of learners in the South African context is not good (Department of Basic Education, 2012). This is shown by the Annual National Assessment (ANA) results in the table below:

Table 1: Average National Mathematics ANA results

Grade	Mathematics 2012	Mathematics 2011
1	68	63
2	57	55
3	41	28
4	37	28
5	30	28
6	27	30

This table indicates the dire straits of mathematics proficiency in South Africa. A closer study of the table reveals that as the learner progresses up to the next grade their level of proficiency, according to the ANA's, deteriorates. This is shown by the average percentage figures indicated in the table declining from 68% to 27% from grade 1 to grade 6 respectively in 2012. It seems hard to imagine that the average learner in grade 6 knows and understands only 27% of the work that they are supposed to. This situation even worse when we take a look at the percentage of learners who obtained at least 50% in the mathematics ANA's:

Table 2: National Percentage of learners who obtained at least 50%

Grade	2012	2011
3	36	17
6	11	12

As can be seen from the above table in 2011 only 12% of the learners who sat the ANA's that year achieved 50% and above. This figure drops in 2012 where the figure is only 11%. If we look closer still at the provincial level (shown in the table below), the picture is even bleaker for the Eastern Cape with only 8.1% of learners achieving 50% or more.

Table 3: Average of Grade 6 Mathematics ANA's by province

Province	Average mark (%)	Percentage of learners achieving 50% or more
Grade 6		
Eastern Cape	24.9	8.1
Free State	28.4	11.7
Gauteng Province	30.9	16.4
Kwa-Zulu-Natal	26.1	11.8
Limpopo Province	21.4	4.6
Mpumalanga	23.4	5.7
Northern Cape	23.8	7.6
North West	23.6	7.1
Western Cape	32.7	19.9
National	26.7	10.6

These figures indicate that at the provincial level the Eastern Cape falls under the national averages for grade 6 mathematics ANA's. The average ANA results for 2012 in some of the districts in the Eastern Cape are as follows:

Table 4: Average ANA results by district

District	Grade 6 Average Mathematics ANA results (%)
Butterworth	29.4
Cradock	23.5
Dutywa	25.7
East London	29.4
Fort Beaufort	26.9
Graaff – Reinet	23.5
Grahamstown	25.4
King Williams Town	24.9
Port Elizabeth	26.4

Concern about these results especially in the Grahamstown area, provide further motivation for my research. They indicate that nationally the standard of mathematics is very poor and provincially the Eastern Cape and Grahamstown is even weaker.

At my school the average Grade 6 mathematics ANA result for 2012 was 48%. Although this is much higher than the national and provincial average there is room for improvement. Of greater concern is the very poor performance at Mathematics Challenge level. Performance on the AMESA Mathematics Challenge, seem to bear no relation to the relatively good overall ANA results. The learners at the school generally do very poorly and few learners progress to the next round. This seems to indicate that even though the learners are able to do 'classroom maths' that require application of work covered they are unable to do unfamiliar questions with non-routine application of concepts such as those in the mathematics Olympiads and in the AMESA Mathematics Challenges. These Challenges are the focal point of my research. The types of questions in the AMESA Mathematics Challenges are 'out of the box' problem solving questions some of which require a multi-step strategy before a solution can be found. So before I go further it is important to define how a problem is conceptualised for this study.

CONCEPTUALISING PROBLEM SOLVING FOR THIS STUDY

Polya (1981, p.117), describes a problem in everyday terms that can easily be applied to mathematics problems:

> *"Getting food is usually no problem in modern life. If I get hungry at home, I grab something in the refrigerator….it is a different matter if the refrigerator is empty… in such a case getting food becomes a problem".*

Polya (1981) states that a problem becomes a problem when we have to 'search consciously for some action appropriate to attain a clearly conceived, but not immediately attainable aim' rather like the AMESA Mathematics Challenges. This suggests that a problem only becomes a problem when the outcome or solution is not immediately obvious and some 'searching' or solving is required. A problem becomes a major problem if it is indeed very difficult and the solution pathways are not clear and is a little problem if the solutions are easily found or immediately evident. Polya (1981) goes on to state, that a 'problem', in essence, must have some level of difficulty.

Jonassen (2000, p. 65) defines a problem as:

> *'An unknown entity in some situation (the difference between a goal state and a current state). Those situations vary from algorithmic maths problems to complex social problems... Finding a value of the unknown entity must have some social, cultural or intellectual worth".*

He describes problems as being well-structured or ill-structured. A well-structured problem is one that is constrained by the content and therefore has pre-determined values and solutions which are clearly defined within the content knowledge required. An ill-structured problem is usually one that is encountered in everyday life; it usually does not have specific content knowledge requirements. Solutions are highly unpredictable. He argues that problem solving in schools is usually well-structured and has pre-determined answers or content knowledge that the teacher wishes the learners to explore.

Kotzé and Strauss (2007) identify eight levels of understanding that a learner has in relation to specific mathematical competencies, this is an extension of Rasch's (citied in Kotzé & Strauss, 2007) 6 level model. The levels are as follows:

Levels of Difficulty of questions

Table 5: Levels of difficulty of questions

Level	Description
1	Pre-numeracy
2	Emergent numeracy
3	Basic numeracy
4	Beginning numeracy
5	Competent numeracy
6	Mathematically skilled
7	Problem solving
8	Abstract problem solving

For the purpose of this research I am interested in levels 5 to 8, as these are the ones that are commonly represented by the types of questions that appear in the AMESA Mathematics Challenges and Olympiads. Level 5 and 6 is where (according to our school's Grade 7 ANA results) most of the learners are at. They are able to perform multiple operations. At this stage the sequence and the ability to manipulate the information is important (Kotzé & Strauss, 2007). This is the level at which that most of the 'classroom' mathematics problem solving in my school is pitched at. For example:

Use the price list to find the total cost of 5 litres of milk, 3 kg of tomatoes and 2 chickens. (Circle the correct answer)

Price List		
Item	**Unit**	**Cost (R)**
Milk	1 L	0,60
Tomatoes	1 Kg	0,50
Chicken	each	6,70

A R7,80 B R16,90 C R17,90 D R18,90

Figure 2: Kotze and Strauss, 2007

This, although a problem in the sense that the solution is not immediately obvious, is a fairly well-structured problem in that there is only one solution and solution path and the content knowledge is pre-determined by the teacher.

Most of the AMESA Mathematics Challenge questions are at levels higher than this; they are at level 7 and 8. These levels require different skills and strategies to be used. For example at level 7 the learner is required to extract information from tables, charts visual and symbolic presentations before they can solve problems that require multiple steps. For example:

The table represents a relation between *x* and *y*. What is the missing number in the table?

x	1	2	4	5
y	3		9	11

A	B	C	D
4	5	6	7

Figure 3: Kotze and Strauss, 2007

Here the learner is required to first extract the information from the table. (I.e. the value of the first term in this sequence is 3, the fourth term is 9 and the fifth term is 11). Using the same pattern find the missing term.

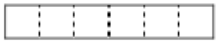
At level eight, the learner needs to be able to identify the:

> *"Nature of an unstated mathematical problem embedded within verbal or graphic information"*. (p. 27)

The learner is required to transform the information presented in the question to such an extent that it can now be approached using a mathematical interpretation. Once this is done the learner needs to identify mathematical strategies that will lead to a correct solution. At this level the learner is required to solve a problem for which the skills, strategies and mathematical knowledge are not immediately obvious.

Example

A figure consists of 6 equal squares. The area of the whole figure is 384 cm^2. Find the perimeter of the figure.



A	B	C	D
64 cm	81 cm	112 cm	192 cm

Figure 4: Kotze and Strauss, 2007

This question for example requires the learner to not just know what a perimeter is but also the relationship between area and perimeter.

Kantowski (1977) states that it is generally accepted amongst mathematics teachers that the development of the ability to solve complex, non-routine problems is an important aim of mathematics instruction and teaching. A learner is faced with a problem when they come across a question that cannot be answered easily, because the knowledge is not immediately available to them. The learner is then forced to think of alternate ways to solve the problem.

Wickelgren (1974) defines a problem as being comprised of three different types of information that are required to be known in order to solve a problem. These three being:

- Information about the 'givens' - a set of expressions that we consider to be present in the problem at the start of the problem.
- Information regarding the operations that shape one or more expressions into new expressions. This refers to the actions that need to be performed on the given expressions, in order to move closer to the solution.
- Information regarding the goals – in its' simplest form, this refers to getting the answer.

In the AMESA Mathematics Challenges, the learner is required to find from all the information provided in the question, the particular information that will help him/her. They

are then required to think about the operations and steps that they would need to take in order to find a suitable solution to the problem posed.

I have chosen the AMESA Mathematics Challenge questions for my study as I have a working knowledge of them and the learners at my school and the learners in my research group will have some experience of them since they begin writing these in Grade 4.

The nature of the mathematics challenges are such that the questions are aimed at conceptual understanding, its application, problem solving, reasoning, communication and general mathematical thinking. So it can be seen that the nature of the challenges incorporate the need for proficiency in Kilpatrick et al's (2001) five strands. The questions are designed in such a way that they address different concepts of mathematics which learners are not exposed to at school with the traditional curriculum that we follow. The organisers of the AMESA Mathematics Challenges follow a matrix as a guide in designing questions for the challenges, this matrix is illustrated below:

Type of questions	Non-creative	Creative
Easy	No	Yes
Difficult	No	No

Figure 5: Types of questions used in AMESA Mathematics Challenges - a guide

According to the figure above, the organisers will actively try to design questions that are easy but creative rather than difficult and non-creative.

The following is an example of a difficult non-creative question that the organisers would try to avoid:

$$\text{Calculate: } \frac{3}{5} \text{ of } (\frac{7}{8} \div 2,7) - 13,2\% \times 0,42 \times (\frac{5}{12} + \frac{7}{11})$$

Figure 6: (AMESA n.d.)

Whereas the following question is an example of a relatively easy creative question:

This flag has 7 regions. You want to colour the flag so that no two touching regions are the same colour. What is the least number of colours you need?

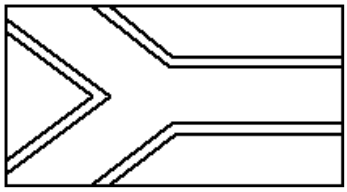


Figure 7: AMESA first round grade 6 paper 2001

The objectives of the AMESA Mathematics Challenges are to give the learners experience of such non-routine questions, the more experience they have of such problems and questions the more successful they will be in the challenge. (AMESA, n.d.)

Polya (1973) refers to the word 'heuristics' for understanding the processes that a learner goes through, whether in their head or on paper, before a problem is solvable. He states that there are four phases or steps that a learner goes through before they are able to solve problems. I elaborate on these as they will provide a useful framework for describing and analysing my data:

- Firstly the learner needs to understand the problem, understand what the problem is asking, and see clearly what is required.
- Secondly the learner would need to devise a plan of action, by seeing how the various parts of the question are connected. Devising a plan can only happen when the learner knows at least in principle which calculations need to be performed in order to obtain the solution.
- Thirdly, once a plan has been made, it now needs to be carried out. This will include the use of knowledge that has been acquired previously and good mental arithmetic skills. To carry out a plan is much easier than to devise a plan.
- Finally, calculations have been made, and solutions need to be checked to ensure calculation errors have not been made and to check the sensibility of the solution. This takes place in a problem solving cycle. If checking the solution leads to a rejection of it, then it is necessary for the learner to revisit their working and strategy used and possibly restarting from scratch.

I will explain the problem solving cycle using the example: a piece of land measuring 128m by 100m is to be made into soccer fields. How many soccer fields measuring 80m by 40m can be made on this piece of land? If learners use the 'area' formula and divide total area by the area of each soccer field, then they will get a solution of 4 soccer fields. But logistically and realistically, four soccer fields can't fit onto the land (as they would be the wrong shape). The learner could realise this when assessing the solution by trying to draw the fields onto the land. The learner will then have to re-visit the original problem, come up with a new plan of action, carry out this new plan and evaluate once again. Thus the problem solving cycle can be revisited several times. The problem designers of AMESA Mathematics Challenge questions often use these types of questions with figures that entice the learner to use the standard algorithms or formulaic interpretation but then do not provide the correct solutions.

Heuristics have been explained by Schoenfeld (1985) as:

> *'Rule of thumb strategies that are used for successful problem solving, general suggestions that help an individual to understand a problem better or to make progress toward its solution'* (p. 23)

This implies that a learner needs to have a set of strategies to follow as they try to find the solution, very similar to Polya's four steps as mentioned above.

Hardin (2002) defines problem solving and problems as having certain characteristics.

Each problem has a certain characteristic in terms of how they are defined for example; are the problems well defined? Do they have any desired output or a set of actions that need to be followed in order to be solved? Do they have fruitful and meaningful solutions?

The knowledge that is required to solve the problems, is it 'declarative knowledge'? i.e. knowledge that relates to facts e.g. area of a square is length of side squared. Or is it 'procedural knowledge'? i.e. knowing how to do something and it includes motor skills, cognitive skills and cognitive strategies.

We can go on to state specifically that mathematical word problems (or story sums as referred to by many primary school teachers) have particular characteristics. These being, as defined by Verschaffel, Greer and De Corte (cited in Sepeng & Webb, 2012, p. 1) as 'textual descriptions of situations' which can then be used to contextualise mathematical questions. In their study Sepeng and Webb (2012) argue that the use of mathematical discussion in the classroom to assist learners with mathematical reasoning and application is very important. My experience indicates however, that discussion is only useful if the discussion is planned in order to act as a platform from which support is provided for the learners. If learners are no longer required to just follow algorithms and other procedural rules, but are instead required to use reasoning and thinking to figure out for themselves the paths that lead to solutions then, through mathematical discussion and collaboration learners might be more willing to try to 'problem solve' and find the appropriate strategies.

Much of what learners do and what learners know is based on their own personal prior experiences (Posamentier & Krulik, 2008). This suggests that the level at which the learners are able to solve problems will undoubtedly vary with the individual. Problems by their very nature hardly ever have just one strategy for the correct solution. Some problems lend themselves to a wider variety of solution methods, as a result the learners need to be

encouraged to try to use alternate solutions to a problem. This could mean listening to a classmate and comparing strategies. This can be facilitated using a co-operative peer-to-peer learning environment.

CHAPTER 3 - RESEARCH METHODOLOGY

INTRODUCTION

The Empirical Field and Sampling

My study was a case study of seven grade 7 learners voluntarily participating in a series of five after school sessions that focused on discussion and collaboration in problem solving. A case study has several characteristics which fulfil my needs as mentioned earlier. I ran these sessions and as I am the HoD of Mathematics at the school and also teach the entire grade 7 class Mathematics, this was a purposeful sample.

I chose the grade 7 pupils as the entry point of AMESA Mathematics Challenges is grade 4 so these learners would have had 3 years of experience in participating in these Challenges. My research sample was seven grade 7's (one of whom participated on an ad hoc basis) who voluntarily agreed to participate in these after-school sessions.

RESEARCH METHODOLOGY

> *"Qualitative data analysis involves organizing, accounting for and explaining the data; in short, making sense of data in terms of the participants' definitions of the situation, noting patterns, themes, categories and regularities."*
>
> (Cohen, Manion, & Morrison, 2007, P.461)

My research framework and methods chosen point to a qualitative research method with quantifiable aspects such as learner scores on problem solving activities. These methods include interviews, questionnaires, observations of learners working and communicating, video recordings of club sessions and field notes.

I have chosen a case study approach for my study. A case study has several specific characteristics, (Cohen, Manion & Morrison, 2007. p253). Those characteristics most pertinent to this study are listed below:

- It looks specifically at an event which contains rich and vivid data pertaining to the study i.e. strategies that may or may not be developed for problem solving
- It allows for a step by step chronological narrative of the events that pertain to the study

- It highlights and draws attention to specific events that occurred that are relevant to the study.
- Finally it allows for the fusion of the description of the events together with the analysis of the qualitative data obtained.

Qualitative research is used to try and understand and make sense of incidences that have occurred from the participants' standpoint (Cohen, Manion & Morrison, 2007).

The methods I chose to collect information and data are outlined below:

- Baseline problem solving assessments

  These sessions were in the form of an individually administered problem solving test used to gain background information on learners and their problem solving abilities

- Questionnaires These were designed to ascertain the learners' disposition towards mathematical problem solving, also to find out a little about the views that the learners themselves hold on problem solving and their beliefs about their own problem solving abilities

- Interviews

  These were semi-structured interviews designed to delve deeper into the thinking behind their attitudes regarding mathematical problem solving.

- Video recordings and observations of sessions with selected transcription.

  These were repeatedly viewed and critical incidents were transcribed for more detailed analysis.

These multiple methods of collecting data allowed for rich data with a thick description which permitted triangulations to make valid and creditable conclusions.

Research Process

The process involved selected learners from my school. This process can be broken down into five main stages;

Stage 1: Baseline data collection
Stage 2: Individual disposition questionnaire
Stage 3: Gathering video and observation on collaborative problem solving
Stage 4: Post club assessments

Stage 5: Post club session interviews

STAGE 1: BASELINE DATA

Problem Solving Assessment

Once these learners were identified they were assessed using a sample of 10 questions to establish baseline individual problem solving abilities. Baseline assessment was conducted and scores collated. For the baseline assessment questions were taken from a grade 6 2012 final round AMESA Mathematics Challenge paper. Scripts were marked and these provided me with the baseline scores and confirmed the range of performance of the 2012 scores.

STAGE 2: INDIVIDUAL DISPOSITION QUESTIONNAIRE

After the individually administered baseline assessment the learners were asked to complete a questionnaire designed to discover and ascertain their attitude and experiences of previous participation in AMESA Mathematics Challenges. The questions for the questionnaires were taken from Graven's (2013, p.55) instrument given below:

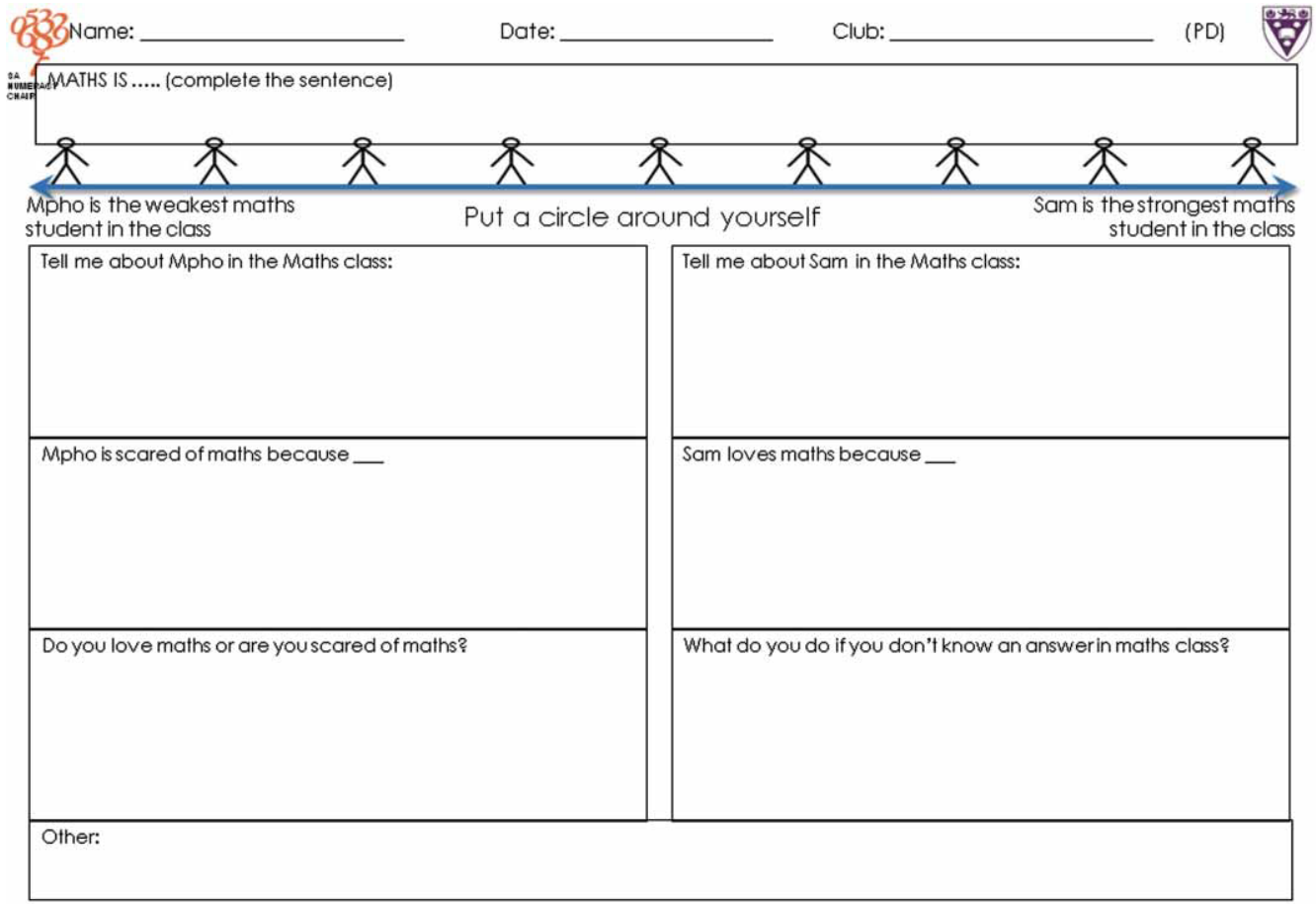


Figure 8: Graven et al'., disposition instrument (Graven, 2013, p.55)

Graven (2012) and Graven, Hewana & Stott (2013) use the questions in her instrument to gather information pertaining to the mathematical dispositions of learners. I adapted many of the questions so that they specifically pertained to mathematical problem solving, both their perceived ability and disposition towards it, see Appendix 1. Thus for example I said *"Mpho is scared of problem solving because...."* and *"Thando enjoys and loves problem solving because..."*. Previous performance histories were obtained from school records. This provided me with background information for the learners in my research, refer to appendix 1.

STAGE 3: GATHERING VIDEO DATA AND OBSERVATION DATA ON COLLABORATIVE PROBLEM SOLVING

During the after school sessions I focused on problem solving strategies, providing an opportunity for active engagement, open discussion and collaboration in order to observe how learners problem solve, the nature of the difficulties they encountered and the problem solving strategies they developed (if at all). In the five collaborative sessions I gathered the following:

- Video recordings of five sessions. (With a video camera focussed on each group of three learners. These were later transcribed by me).
- The learners written mathematical workings in each session were gathered to gain insight into the difficulties and strategies developed. This also aided transcriptions of the above.

During these club sessions the learners worked in pairs and sometimes threes to solve problems. Past AMESA mathematics challenge papers were used. The learners were encouraged to talk out their strategies and show working out. These sessions were informal. They provided a space for volunteer learners to meet once a week and problem solve.

STAGE 4: POST CLUB ASSESSMENTS

Once the five after school sessions were completed the learners were once again assessed. The learners were assessed, individually after the five after school sessions with a test that consisted of the same 10 questions given for the baseline assessment, to see if there was any change:

- In their performance
- Their inclination to problem solve

STAGE 5: POST CLUB SESSION INTERVIEWS

After the five sessions semi structured interviews were conducted individually with learners, response to which are transcribed in Appendix 6. The learners were given the opportunity to reflect on their problem solving strategies and how they developed these as well as their attitude towards problem solving.

Semi-structured individual interviews were conducted. This allowed me the opportunity to gain a deeper understanding of the problem solving strategies that the learners used and allowed me to probe what benefits, if any, were gained from the after school problem solving sessions.

ETHICS

I was aware importance of dealing with moral and ethical issues with respect to the participants especially when dealing with children. Cohen et al. (2007) emphasise that dealing with children is a sensitive matter. As such I made sure the learners had the freedom to participate and were able to withdraw at any time. I explained to the learners, both verbally and via a letter to parents, that their participation was completely voluntary and that there would be no prejudice if they choose not to participate and that they had the right to withdraw at any time. I received permission from the principal of the school and the School Governing Body (see Appendix 3). Additionally I sought permission from the parents of the children concerned. I received signed consent from the parents for their children to participate (see Appendix 4). I assured the parents and the learners that their identity was confidential and anonymity was guaranteed in the reporting of the data.

DATA ANALYSIS

I began my data analysis by categorising what strategies the learners used in the sessions based on what they did. Which of the strategies were recurrent and which seemed to change or emerge. I used the heuristics proposed by Polya (1973) to aid description and analysis of learners strategies and learner difficulties. I record the steps that the learners used, the amount of time spent on each step and which steps seemed most challenging for the learners. These steps are as follows although they continue in a cycle if evaluation led to the need for revising steps 1, 2 and 3

1. Understanding
2. Planning
3. Executing
4. Evaluating

Polya states that before a problem can be solved the learner needs to understand what the problem is asking and what needs to be solved. Then the learners need to plan a way forward with possible strategies that may or may not work. Once a plan has been devised the learners need to now execute the plan with whatever operations they feel are necessary. Once answers (solutions) have been found the learner should evaluate his/her solution to check if it is plausible, whether any errors have been made, and whether it makes sense? Polya's (1973) stages would assist in analysis.

Similarly Kilpatrick et al.'s (2001) five strands will provide me with a lens for analysing proficiency in problem solving. Conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive dispositions are all required when solving problems and a weakness in any one of these areas can create obstacles in the solution process.

Mason, Burton & Stacey, (1985) look at three phases of work: the entry phase, the attack phase and the review phase. They suggest that when confronted with a problem we firstly look at the information and see what we know; what we need to know in order to answer and how it can be solved. Secondly the learner needs to carry out their plan of action involving strategies in order to solve problems. Finally the learner needs to review the solution obtained to ensure that it makes sense. These steps are used as a lens to analyze and describe the problem solving cycle of learners and to note the strengths and weaknesses of learners in the process. This is very much in line with Polya and his steps. Additionally Kilpatrick et al.'s (2001) five strands of mathematical proficiency are used to frame a language of description (as discussed in the previous chapter).

VALIDITY AND RELIABILITY

A qualitative study such as this needs to illustrate that there is a reason why the study is being done, i.e. is it credible?, (Creswell and Miller , 2000). Validity refers to the inferences drawn from the data that has been collected. One procedure used in validating research is 'triangulation'. Triangulation refers to a procedure where a general common point is reached by more than one data source. This overlap of common ground regarding research findings is

a ‘tool’ used by researchers to systematically sort through data collated in an attempt to find common categories. It is also a way of making sense of the various data collected and using triangulation to try and explain the complexities of human behaviour by analysing it from more than one angle; for example videos recordings, interviews, questionnaires and assessments (Cohen et al., 2007). My multiple data sources provided me with the data needed to support statements made and provided richness to the data in my research findings. Maxwell (2004) argues that when looking at qualitative research validity threats have to be kept in mind and strategies to deal with them as they occur need to be found. The structured interviews conducted along with the transcripts of the video recordings provided me with data that was used for my research. Maxwell (2004) suggests that data that is credible can be used to support validity. Member checks are also suggested as a means of addressing validity issues although since learners are young I did not provided them with transcriptions of what was said.

CHAPTER 4 - DATA ANALYSIS

INTRODUCTION

For my research I collected various types of data. I started off my research by conducting a baseline assessment of the learners in the maths club. This was done to ascertain where the learners were at the start of my research. I was interested in knowing what the learners' mathematical proficiency level was and also to get an idea of the learners' abilities in problem solving. Once this was completed I continued with a written, but orally administered questionnaire in order to delve deeper into the thoughts and feelings towards problem solving of the learners. Once I had marked the written answer scripts I had a few follow-up questions with the learners. These were derived from the learners' responses to the questions.

After the baseline assessment the learners spent five 1 hour a week sessions in a maths club practising and solving problems, this club was facilitated by me. The learners worked in pairs and opportunity for discussions was made available to the learners, where they were encouraged to talk and discuss. The learners used this opportunity to work in a collaborative and co-operative manner, often talking out loud about the steps and strategies they were using to find solutions for problems as well as verbalising their thought processes. I used this opportunity to capture video data of learners working collaboratively in the sessions, which I then transcribed. I fully transcribed critical incidents and have provided these in the Appendix 5.

I present the findings and data analysis of my study in the order in which data was collected as described above.

- Stage 1: Baseline data
- Stage 2: Orally administered individual disposition questionnaire
- Stage 3: Gathering video data and observation on collaborative problem solving
- Stage 4: Post club assessment
- Stage 5: Post club sessions interview

STAGE 1: BASE-LEVEL ASSESSMENT

In this section I share my data on the baseline assessment. The six learners (as described in the methodology chapter) were given a baseline assessment. The assessment comprised 15 questions taken from the 2001 and 2002 grade 6 Mathematics Challenge final round papers,

(AMESA Mathematics Challenge. Grade 6 First Round September 2002 and AMESA Mathematics Challenge. Grade 6 First Round September 2001), see Appendix 1. For this baseline assessment the learners were required to work on their own, within a time limit of 1 hour and had the opportunity to use a calculator if they so wished. The results are tabulated below:

Table 4: Baseline assessment results

Learner Script	Baseline Assessment Score	
	Mark out of 15	%
C	3	20
H	3	20
S	4	27
K	5	33
U	8	53
T	9	60
L	Absent	-

The above table shows the results of the initial baseline assessment that was undertaken. From the table it can be seen that there was a range of scores from 20% to 60% and that four of the six learners performed below 50%. The average mark for the six learners for this assessment was 35,5%, which is roughly the average annual score on the first round mathematical challenges. This indicates relatively low levels of mathematical problem solving proficiency, as these questions require the combination of all five strands of mathematical proficiency. The four learners, thus seemed to not possess the necessary 'five strands of mathematical proficiency' (Kilpatrick et al., 2001). Only two of these learners (U and T) displayed a basic level of mathematical proficiency as indicated by their scores 53% and 60%. The baseline data confirmed my earlier experience that learners are generally weak at solving AMESA mathematics challenge problems.

This data provided the background information on learners that will be used in informing and making sense of data collected in subsequent phases of data collection.

STAGE 2: INDIVIDUAL DISPOSITION QUESTIONNAIRE

In order to gather some pre-club data on learners' attitudes I asked the learners to complete an orally administered questionnaire. The questions for this adapted from Graven (2012) discussed in chapter 3 above.

The data from the baseline assessment questionnaire conducted before the start of maths club is given across the six learners per question below:

Table 6: Pre-baseline questionnaire responses

Question number	Question	Responses	Frequency of repeated themes if any.
1	**Complete the sentence: Mathematical problem solving is…**	• Sometimes hard but most of the time easy. – H • Easy for me but sometimes it becomes too much and I lose concentration - T	Sometimes hard, sometimes easy. (2/6) -H; T
		• Solving sums and find different answers - K • When we solve a sum to find the correct answer - C • Is solving problems mathematically - U • About solving the questions – S	To solve for something. (4/6) -K; C; U; S
2	**What do you think makes someone good at problem solving?**	• They know how to work it out and they know their adding, subtracting, timesing and so on – H • If you work hard in maths and if you know your times tables –C • Being able to work things out and being good at their times tables - U	Being good at times tables and the four operations (3/6) -H; C; U
		• Read the sentence, righting (writing) down all the steps, double-checking the answers – K	Reading (1/6) -K
		• I think practise, a lot of practise makes you good at problem solving but not practising continuously with no stop, - T	Practise (1/6) -T
		• I think that they must be very clever at maths to do it and they must understand the questions correctly. – S	Must be clever (1/6) -S
3	**What do think makes someone bad at problem solving?**	• Not reading sentences properly and rushing.- K • What makes someone bad at problem solving is to read things too fast and not checking it – T • Not reading right and not thinking properly – S	Not reading carefully (3/6) -K; T; S
		• They don't know what they are doing and they are not so good in maths - H • Not being that good at maths - U	Not being good at maths (2/6) -H;U

4	**Mpho is scared of problem solving because…**	• She/he does not understand it, finds it hard to do and is sometimes bad in maths - H • He doesn't know how to do it and he doesn't learn - C • He is not that good at it – U	Lack of understanding (3/6) -H; C; U
		• He works fast and doesn't communicate??? - T	Works fast without communication (1/6) -T
		• He is the weakest student in the class - K • He is the weakest student in the class – S	Being the weakest student in the class (2/6) -K; S
5	**Thando enjoys problem solving because…**	• He is the strongest student in the class – K •	Strongest student in the class (1/6) -K
		• She can connect / communicate with partners/peers - U	Good at communication (1/6) -U
		• She/he understands it, finds it easy and is good at maths - H • He knows how to do it and he likes to learn - C • He is very good at it - T	Good at maths (3/6) -H; C; T
		• No response – S	No response (1/6) -S
6	**How do you feel about problem solving in mathematics?**	• I find it easy sometimes – H • I feel fine with problem solving - T • I feel confident - U	Easy, Fine, Confident (3/6) -H; T; U
		• I feel very nervous because sometimes I really don't know my tables - C	Nervous (1/6) -C
		• It needs more thinking – K	More thinking is required (1/6) -K
		• No response – S	No response (1/6) -S
7	**What do you do when you get a problem and you don't know where to begin?**	• I leave it out and finish the rest and then I go back to that one – H	Re-try the question later (1/6) -H
		• I try to work it out or I ask for help - C • I normally ask one of my friends – T	Ask for help (2/6) -C; T
		• I leave it out - U	Leave it out (1/6) -U
		• You start with the number sentence –K	Start with a number sentence (1/6) -K

8	**How are you at problem solving?**	• I would say average – C	Average (1/6) -C
		• I'm good at problem solving – H • I am good but struggle a bit - K • I feel I am pretty good at it - T • I'm good at problem solving but I always make silly mistakes either because I lose concentration or not enough communication - U	Good (4/6) -H; K; T; U
		• No response – S	No response (1/6) -S

Questions were adapted from Graven's (2012) instrument for assessing young learners' mathematical dispositions. Graven's instrument was more general and focused on learners' dispositions about mathematics generally. I adapted it to suit my needs to find out about learners' dispositions focusing on mathematical problem solving. Graven et al. (2013) argue that this instrument mentioned earlier, is limited although it is useful for illuminating emerging themes. These emerging themes are listed below:

Table 7: Summary of Responses to the pre-baseline questionnaire

Question number	Emerging themes	Number of learners (out of 6)
1	Solving for a something	4
	Sometimes hard sometimes easy	2
2	Being good at the four basic operations	3
	Practise	1
	Reading	1
	Must be clever	1
3	Not enough practise	1
	Not reading carefully	3
	Not being good at maths	2
4	Lack of understanding	3
	Weakest student in the class	2
	Works fast without communicating	1
5	Good communication skills	1
	Good at maths	3
	Strongest student in class	1
6	Easy, fine, confident	3
	More thinking required	1
	Nervous	1
7	Re-try and persevere	1
	Start with a number sentence	1
	Leave it out and move on	1
	Ask for help	2
8	Good	4
	Average	1

As you can see from the summary table; when the learners were asked what they thought mathematical problem solving was about they responded with a variation of "s*olving for something"* or in terms of "*how easy or hard it was"*. When asked specifically about problem solving and what skills they needed in order to be good at problem solving most of the learners (5 out of 6), responded with variations of *"being good at the four operations"*. When asked why 'Mpho would be scared of problem solving" (question 4) half of the responses pointed to "*Lack of understanding ..."* as the main reason. Whereas only one learner stated that Mpho is scared because he/she *"works fast without communicating"*. So in a sense without intent the learners have suggested that communication is an important part of problem solving and finding strategies to problem solve. Half of the learners (50%) stated that they failed to read the questions carefully when asked what made someone bad at problem solving (question 3). Of interest, only one learner mentioned (learner U), communication as their response to "Thando enjoys problem solving because".

The response frequency that stands out the most, (aside from solving for something in question 1), is the response for question 8, (How are you at problem solving?). Here 4 out of the 6 or 67% of the learners stated that they are "*good at problem solving"*, although this is not supported by the results of the baseline assessment conducted at the start of my research, where the average mark of these 4 learners is 41,5%. This statement, "*good at problem solving"*, only concurs with the performance for two of the learners, (learner T and U, who obtained 60% and 53% respectively), and not for the other two learners, (learner K and H, who both obtained 20% and 33% respectively). Noticeably aspects of productive disposition such as 'sense-making' and effort (perseverance) only occur in a few of the statements. I.e one learner on question 6 who says "*more thinking required"* and one learner in question 7 who says *"retry and persevere"*

STAGE 3: GATHERING VIDEO AND OBSERVATION ON COLLABORATIVE PROBLEM SOLVING

As described in the methodology chapter learners worked in pairs on mathematical problems over five weekly one hour sessions.

I recorded each pair which resulted in about 5 hours of recordings per pair. Following observations and field notes from these hours of recordings I transcribed critical incidents in relation to my research questions. In this thesis, a critical incident is any incident which was

of significance; it was an event or part of a conversation/discussion that took place that pointed towards my research questions.

Transcribing all the dialogue in sessions would have been beyond the scope and time available for this half thesis. It was also deemed unnecessary as much of the transcription was not of direct relevance to my critical question.

The critical incidents were identified in relation to the following:

- Where the learners were articulate in expressing their thoughts and 'caught each others' thoughts (Meira & Lerman, 2009).
- Where there were indications of the development of emergent strategies.
- Where discussion led to joint strategy or indications of learning for an individual or the pair

In observing and reading my notes of all the pairs I noted that the quality of discussion that occurred was critical to whether paired learning enabled problem solving. Quality discussions tended to have the identifying feature of critical incidents present.

For my detailed analysis I chose two pairs of learners. These two pairs were chosen from the seven learners. This is because over the time frame of my data collection process and due to logistical constraints such as the school production being rehearsed and performed during this term, not all the learners came to all the maths club sessions. My choice of the two pairs was based on the learners who attended regularly to all sessions. From the two pairs of learners I noted excerpts in each case which illuminated both the process of problem solving (and the problem solving cycle as mentioned by Polya,1981) and the role collaboration played in this.

The first pair was K and U (this pair proved to be articulate). These two achieved averages of 33% and 53% respectively in their baseline problem solving assessment.

The second pair was T and C and a third learner (L) joined this pair at times when she was able to attend. (This group were not as articulate as pair 1 and for all of them English was their second language). They achieved 60%, 20% in their baseline assessment.

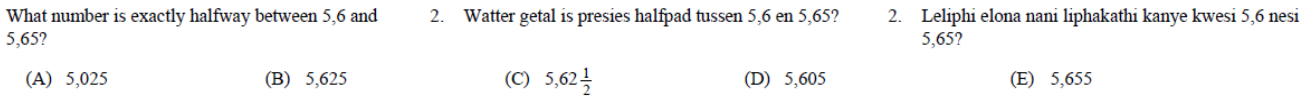
I begin with an analysis of the transcripts for pair 1. I have chosen 6 excerpts of critical incidences involving this pair in order to gather data in relation to my research questions.

Full transcripts of all the critical incidents of the first and second chosen pairs are given in appendix 5.

EXCERPTS OF PAIR 1: LEARNER K AND U

PROBLEM SOLVING CYCLE

During the first session (in excerpt 1 below) the learners worked on a problem that required them to find a number that lies exactly halfway between 5,60 and 5,65.



Source (AMESA Mathematics Challenge., 2002b)

Excerpt 1: Learner U and K

Lines 1, 2 and 3 indicate that the learners are trying to understand the problem.

1	U	it's between 5.60 and 5.65
2	K	won't it be this (*pointing at an answer)* because maybe you need to add that and that.
3	U	it has to be 5.62 and a half. Let's work it out 5.6 is like 5.6 and 5.65 is like 5.6 and a half

Discussion around what it means to be halfway between and what needs to be done to solve it, centres on the numbers 5,60 and 5,65. U and K are trying to sort out what is required verbalise their thoughts so that there is no ambiguity. Learner U says, 'it has to be 5.62 and a half.' Although this is not the correct way of saying the answer the logic behind it is correct. Lines 4 to 9 indicate a plan that learner K and U arrive at.

4	K	Lets' just see. Must we add or must we minus if we want to work it out?
5	U	we minus that to that *(pointing)*
6	K	no that to that *(pointing)*
7		*Furiously working out. Lots of rubbing out and starting out.*
8		*Learner K is writing out the question 5.65 – 5.6. The numbers are not exactly underneath each other in the correct columns, this apparently causes distress for learner U*
9	U	Doesn't the 6 go there? *(pointing to the correct column)*

They are planning their strategy to try and solve the question. Lines 10 – 15 indicate that the learners have found a strategy that may work they have started work towards it.

10		*U ignores K and continues with subtraction question. And gets an answer of 0,05*
11	K	now what do we do
12	U	we half it
13	K	yeah, lets half it and add it to six *(pointing to 5,6)* so that would then (shuffles paper) 5 divided by 2 you have 2 and a half and then
14	K	*Writes on paper (2 ½)*
15	K	5.60 plus 2,5 equals *(interesting that although the number sentence is incorrect the writer (K) seemingly knows what she is doing and now explains to child A what is happening and why it is 2,5 although it is a half)*

The lines above illustrate that the learners are confident that they have found a strategy that they can now utilise to solve the problem. Lines 17 to 21 show that not only have they found an answer that the learners are fairly confident is correct but are now reflecting on their answer.

17	K	its half because we have to add it to 60. Because ...or its going to be, how else because 5.60 and half is not exactly between 5.6 and 5.65
18	U	OK
19	K	5 divided by 2 is 2 and half (*goes back to her written number sentence 5,60 + 2½ = 5,62 ½*
20	U	equals our answer
21	K	Answer C

Although the working out that this pair did in the second part was mathematically incorrect in its written form (i.e. 5,60+ –) see figure 2, their logic behind it was correct. Their failure to connect 5,62½ to 5,625 leads them to selecting the distracter answer C.

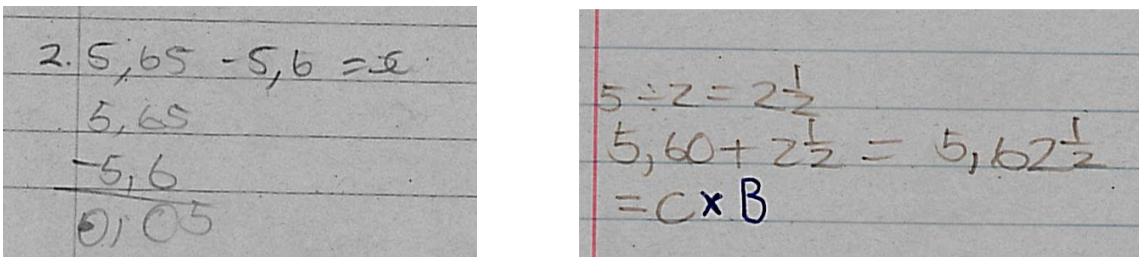


Figure 9: Working out learner K and U

This shows that although the written work on the right hand side was incorrect; they had enough conceptual understanding to make sense of the problem and almost reach the correct

answer. Learner K did arrive at an answer that I think is at least to some extent correct i.e. 5,62½. The pair between them, with all the discussion that they had was unable to re-write the 0,02½ as a decimal number 0,025 with the digits in the correct place value columns, therefore arrived at an answer of 5,62½ as opposed to 5,625. Unfortunately for them, both of these were options to choose from and they chose the incorrect one. This illustrates the need for all five strands of Kilpatrick et al. (2001) for mathematical proficiency to be present as indicated below:

- Conceptual understanding – this is the understanding of the concepts such as the four basic operations and relations. In this case what halfway means and how to solve it. Learner K does have this to some extent as she knows that half of 5 is 2½ therefore she surmises that half way between 5,6 and 5,65 is 5,62½
- Procedural fluency – refers to the skill, flexibility and accuracy with which mathematical procedures are carried out. Learner K in this instance has a certain amount of procedural fluency but not enough to arrive at the correct format of the solution. She does not connect the concept of 5,62½ with the correct mathematical representation of 5,625. Perhaps because the procedural fluency of place value to the right hand side of the decimal point is not established
- Strategic competence – refers to the ability to formulate ideas, represent and solve mathematical problems. Learner K is able to formulate her ideas to solve the solution.
- Adaptive reasoning – the ability to reflect on outcomes, possess logical thought and explain and justify findings. Learner K clearly reflects on her outcomes and is able to explain how she arrived at the solution that she did. For example see lines 13 and 19.
- Productive disposition – refers to the ability of the individual to see mathematics as worthwhile. Both learners illustrate this as they steadily work towards a solution.

In this sense the lack of knowledge of how to represent fractional concepts in decimal form leads to the non-attainment of the solution. This concurs with Kilpatrick et al.'s (2001) comment that mathematical proficiency cannot develop with only one of these strands and that all five strands are needed to be present in order for a learner to be proficient at mathematics.

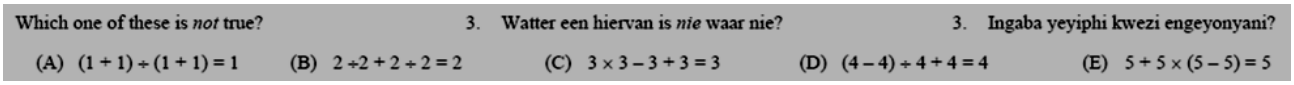
The dynamics within this pair suggest that they are organised into delegating roles which change as will be seen with later dialogues, in that one of the pair will write down while the other thinks aloud and solves. It is worth noticing that it is not always the same learner that writes and solves. The pair seemed to evaluate the question and then decide who would be the best to solve depending on their strengths and weaknesses in that question. There was some informed negotiation that took place, this is illustrated in the dialogue on lines 11 -17.

Here U is unsure that K is doing the correct working out, and is therefore asking and re-affirming her knowledge by asking "*but that's half not 2 and a half?"* K seems to know what she is doing and is maybe not able to express herself in a manner that makes sense to U. Even her reasoning in line 17 doesn't easily make sense.

17	K	It's half because we have to add it to 60. Because ...or its going to be, how else because 5.60 and half is not exactly between 5.6 and 5.65

Excerpt 2: Learner K & U

In the second excerpt learner U is reading the question while learner K writes. The question is as follows:


Which one of these is *not* true? 3. Watter een hiervan is *nie* waar nie? 3. Ingaba yeyiphi kwezi engeyonyani?

(A) $(1+1)\div(1+1)=1$ (B) $2\div2+2\div2=2$ (C) $3\times3-3+3=3$ (D) $(4-4)\div4+4=4$ (E) $5+5\times(5-5)=5$

Source: (AMESA Mathematics Challenge., 2002b)

U is reading out loud and working things out in her head, while K is trying to keep up with the writing of her working out and trying to make sense of the problem. Although K has taken on the role of unofficial scribe for this question she is still trying to contribute to solving it. This is indicated by her comment in line 31 where she interrupts U and says '*no, no that's'*... U quickly realises that K has forgotten an important concept of BODMAS and the rules of BODMAS and reminds K of the rules see lines 30 to 34)

30		equals 1 2 divided 2 equals 1
31	K	*(Interrupts)* no, no that's..
32	U	no BODMAS darling BODMAS plus comes after divide equals 1
33	K	and then
34	U	1 plus 1 equals 2 1 plus 1 equals 2 and then that's right

K then continues with the writing and is not perturbed by the way she is corrected and seems to see it as a helpful reminder. She then continues the way they were. It is plausible to assume at this stage that had they not been working in pairs and had the opportunity to communicate and collaborate then it seems reasonable to assume at this stage that K would have arrived at an incorrect solution (since following order of operations is essential for a correct solution). But because they were given the opportunity to talk and K was given the opportunity to learn from a 'more knowledgeable other' she learnt something or was reminded of something that was an important concept (i.e. BODMAS). In Kilpatrick's terms of the instructional triangle and definition learner U, in this excerpt, is the 'more knowledgeable other' and explains the procedural knowledge required to solve the problem i.e. the order of operations rule to learner K, which is essential to solving this particular problem correctly.

29	K	Then write B 2 divided by 2 . You've got a piece of paper U ... (*K interrupts U)*

This was before U reminded K of the use of 'BODMAS' rules for calculations with more than one operation. The pair methodically approached each of the 5 number sentences and solved them, using 'BODMAS' with their strategy in order to arrive at the correct solution. This question also used the four steps of problem solving as outlined by Polya (1973). They were reading and understanding the problem, i.e. lines 22 – 24.

22	U	1 plus 1 divided by 1plus 1 equals 1 that's true, no, Then B 2 divided by 2 plus 2 no sorry let me start again *(rubs out and starts again)*. This doesn't make sense
23	U	Ok you rub it out.
24		*They start all over*

Line 22 illustrates that learner U has made an error and therefore needed to start over. This shows that whilst working through the problem learner U was able to reflect on her answer and its suitability.

The pair then moves on to solve each of the questions and evaluate them against their original problem of finding the one that is ***not*** true.

Excerpt 3: Learner K and U using manipulatives to solve problems

. A and B are two circles of the same size. If A is fixed and B is rolled around A, how many revolutions will B make before returning to its original position?

(A) $\frac{1}{2}$ (B) $1\frac{1}{2}$

10. A en B is twee ewe-groot sirkels. As A vas is en B word rondom A gerol, hoeveel omwentelings sal B maak voordat dit weer op die oorspronklike posisie is?

(C) 1 (D) 2

10. Imizobo u A kunye no B zizangqa ezinedayametha efanayo. Ukuba u A uye wacinezelwa waza u B waqengwa ejikeleziswa kwisangqa sika A, uya kuqengwa kangaphi ngokupheleleyo u B phambi kokuba abuyele kwindawo abe ekuyo ekuqaleni?

(E) $2\frac{1}{2}$

Source: (AMESA Mathematics Challenge., 2002b)

Solving this problem was enabled by K and U's use of objects to assist them. They decided to use two prit glue sticks as shown in figure 3. Polya's four steps of problem solving are once again highlighted here.

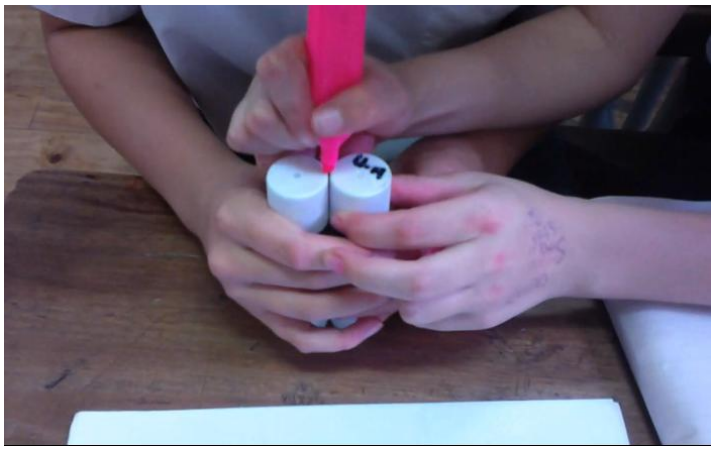


Figure 10: Learner U and K working out

POLYA'S STEPS 1, 2 AND 3

This excerpt is composed of mainly observation notes and their was little dialogue and more action.

44		*K is busy marking on a two Pritt glue sticks in blue in preparation for solving the question. The marking is altered by her partner with a bright pink highlighter. (Q is to see if the circumferences of two circles are the same). They are working on the desk.*
45	K	*She starts turning the pritt stick so that it moves around the edge of the second glue stick to see how many times it goes around it. She starts counting but it is not correct* 1 2 3 Is that right???
46	U	Ok you turn it you are better at turning it
47	K	How??
48	U	Just turn it and I'll just count
49	K	Wait
50		*They both fumble to get grips with the two glue sticks and eventually decide that they should go and work on the floor....*
51		*They move onto floor by the black board and continue with their solution.*
52		*They start to turn the glue sticks again and start to count*
53	U	Just turn it
54	K	Ok
55	K	that's 1 right *(after turning one 'twist' of the hand to indicate 1 instead of 1 rotation around the prit stick)*
56	U	No

57	K	that's 1
58	K	so it goes around once

Line 44; indicates that the pair seem to know what to do practically with the two prit glue sticks. This was done as soon as the problem was read and U quickly started off with what needed to be done. This illustrates Polya's steps 1, 2 and 3 happening at a very fast pace that may have been missed completely. Unfortunately U may understand the problem better than K as K has started turning and counting in what seems like random numbers. U has obviously realised K's error and is now telling K to turn and that she (U) will count. This situation has developed because they are working in pairs and communicating. They have benefited from working and talking together about mathematics and have illustrated the three conditions stipulated by Johnson et al, (1981), those being that within a co-operative working group higher cognitive developments are made through the discussion and interaction that exists within the group. They also exchanged information that benefitted both members of the group and finally there existed an incentive to work for the lower achiever within the group to find possible solutions. This is illustrated by K, who is the weaker mathematically of the two as she did not just give up but was willing to carry on and was determined to understand and ultimately reached her goal of arriving at a solution. (Johnson et al. 1980).

Excerpt 4: Learner U and K

A book and a CD together cost R230. If the CD costs R60 more than the book, how much does the book cost?

(A) R80 (B) R115 (C) R110 (D) R170 (E) R85

13. 'n Boek en 'n CD saam kos R230. As die CD R60 meer kos as die boek, hoeveel kos die boek?

13. Incwadi kunye ne CD zizombini zixabisa ama R230. Ukuba I CD ixabisa ama R60 ngaphezu kwencwadi, ingaba incwadi ixabisa malini?

Source: (AMESA Mathematics Challenge., 2002b)

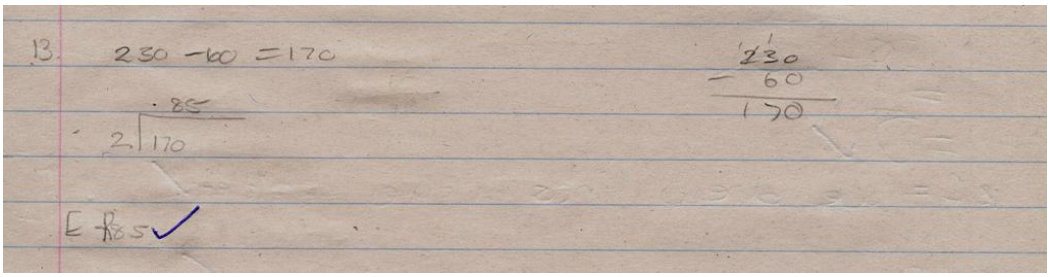


Figure 11: Working out Q13

Lines 59 – 65 illuminate Meira and Lerman's concept of 'catching each others' thoughts.

59	U	If the CD costs R60 more than the book how much does the book cost?
60	K	Then say we times or divide?
61	U	I honestly don't know like dividing that by 2 and then working it with the..

		That's the part because CD costs R60 more than the book. So we need to work out what the CD costs.
62	K	wait that part so then 60 plus 30 cos yes, it 6 .it ... so it costs R60 more than that so you'll say
63	U	I know K I know what about if...
64	K	60 and then you'll say plus .... (*Muffled sound not clear ...inaudible)*
65	U	or even like if you like what was I gonna say if we minus 60 from and the we divide that by 2 then we find out what the book costs. Yeh that should work. You can write it out
66	K	so its 230 – 60 right which
67	U	equals 170 we say
68	K	we divide that by 2
69	U	we say 170 divided by 2 which I think is 75 no not 75 sorry wait I think its 85 I'll just make sure *(punches into calculator)*its 85 and then so that how much the book costs I think. Yeah that's how much it is.
70	K	yes its E answer is E *(pointing to the question sheet)* the book costs R85

There seems to be unspoken communication between the learners, but the learners know what each other is talking and thinking about, since they are able to continue on from each others sentences. The last few lines line (66-70) illustrate that both learners are 'thinking as one' as they seem to be finishing off each others' sentences.

Excerpt 5: Learner U and K

Which of these fractions is the largest?

(A) $\frac{7}{15}$ (B) $\frac{8}{17}$ (C) $\frac{11}{23}$ (D) $\frac{13}{27}$ (E) $\frac{5}{11}$

14. Watter van hierdie breuke is die grootste?

14. Ingaba kula maqhezu leliphi elona likhulu?

Source: (AMESA Mathematics Challenge., 2002b)

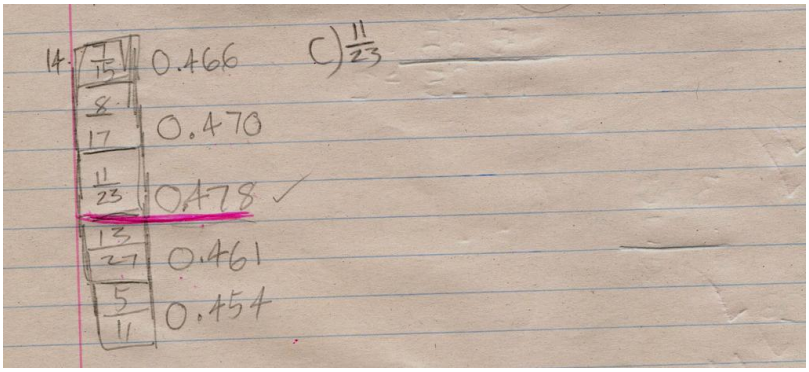


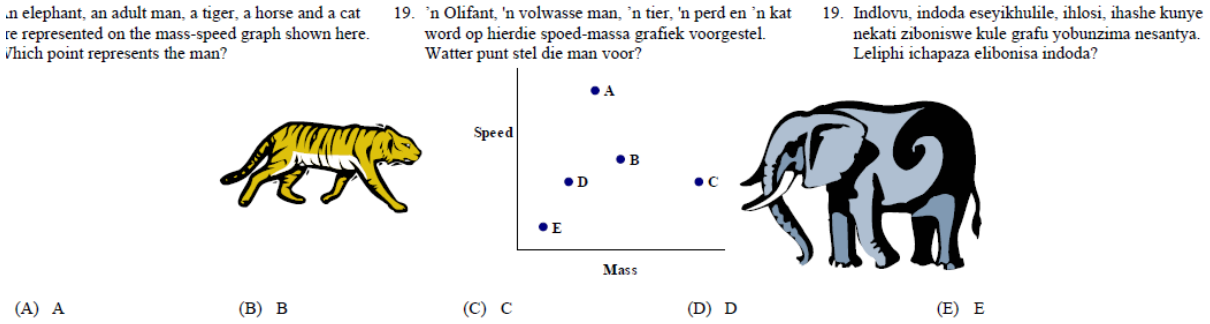
Figure 12: Working out Q14

101	U	LCM ok so let's see 7 divided by 15 8 divided by...
102		*They are trying to solve which fraction is the largest. After some time being*

		used to find the LCM the learners decided there had to be an easier way as the LCM was a large number. After much discussion they decided to use their calc
103	K	7 divided by 15 equals 0.46666..
104	U	so then how do find what oh we work which is the highest
105	K	Kappish
106	U	OHHH! *(a aha moment has passed the learner when she really understands what is happening now)*
107	K	oh so for A I'll write it here
108	U	take your eraser And write it here, actually you can keep it there we'll just write the the whats it under it
109	K	writing 0,46666 it's gonna carry on I'm not gonna write it down
110	U	just write a few dots so then we'll know
111	U	8 divided by 17 equals 0.47058823529 de dede dede
112	K	0.47508823529
113	U	and the 11 divided by 13 equals 0.47826086956
114	K	Which one are we looking for the largest?
115	U	do you have all of them *(pointing to her calc)*
116	K	Yeah
117	U	then what's the next one
118	K	its 13 over 27
119	U	13 divided by 27 equals 0.48148148 ok this is the largest one so far
120	K	so now its 5 over 11
121	U	5 divided by 11 equals 0.4545454545
122	K	ok I don't need to do that
123	U	just write
124	K	nought four five, yeah so number 13 over 27 so that's D
125	U	Yeah

In this excerpt we can see how a strategy has been abandoned in favour of another one which to the learners seemed 'easier'. Careful studying of lines 101 and 102 suggest that this points towards the learners being able to reflect on the problem at hand and try to find a suitable solution with a suitable, workable strategy that they can follow. The fact they were able to use an alternative strategy that, in this case, works shows that they have conceptual understanding as described by Kilpatrick et al. (2001). It also illustrates adaptive reasoning and strategic competence. A critical "*aha"* moment occurred for learner U (line 106) as she suddenly realized what was expected of her in order to solve the problem.

Excerpt 6: Learner U and K



Source: (AMESA Mathematics Challenge., 2002b)

133	U	What does it mean speed and mass?
134	K	this is the speed and then that's the mass I think (*pointing to the graph)*
135	U	the mass is the one that goes here
136	K	Isn't this the adult man*? (pointing to a point on the paper)*
137	U	Why would the man be here. Because the man is the heaviest he's not there
138	K	The elephant is the heaviest. The elephant should be there, but this is speed U
139	U	that's speed and that's mass *(draws a cross on the graph)*
140	K	That's downward speed. Pointing to the speed axis
141	U	that's the elephant and then the horse is obviously ...
142	K	Wait you must write it here. The elephant is there
143	U	this is the horsey, the B
144	K	ok this is horse, the tiger will be up there *(pointing to the a point on the graph)*
145	U	then there's a cat
146	K	a cat too
147	U	the cat is going to be lighter than the man, so that must be the cat and that must be the man
148	K	so then
149	U	it's D

For pair 1, the findings for problem solving confirms Polya's heuristics for problem solving, where the pair tried to firstly understand the question i.e. (line 133) U asks 'what does it mean?" K then explains to U her understanding.

133	U	What does it mean speed and mass?

Once the question is understood they then plan a way forward to see if how they can solve the problem.

The above excerpt illuminates how paired discussion increased reflection on the process they used to solve the problems and also the answers they chose. Polya's four stages of problem

solving as indicated above namely; understanding, planning, executing and reflecting are illustrated in the strategies used and the discussion that followed. For this question hardly any notes were made and discussion seemed to centre on the graph and the possible objects or animals that the points on the graph could represent.

Polya's four steps of problem solving are clearly shown by the strategy that developed. This is shown in the figure below:

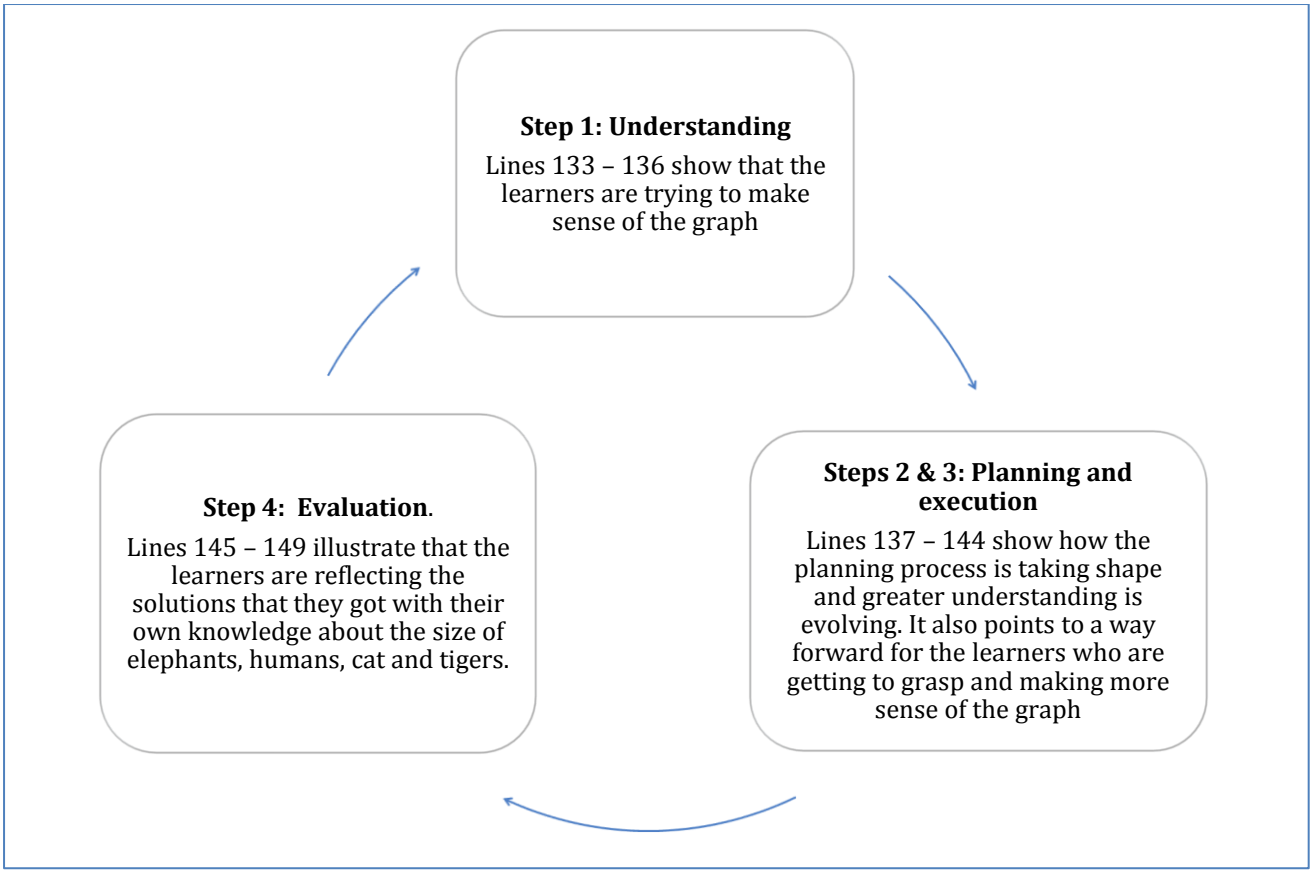


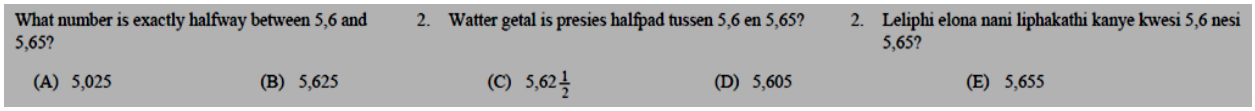
Figure 13: Polya's steps for problem solving illustrated in excerpt 6

This pair showed adaptive reasoning in relation to their knowledge about the mass of certain animals in comparison to others. This is illustrated by their discussion on lines 137 to lines 147. Initially the pair was unsure of what to do and this is shown by the comment U made in line 133. U is asking K what "speed and mass" means. This in itself is quite unclear, it is difficult to see if she wants to know what speed and mass actually are or where they are on the graph. Their communication however is not deterred by this ambiguity as K answers in relation to the graph.

EXCERPTS OF PAIR 2: LEARNER L, C AND T

Pair two (joined by a third learner on a ad hoc basis). This pair is not as articulate as the first pair and are not as expressive verbally. All of the learners who make up this pair are English second language speakers.

Excerpt 7: Learner L, C and T



Source: (AMESA Mathematics Challenge., 2002b)

Here the working out and solving seems to have been done on an individual basis and solutions are compared at the end. It is almost as if T needs reassurance that her solution and the strategy that she used are correct. Interestingly L has used her calculator to work out the answers while T and C are doing the working out on paper.

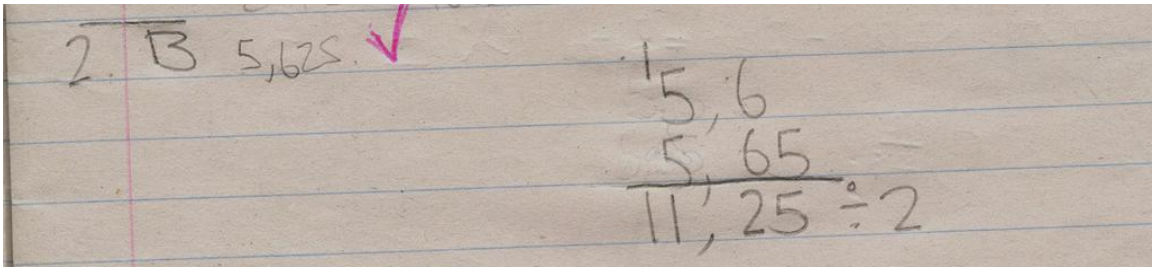


Figure 14: Working out Q2

For this question one member of the group has already solved the problem and is now attempting to explain to the other members what steps and logic she used. This group, different from the U and K pair used a calculator to solve for the solution. As a result it indicates proficiency in calculator use and interpretation as they were not only able to follow the procedure but they also seemed to understood the concept behind the maths that they were doing.

This is evident in the discussion that followed and the explanations that were given. Here L is explaining what she did as C and T try to follow. C is the weakest member of this group mathematically (as discussed in the baseline assessment and school assessment records) and as can be seen from their script above that although the digits were all placed in the correct columns C is still trying to add them up incorrectly (line 81). She is quickly corrected by T who reminds her that the 5 is not in the same column as the 6 (line 82).

71	L	I divided by 2 so that its half way by 2.
72	C	what did you do
73	L	I added the two
74	T	Yeh
75	L	I added the two
76	C	I want to see what you got.
77	L	I added the two and then after that I divided the answer by 2 *(Background noise)*
78	C	Where must the comma be?
79	T	put the 5 here the comma stays in the same place
80		*(L is typing numbers into the calculator)*
81	C	here's the 6 add the 5 is eleven
82	T	no wait that 5 isn't under there that's a zero
83	L	that's 5,65 (she's finished punching in her numbers onto the calc) *(They are adding 5,6 and 5,65)*
84	T	5 plus 0 is 5. 6 plus 6 is 12 carry the 1 5 and 5 is ten and the one 11. Is that what you got (she is now asking L)
85	L	*Is again punching numbers into her calculator*. Yes I did that and then divide it by 2
86	C	11,25 divided by 2 equals 5,625 B *(worked out on the calculator)*
87	L	I also got B

L is the learner who used the calculator and explains her strategy to the rest of her group. She is able to explain step by step what she has done and why she has done it. (Lines 71, 73, 75 and 77) T on the other hand is busy working out the question on paper and showing her working out. Although this group solved the problem correctly whether they had more conceptual understanding than the previous group is debatable.

Excerpt 8: Learner C, T and L

5. A lorry with a load of maize has a mass of 4 653 kg. The mass of the empty lorry is 2 583 kg. One bag of maize has a mass of 90 kg. How many bags of maize are on the lorry?

(A) 20 (B) 21 (C) 22 (D) 23 (E) 24

5. 'n Vragmotor met 'n vrag mielies het 'n massa van 4 653 kg. Die massa van die leë vragmotor is 2 583 kg en een sak mielies het 'n massa van 90 kg. Hoeveel sakke mielies is daar op die vragmotor?

5. I lori enomthwalo wengqolowa inobunzima obungama 4653 kg. Ubunzima belori engenamthwalo bungama 2 583 kg. Ingxowa nganye yengqolowa inobunzima obungama 90 kg. Zingaphi iingxowa zengqolowa ezikule lori?

Source: (AMESA Mathematics Challenge., 2002b)

126	C	L said twenty three
127	T	How did you do it?
128	L	I don't know how I did it. But then I had a paper with working out on it .
129		*(T is busy punching numbers into the calculator) (whispered discussion)*
130	L	um, then I said Mrs X I said I said I said um, the load with the mass of the thingy , with this minus this divided by that is 23.
131	L	this minus this divided by that is 23
132	T	4653 – two hundred and two thousand five hundred and eighty three is 2070 and then we have to say 2070 divided by 90 is 23. You were right.

Once again, one member of this group who struggled to work and co-operate as a team player, had already solved the question and is now busy explaining the strategy that she used. Notice how learner L cannot remember how she arrived at a solution as she has misplaced the paper with her working out on it; (line 128). L eventually finds the paper and is now explaining how she solved it, (line 130) T is busy "double checking" L's solutions and is finally satisfied that it is correct (line 132). Once again we see the learners proceeding through Polya's heuristics for problem solving as explained in an earlier chapter and exemplified in earlier excerpts.

DISCUSSION OF FINDINGS

A key observation across sessions and transcripts was that communication helped with arriving at strategies for solutions to problem solve. The way in which learner talk enabled progress has been shown in the above excerpts. By providing an arena for the learners to be able to communicate meant that together they were able to solve more problems than they were able to do individually. This is because they were able to 'catch each other's thoughts' as mentioned by Meira and Lerman (2001). It could also be attributed to the fact that a 'space' or in Vygotsky's (1978) term's the ZPD was created for the learners so that collaborative learning could take place.

My observations and analysis also indicate that all five strands for mathematical proficiency are necessary and need to be developed at the same time. For example this was seen in my learners, pair 1, who seemed to have the conceptual understanding regarding the strategy that worked towards a solution, but because one of the strands for mathematical proficiency was absent (ie. Procedural knowledge of written form of decimal place value), as a result the solution they circled was the incorrect answer. (See page 44 and 45). This indicates that not only do the learners require procedural knowledge but also the knowledge of conventions i.e. the correct accepted way of writing decimal fractions (as in pair 1, Excerpt 1).

The collaboration and communication also allowed the learners to move forward. For example if they had not been given the opportunity to communicate and collaborate then it is likely that learner K would not have got the correct answer, (Excerpt 2). This allowed learner U to correct learner K in a way that was not enabling of progress and as a result they both moved forward.

Polya's (1981) heuristics have been displayed and illustrated by the conversations that took place. At times these four steps moved at a rapid pace as in Excerpts 3 and 6 (these were difficult to see and could easily be missed) while at other times these were more clear and paced.

STAGE 4: POST CLUB SESSION ASSESSMENT

Once the afterschool collaborative sessions were over after five weeks, the learners were once again assessed on their ability to problem solve. They were given the same assessment paper as the one given for the baseline assessment. Once the baseline assessment was completed the learners were not given back the papers and I did not go through any of the questions with them. With this in mind I was interested to see if the ability to problem solve

the same questions had changed once they had participated in the after school collaborative problem solving sessions. Table 7 below shows learner results across the baseline assessment and post assessment.

Table 8: Post club assessment scores

Learner Script	Baseline Assessment Score		Final Assessment Score	
	Mark out of 15	%	Mark out of 15	%
C	3	20	6	40
H	3	20	7	47
S	4	27	5	33
K	5	33	6	40
U	8	53	10	67
T	9	60	10	67
L	Absent	-	4	27

As can be seen from the results in the table, each of the 6 learners improved in their performance on the test. The average score was no longer 35,5% and it had risen to 45,9%. The range of percentage point increase was from 7% to 27%. This could indicate that providing an environment where mathematical communication is permissible the learners do, to some extent, seem to develop their problem solving strategies. However I cannot say the extent to which such an improvement may have occurred had learners spent five hours working individually on problem in the after school sessions.

Excerpts, however, show that learner C and H who were by far the weakest members of the group certainly seemed to have benefitted from the collaborative environment and peer-to-peer discussions that occurred. This reiterates the findings of Johnson et al (1981) where they state that low achievers benefit more from a co-operative and interactive environment. These two learners doubled their scores and benefitted the most. The other learners also benefitted from the peer-to-peer communication that was made available.

STAGE 5: POST INTERVIEW

A post interview questionnaire (as discussed in chapter 3) was orally administered once the after-school problem solving sessions ended. The initial questionnaire had 10 questions, it was a semi structured interview in that I had prepared certain questions and I followed up on remarks made by the learners if I felt that they would lead to some interesting findings. A copy of the full transcript of these questions and answers can be found in Appendix 6. Table 9 shows my analysis of emergent themes in the responses to specific questions (questions 3,

4, 8 and 9). These questions were more specific to my research while the other questions were for my own interest as a mathematics teacher. I have grouped the responses according to the themes that emerged and provided the frequency (accompanied by learner initials) for each theme.

Table 9: Orally Administered Post Interview

Question number	Emerging themes	Number of responses out of 6
3 **What other factors/skills do you think would help you to become a good problem solver?**	Communicating/Discussion/Talking	1 (K)
	Basic operations	2 (U,H)
	Reading the questions	2 (C, S)
	Working hard	1 (T)
4 **What do you think you have learnt from the maths club that will help you in maths lessons?**	Think a bit more, write things out.	1 (K)
	Read questions carefully	1 (S)
	Talking	1 (T)
	No response	3 (U,C,H)
8 **Did the discussions that you had with your partner help?**	Yes	6 (K,U,T,C,S,H)
9 **In what way did the discussions help?**	Made the question clearer/easier to understand	3 (K, C, H)
	Shared and compared answers	2 (S, U)
	Shared and compared ideas	1 (T)

Table 10: Summary table of results – post assessment interviews after maths club sessions

Question number	Emerging themes	Percentage of learners
3	Communicating/Discussion/Talking	17
	Basic operations	33
	Reading the questions	33
	If you know what to do	17
4	Think a bit more, write things out	17
	Read questions carefully	17
	Talking	17
	No response	50
8	Yes	100
9	Made the question clearer/easier to understand	50
	Shared and compared answers	33
	Shared and compared ideas	17

From this interview it can be seen that all of the learners said they benefited from some form of communication and discussion centred on the problem (question 8). Kilpatrick et al. (2001) as well as Vygotsky (1978), both state that any kind of interaction that takes place within a collaborative space, lends itself to the active participation, collaboration and co-operation between learners. This is illustrated in the ZPD described by Vygotsky (1978) and the ‘instructional triangle described by Kilpatrick et al (2001, p. 314). They both state that a more knowledgeable other (in this case a fellow learner) can help understanding and improve mathematical proficiency.

The post interviews indicated that learners perceived the discussions as helping them to find alternative strategies for problems that they initially struggled with. It also assisted the learners in finding a solution that seemed correct. They were able to reflect on the solution

that they arrived at and test to see if it was correct. This is one of the steps of Polya's heuristics on problem solving (Polya, 1973).

According to 3 out of the 6 learners in my sample, the discussions that took place helped the learners to understand the different strategies used to solve the problems (question 9). The other 3 out of 6 learners stated that the discussions they had with their partner helped them to understand the problem and made it clearer. The learners used this opportunity to compare the different answers that they obtained and discussed how those answers were derived. The discussion that took place whilst "sharing and comparing" answers was also important and forms part of the communication that took place within this 'instructional triangle'. This also helped the learners to make meaning of the problem. The discussions also helped the learners to explore and talk about different strategies that they used and to justify mathematically why they chose the strategy that they did. This is a clear example of the instructional triangle theory put forward by Kilpatrick et al. (2001). As indicated previously the 'instructional triangle' states that mathematical proficiency can improve if there is communication centred on mathematics not just between adults (usually teachers) but also between students (learners). The learners in my study were given the opportunity for collaboration and co-operation and discussion and this has enabled them to improve their mathematical problem solving skills.

Comparing the responses on these questions indicated that while not one learner expressed communication as being a factor that would help them in becoming a good problem solver in the initial questionnaire (question 3) all six learners said that the discussions that they had with their partner did help them to find strategies and ultimately solutions to the problems faced. This contrasts with the baseline assessment where only one learner mentioned 'communicating' as a reason why Thando enjoyed problem solving (question 5)

The data across excerpts and interviews suggests that discussions helped the learners to find alternative strategies for problems that they initially struggled with. The discussions also assisted the learners in finding solutions. They were able to reflect on the solution that they arrived at critically. This is one of the steps of Polya's heuristics on problem solving (Polya, 1973).

CHAPTER 5 – CONCLUSION

The data presented in the previous chapter suggests that the learners largely benefitted from the interaction and peer-to-peer communication and collaboration that were provided for them. They interacted in a positive way within the Zone of Proximal Development as defined by Vygotsky (1978) moving from their level of actual problem solving to a higher potential level through this interaction. Meira and Lerman, (2001) define the ZPD as a 'space' that can be created where interaction and communication take place. Forman and McPhail (citied in Meira & Lerman, 2009) suggest that peer-to-peer collaboration leads to changes in the learners' communication, goals and social interactions as well as their ability to solve a particular problem. Forman and McPhail (citied in Meira & Lerman, 2009) also state that there are potential benefits of peer-to-peer collaboration but in order to analyse this fully, it needs to incorporate how learners' communication, goals and social interactions may have changed over time along with their problem solving ability. This change in their social interactions and its effect on problem solving has been illuminated in this research study. In my research one learner in each pair took on the role of a more knowledgeable other. Although it was not necessarily the same learner and roles shifted depending on the problem at hand. This indicates that by allowing the learners to communicate a positive outcome could be that the learners over time become better at problem solving.

Indeed for me as a teacher it is important to see what has or hasn't changed as a result of this research. One of my key findings is that the quality of discussion that takes place is critical in whether paired learning enables problem solving. Thus the nature of discussion differed qualitatively across pairs and problems. My own teaching practise has undergone some changes and the way that I approach some topics is very different to the way I approached teaching problem solving before this research. It has highlighted to me the importance of communication, not just in terms of the learners discussing with each other but also the need to allow learners to communicate in this way. It is sometimes easier to let learners make errors and learn from those errors themselves than to constantly correct them where there is little 'two –way' communication. As a researcher and teacher I have understood how all five strands of Kilpatrick et al,'s (2001) five strand model of mathematical proficiency can be used to illuminate and uncover what is lacking in order to be mathematically proficient.

One of the difficulties encountered by learners is one of their own doing and that is that they admit they do not read problems carefully enough, and as a result their understanding of the

problem can be different to that of the intended meaning. Communication and the need to come to a common understanding of the problem seem to help address this issue.

Furthermore the study showed that strategies for problem solving evolve over time and this can be structured into the teaching programme. This rate of development of problem solving strategies is more noticeable when learners are working in a collaborative environment where peer-to-peer discussion can take place and can be observed by the teacher. What this research revealed that was of interest to me was that only one of the learners sampled consciously mentioned communication as a factor or skill that could be related to problem solving at the start of the research. However, when asked directly after the five collaborative sessions if discussions helped (question 8 of post interviews) all the learners responded 'yes'.

The collaborative sessions and the discussions held by the learners' supports the findings that co-operative learning does indeed support proficiency in problem solving and over time will lead to improved techniques and ability. The findings in this study cohere with other local and international findings that suggest that mathematical discussion, enabled by collaborative working, is central for learning to problem solve and can support reasoning (e.g. Sepeng and Webb, 2012: Cohen and Ball, 2001; Kilpatrick et al, 2001; Meira and Lerman, 2009) discussed in the literature review above.

The study has provided data on the way in which discussions can support joint problem solving. In particular I have illuminated in two detailed excerpts the frameworks of Kilpatrick et al's (2001) intertwined strands of mathematical proficiency (figure 9) and Polya's (1973) heuristics for problem solving (figure 10). These are provided in summary below:

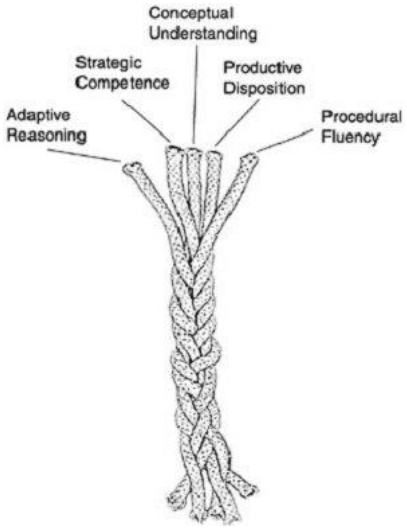


Figure 15: Interwoven strands of mathematical proficiency (Kilpatrick et al,. 2001)

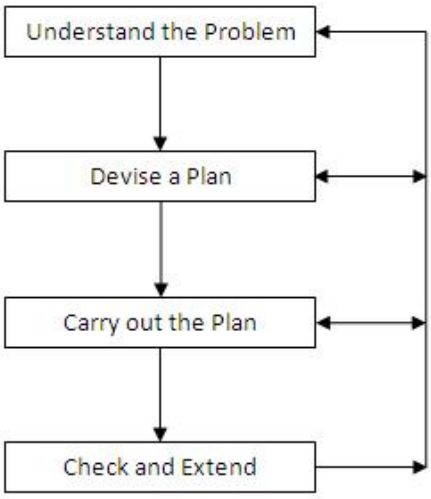


Figure 16: Polya's problem solving steps (Polya,. 1973)

The

limitations of this study are that it gathered data from just one school in the Eastern Cape and only seven learners were involved. This means that the results are not generalisable to the whole of South Africa or even the Eastern Cape. Although the school in the sample is not a private school it is not reflective of the public schools in the Eastern Cape. The school is an ex-model C school. However, the hope is that the study is 'generative' of examples of how collaborative learning can support problem solving. Further research could explore a similar design in other schools and the inclusion of clubs where learners work individually rather than collaboratively could be used to compare possible improvements across groups of learners.

The way that my research area focussed specifically on mathematical problem solving is also a limiting factor in the sense that only AMESA Mathematics Challenge questions were used for this research and not 'the textbook style' word problems that usually occur in a mathematics lesson. The lack of problem solving skills and both conceptual understanding and procedural fluency together with strategic competence and adaptive reasoning is illuminated by AMESA Mathematics Challenge questions and is reflective of the difficulties facing problem solving skills generally.

The study does however point to the power of collaborative working and teaching for my own teaching of problem solving and I will share my experience of this with my colleagues. The learner improvement in results, while not the aim of my study; has helped to convince me of the usefulness of collaborative learning. Additionally I have realised that much can be gained from after school clubs and I thus plan to run another club of this nature next year.

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APPENDICES

APPENDIX 1

What do you think makes someone good at problem solving?	What do you think makes someone bad at problem solving?
Mpho is scared of problem solving because…..	Thando enjoys and loves problem solving because….

How do you feel about problem solving in mathematics?	What do you do when you get a problem and you don't know where to begin?
When you are given problem solving to do in maths what is your first reaction?	How are you at problem solving?

Other comments about problem solving

APPENDIX 2

21st January 2013

Dear Mrs Abrahams and GB Members

I am currently in the midst of my M Ed (Masters in Mathematics Education) as part of my thesis for this year I will be conducting interviews as well as workshops (through the maths club) in order to gather information.

In order for me to obtain this information I would like to run a Maths Club at Victoria Primary School with a select group of Grade 7's. The learners involved will hopefully gain as much from participating as I will, whilst conducting my research.

The identity of the learners will be known only to me and only pseudonyms will be used. The identity of the school will also be confidential and known only to me.

The purpose of the study is:

1. To understand how learners problem solve.
2. To understand why they struggle to problem solve.
3. To understand why they use the strategies they use.
4. To understand how attitudes towards and strategies in problem solving might evolve given the opportunity for problem solving through discussion and interaction with peers.

I am hoping that my findings will be of benefit to the school in future.

My supervisor at Rhodes University is Prof Mellony Graven and she can be contacted on

046 603 7268

Yours sincerely

Mrs Anita Sonne

APPENDIX 3

21st January 2013

Dear parents

I am currently in the midst of my M Ed (Masters in Mathematics Education) as part of my thesis for this year I will be conducting interviews as well as workshops (through the maths club) in order to gather information. Your daughter has expressed an interest in part of this. I will need to gain permission from you in order for your daughter to be allowed to participate.

The results obtained from the research will be strictly confidential and will only be used for the basis of my thesis. Your daughter may withdraw from the maths club at anytime should she wish. Only pseudonyms will be used.

The purpose of the study is:

1. To understand how learners problem solve.
2. To understand why they struggle to problem solve.
3. To understand why they use the strategies they use.
4. To understand how attitudes towards and strategies in problem solving might evolve given the opportunity for problem solving through discussion and interaction with peers.

This is what we will be working on within the maths club for the duration of this term.

If you would like your daughter to participate please complete the reply slip below and return by Friday 25th January 2013

Yours sincerely

Mrs Anita Sonne

I give permission for my daughter _____ to participate in the maths club and be a part of a half thesis towards a Med.

Signed_____ Name:_____

APPENDIX 4

Baseline assessment: (8 April 2013)

Read carefully. Show all steps in your working out.

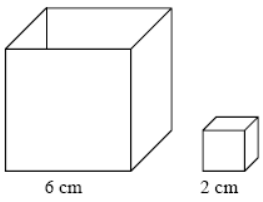
1 Which of the following fractions is closest to −?

A —— B — C — D — E —

2 The sum of seven consecutive numbers is 63. Which is the largest of the seven numbers?

A 63 B 9 C 13 D 12 E 10

3 How many of the small cubes fit exactly into the big cube?



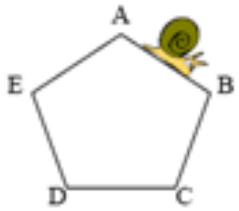
A 9 B 12 C 20 D 21 E 27

4 What is the next number in this number pattern?

302 400; 50 400; 7 200; 900; 100....

A 50 B 1 C 8 D 10 E 0

5 Shane the snail starts at corner A and crawls clockwise once around the regular pentagon (a figure with 5 sides of equal length) what side will he be on when he has crawled — of the distance around the pentagon?



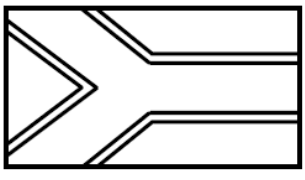
A AB B BC C CD D DE E EA

6 Refer to the previous question. If each side of the pentagon is 5cm long, how far has he then still got to go?

A 16cm B 16,25cm C 9cm D 8,75cm E 7cm

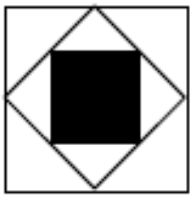
7 This flag has 7 regions. You want to colour the flag so that no two touching regions are the same colour. What is the least number of colours you need?

A 2 B 3 C 4 D 5 E 6

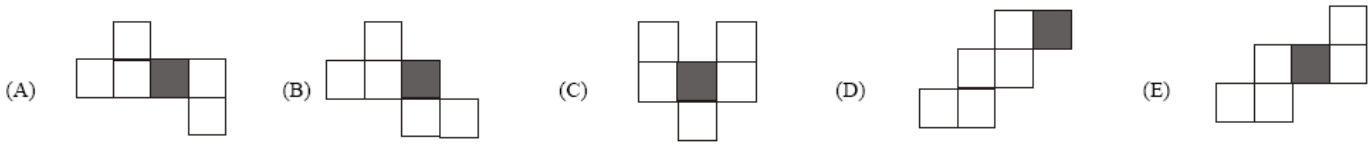


8 The figure is formed by successively joining the midpoints of the sides of a square. What fraction of the whole figure is shaded?

A – B – C – D – E –



9 Which one of the following figures below *cannot* be folded along the lines to form a cube with the shaded square at the base?



10 The sketch shows three ways in which certain objects can be balanced. How many □s are needed to balance the two ☺s?

A 11 B 5 C 10 D 12 E 8

11 Yolanda found that the warehouse was full, so she stacked a truckload of boxes outside. The stack was 6m high, 10m wide and 8m long. Yolanda now needs a rectangular tarp (sheet) to sheet the boxes for protection. What size tarp would cover the stack exactly?

A 14m x 16m B 24m x 24m C 20m x 20 D 22m x 22m E 20m x 22m

12 The following table shows the readings of the mass hung on a spring and the corresponding length of the spring. What will the length of the spring be if a mass of 15kg is hung on it?

Mass (kg) Massa (kg) Ubunzima (kg)	0	1	2	3	4	5
Length (cm) Lengte (cm) Ubude (cm)	15	17	19	21	23	25

A 45cm B 27cm C 30cm D 40cm E 75cm

13 The sum of two numbers is 18. What is the greatest possible product of the two numbers?

A 36 B 81 C 80 D 100 E None of these

14 In a banana eating competition a competitor ate 90 bananas in 4 hours. Each hour he ate 5 less bananas than in the previous hour. Hour many bananas did he eat in the last hour?

A 70 B 15 C 86 D 60 E 22

15 Look at this menu. How much should be charged for egg and mash?

Menu	
Egg & chips	R4,90
Sausage & chips	R6,00
Sausage & mash	R5,70

A R4,60 B R5,53 C R5,30 D R1,80 E None of these

APPENDIX 5

Video transcripts of critical moments.

Excerpt 1(16th April 2013)

1	U	it's between 5.60 and 5.65
2	K	Won't it be this *(pointing at an answer)*? because maybe you need to add that and that.
3	U	it has to be 5.62 and a half. Let's work it out 5.6 is like 5.6 and 5.65 is like 5.6 and a half
4	K	Lets' just see. Must we add or must we minus if we want to work it out?
5	U	we minus that to that *(pointing)*
6	K	no that to that *(pointing)*
7		*Furiously working out. Lots of rubbing out and starting out.*
8		*Learner K is writing out the question 5.65 – 5.6. The numbers are not exactly underneath each other in the correct columns, this causes distress for learner U*
9	U	Doesn't the 6 go there? *(pointing to the correct column)*
10		*U ignores K and continues with subtraction question. And gets an answer of 0,05*
11	K	now what do we do?
12	U	we half it
13	K	yeah, lets half it and add it to six *(pointing to 5,6)* so that would then *(shuffles paper)* 5 divided by 2 you have 2 and a half and then
14		*Writes on paper*
15		5.60 + 2,5 equals *(interesting that although the number sentence is incorrect the writer knows what she is doing and now explains to learner U what is happening and why it is 2,5 although it is a half)*
16	U	buts that half not 2 and half K I know what I'm doing)
17	K	It's half because we have to add it to 60. Because ...or its going to be, how else because 5.60 and half is not exactly between 5.6 and 5.65
18	U	OK
19	K	5 divided by 2 is 2 and half (goes back to her written number sentence 5,60 + 2,5 equals 5,62½
20	U	equals our answer
21	K	answer C

Excerpt 2 (16th April 2013)

22	U	1 plus 1 divided by 1plus 1 equals 1 that's true (no) Then B 2 divided by 2 plus 2 no sorry let me start again *(rubs out and starts again).* This doesn't make sense
23	U	Ok you rub it out.
24		*They start all over*
25	U	Can you write it out?
26	U	So 1plus 1 write it
27	K	Wait
28	U	write A 1plus 1 equals 2 and write 1plus 1 equals 2 again and 2 divided by 2 equals 1 (writing down the working out)
29	U	Then write B 2 divided by 2

		K - you've got a piece of paper U!!! *...(interrupts U)*
30		equals 1 2 divided 2 equals 1
31	K	(Interrupts) no, no that's..
32	U	no BODMAS darling BODMAS plus comes after divide equals 1
33	K	and then
34	U	1 plus 1 equals 2 1 plus 1 equals 2 and then that's right
35	K	B is not true
36	U	Then C 3 times3 equals 9 and then you say 3 plus 3 (no you say 9-3 equals and then you just carry on ok)
37	K	9 minus 3 equals 6
38	U	plus 3 equals
39	K	plus 3 is 9
40	U	The answer is C
41	K	Ok its C
42	U	Write it equals C
43	K	Ok equals C

Excerpt 3(16th April 2013)

Working out the answer to another problem. Needed practical objects to help.

44		*K is busy marking on a two prit glue sticks in blue in preparation for solving the question. The marking is altered by her partner with a bright pink highlighter. (Q is to see if the circumferences of two circles are the same). They are working on the desk.*
45	K	*She starts turning the pritt stick so that it moves around the edge of the second glue stick to see how many times it goes around it. She starts counting but it is not correct* 1 2 3 Is that right???
46	U	Ok you turn it you are better at turning it
47	K	How??
48	U	Just turn it and I'll just count
49	K	Wait
50		*They both fumble to get grips with the two glue sticks and eventually decide that they should go and work on the floor....*
51		*They move onto floor by the black board and continue with their solution.*
52		*They start to turn the glue sticks again and start to count*
53	U	Just turn it
54	K	Ok
55	K	that's 1 right *(after turning one 'twist' of the hand to indicate 1 instead of 1 rotation around the prit stick)*
56	U	No
57	K	that's 1
58	K	so it goes around once

Excerpt 4(16th April 2013)

59	U	If the CD costs R60 more than the book how much does the book cost?
60	K	Then say we times or divide?
61	U	I honestly don't know like dividing that by 2 and then working it with the.. That's the part because CD costs R60 more than the book. So we need to work out what the CD costs.
62	K	wait that part so then 60 plus 30 cos yes, it 6 .it ... so it costs R60 more than that so you'll say
63	U	I know k I know what about if...
64	K	60 and then you'll say plus .... (*Muffled sound not clear)*
65	U	or even like if you like what was I gonna say if we minus 60 from and the we divide that by 2 then we find out what the book costs. Yeh that should work. You can write it out
66	K	so its 230 minus 60 right which
67	U	equals 170 we say
68	K	we divide that by 2
69	U	we say 170 divided by 2 which I think is 75 no not 75 sorry wait I think its 85 I'll just make sure *(punches into calculator)*its 85 and then so that how much the book costs I think. Yeah that's how much it is.
70	K	yes its E answer is E *(pointing to the question sheet*) the book costs R85

Excerpt 5 (4th May 2013)

101	U	LCM ok so let's see 7/15 8/...
102		*They are trying to solve which fraction is the largest. After some time being used to find the LCM the learners decided there had to be an easier way as the LCM was a large number. After much discussion they decided to use their calc*
103	K	7 divided by 15 equals 0.46666..
104	U	so then how do find what what, oh we work which is the highest
105	K	Kappish
106	U	OHHH! *(a aha moment has passed the learner when she really understands what is happening now)*
107	K	oh so for A I'll write it here
108	U	take your eraser And write it here, actually you can keep it there we'll just write the the whats it under it
109	K	writing 0,46666 it's gonna carry on I'm not gonna write it down
110	U	just write a few dots so then we'll know
111	U	8 divided by 17 equals 0.47058823529 de dede dede
112	K	0.47508823529
113	U	and the 11 divided by 13 equals 0.47826086956
114	K	Which one are we looking for the largest?
115	U	do you have all of them *(pointing to her calc)*
116	K	Yeah
117	U	then what's the next one
118	K	its 13 over 27

119	U	13 divided by 27 equals 0.48148148 ok this is the largest one so far
120	K	so now its 5 over 11
121	U	5 divided by 11 equals 0.4545454545
122	K	ok I don't need to do that
123	U	just write
124	K	nought four five, yeah so number 13 over 27 so that's D
125	U	Yeah

Excerpt 6 (4th May 2013)

133	U	What does it mean speed and mass?
134	K	this is the speed and then that's the mass I think (*pointing to the graph)*
135	U	the mass is the one that goes here
136	K	Isn't this the adult man? *(pointing to a point on the paper)*
137	U	Why would the man be here. Because the man is the heaviest he's not there
138	K	The elephant is the heaviest. The elephant should be there, but this is speed U
139	U	that's speed and that's mass *(draws a cross on the graph)*
140	K	That's downward speed. *Pointing to the speed axis*
141	U	that's the elephant and then the horse is obviously ...
142	K	Wait you must write it here. The elephant is there
143	U	this is the horsey, the B
144	K	ok this is horse, the tiger will be up there *(pointing to the a point on the graph)*
145	U	then there's a cat
146	K	a cat too
147	U	the cat is going to be lighter than the man, so that must be the cat and that must be the man
148	K	so then
149	U	it's D

Excerpt 7 (4th May 2013)

In this clip. One learner L already has the answer and she is now trying to convince the others in her group how she arrived at the answer.

71	L	I divided by 2 so that its half way by 2.
72	C	what did you do
73	L	I added the two
74	T	Yeh
75	L	I added the two
76	C	I want to see what you got.
77	L	I added the two and then after that I divided the answer by 2 *(Background noise)*
78	C	Where must the comma be?
79	T	put the 5 here the comma stays in the same place
80		*(L is typing numbers into the calculator)*
81	C	here's the 6 add the 5 is eleven

82	T	no wait that 5 isn't under there that's a zero
83	T	that's 5,65 *(she's finished punching in her numbers onto the calc) (They are adding 5,6 and 5,65)*
84	T	5 plus 0 is 5. 6 plus 6 is 12 carry the 1 5 and 5 is ten and the one 11. Is that what you got (*she is now asking L*)
85	L	Is again punching numbers into her calculator. Yes I did that and then divide it by 2
86	C	11,25 divided by 2 equals 5,625 B
87	L	I also got B

Excerpt 8 (4th May 2013)

T is reading out the question aloud

126	C	L said 23
127	T	How did you do it?
128	L	I don't know how I did it. But then I had a paper with working out on it .
129		*(T is busy punching numbers into the calculator) (whispered discussion)*
130	L	um, then I said Mrs X I said I said I said um, the load with the mass of the thingy , with this minus this divided by that is 23.
131	L	this minus this div by that is 23
132	T	4653 minus two hundred and 2583 is 2070 and then we have to say 2070 divided by 90 is 23. You were right.

APPENDIX 6

Orally Administered Interview conducted 7th May 2013

1 Learner K

Question		Response
1	I	Look at your score, you have made an improvement in your problem solving abilities. How do you feel about that?
	K	I feel good about it, cos I know, cos we discussed the questions on the old piece of paper together and then I did my own ones and then I got a better mark
2	I	Ok if you look back at the questions you just completed do you feel that if you know your times tables then you can do problem solving?
	K	Yes Mrs. Sonne
3	I	You still believe that if you know your timestables then you can do problem solving ok. What other factors or skills do you think will help you become a good problem solver? What sort of things do you think you need
	K	Communicate with people
3a	I	In what way would that help you?
	K	So that we will work it out together and communicate
3b	I	Almost like discuss
	K	Discuss
4	I	What do you think you have learnt from the maths club that will help you in your maths lessons not just the problem solving but with maths generally
	K	We have to think a bit more and if you're doing times tables you have to write them down.
5	I	Previously I'm sure you were one of the children who said that you were not keen on problem solving and how do you feel about them now?
	K	I still struggle a bit I'm still a bit bad at it
5a	I	Ok why do you think that you struggle? What do you think makes it a struggle for you ?
	K	Like I don't, sometimes I don't read properly and I don't understand the questions
5b	I	How do you think or in what way could the question be written that would make you understand them more
	K	I'm not sure
6	I	Ok, were you expecting your marks to improve?
	K	Yes Mrs. Sonne
7	I	Ok. What specifically about the maths club in general helped you with problem solving. What did you learn through it and how did it help?
	K	*Mumble on tape not clear? To d equals work through it to draw*
8	I	Did the discussions you had with your partner help?
	K	Yes Mrs. Sonne
8a	I	Ok so you've got the picture which helped that you translated from the question and then talking about it with your partner
	K	Yeah
9	I	In what way did they help? How did they help?

		Very quiet doesn't say anything
9a	I	What made them helpful in other words? Was it more clarity on the question, was it just the two of you trying to you know finding your way to the end or what was is?
	K	It was more clearer.
10	I	Finally now how have your attitudes towards problem solving changed through being able to work in pairs and being given the opportunity to do so?
	K	Its changed quite a bit, I don't get too nervous and say I can't do this. I say I can do this and try.
	I	Is there anything else you'd like to say now
	K	No

2 Learner U

Question number		Response
1	I	If you look at your score you have made an improvement in your problem solving abilities. How do you feel about that?
	U	I feel happy but like confused
2	I	If you look back at the questions think back to the questions at the beginning of the maths club, one of the questions on there was what do you think you need to become a good problem solver? Some of your answers were if you know your times tables then you can d problem solving. Do you still feel that?
	U	Yes
2a	I	So you don't need anything else besides times tables?
	U	You need like um not sure...
2b	I	So if you were doing some problem solving if you know your times tables then you can answer all of these questions?
	U	Er, um no
3	I	So what other things do you need to be able to do?
	U	Divide, multiply
3a	I	Ok so the four operations what else besides those four operations?
	U	Um
	I	When you look at the question what makes you understand that question? What is it? Do you think reading and understanding would also be a good plan?
	U	Oh yeah. If you don't like read it properly then you won't get it properly
5	I	You also said previously that you were not so keen on problem solving, actually you actually enjoyed problem solving, how do you feel about them now?
	U	I still enjoy them
6	I	I'm sure thats what you said. Were you expecting your marks to improve
	U	Umm
	I	By more by less or no
	U	Yes I was expecting by slightly more because I practised more and got better at it.
8	I	Ok. Did the discussions that you had with your group with your partner

		help you at all?
	U	Yes
9	I	In what way did that help
	U	Um well we had like different like um answers to um helped us like comparing like comparing each other's answers and like how to find out which one is the final answer
9a	I	How did you decide which answers were correct and which one to go with?
	U	Well we went through both of them like a few times and sometimes we had the same kind of answers but if we couldn't decide then like we showed each other how like how we worked it out and then together we would work out the actual answer.
12	I	Ok last question. How has your attitude towards problem solving changed specifically through being to discuss in pairs and given a chance to work in pairs? Obviously in the past you have only done problem solving on your own this year we introducing working in pairs so how has your attitude changed towards problem solving. Is it still I like it but I now maybe I can be better with it or is it I don't like it and I cannot do better? What do you think?
	U	Well I do still like it and I try to do better and it has like it is I'm getting better at it and also don't because I get a bit bored during problem solving during the day
12a	I	Why do you get bored?
	U	I don't know I just get bored.

3 Learner T

Question number		Response
1	I	If you look at your scores you've made improvements in your problem solving abilities. How do you feel about that?
	T	I feel um like I feel like not proud but like um happy that I could understand a bit more better.
2	I	That's fine. If you are looking, if you look back at these questions on this paper that you've just done, do you, in one of the questions that I asked you earlier on, quite a number of you girls said that if you know your times tables then you can do problem solving. Do you still believe that, that if you know your times tables then you can do problem solving, do you think that that is the only thing that you need to know
	T	I don't think so because you need to know your division and all your other stuff
3	I	Ok what other factors do you think will help you to become a good problem solver?
	T	Umm,
3a	I	So besides the four operations what else do you think you need to know, or do you think will help you to become good at problem solving?
	T	Like maybe getting extra classes and like putting the extra hours to study
3b	I	Ok what about things like reading the question and trying to understand it.
	T	Like all the working out and stuff

3c	I	So basically there are other things that you need so what I'm getting at is that there are other things that you need to do problem solving besides just your times tables which is what a lot of you wrote. Ok you said you were not so keen on the problem solving. How do you feel about them now?
	T	Umm, like , umm I think I'm getting better than before so I think I don't have so much of a problem with problem solving anymore
4	I	What specifically about the maths club problem solving helped you?
	T	Umm, talking and ... talking
4a	I	You mean just being able to talk about the problems and not just talking in general.
	T	Yeah
8	I	Did the discussions that you had with you partner help?
	T	Umm,
8a	I	When you were doing maths club problem solving questions did the discussion that you had with your partner did that help you?
	T	Normally I don't really have discussions so then I would not really discuss it but it did kind of in a way.
9	I	In what way did that help?
	T	Like, giving the ideas what the other person thinks and their idea of something and then my idea as well.
9a	I	Ok how did you decide whose idea was the best?
	T	Umm, well we both worked it out and then decided and like then agree what we both think it should be.(*not sure)*
12	I	Umm ok last question, how have you attitudes towards problem solving changed, if it changed at all, through being able to work together in pairs and being given the chance to do so?
	T	Umm, sorry can you please ask that question again.
12a	I	Ok so how has your attitude towards problem solving changed so if your attitude towards PS before was like I hate PS or I love PS how has that changed because you are able to work in pairs or you are given the chance to work in pairs.
	T	Umm, its changed because like, its good to know what another person thinks and yeah.
	I	Ok.

4 Learner C

Question number		Response
1	I	Kerrigan if you look at your score you have an improvement, how do you feel about your improvement in problem solving?
	C	I feel happy,
1a	I	Why do you feel happy?
	C	Because I think I worked hard during the extra maths club.
2	I	Now if you, if we go back to those, remember those questions that you filled in there was a question, on there; there was quite a lot of you that mentioned that if you know your times tables then you can do problem solving. Do you still believe that?
	C	Yes
2a	I	So you still believe that if you know your times tables then you can do all this PS thats all you need to know.

	C	Yes Mrs. S
3	I	What other factors do you think would help you to become good at problem solving.
	C	I think if you know how to do it
3a	I	Ok by knowing how to do it, what do you need to know how to do??
	C	Silence
3b	I	If you take this question for example ok, by knowing what to do , what do you need to be able to do for that question before you know what you're doing?
	C	Count
3c	I	Not only count but read it. Is knowing what to do part of your reading.
	C	Yes
3d	I	Theres no right answer my darling you must just say whatever you think is right ok? What I'm asking is if you read a question do you automatically know what you are doing or do you need to look at the question again or does someone need to explain it. Thats what I'm asking.
	C	I need to read the question again.
4	I	Thats what I'm asking ok. So what do you think you learnt from the maths club that will help you in maths lessons?
	C	Um, can you repeat the question
4a	I	What have you learnt. What do you think you've learnt from the maths club that will help you in maths lessons? Do you think that there's anything that you've done in the maths club as part of the maths club whether it's working together or solving problems whatever that will help you in your maths lessons.
	C	*Silence said nothing*
5	I	You also said in one of your other questions that you were not so keen on PS, you didn't like it ok, how do you feel about them now?
	C	I'm starting to like it, because I know better
5a	I	Because you know better?
5b	I	What were you expecting...Were you expecting your marks to improve so much? You actually doubled your marks. Were you expecting them to be doubled?
	C	No
5c	I	You really work and made some improvement somewhere so you can do it. Your mom will be happy know that won't she?
	C	Yes
8	I	What specifically .. not asking that, did the discussions you had with your partner help you?
	C	Yes
9	I	In what way? How did it help you?
	C	In a good way
9a	I	In the sense that did it help you understand the question, did it help you see the problem clearer, did it help you know which operations to use, in what way did it help you?
	C	Understand the question and the way you supposed to do it, working out and like that.
11	I	You also said earlier, the girls also said that when you were told to do PS in class you used to not really look forward to them because they were

		hard and difficult and impossible to do. How do you feel about them now.
	C	They easy kind of easier.
11a	I	Why do you think they have become easier?
	C	Because I now understand them
12	I	Last question. How have your attitudes towards PS changed through being able to work in pairs and given the opportunity to do that. How do you look at PS, your attitudes towards PS, whether you find them difficult or you don't like doing them or you love doing them? How has that changed because now you've been able to work in pairs or you are able to discuss.
	C	I like doing them
12a	I	You like doing them had it changed, you look forward to doing them or you just like doing them?
	C	I look forward to doing them
	I	Thank you

5 Learner S

Question number		Response
1	I	If you look at your scores of the two assessments you've made some improvements in your problem solving abilities. How do you feel about that?
	S	I feel kind of good but I think I can do much better
1a	I	Ok how do you think you could have done better?
	S	By showing my working out um and practising more
2	I	If you look back at the questions you've just done, these two papers that you've just done. Do you still feel that if you know your times tables then you can do problem solving?
	S	Yes
2a	I	So knowing your times tables is the only thing you need to know to do problem solving?
	S	No I don't think err
3	I	So what other factors, what other skills do you think you would need to become good at problem solving?
	S	Um, by reading the questions more carefully and I think that's it.
4	I	What do you think you have learnt in the maths club that will help you in your maths lessons?
	S	Um I've learnt that we all have to set out the work properly and reading questions carefully so you can, so you can um do your working out properly
5	I	Ok you said previously on one of the interview that I gave you that you were not keen on problem solving you didn't used to like them much how do you feel about them now?
	S	I feel much more better
5a	I	In what sense do you feel better?
	S	Because um ....
5b	I	What makes them, what make sense so that you don't feel like "I don't like problem solving" What is it about them that makes..... it what way does it make you not feel

		"I hate problem solving"
	S	Improving and I understand them
8	I	Ok with the discussions you had with your partner, because I know you had some discussions with your partner in your group did that help?
	S	Yes
8a	I	In what way did that help?
	S	Because we shared our answers and looked at different answers and then we a proper answer
	I	Ok
11	I	Do you think that when we do problem solving in class at one point you never used to look forward to them do you still feel like that?
	S	No
12	I	Last question how have your attitudes towards problem solving changed because you are now able to work in pairs or that you were given a chance to work in pairs. How have they changed, do you still hate problem solving do you still love problem solving is it nicer because you are to discuss them. What is it?
	S	It's nicer because I'm able to discuss it.
12a	I	So how has your attitude changed towards it?
	S	At first it wasn't good, cos I didn't understand problem solving but know I understand it its better.

6 Learner H

Question number		Response
1	I	If you look at your scores you have made such an improvement in scores here from your baseline assessment to your new assessment, how do you feel about that?
	H	I feel really happy because my other marks were low.
2	I	If you think back to when you filled in question when you filled in that questionnaire. There was a question that you had to fill in and one of the questions was what do you need what do you think you need to know in order to do problem solving, your answer was that you need to know your times tables do you think that that is still what you need to do problem solving?
	H	Not as much
3	I	What other things do you think you will need?
	H	Probably dividing, adding, subtracting plus all of the stuff that we have learnt.
3a	I	Ok. What other, anything else you need to be able to do in order to do problem solving besides just the four operations.
	H	I'm not sure
3b	I	Do you think that you need to be able to read and understand?
	H	Yes
4	I	Ok so that would be one that you would need. How do you think the maths club has helped
	H	I think it's helped, because I've improved
4a	I	Do you think it's helped just in problem solving or generally in maths

	H	Generally in maths
4b	I	Did the discussions you had with your partner help? did that help?
	H	Yes
4c	I	Ok in what way? How did it help did it make things clearer did it... did you suddenly understand the question, did you know what to do, how did it help?
	H	It made some of them a bit more clearer to me so I can understand the question.
5	I	Also in one of the earlier papers, in one of the earlier questions that you answered before you said that when you were told to do problem solving in class you never used to look forward to that.
	H	Yes
5a	I	Do you still feel that way?
	H	No not as much
5b	I	Why?
	H	Because it has gotten easier for me and I enjoy doing problem solving.
12	I	So then how would you how.. last question how have your attitudes towards problem solving changed because your are now working in pairs or because you are given the chance to work in pairs. How has it changed?
	H	*No answer*
11	I	Do you still think oh no or do you think problem solving ok I can do them how do you feel?
	H	It makes me feel that I'm comfortable to work with a partner because when I get stuck, they can help me.
11a	I	Problem solving is not such a difficult thing for you now
	H	No